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FREEFALL MATHEMATICS

VELOCITY

TEACHER EDITION

Michael Farlam



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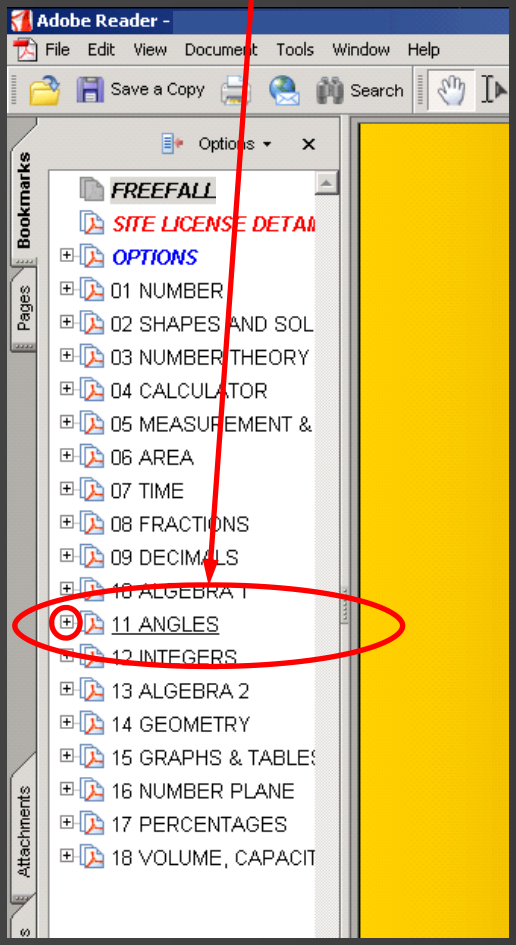
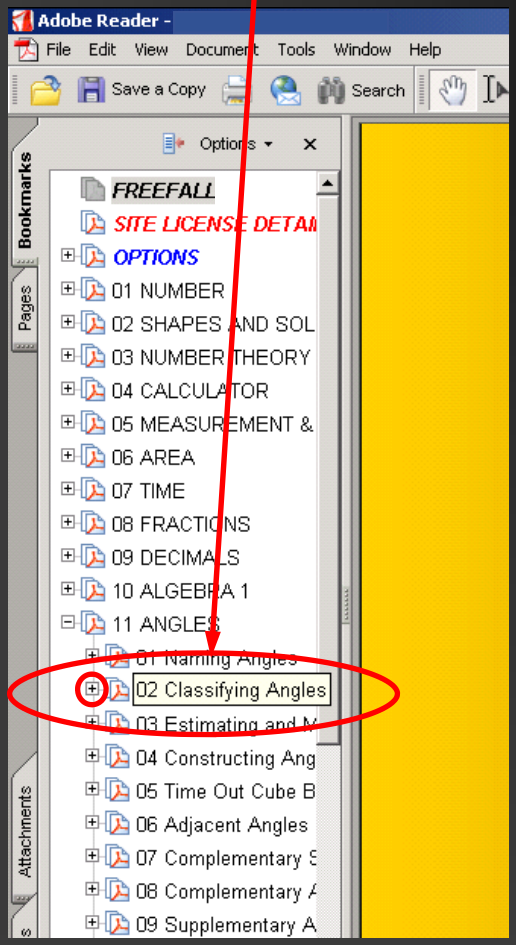
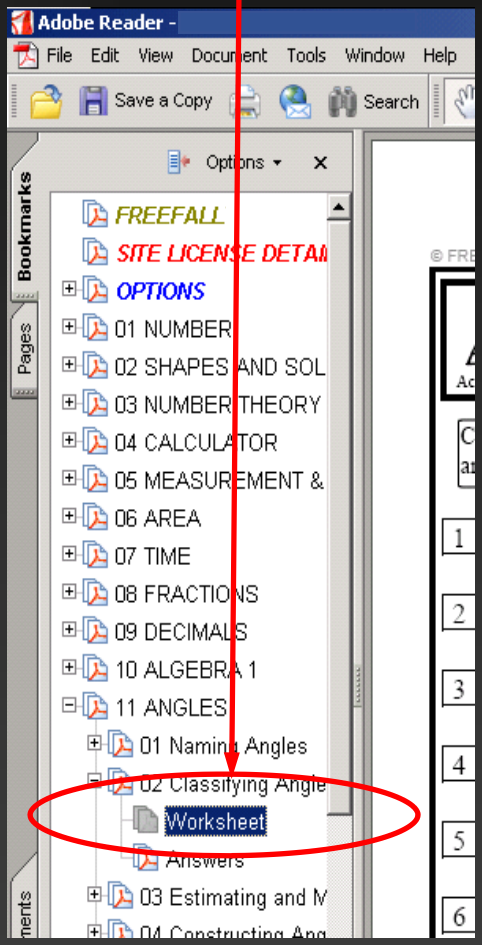
E-book Navigation

Freefall uses 'bookmarks' to navigate through its contents, every page has its own bookmark and simply requires you to click on the bookmark to jump to that page. If for example "02 Classifying Angles" is required:

1	2	3
Open the ebook then single click on the '+' sign in front of the "11 Angles" chapter .	Single click on the + sign in front of 02 Classifying angles.	Then click on worksheet and the sheet is on screen. Zoom in for a better view.







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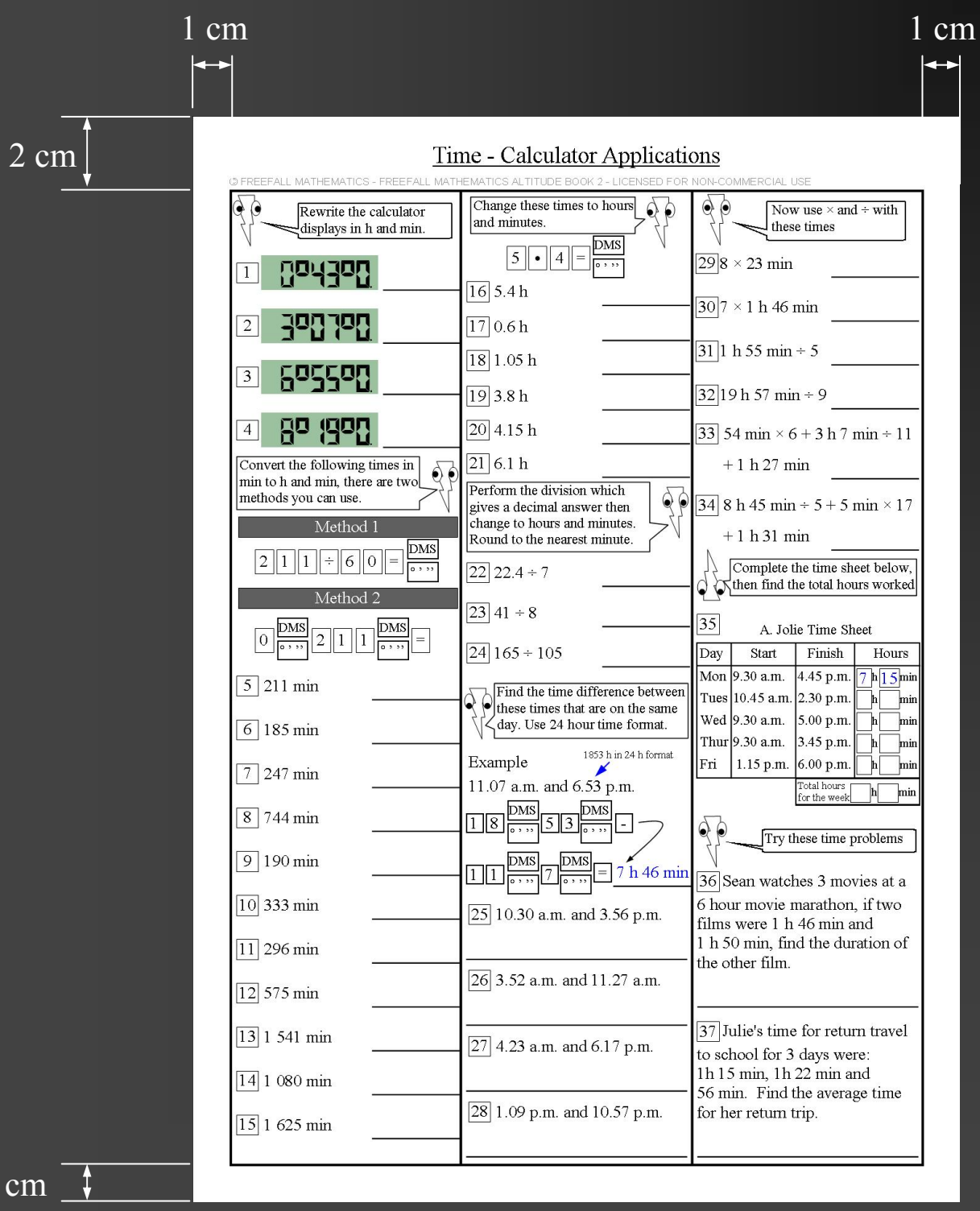
Page Specifications

Freefall worksheets are designed to have gaps of 1 cm on the bottom and sides of the worksheet. The top line is 2 cm down from the top edge of the page.

It is essential that you understand that when you print a worksheet, it has to come out this way otherwise sheets which use a ruler or protractor or grid will have answers that don't match those in the software. So read the next pages to understand the importance of 'No Scaling' when you print worksheets



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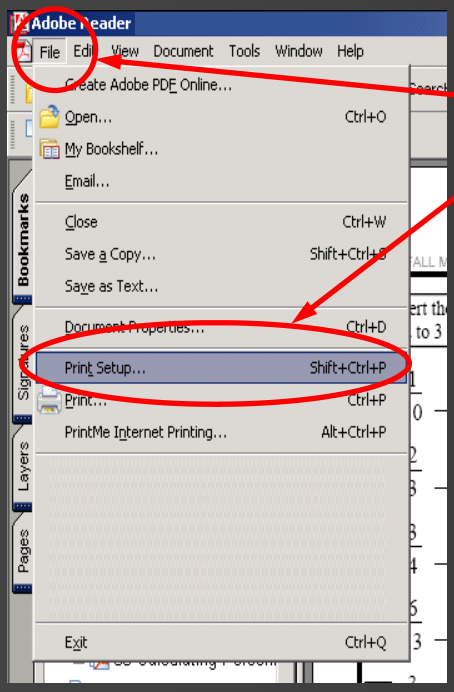
Printer Setup - These worksheets are designed to be printed on A4 paper.

Adobe Reader is usually set in US 'Letter' format as its default setting. Adobe Reader can print on any page size but Freefall worksheets are designed to be printed on A4 paper. To change the paper setting follow the steps below. This step only has to be done once.

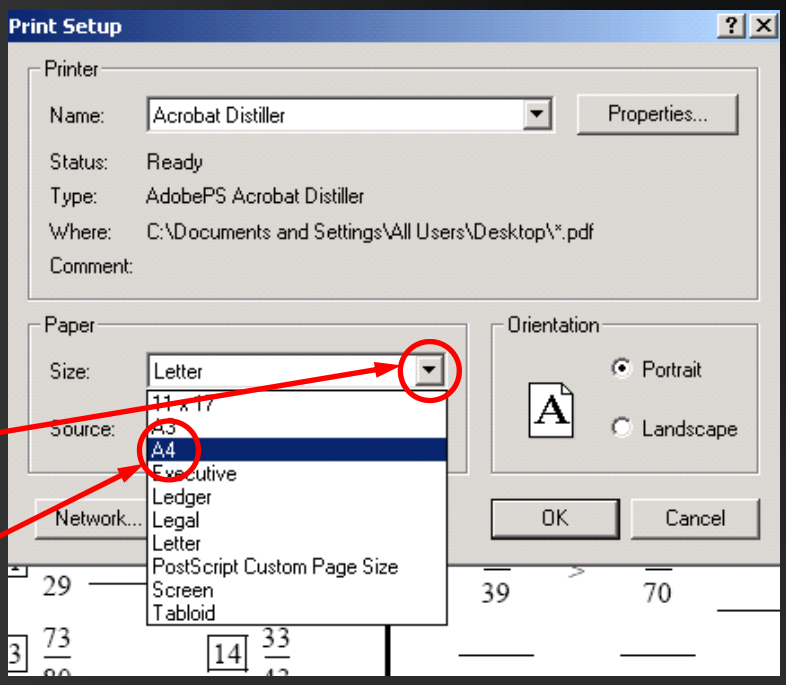


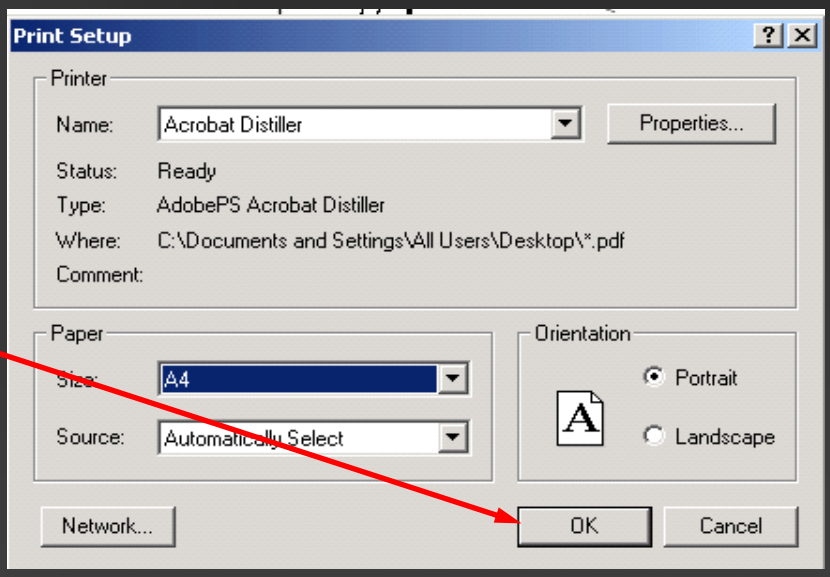
1

Select the 'File' menu and then 'Print Setup'.

2

Press the down arrow & select 'A4' size paper.





3

Press 'OK'. The format is now set to A4.

Setting will stay as A4 unless it is changed.

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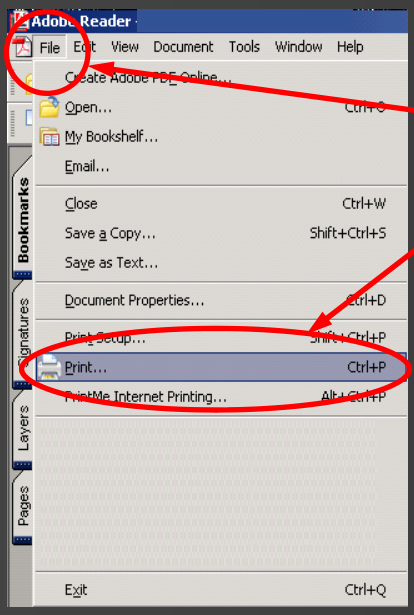
Printing Worksheets - These worksheets are designed to be printed on A4 paper.

NOTE THAT IF PAGE SCALING ISN'T SET TO 'NO SCALING' THEN ANSWERS WILL BE INCORRECT IF A RULER OR PROTRACTOR IS REQUIRED.

To print a worksheet navigate to the sheet using the bookmarks to the left of screen until the page required is on screen. Ensure that only one page is being displayed (not two) and then follow the steps outlined below.

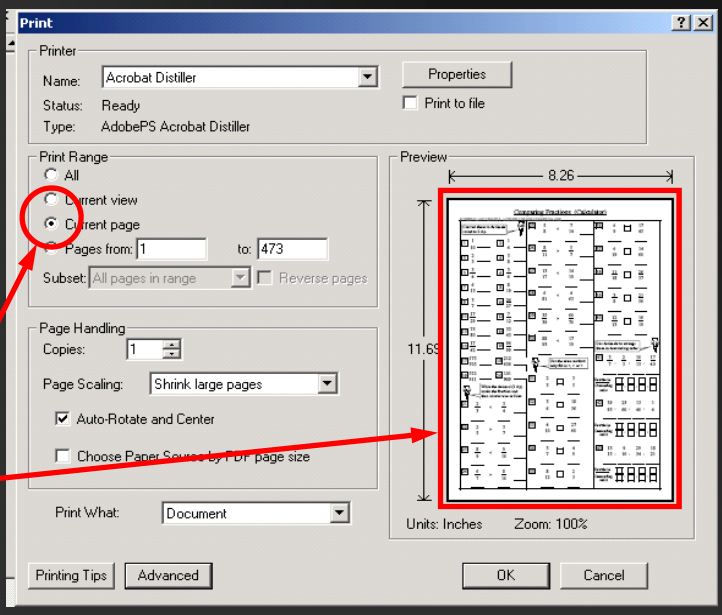


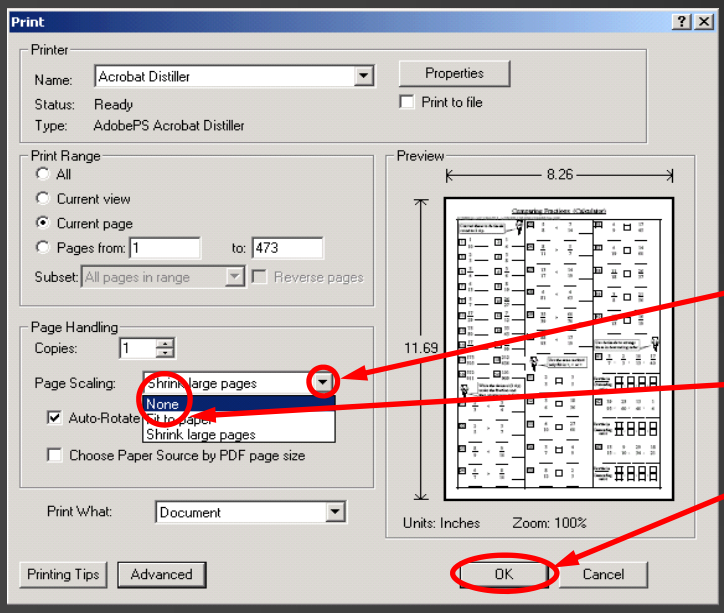
1

Select the 'File' menu and then 'Print'.

2

Select 'Current page'. The page is then displayed.





3

At 'Paper Scaling' press the down arrow and select 'None' then press the 'OK' button.

Your page should then print.

Repeat for answer sheet

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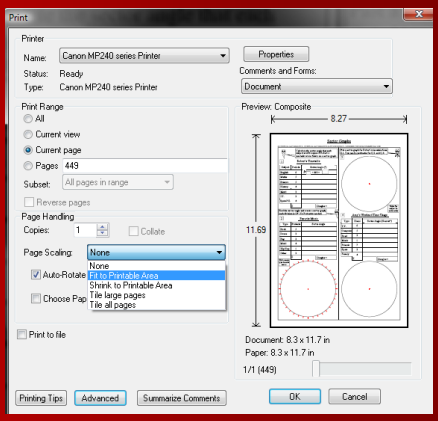
Problem - My printer misses out information at the base of the page

Some older printers are unable to print within 1 cm of the base of the page, which is where Freefall worksheets end. If you are printing sheets using "No Scaling" and the bottom of the page is not being fully printed you have two options:

Select "Fit to Printable Area" as shown in the image to the right, this will shrink the worksheet slightly.

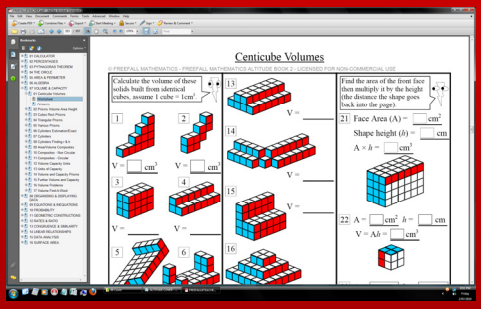


REALISE THAT THE SCALE OF THE WORKSHEET CHANGES IF YOU CHOOSE THIS PRINT OPTION.

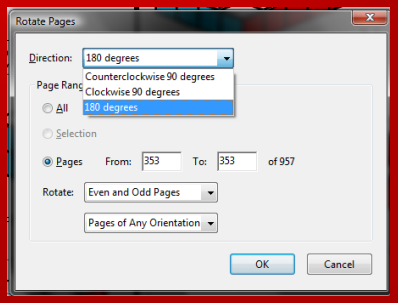
*If a **RULER** or a **PROTRACTOR** is used to answer the sheet, or grid squares of a specific size are used then your **ANSWERS WILL BE WRONG**.*
Note also that the amount of space to write your answer is reduced as the sheet is smaller.

The other method is to turn the page upside-down and print it that way. The title will be affected rather than the bottom of the worksheet. To do this:

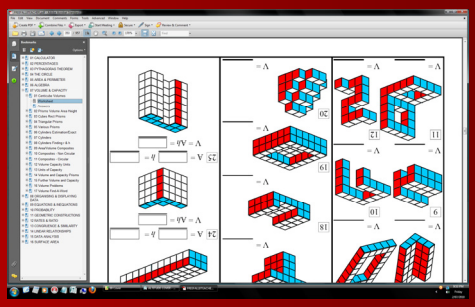
Navigate to the page you want to print



Hold CTRL+ SHIFT and Press R SELECT 180°



The page is now upside-down.



You can also use the View menu & Select Rotate View

Select "No Scaling" when you print the page, so any measurements on the page stay to scale. You will lose the top of your title when you do this, as it is printed last. So write the title in yourself after the page has been printed.

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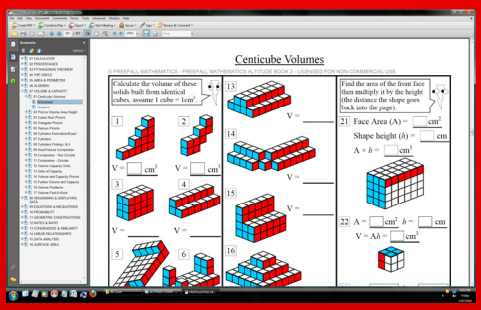
Problem - My printer doesn't print the coloured writing on the page

Some sheets use colour on the page, some examples are in blue and will vanish if you have no colour left in your colour printer. If you have a monochrome (or black only) printer this isn't a problem, because the printer automatically changes the colour to a shade of gray. But if you have a colour printer and either:

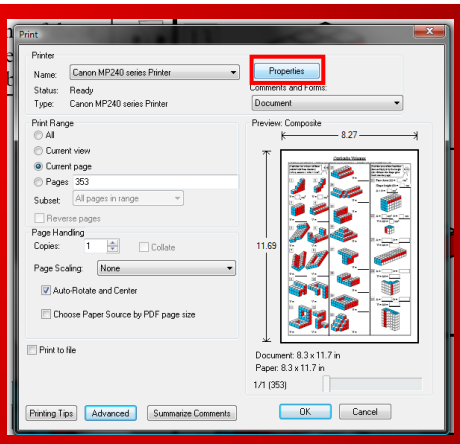
- *Don't want to waste the colour ink on the worksheet, or*
- *Are out of colour ink and you still want to print the coloured parts of the sheet*

Then you need to grayscale your printing. This is how you do it.

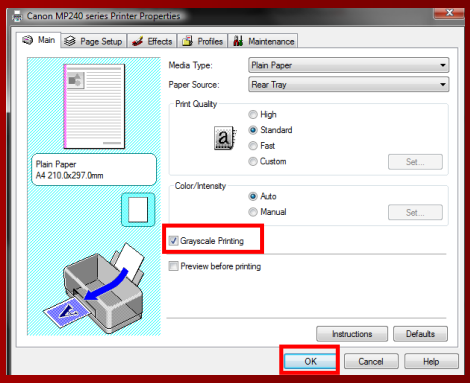
Navigate to the page you want to print



Select Properties from the Print Menu



Select Grayscale Printing and then press OK



Note that the printing menu varies on brand and model of printer. It may be that you have to select a tab such as the 'effects' tab to find the grayscale option, but it will be there. The result will have the colours changed to gray and they will be visible without the need to use your coloured ink.

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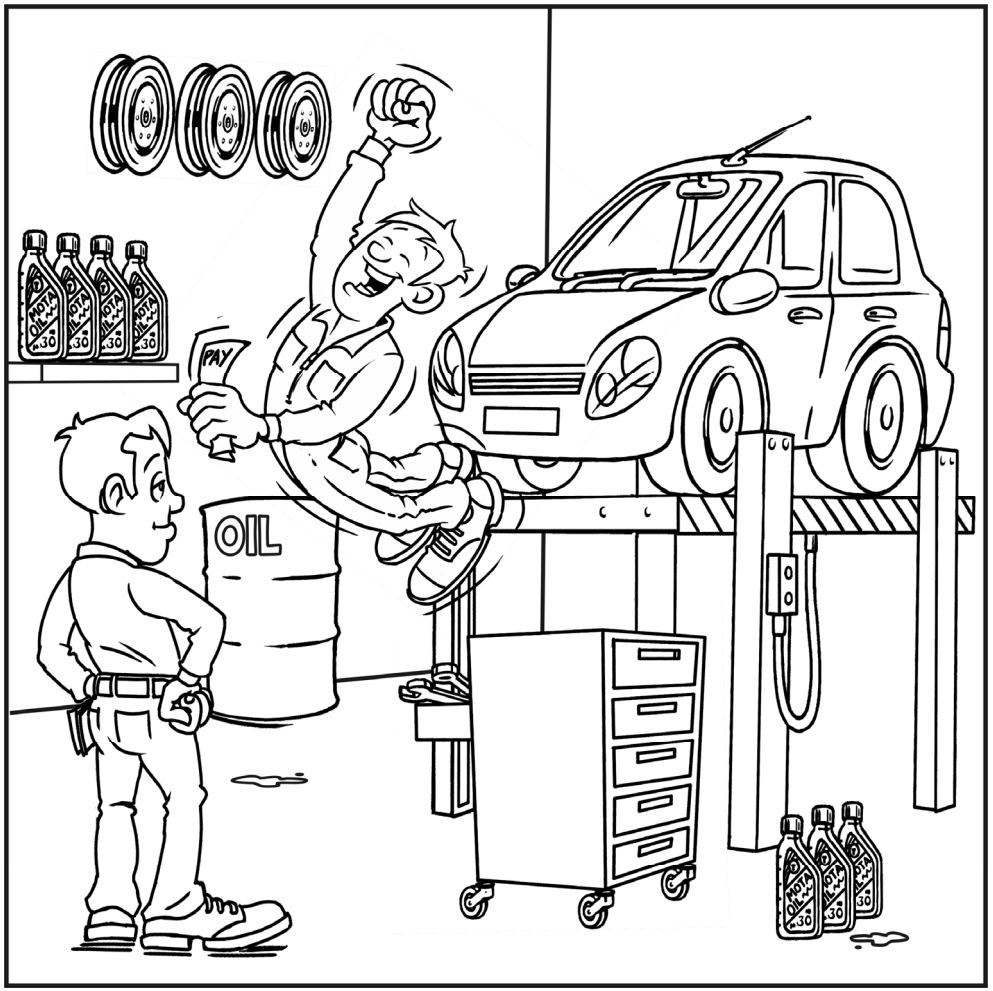
EARNING MONEY

Earning Money



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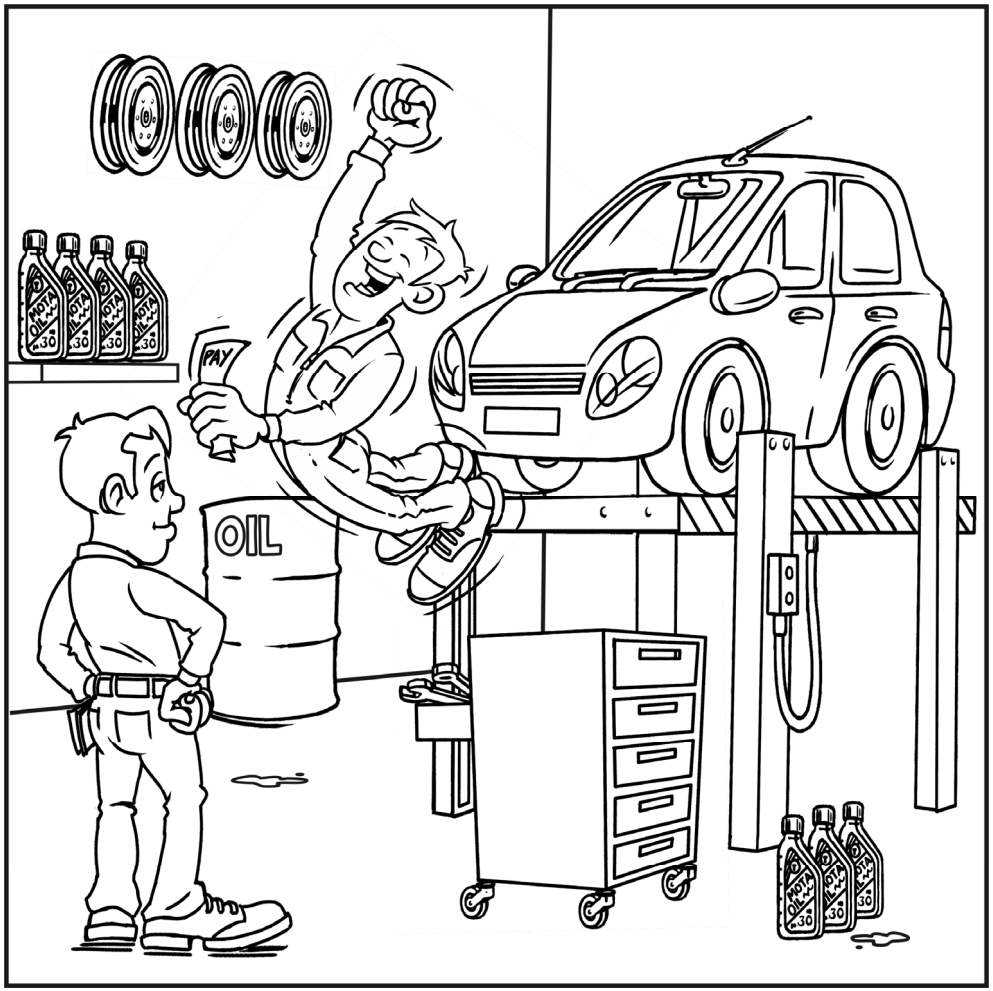
Earning Money



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Earning Money - Hourly Rates

To find how much a person earns in a week, you multiply the hourly rate by the number of hours worked in a day. So if you earn $19.40/h and work a 36 hour week then your pay is:

Example

$19.40/h for a 36 hour week

Weekly Pay = 19.40×36

Weekly Pay = $698.40

Remember dollar sign and always write cents to 2 decimal places

The first sheet gives you hourly rates and you find the weekly pay. Column 3 asks you to compare people's wages or calculate a percentage that a person's wage is from the total of the wages paid.

The second sheet also deals with hourly rates but it asks you to find the daily rate as well as the weekly rate.

Example

Find daily pay by Daily hours × hourly rate

Find weekly pay by Daily pay × days per week

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Ron	6	$17.35	$104.10	4	$416.40
Don	8	$19.72	$157.76	5	$788.80
John	7	$23.11	$161.77	3	$485.31
		Total Daily	$423.63	*Total Weekly*	$1 690.51

Add columns for totals

The third sheet has worded questions testing all the above skills.

Earning Money - Hourly Rates (Sheet 1)

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To convert an hourly rate to a weekly wage, multiply the rate by the hours.

Example: Find the weekly pay on $23/h for a 38 hour week.

$23 \times 38 = \$874$

1 $19/h for a 27 hour week.

Weekly Pay = ☐ × ☐

Weekly Pay = $ ☐☐☐

2 $24/h for a 32 hour week.

Weekly Pay = ☐ × ☐

Weekly Pay = $ ☐☐☐

3 $17.17/h for a 41 hour week

Weekly Pay = ☐ × ☐

Weekly Pay = $ ☐☐☐.☐☐

4 $13.95/h for an 8 hour week

Weekly Pay = ☐ × ☐

Weekly Pay = $ ☐☐☐.☐☐

5 $16.80/h for a 18 hour week

Weekly Pay =

Weekly Pay =

Show the $ sign and cents to 2 decimal places

6 $14.45/h for a 26 hour week

Weekly Pay =

Weekly Pay =

Show the $ sign and cents to 2 decimal places

7 $38.17/h for a 38 hour week

Weekly Pay =

Weekly Pay =

Show the $ sign and cents to 2 decimal places

Find the wages, then add for the total wages paid.

8

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Ted	38	$15.25	
Fred	32	$18.32	
Ned	35	$16.46	
		Total Paid	

9

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Bill	26	$17.45	
Phil	38	$19.26	
Jill	38	$22.90	
		Total Paid	

10

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Anne	42	$28.75	
Stan	25	$27.40	
Jan	38.5	$46.82	
		Total Paid	

11

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Rick	11.5	$21.20	
Mick	9.25	$38.16	
Nick	6.75	$26.32	
		Total Paid	

For the table in Question 8:

12 Who is paid more, Ned or Ted? ☐

13 Find the range of the pays.

Range = ☐ - ☐

Range = ☐

Largest amount minus (-) the Lowest amount

14 Find, to 1 d.p, Fred's pay as a percentage of the total pay.

$$\text{Fred \%} = \frac{\text{Fred's pay}}{\text{Total pay}} \times 100$$

Fred's pay ÷ Total Pay × 100

Fred % = ☐

Show the % sign and round to 1 decimal place

For the table in Question 10:

15 Who is paid more, Anne or Jan? ☐

16 Find the range of the pays.

Range = ☐ - ☐

Range = ☐

17 Find, to 1 d.p, Stan's pay as a percentage of the total pay.

$$\text{Stan \%} = \frac{\text{Stan's pay}}{\text{Total pay}} \times 100$$

Stan % = ☐

For the table in Question 11:

18 How much more than Nick, is Mick paid?

Earning Money - Hourly Rates (Sheet 2)

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Find the daily pay then the weekly pay given the hourly rate.



Example:

Rate: $19.35/h

Conditions: 7 h day, 5 day week

Daily Pay = 19.35 × 7

Daily Pay = $ 135.45

Weekly Pay = 135.45 × 5

Weekly Pay = $ 677.25

1 Rate: $18.45/h

Conditions: 4 h day, 3 day week

Daily Pay = ____ × ____

Daily Pay = $ ____

Show cents to 2 decimal places

Weekly Pay = ____ × ____

Weekly Pay = $ ____

2 Rate: $23.15/h

Conditions: 8 h day, 5 day week

Daily Pay = ____ × ____

Daily Pay = ____

Show the $ sign and cents to 2 decimal places

Weekly Pay = ____ × ____

Weekly Pay = ____

3 Rate: $19.06/h

Conditions: 5 h day, 4 day week

Daily Pay = ____ × ____

Daily Pay = ____

Weekly Pay = ____ × ____

Weekly Pay = ____

Given the hourly rate and hours worked, complete the tables.



Show the $ sign and cents to 2 decimal places

4

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Kane	3	$15.30		6	
Jane	5	$19.15		5	
Zane	4	$26.75		3	
		Total Daily		*Total Weekly*	

5

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Dawn	5	$17.55		5	
Shaun	7	$20.92		3	
Lorne	6	$16.88		4	
		Total Daily		*Total Weekly*	

Use decimals to show half hours, so for 7½ hours use 7.5 instead.

6

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Jo	7½	$17.60		4	
Flo	5	$38.42		6	
Beau	6½	$26.18		5	
		Total Daily		*Total Weekly*	

When you get answers with 3 decimal places, round cents to 2 decimal places

7

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Cam	3	$20.43		5	
Ram	8½	$29.47		3	
Sam	4½	$18.33		6	
Pam	5½	$26.51		4	
		Total Daily		*Total Weekly*	

Earning Money - Hourly Rates (Sheet 3)

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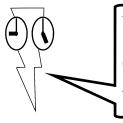

Use the hourly rate to calculate the pay for these employees.

1 Rami earns $28.20 per hour. He works 7 hours one day, and 5 hours the next. Find:

a) Total hours worked

Hours = ☐ + ☐ = ☐ h

b) Total pay earned

Pay = ☐ × ☐

Pay = ☐

Show the $ sign and cents to 2 decimal places

2 Jess earns $37.35 per hour. She works 8 hours per day, and 4 days each week. Find:

a) Her daily pay

Pay =

Pay =

b) Her weekly pay

Pay =

Pay =

3 James earns $21.90/h. If he works a 38 hour week and pays $230/wk rent and has $275/wk in living expenses. Find:

a) His weekly pay

Pay =

Pay =

b) His savings for the week.

Savings =

Savings =

4 Ellen earns $43.85/h while Ben earns $27.90/h. If both work a 38 hour week, find:

a) Ellen's weekly pay

b) Ben's weekly pay

c) The difference between Ellen's and Ben's weekly pay.

5 Travis earns $28.63/h for a 16 h week, while Lewis earns $31.55/h for a 19 h week, find:

a) Travis' weekly pay

b) Lewis' weekly pay

c) The difference between Travis' and Lewis' weekly pay.

d) Express Travis' pay as a percentage of Lewis' pay. (1 dp)

$$\text{Travis \%} = \frac{\text{Travis' pay}}{\text{Lewis' pay}} \times 100$$

Travis % = ☐ %

6 Kelly earns $33.15/h, Toby earns $29.16/h and Ahmed earns $30.25/h. If all work a 38 hour week, find:

a) Kelly's weekly pay

b) Toby's weekly pay

c) Ahmed's weekly pay

d) The average wage earned.

$$\text{Average Wage} = \frac{\square}{3} \leftarrow \text{Total Wages}$$

Add the 3 wages, press '=', then press '÷3'.

Average Wage = ☐

7 Tye earns $23.45/h for a 16 h week, while Kwan earns $22.85/h for a 38 h week, find:

a) Tye's weekly pay

b) Kwan's weekly pay

c) Express Tye's pay as a percentage of Kwan's pay. (1dp)

$$\text{Tye's \%} = \frac{\square}{\square} \times 100$$

Tye's % = ☐

Earning Money - Hourly Rates (Sheet 1)

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To convert an hourly rate to a weekly wage, multiply the rate by the hours.

Example: Find the weekly pay on $23/h for a 38 hour week.

$23 \times 38 =$ $874

1 $19/h for a 27 hour week.

Weekly Pay = 19 × 27

Weekly Pay = $ 5 1 3

2 $24/h for a 32 hour week.

Weekly Pay = 24 × 32

Weekly Pay = $ 7 6 8

3 $17.17/h for a 41 hour week

Weekly Pay = 17.17 × 41

Weekly Pay = $ 7 0 3 . 9 7

4 $13.95/h for an 8 hour week

Weekly Pay = 13.95 × 8

Weekly Pay = $ 1 1 1 . 6 0

5 $16.80/h for a 18 hour week

Weekly Pay = 16.80 × 18

Weekly Pay = $302.40

Show the $ sign and cents to 2 decimal places

6 $14.45/h for a 26 hour week

Weekly Pay = 14.45 × 26

Weekly Pay = $375.70

Show the $ sign and cents to 2 decimal places

7 $38.17/h for a 38 hour week

Weekly Pay = 38.17 × 38

Weekly Pay = $1 450.46

Show the $ sign and cents to 2 decimal places

Find the wages, then add for the total wages paid.

8

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Ted	38	$15.25	$579.50
Fred	32	$18.32	$586.24
Ned	35	$16.46	$576.10
		Total Paid	$1 741.84

9

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Bill	26	$17.45	$453.70
Phil	38	$19.26	$731.88
Jill	38	$22.90	$870.20
		Total Paid	$2 055.78

10

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Anne	42	$28.75	$1 207.50
Stan	25	$27.40	$685
Jan	38.5	$46.82	$1 802.57
		Total Paid	$3 695.07

11

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Rick	11.5	$21.20	$243.80
Mick	9.25	$38.16	$352.98
Nick	6.75	$26.32	$177.66
		Total Paid	$774.44

For the table in Question 8:

12 Who is paid more, Ned or Ted? Ted

13 Find the range of the pays.

Range = 579.50 - 576.10

Range = $3.40

Largest amount minus (-) the Lowest amount

14 Find, to 1 d.p, Fred's pay as a percentage of the total pay.

$$\text{Fred \%} = \frac{586.24}{1741.84} \times 100$$

Fred's pay ÷ Total Pay × 100

Fred % = 33.7%

Show the % sign and round to 1 decimal place

For the table in Question 10:

15 Who is paid more, Anne or Jan? Jan

16 Find the range of the pays.

Range = 1 802.57 - 1 207.50

Range = $595.07

17 Find, to 1 d.p, Stan's pay as a percentage of the total pay.

$$\text{Stan \%} = \frac{685}{3695.07} \times 100$$

Stan % = 18.5%

For the table in Question 11:

18 How much more than Nick, is Mick paid?

Amount = 352.98 - 177.66

Amount = $175.32

Earning Money - Hourly Rates (Sheet 2)

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Find the daily pay then the weekly pay given the hourly rate.



Example:

Rate: $19.35/h

Conditions: 7 h day, 5 day week

Daily Pay = 19.35 × 7

Daily Pay = $ 135.45

Weekly Pay = 135.45 × 5

Weekly Pay = $ 677.25

1 Rate: $18.45/h

Conditions: 4 h day, 3 day week

Daily Pay = 18.45 × 4

Daily Pay = $ 73.80

Show cents to 2 decimal places

Weekly Pay = 73.80 × 3

Weekly Pay = $ 221.40

2 Rate: $23.15/h

Conditions: 8 h day, 5 day week

Daily Pay = 23.15 × 8

Daily Pay = $185.20

Show the $ sign and cents to 2 decimal places

Weekly Pay = 185.20 × 5

Weekly Pay = $926

3 Rate: $19.06/h

Conditions: 5 h day, 4 day week

Daily Pay = 19.06 × 5

Daily Pay = $95.30

Weekly Pay = 95.30 × 4

Weekly Pay = $381.20

Given the hourly rate and hours worked, complete the tables.



Show the $ sign and cents to 2 decimal places

4

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Kane	3	$15.30	$45.90	6	$275.40
Jane	5	$19.15	$95.75	5	$478.75
Zane	4	$26.75	$107	3	$321
		Total Daily	$248.65	*Total Weekly*	$1 075.15

5

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Dawn	5	$17.55	$87.75	5	$438.75
Shaun	7	$20.92	$146.44	3	$439.32
Lorne	6	$16.88	$101.28	4	$405.12
		Total Daily	$335.47	*Total Weekly*	$1 283.19

Use decimals to show half hours, so for 7½ hours use 7.5 instead.

6

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Jo	7½	$17.60	$132	4	$528
Flo	5	$38.42	$192.10	6	$1 152.60
Beau	6½	$26.18	$170.17	5	$850.85
		Total Daily	$494.27	*Total Weekly*	$2 531.45

When you get answers with 3 decimal places, round cents to 2 decimal places

7

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Cam	3	$20.43	$61.29	5	$306.45
Ram	8½	$29.47	$250.50	3	$751.50
Sam	4½	$18.33	$82.49	6	$494.94
Pam	5½	$26.51	$145.81	4	$583.24
		Total Daily	$540.09	*Total Weekly*	$2 136.13

Earning Money - Hourly Rates (Sheet 3)

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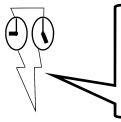

Use the hourly rate to calculate the pay for these employees.

1 Rami earns $28.20 per hour. He works 7 hours one day, and 5 hours the next. Find:

a) Total hours worked

Hours = 7 + 5 = 12 h

b) Total pay earned

Pay = 28.20 × 12

Pay = $338.40

Show the $ sign and cents to 2 decimal places

2 Jess earns $37.35 per hour. She works 8 hours per day, and 4 days each week. Find:

a) Her daily pay

Pay = 37.35 × 8

Pay = $298.80

b) Her weekly pay

Pay = 298.80 × 4

Pay = $1 195.20

3 James earns $21.90/h. If he works a 38 hour week and pays $230/wk rent and has $275/wk in living expenses. Find:

a) His weekly pay

Pay = 21.90 × 38

Pay = $832.20

b) His savings for the week.

Savings = 832.20 - 230 - 275

Savings = $327.20

4 Ellen earns $43.85/h while Ben earns $27.90/h. If both work a 38 hour week, find:

a) Ellen's weekly pay

Pay = 43.85 × 38

Pay = $1 666.30

b) Ben's weekly pay

Pay = 27.90 × 38

Pay = $1 060.20

c) The difference between Ellen's and Ben's weekly pay.

Difference = 1 666.30 - 1060.20

Difference = $606.10

5 Travis earns $28.63/h for a 16 h week, while Lewis earns $31.55/h for a 19 h week, find:

a) Travis' weekly pay

Pay = 28.63 × 16

Pay = $458.08

b) Lewis' weekly pay

Pay = 31.55 × 19

Pay = $599.45

c) The difference between Travis' and Lewis' weekly pay.

Difference = 599.45 - 458.08

Difference = $141.37

d) Express Travis' pay as a percentage of Lewis' pay. (1 dp)

$$\text{Travis \%} = \frac{458.08}{599.45} \times 100$$

Travis % = 76.4 %

6 Kelly earns $33.15/h, Toby earns $29.16/h and Ahmed earns $30.25/h. If all work a 38 hour week, find:

a) Kelly's weekly pay

Pay = 33.15 × 38

Pay = $1 259.70

b) Toby's weekly pay

Pay = 29.16 × 38

Pay = $1 108.08

c) Ahmed's weekly pay

Pay = 30.25 × 38

Pay = $1 149.50

d) The average wage earned.

$$\text{Average Wage} = \frac{3\ 527.28}{3}$$ ← Total Wages

Add the 3 wages, press '=', then press '÷3'.

Average Wage = $1 172.43

7 Tye earns $23.45/h for a 16 h week, while Kwan earns $22.85/h for a 38 h week, find:

a) Tye's weekly pay

Pay = 23.45 × 16

Pay = $375.20

b) Kwan's weekly pay

Pay = 22.85 × 38

Pay = $868.30

c) Express Tye's pay as a percentage of Kwan's pay. (1dp)

$$\text{Tye's \%} = \frac{375.20}{868.30} \times 100$$

Tye's % = 43.2%

Earning Money - Hourly Rates (Homework Strip)

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Date Due:

Find the pay

1 $13.12/h for a 9 hour day

Weekly Pay = ×

Weekly Pay = $.

2 $17.20/h for a 9 hour day

Weekly Pay = ×

Weekly Pay = $.

3 $24.65/h for a 7 hour day

Weekly Pay = ×

Weekly Pay = $.

4 $19.17/h for a 40 hour week

Weekly Pay = ×

Weekly Pay = $.

5 $25.06/h for a 38 hour week

Weekly Pay = ×

Weekly Pay = $.

Date Due:

Find the pay

1 $13.12/h for a 9 hour day

Weekly Pay = ×

Weekly Pay = $.

2 $17.20/h for a 9 hour day

Weekly Pay = ×

Weekly Pay = $.

3 $24.65/h for a 7 hour day

Weekly Pay = ×

Weekly Pay = $.

4 $19.17/h for a 40 hour week

Weekly Pay = ×

Weekly Pay = $.

5 $25.06/h for a 38 hour week

Weekly Pay = ×

Weekly Pay = $.

Date Due:

Find the pay

1 $13.12/h for a 9 hour day

Weekly Pay = ×

Weekly Pay = $.

2 $17.20/h for a 9 hour day

Weekly Pay = ×

Weekly Pay = $.

3 $24.65/h for a 7 hour day

Weekly Pay = ×

Weekly Pay = $.

4 $19.17/h for a 40 hour week

Weekly Pay = ×

Weekly Pay = $.

5 $25.06/h for a 38 hour week

Weekly Pay = ×

Weekly Pay = $.

Date Due:

Find the pay

1 $13.12/h for a 9 hour day

Weekly Pay = ×

Weekly Pay = $.

2 $17.20/h for a 9 hour day

Weekly Pay = ×

Weekly Pay = $.

3 $24.65/h for a 7 hour day

Weekly Pay = ×

Weekly Pay = $.

4 $19.17/h for a 40 hour week

Weekly Pay = ×

Weekly Pay = $.

5 $25.06/h for a 38 hour week

Weekly Pay = ×

Weekly Pay = $.

Earning Money - Hourly Rates (Homework Strip)

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Date Due:

Find the pay

1 $13.12/h for a 9 hour day

Weekly Pay = 13.12 × 9

Weekly Pay = $ 1 1 8 . 0 8

2 $17.20/h for a 9 hour day

Weekly Pay = 17.20 × 9

Weekly Pay = $ 1 5 4 . 8 0

3 $24.65/h for a 7 hour day

Weekly Pay = 24.65 × 7

Weekly Pay = $ 1 7 2 . 5 5

4 $19.17/h for a 40 hour week

Weekly Pay = 19.17 × 40

Weekly Pay = $ 7 6 6 . 8 0

5 $25.06/h for a 38 hour week

Weekly Pay = 25.06 × 38

Weekly Pay = $ 9 5 2 . 2 8

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods

To find how much a person earns in a fortnight, you multiply the hourly rate by the number of hours worked in a week, then double it (× by 2). So if you earn $19.40/h and work a 36 hour week then your fortnightly pay is:

Example

$19.40/h for a 36 hour week

F'tnightly Pay = 19.40 × 36 × 2

F'tnightly Pay = $1 396.80 ← Remember dollar sign and always write cents to 2 decimal places

The second column gives you a weekly wage and hours worked, you have to calculate the hourly rate. If you divide the weekly wage by the hours worked, you find the hourly rate.

Column 3 asks you to find the hourly rate but this time the data is in a table. After you complete the table, calculate the mean hourly rate. The mean hourly rate is found by adding the hourly rates for each person, then dividing by the number of people.

The second sheet also deals with hourly rates but it asks you to find weekly pay, fortnightly pay or divide to find the hourly rate.

The third sheet is a mixture of the above skills with worded questions.

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 1)

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Convert an hourly rate to a fortnightly wage. Multiply the rate by the 2 × weekly hours.

Example: Find the fortnightly pay on $21/h for a 38 hour week.

F'tnightly Pay = 38 × 2 × 21

Hours × 2 × Hourly rate

F'tnightly Pay = $1 596

1 $17/h for a 34 hour week.

F'tnightly Pay = ____ × 2 × ____

F'tnightly Pay = $ ____

2 $18.35 /h for a 34 h week.

F'tn Pay =

F'tn Pay =

3 $23.72 /h for an 18 h week.

F'tn Pay =

F'tn Pay =

4 $35.67 /h for a 38 h week.

5 $29.55 /h for a 25 h week.

6

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Ftn'ly Pay*
Zoe	26	$17.45	
Chloe	25	$19.26	
Joey	16	$22.90	
		Total Paid	

Find the hourly rate by dividing the pay by the hours worked.

Example: Find the hourly rate if paid $790 for a 25 hour week.

Hourly rate = 790 ÷ 25

Pay ÷ hours

Hourly rate = $31.60 / h

7 $456.30 for a 26 hour week.

Hourly rate = ____ ÷ ____

Hourly rate = $ ____ / h

8 $919.60 for a 38 hour week.

Hourly rate =

Hourly rate = $ ____ / h

Show the $ sign and cents to 2 decimal places

9 $340 for a 16 hour week.

Hourly rate =

Hourly rate =

10 $1 512 for a 35 hour week.

11 $162 for an 8 hour week.

Find the hourly rate, then calculate the mean hourly rate.

12

Name	*Weekly Wage*	*Weekly Hours*	*Hourly rate*
Ben	$427.20	16	
Jen	$1 232	35	
Len	$883.40	28	
		Total	

13 Find the mean hourly rate.

Mean Hourly Rate = Total / 3

Total ÷ Number of employees

= ____ / h

14

Name	*Weekly Wage*	*Weekly Hours*	*Hourly rate*
Milly	$787.20	32	
Billy	$207.99	9	
Willy	$646.40	16	
Tilly	$339.48	12	

15 Mean Hourly Rate = Total / ____

= ____

16

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Speedy	32		$764.80	
Pablo		$28.60		$2 173.60
José	38	$21.42		
Manuel	28			$1 822.80

Complete the table for wages at the Ándele! Ándele! Epa! Epa! Café.

Hint! Convert fortnightly wages to weekly by ÷ 2.

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 2)

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Given an hourly rate, or weekly wage, or fortnightly wage, find these wages/rates/hours worked.

1 Jim earns $38.15 / h for a 38 hour week. Find his:

Weekly Pay _____

Multiply the hourly rate by hours worked

_____

F'nightly Pay _____

Hourly rate × hours × 2, **or** weekly pay × 2

_____

2 Kim earns $43.82 / h for a 40 hour week. Find her:

Weekly Pay _____

_____

F'nightly Pay _____

_____

3 Tim earns $1 061.34 for a 38 hour week. Find his:

Hourly Rate _____

_____

F'nightly Pay _____

_____

4 Lim earns $1 408 / f'night working a 25 hour week. Find:

Weekly Pay _____

_____

Hourly Rate _____

_____

5 Tom earns $21.96 / h earning $746.64 / week. Find:

Hours Worked _____

_____

F'nightly Pay _____

_____

6 Dom earns $25.65 / h for a 15 hour week. Find his:

Weekly Pay _____

_____

F'nightly Pay _____

_____

7 Jess earns $1 634.36/f'night She is paid $31.43 / h. Find her:

Weekly Pay _____

_____

Weekly Hours Worked _____

Weekly pay ÷ Hourly rate

_____

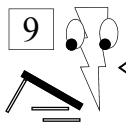
8 Tess earns $20.72 / h for a 56 hour f'tnight. Find her:

F'nightly Pay _____

_____

Weekly Hours Worked _____

_____



9 Complete these tables and then find the mean (average) hours worked and mean weekly pay for the employees listed.

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Ralph	35	$23.22		
Joanie		$27.65	$1 050.70	
Potsie	12			$476.40
Ritchie		$23.88	$644.76	
Total		*Total*		

Mean Weekly Hours = Total hours worked ÷ Number of employees

= _____ / week

Mean Weekly Pay = Total weekly wage ÷ _____

= _____ / week

10

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Legolas		$28.20	$592.20	
Aragon	22	$36.15		
Gimli	11			$620.40
Total		*Total*		

Mean Weekly Hours = _____ ÷ _____

= _____ / week

Mean Weekly Pay = _____ ÷ _____

= _____ / week

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 3)

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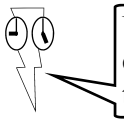

Use the hourly rate to calculate the pay for these employees.

1 Tyrone works 17 hours a week and earns $328.10, find:

a) His hourly rate.

Rate = ☐ ÷ ☐

Rate = ☐ /h

Write '$' sign and show money to 2 d.p.

b) His fortnightly pay.

Pay = ☐ × ☐

Pay = ☐

2 Thien earns $24.95 per hour. If he earned $798.40 for the week, calculate:

a) The hours worked this week

Hours =

Hours =

b) His fortnightly pay.

Pay =

Pay =

3 Bassam earns $27.20 per hour. If he works 38 h per week, calculate:

a) His weekly pay

Pay =

Pay =

b) His fortnightly pay.

Pay =

Pay =

3 Jemma earns $21.40/h and has recorded her hours below.

Day	M	T	W	T	F	Total
Hours	4	7	7	4	8	

a) Complete the total column Finding the total hours worked.

b) Find Jemma's weekly pay.

Pay =

Pay =

4 Simon earns $21.45/h and has a weekly pay of $729.30

a) Calculate the total hours Simon has worked for the week.

Hours =

Hours =

b) Complete the table below showing Simon's daily hours.

Day	M	T	W	T	F	Total
Hours	8	4		7	8	

5 Chloe earns $19.55/h and has a weekly pay of $625.60

a) Calculate the hours Chloe has worked for the week.

Hours =

Hours =

b) Complete the table below showing Chloe's daily hours.

Day	M	T	W	T	F	Total
Hours	7	8	6		8	

c) Find her Thursday earnings.

Pay =

Pay =

6 Hollie earns $23.30/h. If she works 25½ h each week, find:

a) Her weekly pay

Pay =

Pay =

b) Her fortnightly pay.

Pay =

Pay =

7 Faith earns $38.80 per hour. If she earned $2 754.80 for the fortnight, calculate her:

a) Fortnightly hours worked

Hours =

Hours =

b) Her weekly pay.

Pay =

Pay =

8 Georgia earns $84.15 when she works a 5½ h day. How much does she earn for a 38 hour week?

9 Robert earns $21.20/h and is paid $805.60 weekly. How many hours does he work per fortnight?

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 1)

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Convert an hourly rate to a fortnightly wage. Multiply the rate by the 2 × weekly hours.

Example: Find the fortnightly pay on $21/h for a 38 hour week.

F'tnightly Pay = 38 × 2 × 21

Hours × 2 × Hourly rate

F'tnightly Pay = $1 596

1 $17/h for a 34 hour week.

F'tnightly Pay = 17 × 2 × 34

F'tnightly Pay = $ 1 156

2 $18.35 /h for a 34 h week.

F'tn Pay = 18.35 × 2 × 34

F'tn Pay = $1 247.80

3 $23.72 /h for an 18 h week.

F'tn Pay = 23.72 × 2 × 18

F'tn Pay = $853.92

4 $35.67 /h for a 38 h week.

Fnt'ly Pay = 35.67 × 2 × 38

Fnt'ly Pay = $2 710.92

5 $29.55 /h for a 25 h week.

Fnt'ly Pay = 29.55 × 2 × 25

Fnt'ly Pay = $1 477.50

6

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Ftn'ly Pay*
Zoe	26	$17.45	$907.40
Chloe	25	$19.26	$963
Joey	16	$22.90	$732.80
		Total Paid	$2 603.20

Find the hourly rate by dividing the pay by the hours worked.

Example: Find the hourly rate if paid $790 for a 25 hour week.

Hourly rate = 790 ÷ 25

Pay ÷ hours

Hourly rate = $31.60 / h

7 $456.30 for a 26 hour week.

Hourly rate = 456.30 ÷ 26

Hourly rate = $ 17.55 / h

8 $919.60 for a 38 hour week.

Hourly rate = 919.60 ÷ 38

Hourly rate = $ 24.20 / h

Show the $ sign and cents to 2 decimal places

9 $340 for a 16 hour week.

Hourly rate = 340 ÷ 16

Hourly rate = $21.25 / h

10 $1 512 for a 35 hour week.

Hourly rate = 1 512 ÷ 35

Hourly rate = $43.20 / h

11 $162 for an 8 hour week.

Hourly rate = 162 ÷ 8

Hourly rate = $20.25 / h

Find the hourly rate, then calculate the mean hourly rate.

12

Name	*Weekly Wage*	*Weekly Hours*	*Hourly rate*
Ben	$427.20	16	$26.70
Jen	$1 232	35	$35.20
Len	$883.40	28	$31.55
		Total	$93.45

13 Find the mean hourly rate.

Mean Hourly Rate = 93.45 / 3

Total ÷ Number of employees

= $31.15 / h

14

Name	*Weekly Wage*	*Weekly Hours*	*Hourly rate*
Milly	$787.20	32	$24.60
Billy	$207.99	9	$23.11
Willy	$646.40	16	$40.40
Tilly	$339.48	12	$28.29

15 Mean Hourly Rate = 116.40 / 4

= $29.10 / h

16

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Speedy	32	$23.90	$764.80	$1 529.60
Pablo	38	$28.60	$1 086.80	$2 173.60
José	38	$21.42	$813.96	$1 627.92
Manuel	28	$32.55	$911.40	$1 822.80

Complete the table for wages at the Ándele! Ándele!Arriba!Arriba! Epa!Epa! Café.

Hint! Convert fortnightly wages to weekly by ÷ 2.

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 2)

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Given an hourly rate, or weekly wage, or fortnightly wage, find these wages/rates/hours worked.

1 Jim earns $38.15 / h for a 38 hour week. Find his:

Weekly Pay = 38.15 × 38

Multiply the hourly rate by hours worked

= $1 449.70

F'nightly Pay = 1 449.70 × 2

Hourly rate × hours × 2, **or** weekly pay × 2

= $2 899.40

2 Kim earns $43.82 / h for a 40 hour week. Find her:

Weekly Pay = 43.82 × 40

= $1 752.80

F'nightly Pay = 1 752.80 × 2

= $3 505.60

3 Tim earns $1 061.34 for a 38 hour week. Find his:

Hourly Rate = 1 061.34 ÷ 38

= $27.93 / h

F'nightly Pay = 1 061.34 × 2

= $2 122.68

4 Lim earns $1 408 / f'night working a 25 hour week. Find:

Weekly Pay = 1 408 ÷ 2

= $704

Hourly Rate = 704 ÷ 25

=$28.16 / h

5 Tom earns $21.96 / h earning $746.64 / week. Find:

Hours Worked = 746.64 ÷ 21.96

= 34 hours

F'nightly Pay = 746.64 × 2

= $1 493.28

6 Dom earns $25.65 / h for a 15 hour week. Find his:

Weekly Pay = 25.65 × 15

= $384.75

F'nightly Pay = 384.75 × 2

= $769.50

7 Jess earns $1 634.36/f'night She is paid $31.43 / h. Find her:

Weekly Pay = 1 634.36 ÷ 2

= $817.18

Weekly Hours Worked = 817.18 ÷ 31.43

= 26 hours

8 Tess earns $20.72 / h for a 56 hour f'tnight. Find her:

F'nightly Pay = 20.72 × 56

= $1 160.32

Weekly Hours Worked = 56 ÷ 2

= 28 hours

9 Complete these tables and then find the mean (average) hours worked and mean weekly pay for the employees listed.

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Ralph	35	$23.22	$812.70	$1 625.40
Joanie	38	$27.65	$1 050.70	$2 101.40
Potsie	12	$19.85	$238.20	$476.40
Ritchie	27	$23.88	$644.76	$1 289.52
Total	112	*Total*	$2 746.36	

Mean Weekly Hours = Total hours worked 112 / Number of employees 4

= 28 h / week

Mean Weekly Pay = Total weekly wage 2 746.36 / 4

= $686.59 / week

10

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Legolas	21	$28.20	$592.20	$1 184.40
Aragon	22	$36.15	$795.30	$1 590.60
Gimli	11	$28.20	$310.20	$620.40
Total	54	*Total*	$1 697.70	

Mean Weekly Hours = 54 / 3

= 18 h / week

Mean Weekly Pay = 1 697.70 / 3

= $565.90 / week

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 3)

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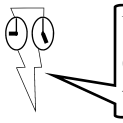

Use the hourly rate to calculate the pay for these employees.

1 Tyrone works 17 hours a week and earns $328.10, find:

a) His hourly rate.

Rate = 328.10 ÷ 17

Rate = $19.30 /h

Write '$' sign and show money to 2 d.p.

b) His fortnightly pay.

Pay = 328.10 × 2

Pay = $656.20

2 Thien earns $24.95 per hour. If he earned $798.40 for the week, calculate:

a) The hours worked this week

Hours = 798.40 ÷ 24.95

Hours = 32 hours

b) His fortnightly pay.

Pay = 798.40 × 2

Pay = $1 596.80

3 Bassam earns $27.20 per hour. If he works 38 h per week, calculate:

a) His weekly pay

Pay = 27.20 × 38

Pay = $1 033.60

b) His fortnightly pay.

Pay = 1 033.60 × 2

Pay = $2 067.20

3 Jemma earns $21.40/h and has recorded her hours below.

Day	M	T	W	T	F	Total
Hours	4	7	7	4	8	30

a) Complete the total column Finding the total hours worked.

b) Find Jemma's weekly pay.

Pay = 21.40 × 30

Pay = $642

4 Simon earns $21.45/h and has a weekly pay of $729.30

a) Calculate the total hours Simon has worked for the week.

Hours = 729.30 ÷ 21.45

Hours = 34 hours

b) Complete the table below showing Simon's daily hours.

Day	M	T	W	T	F	Total
Hours	8	4	7	7	8	34

5 Chloe earns $19.55/h and has a weekly pay of $625.60

a) Calculate the hours Chloe has worked for the week.

Hours = 625.60 ÷ 19.55

Hours = 32 hours

b) Complete the table below showing Chloe's daily hours.

Day	M	T	W	T	F	Total
Hours	7	8	6	3	8	32

c) Find her Thursday earnings.

Pay = 19.55 × 3

Pay = $58.65

6 Hollie earns $23.30/h. If she works 25½ h each week, find:

a) Her weekly pay

Pay = 23.3 × 25.5

Pay = $594.15

b) Her fortnightly pay.

Pay = 594.15× 2

Pay = $1 188.30

7 Faith earns $38.80 per hour. If she earned $2 754.80 for the fortnight, calculate her:

a) Fortnightly hours worked

Hours = 2 754.80 ÷ 38.80

Hours = 71 hours

b) Her weekly pay.

Pay = 2 754.80 ÷ 2

Pay = $1 377.40

8 Georgia earns $84.15 when she works a 5½ h day. How much does she earn for a 38 hour week?

Rate = 84.15 ÷ 5.5

Rate = $15.30/h

Pay = 15.30 × 38

Pay = $581.40/week

9 Robert earns $21.20/h and is paid $805.60 weekly. How many hours does he work per fortnight?

Hours = 805.60 ÷ 21.20

Hours = 38 hours/week

Hours = 38 × 2 = 76 h/fortnight

Earning Money - Calculating Hourly Rates (Homework Strip)

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Date Due:

Find the hourly rate

1 $146 for an 8 hour day.

Hourly rate = ÷

Hourly rate = $.

2 $568.48 for a 34 hour week.

Hourly rate = ÷

Hourly rate = $.

3 $926.40 for a 32 hour week

Hourly rate = ÷

Hourly rate = $.

4 $315.52 for a 17 hour week.

Hourly rate = ÷

Hourly rate = $.

5 $121.52 for a 7 hour day.

Hourly rate = ÷

Hourly rate = $.

Date Due:

Find the hourly rate

1 $146 for an 8 hour day.

Hourly rate = ÷

Hourly rate = $.

2 $568.48 for a 34 hour week.

Hourly rate = ÷

Hourly rate = $.

3 $926.40 for a 32 hour week

Hourly rate = ÷

Hourly rate = $.

4 $315.52 for a 17 hour week.

Hourly rate = ÷

Hourly rate = $.

5 $121.52 for a 7 hour day.

Hourly rate = ÷

Hourly rate = $.

Date Due:

Find the hourly rate

1 $146 for an 8 hour day.

Hourly rate = ÷

Hourly rate = $.

2 $568.48 for a 34 hour week.

Hourly rate = ÷

Hourly rate = $.

3 $926.40 for a 32 hour week

Hourly rate = ÷

Hourly rate = $.

4 $315.52 for a 17 hour week.

Hourly rate = ÷

Hourly rate = $.

5 $121.52 for a 7 hour day.

Hourly rate = ÷

Hourly rate = $.

Date Due:

Find the hourly rate

1 $146 for an 8 hour day.

Hourly rate = ÷

Hourly rate = $.

2 $568.48 for a 34 hour week.

Hourly rate = ÷

Hourly rate = $.

3 $926.40 for a 32 hour week

Hourly rate = ÷

Hourly rate = $.

4 $315.52 for a 17 hour week.

Hourly rate = ÷

Hourly rate = $.

5 $121.52 for a 7 hour day.

Hourly rate = ÷

Hourly rate = $.

Earning Money - Calculating Hourly Rates (Homework Strip)

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Date Due:

Find the hourly rate

1 $146 for an 8 hour day.

Hourly rate = 146 ÷ 8

Hourly rate = $ 18.25

2 $568.48 for a 34 hour week.

Hourly rate = 568.48 ÷ 34

Hourly rate = $ 16.72

3 $926.40 for a 32 hour week

Hourly rate = 926.40 ÷ 32

Hourly rate = $ 28.95

4 $315.52 for a 17 hour week.

Hourly rate = 315.52 ÷ 17

Hourly rate = $ 18.56

5 $121.52 for a 7 hour day.

Hourly rate = 121.52 ÷ 7

Hourly rate = $ 17.36

Pay Period Calculations - Annual Wages

An annual wage is a yearly wage. It is called ‘per annum’ which is shortened to ‘p.a.’. To find an annual wage follow these steps:

Column 1 - Convert weekly to yearly by multiplying by 52.
(as there are 52 weeks in a year)

Example $595 weekly

Wage p.a. = 595 × 52

= $30 940

Column 1 - Convert daily to yearly by multiplying the daily wage by the days worked per week, then 52.

Example $206 daily, 4 days per week.

Wage p.a. = 206 × 4 × 52

= $42 848

Column 2 - Convert hourly rate and hours worked to annual. Multiply hourly rate by hours worked per week, then multiply by 52.

Example $19.85 / h, 22 hour week.

Wage p.a. = 19.85 × 22 × 52

= $436.70

Column 3 - Convert fortnightly to yearly by multiplying the fortnightly wage by 26. Or divide by 2 to get weekly then multiply by 52.

Example $2 588 fortnightly

Wage p.a. = 2 588 ÷ 2 × 52

= $67 288

Below Columns 2 and 3 are two tables to complete. These use the same skills as above, only they are answered in stages. Finding daily pay first, then weekly wage, then annually. Total the annual wage column for the full pay details for all the listed employees.

The second sheet is the reverse of the first. Converting from an annual wage to weekly, Fortnightly and hourly rates. As this is the opposite action, instead of multiplying you divide.

Column 1 - Convert annual to weekly by dividing by 52.
(as there are 52 weeks in a year)

Example $24 551.80 p.a. (annually)

Weekly wage = 25 551.80 ÷ 52

= $472.15

Column 2 - Convert an annual wage to an hourly rate by dividing the annual wage by 52, then divide again by the number of weekly hours worked.

Example $38 038 p.a., for a 38 hour week

Hourly rate = 38 038 ÷ 52 ÷ 38

= $19.25

Column 3 asks you to change an annual wage to a fortnightly wage. You can find the weekly wage by dividing by 52 and doubling it. Or you can divide by 26, as there are 26 fortnights in a year. Then complete the tables, which use the same skills as above.

Pay Period Calculations - Annual Wages (Sheet 1)

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Convert a weekly wage to an annual wage (× by 52).

1 $358 weekly.

Wage p.a. = ☐ × 52

= $ ☐

2 $777 weekly.

Wage p.a. =

=

3 $443.58 weekly.

4 $1 952.37 weekly.

Convert a daily wage to an annual wage, (× days × 52).

5 $147 / day, 3 days per week.

Wage p.a. = ☐ × 3 × 52

= $ ☐

6 $165.20 / day, 2 days / week

Wage p.a. =

=

7 $207.45 / day, 4 days / week

8 $192.72 / day, 3 days / week

Convert an hourly rate to an annual wage (× hours × 52).

9 $24.20 / h, 38 hour week.

Wage p.a. = ☐ × 38 × 52

= $ ☐

10 $17.65 / h, 14 hour week.

Wage p.a. =

=

11 $32.75 / h, 25 hour week.

12 $20.42 / h, 38 hour week.

Convert a fortnightly wage to an annual wage, either (× 26) or you can (÷ 2 × 52).

13 $858 fortnightly.

Wage p.a. = ☐ ÷ 2 × 52

= $ ☐

14 $2 785 fortnightly.

15 $1 877 fortnightly.

16 $1 032 fortnightly.

Complete the tables and then calculate the total annual wages paid.

17

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*	*Annual Pay*
Jen	3	$25.65		2		
Len	5	$32.50		6		
Ben	8	$19.72		5		
Ken	6½	$38.12		4		
				Total Annual Wages		

18

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*	*Annual Pay*
Clay	6	$19.17		3		
May	7	$27.10		5		
Faye	5	$22.78		2		
				Total Annual Wages		

Pay Period Calculations - Annual Wages (Sheet 2)

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Convert an annual wage to a weekly wage (÷ by 52).

1 $46 644 p.a.

W'kly pay = ☐ ÷ 52

= $ ☐

2 $68 484 p.a.

W'kly pay =

=

3 $51 090 p.a.

Show the $ sign and cents to 2 decimal places

4 $73 655.40 p.a.

5 $26 355.68 p.a.

6 $7 587.84 p.a.

Questions 7 - 9 require you to round the cents.

7 $44 901.22 p.a.

8 $28 700 p.a.

9 $73 189.74 p.a.

Convert an annual wage to an hourly rate (÷ 52 ÷ hours).

10 $41 496 p.a., 38 hour week.

H'rly Rate= ☐ ÷ 52 ÷ 38

= $ ☐ / h

11 $38 532 p.a., 38 hour week.

H'rly Rate=

=

Show $ sign, cents to 2 d.p., and '/ h' at end.

12 $51 251.20 p.a., 28 hr week.

13 $21 996 p.a., 15 hour week.

Convert an annual wage to a fortnightly wage, (÷ 26) or you can (÷ 52 × 2).

14 $75 400 p.a.

Wage p.a. = ☐ ÷ 52 × 2

= $ ☐

15 $35 734.40 p.a.

16 $88 725 p.a.

17 $47 120.84 p.a.

Complete the tables and then calculate the average fortnightly pay.

18

Name	*Annual Wage*	*Weekly Wage*	*Fortnightly Wage*	*Fortnightly Hours*	*Hourly Rate*
Don	$41 693.60			38	
Bon	$157 170			78	
Con	$15 741.44			22	
Ron	$20 217.60			48	

Add the fortnight wages, press '=', then divide by 4. (the number of employees). ☐ ← *Average fortnightly wage* (2 d.p.)

19

Name	*Annual Wage*	*Fortnightly Wage*	*Weekly Wage*	*Weekly hours*	*Hourly Rate*
Edward	$39 203.84			32	
Bella	$57 205.20			38	
Jacob	$75 379.20			40	

☐ ← *Average fortnightly wage* (2 d.p.)

Pay Period Calculations - Annual Wages (Sheet 3)

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Convert to an annual wage by multiplying by 52. (× 52).

1 Brianna earns $16.65/h, for 15 hours/week part-time work. How much does she earn in:

a) A week

Pay = [ ] × [ ]

Pay = [ ]

Show the $ sign and cents to 2 decimal places

b) A year

Pay = [ ] × [ ]

Pay = [ ]

2 Dave works a 38 hour week earning $32.90/h. Find:

a) His weekly pay

Pay =

Pay =

b) His annual wage

Pay =

Pay =

3 Eliza earns $1 271.67/week. She is offered a wage increase of $138 per week. Calculate:

a) Her current annual salary

Pay =

Pay =

b) Her increased annual salary

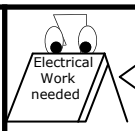
Pay =

Pay =

c) Find the annual increase

Increase =

Increase =



Look at the ads below and compare the wages.

Forklift Driver
Pay: $1 095/ week
Hours: 38 h/week
Apply within

Backhoe Operator
Pay: $28.27/h
Hours: 38 h/week
Call Us!!

4 Convert the Forklift job to an annual wage.

5 Convert the Backhoe job to an annual wage.

6 Calculate the annual pay difference between the jobs.

HR Specialist
Pay: $1 390/ week
Hours: 40 h/week
Ask inside

Recruitment Officer
Pay: $38.40/h
Hours: 36 h/week
Great Job!!

7 Convert the HR weekly wage to an annual wage.

8 Convert the Recruitment hourly rate to an annual wage.

9 Which job would you take?

Courier
Pay: $83 993 p.a.
Hours: 38 h/week
Must know Area

Delivery Driver
Pay: $44.10/h
Hours: 38 h/week
Licence Essential

10 Convert the courier job to a weekly wage.

11 Convert the Delivery Driver job to a weekly wage.

12 Calculate the weekly pay difference between the jobs.

Shop Assistant
Pay: $1 097.50
per fortnight
Hours: 50 h/f'tnight

Customer Service
Pay: $23.54/h
Hours: 25 h/week
Apply within

13 Convert the shop assistant job to a weekly wage.

14 Convert the Customer Service job to a weekly wage.

15 Calculate the weekly pay difference between the two jobs.

Pay Period Calculations - Annual Wages (Sheet 1)

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Convert a weekly wage to an annual wage (× by 52).

1 $358 weekly.

Wage p.a. = 358 × 52

= $ 18 616

2 $777 weekly.

Wage p.a. = 777 × 52

= $40 404

3 $443.58 weekly.

Wage p.a. = 443.58 × 52

= $23 066.16

4 $1 952.37 weekly.

Wage p.a. = 1 952.37 × 52

= $101 523.24

Convert a daily wage to an annual wage, (× days × 52).

5 $147 / day, 3 days per week.

Wage p.a. = 147 × 3 × 52

= $ 22 932

6 $165.20 / day, 2 days / week

Wage p.a. = 165.20 × 2 × 52

= $17 180.80

7 $207.45 / day, 4 days / week

Wage p.a. = 207.45 × 4 × 52

= $43 149.60

8 $192.72 / day, 3 days / week

Wage p.a. = 192.72 × 3 × 52

= $30 064.32

Convert an hourly rate to an annual wage (× hours × 52).

9 $24.20 / h, 38 hour week.

Wage p.a. = 24.20 × 38 × 52

= $ 47 819.20

10 $17.65 / h, 14 hour week.

Wage p.a. = 17.65 × 14 × 52

= $12 849.20

11 $32.75 / h, 25 hour week.

Wage p.a. = 32.75 × 25 × 52

= $42 575

12 $20.42 / h, 38 hour week.

Wage p.a. = 20.42 × 38 × 52

= $40 349.92

Convert a fortnightly wage to an annual wage, either (× 26) or you can (÷ 2 × 52).

13 $858 fortnightly.

Wage p.a. = 858 ÷ 2 × 52

= $ 22 307

14 $2 785 fortnightly.

Wage p.a. = 2 785 ÷ 2 × 52

= $72 410

15 $1 877 fortnightly.

Wage p.a. = 1 877 ÷ 2 × 52

= $48 802

16 $1 032 fortnightly.

Wage p.a. = 1 032 ÷ 2 × 52

= $26 832

Complete the tables and then calculate the total annual wages paid.

17

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*	*Annual Pay*
Jen	3	$25.65	$76.95	2	$153.90	$8 002.80
Len	5	$32.50	$162.50	6	$975	$50 700
Ben	8	$19.72	$157.76	5	$788.80	$41 017.60
Ken	6½	$38.12	$247.78	4	$991.12	$51 538.24
					Total Annual Wages	$151 258.64

18

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*	*Annual Pay*
Clay	6	$19.17	$115.02	3	$345.06	$17 943.12
May	7	$27.10	$189.70	5	$948.50	$49 322
Faye	5	$22.78	$113.90	2	$227.80	$11 845.60
					Total Annual Wages	$79 110.72

Pay Period Calculations - Annual Wages (Sheet 2)

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Convert an annual wage to a weekly wage (÷ by 52).

1 $46 644 p.a.

W'kly pay = 46 644 ÷ 52

= $ 897

2 $68 484 p.a.

W'kly pay = 68 484 ÷ 52

= $1 317

3 $51 090 p.a.

W'kly pay = 51 090 ÷ 52

= $982.50

Show the $ sign and cents to 2 decimal places

4 $73 655.40 p.a.

W'kly pay = 73 655.40 ÷ 52

= $1 416.45

5 $26 355.68 p.a.

W'kly pay = 26 355.68 ÷ 52

= $506.84

6 $7 587.84 p.a.

W'kly pay = 7 587.84 ÷ 52

= $145.92

Questions 7 - 9 require you to round the cents.

7 $44 901.22 p.a.

W'kly pay = 44 901.22 ÷ 52

= $863.49

8 $28 700 p.a.

W'kly pay = 28 700 ÷ 52

= $551.92

9 $73 189.74 p.a.

W'kly pay = 73 189.74 ÷ 52

= $1 407.50

Convert an annual wage to an hourly rate (÷ 52 ÷ hours).

10 $41 496 p.a., 38 hour week.

H'rly Rate= 41 496 ÷ 52 ÷ 38

= $ 21 / h

11 $38 532 p.a., 38 hour week.

H'rly Rate= 38 532 ÷ 52 ÷ 38

= $19.50 / h

Show $ sign, cents to 2 d.p., and '/ h' at end.

12 $51 251.20 p.a., 28 hr week.

H'rly Rate= 51 251.20 ÷ 52 ÷ 28

= $35.20 / h

13 $21 996 p.a., 15 hour week.

H'rly Rate= 21 996 ÷ 52 ÷ 15

= $28.20 / h

Convert an annual wage to a fortnightly wage, (÷ 26) or you can (÷ 52 × 2).

14 $75 400 p.a.

Wage p.a. = 75 400 ÷ 52 × 2

= $ 2 900

15 $35 734.40 p.a.

Wage p.a. = 35 734.40 ÷ 26

= $1 374.40

16 $88 725 p.a.

Wage p.a. = 88 725 ÷ 26

= $3 412.50

17 $47 120.84 p.a.

Wage p.a. = 47 120.84 ÷ 26

= $1 812.34

Complete the tables and then calculate the average fortnightly pay.

18

Name	*Annual Wage*	*Weekly Wage*	*Fortnightly Wage*	*Fortnightly Hours*	*Hourly Rate*
Don	$41 693.60	$801.80	$1 603.60	38	$42.20 / h
Bon	$157 170	$3 022.50	$6 045	78	$77.50 / h
Con	$15 741.44	$302.72	$605.44	22	$27.52 / h
Ron	$20 217.60	$388.80	$777.60	48	$16.20 / h

Add the fortnight wages, press '=', then divide by 4. (the number of employees).

$2 257.91 ← *Average fortnightly wage* (2 d.p.)

19

Name	*Annual Wage*	*Fortnightly Wage*	*Weekly Wage*	*Weekly hours*	*Hourly Rate*
Edward	$39 203.84	$1 507.84	$753.92	32	$23.56 / h
Bella	$57 205.20	$2 200.20	$1 100.10	38	$28.95 / h
Jacob	$75 379.20	$2 899.20	$1 449.60	40	$36.24 / h

$2 202.41 ← *Average fortnightly wage* (2 d.p.)

Pay Period Calculations - Annual Wages (Sheet 3)

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Convert to an annual wage by multiplying by 52. (× 52).

1 Brianna earns $16.65/h, for 15 hours/week part-time work. How much does she earn in:

a) A week

Pay = 16.65 × 15

Pay = $249.75

Show the $ sign and cents to 2 decimal places

b) A year

Pay = 249.75 × 52

Pay = $12 987

2 Dave works a 38 hour week earning $32.90/h. Find:

a) His weekly pay

Pay = 32.90 × 38

Pay = $1 250.20

b) His annual wage

Pay = 1 250.20 × 52

Pay = $65 010.40

3 Eliza earns $1 271.67/week. She is offered a wage increase of $138 per week. Calculate:

a) Her current annual salary

Pay = 1 271.67 × 52

Pay = $66 126.84

b) Her increased annual salary

Pay = (1 271.67 + 138) × 52

Pay = $73 302.84

c) Find the annual increase

Increase = 73 302.84 - 66 126.84

Increase = $7 176

Electrical Work needed

Look at the ads below and compare the wages.

Forklift Driver

Pay: $1 095/ week
Hours: 38 h/week
Apply within

Backhoe Operator

Pay: $28.27/h
Hours: 38 h/week
Call Us!!

4 Convert the Forklift job to an annual wage.

Pay = 1 095 × 52

Pay = $56 940

5 Convert the Backhoe job to an annual wage.

Pay = 28.27 × 38 × 52

Pay = $55 861.52

6 Calculate the annual pay difference between the jobs.

Difference = 56 940 - 55 861.52

Difference = $1 078.48

HR Specialist

Pay: $1 390/ week
Hours: 40 h/week
Ask inside

Recruitment Officer

Pay: $38.40/h
Hours: 36 h/week
Great Job!!

7 Convert the HR weekly wage to an annual wage.

Pay = 1 390 × 52

Pay = $72 280

8 Convert the Recruitment hourly rate to an annual wage.

Pay = 38.40 × 36 × 52

Pay = $71 884.80

9 Which job would you take?

HR pays more, but 208 h extra per year. So Recruitment job is a better job (if hours matter).

Courier

Pay: $83 993 p.a.
Hours: 38 h/week
Must know Area

Delivery Driver

Pay: $44.10/h
Hours: 38 h/week
Licence Essential

10 Convert the courier job to a weekly wage.

Pay = 83 993 ÷ 52

Pay = $1 615.25

11 Convert the Delivery Driver job to a weekly wage.

Pay = 44.10 × 38

Pay = $1 675.80

12 Calculate the weekly pay difference between the jobs.

Difference = 1 675.80 - 1 615.25

Difference = $60.55

Shop Assistant

Pay: $1 097.50
per fortnight
Hours: 50 h/f'tnight

Customer Service

Pay: $23.54/h
Hours: 25 h/week
Apply within

13 Convert the shop assistant job to a weekly wage.

Pay = 1 097.50 ÷ 2

Pay = $548.75

14 Convert the Customer Service job to a weekly wage.

Pay = 23.54 × 25

Pay = $588.50

15 Calculate the weekly pay difference between the two jobs.

Difference = 588.50 - 548.75

Difference = $39.75

Pay Period Calculations - Annual Wages (Homework Strip)

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Date Due: _____

Find the weekly wage

1 $40 474.20 p.a.

Weekly Pay = _____ ÷ _____

Weekly Pay = $_____._____

2 $49 666.24 p.a.

Weekly Pay = _____ ÷ _____

Weekly Pay = $_____._____

Find the annual wage

3 $1 265 per week

Annual Pay = _____ × _____

Annual Pay = _____

Show the $ sign

4 $2 055 per week

Annual Pay = _____ × _____

Annual Pay = _____

5 $26.20/h for a 23 h week

Annual Pay = _____ × _____ × _____

Annual Pay = _____

Date Due: _____

Find the weekly wage

1 $40 474.20 p.a.

Weekly Pay = _____ ÷ _____

Weekly Pay = $_____._____

2 $49 666.24 p.a.

Weekly Pay = _____ ÷ _____

Weekly Pay = $_____._____

Find the annual wage

3 $1 265 per week

Annual Pay = _____ × _____

Annual Pay = _____

Show the $ sign

4 $2 055 per week

Annual Pay = _____ × _____

Annual Pay = _____

5 $26.20/h for a 23 h week

Annual Pay = _____ × _____ × _____

Annual Pay = _____

Date Due: _____

Find the weekly wage

1 $40 474.20 p.a.

Weekly Pay = _____ ÷ _____

Weekly Pay = $_____._____

2 $49 666.24 p.a.

Weekly Pay = _____ ÷ _____

Weekly Pay = $_____._____

Find the annual wage

3 $1 265 per week

Annual Pay = _____ × _____

Annual Pay = _____

Show the $ sign

4 $2 055 per week

Annual Pay = _____ × _____

Annual Pay = _____

5 $26.20/h for a 23 h week

Annual Pay = _____ × _____ × _____

Annual Pay = _____

Date Due: _____

Find the weekly wage

1 $40 474.20 p.a.

Weekly Pay = _____ ÷ _____

Weekly Pay = $_____._____

2 $49 666.24 p.a.

Weekly Pay = _____ ÷ _____

Weekly Pay = $_____._____

Find the annual wage

3 $1 265 per week

Annual Pay = _____ × _____

Annual Pay = _____

Show the $ sign

4 $2 055 per week

Annual Pay = _____ × _____

Annual Pay = _____

5 $26.20/h for a 23 h week

Annual Pay = _____ × _____ × _____

Annual Pay = _____

Pay Period Calculations - Annual Wages (Homework Strip)

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Date Due:

Find the weekly wage

1 $40 474.20 p.a.

Weekly Pay = 40 474.20 ÷ 52

Weekly Pay = $ 7 7 8 . 3 5

2 $49 666.24 p.a.

Weekly Pay = 49 666.24 ÷ 52

Weekly Pay = $ 9 5 5 . 1 2

Find the annual wage

3 $1 265 per week

Annual Pay = 1 265 × 52

Annual Pay = $ 65780

4 $2 055 per week

Annual Pay = 2 055 × 52

Annual Pay = $ 106 860

5 $26.20/h for a 23 h week

Annual Pay = 26.20 × 23 × 52

Annual Pay = $ 31 335.20

Earning Money - Monthly Wages

To change a person's annual wage to a monthly wage, you divide by 12 because there are 12 months in a year. To convert monthly to yearly, you multiply by 12:

Example — Annual to Monthly

$63 327.60 p.a.

Monthly Pay = 63 327.60 ÷ 12

Monthly Pay = $5 277.30

Remember the '$' sign and always write cents to 2 decimal places

Example — Monthly to Annual

$2 966.10/month

Annual Pay = 2 966.10 × 12

Annual Pay = $35 593.20

Remember the '$' sign and always write cents to 2 decimal places

Annual wages are always larger than monthly wages. So if you get a monthly wage that is larger than the annual wage after a conversion, then you have used '×' instead of '÷'. Always look at your answer, then ask yourself 'Does that make sense?'

There may be 4 weeks in a month but no month has exactly 4 weeks in it, except February every leap year. So forget using 4 weeks = 1 month. YOU WILL NEVER USE THIS. To convert weekly to monthly (or the reverse) you always change to annual first.

Example — Weekly to Monthly

$567/week

Monthly Pay = 567 × 52 ÷ 12

× 52 to get annual, then ÷ 12 for monthly

Monthly Pay = $2 457

Example — Monthly to Weekly

$3 718.52/month

Weekly Pay = 3 718.52 ×12 ÷ 52

× 12 to get annual, then ÷ 52 for weekly

Weekly Pay = $858.12

The largest conversion is to convert from an hourly rate to a monthly wage and the reverse.

Example — Hourly to Monthly

$24.96/h for a 36 hour week

Monthly Pay = 24.96 × 36 × 52 ÷ 12

× hours (weekly), × 52 (annual) then ÷ 12 (monthly)

Monthly Pay = $3 893.76

Example — Monthly to hourly

$5 987.26/month for a 38 hour week

Hourly rate = 5 987.26 × 12 ÷ 52 ÷ 38

× 12 (annual), ÷ 52 (weekly) then ÷ hours (hourly)

Hourly rate = $36.36/h

Earning Money - Monthly Wages (Sheet 1)

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Convert a monthly wage to an annual wage, (× by 12).

1 $5 126 monthly.

Wage p.a. = ☐ × 12

= $ ☐

2 $3 556 monthly.

Wage p.a. =

=

3 $7 091 monthly.

4 $6 225.23 monthly.

Now the reverse. Convert an annual wage to a monthly wage, by division. (÷ by 12).

5 $97 286.40 p.a.

Monthly = ☐ ÷ 12

= $ ☐

6 $50 190 p.a.

Monthly =

=

Show the $ sign and cents to 2 decimal places

7 $71 952.60 p.a.

8 $88 528 p.a.

Round cents to 2 decimal places

Convert a monthly wage to a weekly wage. Get yearly first then weekly. (×12 ÷ 52).

This column has been designated a '÷ 4'free zone. Use of '÷ 4' in these questions is banned!

9 $2 937.60 monthly.

Weekly = ☐ × 12 ÷ 52

= $ ☐

10 $8 165.30 monthly.

Weekly = ☐ × 12 ÷ 52

= $ ☐

11 $4 006.08 monthly.

Weekly =

12 $2 817.75 monthly.

Weekly =

These require you to round.

13 $5 600 monthly.

Weekly =

14 $7 128 monthly.

Weekly =

15 $1 921 monthly.

Weekly =

16 $11 877 monthly.

Weekly =

17 $3 333 monthly.

Weekly =

Change the monthly wages (left) to annual wages, if there's no match on right change them to weekly wages (annual÷52) and match with weekly wages.

18	$3 200 monthly	A	$21 000 p.a.
19	$1 750 monthly	B	$58 002 p.a.
20	$4 412.20 monthly	C	$3 859.62 weekly
21	$4 833.50 monthly	D	$738.46 weekly
22	$16 725 monthly	E	$44 550 p.a.
23	$3 712.50 monthly	F	$1 018.20 weekly

Earning Money - Monthly Wages (Sheet 2)

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Convert a weekly wage to a monthly wage, (× by 52 ÷ 12).

1 $480 weekly.

M'thly pay = ☐ × 52 ÷ 12

= $ ☐

2 $510 weekly.

M'thly pay =

=

3 $624 weekly.

M'thly pay =

=

4 $1 123.20 weekly.

M'thly pay =

=

Write your answer to 2 decimal places.

5 $842.40 weekly.

M'thly pay =

=

These have to be rounded.

6 $712.22 weekly.

M'thly pay =

=

7 $295.70 weekly.

M'thly pay =

=

8 $356.84 weekly.

M'thly pay =

=

Given the weekly hours worked, convert an hourly rate to a monthly wage.
(hourly rate × hours × 52 ÷ 12)

9 $24/h for a 16 hour week.

Monthly Pay = ☐ × ☐ × 52 ÷ 12

= $ ☐

10 $18/h for a 35 hour week.

Monthly Pay = ☐ × ☐ × 52 ÷ 12

= $ ☐

11 $19/h for a 38 hour week.

Monthly Pay =

= $ ☐

Write your answer to 2 decimal places.

12 $17/h for an 8 hour week.

Monthly Pay =

= $ ☐

Reverse it! Given the monthly wage find the hourly rate.
(monthly wage × 12 ÷ 52 ÷ hrs)

13 $3 250/mth at 25 hours/wk.

Hourly Rate = ☐ × 12 ÷ 52 ÷ 25

= $ ☐ /h

14 $2 173.60/mth at 38 hrs/wk.

Hourly Rate =

= $ ☐ /h

Write your answer to 2 decimal places.

15 $2 372.50/mth at 30 hrs/wk.

Hourly Rate =

= $ ☐

16 $7 657/mth at 38 hrs/wk.

Hourly Rate =

= $ ☐

Convert all wages to annual wages then rewrite the wages in ascending order. Rewrite the actual wages, not the annual wages.

17 $2 665/week, $11 650/mth, $57/h (38 h week), $5 888/f'tnight.

Annual Wages: ☐ ☐ ☐ ☐

Ascending Wages:

Write the wages from the question, not the equivalent annual wages.

18 $7 434/mth, $24.20/h (34 h week), $3 256 f'tnight, $1 798/week.

Annual Wages: ☐ ☐ ☐ ☐

Ascending Wages:

Write the wages from the question, not the equivalent annual wages.

19 $32.85/h (28 h week), $1 838/f'tnight, $935/week, $4 050/mth.

Annual Wages: ☐ ☐ ☐ ☐

Ascending Wages:

Write the wages from the question, not the equivalent annual wages.

Earning Money - Monthly Wages (Sheet 3)

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Answer these questions involving monthly wages.

1 James earns $32.96/h. If he works a 40 hour week, find:

a) His weekly wage

Pay = ☐ × ☐

Pay = ☐

Show the $ sign and cents to 2 decimal places

b) His annual wage

Pay = ☐ × ☐

Pay = ☐

c) His monthly wage

Pay = ☐ ÷ ☐

Pay = ☐

Round your answer to the nearest cent

2 Juanita earns $2 709.98 per month. Find her weekly pay.

Pay =

Pay =

3 If Juanita (Q.2) has living expenses of $575 per week, find:

a) Her weekly savings

Savings =

Savings =

b) Her monthly savings

Savings =

Savings =

4 Josh earns $40 260 p.a., calculate his monthly wage.

Pay =

Pay =

5 Kylie works as a computer programmer earning $92/hour. If she works a 38 hour week:

a) Calculate her weekly wage.

b) Find her monthly wage.

Compare these jobs by converting them to monthly pay periods.

Optical Assistant

Pay: $1 315/ week
See you inside!

Medical Technician

Pay: $6 350/mth
Watch the step inside!

6 Convert the Optical position wage to a monthly wage.

7 Which job pays the most?

○ Optical Assistant

○ Medical Technician

Kitchen hand

Pay: $22.60/h
Hours: 38h week

Baker's Assistant

Pay: $4 130/mth
No Loafers!

8 Convert the Kitchen hand wage to a monthly wage.

Pay = ☐ × ☐ × ☐ ÷ ☐

9 Calculate the difference between the two monthly wages

10 Jamie earns $3 776/month as salesperson and $220/week as a waiter in a second job.

a) Convert the waiter's wage to a monthly wage.

b) Find his total monthly pay.

11 Three flat-mates are paid in different pay periods. Ted earns $1 170/week, Fred earns $28.38/h for a 38 h week, while Ned earns $95 628 p.a.

a) Convert Ted's weekly wage to a monthly wage.

b) Convert Fred's hourly wage to a monthly wage.

c) Convert Ned's annual salary to a monthly salary.

d) Rank the flat-mates in order of their income.

1. ☐

Write person's name with highest pay

2. ☐

3. ☐

Earning Money - Monthly Wages (Sheet 1)

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Convert a monthly wage to an annual wage, (× by 12).

1 $5 126 monthly.

Wage p.a. = 5 126 × 12

= $ 61 512

2 $3 556 monthly.

Wage p.a. = 3 556 × 12

= $42 672

3 $7 091 monthly.

Wage p.a. = 7 091 × 12

= $85 092

4 $6 225.23 monthly.

Wage p.a. = 6 225.23 × 12

= $74 702.76

Now the reverse. Convert an annual wage to a monthly wage, by division. (÷ by 12).

5 $97 286.40 p.a.

Monthly = 97 286.40 ÷ 12

= $ 8 107.20

6 $50 190 p.a.

Monthly = 50 190 ÷ 12

= $4 182.50

Show the $ sign and cents to 2 decimal places

7 $71 952.60 p.a.

Monthly = 71 952.60 ÷ 12

= $5 996.05

8 $88 528 p.a.

Monthly = 88 528 ÷ 12

= $7 377.33

Round cents to 2 decimal places

Convert a monthly wage to a weekly wage. Get yearly first then weekly. (×12 ÷ 52).

This column has been designated a '÷ 4' free zone. Use of '÷ 4' in these questions is banned!

9 $2 937.60 monthly.

Weekly = 2 937.60 × 12 ÷ 52

= $ 677.91

10 $8 165.30 monthly.

Weekly = 8 165.30 × 12 ÷ 52

= $ 1 884.30

11 $4 006.08 monthly.

Weekly = 4 006.08 × 12 ÷ 52

= $924.48

12 $2 817.75 monthly.

Weekly = 2 817.75 × 12 ÷ 52

= $650.25

These require you to round.

13 $5 600 monthly.

Weekly = 5 600 × 12 ÷ 52

= $1 292.31

14 $7 128 monthly.

Weekly = 7 128 × 12 ÷ 52

= $1 644.92

15 $1 921 monthly.

Weekly = 1 921 × 12 ÷ 52

= $443.31

16 $11 877 monthly.

Weekly = 11 877 × 12 ÷ 52

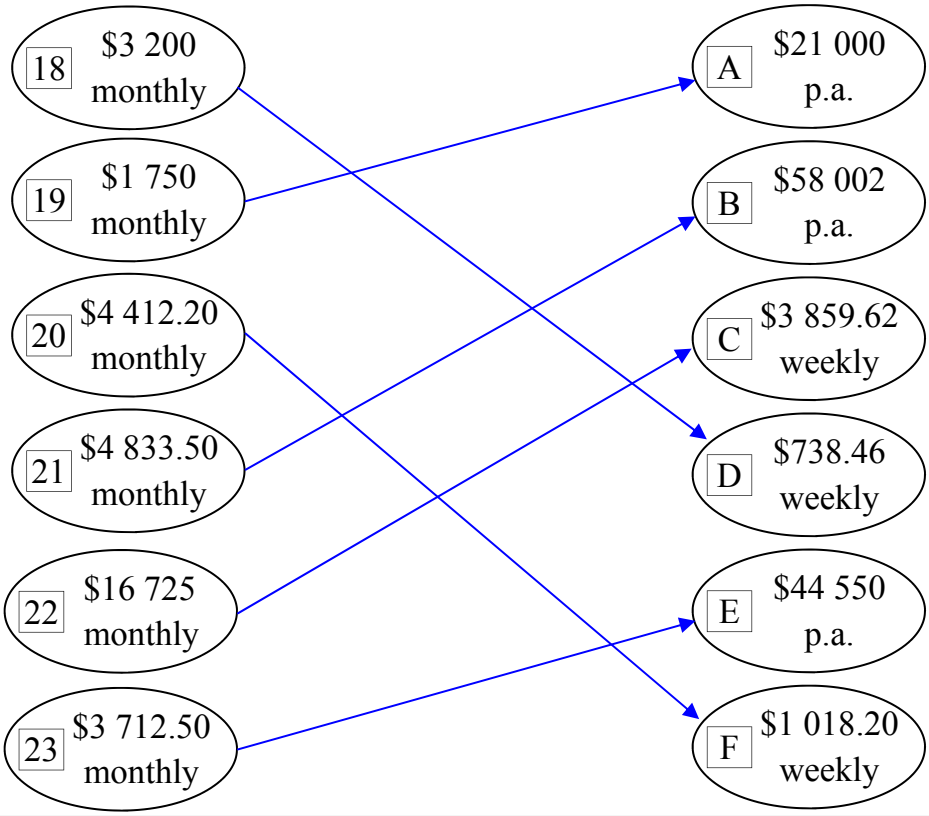
= $2 740.85

17 $3 333 monthly.

Weekly = 3 333 × 12 ÷ 52

= $769.15

Change the monthly wages (left) to annual wages, if there's no match on right change them to weekly wages (annual÷52) and match with weekly wages.



Earning Money - Monthly Wages (Sheet 2)

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Convert a weekly wage to a monthly wage, (× by 52 ÷ 12).

1 $480 weekly.

M'thly pay = 480 × 52 ÷ 12

= $ 2 080

2 $510 weekly.

M'thly pay = 510 × 52 ÷ 12

= $2 210

3 $624 weekly.

M'thly pay = 624 × 52 ÷ 12

= $2 704

4 $1 123.20 weekly.

M'thly pay = 1 123.20 × 52 ÷ 12

= $4 867.20

Write your answer to 2 decimal places.

5 $842.40 weekly.

M'thly pay = 842.40 × 52 ÷ 12

= $3 650.40

These have to be rounded.

6 $712.22 weekly.

M'thly pay = 712.22 × 52 ÷ 12

= $3 086.29

7 $295.70 weekly.

M'thly pay = 295.70 × 52 ÷ 12

= $1 281.37

8 $356.84 weekly.

M'thly pay = 356.84 × 52 ÷ 12

= $1 546.31

Given the weekly hours worked, convert an hourly rate to a monthly wage.
(hourly rate × hours × 52 ÷ 12)

9 $24/h for a 16 hour week.

Monthly Pay = 24 × 16 × 52 ÷ 12

= $ 1 664

10 $18/h for a 35 hour week.

Monthly Pay = 18 × 35 × 52 ÷ 12

= $ 2 730

11 $19/h for a 38 hour week.

Monthly Pay = 19 × 38 × 52 ÷ 12

= $ 3 128.67

Write your answer to 2 decimal places.

12 $17/h for an 8 hour week.

Monthly Pay = 17 × 8 × 52 ÷ 12

= $ 589.33

Reverse it! Given the monthly wage find the hourly rate.
(monthly wage × 12 ÷ 52 ÷ hrs)

13 $3 250/mth at 25 hours/wk.

Hourly Rate = 3 250 × 12 ÷52÷25

= $ 30/h

14 $2 173.60/mth at 38 hrs/wk.

Hourly Rate = 2 173.60 ×12 ÷ 52 ÷ 38

= $ 13.20/h

Write your answer to 2 decimal places.

15 $2 372.50/mth at 30 hrs/wk.

Hourly Rate = 2 372.50 ×12 ÷ 52 ÷ 30

= $ 18.25/h

16 $7 657/mth at 38 hrs/wk.

Hourly Rate = 7 657 ×12 ÷ 52 ÷ 38

= $ 46.50/h

Convert all wages to annual wages then rewrite the wages in ascending order. Rewrite the actual wages, not the annual wages.

17 $2 665/week, $11 650/mth, $57/h (38 h week), $5 888/f'tnight.

Annual Wages: $138 580 | $139 800 | $112 632 | $153 088

Ascending Wages: $57/h, $2 665/week, $11 650/mth, $5 888/fortnight.

Write the wages from the question, not the equivalent annual wages.

18 $7 434/mth, $24.20/h (34 h week), $3 256 f'tnight, $1 798/week.

Annual Wages: $89 208 | $42 785.60 | $84 656 | $93 496

Ascending Wages: $24.20/h, $3 256 fortnight, $7 434/mth, $1 798/week.

Write the wages from the question, not the equivalent annual wages.

19 $1 838/f'tnight, $32.85/h (28 h week), $935/week, $4 050/mth.

Annual Wages: $47 788 | $47 829.60 | $48 620 | $48 600

Ascending Wages: $4 050/mth, $935/week, $1 838/fortnight, $32.85/h.

Write the wages from the question, not the equivalent annual wages.

Earning Money - Monthly Wages (Sheet 3)

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Answer these questions involving monthly wages.

1 James earns $32.96/h. If he works a 40 hour week, find:

a) His weekly wage

Pay = 32.96 × 40

Pay = $1 318.40

Show the $ sign and cents to 2 decimal places

b) His annual wage

Pay = 1 318.40 × 52

Pay = $68 556.80

c) His monthly wage

Pay = 68 556.80 ÷ 12

Pay = $5 713.07

Round your answer to the nearest cent

2 Juanita earns $2 709.98 per month. Find her weekly pay.

Pay = 2 709.98 × 12 ÷ 52

Pay = $625.38

3 If Juanita (Q.2) has living expenses of $575 per week, find:

a) Her weekly savings

Savings = 625.38 - 575

Savings = $50.38

b) Her monthly savings

Savings = 50.38 × 52 ÷ 12

Savings = $218.31

4 Josh earns $40 260 p.a., calculate his monthly wage.

Pay = 40 260 ÷ 12

Pay = $3 355

5 Kylie works as a computer programmer earning $92/hour. If she works a 38 hour week:

a) Calculate her weekly wage.

Pay = 92 × 38

Pay = $3 496

b) Find her monthly wage.

Pay = 3 496 × 52 ÷ 12

Pay = $15 149.33

Compare these jobs by converting them to monthly pay periods.

Optical Assistant

Pay: $1 315/ week
See you inside!

Medical Technician

Pay: $6 350/mth
Watch the step inside!

6 Convert the Optical position wage to a monthly wage.

Pay = 1 315 × 52 ÷ 12

Pay = $5 698.33

7 Which job pays the most?

○ Optical Assistant

● Medical Technician

Kitchen hand

Pay: $22.60/h
Hours: 38h week

Baker's Assistant

Pay: $4 130/mth
No Loafers!

8 Convert the Kitchen hand wage to a monthly wage.

Pay = 22.60 × 38 × 52 ÷ 12

Pay = $3 721.47

9 Calculate the difference between the two monthly wages

Difference = 4 130 - 3 721.47

Difference = $408.53

10 Jamie earns $3 776/month as salesperson and $220/week as a waiter in a second job.

a) Convert the waiter's wage to a monthly wage.

Pay = 220 × 52 ÷ 12

Pay = $953.33

b) Find his total monthly pay.

Pay = 953.33 + 3 776

Pay = $4 729.33

11 Three flat-mates are paid in different pay periods. Ted earns $1 170/week, Fred earns $28.38/h for a 38 h week, while Ned earns $95 628 p.a.

a) Convert Ted's weekly wage to a monthly wage.

Pay = 1 170 × 52 ÷ 12

Pay = $5 070

b) Convert Fred's hourly wage to a monthly wage.

Pay = 28.38 × 38 × 52 ÷ 12

Pay = $4 673.24

c) Convert Ned's annual salary to a monthly salary.

Pay = 95 628 ÷ 12

Pay = $7 969

d) Rank the flat-mates in order of their income.

1. Ned ($7 969/mth)

Write person's name with highest pay

2. Ted ($5 070/mth)

3. Fred ($4 673.24/mth)

Pay Period Calculations - Monthly Wages (Homework Strip)

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Date Due:

Convert these to monthly wages

1 $48 660 p.a.

Pay = ÷

Pay = Show the $ sign

2 $94 207.20 p.a.

Pay = ÷

Pay = Cents to 2 d.p.

3 $30 464.40 p.a.

Pay = ÷

Pay =

Convert these to annual wages

4 $3 888.20 per month.

Pay = ×

Pay =

5 $2 711.34/month.

Pay = ×

Pay =

Date Due:

Convert these to monthly wages

1 $48 660 p.a.

Pay = ÷

Pay = Show the $ sign

2 $94 207.20 p.a.

Pay = ÷

Pay = Cents to 2 d.p.

3 $30 464.40 p.a.

Pay = ÷

Pay =

Convert these to annual wages

4 $3 888.20 per month.

Pay = ×

Pay =

5 $2 711.34/month.

Pay = ×

Pay =

Date Due:

Convert these to monthly wages

1 $48 660 p.a.

Pay = ÷

Pay = Show the $ sign

2 $94 207.20 p.a.

Pay = ÷

Pay = Cents to 2 d.p.

3 $30 464.40 p.a.

Pay = ÷

Pay =

Convert these to annual wages

4 $3 888.20 per month.

Pay = ×

Pay =

5 $2 711.34/month.

Pay = ×

Pay =

Date Due:

Convert these to monthly wages

1 $48 660 p.a.

Pay = ÷

Pay = Show the $ sign

2 $94 207.20 p.a.

Pay = ÷

Pay = Cents to 2 d.p.

3 $30 464.40 p.a.

Pay = ÷

Pay =

Convert these to annual wages

4 $3 888.20 per month.

Pay = ×

Pay =

5 $2 711.34/month.

Pay = ×

Pay =

Pay Period Calculations - Annual Wages (Homework Strip)

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Date Due:

Convert these to monthly wages

1 $48 660 p.a.

Pay = 48 660 ÷ 12

Pay = $4 055 (Show the $ sign)

2 $94 207.20 p.a.

Pay = 94 207.20 ÷ 12

Pay = $7 850.60 (Cents to 2 d.p.)

3 $30 464.40 p.a.

Pay = 30 464.40 ÷ 12

Pay = $2 538.70

Convert these to annual wages

4 $3 888.20 per month.

Pay = 3 888.20 × 12

Pay = $46 658.40

5 $2 711.34/month.

Pay = 2 711.34 × 12

Pay = $32 536.08

Earning Money - Penalty Rates - Time-and-a-half

When an employee exceeds a certain number of hours in a day (varies with contract) they are paid overtime. This often starts with a penalty rate of 1.5×, so they earn 50% more each hour.

Sheet 1 Column 1 asks you to change hourly rates to time-and-a-half rates, just multiply by 1.5

Example

Convert $10.70/h to a Time+½ rate

Time+½ rate = 10.70×1.5

Time+½ rate = $16.05

Column 2 and 3 ask you to calculate the overtime earned when given the hours worked and the standard hourly rate. Multiply the hourly rate by 1.5, then by the number of hours worked.

Example

Normal Rate: $17.80/h, Overtime Hours: 6

Pay = $17.80 \times 1.5 \times 6$

Pay = $160.20

At the base of the two columns is a table to complete. Find the normal pay, then the overtime pay, then add the two together for the total.

Sheet two asks you to find the total pay by working out the normal pay, the overtime, then adding the two together.

Example

20 h week at $31.76/h plus 7 hours overtime.

Pay = 20 × 31.76 + ← This is the normal hours worked.

= 7 × 1.5 × 31.76 ← This is the overtime hours worked.

Pay = $333.48 ← Add the two for the total

The third sheet has you completing tables and payslips to calculate the weekly pay earned.

Earning Money - Penalty Rates - Time-and-a-half (Sheet 1)

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Convert these hourly rates to an overtime rate of time-and-a-half, multiply by 1.5.

Example: Convert $11.60/h to a rate paid at time-and-a-half.

$11.60 = 11.60 × 1.5

Time+½ rate = $17.40/h

1 $14.90 = ☐ × 1.5

Time+½ rate = ☐ /h

Don't forget the '$' sign

2 $18.20 = ☐ × 1.5

Time+½ rate = ☐ /h

Always write cents with 2 decimal places

3 $35.40 = ☐ × ☐

Time+½ rate = ☐ /h

4 $22.10 = ☐ × ☐

Time+½ rate = ☐

5 $45.76 = ☐ × ☐

Time+½ rate = ☐

Now on you have to round to the nearest cent

6 $25.15 = ☐ × ☐

Time+½ rate = ☐

7 $17.05 = ☐ × ☐

Time+½ rate = ☐

8 $12.95 = ☐ × ☐

Time+½ rate = ☐

9 $48.19 = ☐ × ☐

Time+½ rate = ☐

10 $30.35 = ☐ × ☐

Time+½ rate = ☐

Find the overtime paid when given the hours worked.

Example:

Normal rate: $18.80/hour

Overtime Hours: 7

Pay = 18.80 × 1.5 × 7

Pay = $197.40

11 Normal Rate: $16.40/h
Overtime Hours: 4

Pay = ☐ × 1.5 × 4

Pay = ☐

12 Normal Rate: $15.88/h
Overtime Hours: 6

Pay = ☐ × ☐ × ☐

Pay = ☐

13 Normal Rate: $27.50/h
Overtime Hours: 8.5

Pay = ☐ × ☐ × ☐

Pay = ☐

You have to round these to the nearest cent

14 Normal Rate: $17.65/h
Overtime Hours: 2.5

Pay = ☐ × ☐ × ☐

Pay = ☐

15 Normal Rate: $11.99/h
Overtime Hours: 5

Pay = ☐ × ☐ × ☐

Pay = ☐

16 Normal Rate: $15.09/h
Overtime Hours: 1.5

Pay = ☐ × ☐ × ☐

Pay = ☐

17 Normal Rate: $37.37/h
Overtime Hours: 0.5

Pay = ☐ × ☐ × ☐

Pay = ☐

Complete these tables to calculate the daily pay for the employees.

18

	A	B	C = A×B	D	E = B×1.5	F = D×E	G = C+F
Name	*Normal Hours*	*Hourly Rate*	*Normal Pay*	*Time+½ Hours*	*Time+½ Rate*	*Overtime Pay*	*Total Pay*
Bill	8	$24	$192 8 × 24	2	$36 1.5 × 24	$72 2 × 36	$264 192 + 72
Jill	7	$28.20		0			
Lill	8	$19.20		3			
Dill	6	$12.80		2.5			
Phill	6	$15.75		0.5			
Gill	7.5	$19.80		1			
Will	5	$17.45		3.5			

Earning Money - Penalty Rates - Time-and-a-half (Sheet 2)

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Calculate the weekly pay, then the overtime (at time-and-a-half), then the total pay earned

1 Jane worked a 38 h week and also 6 h overtime. Her normal hourly rate is $23.10/h.

a) Calculate her standard pay

Pay = 38 × ☐

Pay = ☐ ← $ sign, cents to 2 d.p.

b) Calculate her overtime

Pay = ☐ × 1.5 × ☐

Pay = ☐

c) Calculate her total pay

Pay = ☐ + ☐

Pay = ☐

2 Geoff worked a 35 h week and also 7 h overtime. His normal hourly rate is $32.60/h.

a) Calculate his standard pay

Pay = ☐ × ☐

Pay = ☐

b) Calculate his overtime

Pay = ☐ × ☐ × ☐

Pay = ☐

c) Calculate his total pay

Pay = ☐ + ☐

Pay = ☐

d) Convert c) to an annual wage

Pay = ☐ × ☐

Pay = ☐

Calculate the weekly pay which includes overtime paid at time-and-a-half.

Example: 38 h week at $19.20 per hour plus 4 hours overtime.

Pay = 38 × 19.20 + [Normal Pay]

4 × 1.5 ×19.20 [Overtime]

Pay = $844.80 [Total]

3 18 h week at $13.48/h plus 3 hours overtime.

Pay = ☐ × ☐ +

= ☐ × 1.5 × ☐

Pay = ☐ ← $ sign, cents to 2 d.p.

4 25 h week at $38.40/h plus 5 hours overtime.

Pay = ☐ × ☐ +

= ☐ × ☐ × ☐

Pay = ☐

5 34 h week at $26.65/h plus 2 hours overtime.

Pay = ☐ × ☐ +

= ☐ × ☐ × ☐

Pay = ☐

6 17 h week at $15.55/h plus 2.5 hours overtime.

Pay = ☐ × ☐ +

= ☐ × ☐ × ☐

Pay = ☐ ← Round cents to 2 d.p.

7 5½ h week at $11.95/h plus 30 minutes overtime.

Pay = ☐ × ☐ +

= ☐ × ☐ × ☐

Pay = ☐

8 Convert 7 to an annual wage

Pay = ☐ × ☐

Pay = ☐

Complete Rebecca's pay for the week. She earns $18.50/h for her normal rostered hours, overtime for time above her rostered hours.

9

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	9	7 $129.50	2 $55.50	$185
Tues	9	5 ☐	☐ ☐	☐
Wed	8	6 ☐	☐ ☐	☐
Thur	10	7 ☐	☐ ☐	☐
Fri	9½	8 ☐	☐ ☐	☐
			Total Pay (Week)	☐

Earning Money - Penalty Rates - Time-and-a-half (Sheet 3)

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Complete these tables to find the weekly pay. Hours after rostered (normal) hours are paid overtime at a rate of time-and-a-half.

1 Hourly Rate = $23.40

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	6	5		
Tues	7	5		
Wed	7	7		
Thur	10	8		
Fri	8½	7		
			Total Pay (Week)	

2 Hourly Rate = $21.90

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	5½	5		
Tues	6	5		
Wed	6	5		
Thur	7	5		
Fri	7½	5		
Sat	6	4		
			Total Pay (Week)	



Complete the pay slips to calculate the gross pay (pay before tax) for these employees.

3

Name: B. Honey	Date From	Date From	Units	Rate	Amount	
Normal Hours	19-Jun-2019	25-Jun-2019	38	$24.30		Units × Rate
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	9			Units × Rate
Total						Add for total

Units = Hours

1.5 × 24.30

4

Name: T. Cake	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	27	$19.48	
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	4		
Total					

5

Name: F. Sharp	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	36	$25.80	
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	7½		
Total					

6

Name: I. Pod	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	38	$38.22	
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	10.25		
Total					

Earning Money - Penalty Rates - Time-and-a-half (Sheet 1)

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Convert these hourly rates to an overtime rate of time-and-a-half, multiply by 1.5.

Example: Convert $11.60/h to a rate paid at time-and-a-half.

$11.60 = 11.60 × 1.5

Time+½ rate = $17.40/h

1 $14.90 = 14.90 × 1.5

Time+½ rate = $22.35 /h

Don't forget the '$' sign

2 $18.20 = 18.20 × 1.5

Time+½ rate = $27.30 /h

Always write cents with 2 decimal places

3 $35.40 = 35.40 × 1.5

Time+½ rate = $53.10 /h

4 $22.10 = 22.10 × 1.5

Time+½ rate = $33.15/h

5 $45.76 = 45.76 × 1.5

Time+½ rate = $68.64/h

Now on you have to round to the nearest cent

6 $25.15 = 25.15 × 1.5

Time+½ rate = $37.73/h

7 $17.05 = 17.05 × 1.5

Time+½ rate = $25.58/h

8 $12.95 = 12.95 × 1.5

Time+½ rate = $19.43/h

9 $48.19 = 48.19 × 1.5

Time+½ rate = $72.29/h

10 $30.35 = 30.35 × 1.5

Time+½ rate = $45.53

Find the overtime paid when given the hours worked.

Example:

Normal rate: $18.80/hour

Overtime Hours: 7

Pay = 18.80 × 1.5 × 7

Pay = $197.40

11 Normal Rate: $16.40/h

Overtime Hours: 4

Pay = 16.40 × 1.5 × 4

Pay = $98.40

12 Normal Rate: $15.88/h

Overtime Hours: 6

Pay = 15.88 × 1.5 × 6

Pay = $142.92

13 Normal Rate: $27.50/h

Overtime Hours: 8.5

Pay = 27.50 × 1.5 × 8.5

Pay = $350.63

You have to round these to the nearest cent

14 Normal Rate: $17.65/h

Overtime Hours: 2.5

Pay = 17.65 × 1.5 × 2.5

Pay = $66.19

15 Normal Rate: $11.99/h

Overtime Hours: 5

Pay = 11.99 × 1.5 × 5

Pay = $89.93

16 Normal Rate: $15.09/h

Overtime Hours: 1.5

Pay = 15.09 × 1.5 × 1.5

Pay = $33.95

17 Normal Rate: $37.37/h

Overtime Hours: 0.5

Pay = 37.37 × 1.5 × 0.5

Pay = $28.03

Complete these tables to calculate the daily pay for the employees.

18

	A	B	C = A×B	D	E = B×1.5	F = D×E	G = C+F
Name	*Normal Hours*	*Hourly Rate*	*Normal Pay*	*Time+½ Hours*	*Time+½ Rate*	*Overtime Pay*	*Total Pay*
Bill	8	$24	$192 (8 × 24)	2	$36 (1.5 × 24)	$72 (2 × 36)	$264 (192 + 72)
Jill	7	$28.20	$197.40	0	$42.30	0	$197.40
Lill	8	$19.20	$153.60	3	$28.80	$86.40	$240
Dill	6	$12.80	$76.80	2.5	$19.20	$48	$124.80
Phill	6	$15.75	$94.50	0.5	$23.63	$11.82	$106.32
Gill	7.5	$19.80	$148.50	1	$29.70	$29.70	$178.20
Will	5	$17.45	$87.25	3.5	$26.18	$91.63	$178.88

Earning Money - Penalty Rates - Time-and-a-half (Sheet 2)

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Calculate the weekly pay, then the overtime (at time-and-a-half), then the total pay earned

1 Jane worked a 38 h week and also 6 h overtime. Her normal hourly rate is $23.10/h.

a) Calculate her standard pay

Pay = 38 × 23.10

Pay = $877.80 ← $ sign, cents to 2 d.p.

b) Calculate her overtime

Pay = 6 × 1.5 × 23.10

Pay = $207.90

c) Calculate her total pay

Pay = 877.80 + 207.90

Pay = $1 085.70

2 Geoff worked a 35 h week and also 7 h overtime. His normal hourly rate is $32.60/h.

a) Calculate his standard pay

Pay = 35 × 32.60

Pay = $1 141

b) Calculate his overtime

Pay = 7 × 1.5 × 32.60

Pay = $342.30

c) Calculate his total pay

Pay = 1 141 + 342.30

Pay = $1 483.30

d) Convert c) to an annual wage

Pay = $1 483.30 × 52

Pay = $77 131.60

Calculate the weekly pay which includes overtime paid at time-and-a-half.

Example: 38 h week at $19.20 per hour plus 4 hours overtime.

Pay = 38 × 19.20 + [Normal Pay]

4 × 1.5 ×19.20 [Overtime]

Pay = $844.80 [Total]

3 18 h week at $13.48/h plus 3 hours overtime.

Pay = 18 × 13.48 +

= 3 × 1.5 × 13.48

Pay = $303.30 ← $ sign, cents to 2 d.p.

4 25 h week at $38.40/h plus 5 hours overtime.

Pay = 25 × 38.40 +

= 5 × 1.5 × 38.40

Pay = $1 248

5 34 h week at $26.65/h plus 2 hours overtime.

Pay = 34 × 26.65 +

= 2 × 1.5 × 26.65

Pay = $986.05

6 17 h week at $15.55/h plus 2.5 hours overtime.

Pay = 17 × 15.55 +

= 2.5 × 1.5 × 15.55

Pay = $322.66 ← Round cents to 2 d.p.

7 5½ h week at $11.95/h plus 30 minutes overtime.

Pay = 5.5 × 11.95 +

= 0.5 × 1.5 × 11.95

Pay = $74.69

8 Convert 7 to an annual wage

Pay = 74.69 × 52

Pay = $3 883.88

Complete Rebecca's pay for the week. She earns $18.50/h for her normal rostered hours, overtime for time above her rostered hours.

9

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	9	7 $129.50	2 $55.50	$185
Tues	9	5 $92.50	4 $111	$203.50
Wed	8	6 $111	2 $55.50	$166.50
Thur	10	7 $129.50	3 $83.25	$212.75
Fri	9½	8 $148	1.5 $41.63	$189.63
			Total Pay (Week)	$957.38

Earning Money - Penalty Rates - Time-and-a-half (Sheet 3)

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Complete these tables to find the weekly pay. Hours after rostered (normal) hours are paid overtime at a rate of time-and-a-half.

1 Hourly Rate = $23.40

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	6	5 $117	1 $35.10	$152.10
Tues	7	5 $117	2 $70.20	$187.20
Wed	7	7 $163.80	0 -	$163.80
Thur	10	8 $187.20	2 $70.20	$257.40
Fri	8½	7 $163.80	1.5 $52.65	$216.45
			Total Pay (Week)	$976.95

2 Hourly Rate = $21.90

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	5½	5 $109.50	0.5 $16.43	$125.93
Tues	6	5 $109.50	1 $32.85	$142.35
Wed	6	5 $109.50	1 $32.85	$142.35
Thur	7	5 $109.50	2 $65.70	$175.20
Fri	7½	5 $109.50	2.5 $82.13	$191.63
Sat	6	4 $87.60	2 $65.70	$153.30
			Total Pay (Week)	$930.76



Complete the pay slips to calculate the gross pay (pay before tax) for these employees.

Units = Hours

3

Name: B. Honey	Date From	Date From	Units	Rate	Amount	
Normal Hours	19-Jun-2019	25-Jun-2019	38	$24.30	$923.40	Units × Rate
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	9	$36.45	$328.05	Units × Rate
Total					$1 251.45	Add for total

1.5 × 24.30

4

Name: T. Cake	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	27	$19.48	$525.96
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	4	$29.22	$116.88
Total					$642.84

5

Name: F. Sharp	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	36	$25.80	$928.80
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	7½	$38.70	$290.25
Total					$1 219.05

6

Name: I. Pod	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	38	$38.22	$1 452.36
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	10.25	$57.33	$587.63
Total					$2 039.99

Earning Money - Penalty Rates - Time-and-a-half (Homework Strip)

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Date Due: _____

Change these rates to time-and-a-half

1 $16.30 = _____ × 1.5

Time+½ rate = _____ /h

2 $18.96 = _____ × 1.5

Time+½ rate = _____ /h

Calculate the overtime earned

3 Normal Rate: $12.80/h
Overtime Hours: 6

Pay = _____ × 1.5 × 6

Pay = _____

4 Normal Rate: $15.34/h
Overtime Hours: 4

Pay = _____ × _____ × _____

Pay = _____

5 Normal Rate: $28.54/h
Overtime Hours: 3

Pay = _____ × _____ × _____

Pay = _____

Date Due: _____

Change these rates to time-and-a-half

1 $16.30 = _____ × 1.5

Time+½ rate = _____ /h

2 $18.96 = _____ × 1.5

Time+½ rate = _____ /h

Calculate the overtime earned

3 Normal Rate: $12.80/h
Overtime Hours: 6

Pay = _____ × 1.5 × 6

Pay = _____

4 Normal Rate: $15.34/h
Overtime Hours: 4

Pay = _____ × _____ × _____

Pay = _____

5 Normal Rate: $28.54/h
Overtime Hours: 3

Pay = _____ × _____ × _____

Pay = _____

Date Due: _____

Change these rates to time-and-a-half

1 $16.30 = _____ × 1.5

Time+½ rate = _____ /h

2 $18.96 = _____ × 1.5

Time+½ rate = _____ /h

Calculate the overtime earned

3 Normal Rate: $12.80/h
Overtime Hours: 6

Pay = _____ × 1.5 × 6

Pay = _____

4 Normal Rate: $15.34/h
Overtime Hours: 4

Pay = _____ × _____ × _____

Pay = _____

5 Normal Rate: $28.54/h
Overtime Hours: 3

Pay = _____ × _____ × _____

Pay = _____

Date Due: _____

Change these rates to time-and-a-half

1 $16.30 = _____ × 1.5

Time+½ rate = _____ /h

2 $18.96 = _____ × 1.5

Time+½ rate = _____ /h

Calculate the overtime earned

3 Normal Rate: $12.80/h
Overtime Hours: 6

Pay = _____ × 1.5 × 6

Pay = _____

4 Normal Rate: $15.34/h
Overtime Hours: 4

Pay = _____ × _____ × _____

Pay = _____

5 Normal Rate: $28.54/h
Overtime Hours: 3

Pay = _____ × _____ × _____

Pay = _____

Earning Money - Penalty Rates - Time-and-a-half (Homework Strip)

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Date Due:

Change these rates to time-and-a-half

1 $16.30 = 16.30 × 1.5

Time+½ rate = $24.45 /h

2 $18.96 = 18.96 × 1.5

Time+½ rate = 28.44 /h

Calculate the overtime earned

3 Normal Rate: $12.80/h
Overtime Hours: 6

Pay = 12.80 × 1.5 × 6

Pay = $115.20

4 Normal Rate: $15.34/h
Overtime Hours: 4

Pay = 15.34 × 1.5 × 4

Pay = $92.04

5 Normal Rate: $28.54/h
Overtime Hours: 3

Pay = 28.54 × 1.5 × 3

Pay = $128.43

Earning Money - Penalty Rates - Double Time

When an employee exceeds a certain number of hours in a day (varies with contract) they are paid overtime. This often starts with a penalty rate of 1.5×, then after 3 hours (again, varies with contract) they earn double time. Double time is 2× the normal hourly rate.

But when an employee works on days such as:

- Public Holidays;
- Sundays (some industries);
- Or they start another shift within a certain number of hours of finishing their previous shift;

They are often paid double time for the entire shift. This is simply 2× the normal rate.

Double Time Rate = 2 × Normal Hourly Rate

Column 1 asks you to change hourly rates to double time rates, just multiply by 2.

Column 1, then columns 2 and 3 ask you to calculate the double time pay. Multiply the hourly rate by 2 then by the hours.

Example

Normal rate: \$23.25/hour, Double-time hours: 5

Pay = $23.25 \times 2 \times 5$

Pay = \$232.50

At the base of the two columns asks you to calculate the double time earned and match it with the answer on the right.

The second sheet asks you to calculate the pay earned when an employee works normal hours and double time hours. Below is an example.

38 h week at \$28.05/h plus 4 hours on Sunday at double-time rate.

Pay = 38 (Normal Hours) × 28.05 (Normal Rate) + 4 (Double Hours) × 2 × 28.05 (Normal Rate)

Pay = \$1 209.30

'\$' sign and cents to 2 decimal places

There are 4 pay slips at the base of the second sheet, this is the same as above except you calculate the double time rate (double normal rate), normal pay, double time pay, then add them.

Earning Money - Penalty Rates - Double Time (Sheet 1)

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Convert these hourly rates to an overtime rate of double time, so multiply by 2.

Example: Convert $13.85/h to a rate paid at double time

$13.85 = 13.85 × 2

D.T rate = $27.70

1 $17.60 = ____ × 2

D.T rate = ____ /h

'$' sign and cents to 2 d.p.

2 $16.63 = ____ × 2

D.T rate = ____ /h

3 $35.40 = ____ × ____

D.T rate = ____ /h

4 $23.11 = ____ × ____

D.T rate = ____

Include the /h to show it is a rate.

5 $25.73 = ____ × ____

D.T rate = ____

An earthmoving business pays its employees double-time on Sundays. Find the pay earned.

Example:

Normal rate: $19.10/hour

Double-time hours: 7

Pay = 19.10 × 2 × 7

Pay = $267.40

6 Normal Rate: $16.40/h
Double-time hours: 6

Pay = ____ × 2 × 6

Pay = ____

7 Normal Rate: $19.25/h
Double-time hours: 7

Pay = ____ × ____ × ____

Pay = ____

8 Normal Rate: $25.60/h
Double-time hours: 5

Pay = ____ × ____ × ____

Pay = ____

9 Normal Rate: $19.45/h
Double-time hours: 8

Pay =

Pay =

10 Normal Rate: $13.07/h
Double-time hours: 4.5

Pay =

Pay =

11 Normal Rate: $35.80/h
Double-time hours: 6.25

12 Normal Rate: $38.63/h
Double-time hours: 4.75

Cents to 2 d.p.

13 Normal Rate: $22.53/h
Double-time hours: 5.25

14 Normal Rate: $31.95/h
Double-time hours: 7.75

Calculate the double time earnings given the normal rate and the hours worked, then match it up with a line to the correct answer.

15 Normal Rate: $13.22/h D.T. Hours: 5 •

16 Normal Rate: $10.15/h D.T. Hours: 2 •

17 Normal Rate: $17.11/h D.T. Hours: 6.5 •

18 Normal Rate: $36.90/h D.T. Hours: 4 •

19 Normal Rate: $19.58/h D.T. Hours: 8 •

20 Normal Rate: $19.30/h D.T. Hours: 4.25 •

21 Normal Rate: $26.35/h D.T. Hours: 7.25 •

- A $222.43
- B $295.20
- C $40.60
- D $313.28
- E $164.05
- F $132.20
- G $382.08

Earning Money - Penalty Rates - Double Time (Sheet 2)

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Find the pay when these employees work Sunday and earn double time on top of their normal weekly wage.

1 38 h week at $19.25/h plus 5 hours on Sunday at double time rate.

Pay = ☐ × ☐ + ☐ × 2 × ☐

(☐: Normal hours × Hourly Rate; ☐: Double hours × 2 × Hourly Rate)

Pay = ☐

2 30 h week at $23.90/h plus 7 hours on Sunday at double-time rate.

Pay = ☐ × ☐ + ☐ × 2 × ☐

Pay = ☐

3 34 h week at $34.70/h plus 4 hours on Sunday at double-time rate.

Pay = ☐ × ☐ + ☐ × ☐ × ☐

Pay = ☐

4 16 h week at $13.20/h plus 5 hours on Sunday at double-time rate.

Pay =

Pay =

5 25 h week at $20.15/h plus 4.5 hours on a public holiday at double-time rate.

Pay =

Pay =

6 16 h week at $42.16/h plus 7½ hours on a Sunday at double-time rate.

Pay =

Pay =

7 30 h week at $18.10/h plus 6.25 hours on a public holiday at double-time rate.

Pay =

Pay =

Complete the pay slips to calculate the gross pay (pay before tax) for these employees.

8

Name: C. Clearly	Date From	Date From	Units (Units = Hours)	Rate	Amount	
Normal Hours	19-Jun-2019	25-Jun-2019	34	$31.30		Units × Rate
Double Time	19-Jun-2019	25-Jun-2019	6			Units × Rate
Total						Add for total

2 × 31.30

9

Name: D. Light	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	30	$19.48	
Double Time	19-Jun-2019	25-Jun-2019	5		
Total					

10

Name: J. Walker	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	38	$25.80	
Double Time	19-Jun-2019	25-Jun-2019	4½		
Total					

11

Name: M. Power	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	15	$38.22	
Double Time	19-Jun-2019	25-Jun-2019	3		
Total					

Earning Money - Penalty Rates - Double Time (Sheet 1)

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Convert these hourly rates to an overtime rate of double time, so multiply by 2.

Example: Convert $13.85/h to a rate paid at double time

$13.85 = 13.85 × 2

D.T rate = $27.70

1 $17.60 = 17.60 × 2

D.T rate = $35.20 /h

'$' sign and cents to 2 d.p.

2 $16.63 = 16.63 × 2

D.T rate = $33.26 /h

3 $35.40 = 35.40 × 2

D.T rate = $70.80 /h

4 $23.11 = 23.11 × 2

D.T rate = $46.22/h

Include the /h to show it is a rate.

5 $25.73 = 25.73 × 2

D.T rate = $51.46/h

An earthmoving business pays its employees double-time on Sundays. Find the pay earned.

Example:

Normal rate: $19.10/hour

Double-time hours: 7

Pay = 19.10 × 2 × 7

Pay = $267.40

6 Normal Rate: $16.40/h

Double-time hours: 6

Pay = 16.40 × 2 × 6

Pay = $196.80

7 Normal Rate: $19.25/h

Double-time hours: 7

Pay = 19.25 × 2 × 7

Pay = $269.50

8 Normal Rate: $25.60/h

Double-time hours: 5

Pay = 25.60 × 2 × 5

Pay = $256

9 Normal Rate: $19.45/h

Double-time hours: 8

Pay = 19.45 × 2 × 8

Pay = $311.20

10 Normal Rate: $13.07/h

Double-time hours: 4.5

Pay = 13.07 × 2 × 4.5

Pay = $117.63

11 Normal Rate: $35.80/h

Double-time hours: 6.25

Pay = 35.80 × 2 × 6.25

Pay = $447.50

12 Normal Rate: $38.63/h

Double-time hours: 4.75

Pay = 38.63 × 2 × 4.75

Pay = $366.99

Cents to 2 d.p.

13 Normal Rate: $22.53/h

Double-time hours: 5.25

Pay = 22.53 × 2 × 5.25

Pay = $236.57

14 Normal Rate: $31.95/h

Double-time hours: 7.75

Pay = 31.95 × 2 × 7.75

Pay = $495.23

Calculate the double time earnings given the normal rate and the hours worked, then match it up with a line to the correct answer.

15 Normal Rate: $13.22/h D.T. Hours: 5

16 Normal Rate: $10.15/h D.T. Hours: 2

17 Normal Rate: $17.11/h D.T. Hours: 6.5

18 Normal Rate: $36.90/h D.T. Hours: 4

19 Normal Rate: $19.58/h D.T. Hours: 8

20 Normal Rate: $19.30/h D.T. Hours: 4.25

21 Normal Rate: $26.35/h D.T. Hours: 7.25

A $222.43

B $295.20

C $40.60

D $313.28

E $164.05

F $132.20

G $382.08

Earning Money - Penalty Rates - Double Time (Sheet 2)

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Find the pay when these employees work Sunday and earn double time on top of their normal weekly wage.

1 38 h week at $19.25/h plus 5 hours on Sunday at double time rate.

Pay = 38 × 19.25 + 5 × 2 × 19.25

(38: Normal hours; 19.25: Hourly Rate; 5: Double hours; 2: × 2; 19.25: Hourly Rate)

Pay = $924

2 30 h week at $23.90/h plus 7 hours on Sunday at double-time rate.

Pay = 30 × 23.90 + 7 × 2 × 23.90

Pay = $1 051.60

3 34 h week at $34.70/h plus 4 hours on Sunday at double-time rate.

Pay = 34 × 34.70 + 4 × 2 × 34.70

Pay = $1 457.40

4 16 h week at $13.20/h plus 5 hours on Sunday at double-time rate.

Pay = 16 × 13.20 + 5 × 2 × 13.20

Pay = $343.20

5 25 h week at $20.15/h plus 4.5 hours on a public holiday at double-time rate.

Pay = 25 × 20.15 + 4.5 × 2 × 20.15

Pay = $685.10

6 16 h week at $42.16/h plus 7½ hours on a Sunday at double-time rate.

Pay = 16 × 42.16 + 7.5 × 2 × 42.16

Pay = $1 306.96

7 30 h week at $18.10/h plus 6.25 hours on a public holiday at double-time rate.

Pay = 30 × 18.10 + 6.25 × 2 × 18.10

Pay = $769.25



Complete the pay slips to calculate the gross pay (pay before tax) for these employees.

8

Name: C. Clearly	Date From	Date From	Units (Units = Hours)	Rate	Amount	
Normal Hours	19-Jun-2019	25-Jun-2019	34	$31.30	$1 064.20	Units × Rate
Double Time	19-Jun-2019	25-Jun-2019	6	$62.60	$375.60	Units × Rate
Total					$1 439.80	Add for total

($62.60: 2 × 31.30)

9

Name: D. Light	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	30	$19.48	$584.40
Double Time	19-Jun-2019	25-Jun-2019	5	$38.96	$194.80
Total					$779.20

10

Name: J. Walker	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	38	$25.80	$980.40
Double Time	19-Jun-2019	25-Jun-2019	4½	$51.60	$232.20
Total					$1 212.60

11

Name: M. Power	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	15	$38.22	$573.30
Double Time	19-Jun-2019	25-Jun-2019	3	$76.44	$229.32
Total					$802.62

Earning Money - Penalty Rates - Double Time (Homework Strip)

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Date Due: _____

Convert these rates to Double time

1 $21.75 = _____ × 2

Double time rate = _____ /h

2 $19.95 = _____ × _____

Double time rate = _____ /h

Calculate the Double time earned

3 Normal Rate: $15.40/h
Double Time Hours: 4

D.T. = _____ × 2 × 4

D.T. = _____

4 Normal Rate: $17.65/h
Double Time Hours: 5

Pay = _____ × _____ × _____

Pay = _____

5 Normal Rate: $25.92/h
Double Time Hours: 7

Pay = _____ × _____ × _____

Pay = _____

Date Due: _____

Convert these rates to Double time

1 $21.75 = _____ × 2

Double time rate = _____ /h

2 $19.95 = _____ × _____

Double time rate = _____ /h

Calculate the Double time earned

3 Normal Rate: $15.40/h
Double Time Hours: 4

D.T. = _____ × 2 × 4

D.T. = _____

4 Normal Rate: $17.65/h
Double Time Hours: 5

Pay = _____ × _____ × _____

Pay = _____

5 Normal Rate: $25.92/h
Double Time Hours: 7

Pay = _____ × _____ × _____

Pay = _____

Date Due: _____

Convert these rates to Double time

1 $21.75 = _____ × 2

Double time rate = _____ /h

2 $19.95 = _____ × _____

Double time rate = _____ /h

Calculate the Double time earned

3 Normal Rate: $15.40/h
Double Time Hours: 4

D.T. = _____ × 2 × 4

D.T. = _____

4 Normal Rate: $17.65/h
Double Time Hours: 5

Pay = _____ × _____ × _____

Pay = _____

5 Normal Rate: $25.92/h
Double Time Hours: 7

Pay = _____ × _____ × _____

Pay = _____

Date Due: _____

Convert these rates to Double time

1 $21.75 = _____ × 2

Double time rate = _____ /h

2 $19.95 = _____ × _____

Double time rate = _____ /h

Calculate the Double time earned

3 Normal Rate: $15.40/h
Double Time Hours: 4

D.T. = _____ × 2 × 4

D.T. = _____

4 Normal Rate: $17.65/h
Double Time Hours: 5

Pay = _____ × _____ × _____

Pay = _____

5 Normal Rate: $25.92/h
Double Time Hours: 7

Pay = _____ × _____ × _____

Pay = _____

Earning Money - Penalty Rates - Double Time (Homework Strip)

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Date Due:

Convert these rates to Double time

1 $21.75 = 21.75 × 2

Double time rate = $43.50 /h

2 $19.95 = 19.95 × 2

Double time rate = $39.90 /h

Calculate the Double time earned

3 Normal Rate: $15.40/h
Double Time Hours: 4

D.T. = 15.40 × 2 × 4

D.T. = $123.20

4 Normal Rate: $17.65/h
Double Time Hours: 5

Pay = 17.65 × 2 × 5

Pay = $176.50

5 Normal Rate: $25.92/h
Double Time Hours: 7

Pay = 25.92 × 2 × 7

Pay = $362.88

Earning Money - Penalty Rates

When an employee exceeds a certain number of hours in a day (varies with contract) they are paid overtime. This often starts with a penalty rate of 1.5×, then after two or three hours at time-and-a-half, double time begins.

Sheet 1, Columns 1 & 2 lists the hours that an employee has worked, in three bands. Normal hours, time-and-a-half hours and double time hours. Work out the pay for each and then add them together.

Example

Erika earns $17.08/h and works:

Normal Time:	30 h
Time+½:	9 h
Double Time:	3 h

Calculate Erika's earnings for:

a) Her normal hours.

Pay = 17.08×30

Pay = $512.40

b) Her Time+½ hours.

Pay = $17.08 \times 1.5 \times 9$

Pay = $230.58

c) Her Double time hours.

Pay = $17.08 \times 2 \times 3$

Pay = $102.48

d) Calculate Erika's total wage:

Pay = Normal 512.40 + Time+½ 230.58 + Double 102.48 = $845.46

Column 2 and Column 1 of sheet 2 ask you to perform the same steps above, but this time do it all at once.

Example

Adam earns $23.25/h and works:

Normal Time:	19 h
Time+½:	8 h
Double Time:	3 h

Calculate Adam's total pay.

Pay = 23.25×19 ← Normal hours worked.

\+ $23.25 \times 1.5 \times 8$ ← Time+½ hours worked.

\+ $23.25 \times 2 \times 3$ ← Double time hours worked.

Pay = $860.25

The last column of sheet 2 has two tables. The first table is a weekly time sheet for an employee, note that the rate is at the top of the table. Calculate the normal, time-and-a-half and double time wages writing them below the hours in each cell of the table. Then add the three for the final column.

The second table is for 7 employees, they have had their hours totaled in each column. Again get totals for each cell and then add them for the final column.

Earning Money - Penalty Rates (Sheet 1)

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Calculate the pay, in steps.

Dave earns $14.60/h and works:

Normal Time:	38 h
Time+½:	7 h
Double Time:	3 h

Calculate Dave's earnings from:

1 His normal hours.

Pay = [Rate] × 38

Pay = [ ] ← '$' sign and cents to 2 d.p.

2 His time-and-a-half hours:

Pay = [Rate] × 1.5 × [Hrs]

Pay = [ ]

3 His double time hours:

Pay = [Rate] × 2 × [Hrs]

Pay = [ ]

4 Calculate Dave's total pay.

Pay = [Normal] + [Time+½]

\+ [Double]

Pay = [ ]

Jess earns $16.70/h and works:

Normal Time:	25 h
Time+½:	4 h
Double Time:	2 h

Calculate Jess' earnings from:

5 Her normal hours.

Pay = [ ] × [ ]

Pay = [ ]

6 Her time-and-a-half hours:

Pay = [ ] × [ ] × [ ]

Pay = [ ]

7 Her double time hours:

Pay = [ ] × [ ] × [ ]

Pay = [ ]

8 Calculate Jess' total pay.

Pay = [ ] + [ ]

\+ [ ]

Pay = [ ]

Toby earns $23.80/h and in a week, he works:

Normal Time:	38 h
Time+½:	8 h
Double Time:	3 h

Calculate Toby's earnings from:

9 His normal hours.

Pay =

Pay =

10 His time-and-a-half hours:

Pay =

Pay =

11 His double time hours:

Pay =

Pay =

12 Calculate Toby's total pay.

Pay =

13 Convert Toby's total weekly pay to an annual wage.

Find total pay in one step

Kwan earns $22.90/h and works:

Normal Time:	34 h
Time+½:	3 h
Double Time:	2 h

14 Calculate Kwan's total pay.

Pay = [ ] × [ ]

Rate × Normal hours

\+ [ ] × 1.5 × [ ]

Rate × 1.5 × Time+½ hours

\+ [ ] × 2 × [ ]

Rate × 2 × Double time hours

Pay = [ ]

Caleb earns $27.15/h and works:

Normal Time:	19 h
Time+½:	8 h
Double Time:	3 h

15 Calculate Caleb's total pay.

Pay = [ ] × [ ]

\+ [ ] × [ ] × [ ]

\+ [ ] × [ ] × [ ]

Pay = [ ]

Rose earns $31.80/h and works:

Normal Time:	38 h
Time+½:	6 h
Double Time:	1.5 h

14 Calculate Rose's total pay.

Pay =

Earning Money - Penalty Rates (Sheet 2)

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Calculate the total pay earned.

Josh earns $26.50/h and works a 38 hour week, with 6 hours at time-and-a-half and 2 hours double time.

1 Calculate Josh's total pay.

Pay =

Rate × Normal Hours

+ Rate × 1.5 × Time+½ Hours

+ Rate × 2 × Double Time Hours

Pay =

Maysun earns $22.85/h and works a 30 hour week, with 4 hours at time-and-a-half and 1.5 hours double time.

2 Find Maysun's total pay.

Pay =

Pay =

Connie earns $30.45/h and works a 36 hour week, with 7 hours at time-and-a-half and 4.5 hours double time.

3 Find Connie's total pay.

Round cents to 2 decimal places

Calculate the daily earnings, then total them for the week.

4

Normal hourly rate = $28.40/h

Day	*Normal hrs*	*Time+½ hrs*	*Double Time hrs*	*Total Pay*
Mon	7	2	3	
Tues	6	-	-	
Wed	6	2	2	
Thur	6	1	-	
Fri	7	2	1.5	
			Total Pay (Week)	

Calculate each employee's weekly earnings

5

Name	*Hourly rate*	*Normal hrs*	*Time+½ hrs*	*Double Time hrs*	*Total Pay*
Ben	$19.70	16	2	1	
Ken	$28.50	26	7	2	
Len	$34.65	38	10	-	
Thien	$37.20	38	5	-	
Glen	$18.60	28	11	5	
Wen	$25.80	30	5	1.5	
Zhen	$42	38	3	0.5	
				Total Employee Wages	

Earning Money - Penalty Rates (Sheet 1)

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Calculate the pay, in steps.

Dave earns $14.60/h and works:

Normal Time:	38 h
Time+½:	7 h
Double Time:	3 h

Calculate Dave's earnings from:

1 His normal hours.

Pay = 14.60 × 38

Pay = $554.80 ← '$' sign and cents to 2 d.p.

2 His time-and-a-half hours:

Pay = 14.60 × 1.5 × 7

Pay = $153.30

3 His double time hours:

Pay = 14.60 × 2 × 3

Pay = $87.60

4 Calculate Dave's total pay.

Pay = 554.80 + 153.30

\+ 87.60

Pay = $795.70

Jess earns $16.70/h and works:

Normal Time:	25 h
Time+½:	4 h
Double Time:	2 h

Calculate Jess' earnings from:

5 Her normal hours.

Pay = 16.70 × 25

Pay = $417.50

6 Her time-and-a-half hours:

Pay = 16.70 × 1.5 × 4

Pay = $100.20

7 Her double time hours:

Pay = 16.70 × 2 × 2

Pay = $66.80

8 Calculate Jess' total pay.

Pay = 417.50 + 100.20

\+ 66.80

Pay = $584.50

Toby earns $23.80/h and in a week, he works:

Normal Time:	38 h
Time+½:	8 h
Double Time:	3 h

Calculate Toby's earnings from:

9 His normal hours.

Pay = 23.80 × 38

Pay = $904.40

10 His time-and-a-half hours:

Pay = 23.80 × 1.5 × 8

Pay = $285.60

11 His double time hours:

Pay = 23.80 × 2 × 3

Pay = $142.80

12 Calculate Toby's total pay.

Pay = 904.40 + 285.60

\+ 142.80

Pay = $1 332.80

13 Convert Toby's total weekly pay to an annual wage.

Pay = 1 332.80 × 52

Pay = $69 305.60

Find total pay in one step

Kwan earns $22.90/h and works:

Normal Time:	34 h
Time+½:	3 h
Double Time:	2 h

14 Calculate Kwan's total pay.

Pay = 22.90 × 34

Rate × Normal hours

\+ 22.90 × 1.5 × 3

Rate × 1.5 × Time+½ hours

\+ 22.90 × 2 × 2

Rate × 2 × Double time hours

Pay = $973.25

Caleb earns $27.15/h and works:

Normal Time:	19 h
Time+½:	8 h
Double Time:	3 h

15 Calculate Caleb's total pay.

Pay = 27.15 × 19

\+ 27.15 × 1.5 × 8

\+ 27.15 × 2 × 3

Pay = $1 004.55

Rose earns $31.80/h and works:

Normal Time:	38 h
Time+½:	6 h
Double Time:	1.5 h

14 Calculate Rose's total pay.

Pay = 31.80 × 38

31.80 × 1.5 × 6

31.80 × 2 × 1.5

Pay = $1 590

Earning Money - Penalty Rates (Sheet 2)

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Calculate the total pay earned.

Josh earns $26.50/h and works a 38 hour week, with 6 hours at time-and-a-half and 2 hours double time.

1 Calculate Josh's total pay.

Pay = 26.50 × 38
Rate × Normal Hours

\+ 26.50 × 1.5 × 6
+ Rate × 1.5 × Time+½ Hours

\+ 26.50 × 2 × 2
+ Rate × 2 × Double Time Hours

Pay = $1 351.50

Maysun earns $22.85/h and works a 30 hour week, with 4 hours at time-and-a-half and 1.5 hours double time.

2 Find Maysun's total pay.

Pay = 22.85 × 30

\+ 22.85 × 1.5 × 4

\+ 22.85 × 2 × 1.5

Pay = $891.15

Connie earns $30.45/h and works a 36 hour week, with 7 hours at time-and-a-half and 4.5 hours double time.

3 Find Connie's total pay.

Pay = 30.45 × 36

\+ 30.45 × 1.5 × 7

\+ 30.45 × 2 × 4.5

Pay = $1 689.98

Round cents to 2 decimal places

Calculate the daily earnings, then total them for the week.

4 Normal hourly rate = $28.40/h

Day	*Normal hrs*	*Time+½ hrs*	*Double Time hrs*	*Total Pay*
Mon	7 $198.80	2 $85.20	3 $170.40	$454.40
Tues	6 $170.40	- -	- -	$170.40
Wed	6 $170.40	2 $85.20	2 $113.60	$369.20
Thur	6 $170.40	1 $42.60	- -	$213
Fri	7 $198.80	2 $85.20	1.5 $85.20	$369.20
			Total Pay (Week)	$1 576.20

Calculate each employee's weekly earnings

5

Name	*Hourly rate*	*Normal hrs*	*Time+½ hrs*	*Double Time hrs*	*Total Pay*
Ben	$19.70	16 $315.20	2 $59.10	1 $39.40	$413.70
Ken	$28.50	26 $741	7 $299.25	2 $114	$1 154.25
Len	$34.65	38 $1 316.70	10 $519.75	- -	$1 836.45
Thien	$37.20	38 $1 413.60	5 $279	- -	$1 692.60
Glen	$18.60	28 $520.80	11 $306.90	5 $186	$1 013.70
Wen	$25.80	30 $774	5 $193.50	1.5 $77.40	$1 044.90
Zhen	$42	38 $1 596	3 $189	0.5 $42	$1 827
				Total Employee Wages	$8 982.60

Earning Money - Penalty Rates (Homework Strip)

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Date Due: _____

Find the normal and overtime pay.

Oscar earns $15.20/h and works:

Normal Time:	35 h
Time+½:	7 h
Double Time:	3 h

Calculate Oscar's earnings from:

1 His normal hours.

Pay = [Rate] × 35

Pay = [ ] ← '$' sign and cents to 2 d.p.

2 His time-and-a-half hours:

Pay = [Rate] × 1.5 × [Hrs]

Pay = [ ]

3 His double time hours:

Pay = [Rate] × 2 × [Hrs]

Pay = [ ]

4 Calculate Oscar's total pay.

Pay = [Normal] + [Time+½]

\+ [Double]

Pay = [ ]

Date Due: _____

Find the normal and overtime pay.

Oscar earns $15.20/h and works:

Normal Time:	35 h
Time+½:	7 h
Double Time:	3 h

Calculate Oscar's earnings from:

1 His normal hours.

Pay = [Rate] × 35

Pay = [ ] ← '$' sign and cents to 2 d.p.

2 His time-and-a-half hours:

Pay = [Rate] × 1.5 × [Hrs]

Pay = [ ]

3 His double time hours:

Pay = [Rate] × 2 × [Hrs]

Pay = [ ]

4 Calculate Oscar's total pay.

Pay = [Normal] + [Time+½]

\+ [Double]

Pay = [ ]

Date Due: _____

Find the normal and overtime pay.

Oscar earns $15.20/h and works:

Normal Time:	35 h
Time+½:	7 h
Double Time:	3 h

Calculate Oscar's earnings from:

1 His normal hours.

Pay = [Rate] × 35

Pay = [ ] ← '$' sign and cents to 2 d.p.

2 His time-and-a-half hours:

Pay = [Rate] × 1.5 × [Hrs]

Pay = [ ]

3 His double time hours:

Pay = [Rate] × 2 × [Hrs]

Pay = [ ]

4 Calculate Oscar's total pay.

Pay = [Normal] + [Time+½]

\+ [Double]

Pay = [ ]

Date Due: _____

Find the normal and overtime pay.

Oscar earns $15.20/h and works:

Normal Time:	35 h
Time+½:	7 h
Double Time:	3 h

Calculate Oscar's earnings from:

1 His normal hours.

Pay = [Rate] × 35

Pay = [ ] ← '$' sign and cents to 2 d.p.

2 His time-and-a-half hours:

Pay = [Rate] × 1.5 × [Hrs]

Pay = [ ]

3 His double time hours:

Pay = [Rate] × 2 × [Hrs]

Pay = [ ]

4 Calculate Oscar's total pay.

Pay = [Normal] + [Time+½]

\+ [Double]

Pay = [ ]

Earning Money - Penalty Rates (Homework Strip)

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Date Due:

Find the normal and overtime pay.

Oscar earns $15.20/h and works:

Normal Time:	35 h
Time+½:	7 h
Double Time:	3 h

Calculate Oscar's earnings from:

1 His normal hours.

Pay = 15.20 × 35

Pay = $532 ← '$' sign and cents to 2 d.p.

2 His time-and-a-half hours:

Pay = 15.20 × 1.5 × 7

Pay = $159.60

3 His double time hours:

Pay = 15.20 × 2 × 3

Pay = $91.20

4 Calculate Oscar's total pay.

Pay = 532 + 159.60

\+ 91.20

Pay = $782.80

24 Hour Time and Time Sheet Calculations

While time in employment is normally a.m./p.m. format, you can use 24 hour time to make using a calculator possible.

The first 2 digits of 24 hour time are the hours, the second 2 digits are the minutes.
The time starts at 0000 hours (midnight) and ends at 2359 (1 minute before midnight). Note that there is no full stop separating hours and minutes. So:

- The time is 0000 hours at 12 a.m. (midnight), 12.30 a.m. is 0030 hours
- Then each hour adds 100 so 1 a.m. is 0100 hours, 2 a.m. is 0200 hours etc
- Noon is 1200 hours and 1 p.m. is 1300 hours

So once the time reaches 1 p.m. add 12 hours to the normal time to get your answer in 24 hr time format, e.g. 4.45 p.m. = 1645 hours (hours = 4 + 12 = 16, minutes stay the same)

Note that the word 'hours' or 'h' can be written after the time.

Sheet 1 Column 1 starts with asking you to change the time from a.m/p.m. to 24 hour format. If the time is 1 p.m. or later add 12 to the hours.

Column 2 asks you to calculate the time worked by giving you a start and finishing time. Convert the times to 24 hr format first, then using your calculator, subtract the two times, the latest time minus the earlier time.

Example

5.45 p.m. in 24 hour time

9.25 a.m. until 5.45 p.m.

= 1 7 4 5 h - 0 9 2 5 h

= 8h 20 m

9.25 a.m. in 24 hour time

Your calculator's DMS key could have DMS on it or these symbols



Column 3 asks you to round times. When time sheets are used the times are usually rounded to the nearest 15 minutes.

Example

7h 20 min = 7h 15 min

7h 20m is in between 7h 15m and 7h 30m. It is closer to 7h 15m (5 minutes) rather than 7h 30m (10 minutes). So it becomes 7h 15 m after rounding.

The last part of Column 3 asks you to perform a subtraction to find the time difference, then round the answer as above. So it joins the two skills.

The last two sheets involve completing time sheets. The above skills of time subtraction and rounding are used, but the last step asks you to multiply the time and the hourly rate. There are examples in Q.2 and Q.8 to show you the calculator steps.

24 Hour Time and Time Calculations

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Change these times to 24 hour time format, (from 1 p.m. add 12 hours, keep minutes as is).

Example

4.45 p.m. 1645 h

1 3 a.m. = ☐☐☐☐ h

2 8 a.m. = ☐☐☐☐ h

3 4 p.m. = ☐☐☐☐ h

4 11.25 a.m.

5 3.52 p.m.

6 5.17 p.m.

7 8.47 p.m.

8 4.56 a.m.

9 12.33 p.m.

10 10.25 p.m.

11 12.19 a.m.

12 7.39 p.m

13 4.06 a.m.

14 11.39 a.m.

15 6.26 p.m

16 11.08 p.m

17 10.43 a.m.

18 9.43 p.m

19 5.56 a.m.

20 11.48 a.m.

Find the time worked, use 24 hour time and subtract the earlier time from the later time.

Example

9.15 a.m. until 4.35 p.m.

= 1635 h - 0915 h

16 DMS 35 DMS - 9 DMS 15 DMS

= ☐ h ☐ min

21 11.25 a.m. until 6.15 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

22 8.35 a.m. until 4.45 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

23 7.50 a.m. until 3.30 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

24 6.10 a.m. until 2.25 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

25 7.05 a.m. until 11.50 a.m.

= ☐☐☐☐ h - ☐☐☐☐ h

26 12.07 p.m. until 8.53 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

27 9.13 a.m. until 5.55 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

Round these times to the nearest 15 mins. So minutes will be 0 or 15 or 30 or 45.

Examples

7h 20 min = 7h 15 min

5h 38 min = 5h 45 min

28 6h 4 min = ☐ h ☐ min

29 4h 19 min = ☐ h ☐ min

30 8h 42 min = ☐ h ☐ min

31 5h 54 min = ☐ h ☐ min

32 7h 38 min = ☐ h ☐ min

33 6h 52 min = ☐ h ☐ min

34 4h 36 min = ☐ h ☐ min

35 3h 50 min = ☐ h ☐ min

Find the time worked, but this time get your answer then round it to the nearest 15 min.

36 9.17 a.m. until 5.38 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min = ☐ h ☐ min

Actual Rounded

37 7.41 a.m. until 2.17 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min = ☐ h ☐ min

38 6.07 a.m. until 1.34 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min = ☐ h ☐ min

39 10.43 a.m. until 4.21 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min = ☐ h ☐ min

Time Sheet Calculations (Sheet 1)

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Find the time worked and the pay earned for the shift, given the hourly rate.

1 9.45 a.m. until 5.15 p.m.

17 DMS 15 DMS - 9 DMS 45 DMS =

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min

2 Hourly rate is $21.40/h.

Using the answer on your calculator screen, then do this: × 21.40 = DMS

= ☐ h ☐ min × ☐

= $ ☐

3 10.45 a.m. until 7.30 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min

4 Hourly rate is $25.60/h.

= ☐ h ☐ min × ☐

= $ ☐

5 7.30 a.m. until 3.15 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min

6 Hourly rate is $31.90/h.

= ☐ h ☐ min × ☐

= $ ☐

Convert the answer in Q.6 to an annual wage if the employee works that shift 4 days/week.

7

Pay =

Pay =

Now complete the table, converting to 24 h time and rounding to the nearest 15 minutes. You have to subtract the employees' lunch time.

8 Hourly rate for all employees is $28.12

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Tim	9.35 0935	4.15 1615	30 m	☐h ☐☐m	☐h ☐☐m	
Actual hours = 16 DMS 15 DMS - 9 DMS 35 DMS - 0 DMS 30 DMS =						
Jim	7.05 ☐☐☐☐	3.42 ☐☐☐☐	45m	☐h ☐☐m	☐h ☐☐m	
Lim	8.19 ☐☐☐☐	5.29 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Kim	6.37 ☐☐☐☐	4.08 ☐☐☐☐	45m	☐h ☐☐m	☐h ☐☐m	
Yim	10.09 ☐☐☐☐	4.57 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Alim	8.19 ☐☐☐☐	5.08 ☐☐☐☐	1h	☐h ☐☐m	☐h ☐☐m	

9 Hourly rate for all employees is $24.64

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Jen	7.52 ☐☐☐☐	3.43 ☐☐☐☐	45m	☐h ☐☐m	☐h ☐☐m	
Ken	9.07 ☐☐☐☐	4.35 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Ben	11.10 ☐☐☐☐	3.27 ☐☐☐☐	1h	☐h ☐☐m	☐h ☐☐m	
Len	8.55 ☐☐☐☐	5.18 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Wen	10.36 ☐☐☐☐	4.09 ☐☐☐☐	1h	☐h ☐☐m	☐h ☐☐m	
Thien	7.16 ☐☐☐☐	4.57 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Zen	8.39 ☐☐☐☐	6.06 ☐☐☐☐	45m	☐h ☐☐m	☐h ☐☐m	

Time Sheet Calculations (Sheet 2)

Complete these time sheets for the employees shown. Round the time worked to the nearest 15 minutes, then calculate the wages.



1

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Hourly Rate*	*Daily Wage*
Luke	8.30 0830h	4.41 1641h	45 m	7h 26m	7h 30m	$19.50	$146.25
Leia	7.15	2.26	30 m	Subtract lunch!		$23.80	
Han	6.45	3.50	45 m	Subtract lunch!		$33.15	
Chewy	5.45	12.06	30 m			$27.28	
Ben	9.15	2.57	45 m			$33.15	
Darth	8.30	1.27	-			$23.85	
Padme	7.45	2.30	30 m			$19.50	Rounding needed
Jabba	10.50	5.18	1h			$33.15	
Lando	8.30	4.45	45 m			$17.55	
Yoda	6.10	3.48	30 m			$23.85	

2

Employee: A. Mouak Hourly Rate: $23.80/h

Day	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	5.40	2.07	45 m			
Tues	6.15	1.23	45 m			
Wed	6.15	12.09	45 m			
Thurs	6.15	2.53	45 m			
Fri	5.40	1.18	45 m			
					Total Weekly Wage	

Note! Not a.m.

3

Employee: W. Zhiang Hourly Rate: $15.64/h

Day	*Time In p.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	3.20	9.36	30 m			
Wed	3.20	10.43	45 m			
Fri	3.20	9.49	30 m			
					Total Weekly Wage	

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24 Hour Time and Time Calculations

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Change these times to 24 hour time format, (from 1 p.m. add 12 hours, keep minutes as is).

Example

4.45 p.m. 1645 h

1. 3 a.m. = 0300 h
2. 8 a.m. = 0800 h
3. 4 p.m. = 1600 h
4. 11.25 a.m. 1125h
5. 3.52 p.m. 1552h
6. 5.17 p.m. 1717h
7. 8.47 p.m. 2047h
8. 4.56 a.m. 0456h
9. 12.33 p.m. 1233h
10. 10.25 p.m. 2225h
11. 12.19 a.m. 0019h
12. 7.39 p.m 1939h
13. 4.06 a.m. 0406h
14. 11.39 a.m. 1139h
15. 6.26 p.m 1826h
16. 11.08 p.m 2308h
17. 10.43 a.m. 1043h
18. 9.43 p.m 2143h
19. 5.56 a.m. 0556h
20. 11.48 a.m. 1148h

Find the time worked, use 24 hour time and subtract the earlier time from the later time.

Example

9.15 a.m. until 4.35 p.m.

= 1635 h - 0915 h

16 DMS 35 DMS - 9 DMS 15 DMS =

= 7 h 20 min

21. 11.25 a.m. until 6.15 p.m.

= 1815 h - 1125 h

= 6h 50m

22. 8.35 a.m. until 4.45 p.m.

= 1645 h - 0835 h

= 8h 10m

23. 7.50 a.m. until 3.30 p.m.

= 1530 h - 0750 h

= 7h 40m

24. 6.10 a.m. until 2.25 p.m.

= 1425 h - 0610 h

= 8h 15h

25. 7.05 a.m. until 11.50 a.m.

= 1150 h - 0705 h

= 4h 45m

26. 12.07 p.m. until 8.53 p.m.

= 2053 h - 1207 h

= 8h 46m

27. 9.13 a.m. until 5.55 p.m.

= 1755 h - 0913 h

= 8h 42m

Round these times to the nearest 15 mins. So minutes will be 0 or 15 or 30 or 45.

Examples

7h 20 min = 7h 15 min

5h 38 min = 5h 45 min

28. 6h 4 min = 6 h 0 min
29. 4h 19 min = 4 h 15 min
30. 8h 42 min = 8 h 45 min
31. 5h 54 min = 6 h 0 min
32. 7h 38 min = 7 h 45 min
33. 6h 52 min = 6 h 45 min
34. 4h 36 min = 4 h 30 min
35. 3h 50 min = 3 h 45 min

Find the time worked, but this time get your answer then round it to the nearest 15 min.

36. 9.17 a.m. until 5.38 p.m.

= 1738 h - 0917 h

= 8 h 21 min = 8 h 15 min

Actual Rounded

37. 7.41 a.m. until 2.17 p.m.

= 1417 h - 0741 h

= 6 h 36 min = 6 h 30 min

38. 6.07 a.m. until 1.34 p.m.

= 1334 h - 0607 h

= 7 h 27 min = 7 h 30 min

39. 10.43 a.m. until 4.21 p.m.

= 1621 h - 1043 h

= 5 h 38 min = 5 h 45 min

Time Sheet Calculations (Sheet 1)

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Find the time worked and the pay earned for the shift, given the hourly rate.

1 9.45 a.m. until 5.15 p.m.

17 DMS 15 DMS - 9 DMS 45 DMS =

= 17 15 h - 09 45 h

= 7 h 30 min

2 Hourly rate is $21.40/h.

Using the answer on your calculator screen, then do this: × 21.40 = DMS

= 7 h 30 min × 21.40

= $ 160.50

3 10.45 a.m. until 7.30 p.m.

= 19 30 h - 10 45 h

= 8 h 45 min

4 Hourly rate is $25.60/h.

= 8 h 45 min × 25.60

= $ 224

5 7.30 a.m. until 3.15 p.m.

= 15 15 h - 07 30 h

= 7 h 45 min

6 Hourly rate is $31.90/h.

= 7 h 45 min × 31.90

= $ 247.23

Convert the answer in Q.6 to an annual wage if the employee works that shift 4 days/week.

7

Pay = 247.23 × 4 × 52

Pay = $51 423.84

Now complete the table, converting to 24 h time and rounding to the nearest 15 minutes. You have to subtract the employees' lunch time.

8 Hourly rate for all employees is $28.12

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Tim	9.35 → 0935	4.15 → 1615	30 m	6 h 10 m	6 h 15 m	$175.75
Jim	7.05 → 0705	3.42 → 1542	45m	7 h 52 m	7 h 45 m	$217.93
Lim	8.19 → 0819	5.29 → 1729	30m	9 h 10 m	9 h 15 m	$260.11
Kim	6.37 → 0637	4.08 → 1608	45m	8 h 46 m	8 h 45 m	$246.05
Yim	10.09 → 1009	4.57 → 1657	30m	6 h 18 m	6 h 15 m	$175.75
Alim	8.19 → 0819	5.08 → 1708	1h	7 h 49 m	7 h 45 m	$217.93

Actual hours = 16 DMS 15 DMS - 9 DMS 35 DMS - 0 DMS 30 DMS =

9 Hourly rate for all employees is $24.64

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Jen	7.52 → 0752	3.43 → 1543	45m	7 h 06 m	7 h 00 m	$172.48
Ken	9.07 → 0907	4.35 → 1635	30m	6 h 58 m	7 h 00 m	$172.48
Ben	11.10 → 1110	3.27 → 1527	1h	3 h 17 m	3 h 15 m	$80.08
Len	8.55 → 0855	5.18 → 1718	30m	7 h 53 m	8 h 00 m	$197.12
Wen	10.36 → 1036	4.09 → 1609	1h	4 h 33 m	4 h 30 m	$110.88
Thien	7.16 → 0716	4.57 → 1657	30m	9 h 11 m	9 h 15 m	$227.92
Zen	8.39 → 0839	6.06 → 1806	45m	8 h 42 m	8 h 45 m	$215.64

Time Sheet Calculations (Sheet 2)

Complete these time sheets for the employees shown. Round the time worked to the nearest 15 minutes, then calculate the wages.



1

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Hourly Rate*	*Daily Wage*
Luke	8.30 0830h	4.41 1641h	45 m	7h 26m	7h 30m	$19.50	$146.25
Leia	7.15 0715h	2.26 1426h	30 m	6h 41m	6h 45m	$23.80	$160.65
Han	6.45 0645h	3.50 1530h	45 m	8h	8h	$33.15	$265.20
Chewy	5.45 0545h	12.06 1206h	30 m	5h 51m	5h 45m	$27.28	$156.86
Ben	9.15 0915h	2.57 1457h	45 m	4h 57m	5h	$33.15	$165.75
Darth	8.30 0830h	1.27 1327h	-	4h 57m	5h	$23.85	$119.25
Padme	7.45 0745h	2.30 1430h	30 m	6h 15m	6h 15m	$19.50	$121.88
Jabba	10.50 1050h	5.18 1718h	1h	5h 28m	5h 30m	$33.15	$182.33
Lando	8.30 0830h	4.45 1645h	45 m	7h 30m	7h 30m	$17.55	$131.63
Yoda	6.10 0610h	3.48 1548h	30 m	9h 8m	9h 15m	$23.85	$220.61

2

Employee: A. Mouak Hourly Rate: $23.80/h

Day	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	5.40 0540h	2.07 1407h	45 m	7h 42m	7h 45m	$184.45
Tues	6.15 0615h	1.23 1323h	45 m	6h 23m	6h 30m	$154.70
Wed	6.15 0615h	12.09 1209h	45 m	5h 9m	5h 15m	$124.95
Thurs	6.15 0615h	2.53 1453h	45 m	7h 53m	8h	$190.40
Fri	5.40 0615h	1.18 1318h	45 m	6h 18m	6h 15m	$148.75
					Total Weekly Wage	$803.25

3

Employee: W. Zhiang Hourly Rate: $15.64/h

Day	*Time In p.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	3.20 1520h	9.36 2136h	30 m	6h 18m	6h 15m	$97.75
Wed	3.20 1520h	10.43 2243h	45 m	6h 38m	6h 45m	$105.57
Fri	3.20 1520h	9.49 2149h	30 m	5h 59m	6h	$93.84
					Total Weekly Wage	$297.16

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Earning Money - Time Sheets (Homework Strip)

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Date Due:

Find the time in 24 hour time format

1 7 a.m. = h

2 9 a.m. = h

3 8 p.m. = h

4 11.17 p.m. = h

5 4.08 p.m. = h

Calculate the hours worked

6 9.06 a.m. until 4.45 p.m.

= h - h

7 10.17 a.m. until 6.05 p.m.

= h - h

8 7.53 a.m. until 4.37 p.m.

= h - h

Date Due:

Find the time in 24 hour time format

1 7 a.m. = h

2 9 a.m. = h

3 8 p.m. = h

4 11.17 p.m. = h

5 4.08 p.m. = h

Calculate the hours worked

6 9.06 a.m. until 4.45 p.m.

= h - h

7 10.17 a.m. until 6.05 p.m.

= h - h

8 7.53 a.m. until 4.37 p.m.

= h - h

Date Due:

Find the time in 24 hour time format

1 7 a.m. = h

2 9 a.m. = h

3 8 p.m. = h

4 11.17 p.m. = h

5 4.08 p.m. = h

Calculate the hours worked

6 9.06 a.m. until 4.45 p.m.

= h - h

7 10.17 a.m. until 6.05 p.m.

= h - h

8 7.53 a.m. until 4.37 p.m.

= h - h

Date Due:

Find the time in 24 hour time format

1 7 a.m. = h

2 9 a.m. = h

3 8 p.m. = h

4 11.17 p.m. = h

5 4.08 p.m. = h

Calculate the hours worked

6 9.06 a.m. until 4.45 p.m.

= h - h

7 10.17 a.m. until 6.05 p.m.

= h - h

8 7.53 a.m. until 4.37 p.m.

= h - h

Earning Money - Time Sheets (Homework Strip)

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Date Due:

Find the time in 24 hour time format

1 7 a.m. = 0700 h

2 9 a.m. = 0900 h

3 8 p.m. = 2000 h

4 11.17 p.m. = 2317 h

5 4.08 p.m. = 1608 h

Calculate the hours worked

6 9.06 a.m. until 4.45 p.m.

= 1645 h - 0906 h

= 7h 39m

7 10.17 a.m. until 6.05 p.m.

= 1805 h - 1017 h

= 7h 48 m

8 7.53 a.m. until 4.37 p.m.

= 1637 h - 0753 h

= 8h 44m

Earning Money - Percentage Applications

Percentages are regularly used with wages. It may be that you get a raise in your hourly rate, for undertaking uncomfortable or more difficult work. Or it may be that your wage is annually reviewed and each year you get an increase in your pay.

Sheet 1 Column 1 asks you to find a percentage of a wage, which is the amount a wage will be increased by. Change the percentage to a fraction by putting it 'over 100', then multiply it with the wage.

Example

Find 8% of $49 300

$$49\,300 \times \frac{8}{100} = \$3\,944$$

Using your calculator's fraction key

4 9 3 0 0 × 8 a b/c 1 0 0 =

Columns 2 and 3 ask you to find the new wage. So it is the same as above, except you then add the increase to the original wage.

Example

Increase $49 300 by 8%

Increase:

$$49\,300 \times \frac{8}{100} = \$3\,944$$

Same as before

New Wage:

$$= 49\,300 + 3\,944$$

$$= \$53\,244$$

Add to original wage

Sheet 2 deals with getting an increased hourly rate when you undertake different work. This may be a height allowance, a wet workplace allowance, dust allowances … there are many different possibilities.

Column 1 asks you to calculate a person's wage after considering their hourly rate being increased for some of the hours worked. These questions are done in stages, finding the increase, finding the new rate, then finding the pay.

Example

Helen earns $28.70/h as window cleaner. When she works above 4 storeys she gets a 15% height allowance. Calculate:
- her hourly rate increase
- her increased hourly rate
- her weekly pay when she works 18h at the normal rate and 20h at the increased rate.

1 The hourly rate increase

$$28.70 \times \frac{15}{100} = \$4.31$$

Hourly rate × percentage (as a fraction)

2 The increased hourly rate

$$28.70 + 4.31 = \$33.01/h$$

Original rate + the increase

3

$$\underset{\text{Weekly Wage}}{} = \underset{\text{Rate}}{28.70} \times \underset{\text{Hours}}{18} +$$

$$\underset{\text{Increased Rate}}{33.01} \times \underset{\text{Hours}}{20}$$

$$\text{Weekly Wage} = \$1\,176.80$$

Rates × hrs added together

Earning Money - Percentage Applications (Sheet 1)

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If you get an increase in pay, it is often by a percentage. Find these percentages.

Example: Find 7% of $17 800

$$17\,800 \times \frac{7}{100} = \$1\,246$$

Using your calculator's fraction key

1 7 8 0 0 × ▣ 7 ◯ 1 0 0 =

1 Find 4% of $48 000

$$\square \times \frac{\square}{100} = \$\square$$

2 Find 6% of $82 000

$$\square \times \frac{\square}{\square} = \square$$

'$' sign

3 Find 5.4% of $63 200

$$\square \times \frac{\square}{\square} = \square$$

Write answer to 2 decimal places.

4 Find 2.6% of $45 670

$$\square \times \frac{\square}{\square} = \square$$

5 Find 6.1% of $104 600

$$\square \times \frac{\square}{\square} = \square$$

6 Find 3.35% of $88 777

$$\square \times \frac{\square}{\square} = \square$$

Round answer to 2 decimal places.

7 Find 5.15% of $78 441

$$\square \times \frac{\square}{\square} = \square$$

8 Find 8.28% of $138 881

$$\square \times \frac{\square}{\square} = \square$$

Increase these annual wages by the percentage given.

Example: Find the new annual wage after $53 000 is increased by 8%.

Increase:

$$53\,000 \times \frac{8}{100} = \$4\,240$$

New Wage:

$= 53\,000 + 4\,240$

$= \$57\,240$

9 Increase $45 000 by 6%

Increase:

$$\square \times \frac{\square}{100} = \square$$

New Wage:

$= \square + \square$

$= \square$

10 Increase $125 500 by 4%

Increase:

$$\square \times \frac{\square}{\square} = \square$$

New Wage:

$= \square + \square$

$= \square$

11 Increase $78 400 by 3%

Increase:

$$\square \times \frac{\square}{\square} = \square$$

New Wage:

$= \square + \square$

$= \square$

Increase the annual salary, then find the new weekly salary.

12 Increase $38 300 by 4.2%

$$\square \times \frac{\square}{\square} = \square$$

Wage = $\square + \square$

Wage = $\square$

13 Find the new weekly wage.

Weekly Wage = $\square \div 52$

Weekly Wage = $\square$

14 Increase $143 558 by 6.5%

Wage = $\square + \square$

Wage = $\square$

15 Find the new weekly wage.

Weekly Wage =

Weekly Wage =

16 Increase $105 600 by 2.17%

17 Find the new weekly wage.

Weekly Wage =

Weekly Wage =

Earning Money - Percentage Applications (Sheet 2)

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Work in a different position and you might get paid extra for it. Find the increased rates.

David earns $28.20/h as a welder. When he works on elevators he gets a 15% height allowance. Calculate:

1 The hourly rate increase

$\square \times \frac{\square}{100} = \$\square$

2 The increased hourly rate

$\square + \square = \$\quad /h$

3 If David works 17 hours at the normal rate and 21 hours at the increased rate, calculate his weekly pay.

Weekly Wage $= \square$ (Rate) $\times \square$ (Hours) $+$

$\square$ (Increased Rate) $\times \square$ (Hours)

Weekly Wage $= \square$

Mischa is a power pole contractor earning $38.10/h. When she works in a hoist she is paid 20% extra per hour. Calculate:

4 Her hourly rate increase

$\square \times \frac{\square}{\square} = \square$

5 Her increased hourly rate

$\square + \square = \square$

6 If Mischa works 25 hours at the normal rate and 13 hours at the increased rate, calculate her weekly pay.

Weekly Wage $= \square \times \square +$

$\square \times \square$

Weekly Wage $= \square$

Chloe earns $32.80/h as a car smash repairer. When she works in the spray booth she gets an 18% increase on her hourly rate. In a 38 hour week she works 17 h in the booth. Find:

7 Her hourly rate increase

$\square \times \frac{\square}{\square} = \square$ (2 d.p.)

8 Her increased hourly rate

$\square + \square = \square$

9 The number of hours Chloe works out of the spray booth.

10 Her weekly pay.

Weekly Wage $= \square \times \square +$

$\square \times \square$

Weekly Wage $= \square$

Sasha earns $45.90/h as a computer programmer. He earns 25% more per hour when he works off-site. If he works a 40 hour week of which 17h is off-site, calculate:

11 His off-site hourly rate

12 The number of hours Sasha works on-site (in his office).

13 His weekly pay.

Raj earns $24.90/h as a baker. He is paid an additional 8% for early hours before 6 a.m. If he works a 38 h week and 10 of those hours are 'early hours', find:

14 His 'early hours' rate

15 The number of standard hours Raj works

16 His weekly pay.

17 His annual wage

Rachel is a Human Resource consultant earning $85.90/h. When she takes on team leader status she earns 15% more. Calculate:

18 Rachel's team leader rate.

19 Her weekly pay given she works 27h at the normal rate and 8h at the team leader rate.

Earning Money - Percentage Applications (Sheet 1)

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If you get an increase in pay, it is often by a percentage. Find these percentages.

Example: Find 7% of $17 800

$$17\,800 \times \frac{7}{100} = \$1\,246$$

Using your calculator's fraction key

1 7 8 0 0 × ▣ 7 ◯ 1 0 0 =

1 Find 4% of $48 000

48 000 × $\frac{4}{100}$ = $1 920

2 Find 6% of $82 000

82 000 × $\frac{6}{100}$ = $4 920

3 Find 5.4% of $63 200

63 200 × $\frac{5.4}{100}$ = $3 412.80

Write answer to 2 decimal places.

4 Find 2.6% of $45 670

45 670 × $\frac{2.6}{100}$ = $1 187.42

5 Find 6.1% of $104 600

104 600 × $\frac{6.1}{100}$ = $6 380.60

6 Find 3.35% of $88 777

88 777 × $\frac{3.35}{100}$ = $2 974.03

Round answer to 2 decimal places.

7 Find 5.15% of $78 441

78 441 × $\frac{5.15}{100}$ = $4 039.71

8 Find 8.28% of $138 881

138 881 × $\frac{8.28}{100}$ = $11 499.35



Increase these annual wages by the percentage given.

Example: Find the new annual wage after $53 000 is increased by 8%.

Increase:

$$53\,000 \times \frac{8}{100} = \$4\,240$$

Using your calculator's fraction key

5 3 0 0 0 × 8 $a^b/_c$ 1 0 0 =

New Wage:

= 53 000 + 4 240

= $57 240

9 Increase $45 000 by 6%

Increase:

45 000 × $\frac{6}{100}$ = $2 700

New Wage:

= 45 000 + 2 700

= $47 700

10 Increase $125 500 by 4%

Increase:

125 500 × $\frac{4}{100}$ = $5 020

New Wage:

= 125 500 + 5 020

= $130 520

11 Increase $78 400 by 3%

Increase:

78 400 × $\frac{3}{100}$ = $2 352

New Wage:

= 78 400 + 2 352

= $80 752

12 Increase $38 300 by 4.2%

Increase:

38 300 × $\frac{4.2}{100}$ = $1 608.60

New Wage:

= 38 300 + 1 608.60

= $39 908.60

13 Increase $98 278 by 3.5%

Increase:

98 278 × $\frac{3.5}{100}$ = $3 439.73

New Wage:

= 98 278 + 3 439.73

= $101 717.73

14 Increase $143 558 by 6.5%

Increase:

143 558 × $\frac{6.5}{100}$ = $9 331.27

New Wage:

= 143 558 + 9 331.27

= $152 889.27

15 Increase $78 200 by 8.3%

Increase:

78 200 × $\frac{8.3}{100}$ = $6 490.60

New Wage:

= 78 200 + 6 490.60

= $84 690.60

16 Increase $105 600 by 2.17%

Increase:

105 600 × $\frac{2.17}{100}$ = $2 291.52

New Wage:

= 105 600 + 2 291.52

= $107 891.52

Earning Money - Percentage Applications (Sheet 2)

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Work in a different position and you might get paid extra for it. Find the increased rates.

David earns $28.20/h as a welder. When he works on elevators he gets a 15% height allowance. Calculate:

1 The hourly rate increase

$28.20 \times \frac{15}{100} = \$4.23$

2 The increased hourly rate

$28.20 + 4.23 = \$32.43/h$

3 If David works 17 hours at the normal rate and 21 hours at the increased rate, calculate his weekly pay.

Weekly Wage $= 28.20$ (Rate) $\times 17$ (Hours) $+$

32.43 (Increased Rate) $\times 21$ (Hours)

Weekly Wage $= \$1\ 160.43$

Mischa is a power pole contractor earning $38.10/h. When she works in a hoist she is paid 20% extra per hour. Calculate:

4 Her hourly rate increase

$38.10 \times \frac{20}{100} = \$7.62$

5 Her increased hourly rate

$38.10 + 7.62 = \$45.72$

6 If Mischa works 25 hours at the normal rate and 13 hours at the increased rate, calculate her weekly pay.

Weekly Wage $= 38.10 \times 25 +$

45.72×13

Weekly Wage $= \$1\ 546.86$

Chloe earns $32.80/h as a car smash repairer. When she works in the spray booth she gets an 18% increase on her hourly rate. In a 38 hour week she works 17 h in the booth. Find:

7 Her hourly rate increase

$32.80 \times \frac{18}{100} = \$5.90$ (2 d.p.)

8 Her increased hourly rate

$32.80 + 5.90 = \$38.70$

9 The number of hours Chloe works out of the spray booth.

$38 - 17 = 21$ h

10 Her weekly pay.

Weekly Wage $= 32.80 \times 21 +$

38.70×17

Weekly Wage $= \$1\ 346.70$

Sasha earns $45.90/h as a computer programmer. He earns 25% more per hour when he works off-site. If he works a 40 hour week of which 17h is off-site, calculate:

11 His off-site hourly rate

$45.90 \times \frac{25}{100} = \$11.48$

$45.90 + 11.48 = \$57.38$

12 The number of hours Sasha works on-site (in his office).

$40 - 17 = 23$h

13 His weekly pay.

Weekly Wage $= 23 \times 45.90 +$

17×57.38

Weekly Wage $= \$2\ 031.16$

Raj earns $24.90/h as a baker. He is paid an additional 8% for early hours before 6 a.m. If he works a 38 h week and 10 of those hours are 'early hours', find:

14 His 'early hours' rate

$24.90 \times \frac{8}{100} = \$1.99$

$24.90 + 1.99 = \$26.89$

15 The number of standard hours Raj works

$38 - 10 = 28$h

16 His weekly pay.

Weekly Wage $= 28 \times 24.90 +$

10×26.89

Weekly Wage $= \$966.10$

17 His annual wage

Annual Wage $= 966.10 \times 52$

Annual Wage $= \$50\ 237.20$

Rachel is a Human Resource consultant earning $85.90/h. When she takes on team leader status she earns 15% more. Calculate:

18 Rachel's team leader rate.

$85.90 \times \frac{15}{100} = \$12.89$

$85.90 + 12.89 = \$98.79$

19 Her weekly pay given she works 27h at the normal rate and 8h at the team leader rate.

Weekly Wage $= 27 \times 85.90 +$

8×98.79

Weekly Wage $= \$3\ 109.62$

Earning Money - Percentage Applications (Homework Strip)

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Date Due:

Find these percentages

1 Find 3% of $76 600

$\square \times \frac{3}{100} = \square$

2 Find 9% of $46 150

$\square \times \frac{\square}{100} = \square$

3 Find 4.5% of $81 070

$\square \times \frac{\square}{\square} = \square$

Increase the wage by 8%

4 Increase $58 500 by 8%

Increase:

$\square \times \frac{\square}{\square} = \square$

New Wage:

$= \square + \square$

$= \square$

Date Due:

Find these percentages

1 Find 3% of $76 600

2 Find 9% of $46 150

3 Find 4.5% of $81 070

Increase the wage by 8%

4 Increase $58 500 by 8%

Increase:

New Wage:

Date Due:

Find these percentages

1 Find 3% of $76 600

2 Find 9% of $46 150

3 Find 4.5% of $81 070

Increase the wage by 8%

4 Increase $58 500 by 8%

Increase:

New Wage:

Date Due:

Find these percentages

1 Find 3% of $76 600

2 Find 9% of $46 150

3 Find 4.5% of $81 070

Increase the wage by 8%

4 Increase $58 500 by 8%

Increase:

New Wage:

Earning Money - Percentage Applications (Homework Strip)

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Date Due:

Find these percentages

1 Find 3% of $76 600

$$76\,600 \times \frac{3}{100} = \$2\,298$$

2 Find 9% of $46 150

$$46\,150 \times \frac{9}{100} = \$4\,153.50$$

3 Find 4.5% of $81 070

$$81\,070 \times \frac{4.5}{100} = \$3\,648.15$$

Increase the wage by 8%

4 Increase $58 500 by 8%

Increase:

$$58\,500 \times \frac{8}{100} = \$4\,680$$

New Wage:

$$= 58\,500 + 4\,680$$

$$= \$63\,180$$

Earning Money - Piecework

Not all people earn a salary or wage, others are paid directly on what they produce. The more you make the more you earn. This type of work is called Piecework, and it allows workers to be as fast or slow as they like. They can stop and do something else then come back to the job. It is a flexible form of employment that has both advantages and disadvantages.

Sheet 1 Column 1 starts with some questions on rates. If you are paid, say $10 to produce something, how much do you earn to produce 5? $5 \times 10 = \$50$. just multiply the rate by the number of items.

At the base of the column is a daily table. A record of daily production is kept and this allows for a end-of-week payment to be received. Total up the table, then multiply by the rate.

Example

Horses reshoed at $58.30/horse

Day	M	Tu	W	Th	F	Total
Sheep	4	6	5	3	5	23

Add along the row

Rate: 58.30 × No. of horses: 23

Payment = 58.30 × 23

Payment = $1 340.90

Question 8 in column 2 is like the above example except there are two rates. The usual rate and a higher rate to motivate more daily production. You need to split the production into two numbers. One at the normal rate, the other at the higher rate. Question 8 has been split up for you already.

Sheet 2 deals with quoting for jobs by trades people. When a trades person is hired they supply a quote or 'price estimate' for the job. The prices vary from person to person, so you can get several quotes and select the best quote.

Column 1 starts with a roof tiler being offered a fixed price on a job. He has to compare the price with what he charges to decide on whether to take the job. These questions vary and so the first column has the operations that you need to do, so follow the steps.

Earning Money - Piecework (Sheet 1)

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Calculate the payment for these items, use multiplication

1 15 drawings at $17 each.

Payment = ☐ × ☐

Payment = $ ☐

2 30 T-shirts at $14.50 each.

Payment = ☐ × ☐

Payment = ☐

Show the $ sign

3 22 Jackets at $56.80 each.

Payment = ☐ × ☐

Payment = ☐

Show the $ sign and cents to 2 decimal places

4 38 m^2 of tiling at $108/$m^2$.

Payment =

Payment =

The tables below show daily production. Get the total, then multiply by the rate.

5 Sheep shorn at $9.30/sheep

Day	M	Tu	W	Th	F	Total
Sheep	15	24	19	32	25	

Payment = ☐ × ☐

Payment = ☐

6 Cars detailed at $64.20/car

Day	M	Tu	W	Th	F	Total
Cars	4	5	3	4	6	

Payment = ☐ × ☐

Payment = ☐

7 Parcels delivered at $2.37 for each parcel.

Day	M	Tu	W	Th	F	Total
No.	208	95	167	198	153	

Payment = ☐ × ☐

Payment = ☐

Production can have different rates to motivate. Split the production into the two rates.

Alison produces flower arrangements, she is paid $6.80/arrangement for the first 20 supplied. If she supplies more than 20 she is paid $8.90/arrangement.

8

Day	M	Tu	W	Th	F	S	Total
No.	19	15	25	16	28	32	
$6.80:	19	15	20	16	20	20	
$8.90:	0	0	5	0	8	12	

Payment = ☐ × ☐ +

☐ × ☐

Payment = ☐

James frames photographs. He is paid $7.95/frame for the first 30 frames, after that he is paid $9.50 per frame.

9

Day	M	Tu	W	Th	F	Total
No.	42	38	19	56	43	
$7.95:						
$9.50:						

Payment = ☐ × ☐ +

☐ × ☐

Payment = ☐

Malik is a fruit-picker. He is paid $3.50 per tub of oranges picked, for the first 35 tubs. He is paid $4.25 per tub after the initial 35.

10

Day	W	Th	F	Sa	Su	Total
No.	62	43	52	28	37	
$3.50:						
$4.25:						

Payment = ☐ × ☐ +

☐ × ☐

Payment = ☐

Troy sells electricity packages by walking house to house. He is paid $65.70 for every sale he makes. If he makes more than 5 sales in a day he is paid an additional $23.95 per sale.

11 Find the rate for extra sales.

= ☐ + ☐

= ☐

Write this below the table under '$65.70'

12

Day	M	Tu	W	Th	F	S	Total
No.	3	7	4	8	5	12	
$65.70:							

Payment = ☐ × ☐ +

☐ × ☐

Payment = ☐

Earning Money - Piecework (Sheet 2)

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A tradesperson needs to know whether a job is worthwhile. Compare the quotes for these.

James is a roof-tile layer. He charges a minimum of \$18/m^2. He is offered a job of laying 205 m^2 for \$3 198.

1 Find the amount offered per m^2 for the \$3 198 job.

Rate = ☐ ÷ ☐

Rate = ☐ /m^2

Show the \$ sign and cents to 2 decimal places

2 Is the rate acceptable?

◯ Yes, it's more than \$18/m^2

◯ No, it's less than \$18/m^2

3 How much would James quote for laying 205 m^2?

Cost = ☐ × ☐

Cost = ☐

4 What is the difference between the \$3 198 offered and James' quote for that job?

Amount = ☐ - ☐

Amount = ☐

Lauren is another roof-tile layer. She charges a \$300 fee + \$14/m^2. What is her quote for laying the 205 m^2?

5

Quote = ☐ + ☐ × ☐

Quote = ☐

6 Should she take the 205 m^2 job, based on her quote?

◯ Yes, her quote is below the \$3 198

◯ No, her quote is above the \$3 198

Clara is a landscape designer. She charges a \$180 fee + \$7.70/m^2 for laying turf.

7 Give a quote for Clara to lay 95 m^2 of turf.

Cost =

Cost =

8 Give a quote for Clara to lay 175 m^2 of turf.

Cost =

Cost =

Tisharna is also a landscape designer. She charges a \$9/m^2 for laying turf.

8 Give a quote for Tisharna to lay 175 m^2 of turf.

Cost =

Cost =

9 Who offers the cheaper quote for the 175 m^2?

◯ Clara ◯ Tisharna

How much turf does Tisharna lay for these quotes listed. Her rate is \$9/m^2.

10 \$504

Area =

Area = ← Answer in m^2

11 \$4 140

Area =

Area =

12 \$1 080

Area =

Area =

Toby fits glass splash-backs in kitchens. He charges \$550 for fitting a standard splash-back, which takes him a day to fit.

13 How much does Toby earn for a 5 day week?

Pay =

Pay =

14 Convert this wage to an annual wage.

Pay =

Pay =

15 Toby is offered a salary of \$142 000 p.a. from a kitchen company for the same hours. Should he take the position?

Glen fits T.V. antenna masts for \$450 and fits sockets for \$35 per room. Give a quote for these jobs.

16 1 mast and 2 sockets

17 1 mast and 4 sockets

18 2 masts and 5 sockets

Earning Money - Piecework (Sheet 1)

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Calculate the payment for these items, use multiplication

1 15 drawings at $17 each.

Payment = 17 × 15

Payment = $ 255

2 30 T-shirts at $14.50 each.

Payment = 14.50 × 30

Payment = $435

Show the $ sign

3 22 Jackets at $56.80 each.

Payment = 56.80 × 22

Payment = $1 249.60

Show the $ sign and cents to 2 decimal places

4 38 m^2 of tiling at $108/$m^2$.

Payment = 108 × 38

Payment = $4 104

The tables below show daily production. Get the total, then multiply by the rate.

5 Sheep shorn at $9.30/sheep

Day	M	Tu	W	Th	F	Total
Sheep	15	24	19	32	25	115

Payment = 9.30 × 115

Payment = $1 069.50

6 Cars detailed at $64.20/car

Day	M	Tu	W	Th	F	Total
Cars	4	5	3	4	6	22

Payment = 64.20 × 22

Payment = $1 412.40

7 Parcels delivered at $2.37 for each parcel.

Day	M	Tu	W	Th	F	Total
No.	208	95	167	198	153	821

Payment = 2.37 × 821

Payment = $1 945.77

Production can have different rates to motivate. Split the production into the two rates.

Alison produces flower arrangements, she is paid $6.80/arrangement for the first 20 supplied. If she supplies more than 20 she is paid $8.90/arrangement.

8

Day	M	Tu	W	Th	F	S	Total
No.	19	15	25	16	28	32	
$6.80:	19	15	20	16	20	20	110
$8.90:	0	0	5	0	8	12	25

Payment = 6.80 × 110 +

8.90 × 25

Payment = $970.50

James frames photographs. He is paid $7.95/frame for the first 30 frames, after that he is paid $9.50 per frame.

9

Day	M	Tu	W	Th	F	Total
No.	42	38	19	56	43	
$7.95:	30	30	19	30	30	139
$9.50:	12	8	0	26	13	59

Payment = 7.95 × 139 +

9.50 × 59

Payment = $1 665.55

Malik is a fruit-picker. He is paid $3.50 per tub of oranges picked, for the first 35 tubs. He is paid $4.25 per tub after the initial 35.

10

Day	W	Th	F	Sa	Su	Total
No.	62	43	52	28	37	
$3.50:	35	35	35	28	35	168
$4.25:	27	8	17	0	2	54

Payment = 3.50 × 168 +

4.25 × 54

Payment = $817.50

Troy sells electricity packages by walking house to house. He is paid $65.70 for every sale he makes. If he makes more than 5 sales in a day he is paid an additional $23.95 per sale.

11 Find the rate for extra sales.

= 65.70 + 23.95

= $89.65

Write this below the table under '$65.70'

12

Day	M	Tu	W	Th	F	S	Total
No.	3	7	4	8	5	12	
$65.70:	3	5	4	5	5	5	27
$89.65	0	2	0	3	0	7	12

Payment = 65.70 × 27 +

89.65 × 12

Payment = $2 849.70

Earning Money - Piecework (Sheet 2)

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A tradesperson needs to know whether a job is worthwhile. Compare the quotes for these.

James is a roof-tile layer. He charges a minimum of \$18/m^2. He is offered a job of laying 205 m^2 for \$3 198.

1 Find the amount offered per m^2 for the \$3 198 job.

Rate = 3 198 ÷ 205

Rate = \$15.60 /m^2

Show the \$ sign and cents to 2 decimal places

2 Is the rate acceptable?

- ○ Yes, it's more than \$18/m^2
- ● No, it's less than \$18/m^2

3 How much would James quote for laying 205 m^2?

Cost = 205 × 18

Cost = \$3 690

4 What is the difference between the \$3 198 offered and James' quote for that job?

Amount = 3 690 - 3 198

Amount = \$492

Lauren is another roof-tile layer. She charges a \$300 fee + \$14/m^2. What is her quote for laying the 205 m^2?

5

Quote = 300 + 205 × 14

Quote = \$3 170

6 Should she take the 205 m^2 job, based on her quote?

- ● Yes, her quote is below the \$3 198
- ○ No, her quote is above the \$3 198

Clara is a landscape designer. She charges a \$180 fee + \$7.70/m^2 for laying turf.

7 Give a quote for Clara to lay 95 m^2 of turf.

Cost = 180 + 7.70 × 95

Cost = \$911.50

8 Give a quote for Clara to lay 175 m^2 of turf.

Cost = 180 + 7.70 × 175

Cost = \$1 527.50

Tisharna is also a landscape designer. She charges a \$9/m^2 for laying turf.

8 Give a quote for Tisharna to lay 175 m^2 of turf.

Cost = 175 × 9

Cost = \$1 575

9 Who offers the cheaper quote for the 175 m^2?

- ● Clara
- ○ Tisharna

How much turf does Tisharna lay for these quotes listed. Her rate is \$9/m^2.

10 \$504

Area = 504 ÷ 9

Area = 56 m^2 ← Answer in m^2

11 \$4 140

Area = 4 140 ÷ 9

Area = 460 m^2

12 \$1 080

Area = 1 080 ÷ 9

Area = 120 m^2

Toby fits glass splash-backs in kitchens. He charges \$550 for fitting a standard splash-back, which takes him a day to fit.

13 How much does Toby earn for a 5 day week?

Pay = 550 × 5

Pay = \$2 750

14 Convert this wage to an annual wage.

Pay = 2 750 × 52

Pay = \$143 000

15 Toby is offered a salary of \$142 000 p.a. from a kitchen company for the same hours. Should he take the position?

Yes, he gets 4 weeks holidays, he might not get 5 days every week in his current job.

Glen fits T.V. antenna masts for \$450 and fits sockets for \$35 per room. Give a quote for these jobs.

16 1 mast and 2 sockets

Cost = 450 + 2 × 35

Cost = \$520

17 1 mast and 4 sockets

Cost = 450 + 4 × 35

Cost = \$590

18 2 masts and 5 sockets

Cost = 2 × 450 + 5 × 35

Cost = \$1 075

Earning Money - Piecework (Homework Strip)

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Date Due: _____

Calculate the amount earned

1 16 visits at $27.95 each.

Payment = _____ × _____

Payment = _____

2 8 tyre rotations at $35 each.

Payment = _____ × _____

Payment = _____

3 6 lawns mowed at $33 each

Payment = _____ × _____

Payment = _____

Find the total, then multiply the rate

4 Tubs of laundry ironed at $14.20 per tub.

Day	M	Tu	W	Th	F	Total
Tubs	8	13	9	7	11	

Payment = _____ × _____

Payment = _____

Date Due: _____

Calculate the amount earned

1 16 visits at $27.95 each.

Payment = _____ × _____

Payment = _____

2 8 tyre rotations at $35 each.

Payment = _____ × _____

Payment = _____

3 6 lawns mowed at $33 each

Payment = _____ × _____

Payment = _____

Find the total, then multiply the rate

4 Tubs of laundry ironed at $14.20 per tub.

Day	M	Tu	W	Th	F	Total
Tubs	8	13	9	7	11	

Payment = _____ × _____

Payment = _____

Date Due: _____

Calculate the amount earned

1 16 visits at $27.95 each.

Payment = _____ × _____

Payment = _____

2 8 tyre rotations at $35 each.

Payment = _____ × _____

Payment = _____

3 6 lawns mowed at $33 each

Payment = _____ × _____

Payment = _____

Find the total, then multiply the rate

4 Tubs of laundry ironed at $14.20 per tub.

Day	M	Tu	W	Th	F	Total
Tubs	8	13	9	7	11	

Payment = _____ × _____

Payment = _____

Date Due: _____

Calculate the amount earned

1 16 visits at $27.95 each.

Payment = _____ × _____

Payment = _____

2 8 tyre rotations at $35 each.

Payment = _____ × _____

Payment = _____

3 6 lawns mowed at $33 each

Payment = _____ × _____

Payment = _____

Find the total, then multiply the rate

4 Tubs of laundry ironed at $14.20 per tub.

Day	M	Tu	W	Th	F	Total
Tubs	8	13	9	7	11	

Payment = _____ × _____

Payment = _____

Earning Money - Piecework (Homework Strip)

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Date Due:

Calculate the amount earned

1 16 visits at $27.95 each.

Payment = 27.95 × 16

Payment = $447.20

2 8 tyre rotations at $35 each.

Payment = 35 × 8

Payment = $280

3 6 lawns mowed at $33 each

Payment = 33 × 6

Payment = $198

Find the total, then multiply the rate

4 Tubs of laundry ironed at $14.20 per tub.

Day	M	Tu	W	Th	F	Total
Tubs	8	13	9	7	11	48

Payment = 14.20 × 48

Payment = $681.60

Earning Money - Commission

Commission is when you are paid a percentage of your sales. This means the more you sell, the more you make. So it is a motivational method of payment to pay you more if you make more money for the business. It can result in low pay also for people who aren't good in sales, so commission can be a poor payment option for some people.

Commission is the same as finding a percentage of a quantity, except it is easier because the units are always $, and are always rounded to 2 decimal places.

Sheet 1 Column 1 is completed by multiplying the sales amount by the percentage as a fraction 'over 100'.

Example

Sales = $17 315, Rate = 6%

Commission = $17\,315 \times \frac{6}{100}$

Commission = $1 038.90

6% as a fraction is 6 over 100.

Column 2 starts with a table. The table has a list of sales employees. They are paid different rates because they sell different products. This is the same as the above method, just no working out is required.

The middle column asks you to compare two employees to see who earns the most. The last column asks you to match commission questions with their answers, use different colours so you don't make a webby mess.

Sheet 2 is about retainers, or base salary. A retainer is an amount paid to an employee regardless of how much they sell. This is used to give employees a wage as a safety net, against quiet times (such as after Christmas). Regardless if on a commission-based wage, an employee must be paid at least the award's minimum hourly rate, so a retainer assists employers in meeting this legal requirement.

Example

Base salary = $310/wk with
Sales = $8 175, Rate = 12%.

Pay = $310 + 8\,175 \times \frac{12}{100}$

Pay = $1 291

The only difference is you add the retainer

Earning Money - Commission (Sheet 1)

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Find the commission on these sales. Multiply by the percentage as a fraction written 'over 100'.

Example

Sales = \$15 540, Rate = 12%

Commission = $15\,540 \times \frac{12}{100}$

$= \$1\,864.80$

1 Sales = \$23 016, Rate = 11%

Commission = $\square \times \frac{\square}{100}$

$=$

2 Sales = \$17 880, Rate = 9%

Commission = $\square \times \frac{\square}{100}$

$=$

3 Sales = \$78 107, Rate = 4%

Commission = $\square \times \frac{\square}{\square}$

$=$

4 Sales = \$12 050, Rate = 8%

Commission =

$=$

5 Sales = \$9 300, Rate = 15%

Commission =

$=$

6 Sales = \$87 600, Rate = 6.5%

Commission =

$=$

Complete the table by finding the commission payable. Then rank the earnings from highest (1) to the lowest (7).

	Name	Rate	Sales	Commission	Rank
7	Gwen	5%	\$40 555		
8	Sven	11%	\$16 931		
9	Len	18%	\$15 676		
10	Wen	4%	\$46 855		
11	Ben	7%	\$25 910		
12	Jen	9.5%	\$23 166		
13	Toby	13%	\$17 582		

Calculate the commission and fill the circle for who earns the most per week.

14 Steve 5% on \$38 960 sales, Georgie 8% on sales of \$18 777.

Steve =

$=$

Georgie =

$=$

◯ Steve ◯ Georgie

15 Nicki 12% on \$43 110 sales, Zaina 8% on sales of \$54 550.

◯ Nicki ◯ Zaina

16 James 13% on \$11 522 sales, Jessie 17% on sales of \$14 070.

◯ James ◯ Jessie

Find the commission and connect it to the answer.

Use a different colour for each line

17 Commission: 5% Sales: \$41 500	•	A • \$1 413.60
18 Commission: 15% Sales: \$22 940	•	B • \$3524.40
19 Commission: 8% Sales: \$17 670	•	C • \$3 441
20 Commission: 1.2% Sales: \$135 200	•	D • \$1 922.50
21 Commission: 7% Sales: \$29 612	•	E • \$2 072.84
22 Commission: 2.5% Sales: \$76 900	•	F • \$4 199.76
23 Commission: 18% Sales: \$19 580	•	G • \$2 075
24 Commission: 12% Sales: \$34 998	•	H • \$1 622.40

Earning Money - Commission (Sheet 2)

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A retainer is a base salary paid with a commission payment. Calculate the pay earned.

Example Base salary = $200/wk
Sales = $13 700, Rate = 9.2%

Pay = $200 + 13\,700 \times \frac{9.2}{100}$

Pay = $1 460.40

1 Base salary = $370/wk with Sales = $8 295, Rate = 25%.

Pay = ☐ + ☐ × ☐/100

Pay = ☐

2 Base salary = $440/wk with Sales = $28 040, Rate = 6%.

Pay = ☐ + ☐ × ☐/100

Pay = ☐

3 Base salary = $290/wk with Sales = $158 900, Rate = 1.5%.

Pay = ☐ + ☐ × ☐/☐

Pay = ☐

4 Retainer of $340/wk with Sales = $38 775, Rate = 4.5%.

Pay =

Pay = ← Cents to 2 d.p.

5 Base salary of $550/wk with Sales = $73 145, Rate = 3.5%.

Pay =

Pay =

6 Base salary = $225/wk with Sales = $899 600, Rate = 0.3%.

Pay =

Pay =

Add the base salary to the commission earned to find the pay. Then rank the earnings from highest (1) to the lowest (7).

	Name	Base Salary	Rate	Sales	Weekly pay	Rank
7	Katniss	$800	7%	$38 900		
8	Cinna	$220	5%	$50 052		
9	Gale	$350	8%	$41 776		
10	Marvel	$150	10%	$50 735		
11	Thresh	$500	5.5%	$27 846		
12	Seneca	$280	4%	$37 200		
13	Peeta	$640	8.5%	$29 670		

Compare these wages to get the best deal.

Hakim sells home loan products. He has average weekly sales of $780 000. He is offered two weekly pay options:
1. $600 + 0.2% commission on sales
2. Flat 0.3% commission on sales, find:

14 His weekly pay for Option 1.

Pay =

Pay =

15 His weekly pay for Option 2.

Pay =

Pay =

16 Which is the best offer?

◯ Option 1 Option 2 ◯

Francis works with Hakim, his sales average $555 000/wk. Find:

17 His weekly pay for Option 1.

Pay =

Pay =

18 His weekly pay for Option 2.

Pay =

Pay =

Best? ◯ Option 1 ◯ Option 2

Rosie sells life insurance. She has average weekly sales of $43 000. She is offered two weekly pay options:
1. $300 + 4% commission on sales
2. An annual salary of $94 952, find:

19 Her weekly pay for Option 1.

Pay =

Pay =

20 Her weekly pay for Option 2.

Pay =

Pay =

21 Which is the best offer?

◯ Option 1 Option 2 ◯

Jane works with Rosie, her sales average $39 200/wk. Find:

22 Her weekly pay for Option 1.

Pay =

Pay =

23 Her weekly pay for Option 2.

Pay = ☐

Same answer as Question 20

24 Which is the best offer?

◯ Option 1 Option 2 ◯

Earning Money - Commission (Sheet 1)

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Find the commission on these sales. Multiply by the percentage as a fraction written 'over 100'.

Example

Sales = \$15 540, Rate = 12%

Commission = $15\,540 \times \frac{12}{100}$

= \$1 864.80

1 Sales = \$23 016, Rate = 11%

Commission = $23\,016 \times \frac{11}{100}$

= \$2 531.76

2 Sales = \$17 880, Rate = 9%

Commission = $17\,880 \times \frac{9}{100}$

= \$1 609.20

3 Sales = \$78 107, Rate = 4%

Commission = $78\,107 \times \frac{4}{100}$

= \$3 124.28

4 Sales = \$12 050, Rate = 8%

Commission = $12\,050 \times \frac{8}{100}$

= \$964

5 Sales = \$9 300, Rate = 15%

Commission = $9\,300 \times \frac{15}{100}$

= \$1 395

6 Sales = \$87 600, Rate = 6.5%

Commission = $87\,600 \times \frac{6.5}{100}$

= \$5 694

Complete the table by finding the commission payable. Then rank the earnings from highest (1) to the lowest (7).

	Name	Rate	Sales	Commission	Rank
7	Gwen	5%	\$40 555	\$2 027.75	4
8	Sven	11%	\$16 931	\$1 862.41	6
9	Len	18%	\$15 676	\$2 821.68	1
10	Wen	4%	\$46 855	\$1 874.20	5
11	Ben	7%	\$25 910	\$1 813.70	7
12	Jen	9.5%	\$23 166	\$2 200.77	3
13	Toby	13%	\$17 582	\$2 285.66	2

Calculate the commission and fill the circle for who earns the most per week.

14 Steve 5% on \$38 960 sales, Georgie 8% on sales of \$18 777.

Steve = $38\,960 \times \frac{5}{100}$

= \$1 948

Georgie = $18\,777 \times \frac{8}{100}$

= \$1 502.16

● Steve ○ Georgie

15 Nicki 12% on \$43 110 sales, Zaina 8% on sales of \$54 550.

Nicki = $43\,110 \times \frac{12}{100}$

= \$5 173.20

Zaina = $54\,550 \times \frac{8}{100}$

= \$4 364

● Nicki ○ Zaina

16 James 13% on \$11 522 sales, Jessie 17% on sales of \$14 070.

James = $11\,522 \times \frac{13}{100}$

= \$1 497.86

Jessie = $14\,070 \times \frac{17}{100}$

= \$2 391.90

○ James ● Jessie

Find the commission and connect it to the answer.

Use a different colour for each line

Commission: 17 5% Sales: \$41 500

Commission: 18 15% Sales: \$22 940

Commission: 19 8% Sales: \$17 670

Commission: 20 1.2% Sales: \$135 200

Commission: 21 7% Sales: \$29 612

Commission: 22 2.5% Sales: \$76 900

Commission: 23 18% Sales: \$19 580

Commission: 24 12% Sales: \$34 998

A \$1 413.60

B \$3524.40

C \$3 441

D \$1 922.50

E \$2 072.84

F \$4 199.76

G \$2 075

H \$1 622.40

ANSWERS
17—G
18—C
19—A
20—H
21—E
22—D
23—B
24—F

Earning Money - Commission (Sheet 2)

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A retainer is a base salary paid with a commission payment. Calculate the pay earned.

Example Base salary = $200/wk
Sales = $13 700, Rate = 9.2%

Pay = $200 + 13\,700 \times \frac{9.2}{100}$

Pay = $1 460.40

1 Base salary = $370/wk with Sales = $8 295, Rate = 25%.

Pay = 370 + 8 295 × $\frac{25}{100}$

Pay = $2 443.75

2 Base salary = $440/wk with Sales = $28 040, Rate = 6%.

Pay = 440 + 28 040 × $\frac{6}{100}$

Pay = $2 122.40

3 Base salary = $290/wk with Sales = $158 900, Rate = 1.5%.

Pay = 290 + 158 900 × $\frac{1.5}{100}$

Pay = $2 673.50

4 Retainer of $340/wk with Sales = $38 775, Rate = 4.5%.

Pay = $340 + 38\,775 \times \frac{4.5}{100}$

Pay = $2 084.88 ← Cents to 2 d.p.

5 Base salary of $550/wk with Sales = $73 145, Rate = 3.5%.

Pay = $550 + 73\,145 \times \frac{3.5}{100}$

Pay = $3 110.08

6 Base salary = $225/wk with Sales = $899 600, Rate = 0.3%.

Pay = $225 + 899\,600 \times \frac{0.3}{100}$

Pay = $2 923.80

Add the base salary to the commission earned to find the pay. Then rank the earnings from highest (1) to the lowest (7).

	Name	Base Salary	Rate	Sales	Weekly pay	Rank
7	Katniss	$800	7%	$38 900	$3 523	3
8	Cinna	$220	5%	$50 052	$2 722.60	5
9	Gale	$350	8%	$41 776	$3 692.08	2
10	Marvel	$150	10%	$50 735	$5 223.50	1
11	Thresh	$500	5.5%	$27 846	$2 031.53	6
12	Seneca	$280	4%	$37 200	$1 768	7
13	Peeta	$640	8.5%	$29 670	$3 161.95	4

Compare these wages to get the best deal.

Hakim sells home loan products. He has average weekly sales of $780 000. He is offered two weekly pay options:
1. $600 + 0.2% commission on sales
2. Flat 0.3% commission on sales, find:

14 His weekly pay for Option 1.

Pay = $600 + 780\,000 \times \frac{0.2}{100}$

Pay = $2 024.20

15 His weekly pay for Option 2.

Pay = $780\,000 \times \frac{0.3}{100}$

Pay = $2 340

16 Which is the best offer?

○ Option 1 ● Option 2

Francis works with Hakim, his sales average $555 000/wk. Find:

17 His weekly pay for Option 1.

Pay = $600 + 555\,000 \times \frac{0.2}{100}$

Pay = $1 710

18 His weekly pay for Option 2.

Pay = $555\,000 \times \frac{0.3}{100}$

Pay = $1 665

Best? ● Option 1 ○ Option 2

Rosie sells life insurance. She has average weekly sales of $43 000. She is offered two weekly pay options:
1. $300 + 4% commission on sales
2. An annual salary of $94 952, find:

19 Her weekly pay for Option 1.

Pay = $300 + 43\,000 \times \frac{4}{100}$

Pay = $2 020

20 Her weekly pay for Option 2.

Pay = 94 952 ÷ 52

Pay = $1 826

21 Which is the best offer?

● Option 1 ○ Option 2

Jane works with Rosie, her sales average $39 200/wk. Find:

22 Her weekly pay for Option 1.

Pay = $300 + 39\,200 \times \frac{4}{100}$

Pay = $1 868

23 Her weekly pay for Option 2.

Pay = $1 826

Same answer as Question 20

24 Which is the best offer?

● Option 1 ○ Option 2

Earning Money - Commission (Homework Strip)

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Date Due: _____

Calculate the commission earned

1 Sales = $25 600, Rate = 12%

Commission = _____ × $\frac{\square}{100}$

= _____

2 Sales = $98 120, Rate = 6%

Commission = _____ × $\frac{\square}{100}$

= _____

3 Sales = $224 000, Rate = 2%

Commission = _____ × $\frac{\square}{\square}$

= _____

4 Sales = $19 300, Rate = 13%

Commission = _____ × $\frac{\square}{\square}$

= _____

5 Sales = $47 920, Rate = 6.5%

Commission = _____ × $\frac{\square}{\square}$

= _____

Date Due: _____

Calculate the commission earned

1 Sales = $25 600, Rate = 12%

Commission = _____ × $\frac{\square}{100}$

= _____

2 Sales = $98 120, Rate = 6%

Commission = _____ × $\frac{\square}{100}$

= _____

3 Sales = $224 000, Rate = 2%

Commission = _____ × $\frac{\square}{\square}$

= _____

4 Sales = $19 300, Rate = 13%

Commission = _____ × $\frac{\square}{\square}$

= _____

5 Sales = $47 920, Rate = 6.5%

Commission = _____ × $\frac{\square}{\square}$

= _____

Date Due: _____

Calculate the commission earned

1 Sales = $25 600, Rate = 12%

Commission = _____ × $\frac{\square}{100}$

= _____

2 Sales = $98 120, Rate = 6%

Commission = _____ × $\frac{\square}{100}$

= _____

3 Sales = $224 000, Rate = 2%

Commission = _____ × $\frac{\square}{\square}$

= _____

4 Sales = $19 300, Rate = 13%

Commission = _____ × $\frac{\square}{\square}$

= _____

5 Sales = $47 920, Rate = 6.5%

Commission = _____ × $\frac{\square}{\square}$

= _____

Date Due: _____

Calculate the commission earned

1 Sales = $25 600, Rate = 12%

Commission = _____ × $\frac{\square}{100}$

= _____

2 Sales = $98 120, Rate = 6%

Commission = _____ × $\frac{\square}{100}$

= _____

3 Sales = $224 000, Rate = 2%

Commission = _____ × $\frac{\square}{\square}$

= _____

4 Sales = $19 300, Rate = 13%

Commission = _____ × $\frac{\square}{\square}$

= _____

5 Sales = $47 920, Rate = 6.5%

Commission = _____ × $\frac{\square}{\square}$

= _____

Earning Money - Commission (Homework Strip)

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Date Due:

Calculate the commission earned

1 Sales = $25 600, Rate = 12%

Commission = 25 600 × $\frac{12}{100}$

= $3 072

2 Sales = $98 120, Rate = 6%

Commission = 98 120 × $\frac{6}{100}$

= $5 887.20

3 Sales = $224 000, Rate = 2%

Commission = 224 000 × $\frac{2}{100}$

= $4 480

4 Sales = $19 300, Rate = 13%

Commission = 19 300 × $\frac{13}{100}$

= $2 509

5 Sales = $47 920, Rate = 6.5%

Commission = 47 920 × $\frac{2}{100}$

= $3 114.80

Earning Money - Leave Loading

An employee is entitled to 4 weeks holiday per year, called annual leave. This is paid leave but on yop of the normal 4 weeks pay, the employee also receives an extra 17½% on top of that pay. This is called leave loading.

To find leave loading, you first have to calculate 4 weeks pay. If you earn a weekly pay, then multiply by 4. But you may earn a monthly or yearly salary, or an hourly rate. Sheet 1, Column 1 asks you to convert weekly and fortnightly wages to 4 week's pay.

Examples

Weekly wage of $568.20

4 weeks pay = 568.20 × 4

4 weeks pay = $2 272.80

Multiply weekly wage by 4

Fortnightly wage of $1 317.60

4 weeks pay = 1 317.60 × 2

4 weeks pay = $2 635.20

Multiply fortnightly wage by 2

Columns 2 and 3 ask you to convert annual and monthly wages to 4 weeks pay. You may need to round the cents in later questions. Remember that 4 weeks doesn't equal a month.

Examples

Annual wage of $35 776

4 weeks pay = 35 776 ÷ 52 × 4

4 weeks pay = $2 752

Divide annual wage by 52 to get a weekly wage, then multiply by 4.

Monthly wage of $5 616

4 weeks pay = 5 616 × 12 ÷ 52 × 4

4 weeks pay = $5 184

Multiply monthly wage by 12 to get an annual wage, then divide by 52 for weekly and finally multiply by 4.

Sheets 2 and 3 ask you to calculate leave loading and holiday pay. This is a three step process:

- Calculate 4 weeks pay first (as above)
- Calculate the leave loading (17½% of 4 weeks pay)
- Add the two answers together. Holiday Pay = 4 weeks pay + Loading

Examples

Tamika earns $59 020 p.a. Find: 4 weeks pay; the leave loading; her holiday pay.

4 weeks pay = 59 020 ÷ 52 × 4

4 weeks pay = $4 540

First convert Annual pay to 4 weeks pay

L. Loading $= 4\,540 \times \frac{17.5}{100}$

$= \$794.50$

Then find the leave loading, Which is always 17½%.

H. Pay = 4 540 + 794.50

= $5 334.50

Then find the holiday pay, Which is 4 weeks pay + The leave loading

Earning Money - Leave Loading (Sheet 1)

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Convert these wages to 4 weeks pay.

Example: $1 365.20/week

4 weeks Pay = 1 365.20 × 4

= $5 460.80

1 $623.17/week

4 weeks Pay = ☐ × ☐

= $ ☐

2 $435.76/week

4 weeks Pay = ☐ × ☐

= $ ☐

3 $1 738.15/week

4 weeks Pay =

=

Show the $ sign and cents to 2 decimal places

4 $1 674.83/week

4 weeks Pay =

=

To find 4 weeks pay, multiply the fortnightly rate by 2 (So, × 2)

5 $2 560.77/fortnight

4 weeks Pay = ☐ × ☐

= ☐

6 $1 363.45/fortnight

4 weeks Pay =

=

7 $425.65/fortnight

4 weeks Pay =

=

Convert an annual wage to 4 weeks pay (÷ 52 × 4).

8 $62 774.40 p.a.

4 weeks pay = ☐ ÷ ☐ × ☐

= ☐

9 $28 605.20 p.a.

4 weeks pay = ☐ ÷ ☐ × ☐

= ☐

10 $45 705.63 p.a.

4 weeks pay =

= ← Round to 2 d.p.

11 $94 771 p.a.

4 weeks pay =

=

12 $59 883 p.a.

4 weeks pay =

=

Convert a monthly wage to 4 weeks pay (× 12 ÷ 52 × 4).

13 $4 680 per month.

4 weeks pay = ☐ × ☐ ÷ ☐ × ☐

= ☐

14 Is your answer above just less than the $4 680 per month?

◯ Yes, because 4 weeks while close, is slightly less than a month.

◯ No, I think I have to do it again.

15 $3 302 per month.

4 weeks pay = ☐ × ☐ ÷ ☐ × ☐

= ☐

16 $1 102.40 per month.

4 weeks pay =

=

17 $4 435 per month.

4 weeks pay =

= ← Round to 2 d.p.

Change the daily hours and rate to weekly pay to a 4 weekly pay.

Daily hours × Hourly rate × Days = Weekly Pay

	Name	*Daily Hours*	*Hourly Rate*	*Days per Week*	*Weekly Pay*	*4 Weeks Pay*
18	E. Claire	7	$24.85	5		
19	I. Sore	6	$19.40	6		
20	D. Light	5½	$25.60	4		
21	C. Saw	7	$15.40	5		
22	G. Whizz	8	$32.80	5		
23	T. Time	6½	$38.90	3		
24	X. Tract	4	$28.90	2		

Earning Money - Leave Loading (Sheet 2)

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Leave loading is 17½%. So multiply by 17.5 'over 100' to find the leave loading.

Example: Find 17½% of $560

L. Loading $= 560 \times \frac{17.5}{100}$

$= \$98$

1 Find 17½% of $2 186

L. Loading = ☐ × ☐/100

= ☐

2 Find 17½% of $778

L. Loading = ☐ × ☐/100

= ☐

3 Find 17½% of $1 412

L. Loading = ☐ × ☐/☐

= ☐

4 Find 17½% of $895.70

L. Loading = ☐ × ☐/☐

= ☐

5 Find 17½% of $1 033

L. Loading =

← Round to 2 d.p.

6 Find 17½% of $1 043.52

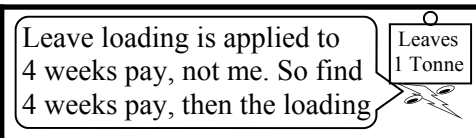
L. Loading =

=

7 Find 17½% of $358.38

L. Loading =

=



Leave loading is applied to 4 weeks pay, not me. So find 4 weeks pay, then the loading

Jodie earns $850 per week. Find:

8 4 Weeks pay.

4 weeks Pay = ☐ × ☐

= $ ☐

9 17½% of 4 weeks pay

L. Loading = ☐ × ☐/☐ (Q.8)

= ☐

10 Jodie's holiday pay.

H. Pay = ☐ (Q.8) + ☐ (Q.9)

= ☐

Iskandar earns $1 055 per week. Find:

11 4 Weeks pay.

4 weeks Pay = ☐ × ☐

= $ ☐

12 17½% of 4 weeks pay

L. Loading = ☐ (Q.11) × ☐/☐

= ☐

13 Iskandar's holiday pay.

H. Pay = ☐ (Q.11) + ☐ (Q.12)

= ☐

Carla earns $777.70 per week. Find:

14 4 Weeks pay.

4 weeks Pay = ☐ × ☐

= ☐

15 17½% of 4 weeks pay

L. Loading = ☐ (Q.14) × ☐/☐

= ☐

16 Carla's holiday pay.

H. Pay = ☐ + ☐

= ☐

Tegan earns $81 120 p.a. Find:

17 4 Weeks pay.

4 weeks pay = ☐ ÷ ☐ × ☐

= ☐

18 17½% of 4 weeks pay

L. Loading = ☐ × ☐/☐

= ☐

19 Tegan's holiday pay.

H. Pay = ☐ + ☐

= ☐

William earns $49 527.92 p.a. Find:

20 4 Weeks pay.

4 weeks pay = ☐ ÷ ☐ × ☐

= ☐

21 17½% of 4 weeks pay

L. Loading = ☐ × ☐/☐

= ☐ 2 d.p.

22 William's holiday pay.

H. Pay = ☐ + ☐

= ☐

Earning Money - Leave Loading (Sheet 3)

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Find 4 weeks pay, the leave loading (at 17½%), then find the holiday pay.

William earns $80 080 p.a. Find:

1 4 Weeks pay.

4 weeks pay = ____ ÷ ____ × ____

= ____

2 17½% of 4 weeks pay

L. Loading = ____ × ____/____

= ____

3 William's holiday pay.

H. Pay = ____ + ____

= ____

Jaycee earns $5 304/month. Find:

4 4 Weeks pay.

4 weeks pay = ____ × ____ ÷ ____ × ____

= ____

5 17½% of 4 weeks pay

L. Loading = ____ × ____/____

= ____

6 Jaycee's holiday pay.

H. Pay = ____ + ____

= ____

Nathan earns $19.30/h for a 12 hour week. Find:

7 4 Weeks pay.

4 weeks pay = ____ × ____ × ____

= ____

8 17½% of 4 weeks pay

L. Loading = ____ × ____/____

= ____

9 Nathan's holiday pay.

H. Pay = ____ + ____

= ____

Tony earns $2 444/month. Find:

10 4 Weeks pay.

4 weeks pay =

=

11 17½% of 4 weeks pay

L. Loading =

=

12 Tony's holiday pay.

H. Pay =

=

Brad earns $76 492 p.a. Find:

13 4 Weeks pay.

4 weeks pay =

=

14 17½% of 4 weeks pay

L. Loading =

=

15 Brad's holiday pay.

H. Pay =

=

Fei Yen earns $24.90/h for a 38 hour week. Find:

16 4 Weeks pay.

4 weeks pay =

=

17 17½% of 4 weeks pay

L. Loading =

=

18 Fei Yen's holiday pay.

H. Pay =

=

Alena earns commission at 14% of her weekly sales. Find:

19 Her weekly commission, given she has average weekly sales of $12 800.

Commission =

Use commission rate here - not 17.5%

=

20 4 Weeks pay.

4 weeks pay =

=

21 17½% of 4 weeks pay

L. Loading =

=

22 Alena's holiday pay.

H. Pay =

=

Earning Money - Leave Loading (Sheet 1)

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Convert these wages to 4 weeks pay.

Example: $1 365.20/week

4 weeks Pay = 1 365.20 × 4

= $5 460.80

1 $623.17/week

4 weeks Pay = 623.17 × 4

= $ 2 492.68

2 $435.76/week

4 weeks Pay = 435.76 × 4

= $ 1 743.04

3 $1 738.15/week

4 weeks Pay = 1 738.15 × 4

= $6 952.60

Show the $ sign and cents to 2 decimal places

4 $1 674.83/week

4 weeks Pay = 1 674.83 × 4

= $6 699.32

To find 4 weeks pay, multiply the fortnightly rate by 2 (So, × 2)

5 $2 560.77/fortnight

4 weeks Pay = 2 560.77 × 2

= $5 121.54

6 $1 363.45/fortnight

4 weeks Pay = 1 363.45 × 2

= $2 726.90

7 $425.65/fortnight

4 weeks Pay = 425.65 × 2

= $851.30

Convert an annual wage to 4 weeks pay (÷ 52 × 4).

8 $62 774.40 p.a.

4 weeks pay = 62 774.40 ÷ 52 × 4

= $4 828.80

9 $28 605.20 p.a.

4 weeks pay = 28 605.20 ÷ 52 × 4

= $2 200.40

10 $45 705.63 p.a.

4 weeks pay = 45 705.63 ÷ 52 × 4

= $3 515.62 ← Round to 2 d.p.

11 $94 771 p.a.

4 weeks pay = 94 771 ÷ 52 × 4

= $7 290.08

12 $59 883 p.a.

4 weeks pay = 59 883 ÷ 52 × 4

= $4 606.38

Convert a monthly wage to 4 weeks pay (× 12 ÷ 52 × 4).

13 $4 680 per month.

4 weeks pay = 4 680 × 12 ÷ 52 × 4

= $4 320

14 Is your answer above just less than the $4 680 per month?

● Yes, because 4 weeks while close, is slightly less than a month.

○ No, I think I have to do it again.

15 $3 302 per month.

4 weeks pay = 3 302 × 12 ÷ 52 × 4

= $3 048

16 $1 102.40 per month.

4 weeks pay = 1 102.40 × 12 ÷ 52 × 4

= $1 017.60

17 $4 435 per month.

4 weeks pay = 4 435 × 12 ÷ 52 × 4

= $4 093.85 ← Round to 2 d.p.

Change the daily hours and rate to weekly pay to a 4 weekly pay.

Daily hours × Hourly rate × Days = Weekly Pay

	Name	*Daily Hours*	*Hourly Rate*	*Days per Week*	*Weekly Pay*	*4 Weeks Pay*
18	E. Claire	7	$24.85	5	$869.75	$3 479
19	I. Sore	6	$19.40	6	$698.40	$2 793.60
20	D. Light	5½	$25.60	4	$563.20	$2 252.80
21	C. Saw	7	$15.40	5	$539	$2 156
22	G. Whizz	8	$32.80	5	$1 312	$5 248
23	T. Time	6½	$38.90	3	$758.55	$3 034.20
24	X. Tract	4	$28.90	2	$231.20	$924.80

Earning Money - Leave Loading (Sheet 2)

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Leave loading is 17½%. So multiply by 17.5 'over 100' to find the leave loading.

Example: Find 17½% of $560

L. Loading = $560 \times \frac{17.5}{100}$

= $98

1 Find 17½% of $2 186

L. Loading = 2 186 × $\frac{17.5}{100}$

= $382.55

2 Find 17½% of $778

L. Loading = 778 × $\frac{17.5}{100}$

= $136.15

3 Find 17½% of $1 412

L. Loading = 1 412 × $\frac{17.5}{100}$

= $247.10

4 Find 17½% of $895.70

L. Loading = 895.70 × $\frac{17.5}{100}$

= $156.75

5 Find 17½% of $1 033

L. Loading = 1 033 × $\frac{17.5}{100}$

= $180.78 ← Round to 2 d.p.

6 Find 17½% of $1 043.52

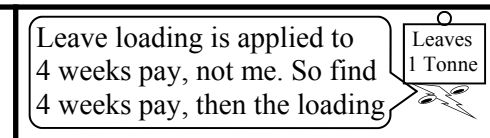
L. Loading = 1 043.52 × $\frac{17.5}{100}$

= $182.62

7 Find 17½% of $358.38

L. Loading = 358.38 × $\frac{17.5}{100}$

= $62.72



Leave loading is applied to 4 weeks pay, not me. So find 4 weeks pay, then the loading

Jodie earns $850 per week. Find:

8 4 Weeks pay.

4 weeks Pay = 850 × 4

= $ 3 400

9 17½% of 4 weeks pay

L. Loading = 3 400 (Q.8) × $\frac{17.5}{100}$

= $595

10 Jodie's holiday pay.

H. Pay = 3 400 (Q.8) + 595 (Q.9)

= $3 995

Iskandar earns $1 055 per week. Find:

11 4 Weeks pay.

4 weeks Pay = 1 055 × 4

= $ 4 220

12 17½% of 4 weeks pay

L. Loading = 4 220 (Q.11) × $\frac{17.5}{100}$

= $738.50

13 Iskandar's holiday pay.

H. Pay = 4 220 (Q.11) + 738.50 (Q.12)

= $4 958.50

Carla earns $777.70 per week. Find:

14 4 Weeks pay.

4 weeks Pay = 777.70 × 4

= $3 110.80

15 17½% of 4 weeks pay

L. Loading = 3 110.80 (Q.14) × $\frac{17.5}{100}$

= $544.39

16 Carla's holiday pay.

H. Pay = 3 110.80 + 544.39

= $3 655.19

Tegan earns $81 120 p.a. Find:

17 4 Weeks pay.

4 weeks pay = 81 120 ÷ 52 × 4

= $6 240

18 17½% of 4 weeks pay

L. Loading = 6 240 × $\frac{17.5}{100}$

= $1 092

19 Tegan's holiday pay.

H. Pay = 6 240 + 1 092

= $7 332

William earns $49 527.92 p.a. Find:

20 4 Weeks pay.

4 weeks pay = 49 527.92 ÷ 52 × 4

= $3 809.84

21 17½% of 4 weeks pay

L. Loading = 3 809.84 × $\frac{17.5}{100}$

= $666.72 2 d.p.

22 William's holiday pay.

H. Pay = 3 809.84 + 666.72

= $4 476.56

Earning Money - Leave Loading (Sheet 3)

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Find 4 weeks pay, the leave loading (at 17½%), then find the holiday pay.

William earns $80 080 p.a. Find:

1 4 Weeks pay.

$\text{4 weeks pay} = 80\,080 \div 52 \times 4$

$= \$6\,160$

2 17½% of 4 weeks pay

$\text{L. Loading} = 6\,160 \times \frac{17.5}{100}$

$= \$1\,078$

3 William's holiday pay.

$\text{H. Pay} = 6\,160 + 1\,078$

$= \$7\,238$

Jaycee earns $5 304/month. Find:

4 4 Weeks pay.

$\text{4 weeks pay} = 5\,304 \times 12 \div 52 \times 4$

$= \$4\,896$

5 17½% of 4 weeks pay

$\text{L. Loading} = 4\,896 \times \frac{17.5}{100}$

$= \$856.80$

6 Jaycee's holiday pay.

$\text{H. Pay} = 4\,896 + 856.80$

$= \$5\,752.80$

Nathan earns $19.30/h for a 12 hour week. Find:

7 4 Weeks pay.

$\text{4 weeks pay} = 19.30 \times 12 \times 4$

$= \$926.40$

8 17½% of 4 weeks pay

$\text{L. Loading} = 926.40 \times \frac{17.5}{100}$

$= \$162.12$

9 Nathan's holiday pay.

$\text{H. Pay} = 926.40 + 162.12$

$= \$1\,088.52$

Tony earns $2 444/month. Find:

10 4 Weeks pay.

$\text{4 weeks pay} = 2\,444 \times 12 \div 52 \times 4$

$= \$2\,256$

11 17½% of 4 weeks pay

$\text{L. Loading} = 2\,256 \times \frac{17.5}{100}$

$= \$394.80$

12 Tony's holiday pay.

$\text{H. Pay} = 2\,256 + 394.80$

$= \$2\,650.80$

Brad earns $76 492 p.a. Find:

13 4 Weeks pay.

$\text{4 weeks pay} = 76\,492 \div 52 \times 4$

$= \$5\,884$

14 17½% of 4 weeks pay

$\text{L. Loading} = 5\,884 \times \frac{17.5}{100}$

$= \$1\,029.70$

15 Brad's holiday pay.

$\text{H. Pay} = 5\,884 + 1\,029.70$

$= \$6\,913.70$

Fei Yen earns $24.90/h for a 38 hour week. Find:

16 4 Weeks pay.

$\text{4 weeks pay} = 24.90 \times 38 \times 4$

$= \$3\,784.80$

17 17½% of 4 weeks pay

$\text{L. Loading} = 3\,784.80 \times \frac{17.5}{100}$

$= \$662.34$

18 Fei Yen's holiday pay.

$\text{H. Pay} = 3\,784.80 + 662.34$

$= \$4\,447.14$

Alena earns commission at 14% of her weekly sales. Find:

19 Her weekly commission, given she has average weekly sales of $12 800.

$\text{Commission} = 12\,800 \times \frac{14}{100}$

Use commission rate here - not 17.5%

$= \$1\,792$

20 4 Weeks pay.

$\text{4 weeks pay} = 1\,792 \times 4$

$= \$7\,168$

21 17½% of 4 weeks pay

$\text{L. Loading} = 7\,168 \times \frac{17.5}{100}$

$= \$1\,254.40$

22 Alena's holiday pay.

$\text{H. Pay} = 7\,168 + 1\,254.40$

$= \$8\,422.40$

Earning Money - Leave Loading (Homework Strip)

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Date Due: _____

Calculate the Leave Loading
All wages shown are 4 weeks pay.

1 Find 17½% of $5 200

L. Loading = _____ × 17.5 / 100

= _____

2 Find 17½% of $3 180

L. Loading = _____ × _____ / 100

= _____

Now find the Holiday Pay

3 Find 17½% of $6 900

L. Loading = _____ × _____ / 100

= _____

H. Pay = 6 900 + _____

= _____

4 Find 17½% of $3 710

L. Loading = _____ × _____ / 100

= _____

H. Pay = _____ + _____

= _____

Date Due: _____

Calculate the Leave Loading
All wages shown are 4 weeks pay.

1 Find 17½% of $5 200

L. Loading = _____ × 17.5 / 100

= _____

2 Find 17½% of $3 180

L. Loading = _____ × _____ / 100

= _____

Now find the Holiday Pay

3 Find 17½% of $6 900

L. Loading = _____ × _____ / 100

= _____

H. Pay = 6 900 + _____

= _____

4 Find 17½% of $3 710

L. Loading = _____ × _____ / 100

= _____

H. Pay = _____ + _____

= _____

Date Due: _____

Calculate the Leave Loading
All wages shown are 4 weeks pay.

1 Find 17½% of $5 200

L. Loading = _____ × 17.5 / 100

= _____

2 Find 17½% of $3 180

L. Loading = _____ × _____ / 100

= _____

Now find the Holiday Pay

3 Find 17½% of $6 900

L. Loading = _____ × _____ / 100

= _____

H. Pay = 6 900 + _____

= _____

4 Find 17½% of $3 710

L. Loading = _____ × _____ / 100

= _____

H. Pay = _____ + _____

= _____

Date Due: _____

Calculate the Leave Loading
All wages shown are 4 weeks pay.

1 Find 17½% of $5 200

L. Loading = _____ × 17.5 / 100

= _____

2 Find 17½% of $3 180

L. Loading = _____ × _____ / 100

= _____

Now find the Holiday Pay

3 Find 17½% of $6 900

L. Loading = _____ × _____ / 100

= _____

H. Pay = 6 900 + _____

= _____

4 Find 17½% of $3 710

L. Loading = _____ × _____ / 100

= _____

H. Pay = _____ + _____

= _____

Earning Money - Leave Loading (Homework Strip)

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Date Due:

Calculate the Leave Loading
All wages shown are 4 weeks pay.

1 Find 17½% of $5 200

L. Loading = 5 200 × $\frac{17.5}{100}$

= $910

2 Find 17½% of $3 180

L. Loading = 3 180 × $\frac{17.5}{100}$

= $556.50

Now find the Holiday Pay

3 Find 17½% of $6 900

L. Loading = 6 900 × $\frac{17.5}{100}$

= $1 207.50

H. Pay = 6 900 + 1 207.50

= $8 107.50

4 Find 17½% of $3 710

L. Loading = 3 710 × $\frac{17.5}{100}$

= $649.25

H. Pay = 3 710 + 649.25

= $4 359.25

Earning Money - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Earning Money - Find a Word

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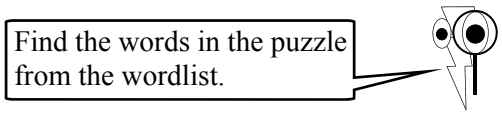

E	K	C	M	N	O	I	S	S	I	M	M	O	C	C
M	O	N	T	H	L	Y	Y	A	D	I	L	O	H	O
E	Q	T	H	G	I	N	T	R	O	F	H	L	S	H
M	O	V	E	R	T	I	M	E	N	U	U	O	A	H
P	W	X	L	A	U	T	C	N	U	P	P	H	L	B
L	R	S	S	E	T	A	R	I	G	G	K	B	A	A
O	E	H	E	T	S	T	R	A	T	S	R	B	R	S
Y	S	I	T	E	R	A	R	T	G	G	O	A	Y	P
E	L	F	L	T	V	S	E	E	G	A	W	S	Y	E
E	O	T	F	A	R	A	E	R	W	A	E	E	E	N
M	A	A	P	N	R	E	E	G	C	Y	C	E	N	A
P	D	A	G	N	V	E	V	L	V	N	E	O	O	L
E	I	N	I	U	H	S	I	N	I	F	I	Y	M	T
D	N	N	N	A	R	R	E	Y	O	L	P	M	E	Y
I	G	N	I	L	Y	S	T	N	E	C	H	O	U	R

WORDLIST

RATE	PENALTY	HOLIDAY	MONEY	EMPLOYER
WAGE	BASE	OVERTIME	FINISH	FORTNIGHT
HOUR	START	MONTHLY	RETAINER	CENTS
SHIFT	LOADING	COMMISSION	LEAVE	PAID
INCREASE	EMPLOYEE	EARNING	SALARY	ANNUAL
	PIECEWORK	CONVERT	PUNCTUAL	

Earning Money - Find a Word

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Find the words in the puzzle from the wordlist.



E	K	C	M	N	O	I	S	S	I	M	M	O	C	C
M	O	N	T	H	L	Y	Y	A	D	I	L	O	H	O
E	Q	T	H	G	I	N	T	R	O	F	H	L	S	H
M	O	V	E	R	T	I	M	E	N	U	U	O	A	H
P	W	X	L	A	U	T	C	N	U	P	P	H	L	B
L	R	S	S	E	T	A	R	I	G	G	K	B	A	A
O	E	H	E	T	S	T	R	A	T	S	R	B	R	S
Y	S	I	T	E	R	A	R	T	G	G	O	A	Y	P
E	L	F	L	T	V	S	E	E	G	A	W	S	Y	E
E	O	T	F	A	R	A	E	R	W	A	E	E	E	N
M	A	A	P	N	R	E	E	G	C	Y	C	E	N	A
P	D	A	G	N	V	E	V	L	V	N	E	O	O	L
E	I	N	I	U	H	S	I	N	I	F	I	Y	M	T
D	N	N	N	A	R	R	E	Y	O	L	P	M	E	Y
I	G	N	I	L	Y	S	T	N	E	C	H	O	U	R

WORDLIST

~~RATE~~
~~WAGE~~
~~HOUR~~
~~SHIFT~~
~~INCREASE~~
~~PENALTY~~
~~BASE~~
~~START~~
~~LOADING~~
~~EMPLOYEE~~
~~PIECEWORK~~
~~HOLIDAY~~
~~OVERTIME~~
~~MONTHLY~~
~~COMMISSION~~
~~EARNING~~
~~CONVERT~~
~~MONEY~~
~~FINISH~~
~~RETAINER~~
~~LEAVE~~
~~SALARY~~
~~PUNCTUAL~~
~~EMPLOYER~~
~~FORTNIGHT~~
~~CENTS~~
~~PAID~~
~~ANNUAL~~

Earning Money Review

This is the review of the Earning Money Topic.

Earning Money - Topic Review

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Calculate the amount earned.

1 $96/day, for 3 days.

2 $26/h, for 4 hours.

3 $795.20/week, for 2 weeks.

Calculate the weekly wage.

4 $25.45/h for a 38 h week.

5 $18.75/h for a 20 h week.

6 $135.65/day for a 3 day week.

Find the weekly wage earned.

7 $46 498.40 p.a.

8 $4 992/month.

Frances is paid $473.10 for a 38 hour week. Convert this rate of pay to:

11 An annual wage

12 A monthly wage

13 An hourly rate

Convert these wages to monthly

9 $61 034 p.a.

10 $775.20/week.

11 $24.90/h for a 20 h week.

Convert these normal hourly rates to rates paid at time-and-a-half

12 $12.50 _____

13 $18.60 _____

Convert these normal hourly rates to rates paid at double time

14 $13.55 _____

15 $9.95 _____

Calculate the penalty rate payments.

16 6 h at time-and-a-half.
(Normal hourly rate is $18.40 per hour.)

17 4 h at double time.
(Normal hourly rate is $23.70 per hour.)

18 3 h at time-and-a-half and 1½ h at double time.
(Normal hourly rate is $21.20 per hour.)

19 **Complete the table using a normal hourly rate of $16.50/hour.**

Name	*Normal Hours*	*Time and a half*	*Double Time*	*Total*
Jane	35	4	-	
Tina	17	7	4	
Brad	38	3	1	
David	25	8	2	
Mel	30	6½	2½	

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James earns $584 per week. If he receives a pay increase of 7.5%, find:

20 The increase in pay

21 His new annual salary

Jaegar is paid 15% commission on sales. Find the commission paid on:

22 Sales of $4 670

Kieran is paid a base salary of $760 per week, as well as 4% commission on sales. Find his earnings if he makes sales of $38 500 for the week.

23 Sales of $38 500

Ahmad earns fits smoke alarms and fire extinguishers. He is paid $9 per smoke alarm and $26 per fire Extinguisher. Find his earnings when he fits the following:

24 6 smoke alarms, and
4 fire extinguishers

25 11 smoke alarms, and
7 fire extinguishers

Husna applies silkscreen artwork to T-shirts. She is paid $16 per shirt, for up to 20 shirts. The rate increases to $19 per shirt on those shirts she produces after the first 20. Her supply figures are in the table below, calculate her earnings for the week.

26

Day	M	T	W	T	F
Quantity	15	20	28	11	32

27 Complete the table

Hourly rate is $20.40 per hour.
Round time worked to nearest 15 minutes.
Lunch times are unpaid.

Day	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	8.20	2.25	30 m			
Tues	8.35	4.10	45 m			

Izolde earns $850.90 per week. She is about to go on 4 weeks holiday. Calculate the following:

28 4 Weeks pay.

29 Leave Loading at 17½% of 4 weeks pay.

30 Izolde's holiday pay.

Carmen earns $70 418.40 p. a. She is about to go on 4 weeks holiday. Calculate the following:

31 4 Weeks pay.

32 Leave Loading at 17½% of 4 weeks pay.

33 Carmen's holiday pay.

Earning Money - Topic Review

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Calculate the amount earned.

1 $96/day, for 3 days.

Pay = 96 × 3

Pay = $288

2 $26/h, for 4 hours.

Pay = 26 × 4

Pay = $104

3 $795.20/week, for 2 weeks.

Pay = 795.20 × 2

Pay = $1 590.40

Calculate the weekly wage.

4 $25.45/h for a 38 h week.

Pay = 25.45 × 38

Pay = $967.10

5 $18.75/h for a 20 h week.

Pay = 18.75 × 20

Pay = $375

6 $135.65/day for a 3 day week.

Pay = 135.65 × 3

Pay = $406.95

Find the weekly wage earned.

7 $46 498.40 p.a.

Weekly = 46 498.40 ÷ 52

Weekly = $894.20

8 $4 992/month.

Weekly = 4 992 × 12 ÷ 52

Weekly = $1 152

Frances is paid $473.10 for a 38 hour week. Convert this rate of pay to:

11 An annual wage

Pay = 473.10 × 52

Pay = $24 601.20

12 A monthly wage

Pay = 24 601.20 ÷ 12

Pay = $2 050.10

13 An hourly rate

Rate = 473.10 ÷ 38

Rate = $12.45/hour

Convert these wages to monthly

9 $61 034 p.a.

Pay = 61 034 ÷ 12

Pay = $5 086.17

10 $775.20/week.

Pay = 775.20 × 52 ÷ 12

Pay = $3 359.20

11 $24.90/h for a 20 h week.

Pay = 24.90 × 20 × 52 ÷ 12

Pay = $2 158

Convert these normal hourly rates to rates paid at time-and-a-half

12 $12.50 — $18.75

13 $18.60 — $27.90

Convert these normal hourly rates to rates paid at double time

14 $13.55 — $27.10

15 $9.95 — $19.90

Calculate the penalty rate payments.

16 6 h at time-and-a-half.

(Normal hourly rate is $18.40 per hour.)

Pay = 6 × 1.5 × 18.40

Pay = $165.60

17 4 h at double time.

(Normal hourly rate is $23.70 per hour.)

Pay = 4 × 2 × 23.70

Pay = $189.60

18 3 h at time-and-a-half and 1½ h at double time.

(Normal hourly rate is $21.20 per hour.)

Pay = 3 × 1.5 × 21.20

+ 1.5 × 2 × 21.20

Pay = $159

19 Complete the table using a normal hourly rate of $16.50/hour.

Name	*Normal Hours*	*Time and a half*	*Double Time*	*Total*
Jane	35 577.50	4 99	- 0	$676.50
Tina	17 280.50	7 173.25	4 132	$585.75
Brad	38 627	3 74.25	1 33	$734.25
David	25 412.50	8 198	2 66	$676.50
Mel	30 495	6½ 160.88	2½ 82.50	$738.38

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James earns $584 per week. If he receives a pay increase of 7.5%, find:

20 The increase in pay

$\text{Increase} = 584 \times \frac{7.5}{100}$

$\text{Increase} = \$43.80$

21 His new annual salary

$\text{Weekly} = 584 + 43.80$

$\text{Weekly} = \$627.80$

$\text{Annual} = 627.80 \times 52$

$\text{Annual} = \$32\,645.60$

Jaegar is paid 15% commission on sales. Find the commission paid on:

22 Sales of $4 670

$\text{Commission} = 4\,670 \times \frac{15}{100}$

$\text{Commission} = \$700.50$

Kieran is paid a base salary of $760 per week, as well as 4% commission on sales. Find his earnings if he makes sales of $38 500 for the week.

23 Sales of $38 500

$\text{Earnings} = 760 + 38\,500 \times \frac{4}{100}$

$\text{Earnings} = \$2\,300$

Ahmad earns fits smoke alarms and fire extinguishers. He is paid $9 per smoke alarm and $26 per fire Extinguisher. Find his earnings when he fits the following:

24 6 smoke alarms, and 4 fire extinguishers

$\text{Earnings} = 6 \times 9 + 4 \times 26$

$\text{Earnings} = \$158$

25 11 smoke alarms, and 7 fire extinguishers

$\text{Earnings} = 11 \times 9 + 7 \times 26$

$\text{Earnings} = \$281$

Husna applies silkscreen artwork to T-shirts. She is paid $16 per shirt, for up to 20 shirts. The rate increases to $19 per shirt on those shirts she produces after the first 20. Her supply figures are in the table below, calculate her earnings for the week.

26

Day	M	T	W	T	F	
Quantity	15	20	28	11	32	
$16:	15	20	20	11	20	86
$19:	-	-	8	-	12	20

$\text{Earnings} = 86 \times 16 + 20 \times 19$

$\text{Earnings} = \$1\,756$

Izolde earns $850.90 per week. She is about to go on 4 weeks holiday. Calculate the following:

28 4 Weeks pay.

$\text{4 weeks pay} = 850.90 \times 4$

$\text{4 weeks pay} = \$3\,403.60$

29 Leave Loading at 17½% of 4 weeks pay.

$\text{L.Loading} = 3\,403.60 \times \frac{17.5}{100}$

$\text{L.Loading} = \$595.63$

30 Izolde's holiday pay.

$\text{H. Pay} = 3\,403.60 + 595.63$

$\text{H. Pay} = \$3\,999.23$

Carmen earns $70 418.40 p. a. She is about to go on 4 weeks holiday. Calculate the following:

31 4 Weeks pay.

$\text{4 weeks Pay} = 70\,418.40 \div 52 \times 4$

$\text{4 weeks Pay} = \$5\,416.80$

32 Leave Loading at 17½% of 4 weeks pay.

$\text{L.Loading} = 5\,416.80 \times \frac{17.5}{100}$

$\text{L.Loading} = \$947.94$

33 Carmen's holiday pay.

$\text{H. Pay} = 5\,416.80 + 947.94$

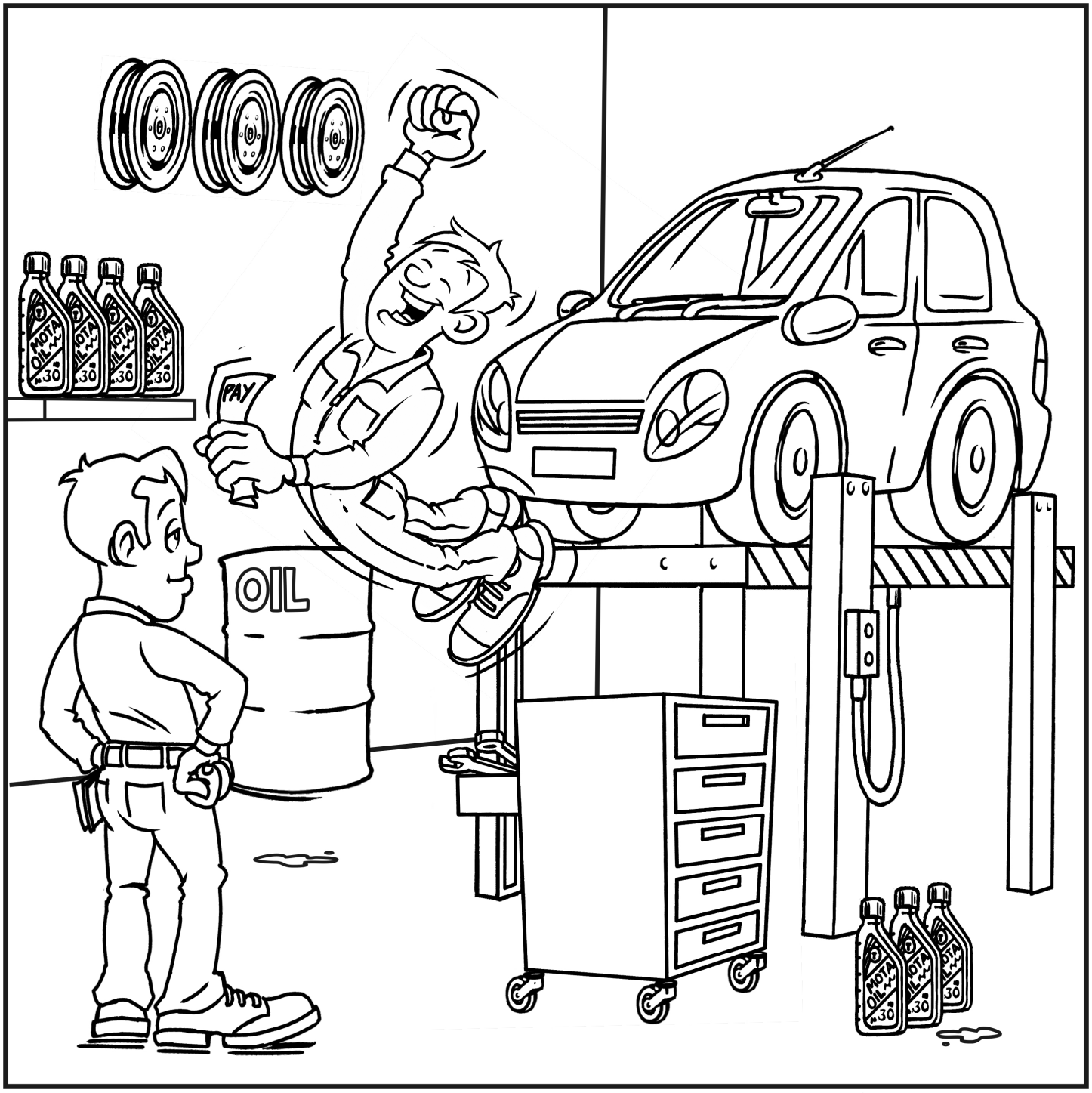
$\text{H. Pay} = \$6\,364.74$

27 Complete the table

Hourly rate is $20.40 per hour.
Round time worked to nearest 15 minutes.
Lunch times are unpaid.

Day	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	8.20 0820	2.25 1425	30 m	5h 35m	5h 30m	$112.20
Tues	8.35 0835	4.10 1610	45 m	6h 50m	6h 45m	$137.70

Earning Money



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Earning Money - Hourly Rates (Sheet 1)

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To convert an hourly rate to a weekly wage, multiply the rate by the hours.

Example: Find the weekly pay on $23/h for a 38 hour week.

$23 \times 38 = \$874$

1 $19/h for a 27 hour week.

Weekly Pay = ☐ × ☐

Weekly Pay = $ ☐☐☐

2 $24/h for a 32 hour week.

Weekly Pay = ☐ × ☐

Weekly Pay = $ ☐☐☐

3 $17.17/h for a 41 hour week

Weekly Pay = ☐ × ☐

Weekly Pay = $ ☐☐☐.☐☐

4 $13.95/h for an 8 hour week

Weekly Pay = ☐ × ☐

Weekly Pay = $ ☐☐☐.☐☐

5 $16.80/h for a 18 hour week

Weekly Pay =

Weekly Pay =

Show the $ sign and cents to 2 decimal places

6 $14.45/h for a 26 hour week

Weekly Pay =

Weekly Pay =

Show the $ sign and cents to 2 decimal places

7 $38.17/h for a 38 hour week

Weekly Pay =

Weekly Pay =

Show the $ sign and cents to 2 decimal places

Find the wages, then add for the total wages paid.

8

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Ted	38	$15.25	
Fred	32	$18.32	
Ned	35	$16.46	
		Total Paid	

9

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Bill	26	$17.45	
Phil	38	$19.26	
Jill	38	$22.90	
		Total Paid	

10

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Anne	42	$28.75	
Stan	25	$27.40	
Jan	38.5	$46.82	
		Total Paid	

11

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Pay*
Rick	11.5	$21.20	
Mick	9.25	$38.16	
Nick	6.75	$26.32	
		Total Paid	

For the table in Question 8:

12 Who is paid more, Ned or Ted? ☐

13 Find the range of the pays.

Range = ☐ - ☐

Range = ☐

Largest amount minus (-) the Lowest amount

14 Find, to 1 d.p, Fred's pay as a percentage of the total pay.

$$\text{Fred \%} = \frac{\text{Fred's pay}}{\text{Total pay}} \times 100$$

Fred's pay ÷ Total Pay × 100

Fred % = ☐

Show the % sign and round to 1 decimal place

For the table in Question 10:

15 Who is paid more, Anne or Jan? ☐

16 Find the range of the pays.

Range = ☐ - ☐

Range = ☐

17 Find, to 1 d.p, Stan's pay as a percentage of the total pay.

$$\text{Stan \%} = \frac{\text{Stan's pay}}{\text{Total pay}} \times 100$$

Stan % = ☐

For the table in Question 11:

18 How much more than Nick, is Mick paid?

Earning Money - Hourly Rates (Sheet 2)

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Find the daily pay then the weekly pay given the hourly rate.



Example:

Rate: $19.35/h

Conditions: 7 h day, 5 day week

Daily Pay = 19.35 × 7

Daily Pay = $ 135.45

Weekly Pay = 135.45 × 5

Weekly Pay = $ 677.25

1 Rate: $18.45/h

Conditions: 4 h day, 3 day week

Daily Pay = ____ × ____

Daily Pay = $ ____

Show cents to 2 decimal places

Weekly Pay = ____ × ____

Weekly Pay = $ ____

2 Rate: $23.15/h

Conditions: 8 h day, 5 day week

Daily Pay = ____ × ____

Daily Pay = ____

Show the $ sign and cents to 2 decimal places

Weekly Pay = ____ × ____

Weekly Pay = ____

3 Rate: $19.06/h

Conditions: 5 h day, 4 day week

Daily Pay = ____ × ____

Daily Pay = ____

Weekly Pay = ____ × ____

Weekly Pay = ____

Given the hourly rate and hours worked, complete the tables.



Show the $ sign and cents to 2 decimal places

4

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Kane	3	$15.30		6	
Jane	5	$19.15		5	
Zane	4	$26.75		3	
		Total Daily		*Total Weekly*	

5

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Dawn	5	$17.55		5	
Shaun	7	$20.92		3	
Lorne	6	$16.88		4	
		Total Daily		*Total Weekly*	

Use decimals to show half hours, so for 7½ hours use 7.5 instead.

6

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Jo	7½	$17.60		4	
Flo	5	$38.42		6	
Beau	6½	$26.18		5	
		Total Daily		*Total Weekly*	

When you get answers with 3 decimal places, round cents to 2 decimal places

7

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*
Cam	3	$20.43		5	
Ram	8½	$29.47		3	
Sam	4½	$18.33		6	
Pam	5½	$26.51		4	
		Total Daily		*Total Weekly*	

Earning Money - Hourly Rates (Sheet 3)

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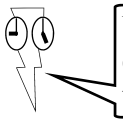

Use the hourly rate to calculate the pay for these employees.

1 Rami earns $28.20 per hour. He works 7 hours one day, and 5 hours the next. Find:

a) Total hours worked

Hours = ☐ + ☐ = ☐ h

b) Total pay earned

Pay = ☐ × ☐

Pay = ☐

Show the $ sign and cents to 2 decimal places

2 Jess earns $37.35 per hour. She works 8 hours per day, and 4 days each week. Find:

a) Her daily pay

Pay =

Pay =

b) Her weekly pay

Pay =

Pay =

3 James earns $21.90/h. If he works a 38 hour week and pays $230/wk rent and has $275/wk in living expenses. Find:

a) His weekly pay

Pay =

Pay =

b) His savings for the week.

Savings =

Savings =

4 Ellen earns $43.85/h while Ben earns $27.90/h. If both work a 38 hour week, find:

a) Ellen's weekly pay

b) Ben's weekly pay

c) The difference between Ellen's and Ben's weekly pay.

5 Travis earns $28.63/h for a 16 h week, while Lewis earns $31.55/h for a 19 h week, find:

a) Travis' weekly pay

b) Lewis' weekly pay

c) The difference between Travis' and Lewis' weekly pay.

d) Express Travis' pay as a percentage of Lewis' pay. (1 dp)

$$\text{Travis \%} = \frac{\text{Travis' pay}}{\text{Lewis' pay}} \times 100$$

Travis % = ☐ %

6 Kelly earns $33.15/h, Toby earns $29.16/h and Ahmed earns $30.25/h. If all work a 38 hour week, find:

a) Kelly's weekly pay

b) Toby's weekly pay

c) Ahmed's weekly pay

d) The average wage earned.

$$\text{Average Wage} = \frac{\square}{3} \leftarrow \text{Total Wages}$$

Add the 3 wages, press '=', then press '÷3'.

Average Wage = ☐

7 Tye earns $23.45/h for a 16 h week, while Kwan earns $22.85/h for a 38 h week, find:

a) Tye's weekly pay

b) Kwan's weekly pay

c) Express Tye's pay as a percentage of Kwan's pay. (1dp)

$$\text{Tye's \%} = \frac{\square}{\square} \times 100$$

Tye's % = ☐

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 1)

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Convert an hourly rate to a fortnightly wage. Multiply the rate by the 2 × weekly hours.

Example: Find the fortnightly pay on $21/h for a 38 hour week.

F'tnightly Pay = 38 × 2 × 21

Hours × 2 × Hourly rate

F'tnightly Pay = $1 596

1 $17/h for a 34 hour week.

F'tnightly Pay = ____ × 2 × ____

F'tnightly Pay = $ ____

2 $18.35 /h for a 34 h week.

F'tn Pay =

F'tn Pay =

3 $23.72 /h for an 18 h week.

F'tn Pay =

F'tn Pay =

4 $35.67 /h for a 38 h week.

5 $29.55 /h for a 25 h week.

6

Show the $ sign and cents to 2 decimal places

Name	*Weekly Hours*	*Hourly Rate*	*Ftn'ly Pay*
Zoe	26	$17.45	
Chloe	25	$19.26	
Joey	16	$22.90	
		Total Paid	

Find the hourly rate by dividing the pay by the hours worked.

Example: Find the hourly rate if paid $790 for a 25 hour week.

Hourly rate = 790 ÷ 25

Pay ÷ hours

Hourly rate = $31.60 / h

7 $456.30 for a 26 hour week.

Hourly rate = ____ ÷ ____

Hourly rate = $ ____ / h

8 $919.60 for a 38 hour week.

Hourly rate =

Hourly rate = $ ____ / h

Show the $ sign and cents to 2 decimal places

9 $340 for a 16 hour week.

Hourly rate =

Hourly rate =

10 $1 512 for a 35 hour week.

11 $162 for an 8 hour week.

Find the hourly rate, then calculate the mean hourly rate.

12

Name	*Weekly Wage*	*Weekly Hours*	*Hourly rate*
Ben	$427.20	16	
Jen	$1 232	35	
Len	$883.40	28	
		Total	

13 Find the mean hourly rate.

Mean Hourly Rate = Total / 3

Total ÷ Number of employees

= ____ / h

14

Name	*Weekly Wage*	*Weekly Hours*	*Hourly rate*
Milly	$787.20	32	
Billy	$207.99	9	
Willy	$646.40	16	
Tilly	$339.48	12	

15 Mean Hourly Rate = Total / ____

= ____

16

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Speedy	32		$764.80	
Pablo		$28.60		$2 173.60
José	38	$21.42		
Manuel	28			$1 822.80

Complete the table for wages at the Ándele! Ándele! Epa! Epa! Café.

Hint! Convert fortnightly wages to weekly by ÷ 2.

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 2)

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Given an hourly rate, or weekly wage, or fortnightly wage, find these wages/rates/hours worked.

1 Jim earns $38.15 / h for a 38 hour week. Find his:

Weekly Pay _____

Multiply the hourly rate by hours worked

_____

F'nightly Pay _____

Hourly rate × hours × 2, **or** weekly pay × 2

_____

2 Kim earns $43.82 / h for a 40 hour week. Find her:

Weekly Pay _____

_____

F'nightly Pay _____

_____

3 Tim earns $1 061.34 for a 38 hour week. Find his:

Hourly Rate _____

_____

F'nightly Pay _____

_____

4 Lim earns $1 408 / f'night working a 25 hour week. Find:

Weekly Pay _____

_____

Hourly Rate _____

_____

5 Tom earns $21.96 / h earning $746.64 / week. Find:

Hours Worked _____

_____

F'nightly Pay _____

_____

6 Dom earns $25.65 / h for a 15 hour week. Find his:

Weekly Pay _____

_____

F'nightly Pay _____

_____

7 Jess earns $1 634.36/f'night She is paid $31.43 / h. Find her:

Weekly Pay _____

_____

Weekly Hours Worked _____

Weekly pay ÷ Hourly rate

_____

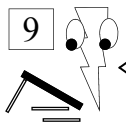
8 Tess earns $20.72 / h for a 56 hour f'tnight. Find her:

F'nightly Pay _____

_____

Weekly Hours Worked _____

_____



9 Complete these tables and then find the mean (average) hours worked and mean weekly pay for the employees listed.

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Ralph	35	$23.22		
Joanie		$27.65	$1 050.70	
Potsie	12			$476.40
Ritchie		$23.88	$644.76	
Total		*Total*		

Mean Weekly Hours = Total hours worked ÷ Number of employees

= _____ / week

Mean Weekly Pay = Total weekly wage ÷ _____

= _____ / week

10

Name	*Weekly Hours*	*Hourly Rate*	*Weekly Wage*	*Fortnightly Wage*
Legolas		$28.20	$592.20	
Aragon	22	$36.15		
Gimli	11			$620.40
Total		*Total*		

Mean Weekly Hours = _____ ÷ _____

= _____ / week

Mean Weekly Pay = _____ ÷ _____

= _____ / week

Calculating Hourly Rates and Weekly/Fortnightly Pay Periods (Sheet 3)

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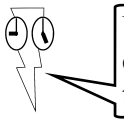

Use the hourly rate to calculate the pay for these employees.

1 Tyrone works 17 hours a week and earns $328.10, find:

a) His hourly rate.

Rate = ☐ ÷ ☐

Rate = ☐ /h

Write '$' sign and show money to 2 d.p.

b) His fortnightly pay.

Pay = ☐ × ☐

Pay = ☐

2 Thien earns $24.95 per hour. If he earned $798.40 for the week, calculate:

a) The hours worked this week

Hours =

Hours =

b) His fortnightly pay.

Pay =

Pay =

3 Bassam earns $27.20 per hour. If he works 38 h per week, calculate:

a) His weekly pay

Pay =

Pay =

b) His fortnightly pay.

Pay =

Pay =

3 Jemma earns $21.40/h and has recorded her hours below.

Day	M	T	W	T	F	Total
Hours	4	7	7	4	8	

a) Complete the total column Finding the total hours worked.

b) Find Jemma's weekly pay.

Pay =

Pay =

4 Simon earns $21.45/h and has a weekly pay of $729.30

a) Calculate the total hours Simon has worked for the week.

Hours =

Hours =

b) Complete the table below showing Simon's daily hours.

Day	M	T	W	T	F	Total
Hours	8	4		7	8	

5 Chloe earns $19.55/h and has a weekly pay of $625.60

a) Calculate the hours Chloe has worked for the week.

Hours =

Hours =

b) Complete the table below showing Chloe's daily hours.

Day	M	T	W	T	F	Total
Hours	7	8	6		8	

c) Find her Thursday earnings.

Pay =

Pay =

6 Hollie earns $23.30/h. If she works 25½ h each week, find:

a) Her weekly pay

Pay =

Pay =

b) Her fortnightly pay.

Pay =

Pay =

7 Faith earns $38.80 per hour. If she earned $2 754.80 for the fortnight, calculate her:

a) Fortnightly hours worked

Hours =

Hours =

b) Her weekly pay.

Pay =

Pay =

8 Georgia earns $84.15 when she works a 5½ h day. How much does she earn for a 38 hour week?

9 Robert earns $21.20/h and is paid $805.60 weekly. How many hours does he work per fortnight?

Pay Period Calculations - Annual Wages (Sheet 1)

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Convert a weekly wage to an annual wage (× by 52).

1 $358 weekly.

Wage p.a. = ☐ × 52

= $ ☐

2 $777 weekly.

Wage p.a. =

=

3 $443.58 weekly.

4 $1 952.37 weekly.

Convert a daily wage to an annual wage, (× days × 52).

5 $147 / day, 3 days per week.

Wage p.a. = ☐ × 3 × 52

= $ ☐

6 $165.20 / day, 2 days / week

Wage p.a. =

=

7 $207.45 / day, 4 days / week

8 $192.72 / day, 3 days / week

Convert an hourly rate to an annual wage (× hours × 52).

9 $24.20 / h, 38 hour week.

Wage p.a. = ☐ × 38 × 52

= $ ☐

10 $17.65 / h, 14 hour week.

Wage p.a. =

=

11 $32.75 / h, 25 hour week.

12 $20.42 / h, 38 hour week.

Convert a fortnightly wage to an annual wage, either (× 26) or you can (÷ 2 × 52).

13 $858 fortnightly.

Wage p.a. = ☐ ÷ 2 × 52

= $ ☐

14 $2 785 fortnightly.

15 $1 877 fortnightly.

16 $1 032 fortnightly.

Complete the tables and then calculate the total annual wages paid.

17

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*	*Annual Pay*
Jen	3	$25.65		2		
Len	5	$32.50		6		
Ben	8	$19.72		5		
Ken	6½	$38.12		4		
				Total Annual Wages		

18

Name	*Daily Hours*	*Hourly Rate*	*Daily Pay*	*Days per Week*	*Weekly Pay*	*Annual Pay*
Clay	6	$19.17		3		
May	7	$27.10		5		
Faye	5	$22.78		2		
				Total Annual Wages		

Pay Period Calculations - Annual Wages (Sheet 2)

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Convert an annual wage to a weekly wage (÷ by 52).

1 $46 644 p.a.

W'kly pay = ☐ ÷ 52

= $ ☐

2 $68 484 p.a.

W'kly pay =

=

3 $51 090 p.a.

Show the $ sign and cents to 2 decimal places

4 $73 655.40 p.a.

5 $26 355.68 p.a.

6 $7 587.84 p.a.

Questions 7 - 9 require you to round the cents.

7 $44 901.22 p.a.

8 $28 700 p.a.

9 $73 189.74 p.a.

Convert an annual wage to an hourly rate (÷ 52 ÷ hours).

10 $41 496 p.a., 38 hour week.

H'rly Rate= ☐ ÷ 52 ÷ 38

= $ ☐ / h

11 $38 532 p.a., 38 hour week.

H'rly Rate=

=

Show $ sign, cents to 2 d.p., and '/ h' at end.

12 $51 251.20 p.a., 28 hr week.

13 $21 996 p.a., 15 hour week.

Convert an annual wage to a fortnightly wage, (÷ 26) or you can (÷ 52 × 2).

14 $75 400 p.a.

Wage p.a. = ☐ ÷ 52 × 2

= $ ☐

15 $35 734.40 p.a.

16 $88 725 p.a.

17 $47 120.84 p.a.

Complete the tables and then calculate the average fortnightly pay.

18

Name	*Annual Wage*	*Weekly Wage*	*Fortnightly Wage*	*Fortnightly Hours*	*Hourly Rate*
Don	$41 693.60			38	
Bon	$157 170			78	
Con	$15 741.44			22	
Ron	$20 217.60			48	

Add the fortnight wages, press '=', then divide by 4. (the number of employees). ☐ ← *Average fortnightly wage* (2 d.p.)

19

Name	*Annual Wage*	*Fortnightly Wage*	*Weekly Wage*	*Weekly hours*	*Hourly Rate*
Edward	$39 203.84			32	
Bella	$57 205.20			38	
Jacob	$75 379.20			40	

☐ ← *Average fortnightly wage* (2 d.p.)

Pay Period Calculations - Annual Wages (Sheet 3)

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Convert to an annual wage by multiplying by 52. (× 52).

1 Brianna earns $16.65/h, for 15 hours/week part-time work. How much does she earn in:

a) A week

Pay = [ ] × [ ]

Pay = [ ]

Show the $ sign and cents to 2 decimal places

b) A year

Pay = [ ] × [ ]

Pay = [ ]

2 Dave works a 38 hour week earning $32.90/h. Find:

a) His weekly pay

Pay =

Pay =

b) His annual wage

Pay =

Pay =

3 Eliza earns $1 271.67/week. She is offered a wage increase of $138 per week. Calculate:

a) Her current annual salary

Pay =

Pay =

b) Her increased annual salary

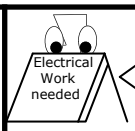
Pay =

Pay =

c) Find the annual increase

Increase =

Increase =



Look at the ads below and compare the wages.

Forklift Driver
Pay: $1 095/ week
Hours: 38 h/week
Apply within

Backhoe Operator
Pay: $28.27/h
Hours: 38 h/week
Call Us!!

4 Convert the Forklift job to an annual wage.

5 Convert the Backhoe job to an annual wage.

6 Calculate the annual pay difference between the jobs.

HR Specialist
Pay: $1 390/ week
Hours: 40 h/week
Ask inside

Recruitment Officer
Pay: $38.40/h
Hours: 36 h/week
Great Job!!

7 Convert the HR weekly wage to an annual wage.

8 Convert the Recruitment hourly rate to an annual wage.

9 Which job would you take?

Courier
Pay: $83 993 p.a.
Hours: 38 h/week
Must know Area

Delivery Driver
Pay: $44.10/h
Hours: 38 h/week
Licence Essential

10 Convert the courier job to a weekly wage.

11 Convert the Delivery Driver job to a weekly wage.

12 Calculate the weekly pay difference between the jobs.

Shop Assistant
Pay: $1 097.50
per fortnight
Hours: 50 h/f'tnight

Customer Service
Pay: $23.54/h
Hours: 25 h/week
Apply within

13 Convert the shop assistant job to a weekly wage.

14 Convert the Customer Service job to a weekly wage.

15 Calculate the weekly pay difference between the two jobs.

Earning Money - Monthly Wages (Sheet 1)

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Convert a monthly wage to an annual wage, (× by 12).

1 $5 126 monthly.

Wage p.a. = ☐ × 12

= $ ☐

2 $3 556 monthly.

Wage p.a. =

=

3 $7 091 monthly.

4 $6 225.23 monthly.

Now the reverse. Convert an annual wage to a monthly wage, by division. (÷ by 12).

5 $97 286.40 p.a.

Monthly = ☐ ÷ 12

= $ ☐

6 $50 190 p.a.

Monthly =

=

Show the $ sign and cents to 2 decimal places

7 $71 952.60 p.a.

8 $88 528 p.a.

Round cents to 2 decimal places

Convert a monthly wage to a weekly wage. Get yearly first then weekly. (×12 ÷ 52).

This column has been designated a '÷ 4'free zone. Use of '÷ 4' in these questions is banned!

9 $2 937.60 monthly.

Weekly = ☐ × 12 ÷ 52

= $ ☐

10 $8 165.30 monthly.

Weekly = ☐ × 12 ÷ 52

= $ ☐

11 $4 006.08 monthly.

Weekly =

12 $2 817.75 monthly.

Weekly =

These require you to round.

13 $5 600 monthly.

Weekly =

14 $7 128 monthly.

Weekly =

15 $1 921 monthly.

Weekly =

16 $11 877 monthly.

Weekly =

17 $3 333 monthly.

Weekly =

Change the monthly wages (left) to annual wages, if there's no match on right change them to weekly wages (annual÷52) and match with weekly wages.

18	$3 200 monthly	A	$21 000 p.a.
19	$1 750 monthly	B	$58 002 p.a.
20	$4 412.20 monthly	C	$3 859.62 weekly
21	$4 833.50 monthly	D	$738.46 weekly
22	$16 725 monthly	E	$44 550 p.a.
23	$3 712.50 monthly	F	$1 018.20 weekly

Earning Money - Monthly Wages (Sheet 2)

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Convert a weekly wage to a monthly wage, (× by 52 ÷ 12).

1 $480 weekly.

M'thly pay = ☐ × 52 ÷ 12

= $ ☐

2 $510 weekly.

M'thly pay =

=

3 $624 weekly.

M'thly pay =

=

4 $1 123.20 weekly.

M'thly pay =

=

Write your answer to 2 decimal places.

5 $842.40 weekly.

M'thly pay =

=

These have to be rounded.

6 $712.22 weekly.

M'thly pay =

=

7 $295.70 weekly.

M'thly pay =

=

8 $356.84 weekly.

M'thly pay =

=

Given the weekly hours worked, convert an hourly rate to a monthly wage.
(hourly rate × hours × 52 ÷ 12)

9 $24/h for a 16 hour week.

Monthly Pay = ☐ × ☐ × 52 ÷ 12

= $ ☐

10 $18/h for a 35 hour week.

Monthly Pay = ☐ × ☐ × 52 ÷ 12

= $ ☐

11 $19/h for a 38 hour week.

Monthly Pay =

= $ ☐

Write your answer to 2 decimal places.

12 $17/h for an 8 hour week.

Monthly Pay =

= $ ☐

Reverse it! Given the monthly wage find the hourly rate.
(monthly wage × 12 ÷ 52 ÷ hrs)

13 $3 250/mth at 25 hours/wk.

Hourly Rate = ☐ × 12 ÷ 52 ÷ 25

= $ ☐ /h

14 $2 173.60/mth at 38 hrs/wk.

Hourly Rate =

= $ ☐ /h

Write your answer to 2 decimal places.

15 $2 372.50/mth at 30 hrs/wk.

Hourly Rate =

= $ ☐

16 $7 657/mth at 38 hrs/wk.

Hourly Rate =

= $ ☐

Convert all wages to annual wages then rewrite the wages in ascending order. Rewrite the actual wages, not the annual wages.

17 $2 665/week, $11 650/mth, $57/h (38 h week), $5 888/f'tnight.

Annual Wages: ☐ ☐ ☐ ☐

Ascending Wages:

Write the wages from the question, not the equivalent annual wages.

18 $7 434/mth, $24.20/h (34 h week), $3 256 f'tnight, $1 798/week.

Annual Wages: ☐ ☐ ☐ ☐

Ascending Wages:

Write the wages from the question, not the equivalent annual wages.

19 $32.85/h (28 h week), $1 838/f'tnight, $935/week, $4 050/mth.

Annual Wages: ☐ ☐ ☐ ☐

Ascending Wages:

Write the wages from the question, not the equivalent annual wages.

Earning Money - Monthly Wages (Sheet 3)

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Answer these questions involving monthly wages.

1 James earns $32.96/h. If he works a 40 hour week, find:

a) His weekly wage

Pay = ☐ × ☐

Pay = ☐

Show the $ sign and cents to 2 decimal places

b) His annual wage

Pay = ☐ × ☐

Pay = ☐

c) His monthly wage

Pay = ☐ ÷ ☐

Pay = ☐

Round your answer to the nearest cent

2 Juanita earns $2 709.98 per month. Find her weekly pay.

Pay =

Pay =

3 If Juanita (Q.2) has living expenses of $575 per week, find:

a) Her weekly savings

Savings =

Savings =

b) Her monthly savings

Savings =

Savings =

4 Josh earns $40 260 p.a., calculate his monthly wage.

Pay =

Pay =

5 Kylie works as a computer programmer earning $92/hour. If she works a 38 hour week:

a) Calculate her weekly wage.

b) Find her monthly wage.

Compare these jobs by converting them to monthly pay periods.

Optical Assistant

Pay: $1 315/ week
See you inside!

Medical Technician

Pay: $6 350/mth
Watch the step inside!

6 Convert the Optical position wage to a monthly wage.

7 Which job pays the most?

○ Optical Assistant

○ Medical Technician

Kitchen hand

Pay: $22.60/h
Hours: 38h week

Baker's Assistant

Pay: $4 130/mth
No Loafers!

8 Convert the Kitchen hand wage to a monthly wage.

Pay = ☐ × ☐ × ☐ ÷ ☐

9 Calculate the difference between the two monthly wages

10 Jamie earns $3 776/month as salesperson and $220/week as a waiter in a second job.

a) Convert the waiter's wage to a monthly wage.

b) Find his total monthly pay.

11 Three flat-mates are paid in different pay periods. Ted earns $1 170/week, Fred earns $28.38/h for a 38 h week, while Ned earns $95 628 p.a.

a) Convert Ted's weekly wage to a monthly wage.

b) Convert Fred's hourly wage to a monthly wage.

c) Convert Ned's annual salary to a monthly salary.

d) Rank the flat-mates in order of their income.

1. ☐
   Write person's name with highest pay
2. ☐
3. ☐

Earning Money - Penalty Rates - Time-and-a-half (Sheet 1)

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Convert these hourly rates to an overtime rate of time-and-a-half, multiply by 1.5.

Example: Convert $11.60/h to a rate paid at time-and-a-half.

$11.60 = 11.60 × 1.5

Time+½ rate = $17.40/h

1 $14.90 = ☐ × 1.5

Time+½ rate = ☐ /h

Don't forget the '$' sign

2 $18.20 = ☐ × 1.5

Time+½ rate = ☐ /h

Always write cents with 2 decimal places

3 $35.40 = ☐ × ☐

Time+½ rate = ☐ /h

4 $22.10 = ☐ × ☐

Time+½ rate = ☐

5 $45.76 = ☐ × ☐

Time+½ rate = ☐

Now on you have to round to the nearest cent

6 $25.15 = ☐ × ☐

Time+½ rate = ☐

7 $17.05 = ☐ × ☐

Time+½ rate = ☐

8 $12.95 = ☐ × ☐

Time+½ rate = ☐

9 $48.19 = ☐ × ☐

Time+½ rate = ☐

10 $30.35 = ☐ × ☐

Time+½ rate = ☐

Find the overtime paid when given the hours worked.

Example:

Normal rate: $18.80/hour

Overtime Hours: 7

Pay = 18.80 × 1.5 × 7

Pay = $197.40

11 Normal Rate: $16.40/h
Overtime Hours: 4

Pay = ☐ × 1.5 × 4

Pay = ☐

12 Normal Rate: $15.88/h
Overtime Hours: 6

Pay = ☐ × ☐ × ☐

Pay = ☐

13 Normal Rate: $27.50/h
Overtime Hours: 8.5

Pay = ☐ × ☐ × ☐

Pay = ☐

You have to round these to the nearest cent

14 Normal Rate: $17.65/h
Overtime Hours: 2.5

Pay = ☐ × ☐ × ☐

Pay = ☐

15 Normal Rate: $11.99/h
Overtime Hours: 5

Pay = ☐ × ☐ × ☐

Pay = ☐

16 Normal Rate: $15.09/h
Overtime Hours: 1.5

Pay = ☐ × ☐ × ☐

Pay = ☐

17 Normal Rate: $37.37/h
Overtime Hours: 0.5

Pay = ☐ × ☐ × ☐

Pay = ☐

Complete these tables to calculate the daily pay for the employees.

18

	A	B	C = A×B	D	E = B×1.5	F = D×E	G = C+F
Name	*Normal Hours*	*Hourly Rate*	*Normal Pay*	*Time+½ Hours*	*Time+½ Rate*	*Overtime Pay*	*Total Pay*
Bill	8	$24	$192 8 × 24	2	$36 1.5 × 24	$72 2 × 36	$264 192 + 72
Jill	7	$28.20		0			
Lill	8	$19.20		3			
Dill	6	$12.80		2.5			
Phill	6	$15.75		0.5			
Gill	7.5	$19.80		1			
Will	5	$17.45		3.5			

Earning Money - Penalty Rates - Time-and-a-half (Sheet 2)

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Calculate the weekly pay, then the overtime (at time-and-a-half), then the total pay earned

1 Jane worked a 38 h week and also 6 h overtime. Her normal hourly rate is $23.10/h.

a) Calculate her standard pay

Pay = 38 × ☐

Pay = ☐ ← $ sign, cents to 2 d.p.

b) Calculate her overtime

Pay = ☐ × 1.5 × ☐

Pay = ☐

c) Calculate her total pay

Pay = ☐ + ☐

Pay = ☐

2 Geoff worked a 35 h week and also 7 h overtime. His normal hourly rate is $32.60/h.

a) Calculate his standard pay

Pay = ☐ × ☐

Pay = ☐

b) Calculate his overtime

Pay = ☐ × ☐ × ☐

Pay = ☐

c) Calculate his total pay

Pay = ☐ + ☐

Pay = ☐

d) Convert c) to an annual wage

Pay = ☐ × ☐

Pay = ☐

Calculate the weekly pay which includes overtime paid at time-and-a-half.

Example: 38 h week at $19.20 per hour plus 4 hours overtime.

Pay = 38 × 19.20 + [Normal Pay]

4 × 1.5 ×19.20 [Overtime]

Pay = $844.80 [Total]

3 18 h week at $13.48/h plus 3 hours overtime.

Pay = ☐ × ☐ +

= ☐ × 1.5 × ☐

Pay = ☐ ← $ sign, cents to 2 d.p.

4 25 h week at $38.40/h plus 5 hours overtime.

Pay = ☐ × ☐ +

= ☐ × ☐ × ☐

Pay = ☐

5 34 h week at $26.65/h plus 2 hours overtime.

Pay = ☐ × ☐ +

= ☐ × ☐ × ☐

Pay = ☐

6 17 h week at $15.55/h plus 2.5 hours overtime.

Pay = ☐ × ☐ +

= ☐ × ☐ × ☐

Pay = ☐ ← Round cents to 2 d.p.

7 5½ h week at $11.95/h plus 30 minutes overtime.

Pay = ☐ × ☐ +

= ☐ × ☐ × ☐

Pay = ☐

8 Convert 7 to an annual wage

Pay = ☐ × ☐

Pay = ☐

Complete Rebecca's pay for the week. She earns $18.50/h for her normal rostered hours, overtime for time above her rostered hours.

9

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	9	7 $129.50	2 $55.50	$185
Tues	9	5 ☐	☐ ☐	☐
Wed	8	6 ☐	☐ ☐	☐
Thur	10	7 ☐	☐ ☐	☐
Fri	9½	8 ☐	☐ ☐	☐
			Total Pay (Week)	☐

Earning Money - Penalty Rates - Time-and-a-half (Sheet 3)

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Complete these tables to find the weekly pay. Hours after rostered (normal) hours are paid overtime at a rate of time-and-a-half.

1 Hourly Rate = $23.40

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	6	5		
Tues	7	5		
Wed	7	7		
Thur	10	8		
Fri	8½	7		
			Total Pay (Week)	

2 Hourly Rate = $21.90

Day	*Hours Worked*	*Normal hrs*	*Time+½ hrs*	*Total Pay*
Mon	5½	5		
Tues	6	5		
Wed	6	5		
Thur	7	5		
Fri	7½	5		
Sat	6	4		
			Total Pay (Week)	



Complete the pay slips to calculate the gross pay (pay before tax) for these employees.

3

Name: B. Honey	Date From	Date From	Units	Rate	Amount	
Normal Hours	19-Jun-2019	25-Jun-2019	38	$24.30		Units × Rate
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	9			Units × Rate
Total						Add for total

Units = Hours

1.5 × 24.30

4

Name: T. Cake	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	27	$19.48	
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	4		
Total					

5

Name: F. Sharp	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	36	$25.80	
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	7½		
Total					

6

Name: I. Pod	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	38	$38.22	
Overtime (1.5×)	19-Jun-2019	25-Jun-2019	10.25		
Total					

Earning Money - Penalty Rates - Double Time (Sheet 1)

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Convert these hourly rates to an overtime rate of double time, so multiply by 2.

Example: Convert $13.85/h to a rate paid at double time

$13.85 = 13.85 × 2

D.T rate = $27.70

1 $17.60 = ____ × 2

D.T rate = ____ /h

'$' sign and cents to 2 d.p.

2 $16.63 = ____ × 2

D.T rate = ____ /h

3 $35.40 = ____ × ____

D.T rate = ____ /h

4 $23.11 = ____ × ____

D.T rate = ____

Include the /h to show it is a rate.

5 $25.73 = ____ × ____

D.T rate = ____

An earthmoving business pays its employees double-time on Sundays. Find the pay earned.

Example:

Normal rate: $19.10/hour

Double-time hours: 7

Pay = 19.10 × 2 × 7

Pay = $267.40

6 Normal Rate: $16.40/h
Double-time hours: 6

Pay = ____ × 2 × 6

Pay = ____

7 Normal Rate: $19.25/h
Double-time hours: 7

Pay = ____ × ____ × ____

Pay = ____

8 Normal Rate: $25.60/h
Double-time hours: 5

Pay = ____ × ____ × ____

Pay = ____

9 Normal Rate: $19.45/h
Double-time hours: 8

Pay =

Pay =

10 Normal Rate: $13.07/h
Double-time hours: 4.5

Pay =

Pay =

11 Normal Rate: $35.80/h
Double-time hours: 6.25

12 Normal Rate: $38.63/h
Double-time hours: 4.75

Cents to 2 d.p.

13 Normal Rate: $22.53/h
Double-time hours: 5.25

14 Normal Rate: $31.95/h
Double-time hours: 7.75

Calculate the double time earnings given the normal rate and the hours worked, then match it up with a line to the correct answer.

15 Normal Rate: $13.22/h D.T. Hours: 5 •

16 Normal Rate: $10.15/h D.T. Hours: 2 •

17 Normal Rate: $17.11/h D.T. Hours: 6.5 •

18 Normal Rate: $36.90/h D.T. Hours: 4 •

19 Normal Rate: $19.58/h D.T. Hours: 8 •

20 Normal Rate: $19.30/h D.T. Hours: 4.25 •

21 Normal Rate: $26.35/h D.T. Hours: 7.25 •

- A $222.43
- B $295.20
- C $40.60
- D $313.28
- E $164.05
- F $132.20
- G $382.08

Earning Money - Penalty Rates - Double Time (Sheet 2)

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Find the pay when these employees work Sunday and earn double time on top of their normal weekly wage.

1 38 h week at $19.25/h plus 5 hours on Sunday at double time rate.

Pay = ☐ × ☐ + ☐ × 2 × ☐

(☐: Normal hours × Hourly Rate; ☐: Double hours × 2 × Hourly Rate)

Pay = ☐

2 30 h week at $23.90/h plus 7 hours on Sunday at double-time rate.

Pay = ☐ × ☐ + ☐ × 2 × ☐

Pay = ☐

3 34 h week at $34.70/h plus 4 hours on Sunday at double-time rate.

Pay = ☐ × ☐ + ☐ × ☐ × ☐

Pay = ☐

4 16 h week at $13.20/h plus 5 hours on Sunday at double-time rate.

Pay =

Pay =

5 25 h week at $20.15/h plus 4.5 hours on a public holiday at double-time rate.

Pay =

Pay =

6 16 h week at $42.16/h plus 7½ hours on a Sunday at double-time rate.

Pay =

Pay =

7 30 h week at $18.10/h plus 6.25 hours on a public holiday at double-time rate.

Pay =

Pay =

Complete the pay slips to calculate the gross pay (pay before tax) for these employees.

8

Name: C. Clearly	Date From	Date From	Units (Units = Hours)	Rate	Amount	
Normal Hours	19-Jun-2019	25-Jun-2019	34	$31.30		Units × Rate
Double Time	19-Jun-2019	25-Jun-2019	6			Units × Rate
Total						Add for total

2 × 31.30

9

Name: D. Light	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	30	$19.48	
Double Time	19-Jun-2019	25-Jun-2019	5		
Total					

10

Name: J. Walker	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	38	$25.80	
Double Time	19-Jun-2019	25-Jun-2019	4½		
Total					

11

Name: M. Power	Date From	Date From	Units	Rate	Amount
Normal Hours	19-Jun-2019	25-Jun-2019	15	$38.22	
Double Time	19-Jun-2019	25-Jun-2019	3		
Total					

Earning Money - Penalty Rates (Sheet 1)

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Calculate the pay, in steps.

Dave earns $14.60/h and works:

Normal Time:	38 h
Time+½:	7 h
Double Time:	3 h

Calculate Dave's earnings from:

1 His normal hours.

Pay = [Rate] × 38

Pay = [ ] ← '$' sign and cents to 2 d.p.

2 His time-and-a-half hours:

Pay = [Rate] × 1.5 × [Hrs]

Pay = [ ]

3 His double time hours:

Pay = [Rate] × 2 × [Hrs]

Pay = [ ]

4 Calculate Dave's total pay.

Pay = [Normal] + [Time+½]

\+ [Double]

Pay = [ ]

Jess earns $16.70/h and works:

Normal Time:	25 h
Time+½:	4 h
Double Time:	2 h

Calculate Jess' earnings from:

5 Her normal hours.

Pay = [ ] × [ ]

Pay = [ ]

6 Her time-and-a-half hours:

Pay = [ ] × [ ] × [ ]

Pay = [ ]

7 Her double time hours:

Pay = [ ] × [ ] × [ ]

Pay = [ ]

8 Calculate Jess' total pay.

Pay = [ ] + [ ]

\+ [ ]

Pay = [ ]

Toby earns $23.80/h and in a week, he works:

Normal Time:	38 h
Time+½:	8 h
Double Time:	3 h

Calculate Toby's earnings from:

9 His normal hours.

Pay =

Pay =

10 His time-and-a-half hours:

Pay =

Pay =

11 His double time hours:

Pay =

Pay =

12 Calculate Toby's total pay.

Pay =

13 Convert Toby's total weekly pay to an annual wage.

Find total pay in one step

Kwan earns $22.90/h and works:

Normal Time:	34 h
Time+½:	3 h
Double Time:	2 h

14 Calculate Kwan's total pay.

Pay = [ ] × [ ]

Rate × Normal hours

\+ [ ] × 1.5 × [ ]

Rate × 1.5 × Time+½ hours

\+ [ ] × 2 × [ ]

Rate × 2 × Double time hours

Pay = [ ]

Caleb earns $27.15/h and works:

Normal Time:	19 h
Time+½:	8 h
Double Time:	3 h

15 Calculate Caleb's total pay.

Pay = [ ] × [ ]

\+ [ ] × [ ] × [ ]

\+ [ ] × [ ] × [ ]

Pay = [ ]

Rose earns $31.80/h and works:

Normal Time:	38 h
Time+½:	6 h
Double Time:	1.5 h

14 Calculate Rose's total pay.

Pay =

Earning Money - Penalty Rates (Sheet 2)

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Calculate the total pay earned.

Josh earns $26.50/h and works a 38 hour week, with 6 hours at time-and-a-half and 2 hours double time.

1 Calculate Josh's total pay.

Pay =

Rate × Normal Hours

+ Rate × 1.5 × Time+½ Hours

+ Rate × 2 × Double Time Hours

Pay =

Maysun earns $22.85/h and works a 30 hour week, with 4 hours at time-and-a-half and 1.5 hours double time.

2 Find Maysun's total pay.

Pay =

Pay =

Connie earns $30.45/h and works a 36 hour week, with 7 hours at time-and-a-half and 4.5 hours double time.

3 Find Connie's total pay.

Round cents to 2 decimal places

Calculate the daily earnings, then total them for the week.

4

Normal hourly rate = $28.40/h

Day	*Normal hrs*	*Time+½ hrs*	*Double Time hrs*	*Total Pay*
Mon	7	2	3	
Tues	6	-	-	
Wed	6	2	2	
Thur	6	1	-	
Fri	7	2	1.5	
			Total Pay (Week)	

Calculate each employee's weekly earnings

5

Name	*Hourly rate*	*Normal hrs*	*Time+½ hrs*	*Double Time hrs*	*Total Pay*
Ben	$19.70	16	2	1	
Ken	$28.50	26	7	2	
Len	$34.65	38	10	-	
Thien	$37.20	38	5	-	
Glen	$18.60	28	11	5	
Wen	$25.80	30	5	1.5	
Zhen	$42	38	3	0.5	
				Total Employee Wages	

24 Hour Time and Time Calculations

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Change these times to 24 hour time format, (from 1 p.m. add 12 hours, keep minutes as is).

Example

4.45 p.m. 1645 h

1 3 a.m. = ☐☐☐☐ h

2 8 a.m. = ☐☐☐☐ h

3 4 p.m. = ☐☐☐☐ h

4 11.25 a.m.

5 3.52 p.m.

6 5.17 p.m.

7 8.47 p.m.

8 4.56 a.m.

9 12.33 p.m.

10 10.25 p.m.

11 12.19 a.m.

12 7.39 p.m

13 4.06 a.m.

14 11.39 a.m.

15 6.26 p.m

16 11.08 p.m

17 10.43 a.m.

18 9.43 p.m

19 5.56 a.m.

20 11.48 a.m.

Find the time worked, use 24 hour time and subtract the earlier time from the later time.

Example

9.15 a.m. until 4.35 p.m.

= 1635 h - 0915 h

16 DMS 35 DMS - 9 DMS 15 DMS

= ☐ h ☐ min

21 11.25 a.m. until 6.15 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

22 8.35 a.m. until 4.45 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

23 7.50 a.m. until 3.30 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

24 6.10 a.m. until 2.25 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

25 7.05 a.m. until 11.50 a.m.

= ☐☐☐☐ h - ☐☐☐☐ h

26 12.07 p.m. until 8.53 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

27 9.13 a.m. until 5.55 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

Round these times to the nearest 15 mins. So minutes will be 0 or 15 or 30 or 45.

Examples

7h 20 min = 7h 15 min

5h 38 min = 5h 45 min

28 6h 4 min = ☐ h ☐ min

29 4h 19 min = ☐ h ☐ min

30 8h 42 min = ☐ h ☐ min

31 5h 54 min = ☐ h ☐ min

32 7h 38 min = ☐ h ☐ min

33 6h 52 min = ☐ h ☐ min

34 4h 36 min = ☐ h ☐ min

35 3h 50 min = ☐ h ☐ min

Find the time worked, but this time get your answer then round it to the nearest 15 min.

36 9.17 a.m. until 5.38 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min = ☐ h ☐ min

Actual Rounded

37 7.41 a.m. until 2.17 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min = ☐ h ☐ min

38 6.07 a.m. until 1.34 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min = ☐ h ☐ min

39 10.43 a.m. until 4.21 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min = ☐ h ☐ min

Time Sheet Calculations (Sheet 1)

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Find the time worked and the pay earned for the shift, given the hourly rate.

1 9.45 a.m. until 5.15 p.m.

17 DMS 15 DMS - 9 DMS 45 DMS =

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min

2 Hourly rate is $21.40/h.

Using the answer on your calculator screen, then do this: × 21.40 = DMS

= ☐ h ☐ min × ☐

= $ ☐

3 10.45 a.m. until 7.30 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min

4 Hourly rate is $25.60/h.

= ☐ h ☐ min × ☐

= $ ☐

5 7.30 a.m. until 3.15 p.m.

= ☐☐☐☐ h - ☐☐☐☐ h

= ☐ h ☐ min

6 Hourly rate is $31.90/h.

= ☐ h ☐ min × ☐

= $ ☐

Convert the answer in Q.6 to an annual wage if the employee works that shift 4 days/week.

7

Pay =

Pay =

Now complete the table, converting to 24 h time and rounding to the nearest 15 minutes. You have to subtract the employees' lunch time.

8 Hourly rate for all employees is $28.12

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Tim	9.35 0935	4.15 1615	30 m	☐h ☐☐m	☐h ☐☐m	
Actual hours = 16 DMS 15 DMS - 9 DMS 35 DMS - 0 DMS 30 DMS =						
Jim	7.05 ☐☐☐☐	3.42 ☐☐☐☐	45m	☐h ☐☐m	☐h ☐☐m	
Lim	8.19 ☐☐☐☐	5.29 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Kim	6.37 ☐☐☐☐	4.08 ☐☐☐☐	45m	☐h ☐☐m	☐h ☐☐m	
Yim	10.09 ☐☐☐☐	4.57 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Alim	8.19 ☐☐☐☐	5.08 ☐☐☐☐	1h	☐h ☐☐m	☐h ☐☐m	

9 Hourly rate for all employees is $24.64

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Jen	7.52 ☐☐☐☐	3.43 ☐☐☐☐	45m	☐h ☐☐m	☐h ☐☐m	
Ken	9.07 ☐☐☐☐	4.35 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Ben	11.10 ☐☐☐☐	3.27 ☐☐☐☐	1h	☐h ☐☐m	☐h ☐☐m	
Len	8.55 ☐☐☐☐	5.18 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Wen	10.36 ☐☐☐☐	4.09 ☐☐☐☐	1h	☐h ☐☐m	☐h ☐☐m	
Thien	7.16 ☐☐☐☐	4.57 ☐☐☐☐	30m	☐h ☐☐m	☐h ☐☐m	
Zen	8.39 ☐☐☐☐	6.06 ☐☐☐☐	45m	☐h ☐☐m	☐h ☐☐m	

Time Sheet Calculations (Sheet 2)

Complete these time sheets for the employees shown. Round the time worked to the nearest 15 minutes, then calculate the wages.



1

Name	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Hourly Rate*	*Daily Wage*
Luke	8.30 0830h	4.41 1641h	45 m	7h 26m	7h 30m	$19.50	$146.25
Leia	7.15	2.26	30 m	Subtract lunch!		$23.80	
Han	6.45	3.50	45 m	Subtract lunch!		$33.15	
Chewy	5.45	12.06	30 m			$27.28	
Ben	9.15	2.57	45 m			$33.15	
Darth	8.30	1.27	-			$23.85	
Padme	7.45	2.30	30 m			$19.50	Rounding needed
Jabba	10.50	5.18	1h			$33.15	
Lando	8.30	4.45	45 m			$17.55	
Yoda	6.10	3.48	30 m			$23.85	

2

Employee: A. Mouak Hourly Rate: $23.80/h

Day	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	5.40	2.07	45 m			
Tues	6.15	1.23	45 m			
Wed	6.15	12.09	45 m			
Thurs	6.15	2.53	45 m			
Fri	5.40	1.18	45 m			
					Total Weekly Wage	

Note! Not a.m.

3

Employee: W. Zhiang Hourly Rate: $15.64/h

Day	*Time In p.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	3.20	9.36	30 m			
Wed	3.20	10.43	45 m			
Fri	3.20	9.49	30 m			
					Total Weekly Wage	

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Earning Money - Percentage Applications (Sheet 1)

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If you get an increase in pay, it is often by a percentage. Find these percentages.

Example: Find 7% of $17 800

$$17\,800 \times \frac{7}{100} = \$1\,246$$

Using your calculator's fraction key

1 7 8 0 0 × ▣ 7 ◯ 1 0 0 =

1 Find 4% of $48 000

$$\square \times \frac{\square}{100} = \$\square$$

2 Find 6% of $82 000

$$\square \times \frac{\square}{\square} = \square$$

'$' sign

3 Find 5.4% of $63 200

$$\square \times \frac{\square}{\square} = \square$$

Write answer to 2 decimal places.

4 Find 2.6% of $45 670

$$\square \times \frac{\square}{\square} = \square$$

5 Find 6.1% of $104 600

$$\square \times \frac{\square}{\square} = \square$$

6 Find 3.35% of $88 777

$$\square \times \frac{\square}{\square} = \square$$

Round answer to 2 decimal places.

7 Find 5.15% of $78 441

$$\square \times \frac{\square}{\square} = \square$$

8 Find 8.28% of $138 881

$$\square \times \frac{\square}{\square} = \square$$

Increase these annual wages by the percentage given.

Example: Find the new annual wage after $53 000 is increased by 8%.

Increase:

$$53\,000 \times \frac{8}{100} = \$4\,240$$

New Wage:

$= 53\,000 + 4\,240$

$= \$57\,240$

9 Increase $45 000 by 6%

Increase:

$$\square \times \frac{\square}{100} = \square$$

New Wage:

$= \square + \square$

$= \square$

10 Increase $125 500 by 4%

Increase:

$$\square \times \frac{\square}{\square} = \square$$

New Wage:

$= \square + \square$

$= \square$

11 Increase $78 400 by 3%

Increase:

$$\square \times \frac{\square}{\square} = \square$$

New Wage:

$= \square + \square$

$= \square$

Increase the annual salary, then find the new weekly salary.

12 Increase $38 300 by 4.2%

$$\square \times \frac{\square}{\square} = \square$$

Wage = $\square + \square$

Wage = $\square$

13 Find the new weekly wage.

Weekly Wage = $\square \div 52$

Weekly Wage = $\square$

14 Increase $143 558 by 6.5%

Wage = $\square + \square$

Wage = $\square$

15 Find the new weekly wage.

Weekly Wage =

Weekly Wage =

16 Increase $105 600 by 2.17%

17 Find the new weekly wage.

Weekly Wage =

Weekly Wage =

Earning Money - Percentage Applications (Sheet 2)

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Work in a different position and you might get paid extra for it. Find the increased rates.

David earns $28.20/h as a welder. When he works on elevators he gets a 15% height allowance. Calculate:

1 The hourly rate increase

$\square \times \frac{\square}{100} = \$\square$

2 The increased hourly rate

$\square + \square = \$\quad /h$

3 If David works 17 hours at the normal rate and 21 hours at the increased rate, calculate his weekly pay.

Weekly Wage $= \square$ (Rate) $\times \square$ (Hours) $+$

$\square$ (Increased Rate) $\times \square$ (Hours)

Weekly Wage $= \square$

Mischa is a power pole contractor earning $38.10/h. When she works in a hoist she is paid 20% extra per hour. Calculate:

4 Her hourly rate increase

$\square \times \frac{\square}{\square} = \square$

5 Her increased hourly rate

$\square + \square = \square$

6 If Mischa works 25 hours at the normal rate and 13 hours at the increased rate, calculate her weekly pay.

Weekly Wage $= \square \times \square +$

$\square \times \square$

Weekly Wage $= \square$

Chloe earns $32.80/h as a car smash repairer. When she works in the spray booth she gets an 18% increase on her hourly rate. In a 38 hour week she works 17 h in the booth. Find:

7 Her hourly rate increase

$\square \times \frac{\square}{\square} = \square$ (2 d.p.)

8 Her increased hourly rate

$\square + \square = \square$

9 The number of hours Chloe works out of the spray booth.

10 Her weekly pay.

Weekly Wage $= \square \times \square +$

$\square \times \square$

Weekly Wage $= \square$

Sasha earns $45.90/h as a computer programmer. He earns 25% more per hour when he works off-site. If he works a 40 hour week of which 17h is off-site, calculate:

11 His off-site hourly rate

12 The number of hours Sasha works on-site (in his office).

13 His weekly pay.

Raj earns $24.90/h as a baker. He is paid an additional 8% for early hours before 6 a.m. If he works a 38 h week and 10 of those hours are 'early hours', find:

14 His 'early hours' rate

15 The number of standard hours Raj works

16 His weekly pay.

17 His annual wage

Rachel is a Human Resource consultant earning $85.90/h. When she takes on team leader status she earns 15% more. Calculate:

18 Rachel's team leader rate.

19 Her weekly pay given she works 27h at the normal rate and 8h at the team leader rate.

Earning Money - Piecework (Sheet 1)

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Calculate the payment for these items, use multiplication

1 15 drawings at $17 each.

Payment = ☐ × ☐

Payment = $ ☐

2 30 T-shirts at $14.50 each.

Payment = ☐ × ☐

Payment = ☐

Show the $ sign

3 22 Jackets at $56.80 each.

Payment = ☐ × ☐

Payment = ☐

Show the $ sign and cents to 2 decimal places

4 38 m^2 of tiling at $108/$m^2$.

Payment =

Payment =

The tables below show daily production. Get the total, then multiply by the rate.

5 Sheep shorn at $9.30/sheep

Day	M	Tu	W	Th	F	Total
Sheep	15	24	19	32	25	

Payment = ☐ × ☐

Payment = ☐

6 Cars detailed at $64.20/car

Day	M	Tu	W	Th	F	Total
Cars	4	5	3	4	6	

Payment = ☐ × ☐

Payment = ☐

7 Parcels delivered at $2.37 for each parcel.

Day	M	Tu	W	Th	F	Total
No.	208	95	167	198	153	

Payment = ☐ × ☐

Payment = ☐

Production can have different rates to motivate. Split the production into the two rates.

Alison produces flower arrangements, she is paid $6.80/arrangement for the first 20 supplied. If she supplies more than 20 she is paid $8.90/arrangement.

8

Day	M	Tu	W	Th	F	S	Total
No.	19	15	25	16	28	32	
$6.80:	19	15	20	16	20	20	
$8.90:	0	0	5	0	8	12	

Payment = ☐ × ☐ +

☐ × ☐

Payment = ☐

James frames photographs. He is paid $7.95/frame for the first 30 frames, after that he is paid $9.50 per frame.

9

Day	M	Tu	W	Th	F	Total
No.	42	38	19	56	43	
$7.95:						
$9.50:						

Payment = ☐ × ☐ +

☐ × ☐

Payment = ☐

Malik is a fruit-picker. He is paid $3.50 per tub of oranges picked, for the first 35 tubs. He is paid $4.25 per tub after the initial 35.

10

Day	W	Th	F	Sa	Su	Total
No.	62	43	52	28	37	
$3.50:						
$4.25:						

Payment = ☐ × ☐ +

☐ × ☐

Payment = ☐

Troy sells electricity packages by walking house to house. He is paid $65.70 for every sale he makes. If he makes more than 5 sales in a day he is paid an additional $23.95 per sale.

11 Find the rate for extra sales.

= ☐ + ☐

= ☐

Write this below the table under '$65.70'

12

Day	M	Tu	W	Th	F	S	Total
No.	3	7	4	8	5	12	
$65.70:							

Payment = ☐ × ☐ +

☐ × ☐

Payment = ☐

Earning Money - Piecework (Sheet 2)

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A tradesperson needs to know whether a job is worthwhile. Compare the quotes for these.

James is a roof-tile layer. He charges a minimum of \$18/m^2. He is offered a job of laying 205 m^2 for \$3 198.

1 Find the amount offered per m^2 for the \$3 198 job.

Rate = ☐ ÷ ☐

Rate = ☐ /m^2

Show the \$ sign and cents to 2 decimal places

2 Is the rate acceptable?

◯ Yes, it's more than \$18/m^2

◯ No, it's less than \$18/m^2

3 How much would James quote for laying 205 m^2?

Cost = ☐ × ☐

Cost = ☐

4 What is the difference between the \$3 198 offered and James' quote for that job?

Amount = ☐ - ☐

Amount = ☐

Lauren is another roof-tile layer. She charges a \$300 fee + \$14/m^2. What is her quote for laying the 205 m^2?

5

Quote = ☐ + ☐ × ☐

Quote = ☐

6 Should she take the 205 m^2 job, based on her quote?

◯ Yes, her quote is below the \$3 198

◯ No, her quote is above the \$3 198

Clara is a landscape designer. She charges a \$180 fee + \$7.70/m^2 for laying turf.

7 Give a quote for Clara to lay 95 m^2 of turf.

Cost =

Cost =

8 Give a quote for Clara to lay 175 m^2 of turf.

Cost =

Cost =

Tisharna is also a landscape designer. She charges a \$9/m^2 for laying turf.

8 Give a quote for Tisharna to lay 175 m^2 of turf.

Cost =

Cost =

9 Who offers the cheaper quote for the 175 m^2?

◯ Clara ◯ Tisharna

How much turf does Tisharna lay for these quotes listed. Her rate is \$9/m^2.

10 \$504

Area =

Area = ← Answer in m^2

11 \$4 140

Area =

Area =

12 \$1 080

Area =

Area =

Toby fits glass splash-backs in kitchens. He charges \$550 for fitting a standard splash-back, which takes him a day to fit.

13 How much does Toby earn for a 5 day week?

Pay =

Pay =

14 Convert this wage to an annual wage.

Pay =

Pay =

15 Toby is offered a salary of \$142 000 p.a. from a kitchen company for the same hours. Should he take the position?

Glen fits T.V. antenna masts for \$450 and fits sockets for \$35 per room. Give a quote for these jobs.

16 1 mast and 2 sockets

17 1 mast and 4 sockets

18 2 masts and 5 sockets

Earning Money - Commission (Sheet 1)

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Find the commission on these sales. Multiply by the percentage as a fraction written 'over 100'.

Example

Sales = \$15 540, Rate = 12%

Commission = $15\,540 \times \frac{12}{100}$

$= \$1\,864.80$

1 Sales = \$23 016, Rate = 11%

Commission = $\square \times \frac{\square}{100}$

$=$

2 Sales = \$17 880, Rate = 9%

Commission = $\square \times \frac{\square}{100}$

$=$

3 Sales = \$78 107, Rate = 4%

Commission = $\square \times \frac{\square}{\square}$

$=$

4 Sales = \$12 050, Rate = 8%

Commission =

$=$

5 Sales = \$9 300, Rate = 15%

Commission =

$=$

6 Sales = \$87 600, Rate = 6.5%

Commission =

$=$

Complete the table by finding the commission payable. Then rank the earnings from highest (1) to the lowest (7).

	Name	Rate	Sales	Commission	Rank
7	Gwen	5%	\$40 555		
8	Sven	11%	\$16 931		
9	Len	18%	\$15 676		
10	Wen	4%	\$46 855		
11	Ben	7%	\$25 910		
12	Jen	9.5%	\$23 166		
13	Toby	13%	\$17 582		

Calculate the commission and fill the circle for who earns the most per week.

14 Steve 5% on \$38 960 sales, Georgie 8% on sales of \$18 777.

Steve =

$=$

Georgie =

$=$

◯ Steve ◯ Georgie

15 Nicki 12% on \$43 110 sales, Zaina 8% on sales of \$54 550.

◯ Nicki ◯ Zaina

16 James 13% on \$11 522 sales, Jessie 17% on sales of \$14 070.

◯ James ◯ Jessie

Find the commission and connect it to the answer.

Use a different colour for each line

17 Commission: 5% Sales: \$41 500	•	A • \$1 413.60
18 Commission: 15% Sales: \$22 940	•	B • \$3524.40
19 Commission: 8% Sales: \$17 670	•	C • \$3 441
20 Commission: 1.2% Sales: \$135 200	•	D • \$1 922.50
21 Commission: 7% Sales: \$29 612	•	E • \$2 072.84
22 Commission: 2.5% Sales: \$76 900	•	F • \$4 199.76
23 Commission: 18% Sales: \$19 580	•	G • \$2 075
24 Commission: 12% Sales: \$34 998	•	H • \$1 622.40

Earning Money - Commission (Sheet 2)

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A retainer is a base salary paid with a commission payment. Calculate the pay earned.

Example Base salary = $200/wk
Sales = $13 700, Rate = 9.2%

Pay = $200 + 13\,700 \times \frac{9.2}{100}$

Pay = $1 460.40

1 Base salary = $370/wk with Sales = $8 295, Rate = 25%.

Pay = ☐ + ☐ × ☐/100

Pay = ☐

2 Base salary = $440/wk with Sales = $28 040, Rate = 6%.

Pay = ☐ + ☐ × ☐/100

Pay = ☐

3 Base salary = $290/wk with Sales = $158 900, Rate = 1.5%.

Pay = ☐ + ☐ × ☐/☐

Pay = ☐

4 Retainer of $340/wk with Sales = $38 775, Rate = 4.5%.

Pay =

Pay = ← Cents to 2 d.p.

5 Base salary of $550/wk with Sales = $73 145, Rate = 3.5%.

Pay =

Pay =

6 Base salary = $225/wk with Sales = $899 600, Rate = 0.3%.

Pay =

Pay =

Add the base salary to the commission earned to find the pay. Then rank the earnings from highest (1) to the lowest (7).

	Name	Base Salary	Rate	Sales	Weekly pay	Rank
7	Katniss	$800	7%	$38 900		
8	Cinna	$220	5%	$50 052		
9	Gale	$350	8%	$41 776		
10	Marvel	$150	10%	$50 735		
11	Thresh	$500	5.5%	$27 846		
12	Seneca	$280	4%	$37 200		
13	Peeta	$640	8.5%	$29 670		

Compare these wages to get the best deal.

Hakim sells home loan products. He has average weekly sales of $780 000. He is offered two weekly pay options:
1. $600 + 0.2% commission on sales
2. Flat 0.3% commission on sales, find:

14 His weekly pay for Option 1.

Pay =

Pay =

15 His weekly pay for Option 2.

Pay =

Pay =

16 Which is the best offer?

◯ Option 1 Option 2 ◯

Francis works with Hakim, his sales average $555 000/wk. Find:

17 His weekly pay for Option 1.

Pay =

Pay =

18 His weekly pay for Option 2.

Pay =

Pay =

Best? ◯ Option 1 ◯ Option 2

Rosie sells life insurance. She has average weekly sales of $43 000. She is offered two weekly pay options:
1. $300 + 4% commission on sales
2. An annual salary of $94 952, find:

19 Her weekly pay for Option 1.

Pay =

Pay =

20 Her weekly pay for Option 2.

Pay =

Pay =

21 Which is the best offer?

◯ Option 1 Option 2 ◯

Jane works with Rosie, her sales average $39 200/wk. Find:

22 Her weekly pay for Option 1.

Pay =

Pay =

23 Her weekly pay for Option 2.

Pay = ☐

Same answer as Question 20

24 Which is the best offer?

◯ Option 1 Option 2 ◯

Earning Money - Leave Loading (Sheet 1)

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Convert these wages to 4 weeks pay.

Example: $1 365.20/week

4 weeks Pay = 1 365.20 × 4

= $5 460.80

1 $623.17/week

4 weeks Pay = ☐ × ☐

= $ ☐

2 $435.76/week

4 weeks Pay = ☐ × ☐

= $ ☐

3 $1 738.15/week

4 weeks Pay =

=

Show the $ sign and cents to 2 decimal places

4 $1 674.83/week

4 weeks Pay =

=

To find 4 weeks pay, multiply the fortnightly rate by 2 (So, × 2)

5 $2 560.77/fortnight

4 weeks Pay = ☐ × ☐

= ☐

6 $1 363.45/fortnight

4 weeks Pay =

=

7 $425.65/fortnight

4 weeks Pay =

=

Convert an annual wage to 4 weeks pay (÷ 52 × 4).

8 $62 774.40 p.a.

4 weeks pay = ☐ ÷ ☐ × ☐

= ☐

9 $28 605.20 p.a.

4 weeks pay = ☐ ÷ ☐ × ☐

= ☐

10 $45 705.63 p.a.

4 weeks pay =

= ← Round to 2 d.p.

11 $94 771 p.a.

4 weeks pay =

=

12 $59 883 p.a.

4 weeks pay =

=

Convert a monthly wage to 4 weeks pay (× 12 ÷ 52 × 4).

13 $4 680 per month.

4 weeks pay = ☐ × ☐ ÷ ☐ × ☐

= ☐

14 Is your answer above just less than the $4 680 per month?

◯ Yes, because 4 weeks while close, is slightly less than a month.

◯ No, I think I have to do it again.

15 $3 302 per month.

4 weeks pay = ☐ × ☐ ÷ ☐ × ☐

= ☐

16 $1 102.40 per month.

4 weeks pay =

=

17 $4 435 per month.

4 weeks pay =

= ← Round to 2 d.p.

Change the daily hours and rate to weekly pay to a 4 weekly pay.

Daily hours × Hourly rate × Days = Weekly Pay

	Name	*Daily Hours*	*Hourly Rate*	*Days per Week*	*Weekly Pay*	*4 Weeks Pay*
18	E. Claire	7	$24.85	5		
19	I. Sore	6	$19.40	6		
20	D. Light	5½	$25.60	4		
21	C. Saw	7	$15.40	5		
22	G. Whizz	8	$32.80	5		
23	T. Time	6½	$38.90	3		
24	X. Tract	4	$28.90	2		

Earning Money - Leave Loading (Sheet 2)

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Leave loading is 17½%. So multiply by 17.5 'over 100' to find the leave loading.

Example: Find 17½% of $560

L. Loading $= 560 \times \frac{17.5}{100}$

$= \$98$

1 Find 17½% of $2 186

L. Loading = ☐ × ☐/100

= ☐

2 Find 17½% of $778

L. Loading = ☐ × ☐/100

= ☐

3 Find 17½% of $1 412

L. Loading = ☐ × ☐/☐

= ☐

4 Find 17½% of $895.70

L. Loading = ☐ × ☐/☐

= ☐

5 Find 17½% of $1 033

L. Loading =

← Round to 2 d.p.

6 Find 17½% of $1 043.52

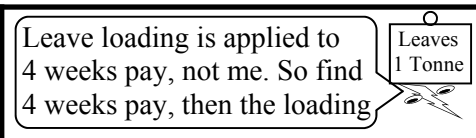
L. Loading =

=

7 Find 17½% of $358.38

L. Loading =

=



Leave loading is applied to 4 weeks pay, not me. So find 4 weeks pay, then the loading

Jodie earns $850 per week. Find:

8 4 Weeks pay.

4 weeks Pay = ☐ × ☐

= $ ☐

9 17½% of 4 weeks pay

L. Loading = ☐ × ☐/☐ (Q.8)

= ☐

10 Jodie's holiday pay.

H. Pay = ☐ (Q.8) + ☐ (Q.9)

= ☐

Iskandar earns $1 055 per week. Find:

11 4 Weeks pay.

4 weeks Pay = ☐ × ☐

= $ ☐

12 17½% of 4 weeks pay

L. Loading = ☐ (Q.11) × ☐/☐

= ☐

13 Iskandar's holiday pay.

H. Pay = ☐ (Q.11) + ☐ (Q.12)

= ☐

Carla earns $777.70 per week. Find:

14 4 Weeks pay.

4 weeks Pay = ☐ × ☐

= ☐

15 17½% of 4 weeks pay

L. Loading = ☐ (Q.14) × ☐/☐

= ☐

16 Carla's holiday pay.

H. Pay = ☐ + ☐

= ☐

Tegan earns $81 120 p.a. Find:

17 4 Weeks pay.

4 weeks pay = ☐ ÷ ☐ × ☐

= ☐

18 17½% of 4 weeks pay

L. Loading = ☐ × ☐/☐

= ☐

19 Tegan's holiday pay.

H. Pay = ☐ + ☐

= ☐

William earns $49 527.92 p.a. Find:

20 4 Weeks pay.

4 weeks pay = ☐ ÷ ☐ × ☐

= ☐

21 17½% of 4 weeks pay

L. Loading = ☐ × ☐/☐

= ☐ 2 d.p.

22 William's holiday pay.

H. Pay = ☐ + ☐

= ☐

Earning Money - Leave Loading (Sheet 3)

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Find 4 weeks pay, the leave loading (at 17½%), then find the holiday pay.

William earns $80 080 p.a. Find:

1 4 Weeks pay.

4 weeks pay = ____ ÷ ____ × ____

= ____

2 17½% of 4 weeks pay

L. Loading = ____ × ____/____

= ____

3 William's holiday pay.

H. Pay = ____ + ____

= ____

Jaycee earns $5 304/month. Find:

4 4 Weeks pay.

4 weeks pay = ____ × ____ ÷ ____ × ____

= ____

5 17½% of 4 weeks pay

L. Loading = ____ × ____/____

= ____

6 Jaycee's holiday pay.

H. Pay = ____ + ____

= ____

Nathan earns $19.30/h for a 12 hour week. Find:

7 4 Weeks pay.

4 weeks pay = ____ × ____ × ____

= ____

8 17½% of 4 weeks pay

L. Loading = ____ × ____/____

= ____

9 Nathan's holiday pay.

H. Pay = ____ + ____

= ____

Tony earns $2 444/month. Find:

10 4 Weeks pay.

4 weeks pay =

=

11 17½% of 4 weeks pay

L. Loading =

=

12 Tony's holiday pay.

H. Pay =

=

Brad earns $76 492 p.a. Find:

13 4 Weeks pay.

4 weeks pay =

=

14 17½% of 4 weeks pay

L. Loading =

=

15 Brad's holiday pay.

H. Pay =

=

Fei Yen earns $24.90/h for a 38 hour week. Find:

16 4 Weeks pay.

4 weeks pay =

=

17 17½% of 4 weeks pay

L. Loading =

=

18 Fei Yen's holiday pay.

H. Pay =

=

Alena earns commission at 14% of her weekly sales. Find:

19 Her weekly commission, given she has average weekly sales of $12 800.

Commission =

Use commission rate here - not 17.5%

=

20 4 Weeks pay.

4 weeks pay =

=

21 17½% of 4 weeks pay

L. Loading =

=

22 Alena's holiday pay.

H. Pay =

=

Earning Money - Find a Word

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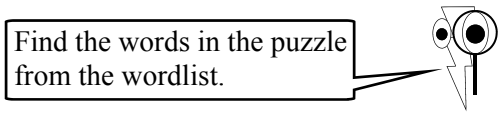

E	K	C	M	N	O	I	S	S	I	M	M	O	C	C
M	O	N	T	H	L	Y	Y	A	D	I	L	O	H	O
E	Q	T	H	G	I	N	T	R	O	F	H	L	S	H
M	O	V	E	R	T	I	M	E	N	U	U	O	A	H
P	W	X	L	A	U	T	C	N	U	P	P	H	L	B
L	R	S	S	E	T	A	R	I	G	G	K	B	A	A
O	E	H	E	T	S	T	R	A	T	S	R	B	R	S
Y	S	I	T	E	R	A	R	T	G	G	O	A	Y	P
E	L	F	L	T	V	S	E	E	G	A	W	S	Y	E
E	O	T	F	A	R	A	E	R	W	A	E	E	E	N
M	A	A	P	N	R	E	E	G	C	Y	C	E	N	A
P	D	A	G	N	V	E	V	L	V	N	E	O	O	L
E	I	N	I	U	H	S	I	N	I	F	I	Y	M	T
D	N	N	N	A	R	R	E	Y	O	L	P	M	E	Y
I	G	N	I	L	Y	S	T	N	E	C	H	O	U	R

WORDLIST

RATE	PENALTY	HOLIDAY	MONEY	EMPLOYER
WAGE	BASE	OVERTIME	FINISH	FORTNIGHT
HOUR	START	MONTHLY	RETAINER	CENTS
SHIFT	LOADING	COMMISSION	LEAVE	PAID
INCREASE	EMPLOYEE	EARNING	SALARY	ANNUAL
	PIECEWORK	CONVERT	PUNCTUAL	

Earning Money - Topic Review

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Calculate the amount earned.

1 $96/day, for 3 days.

2 $26/h, for 4 hours.

3 $795.20/week, for 2 weeks.

Calculate the weekly wage.

4 $25.45/h for a 38 h week.

5 $18.75/h for a 20 h week.

6 $135.65/day for a 3 day week.

Find the weekly wage earned.

7 $46 498.40 p.a.

8 $4 992/month.

Frances is paid $473.10 for a 38 hour week. Convert this rate of pay to:

11 An annual wage

12 A monthly wage

13 An hourly rate

Convert these wages to monthly

9 $61 034 p.a.

10 $775.20/week.

11 $24.90/h for a 20 h week.

Convert these normal hourly rates to rates paid at time-and-a-half

12 $12.50 _____

13 $18.60 _____

Convert these normal hourly rates to rates paid at double time

14 $13.55 _____

15 $9.95 _____

Calculate the penalty rate payments.

16 6 h at time-and-a-half.
(Normal hourly rate is $18.40 per hour.)

17 4 h at double time.
(Normal hourly rate is $23.70 per hour.)

18 3 h at time-and-a-half and 1½ h at double time.
(Normal hourly rate is $21.20 per hour.)

19 Complete the table using a normal hourly rate of $16.50/hour.

Name	*Normal Hours*	*Time and a half*	*Double Time*	*Total*
Jane	35	4	-	
Tina	17	7	4	
Brad	38	3	1	
David	25	8	2	
Mel	30	6½	2½	

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James earns $584 per week. If he receives a pay increase of 7.5%, find:

20 The increase in pay

21 His new annual salary

Jaegar is paid 15% commission on sales. Find the commission paid on:

22 Sales of $4 670

Kieran is paid a base salary of $760 per week, as well as 4% commission on sales. Find his earnings if he makes sales of $38 500 for the week.

23 Sales of $38 500

Ahmad earns fits smoke alarms and fire extinguishers. He is paid $9 per smoke alarm and $26 per fire Extinguisher. Find his earnings when he fits the following:

24 6 smoke alarms, and
4 fire extinguishers

25 11 smoke alarms, and
7 fire extinguishers

Husna applies silkscreen artwork to T-shirts. She is paid $16 per shirt, for up to 20 shirts. The rate increases to $19 per shirt on those shirts she produces after the first 20. Her supply figures are in the table below, calculate her earnings for the week.

26

Day	M	T	W	T	F
Quantity	15	20	28	11	32

Izolde earns $850.90 per week. She is about to go on 4 weeks holiday. Calculate the following:

28 4 Weeks pay.

29 Leave Loading at 17½% of 4 weeks pay.

30 Izolde's holiday pay.

Carmen earns $70 418.40 p. a. She is about to go on 4 weeks holiday. Calculate the following:

31 4 Weeks pay.

32 Leave Loading at 17½% of 4 weeks pay.

33 Carmen's holiday pay.

27 Complete the table

Hourly rate is $20.40 per hour.
Round time worked to nearest 15 minutes.
Lunch times are unpaid.

Day	*Time In a.m.*	*Time Out p.m.*	*Lunch Time*	*Actual hours*	*Rounded hours*	*Daily Wage*
Mon	8.20	2.25	30 m			
Tues	8.35	4.10	45 m			

VELOCITY

AREA

Area



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Area

2



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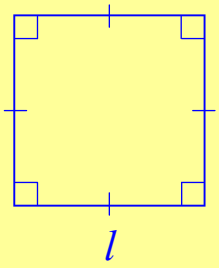
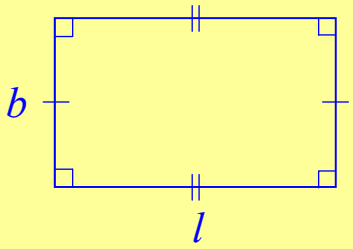
Area of Squares and Rectangles

The area of a square and rectangle is found in the same way. Multiply one side by the other. As squares have sides all the same length, this means you just 'square' the side length.

Square $A = l^2$

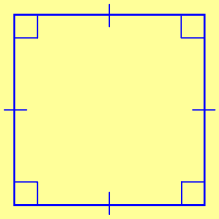


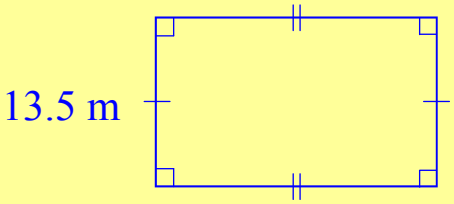
Rectangle $A = lb$



While the answer isn't affected, the length is usually considered the longest length in a rectangle.

Examples - Find the area to 1 decimal place.





$A = l^2$	Write the formula	$A = lb$
$= 22^2$	Substitute l (or l & b)	$= 22 \times 13.5$
$A = 484\ cm^2$	Solve showing square units.	$A = 297\ m^2$

The first sheet is all squares, the second all rectangles, while the third sheet is a combination of both. The third sheet also deals with changing units.

You may measure a length in a unit such as mm, but wish to express the area in a square unit which is more relevant to the material, such as cm^2. Convert the length units and then approach the question in the same way as previous questions.

Area of Squares

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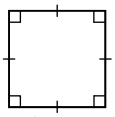
Use $A=l^2$ to find the area of these squares, to 1 d.p. Show units with a ^{2} after them.

1



$A = l^2$

$= \square^2$

Calc: 9 x^2 =

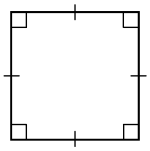
$A = \square$ cm^2

2

$A = l^2$

$= \square^2$

$A = \square$ cm^2



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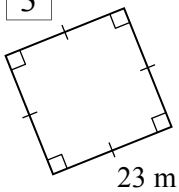


$A = l^2$

$= \square^2$

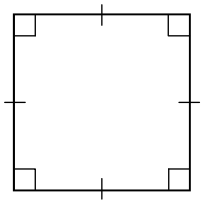
$A = \square$ m^2

4

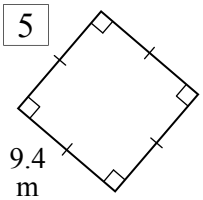
$A = l^2$

$= \square^2$

$A = \square$ m^2

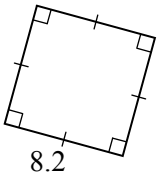


5



6

Remember Square Units



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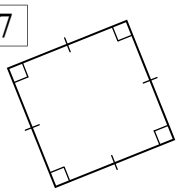

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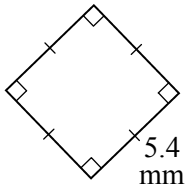
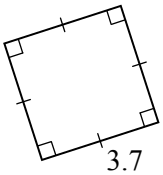


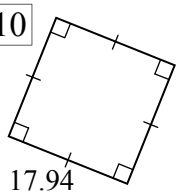
Keep going with these, but round your answer to 1d.p.

9



Round answer to 1 d.p.

10



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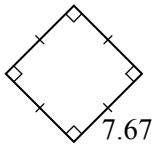

Find the price of these square areas. Find the area of the square, then multiply by the cost.

Thomas has a square roof on a shed that has a side length of 3.4 m. Find:

12 The area of the square roof

No Rounding

13 Rewrite your answer above rounded **UP** to whole m^2.

$A =$

14 The cost to replace the roof if iron sheeting costs \$41.80/m^2

Cost = $\square \times \square$

Area (Q13) | Iron Cost

Cost = $\square$

Show the \$ sign and cents to 2 d.p.

Phom is constructing a raised 6.5 m square platform for use in his gym.

15 Find the platform area.

16 Rewrite your answer above rounded **UP** to whole m^2.

17 Find the build cost for the platform, MDF costs \$17.80/m^2

Cost =

Cost =

18 Draw a square that has sides 4.1 cm length below, show perpendicular symbols and dashes on the sides. Write the side length on one side Make sure you use a ruler.

19 Calculate the area of the square you have just drawn.

Area of Rectangles

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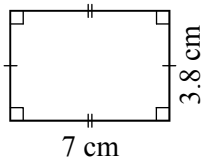
Now rectangles. Use A=*lb* to find the area, round to 1 d.p. when required.

Example



$A = lb$

$= 7 \times 3.8$

$A = 26.6\ cm^2$

Show square units

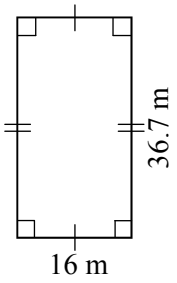
1

$A = lb$

$= \square \times \square$

Multiply, in any order

$A = \square\ m^2$



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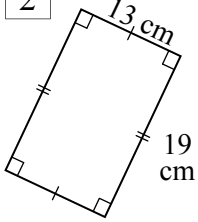


$A = lb$

$= \square \times \square$

$A = \square\ cm^2$

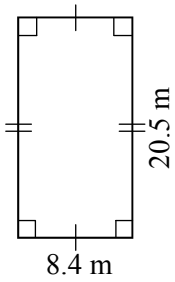
3

$A = lb$

$= \square \times \square$

$A = \square$

Show square units



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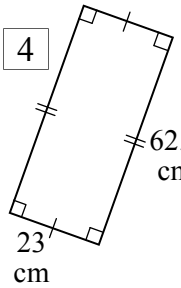

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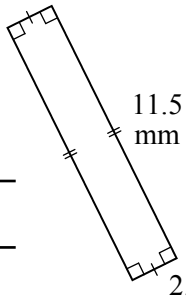

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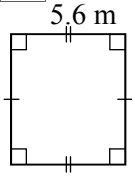
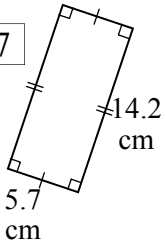


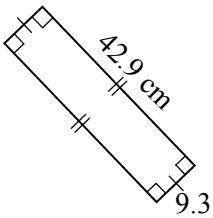
Find the area, but round your answer to 1d.p.

7



Round to 1 d.p.

8



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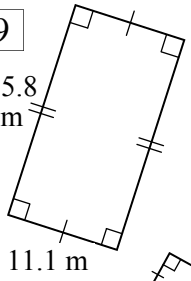

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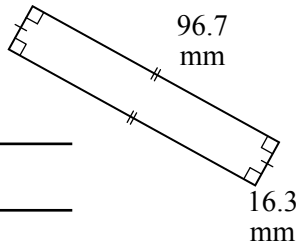

Draw these rectangles then find their area. Mark matching side lengths, show right angles and write side lengths on two sides.

RULER!

11 3.2 cm by 2.5 cm

12 Calculate the area of the rectangle you have just drawn.

13 5.3 cm by 3.7 cm

Long side across

14 Calculate the area of the rectangle you have just drawn.

Now find the area, then calculate the cost.

Jason is painting his lounge room's rectangular ceiling. It is 4.7 × 3.4 m.

15 Find the ceiling's area

16 Rewrite your answer above rounded **UP** to whole m^2.

17 If 1 litre of paint covers 9 m^2 how many whole litres of paint are required.

Paint =

18 If 1 litre of paint costs $28.90 calculate the cost to paint the ceiling.

Cost =

Cost =

Area of Squares and Rectangles - Unit Conversion

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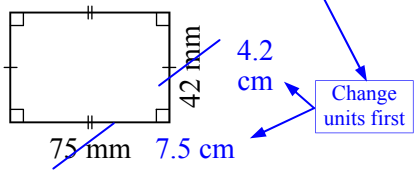
Change the units first, then use A=*lb* or A = l^2 to find the area.

Example Answer in cm^2.

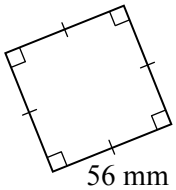


A = *lb*

= 7.5 × 4.2

A = 31.5 cm^2 ← Show square units

1 Answer in cm^2.



A = l^2

= ☐^{2}

A = ☐ cm^2

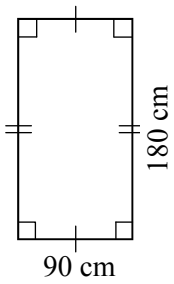
2 Answer in m^2.

A = *lb*

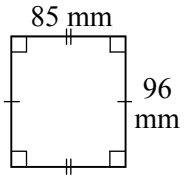
= ☐ × ☐

A = ☐

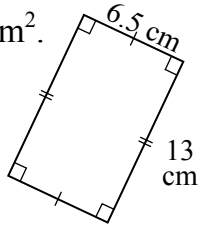
Show square units



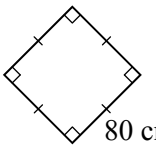
3 Answer in cm^2.



5 Answer in mm^2.

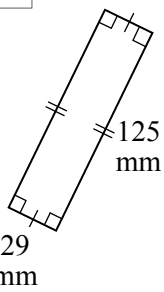


6 in m^2.

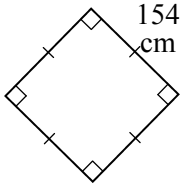


Keep going, but round your answer to 1 decimal place.

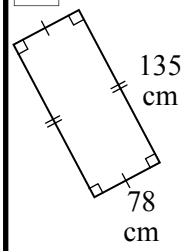
7 Answer in cm^2.



8 Answer in m^2.



9 Answer in m^2.



Draw a rectangle, mark the matching side lengths, show right angles and write the side lengths on two sides. Ruler!

10 4.5 cm by 2.8 cm

11 Calculate the area of the rectangle you have just drawn, but give your answer in mm^2.

Now find the area, to answer the questions.

A rectangular farm paddock is 1.7 km by 850 m wide.

12 Convert 1.7 km to metres.

1.7 km = ☐ m

13 Find the area of the field in square metres.

14 1 ha = 10 000 m^2, rewrite the area in ha

A = ☐ ÷ ☐

Divide Q. 13 by 10 000

A = ☐ ha

A square farm paddock has sides of 1.25 km.

15 Convert 1.25 km to metres.

1.25 km = ☐ m

16 Find the area of the field in square metres.

17 Convert the area to ha, rounding to 1 decimal place.

A =

A =

Area of Squares

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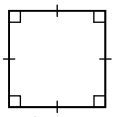
Use $A=l^2$ to find the area of these squares, to 1 d.p. Show units with a ^{2} after them.

1



$A = l^2$

$= 9^2$

Calc: 9 x^2 =

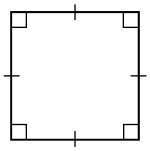
$A = 81$ cm^2

2

$A = l^2$

$= 17^2$

$A = 289$ cm^2



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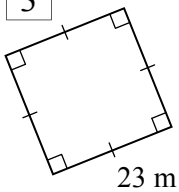


$A = l^2$

$= 23^2$

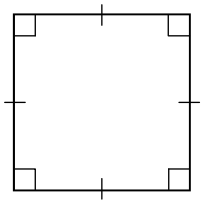
$A = 529$ m^2

4

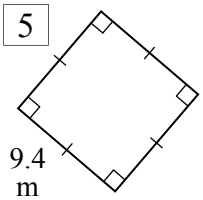
$A = l^2$

$= 4.5^2$

$A = 20.25$ m^2



5



$A = l^2$

$= 9.4^2$

$A = 88.36$ m^2

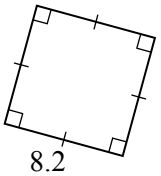
Remember Square Units

6

$A = l^2$

$= 8.2^2$

$A = 67.24$ cm^2



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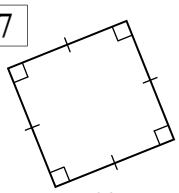


$A = l^2$

$= 64^2$

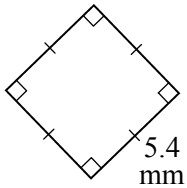
$A = 4\,096$ cm^2

8

$A = l^2$

$= 5.4^2$

$A = 29.16$ mm^2



Keep going with these, but round your answer to 1d.p.

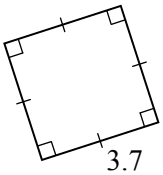
9

$A = l^2$

$= 3.7^2$

$A = 13.7$ cm^2

Round answer to 1 d.p.



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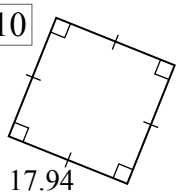
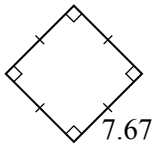


$A = l^2$

$= 17.94^2$

$A = 321.8$ mm^2

11



$A = l^2$

$= 7.67^2$

$A = 58.8$ m^2

Find the price of these square areas. Find the area of the square, then multiply by the cost.

Thomas has a square roof on a shed that has a side length of 3.4 m. Find:

12 The area of the square roof

$A = l^2$

$= 3.4^2$

$A = 11.56$ m^2 ← No Rounding

13 Rewrite your answer above rounded **UP** to whole m^2.

$A = 12$ m^2

14 The cost to replace the roof if iron sheeting costs \$41.80/m^2

Cost = 12 (Area (Q13)) × 41.80 (Iron Cost)

Cost = \$501.60

Show the \$ sign and cents to 2 d.p.

Phom is constructing a raised 6.5 m square platform for use in his gym.

15 Find the platform area.

$A = l^2$

$= 6.5^2$

$A = 42.25$ m^2

16 Rewrite your answer above rounded **UP** to whole m^2.

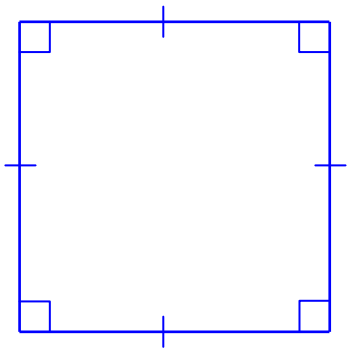
Area = 43 m^2

17 Find the build cost for the platform, MDF costs \$17.80/m^2

Cost = 43×17.80

Cost = \$765.40

18 Draw a square that has sides 4.1 cm length below, show perpendicular symbols and dashes on the sides. Write the side length on one side Make sure you use a ruler.



19 Calculate the area of the square you have just drawn.

$A = l^2$

$= 4.1^2$

$A = 16.81$ cm^2

Area of Rectangles

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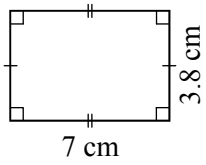
Now rectangles. Use A=*lb* to find the area, round to 1 d.p. when required.

Example



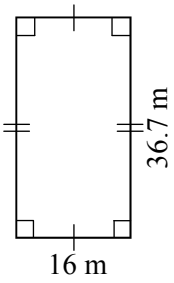
$A = lb$

$= 7 \times 3.8$

$A = 26.6 \text{ cm}^2$

Show square units

1



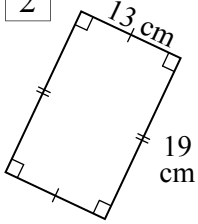
$A = lb$

$= 36.7 \times 16$

Multiply, in any order

$A = 587.2 \text{ m}^2$

2

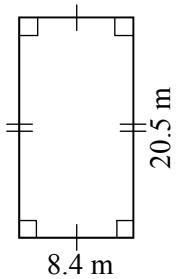


$A = lb$

$= 13 \times 19$

$A = 247 \text{ cm}^2$

3



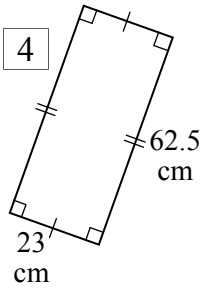
$A = lb$

$= 20.5 \times 8.4$

$A = 172.2 \text{ m}^2$

Show square units

4

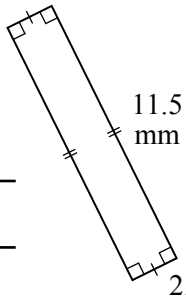


$A = lb$

$= 62.5 \times 23$

$A = 1\,437.5 \text{ cm}^2$

5

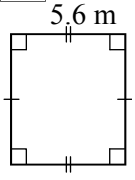


$A = lb$

$= 11.5 \times 2.8$

$A = 32.2 \text{ mm}^2$

6



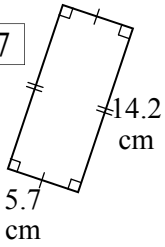
$A = lb$

$= 7.8 \times 5.6$

$A = 43.68 \text{ m}^2$

Find the area, but round your answer to 1d.p.

7



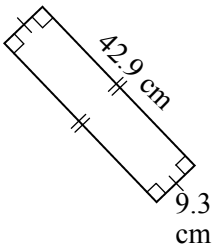
$A = lb$

$= 14.2 \times 5.7$

$A = 80.9 \text{ cm}^2$

Round to 1 d.p.

8

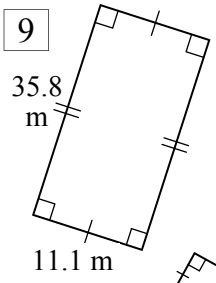


$A = lb$

$= 42.9 \times 9.3$

$A = 399.0 \text{ cm}^2$

9

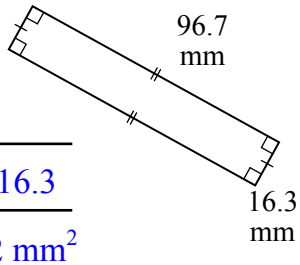


$A = lb$

$= 35.8 \times 11.1$

$A = 397.4 \text{ m}^2$

10



$A = lb$

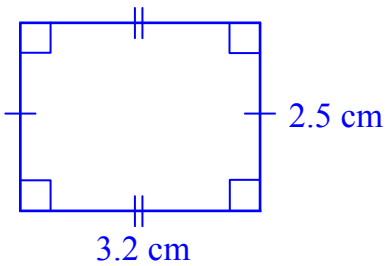
$= 96.7 \times 16.3$

$A = 1\,576.2 \text{ mm}^2$

Draw these rectangles then find their area. Mark matching side lengths, show right angles and write side lengths on two sides.

RULER!

11 3.2 cm by 2.5 cm



12 Calculate the area of the rectangle you have just drawn.

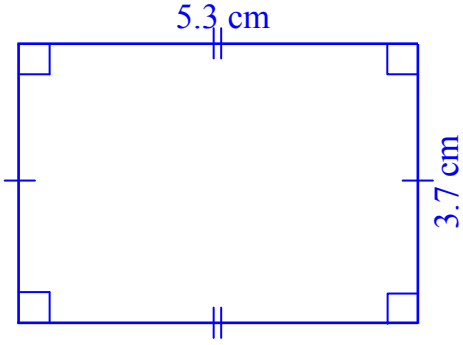
$A = lb$

$= 3.2 \times 2.5$

$A = 8 \text{ m}^2$

13 5.3 cm by 3.7 cm

Long side across



14 Calculate the area of the rectangle you have just drawn.

$A = lb$

$= 5.3 \times 3.7$

$A = 19.61 \text{ cm}^2$

Now find the area, then calculate the cost.

Jason is painting his lounge room's rectangular ceiling. It is 4.7 × 3.4 m.

15 Find the ceiling's area

$A = lb$

$= 4.7 \times 3.4$

$A = 15.98 \text{ m}^2$

16 Rewrite your answer above rounded **UP** to whole m^2.

$A = 16 \text{ m}^2$

17 If 1 litre of paint covers 9 m^2 how many whole litres of paint are required.

Paint $16 \div 9 = 1.\dot{7} = 2$ L

18 If 1 litre of paint costs $28.90 calculate the cost to paint the ceiling.

Cost = 2×28.90

Cost = $57.80

Area of Squares and Rectangles - Unit Conversion

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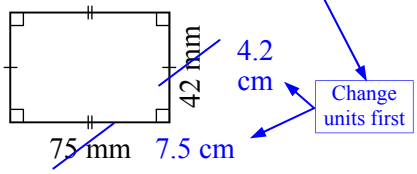
Change the units first, then use A=*lb* or A = l^2 to find the area.

Example Answer in cm^2.

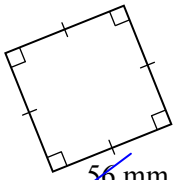


A = *lb*

= 7.5 × 4.2

A = 31.5 cm^2 ← Show square units

1 Answer in cm^2.



A = l^2

= 5.6^2

A = 31.36 cm^2

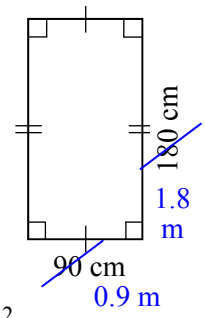
2 Answer in m^2.

A = *lb*

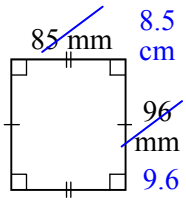
= 1.8 × 0.9

A = 1.62 m^2

Show square units



3 Answer in cm^2.

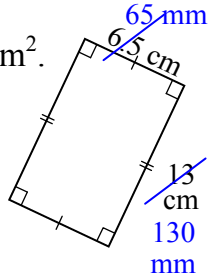


A = *lb*

= 9.6 × 8.5

A = 81.6 cm^2

5 Answer in mm^2.

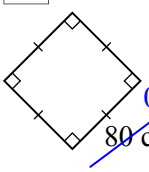


A = *lb*

= 130 × 65

A = 8 450 mm^2

6 in m^2.



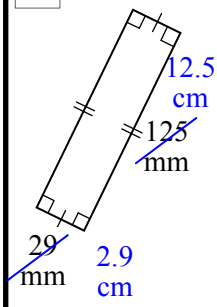
A = l^2

= 0.8^2

A = 0.64 m^2

Keep going, but round your answer to 1 decimal place.

7 Answer in cm^2.

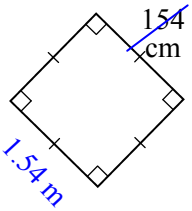


A = *lb*

= 12.5 × 2.9

A = 36.3 cm^2

8 Answer in m^2.

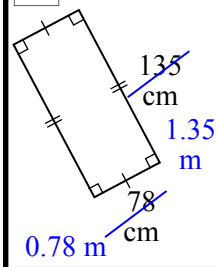


A = l^2

= 1.54^2

A = 2.4 m^2

9 Answer in m^2.



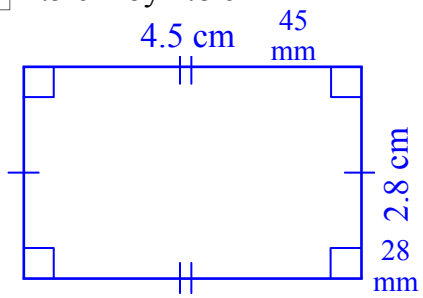
A = *lb*

= 1.35 × 0.78

A = 1.1 m^2

Draw a rectangle, mark the matching side lengths, show right angles and write the side lengths on two sides. Ruler!

10 4.5 cm by 2.8 cm



11 Calculate the area of the rectangle you have just drawn, but give your answer in mm^2.

A = *lb*

= 45 × 28

A = 1 260 mm^2

Now find the area, to answer the questions.

A rectangular farm paddock is 1.7 km by 850 m wide.

12 Convert 1.7 km to metres.

1.7 km = 1 700 m

13 Find the area of the field in square metres.

A = *lb*

= 1 700 × 850

A = 1 445 000 m^2

14 1 ha = 10 000 m^2, rewrite the area in ha

A = 1 445 000 ÷ 10 000

Divide Q. 13 by 10 000

A = 144.5 ha

A square farm paddock has sides of 1.25 km.

15 Convert 1.25 km to metres.

1.25 km = 1 250 m

16 Find the area of the field in square metres.

A = l^2

= 1 250^2

A = 1 562 500 m^2

17 Convert the area to ha, rounding to 1 decimal place.

A = 1 562 500 ÷ 10 000

A = 156.3 ha

Area - Area of Squares and Rectangles (Homework Strip)

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Date Due: _____

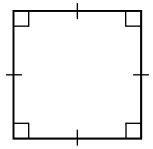
Find the area of these squares

1

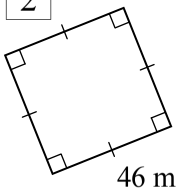
$A = l^2$

$= \square^2$

$A = \square$ cm^2



2



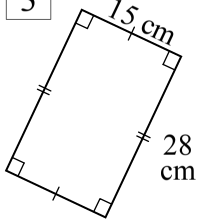
$A = l^2$

$= \square^2$

$A = \square$ m^2

Find the area of these rectangles

3



$A = lb$

$= \square \times \square$

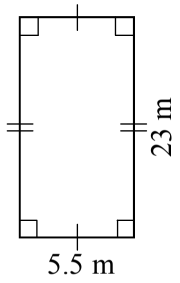
$A = \square$ cm^2

4

$A = lb$

$= \square \times \square$

$A = \square$



Date Due: _____

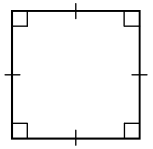
Find the area of these squares

1

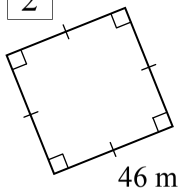
$A = l^2$

$= \square^2$

$A = \square$ cm^2



2



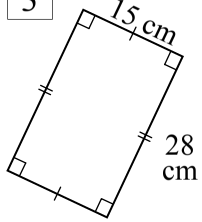
$A = l^2$

$= \square^2$

$A = \square$ m^2

Find the area of these rectangles

3



$A = lb$

$= \square \times \square$

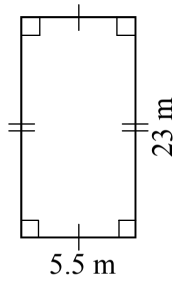
$A = \square$ cm^2

4

$A = lb$

$= \square \times \square$

$A = \square$



Date Due: _____

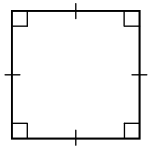
Find the area of these squares

1

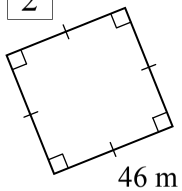
$A = l^2$

$= \square^2$

$A = \square$ cm^2



2



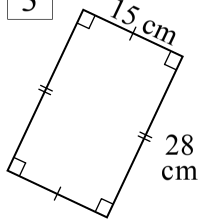
$A = l^2$

$= \square^2$

$A = \square$ m^2

Find the area of these rectangles

3



$A = lb$

$= \square \times \square$

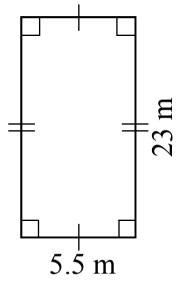
$A = \square$ cm^2

4

$A = lb$

$= \square \times \square$

$A = \square$



Date Due: _____

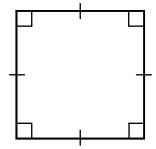
Find the area of these squares

1

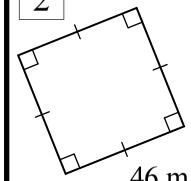
$A = l^2$

$= \square^2$

$A = \square$ cm^2



2



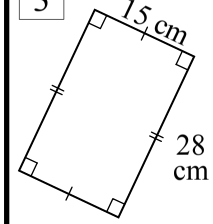
$A = l^2$

$= \square^2$

$A = \square$ m^2

Find the area of these rectangles

3



$A = lb$

$= \square \times \square$

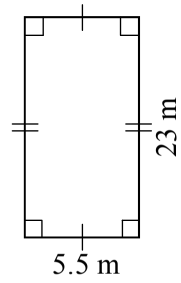
$A = \square$ cm^2

4

$A = lb$

$= \square \times \square$

$A = \square$



Area - Area of Squares and Rectangles (Homework Strip)

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Date Due:

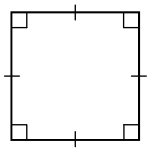
Find the area of these squares

1

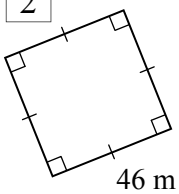
$A = l^2$

$= 35^2$

$A = 1\,225 \text{ cm}^2$



2



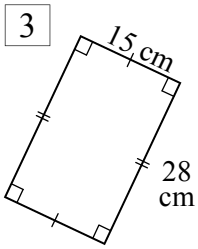
$A = l^2$

$= 46^2$

$A = 2\,116 \text{ m}^2$

Find the area of these rectangles

3



$A = lb$

$= 28 \times 15$

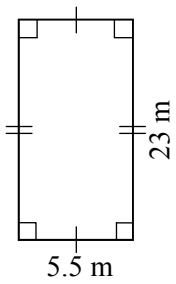
$A = 420 \text{ cm}^2$

4

$A = lb$

$= 23 \times 5.5$

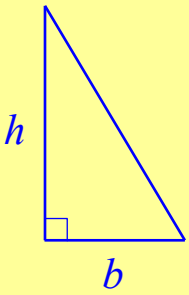
$A = 126.5 \text{ m}^2$



Area of Triangles

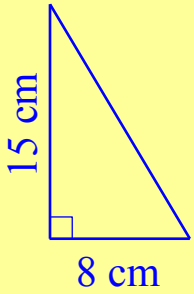
The area of a triangle is found by "half times the base times the perpendicular height." A triangle is half of a rectangle, so it makes sense that the formula is the same as that of a rectangle except it is halved.

Triangle $A = \frac{1}{2}bh$



Where:
b = base length
h = perpendicular height

Note that the height is the perpendicular height, the base and the height must always be at right angles to each other. Below is an example of finding the area of a triangle.



$A = \frac{1}{2}bh$ ← Write the formula

$= \frac{1}{2} \times 8 \times 15$ ← Substitute b and h

$A = 60\ cm^2$ ← Solve showing square units. Round answer if required.

Sheet 1 Column 1 asks you to find the area for a triangle. You may find it hard to decide which is the height and which is the base. It won't affect the answer, whichever you choose. Remember to watch for units and it is quicker to use 0.5 rather than a ½ on your calculator.

Column 2 continues finding the area of a triangle but this time you have to select which numbers you multiply, remember that the height must be perpendicular to the base length. So cross out the length you don't need, from Q 11, you remove 2 lengths.

Column 3 continues with finding the area, but this time you need to round your answer to 1 d.p.

Sheet 2 starts with asking you to draw triangles and calculate the area, rounding to 1 d.p. There are marks on the sheet to show you where to start the construction. Column 2 are some worded questions on area. Column 3 asks you to change units to match a given area unit. So you have to convert one of the side lengths. Except with the last question which requires a double conversion.

Area of Triangles (Sheet 1)

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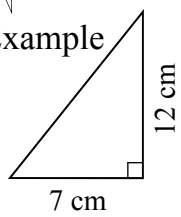
Find the area of these triangles.
Units are square units, e.g. cm^2.

Example



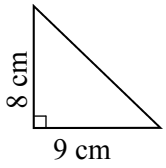
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 7 \times 12$

$A = 42\ cm^2$

Show square units

1



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

Calc: 0 . 5 × 9 × 8 =

$A = \square\ cm^2$

2

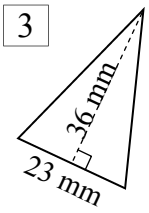
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square\ m^2$



3



$A = \frac{1}{2}bh$

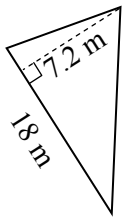
$= \frac{1}{2} \times \square \times \square$

$A = \square$

Show square units

4

$A = \frac{1}{2}bh$



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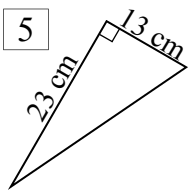

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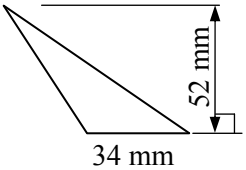
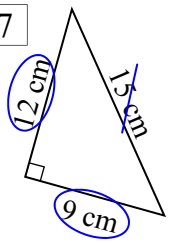


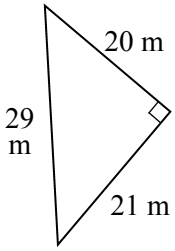
Find the area of these triangles.
Cross out the lengths you don't use, circle the two that you do.

7



Show square units

8



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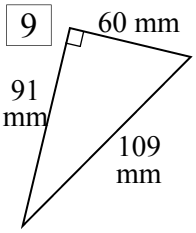

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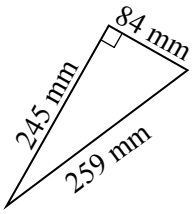

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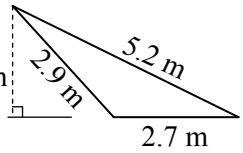
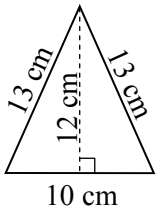


Circle two
Cross out two
Next 3 Q's

12



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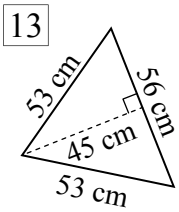
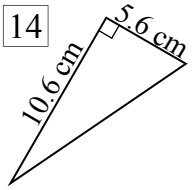


Round your answers to 1 d.p.
Find the area of these triangles.

14



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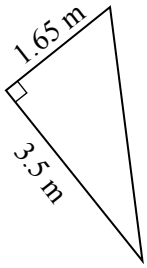

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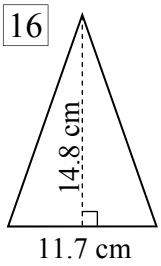

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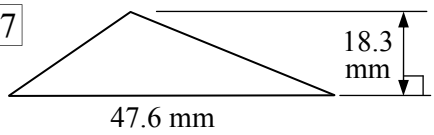

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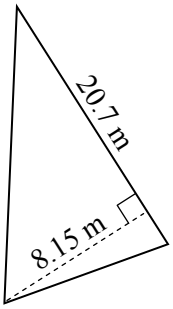

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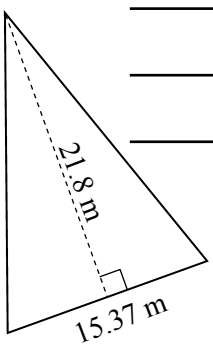

Area of Triangles (Sheet 2)

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Draw a right-angled triangle label the two side lengths and show the right angle. Ruler!

1 Base: 4.3 cm, Height: 3.7 cm

2 Calculate the area of the triangle above, round to 1 d.p.

3 Base: 3.3 cm, Height: 2.8 cm

4 Calculate the area of the triangle above, round to 1 d.p.

5 Base: 8.2 cm, Height: 3.8 cm

Now apply your skills to these.

7 A triangular advertising sail is 8.5 m tall and has a base of 1.2 m. Find its area.

8 If space on the sail is sold for \$85/m^2, how much will it cost to advertise on the whole sail?

Cost =

Cost =

9 A triangular courtyard has base 6.8 m and height 10 m. Find its area.

10 The courtyard is to be tiled and needs 16 tiles/m^2. How many tiles will be used to tile the entire courtyard floor?

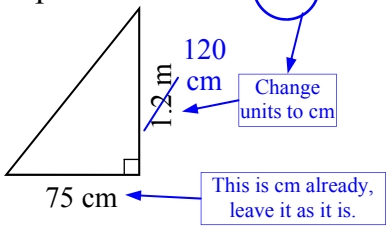
Tiles =

Tiles =

6 Calculate the area of the triangle you have drawn, 1 d.p.

Change the units to match the required area measurement, then find the area, to 1 d.p.

Example Answer in cm^2.

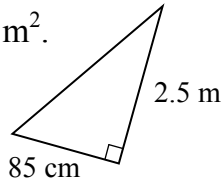


$A = \frac{1}{2}bh$

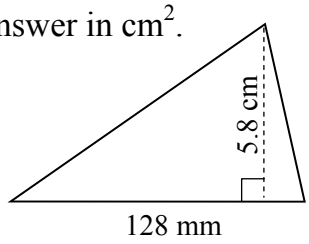
$= \frac{1}{2} \times 75 \times 120$

$A = 4\,500$ cm^2

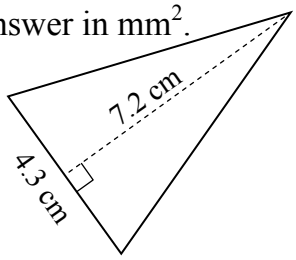
11 Answer in m^2.



12 Answer in cm^2.



13 Answer in mm^2.



Area of Triangles (Sheet 1)

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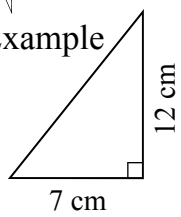
Find the area of these triangles.
Units are square units, e.g. cm^2.

Example



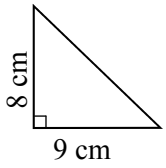
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 7 \times 12$

$A = 42\ cm^2$

Show square units

1



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 9 \times 8$

Calc: 0 . 5 × 9 × 8 =

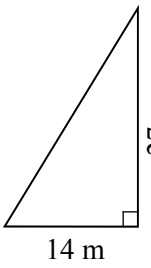
$A =$ 36 cm^2

2

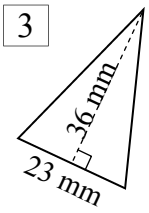
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 14 \times 27$

$A =$ 189 m^2



3



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 23 \times 36$

$A =$ 414 mm^2

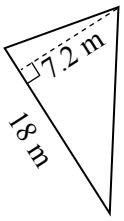
Show square units

4

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 7.2$

$A = 64.8\ m^2$



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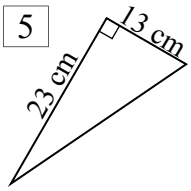
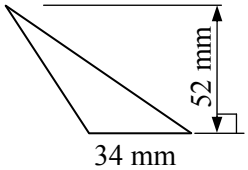


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 13 \times 23$

$A = 149.5\ cm^2$

6



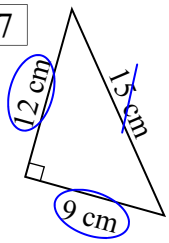
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 34 \times 52$

$A = 884\ cm^2$

Find the area of these triangles.
Cross out the lengths you don't use, circle the two that you do.

7



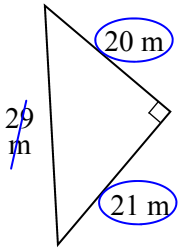
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 13 \times 23$

$A = 149.5\ cm^2$

Show square units

8

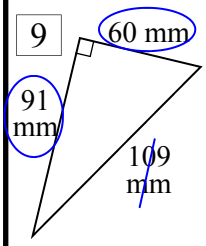


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 20 \times 21$

$A = 210\ m^2$

9



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 60 \times 91$

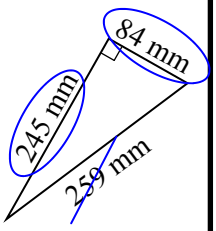
$A = 2\ 730\ mm^2$

10

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 84 \times 245$

$A = 10\ 290\ mm^2$



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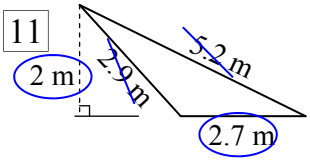


Circle two
Cross out two
Next 3 Q's

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 2.7 \times 2$

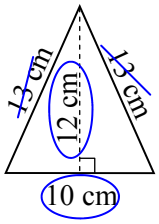
$A = 2.7\ m^2$

12

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 10 \times 12$

$A = 60\ cm^2$



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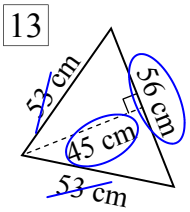


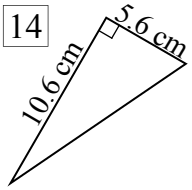
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 56 \times 45$

$A = 1\ 260\ cm^2$

Round your answers to 1 d.p.
Find the area of these triangles.

14



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5.6 \times 10.6$

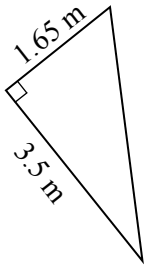
$A = 29.7\ cm^2$

15

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3.5 \times 1.65$

$A = 2.9\ m^2$



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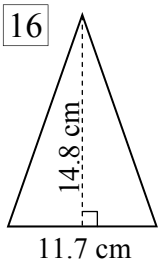
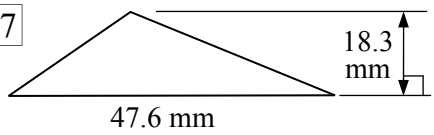


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 11.7 \times 14.8$

$A = 86.6\ cm^2$

17



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 47.6 \times 18.3$

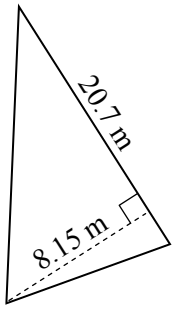
$A = 435.54\ mm^2$

18

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 20.7 \times 8.15$

$A = 84.4\ m^2$



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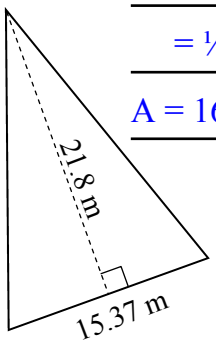

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 15.37 \times 21.8$

$A = 167.5\ m^2$

Area of Triangles (Sheet 2)

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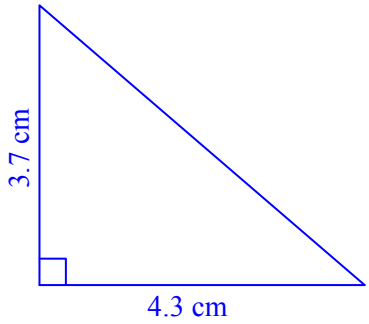
Draw a right-angled triangle label the two side lengths and show the right angle. Ruler!

1 Base: 4.3 cm, Height: 3.7 cm



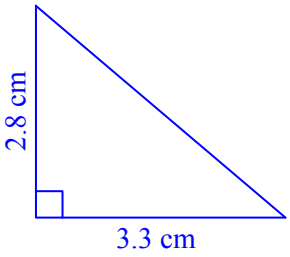
2 Calculate the area of the triangle above, round to 1 d.p.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4.3 \times 3.7$

$A = 8.0 \text{ cm}^2$

3 Base: 3.3 cm, Height: 2.8 cm



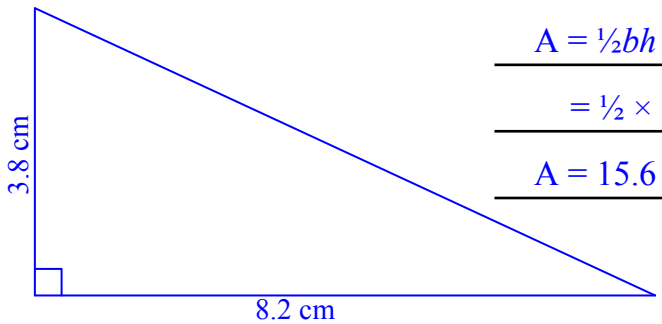
4 Calculate the area of the triangle above, round to 1 d.p.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3.3 \times 2.8$

$A = 4.6 \text{ cm}^2$

5 Base: 8.2 cm, Height: 3.8 cm



6 Calculate the area of the triangle you have drawn, 1 d.p.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8.2 \times 3.8$

$A = 15.6 \text{ cm}^2$

Now apply your skills to these.

7 A triangular advertising sail is 8.5 m tall and has a base of 1.2 m. Find its area.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 1.2 \times 8.5$

$A = 5.1 \text{ m}^2$

8 If space on the sail is sold for \$85/m^2, how much will it cost to advertise on the whole sail?

Cost = 5.1×85

Cost = \$433.50

9 A triangular courtyard has base 6.8 m and height 10 m. Find its area.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6.8 \times 10$

$A = 34 \text{ m}^2$

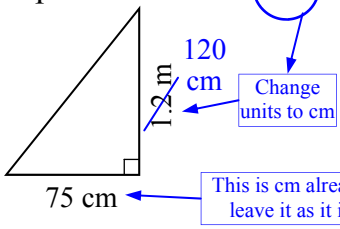
10 The courtyard is to be tiled and needs 16 tiles/m^2. How many tiles will be used to tile the entire courtyard floor?

Tiles = 34×16

Tiles = 544 tiles

Change the units to match the required area measurement, then find the area, to 1 d.p.

Example Answer in cm^2.

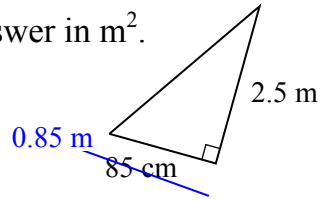


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 75 \times 120$

$A = 4\,500 \text{ cm}^2$

11 Answer in m^2.

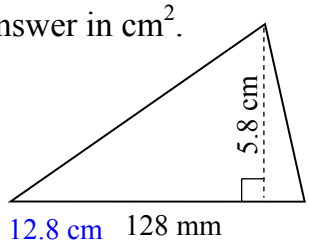


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 0.85 \times 2.5$

$A = 1.1 \text{ m}^2$

12 Answer in cm^2.

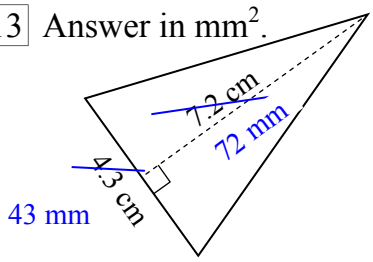


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 12.8 \times 5.8$

$A = 37.1 \text{ cm}^2$

13 Answer in mm^2.



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 43 \times 72$

$A = 1\,548 \text{ mm}^2$

Area - Area of Triangles (Homework Strip)

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Date Due:

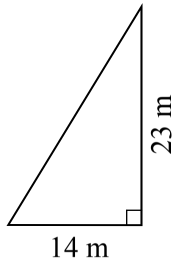
Find the area of these triangles

1

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square$ m^2



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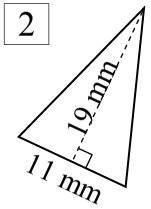


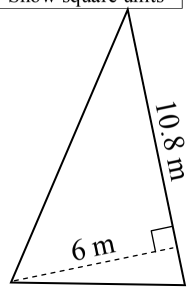
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

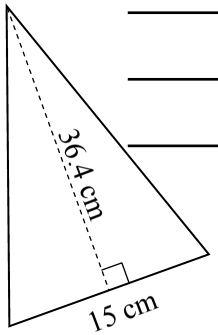
$A = \square$

Show square units

3



4



Date Due:

Find the area of these triangles

1

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square$ m^2

23 m

14 m

2

19 mm

11 mm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square$

Show square units

3

10.8 m

6 m

4

36.4 cm

15 cm

Date Due:

Find the area of these triangles

1

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square$ m^2

23 m

14 m

2

19 mm

11 mm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square$

Show square units

3

10.8 m

6 m

4

36.4 cm

15 cm

Date Due:

Find the area of these triangles

1

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square$ m^2

23 m

14 m

2

19 mm

11 mm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square$

Show square units

3

10.8 m

6 m

4

36.4 cm

15 cm

Area - Area of Triangles (Homework Strip)

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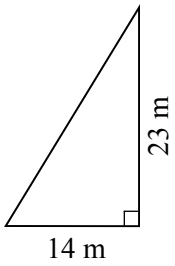
Date Due:

Find the area of these triangles

1

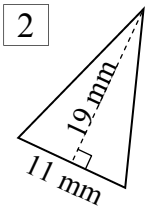


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 14 \times 23$

$A = 161 \text{ m}^2$

2



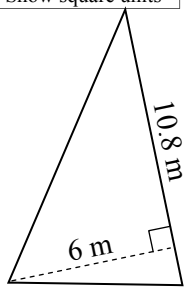
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 11 \times 19$

$A = 104.5 \text{ mm}^2$

Show square units

3

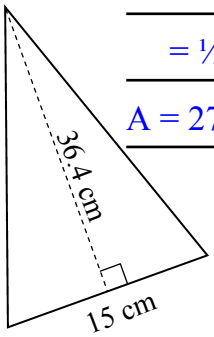


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 10.8 \times 6$

$A = 32.4 \text{ m}^2$

4



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 15 \times 36.4$

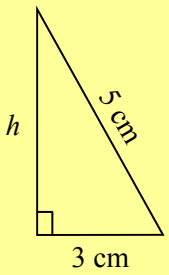
$A = 273 \text{ cm}^2$

Finding a Short Side - Whole Number Answers

Pythagoras' Theorem $c^2 = a^2 + b^2$ can be used to find the length of the hypotenuse given the two short sides of the right-angled triangle. The process can be reversed when given the length of the hypotenuse and one of the short sides of the triangle.

$$b^2 = c^2 - a^2$$

In other words, use subtraction instead of addition. Just remember to use the hypotenuse length as '*c*' (the first number) as if you don't you will get a calculator error.



$h^2 = 5^2 - 3^2$ ← Remember to use '-' not '+' and it is the hypotenuse squared minus the short side squared.

$h^2 = 16$ ← Rewrite h^2 then subtract the two squared numbers.

$h = \sqrt{16}$ ← Rewrite the line above but change h^2 to h and write the number in a square root sign.

$h = 4$ cm ← Solve the square root, on this sheet all the answers are whole numbers, include the units.

Calculator Steps:

5 x^2 - 4 x^2 =

$\sqrt{\ }$ ANS =

The entire sheet is answered in the same way as the method above. Two of the diagrams in Column 3 include isosceles triangles, these are solved by bisecting the triangle to create a right -angled triangle.

The second sheet involves using Pythagoras Theorem to first find a required side of the triangle, then use the area formula. Remember that to find the area, you require the base and the perpendicular height. All these triangles require Pythagoras Theorem (subtraction) first.

Column 1 has an example for you to get you started.

Finding a Short Side - Whole Number Answers

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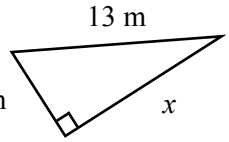
Find the length of the short side, the answer will be a whole number.

Example



$x^2 = 13^2 - 5^2$

Calc: 1 3 x^2 - 5 x^2 =

$x^2 = 144$

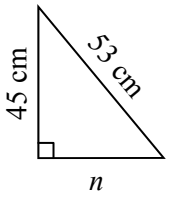
$x = \sqrt{144}$

Calc: $\sqrt{}$ Ans =

Write the units

$x = 12$ m

1



$n^2 = 53^2 - \square^2$

$n^2 = \square$

$n = \sqrt{\square}$

$n = \square$ cm

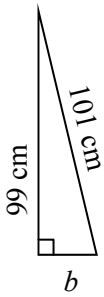
2

$b^2 = \square^2 - \square^2$

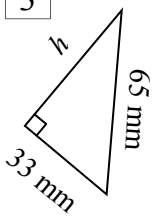
$b^2 = \square$

$b = \sqrt{\square}$

$b = \square$ cm



3



$h^2 =$

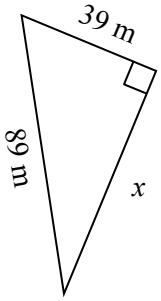
$h^2 =$

$h =$

$h =$

Remember the units

4



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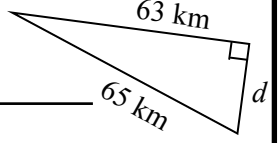

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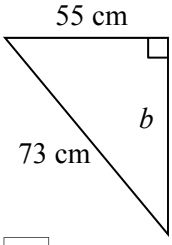

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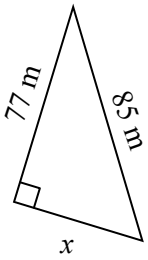

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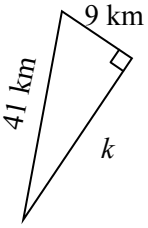

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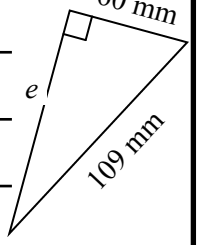
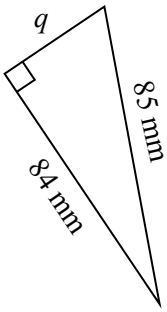
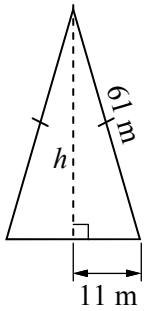


10

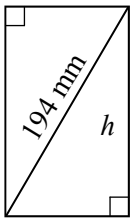


Find '*h*' the unknown height, the answer will be a whole number.

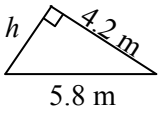
11



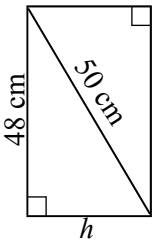
12



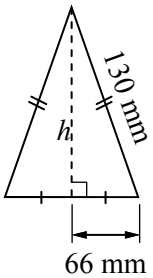
13



14



15



Area of Triangles - Pythagoras

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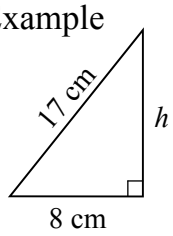
Use Pythagoras Theorem and subtraction for these. Find the length, then find the area. Answers are all whole numbers.

Example



$h^2 = 17^2 - 8^2$

$h^2 = 225$

$h = \sqrt{225}$

$h = 15$ cm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 15$

$A = 60$ cm^2

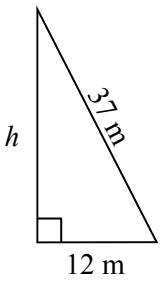
1

$h^2 =$ [37]^2 - [ ]^2

$h^2 =$ [ ]

$h = \sqrt{\ }$ [ ]

$h =$ [ ] m



This answer should be a whole number and less than 37.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ [ ] $\times$ [ ]

Use the base of '12' and the height, that you calculated.

$A =$ [ ] m^2

This answer is an area, use square units.

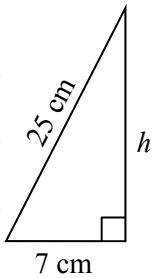
2

$h^2 =$ [ ]^2 - [ ]^2

$h^2 =$ [ ]

$h = \sqrt{\ }$ [ ]

$h =$ [ ] cm



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ [ ] $\times$ [ ]

$A =$ [ ] cm^2

3

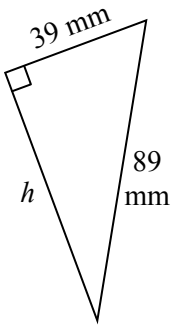
$h^2 =$ [ ]^2 - [ ]^2

$h^2 =$ [ ]

$h = \sqrt{\ }$ [ ]

$h =$ [ ]

Length, so no square units.



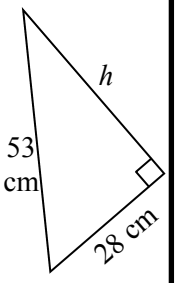
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ [ ] $\times$ [ ]

$A =$ [ ]

Area, so use square units.

4



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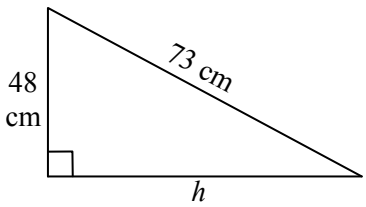
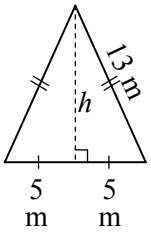


With isosceles triangles use the same method, except when you find the area, use the whole base length, not half.

6



$h^2 = 13^2 - 5^2$

$h^2 =$ [ ]

$h = \sqrt{\ }$ [ ]

$h =$ [ ] m

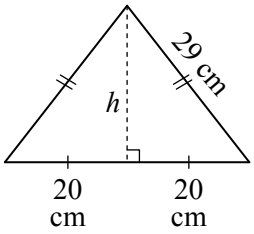
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ [ ] $\times$ [ ]

Use $b = 10$ (2×5) not 5!

$A =$ [ ] m^2

7



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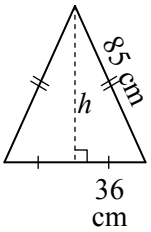

Finding a Short Side - Whole Number Answers

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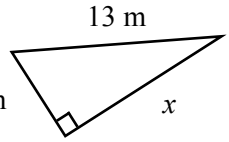
Find the length of the short side, the answer will be a whole number.

Example



$x^2 = 13^2 - 5^2$

Calc: 1 3 x^2 - 5 x^2 =

$x^2 = 144$

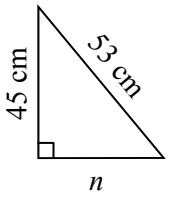
$x = \sqrt{144}$

Calc: $\sqrt{\ }$ Ans =

$x = 12$ m

Write the units

1



$n^2 = 53^2 - 45^2$

$n^2 = 784$

$n = \sqrt{784}$

$n = 28$ cm

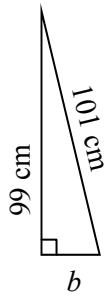
2

$b^2 = 101^2 - 99^2$

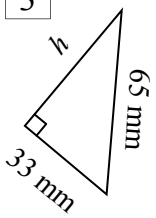
$b^2 = 400$

$b = \sqrt{400}$

$b = 20$ cm



3



$h^2 = 65^2 - 33^2$

$h^2 = 3\,136$

$h = \sqrt{3\,136}$

$h = 56$ mm

Remember the units

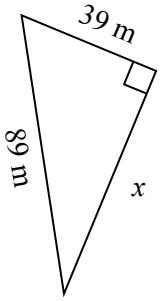
4

$x^2 = 89^2 - 39^2$

$x^2 = 6\,400$

$x = \sqrt{6\,400}$

$x = 80$ m



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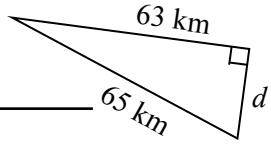


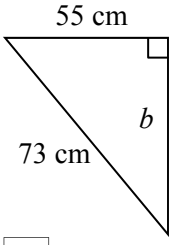
$d^2 = 65^2 - 63^2$

$d^2 = 256$

$d = \sqrt{256}$

$d = 16$ km

6



$b^2 = 73^2 - 55^2$

$b^2 = 2\,304$

$b = \sqrt{2\,304}$

$b = 48$ cm

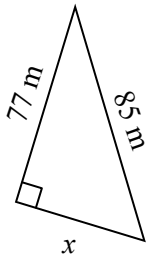
7

$x^2 = 85^2 - 77^2$

$x^2 = 1\,296$

$x = \sqrt{1\,296}$

$x = 36$ m



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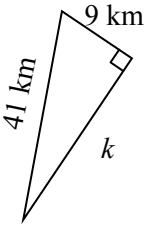


$k^2 = 41^2 - 9^2$

$k^2 = 1\,600$

$k = \sqrt{1\,600}$

$k = 40$ km

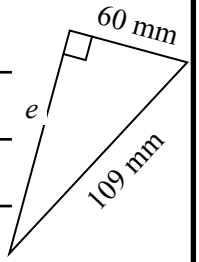
9

$e^2 = 109^2 - 60^2$

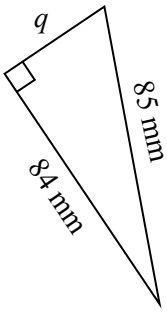
$e^2 = 8\,281$

$e = \sqrt{8\,281}$

$e = 91$ mm



10



$q^2 = 85^2 - 84^2$

$q^2 = 169$

$q = \sqrt{169}$

$q = 13$ mm

Find 'h' the unknown height, the answer will be a whole number.

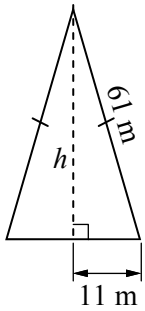
11

$h^2 = 61^2 - 11^2$

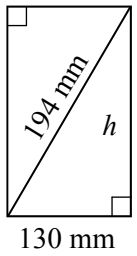
$h^2 = 3\,600$

$h = \sqrt{3\,600}$

$h = 60$ m



12



$h^2 = 194^2 - 130^2$

$h^2 = 20\,736$

$h = \sqrt{20\,736}$

$h = 144$ mm

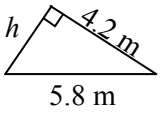
13

$h^2 = 5.8^2 - 4.2^2$

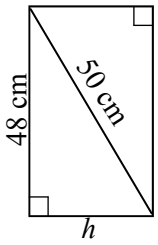
$h^2 = 16$

$h = \sqrt{16}$

$h = 4$ m



14



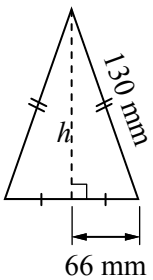
$h^2 = 50^2 - 48^2$

$h^2 = 196$

$h = \sqrt{196}$

$h = 14$ cm

15



$h^2 = 130^2 - 66^2$

$h^2 = 12\,544$

$h = \sqrt{12\,544}$

$h = 112$ mm

Area of Triangles - Pythagoras

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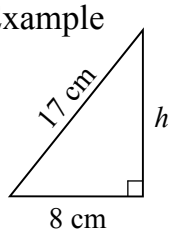
Use Pythagoras Theorem and subtraction for these. Find the length, then find the area. Answers are all whole numbers.

Example



$h^2 = 17^2 - 8^2$

$h^2 = 225$

$h = \sqrt{225}$

$h = 15$ cm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 15$

$A = 60$ cm^2

1

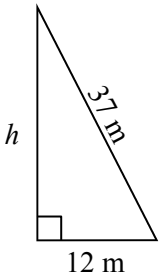
$h^2 =$ 37 ^{2} - 12 ^{2}

$h^2 =$ 1 225

$h = \sqrt{1\ 225}$

$h =$ 35 m

This answer should be a whole number and less than 37.



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ 12 $\times$ 35

Use the base of '12' and the height, that you calculated.

$A =$ 210 m^2

This answer is an area, use square units.

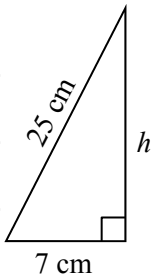
2

$h^2 =$ 25 ^{2} - 7 ^{2}

$h^2 =$ 576

$h = \sqrt{576}$

$h =$ 24 cm



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ 7 $\times$ 24

$A =$ 84 cm^2

3

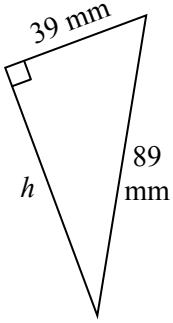
$h^2 =$ 89 ^{2} - 39 ^{2}

$h^2 =$ 6 400

$h = \sqrt{6\ 400}$

$h =$ 80 mm

Length, so no square units.



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ 39 $\times$ 80

$A =$ 1 560 mm^2

Area, so use square units.

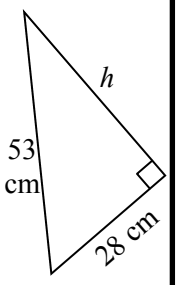
4

$h^2 = 53^2 - 28^2$

$h^2 = 2\ 025$

$h = \sqrt{2\ 025}$

$h = 45$ cm

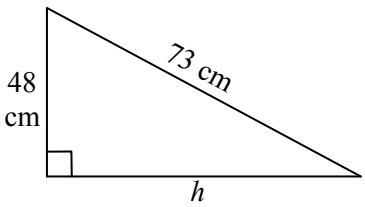


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 28 \times 45$

$A = 630$ cm^2

5



$h^2 = 73^2 - 48^2$

$h^2 = 3\ 025$

$h = \sqrt{3\ 025}$

$h = 55$ cm

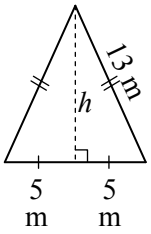
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 55 \times 48$

$A = 1\ 320$ cm^2

With isosceles triangles use the same method, except when you find the area, use the whole base length, not half.

6



$h^2 = 13^2 - 5^2$

$h^2 =$ 144

$h = \sqrt{144}$

$h =$ 12 m

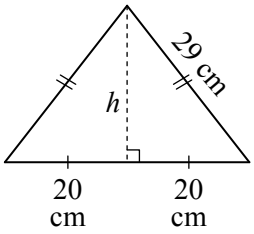
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ 10 $\times$ 12

Use $b = 10$ (2 × 5) not 5!

$A =$ 60 m^2

7



$h^2 = 29^2 - 20^2$

$h^2 = 441$

$h = \sqrt{441}$

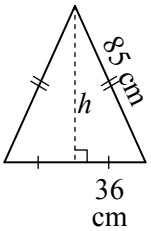
$h = 21$ cm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 40 \times 21$

$A = 420$ cm^2

8



$h^2 = 85^2 - 36^2$

$h^2 = 5\ 929$

$h = \sqrt{5\ 929}$

$h = 77$ cm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 72 \times 77$

$A = 2\ 772$ cm^2

Area - Area of Triangles Using Pythagoras (Homework Strip)

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Date Due: _____

Find the area of these triangles

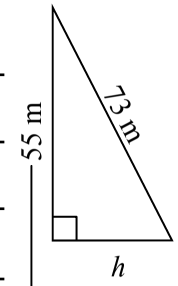
1

$h^2 = 73^2 - 55^2$

$h^2 =$ _____

$h = \sqrt{}$ _____

$h =$ _____ m



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ _____ $\times$ _____

$A =$ _____ m^2

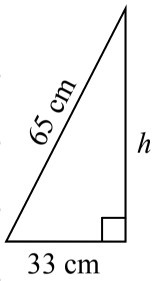
2

$h^2 =$ _____$^2 -$ _____^{2}

$h^2 =$ _____

$h = \sqrt{}$ _____

$h =$ _____ cm



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ _____ $\times$ _____

$A =$ _____ cm^2

Date Due: _____

Find the area of these triangles

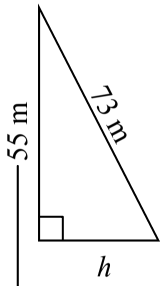
1

$h^2 = 73^2 - 55^2$

$h^2 =$ _____

$h = \sqrt{}$ _____

$h =$ _____ m



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ _____ $\times$ _____

$A =$ _____ m^2

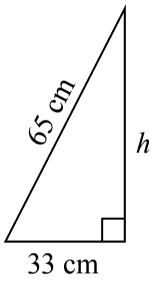
2

$h^2 =$ _____$^2 -$ _____^{2}

$h^2 =$ _____

$h = \sqrt{}$ _____

$h =$ _____ cm



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ _____ $\times$ _____

$A =$ _____ cm^2

Date Due: _____

Find the area of these triangles

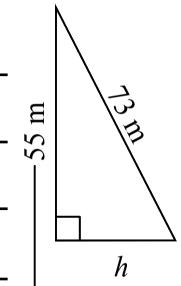
1

$h^2 = 73^2 - 55^2$

$h^2 =$ _____

$h = \sqrt{}$ _____

$h =$ _____ m



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ _____ $\times$ _____

$A =$ _____ m^2

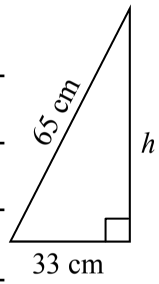
2

$h^2 =$ _____$^2 -$ _____^{2}

$h^2 =$ _____

$h = \sqrt{}$ _____

$h =$ _____ cm



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ _____ $\times$ _____

$A =$ _____ cm^2

Date Due: _____

Find the area of these triangles

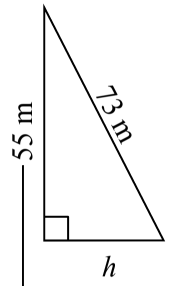
1

$h^2 = 73^2 - 55^2$

$h^2 =$ _____

$h = \sqrt{}$ _____

$h =$ _____ m



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ _____ $\times$ _____

$A =$ _____ m^2

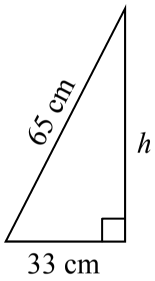
2

$h^2 =$ _____$^2 -$ _____^{2}

$h^2 =$ _____

$h = \sqrt{}$ _____

$h =$ _____ cm



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ _____ $\times$ _____

$A =$ _____ cm^2

Area - Area of Triangles Using Pythagoras (Homework Strip)

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Date Due:

Find the area of these triangles

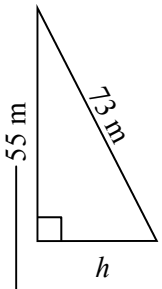
1

$h^2 = 73^2 - 55^2$

$h^2 = 2\,304$

$h = \sqrt{2\,304}$

$h = 48$ m



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 48 \times 55$

$A = 1\,320$ m^2

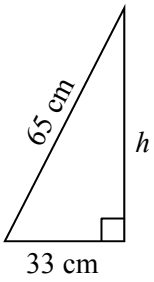
2

$h^2 = 65^2 - 33^2$

$h^2 = 3\,136$

$h = \sqrt{3\,136}$

$h = 56$ cm



$A = \frac{1}{2}bh$

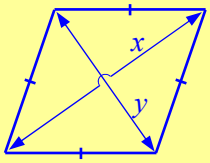
$= \frac{1}{2} \times 33 \times 56$

$A = 924$ cm^2

Area of Parallelograms and Rhombuses

A parallelogram looks like a rectangle that has been pushed. A rhombus, also called a diamond, looks like a square that has been pushed over. So a parallelogram has two pairs of parallel sides of the same length, like a rectangle. A rhombus has four sides all the same length, with two pairs of parallel sides.

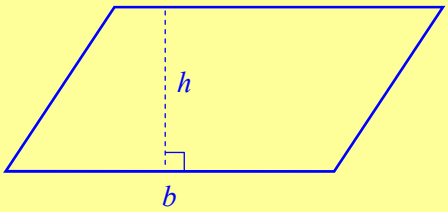
Rhombus $A = \frac{1}{2}xy$



Where:

x and y are the lengths of the diagonals.

Parallelogram $A = bh$

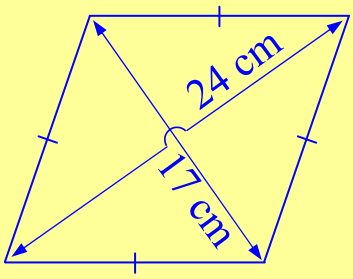


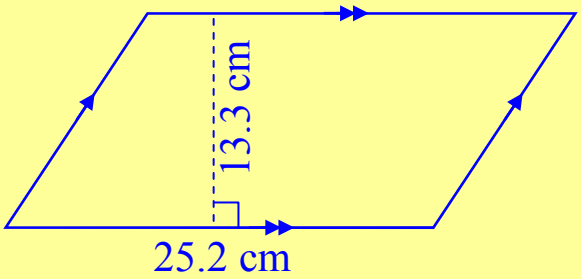
Where:

b = base length

h = perpendicular height

Below are two examples.





$A = \frac{1}{2}xy$	Write the formula	$A = bh$
$= \frac{1}{2} \times 17 \times 24$	Substitute x & y (or b & h)	$= 25.2 \times 13.3$
$A = 204$ cm^2	Solve showing square units. Round to 1 decimal place.	$A = 335.2$ m^2 (1 d.p.)

What happens if you aren't given the diagonals of a rhombus, but instead base and its perpendicular height? Solve just like it is a parallelogram. By definition all rhombuses (or rhombi) are parallelograms.

Area of Parallelograms

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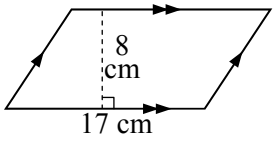
Find the area of these parallelograms.

Example

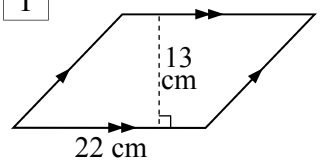


$A = bh$

$= 17 \times 8$

$A = 136 \text{ cm}^2$

1

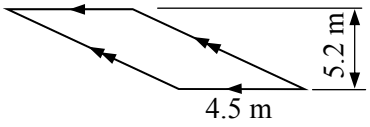


$A = bh$

$= \square \times \square$

$A = \square \text{ cm}^2$

2



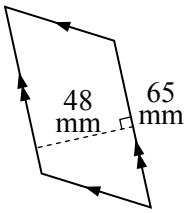
$A = bh$

$= \square \times \square$

$A = \square$

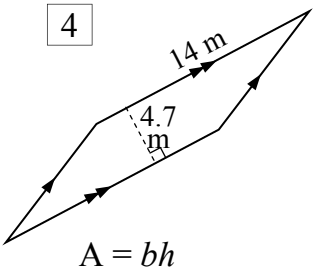
Remember square units

3



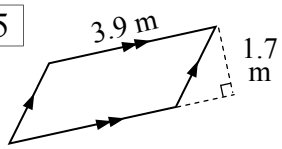
$A = bh$

4



$A = bh$

5



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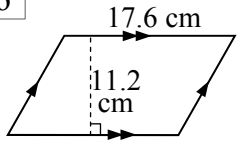

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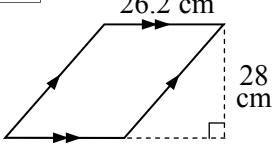

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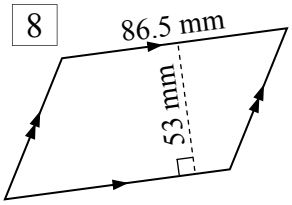
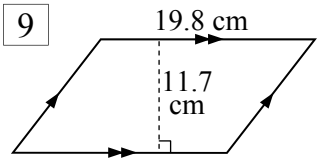


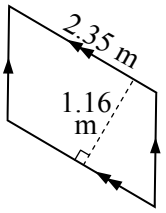
Find the area, rounding your answer to 1 d.p.

9



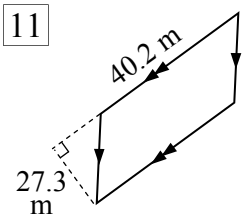
$A = bh$

10

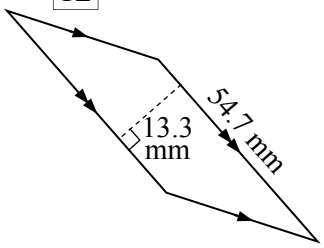


Square units to 1 d.p.

11

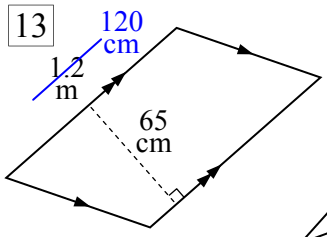


12



Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

13

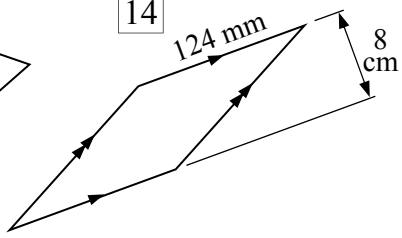


$A = bh$

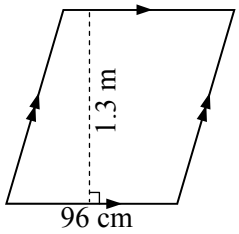
$= \square \times \square$

$A = \square \text{ cm}^2$

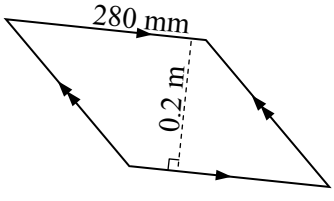
14



15



16



Area of Rhombuses (Rhombi)

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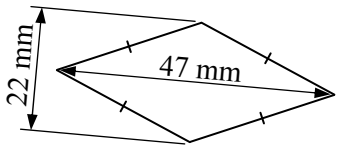
A rhombus is like a square that has been pushed over. Find the area, using $A = \frac{1}{2}xy$.

Example



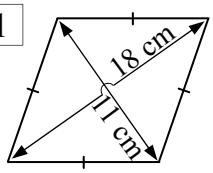
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 2 2 × 4 7 =

$= \frac{1}{2} \times 47 \times 22$

$A = 517 \text{ mm}^2$

1

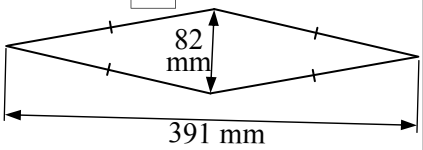


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

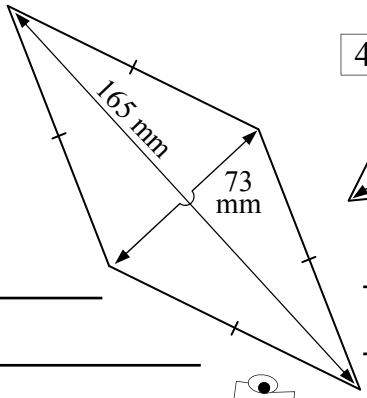
$A = \square \text{ cm}^2$

2



$A = \frac{1}{2}xy$

3



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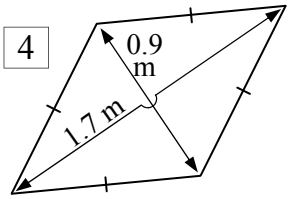
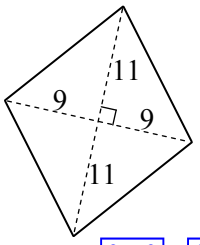


Add for the lengths first, all measurements are in cm.

5



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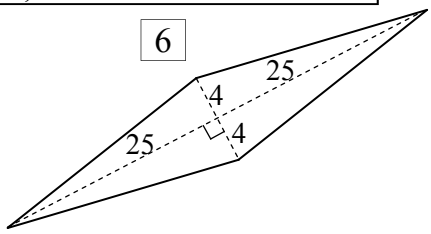
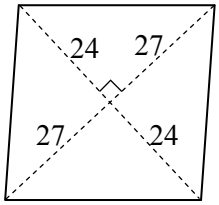


$A = \frac{1}{2}xy$ (9 + 9) (11 + 11)

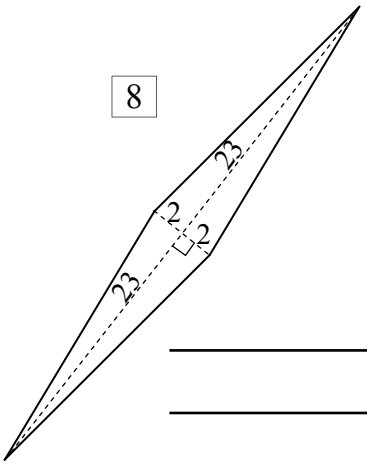
$= \frac{1}{2} \times \square \times \square$

$A = \square \text{ cm}^2$

7

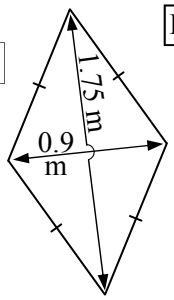


8

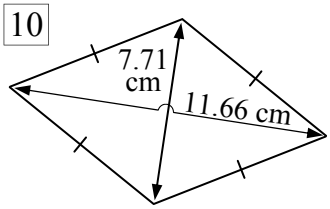


Round 'em up! To 1 d.p. that is.

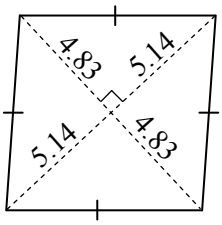
9



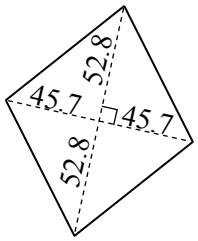
10



11 All cm



12 All mm



Join the dots to form a rhombus and measure the axis lengths in cm to 1 d.p. Then calculate the area in cm^2. Give the area to 1 decimal place.

13

Measure the axes not the sides!

14

Measure the axes not the sides!

Area of Parallelograms

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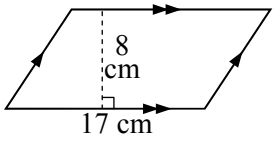
Find the area of these parallelograms.

Example

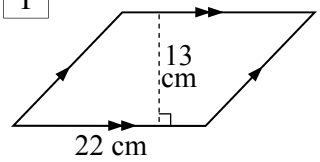


$A = bh$

$= 17 \times 8$

$A = 136 \text{ cm}^2$

1

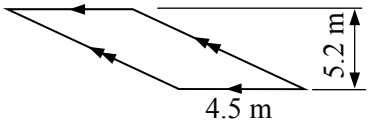


$A = bh$

$= 22 \times 13$

$A = 286 \text{ cm}^2$

2



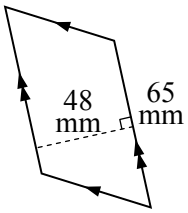
$A = bh$

$= 4.5 \times 5.2$

$A = 23.4 \text{ m}^2$

Remember square units

3

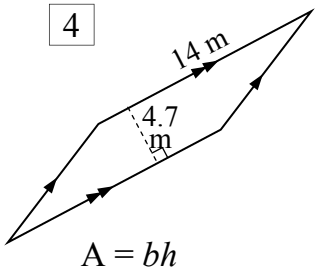


$A = bh$

$= 65 \times 48$

$A = 3\,120 \text{ mm}^2$

4

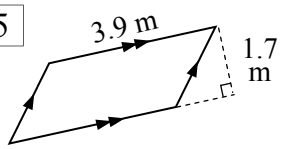


$A = bh$

$= 14 \times 4.7$

$A = 65.8 \text{ m}^2$

5

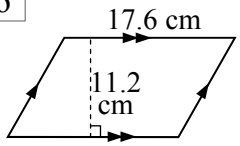


$A = bh$

$= 3.9 \times 1.7$

$A = 6.63 \text{ m}^2$

6

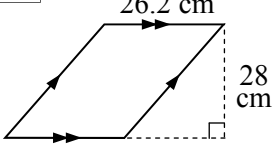


$A = bh$

$= 17.6 \times 11.2$

$A = 197.12 \text{ cm}^2$

7

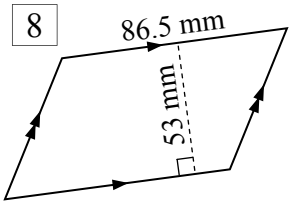


$A = bh$

$= 26.2 \times 28$

$A = 733.6 \text{ cm}^2$

8



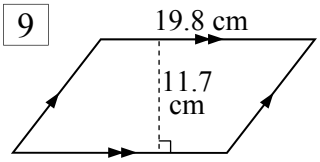
$A = bh$

$= 86.5 \times 53$

$A = 4\,584.5 \text{ cm}^2$

Find the area, rounding your answer to 1 d.p.

9



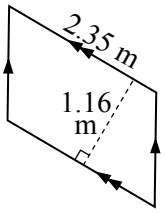
$A = bh$

$= 19.8 \times 11.7$

$A = 231.7 \text{ cm}^2$

Square units to 1 d.p.

10

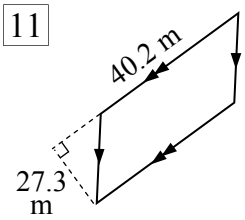


$A = bh$

$= 2.35 \times 1.16$

$A = 2.7 \text{ m}^2$

11

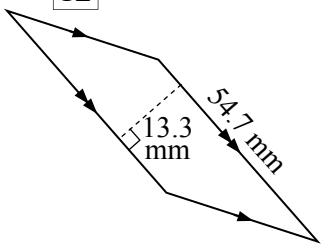


$A = bh$

$= 40.2 \times 27.3$

$A = 1\,097.5 \text{ m}^2$

12



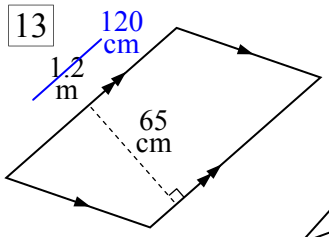
$A = bh$

$= 54.7 \times 13.3$

$A = 727.51 \text{ mm}^2$

Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

13

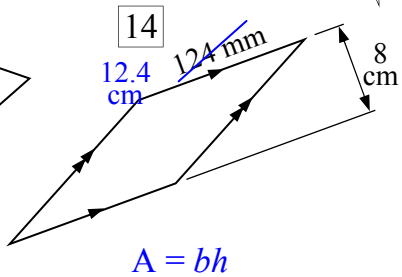


$A = bh$

$= 120 \times 65$

$A = 7\,800 \text{ cm}^2$

14

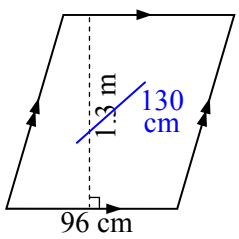


$A = bh$

$= 12.4 \times 8$

$A = 99.2 \text{ cm}^2$

15

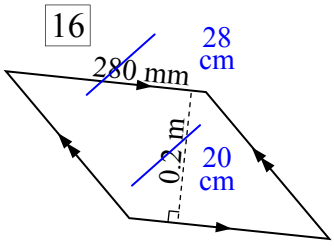


$A = bh$

$= 130 \times 96$

$A = 12\,480 \text{ cm}^2$

16



$A = bh$

$= 28 \times 20$

$A = 560 \text{ cm}^2$

Area of Rhombuses (Rhombi)

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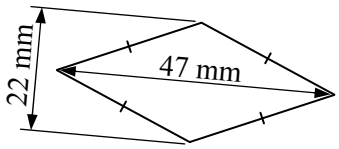
A rhombus is like a square that has been pushed over. Find the area, using $A = \frac{1}{2}xy$.

Example



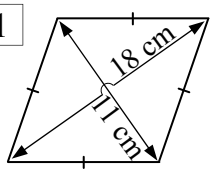
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 2 2 × 4 7 =

$= \frac{1}{2} \times 47 \times 22$

$A = 517 \text{ mm}^2$

1

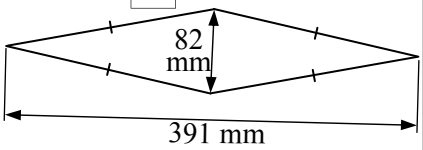


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 18 \times 11$

$A = 99 \text{ cm}^2$

2

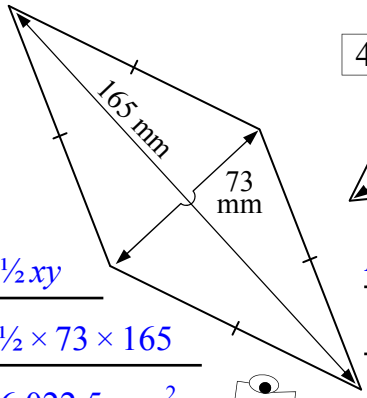


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 391 \times 82$

$A = 16\,031 \text{ mm}^2$

3

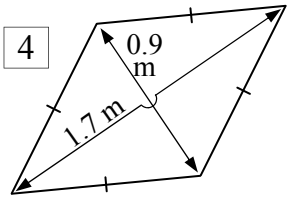


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 73 \times 165$

$A = 6\,022.5 \text{ mm}^2$

4



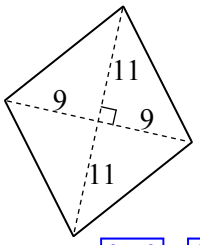
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 0.9 \times 1.7$

$A = 0.765 \text{ m}^2$

Add for the lengths first, all measurements are in cm.

5

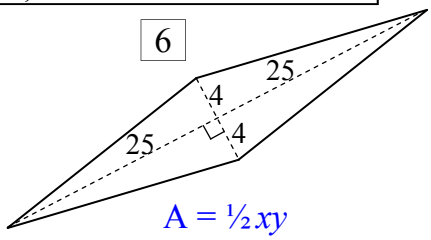


$A = \frac{1}{2}xy$ (9 + 9) (11 + 11)

$= \frac{1}{2} \times 18 \times 22$

$A = 198 \text{ cm}^2$

6

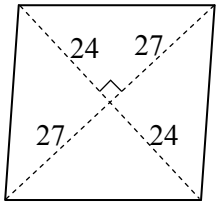


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 8 \times 50$

$A = 200 \text{ cm}^2$

7

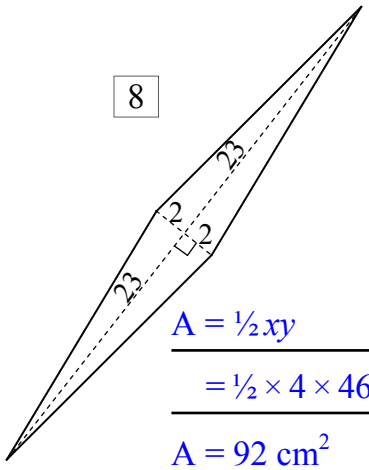


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 54 \times 48$

$A = 1\,296 \text{ cm}^2$

8



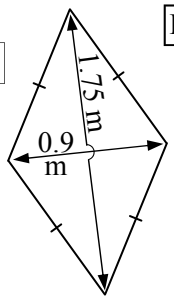
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 4 \times 46$

$A = 92 \text{ cm}^2$

Round 'em up! To 1 d.p. that is.

9

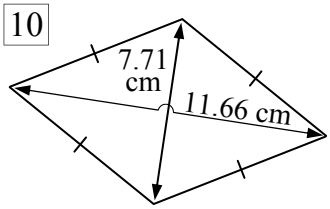


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 1.8 \times 3.5$

$A = 3.2 \text{ m}^2$

10



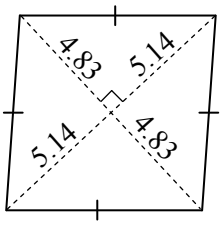
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 7.71 \times 11.66$

$A = 44.9 \text{ cm}^2$

11

All cm



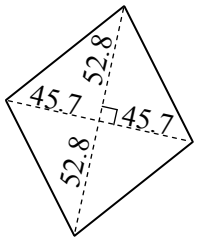
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 10.28 \times 9.66$

$A = 49.7 \text{ cm}^2$

12

All mm



$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 91.4 \times 105.6$

$A = 4\,825.9 \text{ mm}^2$

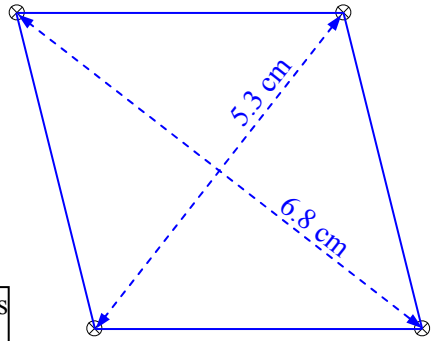
Join the dots to form a rhombus and measure the axis lengths in cm to 1 d.p. Then calculate the area in cm^2. Give the area to 1 decimal place.

13

$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 6.8 \times 5.3$

$A = 18.0 \text{ cm}^2$



Measure the axes not the sides!

Both rhombi have same side length of 4.3 cm.

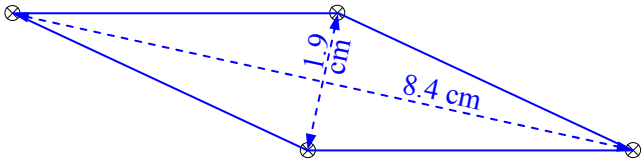
14

$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 8.4 \times 1.9$

$A = 8.0 \text{ cm}^2$

Measure the axes not the sides!



Area - Area of Parallelograms and Rhombuses (Homework Strip)

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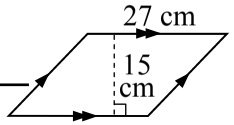
Date Due: _____

Find the area of these parallelograms

1

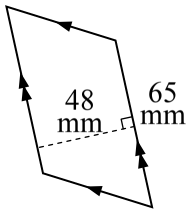


$A = bh$

$= \square \times \square$

$A = \square$ cm^2

2

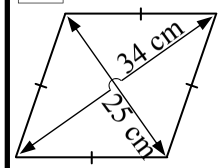


$A = bh$

$A = \square$ mm^2

Find the area of these rhombuses

3



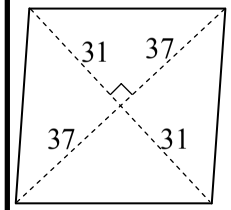
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

$A = \square$ cm^2

4

All cm

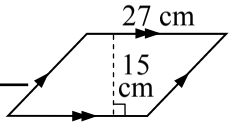


Find 31 + 31 and 37 + 37 first!

Date Due: _____

Find the area of these parallelograms

1

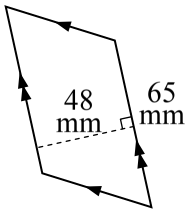


$A = bh$

$= \square \times \square$

$A = \square$ cm^2

2

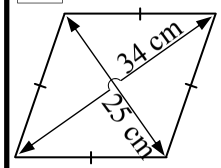


$A = bh$

$A = \square$ mm^2

Find the area of these rhombuses

3



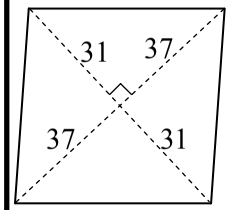
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

$A = \square$ cm^2

4

All cm

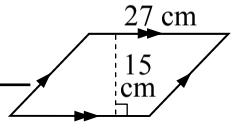


Find 31 + 31 and 37 + 37 first!

Date Due: _____

Find the area of these parallelograms

1

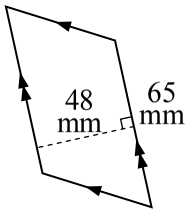


$A = bh$

$= \square \times \square$

$A = \square$ cm^2

2

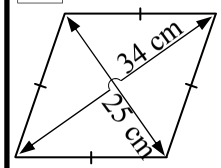


$A = bh$

$A = \square$ mm^2

Find the area of these rhombuses

3



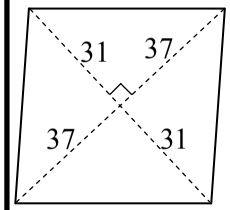
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

$A = \square$ cm^2

4

All cm

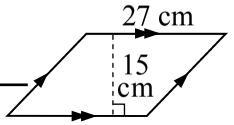


Find 31 + 31 and 37 + 37 first!

Date Due: _____

Find the area of these parallelograms

1

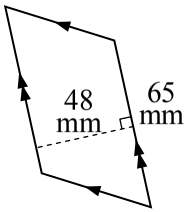


$A = bh$

$= \square \times \square$

$A = \square$ cm^2

2

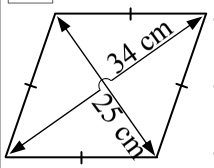


$A = bh$

$A = \square$ mm^2

Find the area of these rhombuses

3



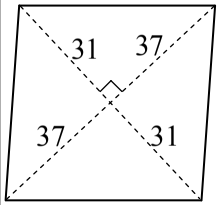
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

$A = \square$ cm^2

4

All cm



Find 31 + 31 and 37 + 37 first!

Area - Area of Parallelograms and Rhombuses (Homework Strip)

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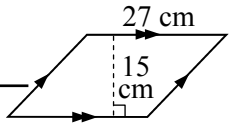
Date Due: _____

Find the area of these parallelograms

1

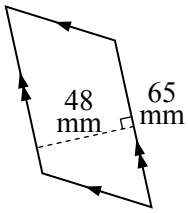


$A = bh$

$= 27 \times 15$

$A = 405 \text{ cm}^2$

2



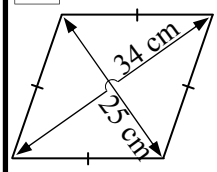
$A = bh$

$= 65 \times 48$

$A = 3\,120 \text{ mm}^2$

Find the area of these rhombuses

3



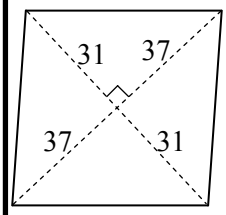
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 25 \times 34$

$A = 425 \text{ cm}^2$

4

All cm



$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 62 \times 74$

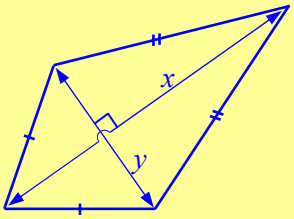
$A = 2\,294 \text{ cm}^2$

Find 31 + 31
and
37 + 37 first!

Area of Kites and Trapeziums

A kite is the easiest shape to remember because its name is the same as the kite that you see flying. To find its area the same formula as the rhombus formula is used, that is, its internal diagonal lengths are multiplied together and by a half. Trapeziums are similar to rectangles in a way. The height is the perpendicular height like in a rectangle, but the base is the average of the top and bottom lengths. This average is then multiplied by the height. The formulae are below:

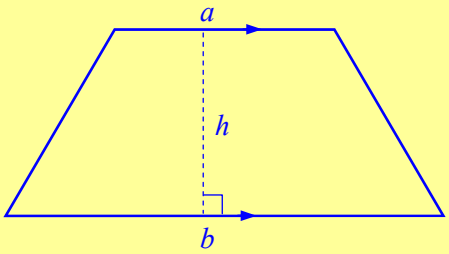
Kite $A = \frac{1}{2}xy$



Where:

x and y are the lengths of the diagonals.

Trapezium $A = \frac{1}{2}(a + b)h$

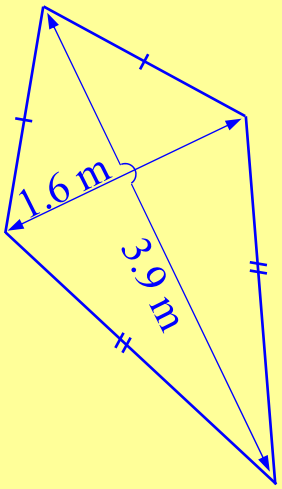


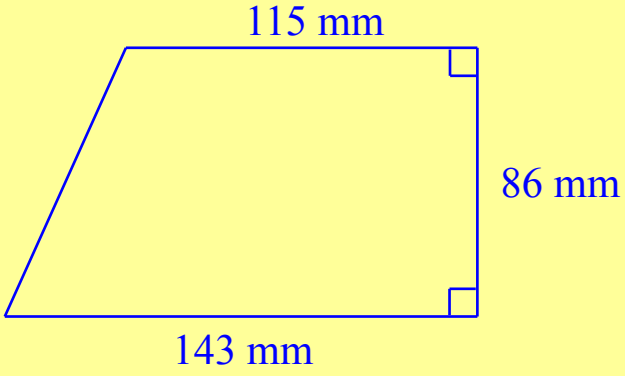
Where:

a, b = parallel side lengths

h = perpendicular height

Here are two examples. With trapeziums you need to use the brackets keys on your calculator.





$A = \frac{1}{2}xy$	Write the formula	$A = \frac{1}{2}(a + b)h$
$= \frac{1}{2} \times 3.9 \times 1.6$	Substitute x & y (or a, b & h)	$= \frac{1}{2} \times (115 + 143) \times 86$
Calc: 0 . 5 × 3 . 9 × 1 . 6 =		Calc: 0 . 5 × (1 1 5 + 1 4 3) × 8 6 =
$A = 3.1$ m^2 (1 d.p.)	Solve showing square units. Round to 1 decimal place.	$A = 11\,094$ mm^2

Area of Kites

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Find the area of these kites.

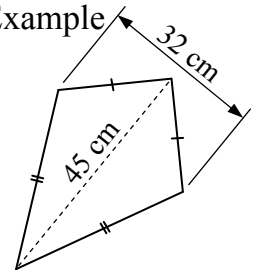
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 3 2 × 4 5 =

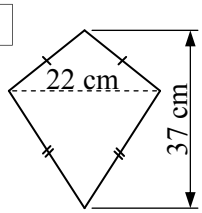
$= \frac{1}{2} \times 32 \times 45$

$A = 720 \text{ cm}^2$

Example



1

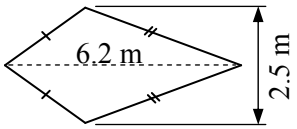


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

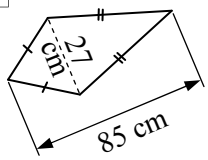
$A = \square \text{ cm}^2$

2

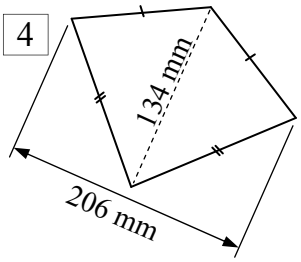


$A = \frac{1}{2}xy$

3

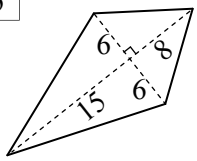


4



From Q.5 all units are in cm.

5

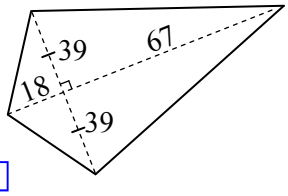


$A = \frac{1}{2}xy$ (6 + 6) (8 + 15)

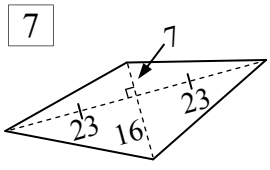
$= \frac{1}{2} \times \square \times \square$

$A = \square \text{ cm}^2$

6



7



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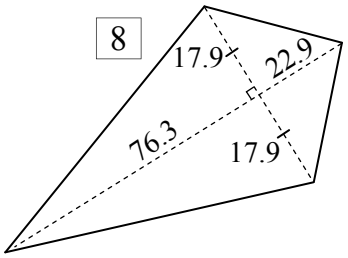
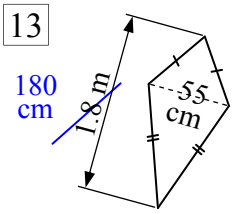


Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

13

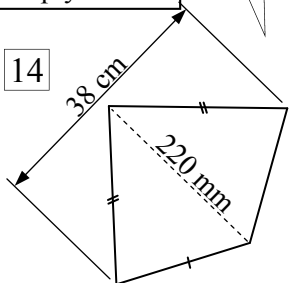


$A = \frac{1}{2}xy$

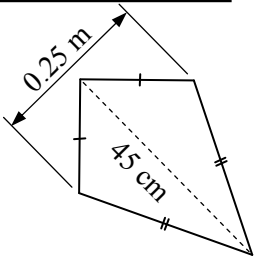
$= \frac{1}{2} \times \square \times \square$

$A = \square \text{ cm}^2$

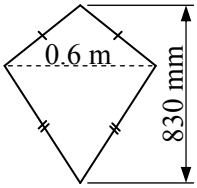
14



15



16



Join the dots to form a kite and measure the axis lengths in cm to 1 d.p. Then calculate the area in cm^2. Give the area to 1 decimal place.

13

Measure the axes not the sides!

14

Area of Trapeziums

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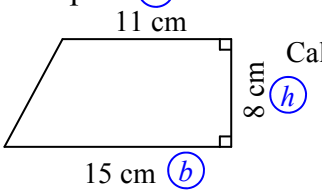
Find the area of these trapeziums. '*a*' and '*b*' are the parallel sides, '*h*' is the distance between them.

Example



$A = \frac{1}{2}(a + b)\,h$

Calc: 0 . 5 × (1 1 + 1 5) × 8 =

$= \frac{1}{2} \times (11 + 15) \times 8$

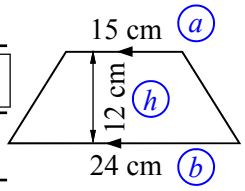
$A = 104 \text{ cm}^2$

1

$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (\square + \square) \times \square$

$A = \square \text{ cm}^2$



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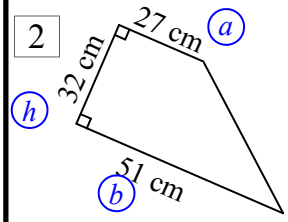
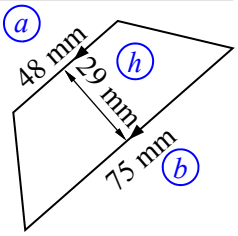


$A = \frac{1}{2}(a + b)\,h$

3



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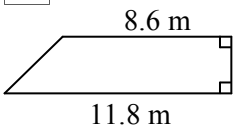

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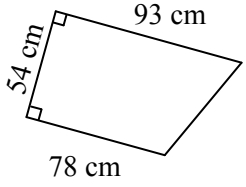

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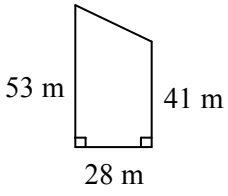

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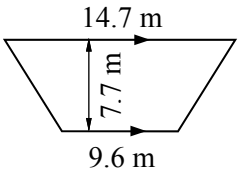

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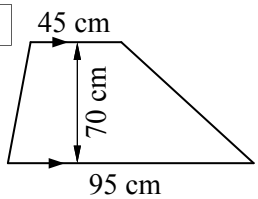


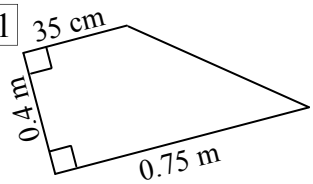
Join the dots to form a trapezium and measure the two parallel sides and the height. Measure the lengths to 1 decimal place. Then calculate the area (in cm^2).

9

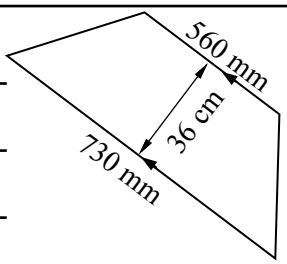
10

Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

11



12



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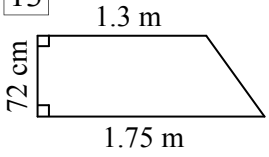
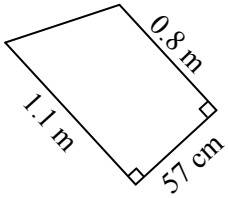


14



Area of Kites

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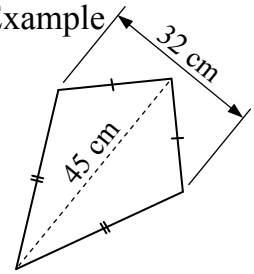
Find the area of these kites.

Example



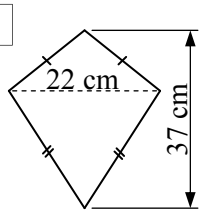
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 3 2 × 4 5 =

$= \frac{1}{2} \times 32 \times 45$

$A = 720 \text{ cm}^2$

1

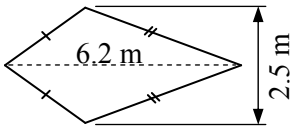


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 22 \times 37$

$A = 407 \text{ cm}^2$

2

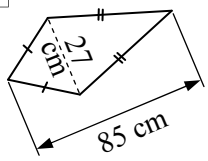


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 2.5 \times 6.2$

$A = 7.75 \text{ m}^2$

3

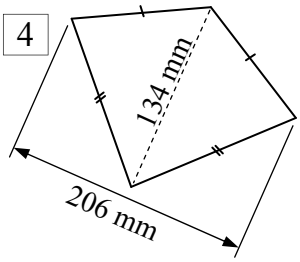


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 27 \times 85$

$A = 1\,147.5 \text{ cm}^2$

4



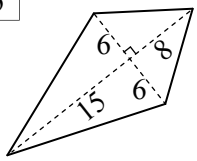
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 206 \times 134$

$A = 13\,802 \text{ m}^2$

From Q.5 all units are in cm.

5

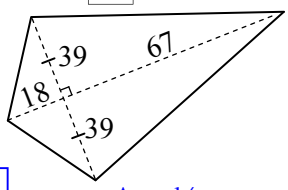


$A = \frac{1}{2}xy$ (6 + 6, 8 + 15)

$= \frac{1}{2} \times 12 \times 23$

$A = 138 \text{ cm}^2$

6

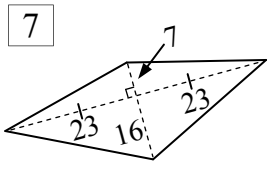


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 78 \times 85$

$A = 3\,315 \text{ cm}^2$

7

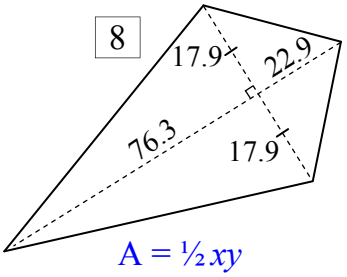


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 23 \times 46$

$A = 529 \text{ cm}^2$

8



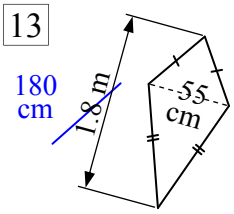
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 35.8 \times 99.2$

$A = 1\,775.68 \text{ cm}^2$

Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

13

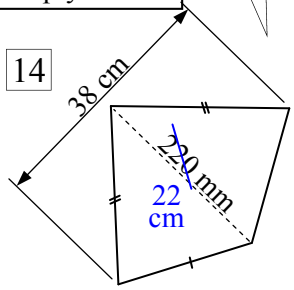


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 180 \times 180$

$A = 16\,200 \text{ cm}^2$

14

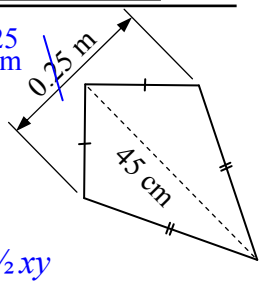


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 22 \times 38$

$A = 418 \text{ cm}^2$

15

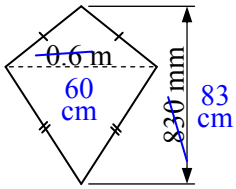


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 25 \times 45$

$A = 562.5 \text{ cm}^2$

16



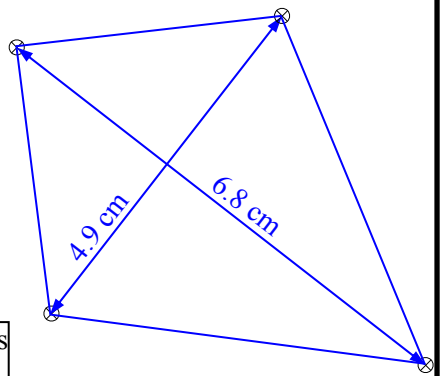
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 83 \times 60$

$A = 2\,490 \text{ cm}^2$

Join the dots to form a kite and measure the axis lengths in cm to 1 d.p. Then calculate the area in cm^2. Give the area to 1 decimal place.

13



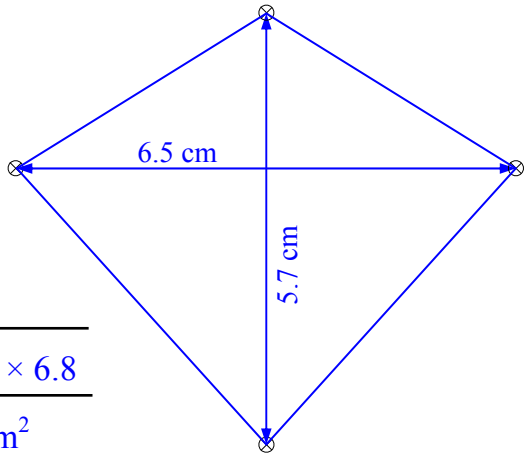
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 4.9 \times 6.8$

$A = 16.7 \text{ cm}^2$

Measure the axes not the sides!

14



$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 5.7 \times 6.8$

$A = 19.38 \text{ cm}^2$

Area of Trapeziums

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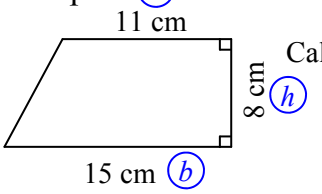
Find the area of these trapeziums. '*a*' and '*b*' are the parallel sides, '*h*' is the distance between them.

Example



$A = \frac{1}{2}(a + b)\,h$

Calc: 0 . 5 × (1 1 + 1 5) × 8 =

$= \frac{1}{2} \times (11 + 15) \times 8$

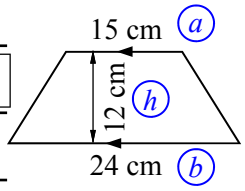
$A = 104 \text{ cm}^2$

1

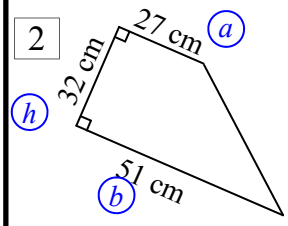
$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (\boxed{15} + \boxed{24}) \times \boxed{12}$

$A = \boxed{234} \text{ cm}^2$



2



$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (27 + 51) \times 32$

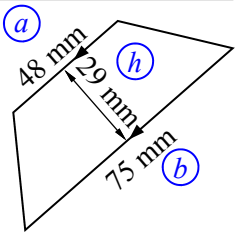
$A = 1\,248 \text{ cm}^2$

3

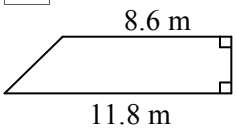
$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (48 + 75) \times 29$

$A = 1\,783.5 \text{ mm}^2$



4



$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (8.6 + 11.8) \times 2.2$

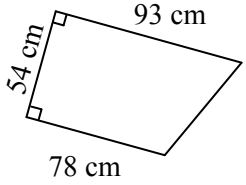
$A = 22.44 \text{ m}^2$

5

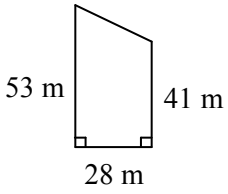
$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (78 + 93) \times 54$

$A = 4\,617 \text{ cm}^2$



6



$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (53 + 41) \times 28$

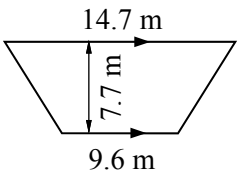
$A = 1\,316 \text{ m}^2$

7

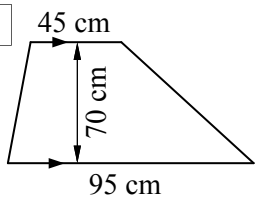
$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (9.6 + 14.7) \times 7.7$

$A = 93.555 \text{ m}^2$



8



$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (45 + 95) \times 70$

$A = 4\,900 \text{ m}^2$

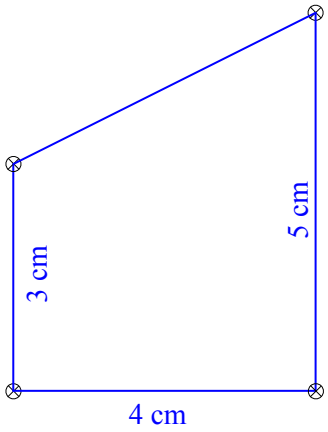
Join the dots to form a trapezium and measure the two parallel sides and the height. Measure the lengths to 1 decimal place. Then calculate the area (in cm^2).

9

$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (3 + 5) \times 4$

$A = 16 \text{ cm}^2$

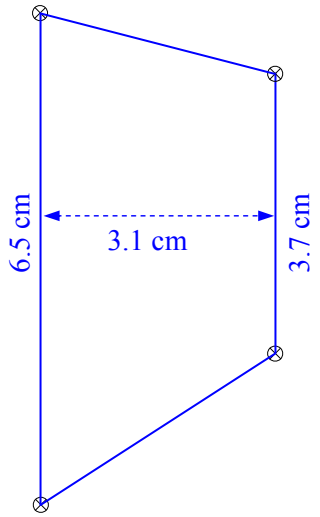


10

$A = \frac{1}{2}(a + b)\,h$

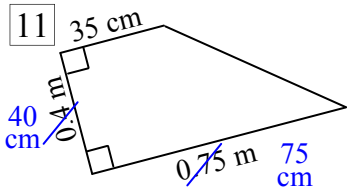
$= \frac{1}{2} \times (3.7 + 6.5) \times 3.1$

$A = 15.81 \text{ cm}^2$



Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

11



$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (35 + 75) \times 40$

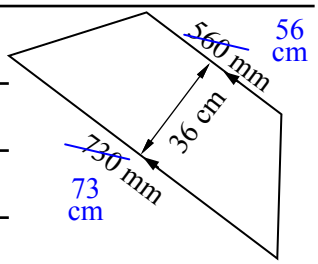
$A = 2\,200 \text{ cm}^2$

12

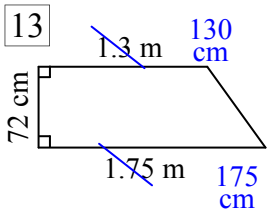
$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (56 + 73) \times 36$

$A = 2\,322 \text{ cm}^2$



13



$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (130 + 175) \times 72$

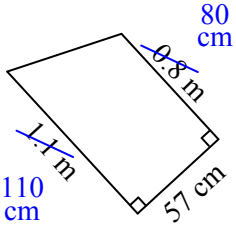
$A = 10\,980 \text{ cm}^2$

14

$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (80 + 110) \times 57$

$A = 5\,415 \text{ cm}^2$



Area - Area of Kites and Trapeziums (Homework Strip)

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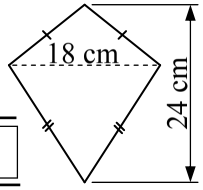
Date Due: _____

Find the area of these parallelograms

1

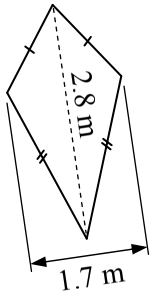


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

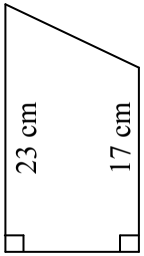
$A = \square$ cm^2

2



$A = \frac{1}{2}xy$

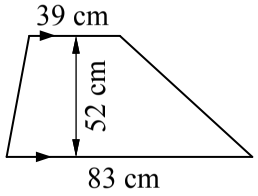
3



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (\square + \square) \times \square$

4

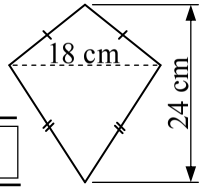


$A = \frac{1}{2}(a + b)h$

Date Due: _____

Find the area of these parallelograms

1

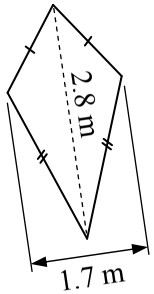


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

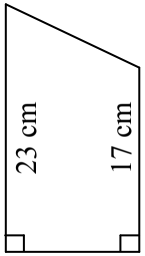
$A = \square$ cm^2

2



$A = \frac{1}{2}xy$

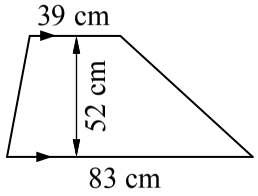
3



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (\square + \square) \times \square$

4

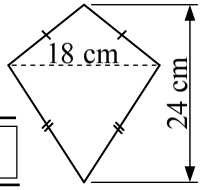


$A = \frac{1}{2}(a + b)h$

Date Due: _____

Find the area of these parallelograms

1

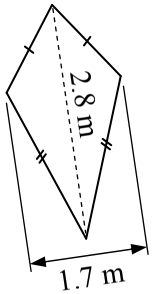


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

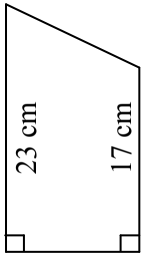
$A = \square$ cm^2

2



$A = \frac{1}{2}xy$

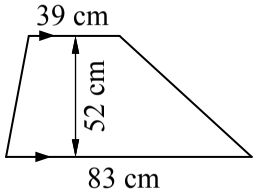
3



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (\square + \square) \times \square$

4

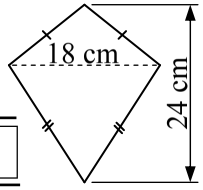


$A = \frac{1}{2}(a + b)h$

Date Due: _____

Find the area of these parallelograms

1

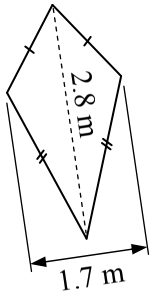


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

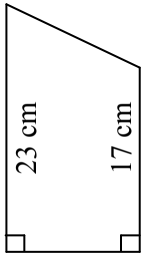
$A = \square$ cm^2

2



$A = \frac{1}{2}xy$

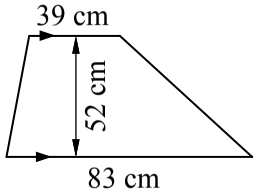
3



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (\square + \square) \times \square$

4



$A = \frac{1}{2}(a + b)h$

Area - Area of Kites and Trapeziums (Homework Strip)

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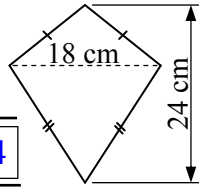
Date Due:

Find the area of these parallelograms

1

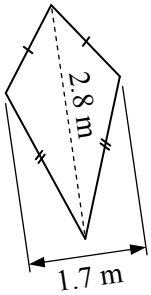


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 18 \times 24$

$A = 216 \text{ cm}^2$

2

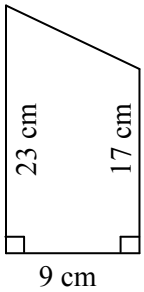


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 1.7 \times 2.8$

$A = 2.38 \text{ m}^2$

3

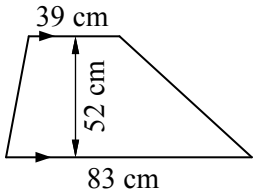


$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (17 + 23) \times 9$

$A = 180 \text{ cm}^2$

4



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (39 + 83) \times 52$

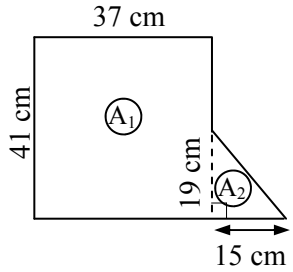
$A = 3\,172 \text{ cm}^2$

Area - Composite Shapes

The first sheet has two columns, the left column are questions finding the area by adding areas. The right column is finding the area using subtraction of areas. You need to know how to find the area for squares, rectangles and triangles. On sheet 2 there is a trapezium also.

Sheet 1 Column 1 starts with questions that have already divided up the composite area in to parts. Find the area of the parts and then add them. It is just the same as finding the area, but doing the process twice (or more) and then adding them.

Example



$A_1 = lb$

$= 41 \times 37$

$A_1 = 1\ 517\ cm^2$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 15 \times 19$

$A_2 = 142.5\ cm^2$

$A = A_1 + A_2 = 1\ 517 + 142.5 = 1\ 659.50\ cm^2$

Column 2 has shapes within others, so that means that you subtract. So calculate the individual areas, then subtract the smaller area(s) from the larger area.

Sheet 2 are worded questions and involve real world applications such as metal fabrication, painting or landscape gardening.

Sheet 3 is more difficult as it involved finding missing lengths first and doesn't help you in dividing the shape up.

Area - Composite Shapes (Sheet 1)

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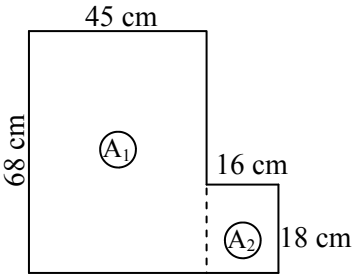
Calculate the area of these shapes by breaking them up into parts first. All angles that look like right angles, are right angles!

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square = \square$ cm^2

2

$A_1 = lb$

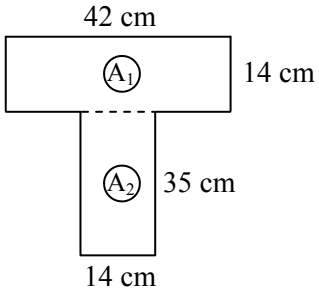
$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

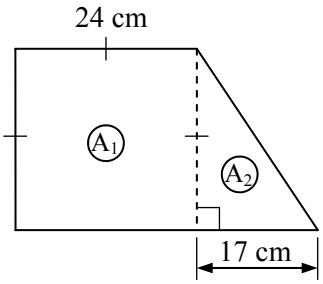
$= \square \times \square$

$A_2 = \square$ cm^2



$A = A_1 + A_2 = \square + \square = \square$ cm^2

3



$A_1 = l^2$

$= \square^2$

$A_1 = \square$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A_2 = \square$

$A = A_1 + A_2 = \square + \square = \square$

4

$A_1 = l^2$

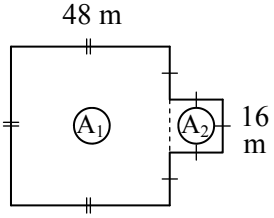
$= \square^2$

$A_1 = \square$

$A_2 = l^2$

$= \square^2$

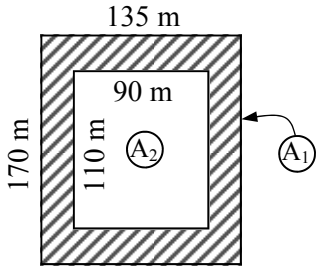
$A_2 = \square$



$A = A_1 + A_2 = \square + \square = \square$

Finding the shaded area is just the same, except instead of adding the areas, you subtract. So it is $A_1 - A_2$, the biggest area minus the smaller one.

5



$A_1 =$

$A_1 =$

$A_2 =$

$A_2 =$

$A = A_1 - A_2 = \square - \square = \square$ m^2

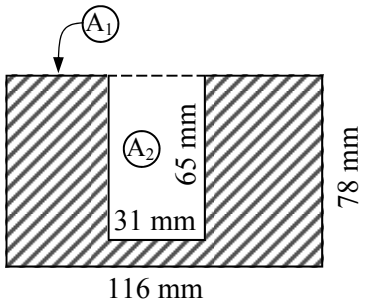
6

$A_1 =$

$A_1 =$

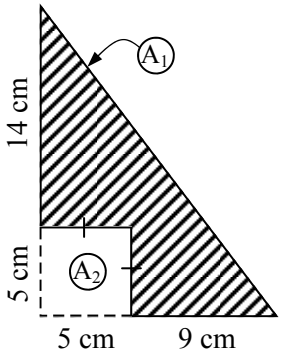
$A_2 =$

$A_2 =$



$A = A_1 - A_2 =$

7



$A_1 =$

Add 2 lengths for the base and also the height of the triangle.

$A_1 =$

$A_2 =$

$A_2 =$

$A = A_1 - A_2 =$

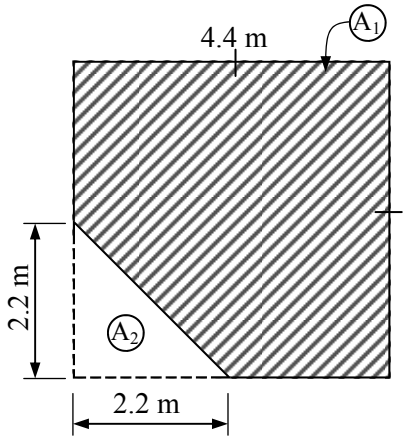
8

$A_1 =$

$A_1 =$

$A_2 =$

$A_2 =$



$A = A_1 - A_2 =$

Area - Composite Shapes (Sheet 2)

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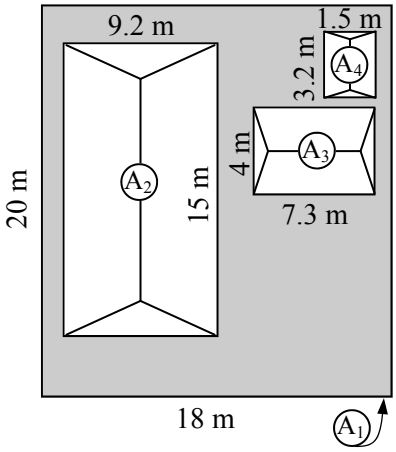
Use your composite area skills to find these.

A block of land has a house, garage and garden shed on the property. The diagram shows a plan (top) view.

Find the:



1 Block Area (A_1)

$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ m^2

2 House Area (A_2)

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ m^2

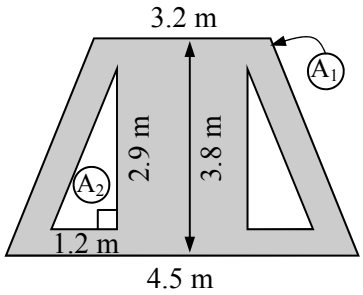
3 Garage Area (A_3)

4 Shed Area (A_4)

5 Grass Area (shaded area).

6 Cost of laying turf on the block if turf costs \$5.60 per m^2. Turf is only sold in whole m^2.

A foot of a crane is shown. It is fabricated from plate steel in the shape of a trapezium with two identical triangular sections removed. Find the:



7 Trapezium Area (A_1)

$A_1 = ½ (a + b) \times h$

8 Triangle Area (A_2)

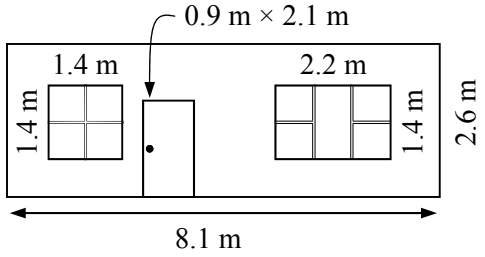
$A_2 = ½bh$

9 Steel area (shaded area).

Remember that there are two triangles!

10 The weight of the footing, if the plate steel weighs 285 kg per m^2.

Dave wants to repaint the front wall of his house. It features a doorway and two windows. The front view is shown below. Find the area of the:



11 Front wall

12 Square window

13 Doorway

14 Rectangular window

15 Paintable Area of the wall.

16 Number of litres required for one coat, if 1 m^2 needs 0.3 L.

17 The paint is sold in 2 L cans, with each can costing \$38.20. How much does it cost to paint the wall?

Area - Further Composite Shapes (Sheet 3)

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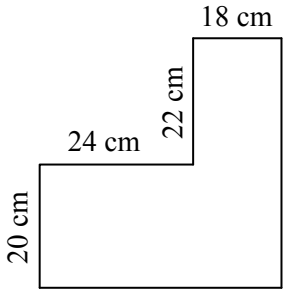
Find the missing sides, then break up the area into more simple shapes. All angles that look like right angles, are right angles!

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

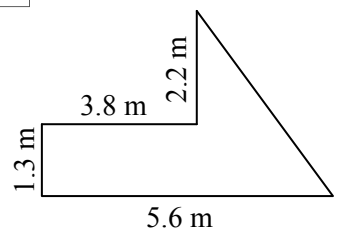
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square = \square$ cm^2

2



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ m^2

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A_2 = \square$ m^2

$A = A_1 + A_2 = \square + \square = \square$ m^2

3

$A_1 =$

$A_1 =$

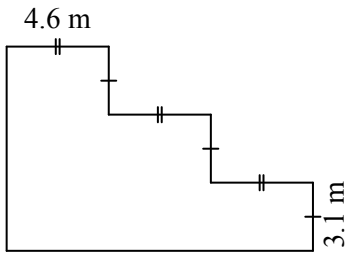
$A_2 =$

$A_2 =$

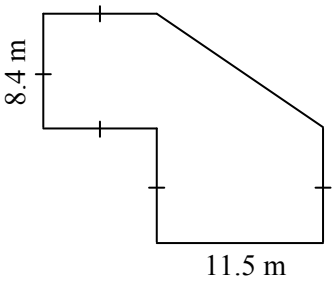
$A_3 =$

$A_3 =$

$A =$



4 *Solve with 2 or 3 Areas



$A_1 =$

$A_1 =$

$A_2 =$

$A_2 =$

$A_3 =$

$A_3 =$

$A =$

5

$A_1 =$

$A_1 =$

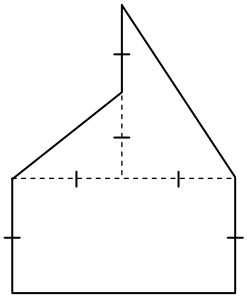
$A_2 =$

$A_2 =$

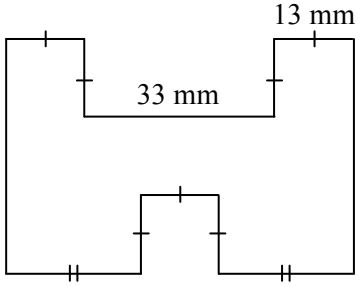
$A_3 =$

$A_3 =$

$A =$



6 *Use Subtraction Or Addition



$A_1 =$

$A_1 =$

$A_2 =$

$A_2 =$

$A_3 =$

$A_3 =$

$A =$

Area - Composite Shapes (Sheet 1)

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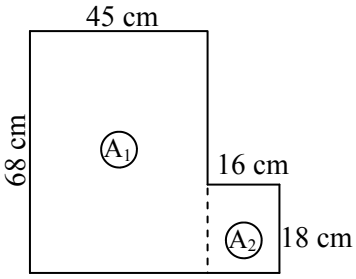
Calculate the area of these shapes by breaking them up into parts first. All angles that look like right angles, are right angles!

1



$A_1 = lb$

$= 68 \times 45$

$A_1 = 3\,060 \text{ cm}^2$

$A_2 = lb$

$= 16 \times 18$

$A_2 = 288 \text{ cm}^2$

$A = A_1 + A_2 = 3\,060 + 288 = 3\,348 \text{ cm}^2$

2

$A_1 = lb$

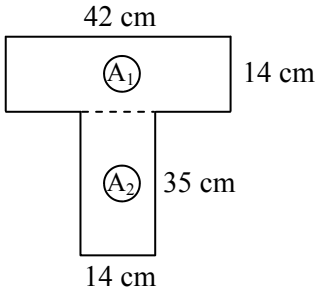
$= 42 \times 14$

$A_1 = 588 \text{ cm}^2$

$A_2 = lb$

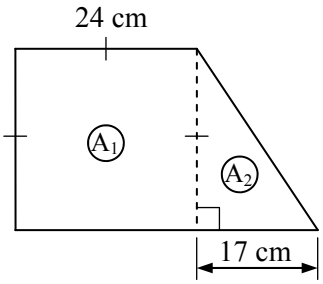
$= 35 \times 14$

$A_2 = 490 \text{ cm}^2$



$A = A_1 + A_2 = 588 + 490 = 1\,078 \text{ cm}^2$

3



$A_1 = l^2$

$= 24^2$

$A_1 = 576 \text{ cm}^2$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 17 \times 24$

$A_2 = 204 \text{ cm}^2$

$A = A_1 + A_2 = 576 + 204 = 780 \text{ cm}^2$

4

$A_1 = l^2$

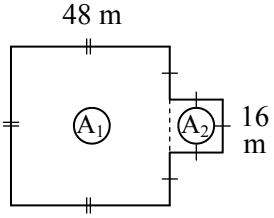
$= 48^2$

$A_1 = 2\,304 \text{ m}^2$

$A_2 = l^2$

$= 16^2$

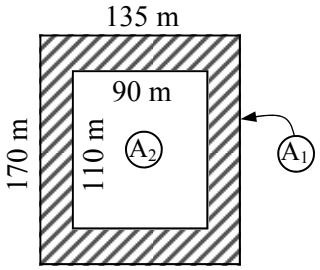
$A_2 = 256 \text{ m}^2$



$A = A_1 + A_2 = 2\,304 + 256 = 2\,560 \text{ m}^2$

Finding the shaded area is just the same, except instead of adding the areas, you subtract. So it is $A_1 - A_2$, the biggest area minus the smaller one.

5



$A_1 = lb$

$= 170 \times 135$

$A_1 = 22\,950 \text{ m}^2$

$A_2 = lb$

$= 110 \times 90$

$A_2 = 9\,900 \text{ m}^2$

$A = A_1 - A_2 = 22\,950 - 9\,900 = 13\,050 \text{ m}^2$

6

$A_1 = lb$

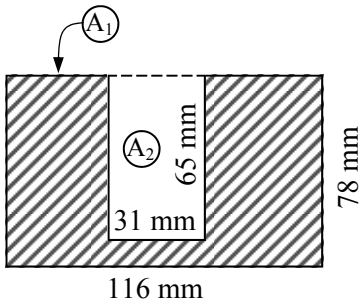
$= 116 \times 78$

$A_1 = 9\,048 \text{ mm}^2$

$A_2 = lb$

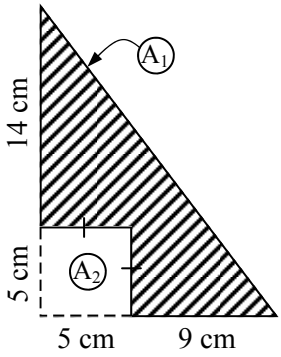
31×65

$A_2 = 2\,015 \text{ mm}^2$



$A = A_1 - A_2 = 9\,048 - 2\,015 = 7\,033 \text{ mm}^2$

7



$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 14 \times 19$

Add 2 lengths for the base and also the height of the triangle.

$A_1 = 133 \text{ cm}^2$

$A_2 = l^2$

$= 5^2$

$A_2 = 25 \text{ cm}^2$

$A = A_1 - A_2 = 133 - 25 = 108 \text{ cm}^2$

8

$A_1 = l^2$

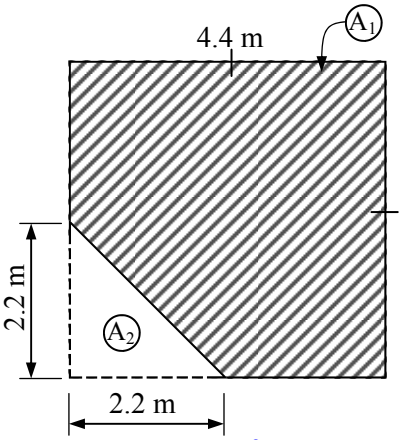
$= 4.4^2$

$A_1 = 19.36 \text{ m}^2$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 2.2 \times 2.2$

$A_2 = 2.42 \text{ m}^2$



$A = A_1 - A_2 = 19.36 - 2.42 = 16.94 \text{ m}^2$

Area - Composite Shapes (Sheet 2)

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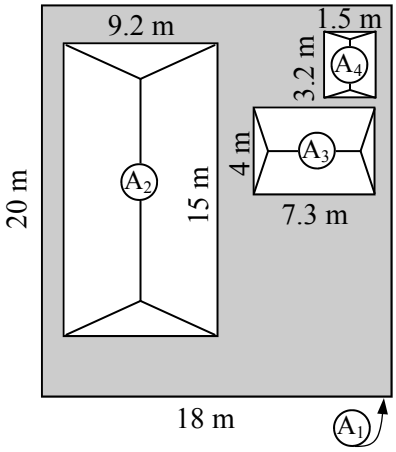
Use your composite area skills to find these.

A block of land has a house, garage and garden shed on the property. The diagram shows a plan (top) view.

Find the:



1 Block Area (A_1)

$A_1 = lb$

$= 20 \times 18$

$A_1 = 360$ m^2

2 House Area (A_2)

$A_2 = lb$

$= 15 \times 9.2$

$A_2 = 138$ m^2

3 Garage Area (A_3)

$A_3 = lb$

$= 7.3 \times 4$

$A_3 = 29.2$ m^2

4 Shed Area (A_4)

$A_4 = lb$

$= 3.2 \times 1.4$

$A_4 = 4.48$ m^2

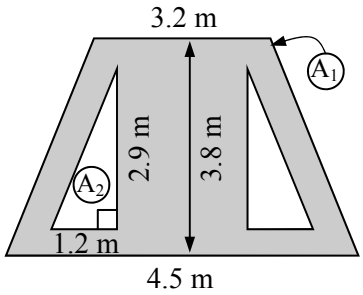
5 Grass Area (shaded area).

$A = 360 - 138 - 29.2 - 4.48 = 188.32$ m^2

6 Cost of laying turf on the block if turf costs \$5.60 per m^2. Turf is only sold in whole m^2.

Cost = $189 \times 5.60 = \$1\,058.40$

A foot of a crane is shown. It is fabricated from plate steel in the shape of a trapezium with two identical triangular sections removed. Find the:



7 Trapezium Area (A_1)

$A_1 = \frac{1}{2}(a + b) \times h$

$= \frac{1}{2} \times (3.2 + 4.5) \times 3.8$

$A_1 = 14.63$ m^2

8 Triangle Area (A_2)

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 1.2 \times 2.9$

$A_2 = 1.74$ m^2

9 Steel area (shaded area).

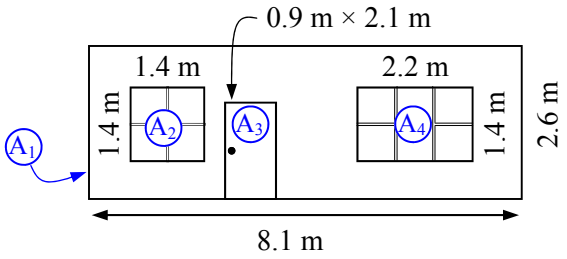
$A = 14.63 - 2 \times 1.74 = 11.15$ m^2

Remember that there are two triangles!

10 The weight of the footing, if the plate steel weighs 285 kg per m^2.

Weight = $11.15 \times 285 = 3\,177.75$ kg

Dave wants to repaint the front wall of his house. It features a doorway and two windows. The front view is shown below. Find the area of the:



11 Front wall

$A_1 = lb$

$= 8.1 \times 2.6$

$A_1 = 21.06$ m^2

12 Square window

$A_2 = l^2$

$= 1.4^2$

$A_2 = 1.96$ m^2

13 Doorway

$A_3 = lb$

$= 2.1 \times 0.9$

$A_3 = 1.89$ m^2

14 Rectangular window

$A_4 = lb$

$= 2.2 \times 1.4$

$A_4 = 3.08$ m^2

15 Paintable Area of the wall.

$A = 21.06 - 1.96 - 1.89 - 3.08 = 14.13$ m^2

16 Number of litres required for one coat, if 1 m^2 needs 0.3 L.

Paint = $14.13 \times 0.3 = 4.239$ L

17 The paint is sold in 2 L cans, with each can costing \$38.20. How much does it cost to paint the wall?

Need 3 cans because 4 L (2 cans) not enough.

Cost = $3 \times 38.20 = \$114.60$

Area - Further Composite Shapes (Sheet 3)

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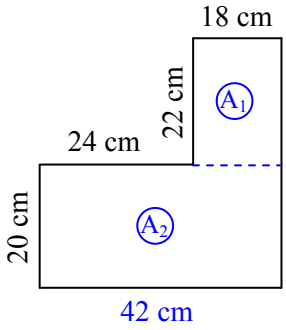
Find the missing sides, then break up the area into more simple shapes. All angles that look like right angles, are right angles!

1



$A_1 = lb$

$= 22 \times 18$

$A_1 = 396 \text{ cm}^2$

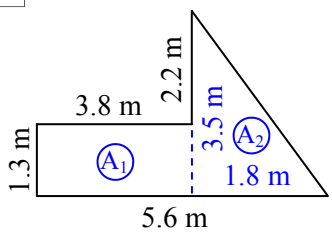
$A_2 = lb$

$= 42 \times 20$

$A_2 = 840 \text{ cm}^2$

$A = A_1 + A_2 = 396 + 840 = 1\,236 \text{ cm}^2$

2



$A_1 = lb$

$= 3.8 \times 1.3$

$A_1 = 4.94 \text{ m}^2$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 1.8 \times 3.5$

$A_2 = 3.15 \text{ m}^2$

$A = A_1 + A_2 = 4.94 + 3.15 = 8.09 \text{ m}^2$

3

$A_1 = lb$

$= 9.3 \times 4.6$

$A_1 = 42.78 \text{ m}^2$

$A_2 = lb$

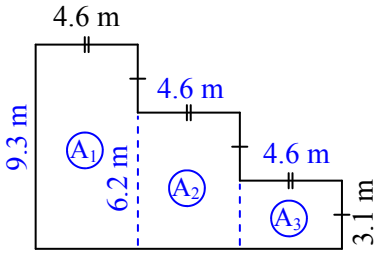
$= 6.2 \times 4.6$

$A_2 = 28.52 \text{ m}^2$

$A_3 = lb$

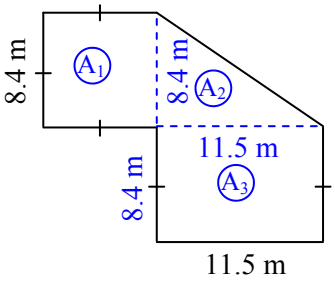
$= 3.1 \times 4.6$

$A_3 = 14.26 \text{ m}^2$



$A = 42.78 + 28.52 + 14.26 = 85.56 \text{ m}^2$

4 *Solve with 2 or 3 Areas



$A_1 = l^2$

$= 8.4^2$

$A_1 = 70.56 \text{ m}^2$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 11.5 \times 8.4$

$A_2 = 48.3 \text{ m}^2$

$A_3 = lb$

$= 11.5 \times 8.4$

$A_3 = 96.6 \text{ m}^2$

$A = 70.56 + 48.3 + 96.6 = 215.46 \text{ m}^2$

5

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 6.2^2$

$A_1 = 19.22 \text{ cm}^2$

$A_2 = \frac{1}{2}bh$

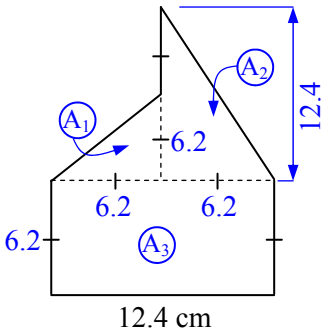
$= \frac{1}{2} \times 6.2 \times 12.4$

$A_2 = 38.44 \text{ cm}^2$

$A_3 = lb$

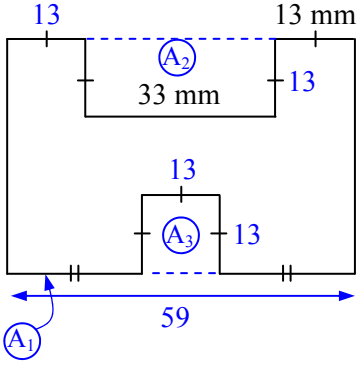
$= 6.2 \times 12.4$

$A_3 = 76.88 \text{ cm}^2$



$A = 19.22 + 38.44 + 76.88 = 134.54 \text{ cm}^2$

6 *Use Subtraction Or Addition



$A_1 = lb$

$= 59 \times 39$

$A_1 = 2\,301 \text{ mm}^2$

$A_2 = lb$

$= 33 \times 13$

$A_2 = 429 \text{ mm}^2$

$A_3 = l^2$

$= 13^2$

$A_3 = 169 \text{ mm}^2$

$A = 2\,301 - 429 - 169 = 1\,703 \text{ mm}^2$

Area - Composite Areas (Homework Strip)

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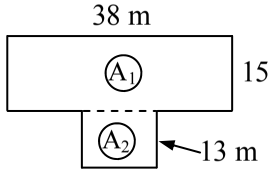
Date Due: _____

Find the area of these shapes

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

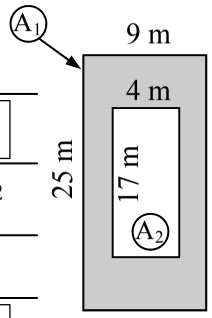
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ m^2

2



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

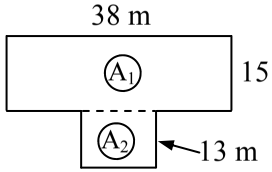
$A_2 = \square$ cm^2

$A = \square - \square = \square$ m^2

Date Due: _____

Find the area of these shapes

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

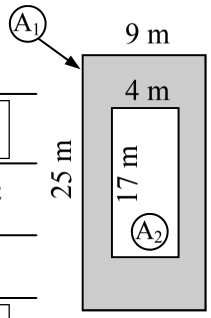
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ m^2

2



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

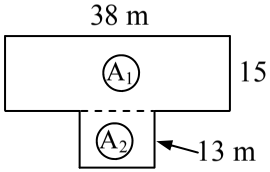
$A_2 = \square$ cm^2

$A = \square - \square = \square$ m^2

Date Due: _____

Find the area of these shapes

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

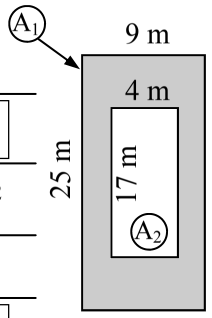
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ m^2

2



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

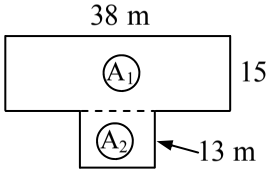
$A_2 = \square$ cm^2

$A = \square - \square = \square$ m^2

Date Due: _____

Find the area of these shapes

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

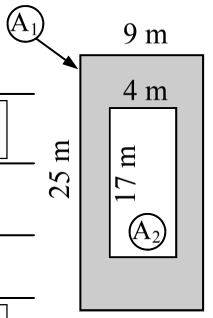
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ m^2

2



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = \square - \square = \square$ m^2

Area - Composite Areas (Homework Strip)

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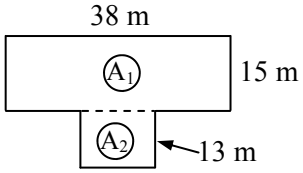
Date Due:

Find the area of these shapes

1



$A_1 = lb$

$= 38 \times 15$

$A_1 = 570$ cm^2

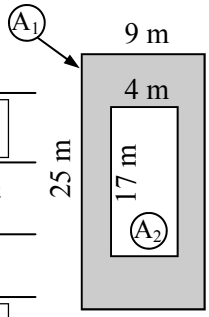
$A_2 = lb$

$= 13 \times 12$

$A_2 = 156$ cm^2

$A = 570 + 156 = 726$ m^2

2



$A_1 = lb$

$= 25 \times 9$

$A_1 = 225$ cm^2

$A_2 = lb$

$= 17 \times 4$

$A_2 = 68$ cm^2

$A = 225 - 68 = 157$ m^2

Area of a Circle

The formula for calculating the area of a circle is:

$$A = \pi r^2$$

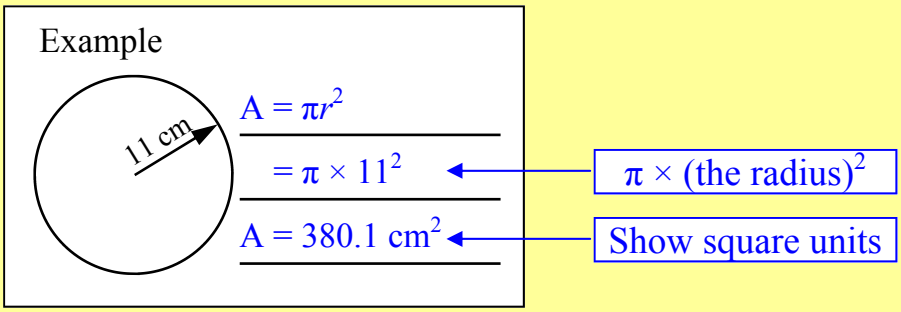
Where:

A = Area of the circle, in the same units as the radius, but squared. (cm^2, m^2, mm^2)
r = The radius of the circle (distance from centre to circumference).

Sheet 1 starts with you being asked to estimate the area of a circle. This is achieved by placing the circle in a cm^2 grid, then counting the grids within the circle. Question 1 is started for you to show you how it is done. Note the symmetry, this will make it easier for you.

After you have finished estimating the areas, use $\pi = 3.14$ and calculate the areas of the circles more accurately. See how you went.

Sheet 2 is finding the area of a circle using the 'Pi' key on your calculator. The location of π will vary, it can have its own button or require the pressing of the SHIFT key or 2ndF key. Column 1 starts with some questions on calculator use. Then the area of circles questions Commence. The process I straight forward, simply multiply π with the square of the radius. Below is an example.



Column 2 is the same as Column 1 except for one additional step. Instead of the radius you are given the diameter. So convert the diameter to a radius by dividing it by 2, write the radius in the space provided at the top of the question. Then solve the questions the same way, using the radius in your calculation.

Area of a Circle (Sheet 1)

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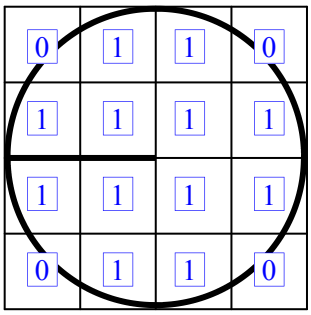
Estimate the area of these circles, using the 1 cm^2 grid squares. If square is close to a ½, call it ½. Less than ½ filled, call it zero. If it is close to full, call it 1 cm^2. Add them up and estimate the area.

1

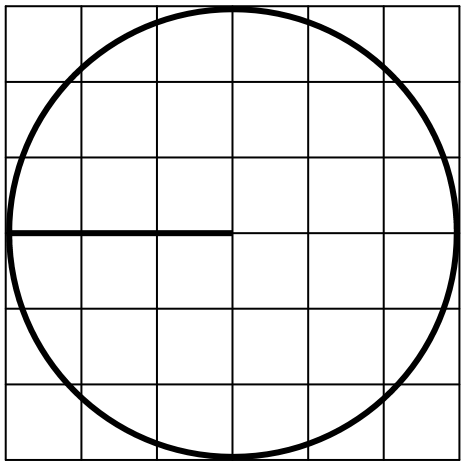


A = 12 × 1

\+ 0 × ½

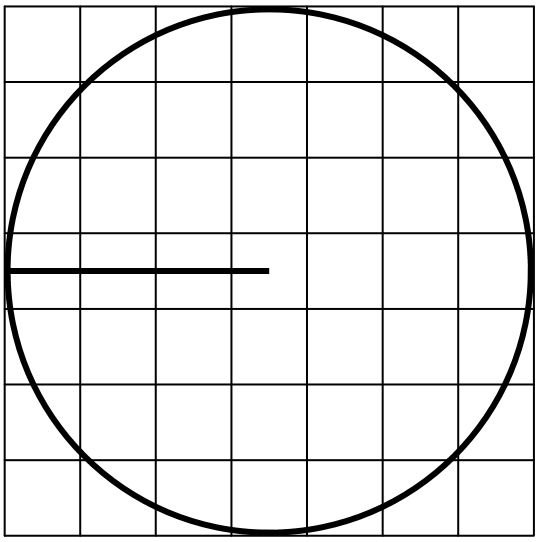
A = ____ cm^2

2



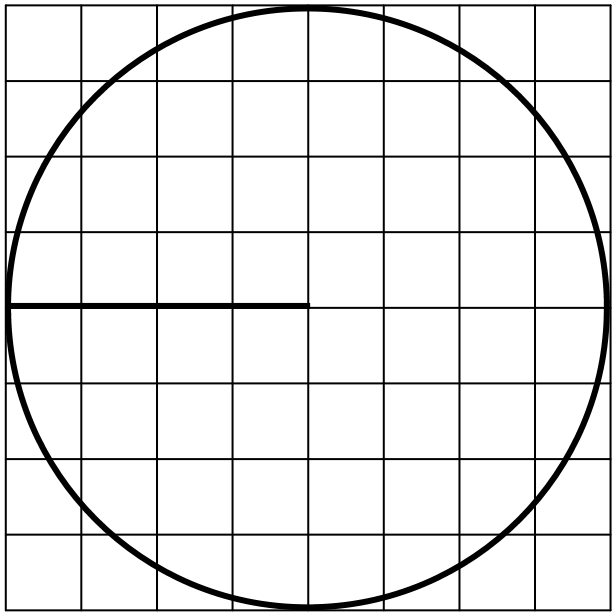
A = ____ × 1 + ____ × ½ = ____ cm^2

3



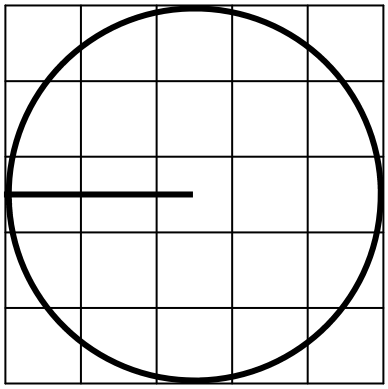
A = ____ × 1 + ____ × ½ = ____ cm^2

4



A = ____ × 1 + ____ × ½ = ____ cm^2

5



A = ____ × 1 + ____ × ½ = ____ cm^2

Now more accurately estimate the area by using $A = 3.14 \times r^2$. Compare the answer with your grid answer you calculated for the first four circles.

6 Circle in Q.1.

$A = 3.14 \times r^2$

$= 3.14 \times$ ____^{2}

Radius = length of black line

A = ____ cm^2

Answer to nearest half cm.

7 Circle in Q.2.

$A = 3.14 \times r^2$

$= 3.14 \times$ ____^{2}

A = ____ cm^2

8 Circle in Q.3.

$A = 3.14 \times r^2$

$= 3.14 \times$ ____^{2}

A = ____ cm^2

9 Circle in Q.4.

$A = 3.14 \times r^2$

$= 3.14 \times$ ____^{2}

A = ____ cm^2

Area of a Circle (Sheet 2)

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To calculate area of a circle you need to use your 'π' key. Calculate these to 1 d.p.

1 $\pi \times 15^2 =$ ____ Round to 1 d.p.

Calc: SHIFT $\times 10^x$ × 1 5 x^2 =

2 $\pi \times 24^2 =$ ____

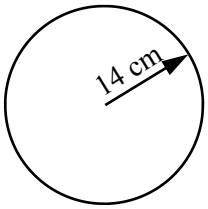
3 $\pi \times 16.5^2 =$ ____

4 $\pi \times 35.6^2 =$ ____

5 $\pi \times 11.2^2 =$ ____

Calculate the area of these circles, round to 1 d.p.

Example

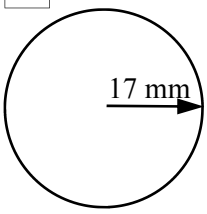


$A = \pi r^2$

$= \pi \times 14^2$

$A = 615.8 \text{ cm}^2$

6



$A = \pi r^2$

$= \pi \times$ ____ ^{2}

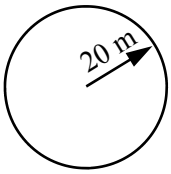
$A =$ ____ mm^2

7

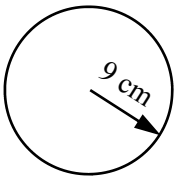
$A = \pi r^2$

$= \pi \times$ ____ ^{2}

$A =$ ____ m^2



8



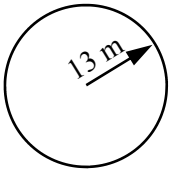
$A = \pi r^2$

$A =$ ____

Show square units

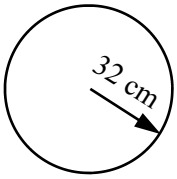
9

$A = \pi r^2$

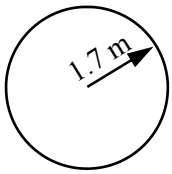


$A =$ ____

10



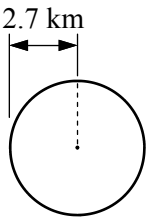
11



12



13



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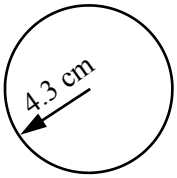
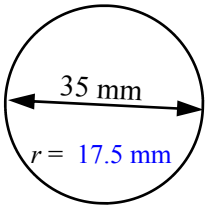


Complete the table below, answering to 2 d.p.

Q.	Radius	Area
15	9.2 m	
16	30 cm	
17	117 m	
18	45.9 mm	
19	72 cm	
20	5.4 km	
21	60.8 cm	
22	40.2 m	
23	8.92 m	

When given a diameter, find the radius first by dividing by 2, then find the area to 1 d.p.

Example



$A = \pi r^2$

$= \pi \times 17.5^2$

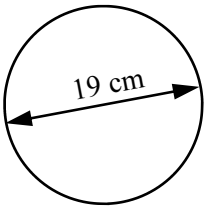
$A = 962.1 \text{ mm}^2$

24 $r =$ ____ cm

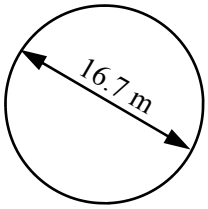
$A = \pi r^2$

$= \pi \times$ ____ ^{2}

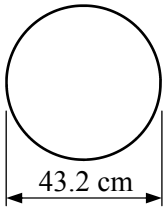
$A =$



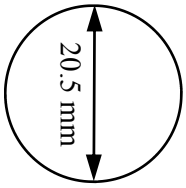
25 $r =$ ____ m



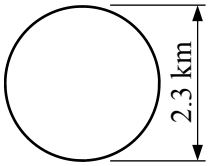
26 $r =$ ____



27 $r =$ ____



28 $r =$ ____



Area of a Circle (Sheet 1)

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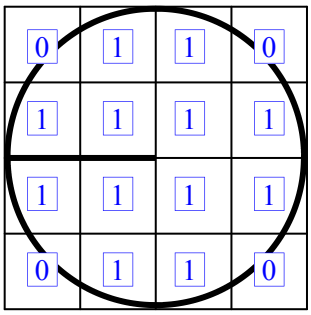
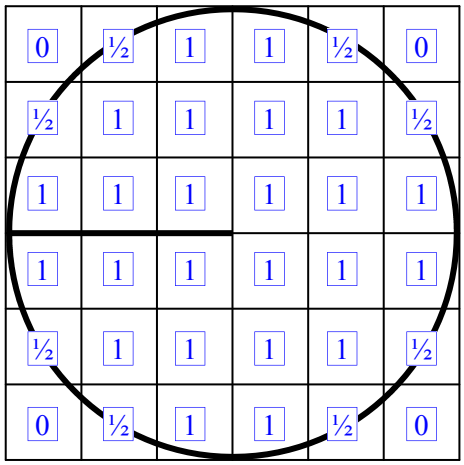
Estimate the area of these circles, using the 1 cm^2 grid squares. If square is close to a ½, call it ½. Less than ½ filled, call it zero. If it is close to full, call it 1 cm^2. Add them up and estimate the area.

1



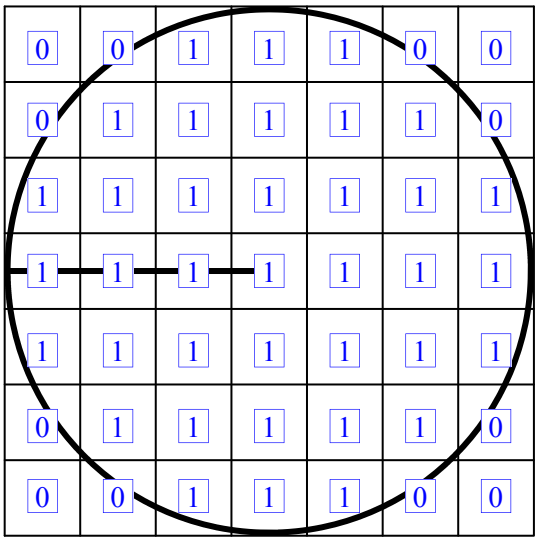
A = 12 × 1
+ 0 × ½
A = 12 cm^2

2



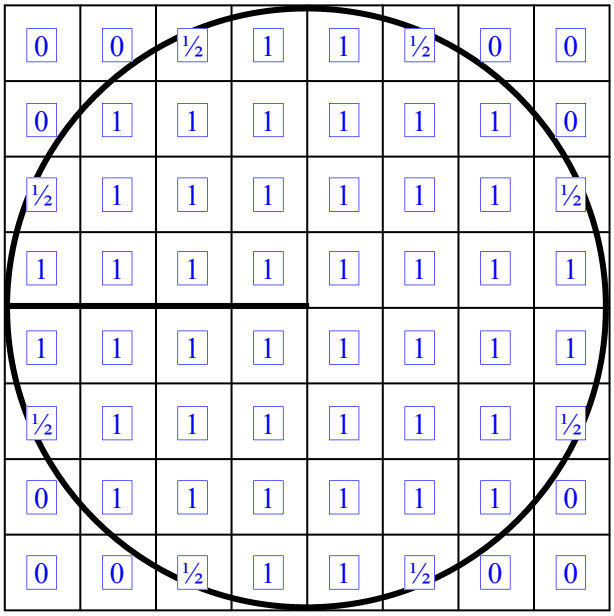
A = 24 × 1 + 8 × ½ = 28 cm^2

3



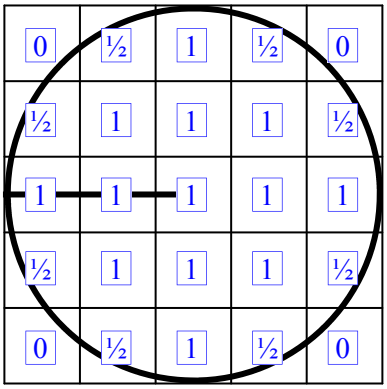
A = 40 × 1 + 0 × ½ = 40 cm^2

4



A = 44 × 1 + 8 × ½ = 48 cm^2

5



A = 13 × 1 + 8 × ½ = 17 cm^2

Now more accurately estimate the area by using $A = 3.14 \times r^2$. Compare the answer with your grid answer you calculated for the first four circles.

6 Circle in Q.1.

$A = 3.14 \times r^2$

$= 3.14 \times 2^2$

Radius = length of black line

A = 12.5 cm^2

Answer to nearest half cm.

7 Circle in Q.2.

$A = 3.14 \times r^2$

$= 3.14 \times 3^2$

A = 28.5 cm^2

8 Circle in Q.3.

$A = 3.14 \times r^2$

$= 3.14 \times 3.5^2$

A = 38.5 cm^2

9 Circle in Q.4.

$A = 3.14 \times r^2$

$= 3.14 \times 4^2$

A = 50.0 cm^2

Area of a Circle (Sheet 2)

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To calculate area of a circle you need to use your 'π' key. Calculate these to 1 d.p.

1 $\pi \times 15^2 =$ **706.9** Round to 1 d.p.

Calc: SHIFT $\times 10^x$ × 1 5 x^2 =

2 $\pi \times 24^2 =$ **1 809.6**

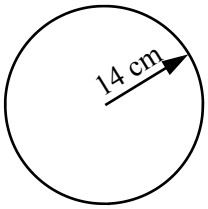
3 $\pi \times 16.5^2 =$ **855.3**

4 $\pi \times 35.6^2 =$ **3 981.5**

5 $\pi \times 11.2^2 =$ **394.1**

Calculate the area of these circles, round to 1 d.p.

Example

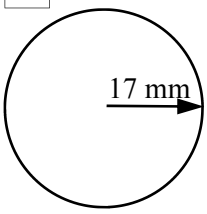


$A = \pi r^2$

$= \pi \times 14^2$

$A = 615.8 \text{ cm}^2$

6



$A = \pi r^2$

$= \pi \times$ **17** ^{2}

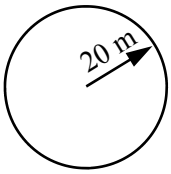
$A =$ **907.9** mm^2

7

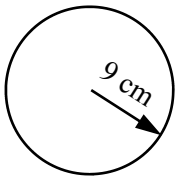
$A = \pi r^2$

$= \pi \times$ **20** ^{2}

$A =$ **1 256.6** m^2



8



$A = \pi r^2$

$= \pi \times 9^2$

$A =$ **254.5 cm²**

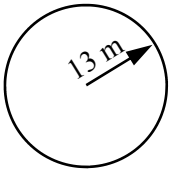
Show square units

9

$A = \pi r^2$

$= \pi \times 13^2$

$A =$ **530.9 m²**



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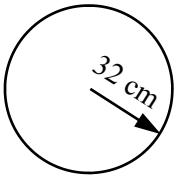


$A = \pi r^2$

$= \pi \times 32^2$

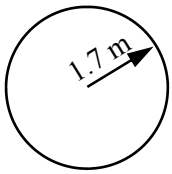
$A = 3\,217.0 \text{ cm}^2$

11

$A = \pi r^2$

$= \pi \times 1.7^2$

$A = 9.1 \text{ m}^2$



12



$A = \pi r^2$

$= \pi \times 28^2$

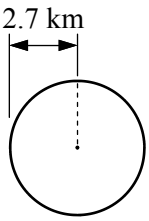
$A = 2\,463.0 \text{ mm}^2$

13

$A = \pi r^2$

$= \pi \times 2.7^2$

$A = 22.9 \text{ km}^2$



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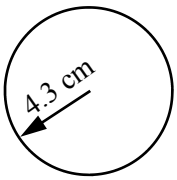


$A = \pi r^2$

$= \pi \times 4.3^2$

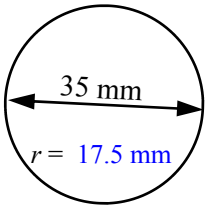
$A = 58.1 \text{ cm}^2$

Complete the table below, answering to 2 d.p.

Q.	Radius	Area
15	9.2 m	265.90 m^2
16	30 cm	2 827.43 cm^2
17	117 m	43 005.26 m^2
18	45.9 mm	6 618.74 mm^2
19	72 cm	16 286.02 cm^2
20	5.4 km	91.61 km^2
21	60.8 cm	11 613.34 cm^2
22	40.2 m	5 076.94 m^2
23	8.92 m	249.97 m^2

When given a diameter, find the radius first by dividing by 2, then find the area to 1 d.p.

Example



$A = \pi r^2$

$= \pi \times 17.5^2$

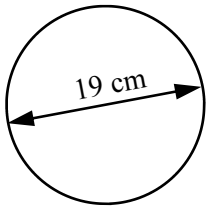
$A = 962.1 \text{ mm}^2$

24 $r =$ **9.5** cm

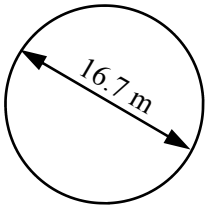
$A = \pi r^2$

$= \pi \times$ **9.5** ^{2}

$A = 283.5 \text{ cm}^2$



25 $r =$ **8.35** m



$A = \pi r^2$

$= \pi \times 8.35^2$

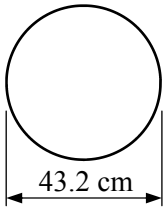
$A = 219.0 \text{ m}^2$

26 $r =$ **21.6 cm**

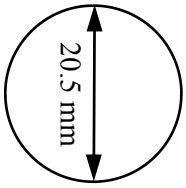
$A = \pi r^2$

$= \pi \times 21.6^2$

$A = 1\,465.7 \text{ cm}^2$



27 $r =$ **10.25 mm**



$A = \pi r^2$

$= \pi \times 10.25^2$

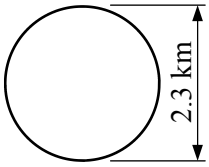
$A = 330.1 \text{ mm}^2$

28 $r =$ **1.15 km**

$A = \pi r^2$

$= \pi \times 1.15^2$

$A = 4.2 \text{ km}^2$



Area - Area of Circles (Homework Strip)

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Date Due: _____

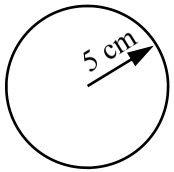
Find the area of these circles (1 d.p.)

1

$A = \pi r^2$

$= \pi \times \square^2$

$A = \square$ cm^2



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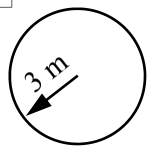


$A = \pi r^2$

$= \pi \times \square^2$

$A = \square$ m^2

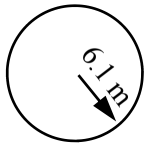
3

$A = \pi r^2$

$= \pi \times \square$

$A = \square$

Write the square units



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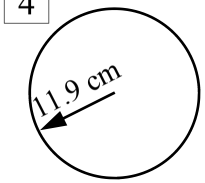

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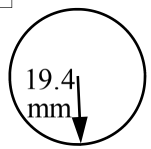


Date Due: _____

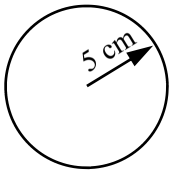
Find the area of these circles (1 d.p.)

1

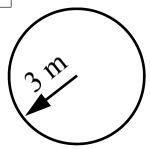
$A = \pi r^2$

$= \pi \times \square^2$

$A = \square$ cm^2



2



$A = \pi r^2$

$= \pi \times \square^2$

$A = \square$ m^2

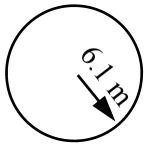
3

$A = \pi r^2$

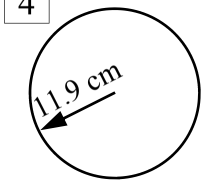
$= \pi \times \square$

$A = \square$

Write the square units



4



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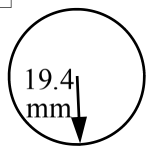


Date Due: _____

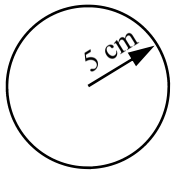
Find the area of these circles (1 d.p.)

1

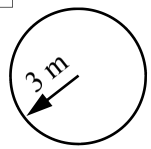
$A = \pi r^2$

$= \pi \times \square^2$

$A = \square$ cm^2



2



$A = \pi r^2$

$= \pi \times \square^2$

$A = \square$ m^2

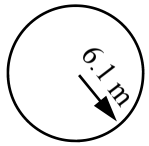
3

$A = \pi r^2$

$= \pi \times \square$

$A = \square$

Write the square units



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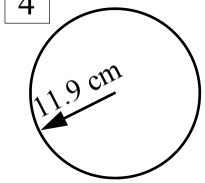

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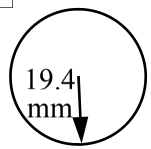


Date Due: _____

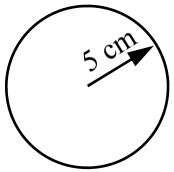
Find the area of these circles (1 d.p.)

1

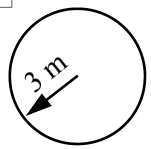
$A = \pi r^2$

$= \pi \times \square^2$

$A = \square$ cm^2



2



$A = \pi r^2$

$= \pi \times \square^2$

$A = \square$ m^2

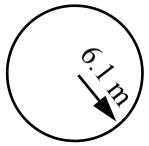
3

$A = \pi r^2$

$= \pi \times \square$

$A = \square$

Write the square units



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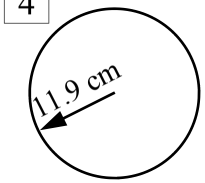

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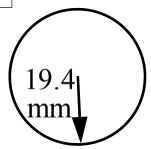

Area - Area of Circles (Homework Strip)

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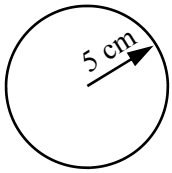
Date Due:

Find the area of these circles (1 d.p.)

1

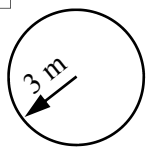


$A = \pi r^2$

$= \pi \times 5^2$

$A = 78.5 \text{ cm}^2$

2

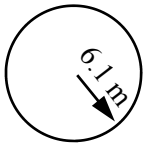


$A = \pi r^2$

$= \pi \times 3^2$

$A = 28.3 \text{ m}^2$

3



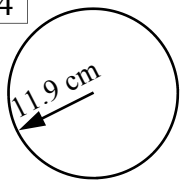
$A = \pi r^2$

$= \pi \times 6.1^2$

$A = 116.9 \text{ m}^2$

Write the square units

4

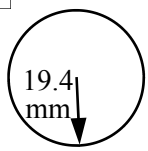


$A = \pi r^2$

$= \pi \times 11.9^2$

$A = 444.9 \text{ cm}^2$

5



$A = \pi r^2$

$= \pi \times 19.4^2$

$A = 1\,182.4 \text{ mm}^2$

Area of Semi-Circles and Quadrants

The formula for calculating the area of a semi-circle is:

$$A = \tfrac{1}{2}\pi r^2$$

Where:

A = Area of the semi-circle, in the same units as the radius, but squared. (cm^2, etc)
r = The radius of the semi-circle (distance from centre to circumference).

Column 1 features semi-circles with the radius shown. Use the area formula above to calculate the answer. The answer should be rounded to 1 decimal place and the units should be shown as square units. There is an example at the top of the column.

Column 2 and column 3 are the same as Column 1 except for one additional step. Instead of the radius you are given the diameter. So convert the diameter to a radius by dividing it by 2, write the radius in the space provided, next to the question number. Then solve the questions the same way.

At the bottom of the page are three semi-circles. Use a ruler to measure the diameter, then divide the length by two to get the radius. Then calculate the area

The formula for calculating the area of a quadrant is:

$$A = \tfrac{1}{4}\pi r^2$$

Where:

A = Area of the quadrant, in the same units as the radius, but squared. (cm^2, etc)
r = The radius of the quadrant (distance from centre to circumference).

Column 1 starts off with some calculator questions, realise that ¼ can be used on your calculator with the fraction key but it may be easier to use 0.25

The rest of the questions are using the quadrant formula. There is an example before Q.7. Note that you are generally given a radius with a quadrant unlike diameters which could feature a radius or diameter.

At the bottom of the page are three quadrants. Use a ruler to measure the radius, then calculate the area

Area of Semi-Circles and Quadrants (Sheet 1)

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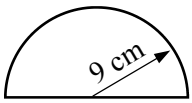
A semi-circle is half a circle, so use $A=\frac{1}{2}\pi r^2$, find the area to 1 d.p.

Example



$A = \frac{1}{2}\pi r^2$

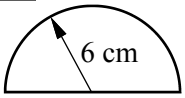
$= \frac{1}{2} \times \pi \times 9^2$

Calc: 0 . 5 × SHIFT ×10^x × 9 x^2 =

$A = 127.2\ cm^2$

Show square units

1



$A = \frac{1}{2}\pi r^2$

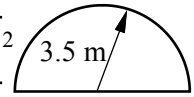
$= \frac{1}{2} \times \pi \times$ ☐^{2}

$A =$ ☐ cm^2

2

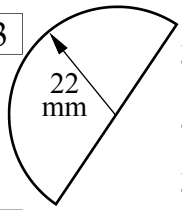
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times$ ☐^{2}



$A =$ ☐ m^2

3

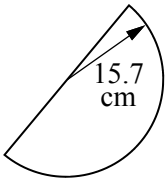


$A = \frac{1}{2}\pi r^2$

$=$

$A =$ ☐

4



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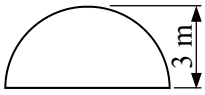

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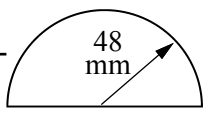

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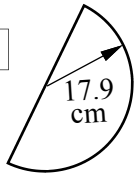


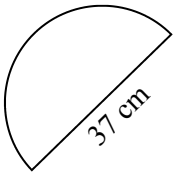
Find the area, but find the radius first by halving the diameter (÷2). Answer to 1 d.p.

8 $r =$ ☐ cm

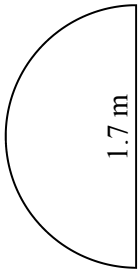
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times$ ☐^{2}

$A =$ ☐ cm^2



10 $r =$ ☐ m

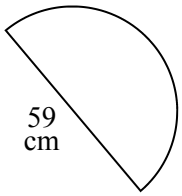


$A = \frac{1}{2}\pi r^2$

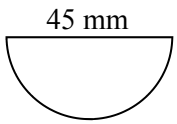
$= \frac{1}{2} \times \pi \times$ ☐^{2}

$A =$ ☐ m^2

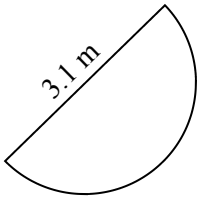
11 $r =$ ☐ cm



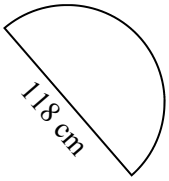
12 $r =$ ☐



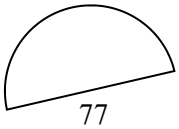
13 $r =$ ☐



12 $r =$ ☐



13 $r =$ ☐



Measure the diameters, halve for the radius, then find the area. Ruler needed! To 1 d.p.

14 ☐ radius

15 ☐ radius

16 ☐ radius

14

15

16

Area of Semi-Circles and Quadrants (Sheet 2)

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A quarter as a decimal is 0.25. Calculate these to 1 d.p., either use ¼ or 0.25.

1 $\frac{1}{4} \times \pi \times 5^2 =$

Calc: 0 . 2 5 × SHIFT ×10^x × 5 x^2 =

or

Calc: 1 ▪/□ 4 ◯ × SHIFT ×10^x × 5 x^2 =

2 $\frac{1}{4} \times \pi \times 13^2 =$

3 $\frac{1}{4} \times \pi \times 9.4^2 =$

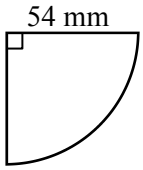
4 $\frac{1}{4} \times \pi \times 21^2 =$

5 $\frac{1}{4} \times \pi \times 74^2 =$

6 $\frac{1}{4} \times \pi \times 145^2 =$

A quadrant is a quarter of a circle, so use $A=\frac{1}{4}\pi r^2$ to find the area, rounding to 1 d.p.

Example



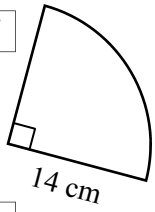
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 54^2$

$A = 2\,290.2 \text{ mm}^2$

Show square units for area

7



$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ ☐^{2}

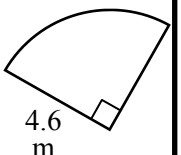
$A =$ ☐ cm^2

8

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ ☐^{2}

$A =$ ☐ m^2

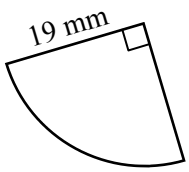


9

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ ☐^{2}

$A =$ ☐ mm^2



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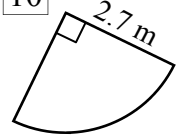
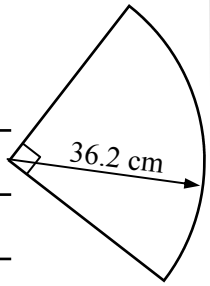


$A = \frac{1}{4}\pi r^2$

$A =$

11



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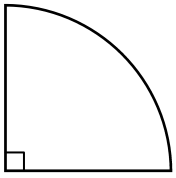

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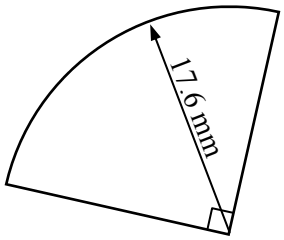
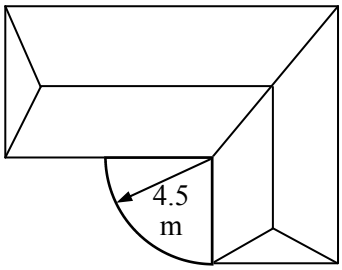


A plan view is a view from above. Look at the plan of a courtyard of a house below.

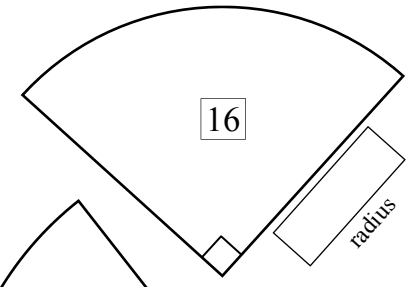


14 Calculate the area of the quadrant shaped courtyard, round your answer to 1 d.p.

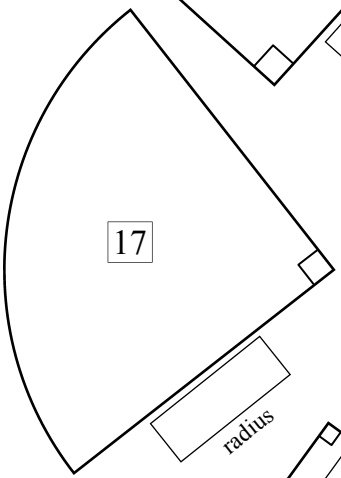
15 Estimate the cost of tiling the area if tiles cost \$56.20/m^2 and are sold in whole m^2 only.

Measure the radius with a ruler then find the area. Round your answer to 1 d.p.

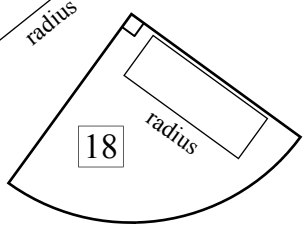
16



17



18



Area of Semi-Circles and Quadrants (Sheet 1)

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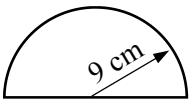
A semi-circle is half a circle, so use $A=\frac{1}{2}\pi r^2$, find the area to 1 d.p.

Example



$A = \frac{1}{2}\pi r^2$

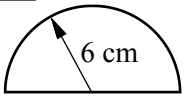
$= \frac{1}{2} \times \pi \times 9^2$

Calc: 0 . 5 × SHIFT ×10^x × 9 x^2 =

$A = 127.2 \text{ cm}^2$

Show square units

1



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 6^2$

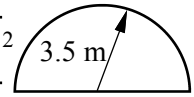
$A = 56.5 \text{ cm}^2$

2

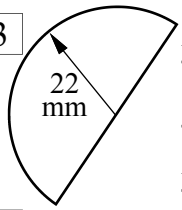
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 3.5^2$

$A = 19.2 \text{ m}^2$



3



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 22^2$

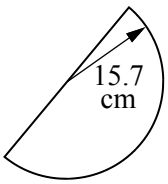
$A = 760.3 \text{ mm}^2$

4

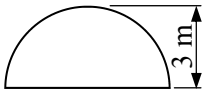
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 15.7^2$

$A = 387.2 \text{ cm}^2$



5



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 3^2$

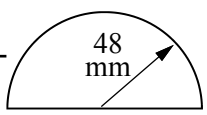
$A = 14.1 \text{ m}^2$

6

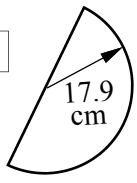
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 48^2$

$A = 3\,619.1 \text{ mm}^2$



7



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 17.9^2$

$A = 503.3 \text{ cm}^2$

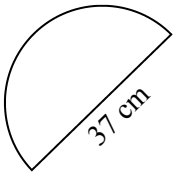
Find the area, but find the radius first by halving the diameter (÷2). Answer to 1 d.p.

8 $r = 18.5$ cm

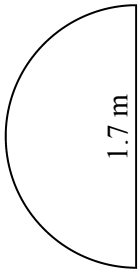
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 18.5^2$

$A = 537.6 \text{ cm}^2$



10 $r = 0.85$ m



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 0.85^2$

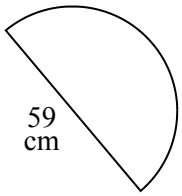
$A = 1.1 \text{ m}^2$

11 $r = 29.5$ cm

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 29.5^2$

$A = 1\,367.0 \text{ cm}^2$

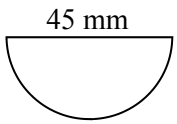


12 $r = 22.5$ mm

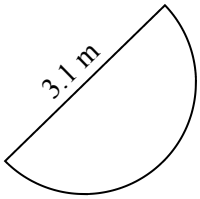
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 22.5^2$

$A = 795.2 \text{ mm}^2$



13 $r = 1.55$ m



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 1.55^2$

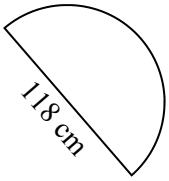
$A = 3.8 \text{ m}^2$

12 $r = 59$ cm

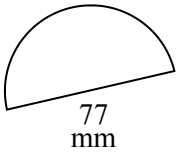
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 59^2$

$A = 5\,467.9 \text{ cm}^2$



13 $r = 38.5$ mm



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 38.5^2$

$A = 2\,328.3 \text{ mm}^2$

Measure the diameters, halve for the radius, then find the area. Ruler needed! To 1 d.p.

14 $A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 2.1^2$

$A = 6.9 \text{ cm}^2$

15 $A = \frac{1}{2}\pi r^2$

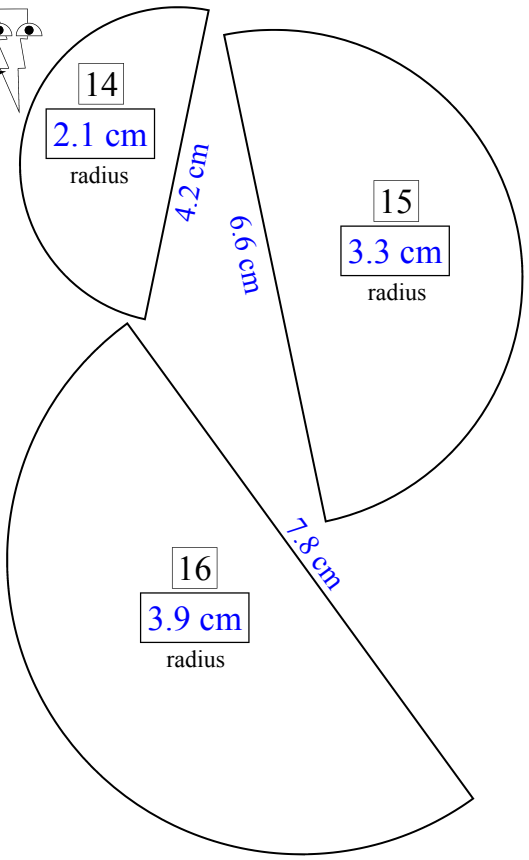
$= \frac{1}{2} \times \pi \times 3.3^2$

$A = 17.1 \text{ cm}^2$

16 $A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 3.9^2$

$A = 23.9 \text{ cm}^2$



Area of Semi-Circles and Quadrants (Sheet 2)

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A quarter as a decimal is 0.25. Calculate these to 1 d.p., either use ¼ or 0.25.

1 $\frac{1}{4} \times \pi \times 5^2 =$ 19.6

Calc: 0 . 2 5 × SHIFT ×10^x × 5 x² =

or

Calc: 1 ■/□ 4 ◯ × SHIFT ×10^x × 5 x² =

2 $\frac{1}{4} \times \pi \times 13^2 =$ 132.7

3 $\frac{1}{4} \times \pi \times 9.4^2 =$ 69.4

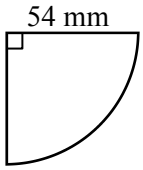
4 $\frac{1}{4} \times \pi \times 21^2 =$ 346.4

5 $\frac{1}{4} \times \pi \times 74^2 =$ 4 300.8

6 $\frac{1}{4} \times \pi \times 145^2 =$ 16 513.0

A quadrant is a quarter of a circle, so use $A=\frac{1}{4}\pi r^2$ to find the area, rounding to 1 d.p.

Example



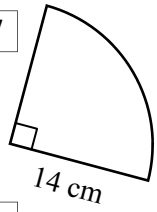
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 54^2$

$A = 2\,290.2 \text{ mm}^2$

Show square units for area

7



$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ 14 ^{2}

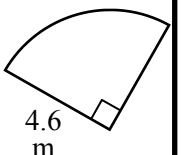
$A =$ 153.9 cm^2

8

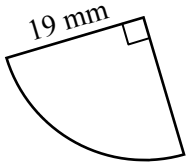
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ 4.6 ^{2}

$A =$ 16.6 m^2



9

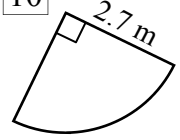


$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ 19 ^{2}

$A =$ 283.5 mm^2

10



$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 2.7^2$

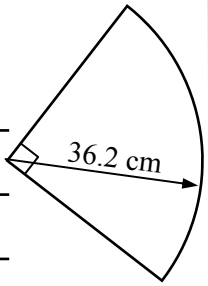
$A =$ 5.7 m^2

11

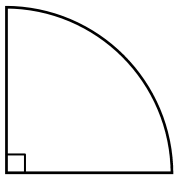
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 36.2^2$

$A = 1\,029.2 \text{ cm}^2$



12

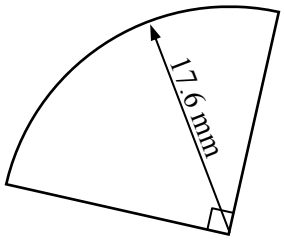


$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 0.28^2$

$A = 0.1 \text{ m}^2$

13

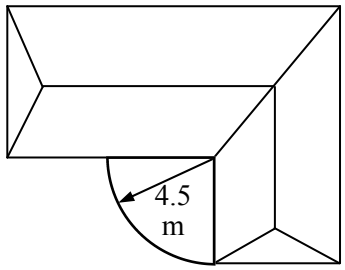


$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 17.6^2$

$A = 243.3 \text{ mm}^2$

A plan view is a view from above. Look at the plan of a courtyard of a house below.



14 Calculate the area of the quadrant shaped courtyard, round your answer to 1 d.p.

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 4.5^2$

$A = 15.9 \text{ m}^2$

15 Estimate the cost of tiling the area if tiles cost \$56.20/m^2 and are sold in whole m^2 only.

Cost = 16 × 56.20

Cost = \$899.20

Measure the radius with a ruler then find the area. Round your answer to 1 d.p.

16 $A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 3.6^2$

$A = 10.2 \text{ cm}^2$

17 $A = \frac{1}{4}\pi r^2$

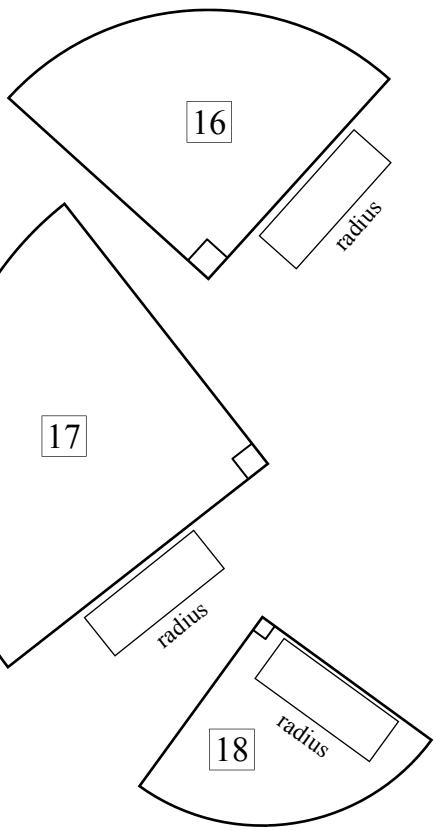
$= \frac{1}{4} \times \pi \times 4.4^2$

$A = 15.2 \text{ cm}^2$

18 $A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 2.8^2$

$A = 6.2 \text{ cm}^2$



Area - Area of Semi-circles and Quadrants (Homework Strip)

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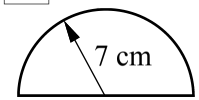
Date Due: _____

Find the area of the semi-circles (1 d.p.)

1



$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

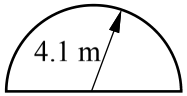
$A = \square$ cm^2

2

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A = \square$ m^2

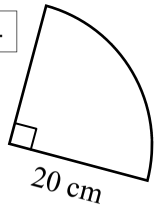


3 $r = \square$ m



Find the area of the quadrants (1 d.p.)

4



$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

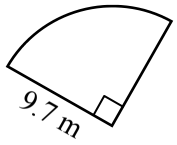
$A = \square$ cm^2

5

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A = \square$



Date Due: _____

Find the area of the semi-circles (1 d.p.)

1

7 cm

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A = \square$ cm^2

2

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A = \square$ m^2

4.1 m

3 $r = \square$ m

16 m

Find the area of the quadrants (1 d.p.)

4

20 cm

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A = \square$ cm^2

5

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A = \square$

9.7 m

Date Due: _____

Find the area of the semi-circles (1 d.p.)

1

7 cm

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A = \square$ cm^2

2

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A = \square$ m^2

4.1 m

3 $r = \square$ m

16 m

Find the area of the quadrants (1 d.p.)

4

20 cm

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A = \square$ cm^2

5

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A = \square$

9.7 m

Date Due: _____

Find the area of the semi-circles (1 d.p.)

1

7 cm

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A = \square$ cm^2

2

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A = \square$ m^2

4.1 m

3 $r = \square$ m

16 m

Find the area of the quadrants (1 d.p.)

4

20 cm

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A = \square$ cm^2

5

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A = \square$

9.7 m

Area - Area of Semi-circles and Quadrants (Homework Strip)

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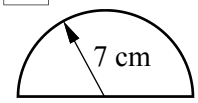
Date Due:

Find the area of the semi-circles (1 d.p.)

1



$A = \frac{1}{2}\pi r^2$

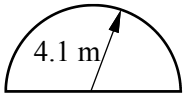
$= \frac{1}{2} \times \pi \times 7^2$

$A =$ 77.0 cm^2

2

$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 4.1^2$



$A =$ 26.4 m^2

3 $r =$ 8 m $A = \frac{1}{2}\pi r^2$

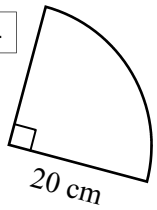


$= \frac{1}{2} \times \pi \times 8^2$

$A = 100.5\ m^2$

Find the area of the quadrants (1 d.p.)

4



$A = \frac{1}{4}\pi r^2$

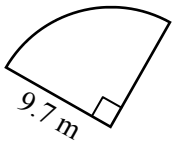
$= \frac{1}{4} \times \pi \times 20^2$

$A =$ 314.2 cm^2

5

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 9.7^2$



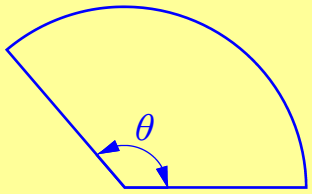
$A =$ 73.9 m^2

Area of Sectors

If you know how to multiply by a fraction and how to find the area of a circle, then these worksheets simply blends the two skills.

The formula for calculating the area of a circle is $A = \pi r^2$. But with sectors the circle formula is multiplied by a fraction. The formula for the Area of a sector is:

$$A = \frac{\theta}{360} \times \pi r^2$$



Where:

A = Area of the circle, in the same units as the radius, but squared. (cm^2, m^2, mm^2)
θ = The internal angle of the sector (measured in degrees '°')
r = The radius of the circle (distance from centre to circumference).

Column 1 starts with fractions of a circle. This is the internal angle over 360°. So using your calculator's fraction key, enter the fraction and press '=' to simplify it. Note that you may be given an external angle, that is, the angle outside the sector. If given an external angle, subtract it from 360° first, (Internal angle = 360° - external angle), before calculating the fraction.

Column 2 introduces the calculation process by giving calculations that are the same as the area calculation. There is a calculator key step process shown underneath Q.10. Then from Q.16 you are asked to calculate the area of a sector. There is an example above Q.16. All these questions feature internal angles, remember to show square units (mm^2, cm^2, m^2, etc.)

The second sheet deals firstly with external angle questions. So these have a 2-step process. The first step is to subtract the angle from 360° to get an internal angle, then solve the same as before.

The second column is four questions on Area of Sectors, but this time you have to measure the angle with a protractor and measure the radius with a ruler. Then using the measurements, calculate the area using the spaces at the base of the column.

Area of Sectors (Sheet 1)

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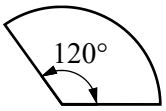
Write the fraction of a circle with a denominator of 360, then simplify with your calculator.

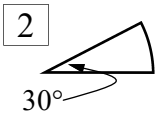
1



$\frac{\square}{360} = \frac{\square}{\square}$

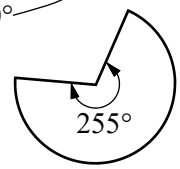
Calc: 1 2 0 ▪ 3 6 0 =

2



$\frac{\square}{\square} = \frac{\square}{\square}$

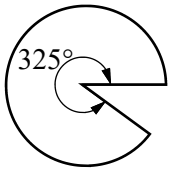
3



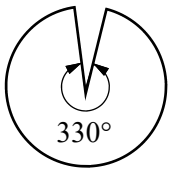
$\frac{\square}{\square} = \frac{\square}{\square}$

4

$\frac{\square}{\square} = \frac{\square}{\square}$



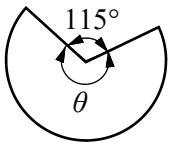
5



$\frac{\square}{\square} = \frac{\square}{\square}$

Find the internal angle θ, by subtracting from 360°. Then solve for the fraction as above.

6



$\theta = 360 - 115$

$\theta = \square°$

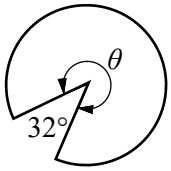
$\frac{\square}{360} = \frac{\square}{\square}$

7

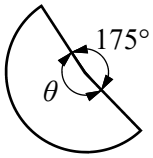
$\theta = 360 - \square$

$\theta = \square°$

$\frac{\square}{\square} = \frac{\square}{\square}$



8



$\theta =$

$\theta =$

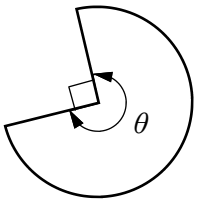
$\frac{\square}{\square} = \frac{\square}{\square}$

9

$\theta =$

$\theta =$

$\frac{\square}{\square} = \frac{\square}{\square}$



Evaluate using your calculator. Round to 1 d.p.

10 $\frac{105}{360} \times \pi \times 16^2 =$

1 0 5 ▪ 3 6 0 ◯ × SHIFT ×10^x × 1 6 x^2 =

11 $\frac{52}{360} \times \pi \times 7^2 =$

12 $\frac{212}{360} \times \pi \times 25^2 =$

13 $\frac{90}{360} \times \pi \times 34.3^2 =$

14 $\frac{322}{360} \times \pi \times 40.8^2 =$

15 $\frac{159}{360} \times \pi \times 19.6^2 =$

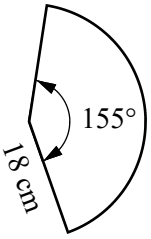
Calculate the area of these sectors, answering to 1 d.p.

Example

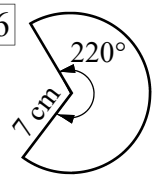
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{155}{360} \times \pi \times 18^2$

$A = 438.3 \text{ cm}^2$



16



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{220}{360} \times \pi \times \square^2$

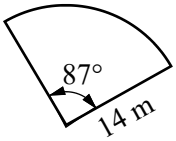
$A = \square \text{ cm}^2$

17

$A = \frac{\theta}{360} \times \pi r^2$

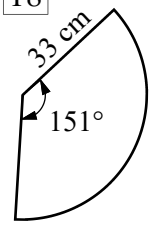
$= \frac{87}{360} \times \pi \times \square^2$

$A = \square \text{ m}^2$



Remember to square the radius!

18

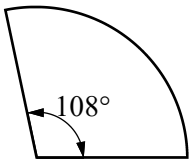


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A = \square \text{ cm}^2$

19



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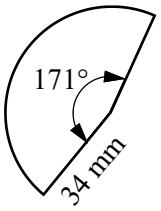

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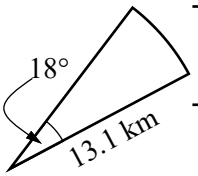

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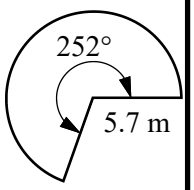

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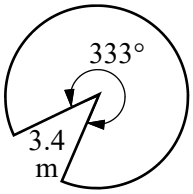

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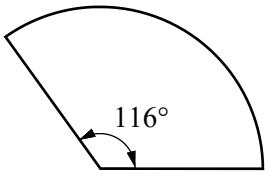

Area of Sectors (Sheet 2)

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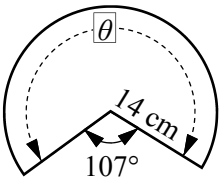
Find the internal angle θ, then calculate the area of the sectors, rounding your answer to 1 d.p.

1



$\theta = 360 - 107$

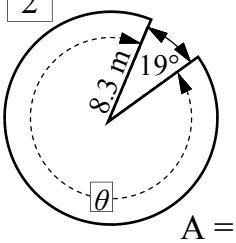
$\theta =$ ☐ °

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A =$ ☐ cm^2

2



$\theta =$

$\theta =$

$A = \frac{\theta}{360} \times \pi r^2$

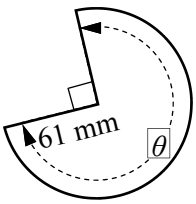
$= \frac{\square}{360} \times \pi \times \square^2$

$A =$ ☐ m^2

3

$\theta =$

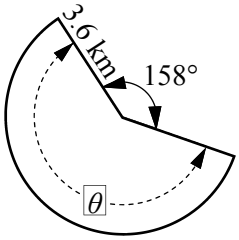
$\theta =$



4

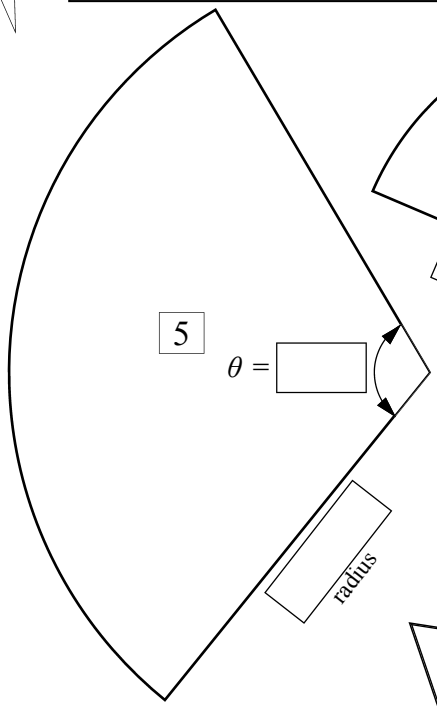
$\theta =$

$\theta =$



Measure the radius in cm to 1 decimal place, measure the angle to the nearest degree, then find the area (1 d.p.). Ruler and protractor needed!

5



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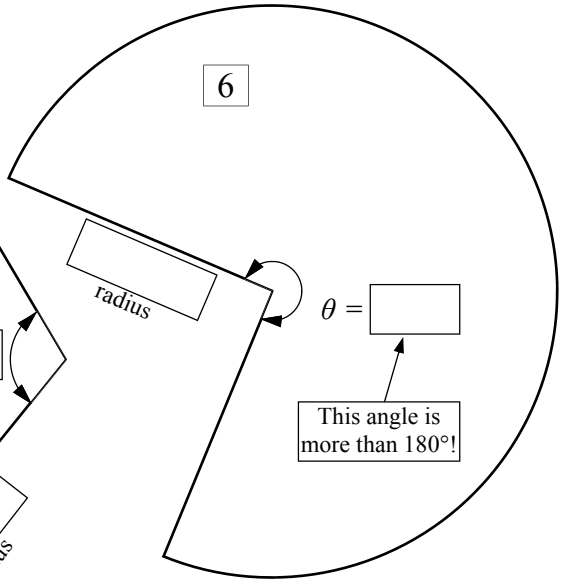

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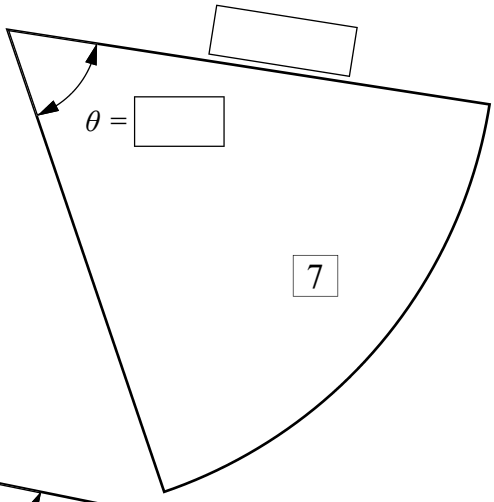

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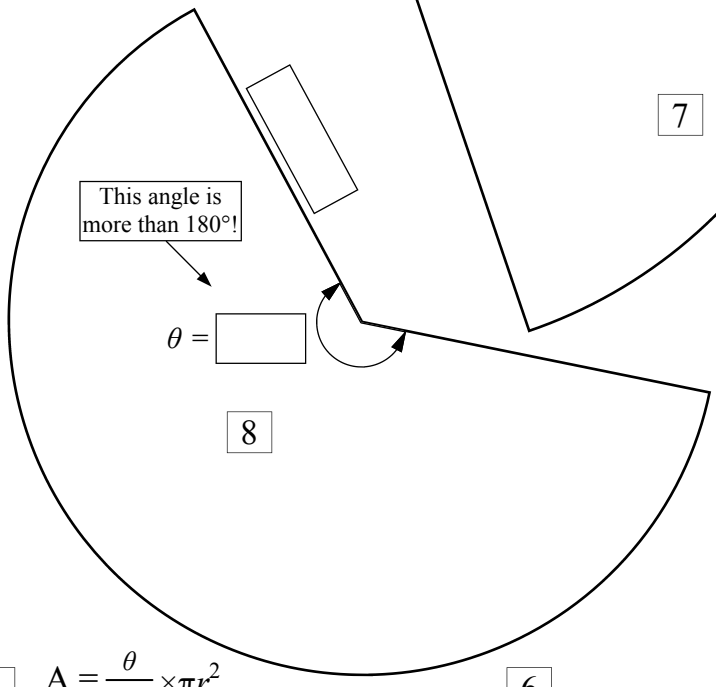

5 $A = \frac{\theta}{360} \times \pi r^2$

6

7

8

Area of Sectors (Sheet 1)

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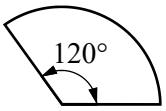
Write the fraction of a circle with a denominator of 360, then simplify with your calculator.

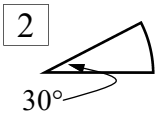
1



$\frac{120}{360} = \frac{1}{3}$

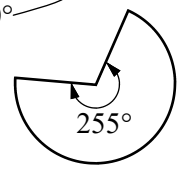
Calc: 1 2 0 ▪ 3 6 0 =

2



$\frac{30}{360} = \frac{1}{12}$

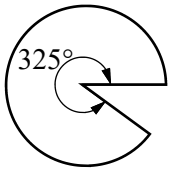
3



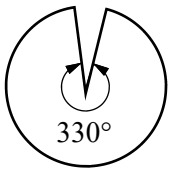
$\frac{255}{360} = \frac{17}{24}$

4

$\frac{325}{360} = \frac{65}{72}$



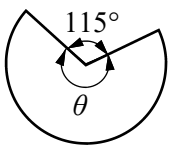
5



$\frac{330}{360} = \frac{11}{12}$

Find the internal angle θ, by subtracting from 360°. Then solve for the fraction as above.

6



$\theta = 360 - 115$

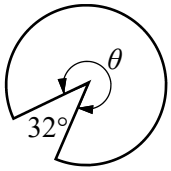
$\theta = 245°$

$\frac{245}{360} = \frac{49}{72}$

7

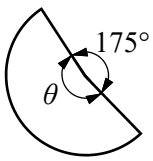
$\theta = 360 - 32$

$\theta = 328°$



$\frac{328}{360} = \frac{41}{45}$

8



$\theta = 360 - 175$

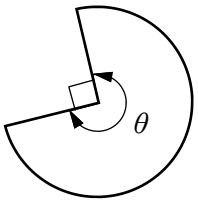
$\theta = 185°$

$\frac{185}{360} = \frac{37}{72}$

9

$\theta = 360 - 90$

$\theta = 270°$



$\frac{270}{360} = \frac{3}{4}$

Evaluate using your calculator. Round to 1 d.p.

10 $\frac{105}{360} \times \pi \times 16^2 =$ **234.6**

1 0 5 ▪ 3 6 0 × SHIFT ×10^x × 1 6 x^2 =

11 $\frac{52}{360} \times \pi \times 7^2 =$ **22.2**

12 $\frac{212}{360} \times \pi \times 25^2 =$ **1 156.3**

13 $\frac{90}{360} \times \pi \times 34.3^2 =$ **924.0**

14 $\frac{322}{360} \times \pi \times 40.8^2 =$ **4 677.6**

15 $\frac{159}{360} \times \pi \times 19.6^2 =$ **533.0**

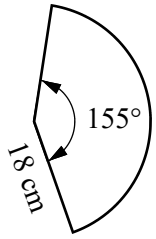
Calculate the area of these sectors, answering to 1 d.p.

Example

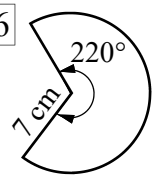
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{155}{360} \times \pi \times 18^2$

$A = 438.3 \text{ cm}^2$



16



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{220}{360} \times \pi \times 7^2$

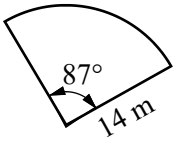
$A = 94.1 \text{ cm}^2$

17

$A = \frac{\theta}{360} \times \pi r^2$

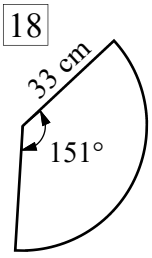
$= \frac{87}{360} \times \pi \times 14^2$

$A = 148.8 \text{ m}^2$



Remember to square the radius!

18

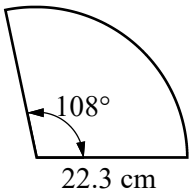


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{151}{360} \times \pi \times 33^2$

$A = 1\,435.0 \text{ cm}^2$

19



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{108}{360} \times \pi \times 22.3^2$

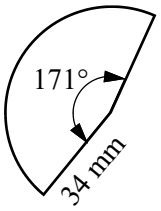
$A = 468.7 \text{ cm}^2$

20

$A = \frac{\theta}{360} \times \pi r^2$

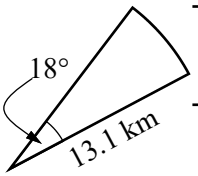
$= \frac{171}{360} \times \pi \times 34^2$

$A = 1\,725.0 \text{ mm}^2$



21

$A = \frac{\theta}{360} \times \pi r^2$



$= \frac{18}{360} \times \pi \times 13.1^2$

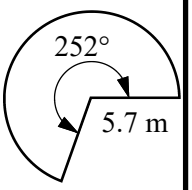
$A = 27.0 \text{ km}^2$

22

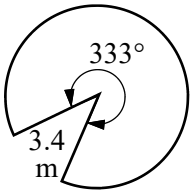
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{252}{360} \times \pi \times 5.7^2$

$A = 71.4 \text{ m}^2$



23

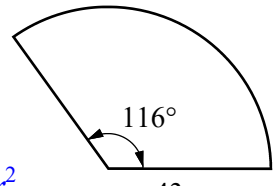


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{333}{360} \times \pi \times 3.4^2$

$A = 33.6 \text{ m}^2$

24



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{116}{360} \times \pi \times 43^2$

$A = 1\,871.7 \text{ cm}^2$

Area of Sectors (Sheet 2)

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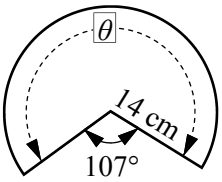
Find the internal angle θ, then calculate the area of the sectors, rounding your answer to 1 d.p.

1



$\theta = 360 - 107$

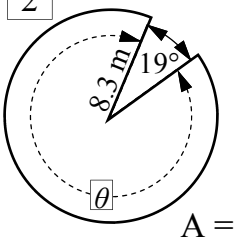
$\theta = 253°$

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{253}{360} \times \pi \times 14^2$

$A = 432.7 \text{ cm}^2$

2

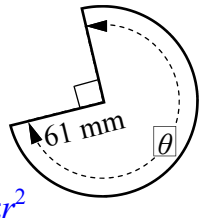


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{341}{360} \times \pi \times 8.3^2$

$A = 205.0 \text{ m}^2$

3



$\theta = 360 - 90$

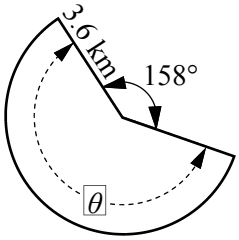
$\theta = 270°$

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{270}{360} \times \pi \times 61^2$

$A = 8\,767.4 \text{ mm}^2$

4



$\theta = 360 - 158$

$\theta = 202°$

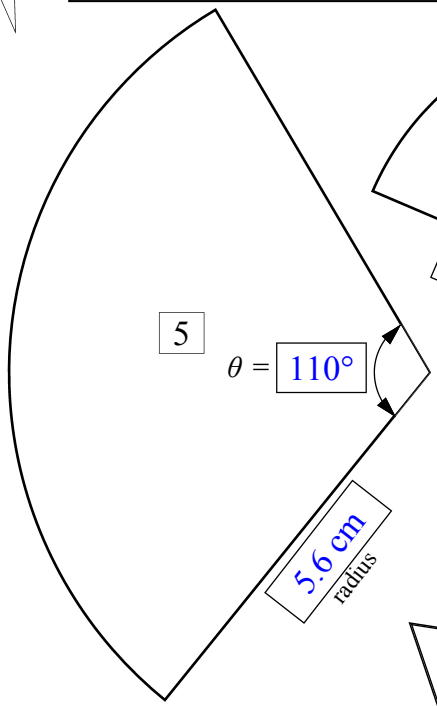
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{158}{360} \times \pi \times 3.6^2$

$A = 17.9 \text{ km}^2$

Measure the radius in cm to 1 decimal place, measure the angle to the nearest degree, then find the area (1 d.p.). Ruler and protractor needed!

5

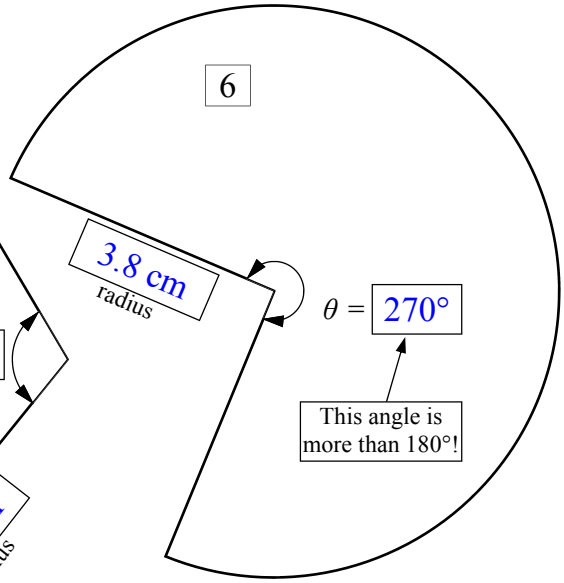


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{110}{360} \times \pi \times 5.6^2$

$A = 30.1 \text{ cm}^2$

6

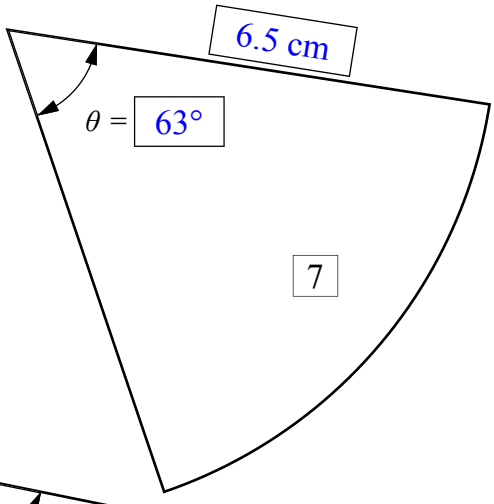


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{270}{360} \times \pi \times 3.8^2$

$A = 34.0 \text{ cm}^2$

7

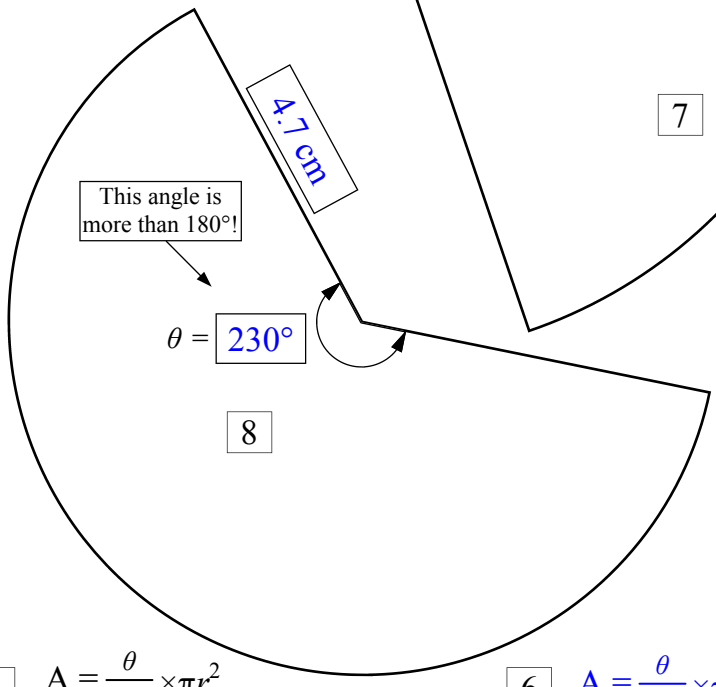


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{63}{360} \times \pi \times 6.5^2$

$A = 23.2 \text{ cm}^2$

8



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{230}{360} \times \pi \times 4.7^2$

$A = 44.3 \text{ km}^2$

Area - Area of Sectors (Homework Strip)

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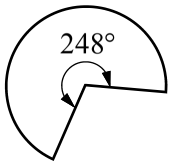
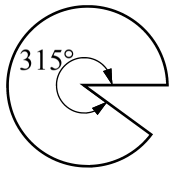
Date Due: _____

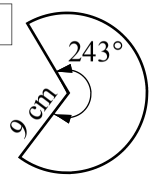
Find the fraction of a circle shown

1 

$\frac{\square}{360} = \frac{\square}{\square}$

2 $\frac{\square}{\square} = \frac{\square}{\square}$ 

Find the area of these sectors (1 d.p.)

3 

$A = \frac{\theta}{360} \times \pi r^2$

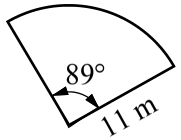
$= \frac{\square}{360} \times \pi \times \square^2$

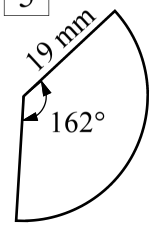
$A = \square$ cm^2

4 $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A = \square$ m^2



5 

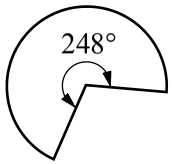
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

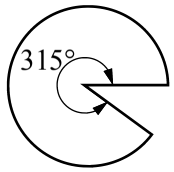
$A = \square$ mm^2

Date Due: _____

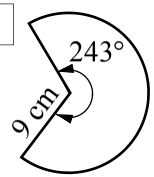
Find the fraction of a circle shown

1 

$\frac{\square}{360} = \frac{\square}{\square}$

2 $\frac{\square}{\square} = \frac{\square}{\square}$ 

Find the area of these sectors (1 d.p.)

3 

$A = \frac{\theta}{360} \times \pi r^2$

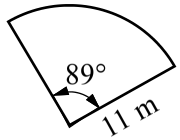
$= \frac{\square}{360} \times \pi \times \square^2$

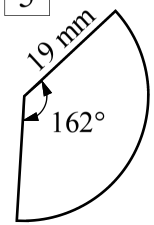
$A = \square$ cm^2

4 $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A = \square$ m^2



5 

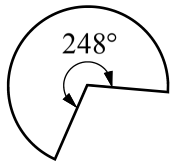
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

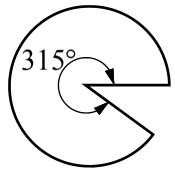
$A = \square$ mm^2

Date Due: _____

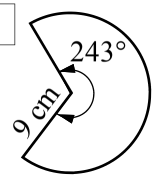
Find the fraction of a circle shown

1 

$\frac{\square}{360} = \frac{\square}{\square}$

2 $\frac{\square}{\square} = \frac{\square}{\square}$ 

Find the area of these sectors (1 d.p.)

3 

$A = \frac{\theta}{360} \times \pi r^2$

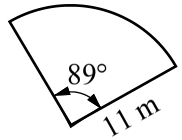
$= \frac{\square}{360} \times \pi \times \square^2$

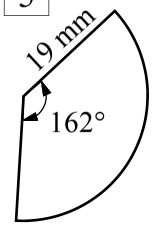
$A = \square$ cm^2

4 $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A = \square$ m^2



5 

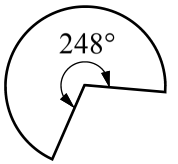
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

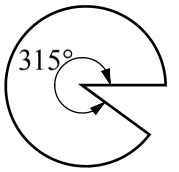
$A = \square$ mm^2

Date Due: _____

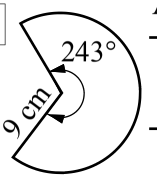
Find the fraction of a circle shown

1 

$\frac{\square}{360} = \frac{\square}{\square}$

2 $\frac{\square}{\square} = \frac{\square}{\square}$ 

Find the area of these sectors (1 d.p.)

3 

$A = \frac{\theta}{360} \times \pi r^2$

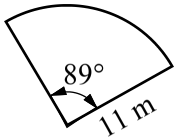
$= \frac{\square}{360} \times \pi \times \square^2$

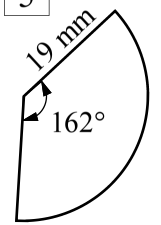
$A = \square$ cm^2

4 $A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A = \square$ m^2



5 

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A = \square$ mm^2

Area - Area of Sectors (Homework Strip)

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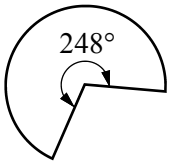
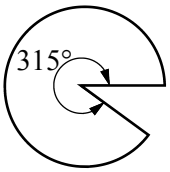
Date Due: _____

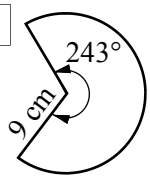
Find the fraction of a circle shown

1 

$\frac{248}{360} = \frac{31}{45}$

2 $\frac{315}{360} = \frac{7}{8}$ 

Find the area of these sectors (1 d.p.)

3 

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{243}{360} \times \pi \times 9^2$

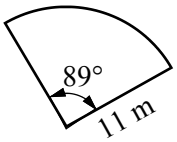
$A = 171.8 \text{ cm}^2$

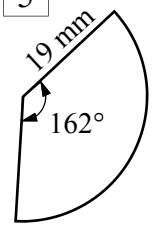
4

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{89}{360} \times \pi \times 11^2$

$A = 94.0 \text{ m}^2$



5 

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{162}{360} \times \pi \times 19^2$

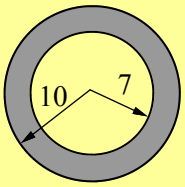
$A = 510.4 \text{ mm}^2$

The Annulus

An annulus is a ring-like figure, in other words a circle within another circle. Most students recognise the similarity with the word ‘anus’, as well as the similar appearance. For this reason it is usually an easy mathematical word to remember!

Column 1 deals with calculating the area of an annulus. The process is to calculate the outside circle’s area, then calculate the inside circle’s area, then subtract them. An example is below.

Example - Area of an Annulus



Lengths are in cm

$A_1 = \pi R^2$

$= \pi \times 10^2$

$A_1 = 314.2 \text{ cm}^2$ (1 dp)

Capital R, means ‘big’ radius.

So A_1 is the area of the outside circle.

$A_2 = \pi r^2$

$= \pi \times 7^2$

$A_2 = 153.9 \text{ cm}^2$ (1 dp)

Lower case r, means ‘small’ radius.

So A_2 is the area of the inside circle.

$A = A_1 - A_2 = 314.2 - 153.9$

$A = 160.3 \text{ cm}^2$

Then subtract for the annulus area. The larger area - smaller area.

Column 2 of the first sheet adds a step by giving some measurements as diameters. Change any diameters to radii (plural of radius) first. Then solve the same way.

Sheet 2 introduces the annulus formula. This is a faster way of solving exercises but it is important that you understand the process above as well. The formula is:

$$A = \pi (R^2 - r^2)$$

Where:

A = Area of annulus
R = outside circle’s radius
r = inside circle’s radius

This formula uses the radius, so when you are given diameters (most of the second column) you have to divide by 2 first.

The Annulus (Circle Formula) - Sheet 1

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Find the outside area and the inside area, then subtract to find the shaded area. (1 d.p.)

All measurements are in cm

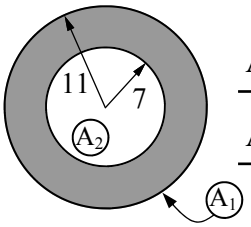
1 $A_1 = \pi R^2$ $A_2 = \pi r^2$

$= \pi \times 11^2$ $= \pi \times 7^2$

$A_1 =$ ____ cm^2 $A_2 =$ ____ cm^2

$A =$ ____ - ____

$A =$ ____ cm^2



2 $A_1 = \pi R^2$ $A_2 = \pi r^2$

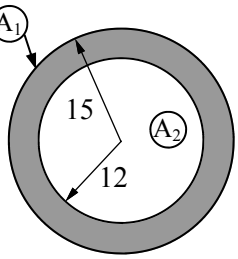
$= \pi \times$ ____ ^{2} $= \pi \times$ ____ ^{2}

This Annulus

$A_1 =$ ____ cm^2 $A_2 =$ ____ cm^2

$A =$ ____ - ____

$A =$ ____ cm^2



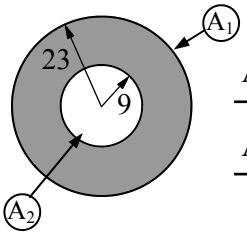
3 $A_1 = \pi R^2$ $A_2 = \pi r^2$

$= \pi \times$ ____ ^{2} $= \pi \times$ ____ ^{2}

$A_1 =$ ____ cm^2 $A_2 =$ ____ cm^2

$A =$ ____ - ____

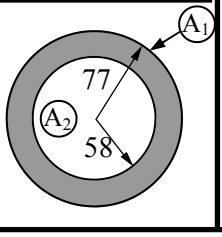
$A =$ ____ cm^2



4

$A_1 = \pi R^2$ $A_2 = \pi r^2$

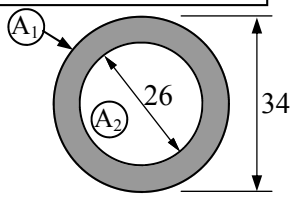
This Annulus



Change diameters to radii first, before calculation of the shaded area. All lengths are in cm. (1 d.p.)

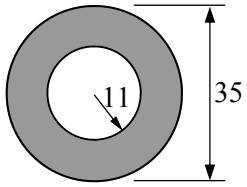
5 $R =$ ____ cm

$r =$ ____ cm



$A_1 = \pi R^2$ $A_2 = \pi r^2$

6 $R =$ ____ cm

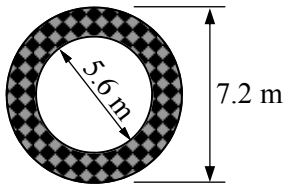


A plan view of a proposed circular swimming pool is shown below. The pool is to have a circular tiled area surrounding it with a fence around the tiled area.

7 Find the tiled (shaded) area (1 d.p.)

$R =$ ____ m

$r =$ ____ m



8 Find the cost of tiling the area given that the tiles cost \$47.55/m^2 and are sold in whole m^2 only.

Cost =

The Annulus (Annulus Formula) - Sheet 2

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Calculate these using your 'brackets' key. Answer to 2 decimal places.

1 $\pi (15^2 - 9^2) =$ ____

Calc: SHIFT $\times 10^x$ × (1 5 x^2 - 9 x^2) =

2 $\pi (27^2 - 18^2) =$ ____

3 $\pi (32^2 - 26^2) =$ ____

4 $\pi (41^2 - 22^2) =$ ____

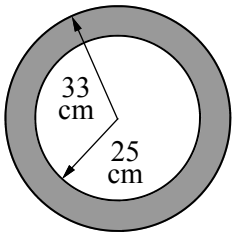
5 $\pi (56.7^2 - 38.3^2) =$ ____

Use the annulus formula, $A = \pi(R^2 - r^2)$ to find the shaded area. Answer to 1 decimal place.

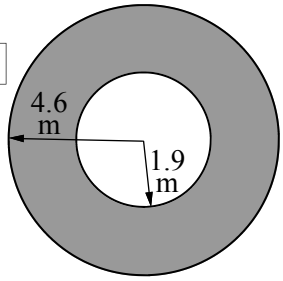
6 $A = \pi(R^2 - r^2)$

$= \pi \times (\square^2 - \square^2)$

$A =$ ____ cm^2



7



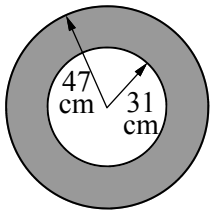
$A = \pi(R^2 - r^2)$

$= \pi \times (\square^2 - \square^2)$

$A =$ ____

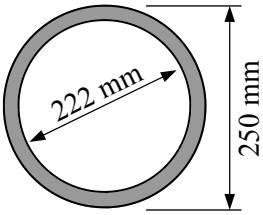
8

$A = \pi(R^2 - r^2)$

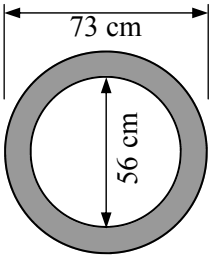


These questions have their diameter measurements, so divide by 2 to get the radius first. (÷2)

9 $R =$ ____ mm $r =$ ____ mm

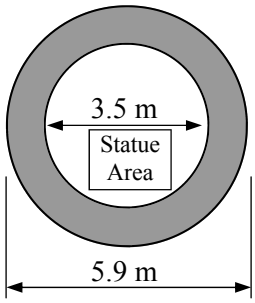


10 $R =$ ____ cm $r =$ ____ cm



A roundabout has a central area with a statue that has a 3.5 m diameter. It has a garden bed around it, shaded area on the plan. The total diameter of the roundabout is 5.9 m

11 Find the garden bed area, answer to nearest m^2.

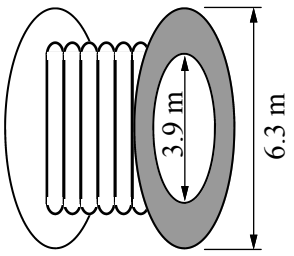


12 If the gardener buys 9 plants for every square metre, how many plants will be required?

13 What is the cost of the plants if the first 50 plants cost $4.50 each, then the remaining plants cost $4.15 each

A mineshaft cable pulley, shown below, has a plate steel circular collar with an inside diameter of 3.9 m and an outside diameter of 6.3 m.

14 Find the collar area (shaded area), to 1 d.p.



15 Find the weight of the collar, if the plate steel weighs 53 kg per m^2. Answer in kg, to 1 d.p.

The Annulus (Circle Formula) - Sheet 1

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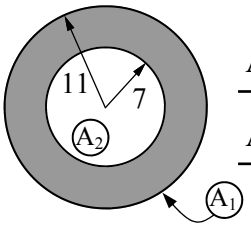
Find the outside area and the inside area, then subtract to find the shaded area. (1 d.p.)

All measurements are in cm

1 $A_1 = \pi R^2$ $= \pi \times 11^2$ $A_1 =$ 380.1 cm²

$A_2 = \pi r^2$ $= \pi \times 7^2$ $A_2 =$ 153.9 cm²



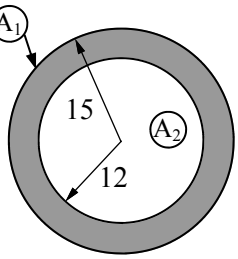
$A =$ 380.1 - 153.9

$A =$ 226.2 cm²

2 $A_1 = \pi R^2$ $= \pi \times$ 15 ² $A_1 =$ 706.9 cm²

This Annulus

$A_2 = \pi r^2$ $= \pi \times$ 12 ² $A_2 =$ 452.4 cm²

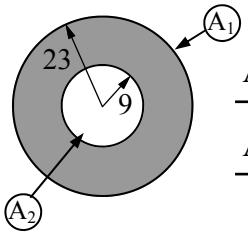


$A =$ 706.9 - 452.4

$A =$ 254.5 cm²

3 $A_1 = \pi R^2$ $= \pi \times$ 23 ² $A_1 =$ 1 661.9 cm²

$A_2 = \pi r^2$ $= \pi \times$ 9 ² $A_2 =$ 254.5 cm²



$A =$ 1 661.9 - 254.5

$A =$ 1 407.4 cm²

4

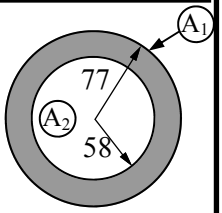
$A_1 = \pi R^2$ $= \pi \times 77^2$ $A_1 = 18\,626.5$ cm²

This Annulus

$A_2 = \pi r^2$ $= \pi \times 58^2$ $A_2 = 10\,568.3$ cm²

$A = 18\,626.5 - 10\,568.3$

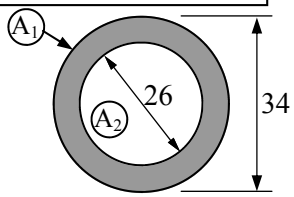
$A = 8\,058.2$ cm²



Change diameters to radii first, before calculation of the shaded area. All lengths are in cm. (1 d.p.)

5 $R =$ 17 cm

$r =$ 13 cm

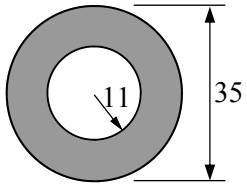


$A_1 = \pi R^2$ $= \pi \times 17^2$ $A_1 = 907.9$ cm²

$A_2 = \pi r^2$ $= \pi \times 13^2$ $A_1 = 530.9$ cm²

$A = 907.9 - 530.9 = 377.0$ cm²

6 $R =$ 17.5 cm



$A_1 = \pi R^2$ $= \pi \times 17.5^2$ $A_1 = 962.1$ cm²

$A_2 = \pi r^2$ $= \pi \times 11^2$ $A_2 = 380.1$ cm²

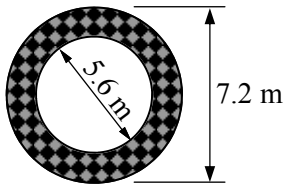
$A = 962.1 - 380.1 = 582.0$ cm²

A plan view of a proposed circular swimming pool is shown below. The pool is to have a circular tiled area surrounding it with a fence around the tiled area.

7 Find the tiled (shaded) area (1 d.p.)

$R =$ 3.6 m

$r =$ 2.8 m



$A_1 = \pi R^2$ $= \pi \times 3.6^2$ $A_1 = 40.7$ m²

$A_2 = \pi r^2$ $= \pi \times 2.8^2$ $A_2 = 24.6$ m²

$A = 40.7 - 24.6 = 16.1$ m²

8 Find the cost of tiling the area given that the tiles cost \$47.55/m² and are sold in whole m² only.

Cost = 47.55 × 17 (17 m² required)

Cost = \$808.35

The Annulus (Annulus Formula) - Sheet 2

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Calculate these using your 'brackets' key. Answer to 2 decimal places.

1 $\pi (15^2 - 9^2) =$ 452.39

Calc: SHIFT $\times 10^x$ × (1 5 x^2 - 9 x^2) =

2 $\pi (27^2 - 18^2) =$ 1 272.35

3 $\pi (32^2 - 26^2) =$ 1 093.27

4 $\pi (41^2 - 22^2) =$ 3 760.49

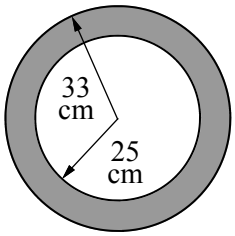
5 $\pi (56.7^2 - 38.3^2) =$ 5 491.50

Use the annulus formula, $A = \pi(R^2 - r^2)$ to find the shaded area. Answer to 1 decimal place.

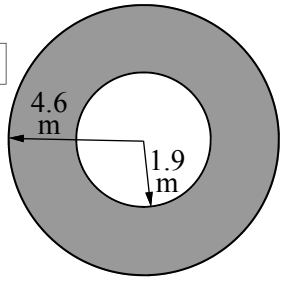
6 $A = \pi(R^2 - r^2)$

$= \pi \times ($ 33 ^{2} - 25 ^{2} $)$

$A =$ 1 457.7 cm^2



7



$A = \pi(R^2 - r^2)$

$= \pi \times ($ 4.6 ^{2} - 1.9 ^{2} $)$

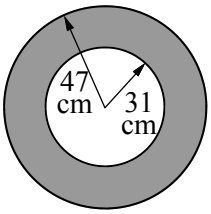
$A =$ 55.1 m^2

8

$A = \pi(R^2 - r^2)$

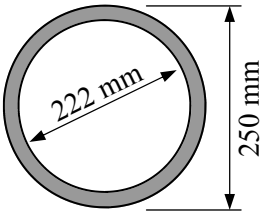
$= \pi \times (47^2 - 31^2)$

$A = 3\,920.7\ cm^2$



These questions have their diameter measurements, so divide by 2 to get the radius first. (÷2)

9 $R =$ 125 mm $r =$ 111 mm

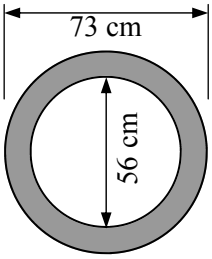


$A = \pi(R^2 - r^2)$

$= \pi \times (125^2 - 111^2)$

$A = 10\,379.8\ mm^2$

10 $R =$ 36.5 cm $r =$ 28 cm



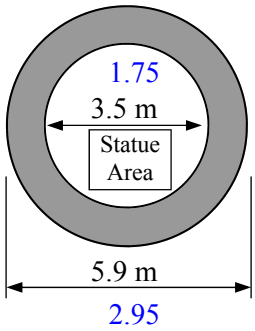
$A = \pi(R^2 - r^2)$

$= \pi \times (36.5^2 - 28^2)$

$A = 1\,722.4\ cm^2$

A roundabout has a central area with a statue that has a 3.5 m diameter. It has a garden bed around it, shaded area on the plan. The total diameter of the roundabout is 5.9 m

11 Find the garden bed area, answer to nearest m^2.



$A = \pi(R^2 - r^2)$

$= \pi \times (2.95^2 - 1.75^2)$

$A = 18\ m^2 \quad (17.7\ m^2)$

12 If the gardener buys 9 plants for every square metre, how many plants will be required?

Plants = 18 × 9 = 162 plants

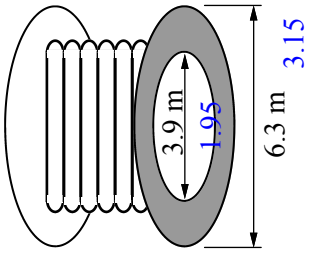
13 What is the cost of the plants if the first 50 plants cost $4.50 each, then the remaining plants cost $4.15 each

Cost = 50 × 4.50 + 112 × 4.15

Cost = $689.80

A mineshaft cable pulley, shown below, has a plate steel circular collar with an inside diameter of 3.9 m and an outside diameter of 6.3 m.

14 Find the collar area (shaded area), to 1 d.p.



$A = \pi(R^2 - r^2)$

$= \pi \times (3.15^2 - 1.95^2)$

$A = 19.2\ m^2$

15 Find the weight of the collar, if the plate steel weighs 53 kg per m^2. Answer in kg, to 1 d.p.

Weight = 19.2 × 53 = 1 017.6 kg

Area - Area of an Annulus (Homework Strip)

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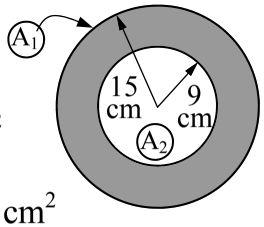
Date Due: _____

Find the area of these Annuli, to 1 d.p.

1



$A_1 = \pi R^2$

$= \pi \times$ ____^{2}

$A_1 =$ ____ cm^2

$A_2 = \pi r^2$

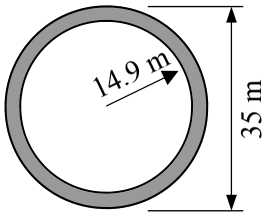
$= \pi \times$ ____^{2}

$A_2 =$ ____ cm^2

$A =$ ____ - ____

$A =$ ____

2 $R =$ ____ m ← Find $35 \div 2$



$A = \pi(R^2 - r^2)$

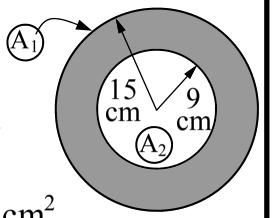
$= \pi \times ($ ____^{2} - ____^{2} $)$

$A =$ ____

Date Due: _____

Find the area of these Annuli, to 1 d.p.

1



$A_1 = \pi R^2$

$= \pi \times$ ____^{2}

$A_1 =$ ____ cm^2

$A_2 = \pi r^2$

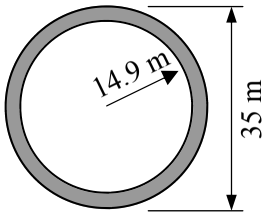
$= \pi \times$ ____^{2}

$A_2 =$ ____ cm^2

$A =$ ____ - ____

$A =$ ____

2 $R =$ ____ m ← Find $35 \div 2$



$A = \pi(R^2 - r^2)$

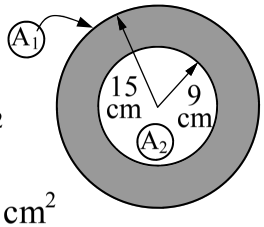
$= \pi \times ($ ____^{2} - ____^{2} $)$

$A =$ ____

Date Due: _____

Find the area of these Annuli, to 1 d.p.

1



$A_1 = \pi R^2$

$= \pi \times$ ____^{2}

$A_1 =$ ____ cm^2

$A_2 = \pi r^2$

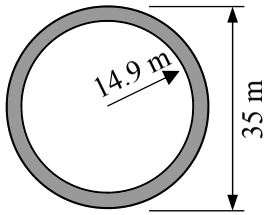
$= \pi \times$ ____^{2}

$A_2 =$ ____ cm^2

$A =$ ____ - ____

$A =$ ____

2 $R =$ ____ m ← Find $35 \div 2$



$A = \pi(R^2 - r^2)$

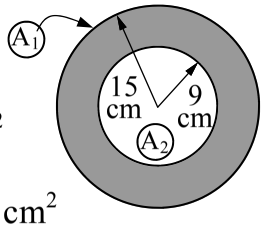
$= \pi \times ($ ____^{2} - ____^{2} $)$

$A =$ ____

Date Due: _____

Find the area of these Annuli, to 1 d.p.

1



$A_1 = \pi R^2$

$= \pi \times$ ____^{2}

$A_1 =$ ____ cm^2

$A_2 = \pi r^2$

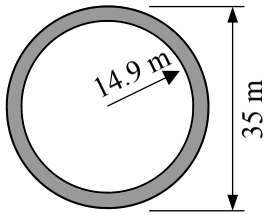
$= \pi \times$ ____^{2}

$A_2 =$ ____ cm^2

$A =$ ____ - ____

$A =$ ____

2 $R =$ ____ m ← Find $35 \div 2$



$A = \pi(R^2 - r^2)$

$= \pi \times ($ ____^{2} - ____^{2} $)$

$A =$ ____

Area - Area of an Annulus (Homework Strip)

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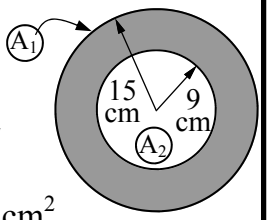
Date Due: _____

Find the area of these Annuli, to 1 d.p.

1



$A_1 = \pi R^2$

$= \pi \times 15^2$

$A_1 = 706.9 \text{ cm}^2$

$A_2 = \pi r^2$

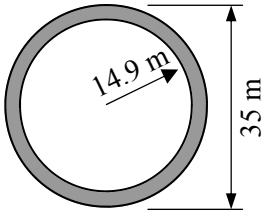
$= \pi \times 9^2$

$A_2 = 254.5 \text{ cm}^2$

$A = 706.9 - 254.5$

$A = 452.4 \text{ cm}^2$

2 $R = 17.5$ m ← Find $35 \div 2$



$A = \pi(R^2 - r^2)$

$= \pi \times (17.5^2 - 14.9^2)$

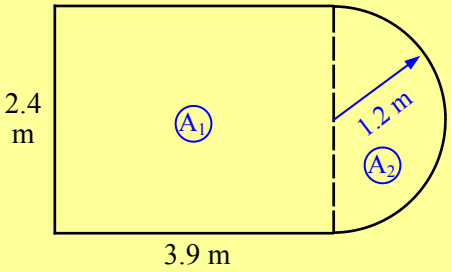
$A = 264.6 \text{ m}^2$

Area of Composite Shapes

A composite shape is a shape that is composed of other plane shapes. The shape can be broken up into these shapes and the area calculated by either adding the shapes together, or subtracting them.

Areas on the diagram are labeled, such as A_1, A_2 etc. Below is an example.

Example - Find the area to 1 d.p.



The radius is 1.2 m because it is half of the 2.4 m side.

The shape consists of a rectangle and a semi-circle.

$A_1 = lb$

$= 2.4 \times 3.9$

$A_1 = 9.4 \text{ m}^2$

$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 1.2^2$

$A_2 = 2.3 \text{ m}^2$

$A = A_1 + A_2 = 9.4 + 2.3 = 11.7 \text{ m}^2$ ← Add the two areas together for the answer

Area - Composite Shapes (Circular) - Sheet 1

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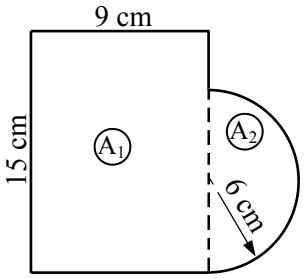
Calculate the area of these shapes by breaking the shapes into parts first. All angles that look like right angles, are right angles! Answer to 1 d.p.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = \frac{1}{2}\pi r^2$

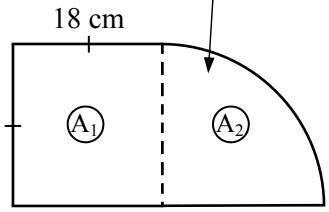
$= \frac{1}{2} \times \pi \times \square^2$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ cm^2

2

Radius of the Quadrant is the same as the side of the square



$A_1 = l^2$

$= \square^2$

$A_1 = \square$ cm^2

$A_2 = \frac{1}{4}\pi r^2$

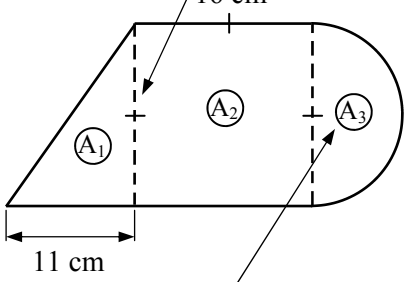
$= \frac{1}{4} \times \pi \times \square^2$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ cm^2

3

Height of the triangle is the same as square side length.



Radius of semi-circle is half the side length of the square.

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A_1 = \square$ cm^2

$A_2 = l^2$

$= \square^2$

$A_2 = \square$ cm^2

$A_3 = \frac{1}{2}\pi r^2$

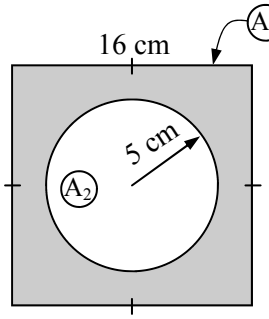
$= \frac{1}{2} \times \pi \times \square^2$

$A_3 = \square$ cm^2

$A =$

Find the shaded area, subtract the smaller area from the larger one. Answer to 1 d.p.

4



$A_1 = l^2$

$= \square^2$

$A_1 = \square$ cm^2

$A_2 = \pi r^2$

$= \pi \times \square^2$

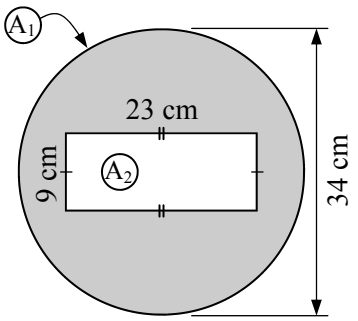
$A_2 = \square$ cm^2

$A = \square - \square = \square$ cm^2

5

Find the radius of the circle first

$r = \square$ cm



$A_1 = \pi r^2$

$= \pi \times \square^2$

$A_1 = \square$ cm^2

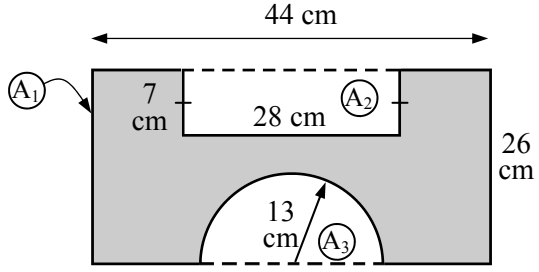
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A =$

6



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A_3 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_3 = \square$ cm^2

$A =$

Area - Composite Shapes (Circular) - Sheet 1

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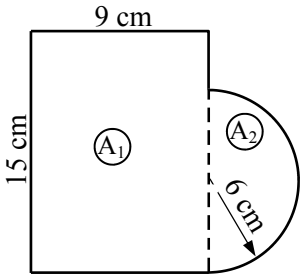
Calculate the area of these shapes by breaking the shapes into parts first. All angles that look like right angles, are right angles! Answer to 1 d.p.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_2 = \square$ cm^2

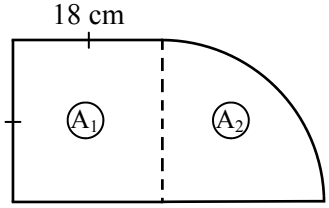
$A = \square + \square = \square$ cm^2

2

$A_1 = l^2$

$= \square^2$

$A_1 = \square$ cm^2



$A_2 = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A_2 = \square$ cm^2

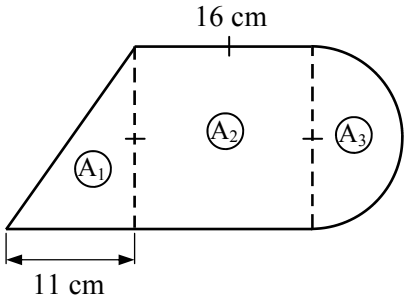
$A = \square + \square = \square$ cm^2

3

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A_1 = \square$ cm^2



$A_2 = l^2$

$= \square^2$

$A_2 = \square$ cm^2

$A_3 = \frac{1}{2}\pi r^2$

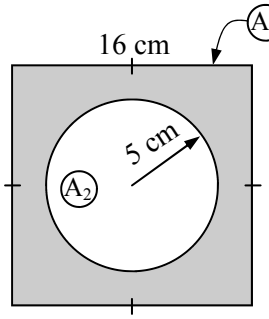
$= \frac{1}{2} \times \pi \times \square^2$

$A_3 = \square$ cm^2

$A =$

Find the shaded area, subtract the smaller area from the larger one. Answer to 1 d.p.

4



$A_1 = l^2$

$= \square^2$

$A_1 = \square$ cm^2

$A_2 = \pi r^2$

$= \pi \times \square^2$

$A_2 = \square$ cm^2

$A = \square - \square = \square$ cm^2

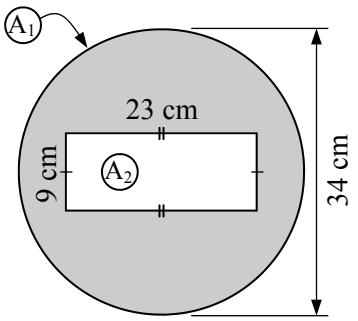
5

$A_1 = \pi r^2$

$= \pi \times \square^2$

$A_1 = \square$ cm^2

$r = \square$ cm



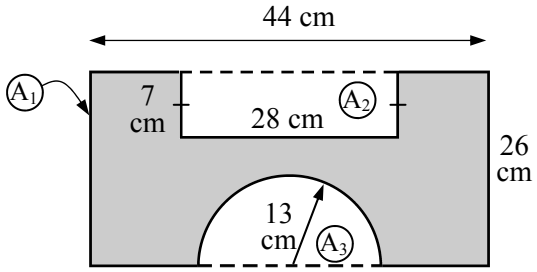
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A =$

6



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A_3 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_3 = \square$ cm^2

$A =$

Area - Composite Shapes (Circular) - Sheet 2

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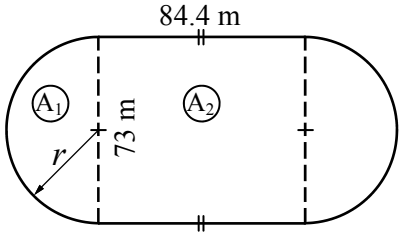
Use your composite area skills to find these. Answer to 1 decimal place when required.

An athletics field is shown. Find the:



1 Radius of the semi-circle $r =$ ____

2 Area (A_1)

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times$ ____^{2}

$A_1 =$ ____ m^2

3 Area (A_2)

$A_2 = lb$

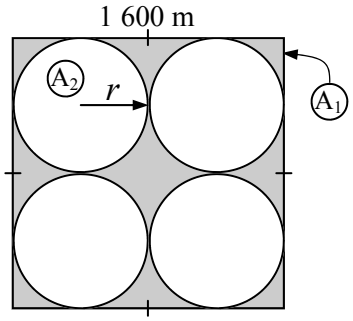
$=$ ____ $\times$ ____

$A_2 =$ ____ m^2

4 Total field area.

5 If 1 m^2 requires 2.2 L of water, how much water is required to irrigate the entire field.

A square field has four identical circular fields fitting perfectly within it. Find the:



6 Radius of the circular field $r =$ ____

7 Area (A_1)

$A_1 = l^2$

8 Area (A_2)

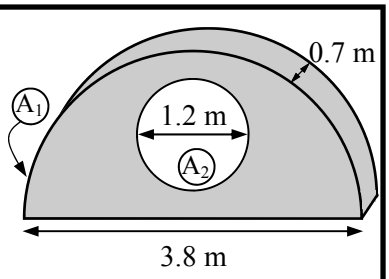
$A_2 = \pi r^2$

9 Amount of the square field which is unused.

10 The percentage of the square field unused.

% Unused = $\frac{\text{Q9}}{\text{Q7}} \times 100 =$ ____

A concrete anchor block for bridge cabling is shown. It is semi-circular with a circular opening. Find the:



11 Radius of the circle $r =$ ____

12 Radius of the semi-circle $R =$ ____

13 Area (A_1)

$A_1 = \frac{1}{2}\pi R^2$

14 Area (A_2)

$A_2 = \pi r^2$

15 Area of the concrete (shaded) face.

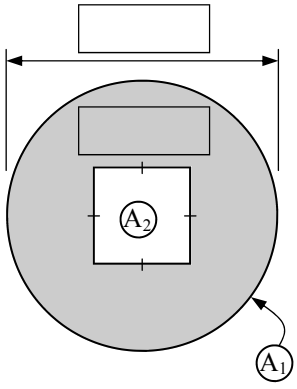
16 Use: $V = Ah$ to find the volume of concrete.

Volume is the face area (Q.15) multiplied by the block's depth (0.7 m).

$V = Ah$

$=$ ____ $\times$ ____

$V =$ ____ m^3



A circular washer with a 25 mm diameter has a square hole with 9 mm sides through its centre. Write these lengths in the boxes on the diagram, then find:

17 Radius of the circle $r =$ ____

18 Area (A_1)

$A_1 = \pi r^2$

19 Area (A_2)

$A_2 = l^2$

20 Face area of the steel washer (shaded area).

Area - Composite Shapes (Circular) - Sheet 1

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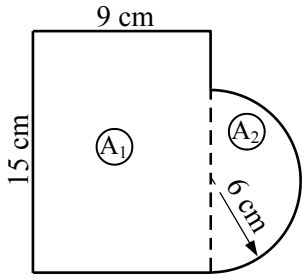
Calculate the area of these shapes by breaking the shapes into parts first. All angles that look like right angles, are right angles! Answer to 1 d.p.

1



$A_1 = lb$

$= 15 \times 9$

$A_1 = 135$ cm^2

$A_2 = \frac{1}{2}\pi r^2$

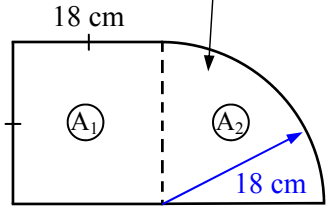
$= \frac{1}{2} \times \pi \times 6^2$

$A_2 = 56.5$ cm^2

$A = 135 + 56.5 = 191.5$ cm^2

2

Radius of the Quadrant is the same as the side of the square



$A_1 = l^2$

$= 18^2$

$A_1 = 324$ cm^2

$A_2 = \frac{1}{4}\pi r^2$

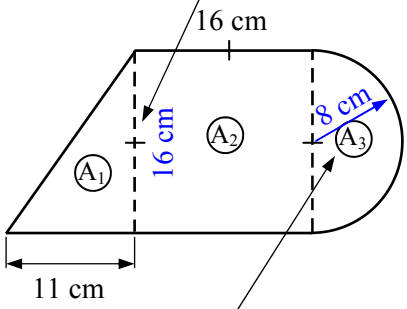
$= \frac{1}{4} \times \pi \times 18^2$

$A_2 = 254.5$ cm^2

$A = 324 + 254.5 = 578.5$ cm^2

3

Height of the triangle is the same as square side length.



Radius of semi-circle is half the side length of the square.

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 11 \times 16$

$A_1 = 88$ cm^2

$A_2 = l^2$

$= 16^2$

$A_2 = 256$ cm^2

$A_3 = \frac{1}{2}\pi r^2$

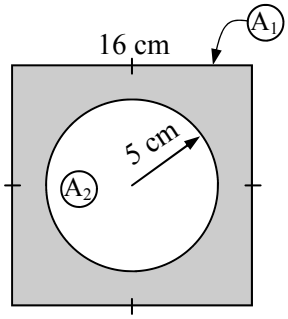
$= \frac{1}{2} \times \pi \times 8^2$

$A_3 = 100.5$ cm^2

$A = 88 + 256 + 100.5 = 444.5$ cm^2

Find the shaded area, subtract the smaller area from the larger one. Answer to 1 d.p.

4



$A_1 = l^2$

$= 16^2$

$A_1 = 256$ cm^2

$A_2 = \pi r^2$

$= \pi \times 5^2$

$A_2 = 78.5$ cm^2

$A = 256 - 78.5 = 177.5$ cm^2

5

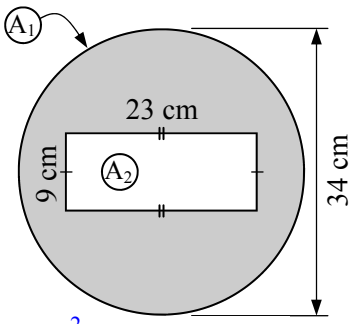
Find the radius of the circle first

$r = 17$ cm

$A_1 = \pi r^2$

$= \pi \times 17^2$

$A_1 = 907.9$ cm^2



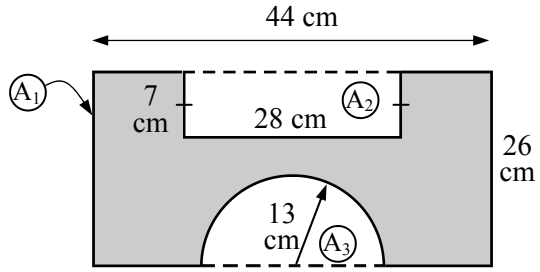
$A_2 = lb$

$= 23 \times 9$

$A_2 = 207$ cm^2

$A = 907.9 - 207 = 700.9$ cm^2

6



$A_1 = lb$

$= 44 \times 26$

$A_1 = 1\,144$ cm^2

$A_2 = lb$

$= 28 \times 7$

$A_2 = 196$ cm^2

$A_3 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 13^2$

$A_3 = 265.5$ cm^2

$A = 1\,144 - 196 - 265.5 = 682.5$ cm^2

Area - Composite Shapes (Circular) - Sheet 2

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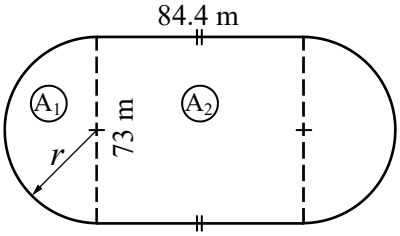
Use your composite area skills to find these. Answer to 1 decimal place when required.

An athletics field is shown. Find the:



1 Radius of the semi-circle $r =$ 36.5 m

2 Area (A_1)

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times$ 36.5 ^{2}

$A_1 =$ 2 092.7 m^2

3 Area (A_2)

$A_2 = lb$

$=$ 84.4 $\times$ 73

$A_2 =$ 6 161.2 m^2

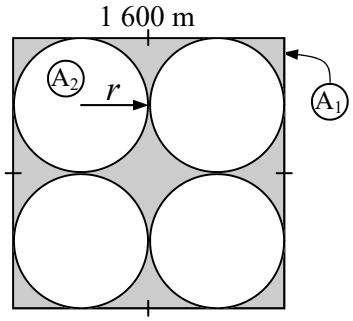
4 Total field area.

$A = 2 \times 2\,092.7 + 6\,161.2 = 10\,346.6\ m^2$

5 If 1 m^2 requires 2.2 L of water, how much water is required to irrigate the entire field.

Water = 10 346.6 × 2.2 = 22 762.52 L

A square field has four identical circular fields fitting perfectly within it. Find the:



6 Radius of the circular field $r =$ 400 m

7 Area (A_1)

$A_1 = l^2$

$= 1\,600^2$

$A_1 = 2\,560\,000\ m^2$

8 Area (A_2)

$A_2 = \pi r^2$

$= \pi \times 400^2$

$A_2 = 502\,654.8\ m^2$

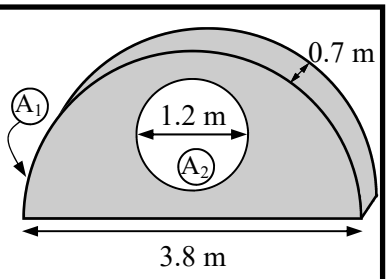
9 Amount of the square field which is unused.

$A = 2\,560\,000 - 4 \times 502\,654.8 = 549\,380.8\ m^2$

10 The percentage of the square field unused.

$$\%\ \text{Unused} = \frac{549\,380.8}{2\,560\,000} \times 100 = 21.5\%$$

A concrete anchor block for bridge cabling is shown. It is semi-circular with a circular opening. Find the:



11 Radius of the circle $r =$ 0.6 m

12 Radius of the semi-circle $R =$ 1.9 m

13 Area (A_1)

$A_1 = \frac{1}{2}\pi R^2$

$= \frac{1}{2} \times \pi \times 1.9^2$

$A_1 = 5.7\ m^2$

14 Area (A_2)

$A_2 = \pi r^2$

$= \pi \times 0.6^2$

$A_2 = 1.1\ m^2$

15 Area of the concrete (shaded) face.

$A = 5.7 - 1.1 = 4.6\ m^2$

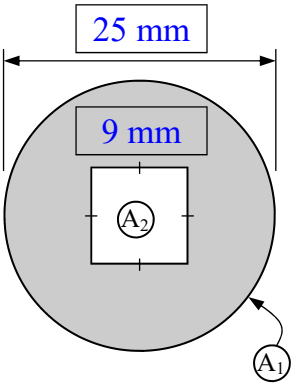
16 Use: $V = Ah$ to find the volume of concrete.

Volume is the face area (Q.15) multiplied by the block's depth (0.7 m).

$V = Ah$

$=$ 4.6 $\times$ 0.7

$V =$ 3.2 m^3



A circular washer with a 25 mm diameter has a square hole with 9 mm sides through its centre. Write these lengths in the boxes on the diagram, then find:

17 Radius of the circle $r =$ 12.5 mm

18 Area (A_1)

$A_1 = \pi r^2$

$= \pi \times 12.5^2$

$A_1 = 490.9\ mm^2$

19 Area (A_2)

$A_2 = l^2$

$= 9^2$

$A_2 = 81\ mm^2$

20 Face area of the steel washer (shaded area).

$A = 490.9 - 81 = 409.9\ mm^2$

Area - Area of Circular Composite Shapes (Homework Strip)

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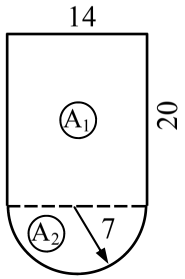
Date Due: _____

Find the area of these shapes, to 1 d.p.
All lengths are in centimetres.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

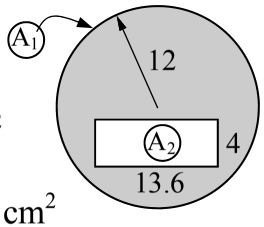
$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ cm^2

2



$A_1 = \pi r^2$

$= \pi \times \square^2$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

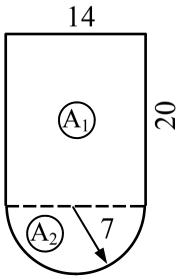
$A = \square - \square$

$A = \square$

Date Due: _____

Find the area of these shapes, to 1 d.p.
All lengths are in centimetres.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

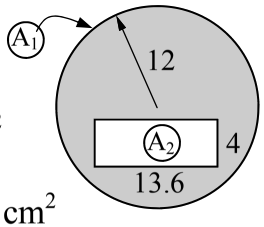
$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ cm^2

2



$A_1 = \pi r^2$

$= \pi \times \square^2$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

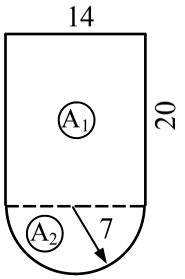
$A = \square - \square$

$A = \square$

Date Due: _____

Find the area of these shapes, to 1 d.p.
All lengths are in centimetres.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

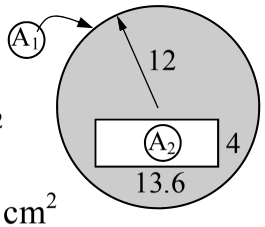
$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ cm^2

2



$A_1 = \pi r^2$

$= \pi \times \square^2$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

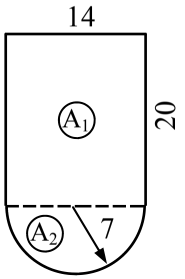
$A = \square - \square$

$A = \square$

Date Due: _____

Find the area of these shapes, to 1 d.p.
All lengths are in centimetres.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

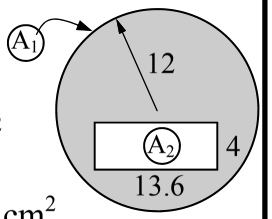
$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_2 = \square$ cm^2

$A = \square + \square = \square$ cm^2

2



$A_1 = \pi r^2$

$= \pi \times \square^2$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = \square - \square$

$A = \square$

Area - Area of Circular Composite Shapes (Homework Strip)

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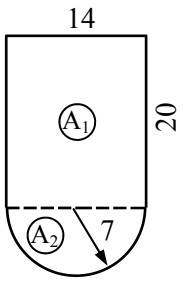
Date Due:

Find the area of these shapes, to 1 d.p.
All lengths are in centimetres.

1



$A_1 = lb$

$= 20 \times 14$

$A_1 = 280 \text{ cm}^2$

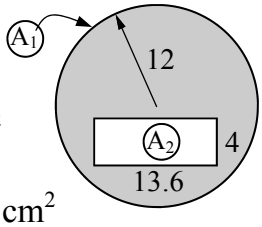
$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 7^2$

$A_2 = 77.0 \text{ cm}^2$

$A = 280 + 77 = 357 \text{ cm}^2$

2



$A_1 = \pi r^2$

$= \pi \times 12^2$

$A_1 = 452.4 \text{ cm}^2$

$A_2 = lb$

$= 13.6 \times 4$

$A_2 = 54.4 \text{ cm}^2$

$A = 452.4 - 54.4$

$A = 398 \text{ cm}^2$

Area - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Area - Find a Word

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Find the words in the puzzle from the wordlist.



I	T	M	A	R	G	O	L	E	L	L	A	R	A	P
I	H	I	T	K	S	C	S	U	B	T	R	A	C	T
K	K	T	I	E	R	C	Y	L	P	I	T	L	U	M
E	I	E	C	I	C	O	M	B	I	N	E	T	E	I
L	E	T	I	S	O	P	M	O	C	E	M	O	T	N
C	O	T	E	A	D	R	E	A	N	N	U	L	U	S
R	K	R	O	I	R	E	T	N	I	N	I	E	S	I
I	I	E	E	R	R	N	R	P	I	E	Z	S	U	D
C	R	L	I	C	A	A	Y	T	L	P	E	T	B	E
E	A	G	L	R	A	R	S	G	R	A	P	H	M	H
R	D	N	D	O	A	P	N	A	E	R	A	G	O	T
A	I	A	I	R	G	A	S	E	Q	A	R	I	H	G
U	U	I	R	O	A	D	I	A	M	E	T	E	R	N
Q	S	R	A	D	J	D	X	D	E	D	A	H	S	E
S	A	T	R	E	C	T	A	N	G	L	E	S	E	L

WORDLIST

RHOMBUS	LENGTH	UNITS	ADD	RADIUS
SYMMETRY	SUBTRACT	AXIS	ANNULUS	QUADRANT
SQUARE	CIRCLE	KITE	COMPOSITE	MULTIPLY
SHADED	SECTOR	RECTANGLE	SPACE	INSIDE
DIAMETER	TRIANGLE	AREA	COMBINE	TRAPEZIUM
JOIN	ANGLE	INTERIOR	HEIGHT	PARALLELOGRAM

Angle appears separately to the words RECTANGLE and TRIANGLE

Area - Find a Word

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Find the words in the puzzle from the wordlist.



I	T	M	A	R	G	O	L	E	L	L	A	R	A	P
I	H	I	T	K	S	C	S	U	B	T	R	A	C	T
K	K	T	I	E	R	C	Y	L	P	I	T	L	U	M
E	I	E	C	I	C	O	M	B	I	N	E	T	E	I
L	E	T	I	S	O	P	M	O	C	E	M	O	T	N
C	O	T	E	A	D	R	E	A	N	N	U	L	U	S
R	K	R	O	I	R	E	T	N	I	N	I	E	S	I
I	I	E	E	R	R	N	R	P	I	E	Z	S	U	D
C	R	L	I	C	A	A	Y	T	L	P	E	T	B	E
E	A	G	L	R	A	R	S	G	R	A	P	H	M	H
R	D	N	D	O	A	P	N	A	E	R	A	G	O	T
A	I	A	I	R	G	A	S	E	Q	A	R	I	H	G
U	U	I	R	O	A	D	I	A	M	E	T	E	R	N
Q	S	R	A	D	J	D	X	D	E	D	A	H	S	E
S	A	T	R	E	C	T	A	N	G	L	E	S	E	L

WORDLIST

~~RHOMBUS~~
~~SYMMETRY~~
~~SQUARE~~
~~SHADED~~
~~DIAMETER~~
~~JOIN~~

~~LENGTH~~
~~SUBTRACT~~
~~CIRCLE~~
~~SECTOR~~
~~TRIANGLE~~
~~ANGLE~~

Angle appears separately to the words RECTANGLE and TRIANGLE

~~UNITS~~
~~AXIS~~
~~KITE~~
~~RECTANGLE~~
~~AREA~~
~~INTERIOR~~

~~ADD~~
~~ANNULUS~~
~~COMPOSITE~~
~~SPACE~~
~~COMBINE~~
~~HEIGHT~~

~~RADIUS~~
~~QUADRANT~~
~~MULTIPLY~~
~~INSIDE~~
~~TRAPEZIUM~~
~~PARALLELOGRAM~~

Area Review

This is the review of the Area Topic.

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Formula List

Squares: $A = l^2$
Rectangles: $A = lb$
Triangles: $A = \frac{1}{2}bh$
Circles: $A = \pi r^2$
Annulus: $A = \pi(R^2 - r^2)$
Sector: $A = \frac{\theta}{360}\ \pi r^2$
Parallelograms: $A = bh$
Rhombus and Kites: $A = \frac{1}{2}xy$
Trapeziums: $A = \frac{1}{2}(a + b)h$

Pythagoras Theorem:
$c^2 = a^2 + b^2$

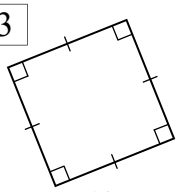
Use one of these words to make the two sentences correct:
Square, length, base or space.

1 Area is the measurement of _____ inside a shape.

2 Area is measured in _____ units.

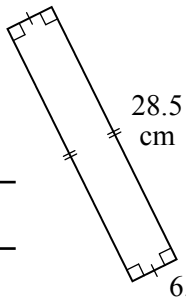
Find the area of these shapes, answer to 1 decimal place.

3

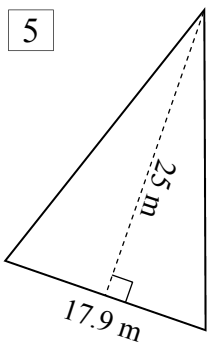


19 cm

4

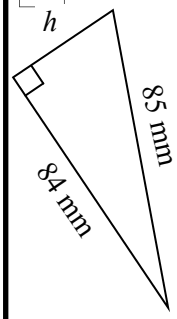


5



Use Pythagoras Theorem to find the height of this triangle, 'h'.

6

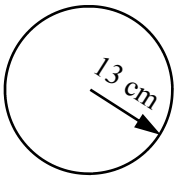


Find the area of the triangle above.

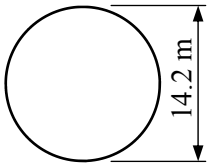
7

Calculate the area of the two circles below, answering to 1 decimal place.

8

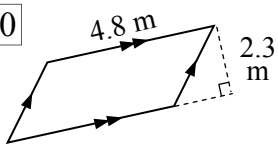


9



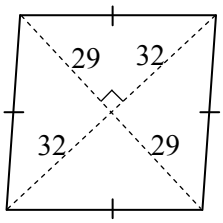
Calculate the area of these shapes, answering to 1 d.p. when necessary.

10

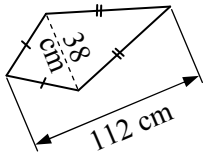


11

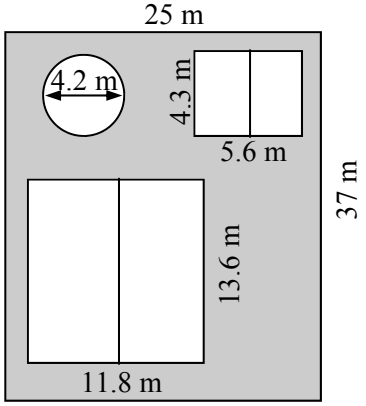
Lengths are in cm



12



13 The plan view shows a house block with a house, garage and pool. Find the undeveloped area (shaded area), answering to 1 d.p.



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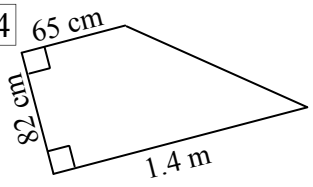
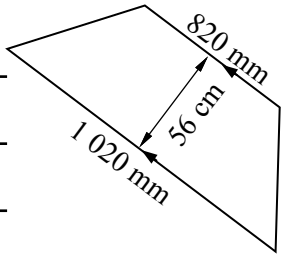
Find the area of the trapeziums shown.
Give the area in cm^2.

14



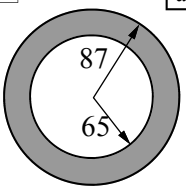
15



Find the area of the annulus shown, answering to 1 d.p.

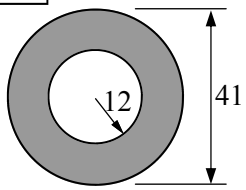
16

Lengths are in cm



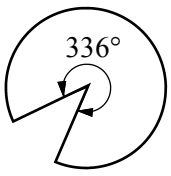
17

Lengths are in cm



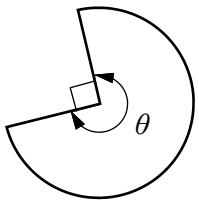
Calculate the fraction of a circle these sectors represent.
Fully simplify your answer.

18



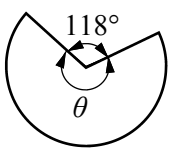
$\frac{\square}{\square} = \frac{\square}{\square}$

19



$\frac{\square}{\square} = \frac{\square}{\square}$

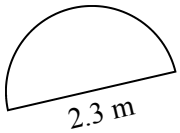
20



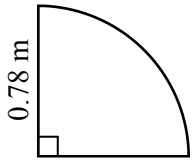
$\frac{\square}{\square} = \frac{\square}{\square}$

Find the area of the semi-circle and quadrant shown.
Answer to 2 d.p.

21

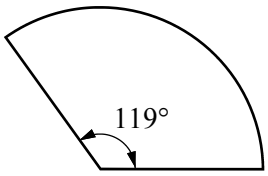


22

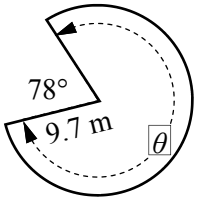


Find the area of the sectors shown. Answering to 1 d.p.

23

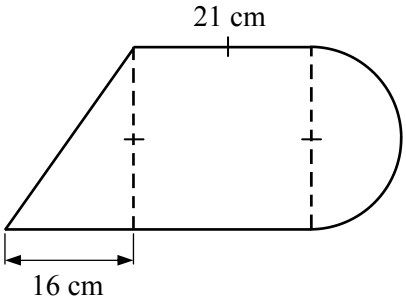


24



Find the area of the shape below, answering to 1 d.p.

25



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Rectangles: $A = lb$
Triangles: $A = \frac{1}{2}bh$
Circles: $A = \pi r^2$
Annulus: $A = \pi(R^2 - r^2)$
Sector: $A = \frac{\theta}{360}\pi r^2$
Parallelograms: $A = bh$
Rhombus and Kites: $A = \frac{1}{2}xy$
Trapeziums: $A = \frac{1}{2}(a + b)h$

Pythagoras Theorem:
$c^2 = a^2 + b^2$

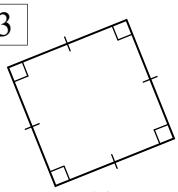
Use one of these words to make the two sentences correct:
Square, length, base or space.

1 Area is the measurement of SPACE inside a shape.

2 Area is measured in SQUARE units.

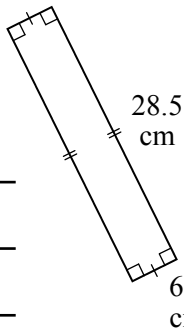
Find the area of these shapes, answer to 1 decimal place.

3



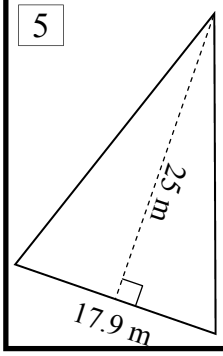
$A = l^2$
$= 19^2$
$A = 361 \text{ cm}^2$

4



$A = lb$
$= 28.5 \times 6.3$
$A = 179.6 \text{ cm}^2$

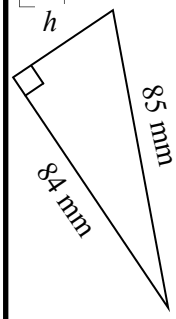
5



$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 17.9 \times 25$
$A = 223.8 \text{ m}^2$

Use Pythagoras Theorem to find the height of this triangle, 'h'.

6



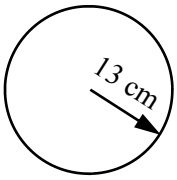
$h^2 = 85^2 - 84^2$
$h^2 = 169$
$h = \sqrt{169}$
$h = 13 \text{ mm}$

Find the area of the triangle above.

7 $A = \frac{1}{2}bh$
$= \frac{1}{2} \times 84 \times 13$
$A = 546 \text{ mm}^2$

Calculate the area of the two circles below, answering to 1 decimal place.

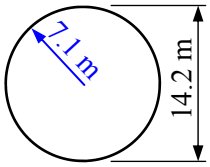
8



$A = \pi r^2$
$= \pi \times 13^2$
$A = 530.9 \text{ cm}^2$

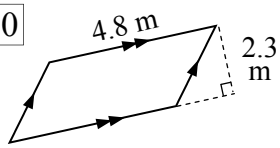
9

$A = \pi r^2$
$= \pi \times 7.1^2$
$A = 158.4 \text{ m}^2$



Calculate the area of these shapes, answering to 1 d.p. when necessary.

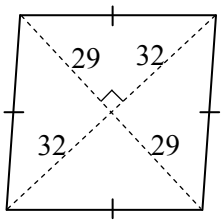
10



$A = bh$
$= 4.8 \times 2.3$
$A = 11.0 \text{ m}^2$

11

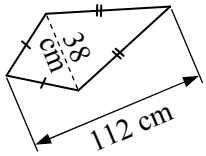
Lengths are in cm



32 + 32 = 64
29 + 29 = 58

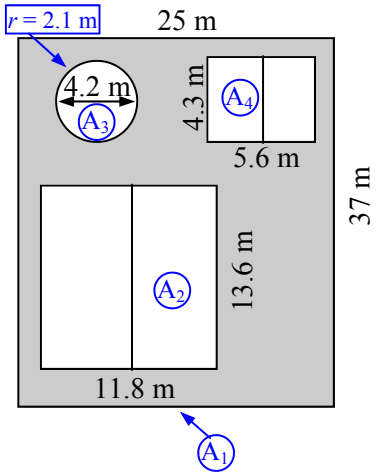
$A = \frac{1}{2}xy$
$= \frac{1}{2} \times 64 \times 58$
$A = 1\,856 \text{ cm}^2$

12



$A = \frac{1}{2}xy$
$= \frac{1}{2} \times 38 \times 112$
$A = 2\,128 \text{ m}^2$

13 The plan view shows a house block with a house, garage and pool. Find the undeveloped area (shaded area), answering to 1 d.p.



$A_1 = lb$
$= 37 \times 25$
$A_1 = 925 \text{ m}^2$

$A_2 = lb$
$= 11.8 \times 13.6$
$A_2 = 160.5 \text{ m}^2$

$A_3 = \pi r^2$
$= \pi \times 2.1^2$
$A_1 = 13.9 \text{ m}^2$

$A_4 = lb$
$= 5.6 \times 4.3$
$A_4 = 24.1 \text{ m}^2$

$A = 925 - 160.5 - 13.9 - 24.1$
$A = 726.5 \text{ m}^2$

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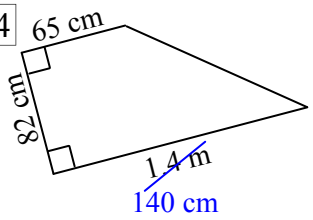
Find the area of the trapeziums shown.
Give the area in cm^2.

14



$A = \frac{1}{2}(a + b)h$

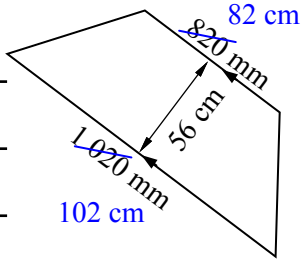
$= \frac{1}{2} \times (65 + 140) \times 82$

$A = 8\,405 \text{ cm}^2$

15 $A = \frac{1}{2}(a + b)h$

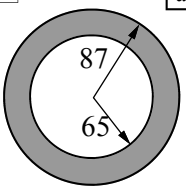
$= \frac{1}{2} \times (82 + 102) \times 56$

$A = 5\,152 \text{ cm}^2$



Find the area of the annulus shown, answering to 1 d.p.

16

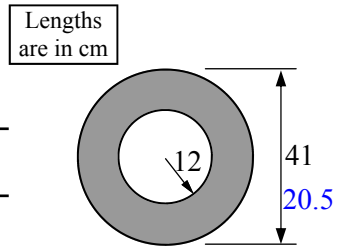


$A = \pi(R^2 - r^2)$

$= \pi \times (87^2 - 65^2)$

$A = 10\,505.5 \text{ cm}^2$

17



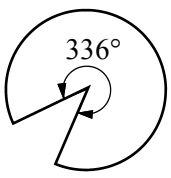
$A = \pi(R^2 - r^2)$

$= \pi \times (20.5^2 - 12^2)$

$A = 867.9 \text{ cm}^2$

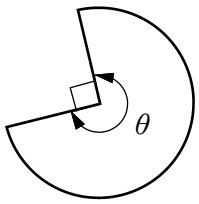
Calculate the fraction of a circle these sectors represent.
Fully simplify your answer.

18



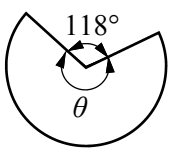
$\frac{336}{360} = \frac{14}{15}$

19



$\frac{270}{360} = \frac{3}{4}$

20



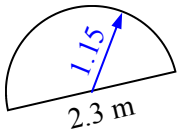
$\theta = 360 - 118$

$\theta = 242°$

$\frac{242}{360} = \frac{121}{180}$

Find the area of the semi-circle and quadrant shown.
Answer to 2 d.p.

21

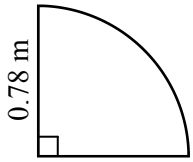


$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 1.15^2$

$A = 2.08 \text{ m}^2$

22



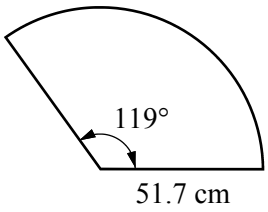
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 0.78^2$

$A = 0.48 \text{ m}^2$

Find the area of the sectors shown. Answering to 1 d.p.

23



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{119}{360} \times \pi \times 51.7^2$

$A = 2\,775.7 \text{ cm}^2$

24

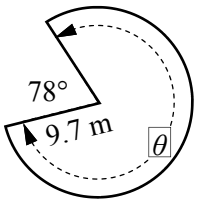
$\theta = 360 - 78$

$\theta = 282°$

$A = \frac{\theta}{360} \times \pi r^2$

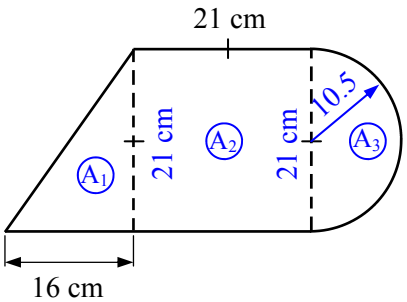
$= \frac{282}{360} \times \pi \times 9.7^2$

$A = 231.5 \text{ m}^2$



Find the area of the shape below, answering to 1 d.p.

25



$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 16 \times 21$

$A_1 = 168 \text{ cm}^2$

$A_2 = l^2$

$= 21^2$

$A_2 = 441 \text{ cm}^2$

$A_3 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 10.5^2$

$A_3 = 173.2 \text{ cm}^2$

$A = 168 + 441 + 173.2$

$A = 782.2 \text{ cm}^2$

Area



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Area of Squares

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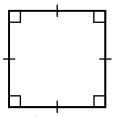
Use $A=l^2$ to find the area of these squares, to 1 d.p. Show units with a ^{2} after them.

1



$A = l^2$

$= \square^2$

Calc: 9 x^2 =

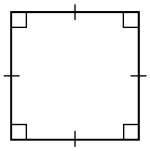
$A = \square$ cm^2

2

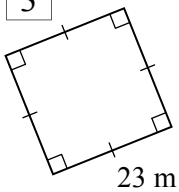
$A = l^2$

$= \square^2$

$A = \square$ cm^2



3



$A = l^2$

$= \square^2$

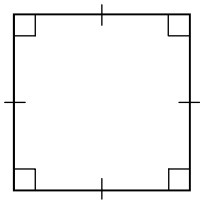
$A = \square$ m^2

4

$A = l^2$

$= \square^2$

$A = \square$ m^2



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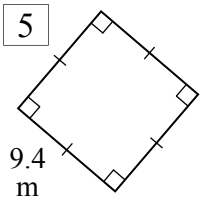
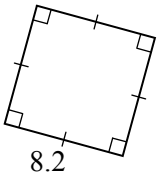


6

Remember Square Units



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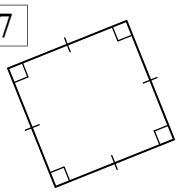
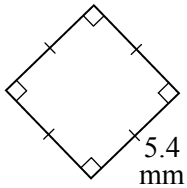
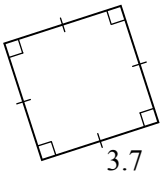


8



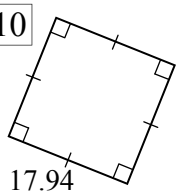
Keep going with these, but round your answer to 1d.p.

9

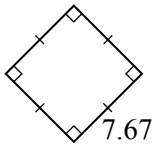


Round answer to 1 d.p.

10



11



Find the price of these square areas. Find the area of the square, then multiply by the cost.

Thomas has a square roof on a shed that has a side length of 3.4 m. Find:

12 The area of the square roof

No Rounding

13 Rewrite your answer above rounded **UP** to whole m^2.

$A =$

14 The cost to replace the roof if iron sheeting costs \$41.80/m^2

Cost = $\square \times \square$

Area (Q13) | Iron Cost

Cost = $\square$

Show the \$ sign and cents to 2 d.p.

Phom is constructing a raised 6.5 m square platform for use in his gym.

15 Find the platform area.

16 Rewrite your answer above rounded **UP** to whole m^2.

17 Find the build cost for the platform, MDF costs \$17.80/m^2

Cost =

Cost =

18 Draw a square that has sides 4.1 cm length below, show perpendicular symbols and dashes on the sides. Write the side length on one side Make sure you use a ruler.

19 Calculate the area of the square you have just drawn.

Area of Rectangles

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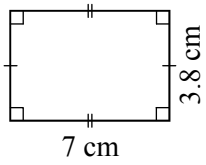
Now rectangles. Use A=*lb* to find the area, round to 1 d.p. when required.

Example



$A = lb$

$= 7 \times 3.8$

$A = 26.6\ cm^2$

Show square units

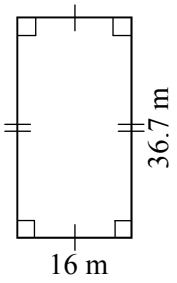
1

$A = lb$

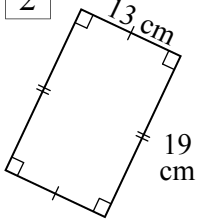
$= \square \times \square$

Multiply, in any order

$A = \square\ m^2$



2



$A = lb$

$= \square \times \square$

$A = \square\ cm^2$

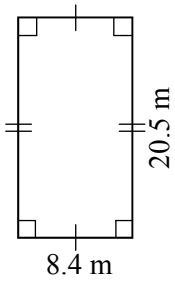
3

$A = lb$

$= \square \times \square$

$A = \square$

Show square units



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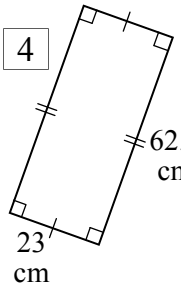

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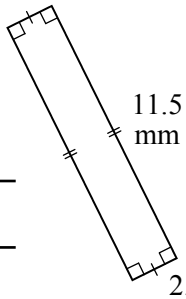

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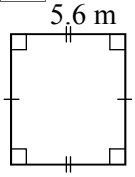
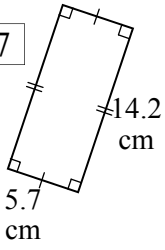


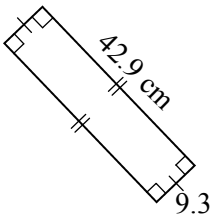
Find the area, but round your answer to 1d.p.

7



Round to 1 d.p.

8



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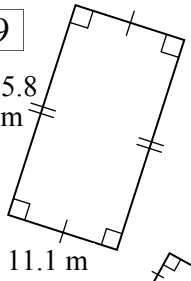

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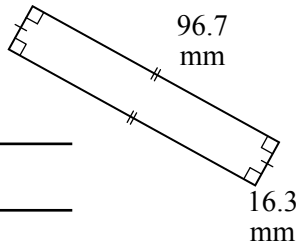

Draw these rectangles then find their area. Mark matching side lengths, show right angles and write side lengths on two sides.

RULER!

11 3.2 cm by 2.5 cm

12 Calculate the area of the rectangle you have just drawn.

13 5.3 cm by 3.7 cm

Long side across

14 Calculate the area of the rectangle you have just drawn.

Now find the area, then calculate the cost.

Jason is painting his lounge room's rectangular ceiling. It is 4.7 × 3.4 m.

15 Find the ceiling's area

16 Rewrite your answer above rounded **UP** to whole m^2.

17 If 1 litre of paint covers 9 m^2 how many whole litres of paint are required.

Paint =

18 If 1 litre of paint costs $28.90 calculate the cost to paint the ceiling.

Cost =

Cost =

Area of Squares and Rectangles - Unit Conversion

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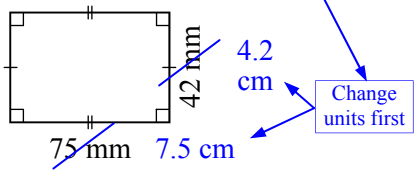
Change the units first, then use A=*lb* or A = l^2 to find the area.

Example Answer in cm^2.

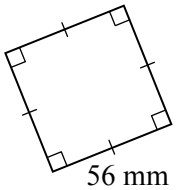


A = *lb*

= 7.5 × 4.2

A = 31.5 cm^2 ← Show square units

1 Answer in cm^2.



A = l^2

= ☐^{2}

A = ☐ cm^2

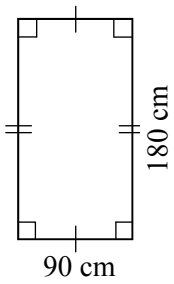
2 Answer in m^2.

A = *lb*

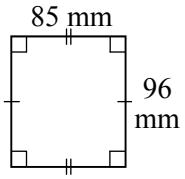
= ☐ × ☐

A = ☐

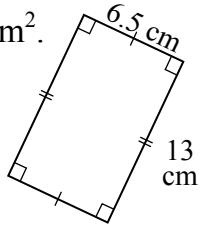
Show square units



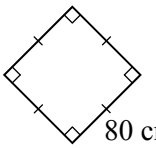
3 Answer in cm^2.



5 Answer in mm^2.

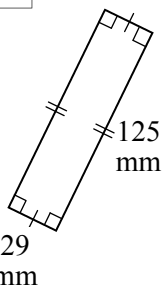


6 in m^2.

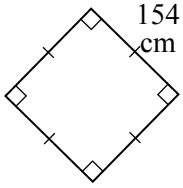


Keep going, but round your answer to 1 decimal place.

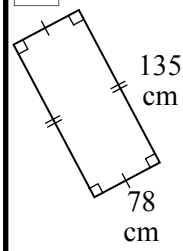
7 Answer in cm^2.



8 Answer in m^2.



9 Answer in m^2.



Draw a rectangle, mark the matching side lengths, show right angles and write the side lengths on two sides. Ruler!

10 4.5 cm by 2.8 cm

11 Calculate the area of the rectangle you have just drawn, but give your answer in mm^2.

Now find the area, to answer the questions.

A rectangular farm paddock is 1.7 km by 850 m wide.

12 Convert 1.7 km to metres.

1.7 km = ☐ m

13 Find the area of the field in square metres.

14 1 ha = 10 000 m^2, rewrite the area in ha

A = ☐ ÷ ☐

Divide Q. 13 by 10 000

A = ☐ ha

A square farm paddock has sides of 1.25 km.

15 Convert 1.25 km to metres.

1.25 km = ☐ m

16 Find the area of the field in square metres.

17 Convert the area to ha, rounding to 1 decimal place.

A =

A =

Area of Triangles (Sheet 1)

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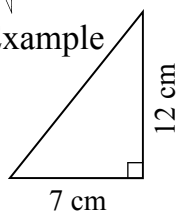
Find the area of these triangles.
Units are square units, e.g. cm^2.

Example



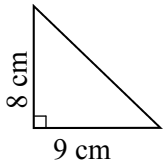
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 7 \times 12$

$A = 42\ cm^2$

Show square units

1



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

Calc: 0 . 5 × 9 × 8 =

$A = \square\ cm^2$

2

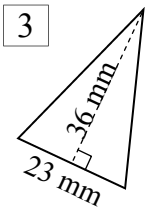
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A = \square\ m^2$



3



$A = \frac{1}{2}bh$

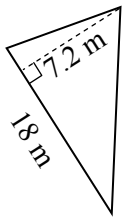
$= \frac{1}{2} \times \square \times \square$

$A = \square$

Show square units

4

$A = \frac{1}{2}bh$



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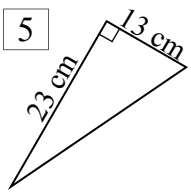

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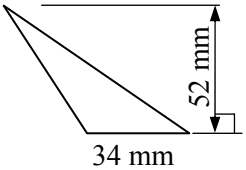
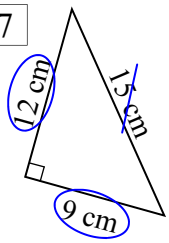


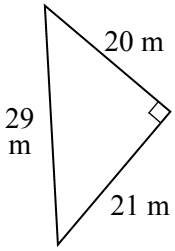
Find the area of these triangles.
Cross out the lengths you don't use, circle the two that you do.

7



Show square units

8



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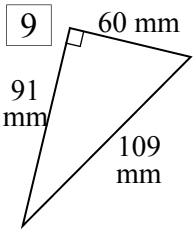

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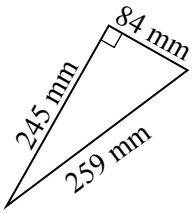

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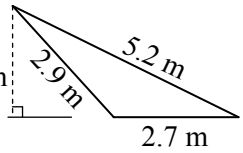
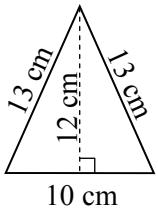


Circle two
Cross out two
Next 3 Q's

12



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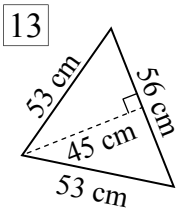
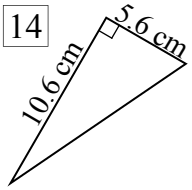


Round your answers to 1 d.p.
Find the area of these triangles.

14



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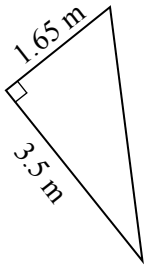

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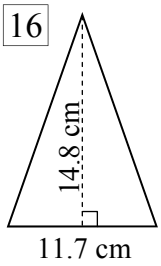

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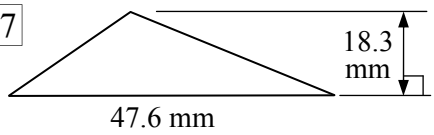

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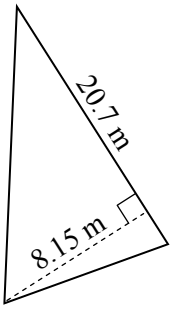

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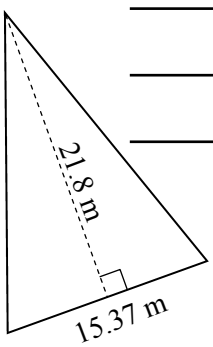

Area of Triangles (Sheet 2)

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Draw a right-angled triangle label the two side lengths and show the right angle. Ruler!

1 Base: 4.3 cm, Height: 3.7 cm

2 Calculate the area of the triangle above, round to 1 d.p.

3 Base: 3.3 cm, Height: 2.8 cm

4 Calculate the area of the triangle above, round to 1 d.p.

5 Base: 8.2 cm, Height: 3.8 cm

Now apply your skills to these.

7 A triangular advertising sail is 8.5 m tall and has a base of 1.2 m. Find its area.

8 If space on the sail is sold for \$85/m^2, how much will it cost to advertise on the whole sail?

Cost =

Cost =

9 A triangular courtyard has base 6.8 m and height 10 m. Find its area.

10 The courtyard is to be tiled and needs 16 tiles/m^2. How many tiles will be used to tile the entire courtyard floor?

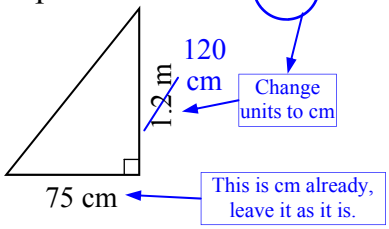
Tiles =

Tiles =

6 Calculate the area of the triangle you have drawn, 1 d.p.

Change the units to match the required area measurement, then find the area, to 1 d.p.

Example Answer in cm^2.

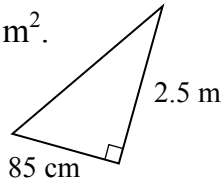


$A = \frac{1}{2}bh$

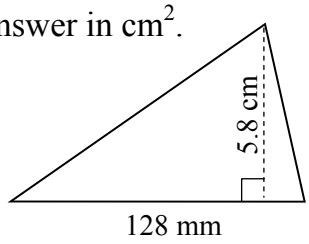
$= \frac{1}{2} \times 75 \times 120$

$A = 4\,500$ cm^2

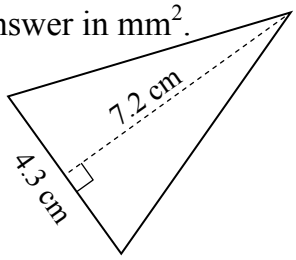
11 Answer in m^2.



12 Answer in cm^2.



13 Answer in mm^2.



Finding a Short Side - Whole Number Answers

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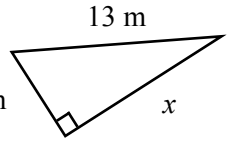
Find the length of the short side, the answer will be a whole number.

Example



$x^2 = 13^2 - 5^2$

Calc: 1 3 x^2 - 5 x^2 =

$x^2 = 144$

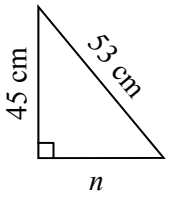
$x = \sqrt{144}$

Calc: $\sqrt{}$ Ans =

Write the units

$x = 12$ m

1



$n^2 = 53^2 - \square^2$

$n^2 = \square$

$n = \sqrt{\square}$

$n = \square$ cm

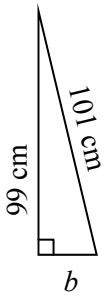
2

$b^2 = \square^2 - \square^2$

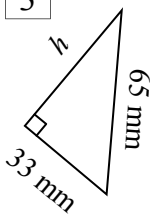
$b^2 = \square$

$b = \sqrt{\square}$

$b = \square$ cm



3



$h^2 =$

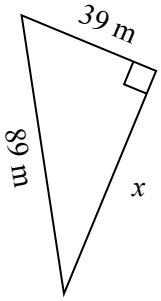
$h^2 =$

$h =$

$h =$

Remember the units

4



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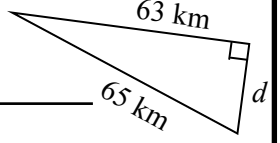

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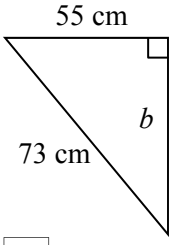

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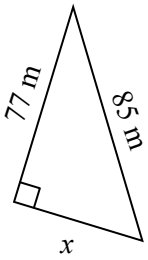

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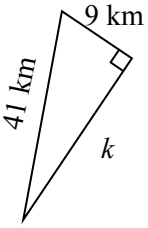

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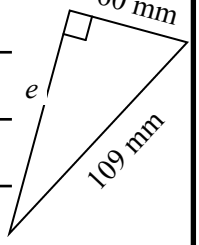
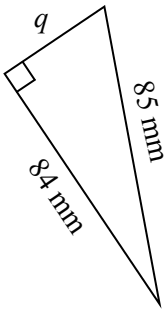
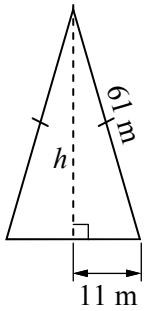


10



Find '*h*' the unknown height, the answer will be a whole number.

11



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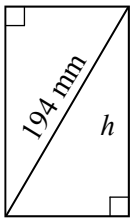
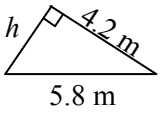
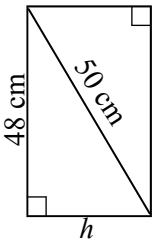


13



14



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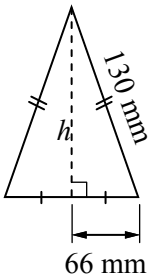

Area of Triangles - Pythagoras

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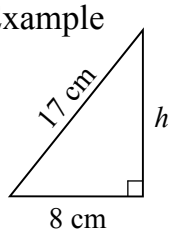
Use Pythagoras Theorem and subtraction for these. Find the length, then find the area. Answers are all whole numbers.

Example



$h^2 = 17^2 - 8^2$

$h^2 = 225$

$h = \sqrt{225}$

$h = 15$ cm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 15$

$A = 60$ cm^2

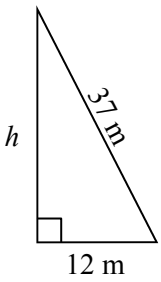
1

$h^2 =$ [37]^2 - [ ]^2

$h^2 =$ [ ]

$h = \sqrt{\ }$ [ ]

$h =$ [ ] m



This answer should be a whole number and less than 37.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ [ ] $\times$ [ ]

Use the base of '12' and the height, that you calculated.

$A =$ [ ] m^2

This answer is an area, use square units.

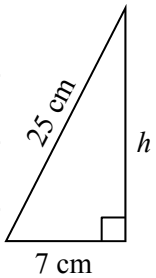
2

$h^2 =$ [ ]^2 - [ ]^2

$h^2 =$ [ ]

$h = \sqrt{\ }$ [ ]

$h =$ [ ] cm



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ [ ] $\times$ [ ]

$A =$ [ ] cm^2

3

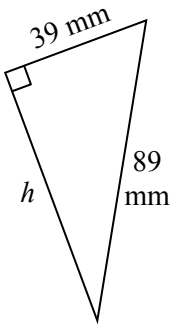
$h^2 =$ [ ]^2 - [ ]^2

$h^2 =$ [ ]

$h = \sqrt{\ }$ [ ]

$h =$ [ ]

Length, so no square units.



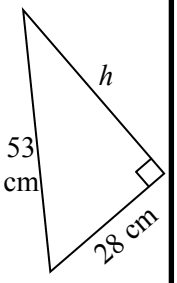
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ [ ] $\times$ [ ]

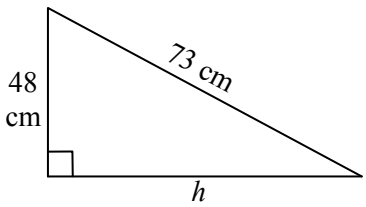
$A =$ [ ]

Area, so use square units.

4

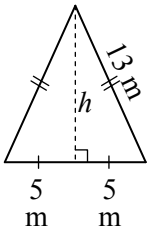


5



With isosceles triangles use the same method, except when you find the area, use the whole base length, not half.

6



$h^2 = 13^2 - 5^2$

$h^2 =$ [ ]

$h = \sqrt{\ }$ [ ]

$h =$ [ ] m

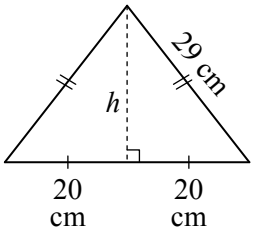
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ [ ] $\times$ [ ]

Use $b = 10$ (2×5) not 5!

$A =$ [ ] m^2

7



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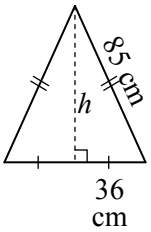

Area of Parallelograms

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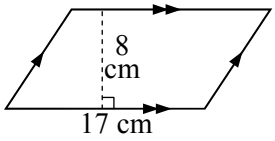
Find the area of these parallelograms.

Example

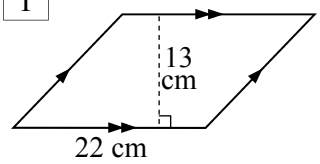


$A = bh$

$= 17 \times 8$

$A = 136 \text{ cm}^2$

1

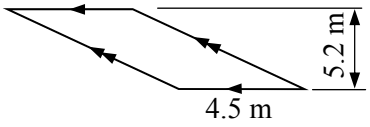


$A = bh$

$= \square \times \square$

$A = \square \text{ cm}^2$

2



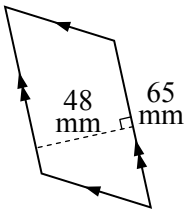
$A = bh$

$= \square \times \square$

$A = \square$

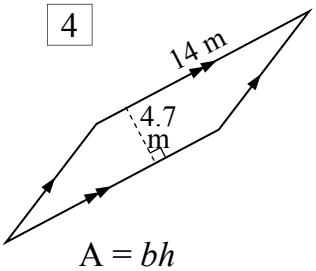
Remember square units

3



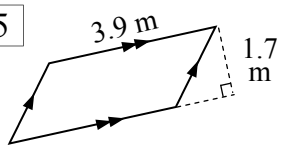
$A = bh$

4



$A = bh$

5



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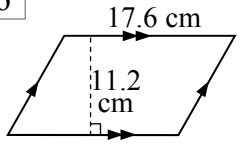

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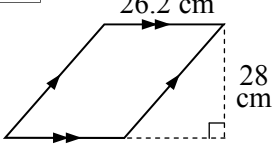

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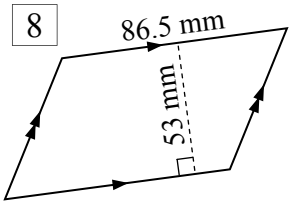
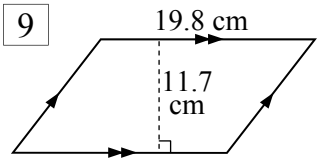


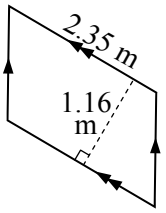
Find the area, rounding your answer to 1 d.p.

9



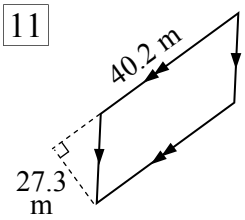
$A = bh$

10

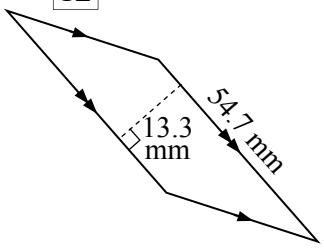


Square units to 1 d.p.

11

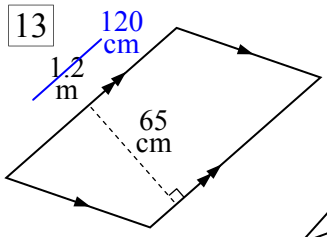


12



Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

13

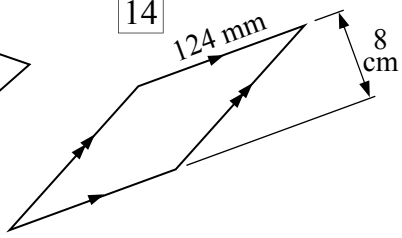


$A = bh$

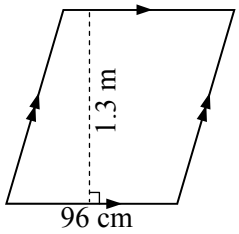
$= \square \times \square$

$A = \square \text{ cm}^2$

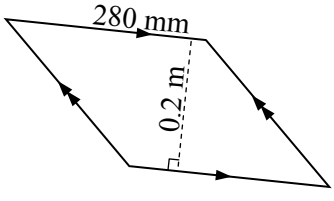
14



15



16



Area of Rhombuses (Rhombi)

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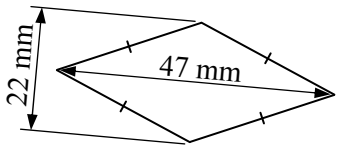
A rhombus is like a square that has been pushed over. Find the area, using $A = \frac{1}{2}xy$.

Example



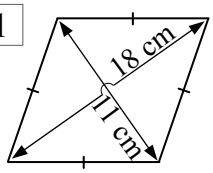
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 2 2 × 4 7 =

$= \frac{1}{2} \times 47 \times 22$

$A = 517 \text{ mm}^2$

1

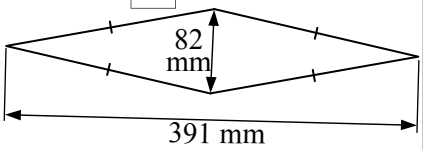


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

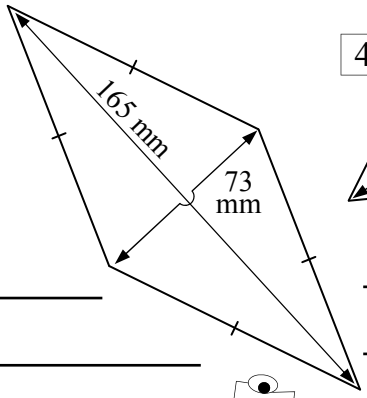
$A = \square \text{ cm}^2$

2

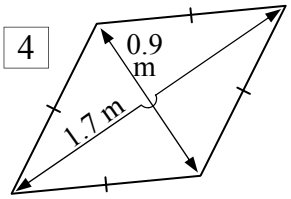


$A = \frac{1}{2}xy$

3

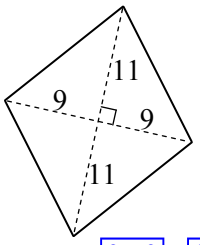


4

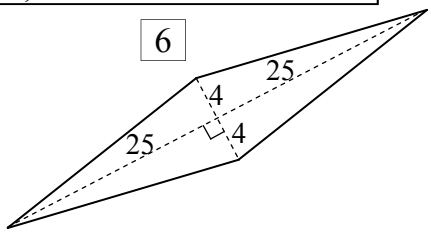


Add for the lengths first, all measurements are in cm.

5



6

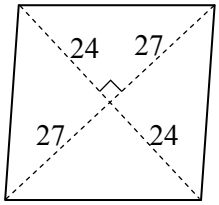


$A = \frac{1}{2}xy$ (9 + 9) (11 + 11)

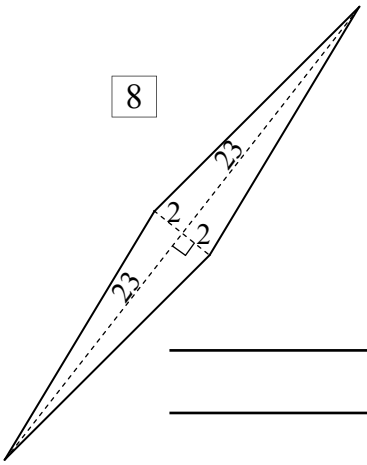
$= \frac{1}{2} \times \square \times \square$

$A = \square \text{ cm}^2$

7

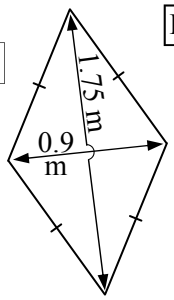


8

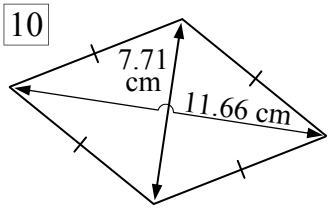


Round 'em up! To 1 d.p. that is.

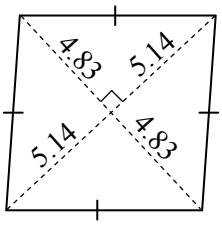
9



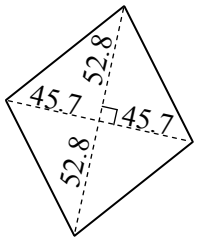
10



11 All cm



12 All mm



Join the dots to form a rhombus and measure the axis lengths in cm to 1 d.p. Then calculate the area in cm^2. Give the area to 1 decimal place.

13

Measure the axes not the sides!

14

Measure the axes not the sides!

Area of Kites

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Find the area of these kites.

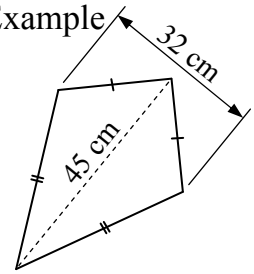
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 3 2 × 4 5 =

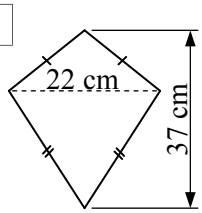
$= \frac{1}{2} \times 32 \times 45$

$A = 720 \text{ cm}^2$

Example



1

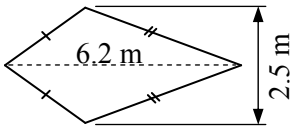


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

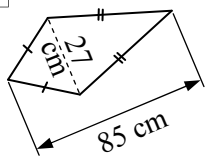
$A = \square \text{ cm}^2$

2



$A = \frac{1}{2}xy$

3



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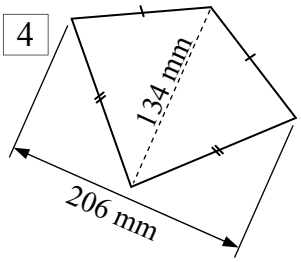
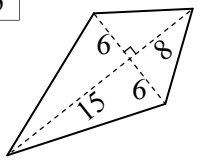


From Q.5 all units are in cm.

5

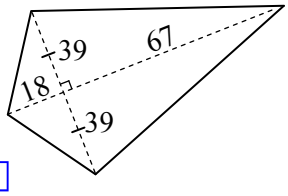


$A = \frac{1}{2}xy$ (6 + 6) (8 + 15)

$= \frac{1}{2} \times \square \times \square$

$A = \square \text{ cm}^2$

6



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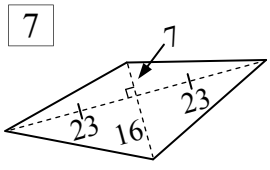
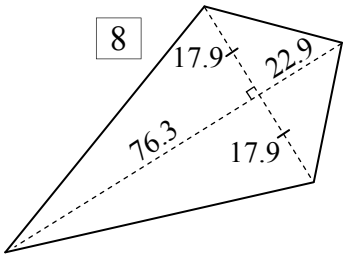
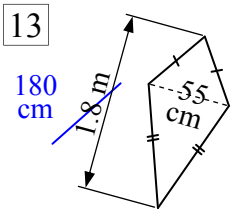


8



Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

13

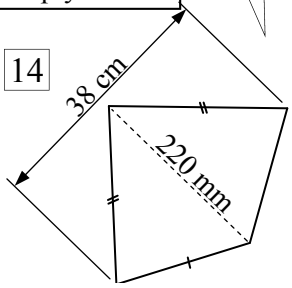


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times \square \times \square$

$A = \square \text{ cm}^2$

14



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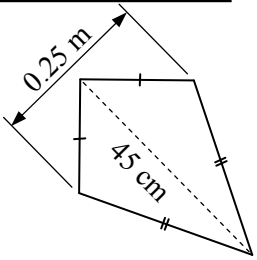
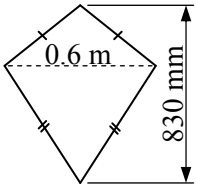


16



Join the dots to form a kite and measure the axis lengths in cm to 1 d.p. Then calculate the area in cm^2. Give the area to 1 decimal place.

13

Measure the axes not the sides!

14

Area of Trapeziums

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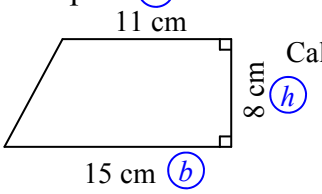
Find the area of these trapeziums. '*a*' and '*b*' are the parallel sides, '*h*' is the distance between them.

Example



$A = \frac{1}{2}(a + b)\,h$

Calc: 0 . 5 × (1 1 + 1 5) × 8 =

$= \frac{1}{2} \times (11 + 15) \times 8$

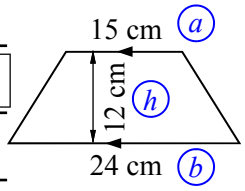
$A = 104 \text{ cm}^2$

1

$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (\square + \square) \times \square$

$A = \square \text{ cm}^2$



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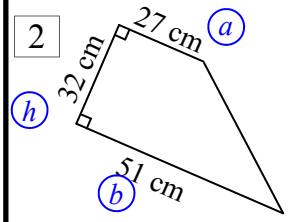
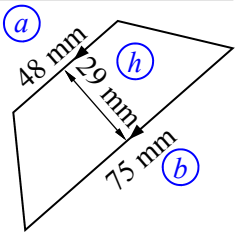


$A = \frac{1}{2}(a + b)\,h$

3



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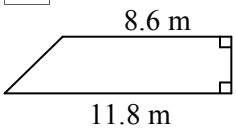

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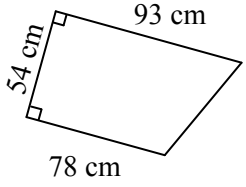

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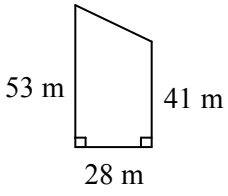

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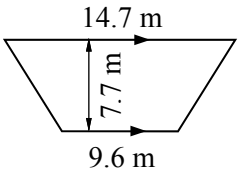

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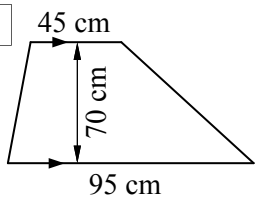


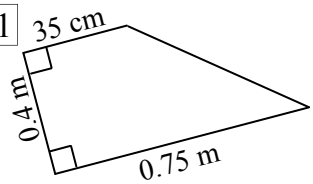
Join the dots to form a trapezium and measure the two parallel sides and the height. Measure the lengths to 1 decimal place. Then calculate the area (in cm^2).

9

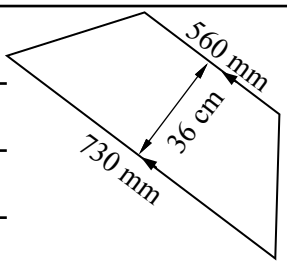
10

Find the area in cm^2. Change all the lengths that aren't in cm, to cm, before you multiply them.

11



12



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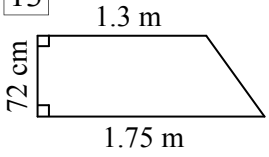

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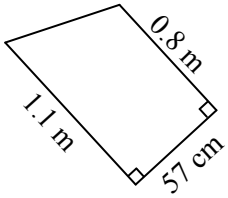

Area - Composite Shapes (Sheet 1)

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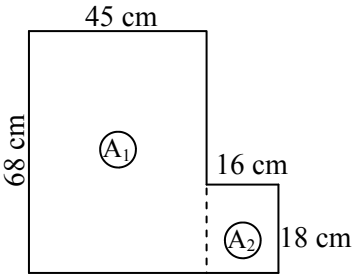
Calculate the area of these shapes by breaking them up into parts first. All angles that look like right angles, are right angles!

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square = \square$ cm^2

2

$A_1 = lb$

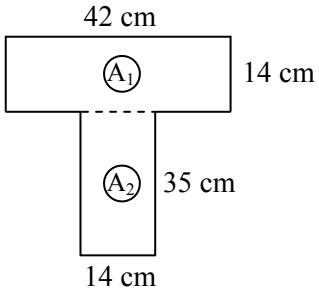
$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

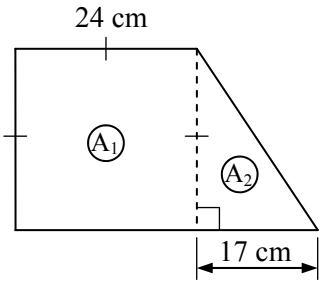
$= \square \times \square$

$A_2 = \square$ cm^2



$A = A_1 + A_2 = \square + \square = \square$ cm^2

3



$A_1 = l^2$

$= \square^2$

$A_1 = \square$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A_2 = \square$

$A = A_1 + A_2 = \square + \square = \square$

4

$A_1 = l^2$

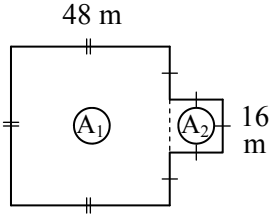
$= \square^2$

$A_1 = \square$

$A_2 = l^2$

$= \square^2$

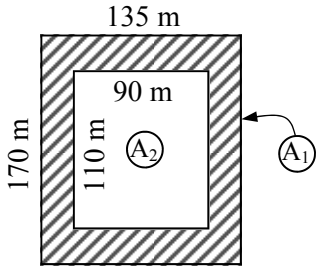
$A_2 = \square$



$A = A_1 + A_2 = \square + \square = \square$

Finding the shaded area is just the same, except instead of adding the areas, you subtract. So it is $A_1 - A_2$, the biggest area minus the smaller one.

5



$A_1 =$

$A_1 =$

$A_2 =$

$A_2 =$

$A = A_1 - A_2 = \square - \square = \square$ m^2

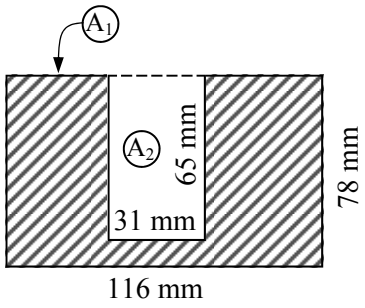
6

$A_1 =$

$A_1 =$

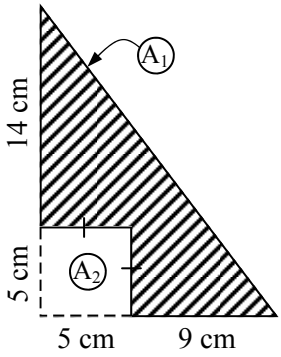
$A_2 =$

$A_2 =$



$A = A_1 - A_2 =$

7



$A_1 =$

Add 2 lengths for the base and also the height of the triangle.

$A_1 =$

$A_2 =$

$A_2 =$

$A = A_1 - A_2 =$

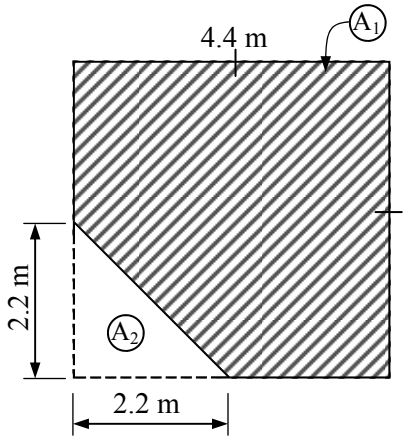
8

$A_1 =$

$A_1 =$

$A_2 =$

$A_2 =$



$A = A_1 - A_2 =$

Area - Composite Shapes (Sheet 2)

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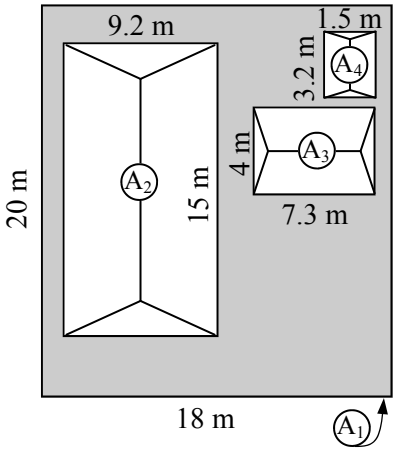
Use your composite area skills to find these.

A block of land has a house, garage and garden shed on the property. The diagram shows a plan (top) view.

Find the:



1 Block Area (A_1)

$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ m^2

2 House Area (A_2)

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ m^2

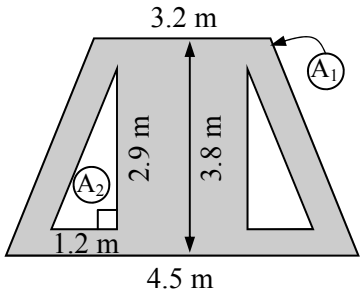
3 Garage Area (A_3)

4 Shed Area (A_4)

5 Grass Area (shaded area).

6 Cost of laying turf on the block if turf costs \$5.60 per m^2. Turf is only sold in whole m^2.

A foot of a crane is shown. It is fabricated from plate steel in the shape of a trapezium with two identical triangular sections removed. Find the:



7 Trapezium Area (A_1)

$A_1 = ½ (a + b) \times h$

8 Triangle Area (A_2)

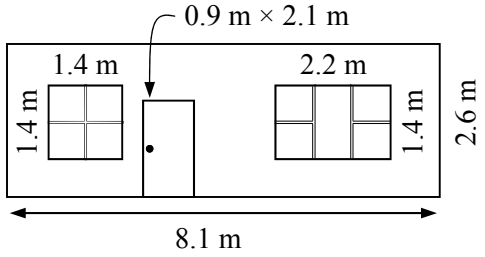
$A_2 = ½bh$

9 Steel area (shaded area).

Remember that there are two triangles!

10 The weight of the footing, if the plate steel weighs 285 kg per m^2.

Dave wants to repaint the front wall of his house. It features a doorway and two windows. The front view is shown below. Find the area of the:



11 Front wall

12 Square window

13 Doorway

14 Rectangular window

15 Paintable Area of the wall.

16 Number of litres required for one coat, if 1 m^2 needs 0.3 L.

17 The paint is sold in 2 L cans, with each can costing \$38.20. How much does it cost to paint the wall?

Area - Further Composite Shapes (Sheet 3)

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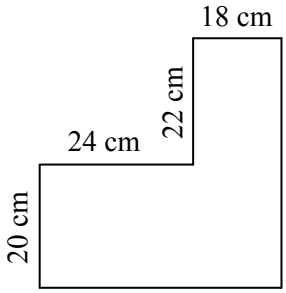
Find the missing sides, then break up the area into more simple shapes. All angles that look like right angles, are right angles!

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

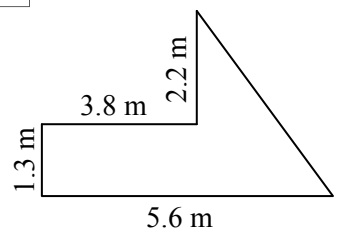
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square = \square$ cm^2

2



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ m^2

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A_2 = \square$ m^2

$A = A_1 + A_2 = \square + \square = \square$ m^2

3

$A_1 =$

$A_1 =$

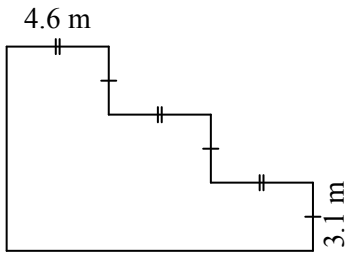
$A_2 =$

$A_2 =$

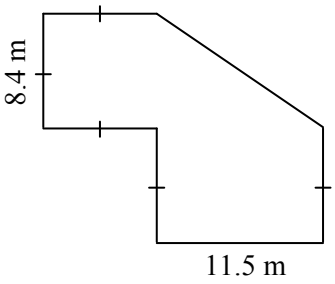
$A_3 =$

$A_3 =$

$A =$



4 *Solve with 2 or 3 Areas



$A_1 =$

$A_1 =$

$A_2 =$

$A_2 =$

$A_3 =$

$A_3 =$

$A =$

5

$A_1 =$

$A_1 =$

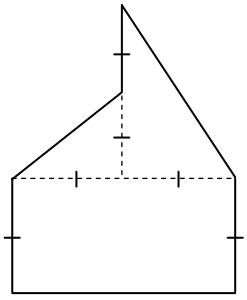
$A_2 =$

$A_2 =$

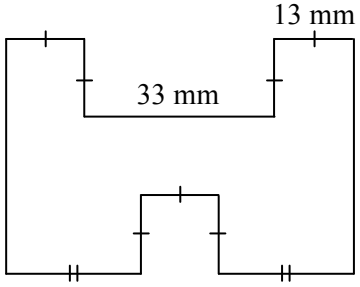
$A_3 =$

$A_3 =$

$A =$



6 *Use Subtraction Or Addition



$A_1 =$

$A_1 =$

$A_2 =$

$A_2 =$

$A_3 =$

$A_3 =$

$A =$

Area of a Circle (Sheet 1)

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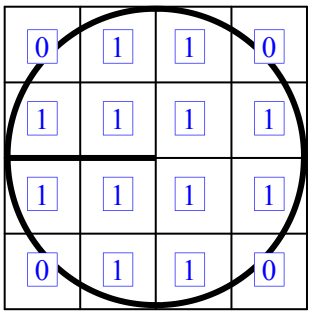
Estimate the area of these circles, using the 1 cm^2 grid squares. If square is close to a ½, call it ½. Less than ½ filled, call it zero. If it is close to full, call it 1 cm^2. Add them up and estimate the area.

1

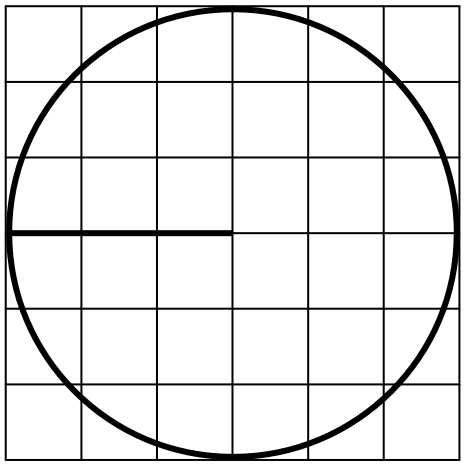


A = 12 × 1

\+ 0 × ½

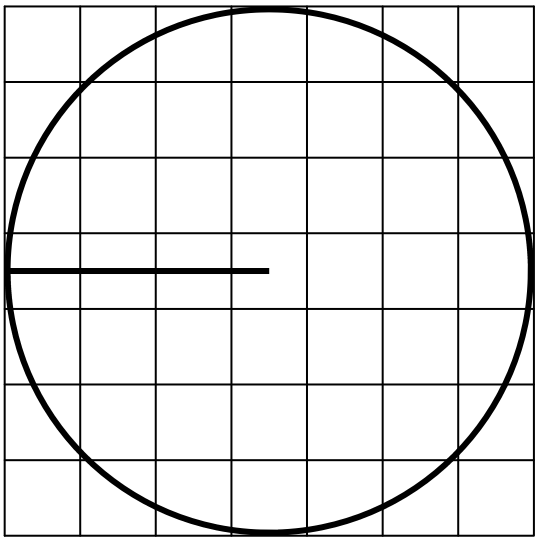
A = ____ cm^2

2



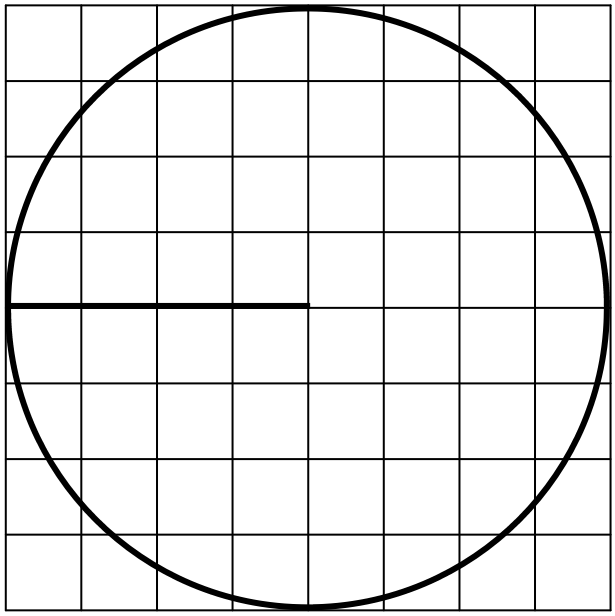
A = ____ × 1 + ____ × ½ = ____ cm^2

3



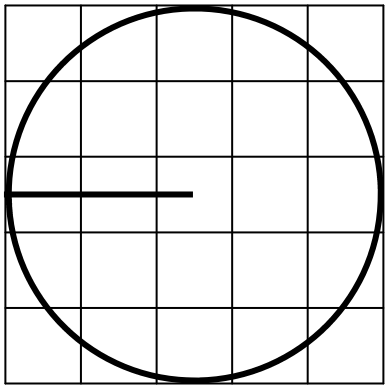
A = ____ × 1 + ____ × ½ = ____ cm^2

4



A = ____ × 1 + ____ × ½ = ____ cm^2

5



A = ____ × 1 + ____ × ½ = ____ cm^2

Now more accurately estimate the area by using $A = 3.14 \times r^2$. Compare the answer with your grid answer you calculated for the first four circles.

6 Circle in Q.1.

$A = 3.14 \times r^2$

$= 3.14 \times$ ____^{2}

Radius = length of black line

A = ____ cm^2

Answer to nearest half cm.

7 Circle in Q.2.

$A = 3.14 \times r^2$

$= 3.14 \times$ ____^{2}

A = ____ cm^2

8 Circle in Q.3.

$A = 3.14 \times r^2$

$= 3.14 \times$ ____^{2}

A = ____ cm^2

9 Circle in Q.4.

$A = 3.14 \times r^2$

$= 3.14 \times$ ____^{2}

A = ____ cm^2

Area of a Circle (Sheet 2)

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To calculate area of a circle you need to use your 'π' key. Calculate these to 1 d.p.

1 $\pi \times 15^2 =$ ____ Round to 1 d.p.

Calc: SHIFT $\times 10^x$ × 1 5 x^2 =

2 $\pi \times 24^2 =$ ____

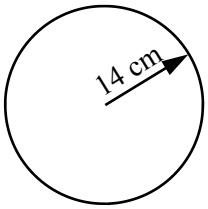
3 $\pi \times 16.5^2 =$ ____

4 $\pi \times 35.6^2 =$ ____

5 $\pi \times 11.2^2 =$ ____

Calculate the area of these circles, round to 1 d.p.

Example

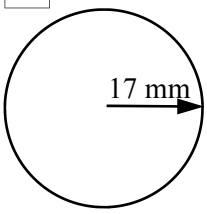


$A = \pi r^2$

$= \pi \times 14^2$

$A = 615.8 \text{ cm}^2$

6



$A = \pi r^2$

$= \pi \times$ ____ ^{2}

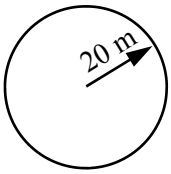
$A =$ ____ mm^2

7

$A = \pi r^2$

$= \pi \times$ ____ ^{2}

$A =$ ____ m^2



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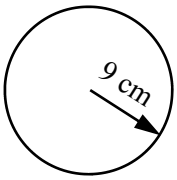


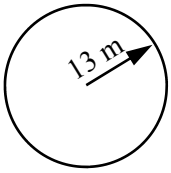
$A = \pi r^2$

$A =$ ____

Show square units

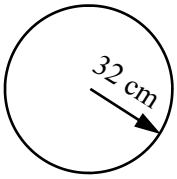
9

$A = \pi r^2$



$A =$ ____

10



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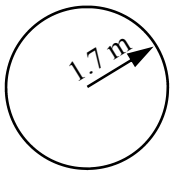
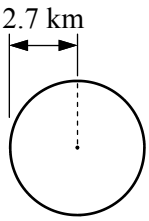


12



13



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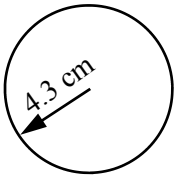
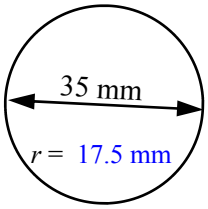


Complete the table below, answering to 2 d.p.

Q.	Radius	Area
15	9.2 m	
16	30 cm	
17	117 m	
18	45.9 mm	
19	72 cm	
20	5.4 km	
21	60.8 cm	
22	40.2 m	
23	8.92 m	

When given a diameter, find the radius first by dividing by 2, then find the area to 1 d.p.

Example



$A = \pi r^2$

$= \pi \times 17.5^2$

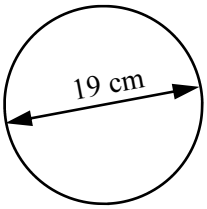
$A = 962.1 \text{ mm}^2$

24 $r =$ ____ cm

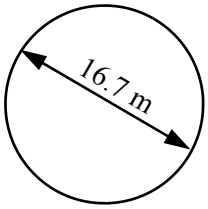
$A = \pi r^2$

$= \pi \times$ ____ ^{2}

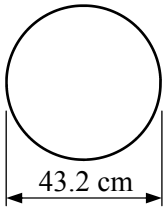
$A =$



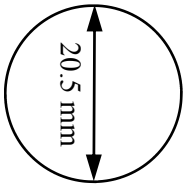
25 $r =$ ____ m



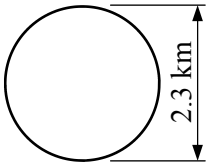
26 $r =$ ____



27 $r =$ ____



28 $r =$ ____



Area of Semi-Circles and Quadrants (Sheet 1)

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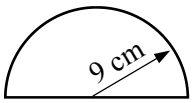
A semi-circle is half a circle, so use $A=\frac{1}{2}\pi r^2$, find the area to 1 d.p.

Example



$A = \frac{1}{2}\pi r^2$

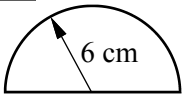
$= \frac{1}{2} \times \pi \times 9^2$

Calc: 0 . 5 × SHIFT ×10^x × 9 x^2 =

$A = 127.2\ cm^2$

Show square units

1



$A = \frac{1}{2}\pi r^2$

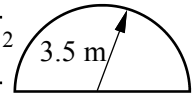
$= \frac{1}{2} \times \pi \times$ ☐^{2}

$A =$ ☐ cm^2

2

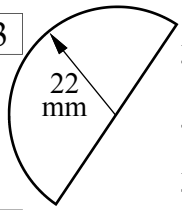
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times$ ☐^{2}



$A =$ ☐ m^2

3

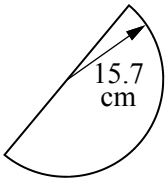


$A = \frac{1}{2}\pi r^2$

$=$

$A =$ ☐

4



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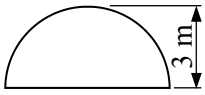

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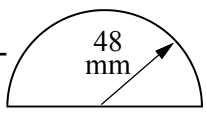

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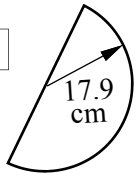


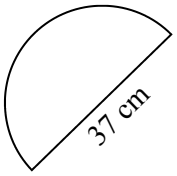
Find the area, but find the radius first by halving the diameter (÷2). Answer to 1 d.p.

8 $r =$ ☐ cm

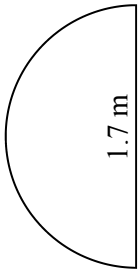
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times$ ☐^{2}

$A =$ ☐ cm^2



10 $r =$ ☐ m

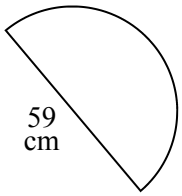


$A = \frac{1}{2}\pi r^2$

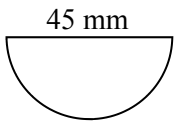
$= \frac{1}{2} \times \pi \times$ ☐^{2}

$A =$ ☐ m^2

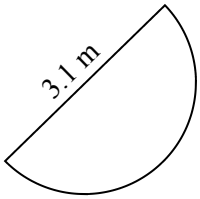
11 $r =$ ☐ cm



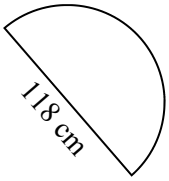
12 $r =$ ☐



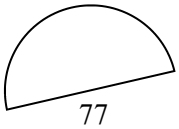
13 $r =$ ☐



12 $r =$ ☐



13 $r =$ ☐



Measure the diameters, halve for the radius, then find the area. Ruler needed! To 1 d.p.

14 ☐ radius

15 ☐ radius

16 ☐ radius

14

15

16

Area of Semi-Circles and Quadrants (Sheet 2)

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A quarter as a decimal is 0.25. Calculate these to 1 d.p., either use ¼ or 0.25.

1 $\frac{1}{4} \times \pi \times 5^2 =$

Calc: 0 . 2 5 × SHIFT ×10^x × 5 x^2 =

or

Calc: 1 ▪/□ 4 ◯ × SHIFT ×10^x × 5 x^2 =

2 $\frac{1}{4} \times \pi \times 13^2 =$

3 $\frac{1}{4} \times \pi \times 9.4^2 =$

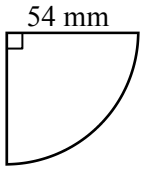
4 $\frac{1}{4} \times \pi \times 21^2 =$

5 $\frac{1}{4} \times \pi \times 74^2 =$

6 $\frac{1}{4} \times \pi \times 145^2 =$

A quadrant is a quarter of a circle, so use $A=\frac{1}{4}\pi r^2$ to find the area, rounding to 1 d.p.

Example



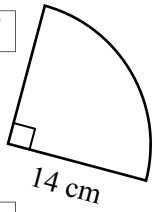
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 54^2$

$A = 2\,290.2 \text{ mm}^2$

Show square units for area

7



$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ ☐^{2}

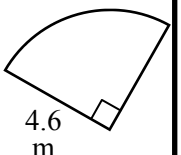
$A =$ ☐ cm^2

8

$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ ☐^{2}

$A =$ ☐ m^2

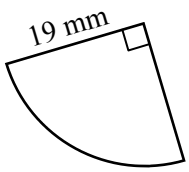


9

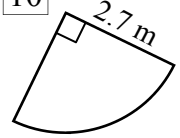
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times$ ☐^{2}

$A =$ ☐ mm^2



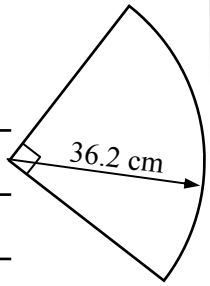
10



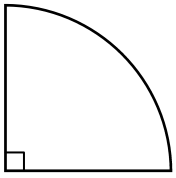
$A = \frac{1}{4}\pi r^2$

$A =$

11



12



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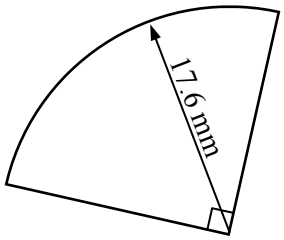
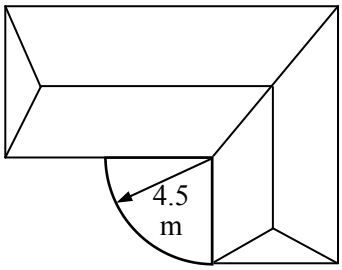


A plan view is a view from above. Look at the plan of a courtyard of a house below.

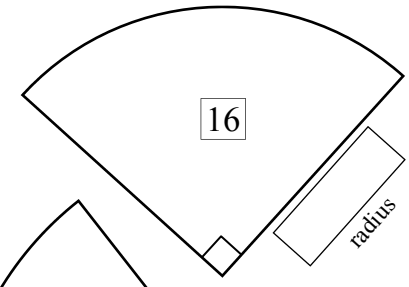


14 Calculate the area of the quadrant shaped courtyard, round your answer to 1 d.p.

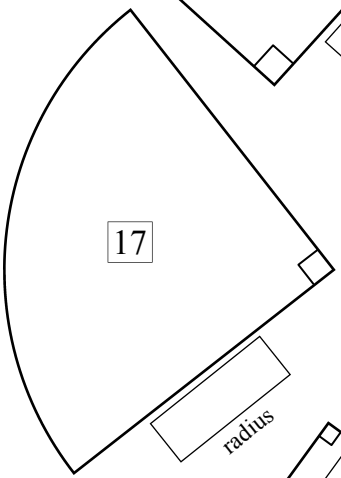
15 Estimate the cost of tiling the area if tiles cost \$56.20/m^2 and are sold in whole m^2 only.

Measure the radius with a ruler then find the area. Round your answer to 1 d.p.

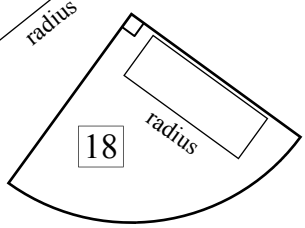
16



17



18



Area of Sectors (Sheet 1)

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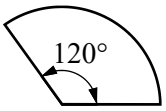
Write the fraction of a circle with a denominator of 360, then simplify with your calculator.

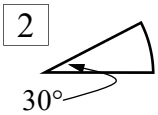
1



$\frac{\square}{360} = \frac{\square}{\square}$

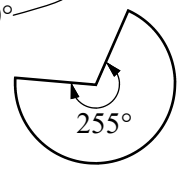
Calc: 1 2 0 ▪ 3 6 0 =

2



$\frac{\square}{\square} = \frac{\square}{\square}$

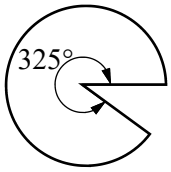
3



$\frac{\square}{\square} = \frac{\square}{\square}$

4

$\frac{\square}{\square} = \frac{\square}{\square}$



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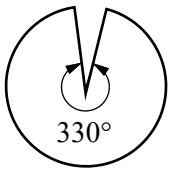
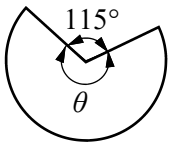


$\frac{\square}{\square} = \frac{\square}{\square}$

Find the internal angle θ, by subtracting from 360°. Then solve for the fraction as above.

6



$\theta = 360 - 115$

$\theta = \square°$

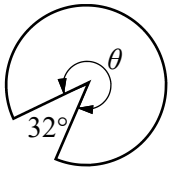
$\frac{\square}{360} = \frac{\square}{\square}$

7

$\theta = 360 - \square$

$\theta = \square°$

$\frac{\square}{\square} = \frac{\square}{\square}$



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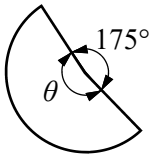


$\theta =$

$\theta =$

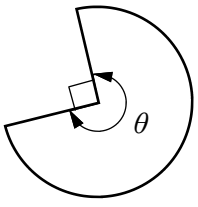
$\frac{\square}{\square} = \frac{\square}{\square}$

9

$\theta =$

$\theta =$

$\frac{\square}{\square} = \frac{\square}{\square}$



Evaluate using your calculator. Round to 1 d.p.

10 $\frac{105}{360} \times \pi \times 16^2 =$

1 0 5 ▪ 3 6 0 ◯ × SHIFT ×10^x × 1 6 x^2 =

11 $\frac{52}{360} \times \pi \times 7^2 =$

12 $\frac{212}{360} \times \pi \times 25^2 =$

13 $\frac{90}{360} \times \pi \times 34.3^2 =$

14 $\frac{322}{360} \times \pi \times 40.8^2 =$

15 $\frac{159}{360} \times \pi \times 19.6^2 =$

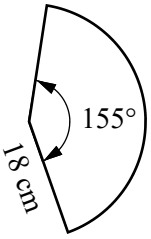
Calculate the area of these sectors, answering to 1 d.p.

Example

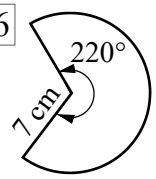
$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{155}{360} \times \pi \times 18^2$

$A = 438.3 \text{ cm}^2$



16



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{220}{360} \times \pi \times \square^2$

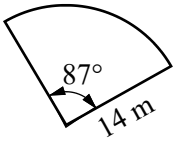
$A = \square \text{ cm}^2$

17

$A = \frac{\theta}{360} \times \pi r^2$

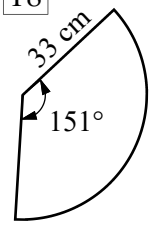
$= \frac{87}{360} \times \pi \times \square^2$

$A = \square \text{ m}^2$



Remember to square the radius!

18

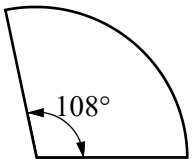


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A = \square \text{ cm}^2$

19



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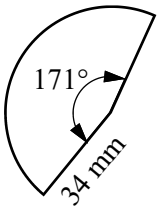

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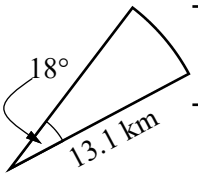

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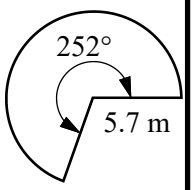

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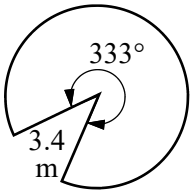

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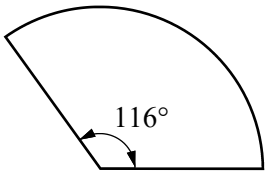

Area of Sectors (Sheet 2)

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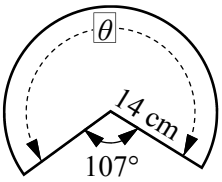
Find the internal angle θ, then calculate the area of the sectors, rounding your answer to 1 d.p.

1



$\theta = 360 - 107$

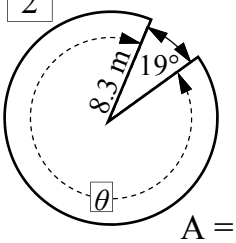
$\theta =$ ☐ °

$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{360} \times \pi \times \square^2$

$A =$ ☐ cm^2

2



$\theta =$

$\theta =$

$A = \frac{\theta}{360} \times \pi r^2$

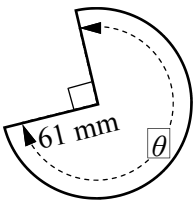
$= \frac{\square}{360} \times \pi \times \square^2$

$A =$ ☐ m^2

3

$\theta =$

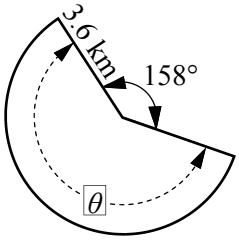
$\theta =$



4

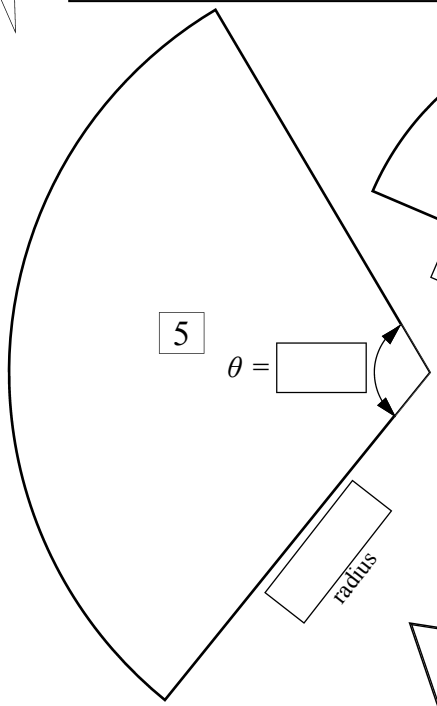
$\theta =$

$\theta =$

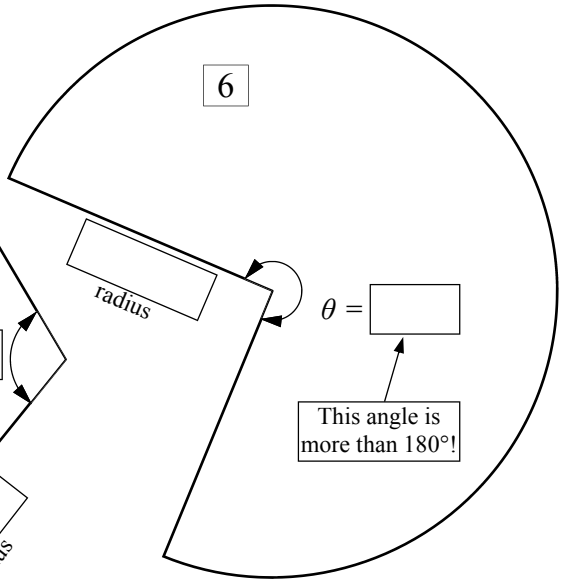


Measure the radius in cm to 1 decimal place, measure the angle to the nearest degree, then find the area (1 d.p.). Ruler and protractor needed!

5



6



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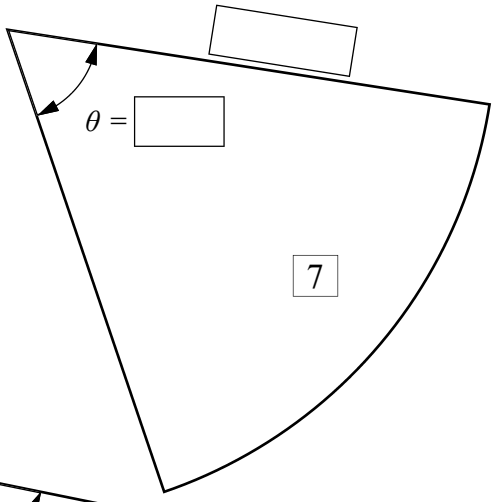

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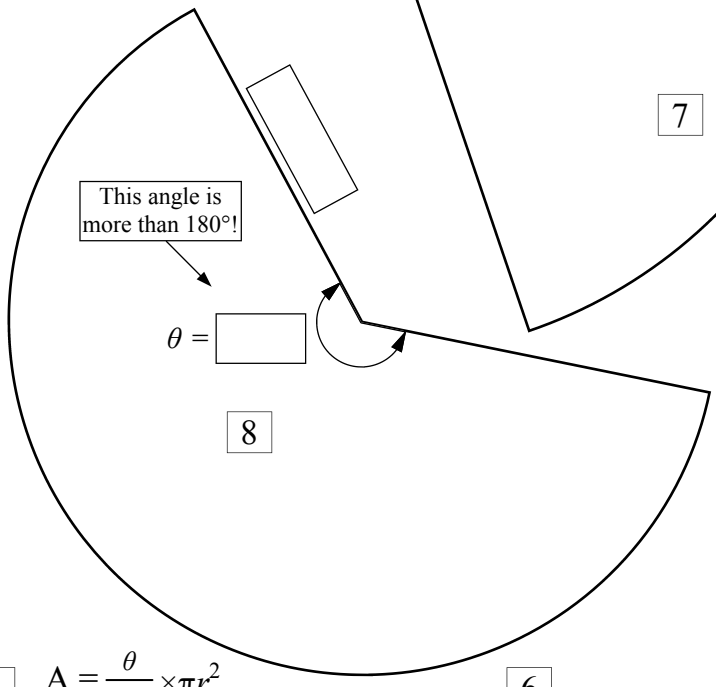

5 $A = \frac{\theta}{360} \times \pi r^2$

6

7

8

The Annulus (Circle Formula) - Sheet 1

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Find the outside area and the inside area, then subtract to find the shaded area. (1 d.p.)

All measurements are in cm

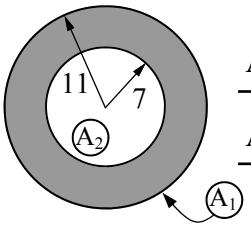
1 $A_1 = \pi R^2$ $A_2 = \pi r^2$

$= \pi \times 11^2$ $= \pi \times 7^2$

$A_1 =$ ____ cm^2 $A_2 =$ ____ cm^2

$A =$ ____ - ____

$A =$ ____ cm^2



2 $A_1 = \pi R^2$ $A_2 = \pi r^2$

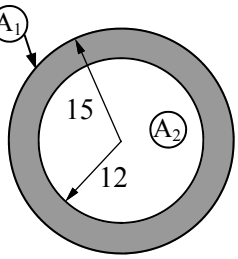
$= \pi \times$ ____ ^{2} $= \pi \times$ ____ ^{2}

This Annulus

$A_1 =$ ____ cm^2 $A_2 =$ ____ cm^2

$A =$ ____ - ____

$A =$ ____ cm^2



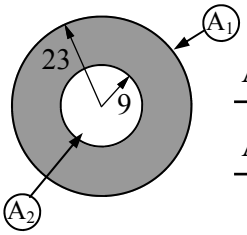
3 $A_1 = \pi R^2$ $A_2 = \pi r^2$

$= \pi \times$ ____ ^{2} $= \pi \times$ ____ ^{2}

$A_1 =$ ____ cm^2 $A_2 =$ ____ cm^2

$A =$ ____ - ____

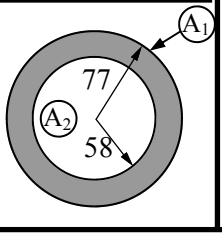
$A =$ ____ cm^2



4

$A_1 = \pi R^2$ $A_2 = \pi r^2$

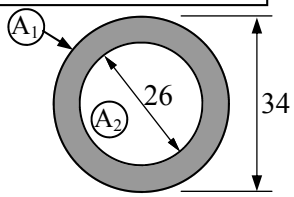
This Annulus



Change diameters to radii first, before calculation of the shaded area. All lengths are in cm. (1 d.p.)

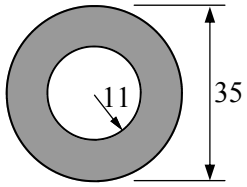
5 $R =$ ____ cm

$r =$ ____ cm



$A_1 = \pi R^2$ $A_2 = \pi r^2$

6 $R =$ ____ cm

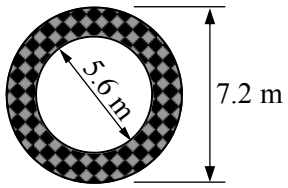


A plan view of a proposed circular swimming pool is shown below. The pool is to have a circular tiled area surrounding it with a fence around the tiled area.

7 Find the tiled (shaded) area (1 d.p.)

$R =$ ____ m

$r =$ ____ m



8 Find the cost of tiling the area given that the tiles cost \$47.55/m^2 and are sold in whole m^2 only.

Cost =

The Annulus (Annulus Formula) - Sheet 2

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Calculate these using your 'brackets' key. Answer to 2 decimal places.

1 $\pi (15^2 - 9^2) =$ ____

Calc: SHIFT $\times 10^x$ × (1 5 x^2 - 9 x^2) =

2 $\pi (27^2 - 18^2) =$ ____

3 $\pi (32^2 - 26^2) =$ ____

4 $\pi (41^2 - 22^2) =$ ____

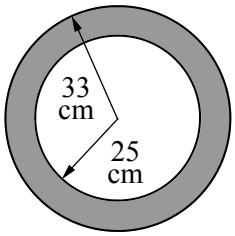
5 $\pi (56.7^2 - 38.3^2) =$ ____

Use the annulus formula, $A = \pi(R^2 - r^2)$ to find the shaded area. Answer to 1 decimal place.

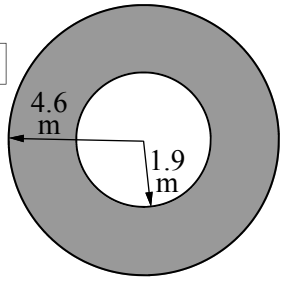
6 $A = \pi(R^2 - r^2)$

$= \pi \times (\square^2 - \square^2)$

$A =$ ____ cm^2



7



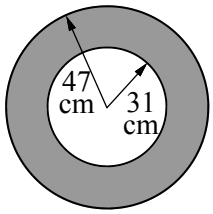
$A = \pi(R^2 - r^2)$

$= \pi \times (\square^2 - \square^2)$

$A =$ ____

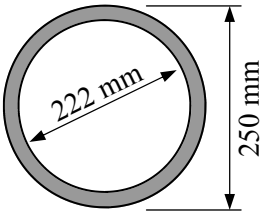
8

$A = \pi(R^2 - r^2)$

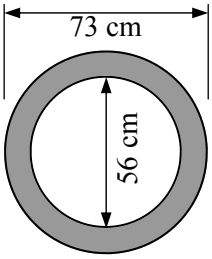


These questions have their diameter measurements, so divide by 2 to get the radius first. (÷2)

9 $R =$ ____ mm $r =$ ____ mm

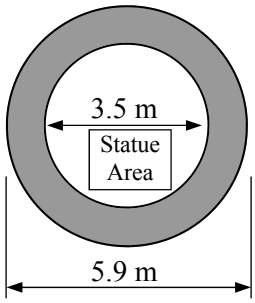


10 $R =$ ____ cm $r =$ ____ cm



A roundabout has a central area with a statue that has a 3.5 m diameter. It has a garden bed around it, shaded area on the plan. The total diameter of the roundabout is 5.9 m

11 Find the garden bed area, answer to nearest m^2.

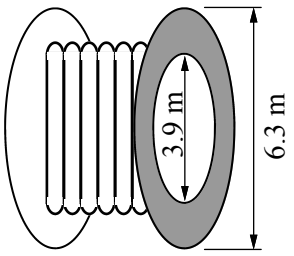


12 If the gardener buys 9 plants for every square metre, how many plants will be required?

13 What is the cost of the plants if the first 50 plants cost $4.50 each, then the remaining plants cost $4.15 each

A mineshaft cable pulley, shown below, has a plate steel circular collar with an inside diameter of 3.9 m and an outside diameter of 6.3 m.

14 Find the collar area (shaded area), to 1 d.p.



15 Find the weight of the collar, if the plate steel weighs 53 kg per m^2. Answer in kg, to 1 d.p.

Area - Composite Shapes (Circular) - Sheet 1

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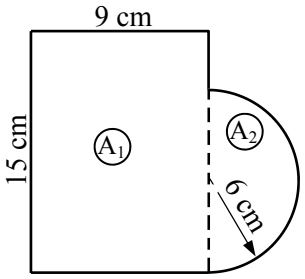
Calculate the area of these shapes by breaking the shapes into parts first. All angles that look like right angles, are right angles! Answer to 1 d.p.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_2 = \square$ cm^2

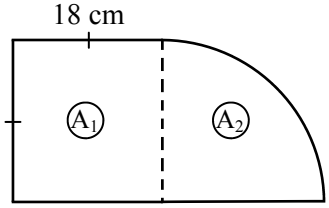
$A = \square + \square = \square$ cm^2

2

$A_1 = l^2$

$= \square^2$

$A_1 = \square$ cm^2



$A_2 = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A_2 = \square$ cm^2

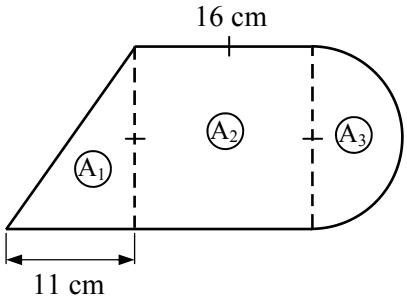
$A = \square + \square = \square$ cm^2

3

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A_1 = \square$ cm^2



$A_2 = l^2$

$= \square^2$

$A_2 = \square$ cm^2

$A_3 = \frac{1}{2}\pi r^2$

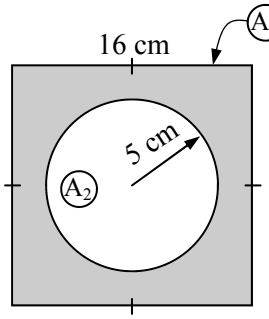
$= \frac{1}{2} \times \pi \times \square^2$

$A_3 = \square$ cm^2

$A =$

Find the shaded area, subtract the smaller area from the larger one. Answer to 1 d.p.

4



$A_1 = l^2$

$= \square^2$

$A_1 = \square$ cm^2

$A_2 = \pi r^2$

$= \pi \times \square^2$

$A_2 = \square$ cm^2

$A = \square - \square = \square$ cm^2

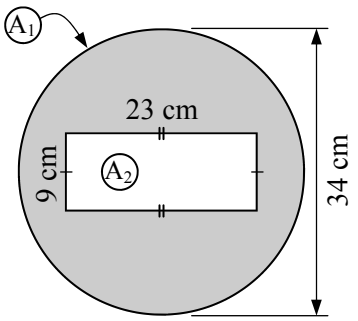
5

$A_1 = \pi r^2$

$= \pi \times \square^2$

$A_1 = \square$ cm^2

$r = \square$ cm



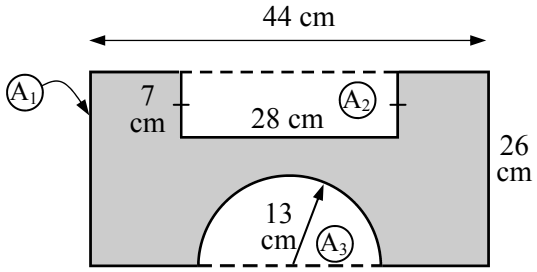
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A =$

6



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A_3 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times \square^2$

$A_3 = \square$ cm^2

$A =$

Area - Composite Shapes (Circular) - Sheet 2

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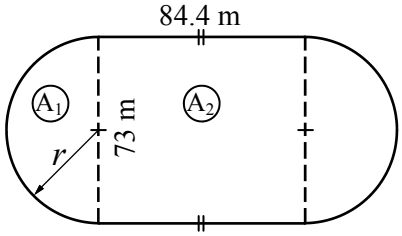
Use your composite area skills to find these. Answer to 1 decimal place when required.

An athletics field is shown. Find the:



1 Radius of the semi-circle $r =$ ____

2 Area (A_1)

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times$ ____^{2}

$A_1 =$ ____ m^2

3 Area (A_2)

$A_2 = lb$

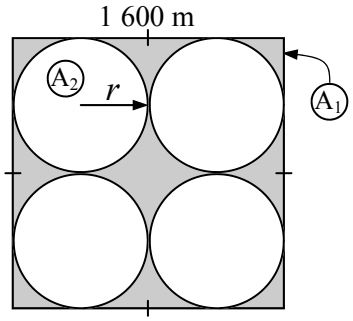
$=$ ____ $\times$ ____

$A_2 =$ ____ m^2

4 Total field area.

5 If 1 m^2 requires 2.2 L of water, how much water is required to irrigate the entire field.

A square field has four identical circular fields fitting perfectly within it. Find the:



6 Radius of the circular field $r =$ ____

7 Area (A_1)

$A_1 = l^2$

8 Area (A_2)

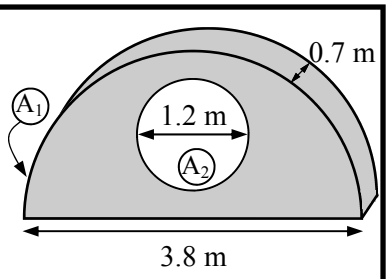
$A_2 = \pi r^2$

9 Amount of the square field which is unused.

10 The percentage of the square field unused.

% Unused = $\frac{\text{Q9}}{\text{Q7}} \times 100 =$ ____

A concrete anchor block for bridge cabling is shown. It is semi-circular with a circular opening. Find the:



11 Radius of the circle $r =$ ____

12 Radius of the semi-circle $R =$ ____

13 Area (A_1)

$A_1 = \frac{1}{2}\pi R^2$

14 Area (A_2)

$A_2 = \pi r^2$

15 Area of the concrete (shaded) face.

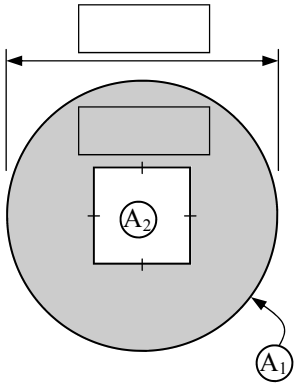
16 Use: $V = Ah$ to find the volume of concrete.

Volume is the face area (Q.15) multiplied by the block's depth (0.7 m).

$V = Ah$

$=$ ____ $\times$ ____

$V =$ ____ m^3



A circular washer with a 25 mm diameter has a square hole with 9 mm sides through its centre. Write these lengths in the boxes on the diagram, then find:

17 Radius of the circle $r =$ ____

18 Area (A_1)

$A_1 = \pi r^2$

19 Area (A_2)

$A_2 = l^2$

20 Face area of the steel washer (shaded area).

Area - Find a Word

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Find the words in the puzzle from the wordlist.



I	T	M	A	R	G	O	L	E	L	L	A	R	A	P
I	H	I	T	K	S	C	S	U	B	T	R	A	C	T
K	K	T	I	E	R	C	Y	L	P	I	T	L	U	M
E	I	E	C	I	C	O	M	B	I	N	E	T	E	I
L	E	T	I	S	O	P	M	O	C	E	M	O	T	N
C	O	T	E	A	D	R	E	A	N	N	U	L	U	S
R	K	R	O	I	R	E	T	N	I	N	I	E	S	I
I	I	E	E	R	R	N	R	P	I	E	Z	S	U	D
C	R	L	I	C	A	A	Y	T	L	P	E	T	B	E
E	A	G	L	R	A	R	S	G	R	A	P	H	M	H
R	D	N	D	O	A	P	N	A	E	R	A	G	O	T
A	I	A	I	R	G	A	S	E	Q	A	R	I	H	G
U	U	I	R	O	A	D	I	A	M	E	T	E	R	N
Q	S	R	A	D	J	D	X	D	E	D	A	H	S	E
S	A	T	R	E	C	T	A	N	G	L	E	S	E	L

WORDLIST

RHOMBUS	LENGTH	UNITS	ADD	RADIUS
SYMMETRY	SUBTRACT	AXIS	ANNULUS	QUADRANT
SQUARE	CIRCLE	KITE	COMPOSITE	MULTIPLY
SHADED	SECTOR	RECTANGLE	SPACE	INSIDE
DIAMETER	TRIANGLE	AREA	COMBINE	TRAPEZIUM
JOIN	ANGLE	INTERIOR	HEIGHT	PARALLELOGRAM

Angle appears separately to the words RECTANGLE and TRIANGLE

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Formula List

Squares: $A = l^2$
Rectangles: $A = lb$
Triangles: $A = \frac{1}{2}bh$
Circles: $A = \pi r^2$
Annulus: $A = \pi(R^2 - r^2)$
Sector: $A = \frac{\theta}{360}\ \pi r^2$
Parallelograms: $A = bh$
Rhombus and Kites: $A = \frac{1}{2}xy$
Trapeziums: $A = \frac{1}{2}(a + b)h$

Pythagoras Theorem:
$c^2 = a^2 + b^2$

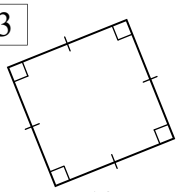
Use one of these words to make the two sentences correct:
Square, length, base or space.

1 Area is the measurement of _____ inside a shape.

2 Area is measured in _____ units.

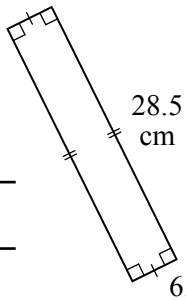
Find the area of these shapes, answer to 1 decimal place.

3

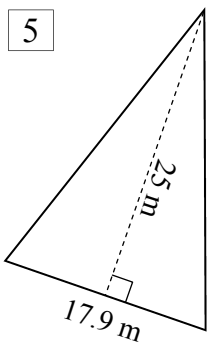


19 cm

4

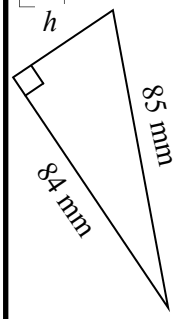


5



Use Pythagoras Theorem to find the height of this triangle, 'h'.

6

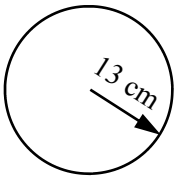


Find the area of the triangle above.

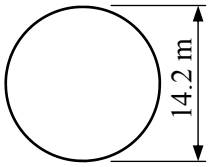
7

Calculate the area of the two circles below, answering to 1 decimal place.

8

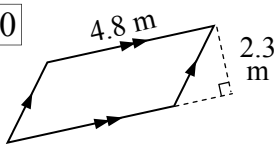


9



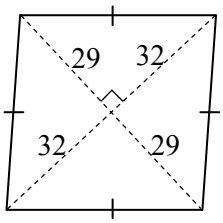
Calculate the area of these shapes, answering to 1 d.p. when necessary.

10

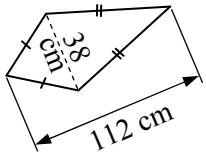


11

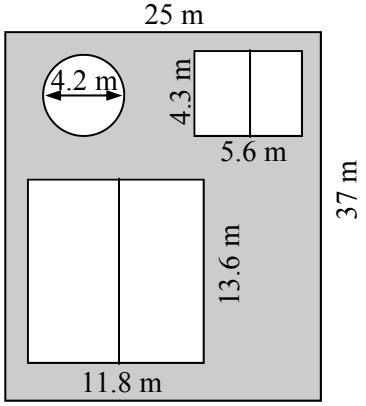
Lengths are in cm



12



13 The plan view shows a house block with a house, garage and pool. Find the undeveloped area (shaded area), answering to 1 d.p.



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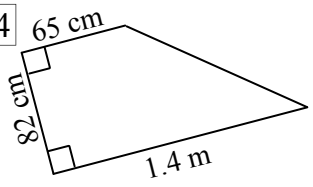
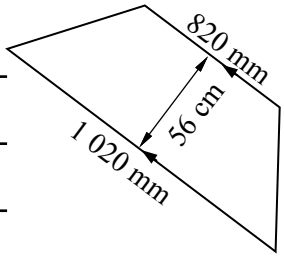
Find the area of the trapeziums shown.
Give the area in cm^2.

14



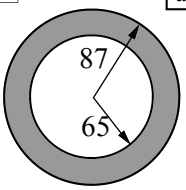
15



Find the area of the annulus shown, answering to 1 d.p.

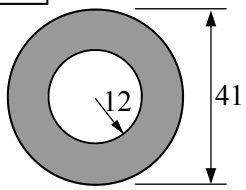
16

Lengths are in cm



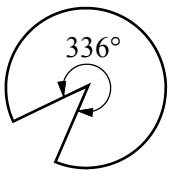
17

Lengths are in cm



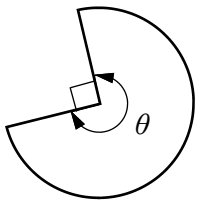
Calculate the fraction of a circle these sectors represent.
Fully simplify your answer.

18



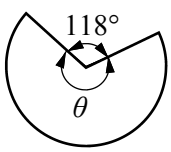
$\frac{\square}{\square} = \frac{\square}{\square}$

19



$\frac{\square}{\square} = \frac{\square}{\square}$

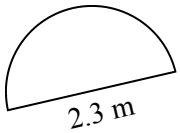
20



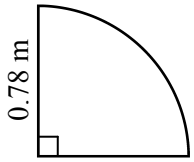
$\frac{\square}{\square} = \frac{\square}{\square}$

Find the area of the semi-circle and quadrant shown.
Answer to 2 d.p.

21

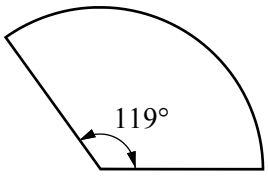


22

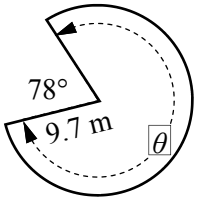


Find the area of the sectors shown. Answering to 1 d.p.

23

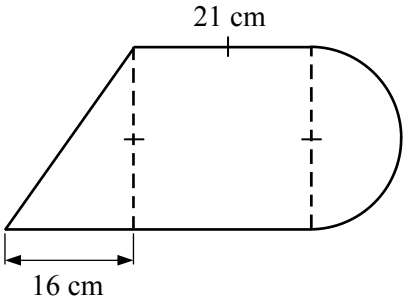


24



Find the area of the shape below, answering to 1 d.p.

25



VELOCITY

PROBABILITY

Probability



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Probability



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Probability Devices

These are spinners for probability applications. Just add toothpicks and tape.

There are also two grids for dropping an object, such as a paperclip on.

Multiple Probability Devices - (Sheet 1)

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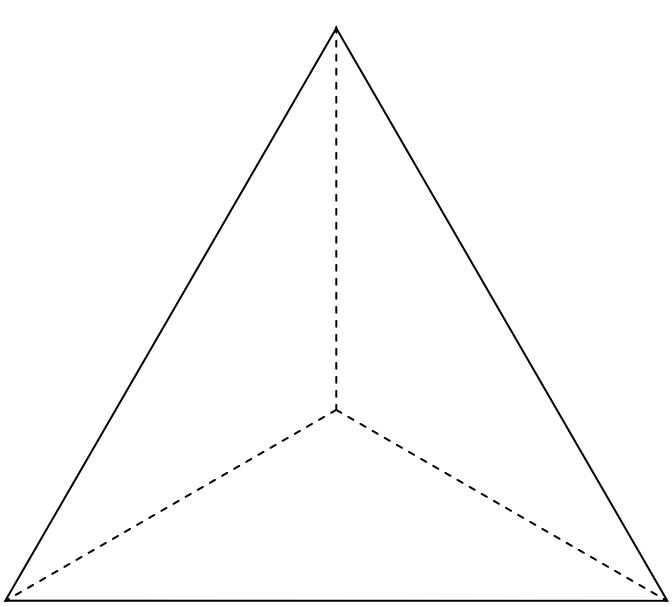

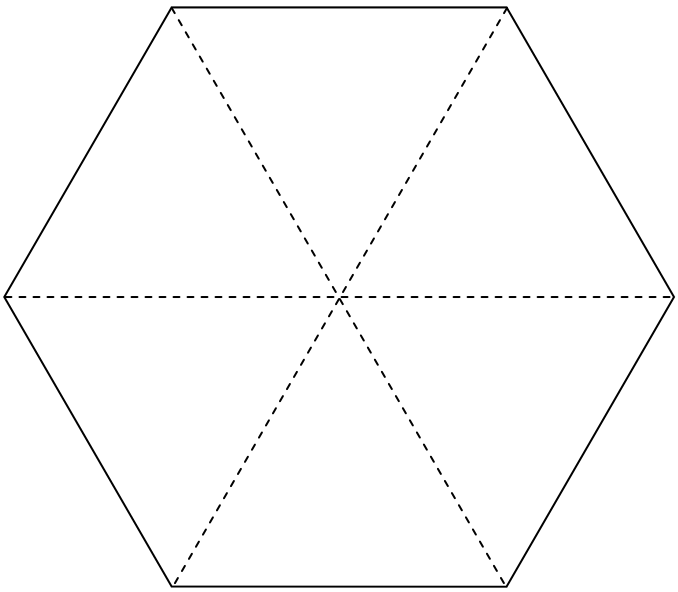

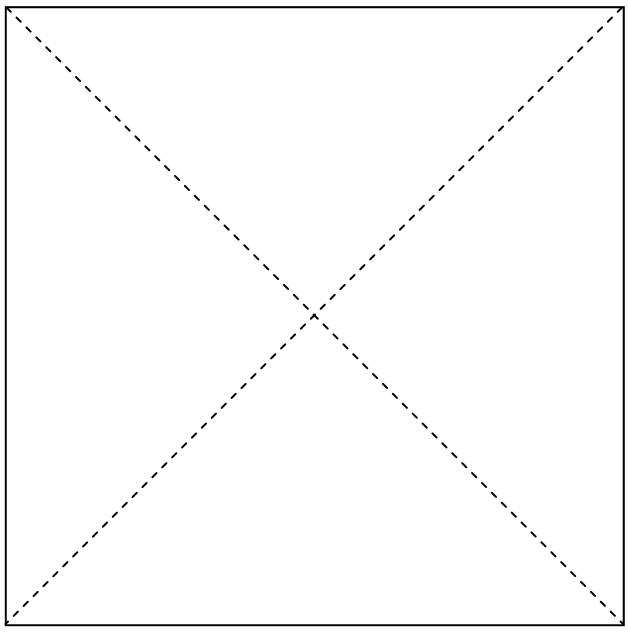

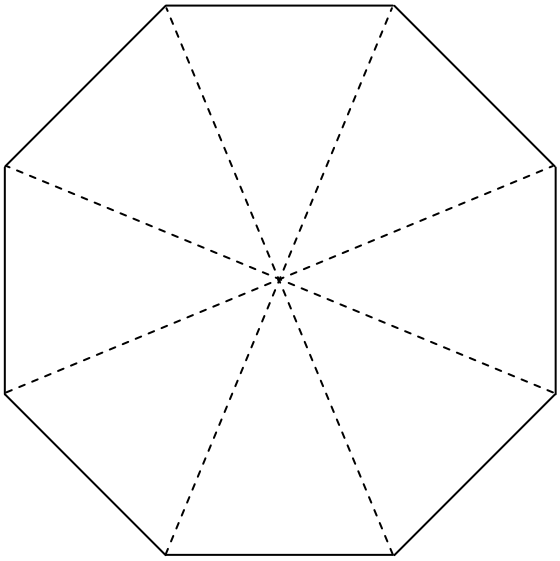

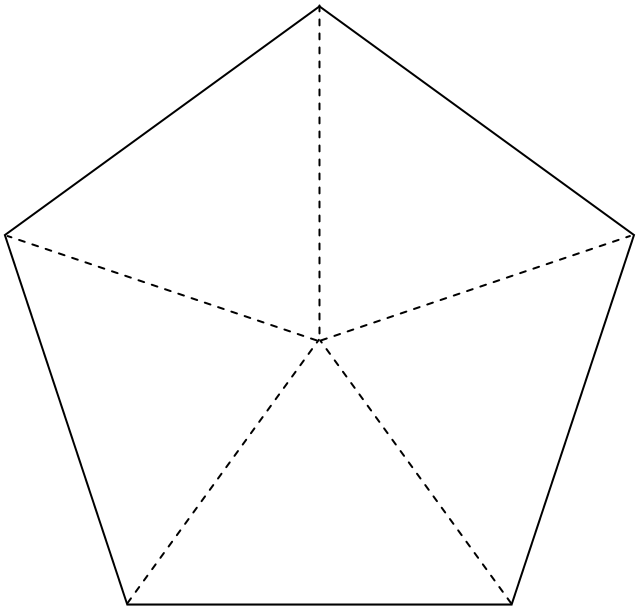

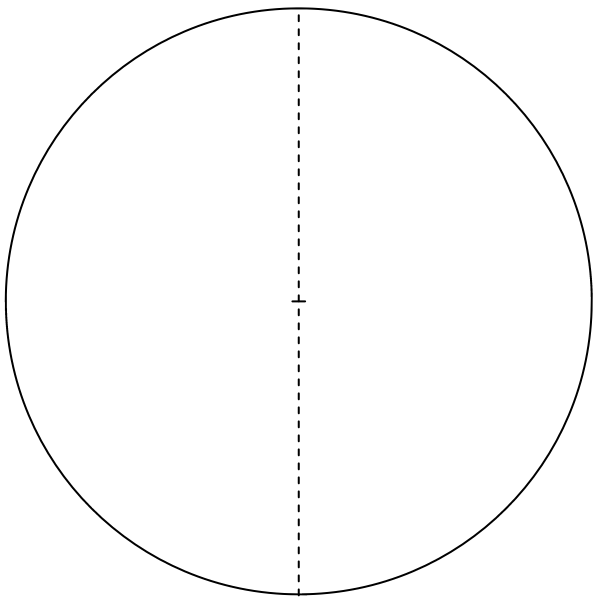

Multiple Probability Devices - (Sheet 2)

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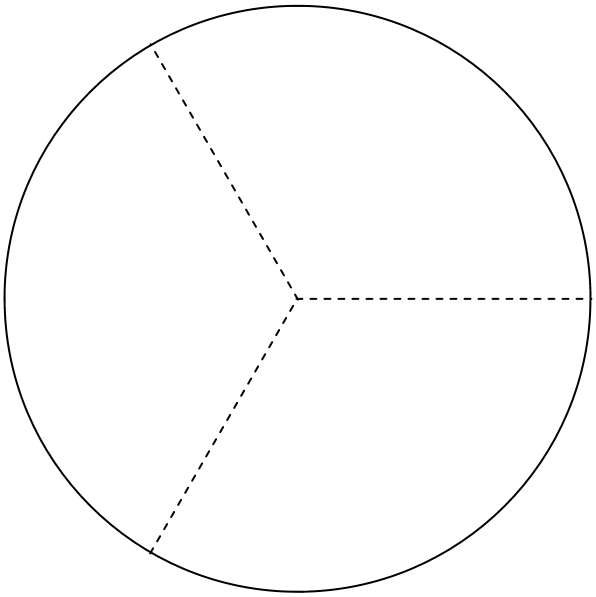

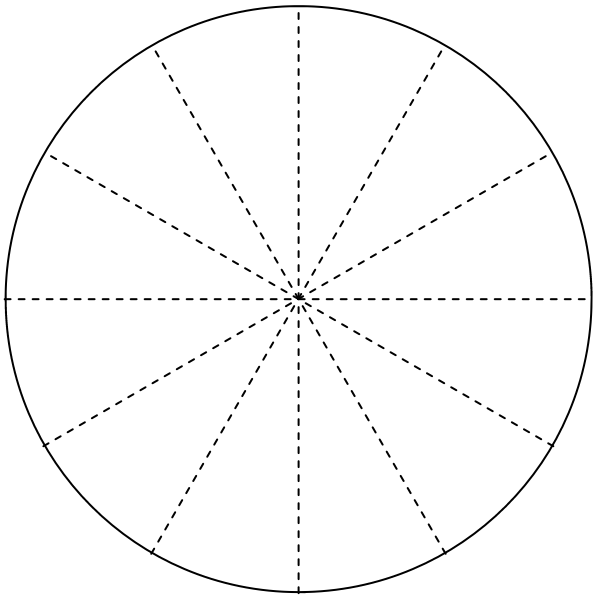

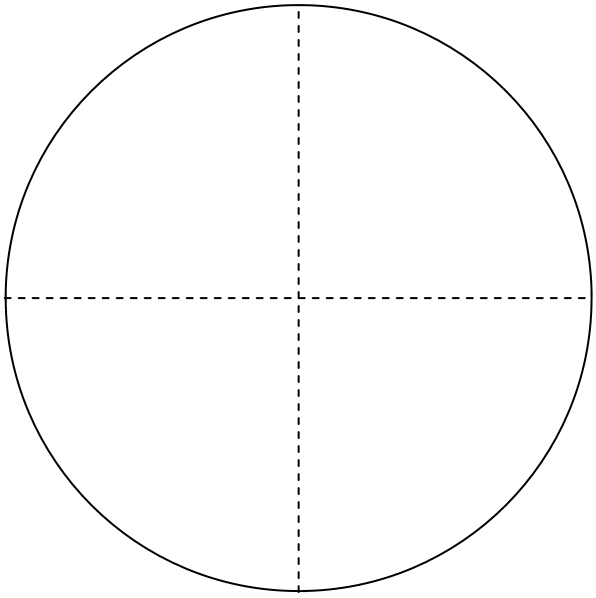

1	2	3	4	5	6
7	8	9	10	11	12
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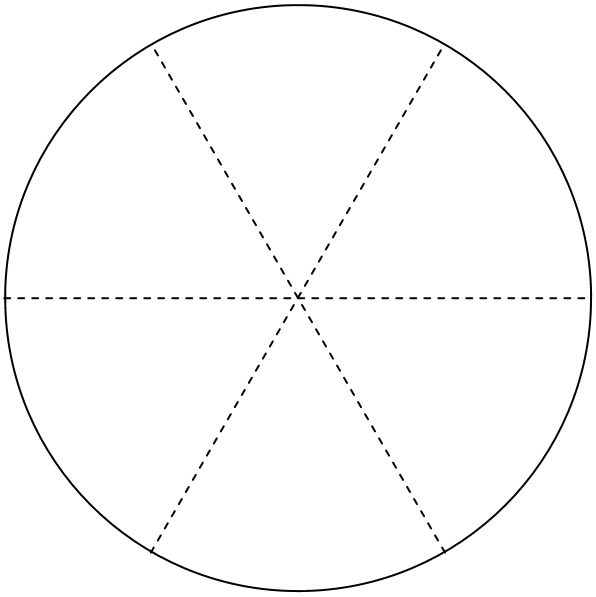

1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

Probability Devices - Triangular Spinners

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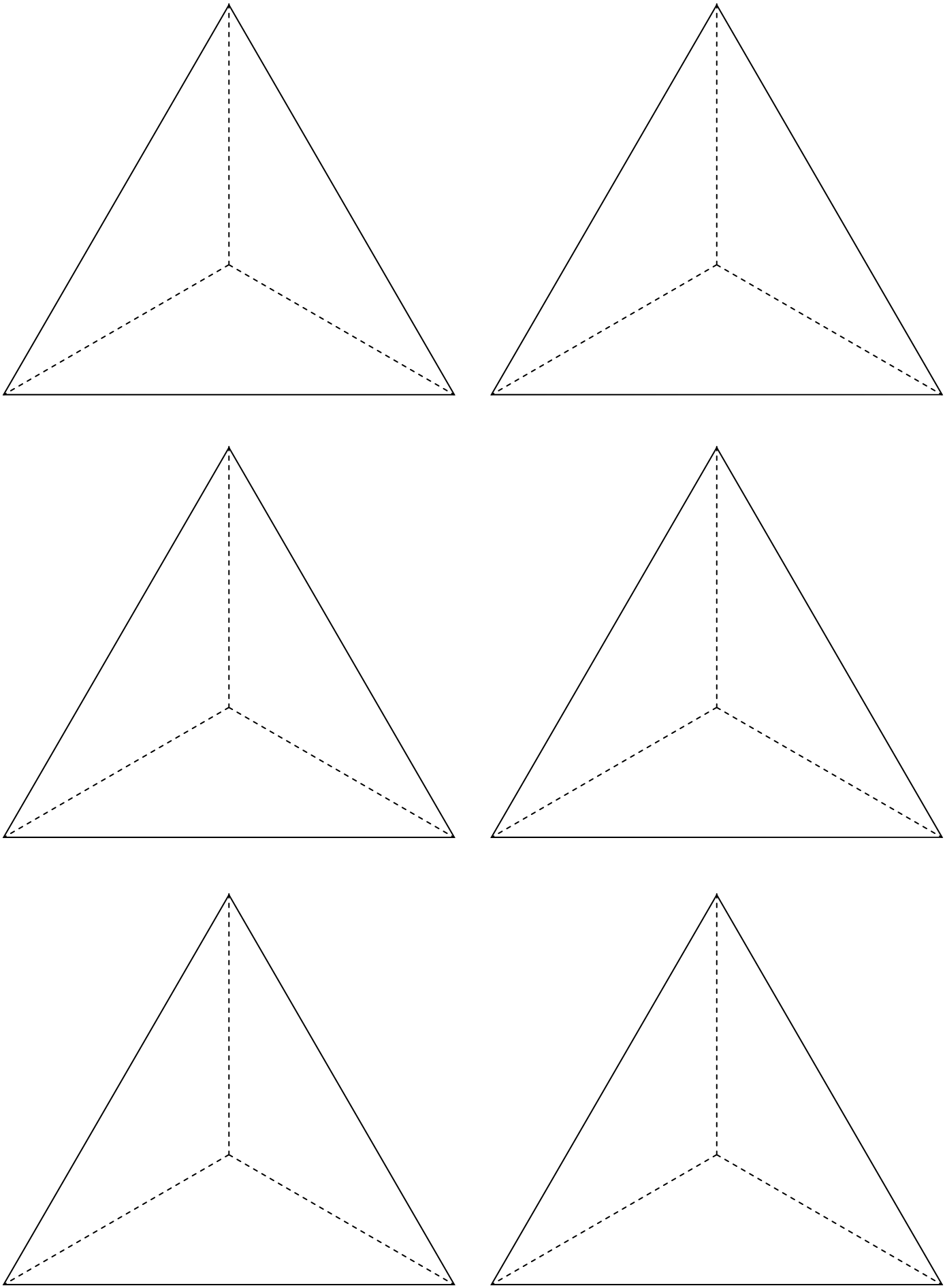

Probability Devices - Square Spinners

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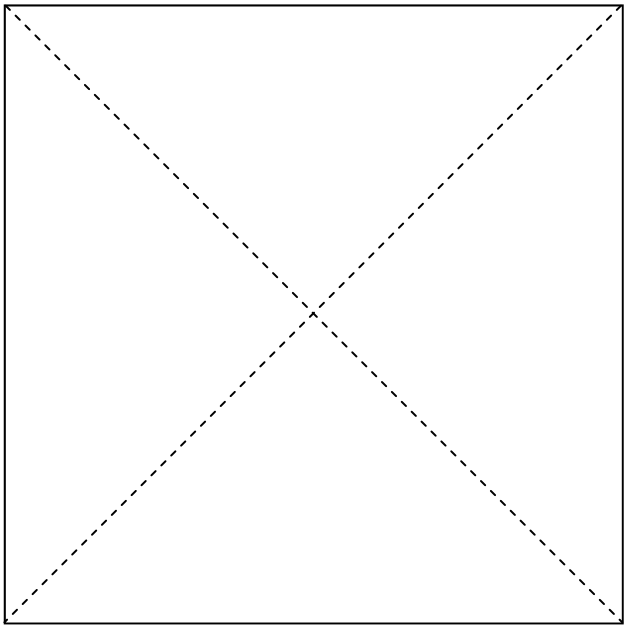

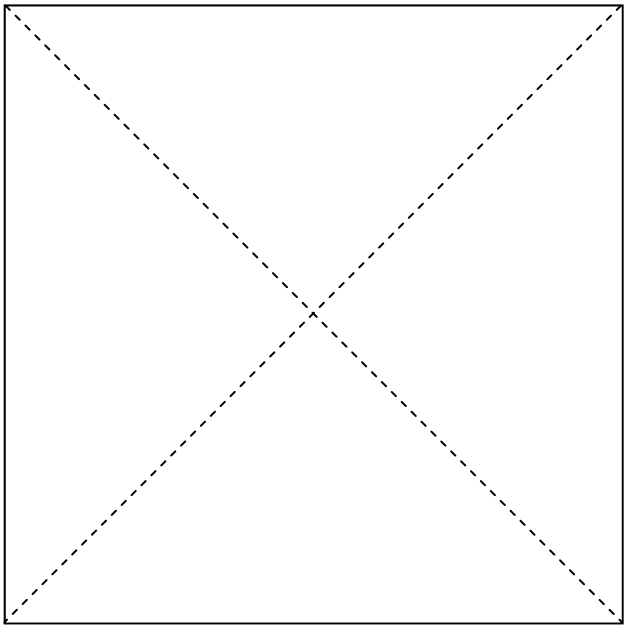

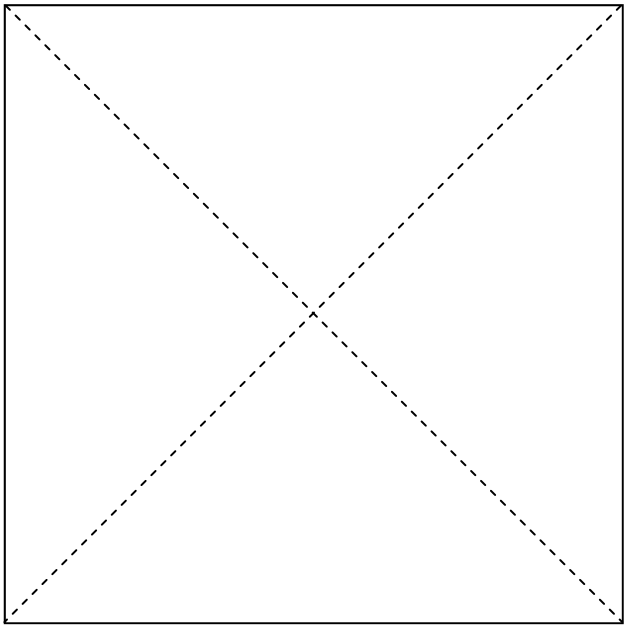

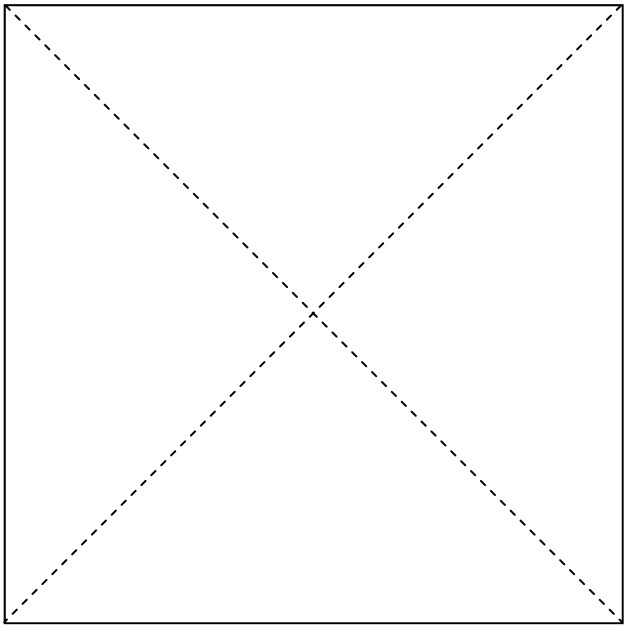

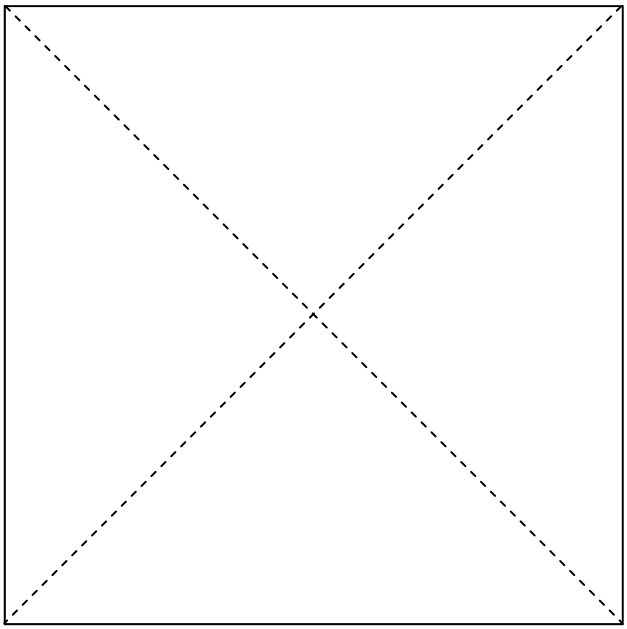

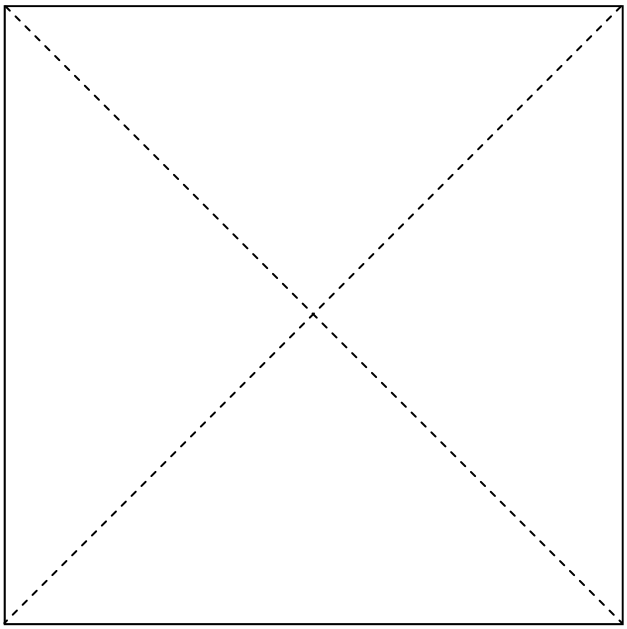

Probability Devices - Pentagonal Spinners

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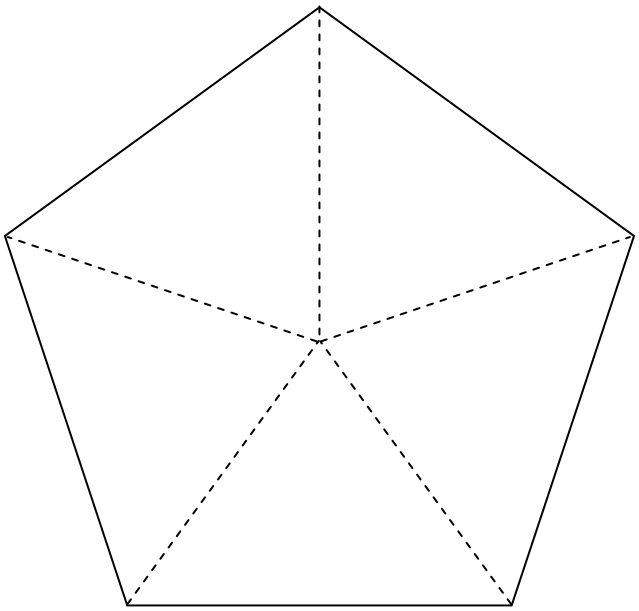
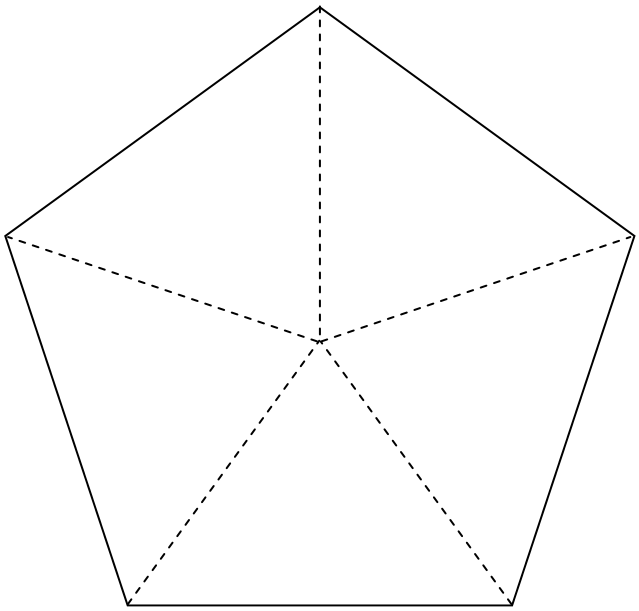
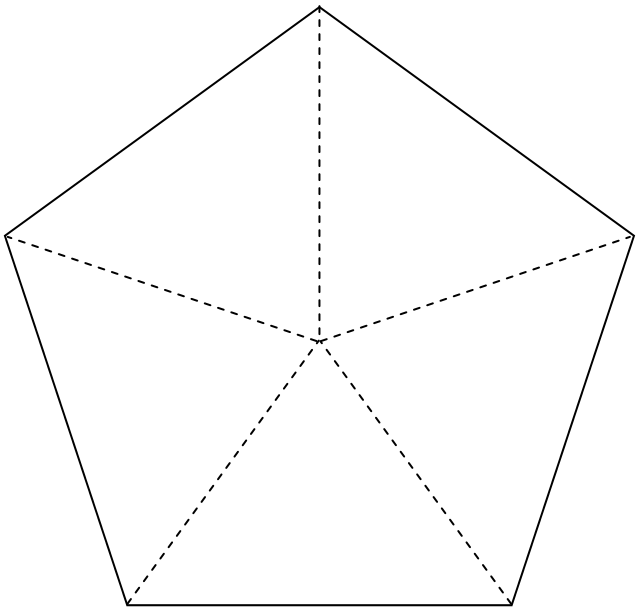
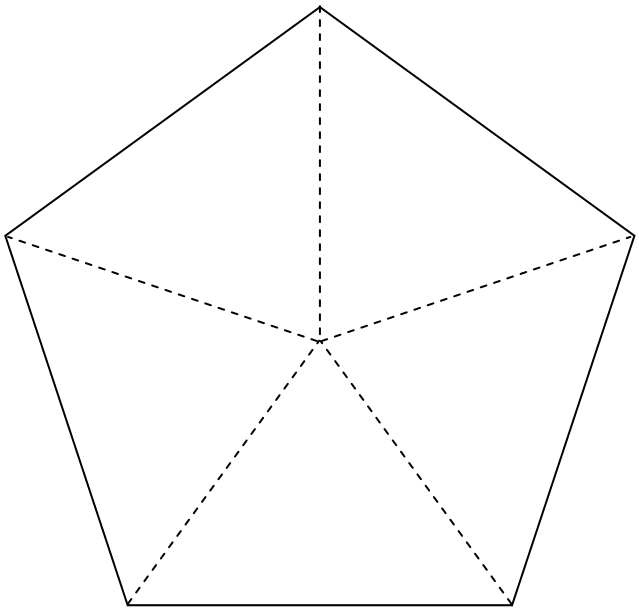
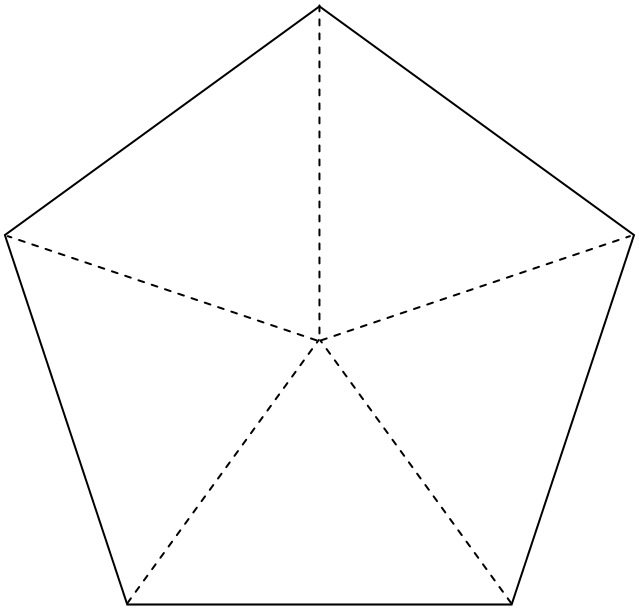
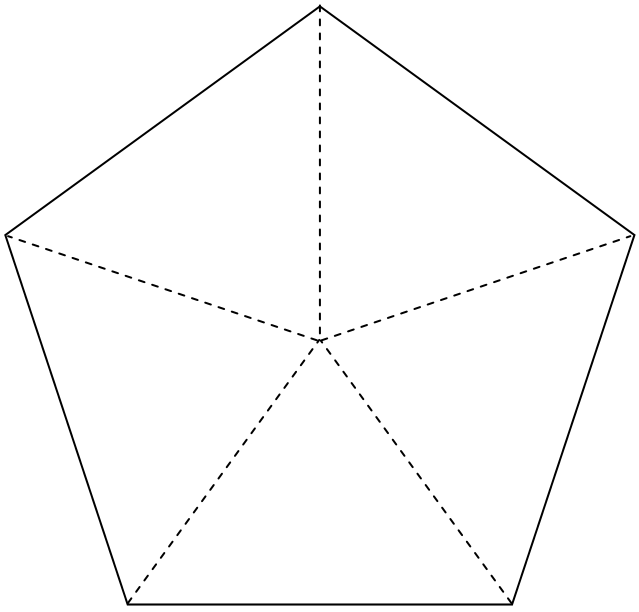

Probability Devices - Hexagonal Spinners

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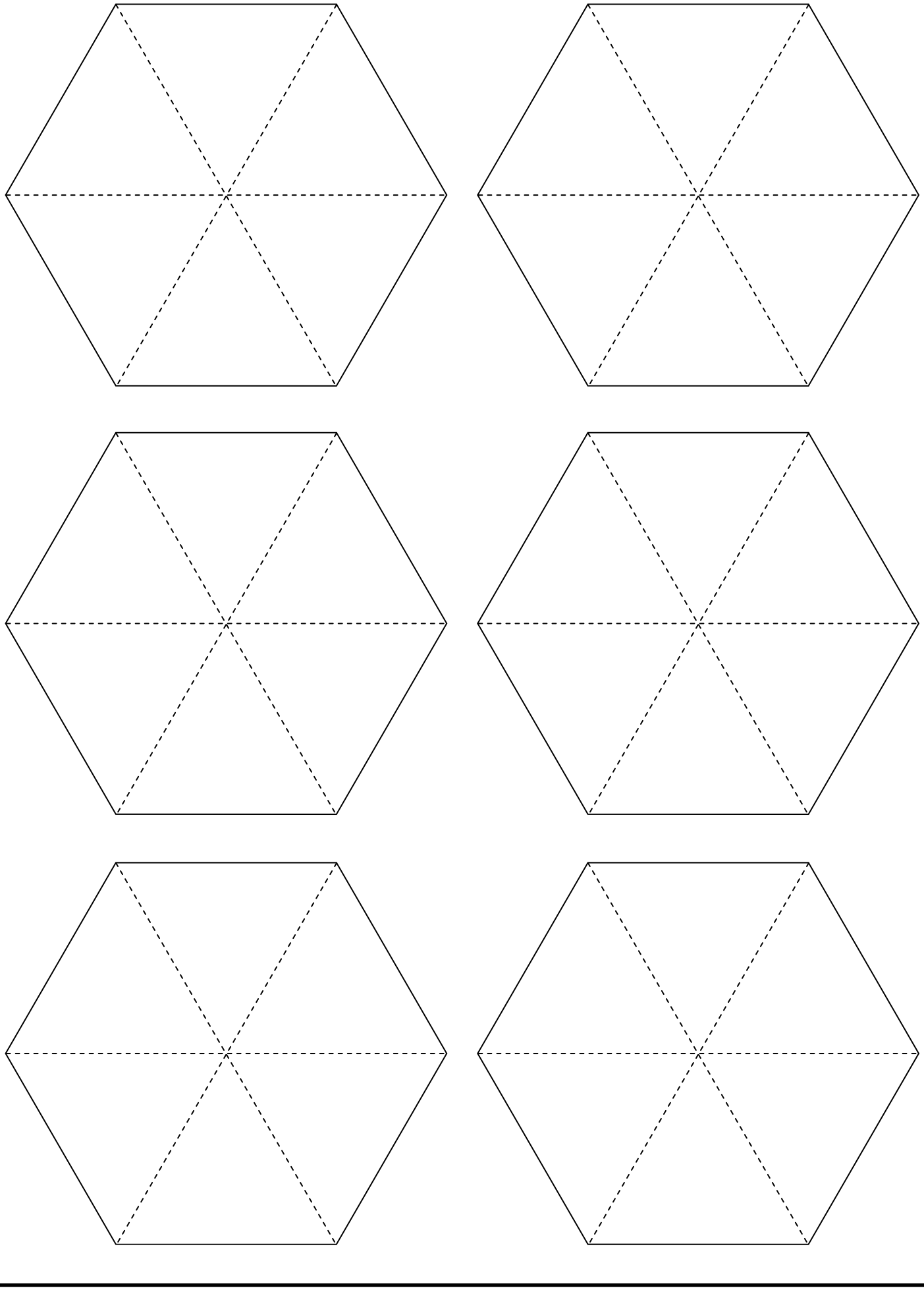

Probability Devices - Octagonal Spinners

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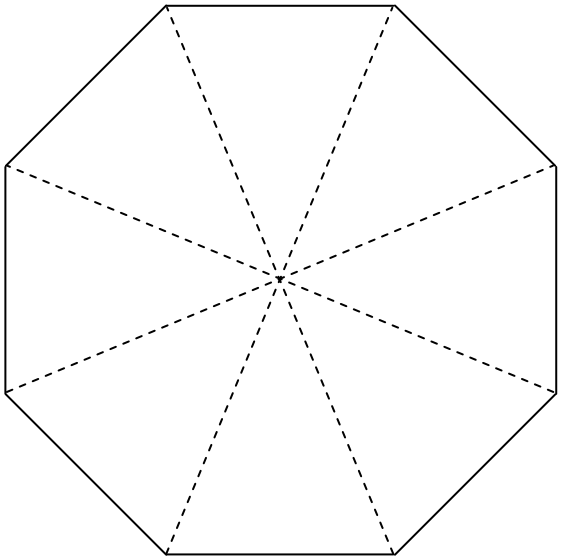

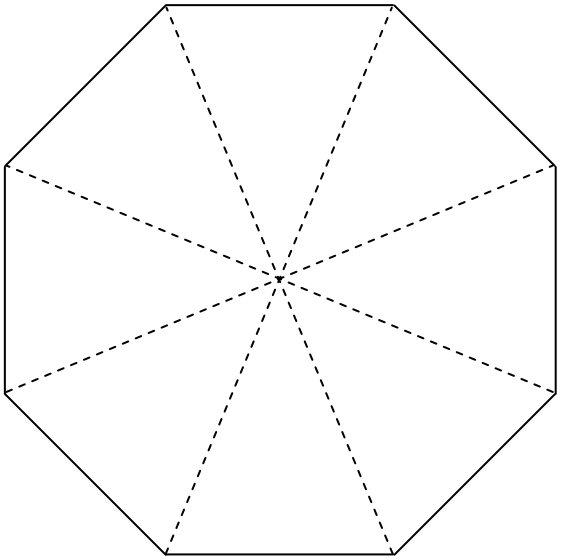

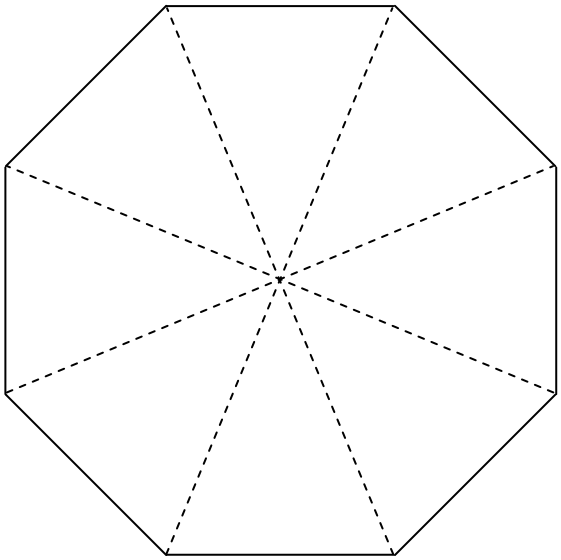

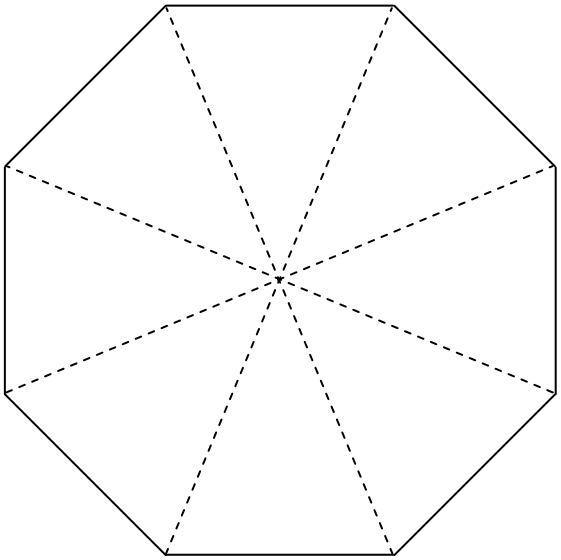

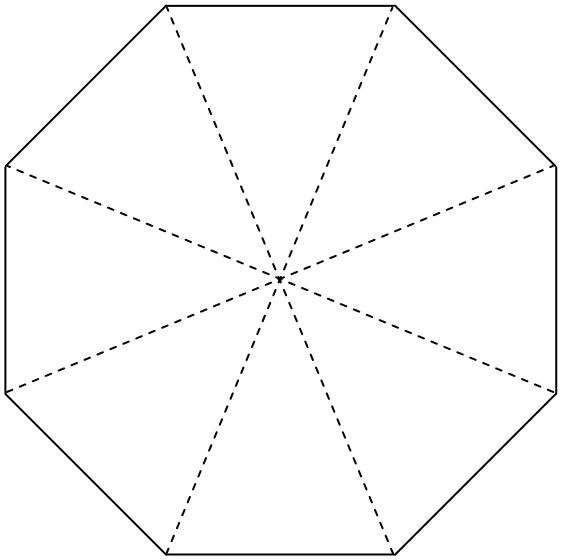

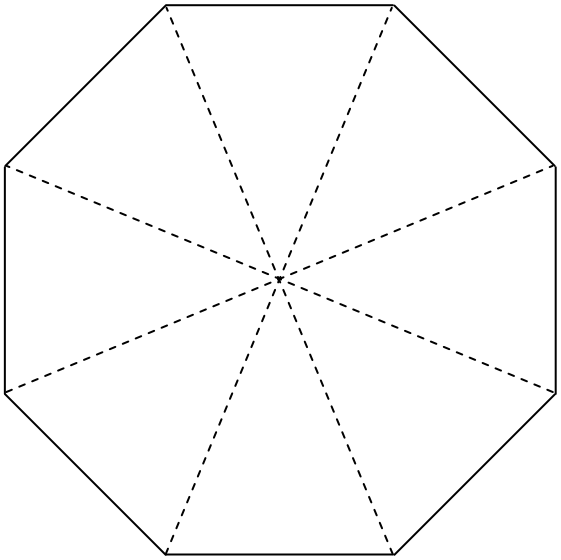

Probability Devices - Circular Spinners 2 sectors

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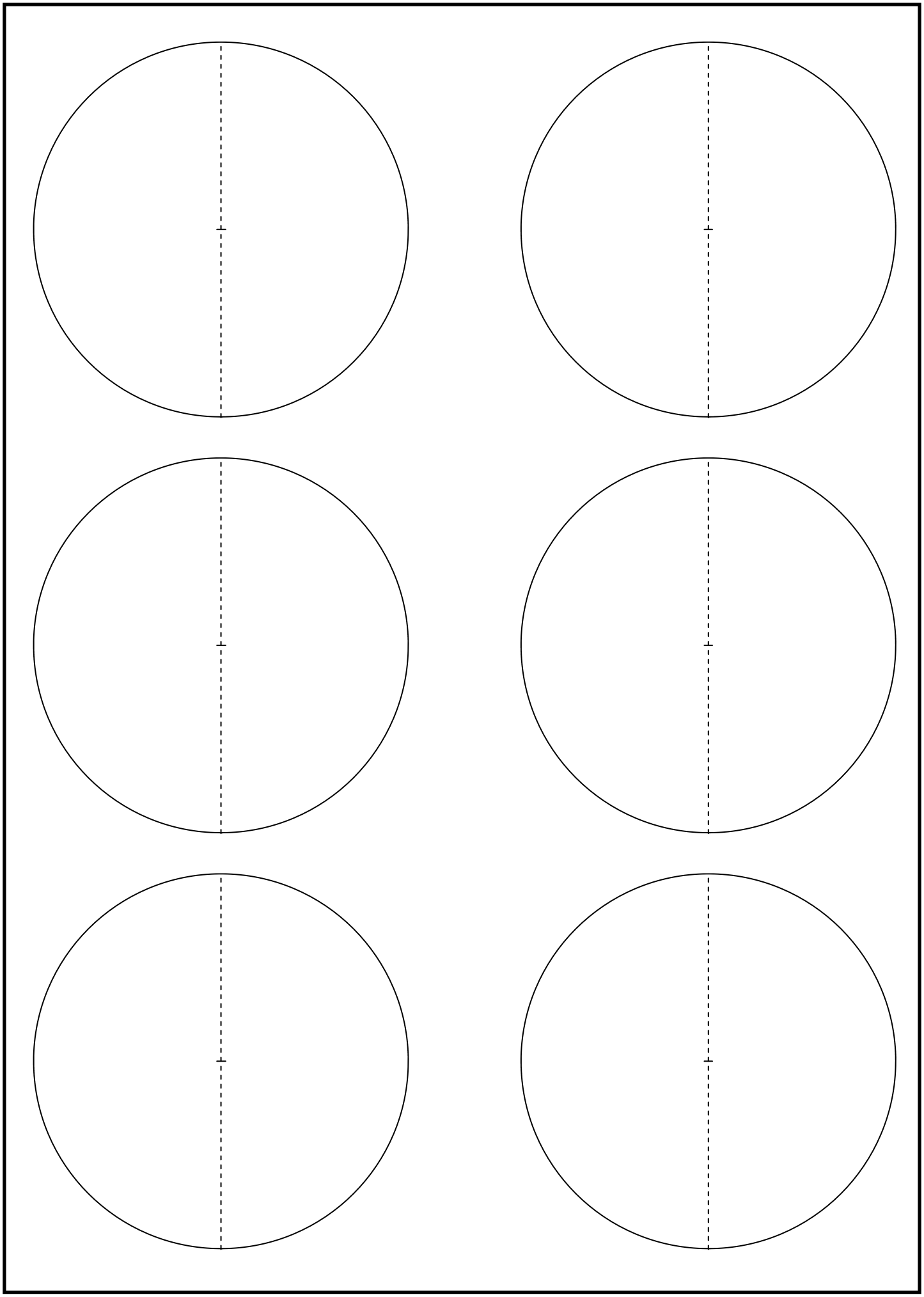

Probability Devices - Circular Spinners 3 sectors

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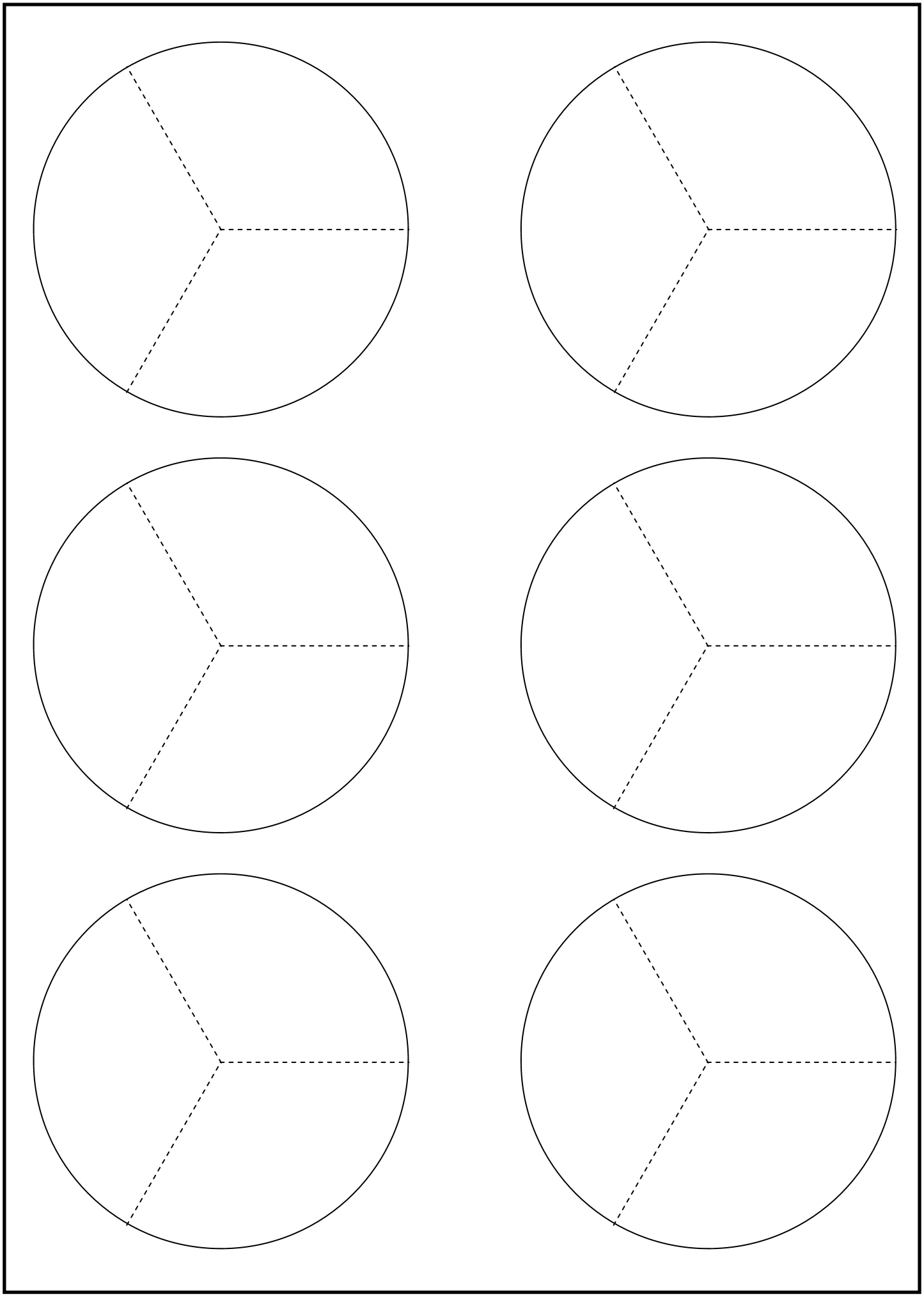

Probability Devices - Circular Spinners 4 sectors

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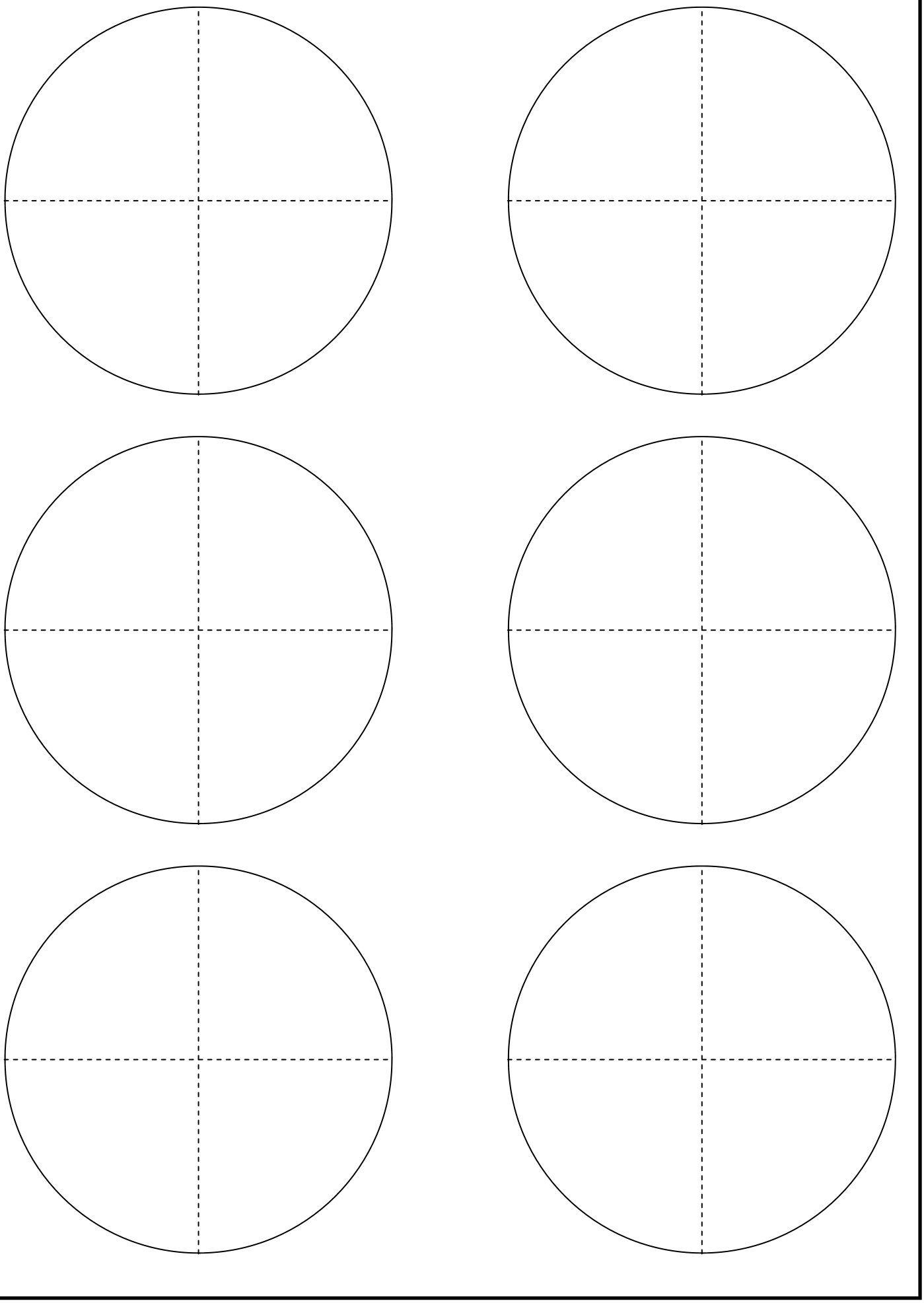

Probability Devices - Circular Spinners 6 sectors

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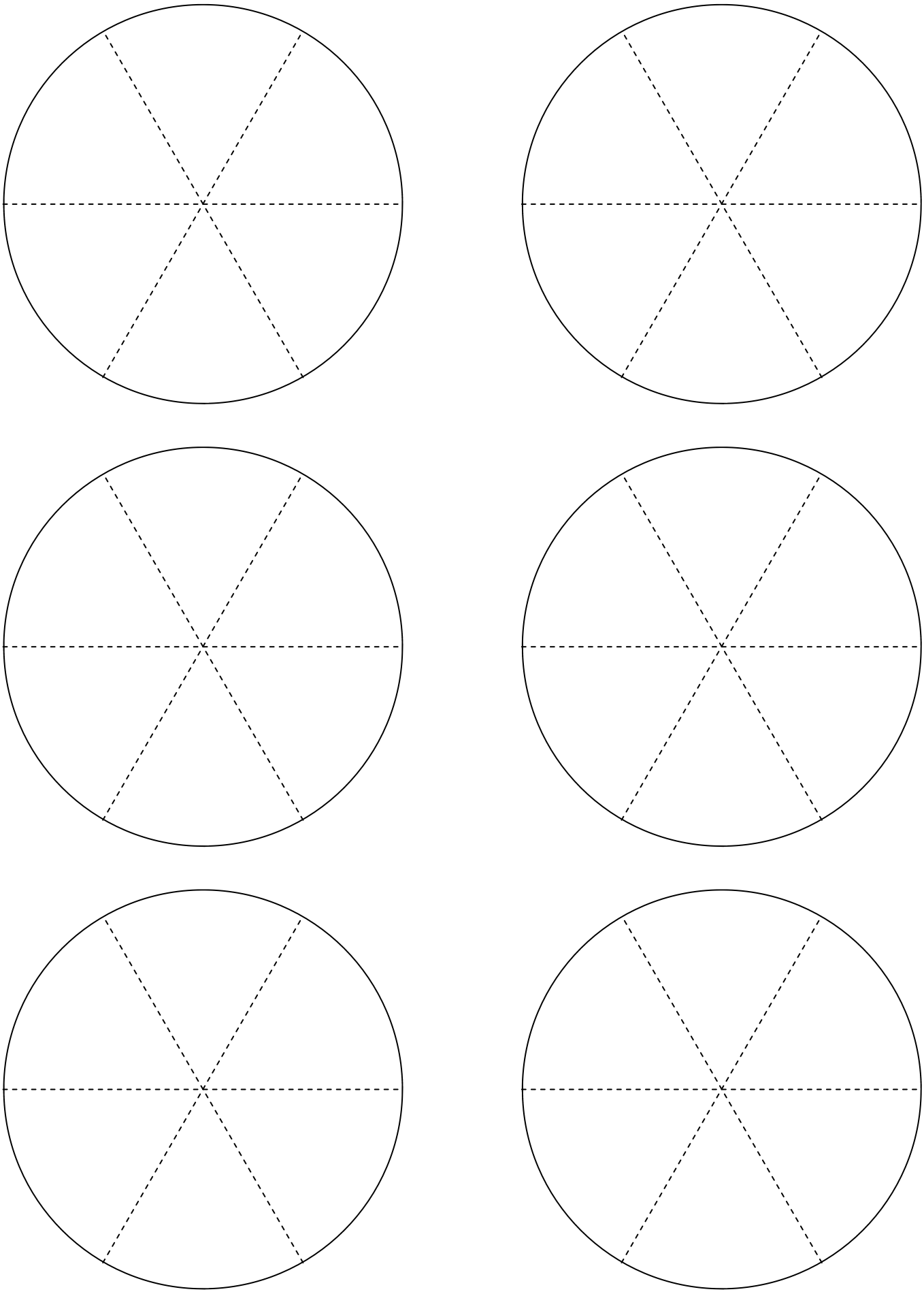

Probability Devices - Circular Spinners 12 sectors

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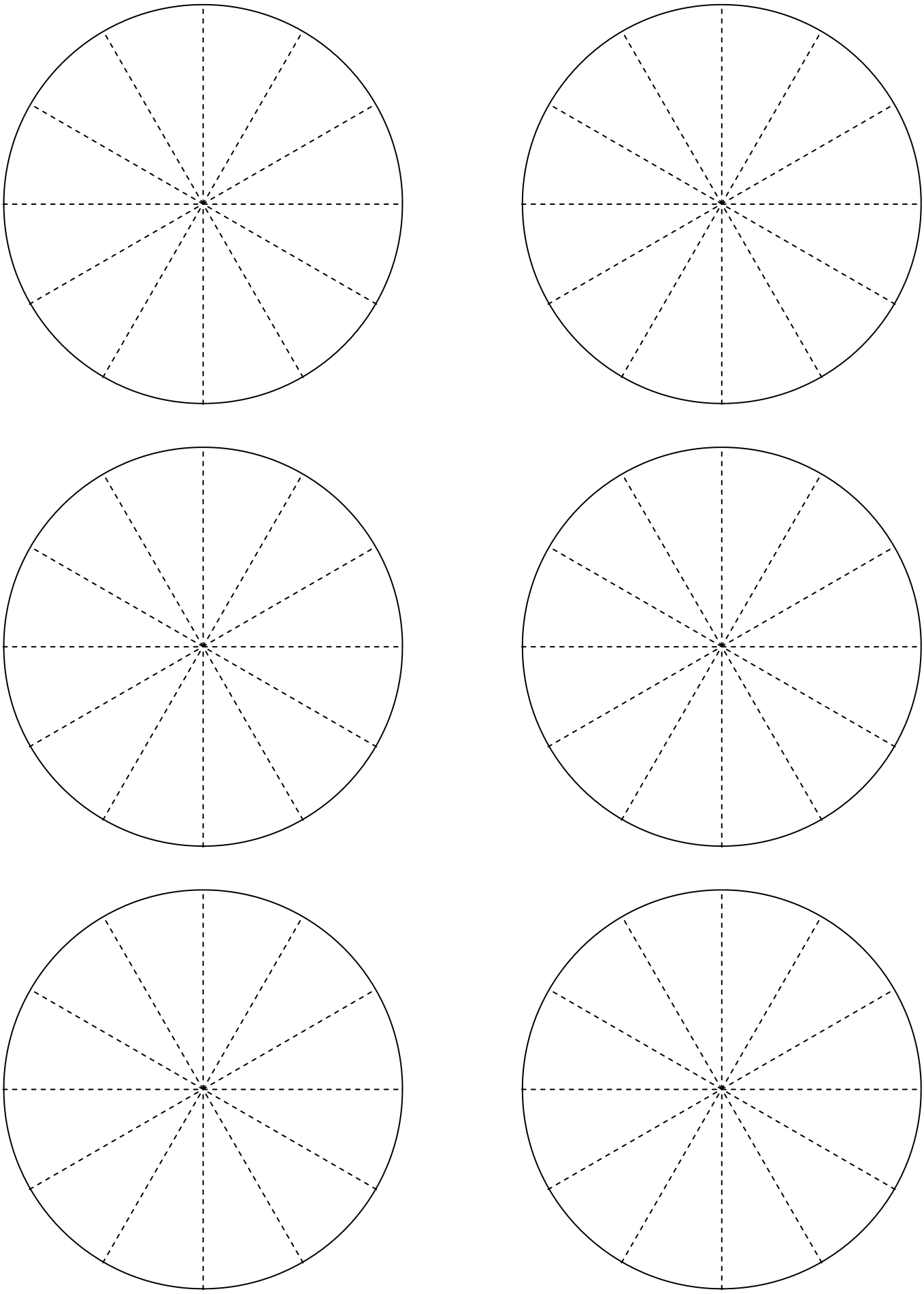

Probability Devices - Number grid

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1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
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31	32	33	34	35	36

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

Probability Devices - Number grid

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1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

Theoretical Probability Review

Sample space is a list of all the possible outcomes for an event. The sample space is normally listed using braces and commas, eg. Sample space of a coin = {H, T} or {Head, Tail}.
So look at all the outcomes and list them. If one outcome could happen twice then list it twice.

Theoretical probability is the theoretical chance of an event occurring it is found by:

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

If the above is a bit confusing then think of it as:

$$P(E) = \frac{\text{How many of what you are after}}{\text{Size of sample space}}$$

Example - Find the probability of rolling an even number with a die.

There are 3 numbers, 2, 4 and 6 are even.

$$P(\text{Even}) = \frac{3}{6} = \frac{1}{2}$$

Simplify if you can. Divide 3 and 6 by 3 and you are left with 1 over 2 (= ½).

Write in the brackets what you are after

There are 6 numbers on a die.

Column 1 starts with finding the sample space. Write 'Sample Space = {' then list all the possibilities, separated by commas. At the top of column 2 are a few questions on simplifying. To answer these put the fraction into your calculator and press '=' or divide the top and bottom (numerator and denominator) by the largest number that goes into both, (HCF).

Column 2 and 3 and all of sheet 2 are all probability questions. Remember to write what you are looking for in the brackets, then write the fraction and simplify if possible.

Theoretical Probability Review (Sheet 1)

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List the sample space for the following. Which is a list of all the possible outcomes

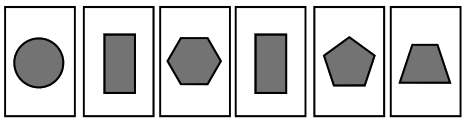
1 A coin.

Sample Space is { ____ }

2 A die.

Sample Space is { ____ }

3 These cards.

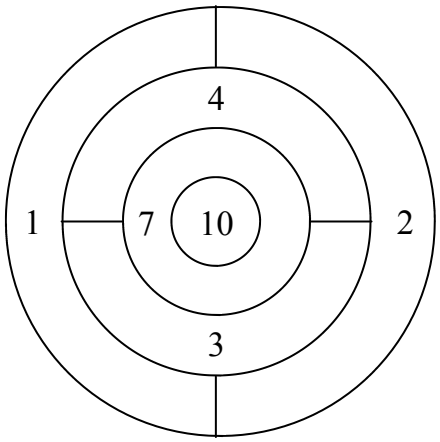


Sample Space is { ____

____

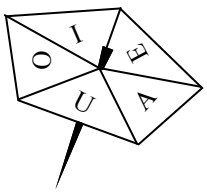
____ }

4 This stickball board



Sample Space is { ____ }

5 This spinner



Sample Space is { ____ }

Use your calculator's fraction key, or your mathematical skills to simplify these fractions.

6 $\frac{2}{8} = \frac{\square}{\square}$ 7 $\frac{10}{15} = \frac{\square}{\square}$

8 $\frac{8}{10} = \frac{\square}{\square}$ 9 $\frac{9}{12} = \frac{\square}{\square}$

10 $\frac{12}{16} = \frac{\square}{\square}$ 11 $\frac{12}{20} = \frac{\square}{\square}$

Find the probability of these events occurring.

Jay's pencil case contains 4 blue pens, 3 red pens, 2 black pens, 2 green pens and a purple pen.

12 How many pens in Jay's pencil case? $\square$

If he reaches in without looking what is the chance of the selected pen being:

13 Green $P(Gr) = \frac{\square}{\square} = \frac{\square}{\square}$ Simplify

14 Red $P(Red) = \frac{\square}{\square} = \frac{\square}{\square}$

15 Red or Purple

$P(R,P) = \frac{\square}{\square} = \frac{\square}{\square}$

16 Any colour but blue

$P(\widetilde{Blue}) = \frac{\square}{\square} = \frac{\square}{\square}$

17 A Green or Blue Pen

____ $= \frac{\square}{\square} = \frac{\square}{\square}$

18 An orange pen

____ $= \frac{\square}{\square} = \square$

A bag has 8 discs inside. The discs are labeled 1 through to 8. If one disc is randomly selected:

19 Find P (Even)

____ $= \frac{\square}{\square} = \frac{\square}{\square}$

20 Find P (< 7)

____ $= \frac{\square}{\square} = \frac{\square}{\square}$

21 Find P (Factor of 12)

Factor means what numbers go into 12

____ $= \frac{\square}{\square}$

22 Find P (Prime number)

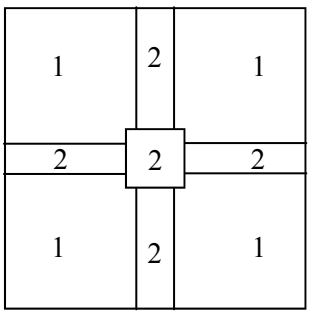
A prime no. has no factors other than 1 & itself
Note: 1 is considered not prime or composite

____ $= \frac{\square}{\square} = \frac{\square}{\square}$

23 Find P (>4)

____ $= \frac{\square}{\square} = \frac{\square}{\square}$

A sideshow has a large dartboard displayed (below). If you land the dart on a 2, you double your money. Get a 1 and you lose your money.
There are more 2's than 1's, so there is more chance of winning than losing.
Discuss this last sentence.



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Theoretical Probability Review (Sheet 2)

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A bag contains the letters that spell P-R-O-B-A-B-I-L-I-T-Y.

If a student randomly selects a single letter, what is the chance it is:

1 An 'L'. P(L) = $\frac{\square}{\square}$

2 An 'I'. P(I) = $\frac{\square}{\square}$

3 An 'I' or 'R'. P(I,R) = $\frac{\square}{\square}$

4 A vowel. P(vowel) = $\frac{\square}{\square}$

5 Not an 'I' or 'B'. $P(\widetilde{I,B}) = \frac{\square}{\square}$

If 25 ping-pong balls are labeled 1 to 25, find the chance of selecting:

6 A '25'. P(25) = $\frac{\square}{\square}$

7 A '19'. P(19) = $\frac{\square}{\square}$

8 A ball with a '5' digit. P(5 digit) = $\frac{\square}{\square}$

9 A ball with a two-digit number, greater than 18.

_____ = $\frac{\square}{\square}$

10 A ball without a '2' in the number.

_____ = $\frac{\square}{\square}$

11 A ball with a '1' in the number.

_____ = $\frac{\square}{\square}$

A lottery has 100 tickets, numbered 1 to 100. There is a single prize awarded. Find the probability of:

12 Buying a single ticket and winning the prize.

_____ = $\frac{\square}{\square}$

13 Buying 20 tickets and winning the prize.

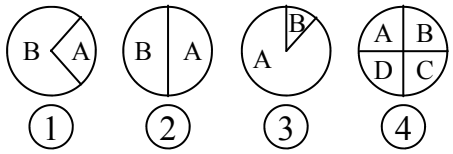
_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

14 Buying 15 tickets and not winning the prize.

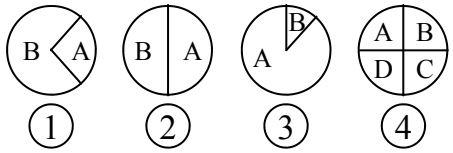
_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

Which spinner is most likely to generate the following results? Fill in the circle of your choice.

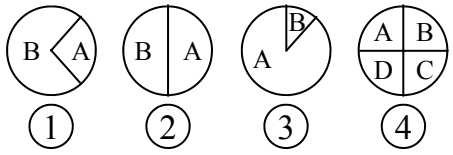
15 10 A's and 2 B's



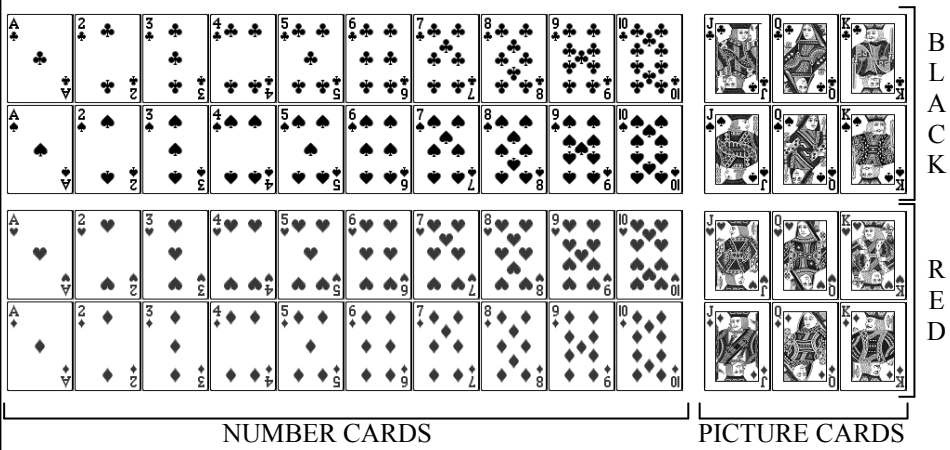
16 20 A's and 21 B's



17 5 A's and 15 B's



A deck of cards has 4 suits. Two suits are black, two are red. Each suit has 10 number cards: Ace through to 10, and 3 picture cards: Jack, Queen and King.



If one card from a standard deck is chosen, find the probability of selecting:

18 A queen

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

19 A red picture card

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

20 A number card

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

21 A 5, 8 or a jack.

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

22 A number card less than 7

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

23 Any card but a red king

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

Theoretical Probability Review (Sheet 1)

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List the sample space for the following. Which is a list of all the possible outcomes

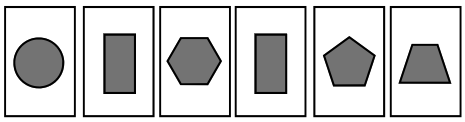
1 A coin.

Sample Space is { H, T. }

2 A die.

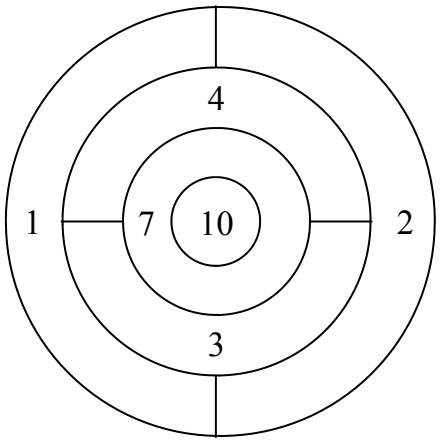
Sample Space is { 1, 2, 3, 4, 5, 6. }

3 These cards.



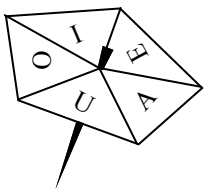
Sample Space is { Circle, rectangle, hexagon, rectangle, pentagon, trapezium. }

4 This stickball board



Sample Space is { 1, 2, 3, 4, 7, 10. }

5 This spinner



Sample Space is { A, E, I, O, U. }

Use your calculator's fraction key, or your mathematical skills to simplify these fractions.

6 $\frac{2}{8} = \frac{1}{4}$

7 $\frac{10}{15} = \frac{2}{3}$

8 $\frac{8}{10} = \frac{4}{5}$

9 $\frac{9}{12} = \frac{3}{4}$

10 $\frac{12}{16} = \frac{3}{4}$

11 $\frac{12}{20} = \frac{3}{5}$

Find the probability of these events occurring.

Jay's pencil case contains 4 blue pens, 3 red pens, 2 black pens, 2 green pens and a purple pen.

12 How many pens in Jay's pencil case? 12

If he reaches in without looking what is the chance of the selected pen being:

13 Green $P(Gr) = \frac{2}{12} = \frac{1}{6}$ Simplify

14 Red $P(Red) = \frac{3}{12} = \frac{1}{4}$

15 Red or Purple

$P(R,P) = \frac{4}{12} = \frac{1}{3}$

16 Any colour but blue

$P(\widetilde{Blue}) = \frac{8}{12} = \frac{2}{3}$

17 A Green or Blue Pen

$P(Gr,Blue) = \frac{6}{12} = \frac{1}{2}$

18 An orange pen

$P(Orange) = \frac{0}{12} = 0$

A bag has 8 discs inside. The discs are labeled 1 through to 8. If one disc is randomly selected:

19 Find P (Even)

$P(Even) = \frac{4}{8} = \frac{1}{2}$

20 Find P (< 7)

$P(<7) = \frac{6}{8} = \frac{3}{4}$

21 Find P (Factor of 12)

Factor means what numbers go into 12

$P(\text{Factor of 12}) = \frac{5}{8}$

22 Find P (Prime number)

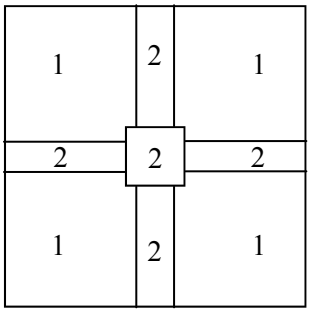
A prime no. has no factors other than 1 & itself
Note: 1 is considered not prime or composite

$P(\text{Prime}) = \frac{4}{8} = \frac{1}{2}$

23 Find P (>4)

$P(>4) = \frac{4}{8} = \frac{1}{2}$

A sideshow has a large dartboard displayed (below). If you land the dart on a 2, you double your money. Get a 1 and you lose your money.
There are more 2's than 1's, so there is more chance of winning than losing.
Discuss this last sentence.



While there are more '2' areas, the combined area of the '2' areas is far less than '1' areas. So it is a false statement.

Theoretical Probability Review (Sheet 2)

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A bag contains the letters that spell P-R-O-B-A-B-I-L-I-T-Y.

If a student randomly selects a single letter, what is the chance it is:

1 An 'L'. $P(L) = \frac{1}{11}$

2 An 'I'. $P(I) = \frac{2}{11}$

3 An 'I' or 'R'. $P(I,R) = \frac{3}{11}$

4 A vowel. $P(\text{vowel}) = \frac{4}{11}$

5 Not an 'I' or 'B'. $P(\widetilde{I,B}) = \frac{4}{11}$

If 25 ping-pong balls are labeled 1 to 25, find the chance of selecting:

6 A '25'. $P(25) = \frac{1}{25}$

7 A '19'. $P(19) = \frac{19}{25}$

8 A ball with a '5' digit. $P(5\text{ digit}) = \frac{3}{25}$

9 A ball with a two-digit number, greater than 18. $P(> 18) = \frac{7}{25}$

10 A ball without a '2' in the number. $P(\text{No 2}) = \frac{17}{25}$

11 A ball with a '1' in the number. $P(1\text{ digit}) = \frac{12}{25}$

A lottery has 100 tickets, numbered 1 to 100. There is a single prize awarded. Find the probability of:

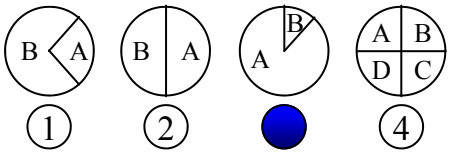
12 Buying a single ticket and winning the prize. $P(\text{Prize}) = \frac{1}{100}$

13 Buying 20 tickets and winning the prize. $P(\text{Prize}) = \frac{20}{100} = \frac{1}{5}$

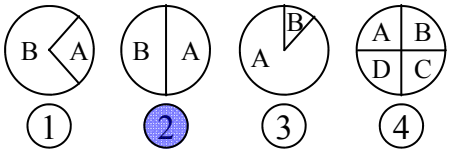
14 Buying 15 tickets and not winning the prize. $P(\text{No Prize}) = \frac{85}{100} = \frac{17}{20}$

Which spinner is most likely to generate the following results? Fill in the circle of your choice.

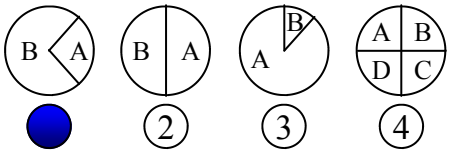
15 10 A's and 2 B's



16 20 A's and 21 B's



17 5 A's and 15 B's



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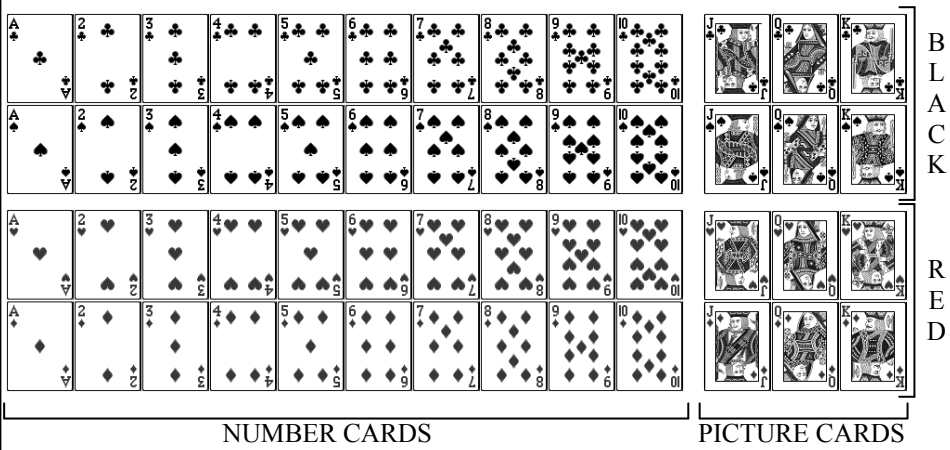

If one card from a standard deck is chosen, find the probability of selecting:

18 A queen $P(\text{Queen}) = \frac{4}{52} = \frac{1}{13}$

19 A red picture card $P(\text{R.Pic}) = \frac{6}{52} = \frac{3}{26}$

20 A number card $P(\text{Num}) = \frac{40}{52} = \frac{10}{13}$

21 A 5, 8 or a jack. $P(5,8,J) = \frac{12}{52} = \frac{3}{13}$

22 A number card less than 7 $P(\text{Num}<7) = \frac{24}{52} = \frac{6}{13}$

23 Any card but a red king $P(\widetilde{\text{R.King}}) = \frac{50}{52} = \frac{25}{26}$

Probability - Theoretical Probability (Homework Strip)

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Date Due: _____

A bag contains 6 Blue marbles, 2 Red marbles and 10 Yellow marbles. If a single marble is randomly selected, find the following probabilities.

1. P(Yellow) = $\frac{\square}{\square} = \frac{\square}{\square}$
2. P(Red) = $\frac{\square}{\square} = \frac{\square}{\square}$
3. P(Red or Blue) = $\frac{\square}{\square} = \frac{\square}{\square}$
4. P(Black) = $\frac{\square}{\square} = \square$
5. P(Not Blue) = $\frac{\square}{\square} = \frac{\square}{\square}$

If a marble was selected and replaced 50 times. How many times would you expect to select a marble that is:

6. Red $\frac{\square}{\square} \times \square = \square$ (Whole Number)
7. Not Blue $\frac{\square}{\square} \times \square = \square$ (Whole Number)

Date Due: _____

A bag contains 6 Blue marbles, 2 Red marbles and 10 Yellow marbles. If a single marble is randomly selected, find the following probabilities.

1. P(Yellow) = $\frac{\square}{\square} = \frac{\square}{\square}$
2. P(Red) = $\frac{\square}{\square} = \frac{\square}{\square}$
3. P(Red or Blue) = $\frac{\square}{\square} = \frac{\square}{\square}$
4. P(Black) = $\frac{\square}{\square} = \square$
5. P(Not Blue) = $\frac{\square}{\square} = \frac{\square}{\square}$

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3. P(Red or Blue) = $\frac{\square}{\square} = \frac{\square}{\square}$
4. P(Black) = $\frac{\square}{\square} = \square$
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5. P(Not Blue) = $\frac{\square}{\square} = \frac{\square}{\square}$

If a marble was selected and replaced 50 times. How many times would you expect to select a marble that is:

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Probability - Theoretical Probability (Homework Strip)

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Date Due:

A bag contains 6 Blue marbles, 2 Red marbles and 10 Yellow marbles. If a single marble is randomly selected, find the following probabilities.

1 P(Yellow) = $\frac{10}{18} = \frac{5}{9}$

2 P(Red) = $\frac{2}{18} = \frac{1}{9}$

3 P(Red or Blue) = $\frac{8}{18} = \frac{4}{9}$

4 P(Black) = $\frac{0}{18} = 0$

5 P(Not Blue) = $\frac{12}{18} = \frac{2}{3}$

If a marble was selected and replaced 50 times. How many times would you expect to select a marble that is:

6 Red $\frac{2}{18} \times 50 = 6$ Whole Number

7 Not Blue $\frac{12}{18} \times 50 = 33$ Whole Number

Probability Experiment Sheet

This sheet is designed to compare experimental results with theoretical results. There are two sheets as the first sheet has the "Sum of two Dice" grid left blank, the second grid has the numbers filled in, to avoid mistakes if this is an issue. As this is an experiment sheet there are no answers.

Column 1 has a table for recording the tossing of two coins. This requires either 2 different coins so that one is called 'Coin 1' and the other 'Coin 2', or use a single coin and toss it twice. The table has a tally and frequency column to complete, find $\sum f$ at the base of the column to verify the correct number of tosses has occurred. The data can then be transferred to the column graph below the table which has the theoretical results already graphed.

Compare the results and describe how your results related to the theoretical, indicate how the sample space size (the number of times the coins are tossed) affects the results.

Column 2 requires you to roll two dice 72 times. The dice can be rolled at the same time. Once rolled, add the two upper face numbers together to get a total and record the sum in the table using the tally column, converting to a frequency on completion. Again find the two $\sum f$'s and add them to verify that you have rolled the dice 72 times.

Now complete the theoretical probability grid (if blank) by adding the two numbers. You should notice a pattern along the diagonals. Plot the experimental result, then plot the theoretical result by counting all the same numbers in the grid, but because you rolled the dice 72 times then you have to double this number. Use different coloured columns for experimental and theoretical results.

Compare the results and describe how your results related to the theoretical, indicate how the sample space size (the number of times the dice are rolled) affects the results.

Probability Experiment Sheet

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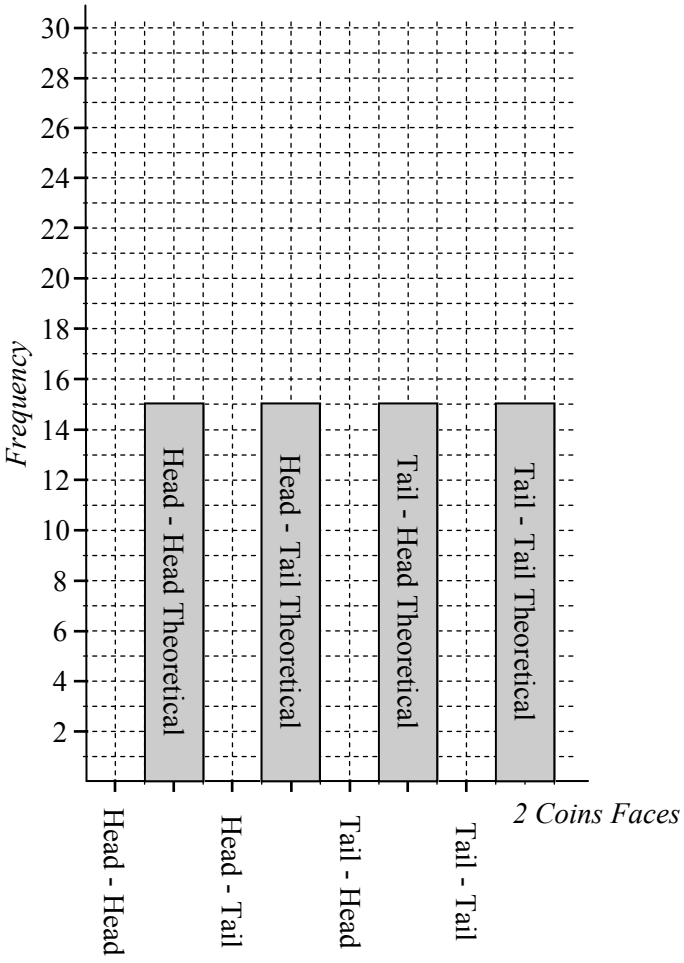
Toss two coins 60 times and record the results below. Toss one coin first then the second, or use two different value coins and call one Coin 1 and the other Coin 2 and toss them together.

Coin 1	Coin 2	Tally	*f*
Head	Head		
Head	Tail		
Tail	Head		
Tail	Tail		
			$\Sigma f =$

Column Graph: Two Tossed Coins Theoretical and Experimental



Explain how the experimental results compared with the theoretical results. Was the sample space large enough to get the expected result?

Now dice. Roll two dice 72 times adding the two face numbers together to get a total. Complete the table and grid, refer to grid to graph theoretical results. Compare the results with the theoretical.

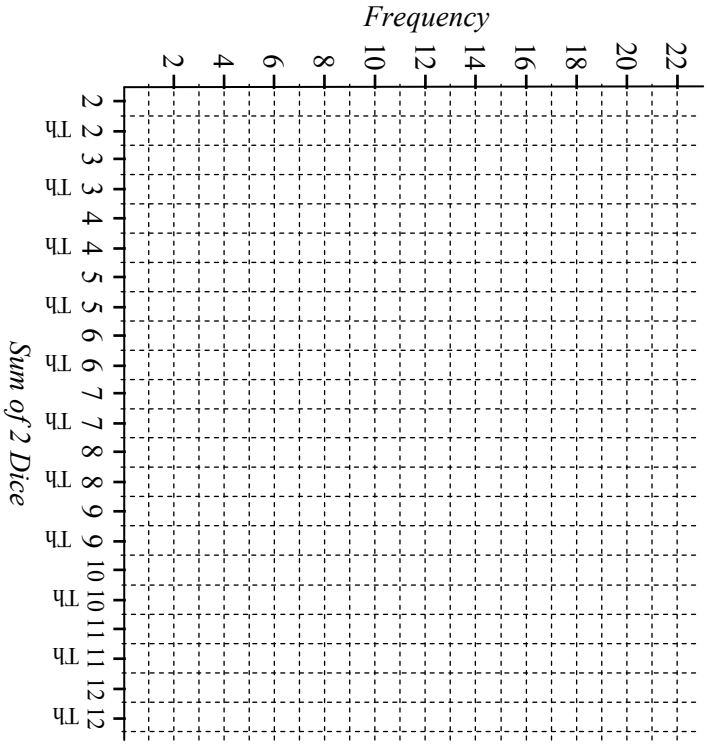
Total	Tally	*f*
2		
3		
4		
5		
6		
	$\Sigma f =$	

Total	Tally	*f*
7		
8		
9		
10		
11		
12		
	$\Sigma f =$	

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Theoretical Probability Grid

This grid has 36 outcomes, so with 72 rolls, double the frequency of each number in the grid and use that as your expected theoretical frequency for the columns below.



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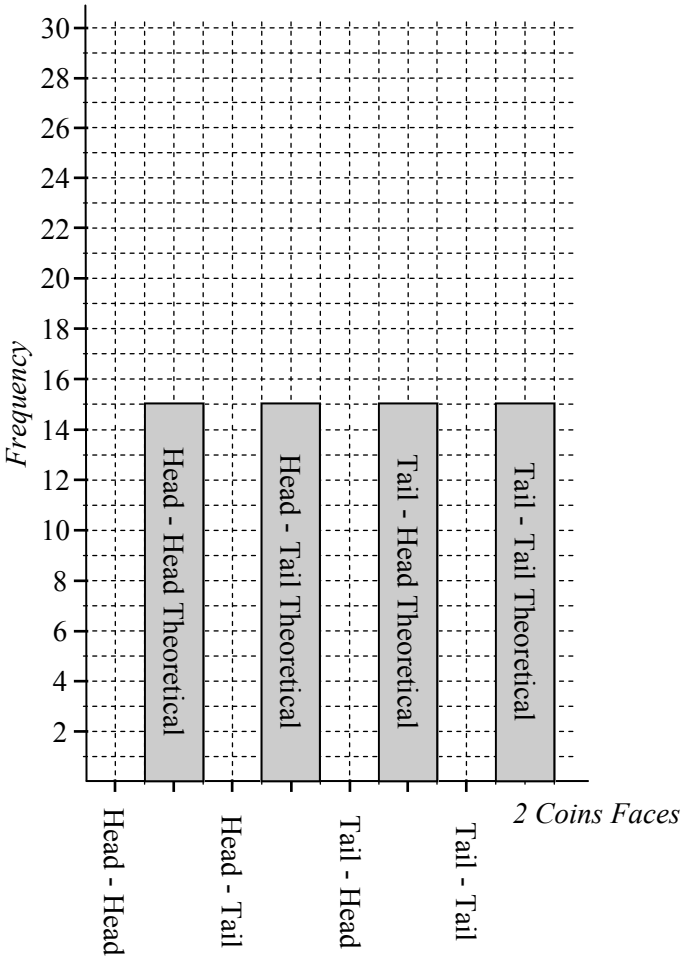
Head	Head		
Head	Tail		
Tail	Head		
Tail	Tail		
			$\Sigma f =$

Column Graph: Two Tossed Coins Theoretical and Experimental



Explain how the experimental results compared with the theoretical results. Was the sample space large enough to get the expected result?

Now dice. Roll two dice 72 times adding the two face numbers together to get a total. Complete the table then refer to grid to graph the theoretical results. Compare the results with the theoretical.

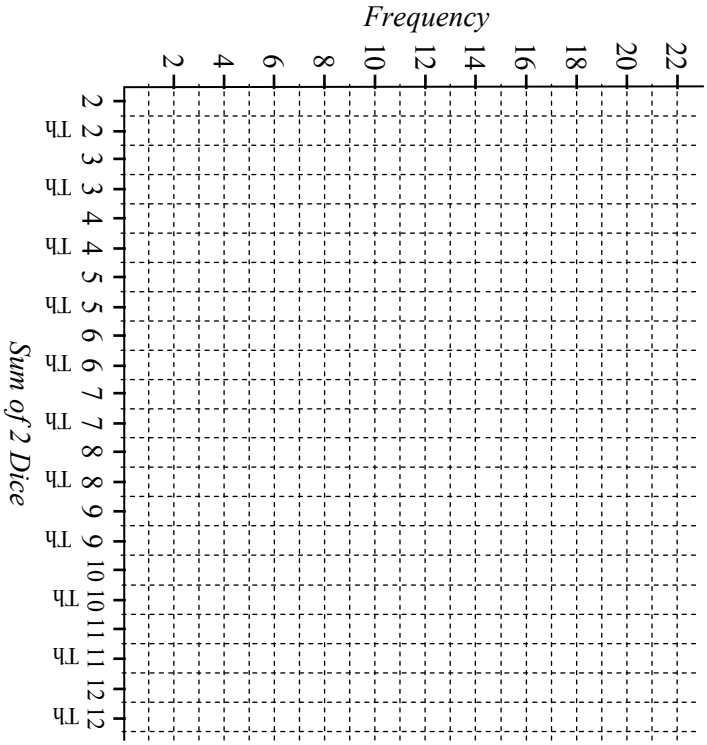
Total	Tally	*f*
2		
3		
4		
5		
6		
	$\Sigma f =$	

Total	Tally	*f*
7		
8		
9		
10		
11		
12		
	$\Sigma f =$	

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Theoretical Probability Grid

This grid has 36 outcomes, so with 72 rolls, double the frequency of each number in the grid and use that as your expected theoretical frequency for the columns below.



Explain how the experimental results compared with the theoretical results.

Generating Random Numbers

A calculator can be used to generate decimals and whole numbers. You can use this function to simulate real world situations such as maths class experiments like rolling a die or tossing a coin or they applied in processes like selecting a team in a sports draw. In maths, it is important to understand the relationship with sample size, the more data you have the more the data will behave like theory. The less sample size, the more unpredictable the result.

The first two sheets ask you to generate decimals, the key strokes required are on the sheet, these are for a Casio FX-82AU Plus. If you have a different calculator, the keys required may not be the same.

Sheet 1 asks you to generate decimals in two sample sizes. The first is 5 decimals, the second is 50 decimals, so 10 times larger. By graphing these you should see that the larger the data, the more it tends to theoretical. This may not be the case! You may find the data is more Balanced from 5 samples. But if you look at other class member's results, the 5 samples will vary wildly, but the 50 will show a greater similarity.

Sheet 2 is also about comparing results with increasing sample size. But this time instead of comparing a graph of the results, you are comparing them mathematically. By finding the mean of the data and seeing which is closest to 0.5, you can see if the data tends toward the mean with more samples. Again, this is an experiment and answers will vary. Look at your class members results and see that the answer will change greatly for the 5 number samples, but be similar for the largest data set.

Sheet 3 asks you to generate whole numbers (integers) to represent a coin and a die. By graphing your results on a column graph with a proportioned scale, you can compare results to check whether your data is getting closer to theoretical results with more iterations.

Generating Random Numbers - Decimals (Sheet 1)

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Use your calculator to generate random decimals.

Simulation 1

1 Generate 10 decimals (3 d.p.)

Calc: SHIFT . =
For the 1st decimal

Calc: =
For the next decimal

_____, _____, _____, _____, _____, _____, _____, _____, _____, _____.

Simulation 2

2 Generate 50 decimals (3 d.p.)

_____, _____, _____, _____, _____, _____, _____, _____, _____, _

Generating Random Numbers - Decimals (Sheet 2)

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Use your calculator to generate random decimals, find the mean.

, , , , .

Total the 5 scores

Calculate the mean:

$\bar{x} = \frac{\square}{5} = \square$ (3 d.p.)

, , , , , , , , , , , , , , .

Total these 15 scores

$\bar{x} = \frac{\square}{15} = \square$ (3 d.p.)

, , , , , , , , , , , , , ,

Generating Random Numbers - Integers and Intervals (Sheet 3)

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Use your calculator to generate random numbers, to represent a tossed coin, a 1 = tail, a 2 = head

1 Generate 4 coin results.

Calc: ALPHA . 1 SHIFT) 2) =
For the 1st integer from 1 to 2

Calc: =
For the next integer

Write 'H' or 'T'

____, ____, ____, ____.
Write 'H' or 'T', NOT '1' OR '2'

2 Generate 10 coin results.

____, ____, ____, ____, ____,
____, ____, ____, ____, ____.

3 Generate 36 coin results.

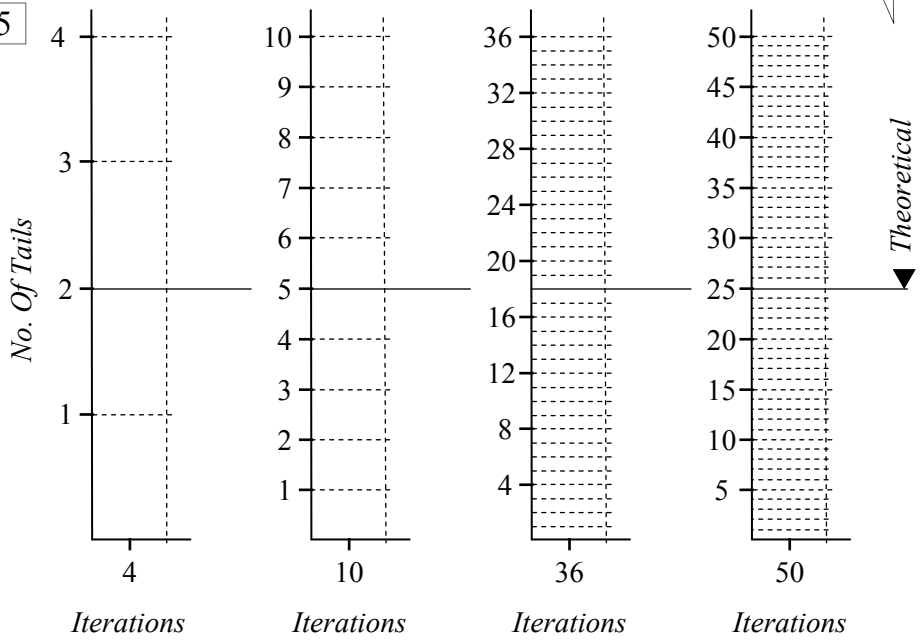
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____.

4 Count the number of 'tails' results you recorded and display them in the table below.

Iterations	*No. of Tails*
4	
10	
36	
50	

Graph your results for question 4 on the grid, just a single column for each. All axes are scaled to the same height so you can compare them.

5



6 Compare the results. _____

Use your calculator to generate random numbers, to represent a die.

Calc: ALPHA . 1 SHIFT) 6) =
For the 1st integer from 1 to 6

Write '1' or '2' etc. up to 6

Calc: =
For the next integer

7 Generate 6 die results. ____, ____, ____, ____, ____, ____.

8 Generate 18 die results. ____, ____, ____, ____, ____, ____, ____,
____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____.

9 Generate 30 die results. ____, ____, ____, ____, ____, ____, ____,
____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____,
____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____.

If you rolled a die 6 times and achieved theoretical probability, you get 1 of each number. See if experimental probability gets closer to theoretical with more rolls! Complete the table using your experimental data above.

Iterations	1*'s*	2*'s*	3*'s*	4*'s*	5*'s*	6*'s*	*Theoretical*
6							1
18							3
30							5
54							9

Generating Random Numbers - Decimals (Sheet 1)

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Use your calculator to generate random decimals.

Simulation 1

1 Generate 10 decimals (3 d.p.)

Calc: SHIFT . =
For the 1st decimal

Calc: =
For the next decimal

0.530,

0.342,	0.086,	0.183,
0.763,	0.523,	0.962,
0.726,	0.371,	0.339.

Simulation 2

2 Generate 50 decimals (3 d.p.)

0.009,	0.222,	0.871,
0.687,	0.140,	0.260,
0.791,	0.825,	0.758.
0.338,	0.247,	0.620,
0.011,	0.649,	0.512,
0.873,	0.962,	0.665.
0.794,	0.174,	0.123,
0.660,	0.079,	0.574,
0.658,	0.120,	0.569.
0.594,	0.763,	0.262,
0.464,	0.623,	0.601,
0.404,	0.314,	0.553.
0.483,	0.380,	0.198,
0.943,	0.279,	0.406,
0.061,	0.772,	0.015.
0.580,	0.039,	0.091,
0.703,	0.698.	

The more data used in experiments, the more the results will expect to approach theoretical probability. One way to show this, is by graphing the results and comparing to theoretical probability.

3 Simulation 1

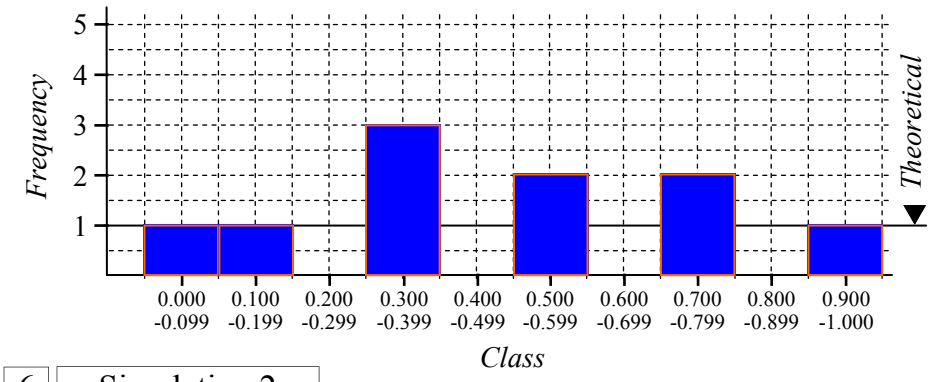
Class	*Tally*	*f*
0.000 - 0.099	|	1
0.100 - 0.199	|	1
0.200 - 0.299		0
0.300 - 0.399	|||	3
0.400 - 0.499		0
0.500 - 0.599	||	2
0.600 - 0.699		0
0.700 - 0.799	||	2
0.800 - 0.899		0
0.900 - 1.000*	|	1

4 Simulation 2

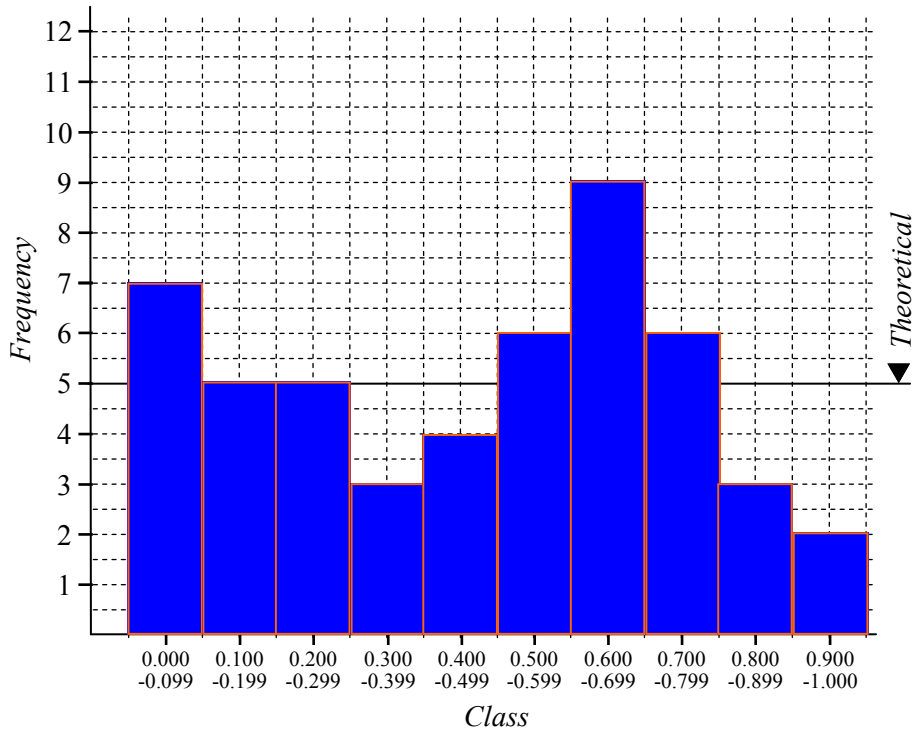
Class	*Tally*	*f*
0.000 - 0.099	𝍸 ||	7
0.100 - 0.199	𝍸	5
0.200 - 0.299	𝍸	5
0.300 - 0.399	|||	3
0.400 - 0.499	||||	4
0.500 - 0.599	𝍸 |	6
0.600 - 0.699	𝍸 ||||	9
0.700 - 0.799	𝍸 |	6
0.800 - 0.899	|||	3
0.900 - 1.000*	||	2

* This class has 101 possible scores compared with 100 in other classes

5 Simulation 1



6 Simulation 2



Generating Random Numbers - Decimals (Sheet 2)

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Use your calculator to generate random decimals, find the mean.

0.096, 0.293, 0.491, 0.511, 0.803.

Total the 5 scores 2.194

Calculate the mean:

$\bar{x} = \frac{2.194}{5} = 0.439$ (3 d.p.)

0.432, 0.749, 0.978, 0.121, 0.725, 0.454, 0.760, 0.740, 0.333, 0.361, 0.97, 0.630, 0.825, 0.608, 0.359.

Total these 15 scores 9.045

$\bar{x} = \frac{9.045}{15} = 0.603$ (3 d.p.)

0.605, 0.185, 0.467, 0.398, 0.032, 0.743, 0.909, 0.439, 0.415, 0.612, 0.005, 0.506, 0.060, 0.718, 0.888, 0.850, 0.327, 0.611, 0.708, 0.269, 0.368, 0.854, 0.684, 0.897, 0.840.

Total : 13.390
25 scores

$\bar{x} = \frac{13.390}{25} = 0.536$ (3 d.p.)

0.385, 0.015, 0.217, 0.200, 0.935, 0.932, 0.225, 0.829, 0.260, 0.754, 0.451, 0.393, 0.959, 0.433, 0.921, 0.747, 0.853, 0.45, 0.242, 0.692, 0.309, 0.492, 0.814, 0.757, 0.850, 0.506, 0.694, 0.34, 0.787, 0.875, 0.585, 0.373, 0.017, 0.423, 0.304.

Total these 35 scores 19.019 $\bar{x} = \frac{19.019}{35} = 0.543$

Now find the mean for all 80 scores. Is it closer to 0.500? If not, remember that this is an experiment! Not theory!

$\sum x = 2.194 + 9.045 + 13.390 + 19.019$

5 score total, 15 score total, 25 score total, 35 score total

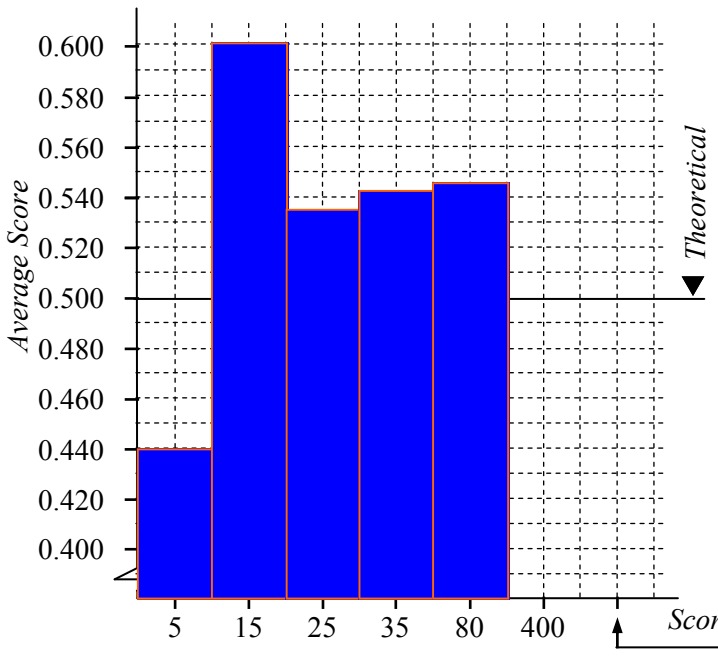
$\sum x = 43.648$ (80 score total) $\bar{x} = \frac{\sum x}{n} = \frac{43.648}{80} = 0.545$

You can further refine your experiment by getting 4 other student's '80 score totals' adding them to yours and finding the mean by ÷ 400. Plot results as a column graph, include a class average if you have one.

$\sum x = \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_ + \_\_\_$

Your 80 total, name

$\sum x = \_\_\_$ (400 score total) $\bar{x} = \frac{\sum x}{n} = \frac{\_\_\_}{\_\_\_} = \_\_\_$



If you have a class average, plot it here

Generating Random Numbers - Integers and Intervals (Sheet 3)

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Use your calculator to generate random numbers, to represent a tossed coin, a 1 = tail, a 2 = head

1 Generate 4 coin results.

Calc: ALPHA . 1 SHIFT) 2) = For the 1st integer from 1 to 2

Calc: = For the next integer

Write 'H' or 'T'

T, T, H, T.

Write 'H' or 'T', NOT '1' OR '2'

2 Generate 10 coin results.

T, H, T, T, T,
H, T, H, H, T.

3 Generate 36 coin results.

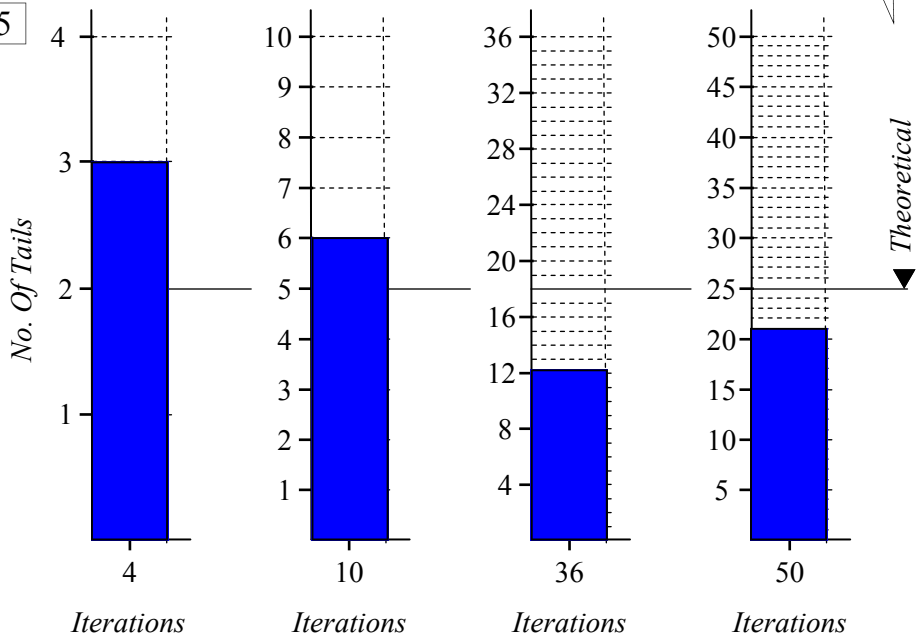
H, T, H, T, H,
H, H, H, T, T,
H, H, T, T, H,
H, H, T, H, H,
H, T, T, T, H,
H, H, H, T, H,
H, H, H, H, T,
H.

4 Count the number of 'tails' results you recorded and display them in the table below.

Iterations	*No. of Tails*
4	3
10	6
36	12
50	21

Graph your results for question 4 on the grid, just a single column for each. All axes are scaled to the same height so you can compare them.

5



6 Compare the results. End result is closest, 36 iterations was a low result dragged total down. Note if 4 iterations was 2 tails?

Use your calculator to generate random numbers, to represent a die.

Calc: ALPHA . 1 SHIFT) 6) = For the 1st integer from 1 to 6

Write '1' or '2' etc. up to 6

Calc: = For the next integer

7 Generate 6 die results. 6, 4, 2, 5, 6, 1.

8 Generate 18 die results. 1, 4, 2, 2, 6, 1, 5,
5, 5, 2, 4, 6, 6, 2, 3, 4, 3, 2.

9 Generate 30 die results. 4, 3, 2, 2, 4, 5, 4,
2, 6, 1, 2, 4, 2, 6, 6, 6, 4, 2,
5, 6, 5, 4, 6, 2, 5, 4, 2, 2, 1, 2.

If you rolled a die 6 times and achieved theoretical probability, you get 1 of each number. See if experimental probability gets closer to theoretical with more rolls! Complete the table using your experimental data above.

Iterations	1's	2's	3's	4's	5's	6's	*Theoretical*
6	1	1	0	1	1	2	1
18	2	5	2	3	3	3	3
30	2	10	1	7	4	6	5
54	5	16	3	11	8	11	9

Relative Frequency

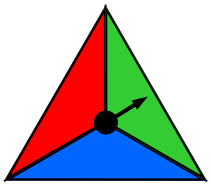
Relative frequency is the same as experimental probability. It is frequently used to get an idea of sales, usage or patronage by taking a sample and then applying the relative frequencies to different models. The larger the sample, the more likely it is to reflect actual numbers, but it is just an estimate regardless of the size of the sample.

To find the relative frequency, total the frequency column, then write the frequency of the target over that total. Here is an example for a three-coloured spinner being spun 120 times.

Example

Face	*f*	*r.f.*
R	38	$\frac{38}{120} = \frac{19}{60}$
G	46	$\frac{46}{120} = \frac{23}{60}$
B	36	$\frac{36}{120} = \frac{3}{10}$
$\Sigma f =$	120	

Step 2:Write the frequency 'over' the total, then simplify.



Step 1:Total the frequency column

From the above it appears that the spinner is biased towards landing on red. This may not be the case, but from the 120 spins the relative frequency suggests that it is.

To use relative frequency, you are usually asked to apply your results to another model.

Example If the above spinner was spun 1 000 times how many times is it likely to land on Green?

$\frac{23}{60} \times 1\,000 = 383$ times

Round to the nearest whole number

Use the green relative frequency

Multiply by the new number of spins

Relative Frequency (Sheet 1)

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Simplify these fractions

1 $\frac{150}{195} = \frac{\square}{\square}$

2 $\frac{171}{285} = \frac{\square}{\square} = \frac{\square}{\square}$

3 $\frac{46}{138}$

Find the relative frequency for the data below. Express your answer as a simplified fraction.

- Total the frequency '*f*' column
- Write the 49, heads 'H' frequency in top box and Σf number in the bottom box
- simplify the fraction.

4 Coin Results

Face	*f*	*r.f.*
H	49	$\frac{\square}{\square} = \frac{\square}{\square}$
T	56	$\frac{\square}{\square} = \frac{\square}{\square}$
$\Sigma f =$		

5 Di Results

Face	*f*	*r.f.*
1	45	$\frac{\square}{\square} = \frac{\square}{\square}$
2	33	$\frac{\square}{\square} = \frac{\square}{\square}$
3	42	$\frac{\square}{\square} = \frac{\square}{\square}$
4	57	$\frac{\square}{\square} = \frac{\square}{\square}$
5	39	$\frac{\square}{\square} = \frac{\square}{\square}$
6	51	$\frac{\square}{\square} = \frac{\square}{\square}$
$\Sigma f =$		

James is recording data for 10 mins of the school parent-teacher evening. He records M=Man, W=Woman, G=Female Student, B=Male student. Put the data into a frequency table and calculate the relative frequencies.

6 Data:

WWMWB	MWGWM	WBWGM	BBMWW	
WMBWM	WMGGM	MMGMW	WWBMG	WMGMG
MMWWW	WMBWM	WMMWG	WMMWG	WWMMW

Type	*Tally*	*f*	*r.f.*
W	----------		$\frac{\square}{\square} = \frac{\square}{\square}$
M	----------		$\frac{\square}{\square} = \frac{\square}{\square}$
B			$\frac{\square}{\square} = \frac{\square}{\square}$
G			$\frac{\square}{\square} = \frac{\square}{\square}$
		$\Sigma f =$	

Deb works in a childcare centre, she asks parents how many children are in their family. Place the data into a frequency table and calculate the relative frequencies.

7 Data:

1, 2, 2, 1, 3,	2, 1, 1, 2, 1,	1, 3, 1, 2, 1	1, 2, 1, 2, 3,	2, 1, 1, 2, 4,
1, 4, 2, 2, 1,	1, 1, 1, 2, 1,	2, 3, 2, 1, 1,	1, 4, 1, 3, 1,	1, 2, 2, 2, 1,
2, 2, 1, 2, 1,	2, 1, 1, 3, 1,	1, 2, 1, 2, 1,	1, 3, 2, 2, 2,	2, 4, 2, 1, 2,
3, 2, 1, 1, 2,	3, 2, 3, 2, 2,	1, 4, 1, 2, 3,	2, 3, 1, 1, 2,	1, 1, 2, 1, 1,

Children	*Tally*	*f*	*r.f.*
	----------		$\frac{\square}{\square} = \frac{\square}{\square}$
	----------		$\frac{\square}{\square} = \frac{\square}{\square}$
			$\frac{\square}{\square} = \frac{\square}{\square}$
			$\frac{\square}{\square} = \frac{\square}{\square}$
		$\Sigma f =$	

Relative Frequency (Sheet 2)

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Find the relative frequency for the data for sales at Dave's Electrical. Then answer the questions.

Product	*f*	*r.f.*
Toaster	15	$\frac{\square}{\square} = \frac{\square}{\square}$
Kettle	18	$\frac{\square}{\square} = \frac{\square}{\square}$
Phone	12	$\frac{\square}{\square} = \frac{\square}{\square}$
PC/ Tablet	8	$\frac{\square}{\square} = \frac{\square}{\square}$
Other	17	$\frac{\square}{\square}$
$\Sigma f =$		

Based on the sales figures above apply relative frequencies to estimate sales. If 350 products were sold, what would be the sales of:

1 Toasters?

$\frac{\square}{\square} \times \square = \square$

2 Phones?

$\frac{\square}{\square} \times \square = \square$

3 Kettles or Toasters?

Add the fractions for kettles and toasters first

$\frac{\square}{\square} \times \square = \square$

4 Everything but toasters?

Subtract the toasters fraction from 1

$\frac{\square}{\square} \times \square = \square$

5 PCs, Tablets or phones?

$\frac{\square}{\square} \times \square = \square$

Find the relative frequency for the daily sales at Pete's Pies. Then answer the questions.

Product	*f*	*r.f.*
Pies	95	
Sausage Rolls	66	
Pastries	125	
Bread	106	
Other	48	
$\Sigma f =$		

Based on the sales figures above apply relative frequencies to estimate sales. Estimate the number of pies sold if total sales were:

6 500?

$\frac{\square}{\square} \times \square = \square$

Round to nearest pie!

7 800?

$\frac{\square}{\square} \times \square = \square$

8 300?

$\frac{\square}{\square} \times \square = \square$

Pete has run a promotion and estimates sales will be 1 300 items on the day. Estimate the number of:

9 Pies that Pete should bake.

10 Products other than bread he should bake.

Write the relative frequencies as decimals. This time it is weekly car sales. No need to round.

Car Type	*f*	*r.f.*
Family	6	0.15
Hatch	5	
4WD	16	
Hybrid	2	
Sports	4	
Wagon	3	
Other	4	
$\Sigma f =$		

Multiply relative frequencies by 100 to answer these questions.
What percentage of sales were:

11 Sports cars?

12 Family cars or wagons?

13 Not a hatchback?

14 Hatchbacks or hybrids?

If a person entered the sales area, what three types of vehicle are they most likely to buy? List them in order of popularity, most popular first.

15

The showroom floor has room for 5 cars. If the showroom cars are to reflect the above sales figures, how many:

16 4WDs should be displayed?

Relative Frequency (Sheet 3)

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A bag contains red, green and blue marbles. Apply your relative frequency knowledge.

A marble is selected and replaced from the bag 20 times, the results are:
Red 8, Green 10 and Blue 2.

1 Are there more red marbles than blue? ○ Yes ○ No ○ Can't tell

2 Are there 20 marbles in the bag? ○ Yes ○ No ○ Can't tell

3 Is it possible that there are more blue marbles than green or red? ○ Yes ○ No

4 Are there definitely more green marbles than blue marbles? ○ Yes ○ No

A computer monitor manufacturer sorts its products as follows:
Class A - Zero Dead Pixels
Class B - 1 to 2 Dead Pixels
Class C - More than 2 Dead Pixels
After taking samples it is found:
Zero Dead Pixels have *r.f.* of 0.87
One Dead Pixel have an *r.f.* of 0.08
Two Dead Pixels have an *r.f.* of 0.03

5 What is the *r.f.* for a monitor to be 'Class A'.

P(Class A) = ☐

6 Calculate the *r.f.* for a monitor to be 'Class B'.

7 Calculate the *r.f.* for a monitor to be 'Class C'.

If 800 monitors are produced in a day, how many would be:

8 'Class A' monitors?

9 'Class B' monitors?

10 'Class C' monitors?

Monitor wholesale prices are:
Class A - $350
Class B - $185
Class C - $55

11 Calculate the income for the 800 monitors.

A fire station has analysed its response times. They are:
Arrival Under 5 mins: 77%
Arrival within 5 to 10 mins: 20%
Arrival more than 10 mins: 3%

12 Convert the percentages to decimals.

77% = ☐ 20% = ☐

3% = ☐

If the station responds to an average of 56 calls per year, calculate (nearest whole number) the number of arrivals:

13 In under 5 minutes

14 In 10 minutes or less.

15 Within 5 to 10 minutes.

16 In 5 minutes or more.

The Ginormous Egg Company only sells eggs larger than 80 g. It sells the smaller eggs to wholesalers.
Its production statistics are:
Saleable eggs 80 g and over: 0.67
Wholesale eggs less than 80 g: 0.27
All other eggs are discarded.

17 Calculate the *r.f.* for an egg to be discarded.

If production averages 17 000 eggs per day, calculate the number of:

18 Daily wholesaled eggs.

19 Weekly discarded eggs.

20 Yearly company-sold eggs.

Relative Frequency (Sheet 1)

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Simplify these fractions

1 $\frac{150}{195} = \frac{10}{13}$

2 $\frac{171}{285} = \frac{3}{5} = \frac{1}{3}$

3 $\frac{46}{138}$

Find the relative frequency for the data below. Express your answer as a simplified fraction.

- Total the frequency '*f*' column
- Write the 49, heads 'H' frequency in top box and Σf number in the bottom box
- simplify the fraction.

4 Coin Results

Face	*f*	*r.f.*
H	49	$\frac{49}{105} = \frac{7}{15}$
T	56	$\frac{56}{105} = \frac{8}{15}$
$\Sigma f =$	105	

5 Di Results

Face	*f*	*r.f.*
1	45	$\frac{45}{267} = \frac{15}{89}$
2	33	$\frac{33}{267} = \frac{11}{89}$
3	42	$\frac{42}{267} = \frac{14}{89}$
4	57	$\frac{57}{267} = \frac{19}{89}$
5	39	$\frac{39}{267} = \frac{13}{89}$
6	51	$\frac{51}{267} = \frac{17}{89}$
$\Sigma f =$	267	

James is recording data for 10 mins of the school parent-teacher evening. He records M=Man, W=Woman, G=Female Student, B=Male student. Put the data into a frequency table and calculate the relative frequencies.

6 Data:

WWMWB MWGWM WBWGM BBMWW
WMBWM WMGGM MMGMW WWBMG WMGMG
MMWWW WMBWM WMMWG WMMWG WWMMW

Type	*Tally*	*f*	*r.f.*
W	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 | | |	28	$\frac{28}{70} = \frac{2}{5}$
M	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	25	$\frac{25}{70} = \frac{5}{14}$
B	𝍸 | |	7	$\frac{7}{70} = \frac{1}{10}$
G	𝍸 𝍸	10	$\frac{10}{70} = \frac{1}{7}$
	$\Sigma f =$	70	

Deb works in a childcare centre, she asks parents how many children are in their family. Place the data into a frequency table and calculate the relative frequencies.

7 Data:

1, 2, 2, 1, 3, 2, 1, 1, 2, 1, 1, 3, 1, 2, 1 1, 2, 1, 2, 3, 2, 1, 1, 2, 4,
1, 4, 2, 2, 1, 1, 1, 1, 2, 1, 2, 3, 2, 1, 1, 1, 4, 1, 3, 1, 1, 2, 2, 2, 1,
2, 2, 1, 2, 1, 2, 1, 1, 3, 1, 1, 2, 1, 2, 1, 1, 3, 2, 2, 2, 2, 4, 2, 1, 2,
3, 2, 1, 1, 2, 3, 2, 3, 2, 2, 1, 4, 1, 2, 3, 2, 3, 1, 1, 2, 1, 1, 2, 1, 1,

Children	*Tally*	*f*	*r.f.*
1	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	45	$\frac{45}{100} = \frac{9}{20}$
2	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 | | |	38	$\frac{38}{100} = \frac{19}{50}$
3	𝍸 𝍸 | |	12	$\frac{12}{100} = \frac{3}{25}$
4	𝍸	5	$\frac{5}{100} = \frac{1}{20}$
	$\Sigma f =$		

Relative Frequency (Sheet 2)

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Find the relative frequency for the data for sales at Dave's Electrical. Then answer the questions.

Product	*f*	*r.f.*
Toaster	15	$\frac{15}{70} = \frac{3}{14}$
Kettle	18	$\frac{18}{70} = \frac{9}{35}$
Phone	12	$\frac{12}{70} = \frac{6}{35}$
PC/ Tablet	8	$\frac{8}{70} = \frac{4}{35}$
Other	17	$\frac{17}{70}$
$\Sigma f =$	70	

Based on the sales figures above apply relative frequencies to estimate sales. If 350 products were sold, what would be the sales of:

1 Toasters?

$\frac{3}{14} \times 350 =$ 75 toasters

2 Phones?

$\frac{6}{35} \times 350 =$ 60 phones

3 Kettles or Toasters?

Add the fractions for kettles and toasters first

$\frac{33}{70} \times 350 =$ 165 products

4 Everything but toasters?

Subtract the toasters fraction from 1

$\frac{11}{14} \times 350 =$ 275 products

5 PCs, Tablets or phones?

$\frac{2}{7} \times 350 =$ 100 products

Find the relative frequency for the daily sales at Pete's Pies. Then answer the questions.

Product	*f*	*r.f.*
Pies	95	$\frac{95}{440} = \frac{19}{88}$
Sausage Rolls	66	$\frac{66}{440} = \frac{3}{20}$
Pastries	125	$\frac{125}{440} = \frac{25}{88}$
Bread	106	$\frac{106}{440} = \frac{53}{220}$
Other	48	$\frac{48}{440} = \frac{6}{55}$
$\Sigma f =$	440	

Based on the sales figures above apply relative frequencies to estimate sales. Estimate the number of pies sold if total sales were:

6 500?

$\frac{19}{88} \times 500 =$ 108 pies

Round to nearest pie!

7 800?

$\frac{19}{88} \times 800 =$ 173 pies

8 300?

$\frac{19}{88} \times 300 =$ 65 pies

Pete has run a promotion and estimates sales will be 1 300 items on the day. Estimate the number of:

9 Pies that Pete should bake.

$\frac{19}{88} \times 1\,300 = 281$ pies

10 Products other than bread he should bake.

$\frac{167}{220} \times 1\,300 = 987$ products

Write the relative frequencies as decimals. This time it is weekly car sales. No need to round.

Car Type	*f*	*r.f.*
Family	6	0.15
Hatch	5	0.125
4WD	16	0.4
Hybrid	2	0.05
Sports	4	0.1
Wagon	3	0.075
Other	4	0.1
$\Sigma f =$	40	

Multiply relative frequencies by 100 to answer these questions. What percentage of sales were:

11 Sports cars?

$0.1 \times 100 = 10\%$

12 Family cars or wagons?

$0.225 \times 100 = 22.5\%$

13 Not a hatchback?

$0.875 \times 100 = 87.5\%$

14 Hatchbacks or hybrids?

$0.175 \times 100 = 17.5\%$

If a person entered the sales area, what three types of vehicle are they most likely to buy? List them in order of popularity, most first.

15 4WD, Family cars and Hatchbacks.

The showroom floor has room for 5 cars. If the showroom cars are to reflect the above sales figures, how many:

16 4WDs should be displayed?

$0.4 \times 5 = 2$

There should be two 4WDs.

Relative Frequency (Sheet 3)

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A bag contains red, green and blue marbles. Apply your relative frequency knowledge.

A marble is selected and replaced from the bag 20 times, the results are:
Red 8, Green 10 and Blue 2.

1 Are there more red marbles than blue?

○ Yes ○ No ● Can't tell

2 Are there 20 marbles in the bag?

○ Yes ○ No ● Can't tell

3 Is it possible that there are more blue marbles than green or red?

● Yes ○ No

4 Are there definitely more green marbles than blue marbles?

○ Yes ● No

A computer monitor manufacturer sorts its products as follows:
Class A - Zero Dead Pixels
Class B - 1 to 2 Dead Pixels
Class C - More than 2 Dead Pixels
After taking samples it is found:
Zero Dead Pixels have *r.f.* of 0.87
One Dead Pixel have an *r.f.* of 0.08
Two Dead Pixels have an *r.f.* of 0.03

5 What is the *r.f.* for a monitor to be 'Class A'.

P(Class A) = 0.87

6 Calculate the *r.f.* for a monitor to be 'Class B'.

P(Class B) = 0.08 + 0.03

P(Class B) = 0.11

7 Calculate the *r.f.* for a monitor to be 'Class C'.

P(Class C) = 1 - 0.87 - 0.11

P(Class C) = 0.02

If 800 monitors are produced in a day, how many would be:

8 'Class A' monitors?

Class A = 0.87 × 800

Class A = 696 monitors

9 'Class B' monitors?

Class B = 0.11 × 800

Class B = 88 monitors

10 'Class C' monitors?

Class C = 0.02 × 800

Class C = 16 monitors

Monitor wholesale prices are:
Class A - $350
Class B - $185
Class C - $55

11 Calculate the income for the 800 monitors.

Income = 696 × 350 + 88 × 185

+16 × 55 = $260 760

A fire station has analysed its response times. They are:
Arrival Under 5 mins: 77%
Arrival within 5 to 10 mins: 20%
Arrival more than 10 mins: 3%

12 Convert the percentages to decimals.

77% = 0.77 20% = 0.2

3% = 0.03

If the station responds to an average of 56 calls per year, calculate (nearest whole number) the number of arrivals:

13 In under 5 minutes

Under 5 arrivals = 0.77 × 56

= 43 arrivals

14 In 10 minutes or less.

10 min or less = 0.97 × 56

= 54 arrivals

15 Within 5 to 10 minutes.

5 to 10 min = 0.2 × 56

= 11 arrivals

16 In 5 minutes or more.

5 min or more = 0.23 × 56

= 13 arrivals

The Ginormous Egg Company only sells eggs larger than 80 g. It sells the smaller eggs to wholesalers.
Its production statistics are:
Saleable eggs 80 g and over: 0.67
Wholesale eggs less than 80 g: 0.27
All other eggs are discarded.

17 Calculate the *r.f.* for an egg to be discarded.

P(Discard) = 1 - 0.67 - 0.27

P(Discard) = 0.06

If production averages 17 000 eggs per day, calculate the number of:

18 Daily wholesaled eggs.

WS eggs = 0.27 × 17 000

WS eggs = 4 590 eggs

19 Weekly discarded eggs.

Disc. eggs = 0.03 × 17 000 × 7

Disc. eggs = 3 570 eggs

20 Yearly company-sold eggs.

Sold eggs = 0.67 × 17 000 × 365

Sold eggs = 4 157 350 eggs

Probability - Relative Frequency (Homework Strip)

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Date Due: _____

Opinion on a bus service is displayed in the table below. Responses can be (G)ood, (S)atisfactory or (B)ad. Complete table and questions below.

Opinion	*f*	*r.f.*
G	48	$\frac{\square}{\square} = \frac{\square}{\square}$
S	27	$\frac{\square}{\square} = \frac{\square}{\square}$
B	15	$\frac{\square}{\square} = \frac{\square}{\square}$
$\Sigma f =$	$\square$	

If 4 230 people use the bus service each day, estimate how many people find:

1 The service good.

$\frac{\square}{\square} \times \square = \underline{\square}$

2 The service good or satisfactory.

$\frac{\square}{\square} \times \square = \underline{\square}$

Date Due: _____

Opinion on a bus service is displayed in the table below. Responses can be (G)ood, (S)atisfactory or (B)ad. Complete table and questions below.

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$\Sigma f =$	$\square$	

If 4 230 people use the bus service each day, estimate how many people find:

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B	15	$\frac{\square}{\square} = \frac{\square}{\square}$
$\Sigma f =$	$\square$	

If 4 230 people use the bus service each day, estimate how many people find:

1 The service good.

$\frac{\square}{\square} \times \square = \underline{\square}$

2 The service good or satisfactory.

$\frac{\square}{\square} \times \square = \underline{\square}$

Probability - Relative Frequency (Homework Strip)

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Date Due:

Opinion on a bus service is displayed in the table below. Responses can be (G)ood, (S)atisfactory or (B)ad. Complete table and questions below.

Opinion	*f*	*r.f.*
G	48	$\frac{48}{90} = \frac{8}{15}$
S	27	$\frac{27}{90} = \frac{3}{10}$
B	15	$\frac{15}{90} = \frac{1}{6}$
$\Sigma f =$	90	

If 4 230 people use the bus service each day, estimate how many people find:

1 The service good.

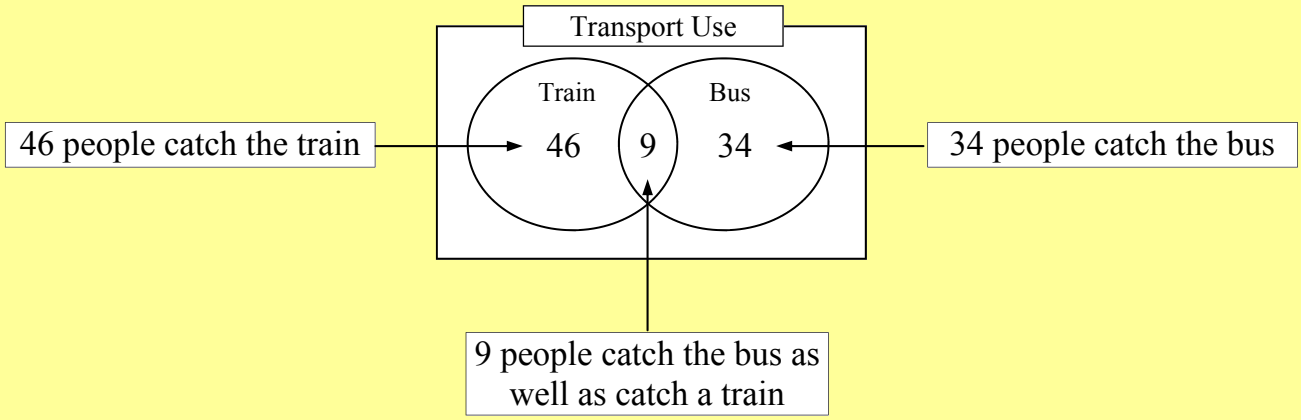
$\frac{8}{15} \times 4\,230 = 2\,256$

2 The service good or satisfactory.

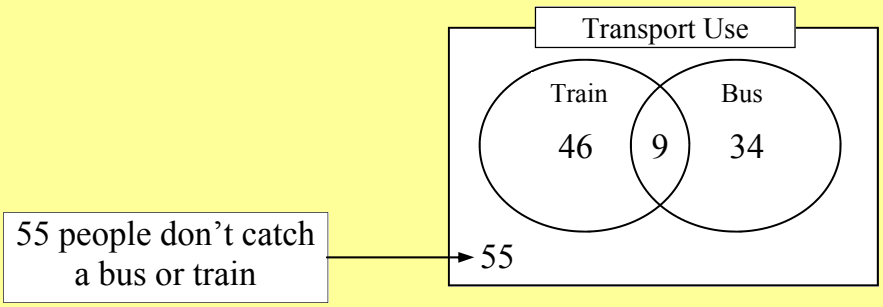
$\frac{5}{6} \times 4\,230 = 3\,525$

Venn Diagrams

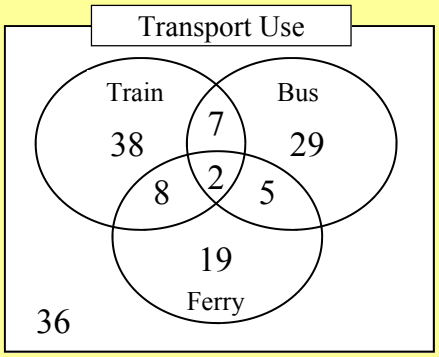
A Venn Diagram is an effective way of showing data when two groups (or more) have some interaction, or in other words, something is common or shared between the groups of data. So you wouldn't use a Venn Diagram to compare unrelated data, such as 'Number of People on a Bus' and 'Shoe Size'. But 'People who catch the bus', 'People who catch the train' and an overlap of 'People who catch a train and a bus' could be shown, see below.



The above could result from separating people who either catch a train, or a bus, and asking them which they use. But what if you asked anybody if they catch a train or bus? As people would drive, walk, bike ride etc. They don't fit in the rings, they either need another ring, or they are outside of the rings.



There can be more than two rings, what if the travel was in a city with a harbor and ferry travel was an option.



32 people use a ferry, but 15 of those ferry users were already in the rings using trains and buses. You can now see that 2 people use all three transport methods.

Venn Diagrams (Sheet 1)

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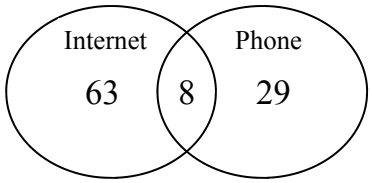
A web-based bank account offers account access by either telephone or internet. Answer these questions.

Customer Usage (%)



What percentage of people access their account using:

1 A phone only? ____

2 A phone? ____

3 The internet? ____

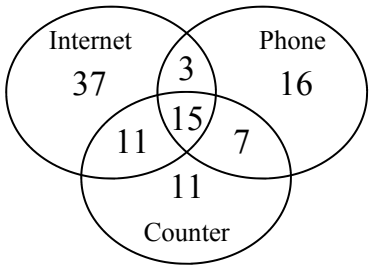
4 The internet only? ____

5 One method only? ____

6 Both methods? ____

The 'extras' bank account includes branch counter service, answer these.

Extras Customer Usage (%)



What percentage of people access their 'Extras' account:

7 Using a phone only? ____

8 Using counter services? ____

9 Using all three methods? ____

10 Not using the internet? ____

What percentage of people access their 'Extras' account using:

11 Both counter and internet services? ____

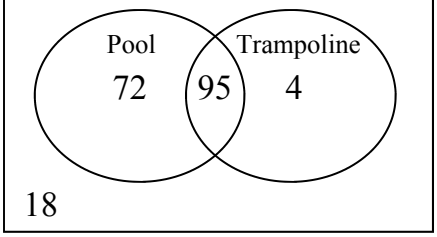
12 Both counter and internet services only? ____

13 A phone but not the internet? ____

14 Counter services but not phone services? ____

A swimming complex offers two pools and six trampolines. Answer these questions.

Monday's Entry Figures (people)



How many people entering the complex on Monday:

15 Only used the pool? ____

16 Used nothing? ____

17 Used only the trampolines? ____

18 Used only one service? ____

19 Didn't enter a pool? ____

20 Didn't use the trampolines? ____

21 Used both services? ____

22 Used the pool? ____

23 Used a pool or a trampoline? ____

Tickets cost: \$2 for the trampolines, \$3 for the pool, \$4 for both and \$1 for spectating care-givers. What were the gate takings?

24 ____

Calculate the relative frequency, as a percentage (to 1 d.p.) for a person to enter the complex and:

25 Use the Pool

All people Using pool

$P(\text{pool}) = \frac{\square}{\square} \times 100$

All People

$P(\text{pool}) =$ ____

Show % sign, round to 1 d.p.

26 Use only the pool or only the trampoline.

$P(\text{tr,pl}) = \frac{\square}{\square} \times \square$

$P(\text{tr,pl}) =$ ____ % sign!

27 Not use the trampoline.

28 Not use the pool.

29 Use the pool only.

Venn Diagrams (Sheet 2)

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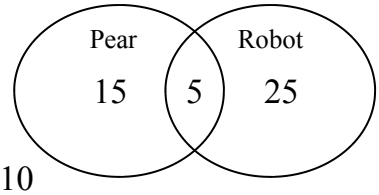
A survey of people that own a mobile phone has had the results shown in the Venn Diagram. Answer these questions.

Mobile Phone Operating Systems



How many people own:

1 A Pear-based phone? ____ (15 + 5)

2 A Robot-based phone? ____

3 A phone that is not Pear or Robot based? ____

4 Both Pear and Robot based phones? ____

Calculate the relative frequency, expressed as a simplified fraction, for a person having a:

5 Pear-based phone.

$P(\text{Pear}) = \frac{15+5}{15+5+25+10} = \frac{\square}{\square}$ (Simplify Using Fraction Key)

6 Phone being neither Pear or Robot based.

$P(\widetilde{P,R}) = \frac{\square}{\square} = \frac{\square}{\square}$

7 Robot-based phone only.

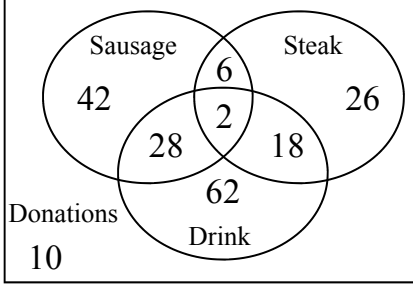
$P(\text{R only}) = \frac{\square}{\square} = \frac{\square}{\square}$

8 Phone that isn't Pear-based.

$P(\widetilde{\text{Pear}}) = \frac{\square}{\square} = \frac{\square}{\square}$

A school stall sells sausage and steak sandwiches as well as drinks. Some people donate money without buying anything.

Sausage Sizzle School-Fund Raiser



How many people bought:

9 A sausage sandwich? ____

10 Only a Sausage sandwich? ____

11 Nothing but donated? ____

12 All three products? ____

13 Both a drink and a sausage sandwich? ____

14 Two products only? ____

15 A steak sandwich and a drink only? ____

Calculate the total number of:

16 Visitors ____ 17 Purchasers ____

How many people came to the stall but didn't buy:

18 A sausage sandwich? ____

Include people making a donation

19 A sandwich? ____

20 A drink? ____

21 A drink or a steak sandwich? ____

22 Two products or more? ____

It's time to analyse the school sales. Use the Sausage Sizzle Venn Diagram to find these.

Calculate the relative frequency as a percentage (1 d.p.) for a person:

23 Visiting the stall and buying any product or products.

$P(\text{buy}) = \frac{\square}{\square} \times 100$ (All numbers in rings Q.17 / All numbers)

$P(\text{buy}) =$ ____

Show % sign, round to 1 d.p.

24 Visiting the stall and buying a drink.

$P(\text{drink}) = \frac{\square}{\square} \times \square$

$P(\text{drink}) =$ ____ (% sign!)

25 Buying from the stall and buying a steak sandwich.

Don't include people giving donation - Buying

26 Buying from the stall and buying both a drink and a sandwich.

27 Visiting the stall and not buying a drink.

28 Buying from the stall and not buying a sausage sandwich.

Venn Diagrams (Sheet 1)

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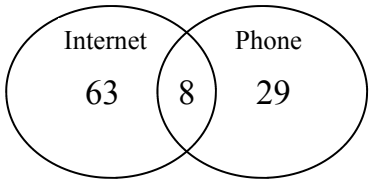
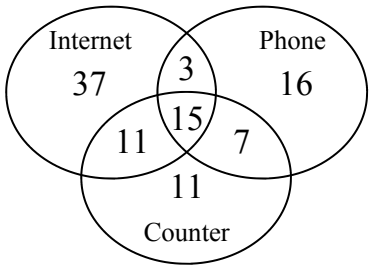


What percentage of people access their account using:

1. A phone only? **29**
2. A phone? **37**
   29 + 8
3. The internet? **71**
   63 + 8
4. The internet only? **63**
5. One method only? **92**
   63 + 29
6. Both methods? **8**

The 'extras' bank account includes branch counter service, answer these.

Extras Customer Usage (%)



What percentage of people access their 'Extras' account:

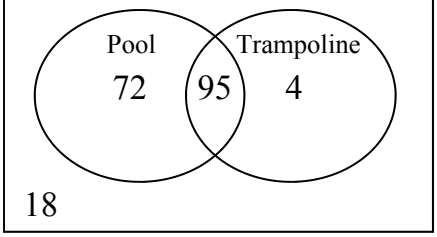
7. Using a phone only? **16**
8. Using counter services? **44**
   2 ×11 + 7 + 15
9. Using all three methods? **15**
10. Not using the internet? **34**
    11 + 7 + 16

What percentage of people access their 'Extras' account using:

11. Both counter and internet services? **26**
    11 + 15
12. Both counter and internet services only? **11**
13. A phone but not the internet? **23**
    16 + 7
14. Counter services but not phone services? **22**
    11 + 11

A swimming complex offers two pools and six trampolines. Answer these questions.

Monday's Entry Figures (people)



How many people entering the complex on Monday:

15. Only used the pool? **72**
16. Used nothing? **18**
17. Used only the trampolines? **4**
18. Used only one service? **76**
    72 + 4
19. Didn't enter a pool? **22**
    18 + 4
20. Didn't use the trampolines? **90**
    72 + 18
21. Used both services? **95**
22. Used the pool? **167**
    95 + 72
23. Used a pool or a trampoline? **171**
    95 + 72 + 4

Tickets cost: \$2 for the trampolines, \$3 for the pool, \$4 for both and \$1 for spectating care-givers. What were the gate takings?

24. Takings = 4 × 2 + 72 × 3 + 95 × 4 + 18 × 1

Takings = \$622

Calculate the relative frequency, as a percentage (to 1 d.p.) for a person to enter the complex and:

25. Use the Pool

$$P(\text{pool}) = \frac{167}{189} \times 100$$

(167: All people Using pool; 189: All People)

P (pool) = 88.4%

Show % sign, round to 1 d.p.

26. Use only the pool or only the trampoline.

$$P(\text{tr,pl}) = \frac{76}{189} \times 100$$

P (tr,pl) = 40.2% % sign!

27. Not use the trampoline.

$$P(\text{no tr}) = \frac{90}{189} \times 100$$

P (no tr) = 47.6%

28. Not use the pool.

$$P(\text{no pool}) = \frac{22}{189} \times 100$$

P (no pool) = 11.6%

29. Use the pool only.

$$P(\text{pool only}) = \frac{72}{189} \times 100$$

P (pool only) = 38.1%

Venn Diagrams (Sheet 2)

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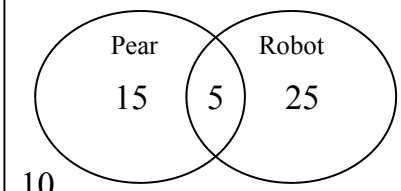


How many people own:

1. A Pear-based phone? **20** (15 + 5)
2. A Robot-based phone? **30**
3. A phone that is not Pear or Robot based? **10**
4. Both Pear and Robot based phones? **5**

Calculate the relative frequency, expressed as a simplified fraction, for a person having a:

5. Pear-based phone.

$$P(\text{Pear}) = \frac{20}{55} = \frac{4}{11}$$ (15 + 5; 15 + 5 + 25 + 10; Simplify Using Fraction Key)

6. Phone being neither Pear or Robot based.

$$P(\widetilde{P,R}) = \frac{10}{55} = \frac{2}{11}$$

7. Robot-based phone only.

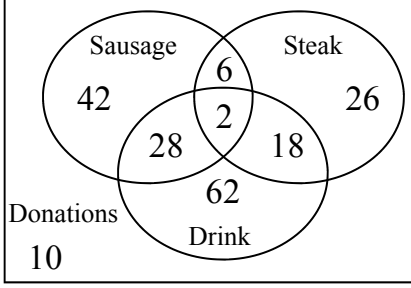
$$P(\text{R only}) = \frac{25}{55} = \frac{5}{11}$$

8. Phone that isn't Pear-based.

$$P(\widetilde{\text{Pear}}) = \frac{35}{55} = \frac{7}{11}$$

A school stall sells sausage and steak sandwiches as well as drinks. Some people donate money without buying anything.

Sausage Sizzle School-Fund Raiser



How many people bought:

9. A sausage sandwich? **78** (42 + 28 + 6 + 2)
10. Only a Sausage sandwich? **42**
11. Nothing but donated? **10**
12. All three products? **2**
13. Both a drink and a sausage sandwich? **30** (28 + 2)
14. Two products only? **52** (28 + 18 + 6)
15. A steak sandwich and a drink only? **18**

Calculate the total number of:

16. Visitors **194**
17. Purchasers **184**

How many people came to the stall but didn't buy:

18. A sausage sandwich? **88** (10 + 62 + 18 + 26)
19. A sandwich? **72** (62 + 10)
20. A drink? **84** (10 + 42 + 6 + 26)
21. A drink or a steak sandwich? **52** (10 + 42)
22. Two products or more? **140** (10 + 62 + 42 + 26)

It's time to analyse the school sales. Use the Sausage Sizzle Venn Diagram to find these.

Calculate the relative frequency as a percentage (1 d.p.) for a person:

23. Visiting the stall and buying any product or products. (All numbers in rings Q.17; All numbers)

$$P(\text{buy}) = \frac{184}{194} \times 100$$

$$P(\text{buy}) = 94.8\%$$

Show % sign, round to 1 d.p.

24. Visiting the stall and buying a drink. (62 + 28 + 2 + 18)

$$P(\text{drink}) = \frac{110}{194} \times 100$$

$$P(\text{drink}) = 56.7\%$$ (% sign!)

25. Buying from the stall and buying a steak sandwich.

Don't include people giving donation - Buying

$$P(\text{ss}) = \frac{90}{184} \times 100$$

$$P(\text{ss}) = 48.9\%$$

26. Buying from the stall and buying both a drink and a sandwich. (28 + 18 + 2)

$$P(\text{d,s}) = \frac{48}{184} \times 100$$

$$P(\text{d,s}) = 26.1\%$$

27. Visiting the stall and not buying a drink. (10 + 42 + 6 + 26)

$$P(\text{no drink}) = \frac{84}{194} \times 100$$

$$P(\text{no drink}) = 43.3\%$$

28. Buying from the stall and not buying a sausage sandwich. (62+18+26)

$$P(\text{no saus}) = \frac{106}{184} \times 100$$

$$P(\text{no saus}) = 57.6\%$$

Probability - Venn Diagrams (Homework Strip)

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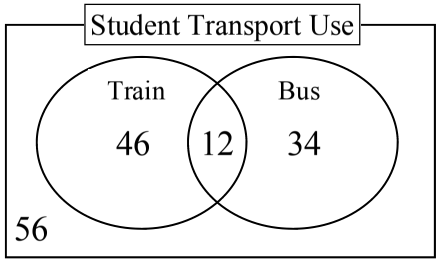
Date Due: _____



How many students:

1. Only catch a train? _____
2. Catch a train? _____
3. Don't catch a bus? _____
4. Don't catch a bus or a train? _____
5. Don't catch a train? _____
6. What is the relative frequency for a student catching a train? (Fraction)

Train users

P (train) = _____ = _____

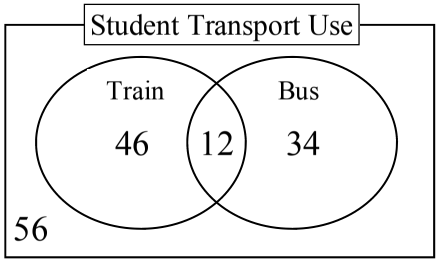
All students

Simplify

7. What is the relative frequency for a student not catching a bus?

P (no bus) = _____ = _____

Date Due: _____



How many students:

1. Only catch a train? _____
2. Catch a train? _____
3. Don't catch a bus? _____
4. Don't catch a bus or a train? _____
5. Don't catch a train? _____
6. What is the relative frequency for a student catching a train? (Fraction)

Train users

P (train) = _____ = _____

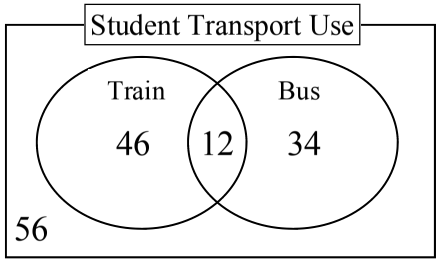
All students

Simplify

7. What is the relative frequency for a student not catching a bus?

P (no bus) = _____ = _____

Date Due: _____



How many students:

1. Only catch a train? _____
2. Catch a train? _____
3. Don't catch a bus? _____
4. Don't catch a bus or a train? _____
5. Don't catch a train? _____
6. What is the relative frequency for a student catching a train? (Fraction)

Train users

P (train) = _____ = _____

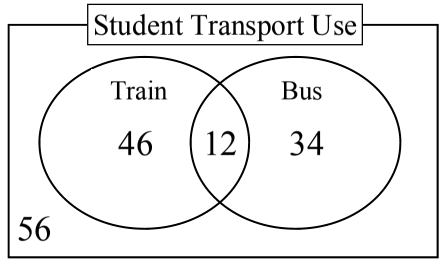
All students

Simplify

7. What is the relative frequency for a student not catching a bus?

P (no bus) = _____ = _____

Date Due: _____



How many students:

1. Only catch a train? _____
2. Catch a train? _____
3. Don't catch a bus? _____
4. Don't catch a bus or a train? _____
5. Don't catch a train? _____
6. What is the relative frequency for a student catching a train? (Fraction)

Train users

P (train) = _____ = _____

All students

Simplify

7. What is the relative frequency for a student not catching a bus?

P (no bus) = _____ = _____

Probability - Venn Diagrams (Homework Strip)

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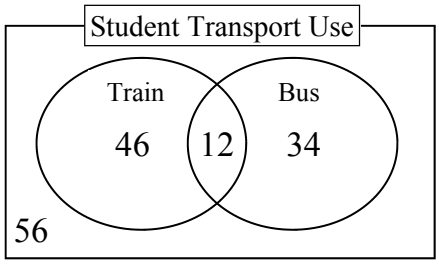
Date Due:



How many students:

1. Only catch a train? **46**
2. Catch a train? **58**
3. Don't catch a bus? **102**
   46 + 56
4. Don't catch a bus or a train? **56**
5. Don't catch a train? **90**
   34 + 56
6. What is the relative frequency for a student catching a train? (Fraction)

Train users / All students

$$P(\text{train}) = \frac{58}{148} = \frac{29}{74}$$

7. What is the relative frequency for a student not catching a bus?

$$P(\text{no bus}) = \frac{102}{148} = \frac{51}{74}$$

Two-Way Tables

A two-way table compares two or more outcomes against another two (or more) outcomes. The outcomes may be related, like ‘Inadequate sleep’ & ‘Normal sleep’ against ‘Good Driver’ & ‘Poor Driver’. The lack of rest may affect the driving skill. But you can have unrelated comparisons like: ‘Right-handed’ & ‘Left-handed’ against ‘Male’ and ‘Female’. Your sex and the hand you write with may be unrelated, but can relevant for statistics. For example by Ordering more specialised handed equipment.

Sheet 1 asks you to complete the table first, by finding the totals. So add across and add down, check that the vertical totals add to the bottom left hand cell, then repeat for the horizontal totals. You are then given two pieces of information. The first piece of information selects the row or column, the second piece of information selects the cell within that column/row.

Example

	Right-H	Left-H	Total
Female	45	6	51
Male	52	3	55
Total	97	9	106

Add Down

Add across

How many female students are left handed? 6

Female so 1st row

Left handed so 2nd col

But these questions can also include probability based questions. Relative frequency is used because the results are based on experimental results, not necessarily theoretical results. For example the probabilities would change if students were absent.

Example

	Right-H	Left-H	Total
Female	45	6	51
Male	52	3	55
Total	97	9	106

What is the relative frequency of selecting from the male students, a male who is right handed?

Female so 2nd row

Right handed so 1st col

$$P(\text{male,right-h}) = \frac{52}{55}$$

There are 55 male students

Probability - Two-Way Tables (Sheet 1)

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Two-way tables can be read horizontally or vertically, answer the questions.

Year 9 Survey: Weather vs. Type of Clothing

	Shorts	Trousers	Total
Hot	198	24	A
Cold	36	162	B
Total	C	D	E

1 Complete the total boxes in the table above.

For the next 5 questions, how many Year 9 students:

2 Wore shorts on the hot day? ____

3 Wore shorts on the cold day? ____

4 Wore trousers on the cold day? ____

5 Wore trousers? ____ 6 Were surveyed? ____

7 An excursion was on one of the two survey days, which day, hot or cold, do you think it was on. Explain your answer.

_____

_____

Writing your answer as a fraction, then simplify it. Calculate the relative frequency of a Year 9 student:

8 On a cold day, wearing shorts. $P(sh,cold) = \frac{\square}{\square} = \frac{\square}{\square}$

9 On a hot day, not wearing shorts. $P(\widetilde{sh},hot) = \frac{\square}{\square} = \frac{\square}{\square}$

10 Wearing trousers. (On a cold or hot day). $P(trous) = \frac{\square}{\square} = \frac{\square}{\square}$ Careful! →

Writing your answer as a percentage (to 1 d.p.), calculate the relative frequency of a Year 9 student:

11 On a cold day, wearing trousers. $P(cold,tr) = \frac{\square}{\square} \times 100$

$= \frac{\square}{\square}$ 1 d.p. Show %

Vet Survey: Pets vs. Flea Infestation

	Cats	Dogs	Total
No fleas	188	285	A
Fleas	18	20	B
Total	C	D	E

12 Complete the total boxes in the table above.

13 How many cats had fleas? ____

14 How many dogs had fleas? ____

15 Calculate the relative frequency for a cat to have fleas, express your answer as a percentage (1 d.p.).

$P(cat,fleas) = \frac{\square}{\square} \times \square = \frac{\square}{\square}$ Show % sign

16 For a dog to have fleas.

$P(dog,fleas) = \frac{\square}{\square} \times \square = \square$

17 For a pet with fleas, to be a dog.

$P(fleas,dog) = \frac{\square}{\square} \times \square = \square$

Bus Transport Survey: Weather vs. Punctuality

	On-time	Late	Total
Raining	50	A	65
Dry	B	5	65
Total	D	E	C

18 Complete the lettered boxes in the table above.

19 How many buses were late? ____

Find the relative frequency as a simplified fraction for:

20 On a dry day, a bus being on-time . $P(dry,on\text{-}time) = \frac{\square}{\square} = \frac{\square}{\square}$

21 On a rainy day, a bus being late. $P(rain,late) = \frac{\square}{\square} = \frac{\square}{\square}$

Probability - Two-Way Tables (Sheet 2)

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The key to car wash success is to never lose your drive. Answer these questions on two-way tables.

Car Wash Survey: Wash vs. Vacuum

	Wash	No Wash	Total
Vacuum	78	4	A
No Vacuum	36	0	B
Total	C	D	E

1 Complete the total boxes in the table above.

Find the relative frequency as a percentage (1 d.p.) from:

2 Cars having a car wash, cars that don't get vacuumed.

$P(\text{wash,no vac}) = \frac{\square}{\square} \times \square = \square$

3 Cars having a vacuum, cars not getting a car wash.

$P(\text{vac,no wash}) = \frac{\square}{\square} \times \square = \square$

4 Cars not having a car wash, cars getting a vacuum.

5 Cars entering the car wash, cars getting a car wash.

Year 9 Hair Survey: Length vs. Curliness

	Straight	Curly	Total
Short		4	125
Long	62		80
Total			

6 Complete all the missing cells in the table above.

Writing your answer as a fraction, then simplify it. Calculate the relative frequency of a Year 9 student:

7 With curly hair, having short hair. $P(\text{curl,sh}) = \frac{\square}{\square} = \frac{\square}{\square}$

8 Having curly short hair. $P(\text{curl,sh}) = \frac{\square}{\square}$



We are still dealing with hair, well you are, I'm not!

Calculate the relative frequency of a Year 9 student:

9 With long hair, having curly hair. $P(\text{lng,curl}) = \frac{\square}{\square} = \frac{\square}{\square}$

10 With curly hair, having long hair. $P(\text{curl,lng}) = \frac{\square}{\square} = \frac{\square}{\square}$

11 Having short hair. $P(\text{short}) = \frac{\square}{\square} = \frac{\square}{\square}$

12 Having straight hair. $P(\text{strgt}) = \frac{\square}{\square}$

Year 9 Language Survey: Language vs. Sex

	French	German	Total
Male		16	30
Female	12		
Total			56

13 Complete all the missing cells in the table above.

14 The survey was taken on a German class and a French class, how many students in each class?

German Students = $\square$ French Students = $\square$

15 If 12 language students were interviewed, how many would you expect to be male and studying French?

16 If there are 95 Year 9 students, how many don't study a language?

Find the relative frequency, as a % to 1 d.p., that a Year 9 student (full year group) chosen randomly, would be:

17 Male and study German.

18 Female and studies a language

Probability - Two-Way Tables (Sheet 1)

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Two-way tables can be read horizontally or vertically, answer the questions.

Year 9 Survey: Weather vs. Type of Clothing

	Shorts	Trousers	Total
Hot	198	24	A 222
Cold	36	162	B 198
Total	C 234	D 186	E 420

1 Complete the total boxes in the table above.

For the next 5 questions, how many Year 9 students:

2 Wore shorts on the hot day? 198

3 Wore shorts on the cold day? 36

4 Wore trousers on the cold day? 162

5 Wore trousers? 186 6 Were surveyed? 420

7 An excursion was on one of the two survey days, which day, hot or cold, do you think it was on. Explain your answer. Most likely the cold day because the number of students was 24 less

Writing your answer as a fraction, then simplify it. Calculate the relative frequency of a Year 9 student:

8 On a cold day, wearing shorts. $P(sh,cold) = \frac{36}{198} = \frac{2}{11}$

9 On a hot day, not wearing shorts. $P(\widetilde{sh},hot) = \frac{24}{222} = \frac{4}{37}$

10 Wearing trousers. (On a cold or hot day). $P(trous) = \frac{186}{420} = \frac{31}{70}$ Careful!

Writing your answer as a percentage (to 1 d.p.), calculate the relative frequency of a Year 9 student:

11 On a cold day, wearing trousers. $P(tr,cold) = \frac{162}{198} \times 100 = 81.8\%$ (1 d.p. Show %)

Vet Survey: Pets vs. Flea Infestation

	Cats	Dogs	Total
No fleas	188	285	A 473
Fleas	18	20	B 38
Total	C 206	D 305	E 473

12 Complete the total boxes in the table above.

13 How many cats had fleas? 18

14 How many dogs had fleas? 20

15 Calculate the relative frequency for a cat to have fleas, express your answer as a percentage (1 d.p.).

$P(cat,fleas) = \frac{18}{206} \times 100 = 8.7\%$ (Show % sign)

16 For a dog to have fleas.

$P(dog,fleas) = \frac{20}{305} \times 100 = 6.6\%$

17 For a pet with fleas, to be a dog.

$P(fleas,dog) = \frac{20}{38} \times 100 = 52.6\%$

Bus Transport Survey: Weather vs. Punctuality

	On-time	Late	Total
Raining	50	A 15	65
Dry	B 60	5	65
Total	D 110	E 20	C 130

18 Complete the lettered boxes in the table above.

19 How many buses were late? 20

Find the relative frequency as a simplified fraction for:

20 On a dry day, a bus being on-time . $P(dry,on\text{-}time) = \frac{60}{65} = \frac{12}{13}$

21 On a rainy day, a bus being late. $P(rain,late) = \frac{15}{65} = \frac{3}{13}$

Probability - Two-Way Tables (Sheet 2)

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The key to car wash success is to never lose your drive. Answer these questions on two-way tables.

Car Wash Survey: Wash vs. Vacuum

	Wash	No Wash	Total
Vacuum	78	4	A 82
No Vacuum	36	0	B 36
Total	C 114	D 4	E 118

1 Complete the total boxes in the table above.

Find the relative frequency as a percentage (1 d.p.) from:

2 Cars having a car wash, cars that don't get vacuumed.

$$P(\text{wash,no vac}) = \frac{36}{114} \times 100 = 31.6\%$$

3 Cars having a vacuum, cars not getting a car wash.

$$P(\text{vac,no wash}) = \frac{4}{82} \times 100 = 4.9\%$$

4 Cars not having a car wash, cars getting vacuumed.

$$P(\text{no wash,vac}) = \frac{4}{4} \times 100 = 100\%$$

5 Cars entering the car wash, cars getting a car wash.

$$P(\text{no wash,vac}) = \frac{114}{118} \times 100 = 96.6\%$$

Year 9 Hair Survey: Length vs. Curliness

	Straight	Curly	Total
Short	121	4	125
Long	62	18	80
Total	183	22	205

6 Complete all the missing cells in the table above.

Writing your answer as a fraction, then simplify it. Calculate the relative frequency of a Year 9 student:

7 With curly hair, having short hair.

$$P(\text{curl,sh}) = \frac{4}{22} = \frac{2}{11}$$

8 Having curly short hair.

$$P(\text{curl,sh}) = \frac{4}{205}$$

We are still dealing with hair, well you are, I'm not!

Calculate the relative frequency of a Year 9 student:

9 With long hair, having curly hair.

$$P(\text{lng,curl}) = \frac{18}{80} = \frac{9}{40}$$

10 With curly hair, having long hair.

$$P(\text{curl,lng}) = \frac{18}{22} = \frac{9}{11}$$

11 Having short hair.

$$P(\text{short}) = \frac{125}{205} = \frac{25}{41}$$

12 Having straight hair.

$$P(\text{strgt}) = \frac{183}{205}$$

Year 9 Language Survey: Language vs. Sex

	French	German	Total
Male	14	16	30
Female	12	14	26
Total	26	30	56

13 Complete all the missing cells in the table above.

14 The survey was taken on a German class and a French class, how many students in each class?

German Students = 30 French Students = 26

15 If 12 language students were interviewed, how many would you expect to be male and studying French?

$$P(\text{male,french}) = \frac{14}{56} \times 12 = 25\%$$

16 If there are 95 Year 9 students, how many don't study a language? 95 - 56 = 39 students

Find the relative frequency, as a % to 1 d.p., that a Year 9 student (full year group) chosen randomly, would be:

17 Male and study German.

$$P(\text{male,german}) = \frac{16}{95} \times 100 = 16.8\%$$

18 Female and studies a language

$$P(\text{female,lang}) = \frac{26}{95} \times 100 = 27.4\%$$

Probability - Two-way Tables (Homework Strip)

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Date Due: _____

Two computer-generated squares are beside each other (left/right). Each square can be black or white, the results of 100 simulations are tabled.

Two Squares: Colour vs. Position

	Black	White	Total
Left	55	45	
Right	42	58	
Total			

1 Complete the totals in the table.

2 How many black squares were generated? _____

3 How many left squares were white? _____

4 What is the relative frequency of generating a black square?

P (black) = _____

5 What is the relative frequency of generating a white square?

P (white) = _____

Date Due: _____

Two computer-generated squares are beside each other (left/right). Each square can be black or white, the results of 100 simulations are tabled.

Two Squares: Colour vs. Position

	Black	White	Total
Left	55	45	
Right	42	58	
Total			

1 Complete the totals in the table.

2 How many black squares were generated? _____

3 How many left squares were white? _____

4 What is the relative frequency of generating a black square?

P (black) = _____

5 What is the relative frequency of generating a white square?

P (white) = _____

Date Due: _____

Two computer-generated squares are beside each other (left/right). Each square can be black or white, the results of 100 simulations are tabled.

Two Squares: Colour vs. Position

	Black	White	Total
Left	55	45	
Right	42	58	
Total			

1 Complete the totals in the table.

2 How many black squares were generated? _____

3 How many left squares were white? _____

4 What is the relative frequency of generating a black square?

P (black) = _____

5 What is the relative frequency of generating a white square?

P (white) = _____

Date Due: _____

Two computer-generated squares are beside each other (left/right). Each square can be black or white, the results of 100 simulations are tabled.

Two Squares: Colour vs. Position

	Black	White	Total
Left	55	45	
Right	42	58	
Total			

1 Complete the totals in the table.

2 How many black squares were generated? _____

3 How many left squares were white? _____

4 What is the relative frequency of generating a black square?

P (black) = _____

5 What is the relative frequency of generating a white square?

P (white) = _____

Probability - Two-way Tables (Homework Strip)

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Date Due:

Two computer-generated squares are beside each other (left/right). Each square can be black or white, the results of 100 simulations are tabled.

Two Squares: Colour vs. Position

	Black	White	Total
Left	55	45	100
Right	42	58	100
Total	97	103	200

1 Complete the totals in the table.

2 How many black squares were generated? 97

3 How many left squares were white? 45

4 What is the relative frequency of generating a black square?

$$P(\text{black}) = \frac{97}{200}$$

5 What is the relative frequency of generating a white square?

$$P(\text{white}) = \frac{103}{200}$$

Probability - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Probability - Find a Word

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Find the words in the puzzle from the wordlist.



T	H	E	O	R	E	T	I	C	A	L	X	P	E	P
A	S	V	U	E	G	A	T	N	E	C	R	E	P	R
L	U	R	E	Y	L	L	G	X	X	P	E	R	R	O
L	R	V	E	N	A	D	E	L	P	M	A	S	E	B
A	V	E	L	L	N	E	N	Y	L	L	A	T	D	A
T	E	E	B	L	T	C	E	S	R	E	T	N	I	B
N	Y	V	I	L	E	I	R	E	G	D	M	Y	C	I
E	L	U	S	I	G	M	A	R	N	I	A	C	T	L
M	A	R	S	K	N	A	T	G	I	A	R	N	L	I
I	C	U	O	E	R	L	E	Y	R	G	G	E	R	T
R	I	W	P	L	E	L	I	L	U	R	O	U	A	Y
E	T	L	K	Y	B	V	K	K	C	A	T	Q	N	T
P	R	O	B	A	B	L	E	E	C	M	S	E	D	W
X	E	K	T	W	L	A	T	N	O	Z	I	R	O	H
E	V	I	T	A	L	E	R	A	T	E	H	F	M	O

WORDLIST

VENN	EVENT	POSSIBLE	TABLE	LIKELY
PERCENTAGE	RELATIVE	INTERSECT	HORIZONTAL	HISTOGRAM
SIGMA	PROBABILITY	TWO	SAMPLE	WAY
FREQUENCY	VERTICAL	DECIMAL	DIAGRAM	GENERATE
SURVEY	OCCURING	PREDICT	RANDOM	PROBABLE
	EXPERIMENTAL	TALLY	THEORETICAL	

Probability - Find a Word

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Find the words in the puzzle from the wordlist.



T	H	E	O	R	E	T	I	C	A	L	X	P	E	P
A	S	V	U	E	G	A	T	N	E	C	R	E	P	R
L	U	R	E	Y	L	L	G	X	X	P	E	R	R	O
L	R	V	E	N	A	D	E	L	P	M	A	S	E	B
A	V	E	L	L	N	E	N	Y	L	L	A	T	D	A
T	E	E	B	L	T	C	E	S	R	E	T	N	I	B
N	Y	V	I	L	E	I	R	E	G	D	M	Y	C	I
E	L	U	S	I	G	M	A	R	N	I	A	C	T	L
M	A	R	S	K	N	A	T	G	I	A	R	N	L	I
I	C	U	O	E	R	L	E	Y	R	G	G	E	R	T
R	I	W	P	L	E	L	I	L	U	R	O	U	A	Y
E	T	L	K	Y	B	V	K	K	C	A	T	Q	N	T
P	R	O	B	A	B	L	E	E	C	M	S	E	D	W
X	E	K	T	W	L	A	T	N	O	Z	I	R	O	H
E	V	I	T	A	L	E	R	A	T	E	H	F	M	O

WORDLIST

~~VENN~~
~~EVENT~~
~~POSSIBLE~~
~~TABLE~~
~~LIKELY~~
~~PERCENTAGE~~
~~RELATIVE~~
~~INTERSECT~~
~~HORIZONTAL~~
~~HISTOGRAM~~
~~SIGMA~~
~~PROBABILITY~~
~~TWO~~
~~SAMPLE~~
~~WAY~~
~~FREQUENCY~~
~~VERTICAL~~
~~DECIMAL~~
~~DIAGRAM~~
~~GENERATE~~
~~SURVEY~~
~~OCCURING~~
~~PREDICT~~
~~RANDOM~~
~~PROBABLE~~
~~EXPERIMENTAL~~
~~TALLY~~
~~THEORETICAL~~

Probability Review

This is the review of the Probability Topic.

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10 cards numbered 1 to 10 are placed in a hat, then one card is randomly selected. Find the probability of:

1 Selecting an '8' card.

$P(8) = \frac{\square}{\square}$

2 Selecting an odd numbered card.

$\square = \frac{\square}{\square} = \frac{\square}{\square}$

3 Selecting a card with a number greater than 4.

$\square = \frac{\square}{\square} = \frac{\square}{\square}$

4 Selecting a card with a number that is a factor of 8.

$\square = \frac{\square}{\square} = \frac{\square}{\square}$

5 Not Selecting a card with a number less than 3.

$\square = \frac{\square}{\square} = \frac{\square}{\square}$

15 discs are in a bag. A single disc is selected, its colour recorded, then replaced. After 25 selections there have been 10 black discs, 3 red discs and 12 green discs selected.

Answer True or False

6 There are definitely more Green discs in the bag. (True) (False)

7 The bag may contain 12 purple discs. (True) (False)

8 There are definitely more green discs than red discs. (True) (False)

(These questions are on the discs still). If a disc was selected and replaced 325 times, estimate the:

9 Number of times a red disc would be selected.

_____

10 Number of times a black disc would be selected.

_____

11 Number of times a selected disc would be green or red.

_____

If a card is chosen from a standard deck of cards. Calculate the probability of:

12 Selecting a red card.

_____

13 Selecting a black picture card.

_____

14 Selecting a '5', '7' or an ace.

_____

15 Not selecting a picture card or an ace.

_____

Three Year 9 boys have nominated for the male SRC position, Joe, Omar and Lance. 70 students vote by selecting one of these students. Results are below.

Data:				
LJLOJ	LLOOJ	JOLLO	OOJLL	
LLLOJ	OOJLO	LOOLL	JLLLO	OLLLJ
OLJLO	JJLLL	JOLOO	LLLLJ	OLLLJ

16 Complete the table, simplifying the relative frequencies.

Student	*Tally*	*f*	*r.f.*
Joe	----------	$\square$	$\frac{\square}{\square} = \frac{\square}{\square}$
Omar	----------	$\square$	$\frac{\square}{\square} = \frac{\square}{\square}$
Lance	----------	$\square$	$\frac{\square}{\square} = \frac{\square}{\square}$
		$\Sigma f = \square$	

25 students were on an excursion on the day, but will vote the next day. Based on the relative frequencies above. How many of the 25 students:

17 Would vote for Omar.

_____

18 Would not vote for Lance.

_____

19 Who would get the most of the total vote and how many would they get?

_____

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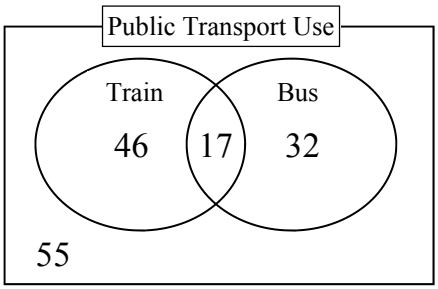
A business asked its staff who use public transport which type of transport they use. Results are shown



How many people:

20 Used the train only? ____

21 Used a both a bus and a train? ____

22 Didn't use a train? ____

23 Were surveyed? ____

Calculate the relative frequency and simplify the fraction if possible, for:

24 A person catching a train.

____ = ____ = ____

25 A person catching only a train, or only a bus.

____ = ____ = ____

26 A person not catching a train

____ = ____ = ____

Calculate the relative frequency as a percentage (1 d.p.), for:

27 A person not catching a train, or a bus.

The results for a survey of Year 9 students are shown below. Complete the spaces in the table and then answer the questions.

	Right-H	Left-H	Total
Female	45	6	
Male	52	3	
Total			

How many Year 9 students:

28 Are left-handed? ____

29 Were surveyed? ____

30 Are male? ____

Find the relative frequencies for these, answer as a percentage (to 1 d.p.)

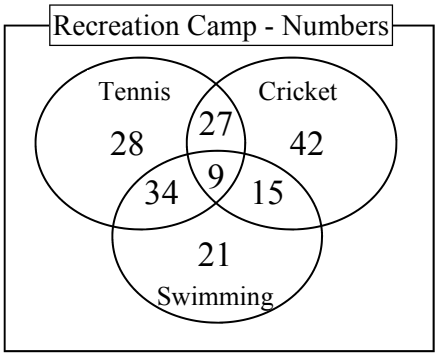
31 That a right-handed student, would be male.

32 That a female student, would would be left-handed.

33 That a Year 9 student, would be male and left-handed.

34 That a Year 9 student, would be right-handed.

35 That a Year 9 student, would be female.



A recreation camp offers a choice of three sports, which students can do individually or in conjunction with other sports. Answer these questions

How many students have chosen:

36 Just swimming? ____

37 Swimming? ____

38 All three sports? ____

39 Both tennis and cricket? ____

40 One sport only? ____

41 Not to do swimming? ____

Calculate the relative frequency as a percentage (1 d.p.), for:

42 A student to choose both swimming and cricket.

43 A student who has chosen cricket, to also choose swimming.

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10 cards numbered 1 to 10 are placed in a hat, then one card is randomly selected. Find the probability of:

1 Selecting an '8' card.

$P(8) = \frac{1}{10}$

2 Selecting an odd numbered card.

$P(\text{odd}) = \frac{5}{10} = \frac{1}{2}$

3 Selecting a card with a number greater than 4.

$P(>4) = \frac{6}{10} = \frac{3}{5}$

4 Selecting a card with a number that is a factor of 8.

$P(\text{factor 8}) = \frac{4}{10} = \frac{2}{5}$

5 Not Selecting a card with a number less than 3.

$P(\widetilde{<3}) = \frac{8}{10} = \frac{4}{5}$

15 discs are in a bag. A single disc is selected, its colour recorded, then replaced. After 25 selections there have been 10 black discs, 3 red discs and 12 green discs selected.

Answer True or False

6 There are definitely more Green discs in the bag. True

7 The bag may contain 12 purple discs. False

8 There are definitely more green discs than red discs. True

(These questions are on the discs still). If a disc was selected and replaced 325 times, estimate the:

9 Number of times a red disc would be selected.

$\frac{3}{25} \times 325 = 39$ times

10 Number of times a black disc would be selected.

$\frac{10}{25} \times 325 = 130$ times

11 Number of times a selected disc would be green or red.

$\frac{15}{25} \times 325 = 195$ times

If a card is chosen from a standard deck of cards. Calculate the probability of:

12 Selecting a red card.

$P(\text{red}) = \frac{26}{52} = \frac{1}{2}$

13 Selecting a black picture card.

$P(\text{black pic}) = \frac{6}{52} = \frac{3}{26}$

14 Selecting a '5', '7' or an ace.

$P(5,7,\text{Ace}) = \frac{12}{52} = \frac{3}{13}$

15 Not selecting a picture card or an ace.

$P(\widetilde{\text{Pic,Ace}}) = \frac{36}{52} = \frac{9}{13}$

Three Year 9 boys have nominated for the male SRC position, Joe, Omar and Lance. 70 students vote by selecting one of these students. Results are below.

Data:				
LJLOJ	LLOOJ	JOLLO	OOJLL	
LLLOJ	OOJLO	LOOLL	JLLLO	OLLLJ
OLJLO	JJLLL	JOLOO	LLLLJ	OLLLJ

16 Complete the table, simplifying the relative frequencies.

Student	*Tally*	*f*	*r.f.*
Joe	𝍸 𝍸 𝍸	15	$\frac{15}{70} = \frac{3}{14}$
Omar	𝍸 𝍸 𝍸 𝍸 |	21	$\frac{21}{70} = \frac{3}{10}$
Lance	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||||	34	$\frac{34}{70} = \frac{17}{35}$
	$\Sigma f =$	70	

25 students were on an excursion on the day, but will vote the next day. Based on the relative frequencies above. How many of the 25 students:

17 Would vote for Omar.

$\frac{3}{10} \times 25 \approx 8$ students

18 Would not vote for Lance.

$\frac{18}{35} \times 25 \approx 13$ students

19 Who would get the most of the total vote and how many would they get?

Lance, $\frac{17}{35} \times 95 \approx 46$ students

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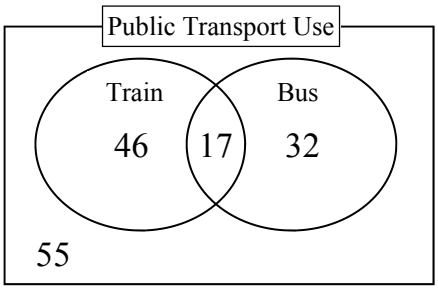
A business asked its staff who use public transport which type of transport they use. Results are shown



How many people:

20 Used the train only? 46

21 Used a both a bus and a train? 15

22 Didn't use a train? 89

23 Were surveyed? 150

Calculate the relative frequency and simplify the fraction if possible, for:

24 A person catching a train.

$P(\text{train}) = \frac{63}{150} = \frac{21}{50}$

25 A person catching only a train, or only a bus.

$P(\text{train,bus}) = \frac{78}{150} = \frac{13}{25}$

26 A person not catching a train

$P(\widetilde{\text{train}}) = \frac{87}{150} = \frac{29}{50}$

Calculate the relative frequency as a percentage (1 d.p.), for:

27 A person not catching a train, or a bus.

$P(\text{no bus or train}) = \frac{55}{150} \times 100$

$P(\text{no bus or train}) = 36.7\%$

The results for a survey of Year 9 students are shown below. Complete the spaces in the table and then answer the questions.

	Right-H	Left-H	Total
Female	45	6	51
Male	52	3	55
Total	97	9	106

How many Year 9 students:

28 Are left-handed? 9

29 Were surveyed? 106

30 Are male? 55

Find the relative frequencies for these, answer as a percentage (to 1 d.p.)

31 That a right-handed student, would be male.

$P(\text{r-h,male}) = \frac{52}{97} \times 100$

$P(\text{r-h,male}) = 53.6\%$

32 That a female student, would would be left-handed.

$P(\text{l-h,female}) = \frac{6}{51} \times 100$

$P(\text{l-h,female}) = 11.8\%$

33 That a Year 9 student, would be male and left-handed.

$P(\text{l-h,male}) = \frac{3}{106} \times 100$

$P(\text{l-h,male}) = 2.8\%$

34 That a Year 9 student, would be right-handed.

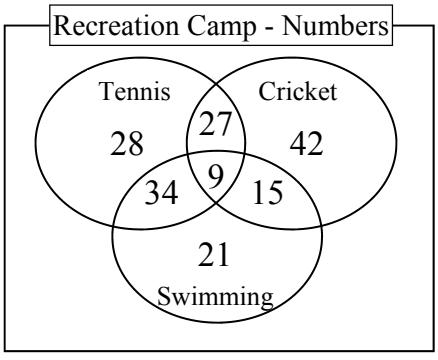
$P(\text{r-h}) = \frac{97}{106} \times 100$

$P(\text{r-h}) = 91.5\%$

35 That a Year 9 student, would be female.

$P(\text{female}) = \frac{51}{106} \times 100$

$P(\text{female}) = 48.1\%$



A recreation camp offers a choice of three sports, which students can do individually or in conjunction with other sports. Answer these questions

How many students have chosen:

36 Just swimming? 21

37 Swimming? 79

38 All three sports? 9

39 Both tennis and cricket? 36

40 One sport only? 91

41 Not to do swimming? 97

Calculate the relative frequency as a percentage (1 d.p.), for:

42 A student to choose both swimming and cricket.

$P(\text{swim,crick}) = \frac{24}{176} \times 100$

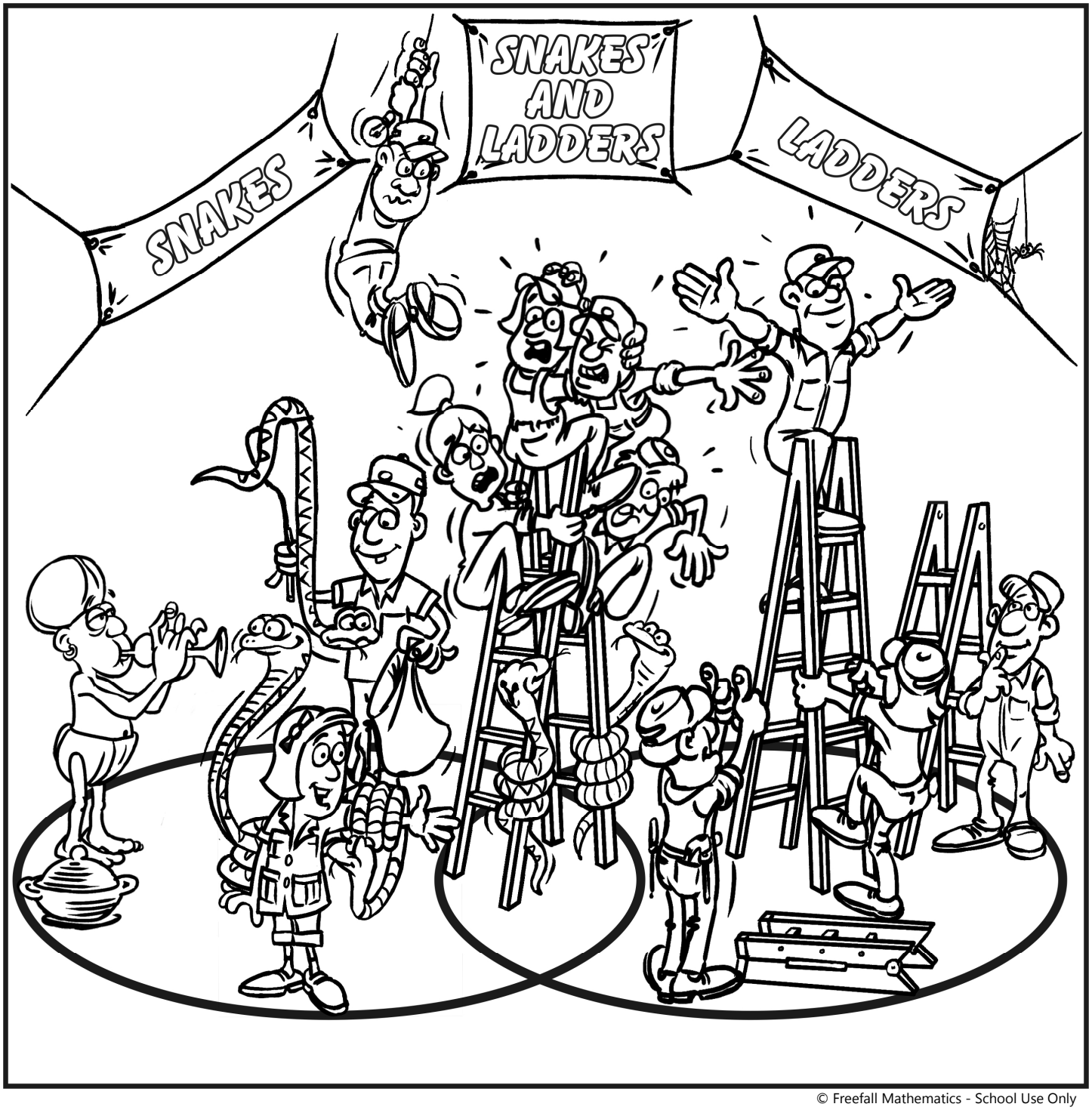
$P(\text{swim,crick}) = 13.6\%$

43 A student who has chosen cricket, to also choose swimming.

$P(\text{swim,crick}) = \frac{24}{93} \times 100$

$P(\text{swim,crick}) = 25.8\%$

Probability



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Multiple Probability Devices - (Sheet 1)

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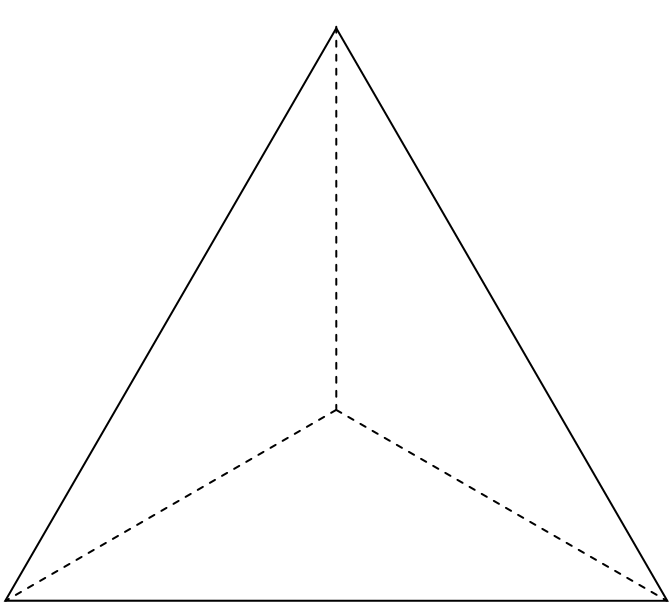

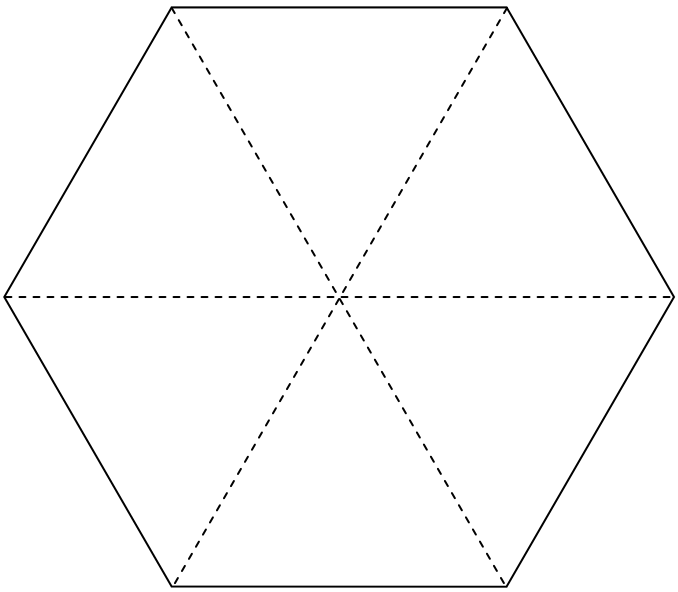

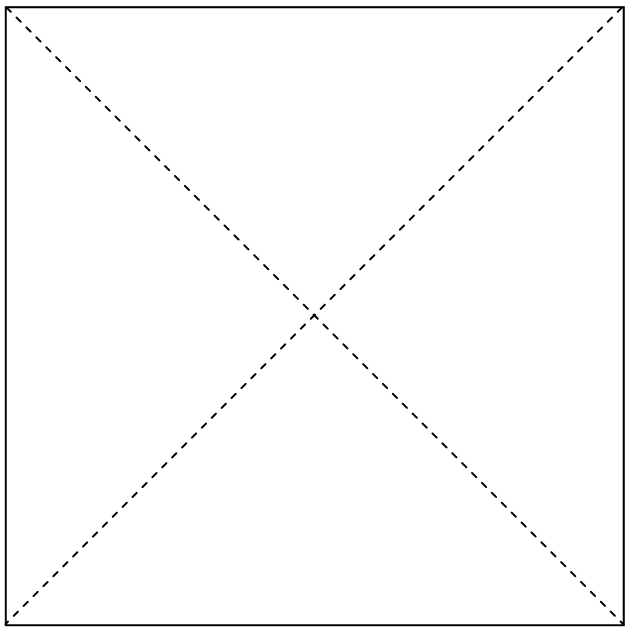

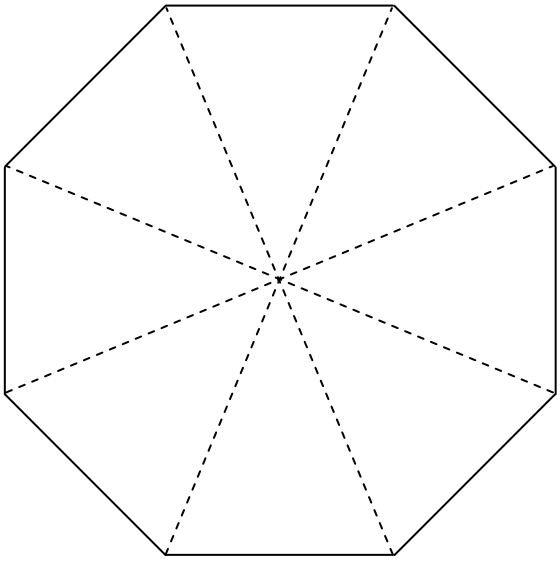

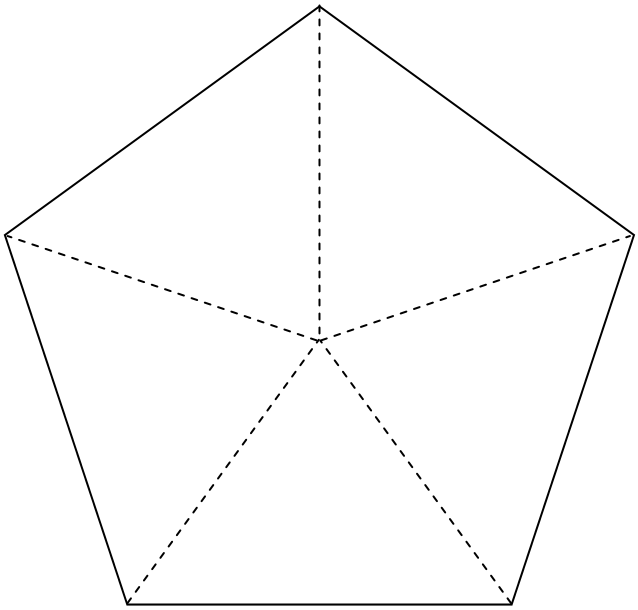

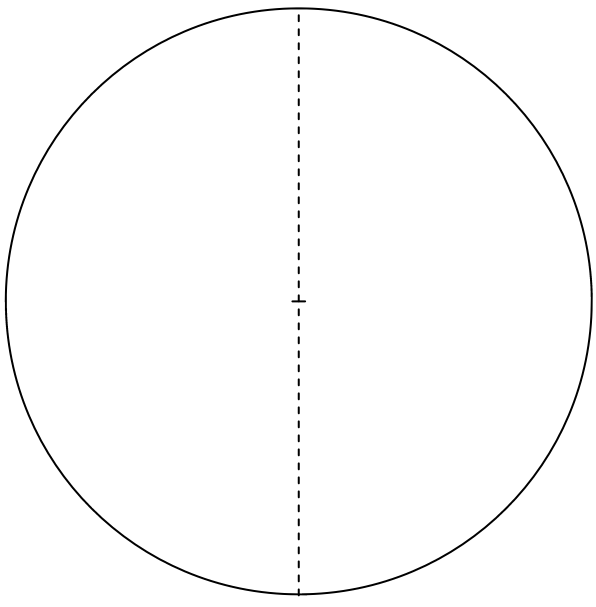

Multiple Probability Devices - (Sheet 2)

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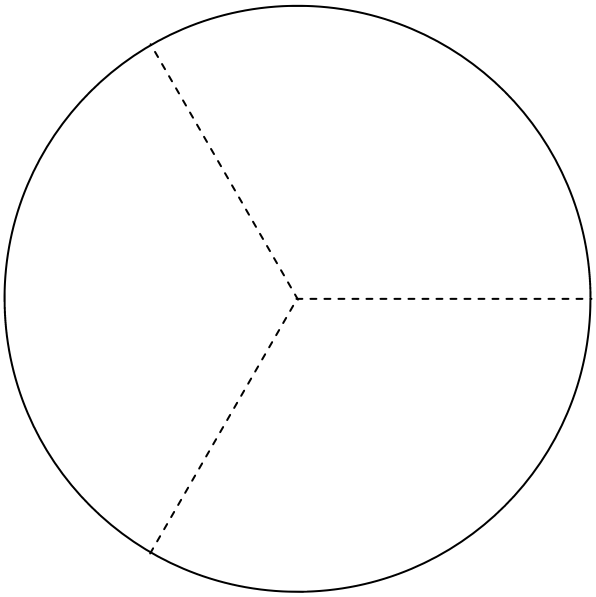
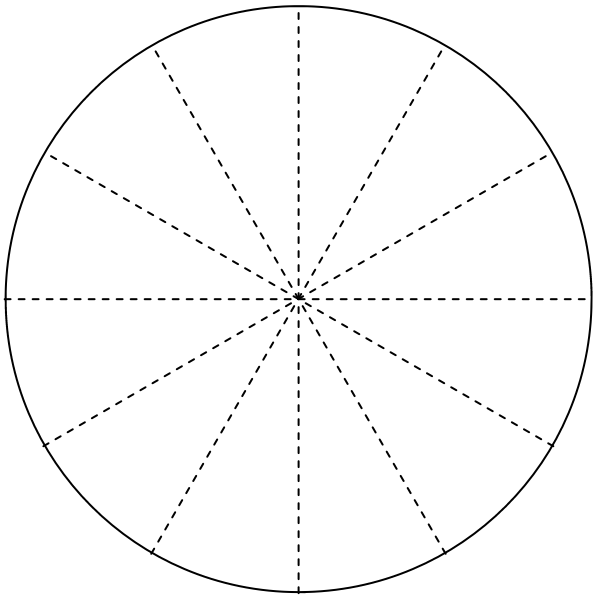
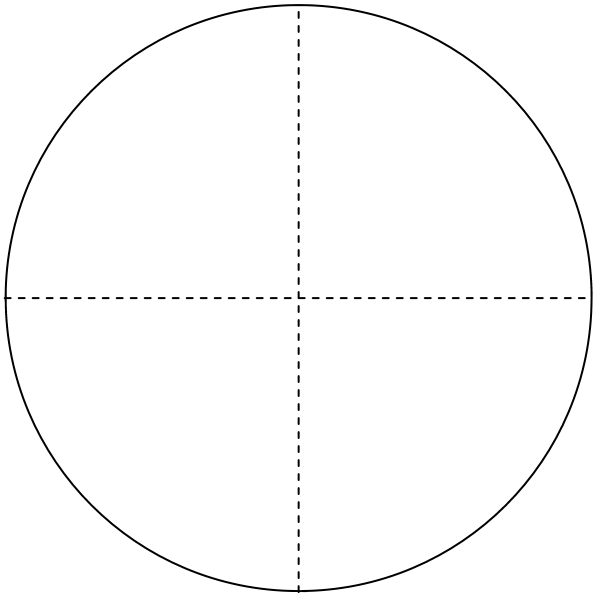
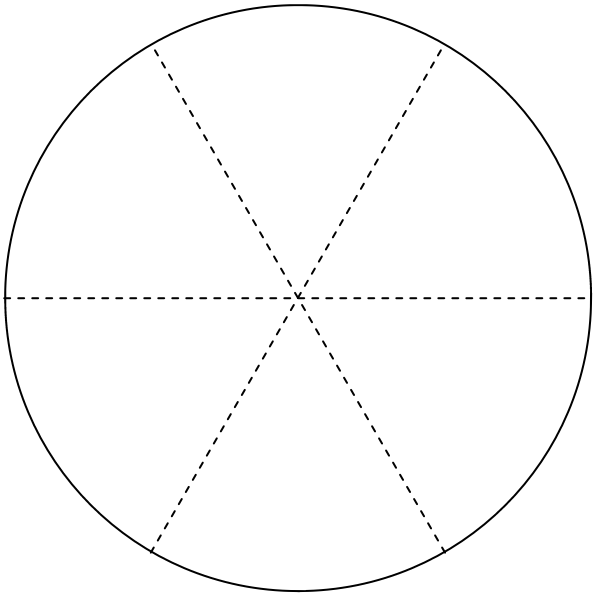






1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36



1	2	3	4	5	6
6	1	2	3	4	5
5	6	1	2	3	4
4	5	6	1	2	3
3	4	5	6	1	2
2	3	4	5	6	1

Probability Review (Sheet 1)

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List the sample space for the following. Which is a list of all the possible outcomes

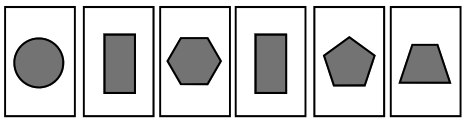
1 A coin.

Sample Space is { ____ }

2 A die.

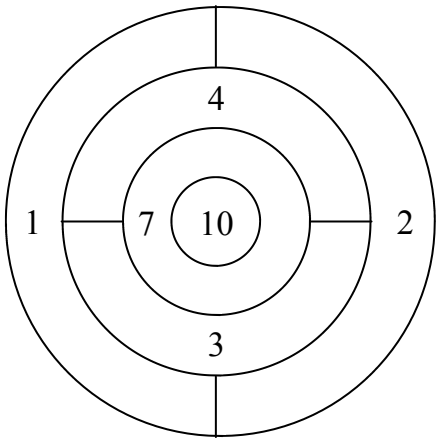
Sample Space is { _____ }

3 These cards.



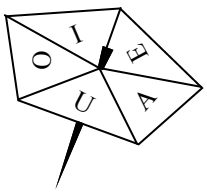
Sample Space is { _____ _____ }

4 This stickball board



Sample Space is { _____ }

5 This spinner



Sample Space is { _____ }

Use your calculator's fraction key, or your mathematical skills to simplify these fractions.

6 $\frac{2}{8} = \frac{\square}{\square}$

7 $\frac{10}{15} = \frac{\square}{\square}$

8 $\frac{8}{10} = \frac{\square}{\square}$

9 $\frac{9}{12} = \frac{\square}{\square}$

10 $\frac{12}{16} = \frac{\square}{\square}$

11 $\frac{12}{20} = \frac{\square}{\square}$

Find the probability of these events occurring.

Jay's pencil case contains 4 blue pens, 3 red pens, 2 black pens, 2 green pens and a purple pen.

12 How many pens in Jay's pencil case? $\square$

If he reaches in without looking what is the chance of the selected pen being:

13 Green $P(Gr) = \frac{\square}{\square} = \frac{\square}{\square}$ Simplify

14 Red $P(Red) = \frac{\square}{\square} = \frac{\square}{\square}$

15 Red or Purple

$P(R,P) = \frac{\square}{\square} = \frac{\square}{\square}$

16 Any colour but blue

$P(\widetilde{Blue}) = \frac{\square}{\square} = \frac{\square}{\square}$

17 A Green or Blue Pen

_____ $= \frac{\square}{\square} = \frac{\square}{\square}$

18 An orange pen

_____ $= \frac{\square}{\square} = \square$

A bag has 8 discs inside. The discs are labeled 1 through to 8. If one disc is randomly selected:

19 Find P (Even)

_____ $= \frac{\square}{\square} = \frac{\square}{\square}$

20 Find P (< 7)

_____ $= \frac{\square}{\square} = \frac{\square}{\square}$

21 Find P (Factor of 12)

Factor means what numbers go into 12

_____ $= \frac{\square}{\square}$

22 Find P (Prime number)

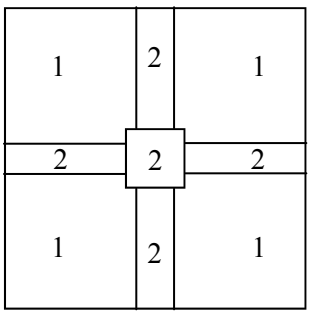
A prime no. has no factors other than 1 & itself
Note: 1 is considered not prime or composite

_____ $= \frac{\square}{\square} = \frac{\square}{\square}$

23 Find P (>4)

_____ $= \frac{\square}{\square} = \frac{\square}{\square}$

A sideshow has a large dartboard displayed (below). If you land the dart on a 2, you double your money. Get a 1 and you lose your money.
There are more 2's than 1's, so there is more chance of winning than losing.
Discuss this last sentence.



_____

_____

_____

_____

Probability Review (Sheet 2)

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A bag contains the letters that spell P-R-O-B-A-B-I-L-I-T-Y.

If a student randomly selects a single letter, what is the chance it is:

1 An 'L'. P(L) = $\frac{\square}{\square}$

2 An 'I'. P(I) = $\frac{\square}{\square}$

3 An 'I' or 'R'. P(I,R) = $\frac{\square}{\square}$

4 A vowel. P(vowel) = $\frac{\square}{\square}$

5 Not an 'I' or 'B'. $P(\widetilde{I,B}) = \frac{\square}{\square}$

If 25 ping-pong balls are labeled 1 to 25, find the chance of selecting:

6 A '25'. P(25) = $\frac{\square}{\square}$

7 A '19'. P(19) = $\frac{\square}{\square}$

8 A ball with a '5' digit. P(5 digit) = $\frac{\square}{\square}$

9 A ball with a two-digit number, greater than 18.

_____ = $\frac{\square}{\square}$

10 A ball without a '2' in the number.

_____ = $\frac{\square}{\square}$

11 A ball with a '1' in the number.

_____ = $\frac{\square}{\square}$

A lottery has 100 tickets, numbered 1 to 100. There is a single prize awarded. Find the probability of:

12 Buying a single ticket and winning the prize.

_____ = $\frac{\square}{\square}$

13 Buying 20 tickets and winning the prize.

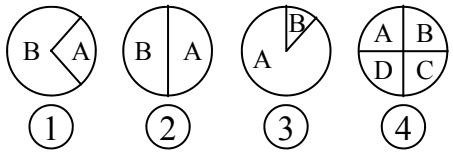
_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

14 Buying 15 tickets and not winning the prize.

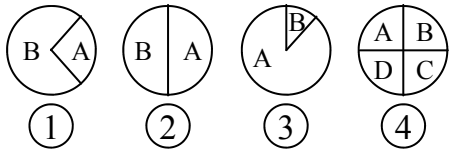
_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

Which spinner is most likely to generate the following results? Fill in the circle of your choice.

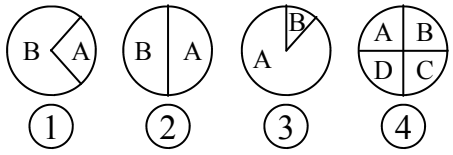
15 10 A's and 2 B's



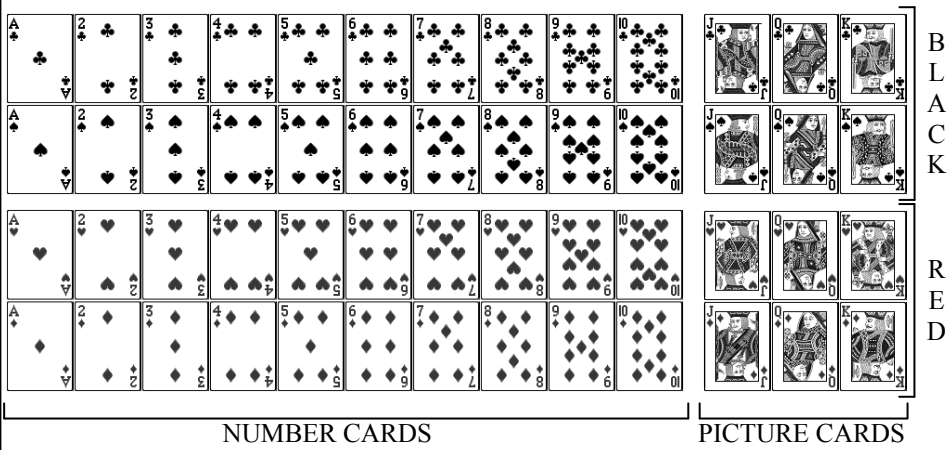
16 20 A's and 21 B's



17 5 A's and 15 B's



A deck of cards has 4 suits. Two suits are black, two are red. Each suit has 10 number cards: Ace through to 10, and 3 picture cards: Jack, Queen and King.



If one card from a standard deck is chosen, find the probability of selecting:

18 A queen

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

19 A red picture card

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

20 A number card

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

21 A 5, 8 or a jack.

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

22 A number card less than 7

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

23 Any card but a red king

_____ = $\frac{\square}{\square}$ = $\frac{\square}{\square}$

Probability Experiment Sheet

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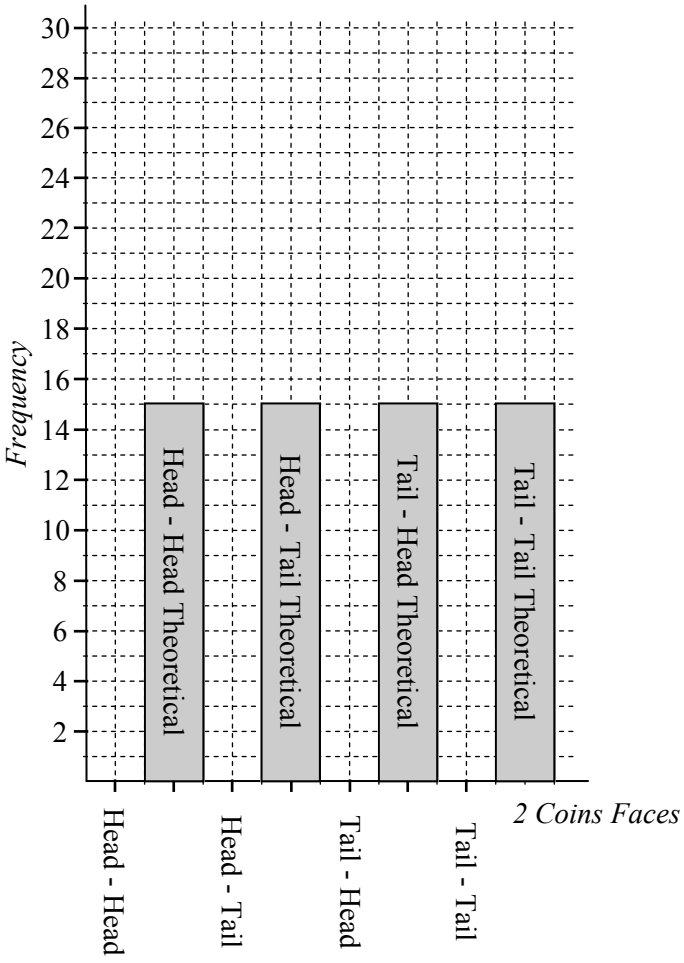
Toss two coins 60 times and record the results below. Toss one coin first then the second, or use two different value coins and call one Coin 1 and the other Coin 2 and toss them together.

Coin 1	Coin 2	Tally	*f*
Head	Head		
Head	Tail		
Tail	Head		
Tail	Tail		
			$\Sigma f =$

Column Graph: Two Tossed Coins Theoretical and Experimental



Explain how the experimental results compared with the theoretical results. Was the sample space large enough to get the expected result?

Now dice. Roll two dice 72 times adding the two face numbers together to get a total. Complete the table and grid, refer to grid to graph theoretical results. Compare the results with the theoretical.

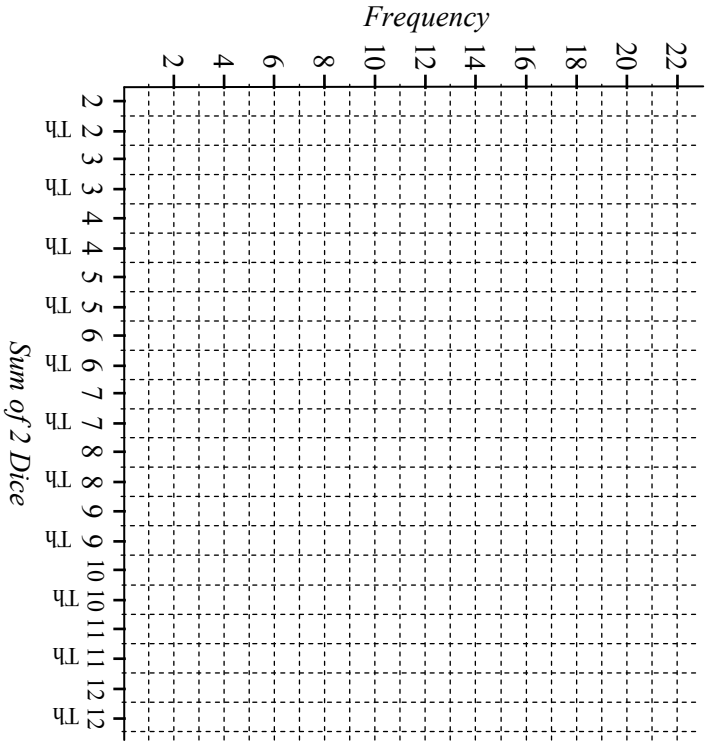
Total	Tally	*f*
2		
3		
4		
5		
6		
	$\Sigma f =$	

Total	Tally	*f*
7		
8		
9		
10		
11		
12		
	$\Sigma f =$	

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Theoretical Probability Grid

This grid has 36 outcomes, so with 72 rolls, double the frequency of each number in the grid and use that as your expected theoretical frequency for the columns below.



Explain how the experimental results compared with the theoretical results.

Generating Random Numbers - Decimals (Sheet 1)

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Use your calculator to generate random decimals.

Simulation 1

1 Generate 10 decimals (3 d.p.)

Calc: SHIFT . =
For the 1st decimal

Calc: =
For the next decimal

_____, _____, _____, _____, _____, _____, _____, _____, _____, _____.

Simulation 2

2 Generate 50 decimals (3 d.p.)

_____, _____, _____, _____, _____, _____, _____, _____, _____, _

Generating Random Numbers - Decimals (Sheet 2)

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Use your calculator to generate random decimals, find the mean.

, , , , .

Total the 5 scores

Calculate the mean:

$\bar{x} = \frac{\square}{5} = \square$ (3 d.p.)

, , , , , , , , , , , , , , .

Total these 15 scores

$\bar{x} = \frac{\square}{15} = \square$ (3 d.p.)

, , , , , , , , , , , , , ,

Generating Random Numbers - Integers and Intervals (Sheet 3)

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Use your calculator to generate random numbers, to represent a tossed coin, a 1 = tail, a 2 = head

1 Generate 4 coin results.

Calc: ALPHA . 1 SHIFT) 2) =
For the 1st integer from 1 to 2

Calc: =
For the next integer

Write 'H' or 'T'

____, ____, ____, ____.
Write 'H' or 'T', NOT '1' OR '2'

2 Generate 10 coin results.

____, ____, ____, ____, ____,
____, ____, ____, ____, ____.

3 Generate 36 coin results.

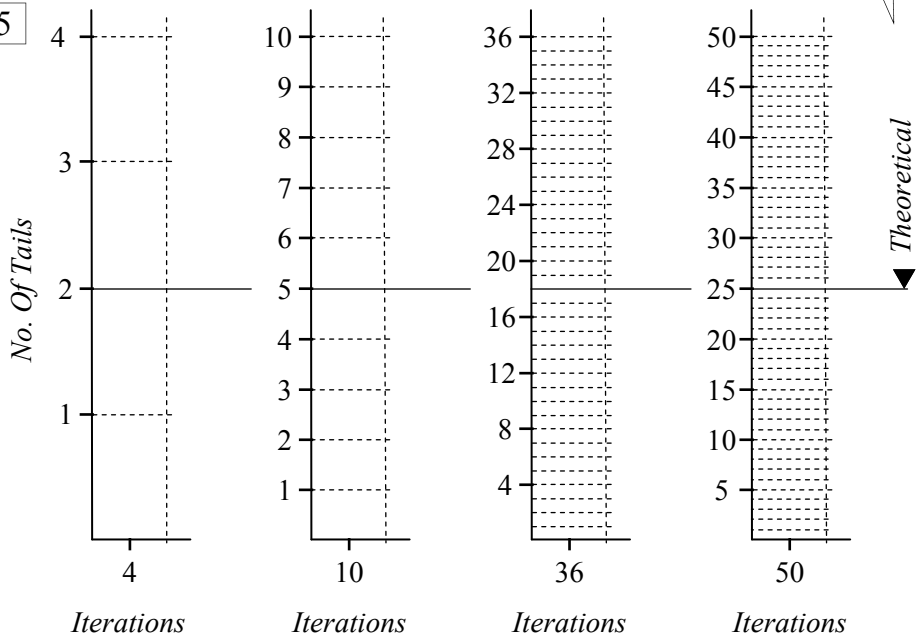
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____, ____, ____, ____, ____,
____.

4 Count the number of 'tails' results you recorded and display them in the table below.

Iterations	*No. of Tails*
4	
10	
36	
50	

Graph your results for question 4 on the grid, just a single column for each. All axes are scaled to the same height so you can compare them.

5



6 Compare the results. _____

Use your calculator to generate random numbers, to represent a die.

Calc: ALPHA . 1 SHIFT) 6) =
For the 1st integer from 1 to 6

Write '1' or '2' etc. up to 6

Calc: =
For the next integer

7 Generate 6 die results. ____, ____, ____, ____, ____, ____.

8 Generate 18 die results. ____, ____, ____, ____, ____, ____, ____,
____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____.

9 Generate 30 die results. ____, ____, ____, ____, ____, ____, ____,
____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____,
____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____.

If you rolled a die 6 times and achieved theoretical probability, you get 1 of each number. See if experimental probability gets closer to theoretical with more rolls! Complete the table using your experimental data above.

Iterations	1*'s*	2*'s*	3*'s*	4*'s*	5*'s*	6*'s*	*Theoretical*
6							1
18							3
30							5
54							9

Relative Frequency (Sheet 1)

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Simplify these fractions

1 $\frac{150}{195} = \frac{\square}{\square}$

2 $\frac{171}{285} = \frac{\square}{\square} = \frac{\square}{\square}$

3 $\frac{46}{138}$

Find the relative frequency for the data below. Express your answer as a simplified fraction.

- Total the frequency '*f*' column
- Write the 49, heads 'H' frequency in top box and Σf number in the bottom box
- simplify the fraction.

4 Coin Results

Face	*f*	*r.f.*
H	49	$\frac{\square}{\square} = \frac{\square}{\square}$
T	56	$\frac{\square}{\square} = \frac{\square}{\square}$
$\Sigma f =$		

5 Di Results

Face	*f*	*r.f.*
1	45	$\frac{\square}{\square} = \frac{\square}{\square}$
2	33	$\frac{\square}{\square} = \frac{\square}{\square}$
3	42	$\frac{\square}{\square} = \frac{\square}{\square}$
4	57	$\frac{\square}{\square} = \frac{\square}{\square}$
5	39	$\frac{\square}{\square} = \frac{\square}{\square}$
6	51	$\frac{\square}{\square} = \frac{\square}{\square}$
$\Sigma f =$		

James is recording data for 10 mins of the school parent-teacher evening. He records M=Man, W=Woman, G=Female Student, B=Male student. Put the data into a frequency table and calculate the relative frequencies.

6 Data:

WWMWB	MWGWM	WBWGM	BBMWW	
WMBWM	WMGGM	MMGMW	WWBMG	WMGMG
MMWWW	WMBWM	WMMWG	WMMWG	WWMMW

Type	*Tally*	*f*	*r.f.*
W	----------		$\frac{\square}{\square} = \frac{\square}{\square}$
M	----------		$\frac{\square}{\square} = \frac{\square}{\square}$
B			$\frac{\square}{\square} = \frac{\square}{\square}$
G			$\frac{\square}{\square} = \frac{\square}{\square}$
		$\Sigma f =$	

Deb works in a childcare centre, she asks parents how many children are in their family. Place the data into a frequency table and calculate the relative frequencies.

7 Data:

1, 2, 2, 1, 3,	2, 1, 1, 2, 1,	1, 3, 1, 2, 1	1, 2, 1, 2, 3,	2, 1, 1, 2, 4,
1, 4, 2, 2, 1,	1, 1, 1, 2, 1,	2, 3, 2, 1, 1,	1, 4, 1, 3, 1,	1, 2, 2, 2, 1,
2, 2, 1, 2, 1,	2, 1, 1, 3, 1,	1, 2, 1, 2, 1,	1, 3, 2, 2, 2,	2, 4, 2, 1, 2,
3, 2, 1, 1, 2,	3, 2, 3, 2, 2,	1, 4, 1, 2, 3,	2, 3, 1, 1, 2,	1, 1, 2, 1, 1,

Children	*Tally*	*f*	*r.f.*
	----------		$\frac{\square}{\square} = \frac{\square}{\square}$
	----------		$\frac{\square}{\square} = \frac{\square}{\square}$
			$\frac{\square}{\square} = \frac{\square}{\square}$
			$\frac{\square}{\square} = \frac{\square}{\square}$
		$\Sigma f =$	

Relative Frequency (Sheet 2)

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Find the relative frequency for the data for sales at Dave's Electrical. Then answer the questions.

Product	*f*	*r.f.*
Toaster	15	$\frac{\square}{\square} = \frac{\square}{\square}$
Kettle	18	$\frac{\square}{\square} = \frac{\square}{\square}$
Phone	12	$\frac{\square}{\square} = \frac{\square}{\square}$
PC/ Tablet	8	$\frac{\square}{\square} = \frac{\square}{\square}$
Other	17	$\frac{\square}{\square}$
$\Sigma f =$		

Based on the sales figures above apply relative frequencies to estimate sales. If 350 products were sold, what would be the sales of:

1 Toasters?

$\frac{\square}{\square} \times \square = \square$

2 Phones?

$\frac{\square}{\square} \times \square = \square$

3 Kettles or Toasters?

Add the fractions for kettles and toasters first

$\frac{\square}{\square} \times \square = \square$

4 Everything but toasters?

Subtract the toasters fraction from 1

$\frac{\square}{\square} \times \square = \square$

5 PCs, Tablets or phones?

$\frac{\square}{\square} \times \square = \square$

Find the relative frequency for the daily sales at Pete's Pies. Then answer the questions.

Product	*f*	*r.f.*
Pies	95	
Sausage Rolls	66	
Pastries	125	
Bread	106	
Other	48	
$\Sigma f =$		

Based on the sales figures above apply relative frequencies to estimate sales. Estimate the number of pies sold if total sales were:

6 500?

$\frac{\square}{\square} \times \square = \square$

Round to nearest pie!

7 800?

$\frac{\square}{\square} \times \square = \square$

8 300?

$\frac{\square}{\square} \times \square = \square$

Pete has run a promotion and estimates sales will be 1 300 items on the day. Estimate the number of:

9 Pies that Pete should bake.

10 Products other than bread he should bake.

Write the relative frequencies as decimals. This time it is weekly car sales. No need to round.

Car Type	*f*	*r.f.*
Family	6	0.15
Hatch	5	
4WD	16	
Hybrid	2	
Sports	4	
Wagon	3	
Other	4	
$\Sigma f =$		

Multiply relative frequencies by 100 to answer these questions.
What percentage of sales were:

11 Sports cars?

12 Family cars or wagons?

13 Not a hatchback?

14 Hatchbacks or hybrids?

If a person entered the sales area, what three types of vehicle are they most likely to buy? List them in order of popularity, most popular first.

15

The showroom floor has room for 5 cars. If the showroom cars are to reflect the above sales figures, how many:

16 4WDs should be displayed?

Relative Frequency (Sheet 3)

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A bag contains red, green and blue marbles. Apply your relative frequency knowledge.

A marble is selected and replaced from the bag 20 times, the results are:
Red 8, Green 10 and Blue 2.

1 Are there more red marbles than blue? ○ Yes ○ No ○ Can't tell

2 Are there 20 marbles in the bag? ○ Yes ○ No ○ Can't tell

3 Is it possible that there are more blue marbles than green or red? ○ Yes ○ No

4 Are there definitely more green marbles than blue marbles? ○ Yes ○ No

A computer monitor manufacturer sorts its products as follows:
Class A - Zero Dead Pixels
Class B - 1 to 2 Dead Pixels
Class C - More than 2 Dead Pixels
After taking samples it is found:
Zero Dead Pixels have *r.f.* of 0.87
One Dead Pixel have an *r.f.* of 0.08
Two Dead Pixels have an *r.f.* of 0.03

5 What is the *r.f.* for a monitor to be 'Class A'.

P(Class A) = ☐

6 Calculate the *r.f.* for a monitor to be 'Class B'.

7 Calculate the *r.f.* for a monitor to be 'Class C'.

If 800 monitors are produced in a day, how many would be:

8 'Class A' monitors?

9 'Class B' monitors?

10 'Class C' monitors?

Monitor wholesale prices are:
Class A - $350
Class B - $185
Class C - $55

11 Calculate the income for the 800 monitors.

A fire station has analysed its response times. They are:
Arrival Under 5 mins: 77%
Arrival within 5 to 10 mins: 20%
Arrival more than 10 mins: 3%

12 Convert the percentages to decimals.

77% = ☐ 20% = ☐

3% = ☐

If the station responds to an average of 56 calls per year, calculate (nearest whole number) the number of arrivals:

13 In under 5 minutes

14 In 10 minutes or less.

15 Within 5 to 10 minutes.

16 In 5 minutes or more.

The Ginormous Egg Company only sells eggs larger than 80 g. It sells the smaller eggs to wholesalers.
Its production statistics are:
Saleable eggs 80 g and over: 0.67
Wholesale eggs less than 80 g: 0.27
All other eggs are discarded.

17 Calculate the *r.f.* for an egg to be discarded.

If production averages 17 000 eggs per day, calculate the number of:

18 Daily wholesaled eggs.

19 Weekly discarded eggs.

20 Yearly company-sold eggs.

Venn Diagrams (Sheet 1)

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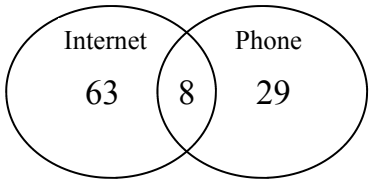
A web-based bank account offers account access by either telephone or internet. Answer these questions.

Customer Usage (%)



What percentage of people access their account using:

1 A phone only? ____

2 A phone? ____

3 The internet? ____

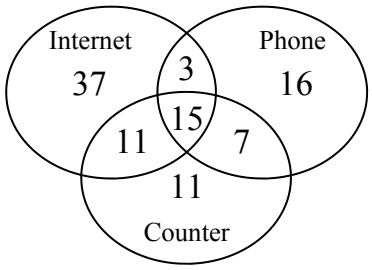
4 The internet only? ____

5 One method only? ____

6 Both methods? ____

The 'extras' bank account includes branch counter service, answer these.

Extras Customer Usage (%)



What percentage of people access their 'Extras' account:

7 Using a phone only? ____

8 Using counter services? ____

9 Using all three methods? ____

10 Not using the internet? ____

What percentage of people access their 'Extras' account using:

11 Both counter and internet services? ____

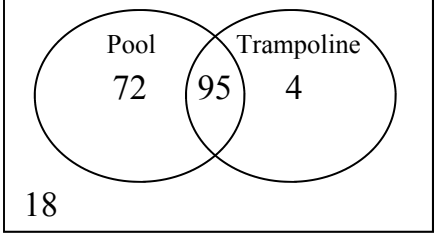
12 Both counter and internet services only? ____

13 A phone but not the internet? ____

14 Counter services but not phone services? ____

A swimming complex offers two pools and six trampolines. Answer these questions.

Monday's Entry Figures (people)



How many people entering the complex on Monday:

15 Only used the pool? ____

16 Used nothing? ____

17 Used only the trampolines? ____

18 Used only one service? ____

19 Didn't enter a pool? ____

20 Didn't use the trampolines? ____

21 Used both services? ____

22 Used the pool? ____

23 Used a pool or a trampoline? ____

Tickets cost: \$2 for the trampolines, \$3 for the pool, \$4 for both and \$1 for spectating care-givers. What were the gate takings?

24 ____

Calculate the relative frequency, as a percentage (to 1 d.p.) for a person to enter the complex and:

25 Use the Pool

All people Using pool

$P(\text{pool}) = \frac{\square}{\square} \times 100$

All People

$P(\text{pool}) =$ ____

Show % sign, round to 1 d.p.

26 Use only the pool or only the trampoline.

$P(\text{tr,pl}) = \frac{\square}{\square} \times \square$

$P(\text{tr,pl}) =$ ____ % sign!

27 Not use the trampoline.

28 Not use the pool.

29 Use the pool only.

Venn Diagrams (Sheet 2)

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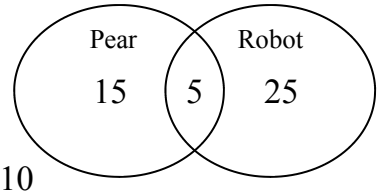
A survey of people that own a mobile phone has had the results shown in the Venn Diagram. Answer these questions.

Mobile Phone Operating Systems



How many people own:

1 A Pear-based phone? ____ (15 + 5)

2 A Robot-based phone? ____

3 A phone that is not Pear or Robot based? ____

4 Both Pear and Robot based phones? ____

Calculate the relative frequency, expressed as a simplified fraction, for a person having a:

5 Pear-based phone.

$P(\text{Pear}) = \frac{15+5}{15+5+25+10} = \frac{\square}{\square}$ (Simplify Using Fraction Key)

6 Phone being neither Pear or Robot based.

$P(\widetilde{P,R}) = \frac{\square}{\square} = \frac{\square}{\square}$

7 Robot-based phone only.

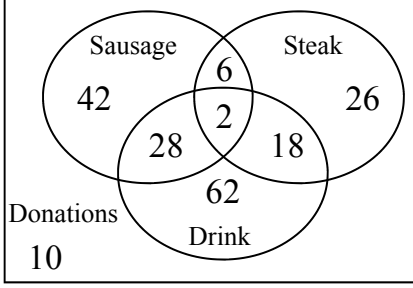
$P(\text{R only}) = \frac{\square}{\square} = \frac{\square}{\square}$

8 Phone that isn't Pear-based.

$P(\widetilde{\text{Pear}}) = \frac{\square}{\square} = \frac{\square}{\square}$

A school stall sells sausage and steak sandwiches as well as drinks. Some people donate money without buying anything.

Sausage Sizzle School-Fund Raiser



How many people bought:

9 A sausage sandwich? ____

10 Only a Sausage sandwich? ____

11 Nothing but donated? ____

12 All three products? ____

13 Both a drink and a sausage sandwich? ____

14 Two products only? ____

15 A steak sandwich and a drink only? ____

Calculate the total number of:

16 Visitors ____ 17 Purchasers ____

How many people came to the stall but didn't buy:

18 A sausage sandwich? ____

Include people making a donation

19 A sandwich? ____

20 A drink? ____

21 A drink or a steak sandwich? ____

22 Two products or more? ____

It's time to analyse the school sales. Use the Sausage Sizzle Venn Diagram to find these.

Calculate the relative frequency as a percentage (1 d.p.) for a person:

23 Visiting the stall and buying any product or products.

$P(\text{buy}) = \frac{\square}{\square} \times 100$ (All numbers in rings Q.17 / All numbers)

$P(\text{buy}) =$ ____

Show % sign, round to 1 d.p.

24 Visiting the stall and buying a drink.

$P(\text{drink}) = \frac{\square}{\square} \times \square$

$P(\text{drink}) =$ ____ (% sign!)

25 Buying from the stall and buying a steak sandwich.

Don't include people giving donation - Buying

26 Buying from the stall and buying both a drink and a sandwich.

27 Visiting the stall and not buying a drink.

28 Buying from the stall and not buying a sausage sandwich.

Probability - Two-Way Tables (Sheet 1)

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Two-way tables can be read horizontally or vertically, answer the questions.

Year 9 Survey: Weather vs. Type of Clothing

	Shorts	Trousers	Total
Hot	198	24	A
Cold	36	162	B
Total	C	D	E

1 Complete the total boxes in the table above.

For the next 5 questions, how many Year 9 students:

2 Wore shorts on the hot day? ____

3 Wore shorts on the cold day? ____

4 Wore trousers on the cold day? ____

5 Wore trousers? ____ 6 Were surveyed? ____

7 An excursion was on one of the two survey days, which day, hot or cold, do you think it was on. Explain your answer.

_____

_____

Writing your answer as a fraction, then simplify it. Calculate the relative frequency of a Year 9 student:

8 On a cold day, wearing shorts. $P(sh,cold) = \frac{\square}{\square} = \frac{\square}{\square}$

9 On a hot day, not wearing shorts. $P(\widetilde{sh},hot) = \frac{\square}{\square} = \frac{\square}{\square}$

10 Wearing trousers. (On a cold or hot day). $P(trous) = \frac{\square}{\square} = \frac{\square}{\square}$ Careful! →

Writing your answer as a percentage (to 1 d.p.), calculate the relative frequency of a Year 9 student:

11 On a cold day, wearing trousers. $P(cold,tr) = \frac{\square}{\square} \times 100$

$= \frac{\square}{\square}$ 1 d.p. Show %

Vet Survey: Pets vs. Flea Infestation

	Cats	Dogs	Total
No fleas	188	285	A
Fleas	18	20	B
Total	C	D	E

12 Complete the total boxes in the table above.

13 How many cats had fleas? ____

14 How many dogs had fleas? ____

15 Calculate the relative frequency for a cat to have fleas, express your answer as a percentage (1 d.p.).

$P(cat,fleas) = \frac{\square}{\square} \times \square = \frac{\square}{\square}$ Show % sign

16 For a dog to have fleas.

$P(dog,fleas) = \frac{\square}{\square} \times \square = \square$

17 For a pet with fleas, to be a dog.

$P(fleas,dog) = \frac{\square}{\square} \times \square = \square$

Bus Transport Survey: Weather vs. Punctuality

	On-time	Late	Total
Raining	50	A	65
Dry	B	5	65
Total	D	E	C

18 Complete the lettered boxes in the table above.

19 How many buses were late? ____

Find the relative frequency as a simplified fraction for:

20 On a dry day, a bus being on-time . $P(dry,on\text{-}time) = \frac{\square}{\square} = \frac{\square}{\square}$

21 On a rainy day, a bus being late. $P(rain,late) = \frac{\square}{\square} = \frac{\square}{\square}$

Probability - Two-Way Tables (Sheet 2)

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The key to car wash success is to never lose your drive. Answer these questions on two-way tables.

Car Wash Survey: Wash vs. Vacuum

	Wash	No Wash	Total
Vacuum	78	4	A
No Vacuum	36	0	B
Total	C	D	E

1 Complete the total boxes in the table above.

Find the relative frequency as a percentage (1 d.p.) from:

2 Cars having a car wash, cars that don't get vacuumed.

$P(\text{wash,no vac}) = \frac{\square}{\square} \times \square = \square$

3 Cars having a vacuum, cars not getting a car wash.

$P(\text{vac,no wash}) = \frac{\square}{\square} \times \square = \square$

4 Cars not having a car wash, cars getting a vacuum.

5 Cars entering the car wash, cars getting a car wash.

Year 9 Hair Survey: Length vs. Curliness

	Straight	Curly	Total
Short		4	125
Long	62		80
Total			

6 Complete all the missing cells in the table above.

Writing your answer as a fraction, then simplify it. Calculate the relative frequency of a Year 9 student:

7 With curly hair, having short hair. $P(\text{curl,sh}) = \frac{\square}{\square} = \frac{\square}{\square}$

8 Having curly short hair. $P(\text{curl,sh}) = \frac{\square}{\square}$



We are still dealing with hair, well you are, I'm not!

Calculate the relative frequency of a Year 9 student:

9 With long hair, having curly hair. $P(\text{lng,curl}) = \frac{\square}{\square} = \frac{\square}{\square}$

10 With curly hair, having long hair. $P(\text{curl,lng}) = \frac{\square}{\square} = \frac{\square}{\square}$

11 Having short hair. $P(\text{short}) = \frac{\square}{\square} = \frac{\square}{\square}$

12 Having straight hair. $P(\text{strgt}) = \frac{\square}{\square}$

Year 9 Language Survey: Language vs. Sex

	French	German	Total
Male		16	30
Female	12		
Total			56

13 Complete all the missing cells in the table above.

14 The survey was taken on a German class and a French class, how many students in each class?

German Students = $\square$ French Students = $\square$

15 If 12 language students were interviewed, how many would you expect to be male and studying French?

16 If there are 95 Year 9 students, how many don't study a language?

Find the relative frequency, as a % to 1 d.p., that a Year 9 student (full year group) chosen randomly, would be:

17 Male and study German.

18 Female and studies a language

Probability - Find a Word

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Find the words in the puzzle from the wordlist.



T	H	E	O	R	E	T	I	C	A	L	X	P	E	P
A	S	V	U	E	G	A	T	N	E	C	R	E	P	R
L	U	R	E	Y	L	L	G	X	X	P	E	R	R	O
L	R	V	E	N	A	D	E	L	P	M	A	S	E	B
A	V	E	L	L	N	E	N	Y	L	L	A	T	D	A
T	E	E	B	L	T	C	E	S	R	E	T	N	I	B
N	Y	V	I	L	E	I	R	E	G	D	M	Y	C	I
E	L	U	S	I	G	M	A	R	N	I	A	C	T	L
M	A	R	S	K	N	A	T	G	I	A	R	N	L	I
I	C	U	O	E	R	L	E	Y	R	G	G	E	R	T
R	I	W	P	L	E	L	I	L	U	R	O	U	A	Y
E	T	L	K	Y	B	V	K	K	C	A	T	Q	N	T
P	R	O	B	A	B	L	E	E	C	M	S	E	D	W
X	E	K	T	W	L	A	T	N	O	Z	I	R	O	H
E	V	I	T	A	L	E	R	A	T	E	H	F	M	O

WORDLIST

VENN	EVENT	POSSIBLE	TABLE	LIKELY
PERCENTAGE	RELATIVE	INTERSECT	HORIZONTAL	HISTOGRAM
SIGMA	PROBABILITY	TWO	SAMPLE	WAY
FREQUENCY	VERTICAL	DECIMAL	DIAGRAM	GENERATE
SURVEY	OCCURING	PREDICT	RANDOM	PROBABLE
	EXPERIMENTAL	TALLY	THEORETICAL	

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10 cards numbered 1 to 10 are placed in a hat, then one card is randomly selected. Find the probability of:

1 Selecting an '8' card.

$P(8) = \frac{\square}{\square}$

2 Selecting an odd numbered card.

$\square = \frac{\square}{\square} = \frac{\square}{\square}$

3 Selecting a card with a number greater than 4.

$\square = \frac{\square}{\square} = \frac{\square}{\square}$

4 Selecting a card with a number that is a factor of 8.

$\square = \frac{\square}{\square} = \frac{\square}{\square}$

5 Not Selecting a card with a number less than 3.

$\square = \frac{\square}{\square} = \frac{\square}{\square}$

15 discs are in a bag. A single disc is selected, its colour recorded, then replaced. After 25 selections there have been 10 black discs, 3 red discs and 12 green discs selected.

Answer True or False

6 There are definitely more Green discs in the bag. (True) (False)

7 The bag may contain 12 purple discs. (True) (False)

8 There are definitely more green discs than red discs. (True) (False)

(These questions are on the discs still). If a disc was selected and replaced 325 times, estimate the:

9 Number of times a red disc would be selected.

_____

10 Number of times a black disc would be selected.

_____

11 Number of times a selected disc would be green or red.

_____

If a card is chosen from a standard deck of cards. Calculate the probability of:

12 Selecting a red card.

_____

13 Selecting a black picture card.

_____

14 Selecting a '5', '7' or an ace.

_____

15 Not selecting a picture card or an ace.

_____

Three Year 9 boys have nominated for the male SRC position, Joe, Omar and Lance. 70 students vote by selecting one of these students. Results are below.

Data:				
LJLOJ	LLOOJ	JOLLO	OOJLL	
LLLOJ	OOJLO	LOOLL	JLLLO	OLLLJ
OLJLO	JJLLL	JOLOO	LLLLJ	OLLLJ

16 Complete the table, simplifying the relative frequencies.

Student	*Tally*	*f*	*r.f.*
Joe	----------	$\square$	$\frac{\square}{\square} = \frac{\square}{\square}$
Omar	----------	$\square$	$\frac{\square}{\square} = \frac{\square}{\square}$
Lance	----------	$\square$	$\frac{\square}{\square} = \frac{\square}{\square}$
		$\Sigma f = \square$	

25 students were on an excursion on the day, but will vote the next day. Based on the relative frequencies above. How many of the 25 students:

17 Would vote for Omar.

_____

18 Would not vote for Lance.

_____

19 Who would get the most of the total vote and how many would they get?

_____

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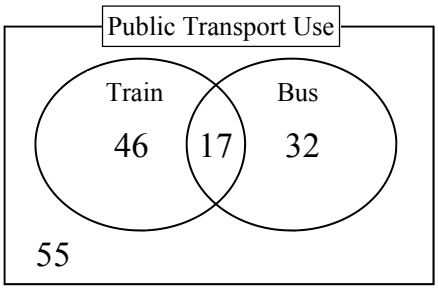
A business asked its staff who use public transport which type of transport they use. Results are shown



How many people:

20 Used the train only? ____

21 Used a both a bus and a train? ____

22 Didn't use a train? ____

23 Were surveyed? ____

Calculate the relative frequency and simplify the fraction if possible, for:

24 A person catching a train.

____ = ____ = ____

25 A person catching only a train, or only a bus.

____ = ____ = ____

26 A person not catching a train

____ = ____ = ____

Calculate the relative frequency as a percentage (1 d.p.), for:

27 A person not catching a train, or a bus.

The results for a survey of Year 9 students are shown below. Complete the spaces in the table and then answer the questions.

	Right-H	Left-H	Total
Female	45	6	
Male	52	3	
Total			

How many Year 9 students:

28 Are left-handed? ____

29 Were surveyed? ____

30 Are male? ____

Find the relative frequencies for these, answer as a percentage (to 1 d.p.)

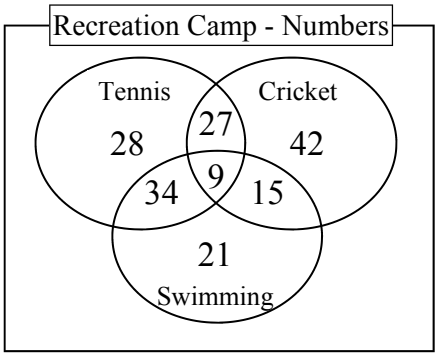
31 That a right-handed student, would be male.

32 That a female student, would would be left-handed.

33 That a Year 9 student, would be male and left-handed.

34 That a Year 9 student, would be right-handed.

35 That a Year 9 student, would be female.



A recreation camp offers a choice of three sports, which students can do individually or in conjunction with other sports. Answer these questions

How many students have chosen:

36 Just swimming? ____

37 Swimming? ____

38 All three sports? ____

39 Both tennis and cricket? ____

40 One sport only? ____

41 Not to do swimming? ____

Calculate the relative frequency as a percentage (1 d.p.), for:

42 A student to choose both swimming and cricket.

43 A student who has chosen cricket, to also choose swimming.

VELOCITY

ORGANISING & DISPLAYING DATA

Organising & Displaying Data



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Organising & Displaying Data



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Data - Frequency Tables

The frequency of something is how often it occurs. So a Frequency Table records how many times scores in a data set occur. There are three columns to a frequency table, when the data is not already entered. These are the 'Score', 'Tally' and 'Frequency'. Score can be replaced with a title when the data is categorical (which means not numbers), for example 'Car Type' or 'Holiday Type', it can also just be called 'x'. The frequency column is regularly written just as 'f'. Note the use of Σf at the bottom of the frequency column. Σ means 'the sum of' so Σf means add all the frequency column up.

Example

Data: 6, 7, 5, 4, 6, 7, 10, 5, 6, 4, 9, 6, 5, 7, 6.

Score	Tally	Frequency
4		
5		
6		
7		
8		
9		
10		
		$\Sigma f =$

STEP 1

Circle the lowest score and the highest score in the data set.

STEP 2

In the score column of the table write in the lowest score at the top, increasing down the column until the highest score.

!

Note that there is no score of '8' in the data set. The number '8' is still included in the score column anyway.

Data: ~~6~~, 7, 5, 4, 6, 7, 10, 5, 6, 4, 9, 6, 5, 7, 6.

Score	Tally	Frequency
4		
5		
6	|	
7		
8		
9		
10		
		$\Sigma f =$

STEP 3

Data is now entered number by number Cross out the 6 and put a mark in the Tally column, then the 7 and so on.

!

Don't count all the 4's then all the 5's. This method is more likely to cause mistakes in large data sets. It also doesn't effectively use the Tally process.

Score	Tally	Frequency
4	||	2
5	|||	3
6	||||	5
7	|||	3
8		0
9	|	1
10	|	1
		$\Sigma f =$ 15

The 5th score is a cross stroke

STEP 4

Now change the Tally into numbers in the Frequency column.

STEP 5

Total the Frequency column to get Σf it has to equal the number of scores in the data set. If it doesn't ….start searching!

Data - Frequency Tables (Sheet 1)

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Complete the frequency column and total for Σf, then answer the questions.

1 Number of Fish in a Tank

Fish	Tally	Frequency
2	𝍸	
3	𝍸 ||	
4	𝍸 𝍸 |||	
5	𝍸 𝍸	
6	𝍸 𝍸 ||||	
7	𝍸 𝍸 |||	
		$\Sigma f =$

2 What number of fish had the highest frequency? _____

3 What number of fish had the same frequency? _____

4 What was the frequency for a tank to have an odd number of fish?

_____

5 What was the frequency for a tank to have less than 5 fish?

_____

6 Soft Drink Preference

Type	Tally	Frequency
A	𝍸 𝍸 𝍸 ||	
B	𝍸 𝍸 |||	
C	||||	
D		
E	𝍸 𝍸 ||	
F	𝍸 𝍸 𝍸	
		$\Sigma f =$ 75

7 D's results were unreadable due to a drink spill, use Σf to calculate them.

8 Rank the drink brands in order from least, to most popular.

_____

9 How many people preferred brand 'A' or brand 'F'? _____

Place the data into the Frequency table, showing the tally, the frequency and write 'Σf= *(total)*', then answer the questions.

10 Away from School - Sports Choices

Sport	Tally	Frequency
Tennis		
Gym		
Bocce		
Archery		

Data:

TTGAA	TTGAA
TGBBA	GBBAT
AAGGG	TTTTA
BTBTA	BTBBA
AAATG	GTBBA
BBGBT	AABTB
TTGBB	BBTBG
ATTTA	ABBTG

11 What was most popular 'away' sport chosen? _____

12 How many people chose 'ball sports'? _____

13 If there are a total of 135 students in the year, calculate the percentage that chose 'away' sports.

$$\%\ \text{Away Sport} = \frac{\square}{135} \times 100$$

$$= \underline{\square}\ \%$$

Round to 1 d.p.

14 A real estate agent is checking rental homes for smoke detectors, if the smoke detector was Battery powered, Mains powered, or Dual (both). If a detector didn't work its type wasn't recorded it was just recorded as Inoperative. If a house had no smoke detector, it was recorded as 'None'.

Smoke Detector Type or Status

Type	Tally	Frequency
Battery		
Mains		
Dual		
Inoperative		
None		

Data:

BBDMB	BBDBM
BIBBM	MNDIB
MBDBI	BBIIN
NBBIM	INDDB
BBNIB	NBBMB
MBBDN	IBBMB
NINBB	NDDMB

15 How many homes had operational smoke detectors? _____

16 What percentage (to 1 d.p.) of homes had inoperative or no smoke detectors?

$$\%\ \text{No S.D} = \frac{\square}{\square} \times 100 = \underline{\square}$$

Round to 1 d.p.

17 What percentage (to 1 d.p.) of homes had Dual-powered smoke detectors?

$$\%\ \text{Dual Powered} = \frac{\square}{\square} \times 100 = \underline{\square}$$

Round to 1 d.p.

Data - Frequency Tables (Sheet 2)

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Place the data below in a frequency table.

Before you start
Circle the smallest number
Circle the largest number

1 Data: ~~0~~, 3, 2, 6, 5, 3, 4, 5, 1, 6, 3, 4, 2, 5, 3, 2, 0, 5, 1, 2, 3, 5, 4, 3, 2, 4, 5, 2, 1, 5, 2, 3, 2, 5, 6, 1, 4, 3, 2, 2.

Score	Tally	Frequency
0	|	
		$\Sigma f =$

Count the numbers in the data, does it equal Σf?

2 Ages of Students on an Excursion.
Data: 15, 17, 12, 17, 16, 17, 12, 18, 13, 16, 17, 15, 11, 13, 13, 14, 12, 15, 17, 17, 15, 14, 13, 15, 16, 17, 12, 16, 14, 13, 15, 17, 17, 17, 16, 12, 14.

Table Title _____

Score	Tally	Frequency
		$\Sigma f =$

3 What was the age that occurred most? _____

4 How many students were older that 14? _____

5 How many teenage students were there? _____

6 What percentage of the students were 16 or older?

_____ = _____

Show working out | Answer to 1 d.p.

Student eye colour was recorded with the data shown, Br = Brown, Bl = Blue, G = Green and H = Hazel. Tabulate the results.

7 Data: Br, Br, Br, G, Bl, Br, Br, Br, Bl, H, H, Br, Br, Br, Br, Bl, H, Bl, Br, Br, H, Bl, G, Br, Br, Br, H, H, Bl.

_____

8 What was most common eye colour? _____

9 How many students had hazel eyes? _____

10 What percentage of the students had blue eyes?

_____ = _____

Answer to 1 d.p.

11 What percentage of the students had either brown or hazel eyes?

_____ = _____

Answer to 1 d.p.

12 Place the spelling test results in the table.
Data: 9, 6, 8, 3, 6, 8, 10, 8, 5, 6, 9, 7, 9, 8, 7, 3, 8, 10, 6, 9, 8, 7, 9, 5, 6, 9, 8, 8, 8, 7, 5, 9, 6, 3, 7, 9, 6, 10, 8, 7, 10, 6, 5, 5, 3, 7, 9, 9, 10, 5.

_____

Data - Frequency Tables (Sheet 1)

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Complete the frequency column and total for Σf, then answer the questions.

1 Number of Fish in a Tank

Fish	Tally	Frequency
2	𝍸	5
3	𝍸 ||	7
4	𝍸 𝍸 |||	13
5	𝍸 𝍸	10
6	𝍸 𝍸 ||||	14
7	𝍸 𝍸 |||	13
		$\Sigma f =$ 62

2 What number of fish had the highest frequency? 6

3 What number of fish had the same frequency? 4 and 7

4 What was the frequency for a tank to have an odd number of fish?

7 + 10 + 13 = 30

5 What was the frequency for a tank to have less than 5 fish?

13 + 7 + 5 = 25

6 Soft Drink Preference

Type	Tally	Frequency
A	𝍸 𝍸 𝍸 ||	17
B	𝍸 𝍸 |||	13
C	||||	4
D	𝍸 𝍸 ||||	14
E	𝍸 𝍸 ||	12
F	𝍸 𝍸 𝍸	15
		$\Sigma f =$ 75

7 D's results were unreadable due to a drink spill, use Σf to calculate them.

8 Rank the drink brands in order from least, to most popular.

C, E, B, D, F, A.

9 How many people preferred brand 'A' or brand 'F'? 17 + 15 = 32

Place the data into the Frequency table, showing the tally, the frequency and write 'Σf= *(total)*', then answer the questions.

10 Away from School - Sports Choices

Sport	Tally	Frequency
Tennis	𝍸 𝍸 𝍸 𝍸 ||||	24
Gym	𝍸 𝍸 |||	13
Bocce	𝍸 𝍸 𝍸 𝍸 |||	23
Archery	𝍸 𝍸 𝍸 𝍸	20
		$\Sigma f = 80$

Data:

TTGAA	TTGAA
TGBBA	GBBAT
AAGGG	TTTTA
BTBTA	BTBBA
AAATG	GTBBA
BBGBT	AABTB
TTGBB	BBTBG
ATTTA	ABBTG

11 What was most popular 'away' sport chosen? Tennis

12 How many people chose 'ball sports'? 47

13 If there are a total of 135 students in the year, calculate the percentage that chose 'away' sports.

$$\%\ \text{Away Sport} = \frac{80}{135} \times 100$$

$$= 59.3\ \%$$

Round to 1 d.p.

14 A real estate agent is checking rental homes for smoke detectors, if the smoke detector was Battery powered, Mains powered, or Dual (both). If a detector didn't work its type wasn't recorded it was just recorded as Inoperative. If a house had no smoke detector, it was recorded as 'None'.

Smoke Detector Type or Status

Type	Tally	Frequency
Battery	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	31
Mains	𝍸 𝍸	10
Dual	𝍸 ||||	9
Inoperative	𝍸 𝍸	10
None	𝍸 𝍸	10
		$\Sigma f = 70$

Data:

BBDMB	BBDBM
BIBBM	MNDIB
MBDBI	BBIIN
NBBIM	INDDB
BBNIB	NBBMB
MBBDN	IBBMB
NINBB	NDDMB

15 How many homes had operational smoke detectors? 31 + 10 + 9 = 50

16 What percentage (to 1 d.p.) of homes had inoperative or no smoke detectors?

$$\%\ \text{No S.D} = \frac{20}{70} \times 100 = 28.6\%$$

Round to 1 d.p.

17 What percentage (to 1 d.p.) of homes had Dual-powered smoke detectors?

$$\%\ \text{Dual Powered} = \frac{9}{70} \times 100 = 12.9\%$$

Round to 1 d.p.

Data - Frequency Tables (Sheet 2)

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Place the data below in a frequency table.

Before you start
Circle the smallest number
Circle the largest number

1 Data: 0, 3, 2, 6, 5, 3, 4, 5, 1, 6, 3, 4, 2, 5, 3, 2, 0, 5, 1, 2, 3, 5, 4, 3, 2, 4, 5, 2, 1, 5, 2, 3, 2, 5, 6, 1, 4, 3, 2, 2.

40 scores

Score	Tally	Frequency
0	||	2
1	||||	4
2	𝍸 𝍸	10
3	𝍸 |||	8
4	𝍸	5
5	𝍸 |||	8
6	|||	3
		$\Sigma f = 40$

check

Count the numbers in the data, does it equal Σf?

2 Ages of Students on an Excursion.

Data: 15, 17, 12, 17, 16, 17, 12, 18, 13, 16, 17, 15, 11, 13, 13, 14, 12, 15, 17, 17, 15, 14, 13, 15, 16, 17, 12, 16, 14, 13, 15, 17, 17, 17, 16, 12, 14.

Table Title: Ages of Students on an Excursion

Score	Tally	Frequency
11	|	1
12	𝍸	5
13	𝍸	5
14	||||	4
15	𝍸 |	6
16	𝍸	5
17	𝍸 𝍸	10
18	|	1
		$\Sigma f = 37$

3 What was the age that occurred most? 17

4 How many students were older that 14? 22

5 How many teenage students were there? 31

6 What percentage of the students were 16 or older?

$\frac{16}{37} \times 100 = 43.2\%$

Show working out — Answer to 1 d.p.

Student eye colour was recorded with the data shown, Br = Brown, Bl = Blue, G = Green and H = Hazel. Tabulate the results.

7 Data: Br, Br, Br, G, Bl, Br, Br, Br, Bl, H, H, Br, Br, Br, Br, Bl, H, Bl, Br, Br, H, Bl, G, Br, Br, Br, H, H, Bl.

Student Eye Colour

Colour	Tally	Frequency
Brown	𝍸 𝍸 𝍸	15
Blue	𝍸 |	6
Green	||	2
Hazel	𝍸 |	6
		$\Sigma f = 29$

8 What was most common eye colour? Brown

9 How many students had hazel eyes? 6

10 What percentage of the students had blue eyes?

$\frac{6}{29} \times 100 = 20.7\%$

Answer to 1 d.p.

11 What percentage of the students had either brown or hazel eyes?

$\frac{21}{29} \times 100 = 72.4\%$

Answer to 1 d.p.

12 Place the spelling test results in the table.

Data: 9, 6, 8, 3, 6, 8, 10, 8, 5, 6, 9, 7, 9, 8, 7, 3, 8, 10, 6, 9, 8, 7, 9, 5, 6, 9, 8, 8, 8, 7, 5, 9, 6, 3, 7, 9, 6, 10, 8, 7, 10, 6, 5, 5, 3, 7, 9, 9, 10, 5.

Spelling Test results

Score	Tally	Frequency
3	||||	4
4		0
5	𝍸 |	6
6	𝍸 |||	8
7	𝍸 ||	7
8	𝍸 𝍸	10
9	𝍸 𝍸	10
10	𝍸	5
		$\Sigma f = 50$

Data - Frequency Tables (Homework Strip)

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Date Due:

Place the data in a Frequency Table

1 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

2 Data: B, B, G, O, R, Y, G, R, R, B, G, Y, G, G, B, O, G, B, Y, O, G, B, R, O, O, Y, G, G, Y, B, G, B, O, Y, B, B, Y, Y, O, B.

Colour	Tally	Frequency
		$\Sigma f =$

Date Due:

Place the data in a Frequency Table

1 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

2 Data: B, B, G, O, R, Y, G, R, R, B, G, Y, G, G, B, O, G, B, Y, O, G, B, R, O, O, Y, G, G, Y, B, G, B, O, Y, B, B, Y, Y, O, B.

Colour	Tally	Frequency
		$\Sigma f =$

Date Due:

Place the data in a Frequency Table

1 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

2 Data: B, B, G, O, R, Y, G, R, R, B, G, Y, G, G, B, O, G, B, Y, O, G, B, R, O, O, Y, G, G, Y, B, G, B, O, Y, B, B, Y, Y, O, B.

Colour	Tally	Frequency
		$\Sigma f =$

Date Due:

Place the data in a Frequency Table

1 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

2 Data: B, B, G, O, R, Y, G, R, R, B, G, Y, G, G, B, O, G, B, Y, O, G, B, R, O, O, Y, G, G, Y, B, G, B, O, Y, B, B, Y, Y, O, B.

Colour	Tally	Frequency
		$\Sigma f =$

Date Due:

Place the data in a Frequency Table

1 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

2 Data: B, B, G, O, R, Y, G, R, R, B, G, Y, G, G, B, O, G, B, Y, O, G, B, R, O, O, Y, G, G, Y, B, G, B, O, Y, B, B, Y, Y, O, B.

Colour	Tally	Frequency
		$\Sigma f =$

Data - Frequency Tables (Homework Strip)

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Date Due:

Place the data in a Frequency Table

1 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1	𝍸	5
2	𝍸 |	6
3	𝍸 |||	8
4	𝍸	5
5	𝍸 ||	7
6	||||	4
		$\Sigma f = 35$

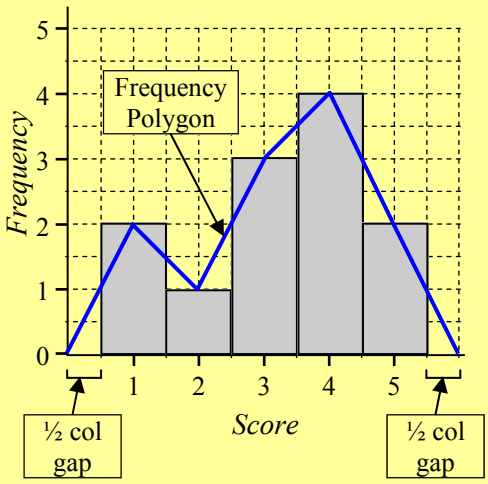
2 Data: B, B, G, O, R, Y, G, R, R, B, G, Y, G, G, B, O, G, B, Y, O, G, B, R, O, O, Y, G, G, Y, B, G, B, O, Y, B, B, Y, Y, O, B.

Colour	Tally	Frequency
B	𝍸 𝍸 |	11
G	𝍸 𝍸	10
O	𝍸 ||	7
R	||||	4
Y	𝍸 |||	8
		$\Sigma f = 40$

Data - Frequency Histograms and Frequency Polygons

Frequency Histograms are very similar to column graphs, the difference being that:

- A half-column gap is left between the first column and the vertical axis.
- The columns touch each other, there is no gap between them.
- A half-column gap is left after the last column.



Frequency Polygons are similar to line graphs, they:

- Always start and finish at the horizontal axis.
- They inter-connect the centres of the top of columns.

Because a half-column gap is required, it makes sense to use columns that are '2 grids' wide so that the grids help you to draw the graph. You shouldn't make a column '1 grid wide' and the gap '½ a grid wide' because your columns won't use the grid lines, it will be harder to create and use.

Sheet 1 starts with looking at a Frequency histogram and extracting the data. Then comparing a histogram with the column graph below it. The second column asks you to create your own frequency histograms/polygons. The first column is drawn so you don't forget the half column gap.

Sheet 2 continues with constructing histograms, the second column asks you to first create a Frequency Table first, as the data is supplied raw instead of in the table, ready to go. More has to be done with these graphs, like labeling the axes.

Data - Frequency Histograms and Frequency Polygons (Sheet 1)

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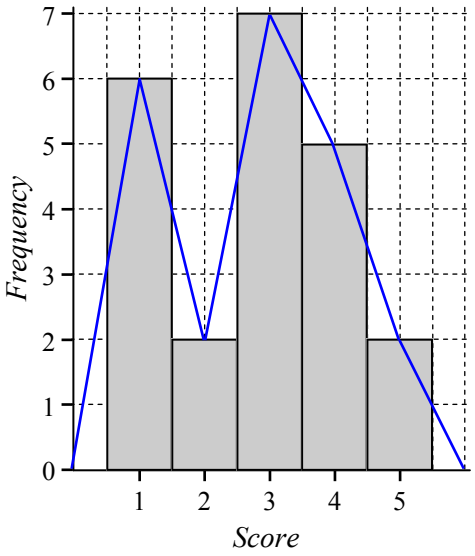
A frequency histogram and polygon are shown. Complete the frequency table, then answer the questions.



Score	*f*
1	
2	
3	
4	
5	

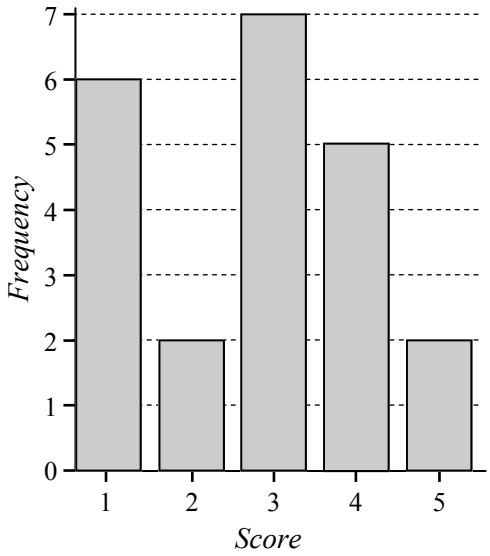
1 Which score occurred the most? _____

2 Which scores have the same frequency? _____

3 Which score occurred five times? _____

4 Total the frequencies. _____

This is a column graph for the same data.



5 List three differences between the column graph and the histogram above it. Write your answer in sentences.

_____

_____

_____

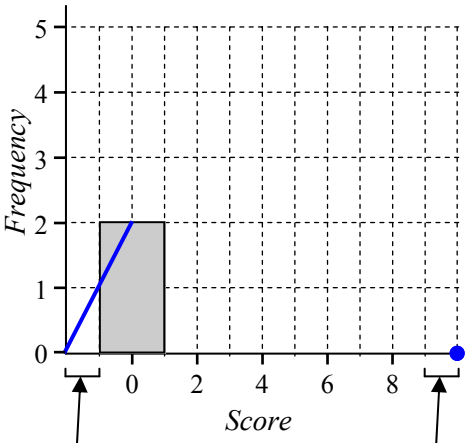
_____

_____

Construct a frequency histogram and frequency polygon for the data in these frequency tables.

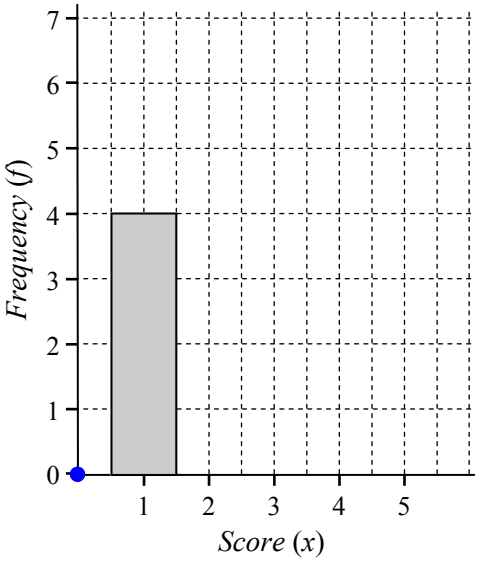
6

Score	*Frequency*
0	2
2	3
4	5
6	1
8	4



Columns touch each other - no gaps between
There is a ½ column gap at the start and at the finish of the columns. Polygon ends at the dot. ●

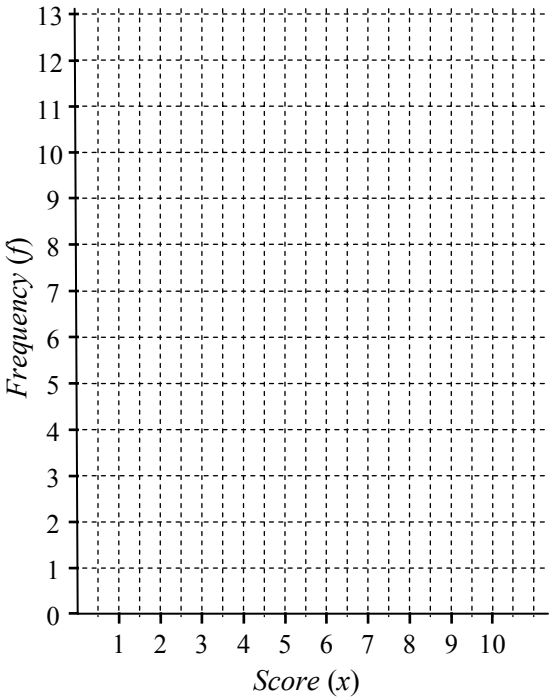
7



x	*f*
1	4
2	2
3	7
4	0
5	6

8

x	*f*
1	2
2	0
3	4
4	6
5	7
6	10
7	13
8	12
9	8
10	5



Data - Frequency Histograms and Frequency Polygons (Sheet 2)

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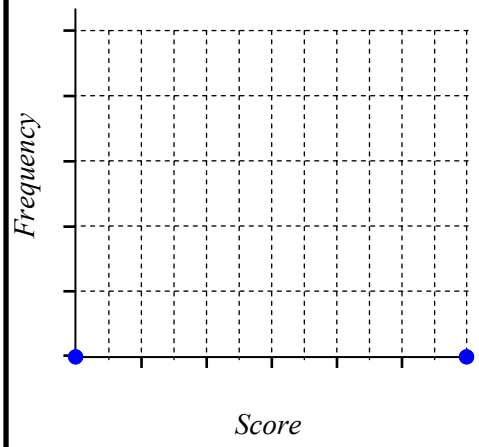
Use a ruler to construct a Frequency Histogram and Polygon for the given data.

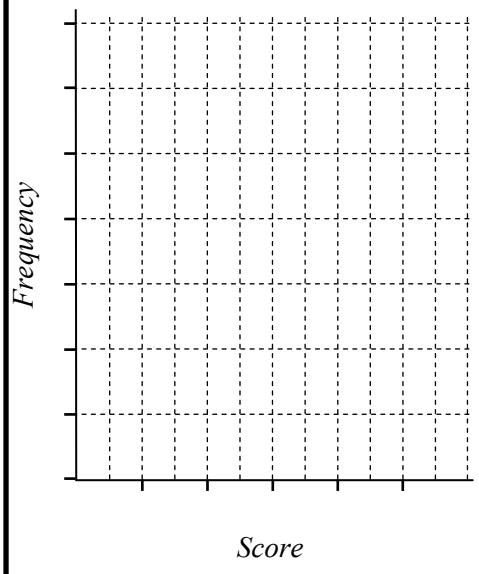
1

Score	*Frequency*
5	5
10	1
15	3
20	0
25	4



2

Score	*Frequency*
0	2
2	5
4	3
6	7
8	1

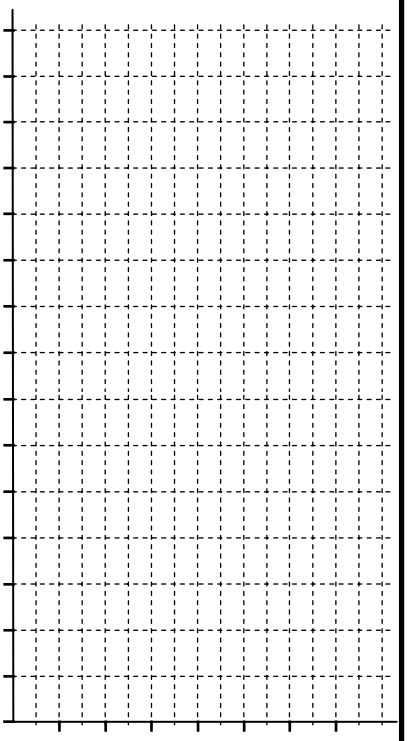


Given a data set, place the data into a Frequency Table then construct a Frequency Histogram and Frequency Polygon. Start with circling the lowest and the highest scores in the data set.

3 Family Size

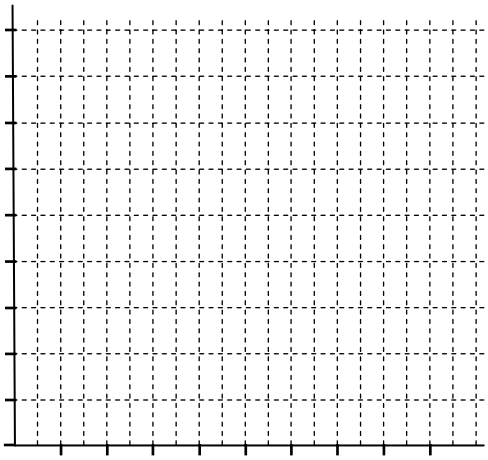
Data: 4, 3, 4, 2, 3, 7, 4, 5, 7, 6, 5, 3, 4, 8, 5, 4, 5, 3, 4, 3, 5, 4, 4, 2, 8, 6, 4, 4, 3, 5, 4, 4, 5, 6, 3, 4, 4, 3, 4.

Score	*Tally*	*Frequency*
Count the scores in the data and compare →		$\Sigma f =$



4 100 m Sprint times (s)

Data:
17, 19, 22, 16, 18, 19, 15, 21, 17, 22, 18, 16, 22, 17, 18, 20, 17, 19, 22, 16, 23, 18, 21, 19, 15, 17, 17, 19, 22, 21, 15, 17, 16, 16, 20, 23, 19, 19, 17, 18.



Data - Frequency Histograms and Frequency Polygons (Sheet 1)

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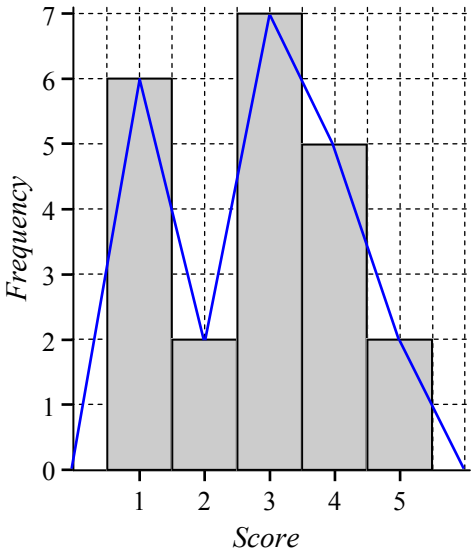
A frequency histogram and polygon are shown. Complete the frequency table, then answer the questions.



Score	f
1	6
2	2
3	7
4	5
5	2

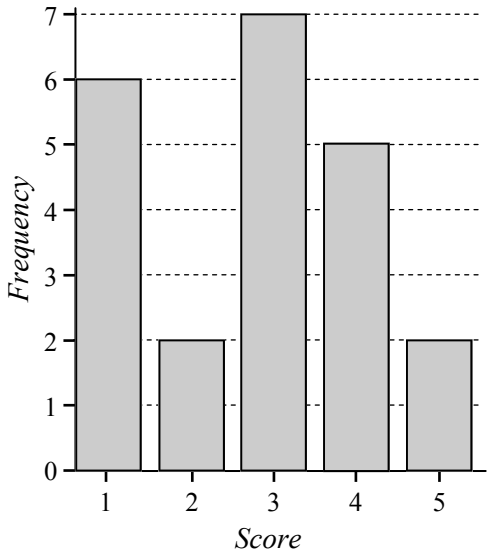
1 Which score occurred the most? 3

2 Which scores have the same frequency? 2 and 5

3 Which score occurred five times? 4

4 Total the frequencies. 6 + 2 + 7 + 5 + 2 = 22

This is a column graph for the same data.



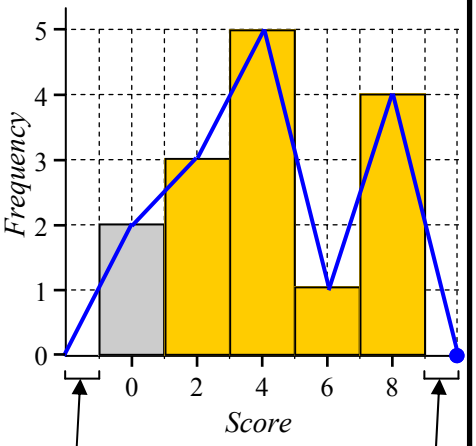
5 List three differences between the column graph and the histogram above it. Write your answer in sentences.

There is no half column gap between the first column and the frequency vertical axis. The columns don't touch each other, there is a gap between them. There is no frequency polygon on a column graph.

Construct a frequency histogram and frequency polygon for the data in these frequency tables.

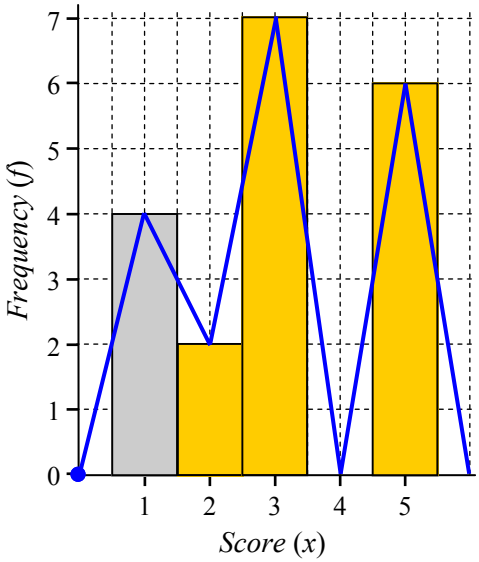
6

Score	Frequency
0	2
2	3
4	5
6	1
8	4



Columns touch each other - no gaps between
There is a ½ column gap at the start and at the finish of the columns. Polygon ends at the dot.

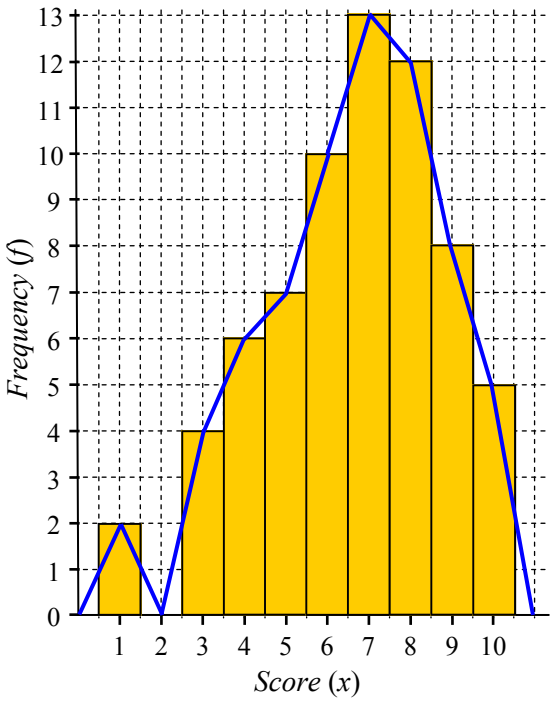
7



x	f
1	4
2	2
3	7
4	0
5	6

8

x	f
1	2
2	0
3	4
4	6
5	7
6	10
7	13
8	12
9	8
10	5



Data - Frequency Histograms and Frequency Polygons (Sheet 2)

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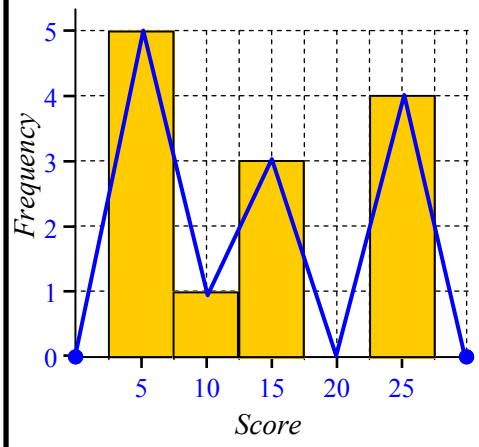
Use a ruler to construct a Frequency Histogram and Polygon for the given data.

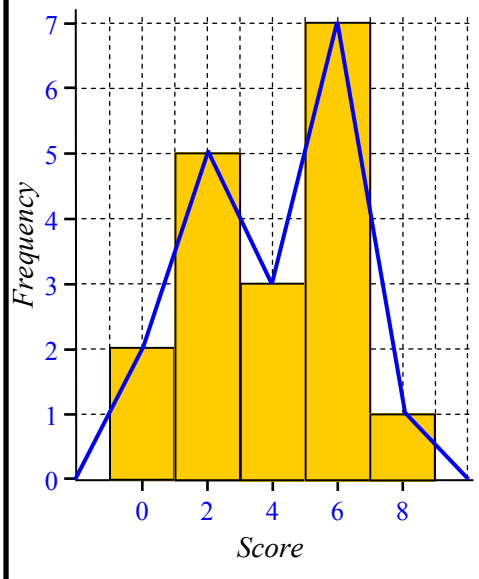
1

Score	*Frequency*
5	5
10	1
15	3
20	0
25	4



2

Score	*Frequency*
0	2
2	5
4	3
6	7
8	1

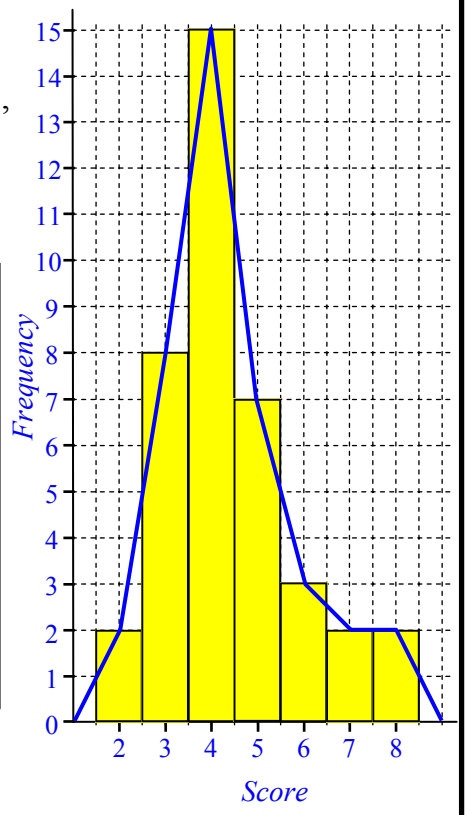


Given a data set, place the data into a Frequency Table then construct a Frequency Histogram and Frequency Polygon. Start with circling the lowest and the highest scores in the data set.

3 Family Size

Data: 4, 3, 4, 2, 3, 7, 4, 5, 7, 6, 5, 3, 4, 8, 5, 4, 5, 3, 4, 3, 5, 4, 4, 2, 8, 6, 4, 4, 3, 5, 4, 4, 5, 6, 3, 4, 4, 3, 4.

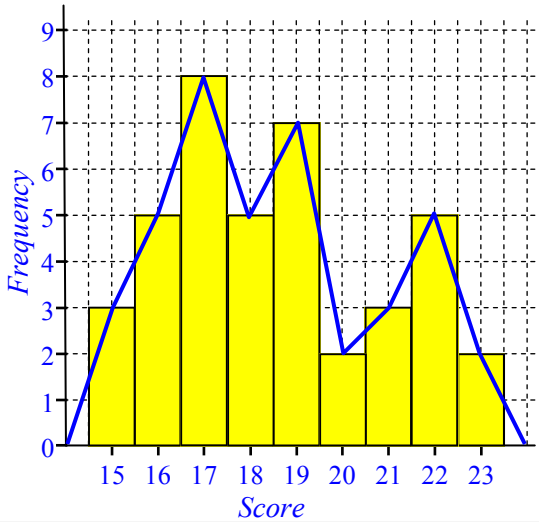
Score	*Tally*	*Frequency*
2	||	2
3	𝍸 |||	8
4	𝍸 𝍸 𝍸	15
5	𝍸 ||	7
6	|||	3
7	||	2
8	||	2
Count the scores in the data and compare →		$\Sigma f = 39$



4 100 m Sprint times (s)

Data:
17, 19, 22, 16, 18, 19, 15, 21, 17, 22, 18, 16, 22, 17, 18, 20, 17, 19, 22, 16, 23, 18, 21, 19, 15, 17, 17, 19, 22, 21, 15, 17, 16, 16, 20, 23, 19, 19, 17, 18.

Time (s)	*Tally*	*Frequency*
15	|||	3
16	𝍸	5
17	𝍸 |||	8
18	𝍸	5
19	𝍸 ||	7
20	||	2
21	|||	3
22	𝍸	5
23	||	2
		$\Sigma f = 40$



Data - Frequency Histograms and Polygons (Homework Strip)

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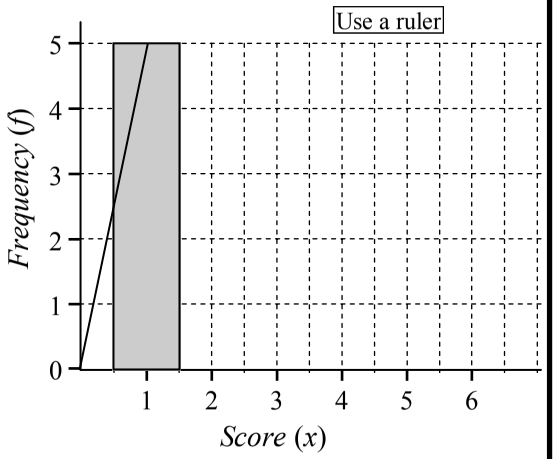
Date Due:

Construct a frequency histogram/polygon from the table

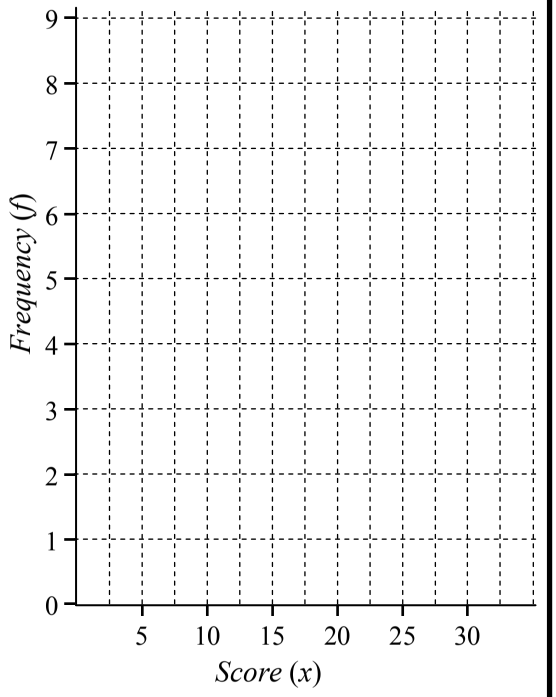
1

x	f
1	5
2	2
3	4
4	1
5	2
6	3



2

x	f
5	3
10	7
15	2
20	5
25	9
30	6



Date Due:

Construct a frequency histogram/polygon from the table

1

x	f
1	5
2	2
3	4
4	1
5	2
6	3

Use a ruler
Frequency (f)
Score (x)

2

x	f
5	3
10	7
15	2
20	5
25	9
30	6

Frequency (f)
Score (x)

Date Due:

Construct a frequency histogram/polygon from the table

1

x	f
1	5
2	2
3	4
4	1
5	2
6	3

Use a ruler
Frequency (f)
Score (x)

2

x	f
5	3
10	7
15	2
20	5
25	9
30	6

Frequency (f)
Score (x)

Data - Frequency Histograms and Polygons (Homework Strip)

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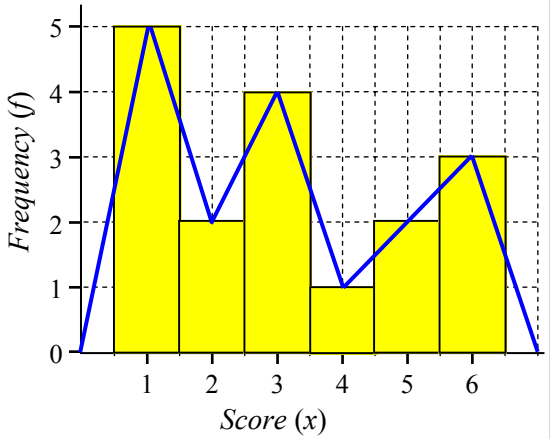
Date Due:

Construct a frequency histogram/polygon from the table

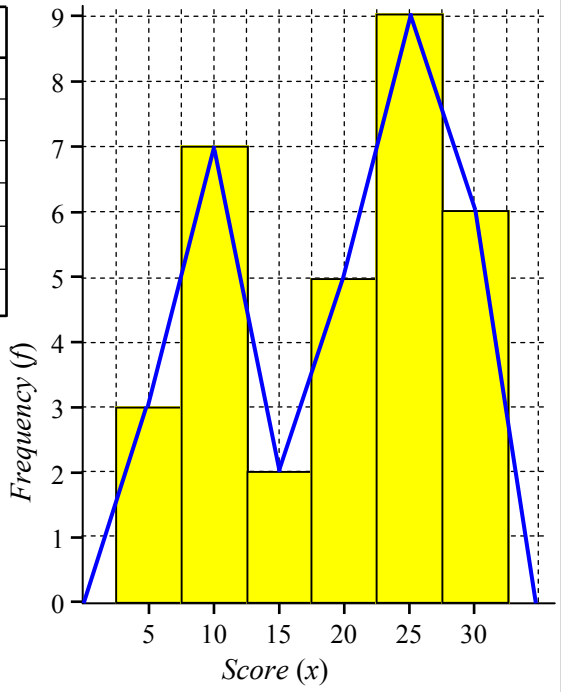
1

x	f
1	5
2	2
3	4
4	1
5	2
6	3



2

x	f
5	3
10	7
15	2
20	5
25	9
30	6



Data - Frequency Histograms with Classes

A frequency table is used when there is high repetition of a certain number of scores. Such as with a die, where there are only 6 outcomes. But when there are a large range of scores then the data is best grouped together in classes.

The process is essentially the same as a frequency table, using a tally column and then changing the tally to a number in the frequency column. The only difference is that the 'Score' column is replaced with the word 'Classes' which cover a range of scores rather than just a single score.

Example. 'BarknFlea' Dog Show - Points listing

Data: ~~15~~, ~~26~~, 11, 31, 27, 44, 28, 31, 46, 23, 5,

Cross out the first score and place a stroke in the tally column, 15 is in the 10-19 class.

Repeat for 26, which is in the 20-29 class.

Class	Tally	Frequency
0 - 9		
10 - 19	||	
20 - 29	||	
30 - 39		
40 - 49		
		$\Sigma f =$

Place all scores into the tally column then change the strokes to numbers in the frequency column.

Class	Tally	Frequency
0 - 9	|	1
10 - 19	||	2
20 - 29	||||	4
30 - 39	||	2
40 - 49	||	2
		$\Sigma f = 11$

Count the number of scores in the data, does it equal Σf?

The display of data in a Histogram/Polygon with classes uses exactly the same method as without classes, with one difference. Instead of displaying the score across the horizontal axis, you display the class centre.

The class centre is found by averaging the two scores shown in the class.

Example

$\frac{20 + 1}{2} = 10.5$

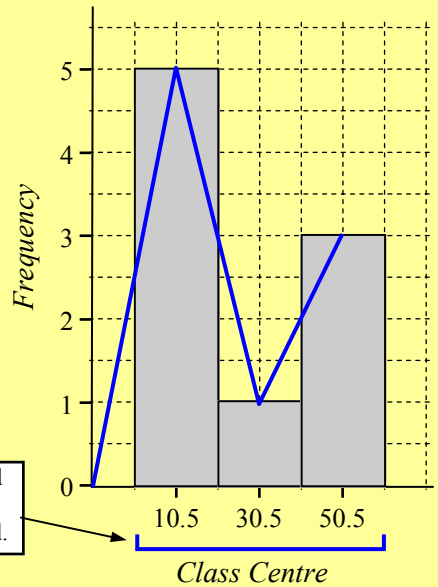
$\frac{21 + 40}{2} = 30.5$

$\frac{41 + 60}{2} = 50.5$

c.c. is short for class centre

Class	*c.c.*	Frequency
1 - 20	10.5	5
21 - 40	30.5	1
41 - 60	50.5	3

Remember that when you divide the sum of the two numbers, press '=' before you '÷2' or you will get the wrong answer, due to order of operations principles.



Plot the data using a histogram and polygon, the only difference being that the class centres are used.

Data - Frequency Histograms with Classes (Sheet 1)

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Place the data below in the frequency table, using the suggested class intervals.

1 Data: 25, 16, 32, 41, 28, 7, 11, 36, 27, 20, 34, 43, 19, 31, 7, 48, 9, 20, 5, 35, 22, 10, 33, 3, 21, 24, 17, 28, 2, 36, 32, 12, 26, 43, 7, 21, 7, 5.

Class	Tally	Frequency
0 - 9		
10 - 19		
20 - 29		
30 - 39		
40 - 49		
		$\Sigma f =$

Count the number of scores in the data, does it equal Σf?

2 'Appleshot' Archery Club Championship

Points: 80, 72, 83, 67, 94, 92, 85, 73, 79, 90, 65, 77, 97, 93, 89, 65, 85, 71, 62, 83, 79, 64, 92, 70, 62, 84, 75, 81, 98, 75, 73, 63, 84, 91, 99, 98, 76, 78, 85, 96, 70, 74, 76, 62, 68, 83, 95, 100, 72, 81.

Points	Tally	Frequency
61 - 65		
66 - 70		
71 - 75		
76 - 80		
81 - 85		
86 - 90		
91 - 95		
96 - 100		

3 How many archers were there? _____

4 How many archers scored 75 or less? _____

5 Which class had the highest frequency? _____

6 What percentage of the archers scored more than 90?

$\Sigma f \rightarrow \frac{\square}{\square} \times 100 =$ _____

7 What percentage scored between 65 and 81?

$\frac{\square}{\square} \times \square =$ _____

8 Time (min) to complete a survey

Data: 6, 15, 20, 11, 19, 22, 18, 5, 11, 10, 18, 24, 16, 23, 4, 18, 9, 14, 28, 9, 14, 13, 25, 14, 17, 29, 4, 15, 5, 23, 19, 12, 17, 25, 10, 14, 19, 24, 16.

Time (min)	Tally	Frequency
1 - 5		
6 - 10		
11 - 15		
16 - 20		
21 - 25		
26 - 30		

9 How many scores between 5 and 16 were there?

_____

10 How many scores were 25 or less? _____

11 Student Height (cm)

Data: 155, 172, 163, 158, 168, 174, 183, 165, 180, 161, 174, 168, 170, 181, 163, 162, 171, 155, 183, 175, 169, 149, 170, 162, 181, 172, 176, 162, 158, 166, 157, 184, 175, 166, 163, 159, 172, 170.

Height (cm)	Tally	Frequency
145 - 149		
150 - 154		
155 - 159		
160 - 164		
165 - 169		
170 - 174		
175 - 179		
180 - 184		

Answer these to 1 decimal place.

12 What percentage of students are 1.8 m or taller?

_____ = _____

13 What percentage of students are less than 1.7 m tall?

_____ = _____

Data - Frequency Histograms with Classes (Sheet 2)

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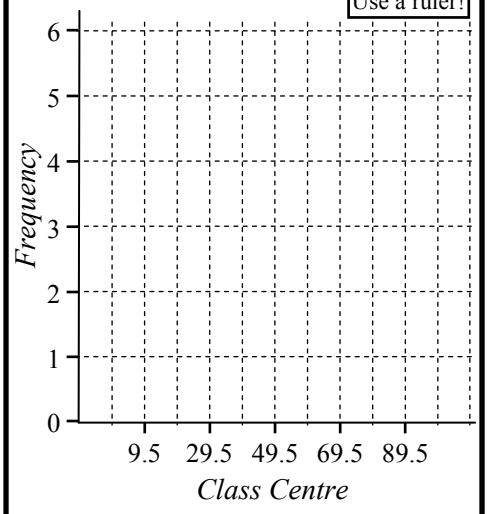
Display the data below using a frequency histogram and polygon

1

Class	*c.c.*	Frequency
0 - 19	9.5	4
20 - 39	29.5	2
40 - 59	49.5	6
60 - 79	69.5	3
80 - 99	89.5	5

Use a ruler!



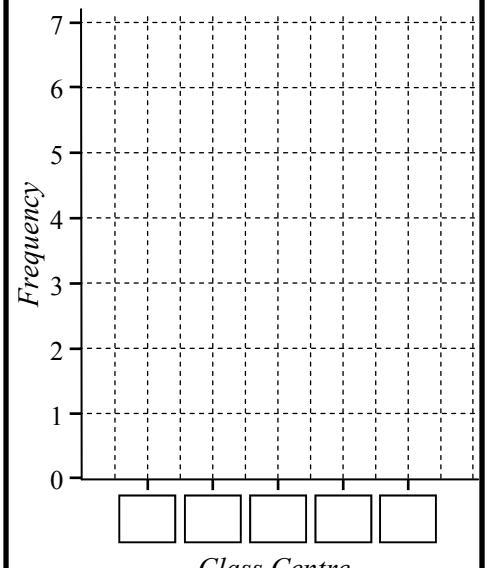
2

Class	*c.c.*	Frequency
1 - 5		4
6 - 10		7
11 - 15		0
16 - 20		2
21 - 25		5

To calculate Class Centre:

Calc: 1 + 5 = ÷ 2 =

Calc: 6 + 1 0 = ÷ 2 = and so on

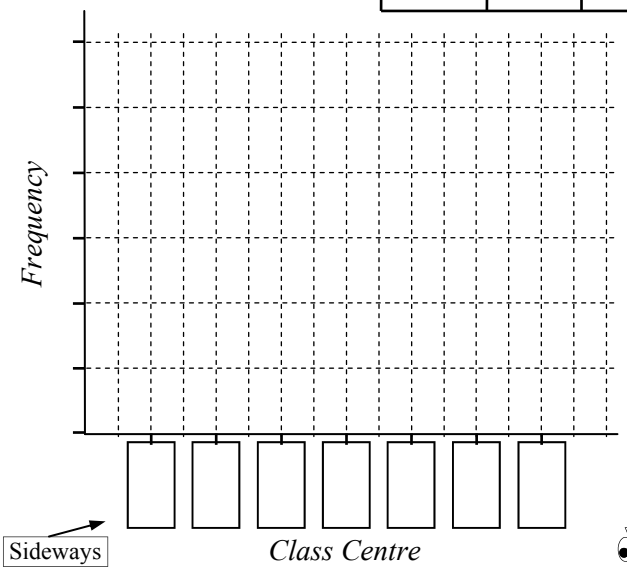


Place the data in the Frequency Table using classes 0 - 9, 10 -19 and so on. Then show the data as a Frequency Histogram and Polygon.

3 Data: 63, 9, 33, 43, 9, 63, 10, 67, 54, 3, 68, 16, 56, 13, 21, 15, 4, 50, 58, 8, 4, 40, 48, 17, 65, 59, 10, 44, 50, 23, 26, 25, 37, 31.

Class	*c.c.*	Tally	*f*
0 - 9			
10 - 19			
			$\Sigma f =$

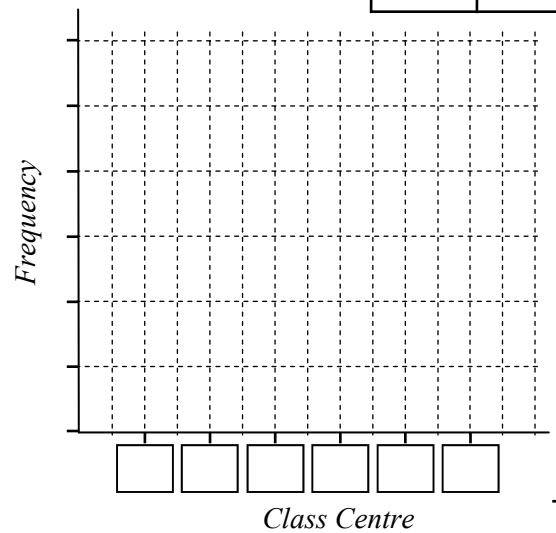
Count the number of scores in the data, does it equal Σf?



Repeat the process except this time use classes 1 - 5, 6 - 10, 11 - 15 and so on.

4 Data: 15, 30, 10, 25, 1, 30, 6, 10, 2, 21, 3, 12, 14, 27, 28, 6, 11, 16, 9, 15, 21, 17, 15, 20, 25.

Class	*c.c.*	Tally	*f*
			$\Sigma f =$



5 How many scores are larger than 15? _____

6 How many scores between 5 and 21? _____

7 What percentage of the scores were 20 or less?

_____

Data - Frequency Histograms with Classes (Sheet 1)

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Place the data below in the frequency table, using the suggested class intervals.

1 Data: 25, 16, 32, 41, 28, 7, 11, 36, 27, 20, 34, 43, 19, 31, 7, 48, 9, 20, 5, 35, 22, 10, 33, 3, 21, 24, 17, 28, 2, 36, 32, 12, 26, 43, 7, 21, 7, 5.

Class	Tally	Frequency
0 - 9	𝍸 ||||	9
10 - 19	𝍸 |	6
20 - 29	𝍸 𝍸 |	11
30 - 39	𝍸 |||	8
40 - 49	||||	4
		$\Sigma f = 38$

Count the number of scores in the data, does it equal Σf?

2 'Appleshot' Archery Club Championship

Points: 80, 72, 83, 67, 94, 92, 85, 73, 79, 90, 65, 77, 97, 93, 89, 65, 85, 71, 62, 83, 79, 64, 92, 70, 62, 84, 75, 81, 98, 75, 73, 63, 84, 91, 99, 98, 76, 78, 85, 96, 70, 74, 76, 62, 68, 83, 95, 100, 72, 81.

Points	Tally	Frequency
61 - 65	𝍸 ||	7
66 - 70	||||	4
71 - 75	𝍸 |||	8
76 - 80	𝍸 ||	7
81 - 85	𝍸 𝍸	10
86 - 90	||	2
91 - 95	𝍸 |	6
96 - 100	𝍸 |	6
		$\Sigma f = 50$

3 How many archers were there? 50

4 How many archers scored 75 or less? 19

5 Which class had the highest frequency? 81 - 85

6 What percentage of the archers scored more than 90?

$\frac{12}{50} \times 100 = 24\%$ ($\Sigma f \rightarrow 50$)

7 What percentage scored between 65 and 81?

$\frac{19}{50} \times 100 = 38\%$

8 Time (min) to complete a survey

Data: 6, 15, 20, 11, 19, 22, 18, 5, 11, 10, 18, 24, 16, 23, 4, 18, 9, 14, 28, 9, 14, 13, 25, 14, 17, 29, 4, 15, 5, 23, 19, 12, 17, 25, 10, 14, 19, 24, 16.

Time (min)	Tally	Frequency
1 - 5	||||	4
6 - 10	𝍸	5
11 - 15	𝍸 𝍸	10
16 - 20	𝍸 𝍸 |	11
21 - 25	𝍸 ||	7
26 - 30	||	2
		$\Sigma f = 39$

9 How many scores between 5 and 16 were there?

$5 + 10 = 15$

10 How many scores were 25 or less? $39 - 2 = 37$

11 Student Height (cm)

Data: 155, 172, 163, 158, 168, 174, 183, 165, 180, 161, 174, 168, 170, 181, 163, 162, 171, 155, 183, 175, 169, 149, 170, 162, 181, 172, 176, 162, 158, 166, 157, 184, 175, 166, 163, 159, 172, 170.

Height (cm)	Tally	Frequency
145 - 149	|	1
150 - 154		0
155 - 159	𝍸 |	6
160 - 164	𝍸 ||	7
165 - 169	𝍸 |	6
170 - 174	𝍸 ||||	9
175 - 179	|||	3
180 - 184	𝍸 |	6
		$\Sigma f = 38$

Answer these to 1 decimal place.

12 What percentage of students are 1.8 m or taller?

$\frac{6}{38} \times 100 = 15.8\%$

13 What percentage of students are less than 1.7 m tall?

$\frac{20}{38} \times 100 = 52.6\%$

Data - Frequency Histograms with Classes (Sheet 2)

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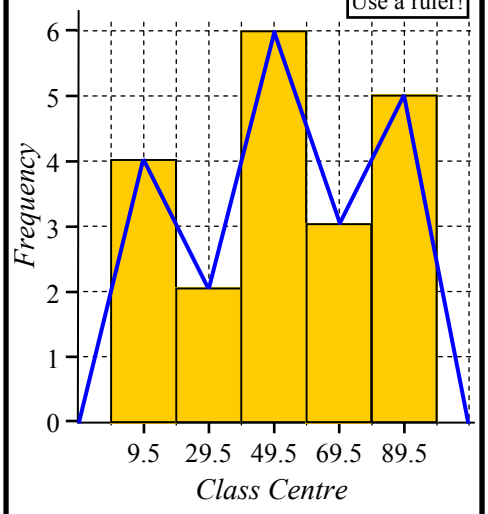
Display the data below using a frequency histogram and polygon

1

Class	*c.c.*	Frequency
0 - 19	9.5	4
20 - 39	29.5	2
40 - 59	49.5	6
60 - 79	69.5	3
80 - 99	89.5	5

Use a ruler!



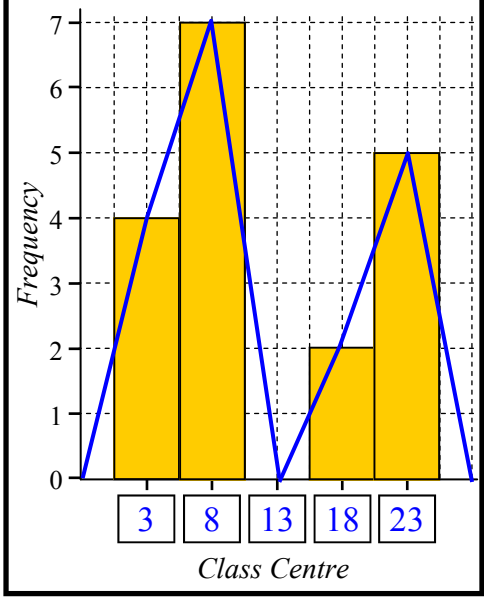
2

Class	*c.c.*	Frequency
1 - 5	3	4
6 - 10	8	7
11 - 15	13	0
16 - 20	18	2
21 - 25	23	5

To calculate Class Centre:

Calc: 1 + 5 = ÷ 2 =

Calc: 6 + 1 0 = ÷ 2 = and so on

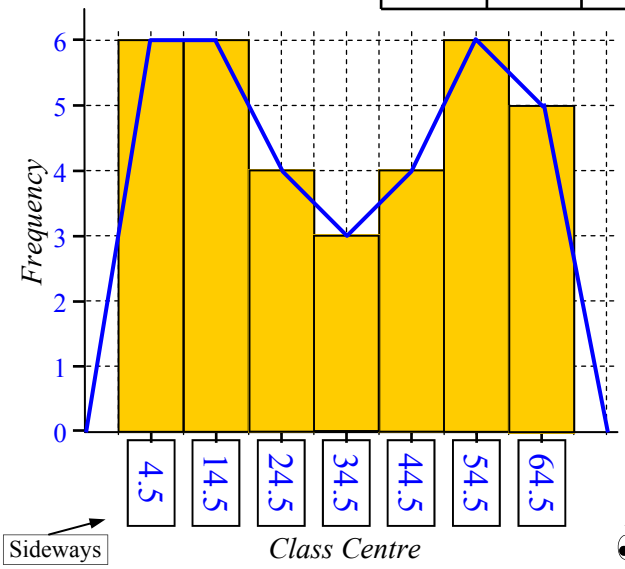


Place the data in the Frequency Table using classes 0 - 9, 10 -19 and so on. Then show the data as a Frequency Histogram and Polygon.

3 Data: 63, 9, 33, 43, 9, 63, 10, 67, 54, 3, 68, 16, 56, 13, 21, 15, 4, 50, 58, 8, 4, 40, 48, 17, 65, 59, 10, 44, 50, 23, 26, 25, 37, 31.

Class	*c.c.*	Tally	*f*
0 - 9	4.5	𝍸 |	6
10 - 19	14.5	𝍸 |	6
20 - 29	24.5	𝍸	4
30 - 39	34.5	𝍸	3
40 - 49	44.5	𝍸	4
50 - 59	54.5	𝍸 |	6
60 - 69	64.5	𝍸	5
			$\Sigma f = 34$

Count the number of scores in the data, does it equal Σf?

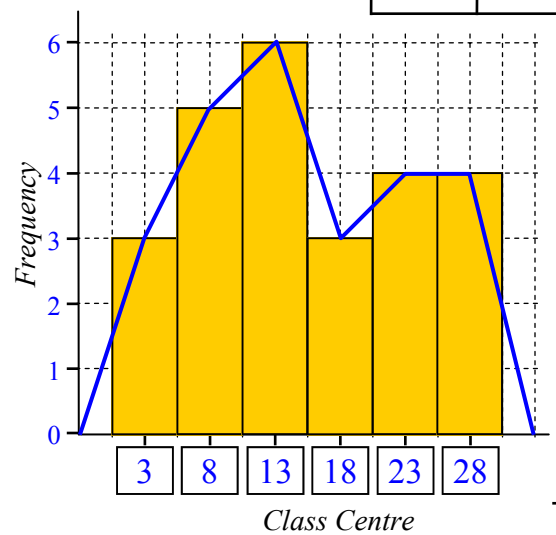


Sideways

Repeat the process except this time use classes 1 - 5, 6 - 10, 11 - 15 and so on.

4 Data: 15, 30, 10, 25, 1, 30, 6, 10, 2, 21, 3, 12, 14, 27, 28, 6, 11, 16, 9, 15, 21, 17, 15, 20, 25.

Class	*c.c.*	Tally	*f*
1 - 5	3	𝍸	3
6 - 10	8	𝍸	5
11 - 15	13	𝍸 |	6
16 - 20	18	𝍸	3
21 - 25	23	𝍸	4
26 - 30	28	𝍸	4
			$\Sigma f = 25$



5 How many scores are larger than 15? 11

6 How many scores between 5 and 21? 14

7 What percentage of the scores were 20 or less?

$\frac{17}{25} \times 100 = 68\%$

Data - Frequency Histograms with Classes (Homework Strip)

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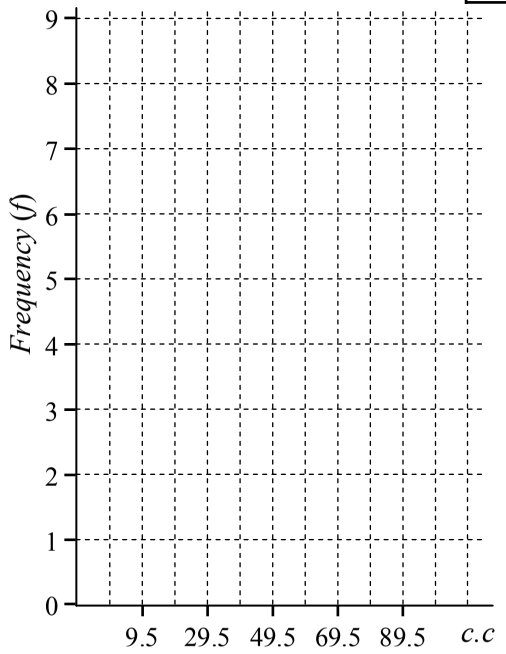
Date Due:

Construct a frequency table, then a histogram/polygon

1 Data: 56, 24, 71, 43, 13, 48, 79, 80, 36, 62, 45, 38, 25, 41, 55, 29, 35, 47, 52, 77, 94, 61, 8, 45, 20, 15, 68, 83, 35, 18, 30, 84, 17.

Class	*c.c.*	Tally	Frequency
0 - 19	9.5		
20 - 39	29.5		
40 - 59	49.5		
60 - 79	69.5		
80 - 99	89.5		
			$\Sigma f =$

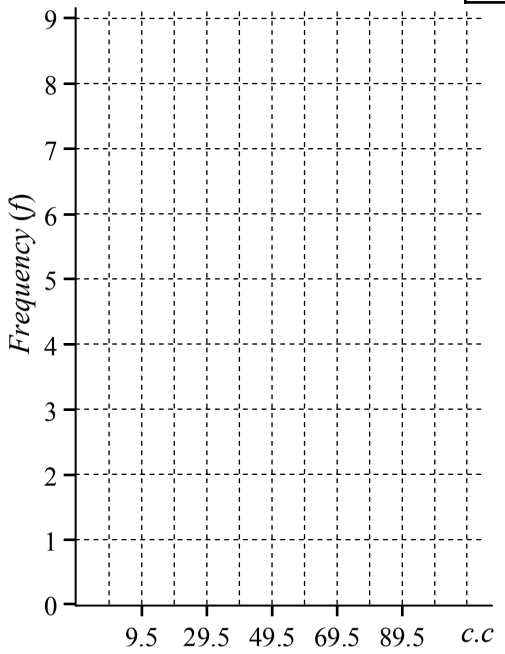


Date Due:

Construct a frequency table, then a histogram/polygon

1 Data: 56, 24, 71, 43, 13, 48, 79, 80, 36, 62, 45, 38, 25, 41, 55, 29, 35, 47, 52, 77, 94, 61, 8, 45, 20, 15, 68, 83, 35, 18, 30, 84, 17.

Class	*c.c.*	Tally	Frequency
0 - 19	9.5		
20 - 39	29.5		
40 - 59	49.5		
60 - 79	69.5		
80 - 99	89.5		
			$\Sigma f =$

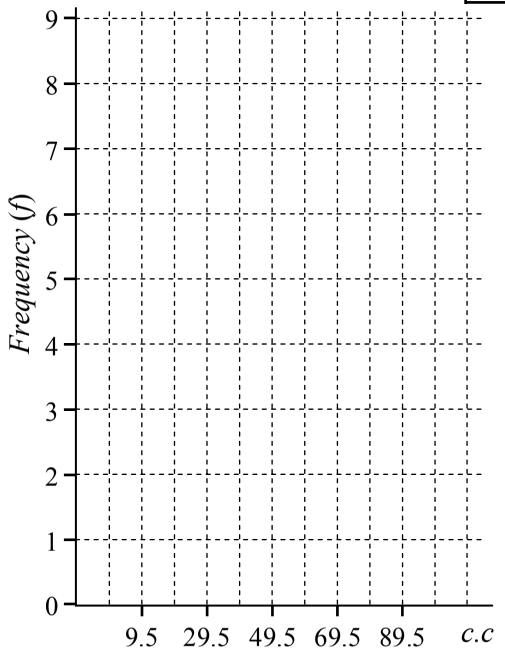


Date Due:

Construct a frequency table, then a histogram/polygon

1 Data: 56, 24, 71, 43, 13, 48, 79, 80, 36, 62, 45, 38, 25, 41, 55, 29, 35, 47, 52, 77, 94, 61, 8, 45, 20, 15, 68, 83, 35, 18, 30, 84, 17.

Class	*c.c.*	Tally	Frequency
0 - 19	9.5		
20 - 39	29.5		
40 - 59	49.5		
60 - 79	69.5		
80 - 99	89.5		
			$\Sigma f =$



Data - Frequency Histograms with Classes (Homework Strip)

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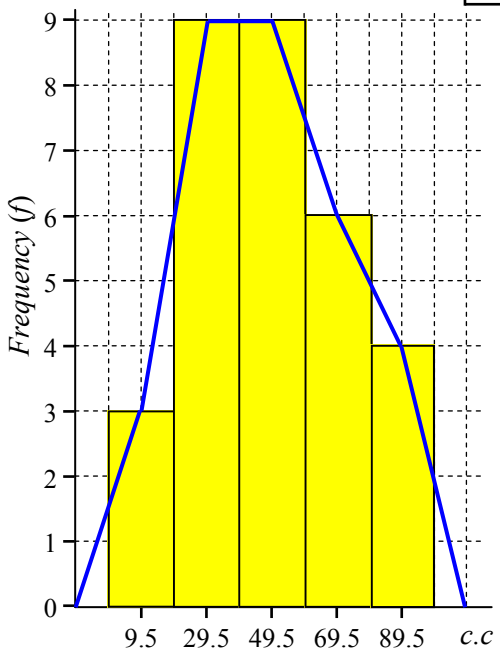
Date Due:

Construct a frequency table, then a histogram/polygon

1 Data: 56, 24, 71, 43, 13, 48, 79, 80, 36, 62, 45, 38, 25, 41, 55, 29, 35, 47, 52, 77, 94, 61, 8, 45, 20, 15, 68, 83, 35, 18, 30, 84, 17.

Class	*c.c.*	Tally	Frequency
0 - 19	9.5	𝍸	5
20 - 39	29.5	𝍸 ||||	9
40 - 59	49.5	𝍸 ||||	9
60 - 79	69.5	𝍸 |	6
80 - 99	89.5	|||	4
			$\Sigma f = 33$



Data - Dot Plots

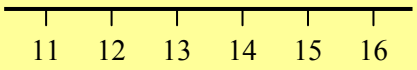
Dot Plots represent data with dots! Amazing! The beauty of a dot plot is it is simple. There is one axis which shows the score, or category and then dots are stacked above them. Dot plots can also be plotted sideways just like a column graph. Two critical features are:

- The dots have to have the same height, for example a column of four dots shouldn't be as high as a column of five dots, so be careful as you plot them.
- If you have a vertical scale then use it effectively. Because dot plots are supposed to be fast to read, a vertical scale can be included so that you don't have to count all the dots in a column.

Example

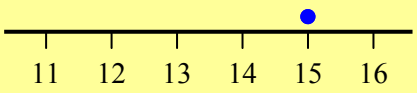
Data: 15, 12, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.

STEP 1 Circle the lowest score and the highest score in the data set.



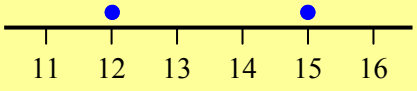
STEP 2 Number the axis from lowest to highest, note there is no 14 score, but it is still included on the axis.

Data: 15, 12, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.



STEP 3 Cross out the first score and place the dot above the number on the axis.

Data: 15, 12, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.



STEP 4 And so on until all the scores are represented with dots

Data - Dot Plots (Sheet 1)

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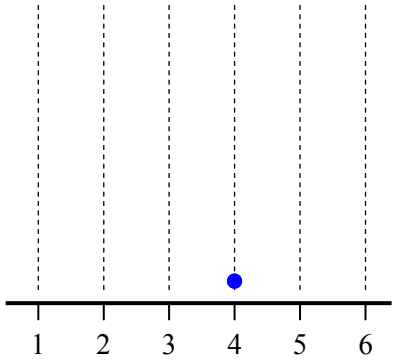
Display the data in a dot plot.

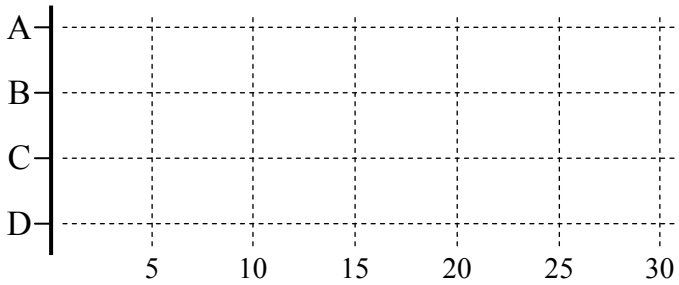
1 Data: 4, 5, 3, 3, 6, 1, 2, 4, 2, 3, 5, 6, 2, 3, 4, 4, 1, 2, 5, 6, 2, 3, 3, 2, 5, 2, 1, 4, 6, 3, 2.



2 An artificial sweetener is comparing itself with three other brands. Graph the results.

Data: BBBDA BABBC ABBBC
BDBAB ABBDB BCBBD
BCABB ABCBD BBADB

sideways! → Artificial Sweetener Preference



3 How many people were sampled? _____

4 How many people preferred Brand B? _____

5 How many people preferred Brand C? _____

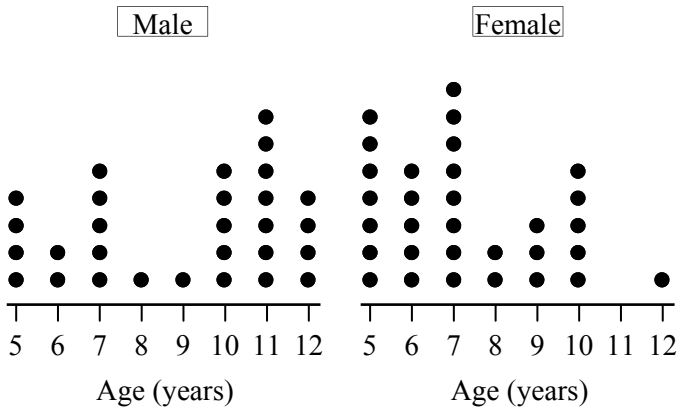
6 What percentage of people prefer Brand B?

Question 4 Ans. → ☐ / ☐ Question 3 Ans. → × 100 = ☐ Round to 1 d.p.

7 What percentage of people prefer Brand C?

Question 5 Ans. → ☐ / ☐ Question 3 Ans. → × 100 = ☐ Round to 1 d.p.

A waterslide at a theme park records the ages of the first 60 children who use the slide, the results are shown in two Dot plots. One male, one female.



8 How many children were male? _____

9 How many children were 7 years old? _____

10 How many children were 11 or older? _____

11 How many more 10 year olds used the slide compared to 6 year olds? _____

12 Are these true or false? There were more ...

Ⓣ Ⓕ Girls using the waterslide

Ⓣ Ⓕ 7 y.o. boys using the slide than 10 y.o. girls

Ⓣ Ⓕ Older male children than female.

13 What percentage of 9 y.o.'s were male?

Male 9 y.o. here → ☐ / ☐ Total 9 y.o. here → × 100 = ☐ Show the % sign

14 What percentage of 5 y.o.'s were female?

☐ / ☐ × 100 = ☐ Round to 1 d.p.

15 What percentage of females were 10 y.o.?

Total females here → ☐ / ☐ × ☐ = ☐ Round to 1 d.p.

16 What percentage of 5 - 7 y.o.'s were male?

☐ / ☐ × ☐ = ☐ Round to 1 d.p.

Data - Dot Plots (Sheet 2)

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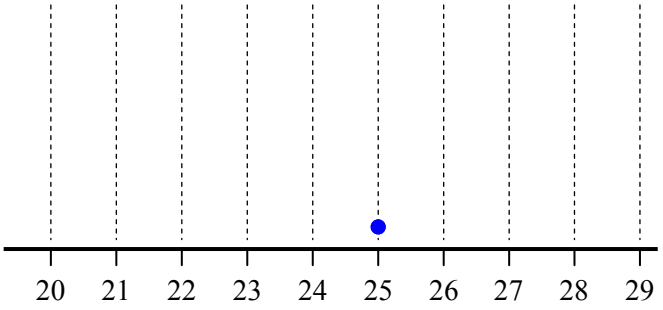
Display the data in a dot plot answering the questions from the plot.



1 Data: 25, 28, 23, 24, 23, 24, 27, 26, 25, 25, 22, 22, 20, 29, 26, 25, 27, 26, 28, 21, 25, 25, 27, 21, 25, 27, 26, 27, 22, 26, 23, 20, 22, 28.

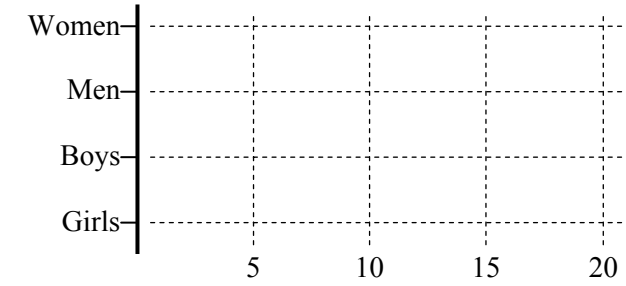


2 Women, Men, Boys and Girls entering a store have been recorded. Display on a dot plot.

Data:
MBGWB GGBWW BBWWW
MMGWB MWGBW GBWBB MMWGW
BGGBW WGMWB MWWGB WWGBM.

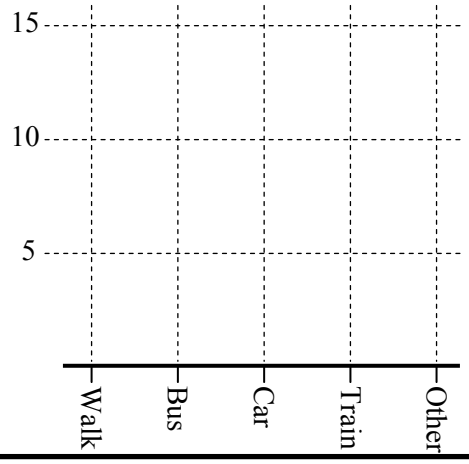
sideways!

Customer Demographic



3 A student survey recorded five different methods of travel to school, they are: Walking, Bus, Car and Train, Other. Display on a dot plot.

Data:
TBWWO BCCTC BCTTT
WWCBB TCBBT TBCWO
BCWOW TWBTT OCTBC
CCBCO TTBCO TTBBB



Method of School Travel

4 How many students were surveyed? _____

5 What percentage of students walked to school?

_____ = _____

Show working out | Answer to 1 d.p.

6 What percentage of students used public transport?

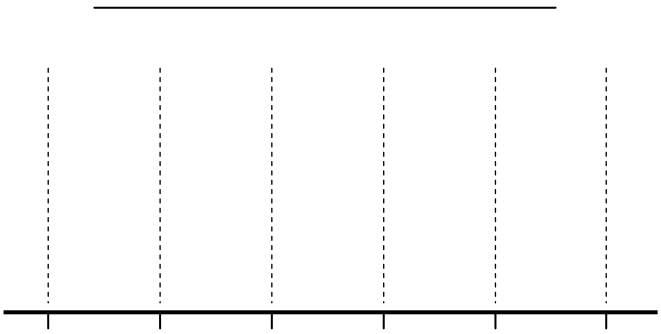
_____ = _____

Answer to 1 d.p.



A die was rolled with the results shown below. Display the data as a dot plot, then in a frequency table.

7 Data: 3, 1, 5, 6, 3, 4, 2, 2, 1, 5, 3, 6, 2, 3, 2, 5, 5, 6, 4, 5, 2, 3, 6, 1, 1, 6, 5, 4, 1, 6, 2, 2, 4, 5.



8

Score	Frequency
	$\Sigma f =$

9 How many times was a number less than 3 rolled? _____

10 How many times was an even number rolled? _____

11 How many times was a prime number rolled? _____

12 What percentage of die rolls resulted in a multiple of 3?

_____ = _____

Answer to 1 d.p.

13 What percentage of die rolls resulted in a factor of 12?

_____ = _____

Answer to 1 d.p.

Data - Dot Plots (Sheet 1)

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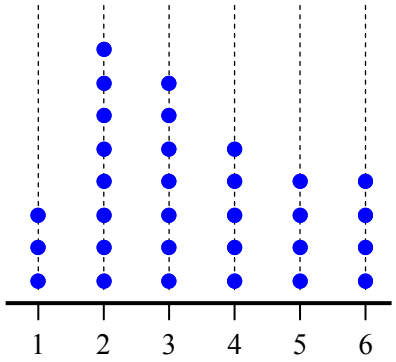
Display the data in a dot plot.

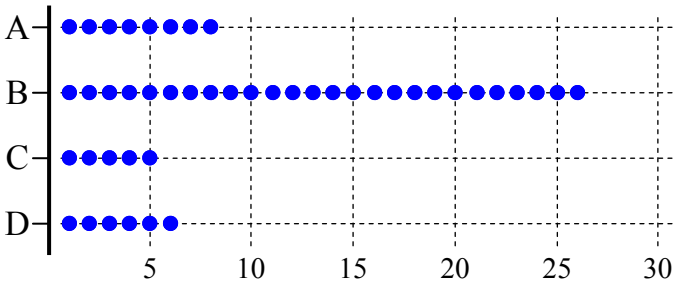
1 Data: ~~4~~, 5, 3, 3, 6, 1, 2, 4, 2, 3, 5, 6, 2, 3, 4, 4, 1, 2, 5, 6, 2, 3, 3, 2, 5, 2, 1, 4, 6, 3, 2.



2 An artificial sweetener is comparing itself with three other brands. Graph the results.

Data: BBBDA BABBC ABBBC
BDBAB ABBDB BCBBD
BCABB ABCBD BBADB

sideways! → Artificial Sweetener Preference



3 How many people were sampled? 45

4 How many people preferred Brand B? 26

5 How many people preferred Brand C? 5

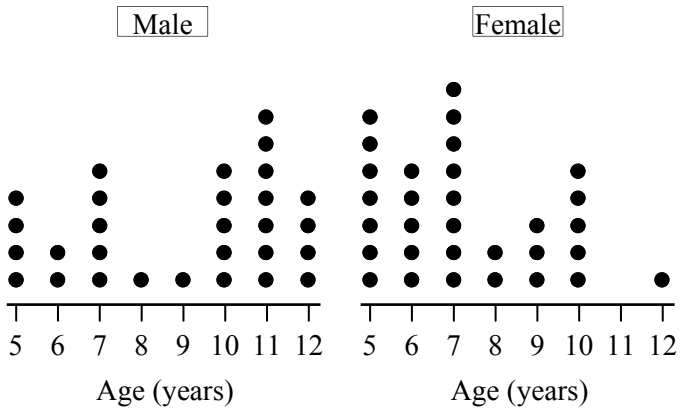
6 What percentage of people prefer Brand B?

Question 4 Ans. → 26 / Question 3 Ans. → 45 × 100 = 57.8%
Round to 1 d.p.

7 What percentage of people prefer Brand C?

Question 5 Ans. → 5 / Question 3 Ans. → 45 × 100 = 11.1%
Round to 1 d.p.

A waterslide at a theme park records the ages of the first 60 children who use the slide, the results are shown in two Dot plots. One male, one female.



8 How many children were male? 29

9 How many children were 7 years old? 13

10 How many children were 11 or older? 12

11 How many more 10 year olds used the slide compared to 6 year olds? 10 - 7 = 3

12 Are these true or false? There were more ...

(T) (F) Girls using the waterslide

(T) (F) 7 y.o. boys using the slide than 10 y.o. girls

(T) (F) Older male children than female.

13 What percentage of 9 y.o.'s were male?

Male 9 y.o. here → 1 / Total 9 y.o. here → 4 × 100 = 25%
Show the % sign

14 What percentage of 5 y.o.'s were female?

7 / 11 × 100 = 63.6%
Round to 1 d.p.

15 What percentage of females were 10 y.o.?

Total females here → 5 / 31 × 100 = 16.1%
Round to 1 d.p.

16 What percentage of 5 - 7 y.o.'s were male?

11 / 31 × 100 = 35.5%
Round to 1 d.p.

Data - Dot Plots (Sheet 2)

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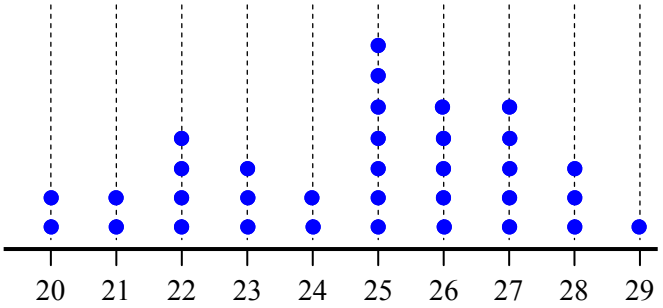
Display the data in a dot plot answering the questions from the plot.

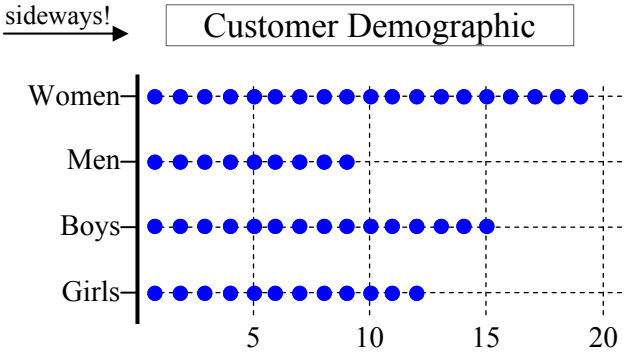
1 Data: 25, 28, 23, 24, 23, 24, 27, 26, 25, 25, 22, 22, 20, 29, 26, 25, 27, 26, 28, 21, 25, 25, 27, 21, 25, 27, 26, 27, 22, 26, 23, 20, 22, 28.



2 Women, Men, Boys and Girls entering a store have been recorded. Display on a dot plot.

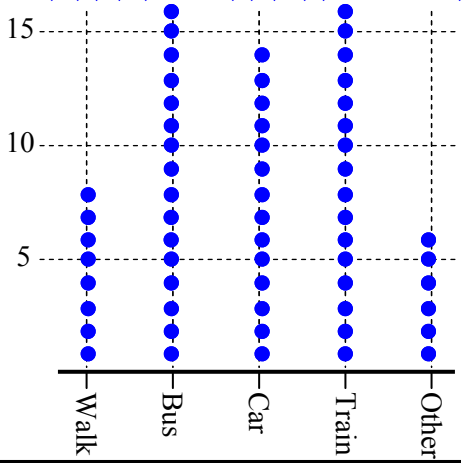
Data:
MBGWB GGBWW BBWWW
MMGWB MWGBW GBWBB MMWGW
BGGBW WGMWB MWWGB WWGBM.

sideways!



3 A student survey recorded five different methods of travel to school, they are: Walking, Bus, Car and Train, Other. Display on a dot plot.

Data:
TBWWO WWCBB BCWOW CCBCO
BCCTC TCBBT TWBTT TTBCO
BCTTT TBCWO OCTBC TTBBB



Method of School Travel

4 How many students were surveyed? 60

5 What percentage of students walked to school?

$\frac{8}{60} \times 100 = 13.3\%$ Answer to 1 d.p.

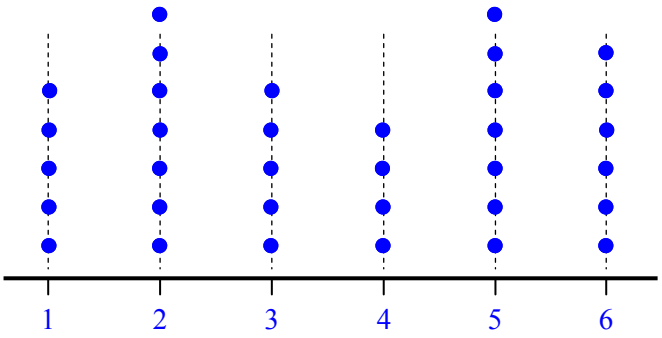
6 What percentage of students used public transport?

$\frac{32}{60} \times 100 = 53.3\%$ Answer to 1 d.p.

A die was rolled with the results shown below. Display the data as a dot plot, then in a frequency table.

7 Data: 3, 1, 5, 6, 3, 4, 2, 2, 1, 5, 3, 6, 2, 3, 2, 5, 5, 6, 4, 5, 2, 3, 6, 1, 1, 6, 5, 4, 1, 6, 2, 2, 4, 5.

Dot Plot of Die Rolling Results



8

Score	Frequency
1	5
2	7
3	5
4	4
5	7
6	6
	$\Sigma f = 34$

9 How many times was a number less than 3 rolled? 12 times

10 How many times was an even number rolled? 17 times

11 How many times was a prime number rolled? 19 times

12 What percentage of die rolls resulted in a multiple of 3?

$\frac{11}{34} \times 100 = 32.4\%$ Answer to 1 d.p.

13 What percentage of die rolls resulted in a factor of 12?

$\frac{27}{34} \times 100 = 79.4\%$ Answer to 1 d.p.

Data - Dot Plots (Homework Strip)

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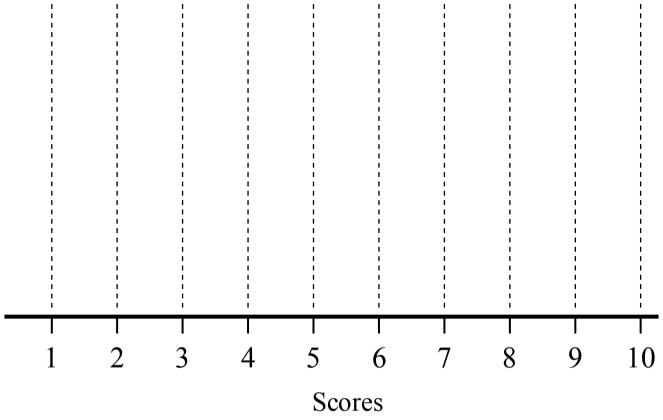
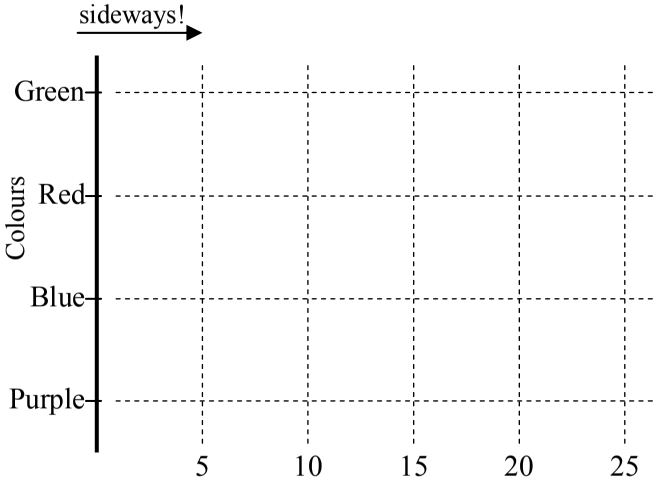
Date Due:

Construct a dot plot using the supplied data

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 9, 8, 7, 2, 6, 3, 10, 9, 7, 3, 4, 1, 9, 8, 3, 5, 7, 10, 5, 3, 2, 8, 3, 1, 3, 5, 7, 9, 6, 4, 2, 3, 9, 10, 1, 4, 7, 7, 5, 7, 8, 9.



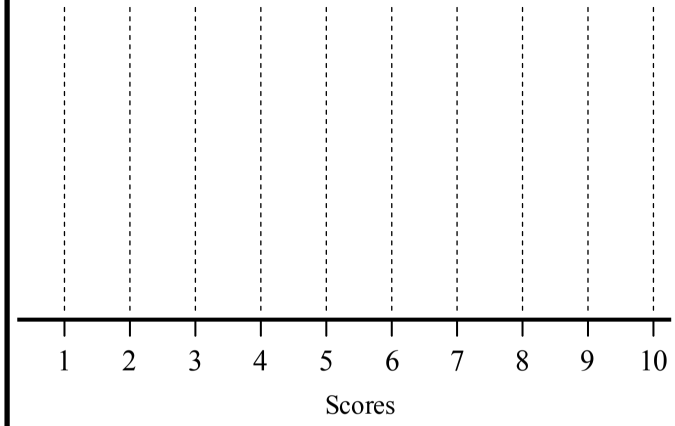
2 Data: GBRBB PRGBG GBBGR RPRGB BBRGB BRPGB PGBBR BRPGB GBBBR GGBBR PRGBP PRGBR RRPGP BRBBR



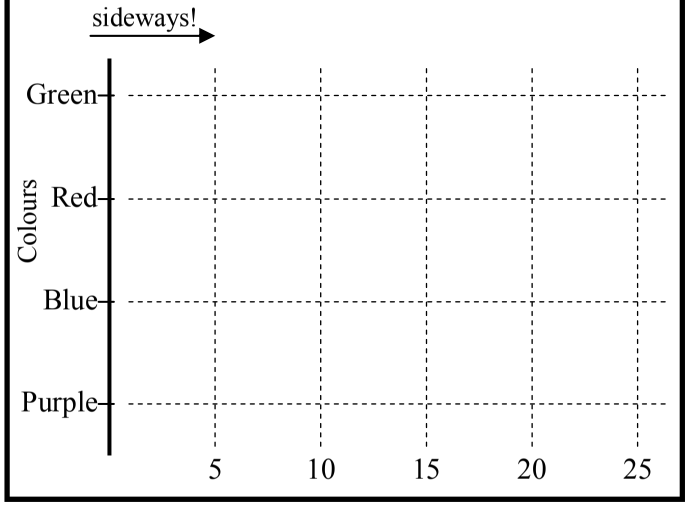
Date Due:

Construct a dot plot using the supplied data

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 9, 8, 7, 2, 6, 3, 10, 9, 7, 3, 4, 1, 9, 8, 3, 5, 7, 10, 5, 3, 2, 8, 3, 1, 3, 5, 7, 9, 6, 4, 2, 3, 9, 10, 1, 4, 7, 7, 5, 7, 8, 9.



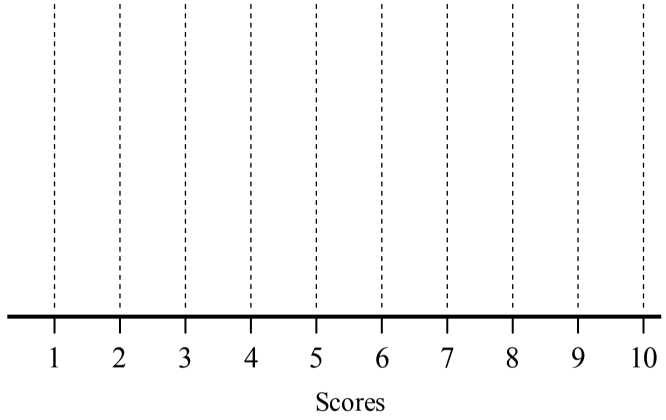
2 Data: GBRBB PRGBG GBBGR RPRGB BBRGB BRPGB PGBBR BRPGB GBBBR GGBBR PRGBP PRGBR RRPGP BRBBR



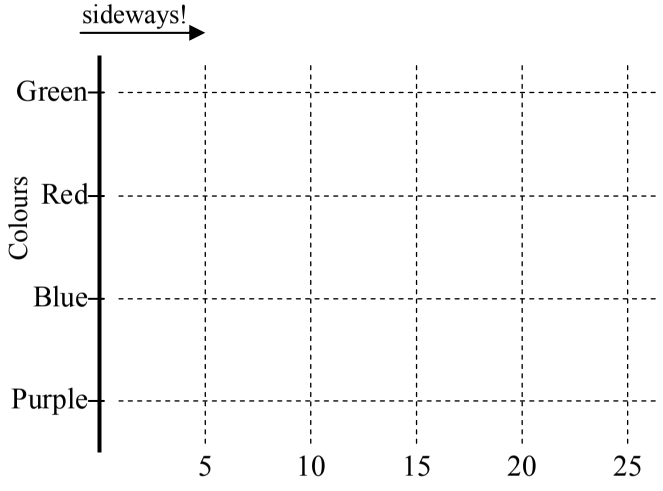
Date Due:

Construct a dot plot using the supplied data

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 9, 8, 7, 2, 6, 3, 10, 9, 7, 3, 4, 1, 9, 8, 3, 5, 7, 10, 5, 3, 2, 8, 3, 1, 3, 5, 7, 9, 6, 4, 2, 3, 9, 10, 1, 4, 7, 7, 5, 7, 8, 9.



2 Data: GBRBB PRGBG GBBGR RPRGB BBRGB BRPGB PGBBR BRPGB GBBBR GGBBR PRGBP PRGBR RRPGP BRBBR



Data - Dot Plots (Homework Strip)

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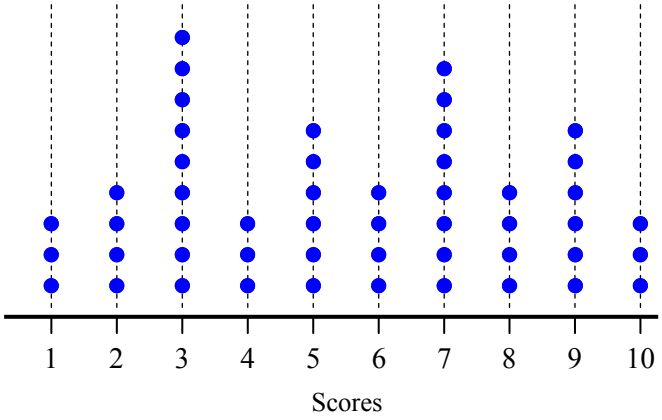
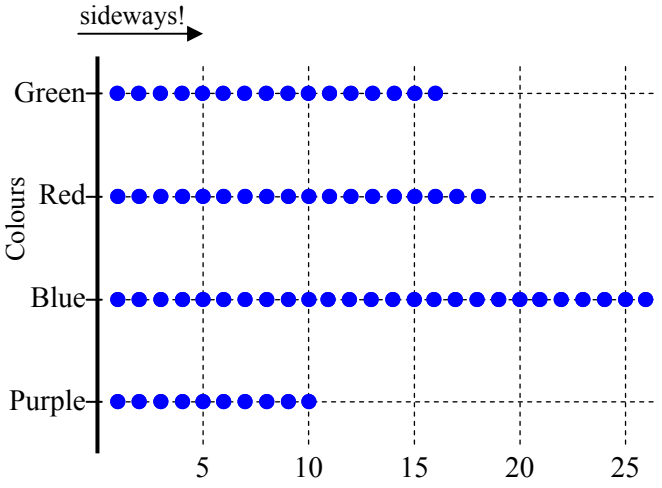
Date Due:

Construct a dot plot using the supplied data

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 9, 8, 7, 2, 6, 3, 10, 9, 7, 3, 4, 1, 9, 8, 3, 5, 7, 10, 5, 3, 2, 8, 3, 1, 3, 5, 7, 9, 6, 4, 2, 3, 9, 10, 1, 4, 7, 7, 5, 7, 8, 9.



2 Data: GBRBB PRGBG GBBGR RPRGB BBRGB BRPGB PGBBR BRPGB GBBBR GGBBR PRGBP PRGBR RRPGP BRBBR



Data - Ordered Stem and Leaf Plots

Stem and leaf plots separate data into its units component and its tens (and above) component. They then display the 'units' on the right in the leaf section and the 'tens' on left in a stem column. The effect is to break the data up into classes and also display a simple column graph (on its side) of numbers showing the way the data is spread.

Numbers have their units removed so:

- 23 becomes 'a 2' and 'a 3'
- 15 becomes 'a 1' and 'a 5'
- 9 (is seen as 09) becomes 'a 0' and 'a 9'

Stem	Leaf
0	9
1	5
2	3

In a plot this looks like the plot shown above. But what about larger numbers?

- 106 becomes 'a 10' and 'a 6'
- 117 becomes 'a 11' and 'a 7'
- 125 becomes 'a 12' and 'a 5'
- 120 becomes 'a 12' and 'a 0'
- 110 becomes 'a 11' and 'a 0'

Stem	Leaf
10	6
11	70
12	50

Because the units have two digits in front, both digits are shown in the Stem column.

When all data has been entered into the plot, the plot then has to be rewritten in order. The stem stays the same, but the leaf is put into ascending order. That is it!

Stem	Leaf
10	6
11	70
12	50

Leaf in order →

Stem	Leaf
10	6
11	07
12	05

Data - Ordered Stem and Leaf Plots (Sheet 1)

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Answer these questions about the Stem and Leaf plots below.

Stem	Leaf
0	2 7
1	4 8 8 ☐
2	5
3	4 7 7 7
4	0 3

1 What is the highest score? ☐

2 Which score occurs twice? (only) ☐

3 Which score has the highest frequency? ☐

4 Suggest two possible scores that could be represented by a digit being in the box in the second row. ☐, ☐

Tens	Units
8	5 8
9	3 3 7 9
10	2 2 5
11	0 0 0
12	7 ☐

5 What is the highest score? ☐

6 Which score has the highest frequency? ☐

7 Suggest three possible scores that could be represented by a digit being in the box in the fifth row.

☐ , ☐ , ☐ .

8 How many scores are above 100? ☐

9 How many scores are between 95 and 105? ☐



These stem and leaf plots aren't in order. Rewrite the leaf section in ascending order. Remember there are no commas in a stem and leaf plot

10

Stem	Leaf
0	8 2 7 3
1	0 5 2 7
2	3 7 1
3	0 9 1 7

IN ORDER →

Stem	Leaf
0	☐ ☐ ☐ ☐
1	☐ ☐ ☐ ☐
2	☐ ☐ ☐
3	☐ ☐ ☐ ☐

11

Tens	Units
9	7 7 2
10	4 0 7 2
11	0 9 1
12	8 8 3 2

IN ORDER →

Tens	Units
9	☐ ☐ ☐
10	☐ ☐ ☐ ☐
11	☐ ☐ ☐
12	☐ ☐ ☐ ☐

You are given data this time. Display the data in two stem and leaf plots. The first one is raw (in the order you enter the data), the second stem and leaf plot is to be ordered.



12 Data: 6, 14, 28, 39, 17, 12, 5, 28, 36, 21, 15, 19, 27, 30, 2, 22, 38, 24.

RAW

Tens	Units
0	
1	
2	
3	

IN ORDER →

ORDERED

Tens	Units
0	
1	
2	
3	

13 Data: 95, 106, 117, 128, 100, 121, 104, 93, 112, 93, 114, 101, 105, 115, 94, 114.

RAW

Tens	Units
9	
10	
11	
12	

IN ORDER →

ORDERED

Tens	Units
9	
10	
11	
12	

Data - Ordered Stem and Leaf Plots (Sheet 2)

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Display the data sets in Raw then Ordered Stem and Leaf plots. Don't use commas!



1 Data: 23, 17, 35, 21, 30, 8, 11, 24, 6, 9, 17, 21, 32, 3, 11, 13.

RAW → ORDERED

Tens	Units
0	
1	
2	
3	

Tens	Units
0	
1	
2	
3	

2 Data: 74, 97, 70, 106, 92, 100, 86, 91, 97, 85, 110, 83, 101, 95, 79, 90, 103, 85.

RAW → ORDERED

Stem	Leaf

Stem	Leaf

3 Data: 998, 971, 1 005, 993, 1 018, 1 015, 985, 994, 1 026, 1 012, 1 002, 994, 978, 993, 970, 1 018, 989, 991, 1 011, 1 021, 986, 1 000, 1 002, 995, 1 012, 977, 983.

Stem	Leaf

Stem	Leaf

4 Data: 220, 195, 206, 212, 184, 177, 209, 195, 191, 204, 173, 168, 188, 228, 219, 176, 192, 213, 201, 180, 171.

Stem	Leaf

→

Stem	Leaf

5 How many scores were over 200? _____

6 How many scores were even? _____

7 What percentage of scores were less than 180?

$$\frac{\square}{\square} \times 100 = \frac{\square}{\text{Answer to 1 d.p.}}$$

8 What percentage of scores were between 190 and 210?

$$\frac{\square}{\square} \times \square = \frac{\square}{\text{Answer to 1 d.p.}}$$

9 Data: 52, 47, 11, 28, 9, 54, 62, 47, 19, 17, 24, 38, 68, 51, 20, 7, 33, 49, 19, 28, 30, 47, 53, 40, 15, 38, 14.

Stem	Leaf

→

Stem	Leaf

Data - Ordered Stem and Leaf Plots (Sheet 1)

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Answer these questions about the Stem and Leaf plots below.

Stem	Leaf
0	2 7
1	4 8 8 ☐
2	5
3	4 7 7 7
4	0 3

1 What is the highest score? 43

2 Which score occurs twice? (only) 18

3 Which score has the highest frequency? 37

4 Suggest two possible scores that could be represented by a digit being in the box in the second row. 18, 19

Tens	Units
8	5 8
9	3 3 7 9
10	2 2 5
11	0 0 0
12	7 ☐

5 What is the highest score? 127

6 Which score has the highest frequency? 110

7 Suggest three possible scores that could be represented by a digit being in the box in the fifth row.

127, 128, 129.

8 How many scores are above 100?

7

9 How many scores are between 95 and 105? 4

These stem and leaf plots aren't in order. Rewrite the leaf section in ascending order. Remember there are no commas in a stem and leaf plot

10

Stem	Leaf
0	8 2 7 3
1	0 5 2 7
2	3 7 1
3	0 9 1 7

IN ORDER

Stem	Leaf
0	2 3 7 8
1	0 2 5 7
2	1 3 7
3	0 1 7 9

11

Tens	Units
9	7 7 2
10	4 0 7 2
11	0 9 1
12	8 8 3 2

IN ORDER

Tens	Units
9	2 7 7
10	0 2 4 7
11	0 1 9
12	2 3 8 8

You are given data this time. Display the data in two stem and leaf plots. The first one is raw (in the order you enter the data), the second stem and leaf plot is to be ordered.

12 Data: 6, 14, 28, 39, 17, 12, 5, 28, 36, 21, 15, 19, 27, 30, 2, 22, 38, 24.

RAW

Tens	Units
0	6 5 2
1	4 7 2 5 9
2	8 8 1 7 2 4
3	9 6 0 8

IN ORDER

ORDERED

Tens	Units
0	2 5 6
1	2 4 5 7 9
2	1 2 4 7 8 8
3	0 6 8 9

13 Data: 95, 106, 117, 128, 100, 121, 104, 93, 112, 93, 114, 101, 105, 115, 94, 114.

RAW

Tens	Units
9	5 3 3 4
10	6 0 4 1 5
11	7 2 4 5 4
12	8 1

IN ORDER

ORDERED

Tens	Units
9	3 3 4 5
10	0 1 4 5 6
11	2 4 4 5 7
12	1 8

Data - Ordered Stem and Leaf Plots (Sheet 2)

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Display the data sets in Raw then Ordered Stem and Leaf plots. Don't use commas!



1 Data: 23, 17, 35, 21, 30, 8, 11, 24, 6, 9, 17, 21, 32, 3, 11, 13.

RAW → ORDERED

Tens	Units
0	8 6 9 3
1	7 1 7 1 3
2	3 1 4 1
3	5 0 2

Tens	Units
0	3 6 8 9
1	1 1 3 7 7
2	1 1 3 4
3	0 2 5

2 Data: 74, 97, 70, 106, 92, 100, 86, 91, 97, 85, 110, 83, 101, 95, 79, 90, 103, 85.

RAW → ORDERED

Stem	Leaf
7	4 0 9
8	6 5 3 5
9	7 2 1 7 5 0
10	6 0 1 3
11	0

Stem	Leaf
7	0 4 9
8	3 5 5 6
9	0 1 2 5 7 7
10	0 1 3 6
11	0

3 Data: 998, 971, 1 005, 993, 1 018, 1 015, 985, 994, 1 026, 1 012, 1 002, 994, 978, 993, 970, 1 018, 989, 991, 1 011, 1 021, 986, 1 000, 1 002, 995, 1 012, 977, 983.

Stem	Leaf
97	1 8 0 7
98	5 9 6 3
99	8 3 4 4 3 1 5
100	5 2 0 2
101	8 5 2 8 1 2
102	6 1

Stem	Leaf
97	0 1 7 8
98	3 5 6 9
99	1 3 3 4 4 5 8
100	0 2 2 5
101	1 2 2 5 8 8
102	1 6

4 Data: 220, 195, 206, 212, 184, 177, 209, 195, 191, 204, 173, 168, 188, 228, 219, 176, 192, 213, 201, 180, 171.

Stem	Leaf
16	8
17	7 3 6 1
18	4 8 0
19	5 5 1 2
20	6 9 4 1
21	2 9 3
22	0 8

Stem	Leaf
16	8
17	1 3 6 7
18	0 4 8
19	1 2 5 5
20	1 4 6 9
21	2 3 9
22	0 8

5 How many scores were over 200? 9

6 How many scores were even? 11

7 What percentage of scores were less than 180?

$$\frac{5}{21} \times 100 = 23.8\%$$

Answer to 1 d.p.

8 What percentage of scores were between 190 and 210?

$$\frac{8}{21} \times 100 = 38.1\%$$

Answer to 1 d.p.

9 Data: 52, 47, 11, 28, 9, 54, 62, 47, 19, 17, 24, 38, 68, 51, 20, 7, 33, 49, 19, 28, 30, 47, 53, 40, 15, 38, 14.

Stem	Leaf
0	9 7
1	1 9 7 9 5 4
2	8 4 0 8
3	8 3 0 8
4	7 7 9 7 0
5	2 4 1 3
6	2 8

Stem	Leaf
0	7 9
1	1 4 5 7 9 9
2	0 4 8 8
3	0 3 8 8
4	0 7 7 7 9
5	1 2 3 4
6	2 8

Data - Stem and Leaf Plots (Homework Strip)

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Date Due: _____

Organise the data in raw & ordered, stem and leaf plots

1 Data: 25, 11, 22, 13, 30, 6, 19, 26, 12, 14, 29, 37, 5, 11, 2, 33, 20, 14, 37, 32.

RAW

Tens	Units
0	
1	
2	
3	

IN ORDER →

ORDERED

Tens	Units
0	
1	
2	
3	

2 Data: 102, 85, 116, 90, 125, 108, 82, 132, 111, 96, 122, 100, 83, 104, 119, 115, 124, 131, 87, 102, 134, 118, 106, 126, 97, 88, 123, 90.

RAW

Stem	Leaf

IN ORDER →

ORDERED

Stem	Leaf

Date Due: _____

Organise the data in raw & ordered, stem and leaf plots

1 Data: 25, 11, 22, 13, 30, 6, 19, 26, 12, 14, 29, 37, 5, 11, 2, 33, 20, 14, 37, 32.

RAW

Tens	Units
0	
1	
2	
3	

IN ORDER →

ORDERED

Tens	Units
0	
1	
2	
3	

2 Data: 102, 85, 116, 90, 125, 108, 82, 132, 111, 96, 122, 100, 83, 104, 119, 115, 124, 131, 87, 102, 134, 118, 106, 126, 97, 88, 123, 90.

RAW

Stem	Leaf

IN ORDER →

ORDERED

Stem	Leaf

Date Due: _____

Organise the data in raw & ordered, stem and leaf plots

1 Data: 25, 11, 22, 13, 30, 6, 19, 26, 12, 14, 29, 37, 5, 11, 2, 33, 20, 14, 37, 32.

RAW

Tens	Units
0	
1	
2	
3	

IN ORDER →

ORDERED

Tens	Units
0	
1	
2	
3	

2 Data: 102, 85, 116, 90, 125, 108, 82, 132, 111, 96, 122, 100, 83, 104, 119, 115, 124, 131, 87, 102, 134, 118, 106, 126, 97, 88, 123, 90.

RAW

Stem	Leaf

IN ORDER →

ORDERED

Stem	Leaf

Data - Stem and Leaf Plots (Homework Strip)

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Date Due:

Organise the data in raw & ordered, stem and leaf plots

1 Data: 25, 11, 22, 13, 30, 6, 19, 26, 12, 14, 29, 37, 5, 11, 2, 33, 20, 14, 37, 32.

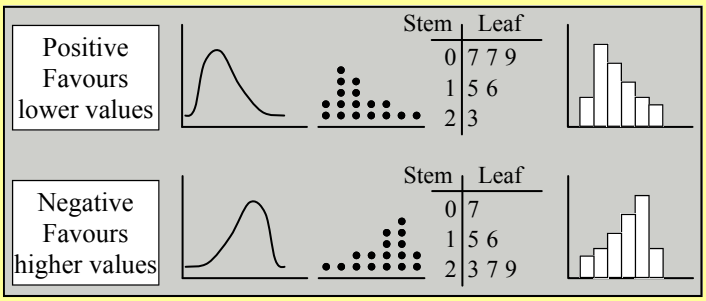
RAW			ORDERED	
Tens	Units		Tens	Units
0	6 5 2		0	2 5 6
1	1 3 9 2 4 1 4	IN ORDER →	1	1 1 2 3 4 4 9
2	5 2 6 9 0		2	0 2 5 6 9
3	0 7 3 7 2		3	0 2 3 7 7

2 Data: 102, 85, 116, 90, 125, 108, 82, 132, 111, 96, 122, 100, 83, 104, 119, 115, 124, 131, 87, 102, 134, 118, 106, 126, 97, 88, 123, 90.

RAW			ORDERED	
Stem	Leaf		Stem	Leaf
8	5 2 3 7 8		8	2 3 5 7 8
9	0 6 7 0		9	0 0 6 7
10	2 8 0 4 2 6	IN ORDER →	10	0 2 2 4 6 8
11	6 1 9 5 8		11	1 5 6 8 9
12	5 2 4 6 3		12	2 3 4 5 6
13	2 1 4		13	1 2 4

Data - Positive and Negative Skew

When scores is displayed in a graph the way the columns, or dots or polygon is spread out is called a distribution. When a distribution is skewed it is like it is pushed to the left or pushed to the right.



Positive skew is when the graph appears to be pushed to the left while Negative skew appears to be pushed to the right. A more involved definition is that Positive skew generally has an order of "mode then median then mean" so the mean has the highest value. While negative is the opposite, but this won't be dealt with, with these sheets. The appearance of the distribution is the deciding factor.

The first column asks you to look at the graphs and decide which way they are skewed, they are all skewed, just determine if it is positive or negative.

The remaining columns require you to organise the data into graphs and then suggest whether they are positively or negatively skewed.

Data - Positive and Negative Skew (Sheet 1)

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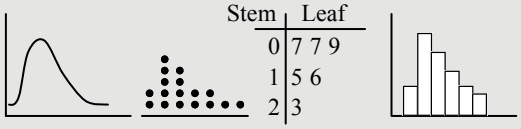
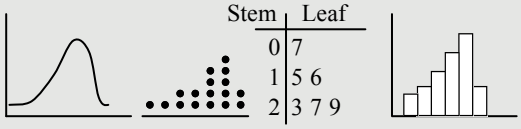
Are these positive or negative skew? Fill the circle for the choice for each of these.

Positive
Favours
lower values



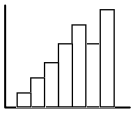
Stem	Leaf
0	7 7 9
1	5 6
2	3

Negative
Favours
higher values



Stem	Leaf
0	7
1	5 6
2	3 7 9

1



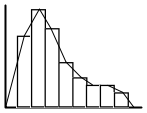
○ Positive
○ Negative

2



○ Positive
○ Negative

3



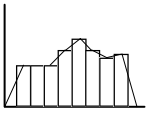
○ Positive
○ Negative

4



○ Positive
○ Negative

5



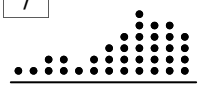
○ Positive
○ Negative

6

Stem	Leaf
5	0
6	5 6
7	3 8 8
8	7 7 7

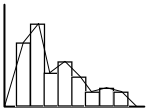
○ Positive ○ Negative

7



○ Positive
○ Negative

8



○ Positive
○ Negative

9



○ Positive
○ Negative

10



○ Positive
○ Negative

11

Stem	Leaf
10	1 5 6 6 8
11	2 3 3
12	4
13	1 5

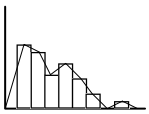
○ Positive ○ Negative

12



○ Positive
○ Negative

13



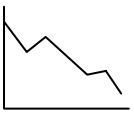
○ Positive
○ Negative

14



○ Positive
○ Negative

15



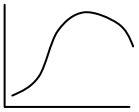
○ Positive
○ Negative

16

Tens	Units
4	1
5	0 3
6	1 8 8

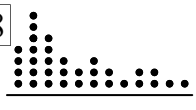
○ Positive
○ Negative

17



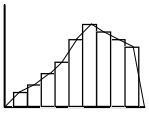
○ Positive
○ Negative

18



○ Positive
○ Negative

19



○ Positive
○ Negative

20



○ Positive
○ Negative

21

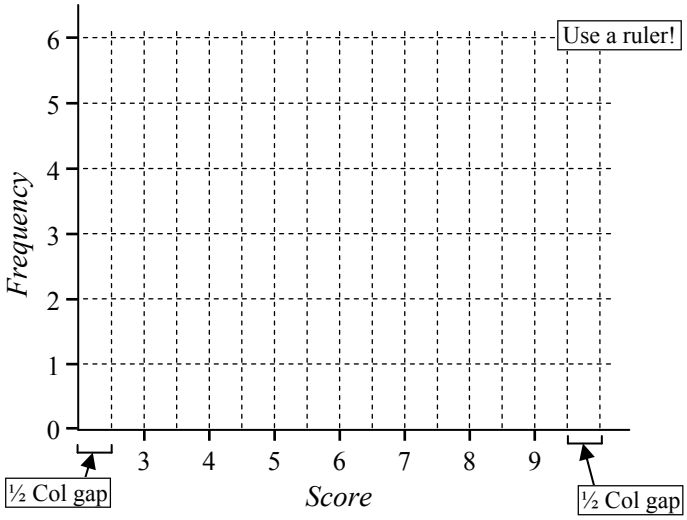
Tens	Units
9	2 3 3 7
10	0 4 9
11	3

○ Positive
○ Negative

Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

22 Data: 3, 4, 7, 5, 6, 3, 3, 4, 4, 5, 9, 4, 3, 5, 4, 6, 5, 3, 4.

Score	Tally	Frequency
		$\Sigma f =$



23 The above distribution is _____ skewed.
Positively/Negatively

Now in a stem and leaf plot. No commas.

24 Data: 67, 72, 51, 75, 40, 63, 75, 67, 42, 60, 59, 66.

Stem	Leaf

IN ORDER

Stem	Leaf

25 The above distribution is _____ skewed.

Data - Positive and Negative Skew (Sheet 2)

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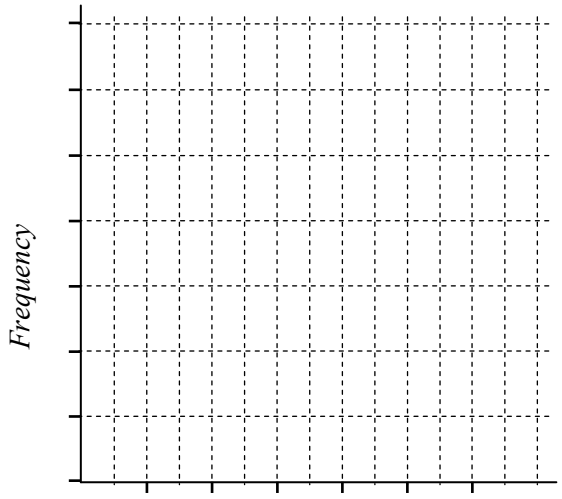
Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

1 Data: 6, 5, 5, 4, 6, 1, 3, 4, 3, 5, 6, 6, 4, 4, 1, 3, 6, 4, 5, 3, 5, 6, 5, 5, 6.

Score	Tally	Frequency



2 The above distribution is _____ skewed.

Display this data in a dot plot. Then comment on the shape of the distribution.



3 Data: 18, 21, 23, 20, 19, 23, 19, 24, 22, 21, 20, 18, 22, 19, 19, 20, 22, 24, 18, 20, 25, 20.



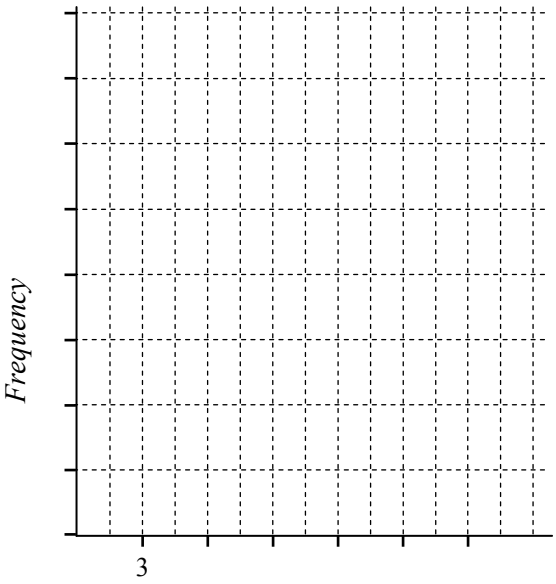
4 The above distribution is _____

Display the data in a frequency table with classes and frequency histogram/polygon. Outline the shape.

5 Data: 6, 20, 11, 25, 9, 7, 30, 4, 12, 6, 29, 15, 1, 17, 3, 13, 9, 23, 14, 8, 5, 2, 27, 14, 6, 7, 20.

Class	*c.c.*	Tally	*f*
1 - 5	3		
6 - 10			
11 - 15			
16 - 20			
21 - 25			
26 - 30			

Add classes and ÷ 2.



6 The above distribution is _____

Display the data in an ordered stem and leaf plot.

7 Data: 756, 778, 750, 761, 783, 760, 785, 772, 767, 772, 764, 783, 775, 753, 780, 773.

Tens	Units

IN ORDER

Tens	Units

8 The above distribution is _____

Data - Positive and Negative Skew (Sheet 1)

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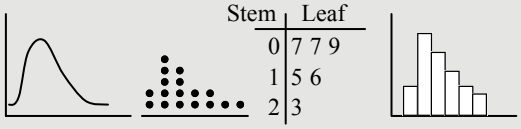
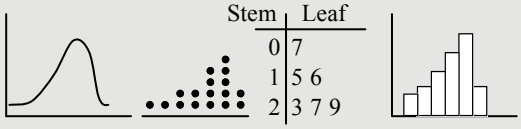
Are these positive or negative skew? Fill the circle for the choice for each of these.

Positive Favours lower values



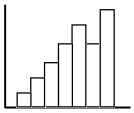
Stem	Leaf
0	7 7 9
1	5 6
2	3

Negative Favours higher values



Stem	Leaf
0	7
1	5 6
2	3 7 9

1



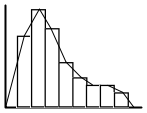
○ Positive
● Negative

2



○ Positive
● Negative

3



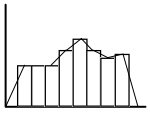
● Positive
○ Negative

4



● Positive
○ Negative

5



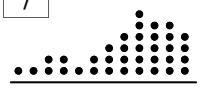
○ Positive
● Negative

6

Stem	Leaf
5	0
6	5 6
7	3 8 8
8	7 7 7

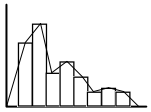
○ Positive ● Negative

7



○ Positive
● Negative

8



● Positive
○ Negative

9



● Positive
○ Negative

10



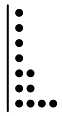
○ Positive
● Negative

11

Stem	Leaf
10	1 5 6 6 8
11	2 3 3
12	4
13	1 5

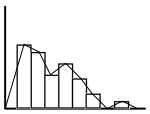
● Positive ○ Negative

12



○ Positive
● Negative

13



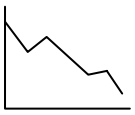
● Positive
○ Negative

14



○ Positive
● Negative

15



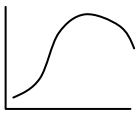
● Positive
○ Negative

16

Tens	Units
4	1
5	0 3
6	1 8 8

○ Positive
● Negative

17



○ Positive
● Negative

18



● Positive
○ Negative

19



○ Positive
● Negative

20



○ Positive
● Negative

21

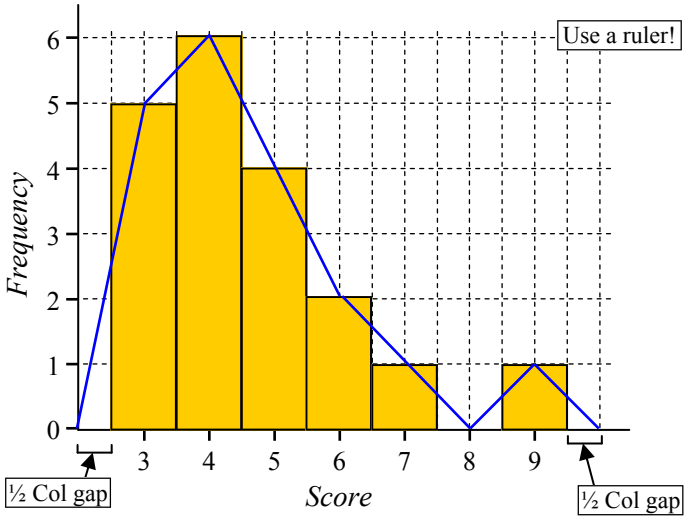
Tens	Units
9	2 3 3 7
10	0 4 9
11	3

● Positive
○ Negative

Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

22 Data: 3, 4, 7, 5, 6, 3, 3, 4, 4, 5, 9, 4, 3, 5, 4, 6, 5, 3, 4.

Score	Tally	Frequency
3	𝍸	5
4	𝍸 |	6
5	𝍸	4
6	||	2
7	|	1
8		0
9	|	1
		$\Sigma f = 19$



23 The above distribution is positively skewed.
Positively/Negatively

Now in a stem and leaf plot. No commas.

24 Data: 67, 72, 51, 75, 40, 63, 75, 67, 42, 60, 59, 66.

Stem	Leaf
4	0 2
5	1 9
6	7 3 7 0 6
7	2 5 5

IN ORDER

Stem	Leaf
4	0 2
5	1 9
6	0 3 6 7 7
7	2 5 5

25 The above distribution is negatively skewed.

Data - Positive and Negative Skew (Sheet 2)

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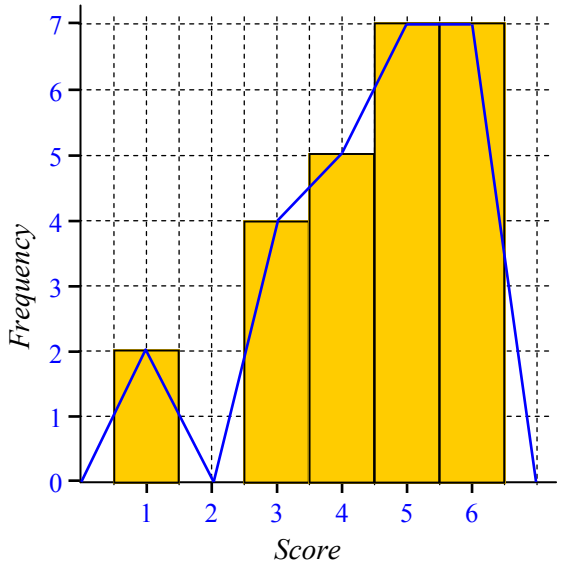
Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

1 Data: 6, 5, 5, 4, 6, 1, 3, 4, 3, 5, 6, 6, 4, 4, 1, 3, 6, 4, 5, 3, 5, 6, 5, 5, 6.

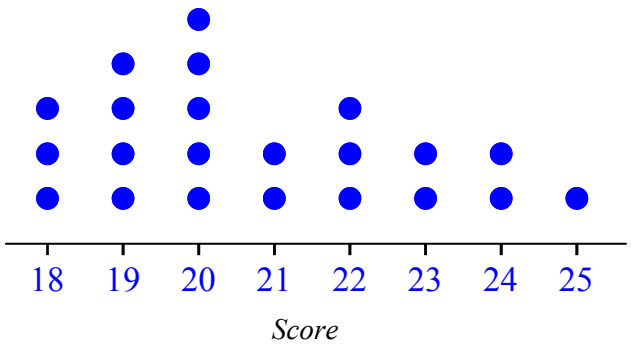
Score	Tally	Frequency
1	||	2
2		0
3	||||	4
4	𝍸	5
5	𝍸 ||	7
6	𝍸 ||	7
		$\Sigma f = 25$



2 The above distribution is negatively skewed.

Display this data in a dot plot. Then comment on the shape of the distribution.

3 Data: 18, 21, 23, 20, 19, 23, 19, 24, 22, 21, 20, 18, 22, 19, 19, 20, 22, 24, 18, 20, 25, 20.



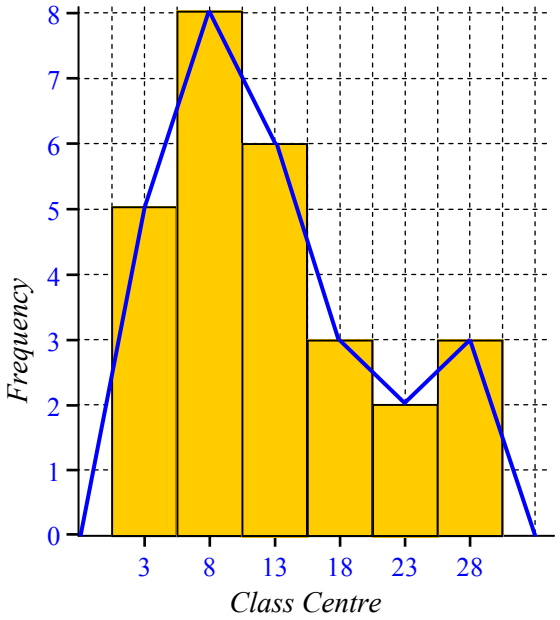
4 The above distribution is positively skewed.

Display the data in a frequency table with classes and frequency histogram/polygon. Outline the shape.

5 Data: 6, 20, 11, 25, 9, 7, 30, 4, 12, 6, 29, 15, 1, 17, 3, 13, 9, 23, 14, 8, 5, 2, 27, 14, 6, 7, 20.

Class	*c.c.*	Tally	*f*
1 - 5	3	𝍸	5
6 - 10	8	𝍸 |||	8
11 - 15	13	𝍸 |	6
16 - 20	18	|||	3
21 - 25	23	||	2
26 - 30	28	|||	3
			$\Sigma f =$

Add classes and ÷ 2.



6 The above distribution is positively skewed.

Display the data in an ordered stem and leaf plot.

7 Data: 756, 778, 750, 761, 783, 760, 785, 772, 767, 772, 764, 783, 775, 753, 780, 773.

Tens	Units
75	6 0 3
76	1 0 7 4
77	8 2 2 5 3
78	3 5 3 0

IN ORDER

Tens	Units
75	0 3 6
76	0 1 4 7
77	2 2 3 5 8
78	0 3 3 5

8 The above distribution is negatively skewed.

Data - Positive and Negative Skew (Homework Strip)

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Date Due:

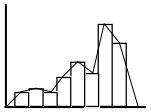
Are these positive or negatively skewed

1



○ Positive
○ Negative

2



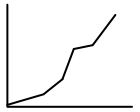
○ Positive
○ Negative

3

Stem	Leaf
2	0 1 3 8
3	5 5 6
4	3
5	7

○ Positive
○ Negative

4



○ Positive
○ Negative

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 41, 42, 38, 36, 40, 41, 42, 40, 39, 39, 40, 41, 41, 42, 38, 40, 42, 40, 41, 39, 41, 38, 42, 39.

Scores

6 The above distribution is:

_____

Date Due:

Are these positive or negatively skewed

1

○ Positive
○ Negative

2

○ Positive
○ Negative

3

Stem	Leaf
2	0 1 3 8
3	5 5 6
4	3
5	7

○ Positive
○ Negative

4

○ Positive
○ Negative

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 41, 42, 38, 36, 40, 41, 42, 40, 39, 39, 40, 41, 41, 42, 38, 40, 42, 40, 41, 39, 41, 38, 42, 39.

Scores

6 The above distribution is:

_____

Date Due:

Are these positive or negatively skewed

1

○ Positive
○ Negative

2

○ Positive
○ Negative

3

Stem	Leaf
2	0 1 3 8
3	5 5 6
4	3
5	7

○ Positive
○ Negative

4

○ Positive
○ Negative

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 41, 42, 38, 36, 40, 41, 42, 40, 39, 39, 40, 41, 41, 42, 38, 40, 42, 40, 41, 39, 41, 38, 42, 39.

Scores

6 The above distribution is:

_____

Date Due:

Are these positive or negatively skewed

1

○ Positive
○ Negative

2

○ Positive
○ Negative

3

Stem	Leaf
2	0 1 3 8
3	5 5 6
4	3
5	7

○ Positive
○ Negative

4

○ Positive
○ Negative

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 41, 42, 38, 36, 40, 41, 42, 40, 39, 39, 40, 41, 41, 42, 38, 40, 42, 40, 41, 39, 41, 38, 42, 39.

Scores

6 The above distribution is:

_____

Data - Positive and Negative Skew (Homework Strip)

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Date Due:

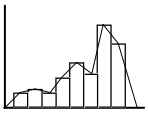
Are these positive or negatively skewed

1



● Positive
○ Negative

2



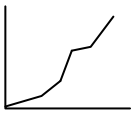
○ Positive
● Negative

3

Stem	Leaf
2	0 1 3 8
3	5 5 6
4	3
5	7

● Positive
○ Negative

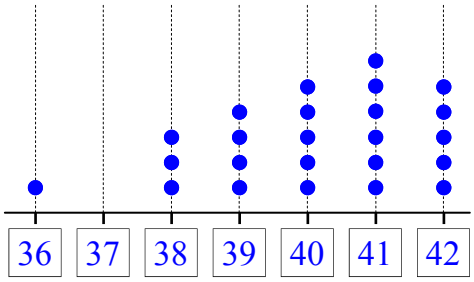
4



○ Positive
● Negative

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 41, 42, 38, 36, 40, 41, 42, 40, 39, 39, 40, 41, 41, 42, 38, 40, 42, 40, 41, 39, 41, 38, 42, 39.



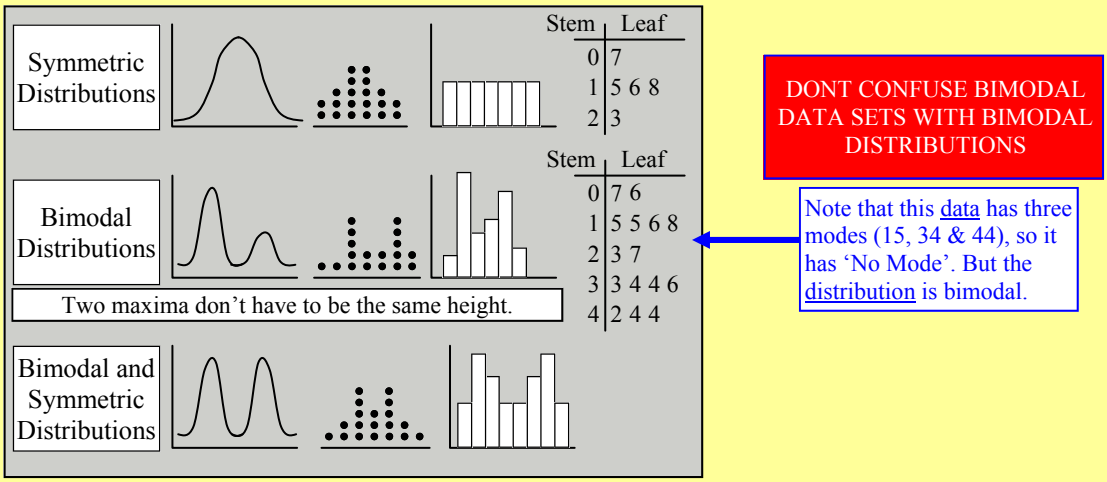
6 The above distribution is:

negatively skewed

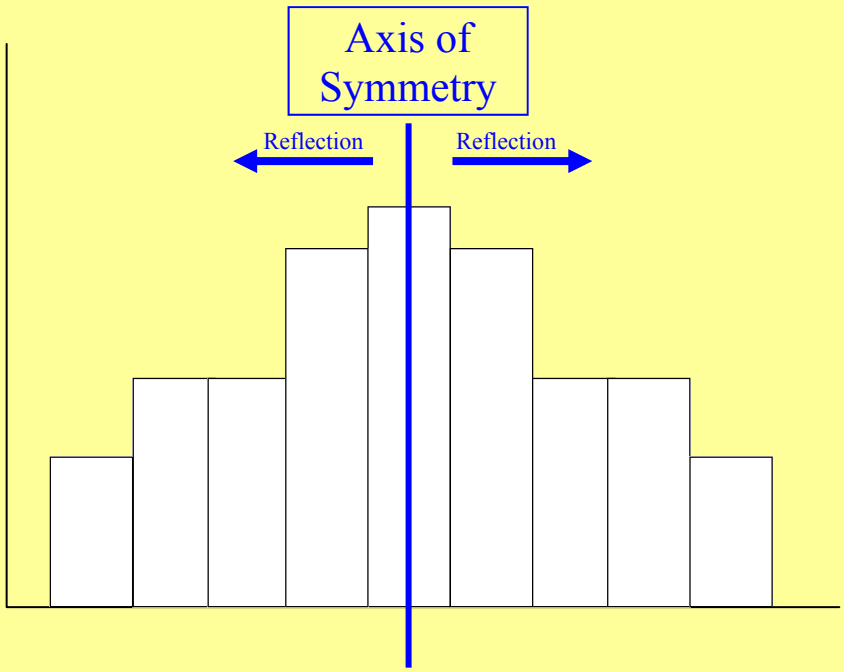
Data - Bimodal and Symmetric

When scores is displayed in a graph the way the columns, or dots or polygon is spread out is called a distribution. Previously ‘skew’ dealt with distributions that favoured lower or higher values in a ‘lop-sided’ distribution.

This worksheet deals with symmetric distributions, which are balanced, which could have an axis of symmetry and be reflected perfectly across it. Bimodal distributions are also covered which is when a distribution has two peaks or ‘maxima’. Don’t confuse Bimodal data with Bimodal distributions. Bimodal data has to have two scores occurring the most, with the same frequency. Bimodal distributions can have different frequency (heights) modes and may not even have a mode in the data set, such as with a stem and leaf plot.



You can also have bimodal and symmetric distributions are shown in the third row above. Bimodal is easy to see as there are two clear peaks, if you want to check for symmetry, find where the middle of the distribution is and check if each side is a reflection of the other.



Data - Bimodal and Symmetric (Sheet 1)

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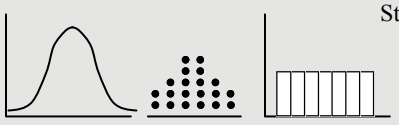
Symmetric Distributions



Stem	Leaf
0	7
1	5 6 8
2	3

Bimodal Distributions



Stem	Leaf
0	7 6
1	5 5 6 8
2	3 7
3	3 4 4 6
4	2 4 4

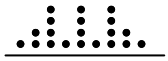
Two maxima don't have to be the same height.

Bimodal and Symmetric Distributions



Are the shape of these distributions symmetric, bimodal, both or neither? Fill in 1 or 2 circles!

1



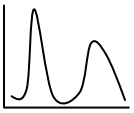
○ Symmetric
○ Bimodal
○ Neither

2



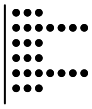
○ Symmetric
○ Bimodal
○ Neither

3



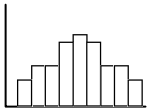
○ Symmetric
○ Bimodal
○ Neither

4



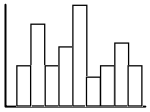
○ Symmetric
○ Bimodal
○ Neither

5



○ Symmetric
○ Bimodal
○ Neither

6



○ Symmetric
○ Bimodal
○ Neither

7



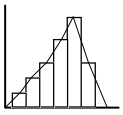
○ Symmetric
○ Bimodal
○ Neither

8

Stem	Leaf
10	1 5
11	2 3 3 3
12	4
13	1 5 5 5
14	2 3

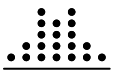
○ Symmetric
○ Bimodal
○ Neither

9



○ Symmetric
○ Bimodal
○ Neither

10



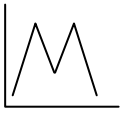
○ Symmetric
○ Bimodal
○ Neither

11



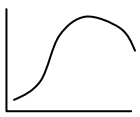
○ Symmetric
○ Bimodal
○ Neither

12



○ Symmetric
○ Bimodal
○ Neither

13



○ Symmetric
○ Bimodal
○ Neither

14

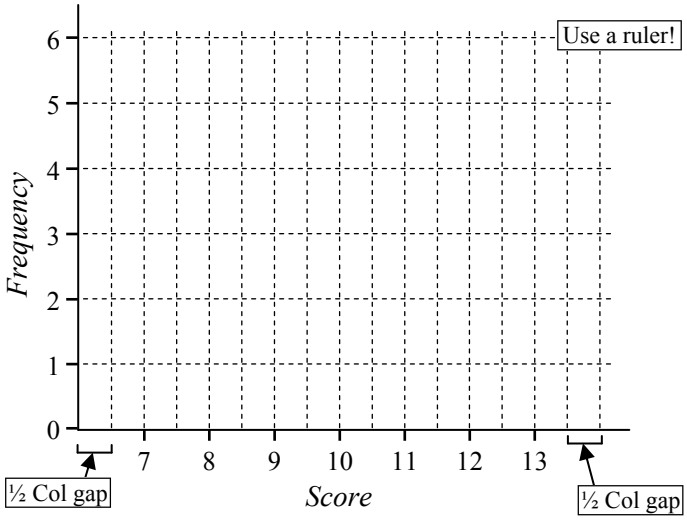


○ Symmetric
○ Bimodal
○ Neither

Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

15 Data: 9, 12, 13, 8, 9, 10, 12, 11, 7, 9, 11, 10, 9, 8, 8, 12, 7, 9, 12, 11, 7, 9.

Score	Tally	Frequency
		$\Sigma f =$



16 The distribution's shape is _____

Bimodal/Symmetric or Bimodal and Symmetric

Now in a stem and leaf plot. No commas.

17 Data: 26, 17, 31, 5, 15, 28, 13, 34, 8, 17, 21, 11, 20, 20.

Stem	Leaf
0	.....
1	.....
2	.....
3	.....

IN ORDER

Stem	Leaf
	.....
	.....
	.....
	.....

18 The distribution's shape is _____

Data - Bimodal and Symmetric (Sheet 2)

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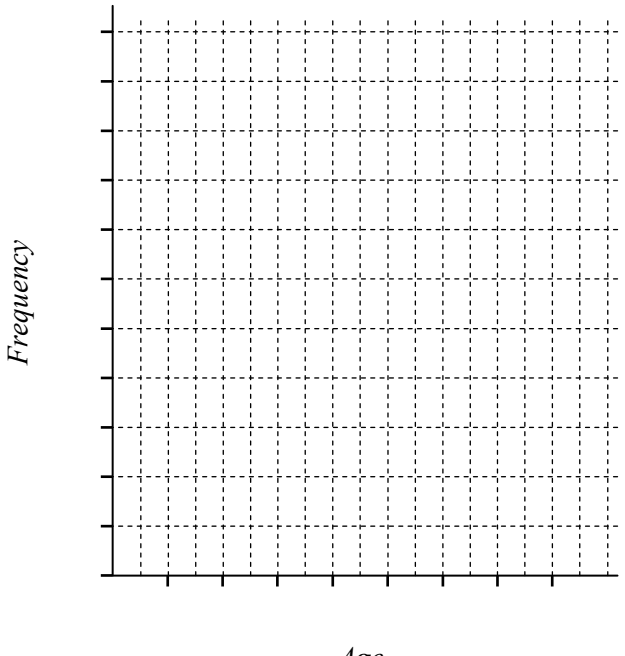
Year 11 students are tutor-reading with 17 Year 7 and 5 Year 8 students, everyone's ages are recorded in a survey. Place the results in a frequency table, then display them in a histogram and polygon.

1 Data: 11, 16, 12, 12, 17, 18, 13, 16, 12, 13, 17, 16, 12, 13, 17, 16, 16, 12, 16, 12, 17, 14, 16, 13, 11, 17, 18, 12, 17, 12, 17, 13, 17, 18, 15, 12, 16, 17, 13, 12, 17, 13, 11, 17, 12, 13.

Age	Tally	Frequency



2 The distribution's shape is: _____

3 Describe why the distribution is the shape shown above

_____

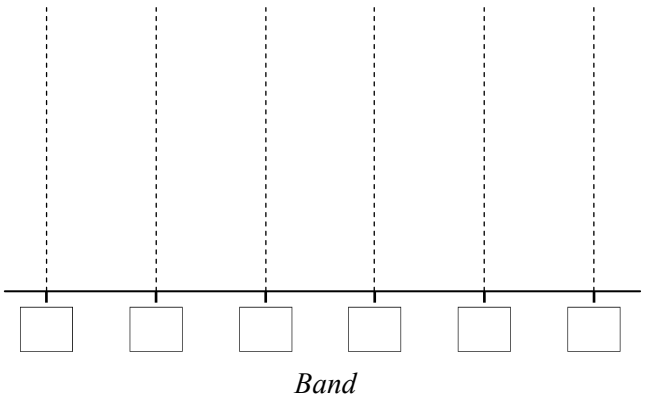
_____

_____

_____

An English teacher selects 30 students randomly from Year 9 to compare their 'Persuasive Writing' band results. Plot their results in a dot plot.

4 Data: 9, 6, 8, 6, 7, 7, 9, 10, 8, 7, 8, 9, 6, 5, 7, 8, 8, 10, 8, 7, 9, 6, 7, 8, 7, 5, 9, 7, 6, 8.



5 The distribution's shape is: _____

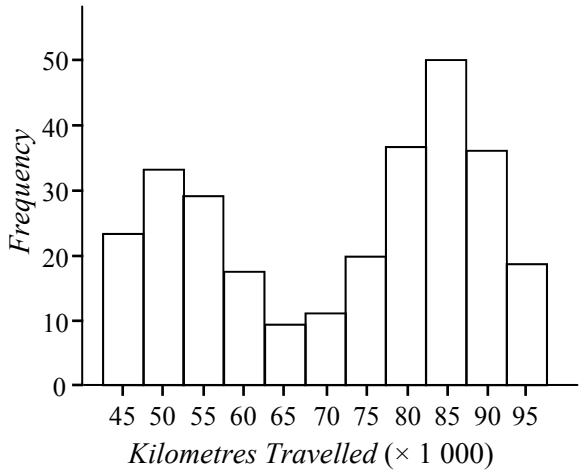
6 If the teacher chose 30 students just from the top English class, how would the results be affected?

_____

_____

_____

A tyre manufacturer has two different tyre types. After surveying its customers, it has plotted the data for the tyre life, to the nearest 5 000 km below.



7 Describe the distribution's shape and suggest a reason why it is shaped in that way.

_____

_____

_____

Data - Bimodal and Symmetric (Sheet 1)

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Symmetric Distributions



Stem	Leaf
0	7
1	5 6 8
2	3

Bimodal Distributions



Stem	Leaf
0	7 6
1	5 5 6 8
2	3 7
3	3 4 4 6
4	2 4 4

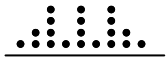
Two maxima don't have to be the same height.

Bimodal and Symmetric Distributions



Are the shape of these distributions symmetric, bimodal, both or neither? Fill in 1 or 2 circles!

1



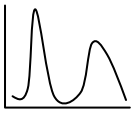
- (●) Symmetric
- ( ) Bimodal
- ( ) Neither

2



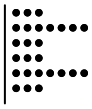
- ( ) Symmetric
- (●) Bimodal
- ( ) Neither

3



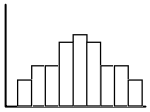
- ( ) Symmetric
- (●) Bimodal
- ( ) Neither

4



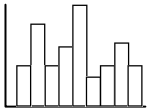
- (●) Symmetric
- (●) Bimodal
- ( ) Neither

5



- (●) Symmetric
- ( ) Bimodal
- ( ) Neither

6



- ( ) Symmetric
- ( ) Bimodal
- (●) Neither

7



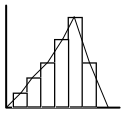
- ( ) Symmetric
- (●) Bimodal
- ( ) Neither

8

Stem	Leaf
10	1 5
11	2 3 3 3
12	4
13	1 5 5 5
14	2 3

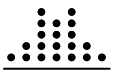
- (●) Symmetric
- (●) Bimodal
- ( ) Neither

9



- ( ) Symmetric
- ( ) Bimodal
- (●) Neither

10



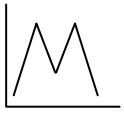
- (●) Symmetric
- (●) Bimodal
- ( ) Neither

11



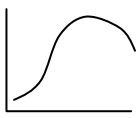
- ( ) Symmetric
- (●) Bimodal
- ( ) Neither

12



- (●) Symmetric
- (●) Bimodal
- ( ) Neither

13



- ( ) Symmetric
- ( ) Bimodal
- (●) Neither

14

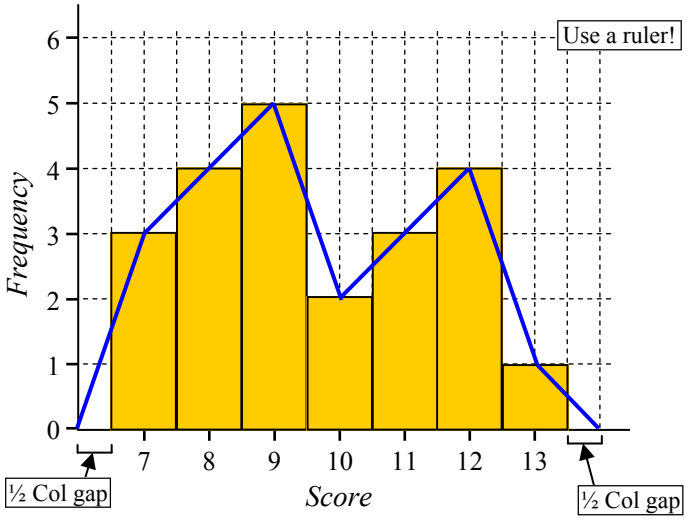


- (●) Symmetric
- ( ) Bimodal
- ( ) Neither

Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

15 Data: 9, 12, 13, 8, 9, 10, 12, 11, 7, 9, 11, 10, 9, 8, 8, 12, 7, 9, 12, 11, 7, 9.

Score	Tally	Frequency
7	|||	3
8	||||	4
9	𝍸 |	5
10	||	2
11	|||	3
12	||||	4
13	|	1
		$\Sigma f = 22$



16 The distribution's shape is Bimodal.

Bimodal/Symmetric Bimodal and Symmetric or

Now in a stem and leaf plot. No commas.

17 Data: 26, 17, 31, 5, 15, 28, 13, 34, 8, 17, 21, 11, 20, 20.

Stem	Leaf
0	5 8
1	7 5 3 7 1
2	6 8 1 0 0
3	1 4

IN ORDER →

Stem	Leaf
0	5 8
1	1 3 5 7 7
2	0 0 1 6 8
3	1 4

18 The distribution's shape is symmetric

Data - Bimodal and Symmetric (Sheet 2)

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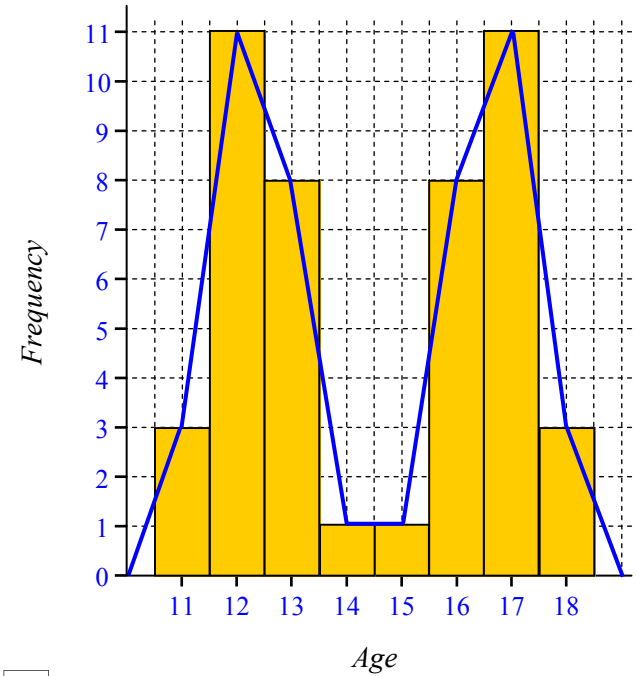
Year 11 students are tutor-reading with 17 Year 7 and 5 Year 8 students, everyone's ages are recorded in a survey. Place the results in a frequency table, then display them in a histogram and polygon.

1 Data: 11, 16, 12, 12, 17, 18, 13, 16, 12, 13, 17, 16, 12, 13, 17, 16, 16, 12, 16, 12, 17, 14, 16, 13, 11, 17, 18, 12, 17, 12, 17, 13, 17, 18, 15, 12, 16, 17, 13, 12, 17, 13, 11, 17, 12, 13.

Age	Tally	Frequency
11	𝍸	3
12	𝍸 𝍸 |	11
13	𝍸 𝍸	8
14	|	1
15	|	1
16	𝍸 𝍸	8
17	𝍸 𝍸 |	11
18	𝍸	3
		$\Sigma f = 46$



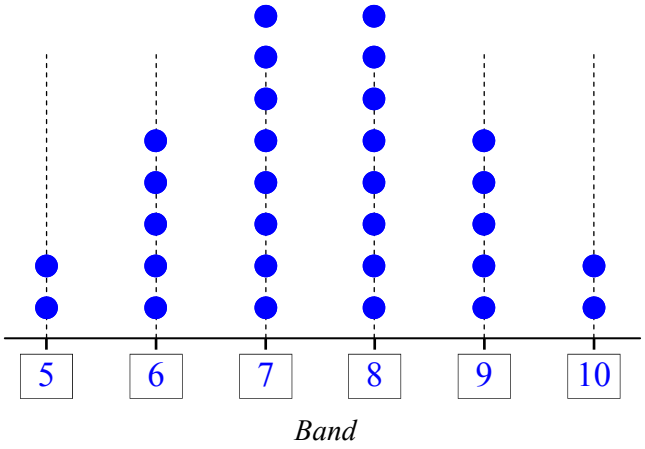
2 The distribution's shape is: Bimodal and symmetric

3 Describe why the distribution is the shape shown above

The distribution is bimodal because of the two distinct age groups. Symmetric by chance. There are no Year 9 and 10 students, so there is a drop in frequency in the 14-15 year age groups.

An English teacher selects 30 students randomly from Year 9 to compare their 'Persuasive Writing' band results. Plot their results in a dot plot.

4 Data: 9, 6, 8, 6, 7, 7, 9, 10, 8, 7, 8, 9, 6, 5, 7, 8, 8, 10, 8, 7, 9, 6, 7, 8, 7, 5, 9, 7, 6, 8.

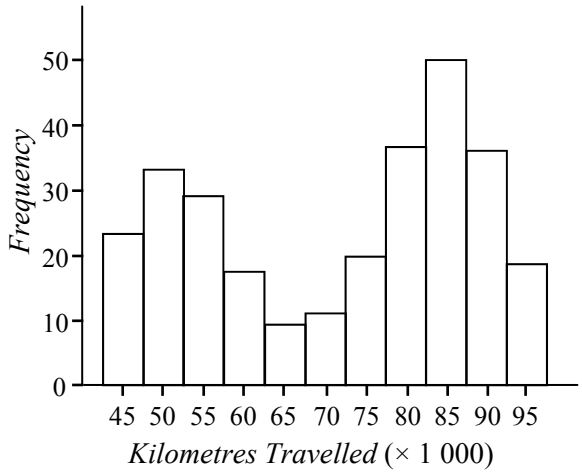


5 The distribution's shape is: symmetric

6 If the teacher chose 30 students just from the top English class, how would the results be affected?

Higher bands would be featured more. Results would be negatively skewed.

A tyre manufacturer has two different tyre types. After surveying its customers, it has plotted the data for the tyre life, to the nearest 5 000 km below.



7 Describe the distribution's shape and suggest a reason why it is shaped in that way.

The distribution is bimodal. There are two types of tyres. A more grip (sport) tyre (lower km) and a more popular longer kilometre (endurance) tyre.

Data - Bimodal and Symmetric (Homework Strip)

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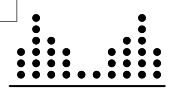
Date Due: _____

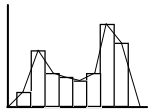
Label as bimodal/symmetrical/both

1



○ Bimodal
○ Symmetric

2



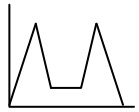
○ Bimodal
○ Symmetric

3

Stem	Leaf
2	0 1 3
3	5 5 6 8
4	0 0 4 7 7
5	7 7 7 7
6	2 2 6

○ Bimodal
○ Symmetric

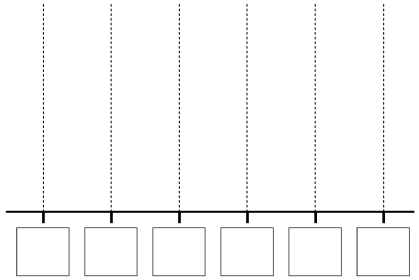
4



○ Bimodal
○ Symmetric

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 5, 4, 3, 2, 2, 5, 6, 1, 4, 1, 4, 2, 1, 3, 1, 3, 2, 4, 2.



Scores

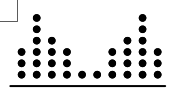
6 The above distribution is:

_____

Date Due: _____

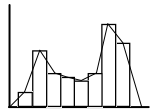
Label as bimodal/symmetrical/both

1



○ Bimodal
○ Symmetric

2



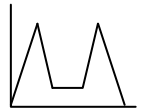
○ Bimodal
○ Symmetric

3

Stem	Leaf
2	0 1 3
3	5 5 6 8
4	0 0 4 7 7
5	7 7 7 7
6	2 2 6

○ Bimodal
○ Symmetric

4



○ Bimodal
○ Symmetric

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 5, 4, 3, 2, 2, 5, 6, 1, 4, 1, 4, 2, 1, 3, 1, 3, 2, 4, 2.

Scores

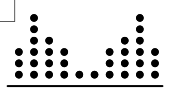
6 The above distribution is:

_____

Date Due: _____

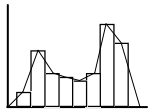
Label as bimodal/symmetrical/both

1



○ Bimodal
○ Symmetric

2



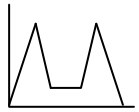
○ Bimodal
○ Symmetric

3

Stem	Leaf
2	0 1 3
3	5 5 6 8
4	0 0 4 7 7
5	7 7 7 7
6	2 2 6

○ Bimodal
○ Symmetric

4



○ Bimodal
○ Symmetric

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 5, 4, 3, 2, 2, 5, 6, 1, 4, 1, 4, 2, 1, 3, 1, 3, 2, 4, 2.

Scores

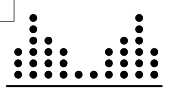
6 The above distribution is:

_____

Date Due: _____

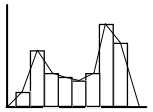
Label as bimodal/symmetrical/both

1



○ Bimodal
○ Symmetric

2



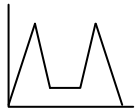
○ Bimodal
○ Symmetric

3

Stem	Leaf
2	0 1 3
3	5 5 6 8
4	0 0 4 7 7
5	7 7 7 7
6	2 2 6

○ Bimodal
○ Symmetric

4



○ Bimodal
○ Symmetric

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 5, 4, 3, 2, 2, 5, 6, 1, 4, 1, 4, 2, 1, 3, 1, 3, 2, 4, 2.

Scores

6 The above distribution is:

_____

Data - Bimodal and Symmetric (Homework Strip)

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Date Due:

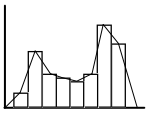
Label as bimodal/symmetric/both

1



● Bimodal
● Symmetric

2



● Bimodal
○ Symmetric

3

Stem	Leaf
2	0 1 3
3	5 5 6 8
4	0 0 4 7 7
5	7 7 7 7
6	2 2 6

○ Bimodal
● Symmetric

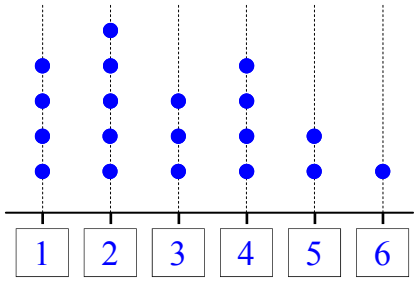
4



● Bimodal
● Symmetric

Place the data in a dot plot and comment on the shape of the distribution.

5 Data: 5, 4, 3, 2, 2, 5, 6, 1, 4, 1, 4, 2, 1, 3, 1, 3, 2, 4, 2.



Scores

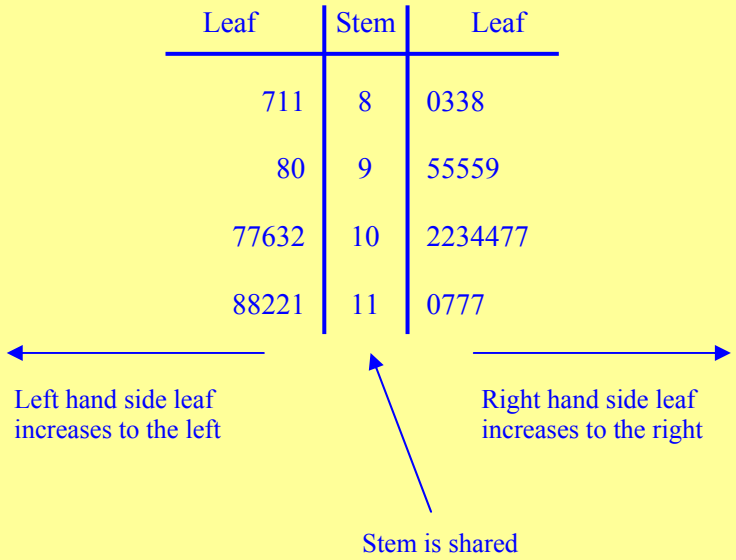
6 The above distribution is:

A bimodal distribution

Data - Back-to-Back Stem and Leaf Plots

Back-to-back stem and leaf plots are used to display 2 different groups of data together in the one plot. The groups are displayed separately but share a common stem. One group is displayed on the left hand side, the other on the R.H.S.

The L.H.S has its leaf in ascending order (going up) with larger numbers to the left, while the R.H.S. leaf has its numbers ascending, as with single leaf stem and leaf plots, up to the right.



Two mistakes are regularly made with Stem and Leaf plots. These are:

- Commas are used in the plots, like data sets. Commas shouldn't be used in a stem and leaf plot.
- A mistake is made finding the smallest 'tens' to put in the stem, so the number is written at the bottom of the stem column. The stem has to be in order, so if you miss the smallest 'tens' then the plot should be redone.

Data - Back-to-Back Stem and Leaf Plots (Sheet 1)

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Rewrite this stem and leaf plot in order. That's an order!



Same data shown in order

65440	0	04456
955	1	559
7200	2	0027

1

RAW Leaf	Stem	Leaf
83026	0	00285
7339	1	716579
780411	2	203884
3986	3	92813

ORDERED Leaf	Stem	Leaf
High ← Low		Low → High
	0	
	1	
	2	
	3	



It's auction time! 'Four n'Fluffy' livestock sales is selling sheep and alpacas. Lot sales are shown below. Place the data in a back-to-back stem and leaf plot, first raw data, then in order.

2 Four n'Fluffy Auction Sales (in Lots)

Sheep: 22, 25, 8, 37,18, 13, 3, 22, 15, 17, 34, 22, 17, 12, 20, 23, 11, 27.

Alpacas: 36, 28, 5, 38, 22, 19, 12, 7, 25, 37, 19, 31, 26, 30, 23, 37, 27, 19, 20.

RAW Alpacas	Stem	Sheep
	0	
	1	
	2	
	3	

ORDERED Alpacas	Stem	Sheep
	0	
	1	
	2	
	3	

3 The distribution's shapes are: Sheep: _____

Alpacas: _____



A restaurant has 30 tables* and records how many bookings are made daily over 4 weeks. Bookings for lunch and dinner are recorded separately. Show the data in a B-2-B stem and leaf plot.

*each table can be used more than once per session

4 Lunch: 90, 74, 105, 103, 60, 117, 96, 102, 83, 117, 82, 58, 74, 87, 95, 91, 100, 82, 60, 107, 96, 119, 102, 63, 91, 95, 66, 112.

Dinner: 105, 72, 89, 65, 67, 50, 93, 88, 77, 64, 60, 77, 57, 64, 111, 78, 54, 63, 98, 72, 64, 55, 77, 84, 66, 67, 52, 57.

Lunch	Stem	Dinner
	5	

Lunch	Stem	Dinner

5 The shape of the distributions are: Lunch: _____

Dinner: _____

6 Suggest a reason for Dinner's distribution.

_____

Data - Back-to-Back Stem and Leaf Plots (Sheet 2)

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Data for 'Public Satisfaction' for government policies has been sourced from 20 electorates. Two samples were taken, one before and one after the May budget. Place the data into a back-to-back stem and leaf plot and then answer the questions.

1 Pre-Budget : 61, 63, 50, 45, 77, 38, 55, 67, 43, 38, 50, 72, 67, 43, 78, 66, 52, 79, 55, 66.

Post-Budget : 55, 68, 34, 49, 40, 49, 56, 37, 66, 30, 49, 70, 35, 43, 55, 37, 43, 32, 38, 51.

Pre-Budget Satisfaction	Stem	Post-Budget Satisfaction



Pre-Budget Satisfaction	Stem	Post-Budget Satisfaction

2 The distribution's shapes are:

Pre-Budget satisfaction _____ Post-Budget satisfaction _____

3 Compare the above data.

_____

_____

_____

_____

_____

30 students have sat two tests, both tests had a maximum mark of 75. Plot the results for the tests in a back-to-back stem and leaf plot.

4 Test 1: 48, 27, 40, 59, 32, 20, 25, 46, 55, 38, 19, 39, 41, 38, 55, 72, 22, 59, 34, 8, 17, 45, 59, 23, 60, 34, 40, 41, 61, 23.

Test 2: 50, 61, 21, 59, 63, 28, 44, 68, 47 20, 17, 50, 63, 28, 3, 62, 11, 71 35, 59, 23, 19, 46, 7, 38, 47, 11, 54, 71, 39.

Test 1	Stem	Test 2



Test 1	Stem	Test 2

5 The distribution's shapes are: Test 1: _____ Test 2: _____

6 Suggest a reason for Test 2's distribution.

_____

_____

Data - Back-to-Back Stem and Leaf Plots (Sheet 1)

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Rewrite this stem and leaf plot in order. That's an order!

Same data shown in order

65440	0	04456
955	1	559
7200	2	0027

1

RAW Leaf	Stem	Leaf
83026	0	00285
7339	1	716579
780411	2	203884
3986	3	92813

ORDERED Leaf (High ← Low)	Stem	Leaf (Low → High)
86320	0	00258
9733	1	156779
874110	2	023488
9863	3	12389

It's auction time! 'Four n'Fluffy' livestock sales is selling sheep and alpacas. Lot sales are shown below. Place the data in a back-to-back stem and leaf plot, first raw data, then in order.

2 Four n'Fluffy Auction Sales (in Lots)

Sheep: 22, 25, 8, 37,18, 13, 3, 22, 15, 17, 34, 22, 17, 12, 20, 23, 11, 27.

Alpacas: 36, 28, 5, 38, 22, 19, 12, 7, 25, 37, 19, 31, 26, 30, 23, 37, 27, 19, 20.

RAW Alpacas	Stem	Sheep
75	0	83
9929	1	8357721
0736528	2	2522037
701786	3	74

ORDERED Alpacas	Stem	Sheep
75	0	38
9992	1	1235778
8765320	2	0222357
877610	3	47

3 The distribution's shapes are:

Sheep: Symmetric

Alpacas: Negatively skewed

A restaurant has 30 tables* and records how many bookings are made daily over 4 weeks. Bookings for lunch and dinner are recorded separately. Show the data in a B-2-B stem and leaf plot.

*each table can be used more than once per session

4

Lunch: 90, 74, 105, 103, 60, 117, 96, 102, 83, 117, 82, 58, 74, 87, 95, 91, 100, 82, 60, 107, 96, 119, 102, 63, 91, 95, 66, 112.

Dinner: 105, 72, 89, 65, 67, 50, 93, 88, 77, 64, 60, 77, 57, 64, 111, 78, 54, 63, 98, 72, 64, 55, 77, 84, 66, 67, 52, 57.

Lunch	Stem	Dinner
8	5	074527
6300	6	5740434 67
44	7	277827
2723	8	984
5161560	9	38
270235	10	5
9772	11	1

Lunch	Stem	Dinner
8	5	024577
6300	6	03444567 7
44	7	227778
7322	8	489
6655110	9	38
753220	10	5
9772	11	1

5 The shape of the distributions are:

Lunch: Bimodal, negative skew.

Dinner: Positive skew

6 Suggest a reason for Dinner's distribution.

Dinner is less popular or sit-down time is longer.

Data - Back-to-Back Stem and Leaf Plots (Sheet 2)

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Data for 'Public Satisfaction' for government policies has been sourced from 20 electorates. Two samples were taken, one before and one after the May budget. Place the data into a back-to-back stem and leaf plot and then answer the questions.

1 Pre-Budget : 61, 63, 50, 45, 77, 38, 55, 67, 43, 38, 50, 72, 67, 43, 78, 66, 52, 79, 55, 66.

Post-Budget : 55, 68, 34, 49, 40, 49, 56, 37, 66, 30, 49, 70, 35, 43, 55, 37, 43, 32, 38, 51.

Pre-Budget Satisfaction	Stem	Post-Budget Satisfaction
88	3	4705728
335	4	909933
52050	5	5651
667731	6	86
9827	7	0



Pre-Budget Satisfaction	Stem	Post-Budget Satisfaction
88	3	0245778
533	4	033999
55200	5	1556
776631	6	68
9872	7	0

2 The distribution's shapes are:

Pre-Budget satisfaction	Post-Budget satisfaction
Negative skew	Positive skew

3 Compare the data in the two leafs.

Satisfaction with the government was high before the budget (mostly >50%). After the budget satisfaction has dropped considerably. Results suggest public are unhappy with the budget, maybe a policy change has occurred.

30 students have sat two tests, both tests had a maximum mark of 75. Plot the results for the tests in a back-to-back stem and leaf plot.

4 Test 1: 48, 27, 40, 59, 32, 20, 25, 46, 55, 38, 19, 39, 41, 38, 55, 72, 22, 59, 34, 8, 17, 45, 59, 23, 60, 34, 40, 41, 61, 33.

Test 2: 50, 61, 21, 59, 63, 28, 44, 68, 47 20, 17, 50, 63, 28, 3, 62, 11, 71 35, 59, 23, 19, 46, 7, 38, 47, 11, 54, 71, 59.

Test 1	Stem	Test 2
8	0	37
79	1	7191
32507	2	18083
3448982	3	58
1051608	4	4767
99559	5	090949
10	6	13832
2	7	11



Test 1	Stem	Test 2
8	0	37
97	1	1179
75320	2	01388
9884432	3	58
8651100	4	4677
99955	5	004999
10	6	12338
2	7	11

5 The distribution's shapes are: Test 1: Symmetric; Test 2: Bimodal

6 Suggest a reason for Test 2's distribution.

Harder test and some students didn't revise for it. A different type of test that some found difficult.

Data - Back-to-Back Stem and Leaf Plots (Homework Strip)

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Date Due:

Place the data in Raw, then Ordered Back-to-Back Stem and Leaf plots

1 CPU 1 Core Temperature (°C)

56, 40, 61, 35, 39, 55, 40, 48,
67, 50, 35, 53, 46, 54, 48, 38.

CPU 2 Core Temperature (°C)

56, 56, 39, 62, 42, 61, 34, 40,
55, 62, 40, 45, 53, 39, 47, 58.

CPU 1	Stem	CPU 2

↓

CPU 1	Stem	CPU 2

CPU 1 Dist. Shape _____ CPU 2 Dist. Shape _____

Date Due:

Place the data in Raw, then Ordered Back-to-Back Stem and Leaf plots

1 CPU 1 Core Temperature (°C)

56, 40, 61, 35, 39, 55, 40, 48,
67, 50, 35, 53, 46, 54, 48, 38.

CPU 2 Core Temperature (°C)

56, 56, 39, 62, 42, 61, 34, 40,
55, 62, 40, 45, 53, 39, 47, 58.

CPU 1	Stem	CPU 2

↓

CPU 1	Stem	CPU 2

CPU 1 Dist. Shape _____ CPU 2 Dist. Shape _____

Date Due:

Place the data in Raw, then Ordered Back-to-Back Stem and Leaf plots

1 CPU 1 Core Temperature (°C)

56, 40, 61, 35, 39, 55, 40, 48,
67, 50, 35, 53, 46, 54, 48, 38.

CPU 2 Core Temperature (°C)

56, 56, 39, 62, 42, 61, 34, 40,
55, 62, 40, 45, 53, 39, 47, 58.

CPU 1	Stem	CPU 2

↓

CPU 1	Stem	CPU 2

CPU 1 Dist. Shape _____ CPU 2 Dist. Shape _____

Date Due:

Place the data in Raw, then Ordered Back-to-Back Stem and Leaf plots

1 CPU 1 Core Temperature (°C)

56, 40, 61, 35, 39, 55, 40, 48,
67, 50, 35, 53, 46, 54, 48, 38.

CPU 2 Core Temperature (°C)

56, 56, 39, 62, 42, 61, 34, 40,
55, 62, 40, 45, 53, 39, 47, 58.

CPU 1	Stem	CPU 2

↓

CPU 1	Stem	CPU 2

CPU 1 Dist. Shape _____ CPU 2 Dist. Shape _____

Data - Back-to-Back Stem and Leaf Plots (Homework Strip)

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Date Due:

Place the data in Raw, then Ordered Back-to-Back Stem and Leaf plots

1 CPU 1 Core Temperature (°C)

56, 40, 61, 35, 39, 55, 40, 48,
67, 50, 35, 53, 46, 54, 48, 38.

CPU 2 Core Temperature (°C)

56, 56, 39, 62, 42, 61, 34, 40,
55, 62, 40, 45, 53, 39, 47, 58.

CPU 1	Stem	CPU 2
8 5 9 5	3	9 4 9
8 6 8 0 0	4	2 0 0 5 7
4 3 0 5 6	5	6 6 5 3 8
7 1	6	2 1 2

↓

CPU 1	Stem	CPU 2
9 8 5 5	3	4 9 9
8 8 6 0 0	4	0 0 2 5 7
6 5 4 3 0	5	3 5 6 6 8
7 1	6	1 2 2

CPU 1 Dist. Shape	CPU 2 Dist. Shape
Positive skew	Symmetric

Data - Types of Graphs - Categorical/Numerical/Misleading

Categorical data is data that involves items instead of numbers. For example the number of cars, trucks, buses and motorcycles that go past an intersection. You are still counting them and have a frequency, but the frequency involves items not numbers.

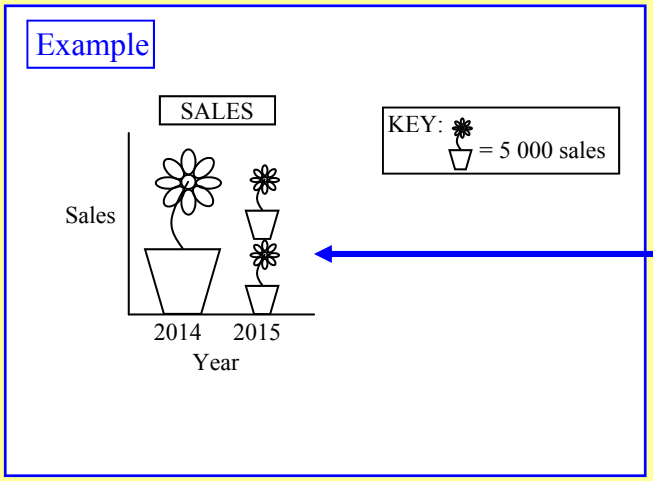
Numerical data is data that involves numbers. Such as rolling a dice or people entering a shop. Numerical data is different not only that it involves numbers, but because there is an order applied. When the data is organised, it will be displayed by the smallest number first, and increase. So the way it is displayed is controlled.

Categorical data can be controlled when the data is 'ordinal', but this isn't covered in this topic.

You are required to state whether the graphs contain categorical or numerical data. If you see an item name, it's simple it is categorical. But don't forget other descriptions like Skew, Bimodal and Symmetric. These are required to further define the graph.

The second sheet involves misleading graphs. Data can be displayed, either by accident or Intentionally, in a way that misleads. To see things in a way that may be the opposite to what is actually being displayed. Often news programs can use misleading graphs to suggest something has a huge difference, when in fact there is virtually no difference.

Example



If a flower in a pot is 5 000 sales then 2015 has 10 000 sales and 2014 has 5 000 sales (using the key). The bigger and taller flower in 2014, makes 2014 appear larger in sales. So selective use of scale has been done in a misleading way.

The second column of the second sheet is also about misleading graphs. But this time the graph itself isn't misleading. By organising data in specific ways you can mislead others. 'Trend' describes 'the path' that figures are heading. Investors look at trends to decide whether to invest in a company. If data is selectively displayed it is possible to suggest a trend that doesn't truly represent a circumstance.

Data - Types of Graphs (Categorical/Numerical)

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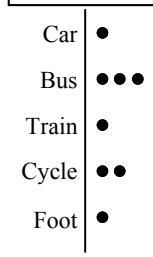
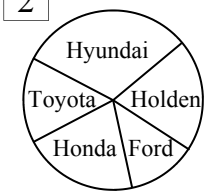
Are these graphs displaying 'Categorical' or 'Numerical' data?

1



○ Categorical
○ Numerical

2



○ Categorical
○ Numerical

3



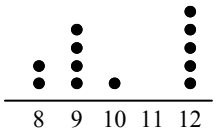
○ Categorical
○ Numerical

4

Stem	Leaf
10	1 5
11	2 3 3 3
12	4
13	1 5 5 5
14	2 3

○ Categorical
○ Numerical

5



○ Categorical
○ Numerical

6

Stem	Leaf
0	7 7
1	0 1 8
2	6
3	3 4 4
4	5

○ Categorical
○ Numerical

Classify the data as either Categorical or Numerical.

7 Cat, Dog, Bird, Fish, Cat. _____

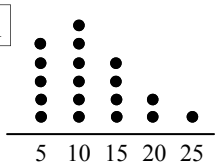
8 114, 270, 196, 414. _____

9 1, 6, 2, 7, 8, 3, 5 _____

10 Red, Green, Red, Blue, Red. _____

Are these graphs displaying Categorical or Numerical data? Then describe the distribution using: Bimodal, Skew (Pos/Neg), Symmetric.

11



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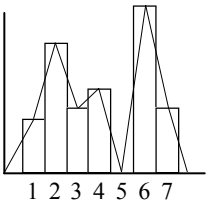
_____

12



_____

_____

13

Leaf	Stem	Leaf
1	0	3
9 5 5 2	1	2 5
6 3	2	1 1 7 9
8 4 3 0	3	8 8
5 1	4	7

Left Leaf _____

_____

Right Leaf _____

_____

14

Leaf	Stem	Leaf
2 0	8	5
6 1	9	2 3 3 3
5 4 4	10	4 9
7 7 3 0	11	3 6 7 7
8 6 2 1	12	8

Left Leaf _____

_____

Right Leaf _____

_____

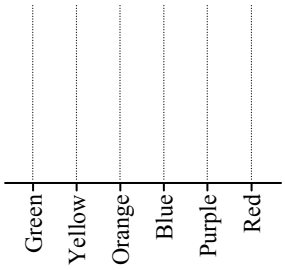
Organise this data into the frequency table below. Plot the results on the different dot plot grids and describe the graphs fully.

15

Data:
BYPYG
OGYPB
OBBYP
ORPYB
YORGB
R

Colour	Tally	Frequency
Red		
Blue		
Orange		
Green		
Yellow		
Purple		

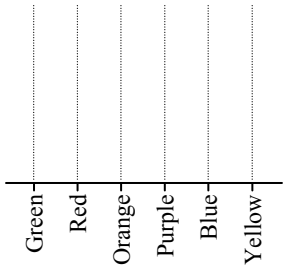
16



_____

_____

17



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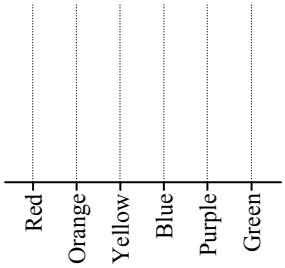
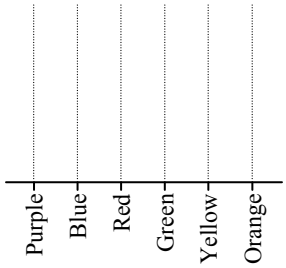
18



_____

_____

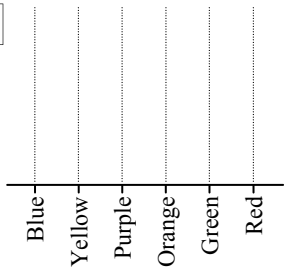
19



_____

_____

20



_____

_____

21 Do you think the words Bimodal, Symmetric and Skew should be used with Categorical data?

_____

_____

Data - Types of Graphs (Misleading Graphs)

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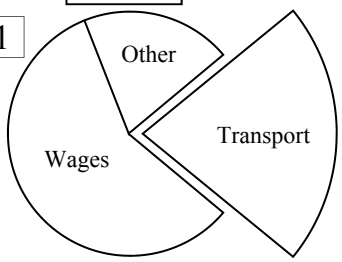
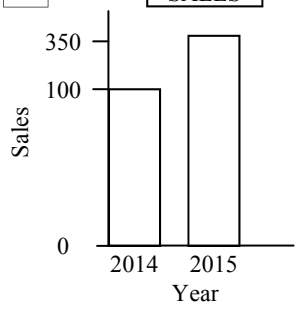
Describe why these graphs are misleading.

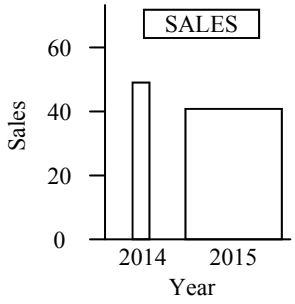
1



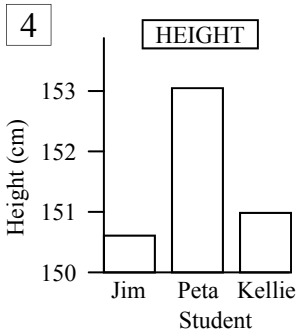
2



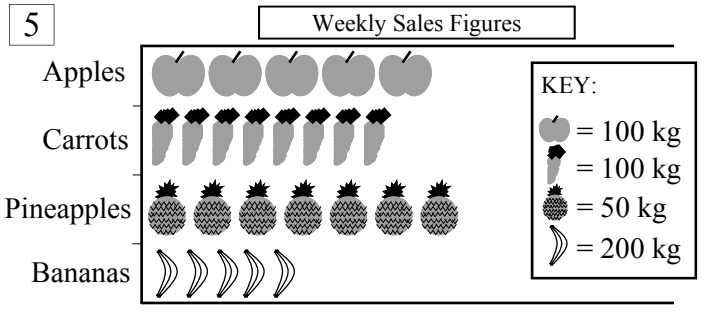
3



4



5



Display the data in the table as a column graph. This isn't a histogram, so no half column gap at the start, no polygon and columns don't touch.

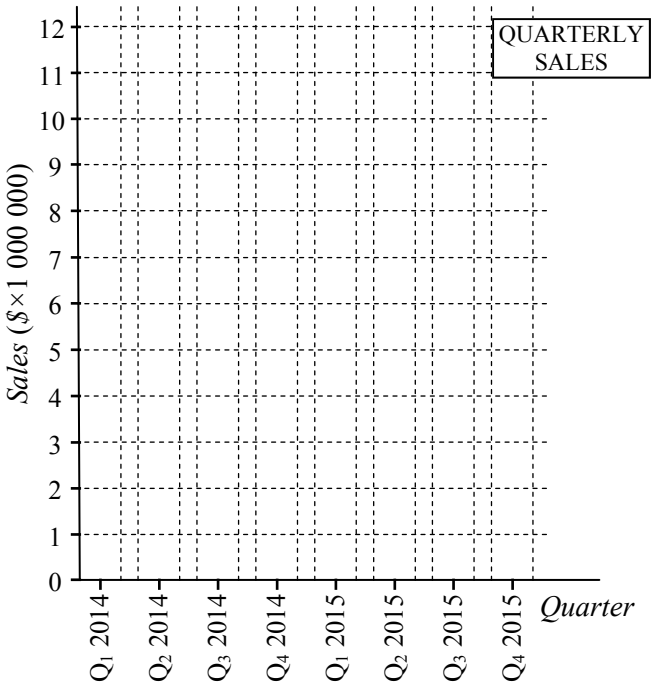
7 Complete the table's 'Total' column

SALES × $1 000 000

Year	Q_1	Q_2	Q_3	Q_4	Annual
2014	1	5	12	11	
2015	10	9	7	4	

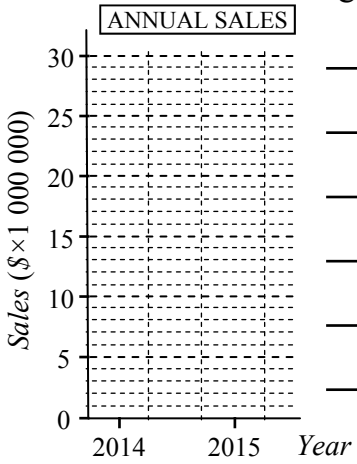
'Q' means Quarter which is a 3 month period

8 Construct a column graph for quarterly sales



9 Describe the quarterly sales above

10 Graph the total yearly sales. Comment on the differences between the graphs. 2016 indications?



Data - Types of Graphs (Categorical/Numerical)

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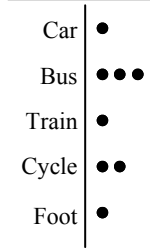
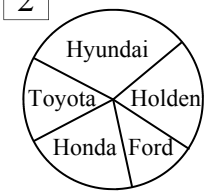
Are these graphs displaying 'Categorical' or 'Numerical' data?

1



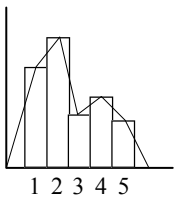
● Categorical ○ Numerical

2



● Categorical ○ Numerical

3



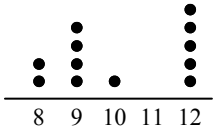
○ Categorical ● Numerical

4

Stem	Leaf
10	1 5
11	2 3 3 3
12	4
13	1 5 5 5
14	2 3

○ Categorical ● Numerical

5



○ Categorical ● Numerical

6

Stem	Leaf
0	7 7
1	0 1 8
2	6
3	3 4 4
4	5

○ Categorical ● Numerical

Classify the data as either Categorical or Numerical.

7 Cat, Dog, Bird, Fish, Cat. — Categorical

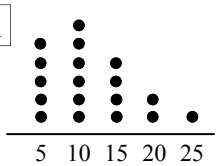
8 114, 270, 196, 414. — Numerical

9 1, 6, 2, 7, 8, 3, 5 — Numerical

10 Red, Green, Red, Blue, Red. — Categorical

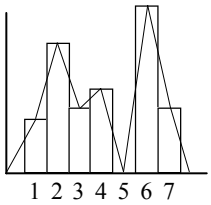
Are these graphs displaying Categorical or Numerical data? Then describe the distribution using: Bimodal, Skew (Pos/Neg), Symmetric.

11



Numerical, Positive Skew

12



Numerical, Bimodal

13

Leaf	Stem	Leaf
1	0	3
9 5 5 2	1	2 5
6 3	2	1 1 7 9
8 4 3 0	3	8 8
5 1	4	7

Left Leaf: Numerical, Bimodal

Right Leaf: Numerical, Symmetric

14

Leaf	Stem	Leaf
2 0	8	5
6 1	9	2 3 3 3
5 4 4	10	4 9
7 7 3 0	11	3 6 7 7
8 6 2 1	12	8

Left Leaf: Numerical, Negative Skew

Right Leaf: Numerical, Symmetric/Bimodal

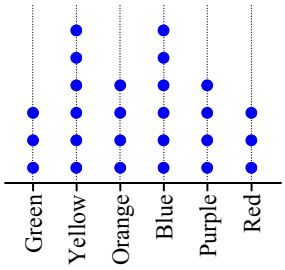
Organise this data into the frequency table below. Plot the results on the different dot plot grids and describe the graphs fully.

15

Data:
BYPYG
OGYPB
OBBYP
ORPYB
YORGB
R

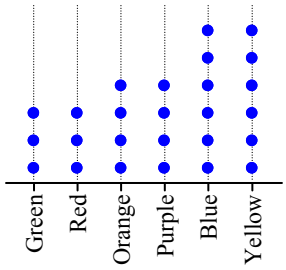
Colour	Tally	Frequency
Red	|||	3
Blue	𝍸 |	6
Orange	||||	4
Green	|||	3
Yellow	𝍸 |	6
Purple	||||	4
		$\Sigma f = 26$

16



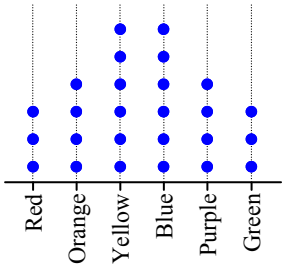
Categorical, Bimodal

17



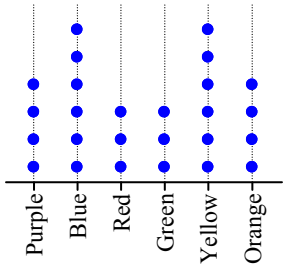
Categorical, Negative Skew

18



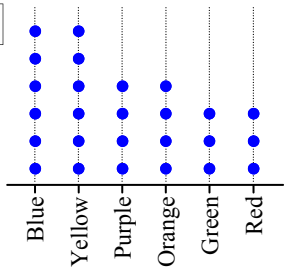
Categorical, Symmetric

19



Categorical, Symmetric/Bimodal

20



Categorical, Positive Skew

21 Do you think the words Bimodal, Symmetric and Skew should be used with Categorical data?

Due to arranging of the categories, skew and symmetric are irrelevant. Bimodal may be relevant

Data - Types of Graphs (Misleading Graphs)

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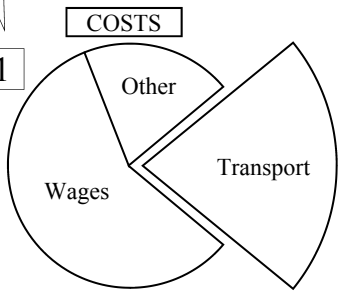
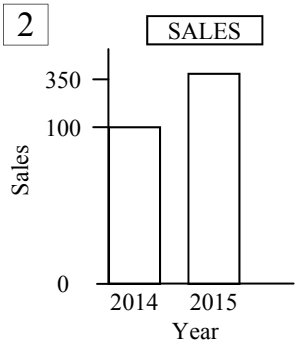
Describe why these graphs are misleading.

1



Transport costs are exaggerated and seem larger than 'other', but is almost the same

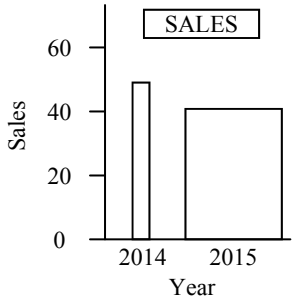
2



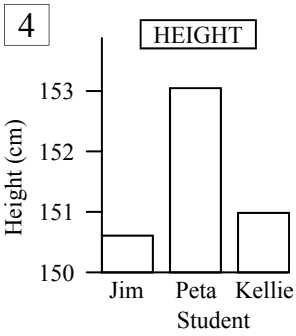
The vertical scale isn't in proportion and doesn't illustrate the increase in sales in 2015 fully.

3

The sales in 2015 are less than 2014 but because the column is wider, it looks larger. They should be of equal width.

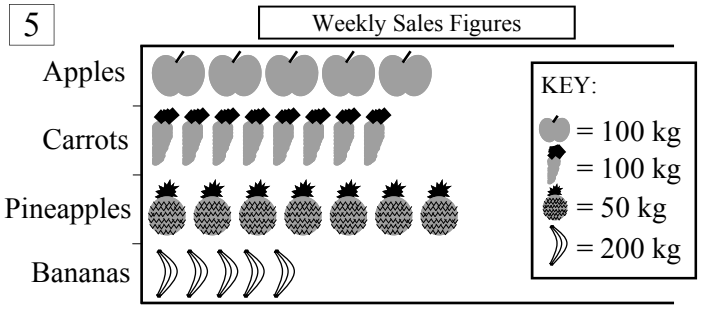


4



The heights are similar for the three students. But because the axis starts at 150 cm, differences are exaggerated.

5



Skinny banana and carrot symbols take less room and should be spaced. Key is inconsistent, makes banana sales look low and pineapple sales high.

Display the data in the table as a column graph. This isn't a histogram, so no half column gap at the start, no polygon and columns don't touch.

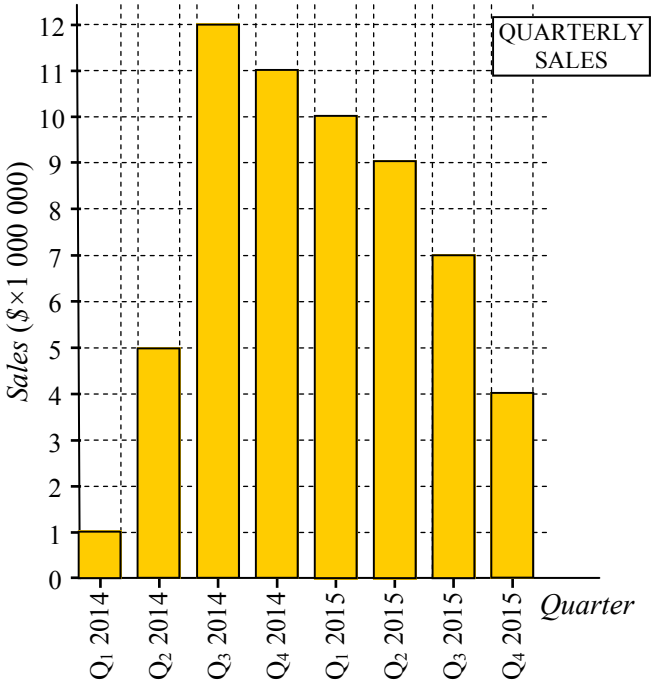
7 Complete the table's 'Total' column

SALES × $1 000 000

Year	Q_1	Q_2	Q_3	Q_4	Annual
2014	1	5	12	11	29
2015	10	9	7	4	30

'Q' means Quarter which is a 3 month period

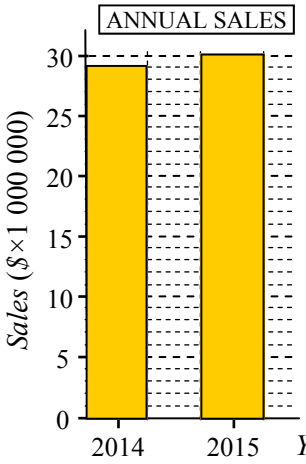
8 Construct a column graph for quarterly sales



9 Describe the quarterly sales above

Sales started in Q_1 2014, increased in Q_2 and peaked in Q_3 2014. Since that peak sales are steadily declining. 2016 looks like low sales.

10 Graph the total yearly sales. Comment on the differences between the graphs. 2016 indications?



Annual sales is misleading suggesting increasing sales. 2016 looks positive, which is the opposite to the first graph. Misleading by compressing information.

Data - Types of Graphs - Categorical/Numerical/Misleading (Homework Strip)

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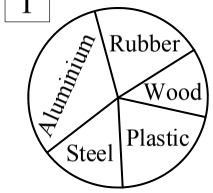
Date Due:

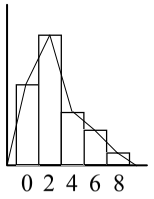
Describe the data in these graphs.

1



○ Categorical
○ Numerical

2



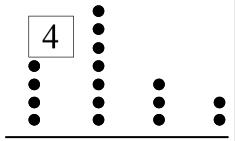
○ Categorical
○ Numerical

Now include skew/bimodal/symmetric

3

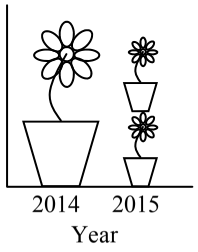
Stem	Leaf
0	5 5 7
1	0 0 6 6
2	4 9
3	1 3 4 4
4	0 7

4



Describe why this graph is misleading

5

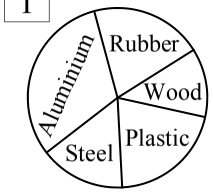


KEY:
= 5 000 sales

Date Due:

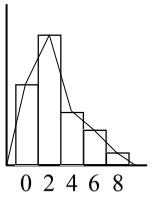
Describe the data in these graphs.

1



○ Categorical
○ Numerical

2



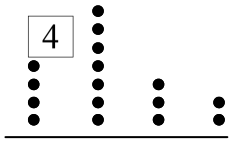
○ Categorical
○ Numerical

Now include skew/bimodal/symmetric

3

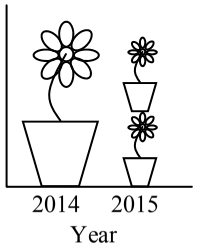
Stem	Leaf
0	5 5 7
1	0 0 6 6
2	4 9
3	1 3 4 4
4	0 7

4



Describe why this graph is misleading

5

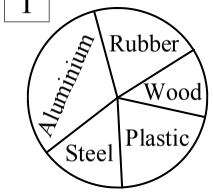


KEY:
= 5 000 sales

Date Due:

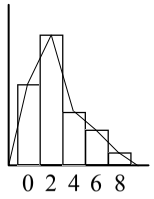
Describe the data in these graphs.

1



○ Categorical
○ Numerical

2



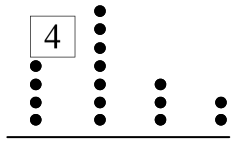
○ Categorical
○ Numerical

Now include skew/bimodal/symmetric

3

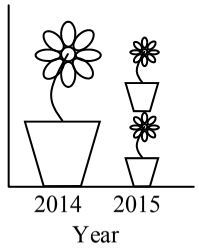
Stem	Leaf
0	5 5 7
1	0 0 6 6
2	4 9
3	1 3 4 4
4	0 7

4



Describe why this graph is misleading

5

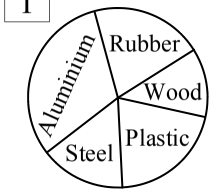


KEY:
= 5 000 sales

Date Due:

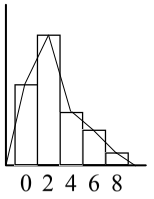
Describe the data in these graphs.

1



○ Categorical
○ Numerical

2



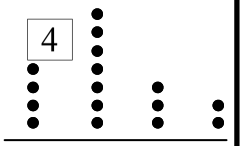
○ Categorical
○ Numerical

Now include skew/bimodal/symmetric

3

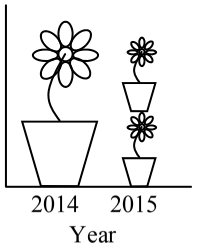
Stem	Leaf
0	5 5 7
1	0 0 6 6
2	4 9
3	1 3 4 4
4	0 7

4



Describe why this graph is misleading

5



KEY:
= 5 000 sales

Data-Types of Graphs-Categorical/Numerical/Misleading (Homework Strip)

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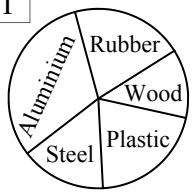
Date Due:

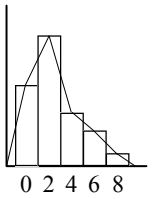
Describe the data in these graphs.

1



● Categorical
○ Numerical

2



○ Categorical
● Numerical

Now include skew/bimodal/symmetric

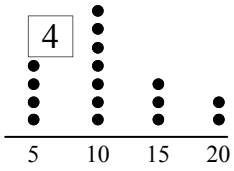
3

Stem	Leaf
0	5 5 7
1	0 0 6 6
2	4 9
3	1 3 4 4
4	0 7

Numerical

Bimodal

4

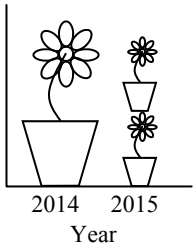


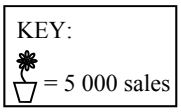
Numerical

Positive skew

Describe why this graph is misleading

5





2014 has less sales but because flower is so big and is taller, it suggests sales are larger.

Data - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Data - Find a Word

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Find the words in the puzzle from the wordlist.



A	T	A	L	L	A	C	I	R	O	G	A	T	A	C
Y	A	N	U	M	E	R	I	C	A	L	H	O	B	N
L	C	E	V	I	T	I	S	O	P	M	I	B	O	I
L	L	N	R	O	R	D	Y	W	E	K	S	I	I	B
A	A	R	E	D	R	O	M	A	E	O	T	M	M	I
Q	S	N	G	U	U	Q	M	T	L	U	O	O	N	Y
G	S	G	N	U	Q	U	E	A	B	T	G	D	E	A
N	E	N	I	Q	U	E	T	I	A	A	R	A	G	L
I	S	H	A	P	E	Y	R	T	T	A	A	L	A	P
D	G	N	A	L	L	T	I	F	W	A	M	E	T	S
N	A	X	T	L	S	T	C	A	L	E	A	F	I	I
E	A	M	A	I	T	P	A	G	P	T	L	O	V	D
C	X	T	D	X	P	O	L	Y	G	O	N	O	E	P
S	A	M	I	X	A	M	D	C	O	L	U	M	N	S
A	O	R	G	A	N	I	S	E	O	P	P	L	L	T

WORDLIST

ORGANISE	LEAF	NUMERICAL	SKEW	NEGATIVE
TABLE	RAW	DATA	MAXIMA	PLOT
HISTOGRAM	DOT	CLASSES	DISPLAY	FREQUENCY
ASCENDING	TALLY	POLYGON	STEM	ORDER
BIMODAL	POSITIVE	SHAPE	GAP	COLUMNS
	CATAGORICAL	SYMMETRICAL	DISTRIBUTION	

Data - Find a Word

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Find the words in the puzzle from the wordlist.



A	T	A	L	L	A	C	I	R	O	G	A	T	A	C
Y	A	N	U	M	E	R	I	C	A	L	H	O	B	N
L	C	E	V	I	T	I	S	O	P	M	I	B	O	I
L	L	N	R	O	R	D	Y	W	E	K	S	I	I	B
A	A	R	E	D	R	O	M	A	E	O	T	M	M	I
Q	S	N	G	U	U	Q	M	T	L	U	O	O	N	Y
G	S	G	N	U	Q	U	E	A	B	T	G	D	E	A
N	E	N	I	Q	U	E	T	I	A	A	R	A	G	L
I	S	H	A	P	E	Y	R	T	T	A	A	L	A	P
D	G	N	A	L	L	T	I	F	W	A	M	E	T	S
N	A	X	T	L	S	T	C	A	L	E	A	F	I	I
E	A	M	A	I	T	P	A	G	P	T	L	O	V	D
C	X	T	D	X	P	O	L	Y	G	O	N	O	E	P
S	A	M	I	X	A	M	D	C	O	L	U	M	N	S
A	O	R	G	A	N	I	S	E	O	P	P	L	L	T

WORDLIST

ORGANISE
TABLE
HISTOGRAM
ASCENDING
BIMODAL
LEAF
RAW
DOT
TALLY
POSITIVE
CATAGORICAL
NUMERICAL
DATA
CLASSES
POLYGON
SHAPE
SYMMETRICAL
SKEW
MAXIMA
DISPLAY
STEM
GAP
DISTRIBUTION
NEGATIVE
PLOT
FREQUENCY
ORDER
COLUMNS

Data Review

This a review of the Organising and Displaying Data topic.

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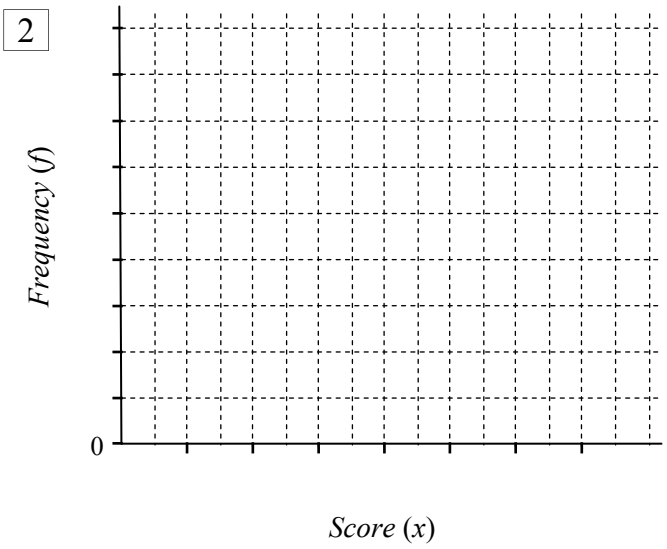
Place the data into a frequency table and then create a frequency histogram and polygon for the data also.

1 Data: 20, 5, 10, 0, 15, 15, 20, 0, 5, 15, 5, 0, 5, 20, 30, 20, 10, 10, 5, 20, 5, 15, 10, 20, 5, 0, 30, 20, 15, 0, 5.

Score	Tally	Frequency
		$\Sigma f =$

2



3 Which score occurred seven times? _____

4 Which score occurred the most? _____

5 How many scores were larger than 15? _____

6 What percentage of scores were 10 or less? (1 d.p.)

$$\frac{\square}{\square} \times 100 = \frac{\square}{\text{Round to 1 d.p.}}$$

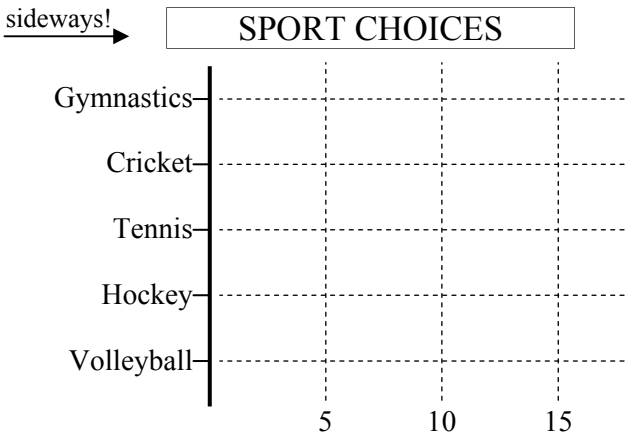
7 What percentage of scores were multiples of 10?

$$\frac{\square}{\square} \times 100 = \frac{\square}{\text{Round to 1 d.p.}}$$

Place the data into a dot plot, then answer the questions

8 Year 9 students can choose one of these sports: Gymnastics, Cricket, Tennis, Hockey or Volleyball. Two classes' selections are:

Data: GTHTT CGCHV TCHHH GVCTT
HCHCV CCCHV TVCCT GHVGC
TGHVH CGCTT VHHGT GCTHV



9 Which sport was the most popular? _____

10 How many students chose Hockey or tennis? _____

Place the data into an ordered stem and leaf plot.

11 Child Allowance ($) for Easter Carnival.

Data: 70, 85, 58, 95, 60, 65, 42, 105, 77, 60, 85, 115, 100, 75, 40, 80, 100, 82, 90, 50, 70, 82, 110, 55, 100, 105, 80, 55, 75, 110, 60, 65.

RAW → ORDERED

Stem	Leaf

Stem	Leaf

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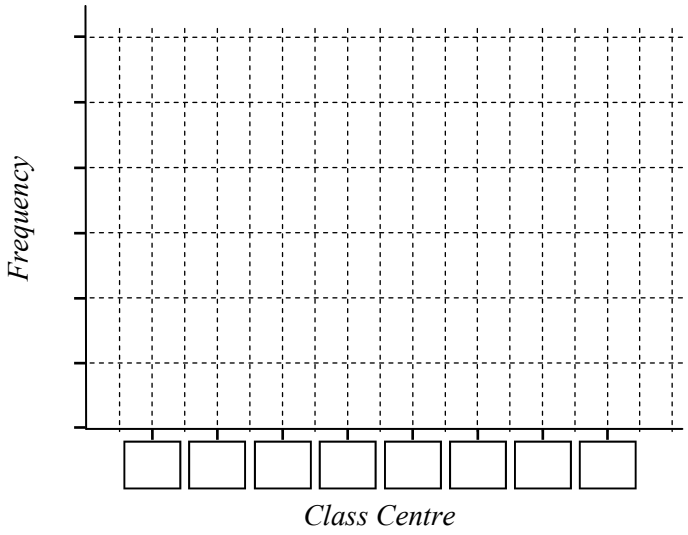
Place the data into a frequency table with classes and then create a frequency histogram and polygon for the data.

12 Data: 21, 26, 31, 8, 30, 30, 13, 2, 38, 10, 18, 12, 33, 15, 5, 13, 9, 33, 24, 6, 2, 32, 7, 27, 30, 8.

Class	*c.c.*	Tally	*f*
1 - 5	3		
6 - 10			
11 - 15			
16 - 20			
21 - 25			
26 - 30			
31 - 35			
35 - 40			
			$\Sigma f =$

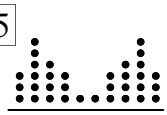


13 Is this data categorical or numerical? _____

14 Describe the shape of the distribution. _____

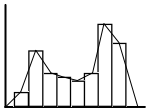
Describe the shape of these distributions

15



○ Bimodal
○ Symmetric

16



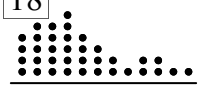
○ Bimodal
○ Symmetric

17

Stem	Leaf
2	0 1 3
3	5 5 6 8
4	0 0 4 7 7
5	7 7 7 7
6	2 2 6

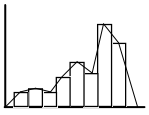
○ Bimodal
○ Symmetric

18



○ Positive Skew
○ Negative Skew

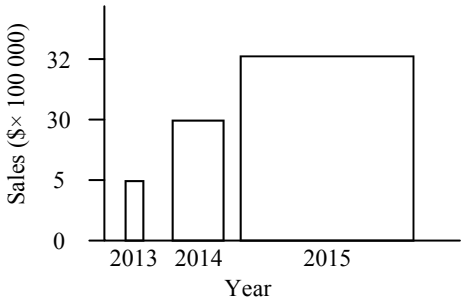
19



○ Positive Skew
○ Negative Skew

Describe the ways in which the graph below is misleading.

20



A cinema manager records the ages of 20 people entering the complex on a weekday. One sample is taken at 2 p.m., another at 9 p.m. Organise the data below into an ordered back-to-back stem and leaf plot.

21 2 p.m. : 58, 67, 43, 76, 69, 27, 77, 38, 43, 67, 67, 40, 79, 35, 59, 78, 58, 70, 59, 65.

9 p.m. : 34, 25, 47, 30, 42, 17, 20, 46, 28, 22, 31, 46, 52, 41, 45, 25, 25, 42, 23, 34.

RAW 2 p.m.	Stem	9 p.m.
	1	

ORDERED 2 p.m.	Stem	9 p.m.

○ Positive Skew ○ Bimodal
○ Negative Skew ○ Symmetric

○ Positive Skew ○ Bimodal
○ Negative Skew ○ Symmetric

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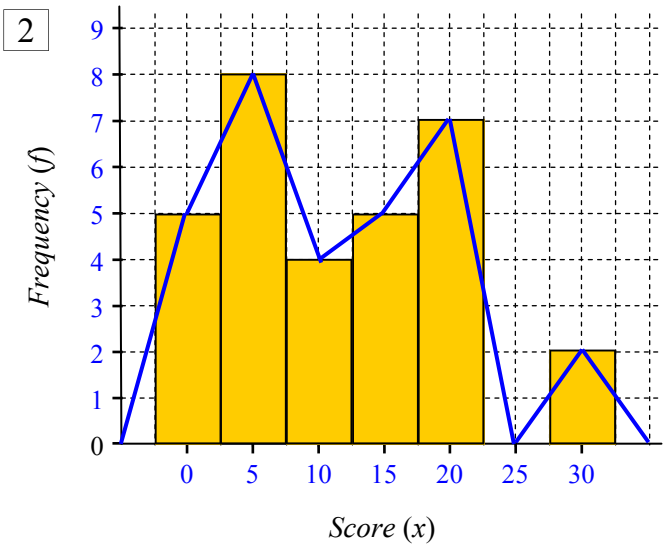
Place the data into a frequency table and then create a frequency histogram and polygon for the data also.

1 Data: 20, 5, 10, 0, 15, 15, 20, 0, 5, 15, 5, 0, 5, 20, 30, 20, 10, 10, 5, 20, 5, 15, 10, 20, 5, 0, 30, 20, 15, 0, 5.

Score	Tally	Frequency
0	𝍸	5
5	𝍸 |||	8
10	||||	4
15	𝍸	5
20	𝍸 ||	7
25		0
30	||	2
		$\Sigma f = 31$

2



3 Which score occurred seven times? 20

4 Which score occurred the most? 5

5 How many scores were larger than 15? 9

6 What percentage of scores were 10 or less? (1 d.p.)

$$\frac{17}{31} \times 100 = 54.8\%$$

Round to 1 d.p.

7 What percentage of scores were multiples of 10?

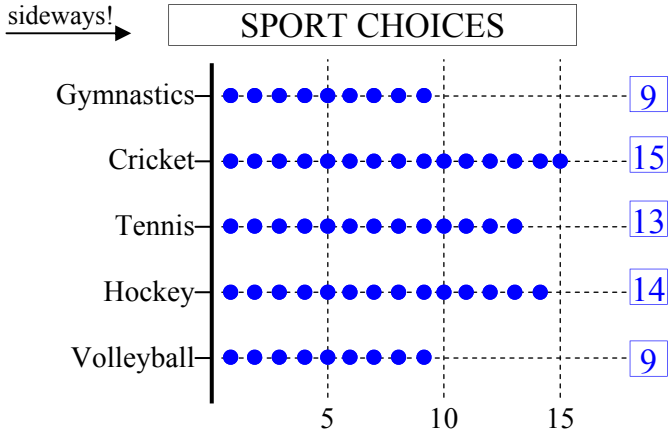
$$\frac{13}{31} \times 100 = 41.9\%$$

Round to 1 d.p.

Place the data into a dot plot, then answer the questions

8 Year 9 students can choose one of these sports: Gymnastics, Cricket, Tennis, Hockey or Volleyball. Two classes' selections are:

Data: GTHTT CGCHV TCHHH GVCTT
HCHCV CCCHV TVCCT GHVGC
TGHVH CGCTT VHHGT GCTHV



9 Which sport was the most popular? Cricket

10 How many students chose Hockey or tennis? 28

Place the data into an ordered stem and leaf plot.

11 Child Allowance ($) for Easter Carnival.

Data: 70, 85, 58, 95, 60, 65, 42, 105, 77, 60, 85, 115, 100, 75, 40, 80, 100, 82, 90, 50, 70, 82, 110, 55, 100, 105, 80, 55, 75, 110, 60, 65.

RAW → ORDERED

Stem	Leaf
4	2 0
5	8 0 5 5
6	0 5 0 0 5
7	0 7 5 0 5
8	5 5 0 2 2 0
9	5 0
10	5 0 0 0 5
11	5 0 0

Stem	Leaf
4	0 2
5	0 5 5 8
6	0 0 0 5 5
7	0 0 5 5 7
8	0 0 2 2 5 5
9	0 5
10	0 0 0 5 5
11	0 0 5

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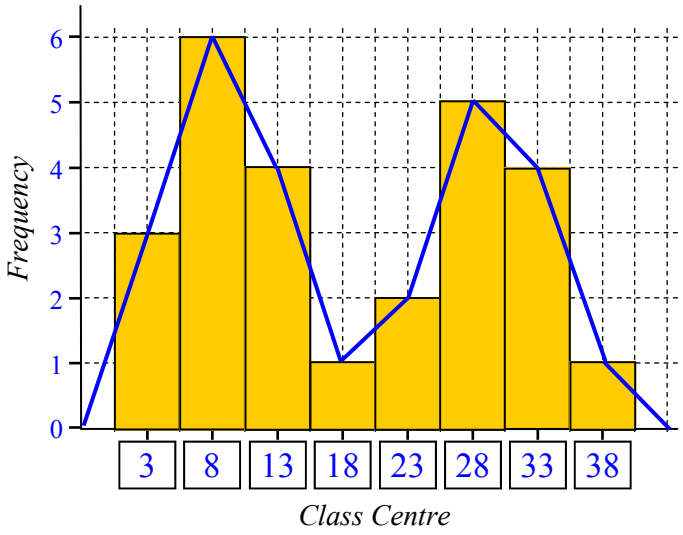
Place the data into a frequency table with classes and then create a frequency histogram and polygon for the data.

12 Data: 21, 26, 31, 8, 30, 30, 13, 2, 38, 10, 18, 12, 33, 15, 5, 13, 9, 33, 24, 6, 2, 32, 7, 27, 30, 8.

Class	*c.c.*	Tally	*f*
1 - 5	3	|||	3
6 - 10	8	𝍸 |	6
11 - 15	13	||||	4
16 - 20	18	|	1
21 - 25	23	||	2
26 - 30	28	𝍸	5
31 - 35	33	||||	4
35 - 40	38	|	1
			$\Sigma f =$

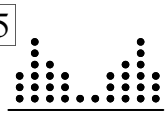


13 Is this data categorical or numerical? Numerical

14 Describe the shape of the distribution. Bimodal

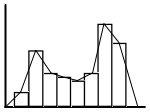
Describe the shape of these distributions

15



● Bimodal
● Symmetric

16



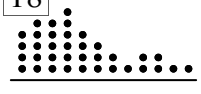
● Bimodal
○ Symmetric

17

Stem	Leaf
2	0 1 3
3	5 5 6 8
4	0 0 4 7 7
5	7 7 7 7
6	2 2 6

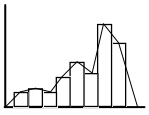
○ Bimodal
● Symmetric

18



● Positive Skew
○ Negative Skew

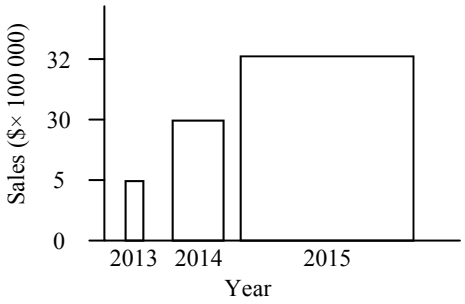
19



○ Positive Skew
● Negative Skew

Describe the ways in which the graph below is misleading.

20



The vertical scale is not linear. Column widths are increasing through the years which exaggerate the 2015 sales.

A cinema manager records the ages of 20 people entering the complex on a weekday. One sample is taken at 2 p.m., another at 9 p.m. Organise the data below into an ordered back-to-back stem and leaf plot.

21 2 p.m. : 58, 67, 43, 76, 69, 27, 77, 38, 43, 67, 67, 40, 79, 35, 59, 78, 58, 70, 59, 65.

9 p.m. : 34, 25, 47, 30, 42, 17, 20, 46, 28, 22, 31, 46, 52, 41, 45, 25, 25, 42, 23, 34.

RAW 2 p.m.	Stem	9 p.m.
	1	7
7	2	5 0 8 2 5 5 3
5 8	3	4 0 1 4
0 3 3	4	7 2 6 6 1 5 2
9 8 9 8	5	2
5 7 7 9 7	6	
0 8 9 7 6	7	

ORDERED 2 p.m.	Stem	9 p.m.
	1	7
7	2	0 2 3 5 5 5 8
8 5	3	0 1 4 4
3 3 0	4	1 2 2 5 6 6 7
9 9 8 8	5	2
9 7 7 7 5	6	
9 8 7 6 0	7	

○ Positive Skew ● Negative Skew
○ Bimodal ○ Symmetric

○ Positive Skew ○ Negative Skew
● Bimodal ● Symmetric

Organising & Displaying Data



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Data - Frequency Tables (Sheet 1)

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Complete the frequency column and total for Σf, then answer the questions.

1 Number of Fish in a Tank

Fish	Tally	Frequency
2	𝍸	
3	𝍸 ||	
4	𝍸 𝍸 |||	
5	𝍸 𝍸	
6	𝍸 𝍸 ||||	
7	𝍸 𝍸 |||	
		$\Sigma f =$

2 What number of fish had the highest frequency? _____

3 What number of fish had the same frequency? _____

4 What was the frequency for a tank to have an odd number of fish?

_____

5 What was the frequency for a tank to have less than 5 fish?

_____

6 Soft Drink Preference

Type	Tally	Frequency
A	𝍸 𝍸 𝍸 ||	
B	𝍸 𝍸 |||	
C	||||	
D		
E	𝍸 𝍸 ||	
F	𝍸 𝍸 𝍸	
		$\Sigma f =$ 75

7 D's results were unreadable due to a drink spill, use Σf to calculate them.

8 Rank the drink brands in order from least, to most popular.

_____

9 How many people preferred brand 'A' or brand 'F'? _____

Place the data into the Frequency table, showing the tally, the frequency and write 'Σf= *(total)*', then answer the questions.

10 Away from School - Sports Choices

Sport	Tally	Frequency
Tennis		
Gym		
Bocce		
Archery		

Data:

TTGAA	TTGAA
TGBBA	GBBAT
AAGGG	TTTTA
BTBTA	BTBBA
AAATG	GTBBA
BBGBT	AABTB
TTGBB	BBTBG
ATTTA	ABBTG

11 What was most popular 'away' sport chosen? _____

12 How many people chose 'ball sports'? _____

13 If there are a total of 135 students in the year, calculate the percentage that chose 'away' sports.

$$\%\ \text{Away Sport} = \frac{\square}{135} \times 100$$

$$= \underline{\square}\ \%$$

Round to 1 d.p.

14 A real estate agent is checking rental homes for smoke detectors, if the smoke detector was Battery powered, Mains powered, or Dual (both). If a detector didn't work its type wasn't recorded it was just recorded as Inoperative. If a house had no smoke detector, it was recorded as 'None'.

Smoke Detector Type or Status

Type	Tally	Frequency
Battery		
Mains		
Dual		
Inoperative		
None		

Data:

BBDMB	BBDBM
BIBBM	MNDIB
MBDBI	BBIIN
NBBIM	INDDB
BBNIB	NBBMB
MBBDN	IBBMB
NINBB	NDDMB

15 How many homes had operational smoke detectors? _____

16 What percentage (to 1 d.p.) of homes had inoperative or no smoke detectors?

$$\%\ \text{No S.D} = \frac{\square}{\square} \times 100 = \underline{\square}$$

Round to 1 d.p.

17 What percentage (to 1 d.p.) of homes had Dual-powered smoke detectors?

$$\%\ \text{Dual Powered} = \frac{\square}{\square} \times 100 = \underline{\square}$$

Round to 1 d.p.

Data - Frequency Tables (Sheet 2)

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Place the data below in a frequency table.

Before you start
Circle the smallest number
Circle the largest number

1 Data: ~~0~~, 3, 2, 6, 5, 3, 4, 5, 1, 6, 3, 4, 2, 5, 3, 2, 0, 5, 1, 2, 3, 5, 4, 3, 2, 4, 5, 2, 1, 5, 2, 3, 2, 5, 6, 1, 4, 3, 2, 2.

Score	Tally	Frequency
0	|	
		$\Sigma f =$

Count the numbers in the data, does it equal Σf?

2 Ages of Students on an Excursion.
Data: 15, 17, 12, 17, 16, 17, 12, 18, 13, 16, 17, 15, 11, 13, 13, 14, 12, 15, 17, 17, 15, 14, 13, 15, 16, 17, 12, 16, 14, 13, 15, 17, 17, 17, 16, 12, 14.

Table Title _____

Score	Tally	Frequency
		$\Sigma f =$

3 What was the age that occurred most? _____

4 How many students were older that 14? _____

5 How many teenage students were there? _____

6 What percentage of the students were 16 or older?

_____ = _____

Show working out | Answer to 1 d.p.

Student eye colour was recorded with the data shown, Br = Brown, Bl = Blue, G = Green and H = Hazel. Tabulate the results.

7 Data: Br, Br, Br, G, Bl, Br, Br, Br, Bl, H, H, Br, Br, Br, Br, Bl, H, Bl, Br, Br, H, Bl, G, Br, Br, Br, H, H, Bl.

_____

8 What was most common eye colour? _____

9 How many students had hazel eyes? _____

10 What percentage of the students had blue eyes?

_____ = _____

Answer to 1 d.p.

11 What percentage of the students had either brown or hazel eyes?

_____ = _____

Answer to 1 d.p.

12 Place the spelling test results in the table.
Data: 9, 6, 8, 3, 6, 8, 10, 8, 5, 6, 9, 7, 9, 8, 7, 3, 8, 10, 6, 9, 8, 7, 9, 5, 6, 9, 8, 8, 8, 7, 5, 9, 6, 3, 7, 9, 6, 10, 8, 7, 10, 6, 5, 5, 3, 7, 9, 9, 10, 5.

_____

Data - Frequency Histograms and Frequency Polygons (Sheet 1)

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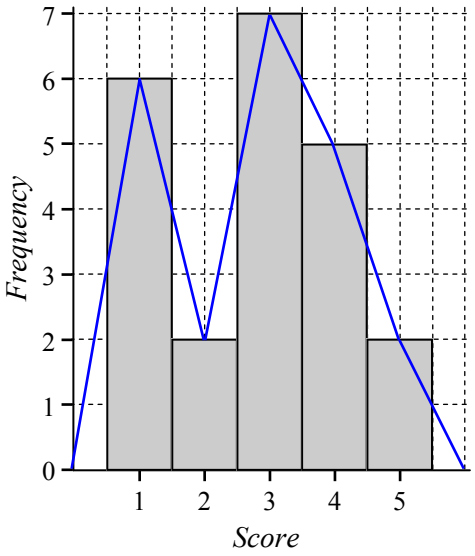
A frequency histogram and polygon are shown. Complete the frequency table, then answer the questions.



Score	*f*
1	
2	
3	
4	
5	

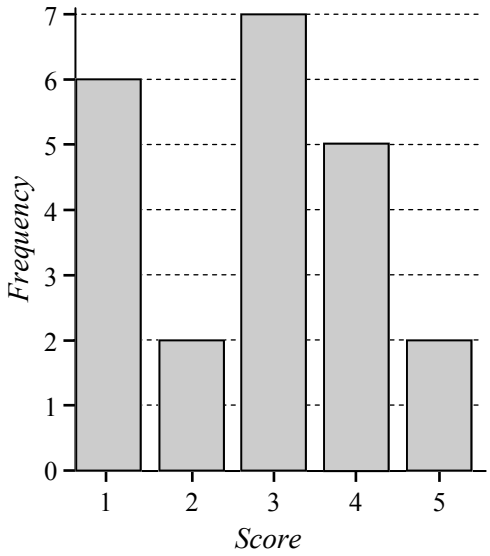
1 Which score occurred the most? _____

2 Which scores have the same frequency? _____

3 Which score occurred five times? _____

4 Total the frequencies. _____

This is a column graph for the same data.



5 List three differences between the column graph and the histogram above it. Write your answer in sentences.

_____

_____

_____

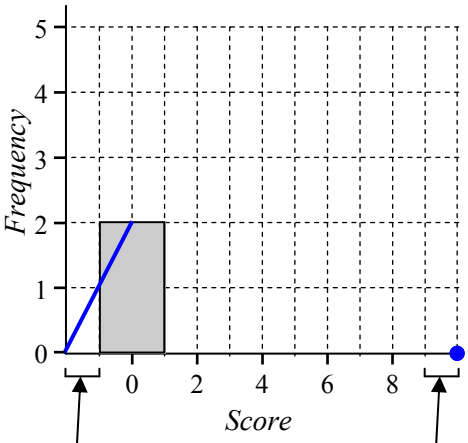
_____

_____

Construct a frequency histogram and frequency polygon for the data in these frequency tables.

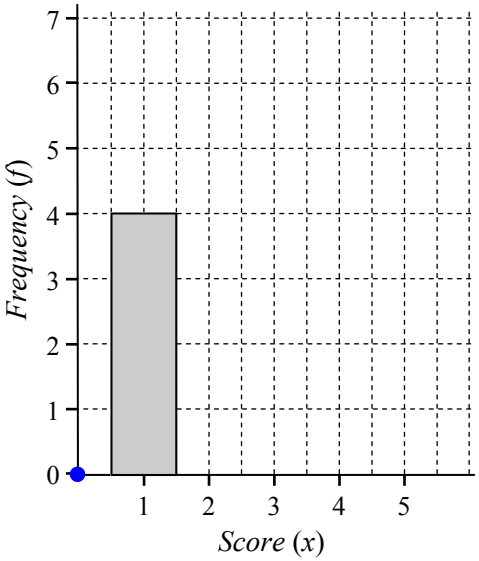
6

Score	*Frequency*
0	2
2	3
4	5
6	1
8	4



Columns touch each other - no gaps between
There is a ½ column gap at the start and at the finish of the columns. Polygon ends at the dot. ●

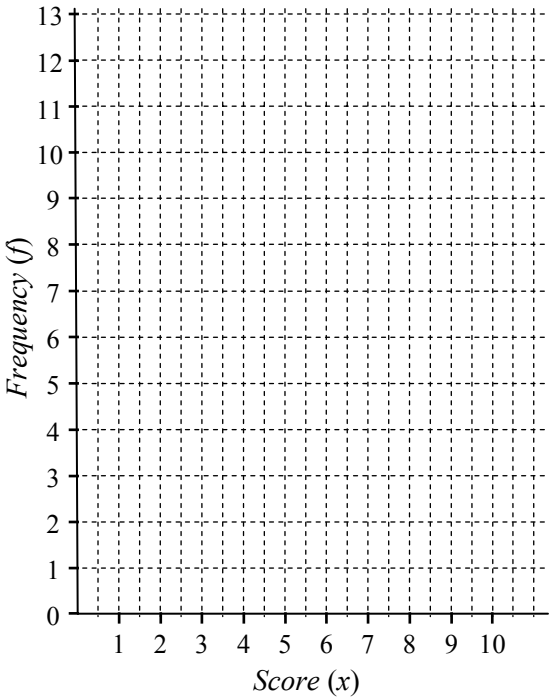
7



x	*f*
1	4
2	2
3	7
4	0
5	6

8

x	*f*
1	2
2	0
3	4
4	6
5	7
6	10
7	13
8	12
9	8
10	5



Data - Frequency Histograms and Frequency Polygons (Sheet 2)

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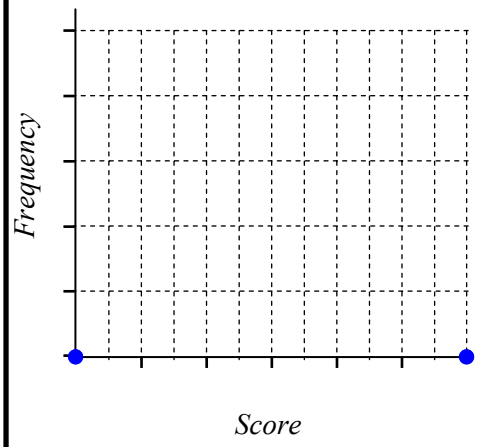
Use a ruler to construct a Frequency Histogram and Polygon for the given data.

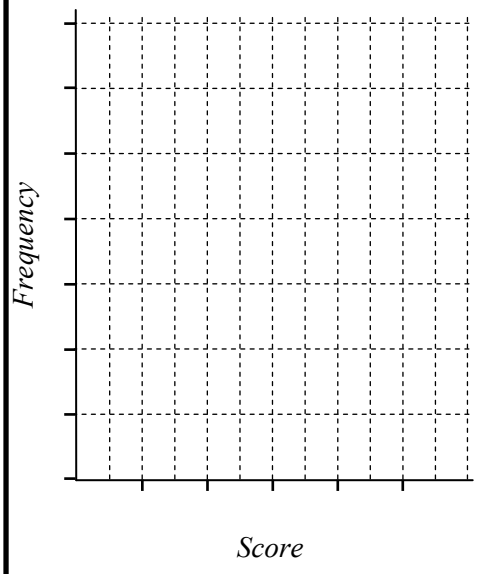
1

Score	*Frequency*
5	5
10	1
15	3
20	0
25	4



2

Score	*Frequency*
0	2
2	5
4	3
6	7
8	1

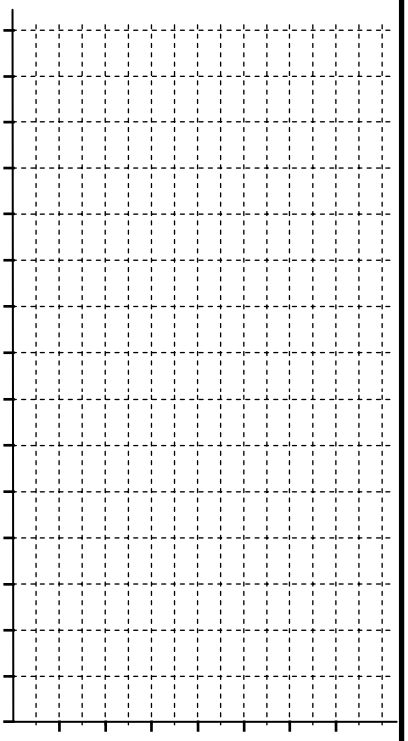


Given a data set, place the data into a Frequency Table then construct a Frequency Histogram and Frequency Polygon. Start with circling the lowest and the highest scores in the data set.

3 Family Size

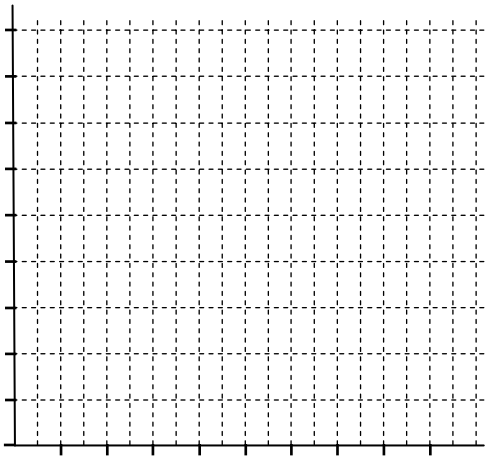
Data: 4, 3, 4, 2, 3, 7, 4, 5, 7, 6, 5, 3, 4, 8, 5, 4, 5, 3, 4, 3, 5, 4, 4, 2, 8, 6, 4, 4, 3, 5, 4, 4, 5, 6, 3, 4, 4, 3, 4.

Score	*Tally*	*Frequency*
Count the scores in the data and compare →		$\Sigma f =$



4 100 m Sprint times (s)

Data:
17, 19, 22, 16, 18, 19, 15, 21, 17, 22, 18, 16, 22, 17, 18, 20, 17, 19, 22, 16, 23, 18, 21, 19, 15, 17, 17, 19, 22, 21, 15, 17, 16, 16, 20, 23, 19, 19, 17, 18.



Data - Frequency Histograms with Classes (Sheet 1)

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Place the data below in the frequency table, using the suggested class intervals.

1 Data: 25, 16, 32, 41, 28, 7, 11, 36, 27, 20, 34, 43, 19, 31, 7, 48, 9, 20, 5, 35, 22, 10, 33, 3, 21, 24, 17, 28, 2, 36, 32, 12, 26, 43, 7, 21, 7, 5.

Class	Tally	Frequency
0 - 9		
10 - 19		
20 - 29		
30 - 39		
40 - 49		
		$\Sigma f =$

Count the number of scores in the data, does it equal Σf?

2 'Appleshot' Archery Club Championship

Points: 80, 72, 83, 67, 94, 92, 85, 73, 79, 90, 65, 77, 97, 93, 89, 65, 85, 71, 62, 83, 79, 64, 92, 70, 62, 84, 75, 81, 98, 75, 73, 63, 84, 91, 99, 98, 76, 78, 85, 96, 70, 74, 76, 62, 68, 83, 95, 100, 72, 81.

Points	Tally	Frequency
61 - 65		
66 - 70		
71 - 75		
76 - 80		
81 - 85		
86 - 90		
91 - 95		
96 - 100		

3 How many archers were there? _____

4 How many archers scored 75 or less? _____

5 Which class had the highest frequency? _____

6 What percentage of the archers scored more than 90?

$\Sigma f \rightarrow \frac{\square}{\square} \times 100 =$ _____

7 What percentage scored between 65 and 81?

$\frac{\square}{\square} \times \square =$ _____

8 Time (min) to complete a survey

Data: 6, 15, 20, 11, 19, 22, 18, 5, 11, 10, 18, 24, 16, 23, 4, 18, 9, 14, 28, 9, 14, 13, 25, 14, 17, 29, 4, 15, 5, 23, 19, 12, 17, 25, 10, 14, 19, 24, 16.

Time (min)	Tally	Frequency
1 - 5		
6 - 10		
11 - 15		
16 - 20		
21 - 25		
26 - 30		

9 How many scores between 5 and 16 were there?

_____

10 How many scores were 25 or less? _____

11 Student Height (cm)

Data: 155, 172, 163, 158, 168, 174, 183, 165, 180, 161, 174, 168, 170, 181, 163, 162, 171, 155, 183, 175, 169, 149, 170, 162, 181, 172, 176, 162, 158, 166, 157, 184, 175, 166, 163, 159, 172, 170.

Height (cm)	Tally	Frequency
145 - 149		
150 - 154		
155 - 159		
160 - 164		
165 - 169		
170 - 174		
175 - 179		
180 - 184		

Answer these to 1 decimal place.

12 What percentage of students are 1.8 m or taller?

_____ = _____

13 What percentage of students are less than 1.7 m tall?

_____ = _____

Data - Frequency Histograms with Classes (Sheet 2)

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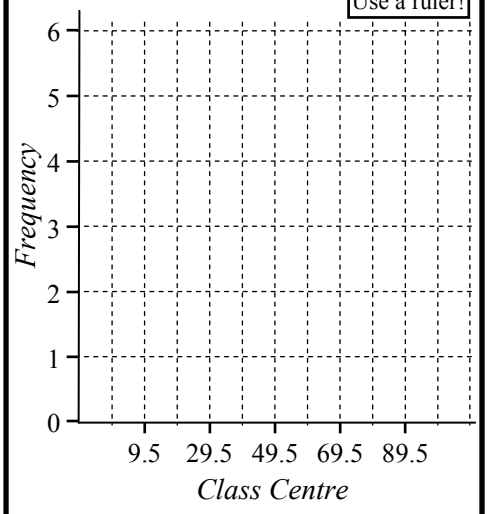
Display the data below using a frequency histogram and polygon

1

Class	*c.c.*	Frequency
0 - 19	9.5	4
20 - 39	29.5	2
40 - 59	49.5	6
60 - 79	69.5	3
80 - 99	89.5	5

Use a ruler!



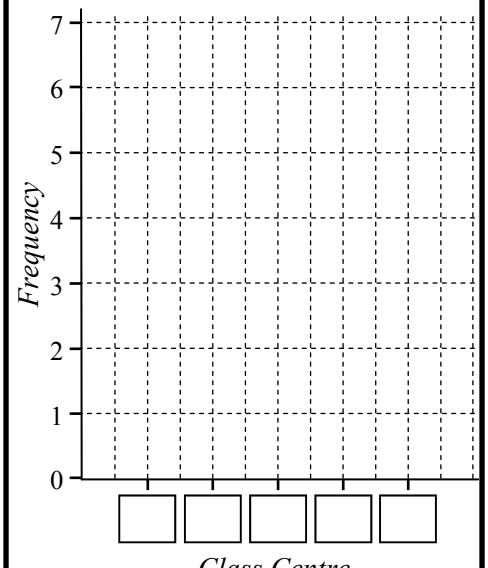
2

Class	*c.c.*	Frequency
1 - 5		4
6 - 10		7
11 - 15		0
16 - 20		2
21 - 25		5

To calculate Class Centre:

Calc: 1 + 5 = ÷ 2 =

Calc: 6 + 1 0 = ÷ 2 = and so on

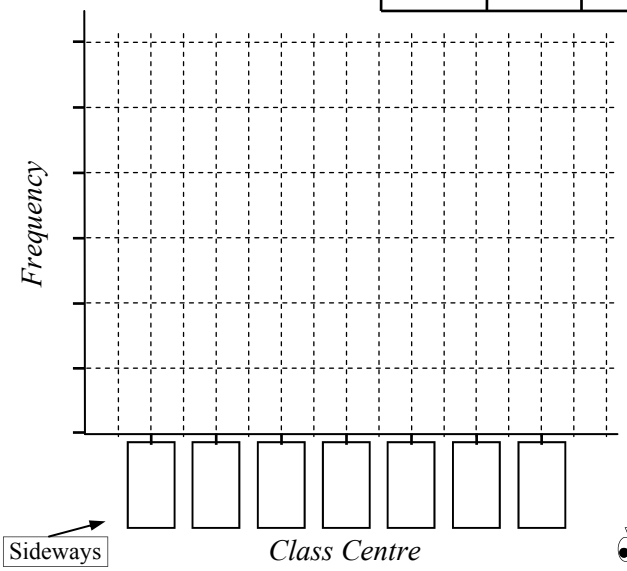


Place the data in the Frequency Table using classes 0 - 9, 10 -19 and so on. Then show the data as a Frequency Histogram and Polygon.

3 Data: 63, 9, 33, 43, 9, 63, 10, 67, 54, 3, 68, 16, 56, 13, 21, 15, 4, 50, 58, 8, 4, 40, 48, 17, 65, 59, 10, 44, 50, 23, 26, 25, 37, 31.

Class	*c.c.*	Tally	*f*
0 - 9			
10 - 19			
			$\Sigma f =$

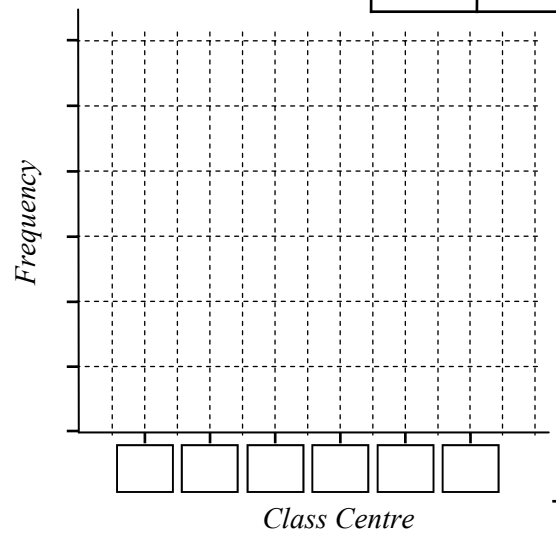
Count the number of scores in the data, does it equal Σf?



Repeat the process except this time use classes 1 - 5, 6 - 10, 11 - 15 and so on.

4 Data: 15, 30, 10, 25, 1, 30, 6, 10, 2, 21, 3, 12, 14, 27, 28, 6, 11, 16, 9, 15, 21, 17, 15, 20, 25.

Class	*c.c.*	Tally	*f*
			$\Sigma f =$



5 How many scores are larger than 15? _____

6 How many scores between 5 and 21? _____

7 What percentage of the scores were 20 or less?

_____

Data - Dot Plots (Sheet 1)

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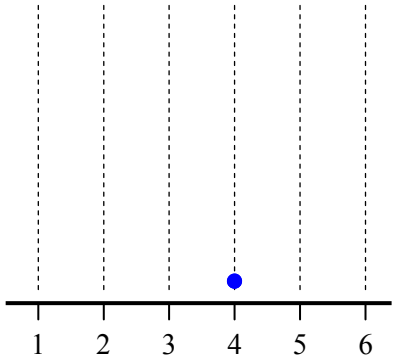
Display the data in a dot plot.

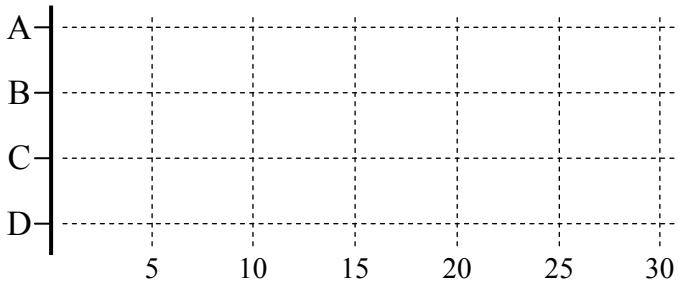
1 Data: 4, 5, 3, 3, 6, 1, 2, 4, 2, 3, 5, 6, 2, 3, 4, 4, 1, 2, 5, 6, 2, 3, 3, 2, 5, 2, 1, 4, 6, 3, 2.



2 An artificial sweetener is comparing itself with three other brands. Graph the results.

Data: BBBDA BABBC ABBBC
BDBAB ABBDB BCBBD
BCABB ABCBD BBADB

sideways! → Artificial Sweetener Preference



3 How many people were sampled? _____

4 How many people preferred Brand B? _____

5 How many people preferred Brand C? _____

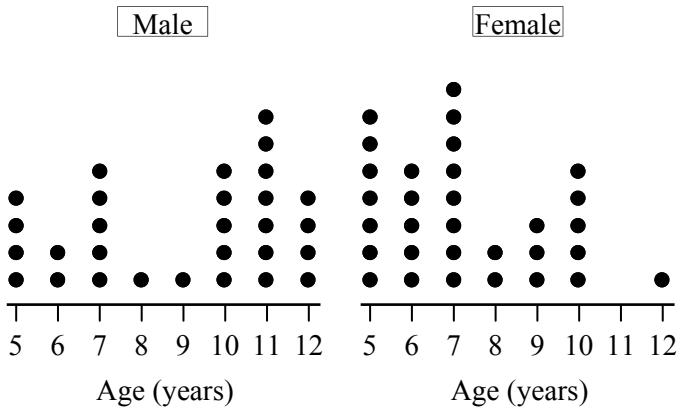
6 What percentage of people prefer Brand B?

Question 4 Ans. → ☐ / ☐ Question 3 Ans. → × 100 = ☐ Round to 1 d.p.

7 What percentage of people prefer Brand C?

Question 5 Ans. → ☐ / ☐ Question 3 Ans. → × 100 = ☐ Round to 1 d.p.

A waterslide at a theme park records the ages of the first 60 children who use the slide, the results are shown in two Dot plots. One male, one female.



8 How many children were male? _____

9 How many children were 7 years old? _____

10 How many children were 11 or older? _____

11 How many more 10 year olds used the slide compared to 6 year olds? _____

12 Are these true or false? There were more ...

Ⓣ Ⓕ Girls using the waterslide

Ⓣ Ⓕ 7 y.o. boys using the slide than 10 y.o. girls

Ⓣ Ⓕ Older male children than female.

13 What percentage of 9 y.o.'s were male?

Male 9 y.o. here → ☐ / ☐ Total 9 y.o. here → × 100 = ☐ Show the % sign

14 What percentage of 5 y.o.'s were female?

☐ / ☐ × 100 = ☐ Round to 1 d.p.

15 What percentage of females were 10 y.o.?

Total females here → ☐ / ☐ × ☐ = ☐ Round to 1 d.p.

16 What percentage of 5 - 7 y.o.'s were male?

☐ / ☐ × ☐ = ☐ Round to 1 d.p.

Data - Dot Plots (Sheet 2)

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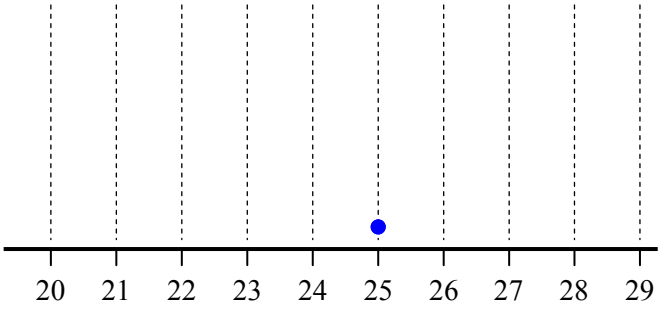
Display the data in a dot plot answering the questions from the plot.



1 Data: 25, 28, 23, 24, 23, 24, 27, 26, 25, 25, 22, 22, 20, 29, 26, 25, 27, 26, 28, 21, 25, 25, 27, 21, 25, 27, 26, 27, 22, 26, 23, 20, 22, 28.

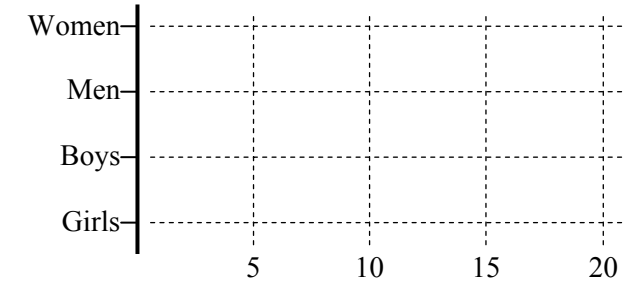


2 Women, Men, Boys and Girls entering a store have been recorded. Display on a dot plot.

Data:
MBGWB GGBWW BBWWW
MMGWB MWGBW GBWBB MMWGW
BGGBW WGMWB MWWGB WWGBM.

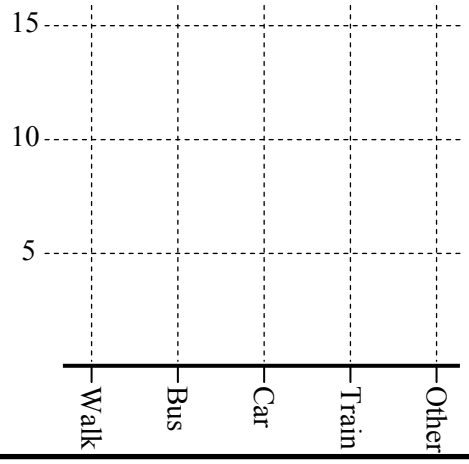
sideways!

Customer Demographic



3 A student survey recorded five different methods of travel to school, they are: Walking, Bus, Car and Train, Other. Display on a dot plot.

Data:
TBWWO BCCTC BCTTT
WWCBB TCBBT TBCWO
BCWOW TWBTT OCTBC
CCBCO TTBCO TTBBB



Method of School Travel

4 How many students were surveyed? _____

5 What percentage of students walked to school?

_____ = _____

Show working out | Answer to 1 d.p.

6 What percentage of students used public transport?

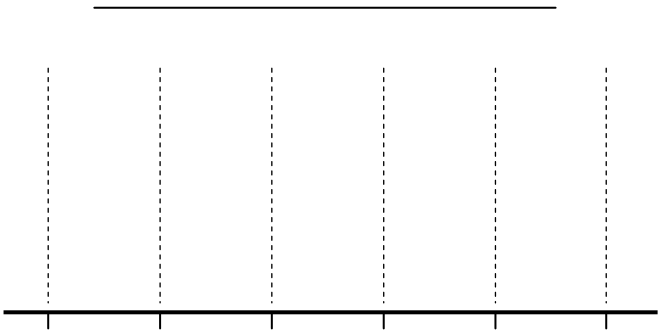
_____ = _____

Answer to 1 d.p.



A die was rolled with the results shown below. Display the data as a dot plot, then in a frequency table.

7 Data: 3, 1, 5, 6, 3, 4, 2, 2, 1, 5, 3, 6, 2, 3, 2, 5, 5, 6, 4, 5, 2, 3, 6, 1, 1, 6, 5, 4, 1, 6, 2, 2, 4, 5.



8

Score	Frequency
	$\Sigma f =$

9 How many times was a number less than 3 rolled? _____

10 How many times was an even number rolled? _____

11 How many times was a prime number rolled? _____

12 What percentage of die rolls resulted in a multiple of 3?

_____ = _____

Answer to 1 d.p.

13 What percentage of die rolls resulted in a factor of 12?

_____ = _____

Answer to 1 d.p.

Data - Ordered Stem and Leaf Plots (Sheet 1)

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Answer these questions about the Stem and Leaf plots below.

Stem	Leaf
0	2 7
1	4 8 8 ☐
2	5
3	4 7 7 7
4	0 3

1 What is the highest score? ☐

2 Which score occurs twice? (only) ☐

3 Which score has the highest frequency? ☐

4 Suggest two possible scores that could be represented by a digit being in the box in the second row. ☐, ☐

Tens	Units
8	5 8
9	3 3 7 9
10	2 2 5
11	0 0 0
12	7 ☐

5 What is the highest score? ☐

6 Which score has the highest frequency? ☐

7 Suggest three possible scores that could be represented by a digit being in the box in the fifth row.

☐ , ☐ , ☐ .

8 How many scores are above 100? ☐

9 How many scores are between 95 and 105? ☐



These stem and leaf plots aren't in order. Rewrite the leaf section in ascending order. Remember there are no commas in a stem and leaf plot

10

Stem	Leaf
0	8 2 7 3
1	0 5 2 7
2	3 7 1
3	0 9 1 7

IN ORDER →

Stem	Leaf
0	☐ ☐ ☐ ☐
1	☐ ☐ ☐ ☐
2	☐ ☐ ☐
3	☐ ☐ ☐ ☐

11

Tens	Units
9	7 7 2
10	4 0 7 2
11	0 9 1
12	8 8 3 2

IN ORDER →

Tens	Units
9	☐ ☐ ☐
10	☐ ☐ ☐ ☐
11	☐ ☐ ☐
12	☐ ☐ ☐ ☐

You are given data this time. Display the data in two stem and leaf plots. The first one is raw (in the order you enter the data), the second stem and leaf plot is to be ordered.



12 Data: 6, 14, 28, 39, 17, 12, 5, 28, 36, 21, 15, 19, 27, 30, 2, 22, 38, 24.

RAW

Tens	Units
0	
1	
2	
3	

IN ORDER →

ORDERED

Tens	Units
0	
1	
2	
3	

13 Data: 95, 106, 117, 128, 100, 121, 104, 93, 112, 93, 114, 101, 105, 115, 94, 114.

RAW

Tens	Units
9	
10	
11	
12	

IN ORDER →

ORDERED

Tens	Units
9	
10	
11	
12	

Data - Ordered Stem and Leaf Plots (Sheet 2)

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Display the data sets in Raw then Ordered Stem and Leaf plots. Don't use commas!



1 Data: 23, 17, 35, 21, 30, 8, 11, 24, 6, 9, 17, 21, 32, 3, 11, 13.

RAW → ORDERED

Tens	Units
0	
1	
2	
3	

Tens	Units
0	
1	
2	
3	

2 Data: 74, 97, 70, 106, 92, 100, 86, 91, 97, 85, 110, 83, 101, 95, 79, 90, 103, 85.

RAW → ORDERED

Stem	Leaf

Stem	Leaf

3 Data: 998, 971, 1 005, 993, 1 018, 1 015, 985, 994, 1 026, 1 012, 1 002, 994, 978, 993, 970, 1 018, 989, 991, 1 011, 1 021, 986, 1 000, 1 002, 995, 1 012, 977, 983.

Stem	Leaf

Stem	Leaf

4 Data: 220, 195, 206, 212, 184, 177, 209, 195, 191, 204, 173, 168, 188, 228, 219, 176, 192, 213, 201, 180, 171.

Stem	Leaf

→

Stem	Leaf

5 How many scores were over 200? _____

6 How many scores were even? _____

7 What percentage of scores were less than 180?

$$\frac{\square}{\square} \times 100 = \frac{\square}{\text{Answer to 1 d.p.}}$$

8 What percentage of scores were between 190 and 210?

$$\frac{\square}{\square} \times \square = \frac{\square}{\text{Answer to 1 d.p.}}$$

9 Data: 52, 47, 11, 28, 9, 54, 62, 47, 19, 17, 24, 38, 68, 51, 20, 7, 33, 49, 19, 28, 30, 47, 53, 40, 15, 38, 14.

Stem	Leaf

→

Stem	Leaf

Data - Positive and Negative Skew (Sheet 1)

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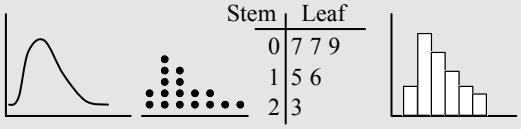
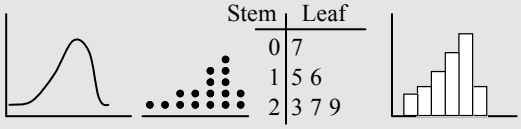
Are these positive or negative skew? Fill the circle for the choice for each of these.

Positive
Favours
lower values



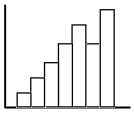
Stem	Leaf
0	7 7 9
1	5 6
2	3

Negative
Favours
higher values



Stem	Leaf
0	7
1	5 6
2	3 7 9

1



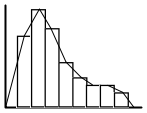
○ Positive
○ Negative

2



○ Positive
○ Negative

3



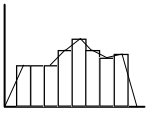
○ Positive
○ Negative

4



○ Positive
○ Negative

5



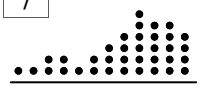
○ Positive
○ Negative

6

Stem	Leaf
5	0
6	5 6
7	3 8 8
8	7 7 7

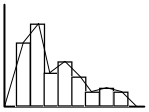
○ Positive ○ Negative

7



○ Positive
○ Negative

8



○ Positive
○ Negative

9



○ Positive
○ Negative

10



○ Positive
○ Negative

11

Stem	Leaf
10	1 5 6 6 8
11	2 3 3
12	4
13	1 5

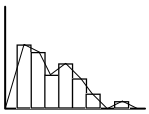
○ Positive ○ Negative

12



○ Positive
○ Negative

13



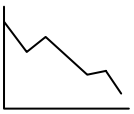
○ Positive
○ Negative

14



○ Positive
○ Negative

15



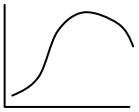
○ Positive
○ Negative

16

Tens	Units
4	1
5	0 3
6	1 8 8

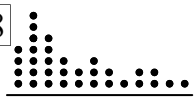
○ Positive
○ Negative

17



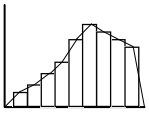
○ Positive
○ Negative

18



○ Positive
○ Negative

19



○ Positive
○ Negative

20



○ Positive
○ Negative

21

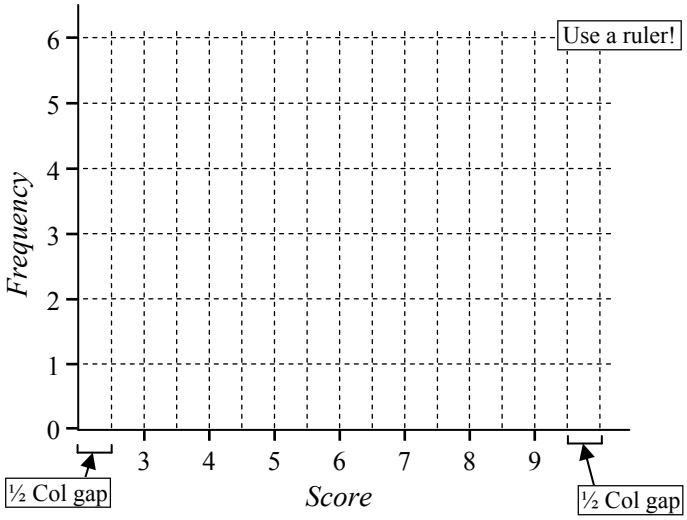
Tens	Units
9	2 3 3 7
10	0 4 9
11	3

○ Positive
○ Negative

Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

22 Data: 3, 4, 7, 5, 6, 3, 3, 4, 4, 5, 9, 4, 3, 5, 4, 6, 5, 3, 4.

Score	Tally	Frequency
		$\Sigma f =$



23 The above distribution is _____ skewed.
Positively/Negatively

Now in a stem and leaf plot. No commas.

24 Data: 67, 72, 51, 75, 40, 63, 75, 67, 42, 60, 59, 66.

Stem	Leaf

IN ORDER

Stem	Leaf

25 The above distribution is _____ skewed.

Data - Positive and Negative Skew (Sheet 2)

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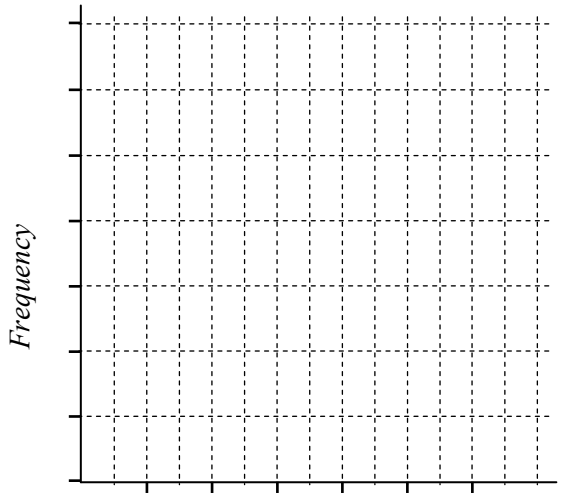
Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

1 Data: 6, 5, 5, 4, 6, 1, 3, 4, 3, 5, 6, 6, 4, 4, 1, 3, 6, 4, 5, 3, 5, 6, 5, 5, 6.

Score	Tally	Frequency



2 The above distribution is _____ skewed.

Display this data in a dot plot. Then comment on the shape of the distribution.



3 Data: 18, 21, 23, 20, 19, 23, 19, 24, 22, 21, 20, 18, 22, 19, 19, 20, 22, 24, 18, 20, 25, 20.



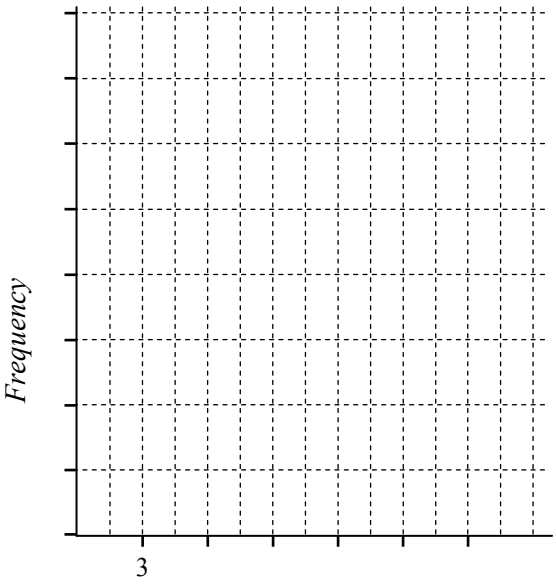
4 The above distribution is _____

Display the data in a frequency table with classes and frequency histogram/polygon. Outline the shape.

5 Data: 6, 20, 11, 25, 9, 7, 30, 4, 12, 6, 29, 15, 1, 17, 3, 13, 9, 23, 14, 8, 5, 2, 27, 14, 6, 7, 20.

Class	*c.c.*	Tally	*f*
1 - 5	3		
6 - 10			
11 - 15			
16 - 20			
21 - 25			
26 - 30			

Add classes and ÷ 2.



6 The above distribution is _____

Display the data in an ordered stem and leaf plot.

7 Data: 756, 778, 750, 761, 783, 760, 785, 772, 767, 772, 764, 783, 775, 753, 780, 773.

Tens	Units

IN ORDER

Tens	Units

8 The above distribution is _____

Data - Bimodal and Symmetric (Sheet 1)

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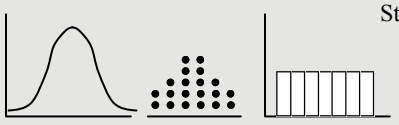
Symmetric Distributions



Stem	Leaf
0	7
1	5 6 8
2	3

Bimodal Distributions



Stem	Leaf
0	7 6
1	5 5 6 8
2	3 7
3	3 4 4 6
4	2 4 4

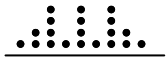
Two maxima don't have to be the same height.

Bimodal and Symmetric Distributions



Are the shape of these distributions symmetric, bimodal, both or neither? Fill in 1 or 2 circles!

1



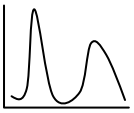
○ Symmetric
○ Bimodal
○ Neither

2



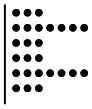
○ Symmetric
○ Bimodal
○ Neither

3



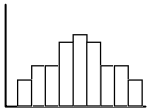
○ Symmetric
○ Bimodal
○ Neither

4



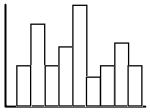
○ Symmetric
○ Bimodal
○ Neither

5



○ Symmetric
○ Bimodal
○ Neither

6



○ Symmetric
○ Bimodal
○ Neither

7



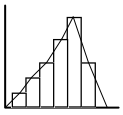
○ Symmetric
○ Bimodal
○ Neither

8

Stem	Leaf
10	1 5
11	2 3 3 3
12	4
13	1 5 5 5
14	2 3

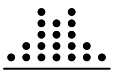
○ Symmetric
○ Bimodal
○ Neither

9



○ Symmetric
○ Bimodal
○ Neither

10



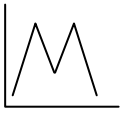
○ Symmetric
○ Bimodal
○ Neither

11



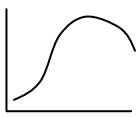
○ Symmetric
○ Bimodal
○ Neither

12



○ Symmetric
○ Bimodal
○ Neither

13



○ Symmetric
○ Bimodal
○ Neither

14

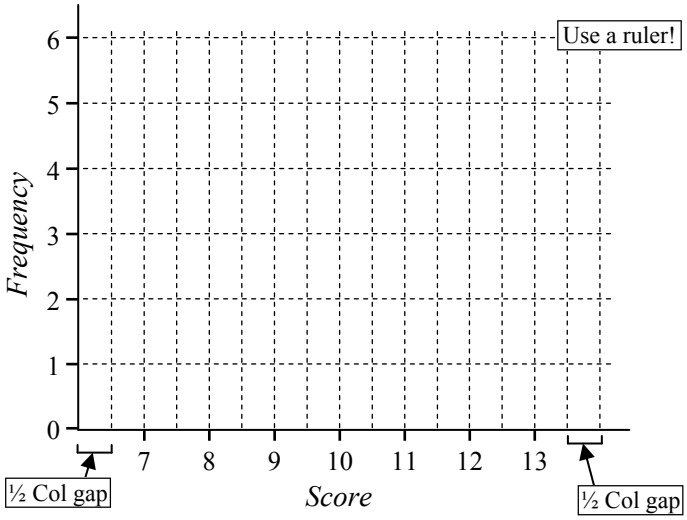


○ Symmetric
○ Bimodal
○ Neither

Display the data in a frequency table and frequency histogram/polygon. Then comment on the shape.

15 Data: 9, 12, 13, 8, 9, 10, 12, 11, 7, 9, 11, 10, 9, 8, 8, 12, 7, 9, 12, 11, 7, 9.

Score	Tally	Frequency
		$\Sigma f =$



16 The distribution's shape is _____

Bimodal/Symmetric or Bimodal and Symmetric

Now in a stem and leaf plot. No commas.

17 Data: 26, 17, 31, 5, 15, 28, 13, 34, 8, 17, 21, 11, 20, 20.

Stem	Leaf
0	.....
1	.....
2	.....
3	.....

IN ORDER

Stem	Leaf
	.....
	.....
	.....
	.....

18 The distribution's shape is _____

Data - Bimodal and Symmetric (Sheet 2)

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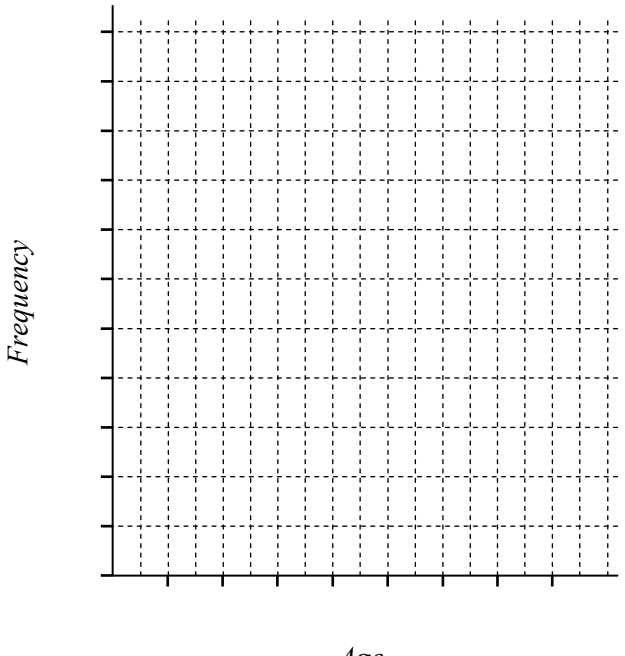
Year 11 students are tutor-reading with 17 Year 7 and 5 Year 8 students, everyone's ages are recorded in a survey. Place the results in a frequency table, then display them in a histogram and polygon.

1 Data: 11, 16, 12, 12, 17, 18, 13, 16, 12, 13, 17, 16, 12, 13, 17, 16, 16, 12, 16, 12, 17, 14, 16, 13, 11, 17, 18, 12, 17, 12, 17, 13, 17, 18, 15, 12, 16, 17, 13, 12, 17, 13, 11, 17, 12, 13.

Age	Tally	Frequency



2 The distribution's shape is: _____

3 Describe why the distribution is the shape shown above

_____

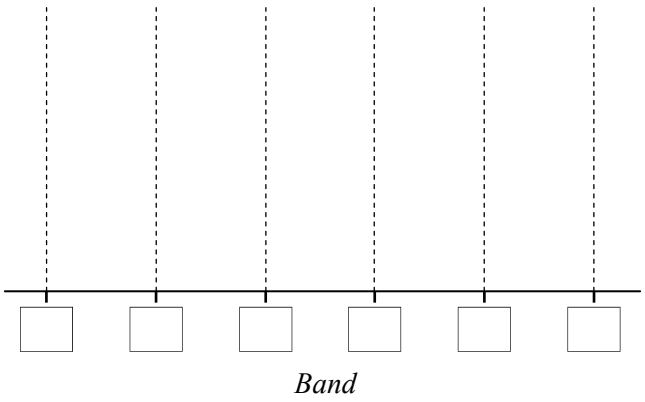
_____

_____

_____

An English teacher selects 30 students randomly from Year 9 to compare their 'Persuasive Writing' band results. Plot their results in a dot plot.

4 Data: 9, 6, 8, 6, 7, 7, 9, 10, 8, 7, 8, 9, 6, 5, 7, 8, 8, 10, 8, 7, 9, 6, 7, 8, 7, 5, 9, 7, 6, 8.



5 The distribution's shape is: _____

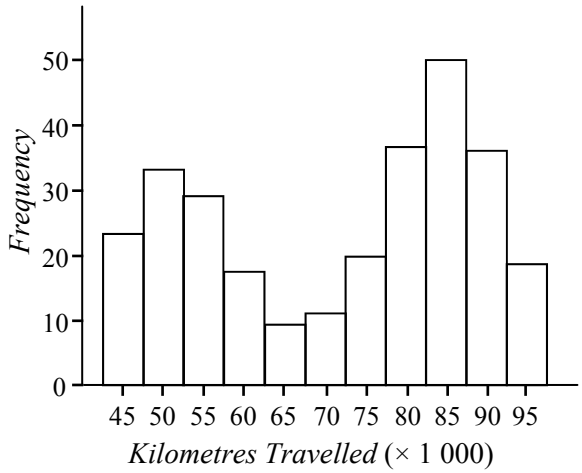
6 If the teacher chose 30 students just from the top English class, how would the results be affected?

_____

_____

_____

A tyre manufacturer has two different tyre types. After surveying its customers, it has plotted the data for the tyre life, to the nearest 5 000 km below.



7 Describe the distribution's shape and suggest a reason why it is shaped in that way.

_____

_____

_____

Data - Back-to-Back Stem and Leaf Plots (Sheet 1)

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Rewrite this stem and leaf plot in order. That's an order!



Same data shown in order

65440	0	04456
955	1	559
7200	2	0027

1

RAW Leaf	Stem	Leaf
83026	0	00285
7339	1	716579
780411	2	203884
3986	3	92813

ORDERED Leaf	Stem	Leaf
High ← Low		Low → High
	0	
	1	
	2	
	3	



It's auction time! 'Four n'Fluffy' livestock sales is selling sheep and alpacas. Lot sales are shown below. Place the data in a back-to-back stem and leaf plot, first raw data, then in order.

2 Four n'Fluffy Auction Sales (in Lots)

Sheep: 22, 25, 8, 37,18, 13, 3, 22, 15, 17, 34, 22, 17, 12, 20, 23, 11, 27.

Alpacas: 36, 28, 5, 38, 22, 19, 12, 7, 25, 37, 19, 31, 26, 30, 23, 37, 27, 19, 20.

RAW Alpacas	Stem	Sheep
	0	
	1	
	2	
	3	

ORDERED Alpacas	Stem	Sheep
	0	
	1	
	2	
	3	

3 The distribution's shapes are: Sheep: _____

Alpacas: _____



A restaurant has 30 tables* and records how many bookings are made daily over 4 weeks. Bookings for lunch and dinner are recorded separately. Show the data in a B-2-B stem and leaf plot.

*each table can be used more than once per session

4 Lunch: 90, 74, 105, 103, 60, 117, 96, 102, 83, 117, 82, 58, 74, 87, 95, 91, 100, 82, 60, 107, 96, 119, 102, 63, 91, 95, 66, 112.

Dinner: 105, 72, 89, 65, 67, 50, 93, 88, 77, 64, 60, 77, 57, 64, 111, 78, 54, 63, 98, 72, 64, 55, 77, 84, 66, 67, 52, 57.

Lunch	Stem	Dinner
	5	

Lunch	Stem	Dinner

5 The shape of the distributions are: Lunch: _____

Dinner: _____

6 Suggest a reason for Dinner's distribution.

_____

Data - Back-to-Back Stem and Leaf Plots (Sheet 2)

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Data for 'Public Satisfaction' for government policies has been sourced from 20 electorates. Two samples were taken, one before and one after the May budget. Place the data into a back-to-back stem and leaf plot and then answer the questions.

1 Pre-Budget : 61, 63, 50, 45, 77, 38, 55, 67, 43, 38, 50, 72, 67, 43, 78, 66, 52, 79, 55, 66.

Post-Budget : 55, 68, 34, 49, 40, 49, 56, 37, 66, 30, 49, 70, 35, 43, 55, 37, 43, 32, 38, 51.

Pre-Budget Satisfaction	Stem	Post-Budget Satisfaction



Pre-Budget Satisfaction	Stem	Post-Budget Satisfaction

2 The distribution's shapes are:

Pre-Budget satisfaction _____ Post-Budget satisfaction _____

3 Compare the above data.

_____

_____

_____

_____

_____

30 students have sat two tests, both tests had a maximum mark of 75. Plot the results for the tests in a back-to-back stem and leaf plot.

4 Test 1: 48, 27, 40, 59, 32, 20, 25, 46, 55, 38, 19, 39, 41, 38, 55, 72, 22, 59, 34, 8, 17, 45, 59, 23, 60, 34, 40, 41, 61, 23.

Test 2: 50, 61, 21, 59, 63, 28, 44, 68, 47 20, 17, 50, 63, 28, 3, 62, 11, 71 35, 59, 23, 19, 46, 7, 38, 47, 11, 54, 71, 39.

Test 1	Stem	Test 2



Test 1	Stem	Test 2

5 The distribution's shapes are: Test 1: _____ Test 2: _____

6 Suggest a reason for Test 2's distribution.

_____

_____

Data - Types of Graphs (Categorical/Numerical)

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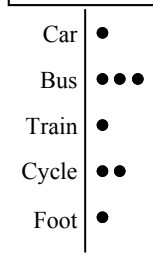
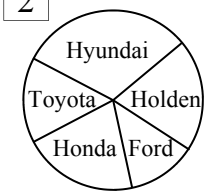
Are these graphs displaying 'Categorical' or 'Numerical' data?

1



○ Categorical
○ Numerical

2



○ Categorical
○ Numerical

3



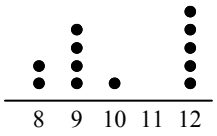
○ Categorical
○ Numerical

4

Stem	Leaf
10	1 5
11	2 3 3 3
12	4
13	1 5 5 5
14	2 3

○ Categorical
○ Numerical

5



○ Categorical
○ Numerical

6

Stem	Leaf
0	7 7
1	0 1 8
2	6
3	3 4 4
4	5

○ Categorical
○ Numerical

Classify the data as either Categorical or Numerical.

7 Cat, Dog, Bird, Fish, Cat. _____

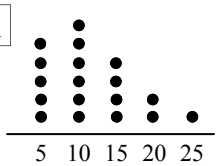
8 114, 270, 196, 414. _____

9 1, 6, 2, 7, 8, 3, 5 _____

10 Red, Green, Red, Blue, Red. _____

Are these graphs displaying Categorical or Numerical data? Then describe the distribution using: Bimodal, Skew (Pos/Neg), Symmetric.

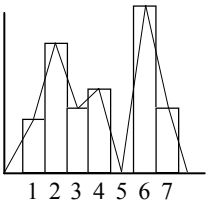
11



_____

_____

12



_____

_____

13

Leaf	Stem	Leaf
1	0	3
9 5 5 2	1	2 5
6 3	2	1 1 7 9
8 4 3 0	3	8 8
5 1	4	7

Left Leaf _____

_____

Right Leaf _____

_____

14

Leaf	Stem	Leaf
2 0	8	5
6 1	9	2 3 3 3
5 4 4	10	4 9
7 7 3 0	11	3 6 7 7
8 6 2 1	12	8

Left Leaf _____

_____

Right Leaf _____

_____

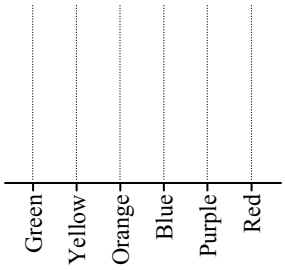
Organise this data into the frequency table below. Plot the results on the different dot plot grids and describe the graphs fully.

15

Data:
BYPYG
OGYPB
OBBYP
ORPYB
YORGB
R

Colour	Tally	Frequency
Red		
Blue		
Orange		
Green		
Yellow		
Purple		

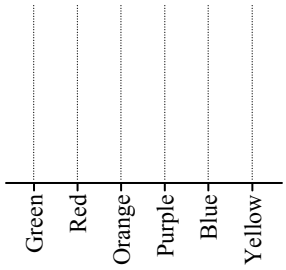
16



_____

_____

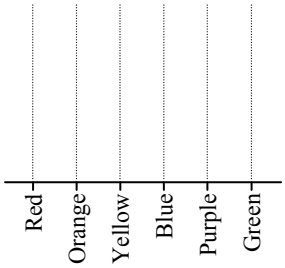
17



_____

_____

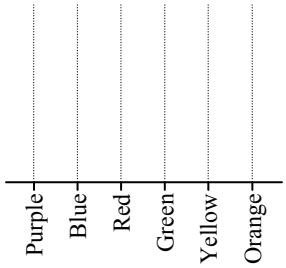
18



_____

_____

19



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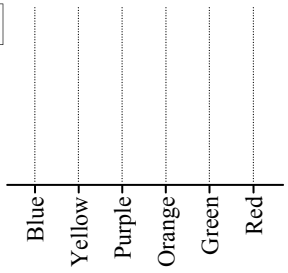
_____

20



_____

_____

21 Do you think the words Bimodal, Symmetric and Skew should be used with Categorical data?

_____

_____

Data - Types of Graphs (Misleading Graphs)

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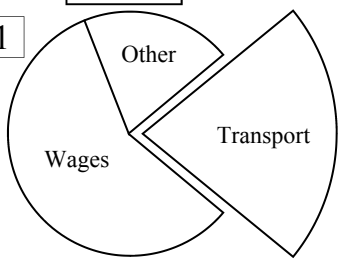
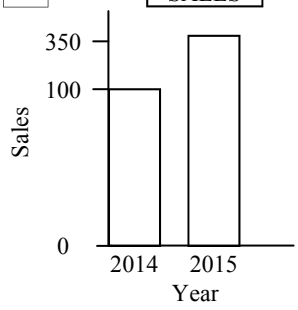
Describe why these graphs are misleading.

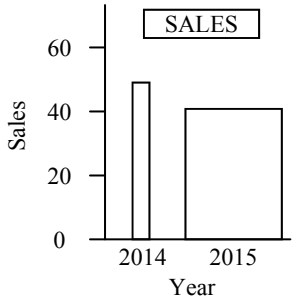
1



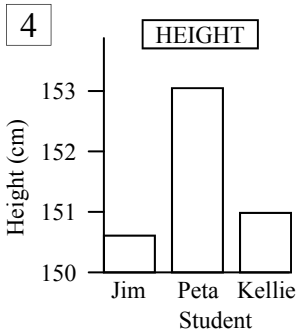
2



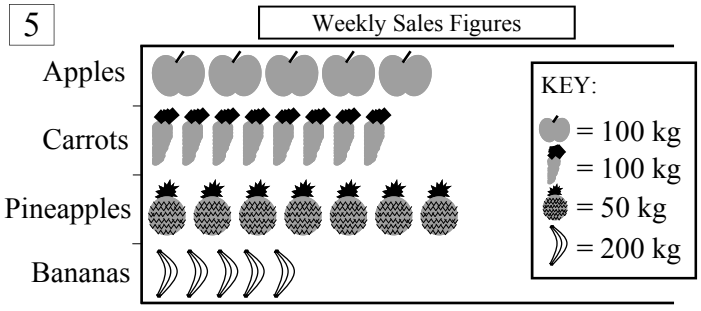
3



4



5



Display the data in the table as a column graph. This isn't a histogram, so no half column gap at the start, no polygon and columns don't touch.

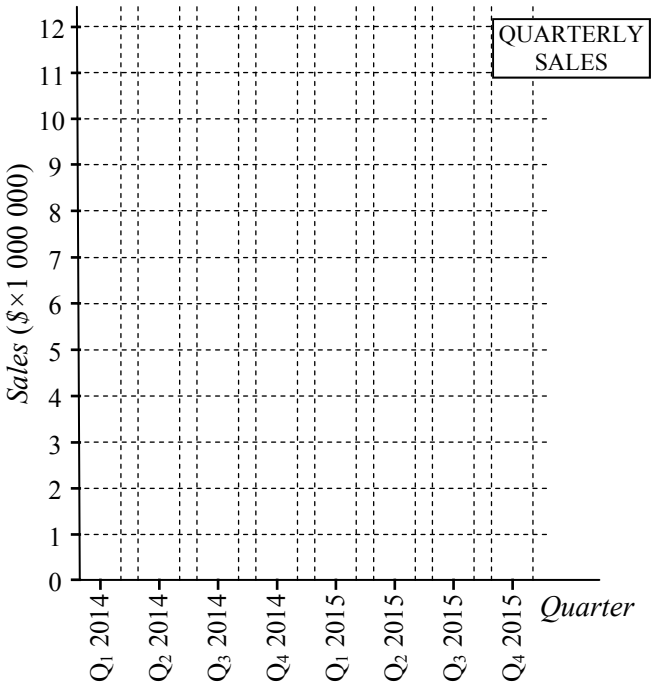
7 Complete the table's 'Total' column

SALES × $1 000 000

Year	Q_1	Q_2	Q_3	Q_4	Annual
2014	1	5	12	11	
2015	10	9	7	4	

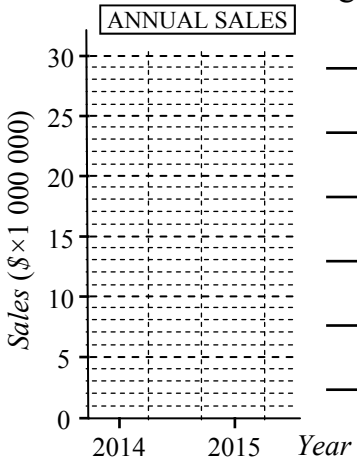
'Q' means Quarter which is a 3 month period

8 Construct a column graph for quarterly sales



9 Describe the quarterly sales above

10 Graph the total yearly sales. Comment on the differences between the graphs. 2016 indications?



Data - Find a Word

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Find the words in the puzzle from the wordlist.



A	T	A	L	L	A	C	I	R	O	G	A	T	A	C
Y	A	N	U	M	E	R	I	C	A	L	H	O	B	N
L	C	E	V	I	T	I	S	O	P	M	I	B	O	I
L	L	N	R	O	R	D	Y	W	E	K	S	I	I	B
A	A	R	E	D	R	O	M	A	E	O	T	M	M	I
Q	S	N	G	U	U	Q	M	T	L	U	O	O	N	Y
G	S	G	N	U	Q	U	E	A	B	T	G	D	E	A
N	E	N	I	Q	U	E	T	I	A	A	R	A	G	L
I	S	H	A	P	E	Y	R	T	T	A	A	L	A	P
D	G	N	A	L	L	T	I	F	W	A	M	E	T	S
N	A	X	T	L	S	T	C	A	L	E	A	F	I	I
E	A	M	A	I	T	P	A	G	P	T	L	O	V	D
C	X	T	D	X	P	O	L	Y	G	O	N	O	E	P
S	A	M	I	X	A	M	D	C	O	L	U	M	N	S
A	O	R	G	A	N	I	S	E	O	P	P	L	L	T

WORDLIST

ORGANISE	LEAF	NUMERICAL	SKEW	NEGATIVE
TABLE	RAW	DATA	MAXIMA	PLOT
HISTOGRAM	DOT	CLASSES	DISPLAY	FREQUENCY
ASCENDING	TALLY	POLYGON	STEM	ORDER
BIMODAL	POSITIVE	SHAPE	GAP	COLUMNS
	CATAGORICAL	SYMMETRICAL	DISTRIBUTION	

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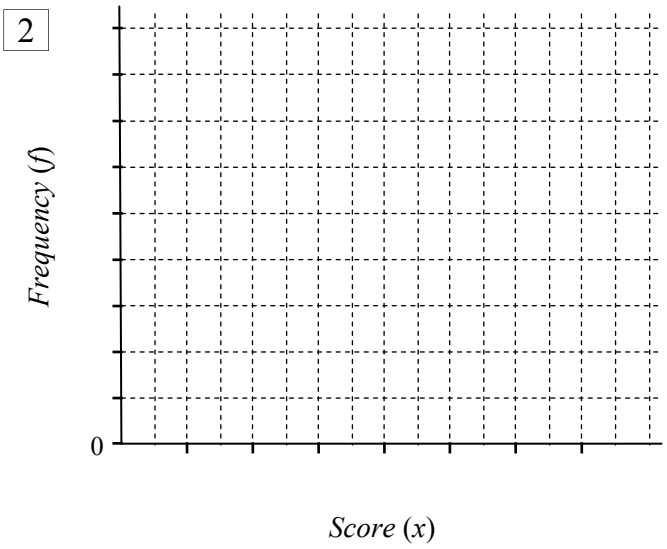
Place the data into a frequency table and then create a frequency histogram and polygon for the data also.

1 Data: 20, 5, 10, 0, 15, 15, 20, 0, 5, 15, 5, 0, 5, 20, 30, 20, 10, 10, 5, 20, 5, 15, 10, 20, 5, 0, 30, 20, 15, 0, 5.

Score	Tally	Frequency
		$\Sigma f =$

2



3 Which score occurred seven times? _____

4 Which score occurred the most? _____

5 How many scores were larger than 15? _____

6 What percentage of scores were 10 or less? (1 d.p.)

$$\frac{\square}{\square} \times 100 = \frac{\square}{\text{Round to 1 d.p.}}$$

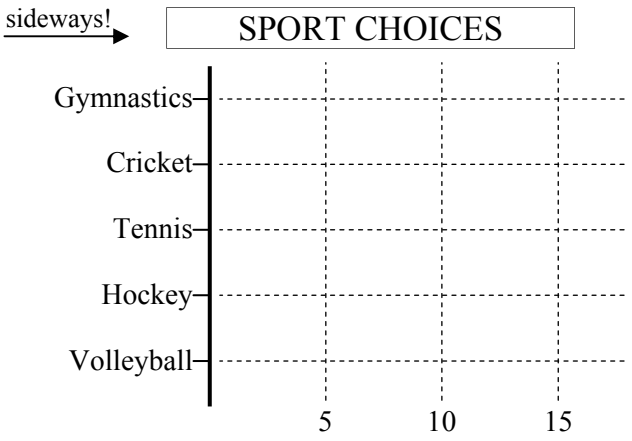
7 What percentage of scores were multiples of 10?

$$\frac{\square}{\square} \times 100 = \frac{\square}{\text{Round to 1 d.p.}}$$

Place the data into a dot plot, then answer the questions

8 Year 9 students can choose one of these sports: Gymnastics, Cricket, Tennis, Hockey or Volleyball. Two classes' selections are:

Data: GTHTT CGCHV TCHHH GVCTT
HCHCV CCCHV TVCCT GHVGC
TGHVH CGCTT VHHGT GCTHV



9 Which sport was the most popular? _____

10 How many students chose Hockey or tennis? _____

Place the data into an ordered stem and leaf plot.

11 Child Allowance ($) for Easter Carnival.

Data: 70, 85, 58, 95, 60, 65, 42, 105, 77, 60, 85, 115, 100, 75, 40, 80, 100, 82, 90, 50, 70, 82, 110, 55, 100, 105, 80, 55, 75, 110, 60, 65.

RAW → ORDERED

Stem	Leaf

Stem	Leaf

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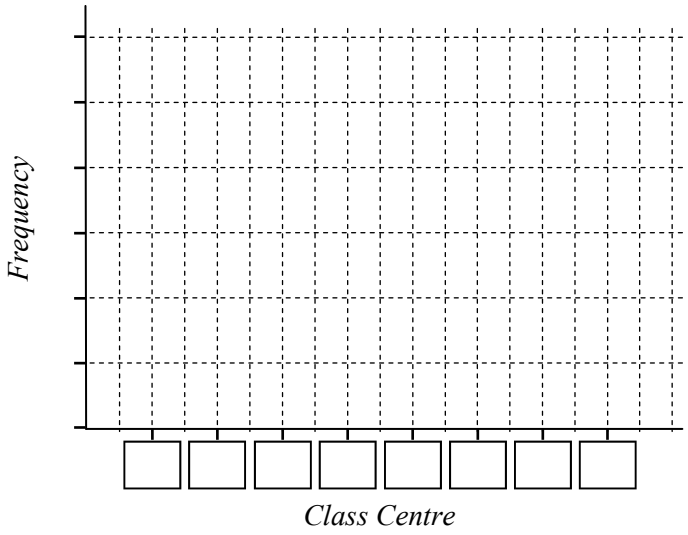
Place the data into a frequency table with classes and then create a frequency histogram and polygon for the data.

12 Data: 21, 26, 31, 8, 30, 30, 13, 2, 38, 10, 18, 12, 33, 15, 5, 13, 9, 33, 24, 6, 2, 32, 7, 27, 30, 8.

Class	*c.c.*	Tally	*f*
1 - 5	3		
6 - 10			
11 - 15			
16 - 20			
21 - 25			
26 - 30			
31 - 35			
35 - 40			
			$\Sigma f =$

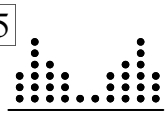


13 Is this data categorical or numerical? _____

14 Describe the shape of the distribution. _____

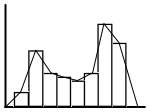
Describe the shape of these distributions

15



○ Bimodal
○ Symmetric

16



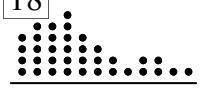
○ Bimodal
○ Symmetric

17

Stem	Leaf
2	0 1 3
3	5 5 6 8
4	0 0 4 7 7
5	7 7 7 7
6	2 2 6

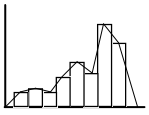
○ Bimodal
○ Symmetric

18



○ Positive Skew
○ Negative Skew

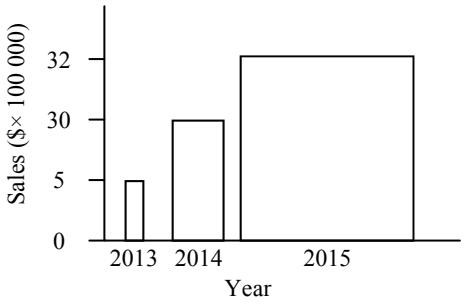
19



○ Positive Skew
○ Negative Skew

Describe the ways in which the graph below is misleading.

20



A cinema manager records the ages of 20 people entering the complex on a weekday. One sample is taken at 2 p.m., another at 9 p.m. Organise the data below into an ordered back-to-back stem and leaf plot.

21 2 p.m. : 58, 67, 43, 76, 69, 27, 77, 38, 43, 67, 67, 40, 79, 35, 59, 78, 58, 70, 59, 65.

9 p.m. : 34, 25, 47, 30, 42, 17, 20, 46, 28, 22, 31, 46, 52, 41, 45, 25, 25, 42, 23, 34.

RAW 2 p.m.	Stem	9 p.m.
	1	

ORDERED 2 p.m.	Stem	9 p.m.

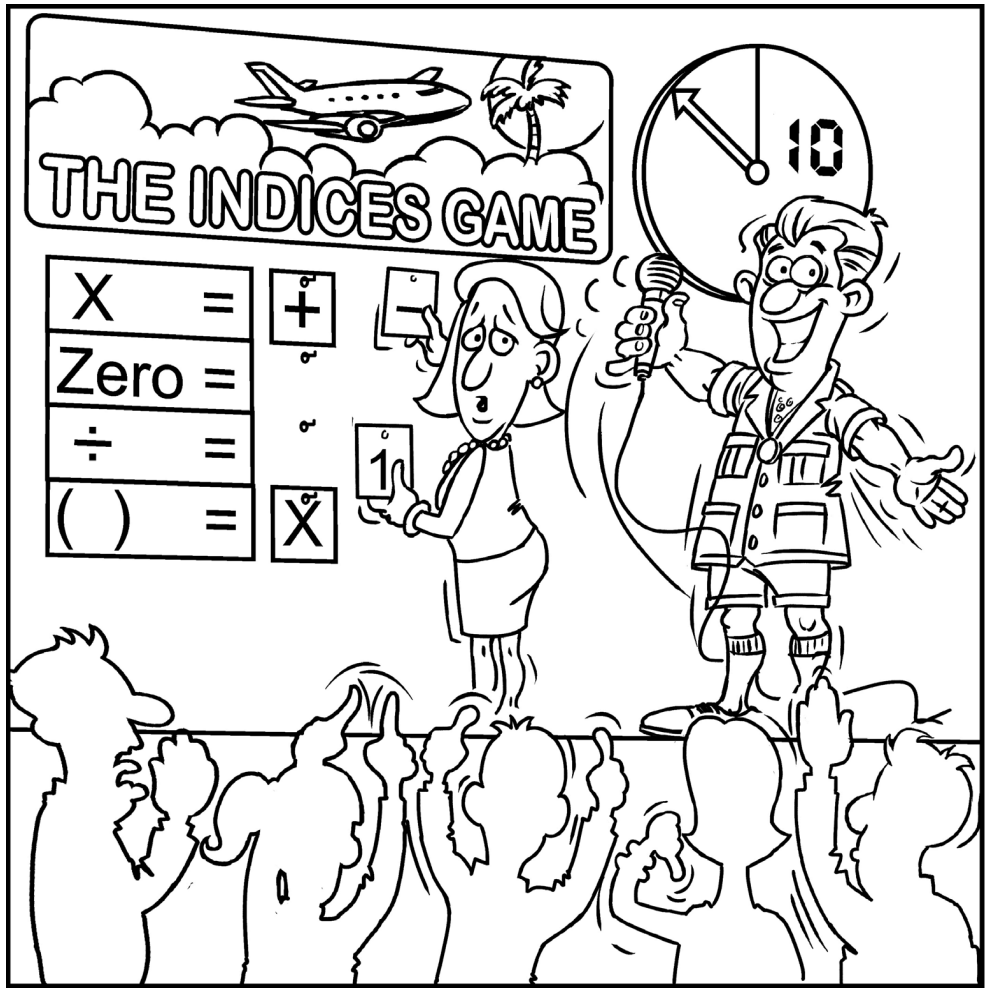
○ Positive Skew ○ Bimodal
○ Negative Skew ○ Symmetric

○ Positive Skew ○ Bimodal
○ Negative Skew ○ Symmetric

VELOCITY

INDICES

Indices



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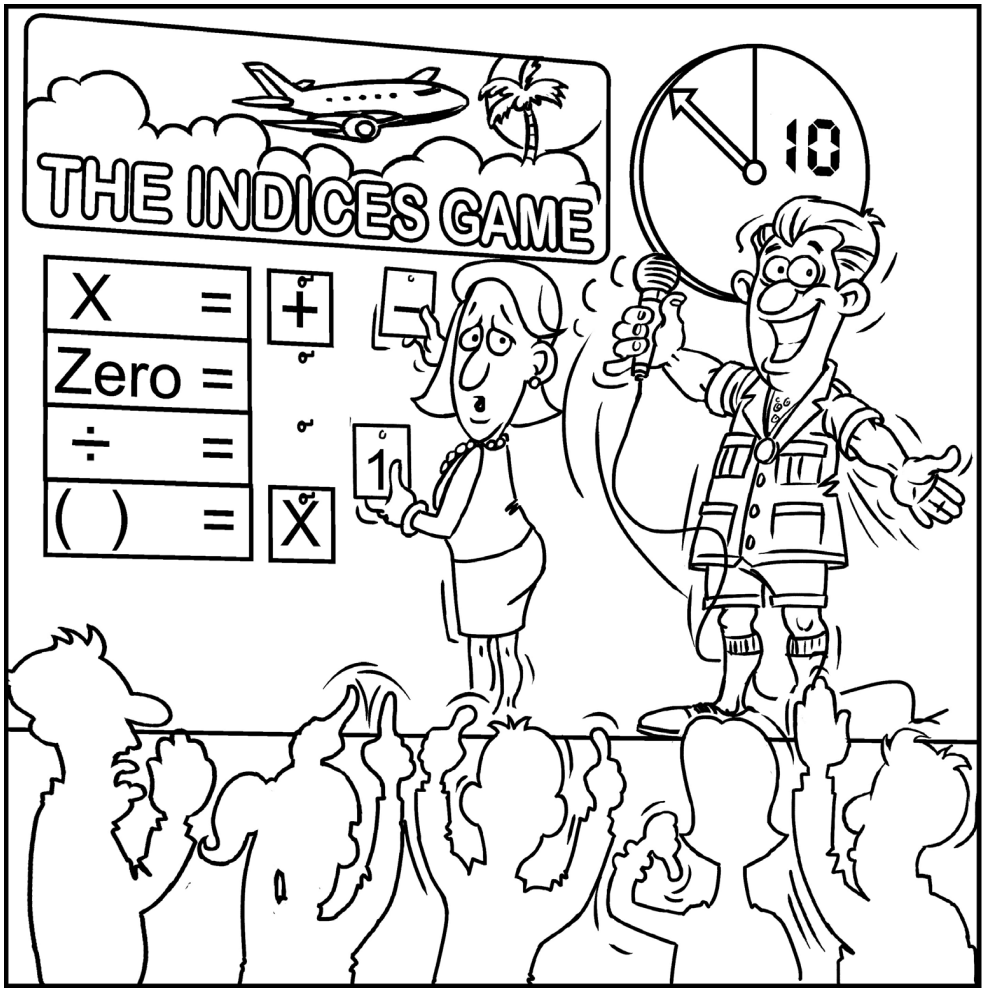
Indices



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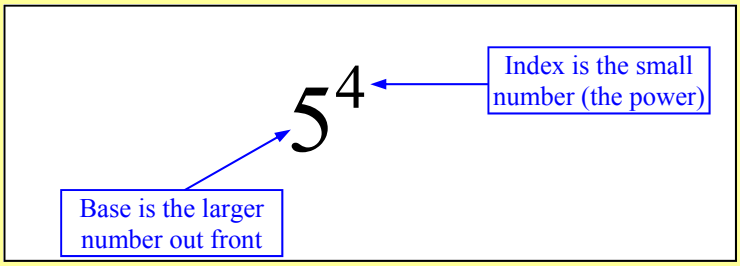
Indices - Power Key Applications

An indice is a number or letter (the base) with a power on it (the index). These sheets just deal with numbers, so examples of indices are: 5^2, 3^6 and 7^3.



This sheet involves use of the power key function of your calculator. While you have a 'Square Key' and a 'Cube key', the power key is used for all powers, not just 2 and 3.

Example Evaluate 5^4.

5^4 Calc: 5 $x^{\blacksquare}$ 4 = $= 625$

The second sheet deals with the properties of negative indices. A negative power is typed into the calculator the same way as above, just press the negative sign before entering the power. But the negative power also means '1 over'.

Example Evaluate 5^4.

5^{-4} Calc: 5 $x^{\blacksquare}$ - 4 = S⇔D $= \frac{1}{625} = \frac{1}{5^4}$

'1 over'

This key writes the answer as a fraction

The last part of the sheet asks you to complete tables. These tables have a pattern in them which allows you to complete them without using the power key each time. It also shows what happens when you have a power of zero.

Indices - Power Key Applications (Sheet 1)

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Use your power key to evaluate (find the value for) these.

Example $5^4 = 625$

Calc: 5 $x^{\blacksquare}$ 4 =

1 $3^3 =$ ____ 2 $2^5 =$ ____

3 $5^2 =$ ____ 4 $7^3 =$ ____

5 $4^4 =$ ____ 6 $3^5 =$ ____

7 $9^5 =$ ____

8 $15^4 =$ ____

9 $10^1 =$ ____ 10 $10^2 =$ ____

11 $10^3 =$ ____

12 $10^4 =$ ____

13 $10^5 =$ ____

14 $10^6 =$ ____

15 $1^1 =$ ____ 16 $1^2 =$ ____

17 $1^3 =$ ____ 18 $1^4 =$ ____

Name the base and the index in these.

Example 5^4 — 5 (Base), 4 (Index)

Base is the larger number out front

Index is the small number (the power)

19 6^7 Base ____ Index ____

20 9^8 Base ____ Index ____

21 7^6 Index ____ Base ____

22 13^5 Index ____ Base ____

Given the base and index, write the indices and evaluate.

Example Base 9, index 4.

$9^4 = 6\,561$

23 Base 4, index 6.

____ = ____

Calc: 4 $x^{\blacksquare}$ 6 =

24 Base 2, index 8.

____ = ____

25 Index 4, base 5.

____ = ____

26 Base 9, index 4.

____ = ____

27 Index 5, base 10.

____ = ____

Use guess, check and revise to find the correct index for these.

28 $2^{\square} = 128$ 29 $3^{\square} = 27$

30 $7^{\square} = 823\,543$

31 $5^{\square} = 48\,828\,125$

32 $13^{\square} = 28\,561$

33 $10^{\square} = 10\,000\,000$

34 $20^{\square} = 8\,000$

35 $2^{\square} = 262\,144$

36 $3^{\square} = 177\,147$

Use guess, check and revise to find the correct base for these.

37 $\square^3 = 64$ 38 $\square^5 = 243$

39 $\square^7 = 128$ 40 $\square^3 = 729$

41 $\square^4 = 83\,521$

42 $\square^9 = 19\,683$

43 $\square^4 = 10\,000$

44 $\square^6 = 64\,000\,000$

45 $\square^3 = 1$ 46 $\square^8 = 1$

Blend the two skills by finding both the base and the index!

47 $2^{\square} = 256 = \square^4$

48 $9^{\square} = 531\,441 = \square^{12}$

49 $\square^4 = 4\,096 = 2^{\square}$

50 $\square^5 = 1\,048\,576 = 2^{\square}$

51 $4^{\square} = 16\,777\,216 = \square^6$

Use '<' or '>' to make these true. Hint: Point the '<' or '>' to the smallest number.

52 4^5 ____ 2^9

53 7^4 ____ 4^7

Indices - Power Key Applications (Sheet 2)

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Use your power key to evaluate these indices, answer these as a fraction.

Example $5^{-2} = \frac{1}{25}$

Calc: 5 $x^{\blacksquare}$ - 2 = S⇔D

1 $3^{-3} =$ ☐/☐ 2 $2^{-5} =$ ☐/☐

3 $5^{-3} =$ ☐/☐ 4 $4^{-4} =$ ☐/☐

5 $2^{-3} =$ ☐/☐ 6 $3^{-5} =$ ☐/☐

7 $10^{-3} =$ ☐/☐

8 $10^{-5} =$ ☐/☐

A negative power means '1 over', so rewrite as a fraction, then evaluate.

Rewrite as '1 over' Use your calculator

9 $2^{-4} = \frac{1}{2^4} =$ ☐/☐

Calc: 1 $\frac{\blacksquare}{\square}$ 2 $x^{\blacksquare}$ 4 =

10 $5^{-3} = \frac{1}{5^3} =$ ☐/☐

Calc: 1 $\frac{\blacksquare}{\square}$ 5 $x^{\blacksquare}$ 3 =

11 $3^{-3} = \frac{1}{\square} = \frac{1}{\square}$

12 $6^{-2} =$ ☐/☐ = ☐/☐

13 $10^{-3} =$ ☐/☐ = ☐/☐

14 $6^{-5} =$ ☐/☐ = ☐/☐

15 $2^{-8} =$ ☐/☐ = ☐/☐

16 $3^{-6} =$ ☐/☐ = ☐/☐

17 $4^{-5} =$ ☐/☐ = ☐/☐

Remove the fraction by writing these as negative indices. No calculator required!

Example $\frac{1}{2^5} = 2^{-5}$

18 $\frac{1}{2^3} =$ ☐☐

19 $\frac{1}{7^2} =$ ☐☐

20 $\frac{1}{5^8} =$ ☐☐

Use your power key to complete these tables.

21

Indice	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}
Number/ Fraction	☐	☐	☐	☐	☐	☐	☐/☐	☐/☐	☐/☐	☐/☐

22 What is the value of 3^0 in the above table? ☐

23

Indice	5^5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}
Number/ Fraction							☐/☐	☐/☐	☐/☐

24 What is the value of 5^0 in the above table? ☐

Use patterns for these tables. For the first table, use your calculator for 2^6, then divide by 2 each time. For the second table use your calculator for 10^4, then divide by 10 each time. Only use the power key once!

25

Indice	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Number/ Fraction											

÷ 2 ÷ 2 and so on, using the pattern to complete the boxes

26 What is the value of 2^0 in the above table? ☐

27 What is the value of any number to the power of zero? ☐

28

Indice	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Number/ Fraction								

÷ 10 ÷ 10 and so on, using the pattern to complete

Indices - Power Key Applications (Sheet 1)

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Use your power key to evaluate (find the value for) these.

Example $5^4 =$ 625

Calc: 5 $x^{\blacksquare}$ 4 =

1 $3^3 =$ 27 2 $2^5 =$ 32

3 $5^2 =$ 25 4 $7^3 =$ 343

5 $4^4 =$ 256 6 $3^5 =$ 243

7 $9^5 =$ 59 049

8 $15^4 =$ 50 625

9 $10^1 =$ 10 10 $10^2 =$ 100

11 $10^3 =$ 1 000

12 $10^4 =$ 10 000

13 $10^5 =$ 100 000

14 $10^6 =$ 1 000 000

15 $1^1 =$ 1 16 $1^2 =$ 1

17 $1^3 =$ 1 18 $1^4 =$ 1

Name the base and the index in these.

Example 5^4 — 5 (Base), 4 (Index)

Base is the larger number out front

Index is the small number (the power)

19 6^7 — 6 (Base), 7 (Index)

20 9^8 — 9 (Base), 8 (Index)

21 7^6 — 6 (Index), 7 (Base)

22 13^5 — 5 (Index), 13 (Base)

Given the base and index, write the indices and evaluate.

Example Base 9, index 4.

$9^4 = 6\,561$

23 Base 4, index 6.

$4^6 =$ 4 096

Calc: 4 $x^{\blacksquare}$ 6 =

24 Base 2, index 8.

$2^8 =$ 256

25 Index 4, base 5.

$5^4 =$ 625

26 Base 9, index 4.

$9^4 =$ 6 561

27 Index 5, base 10.

$10^5 =$ 100 000

?

Use guess, check and revise to find the correct index for these.

28 $2^{\boxed{7}} = 128$ 29 $3^{\boxed{3}} = 27$

30 $7^{\boxed{7}} = 823\,543$

31 $5^{\boxed{11}} = 48\,828\,125$

32 $13^{\boxed{4}} = 28\,561$

33 $10^{\boxed{7}} = 10\,000\,000$

34 $20^{\boxed{3}} = 8\,000$

35 $2^{\boxed{18}} = 262\,144$

36 $3^{\boxed{11}} = 177\,147$

?

Use guess, check and revise to find the correct base for these.

37 $\boxed{4}^3 = 64$ 38 $\boxed{3}^5 = 243$

39 $\boxed{2}^7 = 128$ 40 $\boxed{9}^3 = 729$

41 $\boxed{17}^4 = 83\,521$

42 $\boxed{3}^9 = 19\,683$

43 $\boxed{10}^4 = 10\,000$

44 $\boxed{20}^6 = 64\,000\,000$

45 $\boxed{1}^3 = 1$ 46 $\boxed{1}^8 = 1$

Blend the two skills by finding both the base and the index!

47 $2^{\boxed{8}} = 256 = \boxed{4}^4$

48 $9^{\boxed{8}} = 531\,441 = \boxed{3}^{12}$

49 $\boxed{8}^4 = 4\,096 = 2^{\boxed{12}}$

50 $\boxed{16}^5 = 1\,048\,576 = 2^{\boxed{20}}$

51 $4^{\boxed{12}} = 16\,777\,216 = \boxed{16}^6$

Use '<' or '>' to make these true. Hint: Point the '<' or '>' to the smallest number.

52 4^5 > 2^9

1 024 512

53 7^4 < 4^7

2 401 16 384

Indices - Power Key Applications (Sheet 2)

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Use your power key to evaluate these indices, answer these as a fraction.

Example $5^{-2} = \frac{1}{25}$

Calc: 5 $x^{\blacksquare}$ - 2 = S⇔D

1 $3^{-3} = \frac{1}{27}$

2 $2^{-5} = \frac{1}{32}$

3 $5^{-3} = \frac{1}{125}$

4 $4^{-4} = \frac{1}{256}$

5 $2^{-3} = \frac{1}{8}$

6 $3^{-5} = \frac{1}{243}$

7 $10^{-3} = \frac{1}{1\,000}$

8 $10^{-5} = \frac{1}{100\,000}$

A negative power means '1 over', so rewrite as a fraction, then evaluate.

Rewrite as '1 over'

Use your calculator

9 $2^{-4} = \frac{1}{2^4} = \frac{1}{16}$

Calc: 1 $\frac{\blacksquare}{\square}$ 2 $x^{\blacksquare}$ 4 =

10 $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

Calc: 1 $\frac{\blacksquare}{\square}$ 5 $x^{\blacksquare}$ 3 =

11 $3^{-3} = \frac{1}{3^3} = \frac{1}{81}$

12 $6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

13 $10^{-3} = \frac{1}{10^3} = \frac{1}{1\,000}$

14 $6^{-5} = \frac{1}{6^5} = \frac{1}{7\,776}$

15 $2^{-8} = \frac{1}{2^8} = \frac{1}{256}$

16 $3^{-6} = \frac{1}{3^6} = \frac{1}{729}$

17 $4^{-5} = \frac{1}{4^5} = \frac{1}{1\,024}$

Remove the fraction by writing these as negative indices. No calculator required!

Example $\frac{1}{2^5} = 2^{-5}$

18 $\frac{1}{2^3} = 2^{-3}$

19 $\frac{1}{7^2} = 7^{-2}$

20 $\frac{1}{5^8} = 5^{-8}$

Use your power key to complete these tables.

21

Indice	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}
Number/ Fraction	243	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$

22 What is the value of 3^0 in the above table? 1

23

Indice	5^5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}
Number/ Fraction	3 125	625	125	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$

24 What is the value of 5^0 in the above table? 1

Use patterns for these tables. For the first table, use your calculator for 2^6, then divide by 2 each time. For the second table use your calculator for 10^4, then divide by 10 each time. Only use the power key once!

25

Indice	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Number/ Fraction	64	32	16	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$

÷ 2 ÷ 2 and so on, using the pattern to complete the boxes

26 What is the value of 2^0 in the above table? 1

27 What is the value of any number to the power of zero? 1

28

Indice	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Number/ Fraction	10 000	1 000	100	10	1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1\,000}$

÷ 10 ÷ 10 and so on, using the pattern to complete

Indices - Power Key Applications (Homework Strip)

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Date Due:

Use your power key to find these.

1 $4^3 =$ 2 $2^4 =$

3 $5^3 =$ 4 $6^3 =$

5 $2^9 =$ 6 $3^4 =$

Write these indices

7 Base 2, index 5.

8 Base 4, index 3.

9 Index 6, base 2.

Use your power key to find these.

10 $3^{-4} =$ 11 $2^{-6} =$

12 $4^{-3} =$ 13 $2^{-4} =$

Rewrite with a negative power

14 $\frac{1}{2^4} =$

15 $\frac{1}{3^5} =$

Date Due:

Use your power key to find these.

1 $4^3 =$ 2 $2^4 =$

3 $5^3 =$ 4 $6^3 =$

5 $2^9 =$ 6 $3^4 =$

Write these indices

7 Base 2, index 5.

8 Base 4, index 3.

9 Index 6, base 2.

Use your power key to find these.

10 $3^{-4} =$ 11 $2^{-6} =$

12 $4^{-3} =$ 13 $2^{-4} =$

Rewrite with a negative power

14 $\frac{1}{2^4} =$

15 $\frac{1}{3^5} =$

Date Due:

Use your power key to find these.

1 $4^3 =$ 2 $2^4 =$

3 $5^3 =$ 4 $6^3 =$

5 $2^9 =$ 6 $3^4 =$

Write these indices

7 Base 2, index 5.

8 Base 4, index 3.

9 Index 6, base 2.

Use your power key to find these.

10 $3^{-4} =$ 11 $2^{-6} =$

12 $4^{-3} =$ 13 $2^{-4} =$

Rewrite with a negative power

14 $\frac{1}{2^4} =$

15 $\frac{1}{3^5} =$

Date Due:

Use your power key to find these.

1 $4^3 =$ 2 $2^4 =$

3 $5^3 =$ 4 $6^3 =$

5 $2^9 =$ 6 $3^4 =$

Write these indices

7 Base 2, index 5.

8 Base 4, index 3.

9 Index 6, base 2.

Use your power key to find these.

10 $3^{-4} =$ 11 $2^{-6} =$

12 $4^{-3} =$ 13 $2^{-4} =$

Rewrite with a negative power

14 $\frac{1}{2^4} =$

15 $\frac{1}{3^5} =$

Indices - Power Key Applications (Homework Strip)

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Date Due:

Use your power key to find these.

1 $4^3 = 64$

2 $2^4 = 16$

3 $5^3 = 125$

4 $6^3 = 216$

5 $2^9 = 512$

6 $3^4 = 81$

Write these indices

7 Base 2, index 5. 2^5

8 Base 4, index 3. 4^3

9 Index 6, base 2. 2^6

Use your power key to find these.

10 $3^{-4} = \frac{1}{81}$

11 $2^{-6} = \frac{1}{64}$

12 $4^{-3} = \frac{1}{64}$

13 $2^{-4} = \frac{1}{16}$

Rewrite with a negative power

14 $\frac{1}{2^4} = 2^{-4}$

15 $\frac{1}{3^5} = 3^{-5}$

Indices - Multiplying Indices

An indice is a number or letter (the base) with a power on it (the index). A power on a number means '×' itself that many times. So $3^4 = 3 \times 3 \times 3 \times 3$, or with letters $x^3 = x \times x \times x$. When you write indices in this way it is called 'Expanded form'.

Example Write 2^5 in expanded form.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

Example Write y^4 in expanded form.

$$y^4 = y \times y \times y \times y$$

This sheet starts with asking you to write expressions in expanded form. You can use expansion to multiply indices and to prove the rule that when you multiply 'like base' indices that you add the index.

Rule: When you multiply like base indices, you add the powers

$$a^m \times a^n = a^{m+n}$$

Here are two examples, applying the above rule:

Example Simplify $x^2 \times x^5$.

$$x^2 \times x^5 = x^7$$

2 + 5 = 7

Example Simplify $ax^2 \times a^5x^5$.

$$ax^2 \times a^5x^5 = a^6x^7$$

2 + 5 = 7

No power means that it has a power of '1'

1 + 5 = 6

A co-efficient is the number in front of the letter in an algebraic term. When you multiply terms, the co-efficient are multiplied:

Example Simplify $2x^2 \times 4x^5$.

$$2x^2 \times 4x^5 = 8x^7$$

2 × 4 = 8

2 + 5 = 7

Indices - Multiplying Indices (Sheet 1)

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Rewrite in expanded form.

Example $5^4 = 5 \times 5 \times 5 \times 5$

1 $3^5 = \square \times \square \times \square \times \square \times \square$

2 $15^3 =$

3 $9^4 =$

4 $1^6 =$

Now algebraic indices - same process

5 $y^4 = \square \times \square \times \square \times \square$

6 $a^5 =$

7 $k^6 =$

8 $b^2 =$

Use expansion to answer these in index form. No calculator.

Example $5^2 \times 5^4 = \overbrace{5 \times 5}^{5^2} \times \overbrace{5 \times 5 \times 5 \times 5}^{5^4} = 5^6$

9 $7^2 \times 7^2 = \square \times \square \times \square \times \square$

$7^2 \times 7^2 = \square^{\square}$

10 $3^3 \times 3^2 =$

$3^3 \times 3^2 = \square^{\square}$

11 $4^2 \times 4^4 =$

$4^2 \times 4^4 = \square^{\square}$

Now algebraic indices - same process

12 $x^2 \times x^3 =$

$x^2 \times x^3 = \square^{\square}$

13 $a \times a^3 =$

$a \times a^3 = \square^{\square}$

You can multiply indices by adding the indexes. Multiply these, leave answer in index form

Example $5^2 \times 5^4 = 5^{2+4} = 5^6$

14 $7^4 \times 7^2 = \square^{\square+\square} = \square^{\square}$

15 $9^3 \times 9^5 = \square^{\square+\square} = \square^{\square}$

No power - it's like there is a '1' there

16 $4^6 \times 4 = \square^{\square+\square} = \square^{\square}$

17 $5 \times 5^2 = \square^{\square+\square} = \square^{\square}$

18 $7 \times 7 = \square^{\square+\square} = \square^{\square}$

19 $3^9 \times 3^7 = \square^{\square+\square} = \square^{\square}$

Now algebraic indices - same process

20 $x^3 \times x^3 = \square^{\square+\square} = \square^{\square}$

21 $n^5 \times n = \square^{\square+\square} = \square^{\square}$

22 $a \times a^3 = \square^{\square+\square} = \square^{\square}$

23 $h^8 \times h^4 = \square^{\square+\square} = \square^{\square}$

Forget the middle step, answer only. Triples are just the same.

24 $5^8 \times 5^4 = \square^{\square}$

25 $7^3 \times 7^9 = \square^{\square}$

26 $7^3 \times 7^9 \times 7^2 = \square^{\square}$

27 $3 \times 3^6 \times 3^3 = \square^{\square}$

28 $e^4 \times e^3 \times e^5 = \square^{\square}$

29 $k \times k^5 \times k = \square^{\square}$

30 $h^2 \times h \times h^7 = \square^{\square}$

We now have unlike terms. At last, maths questions that have two correct answers! Simplify.

Example $a^2 \times b^2 \times a^4 \times b = a^6b^3$ *or* b^3a^6

31 $a^6 \times b =$ ____ *or* ____

32 $c^4 \times b^5 =$ ____ *or* ____

33 $k^9 \times t^3 =$ ____ *or* ____

Just one answer from now on

34 $x^3 \times a^5 \times x^2 \times a =$

35 $y^2 \times c^3 \times y^4 \times y^5 =$

36 $a^5 \times b^2 \times a \times b =$

37 $d \times d^2 \times c \times d^5 =$

Less '×' signs - but the same process

38 $bc \times b^2 =$ ____ *or* ____

39 $ay \times y^3 =$ ____ *or* ____

40 $g^3k \times k^3g^2 =$

41 $ab \times ab =$

42 $abc \times abc =$

43 $b^2ac^3 \times acb =$

44 $c^4a^2b^3 \times b^4c^3a^6 =$

45 $ab^3c \times a^2b \times ab^2 =$

Which of these is true?

46 $m^3 \times m^2 =$ ○ m^6 ○ m^5

47 $e^6 \times e =$ ○ e^6 ○ e^7 ○ e^0 ○ e

48 $a^2 \times a^5b =$ ○ $a^{10}b$ ○ a^7b^5 ○ a^2b^5 ○ a^7b

Indices - Multiplying Indices (Sheet 2)

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Coefficients are numbers in front of algebraic terms. Multiply the coefficients, but add the powers.

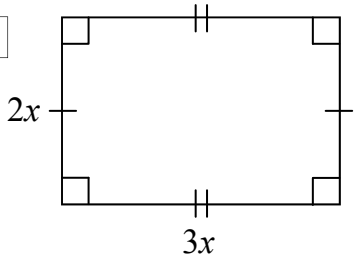
Example $2x \times 3x^2 = 6x^3$

Simplify these.

1. $2a^5 \times a^2 =$ _____
2. $c^2 \times 4c =$ _____
3. $a^3b^2 \times 3a^2 =$ _____
4. $5x^4y^2 \times xy =$ _____
5. $4n^3 \times 5n^2 =$ _____
6. $6j^7 \times 4j =$ _____
7. $2h^3 \times 9ah^2 =$ _____
8. $3s^2 \times 4sv^3 =$ _____
9. $2e^2h \times 8h^2e^4 =$ _____
10. $3x^2 \times 2x^3 \times 4y^2 =$ _____

The area of a rectangle is found by multiplying the two sides. Find the area of these.

11.

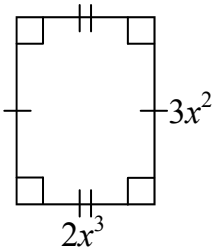


A = lb

= ____ × ____

A = ____ units^2

12.

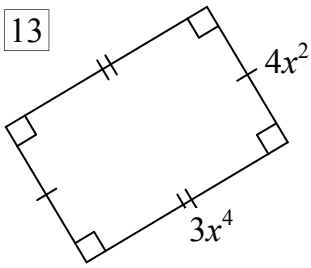


A = lb

= ____ × ____

A = ____ units^2

13.

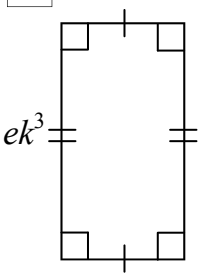


A = lb

= ____ × ____

A = ____ units^2

14.

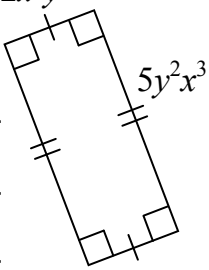


_____

_____

_____

15.



_____

_____

_____

When you square a term you multiply it by itself. Square these terms.

Example

$(3x)^2 = 3x \times 3x = 9x^2$

16. $(4y)^2 =$ ____ × ____ = ____
17. $(2a)^2 =$ ____ × ____ = ____
18. $(2a^2)^2 =$ ____ × ____ = ____
19. $(3a^3)^2 =$ ____ × ____ = ____
20. $(5a^2b)^2 =$ ____ × ____

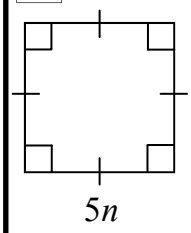
= ____

21. $(2x^3y^5)^2 =$ ____ × ____

= ____

The area of a square is found by squaring the sides. Find the area of these squares.

22.

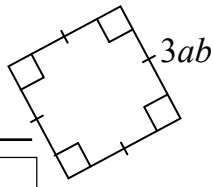


A = l^2

= ____ × ____

A = ____ units^2

23.

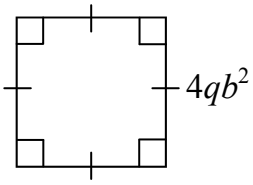


A = l^2

= ____ × ____

A = ____ units^2

24.



_____

_____

_____

Complete the two times tables below.

25.

×	a	a^2b	ab^2	a^2b^2
a				
$5a$				
$2ab$				

26.

×	y^3	$3x^2$	$2x$
$2x$			
y^3			
xy			
x^2y^2			
$3x^3y^2$			

Indices - Multiplying Indices (Sheet 1)

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Rewrite in expanded form.

Example $5^4 = 5 \times 5 \times 5 \times 5$

1 $3^5 = \square \times \square \times \square \times \square \times \square$

2 $15^3 = 15 \times 15 \times 15$

3 $9^4 = 9 \times 9 \times 9 \times 9$

4 $1^6 = 1 \times 1 \times 1 \times 1 \times 1 \times 1$

Now algebraic indices - same process

5 $y^4 = y \times y \times y \times y$

6 $a^5 = a \times a \times a \times a \times a$

7 $k^6 = k \times k \times k \times k \times k \times k$

8 $b^2 = b \times b$

Use expansion to answer these in index form. No calculator.

Example $5^2 \quad 5^4$

$5^2 \times 5^4 = 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$

9 $7^2 \times 7^2 = 7 \times 7 \times 7 \times 7$

$7^2 \times 7^2 = 7^{4}$

10 $3^3 \times 3^2 = 3 \times 3 \times 3 \times 3 \times 3$

$3^3 \times 3^2 = 3^{5}$

11 $4^2 \times 4^4 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$

$4^2 \times 4^4 = 4^{6}$

Now algebraic indices - same process

12 $x^2 \times x^3 = x \times x \times x \times x \times x$

$x^2 \times x^3 = x^{5}$

13 $a \times a^3 = a \times a \times a \times a$

$a \times a^3 = a^{4}$

You can multiply indices by adding the indexes. Multiply these, leave answer in index form

Example $5^2 \times 5^4 = 5^{2+4} = 5^6$

14 $7^4 \times 7^2 = 7^{4+2} = 7^{6}$

15 $9^3 \times 9^5 = 9^{3+5} = 9^{8}$

No power - it's like there is a '1' there

16 $4^6 \times 4 = 4^{6+1} = 4^{7}$

17 $5 \times 5^2 = 5^{1+2} = 5^{3}$

18 $7 \times 7 = 7^{1+1} = 7^{2}$

19 $3^9 \times 3^7 = 3^{9+7} = 3^{16}$

Now algebraic indices - same process

20 $x^3 \times x^3 = x^{3+3} = x^{6}$

21 $n^5 \times n = n^{5+1} = n^{6}$

22 $a \times a^3 = a^{1+3} = a^{4}$

23 $h^8 \times h^4 = h^{8+4} = h^{12}$

Forget the middle step, answer only. Triples are just the same.

24 $5^8 \times 5^4 = 5^{12}$

25 $7^3 \times 7^9 = 7^{12}$

26 $7^3 \times 7^9 \times 7^2 = 7^{14}$

27 $3 \times 3^6 \times 3^3 = 3^{10}$

28 $e^4 \times e^3 \times e^5 = e^{12}$

29 $k \times k^5 \times k = k^{7}$

30 $h^2 \times h \times h^7 = h^{10}$

We now have unlike terms. At last, maths questions that have two correct answers! Simplify.

Example

$a^2 \times b^2 \times a^4 \times b = a^6b^3$ or b^3a^6

31 $a^6 \times b = a^6b$ or ba^6

32 $c^4 \times b^5 = c^4b^5$ or b^5c^4

33 $k^9 \times t^3 = k^9t^3$ or t^3k^9

Just one answer from now on

34 $x^3 \times a^5 \times x^2 \times a = x^5a^6$

35 $y^2 \times c^3 \times y^4 \times y^5 = y^{11}c^3$

36 $a^5 \times b^2 \times a \times b = a^6b^3$

37 $d \times d^2 \times c \times d^5 = d^8c$

Less '×' signs - but the same process

38 $bc \times b^2 = cb^3$ or b^3c

39 $ay \times y^3 = y^4a$ or ay^4

40 $g^3k \times k^3g^2 = g^5k^4$

41 $ab \times ab = a^2b^2$

42 $abc \times abc = a^2b^2c^2$

43 $b^2ac^3 \times acb = a^2b^3c^4$

44 $c^4a^2b^3 \times b^4c^3a^6 = a^8b^7c^7$

45 $ab^3c \times a^2b \times ab^2 = a^4b^6c$

Which of these is true?

46 $m^3 \times m^2 =$ ○ m^6 ● m^5

47 $e^6 \times e =$ ○ e^6 ● e^7 ○ e^0 ○ e

48 $a^2 \times a^5b =$ ○ $a^{10}b$ ○ a^7b^5 ○ a^2b^5 ● a^7b

Indices - Multiplying Indices (Sheet 2)

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Coefficients are numbers in front of algebraic terms. Multiply the coefficients, but add the powers.

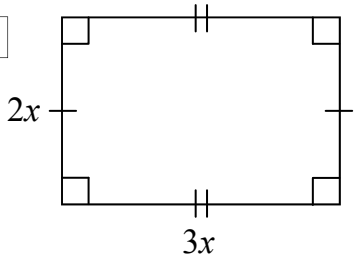
Example $2x \times 3x^2 = 6x^3$

Simplify these.

1. $2a^5 \times a^2 = 2a^7$
2. $c^2 \times 4c = 4c^3$
3. $a^3b^2 \times 3a^2 = 3a^5b^2$
4. $5x^4y^2 \times xy = 5x^5y^3$
5. $4n^3 \times 5n^2 = 20n^5$
6. $6j^7 \times 4j = 24j^8$
7. $2h^3 \times 9ah^2 = 18ah^5$
8. $3s^2 \times 4sv^3 = 12s^3v^3$
9. $2e^2h \times 8h^2e^4 = 16e^6h^3$
10. $3x^2 \times 2x^3 \times 4y^2 = 24x^5y^2$

The area of a rectangle is found by multiplying the two sides. Find the area of these.

11.

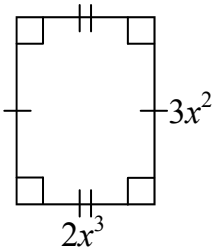


$A = lb$

$= 2x \times 3x$

$A = 6x^2$ units^2

12.

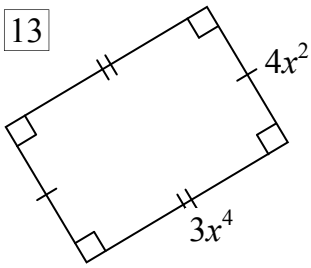


$A = lb$

$= 2x^3 \times 3x^2$

$A = 6x^5$ units^2

13.

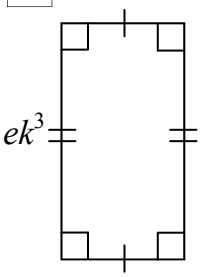


$A = lb$

$= 3x^4 \times 4x^2$

$A = 12x^6$ units^2

14.

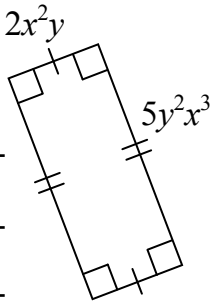


$A = lb$

$= 7e^2 \times ek^3$

$A = 7e^3k^3$ units^2

15.



$A = lb$

$= 2x^2y \times 5y^2x^3$

$A = 10x^5y^3$ units^2

When you square a term you multiply it by itself. Square these terms.

Example

$(3x)^2 = 3x \times 3x = 9x^2$

16. $(4y)^2 = 4y \times 4y = 16y^2$
17. $(2a)^2 = 2a \times 2a = 4a^2$
18. $(2a^2)^2 = 2a^2 \times 2a^2 = 4a^4$
19. $(3a^3)^2 = 3a^3 \times 3a^3 = 9a^6$
20. $(5a^2b)^2 = 5a^2b \times 5a^2b$

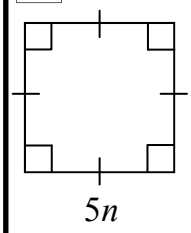
$= 25a^4b^2$

21. $(2x^3y^5)^2 = 2x^3y^5 \times 2x^3y^5$

$= 4x^6y^{10}$

The area of a square is found by squaring the sides. Find the area of these squares.

22.

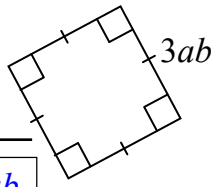


$A = l^2$

$= 5n \times 5n$

$A = 25n^2$ units^2

23.

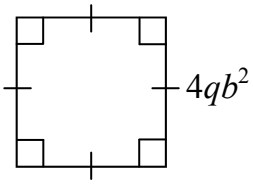


$A = l^2$

$= 3ab \times 3ab$

$A = 9a^2b^2$ units^2

24.



$A = l^2$

$= 4qb^2 \times 4qb^2$

$A = 16q^2b^4$ units^2

Complete the two times tables below.

25.

×	a	a^2b	ab^2	a^2b^2
a	a^2	a^3b	a^2b^2	a^3b^2
$5a$	$5a^2$	$5a^3b$	$5a^2b^2$	$5a^3b^2$
$2ab$	$2a^2b$	$2a^3b^2$	$2a^2b^3$	$2a^3b^3$

26.

×	y^3	$3x^2$	$2x$
$2x$	$2xy^3$	$6x^3$	$4x^2$
y^3	y^6	$3x^2y^3$	$2xy^3$
xy	xy^4	$3x^3y$	$2x^2y$
x^2y^2	x^2y^5	$3x^4y^2$	$2x^3y^2$
$3x^3y^2$	$3x^3y^5$	$9x^5y^2$	$6x^4y^2$

Indices - Multiplying Indices (Homework Strip)

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Date Due: _____

Write these in expanded form

1. $4^2 =$ ___ × ___
2. $7^5 =$ _____
3. $h^4 =$ _____
4. $b^3 =$ _____

Simplify these

5. $a^2 \times a^4 =$ ___
6. $x^3 \times x^5 =$ ___
7. $y^3 \times y^2 \times y^7 =$ ___
8. $a^3 \times b^2 \times a^5 \times b^4 =$ _____
9. $e^3 \times v^3 \times e \times v^2 =$ _____
10. $w^2a^3 \times w^5a =$ _____
11. $g^3te^2 \times t^2g =$ _____
12. $3a^2b^3 \times a^5b =$ _____
13. $4xy^3 \times 5x^2y^2 =$ _____

Date Due: _____

Write these in expanded form

1. $4^2 =$ ___ × ___
2. $7^5 =$ _____
3. $h^4 =$ _____
4. $b^3 =$ _____

Simplify these

5. $a^2 \times a^4 =$ ___
6. $x^3 \times x^5 =$ ___
7. $y^3 \times y^2 \times y^7 =$ ___
8. $a^3 \times b^2 \times a^5 \times b^4 =$ _____
9. $e^3 \times v^3 \times e \times v^2 =$ _____
10. $w^2a^3 \times w^5a =$ _____
11. $g^3te^2 \times t^2g =$ _____
12. $3a^2b^3 \times a^5b =$ _____
13. $4xy^3 \times 5x^2y^2 =$ _____

Date Due: _____

Write these in expanded form

1. $4^2 =$ ___ × ___
2. $7^5 =$ _____
3. $h^4 =$ _____
4. $b^3 =$ _____

Simplify these

5. $a^2 \times a^4 =$ ___
6. $x^3 \times x^5 =$ ___
7. $y^3 \times y^2 \times y^7 =$ ___
8. $a^3 \times b^2 \times a^5 \times b^4 =$ _____
9. $e^3 \times v^3 \times e \times v^2 =$ _____
10. $w^2a^3 \times w^5a =$ _____
11. $g^3te^2 \times t^2g =$ _____
12. $3a^2b^3 \times a^5b =$ _____
13. $4xy^3 \times 5x^2y^2 =$ _____

Date Due: _____

Write these in expanded form

1. $4^2 =$ ___ × ___
2. $7^5 =$ _____
3. $h^4 =$ _____
4. $b^3 =$ _____

Simplify these

5. $a^2 \times a^4 =$ ___
6. $x^3 \times x^5 =$ ___
7. $y^3 \times y^2 \times y^7 =$ ___
8. $a^3 \times b^2 \times a^5 \times b^4 =$ _____
9. $e^3 \times v^3 \times e \times v^2 =$ _____
10. $w^2a^3 \times w^5a =$ _____
11. $g^3te^2 \times t^2g =$ _____
12. $3a^2b^3 \times a^5b =$ _____
13. $4xy^3 \times 5x^2y^2 =$ _____

Indices - Multiplying Indices (Homework Strip)

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Date Due:

Write these in expanded form

1 $4^2 =$ 4 × 4

2 $7^5 = 7 \times 7 \times 7 \times 7 \times 7$

3 $h^4 = h \times h \times h \times h$

4 $b^3 = b \times b \times b$

Simplify these

5 $a^2 \times a^4 = a^6$

6 $x^3 \times x^5 = x^8$

7 $y^3 \times y^2 \times y^7 = y^{12}$

8 $a^3 \times b^2 \times a^5 \times b^4 = a^8b^6$

9 $e^3 \times v^3 \times e \times v^2 = e^4v^5$

10 $w^2a^3 \times w^5a = w^7a^4$

11 $g^3te^2 \times t^2g = g^4t^3e^2$

12 $3a^2b^3 \times a^5b = 3a^7b^4$

13 $4xy^3 \times 5x^2y^2 = 20x^3y^5$

Indices - Dividing Indices

An indice is a number or letter (the base) with a power on it (the index). A power on a number means '×' itself that many times. So $3^4 = 3 \times 3 \times 3 \times 3$, or with letters $x^3 = x \times x \times x$. When you write indices in this way it is called 'Expanded form'.

Example Write 2^5 in expanded form.

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

Example Write y^4 in expanded form.

$$y^4 = y \times y \times y \times y$$

This sheet starts with asking you to write expressions in expanded form. You can use expansion to divide indices and to prove the rule that when you divide 'like base' indices that you subtract the index.

Rule: When you divide like base indices, you subtract the powers

$$a^m \times a^n = a^{m-n}$$

Here are is an example, applying the above rule:

Example Simplify $x^5 \div x^2$.

$$x^5 \div x^2 = x^3$$

5 - 2 = 3

Students often experience difficulty with division. This sheet goes slowly has plenty of examples and increases difficulty, step-by-step. There are too many steps to outline them on this page. So hook in!

Indices - Dividing Indices (Sheet 1)

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Rewrite in expanded form.

Example $5^4 = 5 \times 5 \times 5 \times 5$

1 $7^5 = \square \times \square \times \square \times \square \times \square$

2 $4^6 =$

3 $3^4 =$

4 $2^3 =$

Now algebraic indices - same process

5 $k^3 = \square \times \square \times \square$

6 $b^4 =$

7 $m^5 =$

8 $c^2 =$

Use expansion and cancellation to answer these in index form.

Example

$5^5 \div 5^2 = \dfrac{\cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}{\cancel{5} \times \cancel{5}}$

Cancel bottom with top

$= 5 \times 5 \times 5 = 5^3$

9 $3^6 \div 3^3 =$

10 $\dfrac{y^6}{y^2} =$

11 $a^3 \div a =$

Use expansion and cancellation to answer these, write a '1' on the top line when fully cancelled

Example

$5^2 \div 5^5 = \dfrac{\cancel{5} \times \cancel{5}^{\,1}}{\cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}$

Cancel bottom with top

$= \dfrac{1}{5 \times 5 \times 5} = \dfrac{1}{5^3}$

12 $7^3 \div 7^5 =$

13 $k \div k^5 =$

14 $\dfrac{n^2}{n^4} =$

You can divide indices by subtracting the indexes. Divide these, bigger power - smaller

Example $5^7 \div 5^4 = 5^{7-4} = 5^3$

15 $3^5 \div 3^2 = \square^{\square - \square} = \square^{\square}$

16 $4^9 \div 4^4 = \square^{\square - \square} = \square^{\square}$

No power - it's like there is a '1' there

17 $8^6 \div 8 = \square^{\square - \square} = \square^{\square}$

18 $\dfrac{3^5}{3} = \square^{\square - \square} = \square^{\square}$

19 $6^8 \div 6 = \square^{\square - \square} = \square^{\square}$

Now algebraic indices - same process

20 $x^3 \div x = \square^{\square - \square} = \square^{\square}$

21 $\dfrac{n^7}{n^3} = \square^{\square - \square} = \square^{\square}$

22 $b^8 \div b^6 = \square^{\square - \square} = \square^{\square}$

23 $\dfrac{h^9}{h^4} = \square^{\square - \square} = \square^{\square}$

These will require a '1' up top, as the denominator is larger. Bigger power - smaller power.

Example

$\dfrac{h^4}{h^9} = \dfrac{1}{h^{9-4}} = \dfrac{1}{h^5}$

24 $\dfrac{a^3}{a^7} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

25 $5^5 \div 5^7 = \dfrac{5^5}{5^7} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

26 $\dfrac{n}{n^6} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

27 $\dfrac{e^3}{e^6} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

28 $b^2 \div b^9 = \dfrac{b^2}{b^9} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

29 $\dfrac{q^4}{q^5} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

30 $c \div c^4 = \dfrac{c}{c^4} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

31 $\dfrac{y^6}{y^8} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

Indices - Dividing Indices (Sheet 2)

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Simplify these, larger power minus the smaller power. Use '1 over' when the larger power is the denominator (bottom).

Examples

$\frac{h^{\cancel{9}\,5}}{\cancel{h^4}} = h^5$ $\qquad \frac{\cancel{h^3}}{h^{\cancel{7}\,4}} = \frac{1}{h^4}$

1 $\frac{n^3}{n^5} =$ $\qquad$ 2 $\frac{n^5}{n^3} =$

3 $\frac{a^7}{a^2} =$ $\qquad$ 4 $\frac{a^2}{a^7} =$

5 $\frac{x}{x^4} =$ $\qquad$ 6 $\frac{x^4}{x} =$

7 $\frac{y^5}{y^4} =$ $\qquad$ 8 $\frac{y^4}{y^5} =$

9 $\frac{b^2}{b^9} =$ $\qquad$ 10 $\frac{b^9}{b^2} =$

11 $\frac{c^5}{c^8} =$ $\qquad$ 12 $\frac{k^2}{k^7} =$

13 $\frac{p^9}{p^8} =$ $\qquad$ 14 $\frac{u}{u^2} =$

Two different bases now

Examples

$\frac{\cancel{b^2}\,\cancel{e^2}}{b^{\cancel{3}}\,e^{\cancel{4}\,2}} = \frac{1}{be^2}$ $\qquad \frac{b^{5\,4}\,\cancel{e^2}}{\cancel{be^3}} = b^4e^2$

15 $\frac{ab^9}{b^7} =$ $\qquad$ 16 $\frac{b^9}{ab^7} =$

17 $\frac{x^2y^9}{y^7} =$ $\qquad$ 18 $\frac{x^2y^9}{xy^7} =$

19 $\frac{a^2n^9}{a^2n^8} =$ $\qquad$ 20 $\frac{a^5n^3}{a^2n^9} =$

21 $\frac{bc}{b^2c} =$ $\qquad$ 22 $\frac{c^5b^2}{b^2c^3} =$

23 $\frac{s^4v}{v^2s^2} =$ $\qquad$ 24 $\frac{h^5t^9}{ht^8} =$

25 $\frac{xy^2}{x^2y} =$ $\qquad$ 26 $\frac{c^2f^2}{cf^7} =$

27 $\frac{a^4e}{e^2a^3} =$ $\qquad$ 28 $\frac{u^5v^9}{u^2v^3} =$

Simplify these fractions by dividing by the HCF, or use a calculator.

Examples

Divide both numbers by 5

$\frac{2}{4} = \frac{1}{2}$ $\qquad \frac{10}{15} = \frac{2}{3}$

Divide both numbers by 2

29 $\frac{4}{6} =$ $\qquad$ 30 $\frac{8}{10} =$

31 $\frac{6}{9} =$ $\qquad$ 32 $\frac{2}{8} =$

33 $\frac{10}{12} =$ $\qquad$ 34 $\frac{5}{10} =$

35 $\frac{9}{12} =$ $\qquad$ 36 $\frac{8}{12} =$

Simplify the improper fractions the result will be whole numbers.

Example $\frac{10}{5} = 2$

37 $\frac{10}{2} =$ $\qquad$ 38 $\frac{6}{3} =$

39 $\frac{12}{4} =$ $\qquad$ 40 $\frac{9}{3} =$

41 $\frac{15}{5} =$ $\qquad$ 42 $\frac{12}{3} =$

43 $\frac{8}{2} =$ $\qquad$ 44 $\frac{14}{7} =$

This time the fractions result remains an improper fraction. Simplify these.

Example

Divide both numbers by 5

$\frac{15}{10} = \frac{3}{2}$

or

Calc: 15 ■/□ 10 =

If your calculator says '1½' after trying the above, Press: Shift, Mode, Down arrow, 2.

45 $\frac{12}{8} =$ $\qquad$ 46 $\frac{8}{6} =$

47 $\frac{9}{6} =$ $\qquad$ 48 $\frac{15}{9} =$

Now its numbers and letters. Simplify these.

49 $\frac{6x^2}{x} =$ $\qquad$ 50 $\frac{x^2}{6x} =$

51 $\frac{x}{6x^2} =$ $\qquad$ 52 $\frac{3x}{6x^2} =$

53 $\frac{6x^2}{3x} =$ $\qquad$ 54 $\frac{6x^2}{3x^2} =$

55 $\frac{9x^2}{6x} =$ $\qquad$ 56 $\frac{9x}{6x^2} =$

Rewrite as fractions and then simplify.

Example

$5a \div 15a^2 = \frac{5a}{15a^2} = \frac{1}{3a}$

57 $2y \div 8y^2 =$ $\qquad =$

58 $8y^3 \div 6y =$ $\qquad =$

59 $9y^2 \div 12y^5 =$ $\qquad =$

Indices - Dividing Indices (Sheet 1)

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Rewrite in expanded form.

Example $5^4 = 5 \times 5 \times 5 \times 5$

1 $7^5 = 7 \times 7 \times 7 \times 7 \times 7$

2 $4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$

3 $3^4 = 3 \times 3 \times 3 \times 3$

4 $2^3 = 2 \times 2 \times 2$

Now algebraic indices - same process

5 $k^3 = k \times k \times k$

6 $b^4 = b \times b \times b \times b$

7 $m^5 = m \times m \times m \times m \times m$

8 $c^2 = c \times c$

Use expansion and cancellation to answer these in index form.

Example

$5^5 \div 5^2 = \dfrac{\cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}{\cancel{5} \times \cancel{5}}$

Cancel bottom with top

$= 5 \times 5 \times 5 = 5^3$

9 $3^6 \div 3^3 = \dfrac{\cancel{3} \times \cancel{3} \times \cancel{3} \times 3 \times 3 \times 3}{\cancel{3} \times \cancel{3} \times \cancel{3}}$

$= 3 \times 3 \times 3 = 3^3$

10 $\dfrac{y^6}{y^2} = \dfrac{\cancel{y} \times \cancel{y} \times y \times y \times y \times y}{\cancel{y} \times \cancel{y}}$

$= y \times y \times y \times y = y^4$

11 $a^3 \div a = \dfrac{\cancel{a} \times a \times a}{\cancel{a}}$

$= a \times a = a^2$

Use expansion and cancellation to answer these, write a '1' on the top line when fully cancelled

Example

$5^2 \div 5^5 = \dfrac{\cancel{5} \times \cancel{5}^{\,1}}{\cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}$

Cancel bottom with top

$= \dfrac{1}{5 \times 5 \times 5} = \dfrac{1}{5^3}$

12 $7^3 \div 7^5 = \dfrac{\cancel{7} \times \cancel{7} \times \cancel{7}}{\cancel{7} \times \cancel{7} \times \cancel{7} \times 7 \times 7}$

$= \dfrac{1}{7 \times 7} = \dfrac{1}{7^2}$

13 $k \div k^5 = \dfrac{\cancel{k}}{\cancel{k} \times k \times k \times k \times k}$

$= \dfrac{1}{k \times k \times k \times k} = \dfrac{1}{k^4}$

14 $\dfrac{n^2}{n^4} = \dfrac{\cancel{n} \times \cancel{n}}{\cancel{n} \times \cancel{n} \times n \times n}$

$= \dfrac{1}{n \times n} = \dfrac{1}{n^2}$

You can divide indices by subtracting the indexes. Divide these, bigger power - smaller

Example $5^7 \div 5^4 = 5^{7-4} = 5^3$

15 $3^5 \div 3^2 = 3^{5-2} = 3^3$

16 $4^9 \div 4^4 = 4^{9-4} = 4^5$

No power - it's like there is a '1' there

17 $8^6 \div 8 = 8^{6-1} = 8^5$

18 $\dfrac{3^5}{3} = 3^{5-1} = 3^4$

19 $6^8 \div 6 = 6^{8-1} = 6^7$

Now algebraic indices - same process

20 $x^3 \div x = x^{3-1} = x^2$

21 $\dfrac{n^7}{n^3} = n^{7-3} = n^4$

22 $b^8 \div b^6 = b^{8-6} = b^2$

23 $\dfrac{h^9}{h^4} = h^{9-4} = h^5$

These will require a '1' up top, as the denominator is larger. Bigger power - smaller power.

Example

$\dfrac{h^4}{h^9} = \dfrac{1}{h^{9-4}} = \dfrac{1}{h^5}$

24 $\dfrac{a^3}{a^7} = \dfrac{1}{a^{7-3}} = \dfrac{1}{a^4}$

25 $5^5 \div 5^7 = \dfrac{5^5}{5^7} = \dfrac{1}{5^{7-5}} = \dfrac{1}{5^2}$

26 $\dfrac{n}{n^6} = \dfrac{1}{n^{6-1}} = \dfrac{1}{n^5}$

27 $\dfrac{e^3}{e^6} = \dfrac{1}{e^{6-3}} = \dfrac{1}{e^3}$

28 $b^2 \div b^9 = \dfrac{b^2}{b^9} = \dfrac{1}{b^{9-2}} = \dfrac{1}{b^7}$

29 $\dfrac{q^4}{q^5} = \dfrac{1}{q^{5-4}} = \dfrac{1}{q}$

30 $c \div c^4 = \dfrac{c}{c^4} = \dfrac{1}{c^{4-1}} = \dfrac{1}{c^3}$

31 $\dfrac{y^6}{y^8} = \dfrac{1}{y^{8-6}} = \dfrac{1}{y^2}$

Indices - Dividing Indices (Sheet 2)

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Simplify these, larger power minus the smaller power. Use '1 over' when the larger power is the denominator (bottom).

Examples

$\frac{h^{9}}{h^{4}} = h^{5}$ $\qquad \frac{h^{3}}{h^{7}} = \frac{1}{h^{4}}$

1 $\frac{n^3}{n^5} = \frac{1}{n^2}$ $\qquad$ 2 $\frac{n^5}{n^3} = n^2$

3 $\frac{a^7}{a^2} = a^5$ $\qquad$ 4 $\frac{a^2}{a^7} = \frac{1}{a^5}$

5 $\frac{x}{x^4} = \frac{1}{x^3}$ $\qquad$ 6 $\frac{x^4}{x} = x^3$

7 $\frac{y^5}{y^4} = y$ $\qquad$ 8 $\frac{y^4}{y^5} = \frac{1}{y}$

9 $\frac{b^2}{b^9} = \frac{1}{b^7}$ $\qquad$ 10 $\frac{b^9}{b^2} = b^7$

11 $\frac{c^5}{c^8} = \frac{1}{c^3}$ $\qquad$ 12 $\frac{k^2}{k^7} = \frac{1}{k^5}$

13 $\frac{p^9}{p^8} = p$ $\qquad$ 14 $\frac{u}{u^2} = \frac{1}{u}$

Two different bases now

Examples

$\frac{b^{2}e^{2}}{b^{3}e^{2}} = \frac{1}{be^2}$ $\qquad \frac{b^{4}e^{2}}{b^{3}} = b^4e^2$

15 $\frac{ab^9}{b^7} = ab^2$ $\qquad$ 16 $\frac{b^9}{ab^7} = \frac{b^2}{a}$

17 $\frac{x^2y^9}{y^7} = x^2y^2$ $\qquad$ 18 $\frac{x^2y^9}{xy^7} = xy^2$

19 $\frac{a^2n^9}{a^2n^8} = n$ $\qquad$ 20 $\frac{a^5n^3}{a^2n^9} = \frac{a^2}{n^6}$

21 $\frac{bc}{b^2c} = \frac{1}{b}$ $\qquad$ 22 $\frac{c^5b^2}{b^2c^3} = c^2$

23 $\frac{s^4v}{v^2s^2} = \frac{s^2}{v}$ $\qquad$ 24 $\frac{h^5t^9}{ht^8} = h^4t$

25 $\frac{xy^2}{x^2y} = \frac{y}{x}$ $\qquad$ 26 $\frac{c^2f^2}{cf^7} = \frac{c}{f^5}$

27 $\frac{a^4e}{e^2a^3} = \frac{a}{e}$ $\qquad$ 28 $\frac{u^5v^9}{u^2v^3} = u^3v^6$

Simplify these fractions by dividing by the HCF, or use a calculator.

Examples

Divide both numbers by 5

$\frac{10}{15} = \frac{2}{3}$

$\frac{2}{4} = \frac{1}{2}$

Divide both numbers by 2

29 $\frac{4}{6} = \frac{2}{3}$ $\qquad$ 30 $\frac{8}{10} = \frac{4}{5}$

31 $\frac{6}{9} = \frac{2}{3}$ $\qquad$ 32 $\frac{2}{8} = \frac{1}{4}$

33 $\frac{10}{12} = \frac{5}{6}$ $\qquad$ 34 $\frac{5}{10} = \frac{1}{2}$

35 $\frac{9}{12} = \frac{3}{4}$ $\qquad$ 36 $\frac{8}{12} = \frac{2}{3}$

Simplify the improper fractions the result will be whole numbers.

Example $\frac{10}{5} = 2$

37 $\frac{10}{2} = 5$ $\qquad$ 38 $\frac{6}{3} = 2$

39 $\frac{12}{4} = 3$ $\qquad$ 40 $\frac{9}{3} = 3$

41 $\frac{15}{5} = 3$ $\qquad$ 42 $\frac{12}{3} = 4$

43 $\frac{8}{2} = 4$ $\qquad$ 44 $\frac{14}{7} = 2$

This time the fractions result remains an improper fraction. Simplify these.

Example

Divide both numbers by 5

$\frac{15}{10} = \frac{3}{2}$

or

Calc: 15 ▪/□ 10 =

If your calculator says '1½' after trying the above, Press: Shift, Mode, Down arrow, 2.

45 $\frac{12}{8} = \frac{3}{2}$ $\qquad$ 46 $\frac{8}{6} = \frac{4}{3}$

47 $\frac{9}{6} = \frac{3}{2}$ $\qquad$ 48 $\frac{15}{9} = \frac{5}{3}$

Now its numbers and letters. Simplify these.

49 $\frac{6x^2}{x} = 6x$ $\qquad$ 50 $\frac{x^2}{6x} = \frac{x}{6}$

51 $\frac{x}{6x^2} = \frac{1}{6x}$ $\qquad$ 52 $\frac{3x}{6x^2} = \frac{1}{2x}$

53 $\frac{6x^2}{3x} = 2x$ $\qquad$ 54 $\frac{6x^2}{3x^2} = 2$

55 $\frac{9x^2}{6x} = \frac{3x}{2}$ $\qquad$ 56 $\frac{9x}{6x^2} = \frac{3}{2x}$

Rewrite as fractions and then simplify.

Example

$5a \div 15a^2 = \frac{5a}{15a^2} = \frac{1}{3a}$

57 $2y \div 8y^2 = \frac{2y}{8y^2} = \frac{1}{4y}$

58 $8y^3 \div 6y = \frac{8y^3}{6y} = \frac{4y^2}{3}$

59 $9y^2 \div 12y^5 = \frac{9y^2}{12y^5} = \frac{3}{4y^3}$

Indices - Dividing Indices (Homework Strip)

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Date Due: _____

Simplify these

1 $x^5 \div x^2 =$ ____

2 $y^{10} \div y^5 =$ ____

3 $\frac{y^5}{y^4} =$ ____

4 $\frac{y^4}{y^5} =$ ____

5 $\frac{n}{n^6} =$ ____

6 $\frac{n^6}{n^2} =$ ____

7 $\frac{bn}{b^2} =$ ____

8 $\frac{b^6n^6}{n^2} =$ ____

9 $\frac{z^2y^3}{z^4y^2} =$ ____

10 $\frac{zy}{z^2y} =$ ____

11 $\frac{6y^3}{2} =$ ____

12 $\frac{2}{6y^3} =$ ____

13 $\frac{6y^3}{2y} =$ ____

14 $\frac{2y}{6y^3} =$ ____

15 $\frac{6y^3}{2y^3} =$ ____

16 $\frac{6y^4}{2y^3} =$ ____

Date Due: _____

Simplify these

1 $x^5 \div x^2 =$ ____

2 $y^{10} \div y^5 =$ ____

3 $\frac{y^5}{y^4} =$ ____

4 $\frac{y^4}{y^5} =$ ____

5 $\frac{n}{n^6} =$ ____

6 $\frac{n^6}{n^2} =$ ____

7 $\frac{bn}{b^2} =$ ____

8 $\frac{b^6n^6}{n^2} =$ ____

9 $\frac{z^2y^3}{z^4y^2} =$ ____

10 $\frac{zy}{z^2y} =$ ____

11 $\frac{6y^3}{2} =$ ____

12 $\frac{2}{6y^3} =$ ____

13 $\frac{6y^3}{2y} =$ ____

14 $\frac{2y}{6y^3} =$ ____

15 $\frac{6y^3}{2y^3} =$ ____

16 $\frac{6y^4}{2y^3} =$ ____

Date Due: _____

Simplify these

1 $x^5 \div x^2 =$ ____

2 $y^{10} \div y^5 =$ ____

3 $\frac{y^5}{y^4} =$ ____

4 $\frac{y^4}{y^5} =$ ____

5 $\frac{n}{n^6} =$ ____

6 $\frac{n^6}{n^2} =$ ____

7 $\frac{bn}{b^2} =$ ____

8 $\frac{b^6n^6}{n^2} =$ ____

9 $\frac{z^2y^3}{z^4y^2} =$ ____

10 $\frac{zy}{z^2y} =$ ____

11 $\frac{6y^3}{2} =$ ____

12 $\frac{2}{6y^3} =$ ____

13 $\frac{6y^3}{2y} =$ ____

14 $\frac{2y}{6y^3} =$ ____

15 $\frac{6y^3}{2y^3} =$ ____

16 $\frac{6y^4}{2y^3} =$ ____

Date Due: _____

Simplify these

1 $x^5 \div x^2 =$ ____

2 $y^{10} \div y^5 =$ ____

3 $\frac{y^5}{y^4} =$ ____

4 $\frac{y^4}{y^5} =$ ____

5 $\frac{n}{n^6} =$ ____

6 $\frac{n^6}{n^2} =$ ____

7 $\frac{bn}{b^2} =$ ____

8 $\frac{b^6n^6}{n^2} =$ ____

9 $\frac{z^2y^3}{z^4y^2} =$ ____

10 $\frac{zy}{z^2y} =$ ____

11 $\frac{6y^3}{2} =$ ____

12 $\frac{2}{6y^3} =$ ____

13 $\frac{6y^3}{2y} =$ ____

14 $\frac{2y}{6y^3} =$ ____

15 $\frac{6y^3}{2y^3} =$ ____

16 $\frac{6y^4}{2y^3} =$ ____

Indices - Dividing Indices (Homework Strip)

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Date Due:

Simplify these

1 $x^5 \div x^2 = x^3$

2 $y^{10} \div y^5 = y^5$

3 $\frac{y^5}{y^4} = y$

4 $\frac{y^4}{y^5} = \frac{1}{y}$

5 $\frac{n}{n^6} = \frac{1}{n^5}$

6 $\frac{n^6}{n^2} = n^4$

7 $\frac{bn}{b^2} = \frac{n}{b}$

8 $\frac{b^6n^6}{n^2} = b^6n^4$

9 $\frac{z^2y^3}{z^4y^2} = \frac{y}{z^2}$

10 $\frac{zy}{z^2y} = \frac{1}{z}$

11 $\frac{6y^3}{2} = 3y^3$

12 $\frac{2}{6y^3} = \frac{1}{3y^3}$

13 $\frac{6y^3}{2y} = 3y^2$

14 $\frac{2y}{6y^3} = \frac{1}{3y^2}$

15 $\frac{6y^3}{2y^3} = 3$

16 $\frac{6y^4}{2y^3} = 3y$

Indices - Further Multiplying and Dividing

This sheet is a more difficult sheet as it combines multiplication and division together. The process is to complete the multiplications first, then complete the division.

Examples: Simplify these:

a) $\dfrac{c^4b \times b^2}{c^2 \times b} = \dfrac{c^4b^3}{c^2b} = c^2b^2$

b) $\dfrac{a^2e \times a^2}{ae^2 \times a^2e^5} = \dfrac{a^4e}{a^3e^7} = \dfrac{a}{e^6}$

The last two columns involve coefficients. These can't be simplified in the 2nd column, but in the first column they can be.

Examples: Simplify these:

Column 2 - No simplifying of coefficients.

a) $\dfrac{2c^4b \times 3b^2}{5c^2 \times b} = \dfrac{6c^4b^3}{5c^2b} = \dfrac{6c^2b^2}{5}$

Column 3 - Simplifying of coefficients.

a) $\dfrac{2c^4b \times 3b^2}{4c^2 \times b} = \dfrac{6c^4b^3}{4c^2b} = \dfrac{3c^2b^2}{2}$

Indices - Further Multiplying and Dividing

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Simplify these by multiplying first and then dividing.

Example $\dfrac{v^4 \times v^3}{v^6 \times v} = \dfrac{v^{12}}{v^7} = v^5$

1 $\dfrac{a^5 \times a^4}{a^6} = \dfrac{\quad}{\quad} = \quad$

2 $\dfrac{n^3}{n^7 \times n^2} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

3 $\dfrac{g^5}{g^2 \times g^4} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

4 $\dfrac{p^5}{p^3 \times p^4} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

5 $\dfrac{d^3 \times d^5}{d \times d^4} = \dfrac{\quad}{\quad} = \quad$

6 $\dfrac{c^5 \times c^4}{c^3 \times c^2} = \dfrac{\quad}{\quad} = \quad$

7 $\dfrac{ab \times b^2}{a^2 \times a^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

8 $\dfrac{mx^5 \times m^2}{m^4 \times xm} =$ _____

9 $\dfrac{xy^5 \times y^2}{x^4 \times x^2 y} =$ _____

10 $\dfrac{cn^3 \times cn^2}{c^2 \times cn^2} =$ _____

11 $\dfrac{ac^3 \times a^2}{a^3 c^4 \times c^2} =$ _____

12 $\dfrac{w^2 n^5 \times w^2}{nw \times w^3} =$ _____

13 $\dfrac{k^2 y \times y^2 k^4}{y^3 \times y^2 k^4} =$ _____

14 $\dfrac{q^3 n^2 \times n^4}{qn^6 \times n^4 q^3} =$ _____

15 $\dfrac{b^2 d^3 \times b^2 d}{bd^2 \times b^2 d^2} =$ _____

16 $\dfrac{s^2 u^5 \times u^3 s^2}{s^4 u^3} =$ _____

These feature coefficients. Simplify these.

Example

$\dfrac{v^4 x^5 \times 5v^3}{4vx^3 \times 3xv} = \dfrac{5v^7 x^5}{12v^2 x^4} = \dfrac{5v^5 x}{12}$

17 $\dfrac{3h^2 c \times 3h^4}{2h^3 \times c^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

18 $\dfrac{5a^2 y^2 \times 2y^4}{7y^4 \times a^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

19 $\dfrac{2j^2 k \times 4k^2}{3j^4} =$ _____ $=$ _____

20 $\dfrac{5m^2 e^3 \times m^2}{2m^4 \times e^4 m} =$ _____ $=$ _____

These feature coefficients which can be simplified. Simplify these.

Examples

a) $\dfrac{3v^4 x^5 \times v^3}{2vx^3 \times 6xv} = \dfrac{3v^7 x^5}{12v^2 x^4} = \dfrac{v^5 x}{4}$

b) $\dfrac{3a^2 b^3 \times 4a^3}{2a^3 \times ab} = \dfrac{12a^5 b^3}{2a^4 b} = 6ab^2$

21 $\dfrac{3ve^3 \times 5e}{2e^3 v^5} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

22 $\dfrac{2n^2 w \times 6n^4}{3n^3} = \dfrac{\quad}{\quad} = \quad$

23 $\dfrac{2x^2 y^3 \times 5y^4}{10x \times 2y^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

24 $\dfrac{6h^2 q \times 3q^2}{2hq^4 \times h^2 q^2} =$ _____ $=$ _____

25 $\dfrac{5y^2 x^5 \times 4y^2}{y^4 x^3 \times 10x^2} =$ _____ $=$ _____

Indices - Further Multiplying and Dividing

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Simplify these by multiplying first and then dividing.

Example $\dfrac{v^4 \times v^3}{v^6 \times v} = \dfrac{v^{12}}{v^7} = v^5$

1 $\dfrac{a^5 \times a^4}{a^6} = \dfrac{a^9}{a^6} = a^3$

2 $\dfrac{n^3}{n^7 \times n^2} = \dfrac{n^3}{n^9} = \dfrac{1}{n^6}$

3 $\dfrac{g^5}{g^2 \times g^4} = \dfrac{g^5}{g^6} = \dfrac{1}{g}$

4 $\dfrac{p^5}{p^3 \times p^4} = \dfrac{p^5}{p^7} = \dfrac{1}{p^2}$

5 $\dfrac{d^3 \times d^5}{d \times d^4} = \dfrac{d^8}{d^5} = d^3$

6 $\dfrac{c^5 \times c^4}{c^3 \times c^2} = \dfrac{c^9}{c^5} = c^5$

7 $\dfrac{ab \times b^2}{a^2 \times a^3} = \dfrac{ab^3}{a^5} = \dfrac{b^3}{a^4}$

8 $\dfrac{mx^5 \times m^2}{m^4 \times xm} = \dfrac{m^3x^5}{m^5x} = \dfrac{x^4}{m^2}$

9 $\dfrac{xy^5 \times y^2}{x^4 \times x^2y} = \dfrac{xy^7}{x^6y} = \dfrac{x^4}{m^2}$

10 $\dfrac{cn^3 \times cn^2}{c^2 \times cn^2} = \dfrac{c^2n^5}{c^3n^2} = \dfrac{n^3}{c}$

11 $\dfrac{ac^3 \times a^2}{a^3c^4 \times c^2} = \dfrac{a^3c^3}{a^3c^6} = \dfrac{1}{c^3}$

12 $\dfrac{w^2n^5 \times w^2}{nw \times w^3} = \dfrac{w^4n^5}{w^4n} = n^4$

13 $\dfrac{k^2y \times y^2k^4}{y^3 \times y^2k^4} = \dfrac{k^6y^3}{k^4y^5} = \dfrac{k^2}{y^2}$

14 $\dfrac{q^3n^2 \times n^4}{qn^6 \times n^4q^3} = \dfrac{q^3n^6}{q^4n^{10}} = \dfrac{1}{qn^4}$

15 $\dfrac{b^2d^3 \times b^2d}{bd^2 \times b^2d^2} = \dfrac{b^4d^4}{b^3d^4} = b$

16 $\dfrac{s^2u^5 \times u^3s^2}{s^4u^3} = \dfrac{s^4u^8}{s^4u^3} = u^5$

These feature coefficients. Simplify these.

Example $\dfrac{v^4x^5 \times 5v^3}{4vx^3 \times 3xv} = \dfrac{5v^7x^5}{12v^2x^4} = \dfrac{5v^5x}{12}$

17 $\dfrac{3h^2c \times 3h^4}{2h^3 \times c^3} = \dfrac{9h^6c}{2h^3c^3} = \dfrac{9h^3}{2c^2}$

18 $\dfrac{5a^2y^2 \times 2y^4}{7y^4 \times a^3} = \dfrac{10a^2y^4}{7a^3y^4} = \dfrac{10}{7a}$

19 $\dfrac{2j^2k \times 4k^2}{3j^4} = \dfrac{8j^2k^3}{3j^4} = \dfrac{8k^3}{3j^2}$

20 $\dfrac{5m^2e^3 \times m^2}{2m^4 \times e^4m} = \dfrac{5m^4e^3}{2m^5e^4} = \dfrac{5}{2me}$

These feature coefficients which can be simplified. Simplify these.

Examples

a) $\dfrac{3v^4x^5 \times v^3}{2vx^3 \times 6xv} = \dfrac{3v^7x^5}{12v^2x^4} = \dfrac{v^5x}{4}$

b) $\dfrac{3a^2b^3 \times 4a^3}{2a^3 \times ab} = \dfrac{12a^5b^3}{2a^4b} = 6ab^2$

21 $\dfrac{3ve^3 \times 5e}{2e^3v^5} = \dfrac{15ve^4}{2v^5e^3} = \dfrac{15e}{2v^4}$

22 $\dfrac{2n^2w \times 6n^4}{3n^3} = \dfrac{12n^6w}{3n^3} = 4n^3w$

23 $\dfrac{2x^2y^3 \times 5y^4}{10x \times 2y^3} = \dfrac{10x^2y^7}{20xy^3} = \dfrac{xy^4}{2}$

24 $\dfrac{6h^2q \times 3q^2}{2hq^4 \times h^2q^2} = \dfrac{18h^2q^3}{2h^3q^6} = \dfrac{9}{hq^3}$

25 $\dfrac{5y^2x^5 \times 4y^2}{y^4x^3 \times 10x^2} = \dfrac{20y^4x^5}{10y^4x^5} = 2$

Indices - Further Multiplying and Dividing (Homework Strip)

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Date Due: _____

Simplify these

1. $\frac{n^6 \times n}{n^2} = \frac{n^7}{n^2} = $ _____

2. $\frac{x^3 \times x^2}{x \times x^5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

3. $\frac{ab \times b^2}{b^5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

4. $\frac{n^2v \times v^2}{n^3 \times n^2v} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

5. $\frac{x^2y^4 \times xy^3}{y^2 \times x^3y^3} = \frac{\quad}{\quad} = $ _____

6. $\frac{a^3b^7 \times b^2}{ab^4 \times ab^2} = \frac{\quad}{\quad} = $ _____

7. $\frac{2n^6m^4 \times n}{n^4m^2 \times n^4} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

8. $\frac{6e^6y^2 \times 2e^2y}{5e^4y \times e^4y} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

9. $\frac{4uy^5 \times 2u^2y}{2yu^2 \times y^3u} = \frac{\quad}{\quad} = $ _____

Date Due: _____

Simplify these

1. $\frac{n^6 \times n}{n^2} = \frac{n^7}{n^2} = $ _____

2. $\frac{x^3 \times x^2}{x \times x^5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

3. $\frac{ab \times b^2}{b^5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

4. $\frac{n^2v \times v^2}{n^3 \times n^2v} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

5. $\frac{x^2y^4 \times xy^3}{y^2 \times x^3y^3} = \frac{\quad}{\quad} = $ _____

6. $\frac{a^3b^7 \times b^2}{ab^4 \times ab^2} = \frac{\quad}{\quad} = $ _____

7. $\frac{2n^6m^4 \times n}{n^4m^2 \times n^4} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

8. $\frac{6e^6y^2 \times 2e^2y}{5e^4y \times e^4y} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

9. $\frac{4uy^5 \times 2u^2y}{2yu^2 \times y^3u} = \frac{\quad}{\quad} = $ _____

Date Due: _____

Simplify these

1. $\frac{n^6 \times n}{n^2} = \frac{n^7}{n^2} = $ _____

2. $\frac{x^3 \times x^2}{x \times x^5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

3. $\frac{ab \times b^2}{b^5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

4. $\frac{n^2v \times v^2}{n^3 \times n^2v} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

5. $\frac{x^2y^4 \times xy^3}{y^2 \times x^3y^3} = \frac{\quad}{\quad} = $ _____

6. $\frac{a^3b^7 \times b^2}{ab^4 \times ab^2} = \frac{\quad}{\quad} = $ _____

7. $\frac{2n^6m^4 \times n}{n^4m^2 \times n^4} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

8. $\frac{6e^6y^2 \times 2e^2y}{5e^4y \times e^4y} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

9. $\frac{4uy^5 \times 2u^2y}{2yu^2 \times y^3u} = \frac{\quad}{\quad} = $ _____

Date Due: _____

Simplify these

1. $\frac{n^6 \times n}{n^2} = \frac{n^7}{n^2} = $ _____

2. $\frac{x^3 \times x^2}{x \times x^5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

3. $\frac{ab \times b^2}{b^5} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

4. $\frac{n^2v \times v^2}{n^3 \times n^2v} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

5. $\frac{x^2y^4 \times xy^3}{y^2 \times x^3y^3} = \frac{\quad}{\quad} = $ _____

6. $\frac{a^3b^7 \times b^2}{ab^4 \times ab^2} = \frac{\quad}{\quad} = $ _____

7. $\frac{2n^6m^4 \times n}{n^4m^2 \times n^4} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

8. $\frac{6e^6y^2 \times 2e^2y}{5e^4y \times e^4y} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

9. $\frac{4uy^5 \times 2u^2y}{2yu^2 \times y^3u} = \frac{\quad}{\quad} = $ _____

Indices - Further Multiplying and Dividing (Homework Strip)

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Date Due:

Simplify these

1 $\dfrac{n^6 \times n}{n^2} = \dfrac{n^7}{n^2} = n^5$

2 $\dfrac{x^3 \times x^2}{x \times x^5} = \dfrac{x^5}{x^6} = \dfrac{1}{x}$

3 $\dfrac{ab \times b^2}{b^5} = \dfrac{ab^3}{b^5} = \dfrac{a}{b^2}$

4 $\dfrac{n^2v \times v^2}{n^3 \times n^2v} = \dfrac{n^2v^3}{n^5v} = \dfrac{v^2}{n^3}$

5 $\dfrac{x^2y^4 \times xy^3}{y^2 \times x^3y^3} = \dfrac{x^3y^7}{x^3y^5} = y^2$

6 $\dfrac{a^3b^7 \times b^2}{ab^4 \times ab^2} = \dfrac{a^3b^9}{a^2b^6} = ab^3$

7 $\dfrac{2n^6m^4 \times n}{n^4m^2 \times n^4} = \dfrac{2n^7m^4}{n^8m^2} = \dfrac{2m^2}{n}$

8 $\dfrac{6e^6y^2 \times 2e^2y}{5e^4y \times e^4y} = \dfrac{12e^8y^3}{5e^8y^2} = \dfrac{12y}{5}$

9 $\dfrac{4uy^5 \times 2u^2y}{2yu^2 \times y^3u} = \dfrac{8u^3y^6}{2u^3y^4} = 4y^2$

Indices - Brackets and Indices

When an indice is inside brackets and is raised to a power, it can be expanded to find the solution.

Example Write $(2^5)^3$ without the brackets.

$$(2^5)^3 = 2^5 \times 2^5 \times 2^5 = 2^{15}$$

Example Write $(x^4)^2$ without the brackets.

$$(x^4)^2 = x^4 \times x^4 = x^8$$

This sheet starts with asking you to write expressions in expanded form and then solve them, just like the examples above. Then the rule below is used, where you multiply the powers.

Rule: When you expand bracketed indices, you multiply the powers.

$$(a^m)^n = a^{m \times n}$$

Here are is an example, applying the above rule:

Example Expand and simplify $(x^5)^2$.

$$(x^5)^2 = x^{5\times 2} = x^{10}$$

The bracketed term may include a coefficient as well. A coefficient is a number in front of the algebraic base. When there is a coefficient, the power outside the brackets is applied to the coefficient, you just slide it across.

Example Expand and simplify $(3x^5)^2$.

$$(3x^5)^2 = 3^2x^{5\times 2} = 9x^{10}$$

Indices - Brackets and Indices (Sheet 1)

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Simplify these by expanding first, then adding powers. Leave your answer in index form.

Example $(5^4)^3 = 5^4 \times 5^4 \times 5^4 = 5^{12}$ (4 + 4 + 4 = 12)

1 $(7^3)^2 =$ ☐ × ☐ = ☐

2 $(3^5)^3 =$ ☐ × ☐ × ☐

= ☐

3 $(9^5)^4 =$ ☐ × ☐ × ☐ × ☐

= ☐

Now letters - same process

4 $(y^2)^4 =$ ☐ × ☐ × ☐ × ☐

= ☐

5 $(x^6)^5 =$

=

6 $(k^3)^3 =$

=

Brackets means you multiply the indexes. Simplify these. Leave answer in index form.

$(m^6)^4 = m^{6 \times 4} = m^{24}$

7 $(8^3)^4 =$ ☐^☐×☐ = ☐^☐

8 $(4^3)^5 =$ ☐^☐×☐ = ☐^☐

9 $(y^2)^3 =$ ☐^☐×☐ = ☐^☐

10 $(x^5)^3 =$

11 $(n^8)^2 =$

12 $(k^6)^5 =$

Simplify these. Leave your answer in index form.

13 $(8^2)^3 =$

14 $(5^4)^2 =$

15 $(y^7)^2 =$

16 $(k^3)^2 =$

17 $(m^3)^2 =$

18 $(a^6)^3 =$

19 $(10^4)^3 =$

20 $(h^5)^5 =$

Use your power key to evaluate these.

Example $5^4 = 625$ Calc: 5 $x^{\blacksquare}$ 4 =

21 $2^4 =$

22 $3^6 =$

23 $8^4 =$

24 $2^8 =$

25 $4^5 =$

26 $10^2 =$

27 $10^4 =$

28 $10^3 =$

Simplify these by expanding first, then multiplying. Use your calculator to calculate the numbers with powers.

Example $(2x^4)^3 = 2x^4 \times 2x^4 \times 2x^4 = 2^3x^{12} = 8x^{12}$

29 $(3h^5)^2 =$ ☐ × ☐ = $3^{\square} h^{\square}$ = ☐

30 $(5a^7)^3 =$ ☐ × ☐ × ☐ = $5^{\square} a^{\square}$ = ☐

31 $(2n^6)^4 =$ ☐ × ☐ × ☐ × ☐ = $2^{\square} n^{\square}$ = ☐

32 $(3k^4)^3 =$

33 $(4c^5)^4 =$

Slide the power outside the brackets onto the number inside and multiply the two indexes for the power on the letter.

Example $(2x^4)^3 = 2^3x^{12} = 8x^{12}$

34 $(3e^2)^3 = 3^{\square} e^{\square} =$ ☐

35 $(5n^4)^2 = 5^{\square} n^{\square} =$ ☐

36 $(2c^5)^4 = 2^{\square} c^{\square} =$ ☐

37 $(3x^3)^4 = 3^{\square} x^{\square} =$ ☐

38 $(6v^5)^3 =$ ☐ = ☐

39 $(10a^8)^2 =$ ☐

= ☐

40 $(10y^5)^4 =$ ☐

= ☐

Indices - Brackets and Indices (Sheet 2)

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Rewrite these without the brackets.

Example $(5v^4)^3 = 125v^{12}$

$5^3 = 125$ $3 \times 4 = 12$

1 $(4g^5)^5 =$

2 $(3n^2)^4 =$

3 $(2j^7)^3 =$

4 $(10c^{10})^2 =$

5 $(2b^4)^9 =$

6 $(5g^6)^4 =$

7 $(10m^2)^3 =$

8 $(9x^3)^4 =$

9 $(2h^8)^{12} =$

10 $(10u^6)^5 =$

Multiple bases look harder but the process is the same, no calculator required. Expand.

Example $(ab^4c^2)^5 = a^5b^{20}c^{10}$

$1 \times 5 = 5$ $4 \times 5 = 20$ $2 \times 5 = 10$

11 $(x^3y^2)^4 =$

12 $(a^6k^2)^3 =$

13 $(a^3b)^5 =$

14 $(g^3n^6)^2 =$

15 $(c^{10}e^2)^8 =$

16 $(abc)^3 =$

17 $(a^4b^2c)^5 =$

18 $(a^2bc^3)^7 =$

These include co-efficients (numbers out front). Expand these, calculator required.

Example $(5v^4x^2)^3 = 125v^{12}x^6$

19 $(3me^5)^2 =$

20 $(5w^3x^2)^3 =$

21 $(2r^2d^2)^8 =$

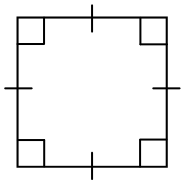
22 $(3q^6e^4)^4 =$

23 $(4q^5xk^4)^5 =$

24 $(10x^2ny^3)^3 =$

Find the area of these squares.

25



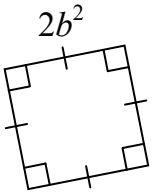
$3a$

$A = l^2$

$= (3a)^2$

$A =$ ☐ units^2

26



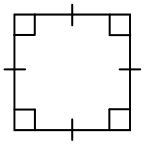
$2b^2$

$A = l^2$

$= (2b^2)^2$

$A =$ ☐ units^2

27



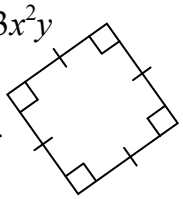
a^2b^3

$A = l^2$

=

$A =$ ☐ units^2

28



$3x^2y$

These are two step questions. Expand and simplify these.

Example

$$\frac{(v^3x)^3}{(v^2x^2)^4} = \frac{v^9x^3}{v^8x^8} = \frac{v}{x^5}$$

Expand Simplify

29

$$\frac{(a^5b^2)^4}{(a^2b^3)^2} = \frac{a^{\square}\,b^{\square}}{a^{\square}\,b^{\square}} = \frac{\square}{\square}$$

30

$$\frac{(n^2a^3)^2}{(n^3a^2)^3} = \frac{n^{\square}\,a^{\square}}{n^{\square}\,a^{\square}} = \frac{\square}{\square}$$

31

$$\frac{(yx^4)^3}{(y^2x^2)^2} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

32

$$\frac{(v^5c^4)^2}{(v^3c^2)^3} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

Now multiplication

Example $(x^2b)^3 \times (x^4b^2)^2$

$= x^6b^3 \times x^8b^4$ Expand

$= x^{14}b^7$ Simplify

33 $(x^3y^2)^4 \times (xy^5)^2$

$=$ $=$

34 $(b^2k)^3 \times (b^2k^2)^4$

$=$ $=$

35 $(u^6v^3)^2 \times (uv^2)^3$

$=$ $=$

36 $(a^2h^3)^5 \times (2h^2)^2$

$=$ $=$

Indices - Brackets and Indices (Sheet 1)

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Simplify these by expanding first, then adding powers. Leave your answer in index form.

Example $(5^4)^3 = 5^4 \times 5^4 \times 5^4 = 5^{12}$ (4 + 4 + 4 = 12)

1 $(7^3)^2 = 7^3 \times 7^3 = 7^6$

2 $(3^5)^3 = 3^5 \times 3^5 \times 3^5 = 3^{15}$

3 $(9^5)^4 = 9^5 \times 9^5 \times 9^5 \times 9^5 = 9^{20}$

Now letters - same process

4 $(y^2)^4 = y^2 \times y^2 \times y^2 \times y^2 = y^8$

5 $(x^6)^5 = x^6 \times x^6 \times x^6 \times x^6 \times x^6 = x^{30}$

6 $(k^3)^3 = k^3 \times k^3 \times k^3 = k^9$

Brackets means you multiply the indexes. Simplify these. Leave answer in index form.

$(m^6)^4 = m^{6\times4} = m^{24}$

7 $(8^3)^4 = 8^{3\times4} = 8^{12}$

8 $(4^3)^5 = 4^{3\times5} = 4^{15}$

9 $(y^2)^3 = y^{2\times3} = y^6$

10 $(x^5)^3 = x^{5\times3} = x^{15}$

11 $(n^8)^2 = n^{8\times2} = n^{16}$

12 $(k^6)^5 = k^{6\times5} = k^{30}$

Simplify these. Leave your answer in index form.

13 $(8^2)^3 = 8^6$

14 $(5^4)^2 = 5^8$

15 $(y^7)^2 = y^{14}$

16 $(k^3)^2 = k^6$

17 $(m^3)^2 = m^6$

18 $(a^6)^3 = a^{18}$

19 $(10^4)^3 = 10^{12}$

20 $(h^5)^5 = h^{25}$

Use your power key to evaluate these.

Example $5^4 = 625$ Calc: 5 $x^{\blacksquare}$ 4 =

21 $2^4 = 16$

22 $3^6 = 729$

23 $8^4 = 4\,096$

24 $2^8 = 256$

25 $4^5 = 1\,024$

26 $10^2 = 100$

27 $10^4 = 10\,000$

28 $10^3 = 1\,000$

Simplify these by expanding first, then multiplying. Use your calculator to calculate the numbers with powers.

Example $(2x^4)^3 = 2x^4 \times 2x^4 \times 2x^4 = 2^3x^{12} = 8x^{12}$

29 $(3h^5)^2 = 3h^5 \times 3h^5 = 3^2 h^{10} = 9h^{10}$

30 $(5a^7)^3 = 5a^7 \times 5a^7 \times 5a^7 = 5^3 a^{21} = 125a^{21}$

31 $(2n^6)^4 = 2n^6 \times 2n^6 \times 2n^6 \times 2n^6 = 2^4 n^{24} = 16n^{24}$

32 $(3k^4)^3 = 3k^4 \times 3k^4 \times 3k^4 = 3^3k^{12} = 27k^{12}$

33 $(4c^5)^4 = 4c^5 \times 4c^5 \times 4c^5 \times 4c^5 = 4^4c^{20} = 256c^{20}$

Slide the power outside the brackets onto the number inside and multiply the two indexes for the power on the letter.

Example $(2x^4)^3 = 2^3x^{12} = 8x^{12}$

34 $(3e^2)^3 = 3^3 e^6 = 27e^6$

35 $(5n^4)^2 = 5^2 n^8 = 25n^8$

36 $(2c^5)^4 = 2^4 c^{20} = 16c^{20}$

37 $(3x^3)^4 = 3^4 x^{12} = 81x^{12}$

38 $(6v^5)^3 = 6^3v^{15} = 216v^{15}$

39 $(10a^8)^2 = 10^2a^{16} = 100a^{16}$

40 $(10y^5)^4 = 10^4y^{20} = 10\,000y^{20}$

Indices - Brackets and Indices (Sheet 2)

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Rewrite these without the brackets.

Example $(5v^4)^3 = 125v^{12}$

$5^3 = 125$; $3 \times 4 = 12$

1 $(4g^5)^5 = 1\,024g^{25}$

2 $(3n^2)^4 = 81n^8$

3 $(2j^7)^3 = 8j^{21}$

4 $(10c^{10})^2 = 100c^{20}$

5 $(2b^4)^9 = 512b^{36}$

6 $(5g^6)^4 = 625g^{24}$

7 $(10m^2)^3 = 1\,000m^6$

8 $(9x^3)^4 = 6\,561x^{12}$

9 $(2h^8)^{12} = 4\,096h^{96}$

10 $(10u^6)^5 = 100\,000u^{30}$

Multiple bases look harder but the process is the same, no calculator required. Expand.

Example $(ab^4c^2)^5 = a^5b^{20}c^{10}$

$1 \times 5 = 5$; $4 \times 5 = 20$; $2 \times 5 = 10$

11 $(x^3y^2)^4 = x^{12}y^8$

12 $(a^6k^2)^3 = a^{18}k^6$

13 $(a^3b)^5 = a^{15}b^5$

14 $(g^3n^6)^2 = g^6n^{12}$

15 $(c^{10}e^2)^8 = c^{80}e^{16}$

16 $(abc)^3 = a^3b^3c^3$

17 $(a^4b^2c)^5 = a^{20}b^{10}c^5$

18 $(a^2bc^3)^7 = a^{14}b^7c^{21}$

These include co-efficients (numbers out front). Expand these, calculator required.

Example $(5v^4x^2)^3 = 125v^{12}x^6$

19 $(3me^5)^2 = 9m^2e^{10}$

20 $(5w^3x^2)^3 = 125w^9x^6$

21 $(2r^2d^2)^8 = 256r^{16}d^{16}$

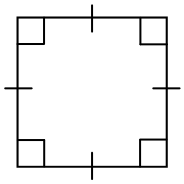
22 $(3q^6e^4)^4 = 81q^{24}e^{16}$

23 $(4q^5xk^4)^5 = 1\,024q^{25}x^5k^{20}$

24 $(10x^2ny^3)^3 = 1\,000x^6n^3y^9$

Find the area of these squares.

25

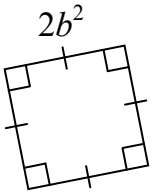


$A = l^2$

$= (3a)^2$

$A = 9a^2$ units^2

26

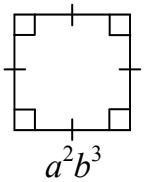


$A = l^2$

$= (2b^2)^2$

$A = 4b^4$ units^2

27

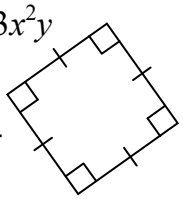


$A = l^2$

$= (a^2b^3)^2$

$A = a^4b^6$ units^2

28



$A = l^2$

$= (3x^2y)^2$

$A = 9x^4y^2$ units^2

These are two step questions. Expand and simplify these.

Example

$$\frac{(v^3x)^3}{(v^2x^2)^4} = \frac{v^9x^3}{v^8x^8} = \frac{v}{x^5}$$

Expand ; Simplify

29

$$\frac{(a^5b^2)^4}{(a^2b^3)^2} = \frac{a^{20}b^8}{a^4b^6} = a^{16}b^2$$

30

$$\frac{(n^2a^3)^2}{(n^3a^2)^3} = \frac{n^4a^6}{n^9a^6} = \frac{1}{n^5}$$

31

$$\frac{(yx^4)^3}{(y^2x^2)^2} = \frac{y^3x^{12}}{y^4x^4} = \frac{x^8}{y}$$

32

$$\frac{(v^5c^4)^2}{(v^3c^2)^3} = \frac{v^{10}c^8}{v^6c^6} = v^4c^2$$

Now multiplication

Example $(x^2b)^3 \times (x^4b^2)^2$

$= x^6b^3 \times x^8b^4$ Expand

$= x^{14}b^7$ Simplify

33 $(x^3y^2)^4 \times (xy^5)^2$

$= x^{12}y^8 \times x^2y^{10} = x^{14}y^{18}$

34 $(b^2k)^3 \times (b^2k^2)^4$

$= b^6k^3 \times b^8k^8 = b^{14}k^{11}$

35 $(u^6v^3)^2 \times (uv^2)^3$

$= u^{12}v^6 \times u^3v^6 = u^{15}v^{12}$

36 $(a^2h^3)^5 \times (2h^2)^2$

$= a^{10}h^{15} \times 4h^4 = 4a^{10}h^{19}$

Indices - Brackets and Indices (Homework Strip)

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Date Due: _____

Expand and simplify these

1 $(a^5)^2 =$ _____ 2 $(b^3)^3 =$ _____

3 $(n^4)^3 =$ _____ 4 $(c^2)^5 =$ _____

5 $(xk^2)^3 =$ _____

6 $(a^3y^2)^5 =$ _____

7 $(b^5v^4)^2 =$ _____

8 $(2c^3)^4 =$ _____

9 $(3e^2)^5 =$ _____

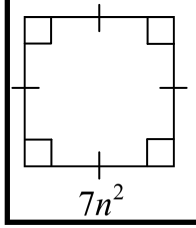
10 $(10w^7)^3 =$ _____

11 $(5a^3)^5 =$ _____

12 $(6n^5)^4 =$ _____

Find the area of the square

13



$A = l^2$

$=$

$A =$ ____ units^2

Date Due: _____

Expand and simplify these

1 $(a^5)^2 =$ _____ 2 $(b^3)^3 =$ _____

3 $(n^4)^3 =$ _____ 4 $(c^2)^5 =$ _____

5 $(xk^2)^3 =$ _____

6 $(a^3y^2)^5 =$ _____

7 $(b^5v^4)^2 =$ _____

8 $(2c^3)^4 =$ _____

9 $(3e^2)^5 =$ _____

10 $(10w^7)^3 =$ _____

11 $(5a^3)^5 =$ _____

12 $(6n^5)^4 =$ _____

Find the area of the square

13

$7n^2$

$A = l^2$

$=$

$A =$ ____ units^2

Date Due: _____

Expand and simplify these

1 $(a^5)^2 =$ _____ 2 $(b^3)^3 =$ _____

3 $(n^4)^3 =$ _____ 4 $(c^2)^5 =$ _____

5 $(xk^2)^3 =$ _____

6 $(a^3y^2)^5 =$ _____

7 $(b^5v^4)^2 =$ _____

8 $(2c^3)^4 =$ _____

9 $(3e^2)^5 =$ _____

10 $(10w^7)^3 =$ _____

11 $(5a^3)^5 =$ _____

12 $(6n^5)^4 =$ _____

Find the area of the square

13

$7n^2$

$A = l^2$

$=$

$A =$ ____ units^2

Date Due: _____

Expand and simplify these

1 $(a^5)^2 =$ _____ 2 $(b^3)^3 =$ _____

3 $(n^4)^3 =$ _____ 4 $(c^2)^5 =$ _____

5 $(xk^2)^3 =$ _____

6 $(a^3y^2)^5 =$ _____

7 $(b^5v^4)^2 =$ _____

8 $(2c^3)^4 =$ _____

9 $(3e^2)^5 =$ _____

10 $(10w^7)^3 =$ _____

11 $(5a^3)^5 =$ _____

12 $(6n^5)^4 =$ _____

Find the area of the square

13

$7n^2$

$A = l^2$

$=$

$A =$ ____ units^2

Indices - Brackets and Indices (Homework Strip)

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Date Due:

Expand and simplify these

1 $(a^5)^2 = a^{10}$

2 $(b^3)^3 = b^9$

3 $(n^4)^3 = n^{12}$

4 $(c^2)^5 = c^{10}$

5 $(xk^2)^3 = x^3k^6$

6 $(a^3y^2)^5 = a^{15}y^{10}$

7 $(b^5v^4)^2 = b^{10}v^8$

8 $(2c^3)^4 = 16c^{12}$

9 $(3e^2)^5 = 243e^{10}$

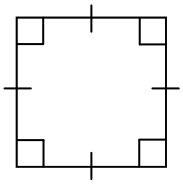
10 $(10w^7)^3 = 1\,000w^{21}$

11 $(5a^3)^5 = 3\,125a^{15}$

12 $(6n^5)^4 = 1\,296n^{20}$

Find the area of the square

13



$A = l^2$

$= (7n^2)^2$

$A = 49n^4$ units^2

Indices - The Zero Index

When a number or a term (letter) is raised to a power of zero, the answer is 1. Note that this doesn't apply to zero itself, the answer for 0^0 is undefined.

Rule: When you raise a number or letter to the power of 0. the answer is 1.

$$a^0 = 1 \qquad a \neq 0$$

Here are some examples of this rule.

Example Evaluate x^0.

$$x^0 = 1$$

Example Evaluate 2^0.

$$2^0 = 1$$

Example Evaluate $(2x)^0$.

$$(2x)^0 = 1$$

Here are some examples where working out can be shown.

Example Evaluate $2x^0$.

$$2x^0 = 2 \times 1$$
$$= 2$$

Example Evaluate $(2^3)^0$.

$$(2n^0)^4 = (2 \times 1)^4$$
$$= 2^4 = 16$$

Example Evaluate $2x^0 + 8$.

$$2x^0 + 8 = 2 \times 1 + 8$$
$$= 10$$

The first sheet starts with showing you how to see how a power of zero can equal 1. After this questions involving the zero index begin. Remember the questions are best solved by replacing everything with a zero power to the number '1'. You can strike out the term and write '1' above it if that makes it easier to work with.

Indices - The Zero Index (Sheet 1)

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Use your calculator to find these values.

Example $5^3 = 125$

Calc: 5 x■ 3 =

1 $7^3 =$ ____ 2 $7^2 =$ ____

3 $7^1 =$ ____ 4 $7^0 =$ ____

5 $9^3 =$ ____ 6 $9^2 =$ ____

7 $9^1 =$ ____ 8 $9^0 =$ ____

9 $4^3 =$ ____ 10 $4^2 =$ ____

11 $4^1 =$ ____ 12 $4^0 =$ ____

13 Look at Q.4, Q.8 and Q.12

The answers were all ____

Now use your calculator to complete these divisions.

14 $7^4 \div 1 =$ ____

Calc: 7 x■ 4 ▶ ÷ 1 =

15 $7^4 \div 7^0 =$ ____

compare

$\therefore 7^0 =$ ____

16 $10^3 \div 1 =$ ____

17 $10^3 \div 10^0 =$ ____

compare

$\therefore 10^0 =$ ____

Now use your division skills with index laws. No calculator

18 $3^5 \div 3^0 =$ $3^{\square - \square} = 3^{\square}$

19 $5^7 \div 5^0 =$ $\square^{\square - \square} = \square^{\square}$

When you divide a number (not 0) by '1', the number is also the answer.

$\therefore 3^0 = 5^0 =$ ____

Answer these divisions in index form, then as numbers.

Example

Index Form: $7^4 \div 7^4 = 7^0$ Number: $= 1$

20 $5^2 \div 5^2 =$ $\square^{\square} =$ ____

21 $4^8 \div 4^8 =$ ____ = ____

22 $\frac{10^3}{10^3} =$ ____ = ____

23 $14^5 \div 14^5 =$ ____ = ____

24 $\frac{9^{11}}{9^{11}} =$ ____ = ____

Algebra is answered the same way.

25 $x^7 \div x^7 =$ $\square^{\square} =$ ____

26 $\frac{m^4}{m^4} =$ ____ = ____

27 $y^8 \div y^8 =$ ____ = ____

28 $\frac{n^{13}}{n^{13}} =$ ____ = ____

Multiply these answering in index form, then as numbers.

Example

Index Form: $5^3 \times 5^0 = 5^3$ Number: $= 125$

Calc: 5 x■ 3 =

29 $4^6 \times 4^0 =$ $\square^{\square} =$ ____

30 $2^9 \times 2^0 =$ ____ = ____

31 $3^7 \times 3^0 =$ ____ = ____

Algebra same, but leave in index form

32 $v^7 \times v^0 =$ $\square^{\square}$

33 $k^9 \times k^0 =$ ____

34 $n^3 \times n^0 =$ ____

Remember with brackets you multiply the powers. Expand these.

Example

Index Form: $(7^3)^0 = 7^0$ Number: $= 1$

35 $(5^6)^0 =$ $\square^{\square} =$ ____

36 $(2^4)^0 =$ ____ = ____

37 $(3^0)^6 =$ ____ = ____

38 $(x^0)^6 =$ ____ = ____

39 $(a^0)^3 =$ ____ = ____

40 $(q^5)^0 =$ ____ = ____

These have co-efficient and so are a little harder.

Example

$x^0 = 1$

$(3x^0)^3 = (3\times1)^3$

$= (3)^3$

Calc: 3 x■ 3 =

$= 27$

41 $(5n^0)^4 = (\square \times \square)^4$

$= \square^4 =$ ____

42 $(8p^0)^3 = (\square \times \square)^3$

$= \square^3 =$ ____

43 $(2h^0)^9 = (\square \times \square)^9$

$= \square^9 =$ ____

When the '0' is outside, answer is 1

44 $(2h^3)^0 =$ ____

45 $(8k^5)^0 =$ ____

46 $(a^2b^5c^3)^0 =$ ____

Indices - The Zero Index (Sheet 2)

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Anything (except 0) with the power of zero is 1. Evaluate.

Example $5m^0 + 7$

$= 5 \times 1 + 7 = 12$

1 $2v^0 =$ ☐ × ☐ = ☐

2 $8h^0 =$ ☐ × ☐ = ☐

3 $h^0 + 7 =$ ☐ =

4 $7h^0 - 4 =$ ☐ =

5 $2^0 =$ ☐ 6 $x^0 =$ ☐

7 $2x^0 =$ ☐ = ☐

8 $2x^0 + 2 =$ ☐ × ☐ + ☐

= ☐

9 $x^0 \times 2x^0 =$ ☐ × ☐ × ☐

= ☐

10 $n^0 =$ ☐ 11 $4^0 =$ ☐

12 $4n^0 =$ ☐ = ☐

13 $(4n)^0 =$ ☐

14 $4n^0 + 4 =$ ☐ × ☐ + ☐

=

15 $(n^0 + 4)^2 =$ (☐ + ☐)^2

= ☐^{2} = ☐

16 $4n^0 + (4n)^0 =$ ☐ × ☐ + ☐

= ☐

T F Answer true or false to these.

17 $2^0 = 2$ ○ True ○ False

18 $5x^0 = (5x)^0$ ○ True ○ False

19 $3b^0 = b^0 + 2$ ○ True ○ False

20 $4a^0 - 4 = a^0$ ○ True ○ False

21 $3g^0 - (3g)^0 = 2g^0$ ○ True ○ False

22 $5k^0 + 4 = 9k^0$ ○ True ○ False

These will result in an algebraic answer. Simplify these.

Example $2m^0b$

$= 2 \times 1 \times b = 2b$

23 $2v^0h^2 =$ ☐

24 $a^0b^2c^3 =$ ☐

25 $(m^6)^0 \times m =$ ☐

26 $9k^2 \times e^0 =$ ☐

27 $4a^0m^2 - m^2a^0$

=

=

28

×	e^0	$3e^0$	e^0x^2
$2x$			
e^0			
e^2			
$3x^2$			
ex^2			

Complete the tables. Replace terms that have a power of 0 with a '1', before you multiply.

29

×	a^2	a^2b	ab^2	$(a^2)^0$
a^0				
$5a$				
a^0b				

Evaluate these and then arrange them in ascending order, that is from the smallest to largest.

30 $5e^0$, $e^0 + 9$, $(3e^0)^2$, $8e^0 - 6$.

☐ ☐ ☐ ☐

☐ ☐ ☐ ☐

☐

Rewrite the algebraic expressions in ascending order, not the numbers you have calculated

31 $10 - n^0$, $(2n^0)^3$, $4n^0$, $12 \div n^0$.

☐ ☐ ☐ ☐

☐ ☐ ☐ ☐

☐

Indices - The Zero Index (Sheet 1)

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Use your calculator to find these values.

Example $5^3 = 125$

Calc: 5 x■ 3 =

1 $7^3 = 343$ 2 $7^2 = 49$

3 $7^1 = 7$ 4 $7^0 = 1$

5 $9^3 = 729$ 6 $9^2 = 81$

7 $9^1 = 9$ 8 $9^0 = 1$

9 $4^3 = 64$ 10 $4^2 = 16$

11 $4^1 = 4$ 12 $4^0 = 1$

13 Look at Q.4, Q.8 and Q.12

The answers were all 1

Now use your calculator to complete these divisions.

14 $7^4 \div 1 =$ 2 401

Calc: 7 x■ 4 ◯ ÷ 1 =

15 $7^4 \div 7^0 =$ 2 401

compare

$\therefore 7^0 =$ 1

16 $10^3 \div 1 =$ 1 000

17 $10^3 \div 10^0 =$ 1 000

compare

$\therefore 10^0 =$ 1

Now use your division skills with index laws. No calculator

18 $3^5 \div 3^0 = 3^{5-0} = 3^5$

19 $5^7 \div 5^0 = 5^{7-0} = 5^7$

When you divide a number (not 0) by '1', the number is also the answer.

$\therefore 3^0 = 5^0 =$ 1

Answer these divisions in index form, then as numbers.

Example $7^4 \div 7^4 = 7^0 = 1$ (Index Form, Number)

20 $5^2 \div 5^2 = 5^0 = 1$

21 $4^8 \div 4^8 = 4^0 = 1$

22 $\frac{10^3}{10^3} = 10^0 = 1$

23 $14^5 \div 14^5 = 14^0 = 1$

24 $\frac{9^{11}}{9^{11}} = 9^0 = 1$

Algebra is answered the same way.

25 $x^7 \div x^7 = x^0 = 1$

26 $\frac{m^4}{m^4} = m^0 = 1$

27 $y^8 \div y^8 = y^0 = 1$

28 $\frac{n^{13}}{n^{13}} = n^0 = 1$

Multiply these answering in index form, then as numbers.

Example $5^3 \times 5^0 = 5^3 = 125$ (Index Form, Number)

Calc: 5 x■ 3 =

29 $4^6 \times 4^0 = 4^6 = 4\,096$

30 $2^9 \times 2^0 = 2^9 = 512$

31 $3^7 \times 3^0 = 3^7 = 2\,187$

Algebra same, but leave in index form

32 $v^7 \times v^0 = v^7$

33 $k^9 \times k^0 = k^9$

34 $n^3 \times n^0 = n^3$

Remember with brackets you multiply the powers. Expand these.

Example $(7^3)^0 = 7^0 = 1$ (Index Form, Number)

35 $(5^6)^0 = 5^0 = 1$

36 $(2^4)^0 = 2^0 = 1$

37 $(3^0)^6 = 3^0 = 1$

38 $(x^0)^6 = x^0 = 1$

39 $(a^0)^3 = a^0 = 1$

40 $(q^5)^0 = q^0 = 1$

These have co-efficient and so are a little harder.

Example $x^0 = 1$

$(3x^0)^3 = (3\times1)^3$

$= (3)^3$

Calc: 3 x■ 3 = $= 27$

41 $(5n^0)^4 = (5 \times 1)^4$

$= 5^4 = 625$

42 $(8p^0)^3 = (8 \times 1)^3$

$= 8^3 = 512$

43 $(2h^0)^9 = (2 \times 1)^9$

$= 2^9 = 512$

When the '0' is outside, answer is 1

44 $(2h^3)^0 = 1$

45 $(8k^5)^0 = 1$

46 $(a^2b^5c^3)^0 = 1$

Indices - The Zero Index (Sheet 2)

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Anything (except 0) with the power of zero is 1. Evaluate.

Example $5m^0 + 7$

$= 5 \times 1 + 7 = 12$

1 $2v^0 = 2 \times 1 = 2$

2 $8h^0 = 8 \times 1 = 8$

3 $h^0 + 7 = 1 + 7 = 8$

4 $7h^0 - 4 = 7 \times 1 - 4 = 3$

5 $2^0 = 1$ 6 $x^0 = 1$

7 $2x^0 = 2 \times 1 = 2$

8 $2x^0 + 2 = 2 \times 1 + 2$

$= 4$

9 $x^0 \times 2x^0 = 1 \times 2 \times 1$

$= 2$

10 $n^0 = 1$ 11 $4^0 = 1$

12 $4n^0 = 4 \times 1 = 4$

13 $(4n)^0 = 1$

14 $4n^0 + 4 = 4 \times 1 + 4$

$= 8$

15 $(n^0 + 4)^2 = (1 + 4)^2$

$= 5^2 = 25$

16 $4n^0 + (4n)^0 = 4 \times 1 + 1$

$= 5$

Answer true or false to these.

17 $2^0 = 2$ ○ True ● False

18 $5x^0 = (5x)^0$ ○ True ● False

19 $3b^0 = b^0 + 2$ ● True ○ False

20 $4a^0 - 4 = a^0$ ○ True ● False

21 $3g^0 - (3g)^0 = 2g^0$ ● True ○ False

22 $5k^0 + 4 = 9k^0$ ● True ○ False

These will result in an algebraic answer. Simplify these.

Example $2m^0b$

$= 2 \times 1 \times b = 2b$

23 $2v^0h^2 = 2 \times 1 \times h^2 = 2h^2$

24 $a^0b^2c^3 = 1 \times b^2 \times c^3 = b^2c^3$

25 $(m^6)^0 \times m = 1 \times m = m$

26 $9k^2 \times e^0 = 9 \times k^2 \times 1 = 9k^2$

27 $4a^0m^2 - m^2a^0$

$= 4 \times 1 \times m^2 - m^2 \times 1$

$= 4m^2 - m^2 = 3m^2$

28

×	1 ~~e^0~~	3 ~~$3e^0$~~	x^2 ~~e^0x^2~~
$2x$	$2x$	$6x$	$2x^3$
1 ~~e^0~~	1	3	x^2
e^2	e^2	$3e^2$	e^2x^2
$3x^2$	$3x^2$	$9x^2$	$3x^4$
ex^2	ex^2	$3ex^2$	ex^4

Complete the tables. Replace terms that have a power of 0 with a '1', before you multiply.

29

×	1 ~~a^0~~	a^2	a^2b	ab^2	~~$(a^2)^0$~~ 1
1 ~~a^0~~	a^2	a^2b	ab^2	1
$5a$	$5a^3$	$5a^3b$	$5a^2b^2$	$5a$
b ~~a^0b~~	a^2b	a^2b^2	ab^3	b

Evaluate these and then arrange them in ascending order, that is from the smallest to largest.

30 $5e^0$, $e^0 + 9$, $(3e^0)^2$, $8e^0 - 6$.

$= 5 \times 1 = 5$ $= 1 + 9 = 10$ $= (3 \times 1)^2 = 3^2 = 9$ $= 8 \times 1 - 6 = 2$

$8e^0 - 6$, $5e^0$, $(3e^0)^2$, $e^0 + 9$.

Rewrite the algebraic expressions in ascending order, not the numbers you have calculated

31 $10 - n^0$, $(2n^0)^3$, $4n^0$, $12 \div n^0$.

$= 10 - 1 = 9$ $= (2 \times 1)^3 = 2^3 = 8$ $= 4 \times 1 = 4$ $= 12 \div 1 = 12$

$4n^0$, $12 \div n^0$, $10 - n^0$, $(2n^0)^3$.

Indices - The Zero Index (Homework Strip)

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Date Due: _____

Use your calculator for these

1 $2^5 =$ _____ 2 $3^4 =$ _____

3 $9^0 =$ _____ 4 $7^0 =$ _____

Simplify, index form first then number

5 $5^3 \times 5^0 =$ $\square^{\square} =$ _____ (Add powers; Calculator)

6 $3^5 \times 3^0 =$ $\square^{\square} =$ _____

Anything to power of 0 is '1'. Simplify

7 $x^5 \times x^0 =$ $\square^{\square}$

8 $n^2 \times n^0 =$ $\square^{\square}$

9 $k^8 \times k^0 =$ $\square^{\square}$

Add powers when you multiply indices

Anything to power of 0 is '1'. Simplify

10 $\frac{10^5}{10^5} =$ $\square^{\square} =$ _____ (Minus powers)

11 $\frac{8^8}{8^8} =$ $\square^{\square} =$ _____

12 $\frac{3^{12}}{3^{12}} =$ $\square^{\square} =$ _____

Date Due: _____

Use your calculator for these

1 $2^5 =$ _____ 2 $3^4 =$ _____

3 $9^0 =$ _____ 4 $7^0 =$ _____

Simplify, index form first then number

5 $5^3 \times 5^0 =$ $\square^{\square} =$ _____ (Add powers; Calculator)

6 $3^5 \times 3^0 =$ $\square^{\square} =$ _____

Anything to power of 0 is '1'. Simplify

7 $x^5 \times x^0 =$ $\square^{\square}$

8 $n^2 \times n^0 =$ $\square^{\square}$

9 $k^8 \times k^0 =$ $\square^{\square}$

Add powers when you multiply indices

Anything to power of 0 is '1'. Simplify

10 $\frac{10^5}{10^5} =$ $\square^{\square} =$ _____ (Minus powers)

11 $\frac{8^8}{8^8} =$ $\square^{\square} =$ _____

12 $\frac{3^{12}}{3^{12}} =$ $\square^{\square} =$ _____

Date Due: _____

Use your calculator for these

1 $2^5 =$ _____ 2 $3^4 =$ _____

3 $9^0 =$ _____ 4 $7^0 =$ _____

Simplify, index form first then number

5 $5^3 \times 5^0 =$ $\square^{\square} =$ _____ (Add powers; Calculator)

6 $3^5 \times 3^0 =$ $\square^{\square} =$ _____

Anything to power of 0 is '1'. Simplify

7 $x^5 \times x^0 =$ $\square^{\square}$

8 $n^2 \times n^0 =$ $\square^{\square}$

9 $k^8 \times k^0 =$ $\square^{\square}$

Add powers when you multiply indices

Anything to power of 0 is '1'. Simplify

10 $\frac{10^5}{10^5} =$ $\square^{\square} =$ _____ (Minus powers)

11 $\frac{8^8}{8^8} =$ $\square^{\square} =$ _____

12 $\frac{3^{12}}{3^{12}} =$ $\square^{\square} =$ _____

Date Due: _____

Use your calculator for these

1 $2^5 =$ _____ 2 $3^4 =$ _____

3 $9^0 =$ _____ 4 $7^0 =$ _____

Simplify, index form first then number

5 $5^3 \times 5^0 =$ $\square^{\square} =$ _____ (Add powers; Calculator)

6 $3^5 \times 3^0 =$ $\square^{\square} =$ _____

Anything to power of 0 is '1'. Simplify

7 $x^5 \times x^0 =$ $\square^{\square}$

8 $n^2 \times n^0 =$ $\square^{\square}$

9 $k^8 \times k^0 =$ $\square^{\square}$

Add powers when you multiply indices

Anything to power of 0 is '1'. Simplify

10 $\frac{10^5}{10^5} =$ $\square^{\square} =$ _____ (Minus powers)

11 $\frac{8^8}{8^8} =$ $\square^{\square} =$ _____

12 $\frac{3^{12}}{3^{12}} =$ $\square^{\square} =$ _____

Indices - The Zero Index (Homework Strip)

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Date Due:

Use your calculator for these

1 $2^5 =$ 32

2 $3^4 =$ 81

3 $9^0 =$ 1

4 $7^0 =$ 1

Simplify, index form first then number

5 $5^3 \times 5^0 =$ $5^{3} =$ 125

Add powers — Calculator

6 $3^5 \times 3^0 =$ $3^{5} =$ 243

Anything to power of 0 is '1'. Simplify

7 $x^5 \times x^0 =$ x^{5}

8 $n^2 \times n^0 =$ n^{2}

9 $k^8 \times k^0 =$ k^{8}

Add powers when you multiply indices

Anything to power of 0 is '1'. Simplify

10 $\frac{10^5}{10^5} =$ $10^{0} =$ 1

Minus powers

11 $\frac{8^8}{8^8} =$ $8^{0} =$ 1

12 $\frac{3^{12}}{3^{12}} =$ $3^{0} =$ 1

Indices - Substitution

Substitution means that you swap the letters in an algebraic expression or formula, for numbers. Because this topic is indices, these substitutions all have powers in the expressions.

Some examples of this.

Example Evaluate $2x^2$ when $x = 6$.

$2x^2 = 2 \times 6^2$ ← Replace the x with 6. Note that there is a $\times$. When numbers/letters are beside each other then they are multiplied.

$= 72$ ← Use your calculator to solve.

Example Evaluate $2y^3 - 8y$, when $y = 4$.

$2y^3 - 8y = 2 \times 4^3 - 8 \times 4$

$= 96$

Example Evaluate $2y^3 - 8y$, when $y = -6$.

$2y^3 - 8y = 2 \times (-6)^3 - 8 \times (-6)$

$= -384$

When you have a negative number to substitute, always put the number in brackets.

Example Evaluate $ay^3 - a^2y$, when a = 3 and $y = 4$.

$ay^3 - a^2y = 3 \times 4^3 - 3^2 \times 4$

$= 156$

You can also have two numbers to substitute

The second page of these worksheets is the plotting of an indice. These are to the power of 2, and the curve they make is called a parabola. Notice what a negative sign and a number in front of the x^2 term does.

Indices - Substitution (Sheet 1)

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Use your calculator to evaluate these when $a = 5$.

Example $a^3 = 5^3 = 125$

Calc: 5 x^3 =

1 $a^4 =$ 5 ^{4} = ____

2 $2a^3 =$ ____ × ____ $^{\square}$ = ____

3 $3a^3 + 7 =$ ____ × ____ $^{\square}$ + ____

= ____

4 $3a^2 + 7a =$ ____

= ____

Evaluate these using $n = 8$.

5 $n^3 - 10 =$ ____ = ____

6 $4n^2 - n =$ ____

= ____

7 $-n^2 =$ ____ = ____ Calc: - 8 x^2 =

8 $(-n)^2 =$ ____ = ____

Calc: (- 8) x^2 =

Evaluate these using $y = -3$. Make sure you use brackets.

9 $y^2 - 12 =$ ____ = ____

Calc: (- 3) x^2 - 1 2 =

10 $20 - y^3 =$ ____ = ____

11 $3y^4 =$ ____ = ____

12 $4y^2 + 3y =$ ____

= ____

13 $4y^2 \times 3y =$ ____

= ____

Evaluate these using $a = 4$ and $x = 3$.

14 $ax^2 =$ ____ × ____ $^{\square}$ = ____

15 $a^2 + x^2 =$ ____ $^{\square}$ + ____ $^{\square}$ = ____

16 $a^2 - x^2 =$ ____ $^{\square}$ - ____ $^{\square}$ = ____

17 $ax^2 + ax =$ ____

= ____

18 $20x^2 \div a^2 =$ ____

= ____

* 19 $\dfrac{20x^5}{ax} =$ ____ / ____ = ____

∗ Brackets required

A table of values is often used to construct a graph. Complete these tables.

20 $b = a^2 + 2$

a	0	1	2	3	4	5
b						

Calc: 1 x^2 + 2 =

Calc: 0 x^2 + 2 =

21 $y = 2x^2 + 5$

x	0	1	2	3	4	5
y						

22 $y = -3x^2 - 7$

x	0	1	2	3	4	5
y						

A frequent use of substitution is using a formula, such as with science. Substitute the values into the formulae to solve these questions.

The surface area of a cube is found using $SA = 6l^2$. Find the SA:

23 When $l = 15$.

$SA = 6l^2$

$= 6 \times$ ____ ^{2}

$SA =$ ____ u^2

24 When $l = 30$.

25 When $l = 8$.

Power loss in a wire is found using $P = I^2R$. Find the loss, when:

26 $I = 2$ and $R = 0.5$

$P = I^2R$

$=$ ____ $^2 \times$ ____

$P =$ ____ W

27 $I = 10$ and $R = 6$.

28 $I = 8$ and $R = 3$.

Kinetic Energy is found using $KE = \frac{1}{2}mv^2$. Find the energy, when:

29 $m = 80$, $v = 15$.

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times$ ____ $\times$ ____ ^{2}

$KE =$ ____ J

30 $m = 65$, $v = 18$.

31 $m = 90$, $v = 25$.

Indices - Substitution (Sheet 2)

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Complete the tables and then plot the points on the number plane below them.

1 $y = x^2 - 4$ ○

x	-4	-3	-2	-1	0	1	2	3	4
y	12								

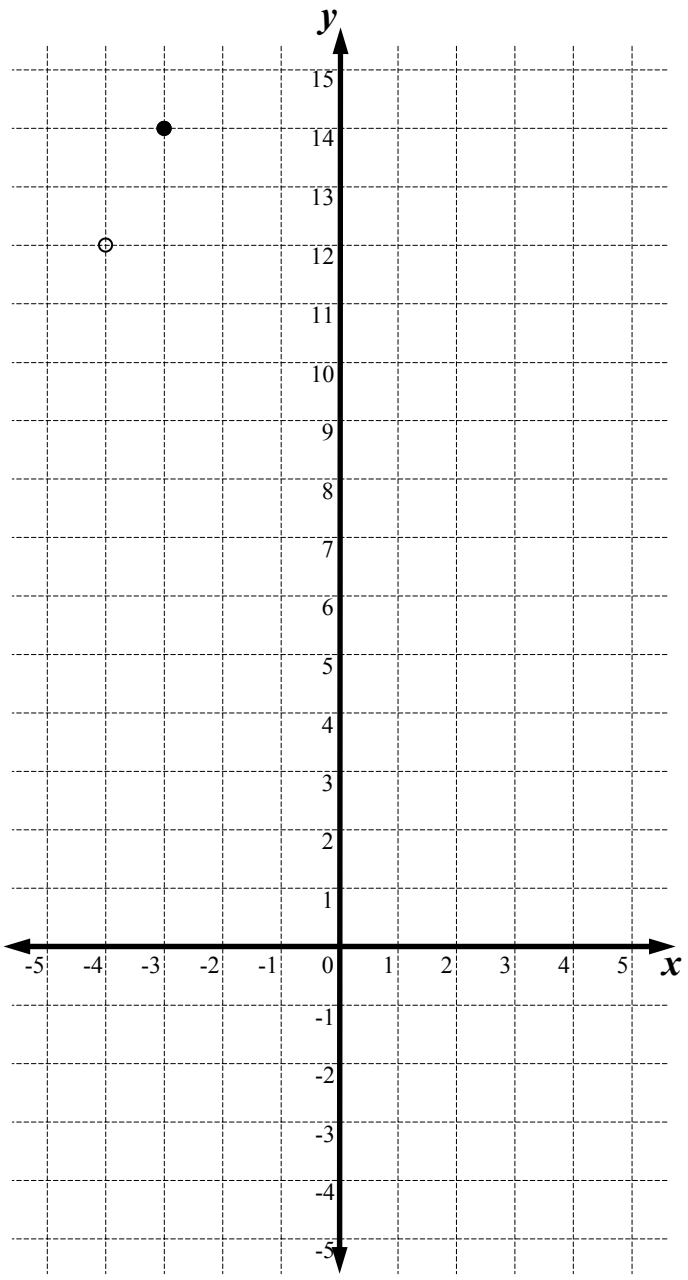
Calc: (- 4) x^2 - 4 =

Use the replay key to change '-4' to '-3' and so on.

2 $y = 2x^2 - 4$ ●

x	-3	-2	-1	0	1	2	3
y	14						

Calc: 2 × (- 3) x^2 - 4 =



3 $y = -x^2 + 6$ ○

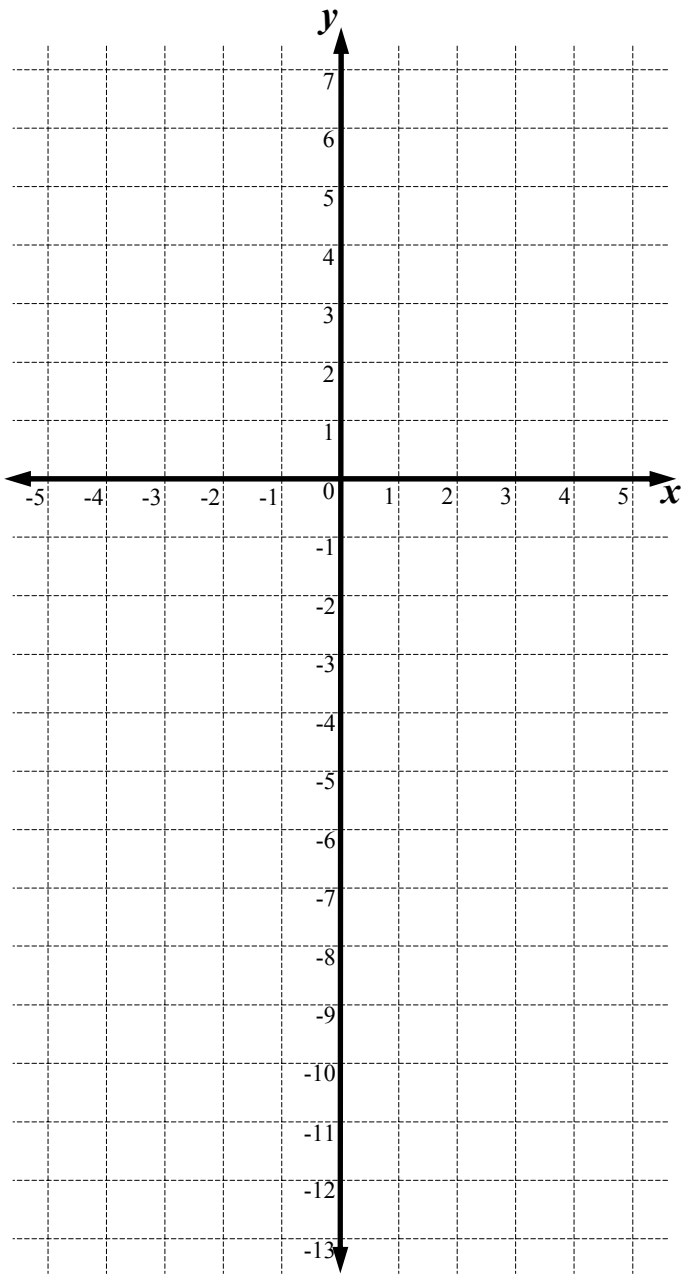
x	-4	-3	-2	-1	0	1	2	3	4
y	-10								

Calc: - (- 4) x^2 + 6 =

4 $y = -2x^2 + 6$ ●

x	-3	-2	-1	0	1	2	3
y	-12						

Calc: - 2 × (- 3) x^2 + 6 =



Indices - Substitution (Sheet 1)

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Use your calculator to evaluate these when $a = 5$.

Example $a^3 = 5^3 = 125$

Calc: 5 x^3 =

1 $a^4 = 5^4 = 625$

2 $2a^3 = 2 \times 5^3 = 250$

3 $3a^3 + 7 = 3 \times 5^3 + 7$

$= 382$

4 $3a^2 + 7a = 3 \times 5^2 + 7 \times 5$

$= 110$

Evaluate these using $n = 8$.

5 $n^3 - 10 = 8^3 - 10 = 502$

6 $4n^2 - n = 4 \times 8^2 - 8$

$= 248$

7 $-n^2 = -8^2 = -64$ Calc: - 8 x^2 =

8 $(-n)^2 = (-8)^2 = 64$

Calc: (- 8) x^2 =

Evaluate these using $y = -3$. Make sure you use brackets.

9 $y^2 - 12 = (-3)^2 - 12 = -3$

Calc: (- 3) x^2 - 1 2 =

10 $20 - y^3 = 20 - (-3)^2 = 47$

11 $3y^4 = 3 \times (-3)^4 = 243$

12 $4y^2 + 3y = 4 \times (-3)^2 + 3 \times (-3)$

$= 27$

13 $4y^2 \times 3y = 4 \times (-3)^2 \times 3 \times (-3)$

$= -324$

Evaluate these using $a = 4$ and $x = 3$.

14 $ax^2 = 4 \times 3^2 = 36$

15 $a^2 + x^2 = 4^2 + 3^2 = 25$

16 $a^2 - x^2 = 4^2 - 3^2 = 7$

17 $ax^2 + ax = 4 \times 3^2 + 4 \times 3$

$= 48$

18 $20x^2 \div a^2 = 20 \times 3^2 \div 4^2$

$= 11.25$

* 19 $\dfrac{20x^5}{ax} = \dfrac{20 \times 3^5}{4 \times 3} = 405$

* Brackets required

A table of values is often used to construct a graph. Complete these tables.

20 $b = a^2 + 2$

a	0	1	2	3	4	5
b	2	3	6	11	18	27

Calc: 1 x^2 + 2 =

Calc: 0 x^2 + 2 =

21 $y = 2x^2 + 5$

x	0	1	2	3	4	5
y	5	7	13	23	37	55

22 $y = -3x^2 - 7$

x	0	1	2	3	4	5
y	-7	-10	-19	-34	-55	-82

A frequent use of substitution is using a formula, such as with science. Substitute the values into the formulae to solve these questions.

The surface area of a cube is found using $SA = 6l^2$. Find the SA:

23 When $l = 15$.

$SA = 6l^2$

$= 6 \times 15^2$

$SA = 1\,350\ u^2$

24 When $l = 30$.

$SA = 6l^2$

$= 6 \times 30^2$

$SA = 5\,400\ u^2$

25 When $l = 8$.

$SA = 6l^2$

$= 6 \times 8^2$

$SA = 384\ u^2$

Power loss in a wire is found using $P = I^2R$. Find the loss, when:

26 $I = 2$ and $R = 0.5$

$P = I^2R$

$= 2^2 \times 0.5$

$P = 2$ W

27 $I = 10$ and $R = 6$.

$P = I^2R$

$= 10^2 \times 6$

$P = 600$ W

28 $I = 8$ and $R = 3$.

$P = I^2R$

$= 8^2 \times 3$

$P = 192$ W

Kinetic Energy is found using $KE = \frac{1}{2}mv^2$. Find the energy, when:

29 $m = 80$, $v = 15$.

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 80 \times 15^2$

$KE = 9\,000$ J

30 $m = 65$, $v = 18$.

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 65 \times 18^2$

$KE = 10\,530$ J

31 $m = 90$, $v = 25$.

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 90 \times 25^2$

$KE = 28\,125$ J

Indices - Substitution (Sheet 2)

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Complete the tables and then plot the points on the number plane below them.

1 $y = x^2 - 4$ ○

x	-4	-3	-2	-1	0	1	2	3	4
y	12	5	0	-3	-4	-3	0	5	12

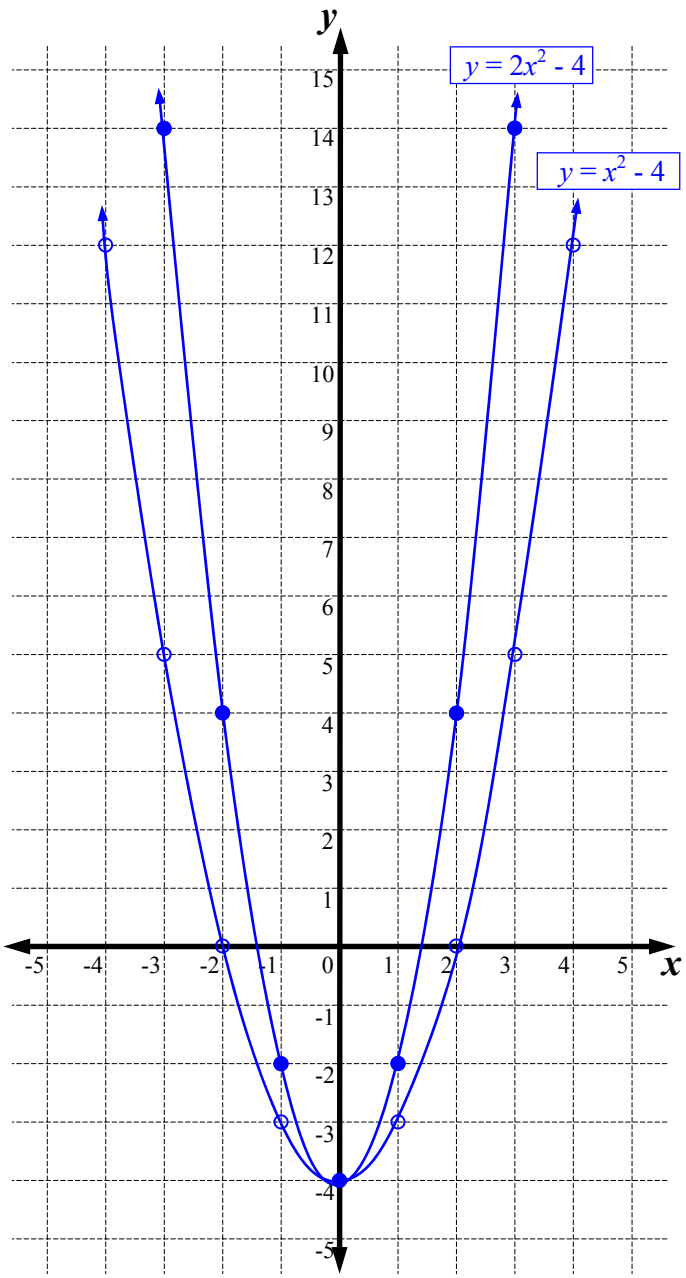
Calc: [(][-][4][)][x^2][-][4][=]

Use the replay key to change '-4' to '-3' and so on.

2 $y = 2x^2 - 4$ ●

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14

Calc: [2][×][(][-][3][)][x^2][-][4][=]



3 $y = -x^2 + 6$ ○

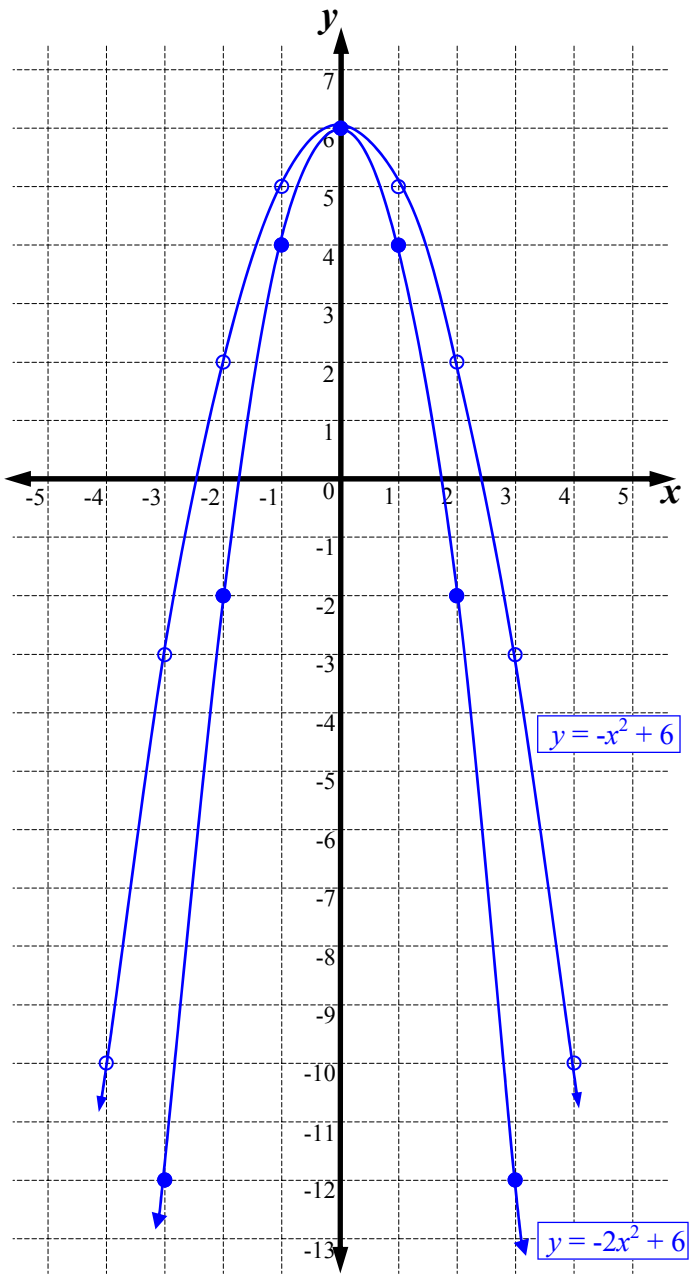
x	-4	-3	-2	-1	0	1	2	3	4
y	-10	-3	2	5	6	5	2	-3	-10

Calc: [-][(][-][4][)][x^2][+][6][=]

4 $y = -2x^2 + 6$ ●

x	-3	-2	-1	0	1	2	3
y	-12	-2	4	6	4	-2	-12

Calc: [-][2][×][(][-][3][)][x^2][+][6][=]



Indices - Substitution (Homework Strip)

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Date Due:

If $a = 4$, evaluate these

1 $a^2 =$ $\square^{\square}$ $=$

2 $5a^2 =$ $\square \times \square^{\square}$ $=$

3 $3a^4 + 20 =$

$=$

4 $7a^5 + 15a =$

$=$

If $a = 4$ and $b = 7$, evaluate these

5 $a^5 + b^2 =$ $=$

6 $10b^4 - 6a^2 =$

$=$

7 $150 - a^3b =$

$=$

8 $a^4b - a^2b^2 =$

$=$

Date Due:

If $a = 4$, evaluate these

1 $a^2 =$ $\square^{\square}$ $=$

2 $5a^2 =$ $\square \times \square^{\square}$ $=$

3 $3a^4 + 20 =$

$=$

4 $7a^5 + 15a =$

$=$

If $a = 4$ and $b = 7$, evaluate these

5 $a^5 + b^2 =$ $=$

6 $10b^4 - 6a^2 =$

$=$

7 $150 - a^3b =$

$=$

8 $a^4b - a^2b^2 =$

$=$

Date Due:

If $a = 4$, evaluate these

1 $a^2 =$ $\square^{\square}$ $=$

2 $5a^2 =$ $\square \times \square^{\square}$ $=$

3 $3a^4 + 20 =$

$=$

4 $7a^5 + 15a =$

$=$

If $a = 4$ and $b = 7$, evaluate these

5 $a^5 + b^2 =$ $=$

6 $10b^4 - 6a^2 =$

$=$

7 $150 - a^3b =$

$=$

8 $a^4b - a^2b^2 =$

$=$

Date Due:

If $a = 4$, evaluate these

1 $a^2 =$ $\square^{\square}$ $=$

2 $5a^2 =$ $\square \times \square^{\square}$ $=$

3 $3a^4 + 20 =$

$=$

4 $7a^5 + 15a =$

$=$

If $a = 4$ and $b = 7$, evaluate these

5 $a^5 + b^2 =$ $=$

6 $10b^4 - 6a^2 =$

$=$

7 $150 - a^3b =$

$=$

8 $a^4b - a^2b^2 =$

$=$

Indices - Substitution (Homework Strip)

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Date Due:

If $a = 4$, evaluate these

1 $a^2 = 4^2 = 16$

2 $5a^2 = 5 \times 4^2 = 80$

3 $3a^4 + 20 = 3 \times 4^4 + 20$

$= 788$

4 $7a^5 + 15a = 7 \times 4^5 + 15 \times 4$

$= 7\,228$

If $a = 4$ and $b = 7$, evaluate these

5 $a^5 + b^2 = 4^5 + 7^2 = 1\,073$

6 $10b^4 - 6a^2 = 10 \times 7^4 - 6 \times 4^2$

$= 23\,914$

7 $150 - a^3b = 150 - 4^3 \times 7$

$= -298$

8 $a^4b - a^2b^2 = 4^4 \times 7 - 4^2 \times 7^2$

$= 1\,008$

Indices - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Indices - Find a Word

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Find the words in the puzzle from the wordlist.



M	E	X	P	P	B	O	L	E	C	I	D	N	I	Y
U	T	X	X	S	I	M	P	L	E	S	T	V	L	R
L	A	A	E	D	I	V	I	D	T	N	M	P	A	E
T	U	L	X	X	C	A	N	C	E	L	U	Y	T	S
I	L	G	P	N	P	E	V	I	R	L	P	U	A	U
P	A	E	Y	L	O	L	C	X	M	L	T	A	V	L
L	V	B	I	Y	S	I	M	P	L	I	F	Y	L	T
Y	E	R	N	L	F	K	S	P	T	T	S	A	E	M
P	X	A	D	F	V	E	E	S	P	C	E	G	V	E
D	R	R	E	W	O	P	B	X	E	A	I	G	I	E
N	E	O	X	P	D	U	E	L	U	R	L	A	T	S
A	C	E	V	E	S	D	L	V	A	T	P	E	A	I
P	P	S	T	E	K	C	A	R	B	B	P	X	G	V
X	L	Y	Z	E	R	O	E	X	P	U	A	P	E	E
E	U	L	A	V	P	L	Y	B	A	S	E	E	N	R

WORDLIST

POWER	VALUE	INDEX	SUBTRACT	TERM
ADD	BASE	RULE	CANCEL	LIKE
ZERO	SIMPLEST	ALGEBRA	RESULT	INDICE
NEGATIVE	BRACKETS	PROVE	SIMPLIFY	EVALUATE
EXPAND	REVISE	MULTIPLY	APPLIES	DIVIDE
	SUBSTITUTE	EXPRESSION	COEFFICIENT	

Indices - Find a Word

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Find the words in the puzzle from the wordlist.



M	E	X	P	P	B	O	L	E	C	I	D	N	I	Y
U	T	X	X	S	I	M	P	L	E	S	T	V	L	R
L	A	A	E	D	I	V	I	D	T	N	M	P	A	E
T	U	L	X	X	C	A	N	C	E	L	U	Y	T	S
I	L	G	P	N	P	E	V	I	R	L	P	U	A	U
P	A	E	Y	L	O	L	C	X	M	L	T	A	V	L
L	V	B	I	Y	S	I	M	P	L	I	F	Y	L	T
Y	E	R	N	L	F	K	S	P	T	T	S	A	E	M
P	X	A	D	F	V	E	E	S	P	C	E	G	V	E
D	R	R	E	W	O	P	B	X	E	A	I	G	I	E
N	E	O	X	P	D	U	E	L	U	R	L	A	T	S
A	C	E	V	E	S	D	L	V	A	T	P	E	A	I
P	P	S	T	E	K	C	A	R	B	B	P	X	G	V
X	L	Y	Z	E	R	O	E	X	P	U	A	P	E	E
E	U	L	A	V	P	L	Y	B	A	S	E	E	N	R

WORDLIST

POWER
ADD
ZERO
NEGATIVE
EXPAND
VALUE
BASE
SIMPLEST
BRACKETS
REVISE
SUBSTITUTE
INDEX
RULE
ALGEBRA
PROVE
MULTIPLY
EXPRESSION
SUBTRACT
CANCEL
RESULT
SIMPLIFY
APPLIES
COEFFICIENT
TERM
LIKE
INDICE
EVALUATE
DIVIDE

Indices Review

This a review of the Indices topic.

Indices - Topic Review

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Use your power key to evaluate these indices.

1 $8^4 =$ _____

2 $2^9 =$ _____

3 $5^{-1} =$ _____ Decimal

4 $9^0 =$ _____

Write these indices in expanded form

5 $e^3 =$ _____

6 $5^4 =$ _____

7 $n^6 =$ _____

Write the indice that has a:

8 Base 'a' and an index of '5'
$=$ _____

9 Index of '4' and a base 'm'
$=$ _____

10 Index of '0' and a base '5'
$=$ _____

Multiply these indices

11 $c^2 \times c^3 =$ _____

12 $a \times a^5 =$ _____

13 $b^4 \times b^2 =$ _____

14 $2^7 \times 2^5 =$ _____ Leave as indice

15 $n^5 \times n \times n^2 =$ _____

16 $5 \times 5^3 \times 5^2 =$ _____ Leave as indice

17 $k^5 \times k \times k^3 =$ _____

18 $e^3 \times e^0 \times e^2 =$ _____

19 $n \times n^0 \times n =$ _____

Multiply these indices

20 $ae \times a =$ _____

21 $xy^2 \times x^2 =$ _____

22 $n^3p^2 \times n^4p^2 =$ _____

23 $a^3b^2c \times a^2c =$ _____

21 $w^3n^3e^2 \times w^4ne^5 =$ _____

These include coefficients. Multiply.

22 $6n^2 \times n =$ _____

22 $3a^3 \times a^3 =$ _____

22 $t \times 9t^5 =$ _____

22 $d^2 \times 7d^2 =$ _____

23 $b^2 \times 4c^2b =$ _____

24 $3ab \times ba^2 =$ _____

25 $y^2 \times 7a^3y =$ _____

26 $2p^3d^2 \times 3p =$ _____

27 $4yu^2 \times 2u =$ _____

28 $6h^7v^4 \times 5h^2 =$ _____

29 $5ac^2 \times 2ac =$ _____

30 $2u^3v^2 \times 6uv^4 =$ _____

31 $4e^4k \times 3e^2k^3 =$ _____

Divide these indices.

32 $u^4 \div u^2 =$ _____

33 $a^5 \div a =$ _____

34 $7^5 \div 7^2 =$ _____ Leave as indice

35 $\frac{p^9}{p^2} =$ _____

36 $\frac{u^5}{u^2} =$ _____

These require '1 over' answers.

37 $\frac{n^2}{n^4} =$ _____

38 $\frac{v^4}{v^5} =$ _____

39 $\frac{a}{a^5} =$ _____

40 $\frac{g^3}{g^6} =$ _____

41 $\frac{n^2}{2n^5} =$ _____

42 $\frac{c^5}{5c^5} =$ _____

These divisions can be simplified.

43 $\frac{4x^2}{2x^5} =$ _____

44 $\frac{3e^4}{9e} =$ _____

45 $\frac{6x}{8x} =$ _____

46 $\frac{5u^5}{10u^3} =$ _____

47 $\frac{6y^7}{4y^2} =$ _____

48 $\frac{6n^4}{2n} =$ _____

Multiply first then divide.

49 $\frac{a^2 \times a^3}{a} = \frac{\square}{\square} =$ _____

50 $\frac{h^4 \times h}{h^7} = \frac{\square}{\square} =$ _____

51 $\frac{k^3 \times k^2}{k \times k^4} = \frac{\square}{\square} =$ _____

52 $\frac{3a^2 \times a}{a^3 \times 4a^3} = \frac{\square}{\square} =$ _____

53 $\frac{4g^3 \times g^4}{g \times 2g^2} = \frac{\square}{\square} =$ _____

54 $\frac{2n \times 3n^5}{6n^3 \times n^3} = \frac{\square}{\square} =$ _____

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Simplify these.

55 $(d^4)^2 =$ ____ 56 $(x^2)^6 =$ ____

57 $(g^3)^5 =$ ____ 58 $(h^2)^6 =$ ____

59 $(2n^3)^2 =$ ____

60 $(3k^5)^2 =$ ____

Simplify these. Calculator required.

61 $(5m^3)^4 =$ ____

62 $(2d)^9 =$ ____

63 $(8a^6)^3 =$ ____

Simplify these.

64 $(xv^3)^4 =$ ____

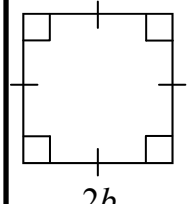
65 $(q^2w^3)^3 =$ ____

66 $(3gk^3)^3 =$ ____

67 $(2e^5c^3)^5 =$ ____

Find the area of these.

68

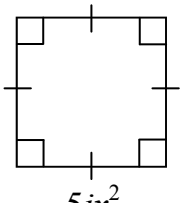


$A = l^2$

$= \square^2$

$A = \square$ units^2

69



Simplify these.

70 $(m^3)^4 \times (m^2)^3 = \square \times \square$

$=$ ____

71 $(ac^2)^5 \times (a^3c^2)^4$

$= \square \times \square$

$=$ ____

72 $(2x^3y^2)^3 \times (3xy^2)^2$

$= \square \times \square$

$=$ ____

Simplify these.

73 $\dfrac{(e^5k^4)^2}{(e^2k)^3} = \dfrac{\square}{\square} =$ ____

74 $\dfrac{(a^3n^4)^3}{(a^5n)^2} = \dfrac{\square}{\square} =$ ____

Simplify these, involving zero index.

75 $\dfrac{a^3}{a^3} = \square = \square$

76 $h^5 \div h^5 = \square = \square$

77 $\dfrac{v^{11}}{v^{11}} = \square = \square$

78 $b^3 \times b^0 = \square$

79 $y^5 \times y^0 = \square$

80 $(b^0)^3 = \square = \square$

81 $(v^0)^4 = \square = \square$

82 $(q^2)^0 = \square = \square$

83 $(2c^4)^0 = \square$

84 $(2a^0k^2)^2 = \square$

Evaluate these.

85 $2v^0 = \square \times \square = \square$

86 $h^0 + 7 = \square + \square = \square$

87 $2a^0 + 3 =$ ____

88 $3q^0 + 5 =$ ____

Evaluate, given $a = 6$.

89 $a^{\square} = \square$

90 $a^3 + 8 =$ ____

91 $3a^3 \div 12 =$ ____

$=$ ____

92 $5a^4 + 7a^2 =$ ____

____

Evaluate, given $x = 2$ and $y = -3$.

93 $x^2y =$ ____

94 $xy^2 =$ ____

95 $a^7 + 7y =$ ____

$=$ ____

Complete the tables.

96 $y = x^2 + 5$

x	0	1	2	3	4	5
y						

97 $y = 2x^2 - 10$

x	0	1	2	3	4	5
y						

Indices - Topic Review

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Use your power key to evaluate these indices.

1 $8^4 = 4\,096$ 2 $2^9 = 512$

3 $5^{-1} = 0.2$ Decimal 4 $9^0 = 1$

Write these indices in expanded form

5 $e^3 = e \times e \times e$

6 $5^4 = 5 \times 5 \times 5 \times 5$

7 $n^6 = n \times n \times n \times n \times n \times n$

Write the indice that has a:

8 Base 'a' and an index of '5' $= a^5$

9 Index of '4' and a base 'm' $= m^4$

10 Index of '0' and a base '5' $= 5^0$

Multiply these indices

11 $c^2 \times c^3 = c^5$

12 $a \times a^5 = a^6$

13 $b^4 \times b^2 = b^6$

14 $2^7 \times 2^5 = 2^{12}$ Leave as indice

15 $n^5 \times n \times n^2 = n^8$

16 $5 \times 5^3 \times 5^2 = 5^6$ Leave as indice

17 $k^5 \times k \times k^3 = k^9$

18 $e^3 \times e^0 \times e^2 = e^5$

19 $n \times n^0 \times n = n^2$

Multiply these indices

20 $ae \times a = a^2e$

21 $xy^2 \times x^2 = x^3y^2$

22 $n^3p^2 \times n^4p^2 = n^7p^4$

23 $a^3b^2c \times a^2c = a^5b^2c^2$

21 $w^3n^3e^2 \times w^4ne^5 = w^7n^4e^7$

These include coefficients. Multiply.

22 $6n^2 \times n = 6n^3$

22 $3a^3 \times a^3 = 3a^6$

22 $t \times 9t^5 = 9t^6$

22 $d^2 \times 7d^2 = 7d^4$

23 $b^2 \times 4c^2b = 4b^3c^2$

24 $3ab \times ba^2 = 3a^3b^2$

25 $y^2 \times 7a^3y = 7a^3y^3$

26 $2p^3d^2 \times 3p = 6p^4d^2$

27 $4yu^2 \times 2u = 8u^3y$

28 $6h^7v^4 \times 5h^2 = 30h^9v^4$

29 $5ac^2 \times 2ac = 10a^2c^3$

30 $2u^3v^2 \times 6uv^4 = 12u^4v^6$

31 $4e^4k \times 3e^2k^3 = 12e^6k^4$

Divide these indices.

32 $u^4 \div u^2 = u^2$

33 $a^5 \div a = a^4$

34 $7^5 \div 7^2 = 7^3$ Leave as indice

35 $\frac{p^9}{p^2} = p^7$ 36 $\frac{u^5}{u^2} = u^3$

These require '1 over' answers.

37 $\frac{n^2}{n^4} = \frac{1}{n^2}$ 38 $\frac{v^4}{v^5} = \frac{1}{v}$

39 $\frac{a}{a^5} = \frac{1}{a^4}$ 40 $\frac{g^3}{g^6} = \frac{1}{g^3}$

41 $\frac{n^2}{2n^5} = \frac{1}{2n^3}$ 42 $\frac{c^5}{5c^5} = \frac{1}{5}$

These divisions can be simplified.

43 $\frac{4x^2}{2x^5} = \frac{2}{x^3}$ 44 $\frac{3e^4}{9e} = \frac{e^3}{3}$

45 $\frac{6x}{8x} = \frac{3}{4}$ 46 $\frac{5u^5}{10u^3} = \frac{u^2}{2}$

47 $\frac{6y^7}{4y^2} = \frac{3y^5}{2}$ 48 $\frac{6n^4}{2n} = 3n^3$

Multiply first then divide.

49 $\frac{a^2 \times a^3}{a} = \frac{a^5}{a} = a^4$

50 $\frac{h^4 \times h}{h^7} = \frac{h^5}{h^7} = \frac{1}{h^2}$

51 $\frac{k^3 \times k^2}{k \times k^4} = \frac{k^5}{k^5} = k^0 = 0$

52 $\frac{3a^2 \times a}{a^3 \times 4a^3} = \frac{3a^3}{4a^6} = \frac{3}{4a^3}$

53 $\frac{4g^3 \times g^4}{g \times 2g^2} = \frac{4g^7}{2g^3} = 2g^4$

54 $\frac{2n \times 3n^5}{6n^3 \times n^3} = \frac{6n^6}{6n^6} = 1$

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Simplify these.

55 $(d^4)^2 = d^8$ 56 $(x^2)^6 = x^{12}$

57 $(g^3)^5 = g^{15}$ 58 $(h^2)^6 = h^{12}$

59 $(2n^3)^2 = 4n^6$

60 $(3k^5)^2 = 9k^{10}$

Simplify these. Calculator required.

61 $(5m^3)^4 = 625m^{12}$

62 $(2d)^9 = 512d^9$

63 $(8a^6)^3 = 512a^{18}$

Simplify these.

64 $(xv^3)^4 = xv^{12}$

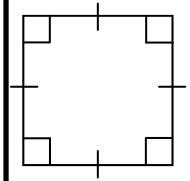
65 $(q^2w^3)^3 = q^6w^9$

66 $(3gk^3)^3 = 27g^3k^9$

67 $(2e^5c^3)^5 = 32e^{25}c^{15}$

Find the area of these.

68



$A = l^2$

$= (2b)^2$

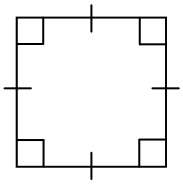
$A = 4b^2$ units^2

69

$A = l^2$

$= (5jx^2)^2$

$A = 25j^2x^4$ units^2



Simplify these.

70 $(m^3)^4 \times (m^2)^3 = m^{12} \times m^6$

$= m^{18}$

71 $(ac^2)^5 \times (a^3c^2)^4$

$= a^5c^{10} \times a^{12}c^8$

$= a^{17}c^{18}$

72 $(2x^3y^2)^3 \times (3xy^2)^2$

$= 8x^9y^6 \times 9x^2y^4$

$= 72x^{11}y^{10}$

Simplify these.

73 $\dfrac{(e^5k^4)^2}{(e^2k)^3} = \dfrac{e^{10}k^8}{e^6k^3} = e^4k^5$

74 $\dfrac{(a^3n^4)^3}{(a^5n)^2} = \dfrac{a^9n^{12}}{a^{10}n^2} = \dfrac{n^{10}}{a}$

Simplify these, involving zero index.

75 $\dfrac{a^3}{a^3} = a^0 = 1$

76 $h^5 \div h^5 = h^0 = 1$

77 $\dfrac{v^{11}}{v^{11}} = v^0 = 1$

78 $b^3 \times b^0 = b^3$

79 $y^5 \times y^0 = y^5$

80 $(b^0)^3 = b^0 = 1$

81 $(v^0)^4 = v^0 = 1$

82 $(q^2)^0 = q^0 = 1$

83 $(2c^4)^0 = 1$

84 $(2a^0k^2)^2 = 4k^4$

Evaluate these.

85 $2v^0 = 2 \times 1 = 2$

86 $h^0 + 7 = 1 + 7 = 8$

87 $2a^0 + 3 = 2 \times 1 + 3 = 5$

88 $3q^0 + 5 = 3 \times 1 + 5 = 8$

Evaluate, given $a = 6$.

89 $a^2 = 6^2 = 36$

90 $a^3 + 8 = 6^3 + 8 = 224$

91 $3a^3 \div 12 = 3 \times 6^3 \div 12$

$= 54$

92 $5a^4 + 7a^2 = 5 \times 6^4 + 7 \times 6^2$

$= 6\,732$

Evaluate, given $x = 2$ and $y = -3$.

93 $x^2y = 2^2 \times (-3) = -12$

94 $xy^2 = 2 \times (-3)^2 = 18$

95 $a^7 + 7y = 2^7 + 7 \times (-3)$

$= 107$

Complete the tables.

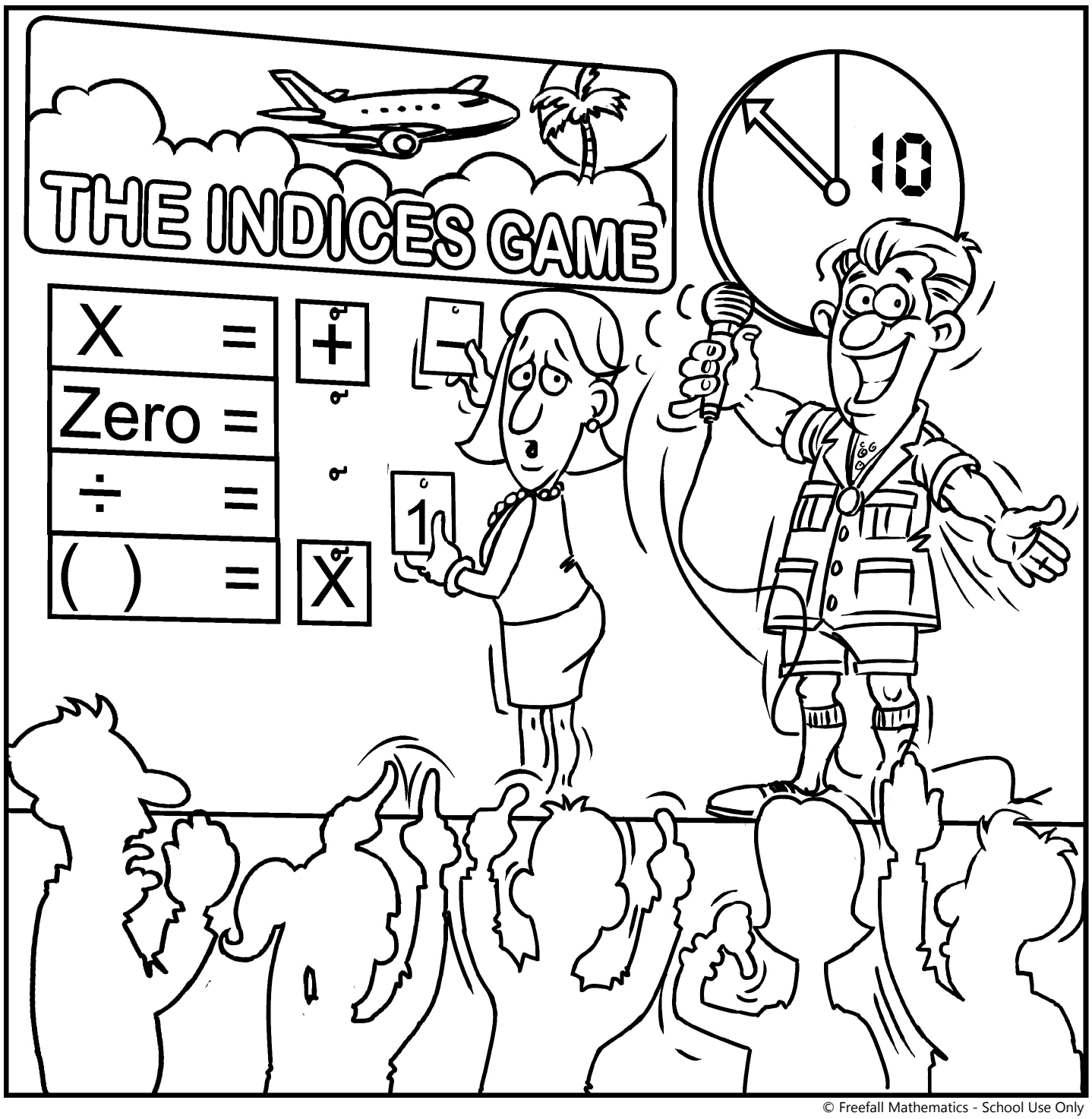
96 $y = x^2 + 5$

x	0	1	2	3	4	5
y	5	6	9	14	21	30

97 $y = 2x^2 - 10$

x	0	1	2	3	4	5
y	-10	-8	-2	8	22	40

Indices



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Indices - Power Key Applications (Sheet 1)

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Use your power key to evaluate (find the value for) these.

Example $5^4 = 625$

Calc: 5 $x^{\blacksquare}$ 4 =

1 $3^3 =$ ____ 2 $2^5 =$ ____

3 $5^2 =$ ____ 4 $7^3 =$ ____

5 $4^4 =$ ____ 6 $3^5 =$ ____

7 $9^5 =$ ____

8 $15^4 =$ ____

9 $10^1 =$ ____ 10 $10^2 =$ ____

11 $10^3 =$ ____

12 $10^4 =$ ____

13 $10^5 =$ ____

14 $10^6 =$ ____

15 $1^1 =$ ____ 16 $1^2 =$ ____

17 $1^3 =$ ____ 18 $1^4 =$ ____

Name the base and the index in these.

Example 5^4 — 5 (Base), 4 (Index)

Base is the larger number out front

Index is the small number (the power)

19 6^7 Base ____ Index ____

20 9^8 Base ____ Index ____

21 7^6 Index ____ Base ____

22 13^5 Index ____ Base ____

Given the base and index, write the indices and evaluate.

Example Base 9, index 4.

$9^4 = 6\,561$

23 Base 4, index 6.

____ = ____

Calc: 4 $x^{\blacksquare}$ 6 =

24 Base 2, index 8.

____ = ____

25 Index 4, base 5.

____ = ____

26 Base 9, index 4.

____ = ____

27 Index 5, base 10.

____ = ____

Use guess, check and revise to find the correct index for these.

28 $2^{\square} = 128$ 29 $3^{\square} = 27$

30 $7^{\square} = 823\,543$

31 $5^{\square} = 48\,828\,125$

32 $13^{\square} = 28\,561$

33 $10^{\square} = 10\,000\,000$

34 $20^{\square} = 8\,000$

35 $2^{\square} = 262\,144$

36 $3^{\square} = 177\,147$

Use guess, check and revise to find the correct base for these.

37 $\square^3 = 64$ 38 $\square^5 = 243$

39 $\square^7 = 128$ 40 $\square^3 = 729$

41 $\square^4 = 83\,521$

42 $\square^9 = 19\,683$

43 $\square^4 = 10\,000$

44 $\square^6 = 64\,000\,000$

45 $\square^3 = 1$ 46 $\square^8 = 1$

Blend the two skills by finding both the base and the index!

47 $2^{\square} = 256 = \square^4$

48 $9^{\square} = 531\,441 = \square^{12}$

49 $\square^4 = 4\,096 = 2^{\square}$

50 $\square^5 = 1\,048\,576 = 2^{\square}$

51 $4^{\square} = 16\,777\,216 = \square^6$

Use '<' or '>' to make these true. Hint: Point the '<' or '>' to the smallest number.

52 4^5 ____ 2^9

53 7^4 ____ 4^7

Indices - Power Key Applications (Sheet 2)

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Use your power key to evaluate these indices, answer these as a fraction.

Example $5^{-2} = \frac{1}{25}$

Calc: 5 $x^{\blacksquare}$ - 2 = S⇔D

1 $3^{-3} =$ ☐/☐ 2 $2^{-5} =$ ☐/☐

3 $5^{-3} =$ ☐/☐ 4 $4^{-4} =$ ☐/☐

5 $2^{-3} =$ ☐/☐ 6 $3^{-5} =$ ☐/☐

7 $10^{-3} =$ ☐/☐

8 $10^{-5} =$ ☐/☐

A negative power means '1 over', so rewrite as a fraction, then evaluate.

Rewrite as '1 over' Use your calculator

9 $2^{-4} = \frac{1}{2^4} =$ ☐/☐

Calc: 1 $\frac{\blacksquare}{\square}$ 2 $x^{\blacksquare}$ 4 =

10 $5^{-3} = \frac{1}{5^3} =$ ☐/☐

Calc: 1 $\frac{\blacksquare}{\square}$ 5 $x^{\blacksquare}$ 3 =

11 $3^{-3} = \frac{1}{\square} = \frac{1}{\square}$

12 $6^{-2} =$ ☐/☐ = ☐/☐

13 $10^{-3} =$ ☐/☐ = ☐/☐

14 $6^{-5} =$ ☐/☐ = ☐/☐

15 $2^{-8} =$ ☐/☐ = ☐/☐

16 $3^{-6} =$ ☐/☐ = ☐/☐

17 $4^{-5} =$ ☐/☐ = ☐/☐

Remove the fraction by writing these as negative indices. No calculator required!

Example $\frac{1}{2^5} = 2^{-5}$

18 $\frac{1}{2^3} =$ ☐☐

19 $\frac{1}{7^2} =$ ☐☐

20 $\frac{1}{5^8} =$ ☐☐

Use your power key to complete these tables.

21

Indice	3^5	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}
Number/ Fraction	☐	☐	☐	☐	☐	☐	☐/☐	☐/☐	☐/☐	☐/☐

22 What is the value of 3^0 in the above table? ☐

23

Indice	5^5	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}
Number/ Fraction							☐/☐	☐/☐	☐/☐

24 What is the value of 5^0 in the above table? ☐

Use patterns for these tables. For the first table, use your calculator for 2^6, then divide by 2 each time. For the second table use your calculator for 10^4, then divide by 10 each time. Only use the power key once!

25

Indice	2^6	2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
Number/ Fraction											

÷ 2 ÷ 2 and so on, using the pattern to complete the boxes

26 What is the value of 2^0 in the above table? ☐

27 What is the value of any number to the power of zero? ☐

28

Indice	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
Number/ Fraction								

÷ 10 ÷ 10 and so on, using the pattern to complete

Indices - Multiplying Indices (Sheet 1)

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Rewrite in expanded form.

Example $5^4 = 5 \times 5 \times 5 \times 5$

1 $3^5 = \square \times \square \times \square \times \square \times \square$

2 $15^3 =$

3 $9^4 =$

4 $1^6 =$

Now algebraic indices - same process

5 $y^4 = \square \times \square \times \square \times \square$

6 $a^5 =$

7 $k^6 =$

8 $b^2 =$

Use expansion to answer these in index form. No calculator.

Example $5^2 \times 5^4 = \overbrace{5 \times 5}^{5^2} \times \overbrace{5 \times 5 \times 5 \times 5}^{5^4} = 5^6$

9 $7^2 \times 7^2 = \square \times \square \times \square \times \square$

$7^2 \times 7^2 = \square^{\square}$

10 $3^3 \times 3^2 =$

$3^3 \times 3^2 = \square^{\square}$

11 $4^2 \times 4^4 =$

$4^2 \times 4^4 = \square^{\square}$

Now algebraic indices - same process

12 $x^2 \times x^3 =$

$x^2 \times x^3 = \square^{\square}$

13 $a \times a^3 =$

$a \times a^3 = \square^{\square}$

You can multiply indices by adding the indexes. Multiply these, leave answer in index form

Example $5^2 \times 5^4 = 5^{2+4} = 5^6$

14 $7^4 \times 7^2 = \square^{\square+\square} = \square^{\square}$

15 $9^3 \times 9^5 = \square^{\square+\square} = \square^{\square}$

No power - it's like there is a '1' there

16 $4^6 \times 4 = \square^{\square+\square} = \square^{\square}$

17 $5 \times 5^2 = \square^{\square+\square} = \square^{\square}$

18 $7 \times 7 = \square^{\square+\square} = \square^{\square}$

19 $3^9 \times 3^7 = \square^{\square+\square} = \square^{\square}$

Now algebraic indices - same process

20 $x^3 \times x^3 = \square^{\square+\square} = \square^{\square}$

21 $n^5 \times n = \square^{\square+\square} = \square^{\square}$

22 $a \times a^3 = \square^{\square+\square} = \square^{\square}$

23 $h^8 \times h^4 = \square^{\square+\square} = \square^{\square}$

Forget the middle step, answer only. Triples are just the same.

24 $5^8 \times 5^4 = \square^{\square}$

25 $7^3 \times 7^9 = \square^{\square}$

26 $7^3 \times 7^9 \times 7^2 = \square^{\square}$

27 $3 \times 3^6 \times 3^3 = \square^{\square}$

28 $e^4 \times e^3 \times e^5 = \square^{\square}$

29 $k \times k^5 \times k = \square^{\square}$

30 $h^2 \times h \times h^7 = \square^{\square}$

We now have unlike terms. At last, maths questions that have two correct answers! Simplify.

Example $a^2 \times b^2 \times a^4 \times b = a^6b^3$ *or* b^3a^6

31 $a^6 \times b =$ ____ *or* ____

32 $c^4 \times b^5 =$ ____ *or* ____

33 $k^9 \times t^3 =$ ____ *or* ____

Just one answer from now on

34 $x^3 \times a^5 \times x^2 \times a =$

35 $y^2 \times c^3 \times y^4 \times y^5 =$

36 $a^5 \times b^2 \times a \times b =$

37 $d \times d^2 \times c \times d^5 =$

Less '×' signs - but the same process

38 $bc \times b^2 =$ ____ *or* ____

39 $ay \times y^3 =$ ____ *or* ____

40 $g^3k \times k^3g^2 =$

41 $ab \times ab =$

42 $abc \times abc =$

43 $b^2ac^3 \times acb =$

44 $c^4a^2b^3 \times b^4c^3a^6 =$

45 $ab^3c \times a^2b \times ab^2 =$

Which of these is true?

46 $m^3 \times m^2 =$ ○ m^6 ○ m^5

47 $e^6 \times e =$ ○ e^6 ○ e^7 ○ e^0 ○ e

48 $a^2 \times a^5b =$ ○ $a^{10}b$ ○ a^7b^5 ○ a^2b^5 ○ a^7b

Indices - Multiplying Indices (Sheet 2)

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Coefficients are numbers in front of algebraic terms. Multiply the coefficients, but add the powers.

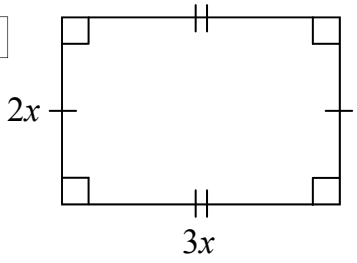
Example $2x \times 3x^2 = 6x^3$

Simplify these.

1. $2a^5 \times a^2 =$ _____
2. $c^2 \times 4c =$ _____
3. $a^3b^2 \times 3a^2 =$ _____
4. $5x^4y^2 \times xy =$ _____
5. $4n^3 \times 5n^2 =$ _____
6. $6j^7 \times 4j =$ _____
7. $2h^3 \times 9ah^2 =$ _____
8. $3s^2 \times 4sv^3 =$ _____
9. $2e^2h \times 8h^2e^4 =$ _____
10. $3x^2 \times 2x^3 \times 4y^2 =$ _____

The area of a rectangle is found by multiplying the two sides. Find the area of these.

11.

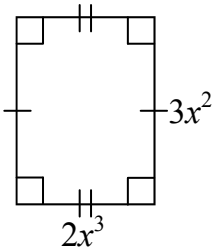


A = lb

= ____ × ____

A = ____ units^2

12.

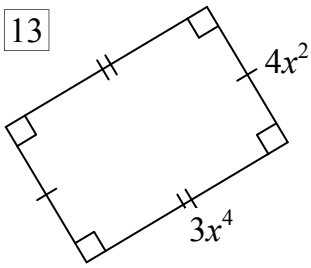


A = lb

= ____ × ____

A = ____ units^2

13.

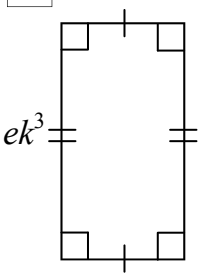


A = lb

= ____ × ____

A = ____ units^2

14.

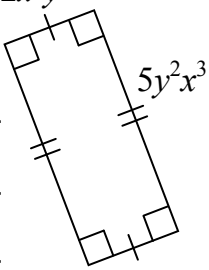


_____

_____

_____

15.



_____

_____

_____

When you square a term you multiply it by itself. Square these terms.

Example

$(3x)^2 = 3x \times 3x = 9x^2$

16. $(4y)^2 =$ ____ × ____ = ____
17. $(2a)^2 =$ ____ × ____ = ____
18. $(2a^2)^2 =$ ____ × ____ = ____
19. $(3a^3)^2 =$ ____ × ____ = ____
20. $(5a^2b)^2 =$ ____ × ____

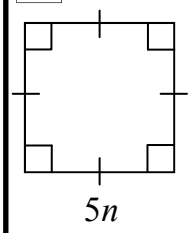
= ____

21. $(2x^3y^5)^2 =$ ____ × ____

= ____

The area of a square is found by squaring the sides. Find the area of these squares.

22.

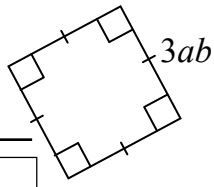


A = l^2

= ____ × ____

A = ____ units^2

23.

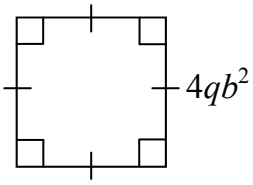


A = l^2

= ____ × ____

A = ____ units^2

24.



_____

_____

_____

Complete the two times tables below.

25.

×	a	a^2b	ab^2	a^2b^2
a				
$5a$				
$2ab$				

26.

×	y^3	$3x^2$	$2x$
$2x$			
y^3			
xy			
x^2y^2			
$3x^3y^2$			

Indices - Dividing Indices (Sheet 1)

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Rewrite in expanded form.

Example $5^4 = 5 \times 5 \times 5 \times 5$

1 $7^5 = \square \times \square \times \square \times \square \times \square$

2 $4^6 =$

3 $3^4 =$

4 $2^3 =$

Now algebraic indices - same process

5 $k^3 = \square \times \square \times \square$

6 $b^4 =$

7 $m^5 =$

8 $c^2 =$

Use expansion and cancellation to answer these in index form.

Example

$5^5 \div 5^2 = \dfrac{\cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}{\cancel{5} \times \cancel{5}}$

Cancel bottom with top

$= 5 \times 5 \times 5 = 5^3$

9 $3^6 \div 3^3 =$

10 $\dfrac{y^6}{y^2} =$

11 $a^3 \div a =$

Use expansion and cancellation to answer these, write a '1' on the top line when fully cancelled

Example

$5^2 \div 5^5 = \dfrac{\cancel{5} \times \cancel{5}^{\,1}}{\cancel{5} \times \cancel{5} \times 5 \times 5 \times 5}$

Cancel bottom with top

$= \dfrac{1}{5 \times 5 \times 5} = \dfrac{1}{5^3}$

12 $7^3 \div 7^5 =$

13 $k \div k^5 =$

14 $\dfrac{n^2}{n^4} =$

You can divide indices by subtracting the indexes. Divide these, bigger power - smaller

Example $5^7 \div 5^4 = 5^{7-4} = 5^3$

15 $3^5 \div 3^2 = \square^{\square - \square} = \square^{\square}$

16 $4^9 \div 4^4 = \square^{\square - \square} = \square^{\square}$

No power - it's like there is a '1' there

17 $8^6 \div 8 = \square^{\square - \square} = \square^{\square}$

18 $\dfrac{3^5}{3} = \square^{\square - \square} = \square^{\square}$

19 $6^8 \div 6 = \square^{\square - \square} = \square^{\square}$

Now algebraic indices - same process

20 $x^3 \div x = \square^{\square - \square} = \square^{\square}$

21 $\dfrac{n^7}{n^3} = \square^{\square - \square} = \square^{\square}$

22 $b^8 \div b^6 = \square^{\square - \square} = \square^{\square}$

23 $\dfrac{h^9}{h^4} = \square^{\square - \square} = \square^{\square}$

These will require a '1' up top, as the denominator is larger. Bigger power - smaller power.

Example

$\dfrac{h^4}{h^9} = \dfrac{1}{h^{9-4}} = \dfrac{1}{h^5}$

24 $\dfrac{a^3}{a^7} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

25 $5^5 \div 5^7 = \dfrac{5^5}{5^7} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

26 $\dfrac{n}{n^6} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

27 $\dfrac{e^3}{e^6} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

28 $b^2 \div b^9 = \dfrac{b^2}{b^9} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

29 $\dfrac{q^4}{q^5} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

30 $c \div c^4 = \dfrac{c}{c^4} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

31 $\dfrac{y^6}{y^8} = \dfrac{\square}{\square^{\square - \square}} = \dfrac{\square}{\square}$

Indices - Dividing Indices (Sheet 2)

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Simplify these, larger power minus the smaller power. Use '1 over' when the larger power is the denominator (bottom).

Examples

$\frac{h^{\cancel{9}\,5}}{\cancel{h^4}} = h^5$ $\qquad \frac{\cancel{h^3}}{h^{\cancel{7}\,4}} = \frac{1}{h^4}$

1 $\frac{n^3}{n^5} =$ $\qquad$ 2 $\frac{n^5}{n^3} =$

3 $\frac{a^7}{a^2} =$ $\qquad$ 4 $\frac{a^2}{a^7} =$

5 $\frac{x}{x^4} =$ $\qquad$ 6 $\frac{x^4}{x} =$

7 $\frac{y^5}{y^4} =$ $\qquad$ 8 $\frac{y^4}{y^5} =$

9 $\frac{b^2}{b^9} =$ $\qquad$ 10 $\frac{b^9}{b^2} =$

11 $\frac{c^5}{c^8} =$ $\qquad$ 12 $\frac{k^2}{k^7} =$

13 $\frac{p^9}{p^8} =$ $\qquad$ 14 $\frac{u}{u^2} =$

Two different bases now

Examples

$\frac{\cancel{b^2}\,\cancel{e^2}}{b^{\cancel{3}}\,e^{\cancel{4}\,2}} = \frac{1}{be^2}$ $\qquad \frac{b^{5\,4}\,\cancel{e^2}}{\cancel{be^3}} = b^4e^2$

15 $\frac{ab^9}{b^7} =$ $\qquad$ 16 $\frac{b^9}{ab^7} =$

17 $\frac{x^2y^9}{y^7} =$ $\qquad$ 18 $\frac{x^2y^9}{xy^7} =$

19 $\frac{a^2n^9}{a^2n^8} =$ $\qquad$ 20 $\frac{a^5n^3}{a^2n^9} =$

21 $\frac{bc}{b^2c} =$ $\qquad$ 22 $\frac{c^5b^2}{b^2c^3} =$

23 $\frac{s^4v}{v^2s^2} =$ $\qquad$ 24 $\frac{h^5t^9}{ht^8} =$

25 $\frac{xy^2}{x^2y} =$ $\qquad$ 26 $\frac{c^2f^2}{cf^7} =$

27 $\frac{a^4e}{e^2a^3} =$ $\qquad$ 28 $\frac{u^5v^9}{u^2v^3} =$

Simplify these fractions by dividing by the HCF, or use a calculator.

Examples

Divide both numbers by 5

$\frac{2}{4} = \frac{1}{2}$ $\qquad \frac{10}{15} = \frac{2}{3}$

Divide both numbers by 2

29 $\frac{4}{6} =$ $\qquad$ 30 $\frac{8}{10} =$

31 $\frac{6}{9} =$ $\qquad$ 32 $\frac{2}{8} =$

33 $\frac{10}{12} =$ $\qquad$ 34 $\frac{5}{10} =$

35 $\frac{9}{12} =$ $\qquad$ 36 $\frac{8}{12} =$

Simplify the improper fractions the result will be whole numbers.

Example $\frac{10}{5} = 2$

37 $\frac{10}{2} =$ $\qquad$ 38 $\frac{6}{3} =$

39 $\frac{12}{4} =$ $\qquad$ 40 $\frac{9}{3} =$

41 $\frac{15}{5} =$ $\qquad$ 42 $\frac{12}{3} =$

43 $\frac{8}{2} =$ $\qquad$ 44 $\frac{14}{7} =$

This time the fractions result remains an improper fraction. Simplify these.

Example

Divide both numbers by 5

$\frac{15}{10} = \frac{3}{2}$

or

Calc: 15 ■/□ 10 =

If your calculator says '1½' after trying the above, Press: Shift, Mode, Down arrow, 2.

45 $\frac{12}{8} =$ $\qquad$ 46 $\frac{8}{6} =$

47 $\frac{9}{6} =$ $\qquad$ 48 $\frac{15}{9} =$

Now its numbers and letters. Simplify these.

49 $\frac{6x^2}{x} =$ $\qquad$ 50 $\frac{x^2}{6x} =$

51 $\frac{x}{6x^2} =$ $\qquad$ 52 $\frac{3x}{6x^2} =$

53 $\frac{6x^2}{3x} =$ $\qquad$ 54 $\frac{6x^2}{3x^2} =$

55 $\frac{9x^2}{6x} =$ $\qquad$ 56 $\frac{9x}{6x^2} =$

Rewrite as fractions and then simplify.

Example

$5a \div 15a^2 = \frac{5a}{15a^2} = \frac{1}{3a}$

57 $2y \div 8y^2 =$ $\qquad =$

58 $8y^3 \div 6y =$ $\qquad =$

59 $9y^2 \div 12y^5 =$ $\qquad =$

Indices - Further Multiplying and Dividing

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Simplify these by multiplying first and then dividing.

Example $\dfrac{v^4 \times v^3}{v^6 \times v} = \dfrac{v^{12}}{v^7} = v^5$

1 $\dfrac{a^5 \times a^4}{a^6} = \dfrac{\quad}{\quad} = \quad$

2 $\dfrac{n^3}{n^7 \times n^2} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

3 $\dfrac{g^5}{g^2 \times g^4} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

4 $\dfrac{p^5}{p^3 \times p^4} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

5 $\dfrac{d^3 \times d^5}{d \times d^4} = \dfrac{\quad}{\quad} = \quad$

6 $\dfrac{c^5 \times c^4}{c^3 \times c^2} = \dfrac{\quad}{\quad} = \quad$

7 $\dfrac{ab \times b^2}{a^2 \times a^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

8 $\dfrac{mx^5 \times m^2}{m^4 \times xm} =$ _____

9 $\dfrac{xy^5 \times y^2}{x^4 \times x^2 y} =$ _____

10 $\dfrac{cn^3 \times cn^2}{c^2 \times cn^2} =$ _____

11 $\dfrac{ac^3 \times a^2}{a^3 c^4 \times c^2} =$ _____

12 $\dfrac{w^2 n^5 \times w^2}{nw \times w^3} =$ _____

13 $\dfrac{k^2 y \times y^2 k^4}{y^3 \times y^2 k^4} =$ _____

14 $\dfrac{q^3 n^2 \times n^4}{qn^6 \times n^4 q^3} =$ _____

15 $\dfrac{b^2 d^3 \times b^2 d}{bd^2 \times b^2 d^2} =$ _____

16 $\dfrac{s^2 u^5 \times u^3 s^2}{s^4 u^3} =$ _____

These feature coefficients. Simplify these.

Example

$\dfrac{v^4 x^5 \times 5v^3}{4vx^3 \times 3xv} = \dfrac{5v^7 x^5}{12v^2 x^4} = \dfrac{5v^5 x}{12}$

17 $\dfrac{3h^2 c \times 3h^4}{2h^3 \times c^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

18 $\dfrac{5a^2 y^2 \times 2y^4}{7y^4 \times a^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

19 $\dfrac{2j^2 k \times 4k^2}{3j^4} =$ _____ $=$ _____

20 $\dfrac{5m^2 e^3 \times m^2}{2m^4 \times e^4 m} =$ _____ $=$ _____

These feature coefficients which can be simplified. Simplify these.

Examples

a) $\dfrac{3v^4 x^5 \times v^3}{2vx^3 \times 6xv} = \dfrac{3v^7 x^5}{12v^2 x^4} = \dfrac{v^5 x}{4}$

b) $\dfrac{3a^2 b^3 \times 4a^3}{2a^3 \times ab} = \dfrac{12a^5 b^3}{2a^4 b} = 6ab^2$

21 $\dfrac{3ve^3 \times 5e}{2e^3 v^5} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

22 $\dfrac{2n^2 w \times 6n^4}{3n^3} = \dfrac{\quad}{\quad} = \quad$

23 $\dfrac{2x^2 y^3 \times 5y^4}{10x \times 2y^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

24 $\dfrac{6h^2 q \times 3q^2}{2hq^4 \times h^2 q^2} =$ _____ $=$ _____

25 $\dfrac{5y^2 x^5 \times 4y^2}{y^4 x^3 \times 10x^2} =$ _____ $=$ _____

Indices - Brackets and Indices (Sheet 1)

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Simplify these by expanding first, then adding powers. Leave your answer in index form.

Example $(5^4)^3 = 5^4 \times 5^4 \times 5^4 = 5^{12}$ (4 + 4 + 4 = 12)

1 $(7^3)^2 =$ ☐ × ☐ = ☐

2 $(3^5)^3 =$ ☐ × ☐ × ☐

= ☐

3 $(9^5)^4 =$ ☐ × ☐ × ☐ × ☐

= ☐

Now letters - same process

4 $(y^2)^4 =$ ☐ × ☐ × ☐ × ☐

= ☐

5 $(x^6)^5 =$

=

6 $(k^3)^3 =$

=

Brackets means you multiply the indexes. Simplify these. Leave answer in index form.

$(m^6)^4 = m^{6 \times 4} = m^{24}$

7 $(8^3)^4 =$ ☐^☐×☐ = ☐^☐

8 $(4^3)^5 =$ ☐^☐×☐ = ☐^☐

9 $(y^2)^3 =$ ☐^☐×☐ = ☐^☐

10 $(x^5)^3 =$

11 $(n^8)^2 =$

12 $(k^6)^5 =$

Simplify these. Leave your answer in index form.

13 $(8^2)^3 =$

14 $(5^4)^2 =$

15 $(y^7)^2 =$

16 $(k^3)^2 =$

17 $(m^3)^2 =$

18 $(a^6)^3 =$

19 $(10^4)^3 =$

20 $(h^5)^5 =$

Use your power key to evaluate these.

Example $5^4 = 625$ Calc: 5 $x^{\blacksquare}$ 4 =

21 $2^4 =$

22 $3^6 =$

23 $8^4 =$

24 $2^8 =$

25 $4^5 =$

26 $10^2 =$

27 $10^4 =$

28 $10^3 =$

Simplify these by expanding first, then multiplying. Use your calculator to calculate the numbers with powers.

Example $(2x^4)^3 = 2x^4 \times 2x^4 \times 2x^4 = 2^3x^{12} = 8x^{12}$

29 $(3h^5)^2 =$ ☐ × ☐ = $3^{\square} h^{\square}$ = ☐

30 $(5a^7)^3 =$ ☐ × ☐ × ☐ = $5^{\square} a^{\square}$ = ☐

31 $(2n^6)^4 =$ ☐ × ☐ × ☐ × ☐ = $2^{\square} n^{\square}$ = ☐

32 $(3k^4)^3 =$

33 $(4c^5)^4 =$

Slide the power outside the brackets onto the number inside and multiply the two indexes for the power on the letter.

Example $(2x^4)^3 = 2^3x^{12} = 8x^{12}$

34 $(3e^2)^3 = 3^{\square} e^{\square} =$ ☐

35 $(5n^4)^2 = 5^{\square} n^{\square} =$ ☐

36 $(2c^5)^4 = 2^{\square} c^{\square} =$ ☐

37 $(3x^3)^4 = 3^{\square} x^{\square} =$ ☐

38 $(6v^5)^3 =$ ☐ = ☐

39 $(10a^8)^2 =$ ☐

= ☐

40 $(10y^5)^4 =$ ☐

= ☐

Indices - Brackets and Indices (Sheet 2)

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Rewrite these without the brackets.

Example $(5v^4)^3 = 125v^{12}$

$5^3 = 125$ $3 \times 4 = 12$

1 $(4g^5)^5 =$

2 $(3n^2)^4 =$

3 $(2j^7)^3 =$

4 $(10c^{10})^2 =$

5 $(2b^4)^9 =$

6 $(5g^6)^4 =$

7 $(10m^2)^3 =$

8 $(9x^3)^4 =$

9 $(2h^8)^{12} =$

10 $(10u^6)^5 =$

Multiple bases look harder but the process is the same, no calculator required. Expand.

Example $(ab^4c^2)^5 = a^5b^{20}c^{10}$

$1 \times 5 = 5$ $4 \times 5 = 20$ $2 \times 5 = 10$

11 $(x^3y^2)^4 =$

12 $(a^6k^2)^3 =$

13 $(a^3b)^5 =$

14 $(g^3n^6)^2 =$

15 $(c^{10}e^2)^8 =$

16 $(abc)^3 =$

17 $(a^4b^2c)^5 =$

18 $(a^2bc^3)^7 =$

These include co-efficients (numbers out front). Expand these, calculator required.

Example $(5v^4x^2)^3 = 125v^{12}x^6$

19 $(3me^5)^2 =$

20 $(5w^3x^2)^3 =$

21 $(2r^2d^2)^8 =$

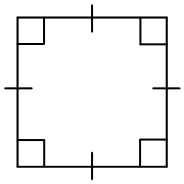
22 $(3q^6e^4)^4 =$

23 $(4q^5xk^4)^5 =$

24 $(10x^2ny^3)^3 =$

Find the area of these squares.

25



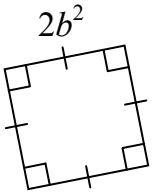
$3a$

$A = l^2$

$= (3a)^2$

$A =$ ☐ units^2

26



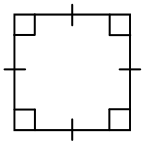
$2b^2$

$A = l^2$

$= (2b^2)^2$

$A =$ ☐ units^2

27



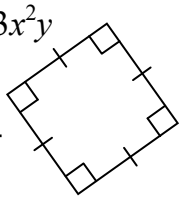
a^2b^3

$A = l^2$

=

$A =$ ☐ units^2

28



$3x^2y$

These are two step questions. Expand and simplify these.

Example

$$\frac{(v^3x)^3}{(v^2x^2)^4} = \frac{v^9x^3}{v^8x^8} = \frac{v}{x^5}$$

Expand Simplify

29

$$\frac{(a^5b^2)^4}{(a^2b^3)^2} = \frac{a^{\square}\,b^{\square}}{a^{\square}\,b^{\square}} = \frac{\square}{\square}$$

30

$$\frac{(n^2a^3)^2}{(n^3a^2)^3} = \frac{n^{\square}\,a^{\square}}{n^{\square}\,a^{\square}} = \frac{\square}{\square}$$

31

$$\frac{(yx^4)^3}{(y^2x^2)^2} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

32

$$\frac{(v^5c^4)^2}{(v^3c^2)^3} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

Now multiplication

Example $(x^2b)^3 \times (x^4b^2)^2$

$= x^6b^3 \times x^8b^4$ Expand

$= x^{14}b^7$ Simplify

33 $(x^3y^2)^4 \times (xy^5)^2$

$=$ $=$

34 $(b^2k)^3 \times (b^2k^2)^4$

$=$ $=$

35 $(u^6v^3)^2 \times (uv^2)^3$

$=$ $=$

36 $(a^2h^3)^5 \times (2h^2)^2$

$=$ $=$

Indices - The Zero Index (Sheet 1)

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Use your calculator to find these values.

Example $5^3 = 125$

Calc: 5 x■ 3 =

1 $7^3 =$ ____ 2 $7^2 =$ ____

3 $7^1 =$ ____ 4 $7^0 =$ ____

5 $9^3 =$ ____ 6 $9^2 =$ ____

7 $9^1 =$ ____ 8 $9^0 =$ ____

9 $4^3 =$ ____ 10 $4^2 =$ ____

11 $4^1 =$ ____ 12 $4^0 =$ ____

13 Look at Q.4, Q.8 and Q.12

The answers were all ____

Now use your calculator to complete these divisions.

14 $7^4 \div 1 =$ ____

Calc: 7 x■ 4 ▶ ÷ 1 =

15 $7^4 \div 7^0 =$ ____

compare

$\therefore 7^0 =$ ____

16 $10^3 \div 1 =$ ____

17 $10^3 \div 10^0 =$ ____

compare

$\therefore 10^0 =$ ____

Now use your division skills with index laws. No calculator

18 $3^5 \div 3^0 =$ $3^{\square - \square} = 3^{\square}$

19 $5^7 \div 5^0 =$ $\square^{\square - \square} = \square^{\square}$

When you divide a number (not 0) by '1', the number is also the answer.

$\therefore 3^0 = 5^0 =$ ____

Answer these divisions in index form, then as numbers.

Example

Index Form: $7^4 \div 7^4 = 7^0$ Number: $= 1$

20 $5^2 \div 5^2 =$ $\square^{\square} =$ ____

21 $4^8 \div 4^8 =$ ____ = ____

22 $\frac{10^3}{10^3} =$ ____ = ____

23 $14^5 \div 14^5 =$ ____ = ____

24 $\frac{9^{11}}{9^{11}} =$ ____ = ____

Algebra is answered the same way.

25 $x^7 \div x^7 =$ $\square^{\square} =$ ____

26 $\frac{m^4}{m^4} =$ ____ = ____

27 $y^8 \div y^8 =$ ____ = ____

28 $\frac{n^{13}}{n^{13}} =$ ____ = ____

Multiply these answering in index form, then as numbers.

Example

Index Form: $5^3 \times 5^0 = 5^3$ Number: $= 125$

Calc: 5 x■ 3 =

29 $4^6 \times 4^0 =$ $\square^{\square} =$ ____

30 $2^9 \times 2^0 =$ ____ = ____

31 $3^7 \times 3^0 =$ ____ = ____

Algebra same, but leave in index form

32 $v^7 \times v^0 =$ $\square^{\square}$

33 $k^9 \times k^0 =$ ____

34 $n^3 \times n^0 =$ ____

Remember with brackets you multiply the powers. Expand these.

Example

Index Form: $(7^3)^0 = 7^0$ Number: $= 1$

35 $(5^6)^0 =$ $\square^{\square} =$ ____

36 $(2^4)^0 =$ ____ = ____

37 $(3^0)^6 =$ ____ = ____

38 $(x^0)^6 =$ ____ = ____

39 $(a^0)^3 =$ ____ = ____

40 $(q^5)^0 =$ ____ = ____

These have co-efficient and so are a little harder.

Example

$x^0 = 1$

$(3x^0)^3 = (3\times1)^3$

$= (3)^3$

Calc: 3 x■ 3 =

$= 27$

41 $(5n^0)^4 = (\square \times \square)^4$

$= \square^4 =$ ____

42 $(8p^0)^3 = (\square \times \square)^3$

$= \square^3 =$ ____

43 $(2h^0)^9 = (\square \times \square)^9$

$= \square^9 =$ ____

When the '0' is outside, answer is 1

44 $(2h^3)^0 =$ ____

45 $(8k^5)^0 =$ ____

46 $(a^2b^5c^3)^0 =$ ____

Indices - The Zero Index (Sheet 2)

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Anything (except 0) with the power of zero is 1. Evaluate.

Example $5m^0 + 7$

$= 5 \times 1 + 7 = 12$

1 $2v^0 =$ ☐ × ☐ = ☐

2 $8h^0 =$ ☐ × ☐ = ☐

3 $h^0 + 7 =$ ☐ =

4 $7h^0 - 4 =$ ☐ =

5 $2^0 =$ ☐ 6 $x^0 =$ ☐

7 $2x^0 =$ ☐ = ☐

8 $2x^0 + 2 =$ ☐ × ☐ + ☐

= ☐

9 $x^0 \times 2x^0 =$ ☐ × ☐ × ☐

= ☐

10 $n^0 =$ ☐ 11 $4^0 =$ ☐

12 $4n^0 =$ ☐ = ☐

13 $(4n)^0 =$ ☐

14 $4n^0 + 4 =$ ☐ × ☐ + ☐

=

15 $(n^0 + 4)^2 =$ (☐ + ☐)^2

= ☐^{2} = ☐

16 $4n^0 + (4n)^0 =$ ☐ × ☐ + ☐

= ☐

T F Answer true or false to these.

17 $2^0 = 2$ ○ True ○ False

18 $5x^0 = (5x)^0$ ○ True ○ False

19 $3b^0 = b^0 + 2$ ○ True ○ False

20 $4a^0 - 4 = a^0$ ○ True ○ False

21 $3g^0 - (3g)^0 = 2g^0$ ○ True ○ False

22 $5k^0 + 4 = 9k^0$ ○ True ○ False

These will result in an algebraic answer. Simplify these.

Example $2m^0b$

$= 2 \times 1 \times b = 2b$

23 $2v^0h^2 =$ ☐

24 $a^0b^2c^3 =$ ☐

25 $(m^6)^0 \times m =$ ☐

26 $9k^2 \times e^0 =$ ☐

27 $4a^0m^2 - m^2a^0$

=

=

28

×	e^0	$3e^0$	e^0x^2
$2x$			
e^0			
e^2			
$3x^2$			
ex^2			

Complete the tables. Replace terms that have a power of 0 with a '1', before you multiply.

29

×	a^2	a^2b	ab^2	$(a^2)^0$
a^0				
$5a$				
a^0b				

Evaluate these and then arrange them in ascending order, that is from the smallest to largest.

30 $5e^0$, $e^0 + 9$, $(3e^0)^2$, $8e^0 - 6$.

☐ ☐ ☐ ☐

☐ ☐ ☐ ☐

☐

Rewrite the algebraic expressions in ascending order, not the numbers you have calculated

31 $10 - n^0$, $(2n^0)^3$, $4n^0$, $12 \div n^0$.

☐ ☐ ☐ ☐

☐ ☐ ☐ ☐

☐

Indices - Substitution (Sheet 1)

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Use your calculator to evaluate these when $a = 5$.

Example $a^3 = 5^3 = 125$

Calc: 5 x^3 =

1 $a^4 =$ 5 ^{4} = ____

2 $2a^3 =$ ____ × ____ $^{\square}$ = ____

3 $3a^3 + 7 =$ ____ × ____ $^{\square}$ + ____

= ____

4 $3a^2 + 7a =$ ____

= ____

Evaluate these using $n = 8$.

5 $n^3 - 10 =$ ____ = ____

6 $4n^2 - n =$ ____

= ____

7 $-n^2 =$ ____ = ____ Calc: - 8 x^2 =

8 $(-n)^2 =$ ____ = ____

Calc: (- 8) x^2 =

Evaluate these using $y = -3$. Make sure you use brackets.

9 $y^2 - 12 =$ ____ = ____

Calc: (- 3) x^2 - 1 2 =

10 $20 - y^3 =$ ____ = ____

11 $3y^4 =$ ____ = ____

12 $4y^2 + 3y =$ ____

= ____

13 $4y^2 \times 3y =$ ____

= ____

Evaluate these using $a = 4$ and $x = 3$.

14 $ax^2 =$ ____ × ____ $^{\square}$ = ____

15 $a^2 + x^2 =$ ____ $^{\square}$ + ____ $^{\square}$ = ____

16 $a^2 - x^2 =$ ____ $^{\square}$ - ____ $^{\square}$ = ____

17 $ax^2 + ax =$ ____

= ____

18 $20x^2 \div a^2 =$ ____

= ____

* 19 $\dfrac{20x^5}{ax} =$ ____ / ____ = ____

∗ Brackets required

A table of values is often used to construct a graph. Complete these tables.

20 $b = a^2 + 2$

a	0	1	2	3	4	5
b						

Calc: 1 x^2 + 2 =

Calc: 0 x^2 + 2 =

21 $y = 2x^2 + 5$

x	0	1	2	3	4	5
y						

22 $y = -3x^2 - 7$

x	0	1	2	3	4	5
y						

A frequent use of substitution is using a formula, such as with science. Substitute the values into the formulae to solve these questions.

The surface area of a cube is found using $SA = 6l^2$. Find the SA:

23 When $l = 15$.

$SA = 6l^2$

$= 6 \times$ ____ ^{2}

$SA =$ ____ u^2

24 When $l = 30$.

25 When $l = 8$.

Power loss in a wire is found using $P = I^2R$. Find the loss, when:

26 $I = 2$ and $R = 0.5$

$P = I^2R$

$=$ ____ $^2 \times$ ____

$P =$ ____ W

27 $I = 10$ and $R = 6$.

28 $I = 8$ and $R = 3$.

Kinetic Energy is found using $KE = \frac{1}{2}mv^2$. Find the energy, when:

29 $m = 80$, $v = 15$.

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times$ ____ $\times$ ____ ^{2}

$KE =$ ____ J

30 $m = 65$, $v = 18$.

31 $m = 90$, $v = 25$.

Indices - Substitution (Sheet 2)

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Complete the tables and then plot the points on the number plane below them.

1 $y = x^2 - 4$ ○

x	-4	-3	-2	-1	0	1	2	3	4
y	12								

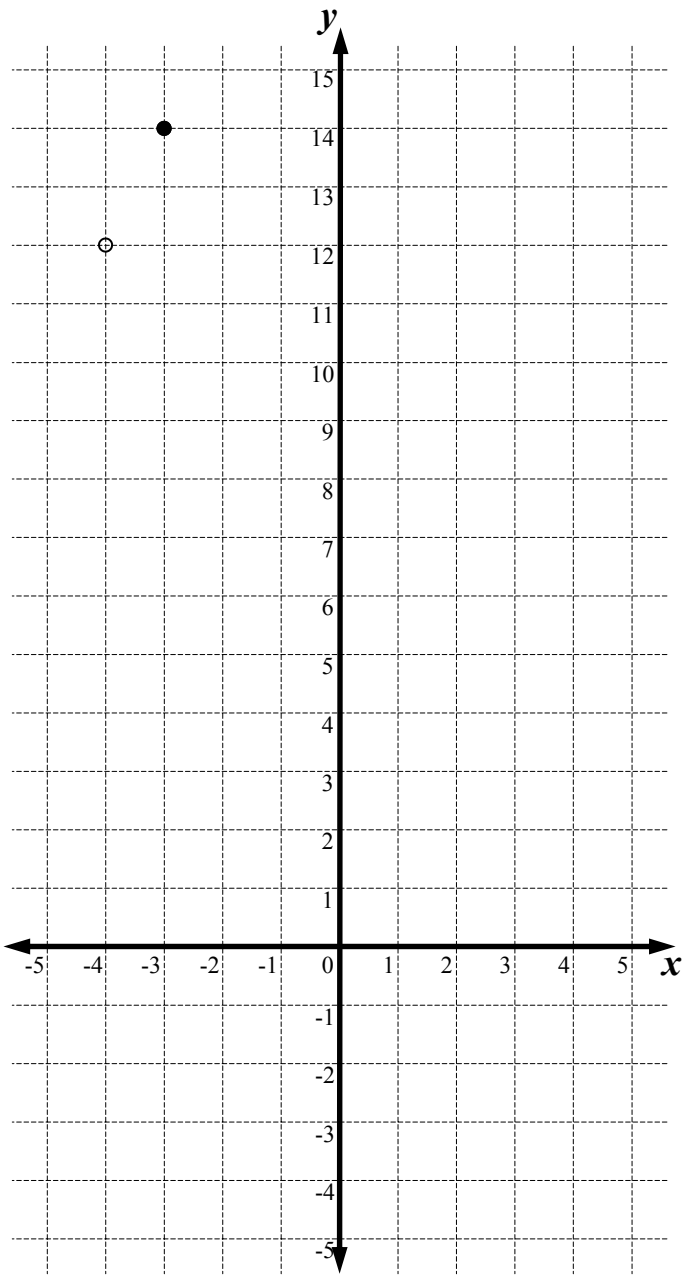
Calc: (- 4) x^2 - 4 =

Use the replay key to change '-4' to '-3' and so on.

2 $y = 2x^2 - 4$ ●

x	-3	-2	-1	0	1	2	3
y	14						

Calc: 2 × (- 3) x^2 - 4 =



3 $y = -x^2 + 6$ ○

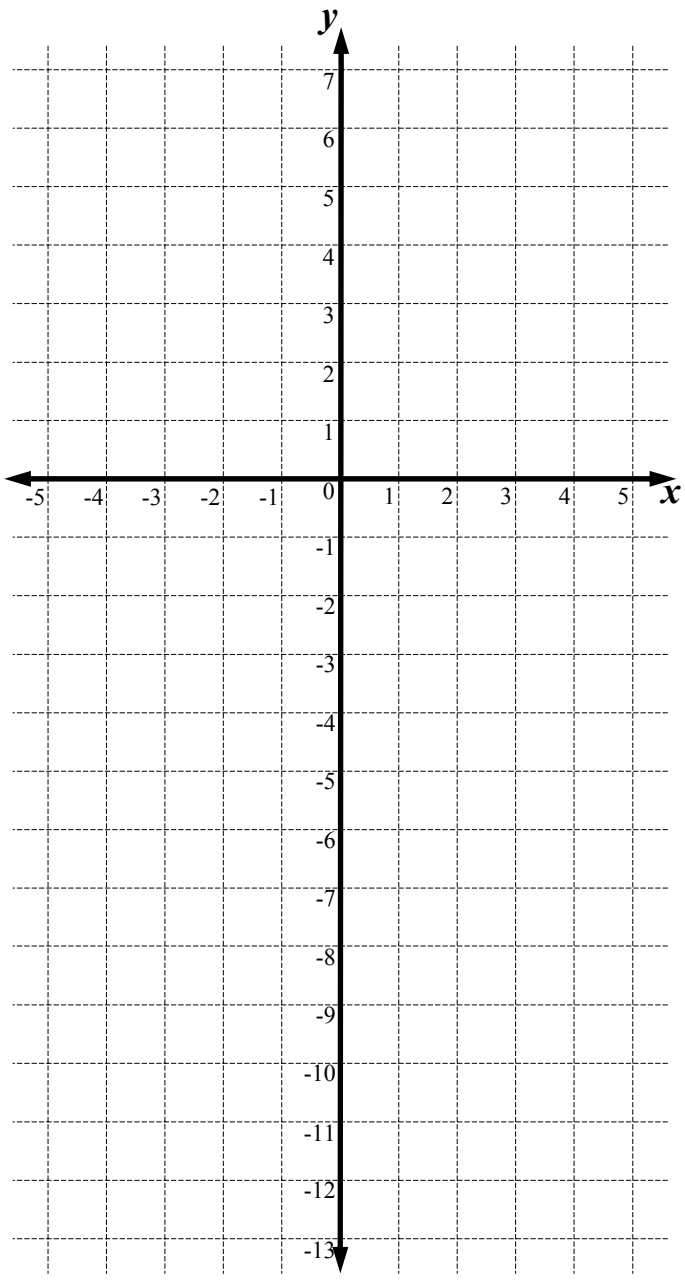
x	-4	-3	-2	-1	0	1	2	3	4
y	-10								

Calc: - (- 4) x^2 + 6 =

4 $y = -2x^2 + 6$ ●

x	-3	-2	-1	0	1	2	3
y	-12						

Calc: - 2 × (- 3) x^2 + 6 =



Indices - Find a Word

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Find the words in the puzzle from the wordlist.



M	E	X	P	P	B	O	L	E	C	I	D	N	I	Y
U	T	X	X	S	I	M	P	L	E	S	T	V	L	R
L	A	A	E	D	I	V	I	D	T	N	M	P	A	E
T	U	L	X	X	C	A	N	C	E	L	U	Y	T	S
I	L	G	P	N	P	E	V	I	R	L	P	U	A	U
P	A	E	Y	L	O	L	C	X	M	L	T	A	V	L
L	V	B	I	Y	S	I	M	P	L	I	F	Y	L	T
Y	E	R	N	L	F	K	S	P	T	T	S	A	E	M
P	X	A	D	F	V	E	E	S	P	C	E	G	V	E
D	R	R	E	W	O	P	B	X	E	A	I	G	I	E
N	E	O	X	P	D	U	E	L	U	R	L	A	T	S
A	C	E	V	E	S	D	L	V	A	T	P	E	A	I
P	P	S	T	E	K	C	A	R	B	B	P	X	G	V
X	L	Y	Z	E	R	O	E	X	P	U	A	P	E	E
E	U	L	A	V	P	L	Y	B	A	S	E	E	N	R

WORDLIST

POWER	VALUE	INDEX	SUBTRACT	TERM
ADD	BASE	RULE	CANCEL	LIKE
ZERO	SIMPLEST	ALGEBRA	RESULT	INDICE
NEGATIVE	BRACKETS	PROVE	SIMPLIFY	EVALUATE
EXPAND	REVISE	MULTIPLY	APPLIES	DIVIDE
	SUBSTITUTE	EXPRESSION	COEFFICIENT	

Indices - Topic Review

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Use your power key to evaluate these indices.

1 $8^4 =$ _____

2 $2^9 =$ _____

3 $5^{-1} =$ _____ Decimal

4 $9^0 =$ _____

Write these indices in expanded form

5 $e^3 =$ _____

6 $5^4 =$ _____

7 $n^6 =$ _____

Write the indice that has a:

8 Base 'a' and an index of '5'
$=$ _____

9 Index of '4' and a base 'm'
$=$ _____

10 Index of '0' and a base '5'
$=$ _____

Multiply these indices

11 $c^2 \times c^3 =$ _____

12 $a \times a^5 =$ _____

13 $b^4 \times b^2 =$ _____

14 $2^7 \times 2^5 =$ _____ Leave as indice

15 $n^5 \times n \times n^2 =$ _____

16 $5 \times 5^3 \times 5^2 =$ _____ Leave as indice

17 $k^5 \times k \times k^3 =$ _____

18 $e^3 \times e^0 \times e^2 =$ _____

19 $n \times n^0 \times n =$ _____

Multiply these indices

20 $ae \times a =$ _____

21 $xy^2 \times x^2 =$ _____

22 $n^3p^2 \times n^4p^2 =$ _____

23 $a^3b^2c \times a^2c =$ _____

21 $w^3n^3e^2 \times w^4ne^5 =$ _____

These include coefficients. Multiply.

22 $6n^2 \times n =$ _____

22 $3a^3 \times a^3 =$ _____

22 $t \times 9t^5 =$ _____

22 $d^2 \times 7d^2 =$ _____

23 $b^2 \times 4c^2b =$ _____

24 $3ab \times ba^2 =$ _____

25 $y^2 \times 7a^3y =$ _____

26 $2p^3d^2 \times 3p =$ _____

27 $4yu^2 \times 2u =$ _____

28 $6h^7v^4 \times 5h^2 =$ _____

29 $5ac^2 \times 2ac =$ _____

30 $2u^3v^2 \times 6uv^4 =$ _____

31 $4e^4k \times 3e^2k^3 =$ _____

Divide these indices.

32 $u^4 \div u^2 =$ _____

33 $a^5 \div a =$ _____

34 $7^5 \div 7^2 =$ _____ Leave as indice

35 $\frac{p^9}{p^2} =$ _____

36 $\frac{u^5}{u^2} =$ _____

These require '1 over' answers.

37 $\frac{n^2}{n^4} =$ _____

38 $\frac{v^4}{v^5} =$ _____

39 $\frac{a}{a^5} =$ _____

40 $\frac{g^3}{g^6} =$ _____

41 $\frac{n^2}{2n^5} =$ _____

42 $\frac{c^5}{5c^5} =$ _____

These divisions can be simplified.

43 $\frac{4x^2}{2x^5} =$ _____

44 $\frac{3e^4}{9e} =$ _____

45 $\frac{6x}{8x} =$ _____

46 $\frac{5u^5}{10u^3} =$ _____

47 $\frac{6y^7}{4y^2} =$ _____

48 $\frac{6n^4}{2n} =$ _____

Multiply first then divide.

49 $\frac{a^2 \times a^3}{a} = \frac{\square}{\square} =$ _____

50 $\frac{h^4 \times h}{h^7} = \frac{\square}{\square} =$ _____

51 $\frac{k^3 \times k^2}{k \times k^4} = \frac{\square}{\square} =$ _____

52 $\frac{3a^2 \times a}{a^3 \times 4a^3} = \frac{\square}{\square} =$ _____

53 $\frac{4g^3 \times g^4}{g \times 2g^2} = \frac{\square}{\square} =$ _____

54 $\frac{2n \times 3n^5}{6n^3 \times n^3} = \frac{\square}{\square} =$ _____

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Simplify these.

55 $(d^4)^2 =$ ____ 56 $(x^2)^6 =$ ____

57 $(g^3)^5 =$ ____ 58 $(h^2)^6 =$ ____

59 $(2n^3)^2 =$ ____

60 $(3k^5)^2 =$ ____

Simplify these. Calculator required.

61 $(5m^3)^4 =$ ____

62 $(2d)^9 =$ ____

63 $(8a^6)^3 =$ ____

Simplify these.

64 $(xv^3)^4 =$ ____

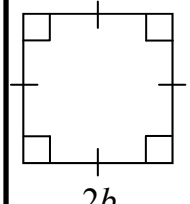
65 $(q^2w^3)^3 =$ ____

66 $(3gk^3)^3 =$ ____

67 $(2e^5c^3)^5 =$ ____

Find the area of these.

68

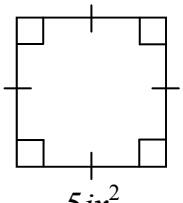


$A = l^2$

$= \square^2$

$A = \square$ units^2

69



Simplify these.

70 $(m^3)^4 \times (m^2)^3 = \square \times \square$

$=$ ____

71 $(ac^2)^5 \times (a^3c^2)^4$

$= \square \times \square$

$=$ ____

72 $(2x^3y^2)^3 \times (3xy^2)^2$

$= \square \times \square$

$=$ ____

Simplify these.

73 $\dfrac{(e^5k^4)^2}{(e^2k)^3} = \dfrac{\square}{\square} =$ ____

74 $\dfrac{(a^3n^4)^3}{(a^5n)^2} = \dfrac{\square}{\square} =$ ____

Simplify these, involving zero index.

75 $\dfrac{a^3}{a^3} = \square = \square$

76 $h^5 \div h^5 = \square = \square$

77 $\dfrac{v^{11}}{v^{11}} = \square = \square$

78 $b^3 \times b^0 = \square$

79 $y^5 \times y^0 = \square$

80 $(b^0)^3 = \square = \square$

81 $(v^0)^4 = \square = \square$

82 $(q^2)^0 = \square = \square$

83 $(2c^4)^0 = \square$

84 $(2a^0k^2)^2 = \square$

Evaluate these.

85 $2v^0 = \square \times \square = \square$

86 $h^0 + 7 = \square + \square = \square$

87 $2a^0 + 3 =$ ____

88 $3q^0 + 5 =$ ____

Evaluate, given $a = 6$.

89 $a^{\square} = \square$

90 $a^3 + 8 =$ ____

91 $3a^3 \div 12 =$ ____

$=$ ____

92 $5a^4 + 7a^2 =$ ____

____

Evaluate, given $x = 2$ and $y = -3$.

93 $x^2y =$ ____

94 $xy^2 =$ ____

95 $a^7 + 7y =$ ____

$=$ ____

Complete the tables.

96 $y = x^2 + 5$

x	0	1	2	3	4	5
y						

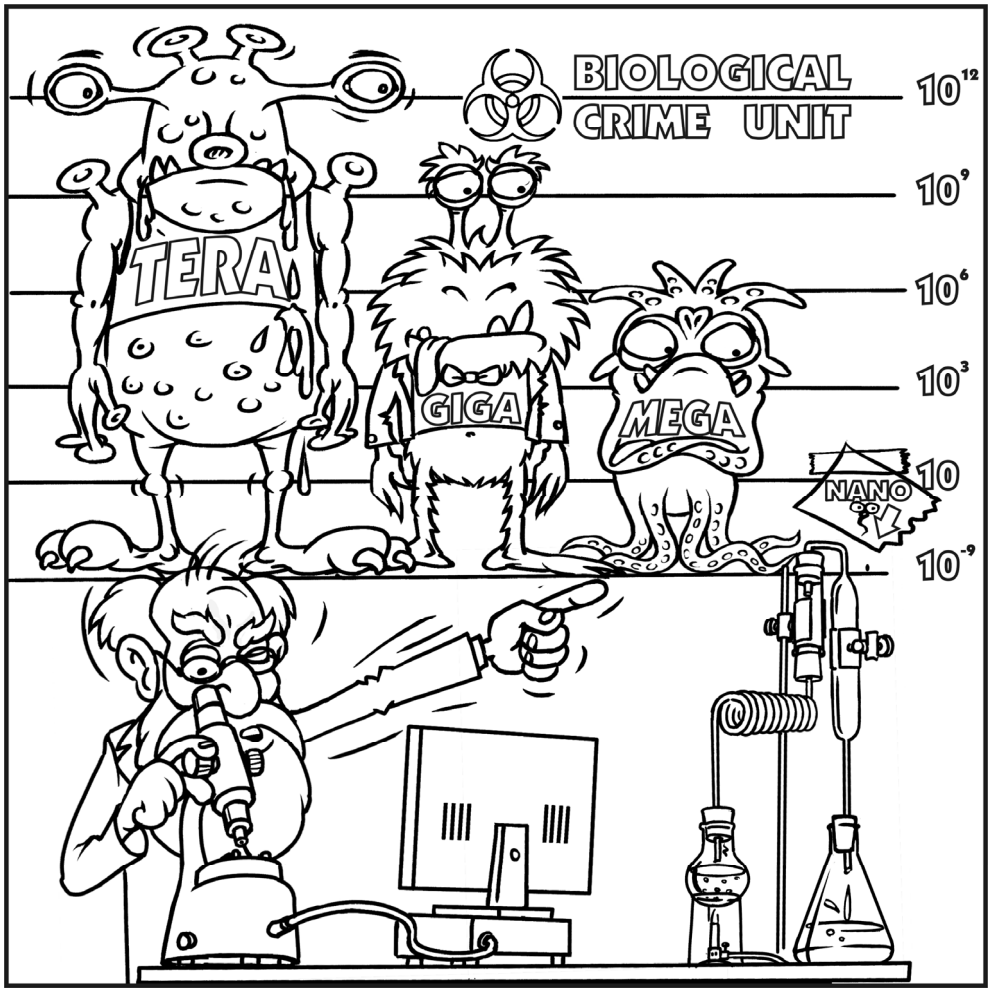
97 $y = 2x^2 - 10$

x	0	1	2	3	4	5
y						

VELOCITY

NUMBERS OF ANY MAGNITUDE

Numbers of Any Magnitude



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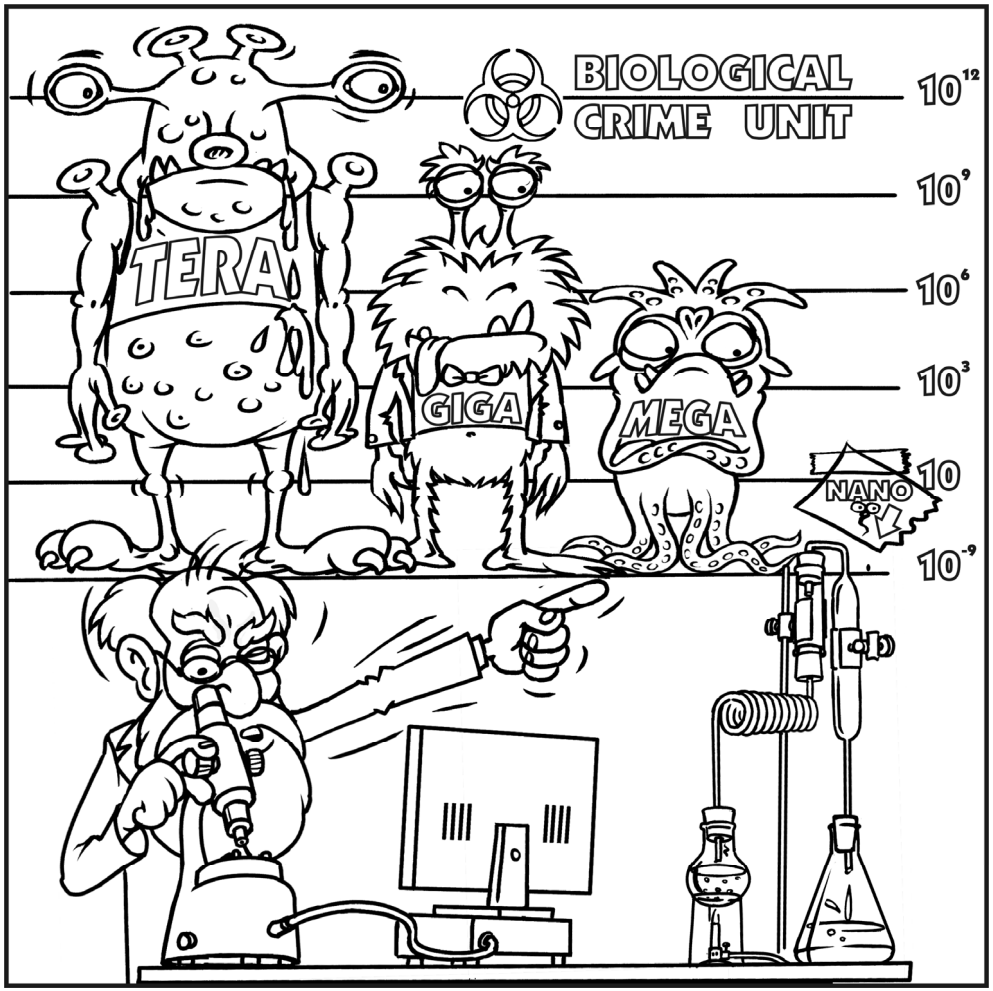
Numbers of Any Magnitude



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Numbers of Any Magnitude - Prefix Tables

These are prefix tables for the Numbers of Any Magnitude topic.

Table 1 - Fully completed table, 2 to a page, to stick into books

Table 2 - Fully blank table, 2 to a page, to stick into books

Table 3 - Cut-out and Match strips and table, 1 to a page, to stick into books
*This table has the prefixes already in position if this is preferred

Table 4 - Cut-out and Match strips and completely blank table, 1 to a page, to stick into books

Numbers of Any Magnitude - Prefix Table

Prefix	Scientific Notation		Numerical Value	Value in Words
Tera	1×10^{12}	LARGEST	1 000 000 000 000	1 trillion
Giga	1×10^{9}		1 000 000 000	1 billion
Mega	1×10^{6}		1 000 000	1 million
Kilo	1×10^{3}		1 000	1 thousand
Milli	1×10^{-3}		$\frac{1}{1\ 000}$	1 thousandth
Micro	1×10^{-6}		$\frac{1}{1\ 000\ 000}$	1 millionth
Nano	1×10^{-9}	SMALLEST	$\frac{1}{1\ 000\ 000\ 000}$	1 billionth
Pico	1×10^{-12}		$\frac{1}{1\ 000\ 000\ 000\ 000}$	1 trillionth

Prefix	Scientific Notation		Numerical Value	Value in Words
Tera	1×10^{12}	LARGEST	1 000 000 000 000	1 trillion
Giga	1×10^{9}		1 000 000 000	1 billion
Mega	1×10^{6}		1 000 000	1 million
Kilo	1×10^{3}		1 000	1 thousand
Milli	1×10^{-3}		$\frac{1}{1\ 000}$	1 thousandth
Micro	1×10^{-6}		$\frac{1}{1\ 000\ 000}$	1 millionth
Nano	1×10^{-9}	SMALLEST	$\frac{1}{1\ 000\ 000\ 000}$	1 billionth
Pico	1×10^{-12}		$\frac{1}{1\ 000\ 000\ 000\ 000}$	1 trillionth

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Numbers of Any Magnitude - Prefix (Blank) Table

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Prefix	Scientific Notation	LARGEST ⟷ SMALLEST	Numerical Value	Value in Words

Prefix	Scientific Notation	LARGEST ⟷ SMALLEST	Numerical Value	Value in Words

Numbers of Any Magnitude - Prefix Match Table - Cut Outs + Table (Prefixes Positioned)

1×10^{9}	1 thousand	1×10^{12}	$\frac{1}{1\ 000\ 000\ 000}$
$\frac{1}{1\ 000}$	1 billionth	1 000 000 000	1×10^{-3}
$\frac{1}{1\ 000\ 000}$	1×10^{-12}	1 thousandth	1 000
1 millionth	1×10^{-6}	1 000 000 000 000	1×10^{3}
1 trillion	$\frac{1}{1\ 000\ 000\ 000\ 000}$	1×10^{-9}	1 million
1×10^{6}	1 000 000	1 billion	1 trillionth

Prefix	Scientific Notation		Numerical Value	Value in Words
Tera		LARGEST		
Giga				
Mega				
Kilo				
Milli				
Micro				
Nano				
Pico		SMALLEST		

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Numbers of Any Magnitude - Prefix Match Table - Cut Outs + Table

1×10^{9}	Milli	1 000 000 000	1 million
Mega	1 thousand	1×10^{12}	$\frac{1}{1\ 000\ 000\ 000}$
$\frac{1}{1\ 000}$	1 billionth	Pico	1×10^{-3}
$\frac{1}{1\ 000\ 000}$	1×10^{-12}	1 thousandth	Kilo
1 millionth	Giga	1 000 000 000 000	1×10^{3}
1 trillion	$\frac{1}{1\ 000\ 000\ 000\ 000}$	1×10^{-9}	Tera
1×10^{6}	1 000 000	Micro	1 trillionth
Nano	1×10^{-6}	1 billion	1 000

Prefix	Scientific Notation		Numerical Value	Value in Words
		LARGEST		
		SMALLEST		

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Numbers of Any Magnitude - Rounding to Nearest 10/100/1 000

When large amounts are involved, it is often easier to round off and make the number easier to handle. For example the annual budget of a hospital might be in the hundreds of millions of dollars, so there is no need for tens or hundreds. That level of accuracy isn't required.

Rounding uses half of the rounding amount to undertake the rounding. So when rounding to the nearest:

- 10, use 5 as the cut-off. 5 or more round up, less than 5 round down
- 100, use 50 as the cut-off. 50 or more round up, less than 50 round down
- 1 000, use 500 as the cut-off. 500 or more round up, less than 500 round down

Note that it is possible to round a number down to zero. For example:

- 4 rounded to the nearest 10, is zero. (It is less than 5)
- 38 rounded to the nearest 100, is zero. (It is less than 50)
- 410 rounded to the nearest 1 000, is zero. (It is less than 500)

Examples

Round to the nearest 10

a) 76 80
6 is more than 5 so up

b) 197 200
7 is more than 5 so up

c) 143 140
3 is less than 5 so down

Round to the nearest 100

a) 352 400
52 is more than 50 so up

b) 42 0
42 is less than 50 so down

c) 377 966 378 000
66 is more than 50 so up

Round to the nearest 1 000

a) 352 0
352 is less than 500 so down

b) 13 866 14 000
866 is more than 500 so up

c) 399 602 400 000
602 is more than 500 so up

Numbers of Any Magnitude - Rounding (Nearest 10)

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Round these numbers to the nearest 10. So if the last digit is 5 or more, round them up.



Examples

47 = 50 273 = 270

1 17 = 2 82 =

3 38 = 4 24 =

5 67 = 6 11 =

7 6 = 8 4 =

Now larger numbers

9 167 =

10 204 =

11 477 =

12 597 =

13 2 036 =

14 1 997 =

15 3 598 =



Calculate these and write the answer, then the answer rounded to the nearest 10.

16 47×28 = Actual

= Rounded

17 $53^2 \times 87$ =

=

18 $36^2 \div (9 \times 6)$ =

=

19 $(9 + 13)^2 \times (8 + 9)^2$

= =

Use the fraction key or bracket both the numerator (top) & denominator

20 $\dfrac{(15 \times 23)^3}{5^2 \times 3}$ =

Bracket denominator

=

Use the $x^{\square}$ key

21 $\dfrac{9^5 \times 4^3}{2^4 \times 3^3}$ =

Use the $x^{\square}$ key =

22 $\dfrac{5^4 \times 6^3}{10^3 \times 3^3}$ = =

Its investigation time. Perform the same operation twice, but the first is rounded after the calculation, the second, before.



23 127×58 = Actual

= Rounded

24 127×58 = 130 × 60 Round the numbers

= Actual

25 232×67 =

= Rounded

26 232×67 = × Round the numbers

=

27 $4\,096 \times 156$ = Actual

= Rounded

28 $4\,096 \times 156$

= ×

=

29 $2\,322 \times 4$ = Actual

= Rounded

30 $2\,322 \times 4$

= ×

=



This time its addition. Round at the end and compare it with rounding at the start.

31 $301 + 494 + 727 + 83$

= Actual = Rounded

32 $309 + 496 + 727 + 83$

= + + +

Round all the numbers to the nearest 10

=

33 $777 + 438 + 219 + 867$

= Actual = Rounded

34 $777 + 438 + 219 + 867$

= + + +

=

Numbers of Any Magnitude - Rounding (Nearest 100/1 000)

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Round these numbers to the nearest 100. If the last two digits are 50 or more, round up

Examples

326 = 300 4 976 = 5 000

1 435 =

2 153 =

3 18 = 4 61 =

5 888 =

6 950 =

Now larger numbers

7 23 478 =

8 5 809 =

9 179 981 =

10 197 090 =

Perform these calculations and give the answer, then the answer rounded to nearest 100

11 76×38 = Actual

= Rounded

12 $24^2 \times 14^2$ =

=

13 $90^2 \div 15^2$ =

=

14 $32^3 + 8^3$ =

=

Round these numbers to the nearest 1 000. If the last three digits are 500 or more, round up

Examples

577 = 1 000 403 = 0

15 1 875 =

16 7 077 =

17 8 498 =

18 3 702 =

19 5 555 =

Now larger numbers

20 87 206 =

21 109 703 =

22 4 528 119 =

23 2 149 944 =

24 7 999 566 =

Calculate the answer then the rounded answer to the nearest thousand.

Use the power key $x^{\blacksquare}$ to answer these

25 $3^6 \times 2^9$ =

Actual

= Rounded to nearest 1 000

26 $5^4 + 6^{10}$ =

=

27 $8^{11} \div 2^6$ =

=

Match answers with questions. The answers have been rounded to the nearest 1 000.

28 7^4 • • 16 000

29 5^6 • • 4 000

30 6^6 • • 66 000

31 4^8 • • 20 000

32 3^9 • • 2 000

33 9^4 • • 47 000

34 4^6 • • 7 000

Perform the calculation and write the answer three times. Rounded to 10, 100 then 1 000

35 $927 \times 43 \times 8^4$ Unrounded Answer Not Required

=

Round to the Nearest 10

=

Round to the Nearest 100

=

Round to the Nearest 1 000

36 $17 \times 11 \times 13 \times 7^3$

=

=

=

37 $3^6 \times 13^4 \div (43 - 16)$

=

=

=

Numbers of Any Magnitude - Rounding (Nearest 10)

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Round these numbers to the nearest 10. So if the last digit is 5 or more, round them up.



Examples

47 = 50 273 = 270

1 17 = 20 2 82 = 80

3 38 = 40 4 24 = 20

5 67 = 70 6 11 = 10

7 6 = 10 8 4 = 0

Now larger numbers

9 167 = 170

10 204 = 200

11 477 = 480

12 597 = 600

13 2 036 = 2 040

14 1 997 = 2 000

15 3 598 = 3 600



Calculate these and write the answer, then the answer rounded to the nearest 10.

16 47×28 = 1 316 Actual

= 1 320 Rounded

17 $53^2 \times 87$ = 244 383

= 244 380

18 $36^2 \div (9 \times 6)$ = 24

= 20

19 $(9 + 13)^2 \times (8 + 9)^2$

= 139 876 = 139 880

Use the fraction key or bracket both the numerator (top) & denominator

20 $\frac{(15 \times 23)^3}{5^2 \times 3}$ = 547 515

Bracket denominator

= 547 520

Use the $\frac{x}{}$ key

21 $\frac{9^5 \times 4^3}{2^4 \times 3^3}$ = 8 748

Use the $x^{\square}$ key = 8 750

22 $\frac{5^4 \times 6^3}{10^3 \times 3^3}$ = 5 = 10

Its investigation time. Perform the same operation twice, but the first is rounded after the calculation, the second, before.



23 127×58 = 7 366 Actual

= 7 370 Rounded

24 127×58 = 130 × 60 Round the numbers

= 7 800 Actual

25 232×67 = 15 544

= 15 540 Rounded

26 232×67 = 230 × 70 Round the numbers

= 16 100

27 $4\,096 \times 156$ = 638 976 Actual

= 638 980 Rounded

28 $4\,096 \times 156$

= 4 100 × 160

= 656 000

29 $2\,322 \times 4$ = 9 288 Actual

= 9 290 Rounded

30 $2\,322 \times 4$

= 2 320 × 0

= 0



This time its addition. Round at the end and compare it with rounding at the start.

31 $301 + 494 + 727 + 83$

= 1 605 Actual = 1 610 Rounded

32 $309 + 496 + 727 + 83$

= 310 + 500 + 730 + 80 Round all the numbers to the nearest 10

= 1 620

33 $777 + 438 + 219 + 867$

= 2 301 Actual = 2 300 Rounded

34 $777 + 438 + 219 + 867$

= 780 + 440 + 220 + 870

= 2 310

Numbers of Any Magnitude - Rounding (Nearest 100/1 000)

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Round these numbers to the nearest 100. If the last two digits are 50 or more, round up

Examples

$326 =$ 300 $4\,976 =$ 5 000

1 $435 =$ 400

2 $153 =$ 200

3 $18 =$ 0 4 $61 =$ 100

5 $888 =$ 900

6 $950 =$ 1 000

Now larger numbers

7 $23\,478 =$ 23 500

8 $5\,809 =$ 5 800

9 $179\,981 =$ 180 000

10 $197\,090 =$ 197 100

Perform these calculations and give the answer, then the answer rounded to nearest 100

11 $76 \times 38 =$ 2 888 Actual

$=$ 2 900 Rounded

12 $24^2 \times 14^2 =$ 112 896

$=$ 112 900

13 $90^2 \div 15^2 =$ 36

$=$ 0

14 $32^3 + 8^3 =$ 33 280

$=$ 33 300

Round these numbers to the nearest 1 000. If the last three digits are 500 or more, round up

Examples

$577 =$ 1 000 $403 =$ 0

15 $1\,875 =$ 2 000

16 $7\,077 =$ 7 000

17 $8\,498 =$ 8 000

18 $3\,702 =$ 4 000

19 $5\,555 =$ 6 000

Now larger numbers

20 $87\,206 =$ 87 000

21 $109\,703 =$ 110 000

22 $4\,528\,119 =$ 4 528 000

23 $2\,149\,944 =$ 2 150 000

24 $7\,999\,566 =$ 8 000 000

Calculate the answer then the rounded answer to the nearest thousand.

Use the power key $x^{\blacksquare}$ to answer these

25 $3^6 \times 2^9 =$ 373 248 Actual

$=$ 373 000 Rounded to nearest 1 000

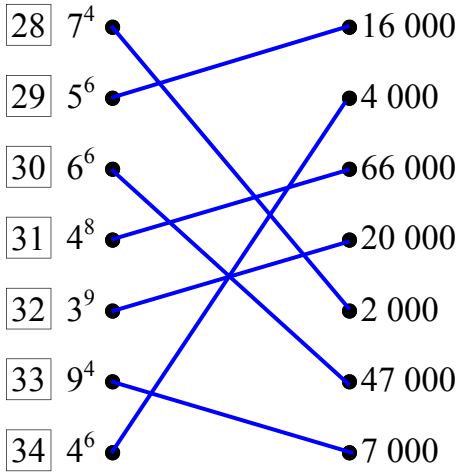
26 $5^4 + 6^{10} =$ 60 466 801

$=$ 60 467 000

27 $8^{11} \div 2^6 =$ 134 217 728

$=$ 134 218 000

Match answers with questions. The answers have been rounded to the nearest 1 000.



Perform the calculation and write the answer three times. Rounded to 10, 100 then 1 000

35 $927 \times 43 \times 8^4$ Unrounded Answer Not Required

$=$ 163 270 660

Round to the Nearest 10

$=$ 163 270 700

Round to the Nearest 100

$=$ 163 271 000

Round to the Nearest 1 000

36 $17 \times 11 \times 13 \times 7^3$

$=$ 833 830

$=$ 833 800

$=$ 834 000

37 $3^6 \times 13^4 \div (43 - 16)$

$=$ 771 150

$=$ 771 100

$=$ 771 000

Numbers of Any Magnitude - Rounding to Nearest 10/100/1 000 (Homework Strip)

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Date Due: _____

Round these to the 'nearest 10'

1. $24 =$ _____
2. $65 =$ _____
3. $56 =$ _____
4. $97 =$ _____
5. $3\,072 =$ _____
6. $9\,896 =$ _____

Round these to the 'nearest 100'

7. $874 =$ _____
8. $26\,774 =$ _____
9. $4^6 =$ _____
10. $7^4 \times 3^3 =$ _____

Round these to the 'nearest 1 000'

11. $217 \times 628 =$ _____
12. $11^3 \times 14^2 =$ _____
13. $16^6 \div 2^3 =$ _____
14. $(27 + 38)^3 =$ _____

Date Due: _____

Round these to the 'nearest 10'

1. $24 =$ _____
2. $65 =$ _____
3. $56 =$ _____
4. $97 =$ _____
5. $3\,072 =$ _____
6. $9\,896 =$ _____

Round these to the 'nearest 100'

7. $874 =$ _____
8. $26\,774 =$ _____
9. $4^6 =$ _____
10. $7^4 \times 3^3 =$ _____

Round these to the 'nearest 1 000'

11. $217 \times 628 =$ _____
12. $11^3 \times 14^2 =$ _____
13. $16^6 \div 2^3 =$ _____
14. $(27 + 38)^3 =$ _____

Date Due: _____

Round these to the 'nearest 10'

1. $24 =$ _____
2. $65 =$ _____
3. $56 =$ _____
4. $97 =$ _____
5. $3\,072 =$ _____
6. $9\,896 =$ _____

Round these to the 'nearest 100'

7. $874 =$ _____
8. $26\,774 =$ _____
9. $4^6 =$ _____
10. $7^4 \times 3^3 =$ _____

Round these to the 'nearest 1 000'

11. $217 \times 628 =$ _____
12. $11^3 \times 14^2 =$ _____
13. $16^6 \div 2^3 =$ _____
14. $(27 + 38)^3 =$ _____

Date Due: _____

Round these to the 'nearest 10'

1. $24 =$ _____
2. $65 =$ _____
3. $56 =$ _____
4. $97 =$ _____
5. $3\,072 =$ _____
6. $9\,896 =$ _____

Round these to the 'nearest 100'

7. $874 =$ _____
8. $26\,774 =$ _____
9. $4^6 =$ _____
10. $7^4 \times 3^3 =$ _____

Round these to the 'nearest 1 000'

11. $217 \times 628 =$ _____
12. $11^3 \times 14^2 =$ _____
13. $16^6 \div 2^3 =$ _____
14. $(27 + 38)^3 =$ _____

Numbers of Any Magnitude - Rounding to Nearest 10/100/1 000 (HWS)

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Date Due:

Round these to the 'nearest 10'

1 $24 =$ 20

2 $65 =$ 70

3 $56 =$ 60

4 $97 =$ 100

5 $3\,072 =$ 3 070

6 $9\,896 =$ 9 900

Round these to the 'nearest 100'

7 $874 =$ 900

8 $26\,774 =$ 26 800

9 $4^6 =$ 4 100

10 $7^4 \times 3^3 =$ 64 800

Round these to the 'nearest 1 000'

11 $217 \times 628 =$ 136 000

12 $11^3 \times 14^2 =$ 261 000

13 $16^6 \div 2^3 =$ 2 097 000

14 $(27 + 38)^3 =$ 275 000

Numbers of Any Magnitude - Rounding Decimals

Rounding is used to shorten decimal answers to a level of accuracy required. Often when answering questions, you are asked to give your answer to a specific number of decimal places. This isn't done by just chopping the decimal off at the number of places required, this method is followed:

- Look at the decimal behind the decimal place asked for. For example if rounding to 2 d.p. (2 decimal places) look at the digit in the 3rd decimal place.
- If this digit is 5 or more then add 1 to the digit in front of it, this means that when increasing a 3 for example, it would become a 4. But it also means that if it is a 9 it will become a 0 and will add 1 to the digit in front of it.
- If the digit is less than 5 the digit is unchanged and the number is just 'chopped' at the required decimal place.
- If there isn't a number in the decimal place asked for, then zeros are added to the number until the decimal place has a zero in the position.

The first sheet involves rounding to the nearest whole number and rounding to 1 decimal place.

The second sheet involves rounding to 2 and 3 decimal places.

The third sheet is all 4 on the one sheet.

Examples of rounding
0.54862
To 1 d.p. : 0.54862 → 0.5 (as 4 < 5)
To 2 d.p. : 0.54862 → 0.55 (as 8 > 5)
To 3 d.p. : 0.54862 → 0.549 (as 6 > 5)
To nearest whole number:
0.54862 → 1 (as 5 = 5)

Example: Whole numbers
65 (adding zeros)
To 1 d.p. : 65.0
To 2 d.p. : 65.00
To 3 d.p. : 65.000

Example:
71.5
To 1 d.p. : 71.5
To 2 d.p. : 71.50
To 3 d.p. : 71.500

Numbers of Any Magnitude - Rounding Decimals - 0 & 1 d.p. (Sheet 1)

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Round these decimals to the nearest whole number. Look at the first decimal digit.



Examples

$3.8 =$ 4 (0.8 more than 0.5)

$7.37 =$ 7 (0.3 less than 0.5)

1 $1.7 =$ ☐

2 $8.3 =$ ☐

3 $6.2 =$ ☐

4 $5.8 =$ ☐

5 $0.6 =$ ☐

6 $0.2 =$ ☐

These have more than 1 decimal place

7 $11.53 =$ ☐ ← Whole number answer

8 $2.07 =$ ☐

9 $15.72 =$ ☐

10 $23.6081 =$ ☐

11 $49.1189 =$ ☐

12 $7.067 =$ ☐



Find these, write the answer, then write the answer rounded to the nearest whole number.

13 $3.7 \times 8.4 =$ ☐ Actual

Whole number answer → $=$ ☐ Rounded

14 $5.16 \times 13.7 =$ ☐ Actual

$=$ ☐ Rounded

15 $7.59 \times 37.8 =$ ☐

$=$ ☐

Its estimation time. Change these to whole numbers to estimate the answer.



Example 3.8×5.2

$\approx 4 \times 5 = 20$

16 $2.9 \times 6.7 \approx$ ☐ (Round 2.9) $\times$ ☐ (Round 6.7) $=$ ☐

17 $5.3 \times 9.8 \approx$ ☐ $\times$ ☐ $=$ ☐

18 $7.8 \div 1.9 \approx$ ☐ $\div$ ☐ $=$ ☐

19 $8.3 \times 4.1 \approx$ ☐ $\times$ ☐ $=$ ☐

20 $9.2 \div 2.7 \approx$ ☐ $\div$ ☐ $=$ ☐



Round these decimals to 1 decimal place. (1 d.p.)

Examples

$3.82 =$ 3.8 (Less than 5)

$7.97 =$ 8.0 (More than 5)

21 $3.16 =$ ☐.☐

22 $8.52 =$ ☐.☐

23 $16.78 =$ ☐

24 $-0.61 =$ ☐

25 $4.95 =$ ☐

Match the decimal with its answer when rounded to 1 d.p.



Use a Ruler!

26	5.26 ●	●6.3
27	6.25 ●	●6.5
28	2.56 ●	●5.6
29	5.62 ●	●5.3
30	6.52 ●	●2.6



These decimals have more decimal places. Round them to 1 decimal place. (1 d.p.)

Examples

Ignore the 5 & 2 (Less than 5): $3.8352 =$ 3.8

Ignore the 2 (More than 5): $7.362 =$ 7.4

Look at the arrowed digit for rounding

31 $9.0671 =$ ☐.☐

32 $0.7155 =$ ☐

33 $48.96051 =$ ☐

34 $145.27 =$ ☐

35 $59.962 =$ ☐

36 $0.3815 =$ ☐

37 $-12.629 =$ ☐

38 $8.0188 =$ ☐

39 $152.3826 =$ ☐

40 $45.56621 =$ ☐

Complete the calculations and give the answer as a whole number and a decimal to 1 d.p.

41 $2.3^3 \times 0.7^2 + 1.8$

$=$ ☐ To 1 d.p. $=$ ☐ Whole Number

42 $1.13^4 \times 3.57^3$

$=$ ☐ To 1 d.p. $=$ ☐ Whole Number

43 $\dfrac{8^2 + 9^3}{4^6 - 2^7} + 0.8$

$=$ ☐ To 1 d.p. $=$ ☐ Whole Number

Numbers of Any Magnitude - Rounding Decimals - 2 & 3 d.p. (Sheet 2)

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Round these decimals to 2 decimal places. Look at the third decimal digit to decide.

Examples

$5.783 = 5.78$ (3 is less than 5)

$0.7283 = 0.73$ (8 is more than 5)

1 $2.916 =$ ☐.☐☐

2 $42.053 =$ ☐

3 $-8.198 =$ ☐

4 $207.371 =$ ☐

5 $1.897 =$ ☐

When a decimal has less than 2 decimal places, add zeros. Write these to 2 d.p.

Examples

$2.6 = 2.60$ (Add a zero)

$14 = 14.00$ (Add two zeros)

6 $8 =$ ☐.☐☐

7 $4.3 =$ ☐

8 $113.7 =$ ☐

9 $344 =$ ☐

These have more decimal places. Use the arrow to help to get you rounding to 2 d.p.

10 $0.1642 =$ ☐.☐☐

11 $0.3884 =$ ☐

12 $17.744823 =$ ☐

13 $5.096273 =$ ☐

14 $83.25199 =$ ☐

15 $-11.99716 =$ ☐

Round these to 2 d.p.

16 $61.37285 =$ ☐

17 $0.115099 =$ ☐

18 $4.797003 =$ ☐

19 $97.98221 =$ ☐

20 $19.99534 =$ ☐

Calculate these and write the answer to 2 d.p.

21 $0.9^2 \times 9.4 =$ ☐ Rounded to 2 d.p.

22 $1.75^3 + 5.2^2 =$ ☐

23 $\dfrac{31.9 + 54.2}{4.7^2} =$ ☐

24 $(-0.762)^5 =$ ☐

Round these to 3 d.p. Use the arrow to help you get started.

Examples

$0.78367 = 0.784$ (6 is more than 5)

$0.21842 = 0.218$ (4 is less than 5)

25 $0.778321 =$ ☐.☐☐☐

26 $6.01596 =$ ☐

27 $-38.09951 =$ ☐

28 $0.047177 =$ ☐

29 $3.141592654 =$ ☐

30 $1.414213562 =$ ☐

Calculate these and write the answer to 3 d.p.

31 $\sqrt{15} =$ ☐.☐☐☐ Rounded to 3 d.p.

32 $\sqrt{624} =$ ☐

33 $\sqrt{5.2^2 - 3.1^2} =$ ☐

34 $\sqrt{24^2 + 17.7^2} =$ ☐

35 $\dfrac{14^3 - 11^3}{5^2} =$ ☐

Match the left hand side with its answer on the right, when rounded to 3 d.p.

Use a Ruler!

36	0.2	0.002
37	0.12^3	0.003
38	$\dfrac{3}{1\,004}$	0.001
39	$3^2 \div 9^4$	0.200
40	$\sqrt{0.08}$	0.283

Round these to the nearest whole number, 1 d.p., 2 d.p. and 3 d.p.

41 0.7

Whole	1 d.p.	2 d.p.	3 d.p.
☐	☐	☐	☐

42 4.8168

Whole	1 d.p.	2 d.p.	3 d.p.
☐	☐	☐	☐

43 0.0957

Whole	1 d.p.	2 d.p.	3 d.p.
☐	☐	☐	☐

44 9.9997

Whole	1 d.p.	2 d.p.	3 d.p.
☐	☐	☐	☐

Numbers of Any Magnitude - Rounding Decimals 0-3 d.p. (Sheet 3)

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Round these numbers to the nearest tenth (1 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.2

1. 0.87
2. 0.43
3. 0.90347
4. 0.05956
5. 18.374
6. 22.128
7. 67
8. 56.942
9. 129.988
10. 99.953

Round these numbers to the nearest hundredth (2 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.16

11. 0.278
12. 0.805
13. 0.4246
14. 0.9793
15. 2.2183
16. 73.2
17. 4.95829
18. 9.99811
19. 67.9942
20. 83
21. 99.9961

Round these numbers to the nearest thousandth (3 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.163

22. 2.7674
23. 0.8436
24. 8.0302
25. 6.9969
26. 6.9996
27. 0.7
28. 0.70
29. 0.707
30. 4
31. 9.9999

Round these numbers to the nearest unit (0 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1

32. 4.7
33. 9.3
34. 18.78
35. 249.499
36. 7 101.835
37. 83.5004
38. 999.278
39. 999.872
40. 0.808
41. 0.080
42. 9 999.5

Round these numbers to the decimal place given in the brackets

43. 29.6751 [2]
44. 1 361.54 [1]
45. 0.4296 [0]
46. 9.1 [3]
47. 19.9971 [2]
48. 6.0606 [3]
49. 86 [2]
50. 56.0438 [1]
51. 11.4099 [0]
52. 8.8997 [3]
53. 5.9949 [2]

Round each decimal to 1, 2 and 3 decimal places

54. 0.1919 [1] _____ [2] _____ [3] _____
55. 3.8225 [1] _____ [2] _____ [3] _____
56. 26.9527 [1] _____ [2] _____ [3] _____
57. 0.3235 [1] _____ [2] _____ [3] _____
58. 9.9994 [1] _____ [2] _____ [3] _____

Numbers of Any Magnitude - Rounding Decimals - 0 & 1 d.p. (Sheet 1)

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Round these decimals to the nearest whole number. Look at the first decimal digit.

Examples

3.8 = 4 (0.8 more than 0.5)

7.37 = 7 (0.3 less than 0.5)

1 1.7 = 2

2 8.3 = 8

3 6.2 = 6

4 5.8 = 6

5 0.6 = 1

6 0.2 = 0

These have more than 1 decimal place

7 11.53 = 12 ← Whole number answer

8 2.07 = 2

9 15.72 = 16

10 23.6081 = 24

11 49.1189 = 49

12 7.067 = 7

Find these, write the answer, then write the answer rounded to the nearest whole number.

13 3.7 × 8.4 = 31.08 (Actual)

Whole number answer → = 31 (Rounded)

14 5.16 × 13.7 = 70.692 (Actual)

= 71 (Rounded)

15 7.59 × 37.8 = 286.902

= 287

Its estimation time. Change these to whole numbers to estimate the answer.

Example 3.8 × 5.2

≈ 4 × 5 = 20

16 2.9 × 6.7 ≈ 3 × 7 = 21 (Round 2.9, Round 6.7)

17 5.3 × 9.8 ≈ 5 × 10 = 50

18 7.8 ÷ 1.9 ≈ 8 ÷ 2 = 4

19 8.3 × 4.1 ≈ 8 × 4 = 32

20 9.2 ÷ 2.7 ≈ 9 ÷ 3 = 3

Round these decimals to 1 decimal place. (1 d.p.)

Examples

3.82 = 3.8 (Less than 5)

7.97 = 8.0 (More than 5)

21 3.16 = 3.2

22 8.52 = 8.5

23 16.78 = 16.8

24 -0.61 = -0.6

25 4.95 = 5.0

Match the decimal with its answer when rounded to 1 d.p.

Use a Ruler!

26	5.26	6.3
27	6.25	6.5
28	2.56	5.6
29	5.62	5.3
30	6.52	2.6

These decimals have more decimal places. Round them to 1 decimal place. (1 d.p.)

Examples

3.8352 = 3.8 (Ignore the 5 & 2; Less than 5)

7.362 = 7.4 (Ignore the 2; More than 5)

Look at the arrowed digit for rounding

31 9.0671 = 9.1

32 0.7155 = 0.8

33 48.96051 = 49.0

34 145.27 = 145.3

35 59.962 = 60.0

36 0.3815 = 0.4

37 -12.629 = -12.6

38 8.0188 = 8.0

39 152.3826 = 152.4

40 45.56621 = 45.6

Complete the calculations and give the answer as a whole number and a decimal to 1 d.p.

41 $2.3^3 \times 0.7^2 + 1.8$

= 7.8 (To 1 d.p.) = 8 (Whole Number)

42 $1.13^4 \times 3.57^3$

= 74.2 (To 1 d.p.) = 74 (Whole Number)

43 $\frac{8^2 + 9^3}{4^6 - 2^7} + 0.8$

= 1.0 (To 1 d.p.) = 1 (Whole Number)

Numbers of Any Magnitude - Rounding Decimals - 2 & 3 d.p. (Sheet 2)

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Round these decimals to 2 decimal places. Look at the third decimal digit to decide.

Examples

5.783 = 5.78 (3 is less than 5)

0.7283 = 0.73 (8 is more than 5)

1 2.916 = 2.92

2 42.053 = 42.05

3 -8.198 = -8.20

4 207.371 = 207.37

5 1.897 = 1.90

When a decimal has less than 2 decimal places, add zeros. Write these to 2 d.p.

Examples

2.6 = 2.60 (Add a zero)

14 = 14.00 (Add two zeros)

6 8 = 8.00

7 4.3 = 4.30

8 113.7 = 113.70

9 344 = 344.00

These have more decimal places. Use the arrow to help to get you rounding to 2 d.p.

10 0.1642 = 0.16

11 0.3884 = 0.39

12 17.744823 = 17.74

13 5.096273 = 5.10

14 83.25199 = 83.25

15 -11.99716 = -12.00

Round these to 2 d.p.

16 61.37285 = 61.37

17 0.115099 = 0.12

18 4.797003 = 4.80

19 97.98221 = 97.98

20 19.99534 = 20.00

Calculate these and write the answer to 2 d.p.

21 $0.9^2 \times 9.4$ = 7.61 (Rounded to 2 d.p.)

22 $1.75^3 + 5.2^2$ = 32.40

23 $\frac{31.9 + 54.2}{4.7^2}$ = 3.90

24 $(-0.762)^5$ = -0.26

Round these to 3 d.p. Use the arrow to help you get started.

Examples

0.78367 = 0.784 (6 is more than 5)

0.21842 = 0.218 (4 is less than 5)

25 0.778321 = 0.778

26 6.01596 = 6.016

27 -38.09951 = -38.100

28 0.047177 = 0.047

29 3.141592654 = 3.142

30 1.414213562 = 1.414

Calculate these and write the answer to 3 d.p.

31 $\sqrt{15}$ = 3.873 (Rounded to 3 d.p.)

32 $\sqrt{624}$ = 24.980

33 $\sqrt{5.2^2 - 3.1^2}$ = 4.175

34 $\sqrt{24^2 + 17.7^2}$ = 29.821

35 $\frac{14^3 - 11^3}{5^2}$ = 56.520

Match the left hand side with its answer on the right, when rounded to 3 d.p.

Use a Ruler!

36 0.2 — 0.200

37 0.12^3 — 0.002

38 $\frac{3}{1\,004}$ — 0.003

39 $3^2 \div 9^4$ — 0.001

40 $\sqrt{0.08}$ — 0.283

Round these to the nearest whole number, 1 d.p., 2 d.p. and 3 d.p.

41 0.7

Whole	1 d.p.	2 d.p.	3 d.p.
1	0.7	0.70	0.700

42 4.8168

Whole	1 d.p.	2 d.p.	3 d.p.
5	4.8	4.82	4.817

43 0.0957

Whole	1 d.p.	2 d.p.	3 d.p.
0	0.1	0.10	0.096

44 9.9997

Whole	1 d.p.	2 d.p.	3 d.p.
10	10.0	10.00	10.000

Numbers of Any Magnitude - Rounding Decimals 0-3 d.p. (Sheet 3)

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Round these numbers to the nearest tenth (1 d.p.)

Look at the digit to the right of the one to be rounded. If it is 5 or more round up, if < 5 the digit is unchanged

Example: 1.1627 → 1.2

1	0.87	0.9
2	0.43	0.4
3	0.90347	0.9
4	0.05956	0.1
5	18.374	18.4
6	22.128	22.1
7	67	67.0
8	56.942	56.9
9	129.988	130.0
10	99.953	100

Round these numbers to the nearest hundredth (2 d.p.)

Look at the digit to the right of the one to be rounded. If it is 5 or more round up, if < 5 the digit is unchanged

Example: 1.1627 → 1.16

11	0.278	0.28
12	0.805	0.81
13	0.4246	0.42
14	0.9793	0.98
15	2.2183	2.22
16	73.2	73.20
17	4.95829	4.96
18	9.99811	10.00
19	67.9942	67.99
20	83	83.00
21	99.9961	100.00

Round these numbers to the nearest thousandth (3 d.p.)

Look at the digit to the right of the one to be rounded. If it is 5 or more round up, if < 5 the digit is unchanged

Example: 1.1627 → 1.163

22	2.7674	2.767
23	0.8436	0.844
24	8.0302	8.030
25	6.9969	6.997
26	6.9996	7.000
27	0.7	0.700
28	0.70	0.700
29	0.707	0.707
30	4	4.000
31	9.9999	10.000

Round these numbers to the nearest unit (0 d.p.)

Look at the digit to the right of the one to be rounded. If it is 5 or more round up, if < 5 the digit is unchanged

Example: 1.1627 → 1

32	4.7	5
33	9.3	9
34	18.78	19
35	249.499	249
36	7 101.835	7 102
37	83.5004	84
38	999.278	999
39	999.872	1 000
40	0.808	1
41	0.080	0
42	9 999.5	1 000

Round these numbers to the decimal place given in the brackets

43	29.6751	[2]	29.68
44	1 361.54	[1]	1 361.5
45	0.4296	[0]	0
46	9.1	[3]	9.100
47	19.9971	[2]	20.00
48	6.0606	[3]	6.061
49	86	[2]	86.00
50	56.0438	[1]	56.0
51	11.4099	[0]	11
52	8.8997	[3]	8.900
53	5.9949	[2]	5.99

Round each decimal to 1, 2 and 3 decimal places

54	0.1919	[1]	0.2	[2]	0.19	[3]	0.192
55	3.8225	[1]	3.8	[2]	3.82	[3]	3.823
56	26.9527	[1]	27.0	[2]	26.95	[3]	26.953
57	0.3235	[1]	0.3	[2]	0.32	[3]	0.324
58	9.9994	[1]	10.0	[2]	10.00	[3]	9.999

Numbers of Any Magnitude - Rounding Decimals (Homework Strip)

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Date Due: _____

Round to the nearest whole number

1 $6.4 =$ _____ 2 $0.82 =$ _____

3 $9.67 =$ _____ 4 $0.48 =$ _____

Calculate, rounding the to nearest unit

5 $\sqrt{29.2} =$ _____

6 $4.33^2 + 7.8^2 =$ _____

Round to one decimal place

7 $0.4283 =$ _____

8 $3.8715 =$ _____

9 $919.9701 =$ _____

Calculate, rounding to 1 d.p.

10 $\sqrt{15.8 + 19.3} =$ _____

11 $7.17^4 =$ _____

Round to two decimal places

12 $0.2163 =$ _____

13 $8.3355 =$ _____

14 $66.66666 =$ _____

Date Due: _____

Round to the nearest whole number

1 $6.4 =$ _____ 2 $0.82 =$ _____

3 $9.67 =$ _____ 4 $0.48 =$ _____

Calculate, rounding the to nearest unit

5 $\sqrt{29.2} =$ _____

6 $4.33^2 + 7.8^2 =$ _____

Round to one decimal place

7 $0.4283 =$ _____

8 $3.8715 =$ _____

9 $919.9701 =$ _____

Calculate, rounding to 1 d.p.

10 $\sqrt{15.8 + 19.3} =$ _____

11 $7.17^4 =$ _____

Round to two decimal places

12 $0.2163 =$ _____

13 $8.3355 =$ _____

14 $66.66666 =$ _____

Date Due: _____

Round to the nearest whole number

1 $6.4 =$ _____ 2 $0.82 =$ _____

3 $9.67 =$ _____ 4 $0.48 =$ _____

Calculate, rounding the to nearest unit

5 $\sqrt{29.2} =$ _____

6 $4.33^2 + 7.8^2 =$ _____

Round to one decimal place

7 $0.4283 =$ _____

8 $3.8715 =$ _____

9 $919.9701 =$ _____

Calculate, rounding to 1 d.p.

10 $\sqrt{15.8 + 19.3} =$ _____

11 $7.17^4 =$ _____

Round to two decimal places

12 $0.2163 =$ _____

13 $8.3355 =$ _____

14 $66.66666 =$ _____

Date Due: _____

Round to the nearest whole number

1 $6.4 =$ _____ 2 $0.82 =$ _____

3 $9.67 =$ _____ 4 $0.48 =$ _____

Calculate, rounding the to nearest unit

5 $\sqrt{29.2} =$ _____

6 $4.33^2 + 7.8^2 =$ _____

Round to one decimal place

7 $0.4283 =$ _____

8 $3.8715 =$ _____

9 $919.9701 =$ _____

Calculate, rounding to 1 d.p.

10 $\sqrt{15.8 + 19.3} =$ _____

11 $7.17^4 =$ _____

Round to two decimal places

12 $0.2163 =$ _____

13 $8.3355 =$ _____

14 $66.66666 =$ _____

Numbers of Any Magnitude - Rounding Decimals (Homework Strip)

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Date Due:

Round to the nearest whole number

1 $6.4 =$ 6

2 $0.82 =$ 1

3 $9.67 =$ 10

4 $0.48 =$ 0

Calculate, rounding the to nearest unit

5 $\sqrt{29.2} =$ 5

6 $4.33^2 + 7.8^2 =$ 80

Round to one decimal place

7 $0.4283 =$ 0.4

8 $3.8715 =$ 3.9

9 $919.9701 =$ 920.0

Calculate, rounding to 1 d.p.

10 $\sqrt{15.8 + 19.3} =$ 5.9

11 $7.17^4 =$ 2 642.9

Round to two decimal places

12 $0.2163 =$ 0.22

13 $8.3355 =$ 8.34

14 $66.66666 =$ 66.67

Numbers of Any Magnitude - Counting Significant Figures

Significant figures are the digits in an answer that are considered relevant in the accuracy of the answer. This sheet deals with counting the number of significant figures in whole numbers and decimals.

The first sheet deals with whole numbers and decimals larger than 1. With whole numbers all digits are significant except zeros at the end of the number, which (due to the possibility of rounding), could be significant.

Examples

45 - 2 significant figures
2 008 - 4 significant figures because the zeros are 'bookended' by non-zero digits
3 100 - 2 or 3 or 4 significant figures because the zeros are not 'bookended' by non-zero digits, so they might or might not be significant. Or possibly significant.

Decimals are easier, especially when they are greater than 1. These are the last 4 questions on the first sheet. With these, all the digits are significant. Just count the digits.

Examples

95.3 - 3 significant figures
10.500 - 5 significant figures
300.00 - Also 5 significant figures. Note that 300 has 1 or 2 or 3 sig. fig., but 300.00 has 5. The reason being that the two decimal places indicate an accuracy of measurement.

The second worksheet deals with decimals less than 1. So '0.__'. With these you don't count the leading zeros. So until you hit a non-zero digit, the digits aren't significant.

Examples

0.005 - 1 significant figure, the first 3 digits are leading zeros and aren't significant
0.10007 - 5 significant figures, bookended zeros are included as they are significant
0.00500 - 3 significant figures, the trailing zeros are considered significant. Note that looking at the decimal you would consider the last two zeros as irrelevant. But their presence indicates the accuracy of the measurement.

Numbers of Any Magnitude - Counting Significant Figures (Sheet 1)

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Break these numbers up into 'significant' and 'possible' significant figures.

Examples 0's at end are possible

2 300 = 2 3 0 0

Significant Possible

0's between non-zero digits are significant

2 003 = 2 0 0 3

Significant

Combination of the two

2 030 = 2 0 3 0

Significant Possible

1 460 = ☐☐☐

2 5 000 = ☐☐☐☐

3 4 705 = ☐☐☐☐

4 71 070 = ☐☐☐☐☐

5 93 600 = ☐☐☐☐☐

6 35 206 000

☐☐☐☐☐☐☐☐

7 409 100

☐☐☐☐☐☐

8 1 480 050

☐☐☐☐☐☐☐

Now count the significant figures. Use the word 'or' between the possible sig figs.

Examples

2 300 = 2 3 0 0

2 3 4

There are 2 or 3 or 4 sig figs.

2 003 = 2 0 0 3

4

There are 4 sig figs.

2 030 = 2 0 3 0

3 4

There are 3 or 4 sig figs.

9 8 030 = ☐☐☐☐ (☐ ☐)

There are ☐ or ☐ sig figs.

10 3 400 = ☐☐☐☐ (☐ ☐ ☐)

There are ☐ or ☐ or ☐ sig figs.

11 5 043 = ☐☐☐☐ (☐)

There are ☐ sig figs.

12 70 900

☐☐☐☐☐

There are ☐ sig figs

13 8 619 000

☐☐☐☐☐☐☐

There are ☐ sig figs

14 406 100

☐☐☐☐☐☐

There are ☐ sig figs

15 9 005 020

☐☐☐☐☐☐☐

There are ☐ sig figs

16 120 000

☐☐☐☐☐☐

There are ☐ sig figs

17 27 000 050

☐☐☐☐☐☐☐☐

There are ☐ sig figs

If a decimal is larger than 1, all the digits in the decimal are significant. Count those sig figs.

Examples

23.00 There are 4 sig figs.

2.0030 There are 5 sig figs.

2000.30 There are 6 sig figs.

18 1.0 There are ☐ sig figs

19 5.100 There are ☐ sig figs

20 75.03 There are ☐ sig figs

21 195.00 There are ☐ sig figs

Numbers of Any Magnitude - Counting Significant Figures (Sheet 2)

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Break these decimals up into 'ignore' and 'significant' figures.

Examples Ignore all leading 0's

$0.007 =$ 0 . 0 0 7

Ignore Significant

0's between non-zero digits are significant

$0.4003 =$ 0 . 4 0 0 3

Ignore Significant

0's to the right are also significant

$0.0600 =$ 0 . 0 6 0 0

Ignore Significant

1 $0.88 =$.

2 $0.027 =$.

3 $0.050 =$.

4 0.010200

.

5 0.000775

.

6 0.009003

.

7 0.50010

.

8 $0.400 =$.

Now underline and then count the significant figures. Ignore all the leading zeros.

Examples

$0.007 =$ 0 . 0 0 7

There is 1 sig fig. 1

$0.4003 =$ 0 . 4 0 0 3

4

There are 4 sig figs.

$0.600 =$ 0 . 6 0 0

3

There are 3 sig figs.

9 $0.020 =$.

There are sig figs.

10 $0.170 =$.

There are sig figs.

11 $0.003 =$.

There is sig fig.

12 $0.900 =$.

13 0.027300

.

14 0.006038

.

Perform these calculations and then give the number of sig figs that are in the answer.

15 $0.32^2 =$ Answer Sig Fig

16 $40.04 \div 50 =$ Answer

Sig Fig

17 $0.02^2 \div 0.5^3$

$=$

18 0.16×0.48^2

$=$

19 $0.2^7 =$

Convert these fractions to decimals and give the number of significant figures in the answer.

20 $\frac{8}{125} =$

21 $\frac{401}{5\,000} =$

22 $\frac{9}{12\,500} =$

23 $\frac{63}{900} =$

24 $\frac{5\,253}{12\,500} =$

25 $\frac{6}{48\,000} =$

26 $\frac{3\,113}{50\,000} =$

Numbers of Any Magnitude - Counting Significant Figures (Sheet 1)

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Break these numbers up into 'significant' and 'possible' significant figures.

Examples 0's at end are possible

2 300 = 2 3 0 0 (Significant: 2 3; Possible: 0 0)

0's between non-zero digits are significant

2 003 = 2 0 0 3 (Significant: 2 0 0 3)

Combination of the two

2 030 = 2 0 3 0 (Significant: 2 0 3; Possible: 0)

1 460 = 4 6 0 (Significant: 4 6; Possible: 0)

2 5 000 = 5 0 0 0 (Significant: 5; Possible: 0 0 0)

3 4 705 = 4 7 0 5 (Significant: 4 7 0 5)

4 71 070 = 7 1 0 7 0 (Significant: 7 1 0 7; Possible: 0)

5 93 600 = 9 3 6 0 0 (Significant: 9 3 6; Possible: 0 0)

6 35 206 000

3 5 2 0 6 0 0 0 (Significant: 3 5 2 0 6; Possible: 0 0 0)

7 409 100

4 0 9 1 0 0 (Significant: 4 0 9 1; Possible: 0 0)

8 1 480 050

1 4 8 0 0 5 0 (Significant: 1 4 8 0 0 5; Possible: 0)

Now count the significant figures. Use the word 'or' between the possible sig figs.

Examples

2 300 = 2 3 0 0 (2 3 4)

There are 2 or 3 or 4 sig figs.

2 003 = 2 0 0 3 (4)

There are 4 sig figs.

2 030 = 2 0 3 0 (3 4)

There are 3 or 4 sig figs.

9 8 030 = 8 0 3 0 (3 4)

There are 3 or 4 sig figs.

10 3 400 = 3 4 0 0 (2 3 4)

There are 2 or 3 or 4 sig figs.

11 5 043 = 5 0 4 3 (4)

There are 4 sig figs.

12 70 900

7 0 9 0 0 (3 4 5)

There are 3 or 4 or 5 sig figs

13 8 619 000

8 6 1 9 0 0 0 (4 5 6 7)

There are 4 or 5 or 6 or 7 sig figs

14 406 100

4 0 6 1 0 0 (5 6)

There are 4 or 5 or 6 sig figs

15 9 005 020

9 0 0 5 0 2 0 (6 7)

There are 6 or 7 sig figs

16 120 000

1 2 0 0 0 0 (2 3 4 5 6)

There are 2 or 3 or 4 or 5 or 6 sig figs

17 27 000 050

2 7 0 0 0 0 5 0 (7 8)

There are 7 or 8 sig figs

If a decimal is larger than 1, all the digits in the decimal are significant. Count those sig figs.

Examples

23.00 There are 4 sig figs.

2.0030 There are 5 sig figs.

2000.30 There are 6 sig figs.

18 1.0 There are 2 sig figs

19 5.100 There are 4 sig figs

20 75.03 There are 4 sig figs

21 195.00 There are 5 sig figs

Numbers of Any Magnitude - Counting Significant Figures (Sheet 2)

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Break these decimals up into 'ignore' and 'significant' figures.

Examples Ignore all leading 0's

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Ignore Significant

0's between non-zero digits are significant

0.4003 = 0 . 4 0 0 3
Ignore Significant

0's to the right are also significant

0.0600 = 0 . 0 6 0 0
Ignore Significant

1 0.88 = 0 . 8 8
Ignore Significant

2 0.027 = 0 . 0 2 7
Ignore Significant

3 0.050 = 0 . 0 5 0
Ignore Significant

4 0.010200

0 . 0 1 0 2 0 0
Ignore Significant

5 0.000775

0 . 0 0 0 7 7 5
Ignore Significant

6 0.009003

0 . 0 0 9 0 0 3
Ignore Significant

7 0.50010

0 . 5 0 0 1 0
Ignore Significant

8 0.400 = 0 . 4 0 0
Ignore Significant

Now underline and then count the significant figures. Ignore all the leading zeros.

Examples

0.007 = 0 . 0 0 7
There is 1 sig fig. 1

0.4003 = 0 . 4 0 0 3
4
There are 4 sig figs.

0.600 = 0 . 6 0 0
3
There are 3 sig figs.

9 0.020 = 0 . 0 2 0
2

There are 2 sig figs.

10 0.170 = 0 . 1 7 0
3

There are 3 sig figs.

11 0.003 = 0 . 0 0 3
1

There is 1 sig fig.

12 0.900 = 0 . 9 0 0
3

There are 3 significant figures

13 0.027300

0 . 0 2 7 3 0 0
5

There are 5 significant figures

14 0.006038

0 . 0 0 6 0 3 8
4

There are 4 significant figures

Perform these calculations and then give the number of sig figs that are in the answer.

15 0.32^2 = 0.1024 (Answer) 4 (Sig Fig)

16 40.04 ÷ 50 = 0.8008 (Answer)
Sig Fig 4

17 $0.02^2 \div 0.5^3$
= 0.0032 2

18 0.16×0.48^2
= 0.036864 5

19 0.2^7 = 0.0000128
3

Convert these fractions to decimals and give the number of significant figures in the answer.

20 $\frac{8}{125}$ = 0.064 2

21 $\frac{401}{5\,000}$ = 0.0802 3

22 $\frac{9}{12\,500}$ = 0.00072 2

23 $\frac{63}{900}$ = 0.07 1

24 $\frac{5\,253}{12\,500}$ = 0.42024 5

25 $\frac{6}{48\,000}$ = 0.000125 3

26 $\frac{3\,113}{50\,000}$ = 0.06226 4

Numbers of Any Magnitude - Counting Significant Figures (Homework Strip)

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Date Due: _____

Underline and count the Sig Figs

1 6 523 ____ 2 145 ____

3 70 ____ or ____

4 300 ____ or ____ or ____

5 15 700 _____

Write the answer and count the sig fig

6 $84^2 =$ _____ Answer ____ Sig Fig

7 $60^2 =$ _____ Answer _____ Sig Fig

Count the significant figures in these

8 3.0 ____ 9 36.20 ____

10 0.5 ____ 11 0.070 ____

12 0.00090200 ____

13 0.01005000 ____

14 1.00040000 ____

Date Due: _____

Underline and count the Sig Figs

1 6 523 ____ 2 145 ____

3 70 ____ or ____

4 300 ____ or ____ or ____

5 15 700 _____

Write the answer and count the sig fig

6 $84^2 =$ _____ Answer ____ Sig Fig

7 $60^2 =$ _____ Answer _____ Sig Fig

Count the significant figures in these

8 3.0 ____ 9 36.20 ____

10 0.5 ____ 11 0.070 ____

12 0.00090200 ____

13 0.01005000 ____

14 1.00040000 ____

Date Due: _____

Underline and count the Sig Figs

1 6 523 ____ 2 145 ____

3 70 ____ or ____

4 300 ____ or ____ or ____

5 15 700 _____

Write the answer and count the sig fig

6 $84^2 =$ _____ Answer ____ Sig Fig

7 $60^2 =$ _____ Answer _____ Sig Fig

Count the significant figures in these

8 3.0 ____ 9 36.20 ____

10 0.5 ____ 11 0.070 ____

12 0.00090200 ____

13 0.01005000 ____

14 1.00040000 ____

Date Due: _____

Underline and count the Sig Figs

1 6 523 ____ 2 145 ____

3 70 ____ or ____

4 300 ____ or ____ or ____

5 15 700 _____

Write the answer and count the sig fig

6 $84^2 =$ _____ Answer ____ Sig Fig

7 $60^2 =$ _____ Answer _____ Sig Fig

Count the significant figures in these

8 3.0 ____ 9 36.20 ____

10 0.5 ____ 11 0.070 ____

12 0.00090200 ____

13 0.01005000 ____

14 1.00040000 ____

Numbers of Any Magnitude - Counting Significant Figures (H'work Strip)

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Date Due:

Underline and count the Sig Figs

1 6 523 (4) — 4

2 145 (3) — 3

3 70 (1 2) — 1 or 2

4 300 (1 2 3) — 1 or 2 or 3

5 15 700 (3 4 5) — 3 or 4 or 5

Write the answer and count the sig fig

6 $84^2 =$ 7 056 (Answer) — 4 (Sig Fig)

7 $60^2 =$ 3 600 (Answer) — 2 or 3 or 4 (Sig Fig)

Count the significant figures in these

8 3.0 — 2

9 36.20 — 4

10 0.5 — 1

11 0.070 — 2

12 0.00090200 — 5

13 0.01005000 — 7

14 1.00040000 — 9

Numbers of Any Magnitude - Significant Figures and Rounding

Significant figures is a way of rounding answers without specifying decimal places. This type of rounding can be used for both numbers or decimals. The first sheet deals with whole numbers (and some decimals greater than 1). The second sheet deals with decimals, which are less than 1.

When you write a number to 1 significant figure, it has only 1 non-zero digit (unless it is 0). All the other digits become '0'.

When you write a number to 2 significant figures, it has only 2 non-zero digits (unless one or both of them are 0). All the other digits become '0'.

Examples of rounding Large Numbers
16 233
To 1 sig.fig. : 16233 → 20 000 (as 6 > 5)

When rounding to 1 sig. fig, use the second digit to round up or down

Examples of rounding Large Numbers
16 233
To 2 sig.fig. : 16233 → 16 000 (as 2 < 5)

When rounding to 2 sig. fig, use the third digit to round up or down

With decimals, the second worksheet, the leading zeros aren't considered significant figures. The first non-zero digit in a decimal is the first significant figure.

Examples of rounding Decimals
0.00073
To 1 sig.fig. : 0.00073 → 0.0007 (as 3 < 5)

When rounding to 1 sig. fig, use the digit after the first significant figure to round up or down. Ignore the leading zeros.

Examples of rounding Decimals
0.0007375
To 2 sig.fig. : 0.0007375 → 0.00074 (as 7 > 5)

When rounding to 2 sig. fig, use the digit after the second significant figure to round up or down. Ignore the leading zeros.

Significant Figures and Rounding - 1 & 2 Sig.Fig. (Sheet 1)

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Round these numbers/decimals to 1 significant figure.

Examples $473 = 500$

7 more than 5 - Round up

$2\,372 = 2\,000$

3 less than 5 - Round down

1 $46 =$ ____ 2 $1.9 =$ ____

3 $5\,800 =$ ____

4 $3\,193 =$ ____

5 $81 =$ ____ 6 $6.6 =$ ____

7 $97\,100 =$ ____

8 $637\,522 =$ ____

Calculate these and give two answers. The actual answer, then rounded to 1 sig. fig.

9 $34^2 + 19^2 =$ ____ Actual

$=$ ____ 1 sig. fig.

10 $31^4 =$ ____ Actual

$=$ ____ 1 sig. fig.

11 $\sqrt{28\,224} =$ ____ Actual $=$ ____ 1 sig. fig.

12 $\dfrac{89 \times 116}{20} =$ ____ Actual

$=$ ____ 1 sig. fig.

13 $227^2 =$ ____ Actual

$=$ ____ 1 sig. fig.

Calculate the area of the rectangles and give the answer to 1 significant fig.

14

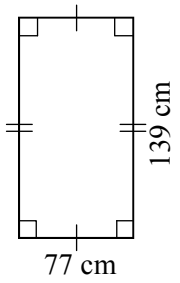
$A = lb$

$=$ ____ $\times$ ____

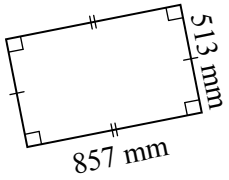
Multiply sides, in any order

$A =$ ____ cm^2

Answer to 1 sig fig



15



$A = lb$

$=$

$A =$ ____ mm^2

Answer to 1 sig fig

Write these to 2 significant figures.

Examples $473 = 470$

3 less than 5 - Round down

$2\,372 = 2\,400$

7 more than 5 - Round up

16 $386 =$ ____

17 $5 =$ ____ 18 $9 =$ ____

19 $25\,677 =$ ____

20 $507\,311 =$ ____

21 $89.7 =$ ____

22 $472.8 =$ ____

23 $3\,777\,529 =$ ____

24 $70\,813 =$ ____

Perform the calculations and answer to 1 and 2 sig. fig.

25 $516 \times 85 =$ ____ Actual

____ 1 sig. fig. ____ 2 sig. fig.

26 $84^3 =$ ____ Actual

____ 1 sig. fig. ____ 2 sig. fig.

27 $15^3 + 37^4 =$ ____ Actual

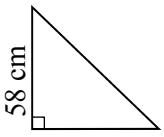
____ 1 sig. fig. ____ 2 sig. fig.

28 $\sqrt{59\,049} =$ ____ Actual

____ 1 sig. fig. ____ 2 sig. fig.

Find the area of these triangles answering to 2 sig. fig.

29



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ ____ $\times$ ____

0 . 5 × 6 3 × 5 8 =

$A =$ ____ cm^2

2 sig. fig.

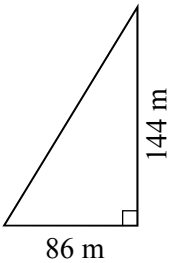
30

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ ____ $\times$ ____

$A =$ ____ m^2

2 sig. fig.



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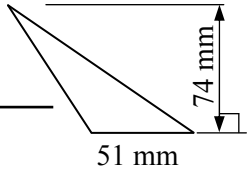

2 sig. fig.

Significant Figures and Rounding Decimals - 1 & 2 Sig.Fig. (Sheet 2)

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Round these decimals to 1 significant figure.

Examples $0.0382 =$ 0.04

8 more than 5 - Round up

$0.0000539 =$ 0.00005

3 less than 5 - Round down

1 $0.0647 =$ ____

2 $0.00231 =$ ____

3 $0.000037 =$ ____

4 $0.00095 =$ ____

*careful

5 $0.849 =$ ____

6 $0.972 =$ ____

7 $0.000033 =$ ____

8 $0.00451388 =$ ____

Perform the calculation and write the answer, then the answer rounded to 1 significant figure.

9 $8 \div 6\,400$

____ Decimal - Actual

$=$ ____ 1 sig. fig.

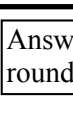
10 $\frac{125}{500\,000} =$ ____ Decimal - Actual

Use the S⇔D key $=$ ____ 1 sig. fig.

11 $\frac{21}{3\,360\,000}$

$=$ ____ Decimal - Actual

$=$ ____ 1 sig. fig.



Answer these as decimals and round to 1 significant figure.

12 $3 \div 1\,290 =$ ____ 1 sig. fig.

13 $\frac{83}{938\,508} =$ ____

14 $9 \div 9\,396 =$ ____

*careful

15 $37 \div 8\,771 =$ ____

16 $\frac{116}{117\,229} =$ ____

17 $\frac{3}{84\,080} =$ ____

18 $2^3 \div 2^{17} =$ ____



Round these decimals to 2 significant figures.

Examples $0.0382 =$ 0.038

2 less than 5 - Round down

$0.0000539 =$ 0.000054

9 more than 5 - Round up

19 $0.4531 =$ ____

20 $0.01039 =$ ____

21 $0.000317 =$ ____

22 $0.000997 =$ ____

*careful

23 $0.02007 =$ ____

24 $0.975 =$ ____

25 $0.068033 =$ ____

26 $0.000566 =$ ____



Calculate these, write the full answer first, then rounded to 2 significant figures.

27 $4 \div 3\,200 =$ ____ Decimal - Actual

$=$ ____ 2 sig. fig.

28 $141 \div 200\,000$

$=$ ____ Decimal - Actual $=$ ____ 2 sig. fig.

29 $281 \div 3\,125$

$=$ ____ Decimal - Actual $=$ ____ 2 sig. fig.



Calculate and write as decimals to both 1 & 2 significant figures.

30 $12 \div 4\,133$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

31 $2^7 \div 7^7$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

32 $\frac{499}{50\,000}$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

33 $5^2 \div 14^5$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

34 $\frac{179}{2\,500}$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

Significant Figures and Rounding - 1 & 2 Sig.Fig. (Sheet 1)

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Round these numbers/decimals to 1 significant figure.

Examples $473 = 500$ (7 more than 5 - Round up)

$2\,372 = 2\,000$ (3 less than 5 - Round down)

1 $46 = 50$ 2 $1.9 = 2$

3 $5\,800 = 6\,000$

4 $3\,193 = 3\,000$

5 $81 = 80$ 6 $6.6 = 7$

7 $97\,100 = 100\,000$

8 $637\,522 = 600\,000$

Calculate these and give two answers. The actual answer, then rounded to 1 sig. fig.

9 $34^2 + 19^2 =$ 1 517 (Actual)

$=$ 2 000 (1 sig. fig.)

10 $31^4 =$ 923 521 (Actual)

$=$ 900 000 (1 sig. fig.)

11 $\sqrt{28\,224} =$ 168 (Actual) $=$ 200 (1 sig. fig.)

12 $\frac{89 \times 116}{20} =$ 516.2 (Actual)

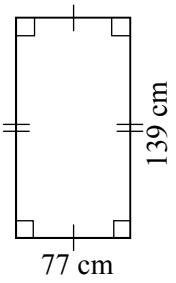
$=$ 500 (1 sig. fig.)

13 $227^2 =$ 51 529 (Actual)

$=$ 50 000 (1 sig. fig.)

Calculate the area of the rectangles and give the answer to 1 significant fig.

14



$A = lb$

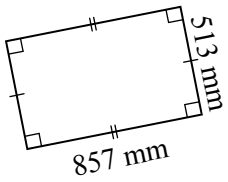
$= 139 \times 77$

Multiply sides, in any order

$A = 10\,000$ cm^2

Answer to 1 sig fig

15



$A = lb$

$= 857 \times 513$

$A = 400\,000$ mm^2

Answer to 1 sig fig

Write these to 2 significant figures.

Examples $473 = 470$ (3 less than 5 - Round down)

$2\,372 = 2\,400$ (7 more than 5 - Round up)

16 $386 = 390$

17 $5 = 5.0$ 18 $9 = 9.0$

19 $25\,677 = 26\,000$

20 $507\,311 = 510\,000$

21 $89.7 = 90$

22 $472.8 = 470$

23 $3\,777\,529 = 3\,800\,000$

24 $70\,813 = 71\,000$

Perform the calculations and answer to 1 and 2 sig. fig.

25 $516 \times 85 =$ 43 860 (Actual)

40 000 (1 sig. fig.) 44 000 (2 sig. fig.)

26 $84^3 =$ 592 704 (Actual)

600 000 (1 sig. fig.) 590 000 (2 sig. fig.)

27 $15^3 + 37^4 =$ 1 877 536 (Actual)

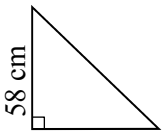
2 000 000 (1 sig. fig.) 1 900 000 (2 sig. fig.)

28 $\sqrt{59\,049} =$ 243 (Actual)

200 (1 sig. fig.) 240 (2 sig. fig.)

Find the area of these triangles answering to 2 sig. fig.

29



$A = \frac{1}{2}bh$

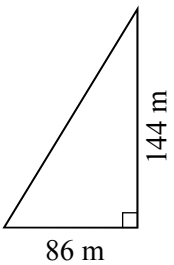
$= \frac{1}{2} \times 63 \times 58$

0 . 5 × 6 3 × 5 8 =

$A = 1\,800$ cm^2

2 sig. fig.

30



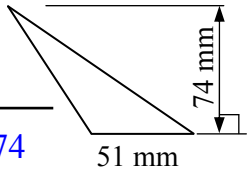
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 86 \times 144$

$A = 6\,200$ m^2

2 sig. fig.

31



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 51 \times 74$

$A = 1\,900$ mm^2

2 sig. fig.

Significant Figures and Rounding Decimals - 1 & 2 Sig.Fig. (Sheet 2)

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Round these decimals to 1 significant figure.

Examples $0.0382 =$ 0.04

8 more than 5 - Round up

$0.0000539 =$ 0.00005

3 less than 5 - Round down

1. $0.0647 =$ 0.06
2. $0.00231 =$ 0.002
3. $0.000037 =$ 0.00004
4. $0.00095 =$ 0.001 *careful
5. $0.849 =$ 0.8
6. $0.972 =$ 1
7. $0.000033 =$ 0.00003
8. $0.00451388 =$ 0.005

Perform the calculation and write the answer, then the answer rounded to 1 significant figure.

9. $8 \div 6\,400$ = 0.00125 Decimal - Actual = 0.001 1 sig. fig.
10. $\frac{125}{500\,000}$ = 0.00025 Decimal - Actual Use the S⇔D key = 0.0003 1 sig. fig.
11. $\frac{21}{3\,360\,000}$ = 0.00000625 Decimal - Actual = 0.000006 1 sig. fig.

Answer these as decimals and round to 1 significant figure.

12. $3 \div 1\,290 =$ 0.002 1 sig. fig.
13. $\frac{83}{938\,508} =$ 0.00009
14. $9 \div 9\,396 =$ 0.001 *careful
15. $37 \div 8\,771 =$ 0.004
16. $\frac{116}{117\,229} =$ 0.001
17. $\frac{3}{84\,080} =$ 0.00004
18. $2^3 \div 2^{17} =$ 0.00006

Round these decimals to 2 significant figures.

Examples $0.0382 =$ 0.038

2 less than 5 - Round down

$0.0000539 =$ 0.000054

9 more than 5 - Round up

19. $0.4531 =$ 0.45
20. $0.01039 =$ 0.010
21. $0.000317 =$ 0.00032
22. $0.000997 =$ 0.0010 *careful
23. $0.02007 =$ 0.020
24. $0.975 =$ 0.98
25. $0.068033 =$ 0.068
26. $0.000566 =$ 0.00057

Calculate these, write the full answer first, then rounded to 2 significant figures.

27. $4 \div 3\,200 =$ 0.00125 Decimal - Actual = 0.0013 2 sig. fig.
28. $141 \div 200\,000$ = 0.000705 Decimal - Actual = 0.00071 2 sig. fig.
29. $281 \div 3\,125$ = 0.08992 Decimal - Actual = 0.090 2 sig. fig.

Calculate and write as decimals to both 1 & 2 significant figures.

30. $12 \div 4\,133$ = 0.003 1 sig. fig. = 0.0029 2 sig. fig.
31. $2^7 \div 7^7$ = 0.0002 1 sig. fig. = 0.00016 2 sig. fig.
32. $\frac{499}{50\,000}$ = 0.01 1 sig. fig. = 0.010 2 sig. fig.
33. $5^2 \div 14^5$ = 0.00005 1 sig. fig. = 0.000046 2 sig. fig.
34. $\frac{179}{2\,500}$ = 0.07 1 sig. fig. = 0.072 2 sig. fig.

Numbers of Any Magnitude - Rounding to 1 & 2 Significant Figures (Homework Strip)

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Date Due: _____

Round these to 1 significant figure

1 98 _____ 2 14 _____

3 84 659 _____

4 467 113 _____

Write the answer and rewrite to 2 s.f.

5 $57^2 =$ _____ (Actual) $=$ _____ (2 sig. fig.)

6 $36^2 + 83^2 =$ _____ (Actual)

$=$ _____ 2 sig. fig.

Round these to 1 & 2 sig figs

7 0.0857 _____ (1 sig. fig.) _____ (2 sig. fig.)

8 1.52 _____ (1 sig. fig.) _____ (2 sig. fig.)

9 0.993 _____ (1 sig. fig.) _____ (2 sig. fig.)

10 0.0996 _____ (1 sig. fig.) _____ (2 sig. fig.)

Date Due: _____

Round these to 1 significant figure

1 98 _____ 2 14 _____

3 84 659 _____

4 467 113 _____

Write the answer and rewrite to 2 s.f.

5 $57^2 =$ _____ (Actual) $=$ _____ (2 sig. fig.)

6 $36^2 + 83^2 =$ _____ (Actual)

$=$ _____ 2 sig. fig.

Round these to 1 & 2 sig figs

7 0.0857 _____ (1 sig. fig.) _____ (2 sig. fig.)

8 1.52 _____ (1 sig. fig.) _____ (2 sig. fig.)

9 0.993 _____ (1 sig. fig.) _____ (2 sig. fig.)

10 0.0996 _____ (1 sig. fig.) _____ (2 sig. fig.)

Date Due: _____

Round these to 1 significant figure

1 98 _____ 2 14 _____

3 84 659 _____

4 467 113 _____

Write the answer and rewrite to 2 s.f.

5 $57^2 =$ _____ (Actual) $=$ _____ (2 sig. fig.)

6 $36^2 + 83^2 =$ _____ (Actual)

$=$ _____ 2 sig. fig.

Round these to 1 & 2 sig figs

7 0.0857 _____ (1 sig. fig.) _____ (2 sig. fig.)

8 1.52 _____ (1 sig. fig.) _____ (2 sig. fig.)

9 0.993 _____ (1 sig. fig.) _____ (2 sig. fig.)

10 0.0996 _____ (1 sig. fig.) _____ (2 sig. fig.)

Date Due: _____

Round these to 1 significant figure

1 98 _____ 2 14 _____

3 84 659 _____

4 467 113 _____

Write the answer and rewrite to 2 s.f.

5 $57^2 =$ _____ (Actual) $=$ _____ (2 sig. fig.)

6 $36^2 + 83^2 =$ _____ (Actual)

$=$ _____ 2 sig. fig.

Round these to 1 & 2 sig figs

7 0.0857 _____ (1 sig. fig.) _____ (2 sig. fig.)

8 1.52 _____ (1 sig. fig.) _____ (2 sig. fig.)

9 0.993 _____ (1 sig. fig.) _____ (2 sig. fig.)

10 0.0996 _____ (1 sig. fig.) _____ (2 sig. fig.)

Numbers of Any Magnitude - Rounding to 1 & 2 Sig Fig (H'work Strip)

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Date Due:

Round these to 1 significant figure

1 98 — 100

2 14 — 10

3 84 659 — 80 000

4 467 113 — 500 000

Write the answer and rewrite to 2 s.f.

5 $57^2 =$ 3 249 (Actual) $=$ 3 200 (2 sig. fig.)

6 $36^2 + 83^2 =$ 8 185 (Actual)

$=$ 8 200 (2 sig. fig.)

Round these to 1 & 2 sig figs

		1 sig. fig.	2 sig. fig.
7	0.0857	0.09	0.086
8	1.52	2	1.5
9	0.993	1	0.99
10	0.0996	0.1	0.10

Numbers of Any Magnitude - Units of Time

This sheet deals with the changing from one time unit to another. Time is measured in: Milliseconds (msec), seconds (s or sec), minutes (min), hours (h), days, weeks, years, decades, centuries, millennia and other units. In business and especially in the construction industry units such as years have to be broken into smaller units like weeks or even days, so that accurate scheduling can occur.

Column 1 in the first worksheet asks you to allocate the correct unit that should be used in a specific case. It may be that there are two answers, choose the most appropriate one.

Column 2 asks you to convert time periods to days, and later to hours. These are commonly used units and you should ensure that you remember key conversions which are:

Key Conversions:

60 seconds in 1 minute
60 minutes in 1 hour
24 hours in 1 day
7 days in 1 week
52 weeks in 1 year
12 months in 1 year
365 days in 1 year (366 for a leap year)

Column 3 features a mixture of time units and specifies what unit to convert them to.

The 2nd worksheet is more calculator focused. Starting with converting from hours to minutes and the reverse. The calculator's Degrees/minutes/seconds button is used to answer these questions, there are examples at the top of the column.

The last column features basic operations with a calculator, remember that when using the button on the calculator that if the question is minutes only, you have to enter a '0' for hours. You can't enter minutes only.

Milliseconds are 1 thousandth of a second. So conversion between milliseconds and seconds will involve multiplying and dividing by 1 000. Note that you can answer these questions without a calculator by moving the decimal place 3 places also.

Numbers of Any Magnitude - Units of Time/Conversions (Sheet 1)

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Use each of these words once to answer the questions. What is the most appropriate time unit to use to measure …….

Wordlist:	seconds	days
years	hours	decade
centuries	millennia	milliseconds
minutes	weeks	Cross them out

1 A person's age _____

2 The time that humans have lived in cities _____

3 A 10 year period _____

4 The time to cook a meal _____

5 The time taken for a pierced balloon to 'pop' _____

6 The time you are allowed to borrow a book from the library _____

7 The amount of time that you sleep daily _____

8 The time a tourist might spend visiting a foreign city _____

9 The time humans have used coal for heating. _____

10 The time taken to cross the street. _____

Convert these larger time periods to days. One year is 365 days, ignoring leap years.

11 2 years = ____ × 365

= ____ days

12 1 decade = ____ × 365

= ____ days

13 17 years = _____

= _____

14 3 centuries = _____

= _____

Convert these times to times measured in hours. 1 day is 24 hours in duration.

15 5 days = ____ × 24

= ____ hours

16 3 weeks = ____ × ____ × ____

= ____ hours

17 July = _____

= _____

18 1 year = _____

= _____

Match the right side with the left.

19

1 year ●	● 3 days
3 weeks ●	● 52 weeks
72 hours ●	● 576 hours
24 days ●	● 21 days

Convert these units of time to the different unit on the right hand side.

20 2 weeks = ____ days

21 1 year = ____ weeks

22 3 days = ____ hours

23 ½ hour = ____ min

24 104 weeks = ____ years

25 6 min = ____ seconds

26 2 h = ____ min

27 1 leap year = ____ days

28 4 weeks = ____ days

29 3 decades = ____ years

30 9 min = ____ seconds

31 24 months = ____ years

32 4 centuries = ____ decades

33 ½ year = ____ months

34 5 days = ____ hours

35 2 fortnights = ____ weeks

36 300 s = ____ min

37 156 weeks = ____ years

38 September = ____ days

39 2 millennia = ____ years

40 March + May = ____ days

Numbers of Any Magnitude - Units of Time/Conversions (Sheet 2)

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Rewrite the calculator displays in h and min.

1  ____ h ____ m

Hours Minutes

2  ____ h ____ m

3  ____ h ____ m

4  ____ h ____ m

Convert the following times from minutes to hours and minutes.

Example

Convert 211 minutes to hours/minutes.

Calc: 0 ° ’ ’’ 2 1 1 ° ’ ’’ =

Display: 

3 h 31 min

5 193 min ____ h ____ min

6 185 min ____ h ____ min

7 247 min ____ h ____ min

8 744 min _____

9 490 min _____

10 333 min _____

11 296 min _____

12 575 min _____

13 1 541 min _____

Now backwards. Change times in hours/minutes to minutes.

Example

Convert 4 h 19 min to minutes.

Calc: 4 ° ’ ’’ 1 9 ° ’ ’’ × 6 0 =

Display: 

259 min

14 3 h 38 min _____ min

15 1 h 43 min _____ min

16 2 h 27 min _____ min

17 6 h 6 min _____ min

18 7 h 25 min _____

19 1 h 19 min _____

20 4 h 53 min _____

21 5 h _____

22 2 h 48 min _____

Now use ×, +, ÷ and - with these time operations.

Example 47 min × 4

Calc: 0 ° ’ ’’ 4 7 ° ’ ’’ × 4 =

If no hours, enter ‘0’ in hours

Display: 

3 h 8 min

23 8 × 23 min ____ h ____ min

24 7 × 1 h 16 min

____ h ____ min

25 19 h 57 min ÷ 9

____ h ____ min

26 2 h 38 min + 1 h 43 min

____ h ____ min

27 5 h 19 min - 2 h 56 min

____ h ____ min

28 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min

____ h ____ min

There are 1 000 milliseconds in a second. Change these times to milliseconds by: × by 1 000.

29 2 sec _____ msec

30 3.2 sec _____ msec

31 0.08 sec _____

32 0.076 sec _____

33 4.4 sec _____

34 0.55 sec _____

35 0.00007 sec _____

Change these times from milliseconds to seconds by ÷ by 1 000.

36 700 msec _____ sec

37 95 msec _____ sec

38 7 100 msec _____

39 0.2 msec _____

40 40 msec _____

41 999 msec _____

42 635.7 msec _____

Numbers of Any Magnitude - Units of Time/Conversions (Sheet 1)

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Use each of these words once to answer the questions. What is the most appropriate time unit to use to measure …….

Wordlist: ~~seconds~~ ~~days~~ ~~years~~ ~~hours~~ ~~decade~~ ~~centuries~~ ~~millennia~~ ~~milliseconds~~ ~~minutes~~ ~~weeks~~

Cross them out

1 A person's age **years**

2 The time that humans have lived in cities **millennia**

3 A 10 year period **decade**

4 The time to cook a meal **minutes**

5 The time taken for a pierced balloon to 'pop' **milliseconds**

6 The time you are allowed to borrow a book from the library **weeks**

7 The amount of time that you sleep daily **hours**

8 The time a tourist might spend visiting a foreign city **days**

9 The time humans have used coal for heating. **centuries**

10 The time taken to cross the street. **seconds**

Convert these larger time periods to days. One year is 365 days, ignoring leap years.

11 2 years = 2 × 365
= 730 days

12 1 decade = 10 × 365
= 3 650 days

13 17 years = 17 × 365
= 6 205 days

14 3 centuries = 300 × 365
= 109 500 days

Convert these times to times measured in hours. 1 day is 24 hours in duration.

15 5 days = 5 × 24
= 120 hours

16 3 weeks = 3 × 7 × 24
= 504 hours

17 July = 31 × 24
= 744 hours

18 1 year = 365 × 24
= 8 760 hours

Match the right side with the left.

19

1 year	3 days
3 weeks	52 weeks
72 hours	576 hours
24 days	21 days

Convert these units of time to the different unit on the right hand side.

20 2 weeks = 14 days

21 1 year = 52 weeks

22 3 days = 72 hours

23 ½ hour = 30 min

24 104 weeks = 2 years

25 6 min = 360 seconds

26 2 h = 120 min

27 1 leap year = 366 days

28 4 weeks = 28 days

29 3 decades = 30 years

30 9 min = 540 seconds

31 24 months = 2 years

32 4 centuries = 40 decades

33 ½ year = 6 months

34 5 days = 120 hours

35 2 fortnights = 4 weeks

36 300 s = 5 min

37 156 weeks = 3 years

38 September = 30 days

39 2 millennia = 2 000 years

40 March + May = 62 days
31 31

Numbers of Any Magnitude - Units of Time/Conversions (Sheet 2)

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Rewrite the calculator displays in h and min.

1  3 h 43 m

Hours Minutes

2  5 h 23 m

3  6 h 54 m

4  8 h 19 m

Convert the following times from minutes to hours and minutes.

Example

Convert 211 minutes to hours/minutes.

Calc: 0 ° ’ ’’ 2 1 1 ° ’ ’’ =

Display: 

3 h 31 min

5 193 min — 3 h 13 min

6 185 min — 3 h 5 min

7 247 min — 4 h 7 min

8 744 min — 12 h 24 min

9 490 min — 8 h 10 min

10 333 min — 5 h 33 min

11 296 min — 4 h 56 min

12 575 min — 9 h 35 min

13 1 541 min — 25 h 41 min

Now backwards. Change times in hours/minutes to minutes.

Example

Convert 4 h 19 min to minutes.

Calc: 4 ° ’ ’’ 1 9 ° ’ ’’ × 6 0 =

Display: 

259 min

14 3 h 38 min — 218 min

15 1 h 43 min — 103 min

16 2 h 27 min — 147 min

17 6 h 6 min — 366 min

18 7 h 25 min — 445 min

19 1 h 19 min — 79 min

20 4 h 53 min — 293 min

21 5 h — 300 min

22 2 h 48 min — 168 min

Now use ×, +, ÷ and - with these time operations.

Example 47 min × 4

Calc: 0 ° ’ ’’ 4 7 ° ’ ’’ × 4 =

If no hours, enter ‘0’ in hours

Display: 

3 h 8 min

23 8 × 23 min — 3 h 4 min

24 7 × 1 h 16 min — 8 h 52 min

25 19 h 57 min ÷ 9 — 2 h 13 min

26 2 h 38 min + 1 h 43 min — 4 h 21 min

27 5 h 19 min - 2 h 56 min — 2 h 23 min

28 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min — 7 h 8 min

There are 1 000 milliseconds in a second. Change these times to milliseconds by: × by 1 000.

29 2 sec — 2 000 msec

30 3.2 sec — 3 200 msec

31 0.08 sec — 80 msec

32 0.076 sec — 76 msec

33 4.4 sec — 4 400 msec

34 0.55 sec — 550 msec

35 0.00007 sec — 0.07 msec

Change these times from milliseconds to seconds by ÷ by 1 000.

36 700 msec — 0.7 sec

37 95 msec — 0.095 sec

38 7 100 msec — 7.1 sec

39 0.2 msec — 0.0002 sec

40 40 msec — 0.04 sec

41 999 msec — 0.999 sec

42 635.7 msec — 0.6357 sec

Numbers of Any Magnitude - Units of Time/Conversions (Homework Strip)

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Date Due:

What word describes a time period of

1. 52 weeks
2. 10 years
3. 24 hours

How many days are there in

4. 3 weeks
5. 2 years
6. March
7. A Leap Year

How many hours are there in

8. 2 days
9. 1 week

Convert these time periods

10. 7 hours = ____ min
11. 9 min = ____ sec

Date Due:

What word describes a time period of

1. 52 weeks
2. 10 years
3. 24 hours

How many days are there in

4. 3 weeks
5. 2 years
6. March
7. A Leap Year

How many hours are there in

8. 2 days
9. 1 week

Convert these time periods

10. 7 hours = ____ min
11. 9 min = ____ sec

Date Due:

What word describes a time period of

1. 52 weeks
2. 10 years
3. 24 hours

How many days are there in

4. 3 weeks
5. 2 years
6. March
7. A Leap Year

How many hours are there in

8. 2 days
9. 1 week

Convert these time periods

10. 7 hours = ____ min
11. 9 min = ____ sec

Date Due:

What word describes a time period of

1. 52 weeks
2. 10 years
3. 24 hours

How many days are there in

4. 3 weeks
5. 2 years
6. March
7. A Leap Year

How many hours are there in

8. 2 days
9. 1 week

Convert these time periods

10. 7 hours = ____ min
11. 9 min = ____ sec

Numbers of Any Magnitude - Units of Time/Conversions (H'work Strip)

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Date Due:

What word describes a time period of

1	52 weeks	year
2	10 years	decade
3	24 hours	day

How many days are there in

4	3 weeks	21 days
5	2 years	730 days
6	March	31 days
7	A Leap Year	366 days

How many hours are there in

8	2 days	48 hours
9	1 week	168 hours

Convert these time periods

10	7 hours	420 min
11	9 min	540 sec

Numbers of Any Magnitude - Scientific Notation (Positive Index)

Scientific Notation is a method of writing very large or very small numbers, without writing all the digits. For example distance of the Earth from the Sun, or the number of molecules in a sample of fluid. The sheets in this sub-topic deal with large numbers, the next sheets deals with decimals.

To convert to scientific notation, you count the number of times you 'hop' the decimal place to the left to get behind the first digit, that number of hops becomes the index on the 10.

Example:

$7\,0\,0\,0\,0\,0\,0\,0 = 7 \times 10^{7}$

7 hops

Note above how much smaller the number is to write. It converts large numbers into more easy to handle sizes.

The front leading digit can be a decimal, for example, you can have:

$$310\,000 = 3.1 \times 10^5 \qquad 312\,000 = 3.12 \times 10^5$$

And like decimals, these leading numbers can be rounded. To the nearest whole number, or a specific decimal place.

Sheets 1 and 2 focus on converting numbers to scientific notation format.

Sheet 3 deals mainly with comparing and multiplying numbers in scientific notation format. When you compare numbers, you have to compare two parts.

The power on the base of 10. The larger the power, the larger the number.

For example: 3.12×10^5 is larger than 3.12×10^4, because the power of 5 is more than the power of 4.

The larger the leading numbers, the larger the number.

For example: 4.12×10^5 is larger than 3.12×10^5, because 4.12 is larger than 3.12.

So there are two things to look at. But the power is the first decider.

The last part of sheet 3 outlines how you can multiply numbers in scientific notation format. You can multiply the leading numbers and using your indice skills, add the powers.

So $3 \times 10^5 \times 2 \times 10^3 = 2 \times 3 \times 10^5 \times 10^3 = 6 \times 10^{5+3} = 6 \times 10^8$

Scientific Notation (Non-Calc) - Positive Index - No Rounding (Sheet 1)

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Convert these numbers into scientific notation format. By hopping the decimal point to be behind the first digit.

Example Convert 4 000.

$4\,000 = 4 \times 10^{3}$

3 places

1 $6\,000 = 6 \times 10^{\square}$

2 $5\,000\,000 = 5 \times 10^{\square}$

3 $70\,000 = 7 \times 10^{\square}$

4 $30\,000\,000 = 3 \times 10^{\square}$

Now you do the hopping, use a pencil

5 $80\,000 = 8 \times 10^{\square}$

6 $1\,000 = 1 \times 10^{\square}$

7 $9\,000\,000 = 9 \times 10^{\square}$

Now the first number and the index

8 $40\,000 = \square \times 10^{\square}$

9 $7\,000\,000 = \square \times 10^{\square}$

10 $30\,000 = \square \times 10^{\square}$

Now the lot, the base is always 10.

11 $300\,000 = \square \times \square^{\square}$

12 $50\,000\,000\,000\,000\,000 = \square \times \square^{\square}$

13 $20\,000\,000\,000\,000 = \square \times \square^{\square}$

14 $600\,000 = \square \times \square^{\square}$

15 $8\,000 = \square \times \square^{\square}$

16 $100\,000\,000 = \square \times \square^{\square}$

17 $20\,000 =$ _____

18 $900\,000\,000 =$ _____

19 $100\,000 =$ _____

20 $4\,000\,000 =$ _____

Any additional non-zero digits are written to the right of the decimal point. Convert these.

Examples

$320 = 3.2 \times 10^{2}$

$570\,000 = 5.7 \times 10^{5}$

21 $75\,000 = \square.\square \times \square^{\square}$

22 $8\,800\,000 = \square.\square \times \square^{\square}$

23 $950\,000\,000 =$ _____

24 $7\,700\,000\,000 =$ _____

25 $4\,500 =$ _____

26 $61\,000\,000 =$ _____

Now there are two decimal places

27 $62\,800\,000 = \square.\square\square \times \square^{\square}$

28 $593\,000\,000\,000\,000 = \square.\square\square \times \square^{\square}$

29 $40\,700\,000\,000 = \square.\square\square \times \square^{\square}$

30 $712\,000\,000 = \square.\square\square \times \square^{\square}$

31 $21\,400 =$ _____

32 $807 =$ _____

33 $9\,090\,000 =$ _____

34 $5\,770\,000\,000\,000\,000\,000 =$ _____

35 $81\,600\,000\,000\,000 =$ _____

These have more than 3 leading significant digits. Include all of them. Convert these.

Examples

$170\,600 = 1.706 \times 10^{5}$

$8\,167\,200\,000 = 8.1672 \times 10^{9}$

36 $88\,760 = \square.\square\square\square \times \square^{\square}$

37 $13\,676\,000\,000 = \square.\square\square\square\square \times \square^{\square}$

38 $50\,255\,800\,000\,000 =$ _____

39 $14\,950\,000 =$ _____

40 $3\,006\,520\,000\,000 =$ _____

41 $804\,370\,000 =$ _____

Scientific Notation (Non-Calc) - Positive Index - Rounding (Sheet 2)

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Round these numbers shown in scientific notation format, to 1 decimal place.

Examples

$4.35 \times 10^9 = 4.4 \times 10^9$

as 5 is 5 or more, round up

$2.9712 \times 10^{14} = 3.0 \times 10^{14}$

as 7 > 5 round up

$3.4218 \times 10^5 = 3.4 \times 10^5$

as 2 < 5 round down

1 4.16×10^5

= ☐.☐ × ☐☐

2 3.71×10^8

= ☐.☐ × ☐☐

3 9.093×10^6

= ☐.☐ × ☐☐

4 1.7504×10^{11} =

5 6.022×10^{23} =

6 5.074×10^{14} =

7 5.9528×10^9 =

8 7.97×10^3 =

Convert and then round to 1 d.p.

9 3 170 000

= ☐.☐☐ × ☐☐

= ☐.☐ × ☐☐

10 856 000

= ☐.☐☐ × ☐☐

= ☐.☐ × ☐☐

11 40 720 000 =

=

12 271 000 =

=

13 859 400 000 =

=

Now go straight to the rounded answer

14 5 183 000

= ☐.☐ × ☐☐

15 70 494

= ☐.☐ × ☐☐

16 8 153

= ☐.☐ × ☐☐

17 1 777 803 225 431 650

= ☐.☐ × ☐☐

18 38 910 583 004

= ☐.☐ × ☐☐

Round these numbers shown in scientific notation format, to 2 decimal places.

19 5.316×10^7

= ☐.☐☐ × ☐☐

20 7.706×10^4

= ☐.☐☐ × ☐☐

21 8.073×10^{12}

= ☐.☐☐ × ☐☐

22 1.60371×10^{19}

=

23 4.04938×10^{13}

=

Convert these numbers to scientific notation format, rounding to 2 decimal places.

24 2 337 000

= ☐.☐☐ × ☐☐

25 49 993 414

= ☐.☐☐ × ☐☐

26 5 660 177 400 812

= ☐.☐☐ × ☐☐

27 914 731 422

= ☐.☐☐ × ☐☐

28 349 961

=

29 79 965 223 000

=

Now to 3 decimal places

30 107 463 020

= ☐.☐☐☐ × ☐☐

31 50 962 851 000

= ☐.☐☐☐ × ☐☐

32 177 966

=

Convert to scientific notation and round to 1 decimal place. These last two are different.

33 9 960 000

=

=

34 99 543 100 080

=

=

Scientific Notation (Non-Calc) - Positive Index (Sheet 3)

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These have a decimal point. Move it to be behind the first digit to express in scientific notation, rounding to 1 d.p.

Examples

$46\,789.7 = 4.7 \times 10^4$

$789\,523.78 = 7.9 \times 10^5$

1 976.4

$= \square.\square \times \square^{\square}$

2 575 009.135

$= \square.\square \times \square^{\square}$

3 4 096 216.88

$= \square.\square \times \square^{\square}$

4 1 143 215.9216

$=$ _____

5 198.636

$=$ _____

6 509 159 608.2816

$=$ _____

7 76 682.0053

$=$ _____

Point the < or > to the smallest numbers, which are expressed in scientific notation.

8 5.3×10^4 ☐ 2.4×10^4

9 8.01×10^9 ☐ 8.10×10^9

10 5.32×10^5 ☐ 5.32×10^7

11 7×10^9 ☐ 7.6×10^8

Rewrite these in ascending order. That means smallest to largest.

12 5.3×10^4, 2.7×10^4, 3.8×10^4, 5.9×10^4.

$=$ _____ (Smallest) , _____ , _____ , _____ (Largest) .

13 8.2×10^7, 8.2×10^5, 8.2×10^6, 8.2×10^3.

$=$ _____ , _____ , _____ , _____ .

14 3.3×10^6, 3.11×10^6, 3.1×10^6, 3.33×10^6.

$=$ _____ , _____ , _____ , _____ .

15 6.7×10^8, 4.02×10^5, 5.8×10^7, 5.89×10^8.

$=$ _____ , _____ , _____ , _____ .

Rewrite these two in descending order, that's largest to smallest.

16 4×10^{11}, 5.6×10^{10}, 2.9×10^{11}, 1.4×10^{12}.

$=$ _____ (Largest) , _____ , _____ , _____ (Smallest) .

17 1.7×10^9, 7.01×10^8, 1.71×10^9, 7.1×10^8.

$=$ _____ , _____ , _____ , _____ .

You can multiply numbers in scientific notation. Multiply the leading numbers and **ADD** the indexes. Multiply these.

Example $2 \times 10^7 \times 3 \times 10^4 = 2 \times 3 \times 10^7 \times 10^4 = 6 \times 10^{11}$.

18 $2 \times 10^2 \times 4 \times 10^5 =$ $\square \times \square \times \square^{\square} \times \square^{\square} = \square \times \square^{\square}$

19 $3 \times 10^8 \times 3 \times 10^6 =$ $\square \times \square \times \square^{\square} \times \square^{\square} = \square \times \square^{\square}$

20 $1 \times 10^7 \times 7 \times 10^4 =$ $\square \times \square \times \square^{\square} \times \square^{\square} = \square \times \square^{\square}$

With these, when you multiply you will get a two digit leading number. Change this back to a decimal and add 1 (+1) to the power.

Example $2 \times 10^7 \times 7 \times 10^4$

$= 2 \times 7 \times 10^7 \times 10^4$

$= 14 \times 10^{11} = 1.4 \times 10^{12}$

21 $3 \times 10^6 \times 5 \times 10^3$

$= \square \times \square \times \square^{\square} \times \square^{\square}$

$= \square \times \square^{\square}$

$= \square.\square \times \square^{\square}$

22 $7 \times 10^8 \times 4 \times 10^9$

$= \square \times \square \times \square^{\square} \times \square^{\square}$

$= \square \times \square^{\square}$

$= \square.\square \times \square^{\square}$

Scientific Notation (Non-Calc) - Positive Index - No Rounding (Sheet 1)

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Convert these numbers into scientific notation format. By hopping the decimal point to be behind the first digit.



Example Convert 4 000.

$4\,000 = 4 \times 10^{3}$

3 places

1 $6\,00 = 6 \times 10^{2}$

2 $5\,000\,000 = 5 \times 10^{6}$

3 $70\,000 = 7 \times 10^{4}$

4 $3\,000\,000\,00 = 3 \times 10^{7}$

Now you do the hopping, use a pencil

5 $80\,000 = 8 \times 10^{4}$

6 $1\,000 = 1 \times 10^{3}$

7 $9\,000\,000 = 9 \times 10^{6}$

Now the first number and the index

8 $40\,000 = 4 \times 10^{4}$

9 $7\,000\,000 = 7 \times 10^{6}$

10 $30\,000 = 3 \times 10^{4}$

Now the lot, the base is always 10.

11 $300\,000 = 3 \times 10^{5}$

12 $50\,000\,000\,000\,000 = 5 \times 10^{13}$

13 $20\,000\,000\,000 = 2 \times 10^{10}$

14 $600\,000 = 6 \times 10^{5}$

15 $8\,000 = 8 \times 10^{3}$

16 $100\,000\,000 = 1 \times 10^{8}$

17 $20\,000 = 2 \times 10^{4}$

18 $900\,000\,000 = 9 \times 10^{8}$

19 $100\,000 = 1 \times 10^{5}$

20 $4\,000\,000 = 4 \times 10^{6}$



Any additional non-zero digits are written to the right of the decimal point. Convert these.

Examples

$320 = 3.2 \times 10^{2}$

$570\,000 = 5.7 \times 10^{5}$

21 $75\,000 = 7.5 \times 10^{4}$

22 $8\,800\,000 = 8.8 \times 10^{6}$

23 $950\,000\,000 = 9.5 \times 10^{8}$

24 $7\,700\,000\,000 = 7.7 \times 10^{9}$

25 $4\,500 = 4.5 \times 10^{3}$

26 $61\,000\,000 = 6.1 \times 10^{7}$

Now there are two decimal places

27 $62\,800\,000 = 6.28 \times 10^{7}$

28 $593\,000\,000\,000\,000 = 5.93 \times 10^{14}$

29 $40\,700\,000\,000 = 4.07 \times 10^{10}$

30 $712\,000\,000 = 7.12 \times 10^{8}$

31 $21\,400 = 2.14 \times 10^{4}$

32 $807 = 8.07 \times 10^{2}$

33 $9\,090\,000 = 9.09 \times 10^{6}$

34 $5\,770\,000\,000\,000\,000\,000 = 5.77 \times 10^{18}$

35 $81\,600\,000\,000\,000 = 8.16 \times 10^{13}$

These have more than 3 leading significant digits. Include all of them. Convert these.



Examples

$170\,600 = 1.706 \times 10^{5}$

$8\,167\,200\,000 = 8.1672 \times 10^{9}$

36 $88\,760 = 8.8760 \times 10^{4}$

37 $13\,676\,000\,000 = 1.36760 \times 10^{10}$

38 $50\,255\,800\,000\,000 = 5.02558 \times 10^{13}$

39 $14\,950\,000 = 1.495 \times 10^{7}$

40 $3\,006\,520\,000\,000 = 3.00652 \times 10^{12}$

41 $804\,370\,000 = 8.0437 \times 10^{8}$

Scientific Notation (Non-Calc) - Positive Index - Rounding (Sheet 2)

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Round these numbers shown in scientific notation format, to 1 decimal place.

Examples

$4.35 \times 10^9 = 4.4 \times 10^9$ as 5 is 5 or more, round up

$2.9712 \times 10^{14} = 3.0 \times 10^{14}$ as 7 > 5 round up

$3.4218 \times 10^5 = 3.4 \times 10^5$ as 2 < 5 round down

1 4.16×10^5

$= 4.2 \times 10^5$ (5)

2 3.71×10^8

$= 3.7 \times 10^8$ (8)

3 9.093×10^6

$= 9.1 \times 10^6$ (6)

4 $1.7504 \times 10^{11} = 1.8 \times 10^{11}$

5 $6.022 \times 10^{23} = 6.0 \times 10^{23}$

6 $5.074 \times 10^{14} = 5.1 \times 10^{14}$

7 $5.9528 \times 10^9 = 6.0 \times 10^9$

8 $7.97 \times 10^3 = 8.0 \times 10^3$

Convert and then round to 1 d.p.

9 3 170 000

$= 3.17 \times 10^6$ (6)

$= 3.2 \times 10^6$ (6)

10 856 000

$= 8.56 \times 10^5$ (5)

$= 8.6 \times 10^5$ (5)

11 40 720 000 $= 4.072 \times 10^7$

$= 4.1 \times 10^7$

12 271 000 $= 2.71 \times 10^5$

$= 2.7 \times 10^5$

13 859 400 000 $= 8.594 \times 10^8$

$= 8.6 \times 10^8$

Now go straight to the rounded answer

14 5 183 000

$= 5.2 \times 10^6$ (6)

15 70 494

$= 7.0 \times 10^4$ (4)

16 8 153

$= 8.2 \times 10^3$ (3)

17 1 777 803 225 431 650

$= 1.8 \times 10^{15}$ (15)

18 38 910 583 004

$= 3.9 \times 10^{10}$ (10)

Round these numbers shown in scientific notation format, to 2 decimal places.

19 5.316×10^7

$= 5.32 \times 10^7$ (7)

20 7.706×10^4

$= 7.71 \times 10^4$ (4)

21 8.073×10^{12}

$= 8.07 \times 10^{12}$ (12)

22 1.60371×10^{19}

$= 1.60 \times 10^{19}$

23 4.04938×10^{13}

$= 4.05 \times 10^{13}$

Convert these numbers to scientific notation format, rounding to 2 decimal places.

24 2 337 000

$= 2.34 \times 10^6$ (6)

25 49 993 414

$= 5.00 \times 10^7$ (7)

26 5 660 177 400 812

$= 5.66 \times 10^{12}$ (12)

27 914 731 422

$= 9.15 \times 10^8$ (8)

28 349 961

$= 3.50 \times 10^5$

29 79 965 223 000

$= 8.00 \times 10^{10}$

Now to 3 decimal places

30 107 463 020

$= 1.075 \times 10^8$ (8)

31 50 962 851 000

$= 5.096 \times 10^{10}$ (10)

32 177 966

$= 1.780 \times 10^5$

Convert to scientific notation and round to 1 decimal place. These last two are different.

33 9 960 000

$= 10.0 \times 10^6$

$= 1.0 \times 10^7$

34 99 543 100 080

$= 10.0 \times 10^{10}$

$= 1.0 \times 10^{11}$

Scientific Notation (Non-Calc) - Positive Index (Sheet 3)

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These have a decimal point. Move it to be behind the first digit to express in scientific notation, rounding to 1 d.p.

Examples

$46\,789.7 = 4.7 \times 10^4$

$789\,523.78 = 7.9 \times 10^5$

1 976.4

$= 9.8 \times 10^{2}$

2 575 009.135

$= 5.8 \times 10^{5}$

3 4 096 216.88

$= 4.1 \times 10^{6}$

4 1 143 215.9216

$= 1.1 \times 10^6$

5 198.636

$= 2.0 \times 10^2$

6 509 159 608.2816

$= 5.1 \times 10^8$

7 76 682.0053

$= 7.7 \times 10^4$

Point the < or > to the smallest numbers, which are expressed in scientific notation.

8 5.3×10^4 [>] 2.4×10^4

9 8.01×10^9 [<] 8.10×10^9

10 5.32×10^5 [<] 5.32×10^7

11 7×10^9 [>] 7.6×10^8

Rewrite these in ascending order. That means smallest to largest.

12 5.3×10^4, 2.7×10^4, 3.8×10^4, 5.9×10^4.

$= 2.7 \times 10^4$ (Smallest), 3.8×10^4, 5.3×10^4, 5.9×10^4 (Largest).

13 8.2×10^7, 8.2×10^5, 8.2×10^6, 8.2×10^3.

$= 8.2 \times 10^3$, 8.2×10^5, 8.2×10^6, 8.2×10^7.

14 3.3×10^6, 3.11×10^6, 3.1×10^6, 3.33×10^6.

$= 3.1 \times 10^6$, 3.11×10^6, 3.3×10^6, 3.33×10^6.

15 6.7×10^8, 4.02×10^5, 5.8×10^7, 5.89×10^8.

$= 4.02 \times 10^5$, 5.8×10^7, 5.89×10^8, 6.7×10^8.

Rewrite these two in descending order, that's largest to smallest.

16 4×10^{11}, 5.6×10^{10}, 2.9×10^{11}, 1.4×10^{12}.

$= 1.4 \times 10^{12}$ (Largest), 4×10^{11}, 2.9×10^{11}, 5.6×10^{10} (Smallest).

17 1.7×10^9, 7.01×10^8, 1.71×10^9, 7.1×10^8.

$= 1.71 \times 10^9$, 1.7×10^9, 7.1×10^8, 7.01×10^8.

You can multiply numbers in scientific notation. Multiply the leading numbers and **ADD** the indexes. Multiply these.

Example $2 \times 10^7 \times 3 \times 10^4 = 2 \times 3 \times 10^7 \times 10^4 = 6 \times 10^{11}$.

18 $2 \times 10^2 \times 4 \times 10^5 = 2 \times 4 \times 10^{2} \times 10^{5} = 8 \times 10^{7}$

19 $3 \times 10^8 \times 3 \times 10^6 = 3 \times 3 \times 10^{8} \times 10^{6} = 9 \times 10^{14}$

20 $1 \times 10^7 \times 7 \times 10^4 = 1 \times 7 \times 10^{7} \times 10^{4} = 7 \times 10^{11}$

With these, when you multiply you will get a two digit leading number. Change this back to a decimal and add 1 (+1) to the power.

Example $2 \times 10^7 \times 7 \times 10^4$

$= 2 \times 7 \times 10^7 \times 10^4$

$= 14 \times 10^{11} = 1.4 \times 10^{12}$

21 $3 \times 10^6 \times 5 \times 10^3$

$= 3 \times 5 \times 10^{6} \times 10^{3}$

$= 15 \times 10^{9}$

$= 1.5 \times 10^{10}$

22 $7 \times 10^8 \times 4 \times 10^9$

$= 7 \times 4 \times 10^{8} \times 10^{9}$

$= 28 \times 10^{17}$

$= 2.8 \times 10^{18}$

Scientific Notation (Non-Calc) - Positive Index (Homework Strip)

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Date Due: _____

Count the hops and write the power

1. $4\,000\,000 = 4 \times 10^{\square}$
2. $7\,000 = 7 \times 10^{\square}$
3. $60\,000 = 6 \times 10^{\square}$

Rewrite in Scientific Notation format

4. $300\,000 = \square \times \square^{\square}$
5. $90\,000\,000 = \square \times \square^{\square}$
6. $10\,000 = \square \times \square^{\square}$

These have decimal places

7. $170\,000 = \square.\square \times \square^{\square}$
8. $960 = \square.\square \times \square^{\square}$
9. $57\,100\,000\,000 = \square.\square\square \times \square^{\square}$
10. $113\,800\,000 = \square.\square\square\square \times \square^{\square}$

Date Due: _____

Count the hops and write the power

1. $4\,000\,000 = 4 \times 10^{\square}$
2. $7\,000 = 7 \times 10^{\square}$
3. $60\,000 = 6 \times 10^{\square}$

Rewrite in Scientific Notation format

4. $300\,000 = \square \times \square^{\square}$
5. $90\,000\,000 = \square \times \square^{\square}$
6. $10\,000 = \square \times \square^{\square}$

These have decimal places

7. $170\,000 = \square.\square \times \square^{\square}$
8. $960 = \square.\square \times \square^{\square}$
9. $57\,100\,000\,000 = \square.\square\square \times \square^{\square}$
10. $113\,800\,000 = \square.\square\square\square \times \square^{\square}$

Date Due: _____

Count the hops and write the power

1. $4\,000\,000 = 4 \times 10^{\square}$
2. $7\,000 = 7 \times 10^{\square}$
3. $60\,000 = 6 \times 10^{\square}$

Rewrite in Scientific Notation format

4. $300\,000 = \square \times \square^{\square}$
5. $90\,000\,000 = \square \times \square^{\square}$
6. $10\,000 = \square \times \square^{\square}$

These have decimal places

7. $170\,000 = \square.\square \times \square^{\square}$
8. $960 = \square.\square \times \square^{\square}$
9. $57\,100\,000\,000 = \square.\square\square \times \square^{\square}$
10. $113\,800\,000 = \square.\square\square\square \times \square^{\square}$

Date Due: _____

Count the hops and write the power

1. $4\,000\,000 = 4 \times 10^{\square}$
2. $7\,000 = 7 \times 10^{\square}$
3. $60\,000 = 6 \times 10^{\square}$

Rewrite in Scientific Notation format

4. $300\,000 = \square \times \square^{\square}$
5. $90\,000\,000 = \square \times \square^{\square}$
6. $10\,000 = \square \times \square^{\square}$

These have decimal places

7. $170\,000 = \square.\square \times \square^{\square}$
8. $960 = \square.\square \times \square^{\square}$
9. $57\,100\,000\,000 = \square.\square\square \times \square^{\square}$
10. $113\,800\,000 = \square.\square\square\square \times \square^{\square}$

Scientific Notation (Non-Calc) - Positive Index (Homework Strip)

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Date Due:

Count the hops and write the power

1. $4\,000\,000 = 4 \times 10^{6}$
2. $7\,000 = 7 \times 10^{3}$
3. $60\,000 = 6 \times 10^{4}$

Rewrite in Scientific Notation format

4. $300\,000 = 3 \times 10^{5}$
5. $90\,000\,000 = 9 \times 10^{7}$
6. $10\,000 = 1 \times 10^{4}$

These have decimal places

7. $170\,000 = 1.7 \times 10^{5}$
8. $960 = 9.6 \times 10^{2}$
9. $57\,100\,000\,000 = 5.71 \times 10^{10}$
10. $113\,800\,000 = 1.138 \times 10^{8}$

Numbers of Any Magnitude - Scientific Notation (Negative Index)

Scientific Notation is a method of writing very large or very small numbers, without writing all the digits. For example distance of the Earth from the Sun, or the thickness of a human hair. The sheets in this sub-topic deal with decimals, which have a negative index.

To convert to scientific notation, you count the number of times you 'hop' the decimal place to the right to get behind the first non-zero digit, that number of hops becomes the index on the 10, but because you are hopping to the right, it is negative.

Example: Convert 0.0007 to scientific notation format.

$0.0007 = 7 \times 10^{-4}$

4 hops

Note above how much smaller the number is to write. It converts very small decimals into more easy to handle sizes.

The front leading digit can be a decimal, for example, you can have:

$$0.0063 = 6.3 \times 10^{-3} \qquad 0.0003025 = 3.025 \times 10^{-4}$$

And like decimals, these leading numbers can be rounded. To the nearest whole number, or a specific decimal place.

Sheets 1 and 2 focus on converting numbers to scientific notation format. The last column on the second sheet deals with comparing decimals in scientific notation format. When you compare them, you have to compare two parts.

The power on the base of 10. The larger the power, the larger the number. However remember that with negative numbers the smaller the number part the larger the number.

So, -3 is larger than -7. So 10^{-4} is larger than 10^{-8}. This can be tricky to remember.

For example: 3.12×10^{-5} is larger than 3.12×10^{-15}.

The larger the leading numbers, the larger the number.

For example: 4.12×10^{-5} is larger than 3.12×10^{-5}, because 4.12 is larger than 3.12.

So there are two things to look at. But the power is the first decider.

Scientific Notation (Non-Calc) - Negative Index - No Rounding (Sheet 1)

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Convert these decimals into scientific notation format. By hopping the decimal point to be behind the first non-zero digit.

Example Convert 0.0007

$0.0007 = 7 \times 10^{-4}$ (4 places)

1 $0.05 = 5 \times 10^{-}$

2 $0.00002 = 2 \times 10^{\square}$

3 $0.009 = 9 \times 10^{\square}$

Now you do the hopping, use a pencil

4 $0.0006 = 6 \times 10^{\square}$

5 $0.00000003 = 3 \times 10^{\square}$

6 $0.00008 = 8 \times 10^{\square}$

Now the first number and the index

7 $0.04 = \square \times 10^{\square}$

8 $0.00005 = \square \times 10^{\square}$

9 $0.000000001 = \square \times 10^{\square}$

Now fill all the boxes

10 $0.00000003 = \square \times \square$

11 $0.000000008 = \square \times \square$

12 $0.00007 = \square \times \square$

13 $0.0004 = \square \times \square$

14 $0.000000000006 = \square \times \square$

15 0.0003 = _____

16 0.0000008 = _____

17 0.009 = _____

18 0.000000004 = _____

Convert these to scientific notation format, the leading numbers to 1 decimal place.

Examples

$0.0083 = 8.3 \times 10^{-3}$

$0.000000096 = 9.6 \times 10^{-8}$

19 $0.0043 = \square.\square \times \square$

20 $0.18 = \square.\square \times \square$

21 $0.00000000092 = \square.\square \times \square$

22 $0.0000058 = \square.\square \times \square$

23 $0.00000000000016 = \square.\square \times \square$

24 0.00033 = _____

25 0.00000075 = _____

26 0.000048 = _____

27 0.00000000096 = _____

These have 3 or more decimal places. Include all the significant digits. Convert these.

28 0.0168 $= \square.\square\square \times \square$

29 0.00000517 $= \square.\square\square \times \square$

30 0.0008818 $= \square.\square\square\square \times \square$

31 0.0000007046 $= \square.\square\square\square \times \square$

32 0.00000000011931 $= \square.\square\square\square\square \times \square$

33 0.00000404 $= \square.\square\square \times \square$

34 0.000217 = _____

35 0.00000004176 = _____

36 0.005705 = _____

37 0.0000119 = _____

38 0.000000627 = _____

39 0.000003012 = _____

40 0.006776 = _____

Scientific Notation (Non-Calc) - Negative Index - Rounding (Sheet 2)

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Round these decimals shown in scientific notation format, to 1 decimal place.

Examples

$2.9712 \times 10^{-14} = 3.0 \times 10^{-14}$

as 7 > 5 round up

$5.4218 \times 10^{-5} = 5.4 \times 10^{-5}$

as 2 < 5 round down

1 2.54×10^{-5}

= ☐.☐ × ☐☐

2 4.06×10^{-3}

= ☐.☐ × ☐☐

3 7.14×10^{-9}

= ☐.☐ × ☐☐

4 3.95×10^{-7}

= ☐.☐ × ☐☐

5 7.77×10^{-4} =

6 2.89×10^{-8} =

7 6.24×10^{-6} =

8 7.11×10^{-9} =

Convert and then round to 1 d.p.

9 0.00417

= ☐.☐☐ × ☐☐

= ☐.☐ × ☐☐

10 0.00008037

= ☐.☐☐☐ × ☐☐

= ☐.☐ × ☐☐

11 0.007379 =

=

12 0.0008461 =

=

13 0.01446 =

=

Now go straight to the rounded answer

14 0.00615099

= ☐.☐ × ☐☐

15 0.0000000220599

= ☐.☐ × ☐☐

16 0.000555004

= ☐.☐ × ☐☐

17 0.0037095

= ☐.☐ × ☐☐

18 0.0909788

= ☐.☐ × ☐☐

19 0.0000053759

=

20 0.00000000074908

=

Round these decimals shown in scientific notation format, to 2 decimal places.

21 3.954×10^{-3}

= ☐.☐☐ × ☐☐

22 1.267×10^{-6}

= ☐.☐☐ × ☐☐

23 4.097×10^{-5}

= ☐.☐☐ × ☐☐

Round to 2 d.p.

24 8.997×10^{-8}

=

25 5.03803×10^{-7}

=

26 8.20399×10^{-3}

=

Convert these to scientific notation format, rounding to 2 decimal places.

27 0.0000703712

= ☐.☐☐ × ☐☐

28 0.000000000903177

= ☐.☐☐ × ☐☐

29 0.6666666

= ☐.☐☐ × ☐☐

30 0.0000090952006

= ☐.☐☐ × ☐☐

Use < or > with these. Point the < to the smallest decimal. 10^{-3} is larger than 10^{-4}.

31 4.3×10^{-7} ☐ 9.4×10^{-7}

32 1.17×10^{-3} ☐ 1.71×10^{-3}

33 3.33×10^{-3} ☐ 3.33×10^{-6}

Rank these, 1 for the largest, 3 for the smallest.

34 4×10^{-2} ☐ 7×10^{-3} ☐ 5×10^{-2} ☐

35 1×10^{-6} ☐ 9×10^{-4} ☐ 6×10^{-7} ☐

Scientific Notation (Non-Calc) - Negative Index - No Rounding (Sheet 1)

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Convert these decimals into scientific notation format. By hopping the decimal point to be behind the first non-zero digit.

Example Convert 0.0007

$0.0007 = 7 \times 10^{-4}$ (4 places)

1 $0.05 = 5 \times 10^{-2}$

2 $0.00002 = 2 \times 10^{-5}$

3 $0.009 = 9 \times 10^{-3}$

Now you do the hopping, use a pencil

4 $0.0006 = 6 \times 10^{-4}$

5 $0.00000003 = 3 \times 10^{-7}$

6 $0.00008 = 8 \times 10^{-5}$

Now the first number and the index

7 $0.04 = 4 \times 10^{-2}$

8 $0.00005 = 5 \times 10^{-5}$

9 $0.000000001 = 1 \times 10^{-9}$

Now fill all the boxes

10 $0.0000003 = 3 \times 10^{-7}$

11 $0.000000008 = 8 \times 10^{-9}$

12 $0.00007 = 7 \times 10^{-5}$

13 $0.0004 = 4 \times 10^{-4}$

14 $0.0000000000006 = 6 \times 10^{-11}$

15 $0.0003 = 3 \times 10^{-4}$

16 $0.0000008 = 8 \times 10^{-7}$

17 $0.009 = 9 \times 10^{-3}$

18 $0.000000004 = 4 \times 10^{-9}$



Convert these to scientific notation format, the leading numbers to 1 decimal place.

Examples

$0.0083 = 8.3 \times 10^{-3}$

$0.000000096 = 9.6 \times 10^{-8}$

19 $0.0043 = 4.3 \times 10^{-3}$

20 $0.18 = 1.8 \times 10^{-1}$

21 $0.00000000092 = 9.2 \times 10^{-10}$

22 $0.0000058 = 5.8 \times 10^{-6}$

23 $0.00000000000016 = 1.6 \times 10^{-13}$

24 $0.00033 = 3.3 \times 10^{-4}$

25 $0.00000075 = 7.5 \times 10^{-7}$

26 $0.000048 = 4.8 \times 10^{-5}$

27 $0.00000000096 = 9.6 \times 10^{-10}$



These have 3 or more decimal places. Include all the significant digits. Convert these.

28 $0.0168 = 1.68 \times 10^{-2}$

29 $0.00000517 = 5.17 \times 10^{-6}$

30 $0.0008818 = 8.818 \times 10^{-4}$

31 $0.0000007046 = 7.046 \times 10^{-7}$

32 $0.00000000011931 = 1.1931 \times 10^{-10}$

33 $0.00000404 = 4.04 \times 10^{-6}$

34 $0.000217 = 2.17 \times 10^{-4}$

35 $0.00000004176 = 4.176 \times 10^{-8}$

36 $0.005705 = 5.705 \times 10^{-4}$

37 $0.0000119 = 1.19 \times 10^{-5}$

38 $0.000000627 = 6.27 \times 10^{-7}$

39 $0.000003012 = 3.012 \times 10^{-6}$

40 $0.006776 = 6.776 \times 10^{-3}$

Scientific Notation (Non-Calc) - Negative Index - Rounding (Sheet 2)

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Round these decimals shown in scientific notation format, to 1 decimal place.

Examples

$2.9712 \times 10^{-14} = 3.0 \times 10^{-14}$ as $7 > 5$ round up

$5.4218 \times 10^{-5} = 5.4 \times 10^{-5}$ as $2 < 5$ round down

1 2.54×10^{-5} = 2.5×10^{-5}

2 4.06×10^{-3} = 4.1×10^{-3}

3 7.14×10^{-9} = 7.1×10^{-9}

4 3.95×10^{-7} = 4.0×10^{-7}

5 7.77×10^{-4} = 7.8×10^{-4}

6 2.89×10^{-8} = 2.9×10^{-8}

7 6.24×10^{-6} = 6.2×10^{-6}

8 7.11×10^{-9} = 7.1×10^{-9}

Convert and then round to 1 d.p.

9 0.00417 = 4.17×10^{-3} = 4.2×10^{-3}

10 0.00008037 = 8.037×10^{-5} = 8.0×10^{-5}

11 0.007379 = 7.379×10^{-3} = 7.4×10^{-3}

12 0.0008461 = 8.461×10^{-4} = 8.5×10^{-4}

13 0.01446 = 1.446×10^{-2} = 1.4×10^{-2}

Now go straight to the rounded answer

14 0.00615099 = 6.2×10^{-3}

15 0.0000000220599 = 2.2×10^{-8}

16 0.000555004 = 5.6×10^{-4}

17 0.0037095 = 3.7×10^{-3}

18 0.0909788 = 9.1×10^{-2}

19 0.0000053759 = 5.4×10^{-6}

20 0.00000000074908 = 7.5×10^{-10}

Round these decimals shown in scientific notation format, to 2 decimal places.

21 3.954×10^{-3} = 3.95×10^{-3}

22 1.267×10^{-6} = 1.27×10^{-6}

23 4.097×10^{-5} = 4.10×10^{-5}

Round to 2 d.p.

24 8.997×10^{-8} = 9.00×10^{-8}

25 5.03803×10^{-7} = 5.04×10^{-7}

26 8.20399×10^{-3} = 8.20×10^{-3}

Convert these to scientific notation format, rounding to 2 decimal places.

27 0.0000703712 = 7.04×10^{-5}

28 0.000000000903177 = 9.03×10^{-10}

29 0.6666666 = 6.67×10^{-1}

30 0.0000090952006 = 9.10×10^{-6}

Use < or > with these. Point the < to the smallest decimal. 10^{-3} is larger than 10^{-4}.

31 4.3×10^{-7} < 9.4×10^{-7}

32 1.17×10^{-3} < 1.71×10^{-3}

33 3.33×10^{-3} > 3.33×10^{-6}

Rank these, 1 for the largest, 3 for the smallest.

34 4×10^{-2} (2) 7×10^{-3} (3) 5×10^{-2} (1)

35 1×10^{-6} (2) 9×10^{-4} (1) 6×10^{-7} (3)

Scientific Notation (Non-Calc) - Negative Index (Homework Strip)

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Date Due: ____

Count the hops and write the power

1. $0.00002 = 2 \times 10^{\square}$
2. $0.00000007 = 7 \times 10^{\square}$
3. $0.004 = 4 \times 10^{\square}$

Rewrite in Scientific Notation format

4. $0.0009 = \square \times \square^{\square}$
5. $0.0000001 = \square \times \square^{\square}$
6. $0.00005 = \square \times \square^{\square}$

These have decimal place answers

7. $0.0073 = \square.\square \times \square^{\square}$
8. $0.0001344 = \square.\square\square\square \times \square^{\square}$
9. $0.0000002962 = \square.\square\square\square \times \square^{\square}$
10. $0.00000000055518 = \square.\square\square\square\square \times \square^{\square}$

Date Due: ____

Count the hops and write the power

1. $0.00002 = 2 \times 10^{\square}$
2. $0.00000007 = 7 \times 10^{\square}$
3. $0.004 = 4 \times 10^{\square}$

Rewrite in Scientific Notation format

4. $0.0009 = \square \times \square^{\square}$
5. $0.0000001 = \square \times \square^{\square}$
6. $0.00005 = \square \times \square^{\square}$

These have decimal place answers

7. $0.0073 = \square.\square \times \square^{\square}$
8. $0.0001344 = \square.\square\square\square \times \square^{\square}$
9. $0.0000002962 = \square.\square\square\square \times \square^{\square}$
10. $0.00000000055518 = \square.\square\square\square\square \times \square^{\square}$

Date Due: ____

Count the hops and write the power

1. $0.00002 = 2 \times 10^{\square}$
2. $0.00000007 = 7 \times 10^{\square}$
3. $0.004 = 4 \times 10^{\square}$

Rewrite in Scientific Notation format

4. $0.0009 = \square \times \square^{\square}$
5. $0.0000001 = \square \times \square^{\square}$
6. $0.00005 = \square \times \square^{\square}$

These have decimal place answers

7. $0.0073 = \square.\square \times \square^{\square}$
8. $0.0001344 = \square.\square\square\square \times \square^{\square}$
9. $0.0000002962 = \square.\square\square\square \times \square^{\square}$
10. $0.00000000055518 = \square.\square\square\square\square \times \square^{\square}$

Date Due: ____

Count the hops and write the power

1. $0.00002 = 2 \times 10^{\square}$
2. $0.00000007 = 7 \times 10^{\square}$
3. $0.004 = 4 \times 10^{\square}$

Rewrite in Scientific Notation format

4. $0.0009 = \square \times \square^{\square}$
5. $0.0000001 = \square \times \square^{\square}$
6. $0.00005 = \square \times \square^{\square}$

These have decimal place answers

7. $0.0073 = \square.\square \times \square^{\square}$
8. $0.0001344 = \square.\square\square\square \times \square^{\square}$
9. $0.0000002962 = \square.\square\square\square \times \square^{\square}$
10. $0.00000000055518 = \square.\square\square\square\square \times \square^{\square}$

Scientific Notation (Non-Calc) - Negative Index (Homework Strip)

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Date Due:

Count the hops and write the power

1 $0.00002 = 2 \times 10^{-5}$

2 $0.0000007 = 7 \times 10^{-7}$

3 $0.004 = 4 \times 10^{-3}$

Rewrite in Scientific Notation format

4 $0.0009 = 9 \times 10^{-4}$

5 $0.0000001 = 1 \times 10^{-7}$

6 $0.00005 = 5 \times 10^{-5}$

These have decimal place answers

7 $0.0073 = 7.3 \times 10^{-3}$

8 $0.0001344 = 1.344 \times 10^{-4}$

9 $0.0000002962 = 2.962 \times 10^{-7}$

10 $0.00000000055518 = 5.5518 \times 10^{-10}$

Converting to Scientific Notation - Calculator

To convert to scientific notation you use the calculator's Scientific Notation function. With the Casio fx-82 AU Plus, use one of these choices:

Calculator Key Use:

To Put into 'Sci' Mode (0 d.p.):	Calc:	Shift Mode 7 1	Example: 5×10^4 or 3×10^{-2}
To Put into 'Sci' Mode (1 d.p.):	Calc:	Shift Mode 7 2	Example: 5.5×10^4
To Put into 'Sci' Mode (2 d.p.):	Calc:	Shift Mode 7 3	Example: 3.35×10^{-4}
To Put into 'Sci' Mode (3 d.p.):	Calc:	Shift Mode 7 4	Example: 3.355×10^{-4}

Remember to take the calculator out of Scientific Notation mode by doing the following:

To return Calculator to Normal Mode:

To leave 'Sci'Mode:	Calc:	Shift Mode 8 2

Converting to Scientific Notation - Calculator

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Use your calculator's 'Sci' Mode to convert these numbers into scientific notation format.

Put into 'Sci' Mode (0 d.p.):
Calc: Shift Mode 7 1

1 $8\,000 = \square \times 10^{\square}$

Calc: 8 0 0 0 =

2 $60\,000\,000 = \square \times \square^{\square}$

3 $400\,000 = \square \times \square^{\square}$

4 $20\,000 =$

5 $9\,000\,000 =$

Put into 'Sci' Mode (1 d.p.):
Calc: Shift Mode 7 2

6 $0.014 =$

7 $0.00035 =$

8 $0.0000073 =$

9 $0.026 =$

10 $0.0058 =$

Put into 'Sci' Mode (2 d.p.):
Calc: Shift Mode 7 3

11 $217\,000 =$

12 $0.0000615 =$

13 $1\,540 =$

14 $0.00000808 =$

15 $81\,900 =$

To Cancel: Shift Mode 8 2

Rewrite these in Scientific Notation, rounded to 3 decimal places.

Put into 'Sci' Mode (3 d.p.):
Calc: Shift Mode 7 4

16 $786\,537 =$

17 0.000000055188
$=$

18 $406\,333\,157$
$=$

19 $0.012345 =$

20 $772\,976\,807\,000$
$=$

To Cancel: Shift Mode 8 2

Give the answer to these in scientific notation form. Rounded to 2 decimal places.

Put into 'Sci' Mode (2 d.p.):
Calc: Shift Mode 7 3

21 $0.00005^3 =$

22 $95^6 =$

23 $\sqrt{0.0009} =$

24 $\dfrac{82^4}{0.3^3} =$

25 $\dfrac{(72 \div 0.08)}{(0.06 \times 0.42)}$
$=$

To Cancel: Shift Mode 8 2

Compare these by answering them in scientific notation to 1 decimal place, then point the < or > to the smallest.

Calc: Shift Mode 7 2

26 $40^3 \square 800^2$

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

27 $\dfrac{5}{25^5} \square \dfrac{(3 \div 19)}{38^3}$

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

28 $0.4^{12} \square 0.3^{11}$

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

29 $112^4 \square 782^3$

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

Rewrite these five in descending order, using scientific notation (to 1 d.p.) to guide you. Descending order means highest to lowest.

30 4^9 10^5 9^7 6^8 3^{12}

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____ Sci. Not. 1 dp ____ Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

Descending Order: ____, ____, ____, ____, ____.

To Cancel: Shift Mode 8 2

Converting to Scientific Notation - Calculator

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Use your calculator's 'Sci' Mode to convert these numbers into scientific notation format.

Put into 'Sci' Mode (0 d.p.):
Calc: Shift Mode 7 1

1 8 000 = 8 × 10^3

Calc: 8 0 0 0 =

2 60 000 000 = 6 × 10^7

3 400 000 = 4 × 10^5

4 20 000 = 2×10^4

5 9 000 000 = 9×10^6

Put into 'Sci' Mode (1 d.p.):
Calc: Shift Mode 7 2

6 0.014 = 1.4×10^{-2}

7 0.00035 = 3.5×10^{-4}

8 0.0000073 = 7.3×10^{-6}

9 0.026 = 2.6×10^{-2}

10 0.0058 = 5.8×10^{-3}

Put into 'Sci' Mode (2 d.p.):
Calc: Shift Mode 7 3

11 217 000 = 2.17×10^5

12 0.0000615 = 6.15×10^{-5}

13 1 540 = 1.54×10^3

14 0.00000808 = 8.08×10^6

15 81 900 = 8.19×10^4

To Cancel: Shift Mode 8 2

Rewrite these in Scientific Notation, rounded to 3 decimal places.

Put into 'Sci' Mode (3 d.p.):
Calc: Shift Mode 7 4

16 786 537 = 2×10^4

17 0.000000055188 = 5.519×10^{-8}

18 406 333 157 = 4.063×10^8

19 0.012345 = 1.235×10^{-2}

20 772 976 807 000 = 7.730×10^{11}

To Cancel: Shift Mode 8 2

Give the answer to these in scientific notation form. Rounded to 2 decimal places.

Put into 'Sci' Mode (2 d.p.):
Calc: Shift Mode 7 3

21 0.00005^3 = 1.25×10^{-13}

22 95^6 = 7.35×10^{11}

23 $\sqrt{0.0009}$ = 3.00×10^{-2}

24 $\frac{82^4}{0.3^3}$ = 1.67×10^9

25 $\frac{(72 \div 0.08)}{(0.06 \times 0.42)}$ = 3.57×10^4

To Cancel: Shift Mode 8 2

Compare these by answering them in scientific notation to 1 decimal place, then point the < or > to the smallest.

Calc: Shift Mode 7 2

26 40^3 < 800^2

6.4×10^4 Sci. Not. 1 dp — 6.4×10^5 Sci. Not. 1 dp

27 $\frac{5}{25^5}$ < $\frac{(3 \div 19)}{38^3}$

5.1×10^{-7} Sci. Not. 1 dp — 2.9×10^{-6} Sci. Not. 1 dp

28 0.4^{12} > 0.3^{11}

1.7×10^{-5} Sci. Not. 1 dp — 1.8×10^{-6} Sci. Not. 1 dp

29 112^4 < 782^3

1.6×10^8 Sci. Not. 1 dp — 4.8×10^8 Sci. Not. 1 dp

Rewrite these five in descending order, using scientific notation (to 1 d.p.) to guide you. Descending order means highest to lowest.

30 4^9 — 10^5 — 9^7 — 6^8 — 3^{12}

2.6×10^5 Sci. Not. 1 dp — 1×10^5 Sci. Not. 1 dp — 4.8×10^6 Sci. Not. 1 dp — 1.7×10^6 Sci. Not. 1 dp — 5.3×10^5 Sci. Not. 1 dp

Descending Order: 9^7, 6^8, 3^{12}, 4^9, 10^5.

To Cancel: Shift Mode 8 2

Converting to Scientific Notation - Calculator (Homework Strip)

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Date Due: _____

Convert to scientific notation (1 d.p.)

Calc: Shift Mode 7 2

1. 140 000 = ☐.☐ × ☐^☐
2. 0.00063 = ☐.☐ × ☐^☐
3. 72 000 000 = _____
4. 0.0000018 = _____
5. 0.00047 = _____

Convert to scientific notation (2 d.p.)

Calc: Shift Mode 7 3

6. 0.09525 = _____
7. 142 554 = _____
8. 24 063 119 = _____
9. 0.006452 = _____

Calculate and write the answer in scientific notation to 2 decimal places

10. 7^5 = _____ ☐ Tick the largest
11. 6^8 = _____ ☐ Tick the largest

Date Due: _____

Convert to scientific notation (1 d.p.)

Calc: Shift Mode 7 2

1. 140 000 = ☐.☐ × ☐^☐
2. 0.00063 = ☐.☐ × ☐^☐
3. 72 000 000 = _____
4. 0.0000018 = _____
5. 0.00047 = _____

Convert to scientific notation (2 d.p.)

Calc: Shift Mode 7 3

6. 0.09525 = _____
7. 142 554 = _____
8. 24 063 119 = _____
9. 0.006452 = _____

Calculate and write the answer in scientific notation to 2 decimal places

10. 7^5 = _____ ☐ Tick the largest
11. 6^8 = _____ ☐ Tick the largest

Date Due: _____

Convert to scientific notation (1 d.p.)

Calc: Shift Mode 7 2

1. 140 000 = ☐.☐ × ☐^☐
2. 0.00063 = ☐.☐ × ☐^☐
3. 72 000 000 = _____
4. 0.0000018 = _____
5. 0.00047 = _____

Convert to scientific notation (2 d.p.)

Calc: Shift Mode 7 3

6. 0.09525 = _____
7. 142 554 = _____
8. 24 063 119 = _____
9. 0.006452 = _____

Calculate and write the answer in scientific notation to 2 decimal places

10. 7^5 = _____ ☐ Tick the largest
11. 6^8 = _____ ☐ Tick the largest

Date Due: _____

Convert to scientific notation (1 d.p.)

Calc: Shift Mode 7 2

1. 140 000 = ☐.☐ × ☐^☐
2. 0.00063 = ☐.☐ × ☐^☐
3. 72 000 000 = _____
4. 0.0000018 = _____
5. 0.00047 = _____

Convert to scientific notation (2 d.p.)

Calc: Shift Mode 7 3

6. 0.09525 = _____
7. 142 554 = _____
8. 24 063 119 = _____
9. 0.006452 = _____

Calculate and write the answer in scientific notation to 2 decimal places

10. 7^5 = _____ ☐ Tick the largest
11. 6^8 = _____ ☐ Tick the largest

Converting to Scientific Notation - Calculator (Homework Strip)

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Date Due:

Convert to scientific notation (1 d.p.)

Calc: Shift Mode 7 2

1. 140 000 = 1.4 × 10^5

2. 0.00063 = 6.3 × 10^-4

3. 72 000 000 = 7.2×10^{7}

4. 0.0000018 = 1.8×10^{6}

5. 0.00047 = 4.7×10^{-4}

Convert to scientific notation (2 d.p.)

Calc: Shift Mode 7 3

6. 0.09525 = 4.7×10^{-4}

7. 142 554 = 1.43×10^{5}

8. 24 063 119 = 2.41×10^{7}

9. 0.006452 = 6.45×10^{-3}

Calculate and write the answer in scientific notation to 2 decimal places

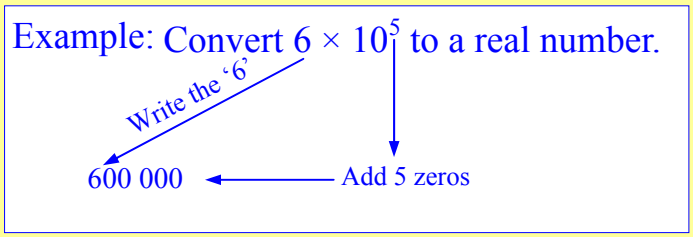
10. 7^5 = 1.7×10^{4} ☐

11. 6^8 = 1.7×10^{6} ☑

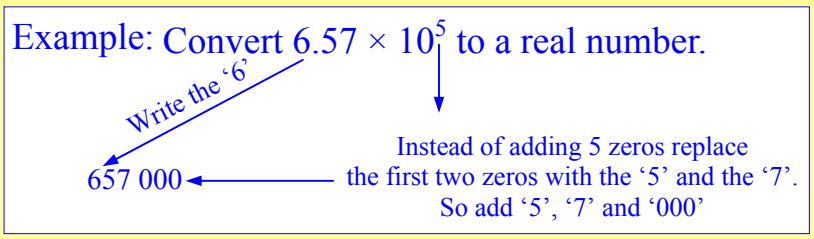
Tick the largest

Scientific Notation - Converting to Real Numbers

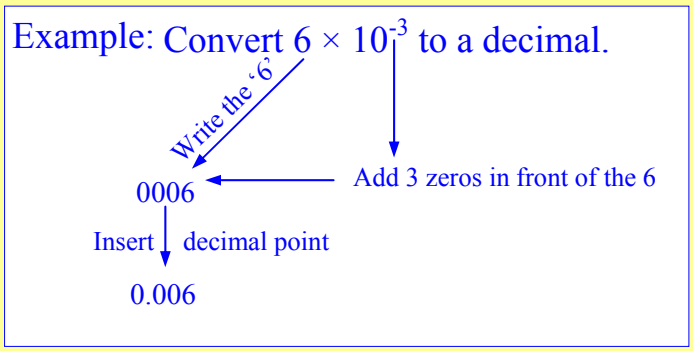
To convert from scientific notation to real numbers, you look at the power on the '10' and add that many zeros to the number.



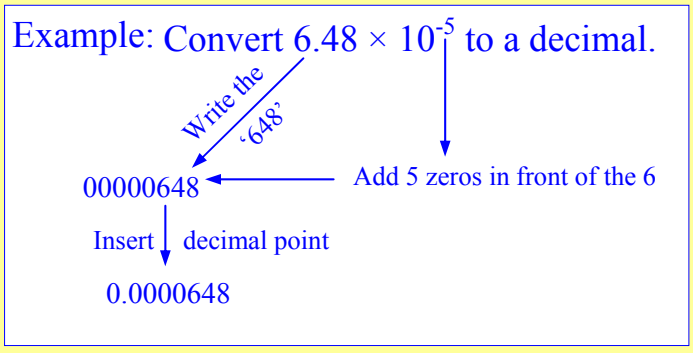
When there is a decimal present in the number in scientific notation, replace the zeros with those numbers.



When there is a negative power on the 10, a decimal will result. These are easier, all you do is write the same number of zeros as the power (positive) in front of the leading number.



When there is a decimal in the scientific notation, answer in the same way.



Scientific Notation (Non-Calc) - Convert to Real Numbers (Sheet 1)

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Convert these numbers from scientific notation format to real numbers.

Example Convert 5×10^4

50 000 ← Add 4 zeros

1 $7 \times 10^5 = 7$ _____

2 $3 \times 10^2 = 3$ _____

3 $2 \times 10^4 =$ _____

4 $9 \times 10^7 =$ _____

5 $6 \times 10^3 =$ _____

6 $4 \times 10^6 =$ _____

These have 1 decimal place, so add one less zero by replacing the first zero with a number

Example Convert 5.6×10^4

56 000 ← Add 4 digits '6' then 3 'zeros'

7 $8.5 \times 10^3 = 8\,5$ _____

8 $1.6 \times 10^4 = 1\,6$ _____

9 $4.7 \times 10^6 =$ _____

10 $9.2 \times 10^2 =$ _____

11 $3.3 \times 10^5 =$ _____

12 1.5×10^8

$=$ _____

13 4.3×10^7

$=$ _____

14 2.1×10^9

$=$ _____

15 5.9×10^{12}

$=$ _____

These have varying decimal places. Convert these to real numbers.

16 $6.35 \times 10^3 = 6\,3\,5$ _____

17 4.237×10^7

$= 4\,2\,3\,7$ _____

18 9.0951×10^9

$= 9\,0\,9\,5\,1$ _____

19 3.417×10^6

$= 3$ _____

20 7.0051×10^8

$=$ _____

21 6.045×10^5

$=$ _____

22 1.22241×10^9

$=$ _____

23 2.0202×10^6

$=$ _____

The next two have decimal answers

24 $5.096 \times 10^2 =$ _____

25 $3.4426 \times 10^3 =$ _____

Convert these decimals from scientific notation format to decimal form.

Example Convert 2×10^{-5}

0.00002 ← Add 5 zeros in front Put a '.' after 1st zero

26 $4 \times 10^{-3} = .\,\square\,\square\,4$ _____

27 $7 \times 10^{-5} = .\,\square\,\square\,\square\,\square\,7$ _____

28 $8 \times 10^{-2} = .\,\square\,8$ _____

29 $3 \times 10^{-4} = .\,\square\,\square\,\square\,3$ _____

30 5×10^{-7}

$=$ _____

31 9×10^{-5}

$=$ _____

32 8×10^{-6}

$=$ _____

33 1×10^{-9}

$=$ _____

34 3×10^{-2}

$=$ _____

34 6×10^{-4}

$=$ _____

Rewrite as decimals. These have more non-zero digits.

Example Convert 2.47×10^{-3}

0.00247 ← Add 3 zeros in front Put a '.' after 1st zero

35 5.2×10^{-4}

$=$ _____

36 4.18×10^{-6}

$=$ _____

37 5.497×10^{-2}

$=$ _____

38 2.4×10^{-9}

$=$ _____

39 3.007×10^{-5}

$=$ _____

40 1.115×10^{-6}

$=$ _____

Scientific Notation (Calculator) - Convert to Real Numbers (Sheet 2)

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Convert these numbers from scientific notation format to real numbers.

Example Convert 5.26×10^3

Calc: 5 . 2 6 ×10^x 3 = 5 260

1 2×10^4 =

Calc: 2 ×10^x 4 =

2 4×10^6 =

3 7×10^4 =

4 8×10^7 =

5 1.3×10^4 =

6 9.7×10^6 =

7 1.1×10^9

=

8 4.047×10^8

=

9 7.1338×10^6

=

10 3.377×10^7

=

The next 4 have decimal answers

11 4.031×10^2 =

12 9.73706×10^4

=

13 4.00788×10^3

=

14 7.8842×10^2

=

Convert these from scientific notation format to decimals.

Example Convert 5.26×10^{-3}

Calc: 5 . 2 6 ×10^x - 3 =

Answer: 0.00526

15 3×10^{-4} =

Calc: 3 ×10^x - 4 =

16 7×10^{-2} =

17 3×10^{-4} =

18 9×10^{-1} =

19 4×10^{-5} =

20 1×10^{-3} =

21 2.3×10^{-4} =

22 7.7×10^{-2} =

23 3.25×10^{-5}

=

24 4.024×10^{-3}

=

25 1.5×10^{-9}

=

26 3.079×10^{-3}

=

27 8.113×10^{-5}

=

28 9.3059×10^{-4}

=

Use your calculator to answer these as numbers or decimals.

29 $3 \times 10^4 \times 5 \times 10^{-2}$

Calc: 3 ×10^x 4 × 5 ×10^x - 2 =

=

30 $8 \times 10^{-9} \div 2 \times 10^{-2}$

=

31 $(4 \times 10^{-3})^2$

=

32 $1.6 \times 10^8 \div 8 \times 10^3$

=

33 $\dfrac{1.05 \times 10^8}{3.5 \times 10^6}$ =

Calculate these and write the answer underneath. Then use <, > or = in the square box.

*With < & > 'point' to the smallest

34 $\dfrac{1.8 \times 10^8}{6 \times 10^2}$ ☐ $(3 \times 10^2)^2$

35 $\dfrac{4 \times 10^3}{2 \times 10^5}$ ☐ $\dfrac{1 \times 10^2}{2 \times 10^4}$

36 $(1 \times 10^3)^2$ ☐ $(1 \times 10^2)^3$

37 $(2 \times 10^2)^2$ ☐ $(4 \times 10^2)^2$

Scientific Notation (Non-Calc) - Convert to Real Numbers (Sheet 1)

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Convert these numbers from scientific notation format to real numbers.

Example Convert 5×10^4

50 000 ← Add 4 zeros

1 $7 \times 10^5 = 7\,0\,0\,0\,0\,0$

2 $3 \times 10^2 = 3\,0\,0$

3 $2 \times 10^4 =$ 20 000

4 $9 \times 10^7 =$ 90 000 000

5 $6 \times 10^3 =$ 6 000

6 $4 \times 10^6 =$ 4 000 000

These have 1 decimal place, so add one less zero by replacing the first zero with a number

Example Convert 5.6×10^4

56 000 ← Add 4 digits '6' then 3 'zeros'

7 $8.5 \times 10^3 = 8\,5\,0\,0$

8 $1.6 \times 10^4 = 1\,6\,0\,0\,0$

9 $4.7 \times 10^6 =$ 4 700 000

10 $9.2 \times 10^2 =$ 920

11 $3.3 \times 10^5 =$ 330 000

12 1.5×10^8
$=$ 150 000 000

13 4.3×10^7
$=$ 43 000 000

14 2.1×10^9
$=$ 2 100 000 000

15 5.9×10^{12}
$=$ 5 900 000 000 000

These have varying decimal places. Convert these to real numbers.

16 $6.35 \times 10^3 = 6\,3\,5\,0$

17 4.237×10^7
$= 4\,2\,3\,7\,0\,0\,0\,0$

18 9.0951×10^9
$= 9\,0\,9\,5\,1\,0\,0\,0\,0\,0$

19 3.417×10^6
$= 3\,4\,1\,7\,0\,0\,0$

20 7.0051×10^8
$= 7\,0\,0\,5\,1\,0\,0\,0\,0$

21 6.045×10^5
$= 6\,0\,4\,5\,0\,0$

22 1.22241×10^9
$=$ 1 222 410 000

23 2.0202×10^6
$=$ 2 020 200

The next two have decimal answers

24 $5.096 \times 10^2 =$ 509.6

25 $3.4426 \times 10^3 =$ 3 442.6

Convert these decimals from scientific notation format to decimal form.

Example Convert 2×10^{-5}

0.00002 ← Add 5 zeros in front Put a '.' after 1st zero

26 $4 \times 10^{-3} = 0.004$

27 $7 \times 10^{-5} = 0.00007$

28 $8 \times 10^{-2} = 0.08$

29 $3 \times 10^{-4} = 0.0003$

30 5×10^{-7}
$= 0.0000005$

31 9×10^{-5}
$= 0.00009$

32 8×10^{-6}
$=$ 0.0000008

33 1×10^{-9}
$=$ 0.000000001

34 $3 \times 10^{-2} =$ 0.03

34 $6 \times 10^{-4} =$ 0.0006

Rewrite as decimals. These have more non-zero digits.

Example Convert 2.47×10^{-3}

0.00247 ← Add 3 zeros in front Put a '.' after 1st zero

35 5.2×10^{-4}
$= 0.00052$

36 4.18×10^{-6}
$= 0.00000418$

37 5.497×10^{-2}
$=$ 0.05497

38 2.4×10^{-9}
$=$ 0.0000000024

39 3.007×10^{-5}
$=$ 0.00003007

40 1.115×10^{-6}
$=$ 0.00000115

Scientific Notation (Calculator) - Convert to Real Numbers (Sheet 2)

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Convert these numbers from scientific notation format to real numbers.

Example Convert 5.26×10^3

Calc: 5 . 2 6 ×10^x 3 = 5 260

1 2×10^4 = 20 000

Calc: 2 ×10^x 4 =

2 4×10^6 = 4 000 000

3 7×10^4 = 70 000

4 8×10^7 = 80 000 000

5 1.3×10^4 = 13 000

6 9.7×10^6 = 9 700 000

7 1.1×10^9

= 1 100 000 000

8 4.047×10^8

= 404 700 000

9 7.1338×10^6

= 7 133 800

10 3.377×10^7

= 33 770 000

The next 4 have decimal answers

11 4.031×10^2 = 403.1

12 9.73706×10^4

= 97 370.6

13 4.00788×10^3

= 4 007.88

14 7.8842×10^2

= 788.42

Convert these from scientific notation format to decimals.

Example Convert 5.26×10^{-3}

Calc: 5 . 2 6 ×10^x - 3 =

Answer: 0.00526

15 3×10^{-4} = 0.0003

Calc: 3 ×10^x - 4 =

16 7×10^{-2} = 0.07

17 3×10^{-4} = 0.0003

18 9×10^{-1} = 0.9

19 4×10^{-5} = 0.00004

20 1×10^{-3} = 0.001

21 2.3×10^{-4} = 0.00023

22 7.7×10^{-2} = 0.077

23 3.25×10^{-5}

= 0.0000325

24 4.024×10^{-3}

= 0.004024

25 1.5×10^{-9}

= 0.0000000015

26 3.079×10^{-3}

= 0.003079

27 8.113×10^{-5}

= 0.00008113

28 9.3059×10^{-4}

= 0.00093059

Use your calculator to answer these as numbers or decimals.

29 $3 \times 10^4 \times 5 \times 10^{-2}$

Calc: 3 ×10^x 4 × 5 ×10^x - 2 =

= 1 500

30 $8 \times 10^{-9} \div 2 \times 10^{-2}$

= 0.0000004

31 $(4 \times 10^{-3})^2$

= 0.000016

32 $1.6 \times 10^8 \div 8 \times 10^3$

= 20 000

33 $\frac{1.05 \times 10^8}{3.5 \times 10^6}$ = 30

Calculate these and write the answer underneath. Then use <, > or = in the square box.

*With < & > 'point' to the smallest

34 $\frac{1.8 \times 10^8}{6 \times 10^2}$ > $(3 \times 10^2)^2$

300 000 90 000

35 $\frac{4 \times 10^3}{2 \times 10^5}$ > $\frac{1 \times 10^2}{2 \times 10^4}$

0.02 0.005

36 $(1 \times 10^3)^2$ = $(1 \times 10^2)^3$

1 000 000 1 000 000

37 $(2 \times 10^2)^2$ < $(4 \times 10^2)^2$

40 000 160 000

Scientific Notation - Convert to Real Numbers (Homework Strip)

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Date Due:

Convert these to real numbers

1. $5 \times 10^4 = 5$ ☐☐☐☐
2. $9 \times 10^2 = 9$ ☐☐
3. $8 \times 10^5 =$ ☐☐☐☐☐☐
4. $4 \times 10^3 =$ ☐☐☐☐
5. $1.6 \times 10^4 =$ ☐☐☐☐☐
6. $5.17 \times 10^3 =$ ☐☐☐☐
7. $4.05 \times 10^6 =$ ☐☐☐☐

Convert these to decimals

8. $2 \times 10^{-4} =$ ☐.☐☐☐2
9. $7 \times 10^{-2} =$ ☐.☐☐
10. $5 \times 10^{-6} =$ ☐☐☐☐☐☐☐
11. $3.2 \times 10^{-3} =$ ☐☐☐☐
12. $9.4 \times 10^{-4} =$ ☐☐☐☐
13. $7.16 \times 10^{-2} =$ ☐☐☐☐

Date Due:

Convert these to real numbers

1. $5 \times 10^4 = 5$ ☐☐☐☐
2. $9 \times 10^2 = 9$ ☐☐
3. $8 \times 10^5 =$ ☐☐☐☐☐☐
4. $4 \times 10^3 =$ ☐☐☐☐
5. $1.6 \times 10^4 =$ ☐☐☐☐☐
6. $5.17 \times 10^3 =$ ☐☐☐☐
7. $4.05 \times 10^6 =$ ☐☐☐☐

Convert these to decimals

8. $2 \times 10^{-4} =$ ☐.☐☐☐2
9. $7 \times 10^{-2} =$ ☐.☐☐
10. $5 \times 10^{-6} =$ ☐☐☐☐☐☐☐
11. $3.2 \times 10^{-3} =$ ☐☐☐☐
12. $9.4 \times 10^{-4} =$ ☐☐☐☐
13. $7.16 \times 10^{-2} =$ ☐☐☐☐

Date Due:

Convert these to real numbers

1. $5 \times 10^4 = 5$ ☐☐☐☐
2. $9 \times 10^2 = 9$ ☐☐
3. $8 \times 10^5 =$ ☐☐☐☐☐☐
4. $4 \times 10^3 =$ ☐☐☐☐
5. $1.6 \times 10^4 =$ ☐☐☐☐☐
6. $5.17 \times 10^3 =$ ☐☐☐☐
7. $4.05 \times 10^6 =$ ☐☐☐☐

Convert these to decimals

8. $2 \times 10^{-4} =$ ☐.☐☐☐2
9. $7 \times 10^{-2} =$ ☐.☐☐
10. $5 \times 10^{-6} =$ ☐☐☐☐☐☐☐
11. $3.2 \times 10^{-3} =$ ☐☐☐☐
12. $9.4 \times 10^{-4} =$ ☐☐☐☐
13. $7.16 \times 10^{-2} =$ ☐☐☐☐

Date Due:

Convert these to real numbers

1. $5 \times 10^4 = 5$ ☐☐☐☐
2. $9 \times 10^2 = 9$ ☐☐
3. $8 \times 10^5 =$ ☐☐☐☐☐☐
4. $4 \times 10^3 =$ ☐☐☐☐
5. $1.6 \times 10^4 =$ ☐☐☐☐☐
6. $5.17 \times 10^3 =$ ☐☐☐☐
7. $4.05 \times 10^6 =$ ☐☐☐☐

Convert these to decimals

8. $2 \times 10^{-4} =$ ☐.☐☐☐2
9. $7 \times 10^{-2} =$ ☐.☐☐
10. $5 \times 10^{-6} =$ ☐☐☐☐☐☐☐
11. $3.2 \times 10^{-3} =$ ☐☐☐☐
12. $9.4 \times 10^{-4} =$ ☐☐☐☐
13. $7.16 \times 10^{-2} =$ ☐☐☐☐

Scientific Notation - Convert to Real Numbers (Homework Strip)

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Date Due:

Convert these to real numbers

1. $5 \times 10^4 = 50000$
2. $9 \times 10^2 = 900$
3. $8 \times 10^5 = 800000$
4. $4 \times 10^3 = 4000$
5. $1.6 \times 10^4 = 16000$
6. $5.17 \times 10^3 = 5\,170$
7. $4.05 \times 10^6 = 4\,050\,000$

Convert these to decimals

8. $2 \times 10^{-4} = 0.0002$
9. $7 \times 10^{-2} = 0.07$
10. $5 \times 10^{-6} = 0.000005$
11. $3.2 \times 10^{-3} = 0.0032$
12. $9.4 \times 10^{-4} = 0.00094$
13. $7.16 \times 10^{-2} = 0.0716$

Numbers of Any Magnitude - Prefixes (Large Numbers)

A prefix is used at the front of a ‘base’ unit to change the unit to larger or small units. For example, using the base ‘byte’:

Base Unit: Byte
1 000 × Base Unit = kB (kilobyte)
1 000 000 × Base Unit = MB (megabyte)
1 000 000 000 × Base Unit = GB (gigabyte)
1 000 000 000 000 × Base Unit = TB (terabyte)

So each increase sees the unit multiplied by 1 000. So to convert from one unit to another is done by:

To move to a larger unit: Divide (÷) by 1 000
To move to a smaller unit: Multiply (÷) by 1 000

The first sheet deals with base units and prefixes of ‘kilo’ and ‘Mega’.

The second sheet deals with prefixes of ‘Mega’, ‘Giga’ and ‘Tera’

Numbers of Any Magnitude - Prefixes ‘Kilo’ & ‘Mega’ (Sheet 1)

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Convert these base units into ‘Kilo’ units. Divide by 1 000 to make the conversion.

Example Convert 3 400 m to km

3 400 ÷ 1 000 = 3.4 km

1 8 500 g = _____ kg

2 43 700 L = _____ kL

3 780 m = _____ km

4 347 500 B = _____ kB

5 220 g = _____ kg

6 4 020 m = _____ km

Ben walks 320 m to the shops, then returns home. His mum forgot that she needs bread so he repeats the trip.

6 How far has Ben walked for the two return trips, in metres.

7 Convert this distance to km.

Jo’s phone downloads at 780 B per second. She downloads a document which takes 5 minutes to download.

8 Convert 5 mins to seconds.

9 How large is the file she downloaded? Answer in B.

10 Convert the file from B to kB.

Convert these ‘Kilo’ units into ‘Mega’ units. Divide by 1 000 to make the conversion.

Example Convert 400 kB to MB

400 ÷ 1 000 = 0.4 MB

11 9 800 kL = _____ ML

12 176 300 kB = _____ MB

13 13 140 kHz = _____ MHz

*Hertz (Hz) is a measure of frequency.

14 490 kL = _____ ML

Josh has taken 527 photos this year. If the average photo is 970 kB. Find:

15 The total size of the folder holding the images in kB.

16 Convert the folder’s size to MB.

An irrigation pump discharges 1.4kL of water per minute. Find the amount:

17 Pumped in an hour, in kL.

18 Pumped in a day. (24 hrs)

19 Pumped annually. (365 days)

20 Convert the amount pumped annually from kL to ML.

To go backwards, multiply by 1000. Mega→Kilo = × 1 000 Kilo→Base unit = × 1 000

Example Convert 3.2 MB to kB

3.2 × 1 000 = 3 200 kB

21 5.4 MB = _____ kB

22 9 km = _____ m

23 4.76 kg = _____ g

24 0.56 ML = _____ kL

25 2.09 km = _____ m

26 1.7 MB = _____ kB

A farm’s diesel storage tank holds 8.2 kL of fuel.

27 Find the volume of fuel in the tank is it is 55% full. (kL)

$\frac{\square}{100} \times \square = \square$

28 Convert the volume to L.

29 Calculate the value of the fuel if diesel costs $1.68/L.

Show the $ sign and cents to 2 d.p.

Karl is training for the 800 m state championship. He runs the 800 m twice each weekday and once on Saturday and Sunday.

30 Calculate the distance Karl runs in a week in m, then km.

Numbers of Any Magnitude - Prefixes ‘Mega’, ‘Giga’ & ‘Tera’ (Sheet 2)

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Convert these ‘Giga’ units into ‘Mega’ units. Multiply by 1 000 to make the conversion.

Example Convert 1.4 GB to MB

1.4 × 1 000 = 1 400 MB

1 3 GB = _____ MB

2 5.6 GW = _____ MW

*Watt (W) is a measure of power.

3 0.64 GL = _____ ML

4 22.07 GB = _____ MB

5 0.034 GL = _____ ML

6 14.8 GB = _____ MB

Chen has 8 files that average 390 MB each. He wants to store them on a DVD which holds 4.7 GB.

7 Convert 4.7 GB to MB.

8 How much memory in MB will the 8 files need?

9 Calculate how much space in MB is left on the DVD?

Ainslie has 2.2 GB available on a USB drive. She wants to put music on the drive that average 4 MB per song.

10 Convert 2.2 GB to MB.

11 How many 4 MB songs fit?

Convert these ‘Mega’ units into ‘Giga’ units. Divide by 1 000 to make the conversion.

Example Convert 720 MB to GB

720 ÷ 1 000 = 0.72 GB

12 4 000 MB = _____ GB

13 125 000 MB = _____ GB

14 280 MB = _____ GB

15 1 080 ML = _____ GL

16 48 MW = _____ GW

17 68 700 ML = _____ GL

David’s PC has 3 old Hard Drives which are 260 MB, 720 MB and 650 MB.

18 Find the total storage (MB).

19 Convert the total storage to GB.

Anthia shoots wedding videos. Once edited the videos average 850 MB in size. She keeps a back up of all the videos and she has 167 of them.

20 Calculate how much space in MB is used by the 167 videos.

21 Convert the storage to GB.

Convert these ‘Giga’ units into ‘Tera’ units. Divide by 1 000 to make the conversion.

Example Convert 480 GB to TB

480 ÷ 1 000 = 0.48 TB

22 6 000 GB = _____ TB

23 300 GB = _____ TB

24 1 020 GB = _____ TB

25 89.2 GB = _____ TB

26 770 GB = _____ TB

27 0.5 GB = _____ TB

Raj has a 64 GB USB drive. His PC has a 4 TB hard drive.

28 Convert 64 GB into TB.

29 How many full USB drives would the hard drive hold?

Convert these ‘Tera’ units into ‘Giga’ units. Multiply by 1 000 to make the conversion.

Example Convert 3.2 TB to GB

3.2 × 1 000 = 3 200 GB

30 5 TB = _____ GB

31 1.7 TB = _____ GB

32 0.06 TB = _____ GB

33 2.85 TB = _____ GB

34 10.6 TB = _____ GB

35 0.0083 TB = _____ GB

Numbers of Any Magnitude - Prefixes ‘Kilo’ & ‘Mega’ (Sheet 1)

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Convert these base units into ‘Kilo’ units. Divide by 1 000 to make the conversion.

Example Convert 3 400 m to km

3 400 ÷ 1 000 = 3.4 km

1 8 500 g = 8.5 kg

2 43 700 L = 43.7 kL

3 780 m = 0.78 km

4 347 500 B = 347.5 kB

5 220 g = 0.22 kg

6 4 020 m = 4.02 km

Ben walks 320 m to the shops, then returns home. His mum forgot that she needs bread so he repeats the trip.

6 How far has Ben walked for the two return trips, in metres.

4 × 320 = 1 280 m

7 Convert this distance to km.

1 280 ÷ 1 000 = 1.28 km

Jo’s phone downloads at 780 B per second. She downloads a document which takes 5 minutes to download.

8 Convert 5 mins to seconds.

5 × 60 = 300

9 How large is the file she downloaded? Answer in B.

300 × 780 = 234 000 B

10 Convert the file from B to kB.

234 000 ÷ 1 000 = 234 kB

Convert these ‘Kilo’ units into ‘Mega’ units. Divide by 1 000 to make the conversion.

Example Convert 400 kB to MB

400 ÷ 1 000 = 0.4 MB

11 9 800 kL = 9.8 ML

12 176 300 kB = 176.3 MB

13 13 140 kHz = 13.14 MHz

*Hertz (Hz) is a measure of frequency.

14 490 kL = 0.49 ML

Josh has taken 527 photos this year. If the average photo is 970 kB. Find:

15 The total size of the folder holding the images in kB.

527 × 970 = 511 190 kB

16 Convert the folder’s size to MB.

511 190 ÷ 1 000 = 511.19 MB

An irrigation pump discharges 1.4kL of water per minute. Find the amount:

17 Pumped in an hour, in kL.

1.4 × 60 = 84 kL

18 Pumped in a day. (24 hrs)

84 × 24 = 2 016 kL

19 Pumped annually. (365 days)

2 016 × 365 = 735 840 kL

20 Convert the amount pumped annually from kL to ML.

735 840 ÷ 1 000 = 735.84 ML

To go backwards, multiply by 1000. Mega→Kilo = × 1 000 Kilo→Base unit = × 1 000

Example Convert 3.2 MB to kB

3.2 × 1 000 = 3 200 kB

21 5.4 MB = 5 400 kB

22 9 km = 9 000 m

23 4.76 kg = 4 760 g

24 0.56 ML = 560 kL

25 2.09 km = 2 090 m

26 1.7 MB = 1 700 kB

A farm’s diesel storage tank holds 8.2 kL of fuel.

27 Find the volume of fuel in the tank is it is 55% full. (kL)

$\frac{55}{100} \times 8.2 = 4.51$ kL

28 Convert the volume to L.

4.51 × 1 000 = 4 510 L

29 Calculate the value of the fuel if diesel costs $1.68/L.

4 510 × 1.68 = $7 576.80

Show the $ sign and cents to 2 d.p.

Karl is training for the 800 m state championship. He runs the 800 m twice each weekday and once on Saturday and Sunday.

30 Calculate the distance Karl runs in a week in m, then km.

(10 + 2) × 800 = 9 600 m

9 600 ÷ 1 000 = 9.6 km

Numbers of Any Magnitude - Prefixes ‘Mega’, ‘Giga’ & ‘Tera’ (Sheet 2)

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Convert these ‘Giga’ units into ‘Mega’ units. Multiply by 1 000 to make the conversion.

Example Convert 1.4 GB to MB

1.4 × 1 000 = 1 400 MB

1 3 GB = 3 000 MB

2 5.6 GW = 5 600 MW

*Watt (W) is a measure of power.

3 0.64 GL = 640 ML

4 22.07 GB = 22 070 MB

5 0.034 GL = 34 ML

6 14.8 GB = 14 800 MB

Chen has 8 files that average 390 MB each. He wants to store them on a DVD which holds 4.7 GB.

7 Convert 4.7 GB to MB.

4.7 × 1 000 = 4 700 MB

8 How much memory in MB will the 8 files need?

8 × 390 = 3 120 MB

9 Calculate how much space in MB is left on the DVD?

4 700 - 3 120 = 1 580 MB

Ainslie has 2.2 GB available on a USB drive. She wants to put music on the drive that average 4 MB per song.

10 Convert 2.2 GB to MB.

2.2 × 1 000 = 2 200 MB

11 How many 4 MB songs fit?

2 200 ÷ 4 = 550 songs

Convert these ‘Mega’ units into ‘Giga’ units. Divide by 1 000 to make the conversion.

Example Convert 720 MB to GB

720 ÷ 1 000 = 0.72 GB

12 4 000 MB = 4 GB

13 125 000 MB = 125 GB

14 280 MB = 0.28 GB

15 1 080 ML = 1.08 GL

16 48 MW = 0.048 GW

17 68 700 ML = 68.7 GL

David’s PC has 3 old Hard Drives which are 260 MB, 720 MB and 650 MB.

18 Find the total storage (MB).

260 + 720 + 650 = 1 630 MB

19 Convert the total storage to GB.

1 630 ÷ 1 000 = 1.63 GB

Anthia shoots wedding videos. Once edited the videos average 850 MB in size. She keeps a back up of all the videos and she has 167 of them.

20 Calculate how much space in MB is used by the 167 videos.

850 × 167 = 141 950 MB

21 Convert the storage to GB.

141 950 ÷ 1 000 = 141.95 GB

Convert these ‘Giga’ units into ‘Tera’ units. Divide by 1 000 to make the conversion.

Example Convert 480 GB to TB

480 ÷ 1 000 = 0.48 TB

22 6 000 GB = 6 TB

23 300 GB = 0.3 TB

24 1 020 GB = 1.02 TB

25 89.2 GB = 0.0892 TB

26 770 GB = 0.77 TB

27 0.5 GB = 0.0005 TB

Raj has a 64 GB USB drive. His PC has a 4 TB hard drive.

28 Convert 64 GB into TB.

64 ÷ 1 000 = 0.064 TB

29 How many full USB drives would the hard drive hold?

4 ÷ 0.064 = 62 USBs (62.5)

Convert these ‘Tera’ units into ‘Giga’ units. Multiply by 1 000 to make the conversion.

Example Convert 3.2 TB to GB

3.2 × 1 000 = 3 200 GB

30 5 TB = 5 000 GB

31 1.7 TB = 1 700 GB

32 0.06 TB = 60 GB

33 2.85 TB = 2 850 GB

34 10.6 TB = 10 600 GB

35 0.0083 TB = 8.3 GB

Numbers of Any Magnitude - Prefixes - Larger Numbers (Homework Strip)

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Date Due: _____

Convert kilo to Mega by ÷ 1 000

1. 6 000 kB = _____ MB
2. 180 kB = _____ MB
3. 1 560 kB = _____ MB

A folder has 342 spreadsheet files that average 165 kB each. Find the:

4. Storage used by the files (kB)

_____ × _____ = _____

5. Convert the storage to MB.

Convert Mega to kilo by × 1 000

6. 2.5 MB = _____ kB
7. 0.75 MB = _____ kB
8. 10.9 MB = _____ kB

A folder has 4 songs totaling 22 MB. Find the average song size in kB.

9. 22 ÷ 4 = _____ MB
10. _____

Now convert your answer to kB

Date Due: _____

Convert kilo to Mega by ÷ 1 000

1. 6 000 kB = _____ MB
2. 180 kB = _____ MB
3. 1 560 kB = _____ MB

A folder has 342 spreadsheet files that average 165 kB each. Find the:

4. Storage used by the files (kB)

_____ × _____ = _____

5. Convert the storage to MB.

Convert Mega to kilo by × 1 000

6. 2.5 MB = _____ kB
7. 0.75 MB = _____ kB
8. 10.9 MB = _____ kB

A folder has 4 songs totaling 22 MB. Find the average song size in kB.

9. 22 ÷ 4 = _____ MB
10. _____

Now convert your answer to kB

Date Due: _____

Convert kilo to Mega by ÷ 1 000

1. 6 000 kB = _____ MB
2. 180 kB = _____ MB
3. 1 560 kB = _____ MB

A folder has 342 spreadsheet files that average 165 kB each. Find the:

4. Storage used by the files (kB)

_____ × _____ = _____

5. Convert the storage to MB.

Convert Mega to kilo by × 1 000

6. 2.5 MB = _____ kB
7. 0.75 MB = _____ kB
8. 10.9 MB = _____ kB

A folder has 4 songs totaling 22 MB. Find the average song size in kB.

9. 22 ÷ 4 = _____ MB
10. _____

Now convert your answer to kB

Date Due: _____

Convert kilo to Mega by ÷ 1 000

1. 6 000 kB = _____ MB
2. 180 kB = _____ MB
3. 1 560 kB = _____ MB

A folder has 342 spreadsheet files that average 165 kB each. Find the:

4. Storage used by the files (kB)

_____ × _____ = _____

5. Convert the storage to MB.

Convert Mega to kilo by × 1 000

6. 2.5 MB = _____ kB
7. 0.75 MB = _____ kB
8. 10.9 MB = _____ kB

A folder has 4 songs totaling 22 MB. Find the average song size in kB.

9. 22 ÷ 4 = _____ MB
10. _____

Now convert your answer to kB

Numbers of Any Magnitude - Prefixes - Larger Numbers (H'work Strip)

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Date Due:

Convert kilo to Mega by ÷ 1 000

1 6 000 kB = 6 MB

2 180 kB = 0.18 MB

3 1 560 kB = 1.56 MB

A folder has 342 spreadsheet files that average 165 kB each. Find the:

4 Storage used by the files (kB)

342 × 165 = 56 430 kB

5 Convert the storage to MB.

56 430 ÷ 1 000 = 56.43 MB

Convert Mega to kilo by × 1 000

6 2.5 MB = 2 500 kB

7 0.75 MB = 750 kB

8 10.9 MB = 10 900 kB

A folder has 4 songs totaling 22 MB. Find the average song size in kB.

9 22 ÷ 4 = 5.5 MB

10 5.5 × 1 000 = 5 500 kB

Now convert your answer to kB

Numbers of Any Magnitude - Prefixes (Small Numbers)

A prefix is used at the front of a ‘base’ unit to change the unit to larger or small units. For example, using the base ‘metre’:

Base Unit: Metre
Base Unit ÷ 100 = cm (centimetre)
Base Unit ÷ 1 000 = mm (millimetre)
Base Unit ÷ 1 000 000 = μm (micrometre or micron)

We use the measurement ‘centi’, mainly with length, centimetres. These are 100th of a metre. Europe frequently uses centi with volume measurements, such as cL (centilitres).

Remember:

To move to a larger unit: Divide
To move to a smaller unit: Multiply

The first sheet deals with length units and prefixes of ‘kilo’, ‘centi’ and ‘milli.’ The top of each column tells you how to convert. You can do this by moving the decimal point, or using a calculator.

The second sheet deals with prefixes of ‘milli’ and ‘micro.’ There are examples at the top of the column also.

Numbers of Any Magnitude - Prefixes & Length milli/centi/kilo (Sheet 1)

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Convert these lengths from mm to cm. There are 10 mm in 1 cm.



Divide by 10 or move 1 place left

1	20 mm	cm
2	50 mm	
3	130 mm	
4	55 mm	
5	92 mm	
6	200 mm	
7	18.6 mm	
8	2 020 mm	
9	435 mm	
10	0.95 mm	
11	5.47 mm	
12	906.2 mm	

Now the reverse. Convert these lengths from cm to mm. There are 10 mm in 1 cm.



Multiply by 10 or move 1 place right

13	5 cm	mm
14	18 cm	
15	6.3 cm	
16	74.8 cm	
17	356.7 cm	
18	8.12 cm	
19	0.4 cm	
20	0.566 cm	
21	155 cm	
22	829.3 cm	
23	1.002 cm	
24	19.91 cm	

Convert these lengths from cm to m. There are 100 cm in 1 m.



Divide by 100 or 2 places left

25	200 cm	m
26	600 cm	
27	150 cm	
28	370 cm	
29	12 cm	
30	86.5 cm	
31	7 300 cm	
32	56.05 cm	
33	3 cm	
34	6.7 cm	
35	8 002 cm	
36	27.08 cm	

Convert these lengths from m to cm



Multiply by 100 or 2 places right

37	3 m	cm
38	7 m	
39	2.2 m	
40	5.9 m	
41	0.3 m	
42	0.75 m	
43	0.363 m	
44	5.02 m	
45	10.7 m	
46	0.006 m	
47	5.071 m	
48	30.262 m	



Convert these lengths from m to km

Divide by 1 000 or 3 places left

49	1 500 m	km
50	1 220 m	
51	4 930 m	
52	110 m	
53	550.6 m	
54	1 003 m	
55	85 m	
56	4.7 m	
57	309.7 m	
58	12 032 m	
59	8 771.3 m	
60	763.2 m	



Convert these lengths from km to m

Multiply by 1 000 3 places right

61	3 km	m
62	8 km	
63	3.2 km	
64	14.7 km	
65	10.6 km	
66	0.9 km	
67	0.673 km	
68	5.12 km	
69	1.007 km	
70	0.4503 km	
71	4.019 km	
72	0.06 km	

Numbers of Any Magnitude - Prefixes ‘milli’ & ‘micro’ (Sheet 2)

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Convert these base units into ‘milli’ units. Multiply by 1 000 to make the conversion.

Example Convert 2.5 m to mm

2.5 × 1 000 = 2 500 mm

1 5 A = _____ mA

*Amps (A) is a measure of electrical current

2 80 g = _____ mg

3 3.08 L = _____ mL

4 0.005 m = _____ mm

5 15.7 g = _____ mg

6 A ceiling is 2.4 m above the floor. Convert the length to mm.

_____ × 1 000 = _____ mm

7 The active ingredient in a antibiotic weighs 0.8 g. Convert this weight to mg.

8 The current across a circuit is 0.0033 A. Convert this to mA.

9 Three boxes have heights: 0.42 m, 0.715 m and 0.505 m. How high are they when stacked on top of each other. Answer in m, then in mm.

10 A standard door is 2.04 m high. Convert the height to mm.

To convert ‘milli’ units to the base unit, divide by 1 000. Change these to ‘milli’ units.

Example Convert 960 mm to m

960 ÷ 1 000 = 0.96 m

11 3 000 mL = _____ L

12 28 900 mg = _____ g

13 31 430 mm = _____ m

14 4 620 mL = _____ L

15 86 mΩ = _____ Ω

*Ohms (Ω) is a measure of electrical resistance

16 A bottle of soft drink contains 1 250 mL of product. Convert this to L.

_____ ÷ 1 000 = _____ L

17 Ceiling fan blades are usually 300 mm below the ceiling. Convert this to m.

18 Two swimmers touch the wall 230 ms apart. Convert this time to seconds.

19 A medicine has 308 mg, 182 mg, 780 mg and 2 750 mg of different active ingredients. Find the total weight in mg, then convert this to g.

Micro is 1 millionth. Convert ‘micro’ to ‘milli’ by dividing by 1 000. Hey … I know mu!

Example Convert 4 440 µm to mm

4 440 ÷ 1 000 = 4.44 mm

20 34 000 µm = _____ mm

21 87 µg = _____ mg

22 8 803 µs = _____ ms

23 40.5 µg = _____ mg

24 A human hair thickness is 120 µm. Convert this to mm.

_____ ÷ 1 000 = _____ mm

25 A dust mite is 275 µm in length. Convert this to mm.

To convert ‘milli’ units to ‘micro’ units, multiply by 1 000. Micrometres can also be called microns.

Example Convert 0.74 mm to µm

0.74 × 1 000 = 740 µm

26 8 mm = _____ µm

27 0.06 mg = _____ µg

28 0.0037 ms = _____ µs

29 5.7 mm = _____ µm

30 A red blood cell is 0.008 mm in diameter. Convert this to µm.

31 The width of an eye of a needle is 1.2 mm Convert to µm.

Numbers of Any Magnitude - Prefixes & Length milli/centi/kilo (Sheet 1)

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Convert these lengths from mm to cm. There are 10 mm in 1 cm.

Divide by 10 or move 1 place left

1	20 mm	2 cm
2	50 mm	5 cm
3	130 mm	13 cm
4	55 mm	5.5 cm
5	92 mm	9.2 cm
6	200 mm	20 cm
7	18.6 mm	1.86 cm
8	2 020 mm	202 cm
9	435 mm	43.5 cm
10	0.95 mm	0.095 cm
11	5.47 mm	0.547 cm
12	906.2 mm	90.62 cm

Convert these lengths from cm to m. There are 100 cm in 1 m.

Divide by 100 or 2 places left

25	200 cm	2 m
26	600 cm	6 m
27	150 cm	1.5 m
28	370 cm	3.7 m
29	12 cm	0.12 m
30	86.5 cm	0.865 m
31	7 300 cm	73 m
32	56.05 cm	0.5605 m
33	3 cm	0.03 m
34	6.7 cm	0.067 m
35	8 002 cm	80.02 m
36	27.08 cm	0.2708 m

Convert these lengths from m to km

Divide by 1 000 or 3 places left

49	1 500 m	1.5 km
50	1 220 m	1.22 km
51	4 930 m	4.93 km
52	110 m	0.11 km
53	550.6 m	0.5506 km
54	1 003 m	1.003 km
55	85 m	0. 085 km
56	4.7 m	0.0047 km
57	309.7 m	0.3097 km
58	12 032 m	12.032 km
59	8 771.3 m	8.7713 km
60	763.2 m	0.7632 km

Now the reverse. Convert these lengths from cm to mm. There are 10 mm in 1 cm.

Multiply by 10 or move 1 place right

13	5 cm	50 mm
14	18 cm	180 mm
15	6.3 cm	63 mm
16	74.8 cm	748mm
17	356.7 cm	3 567 mm
18	8.12 cm	81.2 mm
19	0.4 cm	4 mm
20	0.566 cm	5.66 mm
21	155 cm	1 550 mm
22	829.3 cm	8 293 mm
23	1.002 cm	10.02 mm
24	19.91 cm	199.1 mm

Convert these lengths from m to cm

Multiply by 100 or 2 places right

37	3 m	300 cm
38	7 m	700 cm
39	2.2 m	220 cm
40	5.9 m	590 cm
41	0.3 m	30 cm
42	0.75 m	75 cm
43	0.363 m	36.3 cm
44	5.02 m	502 cm
45	10.7 m	1 070 cm
46	0.006 m	0.6 cm
47	5.071 m	507.1 cm
48	30.262 m	3 026.2 cm

Convert these lengths from km to m

Multiply by 1 000 3 places right

61	3 km	3 000 m
62	8 km	8 000 m
63	3.2 km	3 200 m
64	14.7 km	14 700 m
65	10.6 km	10 600 m
66	0.9 km	900 m
67	0.673 km	673 m
68	5.12 km	5 120 m
69	1.007 km	1 007 m
70	0.4503 km	450.3 m
71	4.019 km	4 019 m
72	0.06 km	60 m

Numbers of Any Magnitude - Prefixes ‘milli’ & ‘micro’ (Sheet 2)

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Convert these base units into ‘milli’ units. Multiply by 1 000 to make the conversion.

Example Convert 2.5 m to mm

2.5 × 1 000 = 2 500 mm

1 5 A = 5 000 mA

*Amps (A) is a measure of electrical current

2 80 g = 80 000 mg

3 3.08 L = 3 080 mL

4 0.005 m = 5 mm

5 15.7 g = 15 700 mg

6 A ceiling is 2.4 m above the floor. Convert the length to mm.

2.4 × 1 000 = 2 400 mm

7 The active ingredient in a antibiotic weighs 0.8 g. Convert this weight to mg.

0.8 × 1 000 = 800 mg

8 The current across a circuit is 0.0033 A. Convert this to mA.

0.0033 × 1 000 = 3.3 mA

9 Three boxes have heights: 0.42 m, 0.715 m and 0.505 m. How high are they when stacked on top of each other. Answer in m, then in mm.

Height = 0.42 + 0.715 + 0.505

Height = 1.64 m

1.64 × 1 000 = 1 640 mm

10 A standard door is 2.04 m high. Convert the height to mm.

2.04 × 1 000 = 2 040 mm

To convert ‘milli’ units to the base unit, divide by 1 000. Change these to ‘milli’ units.

Example Convert 960 mm to m

960 ÷ 1 000 = 0.96 m

11 3 000 mL = 3 L

12 28 900 mg = 28.9 g

13 31 430 mm = 31.43 m

14 4 620 mL = 4.62 L

15 86 mΩ = 0.086 Ω

*Ohms (Ω) is a measure of electrical resistance

16 A bottle of soft drink contains 1 250 mL of product. Convert this to L.

1 250 ÷ 1 000 = 1.25 L

17 Ceiling fan blades are usually 300 mm below the ceiling. Convert this to m.

300 ÷ 1 000 = 0.3 m

18 Two swimmers touch the wall 230 ms apart. Convert this time to seconds.

230 ÷ 1 000 = 0.23 s

19 A medicine has 308 mg, 182 mg, 780 mg and 2 750 mg of different active ingredients. Find the total weight in mg, then convert this to g.

Weight = 182 + 780 + 2 750

Weight = 3 712 mg

3 712 ÷ 1 000 = 3.712 g

Micro is 1 millionth. Convert ‘micro’ to ‘milli’ by dividing by 1 000. Hey … I know mu!

Example Convert 4 440 µm to mm

4 440 ÷ 1 000 = 4.44 mm

20 34 000 µm = 34 mm

21 87 µg = 0.087 mg

22 8 803 µs = 8.803 ms

23 40.5 µg = 0.0405 mg

24 A human hair thickness is 120 µm. Convert this to mm.

120 ÷ 1 000 = 0.12 mm

25 A dust mite is 275 µm in length. Convert this to mm.

275 ÷ 1 000 = 0.275 mm

To convert ‘milli’ units to ‘micro’ units, multiply by 1 000. Micrometres can also be called microns.

Example Convert 0.74 mm to µm

0.74 × 1 000 = 740 µm

26 8 mm = 8 000 µm

27 0.06 mg = 60 µg

28 0.0037 ms = 3.7 µs

29 5.7 mm = 5 700 µm

30 A red blood cell is 0.008 mm in diameter. Convert this to µm.

0.008 × 1 000 = 8 µm

31 The width of an eye of a needle is 1.2 mm Convert to µm.

1.2 × 1 000 = 1 200 µm

Numbers of Any Magnitude - Prefixes - Smaller Numbers (Homework Strip)

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Date Due: _____

Convert base units to milli by × 1 000

1 2 m = _____ mm

2 0.75 g = _____ mg

3 0.007 s = _____ ms

4 A standard door is 0.82 m Wide. Convert the width to mm.

_____ × 1 000 = _____

Convert milli to base units by ÷ 1 000

5 5 500 mm = _____ m

6 95 mg = _____ g

7 375 mL = _____ L

John can fill a bucket by pouring 5 1 250 mL bottles into it.

8 Calculate the volume of the 5 bottles in mL.

_____ × _____ = _____

9 Convert the volume to L.

_____ ÷ _____ = _____

Date Due: _____

Convert base units to milli by × 1 000

1 2 m = _____ mm

2 0.75 g = _____ mg

3 0.007 s = _____ ms

4 A standard door is 0.82 m Wide. Convert the width to mm.

_____ × 1 000 = _____

Convert milli to base units by ÷ 1 000

5 5 500 mm = _____ m

6 95 mg = _____ g

7 375 mL = _____ L

John can fill a bucket by pouring 5 1 250 mL bottles into it.

8 Calculate the volume of the 5 bottles in mL.

_____ × _____ = _____

9 Convert the volume to L.

_____ ÷ _____ = _____

Date Due: _____

Convert base units to milli by × 1 000

1 2 m = _____ mm

2 0.75 g = _____ mg

3 0.007 s = _____ ms

4 A standard door is 0.82 m Wide. Convert the width to mm.

_____ × 1 000 = _____

Convert milli to base units by ÷ 1 000

5 5 500 mm = _____ m

6 95 mg = _____ g

7 375 mL = _____ L

John can fill a bucket by pouring 5 1 250 mL bottles into it.

8 Calculate the volume of the 5 bottles in mL.

_____ × _____ = _____

9 Convert the volume to L.

_____ ÷ _____ = _____

Date Due: _____

Convert base units to milli by × 1 000

1 2 m = _____ mm

2 0.75 g = _____ mg

3 0.007 s = _____ ms

4 A standard door is 0.82 m Wide. Convert the width to mm.

_____ × 1 000 = _____

Convert milli to base units by ÷ 1 000

5 5 500 mm = _____ m

6 95 mg = _____ g

7 375 mL = _____ L

John can fill a bucket by pouring 5 1 250 mL bottles into it.

8 Calculate the volume of the 5 bottles in mL.

_____ × _____ = _____

9 Convert the volume to L.

_____ ÷ _____ = _____

Numbers of Any Magnitude - Prefixes - Smaller Numbers (H'work Strip)

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Date Due:

Convert base units to milli by × 1 000

1 2 m = 2 000 mm

2 0.75 g = 750 mg

3 0.007 s = 7 ms

4 A standard door is 0.82 m Wide. Convert the width to mm.

0.82 × 1 000 = 820 mm

Convert milli to base units by ÷ 1 000

5 5 500 mm = 5.5 m

6 95 mg = 0.095 g

7 375 mL = 0.375 L

John can fill a bucket by pouring 5 1 250 mL bottles into it.

8 Calculate the volume of the 5 bottles in mL.

1 250 × 5 = 6 250 mL

9 Convert the volume to L.

6 250 ÷ 1 000 = 6.25 L

Numbers of Any Magnitude - Error in Measurement

If I measure an object that is 19 cm wide, then it is 19 cm wide!!! Unfortunately when it comes to measurement that isn't the case. You could measure the width using different instruments for example, a laser device ….. a ruler. You would get different results. Also a friend could measure the same object using a different instrument, or even the same ruler, and get a different result. So measurement is never exact.

The Limit of Reading is the smallest unit to which you can measure. So if your ruler is marked in centimetres only, then the L.O.R. is cm. If you can measure to the nearest millimetre then the L.O.R. is mm.

When you read past the halfway point between the divisions on your ruler, you move up, just like rounding a decimal. You use 0.5 as a guide with rounding and you do the same with measurement errors.

Limit of Reading is the smallest unit you can measure on a device

Error = $\pm$ half of the Limit of Reading

Example

A vet's scales measures the weight of a cow as being 665 kg rounded to the nearest 5 kg.

The limit of reading is 5 kg. So that means the error is $\pm$ half that.

Error = $\pm$ 2.5 kg

In the above example, the error was $\pm$2.5 kg. You can calculate the limits on what the actual weight of the cow is, by applying the error.

Lower limit = 665 - 2.5 = 662.5 kg

Upper limit = 665 + 2.5 = 667.5 kg

Numbers of Any Magnitude - Error in Measurement (Sheet 1)

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The error in measurement is half of the limit of reading. Find the error in these measurements.

Example Find the error with measurements made to the nearest:

a) cm ±0.5 cm *c*) 10 kg ±5 kg

b) g ±0.5 g *d*) 5 s ±2.5 s

1 Nearest kg = ± _____ kg

2 Nearest 5 g = ± _____ g

3 Nearest 10 mL = ± _____ mL

4 Nearest 50 m = _____

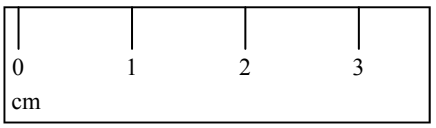
5 Nearest 25 g = _____

6 Nearest 10 cm = _____

7 Nearest 2 mm = _____

These two rulers have different limits of reading, write the limit of reading and the error.

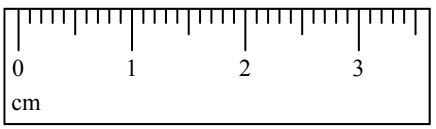
8



Limit of Reading: _____ To the nearest

Error: ± _____

9



Limit of Reading: _____ To the nearest

Error: ± _____

Find the upper and lower limits of these measurements.

Example A parcel weighs 425 g measured to the nearest 5 g.

Error: ± 2.5 g

Lower Limit : 425 - 2.5 = 422.5 g

Upper Limit : 425 + 2.5 = 427.5 g

10 A section of road to be repaired is measured as 870 m long, to the nearest 10 m.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

11 A window has a width of 176 cm, to the nearest cm.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

12 A horse weighs 485 kg, measured to the nearest 5 kg.

Error: _____

Lower Limit : _____

Upper Limit : _____

13 A measuring cylinder contains 115 mL measured to the nearest mL.

Error: _____

Lower Limit : _____

Upper Limit : _____

With decimals, you will add a decimal place.

Example Find the error with measurements made to the nearest:

a) 0.1 cm ±0.05 cm *c*) 0.01 m

b) 0.5 g ±0.25 g ±0.005 m

15 A basket ball hoop is 304.8 cm from the ground, measured to the nearest 0.1 cm.

Error: _____ 0.1 ÷ 2 Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

16 A solution is made by adding 5.72 g of copper sulphate, measured to the nearest 0.01 g.

Error: _____

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

17 A chocolate bar is manufactured to weigh 91.0 g, measured to the nearest 0.5 g.

Error: _____

Lower Limit : _____ = _____

Upper Limit : _____ = _____

Numbers of Any Magnitude - Further Errors Perimeter/Area (Sheet 2)

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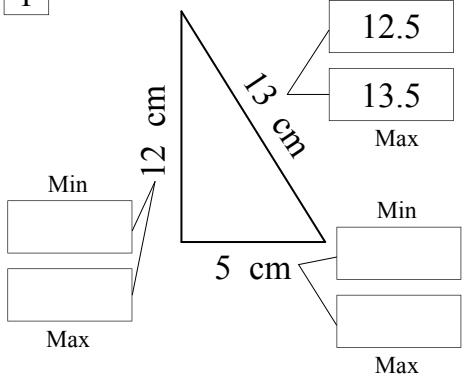
These triangles are measured to the nearest cm. Find the lower and upper limits for each side, then the minimum and maximum perimeter.

1



$P_{MIN} =$ ☐ + ☐ + ☐

Add all the minimum values

$P_{MIN} =$ ☐

Include the units

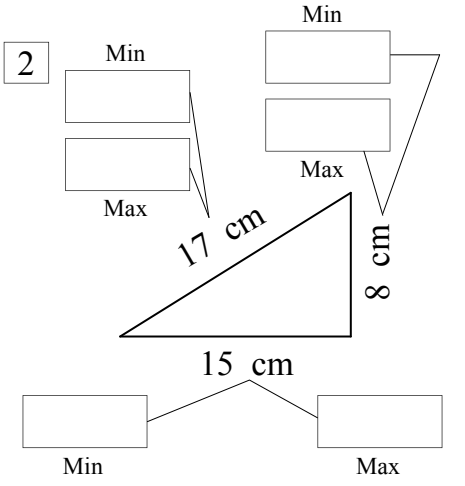
$P_{MAX} <$ ☐ + ☐ + ☐

Add all the maximum values

$P_{MAX} <$ ☐

Include the units

2



$P_{MIN} =$ ☐ + ☐ + ☐

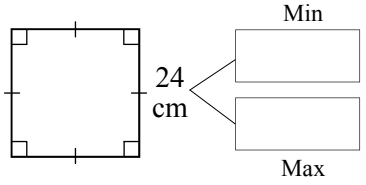
$P_{MIN} =$ ☐

$P_{MAX} <$ ☐ + ☐ + ☐

$P_{MAX} <$ ☐

Squares are easier, get the min and max side length and ×4! Calculate the minimum and maximum perimeter. Lengths are measured to the nearest cm

3



$P_{MIN} =$ ☐ × ☐

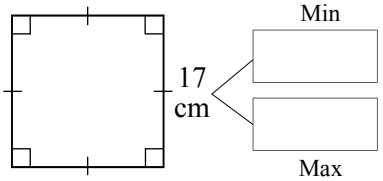
$P_{MIN} =$ ☐

Include the units

$P_{MAX} <$ ☐ × ☐

$P_{MAX} <$ ☐

4



$P_{MIN} =$ ☐

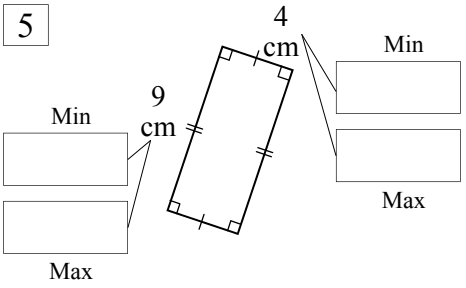
$P_{MIN} =$ ☐

$P_{MAX} <$ ☐

$P_{MAX} <$ ☐

Rectangles, × each side by 2

5



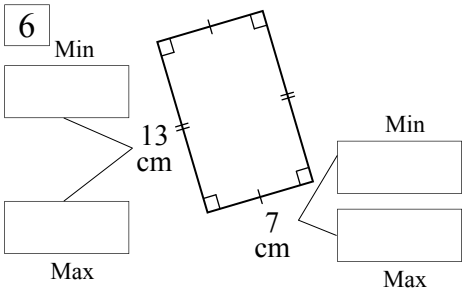
$P_{MIN} =$ ☐ × 2 + ☐ × 2

$P_{MIN} =$ ☐

$P_{MAX} <$ ☐ × ☐ + ☐ × ☐

$P_{MAX} <$ ☐

6



$P_{MIN} =$ ☐ × ☐ + ☐ × ☐

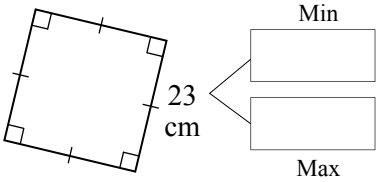
$P_{MIN} =$ ☐

$P_{MAX} <$ ☐

$P_{MAX} <$ ☐

Now find the area, by squaring or multiplying the two sides. All measurements are measured to the nearest cm.

7



$A_{MIN} =$ ☐ ^{2}

Area is found by squaring the min side length

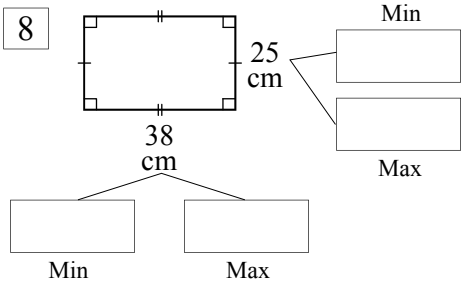
$A_{MIN} =$ ☐

Area units are cm^2, write square units

$A_{MAX} <$ ☐

$A_{MAX} <$ ☐

8



$A_{MIN} =$ ☐ × ☐

$A_{MIN} =$ ☐

$A_{MAX} =$ ☐ × ☐

$A_{MAX} =$ ☐

Numbers of Any Magnitude - Error in Measurement (Sheet 1)

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The error in measurement is half of the limit of reading. Find the error in these measurements.

Example Find the error with measurements made to the nearest:

a) cm ±0.5 cm *c*) 10 kg ±5 kg

b) g ±0.5 g *d*) 5 s ±2.5 s

1 Nearest kg = ± 0.5 kg

2 Nearest 5 g = ± 2.5 g

3 Nearest 10 mL = ± 5 mL

4 Nearest 50 m = ±25 m

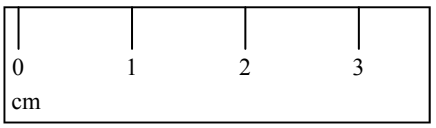
5 Nearest 25 g = ±12.5 g

6 Nearest 10 cm = ±5 cm

7 Nearest 2 mm = ±1 mm

These two rulers have different limits of reading, write the limit of reading and the error.

8

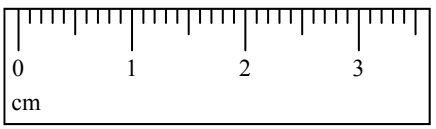


Limit of Reading: cm

To the nearest

Error: ± 0.5 cm

9



Limit of Reading: mm

To the nearest

Error: ± 0.5 mm

Find the upper and lower limits of these measurements.

Example A parcel weighs 425 g measured to the nearest 5 g.

Error: ± 2.5 g

Lower Limit : 425 - 2.5 = 422.5 g

Upper Limit : 425 + 2.5 = 427.5 g

10 A section of road to be repaired is measured as 870 m long, to the nearest 10 m.

Error: ± 5 m

Use '±' and units

Lower Limit : 870 - 5 = 865 m

Upper Limit : 870 + 5 = 875 m

11 A window has a width of 176 cm, to the nearest cm.

Error: ± 0.5 cm

Use '±' and units

Lower Limit : 176 - 0.5 = 175.5 cm

Upper Limit : 176 + 0.5 = 176.5 cm

12 A horse weighs 485 kg, measured to the nearest 5 kg.

Error: ± 2.5 kg

Lower Limit : 485 - 2.5 = 482.5 kg

Upper Limit : 485 + 2.5 = 487.5 kg

13 A measuring cylinder contains 115 mL measured to the nearest mL.

Error: ± 0.5 mL

Lower Limit : 115 - 0.5 = 114.5 mL

Upper Limit : 115 + 0.5 = 115.5 mL

With decimals, you will add a decimal place.

Example Find the error with measurements made to the nearest:

a) 0.1 cm ±0.05 cm *c*) 0.01 m

b) 0.5 g ±0.25 g ±0.005 m

15 A basket ball hoop is 304.8 cm from the ground, measured to the nearest 0.1 cm.

Error: ± 0.05 cm ← 0.1 ÷ 2

Use '±' and units

Lower Limit : 304.8 - 0.05

= 304.75 cm

Upper Limit : 304.8 + 0.05

= 304.85 cm

16 A solution is made by adding 5.72 g of copper sulphate, measured to the nearest 0.01 g.

Error: ±0.005 g

Lower Limit : 5.72 - 0.005

= 5.715 g

Upper Limit : 5.72 + 0.005

= 5.725 g

17 A chocolate bar is manufactured to weigh 91.0 g, measured to the nearest 0.5 g.

Error: ± 0.25 g

Lower Limit : 91 - 0.25

= 90.75 g

Upper Limit : 91 + 0.25

= 91.25 g

Numbers of Any Magnitude - Further Errors Perimeter/Area (Sheet 2)

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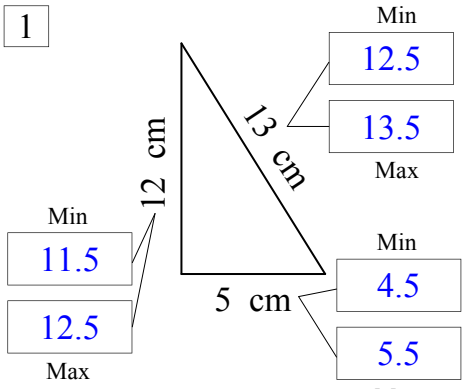
These triangles are measured to the nearest cm. Find the lower and upper limits for each side, then the minimum and maximum perimeter.

1



P_{MIN} = 4.5 + 11.5 + 12.5

Add all the minimum values

P_{MIN} = 28.5 cm

Include the units

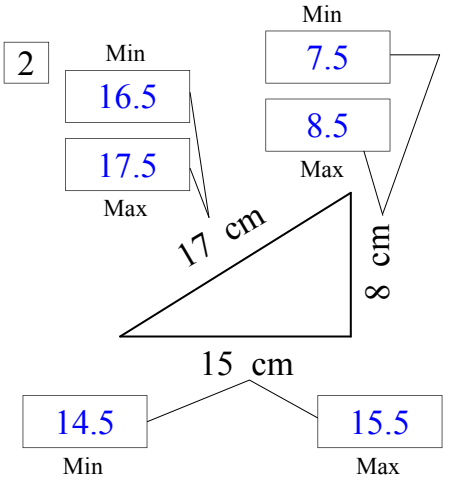
P_{MAX} < 5.5 + 12.5 + 13.5

Add all the maximum values

P_{MAX} < 31.5 cm

Include the units

2



P_{MIN} = 14.5 + 16.5 + 7.5

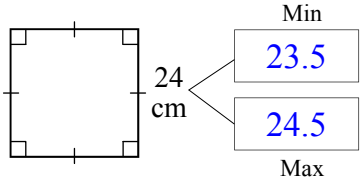
P_{MIN} = 38.5 cm

P_{MAX} < 15.5 + 17.5 + 8.5

P_{MAX} < 41.5 cm

Squares are easier, get the min and max side length and ×4! Calculate the minimum and maximum perimeter. Lengths are measured to the nearest cm

3



P_{MIN} = 23.5 × 4

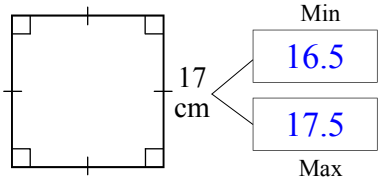
P_{MIN} = 94 cm

Include the units

P_{MAX} < 24.5 × 4

P_{MAX} < 98 cm

4



P_{MIN} = 16.5 × 4

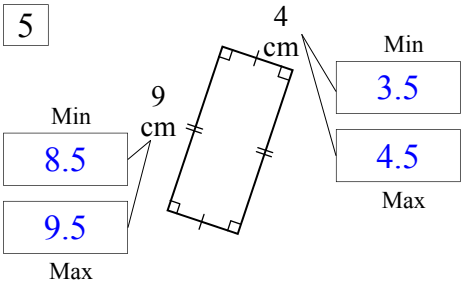
P_{MIN} = 66 cm

P_{MAX} < 17.5 × 4

P_{MAX} < 70 cm

Rectangles, × each side by 2

5



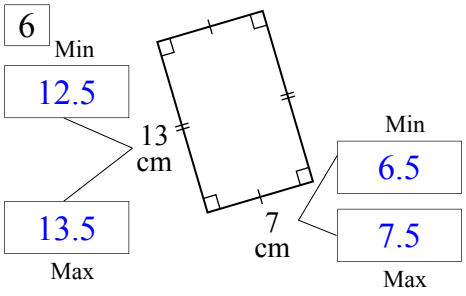
P_{MIN} = 8.5 × 2 + 3.5 × 2

P_{MIN} = 24 cm

P_{MAX} < 9.5 × 2 + 4.5 × 2

P_{MAX} < 28 cm

6



P_{MIN} = 12.5 × 2 + 6.5 × 2

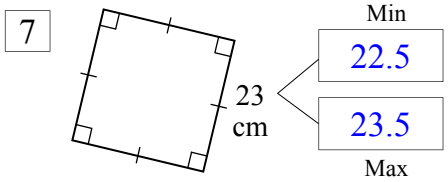
P_{MIN} = 38 cm

P_{MAX} < 13.5 × 2 + 7.5 × 2

P_{MAX} < 42 cm

Now find the area, by squaring or multiplying the two sides. All measurements are measured to the nearest cm.

7



A_{MIN} = 22.5^2

Area is found by squaring the min side length

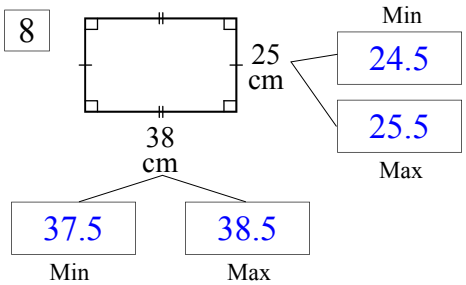
A_{MIN} = 506.25 cm^2

Area units are cm^2, write square units

A_{MAX} < 23.5^2

A_{MAX} < 552.25 cm^2

8



A_{MIN} = 24.5 × 37.5

A_{MIN} = 918.75 cm^2

A_{MAX} = 25.5 × 38.5

A_{MAX} = 981.75 cm^2

Numbers of Any Magnitude - Error in Measurement (Homework Strip)

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Date Due: _____

Find the error in measurements that are made to the nearest ….

1 metre = ± _____ m

2 10 g = ± _____ g

3 5 mL = ± _____

4 2 kg = _____

5 50 g = _____

Find the upper and lower limits.

6 A Boeing 747 is 71 m long measured to the nearest metre.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

7 The amount of petrol in a tank is 7 460 L measured to the nearest 20 L.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

Date Due: _____

Find the error in measurements that are made to the nearest ….

1 metre = ± _____ m

2 10 g = ± _____ g

3 5 mL = ± _____

4 2 kg = _____

5 50 g = _____

Find the upper and lower limits.

6 A Boeing 747 is 71 m long measured to the nearest metre.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

7 The amount of petrol in a tank is 7 460 L measured to the nearest 20 L.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

Date Due: _____

Find the error in measurements that are made to the nearest ….

1 metre = ± _____ m

2 10 g = ± _____ g

3 5 mL = ± _____

4 2 kg = _____

5 50 g = _____

Find the upper and lower limits.

6 A Boeing 747 is 71 m long measured to the nearest metre.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

7 The amount of petrol in a tank is 7 460 L measured to the nearest 20 L.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

Date Due: _____

Find the error in measurements that are made to the nearest ….

1 metre = ± _____ m

2 10 g = ± _____ g

3 5 mL = ± _____

4 2 kg = _____

5 50 g = _____

Find the upper and lower limits.

6 A Boeing 747 is 71 m long measured to the nearest metre.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

7 The amount of petrol in a tank is 7 460 L measured to the nearest 20 L.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

Numbers of Any Magnitude - Error in Measurement (Homework Strip)

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Date Due:

Find the error in measurements that are made to the nearest ….

1 metre = ± 0.5 m

2 10 g = ± 5 g

3 5 mL = ± 2.5 mL

4 2 kg = ± 1 kg

5 50 g = ± 25 g

Find the upper and lower limits.

6 A Boeing 747 is 71 m long measured to the nearest metre.

Error: ± 0.5 m Use '±' and units

Lower Limit: 71 - 0.5 = 70.5 m

Upper Limit: 71 + 0.5 = 71.5 m

7 The amount of petrol in a tank is 7 460 L measured to the nearest 20 L.

Error: ± 10 L Use '±' and units

Lower Limit: 7 460 - 10 = 7 450 L

Upper Limit: 7 460 + 10 = 7 470 L

Numbers of Any Magnitude - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Numbers of Any Magnitude - Find A Word

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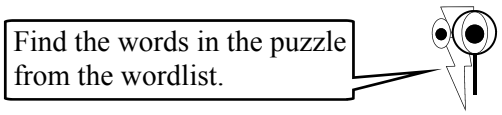

S	C	I	E	N	T	I	F	I	C	R	E	W	O	P
H	X	N	K	D	N	O	C	E	S	I	L	L	I	M
O	N	O	T	A	T	I	O	N	I	N	D	N	I	E
U	X	I	F	E	R	P	B	C	O	N	V	E	R	T
R	R	T	X	X	K	W	A	N	W	E	X	K	T	Y
K	O	A	N	K	D	E	S	O	M	G	T	E	H	B
E	U	M	X	A	W	E	E	I	X	A	Q	T	G	A
R	N	I	Y	E	C	K	C	T	M	T	M	Y	I	G
T	D	X	E	D	N	I	K	A	X	I	K	B	N	I
E	I	O	R	X	E	W	F	M	D	V	C	A	T	G
M	N	R	R	Q	L	A	M	I	C	E	D	R	R	M
O	G	P	O	B	Y	R	U	T	N	E	C	E	O	Q
N	Q	P	R	X	E	E	K	S	X	G	V	T	F	K
A	K	A	F	I	G	U	R	E	S	W	I	T	X	X
N	M	O	N	T	H	T	E	V	I	T	I	S	O	P

WORDLIST

CONVERT	INDEX	PREFIX	NEGATIVE	GIGABYTE
FIGURES	CENTURY	NANOMETRE	DAY	WEEK
FORTNIGHT	POSITIVE	ERROR	MILLISECOND	NOTATION
SCIENTIFIC	TERABYTE	MONTH	POWER	HOUR
MICRO	DECADE	BASE	DECIMAL	ROUNDING
	ESTIMATION	SIGNIFICANT	APPROXIMATION	

Numbers of Any Magnitude - Find A Word

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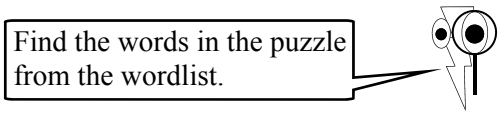

S	C	I	E	N	T	I	F	I	C	R	E	W	O	P
H	X	N	K	D	N	O	C	E	S	I	L	L	I	M
O	N	O	T	A	T	I	O	N	I	N	D	N	I	E
U	X	I	F	E	R	P	B	C	O	N	V	E	R	T
R	R	T	X	X	K	W	A	N	W	E	X	K	T	Y
K	O	A	N	K	D	E	S	O	M	G	T	E	H	B
E	U	M	X	A	W	E	E	I	X	A	Q	T	G	A
R	N	I	Y	E	C	K	C	T	M	T	M	Y	I	G
T	D	X	E	D	N	I	K	A	X	I	K	B	N	I
E	I	O	R	X	E	W	F	M	D	V	C	A	T	G
M	N	R	R	Q	L	A	M	I	C	E	D	R	R	M
O	G	P	O	B	Y	R	U	T	N	E	C	E	O	Q
N	Q	P	R	X	E	E	K	S	X	G	V	T	F	K
A	K	A	F	I	G	U	R	E	S	W	I	T	X	X
N	M	O	N	T	H	T	E	V	I	T	I	S	O	P

WORDLIST

CONVERT
FIGURES
FORTNIGHT
SCIENTIFIC
MICRO
INDEX
CENTURY
POSITIVE
TERABYTE
DECADE
ESTIMATION
PREFIX
NANOMETRE
ERROR
MONTH
BASE
SIGNIFICANT
NEGATIVE
DAY
MILLISECOND
POWER
DECIMAL
APPROXIMATION
GIGABYTE
WEEK
NOTATION
HOUR
ROUNDING

Numbers of Any Magnitude Review

This a review of the Numbers of Any Magnitude topic.

Numbers of any Magnitude - Topic Review

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Round these numbers to the nearest 10

1 75 = _____

2 423 = _____

3 5 796 = _____

Round these numbers to the nearest 100

4 386 = _____

5 67 890 = _____

6 23 = _____

Round these numbers to the nearest 1 000

7 13 170 = _____

8 199 727 = _____

9 473 = _____

Round these decimals to the nearest whole number

10 8.2 = _____ 11 5.7 = _____

12 4.49 = _____ 13 0.67 = _____

Perform the calculations and round to the nearest whole number

14 $148 \div 52$ = _____ (Whole Number)

15 78.6×93.9 = _____

Round these decimals to 1 decimal place (1 d.p.)

16 63.317 = _____

17 0.1178 = _____

18 0.66666666 = _____

19 9.98 = _____

Perform the calculations and round to the nearest decimal place

20 $27 \div 73$ = _____ (1 d.p.)

21 $\sqrt{157}$ = _____

22 $\frac{24^2 - 7^3}{9^2}$ = _____

Round these to 1, 2 and 3 decimal places

23 0.3275 _____ (1 d.p.) _____ (2 d.p.) _____ (3 d.p.)

24 2.9530 _____ (1 d.p.) _____ (2 d.p.) _____ (3 d.p.)

How many significant figures are in these numbers and decimals?

25 45 _____ sig.fig.

26 0.0003 _____ sig.fig.

27 0.07100 _____

28 7 760 _____

29 8 001 _____

30 8 100 _____

Round these to 1 significant figure

31 1 567 _____

32 63 _____

33 0.007539 _____

34 29 555 030 _____

35 0.0000662 _____

Perform the calculations and express the answer to 1 significant figure

36 147^2 = _____ (1 significant figure)

37 $8^9 \div 2^9$ = _____

38 $\frac{(26 + 37)}{(34 - 21)}$ = _____

Round these to 2 significant figures

39 275 _____

40 7 896 _____

41 0.01313 _____

What word describes a time period of

42 7 days _____

43 60 minutes _____

44 10 years _____

45 24 hours _____

46 1 000 years _____

Convert these time periods

47 3 weeks = _____ days

48 1 year = _____ weeks

49 4 days = _____ hours

50 ½ hour = _____ min

51 156 weeks = _____ years

52 4 min = _____ seconds

53 5 h = _____ min

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Change these times to minutes

54 3 hours = _____

55 1 h 19 min = _____

56 4 h 54 min = _____

Change these times to hours and minutes

57 420 min = _____

58 280 min = _____

59 703 min = _____

Change these times from seconds to milliseconds

60 3 sec = _____ msec

61 0.5 sec = _____

62 0.06 sec = _____

Change these times from milliseconds to seconds

63 2000 msec = _____ sec

64 30 msec = _____

65 120 msec = _____

Rewrite these numbers in scientific notation format

66 3 000 = $\square \times \square^{\square}$

67 4 000 000 = _____

68 37 000 = _____

69 80 600 000 = _____

70 64 700 = _____

71 194 000 = _____

Rewrite these decimals in scientific notation format

72 0.006 = $\square \times \square^{\square}$

73 0.00002 = _____

74 0.00037 = _____

75 0.0811 = _____

76 0.108 = _____

Convert these to numbers

77 5×10^4 = _____

78 3×10^2 = _____

79 4.6×10^3 = _____

80 1.02×10^6 = _____

Convert these to decimals

81 6×10^{-2} = _____

82 2×10^{-5} = _____

83 3.7×10^{-1} = _____

84 9.09×10^{-6} = _____

Rank these prefixes from largest (1) to smallest (6)

85

Mega ☐

Milli ☐

Tera ☐

Micro ☐

Giga ☐

Kilo ☐

Change these units to the specified unit

86 4 000 m = _____ [km]

87 3 L = _____ [mL]

88 45 mg = _____ [g]

89 0.7 km = _____ [m]

90 53 mm = _____ [cm]

Change these digital units to the unit specified

91 4 TB = _____ [GB]

92 5 000 kB = _____ [MB]

93 60 GB = _____ [MB]

94 0.3 GB = _____ [MB]

95 3.2 MB = _____ [kB]

What is the error in measurements rounded to the nearest…..

96 cm = ± _____

97 10 mL = _____

98 15 sec = _____

99 100 kg = _____

What is the error, lower and upper limits of this measurement

100 A car weighs 1 075 kg measured to the nearest 5 kg.

Error: _____

Lower Limit : _____

Upper Limit : _____

Numbers of any Magnitude - Topic Review

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Round these numbers to the nearest 10

1 75 = 80

2 423 = 420

3 5 796 = 5 800

Round these numbers to the nearest 100

4 386 = 400

5 67 890 = 67 900

6 23 = 0

Round these numbers to the nearest 1 000

7 13 170 = 13 000

8 199 727 = 200 000

9 473 = 0

Round these decimals to the nearest whole number

10 8.2 = 8 11 5.7 = 6

12 4.49 = 4 13 0.67 = 1

Perform the calculations and round to the nearest whole number

14 $148 \div 52$ = 3 (Whole Number)

15 78.6×93.9 = 7 381

Round these decimals to 1 decimal place (1 d.p.)

16 63.317 = 63.3

17 0.1178 = 0.1

18 0.66666666 = 0.7

19 9.98 = 10.0

Perform the calculations and round to the nearest decimal place

20 $27 \div 73$ = 0.4 (1 d.p.)

21 $\sqrt{157}$ = 12.5

22 $\frac{24^2 - 7^3}{9^2}$ = 2.9

Round these to 1, 2 and 3 decimal places

		1 d.p.	2 d.p.	3 d.p.
23	0.3275	0.3	0.33	0.328
24	2.9530	3.0	2.95	2.953

How many significant figures are in these numbers and decimals?

25 45 — 2 sig.fig.

26 0.0003 — 1 sig.fig.

27 0.07100 — 4 sig. fig.

28 7 760 — 3 or 4 sig. fig.

29 8 001 — 4 sig. fig.

30 8 100 — 2 or 3 or 4 sig. fig.

Round these to 1 significant figure

31 1 567 — 2 000

32 63 — 60

33 0.007539 — 0.008

34 29 555 030 — 30 000 000

35 0.0000662 — 0.00007

Perform the calculations and express the answer to 1 significant figure

36 147^2 = 20 000 (1 significant figure)

37 $8^9 \div 2^9$ = 300 000

38 $\frac{(26 + 37)}{(34 - 21)}$ = 5

Round these to 2 significant figures

39 275 — 280

40 7 896 — 7 900

41 0.01313 — 0.013

What word describes a time period of

42 7 days — week

43 60 minutes — hour

44 10 years — decade

45 24 hours — day

46 1 000 years — millennium

Convert these time periods

47 3 weeks = 21 days

48 1 year = 52 weeks

49 4 days = 96 hours

50 ½ hour = 30 min

51 156 weeks = 3 years

52 4 min = 240 seconds

53 5 h = 300 min

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Change these times to minutes

54 3 hours = 180 min

55 1 h 19 min = 79 min

56 4 h 54 min = 294 min

Change these times to hours and minutes

57 420 min = 7 h

58 280 min = 4 h 40 min

59 703 min = 11 h 43 min

Change these times from seconds to milliseconds

60 3 sec = 3 000 msec

61 0.5 sec = 500 msec

62 0.06 sec = 60 msec

Change these times from milliseconds to seconds

63 2000 msec = 2 sec

64 30 msec = 0.03 msec

65 120 msec = 0.12 msec

Rewrite these numbers in scientific notation format

66 3 000 = 3×10^{3}

67 4 000 000 = 4×10^{6}

68 37 000 = 3.7×10^{4}

69 80 600 000 = 8.06×10^{7}

70 64 700 = 6.47×10^{3}

71 194 000 = 1.94×10^{5}

Rewrite these decimals in scientific notation format

72 0.006 = 6×10^{-3}

73 0.00002 = 2×10^{-5}

74 0.00037 = 3.7×10^{-4}

75 0.0811 = 8.11×10^{-2}

76 0.108 = 1.08×10^{-1}

Convert these to numbers

77 5×10^{4} = 50 000

78 3×10^{2} = 300

79 4.6×10^{3} = 4 600

80 1.02×10^{6} = 1 020 000

Convert these to decimals

81 6×10^{-2} = 0.06

82 2×10^{-5} = 0.00002

83 3.7×10^{-1} = 0.37

84 9.09×10^{-6} = 0.00000909

Rank these prefixes from largest (1) to smallest (6)

85

Mega	3
Milli	5
Tera	1
Micro	6
Giga	2
Kilo	4

Change these units to the specified unit

86 4 000 m = 4 [km]

87 3 L = 3 000 [mL]

88 45 mg = 0.045 [g]

89 0.7 km = 700 [m]

90 53 mm = 5.3 [cm]

Change these digital units to the unit specified

91 4 TB = 4 000 [GB]

92 5 000 kB = 5 [MB]

93 60 GB = 60 000 [MB]

94 0.3 GB = 300 [MB]

95 3.2 MB = 3 200 [kB]

What is the error in measurements rounded to the nearest…..

96 cm = ± 0.5 cm

97 10 mL = ± 5 mL

98 15 sec = ± 7.5 sec

99 100 kg = ± 50 kg

What is the error, lower and upper limits of this measurement

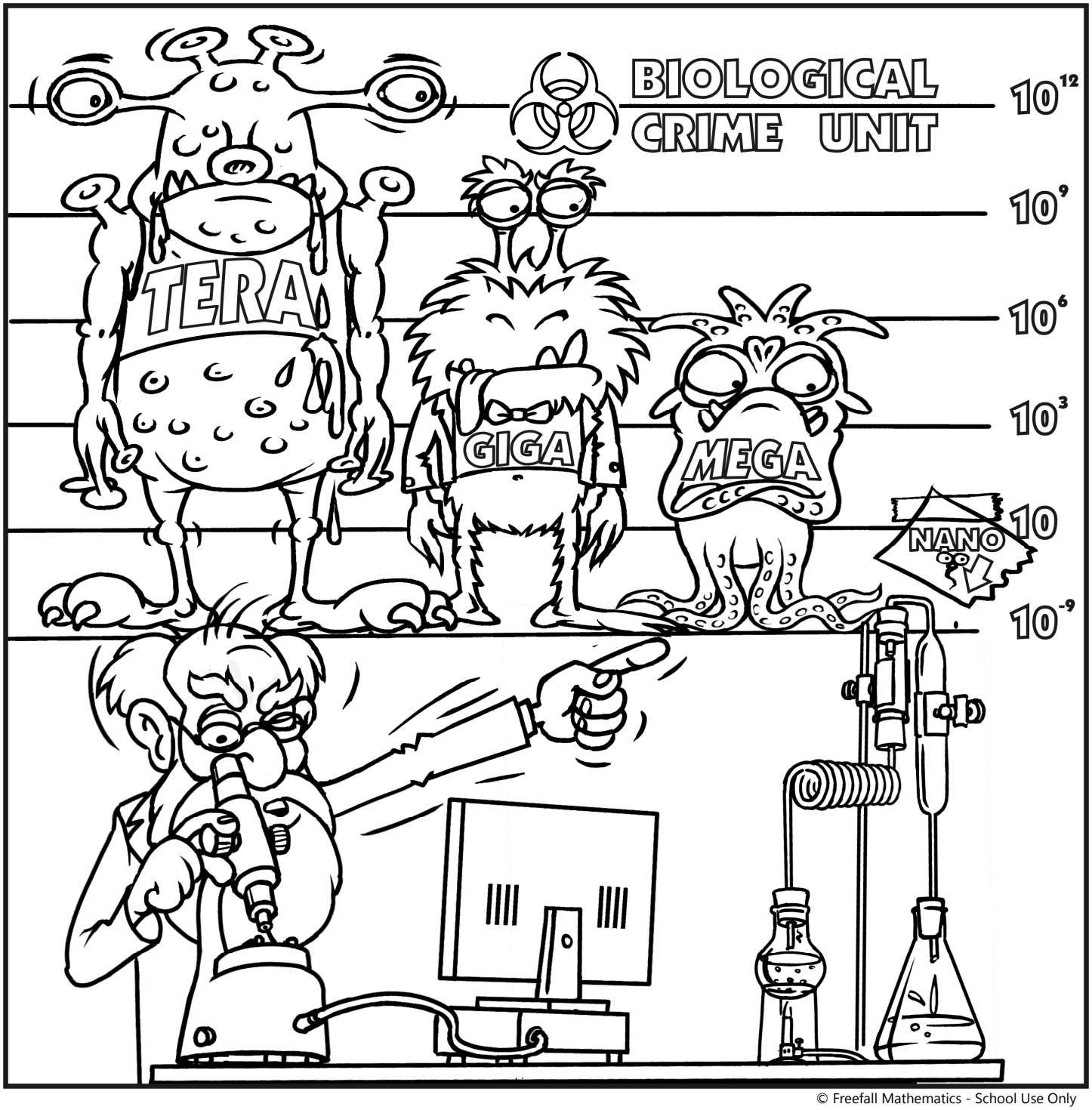
100 A car weighs 1 075 kg measured to the nearest 5 kg.

Error: ± 2.5 kg

Lower Limit : 1 075 - 2.5 = 1 072.5 kg

Upper Limit : 1 075 + 2.5 = 1 077.5 kg

Numbers of Any Magnitude



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Numbers of Any Magnitude - Prefix Table

Prefix	Scientific Notation		Numerical Value	Value in Words
Tera	1×10^{12}	LARGEST	1 000 000 000 000	1 trillion
Giga	1×10^{9}		1 000 000 000	1 billion
Mega	1×10^{6}		1 000 000	1 million
Kilo	1×10^{3}		1 000	1 thousand
Milli	1×10^{-3}		$\frac{1}{1\ 000}$	1 thousandth
Micro	1×10^{-6}		$\frac{1}{1\ 000\ 000}$	1 millionth
Nano	1×10^{-9}	SMALLEST	$\frac{1}{1\ 000\ 000\ 000}$	1 billionth
Pico	1×10^{-12}		$\frac{1}{1\ 000\ 000\ 000\ 000}$	1 trillionth

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Numbers of Any Magnitude - Rounding (Nearest 10)

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Round these numbers to the nearest 10. So if the last digit is 5 or more, round them up.



Examples

47 = 50 273 = 270

1 17 = 2 82 =

3 38 = 4 24 =

5 67 = 6 11 =

7 6 = 8 4 =

Now larger numbers

9 167 =

10 204 =

11 477 =

12 597 =

13 2 036 =

14 1 997 =

15 3 598 =



Calculate these and write the answer, then the answer rounded to the nearest 10.

16 47×28 = Actual

= Rounded

17 $53^2 \times 87$ =

=

18 $36^2 \div (9 \times 6)$ =

=

19 $(9 + 13)^2 \times (8 + 9)^2$

= =

Use the fraction key or bracket both the numerator (top) & denominator

20 $\dfrac{(15 \times 23)^3}{5^2 \times 3}$ =

Bracket denominator

=

Use the $x^{\square}$ key

21 $\dfrac{9^5 \times 4^3}{2^4 \times 3^3}$ =

Use the $x^{\square}$ key =

22 $\dfrac{5^4 \times 6^3}{10^3 \times 3^3}$ = =

Its investigation time. Perform the same operation twice, but the first is rounded after the calculation, the second, before.



23 127×58 = Actual

= Rounded

24 127×58 = 130 × 60 Round the numbers

= Actual

25 232×67 =

= Rounded

26 232×67 = × Round the numbers

=

27 $4\,096 \times 156$ = Actual

= Rounded

28 $4\,096 \times 156$

= ×

=

29 $2\,322 \times 4$ = Actual

= Rounded

30 $2\,322 \times 4$

= ×

=



This time its addition. Round at the end and compare it with rounding at the start.

31 $301 + 494 + 727 + 83$

= Actual = Rounded

32 $309 + 496 + 727 + 83$

= + + +

Round all the numbers to the nearest 10

=

33 $777 + 438 + 219 + 867$

= Actual = Rounded

34 $777 + 438 + 219 + 867$

= + + +

=

Numbers of Any Magnitude - Rounding (Nearest 100/1 000)

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Round these numbers to the nearest 100. If the last two digits are 50 or more, round up

Examples

326 = 300 4 976 = 5 000

1 435 =

2 153 =

3 18 = 4 61 =

5 888 =

6 950 =

Now larger numbers

7 23 478 =

8 5 809 =

9 179 981 =

10 197 090 =

Perform these calculations and give the answer, then the answer rounded to nearest 100

11 76×38 = Actual

= Rounded

12 $24^2 \times 14^2$ =

=

13 $90^2 \div 15^2$ =

=

14 $32^3 + 8^3$ =

=

Round these numbers to the nearest 1 000. If the last three digits are 500 or more, round up

Examples

577 = 1 000 403 = 0

15 1 875 =

16 7 077 =

17 8 498 =

18 3 702 =

19 5 555 =

Now larger numbers

20 87 206 =

21 109 703 =

22 4 528 119 =

23 2 149 944 =

24 7 999 566 =

Calculate the answer then the rounded answer to the nearest thousand.

Use the power key $x^{\blacksquare}$ to answer these

25 $3^6 \times 2^9$ =

Actual

= Rounded to nearest 1 000

26 $5^4 + 6^{10}$ =

=

27 $8^{11} \div 2^6$ =

=

Match answers with questions. The answers have been rounded to the nearest 1 000.

28 7^4 • • 16 000

29 5^6 • • 4 000

30 6^6 • • 66 000

31 4^8 • • 20 000

32 3^9 • • 2 000

33 9^4 • • 47 000

34 4^6 • • 7 000

Perform the calculation and write the answer three times. Rounded to 10, 100 then 1 000

35 $927 \times 43 \times 8^4$ Unrounded Answer Not Required

=

Round to the Nearest 10

=

Round to the Nearest 100

=

Round to the Nearest 1 000

36 $17 \times 11 \times 13 \times 7^3$

=

=

=

37 $3^6 \times 13^4 \div (43 - 16)$

=

=

=

Numbers of Any Magnitude - Rounding Decimals - 0 & 1 d.p. (Sheet 1)

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Round these decimals to the nearest whole number. Look at the first decimal digit.



Examples

$3.8 =$ 4 (0.8 more than 0.5)

$7.37 =$ 7 (0.3 less than 0.5)

1 $1.7 =$ ☐

2 $8.3 =$ ☐

3 $6.2 =$ ☐

4 $5.8 =$ ☐

5 $0.6 =$ ☐

6 $0.2 =$ ☐

These have more than 1 decimal place

7 $11.53 =$ ☐ ← Whole number answer

8 $2.07 =$ ☐

9 $15.72 =$ ☐

10 $23.6081 =$ ☐

11 $49.1189 =$ ☐

12 $7.067 =$ ☐



Find these, write the answer, then write the answer rounded to the nearest whole number.

13 $3.7 \times 8.4 =$ ☐ Actual

Whole number answer → $=$ ☐ Rounded

14 $5.16 \times 13.7 =$ ☐ Actual

$=$ ☐ Rounded

15 $7.59 \times 37.8 =$ ☐

$=$ ☐

Its estimation time. Change these to whole numbers to estimate the answer.



Example 3.8×5.2

$\approx 4 \times 5 = 20$

16 $2.9 \times 6.7 \approx$ ☐ (Round 2.9) $\times$ ☐ (Round 6.7) $=$ ☐

17 $5.3 \times 9.8 \approx$ ☐ $\times$ ☐ $=$ ☐

18 $7.8 \div 1.9 \approx$ ☐ $\div$ ☐ $=$ ☐

19 $8.3 \times 4.1 \approx$ ☐ $\times$ ☐ $=$ ☐

20 $9.2 \div 2.7 \approx$ ☐ $\div$ ☐ $=$ ☐



Round these decimals to 1 decimal place. (1 d.p.)

Examples

$3.82 =$ 3.8 (Less than 5)

$7.97 =$ 8.0 (More than 5)

21 $3.16 =$ ☐.☐

22 $8.52 =$ ☐.☐

23 $16.78 =$ ☐

24 $-0.61 =$ ☐

25 $4.95 =$ ☐

Match the decimal with its answer when rounded to 1 d.p.



Use a Ruler!

26	5.26 ●	●6.3
27	6.25 ●	●6.5
28	2.56 ●	●5.6
29	5.62 ●	●5.3
30	6.52 ●	●2.6



These decimals have more decimal places. Round them to 1 decimal place. (1 d.p.)

Examples

Ignore the 5 & 2 (Less than 5): $3.8352 =$ 3.8

Ignore the 2 (More than 5): $7.362 =$ 7.4

Look at the arrowed digit for rounding

31 $9.0671 =$ ☐.☐

32 $0.7155 =$ ☐

33 $48.96051 =$ ☐

34 $145.27 =$ ☐

35 $59.962 =$ ☐

36 $0.3815 =$ ☐

37 $-12.629 =$ ☐

38 $8.0188 =$ ☐

39 $152.3826 =$ ☐

40 $45.56621 =$ ☐

Complete the calculations and give the answer as a whole number and a decimal to 1 d.p.

41 $2.3^3 \times 0.7^2 + 1.8$

$=$ ☐ To 1 d.p. $=$ ☐ Whole Number

42 $1.13^4 \times 3.57^3$

$=$ ☐ To 1 d.p. $=$ ☐ Whole Number

43 $\dfrac{8^2 + 9^3}{4^6 - 2^7} + 0.8$

$=$ ☐ To 1 d.p. $=$ ☐ Whole Number

Numbers of Any Magnitude - Rounding Decimals - 2 & 3 d.p. (Sheet 2)

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Round these decimals to 2 decimal places. Look at the third decimal digit to decide.

Examples

$5.783 = 5.78$ (3 is less than 5)

$0.7283 = 0.73$ (8 is more than 5)

1 $2.916 =$ ☐.☐☐

2 $42.053 =$ ☐

3 $-8.198 =$ ☐

4 $207.371 =$ ☐

5 $1.897 =$ ☐

When a decimal has less than 2 decimal places, add zeros. Write these to 2 d.p.

Examples

$2.6 = 2.60$ (Add a zero)

$14 = 14.00$ (Add two zeros)

6 $8 =$ ☐.☐☐

7 $4.3 =$ ☐

8 $113.7 =$ ☐

9 $344 =$ ☐

These have more decimal places. Use the arrow to help to get you rounding to 2 d.p.

10 $0.1642 =$ ☐.☐☐

11 $0.3884 =$ ☐

12 $17.744823 =$ ☐

13 $5.096273 =$ ☐

14 $83.25199 =$ ☐

15 $-11.99716 =$ ☐

Round these to 2 d.p.

16 $61.37285 =$ ☐

17 $0.115099 =$ ☐

18 $4.797003 =$ ☐

19 $97.98221 =$ ☐

20 $19.99534 =$ ☐

Calculate these and write the answer to 2 d.p.

21 $0.9^2 \times 9.4 =$ ☐ Rounded to 2 d.p.

22 $1.75^3 + 5.2^2 =$ ☐

23 $\dfrac{31.9 + 54.2}{4.7^2} =$ ☐

24 $(-0.762)^5 =$ ☐

Round these to 3 d.p. Use the arrow to help you get started.

Examples

$0.78367 = 0.784$ (6 is more than 5)

$0.21842 = 0.218$ (4 is less than 5)

25 $0.778321 =$ ☐.☐☐☐

26 $6.01596 =$ ☐

27 $-38.09951 =$ ☐

28 $0.047177 =$ ☐

29 $3.141592654 =$ ☐

30 $1.414213562 =$ ☐

Calculate these and write the answer to 3 d.p.

31 $\sqrt{15} =$ ☐.☐☐☐ Rounded to 3 d.p.

32 $\sqrt{624} =$ ☐

33 $\sqrt{5.2^2 - 3.1^2} =$ ☐

34 $\sqrt{24^2 + 17.7^2} =$ ☐

35 $\dfrac{14^3 - 11^3}{5^2} =$ ☐

Match the left hand side with its answer on the right, when rounded to 3 d.p.

Use a Ruler!

36	0.2	0.002
37	0.12^3	0.003
38	$\dfrac{3}{1\,004}$	0.001
39	$3^2 \div 9^4$	0.200
40	$\sqrt{0.08}$	0.283

Round these to the nearest whole number, 1 d.p., 2 d.p. and 3 d.p.

41 0.7

Whole	1 d.p.	2 d.p.	3 d.p.
☐	☐	☐	☐

42 4.8168

Whole	1 d.p.	2 d.p.	3 d.p.
☐	☐	☐	☐

43 0.0957

Whole	1 d.p.	2 d.p.	3 d.p.
☐	☐	☐	☐

44 9.9997

Whole	1 d.p.	2 d.p.	3 d.p.
☐	☐	☐	☐

Numbers of Any Magnitude - Rounding Decimals 0-3 d.p. (Sheet 3)

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Round these numbers to the nearest tenth (1 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.2

1. 0.87
2. 0.43
3. 0.90347
4. 0.05956
5. 18.374
6. 22.128
7. 67
8. 56.942
9. 129.988
10. 99.953

Round these numbers to the nearest hundredth (2 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.16

11. 0.278
12. 0.805
13. 0.4246
14. 0.9793
15. 2.2183
16. 73.2
17. 4.95829
18. 9.99811
19. 67.9942
20. 83
21. 99.9961

Round these numbers to the nearest thousandth (3 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.163

22. 2.7674
23. 0.8436
24. 8.0302
25. 6.9969
26. 6.9996
27. 0.7
28. 0.70
29. 0.707
30. 4
31. 9.9999

Round these numbers to the nearest unit (0 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1

32. 4.7
33. 9.3
34. 18.78
35. 249.499
36. 7 101.835
37. 83.5004
38. 999.278
39. 999.872
40. 0.808
41. 0.080
42. 9 999.5

Round these numbers to the decimal place given in the brackets

43. 29.6751 [2]
44. 1 361.54 [1]
45. 0.4296 [0]
46. 9.1 [3]
47. 19.9971 [2]
48. 6.0606 [3]
49. 86 [2]
50. 56.0438 [1]
51. 11.4099 [0]
52. 8.8997 [3]
53. 5.9949 [2]

Round each decimal to 1, 2 and 3 decimal places

54. 0.1919 [1] _____ [2] _____ [3] _____
55. 3.8225 [1] _____ [2] _____ [3] _____
56. 26.9527 [1] _____ [2] _____ [3] _____
57. 0.3235 [1] _____ [2] _____ [3] _____
58. 9.9994 [1] _____ [2] _____ [3] _____

Numbers of Any Magnitude - Counting Significant Figures (Sheet 1)

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Break these numbers up into 'significant' and 'possible' significant figures.

Examples 0's at end are possible

2 300 = 2 3 0 0

Significant Possible

0's between non-zero digits are significant

2 003 = 2 0 0 3

Significant

Combination of the two

2 030 = 2 0 3 0

Significant Possible

1 460 = ☐☐☐

2 5 000 = ☐☐☐☐

3 4 705 = ☐☐☐☐

4 71 070 = ☐☐☐☐☐

5 93 600 = ☐☐☐☐☐

6 35 206 000

☐☐☐☐☐☐☐☐

7 409 100

☐☐☐☐☐☐

8 1 480 050

☐☐☐☐☐☐☐

Now count the significant figures. Use the word 'or' between the possible sig figs.

Examples

2 300 = 2 3 0 0

2 3 4

There are 2 or 3 or 4 sig figs.

2 003 = 2 0 0 3

4

There are 4 sig figs.

2 030 = 2 0 3 0

3 4

There are 3 or 4 sig figs.

9 8 030 = ☐☐☐☐ (☐ ☐)

There are ☐ or ☐ sig figs.

10 3 400 = ☐☐☐☐ (☐ ☐ ☐)

There are ☐ or ☐ or ☐ sig figs.

11 5 043 = ☐☐☐☐ (☐)

There are ☐ sig figs.

12 70 900

☐☐☐☐☐

There are ☐ sig figs

13 8 619 000

☐☐☐☐☐☐☐

There are ☐ sig figs

14 406 100

☐☐☐☐☐☐

There are ☐ sig figs

15 9 005 020

☐☐☐☐☐☐☐

There are ☐ sig figs

16 120 000

☐☐☐☐☐☐

There are ☐ sig figs

17 27 000 050

☐☐☐☐☐☐☐☐

There are ☐ sig figs

If a decimal is larger than 1, all the digits in the decimal are significant. Count those sig figs.

Examples

23.00 There are 4 sig figs.

2.0030 There are 5 sig figs.

2000.30 There are 6 sig figs.

18 1.0 There are ☐ sig figs

19 5.100 There are ☐ sig figs

20 75.03 There are ☐ sig figs

21 195.00 There are ☐ sig figs

Numbers of Any Magnitude - Counting Significant Figures (Sheet 2)

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Break these decimals up into 'ignore' and 'significant' figures.

Examples Ignore all leading 0's

$0.007 =$ 0 . 0 0 7

Ignore Significant

0's between non-zero digits are significant

$0.4003 =$ 0 . 4 0 0 3

Ignore Significant

0's to the right are also significant

$0.0600 =$ 0 . 0 6 0 0

Ignore Significant

1 $0.88 =$.

2 $0.027 =$.

3 $0.050 =$.

4 0.010200

.

5 0.000775

.

6 0.009003

.

7 0.50010

.

8 $0.400 =$.

Now underline and then count the significant figures. Ignore all the leading zeros.

Examples

$0.007 =$ 0 . 0 0 7

There is 1 sig fig. 1

$0.4003 =$ 0 . 4 0 0 3

4

There are 4 sig figs.

$0.600 =$ 0 . 6 0 0

3

There are 3 sig figs.

9 $0.020 =$.

There are sig figs.

10 $0.170 =$.

There are sig figs.

11 $0.003 =$.

There is sig fig.

12 $0.900 =$.

13 0.027300

.

14 0.006038

.

Perform these calculations and then give the number of sig figs that are in the answer.

15 $0.32^2 =$ Answer Sig Fig

16 $40.04 \div 50 =$ Answer

Sig Fig

17 $0.02^2 \div 0.5^3$

$=$

18 0.16×0.48^2

$=$

19 $0.2^7 =$

Convert these fractions to decimals and give the number of significant figures in the answer.

20 $\frac{8}{125} =$

21 $\frac{401}{5\,000} =$

22 $\frac{9}{12\,500} =$

23 $\frac{63}{900} =$

24 $\frac{5\,253}{12\,500} =$

25 $\frac{6}{48\,000} =$

26 $\frac{3\,113}{50\,000} =$

Significant Figures and Rounding - 1 & 2 Sig.Fig. (Sheet 1)

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Round these numbers/decimals to 1 significant figure.

Examples $473 = 500$

7 more than 5 - Round up

$2\,372 = 2\,000$

3 less than 5 - Round down

1 $46 =$ ____ 2 $1.9 =$ ____

3 $5\,800 =$ ____

4 $3\,193 =$ ____

5 $81 =$ ____ 6 $6.6 =$ ____

7 $97\,100 =$ ____

8 $637\,522 =$ ____

Calculate these and give two answers. The actual answer, then rounded to 1 sig. fig.

9 $34^2 + 19^2 =$ ____ Actual

$=$ ____ 1 sig. fig.

10 $31^4 =$ ____ Actual

$=$ ____ 1 sig. fig.

11 $\sqrt{28\,224} =$ ____ Actual $=$ ____ 1 sig. fig.

12 $\dfrac{89 \times 116}{20} =$ ____ Actual

$=$ ____ 1 sig. fig.

13 $227^2 =$ ____ Actual

$=$ ____ 1 sig. fig.

Calculate the area of the rectangles and give the answer to 1 significant fig.

14

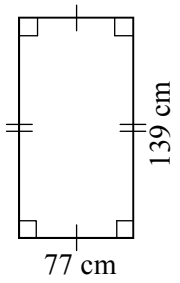
$A = lb$

$=$ ____ $\times$ ____

Multiply sides, in any order

$A =$ ____ cm^2

Answer to 1 sig fig



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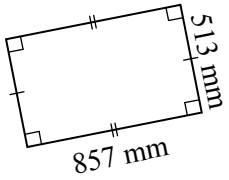


$A = lb$

$=$

$A =$ ____ mm^2

Answer to 1 sig fig

Write these to 2 significant figures.

Examples $473 = 470$

3 less than 5 - Round down

$2\,372 = 2\,400$

7 more than 5 - Round up

16 $386 =$ ____

17 $5 =$ ____ 18 $9 =$ ____

19 $25\,677 =$ ____

20 $507\,311 =$ ____

21 $89.7 =$ ____

22 $472.8 =$ ____

23 $3\,777\,529 =$ ____

24 $70\,813 =$ ____

Perform the calculations and answer to 1 and 2 sig. fig.

25 $516 \times 85 =$ ____ Actual

____ 1 sig. fig. ____ 2 sig. fig.

26 $84^3 =$ ____ Actual

____ 1 sig. fig. ____ 2 sig. fig.

27 $15^3 + 37^4 =$ ____ Actual

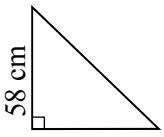
____ 1 sig. fig. ____ 2 sig. fig.

28 $\sqrt{59\,049} =$ ____ Actual

____ 1 sig. fig. ____ 2 sig. fig.

Find the area of these triangles answering to 2 sig. fig.

29



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ ____ $\times$ ____

0 . 5 × 6 3 × 5 8 =

$A =$ ____ cm^2

2 sig. fig.

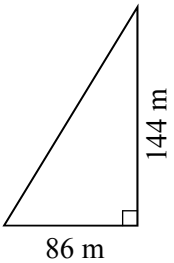
30

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times$ ____ $\times$ ____

$A =$ ____ m^2

2 sig. fig.



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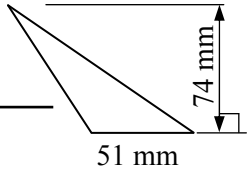

2 sig. fig.

Significant Figures and Rounding Decimals - 1 & 2 Sig.Fig. (Sheet 2)

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Round these decimals to 1 significant figure.

Examples $0.0382 =$ 0.04

8 more than 5 - Round up

$0.0000539 =$ 0.00005

3 less than 5 - Round down

1 $0.0647 =$ ____

2 $0.00231 =$ ____

3 $0.000037 =$ ____

4 $0.00095 =$ ____

*careful

5 $0.849 =$ ____

6 $0.972 =$ ____

7 $0.000033 =$ ____

8 $0.00451388 =$ ____

Perform the calculation and write the answer, then the answer rounded to 1 significant figure.

9 $8 \div 6\,400$

____ Decimal - Actual

$=$ ____ 1 sig. fig.

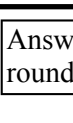
10 $\frac{125}{500\,000} =$ ____ Decimal - Actual

Use the S⇔D key $=$ ____ 1 sig. fig.

11 $\frac{21}{3\,360\,000}$

$=$ ____ Decimal - Actual

$=$ ____ 1 sig. fig.



Answer these as decimals and round to 1 significant figure.

12 $3 \div 1\,290 =$ ____ 1 sig. fig.

13 $\frac{83}{938\,508} =$ ____

14 $9 \div 9\,396 =$ ____

*careful

15 $37 \div 8\,771 =$ ____

16 $\frac{116}{117\,229} =$ ____

17 $\frac{3}{84\,080} =$ ____

18 $2^3 \div 2^{17} =$ ____



Round these decimals to 2 significant figures.

Examples $0.0382 =$ 0.038

2 less than 5 - Round down

$0.0000539 =$ 0.000054

9 more than 5 - Round up

19 $0.4531 =$ ____

20 $0.01039 =$ ____

21 $0.000317 =$ ____

22 $0.000997 =$ ____

*careful

23 $0.02007 =$ ____

24 $0.975 =$ ____

25 $0.068033 =$ ____

26 $0.000566 =$ ____



Calculate these, write the full answer first, then rounded to 2 significant figures.

27 $4 \div 3\,200 =$ ____ Decimal - Actual

$=$ ____ 2 sig. fig.

28 $141 \div 200\,000$

$=$ ____ Decimal - Actual $=$ ____ 2 sig. fig.

29 $281 \div 3\,125$

$=$ ____ Decimal - Actual $=$ ____ 2 sig. fig.



Calculate and write as decimals to both 1 & 2 significant figures.

30 $12 \div 4\,133$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

31 $2^7 \div 7^7$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

32 $\frac{499}{50\,000}$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

33 $5^2 \div 14^5$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

34 $\frac{179}{2\,500}$

$=$ ____ 1 sig. fig. $=$ ____ 2 sig. fig.

Numbers of Any Magnitude - Units of Time/Conversions (Sheet 1)

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Use each of these words once to answer the questions. What is the most appropriate time unit to use to measure …….

Wordlist:	seconds	days
years	hours	decade
centuries	millennia	milliseconds
minutes	weeks	Cross them out

1 A person's age _____

2 The time that humans have lived in cities _____

3 A 10 year period _____

4 The time to cook a meal _____

5 The time taken for a pierced balloon to 'pop' _____

6 The time you are allowed to borrow a book from the library _____

7 The amount of time that you sleep daily _____

8 The time a tourist might spend visiting a foreign city _____

9 The time humans have used coal for heating. _____

10 The time taken to cross the street. _____

Convert these larger time periods to days. One year is 365 days, ignoring leap years.

11 2 years = ____ × 365

= ____ days

12 1 decade = ____ × 365

= ____ days

13 17 years = _____

= _____

14 3 centuries = _____

= _____

Convert these times to times measured in hours. 1 day is 24 hours in duration.

15 5 days = ____ × 24

= ____ hours

16 3 weeks = ____ × ____ × ____

= ____ hours

17 July = _____

= _____

18 1 year = _____

= _____

Match the right side with the left.

19

1 year ●	● 3 days
3 weeks ●	● 52 weeks
72 hours ●	● 576 hours
24 days ●	● 21 days

Convert these units of time to the different unit on the right hand side.

20 2 weeks = ____ days

21 1 year = ____ weeks

22 3 days = ____ hours

23 ½ hour = ____ min

24 104 weeks = ____ years

25 6 min = ____ seconds

26 2 h = ____ min

27 1 leap year = ____ days

28 4 weeks = ____ days

29 3 decades = ____ years

30 9 min = ____ seconds

31 24 months = ____ years

32 4 centuries = ____ decades

33 ½ year = ____ months

34 5 days = ____ hours

35 2 fortnights = ____ weeks

36 300 s = ____ min

37 156 weeks = ____ years

38 September = ____ days

39 2 millennia = ____ years

40 March + May = ____ days

Numbers of Any Magnitude - Units of Time/Conversions (Sheet 2)

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Rewrite the calculator displays in h and min.

1  ____ h ____ m

Hours Minutes

2  ____ h ____ m

3  ____ h ____ m

4  ____ h ____ m

Convert the following times from minutes to hours and minutes.

Example

Convert 211 minutes to hours/minutes.

Calc: 0 ° ’ ’’ 2 1 1 ° ’ ’’ =

Display: 

3 h 31 min

5 193 min ____ h ____ min

6 185 min ____ h ____ min

7 247 min ____ h ____ min

8 744 min _____

9 490 min _____

10 333 min _____

11 296 min _____

12 575 min _____

13 1 541 min _____

Now backwards. Change times in hours/minutes to minutes.

Example

Convert 4 h 19 min to minutes.

Calc: 4 ° ’ ’’ 1 9 ° ’ ’’ × 6 0 =

Display: 

259 min

14 3 h 38 min _____ min

15 1 h 43 min _____ min

16 2 h 27 min _____ min

17 6 h 6 min _____ min

18 7 h 25 min _____

19 1 h 19 min _____

20 4 h 53 min _____

21 5 h _____

22 2 h 48 min _____

Now use ×, +, ÷ and - with these time operations.

Example 47 min × 4

Calc: 0 ° ’ ’’ 4 7 ° ’ ’’ × 4 =

If no hours, enter ‘0’ in hours

Display: 

3 h 8 min

23 8 × 23 min ____ h ____ min

24 7 × 1 h 16 min

____ h ____ min

25 19 h 57 min ÷ 9

____ h ____ min

26 2 h 38 min + 1 h 43 min

____ h ____ min

27 5 h 19 min - 2 h 56 min

____ h ____ min

28 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min

____ h ____ min

There are 1 000 milliseconds in a second. Change these times to milliseconds by: × by 1 000.

29 2 sec _____ msec

30 3.2 sec _____ msec

31 0.08 sec _____

32 0.076 sec _____

33 4.4 sec _____

34 0.55 sec _____

35 0.00007 sec _____

Change these times from milliseconds to seconds by ÷ by 1 000.

36 700 msec _____ sec

37 95 msec _____ sec

38 7 100 msec _____

39 0.2 msec _____

40 40 msec _____

41 999 msec _____

42 635.7 msec _____

Scientific Notation (Non-Calc) - Positive Index - No Rounding (Sheet 1)

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Convert these numbers into scientific notation format. By hopping the decimal point to be behind the first digit.

Example Convert 4 000.

$4\,000 = 4 \times 10^{3}$

3 places

1 $6\,000 = 6 \times 10^{\square}$

2 $5\,000\,000 = 5 \times 10^{\square}$

3 $70\,000 = 7 \times 10^{\square}$

4 $30\,000\,000 = 3 \times 10^{\square}$

Now you do the hopping, use a pencil

5 $80\,000 = 8 \times 10^{\square}$

6 $1\,000 = 1 \times 10^{\square}$

7 $9\,000\,000 = 9 \times 10^{\square}$

Now the first number and the index

8 $40\,000 = \square \times 10^{\square}$

9 $7\,000\,000 = \square \times 10^{\square}$

10 $30\,000 = \square \times 10^{\square}$

Now the lot, the base is always 10.

11 $300\,000 = \square \times \square^{\square}$

12 $50\,000\,000\,000\,000\,000 = \square \times \square^{\square}$

13 $20\,000\,000\,000\,000 = \square \times \square^{\square}$

14 $600\,000 = \square \times \square^{\square}$

15 $8\,000 = \square \times \square^{\square}$

16 $100\,000\,000 = \square \times \square^{\square}$

17 $20\,000 =$ _____

18 $900\,000\,000 =$ _____

19 $100\,000 =$ _____

20 $4\,000\,000 =$ _____

Any additional non-zero digits are written to the right of the decimal point. Convert these.

Examples

$320 = 3.2 \times 10^{2}$

$570\,000 = 5.7 \times 10^{5}$

21 $75\,000 = \square.\square \times \square^{\square}$

22 $8\,800\,000 = \square.\square \times \square^{\square}$

23 $950\,000\,000 =$ _____

24 $7\,700\,000\,000 =$ _____

25 $4\,500 =$ _____

26 $61\,000\,000 =$ _____

Now there are two decimal places

27 $62\,800\,000 = \square.\square\square \times \square^{\square}$

28 $593\,000\,000\,000\,000 = \square.\square\square \times \square^{\square}$

29 $40\,700\,000\,000 = \square.\square\square \times \square^{\square}$

30 $712\,000\,000 = \square.\square\square \times \square^{\square}$

31 $21\,400 =$ _____

32 $807 =$ _____

33 $9\,090\,000 =$ _____

34 $5\,770\,000\,000\,000\,000\,000 =$ _____

35 $81\,600\,000\,000\,000 =$ _____

These have more than 3 leading significant digits. Include all of them. Convert these.

Examples

$170\,600 = 1.706 \times 10^{5}$

$8\,167\,200\,000 = 8.1672 \times 10^{9}$

36 $88\,760 = \square.\square\square\square \times \square^{\square}$

37 $13\,676\,000\,000 = \square.\square\square\square\square \times \square^{\square}$

38 $50\,255\,800\,000\,000 =$ _____

39 $14\,950\,000 =$ _____

40 $3\,006\,520\,000\,000 =$ _____

41 $804\,370\,000 =$ _____

Scientific Notation (Non-Calc) - Positive Index - Rounding (Sheet 2)

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Round these numbers shown in scientific notation format, to 1 decimal place.

Examples

$4.35 \times 10^9 = 4.4 \times 10^9$

as 5 is 5 or more, round up

$2.9712 \times 10^{14} = 3.0 \times 10^{14}$

as 7 > 5 round up

$3.4218 \times 10^5 = 3.4 \times 10^5$

as 2 < 5 round down

1 4.16×10^5

= ☐.☐ × ☐☐

2 3.71×10^8

= ☐.☐ × ☐☐

3 9.093×10^6

= ☐.☐ × ☐☐

4 1.7504×10^{11} =

5 6.022×10^{23} =

6 5.074×10^{14} =

7 5.9528×10^9 =

8 7.97×10^3 =

Convert and then round to 1 d.p.

9 3 170 000

= ☐.☐☐ × ☐☐

= ☐.☐ × ☐☐

10 856 000

= ☐.☐☐ × ☐☐

= ☐.☐ × ☐☐

11 40 720 000 =

=

12 271 000 =

=

13 859 400 000 =

=

Now go straight to the rounded answer

14 5 183 000

= ☐.☐ × ☐☐

15 70 494

= ☐.☐ × ☐☐

16 8 153

= ☐.☐ × ☐☐

17 1 777 803 225 431 650

= ☐.☐ × ☐☐

18 38 910 583 004

= ☐.☐ × ☐☐

Round these numbers shown in scientific notation format, to 2 decimal places.

19 5.316×10^7

= ☐.☐☐ × ☐☐

20 7.706×10^4

= ☐.☐☐ × ☐☐

21 8.073×10^{12}

= ☐.☐☐ × ☐☐

22 1.60371×10^{19}

=

23 4.04938×10^{13}

=

Convert these numbers to scientific notation format, rounding to 2 decimal places.

24 2 337 000

= ☐.☐☐ × ☐☐

25 49 993 414

= ☐.☐☐ × ☐☐

26 5 660 177 400 812

= ☐.☐☐ × ☐☐

27 914 731 422

= ☐.☐☐ × ☐☐

28 349 961

=

29 79 965 223 000

=

Now to 3 decimal places

30 107 463 020

= ☐.☐☐☐ × ☐☐

31 50 962 851 000

= ☐.☐☐☐ × ☐☐

32 177 966

=

Convert to scientific notation and round to 1 decimal place. These last two are different.

33 9 960 000

=

=

34 99 543 100 080

=

=

Scientific Notation (Non-Calc) - Positive Index (Sheet 3)

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These have a decimal point. Move it to be behind the first digit to express in scientific notation, rounding to 1 d.p.

Examples

$46\,789.7 = 4.7 \times 10^4$

$789\,523.78 = 7.9 \times 10^5$

1 976.4

$= \square.\square \times \square^{\square}$

2 575 009.135

$= \square.\square \times \square^{\square}$

3 4 096 216.88

$= \square.\square \times \square^{\square}$

4 1 143 215.9216

$=$ _____

5 198.636

$=$ _____

6 509 159 608.2816

$=$ _____

7 76 682.0053

$=$ _____

Point the < or > to the smallest numbers, which are expressed in scientific notation.

8 5.3×10^4 ☐ 2.4×10^4

9 8.01×10^9 ☐ 8.10×10^9

10 5.32×10^5 ☐ 5.32×10^7

11 7×10^9 ☐ 7.6×10^8

Rewrite these in ascending order. That means smallest to largest.

12 5.3×10^4, 2.7×10^4, 3.8×10^4, 5.9×10^4.

$=$ _____ (Smallest) , _____ , _____ , _____ (Largest) .

13 8.2×10^7, 8.2×10^5, 8.2×10^6, 8.2×10^3.

$=$ _____ , _____ , _____ , _____ .

14 3.3×10^6, 3.11×10^6, 3.1×10^6, 3.33×10^6.

$=$ _____ , _____ , _____ , _____ .

15 6.7×10^8, 4.02×10^5, 5.8×10^7, 5.89×10^8.

$=$ _____ , _____ , _____ , _____ .

Rewrite these two in descending order, that's largest to smallest.

16 4×10^{11}, 5.6×10^{10}, 2.9×10^{11}, 1.4×10^{12}.

$=$ _____ (Largest) , _____ , _____ , _____ (Smallest) .

17 1.7×10^9, 7.01×10^8, 1.71×10^9, 7.1×10^8.

$=$ _____ , _____ , _____ , _____ .

You can multiply numbers in scientific notation. Multiply the leading numbers and **ADD** the indexes. Multiply these.

Example $2 \times 10^7 \times 3 \times 10^4 = 2 \times 3 \times 10^7 \times 10^4 = 6 \times 10^{11}$.

18 $2 \times 10^2 \times 4 \times 10^5 =$ $\square \times \square \times \square^{\square} \times \square^{\square} = \square \times \square^{\square}$

19 $3 \times 10^8 \times 3 \times 10^6 =$ $\square \times \square \times \square^{\square} \times \square^{\square} = \square \times \square^{\square}$

20 $1 \times 10^7 \times 7 \times 10^4 =$ $\square \times \square \times \square^{\square} \times \square^{\square} = \square \times \square^{\square}$

With these, when you multiply you will get a two digit leading number. Change this back to a decimal and add 1 (+1) to the power.

Example $2 \times 10^7 \times 7 \times 10^4$

$= 2 \times 7 \times 10^7 \times 10^4$

$= 14 \times 10^{11} = 1.4 \times 10^{12}$

21 $3 \times 10^6 \times 5 \times 10^3$

$= \square \times \square \times \square^{\square} \times \square^{\square}$

$= \square \times \square^{\square}$

$= \square.\square \times \square^{\square}$

22 $7 \times 10^8 \times 4 \times 10^9$

$= \square \times \square \times \square^{\square} \times \square^{\square}$

$= \square \times \square^{\square}$

$= \square.\square \times \square^{\square}$

Scientific Notation (Non-Calc) - Negative Index - No Rounding (Sheet 1)

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Convert these decimals into scientific notation format. By hopping the decimal point to be behind the first non-zero digit.

Example Convert 0.0007

$0.0007 = 7 \times 10^{-4}$ (4 places)

1 $0.05 = 5 \times 10^{-}$

2 $0.00002 = 2 \times 10^{\square}$

3 $0.009 = 9 \times 10^{\square}$

Now you do the hopping, use a pencil

4 $0.0006 = 6 \times 10^{\square}$

5 $0.00000003 = 3 \times 10^{\square}$

6 $0.00008 = 8 \times 10^{\square}$

Now the first number and the index

7 $0.04 = \square \times 10^{\square}$

8 $0.00005 = \square \times 10^{\square}$

9 $0.000000001 = \square \times 10^{\square}$

Now fill all the boxes

10 $0.00000003 = \square \times \square$

11 $0.000000008 = \square \times \square$

12 $0.00007 = \square \times \square$

13 $0.0004 = \square \times \square$

14 $0.000000000006 = \square \times \square$

15 0.0003 = _____

16 0.0000008 = _____

17 0.009 = _____

18 0.000000004 = _____

Convert these to scientific notation format, the leading numbers to 1 decimal place.

Examples

$0.0083 = 8.3 \times 10^{-3}$

$0.000000096 = 9.6 \times 10^{-8}$

19 $0.0043 = \square.\square \times \square$

20 $0.18 = \square.\square \times \square$

21 $0.00000000092 = \square.\square \times \square$

22 $0.0000058 = \square.\square \times \square$

23 $0.00000000000016 = \square.\square \times \square$

24 0.00033 = _____

25 0.00000075 = _____

26 0.000048 = _____

27 0.00000000096 = _____

These have 3 or more decimal places. Include all the significant digits. Convert these.

28 0.0168 $= \square.\square\square \times \square$

29 0.00000517 $= \square.\square\square \times \square$

30 0.0008818 $= \square.\square\square\square \times \square$

31 0.0000007046 $= \square.\square\square\square \times \square$

32 0.00000000011931 $= \square.\square\square\square\square \times \square$

33 0.00000404 $= \square.\square\square \times \square$

34 0.000217 = _____

35 0.00000004176 = _____

36 0.005705 = _____

37 0.0000119 = _____

38 0.000000627 = _____

39 0.000003012 = _____

40 0.006776 = _____

Scientific Notation (Non-Calc) - Negative Index - Rounding (Sheet 2)

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Round these decimals shown in scientific notation format, to 1 decimal place.

Examples

$2.9712 \times 10^{-14} = 3.0 \times 10^{-14}$

as 7 > 5 round up

$5.4218 \times 10^{-5} = 5.4 \times 10^{-5}$

as 2 < 5 round down

1 2.54×10^{-5}

= ☐.☐ × ☐☐

2 4.06×10^{-3}

= ☐.☐ × ☐☐

3 7.14×10^{-9}

= ☐.☐ × ☐☐

4 3.95×10^{-7}

= ☐.☐ × ☐☐

5 7.77×10^{-4} =

6 2.89×10^{-8} =

7 6.24×10^{-6} =

8 7.11×10^{-9} =

Convert and then round to 1 d.p.

9 0.00417

= ☐.☐☐ × ☐☐

= ☐.☐ × ☐☐

10 0.00008037

= ☐.☐☐☐ × ☐☐

= ☐.☐ × ☐☐

11 0.007379 =

=

12 0.0008461 =

=

13 0.01446 =

=

Now go straight to the rounded answer

14 0.00615099

= ☐.☐ × ☐☐

15 0.0000000220599

= ☐.☐ × ☐☐

16 0.000555004

= ☐.☐ × ☐☐

17 0.0037095

= ☐.☐ × ☐☐

18 0.0909788

= ☐.☐ × ☐☐

19 0.0000053759

=

20 0.00000000074908

=

Round these decimals shown in scientific notation format, to 2 decimal places.

21 3.954×10^{-3}

= ☐.☐☐ × ☐☐

22 1.267×10^{-6}

= ☐.☐☐ × ☐☐

23 4.097×10^{-5}

= ☐.☐☐ × ☐☐

Round to 2 d.p.

24 8.997×10^{-8}

=

25 5.03803×10^{-7}

=

26 8.20399×10^{-3}

=

Convert these to scientific notation format, rounding to 2 decimal places.

27 0.0000703712

= ☐.☐☐ × ☐☐

28 0.000000000903177

= ☐.☐☐ × ☐☐

29 0.6666666

= ☐.☐☐ × ☐☐

30 0.0000090952006

= ☐.☐☐ × ☐☐

Use < or > with these. Point the < to the smallest decimal. 10^{-3} is larger than 10^{-4}.

31 4.3×10^{-7} ☐ 9.4×10^{-7}

32 1.17×10^{-3} ☐ 1.71×10^{-3}

33 3.33×10^{-3} ☐ 3.33×10^{-6}

Rank these, 1 for the largest, 3 for the smallest.

34 4×10^{-2} ☐ 7×10^{-3} ☐ 5×10^{-2} ☐

35 1×10^{-6} ☐ 9×10^{-4} ☐ 6×10^{-7} ☐

Converting to Scientific Notation - Calculator

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Use your calculator's 'Sci' Mode to convert these numbers into scientific notation format.

Put into 'Sci' Mode (0 d.p.):
Calc: Shift Mode 7 1

1 $8\,000 = \square \times 10^{\square}$

Calc: 8 0 0 0 =

2 $60\,000\,000 = \square \times \square^{\square}$

3 $400\,000 = \square \times \square^{\square}$

4 $20\,000 =$ ____

5 $9\,000\,000 =$ ____

Put into 'Sci' Mode (1 d.p.):
Calc: Shift Mode 7 2

6 $0.014 =$ ____

7 $0.00035 =$ ____

8 $0.0000073 =$ ____

9 $0.026 =$ ____

10 $0.0058 =$ ____

Put into 'Sci' Mode (2 d.p.):
Calc: Shift Mode 7 3

11 $217\,000 =$ ____

12 $0.0000615 =$ ____

13 $1\,540 =$ ____

14 $0.00000808 =$ ____

15 $81\,900 =$ ____

To Cancel: Shift Mode 8 2

Rewrite these in Scientific Notation, rounded to 3 decimal places.

Put into 'Sci' Mode (3 d.p.):
Calc: Shift Mode 7 4

16 $786\,537 =$ ____

17 0.000000055188
$=$ ____

18 $406\,333\,157$
$=$ ____

19 $0.012345 =$ ____

20 $772\,976\,807\,000$
$=$ ____

To Cancel: Shift Mode 8 2

Give the answer to these in scientific notation form. Rounded to 2 decimal places.

Put into 'Sci' Mode (2 d.p.):
Calc: Shift Mode 7 3

21 $0.00005^3 =$ ____

22 $95^6 =$ ____

23 $\sqrt{0.0009} =$ ____

24 $\dfrac{82^4}{0.3^3} =$ ____

25 $\dfrac{(72 \div 0.08)}{(0.06 \times 0.42)}$
$=$ ____

To Cancel: Shift Mode 8 2

Compare these by answering them in scientific notation to 1 decimal place, then point the < or > to the smallest.

Calc: Shift Mode 7 2

26 $40^3 \square 800^2$

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

27 $\dfrac{5}{25^5} \square \dfrac{(3 \div 19)}{38^3}$

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

28 $0.4^{12} \square 0.3^{11}$

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

29 $112^4 \square 782^3$

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

Rewrite these five in descending order, using scientific notation (to 1 d.p.) to guide you. Descending order means highest to lowest.

30 4^9 10^5 9^7 6^8 3^{12}

Sci. Not. 1 dp ____ Sci. Not. 1 dp ____ Sci. Not. 1 dp ____ Sci. Not. 1 dp ____ Sci. Not. 1 dp ____

Descending Order: ____, ____, ____, ____, ____.

To Cancel: Shift Mode 8 2

Scientific Notation (Non-Calc) - Convert to Real Numbers (Sheet 1)

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Convert these numbers from scientific notation format to real numbers.

Example Convert 5×10^4

50 000 ← Add 4 zeros

1 $7 \times 10^5 = 7$ _____

2 $3 \times 10^2 = 3$ _____

3 $2 \times 10^4 =$ _____

4 $9 \times 10^7 =$ _____

5 $6 \times 10^3 =$ _____

6 $4 \times 10^6 =$ _____

These have 1 decimal place, so add one less zero by replacing the first zero with a number

Example Convert 5.6×10^4

56 000 ← Add 4 digits '6' then 3 'zeros'

7 $8.5 \times 10^3 = 8\,5$ _____

8 $1.6 \times 10^4 = 1\,6$ _____

9 $4.7 \times 10^6 =$ _____

10 $9.2 \times 10^2 =$ _____

11 $3.3 \times 10^5 =$ _____

12 1.5×10^8

$=$ _____

13 4.3×10^7

$=$ _____

14 2.1×10^9

$=$ _____

15 5.9×10^{12}

$=$ _____

These have varying decimal places. Convert these to real numbers.

16 $6.35 \times 10^3 = 6\,3\,5$ _____

17 4.237×10^7

$= 4\,2\,3\,7$ _____

18 9.0951×10^9

$= 9\,0\,9\,5\,1$ _____

19 3.417×10^6

$= 3$ _____

20 7.0051×10^8

$=$ _____

21 6.045×10^5

$=$ _____

22 1.22241×10^9

$=$ _____

23 2.0202×10^6

$=$ _____

The next two have decimal answers

24 $5.096 \times 10^2 =$ _____

25 $3.4426 \times 10^3 =$ _____

Convert these decimals from scientific notation format to decimal form.

Example Convert 2×10^{-5}

0.00002 ← Add 5 zeros in front Put a '.' after 1st zero

26 $4 \times 10^{-3} = .\,\square\,\square\,4$ _____

27 $7 \times 10^{-5} = .\,\square\,\square\,\square\,\square\,7$ _____

28 $8 \times 10^{-2} = .\,\square\,8$ _____

29 $3 \times 10^{-4} = .\,\square\,\square\,\square\,3$ _____

30 5×10^{-7}

$=$ _____

31 9×10^{-5}

$=$ _____

32 8×10^{-6}

$=$ _____

33 1×10^{-9}

$=$ _____

34 3×10^{-2}

$=$ _____

34 6×10^{-4}

$=$ _____

Rewrite as decimals. These have more non-zero digits.

Example Convert 2.47×10^{-3}

0.00247 ← Add 3 zeros in front Put a '.' after 1st zero

35 5.2×10^{-4}

$=$ _____

36 4.18×10^{-6}

$=$ _____

37 5.497×10^{-2}

$=$ _____

38 2.4×10^{-9}

$=$ _____

39 3.007×10^{-5}

$=$ _____

40 1.115×10^{-6}

$=$ _____

Scientific Notation (Calculator) - Convert to Real Numbers (Sheet 2)

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Convert these numbers from scientific notation format to real numbers.

Example Convert 5.26×10^3

Calc: 5 . 2 6 ×10^x 3 = 5 260

1 2×10^4 =

Calc: 2 ×10^x 4 =

2 4×10^6 =

3 7×10^4 =

4 8×10^7 =

5 1.3×10^4 =

6 9.7×10^6 =

7 1.1×10^9

=

8 4.047×10^8

=

9 7.1338×10^6

=

10 3.377×10^7

=

The next 4 have decimal answers

11 4.031×10^2 =

12 9.73706×10^4

=

13 4.00788×10^3

=

14 7.8842×10^2

=

Convert these from scientific notation format to decimals.

Example Convert 5.26×10^{-3}

Calc: 5 . 2 6 ×10^x - 3 =

Answer: 0.00526

15 3×10^{-4} =

Calc: 3 ×10^x - 4 =

16 7×10^{-2} =

17 3×10^{-4} =

18 9×10^{-1} =

19 4×10^{-5} =

20 1×10^{-3} =

21 2.3×10^{-4} =

22 7.7×10^{-2} =

23 3.25×10^{-5}

=

24 4.024×10^{-3}

=

25 1.5×10^{-9}

=

26 3.079×10^{-3}

=

27 8.113×10^{-5}

=

28 9.3059×10^{-4}

=

Use your calculator to answer these as numbers or decimals.

29 $3 \times 10^4 \times 5 \times 10^{-2}$

Calc: 3 ×10^x 4 × 5 ×10^x - 2 =

=

30 $8 \times 10^{-9} \div 2 \times 10^{-2}$

=

31 $(4 \times 10^{-3})^2$

=

32 $1.6 \times 10^8 \div 8 \times 10^3$

=

33 $\dfrac{1.05 \times 10^8}{3.5 \times 10^6}$ =

Calculate these and write the answer underneath. Then use <, > or = in the square box.

*With < & > 'point' to the smallest

34 $\dfrac{1.8 \times 10^8}{6 \times 10^2}$ ☐ $(3 \times 10^2)^2$

35 $\dfrac{4 \times 10^3}{2 \times 10^5}$ ☐ $\dfrac{1 \times 10^2}{2 \times 10^4}$

36 $(1 \times 10^3)^2$ ☐ $(1 \times 10^2)^3$

37 $(2 \times 10^2)^2$ ☐ $(4 \times 10^2)^2$

Numbers of Any Magnitude - Prefixes ‘Kilo’ & ‘Mega’ (Sheet 1)

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Convert these base units into ‘Kilo’ units. Divide by 1 000 to make the conversion.

Example Convert 3 400 m to km

3 400 ÷ 1 000 = 3.4 km

1 8 500 g = _____ kg

2 43 700 L = _____ kL

3 780 m = _____ km

4 347 500 B = _____ kB

5 220 g = _____ kg

6 4 020 m = _____ km

Ben walks 320 m to the shops, then returns home. His mum forgot that she needs bread so he repeats the trip.

6 How far has Ben walked for the two return trips, in metres.

7 Convert this distance to km.

Jo’s phone downloads at 780 B per second. She downloads a document which takes 5 minutes to download.

8 Convert 5 mins to seconds.

9 How large is the file she downloaded? Answer in B.

10 Convert the file from B to kB.

Convert these ‘Kilo’ units into ‘Mega’ units. Divide by 1 000 to make the conversion.

Example Convert 400 kB to MB

400 ÷ 1 000 = 0.4 MB

11 9 800 kL = _____ ML

12 176 300 kB = _____ MB

13 13 140 kHz = _____ MHz

*Hertz (Hz) is a measure of frequency.

14 490 kL = _____ ML

Josh has taken 527 photos this year. If the average photo is 970 kB. Find:

15 The total size of the folder holding the images in kB.

16 Convert the folder’s size to MB.

An irrigation pump discharges 1.4kL of water per minute. Find the amount:

17 Pumped in an hour, in kL.

18 Pumped in a day. (24 hrs)

19 Pumped annually. (365 days)

20 Convert the amount pumped annually from kL to ML.

To go backwards, multiply by 1000. Mega→Kilo = × 1 000 Kilo→Base unit = × 1 000

Example Convert 3.2 MB to kB

3.2 × 1 000 = 3 200 kB

21 5.4 MB = _____ kB

22 9 km = _____ m

23 4.76 kg = _____ g

24 0.56 ML = _____ kL

25 2.09 km = _____ m

26 1.7 MB = _____ kB

A farm’s diesel storage tank holds 8.2 kL of fuel.

27 Find the volume of fuel in the tank is it is 55% full. (kL)

$\frac{\square}{100} \times \square = \square$

28 Convert the volume to L.

29 Calculate the value of the fuel if diesel costs $1.68/L.

Show the $ sign and cents to 2 d.p.

Karl is training for the 800 m state championship. He runs the 800 m twice each weekday and once on Saturday and Sunday.

30 Calculate the distance Karl runs in a week in m, then km.

Numbers of Any Magnitude - Prefixes ‘Mega’, ‘Giga’ & ‘Tera’ (Sheet 2)

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Convert these ‘Giga’ units into ‘Mega’ units. Multiply by 1 000 to make the conversion.

Example Convert 1.4 GB to MB

1.4 × 1 000 = 1 400 MB

1 3 GB = _____ MB

2 5.6 GW = _____ MW

*Watt (W) is a measure of power.

3 0.64 GL = _____ ML

4 22.07 GB = _____ MB

5 0.034 GL = _____ ML

6 14.8 GB = _____ MB

Chen has 8 files that average 390 MB each. He wants to store them on a DVD which holds 4.7 GB.

7 Convert 4.7 GB to MB.

8 How much memory in MB will the 8 files need?

9 Calculate how much space in MB is left on the DVD?

Ainslie has 2.2 GB available on a USB drive. She wants to put music on the drive that average 4 MB per song.

10 Convert 2.2 GB to MB.

11 How many 4 MB songs fit?

Convert these ‘Mega’ units into ‘Giga’ units. Divide by 1 000 to make the conversion.

Example Convert 720 MB to GB

720 ÷ 1 000 = 0.72 GB

12 4 000 MB = _____ GB

13 125 000 MB = _____ GB

14 280 MB = _____ GB

15 1 080 ML = _____ GL

16 48 MW = _____ GW

17 68 700 ML = _____ GL

David’s PC has 3 old Hard Drives which are 260 MB, 720 MB and 650 MB.

18 Find the total storage (MB).

19 Convert the total storage to GB.

Anthia shoots wedding videos. Once edited the videos average 850 MB in size. She keeps a back up of all the videos and she has 167 of them.

20 Calculate how much space in MB is used by the 167 videos.

21 Convert the storage to GB.

Convert these ‘Giga’ units into ‘Tera’ units. Divide by 1 000 to make the conversion.

Example Convert 480 GB to TB

480 ÷ 1 000 = 0.48 TB

22 6 000 GB = _____ TB

23 300 GB = _____ TB

24 1 020 GB = _____ TB

25 89.2 GB = _____ TB

26 770 GB = _____ TB

27 0.5 GB = _____ TB

Raj has a 64 GB USB drive. His PC has a 4 TB hard drive.

28 Convert 64 GB into TB.

29 How many full USB drives would the hard drive hold?

Convert these ‘Tera’ units into ‘Giga’ units. Multiply by 1 000 to make the conversion.

Example Convert 3.2 TB to GB

3.2 × 1 000 = 3 200 GB

30 5 TB = _____ GB

31 1.7 TB = _____ GB

32 0.06 TB = _____ GB

33 2.85 TB = _____ GB

34 10.6 TB = _____ GB

35 0.0083 TB = _____ GB

Numbers of Any Magnitude - Prefixes & Length milli/centi/kilo (Sheet 1)

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Convert these lengths from mm to cm. There are 10 mm in 1 cm.



Divide by 10 or move 1 place left

1	20 mm	cm
2	50 mm	
3	130 mm	
4	55 mm	
5	92 mm	
6	200 mm	
7	18.6 mm	
8	2 020 mm	
9	435 mm	
10	0.95 mm	
11	5.47 mm	
12	906.2 mm	

Now the reverse. Convert these lengths from cm to mm. There are 10 mm in 1 cm.



Multiply by 10 or move 1 place right

13	5 cm	mm
14	18 cm	
15	6.3 cm	
16	74.8 cm	
17	356.7 cm	
18	8.12 cm	
19	0.4 cm	
20	0.566 cm	
21	155 cm	
22	829.3 cm	
23	1.002 cm	
24	19.91 cm	

Convert these lengths from cm to m. There are 100 cm in 1 m.



Divide by 100 or 2 places left

25	200 cm	m
26	600 cm	
27	150 cm	
28	370 cm	
29	12 cm	
30	86.5 cm	
31	7 300 cm	
32	56.05 cm	
33	3 cm	
34	6.7 cm	
35	8 002 cm	
36	27.08 cm	

Convert these lengths from m to cm



Multiply by 100 or 2 places right

37	3 m	cm
38	7 m	
39	2.2 m	
40	5.9 m	
41	0.3 m	
42	0.75 m	
43	0.363 m	
44	5.02 m	
45	10.7 m	
46	0.006 m	
47	5.071 m	
48	30.262 m	



Convert these lengths from m to km

Divide by 1 000 or 3 places left

49	1 500 m	km
50	1 220 m	
51	4 930 m	
52	110 m	
53	550.6 m	
54	1 003 m	
55	85 m	
56	4.7 m	
57	309.7 m	
58	12 032 m	
59	8 771.3 m	
60	763.2 m	



Convert these lengths from km to m

Multiply by 1 000 3 places right

61	3 km	m
62	8 km	
63	3.2 km	
64	14.7 km	
65	10.6 km	
66	0.9 km	
67	0.673 km	
68	5.12 km	
69	1.007 km	
70	0.4503 km	
71	4.019 km	
72	0.06 km	

Numbers of Any Magnitude - Prefixes ‘milli’ & ‘micro’ (Sheet 2)

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Convert these base units into ‘milli’ units. Multiply by 1 000 to make the conversion.

Example Convert 2.5 m to mm

2.5 × 1 000 = 2 500 mm

1 5 A = _____ mA

*Amps (A) is a measure of electrical current

2 80 g = _____ mg

3 3.08 L = _____ mL

4 0.005 m = _____ mm

5 15.7 g = _____ mg

6 A ceiling is 2.4 m above the floor. Convert the length to mm.

_____ × 1 000 = _____ mm

7 The active ingredient in a antibiotic weighs 0.8 g. Convert this weight to mg.

8 The current across a circuit is 0.0033 A. Convert this to mA.

9 Three boxes have heights: 0.42 m, 0.715 m and 0.505 m. How high are they when stacked on top of each other. Answer in m, then in mm.

10 A standard door is 2.04 m high. Convert the height to mm.

To convert ‘milli’ units to the base unit, divide by 1 000. Change these to ‘milli’ units.

Example Convert 960 mm to m

960 ÷ 1 000 = 0.96 m

11 3 000 mL = _____ L

12 28 900 mg = _____ g

13 31 430 mm = _____ m

14 4 620 mL = _____ L

15 86 mΩ = _____ Ω

*Ohms (Ω) is a measure of electrical resistance

16 A bottle of soft drink contains 1 250 mL of product. Convert this to L.

_____ ÷ 1 000 = _____ L

17 Ceiling fan blades are usually 300 mm below the ceiling. Convert this to m.

18 Two swimmers touch the wall 230 ms apart. Convert this time to seconds.

19 A medicine has 308 mg, 182 mg, 780 mg and 2 750 mg of different active ingredients. Find the total weight in mg, then convert this to g.

Micro is 1 millionth. Convert ‘micro’ to ‘milli’ by dividing by 1 000. Hey … I know mu!

Example Convert 4 440 µm to mm

4 440 ÷ 1 000 = 4.44 mm

20 34 000 µm = _____ mm

21 87 µg = _____ mg

22 8 803 µs = _____ ms

23 40.5 µg = _____ mg

24 A human hair thickness is 120 µm. Convert this to mm.

_____ ÷ 1 000 = _____ mm

25 A dust mite is 275 µm in length. Convert this to mm.

To convert ‘milli’ units to ‘micro’ units, multiply by 1 000. Micrometres can also be called microns.

Example Convert 0.74 mm to µm

0.74 × 1 000 = 740 µm

26 8 mm = _____ µm

27 0.06 mg = _____ µg

28 0.0037 ms = _____ µs

29 5.7 mm = _____ µm

30 A red blood cell is 0.008 mm in diameter. Convert this to µm.

31 The width of an eye of a needle is 1.2 mm Convert to µm.

Numbers of Any Magnitude - Error in Measurement (Sheet 1)

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The error in measurement is half of the limit of reading. Find the error in these measurements.

Example Find the error with measurements made to the nearest:

a) cm ±0.5 cm *c*) 10 kg ±5 kg

b) g ±0.5 g *d*) 5 s ±2.5 s

1 Nearest kg = ± _____ kg

2 Nearest 5 g = ± _____ g

3 Nearest 10 mL = ± _____ mL

4 Nearest 50 m = _____

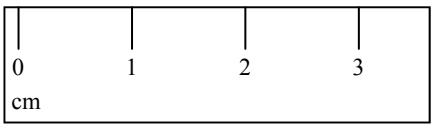
5 Nearest 25 g = _____

6 Nearest 10 cm = _____

7 Nearest 2 mm = _____

These two rulers have different limits of reading, write the limit of reading and the error.

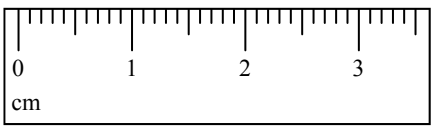
8



Limit of Reading: _____ To the nearest

Error: ± _____

9



Limit of Reading: _____ To the nearest

Error: ± _____

Find the upper and lower limits of these measurements.

Example A parcel weighs 425 g measured to the nearest 5 g.

Error: ± 2.5 g

Lower Limit : 425 - 2.5 = 422.5 g

Upper Limit : 425 + 2.5 = 427.5 g

10 A section of road to be repaired is measured as 870 m long, to the nearest 10 m.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

11 A window has a width of 176 cm, to the nearest cm.

Error: _____ Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

12 A horse weighs 485 kg, measured to the nearest 5 kg.

Error: _____

Lower Limit : _____

Upper Limit : _____

13 A measuring cylinder contains 115 mL measured to the nearest mL.

Error: _____

Lower Limit : _____

Upper Limit : _____

With decimals, you will add a decimal place.

Example Find the error with measurements made to the nearest:

a) 0.1 cm ±0.05 cm *c*) 0.01 m

b) 0.5 g ±0.25 g ±0.005 m

15 A basket ball hoop is 304.8 cm from the ground, measured to the nearest 0.1 cm.

Error: _____ 0.1 ÷ 2 Use '±' and units

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

16 A solution is made by adding 5.72 g of copper sulphate, measured to the nearest 0.01 g.

Error: _____

Lower Limit : _____ - _____ = _____

Upper Limit : _____ + _____ = _____

17 A chocolate bar is manufactured to weigh 91.0 g, measured to the nearest 0.5 g.

Error: _____

Lower Limit : _____ = _____

Upper Limit : _____ = _____

Numbers of Any Magnitude - Further Errors Perimeter/Area (Sheet 2)

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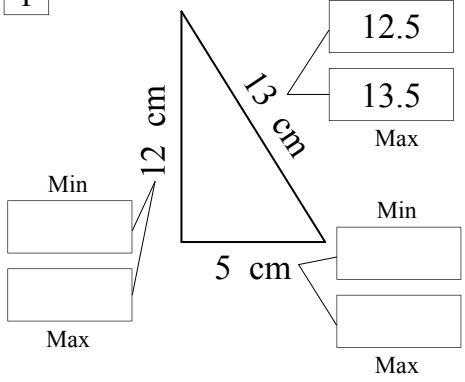
These triangles are measured to the nearest cm. Find the lower and upper limits for each side, then the minimum and maximum perimeter.

1



$P_{MIN} =$ ☐ + ☐ + ☐

Add all the minimum values

$P_{MIN} =$ ☐

Include the units

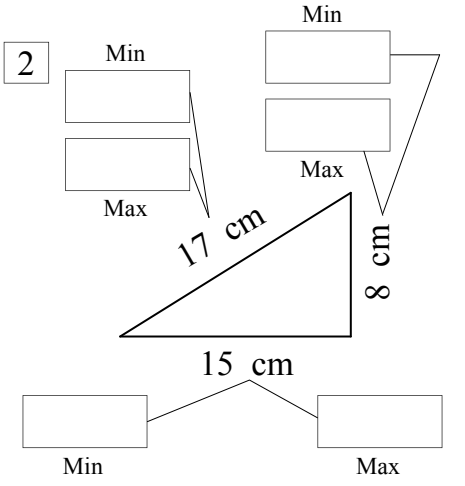
$P_{MAX} <$ ☐ + ☐ + ☐

Add all the maximum values

$P_{MAX} <$ ☐

Include the units

2



$P_{MIN} =$ ☐ + ☐ + ☐

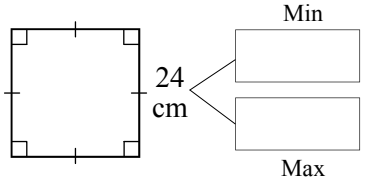
$P_{MIN} =$ ☐

$P_{MAX} <$ ☐ + ☐ + ☐

$P_{MAX} <$ ☐

Squares are easier, get the min and max side length and ×4! Calculate the minimum and maximum perimeter. Lengths are measured to the nearest cm

3



$P_{MIN} =$ ☐ × ☐

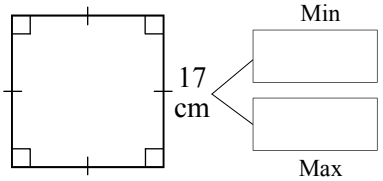
$P_{MIN} =$ ☐

Include the units

$P_{MAX} <$ ☐ × ☐

$P_{MAX} <$ ☐

4



$P_{MIN} =$ ☐

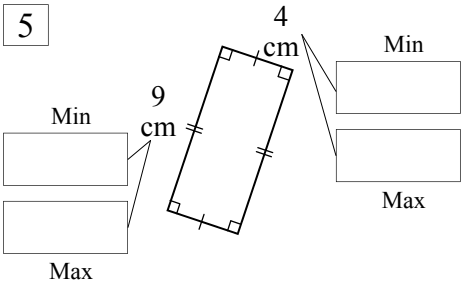
$P_{MIN} =$ ☐

$P_{MAX} <$ ☐

$P_{MAX} <$ ☐

Rectangles, × each side by 2

5



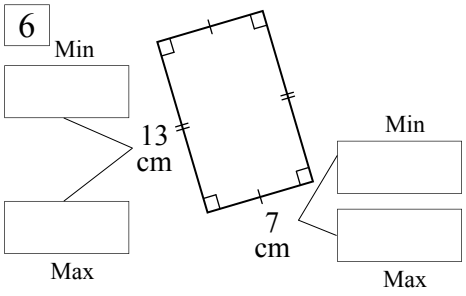
$P_{MIN} =$ ☐ × 2 + ☐ × 2

$P_{MIN} =$ ☐

$P_{MAX} <$ ☐ × ☐ + ☐ × ☐

$P_{MAX} <$ ☐

6



$P_{MIN} =$ ☐ × ☐ + ☐ × ☐

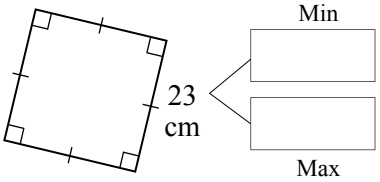
$P_{MIN} =$ ☐

$P_{MAX} <$ ☐

$P_{MAX} <$ ☐

Now find the area, by squaring or multiplying the two sides. All measurements are measured to the nearest cm.

7



$A_{MIN} =$ ☐ ^{2}

Area is found by squaring the min side length

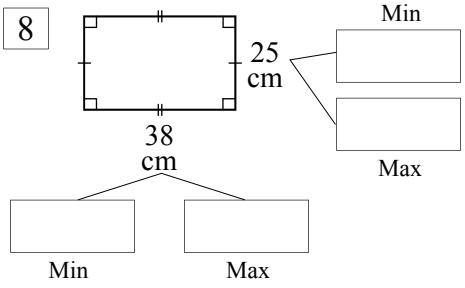
$A_{MIN} =$ ☐

Area units are cm^2, write square units

$A_{MAX} <$ ☐

$A_{MAX} <$ ☐

8



$A_{MIN} =$ ☐ × ☐

$A_{MIN} =$ ☐

$A_{MAX} =$ ☐ × ☐

$A_{MAX} =$ ☐

Numbers of Any Magnitude - Find A Word

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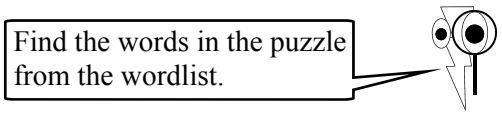

S	C	I	E	N	T	I	F	I	C	R	E	W	O	P
H	X	N	K	D	N	O	C	E	S	I	L	L	I	M
O	N	O	T	A	T	I	O	N	I	N	D	N	I	E
U	X	I	F	E	R	P	B	C	O	N	V	E	R	T
R	R	T	X	X	K	W	A	N	W	E	X	K	T	Y
K	O	A	N	K	D	E	S	O	M	G	T	E	H	B
E	U	M	X	A	W	E	E	I	X	A	Q	T	G	A
R	N	I	Y	E	C	K	C	T	M	T	M	Y	I	G
T	D	X	E	D	N	I	K	A	X	I	K	B	N	I
E	I	O	R	X	E	W	F	M	D	V	C	A	T	G
M	N	R	R	Q	L	A	M	I	C	E	D	R	R	M
O	G	P	O	B	Y	R	U	T	N	E	C	E	O	Q
N	Q	P	R	X	E	E	K	S	X	G	V	T	F	K
A	K	A	F	I	G	U	R	E	S	W	I	T	X	X
N	M	O	N	T	H	T	E	V	I	T	I	S	O	P

WORDLIST

CONVERT	INDEX	PREFIX	NEGATIVE	GIGABYTE
FIGURES	CENTURY	NANOMETRE	DAY	WEEK
FORTNIGHT	POSITIVE	ERROR	MILLISECOND	NOTATION
SCIENTIFIC	TERABYTE	MONTH	POWER	HOUR
MICRO	DECADE	BASE	DECIMAL	ROUNDING
	ESTIMATION	SIGNIFICANT	APPROXIMATION	

Numbers of any Magnitude - Topic Review

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Round these numbers to the nearest 10

1 75 = _____

2 423 = _____

3 5 796 = _____

Round these numbers to the nearest 100

4 386 = _____

5 67 890 = _____

6 23 = _____

Round these numbers to the nearest 1 000

7 13 170 = _____

8 199 727 = _____

9 473 = _____

Round these decimals to the nearest whole number

10 8.2 = _____ 11 5.7 = _____

12 4.49 = _____ 13 0.67 = _____

Perform the calculations and round to the nearest whole number

14 $148 \div 52$ = _____ (Whole Number)

15 78.6×93.9 = _____

Round these decimals to 1 decimal place (1 d.p.)

16 63.317 = _____

17 0.1178 = _____

18 0.66666666 = _____

19 9.98 = _____

Perform the calculations and round to the nearest decimal place

20 $27 \div 73$ = _____ (1 d.p.)

21 $\sqrt{157}$ = _____

22 $\frac{24^2 - 7^3}{9^2}$ = _____

Round these to 1, 2 and 3 decimal places

23 0.3275 _____ (1 d.p.) _____ (2 d.p.) _____ (3 d.p.)

24 2.9530 _____ (1 d.p.) _____ (2 d.p.) _____ (3 d.p.)

How many significant figures are in these numbers and decimals?

25 45 _____ sig.fig.

26 0.0003 _____ sig.fig.

27 0.07100 _____

28 7 760 _____

29 8 001 _____

30 8 100 _____

Round these to 1 significant figure

31 1 567 _____

32 63 _____

33 0.007539 _____

34 29 555 030 _____

35 0.0000662 _____

Perform the calculations and express the answer to 1 significant figure

36 147^2 = _____ (1 significant figure)

37 $8^9 \div 2^9$ = _____

38 $\frac{(26 + 37)}{(34 - 21)}$ = _____

Round these to 2 significant figures

39 275 _____

40 7 896 _____

41 0.01313 _____

What word describes a time period of

42 7 days _____

43 60 minutes _____

44 10 years _____

45 24 hours _____

46 1 000 years _____

Convert these time periods

47 3 weeks = _____ days

48 1 year = _____ weeks

49 4 days = _____ hours

50 ½ hour = _____ min

51 156 weeks = _____ years

52 4 min = _____ seconds

53 5 h = _____ min

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Change these times to minutes

54 3 hours = _____

55 1 h 19 min = _____

56 4 h 54 min = _____

Change these times to hours and minutes

57 420 min = _____

58 280 min = _____

59 703 min = _____

Change these times from seconds to milliseconds

60 3 sec = _____ msec

61 0.5 sec = _____

62 0.06 sec = _____

Change these times from milliseconds to seconds

63 2000 msec = _____ sec

64 30 msec = _____

65 120 msec = _____

Rewrite these numbers in scientific notation format

66 3 000 = $\square \times \square^{\square}$

67 4 000 000 = _____

68 37 000 = _____

69 80 600 000 = _____

70 64 700 = _____

71 194 000 = _____

Rewrite these decimals in scientific notation format

72 0.006 = $\square \times \square^{\square}$

73 0.00002 = _____

74 0.00037 = _____

75 0.0811 = _____

76 0.108 = _____

Convert these to numbers

77 5×10^4 = _____

78 3×10^2 = _____

79 4.6×10^3 = _____

80 1.02×10^6 = _____

Convert these to decimals

81 6×10^{-2} = _____

82 2×10^{-5} = _____

83 3.7×10^{-1} = _____

84 9.09×10^{-6} = _____

Rank these prefixes from largest (1) to smallest (6)

85

Mega ☐

Milli ☐

Tera ☐

Micro ☐

Giga ☐

Kilo ☐

Change these units to the specified unit

86 4 000 m = _____ [km]

87 3 L = _____ [mL]

88 45 mg = _____ [g]

89 0.7 km = _____ [m]

90 53 mm = _____ [cm]

Change these digital units to the unit specified

91 4 TB = _____ [GB]

92 5 000 kB = _____ [MB]

93 60 GB = _____ [MB]

94 0.3 GB = _____ [MB]

95 3.2 MB = _____ [kB]

What is the error in measurements rounded to the nearest…..

96 cm = ± _____

97 10 mL = _____

98 15 sec = _____

99 100 kg = _____

What is the error, lower and upper limits of this measurement

100 A car weighs 1 075 kg measured to the nearest 5 kg.

Error: _____

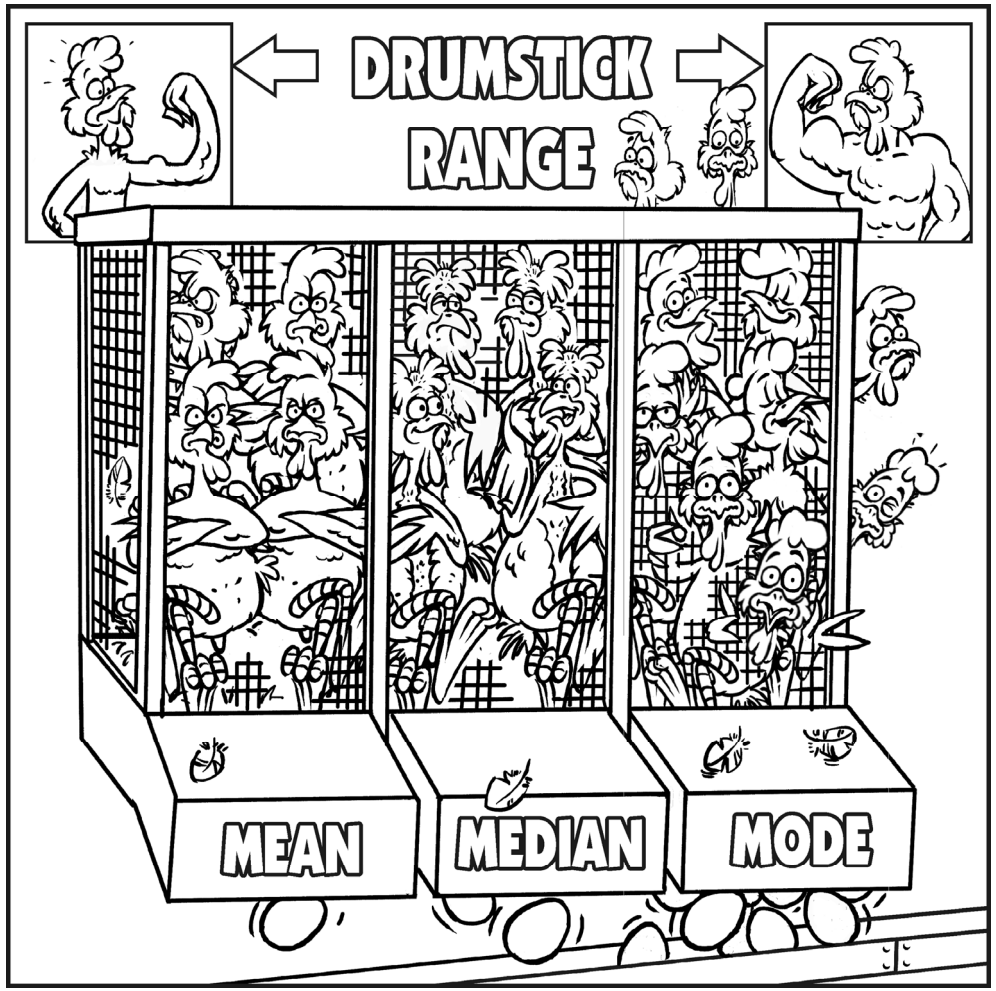
Lower Limit : _____

Upper Limit : _____

VELOCITY

DATA ANALYSIS

Data Analysis



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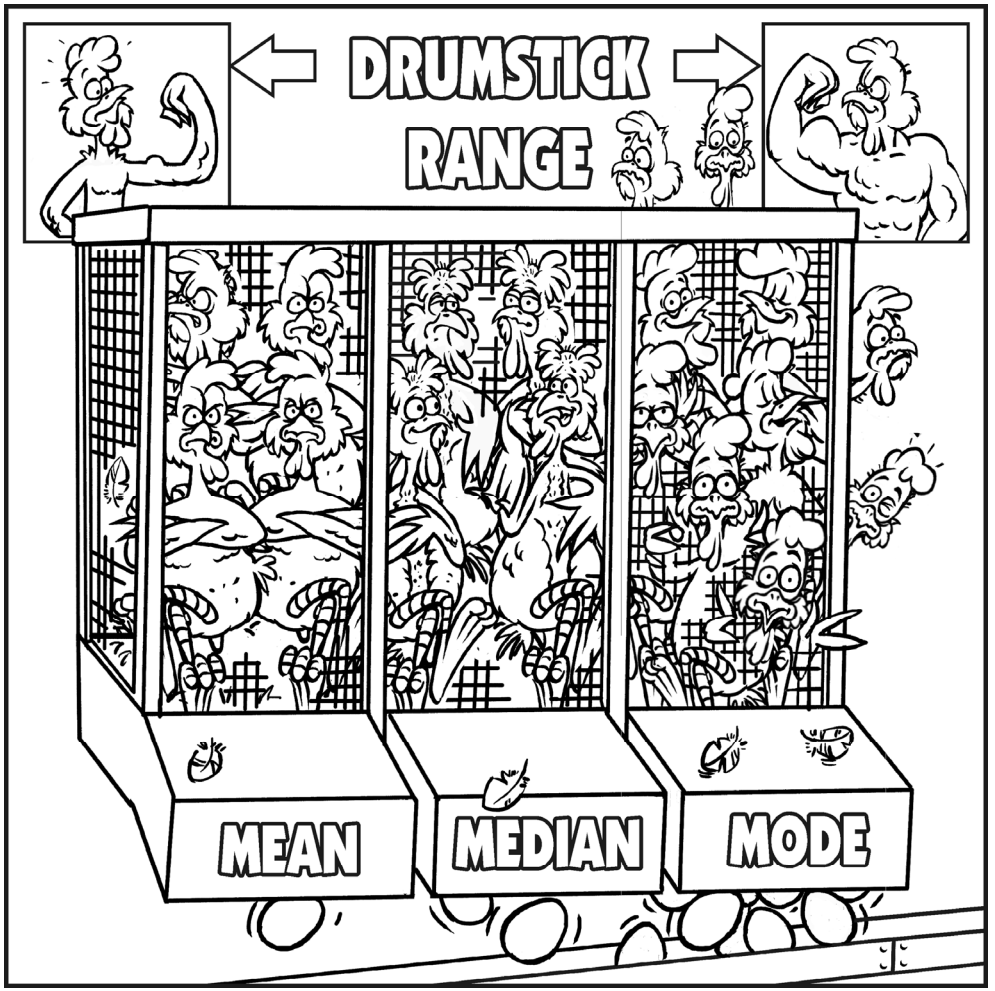
Data Analysis



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Data Analysis - Dot Plots

Dot Plots represent data with dots! Amazing! The beauty of a dot plot is it is simple. There is one axis which shows the score, or category and then dots are stacked above them. Dot plots can also be plotted sideways just like a column graph. Two critical features are:

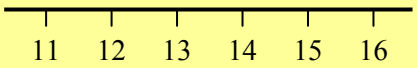
- The dots have to have the same height, for example a column of four dots shouldn't be as high as a column of five dots, so be careful as you plot them.
- If you have a vertical scale then use it effectively. Because dot plots are supposed to be fast to read, a vertical scale can be included so that you don't have to count all the dots in a column.

Example

Data: 15, 12, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.

STEP 1

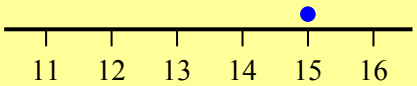
Circle the lowest score and the highest score in the data set.



STEP 2

Number the axis from lowest to highest, note there is no 14 score, but it is still included on the axis.

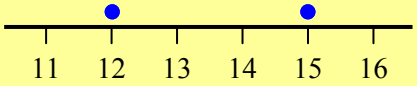
Data: ~~15~~, 12, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.



STEP 3

Cross out the first score and place the dot above the number on the axis.

Data: ~~15~~, ~~12~~, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.



STEP 4

And so on until all the scores are represented with dots

Data Analysis - Frequency Tables

The frequency of something is how often it occurs. So a Frequency Table records how many times scores in a data set occur. There are three columns to a frequency table, when the data is not already entered. These are the 'Score', 'Tally' and 'Frequency'. Score can be replaced with a title when the data is categorical (which means not numbers), for example 'Car Type' or 'Holiday Type', it can also just be called '*x*'. The frequency column is regularly written just as '*f*'. Note the use of Σf at the bottom of the frequency column. Σ means 'the sum of' so Σf means add all the frequency column up.

Example

Data: 6, 7, 5, 4, 6, 7, 10, 5, 6, 4, 9, 6, 5, 7, 6.

Score	Tally	Frequency
4		
5		
6		
7		
8		
9		
10		
		$\Sigma f =$

STEP 1

Circle the lowest score and the highest score in the data set.

STEP 2

In the score column of the table write in the lowest score at the top, increasing down the column until the highest score.

!

Note that there is no score of '8' in the data set. The number '8' is still included in the score column anyway.

Data: ~~6~~, 7, 5, 4, 6, 7, 10, 5, 6, 4, 9, 6, 5, 7, 6.

Score	Tally	Frequency
4		
5		
6	|	
7		
8		
9		
10		
		$\Sigma f =$

STEP 3

Data is now entered number by number Cross out the 6 and put a mark in the Tally column, then the 7 and so on.

!

Don't count all the 4's then all the 5's. This method is more likely to cause mistakes in large data sets. It also doesn't effectively use the Tally process.

Score	Tally	Frequency
4	||	2
5	|||	3
6	||||	5
7	|||	3
8		0
9	|	1
10	|	1
		$\Sigma f =$ 15

The 5th score is a cross stroke

STEP 4

Now change the Tally into numbers in the Frequency column.

STEP 5

Total the Frequency column to get Σf it has to equal the number of scores in the data set. If it doesn't ….start searching!

Data Analysis - Dot Plots and Frequency Tables (Sheet 1)

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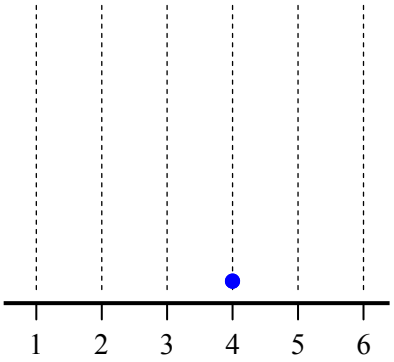
Display the data in a dot plot.

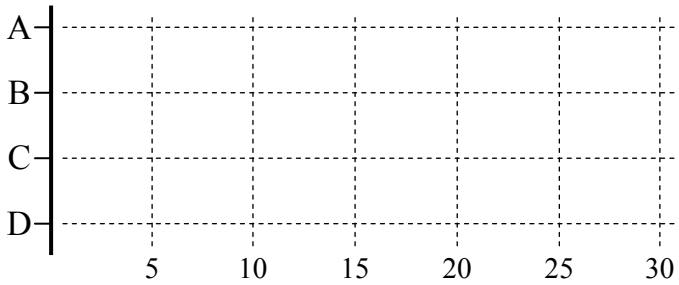
1 Data: 4, 5, 3, 3, 6, 1, 2, 4, 2, 3, 5, 6, 2, 3, 4, 4, 1, 2, 5, 6, 2, 3, 3, 2, 5, 2, 1, 4, 6, 3, 2.



2 An artificial sweetener is comparing itself with three other brands. Graph the results.

Data: BBBDA BABBC ABBBC
BDBAB ABBDB BCBBD
BCABB ABCBD BBADB

sideways! → Artificial Sweetener Preference



3 How many people were sampled? _____

4 How many people preferred Brand B? _____

5 How many people preferred Brand C? _____

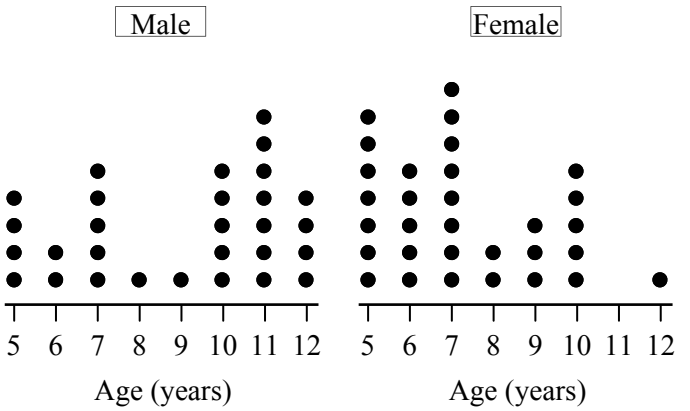
6 What percentage of people prefer Brand B?

Question 4 Ans. → ☐ / ☐ Question 3 Ans. → × 100 = ☐ Round to 1 d.p.

7 What percentage of people prefer Brand C?

Question 5 Ans. → ☐ / ☐ Question 3 Ans. → × 100 = ☐ Round to 1 d.p.

A waterslide at a theme park records the ages of the first 60 children who use the slide, the results are shown in two Dot plots. One male, one female.



8 How many children were male? _____

9 How many children were 7 years old? _____

10 How many children were 11 or older? _____

11 How many more 10 year olds used the slide compared to 6 year olds? _____

12 Are these true or false? There were more ...

(T) (F) Girls using the waterslide

(T) (F) 7 y.o. boys using the slide than 10 y.o. girls

(T) (F) Older male children than female.

13 What percentage of 9 y.o.'s were male?

Male 9 y.o. here → ☐ / ☐ Total 9 y.o. here → × 100 = ☐ Show the % sign

14 What percentage of 5 y.o.'s were female?

☐ / ☐ × 100 = ☐ Round to 1 d.p.

15 What percentage of females were 10 y.o.?

Total females here → ☐ / ☐ × ☐ = ☐ Round to 1 d.p.

16 What percentage of 5 - 7 y.o.'s were male?

☐ / ☐ × ☐ = ☐ Round to 1 d.p.

Data Analysis - Dot Plots and Frequency Tables (Sheet 2)

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Complete the frequency column and total for Σf, then answer the questions.

1 Number of Fish in a Tank

Fish	Tally	Frequency
2	𝍸	
3	𝍸 ||	
4	𝍸 𝍸 |||	
5	𝍸 𝍸	
6	𝍸 𝍸 ||||	
7	𝍸 𝍸 |||	
		$\Sigma f =$

2 What number of fish had the highest frequency? _____

3 What number of fish had the same frequency? _____

4 What was the frequency for a tank to have an odd number of fish?

_____

5 What was the frequency for a tank to have less than 5 fish?

_____

6 Soft Drink Preference

Type	Tally	Frequency
A	𝍸 𝍸 𝍸 ||	
B	𝍸 𝍸 |||	
C	||||	
D		
E	𝍸 𝍸 ||	
F	𝍸 𝍸 𝍸	
		$\Sigma f =$ 75

7 D's results were unreadable due to a drink spill, use Σf to calculate them.

8 Rank the drink brands in order from least, to most popular.

_____

9 How many people preferred brand 'A' or brand 'F'? _____

Place the data into the Frequency table, showing the tally, the frequency and write 'Σf= *(total)*', then answer the questions.

10 Away from School - Sports Choices

Sport	Tally	Frequency
Tennis		
Gym		
Bocce		
Archery		

Data:

TTGAA	TTGAA
TGBBA	GBBAT
AAGGG	TTTTA
BTBTA	BTBBA
AAATG	GTBBA
BBGBT	AABTB
TTGBB	BBTBG
ATTTA	ABBTG

11 What was most popular 'away' sport chosen? _____

12 How many people chose 'ball sports'? _____

13 If there are a total of 135 students in the year, calculate the percentage that chose 'away' sports.

$$\%\ \text{Away Sport} = \frac{\square}{135} \times 100$$

$$= \underline{\square}\ \%$$

Round to 1 d.p.

14 A real estate agent is checking rental homes for smoke detectors, if the smoke detector was Battery powered, Mains powered, or Dual (both). If a detector didn't work its type wasn't recorded it was just recorded as Inoperative. If a house had no smoke detector, it was recorded as 'None'.

Smoke Detector Type or Status

Type	Tally	Frequency
Battery		
Mains		
Dual		
Inoperative		
None		

Data:

BBDMB	BBDBM
BIBBM	MNDIB
MBDBI	BBIIN
NBBIM	INDDB
BBNIB	NBBMB
MBBDN	IBBMB
NINBB	NDDMB

15 How many homes had operational smoke detectors? _____

16 What percentage (to 1 d.p.) of homes had inoperative or no smoke detectors?

$$\%\ \text{No S.D} = \frac{\square}{\square} \times 100 = \underline{\square}$$

Round to 1 d.p.

17 What percentage (to 1 d.p.) of homes had Dual-powered smoke detectors?

$$\%\ \text{Dual Powered} = \frac{\square}{\square} \times 100 = \underline{\square}$$

Round to 1 d.p.

Data Analysis - Dot Plots and Frequency Tables (Sheet 1)

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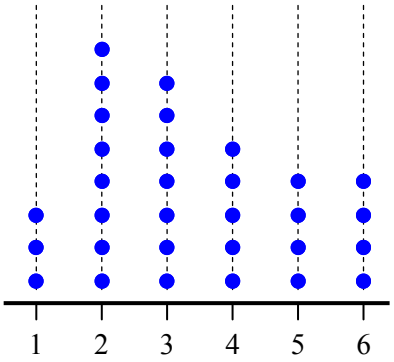
Display the data in a dot plot.

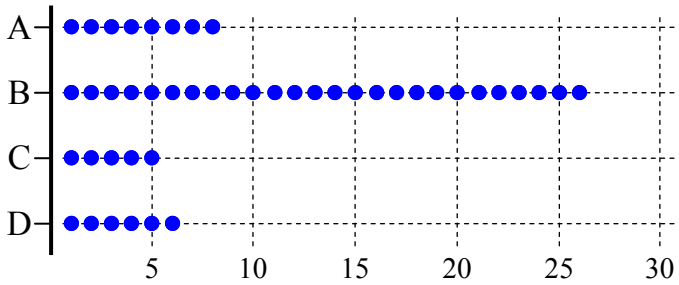
1 Data: ~~4~~, 5, 3, 3, 6, 1, 2, 4, 2, 3, 5, 6, 2, 3, 4, 4, 1, 2, 5, 6, 2, 3, 3, 2, 5, 2, 1, 4, 6, 3, 2.



2 An artificial sweetener is comparing itself with three other brands. Graph the results.

Data: BBBDA BABBC ABBBC
BDBAB ABBDB BCBBD
BCABB ABCBD BBADB

sideways! → Artificial Sweetener Preference



3 How many people were sampled? 45

4 How many people preferred Brand B? 26

5 How many people preferred Brand C? 5

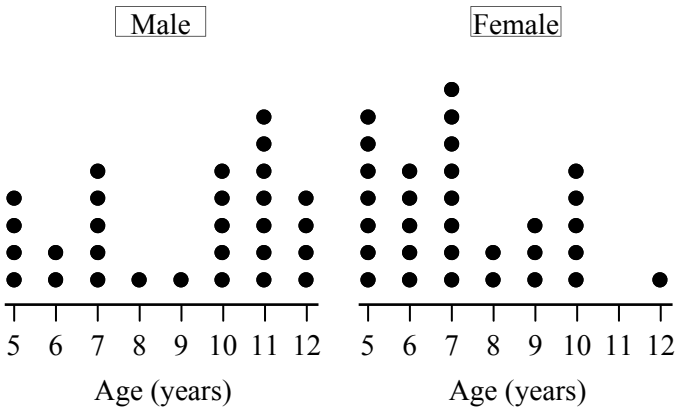
6 What percentage of people prefer Brand B?

Question 4 Ans. → 26 / Question 3 Ans. → 45 × 100 = 57.8%
Round to 1 d.p.

7 What percentage of people prefer Brand C?

Question 5 Ans. → 5 / Question 3 Ans. → 45 × 100 = 11.1%
Round to 1 d.p.

A waterslide at a theme park records the ages of the first 60 children who use the slide, the results are shown in two Dot plots. One male, one female.



8 How many children were male? 29

9 How many children were 7 years old? 13

10 How many children were 11 or older? 12

11 How many more 10 year olds used the slide compared to 6 year olds? 10 - 7 = 3

12 Are these true or false? There were more ...

(T) (F) Girls using the waterslide

(T) (F) 7 y.o. boys using the slide than 10 y.o. girls

(T) (F) Older male children than female.

13 What percentage of 9 y.o.'s were male?

Male 9 y.o. here → 1 / Total 9 y.o. here → 4 × 100 = 25%
Show the % sign

14 What percentage of 5 y.o.'s were female?

7 / 11 × 100 = 63.6%
Round to 1 d.p.

15 What percentage of females were 10 y.o.?

Total females here → 5 / 31 × 100 = 16.1%
Round to 1 d.p.

16 What percentage of 5 - 7 y.o.'s were male?

11 / 31 × 100 = 35.5%
Round to 1 d.p.

Data Analysis - Dot Plots and Frequency Tables (Sheet 2)

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Complete the frequency column and total for Σf, then answer the questions.

1 Number of Fish in a Tank

Fish	Tally	Frequency
2	𝍸	5
3	𝍸 ||	7
4	𝍸 𝍸 |||	13
5	𝍸 𝍸	10
6	𝍸 𝍸 ||||	14
7	𝍸 𝍸 |||	13
	$\Sigma f =$	62

2 What number of fish had the highest frequency? 6

3 What number of fish had the same frequency? 4 and 7

4 What was the frequency for a tank to have an odd number of fish?

7 + 10 + 13 = 30

5 What was the frequency for a tank to have less than 5 fish?

13 + 7 + 5 = 25

6 Soft Drink Preference

Type	Tally	Frequency
A	𝍸 𝍸 𝍸 ||	17
B	𝍸 𝍸 |||	13
C	||||	4
D	𝍸 𝍸 ||||	14
E	𝍸 𝍸 ||	12
F	𝍸 𝍸 𝍸	15
	$\Sigma f =$	75

7 D's results were unreadable due to a drink spill, use Σf to calculate them.

8 Rank the drink brands in order from least, to most popular.

C, E, B, D, F, A.

9 How many people preferred brand 'A' or brand 'F'? 17 + 15 = 32

Place the data into the Frequency table, showing the tally, the frequency and write 'Σf= *(total)*', then answer the questions.

10 Away from School - Sports Choices

Sport	Tally	Frequency
Tennis	𝍸 𝍸 𝍸 𝍸 ||||	24
Gym	𝍸 𝍸 |||	13
Bocce	𝍸 𝍸 𝍸 𝍸 |||	23
Archery	𝍸 𝍸 𝍸 𝍸	20
		$\Sigma f = 80$

Data:

TTGAA	TTGAA
TGBBA	GBBAT
AAGGG	TTTTA
BTBTA	BTBBA
AAATG	GTBBA
BBGBT	AABTB
TTGBB	BBTBG
ATTTA	ABBTG

11 What was most popular 'away' sport chosen? Tennis

12 How many people chose 'ball sports'? 47

13 If there are a total of 135 students in the year, calculate the percentage that chose 'away' sports.

$$\%\ \text{Away Sport} = \frac{80}{135} \times 100$$

$$= 59.3\ \%$$

Round to 1 d.p.

14 A real estate agent is checking rental homes for smoke detectors, if the smoke detector was Battery powered, Mains powered, or Dual (both). If a detector didn't work its type wasn't recorded it was just recorded as Inoperative. If a house had no smoke detector, it was recorded as 'None'.

Smoke Detector Type or Status

Type	Tally	Frequency
Battery	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 |	31
Mains	𝍸 𝍸	10
Dual	𝍸 ||||	9
Inoperative	𝍸 𝍸	10
None	𝍸 𝍸	10
		$\Sigma f = 70$

Data:

BBDMB	BBDBM
BIBBM	MNDIB
MBDBI	BBIIN
NBBIM	INDDB
BBNIB	NBBMB
MBBDN	IBBMB
NINBB	NDDMB

15 How many homes had operational smoke detectors? 31 + 10 + 9 = 50

16 What percentage (to 1 d.p.) of homes had inoperative or no smoke detectors?

$$\%\ \text{No S.D} = \frac{20}{70} \times 100 = 28.6\%$$

Round to 1 d.p.

17 What percentage (to 1 d.p.) of homes had Dual-powered smoke detectors?

$$\%\ \text{Dual Powered} = \frac{9}{70} \times 100 = 12.9\%$$

Round to 1 d.p.

Data Analysis - Dot Plots and Frequency Tables (Homework Strip)

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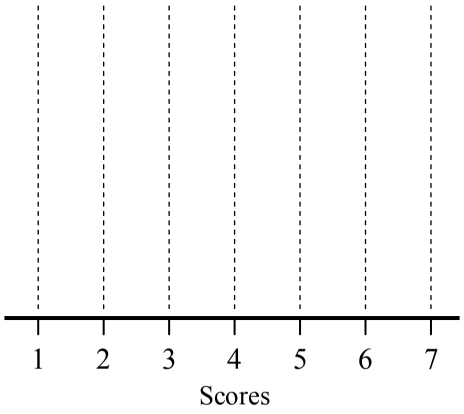
Date Due:

Construct a dot plot for the 35 scores

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 1, 1, 7, 2, 6, 3, 6, 5, 7, 3, 4, 1, 6, 5, 3, 5, 7, 3, 5, 3, 2, 1, 3, 1, 3, 5, 7.



Place the data in a Frequency Table

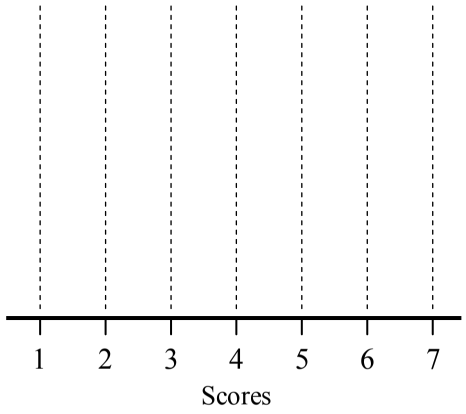
2 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

Date Due:

Construct a dot plot for the 35 scores

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 1, 1, 7, 2, 6, 3, 6, 5, 7, 3, 4, 1, 6, 5, 3, 5, 7, 3, 5, 3, 2, 1, 3, 1, 3, 5, 7.



Place the data in a Frequency Table

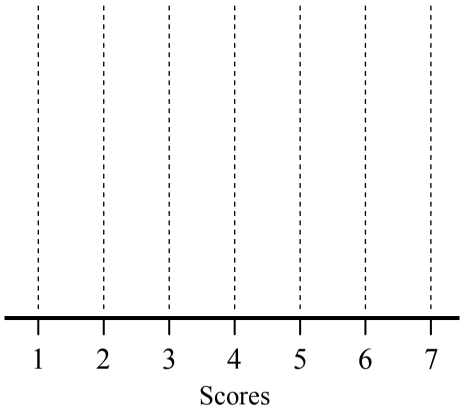
2 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

Date Due:

Construct a dot plot for the 35 scores

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 1, 1, 7, 2, 6, 3, 6, 5, 7, 3, 4, 1, 6, 5, 3, 5, 7, 3, 5, 3, 2, 1, 3, 1, 3, 5, 7.



Place the data in a Frequency Table

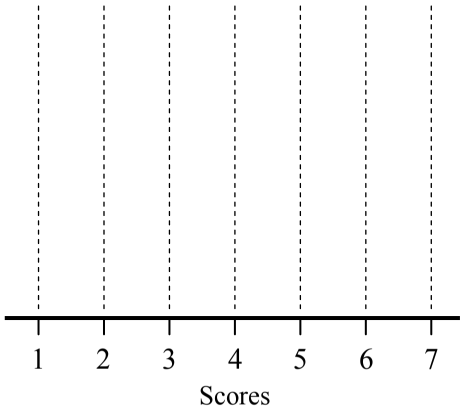
2 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

Date Due:

Construct a dot plot for the 35 scores

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 1, 1, 7, 2, 6, 3, 6, 5, 7, 3, 4, 1, 6, 5, 3, 5, 7, 3, 5, 3, 2, 1, 3, 1, 3, 5, 7.



Place the data in a Frequency Table

2 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
		$\Sigma f =$

Data Analysis - Dot Plots and Frequency Tables (Homework Strip)

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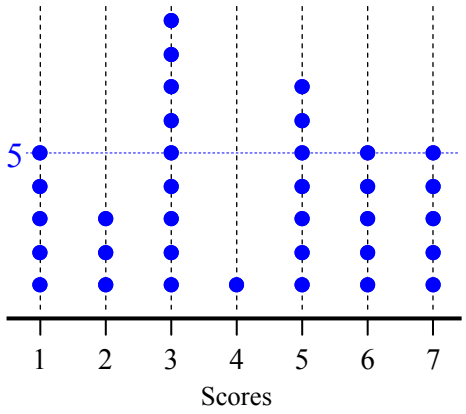
Date Due:

Construct a dot plot for the 35 scores

1 Data: 6, 3, 5, 7, 2, 3, 6, 5, 1, 1, 7, 2, 6, 3, 6, 5, 7, 3, 4, 1, 6, 5, 3, 5, 7, 3, 5, 3, 2, 1, 3, 1, 3, 5, 7.



Place the data in a Frequency Table

2 Data: 1, 6, 3, 4, 5, 5, 3, 5, 3, 4, 5, 1, 2, 4, 6, 3, 2, 1, 5, 5, 4, 3, 2, 2, 3, 4, 1, 5, 6, 3, 2, 2, 3, 1, 6.

Score	Tally	Frequency
1	𝍸	5
2	𝍸 |	6
3	𝍸 |||	8
4	𝍸	5
5	𝍸 ||	7
6						4
		$\Sigma f = 35$

Data Analysis - Ordered Stem and Leaf Plots

Stem and leaf plots separate data into its units component and its tens (and above) component. They then display the 'units' on the right in the leaf section and the 'tens' on left in a stem column. The effect is to break the data up into classes and also display a simple column graph (on its side) of numbers showing the way the data is spread.

Numbers have their units removed so:

- 23 becomes 'a 2' and 'a 3'
- 15 becomes 'a 1' and 'a 5'
- 9 (is seen as 09) becomes 'a 0' and 'a 9'

Stem	Leaf
0	9
1	5
2	3

In a plot this looks like the plot shown above. But what about larger numbers?

- 106 becomes 'a 10' and 'a 6'
- 117 becomes 'a 11' and 'a 7'
- 125 becomes 'a 12' and 'a 5'
- 120 becomes 'a 12' and 'a 0'
- 110 becomes 'a 11' and 'a 0'

Stem	Leaf
10	6
11	70
12	50

Because the units have two digits in front, both digits are shown in the Stem column.

When all data has been entered into the plot, the plot then has to be rewritten in order. The stem stays the same, but the leaf is put into ascending order.

Stem	Leaf
10	6
11	70
12	50

Leaf in order

Stem	Leaf
10	6
11	07
12	05

Back-to-back stem and leaf plots are used to display 2 different groups of data together in the one plot. The groups are displayed separately but share a common stem. One group is displayed on the left hand side, the other on the R.H.S.

The L.H.S has its leaf in ascending order (going up) with larger numbers to the left, while the R.H.S. leaf has its numbers ascending, as with single leaf stem and leaf plots, up to the right.

Leaf	Stem	Leaf
711	8	0338
80	9	55559
77632	10	2234477
88221	11	0777

Left hand side leaf increases to the left

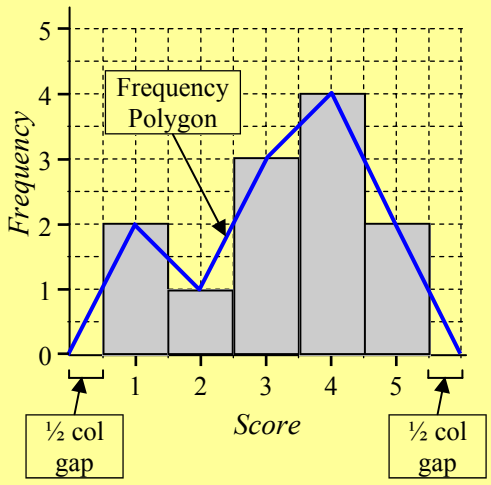
Stem is shared

Right hand side leaf increases to the right

Data Analysis - Frequency Histograms and Frequency Polygons

Frequency Histograms are very similar to column graphs, the difference being that:

- A half-column gap is left between the first column and the vertical axis.
- The columns touch each other, there is no gap between them.
- A half-column gap is left after the last column.



Frequency Polygons are similar to line graphs, they:

- Always start and finish at the horizontal axis.
- They inter-connect the centres of the top of columns.

Because a half-column gap is required, it makes sense to use columns that are '2 grids' wide so that the grids help you to draw the graph. You shouldn't make a column '1 grid wide' and the gap '½ a grid wide' because your columns won't use the grid lines, it will be harder to create and use.

Data Analysis - Ordered Stem and Leaf Plots (Sheet 1)

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Display the data sets in Raw then Ordered Stem and Leaf plots. Don't use commas!

1 Data: 23, 17, 35, 21, 30, 8, 11, 24, 6, 9, 17, 21, 32, 3, 11, 13.

RAW → ORDERED

Tens	Units
0	
1	
2	
3	

Tens	Units
0	
1	
2	
3	

2 Data: 74, 97, 70, 106, 92, 100, 86, 91, 97, 85, 110, 83, 101, 95, 79, 90, 103, 85.

RAW → ORDERED

Stem	Leaf

Stem	Leaf

3 Data: 998, 971, 1 005, 993, 1 018, 1 015, 985, 994, 1 026, 1 012, 1 002, 994, 978, 993, 970, 1 018, 989, 991, 1 011, 1 021, 986, 1 000, 1 002, 995, 1 012, 977, 983.

Stem	Leaf

→

Stem	Leaf



Rewrite this stem and leaf plot in order. That's an order!

Same data shown in order

65440	0	04456
955	1	559
7200	2	0027

4

RAW Leaf	Stem	Leaf
8 3 0 2 6	0	0 0 2 8 5
7 3 3 9	1	7 1 6 5 7 9
7 8 0 4 1 1	2	2 0 3 8 8 4
3 9 8 6	3	9 2 8 1 3

↓

ORDERED Leaf	Stem	Leaf
High ← Low		Low → High
	0	
	1	
	2	
	3	



It's auction time! 'Four n'Fluffy' livestock sales is selling sheep and alpacas. Lot sales are shown below. Place the data in a back-to-back stem and leaf plot, first raw data, then in order.

5 Four n'Fluffy Auction Sales (in Lots)

Sheep: 22, 25, 8, 37,18, 13, 3, 22, 15, 17, 34, 22, 17, 12, 20, 23, 11, 27.

Alpacas: 36, 28, 5, 38, 22, 19, 12, 7, 25, 37, 19, 31, 26, 30, 23, 37, 27, 19, 20.

RAW Alpacas	Stem	Sheep

↓

ORDERED Alpacas	Stem	Sheep

Data Analysis - Frequency Histograms and Frequency Polygons (Sheet 2)

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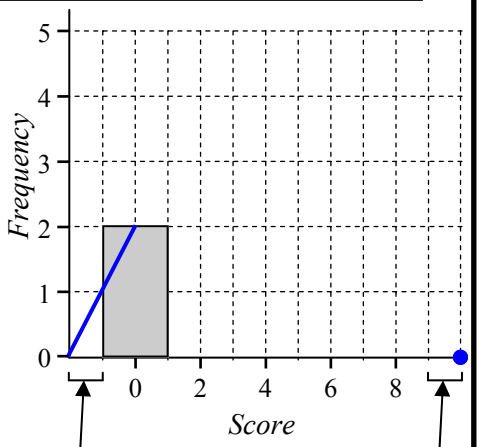
Construct a frequency histogram and frequency polygon for the data in these frequency tables.

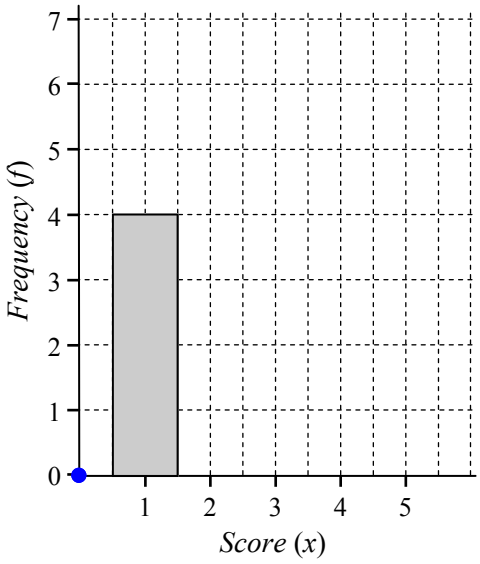
1

Score	*Frequency*
0	2
2	3
4	5
6	1
8	4



Columns touch each other - no gaps between
There is a ½ column gap at the start and at the finish of the columns. Polygon ends at the dot. ●

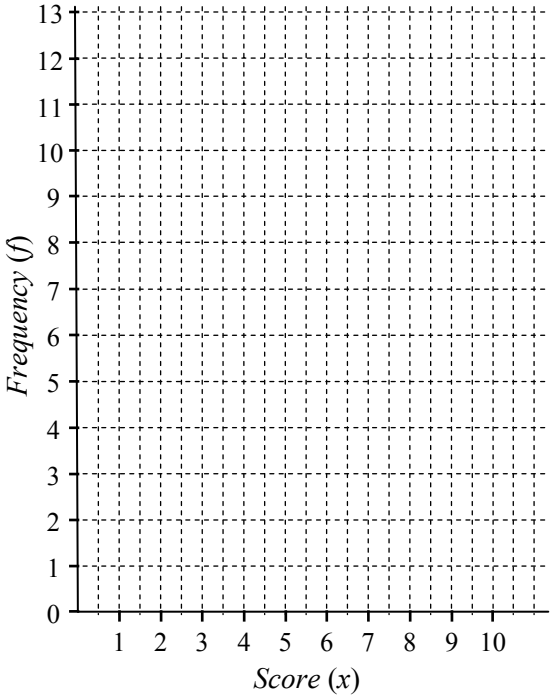
2



x	*f*
1	4
2	2
3	7
4	0
5	6

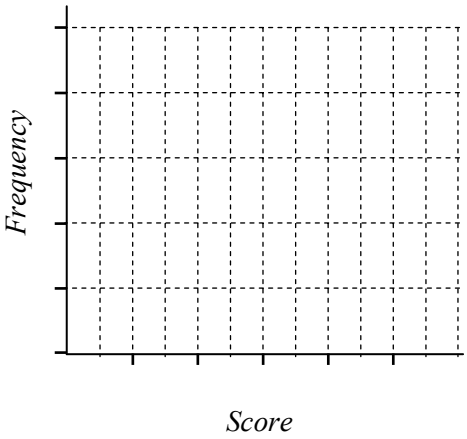
3

x	*f*
1	2
2	0
3	4
4	6
5	7
6	10
7	13
8	12
9	8
10	5



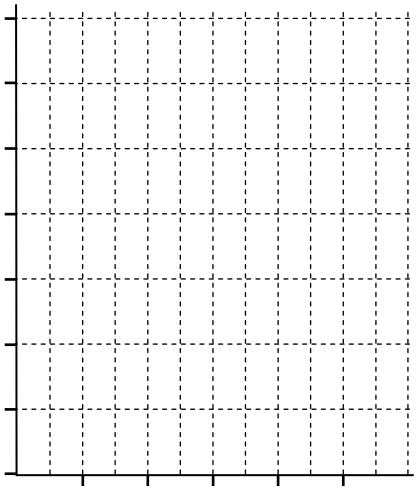
4

Score	*f*
5	5
10	1
15	3
20	0
25	4



5

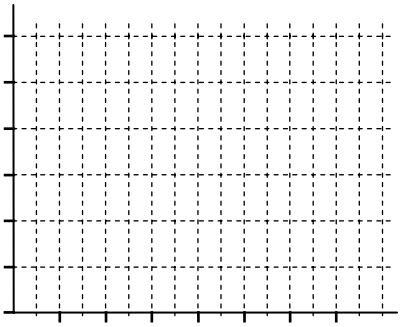
Score	*f*
0	2
2	5
4	3
6	7
8	1



6 Soccer - Home Team - Goals Scored

Data:
1, 2, 5, 2, 0, 3, 2,
3, 4, 1, 1, 3, 1, 2,
3, 3, 0, 2, 1, 2, 2,
1, 4, 0.

Score	*Tally*	*f*
		$\Sigma f =$



Data Analysis - Ordered Stem and Leaf Plots (Sheet 1)

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Display the data sets in Raw then Ordered Stem and Leaf plots. Don't use commas!

1 Data: 23, 17, 35, 21, 30, 8, 11, 24, 6, 9, 17, 21, 32, 3, 11, 13.

RAW → ORDERED

Tens	Units
0	8 6 9 3
1	7 1 7 1 3
2	3 1 4 1
3	5 0 2

Tens	Units
0	3 6 8 9
1	1 1 3 7 7
2	1 1 3 4
3	0 2 5

2 Data: 74, 97, 70, 106, 92, 100, 86, 91, 97, 85, 110, 83, 101, 95, 79, 90, 103, 85.

RAW → ORDERED

Stem	Leaf
7	4 0 9
8	6 5 3 5
9	7 2 1 7 5 0
10	6 0 1 3
11	0

Stem	Leaf
7	0 4 9
8	3 5 5 6
9	0 1 2 5 7 7
10	0 1 3 6
11	0

3 Data: 998, 971, 1 005, 993, 1 018, 1 015, 985, 994, 1 026, 1 012, 1 002, 994, 978, 993, 970, 1 018, 989, 991, 1 011, 1 021, 986, 1 000, 1 002, 995, 1 012, 977, 983.

Stem	Leaf
97	1 8 0 7
98	5 9 6 3
99	8 3 4 4 3 1 5
100	5 2 0 2
101	8 5 2 8 1 2
102	6 1

→

Stem	Leaf
97	0 1 7 8
98	3 5 6 9
99	1 3 3 4 4 5 8
100	0 2 2 5
101	1 2 2 5 8 8
102	1 6



Rewrite this stem and leaf plot in order. That's an order!

Same data shown in order

65440	0	04456
955	1	559
7200	2	0027

4

RAW Leaf	Stem	Leaf
8 3 0 2 6	0	0 0 2 8 5
7 3 3 9	1	7 1 6 5 7 9
7 8 0 4 1 1	2	2 0 3 8 8 4
3 9 8 6	3	9 2 8 1 3

↓

ORDERED Leaf	Stem	Leaf
High ← Low		Low → High
8 6 3 2 0	0	0 0 2 5 8
9 7 3 3	1	1 5 6 7 7 9
8 7 4 1 1 0	2	0 2 3 4 8 8
9 8 6 3	3	1 2 3 8 9



It's auction time! 'Four n'Fluffy' livestock sales is selling sheep and alpacas. Lot sales are shown below. Place the data in a back-to-back stem and leaf plot, first raw data, then in order.

5 Four n'Fluffy Auction Sales (in Lots)

Sheep: 22, 25, 8, 37,18, 13, 3, 22, 15, 17, 34, 22, 17, 12, 20, 23, 11, 27.

Alpacas: 36, 28, 5, 38, 22, 19, 12, 7, 25, 37, 19, 31, 26, 30, 23, 37, 27, 19, 20.

RAW Alpacas	Stem	Sheep
7 5	0	8 3
9 9 2 9	1	8 3 5 7 7 2 1
0 7 3 6 5 2 8	2	2 5 2 2 0 3 7
7 0 1 7 8 6	3	7 4

↓

ORDERED Alpacas	Stem	Sheep
7 5	0	3 8
9 9 9 2	1	1 2 3 5 7 7 8
8 7 6 5 3 2 0	2	0 2 2 2 3 5 7
8 7 7 6 1 0	3	4 7

Data Analysis - Frequency Histograms and Frequency Polygons (Sheet 2)

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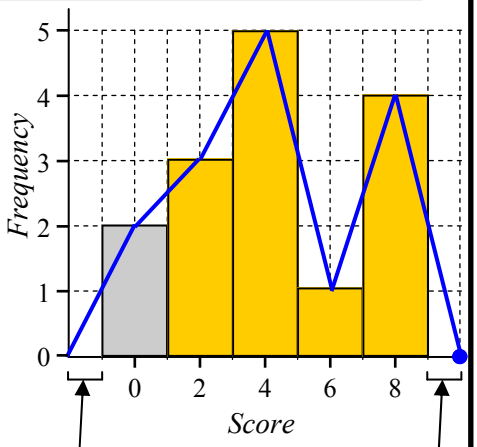
Construct a frequency histogram and frequency polygon for the data in these frequency tables.

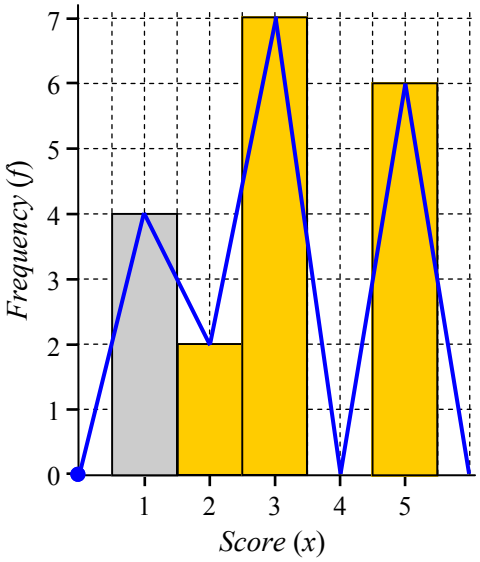
1

Score	Frequency
0	2
2	3
4	5
6	1
8	4



Columns touch each other - no gaps between
There is a ½ column gap at the start and at the finish of the columns. Polygon ends at the dot. ●

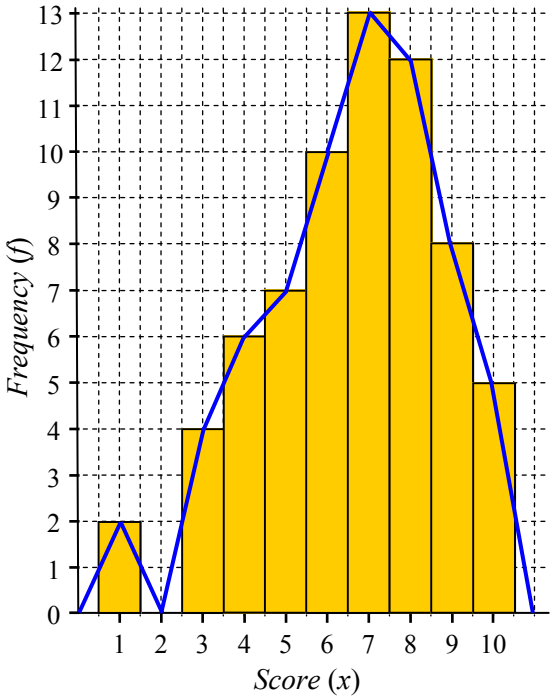
2



x	f
1	4
2	2
3	7
4	0
5	6

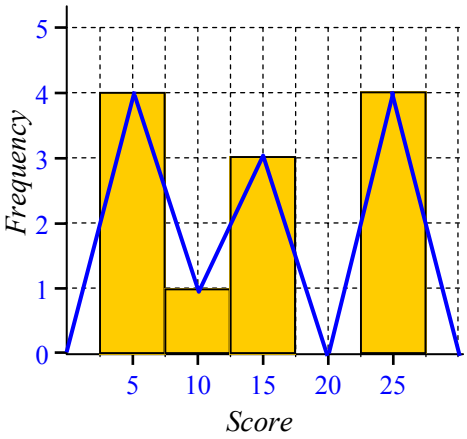
3

x	f
1	2
2	0
3	4
4	6
5	7
6	10
7	13
8	12
9	8
10	5



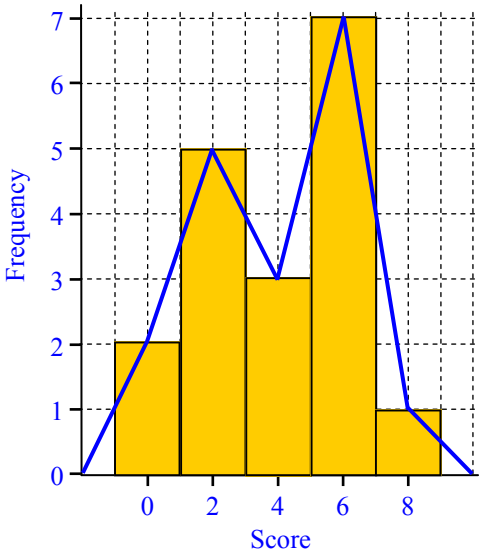
4

Score	f
5	5
10	1
15	3
20	0
25	4



5

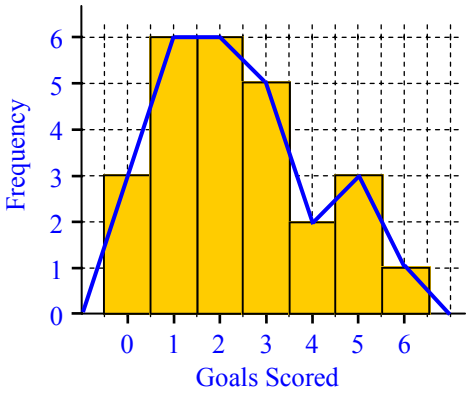
Score	f
0	2
2	5
4	3
6	7
8	1



6 Soccer - Home Team - Goals Scored

Data:
1, 2, 5, 2, 6, 0, 3,
2, 3, 4, 1, 1, 3, 1,
2, 3, 3, 0, 2, 1, 5,
2, 1, 4, 0, 5.

Score	Tally	f
0	|||	3
1	𝍸 |	6
2	𝍸 |	6
3	𝍸	5
4	||	2
5	|||	3
6	|	1
		Σf = 26



Data Analysis - Stem and Leaf Plots and Frequency Histograms (Homework Strip)

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Date Due: _____

Organise the data in raw & ordered, stem and leaf plots

1 Data: 25, 11, 22, 13, 30, 6, 19, 26, 12, 14, 29, 37, 5, 11, 2, 33, 20, 14, 37, 32.

RAW

Tens	Units
0	
1	
2	
3	

IN ORDER →

ORDERED

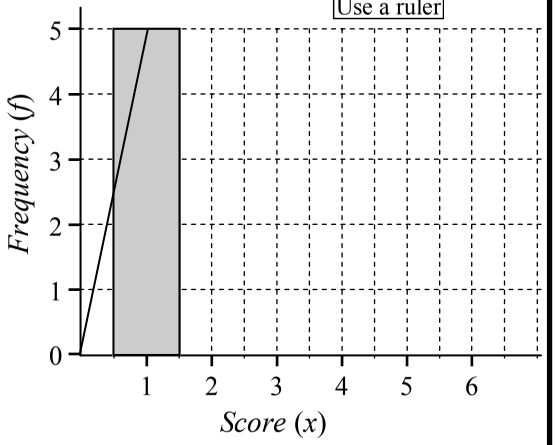
Tens	Units
0	
1	
2	
3	

Construct a frequency histogram/polygon from the table

2

x	f
1	5
2	2
3	4
4	1
5	2
6	3

Use a ruler



Date Due: _____

Organise the data in raw & ordered, stem and leaf plots

1 Data: 25, 11, 22, 13, 30, 6, 19, 26, 12, 14, 29, 37, 5, 11, 2, 33, 20, 14, 37, 32.

RAW

Tens	Units
0	
1	
2	
3	

IN ORDER →

ORDERED

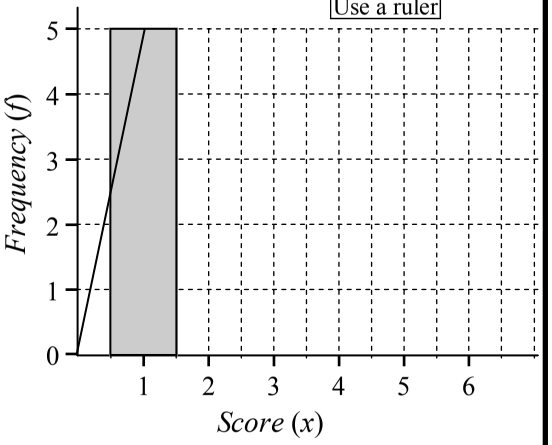
Tens	Units
0	
1	
2	
3	

Construct a frequency histogram/polygon from the table

2

x	f
1	5
2	2
3	4
4	1
5	2
6	3

Use a ruler



Date Due: _____

Organise the data in raw & ordered, stem and leaf plots

1 Data: 25, 11, 22, 13, 30, 6, 19, 26, 12, 14, 29, 37, 5, 11, 2, 33, 20, 14, 37, 32.

RAW

Tens	Units
0	
1	
2	
3	

IN ORDER →

ORDERED

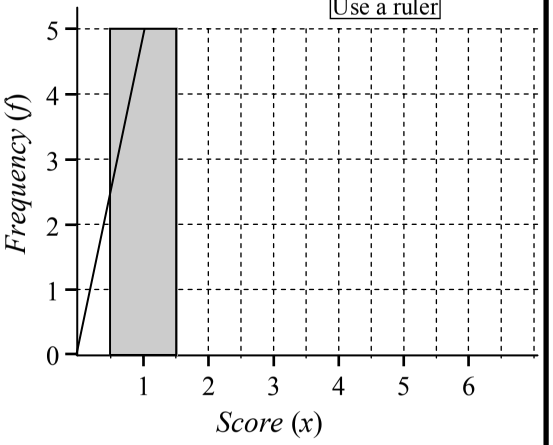
Tens	Units
0	
1	
2	
3	

Construct a frequency histogram/polygon from the table

2

x	f
1	5
2	2
3	4
4	1
5	2
6	3

Use a ruler



Data Analysis - Stem & Leaf Plots/ Frequency Histograms (H'work Strip)

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Date Due: _____

Organise the data in raw & ordered, stem and leaf plots

1 Data: 25, 11, 22, 13, 30, 6, 19, 26, 12, 14, 29, 37, 5, 11, 2, 33, 20, 14, 37, 32.

RAW

Tens	Units
0	6 5 2
1	1 3 9 2 4 1 4
2	5 2 6 9 0
3	0 7 3 7 2

IN ORDER →

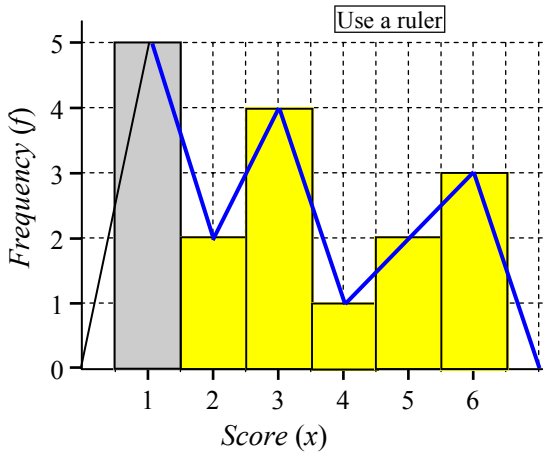
ORDERED

Tens	Units
0	2 5 6
1	1 1 2 3 4 4 9
2	0 2 5 6 9
3	0 2 3 7 7

Construct a frequency histogram/polygon from the table

2

x	f
1	5
2	2
3	4
4	1
5	2
6	3



Data Analysis - The Mean

The Mean is the mathematical term for "Average". So when you find the mean you are finding the average. The mean is found by adding all the scores then dividing the total by the number of scores. As a formula it is written:

The short way of writing the mean is $\bar{x}$. *'x bar'*

$$\bar{x} = \frac{\Sigma x}{n}$$

Σx means the sum of x. So the total of all the scores in the data set.

n means the number of scores in the data set.

Example - Round to 1 decimal place.

Data: 14, 19, 35

$$\bar{x} = \frac{68}{3} = 22.7$$

14 + 19 + 35 = 68

Three scores

Round to 1 d.p.

Calculator Steps:

6 8 ÷ 3 =

Calculator Screen:

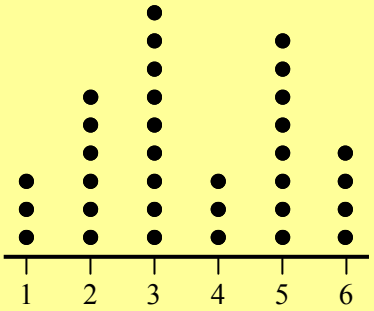
22.6666666….

Column 1 also includes questions involving units. These questions have to be in the same units so look at the question and check what units the answer must have, then change all the data to those units first. Then add and divide for the mean.

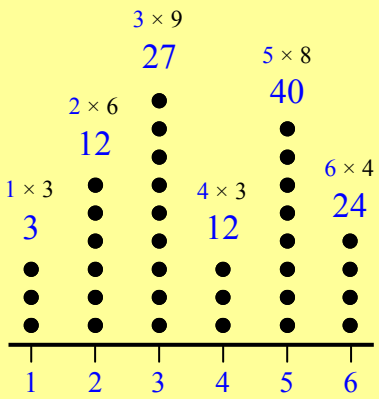
Column 2 features dot plots, the value for *fx* is found by multiplying the score and the frequency directly on the plot. You then add up the columns and divide by the number of dots.

Example - Dot Plots

STEP 1 Here is a dot plot, find the mean.



STEP 2 Calculate *fx* for each column by multiplying *f* & *x*.



STEP 3 Calculate Σfx by adding all the column *fx* together.

3 + 12 + 27 + 12 + 40 + 24

$\Sigma fx =$ 118

STEP 4 Calculate Σf by counting all the dots.

3 + 6 + 9 + 3 + 8 + 4

$\Sigma f =$ 33

STEP 5 Find $\bar{x}$ to 2 d.p.

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{118}{33}$$

$\bar{x} = 3.58$ (2 d.p.)

The second sheet features Stem and Leaf plots, remember that the scores are found by joining the tens column with the units column, an example is below.

Example - Stem and Leaf Plots

Tens	Units
0	3 5
1	0 5 6
2	2 2

$\bar{x} = \frac{93}{7} = 13.3$

$3 + 5 + 10 + 15 + 16 + 22 + 22 = 93$

Seven scores.

Round to 1 d.p.

Back-to-back stem and leaf plots are also featured at the base of the column. These data sets can have the mean calculated for each half of the plot or for the entire plot.

Frequency tables display grouped data similar to a dot plot, except the data is in a table. The formula for the calculation is:

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

Where: $\bar{x}$ is the mean of the scores
Σfx is the sum of the fx column
Σf is the sum of the f column

Example - Finding the mean from a frequency table

STEP 1 Frequency table, find the mean.

Score (x)	*Frequency (f)*
0	6
2	2
4	5
6	8
8	15
10	7

STEP 2 Add an fx column and 'total' boxes at the base.

Score (x)	*Frequency (f)*	*fx*
0	6	
2	2	
4	5	
6	8	
8	15	
10	7	

STEP 3 Multiply the f and the x to get fx.

Score (x)		*Frequency (f)*		*fx*
0	×	6	=	0
2	×	2	=	4
4	×	5	=	20
6	×	8	=	48
8	×	15	=	120
10	×	7	=	70

STEP 4 Add the f column for Σf and the fx column for Σfx.

Score (x)	*Frequency (f)*	*fx*
0	6	0
2	2	4
4	5	20
6	8	48
8	15	120
10	7	70
	$\Sigma f = 43$	$\Sigma fx = 262$

STEP 5 Calculate the mean. (1 d.p.)

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{262}{43} = 6.1 \text{ (1 d.p.)}$$

Data Analysis - The Mean - Data Sets & Dot Plots (Sheet 1)

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Calculate the mean. Add the scores and divide by the number of scores. Answers are whole numbers

1 27, 38, 24, 31, 44, 56, 50, 19, 26.

Add the scores

$\bar{x} = \frac{\square}{9} = \square$ Divide total by 9

Nine scores = 9

2 73, 86, 41, 29, 56. $\bar{x} = \frac{\square}{\square} = \square$

3 -6, 2, -14. $\bar{x} = \frac{\square}{\square} = \square$

4 16 773, 24 919. $\bar{x} = \frac{\square}{\square} = \square$

5 235, 306, 113, 176, 290, 218, 440.

$\square = \frac{\square}{\square} = \square$

This time you get a decimal answer. Round your answer to 1 decimal place.

6 94, 38, 77, 24, 83, 19.

$\square = \frac{\square}{\square} = \square$ (1 d.p.)

7 55, 78, 11, 107.

8 9, 5.4, 4.3, 6, 7, 3.

9 187, 553, 213.

10 57.17, -44.88.

Find the average height of these students. Find the average height in m (2 d.p.), then in cm.

11 1.75 m, 1.8 m, 1.73 m, 1.55 m, 1.77 m.

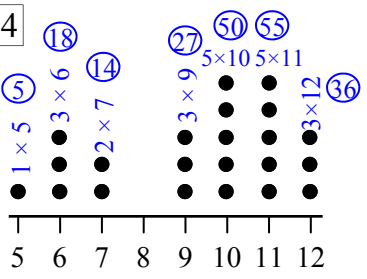
$\square = \frac{\square}{\square} = \square$ m $= \square$ cm

12 1.6 m, 1.74 m, 1.82 m, 1.77 m, 1.83 m, 1.8 m.

13 Find the average height for the 11 students.

Find the mean (1 d.p.) for the data displayed in these dot plots. Find dots × score, add for a total. Then divide the total by the total number of dots.

14



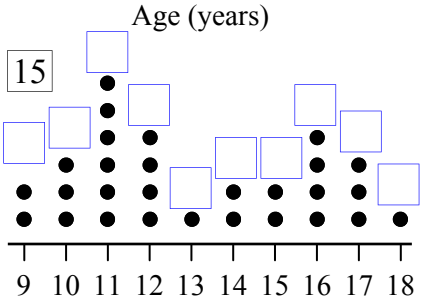
Age (years)

5 + 18 + 14 +...

$\bar{x} = \frac{\square}{\square}$ All Dots

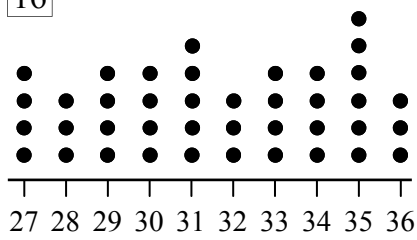
$\bar{x} = \square$ (1 d.p.)

15

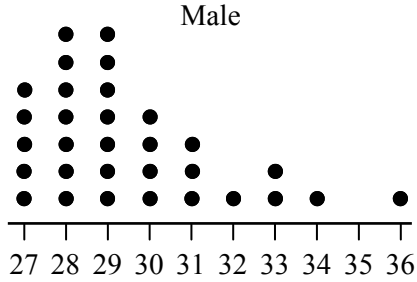


Sales

16



Male



Female

17 Comment on the Male and Female Dot Plots

Compare the mean results and shape of the distribution

Place the data in a dot plot and find the mean.

18 Data: 6, 8, 2, 3, 6, 4, 3, 2, 6, 4, 4, 2, 6, 4, 2, 2, 6, 5, 3, 2, 2, 3, 6, 8, 3.

Data Analysis - The Mean - Stem & Leaf & Freq Tables (Sheet 2)

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Calculate the mean. Add the scores and divide by the number of scores. Answer to 1 d.p. if needed.

1

Stem	Leaf
0	7 8
1	0 4 6
2	8 8 9

Remember the 10's: 7,8 — 10,14,16 — 28,28,29

Add the scores

$\bar{x} =$ ☐ / 8

Divide total

$\bar{x} =$ ☐ by 8

2

Tens	Units
8	4 4 5
9	3
10	0 0 7

$\bar{x} =$ ☐ / ☐

$\bar{x} =$ ☐

3

Tens	Units
18	1 7 7
19	2 6
20	0 3 6
21	1

_____

_____

4

AM.	Tens	PM.
1 1 1	0	5
9 8 7 6	1	8 8 9
0 0	2	1 7 7 8
9 7 3 3	3	0 0 2 2 6

AM: _____ PM: _____

_____ _____

5

Boys	Tens	Girls
8	15	0 3 6 7 7
8 7 5 5	16	2 4 9
9 3	17	1 1
4 1 1	18	

Boys: _____ Girls: _____

_____ _____

Boys & Girls: _____

Complete *fx* columns, find Σfx and divide by the number of scores (Σf) to find the mean to 1 d.p.

6 Use a ruler to go over the dotted lines.

Score (x)		Frequency (f)		fx
0	×	5	=	
2	×	1	=	
4	×	7	=	
6	×	0	=	
8	×	8	=	
10	×	7	=	
Sum the *f* column →		$\Sigma f =$		$\Sigma fx =$ ← Sum the *fx* column

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{\square}{\square}$

$\bar{x} =$ ☐ 1 d.p.

7

Score	f	
0	2	
1	3	
2	7	
3	12	
4	9	
5	14	
6	13	
	$\Sigma f =$	$\Sigma fx =$

8

Score	f	
0	4	
5	2	
10	7	
15	11	
20	6	
25	4	

_____

_____

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{\square}{\square}$

$\bar{x} =$ ☐

Place this data in a frequency table and then find the mean to 1 d.p.

9 Data: 1, 3, 5, 5, 2, 2, 3, 5, 2, 4, 3, 1, 6, 1, 3, 2, 5, 4, 4, 3, 2, 2, 5, 3, 6, 1, 2, 4, 2, 3.

Score	Tally	f	fx

_____

Data Analysis - The Mean - Data Sets & Dot Plots (Sheet 1)

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Calculate the mean. Add the scores and divide by the number of scores. Answers are whole numbers

1 27, 38, 24, 31, 44, 56, 50, 19, 26.

Add the scores

$\bar{x} = \frac{315}{9} = 35$

Divide total by 9

Nine scores = 9

2 73, 86, 41, 29, 56. $\bar{x} = \frac{285}{5} = 57$

3 -6, 2, -14. $\bar{x} = \frac{-18}{3} = -6$

4 16 773, 24 919. $\bar{x} = \frac{41\,692}{2} = 20\,846$

5 235, 306, 113, 176, 290, 218, 440.

$\bar{x} = \frac{1\,778}{7} = 254$

This time you get a decimal answer. Round your answer to 1 decimal place.

6 94, 38, 77, 24, 83, 19.

$\bar{x} = \frac{335}{6} = 55.8$ (1 d.p.)

7 55, 78, 11, 107.

$\bar{x} = \frac{251}{4} = 62.8$

8 9, 5.4, 4.3, 6, 7, 3.

$\bar{x} = \frac{34.7}{6} = 5.8$

9 187, 553, 213.

$\bar{x} = \frac{953}{3} = 317.7$

10 57.17, -44.88.

$\bar{x} = \frac{12.29}{2} = 6.1$

Find the average height of these students. Find the average height in m (2 d.p.), then in cm.

11 1.75 m, 1.8 m, 1.73 m, 1.55 m, 1.77 m.

$\bar{x} = \frac{8.6}{5} = 1.72$ m = 172 cm

12 1.6 m, 1.74 m, 1.82 m, 1.77 m, 1.83 m, 1.8 m.

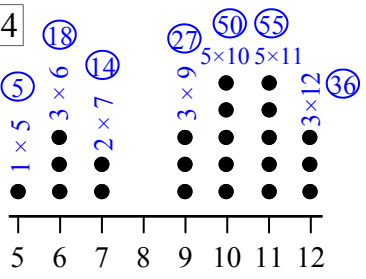
$\bar{x} = \frac{10.56}{6} = 1.76$ m = 176 cm

13 Find the average height for the 11 students.

$\bar{x} = \frac{19.16}{11} = 1.74$ m (2 d.p.) = 174 cm

Find the mean (1 d.p.) for the data displayed in these dot plots. Find dots × score, add for a total. Then divide the total by the total number of dots.

14



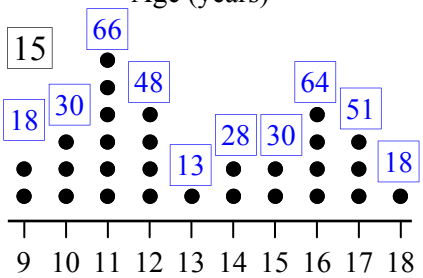
Age (years)

5 + 18 + 14 +...

$\bar{x} = \frac{205}{22}$ All Dots

$\bar{x} = 9.3$ (1 d.p.)

15

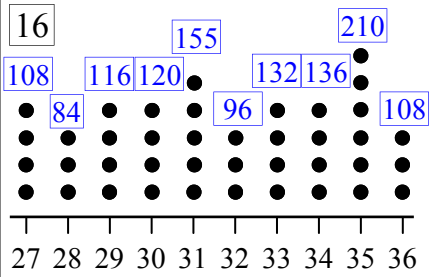


Sales

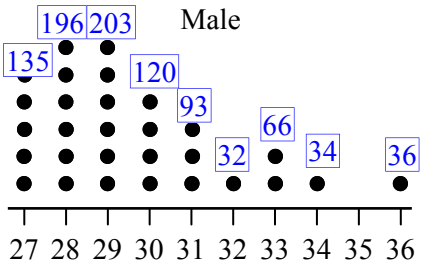
$\bar{x} = \frac{366}{28}$

$\bar{x} = 13.1$

16



Male



Female

$\bar{x} = \frac{1\,265}{40}$

$\bar{x} = 31.6$

$\bar{x} = \frac{915}{31}$

$\bar{x} = 29.5$

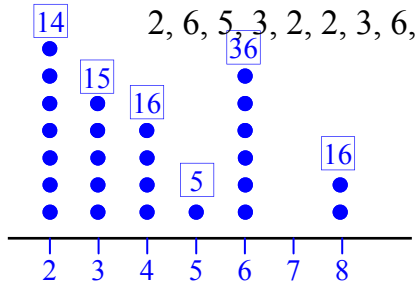
17 Comment on the Male and Female Dot Plots

Compare the mean results and shape of the distribution

Male data had a higher mean. Male data was evenly distributed, female positive skewed.

Place the data in a dot plot and find the mean.

18 Data: 6, 8, 2, 3, 6, 4, 3, 2, 6, 4, 4, 2, 6, 4, 2, 2, 6, 5, 3, 2, 2, 3, 6, 8, 3.



$\bar{x} = \frac{102}{25}$

$\bar{x} = 4.08$

or 4.1 (1 d.p.)

Data Analysis - The Mean - Stem & Leaf & Freq Tables (Sheet 2)

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Calculate the mean. Add the scores and divide by the number of scores. Answer to 1 d.p. if needed.

1

Stem	Leaf
0	7 8
1	0 4 6
2	8 8 9

Remember the 10's: 7,8 — 10,14,16 — 28,28,29

Add the scores: $\bar{x} = \frac{140}{8}$

Divide total: $\bar{x} = 17.5$ by 8

2

Tens	Units
8	4 4 5
9	3
10	0 0 7

$\bar{x} = \frac{653}{7}$

$\bar{x} = 93.3$

3

Tens	Units
18	1 7 7
19	2 6
20	0 3 6
21	1

$\bar{x} = \frac{1\,763}{9}$

$\bar{x} = 195.9$

4

AM.	Tens	PM.
1 1 1	0	5
9 8 7 6	1	8 8 9
0 0	2	1 7 7 8
9 7 3 3	3	0 0 2 2 6

AM: $\bar{x} = \frac{255}{13}$

$\bar{x} = 19.6$

PM: $\bar{x} = \frac{353}{13}$

$\bar{x} = 27.2$

5

Boys	Tens	Girls
8	15	0 3 6 7 7
8 7 5 5	16	2 4 9
9 3	17	1 1
4 1 1	18	

Boys: $\bar{x} = \frac{1\,721}{10}$

$\bar{x} = 172.1$

Girls: $\bar{x} = \frac{1\,610}{10}$

$\bar{x} = 161$

Boys & Girls: $\bar{x} = \frac{3\,331}{20} = 166.6$

Complete fx columns, find Σfx and divide by the number of scores (Σf) to find the mean to 1 d.p.

6 Use a ruler to go over the dotted lines.

Score (x)		Frequency (f)		fx
0	×	5	=	0
2	×	1	=	2
4	×	7	=	28
6	×	0	=	0
8	×	8	=	64
10	×	7	=	70
		$\Sigma f =$ 28		$\Sigma fx =$ 164

Sum the f column. Sum the fx column.

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{164}{28}$

$\bar{x} = 5.9$ 1 d.p.

7

Score	f	fx
0	2	0
1	3	3
2	7	14
3	12	36
4	9	36
5	14	70
6	13	78
	Σf=60	Σfx= 237

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{237}{60}$

$\bar{x} = 4.0$

8

Score	f	fx
0	4	0
5	2	10
10	7	70
15	11	165
20	6	120
25	4	100
	Σf=34	Σfx=465

$\bar{x} = \frac{465}{34}$

$\bar{x} = 13.7$

Place this data in a frequency table and then find the mean to 1 d.p.

9 Data: 1, 3, 5, 5, 2, 2, 3, 5, 2, 4, 3, 1, 6, 1, 3, 2, 5, 4, 4, 3, 2, 2, 5, 3, 6, 1, 2, 4, 2, 3.

Score	Tally	f	fx
1	𝍸	4	4
2	𝍸 | | |	8	16
3	𝍸 | |	7	21
4	𝍸	4	16
5	𝍸	5	25
6	| |	2	12
		Σf=30	Σfx= 94

$\bar{x} = \frac{94}{30} = 3.1$

Data Analysis - The Mean (Homework Strip)

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Date Due: _____

Find the mean to 1 decimal place.

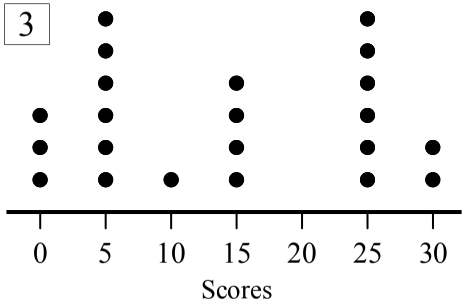
1 206, 194, 335.

$\bar{x} = \frac{\quad}{\quad} =$ _____

2 47, 18, 32, 55, 31.

$\bar{x} = \frac{\quad}{\quad} =$ _____

3



$\bar{x} = \frac{\quad}{\quad} =$ _____

4

Tens	Units
3	0 2 4
4	7
5	3 3 3

$\bar{x} = \frac{\quad}{\quad} =$ _____

Date Due: _____

Find the mean to 1 decimal place.

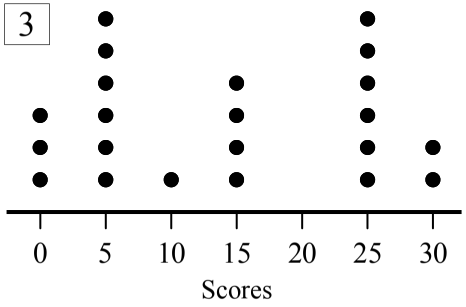
1 206, 194, 335.

$\bar{x} = \frac{\quad}{\quad} =$ _____

2 47, 18, 32, 55, 31.

$\bar{x} = \frac{\quad}{\quad} =$ _____

3



$\bar{x} = \frac{\quad}{\quad} =$ _____

4

Tens	Units
3	0 2 4
4	7
5	3 3 3

$\bar{x} = \frac{\quad}{\quad} =$ _____

Date Due: _____

Find the mean to 1 decimal place.

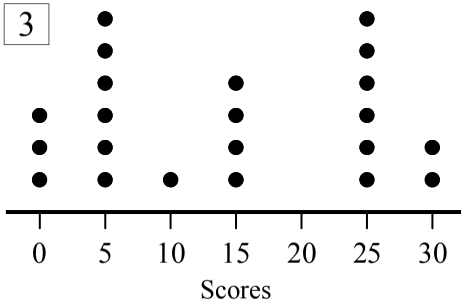
1 206, 194, 335.

$\bar{x} = \frac{\quad}{\quad} =$ _____

2 47, 18, 32, 55, 31.

$\bar{x} = \frac{\quad}{\quad} =$ _____

3



$\bar{x} = \frac{\quad}{\quad} =$ _____

4

Tens	Units
3	0 2 4
4	7
5	3 3 3

$\bar{x} = \frac{\quad}{\quad} =$ _____

Date Due: _____

Find the mean to 1 decimal place.

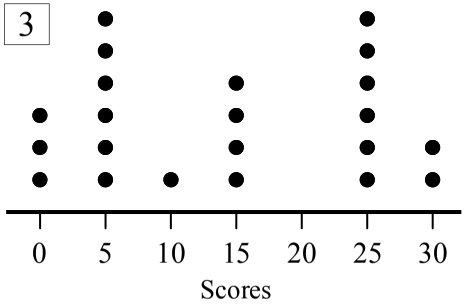
1 206, 194, 335.

$\bar{x} = \frac{\quad}{\quad} =$ _____

2 47, 18, 32, 55, 31.

$\bar{x} = \frac{\quad}{\quad} =$ _____

3



$\bar{x} = \frac{\quad}{\quad} =$ _____

4

Tens	Units
3	0 2 4
4	7
5	3 3 3

$\bar{x} = \frac{\quad}{\quad} =$ _____

Data Analysis - The Mean (Homework Strip)

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Date Due: _____

Find the mean to 1 decimal place.

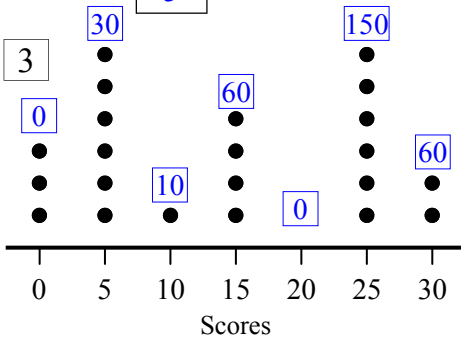
1 206, 194, 335.

$$\bar{x} = \frac{735}{3} = 245.0$$

2 47, 18, 32, 55, 31.

$$\bar{x} = \frac{183}{5} = 36.6$$

3



$$\bar{x} = \frac{310}{22} = 14.1$$

4

Tens	Units
3	0 2 4
4	7
5	3 3 3

$$\bar{x} = \frac{302}{7} = 43.1$$

Data Analysis - Further Mean - Time Applications

The mean is found by adding all the separate times, then dividing the total by the number of times. This sheet uses the DMS key of your calculator, it also requires your skills to understand 24 hour time, so this sheet should be considered more challenging.

Example Find the average time of these times.
1 h 19 m, 2h 33m, 44m.

STEP 1 Add the times.

Calc: 1 º ’ ’’ 1 9 º ’ ’’ + 2 º ’ ’’ 3 3 º ’ ’’ + 0 º ’ ’’ 4 4 º ’ ’’ = 4h 36m

STEP 2 Divide the total by the number of times (÷3)

Leave the previous answer on the screen and ÷3

Calc: ÷ 3 = 1h 32m

The 2nd column of the sheet starts with 24 hour time revision. Remember that when you reach 1 pm, you add 12 hours. So that means:

1 pm = 1300 h, 1.26 pm = 1326 h, 8 pm = 2000h, 10.16 pm = 2216. and so on.

You need 24 hour time to perform calculations with subtraction. For example if you work between 9.38 am and 4.45 pm, how do you work out the time you have worked?

9.38 am becomes 0938 h

4.45 pm becomes 1645 h

Perform a subtraction: 1645 h - 0938 h Calc: 1 6 º ’ ’’ 4 5 º ’ ’’ - 9 º ’ ’’ 3 8 º ’ ’’ = 7h 7m

If you have the hours worked each day, you can calculate the average working hours. Just add all the daily hours and divide by the number of days.

Data Analysis - Further Mean - Time Applications

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Find the average of these times using your calculator's DMS key.

Don't forget!

1 1h 22m, 1h 47m, 54m.

Calc: 1 °’’’ 2 2 °’’’ + 1 °’’’ 4 7 °’’’ + 0 °’’’ 5 4 °’’’ =

Write the total in the top box then:

Calc: ÷ 3 = $\bar{x} = \frac{\square\text{ h }\square\text{ m}}{3} = \square\text{ h }\square\text{ m}$

2 2h 43m, 1h 53m, 2h 16m, 1h 36m.

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m}$

3 57m, 1h 38m, 39m.

57m = 0h 57m!

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m}$

4 23m, 1h 38m, 2h 47m, 48m, 56m, 1h 52m.

$\bar{x} = \frac{\square}{\square} = \square$

These are the same, except the answer is required in both h/min and converted to minutes alone.

5 1h 13m, 2h 6m, 52m, 97m.

97m = 0h 97m!

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$

Calc: × 6 0 =

6 2h 31m, 1h 11m, 67m, 3h 19 m, 43m, 87m.

$\bar{x} = \frac{\square}{\square} = \square = \square$

7 Find the average running time in minutes for these three movies: 1h 19m, 1h 47m, 1h 33m.

8 Find the average running time in minutes for these five movies:

1h 46m, 2h 13m, 86m, 1h 55m, 115m.

Convert these times to 24 hour time format.

Examples:

5.17 am = 0517 h

3 + 12 = 15

3.43 pm = 1543 h

9 5 am = ☐☐☐☐ h

10 7 pm = ☐☐☐☐ h

11 9.23 pm = ☐☐☐☐ h

12 8.08 pm = ☐☐☐☐

13 11.37 pm = ☐☐☐☐

14 2.41 pm = ☐☐☐☐

Find the time difference between these times, use 24 hour time format to make the subtraction.

14 6.25 am until 3.08 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

3.08 pm in 24 h — 6.25 am in 24 h

Calc: 1 5 °’’’ 8 °’’’ - 6 °’’’ 2 5 °’’’ =

15 11.44 am until 7.17 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

7.17 pm in 24 h — 11.44 am in 24 h

Dave works the following times for 5 days. Find the average daily hours he works.

16 Monday: 8.47 am until 5.28 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

5.28 pm in 24 h — 8.47 am in 24 h

17 Tuesday: 9.53 am until 6.42 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

18 Wednesday: 7.36 am until 4.15 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

19 Thursday: 2.39 pm until 11.05 pm

Careful! 2 pm times

20 Friday: 3.48 pm until 10.08 pm

21 Calculate Dave's average hours for the week.

$\bar{x} = \frac{\square}{\square} = \square$

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Find the average of these times using your calculator's DMS key.

1 1h 22m, 1h 47m, 54m.

Don't forget!

Calc: 1 °’’’ 2 2 °’’’ + 1 °’’’ 4 7 °’’’ + 0 °’’’ 5 4 °’’’ = 1 °’’’ 2 2 °’’’

Write the total in the

Calc: ÷ 3 =

$\bar{x} = \frac{4\text{ h }3\text{ m}}{3} = 1\text{ h }21\text{ m}$

2 2h 43m, 1h 53m, 2h 16m, 1h 36m.

$\bar{x} = \frac{8\text{ h }28\text{ m}}{4} = 2\text{ h }7\text{ m}$

3 57m, 1h 38m, 52m.

57m = 0h 57m!

$\bar{x} = \frac{3\text{ h }27\text{ m}}{3} = 1\text{ h }9\text{ m}$

4 23m, 1h 38m, 2h 47m, 48m, 56m, 1h 52m.

$\bar{x} = \frac{8\text{h }24\text{m}}{6} = 1\text{h }24\text{ m}$

These are the same, except the answer is required in both h/min and converted to minutes alone.

5 1h 13m, 2h 6m, 52m, 97m.

97m = 0h 97m!

$\bar{x} = \frac{5\text{ h }48\text{ m}}{4} = 1\text{ h }27\text{ m} = 87\text{ m}$

Calc: × 6 0 =

6 2h 31m, 1h 11m, 67m, 3h 19 m, 43m, 87m.

$\bar{x} = \frac{10\text{h }18\text{m}}{6} = 1\text{h }43\text{m} = 103\text{ m}$

7 Find the average running time in minutes for these three movies: 1h 19m, 1h 47m, 1h 33m.

$\bar{x} = \frac{4\text{h }39\text{m}}{3} = 1\text{h }33\text{m} = 93\text{m}$

8 Find the average running time in minutes for these five movies:

1h 46m, 2h 13m, 86m, 1h 55m, 115m.

$\bar{x} = \frac{9\text{h }15\text{m}}{5} = 1\text{h }51\text{m} = 111\text{m}$

Convert these times to 24 hour time format.

Examples: 3 + 12 = 15

5.17 am = 0517 h 3.43 pm = 1543 h

9 5 am = 0500 h

10 7 pm = 1900 h

11 9.23 pm = 2123 h

12 8.08 pm = 2008h

13 11.37 pm = 2337h

14 2.41 pm = 1441h

Find the time difference between these times, use 24 hour time format to make the subtraction.

14 6.25 am until 3.08 pm

1508 h - 0625 h = 8 h 43 m

3.08 pm in 24 h 6.25 am in 24 h

Calc: 1 5 °’’’ 8 °’’’ - 6 °’’’ 2 5 °’’’ =

15 11.44 am until 7.17 pm

1917 h - 1124 h = 7 h 53 m

7.17 pm in 24 h 11.44 am in 24 h

Dave works the following times for 5 days. Find the average daily hours he works.

16 Monday: 8.47 am until 5.28 pm

1728 h - 0847 h = 8 h 41 m

5.28 pm in 24 h 8.47 am in 24 h

17 Tuesday: 9.53 am until 6.42 pm

1842 h - 0953 h = 8 h 49 m

18 Wednesday: 7.36 am until 4.15 pm

1615 h - 0736 h = 8 h 39 m

19 Thursday: 2.39 pm until 11.05 pm

Careful! 2 pm times

2305h - 1439h = 8h 26m

20 Friday: 3.48 pm until 10.08 pm

2208h - 1548h = 6h 20m

21 Calculate Dave's average hours for the week.

$\bar{x} = \frac{40\text{h }55\text{m}}{5} = 8\text{h }11\text{m}$

Data Analysis - Further Mean - Time Applications (Homework Strip)

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Date Due:

Find the average time for these.

1 1h 58 m, 2h 36 m

Calc: 1 °’’’ 5 8 °’’’ + 2 °’’’ 3 6 °’’’ =

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{2} = \square\text{ h }\square\text{ m}$ Calc: ÷ 2 =

2 1h 26 m, 1h 56 m, 1h 44 m

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m}$

3 55 m, 42 m, 1h 12 m, 1h 19m

Calc: 0 °’’’ 5 5 °’’’ + 0 °’’’ .....

$\bar{x} = \frac{\square}{\square} = \square$

Give the answer in minutes as well

4 1h 38 m, 2h 42 m

$\bar{x} = \frac{\square}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$ Calc: × 6 0 =

5 2h 19 m, 1h 58 m, 85 m.

$\bar{x} = \frac{\square}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$

Date Due:

Find the average time for these.

1 1h 58 m, 2h 36 m

Calc: 1 °’’’ 5 8 °’’’ + 2 °’’’ 3 6 °’’’ =

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{2} = \square\text{ h }\square\text{ m}$ Calc: ÷ 2 =

2 1h 26 m, 1h 56 m, 1h 44 m

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m}$

3 55 m, 42 m, 1h 12 m, 1h 19m

Calc: 0 °’’’ 5 5 °’’’ + 0 °’’’ .....

$\bar{x} = \frac{\square}{\square} = \square$

Give the answer in minutes as well

4 1h 38 m, 2h 42 m

$\bar{x} = \frac{\square}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$ Calc: × 6 0 =

5 2h 19 m, 1h 58 m, 85 m.

$\bar{x} = \frac{\square}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$

Date Due:

Find the average time for these.

1 1h 58 m, 2h 36 m

Calc: 1 °’’’ 5 8 °’’’ + 2 °’’’ 3 6 °’’’ =

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{2} = \square\text{ h }\square\text{ m}$ Calc: ÷ 2 =

2 1h 26 m, 1h 56 m, 1h 44 m

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m}$

3 55 m, 42 m, 1h 12 m, 1h 19m

Calc: 0 °’’’ 5 5 °’’’ + 0 °’’’ .....

$\bar{x} = \frac{\square}{\square} = \square$

Give the answer in minutes as well

4 1h 38 m, 2h 42 m

$\bar{x} = \frac{\square}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$ Calc: × 6 0 =

5 2h 19 m, 1h 58 m, 85 m.

$\bar{x} = \frac{\square}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$

Date Due:

Find the average time for these.

1 1h 58 m, 2h 36 m

Calc: 1 °’’’ 5 8 °’’’ + 2 °’’’ 3 6 °’’’ =

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{2} = \square\text{ h }\square\text{ m}$ Calc: ÷ 2 =

2 1h 26 m, 1h 56 m, 1h 44 m

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m}$

3 55 m, 42 m, 1h 12 m, 1h 19m

Calc: 0 °’’’ 5 5 °’’’ + 0 °’’’ .....

$\bar{x} = \frac{\square}{\square} = \square$

Give the answer in minutes as well

4 1h 38 m, 2h 42 m

$\bar{x} = \frac{\square}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$ Calc: × 6 0 =

5 2h 19 m, 1h 58 m, 85 m.

$\bar{x} = \frac{\square}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$

Data Analysis - Further Mean - Time Applications (Homework Strip)

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Date Due: _____

Find the average time for these.

1 1h 58 m, 2h 36 m

Calc: 1 °’’’ 5 8 °’’’ + 2 °’’’ 3 6 °’’’ =

$\bar{x} = \frac{4 \text{ h } 34 \text{ m}}{2} = 2 \text{ h } 17 \text{ m}$

Calc: ÷ 2 =

2 1h 26 m, 1h 56 m, 1h 44 m

$\bar{x} = \frac{5 \text{ h } 6 \text{ m}}{3} = 1 \text{ h } 42 \text{ m}$

3 55 m, 42 m, 1h 12 m, 1h 19m

Calc: 0 °’’’ 5 5 °’’’ + 0 °’’’ …...

$\bar{x} = \frac{4\text{h } 8 \text{ m}}{4} = 1\text{h } 2\text{m}$

Give the answer in minutes as well

4 1h 38 m, 2h 42 m

$\bar{x} = \frac{4\text{h } 20\text{m}}{2} = 2 \text{ h } 10 \text{ m}$

$= 130 \text{ m}$

Calc: × 6 0 =

5 2h 19 m, 1h 58 m, 85 m.

$\bar{x} = \frac{5\text{h } 42\text{h}}{3} = 1 \text{ h } 54 \text{ m}$

$= 114 \text{ m}$

Data Analysis - Mean - Problems

The Mean is the mathematical term for "Average". So when you find the mean you are finding the average. The mean is found by adding all the scores then dividing the total by the number of scores. As a formula it is written:

The short way of writing the mean is $\bar{x}$. *'x bar'*

$$\bar{x} = \frac{\Sigma x}{n}$$

Σx means the sum of x. So the total of all the scores in the data set.

n means the number of scores in the data set.

Example

Tilly installs glass splash-backs in kitchens. Working a 5 day week, the number of installations is shown in the table below. If she is paid $485 per splash-back, calculate:

- The amount earned each day, by completing the table
- The average amount Tilly earns daily

Day	Splash-backs Installed	Amount Earned
Mon	2	$970
Tues	3	$1 455
Wed	3	$1 455
Thur	5	$2 425
Fri	4	$1 940

485 × 2

Tilly works 5 days, so to find the average amount earned, divide the amount earned by 5.

$$\bar{x} = \frac{8\ 245}{5}$$

Total of Amount earned column

$$= \$1\ 649$$

Remember the $ sign

Data Analysis - Mean - Problems

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Find the mean for these, round to 1 d.p. if required.

1 James' biology experiment results in three bean sprouts at heights of: 11.5 cm, 14.3 cm and 17.1 cm. Find the average height for the sprouts.

$\bar{x} = \frac{\square}{\square} = \square$ cm

2 Jo earns the following amounts over 5 weekends:
$124, $85, $110.80, $96.50 and $135. Find the average amount she earns.

$\bar{x} = \frac{\square}{\square} = \square$

3 Chloe earns $17.60 per hour, working a 4 day week. Calculate the earnings for each day she works.

M: 6 h $6 \times 17.60 = \square$

T: 7 h $\square$

W: 4 h $\square$

F: 9 h $\square$

4 Calculate Chloe's average daily wage.

$\bar{x} = \frac{\square}{\square} = \square$

5 Sam supplies Holland blinds, he makes different profits on each width. Find the profit for sales of each width in the weekly table of sales below.

Width (mm)	Profit rate	Quantity	Profit
900	$42	18	
1 200	$68	7	
1 800	$95	9	
2 400	$76	5	

6 Find total sold and total profit.

Total number of Blinds sold: _____

Total Weekly Profit: _____

7 Find the average profit made per blind sold for the week.

_____

8 Below is Sam's results for the second week. Complete the table to calculate the profit made on each size.

Width (mm)	Profit rate	Quantity	Profit
900	$42	4	
1 200	$68	8	
1 800	$95	12	
2 400	$76	8	

9 Find total sold and total profit.

Total number of Blinds sold: _____

Total Weekly Profit: _____

10 Find the average profit made per blind sold for the second week.

_____

11 Has the average blind profit increased or decreased in the second week? Explain why.

_____

_____

_____

_____

_____

12 David is a real estate agent who has sold three properties this week. Calculate David's total weekly sales.

2 Hillside Dr	$807 700
15/9 Botany St	$795 000
9 Seaview Crt	$2 675 000
TOTAL	

13 Calculate the average sales price for the three properties.

_____

_____

14 Calculate the average sales price just for the first two properties.

_____

_____

15 Explain why the answer to Q.13 is much higher than Q.14.

_____

_____

_____

_____

16 Geoff is managing the digging and walling of a train tunnel. He records the rate of progress below.

Day	Distance Completed
Mon	6.3 m
Tues	3.5 m
Wed	4.9 m
Thur	2.5 m
Fri	9.3 m
Sat	7.7 m

Calculate the average distance completed per day.

_____

Data Analysis - Mean - Problems

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Find the mean for these, round to 1 d.p. if required.

1 James' biology experiment results in three bean sprouts at heights of: 11.5 cm, 14.3 cm and 17.1 cm. Find the average height for the sprouts.

$\bar{x} = \frac{42.9}{3} = 14.3$ cm

2 Jo earns the following amounts over 5 weekends:
$124, $85, $110.80, $96.50 and $135. Find the average amount she earns.

$\bar{x} = \frac{551.30}{5} = \$110.26$

3 Chloe earns $17.60 per hour, working a 4 day week. Calculate the earnings for each day she works.

M: 6 h $6 \times 17.60 = \$105.60$

T: 7 h $7 \times 17.60 = \$123.20$

W: 4 h $4 \times 17.60 = \$70.40$

F: 9 h $9 \times 17.60 = \$158.40$

4 Calculate Chloe's average daily wage.

$\bar{x} = \frac{457.60}{4} = \$114.40$

5 Sam supplies Holland blinds, he makes different profits on each width. Find the profit for sales of each width in the weekly table of sales below.

Width (mm)	Profit rate	Quantity	Profit
900	$42	18	$756
1 200	$68	7	$476
1 800	$95	9	$855
2 400	$76	5	$380

6 Find total sold and total profit.

Total number of Blinds sold: 39 blinds

Total Weekly Profit: $2 467

7 Find the average profit made per blind sold for the week.

$\bar{x} = \frac{2\,467}{39} = \$63.26$

8 Below is Sam's results for the second week. Complete the table to calculate the profit made on each size.

Width (mm)	Profit rate	Quantity	Profit
900	$42	4	$168
1 200	$68	8	$544
1 800	$95	12	$1 140
2 400	$76	8	$608

9 Find total sold and total profit.

Total number of Blinds sold: 33 blinds

Total Weekly Profit: $2 460

10 Find the average profit made per blind sold for the second week.

$\bar{x} = \frac{2\,460}{33} = \$74.55$

11 Has the average blind profit increased or decreased in the second week? Explain why.

The profit per blind has increased even though there was less profit and less sales. Because the quantity of higher profit blind sales increased, the average profit per blind increased.

12 David is a real estate agent who has sold three properties this week. Calculate David's total weekly sales.

2 Hillside Dr	$807 700
15/9 Botany St	$795 000
9 Seaview Crt	$2 675 000
TOTAL	$4 277 700

13 Calculate the average sales price for the three properties.

$\bar{x} = \frac{4\,277\,700}{3}$

$\bar{x} = \$1\,425\,900$

14 Calculate the average sales price just for the first two properties.

$\bar{x} = \frac{1\,602\,700}{2}$

$\bar{x} = \$801\,350$

15 Explain why the answer to Q.13 is much higher than Q.14.

The Seaview Crt house was very Expensive and pushed the average price up. By removing it the average price drops down.

16 Geoff is managing the digging and walling of a train tunnel. He records the rate of progress below.

Day	Distance Completed
Mon	6.3 m
Tues	3.5 m
Wed	4.9 m
Thur	2.5 m
Fri	9.3 m
Sat	7.7 m

Calculate the average distance completed per day.

$\bar{x} = \frac{34.2}{6} = 5.7$ m

Data Analysis - Mean - Problems (Homework Strip)

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Date Due: _____

1 Trey is a carpet layer. He travels: 13.2 km, 22 km, 7.6 km and 16.8 km for 4 quotes. Calculate the average distance he travels for a quote.

$$\bar{x} = \frac{\square}{\square} = \square$$

2 Josh has 3 attempts at the long jump. He jumps: 3.8 m, 4.4 m and 4.1 m. Calculate his average jump distance.

3 Yvonne sells flower arrangements, each arrangement sells for $85. Complete the table and calculate her average daily turnover.

Day	Arrangements Sold	Amount Earned
Mon	14	
Tues	6	
Wed	9	
Thur	8	
Fri	7	

Date Due: _____

1 Trey is a carpet layer. He travels: 13.2 km, 22 km, 7.6 km and 16.8 km for 4 quotes. Calculate the average distance he travels for a quote.

$$\bar{x} = \frac{\square}{\square} = \square$$

2 Josh has 3 attempts at the long jump. He jumps: 3.8 m, 4.4 m and 4.1 m. Calculate his average jump distance.

3 Yvonne sells flower arrangements, each arrangement sells for $85. Complete the table and calculate her average daily turnover.

Day	Arrangements Sold	Amount Earned
Mon	14	
Tues	6	
Wed	9	
Thur	8	
Fri	7	

Date Due: _____

1 Trey is a carpet layer. He travels: 13.2 km, 22 km, 7.6 km and 16.8 km for 4 quotes. Calculate the average distance he travels for a quote.

$$\bar{x} = \frac{\square}{\square} = \square$$

2 Josh has 3 attempts at the long jump. He jumps: 3.8 m, 4.4 m and 4.1 m. Calculate his average jump distance.

3 Yvonne sells flower arrangements, each arrangement sells for $85. Complete the table and calculate her average daily turnover.

Day	Arrangements Sold	Amount Earned
Mon	14	
Tues	6	
Wed	9	
Thur	8	
Fri	7	

Date Due: _____

1 Trey is a carpet layer. He travels: 13.2 km, 22 km, 7.6 km and 16.8 km for 4 quotes. Calculate the average distance he travels for a quote.

$$\bar{x} = \frac{\square}{\square} = \square$$

2 Josh has 3 attempts at the long jump. He jumps: 3.8 m, 4.4 m and 4.1 m. Calculate his average jump distance.

3 Yvonne sells flower arrangements, each arrangement sells for $85. Complete the table and calculate her average daily turnover.

Day	Arrangements Sold	Amount Earned
Mon	14	
Tues	6	
Wed	9	
Thur	8	
Fri	7	

Data Analysis - Mean - Problems (Homework Strip)

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Date Due:

1 Trey is a carpet layer. He travels: 13.2 km, 22 km, 7.6 km and 16.8 km for 4 quotes. Calculate the average distance he travels for a quote.

$$\bar{x} = \frac{59.6}{4} = 14.9 \text{ km}$$

2 Josh has 3 attempts at the long jump. He jumps: 3.8 m, 4.4 m and 4.1 m. Calculate his average jump distance.

$$\bar{x} = \frac{12.3}{3} = 4.1 \text{ m}$$

3 Yvonne sells flower arrangements, each arrangement sells for $85. Complete the table and calculate her average daily turnover.

Day	Arrangements Sold	Amount Earned
Mon	14	$1 190
Tues	6	$510
Wed	9	$765
Thur	8	$680
Fri	7	$595

$$\bar{x} = \frac{3\ 740}{5} = \$748$$

Data Analysis - Range and Mode

The range of a data set is found by subtracting the smallest score from the largest score. So look through the data for the largest score, circle it if you like, then find the smallest score.

Example

Data: 15 , 33 , 21 , 18 , 11 , 15 .

Data: 15 , (33) , 21 , 18 , (11) , 15 .

Find the largest and the smallest scores. Circle them if you like.

Range = 33 - 11 = 22

Subtract them for the range.

Negative numbers can also occur in data sets, the same method is used but when you subtract then the two minus signs become a plus sign.

Example - Negative numbers

Data: -8 , 11 , -3 , 12 , -5 , 7.

Data: (-8) , 11 , -3 , (12) , -5 , 7.

Find the largest and the smallest scores. If circling them helps you, do it!

Range = 12 - -8 = 20

12 - - 8 becomes 12 + 8 which = 20.

Data can also be assembled in Stem and Leaf Plots

Example

Tens	Units
0	5 5 7 8
1	0 0 3 3
2	8 8 9

The smallest score is '05' which is 5

The largest score is '29' which is 29.

Range = 29 - 5 = 24

Calculate the range by the same method.

The mode is the score that occurs the most frequently in a data set. Note that the word 'mode' is similar to the word 'most'. If the data contains no numbers that are repeated then there is 'No Mode', note that you write 'No Mode' not Mode = 0. If one number occurs the most then the mode is that number. If two numbers occur the most and with the same frequency then list both numbers (this is called Bi-modal). However if there are three modes (or more) then the data set is said to have "No Mode". So it is either No mode, a single score, or two scores.

15, 17, 16, 3.	17, 3, 17, 3, 9, 9, 6.	17, 15, 17, 15, 17.	17, 11, 17, 15, 11.
Mode = No Mode	Mode = No Mode	Mode = 17.	Mode = 11, 17.
No score occurs more than once	3, 9, 17 occur the most … No Mode	17 occurs more frequently than 15	Bi-Modal as 11 and 17 both occur twice

Data Analysis - Range (Sheet 1)

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Circle the smallest and largest numbers. Subtract them and you have the range.

1 40 , 53 , 19 , 21 , 33 , 10 .

Range = ☐ - ☐ = ☐

2 71 , 66 , 57 , 42 , 42 , 25 .

Range = ☐ - ☐ = ☐

3 13, 38, 23, 46, 27, 19, 54, 62, 83, 50, 39, 44, 23, 11, 6, 79.

Range = ☐ - ☐ = ☐

4 812, 776, 909, 694, 552, 773, 833, 905, 541, 890.

Range =

Watch out when you subtract negative numbers, - - = + !

5 10, 3, -4, 17, 0, 8, -9, 6, 11.

Range = ☐ - ☐ = ☐

6 52, -37, 65, 24, 39, -7, 19.

7 38, 12, 19, 0, 58, -21, 83, 9, 41, -65, 42, -33, 91, 50, -18.

Find the range for these temperatures.

8 Overnight Minimum: -1°C
Maximum: 11°C

Range =

9 Overnight Minimum: -7°C
Maximum: 17°C

Range =

Now find the range of these Stem & Leaf Plots.

10

Tens	Units	
0	0 7 7	0,7,7
1	8 9	18,19
2	3 3	23,23

Remember the 10's

Range = ☐ - ☐ = ☐

11

Tens	Units
7	0 1 3
8	2
9	5 5 6
10	0 4

Range =

12

Tens	Units
95	2 4 4 7 7
96	0 1 3 6
97	9 9

13

Early	Tens	Late
5 4	2	2
8 3 2	3	8 8 9
7 3	4	1 7 8

Early Range =

Late Range =

14

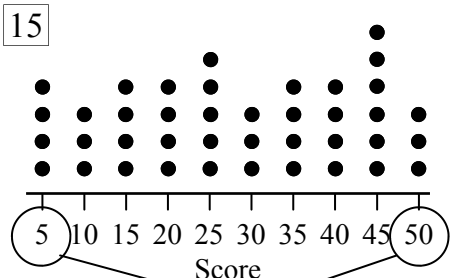
Male	Tens	Female
8 6 2	15	2 2 9
4 0 0	16	0 4 8
7 5	17	6

Male Range =

Female Range =

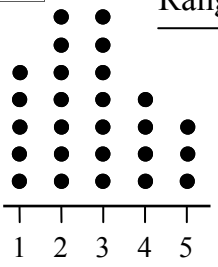
Ignore the dots, ignore the frequency columns, just use the score. Find the range.

15



Range = ☐ - ☐ = ☐

16



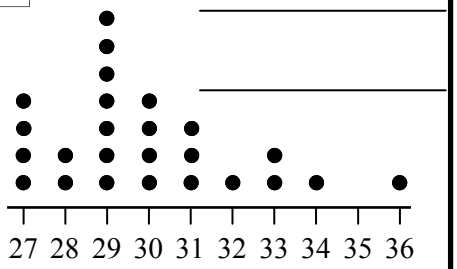
Range = ☐ - ☐

Range = ☐

17

Score	*f*
0	4
5	2
10	7
15	11
20	6
25	4

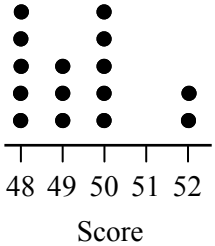
18



19

Score	*f*
10	1
20	6
30	2
40	7

20



Data Analysis - Mode (Sheet 2)

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The MOde is the score that occurs the MOst. Find the mode.

1 1 , 4 , 2 , 2 , 6 , 3 , 2 , 5 .

Mode = ☐

2 8 , 3 , 7 , 2 , 3 , 7 , 2 , 7 .

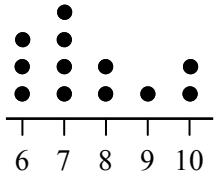
Mode = ☐

3

Tens	Units
9	0 3 3 4
10	5 7 8
11	1 6 6 6

Mode = _____

4



_____
Use the dots to pick a score

5

Score	*f*
2	2
3	8
4	2
5	7
6	7

_____
Use the frequency to pick a score

6

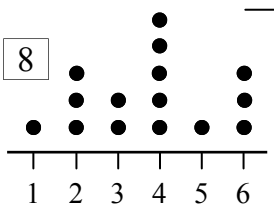
Tens	Units
551	5 6 7
552	9 9
553	1 5

_____

7 5 , 0 , 9 , 4 , 3 , 1 , 7 , 0 .

_____

8



_____

You can have no mode, one mode or two modes. No more than two. 3 Modes = No Mode!

9 32 , 42 , 16 , 32 , 14 , 16 .

Mode = ☐ and ☐

10 25, 13, 12, 18, 12, 16, 13, 5.

Mode = _____

11 11, 15, 9, 14, 17, 25, 23, 6.

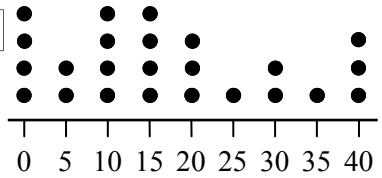
_____

12

Tens	Units
0	3 5 5 7 8
1	2 2 2 6
2	0 1 1
3	3 5 5 5

_____

13



_____

14

Score	*f*
98	5
99	2
100	4
101	5

_____

15

Score	*f*
0	7
2	7
4	3
6	4
8	7
10	5

_____

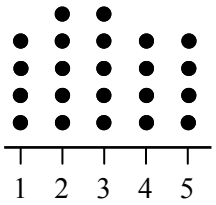
16 95, 97, 86, 97, 90, 84, 95.

_____

17 -6, 23, 17, 5, 11, 17, 5, 17.

_____

18



_____

19

A	Tens	B
9 7	8	1 4 4
3 1 0	9	2 6 6
7 7	10	7 7

Group A Mode = _____

Group B Mode = _____

20

Score	*f*
0	2
1	8
2	3
3	5
4	6
5	0

21

Score	*f*
11	19
12	13
13	14
14	19

_____

_____

The entry and exit modes are both 25. Write a number in the boxes to make this true.

22

Entry	Tens	Exit
5 3	1	0 3
☐ 5 0	2	5 5 5
7 4	3	8 8 ☐

23

Entry	Tens	Exit
7 6	0	3 3
☐ 8	1	1 4 7
5 5	2	5 ☐ ☐

Data Analysis - Range (Sheet 1)

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Circle the smallest and largest numbers. Subtract them and you have the range.

1 40, (53), 19, 21, 33, (10)

Range = 53 - 10 = 43

2 (71), 66, 57, 42, 42, (25), 3

Range = 71 - 25 = 46

3 13, 38, 23, 46, 27, 19, 54, 62, (83), 50, 39, 44, 23, 11, (6), 79.

Range = 83 - 6 = 77

4 812, 776, (909), 694, 552, 773, 833, 905, (541), 890.

Range = 909 - 541 = 368

Watch out when you subtract negative numbers, - - = + !

5 10, 3, -4, (17), 0, 8, (-9), 6, 11.

Range = 17 - -9 = 26

6 52, (-37), (65), 24, 39, -7, 19.

Range = 65 - (-37) = 102

7 38, 12, 19, 0, 58, -21, 83, 9, 41, (-65), 42, -33, (91), 50, -18.

Range = 91 - (-65) = 156

Find the range for these temperatures.

8 Overnight Minimum: -1°C
Maximum: 11°C

Range = 11 - (-1) = 12°C

9 Overnight Minimum: -7°C
Maximum: 17°C

Range = 17 - (-7) = 24°C

Now find the range of these Stem & Leaf Plots.

10

Tens	Units	
0	0 7 7	0,7,7
1	8 9	18,19
2	3 3	23,23

Remember the 10's

Range = 23 - 0 = 23

11

Tens	Units
7	0 1 3
8	2
9	5 5 6
10	0 4

Range = 104 - 70 = 34

12

Tens	Units
95	2 4 4 7 7
96	0 1 3 6
97	9 9

Range = 979 - 952 = 27

13

Early	Tens	Late
5 4	2	2
8 3 2	3	8 8 9
7 3	4	1 7 8

Early Range = 47 - 24 = 23

Late Range = 48 - 22 = 26

14

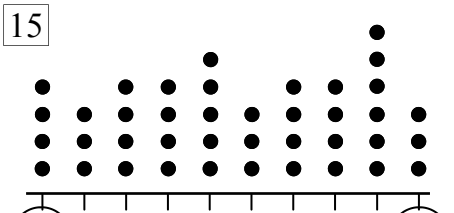
Male	Tens	Female
8 6 2	15	2 2 9
4 0 0	16	0 4 8
7 5	17	6

Male Range = 177 - 152 = 25

Female Range = 176 - 152 = 24

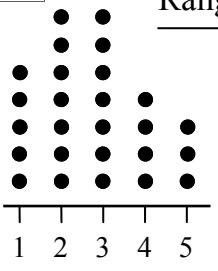
Ignore the dots, ignore the frequency columns, just use the score. Find the range.

15



Range = 50 - 5 = 45

16



Range = 5 - 1

Range = 4

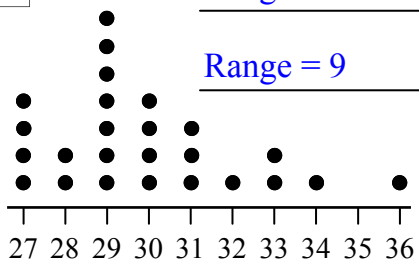
17

Score	f
0	4
5	2
10	7
15	11
20	6
25	4

Range = 25 - 0

Range = 25

18



Range = 36 - 27

Range = 9

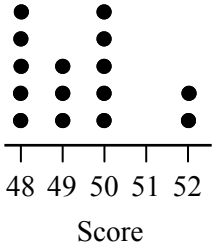
19

Score	f
10	1
20	6
30	2
40	7

Range= 40 - 30

Range = 10

20



Range= 52 - 48

Range = 4

Data Analysis - Mode (Sheet 2)

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The MOde is the score that occurs the MOst. Find the mode.

1 1 , 4 , 2 , 2 , 6 , 3 , 2 , 5 .

Mode = 2

2 8 , 3 , 7 , 2 , 3 , 7 , 2 , 7 .

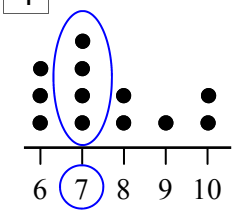
Mode = 7

3

Tens	Units
9	0 3 3 4
10	5 7 8
11	1 6 6 6

Mode = 116

4



Mode = 7

Use the dots to pick a score

5

Score	*f*
2	2
3	8
4	2
5	7
6	7

Mode = 3

Use the frequency to pick a score

6

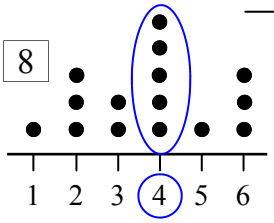
Tens	Units
551	5 6 7
552	9 9
553	1 5

Mode = 5 529

7 5 , 0 , 9 , 4 , 3 , 1 , 7 , 0 .

Mode = 0

8



Mode = 4

You can have no mode, one mode or two modes. No more than two. 3 Modes = No Mode!

9 32 , 42 , 16 , 32 , 14 , 16

Mode = 16 and 32

10 25, 13, 12, 18, 12, 16, 13, 5.

Mode = 12 and 13.

11 11, 15, 9, 14, 17, 25, 23, 6.

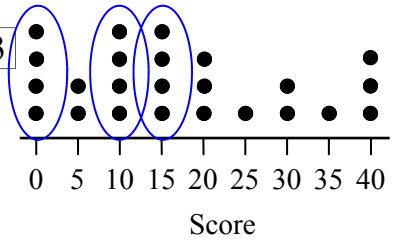
No Mode

12

Tens	Units
0	3 5 5 7 8
1	2 2 2 6
2	0 1 1
3	3 5 5 5

Mode = 12 and 35.

13



No Mode

14

Score	*f*
98	5
99	2
100	4
101	5

Modes = 98,101.

15

Score	*f*
0	7
2	7
4	3
6	4
8	7
10	5

No Mode

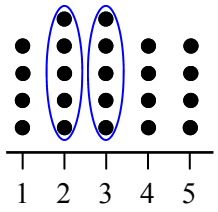
16 95, 97, 86, 97, 90, 84, 95.

Modes = 95, 97.

17 -6, 23, 17, 5, 11, 17, 5, 17.

Mode = 17

18



Mode = 2 and 3

19

A	Tens	B
9 7	8	1 4 4
3 1 0	9	2 6 6
7 7	10	7 7

Group A Mode = 107

Group B Mode = No Mode

20

Score	*f*
0	2
1	8
2	3
3	5
4	6
5	0

Mode = 1

21

Score	*f*
11	19
12	13
13	14
14	19

Mode = 11, 14.

The entry and exit modes are both 25. Write a number in the boxes to make this true.

22

Entry	Tens	Exit
5 3	1	0 3
5 0	2	5 5 5
7 4	3	8 8 9

(5)

23

Entry	Tens	Exit
7 6	0	3 3
9 8	1	1 4 7
5 5	2	5 5 5

Data Analysis - Range and Mode (Homework Strip)

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Date Due: _____

Find the range and mode for these.

1 3, 6, 3, 5, 4, 2, 5, 4, 3, 2.

Range = ___ - ___ = ___

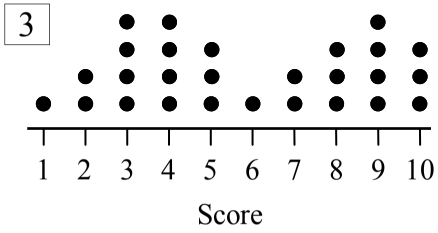
Mode = ___

2 15, 27, 11, 8, 41, 9, 19, 8, 15.

Range = _____

Mode = _____

3



Range = _____

Mode = _____

4

Tens	Units
8	4 4 9
9	2 6
10	5 5 5

Mode = _____

Range = _____

Date Due: _____

Find the range and mode for these.

1 3, 6, 3, 5, 4, 2, 5, 4, 3, 2.

Range = ___ - ___ = ___

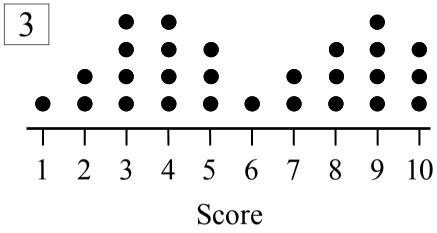
Mode = ___

2 15, 27, 11, 8, 41, 9, 19, 8, 15.

Range = _____

Mode = _____

3



Range = _____

Mode = _____

4

Tens	Units
8	4 4 9
9	2 6
10	5 5 5

Mode = _____

Range = _____

Date Due: _____

Find the range and mode for these.

1 3, 6, 3, 5, 4, 2, 5, 4, 3, 2.

Range = ___ - ___ = ___

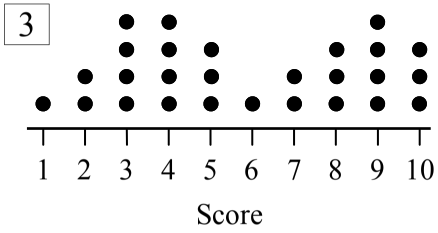
Mode = ___

2 15, 27, 11, 8, 41, 9, 19, 8, 15.

Range = _____

Mode = _____

3



Range = _____

Mode = _____

4

Tens	Units
8	4 4 9
9	2 6
10	5 5 5

Mode = _____

Range = _____

Date Due: _____

Find the range and mode for these.

1 3, 6, 3, 5, 4, 2, 5, 4, 3, 2.

Range = ___ - ___ = ___

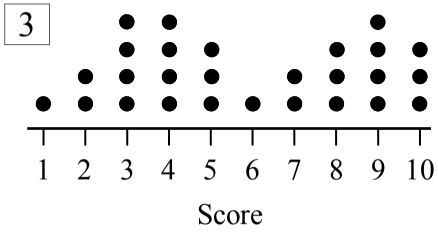
Mode = ___

2 15, 27, 11, 8, 41, 9, 19, 8, 15.

Range = _____

Mode = _____

3



Range = _____

Mode = _____

4

Tens	Units
8	4 4 9
9	2 6
10	5 5 5

Mode = _____

Range = _____

Data Analysis - Range and Mode (Homework Strip)

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Date Due:

Find the range and mode for these.

1 3, 6, 3, 5, 4, 2, 5, 4, 3, 2.

Range = 6 - 2 = 4

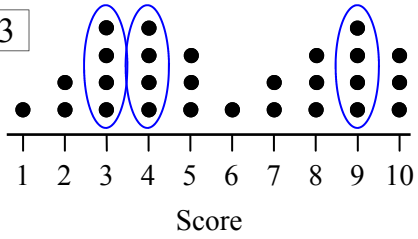
Mode = 3

2 15, 27, 11, 8, 41, 9, 19, 8, 15.

Range = 41 - 8 = 33

Mode = 8 and 15

3



Range = 10 - 1 = 9

Mode = No mode

4

Tens	Units
8	4 4 9
9	2 6
10	5 5 5

Mode = 105

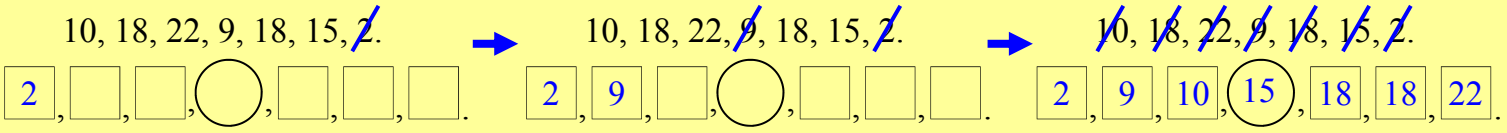
Range = 105 - 84 = 21

Data Analysis - The Median - Introduction

The Median is the middle score in a data set. That is if there are an odd number of scores. Just imagine a see-saw pivoting on the median. Write the scores in order from lowest to highest and circle the middle number … that is the median.

Example - Finding the Median (Odd Number of Scores)

Data: 10, 18, 22, 9, 18, 15, 2.



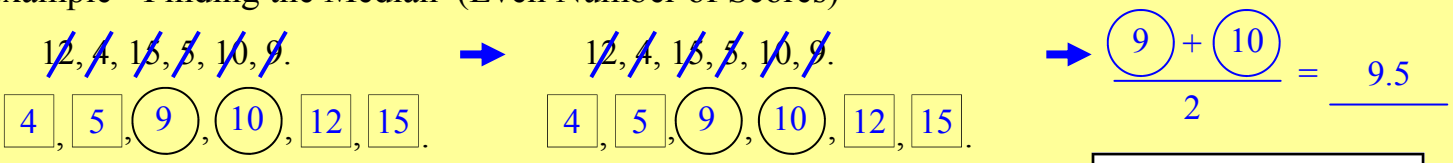
Find the smallest number in the data set and write it down first, then strike out the number to avoid mistakes.

Find the next smallest number and write it down.

Repeat the process until all the data is in ascending order. The middle score (circled) is the Median.

The above is for an odd number of scores. If there are an even number of scores then the Median is halfway between the two middle scores. So find the two middle scores and average them to find the Median.

Example - Finding the Median (Even Number of Scores)



Write the data in ascending order as with odd numbered data sets.

This time there are two middle numbers. Find the average, that is the median.

Calculator Steps:

9 + 1 0 = ÷ 2 =

Data can also be displayed in Ordered Stem and Leaf plots, Dot Plots and Frequency Tables. This is an introduction sheet and so it just asks you to write the scores down in a row, then find the median, just like a data set. So the method above is used for all questions, whether data sets, Stem and Leaf Plots, Dot Plots or Frequency Tables.

Data Analysis - The Median - Introduction (Sheet 1)

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Rewrite these numbers in order into the circle and squares. The median is in the circle.

1 3, 5, 10, 12, 19, 23, 34.

□, □, □, ○, □, □, □.

Median = □

2 6, 14, 14, 22, 37.

□, □, ○, □, □.

Median = □

3 7, 7, 12, 16, 18, 21, 26.

□, □, □, ○, □, □, □.

Median = □

Now the numbers have to be put in order as well. Low to high.

4 12, 8, 19, 36, 17.

□, □, ○, □, □.

Median = □

5 84, 36, 55, 58, 65.

□, □, ○, □, □.

Median = □

6 66, 86, 99, 92, 49, 62, 57.

□, □, □, ○, □, □, □.

Median = □

This time circle the middle number yourself. Count the numbers to make sure you have rewritten all of them.

7 11, 14, 24, 16, 19.

Rewrite numbers in order, there are 5.

Median = □

8 8, 2, 7, 3, 4, 7, 8, 2, 9.

Median = □

9 4, 8, 2, 4, 6, 7, 13, 15, 9, 3, 5.

Median = □

Find the mean for these.

10 12 and 26. Mean = □

Calc: 1 2 + 2 6 = ÷ 2 =

11 25 and 37. Mean = □

12 13 and 18. Mean = □

These have an even number of scores. You have to find the average of the 2 middle scores.

13 11, 24, 28, 36.

□, ○, ○, □.

Add then ÷ 2

Median = □

14 46, 21, 11, 35.

□, ○, ○, □.

Median = □

15 22, 19, 34, 15, 18, 24.

□, □, ○, ○, □, □.

Median = □

16 8, 19, 14, 6, 11, 15.

□, □, ○, ○, □, □.

Median = □

17 82, 71, 48, 55.

Median = □

18 23, 11, 14, 40, 28, 32.

Median = □

19 104, 111, 86, 123, 92, 88.

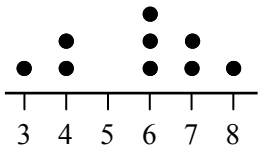
Median = □

20 8, 13, 12, 6, 13, 18, 7, 12.

Median = □

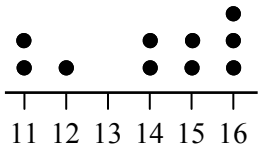
Write these dot plots out as numbers and find the median.

21



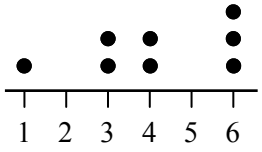
Median = □

22



Median = □

23



Median = □

Data Analysis - The Median - Introduction (Sheet 2)

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Write these stem and leaf plots out as numbers and find the median.

1

Tens	Units
0	3 8 8
1	5 7
2	8 8

3, 8, 8, 15, ____, ____, ____.

Median = ____

2

Stem	Leaf
0	1 2 6
1	4 5
2	7 8 8

Median = ____

3

Stem	Leaf
17	2 6
18	
19	4
20	0 6
21	3

Median = ____

4

Stem	Leaf
8	0 1 5
9	0 6
10	6
11	7 7

Median = ____

Now it is Frequency Tables, write the numbers out on the line below. Find the median.

5

Score	*f*
0	2
2	4
4	1
6	0
8	3
10	1

0, 0, 2, 2, 2, 2, ____

Median = ____

6

Score	*f*
1	4
2	2
3	3
4	1
5	3

Median = ____

7

Score	*f*
0	1
5	0
10	2
15	4
20	1
25	3
30	1

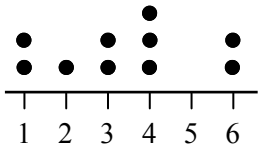
Median = ____

Find the median for these.

8 18, 45, 22, 17, 31, 28, 9.

Median = ____

9



Median = ____

10 24, 12, 18, 31, 25, 50, 38, 27.

Median = ____

11

Stem	Leaf
9	4 5 8
10	2 2 6 6
11	1 3 7

Median = ____

12

Score	*f*
10	2
20	4
30	1
40	2
50	3

Median = ____

Data Analysis - The Median - Introduction (Sheet 1)

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Rewrite these numbers in order into the circle and squares. The median is in the circle.

1 3, 5, 10, 12, 19, 23, 34.

3, 5, 10, (12), 19, 23, 34.

Median = 12

2 6, 14, 14, 22, 37.

6, 14, (14), 22, 37.

Median = 14

3 7, 7, 12, 16, 18, 21, 26.

7, 7, 12, (16), 18, 21, 26.

Median = 16

Now the numbers have to be put in order as well. Low to high.

4 12, 8, 19, 36, 17.

8, 12, (17), 19, 36.

Median = 17

5 84, 36, 55, 58, 65.

36, 55, (58), 65, 84.

Median = 58

6 66, 86, 99, 92, 49, 62, 57.

49, 57, 62, (66), 86, 92, 99.

Median = 66

This time circle the middle number yourself. Count the numbers to make sure you have rewritten all of them.

7 11, 14, 24, 16, 19.

11, 14, (16), 19, 24

Rewrite numbers in order, there are 5.

Median = 16

8 8, 2, 7, 3, 4, 7, 8, 2, 9.

2, 2, 3, 4, (7), 7, 8, 8, 9.

Median = 7

9 4, 8, 2, 4, 6, 7, 13, 15, 9, 3, 5.

2, 3, 4, 4, 5, (6), 7, 8, 9, 13, 15.

Median = 6

Find the mean for these.

10 12 and 26. Mean = 19

Calc: 12 + 26 = ÷ 2 =

11 25 and 37. Mean = 31

12 13 and 18. Mean = 15.5

These have an even number of scores. You have to find the average of the 2 middle scores.

13 11, 24, 28, 36.

11, (24), (28), 36.

Add then ÷ 2

Median = 26

14 46, 21, 11, 35.

11, (21), (35), 46. ← In Order

Median = 28

15 22, 19, 34, 15, 18, 24.

15, 18, (19), (22), 24, 34.

Median = 20.5

16 8, 19, 14, 6, 11, 15.

6, 8, (11), (14), 15, 19.

Median = 12.5

17 82, 71, 48, 55.

48, (55), (71), 82.

Median = 63

18 23, 11, 14, 40, 28, 32.

11, 14, (23), (28), 32, 40.

Median = 25.5

19 104, 111, 86, 123, 92, 88.

86, 88, (92), (104), 111, 123.

Median = 98

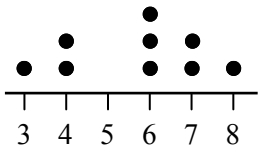
20 8, 13, 12, 6, 13, 18, 7, 12.

6, 7, 8, (12), (12), 13, 13, 18.

Median = 12

Write these dot plots out as numbers and find the median.

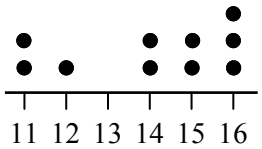
21



3, 4, 4, 6, (6), 6, 7, 7, 8

Median = 6

22

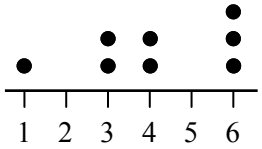


11, 11, 12, 14, (14), (15), 15, 16, 16

16.

Median = 14.5

23



1, 3, 3, (4), (4), 6, 6, 6.

Median = 4

Data Analysis - The Median - Introduction (Sheet 2)

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Write these stem and leaf plots out as numbers and find the median.

1

Tens	Units
0	3 8 8
1	5 7
2	8 8

3, 8, 8, 15, [17], [28], [28].

Median = [15]

2

Stem	Leaf
0	1 2 6
1	4 5
2	7 8 8

1, 2, 6, (14), (15), 27, 28, 28.

Median = [14.5]

3

Stem	Leaf
17	2 6
18	
19	4
20	0 6
21	3

172, 176, (194), (200), 206, 203

Median = [197]

4

Stem	Leaf
8	0 1 5
9	0 6
10	6
11	7 7

80, 81, 85, (90), (96), 106, 117, 117.

Median = [93]

Now it is Frequency Tables, write the numbers out on the line below. Find the median.

5

Score	*f*
0	2
2	4
4	1
6	0
8	3
10	1

0, 0, 2, 2, 2, (2), 4, 8, 8, 8, 10.

Median = [2]

6

Score	*f*
1	4
2	2
3	3
4	1
5	3

1, 1, 1, 1, 2, 2, (3), 3, 3, 4, 5, 5, 5.

Median = [3]

7

Score	*f*
0	1
5	0
10	2
15	4
20	1
25	3
30	1

0, 10, 10, 15, 15, (15), (15), 20, 25, 25, 25, 30.

Median = [15]

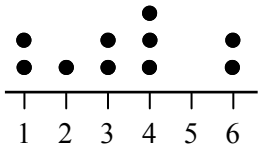
Find the median for these.

8 18, 45, 22, 17, 31, 28, 9.

9, 17, 18, (22), 28, 31, 45.

Median = [22]

9



1, 1, 2, 3, (3), (4), 4, 4, 6, 6.

Median = [3.5]

10 24, 12, 18, 31, 25, 50, 38, 27.

12, 18, 24, (25), (27), 31, 38, 50.

Median = [26]

11

Stem	Leaf
9	4 5 8
10	2 2 6 6
11	1 3 7

94, 95, 98, 102, (102), (106), 106, 111, 113, 117.

Median = [104]

12

Score	*f*
10	2
20	4
30	1
40	2
50	3

10, 10, 20, 20, 20, (20), (30), 40, 40, 50, 50, 50.

Median = [25]

Data Analysis - Introduction to Median (Homework Strip)

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Date Due: _____

Find the median for these.

1 4, 7, 10, 14, 19, 19, 23.

Median = _____

2 8, 15, 17, 21, 24, 24.

Median = _____

Rewrite in order, then find the median.

3 15, 8, 25, 6, 18, 4, 10.

Median = _____

4 11, 18, 14, 6, 12, 27.

Median = _____

5

Tens	Units
8	4 4 9
9	2 6
10	5 5

Median = _____

Date Due: _____

Find the median for these.

1 4, 7, 10, 14, 19, 19, 23.

Median = _____

2 8, 15, 17, 21, 24, 24.

Median = _____

Rewrite in order, then find the median.

3 15, 8, 25, 6, 18, 4, 10.

Median = _____

4 11, 18, 14, 6, 12, 27.

Median = _____

5

Tens	Units
8	4 4 9
9	2 6
10	5 5

Median = _____

Date Due: _____

Find the median for these.

1 4, 7, 10, 14, 19, 19, 23.

Median = _____

2 8, 15, 17, 21, 24, 24.

Median = _____

Rewrite in order, then find the median.

3 15, 8, 25, 6, 18, 4, 10.

Median = _____

4 11, 18, 14, 6, 12, 27.

Median = _____

5

Tens	Units
8	4 4 9
9	2 6
10	5 5

Median = _____

Date Due: _____

Find the median for these.

1 4, 7, 10, 14, 19, 19, 23.

Median = _____

2 8, 15, 17, 21, 24, 24.

Median = _____

Rewrite in order, then find the median.

3 15, 8, 25, 6, 18, 4, 10.

Median = _____

4 11, 18, 14, 6, 12, 27.

Median = _____

5

Tens	Units
8	4 4 9
9	2 6
10	5 5

Median = _____

Data Analysis - Introduction to Median (Homework Strip)

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Date Due:

Find the median for these.

1 4, 7, 10, 14, 19, 19, 23.

Median = 14

2 8, 15, 17, 21, 24, 24.

Median = 19

Rewrite in order, then find the median.

3 15, 8, 25, 6, 18, 4, 10.

4, 6, 8, 10, 15, 18, 25.

Median = 10

4 11, 18, 14, 6, 12, 27.

6, 11, 12, 14, 18, 27.

Median = 13

5

Tens	Units
8	4 4 9
9	2 6
10	5 5

84, 84, 89, 92, 96, 105, 105.

Median = 92

Data Analysis - The Median

Crossing off the first and last score, then the 2nd and 2nd last score ….. and so on. This is the manual way to find the median. But the faster way, especially with larger amounts of data is to use the formula:

$$\text{Median Position} = \frac{n+1}{2}$$

Where n = number of scores in a data set. Note the answer can be a whole number or end with 0.5.

Data Sets

Example - Odd number of scores

Data: 1, 5, 7, 10, 14, 19, 23.

There are 7 scores

$$\text{Position} = \frac{7+1}{2} = 4^{th} \text{ score.}$$

So for the data above, the median is the 4th score.

Data: 1, 5, 7, 10, 14, 19, 23.

Median = 10

Example - Even number of scores

Data: 1, 5, 7, 10, 14, 19, 23, 37, 37, 40.

There are 10 scores

$$\text{Position} = \frac{10+1}{2} = 5.5^{th} \text{ score.}$$

So for the data above, the median is the 5.5th score. In other words it is between the 5th and 6th scores.

Data: 1, 5, 7, 10, 14, 19, 23, 37, 37, 40.

$$\text{Median} = \frac{14+19}{2} = 16.5$$

Stem & Leaf Plots

Example Stem & Leaf (Odd)

Tens	Units
8	4 6
9	0 2 6 6
10	1 3 3 5 8
11	3 3 8 9

There are 15 scores

$$\text{Position} = \frac{15+1}{2} = 8^{th} \text{ score.}$$

Median = 103

Example Stem & Leaf (Even)

Tens	Units
0	1 7
1	0 0 5
2	1 8 8 8 9
3	3 3

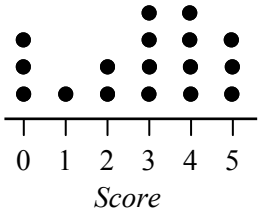
There are 12 scores

$$\text{Position} = \frac{12+1}{2} = 6.5^{th} \text{ score.}$$

$$\text{Median} = \frac{21+28}{2} = 24.5$$

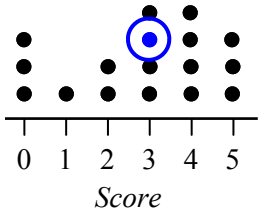
Dot Plots

Example Dot Plots (Odd)



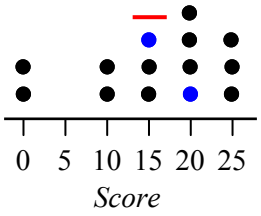
There are 17 scores (dots)

$$\text{Position} = \frac{17+1}{2} = 9^{th} \text{ score.}$$



Median = 3

Example Dot Plots (Even)



There are 14 scores (dots)

$$\text{Position} = \frac{14+1}{2} = 7.5^{th} \text{ score.}$$

The 7.5^{th} score is between the 7th and 8th score. Red line above.

$$\text{Median} = \frac{15+20}{2} = 17.5$$

Frequency Tables

Example Frequency Tables (odd)

Score	*f*	
2	1	2
4	3	4, 4, 4
6	2	6, 6
8	4	8, 8, 8, 8
10	3	10, 10, 10
$\Sigma f =$	13	

Write the scores out

Add the *f* column

$$\text{Position} = \frac{13+1}{2} = 7^{th} \text{ score.}$$

Median = 8

Example Frequency Tables (even)

Score	*f*	
2	1	2
4	3	4, 4, 4
6	3	6, 6, 6
8	4	8, 8, 8, 8
10	3	10, 10, 10
$\Sigma f =$	14	

Write the scores out

Add the *f* column

$$\text{Position} = \frac{14+1}{2} = 7.5^{th} \text{ score.}$$

Median = 8

Data Analysis - The Median (Sheet 1)

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Find the position of the median and the median itself, the data is in ascending order.

1 3, 4, 7, 12, 12, 15, 18, 25, 26, 30, 38, 38, 45, 48, 55, 63, 70.

Position = $\frac{\square + 1}{2}$ (Count the scores)

$= \square^{th}$ score.

The median is $\square$

2 63, 82, 83, 87, 88, 94, 96, 98, 99, 100, 104, 110, 115, 122, 122, 127, 129, 135, 142, 154, 179.

Position = $\frac{\square + 1}{2}$

$= \square^{th}$ score.

The median is $\square$

3 0, 2, 3, 4, 12, 13, 13, 18, 23, 24, 29, 29, 30, 31, 37, 39, 42, 46, 55, 57, 59, 94, 123, 128, 152.

Position =

Median = $\square$

4 105, 127, 185, 209, 234, 276, 362, 368, 391, 412, 419, 438, 446, 502, 535, 572, 590, 590, 612, 616, 623, 655, 662, 720, 739, 750, 750, 773, 781, 782, 810, 834, 880, 881.

Position =

Median = $\frac{\square + \square}{2}$

$= \square$

Arrange the data in ascending order then find both the position of the median and its value. Strike out the numbers as you go.

5 53, 26, 40, 45, 52, 31, 76, 42, 50, 64, 39, 43, 24, 33, 60, 59, 78, 25, 10, 18, 32, 40, 57, 62, 20, 25, 55, 19, 28.

10,

Position = Median =

Find the position and value for the median in the stem and leaf plots.
For Q.8, 9 place the data in the raw and ordered S&L plots first.

6

Tens	Units
0	5 7 7 8 9
1	4 4
2	1 1 1 4 4
3	0 0 3

Position =

Median =

7

Tens	Units
8	3 5
9	2 4 9
10	0 3 3 4 7
11	2 6

Position =

Median =

8 16, 25, 11, 24, 8, 12, 24, 20, 23, 15, 19, 4, 13, 8, 2, 9, 3.

RAW

Stem	Leaf
$\square$	.....
$\square$	.....
$\square$	.....

ORDERED

Stem	Leaf
$\square$	.....
$\square$	.....
$\square$	.....

9 58, 42, 67, 70, 73, 53, 68, 32, 49, 50, 62, 76, 61, 75.

RAW

Tens	Units
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....

ORDERED

Tens	Units
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....

Data Analysis - The Median (Sheet 2)

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Find the median for each leaf of the back-to-back stem and leaf plots below.

1

Male			Stem	Female		
		3	0	0	1	5
9	3	2	1	0	6	
	1	5	2			

Male: Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

Female: Position = $\frac{\square + 1}{2}$ = $\square$ rd score.

The median is $\square$

2

A.M.			Stem	P.M.			
5	5	1	8	2	3	6	8
	6	0	9	1	7	7	
8	8	4	10	4	9	9	

A.M.: Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

P.M.: Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

With Frequency Tables find Σf, write the numbers out beside the column and find the median.

3

Score	*f*	
1	3	1,1,1
2	1	2
3	2	3, 3
4	3	
5	5	
6	3	
Σf=		

Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

4

Score	*f*	
1	2	
2	5	
3	3	
4	4	
5	2	
Σf=		

Position =

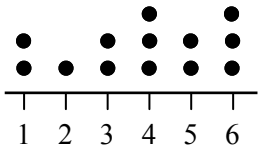
The median is $\square$

5

Score	*f*	
0	1	
10	2	
20	4	
30	3	
40	4	
50	3	
60	3	
Σf=		

Find the median for these.

6

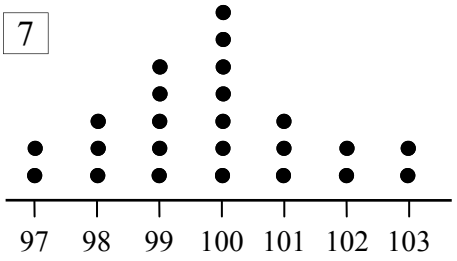


Count dots

Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

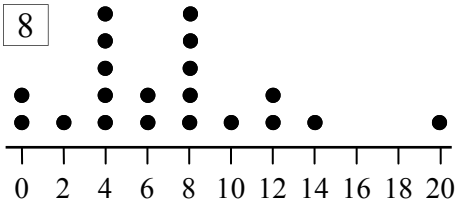
7



Position = $\frac{\square + 1}{2}$ = $\square$ th score.

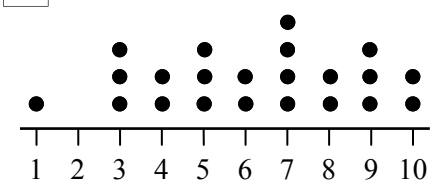
The median is $\square$

8



Position =

9



Data Analysis - The Median (Sheet 1)

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Find the position of the median and the median itself, the data is in ascending order.

1 3, 4, 7, 12, 12, 15, 18, 25, 26, 30, 38, 38, 45, 48, 55, 63, 70.

Position = $\frac{17 + 1}{2}$ (Count the scores)

$= 9^{th}$ score.

The median is 26

2 63, 82, 83, 87, 88, 94, 96, 98, 99, 100, 104, 110, 115, 122, 122, 127, 129, 135, 142, 154, 179.

Position = $\frac{21 + 1}{2}$

$= 11^{th}$ score.

The median is 104

3 0, 2, 3, 4, 12, 13, 13, 18, 23, 24, 29, 29, 30, 31, 37, 39, 42, 46, 55, 57, 59, 94, 123, 128, 152.

Position = $\frac{25 + 1}{2}$

$= 13^{th}$ score

Median = 30

4 105, 127, 185, 209, 234, 276, 362, 368, 391, 412, 419, 438, 446, 502, 535, 572, 590, 590, 612, 616, 623, 655, 662, 720, 739, 750, 750, 773, 781, 782, 810, 834, 880, 881.

Position = $\frac{34 + 1}{2}$

$= 17.5^{th}$ score

Median = $\frac{590 + 590}{2}$

$= 590$

Arrange the data in ascending order then find both the position of the median and its value.

5 53, 26, 40, 45, 52, 31, 76, 42, 50, 64, 39, 43, 24, 33, 60, 59, 78, 25, 10, 18, 32, 40, 57, 62, 20, 25, 55, 19, 28.

10, 18, 19, 20, 24, 25, 25, 26, 28, 31, 32, 33, 39, 40, 40, 42, 43, 45, 50, 52, 53, 55, 57, 59, 60, 62, 64, 76, 78.

Position = $\frac{29 + 1}{2} = 15^{th}$ score

Median = 40.

Find the position and value for the median in the stem and leaf plots.
For Q.8, 9 place the data in the raw and ordered S&L plots first.

6

Tens	Units
0	5 7 7 8 9
1	4 4
2	1 1 1 4 4
3	0 0 3

Position = $\frac{15 + 1}{2} = 8^{th}$ score

Median = 21

7

Tens	Units
8	3 5
9	2 4 9
10	0 | 3 3 4 7
11	2 6

Position = $\frac{12 + 1}{2} = 6.5^{th}$ score

Median = $\frac{100 + 103}{2} = 101.5$

8 16, 25, 11, 24, 8, 12, 24, 20, 23, 15, 19, 4, 13, 8, 2, 9, 3.

RAW

Stem	Leaf
0	848293
1	612593
2	54403

ORDERED

Stem	Leaf
0	234889
1	123569
2	03445

Position = $\frac{17 + 1}{2}$

$= 9^{th}$ score

Median = 13

9 58, 42, 67, 70, 73, 53, 68, 32, 49, 50, 62, 76, 61, 75.

RAW

Tens	Units
3	2
4	29
5	830
6	7821
7	0365

ORDERED

Tens	Units
3	2
4	29
5	038
6	1278
7	0356

Position = $\frac{14 + 1}{2}$

$= 7.5^{th}$ score

Median = $\frac{61 + 62}{2}$

$= 61.5$

Data Analysis - The Median (Sheet 2)

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Find the median for each leaf of the back-to-back stem and leaf plots below.

1

Male			Stem	Female		
		3	0	0	1	5
9	3	2	1	0	6	
	1	5	2			

Male: Position = $\frac{6+1}{2}$ = 3.5th score.

The median is 16

Female: Position = $\frac{5+1}{2}$ = 3rd score.

The median is 5

2

A.M.			Stem	P.M.			
5	5	1	8	2	3	6	8
	6	0	9	1	7	7	
8	8	4	10	4	9	9	

A.M.: Position = $\frac{8+1}{2}$ = 4.5th score.

The median is 93

P.M.: Position = $\frac{10+1}{2}$ = 5.5th score.

The median is 94

With Frequency Tables find Σf, write the numbers out beside the column and find the median.

3

Score	*f*	
1	3	1,1,1
2	1	2
3	2	3, 3
4	3	4, 4, 4
5	5	5, 5, 5, 5, 5
6	3	6, 6, 6
Σf=	17	

Position = $\frac{17+1}{2}$ = 9th score.

The median is 4

4

Score	*f*	
1	2	1, 1
2	5	2, 2, 2, 2, 2
3	3	3, 3, 3
4	4	4, 4, 4, 4
5	2	5, 5
Σf=	16	

Position = $\frac{16+1}{2}$ = 8.5th score

The median is 3

5

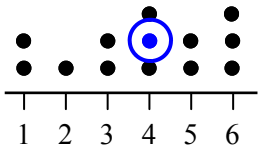
Score	*f*	
0	1	0
10	2	10,10
20	4	20, 20, 20, 20
30	3	30, 30, 30
40	4	40, 40, 40
50	3	50, 50, 50
60	3	60, 60, 60
Σf=	20	

Position = $\frac{20+1}{2}$ = 10.5th score

The median is 35

Find the median for these.

6

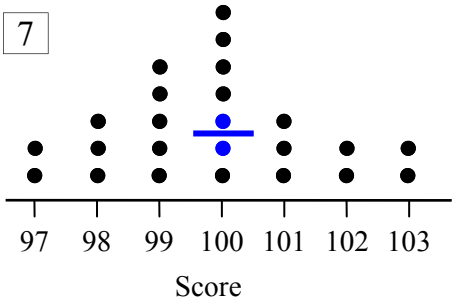


Count dots

Position = $\frac{13+1}{2}$ = 7th score.

The median is 4

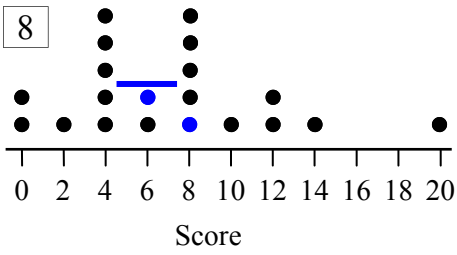
7



Position = $\frac{24+1}{2}$ = 12.5th score.

The median is 100

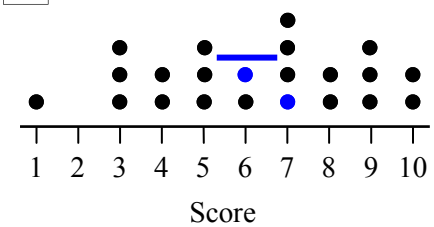
8



Position = $\frac{20+1}{2}$ = 10.5th score

The median is 7

9



Position = $\frac{22+1}{2}$ = 11.5th score

The median is 6.5

Data Analysis - The Median (Homework Strip)

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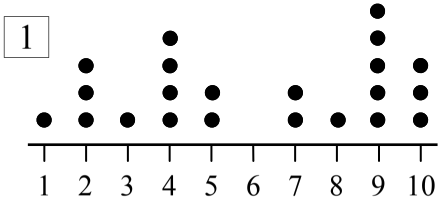
Date Due: _____

Find the median for these.

1



Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

2

Coffee	Tens	Tea
6 2 2	0	5 7 7
9 3 1	1	0 1 4 7
7 4 0	2	3 3 8

Coffee

Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

Tea

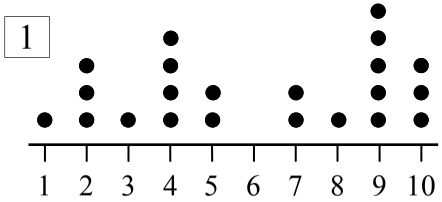
Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

Date Due: _____

Find the median for these.

1



Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

2

Coffee	Tens	Tea
6 2 2	0	5 7 7
9 3 1	1	0 1 4 7
7 4 0	2	3 3 8

Coffee

Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

Tea

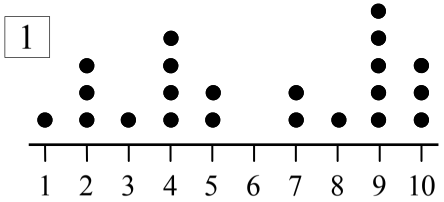
Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

Date Due: _____

Find the median for these.

1



Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

2

Coffee	Tens	Tea
6 2 2	0	5 7 7
9 3 1	1	0 1 4 7
7 4 0	2	3 3 8

Coffee

Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

Tea

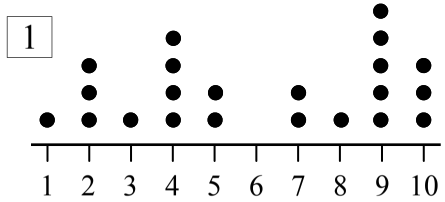
Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

Date Due: _____

Find the median for these.

1



Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

2

Coffee	Tens	Tea
6 2 2	0	5 7 7
9 3 1	1	0 1 4 7
7 4 0	2	3 3 8

Coffee

Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

Tea

Position = $\frac{\square + 1}{2}$ = $\square^{th}$ score.

The median is _____

Data Analysis - The Median (Homework Strip)

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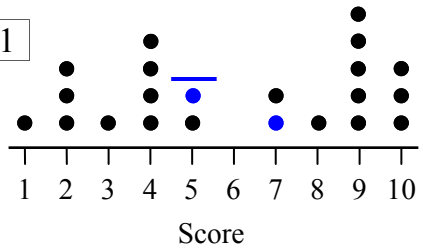
Date Due:

Find the median for these.

1



Position = $\frac{22 + 1}{2}$ = 11.5 th score.

The median is 6

2

Coffee	Tens	Tea
6 2 2	0	5 7 7
9 (3) 1	1	0 1 | 4 7
7 4 0	2	3 3 8

Coffee

Position = $\frac{9 + 1}{2}$ = 5 th score.

The median is 13

Tea

Position = $\frac{10 + 1}{2}$ = 5.5 th score.

The median is 12.5

Data Analysis - Mode, Median, Range and Mean.

These sheets are review sheets and so the method of finding the Mode, Mean, Range and Median won't be outlined below. But here is a reminder:

Mode - Is the score that occurs the most, there can be 1 mode, 2 modes (bi-modal) or No Mode. Write "No Mode" not "0" if there is no mode. If there are 3 or modes then the data is considered to have no mode.

Mean - Is the average of the scores. So add the scores then divide by the number of scores.

Range - Is the highest score minus the lowest score. If the lowest score is negative then when you subtract the two negative signs change to a plus '+'.

Median - Is the middle score when the scores are arranged in ascending order. If there are an odd number of scores it is the middle score. If there are an even number of scores then the median is the average of the two middle scores. So add them and divide by two.

Data Analysis - Mode, Median, Range and Mean. (Sheet 1)

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Calculate the mode, range, median and mean, for the data below. Round to 1 d.p. if required.

1 19 , 34 , 22 , 16 , 27 , 35 , 19 , 12 , 16

Write the scores out in ascending order on this line

Mode = Mode = Most

Median = Middle score (when in order)

Range = Largest - Smallest

=

Mean = Add scores / No. of scores = 1 d.p.

2 68 , 54 , 23 , 42 , 38 , 52 , 65 , 42 .

Mode =

Median = $\frac{\quad + \quad}{2}$ =

Range = - =

Mean = =

3 240, 310, 205, 296, 240, 256, 205, 282, 310.

Mode =

Median =

Range = - =

Mean = =

4

Tens	Units
0	0 7 7
1	2 5 9 9
2	2 3 7
3	1
4	2 7

Mode =

Range = =

Median =

Mean = =

5

Mode =

Range = =

Median = =

Mean = =

Stem	Leaf
7	5 5 9 9
8	0 0 2
9	6 6 6 7
10	
11	3 4 8
12	1 3 5 5

6

TRAIN	Stem	BUS
3 2	29	2 3 7
7 6 4	30	1 1 5 8
5 2 0	31	2 4 9
6 3 0 0	32	0 1
4 2 2	33	

TRAIN

Mode =

Range = =

Median = =

Mean = =

BUS

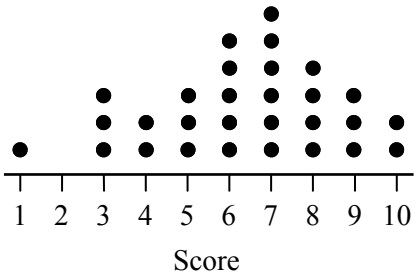
Mode =

Range = =

Median = =

Mean = =

7



Mode =

Range = =

Median = = =

Mean = = =

Data Analysis - Mode, Median, Range and Mean. (Sheet 2)

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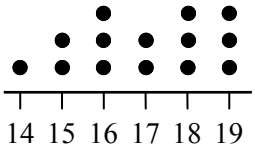
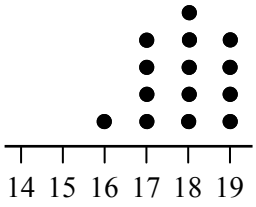
A hot dog stall compares its sale figures for two locations. The dot plots show the number of hot dogs sold per hour, over a 14 hour time period.

Location 1



Hot Dogs Sold per hr

Location 2



Hot Dogs Sold per hr

Calculate the range, mode, median and mean for both locations. Round your answer to 1 d.p., if required.

1

Location 1	Location 2
Mode =	Mode =
Range =	Range =
=	=
Median =	Median =
=	=
Mean =	Mean =
=	=

2 Which stand was more profitable? Would you use the range, mode, median or mean to support your answer?

3 Location 2 has more consistent sales. Use the range and the mode (from both locations) to support this.

Calculate the mode, range, median and mean, for the frequency tables below.

4

Score	*f*	*fx*
10	3	
20	5	
30	2	
40	3	
50	1	
$\Sigma f =$		$\Sigma fx =$

Write the scores out to find the median

Mode = Median =

Range = =

= Mean =

=

Steve is selling units in a proposed unit block. He has sold 5 standard units and one penthouse suite.

5 Find the mean, mode, range and median for the 5 standard units, prices (in $) are below.

677 000, 732 000, 775 000, 790 000, 815 000.

Mode = Median =

Range = =

Mean = =

6 Find the mean, mode, range and median for the 5 standard units and the penthouse, prices (in $) are below.

677 000, 732 000, 775 000, 790 000, 815 000, 3 750 000.

Mode =

Mean = =

Range = =

Median = =

7 Discuss the effect of the penthouse's sale on the range, mean and median.

Data Analysis - Mode, Median, Range and Mean. (Sheet 1)

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Calculate the mode, range, median and mean, for the data below. Round to 1 d.p. if required.

1 19 , 34 , 22 , 16 , 27 , 35 , 19 , 12 , 16

12, 16, 16, 19, 19, 22, 27, 34, 35.

Write the scores out in ascending order on this line

Mode = 16, 19 (Mode = Most)

Median = 19 (Middle score (when in order))

Range = 35 - 12

= 23 (Largest - Smallest)

Mean = 200 / 9 = 22.2 (Add scores; No. of scores; 1 d.p.)

2 68 , 54 , 23 , 42 , 38 , 52 , 65 , 42 .

23, 38, 42, 42, 52, 54, 65, 68.

Mode = 42

Median = (42 + 52) / 2

= 47

Range = 68 - 23

= 45

Mean = 384 / 8 = 48

3 240, 310, 205, 296, 240, 256, 205, 282, 310.

205, 205, 240, 240, 256, 282, 296, 310, 310.

Mode = No mode

Median = 256

Range = 310 - 205

= 105

Mean = 2 344 / 9

= 260.4

4

Tens	Units
0	0 7 7
1	2 5 9 9
2	2 3 7
3	1
4	2 7

Mode = 7 and 19.

Range = 47 - 0

= 47

Median = 19

Mean = $^{271}/_{13}$

= 20.8

5

Mode = 96

Range = 125 - 75

= 50

Median = $^{96 + 96}/_{2}$

= 96

Mean = $^{1\,774}/_{18}$

= 98.6

Stem	Leaf
7	5 5 9 9
8	0 0 2
9	6 6 6 7
10	
11	3 4 8
12	1 3 5 5

6

TRAIN	Stem	BUS
3 2	29	2 3 7
7 6 4	30	1 1 5 8
5 2 0	31	2 4 9
6 3 0 0	32	0 1
4 2 2	33	

TRAIN

Mode = 320 and 332

Range = 334 - 292

= 42

Median = 315

=

Mean = $^{4\,726}/_{15}$

= 315.1

BUS

Mode = 301

Range = 321 - 292

= 29

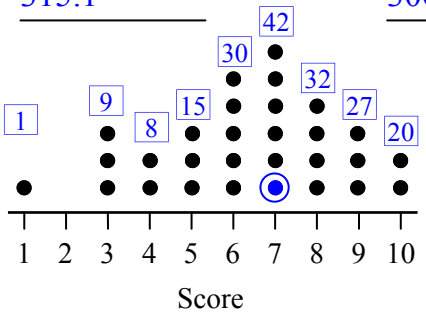
Median = $^{305+308}/_{2}$

= 306.5

Mean = $^{3\,683}/_{12}$

= 306.9

7



Mode = 7

Range = 10 - 1

= 9

Median = 7

=

Mean = $^{184}/_{29}$

= 6.3

Data Analysis - Mode, Median, Range and Mean. (Sheet 2)

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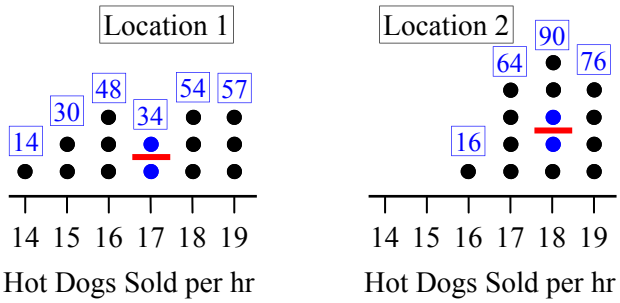
A hot dog stall compares its sale figures for two locations. The dot plots show the number of hot dogs sold per hour, over a 14 hour time period.



Calculate the range, mode, median and mean for both locations. Round your answer to 1 d.p., if required.

1

Location 1		Location 2	
Mode =	No mode	Mode =	18
Range =	19 - 14	Range =	19 - 16
=	5	=	3
Median =	$^{17+17}/_{2}$	Median =	$^{18+18}/_{2}$
=	17	=	18
Mean =	$^{237}/_{14}$	Mean =	$^{246}/_{14}$
=	16.9	=	17.6

2 Which stand was more profitable? Would you use the range, mode, median or mean to support your answer?

Location 2 is more profitable. You would use the mean to support this because it shows that the average number of sales is 0.7 more per hour at Location 2. Median also indicates a higher middle score suggesting a higher sale rate.

3 Location 2 has more consistent sales. Use the range and the mode (from both locations) to support this.

Location 2 has a mode of 18, while Location 1 has no mode due to being more evenly Distributed (less consistent). The range of 5 at location 1 shows wider spread, while the range of 3 at Location1 shows more consistent sales.

Calculate the mode, range, median and mean, for the frequency tables below.

4

Score	*f*	*fx*
10	3	30
20	5	100
30	2	60
40	3	120
50	1	50
$\Sigma f =$	14	$\Sigma fx = 360$

Write the scores out to find the median

10, 10, 10

20, 20, 20, 20, 20

30, 30

40, 40, 40

50

Mode = 20

Range = 50 - 10

= 40

Median = 20

=

Mean = $^{360}/_{14}$

= 25.7

Steve is selling units in a proposed unit block. He has sold 5 standard units and one penthouse suite.

5 Find the mean, mode, range and median for the 5 standard units, prices (in $) are below.

677 000, 732 000, 775 000, 790 000, 815 000.

Mode = No mode Median = $775 000

Range = 815 000 - 677 000 = $138 000

Mean = $^{3\ 789\ 000}/_{5}$ = $757 800

6 Find the mean, mode, range and median for the 5 standard units and the penthouse, prices (in $) are below.

677 000, 732 000, 775 000, 790 000, 815 000, 3 750 000.

Mode = No mode

Mean = $^{7\ 539\ 000}/_{6}$ = $1 256 500

Range = 3 750 000 - 677 000 = $3 073 000

Median = $^{775\ 000+790\ 000}/_{2}$ = $782 500

7 Discuss the effect of the penthouse's sale on the range, mean and median.

The range has substantially increased due to the penthouses high price. The mean has also increased substantially but the median has only increased by $7 500. So the median is less affected by the large price, or outlier.

Data Analysis - Mode, Median, Range and Mean (Homework Strip)

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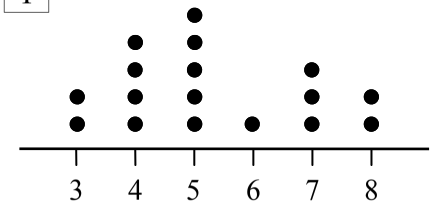
Date Due: _____

Find the range, mode, mean (1 d.p.) and median for these.

1



Mode = ____ Median = ____

Range = ____ - ____ = ____

Mean = ____ / ____ = ____ (1 d.p.)

2

Tens	Units
11	2 5 5
12	0 3 6
13	7 7 7 8
14	1

Mode = ____ Median = ____

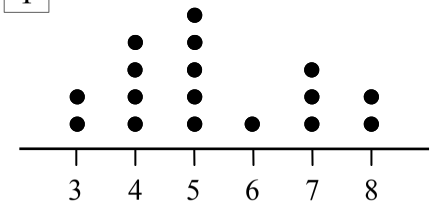
Range = ____ - ____ = ____

Mean = ____ / ____ = ____

Date Due: _____

Find the range, mode, mean (1 d.p.) and median for these.

1



Mode = ____ Median = ____

Range = ____ - ____ = ____

Mean = ____ / ____ = ____ (1 d.p.)

2

Tens	Units
11	2 5 5
12	0 3 6
13	7 7 7 8
14	1

Mode = ____ Median = ____

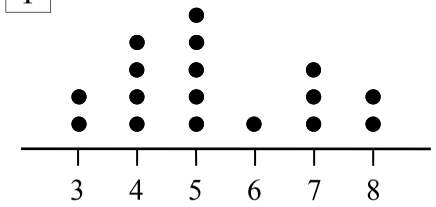
Range = ____ - ____ = ____

Mean = ____ / ____ = ____

Date Due: _____

Find the range, mode, mean (1 d.p.) and median for these.

1



Mode = ____ Median = ____

Range = ____ - ____ = ____

Mean = ____ / ____ = ____ (1 d.p.)

2

Tens	Units
11	2 5 5
12	0 3 6
13	7 7 7 8
14	1

Mode = ____ Median = ____

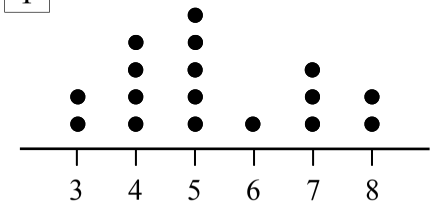
Range = ____ - ____ = ____

Mean = ____ / ____ = ____

Date Due: _____

Find the range, mode, mean (1 d.p.) and median for these.

1



Mode = ____ Median = ____

Range = ____ - ____ = ____

Mean = ____ / ____ = ____ (1 d.p.)

2

Tens	Units
11	2 5 5
12	0 3 6
13	7 7 7 8
14	1

Mode = ____ Median = ____

Range = ____ - ____ = ____

Mean = ____ / ____ = ____

Data Analysis - Mode, Median, Range and Mean (Homework Strip)

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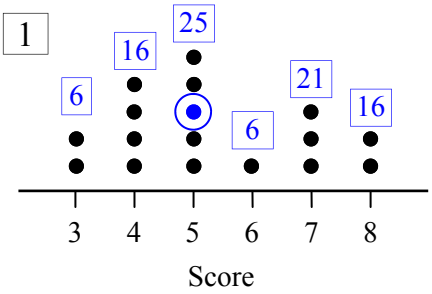
Date Due: _____

Find the range, mode, mean (1 d.p.) and median for these.

1



Mode = 5 Median = 5

Range = 8 - 3 = 5

Mean = $\frac{90}{17}$ = 5.3 (1 d.p.)

2

Tens	Units
11	2 5 5
12	0 3 (6)
13	7 7 7 8
14	1

Mode = 137 Median = 126

Range = 141 - 112 = 29

Mean = $\frac{1\,401}{11}$ = 127.4

Data Analysis - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Data Analysis - Find a Word

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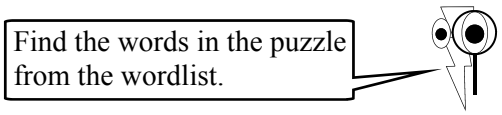

L	O	L	T	E	A	C	E	R	A	P	M	O	C	L
L	O	C	A	T	E	V	T	S	E	W	O	L	P	P
O	E	Q	O	X	W	X	E	H	I	G	H	E	S	T
B	R	X	B	O	H	L	X	O	N	A	I	D	E	M
P	O	S	I	T	I	O	N	I	L	A	B	T	G	V
T	C	A	A	T	S	X	D	O	Q	J	W	W	A	F
O	S	A	R	Q	K	N	W	U	U	U	A	R	R	A
L	U	A	R	U	E	E	M	E	R	T	X	E	E	E
P	U	R	R	C	R	A	O	P	N	U	Q	P	V	L
Q	Q	R	S	T	T	O	D	V	M	U	A	P	A	N
L	E	A	U	V	N	M	E	E	E	M	M	U	S	M
O	L	N	A	X	V	O	G	N	A	X	O	B	T	M
L	B	G	R	O	X	N	C	W	N	O	R	D	E	R
W	A	E	Q	B	A	Y	R	A	M	M	U	S	M	R
O	T	A	U	R	M	A	R	G	O	T	S	I	H	M

WORDLIST

MEDIAN	HIGHEST	SCORE	COMPARE	RANGE
ORDER	HISTOGRAM	DOT	LOCATE	UPPER
LEAF	TABLE	QUARTILES	NUMBER	SUMMARY
EXTREME	POSITION	MODE	LOWER	STEM
MEAN	BOX	WHISKER	LOWEST	ARRANGE
PLOT	FREQUENCY	AVERAGE	ASCENDING	CONTRAST

Data Analysis - Find a Word

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Find the words in the puzzle from the wordlist.



L	O	L	T	E	A	C	E	R	A	P	M	O	C	L
L	O	C	A	T	E	V	T	S	E	W	O	L	P	P
O	E	Q	O	X	W	X	E	H	I	G	H	E	S	T
B	R	X	B	O	H	L	X	O	N	A	I	D	E	M
P	O	S	I	T	I	O	N	I	L	A	B	T	G	V
T	C	A	A	T	S	X	D	O	Q	J	W	W	A	F
O	S	A	R	Q	K	N	W	U	U	U	A	R	R	A
L	U	A	R	U	E	E	M	E	R	T	X	E	E	E
P	U	R	R	C	R	A	O	P	N	U	Q	P	V	L
Q	Q	R	S	T	T	O	D	V	M	U	A	P	A	N
L	E	A	U	V	N	M	E	E	E	M	M	U	S	M
O	L	N	A	X	V	O	G	N	A	X	O	B	T	M
L	B	G	R	O	X	N	C	W	N	O	R	D	E	R
W	A	E	Q	B	A	Y	R	A	M	M	U	S	M	R
O	T	A	U	R	M	A	R	G	O	T	S	I	H	M

WORDLIST

MEDIAN	HIGHEST	SCORE	COMPARE	RANGE
ORDER	HISTOGRAM	DOT	LOCATE	UPPER
LEAF	TABLE	QUARTILES	NUMBER	SUMMARY
EXTREME	POSITION	MODE	LOWER	STEM
MEAN	BOX	WHISKER	LOWEST	ARRANGE
PLOT	FREQUENCY	AVERAGE	ASCENDING	CONTRAST

Data Analysis Review

This is the review of the Data Analysis Topic.

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Display this data in an ordered Stem and Leaf plot.

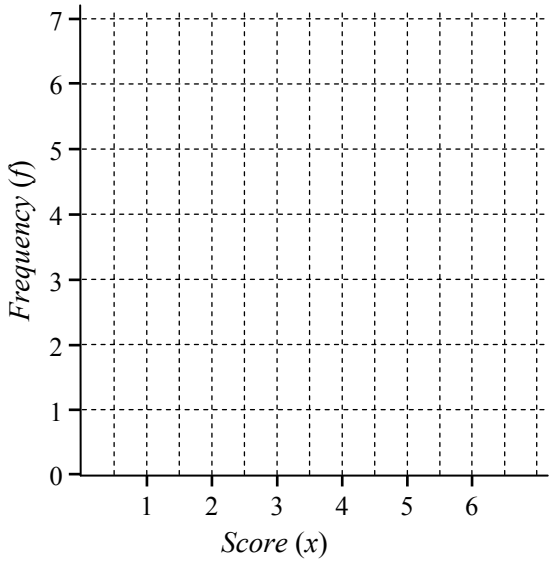
1 19, 25, 11, 23, 34, 8, 18, 21, 30, 15, 15, 3.

2 Display the given data in the frequency table.

Data: 3, 1, 5, 1, 6, 4, 4, 3, 1, 1, 2, 5, 3, 5, 1, 5, 1.

Score	Tally	Frequency
		$\Sigma f =$

3 Display the above data (Q.2.) in a histogram with a frequency polygon.



4 Display the supplied data in a dot plot.

Data: 55, 61, 55, 56, 62, 63, 60, 63, 57, 55, 61, 57, 55, 55, 61, 56, 57, 63, 63.

Scores

Find the mode and range for these

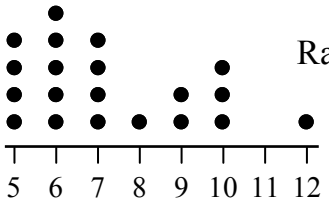
5 7, 13, 19, 13, 22, 18, 5, 13, 23, 16, 19.

Mode = _____ Range = _____

6 24, 0, 38, 33, 22, 19, -10, 5, -2, 33, 7, 22.

Mode = _____ Range = _____

7



Cars per Hour

Range = _____

Mode = _____

8

Tens	Units
8	2 6 6
9	0 7
10	3 3 5
11	1

Range = _____

Mode = _____

9

Score	*f*
0	3
2	1
4	3
6	2
8	2
10	3

Range = _____

Mode = _____

10

Adults	Tens	Children
6 6 3	0	5
4 4 2 1	1	3 3 4
8 4	2	1 1 1 8
9 7 3 0	3	0 1 2 2 2

Adults

Range = _____

Mode = _____

Children

Range = _____

Mode = _____

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Find the mean for these. Round to 1 d.p. when required.

11 18, 53, 55, 94.

12 James scored 52% and 73% in two tests, find his average mark.

13

Score	*f*
10	3
15	1
20	3
25	2
30	2

14

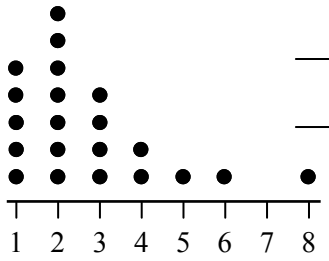
Stem	Leaf
20	4
21	6 8 8
22	1 5 9 9
23	7 7

15 Emily earns $24.75 per hour. In one week she works 19 hours, in the next she works 27 hours. Calculate her pay for each week.

1st Week 2nd Week

16 Calculate Emily's average weekly wage.

17



Children in a Family

Find the median for these.

18 4, 14, 20, 25, 31.

19 9, 14, 25, 25, 35, 44, 48, 55.

20 107, 83, 144, 110, 95, 77, 149, 86.

21

Tens	Units
0	4 8
1	4 7
2	0 6 8
3	1 4

22

Score	*f*
1	2
2	0
3	4
4	2
5	3
6	2

23 Tina auctioned 4 houses on the weekend. Prices are listed. Find the median house price.

$802 000, $915 000, $766 000, $1 877 000.

24 Describe why the median house price is preferred to the mean house price.

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Display this data in an ordered Stem and Leaf plot.

1 19, 25, 11, 23, 34, 8, 18, 21, 30, 15, 15, 3.

Tens	Units
0	8 3
1	9 1 8 5 5
2	5 3 1
3	4 0

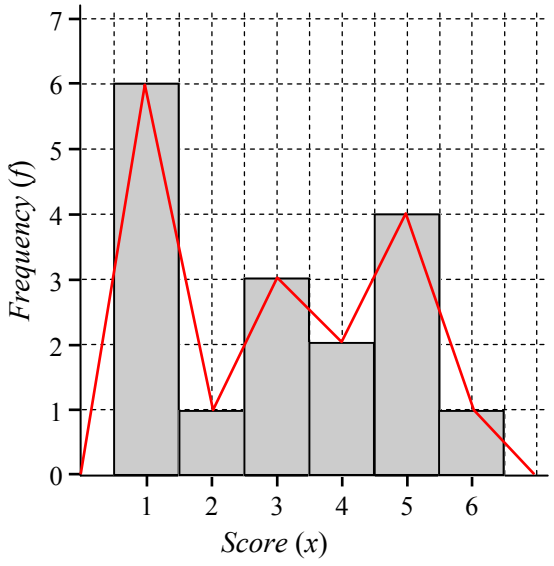
Tens	Units
0	3 8
1	1 5 5 8 9
2	1 3 5
3	0 4

2 Display the given data in the frequency table.

Data: 3, 1, 5, 1, 6, 4, 4, 3, 1, 1, 2, 5, 3, 5, 1, 5, 1.

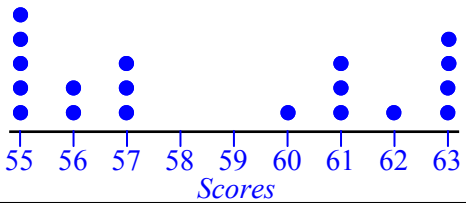
Score	Tally	Frequency
1	𝍸 |	6
2	|	1
3	|||	3
4	||	2
5	𝍸	4
6	|	1
		$\Sigma f =$ 17

3 Display the above data (Q.2.) in a histogram with a frequency polygon.



4 Display the supplied data in a dot plot.

Data: 55, 61, 55, 56, 62, 63, 60, 63, 57, 55, 61, 57, 55, 55, 61, 56, 57, 63, 63.



Find the mode and range for these

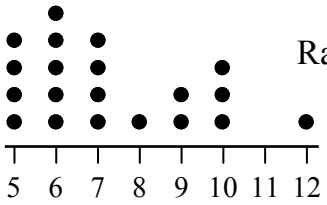
5 7, 13, 19, 13, 22, 18, 5, 13, 23, 16, 19.

Mode = 13 Range = 23 - 5 = 18

6 24, 0, 38, 33, 22, 19, -10, 5, -2, 33, 7, 22.

Mode = 22, 33 Range = 38 -(-10) = 48

7



Range = 12 - 5 = 7

Mode = 6

8

Tens	Units
8	2 6 6
9	0 7
10	3 3 5
11	1

Range = 111 - 82 = 29

Mode = 86, 103.

9

Score	*f*
0	3
2	1
4	3
6	2
8	2
10	3

Range = 10 - 0 = 10

Mode = No mode

10

Adults	Tens	Children
6 6 3	0	5
4 4 2 1	1	3 3 4
8 4	2	1 1 1 8
9 7 3 0	3	0 1 2 2 2

Adults

Range = 39 - 3 = 36

Mode = 6, 14

Children

Range = 32 - 5 = 27

Mode = 21, 32

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Find the mean for these. Round to 1 d.p. when required.

11 18, 53, 55, 94.

$\bar{x} = \frac{220}{4} = 55$

12 James scored 52% and 73% in two tests, find his average mark.

$\bar{x} = \frac{52 + 73}{2} = 62.5\%$

13

Score	*f*	*fx*
10	3	30
15	1	15
20	3	60
25	2	50
30	2	60
	$\Sigma f = 11$	$\Sigma fx = 215$

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{215}{11} = 19.5$

14

Stem	Leaf
20	4
21	6 8 8
22	1 5 9 9
23	7 7

$\bar{x} = \frac{2\,234}{10}$

$= 223.4$

15 Emily earns $24.75 per hour. In one week she works 19 hours, in the next she works 27 hours. Calculate her pay for each week.

1st Week: Pay = 24.75×19

Pay = $470.25

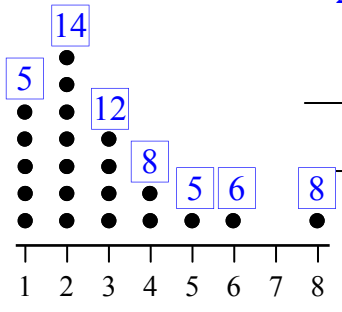
2nd Week: Pay = 24.75×27

Pay = $668.25

16 Calculate Emily's average weekly wage.

Av. weekly wage = $\frac{470.25 + 668.25}{2} = \$569.25$

17



Children in a Family

$\bar{x} = \frac{58}{21}$

$= 2.8$ kids!

Find the median for these.

18 4, 14, 20, 25, 31. Median is 20.

19 9, 14, 25, 25, 35, 44, 48, 55.

Median = $\frac{25 + 35}{2} = 30$

20 107, 83, 144, 110, 95, 77, 149, 86.

77, 83, 86, 95, 107, 110, 144, 149.

Median = $\frac{95 + 107}{2} = 101$

21

Tens	Units
0	4 8
1	4 7
2	0 6 8
3	1 4

Median is 20

22

Score	*f*	
1	2	1, 1
2	0	
3	4	3, 3, 3, 3
4	2	4, 4
5	3	5, 5, 5
6	2	6, 6
	$\Sigma f = 13$	→ Median is the 7th score

Median is 4

23 Tina auctioned 4 houses on the weekend. Prices are listed. Find the median house price.

$802 000, $915 000, $766 000, $1 877 000.

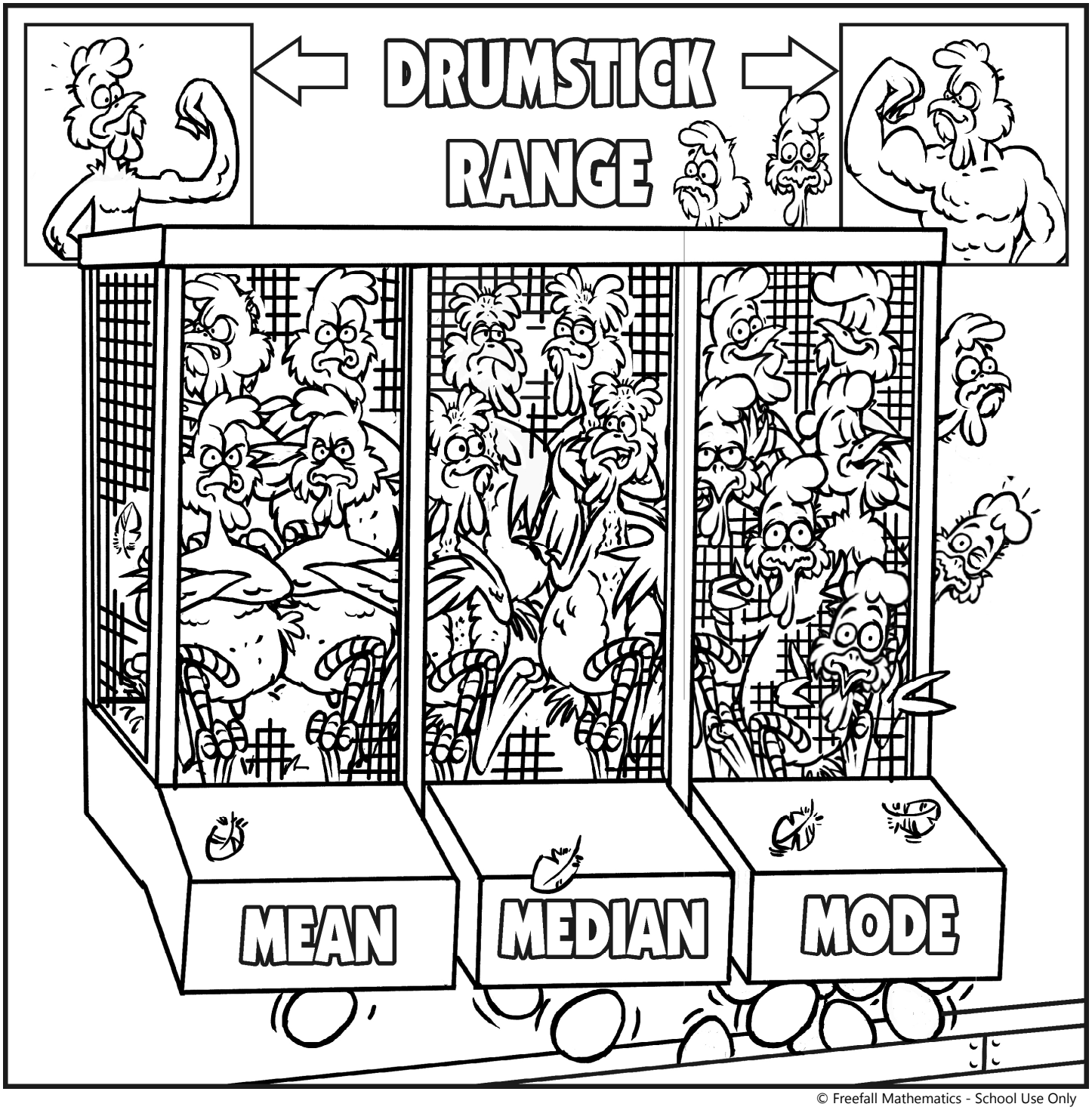
$766 000, $802 000, $915 000, $1 877 000.

Median = $\frac{802\,000 + 915\,000}{2} = \$858\,500$

24 Describe why the median house price is preferred to the mean house price.

The median removes the effect of very cheap and expensive houses. So it gives a better sample of an 'middle' price. The mean would be affected.

Data Analysis



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Data Analysis - Dot Plots and Frequency Tables (Sheet 1)

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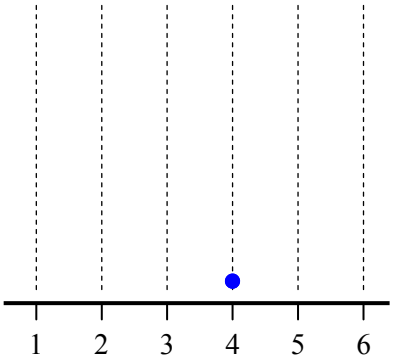
Display the data in a dot plot.

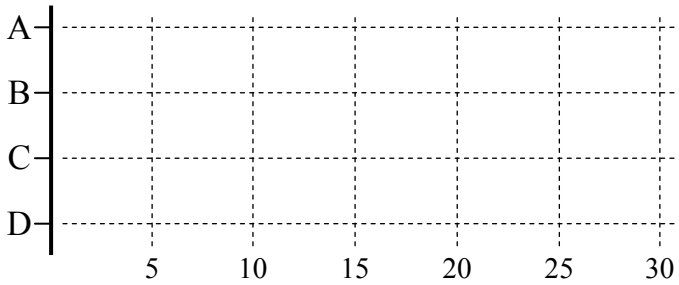
1 Data: 4, 5, 3, 3, 6, 1, 2, 4, 2, 3, 5, 6, 2, 3, 4, 4, 1, 2, 5, 6, 2, 3, 3, 2, 5, 2, 1, 4, 6, 3, 2.



2 An artificial sweetener is comparing itself with three other brands. Graph the results.

Data: BBBDA BABBC ABBBC
BDBAB ABBDB BCBBD
BCABB ABCBD BBADB

sideways! → Artificial Sweetener Preference



3 How many people were sampled? _____

4 How many people preferred Brand B? _____

5 How many people preferred Brand C? _____

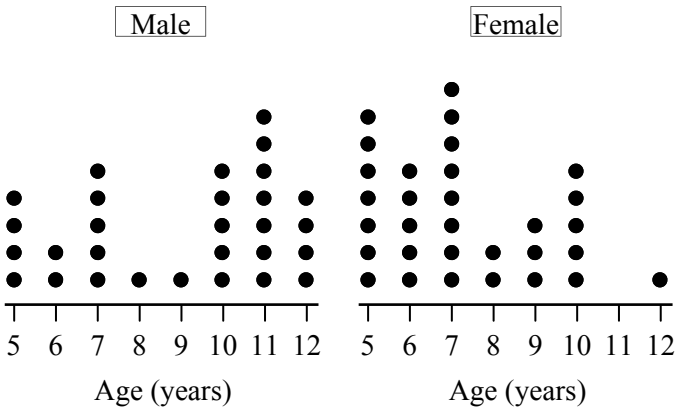
6 What percentage of people prefer Brand B?

Question 4 Ans. → ☐ / ☐ Question 3 Ans. → × 100 = ☐ Round to 1 d.p.

7 What percentage of people prefer Brand C?

Question 5 Ans. → ☐ / ☐ Question 3 Ans. → × 100 = ☐ Round to 1 d.p.

A waterslide at a theme park records the ages of the first 60 children who use the slide, the results are shown in two Dot plots. One male, one female.



8 How many children were male? _____

9 How many children were 7 years old? _____

10 How many children were 11 or older? _____

11 How many more 10 year olds used the slide compared to 6 year olds? _____

12 Are these true or false? There were more ...

(T) (F) Girls using the waterslide

(T) (F) 7 y.o. boys using the slide than 10 y.o. girls

(T) (F) Older male children than female.

13 What percentage of 9 y.o.'s were male?

Male 9 y.o. here → ☐ / ☐ Total 9 y.o. here → × 100 = ☐ Show the % sign

14 What percentage of 5 y.o.'s were female?

☐ / ☐ × 100 = ☐ Round to 1 d.p.

15 What percentage of females were 10 y.o.?

Total females here → ☐ / ☐ × ☐ = ☐ Round to 1 d.p.

16 What percentage of 5 - 7 y.o.'s were male?

☐ / ☐ × ☐ = ☐ Round to 1 d.p.

Data Analysis - Dot Plots and Frequency Tables (Sheet 2)

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Complete the frequency column and total for Σf, then answer the questions.

1 Number of Fish in a Tank

Fish	Tally	Frequency
2	𝍸	
3	𝍸 ||	
4	𝍸 𝍸 |||	
5	𝍸 𝍸	
6	𝍸 𝍸 ||||	
7	𝍸 𝍸 |||	
		$\Sigma f =$

2 What number of fish had the highest frequency? _____

3 What number of fish had the same frequency? _____

4 What was the frequency for a tank to have an odd number of fish?

_____

5 What was the frequency for a tank to have less than 5 fish?

_____

6 Soft Drink Preference

Type	Tally	Frequency
A	𝍸 𝍸 𝍸 ||	
B	𝍸 𝍸 |||	
C	||||	
D		
E	𝍸 𝍸 ||	
F	𝍸 𝍸 𝍸	
		$\Sigma f =$ 75

7 D's results were unreadable due to a drink spill, use Σf to calculate them.

8 Rank the drink brands in order from least, to most popular.

_____

9 How many people preferred brand 'A' or brand 'F'? _____

Place the data into the Frequency table, showing the tally, the frequency and write 'Σf= *(total)*', then answer the questions.

10 Away from School - Sports Choices

Sport	Tally	Frequency
Tennis		
Gym		
Bocce		
Archery		

Data:

TTGAA	TTGAA
TGBBA	GBBAT
AAGGG	TTTTA
BTBTA	BTBBA
AAATG	GTBBA
BBGBT	AABTB
TTGBB	BBTBG
ATTTA	ABBTG

11 What was most popular 'away' sport chosen? _____

12 How many people chose 'ball sports'? _____

13 If there are a total of 135 students in the year, calculate the percentage that chose 'away' sports.

$$\%\ \text{Away Sport} = \frac{\square}{135} \times 100$$

$$= \underline{\square}\ \%$$

Round to 1 d.p.

14 A real estate agent is checking rental homes for smoke detectors, if the smoke detector was Battery powered, Mains powered, or Dual (both). If a detector didn't work its type wasn't recorded it was just recorded as Inoperative. If a house had no smoke detector, it was recorded as 'None'.

Smoke Detector Type or Status

Type	Tally	Frequency
Battery		
Mains		
Dual		
Inoperative		
None		

Data:

BBDMB	BBDBM
BIBBM	MNDIB
MBDBI	BBIIN
NBBIM	INDDB
BBNIB	NBBMB
MBBDN	IBBMB
NINBB	NDDMB

15 How many homes had operational smoke detectors? _____

16 What percentage (to 1 d.p.) of homes had inoperative or no smoke detectors?

$$\%\ \text{No S.D} = \frac{\square}{\square} \times 100 = \underline{\square}$$

Round to 1 d.p.

17 What percentage (to 1 d.p.) of homes had Dual-powered smoke detectors?

$$\%\ \text{Dual Powered} = \frac{\square}{\square} \times 100 = \underline{\square}$$

Round to 1 d.p.

Data Analysis - Ordered Stem and Leaf Plots (Sheet 1)

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Display the data sets in Raw then Ordered Stem and Leaf plots. Don't use commas!

1 Data: 23, 17, 35, 21, 30, 8, 11, 24, 6, 9, 17, 21, 32, 3, 11, 13.

RAW → ORDERED

Tens	Units
0	
1	
2	
3	

Tens	Units
0	
1	
2	
3	

2 Data: 74, 97, 70, 106, 92, 100, 86, 91, 97, 85, 110, 83, 101, 95, 79, 90, 103, 85.

RAW → ORDERED

Stem	Leaf

Stem	Leaf

3 Data: 998, 971, 1 005, 993, 1 018, 1 015, 985, 994, 1 026, 1 012, 1 002, 994, 978, 993, 970, 1 018, 989, 991, 1 011, 1 021, 986, 1 000, 1 002, 995, 1 012, 977, 983.

Stem	Leaf

→

Stem	Leaf



Rewrite this stem and leaf plot in order. That's an order!

Same data shown in order

65440	0	04456
955	1	559
7200	2	0027

4

RAW Leaf	Stem	Leaf
8 3 0 2 6	0	0 0 2 8 5
7 3 3 9	1	7 1 6 5 7 9
7 8 0 4 1 1	2	2 0 3 8 8 4
3 9 8 6	3	9 2 8 1 3

↓

ORDERED Leaf	Stem	Leaf
High ← Low		Low → High
	0	
	1	
	2	
	3	



It's auction time! 'Four n'Fluffy' livestock sales is selling sheep and alpacas. Lot sales are shown below. Place the data in a back-to-back stem and leaf plot, first raw data, then in order.

5 Four n'Fluffy Auction Sales (in Lots)

Sheep: 22, 25, 8, 37,18, 13, 3, 22, 15, 17, 34, 22, 17, 12, 20, 23, 11, 27.

Alpacas: 36, 28, 5, 38, 22, 19, 12, 7, 25, 37, 19, 31, 26, 30, 23, 37, 27, 19, 20.

RAW Alpacas	Stem	Sheep

↓

ORDERED Alpacas	Stem	Sheep

Data Analysis - Frequency Histograms and Frequency Polygons (Sheet 2)

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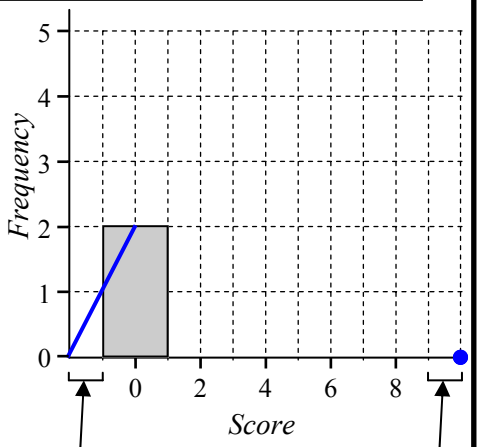
Construct a frequency histogram and frequency polygon for the data in these frequency tables.

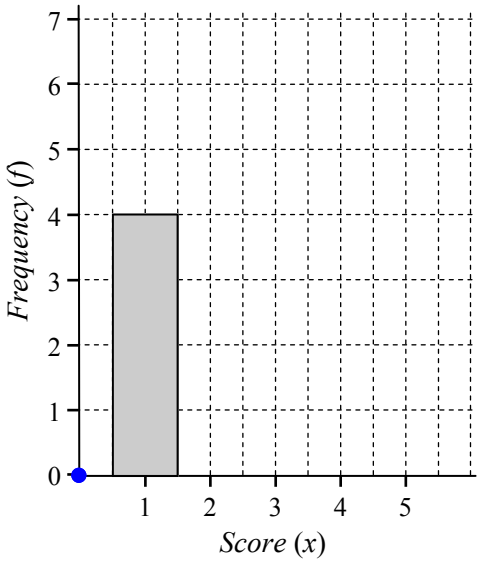
1

Score	*Frequency*
0	2
2	3
4	5
6	1
8	4



Columns touch each other - no gaps between
There is a ½ column gap at the start and at the finish of the columns. Polygon ends at the dot. ●

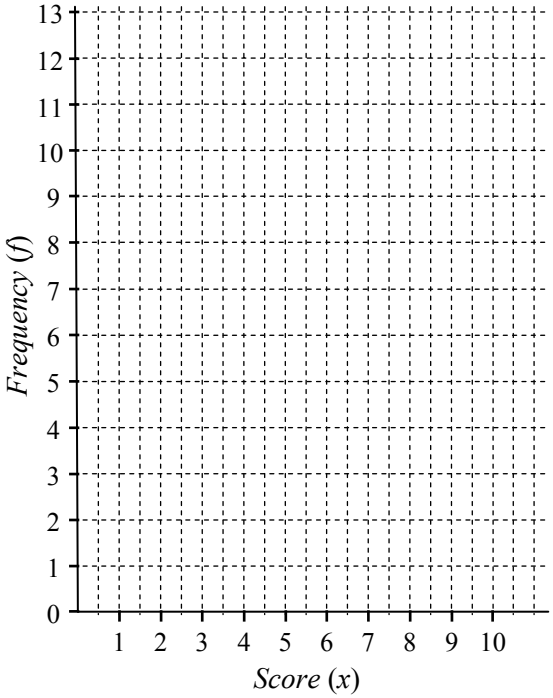
2



x	*f*
1	4
2	2
3	7
4	0
5	6

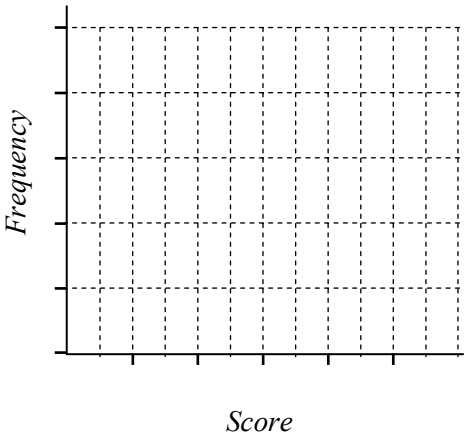
3

x	*f*
1	2
2	0
3	4
4	6
5	7
6	10
7	13
8	12
9	8
10	5



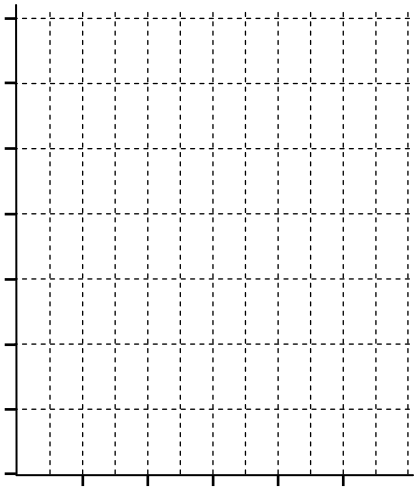
4

Score	*f*
5	5
10	1
15	3
20	0
25	4



5

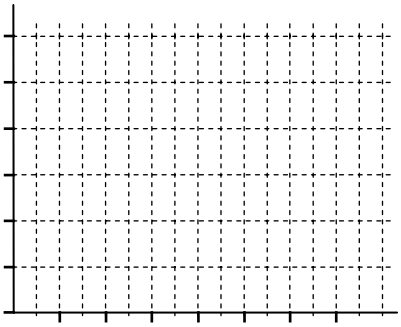
Score	*f*
0	2
2	5
4	3
6	7
8	1



6 Soccer - Home Team - Goals Scored

Data:
1, 2, 5, 2, 0, 3, 2,
3, 4, 1, 1, 3, 1, 2,
3, 3, 0, 2, 1, 2, 2,
1, 4, 0.

Score	*Tally*	*f*
		$\Sigma f =$



Data Analysis - The Mean - Data Sets & Dot Plots (Sheet 1)

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Calculate the mean. Add the scores and divide by the number of scores. Answers are whole numbers

1 27, 38, 24, 31, 44, 56, 50, 19, 26.

Add the scores

$\bar{x} = \frac{\square}{9} = \square$ Divide total by 9

Nine scores = 9

2 73, 86, 41, 29, 56. $\bar{x} = \frac{\square}{\square} = \square$

3 -6, 2, -14. $\bar{x} = \frac{\square}{\square} = \square$

4 16 773, 24 919. $\bar{x} = \frac{\square}{\square} = \square$

5 235, 306, 113, 176, 290, 218, 440.

$\square = \frac{\square}{\square} = \square$

This time you get a decimal answer. Round your answer to 1 decimal place.

6 94, 38, 77, 24, 83, 19.

$\square = \frac{\square}{\square} = \square$ (1 d.p.)

7 55, 78, 11, 107.

8 9, 5.4, 4.3, 6, 7, 3.

9 187, 553, 213.

10 57.17, -44.88.

Find the average height of these students. Find the average height in m (2 d.p.), then in cm.

11 1.75 m, 1.8 m, 1.73 m, 1.55 m, 1.77 m.

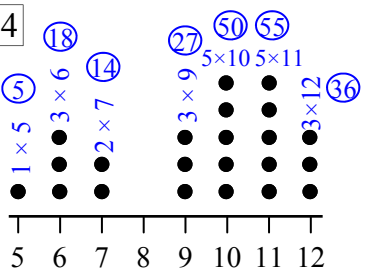
$\square = \frac{\square}{\square} = \square$ m $= \square$ cm

12 1.6 m, 1.74 m, 1.82 m, 1.77 m, 1.83 m, 1.8 m.

13 Find the average height for the 11 students.

Find the mean (1 d.p.) for the data displayed in these dot plots. Find dots × score, add for a total. Then divide the total by the total number of dots.

14



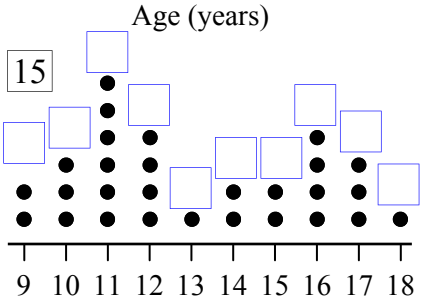
Age (years)

5 + 18 + 14 +...

$\bar{x} = \frac{\square}{\square}$ All Dots

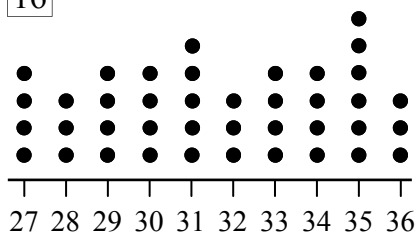
$\bar{x} = \square$ (1 d.p.)

15

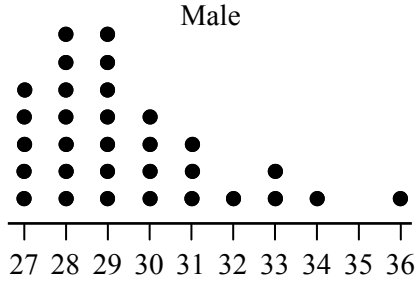


Sales

16



Male



Female

17 Comment on the Male and Female Dot Plots

Compare the mean results and shape of the distribution

Place the data in a dot plot and find the mean.

18 Data: 6, 8, 2, 3, 6, 4, 3, 2, 6, 4, 4, 2, 6, 4, 2, 2, 6, 5, 3, 2, 2, 3, 6, 8, 3.

Data Analysis - The Mean - Stem & Leaf & Freq Tables (Sheet 2)

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Calculate the mean. Add the scores and divide by the number of scores. Answer to 1 d.p. if needed.

1

Stem	Leaf
0	7 8
1	0 4 6
2	8 8 9

Remember the 10's: 7,8 — 10,14,16 — 28,28,29

Add the scores

$\bar{x} =$ ☐ / 8

Divide total

$\bar{x} =$ ☐ by 8

2

Tens	Units
8	4 4 5
9	3
10	0 0 7

$\bar{x} =$ ☐ / ☐

$\bar{x} =$ ☐

3

Tens	Units
18	1 7 7
19	2 6
20	0 3 6
21	1

_____

_____

4

AM.	Tens	PM.
1 1 1	0	5
9 8 7 6	1	8 8 9
0 0	2	1 7 7 8
9 7 3 3	3	0 0 2 2 6

AM: _____ PM: _____

_____ _____

5

Boys	Tens	Girls
8	15	0 3 6 7 7
8 7 5 5	16	2 4 9
9 3	17	1 1
4 1 1	18	

Boys: _____ Girls: _____

_____ _____

Boys & Girls: _____

Complete *fx* columns, find Σfx and divide by the number of scores (Σf) to find the mean to 1 d.p.

6 Use a ruler to go over the dotted lines.

Score (x)		Frequency (f)		fx
0	×	5	=	
2	×	1	=	
4	×	7	=	
6	×	0	=	
8	×	8	=	
10	×	7	=	
Sum the *f* column →		$\Sigma f =$		$\Sigma fx =$ ← Sum the *fx* column

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$

$= \frac{\square}{\square}$

$\bar{x} =$ ☐ 1 d.p.

7

Score	f	
0	2	
1	3	
2	7	
3	12	
4	9	
5	14	
6	13	
	$\Sigma f =$	$\Sigma fx =$

8

Score	f	
0	4	
5	2	
10	7	
15	11	
20	6	
25	4	

_____

_____

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{\square}{\square}$

$\bar{x} =$ ☐

Place this data in a frequency table and then find the mean to 1 d.p.

9 Data: 1, 3, 5, 5, 2, 2, 3, 5, 2, 4, 3, 1, 6, 1, 3, 2, 5, 4, 4, 3, 2, 2, 5, 3, 6, 1, 2, 4, 2, 3.

Score	Tally	f	fx

_____

Data Analysis - Further Mean - Time Applications

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Find the average of these times using your calculator's DMS key.

Don't forget!

1 1h 22m, 1h 47m, 54m.

Calc: 1 °’’’ 2 2 °’’’ + 1 °’’’ 4 7 °’’’ + 0 °’’’ 5 4 °’’’ =

Write the total in the top box then:

Calc: ÷ 3 = $\bar{x} = \frac{\square\text{ h }\square\text{ m}}{3} = \square\text{ h }\square\text{ m}$

2 2h 43m, 1h 53m, 2h 16m, 1h 36m.

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m}$

3 57m, 1h 38m, 39m.

57m = 0h 57m!

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m}$

4 23m, 1h 38m, 2h 47m, 48m, 56m, 1h 52m.

$\bar{x} = \frac{\square}{\square} = \square$

These are the same, except the answer is required in both h/min and converted to minutes alone.

5 1h 13m, 2h 6m, 52m, 97m.

97m = 0h 97m!

$\bar{x} = \frac{\square\text{ h }\square\text{ m}}{\square} = \square\text{ h }\square\text{ m} = \square\text{ m}$

Calc: × 6 0 =

6 2h 31m, 1h 11m, 67m, 3h 19 m, 43m, 87m.

$\bar{x} = \frac{\square}{\square} = \square = \square$

7 Find the average running time in minutes for these three movies: 1h 19m, 1h 47m, 1h 33m.

8 Find the average running time in minutes for these five movies:

1h 46m, 2h 13m, 86m, 1h 55m, 115m.

Convert these times to 24 hour time format.

Examples:

5.17 am = 0517 h

3 + 12 = 15

3.43 pm = 1543 h

9 5 am = ☐☐☐☐ h

10 7 pm = ☐☐☐☐ h

11 9.23 pm = ☐☐☐☐ h

12 8.08 pm = ☐☐☐☐

13 11.37 pm = ☐☐☐☐

14 2.41 pm = ☐☐☐☐

Find the time difference between these times, use 24 hour time format to make the subtraction.

14 6.25 am until 3.08 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

3.08 pm in 24 h — 6.25 am in 24 h

Calc: 1 5 °’’’ 8 °’’’ - 6 °’’’ 2 5 °’’’ =

15 11.44 am until 7.17 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

7.17 pm in 24 h — 11.44 am in 24 h

Dave works the following times for 5 days. Find the average daily hours he works.

16 Monday: 8.47 am until 5.28 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

5.28 pm in 24 h — 8.47 am in 24 h

17 Tuesday: 9.53 am until 6.42 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

18 Wednesday: 7.36 am until 4.15 pm

☐☐☐☐ h - ☐☐☐☐ h = ☐ h ☐☐ m

19 Thursday: 2.39 pm until 11.05 pm

Careful! 2 pm times

20 Friday: 3.48 pm until 10.08 pm

21 Calculate Dave's average hours for the week.

$\bar{x} = \frac{\square}{\square} = \square$

Data Analysis - Mean - Problems

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Find the mean for these, round to 1 d.p. if required.

1 James' biology experiment results in three bean sprouts at heights of: 11.5 cm, 14.3 cm and 17.1 cm. Find the average height for the sprouts.

$\bar{x} = \frac{\square}{\square} = \square$ cm

2 Jo earns the following amounts over 5 weekends:
$124, $85, $110.80, $96.50 and $135. Find the average amount she earns.

$\bar{x} = \frac{\square}{\square} = \square$

3 Chloe earns $17.60 per hour, working a 4 day week. Calculate the earnings for each day she works.

M: 6 h $6 \times 17.60 = \square$

T: 7 h $\square$

W: 4 h $\square$

F: 9 h $\square$

4 Calculate Chloe's average daily wage.

$\bar{x} = \frac{\square}{\square} = \square$

5 Sam supplies Holland blinds, he makes different profits on each width. Find the profit for sales of each width in the weekly table of sales below.

Width (mm)	Profit rate	Quantity	Profit
900	$42	18	
1 200	$68	7	
1 800	$95	9	
2 400	$76	5	

6 Find total sold and total profit.

Total number of Blinds sold: _____

Total Weekly Profit: _____

7 Find the average profit made per blind sold for the week.

_____

8 Below is Sam's results for the second week. Complete the table to calculate the profit made on each size.

Width (mm)	Profit rate	Quantity	Profit
900	$42	4	
1 200	$68	8	
1 800	$95	12	
2 400	$76	8	

9 Find total sold and total profit.

Total number of Blinds sold: _____

Total Weekly Profit: _____

10 Find the average profit made per blind sold for the second week.

_____

11 Has the average blind profit increased or decreased in the second week? Explain why.

_____

_____

_____

_____

_____

12 David is a real estate agent who has sold three properties this week. Calculate David's total weekly sales.

2 Hillside Dr	$807 700
15/9 Botany St	$795 000
9 Seaview Crt	$2 675 000
TOTAL	

13 Calculate the average sales price for the three properties.

_____

_____

14 Calculate the average sales price just for the first two properties.

_____

_____

15 Explain why the answer to Q.13 is much higher than Q.14.

_____

_____

_____

_____

16 Geoff is managing the digging and walling of a train tunnel. He records the rate of progress below.

Day	Distance Completed
Mon	6.3 m
Tues	3.5 m
Wed	4.9 m
Thur	2.5 m
Fri	9.3 m
Sat	7.7 m

Calculate the average distance completed per day.

_____

Data Analysis - Range (Sheet 1)

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Circle the smallest and largest numbers. Subtract them and you have the range.

1 40 , 53 , 19 , 21 , 33 , 10 .

Range = ☐ - ☐ = ☐

2 71 , 66 , 57 , 42 , 42 , 25 .

Range = ☐ - ☐ = ☐

3 13, 38, 23, 46, 27, 19, 54, 62, 83, 50, 39, 44, 23, 11, 6, 79.

Range = ☐ - ☐ = ☐

4 812, 776, 909, 694, 552, 773, 833, 905, 541, 890.

Range =

Watch out when you subtract negative numbers, - - = + !

5 10, 3, -4, 17, 0, 8, -9, 6, 11.

Range = ☐ - ☐ = ☐

6 52, -37, 65, 24, 39, -7, 19.

7 38, 12, 19, 0, 58, -21, 83, 9, 41, -65, 42, -33, 91, 50, -18.

Find the range for these temperatures.

8 Overnight Minimum: -1°C
Maximum: 11°C

Range =

9 Overnight Minimum: -7°C
Maximum: 17°C

Range =

Now find the range of these Stem & Leaf Plots.

10

Tens	Units	
0	0 7 7	0,7,7
1	8 9	18,19
2	3 3	23,23

Remember the 10's

Range = ☐ - ☐ = ☐

11

Tens	Units
7	0 1 3
8	2
9	5 5 6
10	0 4

Range =

12

Tens	Units
95	2 4 4 7 7
96	0 1 3 6
97	9 9

13

Early	Tens	Late
5 4	2	2
8 3 2	3	8 8 9
7 3	4	1 7 8

Early Range =

Late Range =

14

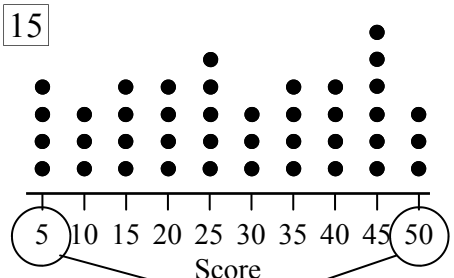
Male	Tens	Female
8 6 2	15	2 2 9
4 0 0	16	0 4 8
7 5	17	6

Male Range =

Female Range =

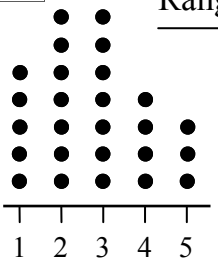
Ignore the dots, ignore the frequency columns, just use the score. Find the range.

15



Range = ☐ - ☐ = ☐

16



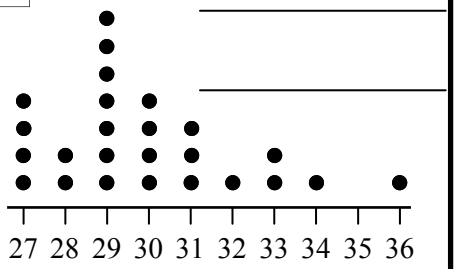
Range = ☐ - ☐

Range = ☐

17

Score	*f*
0	4
5	2
10	7
15	11
20	6
25	4

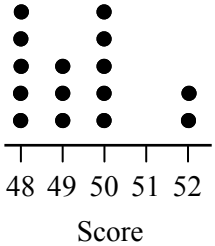
18



19

Score	*f*
10	1
20	6
30	2
40	7

20



Data Analysis - Mode (Sheet 2)

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The MOde is the score that occurs the MOst. Find the mode.

1 1 , 4 , 2 , 2 , 6 , 3 , 2 , 5 .

Mode = ☐

2 8 , 3 , 7 , 2 , 3 , 7 , 2 , 7 .

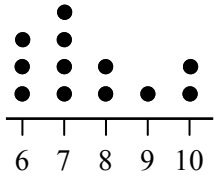
Mode = ☐

3

Tens	Units
9	0 3 3 4
10	5 7 8
11	1 6 6 6

Mode = _____

4



_____
Use the dots to pick a score

5

Score	*f*
2	2
3	8
4	2
5	7
6	7

_____
Use the frequency to pick a score

6

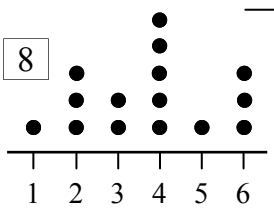
Tens	Units
551	5 6 7
552	9 9
553	1 5

_____

7 5 , 0 , 9 , 4 , 3 , 1 , 7 , 0 .

_____

8



_____

You can have no mode, one mode or two modes. No more than two. 3 Modes = No Mode!

9 32 , 42 , 16 , 32 , 14 , 16 .

Mode = ☐ and ☐

10 25, 13, 12, 18, 12, 16, 13, 5.

Mode = _____

11 11, 15, 9, 14, 17, 25, 23, 6.

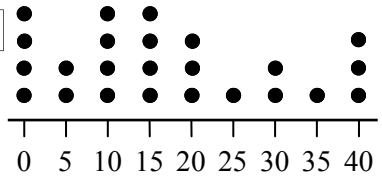
_____

12

Tens	Units
0	3 5 5 7 8
1	2 2 2 6
2	0 1 1
3	3 5 5 5

_____

13



_____

14

Score	*f*
98	5
99	2
100	4
101	5

_____

15

Score	*f*
0	7
2	7
4	3
6	4
8	7
10	5

_____

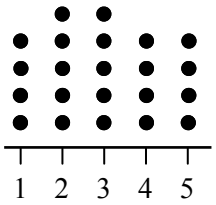
16 95, 97, 86, 97, 90, 84, 95.

_____

17 -6, 23, 17, 5, 11, 17, 5, 17.

_____

18



_____

19

A	Tens	B
9 7	8	1 4 4
3 1 0	9	2 6 6
7 7	10	7 7

Group A Mode = _____

Group B Mode = _____

20

Score	*f*
0	2
1	8
2	3
3	5
4	6
5	0

21

Score	*f*
11	19
12	13
13	14
14	19

_____

_____

The entry and exit modes are both 25. Write a number in the boxes to make this true.

22

Entry	Tens	Exit
5 3	1	0 3
☐ 5 0	2	5 5 5
7 4	3	8 8 ☐

23

Entry	Tens	Exit
7 6	0	3 3
☐ 8	1	1 4 7
5 5	2	5 ☐ ☐

Data Analysis - The Median - Introduction (Sheet 1)

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Rewrite these numbers in order into the circle and squares. The median is in the circle.

1 3, 5, 10, 12, 19, 23, 34.

□, □, □, ○, □, □, □.

Median = □

2 6, 14, 14, 22, 37.

□, □, ○, □, □.

Median = □

3 7, 7, 12, 16, 18, 21, 26.

□, □, □, ○, □, □, □.

Median = □

Now the numbers have to be put in order as well. Low to high.

4 12, 8, 19, 36, 17.

□, □, ○, □, □.

Median = □

5 84, 36, 55, 58, 65.

□, □, ○, □, □.

Median = □

6 66, 86, 99, 92, 49, 62, 57.

□, □, □, ○, □, □, □.

Median = □

This time circle the middle number yourself. Count the numbers to make sure you have rewritten all of them.

7 11, 14, 24, 16, 19.

Rewrite numbers in order, there are 5.

Median = □

8 8, 2, 7, 3, 4, 7, 8, 2, 9.

Median = □

9 4, 8, 2, 4, 6, 7, 13, 15, 9, 3, 5.

Median = □

Find the mean for these.

10 12 and 26. Mean = □

Calc: 1 2 + 2 6 = ÷ 2 =

11 25 and 37. Mean = □

12 13 and 18. Mean = □

These have an even number of scores. You have to find the average of the 2 middle scores.

13 11, 24, 28, 36.

□, ○, ○, □.

Add then ÷ 2

Median = □

14 46, 21, 11, 35.

□, ○, ○, □.

Median = □

15 22, 19, 34, 15, 18, 24.

□, □, ○, ○, □, □.

Median = □

16 8, 19, 14, 6, 11, 15.

□, □, ○, ○, □, □.

Median = □

17 82, 71, 48, 55.

Median = □

18 23, 11, 14, 40, 28, 32.

Median = □

19 104, 111, 86, 123, 92, 88.

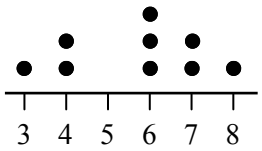
Median = □

20 8, 13, 12, 6, 13, 18, 7, 12.

Median = □

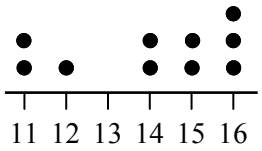
Write these dot plots out as numbers and find the median.

21



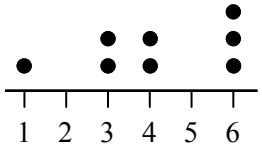
Median = □

22



Median = □

23



Median = □

Data Analysis - The Median - Introduction (Sheet 2)

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Write these stem and leaf plots out as numbers and find the median.

1

Tens	Units
0	3 8 8
1	5 7
2	8 8

3, 8, 8, 15, ____, ____, ____.

Median = ____

2

Stem	Leaf
0	1 2 6
1	4 5
2	7 8 8

Median = ____

3

Stem	Leaf
17	2 6
18	
19	4
20	0 6
21	3

Median = ____

4

Stem	Leaf
8	0 1 5
9	0 6
10	6
11	7 7

Median = ____

Now it is Frequency Tables, write the numbers out on the line below. Find the median.

5

Score	*f*
0	2
2	4
4	1
6	0
8	3
10	1

0, 0, 2, 2, 2, 2, ____

Median = ____

6

Score	*f*
1	4
2	2
3	3
4	1
5	3

Median = ____

7

Score	*f*
0	1
5	0
10	2
15	4
20	1
25	3
30	1

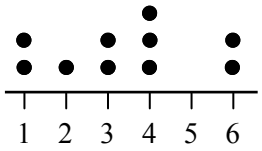
Median = ____

Find the median for these.

8 18, 45, 22, 17, 31, 28, 9.

Median = ____

9



Median = ____

10 24, 12, 18, 31, 25, 50, 38, 27.

Median = ____

11

Stem	Leaf
9	4 5 8
10	2 2 6 6
11	1 3 7

Median = ____

12

Score	*f*
10	2
20	4
30	1
40	2
50	3

Median = ____

Data Analysis - The Median (Sheet 1)

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Find the position of the median and the median itself, the data is in ascending order.

1 3, 4, 7, 12, 12, 15, 18, 25, 26, 30, 38, 38, 45, 48, 55, 63, 70.

Position = $\frac{\square + 1}{2}$ (Count the scores)

$= \square^{th}$ score.

The median is $\square$

2 63, 82, 83, 87, 88, 94, 96, 98, 99, 100, 104, 110, 115, 122, 122, 127, 129, 135, 142, 154, 179.

Position = $\frac{\square + 1}{2}$

$= \square^{th}$ score.

The median is $\square$

3 0, 2, 3, 4, 12, 13, 13, 18, 23, 24, 29, 29, 30, 31, 37, 39, 42, 46, 55, 57, 59, 94, 123, 128, 152.

Position =

Median = $\square$

4 105, 127, 185, 209, 234, 276, 362, 368, 391, 412, 419, 438, 446, 502, 535, 572, 590, 590, 612, 616, 623, 655, 662, 720, 739, 750, 750, 773, 781, 782, 810, 834, 880, 881.

Position =

Median = $\frac{\square + \square}{2}$

$= \square$

Arrange the data in ascending order then find both the position of the median and its value. Strike out the numbers as you go.

5 53, 26, 40, 45, 52, 31, 76, 42, 50, 64, 39, 43, 24, 33, 60, 59, 78, 25, 10, 18, 32, 40, 57, 62, 20, 25, 55, 19, 28.

10,

Position = Median =

Find the position and value for the median in the stem and leaf plots.
For Q.8, 9 place the data in the raw and ordered S&L plots first.

6

Tens	Units
0	5 7 7 8 9
1	4 4
2	1 1 1 4 4
3	0 0 3

Position =

Median =

7

Tens	Units
8	3 5
9	2 4 9
10	0 3 3 4 7
11	2 6

Position =

Median =

8 16, 25, 11, 24, 8, 12, 24, 20, 23, 15, 19, 4, 13, 8, 2, 9, 3.

RAW

Stem	Leaf
$\square$	.....
$\square$	.....
$\square$	.....

ORDERED

Stem	Leaf
$\square$	.....
$\square$	.....
$\square$	.....

9 58, 42, 67, 70, 73, 53, 68, 32, 49, 50, 62, 76, 61, 75.

RAW

Tens	Units
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....

ORDERED

Tens	Units
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....

Data Analysis - The Median (Sheet 2)

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Find the median for each leaf of the back-to-back stem and leaf plots below.

1

Male			Stem	Female		
		3	0	0	1	5
9	3	2	1	0	6	
	1	5	2			

Male: Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

Female: Position = $\frac{\square + 1}{2}$ = $\square$ rd score.

The median is $\square$

2

A.M.			Stem	P.M.			
5	5	1	8	2	3	6	8
	6	0	9	1	7	7	
8	8	4	10	4	9	9	

A.M.: Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

P.M.: Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

With Frequency Tables find Σf, write the numbers out beside the column and find the median.

3

Score	*f*	
1	3	1,1,1
2	1	2
3	2	3, 3
4	3	
5	5	
6	3	
Σf=		

Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

4

Score	*f*	
1	2	
2	5	
3	3	
4	4	
5	2	
Σf=		

Position =

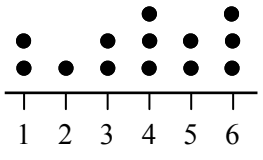
The median is $\square$

5

Score	*f*	
0	1	
10	2	
20	4	
30	3	
40	4	
50	3	
60	3	
Σf=		

Find the median for these.

6

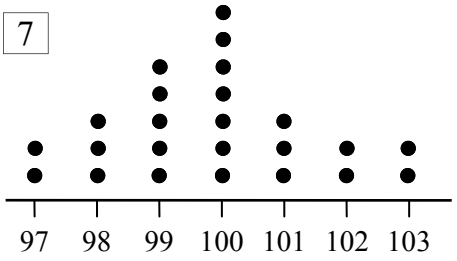


Count dots

Position = $\frac{\square + 1}{2}$ = $\square$ th score.

The median is $\square$

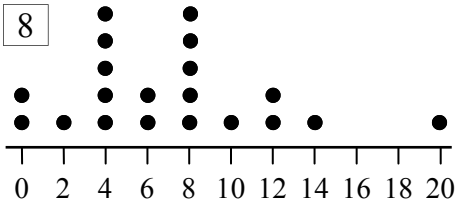
7



Position = $\frac{\square + 1}{2}$ = $\square$ th score.

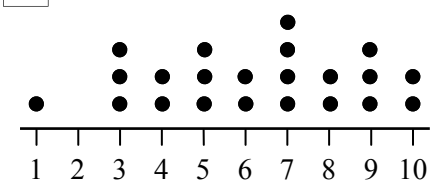
The median is $\square$

8



Position =

9



Data Analysis - Mode, Median, Range and Mean. (Sheet 1)

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Calculate the mode, range, median and mean, for the data below. Round to 1 d.p. if required.

1 19 , 34 , 22 , 16 , 27 , 35 , 19 , 12 , 16

Write the scores out in ascending order on this line

Mode = Mode = Most

Median = Middle score (when in order)

Range = Largest - Smallest

=

Mean = Add scores / No. of scores = 1 d.p.

2 68 , 54 , 23 , 42 , 38 , 52 , 65 , 42 .

Mode =

Median = $\frac{\quad + \quad}{2}$ =

Range = - =

Mean = =

3 240, 310, 205, 296, 240, 256, 205, 282, 310.

Mode =

Median =

Range = - =

Mean = =

4

Tens	Units
0	0 7 7
1	2 5 9 9
2	2 3 7
3	1
4	2 7

Mode =

Range = =

Median =

Mean = =

5

Mode =

Range = =

Median = =

Mean = =

Stem	Leaf
7	5 5 9 9
8	0 0 2
9	6 6 6 7
10	
11	3 4 8
12	1 3 5 5

6

TRAIN	Stem	BUS
3 2	29	2 3 7
7 6 4	30	1 1 5 8
5 2 0	31	2 4 9
6 3 0 0	32	0 1
4 2 2	33	

TRAIN

Mode =

Range = =

Median = =

Mean = =

BUS

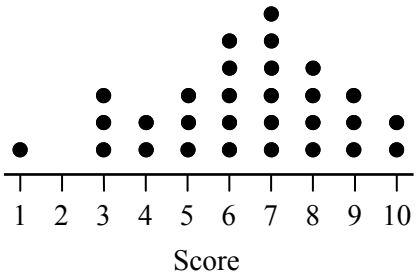
Mode =

Range = =

Median = =

Mean = =

7



Mode =

Range = =

Median = = =

Mean = = =

Data Analysis - Mode, Median, Range and Mean. (Sheet 2)

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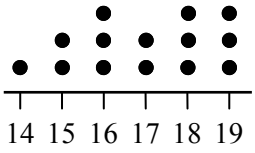
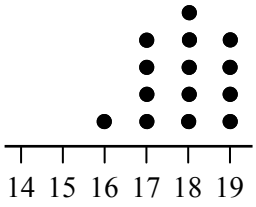
A hot dog stall compares its sale figures for two locations. The dot plots show the number of hot dogs sold per hour, over a 14 hour time period.

Location 1



Hot Dogs Sold per hr

Location 2



Hot Dogs Sold per hr

Calculate the range, mode, median and mean for both locations. Round your answer to 1 d.p., if required.

1

Location 1	Location 2
Mode =	Mode =
Range =	Range =
=	=
Median =	Median =
=	=
Mean =	Mean =
=	=

2 Which stand was more profitable? Would you use the range, mode, median or mean to support your answer?

3 Location 2 has more consistent sales. Use the range and the mode (from both locations) to support this.

Calculate the mode, range, median and mean, for the frequency tables below.

4

Score	*f*	*fx*
10	3	
20	5	
30	2	
40	3	
50	1	
$\Sigma f =$		$\Sigma fx =$

Write the scores out to find the median

Mode = Median =

Range = =

= Mean =

=

Steve is selling units in a proposed unit block. He has sold 5 standard units and one penthouse suite.

5 Find the mean, mode, range and median for the 5 standard units, prices (in $) are below.

677 000, 732 000, 775 000, 790 000, 815 000.

Mode = Median =

Range = =

Mean = =

6 Find the mean, mode, range and median for the 5 standard units and the penthouse, prices (in $) are below.

677 000, 732 000, 775 000, 790 000, 815 000, 3 750 000.

Mode =

Mean = =

Range = =

Median = =

7 Discuss the effect of the penthouse's sale on the range, mean and median.

Data Analysis - Find a Word

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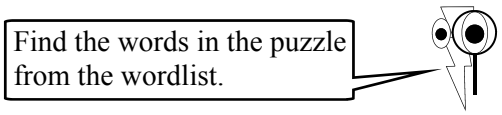

L	O	L	T	E	A	C	E	R	A	P	M	O	C	L
L	O	C	A	T	E	V	T	S	E	W	O	L	P	P
O	E	Q	O	X	W	X	E	H	I	G	H	E	S	T
B	R	X	B	O	H	L	X	O	N	A	I	D	E	M
P	O	S	I	T	I	O	N	I	L	A	B	T	G	V
T	C	A	A	T	S	X	D	O	Q	J	W	W	A	F
O	S	A	R	Q	K	N	W	U	U	U	A	R	R	A
L	U	A	R	U	E	E	M	E	R	T	X	E	E	E
P	U	R	R	C	R	A	O	P	N	U	Q	P	V	L
Q	Q	R	S	T	T	O	D	V	M	U	A	P	A	N
L	E	A	U	V	N	M	E	E	E	M	M	U	S	M
O	L	N	A	X	V	O	G	N	A	X	O	B	T	M
L	B	G	R	O	X	N	C	W	N	O	R	D	E	R
W	A	E	Q	B	A	Y	R	A	M	M	U	S	M	R
O	T	A	U	R	M	A	R	G	O	T	S	I	H	M

WORDLIST

MEDIAN	HIGHEST	SCORE	COMPARE	RANGE
ORDER	HISTOGRAM	DOT	LOCATE	UPPER
LEAF	TABLE	QUARTILES	NUMBER	SUMMARY
EXTREME	POSITION	MODE	LOWER	STEM
MEAN	BOX	WHISKER	LOWEST	ARRANGE
PLOT	FREQUENCY	AVERAGE	ASCENDING	CONTRAST

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Display this data in an ordered Stem and Leaf plot.

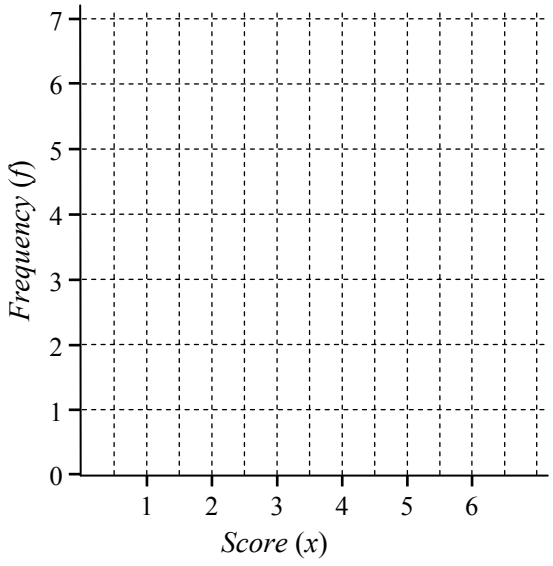
1 19, 25, 11, 23, 34, 8, 18, 21, 30, 15, 15, 3.

2 Display the given data in the frequency table.

Data: 3, 1, 5, 1, 6, 4, 4, 3, 1, 1, 2, 5, 3, 5, 1, 5, 1.

Score	Tally	Frequency
		$\Sigma f =$

3 Display the above data (Q.2.) in a histogram with a frequency polygon.



4 Display the supplied data in a dot plot.

Data: 55, 61, 55, 56, 62, 63, 60, 63, 57, 55, 61, 57, 55, 55, 61, 56, 57, 63, 63.

Scores

Find the mode and range for these

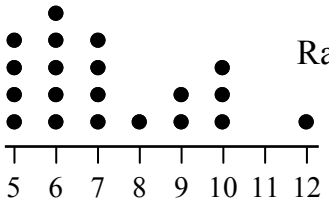
5 7, 13, 19, 13, 22, 18, 5, 13, 23, 16, 19.

Mode = _____ Range = _____

6 24, 0, 38, 33, 22, 19, -10, 5, -2, 33, 7, 22.

Mode = _____ Range = _____

7



Cars per Hour

Range = _____

Mode = _____

8

Tens	Units
8	2 6 6
9	0 7
10	3 3 5
11	1

Range = _____

Mode = _____

9

Score	*f*
0	3
2	1
4	3
6	2
8	2
10	3

Range = _____

Mode = _____

10

Adults	Tens	Children
6 6 3	0	5
4 4 2 1	1	3 3 4
8 4	2	1 1 1 8
9 7 3 0	3	0 1 2 2 2

Adults

Range = _____

Mode = _____

Children

Range = _____

Mode = _____

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Find the mean for these. Round to 1 d.p. when required.

11 18, 53, 55, 94.

12 James scored 52% and 73% in two tests, find his average mark.

13

Score	*f*
10	3
15	1
20	3
25	2
30	2

14

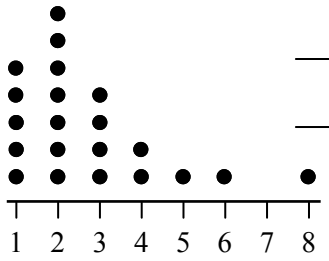
Stem	Leaf
20	4
21	6 8 8
22	1 5 9 9
23	7 7

15 Emily earns $24.75 per hour. In one week she works 19 hours, in the next she works 27 hours. Calculate her pay for each week.

1st Week 2nd Week

16 Calculate Emily's average weekly wage.

17



Children in a Family

Find the median for these.

18 4, 14, 20, 25, 31.

19 9, 14, 25, 25, 35, 44, 48, 55.

20 107, 83, 144, 110, 95, 77, 149, 86.

21

Tens	Units
0	4 8
1	4 7
2	0 6 8
3	1 4

22

Score	*f*
1	2
2	0
3	4
4	2
5	3
6	2

23 Tina auctioned 4 houses on the weekend. Prices are listed. Find the median house price.

$802 000, $915 000, $766 000, $1 877 000.

24 Describe why the median house price is preferred to the mean house price.

VELOCITY

DATA ANALYSIS

EXTENSION

Data Analysis - Introduction to Quartiles

Data can be broken up into quartiles. This breaking up of data allows groups within that data to be analysed. Why would you do this? Imagine a shopping centre wanting to target customers. Customers could be divided into:

- Lower 25% - People who are not going to respond to marketing
- Next 25% - People who respond with significant marketing
- Next 25% - People who respond with a small amount of marketing
- Upper 25% - People who respond without marketing

So by splitting data up, you can allocate resources more effectively, target specific groups or simply understand your data better.

Splitting data up into quartiles requires you to find three dividing points, these are called Q_1, Q_2 and Q_3. Note that Q_2 is the median, which you already know how to locate. Q_1 and Q_2 are like medians also, but just of the lower and upper halves of the data.

Example - Finding Q_1, Q_2 and Q_3

A test out of 30 was taken and the 10 students results are shown:

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.
10 scores

Median Location = $\frac{10 + 1}{2}$ = 5.5th score

STEP 1 Locate the Median.

Data: 9, 14, 16, 19, 20, | 23, 23, 25, 26, 28.

STEP 2 Mark the median. The 5.5th score is between the 5th and 6th score.

Data: 9, 14, 16, 19, 20, | 23, 23, 25, 26, 28.
5 scores 5 scores

Q_1 Location = $\frac{5 + 1}{2}$ = 3rd score

Q_3 Location will be the same as Q_1. (above median)

STEP 3 Locate the position of Q_1 and Q_3. Note that both are identical in their position above and below the median.

Data: 9, 14, (16), 19, 20, | 23, 23, (25), 26, 28.

Q_1 = 16 Q_3 = 25

Q_2 = 21.5

Calc: 20+23=÷2=

STEP 4 Mark Q_1 and Q_3. Note how the data is split up into 4 equal parts, called quartiles.

STEP 5 Write down the values for Q_1, Q_2 and Q_3. You only have to calculate Q_2 in this case.

Data Analysis - Introduction to Quartiles (Sheet 1)

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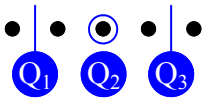
To divide a data set into quartiles, you need three numbers these are called Q_1, Q_2 and Q_3. We will start with a 5 score data set. Find Q_1, Q_2 and Q_3.



1 14, 20, 28, 36, 50.



$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

Calc: 14 + 20 = ÷ 2 = Calc: 36 + 50 = ÷ 2 =

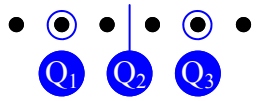
2 8, 13, 18, 23, 37.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

3 52, 64, 115, 144, 200.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

Now it's 6



4 5, 16, 25, 45, 45, 67.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

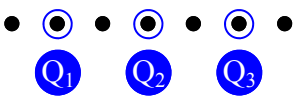
5 38, 48, 55, 75, 82, 93.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

6 185, 241, 386, 405, 412, 440.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

It's 7



7 49, 57, 81, 93, 102, 131, 150.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

8 9, 14, 16, 22, 22, 22, 31.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

Keep going with the same process, but this time apply the skill to stem and leaf plots and dot plots as well. We will kick it off with 8 scores.



9 11, 20, 24, 41, 46, 55, 60, 72.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

10 48, 61, 77, 77, 84, 90, 91, 95.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

11

Tens	Units
0	4 4 | 8
1	3 | 5
2	5 | 7 7

$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

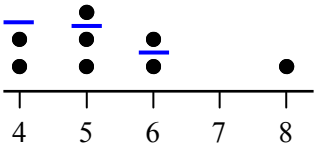
Remember 10's

12

Stem	Leaf
8	0 4
9	
10	2 4 6 6 8
11	5

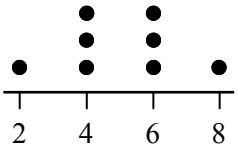
$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

13



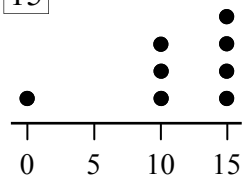
$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

14



$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

15



$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

Data Analysis - Introduction to Quartiles (Sheet 2)

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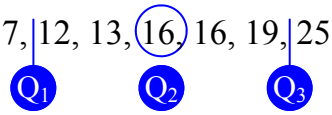
Find Q_1, Q_2 and Q_3. there are 9 scores.



1 1, 7, 12, 13, 16, 16, 19, 25, 28.



$Q_1 =$ $Q_2 =$ $Q_3 =$

2 19, 20, 42, 42, 57, 63, 66, 72, 77.

$Q_1 =$ $Q_2 =$ $Q_3 =$

3 88, 93, 99, 106, 108, 110, 110, 113, 119.

$Q_1 =$ $Q_2 =$ $Q_3 =$

4

Tens	Units
4	1
5	0 3 4 4 7
6	2 5 6

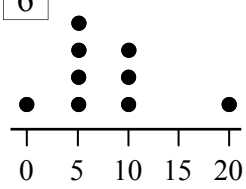
$Q_1 =$
$Q_2 =$
$Q_3 =$

5

Stem	Leaf
98	1 1 4
99	5 8
100	2 2 6
101	5

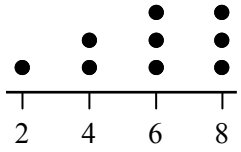
$Q_1 =$
$Q_2 =$
$Q_3 =$

6



$Q_1 =$
$Q_2 =$
$Q_3 =$

7



$Q_1 =$
$Q_2 =$
$Q_3 =$

Now it's 10



8 73, 83, 84, 105, 111, 118, 118, 133, 142, 170.

$Q_1 =$ $Q_2 =$ $Q_3 =$

9 3, 5, 5, 8, 12, 12, 12, 14, 17, 19.

$Q_1 =$ $Q_2 =$ $Q_3 =$

10 38, 46, 58, 62, 67, 73, 88, 88, 90, 94.

$Q_1 =$ $Q_2 =$ $Q_3 =$

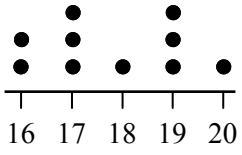
11

Off-Peak	Tens	Peak
8 8 7	7	8
6 6 4 1 1	8	3 8 8
0 0	9	1 2 2 5 5
	10	5

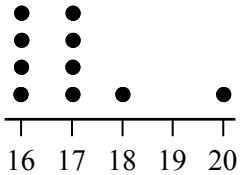
$Q_1 =$
$Q_2 =$
$Q_3 =$

$Q_1 =$
$Q_2 =$
$Q_3 =$

12



$Q_1 =$
$Q_2 =$
$Q_3 =$



$Q_1 =$
$Q_2 =$
$Q_3 =$

Data Analysis - Introduction to Quartiles (Sheet 1)

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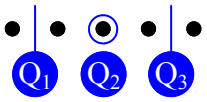
To divide a data set into quartiles, you need three numbers these are called Q_1, Q_2 and Q_3. We will start with a 5 score data set. Find Q_1, Q_2 and Q_3.



1 14, 20, **28**, 36, 50.

Q_1 = 17 Q_2 = 28 Q_3 = 43

Calc: 14 + 20 = ÷ 2 = Calc: 36 + 50 = ÷ 2 =

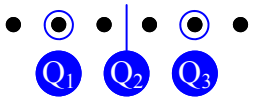
2 8, 13, **18**, 23, 37.

Q_1 = 10.5 Q_2 = 18 Q_3 = 30

3 52, 64, **115**, 144, 200.

Q_1 = 58 Q_2 = 115 Q_3 = 172

Now it's 6



4 5, **16**, 25, 45, **45**, 67.

Q_1 = 16 Q_2 = 35 Q_3 = 45

5 38, **48**, 55, 75, **82**, 93.

Q_1 = 48 Q_2 = 65 Q_3 = 82

6 185, **241**, 386, 405, **412**, 440.

Q_1 = 241 Q_2 = 395.5 Q_3 = 412

It's 7



7 49, **57**, 81, **93**, 102, **131**, 150.

Q_1 = 57 Q_2 = 93 Q_3 = 131

8 9, **14**, 16, **22**, 22, **22**, 31.

Q_1 = 14 Q_2 = 22 Q_3 = 22

Keep going with the same process, but this time apply the skill to stem and leaf plots and dot plots as well. We will kick it off with 8 scores.



9 11, 20, 24, 41, 46, 55, 60, 72.

Q_1 = 22 Q_2 = 43.5 Q_3 = 57.5

10 48, 61, 77, 77, 84, 90, 91, 95.

Q_1 = 69 Q_2 = 80.5 Q_3 = 90.5

11

Tens	Units
0	4 4 | 8
1	3 | 5
2	5 | 7 7

Q_1 = 6
Q_2 = 14
Q_3 = 26

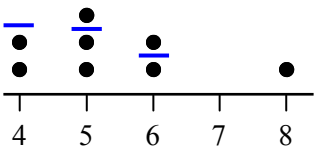
Remember 10's

12

Stem	Leaf
8	0 4 |
9	
10	2 4 | 6 6 | 8
11	5

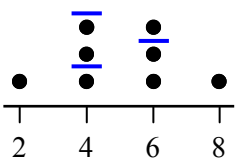
Q_1 = 93
Q_2 = 105
Q_3 = 107

13



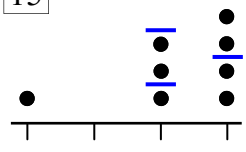
Q_1 = 4
Q_2 = 5
Q_3 = 6

14



Q_1 = 4
Q_2 = 5
Q_3 = 6

15



Q_1 = 10
Q_2 = 12.5
Q_3 = 15

Data Analysis - Introduction to Quartiles (Sheet 2)

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Find Q_1, Q_2 and Q_3. there are 9 scores.



1 1, 7, | 12, 13, (16) 16, 19, | 25, 28.

Q_1 = 9.5 Q_2 = 18 Q_3 = 22

2 19, 20, | 42, 42, (57) 63, 66, | 72, 77.

Q_1 = 31 Q_2 = 57 Q_3 = 119

3 88, 93, | 99, 106, (108) 110, 110, | 113, 119.

Q_1 = 96 Q_2 = 108 Q_3 = 111.5

4

Tens	Units
4	1
5	0 | 3 4 (4) 7
6	2 | 5 6

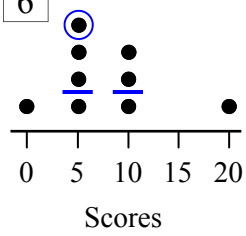
Q_1 = 51.5
Q_2 = 54
Q_3 = 63.5

5

Stem	Leaf
98	1 1 | 4
99	5 (8)
100	2 2 | 6
101	5

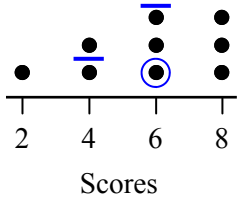
Q_1 = 982.5
Q_2 = 998
Q_3 = 1 004

6



Q_1 = 5
Q_2 = 5
Q_3 = 10

7



Q_1 = 4
Q_2 = 6
Q_3 = 7

Now it's 10



8 73, 83, (84) 105, 111, | 118, 118, (133) 142, 170.

Q_1 = 84 Q_2 = 114.5 Q_3 = 133

9 3, 5, (5) 8, 12, | 12, 12, (14) 17, 19.

Q_1 = 5 Q_2 = 12 Q_3 = 14

10 38, 46, (58) 62, 67, 73, 88, (88) 90, 94.

Q_1 = 58 Q_2 = 70 Q_3 = 88

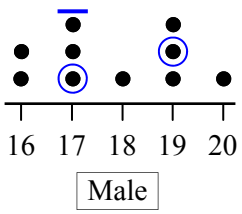
11

Off-Peak	Tens	Peak
(8) 8 7	7	8
(6) 6 4 | 1 1	8	3 (8) 8
0 0	9	1 | 2 2 (5) 5
	10	5

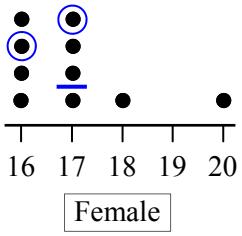
Q_1 = 78
Q_2 = 82.5
Q_3 = 86

Q_1 = 88
Q_2 = 91.5
Q_3 = 95

12



Q_1 = 17
Q_2 = 17.5
Q_3 = 19



Q_1 = 16
Q_2 = 17
Q_3 = 17

Data Analysis - Introduction to Quartiles (Homework Strip)

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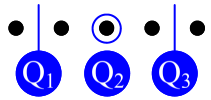
Date Due:

Find Q_1, Q_2 and Q_3 for these.



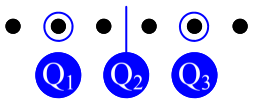
1 19, 37, 46, 55, 63.

Q_1 Q_2 Q_3

2 104, 110, 121, 138, 144.

Q_1 Q_2 Q_3

Now 6



3 4, 7, 11, 15, 15, 23.

Q_1 Q_2 Q_3

4 54, 60, 72, 83, 89, 95.

Q_1 Q_2 Q_3

Now 7

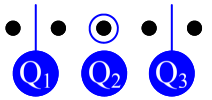


5 26, 32, 43, 55, 61, 77, 80.

Q_1 Q_2 Q_3

Date Due:

Find Q_1, Q_2 and Q_3 for these.



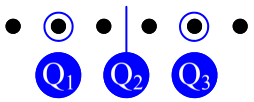
1 19, 37, 46, 55, 63.

Q_1 Q_2 Q_3

2 104, 110, 121, 138, 144.

Q_1 Q_2 Q_3

Now 6



3 4, 7, 11, 15, 15, 23.

Q_1 Q_2 Q_3

4 54, 60, 72, 83, 89, 95.

Q_1 Q_2 Q_3

Now 7

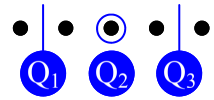


5 26, 32, 43, 55, 61, 77, 80.

Q_1 Q_2 Q_3

Date Due:

Find Q_1, Q_2 and Q_3 for these.



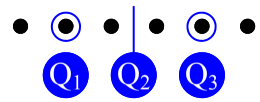
1 19, 37, 46, 55, 63.

Q_1 Q_2 Q_3

2 104, 110, 121, 138, 144.

Q_1 Q_2 Q_3

Now 6



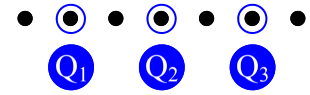
3 4, 7, 11, 15, 15, 23.

Q_1 Q_2 Q_3

4 54, 60, 72, 83, 89, 95.

Q_1 Q_2 Q_3

Now 7

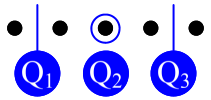


5 26, 32, 43, 55, 61, 77, 80.

Q_1 Q_2 Q_3

Date Due:

Find Q_1, Q_2 and Q_3 for these.



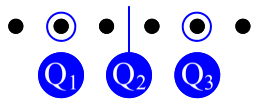
1 19, 37, 46, 55, 63.

Q_1 Q_2 Q_3

2 104, 110, 121, 138, 144.

Q_1 Q_2 Q_3

Now 6



3 4, 7, 11, 15, 15, 23.

Q_1 Q_2 Q_3

4 54, 60, 72, 83, 89, 95.

Q_1 Q_2 Q_3

Now 7



5 26, 32, 43, 55, 61, 77, 80.

Q_1 Q_2 Q_3

Data Analysis - Introduction to Quartiles (Homework Strip)

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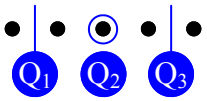
Date Due:

Find Q_1, Q_2 and Q_3 for these.



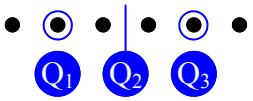
1 19, 37, 46, 55, 63.

Q_1 28 Q_2 46 Q_3 59

2 104, 110, 121, 138, 144.

Q_1 107 Q_2 121 Q_3 141

Now 6



3 4, 7, 11, 15, 15, 23.

Q_1 7 Q_2 13 Q_3 23

4 54, 60, 72, 83, 89, 95.

Q_1 60 Q_2 77.5 Q_3 95

Now 7



5 26, 32, 43, 55, 61, 77, 80.

Q_1 32 Q_2 55 Q_3 77

Data Analysis - Quartiles - Data Sets

Data can be broken up into quartiles. This breaking up of data allows groups within that data to be analysed. Why would you do this? By splitting data up, you can allocate resources more effectively, target specific groups or simply understand your data better.

Splitting data up into quartiles requires you to find three dividing points, these are called Q_1, Q_2 and Q_3. Note that Q_2 is the median, which you already know how to locate. Q_1 and Q_2 are like medians also, but just of the lower and upper halves of the data.

Example - Finding Q_1, Q_2 and Q_3

A test out of 30 was taken and the 10 students results are shown:

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.

10 scores

Median Location = $\frac{10 + 1}{2}$ = 5.5th score

STEP 1 Locate the Median.

Data: 9, 14, 16, 19, 20, | 23, 23, 25, 26, 28.

STEP 2 Mark the median. The 5.5th score is between the 5th and 6th score.

Data: 9, 14, 16, 19, 20, | 23, 23, 25, 26, 28.

5 scores 5 scores

Q_1 Location = $\frac{5 + 1}{2}$ = 3rd score

Q_3 Location will be the same as Q_1. (above median)

STEP 3 Locate the position of Q_1 and Q_3. Note that both are identical in their position above and below the median.

Data: 9, 14, (16), 19, 20, | 23, 23, (25), 26, 28.

Q_1 = 16 Q_3 = 25

Q_2 = 21.5

Calc: 20+23=÷2=

STEP 4 Mark Q_1 and Q_3. Note how the data is split up into 4 equal parts, called quartiles.

STEP 5 Write down the values for Q_1, Q_2 and Q_3. You only have to calculate Q_2 in this case.

Later exercises require you to rewrite the raw data in ascending order, you then repeat the above process on the ordered data.

Data Analysis - Quartiles - Stem & Leaf, Dot Plots

Stem and leaf plots are solved the same way as a data set. The added bonus with S & L and dot plots is the data is already in ascending order.

Example - Find Q_1, Q_2 and Q_3 in the stem and leaf plot

STEP 1 Count the scores...9.
Add 1 and divide by 2.
Locate the Median (Q_2).

Tens	Units
8	4 5 9
9	3 (7)
10	1 1 6
11	5

STEP 2 Locate Q_1 and Q_3.
$(4 + 1) \div 2 = 2.5$th
Between 2nd and 3rd

Tens	Units
8	4 5 | 9
9	3 (7)
10	1 1 | 6
11	5

STEP 3 Solve

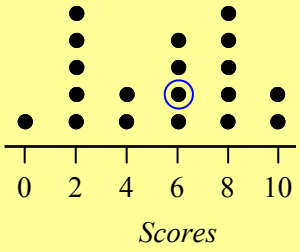
$Q_1 =$ 87

$Q_2 =$ 97

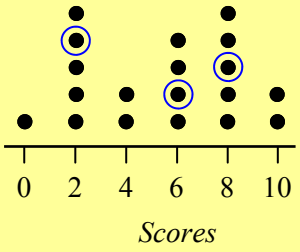
$Q_3 =$ 103.5

Example - Find Q_1, Q_2 and Q_3 in the dot plot

STEP 1 Count the dots...19.
Add 1 and divide by 2.
Locate the Median (Q_2).



STEP 2 Locate Q_1 and Q_3.
$(9 + 1) \div 2 = 5^{th}$



STEP 3 Solve

$Q_1 =$ 2

$Q_2 =$ 6

$Q_3 =$ 8

Data Analysis - Quartiles - Data Sets (Sheet 1)

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Find the median, Q_2, in these data sets.

Examples

14, 20, (28), 36, 50. $Q_2 =$ 28

Calc: 2 4 + 3 3 = ÷ 2 =

15, 18, 24, | 33, 38, 38. $Q_2 =$ 28.5

1 8, 13, 18, 23, 37. $Q_2 =$

2 1, 3, 4, 4, 6, 7, 8, 8. $Q_2 =$

3 29, 36, 44, 70. $Q_2 =$

4 44, 44, 52, 77, 93, 98. $Q_2 =$

Now you are given the median, Q_2, find Q_1 and Q_3 in these data sets.

Example

Q_1 location $= \frac{2+1}{2} = 1.5$th

1.5th means between 1st and 2nd

14, 20, (28), 36, 50.

2 scores 2 scores

14, | 20, (28), 36, | 50. $Q_1 =$ 17 $Q_3 =$ 43

5 8, 13, 18, | 23, 37, 42. Q_1 Q_3

6 11, 16, (27), 33, 33. Q_1 Q_3

7 36, 39, 47, 54, 63, 70, | 70, 74, 79, 83, 88, 89. Q_1 Q_3

8 5, 6, 8, 10, 13, 17, (18), 18, 19, 23, 23, 41, 55. Q_1 Q_3

Keep going with the same process, but this time record Q_1, Q_2 and Q_3. Q_2 is marked for you.

9 2, 2, 3, 4, 5, 5, 6, 6, | 7, 7, 7, 8, 8, 10, 10, 10. $Q_1 =$ $Q_2 =$ $Q_3 =$

10 Data: 24, 31, 34, 40, 40, 46, 49, 50, 51, 53, 55, 56, 58, 62, (64), 65, 68, 71, 77, 78, 82, 84, 87, 89, 92, 95, 95, 95, 97.

$Q_1 =$ $Q_2 =$ $Q_3 =$

Now do it all yourself. Find Q_2 first, then Q_1 and Q_3.

11 76, 83, 94, 104, 155, 176, 183. $Q_1 =$ $Q_2 =$ $Q_3 =$

12 1, 1, 1, 3, 3, 4, 5, 5, 7, 7, 7, 8, 8, 8, 9, 9, 10. $Q_1 =$ $Q_2 =$ $Q_3 =$

13 Data: 15, 17, 17, 21, 24, 26, 33, 36, 38, 48, 52, 63, 65, 68, 74, 75, 81, 85, 93, 94. $Q_1 =$ $Q_2 =$ $Q_3 =$

14 28, 36, 42, 55, 62, 68, 77, 78, 78, 85, 92. $Q_1 =$ $Q_2 =$ $Q_3 =$

Just the same, except this time the data has to be put into ascending order first. So rewrite the data on the line below, lowest to highest. Then find Q_2, Q_1 & Q_3.

15 17, 11, 9, 16, 22, 8, 13, 26, 16, 30, 15.

8,

There are 11 scores, make sure you have written 11 scores

$Q_1 =$ $Q_2 =$ $Q_3 =$

16 4, 9, 2, 4, 6, 11, 15, 8, 7, 6, 10, 3, 16.

$Q_1 =$ $Q_2 =$ $Q_3 =$

17 470, 233, 114, 330, 276, 195, 308, 222, 411.

$Q_1 =$ $Q_2 =$ $Q_3 =$

18 3, 2, 5, 7, 8, 2, 6, 10, 2, 8, 11, 0, 16, 14.

$Q_1 =$ $Q_2 =$ $Q_3 =$

Data Analysis - Quartiles - Stem & Leaf / Dot Plots (Sheet 2)

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Find Q_2 first, then Q_1 and Q_3 for these Stem and leaf plots. The first question has the three located for you.

1

Tens	Units
0	1 7 | 7
1	2 6
2	0 3 | 9
3	1

$Q_1 =$ $Q_2 =$ $Q_3 =$

2

Stem	Leaf
10	0 4 4
11	6 6 8 9 9
12	1 1 6 6 7

$Q_1 =$ $Q_2 =$ $Q_3 =$

3

Stem	Leaf
28	1 7
29	4
30	7 7
31	2 4 4 8
32	1 3 3 4 6
33	0 1 1

$Q_1 =$ $Q_2 =$ $Q_3 =$

4

Tens	Units
8	5 5 6 7 8
9	0 7 9
10	2 2 5 9
11	7 7

$Q_1 =$ $Q_2 =$ $Q_3 =$

5

Early	Tens	Late
7	12	1 6 6 8
8 5 4 4	13	0 4 8 8 8
6 3 3 0 0	14	5 9 9
7 4 1	15	5

Early

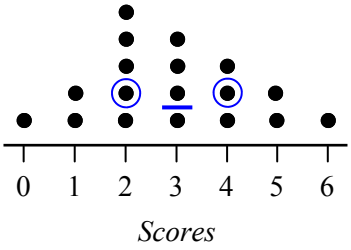
$Q_1 =$ $Q_2 =$ $Q_3 =$

Late

$Q_1 =$ $Q_2 =$ $Q_3 =$

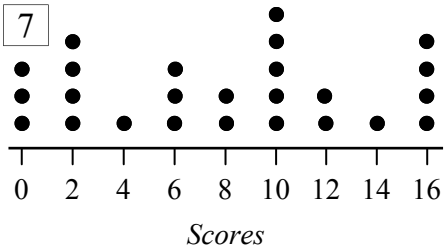
Find Q_2 first, then Q_1 and Q_3 for these dot plots.

6



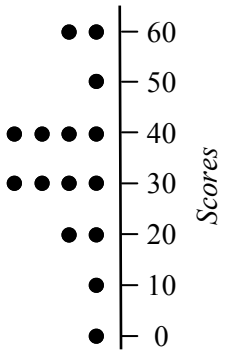
$Q_1 =$ $Q_2 =$ $Q_3 =$

7



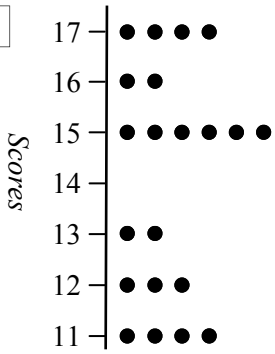
$Q_1 =$ $Q_2 =$ $Q_3 =$

8



$Q_1 =$ $Q_2 =$ $Q_3 =$

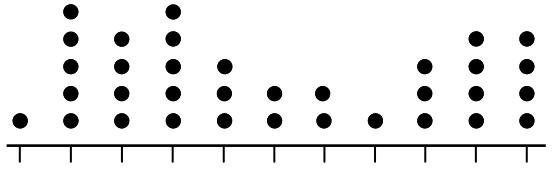
9



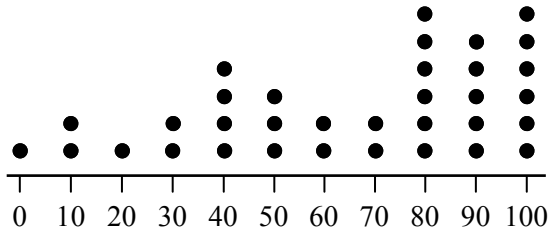
$Q_1 =$ $Q_2 =$ $Q_3 =$

10

Task 1



Task 2



Task 1

$Q_1 =$ $Q_2 =$ $Q_3 =$

Task 2

$Q_1 =$ $Q_2 =$ $Q_3 =$

Data Analysis - Quartiles - Data Sets (Sheet 1)

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Find the median, Q_2, in these data sets.

Examples

14, 20, (28), 36, 50. $Q_2 =$ 28

Calc: 2 4 + 3 3 = ÷ 2 =

15, 18, 24, | 33, 38, 38. $Q_2 =$ 28.5

1 8, 13, (18), 23, 37. $Q_2 =$ 18

2 1, 3, 4, 4, | 6, 7, 8, 8. $Q_2 =$ 5

3 29, 36, | 44, 70. $Q_2 =$ 40

4 44, 44, 52, | 77, 93, 98. $Q_2 =$ 64.5

Now you are given the median, Q_2, find Q_1 and Q_3 in these data sets.

Example

Q_1 location $= \frac{2+1}{2} = 1.5$th

1.5th means between 1st and 2nd

14, 20, (28), 36, 50.

2 scores 2 scores

14, | 20, (28), 36, | 50. $Q_1 =$ 17 $Q_3 =$ 43

5 8, (13), 18, | 23, (37), 42. Q_1 13 Q_3 37

6 11, | 16, (27), 33, | 33. Q_1 13.5 Q_3 33

7 36, 39, 47, | 54, 63, 70, | 70, 74, 79, | 83, 88, 89.

Q_1 50.5 Q_3 81

8 5, 6, 8, | 10, 13, 17, (18), 18, 19, 23, | 23, 41, 55.

Q_1 9 Q_3 23

Keep going with the same process, but this time record Q_1, Q_2 and Q_3. Q_2 is marked for you.

9 2, 2, 3, 4, | 5, 5, 6, 6, | 7, 7, 7, 8, | 8, 10, 10, 10.

$Q_1 =$ 4.5 $Q_2 =$ 6.5 $Q_3 =$ 8

10 Data: 24, 31, 34, 40, 40, 46, 49, | 50, 51, 53, 55, 56, 58, 62, (64), 65, 68, 71, 77, 78, 82, 84, | 87, 89, 92, 95, 95, 95, 97.

$Q_1 =$ 49.5 $Q_2 =$ 64 $Q_3 =$ 85.5

Now do it all yourself. Find Q_2 first, then Q_1 and Q_3.

11 76, (83), 94, (104), 155, (176), 183.

$Q_1 =$ 83 $Q_2 =$ 104 $Q_3 =$ 176

12 1, 1, 1, 3, | 3, 4, 5, 5, (7), 7, 7, 7, 8, 8, | 8, 9, 9, 10.

$Q_1 =$ 3 $Q_2 =$ 7 $Q_3 =$ 8

13 Data: 15, 17, 17, 21, 24, | 26, 33, 36, 38, 48, | 52, 63, 65, 68, 74, | 75, 81, 85, 93, 94.

$Q_1 =$ 25 $Q_2 =$ 50 $Q_3 =$ 74.5

14 28, 36, (42), 55, 62, (68), 77, 78, (78), 85, 92.

$Q_1 =$ 42 $Q_2 =$ 68 $Q_3 =$ 78

Just the same, except this time the data has to be put into ascending order first. So rewrite the data on the line below, lowest to highest. Then find Q_2, Q_1 & Q_3.

15 ~~17, 11, 9, 16, 22, 8, 13, 26, 16, 30, 15.~~

8, 9, (11), 13, 15, (16), 16, 17, (22), 26, 30.

There are 11 scores, make sure you have written 11 scores

$Q_1 =$ 11 $Q_2 =$ 16 $Q_3 =$ 22

16 ~~4, 9, 2, 4, 6, 11, 15, 8, 7, 6, 10, 3, 16.~~

2, 3, 4, | 4, 6, 6, (7), 8, 9, 10, | 11, 15, 16.

$Q_1 =$ 4 $Q_2 =$ 7 $Q_3 =$ 10.5

17 ~~470, 233, 114, 330, 276, 195, 308, 222, 411.~~

114, 195, (222), 233, (276), 308, (330), 411, 470.

$Q_1 =$ 222 $Q_2 =$ 276 $Q_3 =$ 330

18 ~~3, 2, 5, 7, 8, 2, 6, 10, 2, 8, 11, 0, 16, 14.~~

0, 2, 2, (2), 3, 5, 6, | 7, 8, 8, (10), 11, 14, 16.

$Q_1 =$ 2 $Q_2 =$ 6.5 $Q_3 =$ 10

Data Analysis - Quartiles - Stem & Leaf / Dot Plots (Sheet 2)

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Find Q_2 first, then Q_1 and Q_3 for these Stem and leaf plots. The first question has the three located for you.

1

Tens	Units
0	1 7 | 7
1	2 (6)
2	0 3 | 9
3	1

$Q_1 =$ 7
$Q_2 =$ 16
$Q_3 =$ 26

2

Stem	Leaf
10	0 4 4 |
11	6 6 8 (9) 9
12	1 1 | 6 6 7

$Q_1 =$ 110
$Q_2 =$ 119
$Q_3 =$ 123.5

3

Stem	Leaf
28	1 7
29	4
30	7 | 7
31	2 4 4 (8)
32	1 3 3 4 | 6
33	0 1 1

$Q_1 =$ 307
$Q_2 =$ 318
$Q_3 =$ 325

4

Tens	Units
8	5 5 6 (7) 8
9	0 7 | 9
10	2 2 (5) 9
11	7 7

$Q_1 =$ 87
$Q_2 =$ 98
$Q_3 =$ 105

5

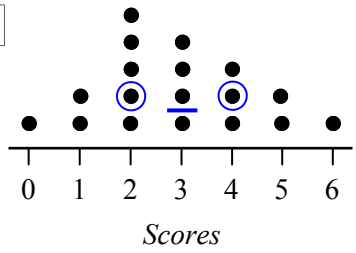
Early	Tens	Late
7	12	1 6 6 | 8
8 5 | 4 4	13	0 4 (8) 8 8
| 6 3 3 (0) 0	14	5 | 9 9
7 4 1	15	5

Early
$Q_1 =$ 134.5 $Q_2 =$ 140 $Q_3 =$ 148.5

Late
$Q_1 =$ 127 $Q_2 =$ 138 $Q_3 =$ 147

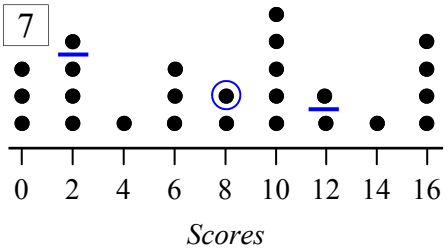
Find Q_2 first, then Q_1 and Q_3 for these dot plots.

6



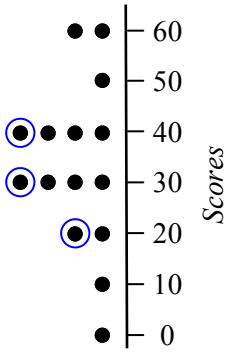
$Q_1 =$ 2
$Q_2 =$ 3
$Q_3 =$ 4

7



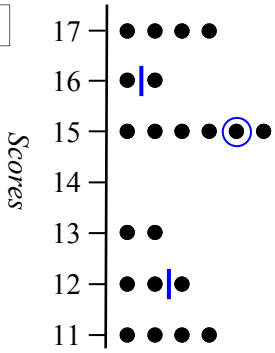
$Q_1 =$ 2
$Q_2 =$ 8
$Q_3 =$ 12

8



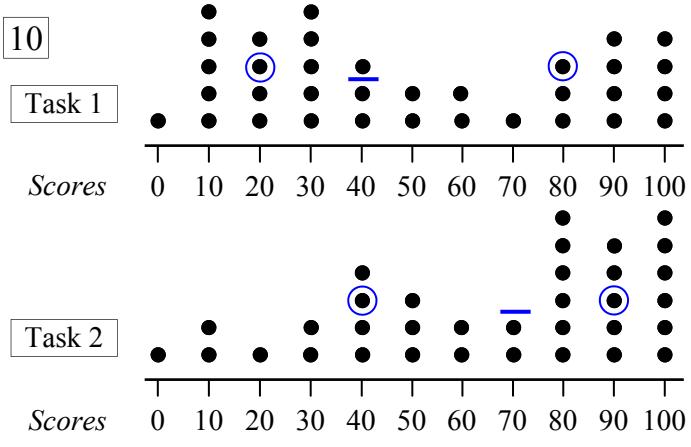
$Q_1 =$ 20
$Q_2 =$ 30
$Q_3 =$ 40

9



$Q_1 =$ 12
$Q_2 =$ 15
$Q_3 =$ 16

10



Task 1
$Q_1 =$ 20 $Q_2 =$ 40 $Q_3 =$ 80

Task 2
$Q_1 =$ 40 $Q_2 =$ 75 $Q_3 =$ 90

Data Analysis - Quartiles (Homework Strip)

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Date Due:

Find Q_1, Q_2 and Q_3 for these.

1 11, (16), 22, | 38, (45), 77.

Q_1 Q_2 Q_3

2 0, 1, 1, 4, 4, 5, 6, 6, 6, 7, 9, 9.

Q_1 Q_2 Q_3

3 15, 8, 24, 6, 33, 21, 14, 9.

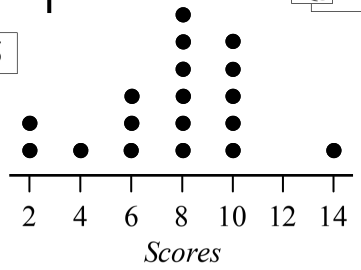
Q_1 Q_2 Q_3

4

Tens	Units
0	3 5
1	1 7 7 7
2	0 1 3
3	8 9

Q_1 Q_2 Q_3

5



Q_1 Q_2 Q_3

Date Due:

Find Q_1, Q_2 and Q_3 for these.

1 11, (16), 22, | 38, (45), 77.

Q_1 Q_2 Q_3

2 0, 1, 1, 4, 4, 5, 6, 6, 6, 7, 9, 9.

Q_1 Q_2 Q_3

3 15, 8, 24, 6, 33, 21, 14, 9.

Q_1 Q_2 Q_3

4

Tens	Units
0	3 5
1	1 7 7 7
2	0 1 3
3	8 9

Q_1 Q_2 Q_3

5

2 4 6 8 10 12 14
Scores

Q_1 Q_2 Q_3

Date Due:

Find Q_1, Q_2 and Q_3 for these.

1 11, (16), 22, | 38, (45), 77.

Q_1 Q_2 Q_3

2 0, 1, 1, 4, 4, 5, 6, 6, 6, 7, 9, 9.

Q_1 Q_2 Q_3

3 15, 8, 24, 6, 33, 21, 14, 9.

Q_1 Q_2 Q_3

4

Tens	Units
0	3 5
1	1 7 7 7
2	0 1 3
3	8 9

Q_1 Q_2 Q_3

5

2 4 6 8 10 12 14
Scores

Q_1 Q_2 Q_3

Date Due:

Find Q_1, Q_2 and Q_3 for these.

1 11, (16), 22, | 38, (45), 77.

Q_1 Q_2 Q_3

2 0, 1, 1, 4, 4, 5, 6, 6, 6, 7, 9, 9.

Q_1 Q_2 Q_3

3 15, 8, 24, 6, 33, 21, 14, 9.

Q_1 Q_2 Q_3

4

Tens	Units
0	3 5
1	1 7 7 7
2	0 1 3
3	8 9

Q_1 Q_2 Q_3

5

2 4 6 8 10 12 14
Scores

Q_1 Q_2 Q_3

Data Analysis - Quartiles (Homework Strip)

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Date Due:

Find Q_1, Q_2 and Q_3 for these.

1 11, (16,) 22, | 38, (45,) 77.

Q_1 16 Q_2 30 Q_3 45

2 0, 1, 1, | 4, 4, 5, | 6, 6, 6, | 7, 9, 9.

Q_1 2.5 Q_2 5.5 Q_3 6.5

3 ~~15, 8, 24, 6, 33, 21, 14, 9.~~

6, 8, | 9, 14, | 15, 21, | 24, 33.

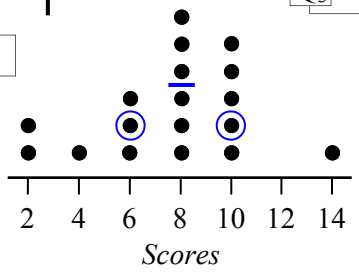
Q_1 8.5 Q_2 14.5 Q_3 22.5

4

Tens	Units
0	3 5
1	(1) 7 7 (7)
2	0 1 (3)
3	8 9

Q_1 11 Q_2 17 Q_3 23

5



Q_1 6 Q_2 8 Q_3 10

Data Analysis - 5-Number Summary/IQR - Data Sets

A 5-number summary is the same as finding quartiles. The only difference is you include the lower extreme value (Min) and the upper extreme value (Max) along with Q_1, Q_2 and Q_3.

The interquartile range is the range for the middle 50% of scores. This is found by subtracting Q_3 from Q_1.

$$IQR = Q_3 - Q_1$$

Example - Find the 5 number summary and IQR for this data set.

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.

10 scores

Median Location = $\frac{10 + 1}{2}$ = 5.5th score

STEP 1

Locate the Median (Q_2).

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.

STEP 2

Mark the median. The 5.5th score is between the 5th and 6th score.

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.

5 scores 5 scores

Q_1 Location = $\frac{5 + 1}{2}$ = 3rd score

Q_3 Location will be the same as Q_1. (above median)

STEP 3

Locate the position of Q_1 and Q_3. Note that both are identical in their position above and below the median.

Data: 9, 14, 16, 19, 20, 23, 23, 25, 26, 28.

STEP 4

Mark Q_1 and Q_3. Note how the data is split up into 4 equal parts, called quartiles.

Min 9

Q_1 16

Q_2 21.5

Q_3 25

Max 28

STEP 5

5 number summary includes the lower extreme and upper extreme values along with Q_1,Q_2 & Q_3.

STEP 6

Calculate the interquartile range. IQR = Q_3 - Q_1

IQR = Q_3 25 - Q_1 16 = 9

Data Analysis - 5-Number Summary/IQR - Stem & Leaf/Dot Plots

Example - Find the 5 number summary and IQR for this stem and leaf plot.

Tens	Units
5	2 5 9
6	6 7
7	0 3 3
8	2 8

Tens	Units
5	2 5 9
6	6 7 |
7	0 3 3
8	2 8

Tens	Units
5	2 5 (9)
6	6 7 |
7	0 3 (3)
8	2 8

STEP 1 Locate the Median (Q_2).

STEP 2 Locate Q_1 and Q_3.

Tens	Units
5	2 5 (9)
6	6 7 |
7	0 3 (3)
8	2 8

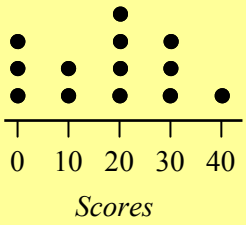
Min 52
Q_1 59
Q_2 68.5
Q_3 73
Max 88

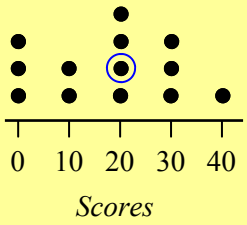
IQR = Q_3 73 - Q_1 59 = 14

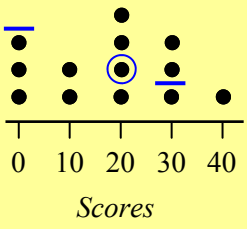
STEP 3 Write the 5 number summary

STEP 4 Calculate the IQR

Example - Find the 5 number summary and IQR for this dot plot.

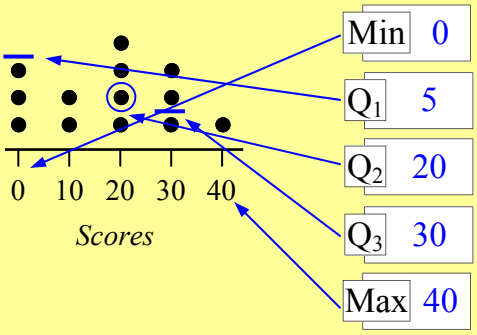






STEP 1 Locate the Median (Q_2).

STEP 2 Locate Q_1 and Q_3.



Min 0
Q_1 5
Q_2 20
Q_3 30
Max 40

IQR = Q_3 30 - Q_1 5 = 25

STEP 3 Write the 5 number summary

STEP 4 Calculate the IQR

Data Analysis - 5-Number Summary/IQR - Data Sets (Sheet 1)

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Find the 5-number summary for these data sets.

Example 12, 16, 19, 20, 28, 36, 44, 49, 50.

12, 16, 19, 20, 28, 36, 44, 49, 50.

Min 12 | Q_1 17.5 | Q_2 28 | Q_3 46.5 | Max 50

1 28, 34, 41, 55, 64, 64, 72, 77, 79, 85, 93.

Min | Q_1 | Q_2 | Q_3 | Max

2 3, 5, 7, 13, 18, 23, 27, 29, 36, 44, 52, 60, 67.

Min | Q_1 | Q_2 | Q_3 | Max

3 82, 106, 120, 127, 145, 145, 163, 178, 191.

Min | Q_1 | Q_2 | Q_3 | Max

4 4, 7, 13, 22, 26, 30, 32, 36, 38, 41, 48, 48.

Min | Q_1 | Q_2 | Q_3 | Max

Find the 5-number summary and calculate the IQR.
The IQR = Q_3 - Q_1.

5 15, 28, 34, 41, 56, 62, 73, 77, 81, 85.

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 (Not Max) - Q_1 (Not Min) =

6 63, 66, 75, 81, 88, 90, 93, 97, 100.

7 0, 0, 3, 3, 4, 5, 5, 5, 6, 6, 7, 7, 8, 8, 8, 10, 10.

8 23, 28, 34, 39, 43, 47, 49, 54, 60, 67, 74, 81.

This time organise the data in ascending order, lowest to highest. Then find a 5-number summary and the IQR.

9 35, 11, 26, 45, 22, 36, 53, 38, 40, 19.

11,

There are 10 scores, make sure you have written 10 scores

Min | Q_1 | Q_2 | Q_3 | Max

IQR = (Not Max) - (Not Min) =

10 108, 117, 94, 103, 125, 107, 129, 105, 100.

Min | Q_1 | Q_2 | Q_3 | Max

IQR = - =

11 40, 27, 17, 33, 5, 21, 40, 38.

12 17, 11, 8, 15, 3, 14, 10, 12, 7, 19, 15, 16.

13 7, 8, 2, 5, 1, 6, 2, 6, 3, 9, 2, 4, 5, 0.

14 856, 955, 778, 915, 713, 798, 808, 930.

Data Analysis - 5-No. Summary/IQR - Stem & Leaf / Dot Plots (Sheet 2)

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Find a 5-number summary and the IQR for these.

1

Tens	Units
0	2 5 | 9
1	3 8
2	0 1 | 5
3	4

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

IQR = Q_3 ☐ - Q_1 ☐ = ☐
(Not Max, Not Min)

2

Stem	Leaf
7	1 2 4 4
8	1 3 3 4 7
9	2 5
10	0 0 6
11	1 4 6 6

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

IQR = ☐ - ☐ = ☐

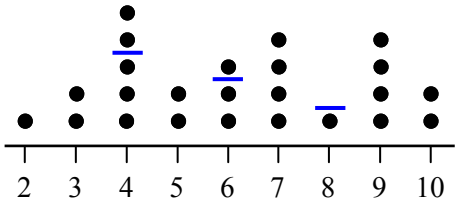
3

Male	Tens	Female
7 4 3 0	27	3 3
5 4 1 1 0	28	
8 8 8	29	1 1 2 4 5
9 4	30	0 1 1 3

Male	Female
Min = ☐	Min = ☐
Q_1 = ☐	Q_1 = ☐
Q_2 = ☐	Q_2 = ☐
Q_3 = ☐	Q_3 = ☐
Max = ☐	Max = ☐
IQR = ☐ - ☐	IQR = ☐ - ☐
IQR = ☐	IQR = ☐

Keep going, now it is dot plots.

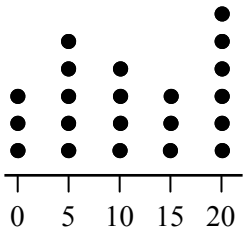
4



Min ☐ Q_1 ☐ Q_2 ☐ Q_3 ☐ Max ☐

IQR = Q_3 ☐ - Q_1 ☐ = ☐

5

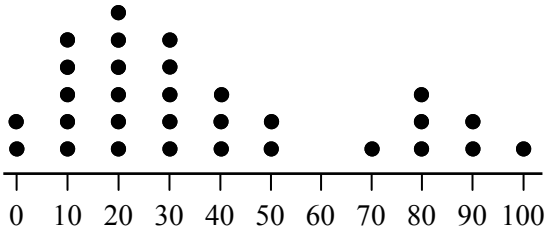


IQR = ☐ - ☐ = ☐

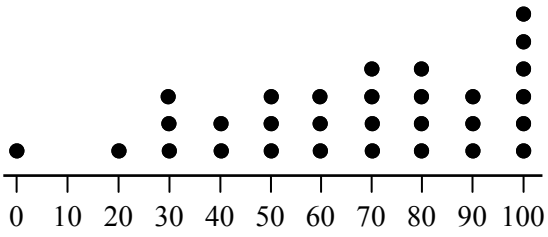
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

6

Sample A



Sample B



Sample A	Sample B
Min = ☐	Min = ☐
Q_1 = ☐	Q_1 = ☐
Q_2 = ☐	Q_2 = ☐
Q_3 = ☐	Q_3 = ☐
Max = ☐	Max = ☐
IQR = ☐ - ☐	IQR = ☐ - ☐
IQR = ☐	IQR = ☐

Data Analysis - 5-Number Summary/IQR - Data Sets (Sheet 1)

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Find the 5-number summary for these data sets.

Example 12, 16, 19, 20, 28, 36, 44, 49, 50.

12, 16, | 19, 20, 28, 36, 44, | 49, 50.

Min 12 | Q_1 17.5 | Q_2 28 | Q_3 46.5 | Max 50

1 28, 34, 41, 55, 64, 64, 72, 77, 79, 85, 93.

Min 28 | Q_1 41 | Q_2 64 | Q_3 79 | Max 93

2 3, 5, 7, | 13, 18, 23, 27, 29, 36, 44, | 52, 60, 67.

Min 3 | Q_1 10 | Q_2 27 | Q_3 48 | Max 67

3 82, 106, | 120, 127, 145, 145, 163, | 178, 191.

Min 82 | Q_1 113 | Q_2 145 | Q_3 170.5 | Max 191

4 4, 7, 13, | 22, 26, 30, | 32, 36, 38, | 41, 48, 48.

Min 4 | Q_1 17.5 | Q_2 31 | Q_3 39.5 | Max 48

Find the 5-number summary and calculate the IQR.
The IQR = Q_3 - Q_1.

5 15, 28, 34, 41, 56, | 62, 73, 77, 81, 85.

Min 15 | Q_1 34 | Q_2 59 | Q_3 77 | Max 85

IQR = Q_3 77 (Not Max) - Q_1 34 (Not Min) = 43

6 63, 66, | 75, 81, 88, 90, 93, | 97, 100.

Min = 63, Q_1 = 70.5, Q_2 = 88, Q_3 = 95, Max = 100.

IQR = 95 - 70.5 = 14.5

7 0, 0, 3, 3, | 4, 5, 5, 5, 6, 6, 7, 7, 8, | 8, 8, 10, 10.

Min = 0, Q_1 = 3.5, Q_2 = 6, Q_3 = 8, Max = 10.

IQR = 8 - 3.5 = 4.5

8 23, 28, 34, | 39, 43, 47, | 49, 54, 60, | 67, 74, 81.

Min = 23, Q_1 = 36.5, Q_2 = 48, Q_3 = 63.5, Max = 81.

IQR = 63.5 - 36.5 = 27

This time organise the data in ascending order, lowest to highest. Then find a 5-number summary and the IQR.

9 ~~35, 11, 26, 45, 22, 36, 53, 38, 40, 19.~~

11, 19, 22, 26, 35, | 36, 38, 40, 45, 53.

There are 10 scores, make sure you have written 10 scores

Min 11 | Q_1 22 | Q_2 35.5 | Q_3 40 | Max 53

IQR = 40 (Not Max) - 22 (Not Min) = 18

10 ~~108, 117, 94, 103, 125, 107, 129, 105, 100.~~

94, 100, | 103, 105, 107, 108, 117, | 125, 129.

Min 94 | Q_1 101.5 | Q_2 107 | Q_3 121 | Max 129

IQR = 121 - 101.5 = 19.5

11 ~~40, 27, 17, 33, 5, 21, 40, 38.~~

5, 17, | 21, 27, | 33, 38, | 40, 40.

Min = 5, Q_1 = 19, Q_2 = 30, Q_3 = 39, Max = 40.

IQR = 39 - 19 = 20

12 ~~17, 11, 8, 15, 3, 14, 10, 12, 7, 19, 15, 16.~~

3, 7, 8, | 10, 11, 12, | 14, 15, 15, | 16, 17, 19.

Min = 3, Q_1 = 9, Q_2 = 13, Q_3 = 15.5, Max = 19.

IQR = 15.5 - 9 = 6.5

13 ~~7, 8, 2, 5, 1, 6, 2, 6, 3, 9, 2, 4, 5, 0.~~

0, 1, 2, 2, 2, 3, 4, | 5, 5, 6, 6, 7, 8, 9.

Min = 0, Q_1 = 2, Q_2 = 4.5, Q_3 = 6, Max = 9.

IQR = 6 - 2 = 4

14 ~~856, 955, 778, 915, 713, 798, 808, 930.~~

713, 778, | 798, 808, | 856, 915, | 930, 955.

Min = 713, Q_1 = 788, Q_2 = 832, Q_3 = 922.5

Max = 955. IQR = 922.5 - 788 = 134.5

Data Analysis - 5-No. Summary/IQR - Stem & Leaf / Dot Plots (Sheet 2)

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Find a 5-number summary and the IQR for these.

1

Tens	Units
0	2 5 | 9
1	3 (8)
2	0 1 | 5
3	4

Min = 2
Q_1 = 7
Q_2 = 18
Q_3 = 23
Max = 34

IQR = Q_3 23 - Q_1 7 = 16
(Not Max, Not Min)

2

Stem	Leaf
7	1 2 4 4
8	(1) 3 3 4 7 |
9	2 5
10	0 0 (6)
11	1 4 6 6

Min = 71
Q_1 = 81
Q_2 = 89.5
Q_3 = 106
Max = 116

IQR = 106 - 81 = 25

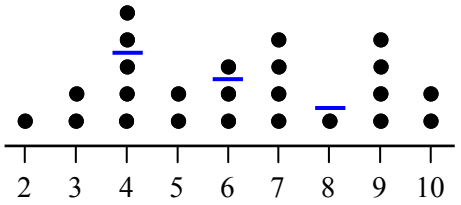
3

Male	Tens	Female
(7) 4 3 0	27	3 3
5 4 | 1 1 0	28	
8 (8) 8	29	(1) 1 2 (4) 5
9 4	30	0 (1) 1 3

Male	Female
Min = 270	Min = 273
Q_1 = 277	Q_1 = 291
Q_2 = 282.5	Q_2 = 294
Q_3 = 298	Q_3 = 301
Max = 309	Max = 303
IQR = 298 - 277	IQR = 301 - 291
IQR = 22	IQR = 10

Keep going, now it is dot plots.

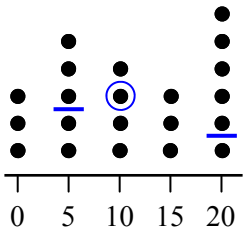
4



Min 2 | Q_1 4 | Q_2 6 | Q_3 8.5 | Max 10

IQR = Q_3 8.5 - Q_1 4 = 4.5

5

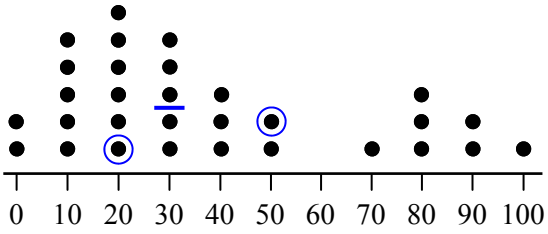


Min = 0
Q_1 = 5
Q_2 = 10
Q_3 = 20
Max = 20

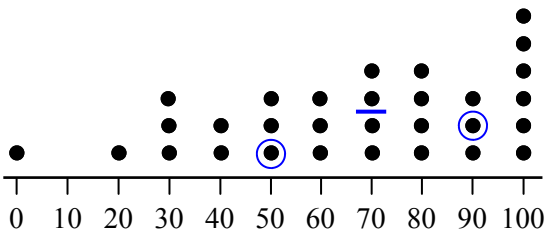
IQR = 20 - 5 = 15

6

Sample A



Sample B



Sample A	Sample B
Min = 0	Min = 0
Q_1 = 20	Q_1 = 50
Q_2 = 30	Q_2 = 70
Q_3 = 50	Q_3 = 90
Max = 100	Max = 100
IQR = 50 - 20	IQR = 90 - 50
IQR = 30	IQR = 40

Data Analysis - 5-Number Summary/IQR (Homework Strip)

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Date Due:

Find the 5-number summary and IQR

1 2, 2, 6, 9, 11, 17, 22, 34, 42.

Min | Q_1 | Q_2

Q_3 | Max

IQR = Q_3 - Q_1 =

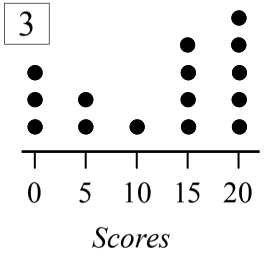
2

Tens	Units
4	0 2 4
5	2 3
6	5 6 6
7	0 1

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

3



Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

Date Due:

Find the 5-number summary and IQR

1 2, 2, 6, 9, 11, 17, 22, 34, 42.

Min | Q_1 | Q_2

Q_3 | Max

IQR = Q_3 - Q_1 =

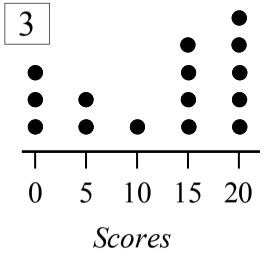
2

Tens	Units
4	0 2 4
5	2 3
6	5 6 6
7	0 1

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

3



Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

Date Due:

Find the 5-number summary and IQR

1 2, 2, 6, 9, 11, 17, 22, 34, 42.

Min | Q_1 | Q_2

Q_3 | Max

IQR = Q_3 - Q_1 =

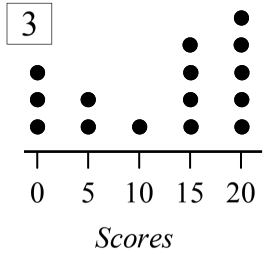
2

Tens	Units
4	0 2 4
5	2 3
6	5 6 6
7	0 1

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

3



Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

Date Due:

Find the 5-number summary and IQR

1 2, 2, 6, 9, 11, 17, 22, 34, 42.

Min | Q_1 | Q_2

Q_3 | Max

IQR = Q_3 - Q_1 =

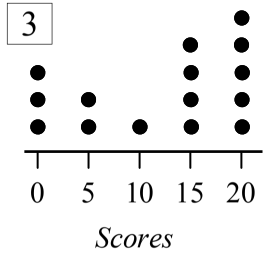
2

Tens	Units
4	0 2 4
5	2 3
6	5 6 6
7	0 1

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

3



Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

Data Analysis - 5-Number Summary/IQR (Homework Strip)

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Date Due: _____

Find the 5-number summary and IQR

1 2, 2, 6, 9, (11), 17, 22, 34, 42.

Min 2 Q_1 4 Q_2 11

Q_3 28 Max 42

IQR = Q_3 28 - Q_1 4 = 24

2

Tens	Units
4	0 2 (4)
5	2 3
6	5 6 (6)
7	0 1

Min 40

Q_1 44

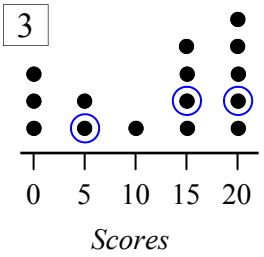
Q_2 59

Q_3 66

Max 71

IQR = Q_3 66 - Q_1 44 = 22

3



Min 0

Q_1 5

Q_2 15

Q_3 20

Max 20

IQR = Q_3 20 - Q_1 5 = 15

Data Analysis - 5-Number Summary/IQR - Frequency Tables

To find a 5-number summary for frequency tables, convert the table to a data set by writing the data out beside the table. Then use the same skills as a data set.

Example - Find the 5-number summary and IQR for the frequency table below.

x	f
0	3
5	4
10	2
15	6
20	4
$\Sigma f =$	

x	f	
0	3	0, 0, 0
5	4	5, 5, 5, 5
10	2	10, 10
15	6	15, 15, 15, 15, 15, 15
20	4	20, 20, 20, 20
$\Sigma f =$	19	

STEP 1

Find Σf and write out the data beside the table

x	f	
0	3	0, 0, 0
5	4	5, 5, 5, 5
10	2	10, 10
15	6	15, 15, 15, 15, 15, 15
20	4	20, 20, 20, 20
$\Sigma f =$	19	

STEP 2

Use Σf to locate the median.

$$\text{Median} = \frac{19+1}{2} = 10^{\text{th}} \text{ score}$$

x	f	
0	3	0, 0, 0
5	4	5, 5, 5, 5
10	2	10, 10
15	6	15, 15, 15, 15, 15, 15
20	4	20, 20, 20, 20
$\Sigma f =$	19	

STEP 3

Locate the position of Q_1 and Q_3. Note that both are identical in their position above and below the median.

9 scores

$$Q_1 \text{ Location} = \frac{9+1}{2} = 5^{\text{th}} \text{ score}$$

Min = 0

Q_1 = 5

Q_2 = 15

Q_3 = 15

Max = 20

IQR = 20 - 0 = 20

STEP 4

Write down the values for Q_1, Q_2 and Q_3, record the upper and lower extreme values and calculate the IQR.

Data Analysis - 5-Number Summary/IQR - Frequency Tables

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With Frequency Tables find Σf, write the scores out beside the frequency column and locate Q_2, the median. Then locate Q_1 and Q_3.

1

Score	*f*	
5	2	5, 5
6	4	6, 6, | 6, 6
7	3	7, 7, ⑦
8	5	8, 8, 8, 8, | 8
9	2	9, 9
10	1	10
$\Sigma f=$	17	

$Q_1 =$ $Q_2 =$ $Q_3 =$

$\Sigma f = 17 \rightarrow$ Median $= \frac{17+1}{2} = 9^{th}$

2

$Q_1 =$ $Q_2 =$ $Q_3 =$

x	*f*	
1	2	
2	4	
3	1	
4	3	
5	3	
$\Sigma f=$		

3

x	*f*	
0	1	
5	5	
10	2	
15	4	
20	3	
25	1	
30	3	
$\Sigma f=$		

$Q_1 =$ $Q_2 =$ $Q_3 =$

4

$Q_1 =$ $Q_2 =$ $Q_3 =$

x	*f*	
0	3	
2	5	
4	3	
6	2	
8	4	
10	4	
12	1	
$\Sigma f=$		

Find the 5 number summary for these frequency tables and calculate the Interquartile range.

5

Min = $Q_1 =$ $Q_2 =$ $Q_3 =$ Max =

x	*f*	
48	2	
49	4	
50	1	
51	3	
52	3	
$\Sigma f=$		

IQR = (Q_3 not Max) - (Q_1 not Min) =

6

Min = $Q_1 =$ $Q_2 =$ $Q_3 =$ Max =

x	*f*	
97	5	
98	4	
99	5	
100	4	
101	2	
102	0	
103	1	
$\Sigma f=$		

IQR = - =

7

Min = $Q_1 =$ $Q_2 =$ $Q_3 =$ Max =

x	*f*	
0	1	
1	3	
2	1	
3	4	
4	5	
5	3	
6	4	
7	3	
8	4	
9	2	
10	1	
$\Sigma f=$		

IQR = - =

Data Analysis - 5-Number Summary/IQR - Frequency Tables

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With Frequency Tables find Σf, write the scores out beside the frequency column and locate Q_2, the median. Then locate Q_1 and Q_3.

1

Score	*f*	
5	2	5, 5
6	4	6, 6, 6, 6
7	3	7, 7, 7
8	5	8, 8, 8, 8, 8
9	2	9, 9
10	1	10
$\Sigma f=$	17	

$Q_1 =$ 6

$Q_2 =$ 7

$Q_3 =$ 8

$\Sigma f = 17 \rightarrow$ Median $= \frac{17+1}{2} = 9^{th}$

2

$Q_1 =$ 2

$Q_2 =$ 3

$Q_3 =$ 4.5

x	*f*	
1	2	1, 1
2	4	2, 2, 2, 2
3	1	3
4	3	4, 4, 4
5	3	5, 5, 5
$\Sigma f=$	13	

3

x	*f*	
0	1	0
5	5	5, 5, 5, 5, 5,
10	2	10, 10
15	4	15, 15, 15, 15
20	3	20, 20, 20
25	1	25
30	3	30, 30, 30
$\Sigma f=$	19	

$Q_1 =$ 5

$Q_2 =$ 15

$Q_3 =$ 20

4

$Q_1 =$ 2

$Q_2 =$ 5

$Q_3 =$ 8

x	*f*	
0	3	0, 0, 0
2	5	2, 2, 2, 2, 2
4	3	4, 4, 4
6	2	6, 6
8	4	8, 8, 8, 8
10	4	10, 10, 10, 10
12	1	12
$\Sigma f=$	22	

Find the 5 number summary for these frequency tables and calculate the Interquartile range.

5

Min = 48

$Q_1 =$ 49

$Q_2 =$ 50

$Q_3 =$ 51.5

Max = 52

x	*f*	
48	2	48, 48
49	4	49, 49, 49, 49
50	1	50
51	3	51, 51, 51
52	3	52, 52, 52
$\Sigma f=$	13	

IQR = 51.5 - 49 = 2.5

(Q_3 not Max) (Q_1 not Min)

6

Min = 97

$Q_1 =$ 97.5

$Q_2 =$ 99

$Q_3 =$ 100

Max = 103

x	*f*	
97	5	97, 97, 97, 97, 97
98	4	98, 98, 98, 98
99	5	99, 99, 99, 99, 99
100	4	100, 100, 100, 100
101	2	101, 101
102	0	
103	1	103
$\Sigma f=$	21	

IQR = 100 - 97.5 = 2.5

7

Min = 0

$Q_1 =$ 3

$Q_2 =$ 5

$Q_3 =$ 7

Max = 10

x	*f*	
0	1	0
1	3	1, 1, 1
2	1	2
3	4	3, 3, 3, 3
4	5	4, 4, 4, 4, 4
5	3	5, 5, 5
6	4	6, 6, 6, 6
7	3	7, 7, 7
8	4	8, 8, 8, 8
9	2	9, 9
10	1	10
$\Sigma f=$	31	

IQR = 7 - 3 = 4

Data Analysis - 5-Number Summary/IQR - Frequency Tables (Homework Strip)

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Date Due:

Find the 5 number summary & IQR.

1

x	f	
0	5	
1	4	
2	2	
3	3	
4	1	
$\Sigma f=$		

Min | Q_1 | Q_2

Q_3 | Max

IQR = - =

2

x	f	
8	2	
9	5	
10	3	
11	3	
12	4	
$\Sigma f=$		

Min | Q_1 | Q_2

Q_3 | Max

IQR = - =

Date Due:

Find the 5 number summary & IQR.

1

x	f	
0	5	
1	4	
2	2	
3	3	
4	1	
$\Sigma f=$		

Min | Q_1 | Q_2

Q_3 | Max

IQR = - =

2

x	f	
8	2	
9	5	
10	3	
11	3	
12	4	
$\Sigma f=$		

Min | Q_1 | Q_2

Q_3 | Max

IQR = - =

Date Due:

Find the 5 number summary & IQR.

1

x	f	
0	5	
1	4	
2	2	
3	3	
4	1	
$\Sigma f=$		

Min | Q_1 | Q_2

Q_3 | Max

IQR = - =

2

x	f	
8	2	
9	5	
10	3	
11	3	
12	4	
$\Sigma f=$		

Min | Q_1 | Q_2

Q_3 | Max

IQR = - =

Date Due:

Find the 5 number summary & IQR.

1

x	f	
0	5	
1	4	
2	2	
3	3	
4	1	
$\Sigma f=$		

Min | Q_1 | Q_2

Q_3 | Max

IQR = - =

2

x	f	
8	2	
9	5	
10	3	
11	3	
12	4	
$\Sigma f=$		

Min | Q_1 | Q_2

Q_3 | Max

IQR = - =

Data Analysis - 5-Number Summary/IQR - Frequency Tables (HWStrip)

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Date Due:

Find the 5 number summary & IQR.

1

x	f	
0	5	0, 0, 0, 0, 0
1	4	1, 1, 1, 1
2	2	2, 2
3	3	3, 3, 3
4	1	4
$\Sigma f=$	15	

Min 0 Q_1 0 Q_2 1

Q_3 3 Max 4

IQR = 3 - 0 = 3

2

x	f	
8	2	8, 8
9	5	9, 9, 9, 9, 9
10	3	10, 10, 10
11	3	11, 11, 11
12	4	12, 12, 12, 12
$\Sigma f=$	17	

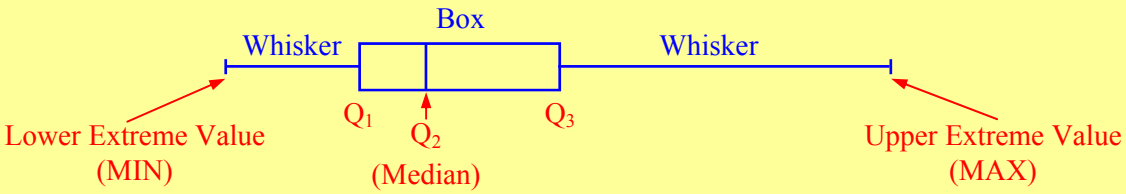
Min 8 Q_1 9 Q_2 10

Q_3 11.5 Max 12

IQR = 11.5 - 9 = 2.5

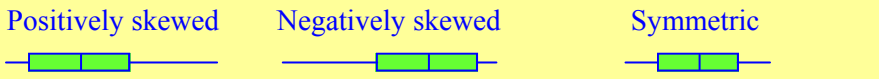
Data Analysis - Box and Whisker Plots

A Box and Whisker plot is a graphical representation of a 5-number summary. So if you know how to create a 5 number summary, box and whisker plots are just a drawing exercise.



Why do we need a box and whisker plot? They convert data into an easy to read, and an easy to compare graph. You can have box and whisker plots for 'before' and 'after' comparisons which allow you to quickly see what has changed, with quartile groups easily compared.

Box and Whisker plots are a graph, so when describing a graph, remember these three:



It is possible to have no Q_2 line in a box. It is also possible for plots to have one whisker and also no whiskers, but these are not featured on these sheets.

Example - Create a Box and Whisker plot for this data set.

Data: 9, 14, 16, 19, 20, 22, 23, 25, 26, 28.

STEP 1

Data: 9, 14, 16, 19, 20, 22, 23, 25, 26, 28.

Locate the Median (Q_2).

STEP 2

Data: 9, 14, (16), 19, 20, 22, 23, (25), 26, 28.

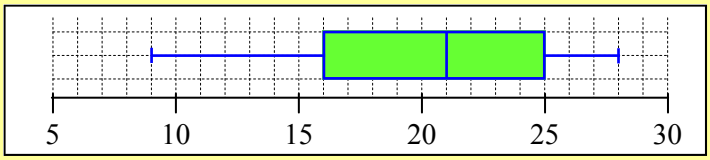
Locate Q_1 and Q_3.

STEP 3

Min	9	Q_1	16	Q_2	21	Q_3	25	Max	28

Create a 5-number summary

STEP 4



Create a Box and Whisker plot

This plot is negatively skewed.

Data Analysis - Box and Whisker Plots - Stem & Leaf/Dot Plots

Example - Create a box and whisker plot for the stem and leaf plot below.

Tens	Units
5	2 5 9
6	6 8
7	2 3 3
8	2 8

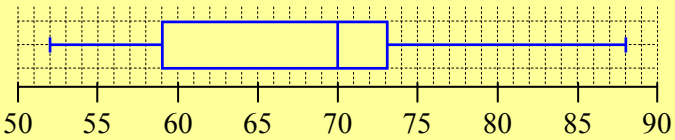
Tens	Units
5	2 5 9
6	6 8 |
7	2 3 3
8	2 8

Tens	Units
5	2 5 (9)
6	6 8 |
7	2 3 (3)
8	2 8

STEP 1 Locate the Median (Q_2).

STEP 2 Locate Q_1 and Q_3.

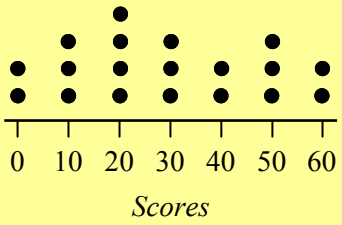
Tens	Units	
5	2 5 (9)	Min 52
6	6 8 |	Q_1 59
7	2 3 (3)	Q_2 70
8	2 8	Q_3 73
		Max 88

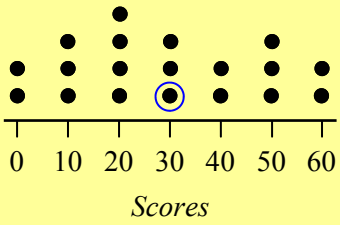


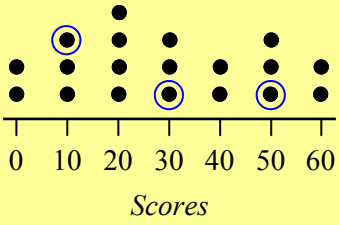
STEP 3 Write the 5 number summary

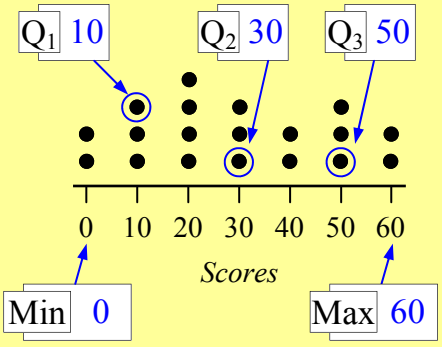
STEP 4 Create the box and whisker plot

Example - Create a box and whisker plot for the dot plot below.



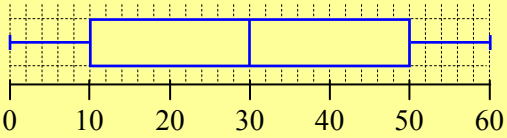






STEP 1 Locate the Median (Q_2).

STEP 2 Locate Q_1 and Q_3.



STEP 3 Write the 5 number summary

STEP 4 Create the box and whisker plot

Data Analysis - Box and Whisker Plots - Data Sets (Sheet 1)

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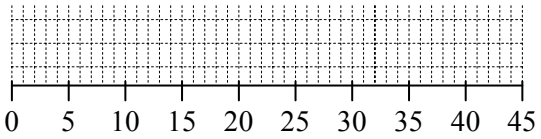
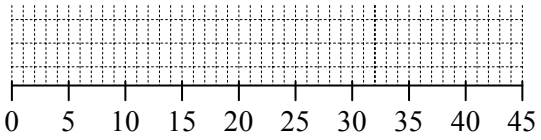
Create box and whisker plots for these 5-number summaries. Then comment on the distribution.

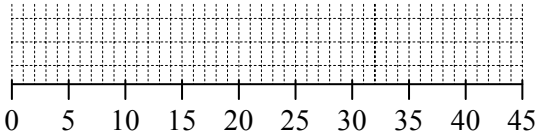
1 Min = 3, Q_1 = 26, Q_2 = 29, Q_3 = 34, Max = 41.



2 Min = 3, Q_1 = 15, Q_2 = 19, Q_3 = 23, Max = 35.

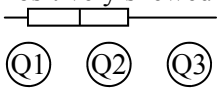


3 Min = 6, Q_1 = 12, Q_2 = 16, Q_3 = 19, Max = 42.

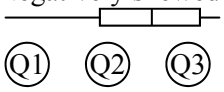


4 Pick which box and whisker plot is :

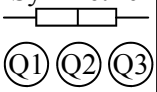
Positively skewed



Negatively skewed



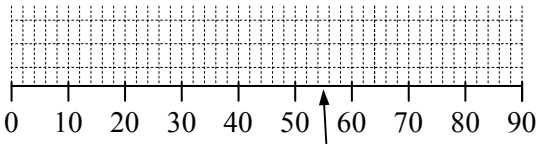
Symmetric



Now find the 5-number summaries for the data sets. Then plot them in a box and whisker plot.

5 4, 17, 25, 27, 38, 44, 55, 58, 76, 76, 84, 86, 88.

Min | Q_1 | Q_2 | Q_3 | Max



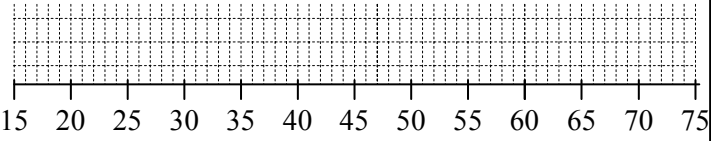
55 is between 54 and 56!

6 The above plot is: _____

Pos skew/neg skew/symmetric

7 16, 21, 35, 41, 47, 58, 62, 72.

Min | Q_1 | Q_2 | Q_3 | Max



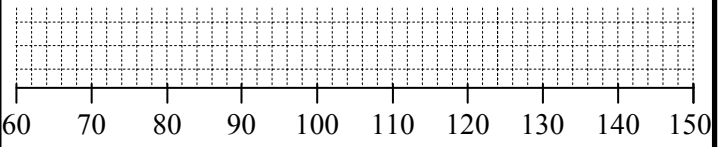
8 The above plot is: _____

Pos skew/neg skew/symmetric

Find the 5-number summaries, then plot them in a box and whisker plot. Finally, calculate the IQR.

9 62, 64, 68, 74, 86, 94, 107, 111, 126, 145.

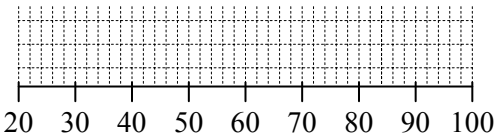
Min | Q_1 | Q_2 | Q_3 | Max



IQR = Q_3 (Not Max) - Q_1 (Not Min) = _____

10 24, 39, 49, 55, 63, 63, 67, 72, 78, 82, 86, 94.

Min | Q_1 | Q_2 | Q_3 | Max



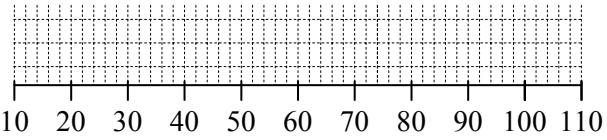
IQR = _____ - _____ = _____

Now rewrite the data in ascending order first.

11 72, 24, 66, 94, 11, 45, 52, 78, 60, 106, 18.

Count the scores to ensure you haven't missed any.

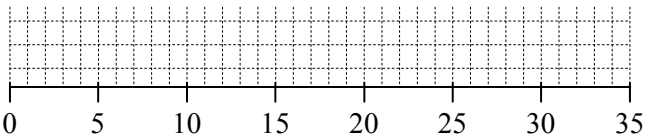
Min | Q_1 | Q_2 | Q_3 | Max



IQR = _____ - _____ = _____

12 14, 6, 21, 19, 2, 28, 11, 34, 30, 16, 25, 31, 7.

Min | Q_1 | Q_2 | Q_3 | Max



IQR = _____ - _____ = _____

Data Analysis - Box and Whisker Plots - Stem & Leaf Plots (Sheet 2)

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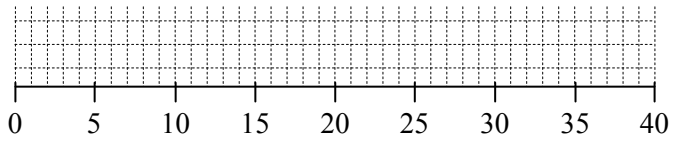
Create box and whisker plots for these stem and leaf plots. Then comment on the distribution.

1

Tens	Units
0	1 4 9 | 9 9
1	3 8 8 9
2	5 | 9
3	2 5

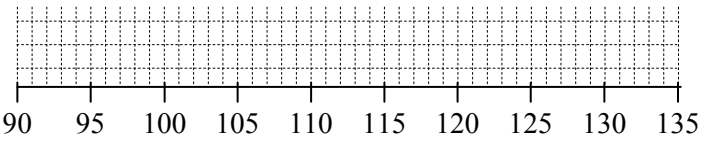
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



2

Stem	Leaf
9	1 8
10	2 6 8
11	4 7 8 9
12	1 8 8
13	0 2 2

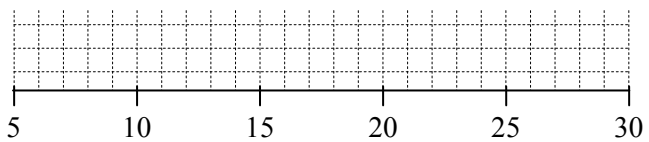
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



3

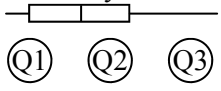
Tens	Units
0	8 8
1	1 1 2 4 4
2	0 0 1 1 4 6 9

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

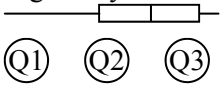


4 Pick which box and whisker plot is :

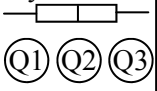
Positively skewed



Negatively skewed



Symmetric

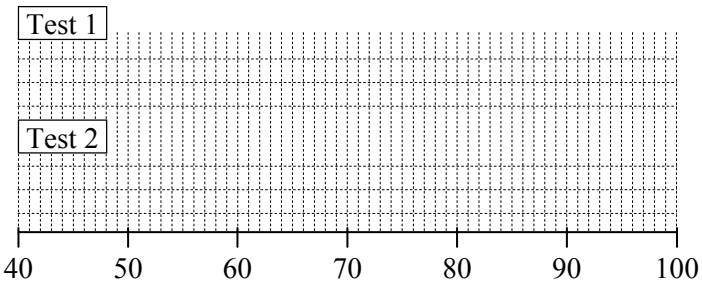


20 students sat Test 1, then the week after, Test 2. Construct B&W plots and calculate the IQR.

5

Test 1	Tens	Test 2
7 2	4	
6 3 0	5	2 3 8
6 6 5 4 4	6	3 3 3
8 2 2 0	7	1 1 2 2 6 8
9 7 6 6	8	3 4 4 6 8
2 2	9	5 6 6

Test 1	Test 2
Min = ☐	Min = ☐
Q_1 = ☐	Q_1 = ☐
Q_2 = ☐	Q_2 = ☐
Q_3 = ☐	Q_3 = ☐
Max = ☐	Max = ☐
IQR = Q_3 ☐ - Q_1 ☐	IQR = ☐ - ☐
IQR = ☐	IQR = ☐



6 Comment on the distributions. Include the shape Eg. skew/symmetric and compare the IQR's and the consistency of the results.

Data Analysis - Box and Whisker Plots - Dot Plots (Sheet 3)

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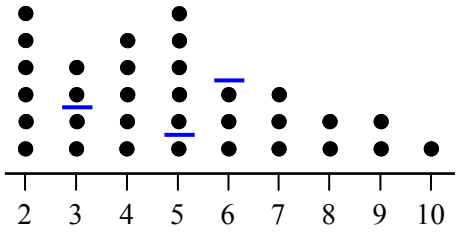
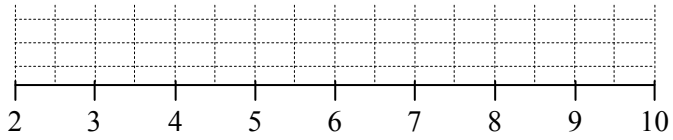
Create box and whisker plots for these dot plots. Then comment on the distribution.

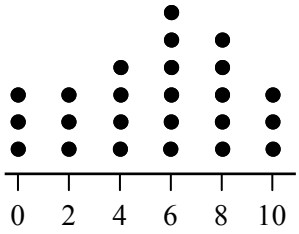
1



Min | Q_1 | Q_2 | Q_3 | Max



2



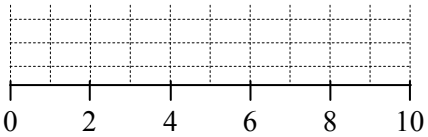
Min =

Q_1 =

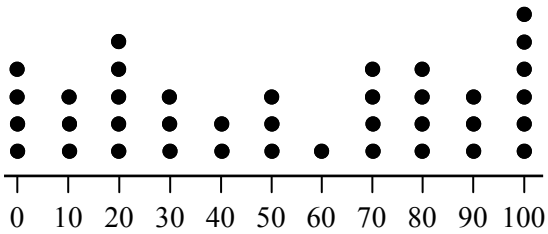
Q_2 =

Q_3 =

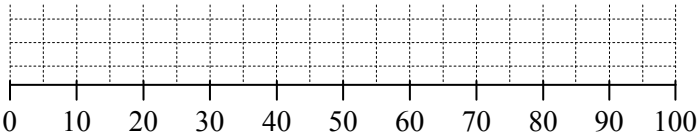
Max =



3

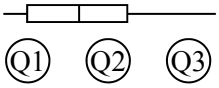


Min | Q_1 | Q_2 | Q_3 | Max

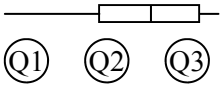


4 Pick which box and whisker plot is :

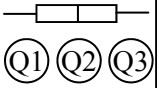
Positively skewed



Negatively skewed



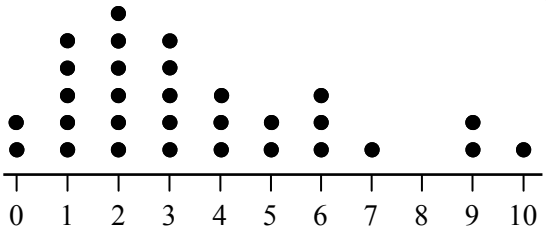
Symmetric



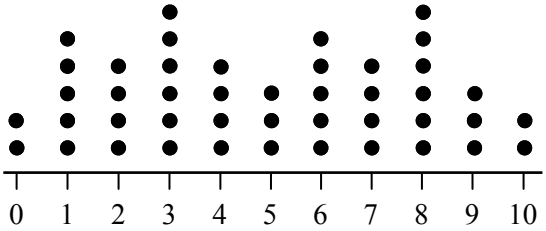
Create box and whisker plots for the dot plots below. Calculate the IQR and compare and contrast.

5

Sample A



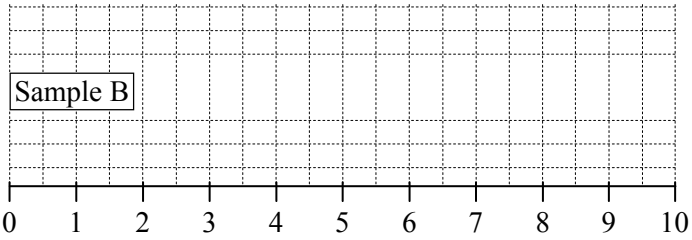
Sample B



Sample A	Sample B
Min =	Min =
Q_1 =	Q_1 =
Q_2 =	Q_2 =
Q_3 =	Q_3 =
Max =	Max =
IQR = ___ - ___	IQR = ___ - ___
IQR =	IQR =

Sample A

Sample B



6 Compare and contrast Sample A and Sample B. Use the distribution shape and IQR in your statement.

Data Analysis - Box and Whisker Plots - Data Sets (Sheet 1)

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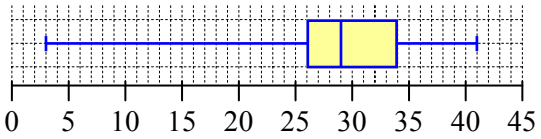
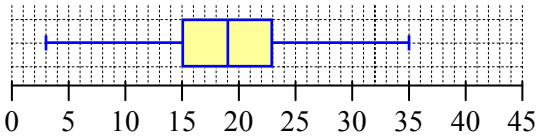
Create box and whisker plots for these 5-number summaries. Then comment on the distribution.

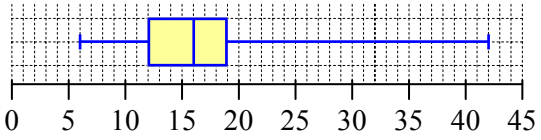
1 Min = 3, Q_1 = 26, Q_2 = 29, Q_3 = 34, Max = 41.



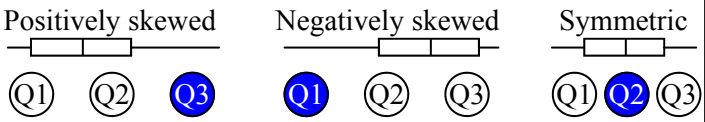
2 Min = 3, Q_1 = 15, Q_2 = 19, Q_3 = 23, Max = 35.



3 Min = 6, Q_1 = 12, Q_2 = 16, Q_3 = 19, Max = 42.



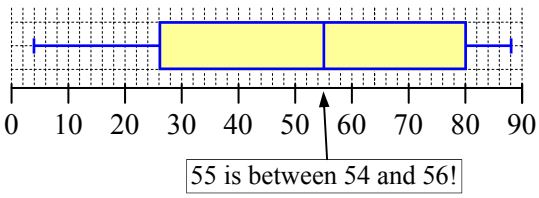
4 Pick which box and whisker plot is :



Now find the 5-number summaries for the data sets. Then plot them in a box and whisker plot.

5 4, 17, 25, | 27, 38, 44, (55), 58, 76, 76, | 84, 86, 88.

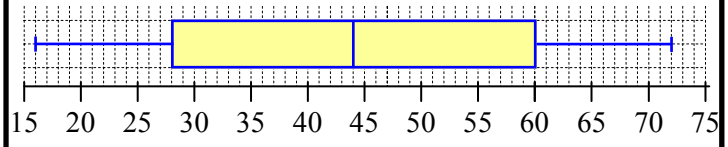
Min 4 | Q_1 26 | Q_2 55 | Q_3 80 | Max 88



6 The above plot is: Negatively skewed.
Pos skew/neg skew/symmetric

7 16, 21, | 35, 41, | 47, 58, | 62, 72.

Min 16 | Q_1 28 | Q_2 44 | Q_3 60 | Max 72

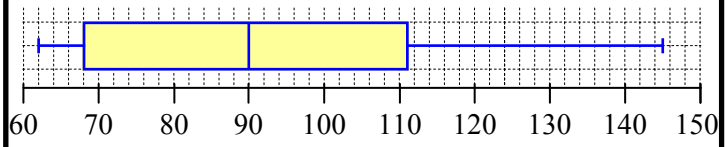


8 The above plot is: Symmetric
Pos skew/neg skew/symmetric

Find the 5-number summaries, then plot them in a box and whisker plot. Finally, calculate the IQR.

9 62, 64, (68), 74, 86, | 94, 107, (111), 126, 145.

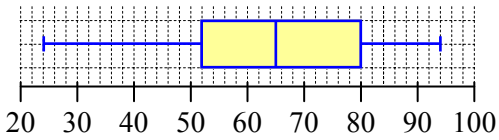
Min 62 | Q_1 68 | Q_2 90 | Q_3 111 | Max 145



IQR = Q_3 111 - Q_1 68 = 43
Not Max Not Min

10 24, 39, 49, | 55, 63, 63, | 67, 72, 78, | 82, 86, 94.

Min 24 | Q_1 52 | Q_2 65 | Q_3 80 | Max 94



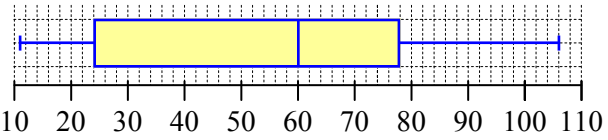
IQR = 80 - 52 = 28

Now rewrite the data in ascending order first.

11 ~~72~~, ~~24~~, ~~66~~, ~~94~~, ~~11~~, ~~45~~, ~~52~~, ~~78~~, ~~60~~, ~~106~~, ~~18~~.
11, 18, (24), 45, 52, (60), 66, 72, (78), 94, 106.

Count the scores to ensure you haven't missed any.

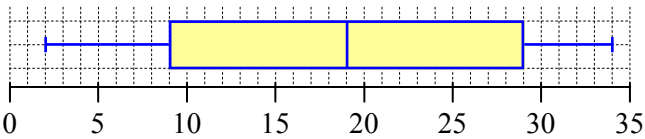
Min 11 | Q_1 24 | Q_2 60 | Q_3 78 | Max 106



IQR = 78 - 24 = 54

12 ~~14~~, ~~6~~, ~~21~~, ~~19~~, ~~2~~, ~~28~~, ~~11~~, ~~34~~, ~~30~~, ~~16~~, ~~25~~, ~~31~~, ~~7~~.
2, 6, 7, | 11, 14, 16, (19), 21, 25, 28, | 30, 31, 34.

Min 2 | Q_1 9 | Q_2 19 | Q_3 29 | Max 34



IQR = 29 - 9 = 20

Data Analysis - Box and Whisker Plots - Stem & Leaf Plots (Sheet 2)

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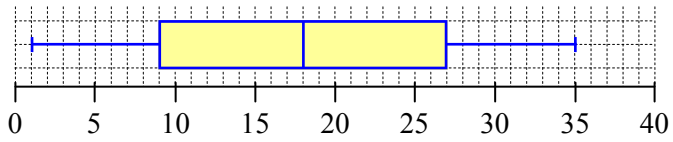
Create box and whisker plots for these stem and leaf plots. Then comment on the distribution.

1

Tens	Units
0	1 4 9 | 9 9
1	3 (8) 8 9
2	5 | 9
3	2 5

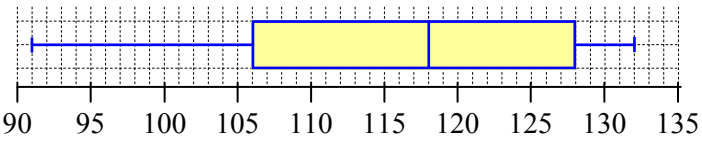
Min = 1
Q_1 = 9
Q_2 = 18
Q_3 = 27
Max = 35



2

Stem	Leaf
9	1 8
10	2 (6) 8
11	4 7 (8) 9
12	1 8 (8)
13	0 2 2

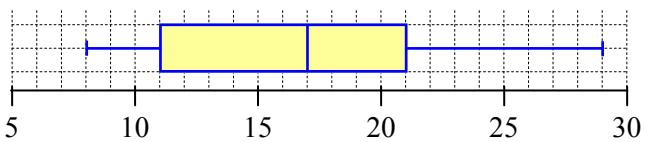
Min = 91
Q_1 = 106
Q_2 = 118
Q_3 = 128
Max = 132



3

Tens	Units
0	8 8
1	1 (1) 2 4 4 |
2	0 0 1 (1) 4 6 9

Min = 8
Q_1 = 11
Q_2 = 17
Q_3 = 21
Max = 29



4 Pick which box and whisker plot is :

Positively skewed — (Q1) (Q2) **(Q3)**

Negatively skewed — (Q1) **(Q2)** (Q3)

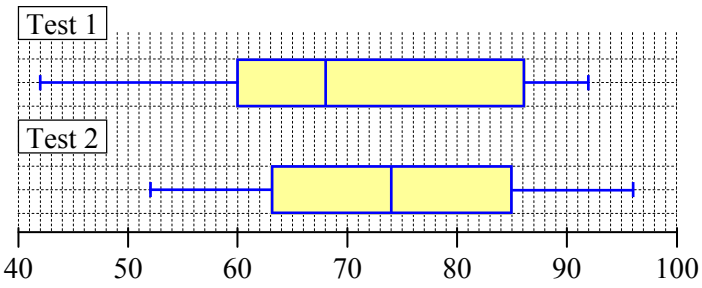
Symmetric — **(Q1)** (Q2) (Q3)

20 students sat Test 1, then the week after, Test 2. Construct B&W plots and calculate the IQR.

5

Test 1	Tens	Test 2
7 2	4	
6 3 0	5	2 3 8
6 6 5 4 4	6	3 3 | 3
8 2 2 0	7	1 1 2 2 | 6 8
9 7 6 | 6	8	3 4 4 | 6 8
2 2	9	5 6 6

	Test 1	Test 2
Min =	42	52
Q_1 =	60	63
Q_2 =	68	74
Q_3 =	86	85
Max =	92	96
IQR =	Q_3 86 - Q_1 60	85 - 63
IQR =	26	22



6 Comment on the distributions. Include the shape Eg. skew/symmetric and compare the IQR's and the consistency of the results.

Test 1 is negatively skewed, has a larger IQR and a larger range. Test 2 is symmetric has a lower IQR and range. Other points are: Test 2 has a higher median, lower and upper extremes.

Data Analysis - Box and Whisker Plots - Dot Plots (Sheet 3)

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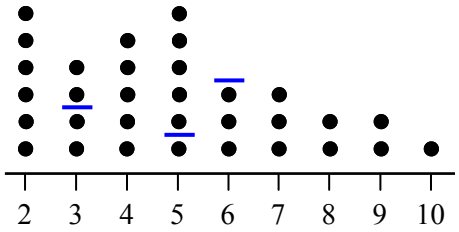
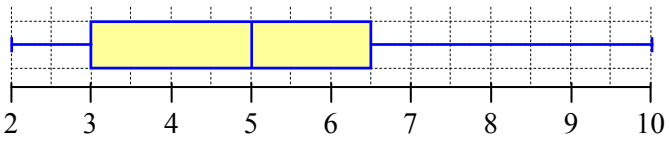
Create box and whisker plots for these dot plots. Then comment on the distribution.

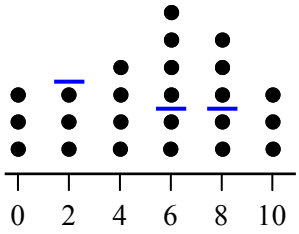
1

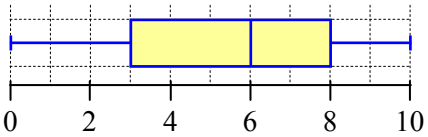


Min 2 | Q_1 3 | Q_2 5 | Q_3 6.5 | Max 10



2





Min = 0

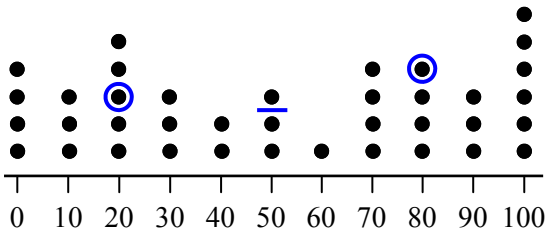
Q_1 = 3

Q_2 = 6

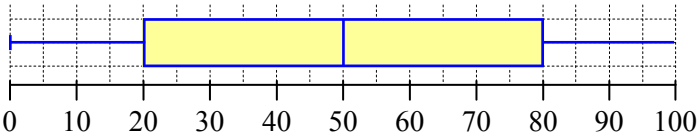
Q_3 = 8

Max = 10

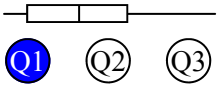
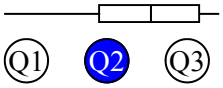
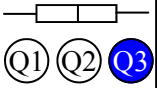
3



Min 0 | Q_1 20 | Q_2 60 | Q_3 80 | Max 100



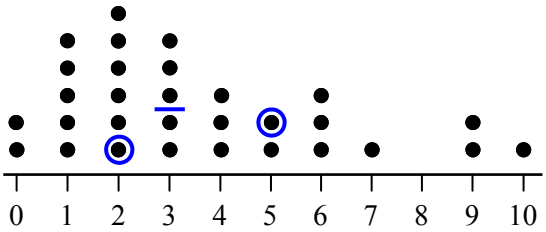
4 Pick which box and whisker plot is :

Positively skewed	Negatively skewed	Symmetric
		
Q1 Q2 Q3	Q1 **Q2** Q3	Q1 Q2 **Q3**

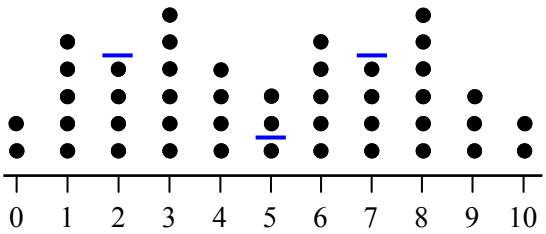
Create box and whisker plots for the dot plots below. Calculate the IQR and compare and contrast.

5

Sample A



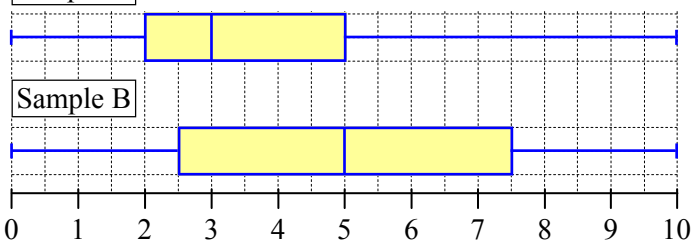
Sample B



Sample A	Sample B
Min = 0	Min = 0
Q_1 = 2	Q_1 = 2.5
Q_2 = 3	Q_2 = 5
Q_3 = 5	Q_3 = 7.5
Max = 10	Max = 10
IQR = 5 - 2	IQR = 7.5 - 2.5
IQR = 3	IQR = 5

Sample A

Sample B



6 Compare and contrast Sample A and Sample B. Use the distribution shape and IQR in your statement.

Sample A is positively skewed with a smaller IQR. Sample B is symmetric with a larger IQR. Both samples have the same range, Sample B has a higher median value.

Data Analysis - Box and Whisker Plots (Homework Strip)

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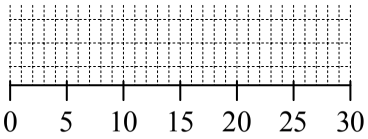
Date Due: _____

Display the following in a box and whisker plot. Describe the distribution.

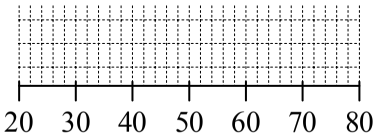
1 | 4 (Min) | 11 (Q_1) | 16 (Q_2) | 21 (Q_3) | 28 (Max)



○ Positive Skew ○ Symmetric ○ Negative Skew

2 | 24, 44, 48, 52, 55, 68, 72.

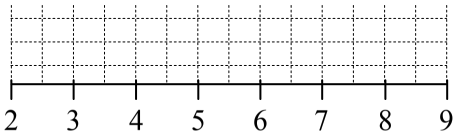
Min	Q_1	Q_2	Q_3	Max



○ Positive Skew ○ Symmetric ○ Negative Skew

3 | 2, 2, 3, 3, 3, 4, 4, 5, 5, 8, 9, 9.

Min	Q_1	Q_2	Q_3	Max



○ Positive Skew ○ Symmetric ○ Negative Skew

Date Due: _____

Display the following in a box and whisker plot. Describe the distribution.

1 | 4 (Min) | 11 (Q_1) | 16 (Q_2) | 21 (Q_3) | 28 (Max)

○ Positive Skew ○ Symmetric ○ Negative Skew

2 | 24, 44, 48, 52, 55, 68, 72.

Min	Q_1	Q_2	Q_3	Max

○ Positive Skew ○ Symmetric ○ Negative Skew

3 | 2, 2, 3, 3, 3, 4, 4, 5, 5, 8, 9, 9.

Min	Q_1	Q_2	Q_3	Max

○ Positive Skew ○ Symmetric ○ Negative Skew

Date Due: _____

Display the following in a box and whisker plot. Describe the distribution.

1 | 4 (Min) | 11 (Q_1) | 16 (Q_2) | 21 (Q_3) | 28 (Max)

○ Positive Skew ○ Symmetric ○ Negative Skew

2 | 24, 44, 48, 52, 55, 68, 72.

Min	Q_1	Q_2	Q_3	Max

○ Positive Skew ○ Symmetric ○ Negative Skew

3 | 2, 2, 3, 3, 3, 4, 4, 5, 5, 8, 9, 9.

Min	Q_1	Q_2	Q_3	Max

○ Positive Skew ○ Symmetric ○ Negative Skew

Date Due: _____

Display the following in a box and whisker plot. Describe the distribution.

1 | 4 (Min) | 11 (Q_1) | 16 (Q_2) | 21 (Q_3) | 28 (Max)

○ Positive Skew ○ Symmetric ○ Negative Skew

2 | 24, 44, 48, 52, 55, 68, 72.

Min	Q_1	Q_2	Q_3	Max

○ Positive Skew ○ Symmetric ○ Negative Skew

3 | 2, 2, 3, 3, 3, 4, 4, 5, 5, 8, 9, 9.

Min	Q_1	Q_2	Q_3	Max

○ Positive Skew ○ Symmetric ○ Negative Skew

Data Analysis - 5-Number Summary/IQR - Frequency Tables

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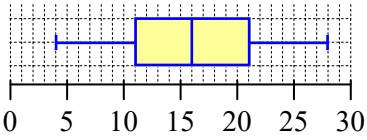
Date Due:

Display the following in a box and whisker plot. Describe the distribution.

1

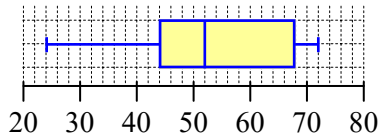
4	11	16	21	28
Min	Q_1	Q_2	Q_3	Max



○ Positive Skew ● Symmetric ○ Negative Skew

2 24, (44), 48, (52), 55, (68), 72.

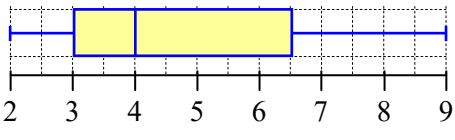
24	44	52	68	72
Min	Q_1	Q_2	Q_3	Max



○ Positive Skew ○ Symmetric ● Negative Skew

3 2, 2, 3, | 3, 3, 4, | 4, 5, 5, | 8, 9, 9.

2	3	4	6.5	9
Min	Q_1	Q_2	Q_3	Max



● Positive Skew ○ Symmetric ○ Negative Skew

Data Analysis - Box and Whisker Plots - Frequency Tables

To construct a Box and Whisker plot first find a 5-number summary for frequency tables. To achieve this, convert the table to a data set by writing the data out beside the table. Then use the same skills as if working with a data set.

Example - Construct a Box and Whisker plot for the frequency table below.

x	f
0	3
5	4
10	3
15	5
20	4
$\Sigma f =$	

x	f	
0	3	0, 0, 0
5	4	5, 5, 5, 5
10	3	10, 10, 10
15	5	15, 15, 15, 15, 15
20	4	20, 20, 20, 20
$\Sigma f =$	19	

STEP 1

Find Σf and write out the data beside the table

x	f	
0	3	0, 0, 0
5	4	5, 5, 5, 5
10	3	10, 10, 10
15	5	15, 15, 15, 15, 15
20	4	20, 20, 20, 20
$\Sigma f =$	19	

STEP 2

Use Σf to locate the median.

$$\text{Median} = \frac{19 + 1}{2} = 10^{\text{th}} \text{ score}$$

x	f	
0	3	0, 0, 0
5	4	5, 5, 5, 5
10	3	10, 10, 10
15	5	15, 15, 15, 15, 15
20	4	20, 20, 20, 20
$\Sigma f =$	19	

9 scores

STEP 3

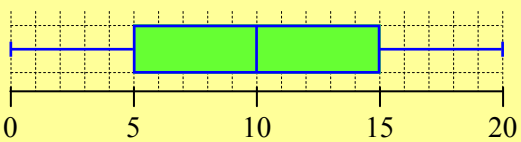
Locate the position of Q_1 and Q_3. Note that both are identical in their position above and below the median.

$$Q_1 \text{ Location} = \frac{9 + 1}{2} = 5^{\text{th}} \text{ score}$$

Min = 0 Q_1 = 5 Q_2 = 10 Q_2 = 15 Max = 20

STEP 4

Write down the values for Q_1, Q_2 and Q_3, record the upper and lower extreme values.



STEP 5

Create the Box and Whisker plot. Note the distribution shape.

○ Positive Skew ○ Negative Skew ● Symmetric

Data Analysis - Box and Whisker Plots - Frequency Tables

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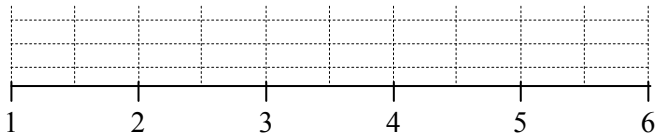
Find Σf, write the scores out beside the frequency column and locate Q_1, Q_2 and Q_3. Create a box and whisker plot. Describe the distribution.

1

Score	*f*	
1	3	1, 1, 1
2	4	2, 2, 2, 2
3	5	3, 3, 3, (3), 3
4	5	4, 4, 4, 4, 4
5	3	5, 5, 5
6	1	6
$\Sigma f =$	21	

$Q_2 = \frac{21+1}{2} = 11^{th}$

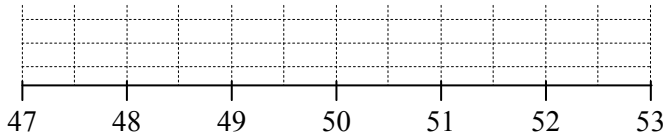
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



2

x	*f*	
47	2	
48	4	
49	1	
50	3	
51	0	
52	3	
53	2	
$\Sigma f =$		

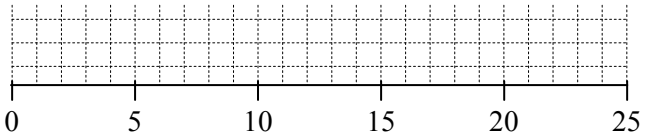
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



3

Score	*f*	
0	2	
5	1	
10	2	
15	5	
20	3	
25	4	
$\Sigma f =$		

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



4 From the 1st column pick which box & whisker plot is:

Positively skewed ☐ Q1 Q2 Q3

Negatively skewed ☐ Q1 Q2 Q3

Symmetric ☐ Q1 Q2 Q3

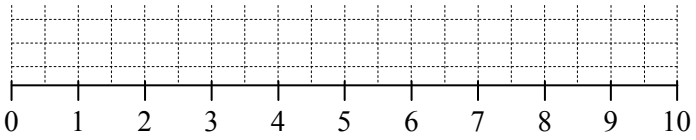
5 Now find the IQR for the data as well.

x	*f*	
0	3	
1	6	
2	2	
3	5	
4	3	
5	0	
6	2	
7	0	
8	1	
9	3	
10	3	
$\Sigma f =$		

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

IQR = ☐ - ☐ = ☐

(Q_3 not Max) (Q_1 not Min)



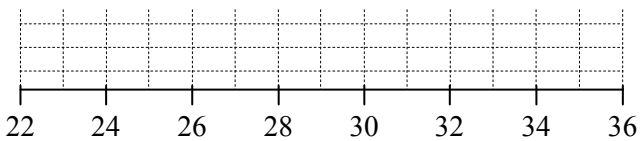
○ Positive Skew ○ Negative Skew ○ Symmetric

6

x	*f*	
22	4	
24	2	
26	3	
28	2	
30	1	
32	3	
34	4	
36	3	
$\Sigma f =$		

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

IQR = ☐ - ☐ = ☐



○ Positive Skew ○ Negative Skew ○ Symmetric

Data Analysis - Box and Whisker Plots - Frequency Tables

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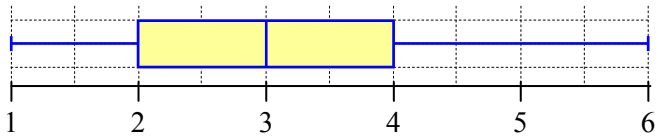
Find Σf, write the scores out beside the frequency column and locate Q_1, Q_2 and Q_3. Create a box and whisker plot. Describe the distribution.

1

Score	*f*	
1	3	1, 1, 1
2	4	2, 2, 2, 2
3	5	3, 3, 3, 3, 3
4	5	4, 4, 4, 4, 4
5	3	5, 5, 5
6	1	6
Σf=	21	

$Q_2 = \frac{21+1}{2} = 11^{th}$

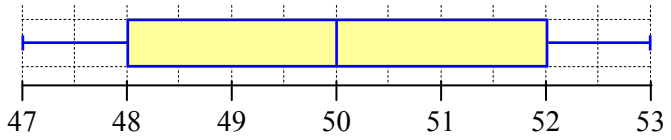
Min = 1
Q_1 = 2
Q_2 = 3
Q_3 = 4
Max = 6



2

x	*f*	
47	2	47, 47
48	4	48, 48, 48, 48
49	1	49
50	3	50, 50, 50
51	0	
52	3	52, 52, 52
53	2	53, 53
Σf=	15	

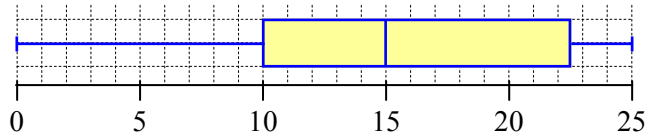
Min = 47
Q_1 = 48
Q_2 = 50
Q_3 = 52
Max = 53



3

Score	*f*	
0	2	0, 0
5	1	5
10	2	10, 10
15	5	15, 15, 15, 15, 15
20	3	20, 20, 20
25	4	25, 25, 25, 25
Σf=	17	

Min = 0
Q_1 = 10
Q_2 = 15
Q_3 = 22.5
Max = 25



4 From the 1st column pick which box & whisker plot is:

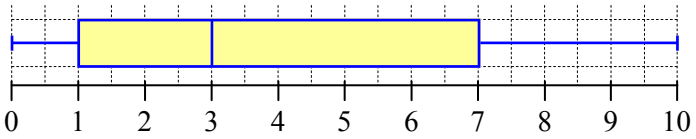
Positively skewed: Q1 Q2 Q3
Negatively skewed: Q1 Q2 Q3
Symmetric: Q1 Q2 Q3

5 Now find the IQR for the data as well.

x	*f*	
0	3	0, 0, 0
1	6	1, 1, 1, 1, 1, 1
2	2	2, 2
3	5	3, 3, 3, 3, 3
4	3	4, 4, 4
5	0	
6	2	6, 6
7	0	
8	1	8
9	3	9, 9, 9
10	3	10, 10, 10
Σf=	28	

Min = 0
Q_1 = 1
Q_2 = 3
Q_3 = 7
Max = 10

IQR = 7 - 1 = 6
Q_3 not Max Q_1 not Min



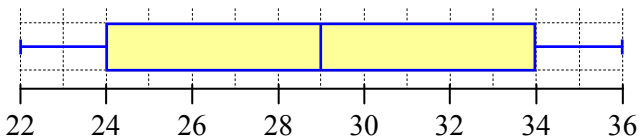
● Positive Skew ○ Negative Skew ○ Symmetric

6

Min = 22
Q_1 = 24
Q_2 = 29
Q_3 = 34
Max = 36

x	*f*	
22	4	22, 22, 22, 22
24	2	24, 24
26	3	26, 26, 26
28	2	28, 28
30	1	30
32	3	32, 32, 32
34	4	34, 34, 34, 34
36	3	36, 36, 36
Σf=	22	

IQR = 34 - 24 = 10



○ Positive Skew ○ Negative Skew ● Symmetric

Data Analysis - Box and Whisker Plots - Frequency Tables (Homework Strip)

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Date Due:

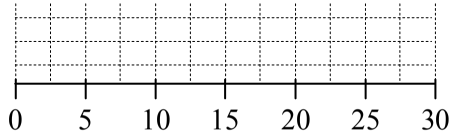
Find the 5 number summary & IQR. Construct a Box and Whisker plot, describe the distribution shape.

1

x	f	
0	3	
5	4	
10	3	
15	0	
20	2	
25	0	
30	3	
$\Sigma f=$		

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =



2 The distribution is:

- ◯ Negatively skewed
- ◯ Positively skewed
- ◯ Symmetric

Date Due:

Find the 5 number summary & IQR. Construct a Box and Whisker plot, describe the distribution shape.

1

x	f	
0	3	
5	4	
10	3	
15	0	
20	2	
25	0	
30	3	
$\Sigma f=$		

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

0 5 10 15 20 25 30

2 The distribution is:

- ◯ Negatively skewed
- ◯ Positively skewed
- ◯ Symmetric

Date Due:

Find the 5 number summary & IQR. Construct a Box and Whisker plot, describe the distribution shape.

1

x	f	
0	3	
5	4	
10	3	
15	0	
20	2	
25	0	
30	3	
$\Sigma f=$		

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

0 5 10 15 20 25 30

2 The distribution is:

- ◯ Negatively skewed
- ◯ Positively skewed
- ◯ Symmetric

Date Due:

Find the 5 number summary & IQR. Construct a Box and Whisker plot, describe the distribution shape.

1

x	f	
0	3	
5	4	
10	3	
15	0	
20	2	
25	0	
30	3	
$\Sigma f=$		

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 - Q_1 =

0 5 10 15 20 25 30

2 The distribution is:

- ◯ Negatively skewed
- ◯ Positively skewed
- ◯ Symmetric

Data Analysis - Box and Whisker Plots - Frequency Tables (H'work Strip)

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Date Due:

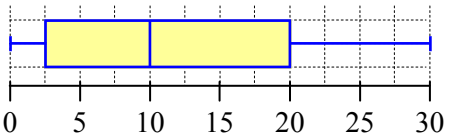
Find the 5 number summary & IQR. Construct a Box and Whisker plot, describe the distribution shape.

1

x	f	
0	3	0, 0, 0
5	4	5, 5, 5, 5
10	3	10, 10, 10
15	0	
20	2	20, 20
25	0	
30	3	30, 30, 30
$\Sigma f =$	15	

0	5	10	20	30
Min	Q_1	Q_2	Q_3	Max

IQR = 20 (Q_3) - 5 (Q_1) = 15

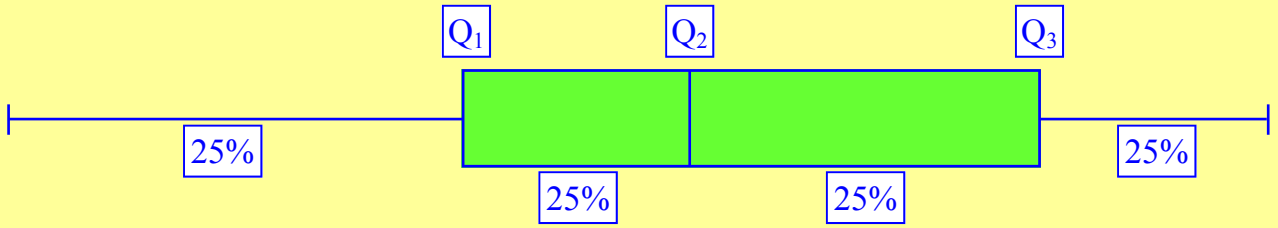


2 The distribution is:

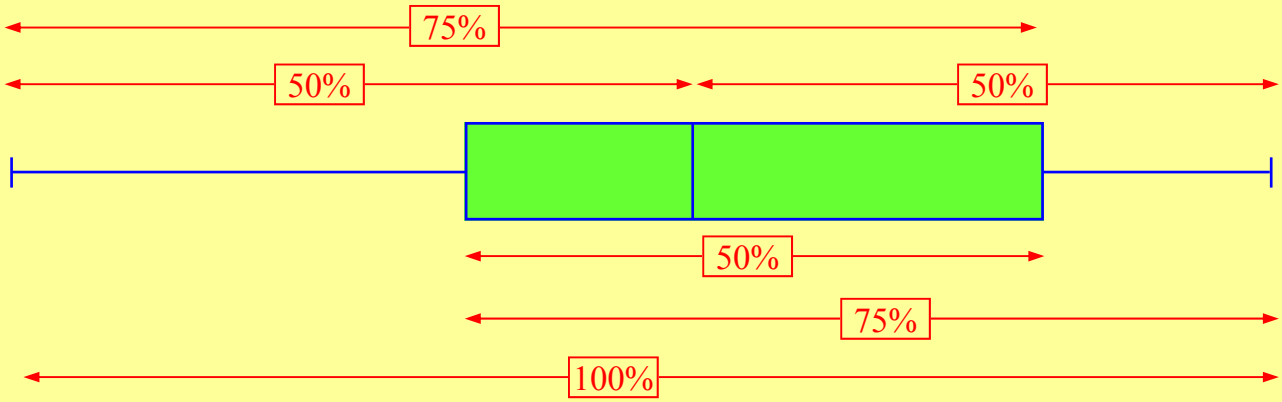
- ○ Negatively skewed
- ● Positively skewed
- ○ Symmetric

Data Analysis - Box and Whisker Applications

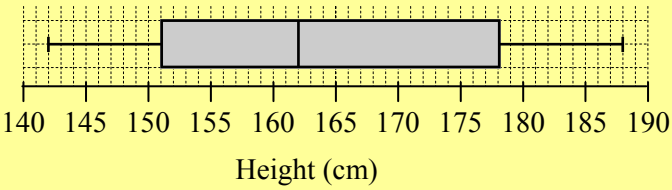
A box and whisker plot breaks data up into quarters, defined by the location of the quartiles.



So if 25% of scores lie between each quartile, you can find different percentages when you combine sections, here are some examples.



Example: The box and whisker plot is based on height measurements of 80 year 12 students.



Calculate the percentage of student with a height between:

a) 151 cm - 188 cm Answer 75%

Because 151 cm to 188 cm is three quarter, that means $3 \times 25\% = 75\%$

b) 162 cm - 178 cm Answer 25%

Because 162 cm to 178 cm is one quarter, that means = 25%

c) How many students have a height between 178 cm and 188 cm? Answer 20 students

Because 178 - 188 cm is one quarer, which is 25%. As there are 80 students:

$$\frac{25}{100} \times 80 = 20 \text{ students}$$

Data Analysis - Box and Whisker Applications

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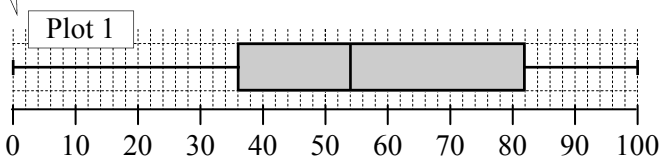
The box and whisker plot below displays results for a regional exam. Examine it & answer the questions.



1 The 1st 25% is between: 0 and ____

2 The 2nd 25% is between: ____ & ____

3 The 3rd 25% is between: ____ & ____

4 The 4th 25% is between: ____ & ____

5 The middle 50% is between: ____ & ____

6 The lower 50% is between: ____ & ____

7 The upper 50% is between: ____ & ____

What percentage of scores lie between:

8 54 and 82? ____ %

9 36 and 100? ____ % sign

10 0 and 54? ____

11 36 and 82? ____

If 86 000 people attempted the test, how many candidates achieved a mark between....

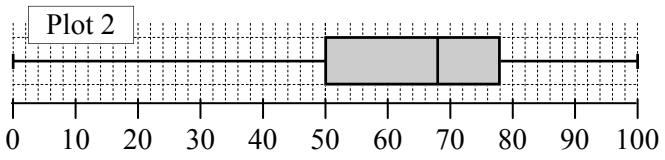
12 0 and 36? $\frac{25}{100} \times 86\,000 =$ ____ people

13 36 and 82? _____

14 0 and 82? _____

15 54 and 82? _____

The same test had the results below from another region.



What percentage of scores in Plot 2, lie between:

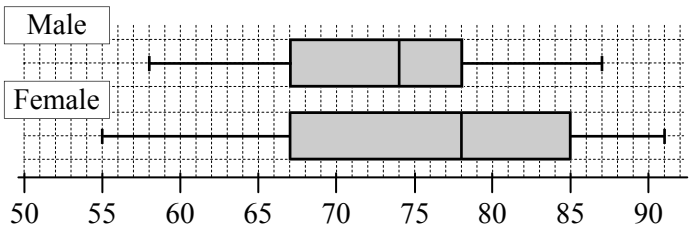
16 50 and 78? ____ %

17 68 and 100? ____

18 0 and 50? ____

19 50 and 100? ____

The ages of people living in a retirement complex is displayed in a double box and whisker plot below. The data has been sorted according to sex.



What percentage of male retirees are aged between:

20 58 and 67? ____ %

21 74 and 87? ____ % sign

What percentage of female retirees are aged between:

22 55 and 85? ____

23 67 and 78? ____

If there are 432 males and 584 females in the complex, calculate the number of ....

24 Females aged between 67 and 85?

$\frac{\square}{100} \times \square = \square$ females

25 Males aged between 67 and 87?

_____

26 Males aged between 67 and 78?

_____

27 Females aged between 67 and 78?

_____

28 Retirees aged between 67 and 78? Use Q26 & Q27

_____

Calculate how many males are older than 78. Then calculate the number of females over 78. How many more females are there than males?

29 _____

_____

_____

_____

Data Analysis - Box and Whisker Applications

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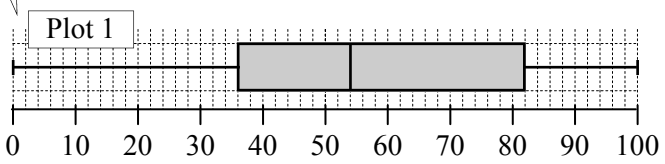
The box and whisker plot below displays results for a regional exam. Examine it & answer the questions.



1 The 1st 25% is between: 0 and **36**

2 The 2nd 25% is between: **36** & **54**

3 The 3rd 25% is between: **54** & **82**

4 The 4th 25% is between: **82** & **100**

5 The middle 50% is between: **36** & **82**

6 The lower 50% is between: **0** & **54**

7 The upper 50% is between: **54** & **100**

What percentage of scores lie between:

8 54 and 82? **25** %

9 36 and 100? **75%** % sign

10 0 and 54? **50%**

11 36 and 82? **50%**

If 86 000 people attempted the test, how many candidates achieved a mark between....

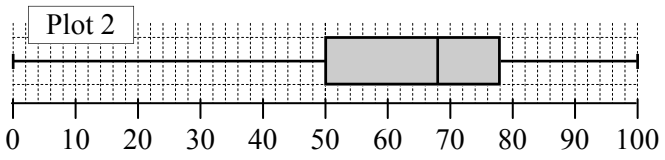
12 0 and 36? $\frac{25}{100} \times 86\,000 =$ **21 500** people

13 36 and 82? $\frac{50}{100} \times 86\,000 = 43\,000$ people

14 0 and 82? $\frac{75}{100} \times 86\,000 = 64\,500$ people

15 54 and 82? $\frac{25}{100} \times 86\,000 = 21\,500$ people

The same test had the results below from another region.



What percentage of scores in Plot 2, lie between:

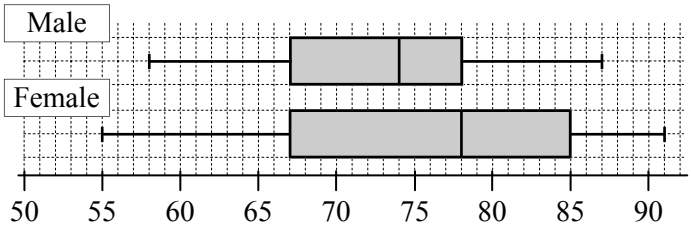
16 50 and 78? **50** %

17 68 and 100? **50%**

18 0 and 50? **25%**

19 50 and 100? **75%**

The ages of people living in a retirement complex is displayed in a double box and whisker plot below. The data has been sorted according to sex.



What percentage of male retirees are aged between:

20 58 and 67? **25** %

21 74 and 87? **50%** % sign

What percentage of female retirees are aged between:

22 55 and 85? **75%**

23 67 and 78? **25%**

If there are 432 males and 584 females in the complex, calculate the number of ....

24 Females aged between 67 and 85?

$\frac{50}{100} \times 584 =$ **292** females

25 Males aged between 67 and 87?

$\frac{75}{100} \times 432 = 324$ males

26 Males aged between 67 and 78?

$\frac{50}{100} \times 432 = 216$ males

27 Females aged between 67 and 78?

$\frac{25}{100} \times 584 = 146$ females

28 Retirees aged between 67 and 78? Use Q26 & Q27

$= 216 + 146 = 362$ retirees

Calculate how many males are older than 78. Then calculate the number of females over 78. How many more females are there than males?

29 $\frac{25}{100} \times 432 = 108$ males

$\frac{50}{100} \times 584 = 292$ females

Difference = 292 - 108 = 184

Data Analysis - Box and Whisker Plots - Applications (Homework Strip)

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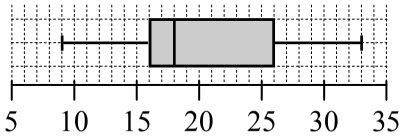
Date Due: _____

The box and whisker plot below is for ages of 348 people who uploaded a video on ewepipe.com.au



What % of people are aged between:

1. 16 and 18? _____ %
2. 9 and 26? _____
3. 16 and 26? _____

How many people were between:

4. 16 and 33?

$$\frac{\square}{100} \times 348 = \square \text{ people}$$

5. 18 and 26?

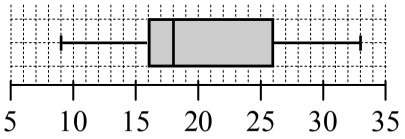
$$\frac{\square}{\square} \times \square = \square$$

6. 16 and 26?

$$\frac{\square}{\square} \times \square = \square$$

Date Due: _____

The box and whisker plot below is for ages of 348 people who uploaded a video on ewepipe.com.au



What % of people are aged between:

1. 16 and 18? _____ %
2. 9 and 26? _____
3. 16 and 26? _____

How many people were between:

4. 16 and 33?

$$\frac{\square}{100} \times 348 = \square \text{ people}$$

5. 18 and 26?

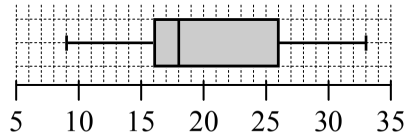
$$\frac{\square}{\square} \times \square = \square$$

6. 16 and 26?

$$\frac{\square}{\square} \times \square = \square$$

Date Due: _____

The box and whisker plot below is for ages of 348 people who uploaded a video on ewepipe.com.au



What % of people are aged between:

1. 16 and 18? _____ %
2. 9 and 26? _____
3. 16 and 26? _____

How many people were between:

4. 16 and 33?

$$\frac{\square}{100} \times 348 = \square \text{ people}$$

5. 18 and 26?

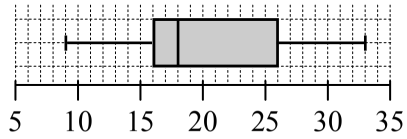
$$\frac{\square}{\square} \times \square = \square$$

6. 16 and 26?

$$\frac{\square}{\square} \times \square = \square$$

Date Due: _____

The box and whisker plot below is for ages of 348 people who uploaded a video on ewepipe.com.au



What % of people are aged between:

1. 16 and 18? _____ %
2. 9 and 26? _____
3. 16 and 26? _____

How many people were between:

4. 16 and 33?

$$\frac{\square}{100} \times 348 = \square \text{ people}$$

5. 18 and 26?

$$\frac{\square}{\square} \times \square = \square$$

6. 16 and 26?

$$\frac{\square}{\square} \times \square = \square$$

Data Analysis - Box and Whisker Plots - Applications (Homework Strip)

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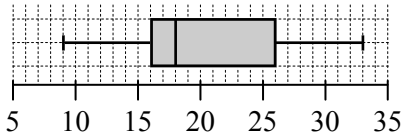
Date Due:

The box and whisker plot below is for ages of 348 people who uploaded a video on ewepipe.com.au



What % of people are aged between:

1 16 and 18? 25 %

2 9 and 26? 75%

3 16 and 26? 50%

How many people were between:

4 16 and 33?

$\frac{75}{100} \times 348 =$ 261 people

5 18 and 26?

$\frac{25}{100} \times 348 =$ 96 people

6 16 and 26?

$\frac{50}{100} \times 348 =$ 174 people

Data Analysis Extension Review

This is the review of the Data Analysis Extension Topic.

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Find Q_1, Q_2 and Q_3 for these data sets.

1 8, 13, 29, 30, 35, 43, 44.

2 35, 39, 46, 52, 58, 66, 78, 82.

3 56, 90, 22, 51, 79, 92, 63, 45, 67.

Find Q_1, Q_2 and Q_3 for these plots.

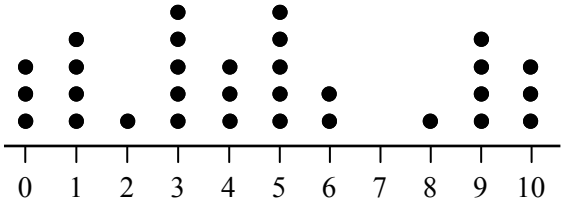
4

Tens	Units
0	2 4 4 5 7
1	3 3 6
2	0 4 6 6
3	8 9 9

5

AM	Tens	PM
2	8	2 7 7 8
8 8 3 0	9	1 5 8 9 9
7 5 5 5 2	10	3 3 3
8 6 1	11	6 7

6



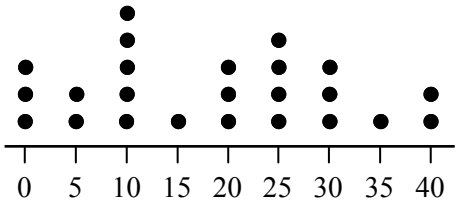
Find a 5-number summary for these. The lower extreme value (Min), Q_1, Q_2, Q_3 and Upper extreme value (Max).

7 16, 25, 44, 48, 52, 63.

8

Stem	Leaf
87	1 3 4 7
88	0 2 3 5 6
89	0 4
90	6 7 9
91	2 5 5 8

9

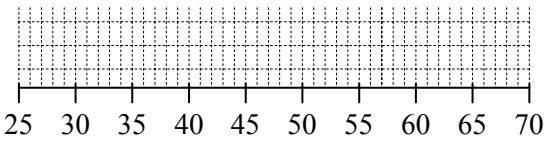


10

x	f
0	5
10	2
20	4
30	5
40	1
50	4
60	2
$\Sigma f =$	

Create a box and whisker plot using the 5-number summary values below.

11 Min = 27, Q_1 = 32, Q_2 = 43, Q_3 = 59, Max = 68.



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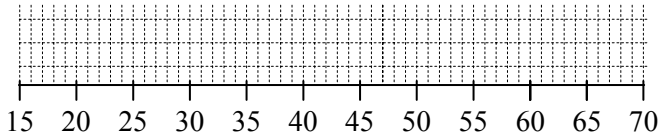
Find the 5-number summary and then display the results in a box and whisker plot. Describe the distribution shape.

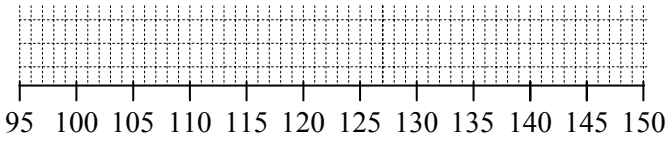
12 18, 24, 36, 41, 45, 54, 58, 68.



◯ Positively skewed ◯ Negative skewed ◯ Symmetric

13

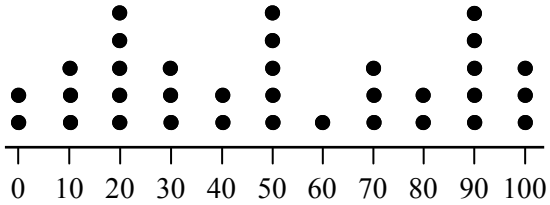
Stem	Leaf
9	6 7
10	2 6 8
11	0 1 2 2
12	4 6
13	7 7 9
14	1 2 2

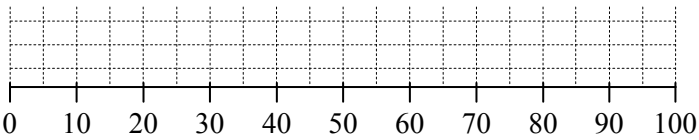


◯ Positively skewed ◯ Negative skewed ◯ Symmetric

Find the 5-number summary, display the results in a box and whisker plot. Then find the IQR.

14





IQR

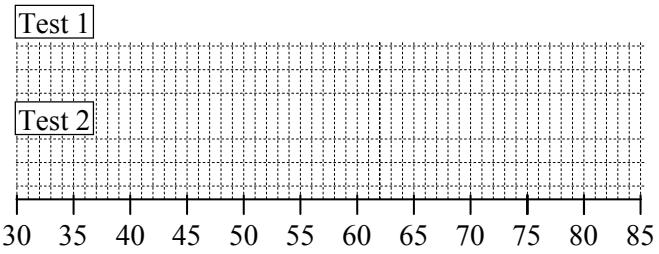
Find the 5-number summaries and display the results in parallel box and whisker plots. Then find the IQR.

16

Test 1	Tens	Test 2
4 1 1 0	3	9
8 5 5 5 2	4	4 4 9
7 3 1	5	2 6 7
9 8	6	0 0 6 7 7 8
8	7	2 3 5 9 9
	8	1 2 4

T_1:

T_2:



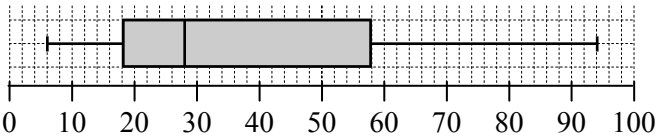
Test 1 IQR:

Test 2 IQR:

Compare the two box and whisker plots. Include comments about the distribution shape, median and IQR.

17

Use the box and whisker plot below to answer these.



What percentage of scores in the plot above, lie between:

18 28 and 94? _____ 19 6 and 58? _____

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Find Q_1, Q_2 and Q_3 for these data sets.

1 8, (13), 29, (30), 35, (43), 44.

$Q_1 = 13$, $Q_2 = 30$, $Q_3 = 43$.

2 35, 39, | 46, 52, | 58, 66, | 78, 82.

$Q_1 = 42.5$, $Q_2 = 55$, $Q_3 = 72$.

3 56, 90, 22, 51, 79, 92, 63, 45, 67.

22, 45, | 51, 56, (63) 67, 79, | 90, 92.

$Q_1 = 48$, $Q_2 = 63$, $Q_3 = 84.5$.

Find Q_1, Q_2 and Q_3 for these plots.

4

Tens	Units
0	2 4 4 (5) 7
1	3 3 (6)
2	0 4 6 (6)
3	8 9 9

$Q_1 = 5$

$Q_2 = 16$

$Q_3 = 26$

5

AM	Tens	PM
2	8	2 7 7 (8)
8 8 | 3 0	9	1 5 8 | 9 9
| 7 5 5 (5) 2	10	3 (3) 3
8 6 1	11	6 7

$Q_1 = 95.5$

$Q_2 = 105$

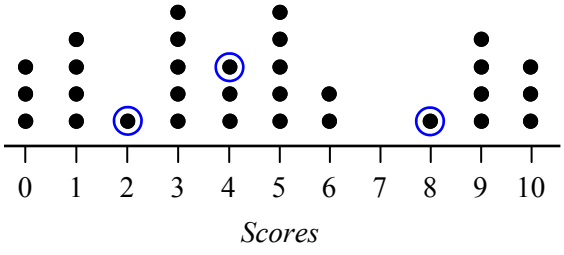
$Q_3 = 109$

$Q_1 = 88$

$Q_2 = 108.5$

$Q_3 = 103$

6



$Q_1 = 2$, $Q_2 = 4$, $Q_3 = 8$.

Find a 5-number summary for these. The lower extreme value (Min), Q_1, Q_2, Q_3 and Upper extreme value (Max).

7 16, (25), 44, | 48, (52), 63.

Min = 16, $Q_1 = 25$, $Q_2 = 46$, $Q_3 = 52$, Max = 63.

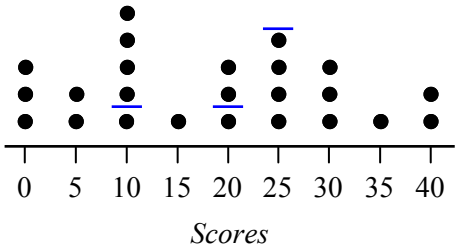
8

Stem	Leaf
87	1 3 4 7
88	(0) 2 3 5 6
89	(0) 4
90	6 7 9
91	(2) 5 5 8

Min = 871, $Q_1 = 880$, $Q_2 = 890$, $Q_3 = 912$,

Max = 918.

9



Min = 0, $Q_1 = 10$, $Q_2 = 20$, $Q_3 = 27.5$, Max = 40.

10

x	f	
0	5	0, 0, 0, 0, 0.
10	2	(10), 10.
20	4	20, 20, 20, 20.
30	5	(30), 30, 30, 30, 30.
40	1	40.
50	4	(50), 50, 50, 50.
60	2	60, 60.
$\Sigma f =$	23	

Min = 0

$Q_1 = 10$

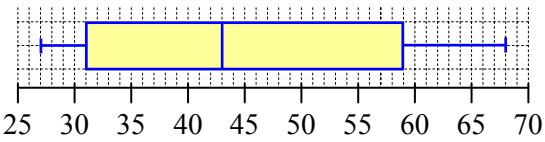
$Q_2 = 30$

$Q_3 = 50$

Max = 60

Create a box and whisker plot using the 5-number summary values below.

11 Min = 27, $Q_1 = 32$, $Q_2 = 43$, $Q_3 = 59$, Max = 68.



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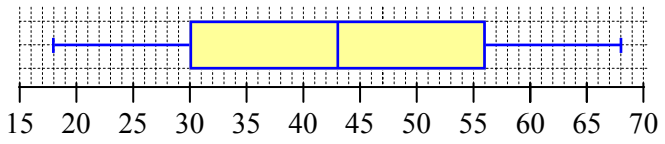
Find the 5-number summary and then display the results in a box and whisker plot. Describe the distribution shape.

12 18, 24, | 36, 41, | 45, 54, | 58, 68.

Min = 18, Q_1 = 30, Q_2 = 43, Q_3 = 56, Max = 68.



○ Positively skewed ○ Negatively skewed ● Symmetric

13

Stem	Leaf
9	6 7
10	2 6 | 8
11	0 1 2 (2)
12	4 6
13	7 7 | 9
14	1 2 2

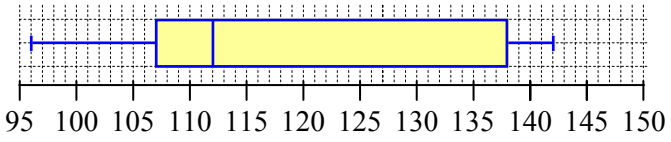
Min = 96

Q_1 = 107

Q_2 = 112

Q_3 = 138

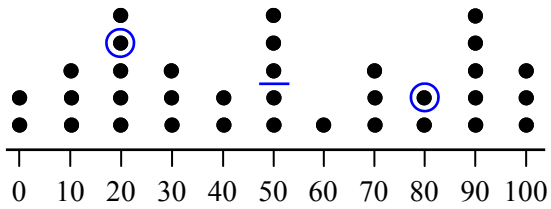
Max = 142.



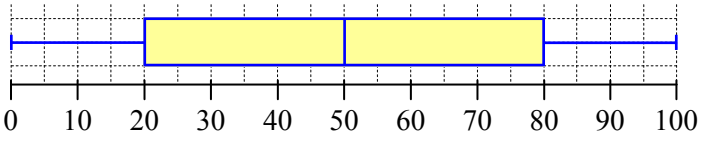
○ Positively skewed ● Negatively skewed ○ Symmetric

Find the 5-number summary, display the results in a box and whisker plot. Then find the IQR.

14



Min = 0, Q_1 = 20, Q_2 = 50, Q_3 = 80, Max = 100.



IQR = 80 - 20 = 60

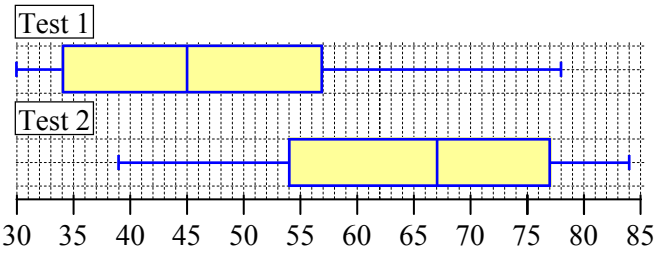
Find the 5-number summaries and display the results in parallel box and whisker plots. Then find the IQR.

16

Test 1	Tens	Test 2
(4) 1 1 0	3	9
8 (5) 5 5 2	4	4 4 9
(7) 3 1	5	2 | 6 7
9 8	6	0 0 6 (7) 7 8
8	7	2 3 5 | 9 9
	8	1 2 4

T_1: Min = 30, Q_1 = 34, Q_2 = 45, Q_3 = 57, Max = 78.

T_2: Min = 39, Q_1 = 54, Q_2 = 67, Q_3 = 77, Max = 84.



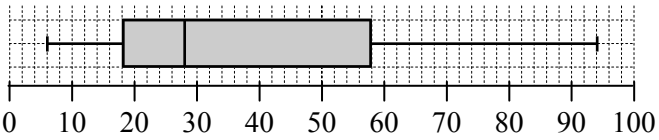
Test 1 IQR: = 57 - 34 = 23

Test 2 IQR: = 77 - 54 = 23

Compare the two box and whisker plots. Include comments about the distribution shape, median and IQR.

17 Test 1 is positively skewed, while Test 2 is negatively skewed. Both IQRs are 23, so the middle 50% was moved up in Test 2. The median in Test 2 is 67 compared with a median of 45 in Test 1. Test 2 has both higher lower Extreme and higher upper extreme values.

Use the box and whisker plot below to answer these.

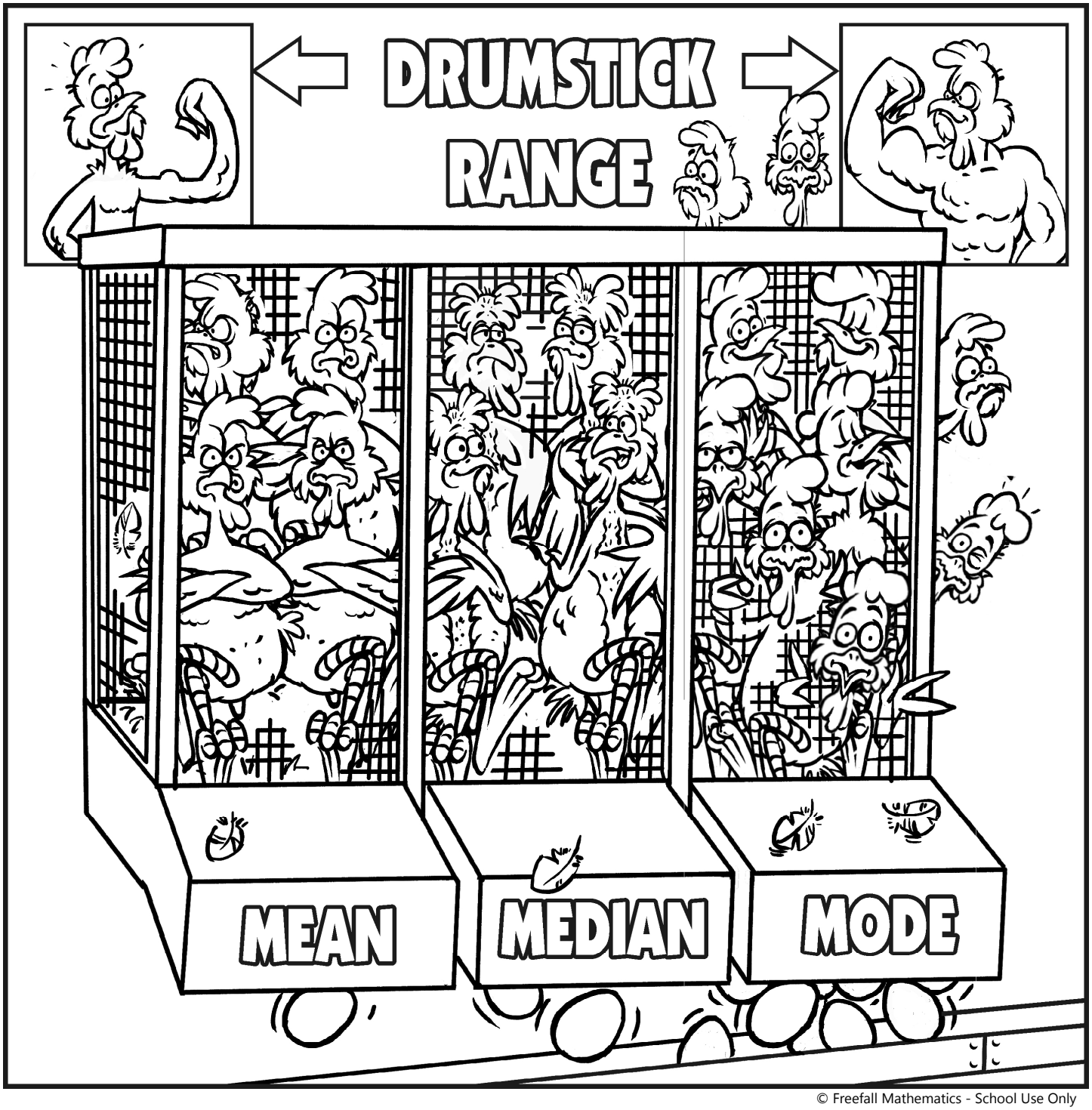


What percentage of scores in the plot above, lie between:

18 28 and 94? 50%

19 6 and 58? 75%

Data Analysis Extension



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Data Analysis - Introduction to Quartiles (Sheet 1)

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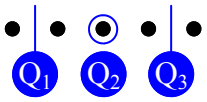
To divide a data set into quartiles, you need three numbers these are called Q_1, Q_2 and Q_3. We will start with a 5 score data set. Find Q_1, Q_2 and Q_3.



1 14, 20, 28, 36, 50.



$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

Calc: 14 + 20 = ÷ 2 = Calc: 36 + 50 = ÷ 2 =

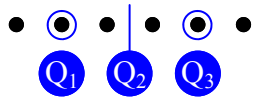
2 8, 13, 18, 23, 37.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

3 52, 64, 115, 144, 200.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

Now it's 6



4 5, 16, 25, 45, 45, 67.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

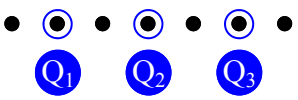
5 38, 48, 55, 75, 82, 93.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

6 185, 241, 386, 405, 412, 440.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

It's 7



7 49, 57, 81, 93, 102, 131, 150.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

8 9, 14, 16, 22, 22, 22, 31.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

Keep going with the same process, but this time apply the skill to stem and leaf plots and dot plots as well. We will kick it off with 8 scores.



9 11, 20, 24, 41, 46, 55, 60, 72.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

10 48, 61, 77, 77, 84, 90, 91, 95.

$Q_1 =$ ____ $Q_2 =$ ____ $Q_3 =$ ____

11

Tens	Units
0	4 4 | 8
1	3 | 5
2	5 | 7 7

$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

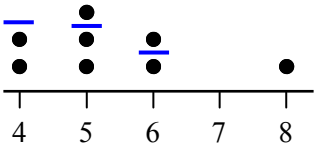
Remember 10's

12

Stem	Leaf
8	0 4
9	
10	2 4 6 6 8
11	5

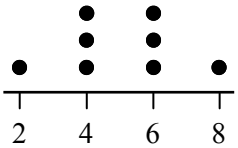
$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

13



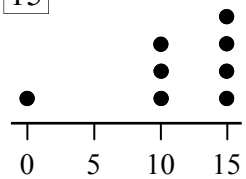
$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

14



$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

15



$Q_1 =$ ____
$Q_2 =$ ____
$Q_3 =$ ____

Data Analysis - Introduction to Quartiles (Sheet 2)

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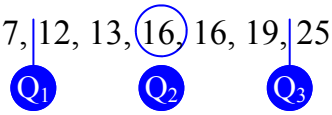
Find Q_1, Q_2 and Q_3. there are 9 scores.



1 1, 7, 12, 13, 16, 16, 19, 25, 28.



$Q_1 =$ $Q_2 =$ $Q_3 =$

2 19, 20, 42, 42, 57, 63, 66, 72, 77.

$Q_1 =$ $Q_2 =$ $Q_3 =$

3 88, 93, 99, 106, 108, 110, 110, 113, 119.

$Q_1 =$ $Q_2 =$ $Q_3 =$

4

Tens	Units
4	1
5	0 3 4 4 7
6	2 5 6

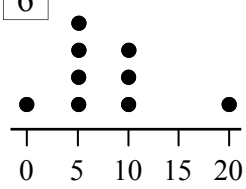
$Q_1 =$
$Q_2 =$
$Q_3 =$

5

Stem	Leaf
98	1 1 4
99	5 8
100	2 2 6
101	5

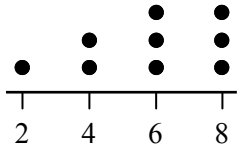
$Q_1 =$
$Q_2 =$
$Q_3 =$

6



$Q_1 =$
$Q_2 =$
$Q_3 =$

7



$Q_1 =$
$Q_2 =$
$Q_3 =$

Now it's 10



8 73, 83, 84, 105, 111, 118, 118, 133, 142, 170.

$Q_1 =$ $Q_2 =$ $Q_3 =$

9 3, 5, 5, 8, 12, 12, 12, 14, 17, 19.

$Q_1 =$ $Q_2 =$ $Q_3 =$

10 38, 46, 58, 62, 67, 73, 88, 88, 90, 94.

$Q_1 =$ $Q_2 =$ $Q_3 =$

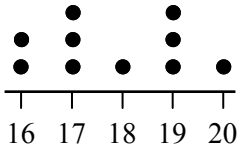
11

Off-Peak	Tens	Peak
8 8 7	7	8
6 6 4 1 1	8	3 8 8
0 0	9	1 2 2 5 5
	10	5

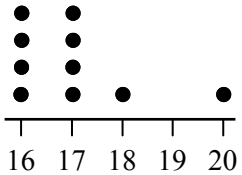
$Q_1 =$
$Q_2 =$
$Q_3 =$

$Q_1 =$
$Q_2 =$
$Q_3 =$

12



$Q_1 =$
$Q_2 =$
$Q_3 =$



$Q_1 =$
$Q_2 =$
$Q_3 =$

Data Analysis - Quartiles - Data Sets (Sheet 1)

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Find the median, Q_2, in these data sets.

Examples

14, 20, (28), 36, 50. $Q_2 =$ 28

Calc: 2 4 + 3 3 = ÷ 2 =

15, 18, 24, | 33, 38, 38. $Q_2 =$ 28.5

1 8, 13, 18, 23, 37. $Q_2 =$

2 1, 3, 4, 4, 6, 7, 8, 8. $Q_2 =$

3 29, 36, 44, 70. $Q_2 =$

4 44, 44, 52, 77, 93, 98. $Q_2 =$

Now you are given the median, Q_2, find Q_1 and Q_3 in these data sets.

Example

Q_1 location $= \frac{2+1}{2} = 1.5$th

1.5th means between 1st and 2nd

14, 20, (28), 36, 50.

2 scores 2 scores

14, | 20, (28), 36, | 50. $Q_1 =$ 17 $Q_3 =$ 43

5 8, 13, 18, | 23, 37, 42. Q_1 Q_3

6 11, 16, (27), 33, 33. Q_1 Q_3

7 36, 39, 47, 54, 63, 70, | 70, 74, 79, 83, 88, 89. Q_1 Q_3

8 5, 6, 8, 10, 13, 17, (18), 18, 19, 23, 23, 41, 55. Q_1 Q_3

Keep going with the same process, but this time record Q_1, Q_2 and Q_3. Q_2 is marked for you.

9 2, 2, 3, 4, 5, 5, 6, 6, | 7, 7, 7, 8, 8, 10, 10, 10. $Q_1 =$ $Q_2 =$ $Q_3 =$

10 Data: 24, 31, 34, 40, 40, 46, 49, 50, 51, 53, 55, 56, 58, 62, (64), 65, 68, 71, 77, 78, 82, 84, 87, 89, 92, 95, 95, 95, 97.

$Q_1 =$ $Q_2 =$ $Q_3 =$

Now do it all yourself. Find Q_2 first, then Q_1 and Q_3.

11 76, 83, 94, 104, 155, 176, 183. $Q_1 =$ $Q_2 =$ $Q_3 =$

12 1, 1, 1, 3, 3, 4, 5, 5, 7, 7, 7, 8, 8, 8, 9, 9, 10. $Q_1 =$ $Q_2 =$ $Q_3 =$

13 Data: 15, 17, 17, 21, 24, 26, 33, 36, 38, 48, 52, 63, 65, 68, 74, 75, 81, 85, 93, 94. $Q_1 =$ $Q_2 =$ $Q_3 =$

14 28, 36, 42, 55, 62, 68, 77, 78, 78, 85, 92. $Q_1 =$ $Q_2 =$ $Q_3 =$

Just the same, except this time the data has to be put into ascending order first. So rewrite the data on the line below, lowest to highest. Then find Q_2, Q_1 & Q_3.

15 17, 11, 9, 16, 22, 8, 13, 26, 16, 30, 15.

8,

There are 11 scores, make sure you have written 11 scores

$Q_1 =$ $Q_2 =$ $Q_3 =$

16 4, 9, 2, 4, 6, 11, 15, 8, 7, 6, 10, 3, 16.

$Q_1 =$ $Q_2 =$ $Q_3 =$

17 470, 233, 114, 330, 276, 195, 308, 222, 411.

$Q_1 =$ $Q_2 =$ $Q_3 =$

18 3, 2, 5, 7, 8, 2, 6, 10, 2, 8, 11, 0, 16, 14.

$Q_1 =$ $Q_2 =$ $Q_3 =$

Data Analysis - Quartiles - Stem & Leaf / Dot Plots (Sheet 2)

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Find Q_2 first, then Q_1 and Q_3 for these Stem and leaf plots. The first question has the three located for you.

1

Tens	Units
0	1 7 | 7
1	2 6
2	0 3 | 9
3	1

$Q_1 =$ $Q_2 =$ $Q_3 =$

2

Stem	Leaf
10	0 4 4
11	6 6 8 9 9
12	1 1 6 6 7

$Q_1 =$ $Q_2 =$ $Q_3 =$

3

Stem	Leaf
28	1 7
29	4
30	7 7
31	2 4 4 8
32	1 3 3 4 6
33	0 1 1

$Q_1 =$ $Q_2 =$ $Q_3 =$

4

Tens	Units
8	5 5 6 7 8
9	0 7 9
10	2 2 5 9
11	7 7

$Q_1 =$ $Q_2 =$ $Q_3 =$

5

Early	Tens	Late
7	12	1 6 6 8
8 5 4 4	13	0 4 8 8 8
6 3 3 0 0	14	5 9 9
7 4 1	15	5

Early

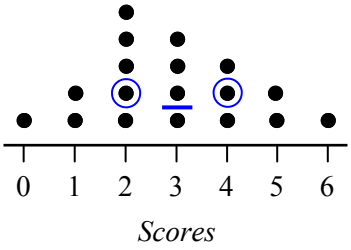
$Q_1 =$ $Q_2 =$ $Q_3 =$

Late

$Q_1 =$ $Q_2 =$ $Q_3 =$

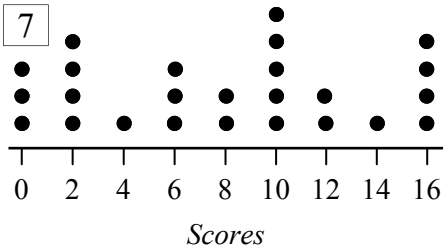
Find Q_2 first, then Q_1 and Q_3 for these dot plots.

6



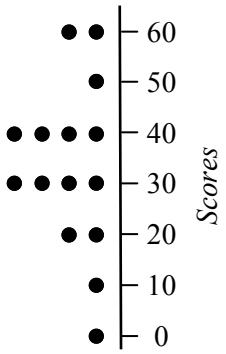
$Q_1 =$ $Q_2 =$ $Q_3 =$

7



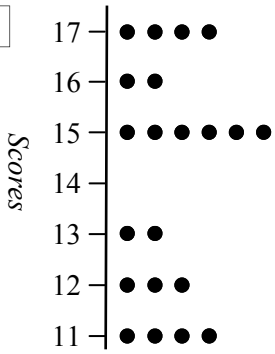
$Q_1 =$ $Q_2 =$ $Q_3 =$

8



$Q_1 =$ $Q_2 =$ $Q_3 =$

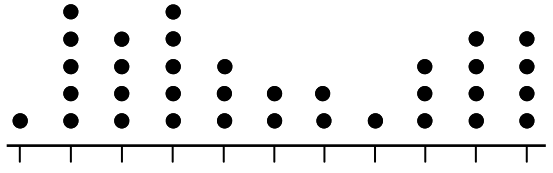
9



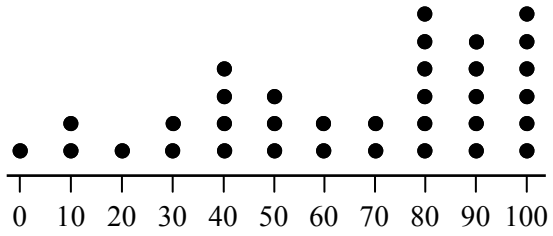
$Q_1 =$ $Q_2 =$ $Q_3 =$

10

Task 1



Task 2



Task 1

$Q_1 =$ $Q_2 =$ $Q_3 =$

Task 2

$Q_1 =$ $Q_2 =$ $Q_3 =$

Data Analysis - 5-Number Summary/IQR - Data Sets (Sheet 1)

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Find the 5-number summary for these data sets.

Example 12, 16, 19, 20, 28, 36, 44, 49, 50.

12, 16, 19, 20, 28, 36, 44, 49, 50.

Min 12 | Q_1 17.5 | Q_2 28 | Q_3 46.5 | Max 50

1 28, 34, 41, 55, 64, 64, 72, 77, 79, 85, 93.

Min | Q_1 | Q_2 | Q_3 | Max

2 3, 5, 7, 13, 18, 23, 27, 29, 36, 44, 52, 60, 67.

Min | Q_1 | Q_2 | Q_3 | Max

3 82, 106, 120, 127, 145, 145, 163, 178, 191.

Min | Q_1 | Q_2 | Q_3 | Max

4 4, 7, 13, 22, 26, 30, 32, 36, 38, 41, 48, 48.

Min | Q_1 | Q_2 | Q_3 | Max

Find the 5-number summary and calculate the IQR.
The IQR = Q_3 - Q_1.

5 15, 28, 34, 41, 56, 62, 73, 77, 81, 85.

Min | Q_1 | Q_2 | Q_3 | Max

IQR = Q_3 (Not Max) - Q_1 (Not Min) =

6 63, 66, 75, 81, 88, 90, 93, 97, 100.

7 0, 0, 3, 3, 4, 5, 5, 5, 6, 6, 7, 7, 8, 8, 8, 10, 10.

8 23, 28, 34, 39, 43, 47, 49, 54, 60, 67, 74, 81.

This time organise the data in ascending order, lowest to highest. Then find a 5-number summary and the IQR.

9 35, 11, 26, 45, 22, 36, 53, 38, 40, 19.

11,

There are 10 scores, make sure you have written 10 scores

Min | Q_1 | Q_2 | Q_3 | Max

IQR = (Not Max) - (Not Min) =

10 108, 117, 94, 103, 125, 107, 129, 105, 100.

Min | Q_1 | Q_2 | Q_3 | Max

IQR = - =

11 40, 27, 17, 33, 5, 21, 40, 38.

12 17, 11, 8, 15, 3, 14, 10, 12, 7, 19, 15, 16.

13 7, 8, 2, 5, 1, 6, 2, 6, 3, 9, 2, 4, 5, 0.

14 856, 955, 778, 915, 713, 798, 808, 930.

Data Analysis - 5-No. Summary/IQR - Stem & Leaf / Dot Plots (Sheet 2)

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Find a 5-number summary and the IQR for these.

1

Tens	Units
0	2 5 | 9
1	3 8
2	0 1 | 5
3	4

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

IQR = Q_3 ☐ - Q_1 ☐ = ☐
(Not Max, Not Min)

2

Stem	Leaf
7	1 2 4 4
8	1 3 3 4 7
9	2 5
10	0 0 6
11	1 4 6 6

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

IQR = ☐ - ☐ = ☐

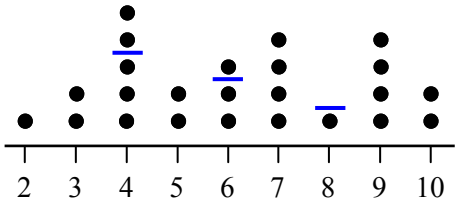
3

Male	Tens	Female
7 4 3 0	27	3 3
5 4 1 1 0	28	
8 8 8	29	1 1 2 4 5
9 4	30	0 1 1 3

Male	Female
Min = ☐	Min = ☐
Q_1 = ☐	Q_1 = ☐
Q_2 = ☐	Q_2 = ☐
Q_3 = ☐	Q_3 = ☐
Max = ☐	Max = ☐
IQR = ☐ - ☐	IQR = ☐ - ☐
IQR = ☐	IQR = ☐

Keep going, now it is dot plots.

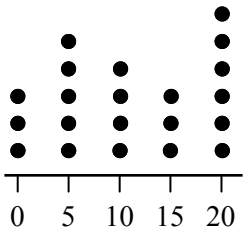
4



Min ☐ Q_1 ☐ Q_2 ☐ Q_3 ☐ Max ☐

IQR = Q_3 ☐ - Q_1 ☐ = ☐

5

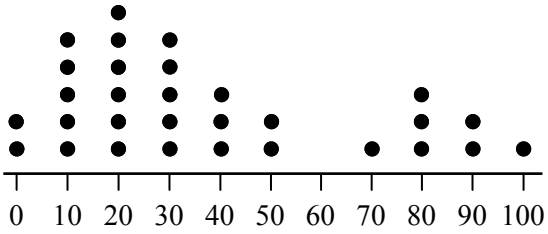


IQR = ☐ - ☐ = ☐

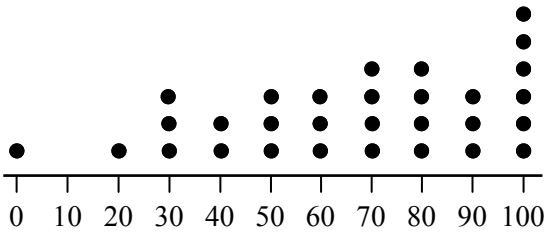
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

6

Sample A



Sample B



Sample A	Sample B
Min = ☐	Min = ☐
Q_1 = ☐	Q_1 = ☐
Q_2 = ☐	Q_2 = ☐
Q_3 = ☐	Q_3 = ☐
Max = ☐	Max = ☐
IQR = ☐ - ☐	IQR = ☐ - ☐
IQR = ☐	IQR = ☐

Data Analysis - 5-Number Summary/IQR - Frequency Tables

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With Frequency Tables find Σf, write the scores out beside the frequency column and locate Q_2, the median. Then locate Q_1 and Q_3.

1

Score	*f*	
5	2	5, 5
6	4	6, 6, | 6, 6
7	3	7, 7, ⑦
8	5	8, 8, 8, 8, | 8
9	2	9, 9
10	1	10
$\Sigma f=$	17	

$Q_1 =$ $Q_2 =$ $Q_3 =$

$\Sigma f = 17 \rightarrow$ Median $= \frac{17+1}{2} = 9^{th}$

2

$Q_1 =$ $Q_2 =$ $Q_3 =$

x	*f*	
1	2	
2	4	
3	1	
4	3	
5	3	
$\Sigma f=$		

3

x	*f*	
0	1	
5	5	
10	2	
15	4	
20	3	
25	1	
30	3	
$\Sigma f=$		

$Q_1 =$ $Q_2 =$ $Q_3 =$

4

$Q_1 =$ $Q_2 =$ $Q_3 =$

x	*f*	
0	3	
2	5	
4	3	
6	2	
8	4	
10	4	
12	1	
$\Sigma f=$		

Find the 5 number summary for these frequency tables and calculate the Interquartile range.

5

Min = $Q_1 =$ $Q_2 =$ $Q_3 =$ Max =

x	*f*	
48	2	
49	4	
50	1	
51	3	
52	3	
$\Sigma f=$		

IQR = (Q_3 not Max) - (Q_1 not Min) =

6

Min = $Q_1 =$ $Q_2 =$ $Q_3 =$ Max =

x	*f*	
97	5	
98	4	
99	5	
100	4	
101	2	
102	0	
103	1	
$\Sigma f=$		

IQR = - =

7

Min = $Q_1 =$ $Q_2 =$ $Q_3 =$ Max =

x	*f*	
0	1	
1	3	
2	1	
3	4	
4	5	
5	3	
6	4	
7	3	
8	4	
9	2	
10	1	
$\Sigma f=$		

IQR = - =

Data Analysis - Box and Whisker Plots - Data Sets (Sheet 1)

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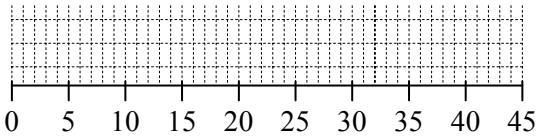
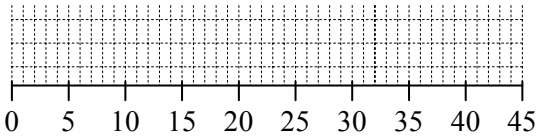
Create box and whisker plots for these 5-number summaries. Then comment on the distribution.

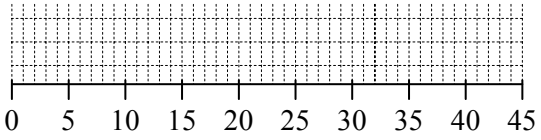
1 Min = 3, Q_1 = 26, Q_2 = 29, Q_3 = 34, Max = 41.



2 Min = 3, Q_1 = 15, Q_2 = 19, Q_3 = 23, Max = 35.

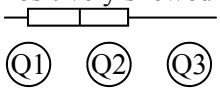


3 Min = 6, Q_1 = 12, Q_2 = 16, Q_3 = 19, Max = 42.

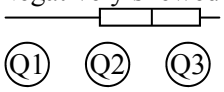


4 Pick which box and whisker plot is :

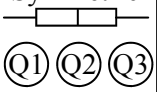
Positively skewed



Negatively skewed



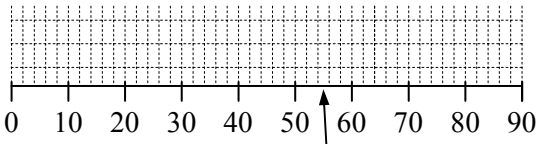
Symmetric



Now find the 5-number summaries for the data sets. Then plot them in a box and whisker plot.

5 4, 17, 25, 27, 38, 44, 55, 58, 76, 76, 84, 86, 88.

Min | Q_1 | Q_2 | Q_3 | Max



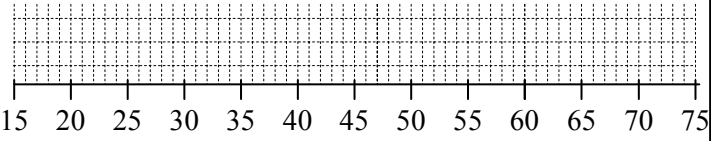
55 is between 54 and 56!

6 The above plot is: _____

Pos skew/neg skew/symmetric

7 16, 21, 35, 41, 47, 58, 62, 72.

Min | Q_1 | Q_2 | Q_3 | Max



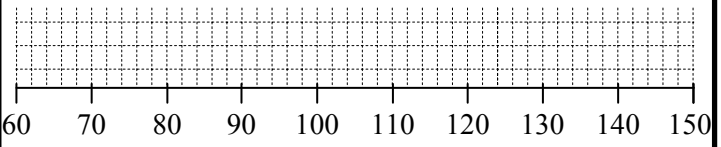
8 The above plot is: _____

Pos skew/neg skew/symmetric

Find the 5-number summaries, then plot them in a box and whisker plot. Finally, calculate the IQR.

9 62, 64, 68, 74, 86, 94, 107, 111, 126, 145.

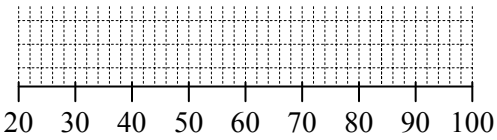
Min | Q_1 | Q_2 | Q_3 | Max



IQR = Q_3 (Not Max) - Q_1 (Not Min) = _____

10 24, 39, 49, 55, 63, 63, 67, 72, 78, 82, 86, 94.

Min | Q_1 | Q_2 | Q_3 | Max



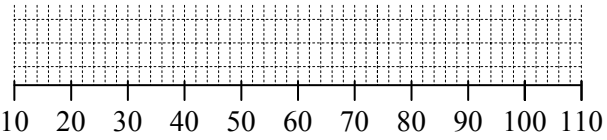
IQR = _____ - _____ = _____

Now rewrite the data in ascending order first.

11 72, 24, 66, 94, 11, 45, 52, 78, 60, 106, 18.

Count the scores to ensure you haven't missed any.

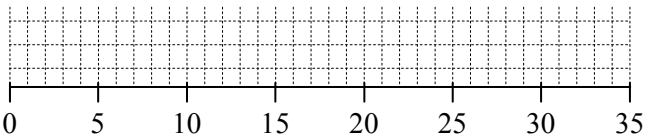
Min | Q_1 | Q_2 | Q_3 | Max



IQR = _____ - _____ = _____

12 14, 6, 21, 19, 2, 28, 11, 34, 30, 16, 25, 31, 7.

Min | Q_1 | Q_2 | Q_3 | Max



IQR = _____ - _____ = _____

Data Analysis - Box and Whisker Plots - Stem & Leaf Plots (Sheet 2)

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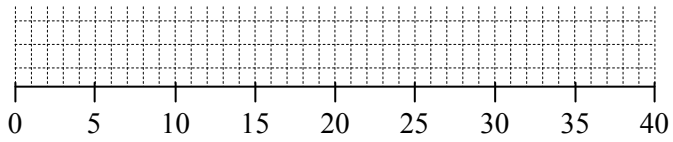
Create box and whisker plots for these stem and leaf plots. Then comment on the distribution.

1

Tens	Units
0	1 4 9 | 9 9
1	3 8 8 9
2	5 | 9
3	2 5

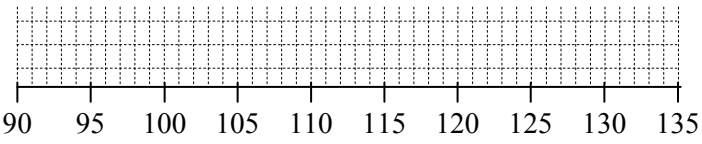
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



2

Stem	Leaf
9	1 8
10	2 6 8
11	4 7 8 9
12	1 8 8
13	0 2 2

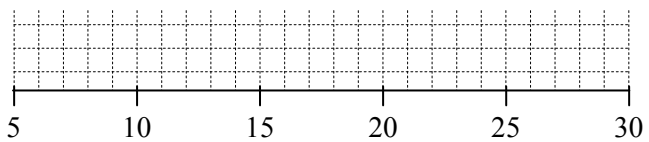
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



3

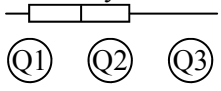
Tens	Units
0	8 8
1	1 1 2 4 4
2	0 0 1 1 4 6 9

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

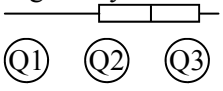


4 Pick which box and whisker plot is :

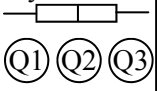
Positively skewed



Negatively skewed



Symmetric

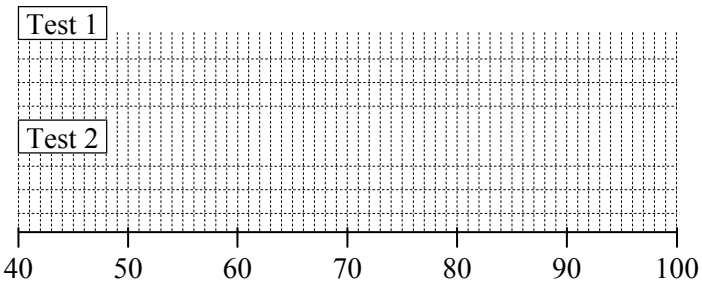


20 students sat Test 1, then the week after, Test 2. Construct B&W plots and calculate the IQR.

5

Test 1	Tens	Test 2
7 2	4	
6 3 0	5	2 3 8
6 6 5 4 4	6	3 3 3
8 2 2 0	7	1 1 2 2 6 8
9 7 6 6	8	3 4 4 6 8
2 2	9	5 6 6

Test 1	Test 2
Min = ☐	Min = ☐
Q_1 = ☐	Q_1 = ☐
Q_2 = ☐	Q_2 = ☐
Q_3 = ☐	Q_3 = ☐
Max = ☐	Max = ☐
IQR = Q_3 ☐ - Q_1 ☐	IQR = ☐ - ☐
IQR = ☐	IQR = ☐



6 Comment on the distributions. Include the shape Eg. skew/symmetric and compare the IQR's and the consistency of the results.

Data Analysis - Box and Whisker Plots - Dot Plots (Sheet 3)

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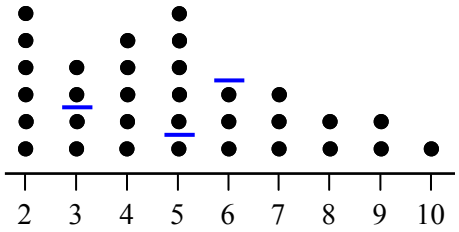
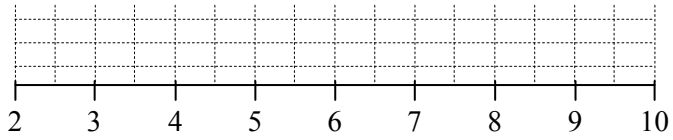
Create box and whisker plots for these dot plots. Then comment on the distribution.

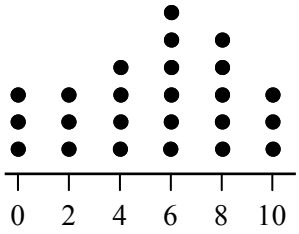
1



Min | Q_1 | Q_2 | Q_3 | Max



2



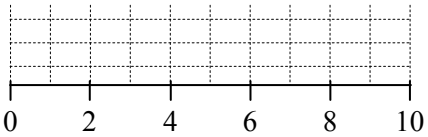
Min =

Q_1 =

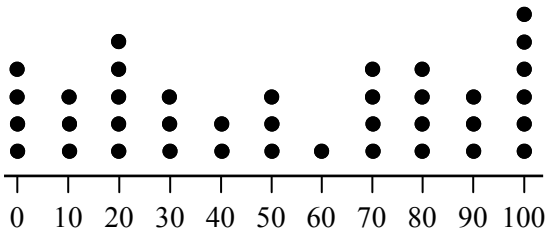
Q_2 =

Q_3 =

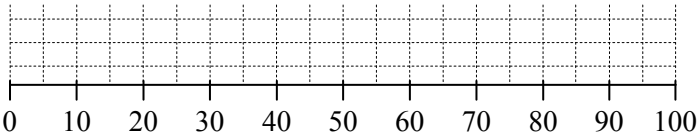
Max =



3

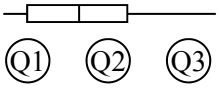


Min | Q_1 | Q_2 | Q_3 | Max

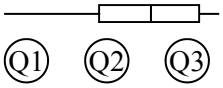


4 Pick which box and whisker plot is :

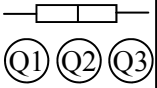
Positively skewed



Negatively skewed



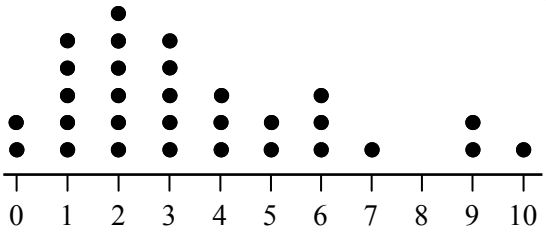
Symmetric



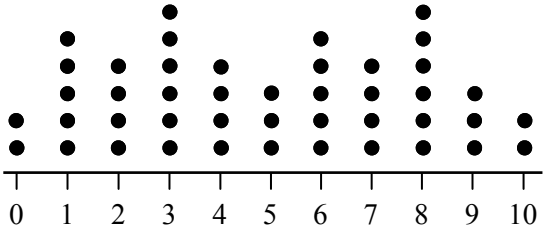
Create box and whisker plots for the dot plots below. Calculate the IQR and compare and contrast.

5

Sample A



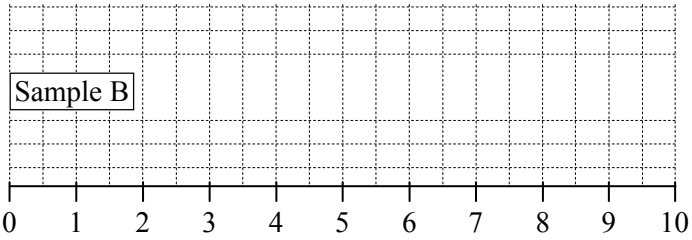
Sample B



Sample A	Sample B
Min =	Min =
Q_1 =	Q_1 =
Q_2 =	Q_2 =
Q_3 =	Q_3 =
Max =	Max =
IQR = ___ - ___	IQR = ___ - ___
IQR =	IQR =

Sample A

Sample B



6 Compare and contrast Sample A and Sample B. Use the distribution shape and IQR in your statement.

Data Analysis - Box and Whisker Plots - Frequency Tables

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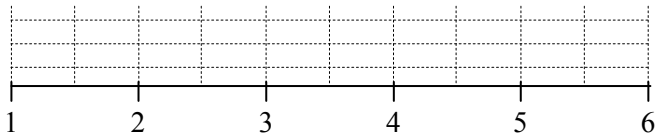
Find Σf, write the scores out beside the frequency column and locate Q_1, Q_2 and Q_3. Create a box and whisker plot. Describe the distribution.

1

Score	*f*	
1	3	1, 1, 1
2	4	2, 2, 2, 2
3	5	3, 3, 3, (3), 3
4	5	4, 4, 4, 4, 4
5	3	5, 5, 5
6	1	6
$\Sigma f =$	21	

$Q_2 = \frac{21+1}{2} = 11^{th}$

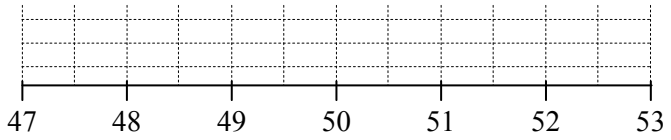
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



2

x	*f*	
47	2	
48	4	
49	1	
50	3	
51	0	
52	3	
53	2	
$\Sigma f =$		

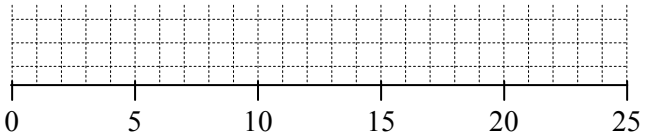
Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



3

Score	*f*	
0	2	
5	1	
10	2	
15	5	
20	3	
25	4	
$\Sigma f =$		

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐



4 From the 1st column pick which box & whisker plot is:

Positively skewed ☐ Q1 Q2 Q3

Negatively skewed ☐ Q1 Q2 Q3

Symmetric ☐ Q1 Q2 Q3

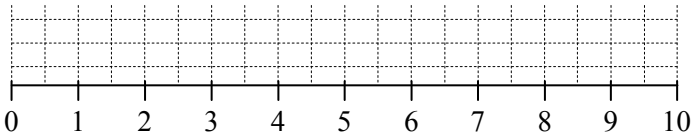
5 Now find the IQR for the data as well.

x	*f*	
0	3	
1	6	
2	2	
3	5	
4	3	
5	0	
6	2	
7	0	
8	1	
9	3	
10	3	
$\Sigma f =$		

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

IQR = ☐ - ☐ = ☐

(Q_3 not Max) (Q_1 not Min)



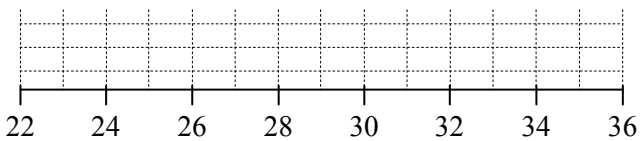
○ Positive Skew ○ Negative Skew ○ Symmetric

6

x	*f*	
22	4	
24	2	
26	3	
28	2	
30	1	
32	3	
34	4	
36	3	
$\Sigma f =$		

Min = ☐
Q_1 = ☐
Q_2 = ☐
Q_3 = ☐
Max = ☐

IQR = ☐ - ☐ = ☐



○ Positive Skew ○ Negative Skew ○ Symmetric

Data Analysis - Box and Whisker Applications

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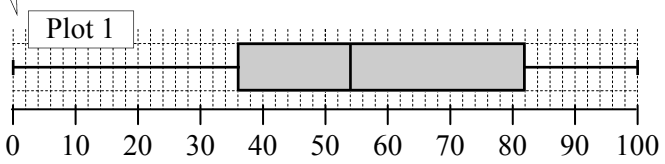
The box and whisker plot below displays results for a regional exam. Examine it & answer the questions.



1 The 1st 25% is between: 0 and ____

2 The 2nd 25% is between: ____ & ____

3 The 3rd 25% is between: ____ & ____

4 The 4th 25% is between: ____ & ____

5 The middle 50% is between: ____ & ____

6 The lower 50% is between: ____ & ____

7 The upper 50% is between: ____ & ____

What percentage of scores lie between:

8 54 and 82? ____ %

9 36 and 100? ____ % sign

10 0 and 54? ____

11 36 and 82? ____

If 86 000 people attempted the test, how many candidates achieved a mark between....

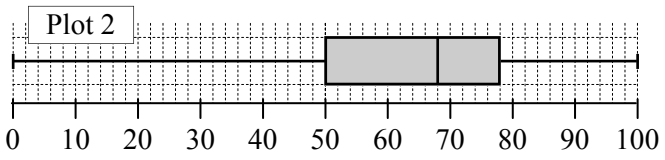
12 0 and 36? $\frac{25}{100} \times 86\,000 =$ ____ people

13 36 and 82? _____

14 0 and 82? _____

15 54 and 82? _____

The same test had the results below from another region.



What percentage of scores in Plot 2, lie between:

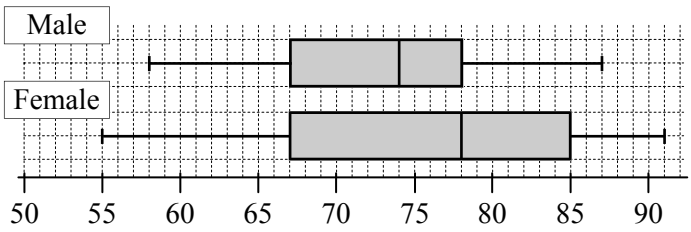
16 50 and 78? ____ %

17 68 and 100? ____

18 0 and 50? ____

19 50 and 100? ____

The ages of people living in a retirement complex is displayed in a double box and whisker plot below. The data has been sorted according to sex.



What percentage of male retirees are aged between:

20 58 and 67? ____ %

21 74 and 87? ____ % sign

What percentage of female retirees are aged between:

22 55 and 85? ____

23 67 and 78? ____

If there are 432 males and 584 females in the complex, calculate the number of ....

24 Females aged between 67 and 85?

$\frac{\square}{100} \times \square = \square$ females

25 Males aged between 67 and 87?

_____

26 Males aged between 67 and 78?

_____

27 Females aged between 67 and 78?

_____

28 Retirees aged between 67 and 78? Use Q26 & Q27

_____

Calculate how many males are older than 78. Then calculate the number of females over 78. How many more females are there than males?

29 _____

_____

_____

_____

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Find Q_1, Q_2 and Q_3 for these data sets.

1 8, 13, 29, 30, 35, 43, 44.

2 35, 39, 46, 52, 58, 66, 78, 82.

3 56, 90, 22, 51, 79, 92, 63, 45, 67.

Find Q_1, Q_2 and Q_3 for these plots.

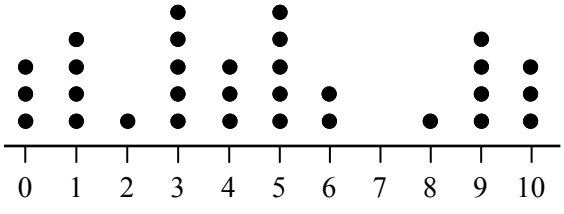
4

Tens	Units
0	2 4 4 5 7
1	3 3 6
2	0 4 6 6
3	8 9 9

5

AM	Tens	PM
2	8	2 7 7 8
8 8 3 0	9	1 5 8 9 9
7 5 5 5 2	10	3 3 3
8 6 1	11	6 7

6



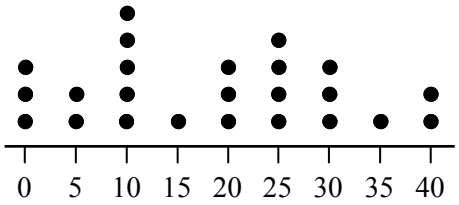
Find a 5-number summary for these. The lower extreme value (Min), Q_1, Q_2, Q_3 and Upper extreme value (Max).

7 16, 25, 44, 48, 52, 63.

8

Stem	Leaf
87	1 3 4 7
88	0 2 3 5 6
89	0 4
90	6 7 9
91	2 5 5 8

9

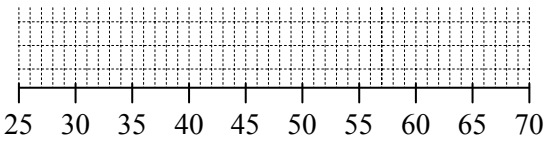


10

x	f
0	5
10	2
20	4
30	5
40	1
50	4
60	2
$\Sigma f =$	

Create a box and whisker plot using the 5-number summary values below.

11 Min = 27, Q_1 = 32, Q_2 = 43, Q_3 = 59, Max = 68.



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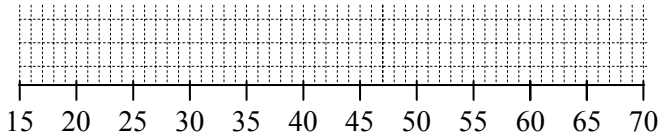
Find the 5-number summary and then display the results in a box and whisker plot. Describe the distribution shape.

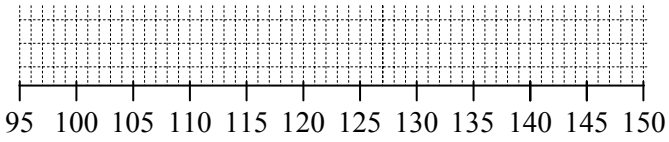
12 18, 24, 36, 41, 45, 54, 58, 68.



◯ Positively skewed ◯ Negative skewed ◯ Symmetric

13

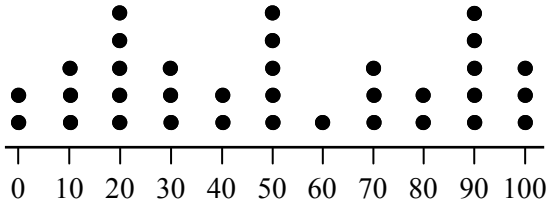
Stem	Leaf
9	6 7
10	2 6 8
11	0 1 2 2
12	4 6
13	7 7 9
14	1 2 2

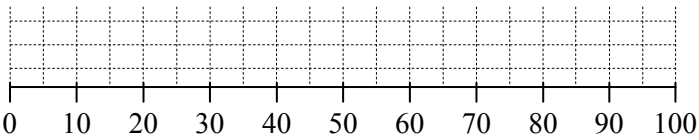


◯ Positively skewed ◯ Negative skewed ◯ Symmetric

Find the 5-number summary, display the results in a box and whisker plot. Then find the IQR.

14





IQR

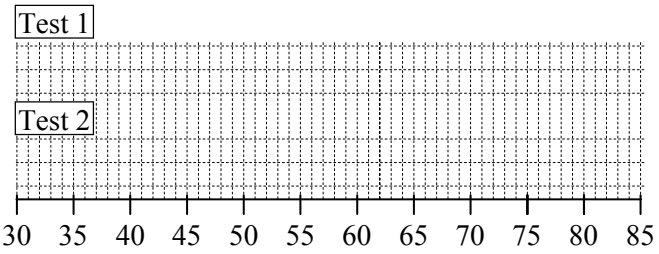
Find the 5-number summaries and display the results in parallel box and whisker plots. Then find the IQR.

16

Test 1	Tens	Test 2
4 1 1 0	3	9
8 5 5 5 2	4	4 4 9
7 3 1	5	2 6 7
9 8	6	0 0 6 7 7 8
8	7	2 3 5 9 9
	8	1 2 4

T_1:

T_2:



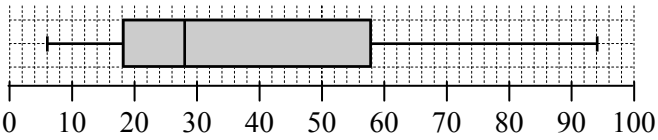
Test 1 IQR:

Test 2 IQR:

Compare the two box and whisker plots. Include comments about the distribution shape, median and IQR.

17

Use the box and whisker plot below to answer these.



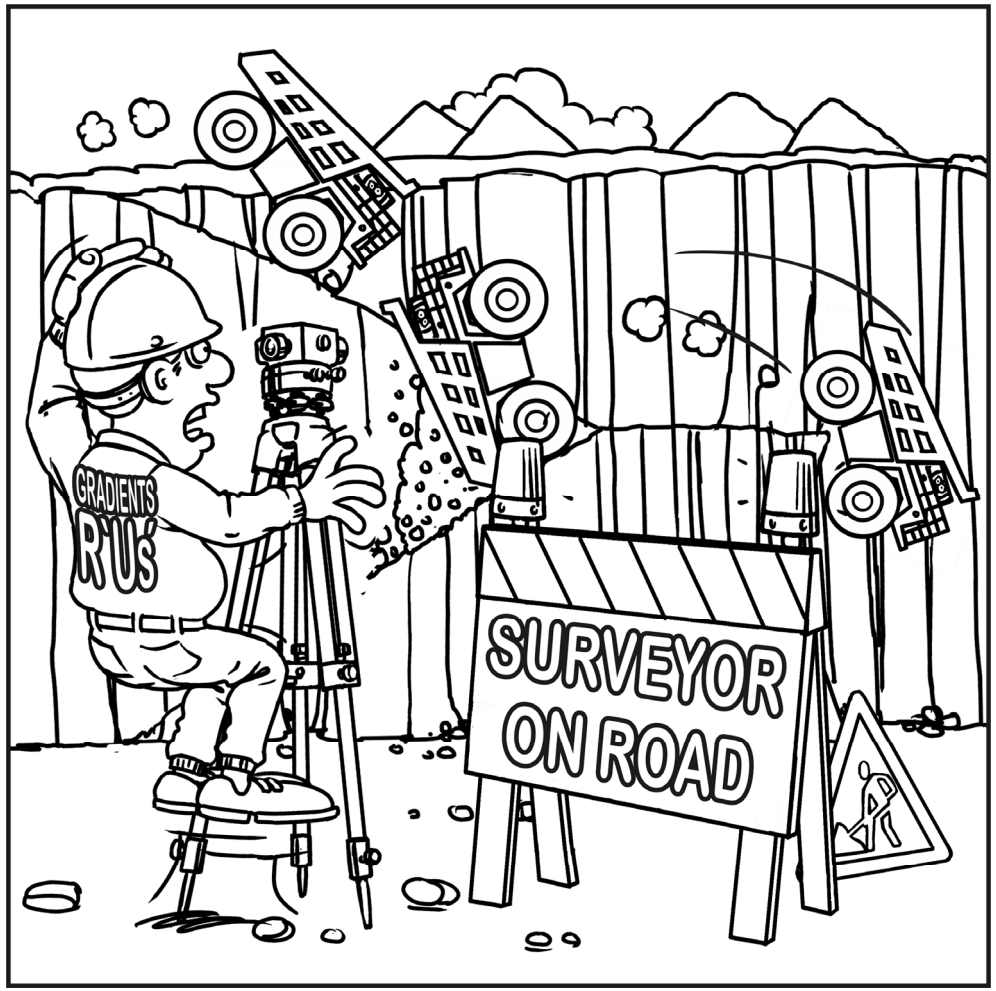
What percentage of scores in the plot above, lie between:

18 28 and 94? _____ 19 6 and 58? _____

VELOCITY

THE CARTESIAN PLANE

The Cartesian Plane



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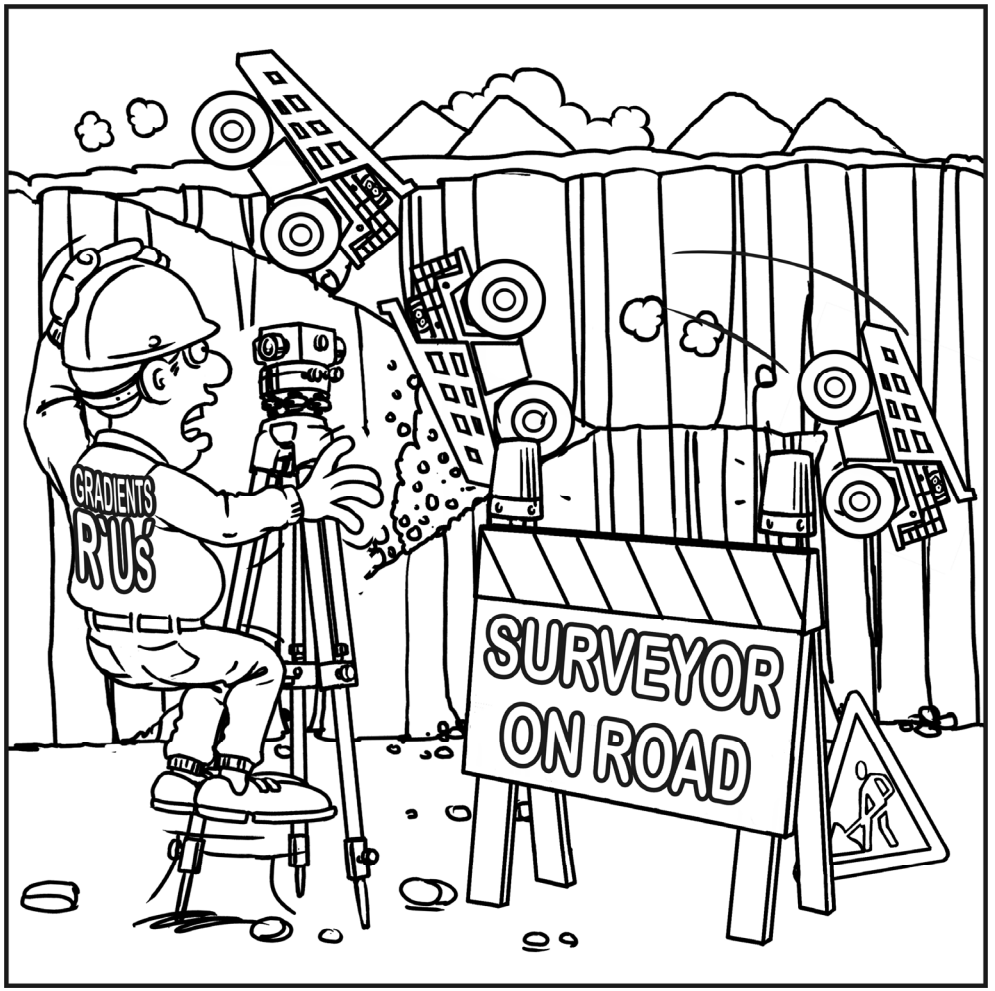
The Cartesian Plane



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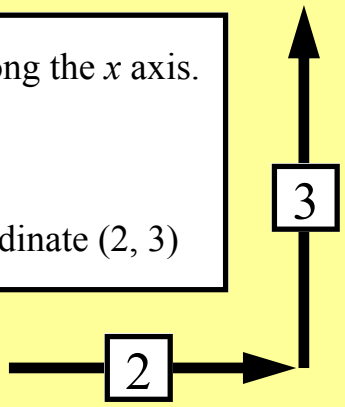
Cartesian Plane - Plotting Points - Positive Quadrant

This sheet involves the plotting of Cartesian co-ordinates. When plotting points on a number plane you are given 2 numbers, referred to as an ordered pair. The first number tells you how much you move across and the second number tells you how much you move up or down. This sheet only moves across to the right and up, negative numbers (not on this sheet) move to the left and down.

Move across first along the *x* axis.

Then move up.

Eg. To plot the co-ordinate (2, 3)



Note that the plane consists of an *x*-axis (horizontal or across) and a *y*-axis which is vertical (up and down). The plural of axis is axes (pronounced ak-seas). The co-ordinate (0, 0) is commonly referred to as the 'origin', the place where measurement starts from. Teachers often ask for the co-ordinates of the origin.

The first column asks you to locate the points and write in their co-ordinates. This is done by opening a bracket writing the *x* co-ordinate, writing a comma, then the *y* co-ordinate and close bracket. You should always place the co-ordinates in brackets.

The second column asks you to plot points. Once the group of points are plotted draw a line between points (in alphabetical order) to form a shape. Then write in the name of the shape plotted in the space provided.

The second sheet uses your skills in plotting points from the previous sheet. The series of points makes a pair of animals when lines are joined between the points. The way you attempt this exercise is, for the first animal:

- Plot one point at a time and then draw a line between it and the previous point. Don't try to plot all the points and then draw the lines afterwards as the drawing is too complicated.
- When you reach stop, restart at the new position and start again, DON'T JOIN THE POINTS BETWEEN WHERE YOU STOPPED AND WHERE YOU RESTARTED.

For the next animal follow the same process except use 2 different colours.

Cartesian Plane - Plotting Points - Positive Quadrant (Sheet 1)

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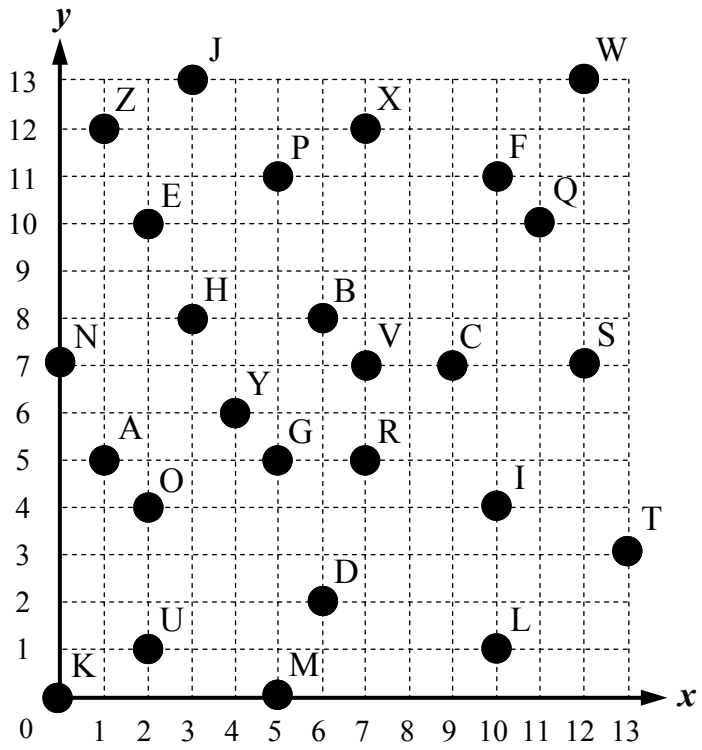
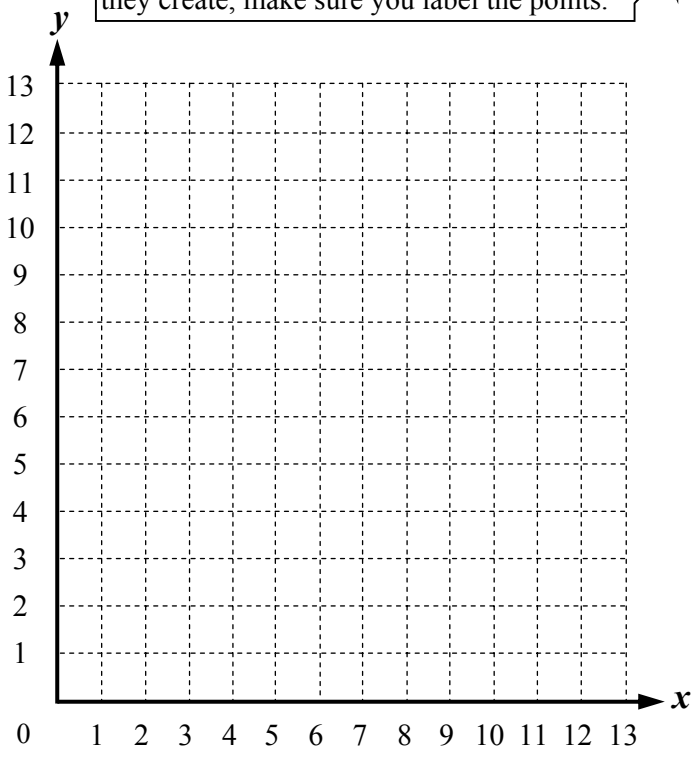
Give the co-ordinates for the following points, place them in brackets separated by a comma



1	Point A (,)	14	Point N
2	Point B (,)	15	Point O
3	Point C	16	Point P
4	Point D	17	Point Q
5	Point E	18	Point R
6	Point F	19	Point S
7	Point G	20	Point T
8	Point H	21	Point U
9	Point I	22	Point V
10	Point J	23	Point W
11	Point K	24	Point X
12	Point L	25	Point Y
13	Point M	26	Point Z

Place the groups of points given below on the grid, join the points and name the shape they create, make sure you label the points.



27 Point A (10, 4) Point B (11, 2)
Point C (12, 4) Point D (11, 5)
Shape formed: _____

28 Point E (3, 5) Point F (0, 8)
Point G (5, 13) Point H (8, 10)
Shape formed: _____

29 Point J (5, 4) Point K (9, 6)
Point L (7, 8)
Shape formed: _____
Two word description

30 Point M (2, 3) Point N (8, 3)
Point O (6, 0) Point P (0, 0)
Shape formed: _____

31 Point R (10, 9) Point S (13, 13)
Point T (13, 9)
Shape formed: _____
Two or three word description

Cartesian Plane - Drawing Animals - Positive Quadrant (Sheet 2)

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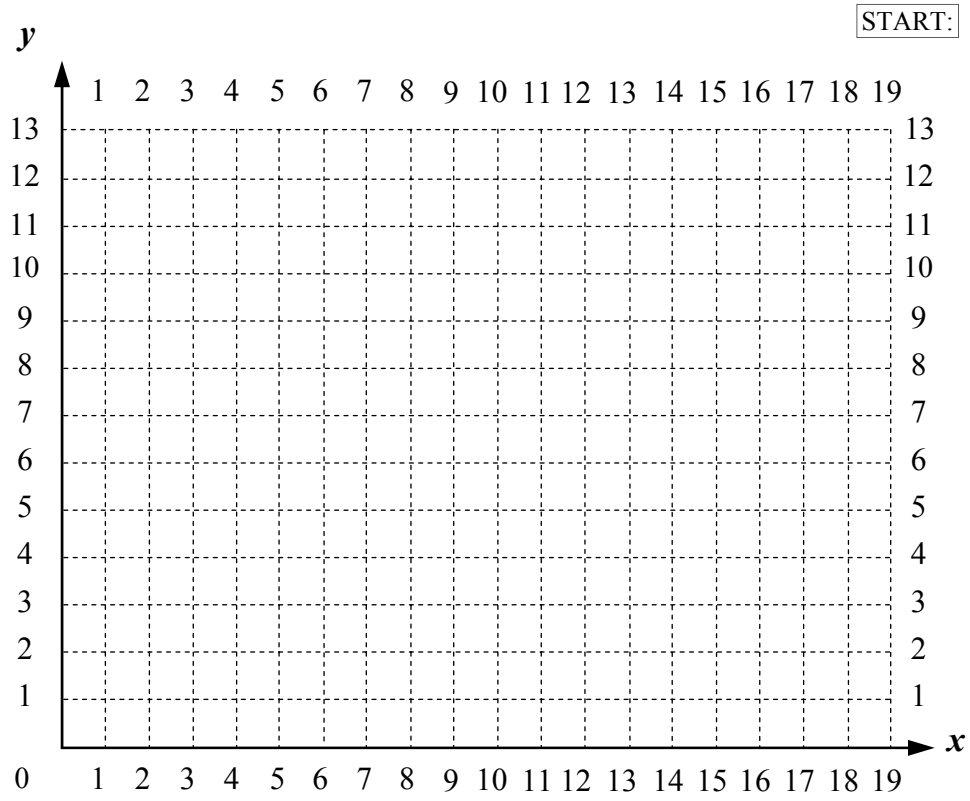
Join the points on the grid with the following co-ordinates and you will find an animal friend who has only one hand.



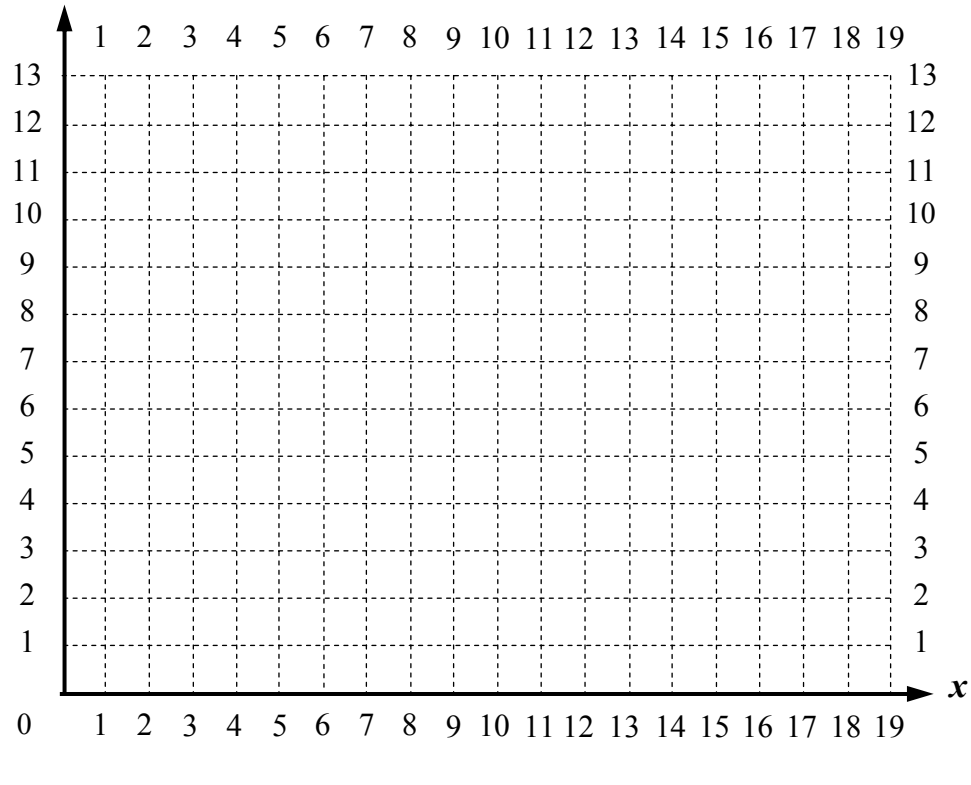
Co-ordinates:

START: (2, 12) , (3, 12) , (4, 11) , (5, 12) ,
(7, 12) , (8, 11) , (9, 12) , (10, 12) ,
(11, 11) , (11, 6) , (10, 3) , (9, 3) ,
(8, 7) , (8, 5) , (7, 7) , (7, 3) ,
(6, 2) , (4, 2) , (4, 3) , (5, 3) ,
(6, 4) , (5, 7) , (4, 5) , (4, 7) ,
(3, 3) , (2, 3) , (1, 6) , (1, 11) ,
(2, 12) STOP!

START:
(11, 10) , (12, 11) , (16, 12) , (18, 10) ,
(19, 7) , (18, 9) , (18, 3) , (15, 3) ,
(16, 4) , (16, 6) , (14, 5) , (11, 5) ,
(11, 4) , (13, 4) , (13, 1) , (12, 2) ,
(9, 2) , (8, 3) , (7, 6) , STOP!



Join the points and this time you will find a friend with which you will have to be very patient.



Co-ordinates:

START: (1, 11) , (2, 10) , (2, 9) , (1, 8) ,
(1, 7) , (2, 6) , (2, 5) , (3, 3) ,
(5, 1) , (7, 0) , (13, 0) , (16, 3) ,
(17, 5) , (15, 3) , (12, 2) , (8, 2) ,
(6, 3) , (4, 5) , (3, 6) , (4, 7) ,
(4, 8) , (3, 9) , (3, 10) , (4, 11) ,
STOP!

START:
(11, 6) , (12, 6) , (11, 7) , (10, 6) ,
(11, 5) , (12, 5) , (14, 6) , (14, 7) ,
(12, 9) , (10, 9) , (8, 7) , (8, 5) ,
(10, 3) , (12, 3) , (14, 4) , (15, 6) ,
(15, 7) , (14, 9) , (13, 10) , (10, 10) ,
(8, 9) , (7, 7) , (8, 3) , (11, 2) ,
(12, 2) , (15, 3) , (17, 6) , (17, 7) ,
(15, 11) , (12, 12) , (10, 12) , (7, 11) ,
(5, 9) , (4, 6) , (4, 5) , STOP!

Cartesian Plane - Plotting Points - Positive Quadrant (Sheet 1)

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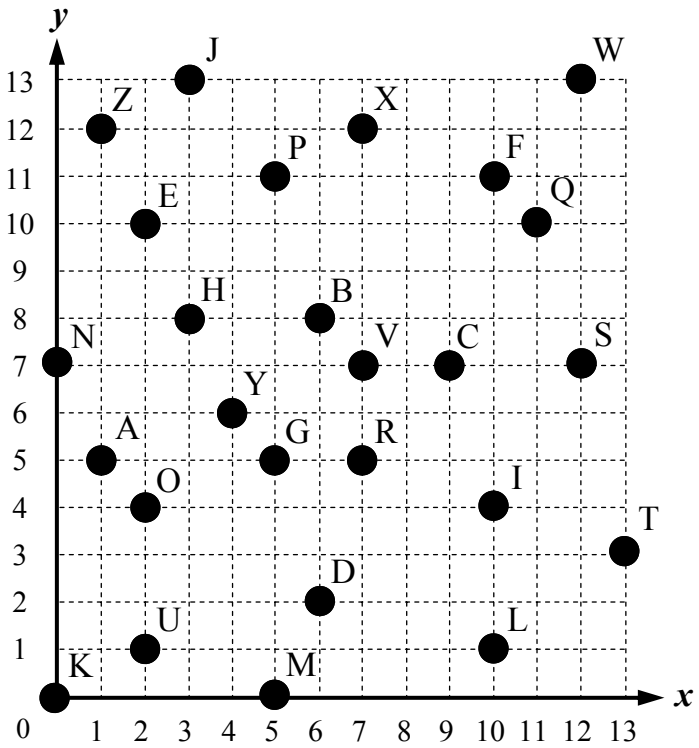
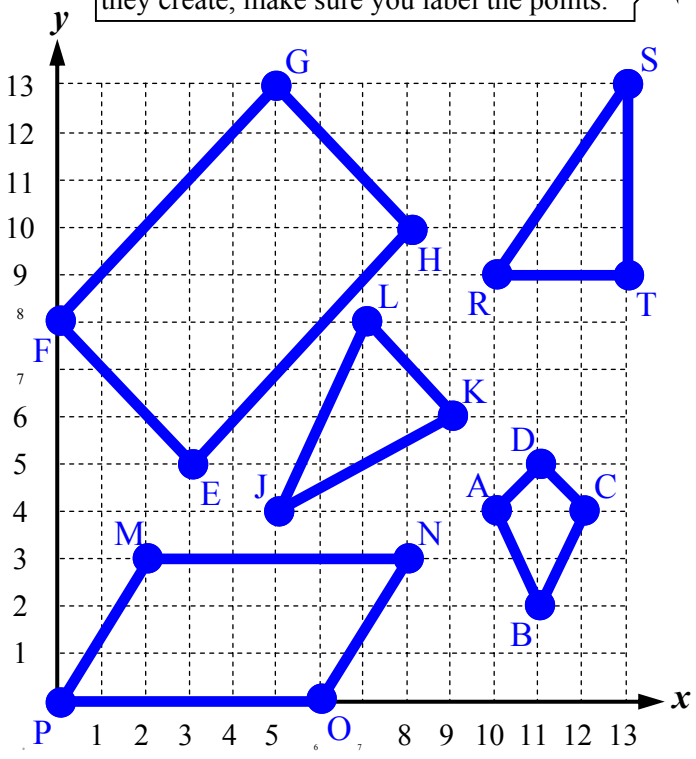
Give the co-ordinates for the following points, place them in brackets separated by a comma



1	Point A	(1, 5)	14	Point N	(0, 7)
2	Point B	(6, 8)	15	Point O	(2, 4)
3	Point C	(9, 7)	16	Point P	(5, 11)
4	Point D	(6, 2)	17	Point Q	(11, 10)
5	Point E	(2, 10)	18	Point R	(7, 5)
6	Point F	(10, 11)	19	Point S	(12, 7)
7	Point G	(5, 5)	20	Point T	(13, 3)
8	Point H	(3, 8)	21	Point U	(2, 1)
9	Point I	(10, 4)	22	Point V	(7, 7)
10	Point J	(3, 13)	23	Point W	(12, 13)
11	Point K	(0, 0)	24	Point X	(7, 12)
12	Point L	(10, 1)	25	Point Y	(4, 6)
13	Point M	(5, 0)	26	Point Z	(1, 12)

Place the groups of points given below on the grid, join the points and name the shape they create, make sure you label the points.



27 Point A (10, 4) Point B (11, 2)
Point C (12, 4) Point D (11, 5)
Shape formed: Kite

28 Point E (3, 5) Point F (0, 8)
Point G (5, 13) Point H (8, 10)
Shape formed: Rectangle

29 Point J (5, 4) Point K (9, 6)
Point L (7, 8)
Shape formed: Isosceles Triangle
Two word description

30 Point M (2, 3) Point N (8, 3)
Point O (6, 0) Point P (0, 0)
Shape formed: Parallelogram

31 Point R (10, 9) Point S (13, 13)
Point T (13, 9)
Shape formed: Right (Angled) Triangle
Two or three word description

Cartesian Plane - Drawing Animals - Positive Quadrant (Sheet 2)

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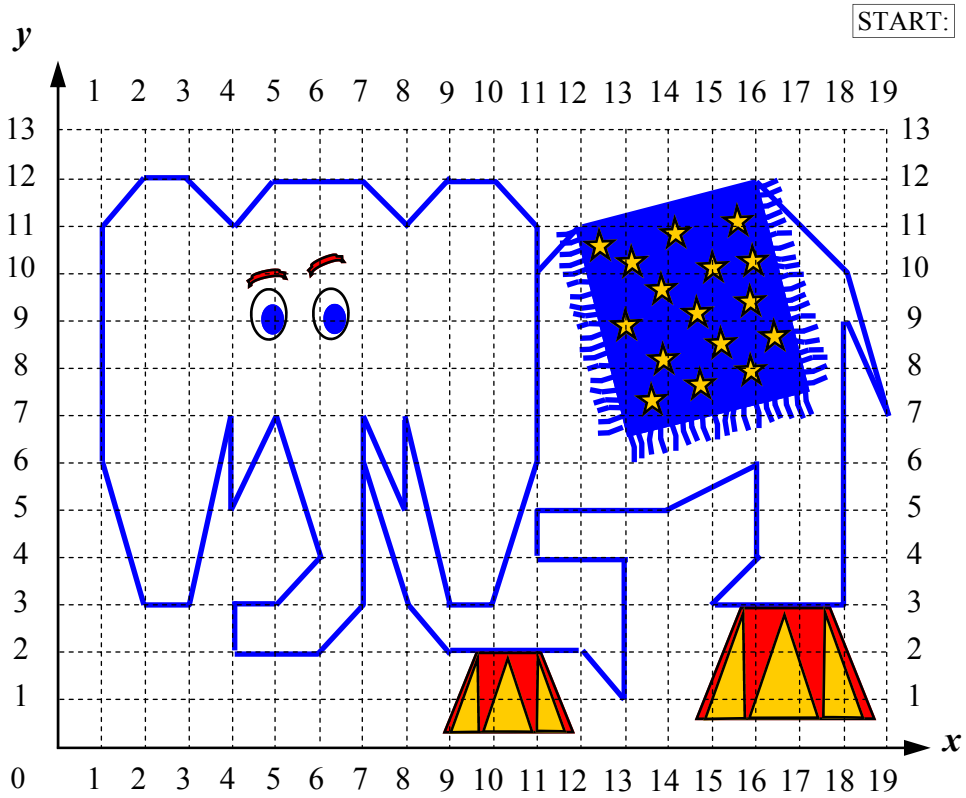
Join the points on the grid with the following co-ordinates and you will find an animal friend who has only one hand.

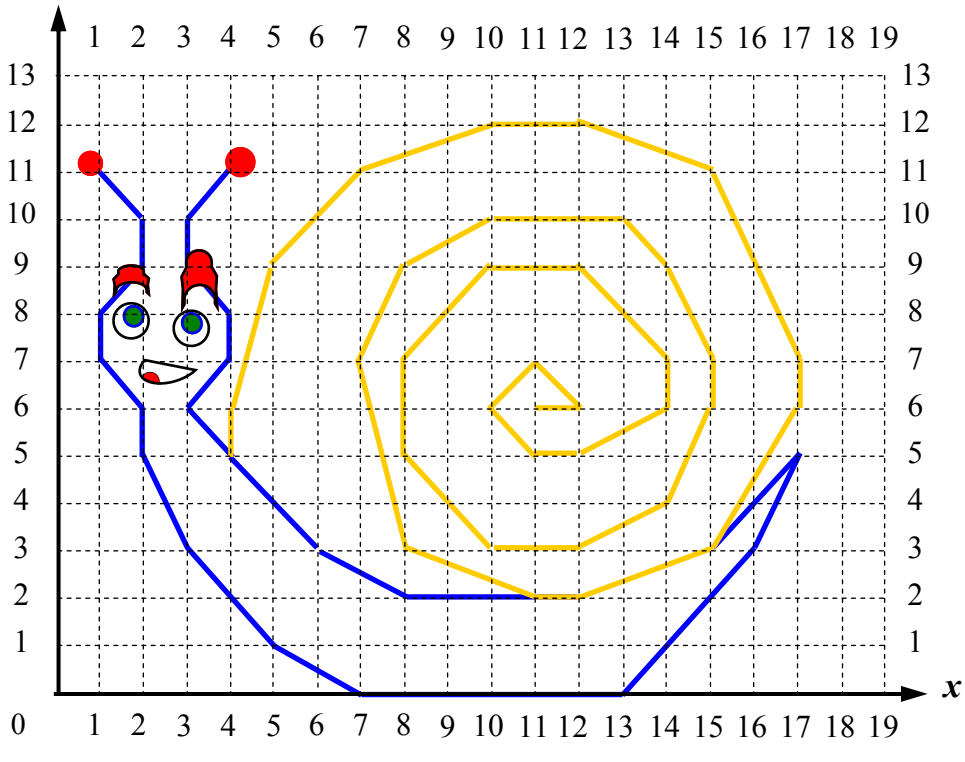


Co-ordinates:

START: (2, 12) , (3, 12) , (4, 11) , (5, 12) ,
(7, 12) , (8, 11) , (9, 12) , (10, 12) ,
(11, 11) , (11, 6) , (10, 3) , (9, 3) ,
(8, 7) , (8, 5) , (7, 7) , (7, 3) ,
(6, 2) , (4, 2) , (4, 3) , (5, 3) ,
(6, 4) , (5, 7) , (4, 5) , (4, 7) ,
(3, 3) , (2, 3) , (1, 6) , (1, 11) ,
(2, 12) STOP!

START:
(11, 10) , (12, 11) , (16, 12) , (18, 10) ,
(19, 7) , (18, 9) , (18, 3) , (15, 3) ,
(16, 4) , (16, 6) , (14, 5) , (11, 5) ,
(11, 4) , (13, 4) , (13, 1) , (12, 2) ,
(9, 2) , (8, 3) , (7, 6) , STOP!

Join the points and this time you will find a friend with which you will have to be very patient.



Co-ordinates:

START: (1, 11) , (2, 10) , (2, 9) , (1, 8) ,
(1, 7) , (2, 6) , (2, 5) , (3, 3) ,
(5, 1) , (7, 0) , (13, 0) , (16, 3) ,
(17, 5) , (15, 3) , (12, 2) , (8, 2) ,
(6, 3) , (4, 5) , (3, 6) , (4, 7) ,
(4, 8) , (3, 9) , (3, 10) , (4, 11) ,
STOP!

START:
(11, 6) , (12, 6) , (11, 7) , (10, 6) ,
(11, 5) , (12, 5) , (14, 6) , (14, 7) ,
(12, 9) , (10, 9) , (8, 7) , (8, 5) ,
(10, 3) , (12, 3) , (14, 4) , (15, 6) ,
(15, 7) , (14, 9) , (13, 10) , (10, 10) ,
(8, 9) , (7, 7) , (8, 3) , (11, 2) ,
(12, 2) , (15, 3) , (17, 6) , (17, 7) ,
(15, 11) , (12, 12) , (10, 12) , (7, 11) ,
(5, 9) , (4, 6) , (4, 5) , STOP!

Cartesian Plane - Plotting Points - Positive Quadrant (Homework Strip)

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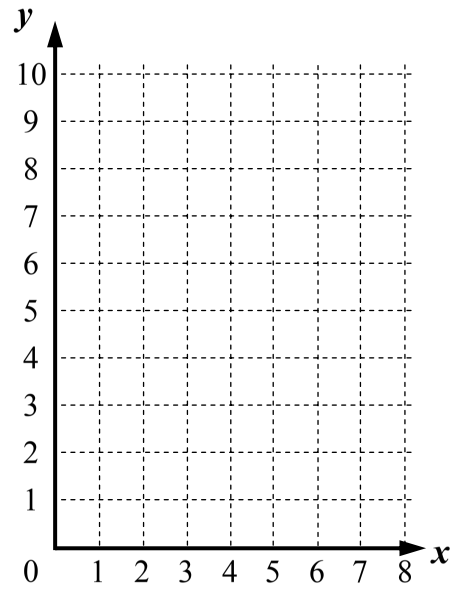
Date Due: _____

Plot and join the points, name the shape



1 (3, 3) ; (3, 9) ; (7, 9) ; (7, 3) .

Shape formed: _____

2 (8, 7) ; (1, 7) ; (1, 10) ; (5, 10) .

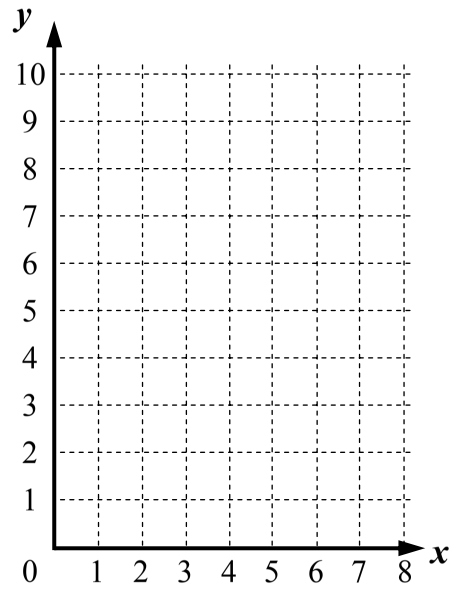
Shape formed: _____

3 (5, 5) ; (6, 3) ; (5, 1) ; (2, 1) ; (1, 3) ; (2, 5) .

Shape formed: _____

Date Due: _____

Plot and join the points, name the shape



1 (3, 3) ; (3, 9) ; (7, 9) ; (7, 3) .

Shape formed: _____

2 (8, 7) ; (1, 7) ; (1, 10) ; (5, 10) .

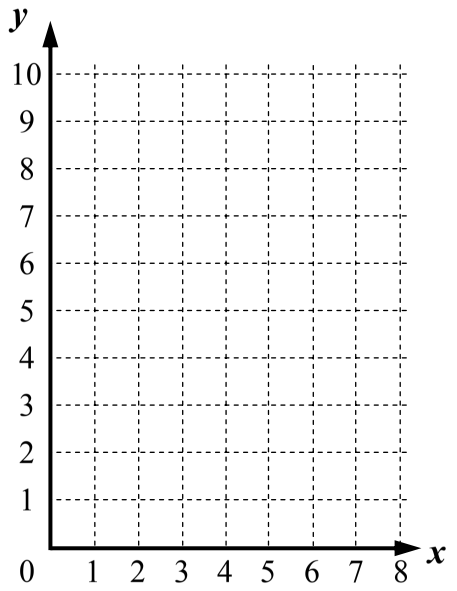
Shape formed: _____

3 (5, 5) ; (6, 3) ; (5, 1) ; (2, 1) ; (1, 3) ; (2, 5) .

Shape formed: _____

Date Due: _____

Plot and join the points, name the shape



1 (3, 3) ; (3, 9) ; (7, 9) ; (7, 3) .

Shape formed: _____

2 (8, 7) ; (1, 7) ; (1, 10) ; (5, 10) .

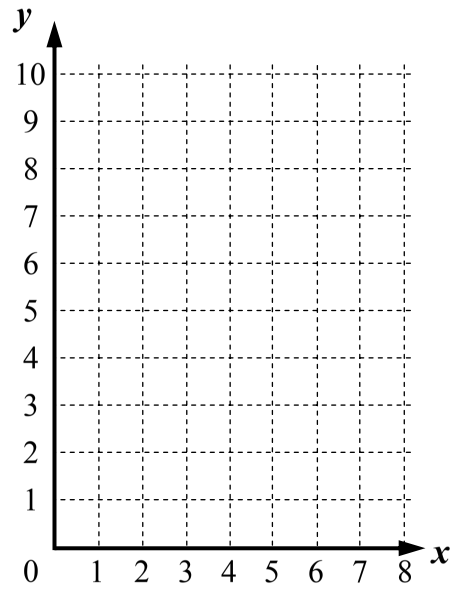
Shape formed: _____

3 (5, 5) ; (6, 3) ; (5, 1) ; (2, 1) ; (1, 3) ; (2, 5) .

Shape formed: _____

Date Due: _____

Plot and join the points, name the shape



1 (3, 3) ; (3, 9) ; (7, 9) ; (7, 3) .

Shape formed: _____

2 (8, 7) ; (1, 7) ; (1, 10) ; (5, 10) .

Shape formed: _____

3 (5, 5) ; (6, 3) ; (5, 1) ; (2, 1) ; (1, 3) ; (2, 5) .

Shape formed: _____

Cartesian Plane - Plotting Points - Positive Quadrant (Homework Strip)

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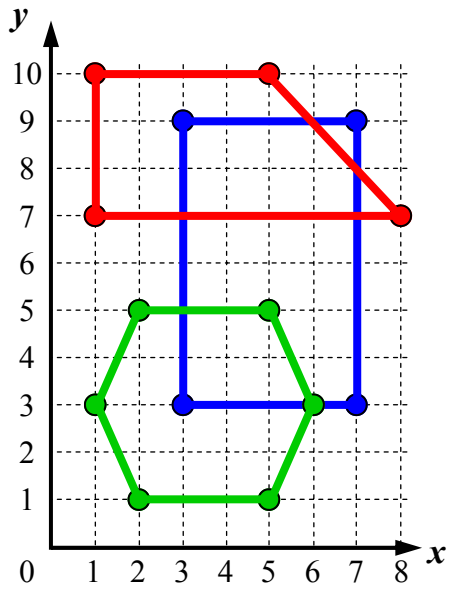
Date Due: _____

Plot and join the points, name the shape



1 (3, 3) ; (3, 9) ; (7, 9) ; (7, 3) .

Shape formed: Rectangle

2 (8, 7) ; (1, 7) ; (1, 10) ; (5, 10) .

Shape formed: Trapezium

3 (5, 5) ; (6, 3) ; (5, 1) ; (2, 1) ; (1, 3) ; (2, 5) .

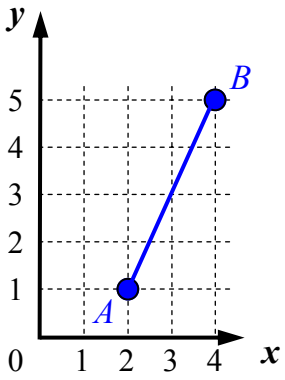
Shape formed: Irregular Hexagon

Cartesian Plane - Midpoint - Positive Quadrant

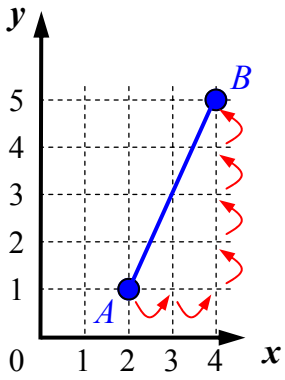
If you draw a line, halfway along that line is the midpoint. There are two common ways of finding the location of the midpoint. You can measure it's location on a number plane 'Graphically', or you can calculate its location 'Analytically'.

Graphical Solution

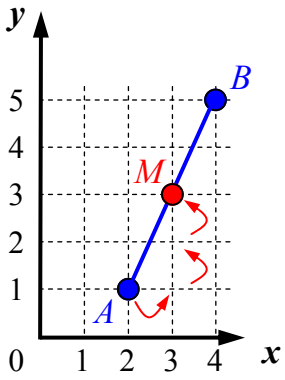
Example: Find the midpoint M on the interval joining A (2, 1) and B (4, 5)



Plot A and B and draw the interval AB



Count the 'hops' and halve them. Horizontal is 2 hops, so midpoint is 1 hop past the 2. So it is 3. Vertical is 4 hops, so midpoint is 2 hops above the 1. So it is 3 also.



Plot the midpoint and label it M.
There is a space in the question to record its coordinates, for the M above, it is (3, 3).

Analytical Method

It may be that the points aren't graphed, or the values are that large that graphing them isn't feasible. Or you just prefer using numbers, either way you can find the midpoint using this formula:

$$M(\bar{x}, \bar{y}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Where one point is (x_1, y_1), and the other point is (x_2, y_2).

What does this mean? It means that you add the x-values and divide them by 2, for the x-coordinate of the midpoint. Then repeat for the y-values, for the y-coordinate of the midpoint. There is an example at the bottom of Column 1 on Sheet 2.

Cartesian Plane - Midpoint (Graphical) - Positive Quadrant (Sheet 1)

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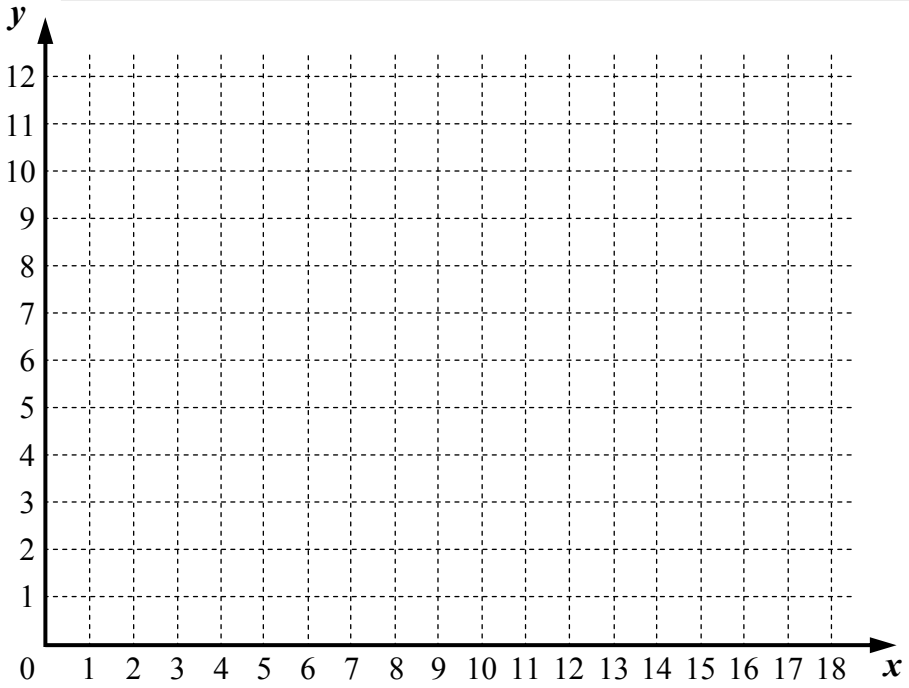
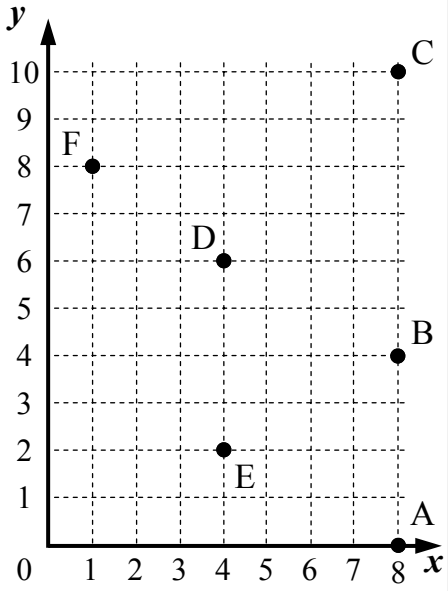
Plot the points and use a ruler to rule a line between the two points and find the midpoint. Plot the midpoint on the line record its co-ordinates. Write letters in pencil, as you may need to move them later!



This time the points are plotted, rule the line, find the midpoint and record all coordinates.



13 Construct DC and find 'M'

D: (☐ , ☐) C: (☐ , ☐)

Midpoint M: (☐ , ☐)

14 Construct FE and find 'M'

F: (☐ , ☐) E: (☐ , ☐)

Midpoint M: (☐ , ☐)

15 Construct FC and find 'M'

F: (☐ , ☐) C: (☐ , ☐)

Midpoint M: (☐ , ☐)

16 Construct BD and find 'M'

B: (☐ , ☐) D: (☐ , ☐)

Midpoint M: (☐ , ☐)

17 Construct AD and find 'M'

A: (☐ , ☐) D: (☐ , ☐)

Midpoint M: (☐ , ☐)

Steps: Plot the points on the number plane.
Rule a line in between the two points.
Plot the midpoint, call it 'M'.
Write the co-ordinate for the midpoint.

1 A (1, 12) and B (3, 4).

Midpoint: (☐ , ☐)

Label the midpoint 'M' on the number plane

2 Y (18, 10) and X (6, 12).

Midpoint: (☐ , ☐)

3 L (6, 11) and N (6, 1).

Midpoint: _____

4 E (0, 4) and F (6, 0).

Midpoint: _____

5 Q (15, 9) and R (5, 3).

Midpoint: _____

'QR' will cross previous line

6 J (9, 7) and K (17, 7).

Midpoint: _____

It is possible to have '½' in a co-ordinate. These all do! Approach these the same way.

Use a different colour for these lines

7 T (18, 4) and B (3, 4).

Midpoint: _____

8 C (10, 0) and D (16, 3).

Midpoint: _____

9 L (6, 11) and J (9, 7).

Midpoint: _____

10 T (18, 4) and K (17, 7).

Midpoint: _____

11 B (3, 4) and L (6, 11).

Midpoint: _____

12 V (10, 10) and Q (15, 9).

Midpoint: _____

Cartesian Plane - Midpoint (Analytical) - Positive Quadrant (Sheet 2)

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Find the average of these two numbers. Add them, press '=' and then divide '÷' by 2. $\bar{x}$ is the mean (pronounced *x*-bar).

1 22 and 38

$$\bar{x} = \frac{\square + \square}{2} = \square$$

Calc: 2 2 + 3 8 = ÷ 2 =

2 26 and 6

$$\bar{x} = \frac{\square + \square}{2} = \square$$

3 5 and 33

$$\bar{x} =$$

4 14 and 21

$$\bar{x} =$$

With halves use '½' not '.5'

Find the midpoint of the interval that joins these two points. Find two averages.

Example

Find the midpoint of the interval joining (2, 7) and (14, 15).

x_1 y_1 x_2 y_2

$$\bar{x} = \frac{2 + 14}{2} = 8$$

$$\bar{y} = \frac{7 + 15}{2} = 11$$

$$M(\bar{x}, \bar{y}) = (8, 11)$$

5 (5, 2) and (23, 14).

$$\bar{x} = \frac{\square + \square}{2} = \square$$

$$\bar{y} = \frac{\square + \square}{2} = \square$$

$$M(\bar{x}, \bar{y}) = (\square, \square)$$

6 (1, 14) and (19, 22).

$$\bar{x} = \frac{\square + \square}{2} = \square$$

$$\bar{y} = \frac{\square + \square}{2} = \square$$

$$M(\bar{x}, \bar{y}) = (\square, \square)$$

7 (14, 2) and (8, 6).

$$\bar{x} =$$

$$\bar{y} =$$

$$M(\bar{x}, \bar{y}) = (\square, \square)$$

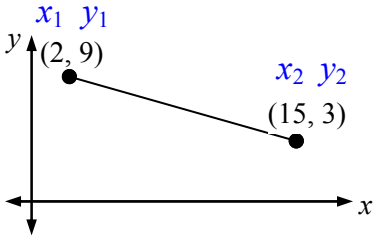
8 (9, 17) and (3, 8).

9 (11, 2) and (12, 9).

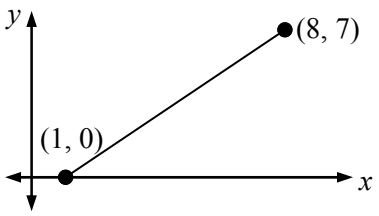
10 (6, 0) and (11, 13).

Use the same method, but this time you have to get your information from a sketch.

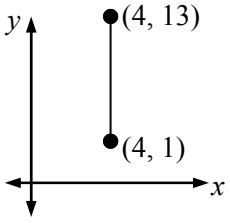
11



12



13



Cartesian Plane - Midpoint (Graphical) - Positive Quadrant (Sheet 1)

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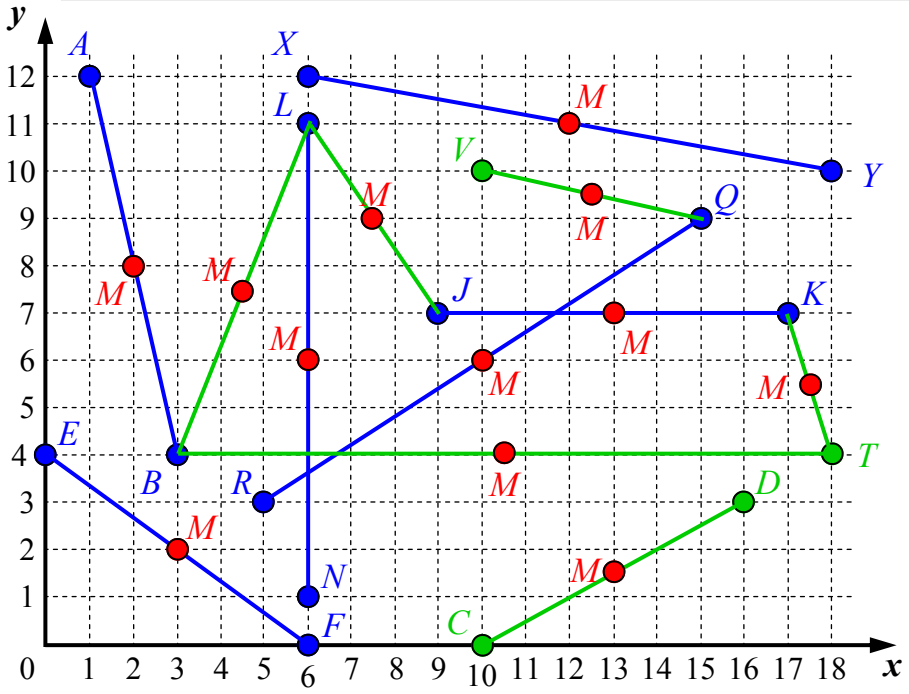
Plot the points and use a ruler to rule a line between the two points and find the midpoint. Plot the midpoint on the line record its co-ordinates. Write letters in pencil, as you may need to move them later!



Steps: Plot the points on the number plane.
Rule a line in between the two points.
Plot the midpoint, call it 'M'.
Write the co-ordinate for the midpoint.

1 A (1, 12) and B (3, 4).

Midpoint: (2 , 8)

Label the midpoint 'M' on the number plane

2 Y (18, 10) and X (6, 12).

Midpoint: (12 , 11)

3 L (6, 11) and N (6, 1).

Midpoint: (6, 6)

4 E (0, 4) and F (6, 0).

Midpoint: (3, 2)

5 Q (15, 9) and R (5, 3).

Midpoint: (10, 6)

'QR' will cross previous line

6 J (9, 7) and K (17, 7).

Midpoint: (13, 7)

It is possible to have '½' in a co-ordinate. These all do! Approach these the same way.

Use a different colour for these lines

7 T (18, 4) and B (3, 4).

Midpoint: (10½, 4)

8 C (10, 0) and D (16, 3).

Midpoint: (13, 1½)

9 L (6, 11) and J (9, 7).

Midpoint: (7½, 9)

10 T (18, 4) and K (17, 7).

Midpoint: (17½, 5½)

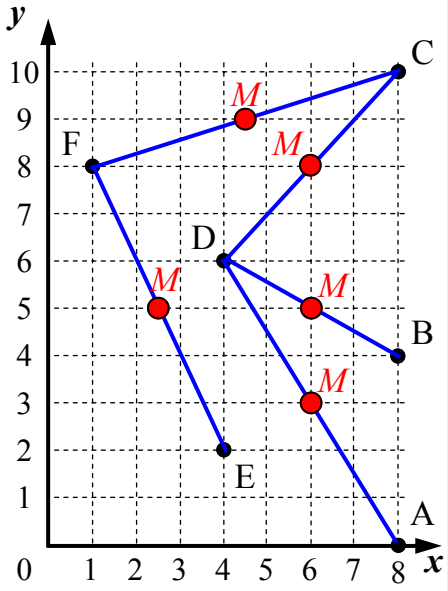
11 B (3, 4) and L (6, 11).

Midpoint: (4½, 7½)

12 V (10, 10) and Q (15, 9).

Midpoint: (12½, 9½)

This time the points are plotted, rule the line, find the midpoint and record all coordinates.



13 Construct DC and find 'M'

D: (4 , 6) C: (8 , 10)

Midpoint M: (6 , 8)

14 Construct FE and find 'M'

F: (1 , 8) E: (4 , 2)

Midpoint M: (2½ , 5)

15 Construct FC and find 'M'

F: (1 , 8) C: (8 , 10)

Midpoint M: (4½ , 9)

16 Construct BD and find 'M'

B: (8 , 4) D: (4 , 6)

Midpoint M: (6 , 5)

17 Construct AD and find 'M'

A: (8 , 0) D: (4 , 6)

Midpoint M: (6 , 3)

Cartesian Plane - Midpoint (Analytical) - Positive Quadrant (Sheet 2)

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Find the average of these two numbers. Add them, press '=' and then divide '÷' by 2. $\bar{x}$ is the mean (pronounced *x*-bar).

1 22 and 38

$$\bar{x} = \frac{22 + 38}{2} = 30$$

Calc: 2 2 + 3 8 = ÷ 2 =

2 26 and 6

$$\bar{x} = \frac{26 + 6}{2} = 16$$

3 5 and 33

$$\bar{x} = \frac{5 + 33}{2} = 19$$

4 14 and 21

$$\bar{x} = \frac{14 + 21}{2} = 17\frac{1}{2}$$

With halves use '½' not '.5'

Find the midpoint of the interval that joins these two points. Find two averages.

Example

Find the midpoint of the interval joining (2, 7) and (14, 15).

$x_1\ y_1 \qquad x_2\ y_2$

$$\bar{x} = \frac{2 + 14}{2} = 8$$

$$\bar{y} = \frac{7 + 15}{2} = 11$$

$$M(\bar{x}, \bar{y}) = (8, 11)$$

5 (5, 2) and (23, 14).

$x_1\ y_1 \qquad x_2\ y_2$

$$\bar{x} = \frac{5 + 23}{2} = 14$$

$$\bar{y} = \frac{2 + 14}{2} = 8$$

$$M(\bar{x}, \bar{y}) = (14, 8)$$

6 (1, 14) and (19, 22).

$x_1\ y_1 \qquad x_2\ y_2$

$$\bar{x} = \frac{1 + 19}{2} = 10$$

$$\bar{y} = \frac{14 + 22}{2} = 18$$

$$M(\bar{x}, \bar{y}) = (10, 18)$$

7 (14, 2) and (8, 6).

$x_1\ y_1 \qquad x_2\ y_2$

$$\bar{x} = \frac{14 + 8}{2} = 11$$

$$\bar{y} = \frac{2 + 6}{2} = 4$$

$$M(\bar{x}, \bar{y}) = (11, 4)$$

8 (9, 17) and (3, 8).

$x_1\ y_1 \qquad x_2\ y_2$

$$\bar{x} = \frac{9 + 3}{2} = 6$$

$$\bar{y} = \frac{17 + 8}{2} = 12\frac{1}{2}$$

$$M(\bar{x}, \bar{y}) = (6, 12\frac{1}{2})$$

9 (11, 2) and (12, 9).

$x_1\ y_1 \qquad x_2\ y_2$

$$\bar{x} = \frac{11 + 12}{2} = 11\frac{1}{2}$$

$$\bar{y} = \frac{2 + 9}{2} = 5\frac{1}{2}$$

$$M(\bar{x}, \bar{y}) = (11\frac{1}{2}, 5\frac{1}{2})$$

10 (6, 0) and (11, 13).

$x_1\ y_1 \qquad x_2\ y_2$

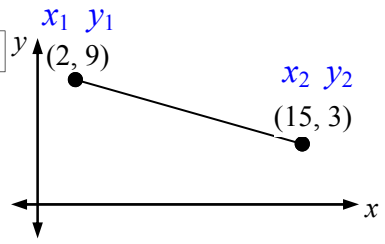
$$\bar{x} = \frac{6 + 11}{2} = 8\frac{1}{2}$$

$$\bar{y} = \frac{0 + 13}{2} = 6\frac{1}{2}$$

$$M(\bar{x}, \bar{y}) = (8\frac{1}{2}, 6\frac{1}{2})$$

Use the same method, but this time you have to get your information from a sketch.

11

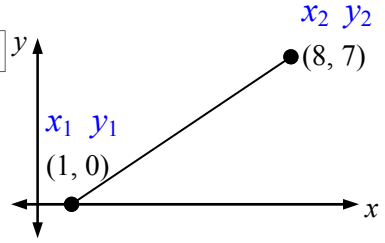


$$\bar{x} = \frac{2 + 15}{2} = 8\frac{1}{2}$$

$$\bar{y} = \frac{9 + 3}{2} = 6$$

$$M(\bar{x}, \bar{y}) = (8\frac{1}{2}, 6)$$

12

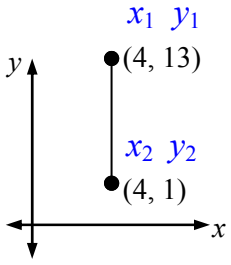


$$\bar{x} = \frac{1 + 8}{2} = 4\frac{1}{2}$$

$$\bar{y} = \frac{0 + 7}{2} = 3\frac{1}{2}$$

$$M(\bar{x}, \bar{y}) = (4\frac{1}{2}, 3\frac{1}{2})$$

13



$$\bar{x} = \frac{4 + 4}{2} = 4$$

$$\bar{y} = \frac{13 + 1}{2} = 7$$

$$M(\bar{x}, \bar{y}) = (4, 7)$$

Cartesian Plane - Midpoint - Positive Quadrant (Homework Strip)

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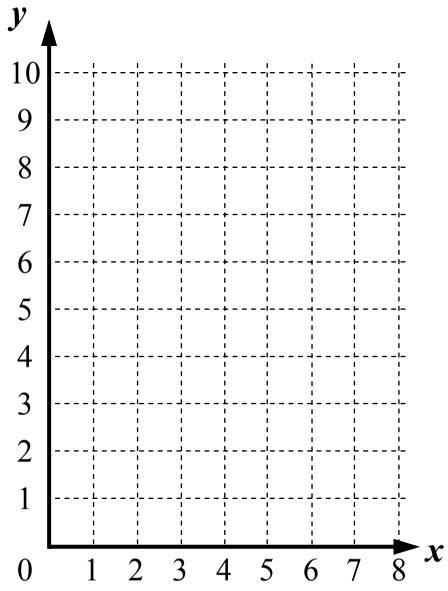
Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 $A\ (0, 10)$ and $B\ (2, 2)$.

Midpoint: (____ , ____)

2 $C\ (8, 3)$ and $D\ (3, 7)$.

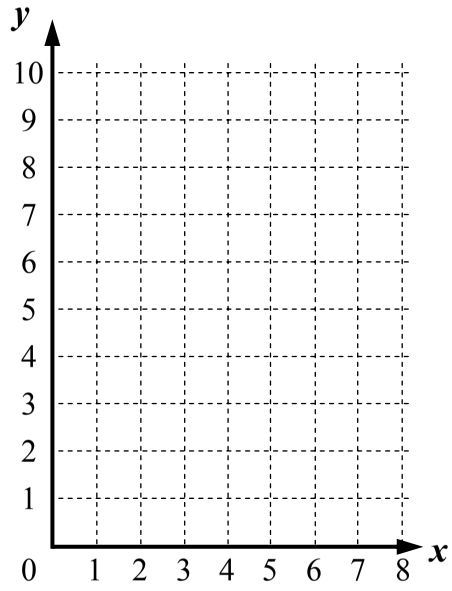
Midpoint: (____ , ____)

3 $E\ (7, 10)$ and $F\ (2, 9)$.

Midpoint: (____ , ____)

Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 $A\ (0, 10)$ and $B\ (2, 2)$.

Midpoint: (____ , ____)

2 $C\ (8, 3)$ and $D\ (3, 7)$.

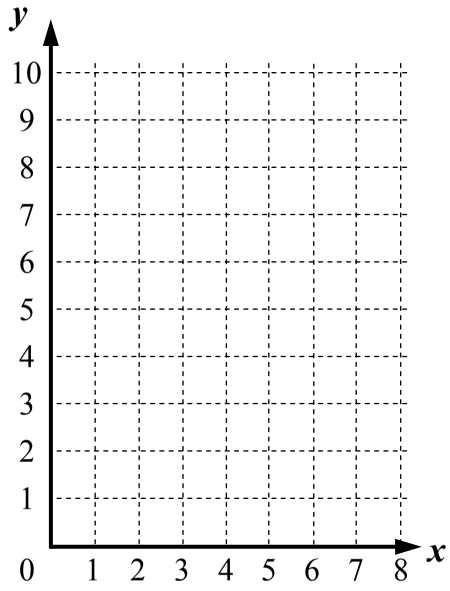
Midpoint: (____ , ____)

3 $E\ (7, 10)$ and $F\ (2, 9)$.

Midpoint: (____ , ____)

Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 $A\ (0, 10)$ and $B\ (2, 2)$.

Midpoint: (____ , ____)

2 $C\ (8, 3)$ and $D\ (3, 7)$.

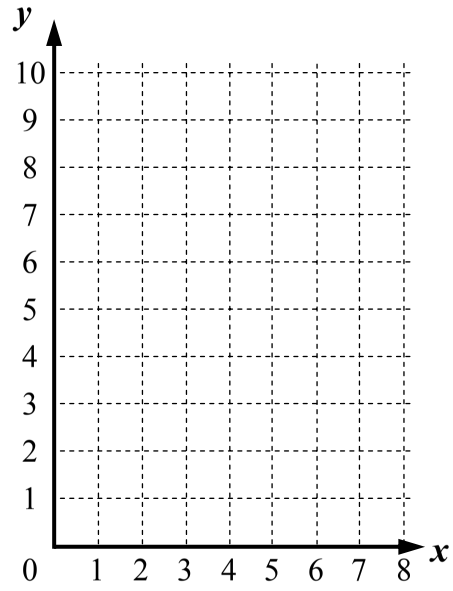
Midpoint: (____ , ____)

3 $E\ (7, 10)$ and $F\ (2, 9)$.

Midpoint: (____ , ____)

Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 $A\ (0, 10)$ and $B\ (2, 2)$.

Midpoint: (____ , ____)

2 $C\ (8, 3)$ and $D\ (3, 7)$.

Midpoint: (____ , ____)

3 $E\ (7, 10)$ and $F\ (2, 9)$.

Midpoint: (____ , ____)

Cartesian Plane - Midpoint - Positive Quadrant (Homework Strip)

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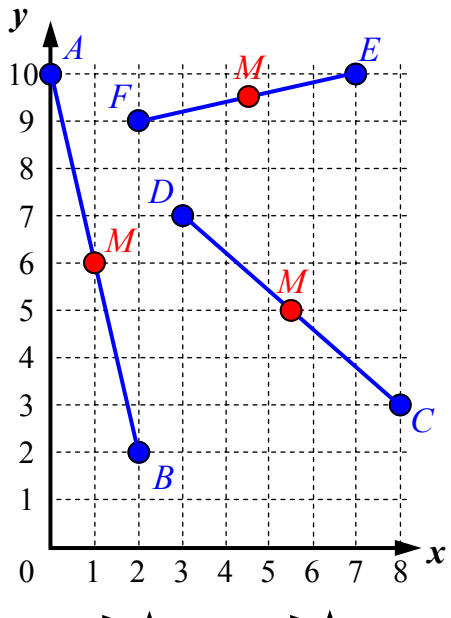
Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 *A* (0, 10) and *B* (2, 2).

Midpoint: (1 , 6)

2 *C* (8, 3) and *D* (3, 7).

Midpoint: (5½ , 5)

3 *E* (7, 10) and *F* (2, 9).

Midpoint: (4½ , 9½)

Cartesian Plane - Plotting Points - All Quadrants

This sheet involves the plotting of Cartesian co-ordinates. When plotting points on a number plane you are given two numbers. The first number tells you how much you move across and the second number tells you how much you move up or down. As the numbers are in order, across first, up/down second, these numbers can be referred to as 'ordered pairs', as their order is important. Positive numbers move across to the right if the first number, negative numbers move to the left if the first number. Positive numbers move up if the second number, negative numbers move down if the second number.

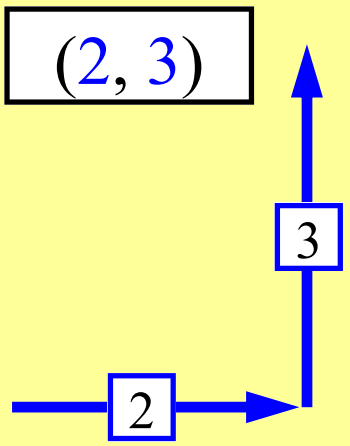
The first column asks you to locate the points and write in their co-ordinates. This is done by opening a bracket writing the *x* co-ordinate, a comma, then the *y* co-ordinate and close bracket. Always place co-ordinates in brackets.

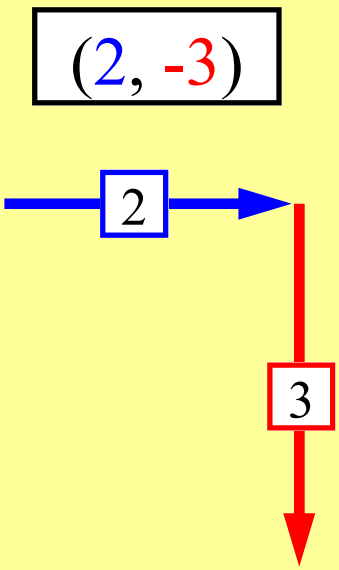
The second column asks you to plot points. Draw filled in circles at the given location and label them with the letter supplied in each question.

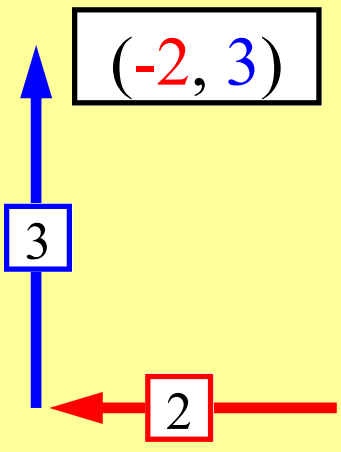
Once you plot the point record down which quadrant it is in. Quadrants start with the 1st Quadrant (top right) and then moving anti-clockwise, are called II (2nd), III (3rd) and IV (4th). If the point is on the axis then record it by filling in the oval around the word 'Axis'.

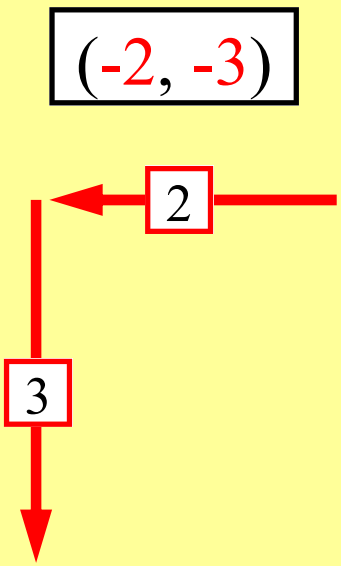
Move across first along the *x* axis.
Negative to the left, Positive to the right.

Then move up or down in the *y* direction.
Negative is down, Positive is up.









Cartesian Plane - Plotting Points - All Quadrants (Sheet 1)

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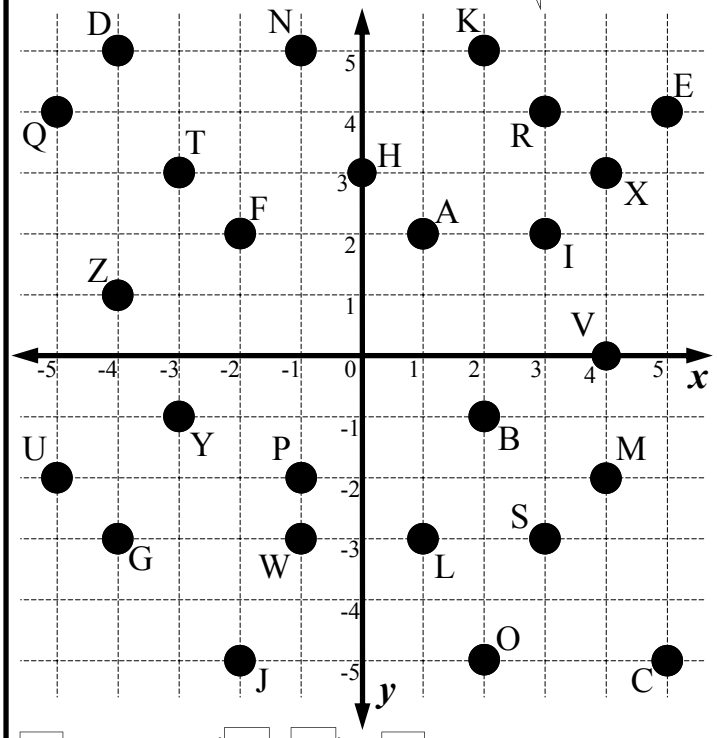
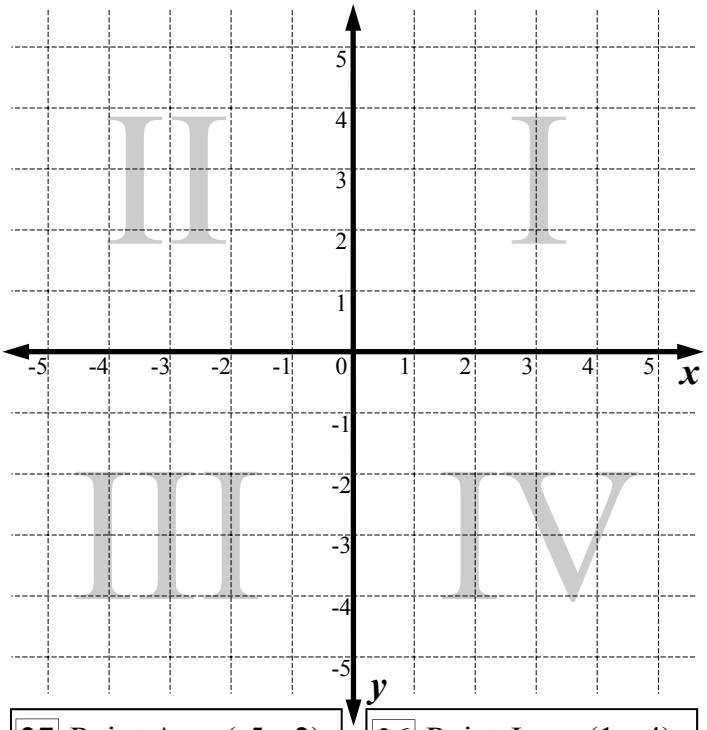
Give the co-ordinates for the points on the number plane below.



1 Point A (,)
2 Point B
3 Point C
4 Point D
5 Point E
6 Point F
7 Point G
8 Point H
9 Point I
10 Point J
11 Point K
12 Point L
13 Point M
14 Point N
15 Point O
16 Point P
17 Point Q
18 Point R
19 Point S
20 Point T
21 Point U
22 Point V
23 Point W
24 Point X
25 Point Y
26 Point Z

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant I II III IV Axis
28 Point B (2, 4) Quadrant I II III IV Axis
29 Point C (4, 0) Quadrant I II III IV Axis
30 Point D (3, -3) Quadrant I II III IV Axis
31 Point E (-3, 5) Quadrant I II III IV Axis
32 Point F (0, 0) Quadrant I II III IV Axis
33 Point G (5, -5) Quadrant I II III IV Axis
34 Point H (-2, 4) Quadrant I II III IV Axis
35 Point I (1, 2) Quadrant I II III IV Axis
36 Point J (1, -4) Quadrant I II III IV Axis
37 Point K (-3, -4) Quadrant I II III IV Axis
38 Point L (-1, 0) Quadrant I II III IV Axis
39 Point M (-4, 2) Quadrant I II III IV Axis
40 Point N (3, -5) Quadrant I II III IV Axis
41 Point O (0, -3) Quadrant I II III IV Axis
42 Point P (-5, -5) Quadrant I II III IV Axis
43 Point R (-2, 1) Quadrant I II III IV Axis
44 Point S (2, -2) Quadrant I II III IV Axis

Cartesian Plane - Drawing Animals - All Quadrants (Sheet 2)

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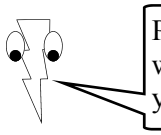
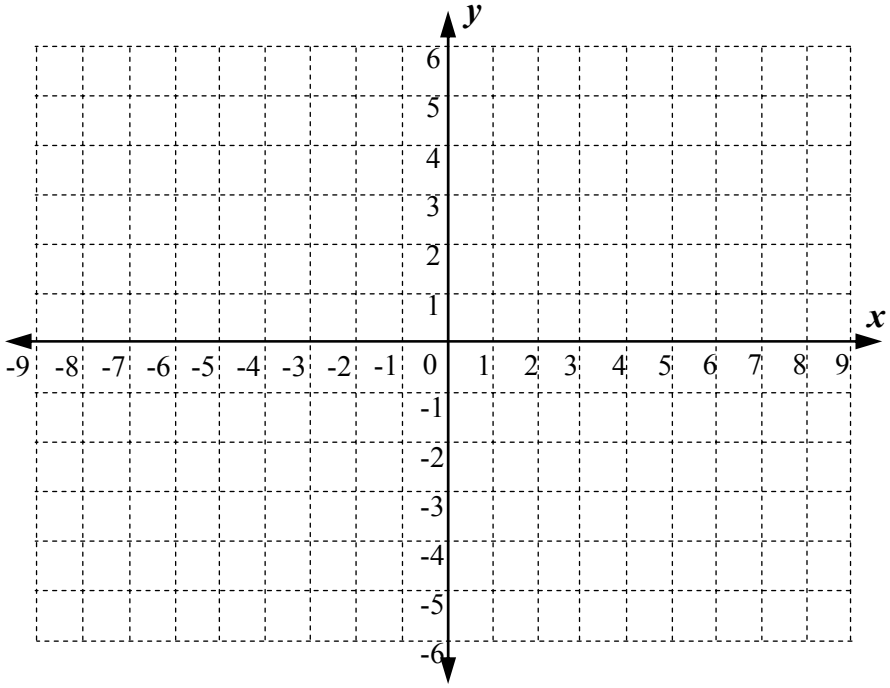


Plot the points and join them with lines to meet one of your more colourful friends

Co-ordinates: START: (-1, 3) , (-1, 4) , (1, 6) , (2, 6) , (6, 5) , (7, 4) , (7, 2) , (6, -1) , (3, -2) , (5,-4) , (5, -5) , (3, -6) , (1, -6) , (-2, -4) , (-4, -6) , (-5, -6) , (-6, -5) , (-6, -4) , (-5, -2) , (-6, -1) , (-8, 2) , (-8, 4) , (-7, 6) , (-6, 6) , (-4, 4) , (-4, 3) , STOP!

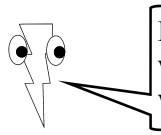


START: (-2, 1) , (1, 4) , (4, 4) , (5, 2) , (3,-1) , (0, -2) , (3, -3) , (3, -5) , (1, -5) , (-1, -4) , (-2, -3) , (-4, -5) , (-5, -5) , (-5, -3) , (-3, -2) , (-5, -1) , (-7, 2) , (-6, 4) , (-5, 4) , (-3,1) , STOP!

START: (-4, 3) , (-3, 4) , (-2, 4) , (-1, 3) , (-2, 2) , (-2, -4) , (-3, 2) , (-4, 3) , STOP!

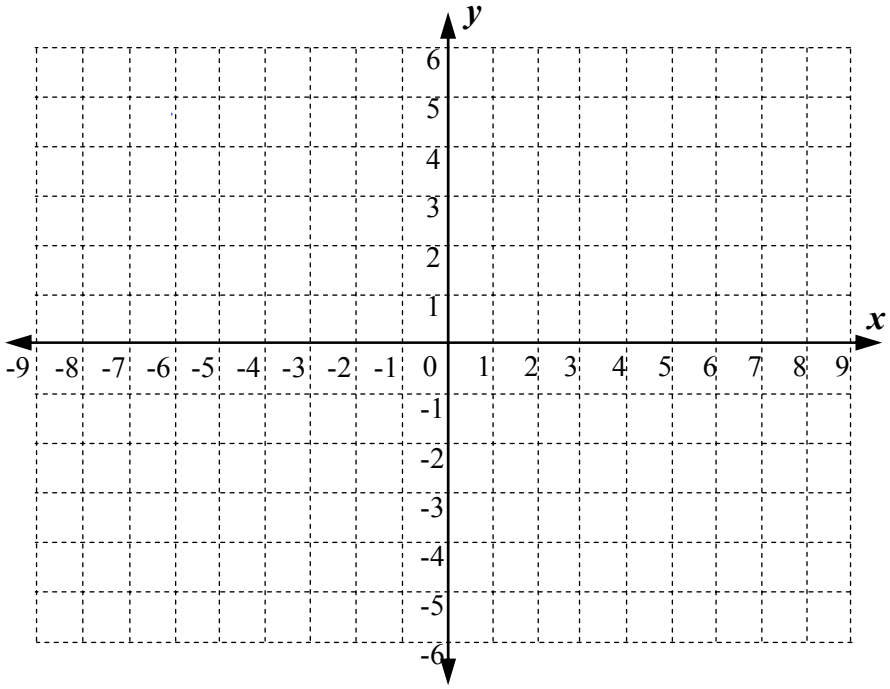
START: (-3, 4) , (-4, 6) , STOP!

START: (-2, 4) , (-1, 6) , STOP!



Plot the points and join them with lines to share your lunch with your vegetarian friend.

Co-ordinates: START: (2, -1) , (3, -3) , (3, -4) , (2, -5) , (0, -6) , (-1, -6) , (-3, -5) , (-4, -4) , (-4, -3) , (-3, -1) , (-6, 2) , (-7, -4) , (-8, 4) , (-7, 6) , (-4, 5) , (-2, -1) , (0, -1) , (3, 4) , (5, 6) , (8, 6) , (9, 4) , (5, 6) , STOP!



START: (7, 5) , (6, 3) , (2, -1) , STOP!

START: (2, 0) , (4, 4) , (6, 5) , (5, 3) , (2, 0) , STOP!

START: (-3, 0) , (-5, 2) , (-6, 4) , (-4, 3) , (-3, 0) , STOP!

START: (-2, -4) , (-6, -4) , STOP!

START: (-2, -4) , (-5, -2) , STOP!

START: (-2, -4) , (-5, -3) , STOP!

START: (1, -4) , (5, -4) , STOP!

START: (1, -4) , (5, -3) , STOP!

START: (1, -4) , (5, -5) , STOP!

Cartesian Plane - Plotting Points - All Quadrants (Sheet 1)

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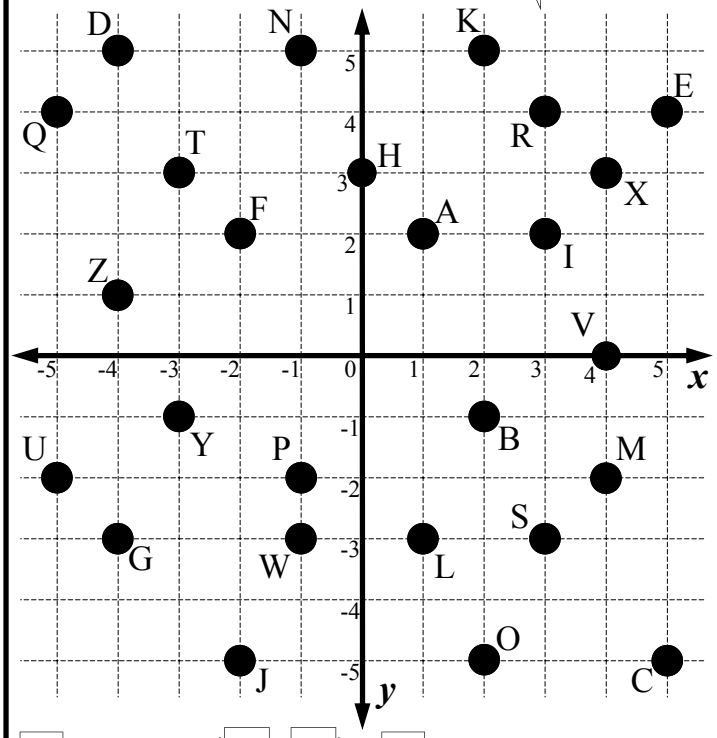
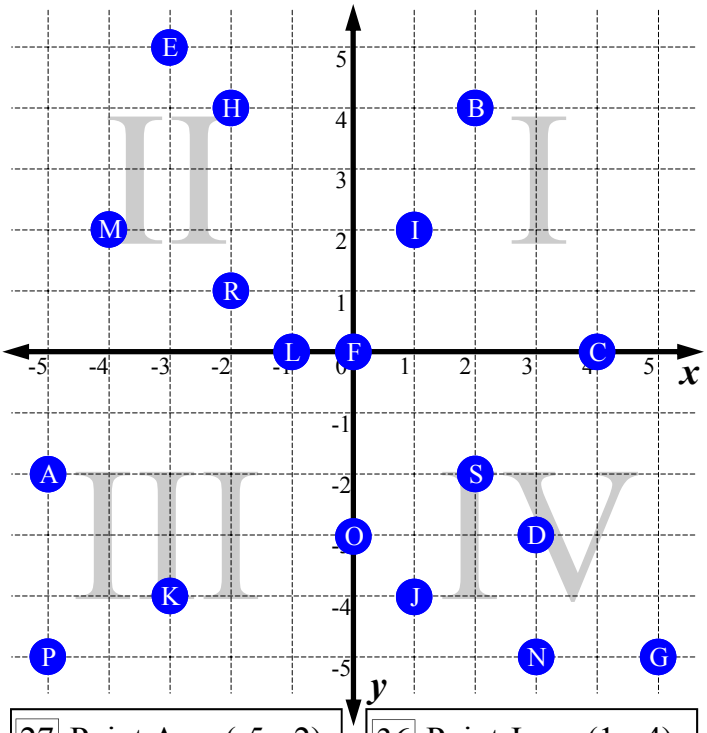
Give the co-ordinates for the points on the number plane below.



1	Point A	(1, 2)	14	Point N	(-1, 5)
2	Point B	(2, -1)	15	Point O	(2, -5)
3	Point C	(5, -5)	16	Point P	(-1, -2)
4	Point D	(-4, 5)	17	Point Q	(-5, 4)
5	Point E	(5, 4)	18	Point R	(3, 4)
6	Point F	(-2, 2)	19	Point S	(3, -3)
7	Point G	(-4, -3)	20	Point T	(-3, 3)
8	Point H	(0, 3)	21	Point U	(-5, -2)
9	Point I	(3, 2)	22	Point V	(4, 0)
10	Point J	(-2, -5)	23	Point W	(-1, -3)
11	Point K	(2, 5)	24	Point X	(4, 3)
12	Point L	(1, -3)	25	Point Y	(-3, -1)
13	Point M	(4, -2)	26	Point Z	(-4, 1)

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant I II III IV **Axis**

28 Point B (2, 4) Quadrant **I** II III IV Axis

29 Point C (4, 0) Quadrant I II III IV **Axis**

30 Point D (3, -3) Quadrant I II III IV **Axis**

31 Point E (-3, 5) Quadrant **II** III IV Axis

32 Point F (0, 0) Quadrant I II III IV **Axis**

33 Point G (5, -5) Quadrant I II III IV **Axis**

34 Point H (-2, 4) Quadrant **II** III IV Axis

35 Point I (1, 2) Quadrant **I** II III IV Axis

36 Point J (1, -4) Quadrant I II III IV **Axis**

37 Point K (-3, -4) Quadrant I II III IV **Axis**

38 Point L (-1, 0) Quadrant I II III IV **Axis**

39 Point M (-4, 2) Quadrant I **II** III IV Axis

40 Point N (3, -5) Quadrant I II III IV **Axis**

41 Point O (0, -3) Quadrant I II III IV **Axis**

42 Point P (-5, -5) Quadrant I II III IV **Axis**

43 Point R (-2, 1) Quadrant I **II** III IV Axis

44 Point S (2, -2) Quadrant I II III IV **Axis**

Cartesian Plane - Drawing Animals - All Quadrants (Sheet 2)

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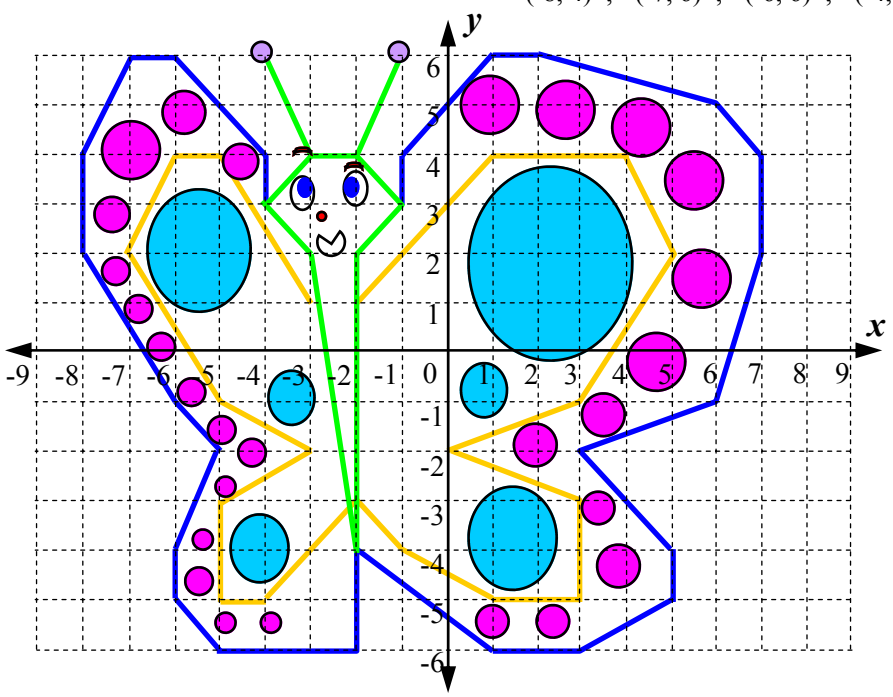
Plot the points and join them with lines to meet one of your more colourful friends

Co-ordinates: START: (-1, 3) , (-1, 4) , (1, 6) , (2, 6) , (6, 5) , (7, 4) , (7, 2) , (6, -1) , (3, -2) , (5,-4) , (5, -5) , (3, -6) , (1, -6) , (-2, -4) , (-4, -6) , (-5, -6) , (-6, -5) , (-6, -4) , (-5, -2) , (-6, -1) , (-8, 2) , (-8, 4) , (-7, 6) , (-6, 6) , (-4, 4) , (-4, 3) , STOP!



START: (-2, 1) , (1, 4) , (4, 4) , (5, 2) , (3,-1) , (0, -2) , (3, -3) , (3, -5) , (1, -5) , (-1, -4) , (-2, -3) , (-4, -5) , (-5, -5) , (-5, -3) , (-3, -2) , (-5, -1) , (-7, 2) , (-6, 4) , (-5, 4) , (-3,1) , STOP!

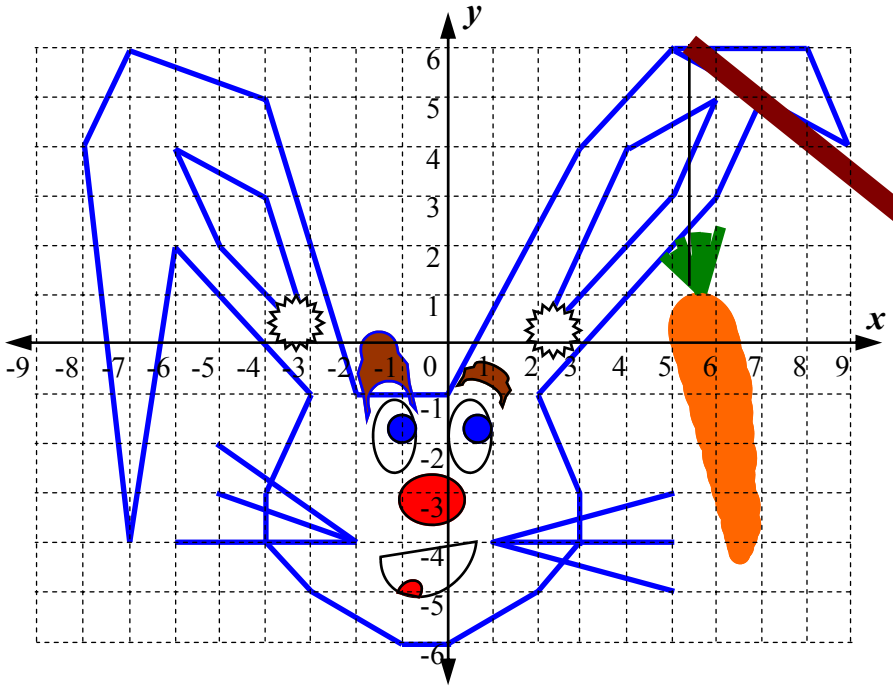
START: (-4, 3) , (-3, 4) , (-2, 4) , (-1, 3) , (-2, 2) , (-2, -4) , (-3, 2) , (-4, 3) , STOP!

START: (-3, 4) , (-4, 6) , STOP!

START: (-2, 4) , (-1, 6) , STOP!

Plot the points and join them with lines to share your lunch with your vegetarian friend.

Co-ordinates: START: (2, -1) , (3, -3) , (3, -4) , (2, -5) , (0, -6) , (-1, -6) , (-3, -5) , (-4, -4) , (-4, -3) , (-3, -1) , (-6, 2) , (-7, -4) , (-8, 4) , (-7, 6) , (-4, 5) , (-2, -1) , (0, -1) , (3, 4) , (5, 6) , (8, 6) , (9, 4) , (5, 6) , STOP!



START: (7, 5) , (6, 3) , (2, -1) , STOP!

START: (2, 0) , (4, 4) , (6, 5) , (5, 3) , (2, 0) , STOP!

START: (-3, 0) , (-5, 2) , (-6, 4) , (-4, 3) , (-3, 0) , STOP!

START: (-2, 4) , (-6, -4) , STOP!

START: (-2, -4) , (5, -2) , STOP!

START: (-2, -4) , (-5, -3) , STOP!

START: (1, -4) , (5, -4) , STOP!

START: (1, -4) , (5, -3) , STOP!

START: (1, -4) , (5, -5) , STOP!

Cartesian Plane - Plotting Points - All Quadrants (Homework Strip)

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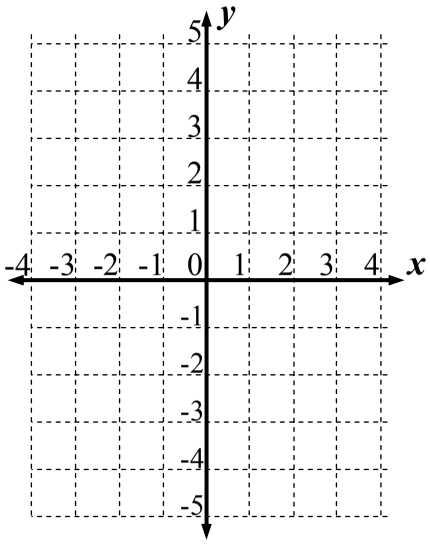
Date Due: _____

Plot and join the points, name the shape



1 (-3, 2) ; (2, 5) ; (-3, 5) .

Shape formed: _____

2 (1, 3) ; (3, 1) ; (0, -2) ; (-2, 0) .

Shape formed: _____

3 (3, -5) ; (-3, -5) ; (-4, -2) ; (2, -2) .

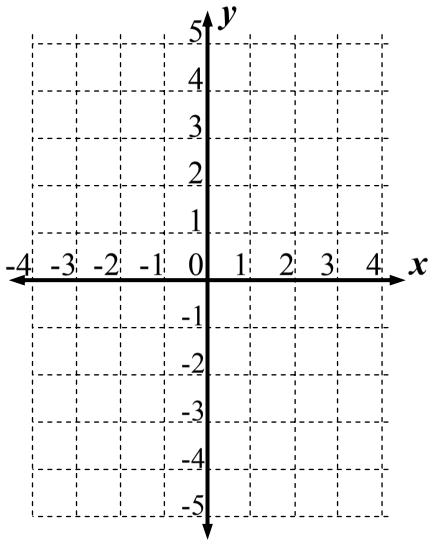
Shape formed: _____

4 (1, 1) ; (4, -3) ; (-3, -3) ; (-3, 1) .

Shape formed: _____

Date Due: _____

Plot and join the points, name the shape



1 (-3, 2) ; (2, 5) ; (-3, 5) .

Shape formed: _____

2 (1, 3) ; (3, 1) ; (0, -2) ; (-2, 0) .

Shape formed: _____

3 (3, -5) ; (-3, -5) ; (-4, -2) ; (2, -2) .

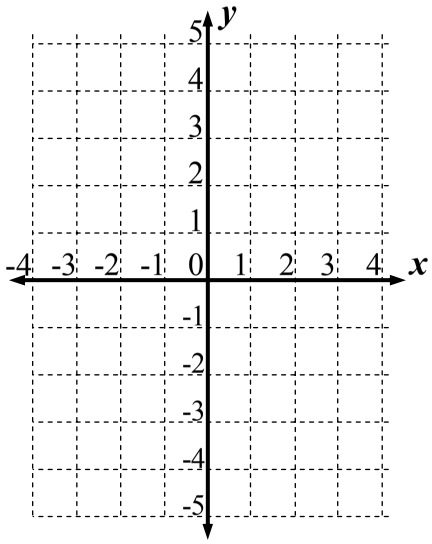
Shape formed: _____

4 (1, 1) ; (4, -3) ; (-3, -3) ; (-3, 1) .

Shape formed: _____

Date Due: _____

Plot and join the points, name the shape



1 (-3, 2) ; (2, 5) ; (-3, 5) .

Shape formed: _____

2 (1, 3) ; (3, 1) ; (0, -2) ; (-2, 0) .

Shape formed: _____

3 (3, -5) ; (-3, -5) ; (-4, -2) ; (2, -2) .

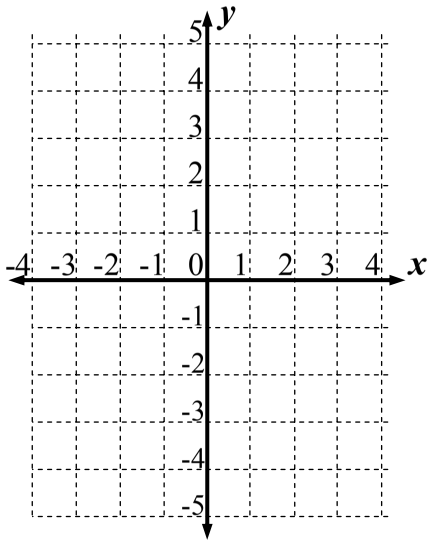
Shape formed: _____

4 (1, 1) ; (4, -3) ; (-3, -3) ; (-3, 1) .

Shape formed: _____

Date Due: _____

Plot and join the points, name the shape



1 (-3, 2) ; (2, 5) ; (-3, 5) .

Shape formed: _____

2 (1, 3) ; (3, 1) ; (0, -2) ; (-2, 0) .

Shape formed: _____

3 (3, -5) ; (-3, -5) ; (-4, -2) ; (2, -2) .

Shape formed: _____

4 (1, 1) ; (4, -3) ; (-3, -3) ; (-3, 1) .

Shape formed: _____

Cartesian Plane - Plotting Points - All Quadrants (Homework Strip)

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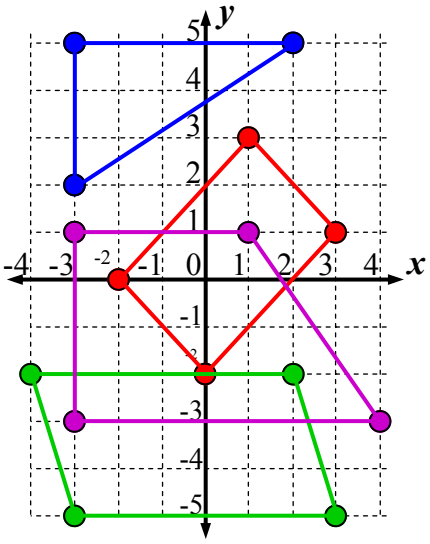
Date Due: _____

Plot and join the points, name the shape



1 (-3, 2) ; (2, 5) ; (-3, 5) .

Shape formed: Right triangle

2 (1, 3) ; (3, 1) ; (0, -2) ; (-2, 0) .

Shape formed: Rectangle

3 (3, -5) ; (-3, -5) ; (-4, -2) ; (2, -2) .

Shape formed: Parallelogram

4 (1, 1) ; (4, -3) ; (-3, -3) ; (-3, 1) .

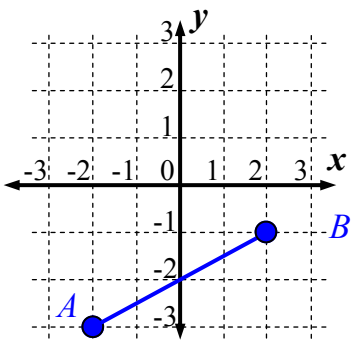
Shape formed: Trapezium

Cartesian Plane - Midpoint - All Quadrants

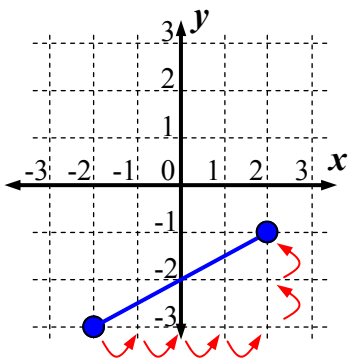
If you draw a line, halfway along that line is the midpoint. There are two common ways of finding the location of the midpoint. You can measure it's location on a number plane 'Graphically', or you can calculate its location 'Analytically'.

Graphical Solution

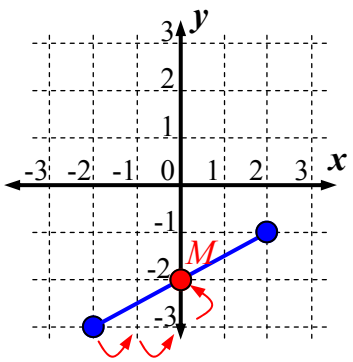
Example: Find the midpoint *M* on the interval joining *A* (-2, -3) and *B* (2, -1)



Plot *A* and *B* and draw the interval *AB*



Count the 'hops' and halve them. Horizontal is 4 hops, so midpoint is 2 hops right of the -2. So it is 0. Vertical is 2 hops, so midpoint is 1 hop above the -3. So it is -2.



Plot the midpoint and label it *M*.
There is a space in the question to record its coordinates, for the *M* above, it is (0, -2).

Analytical Method

It may be that the points aren't graphed, or the values are that large that graphing them isn't feasible. Or you just prefer using numbers, either way you can find the midpoint using this formula:

$$M(\bar{x}, \bar{y}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Where one point is (x_1, y_1), and the other point is (x_2, y_2).

What does this mean? It means that you add the *x*-values and divide them by 2, for the *x* coordinate of the midpoint. Then repeat for the *y*-values, for the *y* coordinate of the midpoint. There is an example at the bottom of Column 1 on Sheet 2.

Cartesian Plane - Midpoint (Graphical) - All Quadrants (Sheet 1)

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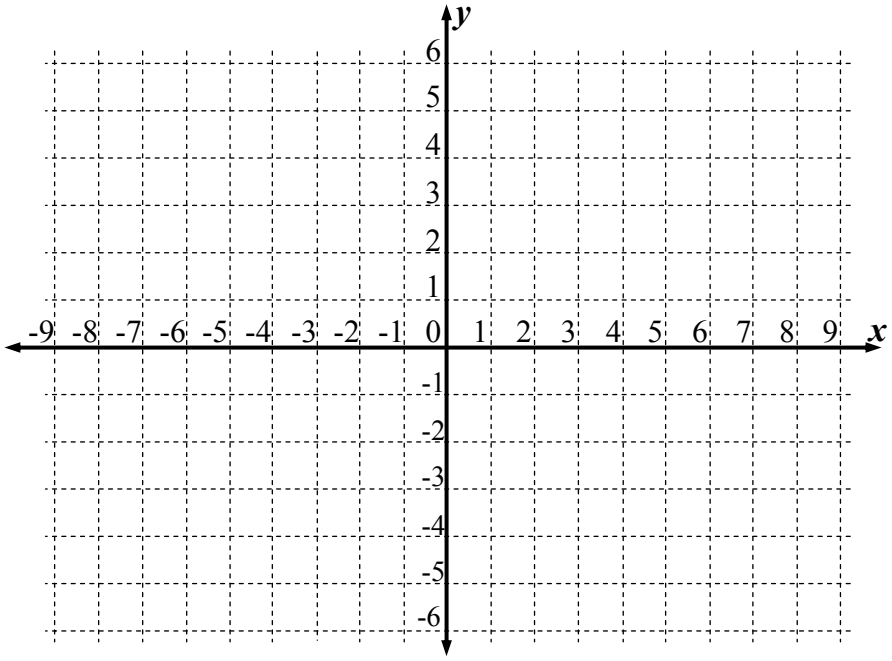
Plot the points and use a ruler to rule a line between the two points and find the midpoint. Plot the midpoint on the line record its co-ordinates. Write letters in pencil, as you may need to move them later!



Steps: Plot the points on the number plane.
Rule a line in between the two points.
Plot the midpoint, call it 'M'.
Write the co-ordinate for the midpoint.

1 *A* (-7, 5) and *H* (-3, 3).

Midpoint: (___ , ___)

Label the midpoint '*M*' on the number plane

2 *G* (8, -2) and *E* (2, -5).

Midpoint: (___ , ___)

3 *L* (-2, 6) and *F* (0, 4).

Midpoint: _____

4 *B* (7, 3) and *D* (-3, -1).

Midpoint: _____

5 *N* (-1, 2) and *P* (5, 6).

Midpoint: _____

6 *K* (-9, 3) and *H* (-3, 3).

Midpoint: _____

7 *C* (-9, -5) and *F* (0, 4).

Midpoint: _____

8 *J* (-6, -6) and *D* (-3, -1).

Midpoint: _____

9 *A* (-7, 5) and *F* (0, 4).

Midpoint: _____

10 *B* (7, 3) and *G* (8, -2).

Midpoint: _____

11 *G* (8, -2) and *D* (-3, -1).

Midpoint: _____

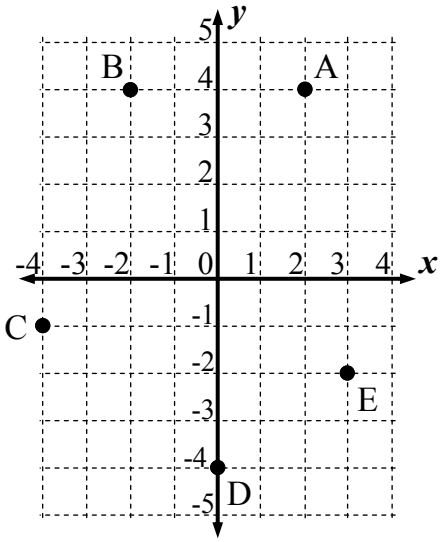
12 *C* (-9, -5) and *K* (-9, 3).

Midpoint: _____

13 *C* (-9, -5) and *E* (2, -5).

Midpoint: _____

This time the points are plotted, rule the line, find the midpoint and record all coordinates.



14 Construct *AB* and find '*M*'

A: (___ , ___) *B*: (___ , ___)

Midpoint *M*: (___ , ___)

15 Construct *AD* and find '*M*'

A: (___ , ___) *D*: (___ , ___)

Midpoint *M*: (___ , ___)

16 Construct *CB* and find '*M*'

C: (___ , ___) *B*: (___ , ___)

Midpoint *M*: (___ , ___)

17 Construct *EA* and find '*M*'

E: (___ , ___) *A*: (___ , ___)

Midpoint *M*: (___ , ___)

18 Construct *DE* and find '*M*'

D: (___ , ___) *E*: (___ , ___)

Midpoint *M*: (___ , ___)

Cartesian Plane - Midpoint (Analytical) - All Quadrants (Sheet 2)

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Find the average of these two numbers. Add them, press '=' and then divide by 2. Always write negatives in brackets.

1 -6 and -12

$$\bar{x} = \frac{(\square) + (\square)}{2} = \square$$

Calc: [−](-) 6 + [−](-) 1 2 = ÷ 2 =

2 -7 and 21

$$\bar{x} = \frac{(\square) + \square}{2} = \square$$

3 6 and -12

$\bar{x} =$

4 -4 and -9

$\bar{x} =$

With halves use '½' not '.5'

Find the midpoint of the interval that joins these two points. Find two averages.

Example

Find the midpoint of the interval joining (-4, -3) and (8, -11).

x_1 y_1 x_2 y_2

$$\bar{x} = \frac{(-4) + 8}{2} = 2$$

$$\bar{y} = \frac{(-3) + (-11)}{2} = -7$$

$M(\bar{x}, \bar{y}) = (2, -7)$

5 (2, -3) and (-6, 9).

$$\bar{x} = \frac{\square + (\square)}{2} = \square$$

$$\bar{y} = \frac{(\square) + \square}{2} = \square$$

$M(\bar{x}, \bar{y}) = (\square, \square)$

6 (-1, -6) and (-3, 10).

$$\bar{x} = \frac{(\square) + (\square)}{2} = \square$$

$$\bar{y} = \frac{(\square) + \square}{2} = \square$$

$M(\bar{x}, \bar{y}) = (\square, \square)$

7 (7, -5) and (-3, 9).

$\bar{x} =$

$\bar{y} =$

$M(\bar{x}, \bar{y}) = (\square, \square)$

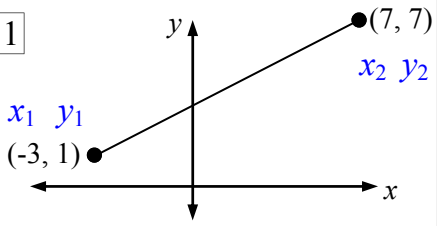
8 (-9, -2) and (-1, -9).

9 (3, 8) and (-3, -6).

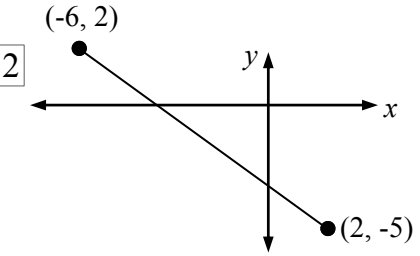
10 (0, -5) and (-10, 8).

Use the same method, but this time get your information from a sketch. Label points (x_1, y_1)…

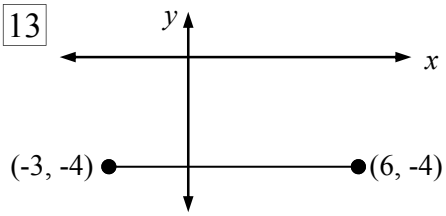
11



12



13



Cartesian Plane - Midpoint (Graphical) - All Quadrants (Sheet 1)

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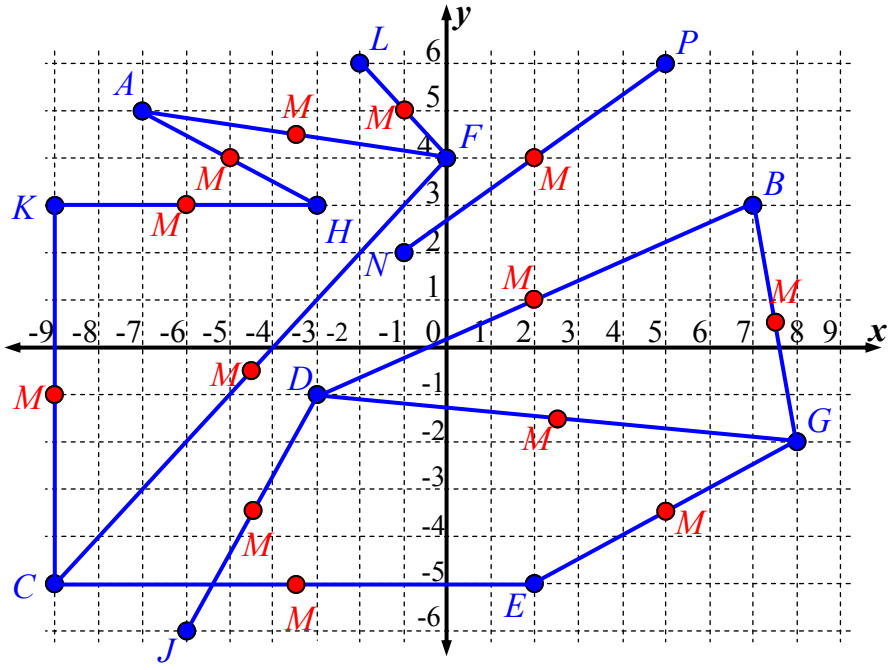
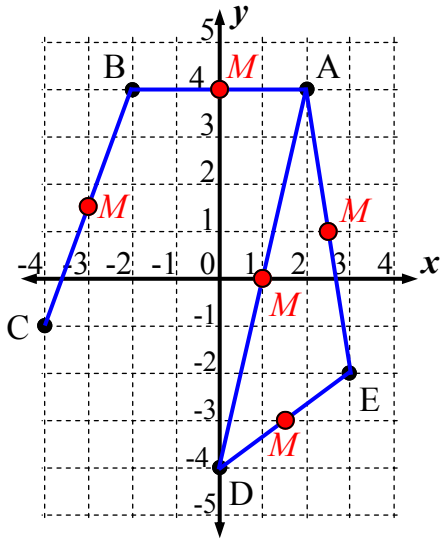
Plot the points and use a ruler to rule a line between the two points and find the midpoint. Plot the midpoint on the line record its co-ordinates. Write letters in pencil, as you may need to move them later!



This time the points are plotted, rule the line, find the midpoint and record all coordinates.



Steps: Plot the points on the number plane.
Rule a line in between the two points.
Plot the midpoint, call it 'M'.
Write the co-ordinate for the midpoint.

1 A (-7, 5) and H (-3, 3).

Midpoint: (-5 , 4)

Label the midpoint 'M' on the number plane

2 G (8, -2) and E (2, -5).

Midpoint: (5 , -3½)

3 L (-2, 6) and F (0, 4).

Midpoint: (-1, 5)

4 B (7, 3) and D (-3, -1).

Midpoint: (2, 1)

5 N (-1, 2) and P (5, 6).

Midpoint: (2, 4)

6 K (-9, 3) and H (-3, 3).

Midpoint: (-6, 3)

7 C (-9, -5) and F (0, 4).

Midpoint: (-4½, -½)

8 J (-6, -6) and D (-3, -1).

Midpoint: (-4½, -3½)

9 A (-7, 5) and F (0, 4).

Midpoint: (-3½, 4½)

10 B (7, 3) and G (8, -2).

Midpoint: (7½, ½)

11 G (8, -2) and D (-3, -1).

Midpoint: (2½, -1½)

12 C (-9, -5) and K (-9, 3).

Midpoint: (-9, -1)

13 C (-9, -5) and E (2, -5).

Midpoint: (-3½, -5)

14 Construct AB and find 'M'

A: (2 , 4) B: (-2 , 4)

Midpoint M: (0 , 4)

15 Construct AD and find 'M'

A: (2 , 4) D: (0 , -4)

Midpoint M: (1 , 0)

16 Construct CB and find 'M'

C: (-4 , -1) B: (-2 , 4)

Midpoint M: (-3 , 1½)

17 Construct EA and find 'M'

E: (3 , -2) A: (2 , 4)

Midpoint M: (2½ , 1)

18 Construct DE and find 'M'

D: (0 , -4) E: (3 , -2)

Midpoint M: (1½ , -3)

Cartesian Plane - Midpoint (Analytical) - All Quadrants (Sheet 2)

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Find the average of these two numbers. Add them, press '=' and then divide by 2. Always write negatives in brackets.

1 -6 and -12

$\bar{x} = \dfrac{(\boxed{-6}) + (\boxed{-12})}{2} = \boxed{-9}$

Calc: [−] [(-)] 6 [+] [−] [(-)] 1 2 [=] [÷] 2 [=]

2 -7 and 21

$\bar{x} = \dfrac{(\boxed{-7}) + \boxed{21}}{2} = \boxed{7}$

3 6 and -12

$\bar{x} = \dfrac{6 + (-12)}{2} = -3$

4 -4 and -9

$\bar{x} = \dfrac{(-4) + (-9)}{2} = -6\frac{1}{2}$

With halves use '½' not '.5'

Find the midpoint of the interval that joins these two points. Find two averages.

Example

Find the midpoint of the interval joining (-4, -3) and (8, -11).

$x_1\ y_1 \qquad x_2\ y_2$

$\bar{x} = \dfrac{(-4) + 8}{2} = 2$

$\bar{y} = \dfrac{(-3) + (-11)}{2} = -7$

$M(\bar{x}, \bar{y}) = (2, -7)$

5 (2, -3) and (-6, 9).

$x_1\ y_1 \qquad x_2\ y_2$

$\bar{x} = \dfrac{\boxed{2} + (\boxed{-6})}{2} = \boxed{-2}$

$\bar{y} = \dfrac{(\boxed{-3}) + \boxed{9}}{2} = \boxed{3}$

$M(\bar{x}, \bar{y}) = (\boxed{-2}, \boxed{3})$

6 (-1, -6) and (-3, 10).

$x_1\ y_1 \qquad x_2\ y_2$

$\bar{x} = \dfrac{(\boxed{-1}) + (\boxed{-3})}{2} = \boxed{-2}$

$\bar{y} = \dfrac{(\boxed{-6}) + \boxed{10}}{2} = \boxed{2}$

$M(\bar{x}, \bar{y}) = (\boxed{-2}, \boxed{2})$

7 (7, -5) and (-3, 9).

$x_1\ y_1 \qquad x_2\ y_2$

$\bar{x} = \dfrac{7 + (-3)}{2} = 2$

$\bar{y} = \dfrac{(-5) + 9}{2} = 2$

$M(\bar{x}, \bar{y}) = (\boxed{2}, \boxed{2})$

8 (-9, -2) and (-1, -9).

$x_1\ y_1 \qquad x_2\ y_2$

$\bar{x} = \dfrac{(-9) + (-1)}{2} = -5$

$\bar{y} = \dfrac{(-2) + (-9)}{2} = -5\frac{1}{2}$

$M(\bar{x}, \bar{y}) = (-5, -5\frac{1}{2})$

9 (3, 8) and (-3, -6).

$x_1\ y_1 \qquad x_2\ y_2$

$\bar{x} = \dfrac{3 + (-3)}{2} = 0$

$\bar{y} = \dfrac{8 + (-6)}{2} = 1$

$M(\bar{x}, \bar{y}) = (0, 1)$

10 (0, -5) and (-10, 8).

$x_1\ y_1 \qquad x_2\ y_2$

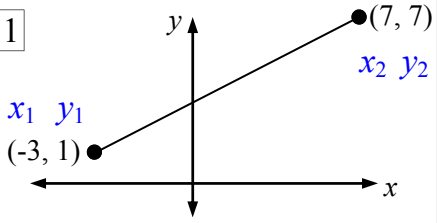
$\bar{x} = \dfrac{0 + (-10)}{2} = -5$

$\bar{y} = \dfrac{(-5) + 8}{2} = 1\frac{1}{2}$

$M(\bar{x}, \bar{y}) = (-5, 1\frac{1}{2})$

Use the same method, but this time get your information from a sketch. Label points (x_1, y_1)…

11

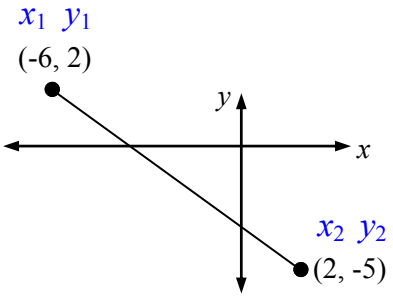


$\bar{x} = \dfrac{(-3) + 7}{2} = 2$

$\bar{y} = \dfrac{1 + 7}{2} = 4$

$M(\bar{x}, \bar{y}) = (2, 4)$

12

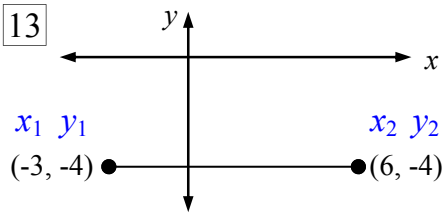


$\bar{x} = \dfrac{(-6) + 2}{2} = -2$

$\bar{y} = \dfrac{2 + (-5)}{2} = -1\frac{1}{2}$

$M(\bar{x}, \bar{y}) = (-2, -1\frac{1}{2})$

13



$\bar{x} = \dfrac{(-3) + 6}{2} = 1\frac{1}{2}$

$\bar{y} = \dfrac{(-4) + (-4)}{2} = -4$

$M(\bar{x}, \bar{y}) = (1\frac{1}{2}, -4)$

Cartesian Plane - Midpoint - All Quadrants (Homework Strip)

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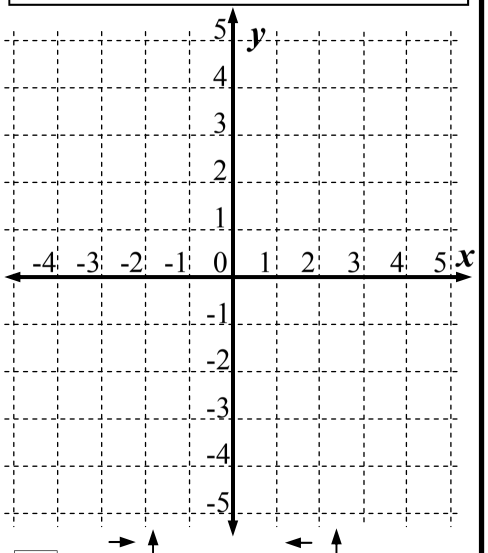
Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 *A* (2, 5) and *B* (-4, 3).

Midpoint: (,)

2 *C* (2, -5) and *D* (-2, -1).

Midpoint: (,)

3 *E* (2, -2) and *F* (5, 4).

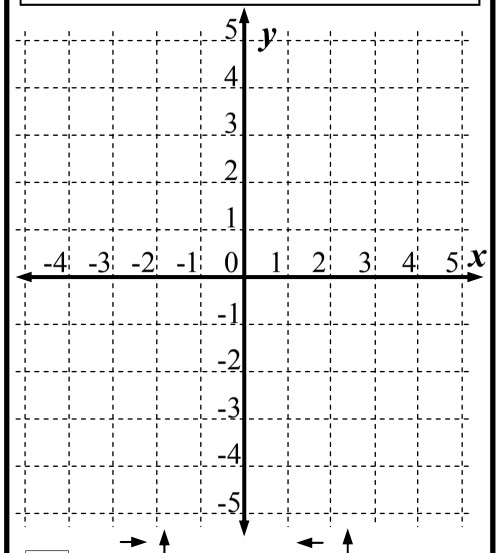
Midpoint: (,)

4 *G* (-3, 2) and *H* (-4, -4).

Midpoint: (,)

Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 *A* (2, 5) and *B* (-4, 3).

Midpoint: (,)

2 *C* (2, -5) and *D* (-2, -1).

Midpoint: (,)

3 *E* (2, -2) and *F* (5, 4).

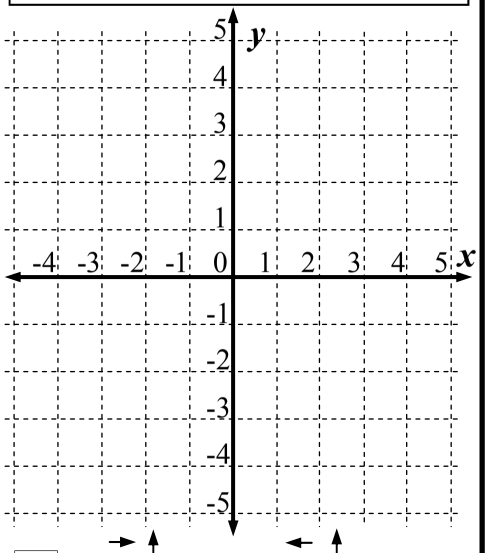
Midpoint: (,)

4 *G* (-3, 2) and *H* (-4, -4).

Midpoint: (,)

Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 *A* (2, 5) and *B* (-4, 3).

Midpoint: (,)

2 *C* (2, -5) and *D* (-2, -1).

Midpoint: (,)

3 *E* (2, -2) and *F* (5, 4).

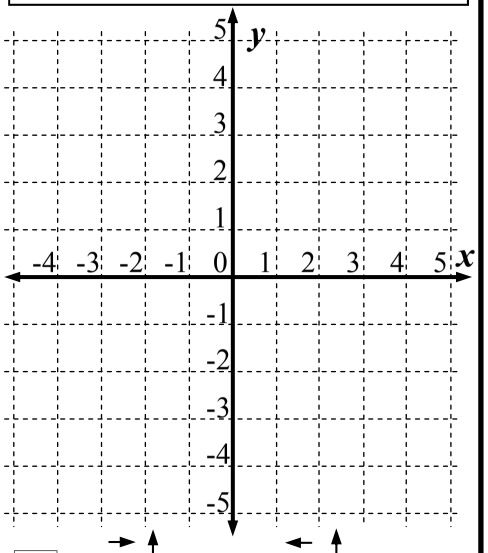
Midpoint: (,)

4 *G* (-3, 2) and *H* (-4, -4).

Midpoint: (,)

Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 *A* (2, 5) and *B* (-4, 3).

Midpoint: (,)

2 *C* (2, -5) and *D* (-2, -1).

Midpoint: (,)

3 *E* (2, -2) and *F* (5, 4).

Midpoint: (,)

4 *G* (-3, 2) and *H* (-4, -4).

Midpoint: (,)

Cartesian Plane - Midpoint - All Quadrants (Homework Strip)

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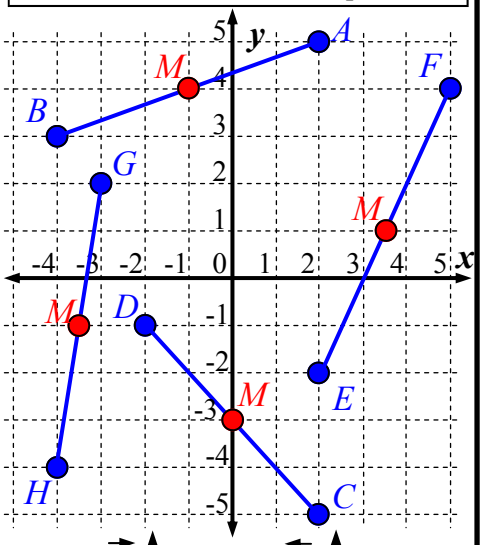
Date Due: _____

Plot and label the points, plot and give the coordinates for the midpoint *M*.



1 *A* (2, 5) and *B* (-4, 3).

Midpoint: (-1 , 4)

2 *C* (2, -5) and *D* (-2, -1).

Midpoint: (0 , -3)

3 *E* (2, -2) and *F* (5, 4).

Midpoint: (3½ , 1)

4 *G* (-3, 2) and *H* (-4, -4).

Midpoint: (-3½ , -1)

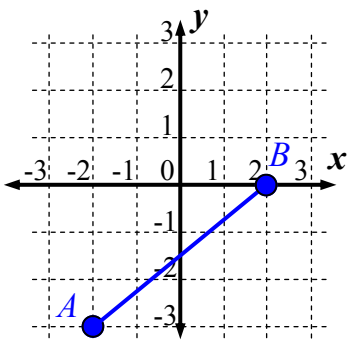
Cartesian Plane - Introduction to Gradient

Gradient is a measure of slope. So the larger the number, the steeper the interval is. A gradient of 5 is much steeper than a gradient of 1. Gradient is found creating a 'gradient triangle' and measuring the rise over the run, the vertical length over the horizontal. Gradient is given the symbol '*m*', it has nothing to do with the midpoint.

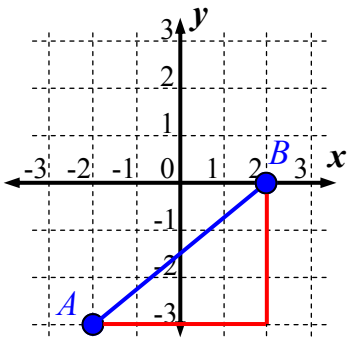
$$m = \frac{Rise}{Run} = \frac{Vertical\ Distance}{Horizontal\ Distance}$$

Here is an example of finding the gradient.

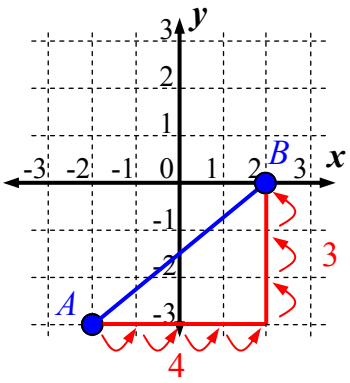
Example: Find the gradient of the interval joining *A* (-2, -3) and *B* (2, 0)



Plot *A* and *B* and draw the interval *AB*



Construct a gradient triangle, which is a right angle triangle which has AB as its hypotenuse.



Count and mark the rise and the run on the sides of the triangle.

$m = \frac{3}{4}$ ← Find the gradient by writing the rise over the run. Remember that the second sheet has all negative gradients

The process for finding the gradient is straight forward. The only issue is you can have positive and negative gradient. Positive Gradient (covered in the first sheet) 'Goes up to the right', Negative Gradient (covered in the second sheet) goes 'up to the left'.

Positive Gradient goes up to the right

Negative Gradient goes up to the left

Note that when two (or more) intervals have the same gradient, they are parallel.

Cartesian Plane - Introduction to Gradient (Positive) (Sheet 1)

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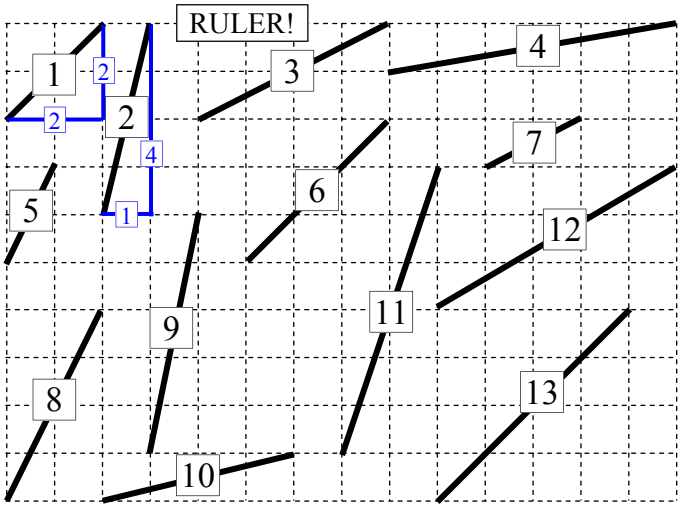
RULER!



Create 'gradient triangles' for the above intervals. Then write the 'rise' and the 'run' on each triangle. The first two are done. Find the gradient for each.

1 $m = \frac{\square}{\square} = \square$

2 $m = \frac{\square}{\square} = \square$

3 $m = \frac{\square}{\square} = \frac{\square}{\square}$

4 $m = \frac{\square}{\square}$

Calc: 2 ▪ 4 =

5 $m = \frac{\square}{\square} = \square$

6 $m = \frac{\square}{\square} = \square$

7 $m = \frac{\square}{\square}$

8 $m = \frac{\square}{\square} = \square$

9 $m = \frac{\square}{\square} = \square$

10 $m = \frac{\square}{\square}$

11 $m = \frac{\square}{\square} = \square$

12 $m = \frac{\square}{\square}$

13 $m = \frac{\square}{\square} = \square$

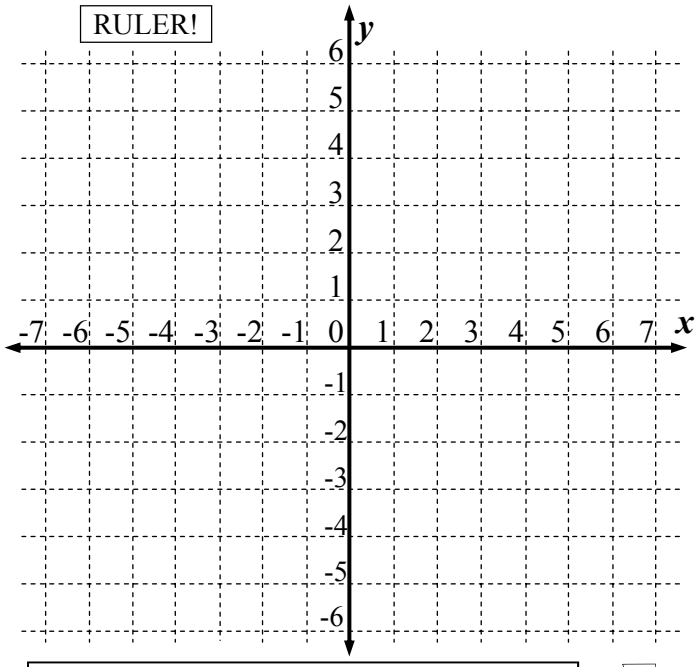
14 Which of the intervals are parallel (have the same gradient)?

a) Q3 || Q□

b) Q5 || Q□

c) Q1 || Q□ || Q□

RULER!



Plot the coordinates, then join them with an interval. Calculate the gradient for the interval by creating a gradient triangle. Use a ruler.

15 *A* (1, 2) and *B* (2, 6). $m = \frac{\square}{\square} = \square$

16 *C* (1, -2) and *D* (7, 4). $m = \frac{\square}{\square} = \square$

17 *E* (-7, -5) and *F* (-5, 3). $m = \frac{\square}{\square} = \square$

18 *G* (-7, 4) and *H* (-3, 6). $m = \frac{\square}{\square} = \frac{\square}{\square}$

19 *J* (-2, -6) and *K* (6, -2). $m = \frac{\square}{\square} = \frac{\square}{\square}$

20 *Q* (-5, -3) and *R* (-1, 1). $m = \frac{\square}{\square} = \square$

21 Which of the intervals are parallel? Use the letters on the interval to complete these:

GH || □ (2 letters) *AB* || □ *CD* || □

Cartesian Plane - Introduction to Gradient (Negative) (Sheet 2)

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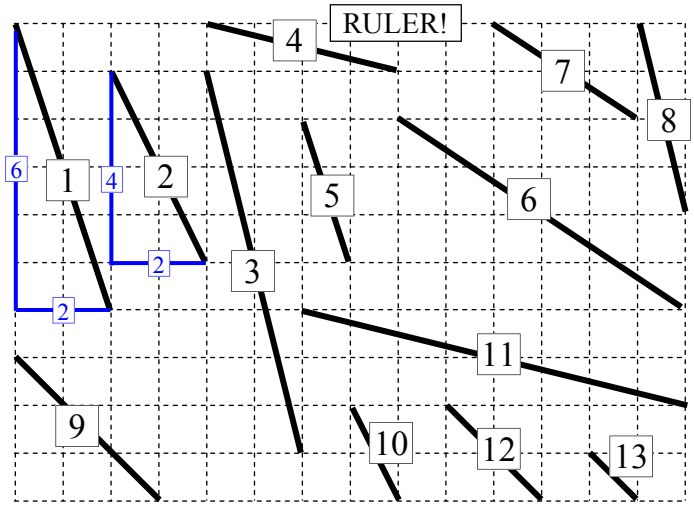
RULER!



Create 'gradient triangles' and write the 'rise' and the 'run' on each triangle, all positive numbers. Find the gradient and write a negative sign '-' in front of your answer.

1 $m =$ ☐/☐ $=$ ☐

2 $m =$ ☐/☐ $=$ ☐

3 $m =$ ☐/☐ $=$ ☐

4 $m =$ ☐/☐

5 $m =$ ☐/☐ $=$ ☐

6 $m =$ ☐/☐ $=$ ☐/☐

7 $m =$ ☐/☐

8 $m =$ ☐/☐ $=$ ☐

9 $m =$ ☐/☐ $=$ ☐

10 $m =$ ☐/☐ $=$ ☐

11 $m =$ ☐/☐ $=$ ☐/☐

12 $m =$ ☐/☐ $=$ ☐

13 $m =$ ☐/☐ $=$ ☐

14 Which of the intervals are parallel?

a) Q1 || Q☐

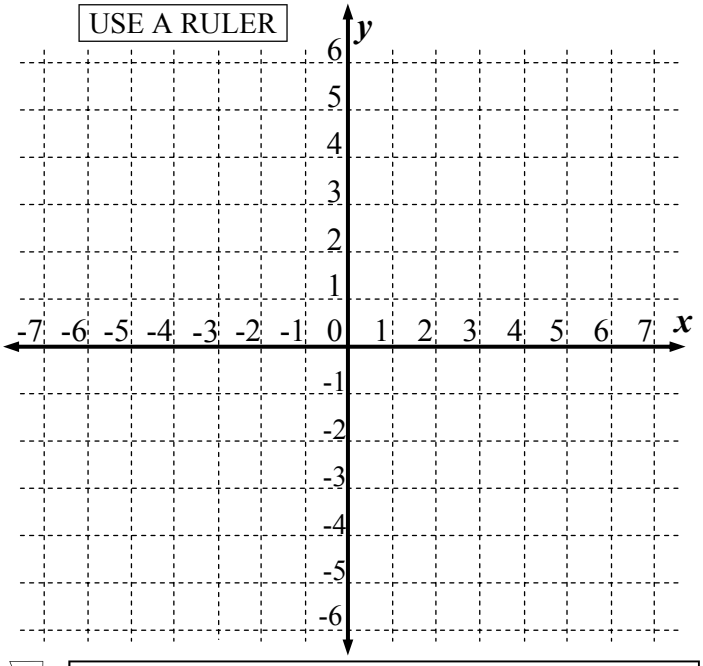
b) Q2 || Q☐

c) Q3 || Q☐

d) Q4 || Q☐

e) Q9 || Q☐ || Q☐

USE A RULER



Plot the coordinates, then join them with an interval. Calculate the gradient for the interval by creating a gradient triangle. **Use positive numbers** on the gradient triangles, but final answers will be negative.

15 A (6, 6) and B (7, 1). $m =$ ☐/☐ $=$ ☐

16 C (0, 5) and D (5, 0). $m =$ ☐/☐ $=$ ☐

17 E (-3, 3) and F (6, -3). $m =$ ☐/☐ $=$ ☐/☐

18 G (-5, 6) and H (-3, -4). $m =$ ☐/☐ $=$ ☐

19 J (-7, -2) and K (-3, -6). $m =$ ☐/☐ $=$ ☐

20 L (1, 6) and P (4, 4). $m =$ ☐/☐

21 Which of the intervals are parallel? Use the letters on the interval to complete these:

AB || ☐ EF || ☐ JK || ☐

Cartesian Plane - Introduction to Gradient (Positive) (Sheet 1)

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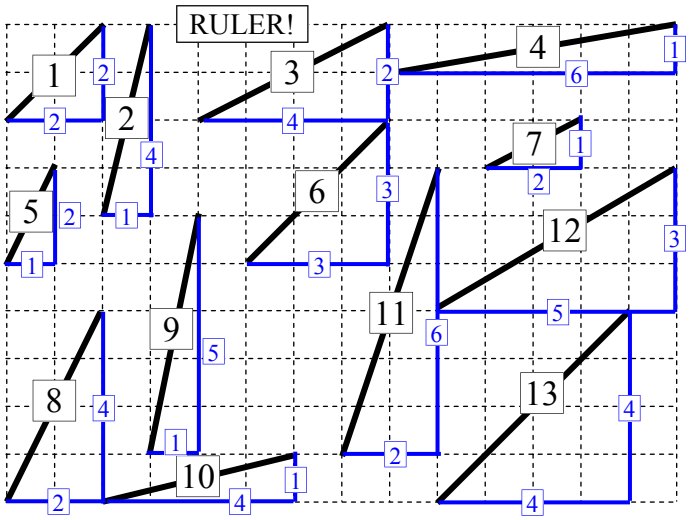


Create 'gradient triangles' for the above intervals. Then write the 'rise' and the 'run' on each triangle. The first two are done. Find the gradient for each.

1 $m = \frac{2}{2} = 1$

2 $m = \frac{4}{1} = 4$

3 $m = \frac{2}{4} = \frac{1}{2}$

4 $m = \frac{1}{6}$

Calc: 2 ▪/□ 4 =

5 $m = \frac{2}{1} = 2$

6 $m = \frac{3}{3} = 1$

7 $m = \frac{1}{2}$

8 $m = \frac{4}{2} = 2$

9 $m = \frac{5}{1} = 5$

10 $m = \frac{1}{4}$

11 $m = \frac{6}{2} = 3$

12 $m = \frac{3}{5}$

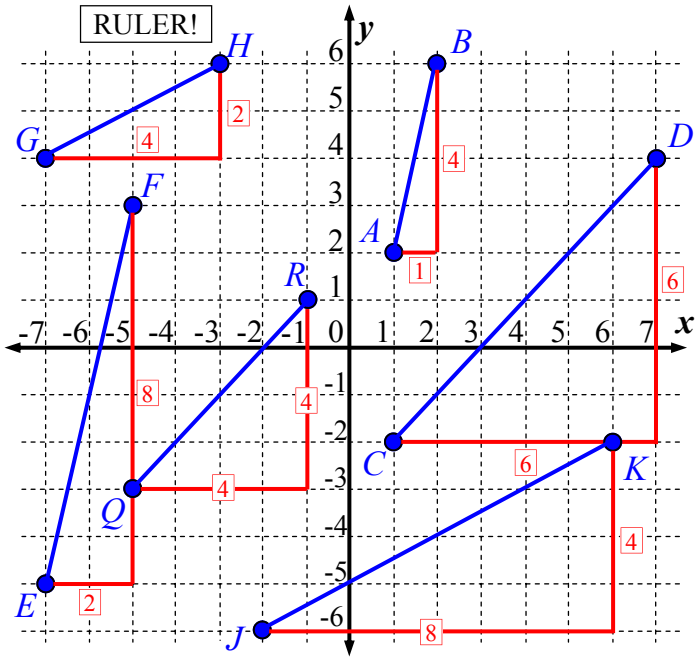
13 $m = \frac{4}{4} = 1$

14 Which of the intervals are parallel (have the same gradient)?

a) Q3 || Q 7

b) Q5 || Q 8

c) Q1 || Q 6 || Q 13



Plot the coordinates, then join them with an interval. Calculate the gradient for the interval by creating a gradient triangle. Use a ruler.

15 *A* (1, 2) and *B* (2, 6). $m = \frac{4}{1} = 4$

16 *C* (1, -2) and *D* (7, 4). $m = \frac{6}{6} = 1$

17 *E* (-7, -5) and *F* (-5, 3). $m = \frac{8}{2} = 4$

18 *G* (-7, 4) and *H* (-3, 6). $m = \frac{2}{4} = \frac{1}{2}$

19 *J* (-2, -6) and *K* (6, -2). $m = \frac{4}{8} = \frac{1}{2}$

20 *Q* (-5, -3) and *R* (-1, 1). $m = \frac{4}{4} = 1$

21 Which of the intervals are parallel? Use the letters on the interval to complete these:

GH || *JK* (2 letters) *AB* || *EF* *CD* || *QR*

Cartesian Plane - Introduction to Gradient (Negative) (Sheet 2)

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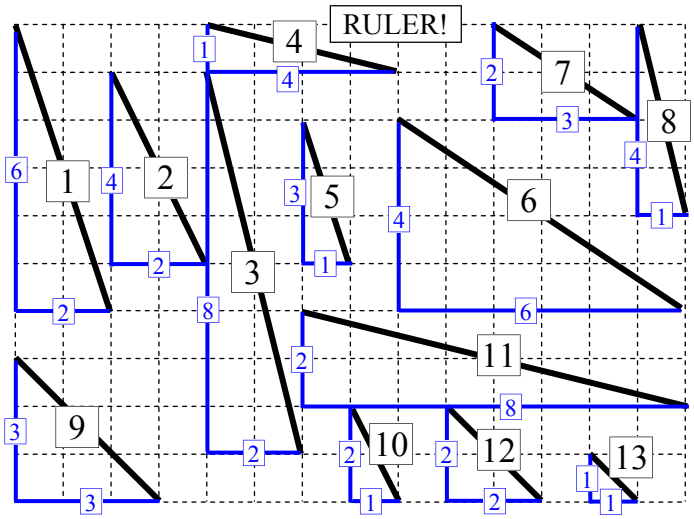


Create 'gradient triangles' and write the 'rise' and the 'run' on each triangle, all positive numbers. Find the gradient and write a negative sign '-' in front of your answer.

1 $m = \frac{6}{2} = -3$

2 $m = \frac{4}{2} = -2$

3 $m = \frac{8}{2} = -4$

4 $m = \frac{-1}{4}$

5 $m = \frac{3}{1} = -3$

6 $m = \frac{4}{6} = \frac{-2}{3}$

7 $m = \frac{-2}{3}$

8 $m = \frac{4}{1} = -4$

9 $m = \frac{3}{3} = -1$

10 $m = \frac{2}{1} = -2$

11 $m = \frac{2}{8} = \frac{-1}{4}$

12 $m = \frac{2}{2} = -1$

13 $m = \frac{1}{1} = -1$

14 Which of the intervals are parallel?

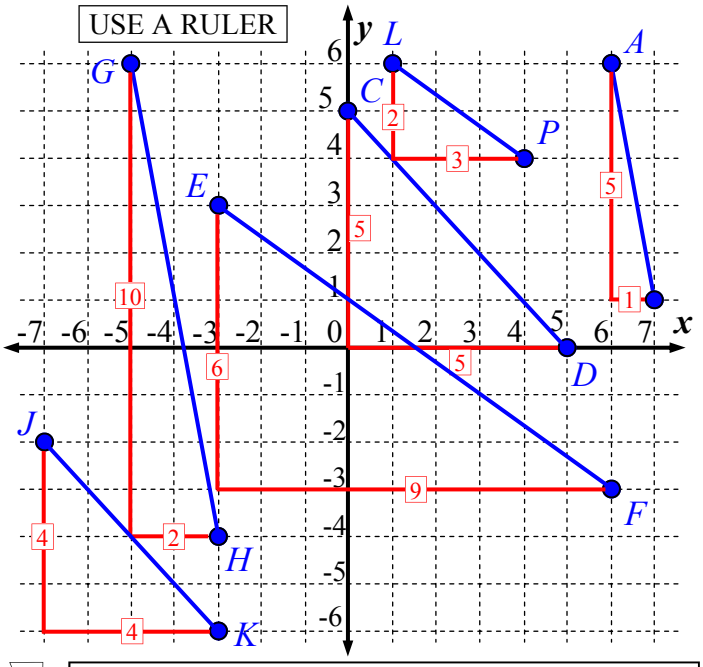
a) Q1 || Q 5

b) Q2 || Q 10

c) Q3 || Q 8

d) Q4 || Q 11

e) Q9 || Q 12 || Q 13



Plot the coordinates, then join them with an interval. Calculate the gradient for the interval by creating a gradient triangle. **Use positive numbers** on the gradient triangles, but final answers will be negative.

15 A (6, 6) and B (7, 1). $m = \frac{5}{1} = -5$

16 C (0, 5) and D (5, 0). $m = \frac{5}{5} = -1$

17 E (-3, 3) and F (6, -3). $m = \frac{6}{9} = \frac{-2}{3}$

18 G (-5, 6) and H (-3, -4). $m = \frac{10}{2} = -5$

19 J (-7, -2) and K (-3, -6). $m = \frac{4}{4} = -1$

20 L (1, 6) and P (4, 4). $m = \frac{-2}{3}$

21 Which of the intervals are parallel? Use the letters on the interval to complete these:

AB || GH EF || LP JK || CD

Cartesian Plane - Introduction to Gradient (Homework Strip)

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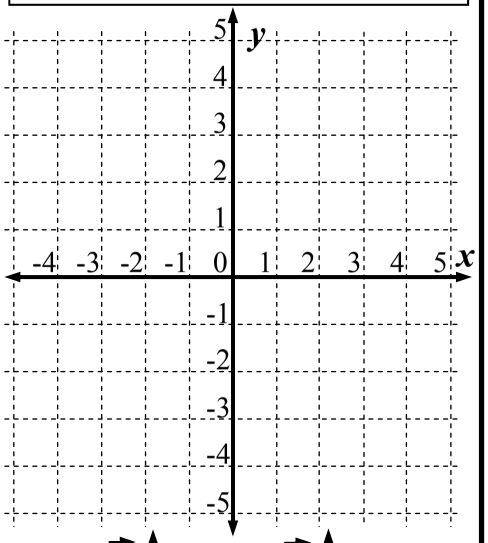
Date Due: _____

Plot and label the points, construct an interval and find its gradient.



1 $A\ (2, 2)$ and $B\ (5, 4)$.

$$m = \frac{\square}{\square}$$

2 $E\ (-3, -4)$ and $F\ (-1, 4)$.

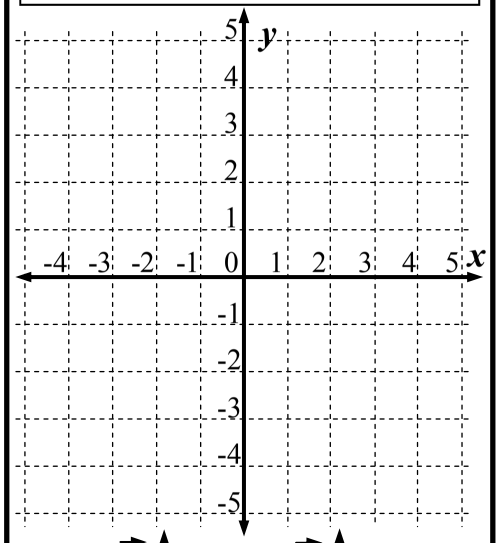
$$m = \frac{\square}{\square} = \square$$

3 $T\ (5, -1)$ and $V\ (1, -5)$.

$$m = \frac{\square}{\square} = \square$$

Date Due: _____

Plot and label the points, construct an interval and find its gradient.



1 $A\ (2, 2)$ and $B\ (5, 4)$.

$$m = \frac{\square}{\square}$$

2 $E\ (-3, -4)$ and $F\ (-1, 4)$.

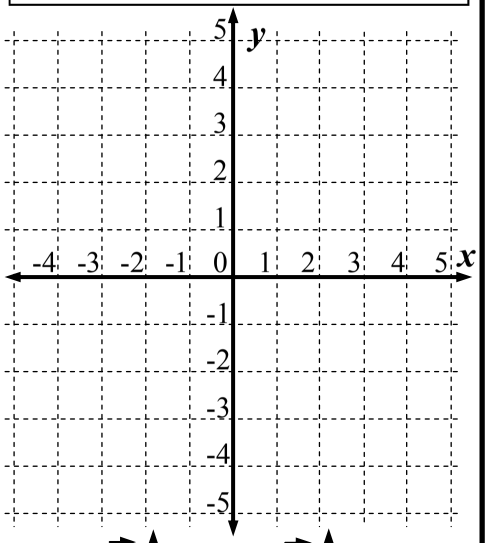
$$m = \frac{\square}{\square} = \square$$

3 $T\ (5, -1)$ and $V\ (1, -5)$.

$$m = \frac{\square}{\square} = \square$$

Date Due: _____

Plot and label the points, construct an interval and find its gradient.



1 $A\ (2, 2)$ and $B\ (5, 4)$.

$$m = \frac{\square}{\square}$$

2 $E\ (-3, -4)$ and $F\ (-1, 4)$.

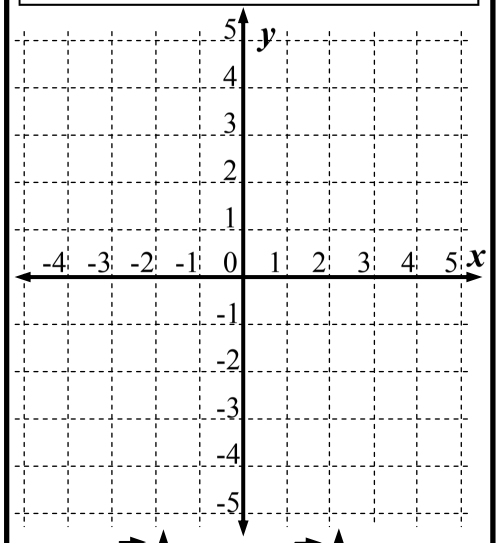
$$m = \frac{\square}{\square} = \square$$

3 $T\ (5, -1)$ and $V\ (1, -5)$.

$$m = \frac{\square}{\square} = \square$$

Date Due: _____

Plot and label the points, construct an interval and find its gradient.



1 $A\ (2, 2)$ and $B\ (5, 4)$.

$$m = \frac{\square}{\square}$$

2 $E\ (-3, -4)$ and $F\ (-1, 4)$.

$$m = \frac{\square}{\square} = \square$$

3 $T\ (5, -1)$ and $V\ (1, -5)$.

$$m = \frac{\square}{\square} = \square$$

Cartesian Plane - Introduction to Gradient (Homework Strip)

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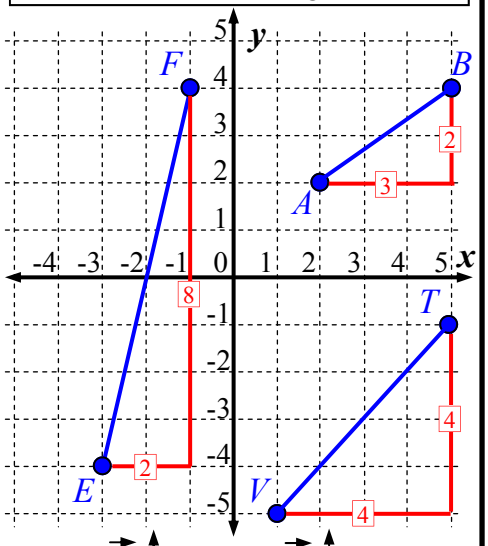
Date Due:

Plot and label the points, construct an interval and find its gradient.



1 $A\ (2, 2)$ and $B\ (5, 4)$.

$$m = \frac{2}{3}$$

2 $E\ (-3, -4)$ and $F\ (-1, 4)$.

$$m = \frac{8}{2} = 4$$

3 $T\ (5, -1)$ and $V\ (1, -5)$.

$$m = \frac{4}{4} = 1$$

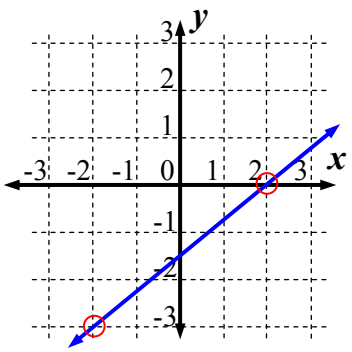
Cartesian Plane - Gradient

Gradient is a measure of slope. So the larger the number, the steeper the line is. A gradient of 5 is much steeper than a gradient of 1. Gradient is found creating a 'gradient triangle' and measuring the rise over the run, the vertical length over the horizontal. Gradient is given the symbol '*m*', it has nothing to do with the midpoint.

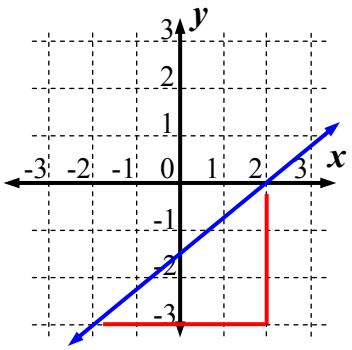
$$m = \frac{Rise}{Run} = \frac{Vertical\ Distance}{Horizontal\ Distance}$$

Here is an example of finding the gradient.

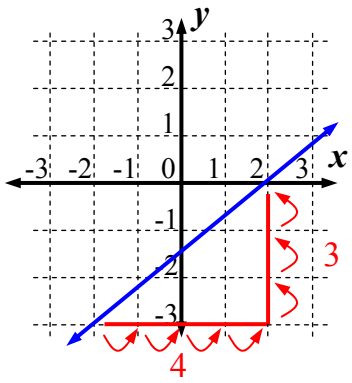
Example: Find the gradient of the line below.



Locate where the line cuts through exact coordinates. (in red)



Construct a gradient triangle, which is a right angle triangle with the line as the hypotenuse



Count and mark the rise and the run on the sides of the triangle.

$$m = \frac{3}{4}$$

Find the gradient by writing the rise over the run. Then finally, check if it is a positive or negative slope.

The process for finding the gradient is straight forward. The only issue is you can have positive and negative gradient. Positive Gradient 'Goes up to the right', Negative Gradient' goes 'up to the left'.

Positive Gradient goes up to the right

Negative Gradient goes up to the left

If the line is flat, it has a gradient of zero ($m = 0$). If the line is vertical (straight up and down), it has a gradient that is undefined (m = undef).

Cartesian Plane - Gradient (Sheet 1)

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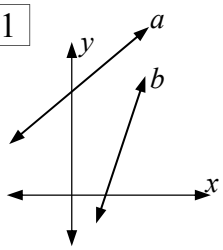
State whether the following have a positive or negative gradient.

Up to the left is -ve

Up to the right is +ve

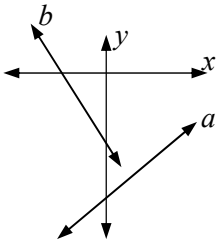
1



a) _____ Positive or Negative

b) _____ Positive or Negative

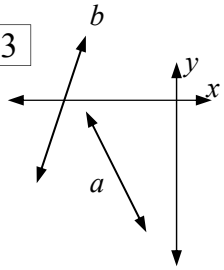
2



a) _____

b) _____

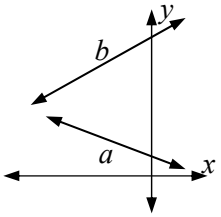
3



a) _____

b) _____

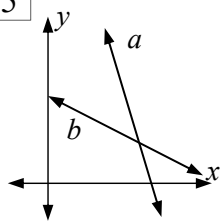
4



a) _____

b) _____

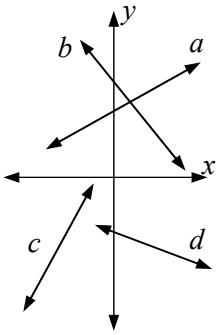
5



a) _____

b) _____

6



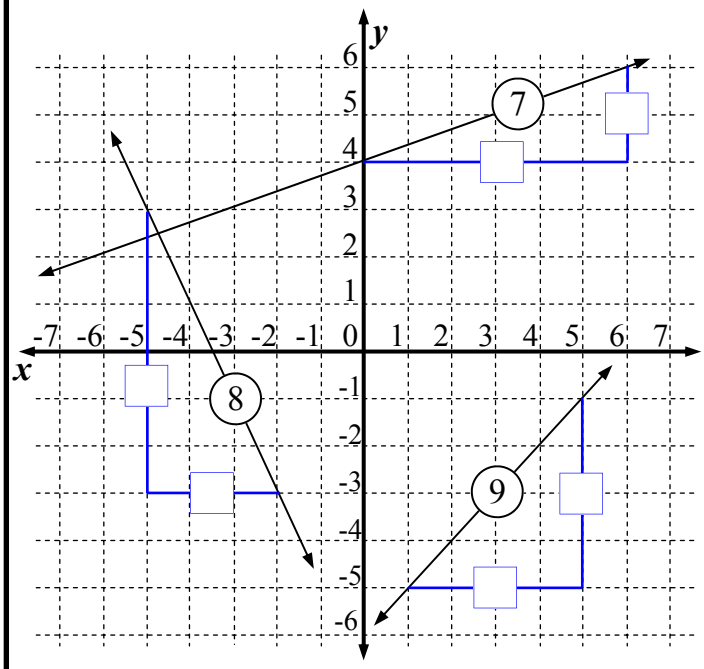
a) _____

b) _____

c) _____

d) _____

These lines have gradient triangles installed! Write the rise and run in the boxes on the triangles sides, then find the gradient. Make sure you remember to check for positive and negative gradient at the end.



7 $m = \frac{\square}{\square} = \frac{\square}{\square}$

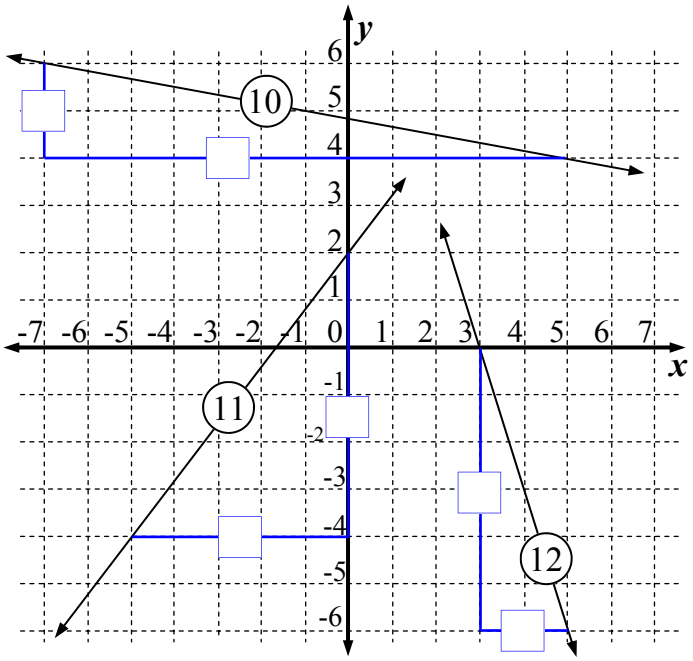
8 $m = \frac{\square}{\square} = \square$

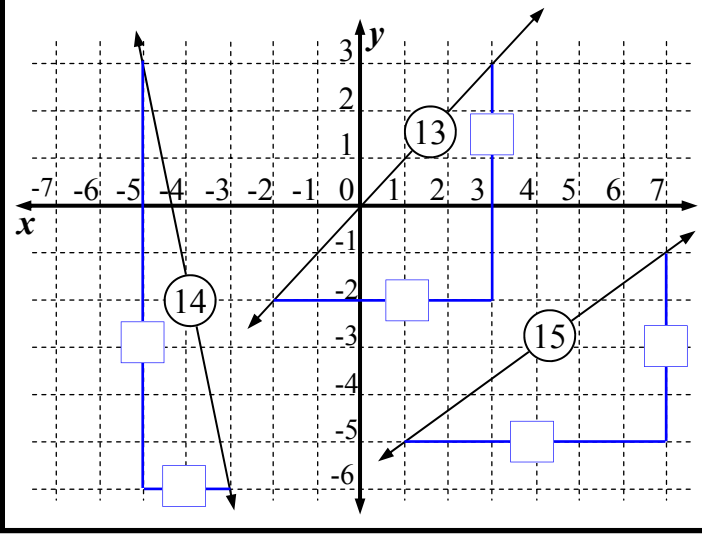
9 $m = \frac{\square}{\square} = \square$

10 $m = \frac{\square}{\square} = \frac{\square}{\square}$

11 $m = \frac{\square}{\square}$

12 $m = \frac{\square}{\square} = \square$





13 $m = \frac{\square}{\square} = \square$

14 $m = \frac{\square}{\square}$

15 $m = \frac{\square}{\square} = \frac{\square}{\square}$

Cartesian Plane - Gradient (Sheet 2)

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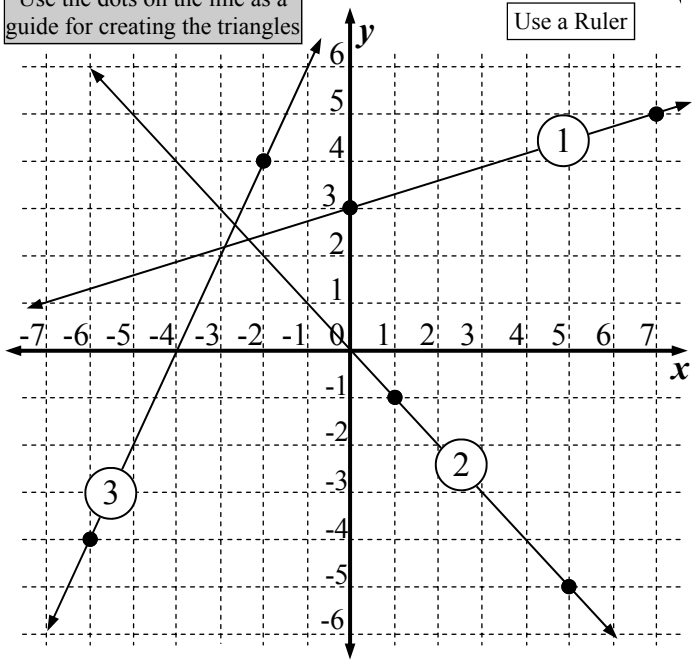
Find the gradient for these lines. Create gradient triangles and remember your positive and negatives.

Use the dots on the line as a guide for creating the triangles

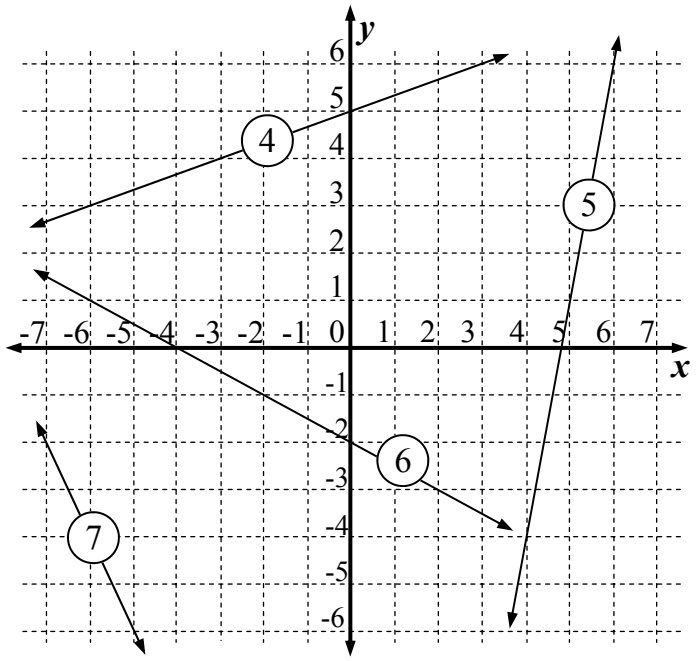
Use a Ruler



1 $m = \frac{\square}{\square}$

2 $m = \frac{\square}{\square} = \square$

3 $m = \frac{\square}{\square} = \square$

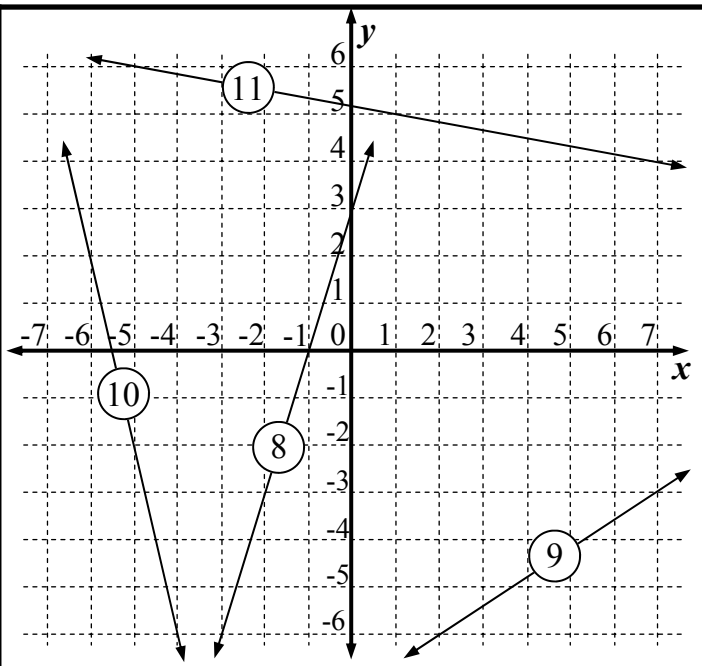


4 $m = \frac{\square}{\square} = \frac{\square}{\square}$

5 $m = \frac{\square}{\square} = \square$

6 $m = \frac{\square}{\square} = \frac{\square}{\square}$

7 $m = \frac{\square}{\square} = \square$

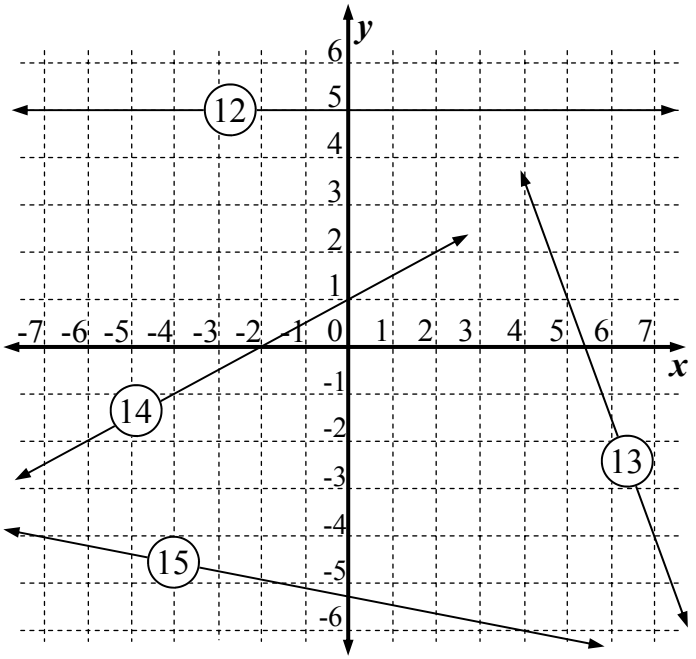


8 $m =$ _____

9 $m =$ _____

10 _____

11 _____



12 _____

13 _____

14 _____

15 _____

Cartesian Plane - Gradient (Sheet 1)

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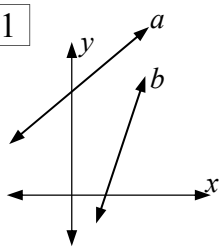
State whether the following have a positive or negative gradient.

Up to the left is -ve

Up to the right is +ve

1



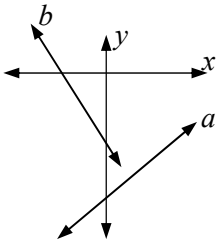
a) Positive

Positive or Negative

b) Positive

Positive or Negative

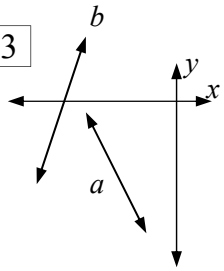
2



a) Positive

b) Negative

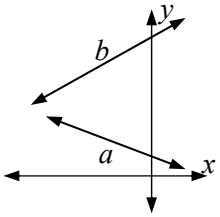
3



a) Negative

b) Positive

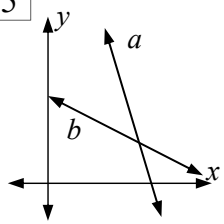
4



a) Negative

b) Positive

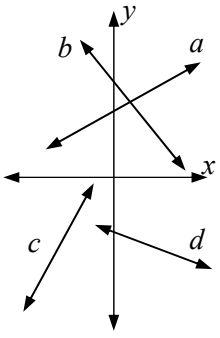
5



a) Negative

b) Negative

6



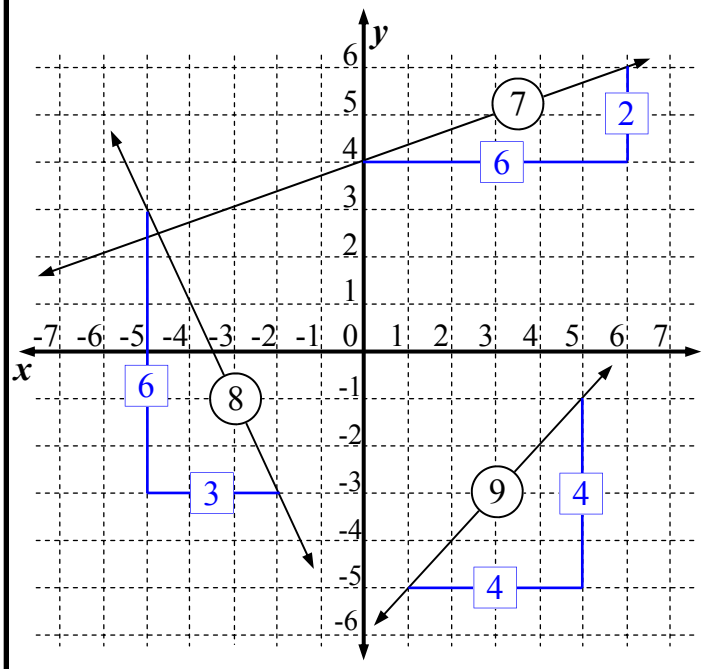
a) Positive

b) Negative

c) Positive

d) Negative

These lines have gradient triangles installed! Write the rise and run in the boxes on the triangles sides, then find the gradient. Make sure you remember to check for positive and negative gradient at the end.



7 $m = \frac{2}{6} = \frac{1}{3}$

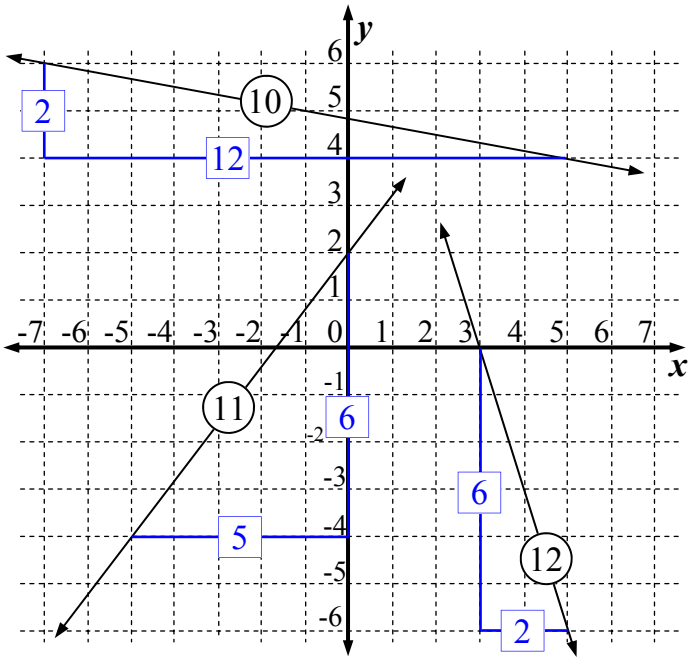
8 $m = \frac{6}{3} = -2$

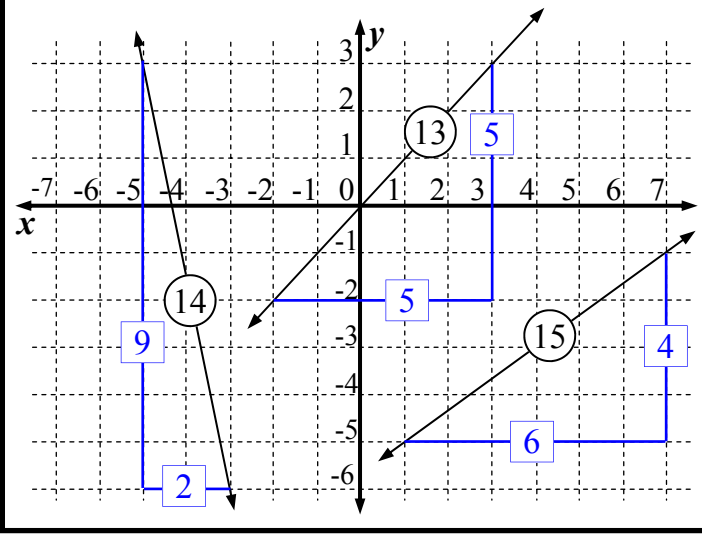
9 $m = \frac{4}{4} = 1$

10 $m = \frac{2}{12} = \frac{-1}{6}$

11 $m = \frac{6}{5}$

12 $m = \frac{6}{2} = -3$





13 $m = \frac{5}{5} = 1$

14 $m = \frac{-9}{2}$

15 $m = \frac{4}{6} = \frac{2}{3}$

Cartesian Plane - Gradient (Sheet 2)

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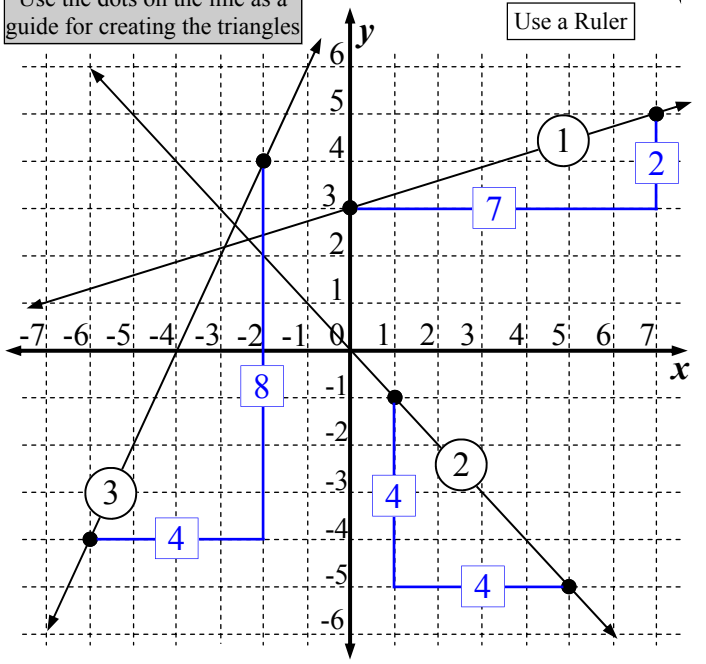
Find the gradient for these lines. Create gradient triangles and remember your positive and negatives.

Use the dots on the line as a guide for creating the triangles

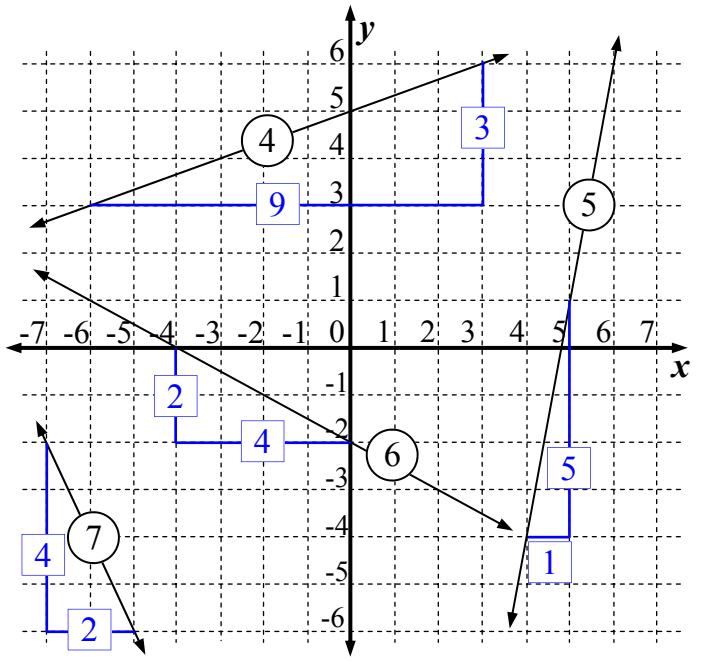
Use a Ruler



1 $m = \frac{2}{7}$

2 $m = \frac{4}{4} = -1$

3 $m = \frac{8}{4} = 2$

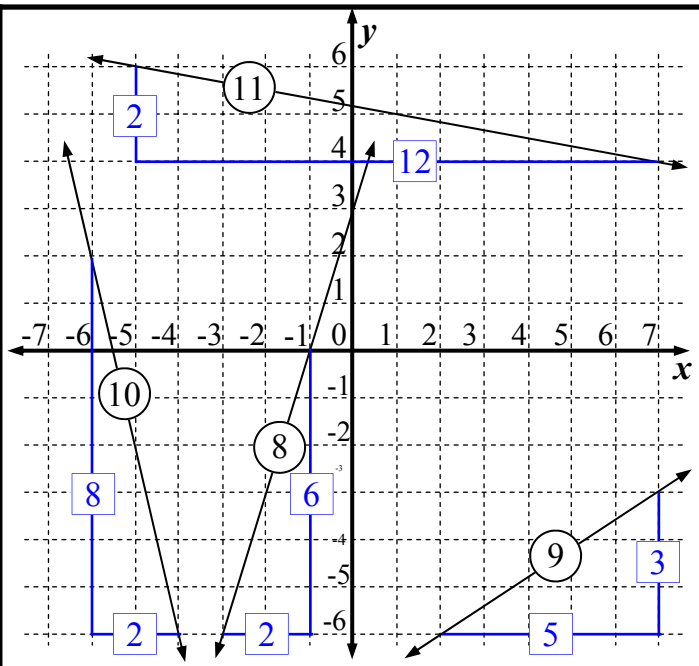


4 $m = \frac{3}{9} = \frac{1}{3}$

5 $m = \frac{5}{1} = 5$

6 $m = \frac{2}{4} = \frac{-1}{2}$

7 $m = \frac{4}{2} = -2$

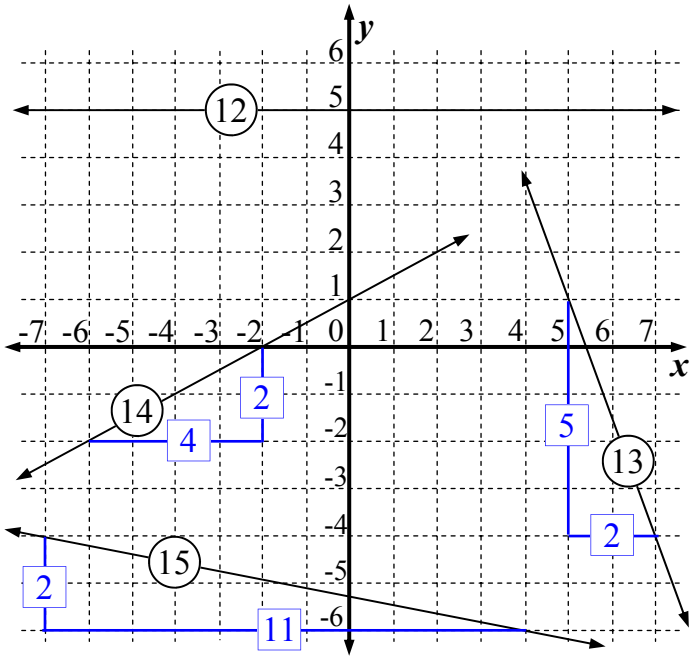


8 $m = \frac{6}{2} = 3$

9 $m = \frac{3}{5}$

10 $m = \frac{8}{2} = -4$

11 $m = \frac{2}{12} = \frac{-1}{6}$



12 $m = 0$

13 $m = \frac{-5}{2}$

14 $m = \frac{2}{4} = \frac{1}{2}$

15 $m = \frac{-2}{11}$

Cartesian Plane - Gradient (Homework Strip)

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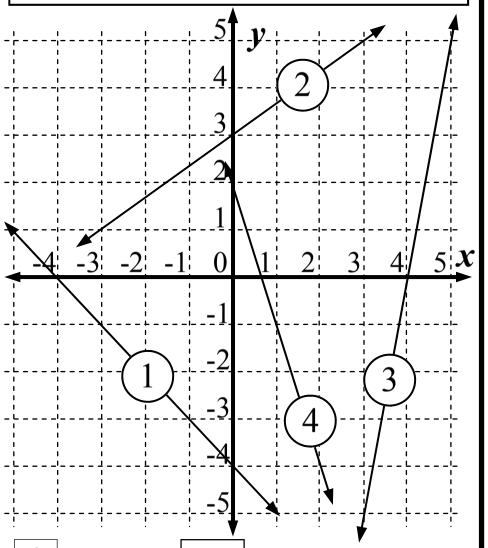
Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{\square}{\square} = \square$ Positive or negative?

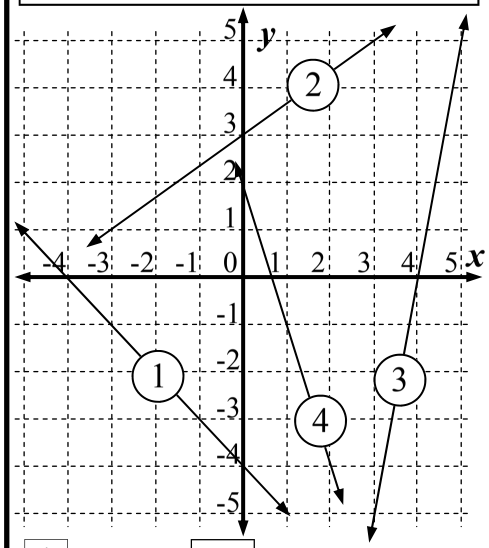
2 $m = \frac{\square}{\square} = \frac{\square}{\square}$ You might not need these boxes

3 $m = \frac{\square}{\square} = \square$

4 $m = \frac{\square}{\square} = \square$

Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{\square}{\square} = \square$ Positive or negative?

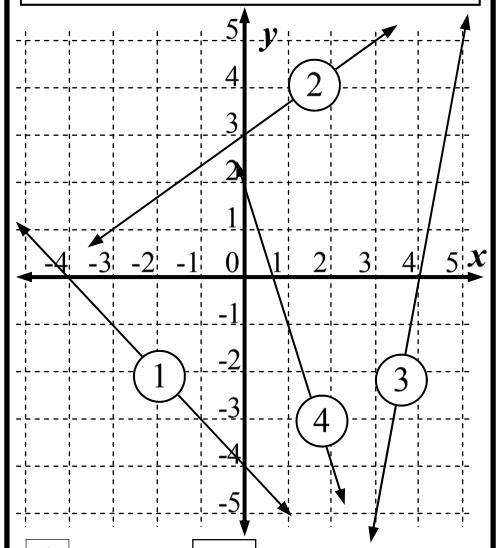
2 $m = \frac{\square}{\square} = \frac{\square}{\square}$ You might not need these boxes

3 $m = \frac{\square}{\square} = \square$

4 $m = \frac{\square}{\square} = \square$

Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{\square}{\square} = \square$ Positive or negative?

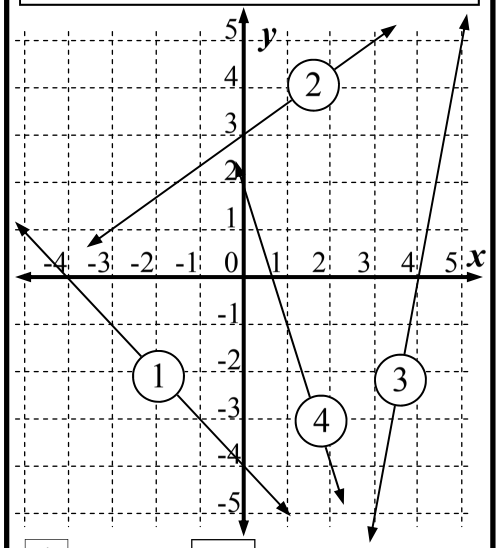
2 $m = \frac{\square}{\square} = \frac{\square}{\square}$ You might not need these boxes

3 $m = \frac{\square}{\square} = \square$

4 $m = \frac{\square}{\square} = \square$

Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{\square}{\square} = \square$ Positive or negative?

2 $m = \frac{\square}{\square} = \frac{\square}{\square}$ You might not need these boxes

3 $m = \frac{\square}{\square} = \square$

4 $m = \frac{\square}{\square} = \square$

Cartesian Plane - Gradient (Homework Strip)

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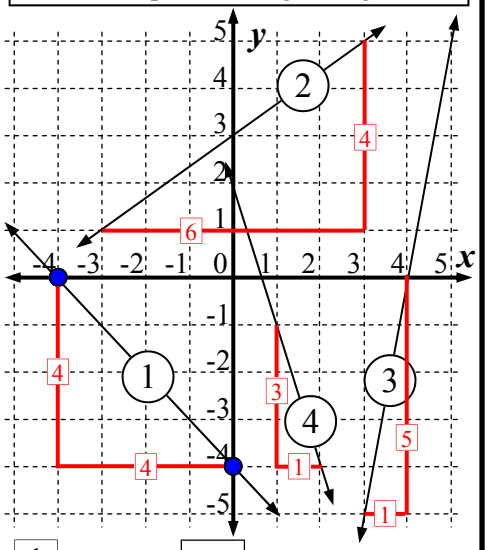
Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{4}{4} = -1$ Positive or negative?

2 $m = \frac{4}{6} = \frac{2}{3}$ You might not need these boxes

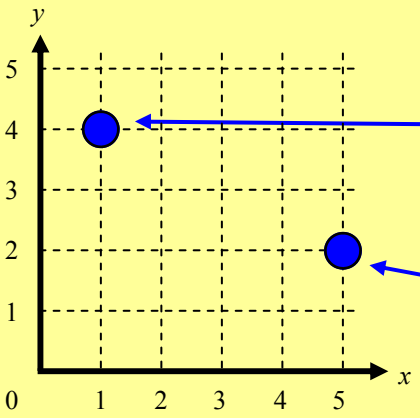
3 $m = \frac{5}{1} = 5$

4 $m = \frac{3}{1} = -3$

Cartesian Plane - Distance (Positive Quadrant)

This sheet uses Pythagoras Theorem to find the distance between two points. When two points are plotted on a cartesian plane a triangle can be created after a third point is plotted. The steps are shown below.

Example (1, 4) and (5, 2).

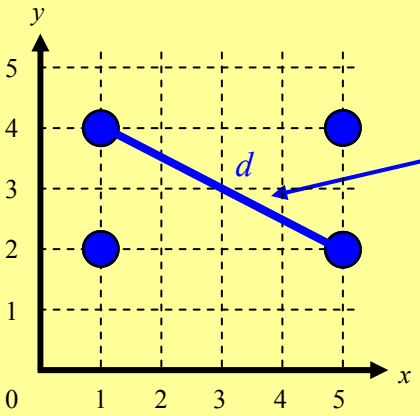


STEP 1

Plot point (1, 4). From the origin (0,0) move across 1 and up 4. Plot the point.

STEP 2

Plot point (5, 2). From the origin (0,0) move across 5 and up 2. Plot the point.

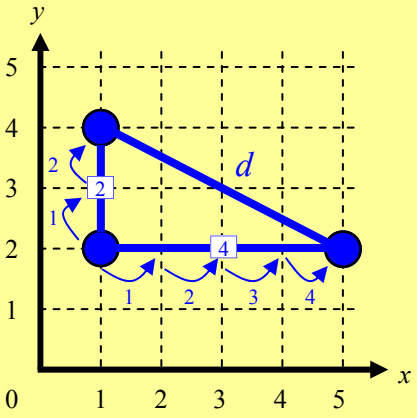


STEP 3

Draw a line between the two points, call its length '*d*'.

STEP 4

Now plot a point to form a right-angle triangle. It can be in either of the two positions shown. (Not both).



STEP 5

Construct the two short sides of the triangle.

STEP 6

Count off the lengths to find the length of both of the short sides. In the example one side is 2 units, the other is 4 units.

STEP 7

Now apply Pythagoras Theorem to the triangle to find the hypotenuse. Answer to 1 d.p. and the length is in 'units'.

$d^2 = 2^2 + 4^2$

$d^2 = 20$

$d = \sqrt{20}$

$d = 4.5$ units (1 d.p.)

Cartesian Plane - Distance - Positive Quadrant (Sheet 1)

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Calculate these square roots, round your answer to 1 d.p.

1 $\sqrt{43} =$ ☐.☐ Round to 1 d.p.

Calc: √ 4 3 =

2 $\sqrt{28} =$ ☐

3 $\sqrt{75} =$ ☐

4 $\sqrt{66} =$ ☐

5 $\sqrt{59} =$ ☐

Calculate the additions of two squared numbers.

6 $8^2 + 4^2 =$ ☐

Calc: 8 x^2 + 4 x^2 =

7 $5^2 + 7^2 =$ ☐

8 $6^2 + 3^2 =$ ☐

9 $4^2 + 10^2 =$ ☐

10 $2^2 + 8^2 =$ ☐

Combine the two operations. Add the squares and square root the answer.

11 $8^2 + 5^2 =$ ☐

Get your answer, then write it here

$\sqrt{\square} =$ ☐

Calc:

Then square root (1 d.p.) √ ANS =

12 $4^2 + 9^2 =$ ☐

$\sqrt{\square} =$ ☐

13 $1^2 + 7^2 =$ ☐

$\sqrt{\square} =$ ☐

14 $6^2 + 6^2 =$ ☐

$\sqrt{\square} =$ ☐

Now use Pythagoras Theorem to calculate the length of an interval. Use 'd' to represent the distance between the two plotted points. (1 d.p.)

15

$d^2 = \square^2 + \square^2$

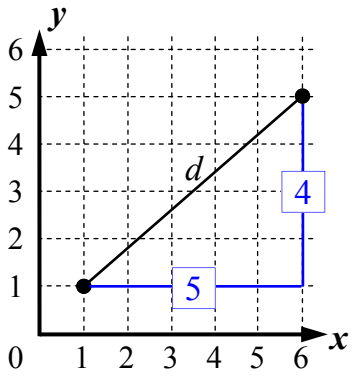
$d^2 = \square$

$d = \sqrt{\square}$ Rewrite

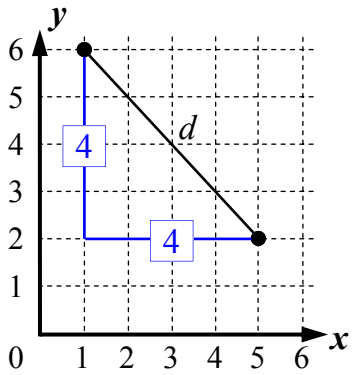
Calc: √ ANS =

$d =$ ☐.☐ *units*

(1 d.p.)



16



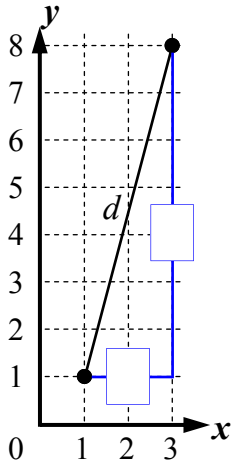
$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d =$ ☐ *units*

17



Now write the side lengths in the boxes.

$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d =$ ☐ *units*

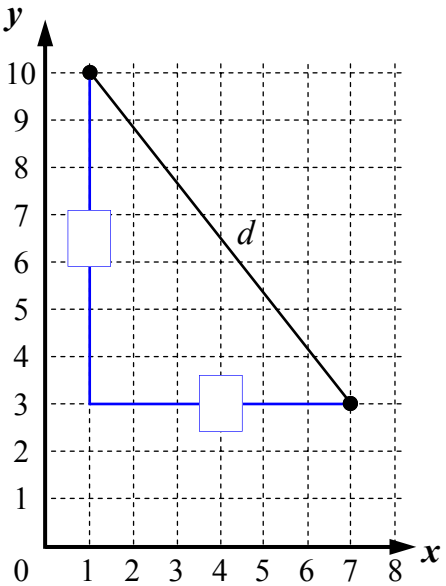
18

$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d =$ ☐ *units*



Cartesian Plane - Distance - Positive Quadrant (Sheet 2)

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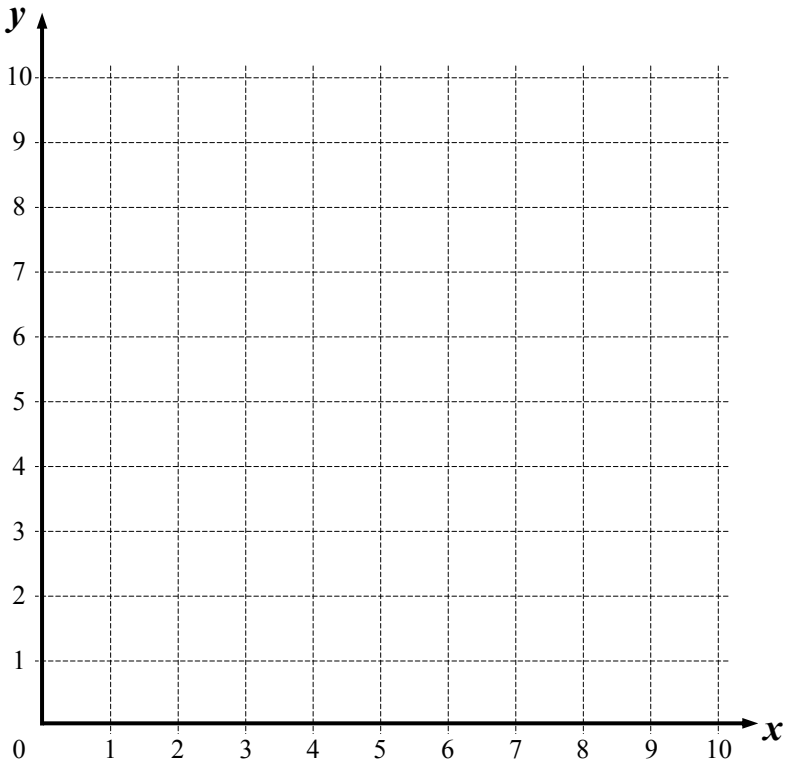
Plot the two points on the graph and rule an interval between them. This will be the hypotenuse of a right angle triangle. Draw the two short sides then use Pythagoras to find the interval length. (1 d.p.)



1 (7, 10) and (10, 6). Plot right angle at (10, 10).

$d^2 = \square^2 + \square^2$

Calc: 3 x^2 + 4 x^2 =

$d^2 =$ ☐

Rewrite

$d = \sqrt{\square}$

Calc: $\sqrt{\blacksquare}$ ANS =

$d =$ ☐ *units*

(1 d.p.)

2 (6, 10) and (1, 9). Plot right angle at (1, 10).

$d^2 = \square^2 + \square^2$

$d^2 =$ ☐

$d = \sqrt{\square}$

$d =$ ☐ *units*

(1 d.p.)

3 (7, 10) and (9, 1). Plot right angle at (7, 1).

$d^2 = \square^2 + \square^2$

$d^2 =$ ☐

$d = \sqrt{\square}$

$d =$ ☐ *units*

4 (2, 4) and (0, 8). Plot right angle at (0, 4).

$d^2 =$

$d^2 =$

$d =$

$d =$

5 (4, 4) and (4, 8).

$d =$

Now you have place the right angle yourself. Find the distance 'd' between these two points.

6 (0, 2) and (7, 0).

Right angle at (☐ , ☐)

$d^2 =$

$d^2 =$

$d =$

$d =$

7 (2, 2) and (6, 9).

Right angle at (☐ , ☐)

8 (9, 5) and (10, 1).

Right angle at (☐ , ☐)

9 (1, 9) and (9, 5).

Right angle at (☐ , ☐)

Cartesian Plane - Distance - Positive Quadrant (Sheet 1)

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Calculate these square roots, round your answer to 1 d.p.

1 $\sqrt{43} = 6.6$ Round to 1 d.p.

Calc: √ 4 3 =

2 $\sqrt{28} = 5.3$

3 $\sqrt{75} = 8.7$

4 $\sqrt{66} = 8.1$

5 $\sqrt{59} = 7.7$

Calculate the additions of two squared numbers.

6 $8^2 + 4^2 = 80$

Calc: 8 x^2 + 4 x^2 =

7 $5^2 + 7^2 = 74$

8 $6^2 + 3^2 = 45$

9 $4^2 + 10^2 = 116$

10 $2^2 + 8^2 = 68$

Combine the two operations. Add the squares and square root the answer.

11 $8^2 + 5^2 = 89$

Get your answer, then write it here

$\sqrt{89} = 9.4$

Calc: Then square root (1 d.p.) √ ANS =

12 $4^2 + 9^2 = 97$

$\sqrt{97} = 9.8$

13 $1^2 + 7^2 = 50$

$\sqrt{50} = 7.1$

14 $6^2 + 6^2 = 72$

$\sqrt{72} = 8.5$

Now use Pythagoras Theorem to calculate the length of an interval. Use 'd' to represent the distance between the two plotted points. (1 d.p.)

15

$d^2 = 4^2 + 5^2$

$d^2 = 41$

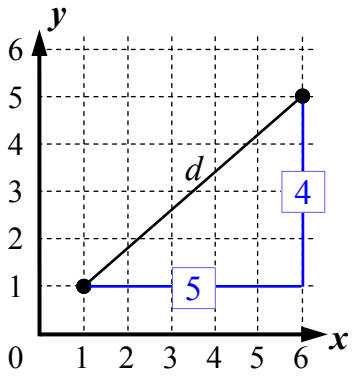
Rewrite

$d = \sqrt{41}$

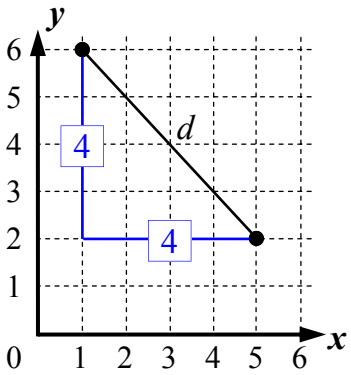
Calc: √ ANS =

$d = 6.4$ *units*

(1 d.p.)



16



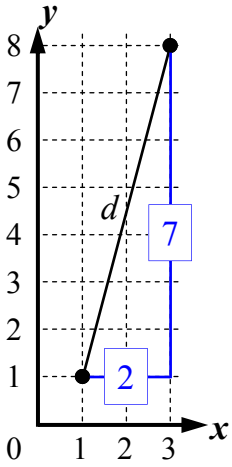
$d^2 = 4^2 + 4^2$

$d^2 = 32$

$d = \sqrt{32}$

$d = 5.7$ *units*

17



Now write the side lengths in the boxes.

$d^2 = 2^2 + 7^2$

$d^2 = 53$

$d = \sqrt{53}$

$d = 7.3$ *units*

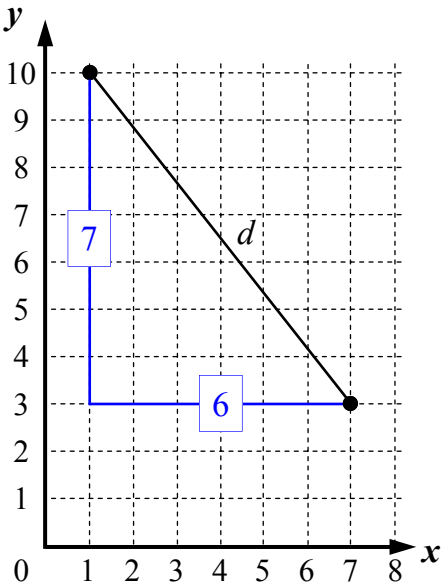
18

$d^2 = 6^2 + 7^2$

$d^2 = 85$

$d = \sqrt{85}$

$d = 9.2$ *units*



Cartesian Plane - Distance - Positive Quadrant (Sheet 2)

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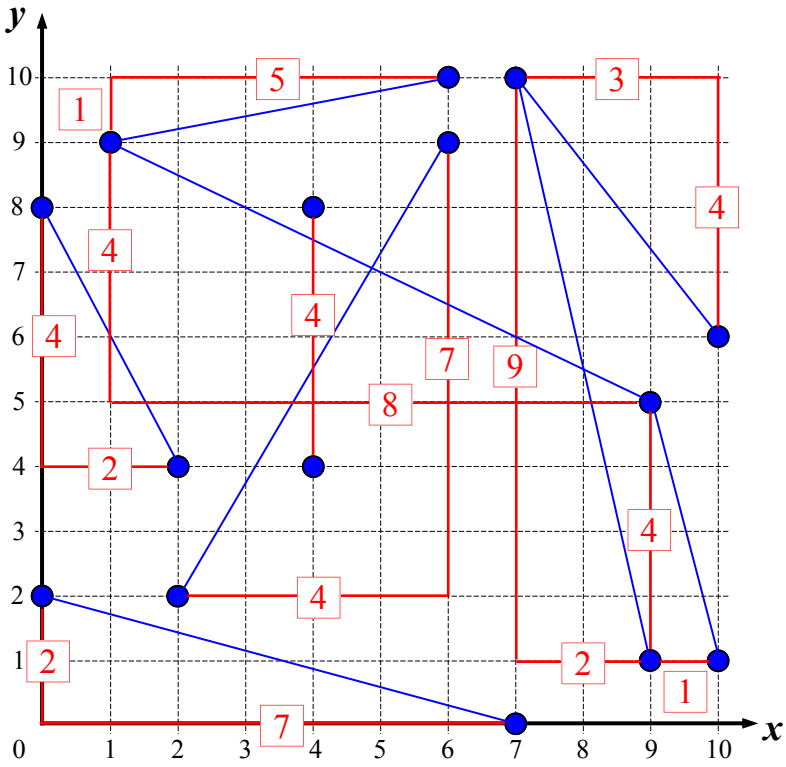
Plot the two points on the graph and rule an interval between them. This will be the hypotenuse of a right angle triangle. Draw the two short sides then use Pythagoras to find the interval length. (1 d.p.)



1 (7, 10) and (10, 6). Plot right angle at (10, 10).

$d^2 =$ 3 ^{2} + 4 ^{2}

Calc: 3 x^2 + 4 x^2 =

$d^2=$ 25

$d =$ $\sqrt{25}$ Rewrite

Calc: $\sqrt{\blacksquare}$ ANS =

$d =$ 5.0 *units*

(1 d.p.)

2 (6, 10) and (1, 9). Plot right angle at (1, 10).

$d^2 =$ 1 ^{2} + 5 ^{2}

$d^2 =$ 26

$d =$ $\sqrt{26}$

$d =$ 5.1 *units*

(1 d.p.)

3 (7, 10) and (9, 1). Plot right angle at (7, 1).

$d^2 =$ 2 ^{2} + 9 ^{2}

$d^2 =$ 85

$d =$ $\sqrt{85}$

$d =$ 9.2 *units*

4 (2, 4) and (0, 8). Plot right angle at (0, 4).

$d^2 = 2^2 + 4^2$

$d^2 = 20$

$d = \sqrt{20}$

$d = 4.5$ units

5 (4, 4) and (4, 8).

$d = 4$ units

Now you have place the right angle yourself. Find the distance 'd' between these two points.

6 (0, 2) and (7, 0).

Right angle at (0 , 0)

$d^2 = 2^2 + 7^2$ or (7, 2)

$d^2 = 53$

$d = \sqrt{53}$

$d = 7.3$ units

7 (2, 2) and (6, 9).

Right angle at (6 , 2)

$d^2 = 4^2 + 7^2$ or (2, 9)

$d^2 = 65$

$d = \sqrt{65}$

$d = 8.1$ units

8 (9, 5) and (10, 1).

Right angle at (9 , 1)

$d^2 = 1^2 + 4^2$ or (10, 5)

$d^2 = 17$

$d = \sqrt{17}$

$d = 4.1$ units

9 (1, 9) and (9, 5).

Right angle at (1 , 5)

$d^2 = 4^2 + 8^2$ or (9, 9)

$d^2 = 80$

$d = \sqrt{80}$

$d = 8.9$ units

Cartesian Plane - Distance - All Quadrants (Homework Strip)

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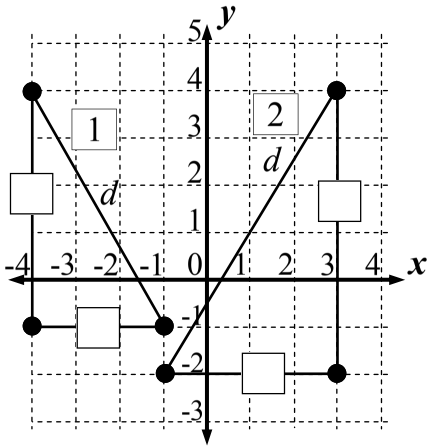
Date Due: _____

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

2 $d^2 = \square^2 + \square^2$

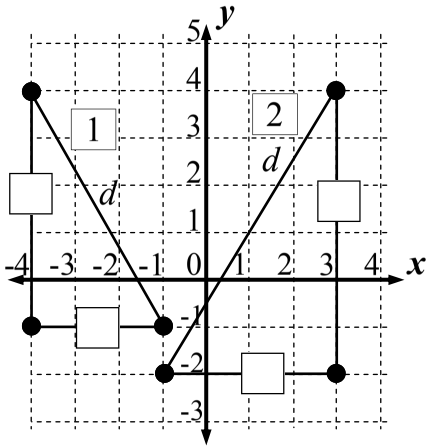
$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

Date Due: _____

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

2 $d^2 = \square^2 + \square^2$

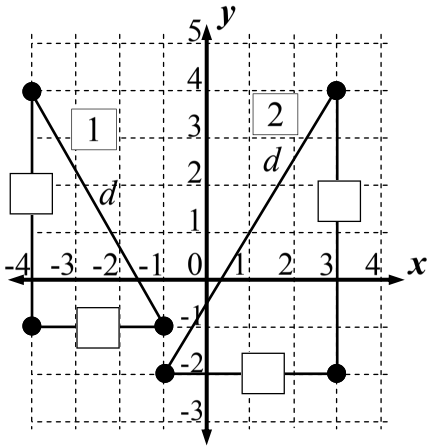
$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

Date Due: _____

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

2 $d^2 = \square^2 + \square^2$

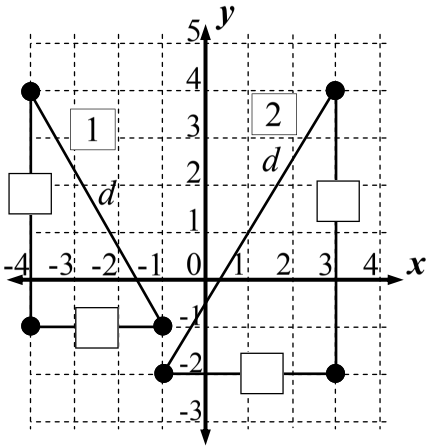
$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

Date Due: _____

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

2 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

Cartesian Plane - Distance - Positive Quadrant (Homework Strip)

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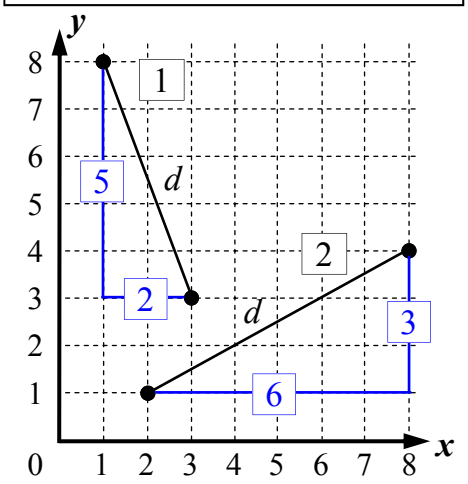
Date Due:

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = 2^2 + 5^2$

$d^2 = 29$

$d = \sqrt{29}$

$d = 5.4$ *units*

2 $d^2 = 3^2 + 6^2$

$d^2 = 45$

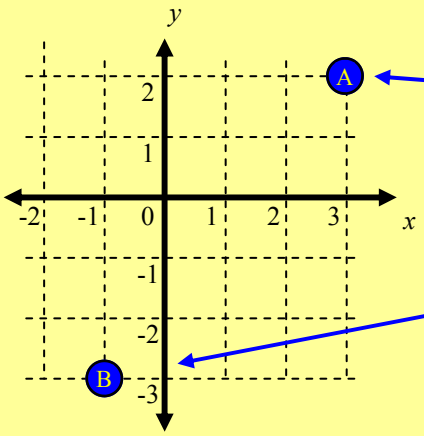
$d = \sqrt{45}$

$d = 6.7$ *units*

Cartesian Plane - Distance - All Quadrants

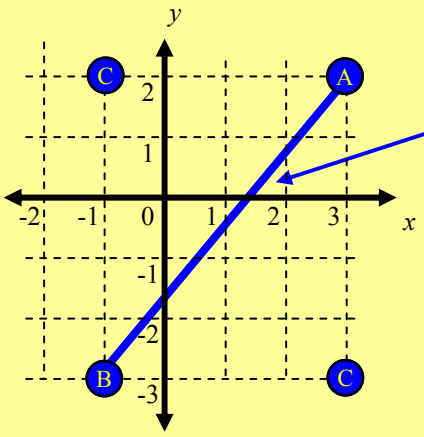
This sheet uses Pythagoras Theorem to find the distance between two points. When two points are plotted on a cartesian plane a right angle triangle can be created. It is assumed that the previous positive quadrant sheet is attempted prior to this sheet.

Example A (3, 2) and B (-1, -3). Label right angle C.



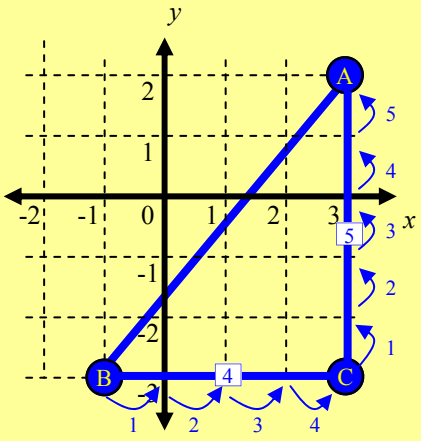
STEP 1 Plot point 'A', from the origin (0,0) move right, across 3, then up 2. Plot the point with an 'A' inside.

STEP 2 Plot point 'B', from the origin (0,0) move left across 1, to -1, and then down 3, to -3. Plot the point with an 'B' inside.



STEP 3 Draw a line between the two points

STEP 4 Now plot point 'C' to form a right-angle triangle. It can be in either of the two positions shown. Not both.



STEP 5 Construct the two short sides of the Triangle.

STEP 6 Count off the lengths to find the length of both of the short sides. In the example one side is 5 units, the other is 4 units.

STEP 7 Now apply Pythagoras Theorem to the triangle to find the hypotenuse. Answer to 1 d.p. and the length is in 'units'.

AC = 5

CB = 4

$AB^2 = 5^2 + 4^2$

$AB^2 = 41$

$AB = \sqrt{41}$

AB = 6.4 units (1 d.p.)

Cartesian Plane - Distance - All Quadrants (Sheet 1)

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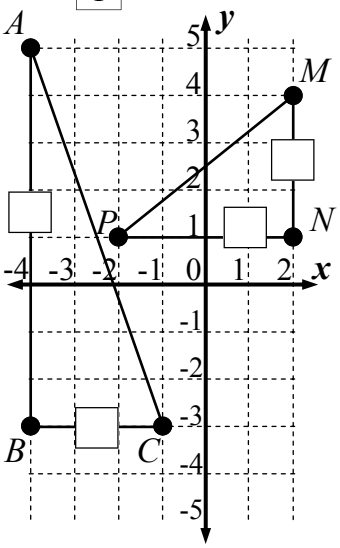
Write the lengths of the short sides in the boxes provided on the number plane.

1



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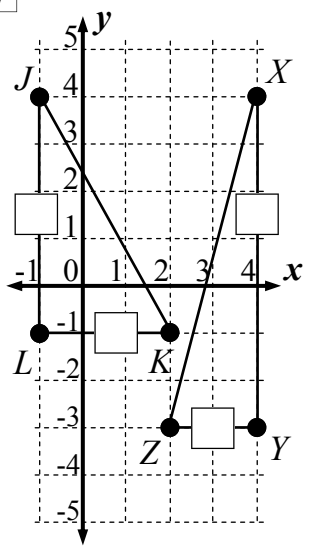

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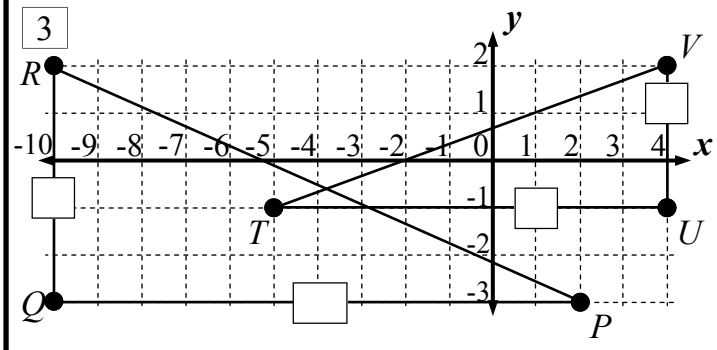
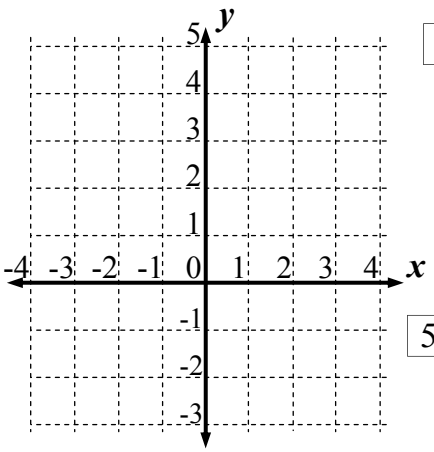


Plot the pair of points, then create a right angle triangle on the number plane. Find the two short side lengths. Plot Q4 and Q5 on the first number plane.



4 (4, 5) & (1, -3).

Vertical Length : ☐

Horizontal Length : ☐

5 (-3, 4) & (-1, -2).

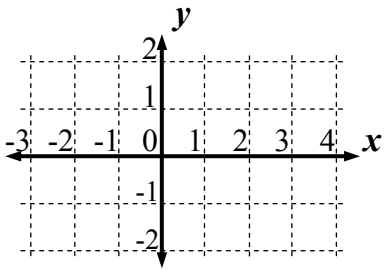
Vertical Length : ☐

Horizontal Length : ☐

6 (4, -2) & (-1, 2).

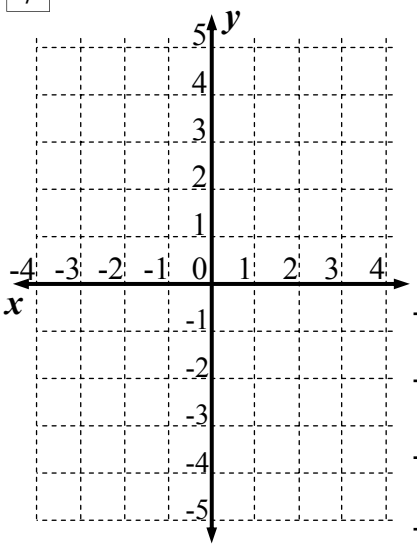
Vertical Length : ☐

Horizontal Length : ☐



Now use Pythagoras to calculate the distance between the two plotted points. Find the short side lengths first. Answer to 1 decimal place.

7



(3, 5) & (-1, -3).

Vertical Length : ☐

Horizontal Length : ☐

$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square.\square$ *units*

8 (-4, -4) & (2, 5).

Vertical Length : ☐

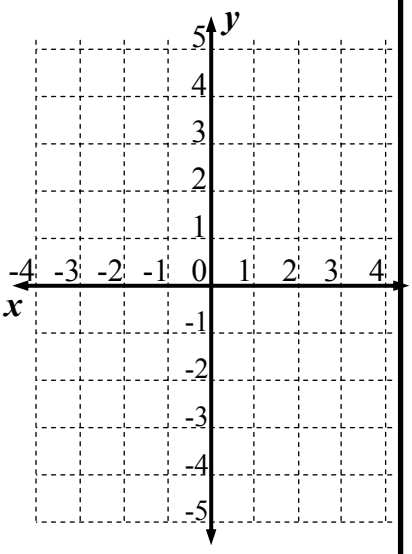
Horizontal Length : ☐

$d^2 = \square^2 + \square^2$

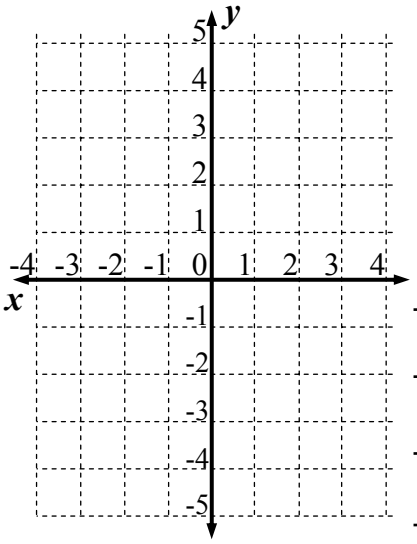
$d^2 = \square$

$d = \sqrt{\square}$

$d = \square\square.\square$ *units*



9



(-4, 2) & (1, -5).

Vertical Length : ☐

Horizontal Length : ☐

$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square.\square$ *units*

Cartesian Plane - Distance - All Quadrants (Sheet 2)

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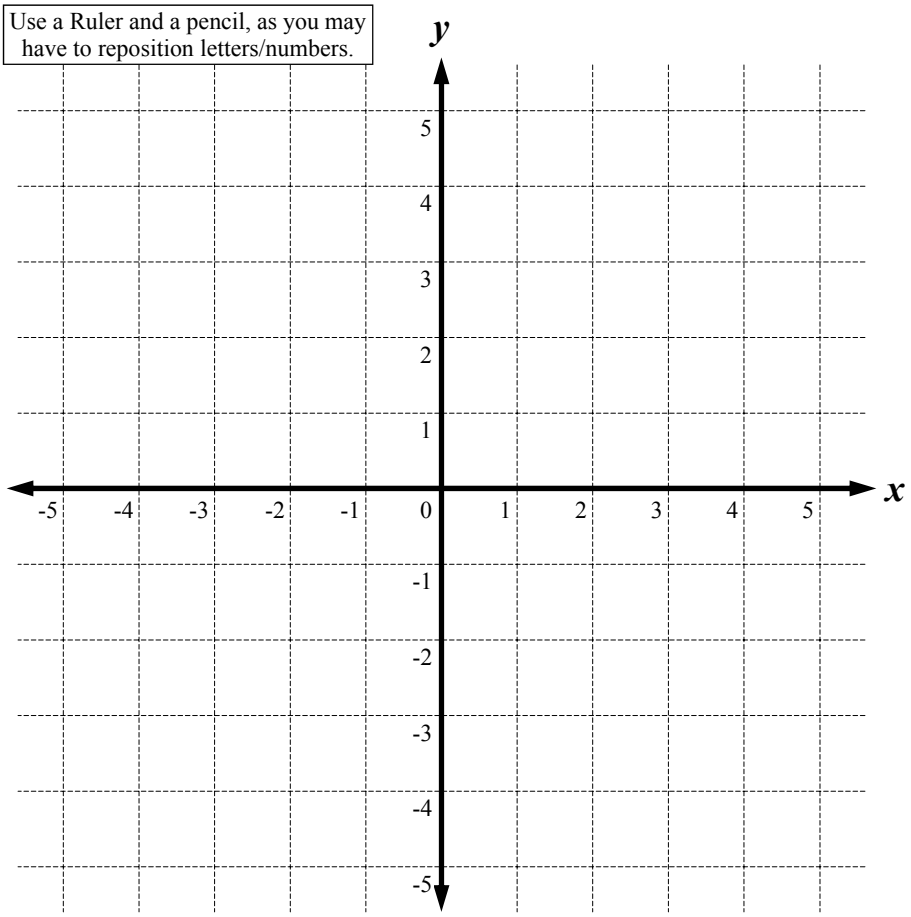
Plot the two points on the graph and draw a line between them, this is the hypotenuse. Draw the two short sides then apply Pythagoras to find the distance between the two points (to 1 d.p.). Note that '*u*' stands for units.

Use a Ruler and a pencil, as you may have to reposition letters/numbers.



1 *K* (-3, 3) and *L* (4, 5). Label right angle *J* at (4, 3).

JK = ☐ *u* *JL* = ☐ *u*

KL^2 = ☐^{2} + ☐^{2}

KL^2 = ☐

KL = $\sqrt{\square}$ Rewrite

KL = ☐.☐ *units*

2 *A* (-5, 4) & *B* (-3, -2). Label right angle *C* at (-5, -2).

AC = ☐ *u* *BC* = ☐ *u*

AB^2 = ☐^{2} + ☐^{2}

AB^2 = ☐

AB = $\sqrt{\square}$ Rewrite

AB = ☐.☐ *units*

3 *S* (-2, -4) and *R* (2, 2). Label right angle *T* at (2, -4).

TR = ☐ *u* *TS* = ☐ *u*

SR^2 =

SR^2 =

SR =

SR =

4 *V* (-5, -4) & *X* (-3, -3). Label right angle *Y* at (-3, -4).

VY = ☐ *u* *XY* = ☐ *u*

VX^2 =

VX^2 =

VX =

VX =

5 *F* (1, 5) & *T* (5, 1). Label right angle *N*.

FN = *TN* =

6 *U* (0, 5) & *Z* (5, -3). Label right angle *D*.

UD = *ZD* =

7 *E* (4, -2) & *G* (2, -5). Label right angle *H*.

EH = *GH* =

8 *M* (-2, 2) & *Q* (3, -1). Label right angle *P*.

MP = *QP* =

Cartesian Plane - Distance - All Quadrants (Sheet 1)

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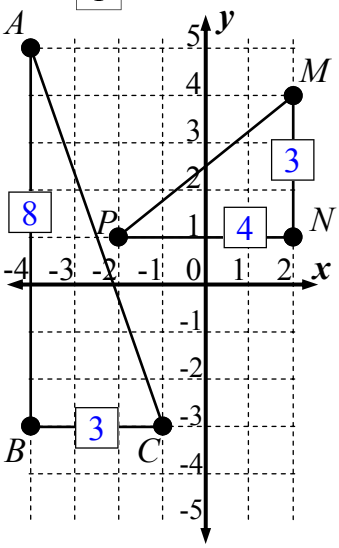
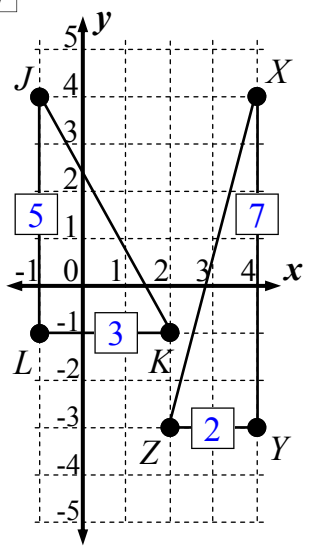
Write the lengths of the short sides in the boxes provided on the number plane.

1



2



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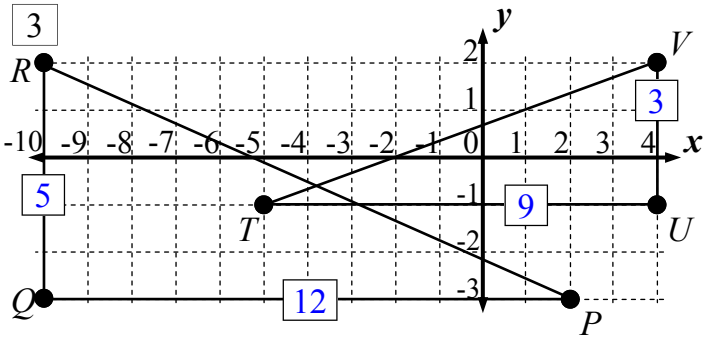
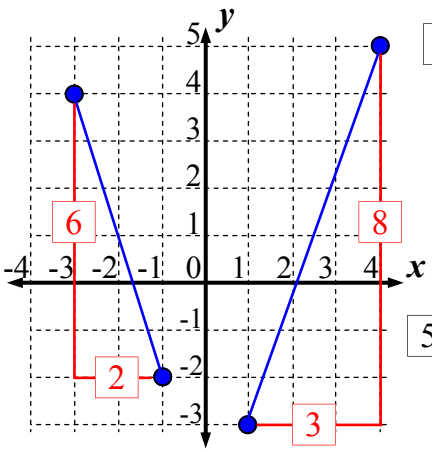


Plot the pair of points, then create a right angle triangle on the number plane. Find the two short side lengths. Plot Q4 and Q5 on the first number plane.

4 (4, 5) & (1, -3).



Vertical Length : 8

Horizontal Length : 3

5 (-3, 4) & (-1, -2).

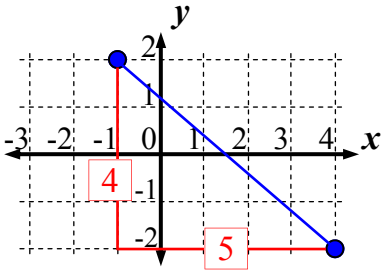
Vertical Length : 6

Horizontal Length : 2

6 (4, -2) & (-1, 2).

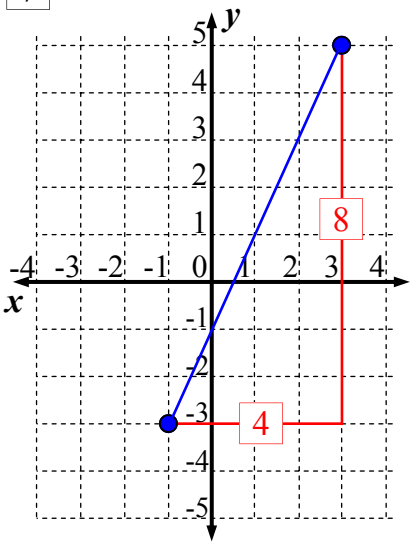
Vertical Length : 4

Horizontal Length : 5



Now use Pythagoras to calculate the distance between the two plotted points. Find the short side lengths first. Answer to 1 decimal place.

7



(3, 5) & (-1, -3).

Vertical Length : 8

Horizontal Length : 4

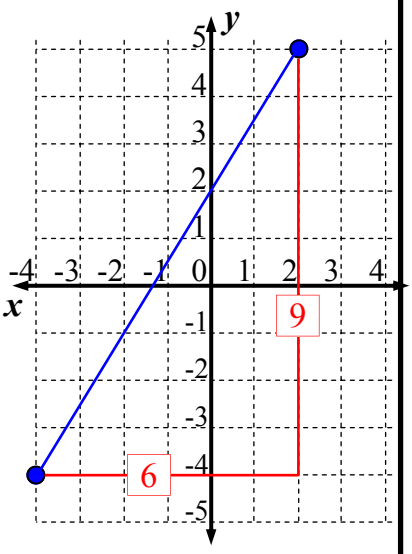
$d^2 = 8^2 + 4^2$

$d^2 = 80$

$d = \sqrt{80}$

$d = 8.9$ units

8 (-4, -4) & (2, 5).



Vertical Length : 9

Horizontal Length : 6

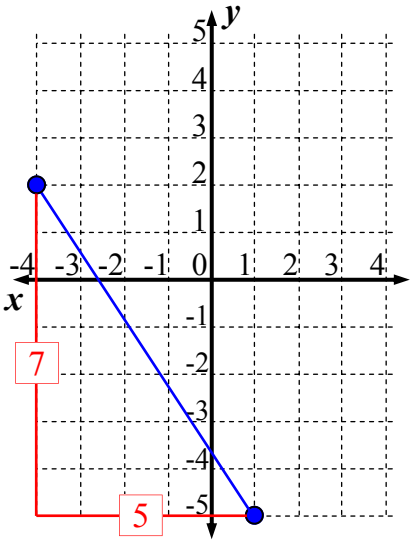
$d^2 = 6^2 + 9^2$

$d^2 = 117$

$d = \sqrt{117}$

$d = 10.8$ units

9



(-4, 2) & (1, -5).

Vertical Length : 7

Horizontal Length : 5

$d^2 = 7^2 + 5^2$

$d^2 = 74$

$d = \sqrt{74}$

$d = 8.6$ units

Cartesian Plane - Distance - All Quadrants (Sheet 2)

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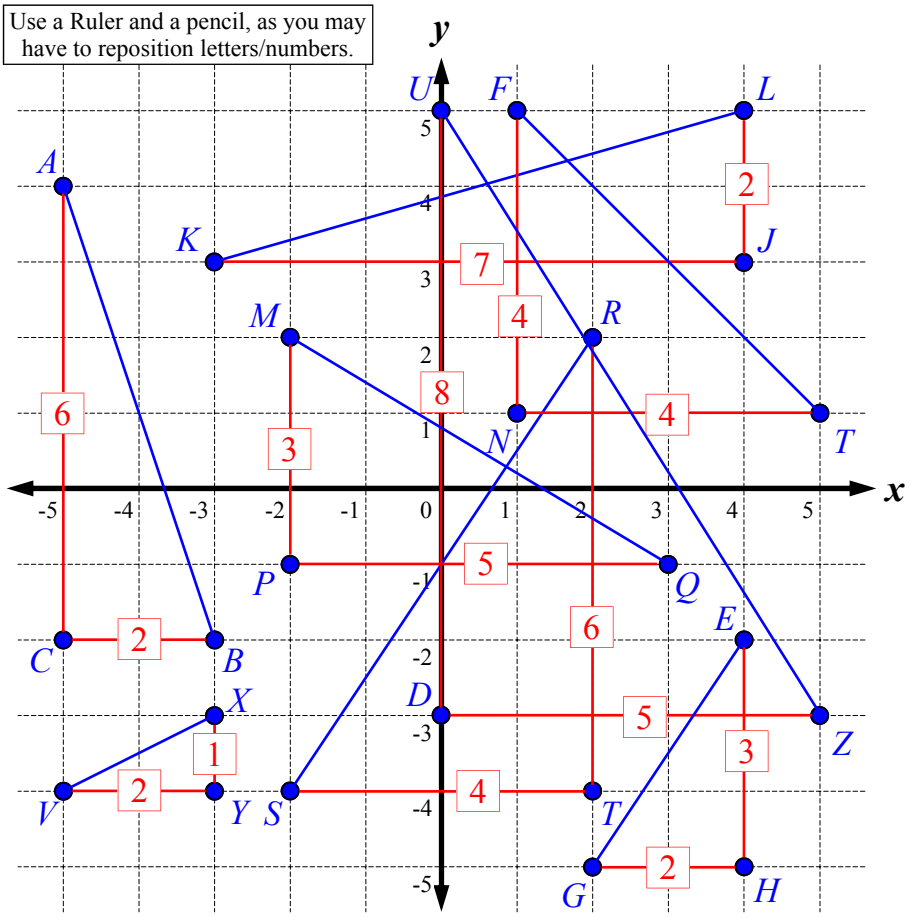
Plot the two points on the graph and draw a line between them, this is the hypotenuse. Draw the two short sides then apply Pythagoras to find the distance between the two points (to 1 d.p.). Note that '*u*' stands for units.

Use a Ruler and a pencil, as you may have to reposition letters/numbers.



1 *K* (-3, 3) and *L* (4, 5). Label right angle *J* at (4, 3).

JK = 7 *u* *JL* = 2 *u*

$KL^2 = 7^2 + 2^2$

$KL^2 = 53$

Rewrite

$KL = \sqrt{53}$

$KL = 7.3$ *units*

2 *A* (-5, 4) & *B* (-3, -2). Label right angle *C* at (-5, -2).

AC = 6 *u* *BC* = 2 *u*

$AB^2 = 6^2 + 2^2$

$AB^2 = 40$

Rewrite

$AB = \sqrt{40}$

$AB = 6.3$ *units*

3 *S* (-2, -4) and *R* (2, 2). Label right angle *T* at (2, -4).

TR = 6 *u* *TS* = 4 *u*

$SR^2 = 6^2 + 4^2$

$SR^2 = 52$

$SR = \sqrt{52}$

$SR = 7.2$ units

4 *V* (-5, -4) & *X* (-3, -3). Label right angle *Y* at (-3, -4).

VY = 2 *u* *XY* = 1 *u*

$VX^2 = 2^2 + 1^2$

$VX^2 = 5$

$VX = \sqrt{5}$

$VX = 2.2$ units

5 *F* (1, 5) & *T* (5, 1). Label right angle *N*.

FN = 4 *units* *TN* = 4 *units*

$FT^2 = 4^2 + 4^2$

$FT^2 = 32$

$FT = \sqrt{32}$

$FT = 5.7$ *units*

6 *U* (0, 5) & *Z* (5, -3). Label right angle *D*.

UD = 8 *units* *ZD* = 5 *units*

$UZ^2 = 8^2 + 5^2$

$UZ^2 = 89$

$UZ = \sqrt{89}$

$UZ = 9.4$ *units*

7 *E* (4, -2) & *G* (2, -5). Label right angle *H*.

EH = 3 *units* *GH* = 2 *units*

$EG^2 = 3^2 + 2^2$

$EG^2 = 13$

$EG = \sqrt{13}$

$EG = 3.6$ *units*

8 *M* (-2, 2) & *Q* (3, -1). Label right angle *P*.

MP = 3 *units* *QP* = 5 *units*

$MQ^2 = 3^2 + 5^2$

$MQ^2 = 34$

$MQ = \sqrt{34}$

$MQ = 5.8$ *units*

Cartesian Plane - Distance - All Quadrants (Homework Strip)

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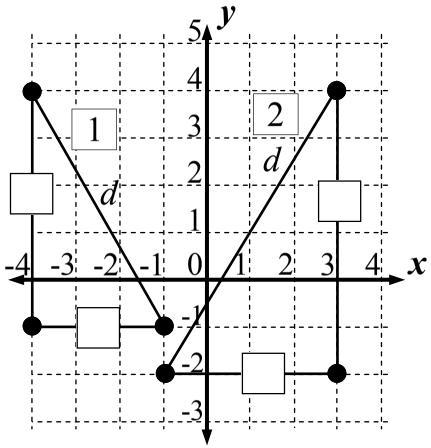
Date Due: _____

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

2 $d^2 = \square^2 + \square^2$

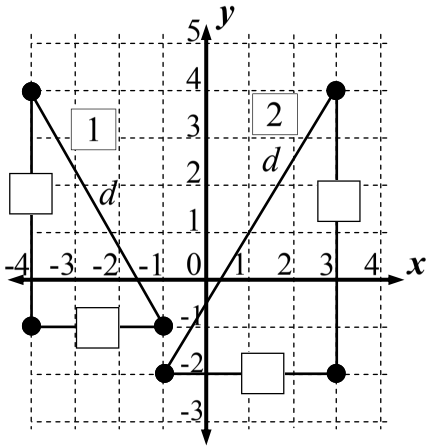
$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

Date Due: _____

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

2 $d^2 = \square^2 + \square^2$

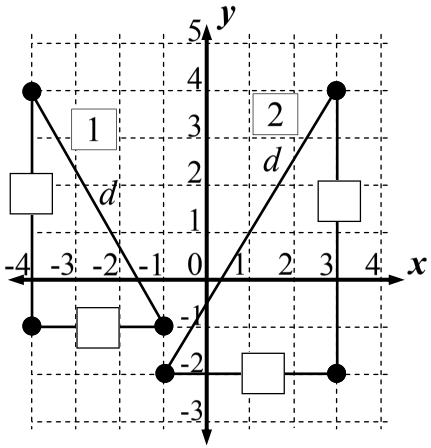
$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

Date Due: _____

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

2 $d^2 = \square^2 + \square^2$

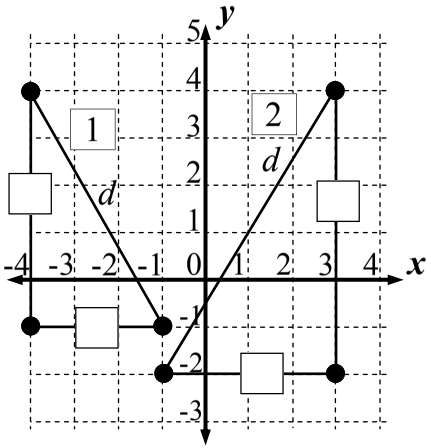
$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

Date Due: _____

Calculate the distance between the two points. Answer to 1 decimal place



1 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

2 $d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square$ *units*

Cartesian Plane - Distance - All Quadrants (Homework Strip)

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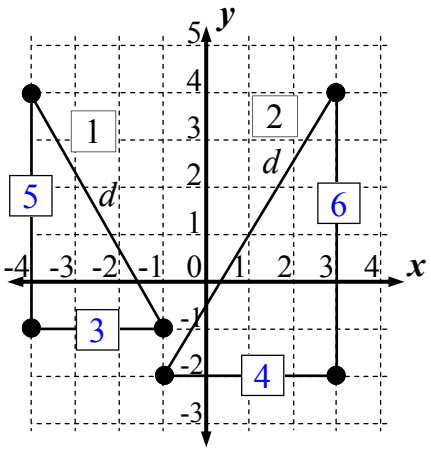
Date Due:

Calculate the distance between the two points. Answer to 1 decimal place



1

$d^2 = 3^2 + 5^2$

$d^2 = 34$

$d = \sqrt{34}$

$d = 5.8$ *units*

2

$d^2 = 4^2 + 6^2$

$d^2 = 52$

$d = \sqrt{52}$

$d = 7.2$ *units*

Cartesian Plane - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Cartesian Plane - Find A Word

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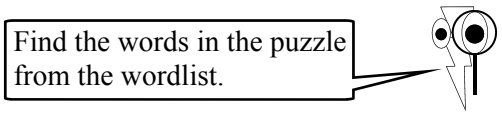

E	H	E	Q	U	A	T	I	O	N	A	X	S	X	A
V	A	T	Q	P	V	W	T	N	I	O	P	D	I	M
I	L	A	Q	L	E	P	Q	E	U	N	A	F	N	X
T	F	N	Q	O	R	E	L	G	N	A	I	R	T	A
I	D	I	I	T	T	U	Q	A	Q	I	N	A	E	M
S	I	D	N	T	I	N	N	V	N	S	A	X	R	A
O	S	R	T	I	C	Q	A	E	A	E	E	X	V	X
P	T	O	E	N	A	X	S	R	O	T	P	X	A	E
X	A	O	R	G	L	E	O	A	D	R	A	O	L	X
A	N	C	C	C	X	G	P	G	X	A	I	J	L	A
X	C	A	E	A	A	O	H	E	E	C	U	G	R	S
E	E	X	P	H	I	N	W	O	D	S	L	Q	I	O
X	L	A	T	N	O	Z	I	R	O	H	I	I	H	N
A	X	Y	T	G	R	A	D	I	E	N	T	R	N	H
E	P	N	E	G	A	T	I	V	E	H	O	R	I	E

WORDLIST

GRADIENT	POINT*	MIDPOINT*	RISE	VERTICAL
POSITIVE	INTERVAL	INTERCEPT	SLOPE	MEAN
HALF	TRIANGLE	PLOTTING	QUADRANT	CARTESIAN
AXES	RUN	NEGATIVE	LINE	UP
DISTANCE	AVERAGE	ACROSS	DOWN	EQUATION
PLANE	HORIZONTAL	PYTHAGORAS	COORDINATE	ORIGIN

*MIDPOINT AND POINT ARE SEPARATE WORDS

Cartesian Plane - Find A Word

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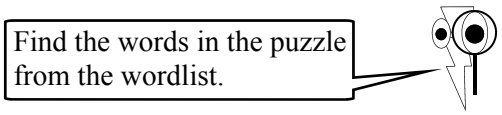

E	H	E	Q	U	A	T	I	O	N	A	X	S	X	A
V	A	T	Q	P	V	W	T	N	I	O	P	D	I	M
I	L	A	Q	L	E	P	Q	E	U	N	A	F	N	X
T	F	N	Q	O	R	E	L	G	N	A	I	R	T	A
I	D	I	I	T	T	U	Q	A	Q	I	N	A	E	M
S	I	D	N	T	I	N	N	V	N	S	A	X	R	A
O	S	R	T	I	C	Q	A	E	A	E	E	X	V	X
P	T	O	E	N	A	X	S	R	O	T	P	X	A	E
X	A	O	R	G	L	E	O	A	D	R	A	O	L	X
A	N	C	C	C	X	G	P	G	X	A	I	J	L	A
X	C	A	E	A	A	O	H	E	E	C	U	G	R	S
E	E	X	P	H	I	N	W	O	D	S	L	Q	I	O
X	L	A	T	N	O	Z	I	R	O	H	I	I	H	N
A	X	Y	T	G	R	A	D	I	E	N	T	R	N	H
E	P	N	E	G	A	T	I	V	E	H	O	R	I	E

WORDLIST

~~GRADIENT~~	~~POINT*~~	~~MIDPOINT*~~	~~RISE~~	~~VERTICAL~~
~~POSITIVE~~	~~INTERVAL~~	~~INTERCEPT~~	~~SLOPE~~	~~MEAN~~
~~HALF~~	~~TRIANGLE~~	~~PLOTTING~~	~~QUADRANT~~	~~CARTESIAN~~
~~AXES~~	~~RUN~~	~~NEGATIVE~~	~~LINE~~	~~UP~~
~~DISTANCE~~	~~AVERAGE~~	~~ACROSS~~	~~DOWN~~	~~EQUATION~~
~~PLANE~~	~~HORIZONTAL~~	~~PYTHAGORAS~~	~~COORDINATE~~	~~ORIGIN~~

*MIDPOINT AND POINT ARE SEPARATE WORDS

Cartesian Plane Review

This is the review of the Cartesian Plane Topic.

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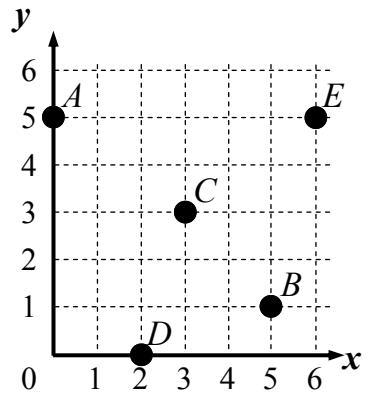
Give the coordinates for the five plotted points.



1 *A* _____

2 *B* _____

3 *C* _____

4 *D* _____

5 *E* _____

Give the coordinates for the six plotted points.

6 *A* _____

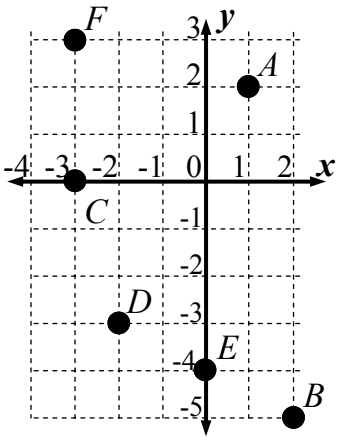
7 *B* _____

8 *C* _____

9 *D* _____

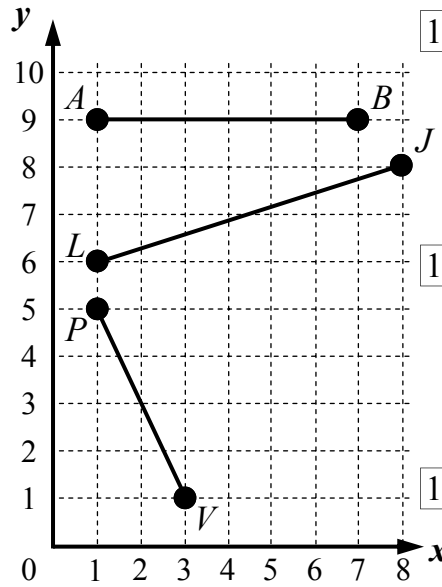
10 *E* _____

11 *F* _____



12 Give the coordinates for the origin _____

Plot the midpoint '*M*' on the intervals, give the coordinates



13 *AB* _____ Midpoint Coordinates

14 *LJ* _____ Midpoint Coordinates

15 *PV* _____ Midpoint Coordinates

Plot the points, find the midpoint coordinates.

16 *A* (-4, -5)
B (-2, 3)

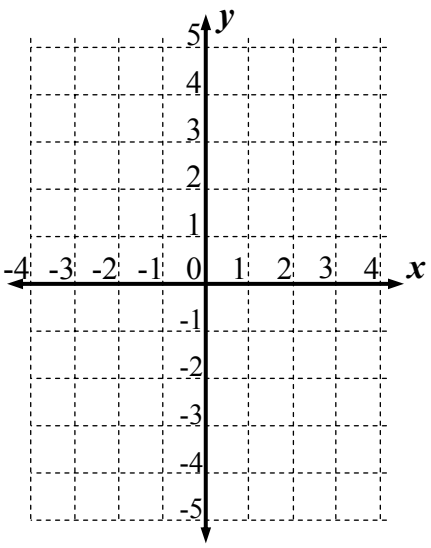
_____ Midpoint of *AB*

17 *F* (-3, 5)
Z (4, 3)

_____ Midpoint of *FZ*

18 *T* (3, -1)
Q (-3, -5)

_____ Midpoint of *TQ*



Calculate the coordinates of the midpoint for these intervals, use:

$$M(\overline{x}, \overline{y}) = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

19 (4, 6) and (10, 12)

_____ _____

_____ _____

_____

20 (-5, -2) and (9, 8)

_____ _____

_____ _____

_____

21 (-9, -8) and (-2, -2)

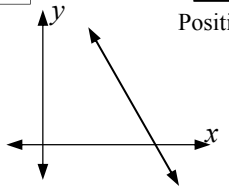
_____ _____

_____ _____

_____

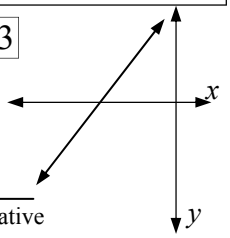
Do these lines have a positive or negative gradient?

22



_____ Positive or negative

23



_____ Positive or negative

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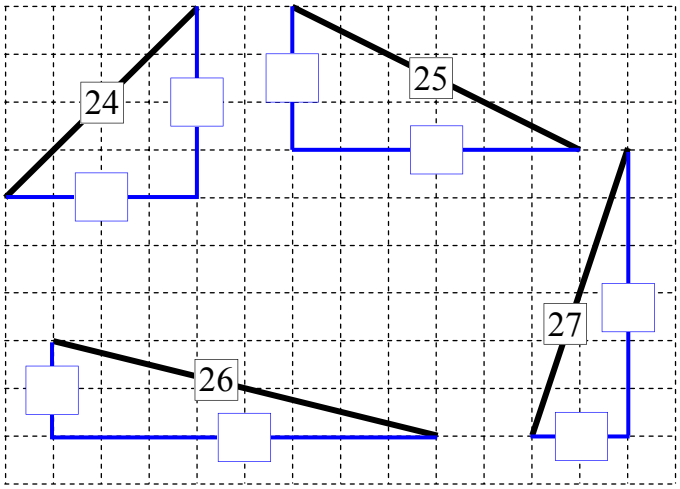
Record the triangle side lengths and calculate the gradient for the following intervals. Simplify when possible.

Use: $m = \frac{rise}{run}$



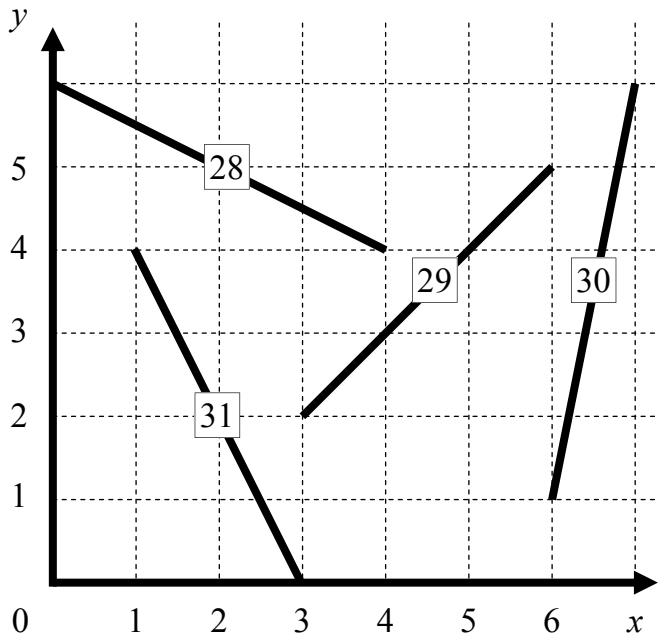
24 $m =$ _____ 25 _____

26 _____ 27 _____

Calculate the gradient for the following intervals.

Use: $m = \frac{rise}{run}$

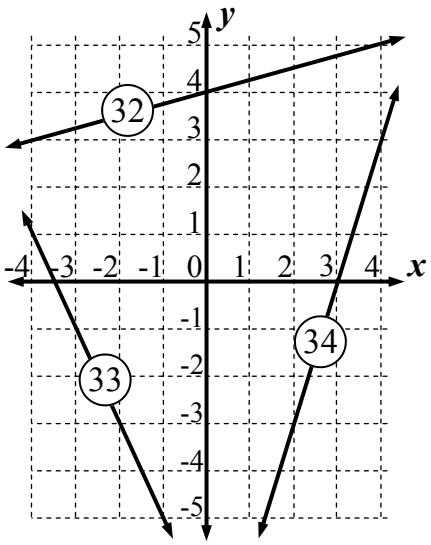
Simplify your answer where possible.



28 _____ 29 _____

30 _____ 31 _____

Calculate the gradient for the following lines. Simplify your answer where possible.



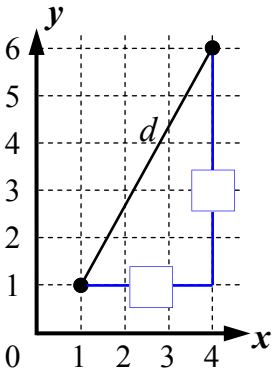
32 _____ 33 _____

34 _____

Calculate the distance between these two points. Use Pythagoras Theorem rounding to 1 d.p.

Use: $c^2 = a^2 + b^2$

35



_____

_____

_____

_____

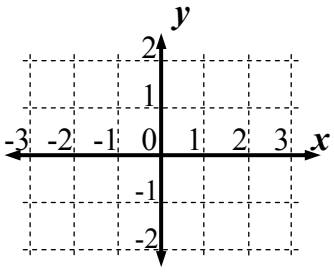
36 (-3, 2) and (3, -2).

_____

_____

_____

_____



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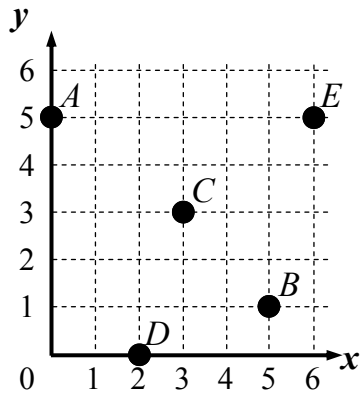
Give the coordinates for the five plotted points.



1 A (0, 5)

2 B (5, 1)

3 C (3, 3)

4 D (2, 0)

5 E (6, 5)

Give the coordinates for the six plotted points.

6 A (1, 2)

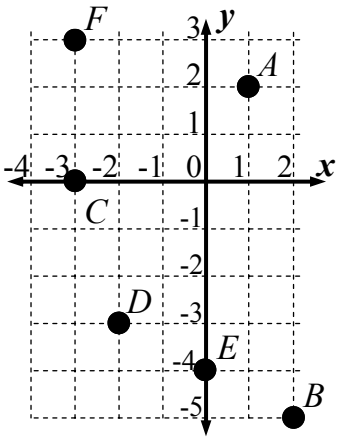
7 B (2, -5)

8 C (-3, -1)

9 D (-2, -3)

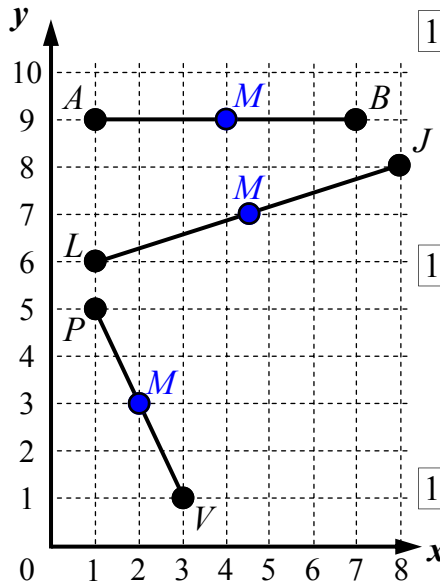
10 E (0, -4)

11 F (-3, 3)



12 Give the coordinates for the origin (0, 0)

Plot the midpoint 'M' on the intervals, give the coordinates



13 AB (4, 9) Midpoint Coordinates

14 LJ (4½, 7) Midpoint Coordinates

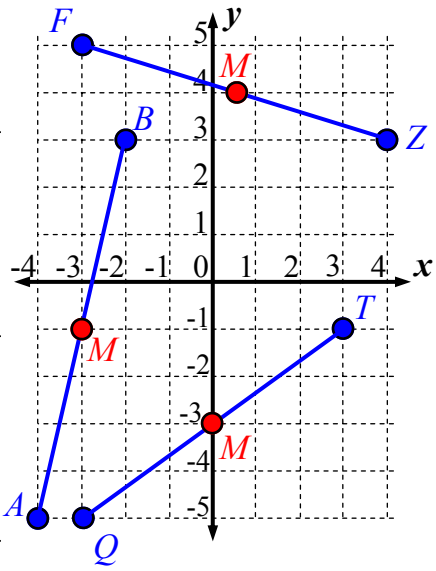
15 PV (2, 3) Midpoint Coordinates

Plot the points, find the midpoint coordinates.

16 A (-4, -5)
B (-2, 3)
(-3, -1)
Midpoint of AB

17 F (-3, 5)
Z (4, 3)
(½, 4)
Midpoint of FZ

18 T (3, -1)
Q (-3, -5)
(0, -3)
Midpoint of TQ



Calculate the coordinates of the midpoint for these intervals, use:

$$M(\bar{x}, \bar{y}) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

19 (4, 6) and (10, 12)

$x_1\ y_1 \quad x_2\ y_2$

$\bar{x} = \frac{4 + 10}{2} = 7$ $\bar{y} = \frac{6 + 12}{2} = 9$

$M(\bar{x}, \bar{y}) = (7, 9)$

20 (-5, -2) and (9, 8)

$x_1\ y_1 \quad x_2\ y_2$

$\bar{x} = \frac{(-5) + 9}{2} = 2$ $\bar{y} = \frac{(-2) + 8}{2} = 3$

$M(\bar{x}, \bar{y}) = (2, 3)$

21 (-9, -8) and (-2, -2)

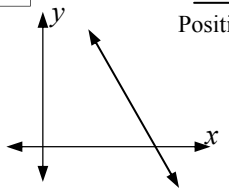
$x_1\ y_1 \quad x_2\ y_2$

$\bar{x} = \frac{(-9) + (-2)}{2} = -5\frac{1}{2}$ $\bar{y} = \frac{(-2) + (-8)}{2} = -5$

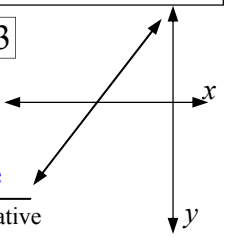
$M(\bar{x}, \bar{y}) = (-5\frac{1}{2}, -5)$

Do these lines have a positive or negative gradient?

22 Negative (Positive or negative)



23 Positive (Positive or negative)



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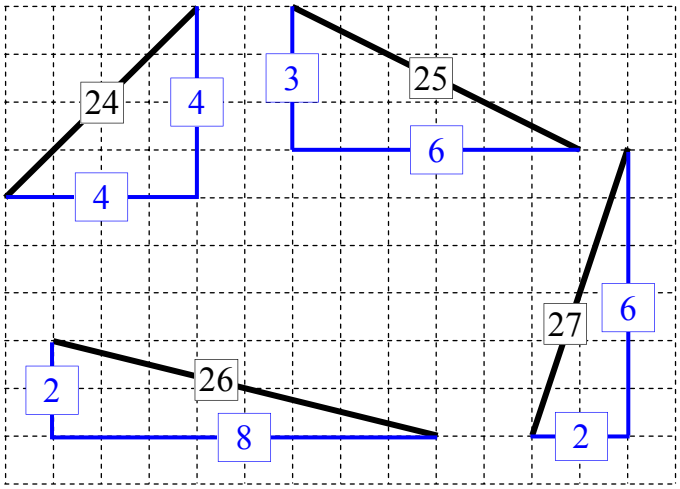
Record the triangle side lengths and calculate the gradient for the following intervals. Simplify when possible.

Use: $m = \frac{rise}{run}$



24 $m = \frac{4}{4} = 1$

25 $m = \frac{3}{6} = \frac{-1}{2}$

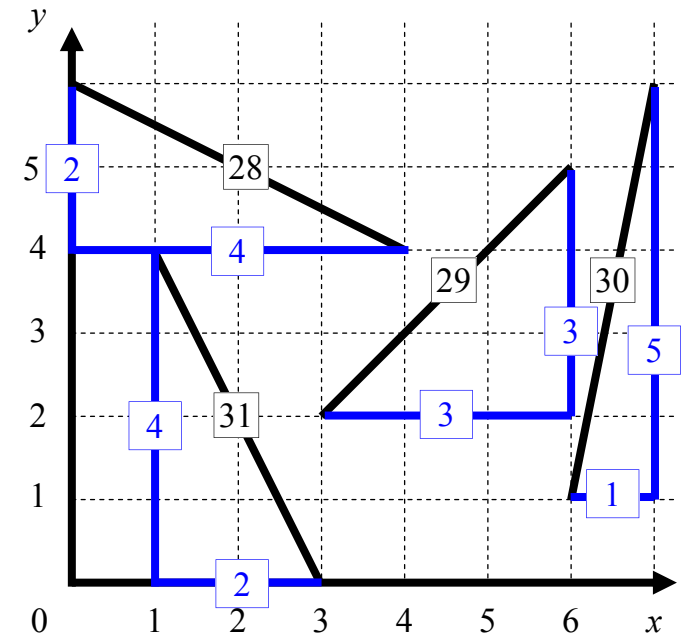
26 $m = \frac{2}{8} = \frac{-1}{4}$

27 $m = \frac{6}{2} = 3$

Calculate the gradient for the following intervals.

Use: $m = \frac{rise}{run}$

Simplify your answer where possible.



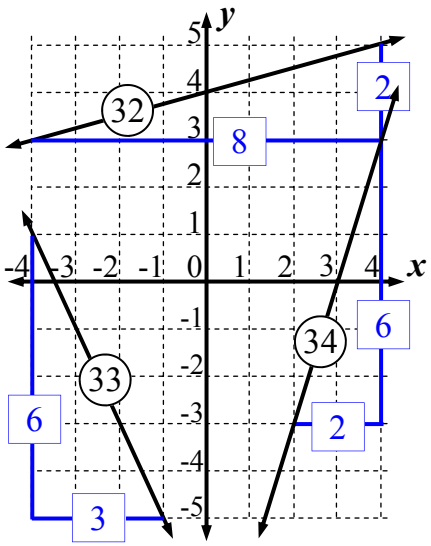
28 $m = \frac{2}{4} = \frac{-1}{2}$

29 $m = \frac{3}{3} = 1$

30 $m = \frac{5}{1} = 5$

31 $m = \frac{4}{2} = -2$

Calculate the gradient for the following lines. Simplify your answer where possible.



32 $m = \frac{2}{8} = \frac{1}{4}$

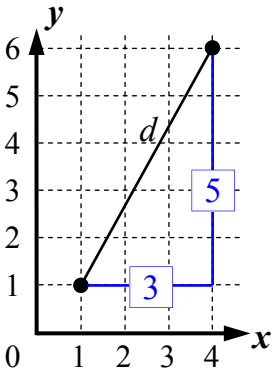
33 $m = \frac{6}{3} = -2$

34 $m = \frac{6}{2} = 3$

Calculate the distance between these two points. Use Pythagoras Theorem rounding to 1 d.p.

Use: $c^2 = a^2 + b^2$

35



$d^2 = 3^2 + 5^2$

$d^2 = 34$

$d = \sqrt{34}$

$d = 5.8$ units

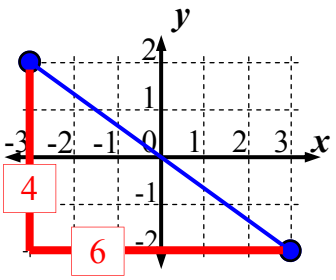
36 (-3, 2) and (3, -2).

$d^2 = 4^2 + 6^2$

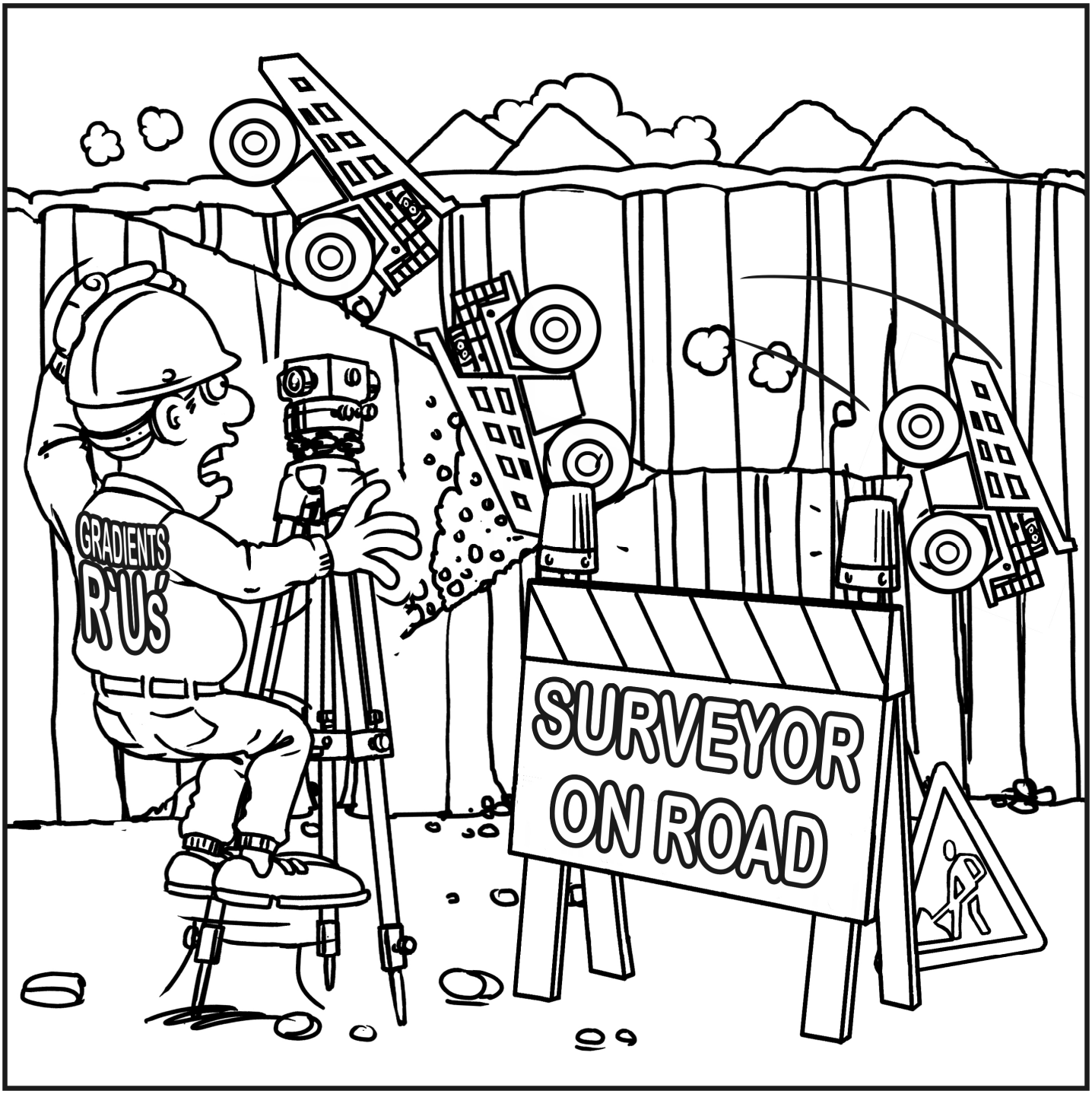
$d^2 = 52$

$d = \sqrt{52}$

$d = 7.2$ units



The Cartesian Plane



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Cartesian Plane - Plotting Points - Positive Quadrant (Sheet 1)

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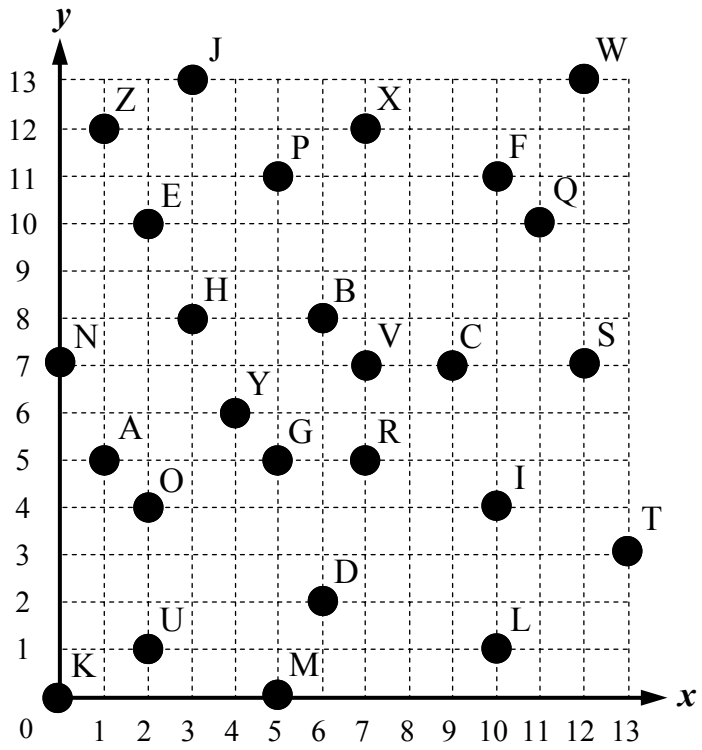
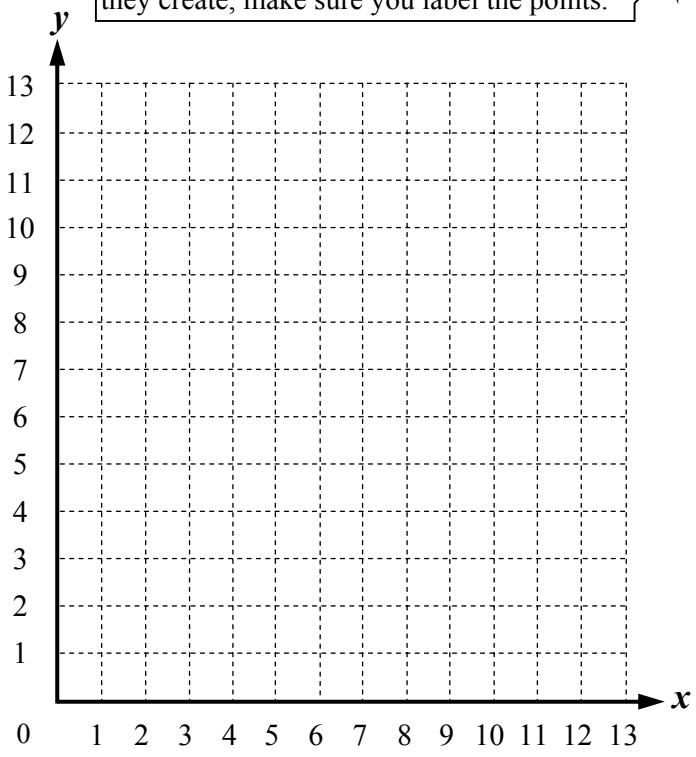
Give the co-ordinates for the following points, place them in brackets separated by a comma



1	Point A (,)	14	Point N
2	Point B (,)	15	Point O
3	Point C	16	Point P
4	Point D	17	Point Q
5	Point E	18	Point R
6	Point F	19	Point S
7	Point G	20	Point T
8	Point H	21	Point U
9	Point I	22	Point V
10	Point J	23	Point W
11	Point K	24	Point X
12	Point L	25	Point Y
13	Point M	26	Point Z

Place the groups of points given below on the grid, join the points and name the shape they create, make sure you label the points.



27 Point A (10, 4) Point B (11, 2)
Point C (12, 4) Point D (11, 5)
Shape formed: _____

28 Point E (3, 5) Point F (0, 8)
Point G (5, 13) Point H (8, 10)
Shape formed: _____

29 Point J (5, 4) Point K (9, 6)
Point L (7, 8)
Shape formed: _____
Two word description

30 Point M (2, 3) Point N (8, 3)
Point O (6, 0) Point P (0, 0)
Shape formed: _____

31 Point R (10, 9) Point S (13, 13)
Point T (13, 9)
Shape formed: _____
Two or three word description

Cartesian Plane - Drawing Animals - Positive Quadrant (Sheet 2)

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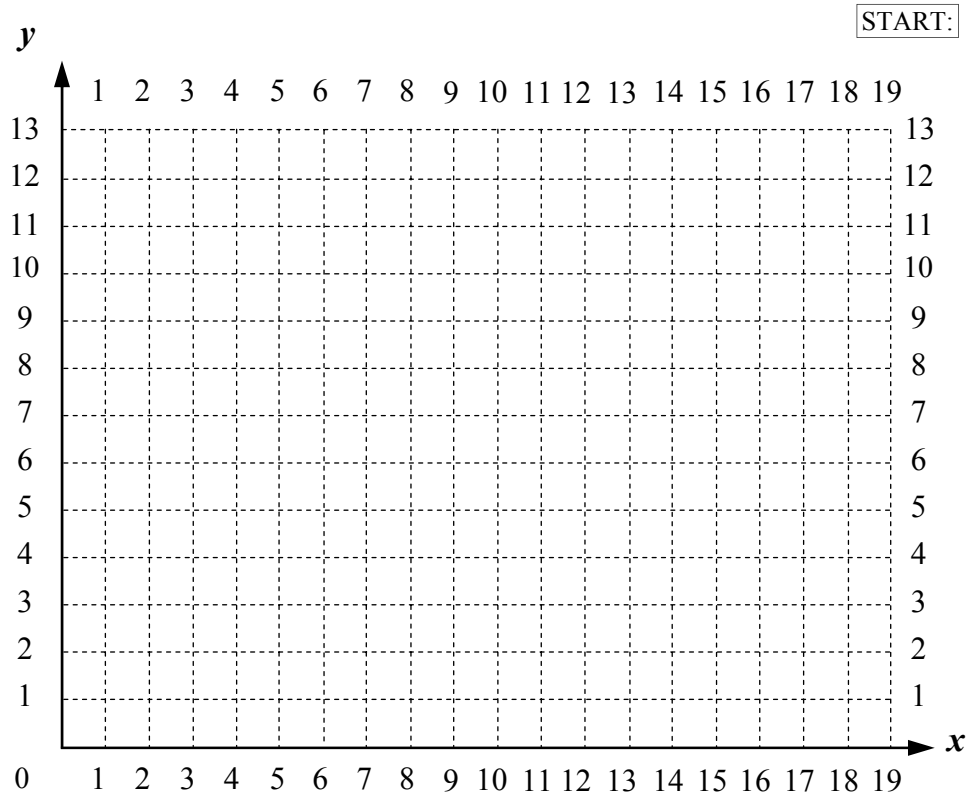
Join the points on the grid with the following co-ordinates and you will find an animal friend who has only one hand.



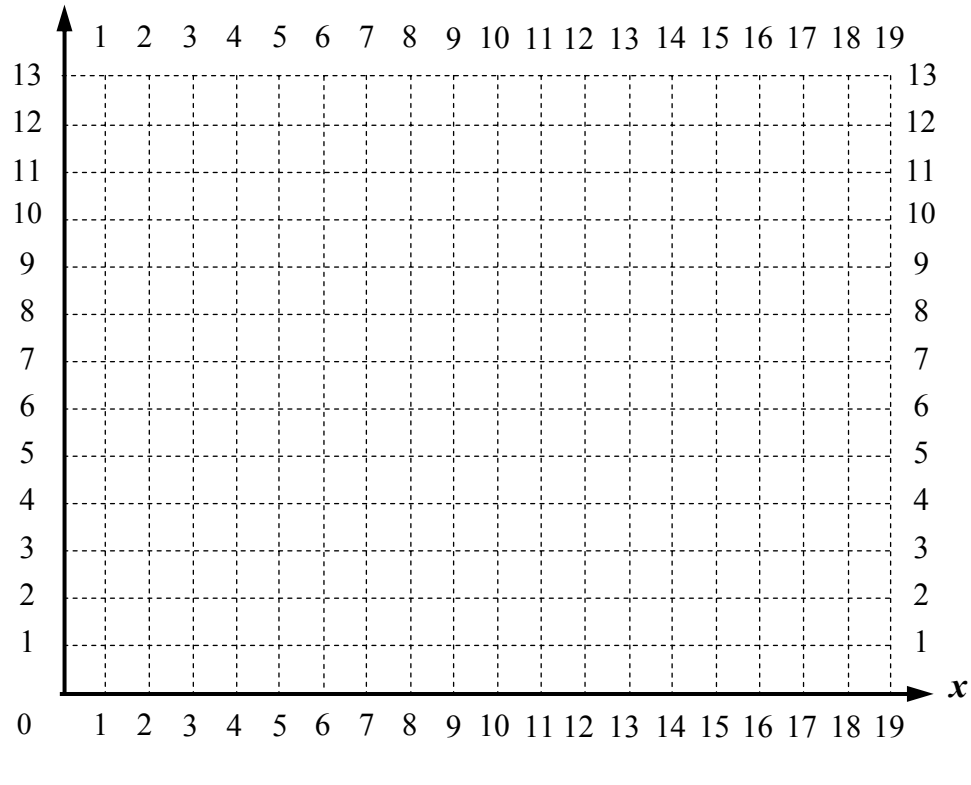
Co-ordinates:

START: (2, 12) , (3, 12) , (4, 11) , (5, 12) ,
(7, 12) , (8, 11) , (9, 12) , (10, 12) ,
(11, 11) , (11, 6) , (10, 3) , (9, 3) ,
(8, 7) , (8, 5) , (7, 7) , (7, 3) ,
(6, 2) , (4, 2) , (4, 3) , (5, 3) ,
(6, 4) , (5, 7) , (4, 5) , (4, 7) ,
(3, 3) , (2, 3) , (1, 6) , (1, 11) ,
(2, 12) STOP!

START:
(11, 10) , (12, 11) , (16, 12) , (18, 10) ,
(19, 7) , (18, 9) , (18, 3) , (15, 3) ,
(16, 4) , (16, 6) , (14, 5) , (11, 5) ,
(11, 4) , (13, 4) , (13, 1) , (12, 2) ,
(9, 2) , (8, 3) , (7, 6) , STOP!



Join the points and this time you will find a friend with which you will have to be very patient.



Co-ordinates:

START: (1, 11) , (2, 10) , (2, 9) , (1, 8) ,
(1, 7) , (2, 6) , (2, 5) , (3, 3) ,
(5, 1) , (7, 0) , (13, 0) , (16, 3) ,
(17, 5) , (15, 3) , (12, 2) , (8, 2) ,
(6, 3) , (4, 5) , (3, 6) , (4, 7) ,
(4, 8) , (3, 9) , (3, 10) , (4, 11) ,
STOP!

START:
(11, 6) , (12, 6) , (11, 7) , (10, 6) ,
(11, 5) , (12, 5) , (14, 6) , (14, 7) ,
(12, 9) , (10, 9) , (8, 7) , (8, 5) ,
(10, 3) , (12, 3) , (14, 4) , (15, 6) ,
(15, 7) , (14, 9) , (13, 10) , (10, 10) ,
(8, 9) , (7, 7) , (8, 3) , (11, 2) ,
(12, 2) , (15, 3) , (17, 6) , (17, 7) ,
(15, 11) , (12, 12) , (10, 12) , (7, 11) ,
(5, 9) , (4, 6) , (4, 5) , STOP!

Cartesian Plane - Midpoint (Graphical) - Positive Quadrant (Sheet 1)

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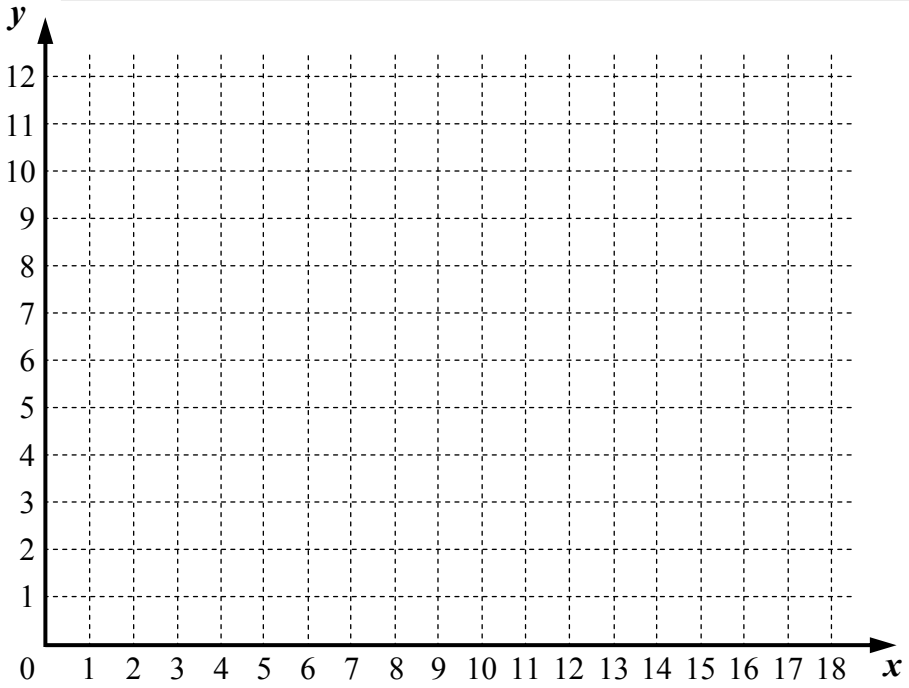
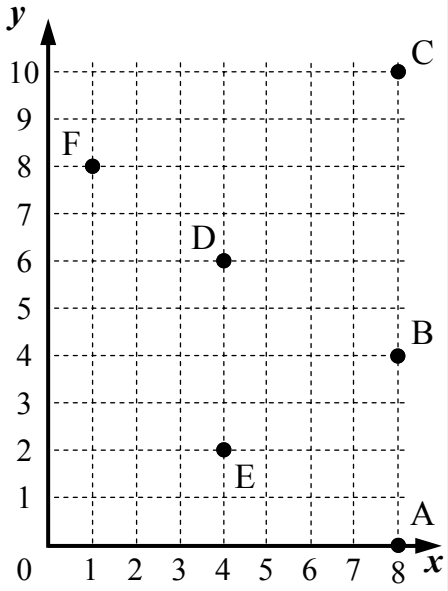
Plot the points and use a ruler to rule a line between the two points and find the midpoint. Plot the midpoint on the line record its co-ordinates. Write letters in pencil, as you may need to move them later!



This time the points are plotted, rule the line, find the midpoint and record all coordinates.



13 Construct DC and find 'M'

D: (☐ , ☐) C: (☐ , ☐)

Midpoint M: (☐ , ☐)

14 Construct FE and find 'M'

F: (☐ , ☐) E: (☐ , ☐)

Midpoint M: (☐ , ☐)

15 Construct FC and find 'M'

F: (☐ , ☐) C: (☐ , ☐)

Midpoint M: (☐ , ☐)

16 Construct BD and find 'M'

B: (☐ , ☐) D: (☐ , ☐)

Midpoint M: (☐ , ☐)

17 Construct AD and find 'M'

A: (☐ , ☐) D: (☐ , ☐)

Midpoint M: (☐ , ☐)

Steps: Plot the points on the number plane.
Rule a line in between the two points.
Plot the midpoint, call it 'M'.
Write the co-ordinate for the midpoint.

1 A (1, 12) and B (3, 4).

Midpoint: (☐ , ☐)

Label the midpoint 'M' on the number plane

2 Y (18, 10) and X (6, 12).

Midpoint: (☐ , ☐)

3 L (6, 11) and N (6, 1).

Midpoint: _____

4 E (0, 4) and F (6, 0).

Midpoint: _____

5 Q (15, 9) and R (5, 3).

Midpoint: _____

'QR' will cross previous line

6 J (9, 7) and K (17, 7).

Midpoint: _____

It is possible to have '½' in a co-ordinate. These all do! Approach these the same way.

Use a different colour for these lines

7 T (18, 4) and B (3, 4).

Midpoint: _____

8 C (10, 0) and D (16, 3).

Midpoint: _____

9 L (6, 11) and J (9, 7).

Midpoint: _____

10 T (18, 4) and K (17, 7).

Midpoint: _____

11 B (3, 4) and L (6, 11).

Midpoint: _____

12 V (10, 10) and Q (15, 9).

Midpoint: _____

Cartesian Plane - Midpoint (Analytical) - Positive Quadrant (Sheet 2)

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Find the average of these two numbers. Add them, press '=' and then divide '÷' by 2. $\bar{x}$ is the mean (pronounced *x*-bar).

1 22 and 38

$$\bar{x} = \frac{\square + \square}{2} = \square$$

Calc: 2 2 + 3 8 = ÷ 2 =

2 26 and 6

$$\bar{x} = \frac{\square + \square}{2} = \square$$

3 5 and 33

$$\bar{x} =$$

4 14 and 21

$$\bar{x} =$$

With halves use '½' not '.5'

Find the midpoint of the interval that joins these two points. Find two averages.

Example

Find the midpoint of the interval joining (2, 7) and (14, 15).

x_1 y_1 x_2 y_2

$$\bar{x} = \frac{2 + 14}{2} = 8$$

$$\bar{y} = \frac{7 + 15}{2} = 11$$

$$M(\bar{x}, \bar{y}) = (8, 11)$$

5 (5, 2) and (23, 14).

$$\bar{x} = \frac{\square + \square}{2} = \square$$

$$\bar{y} = \frac{\square + \square}{2} = \square$$

$$M(\bar{x}, \bar{y}) = (\square, \square)$$

6 (1, 14) and (19, 22).

$$\bar{x} = \frac{\square + \square}{2} = \square$$

$$\bar{y} = \frac{\square + \square}{2} = \square$$

$$M(\bar{x}, \bar{y}) = (\square, \square)$$

7 (14, 2) and (8, 6).

$$\bar{x} =$$

$$\bar{y} =$$

$$M(\bar{x}, \bar{y}) = (\square, \square)$$

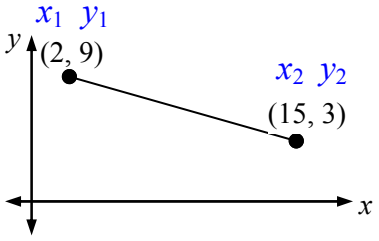
8 (9, 17) and (3, 8).

9 (11, 2) and (12, 9).

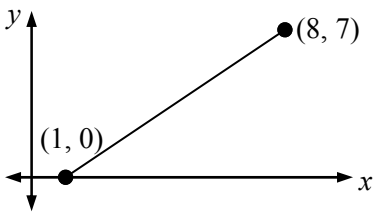
10 (6, 0) and (11, 13).

Use the same method, but this time you have to get your information from a sketch.

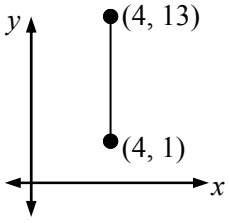
11



12



13



Cartesian Plane - Plotting Points - All Quadrants (Sheet 1)

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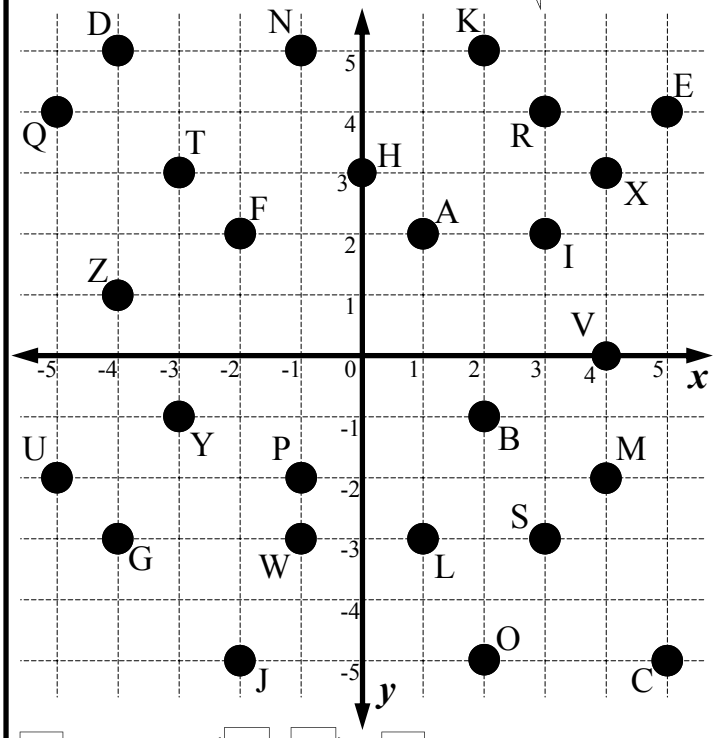
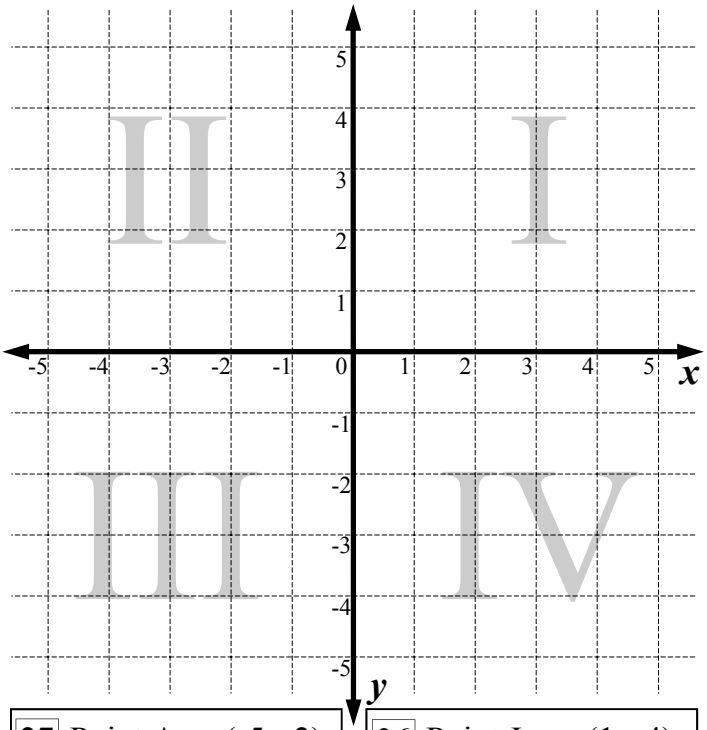
Give the co-ordinates for the points on the number plane below.



1 Point A (,)
2 Point B
3 Point C
4 Point D
5 Point E
6 Point F
7 Point G
8 Point H
9 Point I
10 Point J
11 Point K
12 Point L
13 Point M
14 Point N
15 Point O
16 Point P
17 Point Q
18 Point R
19 Point S
20 Point T
21 Point U
22 Point V
23 Point W
24 Point X
25 Point Y
26 Point Z

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant I II III IV Axis
28 Point B (2, 4) Quadrant I II III IV Axis
29 Point C (4, 0) Quadrant I II III IV Axis
30 Point D (3, -3) Quadrant I II III IV Axis
31 Point E (-3, 5) Quadrant I II III IV Axis
32 Point F (0, 0) Quadrant I II III IV Axis
33 Point G (5, -5) Quadrant I II III IV Axis
34 Point H (-2, 4) Quadrant I II III IV Axis
35 Point I (1, 2) Quadrant I II III IV Axis
36 Point J (1, -4) Quadrant I II III IV Axis
37 Point K (-3, -4) Quadrant I II III IV Axis
38 Point L (-1, 0) Quadrant I II III IV Axis
39 Point M (-4, 2) Quadrant I II III IV Axis
40 Point N (3, -5) Quadrant I II III IV Axis
41 Point O (0, -3) Quadrant I II III IV Axis
42 Point P (-5, -5) Quadrant I II III IV Axis
43 Point R (-2, 1) Quadrant I II III IV Axis
44 Point S (2, -2) Quadrant I II III IV Axis

Cartesian Plane - Drawing Animals - All Quadrants (Sheet 2)

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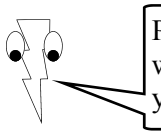
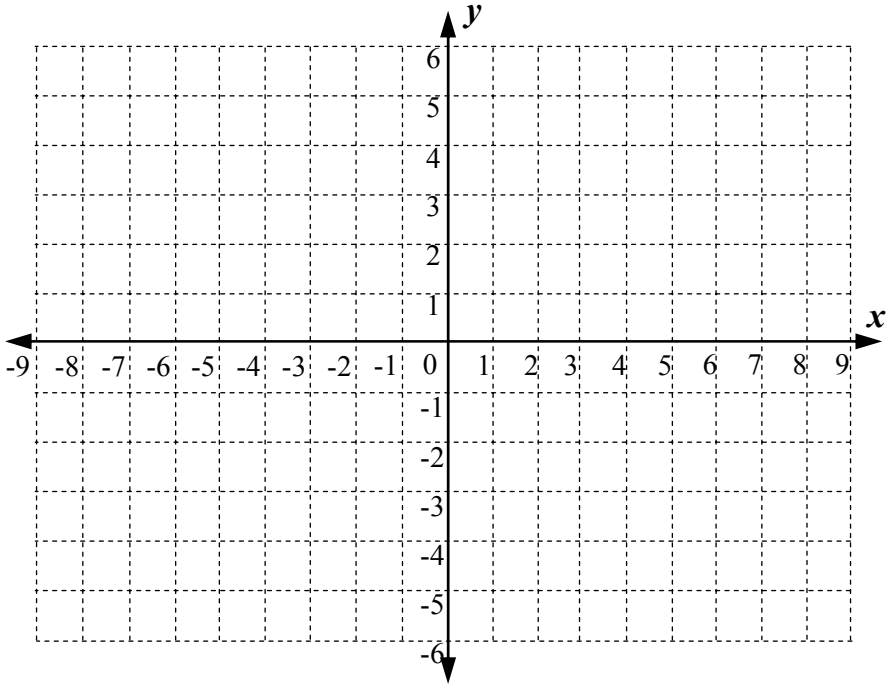


Plot the points and join them with lines to meet one of your more colourful friends

Co-ordinates: START: (-1, 3) , (-1, 4) , (1, 6) , (2, 6) , (6, 5) , (7, 4) , (7, 2) , (6, -1) , (3, -2) , (5,-4) , (5, -5) , (3, -6) , (1, -6) , (-2, -4) , (-4, -6) , (-5, -6) , (-6, -5) , (-6, -4) , (-5, -2) , (-6, -1) , (-8, 2) , (-8, 4) , (-7, 6) , (-6, 6) , (-4, 4) , (-4, 3) , STOP!

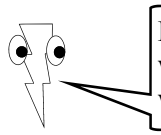


START: (-2, 1) , (1, 4) , (4, 4) , (5, 2) , (3,-1) , (0, -2) , (3, -3) , (3, -5) , (1, -5) , (-1, -4) , (-2, -3) , (-4, -5) , (-5, -5) , (-5, -3) , (-3, -2) , (-5, -1) , (-7, 2) , (-6, 4) , (-5, 4) , (-3,1) , STOP!

START: (-4, 3) , (-3, 4) , (-2, 4) , (-1, 3) , (-2, 2) , (-2, -4) , (-3, 2) , (-4, 3) , STOP!

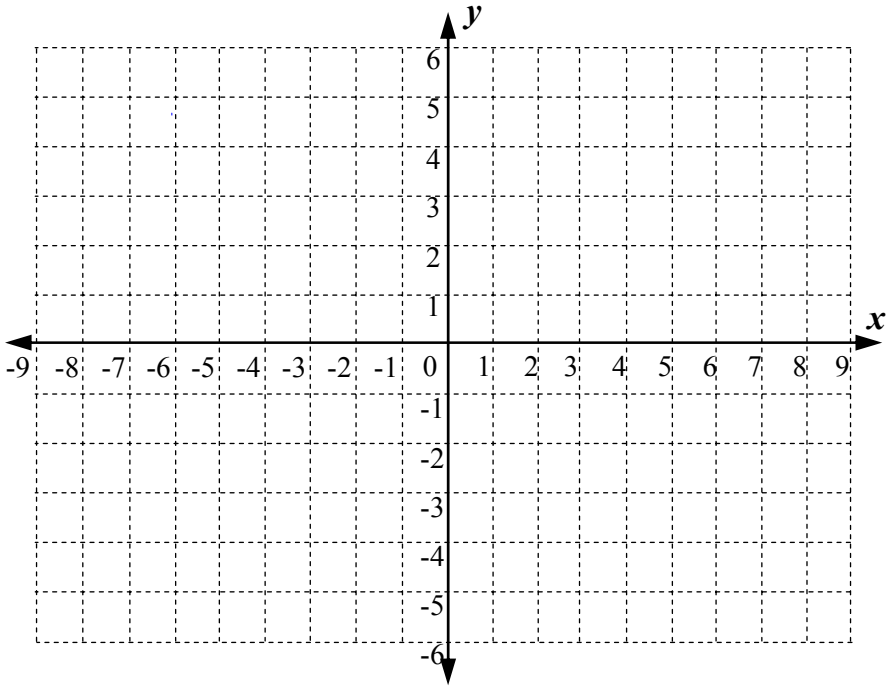
START: (-3, 4) , (-4, 6) , STOP!

START: (-2, 4) , (-1, 6) , STOP!



Plot the points and join them with lines to share your lunch with your vegetarian friend.

Co-ordinates: START: (2, -1) , (3, -3) , (3, -4) , (2, -5) , (0, -6) , (-1, -6) , (-3, -5) , (-4, -4) , (-4, -3) , (-3, -1) , (-6, 2) , (-7, -4) , (-8, 4) , (-7, 6) , (-4, 5) , (-2, -1) , (0, -1) , (3, 4) , (5, 6) , (8, 6) , (9, 4) , (5, 6) , STOP!



START: (7, 5) , (6, 3) , (2, -1) , STOP!

START: (2, 0) , (4, 4) , (6, 5) , (5, 3) , (2, 0) , STOP!

START: (-3, 0) , (-5, 2) , (-6, 4) , (-4, 3) , (-3, 0) , STOP!

START: (-2, -4) , (-6, -4) , STOP!

START: (-2, -4) , (-5, -2) , STOP!

START: (-2, -4) , (-5, -3) , STOP!

START: (1, -4) , (5, -4) , STOP!

START: (1, -4) , (5, -3) , STOP!

START: (1, -4) , (5, -5) , STOP!

Cartesian Plane - Midpoint (Graphical) - All Quadrants (Sheet 1)

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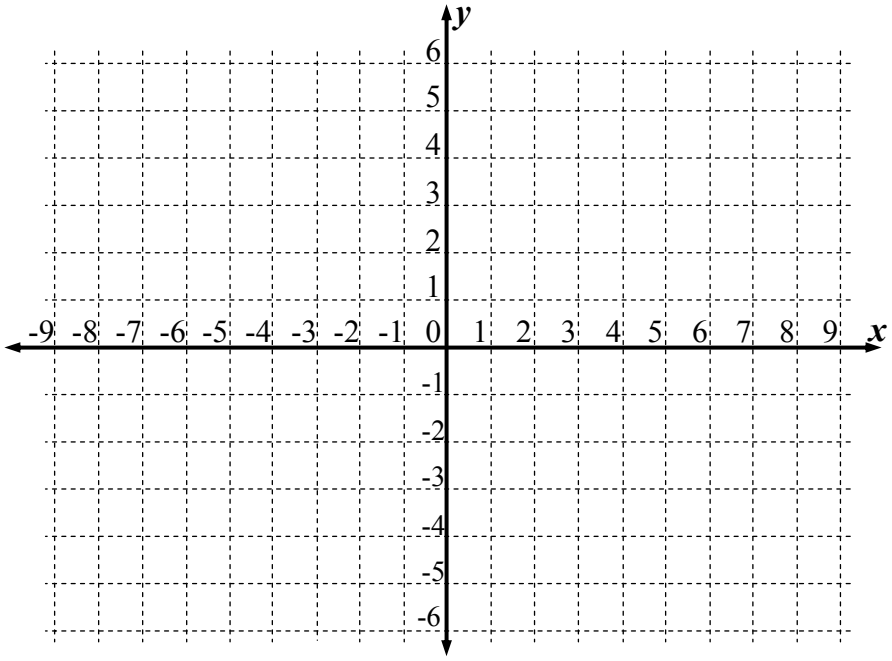
Plot the points and use a ruler to rule a line between the two points and find the midpoint. Plot the midpoint on the line record its co-ordinates. Write letters in pencil, as you may need to move them later!



Steps: Plot the points on the number plane.
Rule a line in between the two points.
Plot the midpoint, call it 'M'.
Write the co-ordinate for the midpoint.

1 *A* (-7, 5) and *H* (-3, 3).

Midpoint: (___ , ___)

Label the midpoint '*M*' on the number plane

2 *G* (8, -2) and *E* (2, -5).

Midpoint: (___ , ___)

3 *L* (-2, 6) and *F* (0, 4).

Midpoint: _____

4 *B* (7, 3) and *D* (-3, -1).

Midpoint: _____

5 *N* (-1, 2) and *P* (5, 6).

Midpoint: _____

6 *K* (-9, 3) and *H* (-3, 3).

Midpoint: _____

7 *C* (-9, -5) and *F* (0, 4).

Midpoint: _____

8 *J* (-6, -6) and *D* (-3, -1).

Midpoint: _____

9 *A* (-7, 5) and *F* (0, 4).

Midpoint: _____

10 *B* (7, 3) and *G* (8, -2).

Midpoint: _____

11 *G* (8, -2) and *D* (-3, -1).

Midpoint: _____

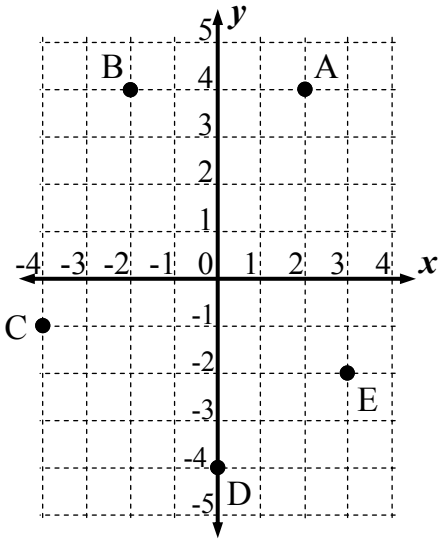
12 *C* (-9, -5) and *K* (-9, 3).

Midpoint: _____

13 *C* (-9, -5) and *E* (2, -5).

Midpoint: _____

This time the points are plotted, rule the line, find the midpoint and record all coordinates.



14 Construct *AB* and find '*M*'

A: (___ , ___) *B*: (___ , ___)

Midpoint *M*: (___ , ___)

15 Construct *AD* and find '*M*'

A: (___ , ___) *D*: (___ , ___)

Midpoint *M*: (___ , ___)

16 Construct *CB* and find '*M*'

C: (___ , ___) *B*: (___ , ___)

Midpoint *M*: (___ , ___)

17 Construct *EA* and find '*M*'

E: (___ , ___) *A*: (___ , ___)

Midpoint *M*: (___ , ___)

18 Construct *DE* and find '*M*'

D: (___ , ___) *E*: (___ , ___)

Midpoint *M*: (___ , ___)

Cartesian Plane - Midpoint (Analytical) - All Quadrants (Sheet 2)

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Find the average of these two numbers. Add them, press '=' and then divide by 2. Always write negatives in brackets.

1 -6 and -12

$$\bar{x} = \frac{(\square) + (\square)}{2} = \square$$

Calc: [−](-) 6 + [−](-) 1 2 = ÷ 2 =

2 -7 and 21

$$\bar{x} = \frac{(\square) + \square}{2} = \square$$

3 6 and -12

$\bar{x} =$

4 -4 and -9

$\bar{x} =$

With halves use '½' not '.5'

Find the midpoint of the interval that joins these two points. Find two averages.

Example

Find the midpoint of the interval joining (-4, -3) and (8, -11).

x_1 y_1 x_2 y_2

$$\bar{x} = \frac{(-4) + 8}{2} = 2$$

$$\bar{y} = \frac{(-3) + (-11)}{2} = -7$$

$M(\bar{x}, \bar{y}) = (2, -7)$

5 (2, -3) and (-6, 9).

$$\bar{x} = \frac{\square + (\square)}{2} = \square$$

$$\bar{y} = \frac{(\square) + \square}{2} = \square$$

$M(\bar{x}, \bar{y}) = (\square, \square)$

6 (-1, -6) and (-3, 10).

$$\bar{x} = \frac{(\square) + (\square)}{2} = \square$$

$$\bar{y} = \frac{(\square) + \square}{2} = \square$$

$M(\bar{x}, \bar{y}) = (\square, \square)$

7 (7, -5) and (-3, 9).

$\bar{x} =$

$\bar{y} =$

$M(\bar{x}, \bar{y}) = (\square, \square)$

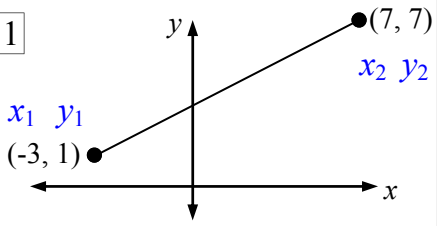
8 (-9, -2) and (-1, -9).

9 (3, 8) and (-3, -6).

10 (0, -5) and (-10, 8).

Use the same method, but this time get your information from a sketch. Label points (x_1, y_1)…

11



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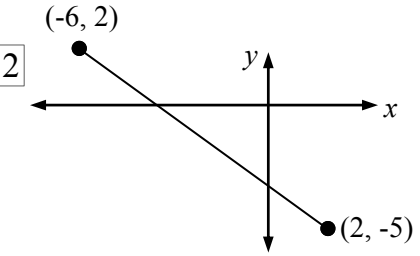

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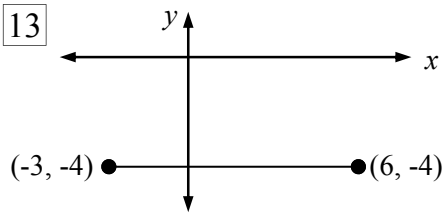

Cartesian Plane - Introduction to Gradient (Positive) (Sheet 1)

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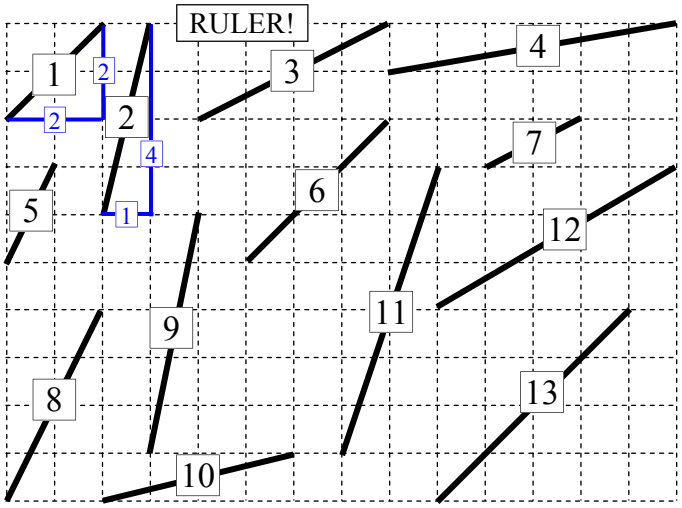
RULER!



Create 'gradient triangles' for the above intervals. Then write the 'rise' and the 'run' on each triangle. The first two are done. Find the gradient for each.

1 $m = \frac{\square}{\square} = \square$

2 $m = \frac{\square}{\square} = \square$

3 $m = \frac{\square}{\square} = \frac{\square}{\square}$

4 $m = \frac{\square}{\square}$

Calc: 2 ▪ 4 =

5 $m = \frac{\square}{\square} = \square$

6 $m = \frac{\square}{\square} = \square$

7 $m = \frac{\square}{\square}$

8 $m = \frac{\square}{\square} = \square$

9 $m = \frac{\square}{\square} = \square$

10 $m = \frac{\square}{\square}$

11 $m = \frac{\square}{\square} = \square$

12 $m = \frac{\square}{\square}$

13 $m = \frac{\square}{\square} = \square$

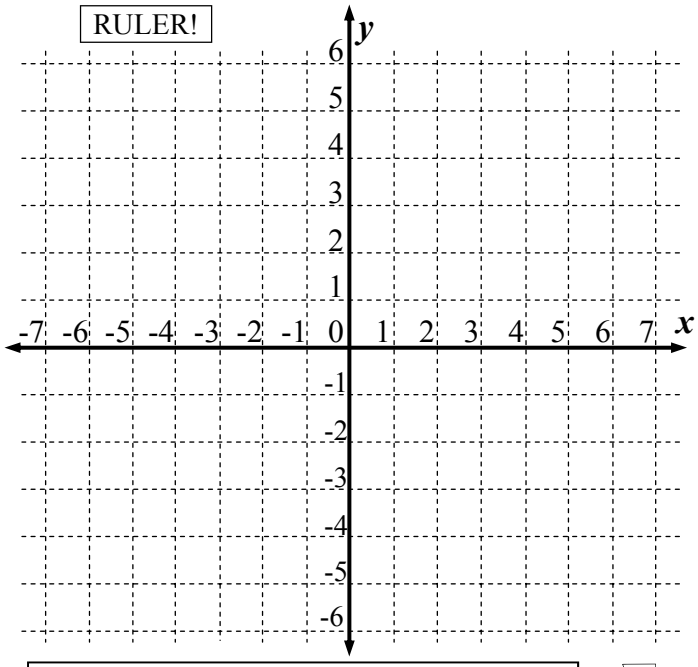
14 Which of the intervals are parallel (have the same gradient)?

a) Q3 || Q□

b) Q5 || Q□

c) Q1 || Q□ || Q□

RULER!



Plot the coordinates, then join them with an interval. Calculate the gradient for the interval by creating a gradient triangle. Use a ruler.

15 *A* (1, 2) and *B* (2, 6). $m = \frac{\square}{\square} = \square$

16 *C* (1, -2) and *D* (7, 4). $m = \frac{\square}{\square} = \square$

17 *E* (-7, -5) and *F* (-5, 3). $m = \frac{\square}{\square} = \square$

18 *G* (-7, 4) and *H* (-3, 6). $m = \frac{\square}{\square} = \frac{\square}{\square}$

19 *J* (-2, -6) and *K* (6, -2). $m = \frac{\square}{\square} = \frac{\square}{\square}$

20 *Q* (-5, -3) and *R* (-1, 1). $m = \frac{\square}{\square} = \square$

21 Which of the intervals are parallel? Use the letters on the interval to complete these:

GH || □ (2 letters) *AB* || □ *CD* || □

Cartesian Plane - Introduction to Gradient (Negative) (Sheet 2)

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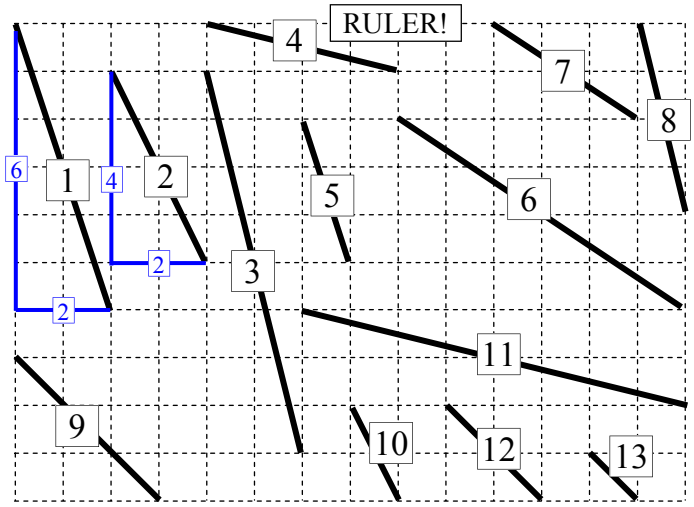
RULER!



Create 'gradient triangles' and write the 'rise' and the 'run' on each triangle, all positive numbers. Find the gradient and write a negative sign '-' in front of your answer.

1 $m =$ ☐/☐ $=$ ☐

2 $m =$ ☐/☐ $=$ ☐

3 $m =$ ☐/☐ $=$ ☐

4 $m =$ ☐/☐

5 $m =$ ☐/☐ $=$ ☐

6 $m =$ ☐/☐ $=$ ☐/☐

7 $m =$ ☐/☐

8 $m =$ ☐/☐ $=$ ☐

9 $m =$ ☐/☐ $=$ ☐

10 $m =$ ☐/☐ $=$ ☐

11 $m =$ ☐/☐ $=$ ☐/☐

12 $m =$ ☐/☐ $=$ ☐

13 $m =$ ☐/☐ $=$ ☐

14 Which of the intervals are parallel?

a) Q1 || Q☐

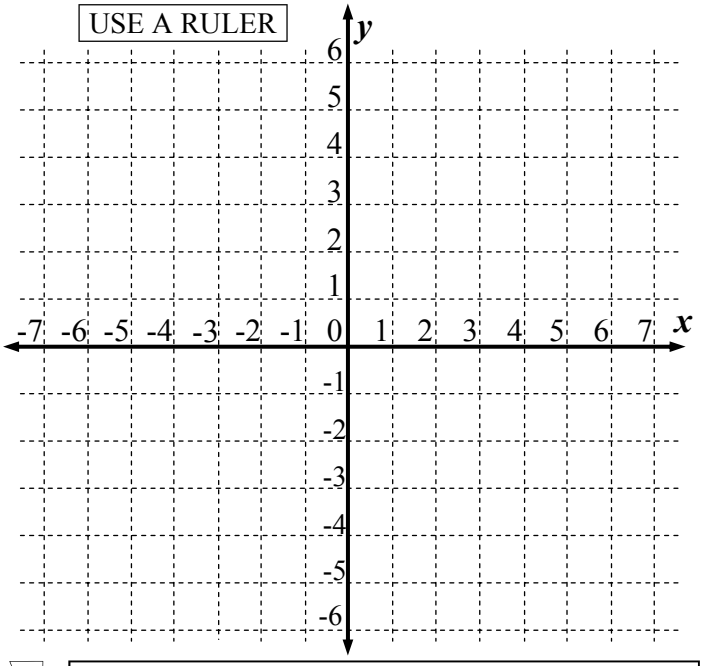
b) Q2 || Q☐

c) Q3 || Q☐

d) Q4 || Q☐

e) Q9 || Q☐ || Q☐

USE A RULER



Plot the coordinates, then join them with an interval. Calculate the gradient for the interval by creating a gradient triangle. **Use positive numbers** on the gradient triangles, but final answers will be negative.

15 A (6, 6) and B (7, 1). $m =$ ☐/☐ $=$ ☐

16 C (0, 5) and D (5, 0). $m =$ ☐/☐ $=$ ☐

17 E (-3, 3) and F (6, -3). $m =$ ☐/☐ $=$ ☐/☐

18 G (-5, 6) and H (-3, -4). $m =$ ☐/☐ $=$ ☐

19 J (-7, -2) and K (-3, -6). $m =$ ☐/☐ $=$ ☐

20 L (1, 6) and P (4, 4). $m =$ ☐/☐

21 Which of the intervals are parallel? Use the letters on the interval to complete these:

AB || ☐ EF || ☐ JK || ☐

Cartesian Plane - Gradient (Sheet 1)

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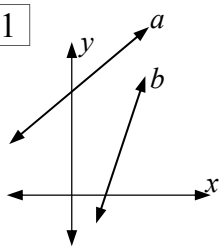
State whether the following have a positive or negative gradient.

Up to the left is -ve

Up to the right is +ve

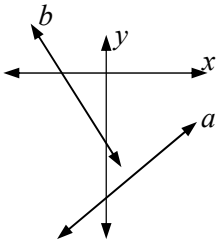
1



a) _____ Positive or Negative

b) _____ Positive or Negative

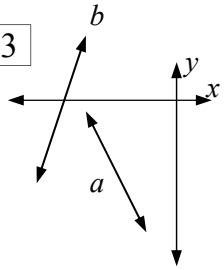
2



a) _____

b) _____

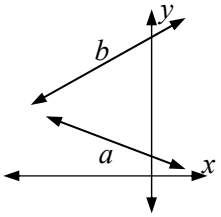
3



a) _____

b) _____

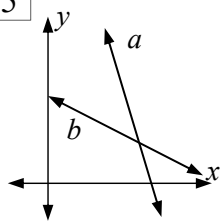
4



a) _____

b) _____

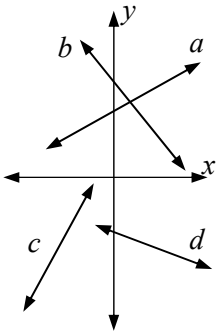
5



a) _____

b) _____

6



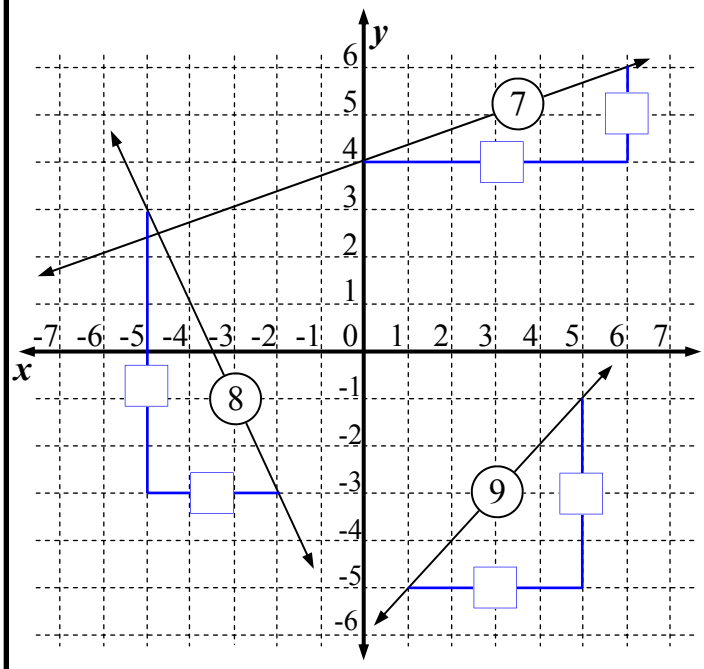
a) _____

b) _____

c) _____

d) _____

These lines have gradient triangles installed! Write the rise and run in the boxes on the triangles sides, then find the gradient. Make sure you remember to check for positive and negative gradient at the end.



7 $m = \frac{\square}{\square} = \frac{\square}{\square}$

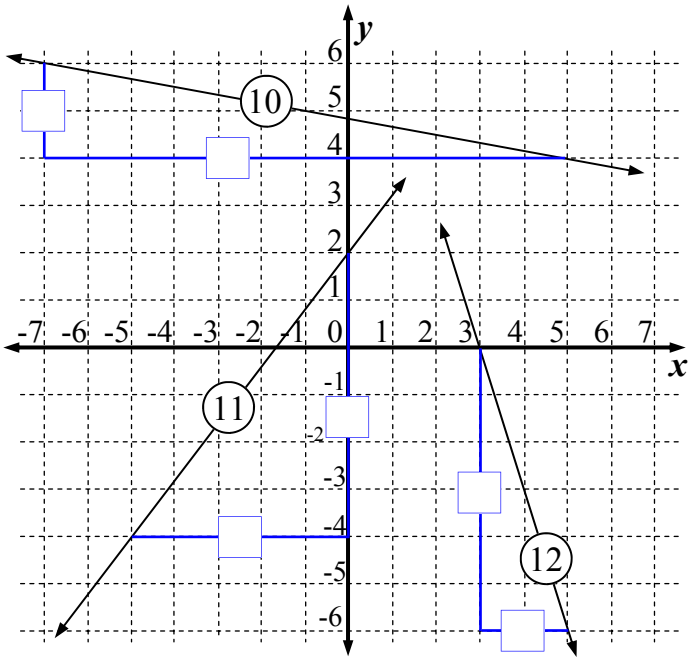
8 $m = \frac{\square}{\square} = \square$

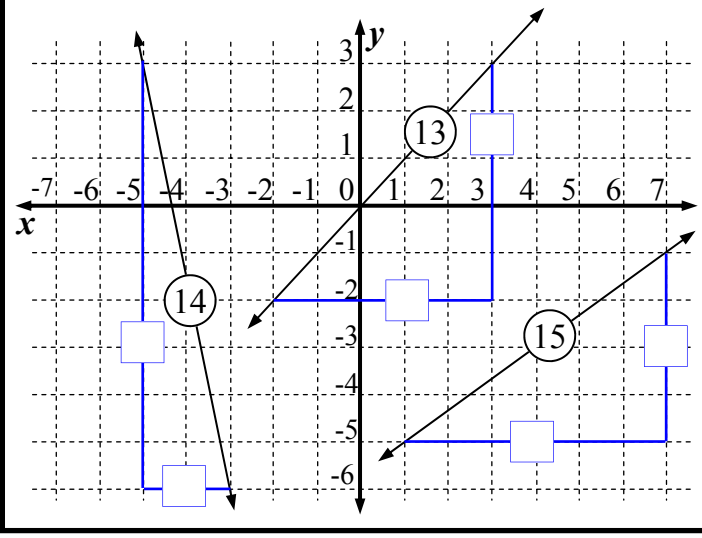
9 $m = \frac{\square}{\square} = \square$

10 $m = \frac{\square}{\square} = \frac{\square}{\square}$

11 $m = \frac{\square}{\square}$

12 $m = \frac{\square}{\square} = \square$





13 $m = \frac{\square}{\square} = \square$

14 $m = \frac{\square}{\square}$

15 $m = \frac{\square}{\square} = \frac{\square}{\square}$

Cartesian Plane - Gradient (Sheet 2)

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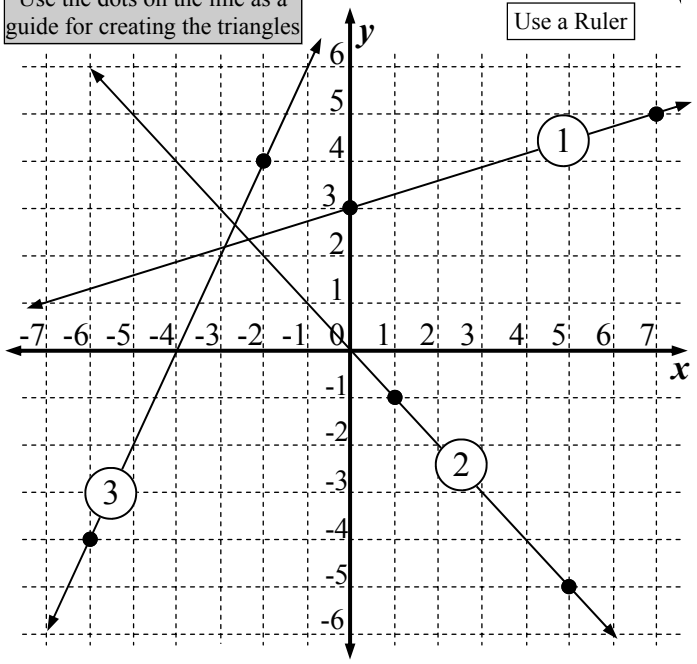
Find the gradient for these lines. Create gradient triangles and remember your positive and negatives.

Use the dots on the line as a guide for creating the triangles

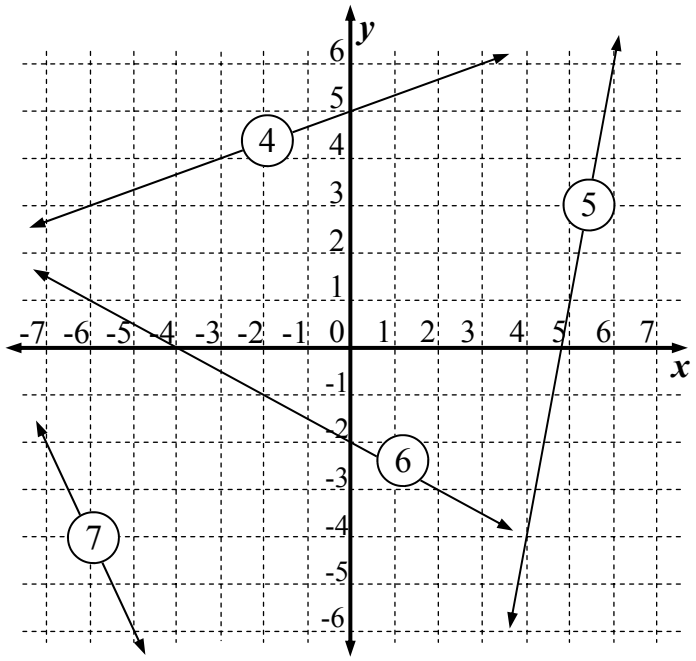
Use a Ruler



1 $m = \frac{\square}{\square}$

2 $m = \frac{\square}{\square} = \square$

3 $m = \frac{\square}{\square} = \square$

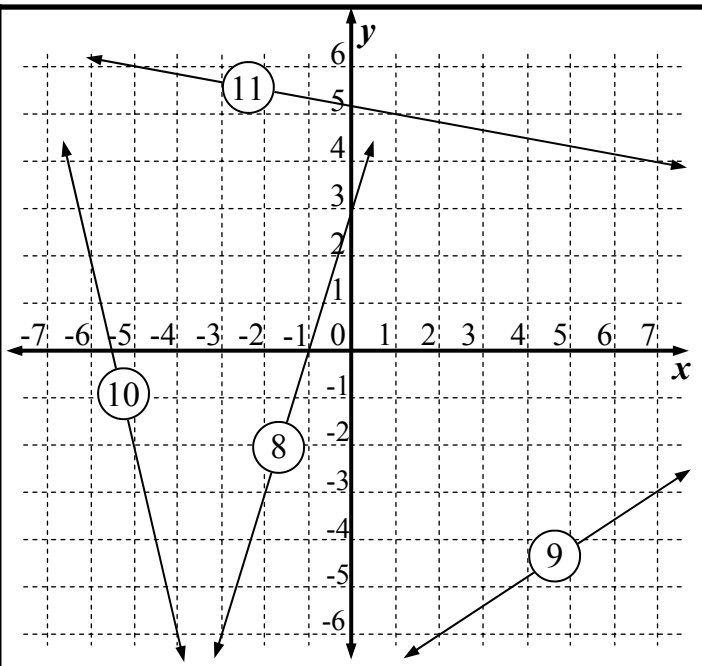


4 $m = \frac{\square}{\square} = \frac{\square}{\square}$

5 $m = \frac{\square}{\square} = \square$

6 $m = \frac{\square}{\square} = \frac{\square}{\square}$

7 $m = \frac{\square}{\square} = \square$

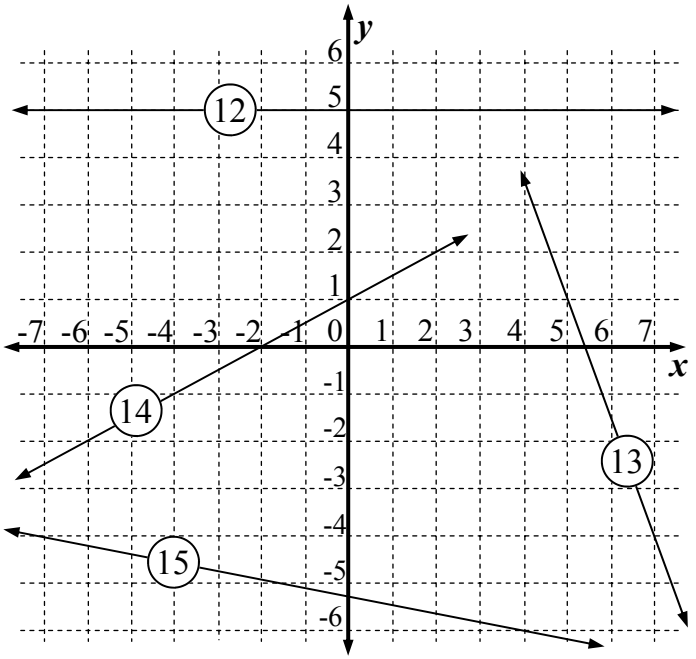


8 $m =$ _____

9 $m =$ _____

10 _____

11 _____



12 _____

13 _____

14 _____

15 _____

Cartesian Plane - Distance - Positive Quadrant (Sheet 1)

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Calculate these square roots, round your answer to 1 d.p.

1 $\sqrt{43} =$ ☐.☐ Round to 1 d.p.

Calc: √ 4 3 =

2 $\sqrt{28} =$ ☐

3 $\sqrt{75} =$ ☐

4 $\sqrt{66} =$ ☐

5 $\sqrt{59} =$ ☐

Calculate the additions of two squared numbers.

6 $8^2 + 4^2 =$ ☐

Calc: 8 x^2 + 4 x^2 =

7 $5^2 + 7^2 =$ ☐

8 $6^2 + 3^2 =$ ☐

9 $4^2 + 10^2 =$ ☐

10 $2^2 + 8^2 =$ ☐

Combine the two operations. Add the squares and square root the answer.

11 $8^2 + 5^2 =$ ☐

Get your answer, then write it here

$\sqrt{\square} =$ ☐

Calc:

Then square root (1 d.p.) √ ANS =

12 $4^2 + 9^2 =$ ☐

$\sqrt{\square} =$ ☐

13 $1^2 + 7^2 =$ ☐

$\sqrt{\square} =$ ☐

14 $6^2 + 6^2 =$ ☐

$\sqrt{\square} =$ ☐

Now use Pythagoras Theorem to calculate the length of an interval. Use 'd' to represent the distance between the two plotted points. (1 d.p.)

15

$d^2 = \square^2 + \square^2$

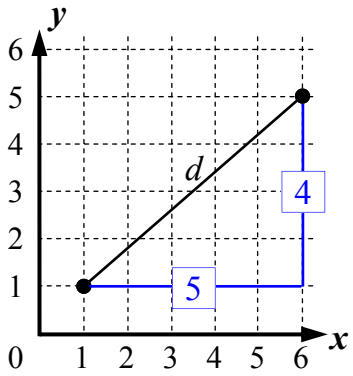
$d^2 = \square$

$d = \sqrt{\square}$ Rewrite

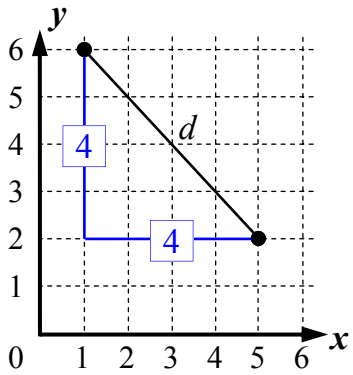
Calc: √ ANS =

$d =$ ☐.☐ *units*

(1 d.p.)



16



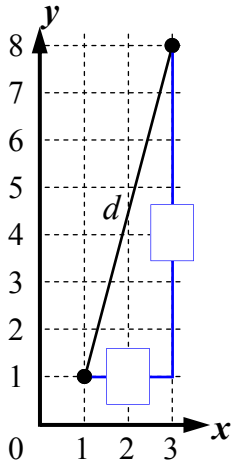
$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d =$ ☐ *units*

17



Now write the side lengths in the boxes.

$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d =$ ☐ *units*

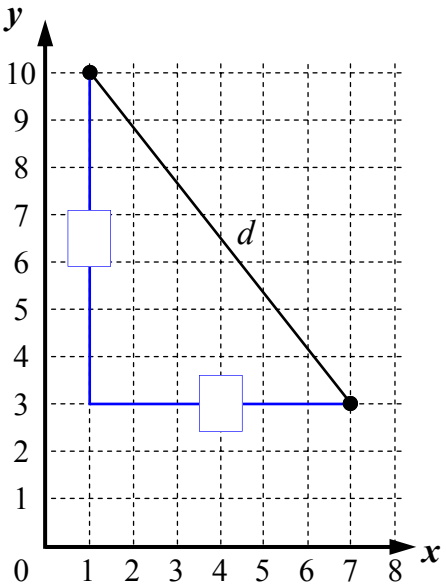
18

$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d =$ ☐ *units*



Cartesian Plane - Distance - Positive Quadrant (Sheet 2)

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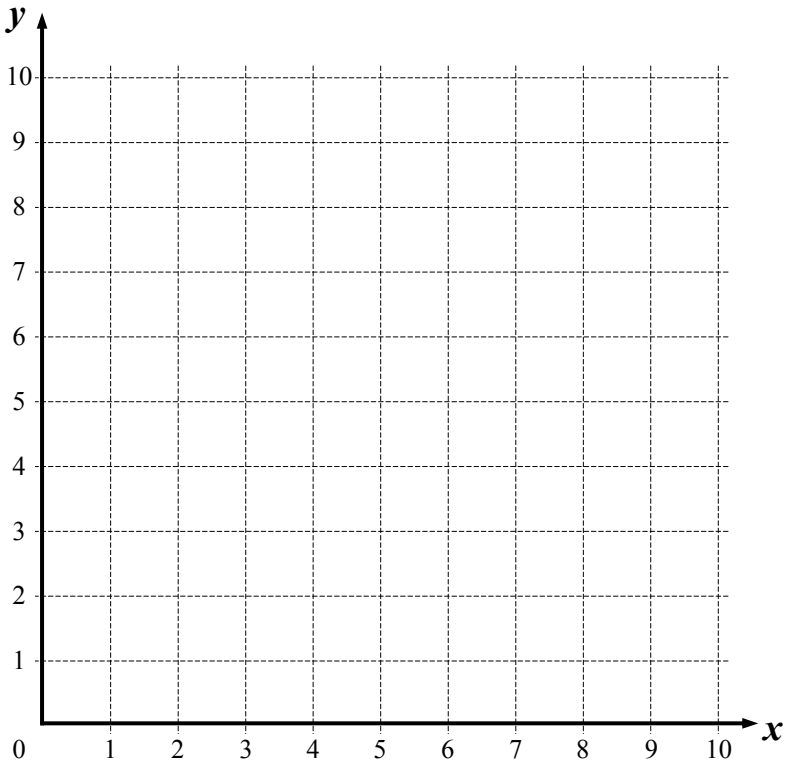
Plot the two points on the graph and rule an interval between them. This will be the hypotenuse of a right angle triangle. Draw the two short sides then use Pythagoras to find the interval length. (1 d.p.)



1 (7, 10) and (10, 6). Plot right angle at (10, 10).

$d^2 = \square^2 + \square^2$

Calc: 3 x^2 + 4 x^2 =

$d^2 =$ ☐

Rewrite

$d = \sqrt{\square}$

Calc: $\sqrt{\blacksquare}$ ANS =

$d =$ ☐ *units*

(1 d.p.)

2 (6, 10) and (1, 9). Plot right angle at (1, 10).

$d^2 = \square^2 + \square^2$

$d^2 =$ ☐

$d = \sqrt{\square}$

$d =$ ☐ *units*

(1 d.p.)

3 (7, 10) and (9, 1). Plot right angle at (7, 1).

$d^2 = \square^2 + \square^2$

$d^2 =$ ☐

$d = \sqrt{\square}$

$d =$ ☐ *units*

4 (2, 4) and (0, 8). Plot right angle at (0, 4).

$d^2 =$

$d^2 =$

$d =$

$d =$

5 (4, 4) and (4, 8).

$d =$

Now you have place the right angle yourself. Find the distance 'd' between these two points.

6 (0, 2) and (7, 0).

Right angle at (☐ , ☐)

$d^2 =$

$d^2 =$

$d =$

$d =$

7 (2, 2) and (6, 9).

Right angle at (☐ , ☐)

8 (9, 5) and (10, 1).

Right angle at (☐ , ☐)

9 (1, 9) and (9, 5).

Right angle at (☐ , ☐)

Cartesian Plane - Distance - All Quadrants (Sheet 1)

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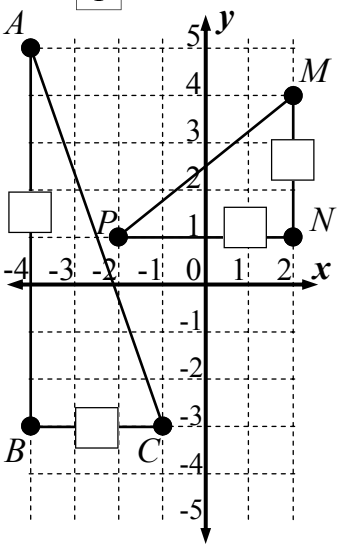
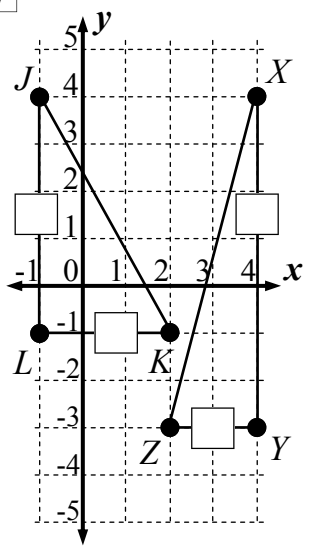
Write the lengths of the short sides in the boxes provided on the number plane.

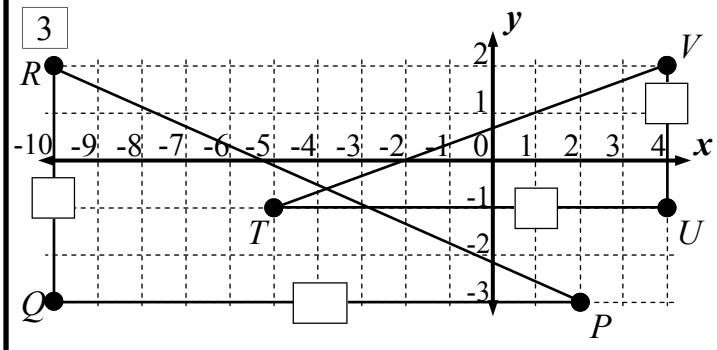
1



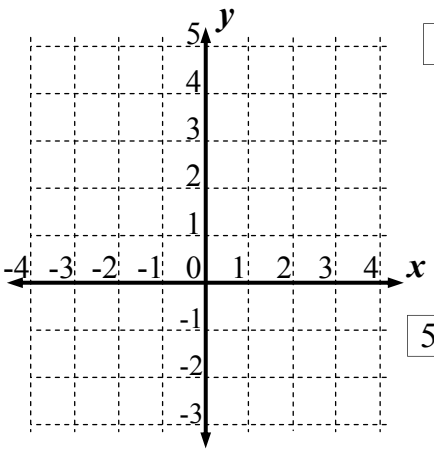
2



3



Plot the pair of points, then create a right angle triangle on the number plane. Find the two short side lengths. Plot Q4 and Q5 on the first number plane.



4 (4, 5) & (1, -3).

Vertical Length : ☐

Horizontal Length : ☐

5 (-3, 4) & (-1, -2).

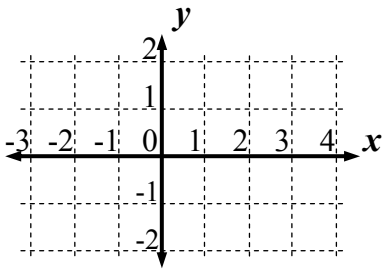
Vertical Length : ☐

Horizontal Length : ☐

6 (4, -2) & (-1, 2).

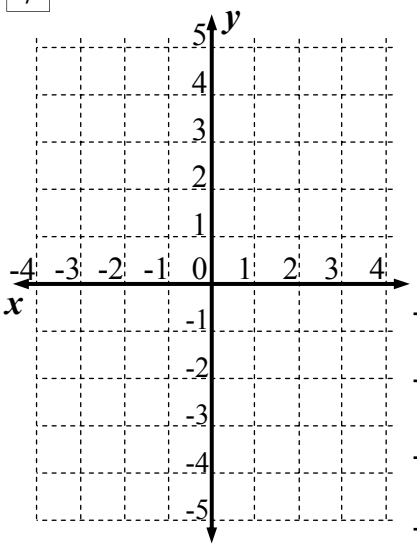
Vertical Length : ☐

Horizontal Length : ☐



Now use Pythagoras to calculate the distance between the two plotted points. Find the short side lengths first. Answer to 1 decimal place.

7



(3, 5) & (-1, -3).

Vertical Length : ☐

Horizontal Length : ☐

$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square.\square$ *units*

8 (-4, -4) & (2, 5).

Vertical Length : ☐

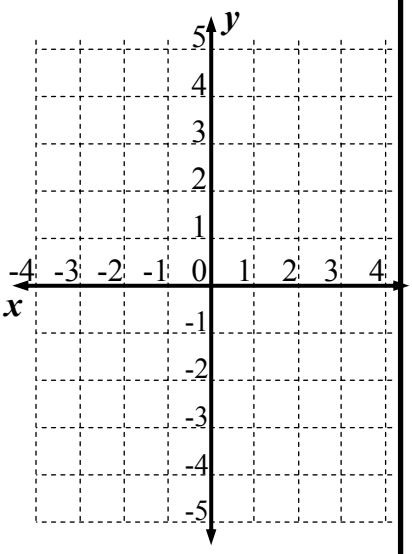
Horizontal Length : ☐

$d^2 = \square^2 + \square^2$

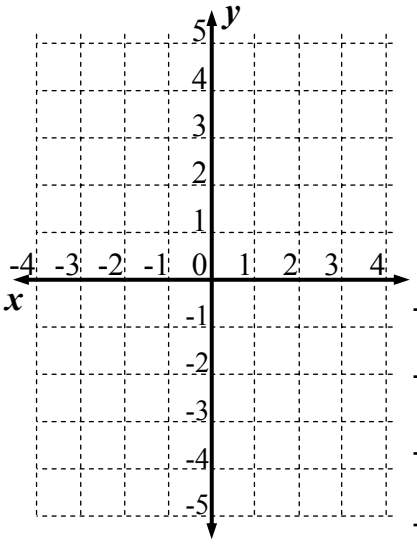
$d^2 = \square$

$d = \sqrt{\square}$

$d = \square\square.\square$ *units*



9



(-4, 2) & (1, -5).

Vertical Length : ☐

Horizontal Length : ☐

$d^2 = \square^2 + \square^2$

$d^2 = \square$

$d = \sqrt{\square}$

$d = \square.\square$ *units*

Cartesian Plane - Distance - All Quadrants (Sheet 2)

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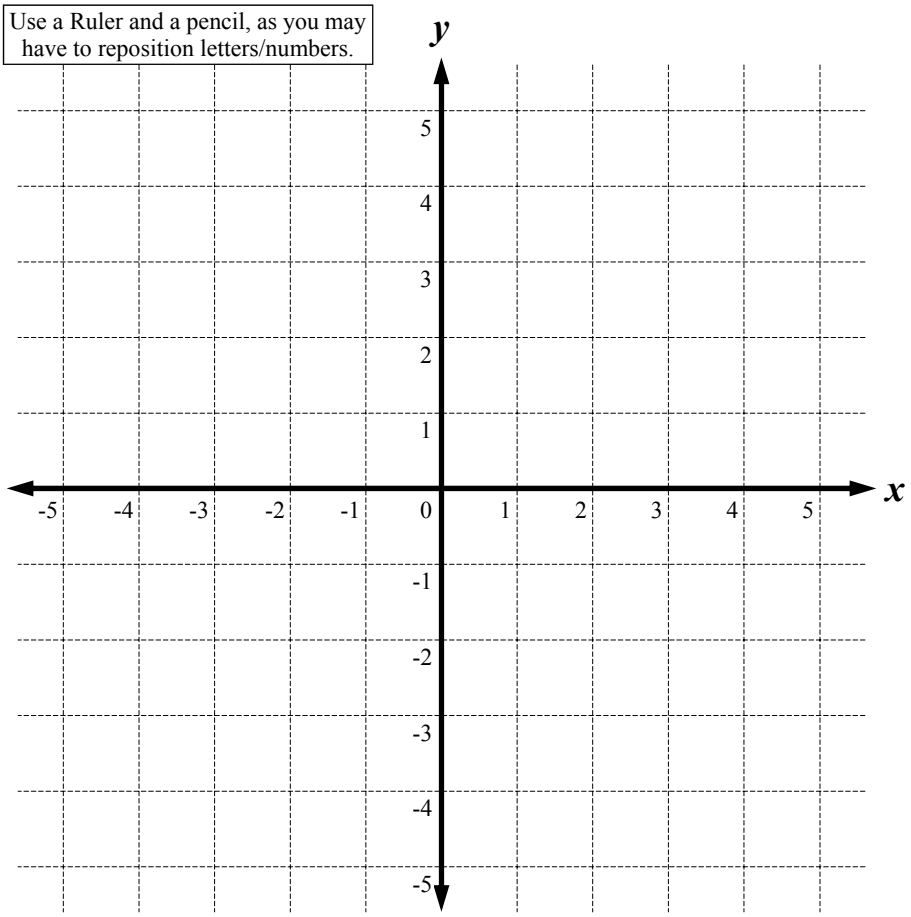
Plot the two points on the graph and draw a line between them, this is the hypotenuse. Draw the two short sides then apply Pythagoras to find the distance between the two points (to 1 d.p.). Note that '*u*' stands for units.

Use a Ruler and a pencil, as you may have to reposition letters/numbers.



1 *K* (-3, 3) and *L* (4, 5). Label right angle *J* at (4, 3).

JK = ☐ *u* *JL* = ☐ *u*

$KL^2 =$ ☐$^2 +$ ☐^{2}

$KL^2 =$ ☐

KL = $\sqrt{\square}$ Rewrite

KL = ☐.☐ *units*

2 *A* (-5, 4) & *B* (-3, -2). Label right angle *C* at (-5, -2).

AC = ☐ *u* *BC* = ☐ *u*

$AB^2 =$ ☐$^2 +$ ☐^{2}

$AB^2 =$ ☐

AB = $\sqrt{\square}$ Rewrite

AB = ☐.☐ *units*

3 *S* (-2, -4) and *R* (2, 2). Label right angle *T* at (2, -4).

TR = ☐ *u* *TS* = ☐ *u*

$SR^2 =$

$SR^2 =$

SR =

SR =

4 *V* (-5, -4) & *X* (-3, -3). Label right angle *Y* at (-3, -4).

VY = ☐ *u* *XY* = ☐ *u*

$VX^2 =$

$VX^2 =$

VX =

VX =

5 *F* (1, 5) & *T* (5, 1). Label right angle *N*.

FN = *TN* =

6 *U* (0, 5) & *Z* (5, -3). Label right angle *D*.

UD = *ZD* =

7 *E* (4, -2) & *G* (2, -5). Label right angle *H*.

EH = *GH* =

8 *M* (-2, 2) & *Q* (3, -1). Label right angle *P*.

MP = *QP* =

Cartesian Plane - Find A Word

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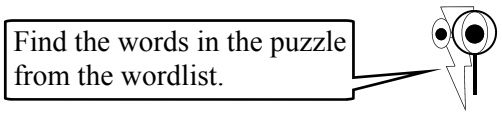

E	H	E	Q	U	A	T	I	O	N	A	X	S	X	A
V	A	T	Q	P	V	W	T	N	I	O	P	D	I	M
I	L	A	Q	L	E	P	Q	E	U	N	A	F	N	X
T	F	N	Q	O	R	E	L	G	N	A	I	R	T	A
I	D	I	I	T	T	U	Q	A	Q	I	N	A	E	M
S	I	D	N	T	I	N	N	V	N	S	A	X	R	A
O	S	R	T	I	C	Q	A	E	A	E	E	X	V	X
P	T	O	E	N	A	X	S	R	O	T	P	X	A	E
X	A	O	R	G	L	E	O	A	D	R	A	O	L	X
A	N	C	C	C	X	G	P	G	X	A	I	J	L	A
X	C	A	E	A	A	O	H	E	E	C	U	G	R	S
E	E	X	P	H	I	N	W	O	D	S	L	Q	I	O
X	L	A	T	N	O	Z	I	R	O	H	I	I	H	N
A	X	Y	T	G	R	A	D	I	E	N	T	R	N	H
E	P	N	E	G	A	T	I	V	E	H	O	R	I	E

WORDLIST

GRADIENT	POINT*	MIDPOINT*	RISE	VERTICAL
POSITIVE	INTERVAL	INTERCEPT	SLOPE	MEAN
HALF	TRIANGLE	PLOTTING	QUADRANT	CARTESIAN
AXES	RUN	NEGATIVE	LINE	UP
DISTANCE	AVERAGE	ACROSS	DOWN	EQUATION
PLANE	HORIZONTAL	PYTHAGORAS	COORDINATE	ORIGIN

*MIDPOINT AND POINT ARE SEPARATE WORDS

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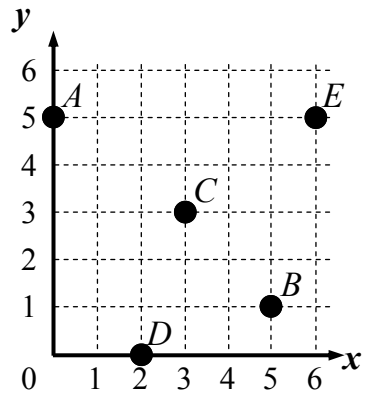
Give the coordinates for the five plotted points.



1 *A* _____

2 *B* _____

3 *C* _____

4 *D* _____

5 *E* _____

Give the coordinates for the six plotted points.

6 *A* _____

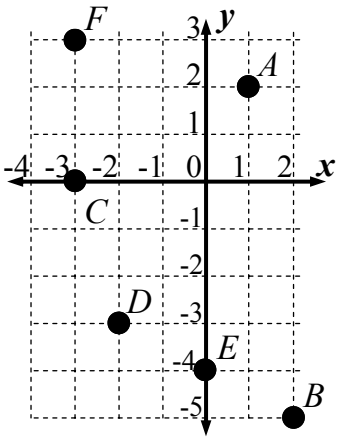
7 *B* _____

8 *C* _____

9 *D* _____

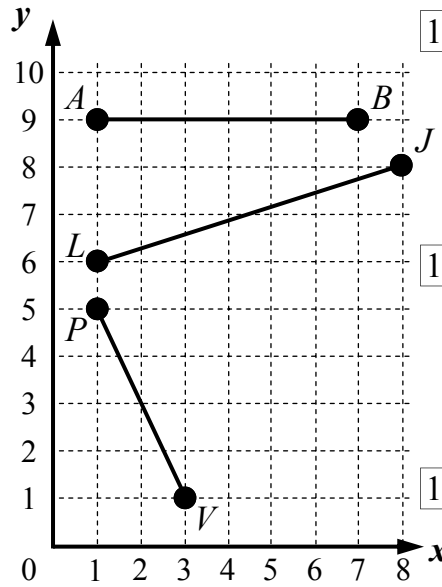
10 *E* _____

11 *F* _____



12 Give the coordinates for the origin _____

Plot the midpoint '*M*' on the intervals, give the coordinates



13 *AB* _____ Midpoint Coordinates

14 *LJ* _____ Midpoint Coordinates

15 *PV* _____ Midpoint Coordinates

Plot the points, find the midpoint coordinates.

16 *A* (-4, -5)
B (-2, 3)

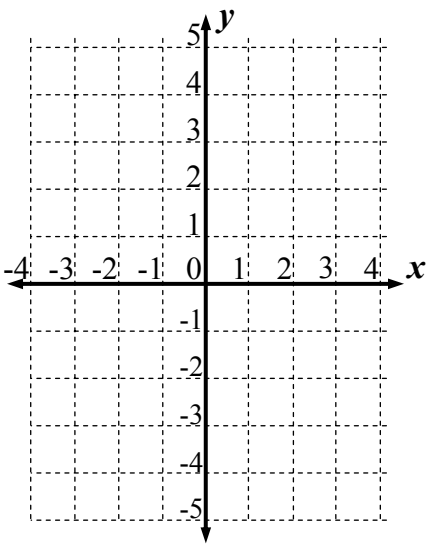
_____ Midpoint of *AB*

17 *F* (-3, 5)
Z (4, 3)

_____ Midpoint of *FZ*

18 *T* (3, -1)
Q (-3, -5)

_____ Midpoint of *TQ*



Calculate the coordinates of the midpoint for these intervals, use:

$$M(\overline{x}, \overline{y}) = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

19 (4, 6) and (10, 12)

_____ _____

_____ _____

_____

20 (-5, -2) and (9, 8)

_____ _____

_____ _____

_____

21 (-9, -8) and (-2, -2)

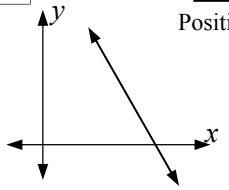
_____ _____

_____ _____

_____

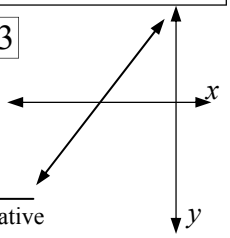
Do these lines have a positive or negative gradient?

22



_____ Positive or negative

23



_____ Positive or negative

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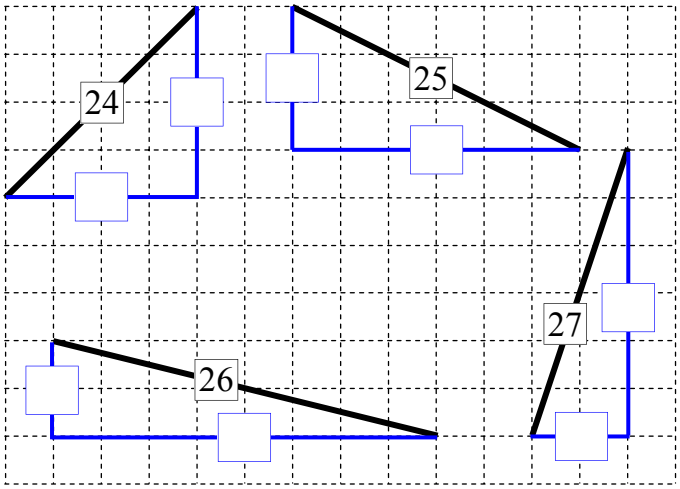
Record the triangle side lengths and calculate the gradient for the following intervals. Simplify when possible.

Use: $m = \frac{rise}{run}$



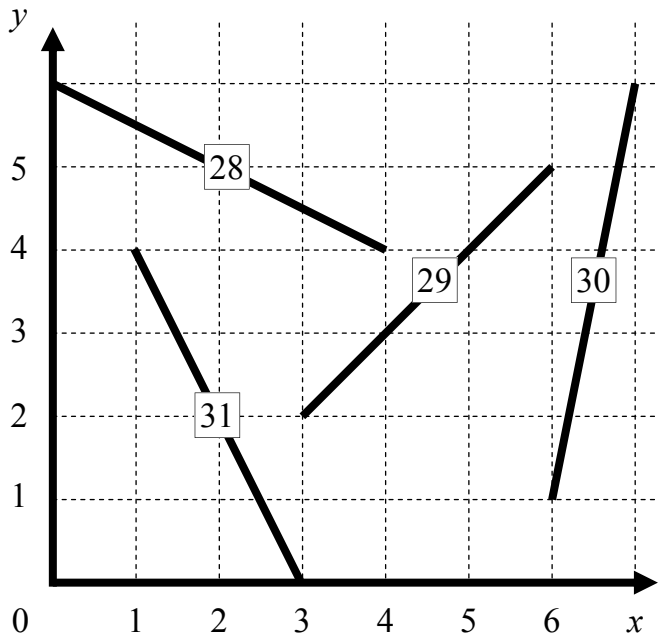
24 $m =$ _____ 25 _____

26 _____ 27 _____

Calculate the gradient for the following intervals.

Use: $m = \frac{rise}{run}$

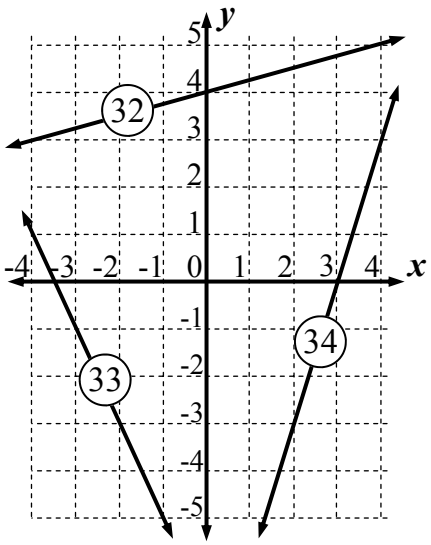
Simplify your answer where possible.



28 _____ 29 _____

30 _____ 31 _____

Calculate the gradient for the following lines. Simplify your answer where possible.



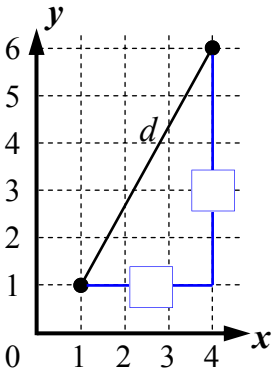
32 _____ 33 _____

34 _____

Calculate the distance between these two points. Use Pythagoras Theorem rounding to 1 d.p.

Use: $c^2 = a^2 + b^2$

35



_____

_____

_____

_____

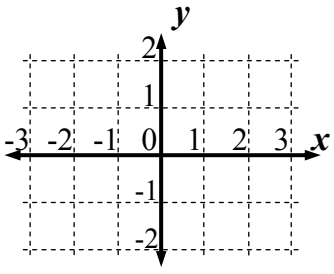
36 (-3, 2) and (3, -2).

_____

_____

_____

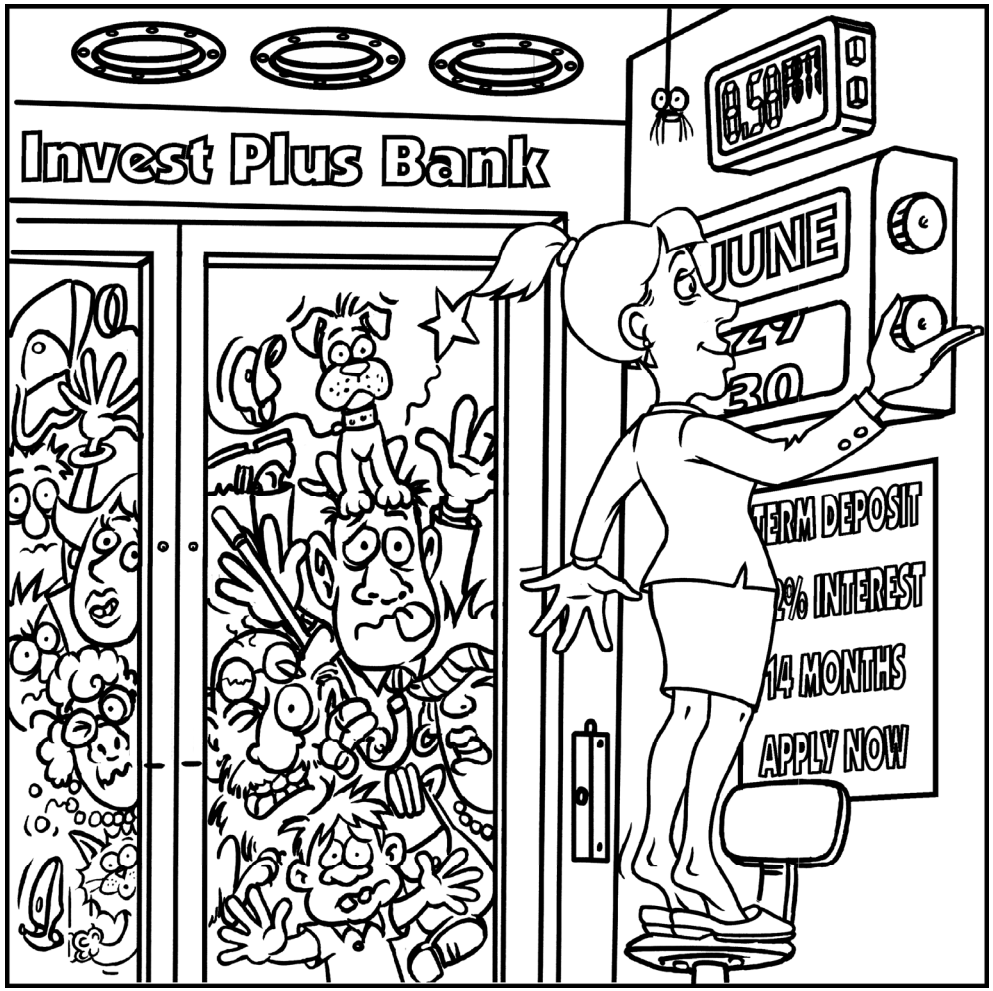
_____



VELOCITY

INVESTING MONEY

Investing Money



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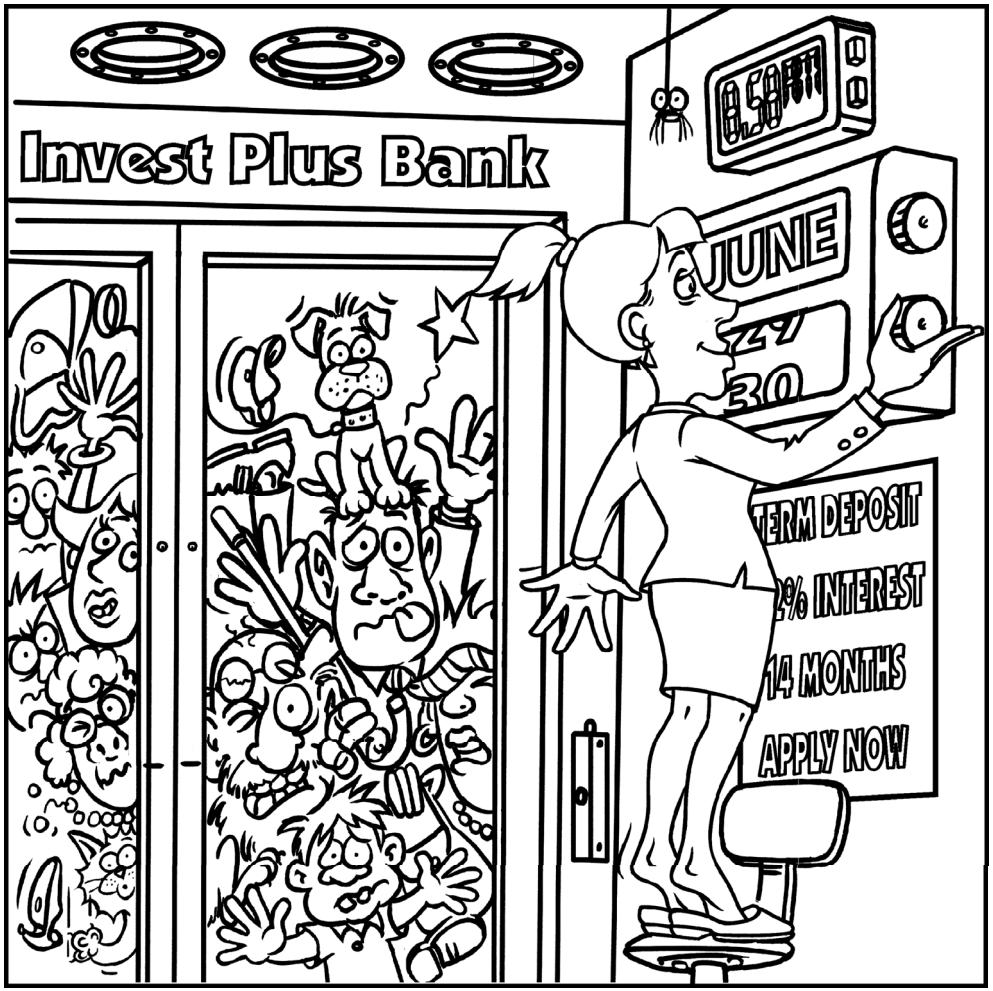
Investing Money



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Investing Money - Simple Interest

Simple Interest is called simple interest because at the end of each year no interest is paid, it is only paid when the total time period of the investment is reached.

$$I = Prn$$

There are 4 variables in the formula, these are:

I = Interest earned on the investment (or payable on a loan)
P = Principal, the amount invested (or borrowed if a loan)
r = interest rate applied to the investment (or loan), expressed as a decimal (not a percentage)
n = the term, which is the time period the investment (or loan) is over

To find the interest payable, multiply the Principal, rate (as a decimal) and term. There is an example below. As you are dealing with money remember to round your answer to 2 decimal places (2 d.p.)

Example: Find the interest earned when \$5 100 is invested for 6 years at 7% p.a.

$r = 7 \div 100 = 0.07$	Convert 7% to a decimal by ÷ 100
$I = Prn$	Write the formula
$= 5\ 100 \times 0.07 \times 6$	Substitute values into formula
$I =$ \$2 142	Answer with '\$' sign and cents to 2 decimal places (if needed)

The second step, if required is finding the account balance (A). Which is the amount you have in the account after the interest is paid. The account balance is found by adding the principal and the interest together. So:

$$A = P + I$$

In the example above. The account balance would be:

$$A = P + I = 5\ 100 + 2\ 142 = \$7\ 242$$

Investing Money - Simple Interest (Sheet 1)

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Convert these interest rates from percentages to decimals. Divide by 100 to get a decimal.

Example:

16% = 16 ÷ 100 = 0.16

1 9% = ☐ ÷ 100 = ☐

2 5% = ☐ ÷ ☐ = ☐

3 10% = ☐ ÷ ☐ = ☐

Answer only, from now on.

4 14.5% = ☐

5 12.7% = ☐

6 4.92% = ☐

7 0.5% = ☐

8 7¼% = ☐

Calc: SHIFT 7 1 4 ÷ 1 0 0 S⇔D =

9 6¾% = ☐

Simple interest is found using $I = Prn$, but first convert the rate R to a decimal. Find I.

Example: Find the interest earned:

P = \$950, R = 8% p.a., n = 5 years

$r = 8 \div 100 = 0.08$

$I = Prn$

$= 950 \times 0.08 \times 5$

I = \$380

P = \$8 300, R = 6% p.a., n = 4 years

10 r = ☐ ÷ 100 = ☐

$I = Prn$

= ☐ × ☐ × ☐

P r n

I = ☐

Show the '$' sign

P = \$970, R = 4.5% p.a., n = 2 years

11 r = ☐ ÷ ☐ = ☐

$I = Prn$

= ☐ × ☐ × ☐

I = ☐ ← Cents are always 2 d.p.

P = \$5 400, R = 6.04% p.a., n = 7 yrs

12 r = ☐

$I = Prn$

= ☐

I = ☐

The next three have smaller interest rates, but you still have to ÷ 100, as they are percentages

P = \$820, R = 0.15%/week, n = 9 wks

13 r = ☐

$I = Prn$

= ☐

I = ☐

P = \$11 600, R = 0.4%/mth, n = 8 mths

14

Convert R to r (% to decimal)

Write Simple Interest Formula

Make the Substitution

Show 'I =', the '$' sign and cents to 2 d.p.

P = \$450, R = 0.05%/wk, n = 19 weeks

15

Round to 2 d.p. NOT nearest 5c.

P = \$47 130, R = 8.4% p.a., n = 6 years

16

R r

Keep going, but do less work. Use 3 lines not 4, Convert R to r, in the substitution line.

P = \$1 720, R = 4% p.a., n = 5 yrs

17 $I = Prn$ *Use r not R*

= ☐ × ☐ × ☐

I =

P = \$9 050, R = 3.75% p.a., n = 4 yrs

18 $I = Prn$ *Use r not R*

= ☐ × ☐ × ☐

I =

P = \$180, R = 6.25% p.a., n = 4.5 yrs

19

P = \$57 000, R = 4.02% p.a., n = 9 yrs

20

P = \$685.12, R = 13.07% p.a., n = 3 yrs

21

Investing Money - Simple Interest - Account Balance (Sheet 2)

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The account balance 'A' is what is in your account after the interest is paid. Calculate A.

Example: P = \$1 300, I = \$275.20

$A = P + I = 1\,300 + 275.20$

$A = \$1\,575.20$

$P = \$1\,150$, $I = \$662.30$

1 $A =$ ☐ + ☐

$A =$ ☐

Show '$' sign and cents to 2 decimal places

$P = \$580$, $I = \$39.65$

2 $A =$ ☐ + ☐

$A =$ ☐

$P = \$56\,970$, $I = \$4\,076.10$

3 $A =$

$A =$

$P = \$8\,030.62$, $I = \$1\,510.18$

4

$P = \$3\,877$, $I = \$772.92$

5

Jen has \$458.08 in her bank account and receives \$14.32 interest. Find her new account balance.

6

Neil has \$2 504.24 in the bank. He then receives interest of \$83.19. Calculate his account balance.

7

Calculate the interest earned then add it to the principal to find the account balance.

Example:

$P = \$1\,400$, $R = 7\%$ p.a., $n = 5$ years

$I = Prn$

$= 1\,400 \times 0.07 \times 5$

$I = \$490$

$A = 1\,400 + 490 = \$1\,890$

$P = \$4\,200$, $R = 5.2\%$ p.a., $n = 3$ yrs

8 $I = Prn$ *Use r not R*

$=$ ☐ × ☐ × ☐

$I =$ ☐

Show '$' sign and cents to 2 decimal places

$A =$ ☐ + ☐

$A =$ ☐

$P = \$19\,050$, $R = 4.8\%$ p.a., $n = 4$ yrs

9 $I = Prn$

$=$ ☐ × ☐ × ☐

$I =$ ☐

$A =$ ☐ + ☐

$A =$ ☐

$P = \$1\,150$, $R = 5.25\%$ p.a., $n = 6$ yrs

10

Interest rates are smaller as the period is shorter. Find the interest and account balance.

$P = \$79\,500$

$R = 0.05\%$ per week $n = 38$ weeks

Remember to convert R to r (% to a decimal)

11

$P = \$10\,200$

$R = 0.7\%$ per month $n = 9$ months

12

Jason invests \$840 in an account for 7 months which pays 0.45% per month. Calculate:

13 The interest Jason has earned

14 His account balance once the interest has been credited.

Investing Money - Simple Interest - Worded Questions (Sheet 3)

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Answer these. Always use the '$' sign and round cents, if there are any, to 2 d.p.

Eli invests $3 500 for 7 years at 5.4% p.a.

1 Convert 5.4% to a decimal.

2 Calculate the interest earned

$I = Prn$

3 His account balance after the interest has been paid.

Russell invests $950 for 8 months at 0.86% per month.

4 Convert 0.86% to a decimal.

5 Calculate the interest Russell earns on his investment.

6 His account balance after the interest is deposited.

Olivia has $13 600 to invest. She goes to two banks and is offered:
Bank 1: 6.05% p.a. on a 2 year term
Bank 2: 5.75% p.a. on a 2 year term

7 Calculate the interest earned if Bank 1 was chosen.

8 Calculate the interest earned if Bank 2 was chosen.

9 Calculate the extra interest earned if Olivia uses Bank 1.

Jiang li wants $38 000 in 3 years time to travel overseas. She invests $29 500 now earning 8.05% p.a.

10 Calculate the interest earned

11 Her account balance.

12 The additional amount req'd.

Yousef's investment of $17 000 for 4 years at 5.6% p.a. is about to mature. If he chooses to re-invest for 6 months he will earn 0.47%/month.

13 Calculate the interest earned from Yousef's first investment.

14 Calculate the account balance after the interest is paid.

Yousef chooses to reinvest all of his account balance for the 6 months

15 Convert 0.47% to a decimal.

16 Calculate the interest earned from Yousef's 2nd investment.

17 Calculate the account balance after the interest is paid.

Yousef is going to live off the money (in Q.17) while he completes a course for 1 year.

18 Convert the amount in Q.17 to a weekly amount.

Investing Money - Simple Interest (Sheet 1)

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Convert these interest rates from percentages to decimals. Divide by 100 to get a decimal.

Example:

16% = 16 ÷ 100 = 0.16

1 9% = 9 ÷ 100 = 0.09

2 5% = 5 ÷ 100 = 0.05

3 10% = 10 ÷ 100 = 0.1

Answer only, from now on.

4 14.5% = 0.145

5 12.7% = 0.127

6 4.92% = 0.0492

7 0.5% = 0.005

8 7¼% = 0.0725

Calc: SHIFT 7 1 4 ÷ 1 0 0 S⇔D =

9 6¾% = 0.0675

Simple interest is found using $I = Prn$, but first convert the rate R to a decimal. Find I.

Example: Find the interest earned:

P = \$950, R = 8% p.a., n = 5 years

$r = 8 \div 100 = 0.08$

$I = Prn$

$= 950 \times 0.08 \times 5$

I = \$380

P = \$8 300, R = 6% p.a., n = 4 years

10 r = 6 ÷ 100 = 0.06

$I = Prn$

$=$ 8 300 (P) × 0.06 (r) × 4 (n)

I = \$1 992

Show the '$' sign

P = \$970, R = 4.5% p.a., n = 2 years

11 r = 4.5 ÷ 100 = 0.045

$I = Prn$

$=$ 970 × 0.045 × 2

I = \$87.30 ← Cents are always 2 d.p.

P = \$5 400, R = 6.04% p.a., n = 7 yrs

12 $r = 6.04 \div 100 = 0.0604$

$I = Prn$

$=$ 5 400 × 0.0604 × 7

I = \$2 283.12

The next three have smaller interest rates, but you still have to ÷ 100, as they are percentages

P = \$820, R = 0.15%/week, n = 9 wks

13 $r = 0.15 \div 100 = 0.0015$

$I = Prn$

$=$ 820 × 0.0015 × 9

I = \$11.07

P = \$11 600, R = 0.4%/mth, n = 8 mths

14 $r = 0.4 \div 100 = 0.004$

Convert R to r (% to decimal)

$I = Prn$

Write Simple Interest Formula

$= 11\,600 \times 0.004 \times 8$

Make the Substitution

I = \$371.20

Show 'I =', the '$' sign and cents to 2 d.p.

P = \$450, R = 0.05%/wk, n = 19 weeks

15 $r = 0.05 \div 100 = 0.0005$

$I = Prn$

$= 450 \times 0.0005 \times 19$

I = \$4.28 ← Round to 2 d.p. NOT nearest 5c.

P = \$47 130, R = 8.4% p.a., n = 6 years

16 $r = 8.4 \div 100 = 0.084$

$I = Prn$

$= 47\,130 \times 0.084 \times 6$

I = \$23 753.52

R r

Keep going, but do less work. Use 3 lines not 4, Convert R to r, in the substitution line.

P = \$1 720, R = 4% p.a., n = 5 yrs

17 $I = Prn$ (Use r not R)

$=$ 1 720 × 0.04 × 5

I = \$344

P = \$9 050, R = 3.75% p.a., n = 4 yrs

18 $I = Prn$ (Use r not R)

$=$ 9 050 × 0.0375 × 4

I = \$1 357.50

P = \$180, R = 6.25% p.a., n = 4.5 yrs

19 $I = Prn$

$= 180 \times 0.0625 \times 4.5$

I = \$50.63

P = \$57 000, R = 4.02% p.a., n = 9 yrs

20 $I = Prn$

$= 57\,000 \times 0.0402 \times 9$

I = \$20 622.60

P = \$685.12, R = 13.07% p.a., n = 3 yrs

21 $I = Prn$

$= 685.12 \times 0.1307 \times 3$

I = \$268.64

Investing Money - Simple Interest - Account Balance (Sheet 2)

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The account balance 'A' is what is in your account after the interest is paid. Calculate A.

Example: P = \$1 300, I = \$275.20

$A = P + I = 1\,300 + 275.20$

$A = \$1\,575.20$

$P = \$1\,150$, $I = \$662.30$

1 $A =$ 1 150 + 662.30

$A =$ \$1 812.30

Show '\$' sign and cents to 2 decimal places

$P = \$580$, $I = \$39.65$

2 $A =$ 580 + 39.65

$A =$ \$619.65

$P = \$56\,970$, $I = \$4\,076.10$

3 $A = 56\,970 + 4\,076.10$

$A = \$61\,046.10$

$P = \$8\,030.62$, $I = \$1\,510.18$

4 $A = 8\,030.62 + 1\,510.18$

$A = \$9\,540.80$

$P = \$3\,877$, $I = \$772.92$

5 $A = 3\,877 + 772.92$

$A = \$4\,649.92$

Jen has \$458.08 in her bank account and receives \$14.32 interest. Find her new account balance.

6 $A = 458.08 + 14.32$

$A = \$472.40$

Neil has \$2 504.24 in the bank. He then receives interest of \$83.19. Calculate his account balance.

7 $A = 2\,504.24 + 83.19$

$A = \$2\,587.43$

Calculate the interest earned then add it to the principal to find the account balance.

Example:

$P = \$1\,400$, $R = 7\%$ p.a., $n = 5$ years

$I = Prn$

$= 1\,400 \times 0.07 \times 5$

$I = \$490$

$A = 1\,400 + 490 = \$1\,890$

$P = \$4\,200$, $R = 5.2\%$ p.a., $n = 3$ yrs

8 $I = Prn$ *Use r not R*

$=$ 4 200 × 0.052 × 3

$I =$ \$655.20

Show '\$' sign and cents to 2 decimal places

$A =$ 4 200 + 655.20

$A =$ \$4 855.20

$P = \$19\,050$, $R = 4.8\%$ p.a., $n = 4$ yrs

9 $I = Prn$

$=$ 19 050 × 0.048 × 4

$I =$ \$3 657.60

$A =$ 19 050 + 3 657.60

$A =$ \$22 707.60

$P = \$1\,150$, $R = 5.25\%$ p.a., $n = 6$ yrs

10 $I = Prn$

$= 1\,150 \times 0.0525 \times 6$

$I = \$362.25$

$A = 1\,150 + 362.25$

$A = \$1\,512.25$

Interest rates are smaller as the period is shorter. Find the interest and account balance.

$P = \$79\,500$

$R = 0.05\%$ per week $n = 38$ weeks

Remember to convert R to r (% to a decimal)

11 $I = Prn$

$= 79\,500 \times 0.0005 \times 38$

$I = \$1\,510.50$

$A = 79\,500 + 1\,510.50$

$A = \$81\,101.50$

$P = \$10\,200$

$R = 0.7\%$ per month $n = 9$ months

12 $I = Prn$

$= 10\,200 \times 0.007 \times 9$

$I = \$642.60$

$A = 10\,200 + 642.60$

$A = \$10\,842.60$

Jason invests \$840 in an account for 7 months which pays 0.45% per month. Calculate:

13 The interest Jason has earned

$I = Prn$

$= 840 \times 0.0045 \times 7$

$I = \$26.46$

14 His account balance once the interest has been credited.

$A = 840 + 26.46$

$A = \$866.46$

Investing Money - Simple Interest - Worded Questions (Sheet 3)

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Answer these. Always use the '$' sign and round cents, if there are any, to 2 d.p.

Eli invests $3 500 for 7 years at 5.4% p.a.

1 Convert 5.4% to a decimal.

$r = 5.4 \div 100 = 0.054$

2 Calculate the interest earned

$I = Prn$

$= 3\ 500 \times 0.054 \times 7$

$I = \$1\ 323$

3 His account balance after the interest has been paid.

$A = 3\ 500 + 1\ 323$

$A = \$4\ 823$

Russell invests $950 for 8 months at 0.86% per month.

4 Convert 0.86% to a decimal.

$r = 0.86 \div 100 = 0.0086$

5 Calculate the interest Russell earns on his investment.

$I = Prn$

$= 950 \times 0.0086 \times 8$

$I = \$65.36$

6 His account balance after the interest is deposited.

$A = 950 + 65.36$

$A = \$1\ 015.36$

Olivia has $13 600 to invest. She goes to two banks and is offered:
Bank 1: 6.05% p.a. on a 2 year term
Bank 2: 5.75% p.a. on a 2 year term

7 Calculate the interest earned if Bank 1 was chosen.

$I = Prn$

$= 13\ 600 \times 0.0605 \times 2$

$I = \$1\ 645.60$

8 Calculate the interest earned if Bank 2 was chosen.

$I = Prn$

$= 13\ 600 \times 0.0575 \times 2$

$I = \$1\ \ 564$

9 Calculate the extra interest earned if Olivia uses Bank 1.

$Extra = 1\ 645.60 - 1\ \ 564$

$Extra = \$81.60$

Jiang li wants $38 000 in 3 years time to travel overseas. She invests $29 500 now earning 8.05% p.a.

10 Calculate the interest earned

$I = Prn$

$= 29\ 500 \times 0.0805 \times 3$

$I = \$7\ 124.25$

11 Her account balance.

$A = 29\ 500 + 7\ 124.25$

$A = \$36\ 624.25$

12 The additional amount req'd.

$Amount = 38\ 000 - 36\ 624.25$

$Amount = \$1\ 375.75$

Yousef's investment of $17 000 for 4 years at 5.6% p.a. is about to mature. If he chooses to re-invest for 6 months he will earn 0.47%/month.

13 Calculate the interest earned from Yousef's first investment.

$I = Prn$

$= 17\ 000 \times 0.056 \times 4$

$I = \$3\ 808$

14 Calculate the account balance after the interest is paid.

$A = 17\ 000 + 3\ 808$

$A = \$20\ 808$

Yousef chooses to reinvest all of his account balance for the 6 months

15 Convert 0.47% to a decimal.

$r = 0.47 \div 100 = 0.0047$

16 Calculate the interest earned from Yousef's 2nd investment.

$I = Prn$

$= 20\ 808 \times 0.0047 \times 6$

$I = \$586.79$

17 Calculate the account balance after the interest is paid.

$A = 20\ 808 + 586.79$

$A = \$21\ 394.79$

Yousef is going to live off the money (in Q.17) while he completes a course for 1 year.

18 Convert the amount in Q.17 to a weekly amount.

$Weekly = 21\ 394.79 \div 52$

$Weekly = \$411.44$

Investing Money - Simple Interest (Homework Strip)

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Date Due: _____

Convert these interest rates to decimals

1 $8\% = 8 \div 100 =$ _____

2 $11\% =$ _____ $\div 100 =$ _____

3 $5.2\% =$ _____

Use $I = Prn$ to find the interest earned

$P = \$850$, $R = 7\%$ p.a., $n = 4$ years

4 $r =$ _____ $\div$ _____ $=$ _____

Divide the interest rate by 100 to get a decimal

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____

Principal | Rate (decimal) | Time

$I =$ _____ ← $ sign

$P = \$1\,300$, $R = 7.8\%$ p.a., $n = 6$ years

5 $r =$ _____ $\div$ _____ $=$ _____

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____

$I =$ _____ ← Cents are always 2 d.p.

Date Due: _____

Convert these interest rates to decimals

1 $8\% = 8 \div 100 =$ _____

2 $11\% =$ _____ $\div 100 =$ _____

3 $5.2\% =$ _____

Use $I = Prn$ to find the interest earned

$P = \$850$, $R = 7\%$ p.a., $n = 4$ years

4 $r =$ _____ $\div$ _____ $=$ _____

Divide the interest rate by 100 to get a decimal

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____

Principal | Rate (decimal) | Time

$I =$ _____ ← $ sign

$P = \$1\,300$, $R = 7.8\%$ p.a., $n = 6$ years

5 $r =$ _____ $\div$ _____ $=$ _____

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____

$I =$ _____ ← Cents are always 2 d.p.

Date Due: _____

Convert these interest rates to decimals

1 $8\% = 8 \div 100 =$ _____

2 $11\% =$ _____ $\div 100 =$ _____

3 $5.2\% =$ _____

Use $I = Prn$ to find the interest earned

$P = \$850$, $R = 7\%$ p.a., $n = 4$ years

4 $r =$ _____ $\div$ _____ $=$ _____

Divide the interest rate by 100 to get a decimal

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____

Principal | Rate (decimal) | Time

$I =$ _____ ← $ sign

$P = \$1\,300$, $R = 7.8\%$ p.a., $n = 6$ years

5 $r =$ _____ $\div$ _____ $=$ _____

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____

$I =$ _____ ← Cents are always 2 d.p.

Date Due: _____

Convert these interest rates to decimals

1 $8\% = 8 \div 100 =$ _____

2 $11\% =$ _____ $\div 100 =$ _____

3 $5.2\% =$ _____

Use $I = Prn$ to find the interest earned

$P = \$850$, $R = 7\%$ p.a., $n = 4$ years

4 $r =$ _____ $\div$ _____ $=$ _____

Divide the interest rate by 100 to get a decimal

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____

Principal | Rate (decimal) | Time

$I =$ _____ ← $ sign

$P = \$1\,300$, $R = 7.8\%$ p.a., $n = 6$ years

5 $r =$ _____ $\div$ _____ $=$ _____

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____

$I =$ _____ ← Cents are always 2 d.p.

Investing Money - Simple Interest (HWS)

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Date Due:

Convert these interest rates to decimals

1 $8\% = 8 \div 100 =$ 0.08

2 $11\% =$ 11 $\div 100 =$ 0.11

3 $5.2\% =$ 5.2 ÷ 100 = 0.052

Use $I = Prn$ to find the interest earned

$P = \$850$, $R = 7\%$ p.a., $n = 4$ years

4 $r =$ 7 ÷ 100 = 0.07

Divide the interest rate by 100 to get a decimal

$I = Prn$

$=$ 850 × 0.07 × 4

Principal | Rate (decimal) | Time

$I =$ $238 ← $ sign

$P = \$1\,300$, $R = 7.8\%$ p.a., $n = 6$ years

5 $r =$ 7.8 ÷ 100 = 0.078

$I = Prn$

$=$ 1 300 × 0.078 × 6

$I =$ $608.40 ← Cents are always 2 d.p.

Investing Money - Simple Interest - Changing the Term

Simple Interest is called simple interest because at the end of each year no interest is paid, it is only paid when the total time period of the investment is reached.

$$I = Prn$$

n = the term, which is the time period the investment (or loan) is over

The term can be any period, but because the interest rate is usually an annual (yearly) rate the term should also be in years. So when you have an investment period of days, weeks or months, these have to be converted to years using fractions.

To convert to years:

Number of Days	Number of Weeks	Number of Months
365	52	12
Period in Days	Period in Weeks	Period in Months

Note that the denominators (bottom numbers) don't change if the period is longer than a year. For example if a period is 15 months (more than 1 year), you still use 12, not 24.

Example: Find the interest earned when \$5 100 is invested for 17 months at 7% p.a.

$r = 7 \div 100 = 0.07$ ← Convert 7% to a decimal by ÷ 100

$I = Prn$ ← Write the formula

$= 5\,100 \times 0.07 \times \frac{17}{12}$ ← Substitute values into formula Changing 17 months into a fraction of a year.

$I = \$505.75$

Answer with '\$' sign and cents to 2 decimal places (if needed)

Investing Money - Simple Interest - Changing the Term (Sheet 1)

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Write these time periods as fractions of a year. Don't simplify the fractions.

Examples:

5 months = $\frac{5}{12}$ 13 weeks = $\frac{13}{52}$

1 25 weeks = $\frac{\square}{\square}$

2 9 months = $\frac{\square}{\square}$

3 117 days = $\frac{\square}{\square}$

4 14 weeks = $\frac{\square}{\square}$

5 15 months = $\frac{\square}{\square}$

Calculate the interest earned on these investments.

P = \$1 040, R = 8% p.a., n = 9 weeks

6 r = $\square$ ÷ 100 = $\square$

Convert the rate to a decimal

$I = Prn$

= $\square$ × $\square$ × $\frac{\square}{\square}$

P r n

Calc: 1 0 4 0 × 0 . 0 8 × 9 ab/c 5 2 =

I = $\square$

Show the '\$' sign and write cents to 2 d.p.

P = \$4 260, R = 9% p.a., n = 7 months

7 r = $\square$ ÷ $\square$ = $\square$

$I = Prn$

= $\square$ × $\square$ × $\frac{\square}{\square}$

I = $\square$

P = \$1 095, R = 5.6% p.a., n = 220 days

8 r = $\square$ ÷ $\square$ = $\square$

$I = Prn$

= $\square$ × $\square$ × $\frac{\square}{\square}$

I = $\square$

P = \$7 176, R = 3.2% p.a., n = 85 wks.

9 r =

$I = Prn$

=

I =

Complete these calculations, rounding to 2 decimal places.

10 $I = 5\,000 \times 0.046 \times \frac{19}{12}$

I = $\square$

11 $I = 83\,560 \times 0.105 \times \frac{96}{365}$

I = $\square$

P = \$2 500, R = 6.3% p.a., n = 177 days

12 $I = Prn$

= $\square$ × $\square$ × $\frac{\square}{\square}$

r as a decimal

I =

P = \$880, R = 13.25% p.a., n = 16 mths

13

Now find the interest and then the account balance.

P = \$700, R = 5% p.a., n = 12 weeks

14

A = $\square$ + $\square$

Principal Interest

A =

P = \$34 900, R = 11.6% p.a., n = 20 months.

15

A =

A =

Ben invests \$8 500 at 12.2% p.a. for 34 weeks. He wants \$10 000 for a new car. Calculate:

16 His interest and Acc. Bal.

17 The amount above or below \$10 000.

Use the words 'above' or 'below'.

Investing Money - Simple Interest - Changing the Term (Sheet 2)

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Find the interest earned and the account balance with these.

Tom invests $17 450 at 6.65% p.a. for 10 months.

1 Convert 6.65% to a decimal.

$r =$ ☐ ÷ ☐ = ☐

2 Calculate the interest earned

$I = Prn$

$= $ ☐ × ☐ × ☐/☐

$I =$

Show the '$' sign and write cents to 2 d.p.

3 Find his account balance after the interest has been paid.

$A =$ ☐ + ☐

$A =$

Josiah invests $4 200 in a term deposit at 8.35% p.a. for 195 days.

4 Convert 8.35% to a decimal.

5 Calculate the interest earned.

6 Find his account balance after the interest is credited.

Bailey is saving for a trip. He invests $11 200 for 15 months at 7.16% p.a.

7 Express 7.16% as a decimal.

8 Calculate the interest earned.

9 Find his account balance after the interest has been paid.

10 If the airfare is $2 120, how much spending money is left?

Julia has been left an inheritance. Her father gives her a choice, she can:
1 - Receive $19 000 now, and invest it at 4.35% p.a. for 75 weeks, or
2 - Receive $21 000 in 75 weeks time.
Which is the best financial option?

11 Calculate the interest earned on the $19 000 investment (1).

12 Find Julia's account balance after the interest has been paid.

13 Which option is the best? 1 or 2 and by how much?

Tayla won a prize of $2 000 and she invested it for 175 days at 4.3% p.a.

14 Calculate the interest Tayla earned on the $2 000 investment.

15 Find Tayla's account balance when the interest was deposited.

16 If she spends 75% of the money on a new laptop, how much did the laptop cost?

Cost $=$ ☐ × ☐/☐

Cost $=$ ☐

Wai received $765 for holiday work and $320 on his birthday. He invests all the money for 17 weeks at 3.3% p.a. Find the interest Wai earned.

17

Investing Money - Simple Interest - Changing the Term (Sheet 1)

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Write these time periods as fractions of a year. Don't simplify the fractions.

Examples:

5 months = $\frac{5}{12}$ 13 weeks = $\frac{13}{52}$

1 25 weeks = $\frac{25}{52}$

2 9 months = $\frac{9}{12}$

3 117 days = $\frac{117}{365}$

4 14 weeks = $\frac{14}{52}$

5 15 months = $\frac{15}{12}$

Calculate the interest earned on these investments.

P = \$1 040, R = 8% p.a., n = 9 weeks

6 r = 8 ÷ 100 = 0.08

Convert the rate to a decimal

$I = Prn$

$= 1\,040 \times 0.08 \times \frac{9}{52}$

P r n

Calc: 1040×0.08×9▪52=

I = \$14.40

Show the '\$' sign and write cents to 2 d.p.

P = \$4 260, R = 9% p.a., n = 7 months

7 r = 9 ÷ 100 = 0.09

$I = Prn$

$= 4\,260 \times 0.09 \times \frac{7}{12}$

I = \$223.65

P = \$1 095, R = 5.6% p.a., n = 220 days

8 r = 5.6 ÷ 100 = 0.056

$I = Prn$

$= 1\,095 \times 0.056 \times \frac{220}{365}$

I = \$36.96

P = \$7 176, R = 3.2% p.a., n = 85 wks.

9 $r = 3.2 \div 100 = 0.032$

$I = Prn$

$= 7\,176 \times 0.032 \times \frac{85}{52}$

I = \$375.36

Complete these calculations, rounding to 2 decimal places.

10 $I = 5\,000 \times 0.046 \times \frac{19}{12}$

I = (\$)364.17

11 $I = 83\,560 \times 0.105 \times \frac{96}{365}$

I = (\$)2 307.63

P = \$2 500, R = 6.3% p.a., n = 177 days

12 $I = Prn$

$= 2\,500 \times 0.063 \times \frac{177}{365}$

r as a decimal

I = \$76.38

P = \$880, R = 13.25% p.a., n = 16 mths

13 $I = Prn$

$= 880 \times 0.1325 \times \frac{16}{12}$

I = \$155.47

Now find the interest and then the account balance.

P = \$700, R = 5% p.a., n = 12 weeks

14 $I = Prn$

$= 700 \times 0.05 \times \frac{12}{52}$

I = \$8.08

A = 700 + 8.08

Principal Interest

A = \$708.08

P = \$34 900, R = 11.6% p.a., n = 20 months.

15 $I = Prn$

$= 34\,900 \times 0.116 \times \frac{20}{12}$

I = \$6 747.33

A = 34 900 + 6 747.33

A = \$41 647.33

Ben invests \$8 500 at 12.2% p.a. for 34 weeks. He wants \$10 000 for a new car. Calculate:

16 His interest and Acc. Bal.

$I = Prn$

$= 8\,500 \times 0.122 \times \frac{34}{52}$

I = \$678.04

A = 8 500 + 678.04

A = \$9 178.04

17 The amount above or below \$10 000.

Amount = 10 000 - 9 178.04

Amount = \$821.96 below

Use the words 'above' or 'below'.

Investing Money - Simple Interest - Changing the Term (Sheet 2)

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Find the interest earned and the account balance with these.

Tom invests $17 450 at 6.65% p.a. for 10 months.

1 Convert 6.65% to a decimal.

$r =$ 6.65 $\div$ 100 $=$ 0.0665

2 Calculate the interest earned

$I = Prn$

$= 17\,450 \times 0.0665 \times \frac{10}{12}$

$I =$ $967.02

Show the '$' sign and write cents to 2 d.p.

3 Find his account balance after the interest has been paid.

$A =$ 17 450 $+$ 967.02

$A =$ $18 417.02

Josiah invests $4 200 in a term deposit at 8.35% p.a. for 195 days.

4 Convert 8.35% to a decimal.

$r = 8.35 \div 100 = 0.0835$

5 Calculate the interest earned.

$I = Prn$

$= 4\,200 \times 0.0835 \times \frac{195}{365}$

$I =$ $187.36

6 Find his account balance after the interest is credited.

$A = 4\,200 + 187.36$

$A =$ $4 387.36

Bailey is saving for a trip. He invests $11 200 for 15 months at 7.16% p.a.

7 Express 7.16% as a decimal.

$r = 7.16 \div 100 = 0.0716$

8 Calculate the interest earned.

$I = Prn$

$= 11\,200 \times 0.0716 \times \frac{15}{12}$

$I =$ $1 002.40

9 Find his account balance after the interest has been paid.

$A = 11\,200 + 1\,002.40$

$A =$ $12 202.40

10 If the airfare is $2 120, how much spending money is left?

$S.\ Money = 12\,202.40 - 2\,120$

$S.\ Money =$ $10 082.40

Julia has been left an inheritance. Her father gives her a choice, she can:
1 - Receive $19 000 now, and invest it at 4.35% p.a. for 75 weeks, or
2 - Receive $21 000 in 75 weeks time.
Which is the best financial option?

11 Calculate the interest earned on the $19 000 investment (1).

$I = Prn$

$= 19\,000 \times 0.0435 \times \frac{75}{52}$

$I =$ $1 192.07

12 Find Julia's account balance after the interest has been paid.

$A = 19\,000 + 1\,192.07$

$A =$ $20 192.07

13 Which option is the best? 1 or 2 and by how much?

Option 2 is the best option

Amount $= 21\,000 - 20\,192.07$

Amount $=$ $807.93

Tayla won a prize of $2 000 and she invested it for 175 days at 4.3% p.a.

14 Calculate the interest Tayla earned on the $2 000 investment.

$I = Prn$

$= 2\,000 \times 0.043 \times \frac{175}{365}$

$I =$ $41.23

15 Find Tayla's account balance when the interest was deposited.

$A = 2\,000 + 41.23$

$A =$ $2 041.23

16 If she spends 75% of the money on a new laptop, how much did the laptop cost?

Cost $=$ 2 041.23 $\times \frac{75}{100}$

Cost $=$ $1 530.92

Wai received $765 for holiday work and $320 on his birthday. He invests all the money for 17 weeks at 3.3% p.a. Find the interest Wai earned.

17 Total $= 765 + 320$

Total $=$ $1 085

$I = Prn$

$= 1\,085 \times 0.033 \times \frac{17}{52}$

$I =$ $11.71

Investing Money - Simple Interest - Changing the Term (Homework Strip)

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Date Due:

Write these times as fractions of a year

1 43 weeks = $\frac{\square}{\square}$ ← How many weeks in a year

2 220 days = $\frac{\square}{\square}$ ← How many days in a year

Calculate the Interest earned

P = \$26 000, R = 9% p.a., n = 37 weeks

3 $r = \frac{\square}{\text{Convert the rate } R \text{ to a decimal}} \div 100 = \square$

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

P r n

$I = \square$

P = \$18 000, R = 6% p.a., n = 8 months

4 $r = \frac{\square}{\square} \div \square = \square$

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I = \square$

Date Due:

Write these times as fractions of a year

1 43 weeks = $\frac{\square}{\square}$ ← How many weeks in a year

2 220 days = $\frac{\square}{\square}$ ← How many days in a year

Calculate the Interest earned

P = \$26 000, R = 9% p.a., n = 37 weeks

3 $r = \frac{\square}{\text{Convert the rate } R \text{ to a decimal}} \div 100 = \square$

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

P r n

$I = \square$

P = \$18 000, R = 6% p.a., n = 8 months

4 $r = \frac{\square}{\square} \div \square = \square$

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I = \square$

Date Due:

Write these times as fractions of a year

1 43 weeks = $\frac{\square}{\square}$ ← How many weeks in a year

2 220 days = $\frac{\square}{\square}$ ← How many days in a year

Calculate the Interest earned

P = \$26 000, R = 9% p.a., n = 37 weeks

3 $r = \frac{\square}{\text{Convert the rate } R \text{ to a decimal}} \div 100 = \square$

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

P r n

$I = \square$

P = \$18 000, R = 6% p.a., n = 8 months

4 $r = \frac{\square}{\square} \div \square = \square$

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I = \square$

Date Due:

Write these times as fractions of a year

1 43 weeks = $\frac{\square}{\square}$ ← How many weeks in a year

2 220 days = $\frac{\square}{\square}$ ← How many days in a year

Calculate the Interest earned

P = \$26 000, R = 9% p.a., n = 37 weeks

3 $r = \frac{\square}{\text{Convert the rate } R \text{ to a decimal}} \div 100 = \square$

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

P r n

$I = \square$

P = \$18 000, R = 6% p.a., n = 8 months

4 $r = \frac{\square}{\square} \div \square = \square$

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I = \square$

Investing Money - Simple Interest - Changing the Term (H'work Strip)

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Date Due:

Write these times as fractions of a year

1 43 weeks = $\frac{43}{52}$ ← How many weeks in a year

2 220 days = $\frac{220}{365}$ ← How many days in a year

Calculate the Interest earned

P = \$26 000, R = 9% p.a., n = 37 weeks

3 $r = 9 \div 100 = 0.09$

Convert the rate R to a decimal

$I = Prn$

$= 26\,000 \times 0.09 \times \frac{37}{52}$

P r n

$I =$ \$1 665

P = \$18 000, R = 6% p.a., n = 8 months

4 $r = 6 \div 100 = 0.06$

$I = Prn$

$= 18\,000 \times 0.06 \times \frac{8}{12}$

$I =$ \$720

Investing Money - Simple Interest - Finding the Term 'n'

Imagine you are saving for something and know how much you need. You can invest an amount now, and later, after the investment has had the interest paid have the amount that you need. The problem is how long do you invest it for?

These sheets deal with finding how long to invest. The answers are all whole years. There are two methods used.

Guess and Check: This means you pick a period of time, put the number into the simple interest formula and see if it gets the answer. If the answer is too low increase the time. Too high, decrease it, until you get the answer.

Example: An investment of \$9 000 at 6% p.a., results in interest of \$4 320. Find the period.

$I = Prn$ ← Simple Interest Formula

Convert rate

$4\,320 = 9\,000 \times 0.06 \times n$ ← Substitute the values, remember to use the interest rate r as a decimal

Interest | Principal | to a decimal

Tried $n =$ ☐ ☐ ☐ ☐

○ Low ○ Low ○ Low ○ Low

○ High ○ High ○ High ○ High

← Try a value of 'n'. Work out $9\,000 \times 0.06 \times$ (the value). If it = 4 320 that's n! If it is less than 4 320 increase n. If it is more than 4 320 decrease n. Keep going until you get 4 320.

$n =$ ☐ years

Algebraic: This method uses equation solving skills. It has the advantage of getting the answer straight away. But you have to know how to solve an equation.

Example: An investment of \$9 000 at 6% p.a., results in interest of \$4 320. Find the period.

17 Interest of \$1 386 is earned from a \$3 300 investment at 7% p.a.

$1\,386 = 3\,300 \times 0.07 \times n$ ← Simple Interest Formula. Substitute the values, remember to use the interest rate r as a decimal

Interest | Principal | Rate (decimal)

$1\,386 = 231 \times n$ ← Multiply P and r

$\div 231$ $\div 231$ ← Divide both sides by the number with n. That gives you your answer.

$n = \frac{1\,386}{231} = 6$ years

Investing Money - Simple Interest - Guess & Check for Term '*n*' (Sheet 1)

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Use 'Guess and Check' to find the value of '*n*' for these. Use the back arrow.



1 $2\,160 = 6\,000 \times 0.09 \times n$

Try '*n*' = 3 ____ (Compare to 2 160)

$6000 \times 0.09 \times 3$

Try '*n*' = 5 ____ (Compare to 2 160)

$6000 \times 0.09 \times 5$

Try '*n*' = ____ ____

2 $336 = 1\,200 \times 0.04 \times n$

Try '*n*' = 5 ____ (Compare to 336)

$1\,200 \times 0.04 \times 5$

Try '*n*' = 9 ____ (Compare to 336)

$1\,200 \times 0.04 \times 9$

Try '*n*' = ____ ____

Keep trying '*n*' values until you get 336

3 $2\,805 = 8\,500 \times 0.03 \times n$

Keep trying '*n*' values until you get 2 805

Tried *n* = ____ ____ ____ ○Low ○Low ○Low ○High ○High ○High $n =$ ____

4 $1\,100 = 1\,250 \times 0.11 \times n$

Keep trying '*n*' values until you get 1 100

Tried *n* = ____ ____ ____ ○Low ○Low ○Low ○High ○High ○High $n =$ ____

5 $6\,600 = 44\,000 \times 0.05 \times n$

Tried *n* = ____ ____ ____ ○Low ○Low ○Low ○High ○High ○High $n =$ ____

6 $934.40 = 730 \times 0.08 \times n$

Tried *n* = ____ ____ ____ ○Low ○Low ○Low ○High ○High ○High $n =$ ____



These are done the same way but this time you first need to build the calculation line. Find '*n*'.

7 An investment of $800 at 7% p.a., results in interest of $224.

____ = ____ × ____ × *n*

Interest | Principal | Convert rate to a decimal

Tried *n* = ____ ____ ____ ____ ○Low ○Low ○Low ○Low ○High ○High ○High ○High

$n =$ ____ years

8 Interest of $9 984 is earned from a $9 600 investment at 8% p.a.

____ = ____ × ____ × *n*

Interest | Principal | Rate (decimal)

Tried *n* = ____ ____ ____ ____ ○Low ○Low ○Low ○Low ○High ○High ○High ○High

$n =$ ____ years

9 $1 450 is invested at 4% p.a. and earns interest of $464.

$I = P r$

____ = ____ × ____ × *n*

Tried *n* = ____ ____ ____ ____ ○Low ○Low ○Low ○Low ○High ○High ○High ○High

$n =$ ____ years

10 Interest of $1 137.50 is earned from a $3 250 investment at 5% p.a.

$I = P r$

____ = ____ × ____ × *n*

Tried *n* = ____ ____ ____ ____ ○Low ○Low ○Low ○Low ○High ○High ○High ○High

$n =$ ____ years



Now write the formula, make the substitution and find '*n*' by Guess and Check methods.

11 An investment of $800 at 7% p.a., results in interest of $224.

Write the formula $I = Prn$

____ = ____ × ____ × *n*

Substitute *I*, *P* and *r* (as a decimal)

$n =$ ____ years

12 Interest of $3 652 is earned from a $4 150 investment at 11% p.a.

____ = ____ × ____ × *n*

$n =$ ____ years

13 An investment of $930 at 6% p.a., results in interest of $167.40.

14 Interest of $1 430 is earned from a $2 200 investment at 13% p.a.

Investing Money - Simple Interest - Algebraic Solution Term 'n' (Sheet 2)

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Find 'n' by solving these. Use the '÷' key or the fraction key.

1 $n = \frac{7\,500}{1\,250} =$ ☐

2 $n = \frac{9\,960}{830} =$ ☐

3 $n = \frac{587}{117.40} =$ ☐

4 $n = \frac{7\,994}{3\,997} =$ ☐

Create the division first, by dividing by the number next to the 'n'. Find the value of 'n'.

Example

$176.40 = 29.4 \times n$

(÷ 29.4) (÷ 29.4)

$n = \frac{176.40}{29.4} = 6$

5 $126 = 18 \times n$

(÷ 18) (÷ 18)

$n = \frac{\square}{\square} = \square$

6 $420 = 70 \times n$

(÷ 70) (÷ 70)

$n = \frac{\square}{\square} = \square$

7 $1\,101.10 = 100.10 \times n$

(÷ 100.10) (÷ 100.10)

$n = \frac{\square}{\square} = \square$

8 $2\,274.80 = 568.70 \times n$

(÷ 568.70) (÷ 568.70)

$n = \frac{\square}{\square} = \square$

9 $3\,080 = 440 \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

10 $414 = 138 \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

11 $1\,336.50 = 148.50 \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

1 extra step, perform a × first, then it's the same process. Find the value of 'n'.

Example

$1\,300 = 6\,500 \times 0.05 \times n$

Multiply these

$1\,300 = 325 \times n$

(÷ 325) (÷ 325)

$n = \frac{1\,300}{325} = 4$

12 $680 = 1\,700 \times 0.08 \times n$

$680 = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

13 $2\,541 = 3\,850 \times 0.06 \times n$

$\square = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

14 $4\,900 = 14\,000 \times 0.05 \times n$

$\square = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

Get the numbers out of the sentence and solve for 'n', which is the term, in years.

15 Interest of $537.60 is earned from a $960 investment at 7% p.a.

☐ = ☐ × ☐ × n

Interest | Principal | Rate (decimal)

$\square = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$ years

16 Interest of $3 354 is earned from a $4 300 investment at 13% p.a.

☐ = ☐ × ☐ × n

Interest | Principal | Rate (decimal)

$\square = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$ years

17 Interest of $590 is earned from a $1 475 investment at 5% p.a.

☐ = ☐ × ☐ × n

Interest | Principal | Rate (decimal)

$\square = \square \times n$

( ) ( )

$n = \frac{\square}{\square} = \square$ years

Investing Money - Simple Interest - Further Finding the Term (Sheet 3)

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Find the term 'n' by solving these. Follow the example.

Example

Interest of \$1 752 is earned from a \$7 300 investment at 4% p.a., find the term of the investment.

$I = Prn$ — Write Formula

$1752 = 7\,300 \times 0.04 \times n$ — Substitute Values (Decimal)

$1\,752 = 292 \times n$ — Simplify The '×'

$\div 292$ $\div 292$

$n = \frac{1\,752}{292} = 6$ years — Divide and solve

1 Interest of \$120 is earned from a \$500 investment at 4% p.a., find the term of the investment.

2 An investment of \$3 250 at 7% p.a., results in interest of \$2 275 being paid. Find the term of the investment.

3 Interest of \$620.40 is received from a \$940 investment at 6% p.a., find the term of the investment.

4 An investment of \$1 700 at 8% p.a., results in interest of \$952 being paid. Find the term of the investment.

5 Interest of \$8 944 is earned from an \$8 600 investment at 13% p.a., find the term of the investment.

6 An investment of \$4 850 at 11% p.a., results in interest of \$4 801.50 being paid. Find the investment's term.

This time you want a total amount in the future, follow the steps to find the term.

David has \$9 000 in the bank but he needs \$12 600 for a future trip.

7 How much interest does he have to earn to have an account balance of \$12 600?

$I =$ [ ] - [ ]

$I =$ [ ]

8 If David's investment pays interest at 5% p.a., find the term of the investment to achieve the interest.

Skye has \$26 000 in the bank but she needs \$36 400 for a home deposit.

9 How much interest does she have to earn to have an account balance of \$36 400?

$I =$

$I =$

10 If Skye earns interest at 8% p.a., find the term of the investment needed to get the deposit.

Investing Money - Simple Interest - Guess & Check for Term 'n' (Sheet 1)

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Use 'Guess and Check' to find the value of 'n' for these. Use the back arrow.

1 $2\,160 = 6\,000 \times 0.09 \times n$

Try 'n' = 3 → 1 620 (Compare to 2 160)
$6000 \times 0.09 \times 3$

Try 'n' = 5 → 2 700 (Compare to 2 160)
$6000 \times 0.09 \times 5$

Try 'n' = 4 → 2 160

2 $336 = 1\,200 \times 0.04 \times n$

Try 'n' = 5 → 240 (Compare to 336)
$1\,200 \times 0.04 \times 5$

Try 'n' = 9 → 432 (Compare to 336)
$1\,200 \times 0.04 \times 9$

Try 'n' = 7 → 336

Keep trying 'n' values until you get 336

3 $2\,805 = 8\,500 \times 0.03 \times n$

Keep trying 'n' values until you get 2 805

Tried n =	3	7	13
	Low	Low	Low
	High	High	High

n = 11

4 $1\,100 = 1\,250 \times 0.11 \times n$

Keep trying 'n' values until you get 1 100

Tried n =	5	9	7
	Low	Low	Low
	High	High	High

n = 8

5 $6\,600 = 44\,000 \times 0.05 \times n$

Tried n =	5	2	4
	Low	Low	Low
	High	High	High

n = 3

6 $934.40 = 730 \times 0.08 \times n$

Tried n =	4	8	14
	Low	Low	Low
	High	High	High

n = 16

These are done the same way but this time you first need to build the calculation line. Find 'n'.

7 An investment of \$800 at 7% p.a., results in interest of \$224.

224	=	800	×	0.07	× n
Interest		Principal		Convert rate to a decimal	

Tried n =	2	6	5	
	Low	Low	Low	Low
	High	High	High	High

n = 4 years

8 Interest of \$9 984 is earned from a \$9 600 investment at 8% p.a.

9 984	=	9 600	×	0.08	× n
Interest		Principal		Rate (decimal)	

Tried n =	8	10	12	14
	Low	Low	Low	Low
	High	High	High	High

n = 13 years

9 \$1 450 is invested at 4% p.a. and earns interest of \$464.

I	=	P		r	
464	=	1 450	×	0.04	× n

Tried n =	5	7	9	
	Low	Low	Low	Low
	High	High	High	High

n = 8 years

10 Interest of \$1 137.50 is earned from a \$3 250 investment at 5% p.a.

I	=	P		r	
1 137.50	=	3 250	×	0.05	× n

Tried n =	4	6	8	
	Low	Low	Low	Low
	High	High	High	High

n = 7 years

Now write the formula, make the substitution and find 'n' by Guess and Check methods.

11 An investment of \$800 at 7% p.a., results in interest of \$224.

$I = Prn$

Write the formula $I = Prn$

$224 = 800 \times 0.07 \times n$

Substitute I, P and r (as a decimal)

n = 4 years

12 Interest of \$3 652 is earned from a \$4 150 investment at 11% p.a.

$I = Prn$

$3\,652 = 4\,150 \times 0.11 \times n$

n = 8 years

13 An investment of \$930 at 6% p.a., results in interest of \$167.40.

$I = Prn$

$167.40 = 930 \times 0.06 \times n$

$n = 3$ years

14 Interest of \$1 430 is earned from a \$2 200 investment at 13% p.a.

$I = Prn$

$1\,430 = 2\,200 \times 0.13 \times n$

$n = 5$ years

Investing Money - Simple Interest - Algebraic Solution Term '*n*' (Sheet 2)

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Find '*n*' by solving these. Use the '÷' key or the fraction key.

1 $n = \frac{7\,500}{1\,250} =$ 6

2 $n = \frac{9\,960}{830} =$ 12

3 $n = \frac{587}{117.40} =$ 5

4 $n = \frac{7\,994}{3\,997} =$ 2

Create the division first, by dividing by the number next to the '*n*'. Find the value of '*n*'.

Example

$176.40 = 29.4 \times n$

÷ 29.4 ÷ 29.4

$n = \frac{176.40}{29.4} =$ 6

5 $126 = 18 \times n$

÷ 18 ÷ 18

$n = \frac{126}{18} =$ 7

6 $420 = 70 \times n$

÷ 70 ÷ 70

$n = \frac{420}{70} =$ 6

7 $1\,101.10 = 100.10 \times n$

÷ 100.10 ÷ 100.10

$n = \frac{1\,101.10}{100.10} =$ 11

8 $2\,274.80 = 568.70 \times n$

÷ 568.70 ÷ 568.70

$n = \frac{2\,274.80}{568.70} =$ 4

9 $3\,080 = 440 \times n$

÷ 440 ÷ 440

$n = \frac{3\,080}{440} =$ 7

10 $414 = 138 \times n$

÷ 138 ÷ 138

$n = \frac{414}{138} =$ 3

11 $1\,336.50 = 148.50 \times n$

÷ 148.50 ÷ 148.50

$n = \frac{1\,336.50}{148.50} =$ 9

1 extra step, perform a × first, then it's the same process. Find the value of '*n*'.

Example

$1\,300 = 6\,500 \times 0.05 \times n$

Multiply these

$1\,300 = 325 \times n$

÷ 325 ÷ 325

$n = \frac{1\,300}{325} =$ 4

12 $680 = 1\,700 \times 0.08 \times n$

$680 =$ 136 $\times n$

÷ 136 ÷ 136

$n = \frac{680}{136} =$ 5

13 $2\,541 = 3\,850 \times 0.06 \times n$

$2\,541 =$ 231 $\times n$

÷ 231 ÷ 231

$n = \frac{2\,541}{231} =$ 11

14 $4\,900 =$ 14 000 $\times 0.05 \times n$

4 900 $=$ 700 $\times n$

÷ 700 ÷ 700

$n = \frac{4\,900}{700} =$ 7

Get the numbers out of the sentence and solve for '*n*', which is the term, in years.

15 Interest of $537.60 is earned from a $960 investment at 7% p.a.

537.60 = 960 × 0.07 × *n*

Interest | Principal | Rate (decimal)

537.60 = 67.2 × *n*

÷ 67.2 ÷ 67.2

$n = \frac{537.60}{67.2} =$ 8 years

16 Interest of $3 354 is earned from a $4 300 investment at 13% p.a.

3 354 = 4 300 × 0.13 × *n*

Interest | Principal | Rate (decimal)

3 354 = 559 × *n*

÷ 559 ÷ 559

$n = \frac{3\,354}{559} =$ 6 years

17 Interest of $590 is earned from a $1 475 investment at 5% p.a.

590 = 1 475 × 0.05 × *n*

Interest | Principal | Rate (decimal)

590 = 73.75 × *n*

÷ 73.75 ÷ 73.75

$n = \frac{590}{73.75} =$ 8 years

Investing Money - Simple Interest - Further Finding the Term (Sheet 3)

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Find the term 'n' by solving these. Follow the example.

Example

Interest of \$1 752 is earned from a \$7 300 investment at 4% p.a., find the term of the investment.

$I = Prn$ — Write Formula

Decimal

$1752 = 7\,300 \times 0.04 \times n$ — Substitute Values

$1\,752 = 292 \times n$ — Simplify The '×'

$\div 292 \quad \div 292$

$n = \frac{1\,752}{292} = 6$ years — Divide and solve

1 Interest of \$120 is earned from a \$500 investment at 4% p.a., find the term of the investment.

$I = Prn$

$120 = 500 \times 0.04 \times n$

$120 = 20 \times n$

$\div 20 \quad \div 20$

$n = \frac{120}{20} = 6$ years

2 An investment of \$3 250 at 7% p.a., results in interest of \$2 275 being paid. Find the term of the investment.

$I = Prn$

$2\,275 = 3\,250 \times 0.07 \times n$

$2\,275 = 227.5 \times n$

$\div 227.5 \quad \div 227.5$

$n = \frac{2\,275}{227.5} = 10$ years

3 Interest of \$620.40 is received from a \$940 investment at 6% p.a., find the term of the investment.

$I = Prn$

$620.40 = 940 \times 0.06 \times n$

$620.40 = 56.4 \times n$

$\div 56.4 \quad \div 56.4$

$n = \frac{620.40}{56.4} = 11$ years

4 An investment of \$1 700 at 8% p.a., results in interest of \$952 being paid. Find the term of the investment.

$I = Prn$

$952 = 1\,700 \times 0.08 \times n$

$952 = 136 \times n$

$\div 136 \quad \div 136$

$n = \frac{952}{136} = 7$ years

5 Interest of \$8 944 is earned from an \$8 600 investment at 13% p.a., find the term of the investment.

$I = Prn$

$8\,944 = 8\,600 \times 0.13 \times n$

$8\,944 = 1\,118 \times n$

$\div 1\,118 \quad \div 1\,118$

$n = \frac{8\,944}{1\,118} = 8$ years

6 An investment of \$4 850 at 11% p.a., results in interest of \$4 801.50 being paid. Find the investment's term.

$I = Prn$

$4\,801.50 = 4\,850 \times 0.11 \times n$

$4\,801.50 = 533.5 \times n$

$\div 533.5 \quad \div 533.5$

$n = \frac{4\,801.50}{533.5} = 9$ years

This time you want a total amount in the future, follow the steps to find the term.

David has \$9 000 in the bank but he needs \$12 600 for a future trip.

7 How much interest does he have to earn to have an account balance of \$12 600?

$I =$ 12 600 - 9 000

$I =$ \$3 600

8 If David's investment pays interest at 5% p.a., find the term of the investment to achieve the interest.

$I = Prn$

$3\,600 = 9\,000 \times 0.05 \times n$

$3\,600 = 450 \times n$

$\div 450 \quad \div 450$

$n = \frac{3\,600}{450} = 8$ years

Skye has \$26 000 in the bank but she needs \$36 400 for a home deposit.

9 How much interest does she have to earn to have an account balance of \$36 400?

$I =$ 36 400 - 26 000

$I =$ \$10 400

10 If Skye earns interest at 8% p.a., find the term of the investment needed to get the deposit.

$I = Prn$

$10\,400 = 26\,000 \times 0.08 \times n$

$10\,400 = 2\,080 \times n$

$\div 2\,080 \quad \div 2\,080$

$n = \frac{10\,400}{2\,080} = 5$ years

Investing Money - Simple Interest - Finding the Term (Homework Strip)

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Date Due: _____

Find the term 'n' by guess & check

1 Interest of $1 200 is earned from a $4 000 investment at 5% p.a.

$I = P \quad 5 \div 100 \quad r$

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

2 Interest of $572 is earned from a $650 investment at 8% p.a.

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

3 Interest of $1 804 is earned from a $9 020 investment at 10% p.a.

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

Date Due: _____

Find the term 'n' by guess & check

1 Interest of $1 200 is earned from a $4 000 investment at 5% p.a.

$I = P \quad 5 \div 100 \quad r$

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

2 Interest of $572 is earned from a $650 investment at 8% p.a.

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

3 Interest of $1 804 is earned from a $9 020 investment at 10% p.a.

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

Date Due: _____

Find the term 'n' by guess & check

1 Interest of $1 200 is earned from a $4 000 investment at 5% p.a.

$I = P \quad 5 \div 100 \quad r$

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

2 Interest of $572 is earned from a $650 investment at 8% p.a.

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

3 Interest of $1 804 is earned from a $9 020 investment at 10% p.a.

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

Date Due: _____

Find the term 'n' by guess & check

1 Interest of $1 200 is earned from a $4 000 investment at 5% p.a.

$I = P \quad 5 \div 100 \quad r$

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

2 Interest of $572 is earned from a $650 investment at 8% p.a.

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

3 Interest of $1 804 is earned from a $9 020 investment at 10% p.a.

_____ = _____ × _____ × n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = _____ years

Investing Money - Simple Interest - Finding the Term (Homework Strip)

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Date Due:

Find the term 'n' by guess & check

1 Interest of $1 200 is earned from a $4 000 investment at 5% p.a.

$I = P \quad 5 \div 100 \quad r$

1 200	=	4 000	×	0.05	× n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = 6 years

2 Interest of $572 is earned from a $650 investment at 8% p.a.

572	=	650	×	0.08	× n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = 11 years

3 Interest of $1 804 is earned from a $9 020 investment at 10% p.a.

1 804	=	9 020	×	0.1	× n

Tried n =				
	○ Low ○ High	○ Low ○ High	○ Low ○ High	○ Low ○ High

n = 2 years

Investing Money - Simple Interest - Finding the Principal '*P*'

Imagine you are saving for something and know how much you need. You can invest an amount now, and later, after the investment has had the interest paid have the amount that you need. The problem is how much do you need to invest now?

These sheets deal with finding how much to invest. There are two methods used.

Guess and Check: This means you an amount, put the number into the simple interest formula and see if it gets the answer. If the answer is too low increase the amount. Too high, decrease it, until you get the answer.

Example: An investment for 3 years at 4% p.a., results in interest of $660. Find the principal.

$I = Prn$ ← Simple Interest Formula

Convert rate

660 = $P \times$ 0.04 $\times$ 3

Interest | to a decimal | Period

← Substitute the values, remember to use the interest rate *r* as a decimal

Tried $P =$			
	○ Low ○ High	○ Low ○ High	○ Low ○ High

← Try a value of '*P*'. Work out (the value) × 9 000 × 0.06
If it = 660 that's *P*!
If it is less than 660 increase *P*.
If it is more than 660 decrease *P*.
Keep going until you get 660.

$P =$

Algebraic: This method uses equation solving skills. It has the advantage of getting the answer straight away. But you have to know how to solve an equation.

Example: An investment of $9 000 at 6% p.a., results in interest of $4 320. Find the period.

17 Interest of $1 386 is earned from a $3 300 investment at 7% p.a.

1 386 = 3 300 × 0.07 × *n*

Interest | Principal | Rate (decimal)

← Simple Interest Formula
Substitute the values, remember to use the interest rate *r* as a decimal

1 386 = 231 × *n*

← Multiply *P* and *r*

÷ 231 ÷ 231

← Divide both sides by the number with *n*.
That gives you your answer.

$n = \frac{1\,386}{231} =$ 6 years

Investing Money - Simple Interest - Guess & Check Principal (Sheet 1)

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Use 'Guess and Check' to find the value of 'P' for these. Use the back arrow. All values are in thousands.

1 $3\,360 = 0.07 \times 8 \times P$

Try 'P' = 3 000 ____ / $0.07 \times 8 \times 3\,000$ — Compare to 3 360

Try 'P' = 5 000 ____ / $0.07 \times 8 \times 5\,000$ — Compare to 3 360

Try 'P' = 7 000 ____ / $0.07 \times 8 \times 7\,000$ — Compare to 3 360

$P =$ ____

2 $1\,440 = 0.04 \times 3 \times P$

Try 'P' = 3 000 ____ / $0.04 \times 3 \times 3\,000$ — Compare to 1 440

Try 'P' = 7 000 ____ / $0.04 \times 3 \times 7\,000$ — Compare to 1 440

Try 'P' = 11 000 ____ / $0.04 \times 3 \times 11\,000$ — Compare to 1 440

$P =$ ____

Keep trying 'P' values until you get 1 440

3 $12\,870 = 0.11 \times 9 \times P$

Try 'P' = 5 000 ____ / $0.11 \times 9 \times 5\,000$ — Compare to 12 870

Try 'P' = 10 000 ____ / $0.11 \times 9 \times 10\,000$ — Compare to 12 870

Try 'P' = 15 000 ____ / $0.11 \times 9 \times 15\,000$ — Compare to 12 870

$P =$ ____

Keep trying 'P' values until you get 12 870

Build the calculation line. But use $I = rnP$, so that the P is the last number on your calc. Find 'P'.

4 An investment at 6% p.a. for 5 years, results in interest of $2 400.

____ = ____ × ____ × P

Interest | Rate (decimal) | Term

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

$P =$ $

5 An investment yields interest of $2 880 after 8 years, at 12% p.a.

____ = ____ × ____ × P

I | r | n

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

$P =$ ____

6 An investment at 11% p.a. for 2 years, results in interest of $5 280.

____ = ____ × ____ × P

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

$P =$ ____

7 An investment at 9% p.a. for 7 years, results in interest of $10 080.

____ = ____ × ____ × P

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

$P =$ ____

Now write the formula, with P at the end. Make the substitution and find 'P'. (whole thousands)

8 An investment at 6% p.a. for 8 years, results in interest of $6 240.

____

Write the formula $I = rnP$, with P at the end

____ = ____ × ____ × P

Substitute I, then r (as a decimal) and then n.

$P =$ ____

Show the $ sign

9 An investment at 10% p.a. for 4 years, results in interest of $1 200.

____

____ = ____ × ____ × P

$P =$ ____

10 An investment yields interest of $3 780 after 7 years, at 3% p.a.

____

____

____

11 An investment yields interest of $1 980 after 3 years, at 6% p.a.

____

____

____

Investing Money - Simple Interest - Algebraic Sol Principal 'P' (Sheet 2)

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Find 'P' by solving these. Use the '÷' key or the fraction key.

1 $P = \dfrac{120}{0.6} =$ ☐

2 $P = \dfrac{868}{0.62} =$ ☐

3 $P = \dfrac{4\ 200}{0.48} =$ ☐

4 $P = \dfrac{111.60}{0.72} =$ ☐

Create the division first, by dividing by the number next to the 'P'. Find the value of 'P'.

Example

$176.40 = P \times 0.42$

÷ 0.42 ÷ 0.42

$P = \dfrac{176.40}{0.42} = 420$

5 $240 = P \times 0.48$

÷ 0.48 ÷ 0.48

$P = \dfrac{\square}{\square} = \square$

6 $975 = P \times 0.65$

÷ 0.65 ÷ 0.65

$P = \dfrac{\square}{\square} = \square$

7 $4\ 968 = P \times 0.54$

÷ 0.54 ÷ 0.54

$P = \dfrac{\square}{\square} = \square$

8 $1\ 344 = P \times 0.24$

÷ 0.24 ÷ 0.24

$P = \dfrac{\square}{\square} = \square$

9 $459.20 = P \times 0.56$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

10 $2\ 343 = P \times 0.33$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

11 $2\ 196 = P \times 0.72$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

Multiply r and n first, then it's the same division process. Find the value of 'P'.

Example

$279.60 = P \times 0.03 \times 8$

Multiply these

$279.60 = P \times 0.24$

÷ 0.24 ÷ 0.24

$P = \dfrac{279.60}{0.24} = 1\ 165$

12 $195 = P \times 0.06 \times 5$

$195 = P \times \square$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

13 $484.20 = P \times 0.04 \times 3$

$484.20 = P \times \square$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

Substitute into the formula to find the value of 'P'.

14 Interest of \$86 is earned from a 5 year investment at 4% p.a.

☐ $= P \times$ ☐ $\times$ ☐

Interest Rate (decimal) Term

☐ $= P \times$ ☐

÷ ÷

$P = \dfrac{\square}{\square} =$ \$ ☐

$ sign

15 Interest of \$14 016 is earned from a 4 year investment at 12% p.a.

☐ $= P \times$ ☐ $\times$ ☐

☐ $= P \times$ ☐

÷ ÷

$P = \dfrac{\square}{\square} = \square$

$ sign

16 Interest of \$799.20 is earned from a 6 year investment at 9% p.a.

☐ $= P \times$ ☐ $\times$ ☐

☐ $= P \times$ ☐

÷ ÷

$P = \dfrac{\square}{\square} = \square$

17 Interest of \$457.20 is earned from a 3 year investment at 6% p.a.

☐ $= P \times$ ☐ $\times$ ☐

☐ $= P \times$ ☐

÷ ÷

$P = \dfrac{\square}{\square} = \square$

Investing Money - Simple Interest - Further Finding Principal (Sheet 3)

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Find the amount invested, the Principal 'P' by solving these. Remember the $ sign.

Example

Interest of $374.50 is earned from a 5 year investment at 7% p.a., find the principal invested.

$I = Prn$ — Write Formula

Decimal: 0.07

$374.50 = P \times 0.07 \times 5$ — Substitute Values

$374.50 = P \times 0.35$ — Simplify The '×'

$\div 0.35$ $\div 0.35$

$P = \frac{374.50}{0.35} = \$1\,070$ — Divide and solve

1 Interest of $914.40 is earned from a 3 year investment at 8% p.a., find the principal invested.

2 Interest of $8 127 is earned from a 7 year investment at 6% p.a., find the principal invested.

3 A 5 year investment which has an interest rate of 4% p.a. yields interest of $1 150. Find the amount invested.

$I = Prn$

$1\,150 = P \times 0.04 \times 5$

$1\,150 = P \times 0.2$

4 Interest of $8 658 is earned from a 6 year investment at 13% p.a., find the principal invested.

5 A 2 year investment which has an interest rate of 9% p.a. yields interest of $129.60. Find the amount invested.

This time find the account balance. So find the principal then add it to the interest.

6 Interest of $1 545 is earned from a 5 year investment at 6% p.a., find the principal invested.

7 What will be the account balance at the end of the investment?

$A =$ [ ] + [ ]

Principal Interest

$A =$ [ ]

8 Interest of $321.60 is earned from a 4 year investment at 12% p.a., find the principal invested.

9 What will be the account balance at the end of the investment?

$A =$ [ ] + [ ]

$A =$ [ ]

Investing Money - Simple Interest - Guess & Check Principal (Sheet 1)

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Use 'Guess and Check' to find the value of 'P' for these. Use the back arrow. All values are in thousands.

1 $3\,360 = 0.07 \times 8 \times P$

Try 'P' = 3 000	1 680	$0.07 \times 8 \times 3\,000$	Compare to 3 360
Try 'P' = 5 000	2 800	$0.07 \times 8 \times 5\,000$	Compare to 3 360
Try 'P' = 7 000	3 920	$0.07 \times 8 \times 7\,000$	Compare to 3 360

$P =$ 6 000

2 $1\,440 = 0.04 \times 3 \times P$

Try 'P' = 3 000	360	$0.04 \times 3 \times 3\,000$	Compare to 1 440
Try 'P' = 7 000	840	$0.04 \times 3 \times 7\,000$	Compare to 1 440
Try 'P' = 11 000	1 320	$0.04 \times 3 \times 11\,000$	Compare to 1 440

$P =$ 12 000

Keep trying 'P' values until you get 1 440

3 $12\,870 = 0.11 \times 9 \times P$

Try 'P' = 5 000	4 950	$0.11 \times 9 \times 5\,000$	Compare to 12 870
Try 'P' = 10 000	9 900	$0.11 \times 9 \times 10\,000$	Compare to 12 870
Try 'P' = 15 000	14 850	$0.11 \times 9 \times 15\,000$	Compare to 12 870

$P =$ 13 000

Keep trying 'P' values until you get 12 870

Build the calculation line. But use $I = rnP$, so that the P is the last number on your calc. Find 'P'.

4 An investment at 6% p.a. for 5 years, results in interest of \$2 400.

2 400 (Interest) = 0.06 (Rate (decimal)) × 5 (Term) × P

Tried $P =$	3 000	● Low ○ High
Tried $P =$	6 000	● Low ○ High
Tried $P =$	9 000	○ Low ● High

$P =$ \$8 000

5 An investment yields interest of \$2 880 after 8 years, at 12% p.a.

2 880 (I) = 0.12 (r) × 8 (n) × P

Tried $P =$	2 000	● Low ○ High
Tried $P =$	2 500	● Low ○ High
Tried $P =$	3 500	○ Low ● High

$P =$ 3 000

6 An investment at 11% p.a. for 2 years, results in interest of \$5 280.

5 280 = 0.11 × 2 × P

Tried $P =$	10 000	● Low ○ High
Tried $P =$	20 000	● Low ○ High

$P =$ \$24 000

7 An investment at 9% p.a. for 7 years, results in interest of \$10 080.

10 080 = 0.09 × 7 × P

Tried $P =$	10 000	● Low ○ High
Tried $P =$	20 000	○ Low ● High

$P =$ \$16 000

Now write the formula, with P at the end. Make the substitution and find 'P'. (whole thousands)

8 An investment at 6% p.a. for 8 years, results in interest of \$6 240.

$I = rnP$

Write the formula $I = rnP$, with P at the end

6 240 = 0.06 × 8 × P

Substitute I, then r (as a decimal) and then n.

$P =$ \$13 000

Show the \$ sign

9 An investment at 10% p.a. for 4 years, results in interest of \$1 200.

$I = rnP$

1 200 = 0.10 × 4 × P

$P =$ \$3 000

10 An investment yields interest of \$3 780 after 7 years, at 3% p.a.

$I = rnP$

$3\,780 = 0.03 \times 7 \times P$

$P =$ \$18 000

11 An investment yields interest of \$1 980 after 3 years, at 6% p.a.

$I = rnP$

$1\,980 = 0.06 \times 3 \times P$

$P =$ \$11 000

Investing Money - Simple Interest - Algebraic Sol Principal 'P' (Sheet 2)

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Find 'P' by solving these. Use the '÷' key or the fraction key.

1 $P = \dfrac{120}{0.6} =$ 200

2 $P = \dfrac{868}{0.62} =$ 1 400

3 $P = \dfrac{4\,200}{0.48} =$ 8 750

4 $P = \dfrac{111.60}{0.72} =$ 155

Create the division first, by dividing by the number next to the 'P'. Find the value of 'P'.

Example

$176.40 = P \times 0.42$

÷ 0.42 ÷ 0.42

$P = \dfrac{176.40}{0.42} =$ 420

5 $240 = P \times 0.48$

÷ 0.48 ÷ 0.48

$P = \dfrac{240}{0.48} =$ 500

6 $975 = P \times 0.65$

÷ 0.65 ÷ 0.65

$P = \dfrac{975}{0.65} =$ 1 500

7 $4\,968 = P \times 0.54$

÷ 0.54 ÷ 0.54

$P = \dfrac{4\,968}{0.54} =$ 9 200

8 $1\,344 = P \times 0.24$

÷ 0.24 ÷ 0.24

$P = \dfrac{1\,344}{0.24} =$ 5 600

9 $459.20 = P \times 0.56$

÷ 0.56 ÷ 0.56

$P = \dfrac{459.20}{0.56} =$ 820

10 $2\,343 = P \times 0.33$

÷ 0.33 ÷ 0.33

$P = \dfrac{2\,343}{0.33} =$ 7 100

11 $2\,196 = P \times 0.72$

÷ 0.72 ÷ 0.72

$P = \dfrac{2\,196}{0.72} =$ 3 050

Multiply r and n first, then it's the same division process. Find the value of 'P'.

Example

$279.60 = P \times 0.03 \times 8$

Multiply these

$279.60 = P \times 0.24$

÷ 0.24 ÷ 0.24

$P = \dfrac{279.60}{0.24} =$ 1 165

12 $195 = P \times 0.06 \times 5$

$195 = P \times$ 0.3

÷ 0.3 ÷ 0.3

$P = \dfrac{195}{0.3} =$ 650

13 $484.20 = P \times 0.04 \times 3$

$484.20 = P \times$ 0.12

÷ 0.12 ÷ 0.12

$P = \dfrac{484.20}{0.12} =$ 4 035

Substitute into the formula to find the value of 'P'.

14 Interest of \$86 is earned from a 5 year investment at 4% p.a.

86 $= P \times$ 0.04 $\times$ 5

Interest; Rate (decimal); Term

86 $= P \times$ 0.2

÷ 0.2 ÷ 0.2

$P = \dfrac{86}{0.2} =$ \$430 ($ sign)

15 Interest of \$14 016 is earned from a 4 year investment at 12% p.a.

14 016 $= P \times$ 0.12 $\times$ 4

14 016 $= P \times$ 0.48

÷ 0.48 ÷ 0.48

$P = \dfrac{14\,016}{0.48} =$ \$29 200 ($ sign)

16 Interest of \$799.20 is earned from a 6 year investment at 9% p.a.

799.20 $= P \times$ 0.09 $\times$ 6

799.20 $= P \times$ 0.54

÷ 0.54 ÷ 0.54

$P = \dfrac{799.20}{0.54} =$ \$1 480

17 Interest of \$457.20 is earned from a 3 year investment at 6% p.a.

457.20 $= P \times$ 0.06 $\times$ 3

457.20 $= P \times$ 0.18

÷ 0.18 ÷ 0.18

$P = \dfrac{457.20}{0.18} =$ \$2 540

Investing Money - Simple Interest - Further Finding Principal (Sheet 3)

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Find the amount invested, the Principal 'P' by solving these. Remember the $ sign.

Example

Interest of $374.50 is earned from a 5 year investment at 7% p.a., find the principal invested.

$I = Prn$ — Write Formula

$374.50 = P \times 0.07 \times 5$ — Substitute Values (Decimal)

$374.50 = P \times 0.35$ — Simplify The '×'

$\div 0.35$ $\div 0.35$

$P = \frac{374.50}{0.35} = \$1\,070$ — Divide and solve

1 Interest of $914.40 is earned from a 3 year investment at 8% p.a., find the principal invested.

$I = Prn$

$914.40 = P \times 0.08 \times 3$

$914.40 = P \times 0.24$

$\div 0.24$ $\div 0.24$

$P = \frac{914.40}{0.24} = \$3\,810$

2 Interest of $8 127 is earned from a 7 year investment at 6% p.a., find the principal invested.

$I = Prn$

$8\,127 = P \times 0.06 \times 7$

$8\,127 = P \times 0.42$

$\div 0.42$ $\div 0.42$

$P = \frac{8\,127}{0.42} = \$19\,350$

3 A 5 year investment which has an interest rate of 4% p.a. yields interest of $1 150. Find the amount invested.

$I = Prn$

$1\,150 = P \times 0.04 \times 5$

$1\,150 = P \times 0.2$

$\div 0.2$ $\div 0.2$

$P = \frac{1\,150}{0.2} = \$5\,750$

4 Interest of $8 658 is earned from a 6 year investment at 13% p.a., find the principal invested.

$I = Prn$

$8\,658 = P \times 0.13 \times 6$

$8\,658 = P \times 0.78$

$\div 0.78$ $\div 0.78$

$P = \frac{8\,658}{0.78} = \$11\,100$

5 A 2 year investment which has an interest rate of 9% p.a. yields interest of $129.60. Find the amount invested.

$I = Prn$

$129.60 = P \times 0.13 \times 6$

$\div 0.78$ $\div 0.78$

$P = \frac{8\,658}{0.78} = \$11\,100$

This time find the account balance. So find the principal then add it to the interest.

6 Interest of $1 545 is earned from a 5 year investment at 6% p.a., find the principal invested.

$I = Prn$

$1\,545 = P \times 0.06 \times 5$

$1\,545 = P \times 0.3$

$\div 0.3$ $\div 0.3$

$P = \frac{1\,545}{0.3} = \$5\,150$

7 What will be the account balance at the end of the investment?

$A =$ 5 150 (Principal) $+$ 1 545 (Interest)

$A =$ $6 695

8 Interest of $321.60 is earned from a 4 year investment at 12% p.a., find the principal invested.

$I = Prn$

$321.60 = P \times 0.12 \times 4$

$321.60 = P \times 0.48$

$\div 0.48$ $\div 0.48$

$P = \frac{321.60}{0.48} = \$670$

9 What will be the account balance at the end of the investment?

$A =$ 670 $+$ 321.60

$A =$ $991.60

Investing Money - Simple Interest - Finding the Principal (Homework Strip)

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Date Due:

Find the Principal 'P' by guess & check

1 An investment at 6% p.a. for 5 years, results in interest of $2 100.

$\square = \square \times \square \times P$

6÷100

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$ $

2 An investment at 12% p.a. for 8 years, results in interest of $576.

$\square = \square \times \square \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$

3 An investment at 8% p.a. for 11 years, results in interest of $1 760.

$\square = \square \times \square \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$

Date Due:

Find the Principal 'P' by guess & check

1 An investment at 6% p.a. for 5 years, results in interest of $2 100.

$\square = \square \times \square \times P$

6÷100

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$ $

2 An investment at 12% p.a. for 8 years, results in interest of $576.

$\square = \square \times \square \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$

3 An investment at 8% p.a. for 11 years, results in interest of $1 760.

$\square = \square \times \square \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$

Date Due:

Find the Principal 'P' by guess & check

1 An investment at 6% p.a. for 5 years, results in interest of $2 100.

$\square = \square \times \square \times P$

6÷100

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$ $

2 An investment at 12% p.a. for 8 years, results in interest of $576.

$\square = \square \times \square \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$

3 An investment at 8% p.a. for 11 years, results in interest of $1 760.

$\square = \square \times \square \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$

Date Due:

Find the Principal 'P' by guess & check

1 An investment at 6% p.a. for 5 years, results in interest of $2 100.

$\square = \square \times \square \times P$

6÷100

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$ $

2 An investment at 12% p.a. for 8 years, results in interest of $576.

$\square = \square \times \square \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$

3 An investment at 8% p.a. for 11 years, results in interest of $1 760.

$\square = \square \times \square \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$

Investing Money - Simple Interest - Finding the Principal (H'work Strip)

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Date Due:

Find the Principal 'P' by guess & check

1 An investment at 6% p.a. for 5 years, results in interest of $2 100.

$2\,100 = 0.06 \times 5 \times P$

6÷100

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$ $7 000

2 An investment at 12% p.a. for 8 years, results in interest of $576.

$576 = 0.12 \times 8 \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$ $600

3 An investment at 8% p.a. for 11 years, results in interest of $1 760.

$1\,760 = 0.08 \times 11 \times P$

Tried $P =$ ○ Low ○ High

Tried $P =$ ○ Low ○ High

$P =$ $2 000

Investing Money - Simple Interest - Repeated Use

Simple interest investments pay their interest at the end of the investment. So if the investment is for 3 years, the interest is paid at the end of that 3 year period. Compound interest investments pay interest during the investment. For example a 3 year investment might have interest credited after the first, second and third years.

This doesn't necessarily mean that you get your interest earlier. The interest is invested with the original principal. So this means that for the second year the principal is higher with an increase in the amount of interest earned. The third year will again see an increase in the Principal with an increase in interest.

So compound investments earn more interest, this sheet deals with calculating the final interest payment without using the Compound Interest formula, which is introduced in the next outcome.

The method is simple, you apply the simple interest formula multiple times. There is a short cut however and this is shown on Sheet 2. Here is an example.

Example: An investment of \$4 000 at 6% p.a., is compounded annually for 3 years.
Find the account balance, A, at the end of the investment.

First Application: Year 1	Second Application: Year 2	Third Application: Year 3
$I = Prn$	$I = Prn$	$I = Prn$
$= 4\,000 \times 0.06 \times 1$	$= 4\,240 \times 0.06 \times 1$	$= 4\,494.40 \times 0.06 \times 1$
$I = \$240$	$I = \$254.40$	$I = \$269.66$
$A = 4\,000 + 240$	$A = 4\,240 + 254.40$	$A = 4\,494.40 + 269.66$
$A = \$4\,240$	$A = \$4\,494.40$	$A = \$4\,764.06$

Note how the principal increases each year. Instead of being \$4 000 for three years, it increases each year. So the interest earned also increases.

As you can see, instead of earning \$240 every year, each year sees an increase in the amount of interest earned. So compound investments offer a better return than simple interest investments. As long as they are for a longer duration than 1 compound period.

Investing Money - Simple Interest - Repeated Use (Sheet 1)

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When an investment compounds, interest is paid during the investment. Calculate the account balance for these.

Principal \$2 000,
Rate = 7% p.a.,
n = 3 yrs, compounding annually.

First Application: Year 1

1 $I = Prn$

$= \square \times \square \times 1$
Principal; Rate (decimal)

$I = \square$ Show '\$' sign

$A = \square + \square$
Principal; Interest (Q1)

$A = \square$

Second Application: Year 2

2 $I = Prn$

$= \square \times \square \times 1$
Principal (Use A-Yr 1); Rate (decimal)

$I = \square$

$A = \square + \square$
Principal (Use A-Yr 1); Interest (Q2)

$A = \square$

Third (Final) Application: Year 3

3 $I = Prn$

$= \square \times \square \times 1$
Principal (Use A-Yr 2); Rate (decimal)

$I = \square$

$A = \square + \square$
Principal (Use A-Yr 2); Interest (Q3)

$A = \square$

Compare the compounding investment (Q 1-3) to a standard simple interest investment.

4 $I = Prn$

$= \square \times \square \times 3$
\$2 000; Rate 7% p.a.

$I = \square$

$A = \square + \square$
Principal; Interest (Q4)

$A = \square$

5 Find the difference between your answer for 'A' in Q3 & Q4

Diff = (A (Q3)) - (A (Q4))

$= \square$

Now try a 2 year compounding investment. Find the account balance.

Principal \$920,
Rate = 8.4% p.a.,
n = 2 yrs, compounding annually

First Application: Year 1

6 $I = Prn$

$= \square \times \square \times 1$

$I = \square$

$A = \square + \square$

$A = \square$

Second (Final) Application: Year 2

7 $I = Prn$

$=$

$I =$

$A =$

$A =$

Compare the compounding investment (Q 6-7) to a standard simple interest investment.

8 $I = Prn$

$= \square \times \square \times 2$
\$920; Rate 8.4% p.a.

$I = \square$

$A = \square + \square$
Principal; Interest (Q8)

$A = \square$

9 Find the difference between your answer for 'A' in Q7 & Q8

Diff = (A (Q7)) - (A (Q8))

$= \square$

Now use a different approach to answering compound questions. Calculate these …..

10 *Principal* \$2 000,
Rate = 7% p.a., [1 + 0.07 = 1.07]
n = 3 yrs, compounding annually.

$A = 2\,000 \times 1.07 \times 1.07 \times 1.07$
Principal; Repeated 3 times as $n = 3$

$A = \square$

11 Is this the same as your answer in Q3? ○ Yes ○ No

12 *Principal* \$920,
Rate = 8.4% p.a., [1 + r = 1.084]
n = 2 yrs, compounding annually

$A = 920 \times 1.084 \times 1.084$
Principal; Repeated 2 times as $n = 2$

$A = \square$

13 Is this the same as your answer in Q7? ○ Yes ○ No

Investing Money - Simple Interest - Repeated Use (Sheet 2)

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Use multiplication to find the account balance more quickly. Change the rate to "1.xx", where 'xx' is the rate as a decimal. Find the account balance, A.

1 Principal \$5 000, *Rate* = 6% p.a., n = 4 yrs, compounding annually.

$A = 5\,000 \times 1.06 \times 1.06 \times 1.06 \times 1.06$

$A =$

2 Principal \$3 100, *Rate* = 9% p.a., n = 5 yrs, compounding annually.

$A = 3\,100 \times 1.09 \times 1.09 \times 1.09 \times 1.09 \times 1.09$

$A =$

3 Principal \$665, *Rate* = 5% p.a., n = 4 yrs, compounding annually.

$A =$ ☐ × ☐ × ☐ × ☐ × ☐

$A =$

Did you use 1.05? Not 0.05.

4 Principal \$10 444, *Rate* = 12% p.a., n = 3 yrs, compounding annually.

$A =$

$A =$

5 Principal \$850.70, *Rate* = 8.2% p.a., n = 2 yrs, compounding annually.

$A =$

$A =$

6 Principal \$72 310, *Rate* = 6.7% p.a., n = 4 yrs, compounding annually.

$A =$

$A =$

7 Principal \$40 700, *Rate* = 11.6% p.a., n = 3 yrs, compounding annually.

$A =$

$A =$

Instead of using "1.xx"… n times, use powers to find A. Once you find A, find I by subtracting the principal from A.

8 Principal \$6 100, *Rate* = 4.8% p.a., n = 7 yrs, compounding annually.

$A = 6\,100 \times 1.048^7 =$ ☐

Calc: 6 1 0 0 × 1 • 0 4 8 $x^{\blacksquare}$ 7 =

$I =$ ☐ (A) - ☐ (P) = ☐

9 Principal \$7 040, *Rate* = 6.6% p.a., n = 4 yrs, compounding annually.

$A = 7\,040 \times 1.066^4 =$ ☐

$I =$ ☐ - ☐ = ☐

10 Principal \$370, *Rate* = 9% p.a., n = 2 yrs, compounding annually.

$A =$ ☐ × 1. ☐☐☐ = ☐

$I =$ ☐ - ☐ = ☐

11 Principal \$24 112, *Rate* = 7.3% p.a., n = 8 yrs, compounding annually.

$A =$ ☐ × ☐☐ = ☐

$I =$ ☐ - ☐ = ☐

12 Principal \$1 770, *Rate* = 6.2% p.a., n = 5 yrs, compounding annually.

$A =$ ☐ × ☐☐ = ☐

$I =$ ☐ - ☐ = ☐

Yu invests \$5 000 at 8.3% p.a. for 6 years, compounded annually. Find his account balance and interest earned.

13

$A =$ ☐ × ☐☐ = ☐

$I =$ ☐ - ☐ = ☐

Investing Money - Simple Interest - Repeated Use (Sheet 1)

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When an investment compounds, interest is paid during the investment. Calculate the account balance for these.

Principal $2 000,
Rate = 7% p.a.,
n = 3 yrs, compounding annually.

First Application: Year 1

1 $I = Prn$

= 2 000 (Principal) × 0.07 (Rate (decimal)) × 1

$I =$ $140 (Show '$' sign)

$A =$ 2 000 (Principal) + 140 (Interest (Q1))

$A =$ $2 140

Second Application: Year 2

2 $I = Prn$

= 2 140 (Principal (Use A-Yr 1)) × 0.07 (Rate (decimal)) × 1

$I =$ $149.80

$A =$ 2 140 (Principal (Use A-Yr 1)) + 149.80 (Interest (Q2))

$A =$ $2 289.80

Third (Final) Application: Year 3

3 $I = Prn$

= 2 289.80 (Principal (Use A-Yr 2)) × 0.07 (Rate (decimal)) × 1

$I =$ $160.29

$A =$ 2 289.80 (Principal (Use A-Yr 2)) + 160.29 (Interest (Q3))

$A =$ $2 450.09

Compare the compounding investment (Q 1-3) to a standard simple interest investment.

4 $I = Prn$

= 2 000 ($2 000) × 0.07 (Rate 7% p.a.) × 3

$I =$ $420

$A =$ 2 000 (Principal) + 420 (Interest (Q4))

$A =$ $2 420

5 Find the difference between your answer for 'A' in Q3 & Q4

Diff = 2 450.09 - 2 420

= $30.09

Now try a 2 year compounding investment. Find the account balance.

Principal $920,
Rate = 8.4% p.a.,
n = 2 yrs, compounding annually

First Application: Year 1

6 $I = Prn$

= 920 × 0.084 × 1

$I =$ $77.28

$A =$ 920 + 77.28

$A =$ $997.28

Second (Final) Application: Year 2

7 $I = Prn$

$= 997.28 \times 0.084 \times 1$

$I =$ $83.77

$A = 997.28 + 83.77$

$A =$ $1 081.05

Compare the compounding investment (Q 6-7) to a standard simple interest investment.

8 $I = Prn$

= 920 ($920) × 0.084 (Rate 8.4% p.a.) × 2

$I =$ $154.56

$A =$ 920 (Principal) + 154.56 (Interest (Q8))

$A =$ $1 074.56

9 Find the difference between your answer for 'A' in Q7 & Q8

Diff = 1 081.05 - 1 074.56

= $6.49

Now use a different approach to answering compound questions. Calculate these .....

10 *Principal* $2 000,
Rate = 7% p.a., [1 + 0.07 = 1.07]
n = 3 yrs, compounding annually.

$A = 2\,000 \times 1.07 \times 1.07 \times 1.07$ (Principal; Repeated 3 times as $n = 3$)

$A =$ $2 450.09

11 Is this the same as your answer in Q3? ● Yes ○ No

12 *Principal* $920,
Rate = 8.4% p.a., [1 + r = 1.084]
n = 2 yrs, compounding annually

$A = 920 \times 1.084 \times 1.084$ (Principal; Repeated 2 times as $n = 2$)

$A =$ $1 081.05

13 Is this the same as your answer in Q7? ● Yes ○ No

Investing Money - Simple Interest - Repeated Use (Sheet 2)

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Use multiplication to find the account balance more quickly. Change the rate to "1.xx", where 'xx' is the rate as a decimal. Find the account balance, A.

1 Principal \$5 000, *Rate* = 6% p.a., n = 4 yrs, compounding annually.

$A = 5\,000 \times 1.06 \times 1.06 \times 1.06 \times 1.06$

$A =$ \$6 312.38

2 Principal \$3 100, *Rate* = 9% p.a., n = 5 yrs, compounding annually.

$A = 3\,100 \times 1.09 \times 1.09 \times 1.09 \times 1.09 \times 1.09$

$A =$ \$4 769.73

3 Principal \$665, *Rate* = 5% p.a., n = 4 yrs, compounding annually.

$A =$ 665 × 1.05 × 1.05 × 1.05 × 1.05

$A =$ \$808.31

Did you use 1.05? Not 0.05.

4 Principal \$10 444, *Rate* = 12% p.a., n = 3 yrs, compounding annually.

$A = 10\,444 \times 1.12 \times 1.12 \times 1.12$

$A =$ \$14 673.07

5 Principal \$850.70, *Rate* = 8.2% p.a., n = 2 yrs, compounding annually.

$A = 850.70 \times 1.082 \times 1.082$

$A =$ \$995.93

6 Principal \$72 310, *Rate* = 6.7% p.a., n = 4 yrs, compounding annually.

$A = 72\,310 \times 1.067 \times 1.067 \times 1.067 \times 1.067$

$A =$ \$93 725.13

7 Principal \$40 700, *Rate* = 11.6% p.a., n = 3 yrs, compounding annually.

$A = 40\,700 \times 1.116 \times 1.116 \times 1.116$

$A =$ \$56 570.11

Instead of using "1.xx"… n times, use powers to find A. Once you find A, find I by subtracting the principal from A.

8 Principal \$6 100, *Rate* = 4.8% p.a., n = 7 yrs, compounding annually.

$A = 6\,100 \times 1.048^7 =$ \$8 469.52

Calc: 6 1 0 0 × 1 • 0 4 8 $x^{\blacksquare}$ 7 =

$I =$ 8 469.52 - 6 100 = \$2 369.52

9 Principal \$7 040, *Rate* = 6.6% p.a., n = 4 yrs, compounding annually.

$A = 7\,040 \times 1.066^4 =$ \$9 090.79

$I =$ 9 090.79 - 7 040 = \$2 050.79

10 Principal \$370, *Rate* = 9% p.a., n = 2 yrs, compounding annually.

$A =$ 370 × 1.0 9 2 = \$439.60

$I =$ 439.60 - 370 = \$69.60

11 Principal \$24 112, *Rate* = 7.3% p.a., n = 8 yrs, compounding annually.

$A =$ 24 112 × 1.073 8 = \$42 367.32

$I =$ 42 367.32 - 24 112 = \$18 255.32

12 Principal \$1 770, *Rate* = 6.2% p.a., n = 5 yrs, compounding annually.

$A =$ 1 770 × 1.062 5 = \$2 391.09

$I =$ 2 391.09 - 1 770 = \$621.09

Yu invests \$5 000 at 8.3% p.a. for 6 years, compounded annually. Find his account balance and interest earned.

13

$A =$ 5 000 × 1.083 6 = \$8 067.53

$I =$ 8 067.53 - 5 000 = \$3 067.53

Investing Money - Simple Interest - Repeated Use (Homework Strip)

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Date Due:

Find the final account balance on a $4 000 investment at 8% p.a., for 2 years, compounded annually

First Application: Year 1

1 $I = Prn$

$= \square \times \square . \square\square \times 1$

Principal; Rate (decimal)

$I =$ ____ Show '$' sign

$A =$ ____ + ____

Principal; Interest (Q1)

$A =$ ____

Second Application: Year 2

2 $I = Prn$

$= \square \times \square \times 1$

Principal (Use A-Yr 1); Rate (decimal)

$I =$ ____

$A =$ ____ + ____

Principal (Use A-Yr 1); Interest (Q2)

$A =$ ____

Date Due:

Find the final account balance on a $4 000 investment at 8% p.a., for 2 years, compounded annually

First Application: Year 1

1 $I = Prn$

$= \square \times \square . \square\square \times 1$

Principal; Rate (decimal)

$I =$ ____ Show '$' sign

$A =$ ____ + ____

Principal; Interest (Q1)

$A =$ ____

Second Application: Year 2

2 $I = Prn$

$= \square \times \square \times 1$

Principal (Use A-Yr 1); Rate (decimal)

$I =$ ____

$A =$ ____ + ____

Principal (Use A-Yr 1); Interest (Q2)

$A =$ ____

Date Due:

Find the final account balance on a $4 000 investment at 8% p.a., for 2 years, compounded annually

First Application: Year 1

1 $I = Prn$

$= \square \times \square . \square\square \times 1$

Principal; Rate (decimal)

$I =$ ____ Show '$' sign

$A =$ ____ + ____

Principal; Interest (Q1)

$A =$ ____

Second Application: Year 2

2 $I = Prn$

$= \square \times \square \times 1$

Principal (Use A-Yr 1); Rate (decimal)

$I =$ ____

$A =$ ____ + ____

Principal (Use A-Yr 1); Interest (Q2)

$A =$ ____

Date Due:

Find the final account balance on a $4 000 investment at 8% p.a., for 2 years, compounded annually

First Application: Year 1

1 $I = Prn$

$= \square \times \square . \square\square \times 1$

Principal; Rate (decimal)

$I =$ ____ Show '$' sign

$A =$ ____ + ____

Principal; Interest (Q1)

$A =$ ____

Second Application: Year 2

2 $I = Prn$

$= \square \times \square \times 1$

Principal (Use A-Yr 1); Rate (decimal)

$I =$ ____

$A =$ ____ + ____

Principal (Use A-Yr 1); Interest (Q2)

$A =$ ____

Investing Money - Simple Interest - Repeated Use (Homework Strip)

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Date Due:

Find the final account balance on a $4 000 investment at 8% p.a., for 2 years, compounded annually

First Application: Year 1

1 $I = Prn$

$= 4\,000 \times 0.08 \times 1$

Principal; Rate (decimal)

$I =$ $320 (Show '$' sign)

$A =$ 4 000 (Principal) + 320 (Interest (Q1))

$A =$ $4 320

Second Application: Year 2

2 $I = Prn$

$= 4\,320 \times 0.08 \times 1$

Principal (Use A-Yr 1); Rate (decimal)

$I =$ $345.60

$A =$ 4 320 (Principal (Use A-Yr 1)) + 345.60 (Interest (Q2))

$A =$ $4 665.60

Investing Money - Compound Interest

Compound Interest is different from simple interest in two key ways:

- Interest is paid during the investment, rather than at the end of the investment only.
- Because interest is paid during the investment and re-invested, the account balance increases and so more interest is paid.

So compound interest offers you more interest on your investment because your account balance grows during the investment and the interest earned increases.

Another difference with compound interest is the formula. Rather than calculating the interest earned, the formula calculates the account balance. So to find the interest earned you have to subtract the principal from the account balance. This is the opposite to simple interest.

Compound Interest Formula: $A = P(1 + r)^n$

Where:
A = Account balance
P = Principal
r = Interest rate expressed as a decimal
n = Number of compound periods

Interest Calculation: $I = A - P$

Where:
I = Interest earned
A = Account balance
P = Principal

Example: Find the interest earned on an investment of \$9 000 at 6% p.a., for 3 years, compounded annually.

$A = P(1 + r)^n$ ← Compound Interest Formula

$= 9\,000 \times (1 + 0.06)^3$ ← Substitute the values, remember to use the interest rate r as a decimal

$A = \$10\,719.14$ ← Round to 2 decimal places

$I = 10\,719.14 - 9\,000$ ← Subtract 'P' from 'A' to find 'I'

$I = \$1\,719.14$

Investing Money - Compound Interest - Account Balance (Sheet 1)

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Use your calculator to evaluate these to 2 d.p.

1 $450 \times (1+0.04)^4$

$=$ ☐ Round to 2 decimal places

Calc: 4 5 0 × (1 + 0 • 0 4) $x^{\blacksquare}$ 4 =

2 $3\,277 \times (1+0.11)^5$

$=$ ☐

3 $940.30 \times (1+0.055)^7$

$=$ ☐

4 $7\,500 \times (1+0.09)^2$

$=$ ☐

Use the compound interest formula to find the account balance for these investments.

Example: Find the account balance:

A \$650 investment, at 6% p.a., for 5 years, compounded annually.

$A = P(1+r)^n$

$= 650 \times (1+0.06)^5$

$A = \$869.85$

5 \$1 450 at 8% p.a., for 3 years, compounded annually.

$A = P(1+r)^n$

$=$ ☐ $\times$ (1 + ☐) ☐

Divide R by 100

$A =$ ☐

6 \$880 at 4% p.a., for 7 years, compounded annually.

$A = P(1+r)^n$

$=$ ☐ $\times$ (1 + ☐) ☐

$A =$ ☐

7 \$566 at 6% p.a., for 4 years, compounded annually.

$A = P(1+r)^n$

$=$ ☐ $\times$ (☐ + ☐) ☐

$A =$ ☐

8 \$275 at 7% p.a., for 3 years, compounded annually.

$A = P(1+r)^n$

$=$ ☐ $\times$ (☐ + ☐) ☐

$A =$ ☐

9 \$19 233 at 5.7% p.a., for 6 years, compounded annually.

$A = P(1+r)^n$

$=$

$A =$

10 \$7 055 at 4.9% p.a., for 5 years, compounded annually.

11 \$1 020 at 8.1% p.a., for 2 years, compounded annually.

12 \$385.20 at 11.3% p.a., for 7 years, compounded annually.

If the rate and compound period and term match, the process is the same. Find A.

13 \$15 000 at 0.7% per month, for 9 months, compounded monthly.

$A = P(1+r)^n$

$P = 15\,000$, $r = 0.7 \div 100$, $n = 9$

$=$ ☐ $\times$ (☐ + ☐) ☐

Divide R by 100

$A =$ ☐

14 \$6 120 at 0.12% per week, for 28 weeks, compounded weekly.

15 \$337 at 0.09% per week, for 47 weeks, compounded weekly.

16 \$706 at 0.3% per month, for 8 months, compounded monthly.

17 \$90 050 at 0.11% per week, for 4 weeks, compounded weekly.

Investing Money - Compound Interest - Calculating the Interest (Sheet 2)

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Calculate the Account balance, then subtract the principal to find the Interest earned.

1 \$2 676 at 5% p.a., for 3 years, compounded annually.

$A = P(1 + r)^n$

$= \square \times (\square + \square)^{\square}$

$A =$

$I =$ (A) - (P)

$I =$

2 \$17 905 at 3.8% p.a., for 5 years, compounded annually.

$A = P(1 + r)^n$

$= \square \times (\square + \square)^{\square}$

$A =$

$I =$ -

$I =$

3 \$6 650 at 8.8% p.a., for 9 years, compounded annually.

$A = P(1 + r)^n$

$A =$

$I =$

$I =$

4 \$930 at 6.2% p.a., for 5 years, compounded annually.

$A =$

$A =$

Continued in the next column

$I =$

$I =$

5 \$31 000 at 0.5% per month for 15 months, compounded monthly.

6 \$76 000 at 0.17% per week for 9 weeks, compounded weekly.

7 \$270 000 at 0.3% per month for 8 months, compounded monthly.

8 \$4 706 at 0.13% per week for 42 weeks, compounded weekly.

9 Complete the table. All the investments are compounded annually.

A = principal invested × (1 + rate as decimal)^{term}

Principal Invested	*Annual Rate*	*Term (years)*	*Account Balance*	*Interest Earned*
\$1 200	6%	2		
\$440	5.5%	7		
\$2 120	7.2%	4		
\$90 300	3.9%	3		
\$4 777	12%	5		
\$610.90	6.6%	3		
\$15 820	12%	4		
\$90 000	10.5%	2		

Investing Money - Compound Interest - Account Balance (Sheet 1)

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Use your calculator to evaluate these to 2 d.p.

1 $450 \times (1+0.04)^4$

$= 526.44$ (Round to 2 decimal places)

Calc: 4 5 0 × (1 + 0 • 0 4) $x^{\blacksquare}$ 4 =

2 $3\,277 \times (1+0.11)^5$

$= 5\,521.94$

3 $940.30 \times (1+0.055)^7$

$= 1\,367.83$

4 $7\,500 \times (1+0.09)^2$

$= 8\,910.75$

Use the compound interest formula to find the account balance for these investments.

Example: Find the account balance:

A \$650 investment, at 6% p.a., for 5 years, compounded annually.

$A = P(1+r)^n$

$= 650 \times (1+0.06)^5$

$A = \$869.85$

5 \$1 450 at 8% p.a., for 3 years, compounded annually.

$A = P(1+r)^n$

$= 1\,450 \times (1 + 0.08)^3$ (Divide R by 100)

$A = \$1\,826.58$

6 \$880 at 4% p.a., for 7 years, compounded annually.

$A = P(1+r)^n$

$= 880 \times (1 + 0.04)^7$

$A = \$1\,158.02$

7 \$566 at 6% p.a., for 4 years, compounded annually.

$A = P(1+r)^n$

$= 566 \times (1 + 0.06)^4$

$A = \$714.56$

8 \$275 at 7% p.a., for 3 years, compounded annually.

$A = P(1+r)^n$

$= 275 \times (1 + 0.07)^3$

$A = \$336.89$

9 \$19 233 at 5.7% p.a., for 6 years, compounded annually.

$A = P(1+r)^n$

$= 19\,233 \times (1 + 0.057)^6$

$A = \$26\,822.36$

10 \$7 055 at 4.9% p.a., for 5 years, compounded annually.

$A = P(1+r)^n$

$= 7\,055 \times (1 + 0.049)^5$

$A = \$8\,961.37$

11 \$1 020 at 8.1% p.a., for 2 years, compounded annually.

$A = P(1+r)^n$

$= 1\,020 \times (1 + 0.081)^2$

$A = \$1\,191.93$

12 \$385.20 at 11.3% p.a., for 7 years, compounded annually.

$A = P(1+r)^n$

$= 385.20 \times (1 + 0.113)^7$

$A = \$814.99$

If the rate and compound period and term match, the process is the same. Find A.

13 \$15 000 at 0.7% per month, for 9 months, compounded monthly.

$A = P(1+r)^n$

$P = 15\,000$, $r = 0.7 \div 100$, $n = 9$

$= 15\,000 \times (1 + 0.007)^9$ (Divide R by 100)

$A = \$15\,971.90$

14 \$6 120 at 0.12% per week, for 28 weeks, compounded weekly.

$A = P(1+r)^n$

$= 6\,120 \times (1 + 0.0012)^{28}$

$A = \$6\,329$

15 \$337 at 0.09% per week, for 47 weeks, compounded weekly.

$A = P(1+r)^n$

$= 337 \times (1 + 0.0009)^{47}$

$A = \$351.55$

16 \$706 at 0.3% per month, for 8 months, compounded monthly.

$A = P(1+r)^n$

$= 706 \times (1 + 0.003)^8$

$A = \$723.12$

17 \$90 050 at 0.11% per week, for 4 weeks, compounded weekly.

$A = P(1+r)^n$

$= 90\,050 \times (1 + 0.0011)^4$

$A = \$90\,446.87$

Investing Money - Compound Interest - Calculating the Interest (Sheet 2)

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Calculate the Account balance, then subtract the principal to find the Interest earned.

1 $2 676 at 5% p.a., for 3 years, compounded annually.

$A = P(1 + r)^n$

$= 2\,676 \times (1 + 0.05)^3$

$A =$ \$3 097.80

$I = 3\,097.80 - 2\,676$

$I =$ \$421.80

2 $17 905 at 3.8% p.a., for 5 years, compounded annually.

$A = P(1 + r)^n$

$= 17\,905 \times (1 + 0.038)^5$

$A =$ \$21 575.51

$I = 21\,575.51 - 17\,905$

$I =$ \$3 670.51

3 $6 650 at 8.8% p.a., for 9 years, compounded annually.

$A = P(1 + r)^n$

$= 6\,650 \times (1 + 0.088)^9$

$A =$ \$14 206.32

$I = 14\,206.32 - 6\,650$

$I =$ \$7 556.32

4 $930 at 6.2% p.a., for 5 years, compounded annually.

$A = P(1 + r)^n$

$= 930 \times (1 + 0.062)^5$

$A =$ \$1 256.34

Continued in the next column

$I = 1\,256.34 - 930$

$I =$ \$326.34

5 $31 000 at 0.5% per month for 15 months, compounded monthly.

$A = P(1 + r)^n$

$= 31\,000 \times (1 + 0.005)^{15}$

$A =$ \$33 408.16

$I = 33\,408.16 - 31\,000$

$I =$ \$2 408.16

6 $76 000 at 0.17% per week for 9 weeks, compounded weekly.

$A = P(1 + r)^n$

$= 76\,000 \times (1 + 0.0017)^9$

$A =$ \$77 170.74

$I = 77\,170.74 - 76\,000$

$I =$ \$1 170.74

7 $270 000 at 0.3% per month for 8 months, compounded monthly.

$A = P(1 + r)^n$

$= 270\,000 \times (1 + 0.003)^8$

$A =$ \$276 548.45

$I = 276\,548.45 - 270\,000$

$I =$ \$6 548.45

8 $4 706 at 0.13% per week for 42 weeks, compounded weekly.

$A = P(1 + r)^n$

$= 4\,706 \times (1 + 0.0013)^{42}$

$A =$ \$4 969.92

$I = 4\,969.92 - 4\,706$

$I =$ \$263.92

9 Complete the table. All the investments are compounded annually.

A = principal invested × (1 + rate as decimal)^{term}

Principal Invested	*Annual Rate*	*Term (years)*	*Account Balance*	*Interest Earned*
\$1 200	6%	2	\$1 348.32	\$148.32
\$440	5.5%	7	\$640.06	\$200.06
\$2 120	7.2%	4	\$2 799.72	\$679.72
\$90 300	3.9%	3	\$101 282.50	\$10 982.50
\$4 777	12%	5	\$8 418.71	\$3 641.71
\$610.90	6.6%	3	\$740.02	\$129.12
\$15 820	12%	4	\$24 893.08	\$9 073.08
\$90 000	10.5%	2	\$109 892.25	\$19 892.25

Investing Money - Compound Interest (Homework Strip)

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Date Due:

John has invested $7 500 at 5% p.a., compounded annually for 6 years.

1 Convert 5% to a decimal

$r =$ ÷ 100 =

2 Find the Account balance.

$A = P(1 + r)^n$

= × (+) □

$A =$

3 Calculate the interest earned

$I =$ -

$I =$

Bella has invested $1 450 at 7% p.a., compounded annually for 3 years. Calculate the interest earned.

4 $A = P(1 + r)^n$

= × (+) □

$A =$

$I =$ -

$I =$

Date Due:

John has invested $7 500 at 5% p.a., compounded annually for 6 years.

1 Convert 5% to a decimal

$r =$ ÷ 100 =

2 Find the Account balance.

$A = P(1 + r)^n$

= × (+) □

$A =$

3 Calculate the interest earned

$I =$ -

$I =$

Bella has invested $1 450 at 7% p.a., compounded annually for 3 years. Calculate the interest earned.

4 $A = P(1 + r)^n$

= × (+) □

$A =$

$I =$ -

$I =$

Date Due:

John has invested $7 500 at 5% p.a., compounded annually for 6 years.

1 Convert 5% to a decimal

$r =$ ÷ 100 =

2 Find the Account balance.

$A = P(1 + r)^n$

= × (+) □

$A =$

3 Calculate the interest earned

$I =$ -

$I =$

Bella has invested $1 450 at 7% p.a., compounded annually for 3 years. Calculate the interest earned.

4 $A = P(1 + r)^n$

= × (+) □

$A =$

$I =$ -

$I =$

Date Due:

John has invested $7 500 at 5% p.a., compounded annually for 6 years.

1 Convert 5% to a decimal

$r =$ ÷ 100 =

2 Find the Account balance.

$A = P(1 + r)^n$

= × (+) □

$A =$

3 Calculate the interest earned

$I =$ -

$I =$

Bella has invested $1 450 at 7% p.a., compounded annually for 3 years. Calculate the interest earned.

4 $A = P(1 + r)^n$

= × (+) □

$A =$

$I =$ -

$I =$

Investing Money - Compound Interest (Homework Strip)

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Date Due:

John has invested $7 500 at 5% p.a., compounded annually for 6 years.

1 Convert 5% to a decimal

$r =$ 5 $\div 100 =$ 0.05

2 Find the Account balance.

$A = P(1 + r)^n$

$=$ 7 500 $\times ($ 1 $+$ 0.05 $)^{6}$

$A =$ $10 050.72

3 Calculate the interest earned

$I =$ 10 050.72 - 7 500

$I =$ $2 550.72

Bella has invested $1 450 at 7% p.a., compounded annually for 3 years. Calculate the interest earned.

4 $A = P(1 + r)^n$

$=$ 1 450 $\times ($ 1 $+$ 0.07 $)^{3}$

$A =$ $1 776.31

$I =$ 1 776.31 - 1 450

$I =$ $326.31

Investing Money - Compound Interest Comparisons

Compound Interest is different from simple interest in two key ways:

- Interest is paid during the investment, rather than at the end of the investment only.
- Because interest is paid during the investment and re-invested, the account balance increases and so more interest is paid.

This worksheet compares simple interest to compound interest, but also compound interest against itself, only with more compound periods. The result is that the more compound periods you have, the more interest you earn. Because the interest you earn is re-invested more regularly so the opportunity to earn 'interest on your interest' is greater.

Compound Interest Formula: $A = P(1 + r)^n$

Where:
A = Account balance
P = Principal
r = Interest rate expressed as a decimal
n = Number of compound periods

Interest Calculation: $I = A - P$

Where:
I = Interest earned
A = Account balance
P = Principal

Example: Find the interest earned on an investment of \$9 000 at 6% p.a., for 3 years, compounded annually.

$A = P(1 + r)^n$ ← Compound Interest Formula

$= 9\,000 \times (1 + 0.06)^3$ ← Substitute the values, remember to use the interest rate r as a decimal

$A = \$10\,719.14$ ← Round to 2 decimal places

$I = 10\,719.14 - 9\,000$ ← Subtract 'P' from 'A' to find 'I'

$I = \$1\,719.14$

Investing Money - Compound Interest Comparisons

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Compare the two compound interest investments, which have the same annual rate.

Ben invests $3 000 at 7.2% p.a., for 3 years, compounded annually.

1 Convert 7.2% to a decimal

$r =$ ☐ $\div 100 =$ ☐

2 Find the account balance

$A = P(1 + r)^n$

$= $ ☐ $\times ($ ☐ $+$ ☐ $)^{\square}$

$A =$ ☐ '$' sign and to 2 d.p.

Eve invests $3 000 at 0.6% per month, for 36 months, compounded monthly.

3 Convert 0.6% to a decimal

$r =$ ☐ $\div 100 =$ ☐

4 Find the account balance

$A = P(1 + r)^n$

$= $ ☐ $\times ($ ☐ $+$ ☐ $)^{\square}$

$A =$ ☐

Calculate the difference in account balances. Who earns the most?

5 Account balance difference

$=$ ☐ $-$ ☐

Largest A Smallest A

$=$ ☐ ○ Ben earns the most ○ Eve earns the most

If 3 years = 36 months and 0.6% per month = 7.2% p.a. Explain why there is a difference between Ben and Eve's account balance.

6

This time compare a compound investment to a simple interest investment. Find the interest.

Rahj invests $900 at 8.2% p.a., for 5 years, compounded annually.

7 Find the interest earned

$A = P(1 + r)^n$

$=$

$A =$

$I =$ ☐ $-$ ☐

$I =$ ☐

Justin invests $900 at 8.2% p.a., for 5 years, in a simple interest investment

8 $I = Prn$

$= $ ☐ $\times$ ☐ $\times$ ☐

$I =$ ☐

Calculate the difference in interest earned. Who earns the most?

9 Difference in interest earned

$=$ ☐ $-$ ☐

$=$ ☐ ○ Rahj earns more ○ Justin earns more

Both investments above are for 5 years. Explain why one investment accrues more interest than the other.

10

Ryan can choose from one of these three, 5 year investments:
Option1: Simple interest: 9% p.a.,
Option 2: Compound interest: 9% p.a. compounding annually.
Option 3: Compound interest: 0.75% per month, compounding monthly for 60 months.
He has $4 000 to invest.

11 Find the interest earned if he chooses Option 1.

$I = Prn$

$=$

$I =$

12 Find the interest earned if he chooses Option 2.

$A = P(1 + r)^n$

$=$

$A =$

$I =$

$I =$

13 Find the interest earned if he chooses Option 3.

$A = P(1 + r)^n$

$=$

$A =$

$I =$

$I =$

Rank the options with a '1' for best performer to '3' for worst performer.

14 Option 1: 'Simple' ☐

Option 2: '5 periods' ☐

Option 3: '60 periods' ☐

Investing Money - Compound Interest Comparisons

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Compare the two compound interest investments, which have the same annual rate.

Ben invests $3 000 at 7.2% p.a., for 3 years, compounded annually.

1 Convert 7.2% to a decimal

$r =$ 7.2 $\div 100 =$ 0.072

2 Find the account balance

$A = P(1 + r)^n$

$=$ 3 000 $\times (1 +$ 0.072 $)^{3}$

$A =$ $3 695.78 '$' sign and to 2 d.p.

Eve invests $3 000 at 0.6% per month, for 36 months, compounded monthly.

3 Convert 0.6% to a decimal

$r =$ 0.6 $\div 100 =$ 0.006

4 Find the account balance

$A = P(1 + r)^n$

$=$ 3 000 $\times (1 +$ 0.006 $)^{36}$

$A =$ $3 720.90

Calculate the difference in account balances. Who earns the most?

5 Account balance difference

$=$ 3 720.90 - 3 695.78

Largest A Smallest A

$=$ $25.12 ○ Ben earns the most ● Eve earns the most

If 3 years = 36 months and 0.6% per month = 7.2% p.a. Explain why there is a difference between Ben and Eve's account balance.

6 Ben has 3 compound periods

Eve has 36 compound periods.

More compound periods leads to larger interest amount earned.

This time compare a compound investment to a simple interest investment. Find the interest.

Rahj invests $900 at 8.2% p.a., for 5 years, compounded annually.

7 Find the interest earned

$A = P(1 + r)^n$

$= 900 (1 + 0.082)^5$

$A =$ $1 334.69

$I =$ 1 334.69 - 900

$I =$ $434.69

Justin invests $900 at 8.2% p.a., for 5 years, in a simple interest investment

8 $I = Prn$

$=$ 900 $\times$ 0.082 $\times$ 5

$I =$ $369

Calculate the difference in interest earned. Who earns the most?

9 Difference in interest earned

$=$ 434.69 - 369

$=$ $65.69 ● Rahj earns more ○ Justin earns more

Both investments above are for 5 years. Explain why one investment accrues more interest than the other.

10 Rahj earns compound interest, so he earns 'interest on his interest'. So he earns more than Justin, who just gets the interest at the end of the investment.

Ryan can choose from one of these three, 5 year investments:
Option1: Simple interest: 9% p.a.,
Option 2: Compound interest: 9% p.a. compounding annually.
Option 3: Compound interest: 0.75% per month, compounding monthly for 60 months.
He has $4 000 to invest.

11 Find the interest earned if he chooses Option 1.

$I = Prn$

$= 4\,000 \times 0.09 \times 5$

$I =$ $1 800

12 Find the interest earned if he chooses Option 2.

$A = P(1 + r)^n$

$= 4\,000 \times (1 + 0.09)^5$

$A =$ $6 154.50

$I =$ 6 154.50 - 4 000

$I =$ $2 154.50

13 Find the interest earned if he chooses Option 3.

$A = P(1 + r)^n$

$= 4\,000 \times (1 + 0.0075)^{60}$

$A =$ $6 262.72

$I =$ 6 262.72 - 4 000

$I =$ $2 262.72

Rank the options with a '1' for best performer to '3' for worst performer.

14 Option 1: 'Simple' 1

Option 2: '5 periods' 2

Option 3: '60 periods' 3

Investing Money - Compound Interest Comparisons (Homework Strip)

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Date Due: _____

Peta can choose from 2 investments:
Option 1: Simple interest investment, $8 000, at 6% p.a. for 5 years. OR
Option 2: Compound int. investment, $8 000, at 6% p.a. for 5 years (Compounded annually). Find:

1 Convert 6% to a decimal

$r =$ ____ $\div 100 =$ ____

2 Find the interest (Option 1)

$I = Prn$

$=$ ____ $\times$ ____ $\times$ ____

$I =$ ____ '$' sign

3 Find the interest (Option 2)

$A = P(1 + r)^n$

$=$ ____ $\times (1 +$ ____ $)^{\square}$

$A =$ ____ '$' sign and to 2 d.p.

$I =$ ____ $(A) -$ ____ (P)

$I =$ ____

4 Rank the options '1' is best

Option 1 ____ Option 2 ____

Date Due: _____

Peta can choose from 2 investments:
Option 1: Simple interest investment, $8 000, at 6% p.a. for 5 years. OR
Option 2: Compound int. investment, $8 000, at 6% p.a. for 5 years (Compounded annually). Find:

1 Convert 6% to a decimal

$r =$ ____ $\div 100 =$ ____

2 Find the interest (Option 1)

$I = Prn$

$=$ ____ $\times$ ____ $\times$ ____

$I =$ ____ '$' sign

3 Find the interest (Option 2)

$A = P(1 + r)^n$

$=$ ____ $\times (1 +$ ____ $)^{\square}$

$A =$ ____ '$' sign and to 2 d.p.

$I =$ ____ $(A) -$ ____ (P)

$I =$ ____

4 Rank the options '1' is best

Option 1 ____ Option 2 ____

Date Due: _____

Peta can choose from 2 investments:
Option 1: Simple interest investment, $8 000, at 6% p.a. for 5 years. OR
Option 2: Compound int. investment, $8 000, at 6% p.a. for 5 years (Compounded annually). Find:

1 Convert 6% to a decimal

$r =$ ____ $\div 100 =$ ____

2 Find the interest (Option 1)

$I = Prn$

$=$ ____ $\times$ ____ $\times$ ____

$I =$ ____ '$' sign

3 Find the interest (Option 2)

$A = P(1 + r)^n$

$=$ ____ $\times (1 +$ ____ $)^{\square}$

$A =$ ____ '$' sign and to 2 d.p.

$I =$ ____ $(A) -$ ____ (P)

$I =$ ____

4 Rank the options '1' is best

Option 1 ____ Option 2 ____

Date Due: _____

Peta can choose from 2 investments:
Option 1: Simple interest investment, $8 000, at 6% p.a. for 5 years. OR
Option 2: Compound int. investment, $8 000, at 6% p.a. for 5 years (Compounded annually). Find:

1 Convert 6% to a decimal

$r =$ ____ $\div 100 =$ ____

2 Find the interest (Option 1)

$I = Prn$

$=$ ____ $\times$ ____ $\times$ ____

$I =$ ____ '$' sign

3 Find the interest (Option 2)

$A = P(1 + r)^n$

$=$ ____ $\times (1 +$ ____ $)^{\square}$

$A =$ ____ '$' sign and to 2 d.p.

$I =$ ____ $(A) -$ ____ (P)

$I =$ ____

4 Rank the options '1' is best

Option 1 ____ Option 2 ____

Investing Money - Compound Interest Comparisons (Homework Strip)

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Date Due:

Peta can choose from 2 investments:
Option 1: Simple interest investment, $8 000, at 6% p.a. for 5 years. OR
Option 2: Compound int. investment, $8 000, at 6% p.a. for 5 years (Compounded annually). Find:

1 Convert 6% to a decimal

$r =$ 6 $\div 100 =$ 0.06

2 Find the interest (Option 1)

$I = Prn$

$=$ 8 000 $\times$ 0.06 $\times$ 5

$I =$ $2 400 '$' sign

3 Find the interest (Option 2)

$A = P(1 + r)^n$

$=$ 8 000 $\times ($ 1 $+$ 0.06 $)^{5}$

$A =$ $10 705.80 '$' sign and to 2 d.p.

$I =$ 10 705.80 - 8 000

$I =$ $2 705.80

4 Rank the options '1' is best

Option 1 2 Option 2 1

Investing Money - Graphing and Comparing Interest

These worksheets compare simple interest to compound interest, by graphing interest earned. When you perform normal interest calculations, the questions are individual and don't form a pattern. But the questions on these sheets are grouped in patterns to create answers that show a relationship.

Simple interest calculations result is a 'straight-line relationship'. So when the values are plotted on a graph you can draw a straight line through them. You will see the relationship between your answers forming a pattern and the nature of the graph.

Compound interest questions result in curves. The curve is only slight, but you will notice, especially as the interest rate and period increases that the curve becomes more pronounced.

The process of these sheets is to find the interest earned for 1 year, 2 years and 3 years. Then plot the points on a graph. All amounts are for $1 000 being invested.

INTEREST RATE* 10% *p.a.

P=\$1 000, R = 10%, n = 1

$I = Prn$

$= 1\,000 \times 0.1 \times 1$

I = \$100

First point uses n = 1

P=\$1 000, R = 10%, n = 2

$I = Prn$

$= 1\,000 \times 0.1 \times 2$

I = \$200

Second point uses n = 2

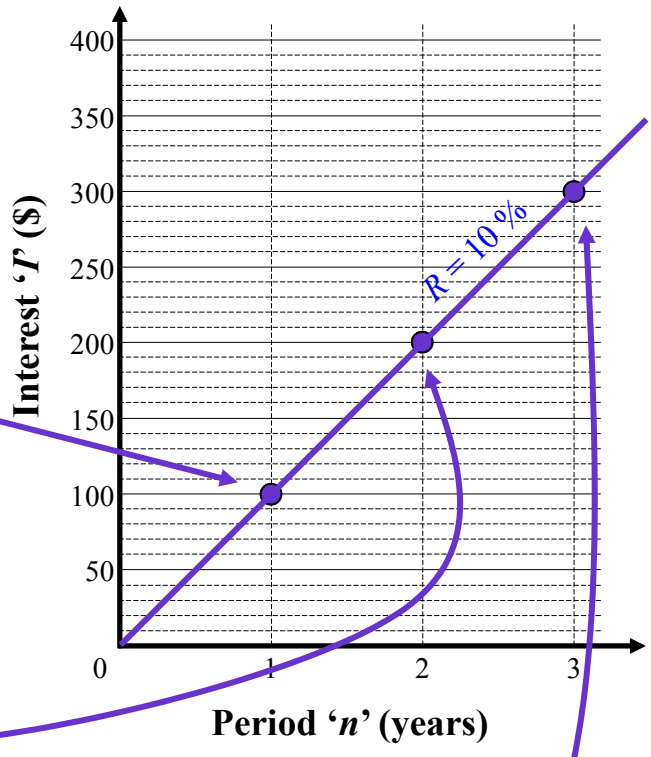
P=\$1 000, R = 10%, n = 3

$I = Prn$

$= 1\,000 \times 0.1 \times 3$

I = \$300

Third point uses n = 3



Investing Money - Graphing and Comparing Simple Interest (Sheet 1)

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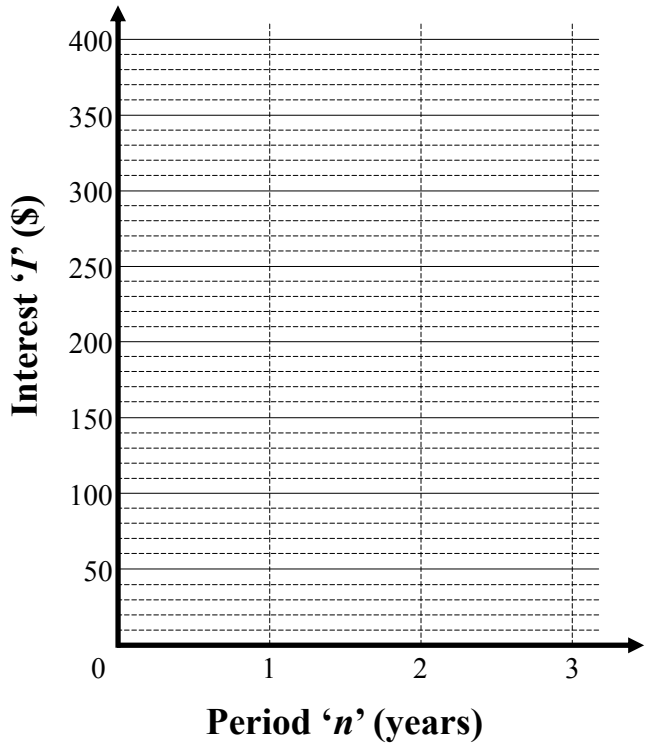
Calculate the interest earned on an investment of \$1 000. Use periods of 1 year, 2 years and 3 years. Use interest rates of 4% p.a., 8% p.a. and 12% p.a. Interest earned at time $n = 0$ is \$0. Calculate first, then graph. Plot the points in pencil first, then use a ruler as well as different coloured pens. If the lines aren't straight …. *PANIC!*



INTEREST RATE 4% p.a.

1 *Use* $P = 1\,000$, $r = 0.04$ *and* 'n' $= 1$.

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

2 *Use* 'n' $= 2$.

$I = Prn$

3 *Use* 'n' $= 3$.

$I = Prn$

Plot the 4% line (own colour). Write '$R = 4\%$' on the line.

INTEREST RATE 8% p.a.

4 $I = Prn$

$= \square \times \square \times \square$

$I = \square$

5 *Use* 'n' $= 2$.

6 *Use* 'n' $= 3$.

Plot the 8% line (own colour). Write '$R = 8\%$' on the line.

INTEREST RATE 12% p.a.

7 $I = Prn$

$= \square \times \square \times \square$

$I = \square$

8 *Use* 'n' $= 2$.

9 *Use* 'n' $= 3$.

Plot the 12% line (own colour). Write '$R = 12\%$' on the line.

Use the graph to find these. Each vertical jump in interest is worth \$10. So count the jumps, then × 10!

10 The difference in interest earned after 3 years, between $R = 4\%$ p.a. and $R = 12\%$ p.a.

$= \square - \square$

I at $n=3$, $R=12\%$ | I at $n=3$, $R=4\%$

$= \square$

11 The difference in interest earned after 2 years, between $R = 8\%$ p.a. and $R = 12\%$ p.a.

$= \square - \square = \square$

12 The difference in interest earned after 1 year, between $R = 4\%$ p.a. and $R = 8\%$ p.a.

$= \square - \square = \square$

13 The difference in interest earned after 2 years, between $R = 4\%$ p.a. and $R = 12\%$ p.a.

$= \square - \square$

$= \square$

Investing Money - Graphing and Comparing Compound Interest (Sheet 2)

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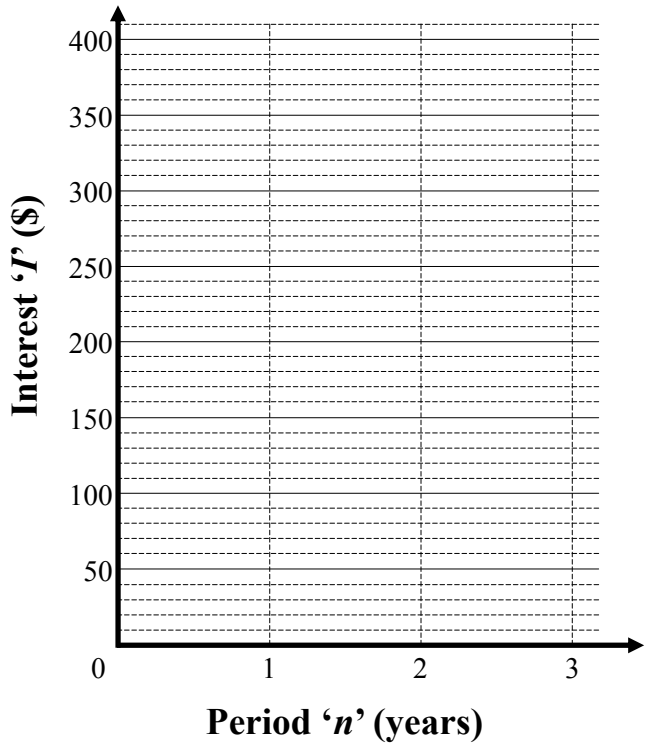
Calculate the compound interest earned on an investment of \$1 000. Use periods of 1 year, 2 years and 3 years. Use interest rates of 4% p.a., 8% p.a. and 12% p.a. Interest earned at time $n = 0$ is \$0. Calculate first, then graph. Plot the points in pencil first, then sketch curves using different coloured pens. Round '*A*' to the nearest \$10.



INTEREST RATE 4% p.a. Compounded Annually

1 *Use* $P = 1\,000$, $r = 0.04$ *and* 'n' $= 1$.

$A = P(1 + r)^n$

$= \square \times (1 + \square)^{\square}$

$A = \square$

$I = \square - 1\,000$

$I = \square$

2 *Use* 'n' $= 2$.

$A = P(1 + r)^n$

$A =$

Round A to nearest \$10

$I =$

$I =$

3 *Use* 'n' $= 3$.

$A = P(1 + r)^n$

$A =$

Round A to nearest \$10

$I =$

$I =$

Plot the 4% curve (own colour). Write '$R = 4\%$' on the curve.

INTEREST RATE 8% p.a. Compounded Annually

Use 'n' $= 1$.

4 $A = P(1 + r)^n$

$= \square \times (1 + \square)^{\square}$

$A = \square$

$I = \square - 1\,000$

$I = \square$

5 *Use* 'n' $= 2$.

$A =$

$A =$

Round A to nearest \$10

$I =$

$I =$

6 *Use* 'n' $= 3$.

Round A to nearest \$10

Plot the 8% curve (own colour). Write '$R = 8\%$' on the curve.

INTEREST RATE 12% p.a. Compounded Annually

7 *Use* 'n' $= 1$.

$A =$

$I =$

$I =$

$A =$

8 *Use* 'n' $= 2$.

9 *Use* 'n' $= 3$.

Plot the 12% curve (own colour). Write '$R = 12\%$' on the curve.

Investing Money - Graphing and Comparing Simple Interest (Sheet 1)

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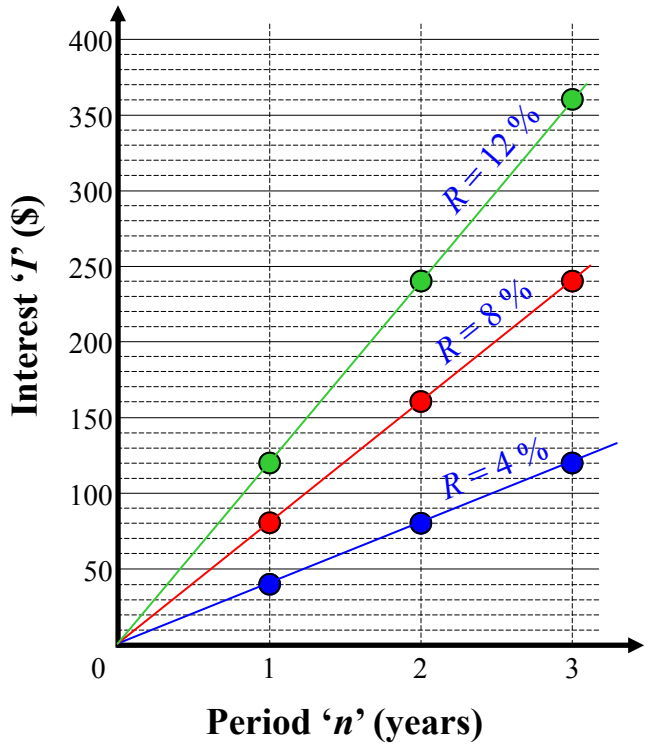
Calculate the interest earned on an investment of \$1 000. Use periods of 1 year, 2 years and 3 years. Use interest rates of 4% p.a., 8% p.a. and 12% p.a. Interest earned at time $n = 0$ is \$0. Calculate first, then graph. Plot the points in pencil first, then use a ruler as well as different coloured pens. If the lines aren't straight …. *PANIC!*



INTEREST RATE 4% p.a.

1 *Use* $P = 1\,000$, $r = 0.04$ *and* 'n' $= 1$.

$I = Prn$

$= 1\,000 \times 0.04 \times 1$

$I =$ \$40

2 *Use* 'n' $= 2$.

$I = Prn$

$= 1\,000 \times 0.04 \times 2$

$I =$ \$80

3 *Use* 'n' $= 3$.

$I = Prn$

$= 1\,000 \times 0.04 \times 3$

$I =$ \$120

Plot the 4% line (own colour). Write '$R = 4\%$' on the line.

INTEREST RATE 8% p.a.

4 $I = Prn$

$= 1\,000 \times 0.08 \times 1$

$I =$ \$80

5 *Use* 'n' $= 2$.

$I = Prn$

$= 1\,000 \times 0.08 \times 2$

$I =$ \$160

6 *Use* 'n' $= 3$.

$I = Prn$

$= 1\,000 \times 0.08 \times 3$

$I =$ \$240

Plot the 8% line (own colour). Write '$R = 8\%$' on the line.

INTEREST RATE 12% p.a.

7 $I = Prn$

$= 1\,000 \times 0.12 \times 1$

$I =$ \$120

8 *Use* 'n' $= 2$.

$I = Prn$

$= 1\,000 \times 0.12 \times 2$

$I =$ \$240

9 *Use* 'n' $= 3$.

$I = Prn$

$= 1\,000 \times 0.12 \times 3$

$I =$ \$360

Plot the 12% line (own colour). Write '$R = 12\%$' on the line.

Use the graph to find these. Each vertical jump in interest is worth \$10. So count the jumps, then × 10!

10 The difference in interest earned after 3 years, between $R = 4\%$ p.a. and $R = 12\%$ p.a.

$= 360 - 120$

(I at $n=3$, $R=12\%$; I at $n=3$, $R=4\%$)

$=$ \$240

11 The difference in interest earned after 2 years, between $R = 8\%$ p.a. and $R = 12\%$ p.a.

$= 240 - 160 =$ \$80

12 The difference in interest earned after 1 year, between $R = 4\%$ p.a. and $R = 8\%$ p.a.

$= 80 - 40 =$ \$40

13 The difference in interest earned after 2 years, between $R = 4\%$ p.a. and $R = 12\%$ p.a.

$= 240 - 80$

$=$ \$160

Investing Money - Graphing and Comparing Compound Interest (Sheet 2)

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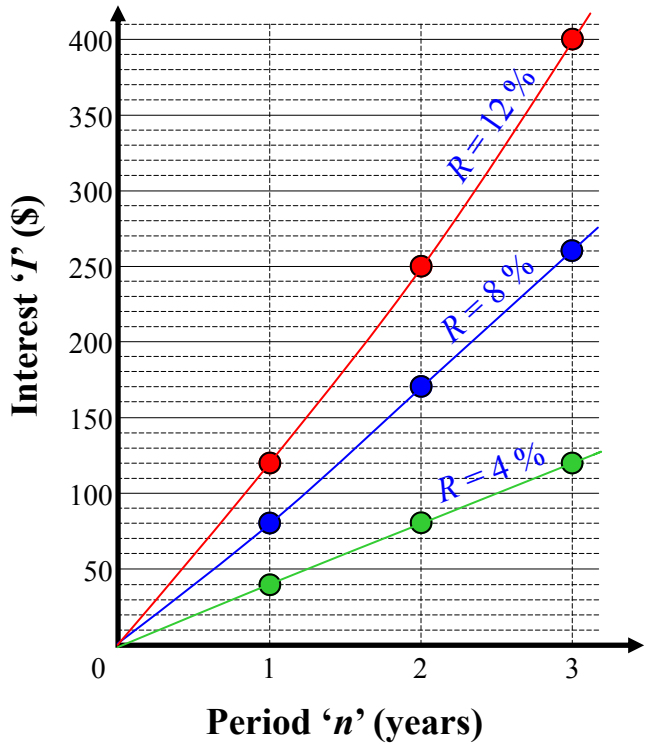
Calculate the compound interest earned on an investment of \$1 000. Use periods of 1 year, 2 years and 3 years. Use interest rates of 4% p.a., 8% p.a. and 12% p.a. Interest earned at time $n = 0$ is \$0. Calculate first, then graph. Plot the points in pencil first, then sketch curves using different coloured pens. Round '*A*' to the nearest \$10.



INTEREST RATE 4% p.a. Compounded Annually

1 *Use* $P = 1\,000$, $r = 0.04$ *and* 'n' $= 1$.

$A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.04)^1$

$A =$ \$1 040

$I =$ 1 040 - 1 000

$I =$ \$40

2 *Use* 'n' $= 2$.

$A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.04)^2$

$A =$ \$1 080

Round A to nearest \$10

$I =$ 1 080 - 1 000

$I =$ \$80

3 *Use* 'n' $= 3$.

$A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.04)^3$

$A =$ \$1 120

Round A to nearest \$10

$I =$ 1 120 - 1 000

$I =$ \$120

Plot the 4% curve (own colour). Write '$R = 4\%$' on the curve.

INTEREST RATE 8% p.a. Compounded Annually

Use 'n' $= 1$.

4 $A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.08)^1$

$A =$ \$1 080

$I =$ 1 080 - 1 000

$I =$ \$80

5 *Use* 'n' $= 2$.

$A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.08)^2$

$A =$ \$1 170

Round A to nearest \$10

$I =$ 1 170 - 1 000

$I =$ \$170

6 *Use* 'n' $= 3$.

$A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.08)^3$

$A =$ \$1 260

Round A to nearest \$10

$I =$ 1 260 - 1 000

$I =$ \$260

Plot the 8% curve (own colour). Write '$R = 8\%$' on the curve.

INTEREST RATE 12% p.a. Compounded Annually

7 *Use* 'n' $= 1$.

$A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.12)^1$

$A =$ \$1 120

$I =$ 1 120 - 1 000

$I =$ \$120

8 *Use* 'n' $= 2$.

$A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.12)^2$

$A =$ \$1 250

$I =$ 1 250 - 1 000

$I =$ \$250

9 *Use* 'n' $= 3$.

$A = P(1 + r)^n$

$= 1\,000 \times (1 + 0.12)^3$

$A =$ \$1 400

$I =$ 1 400 - 1 000

$I =$ \$400

Plot the 12% curve (own colour). Write '$R = 12\%$' on the curve.

Investing Money - Graphing Simple Interest (Homework Strip)

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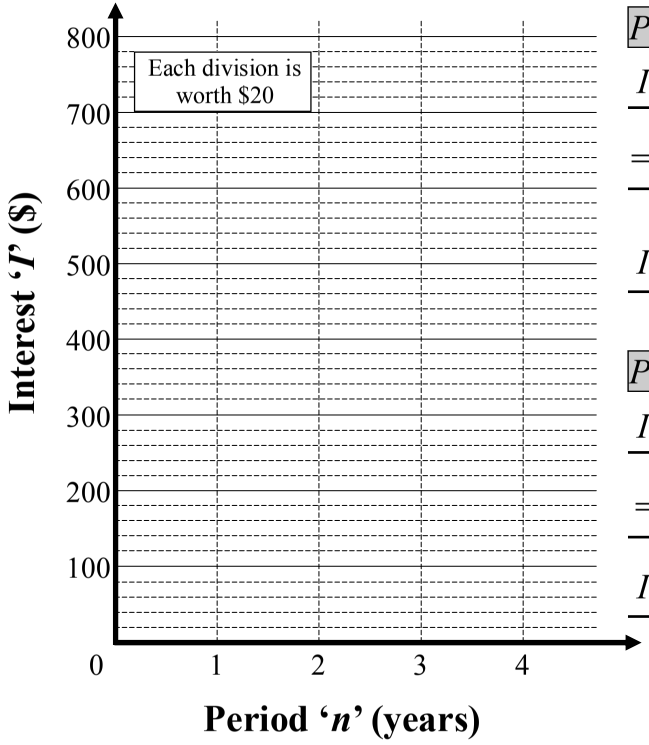
Date Due: _____

Calculate the simple interest earned on a $1 000 investment at 16% p.a. and plot the values on the graph. **Note that every vertical division on the graph is $20.**



$P = \$1\,000$, $R = 16\%$, $n = 1$

$I = Prn$

$= \square \times \square.\square\square \times \square$

P=1 000 | r as decimal | $n = 1$

$I = \$ \square$

$P = \$1\,000$, $R = 16\%$, $n = 2$

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

$P = \$1\,000$, $R = 16\%$, $n = 3$

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

$P = \$1\,000$, $R = 16\%$, $n = 4$

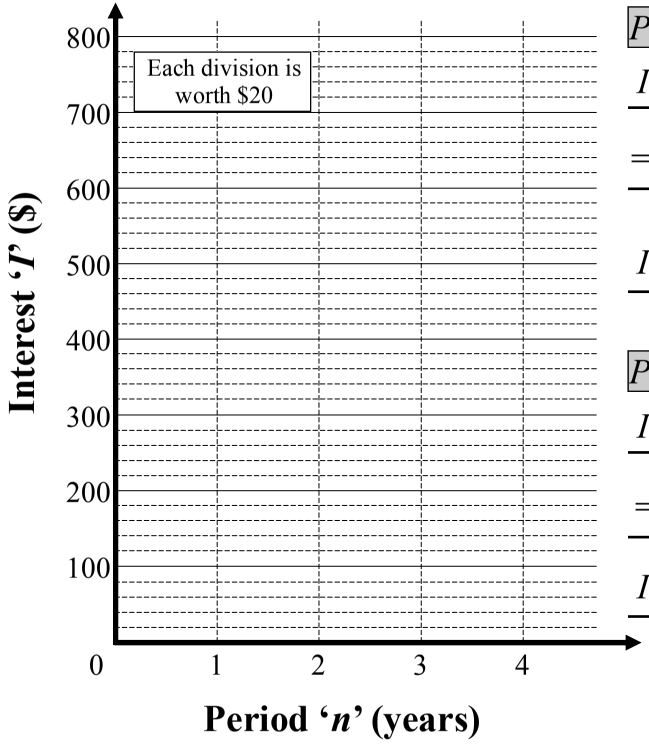
$I = Prn$

$= \square \times \square \times \square$

$I = \square$

Date Due: _____

Calculate the simple interest earned on a $1 000 investment at 16% p.a. and plot the values on the graph. **Note that every vertical division on the graph is $20.**



$P = \$1\,000$, $R = 16\%$, $n = 1$

$I = Prn$

$= \square \times \square.\square\square \times \square$

P=1 000 | r as decimal | $n = 1$

$I = \$ \square$

$P = \$1\,000$, $R = 16\%$, $n = 2$

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

$P = \$1\,000$, $R = 16\%$, $n = 3$

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

$P = \$1\,000$, $R = 16\%$, $n = 4$

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

Investing Money - Graphing Simple Interest (Homework Strip)

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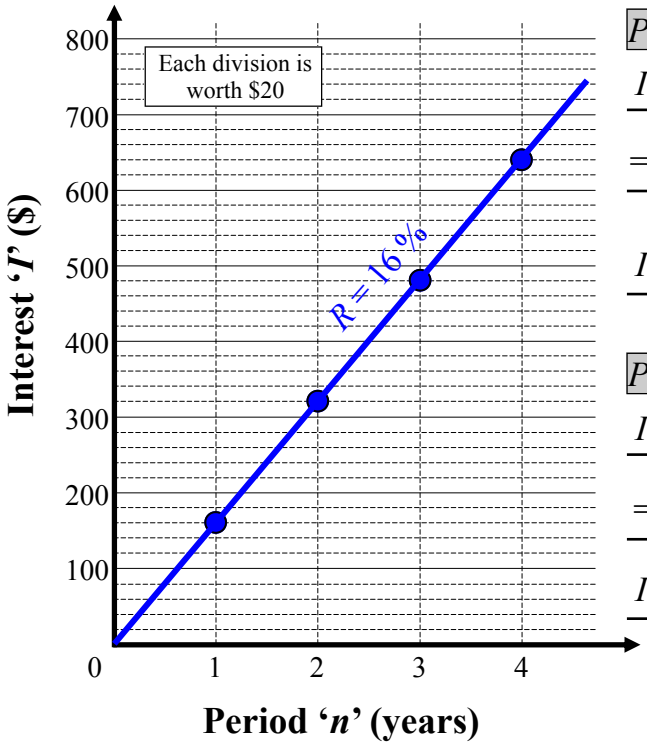
Date Due:

Calculate the simple interest earned on a \$1 000 investment at 16% p.a. and plot the values on the graph. **Note that every vertical division on the graph is \$20.**



P = \$1 000, R=16%, n = 1

$I = Prn$

= 1 000 × 0.16 × 1

P=1 000 | r as decimal | n = 1

I = \$160

P = \$1 000, R=16%, n = 2

$I = Prn$

= 1 000 × 0.16 × 2

I = \$320

P = \$1 000, R=16%, n = 3

$I = Prn$

= 1 000 × 0.16 × 3

I = \$480

P = \$1 000, R=16%, n = 4

$I = Prn$

= 1 000 × 0.16 × 4

I = \$640

Investing Money - Interest Calculation Using Tables

You don't often have a table for calculating interest. But if you work in a bank, you want access to a fast method of calculating interest. Tables allow this, they also allow people who don't know how to calculate interest …… to calculate interest!

The process is to first find the 'multiplier'. This is the amount earned on a $1 investment. Of course people don't invest $1, but once you have this number, you multiply by the amount invested. Here is an example.

Example $4 200 at 8% (simple interest) for 6 years

First go across to the 8%

Then down to 6 years

Periods (n)	Simple Interest Earned ($) on a $1 Investment									
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$	$R = 8\ \%$	$R = 9\ \%$	$R = 10\ \%$
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
2	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.30
4	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40
5	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
6	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60
7	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70
8	0.08	0.16	0.24	0.32	0.40	0.48	0.56	0.64	0.72	0.80
9	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.90
10	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

The multiplier from the table is 0.48

$I = 4\ 200 \times 0.48 \quad = \$2\ 016$ ← Multiply the amount invested by the multiplier

If you want to calculate the account balance

$A = 4\ 200 + 2\ 016$ ← Add the amount invested to the interest earned

$A = \$6\ 216$

The first worksheet is simple interest, the second is compound interest. You will require rounding with the compound interest sheet.

Investing Money - Interest Calculation Using Tables (Sheet 1)

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The table below is for the simple interest investment of $1. Use the table to answer the questions.

Periods (n)	Simple Interest Earned ($) on a $1 Investment									
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$	$R = 8\ \%$	$R = 9\ \%$	$R = 10\ \%$
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
2	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.30
4	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40
5	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
6	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60
7	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70
8	0.08	0.16	0.24	0.32	0.40	0.48	0.56	0.64	0.72	0.80
9	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.90
10	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

Use the table to find the interest earned on these investments.

1 $1 at 4% p.a. for 6 years

$I =$ $

2 $650 at 4% p.a. for 6 years

$I =$

Multiply your Q.1 answer × 650

3 $1 at 7% p.a. for 3 years

$I =$

4 $8 000 at 7% p.a. for 3 years

$I =$

Multiply your Q.3 answer × 8 000

5 $1 at 2% p.a. for 9 years

$I =$

6 $1 250 at 2% p.a. for 9 years

$I =$

7 $1 at 9% p.a. for 4 years

$I =$

8 $97.63 at 9% p.a. for 4 years

$I =$

9 $1 at 5% p.a. for 7 years

$I =$

10 $202.30 at 5% p.a. for 7 yrs

$I =$

Find the interest earned, then calculate the account balance. $A = P + I$

11 $495 at 3% p.a. for 7 yrs

$I =$

$A = 495 +$

$A =$

12 $1 050 at 8% p.a. for 4 yrs

$I =$

$A =$ +

$A =$

13 $305.20 at 6% p.a. for 4 yrs

$I =$

$A =$ +

$A =$

14 $290 000 at 7% p.a. for 4 yrs

$I =$

$A =$

$A =$

You can use the table the same way for other periods

15 $1 at 2% per month, for 7 months.

$I =$

16 $4 300 at 2% per month, for 7 months.

$I =$

$A =$

$A =$

17 $19 676 at 1% per month, for 4 months.

$I =$

$A =$

$A =$

Investing Money - Interest Calculation Using Tables (Sheet 2)

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The table below is for the compound interest investment of $1. Use the table to answer the questions.

Periods (n)	Compound Interest Earned ($) on a $1 Investment									
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$	$R = 8\ \%$	$R = 9\ \%$	$R = 10\ \%$
1	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900	0.1000
2	0.0201	0.0404	0.0609	0.0816	0.1025	0.1236	0.1449	0.1664	0.1881	0.2100
3	0.0303	0.0612	0.0927	0.1249	0.1576	0.1910	0.2250	0.2597	0.2950	0.3310
4	0.0406	0.0824	0.1255	0.1699	0.2155	0.2625	0.3108	0.3605	0.4116	0.4641
5	0.0510	0.1041	0.1593	0.2167	0.2763	0.3382	0.4026	0.4693	0.5386	0.6105
6	0.0615	0.1262	0.1941	0.2653	0.3401	0.4185	0.5007	0.5869	0.6771	0.7716
7	0.0721	0.1487	0.2299	0.3159	0.4071	0.5036	0.6058	0.7138	0.8280	0.9487
8	0.0829	0.1717	0.2668	0.3686	0.4775	0.5938	0.7182	0.8509	0.9926	1.1436
9	0.0937	0.1951	0.3048	0.4233	0.5513	0.6895	0.8385	0.9990	1.1719	1.3579
10	0.1046	0.2190	0.3439	0.4802	0.6289	0.7908	0.9672	1.1589	1.3674	1.5937

Use the table to find the interest earned on these investments. These will require rounding.

1 $1 at 3% p.a. for 8 years

$I =$ $ Round to 2 d.p.

2 $500 at 3% p.a. for 8 years

$I =$

Use unrounded Q.1 answer × 500

3 $1 at 8% p.a. for 5 years

$I =$

4 $920 at 8% p.a. for 5 years

$I =$

Use unrounded Q.3 answer × 920

5 $1 at 6% p.a. for 10 years

$I =$

6 $205 at 6% p.a. for 10 years

$I =$

7 $1 at 2% p.a. for 7 years

$I =$

8 $140.20 at 2% p.a. for 7 yrs

$I =$

You can use the table the same way for other periods

9 $8 150 at 1% per month, for 3 months.

$I =$

10 $66 117 at 2% per month, for 10 months.

$I =$

11 $57.83 at 3% per month, for 6 months.

$I =$

$A = P + I$

Now find the account balance by adding I and P, to get A!

12 $3 088 at 5% p.a. for 4 yrs

$I =$

$A =$ +

$A =$

13 $119.25 at 7% p.a. for 8 yrs

$I =$

$A =$ +

$A =$

14 $771 at 10% p.a. for 3 yrs

$I =$

$A =$

$A =$

15 $340 200 at 4% p.a. for 4 yrs

$I =$

$A =$

$A =$

16 $805.40 at 9% p.a. for 10 yrs

$I =$

$A =$

$A =$

17 $7 111 at 6% p.a. for 2 yrs

$I =$

$A =$

$A =$

Investing Money - Interest Calculation Using Tables (Sheet 1)

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The table below is for the simple interest investment of $1. Use the table to answer the questions.

Periods (n)	Simple Interest Earned ($) on a $1 Investment									
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$	$R = 8\ \%$	$R = 9\ \%$	$R = 10\ \%$
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
2	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.30
4	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40
5	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
6	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60
7	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70
8	0.08	0.16	0.24	0.32	0.40	0.48	0.56	0.64	0.72	0.80
9	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.90
10	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

Use the table to find the interest earned on these investments.

1 $1 at 4% p.a. for 6 years

$I =$ $0.24

2 $650 at 4% p.a. for 6 years

$I =$ $156

Multiply your Q.1 answer × 650

3 $1 at 7% p.a. for 3 years

$I =$ $0.21

4 $8 000 at 7% p.a. for 3 years

$I =$ $1 680

Multiply your Q.3 answer × 8 000

5 $1 at 2% p.a. for 9 years

$I =$ $0.18

6 $1 250 at 2% p.a. for 9 years

$I =$ $225

7 $1 at 9% p.a. for 4 years

$I =$ $0.36

8 $97.63 at 9% p.a. for 4 years

$I =$ $35.15

9 $1 at 5% p.a. for 7 years

$I =$ $0.35

10 $202.30 at 5% p.a. for 7 yrs

$I =$ $70.81

Find the interest earned, then calculate the account balance. $A = P + I$

11 $495 at 3% p.a. for 7 yrs

$I =$ $103.95

$A =$ 495 + 103.95

$A =$ $598.95

12 $1 050 at 8% p.a. for 4 yrs

$I =$ $336

$A =$ 1 050 + 336

$A =$ $1 386

13 $305.20 at 6% p.a. for 4 yrs

$I =$ $73.25

$A =$ 305.20 + 73.25

$A =$ $378.45

14 $290 000 at 7% p.a. for 4 yrs

$I =$ $81 200

$A =$ 290 000 + 81 200

$A =$ $371 200

You can use the table the same way for other periods

15 $1 at 2% per month, for 7 months.

$I =$ $0.14

16 $4 300 at 2% per month, for 7 months.

$I =$ $602

$A =$ 4 300 + 602

$A =$ $4 902

17 $19 676 at 1% per month, for 4 months.

$I =$ $787.04

$A =$ 19 676 + 787.04

$A =$ $20 463.04

Investing Money - Interest Calculation Using Tables (Sheet 2)

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The table below is for the compound interest investment of $1. Use the table to answer the questions.

Periods (n)	Compound Interest Earned ($) on a $1 Investment									
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$	$R = 8\ \%$	$R = 9\ \%$	$R = 10\ \%$
1	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900	0.1000
2	0.0201	0.0404	0.0609	0.0816	0.1025	0.1236	0.1449	0.1664	0.1881	0.2100
3	0.0303	0.0612	0.0927	0.1249	0.1576	0.1910	0.2250	0.2597	0.2950	0.3310
4	0.0406	0.0824	0.1255	0.1699	0.2155	0.2625	0.3108	0.3605	0.4116	0.4641
5	0.0510	0.1041	0.1593	0.2167	0.2763	0.3382	0.4026	0.4693	0.5386	0.6105
6	0.0615	0.1262	0.1941	0.2653	0.3401	0.4185	0.5007	0.5869	0.6771	0.7716
7	0.0721	0.1487	0.2299	0.3159	0.4071	0.5036	0.6058	0.7138	0.8280	0.9487
8	0.0829	0.1717	0.2668	0.3686	0.4775	0.5938	0.7182	0.8509	0.9926	1.1436
9	0.0937	0.1951	0.3048	0.4233	0.5513	0.6895	0.8385	0.9990	1.1719	1.3579
10	0.1046	0.2190	0.3439	0.4802	0.6289	0.7908	0.9672	1.1589	1.3674	1.5937

Use the table to find the interest earned on these investments. These will require rounding.

1 $1 at 3% p.a. for 8 years

$I =$ $0.27 Round to 2 d.p.

2 $500 at 3% p.a. for 8 years

$I =$ $133.40

Use unrounded Q.1 answer × 500

3 $1 at 8% p.a. for 5 years

$I =$ $0.47

4 $920 at 8% p.a. for 5 years

$I =$ $431.76

Use unrounded Q.3 answer × 920

5 $1 at 6% p.a. for 10 years

$I =$ $0.79

6 $205 at 6% p.a. for 10 years

$I =$ $162.11

7 $1 at 2% p.a. for 7 years

$I =$ $0.15

8 $140.20 at 2% p.a. for 7 yrs

$I =$ $20.85

You can use the table the same way for other periods

9 $8 150 at 1% per month, for 3 months.

$I =$ $246.95

10 $66 117 at 2% per month, for 10 months.

$I =$ $14 479.62

11 $57.83 at 3% per month, for 6 months.

$I =$ $11.22

$A = P + I$

Now find the account balance by adding I and P, to get A!

12 $3 088 at 5% p.a. for 4 yrs

$I =$ $665.46

$A =$ 3 088 + 665.46

$A =$ $3 753.46

13 $119.25 at 7% p.a. for 8 yrs

$I =$ $85.65

$A =$ 119.25 + 85.65

$A =$ $204.90

14 $771 at 10% p.a. for 3 yrs

$I =$ $255.20

$A =$ 771 + 255.20

$A =$ $1 026.20

15 $340 200 at 4% p.a. for 4 yrs

$I =$ $57 799.98

$A =$ 340 200 + 57 799.98

$A =$ $397 999.98

16 $805.40 at 9% p.a. for 10 yrs

$I =$ $1 101.30

$A =$ 805.40 + 1 101.30

$A =$ $1 906.70

17 $7 111 at 6% p.a. for 2 yrs

$I =$ $878.92

$A =$ 7 111 + 878.92

$A =$ $7 989.92

Investing Money - Interest Calculation Using Tables (Homework Strip)

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Date Due: _____

Periods (n)	Simple Interest Earned ($) on a $1 Investment						
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07
2	0.02	0.04	0.06	0.08	0.10	0.12	0.14
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21
4	0.04	0.08	0.12	0.16	0.20	0.24	0.28
5	0.05	0.10	0.15	0.20	0.25	0.30	0.35
6	0.06	0.12	0.18	0.24	0.30	0.36	0.42

Find the simple interest earned on these $1 investments, using the table above.

1 $1, 4% p.a., 3 years

$I = \$ \square.\square\square$

2 $1, 6% p.a., 5 years

$I = \$ \square.\square\square$

3 $1, 3% p.a., 2 years

$I = \$ \square.\square\square$

Calculate the simple interest earned on these investments, using the table above.

4 $2 000, at 7% p.a., for 5 years

$I =$ _____

5 $1 130, at 5% p.a., for 4 years

$I =$ _____

6 $165.20, at 2% p.a., for 4 years

$I =$ _____

7 $5 200, at 3% p.a., for 2 years

$I =$ _____

8 $55, at 6% p.a., for 3 years

$I =$ _____

9 $10 500, at 4% p.a., for 1 year

$I =$ _____

Add the interest to the amount invested to find the account balance for these.

10 $500, at 4% p.a., for 5 years

$I =$ _____

$A =$ _____ $\$500 + I$

11 $1 877, at 7% p.a., for 6 years

$I =$ _____

$A =$ _____

12 $605.30, at 2% p.a., for 6 years

$I =$ _____

$A =$ _____

Date Due: _____

Periods (n)	Simple Interest Earned ($) on a $1 Investment						
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07
2	0.02	0.04	0.06	0.08	0.10	0.12	0.14
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21
4	0.04	0.08	0.12	0.16	0.20	0.24	0.28
5	0.05	0.10	0.15	0.20	0.25	0.30	0.35
6	0.06	0.12	0.18	0.24	0.30	0.36	0.42

Find the simple interest earned on these $1 investments, using the table above.

1 $1, 4% p.a., 3 years

$I = \$ \square.\square\square$

2 $1, 6% p.a., 5 years

$I = \$ \square.\square\square$

3 $1, 3% p.a., 2 years

$I = \$ \square.\square\square$

Calculate the simple interest earned on these investments, using the table above.

4 $2 000, at 7% p.a., for 5 years

$I =$ _____

5 $1 130, at 5% p.a., for 4 years

$I =$ _____

6 $165.20, at 2% p.a., for 4 years

$I =$ _____

7 $5 200, at 3% p.a., for 2 years

$I =$ _____

8 $55, at 6% p.a., for 3 years

$I =$ _____

9 $10 500, at 4% p.a., for 1 year

$I =$ _____

Add the interest to the amount invested to find the account balance for these.

10 $500, at 4% p.a., for 5 years

$I =$ _____

$A =$ _____ $\$500 + I$

11 $1 877, at 7% p.a., for 6 years

$I =$ _____

$A =$ _____

12 $605.30, at 2% p.a., for 6 years

$I =$ _____

$A =$ _____

Investing Money - Interest Calculation Using Tables (Homework Strip)

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Date Due: _____

Periods (n)	Simple Interest Earned ($) on a $1 Investment						
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07
2	0.02	0.04	0.06	0.08	0.10	0.12	0.14
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21
4	0.04	0.08	0.12	0.16	0.20	0.24	0.28
5	0.05	0.10	0.15	0.20	0.25	0.30	0.35
6	0.06	0.12	0.18	0.24	0.30	0.36	0.42

Find the simple interest earned on these $1 investments, using the table above.

1 $1, 4% p.a., 3 years

$I = \$ 0.12$

2 $1, 6% p.a., 5 years

$I = \$ 0.30$

3 $1, 3% p.a., 2 years

$I = \$ 0.06$

Calculate the simple interest earned on these investments, using the table above.

4 $2 000, at 7% p.a., for 5 years

$I =$ $700

5 $1 130, at 5% p.a., for 4 years

$I =$ $226

6 $165.20, at 2% p.a., for 4 years

$I =$ $13.22

7 $5 200, at 3% p.a., for 2 years

$I =$ $312

8 $55, at 6% p.a., for 3 years

$I =$ $9.90

9 $10 500, at 4% p.a., for 1 year

$I =$ $420

Add the interest to the amount invested to find the account balance for these.

10 $500, at 4% p.a., for 5 years

$I =$ $100

$A =$ $600 ($500 + I)

11 $1 877, at 7% p.a., for 6 years

$I =$ $788.34

$A =$ $2 665.34

12 $605.30, at 2% p.a., for 6 years

$I =$ $72.64

$A =$ $677.64

Investing Money - Flat Rate Loans

When you want to buy something and don't have enough money to get it, you have two options, either save up for it and hold off the purchase, or take out a loan and get it now. When you take out a loan you pay interest to the finance company. The money you borrowed plus the interest is then paid back in repayments, or installments. The period of these installments could be weekly, monthly, fortnightly etc. You also frequently pay a loan fee and a deposit.

Example:

Fred buys a swimming pool which sells for $14 250. He takes out a flat rate loan, with these conditions:
- $160 loan stamp duty
- $2 137.50 deposit
- $345/month repayments for 4 years

Find the total of the repayments

Total = 4 × 12 × 345 ← Repayments are monthly for 4 years so, 4 × 12

Total = $16 560

Find the full amount paid

Total = 160 + 2 137.50 + 16 560 ← Full amount is stamp duty + deposit + repayments

Total = $18 857.50

Find amount Fred would save if he paid cash instead of taking out a loan

Total = 18 857.50 - 14 250 ← This is found by 'Full Amount Paid' - Selling Price

Total = $4 607.50

Investing Money - Introduction to Flat Rate Loans (Sheet 1)

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Get a total for these repayments.

1 12 repayments of $89.20

Total = ☐ × ☐

Total = $ ☐

Cents to 2 d.p.

2 24 repayments of $117.25

Total = ☐ × ☐

Total = ☐

$ Sign

3 Repayments of $28.30/week for 5 years.

Total = ☐ × ☐ × ☐

There are 52 weeks in a year

Total = ☐

4 Repayments of $59.16/month for 4 years.

Total = ☐ × ☐ × ☐

Total = ☐

5 Repayments of $72.25/week for 3 years.

Total =

Total =

6 Repayments of $30.67/fortnight for 5 years.

There are 26 fortnights in a year

7 Repayments of $44.05/week for 6 years.

Calculate the amount paid for these products.

Leon buys a refrigerator which sells for $1 250. He takes out a flat rate loan, with these conditions:
- $45 loan stamp duty
- $350 deposit
- $36/month repayments for 4 years

8 Find the total of the repayments

Total = ☐ × ☐ × ☐

Total = $ ☐

9 Find the full amount paid

Total = ☐ + ☐ + ☐

Stamp Duty | Deposit | Question 1

Total = $ ☐

10 How much would Leon save if he paid cash, instead of the loan?

Amount Saved = ☐ - ☐

Question 9 | Cash Price

Amount Saved = $ ☐

Lim buys a lounge suite using a flat rate loan, with these conditions:
- $60 loan stamp duty
- $475 deposit
- $93.50/mth repayments for 5 years

The lounge retails for $3 900.

11 Find the total of the repayments

Total = ☐ × ☐ × ☐

Total = ☐

12 Find the full amount paid

Total = ☐ + ☐ + ☐

Total = ☐

13 How much would Lim save if he paid cash, instead of using a loan?

Amount Saved = ☐ - ☐

Amount Saved = ☐

Jess buys a new kitchen using a flat rate loan, with these conditions:
- $72 loan stamp duty
- $2 620 deposit
- $103.15/wk repayments for 3 years

The kitchen retails for $13 100.

14 Find the total of the repayments

Total =

Total =

15 Find the full amount paid

Total =

16 How much would Jess save if she paid cash, instead of using a loan?

Amount Saved =

Amount Saved =

Eliza buys a new fishing boat using a flat rate loan, with these conditions:
- $115 loan stamp duty
- $430 loan insurance
- $3 125 deposit
- $117.50/wk repayments for 5 years

The boat retails for $25 000.

17 Find the total of the repayments

18 Find the full amount paid

19 How much would Eliza save if she paid cash, rather than using a loan?

Investing Money - Introduction to Flat Rate Loans (Sheet 2)

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Calculate the size of these deposits, using your percentage of a quantity skills.



Example: 12% deposit on a $13 400 Jet-ski.

Deposit = 0.12 × 13 400

12% is 0.12 (decimal), 12 ÷ 100 on calculator

Deposit = $1 608

1 15% deposit on a $28 000 car.

Deposit = ☐ × ☐

Deposit = ☐ Show '$' sign

2 5% deposit on a $1 720 TV.

Deposit = 0.☐☐ × ☐

Deposit = ☐

3 8% deposit on a $3 050 cruise.

Deposit =

Deposit =

4 15% deposit on a $47 111 car.

5 20% deposit on a $860 hotel stay.

6 45% deposit on a $164 700 truck.

7 12.5% deposit on a $260 juicer.

Calculate the deposit, amount paid and the amount saved if cash was used for these loans.



Jonathan buys a $1 300 laptop using a flat rate loan, with these conditions:
- $36 loan stamp duty
- 15% deposit
- $11.80/week repayments for 3 years

8 Calculate the deposit paid.

9 Find the total of the repayments

10 Find the full amount paid

11 How much would Jonathan save if he paid cash, rather than use a loan?

Lucy buys a $4 150 wardrobe using a flat rate loan, with these conditions:
- $65 loan stamp duty
- 20% deposit
- $136.45/month repayments for 4 years

12 Calculate the deposit paid.

13 Find the total of the repayments

14 Find the full amount paid

15 How much would Lucy save if she paid cash, rather than use a loan?

Jasmine buys a car for $44 676 using a flat rate loan, with these conditions:
- $65 loan stamp duty
- $580 loan insurance
- 12% deposit
- $388.90/fortnight repayments for 5 years

16 Calculate the deposit paid.

17 Find the total of the repayments

18 Find the full amount paid

19 How much would Jasmine save if she paid cash, rather than use a loan?

Investing Money - Introduction to Flat Rate Loans (Sheet 1)

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Get a total for these repayments.

1 12 repayments of $89.20

Total = 12 × 89.20

Total = $1 070.40

Cents to 2 d.p.

2 24 repayments of $117.25

Total = 24 × 117.25

Total = $2 814

$ Sign

3 Repayments of $28.30/week for 5 years.

Total = 5 × 52 × 28.30

There are 52 weeks in a year

Total = $7 358

4 Repayments of $59.16/month for 4 years.

Total = 4 × 12 × 59.16

Total = $2 839.68

5 Repayments of $72.25/week for 3 years.

Total = 3 × 52 × 72.25

Total = $11 271

6 Repayments of $30.67/fortnight for 5 years.

Total = 5 × 26 × 30.67

There are 26 fortnights in a year

Total = $3 987.10

7 Repayments of $44.05/week for 6 years.

Total = 6 × 52 × 44.05

Total = $13 743.60

Calculate the amount paid for these products.

Leon buys a refrigerator which sells for $1 250. He takes out a flat rate loan, with these conditions:
- $45 loan stamp duty
- $350 deposit
- $36/month repayments for 4 years

8 Find the total of the repayments

Total = 4 × 12 × 36

Total = $1 728

9 Find the full amount paid

Total = 45 + 350 + 1 728

Stamp Duty | Deposit | Question 1

Total = $2 123

10 How much would Leon save if he paid cash, instead of the loan?

Amount Saved = 2 123 - 1 250

Question 9 | Cash Price

Amount Saved = $873

Lim buys a lounge suite using a flat rate loan, with these conditions:
- $60 loan stamp duty
- $475 deposit
- $93.50/mth repayments for 5 years

The lounge retails for $3 900.

11 Find the total of the repayments

Total = 5 × 12 × 93.50

Total = $5 610

12 Find the full amount paid

Total = 60 + 475 + 5 610

Total = $6 145

13 How much would Lim save if he paid cash, instead of using a loan?

Amount Saved = 6 145 - 3 900

Amount Saved = $2 245

Jess buys a new kitchen using a flat rate loan, with these conditions:
- $72 loan stamp duty
- $2 620 deposit
- $103.15/wk repayments for 3 years

The kitchen retails for $13 100.

14 Find the total of the repayments

Total = 3 × 52 × 103.15

Total = $16 091.40

15 Find the full amount paid

Total = 72 + 2 620 + 16 091.40

Total = $18 783.40

16 How much would Jess save if she paid cash, instead of using a loan?

Amount Saved = 18 783.40 - 13 100

Amount Saved = $5 683.40

Eliza buys a new fishing boat using a flat rate loan, with these conditions:
- $115 loan stamp duty
- $430 loan insurance
- $3 125 deposit
- $117.50/wk repayments for 5 years

The boat retails for $25 000.

17 Find the total of the repayments

Total = 5 × 52 × 117.50

Total = $30 550

18 Find the full amount paid

Total = 115 + 430 + 3 125

+ 30 550

Total = $34 220

19 How much would Eliza save if she paid cash, rather than using a loan?

Amount Saved = 34 220 - 25 000

Amount Saved = $9 220

Investing Money - Introduction to Flat Rate Loans (Sheet 2)

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Calculate the size of these deposits, using your percentage of a quantity skills.



Example: 12% deposit on a $13 400 Jet-ski.

Deposit = 0.12 × 13 400

12% is 0.12 (decimal), 12 ÷ 100 on calculator

Deposit = $1 608

1 15% deposit on a $28 000 car.

Deposit = 0.15 × 28 000

Deposit = $4 200 Show '$' sign

2 5% deposit on a $1 720 TV.

Deposit = 0.05 × 1 720

Deposit = $86

3 8% deposit on a $3 050 cruise.

Deposit = 0.08 × 3 050

Deposit = $244

4 15% deposit on a $47 111 car.

Deposit = 0.15 × 47 111

Deposit = $7 066.65

5 20% deposit on a $860 hotel stay.

Deposit = 0.2 × 860

Deposit = $172

6 45% deposit on a $164 700 truck.

Deposit = 0.45 × 164 700

Deposit = $32 940

7 12.5% deposit on a $260 juicer.

Deposit = 0.125 × 260

Deposit = $32.50

Calculate the deposit, amount paid and the amount saved if cash was used for these loans.



Jonathan buys a $1 300 laptop using a flat rate loan, with these conditions:
- $36 loan stamp duty
- 15% deposit
- $11.80/week repayments for 3 years

8 Calculate the deposit paid.

Deposit = 0.15 × 1 300

Deposit = $195

9 Find the total of the repayments

Total = 3 × 52 × 11.80

Total = $1 840.80

10 Find the full amount paid

Total = 36 + 195 + 1 840.80

Total = $2 071.80

11 How much would Jonathan save if he paid cash, rather than use a loan?

Amount Saved = 2 071.80 - 1 300

Amount Saved = $771.80

Lucy buys a $4 150 wardrobe using a flat rate loan, with these conditions:
- $65 loan stamp duty
- 20% deposit
- $136.45/month repayments for 4 years

12 Calculate the deposit paid.

Deposit = 0.2 × 4 150

Deposit = $830

13 Find the total of the repayments

Total = 4 × 12 × 136.45

Total = $6 549.60

14 Find the full amount paid

Total = 65 + 830 + 6 549.60

Total = $7 444.60

15 How much would Lucy save if she paid cash, rather than use a loan?

Amount Saved = 7 444.60 - 4 150

Amount Saved = $3 294.60

Jasmine buys a car for $44 676 using a flat rate loan, with these conditions:
- $65 loan stamp duty
- $580 loan insurance
- 12% deposit
- $388.90/fortnight repayments for 5 years

16 Calculate the deposit paid.

Deposit = 0.12 × 44 676

Deposit = $5 361.12

17 Find the total of the repayments

Total = 5 × 26 × 388.90

Total = $50 557

18 Find the full amount paid

Total = 65 + 580 + 5 361.20 + 50 557

Total = $56 563.20

19 How much would Jasmine save if she paid cash, rather than use a loan?

Amount Saved = 56 563.20 - 44 676

Amount Saved = $11 887.20

Investing Money - Introduction to Flat Rate Loans (Homework Strip)

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Date Due:

Ashley buys a new TV using a flat rate loan, with these conditions:
- $35 loan stamp duty
- $175 deposit
- $23.10/mth repayments for 3 years

The TV retails for $650.

1 Find the total of the repayments

Total = ____ × ____ × ____

(No. of years) (Months in a year) (Repayment)

Total = $ ____

2 Find the full amount paid

Total = ____ + ____ + ____

(Stamp Duty) (Deposit) (Question 1)

Total = $ ____

3 How much would Ashley save if he paid cash, instead of the loan?

Amount Saved = ____ - ____

(Question 2) (Cash Price)

Amount Saved = $ ____

Date Due:

Ashley buys a new TV using a flat rate loan, with these conditions:
- $35 loan stamp duty
- $175 deposit
- $23.10/mth repayments for 3 years

The TV retails for $650.

1 Find the total of the repayments

Total = ____ × ____ × ____

(No. of years) (Months in a year) (Repayment)

Total = $ ____

2 Find the full amount paid

Total = ____ + ____ + ____

(Stamp Duty) (Deposit) (Question 1)

Total = $ ____

3 How much would Ashley save if he paid cash, instead of the loan?

Amount Saved = ____ - ____

(Question 2) (Cash Price)

Amount Saved = $ ____

Date Due:

Ashley buys a new TV using a flat rate loan, with these conditions:
- $35 loan stamp duty
- $175 deposit
- $23.10/mth repayments for 3 years

The TV retails for $650.

1 Find the total of the repayments

Total = ____ × ____ × ____

(No. of years) (Months in a year) (Repayment)

Total = $ ____

2 Find the full amount paid

Total = ____ + ____ + ____

(Stamp Duty) (Deposit) (Question 1)

Total = $ ____

3 How much would Ashley save if he paid cash, instead of the loan?

Amount Saved = ____ - ____

(Question 2) (Cash Price)

Amount Saved = $ ____

Date Due:

Ashley buys a new TV using a flat rate loan, with these conditions:
- $35 loan stamp duty
- $175 deposit
- $23.10/mth repayments for 3 years

The TV retails for $650.

1 Find the total of the repayments

Total = ____ × ____ × ____

(No. of years) (Months in a year) (Repayment)

Total = $ ____

2 Find the full amount paid

Total = ____ + ____ + ____

(Stamp Duty) (Deposit) (Question 1)

Total = $ ____

3 How much would Ashley save if he paid cash, instead of the loan?

Amount Saved = ____ - ____

(Question 2) (Cash Price)

Amount Saved = $ ____

Investing Money - Introduction to Flat Rate Loans (Homework Strip)

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Date Due:

Ashley buys a new TV using a flat rate loan, with these conditions:
- $35 loan stamp duty
- $175 deposit
- $23.10/mth repayments for 3 years

The TV retails for $650.

1 Find the total of the repayments

Total = 3 × 12 × 23.10

(No. of years; Months in a year; Repayment)

Total = $831.60

2 Find the full amount paid

Total = 35 + 175 + 831.60

(Stamp Duty; Deposit; Question 1)

Total = $1 041.60

3 How much would Ashley save if he paid cash, instead of the loan?

Amount Saved = 1 041.60 - 650

(Question 2; Cash Price)

Amount Saved = $391.60

Investing Money - Flat Rate Loans

When you want to buy something and don't have enough money to get it, you have two options, either save up for it and hold off the purchase, or take out a loan and get it now. When you take out a loan you pay interest to the finance company. The money you borrowed plus the interest is then paid back in repayments, or installments. The period could be weekly, monthly, fortnightly etc. These worksheets deal with repaying Flat Rate Loans, without deposits.

Example: Fred buys a swimming pool which sells for $14 250. He takes out a flat rate loan at 7% p.a. over 5 years. He repays the loan in monthly payments. Calculate the monthly repayment.

Solution:

Find the interest payable

$I = Prn$

$= 14\,250 \times 0.07 \times 5$ ← Remember to convert the interest rate to a decimal

$I = \$4\,987.50$

Find the total amount payable

$A = 14\,250 + 4\,987.50$ ← Add the interest to the amount borrowed

$A = \$19\,237.50$ ← This is the amount that has to be paid back

Find the monthly payment

$\text{Payment} = \dfrac{19\,237.50}{5 \times 12}$ ← Divide the amount payable by the number of months in 5 years. So use: (5×12)

Payment = $320.63/month

Investing Money - Flat Rate Loans - Monthly Payments (Sheet 1)

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Convert these amounts to monthly repayments. Either use your fraction key or ÷/brackets.

Example: Convert $2 712 into monthly payments over 5 years.

Payment = $\frac{2\ 712}{5 \times 12}$ = $45.20/mth

Calc: 2 7 1 2 ▪ 5 × 1 2 = S⇔D

1 $1 306.80 into monthly over 3 yrs

Payment = $\frac{\quad}{\quad \times \quad}$

Payment = /month

$ sign + cents to 2 d.p.

2 $950.40 into monthly over 4 yrs

Payment = $\frac{\quad}{\quad \times \quad}$

Payment = /month

3 $3 822 into monthly over 5 yrs

Payment = $\frac{\quad}{\quad \times \quad}$

Payment = /month

The next 3 questions require rounding

4 $5 675 into monthly over 3 yrs

Payment =

Payment =

5 $1 658 into monthly over 6 yrs

Payment =

Payment =

6 $3 047.90 into monthly over 2 yrs

Calculate the interest payable, the total, then convert this to a monthly repayment amount.

$4 850 is borrowed for 3 years at 9% p.a., with monthly repayments.

7 Calculate the interest payable

$I = Prn$

$= \quad \times \quad \times$

Amount | 9% decimal | years

$I =$

8 Calculate the amount owing

$A =$ Amount borrowed + Interest (Q7)

$A =$

9 Convert to monthly payments

Payment = $\frac{\quad \leftarrow (Q8)}{\quad \times \quad \swarrow 3 \times 12}$

Payment = /month

$1 115 is borrowed for 2 years at 5% p.a., with monthly repayments.

10 Calculate the interest payable

$I = Prn$

$= \quad \times \quad \times$

Amount | 5% decimal | years

$I =$

11 Calculate the amount owing

$A = \quad +$

$A =$

12 Convert to monthly payments

Payment = $\frac{\quad}{\quad \times \quad}$

Payment =

A spa costs $7 780 and is bought using a flat rate loan for 4 years at 12% p.a. Calculate the equal monthly repayments over the 4 years.

13 Find the interest payable

$I = Prn$

$= \quad \times \quad \times$

$I =$

14 Calculate the amount owing

$A = \quad +$

$A =$

15 Convert to monthly payments

Payment = $\frac{\quad}{\quad \times \quad}$

Payment =

A lounge costs $2 225 and is bought using a flat rate loan for 3 years at 15% p.a. Calculate the equal monthly repayments over the 3 years.

16 Find the interest payable

$I = Prn$

$= \quad \times \quad \times$

$I =$

17 Calculate the amount owing

$A = \quad +$

$A =$

18 Convert to monthly payments

Payment = $\frac{\quad}{\quad \times \quad}$

Payment =

Investing Money - Flat Rate Loans - Weekly Payments (Sheet 2)

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Convert these amounts to weekly repayments. Either use your fraction key or ÷/brackets.

Example: Convert $3 198 into weekly payments over 3 years.

Payment = $\frac{3\ 198}{3 \times 52}$ = $20.50/week

Calc: 3 1 9 8 ▪□ 3 × 5 2 = S⇔D

1 $6 895.20 into weekly over 2 yrs

Payment = $\frac{\square}{\square \times \square}$

Payment = /week

$ sign + cents to 2 d.p.

2 $3 556.80 into weekly over 4 yrs

Payment = $\frac{\square}{\square \times \square}$

Payment = /week

3 $7 927.92 into weekly over 3 yrs

Payment = $\frac{\square}{\square \times \square}$

Payment = /week

The next 3 questions require rounding

4 $4 200 into weekly over 5 yrs

Payment =

Payment =

5 $5 389 into weekly over 4 yrs

Payment =

Payment =

6 $58 222 into weekly over 6 yrs

Calculate the interest payable, the total, then convert this to a weekly repayment amount.

$12 280 is borrowed for 5 years at 8% p.a., repaid weekly.

7 Calculate the interest payable

$I = Prn$

$= \square \times \square \times \square$

Amount | 8% decimal | years

$I = \square$

8 Calculate the amount owing

$A = \square + \square$

Amount borrowed | Interest (Q7)

$A = \square$

9 Convert to weekly payments

Payment = $\frac{\square \leftarrow (Q8)}{\square \times \square \swarrow 5\times52}$

Payment = /week

$37 612 is borrowed for 3 years at 12% p.a., repaid weekly.

10 Calculate the interest payable

$I = Prn$

$= \square \times \square \times \square$

Amount | 12% decimal | years

$I = \square$

11 Calculate the amount owing

$A = \square + \square$

$A = \square$

12 Convert to weekly payments

Payment = $\frac{\square}{\square \times \square}$

Payment =

A car costs $26 006 and is purchased using a flat rate loan for 4 years at 6% p.a. Calculate the equal weekly repayments over the 4 years.

13 Find the interest payable

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

14 Calculate the amount owing

$A = \square + \square$

$A = \square$

15 Convert to weekly payments

Payment = $\frac{\square}{\square \times \square}$

Payment =

A stove costs $1 960 and is bought using a flat rate loan for 2 years at 18% p.a. Calculate the equal weekly repayments over the 2 years.

16 Find the interest payable

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

17 Calculate the amount owing

$A = \square + \square$

$A = \square$

18 Convert to weekly payments

Payment = $\frac{\square}{\square \times \square}$

Payment =

Investing Money - Flat Rate Loans - Calculating Repayments (Sheet 3)

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Calculate the interest payable, the total, then convert this to the specified repayment period

$18 170 is borrowed for 4 years at 5% p.a., repaid in weekly payments

1 Calculate the interest payable

$I = Prn$

$= \square \times \square \times \square$

Amount | 5% decimal | years

$I = \square$

2 Calculate the amount owing

$A = \square + \square$

Amount borrowed | Interest (Q1)

$A = \square$

3 Convert to weekly payments

$$\text{Payment} = \frac{\square}{\square \times \square}$$

Payment = ____ /week

$40333 is borrowed for 7 years at 16% p.a., repaid in monthly payments

4 Calculate the interest payable

$I = Prn$

$=$

$I =$

5 Calculate the amount owing

$A =$

$A =$

6 Convert to monthly payments

Payment =

Payment =

A pool costs $34 449 and is purchased using a flat rate loan for 6 years at 14% p.a. Calculate the weekly repayments over the 6 years.

7 Calculate the interest payable

8 Calculate the amount owing

9 Find the weekly payments

A tractor costs $60 112 and is bought using a flat rate loan for 3 years at 9% p.a. Calculate the fortnightly repayments over the 3 years.

10 Calculate the interest payable

11 Calculate the amount owing

12 Find the fortnightly payment

A cherry-picker costs $184 290 and is purchased using a flat rate loan over 4 years at 8% p.a. Calculate the monthly repayments over the 4 years.

13 Calculate the interest payable

14 Calculate the amount owing

15 Find the monthly payments

A caravan costs $39 777 and is bought using a flat rate loan for 5 years at 11% p.a. Calculate the fortnightly repayments over the 5 years.

16 Calculate the interest payable

17 Calculate the amount owing

18 Find the fortnightly payment

Investing Money - Flat Rate Loans - Monthly Payments (Sheet 1)

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Convert these amounts to monthly repayments. Either use your fraction key or ÷/brackets.

Example: Convert $2 712 into monthly payments over 5 years.

Payment = $\frac{2\ 712}{5 \times 12}$ = $45.20/mth

Calc: 2 7 1 2 ▪/□ 5 × 1 2 = S⇔D

1 $1 306.80 into monthly over 3 yrs

Payment = $\frac{1\ 306.80}{3 \times 12}$

Payment = $36.30/month

$ sign + cents to 2 d.p.

2 $950.40 into monthly over 4 yrs

Payment = $\frac{950.40}{4 \times 12}$

Payment = $19.80/month

3 $3 822 into monthly over 5 yrs

Payment = $\frac{3\ 822}{5 \times 12}$

Payment = $63.70/month

The next 3 questions require rounding

4 $5 675 into monthly over 3 yrs

Payment = $\frac{5\ 675}{3 \times 12}$

Payment = $157.64/month

5 $1 658 into monthly over 6 yrs

Payment = $\frac{1\ 658}{6 \times 12}$

Payment = $23.03/month

6 $3 047.90 into monthly over 2 yrs

Payment = $\frac{3\ 047.90}{2 \times 12}$

Payment = $127/month

Calculate the interest payable, the total, then convert this to a monthly repayment amount.

$4 850 is borrowed for 3 years at 9% p.a., with monthly repayments.

7 Calculate the interest payable

$I = Prn$

$= 4\ 850 \times 0.09 \times 3$

Amount | 9% decimal | years

I = $1 309.50

8 Calculate the amount owing

$A = 4\ 850 + 1\ 309.50$

Amount borrowed | Interest (Q7)

A = $6 159.50

9 Convert to monthly payments

Payment = $\frac{6\ 159.50}{3 \times 12}$ ← (Q8), 3×12

Payment = $171.10/month

$1 115 is borrowed for 2 years at 5% p.a., with monthly repayments.

10 Calculate the interest payable

$I = Prn$

$= 1\ 115 \times 0.05 \times 2$

Amount | 5% decimal | years

I = $111.50

11 Calculate the amount owing

$A = 1\ 115 + 111.50$

A = $1 226.50

12 Convert to monthly payments

Payment = $\frac{1\ 226.50}{2 \times 12}$

Payment = $51.10/month

A spa costs $7 780 and is bought using a flat rate loan for 4 years at 12% p.a. Calculate the equal monthly repayments over the 4 years.

13 Find the interest payable

$I = Prn$

$= 7\ 780 \times 0.12 \times 4$

I = $3 734.40

14 Calculate the amount owing

$A = 7\ 780 + 3\ 734.40$

A = $11 514.40

15 Convert to monthly payments

Payment = $\frac{11\ 514.40}{4 \times 12}$

Payment = $239.88/month

A lounge costs $2 225 and is bought using a flat rate loan for 3 years at 15% p.a. Calculate the equal monthly repayments over the 3 years.

16 Find the interest payable

$I = Prn$

$= 2\ 225 \times 0.15 \times 3$

I = $1 001.25

17 Calculate the amount owing

$A = 2\ 225 + 1\ 001.25$

A = $3 226.25

18 Convert to monthly payments

Payment = $\frac{3\ 226.25}{3 \times 12}$

Payment = $89.62/month

Investing Money - Flat Rate Loans - Weekly Payments (Sheet 2)

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Convert these amounts to weekly repayments. Either use your fraction key or ÷/brackets.

Example: Convert $3 198 into weekly payments over 3 years.

Payment = $\frac{3\ 198}{3 \times 52}$ = $20.50/week

Calc: 3 1 9 8 ▪ 3 × 5 2 = S⇔D

1 $6 895.20 into weekly over 2 yrs

Payment = $\frac{6\ 895.20}{2 \times 52}$

Payment = $66.30/week

$ sign + cents to 2 d.p.

2 $3 556.80 into weekly over 4 yrs

Payment = $\frac{3\ 556.80}{4 \times 52}$

Payment = $17.10/week

3 $7 927.92 into weekly over 3 yrs

Payment = $\frac{7\ 927.92}{3 \times 12}$

Payment = $220.22/week

The next 3 questions require rounding

4 $4 200 into weekly over 5 yrs

Payment = $\frac{4\ 200}{5 \times 52}$

Payment = $16.15/week

5 $5 389 into weekly over 4 yrs

Payment = $\frac{5\ 389}{4 \times 52}$

Payment = $25.91/week

6 $58 222 into weekly over 6 yrs

Payment = $\frac{58\ 222}{6 \times 52}$

Payment = $186.61/week

Calculate the interest payable, the total, then convert this to a weekly repayment amount.

$12 280 is borrowed for 5 years at 8% p.a., repaid weekly.

7 Calculate the interest payable

$I = Prn$

$= 12\ 280 \times 0.08 \times 5$

(Amount; 8% decimal; years)

I = $4 912

8 Calculate the amount owing

$A = 12\ 280 + 4\ 912$

(Amount borrowed; Interest (Q7))

A = $17 192

9 Convert to weekly payments

Payment = $\frac{17\ 192}{5 \times 52}$ ← (Q8); 5×52

Payment = $66.12/week

$37 612 is borrowed for 3 years at 12% p.a., repaid weekly.

10 Calculate the interest payable

$I = Prn$

$= 37\ 612 \times 0.12 \times 3$

(Amount; 12% decimal; years)

I = $13 540.32

11 Calculate the amount owing

$A = 37\ 612 + 13\ 540.32$

A = $51 152.32

12 Convert to weekly payments

Payment = $\frac{51\ 152.32}{3 \times 52}$

Payment = $327.90/week

A car costs $26 006 and is purchased using a flat rate loan for 4 years at 6% p.a. Calculate the equal weekly repayments over the 4 years.

13 Find the interest payable

$I = Prn$

$= 26\ 006 \times 0.06 \times 4$

I = $6 241.44

14 Calculate the amount owing

$A = 26\ 006 + 6\ 241.44$

A = $32 247.44

15 Convert to weekly payments

Payment = $\frac{32\ 247.44}{4 \times 52}$

Payment = $155.04/week

A stove costs $1 960 and is bought using a flat rate loan for 2 years at 18% p.a. Calculate the equal weekly repayments over the 2 years.

16 Find the interest payable

$I = Prn$

$= 1\ 960 \times 0.18 \times 2$

I = $705.60

17 Calculate the amount owing

$A = 1\ 960 + 705.60$

A = $2 665.60

18 Convert to weekly payments

Payment = $\frac{2\ 665.60}{2 \times 52}$

Payment = $25.63/week

Investing Money - Flat Rate Loans - Calculating Repayments (Sheet 3)

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Calculate the interest payable, the total, then convert this to the specified repayment period

$18 170 is borrowed for 4 years at 5% p.a., repaid in weekly payments

1 Calculate the interest payable

$I = Prn$

$= 18\,170 \times 0.05 \times 4$

(Amount; 5% decimal; years)

$I = \$3\,634$

2 Calculate the amount owing

$A = 18\,170 + 3\,634$

(Amount borrowed; Interest (Q1))

$A = \$21\,804$

3 Convert to weekly payments

$$\text{Payment} = \frac{21\,804}{4 \times 52}$$

Payment = $104.83/week

$40333 is borrowed for 7 years at 16% p.a., repaid in monthly payments

4 Calculate the interest payable

$I = Prn$

$= 40\,333 \times 0.16 \times 7$

$I = \$45\,172.96$

5 Calculate the amount owing

$A = 40\,333 + 45\,172.96$

$A = \$85\,505.96$

6 Convert to monthly payments

$$\text{Payment} = \frac{85\,505.96}{7 \times 12}$$

Payment = $1 017.93/month

A pool costs $34 449 and is purchased using a flat rate loan for 6 years at 14% p.a. Calculate the weekly repayments over the 6 years.

7 Calculate the interest payable

$I = Prn$

$= 34\,449 \times 0.14 \times 6$

$I = \$28\,937.16$

8 Calculate the amount owing

$A = 34\,449 + 28\,937.16$

$A = \$63\,386.16$

9 Find the weekly payments

$$\text{Payment} = \frac{63\,386.16}{6 \times 12}$$

Payment = $880.36/week

A tractor costs $60 112 and is bought using a flat rate loan for 3 years at 9% p.a. Calculate the fortnightly repayments over the 3 years.

10 Calculate the interest payable

$I = Prn$

$= 60\,112 \times 0.09 \times 3$

$I = \$16\,230.24$

11 Calculate the amount owing

$A = 60\,112 + 16\,230.24$

$A = \$76\,342.24$

12 Find the fortnightly payment

$$\text{Payment} = \frac{76\,342.24}{3 \times 26}$$

Payment = $978.75/fortnight

A cherry-picker costs $184 290 and is purchased using a flat rate loan over 4 years at 8% p.a. Calculate the monthly repayments over the 4 years.

13 Calculate the interest payable

$I = Prn$

$= 184\,290 \times 0.08 \times 4$

$I = \$58\,972.80$

14 Calculate the amount owing

$A = 184\,290 + 58\,972.80$

$A = \$243\,262.80$

15 Find the monthly payments

$$\text{Payment} = \frac{243\,262.80}{4 \times 12}$$

Payment = $5 067.98/month

A caravan costs $39 777 and is bought using a flat rate loan for 5 years at 11% p.a. Calculate the fortnightly repayments over the 5 years.

16 Calculate the interest payable

$I = Prn$

$= 39\,777 \times 0.11 \times 5$

$I = \$21\,877.35$

17 Calculate the amount owing

$A = 39\,777 + 21\,877.35$

$A = \$61\,654.35$

18 Find the fortnightly payment

$$\text{Payment} = \frac{61\,654.35}{5 \times 26}$$

Payment = $474.26/fortnight

Investing Money - Flat Rate Loans (Homework Strip)

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Date Due: _____

Troy has bought a $40 500 speedboat using a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments over the 4 year period.

1 Convert 9% p.a. to a decimal

$r =$ _____

2 Find the interest payable

$I = Prn$

$= \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_$

Amount borrowed | 9% decimal | Years

$I = \$$ _____

3 Calculate the amount owing

$A =$ _____ $+$ _____

Amount borrowed | Interest (Q2)

$A =$ _____

4 Convert to monthly payments

$$\text{Payment} = \frac{\_\_\_\_}{\_\_\_\_ \times \_\_\_\_}$$

Payment = _____

Date Due: _____

Troy has bought a $40 500 speedboat using a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments over the 4 year period.

1 Convert 9% p.a. to a decimal

$r =$ _____

2 Find the interest payable

$I = Prn$

$= \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_$

Amount borrowed | 9% decimal | Years

$I = \$$ _____

3 Calculate the amount owing

$A =$ _____ $+$ _____

Amount borrowed | Interest (Q2)

$A =$ _____

4 Convert to monthly payments

$$\text{Payment} = \frac{\_\_\_\_}{\_\_\_\_ \times \_\_\_\_}$$

Payment = _____

Date Due: _____

Troy has bought a $40 500 speedboat using a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments over the 4 year period.

1 Convert 9% p.a. to a decimal

$r =$ _____

2 Find the interest payable

$I = Prn$

$= \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_$

Amount borrowed | 9% decimal | Years

$I = \$$ _____

3 Calculate the amount owing

$A =$ _____ $+$ _____

Amount borrowed | Interest (Q2)

$A =$ _____

4 Convert to monthly payments

$$\text{Payment} = \frac{\_\_\_\_}{\_\_\_\_ \times \_\_\_\_}$$

Payment = _____

Date Due: _____

Troy has bought a $40 500 speedboat using a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments over the 4 year period.

1 Convert 9% p.a. to a decimal

$r =$ _____

2 Find the interest payable

$I = Prn$

$= \_\_\_\_ \times \_\_\_\_ \times \_\_\_\_$

Amount borrowed | 9% decimal | Years

$I = \$$ _____

3 Calculate the amount owing

$A =$ _____ $+$ _____

Amount borrowed | Interest (Q2)

$A =$ _____

4 Convert to monthly payments

$$\text{Payment} = \frac{\_\_\_\_}{\_\_\_\_ \times \_\_\_\_}$$

Payment = _____

Investing Money - Flat Rate Loans (Homework Strip)

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Date Due:

Troy has bought a $40 500 speedboat using a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments over the 4 year period.

1 Convert 9% p.a. to a decimal

$r =$ 0.09

2 Find the interest payable

$I = Prn$

$= 40\,500 \times 0.09 \times 4$

(Amount borrowed) (9% decimal) (Years)

$I =$ $14 580

3 Calculate the amount owing

$A = 40\,500 + 14\,580$

(Amount borrowed) (Interest (Q2))

$A =$ $55 080

4 Convert to monthly payments

$$\text{Payment} = \frac{55\,080}{4 \times 12}$$

Payment = $1 147.50

Investing Money - Further Flat Rate Loans

This sheet involves flat rate loan calculations that include a deposit. This sheet covers a conceptually difficult process because students have to understand the function of a deposit and that it is not a borrowing. There is one additional step from the previous sheets, that is, the subtraction of a deposit from the sale price. But this means that a question is a four step process, five steps when the deposit is given as a percentage.

Example: Fred buys a swimming pool which sells for $14 250. He pays a $2 850 deposit. He takes out a flat rate loan at 7% p.a. over 5 years. He repays the loan in monthly payments. Calculate the monthly repayment.

Solution:

Find the Principal borrowed

$A = 14\,250 - 2\,850$ ← Subtract the deposit from the sale price, this is the Principal, which is the amount borrowed.

$A = \$11\,400$

Find the interest payable

$I = Prn$

$= 11\,400 \times 0.07 \times 5$ ← Remember to convert the interest rate to a decimal. The amount borrowed is the principal, not sale price

$I = \$3\,990$

Find the total amount payable

$A = 11\,400 + 3\,990$ ← Add the interest to the principal

$A = \$15\,390$ ← This is the amount that has to be paid back

Find the monthly payment

$\text{Monthly Payment} = \dfrac{15\,390}{5 \times 12}$ ← Divide the amount payable by the number of months in 5 years. (So Use 5×12)

$= \$256.50$

Investing Money - Further Flat Rate Loans - Deposits (Sheet 1)

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A deposit is your own money, so you don't pay interest on it. Find the size of the monthly repayments on these loans.

A new garage costs $5 600 and is bought using a flat rate loan. A deposit of $1 680 is paid. The loan is for 3 years at 15% p.a. Calculate the monthly repayments over the 3 years.

1 Find the Principal borrowed

$P =$ ☐ - ☐

Sale Price Deposit

$P =$ ☐

2 Calculate the interest payable

$I = Prn$

$=$ ☐ × ☐ × ☐

Principal (Q1) 15% decimal years

$I =$ ☐ $ Sign

3 Calculate the amount owing

$A =$ ☐ + ☐

Principal (Q1) Interest (Q2)

$A =$ ☐

4 Convert to monthly payments

$\text{Monthly Payment} = \dfrac{\square}{\square \times \square}$ ← (Q3), 3×12

$=$ ☐

$ Sign + Cents to 2 d.p.

A new quad-bike costs $9 150 and is bought using a flat rate loan. A deposit of $1 830 is paid. The loan is for 5 years at 8% p.a. Calculate the monthly repayments over the 5 years.

5 Find the Principal borrowed

$P =$ ☐ - ☐

Sale Price Deposit

$P =$ ☐

6 Calculate the interest payable

$I = Prn$

$=$ ☐ × ☐ × ☐

$I =$ ☐

7 Calculate the amount owing

$A =$ ☐ + ☐

Principal (Q5) Interest (Q6)

$A =$ ☐

8 Convert to monthly payments

$\text{Monthly Payment} = \dfrac{\square}{\square \times \square}$

$=$ ☐

Scuba gear costs $3 460 and is bought with a 4 year flat rate loan. A deposit of $519 is paid. The loan interest rate is 12% p.a. Calculate the monthly repayments over the 4 years.

9 Find the Principal borrowed

$P =$

$P =$

10 Calculate the interest payable

$I = Prn$

$=$

$I =$

11 Calculate the amount owing

$A =$

$A =$

12 Convert to monthly payments

$\text{Monthly Payment} =$

$=$

Convert these payments to weekly repayments.

13 $17 565.60 over 6 years.

$\text{Weekly Payment} = \dfrac{\square}{\square \times \square}$

$=$ ☐

14 $8 477 over 3 years.

$\text{Weekly Payment} = \dfrac{\square}{\square \times \square}$

$=$ ☐

This time payments are weekly. Find the size of the weekly repayments on this loan.

A catamaran costs $46 870 and is bought with a 5 year flat rate loan. A deposit of $9 374 is paid. The loan interest rate is 6% p.a. Calculate the weekly repayments over the 5 years.

15 Find the Principal borrowed

16 Calculate the interest payable

17 Calculate the amount owing

18 Convert to weekly payments

Investing Money - Further Flat Rate Loans - Deposits (Sheet 2)

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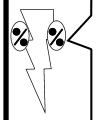

Find the size of the repayments on these loans. Watch out for the percentages, one is for the deposit, the other for the loan.

An office fit-out costs $13 444 and is financed using a flat rate loan. A deposit of 15% is paid. The loan is for 3 years at 8% p.a. Calculate the monthly repayments over the 3 years.

1 Calculate the deposit

$Dep =$ [ ] $\times$ [ ]

Purchase Price | 15% decimal

$Dep =$ [ ]

$ Sign + Cents to 2 d.p.

2 Find the Principal borrowed

$P =$ [ ] - [ ]

Sale Price | Deposit (Q1)

$P =$ [ ]

3 Calculate the interest payable

$I = Prn$

$=$ [ ] $\times$ [ ] $\times$ [ ]

Principal (Q2) | 8% decimal | years

$I =$ [ ]

4 Calculate the amount owing

$A =$ [ ] $+$ [ ]

Principal (Q2) | Interest (Q3)

$A =$ [ ]

5 Convert to monthly payments

$$\text{Monthly Payment} = \frac{\square}{\square \times \square}$$

$=$ [ ]

$ Sign + Cents to 2 d.p.

A trampoline costs $1 720 and is financed using a flat rate loan. A deposit of 10% is paid. The loan is for 2 years at 7% p.a. Calculate the weekly repayments over the 2 years.

6 Calculate the deposit

7 Find the Principal borrowed

8 Calculate the interest payable

9 Calculate the amount owing

10 Convert to weekly payments

An outboard motor costs $8 059 and is financed using a flat rate loan. A deposit of 15% is paid. The loan is for 4 years at 12% p.a. Calculate the weekly repayments over the 4 years.

11 Calculate the deposit

12 Calculate the Principal

13 Calculate the interest payable

14 Calculate the amount owing

15 Convert to weekly payments

A car costs $19 999 and is financed using a flat rate loan. A deposit of 20% is paid. The loan is for 5 years at 6% p.a. Calculate the fortnightly repayments over the 5 years.

16 Calculate the deposit

17 Find the Principal borrowed

18 Calculate the interest payable

19 Calculate the amount owing

20 Find the fortnightly payments

Investing Money - Further Flat Rate Loans - Deposits (Sheet 1)

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A deposit is your own money, so you don't pay interest on it. Find the size of the monthly repayments on these loans.

A new garage costs $5 600 and is bought using a flat rate loan. A deposit of $1 680 is paid. The loan is for 3 years at 15% p.a. Calculate the monthly repayments over the 3 years.

1 Find the Principal borrowed

$P =$ 5 600 - 1 680

(Sale Price) (Deposit)

$P =$ $3 920

2 Calculate the interest payable

$I = Prn$

$=$ 3 920 $\times$ 0.15 $\times$ 3

(Principal (Q1)) (15% decimal) (years)

$I =$ $1 764 ($ Sign)

3 Calculate the amount owing

$A =$ 3 920 + 1 764

(Principal (Q1)) (Interest (Q2))

$A =$ $5 684

4 Convert to monthly payments

$$\text{Monthly Payment} = \frac{5\,684}{3 \times 12}$$ (Q3), 3×12

$=$ $157.89

($ Sign + Cents to 2 d.p.)

A new quad-bike costs $9 150 and is bought using a flat rate loan. A deposit of $1 830 is paid. The loan is for 5 years at 8% p.a. Calculate the monthly repayments over the 5 years.

5 Find the Principal borrowed

$P =$ 9 150 - 1 830

(Sale Price) (Deposit)

$P =$ $7 320

6 Calculate the interest payable

$I = Prn$

$=$ 7 320 $\times$ 0.08 $\times$ 5

$I =$ $2 928

7 Calculate the amount owing

$A =$ 7 320 + 2 928

(Principal (Q5)) (Interest (Q6))

$A =$ $10 248

8 Convert to monthly payments

$$\text{Monthly Payment} = \frac{10\,248}{5 \times 12}$$

$=$ $170.80

Scuba gear costs $3 460 and is bought with a 4 year flat rate loan. A deposit of $519 is paid. The loan interest rate is 12% p.a. Calculate the monthly repayments over the 4 years.

9 Find the Principal borrowed

$P =$ 3 460 - 519

$P =$ $2 941

10 Calculate the interest payable

$I = Prn$

$= 2\,941 \times 0.12 \times 4$

$I =$ $1 411.68

11 Calculate the amount owing

$A =$ 2 941 + 1 411.68

$A =$ $4 352.68

12 Convert to monthly payments

$$\text{Monthly Payment} = \frac{4\,352.68}{4 \times 12}$$

$=$ $90.68

Convert these payments to weekly repayments.

13 $17 565.60 over 6 years.

$$\text{Weekly Payment} = \frac{17\,565.60}{6 \times 52}$$

$=$ $56.30

14 $8 477 over 3 years.

$$\text{Weekly Payment} = \frac{8\,477}{3 \times 52}$$

$=$ $54.34

This time payments are weekly. Find the size of the weekly repayments on this loan.

A catamaran costs $46 870 and is bought with a 5 year flat rate loan. A deposit of $9 374 is paid. The loan interest rate is 6% p.a. Calculate the weekly repayments over the 5 years.

15 Find the Principal borrowed

$P =$ 46 870 - 9 374

$P =$ $37 496

16 Calculate the interest payable

$I = Prn$

$= 37\,496 \times 0.06 \times 5$

$I =$ $11 248.80

17 Calculate the amount owing

$A =$ 37 496 + 11 248.80

$A =$ $48 744.80

18 Convert to weekly payments

$$\text{Weekly Payment} = \frac{48\,744.80}{5 \times 52}$$

$=$ $187.48

Investing Money - Further Flat Rate Loans - Deposits (Sheet 2)

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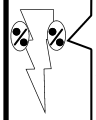

Find the size of the repayments on these loans. Watch out for the percentages, one is for the deposit, the other for the loan.

An office fit-out costs $13 444 and is financed using a flat rate loan. A deposit of 15% is paid. The loan is for 3 years at 8% p.a. Calculate the monthly repayments over the 3 years.

1 Calculate the deposit

$Dep =$ 13 444 (Purchase Price) $\times$ 0.15 (15% decimal)

$Dep =$ $2 016.60 ($ Sign + Cents to 2 d.p.)

2 Find the Principal borrowed

$P =$ 13 444 (Sale Price) - 2 016.60 (Deposit (Q1))

$P =$ $11 427.40

3 Calculate the interest payable

$I = Prn$

$=$ 11 427.40 (Principal (Q2)) $\times$ 0.08 (8% decimal) $\times$ 3 (years)

$I =$ $2 742.58

4 Calculate the amount owing

$A =$ 11 427.40 (Principal (Q2)) + 2 742.58 (Interest (Q3))

$A =$ $14 169.98

5 Convert to monthly payments

$$\text{Monthly Payment} = \frac{14\,169.98}{3 \times 12}$$

$= \$393.61$ ($ Sign + Cents to 2 d.p.)

A trampoline costs $1 720 and is financed using a flat rate loan. A deposit of 10% is paid. The loan is for 2 years at 7% p.a. Calculate the weekly repayments over the 2 years.

6 Calculate the deposit

$Deposit = 1\,720 \times 0.1$

$Deposit = \$172$

7 Find the Principal borrowed

$P = 1\,720 - 172$

$P = \$1\,548$

8 Calculate the interest payable

$I = Prn$

$= 1\,548 \times 0.07 \times 2$

$I = \$216.72$

9 Calculate the amount owing

$A = 1\,548 + 216.72$

$A = \$1\,764.72$

10 Convert to weekly payments

$$\text{Weekly Payment} = \frac{1\,764.72}{2 \times 52}$$

$= \$16.97$

An outboard motor costs $8 059 and is financed using a flat rate loan. A deposit of 15% is paid. The loan is for 4 years at 12% p.a. Calculate the weekly repayments over the 4 years.

11 Calculate the deposit

$Deposit = 8\,059 \times 0.15$

$Deposit = \$1\,208.85$

12 Calculate the Principal

$P = 8\,059 - 1\,208.85$

$P = \$6\,850.15$

13 Calculate the interest payable

$I = Prn$

$= 6\,850.15 \times 0.12 \times 4$

$I = \$3\,288.07$

14 Calculate the amount owing

$A = 6\,850.15 + 3\,288.07$

$A = \$10\,138.22$

15 Convert to weekly payments

$$\text{Weekly Payment} = \frac{10\,138.22}{4 \times 52}$$

$= \$48.74$

A car costs $19 999 and is financed using a flat rate loan. A deposit of 20% is paid. The loan is for 5 years at 6% p.a. Calculate the fortnightly repayments over the 5 years.

16 Calculate the deposit

$Deposit = 19\,999 \times 0.2$

$Deposit = \$3\,999.80$

17 Find the Principal borrowed

$P = 19\,999 - 3\,999.80$

$P = \$15\,999.20$

18 Calculate the interest payable

$I = Prn$

$= 15\,999.20 \times 0.06 \times 5$

$I = \$1\,919.90$

19 Calculate the amount owing

$A = 15\,999.20 + 1\,919.90$

$A = \$17\,919.10$

20 Find the fortnightly payments

$$\text{Fortnightly Payment} = \frac{17\,919.10}{5 \times 26}$$

$= \$137.84$

Investing Money - Further Flat Rate Loans (Homework Strip)

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Date Due: _____

John has bought a $36 500 car. He paid a $6 205 deposit and took out a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments.

1 Find the Principal borrowed

$P =$ _____ - _____
(Sale Price) (Deposit)

$P =$ _____

2 Calculate the interest payable

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____
(Principal (Q1)) (9% decimal) (years)

$I =$ _____

3 Calculate the amount owing

$A =$ _____ $+$ _____

$A =$ _____

4 Convert to monthly payments

$\text{Monthly Payment} = \dfrac{\text{_____}}{\text{_____} \times \text{_____}}$

$=$ _____

Date Due: _____

John has bought a $36 500 car. He paid a $6 205 deposit and took out a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments.

1 Find the Principal borrowed

$P =$ _____ - _____
(Sale Price) (Deposit)

$P =$ _____

2 Calculate the interest payable

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____
(Principal (Q1)) (9% decimal) (years)

$I =$ _____

3 Calculate the amount owing

$A =$ _____ $+$ _____

$A =$ _____

4 Convert to monthly payments

$\text{Monthly Payment} = \dfrac{\text{_____}}{\text{_____} \times \text{_____}}$

$=$ _____

Date Due: _____

John has bought a $36 500 car. He paid a $6 205 deposit and took out a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments.

1 Find the Principal borrowed

$P =$ _____ - _____
(Sale Price) (Deposit)

$P =$ _____

2 Calculate the interest payable

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____
(Principal (Q1)) (9% decimal) (years)

$I =$ _____

3 Calculate the amount owing

$A =$ _____ $+$ _____

$A =$ _____

4 Convert to monthly payments

$\text{Monthly Payment} = \dfrac{\text{_____}}{\text{_____} \times \text{_____}}$

$=$ _____

Date Due: _____

John has bought a $36 500 car. He paid a $6 205 deposit and took out a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments.

1 Find the Principal borrowed

$P =$ _____ - _____
(Sale Price) (Deposit)

$P =$ _____

2 Calculate the interest payable

$I = Prn$

$=$ _____ $\times$ _____ $\times$ _____
(Principal (Q1)) (9% decimal) (years)

$I =$ _____

3 Calculate the amount owing

$A =$ _____ $+$ _____

$A =$ _____

4 Convert to monthly payments

$\text{Monthly Payment} = \dfrac{\text{_____}}{\text{_____} \times \text{_____}}$

$=$ _____

Investing Money - Further Flat Rate Loans (Homework Strip)

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Date Due:

John has bought a $36 500 car. He paid a $6 205 deposit and took out a flat rate loan over 4 years at 9% p.a. Calculate the monthly repayments.

1 Find the Principal borrowed

$P =$ 36 500 (Sale Price) - 6 205 (Deposit)

$P =$ $30 295

2 Calculate the interest payable

$I = Prn$

$=$ 30 295 (Principal (Q1)) $\times$ 0.09 (9% decimal) $\times$ 4 (years)

$I =$ $10 906.20

3 Calculate the amount owing

$A =$ 30 295 $+$ 10 906.20

$A =$ $41 201.20

4 Convert to monthly payments

$$\text{Monthly Payment} = \frac{41\ 201.20}{4 \times 12}$$

$=$ $858.36

Investing Money - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don't colour it in too dark (using a texta for example) so that you can still read it.

Investing Money - Find a Word

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Find the words in the puzzle from the wordlist.



L	A	M	I	C	E	D	B	A	L	A	N	C	E	T
T	C	B	M	A	T	U	R	E	D	R	E	D	E	N
N	C	G	A	L	A	I	O	A	A	T	O	S	S	E
U	R	N	L	B	L	O	D	N	T	I	E	M	E	V
O	U	I	O	A	L	L	S	E	R	G	S	O	F	L
C	E	C	O	M	P	A	R	E	R	E	E	N	S	O
C	C	U	R	Y	L	P	P	A	F	C	I	T	C	S
A	H	D	U	C	C	I	H	Q	F	W	N	H	H	N
D	S	E	S	A	H	C	R	U	P	E	T	L	A	I
E	A	R	L	A	U	N	N	A	M	E	E	Y	R	B
P	C	E	L	P	M	I	S	Y	B	K	R	S	R	O
O	S	N	A	O	L	R	A	T	E	L	E	M	C	R
S	W	D	N	U	O	P	M	O	C	Y	S	R	H	R
I	W	I	N	V	E	S	T	M	E	N	T	E	A	O
T	B	E	D	R	L	A	W	A	R	D	H	T	I	W

WORDLIST

RATE	PERIOD	SIMPLE	PRINCIPAL	CREDIT
INVESTMENT	BORROW	FEES	PURCHASES	CHARGES
ANNUAL	COMPOUND	LOANS	REDUCING	WEEKLY
DEPOSIT	ACCOUNT	TERM	DEBT	INSOLVENT
DECIMAL	BALANCE	INTEREST	MONTHLY	CASH
APPLY	WITHDRAWAL	COMPARE	REPAYMENTS	ACCRUE
		MATURED		

Investing Money - Find a Word

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Find the words in the puzzle from the wordlist.



L	A	M	I	C	E	D	B	A	L	A	N	C	E	T
T	C	B	M	A	T	U	R	E	D	R	E	D	E	N
N	C	G	A	L	A	I	O	A	A	T	O	S	S	E
U	R	N	L	B	L	O	D	N	T	I	E	M	E	V
O	U	I	O	A	L	L	S	E	R	G	S	O	F	L
C	E	C	O	M	P	A	R	E	R	E	E	N	S	O
C	C	U	R	Y	L	P	P	A	F	C	I	T	C	S
A	H	D	U	C	C	I	H	Q	F	W	N	H	H	N
D	S	E	S	A	H	C	R	U	P	E	T	L	A	I
E	A	R	L	A	U	N	N	A	M	E	E	Y	R	B
P	C	E	L	P	M	I	S	Y	B	K	R	S	R	O
O	S	N	A	O	L	R	A	T	E	L	E	M	C	R
S	W	D	N	U	O	P	M	O	C	Y	S	R	H	R
I	W	I	N	V	E	S	T	M	E	N	T	E	A	O
T	B	E	D	R	L	A	W	A	R	D	H	T	I	W

WORDLIST

~~RATE~~
~~INVESTMENT~~
~~ANNUAL~~
~~DEPOSIT~~
~~DECIMAL~~
~~APPLY~~
~~PERIOD~~
~~BORROW~~
~~COMPOUND~~
~~ACCOUNT~~
~~BALANCE~~
~~WITHDRAWAL~~
~~SIMPLE~~
~~FEES~~
~~LOANS~~
~~TERM~~
~~INTEREST~~
~~COMPARE~~
~~MATURED~~
~~PRINCIPAL~~
~~PURCHASES~~
~~REDUCING~~
~~DEBT~~
~~MONTHLY~~
~~REPAYMENTS~~
~~CREDIT~~
~~CHARGES~~
~~WEEKLY~~
~~INSOLVENT~~
~~CASH~~
~~ACCRUE~~

Investing Money Review

This a review of the Investing Money topic.

Investing Money - Topic Review

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Convert these interest rates to decimals.

1 9% = _____

2 12% = _____

3 8.4% = _____

Calculate the simple interest earned on these investments. Use: $I = Prn$

P = \$700, R = 6% p.a., n = 4 years

4 _____

P = \$1 420, R = 8.9% p.a., n = 7 years

5 _____

Calculate the simple interest earned and then the account balance.

An investment of \$47 000 at 4% p.a. for 5 years.

6 _____

An investment of \$6 112 at 1.5% per month for 8 months.

7 _____

Calculate the simple interest earned on these investments.

An investment of \$1 440 at 9% p.a. for 26 weeks.

8 _____

An investment of \$13 900 at 12% p.a. for 145 days.

9 _____

An investment of \$8 055 at 5.3% p.a. for 16 months.

10 _____

Use either 'Guess & Check' or algebraic methods to find the term of these investments

An investment of \$6 000 at 4% p.a. earns interest of \$720.

11 _____

An investment of \$430 at 7.4% p.a. earns interest of \$413.66.

12 _____

Use either 'Guess & Check' or algebraic methods to find the principal invested

An investment at 7% p.a., for 3 years earns interest of \$126. Find the amount invested.

13 _____

An investment has interest paid at the end of each year. Use $I = Prn$ twice to find the account balance at the end of a two year investment.

\$12 000 at 5.5% p.a. for 2 years.

14 _____

Find the account balance after the following compound interest investments have matured. Use $A = P(1 + r)^n$

An investment of \$2 000 at 9% p.a. for 5 years, compounded annually.

15 _____

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Find A, the account balance, for this compound investment. (Continued)

An investment of $6 020 at 1.6% per month for 10 months, compounded monthly.

16

Calculate the interest earned from these compound investments.

An investment of $750 at 8% p.a. for 4 years, compounded annually.

17

An investment of $23 770 at 0.6% per month for 15 months, compounded monthly.

18

Compare the interest earned in these two investment options.
Option 1: $3 500 invested at 6% p.a. for 7 yrs, simple interest investment.
Option 2: $3 500 invested at 6% p.a. for 7 years, compound interest investment, compounding annually.

Calculate the interest earned if Option 1 is chosen. (Simple Interest)

19

Calculate the interest earned if Option 2 is chosen. (Compound Int.)

20

Which option offers the better return and calculate the difference between the two options.

21

Express in your own words, why compound interest investments result in higher interest returns.

22

Calculate the amount paid given the flat rate loan repayments.

23 $240/month for 8 months

24 $85.90/month for 5 years

25 $58.65/week for 2 years

Calculate the amount paid given the deposit and flat rate loan repayments.

Deposit of $430, repayments of $75.36 per month for 2 years.

26

Deposit of $1 715, repayments of $89.95 per week for 4 years.

27

A car costs $28 200. It is bought using a loan charging 8% p.a. simple interest, for 4 years. The repayments are monthly.

28 Calculate the interest payable

29 Calculate the total amount to be repaid.

30 Calculate the repayments.

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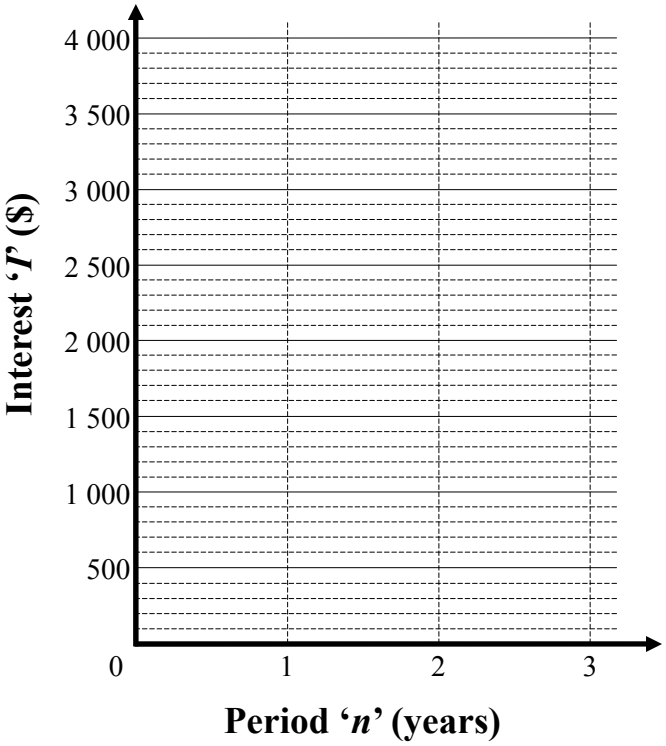
$9 000 is invested for 3 years in a simple interest account which pays 12% p.a. Plot the interest earned on the graph.



31 Calculate I for '$n = 1$'

32 Calculate I for '$n = 2$'

33 Calculate I for '$n = 3$'

34 Plot the line representing the interest earned on the investment, on the graph above.

Use the compound investment table below to answer the questions. All investments are compounded annually.

Periods (n)	Compound Interest Earned ($) on a $1 Investment									
	$R = 1$ %	$R = 2$ %	$R = 3$ %	$R = 4$ %	$R = 5$ %	$R = 6$ %	$R = 7$ %	$R = 8$ %	$R = 9$ %	$R = 10$ %
1	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900	0.1000
2	0.0201	0.0404	0.0609	0.0816	0.1025	0.1236	0.1449	0.1664	0.1881	0.2100
3	0.0303	0.0612	0.0927	0.1249	0.1576	0.1910	0.2250	0.2597	0.2950	0.3310
4	0.0406	0.0824	0.1255	0.1699	0.2155	0.2625	0.3108	0.3605	0.4116	0.4641
5	0.0510	0.1041	0.1593	0.2167	0.2763	0.3382	0.4026	0.4693	0.5386	0.6105
6	0.0615	0.1262	0.1941	0.2653	0.3401	0.4185	0.5007	0.5869	0.6771	0.7716

Calculate the interest earned

35 $1 at 3% p.a. for 4 years

36 $300 at 8% p.a. for 2 years

37 $90 700 at 4% p.a. for 5 yrs

Calculate the interest earned and then the account balance

38 $700 at 7% p.a. for 3 years

39 $4 500 at 5% p.a. for 6 years

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Convert these interest rates to decimals.

1 9% = 0.09

2 12% = 0.12

3 8.4% = 0.084

Calculate the simple interest earned on these investments. Use: $I = Prn$

$P = \$700$, $R = 6\%$ p.a., $n = 4$ years

4 $I = Prn$

$= 700 \times 0.06 \times 4$

$I = \$168$

$P = \$1\,420$, $R = 8.9\%$ p.a., $n = 7$ years

5 $I = Prn$

$= 1\,420 \times 0.089 \times 7$

$I = \$884.66$

Calculate the simple interest earned and then the account balance.

An investment of \$47 000 at 4% p.a. for 5 years.

6 $I = Prn$

$= 47\,000 \times 0.04 \times 5$

$I = \$9\,400$

$A = 47\,000 + 9\,400$

$A = \$56\,400$

An investment of \$6 112 at 1.5% per month for 8 months.

7 $I = Prn$

$= 6\,112 \times 0.015 \times 8$

$I = \$733.44$

$A = 6\,112 + 733.44$

$A = \$6\,845.44$

Calculate the simple interest earned on these investments.

An investment of \$1 440 at 9% p.a. for 26 weeks.

8 $I = Prn$

$= 1\,440 \times 0.09 \times \frac{26}{52}$

$I = \$64.80$

An investment of \$13 900 at 12% p.a. for 145 days.

9 $I = Prn$

$= 13\,900 \times 0.12 \times \frac{145}{365}$

$I = \$662.63$

An investment of \$8 055 at 5.3% p.a. for 16 months.

10 $I = Prn$

$= 8\,055 \times 0.053 \times \frac{16}{12}$

$I = \$569.22$

Use either 'Guess & Check' or algebraic methods to find the term of these investments

An investment of \$6 000 at 4% p.a. earns interest of \$720.

11 $I = Prn$

$720 = 6\,000 \times 0.04 \times n$

$720 = 240 \times n$

$n = \frac{720}{240} = 3$ years

An investment of \$430 at 7.4% p.a. earns interest of \$413.66.

12 $I = Prn$

$413.66 = 430 \times 0.074 \times n$

$413.66 = 31.82 \times n$

$n = \frac{413.66}{31.82} = 13$ years

Use either 'Guess & Check' or algebraic methods to find the principal invested

An investment at 7% p.a., for 3 years earns interest of \$126. Find the amount invested.

13 $I = Prn$

$126 = P \times 0.07 \times 3$

$126 = P \times 0.21$

$P = \frac{126}{0.21} = \$600$

An investment has interest paid at the end of each year. Use $I = Prn$ twice to find the account balance at the end of a two year investment.

\$12 000 at 5.5% p.a. for 2 years.

14 $I = Prn$

$= 12\,000 \times 0.055 \times 1$

$I = \$660$

$A = 12\,000 + 660$

$A = \$12\,660$

$I = Prn$

$= 12\,660 \times 0.055 \times 1$

$I = \$696.30$

$A = 12\,660 + 696.30$

$A = \$13\,356.30$

Find the account balance after the following compound interest investments have matured. Use $A = P(1 + r)^n$

An investment of \$2 000 at 9% p.a. for 5 years, compounded annually.

15 $A = P(1 + r)^n$

$= 2\,000 \times (1 + 0.09)^5$

$A = \$3\,077.25$

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Find A, the account balance, for this compound investment. (Continued)

An investment of \$6 020 at 1.6% per month for 10 months, compounded monthly.

16 $A = P(1 + r)^n$

$= 6\,020 \times (1 + 0.016)^{10}$

$A = \$7\,055.59$

Calculate the interest earned from these compound investments.

An investment of \$750 at 8% p.a. for 4 years, compounded annually.

17 $A = P(1 + r)^n$

$= 750 \times (1 + 0.08)^4$

$A = \$1\,020.37$

$I = 1\,020.37 - 750$

$I = \$270.37$

An investment of \$23 770 at 0.6% per month for 15 months, compounded monthly.

18 $A = P(1 + r)^n$

$= 23\,770 \times (1 + 0.006)^{15}$

$A = \$26\,001.53$

$I = 26\,001.53 - 23\,770$

$I = \$2\,231.53$

Compare the interest earned in these two investment options.
Option 1: \$3 500 invested at 6% p.a. for 7 yrs, simple interest investment.
Option 2: \$3 500 invested at 6% p.a. for 7 years, compound interest investment, compounding annually.

Calculate the interest earned if Option 1 is chosen. (Simple Interest)

19 $I = Prn$

$= 3\,500 \times 0.06 \times 7$

$I = \$1\,470$

Calculate the interest earned if Option 2 is chosen. (Compound Int.)

20 $A = P(1 + r)^n$

$= 3\,500 \times (1 + 0.06)^7$

$A = \$5\,262.71$

$I = 5\,262.71 - 3\,500$

$I = \$1\,762.71$

Which option offers the better return and calculate the difference between the two options.

21 *Compound is better.*

Difference = 1 762.71 - 1 470

Difference = \$292.71

Express in your own words, why compound interest investments result in higher interest returns.

22 *Compound investments pay interest during the investment. That means that your account balance increases and you receive interest on your interest. So you are paid more interest.*

Calculate the amount paid given the flat rate loan repayments.

23 \$240/month for 8 months

Amount = 240 × 8

Amount = \$1 920

24 \$85.90/month for 5 years

Amount = 85.90 × 12 × 5

Amount = \$5 154

25 \$58.65/week for 2 years

Amount = 58.65 × 52 × 2

Amount = \$6 099.60

Calculate the amount paid given the deposit and flat rate loan repayments.

Deposit of \$430, repayments of \$75.36 per month for 2 years.

26 Amount = 75.36 × 12 × 2 + 430

Amount = \$2 238.64

Deposit of \$1 715, repayments of \$89.95 per week for 4 years.

27 Amount = 89.95 × 52 × 4 + 1 715

Amount = \$20 424.60

A car costs \$28 200. It is bought using a loan charging 8% p.a. simple interest, for 4 years. The repayments are monthly.

28 Calculate the interest payable

$I = Prn$

$= 28\,200 \times 0.08 \times 4$

$I = \$9\,024$

29 Calculate the total amount to be repaid.

Amount = 28 200 + 9 024

Amount = \$37 224

30 Calculate the repayments.

Amount = $\dfrac{37\,224}{4 \times 12}$

Amount = \$775.50

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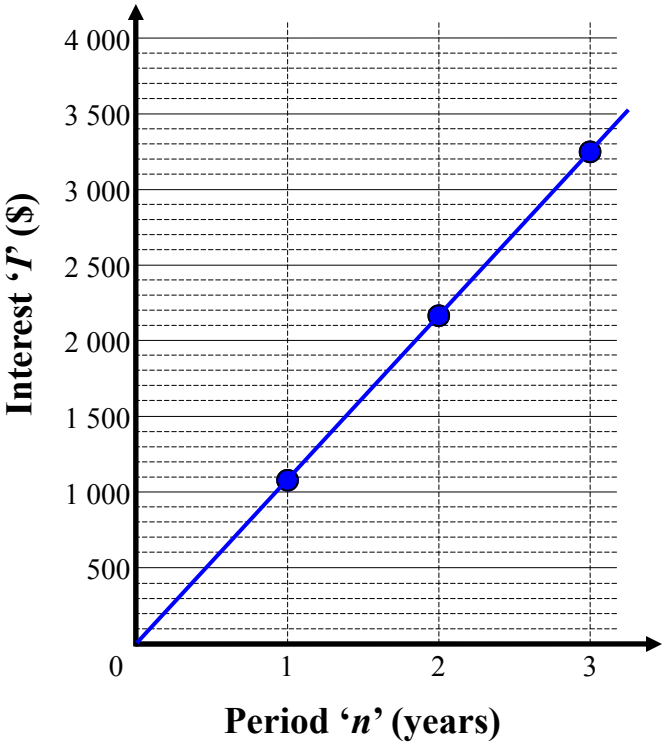


34 Plot the line representing the interest earned on the investment, on the graph above.

31 Calculate I for '$n = 1$'

$I = Prn$

$= 9\,000 \times 0.12 \times 1$

$I = \$1\,080$

32 Calculate I for '$n = 2$'

$I = Prn$

$= 9\,000 \times 0.12 \times 2$

$I = \$2\,160$

33 Calculate I for '$n = 3$'

$I = Prn$

$= 9\,000 \times 0.12 \times 3$

$I = \$3\,240$

Use the compound investment table below to answer the questions. All investments are compounded annually.

Periods (n)	Compound Interest Earned ($) on a $1 Investment									
	$R = 1\,\%$	$R = 2\,\%$	$R = 3\,\%$	$R = 4\,\%$	$R = 5\,\%$	$R = 6\,\%$	$R = 7\,\%$	$R = 8\,\%$	$R = 9\,\%$	$R = 10\,\%$
1	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900	0.1000
2	0.0201	0.0404	0.0609	0.0816	0.1025	0.1236	0.1449	0.1664	0.1881	0.2100
3	0.0303	0.0612	0.0927	0.1249	0.1576	0.1910	0.2250	0.2597	0.2950	0.3310
4	0.0406	0.0824	0.1255	0.1699	0.2155	0.2625	0.3108	0.3605	0.4116	0.4641
5	0.0510	0.1041	0.1593	0.2167	0.2763	0.3382	0.4026	0.4693	0.5386	0.6105
6	0.0615	0.1262	0.1941	0.2653	0.3401	0.4185	0.5007	0.5869	0.6771	0.7716

Calculate the interest earned

35 $1 at 3% p.a. for 4 years

$I = \$0.13$

36 $300 at 8% p.a. for 2 years

$I = \$49.92$

37 $90 700 at 4% p.a. for 5 yrs

$I = \$19\,654.69$

Calculate the interest earned and then the account balance

38 $700 at 7% p.a. for 3 years

$I = \$157.50$

$A = 700 + 157.50$

$A = \$857.50$

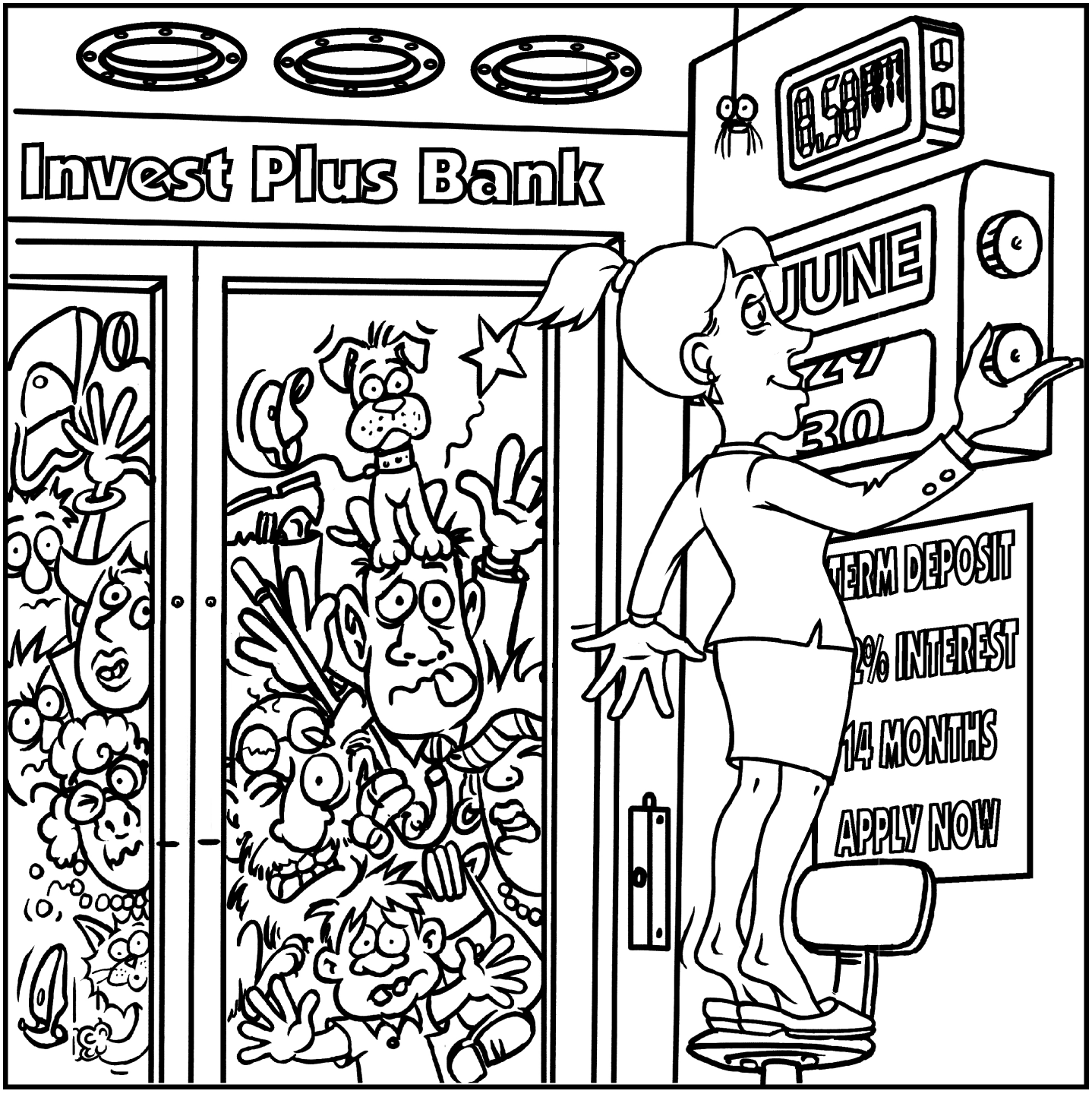
39 $4 500 at 5% p.a. for 6 years

$I = \$1\,530.45$

$A = 4\,500 + 1\,530.45$

$A = \$6\,030.45$

Investing Money



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Investing Money - Simple Interest (Sheet 1)

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Convert these interest rates from percentages to decimals. Divide by 100 to get a decimal.

Example:

16% = 16 ÷ 100 = 0.16

1 9% = ☐ ÷ 100 = ☐

2 5% = ☐ ÷ ☐ = ☐

3 10% = ☐ ÷ ☐ = ☐

Answer only, from now on.

4 14.5% = ☐

5 12.7% = ☐

6 4.92% = ☐

7 0.5% = ☐

8 7¼% = ☐

Calc: SHIFT 7 1 4 ÷ 1 0 0 S⇔D =

9 6¾% = ☐

Simple interest is found using $I = Prn$, but first convert the rate R to a decimal. Find I.

Example: Find the interest earned:

P = \$950, R = 8% p.a., n = 5 years

$r = 8 \div 100 = 0.08$

$I = Prn$

$= 950 \times 0.08 \times 5$

I = \$380

P = \$8 300, R = 6% p.a., n = 4 years

10 r = ☐ ÷ 100 = ☐

$I = Prn$

= ☐ × ☐ × ☐

P r n

I = ☐

Show the '$' sign

P = \$970, R = 4.5% p.a., n = 2 years

11 r = ☐ ÷ ☐ = ☐

$I = Prn$

= ☐ × ☐ × ☐

I = ☐ ← Cents are always 2 d.p.

P = \$5 400, R = 6.04% p.a., n = 7 yrs

12 r = ☐

$I = Prn$

= ☐

I = ☐

The next three have smaller interest rates, but you still have to ÷ 100, as they are percentages

P = \$820, R = 0.15%/week, n = 9 wks

13 r = ☐

$I = Prn$

= ☐

I = ☐

P = \$11 600, R = 0.4%/mth, n = 8 mths

14

Convert R to r (% to decimal)

Write Simple Interest Formula

Make the Substitution

Show 'I =', the '$' sign and cents to 2 d.p.

P = \$450, R = 0.05%/wk, n = 19 weeks

15

Round to 2 d.p. NOT nearest 5c.

P = \$47 130, R = 8.4% p.a., n = 6 years

16

R r

Keep going, but do less work. Use 3 lines not 4, Convert R to r, in the substitution line.

P = \$1 720, R = 4% p.a., n = 5 yrs

17 $I = Prn$ *Use r not R*

= ☐ × ☐ × ☐

I =

P = \$9 050, R = 3.75% p.a., n = 4 yrs

18 $I = Prn$ *Use r not R*

= ☐ × ☐ × ☐

I =

P = \$180, R = 6.25% p.a., n = 4.5 yrs

19

P = \$57 000, R = 4.02% p.a., n = 9 yrs

20

P = \$685.12, R = 13.07% p.a., n = 3 yrs

21

Investing Money - Simple Interest - Account Balance (Sheet 2)

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The account balance 'A' is what is in your account after the interest is paid. Calculate A.

Example: P = \$1 300, I = \$275.20

$A = P + I = 1\,300 + 275.20$

$A = \$1\,575.20$

$P = \$1\,150$, $I = \$662.30$

1 $A =$ ☐ + ☐

$A =$ ☐

Show '$' sign and cents to 2 decimal places

$P = \$580$, $I = \$39.65$

2 $A =$ ☐ + ☐

$A =$ ☐

$P = \$56\,970$, $I = \$4\,076.10$

3 $A =$

$A =$

$P = \$8\,030.62$, $I = \$1\,510.18$

4

$P = \$3\,877$, $I = \$772.92$

5

Jen has \$458.08 in her bank account and receives \$14.32 interest. Find her new account balance.

6

Neil has \$2 504.24 in the bank. He then receives interest of \$83.19. Calculate his account balance.

7

Calculate the interest earned then add it to the principal to find the account balance.

Example:

$P = \$1\,400$, $R = 7\%$ p.a., $n = 5$ years

$I = Prn$

$= 1\,400 \times 0.07 \times 5$

$I = \$490$

$A = 1\,400 + 490 = \$1\,890$

$P = \$4\,200$, $R = 5.2\%$ p.a., $n = 3$ yrs

8 $I = Prn$ *Use r not R*

$=$ ☐ × ☐ × ☐

$I =$ ☐

Show '$' sign and cents to 2 decimal places

$A =$ ☐ + ☐

$A =$ ☐

$P = \$19\,050$, $R = 4.8\%$ p.a., $n = 4$ yrs

9 $I = Prn$

$=$ ☐ × ☐ × ☐

$I =$ ☐

$A =$ ☐ + ☐

$A =$ ☐

$P = \$1\,150$, $R = 5.25\%$ p.a., $n = 6$ yrs

10

Interest rates are smaller as the period is shorter. Find the interest and account balance.

$P = \$79\,500$

$R = 0.05\%$ per week $n = 38$ weeks

Remember to convert R to r (% to a decimal)

11

$P = \$10\,200$

$R = 0.7\%$ per month $n = 9$ months

12

Jason invests \$840 in an account for 7 months which pays 0.45% per month. Calculate:

13 The interest Jason has earned

14 His account balance once the interest has been credited.

Investing Money - Simple Interest - Worded Questions (Sheet 3)

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Answer these. Always use the '$' sign and round cents, if there are any, to 2 d.p.

Eli invests $3 500 for 7 years at 5.4% p.a.

1 Convert 5.4% to a decimal.

2 Calculate the interest earned

$I = Prn$

3 His account balance after the interest has been paid.

Russell invests $950 for 8 months at 0.86% per month.

4 Convert 0.86% to a decimal.

5 Calculate the interest Russell earns on his investment.

6 His account balance after the interest is deposited.

Olivia has $13 600 to invest. She goes to two banks and is offered:
Bank 1: 6.05% p.a. on a 2 year term
Bank 2: 5.75% p.a. on a 2 year term

7 Calculate the interest earned if Bank 1 was chosen.

8 Calculate the interest earned if Bank 2 was chosen.

9 Calculate the extra interest earned if Olivia uses Bank 1.

Jiang li wants $38 000 in 3 years time to travel overseas. She invests $29 500 now earning 8.05% p.a.

10 Calculate the interest earned

11 Her account balance.

12 The additional amount req'd.

Yousef's investment of $17 000 for 4 years at 5.6% p.a. is about to mature. If he chooses to re-invest for 6 months he will earn 0.47%/month.

13 Calculate the interest earned from Yousef's first investment.

14 Calculate the account balance after the interest is paid.

Yousef chooses to reinvest all of his account balance for the 6 months

15 Convert 0.47% to a decimal.

16 Calculate the interest earned from Yousef's 2nd investment.

17 Calculate the account balance after the interest is paid.

Yousef is going to live off the money (in Q.17) while he completes a course for 1 year.

18 Convert the amount in Q.17 to a weekly amount.

Investing Money - Simple Interest - Changing the Term (Sheet 1)

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Write these time periods as fractions of a year. Don't simplify the fractions.

Examples:

5 months = $\frac{5}{12}$ 13 weeks = $\frac{13}{52}$

1 25 weeks = $\frac{\square}{\square}$

2 9 months = $\frac{\square}{\square}$

3 117 days = $\frac{\square}{\square}$

4 14 weeks = $\frac{\square}{\square}$

5 15 months = $\frac{\square}{\square}$

Calculate the interest earned on these investments.

P = \$1 040, R = 8% p.a., n = 9 weeks

6 r = $\square$ ÷ 100 = $\square$

Convert the rate to a decimal

$I = Prn$

= $\square$ × $\square$ × $\frac{\square}{\square}$

P r n

Calc: 1 0 4 0 × 0 . 0 8 × 9 ab/c 5 2 =

I = $\square$

Show the '\$' sign and write cents to 2 d.p.

P = \$4 260, R = 9% p.a., n = 7 months

7 r = $\square$ ÷ $\square$ = $\square$

$I = Prn$

= $\square$ × $\square$ × $\frac{\square}{\square}$

I = $\square$

P = \$1 095, R = 5.6% p.a., n = 220 days

8 r = $\square$ ÷ $\square$ = $\square$

$I = Prn$

= $\square$ × $\square$ × $\frac{\square}{\square}$

I = $\square$

P = \$7 176, R = 3.2% p.a., n = 85 wks.

9 r =

$I = Prn$

=

I =

Complete these calculations, rounding to 2 decimal places.

10 $I = 5\,000 \times 0.046 \times \frac{19}{12}$

I = $\square$

11 $I = 83\,560 \times 0.105 \times \frac{96}{365}$

I = $\square$

P = \$2 500, R = 6.3% p.a., n = 177 days

12 $I = Prn$

= $\square$ × $\square$ × $\frac{\square}{\square}$

r as a decimal

I =

P = \$880, R = 13.25% p.a., n = 16 mths

13

Now find the interest and then the account balance.

P = \$700, R = 5% p.a., n = 12 weeks

14

A = $\square$ + $\square$

Principal Interest

A =

P = \$34 900, R = 11.6% p.a., n = 20 months.

15

A =

A =

Ben invests \$8 500 at 12.2% p.a. for 34 weeks. He wants \$10 000 for a new car. Calculate:

16 His interest and Acc. Bal.

17 The amount above or below \$10 000.

Use the words 'above' or 'below'.

Investing Money - Simple Interest - Changing the Term (Sheet 2)

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Find the interest earned and the account balance with these.

Tom invests $17 450 at 6.65% p.a. for 10 months.

1 Convert 6.65% to a decimal.

$r =$ ☐ ÷ ☐ = ☐

2 Calculate the interest earned

$I = Prn$

$= $ ☐ × ☐ × ☐/☐

$I =$

Show the '$' sign and write cents to 2 d.p.

3 Find his account balance after the interest has been paid.

$A =$ ☐ + ☐

$A =$

Josiah invests $4 200 in a term deposit at 8.35% p.a. for 195 days.

4 Convert 8.35% to a decimal.

5 Calculate the interest earned.

6 Find his account balance after the interest is credited.

Bailey is saving for a trip. He invests $11 200 for 15 months at 7.16% p.a.

7 Express 7.16% as a decimal.

8 Calculate the interest earned.

9 Find his account balance after the interest has been paid.

10 If the airfare is $2 120, how much spending money is left?

Julia has been left an inheritance. Her father gives her a choice, she can:
1 - Receive $19 000 now, and invest it at 4.35% p.a. for 75 weeks, or
2 - Receive $21 000 in 75 weeks time.
Which is the best financial option?

11 Calculate the interest earned on the $19 000 investment (1).

12 Find Julia's account balance after the interest has been paid.

13 Which option is the best? 1 or 2 and by how much?

Tayla won a prize of $2 000 and she invested it for 175 days at 4.3% p.a.

14 Calculate the interest Tayla earned on the $2 000 investment.

15 Find Tayla's account balance when the interest was deposited.

16 If she spends 75% of the money on a new laptop, how much did the laptop cost?

Cost $=$ ☐ × ☐/☐

Cost $=$ ☐

Wai received $765 for holiday work and $320 on his birthday. He invests all the money for 17 weeks at 3.3% p.a. Find the interest Wai earned.

17

Investing Money - Simple Interest - Guess & Check for Term '*n*' (Sheet 1)

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Use 'Guess and Check' to find the value of '*n*' for these. Use the back arrow.



1 $2\,160 = 6\,000 \times 0.09 \times n$

Try '*n*' = 3 ____ (Compare to 2 160)

$6000 \times 0.09 \times 3$

Try '*n*' = 5 ____ (Compare to 2 160)

$6000 \times 0.09 \times 5$

Try '*n*' = ____ ____

2 $336 = 1\,200 \times 0.04 \times n$

Try '*n*' = 5 ____ (Compare to 336)

$1\,200 \times 0.04 \times 5$

Try '*n*' = 9 ____ (Compare to 336)

$1\,200 \times 0.04 \times 9$

Try '*n*' = ____ ____

Keep trying '*n*' values until you get 336

3 $2\,805 = 8\,500 \times 0.03 \times n$

Keep trying '*n*' values until you get 2 805

Tried *n* = ____ ____ ____ ○Low ○Low ○Low ○High ○High ○High $n =$ ____

4 $1\,100 = 1\,250 \times 0.11 \times n$

Keep trying '*n*' values until you get 1 100

Tried *n* = ____ ____ ____ ○Low ○Low ○Low ○High ○High ○High $n =$ ____

5 $6\,600 = 44\,000 \times 0.05 \times n$

Tried *n* = ____ ____ ____ ○Low ○Low ○Low ○High ○High ○High $n =$ ____

6 $934.40 = 730 \times 0.08 \times n$

Tried *n* = ____ ____ ____ ○Low ○Low ○Low ○High ○High ○High $n =$ ____



These are done the same way but this time you first need to build the calculation line. Find '*n*'.

7 An investment of $800 at 7% p.a., results in interest of $224.

____ = ____ × ____ × *n*

Interest | Principal | Convert rate to a decimal

Tried *n* = ____ ____ ____ ____ ○Low ○Low ○Low ○Low ○High ○High ○High ○High

$n =$ ____ years

8 Interest of $9 984 is earned from a $9 600 investment at 8% p.a.

____ = ____ × ____ × *n*

Interest | Principal | Rate (decimal)

Tried *n* = ____ ____ ____ ____ ○Low ○Low ○Low ○Low ○High ○High ○High ○High

$n =$ ____ years

9 $1 450 is invested at 4% p.a. and earns interest of $464.

$I = P r$

____ = ____ × ____ × *n*

Tried *n* = ____ ____ ____ ____ ○Low ○Low ○Low ○Low ○High ○High ○High ○High

$n =$ ____ years

10 Interest of $1 137.50 is earned from a $3 250 investment at 5% p.a.

$I = P r$

____ = ____ × ____ × *n*

Tried *n* = ____ ____ ____ ____ ○Low ○Low ○Low ○Low ○High ○High ○High ○High

$n =$ ____ years



Now write the formula, make the substitution and find '*n*' by Guess and Check methods.

11 An investment of $800 at 7% p.a., results in interest of $224.

Write the formula $I = Prn$

____ = ____ × ____ × *n*

Substitute *I*, *P* and *r* (as a decimal)

$n =$ ____ years

12 Interest of $3 652 is earned from a $4 150 investment at 11% p.a.

____ = ____ × ____ × *n*

$n =$ ____ years

13 An investment of $930 at 6% p.a., results in interest of $167.40.

14 Interest of $1 430 is earned from a $2 200 investment at 13% p.a.

Investing Money - Simple Interest - Algebraic Solution Term 'n' (Sheet 2)

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Find 'n' by solving these. Use the '÷' key or the fraction key.

1 $n = \frac{7\,500}{1\,250} =$ ☐

2 $n = \frac{9\,960}{830} =$ ☐

3 $n = \frac{587}{117.40} =$ ☐

4 $n = \frac{7\,994}{3\,997} =$ ☐

Create the division first, by dividing by the number next to the 'n'. Find the value of 'n'.

Example

$176.40 = 29.4 \times n$

(÷ 29.4) (÷ 29.4)

$n = \frac{176.40}{29.4} = 6$

5 $126 = 18 \times n$

(÷ 18) (÷ 18)

$n = \frac{\square}{\square} = \square$

6 $420 = 70 \times n$

(÷ 70) (÷ 70)

$n = \frac{\square}{\square} = \square$

7 $1\,101.10 = 100.10 \times n$

(÷ 100.10) (÷ 100.10)

$n = \frac{\square}{\square} = \square$

8 $2\,274.80 = 568.70 \times n$

(÷ 568.70) (÷ 568.70)

$n = \frac{\square}{\square} = \square$

9 $3\,080 = 440 \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

10 $414 = 138 \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

11 $1\,336.50 = 148.50 \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

1 extra step, perform a × first, then it's the same process. Find the value of 'n'.

Example

$1\,300 = 6\,500 \times 0.05 \times n$

Multiply these

$1\,300 = 325 \times n$

(÷ 325) (÷ 325)

$n = \frac{1\,300}{325} = 4$

12 $680 = 1\,700 \times 0.08 \times n$

$680 = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

13 $2\,541 = 3\,850 \times 0.06 \times n$

$\square = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

14 $4\,900 = 14\,000 \times 0.05 \times n$

$\square = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$

Get the numbers out of the sentence and solve for 'n', which is the term, in years.

15 Interest of $537.60 is earned from a $960 investment at 7% p.a.

☐ = ☐ × ☐ × n

Interest | Principal | Rate (decimal)

$\square = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$ years

16 Interest of $3 354 is earned from a $4 300 investment at 13% p.a.

☐ = ☐ × ☐ × n

Interest | Principal | Rate (decimal)

$\square = \square \times n$

(÷) (÷)

$n = \frac{\square}{\square} = \square$ years

17 Interest of $590 is earned from a $1 475 investment at 5% p.a.

☐ = ☐ × ☐ × n

Interest | Principal | Rate (decimal)

$\square = \square \times n$

( ) ( )

$n = \frac{\square}{\square} = \square$ years

Investing Money - Simple Interest - Further Finding the Term (Sheet 3)

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Find the term 'n' by solving these. Follow the example.

Example

Interest of \$1 752 is earned from a \$7 300 investment at 4% p.a., find the term of the investment.

$I = Prn$ — Write Formula

$1752 = 7\,300 \times 0.04 \times n$ — Substitute Values (Decimal)

$1\,752 = 292 \times n$ — Simplify The '×'

$\div 292$ $\div 292$

$n = \frac{1\,752}{292} = 6$ years — Divide and solve

1 Interest of \$120 is earned from a \$500 investment at 4% p.a., find the term of the investment.

2 An investment of \$3 250 at 7% p.a., results in interest of \$2 275 being paid. Find the term of the investment.

3 Interest of \$620.40 is received from a \$940 investment at 6% p.a., find the term of the investment.

4 An investment of \$1 700 at 8% p.a., results in interest of \$952 being paid. Find the term of the investment.

5 Interest of \$8 944 is earned from an \$8 600 investment at 13% p.a., find the term of the investment.

6 An investment of \$4 850 at 11% p.a., results in interest of \$4 801.50 being paid. Find the investment's term.

This time you want a total amount in the future, follow the steps to find the term.

David has \$9 000 in the bank but he needs \$12 600 for a future trip.

7 How much interest does he have to earn to have an account balance of \$12 600?

$I =$ [ ] - [ ]

$I =$ [ ]

8 If David's investment pays interest at 5% p.a., find the term of the investment to achieve the interest.

Skye has \$26 000 in the bank but she needs \$36 400 for a home deposit.

9 How much interest does she have to earn to have an account balance of \$36 400?

$I =$

$I =$

10 If Skye earns interest at 8% p.a., find the term of the investment needed to get the deposit.

Investing Money - Simple Interest - Guess & Check Principal (Sheet 1)

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Use 'Guess and Check' to find the value of 'P' for these. Use the back arrow. All values are in thousands.

1 $3\,360 = 0.07 \times 8 \times P$

Try 'P' = 3 000 ____ / $0.07 \times 8 \times 3\,000$ — Compare to 3 360

Try 'P' = 5 000 ____ / $0.07 \times 8 \times 5\,000$ — Compare to 3 360

Try 'P' = 7 000 ____ / $0.07 \times 8 \times 7\,000$ — Compare to 3 360

$P =$ ____

2 $1\,440 = 0.04 \times 3 \times P$

Try 'P' = 3 000 ____ / $0.04 \times 3 \times 3\,000$ — Compare to 1 440

Try 'P' = 7 000 ____ / $0.04 \times 3 \times 7\,000$ — Compare to 1 440

Try 'P' = 11 000 ____ / $0.04 \times 3 \times 11\,000$ — Compare to 1 440

$P =$ ____

Keep trying 'P' values until you get 1 440

3 $12\,870 = 0.11 \times 9 \times P$

Try 'P' = 5 000 ____ / $0.11 \times 9 \times 5\,000$ — Compare to 12 870

Try 'P' = 10 000 ____ / $0.11 \times 9 \times 10\,000$ — Compare to 12 870

Try 'P' = 15 000 ____ / $0.11 \times 9 \times 15\,000$ — Compare to 12 870

$P =$ ____

Keep trying 'P' values until you get 12 870

Build the calculation line. But use $I = rnP$, so that the P is the last number on your calc. Find 'P'.

4 An investment at 6% p.a. for 5 years, results in interest of $2 400.

____ = ____ × ____ × P

Interest | Rate (decimal) | Term

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

$P =$ $

5 An investment yields interest of $2 880 after 8 years, at 12% p.a.

____ = ____ × ____ × P

I | r | n

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

$P =$ ____

6 An investment at 11% p.a. for 2 years, results in interest of $5 280.

____ = ____ × ____ × P

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

$P =$ ____

7 An investment at 9% p.a. for 7 years, results in interest of $10 080.

____ = ____ × ____ × P

Tried $P =$ ____ ○ Low ○ High

Tried $P =$ ____ ○ Low ○ High

$P =$ ____

Now write the formula, with P at the end. Make the substitution and find 'P'. (whole thousands)

8 An investment at 6% p.a. for 8 years, results in interest of $6 240.

____

Write the formula $I = rnP$, with P at the end

____ = ____ × ____ × P

Substitute I, then r (as a decimal) and then n.

$P =$ ____

Show the $ sign

9 An investment at 10% p.a. for 4 years, results in interest of $1 200.

____

____ = ____ × ____ × P

$P =$ ____

10 An investment yields interest of $3 780 after 7 years, at 3% p.a.

____

____

____

11 An investment yields interest of $1 980 after 3 years, at 6% p.a.

____

____

____

Investing Money - Simple Interest - Algebraic Sol Principal 'P' (Sheet 2)

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Find 'P' by solving these. Use the '÷' key or the fraction key.

1 $P = \dfrac{120}{0.6} =$ ☐

2 $P = \dfrac{868}{0.62} =$ ☐

3 $P = \dfrac{4\ 200}{0.48} =$ ☐

4 $P = \dfrac{111.60}{0.72} =$ ☐

Create the division first, by dividing by the number next to the 'P'. Find the value of 'P'.

Example

$176.40 = P \times 0.42$

÷ 0.42 ÷ 0.42

$P = \dfrac{176.40}{0.42} = 420$

5 $240 = P \times 0.48$

÷ 0.48 ÷ 0.48

$P = \dfrac{\square}{\square} = \square$

6 $975 = P \times 0.65$

÷ 0.65 ÷ 0.65

$P = \dfrac{\square}{\square} = \square$

7 $4\ 968 = P \times 0.54$

÷ 0.54 ÷ 0.54

$P = \dfrac{\square}{\square} = \square$

8 $1\ 344 = P \times 0.24$

÷ 0.24 ÷ 0.24

$P = \dfrac{\square}{\square} = \square$

9 $459.20 = P \times 0.56$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

10 $2\ 343 = P \times 0.33$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

11 $2\ 196 = P \times 0.72$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

Multiply r and n first, then it's the same division process. Find the value of 'P'.

Example

$279.60 = P \times 0.03 \times 8$

Multiply these

$279.60 = P \times 0.24$

÷ 0.24 ÷ 0.24

$P = \dfrac{279.60}{0.24} = 1\ 165$

12 $195 = P \times 0.06 \times 5$

$195 = P \times \square$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

13 $484.20 = P \times 0.04 \times 3$

$484.20 = P \times \square$

÷ ÷

$P = \dfrac{\square}{\square} = \square$

Substitute into the formula to find the value of 'P'.

14 Interest of \$86 is earned from a 5 year investment at 4% p.a.

☐ $= P \times$ ☐ $\times$ ☐

Interest Rate (decimal) Term

☐ $= P \times$ ☐

÷ ÷

$P = \dfrac{\square}{\square} =$ \$ ☐

$ sign

15 Interest of \$14 016 is earned from a 4 year investment at 12% p.a.

☐ $= P \times$ ☐ $\times$ ☐

☐ $= P \times$ ☐

÷ ÷

$P = \dfrac{\square}{\square} = \square$

$ sign

16 Interest of \$799.20 is earned from a 6 year investment at 9% p.a.

☐ $= P \times$ ☐ $\times$ ☐

☐ $= P \times$ ☐

÷ ÷

$P = \dfrac{\square}{\square} = \square$

17 Interest of \$457.20 is earned from a 3 year investment at 6% p.a.

☐ $= P \times$ ☐ $\times$ ☐

☐ $= P \times$ ☐

÷ ÷

$P = \dfrac{\square}{\square} = \square$

Investing Money - Simple Interest - Further Finding Principal (Sheet 3)

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Find the amount invested, the Principal 'P' by solving these. Remember the $ sign.

Example

Interest of $374.50 is earned from a 5 year investment at 7% p.a., find the principal invested.

$I = Prn$ — Write Formula

Decimal: 0.07

$374.50 = P \times 0.07 \times 5$ — Substitute Values

$374.50 = P \times 0.35$ — Simplify The '×'

$\div 0.35$ $\div 0.35$

$P = \frac{374.50}{0.35} = \$1\,070$ — Divide and solve

1 Interest of $914.40 is earned from a 3 year investment at 8% p.a., find the principal invested.

2 Interest of $8 127 is earned from a 7 year investment at 6% p.a., find the principal invested.

3 A 5 year investment which has an interest rate of 4% p.a. yields interest of $1 150. Find the amount invested.

$I = Prn$

$1\,150 = P \times 0.04 \times 5$

$1\,150 = P \times 0.2$

4 Interest of $8 658 is earned from a 6 year investment at 13% p.a., find the principal invested.

5 A 2 year investment which has an interest rate of 9% p.a. yields interest of $129.60. Find the amount invested.

This time find the account balance. So find the principal then add it to the interest.

6 Interest of $1 545 is earned from a 5 year investment at 6% p.a., find the principal invested.

7 What will be the account balance at the end of the investment?

$A =$ [ ] + [ ]

Principal Interest

$A =$ [ ]

8 Interest of $321.60 is earned from a 4 year investment at 12% p.a., find the principal invested.

9 What will be the account balance at the end of the investment?

$A =$ [ ] + [ ]

$A =$ [ ]

Investing Money - Simple Interest - Repeated Use (Sheet 1)

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When an investment compounds, interest is paid during the investment. Calculate the account balance for these.

Principal \$2 000,
Rate = 7% p.a.,
n = 3 yrs, compounding annually.

First Application: Year 1

1 $I = Prn$

$= \square \times \square \times 1$
Principal; Rate (decimal)

$I = \square$ Show '\$' sign

$A = \square + \square$
Principal; Interest (Q1)

$A = \square$

Second Application: Year 2

2 $I = Prn$

$= \square \times \square \times 1$
Principal (Use A-Yr 1); Rate (decimal)

$I = \square$

$A = \square + \square$
Principal (Use A-Yr 1); Interest (Q2)

$A = \square$

Third (Final) Application: Year 3

3 $I = Prn$

$= \square \times \square \times 1$
Principal (Use A-Yr 2); Rate (decimal)

$I = \square$

$A = \square + \square$
Principal (Use A-Yr 2); Interest (Q3)

$A = \square$

Compare the compounding investment (Q 1-3) to a standard simple interest investment.

4 $I = Prn$

$= \square \times \square \times 3$
\$2 000; Rate 7% p.a.

$I = \square$

$A = \square + \square$
Principal; Interest (Q4)

$A = \square$

5 Find the difference between your answer for 'A' in Q3 & Q4

Diff = (A (Q3)) - (A (Q4))

$= \square$

Now try a 2 year compounding investment. Find the account balance.

Principal \$920,
Rate = 8.4% p.a.,
n = 2 yrs, compounding annually

First Application: Year 1

6 $I = Prn$

$= \square \times \square \times 1$

$I = \square$

$A = \square + \square$

$A = \square$

Second (Final) Application: Year 2

7 $I = Prn$

$=$

$I =$

$A =$

$A =$

Compare the compounding investment (Q 6-7) to a standard simple interest investment.

8 $I = Prn$

$= \square \times \square \times 2$
\$920; Rate 8.4% p.a.

$I = \square$

$A = \square + \square$
Principal; Interest (Q8)

$A = \square$

9 Find the difference between your answer for 'A' in Q7 & Q8

Diff = (A (Q7)) - (A (Q8))

$= \square$

Now use a different approach to answering compound questions. Calculate these …..

10 *Principal* \$2 000,
Rate = 7% p.a., [1 + 0.07 = 1.07]
n = 3 yrs, compounding annually.

$A = 2\,000 \times 1.07 \times 1.07 \times 1.07$
Principal; Repeated 3 times as $n = 3$

$A = \square$

11 Is this the same as your answer in Q3? ○ Yes ○ No

12 *Principal* \$920,
Rate = 8.4% p.a., [1 + r = 1.084]
n = 2 yrs, compounding annually

$A = 920 \times 1.084 \times 1.084$
Principal; Repeated 2 times as $n = 2$

$A = \square$

13 Is this the same as your answer in Q7? ○ Yes ○ No

Investing Money - Simple Interest - Repeated Use (Sheet 2)

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Use multiplication to find the account balance more quickly. Change the rate to "1.xx", where 'xx' is the rate as a decimal. Find the account balance, A.

1 Principal \$5 000, *Rate* = 6% p.a., n = 4 yrs, compounding annually.

$A = 5\,000 \times 1.06 \times 1.06 \times 1.06 \times 1.06$

$A =$

2 Principal \$3 100, *Rate* = 9% p.a., n = 5 yrs, compounding annually.

$A = 3\,100 \times 1.09 \times 1.09 \times 1.09 \times 1.09 \times 1.09$

$A =$

3 Principal \$665, *Rate* = 5% p.a., n = 4 yrs, compounding annually.

$A =$ ☐ × ☐ × ☐ × ☐ × ☐

$A =$

Did you use 1.05? Not 0.05.

4 Principal \$10 444, *Rate* = 12% p.a., n = 3 yrs, compounding annually.

$A =$

$A =$

5 Principal \$850.70, *Rate* = 8.2% p.a., n = 2 yrs, compounding annually.

$A =$

$A =$

6 Principal \$72 310, *Rate* = 6.7% p.a., n = 4 yrs, compounding annually.

$A =$

$A =$

7 Principal \$40 700, *Rate* = 11.6% p.a., n = 3 yrs, compounding annually.

$A =$

$A =$

Instead of using "1.xx"… n times, use powers to find A. Once you find A, find I by subtracting the principal from A.

8 Principal \$6 100, *Rate* = 4.8% p.a., n = 7 yrs, compounding annually.

$A = 6\,100 \times 1.048^7 =$ ☐

Calc: 6 1 0 0 × 1 • 0 4 8 $x^{\blacksquare}$ 7 =

$I =$ ☐ (A) - ☐ (P) = ☐

9 Principal \$7 040, *Rate* = 6.6% p.a., n = 4 yrs, compounding annually.

$A = 7\,040 \times 1.066^4 =$ ☐

$I =$ ☐ - ☐ = ☐

10 Principal \$370, *Rate* = 9% p.a., n = 2 yrs, compounding annually.

$A =$ ☐ × 1. ☐☐☐ = ☐

$I =$ ☐ - ☐ = ☐

11 Principal \$24 112, *Rate* = 7.3% p.a., n = 8 yrs, compounding annually.

$A =$ ☐ × ☐☐ = ☐

$I =$ ☐ - ☐ = ☐

12 Principal \$1 770, *Rate* = 6.2% p.a., n = 5 yrs, compounding annually.

$A =$ ☐ × ☐☐ = ☐

$I =$ ☐ - ☐ = ☐

Yu invests \$5 000 at 8.3% p.a. for 6 years, compounded annually. Find his account balance and interest earned.

13

$A =$ ☐ × ☐☐ = ☐

$I =$ ☐ - ☐ = ☐

Investing Money - Compound Interest - Account Balance (Sheet 1)

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Use your calculator to evaluate these to 2 d.p.

1 $450 \times (1+0.04)^4$

$=$ ☐ Round to 2 decimal places

Calc: 4 5 0 × (1 + 0 • 0 4) $x^{\blacksquare}$ 4 =

2 $3\,277 \times (1+0.11)^5$

$=$ ☐

3 $940.30 \times (1+0.055)^7$

$=$ ☐

4 $7\,500 \times (1+0.09)^2$

$=$ ☐

Use the compound interest formula to find the account balance for these investments.

Example: Find the account balance:

A \$650 investment, at 6% p.a., for 5 years, compounded annually.

$A = P(1+r)^n$

$= 650 \times (1+0.06)^5$

$A = \$869.85$

5 \$1 450 at 8% p.a., for 3 years, compounded annually.

$A = P(1+r)^n$

$=$ ☐ $\times$ (1 + ☐) ☐

Divide R by 100

$A =$ ☐

6 \$880 at 4% p.a., for 7 years, compounded annually.

$A = P(1+r)^n$

$=$ ☐ $\times$ (1 + ☐) ☐

$A =$ ☐

7 \$566 at 6% p.a., for 4 years, compounded annually.

$A = P(1+r)^n$

$=$ ☐ $\times$ (☐ + ☐) ☐

$A =$ ☐

8 \$275 at 7% p.a., for 3 years, compounded annually.

$A = P(1+r)^n$

$=$ ☐ $\times$ (☐ + ☐) ☐

$A =$ ☐

9 \$19 233 at 5.7% p.a., for 6 years, compounded annually.

$A = P(1+r)^n$

$=$

$A =$

10 \$7 055 at 4.9% p.a., for 5 years, compounded annually.

11 \$1 020 at 8.1% p.a., for 2 years, compounded annually.

12 \$385.20 at 11.3% p.a., for 7 years, compounded annually.

If the rate and compound period and term match, the process is the same. Find A.

13 \$15 000 at 0.7% per month, for 9 months, compounded monthly.

$A = P(1+r)^n$

$P = 15\,000$, $r = 0.7 \div 100$, $n = 9$

$=$ ☐ $\times$ (☐ + ☐) ☐

Divide R by 100

$A =$ ☐

14 \$6 120 at 0.12% per week, for 28 weeks, compounded weekly.

15 \$337 at 0.09% per week, for 47 weeks, compounded weekly.

16 \$706 at 0.3% per month, for 8 months, compounded monthly.

17 \$90 050 at 0.11% per week, for 4 weeks, compounded weekly.

Investing Money - Compound Interest - Calculating the Interest (Sheet 2)

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Calculate the Account balance, then subtract the principal to find the Interest earned.

1 $2 676 at 5% p.a., for 3 years, compounded annually.

$A = P(1 + r)^n$

$= \square \times (\square + \square)^{\square}$

$A =$

$I =$ A - P

$I =$

2 $17 905 at 3.8% p.a., for 5 years, compounded annually.

$A = P(1 + r)^n$

$= \square \times (\square + \square)^{\square}$

$A =$

$I =$ -

$I =$

3 $6 650 at 8.8% p.a., for 9 years, compounded annually.

$A = P(1 + r)^n$

$A =$

$I =$

$I =$

4 $930 at 6.2% p.a., for 5 years, compounded annually.

$A =$

$A =$

Continued in the next column

$I =$

$I =$

5 $31 000 at 0.5% per month for 15 months, compounded monthly.

6 $76 000 at 0.17% per week for 9 weeks, compounded weekly.

7 $270 000 at 0.3% per month for 8 months, compounded monthly.

8 $4 706 at 0.13% per week for 42 weeks, compounded weekly.

9 Complete the table. All the investments are compounded annually.

A = principal invested × (1 + rate as decimal)^{term}

Principal Invested	*Annual Rate*	*Term (years)*	*Account Balance*	*Interest Earned*
$1 200	6%	2		
$440	5.5%	7		
$2 120	7.2%	4		
$90 300	3.9%	3		
$4 777	12%	5		
$610.90	6.6%	3		
$15 820	12%	4		
$90 000	10.5%	2		

Investing Money - Compound Interest Comparisons

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Compare the two compound interest investments, which have the same annual rate.

Ben invests $3 000 at 7.2% p.a., for 3 years, compounded annually.

1 Convert 7.2% to a decimal

$r =$ ☐ $\div 100 =$ ☐

2 Find the account balance

$A = P(1 + r)^n$

$= $ ☐ $\times ($ ☐ $+$ ☐ $)^{\square}$

$A =$ ☐ '$' sign and to 2 d.p.

Eve invests $3 000 at 0.6% per month, for 36 months, compounded monthly.

3 Convert 0.6% to a decimal

$r =$ ☐ $\div 100 =$ ☐

4 Find the account balance

$A = P(1 + r)^n$

$= $ ☐ $\times ($ ☐ $+$ ☐ $)^{\square}$

$A =$ ☐

Calculate the difference in account balances. Who earns the most?

5 Account balance difference

$=$ ☐ $-$ ☐

Largest A Smallest A

$=$ ☐ ○ Ben earns the most ○ Eve earns the most

If 3 years = 36 months and 0.6% per month = 7.2% p.a. Explain why there is a difference between Ben and Eve's account balance.

6

This time compare a compound investment to a simple interest investment. Find the interest.

Rahj invests $900 at 8.2% p.a., for 5 years, compounded annually.

7 Find the interest earned

$A = P(1 + r)^n$

$=$

$A =$

$I =$ ☐ $-$ ☐

$I =$ ☐

Justin invests $900 at 8.2% p.a., for 5 years, in a simple interest investment

8 $I = Prn$

$= $ ☐ $\times$ ☐ $\times$ ☐

$I =$ ☐

Calculate the difference in interest earned. Who earns the most?

9 Difference in interest earned

$=$ ☐ $-$ ☐

$=$ ☐ ○ Rahj earns more ○ Justin earns more

Both investments above are for 5 years. Explain why one investment accrues more interest than the other.

10

Ryan can choose from one of these three, 5 year investments:
Option1: Simple interest: 9% p.a.,
Option 2: Compound interest: 9% p.a. compounding annually.
Option 3: Compound interest: 0.75% per month, compounding monthly for 60 months.
He has $4 000 to invest.

11 Find the interest earned if he chooses Option 1.

$I = Prn$

$=$

$I =$

12 Find the interest earned if he chooses Option 2.

$A = P(1 + r)^n$

$=$

$A =$

$I =$

$I =$

13 Find the interest earned if he chooses Option 3.

$A = P(1 + r)^n$

$=$

$A =$

$I =$

$I =$

Rank the options with a '1' for best performer to '3' for worst performer.

14 Option 1: 'Simple' ☐

Option 2: '5 periods' ☐

Option 3: '60 periods' ☐

Investing Money - Graphing and Comparing Simple Interest (Sheet 1)

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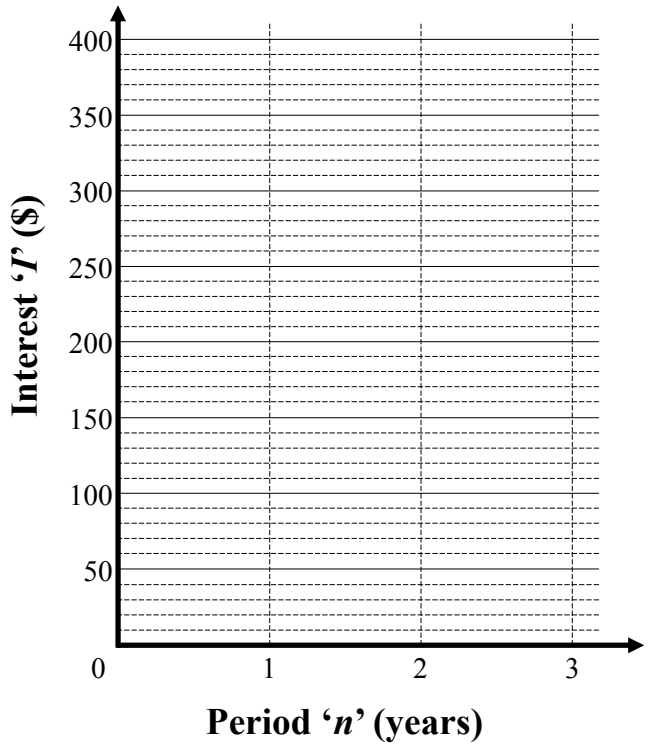
Calculate the interest earned on an investment of \$1 000. Use periods of 1 year, 2 years and 3 years. Use interest rates of 4% p.a., 8% p.a. and 12% p.a. Interest earned at time $n = 0$ is \$0. Calculate first, then graph. Plot the points in pencil first, then use a ruler as well as different coloured pens. If the lines aren't straight …. *PANIC!*



INTEREST RATE 4% p.a.

1 *Use* $P = 1\,000$, $r = 0.04$ *and* 'n' $= 1$.

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

2 *Use* 'n' $= 2$.

$I = Prn$

3 *Use* 'n' $= 3$.

$I = Prn$

Plot the 4% line (own colour). Write '$R = 4\%$' on the line.

INTEREST RATE 8% p.a.

4 $I = Prn$

$= \square \times \square \times \square$

$I = \square$

5 *Use* 'n' $= 2$.

6 *Use* 'n' $= 3$.

Plot the 8% line (own colour). Write '$R = 8\%$' on the line.

INTEREST RATE 12% p.a.

7 $I = Prn$

$= \square \times \square \times \square$

$I = \square$

8 *Use* 'n' $= 2$.

9 *Use* 'n' $= 3$.

Plot the 12% line (own colour). Write '$R = 12\%$' on the line.

Use the graph to find these. Each vertical jump in interest is worth \$10. So count the jumps, then × 10!

10 The difference in interest earned after 3 years, between $R = 4\%$ p.a. and $R = 12\%$ p.a.

$= \square - \square$

I at $n=3$, $R=12\%$ | I at $n=3$, $R=4\%$

$= \square$

11 The difference in interest earned after 2 years, between $R = 8\%$ p.a. and $R = 12\%$ p.a.

$= \square - \square = \square$

12 The difference in interest earned after 1 year, between $R = 4\%$ p.a. and $R = 8\%$ p.a.

$= \square - \square = \square$

13 The difference in interest earned after 2 years, between $R = 4\%$ p.a. and $R = 12\%$ p.a.

$= \square - \square$

$= \square$

Investing Money - Graphing and Comparing Compound Interest (Sheet 2)

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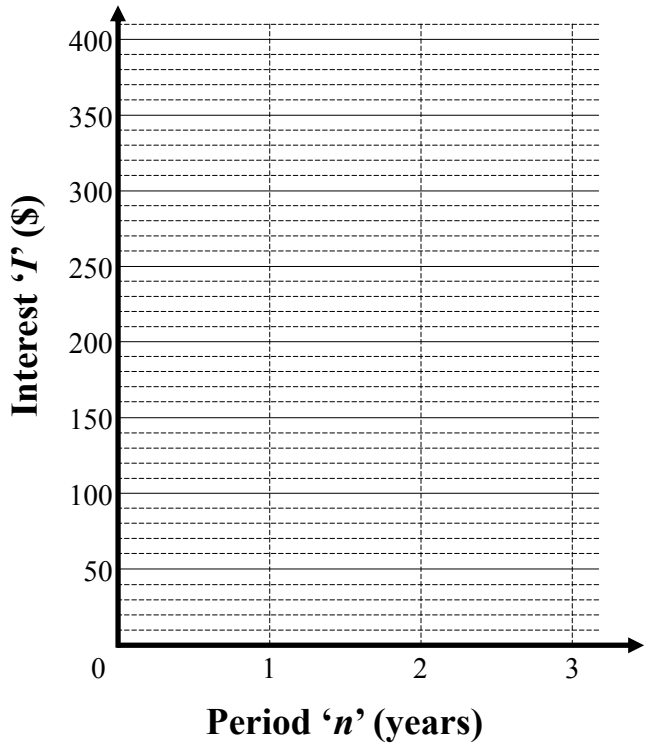
Calculate the compound interest earned on an investment of \$1 000. Use periods of 1 year, 2 years and 3 years. Use interest rates of 4% p.a., 8% p.a. and 12% p.a. Interest earned at time $n = 0$ is \$0. Calculate first, then graph. Plot the points in pencil first, then sketch curves using different coloured pens. Round '*A*' to the nearest \$10.



INTEREST RATE 4% p.a. Compounded Annually

1 *Use* $P = 1\,000$, $r = 0.04$ *and* 'n' $= 1$.

$A = P(1 + r)^n$

$= \square \times (1 + \square)^{\square}$

$A = \square$

$I = \square - 1\,000$

$I = \square$

2 *Use* 'n' $= 2$.

$A = P(1 + r)^n$

$A =$

Round A to nearest \$10

$I =$

$I =$

3 *Use* 'n' $= 3$.

$A = P(1 + r)^n$

$A =$

Round A to nearest \$10

$I =$

$I =$

Plot the 4% curve (own colour). Write '$R = 4\%$' on the curve.

INTEREST RATE 8% p.a. Compounded Annually

Use 'n' $= 1$.

4 $A = P(1 + r)^n$

$= \square \times (1 + \square)^{\square}$

$A = \square$

$I = \square - 1\,000$

$I = \square$

5 *Use* 'n' $= 2$.

$A =$

$A =$

Round A to nearest \$10

$I =$

$I =$

6 *Use* 'n' $= 3$.

Round A to nearest \$10

Plot the 8% curve (own colour). Write '$R = 8\%$' on the curve.

INTEREST RATE 12% p.a. Compounded Annually

7 *Use* 'n' $= 1$.

$A =$

$I =$

$I =$

$A =$

8 *Use* 'n' $= 2$.

9 *Use* 'n' $= 3$.

Plot the 12% curve (own colour). Write '$R = 12\%$' on the curve.

Investing Money - Interest Calculation Using Tables (Sheet 1)

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The table below is for the simple interest investment of $1. Use the table to answer the questions.

Periods (n)	Simple Interest Earned ($) on a $1 Investment									
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$	$R = 8\ \%$	$R = 9\ \%$	$R = 10\ \%$
1	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
2	0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20
3	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.30
4	0.04	0.08	0.12	0.16	0.20	0.24	0.28	0.32	0.36	0.40
5	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
6	0.06	0.12	0.18	0.24	0.30	0.36	0.42	0.48	0.54	0.60
7	0.07	0.14	0.21	0.28	0.35	0.42	0.49	0.56	0.63	0.70
8	0.08	0.16	0.24	0.32	0.40	0.48	0.56	0.64	0.72	0.80
9	0.09	0.18	0.27	0.36	0.45	0.54	0.63	0.72	0.81	0.90
10	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00

Use the table to find the interest earned on these investments.

1 $1 at 4% p.a. for 6 years

$I =$ $

2 $650 at 4% p.a. for 6 years

$I =$

Multiply your Q.1 answer × 650

3 $1 at 7% p.a. for 3 years

$I =$

4 $8 000 at 7% p.a. for 3 years

$I =$

Multiply your Q.3 answer × 8 000

5 $1 at 2% p.a. for 9 years

$I =$

6 $1 250 at 2% p.a. for 9 years

$I =$

7 $1 at 9% p.a. for 4 years

$I =$

8 $97.63 at 9% p.a. for 4 years

$I =$

9 $1 at 5% p.a. for 7 years

$I =$

10 $202.30 at 5% p.a. for 7 yrs

$I =$

Find the interest earned, then calculate the account balance. $A = P + I$

11 $495 at 3% p.a. for 7 yrs

$I =$

$A = 495 +$

$A =$

12 $1 050 at 8% p.a. for 4 yrs

$I =$

$A =$ +

$A =$

13 $305.20 at 6% p.a. for 4 yrs

$I =$

$A =$ +

$A =$

14 $290 000 at 7% p.a. for 4 yrs

$I =$

$A =$

$A =$

You can use the table the same way for other periods

15 $1 at 2% per month, for 7 months.

$I =$

16 $4 300 at 2% per month, for 7 months.

$I =$

$A =$

$A =$

17 $19 676 at 1% per month, for 4 months.

$I =$

$A =$

$A =$

Investing Money - Interest Calculation Using Tables (Sheet 2)

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The table below is for the compound interest investment of $1. Use the table to answer the questions.

Periods (n)	Compound Interest Earned ($) on a $1 Investment									
	$R = 1\ \%$	$R = 2\ \%$	$R = 3\ \%$	$R = 4\ \%$	$R = 5\ \%$	$R = 6\ \%$	$R = 7\ \%$	$R = 8\ \%$	$R = 9\ \%$	$R = 10\ \%$
1	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900	0.1000
2	0.0201	0.0404	0.0609	0.0816	0.1025	0.1236	0.1449	0.1664	0.1881	0.2100
3	0.0303	0.0612	0.0927	0.1249	0.1576	0.1910	0.2250	0.2597	0.2950	0.3310
4	0.0406	0.0824	0.1255	0.1699	0.2155	0.2625	0.3108	0.3605	0.4116	0.4641
5	0.0510	0.1041	0.1593	0.2167	0.2763	0.3382	0.4026	0.4693	0.5386	0.6105
6	0.0615	0.1262	0.1941	0.2653	0.3401	0.4185	0.5007	0.5869	0.6771	0.7716
7	0.0721	0.1487	0.2299	0.3159	0.4071	0.5036	0.6058	0.7138	0.8280	0.9487
8	0.0829	0.1717	0.2668	0.3686	0.4775	0.5938	0.7182	0.8509	0.9926	1.1436
9	0.0937	0.1951	0.3048	0.4233	0.5513	0.6895	0.8385	0.9990	1.1719	1.3579
10	0.1046	0.2190	0.3439	0.4802	0.6289	0.7908	0.9672	1.1589	1.3674	1.5937

Use the table to find the interest earned on these investments. These will require rounding.

1 $1 at 3% p.a. for 8 years

$I =$ $ Round to 2 d.p.

2 $500 at 3% p.a. for 8 years

$I =$

Use unrounded Q.1 answer × 500

3 $1 at 8% p.a. for 5 years

$I =$

4 $920 at 8% p.a. for 5 years

$I =$

Use unrounded Q.3 answer × 920

5 $1 at 6% p.a. for 10 years

$I =$

6 $205 at 6% p.a. for 10 years

$I =$

7 $1 at 2% p.a. for 7 years

$I =$

8 $140.20 at 2% p.a. for 7 yrs

$I =$

You can use the table the same way for other periods

9 $8 150 at 1% per month, for 3 months.

$I =$

10 $66 117 at 2% per month, for 10 months.

$I =$

11 $57.83 at 3% per month, for 6 months.

$I =$

$A = P + I$

Now find the account balance by adding I and P, to get A!

12 $3 088 at 5% p.a. for 4 yrs

$I =$

$A =$ +

$A =$

13 $119.25 at 7% p.a. for 8 yrs

$I =$

$A =$ +

$A =$

14 $771 at 10% p.a. for 3 yrs

$I =$

$A =$

$A =$

15 $340 200 at 4% p.a. for 4 yrs

$I =$

$A =$

$A =$

16 $805.40 at 9% p.a. for 10 yrs

$I =$

$A =$

$A =$

17 $7 111 at 6% p.a. for 2 yrs

$I =$

$A =$

$A =$

Investing Money - Introduction to Flat Rate Loans (Sheet 1)

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Get a total for these repayments.

1 12 repayments of $89.20

Total = ☐ × ☐

Total = $ ☐

Cents to 2 d.p.

2 24 repayments of $117.25

Total = ☐ × ☐

Total = ☐

$ Sign

3 Repayments of $28.30/week for 5 years.

Total = ☐ × ☐ × ☐

There are 52 weeks in a year

Total = ☐

4 Repayments of $59.16/month for 4 years.

Total = ☐ × ☐ × ☐

Total = ☐

5 Repayments of $72.25/week for 3 years.

Total =

Total =

6 Repayments of $30.67/fortnight for 5 years.

There are 26 fortnights in a year

7 Repayments of $44.05/week for 6 years.

Calculate the amount paid for these products.

Leon buys a refrigerator which sells for $1 250. He takes out a flat rate loan, with these conditions:
- $45 loan stamp duty
- $350 deposit
- $36/month repayments for 4 years

8 Find the total of the repayments

Total = ☐ × ☐ × ☐

Total = $ ☐

9 Find the full amount paid

Total = ☐ + ☐ + ☐

Stamp Duty | Deposit | Question 1

Total = $ ☐

10 How much would Leon save if he paid cash, instead of the loan?

Amount Saved = ☐ - ☐

Question 9 | Cash Price

Amount Saved = $ ☐

Lim buys a lounge suite using a flat rate loan, with these conditions:
- $60 loan stamp duty
- $475 deposit
- $93.50/mth repayments for 5 years

The lounge retails for $3 900.

11 Find the total of the repayments

Total = ☐ × ☐ × ☐

Total = ☐

12 Find the full amount paid

Total = ☐ + ☐ + ☐

Total = ☐

13 How much would Lim save if he paid cash, instead of using a loan?

Amount Saved = ☐ - ☐

Amount Saved = ☐

Jess buys a new kitchen using a flat rate loan, with these conditions:
- $72 loan stamp duty
- $2 620 deposit
- $103.15/wk repayments for 3 years

The kitchen retails for $13 100.

14 Find the total of the repayments

Total =

Total =

15 Find the full amount paid

Total =

16 How much would Jess save if she paid cash, instead of using a loan?

Amount Saved =

Amount Saved =

Eliza buys a new fishing boat using a flat rate loan, with these conditions:
- $115 loan stamp duty
- $430 loan insurance
- $3 125 deposit
- $117.50/wk repayments for 5 years

The boat retails for $25 000.

17 Find the total of the repayments

18 Find the full amount paid

19 How much would Eliza save if she paid cash, rather than using a loan?

Investing Money - Introduction to Flat Rate Loans (Sheet 2)

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Calculate the size of these deposits, using your percentage of a quantity skills.



Example: 12% deposit on a $13 400 Jet-ski.

Deposit = 0.12 × 13 400

12% is 0.12 (decimal), 12 ÷ 100 on calculator

Deposit = $1 608

1 15% deposit on a $28 000 car.

Deposit = ☐ × ☐

Deposit = ☐ Show '$' sign

2 5% deposit on a $1 720 TV.

Deposit = 0.☐☐ × ☐

Deposit = ☐

3 8% deposit on a $3 050 cruise.

Deposit =

Deposit =

4 15% deposit on a $47 111 car.

5 20% deposit on a $860 hotel stay.

6 45% deposit on a $164 700 truck.

7 12.5% deposit on a $260 juicer.

Calculate the deposit, amount paid and the amount saved if cash was used for these loans.



Jonathan buys a $1 300 laptop using a flat rate loan, with these conditions:
- $36 loan stamp duty
- 15% deposit
- $11.80/week repayments for 3 years

8 Calculate the deposit paid.

9 Find the total of the repayments

10 Find the full amount paid

11 How much would Jonathan save if he paid cash, rather than use a loan?

Lucy buys a $4 150 wardrobe using a flat rate loan, with these conditions:
- $65 loan stamp duty
- 20% deposit
- $136.45/month repayments for 4 years

12 Calculate the deposit paid.

13 Find the total of the repayments

14 Find the full amount paid

15 How much would Lucy save if she paid cash, rather than use a loan?

Jasmine buys a car for $44 676 using a flat rate loan, with these conditions:
- $65 loan stamp duty
- $580 loan insurance
- 12% deposit
- $388.90/fortnight repayments for 5 years

16 Calculate the deposit paid.

17 Find the total of the repayments

18 Find the full amount paid

19 How much would Jasmine save if she paid cash, rather than use a loan?

Investing Money - Flat Rate Loans - Monthly Payments (Sheet 1)

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Convert these amounts to monthly repayments. Either use your fraction key or ÷/brackets.

Example: Convert $2 712 into monthly payments over 5 years.

Payment = $\frac{2\ 712}{5 \times 12}$ = $45.20/mth

Calc: 2 7 1 2 ▪ 5 × 1 2 = S⇔D

1 $1 306.80 into monthly over 3 yrs

Payment = $\frac{\quad}{\quad \times \quad}$

Payment = /month

$ sign + cents to 2 d.p.

2 $950.40 into monthly over 4 yrs

Payment = $\frac{\quad}{\quad \times \quad}$

Payment = /month

3 $3 822 into monthly over 5 yrs

Payment = $\frac{\quad}{\quad \times \quad}$

Payment = /month

The next 3 questions require rounding

4 $5 675 into monthly over 3 yrs

Payment =

Payment =

5 $1 658 into monthly over 6 yrs

Payment =

Payment =

6 $3 047.90 into monthly over 2 yrs

Calculate the interest payable, the total, then convert this to a monthly repayment amount.

$4 850 is borrowed for 3 years at 9% p.a., with monthly repayments.

7 Calculate the interest payable

$I = Prn$

$= \quad \times \quad \times$

Amount | 9% decimal | years

$I =$

8 Calculate the amount owing

$A =$ Amount borrowed + Interest (Q7)

$A =$

9 Convert to monthly payments

Payment = $\frac{\quad \leftarrow (Q8)}{\quad \times \quad \swarrow 3 \times 12}$

Payment = /month

$1 115 is borrowed for 2 years at 5% p.a., with monthly repayments.

10 Calculate the interest payable

$I = Prn$

$= \quad \times \quad \times$

Amount | 5% decimal | years

$I =$

11 Calculate the amount owing

$A = \quad +$

$A =$

12 Convert to monthly payments

Payment = $\frac{\quad}{\quad \times \quad}$

Payment =

A spa costs $7 780 and is bought using a flat rate loan for 4 years at 12% p.a. Calculate the equal monthly repayments over the 4 years.

13 Find the interest payable

$I = Prn$

$= \quad \times \quad \times$

$I =$

14 Calculate the amount owing

$A = \quad +$

$A =$

15 Convert to monthly payments

Payment = $\frac{\quad}{\quad \times \quad}$

Payment =

A lounge costs $2 225 and is bought using a flat rate loan for 3 years at 15% p.a. Calculate the equal monthly repayments over the 3 years.

16 Find the interest payable

$I = Prn$

$= \quad \times \quad \times$

$I =$

17 Calculate the amount owing

$A = \quad +$

$A =$

18 Convert to monthly payments

Payment = $\frac{\quad}{\quad \times \quad}$

Payment =

Investing Money - Flat Rate Loans - Weekly Payments (Sheet 2)

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Convert these amounts to weekly repayments. Either use your fraction key or ÷/brackets.

Example: Convert $3 198 into weekly payments over 3 years.

Payment = $\frac{3\ 198}{3 \times 52}$ = $20.50/week

Calc: 3 1 9 8 ▪□ 3 × 5 2 = S⇔D

1 $6 895.20 into weekly over 2 yrs

Payment = $\frac{\square}{\square \times \square}$

Payment = /week

$ sign + cents to 2 d.p.

2 $3 556.80 into weekly over 4 yrs

Payment = $\frac{\square}{\square \times \square}$

Payment = /week

3 $7 927.92 into weekly over 3 yrs

Payment = $\frac{\square}{\square \times \square}$

Payment = /week

The next 3 questions require rounding

4 $4 200 into weekly over 5 yrs

Payment =

Payment =

5 $5 389 into weekly over 4 yrs

Payment =

Payment =

6 $58 222 into weekly over 6 yrs

Calculate the interest payable, the total, then convert this to a weekly repayment amount.

$12 280 is borrowed for 5 years at 8% p.a., repaid weekly.

7 Calculate the interest payable

$I = Prn$

$= \square \times \square \times \square$

Amount | 8% decimal | years

$I = \square$

8 Calculate the amount owing

$A = \square + \square$

Amount borrowed | Interest (Q7)

$A = \square$

9 Convert to weekly payments

Payment = $\frac{\square \leftarrow (Q8)}{\square \times \square \swarrow 5\times52}$

Payment = /week

$37 612 is borrowed for 3 years at 12% p.a., repaid weekly.

10 Calculate the interest payable

$I = Prn$

$= \square \times \square \times \square$

Amount | 12% decimal | years

$I = \square$

11 Calculate the amount owing

$A = \square + \square$

$A = \square$

12 Convert to weekly payments

Payment = $\frac{\square}{\square \times \square}$

Payment =

A car costs $26 006 and is purchased using a flat rate loan for 4 years at 6% p.a. Calculate the equal weekly repayments over the 4 years.

13 Find the interest payable

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

14 Calculate the amount owing

$A = \square + \square$

$A = \square$

15 Convert to weekly payments

Payment = $\frac{\square}{\square \times \square}$

Payment =

A stove costs $1 960 and is bought using a flat rate loan for 2 years at 18% p.a. Calculate the equal weekly repayments over the 2 years.

16 Find the interest payable

$I = Prn$

$= \square \times \square \times \square$

$I = \square$

17 Calculate the amount owing

$A = \square + \square$

$A = \square$

18 Convert to weekly payments

Payment = $\frac{\square}{\square \times \square}$

Payment =

Investing Money - Flat Rate Loans - Calculating Repayments (Sheet 3)

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Calculate the interest payable, the total, then convert this to the specified repayment period

$18 170 is borrowed for 4 years at 5% p.a., repaid in weekly payments

1 Calculate the interest payable

$I = Prn$

$= \square \times \square \times \square$

Amount | 5% decimal | years

$I = \square$

2 Calculate the amount owing

$A = \square + \square$

Amount borrowed | Interest (Q1)

$A = \square$

3 Convert to weekly payments

$$\text{Payment} = \frac{\square}{\square \times \square}$$

Payment = ____ /week

$40333 is borrowed for 7 years at 16% p.a., repaid in monthly payments

4 Calculate the interest payable

$I = Prn$

$=$

$I =$

5 Calculate the amount owing

$A =$

$A =$

6 Convert to monthly payments

Payment =

Payment =

A pool costs $34 449 and is purchased using a flat rate loan for 6 years at 14% p.a. Calculate the weekly repayments over the 6 years.

7 Calculate the interest payable

8 Calculate the amount owing

9 Find the weekly payments

A tractor costs $60 112 and is bought using a flat rate loan for 3 years at 9% p.a. Calculate the fortnightly repayments over the 3 years.

10 Calculate the interest payable

11 Calculate the amount owing

12 Find the fortnightly payment

A cherry-picker costs $184 290 and is purchased using a flat rate loan over 4 years at 8% p.a. Calculate the monthly repayments over the 4 years.

13 Calculate the interest payable

14 Calculate the amount owing

15 Find the monthly payments

A caravan costs $39 777 and is bought using a flat rate loan for 5 years at 11% p.a. Calculate the fortnightly repayments over the 5 years.

16 Calculate the interest payable

17 Calculate the amount owing

18 Find the fortnightly payment

Investing Money - Further Flat Rate Loans - Deposits (Sheet 1)

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A deposit is your own money, so you don't pay interest on it. Find the size of the monthly repayments on these loans.

A new garage costs $5 600 and is bought using a flat rate loan. A deposit of $1 680 is paid. The loan is for 3 years at 15% p.a. Calculate the monthly repayments over the 3 years.

1 Find the Principal borrowed

$P =$ ☐ - ☐

Sale Price Deposit

$P =$ ☐

2 Calculate the interest payable

$I = Prn$

$=$ ☐ × ☐ × ☐

Principal (Q1) 15% decimal years

$I =$ ☐ $ Sign

3 Calculate the amount owing

$A =$ ☐ + ☐

Principal (Q1) Interest (Q2)

$A =$ ☐

4 Convert to monthly payments

$\text{Monthly Payment} = \dfrac{\square}{\square \times \square}$ ← (Q3), 3×12

$=$ ☐

$ Sign + Cents to 2 d.p.

A new quad-bike costs $9 150 and is bought using a flat rate loan. A deposit of $1 830 is paid. The loan is for 5 years at 8% p.a. Calculate the monthly repayments over the 5 years.

5 Find the Principal borrowed

$P =$ ☐ - ☐

Sale Price Deposit

$P =$ ☐

6 Calculate the interest payable

$I = Prn$

$=$ ☐ × ☐ × ☐

$I =$ ☐

7 Calculate the amount owing

$A =$ ☐ + ☐

Principal (Q5) Interest (Q6)

$A =$ ☐

8 Convert to monthly payments

$\text{Monthly Payment} = \dfrac{\square}{\square \times \square}$

$=$ ☐

Scuba gear costs $3 460 and is bought with a 4 year flat rate loan. A deposit of $519 is paid. The loan interest rate is 12% p.a. Calculate the monthly repayments over the 4 years.

9 Find the Principal borrowed

$P =$

$P =$

10 Calculate the interest payable

$I = Prn$

$=$

$I =$

11 Calculate the amount owing

$A =$

$A =$

12 Convert to monthly payments

$\text{Monthly Payment} =$

$=$

Convert these payments to weekly repayments.

13 $17 565.60 over 6 years.

$\text{Weekly Payment} = \dfrac{\square}{\square \times \square}$

$=$ ☐

14 $8 477 over 3 years.

$\text{Weekly Payment} = \dfrac{\square}{\square \times \square}$

$=$ ☐

This time payments are weekly. Find the size of the weekly repayments on this loan.

A catamaran costs $46 870 and is bought with a 5 year flat rate loan. A deposit of $9 374 is paid. The loan interest rate is 6% p.a. Calculate the weekly repayments over the 5 years.

15 Find the Principal borrowed

16 Calculate the interest payable

17 Calculate the amount owing

18 Convert to weekly payments

Investing Money - Further Flat Rate Loans - Deposits (Sheet 2)

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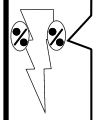

Find the size of the repayments on these loans. Watch out for the percentages, one is for the deposit, the other for the loan.

An office fit-out costs $13 444 and is financed using a flat rate loan. A deposit of 15% is paid. The loan is for 3 years at 8% p.a. Calculate the monthly repayments over the 3 years.

1 Calculate the deposit

$Dep =$ [ ] $\times$ [ ]

Purchase Price | 15% decimal

$Dep =$ [ ]

$ Sign + Cents to 2 d.p.

2 Find the Principal borrowed

$P =$ [ ] - [ ]

Sale Price | Deposit (Q1)

$P =$ [ ]

3 Calculate the interest payable

$I = Prn$

$=$ [ ] $\times$ [ ] $\times$ [ ]

Principal (Q2) | 8% decimal | years

$I =$ [ ]

4 Calculate the amount owing

$A =$ [ ] $+$ [ ]

Principal (Q2) | Interest (Q3)

$A =$ [ ]

5 Convert to monthly payments

$$\text{Monthly Payment} = \frac{\square}{\square \times \square}$$

$=$ [ ]

$ Sign + Cents to 2 d.p.

A trampoline costs $1 720 and is financed using a flat rate loan. A deposit of 10% is paid. The loan is for 2 years at 7% p.a. Calculate the weekly repayments over the 2 years.

6 Calculate the deposit

7 Find the Principal borrowed

8 Calculate the interest payable

9 Calculate the amount owing

10 Convert to weekly payments

An outboard motor costs $8 059 and is financed using a flat rate loan. A deposit of 15% is paid. The loan is for 4 years at 12% p.a. Calculate the weekly repayments over the 4 years.

11 Calculate the deposit

12 Calculate the Principal

13 Calculate the interest payable

14 Calculate the amount owing

15 Convert to weekly payments

A car costs $19 999 and is financed using a flat rate loan. A deposit of 20% is paid. The loan is for 5 years at 6% p.a. Calculate the fortnightly repayments over the 5 years.

16 Calculate the deposit

17 Find the Principal borrowed

18 Calculate the interest payable

19 Calculate the amount owing

20 Find the fortnightly payments

Investing Money - Find a Word

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Find the words in the puzzle from the wordlist.



L	A	M	I	C	E	D	B	A	L	A	N	C	E	T
T	C	B	M	A	T	U	R	E	D	R	E	D	E	N
N	C	G	A	L	A	I	O	A	A	T	O	S	S	E
U	R	N	L	B	L	O	D	N	T	I	E	M	E	V
O	U	I	O	A	L	L	S	E	R	G	S	O	F	L
C	E	C	O	M	P	A	R	E	R	E	E	N	S	O
C	C	U	R	Y	L	P	P	A	F	C	I	T	C	S
A	H	D	U	C	C	I	H	Q	F	W	N	H	H	N
D	S	E	S	A	H	C	R	U	P	E	T	L	A	I
E	A	R	L	A	U	N	N	A	M	E	E	Y	R	B
P	C	E	L	P	M	I	S	Y	B	K	R	S	R	O
O	S	N	A	O	L	R	A	T	E	L	E	M	C	R
S	W	D	N	U	O	P	M	O	C	Y	S	R	H	R
I	W	I	N	V	E	S	T	M	E	N	T	E	A	O
T	B	E	D	R	L	A	W	A	R	D	H	T	I	W

WORDLIST

RATE	PERIOD	SIMPLE	PRINCIPAL	CREDIT
INVESTMENT	BORROW	FEES	PURCHASES	CHARGES
ANNUAL	COMPOUND	LOANS	REDUCING	WEEKLY
DEPOSIT	ACCOUNT	TERM	DEBT	INSOLVENT
DECIMAL	BALANCE	INTEREST	MONTHLY	CASH
APPLY	WITHDRAWAL	COMPARE	REPAYMENTS	ACCRUE
		MATURED		

Investing Money - Topic Review

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Convert these interest rates to decimals.

1 9% = _____

2 12% = _____

3 8.4% = _____

Calculate the simple interest earned on these investments. Use: $I = Prn$

P = \$700, R = 6% p.a., n = 4 years

4 _____

P = \$1 420, R = 8.9% p.a., n = 7 years

5 _____

Calculate the simple interest earned and then the account balance.

An investment of \$47 000 at 4% p.a. for 5 years.

6 _____

An investment of \$6 112 at 1.5% per month for 8 months.

7 _____

Calculate the simple interest earned on these investments.

An investment of \$1 440 at 9% p.a. for 26 weeks.

8 _____

An investment of \$13 900 at 12% p.a. for 145 days.

9 _____

An investment of \$8 055 at 5.3% p.a. for 16 months.

10 _____

Use either 'Guess & Check' or algebraic methods to find the term of these investments

An investment of \$6 000 at 4% p.a. earns interest of \$720.

11 _____

An investment of \$430 at 7.4% p.a. earns interest of \$413.66.

12 _____

Use either 'Guess & Check' or algebraic methods to find the principal invested

An investment at 7% p.a., for 3 years earns interest of \$126. Find the amount invested.

13 _____

An investment has interest paid at the end of each year. Use $I = Prn$ twice to find the account balance at the end of a two year investment.

\$12 000 at 5.5% p.a. for 2 years.

14 _____

Find the account balance after the following compound interest investments have matured. Use $A = P(1 + r)^n$

An investment of \$2 000 at 9% p.a. for 5 years, compounded annually.

15 _____

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Find A, the account balance, for this compound investment. (Continued)

An investment of $6 020 at 1.6% per month for 10 months, compounded monthly.

16

Calculate the interest earned from these compound investments.

An investment of $750 at 8% p.a. for 4 years, compounded annually.

17

An investment of $23 770 at 0.6% per month for 15 months, compounded monthly.

18

Compare the interest earned in these two investment options.
Option 1: $3 500 invested at 6% p.a. for 7 yrs, simple interest investment.
Option 2: $3 500 invested at 6% p.a. for 7 years, compound interest investment, compounding annually.

Calculate the interest earned if Option 1 is chosen. (Simple Interest)

19

Calculate the interest earned if Option 2 is chosen. (Compound Int.)

20

Which option offers the better return and calculate the difference between the two options.

21

Express in your own words, why compound interest investments result in higher interest returns.

22

Calculate the amount paid given the flat rate loan repayments.

23 $240/month for 8 months

24 $85.90/month for 5 years

25 $58.65/week for 2 years

Calculate the amount paid given the deposit and flat rate loan repayments.

Deposit of $430, repayments of $75.36 per month for 2 years.

26

Deposit of $1 715, repayments of $89.95 per week for 4 years.

27

A car costs $28 200. It is bought using a loan charging 8% p.a. simple interest, for 4 years. The repayments are monthly.

28 Calculate the interest payable

29 Calculate the total amount to be repaid.

30 Calculate the repayments.

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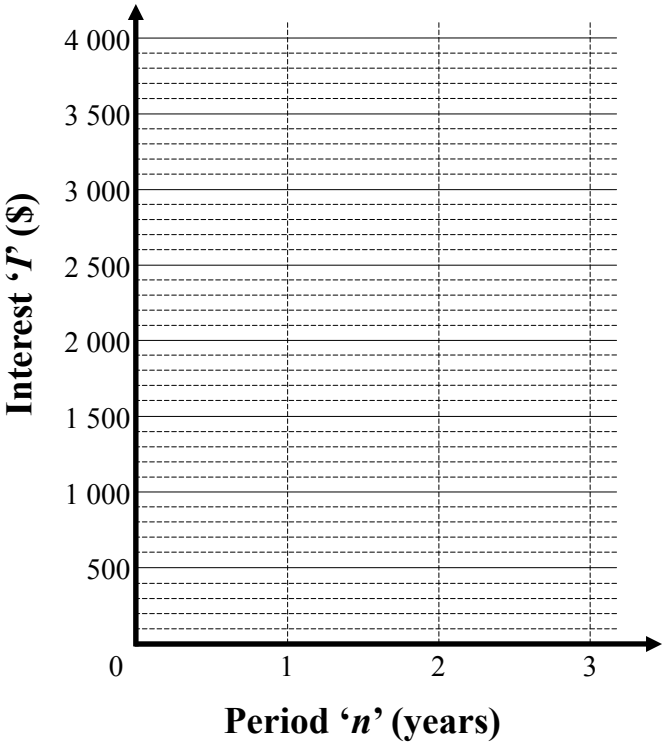
$9 000 is invested for 3 years in a simple interest account which pays 12% p.a. Plot the interest earned on the graph.



31 Calculate I for '$n = 1$'

32 Calculate I for '$n = 2$'

33 Calculate I for '$n = 3$'

34 Plot the line representing the interest earned on the investment, on the graph above.

Use the compound investment table below to answer the questions. All investments are compounded annually.

Periods (n)	Compound Interest Earned ($) on a $1 Investment									
	$R = 1$ %	$R = 2$ %	$R = 3$ %	$R = 4$ %	$R = 5$ %	$R = 6$ %	$R = 7$ %	$R = 8$ %	$R = 9$ %	$R = 10$ %
1	0.0100	0.0200	0.0300	0.0400	0.0500	0.0600	0.0700	0.0800	0.0900	0.1000
2	0.0201	0.0404	0.0609	0.0816	0.1025	0.1236	0.1449	0.1664	0.1881	0.2100
3	0.0303	0.0612	0.0927	0.1249	0.1576	0.1910	0.2250	0.2597	0.2950	0.3310
4	0.0406	0.0824	0.1255	0.1699	0.2155	0.2625	0.3108	0.3605	0.4116	0.4641
5	0.0510	0.1041	0.1593	0.2167	0.2763	0.3382	0.4026	0.4693	0.5386	0.6105
6	0.0615	0.1262	0.1941	0.2653	0.3401	0.4185	0.5007	0.5869	0.6771	0.7716

Calculate the interest earned

35 $1 at 3% p.a. for 4 years

36 $300 at 8% p.a. for 2 years

37 $90 700 at 4% p.a. for 5 yrs

Calculate the interest earned and then the account balance

38 $700 at 7% p.a. for 3 years

39 $4 500 at 5% p.a. for 6 years

VELOCITY

INVESTING MONEY

EXTENSION

Investing Money - Compound Interest Changing Periods

Compound Interest investments aren't always compounded yearly and invested for years. The investments can be for daily, weekly, monthly even quarterly and the compound period may, or may not, match these periods.

Interest rates and time periods must match the compound period

So when you are given an investment duration that doesn't match the compound period you have to change the term and interest rate (if required) to match. Subsequently these questions are more difficult to answer as you have to manipulate numbers before using the compound interest formula.

Compound Interest Formula: $A = P(1 + r)^n$

Where:
A = Account balance
P = Principal
r = Interest rate expressed as a decimal
n = Number of compound periods

Interest Calculation: $I = A - P$

Where:
I = Interest earned
A = Account balance
P = Principal

Example: Find the interest earned on an investment of \$9 000 at 1% per month, for 3 years compounded monthly.

3 years = 3 × 12 (36 month periods), so n = 36 (not 3)
1% per month = 1 ÷ 100 = 0.01

$A = P(1 + r)^n$	Compound Interest Formula
$= 9\,000 \times (1 + 0.01)^{36}$	Substitute the values, remember to use the interest rate r as a decimal
$A = \$12\,876.92$	Round to 2 decimal places
$I = 12\,876.92 - 9\,000$	Subtract 'P' from 'A' to find 'I'
$I = \$3\,876.92$	

Investing Money - Compound Interest Changing Periods (Sheet 1)

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Convert these periods to months.

1 1 year = ☐ months

2 2 years = ☐

3 ½ year = ☐

4 1½ years = ☐

Convert these periods to months and use the new number as the power to evaluate these.

$3 000 invested for **1 year compounded monthly** at 1.5% per month.

5 $A = 3\,000 \times (1 + 0.015)^{\square}$

3 0 0 0 × (1 + 0 • 0 1 5) x▪ 1 2 =

$A = \$$ ☐

Round to 2 d.p.

$1 140 invested for **2 years compounded monthly** at 0.3% per month.

6 $A = 1\,140 \times (1 + 0.003)^{\square}$

$A = \$$ ☐

$895 invested for **1½ years compounded monthly** at 0.5% per month.

7 $A = 895 \times (1 + 0.005)^{\square}$

$4 000 borrowed for 4 years compounded monthly at 1.8% per month.

8 $A = 4\,000 \times (1 + 0.018)^{\square}$

$600 borrowed for ½ a year compounded monthly at 2% per month.

9 $A = 600 \times (1 + 0.02)^{\square}$

Convert the periods to months and calculate the account balance.

Formula → Substitution → Answer $ sign

$5 000 invested for **1 year compounded monthly** at 1% per month.

10 $A = P(1 + r)^n$

$= \square \times (\square + \square)^{\square}$

$A =$ ☐

$750 invested for **2 years compounded monthly** at 0.5 % per month.

11 $A = P(1 + r)^n$

$=$

$A =$ ☐

$16 000 borrowed for 4 years compounded monthly at 3% per month.

12 $A = P(1 + r)^n$

$1 200 invested for ½ year compounded monthly at 0.7% per month.

13

$30 500 invested for 1½ years compounded monthly at 0.2% per month.

14

Now it is weekly. There are 52 weeks in a year, convert these terms to weeks, to find A.

$600 invested for **1 year compounded weekly** at 0.1% per week.

15 $A = P(1 + r)^n$

$= \square \times (\square + \square)^{\square}$

$A =$ ☐

$17 400 invested for 2 years compounded weekly at 0.2% per week.

16

$6 130 invested for 3 years compounded weekly at 0.1% per week.

17

Now biannually. That means compounded every 6 months, or, if you prefer, twice a year.

$4 085 invested for 5 years compounded biannually at 3.7% per 6 months.

18

Semiannually is the same as Biannually

$2 330 invested for 2 years compounded semiannually at 5.4% per 6 months.

19

Investing Money - Compound Interest Changing Periods (Sheet 2)

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Convert the periods to quarters and then calculate the account balance and interest earned.

Formula → Substitution → Answer $ sign

$27 500 invested for **1 year compounded quarterly** at 2% per quarter.

1 $A = P(1 + r)^n$

$= 27\,500 \times (1 + 0.02)$ ☐

$A =$ ☐

Now Calculate the Interest Earned

2 $I =$ ☐ $- 27\,500$

$I =$ ☐

$1 700 invested for **3 years compounded quarterly** at 3% per quarter.

3 ☐

Now Calculate the Interest Earned

4 $I =$ ☐ $- 1\,700$

$I =$ ☐

$3 120 invested for **5 years compounded quarterly** at 2.2% per quarter.

5 ☐

Now Calculate the Interest Earned

6 ☐

Find the interest earned and the account balance with these.

Bella is saving for a home deposit. She invests $25 400 for 3 years which is compounded monthly at 0.7% per month.

7 Express 0.7% as a decimal.

$r =$ ☐ $\div$ ☐ $=$ ☐

8 Calculate the number of compound periods in 3 years.

$n =$ ☐ $\times$ ☐ $=$ ☐

9 Find the account balance.

Dave has two investment options which both are 12.48% p.a.:

- 1.04% per month, compounded monthly
- 0.24% per week, compounded weekly

He has $5 000 to invest for 2 years.

10 Convert 1.04% to a decimal.

11 Convert 2 years to monthly compound periods.

12 Find the account balance for the 1.04% per month option.

13 Convert 0.24% to a decimal.

14 Convert 2 years to weekly compound periods.

15 Find the account balance for the 0.24% per week option.

16 Which option results in the best financial outcome?

○ 1.04% monthly ○ 0.24% weekly

17 By how much?

Georgie invests $3 000 for 4 years at 0.5% per month, compounded monthly.

18 Convert 0.5% to a decimal.

19 Find the account balance.

20 Calculate the interest earned

Investing Money - Compound Interest Changing Periods (Sheet 1)

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Convert these periods to months.

1 1 year = 12 months

2 2 years = 24 months

3 ½ year = 6 months

4 1½ years = 18 months

Convert these periods to months and use the new number as the power to evaluate these.

$3 000 invested for **1 year compounded monthly** at 1.5% per month.

5 $A = 3\,000 \times (1 + 0.015)^{12}$

3 0 0 0 × (1 + 0 • 0 1 5) x^ 1 2 =

$A = \$3\,586.85$

Round to 2 d.p.

$1 140 invested for **2 years compounded monthly** at 0.3% per month.

6 $A = 1\,140 \times (1 + 0.003)^{24}$

$A = \$1\,224.98$

$895 invested for **1½ years compounded monthly** at 0.5% per month.

7 $A = 895 \times (1 + 0.005)^{\square}$

$A = \$979.07$

$4 000 borrowed for 4 years compounded monthly at 1.8% per month.

8 $A = 4\,000 \times (1 + 0.018)^{48}$

$A = \$9\,417.88$

$600 borrowed for ½ a year compounded monthly at 2% per month.

9 $A = 600 \times (1 + 0.02)^{6}$

$A = \$675.70$

Convert the periods to months and calculate the account balance.

Formula → Substitution → Answer $ sign

$5 000 invested for **1 year compounded monthly** at 1% per month.

10 $A = P(1 + r)^n$

$= 5\,000 \times (1 + 0.01)^{12}$

$A = \$5\,634.13$

$750 invested for **2 years compounded monthly** at 0.5 % per month.

11 $A = P(1 + r)^n$

$= 750 \times (1 + 0.005)^{24}$

$A = \$845.37$

$16 000 borrowed for 4 years compounded monthly at 3% per month.

12 $A = P(1 + r)^n$

$= 16\,000 \times (1 + 0.03)^{48}$

$A = \$66\,116.03$

$1 200 invested for ½ year compounded monthly at 0.7% per month.

13 $A = P(1 + r)^n$

$= 1\,200 \times (1 + 0.007)^{6}$

$A = \$1\,251.29$

$30 500 invested for 1½ years compounded monthly at 0.2% per month.

14 $A = P(1 + r)^n$

$= 30\,500 \times (1 + 0.002)^{18}$

$A = \$31\,616.87$

Now it is weekly. There are 52 weeks in a year, convert these terms to weeks, to find A.

$600 invested for **1 year compounded weekly** at 0.1% per week.

15 $A = P(1 + r)^n$

$= 600 \times (1 + 0.001)^{52}$

$A = \$632.01$

$17 400 invested for 2 years compounded weekly at 0.2% per week.

16 $A = P(1 + r)^n$

$= 17\,400 \times (1 + 0.002)^{104}$

$A = \$21\,418.66$

$6 130 invested for 3 years compounded weekly at 0.1% per week.

17 $A = P(1 + r)^n$

$= 6\,130 \times (1 + 0.001)^{156}$

$A = \$7\,164.35$

Now biannually. That means compounded every 6 months, or, if you prefer, twice a year.

$4 085 invested for 5 years compounded biannually at 3.7% per 6 months.

18 $A = P(1 + r)^n$

$= 4\,085 \times (1 + 0.037)^{10}$

$A = \$5\,874.62$

Semiannually is the same as Biannually

$2 330 invested for 2 years compounded semiannually at 5.4% per 6 months.

19 $A = P(1 + r)^n$

$= 2\,330 \times (1 + 0.054)^{4}$

$A = \$2\,875.53$

Investing Money - Compound Interest Changing Periods (Sheet 2)

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Convert the periods to quarters and then calculate the account balance and interest earned.

Formula → Substitution → Answer $ sign

$27 500 invested for **1 year compounded quarterly** at 2% per quarter.

1 $A = P(1 + r)^n$

$= 27\,500 \times (1 + 0.02)$ 4

$A =$ $29 766.88

Now Calculate the Interest Earned

2 $I =$ 29 766.88 - 27 500

$I =$ $2 266.88

$1 700 invested for **3 years compounded quarterly** at 3% per quarter.

3 $A = P(1 + r)^n$

$= 1\,700 \times (1 + 0.03)^{12}$

$A = \$2\,423.79$

Now Calculate the Interest Earned

4 $I =$ 2 423.79 - 1 700

$I =$ $723.79

$3 120 invested for **5 years compounded quarterly** at 2.2% per quarter.

5 $A = P(1 + r)^n$

$= 3\,120 \times (1 + 0.022)^{20}$

$A = \$3\,745.53$

Now Calculate the Interest Earned

6

$I = 3\,745.53 - 3\,120$

$I = \$625.53$



Find the interest earned and the account balance with these.

Bella is saving for a home deposit. She invests $25 400 for 3 years which is compounded monthly at 0.7% per month.

7 Express 0.7% as a decimal.

$r =$ 0.7 ÷ 100 = 0.007

8 Calculate the number of compound periods in 3 years.

$n =$ 3 × 12 = 36

9 Find the account balance.

$A = P(1 + r)^n$

$= 25\,400 \times (1 + 0.007)^{36}$

$A = \$32\,650.86$

Dave has two investment options which both are 12.48% p.a.:

- 1.04% per month, compounded monthly
- 0.24% per week, compounded weekly

He has $5 000 to invest for 2 years.

10 Convert 1.04% to a decimal.

$r = 1.04 \div 100 = 0.0104$

11 Convert 2 years to monthly compound periods.

$n = 2 \times 12 = 24$

12 Find the account balance for the 1.04% per month option.

$A = P(1 + r)^n$

$= 5\,000 \times (1 + 0.0104)^{24}$

$A = \$6\,409.29$

13 Convert 0.24% to a decimal.

$r = 0.24 \div 100 = 0.0024$

14 Convert 2 years to weekly compound periods.

$n = 2 \times 52 = 104$

15 Find the account balance for the 0.24% per week option.

$A = P(1 + r)^n$

$= 5\,000 \times (1 + 0.0024)^{104}$

$A = \$6\,415.64$

16 Which option results in the best financial outcome?

○ 1.04% monthly ● 0.24% weekly

17 By how much?

$Amount = 6\,415.64 - 6\,409.29$

$Amount = \$6.35$

Georgie invests $3 000 for 4 years at 0.5% per month, compounded monthly.

18 Convert 0.5% to a decimal.

$r = 0.5 \div 100 = 0.005$

19 Find the account balance.

$A = P(1 + r)^n$

$= 3\,000 \times (1 + 0.005)^{48}$

$A = \$3\,811.47$

20 Calculate the interest earned

$I = 3\,811.47 - 3\,000$

$I = \$811.47$

Investing Money - Compound Interest Changing Periods (Homework Strip)

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Date Due: ____

$9 000 invested for 2 years compounded monthly at 1 % per month.

1 Convert 1% to a decimal

$r =$ ____ $\div 100 =$ ____

2 Calculate the number of compound periods in 2 years.

$n =$ ____ $\times$ ____ $=$ ____

3 Find the account balance.

$A = P(1 + r)^n$

$= 9\,000 \times (1 +$ ____ $)^{\square}$

$A =$ ____

$4 000 invested for 3 years compounded monthly at 1.5 % per month.

4 Find the account balance.

$A = P(1 + r)^n$

$= 4\,000 \times (1 +$ ____ $)^{\square}$

$A =$ ____

Date Due: ____

$9 000 invested for 2 years compounded monthly at 1 % per month.

1 Convert 1% to a decimal

$r =$ ____ $\div 100 =$ ____

2 Calculate the number of compound periods in 2 years.

$n =$ ____ $\times$ ____ $=$ ____

3 Find the account balance.

$A = P(1 + r)^n$

$= 9\,000 \times (1 +$ ____ $)^{\square}$

$A =$ ____

$4 000 invested for 3 years compounded monthly at 1.5 % per month.

4 Find the account balance.

$A = P(1 + r)^n$

$= 4\,000 \times (1 +$ ____ $)^{\square}$

$A =$ ____

Date Due: ____

$9 000 invested for 2 years compounded monthly at 1 % per month.

1 Convert 1% to a decimal

$r =$ ____ $\div 100 =$ ____

2 Calculate the number of compound periods in 2 years.

$n =$ ____ $\times$ ____ $=$ ____

3 Find the account balance.

$A = P(1 + r)^n$

$= 9\,000 \times (1 +$ ____ $)^{\square}$

$A =$ ____

$4 000 invested for 3 years compounded monthly at 1.5 % per month.

4 Find the account balance.

$A = P(1 + r)^n$

$= 4\,000 \times (1 +$ ____ $)^{\square}$

$A =$ ____

Date Due: ____

$9 000 invested for 2 years compounded monthly at 1 % per month.

1 Convert 1% to a decimal

$r =$ ____ $\div 100 =$ ____

2 Calculate the number of compound periods in 2 years.

$n =$ ____ $\times$ ____ $=$ ____

3 Find the account balance.

$A = P(1 + r)^n$

$= 9\,000 \times (1 +$ ____ $)^{\square}$

$A =$ ____

$4 000 invested for 3 years compounded monthly at 1.5 % per month.

4 Find the account balance.

$A = P(1 + r)^n$

$= 4\,000 \times (1 +$ ____ $)^{\square}$

$A =$ ____

Investing Money - Compound Interest Changing Periods (H'work Strip)

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Date Due:

$9 000 invested for 2 years compounded monthly at 1 % per month.

1 Convert 1% to a decimal

$r =$ 1 $\div 100 =$ 0.01

2 Calculate the number of compound periods in 2 years.

$n =$ 2 $\times$ 12 $=$ 24

3 Find the account balance.

$A = P(1 + r)^n$

$= 9\,000 \times (1 +$ 0.01 $)^{24}$

$A =$ $11 427.61

$4 000 invested for 3 years compounded monthly at 1.5 % per month.

4 Find the account balance.

$A = P(1 + r)^n$

$= 4\,000 \times (1 +$ 0.015 $)^{36}$

$A =$ $6 836.56

Investing Money - Compound Interest Changing Rates

Compound Interest investments aren't always compounded yearly and invested for years. The investments can be for daily, weekly, monthly even quarterly and the interest rate has to match the compound period.

Interest rates must match the compound period

So when you are given an interest rate that doesn't match the compound period you have to change the interest rate to match. Subsequently these questions are more difficult to answer as you have to perform a calculation before using the compound interest formula.

Compound Interest Formula: $A = P(1 + r)^n$

Where:
A = Account balance
P = Principal
r = Interest rate expressed as a decimal
n = Number of compound periods

Interest Calculation: $I = A - P$

Where:
I = Interest earned
A = Account balance
P = Principal

Example: Find the interest earned on an investment of \$3 620 at 9% p.a., for 7 months, compounded monthly.

$r = 9 \div 12 \div 100$ ← First convert the rate to monthly (decimal)

$r = 0.0075$

$A = P(1 + r)^n$ ← Compound Interest Formula

$= 3\,620 \times (1 + 0.0075)^7$ ← Substitute the values, remember to use the interest rate r as a decimal

$A = \$3\,814.38$ ← Round to 2 decimal places

$I = 3\,814.38 - 3\,620$ ← Subtract 'P' from 'A' to find 'I'

$I = \$194.38$

Investing Money - Compound Interest Changing Rates (Sheet 1)

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Change the rates to match the compound period. Then convert them to decimals.

1 10.8% p.a. to monthly

$r =$ ☐ %/month

÷ 10.8% by 12 to get monthly

$r =$ ☐.☐☐☐

÷ 100 to convert to a decimal

2 7.8% p.a. to weekly

$r =$ ☐ %/week

÷ 7.8% by 52 to get weekly

$r =$ ☐.☐☐☐☐

÷ 100 to convert to a decimal

3 6.4% p.a. to quarterly

$r =$ ☐ %/quarter

÷ 6.4% by 4 to get quarterly

$r =$ ☐

÷ 100 to convert to a decimal

4 7.2% p.a. to monthly

$r =$ ☐ %/month

$r =$ ☐

5 9.2% p.a. to quarterly

$r =$ ☐ %/quarter

$r =$ ☐

6 11.44% p.a. to weekly

$r =$ ☐ %/week

$r =$ ☐

7 9.6% p.a. to monthly

$r =$ ☐ %/month

$r =$ ☐

Convert the rates to monthly and calculate the account balance on these investments.

$8 000 is invested for 5 months and is compounded monthly at 9% p.a.

8 Find r as a decimal

$r = 9 \div 12 \div 100$

$r =$ ☐

9 $A = P(1 + r)^n$

$= 8\,000 \times (1 +$ ☐ $)^5$

8 0 0 0 × (1 + 0 • 0 0 7 5) x▪ 5 =

$A = \$$ ☐

Round to 2 d.p.

$3 400 is invested for 8 months and is compounded monthly at 14.4% p.a.

10 Find r as a decimal

$r =$ ☐ ÷ ☐ ÷ 100

$r =$ ☐

11 $A = P(1 + r)^n$

$= 3\,400 \times (1 +$ ☐ $)^8$

$A = \$$ ☐

$23 000 is invested for 15 months and is compounded monthly at 6% p.a.

12 Find r as a decimal

13

Convert the rates to weekly and calculate the account balance.

$2 500 is invested for 14 weeks and is compounded weekly at 7.8% p.a.

14 Find r as a decimal

$r =$ ☐ ÷ 52 ÷ ☐

$r =$ ☐

15

$5 390 is invested for 40 weeks and is compounded weekly at 9.36% p.a.

16

17

$770 is invested for 8 weeks and is compounded weekly at 7.28% p.a.

18

19

Calculate the interest earned on the $770 investment.

20

$I =$ ☐ - 770

$I =$

Investing Money - Compound Interest Changing Rates (Sheet 2)

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Change the rates to a quarterly decimal and then calculate the account balance.



\$650 is invested for 3 quarters and is compounded quarterly at 8% p.a.

1 $r = \square \div 4 \div \square$

$r = \square$

2 $A = P(1 + r)^n$

\$48 000 is invested for 7 quarters and is compounded quarterly at 13.6% p.a.

3

4

Now its daily, so divide the rate by 365. Find the daily rate then calculate the account balance.



\$4 450 is invested for 95 days and is compounded daily at 10.95% p.a.

5

6

\$11 250 is invested for 200 days and is compounded daily at 7.3% p.a.

7

8

Find the rate, the account balance and then the interest earned on these investments.



Eliza invests \$5 000 for 18 weeks at 6.24% p.a., compounded weekly.

9 Convert 6.24% p.a. to weekly, express as a decimal.

10 Find the account balance

11 Calculate the interest earned

$I = \square - 5\,000$

$I = \square$

12 Eliza needs \$8 000 for a car. Calculate the amount she still requires after the investment matures.

Leung invests \$7 200 for 15 months at 5.4% p.a., compounded monthly.

13 Convert 5.4% p.a. to monthly, express as a decimal.

14 Find the account balance

15 Calculate the interest earned

Omar invests \$14 900 for 300 days at 8.76% p.a., compounded daily.

16 Convert 8.76% p.a. to daily, expressing it as a decimal.

17 Find the account balance

18 Calculate the interest earned

Investing Money - Compound Interest Changing Rates (Sheet 1)

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Change the rates to match the compound period. Then convert them to decimals.

1 10.8% p.a. to monthly

$r =$ 0.9 %/month

÷ 10.8% by 12 to get monthly

$r =$ 0.009

÷ 100 to convert to a decimal

2 7.8% p.a. to weekly

$r =$ 0.15 %/week

÷ 7.8% by 52 to get weekly

$r =$ 0.0015

÷ 100 to convert to a decimal

3 6.4% p.a. to quarterly

$r =$ 1.6 %/quarter

÷ 6.4% by 4 to get quarterly

$r =$ 0.016

÷ 100 to convert to a decimal

4 7.2% p.a. to monthly

$r =$ 0.6 %/month

$r =$ 0.006

5 9.2% p.a. to quarterly

$r =$ 2.3 %/quarter

$r =$ 0.023

6 11.44% p.a. to weekly

$r =$ 0.22 %/week

$r =$ 0.0022

7 9.6% p.a. to monthly

$r =$ 0.8 %/month

$r =$ 0.0088

Convert the rates to monthly and calculate the account balance on these investments.

$8 000 is invested for 5 months and is compounded monthly at 9% p.a.

8 Find r as a decimal

$r = 9 \div 12 \div 100$

$r =$ 0.0075

9 $A = P(1 + r)^n$

$= 8\,000 \times (1 +$ 0.0075 $)^5$

8 0 0 0 × (1 + 0 • 0 0 7 5) x^y 5 =

$A = \$$ 8 304.53

Round to 2 d.p.

$3 400 is invested for 8 months and is compounded monthly at 14.4% p.a.

10 Find r as a decimal

$r =$ 14.4 ÷ 12 ÷ 100

$r =$ 0.012

11 $A = P(1 + r)^n$

$= 3\,400 \times (1 +$ 0.012 $)^8$

$A = \$$ 3 740.44

$23 000 is invested for 15 months and is compounded monthly at 6% p.a.

12 Find r as a decimal

$r = 6 \div 12 \div 100$

$r = 0.005$

13 $A = P(1 + r)^n$

$= 23\,000\,(1 + 0.005)^{15}$

$A = \$24\,786.70$

Convert the rates to weekly and calculate the account balance.

$2 500 is invested for 14 weeks and is compounded weekly at 7.8% p.a.

14 Find r as a decimal

$r =$ 7.8 ÷ 52 ÷ 100

$r =$ 0.0015

15 $A = P(1 + r)^n$

$= 2\,500 \times (1 + 0.0015)^{14}$

$A = \$2\,553.01$

$5 390 is invested for 40 weeks and is compounded weekly at 9.36% p.a.

16 $r = 9.36 \div 52 \div 100$

$r = 0.0018$

17 $A = P(1 + r)^n$

$= 5\,390 \times (1 + 0.0018)^{40}$

$A = \$5\,792.02$

$770 is invested for 8 weeks and is compounded weekly at 7.28% p.a.

18 $r = 7.28 \div 52 \div 100$

$r = 0.0014$

19 $A = P(1 + r)^n$

$= 770 \times (1 + 0.0014)^8$

$A = \$778.67$

Calculate the interest earned on the $770 investment.

20 $I =$ 778.67 - 770

$I = \$8.67$

Investing Money - Compound Interest Changing Rates (Sheet 2)

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Change the rates to a quarterly decimal and then calculate the account balance.



$650 is invested for 3 quarters and is compounded quarterly at 8% p.a.

1 $r = 8 \div 4 \div 100$

$r = 0.02$

2 $A = P(1 + r)^n$

$= 650 \times (1 + 0.02)^3$

$A = \$689.79$

$48 000 is invested for 7 quarters and is compounded quarterly at 13.6% p.a.

3 $r = 13.6 \div 4 \div 100$

$r = 0.034$

4 $A = P(1 + r)^n$

$= 48\,000 \times (1 + 0.034)^7$

$A = \$60\,657.57$



Now its daily, so divide the rate by 365. Find the daily rate then calculate the account balance.

$4 450 is invested for 95 days and is compounded daily at 10.95% p.a.

5 $r = 10.95 \div 365 \div 100$

$r = 0.0003$

6 $A = P(1 + r)^n$

$= 4\,450 \times (1 + 0.0003)^{95}$

$A = \$4\,578.63$

$11 250 is invested for 200 days and is compounded daily at 7.3% p.a.

7 $r = 7.3 \div 365 \div 100$

$r = 0.0002$

8 $A = P(1 + r)^n$

$= 11\,250 \times (1 + 0.0002)^{200}$

$A = \$11\,709.07$

Find the rate, the account balance and then the interest earned on these investments.



Eliza invests $5 000 for 18 weeks at 6.24% p.a., compounded weekly.

9 Convert 6.24% p.a. to weekly, express as a decimal.

$r = 6.24 \div 52 \div 100$

$r = 0.0012$

10 Find the account balance

$A = P(1 + r)^n$

$= 5\,000 \times (1 + 0.0012)^{18}$

$A = \$5\,109.11$

11 Calculate the interest earned

$I =$ 5 109.11 $- 5\,000$

$I =$ $109.11

12 Eliza needs $8 000 for a car. Calculate the amount she still requires after the investment matures.

$Amount = 8\,000 - 5\,109.11$

$Amount = \$2\,890.89$

Leung invests $7 200 for 15 months at 5.4% p.a., compounded monthly.

13 Convert 5.4% p.a. to monthly, express as a decimal.

$r = 5.4 \div 12 \div 100$

$r = 0.0045$

14 Find the account balance

$A = P(1 + r)^n$

$= 7\,200 \times (1 + 0.0045)^{15}$

$A = \$7\,701.61$

15 Calculate the interest earned

$I = 7\,701.61 - 7\,200$

$I = \$501.61$

Omar invests $14 900 for 300 days at 8.76% p.a., compounded daily.

16 Convert 8.76% p.a. to daily, expressing it as a decimal.

$r = 8.76 \div 365 \div 100$

$r = 0.00024$

17 Find the account balance

$A = P(1 + r)^n$

$= 14\,900 \times (1 + 0.00024)^{300}$

$A = \$16\,012.23$

18 Calculate the interest earned

$I = 16\,012.23 - 14\,900$

$I = \$1\,112.23$

Investing Money - Compound Interest Changing Rates (Homework Strip)

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Date Due: _____

$7 500 is invested for 8 months and is compounded monthly at 15.6% p.a.

1 Find r as a decimal

$r =$ _____ $\div$ _____ $\div 100$

$r =$ _____

2 $A = P(1 + r)^n$

$= 7\,500 \times (1 +$ _____ $)^{8}$

$A = \$$ _____

$9 100 is invested for 3 months and is compounded monthly at 8.4% p.a. Find the account balance and interest earned.

3 Find r as a decimal

$r =$

$r =$

4 $A = P(1 + r)^n$

$=$

$A =$

5 $I =$ _____ $- 9\,100$

$I =$

Date Due: _____

$7 500 is invested for 8 months and is compounded monthly at 15.6% p.a.

1 Find r as a decimal

$r =$ _____ $\div$ _____ $\div 100$

$r =$ _____

2 $A = P(1 + r)^n$

$= 7\,500 \times (1 +$ _____ $)^{8}$

$A = \$$ _____

$9 100 is invested for 3 months and is compounded monthly at 8.4% p.a. Find the account balance and interest earned.

3 Find r as a decimal

$r =$

$r =$

4 $A = P(1 + r)^n$

$=$

$A =$

5 $I =$ _____ $- 9\,100$

$I =$

Date Due: _____

$7 500 is invested for 8 months and is compounded monthly at 15.6% p.a.

1 Find r as a decimal

$r =$ _____ $\div$ _____ $\div 100$

$r =$ _____

2 $A = P(1 + r)^n$

$= 7\,500 \times (1 +$ _____ $)^{8}$

$A = \$$ _____

$9 100 is invested for 3 months and is compounded monthly at 8.4% p.a. Find the account balance and interest earned.

3 Find r as a decimal

$r =$

$r =$

4 $A = P(1 + r)^n$

$=$

$A =$

5 $I =$ _____ $- 9\,100$

$I =$

Date Due: _____

$7 500 is invested for 8 months and is compounded monthly at 15.6% p.a.

1 Find r as a decimal

$r =$ _____ $\div$ _____ $\div 100$

$r =$ _____

2 $A = P(1 + r)^n$

$= 7\,500 \times (1 +$ _____ $)^{8}$

$A = \$$ _____

$9 100 is invested for 3 months and is compounded monthly at 8.4% p.a. Find the account balance and interest earned.

3 Find r as a decimal

$r =$

$r =$

4 $A = P(1 + r)^n$

$=$

$A =$

5 $I =$ _____ $- 9\,100$

$I =$

Investing Money - Compound Interest Changing Rates (Homework Strip)

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Date Due:

$7 500 is invested for 8 months and is compounded monthly at 15.6% p.a.

1 Find r as a decimal

$r = 15.6 \div 12 \div 100$

$r = 0.013$

2 $A = P(1 + r)^n$

$= 7\,500 \times (1 + 0.013)^8$

$A = \$\ 8\,316.43$

$9 100 is invested for 3 months and is compounded monthly at 8.4% p.a. Find the account balance and interest earned.

3 Find r as a decimal

$r = 8.4 \div 12 \div 100$

$r = 0.007$

4 $A = P(1 + r)^n$

$= 9\,100 \times (1 + 0.007)^3$

$A = \$9\,292.44$

5 $I = 9\,292.44 - 9\,100$

$I = \$192.44$

Investing Money - Compound Interest - Calculating *P*

Compound Interest investments involve making an investment and getting your money back later with interest. If you are aiming to save for something in the future then you can determine how much to deposit now to get a future amount. Or in maths speak: Given A, what is P?

These questions are solved by substituting into the compound interest formula, but using division as the last step, as you are solving a one-step equation. An example is below.

Compound Interest Formula: $A = P(1 + r)^n$

Where:
A = Account balance
P = Principal
r = Interest rate expressed as a decimal
n = Number of compound periods

Example: Geoff wants $15 000 in three years time. An investment offers 5% p.a. for 3 years compounded annually. Calculate the amount Geoff needs to invest now.

$r = 5 \div 100$ ← First convert the rate to a decimal

$r = 0.05$

$A = P(1 + r)^n$ ← Compound Interest Formula

$15\,000 = P \times (1 + 0.05)^3$ ← Substitute the values, remember to use the interest rate r as a decimal

$15\,000 = P \times 1.157625$ ← Calculate the bracketed part

÷1.15... ÷1.15... ← Divide 'A' by the decimal

Calc: 1 5 0 0 0 ÷ Ans =

$P = \$12\,957.56$ ← Show the amount with a $ sign and to 2 d.p.

Investing Money - Compound Interest - Calculating P (Sheet 1)

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Solve these 1-step equations using division.

Example:

$1\,350 = P \times 1.2$

÷1.2 ÷1.2

$P = 1\,125$

1 $448 = P \times 1.6$

÷1.6 ÷1.6

$P =$ ____

2 $823.50 = P \times 1.35$

$P =$ ____

3 $4\,830 = P \times 1.15$

$P =$ ____

4 $39\,750 = P \times 1.06$

5 $20\,886 = P \times 1.77$

The next two have to be rounded to 2 d.p.

6 $15\,660 = P \times 1.033$

Round to 2 d.p.

7 $2\,200 = P \times 1.813$

Round to 2 d.p.

Find P, but first perform a calculation. Show the $ sign and round to 2 d.p.

Example:

$3\,500 = P \times (1 + 0.1)^4$

Calc: (1 + 0 • 1) $x^{\blacksquare}$ 4 =

$3\,500 = P \times 1.4641$

÷1.4641 ÷1.4641

$P = \$2\,390.55$

Round to 2 d.p. and show the $ sign

8 $5\,600 = P \times (1 + 0.15)^2$

Calc: (1 + 0 • 1 5) x^2 =

$5\,600 = P \times$ ____

÷1.3225 ÷1.3225

$P = \$$ ____

Round to 2 d.p.

9 $850 = P \times (1 + 0.04)^3$

$850 = P \times$ ____

÷1.12…. ÷1.12….

Calc: 8 5 0 ÷ Ans =

$P =$ ____

Round to 2 d.p. and show the $ sign

10 $4\,440 = P \times (1 + 0.06)^3$

$4\,440 = P \times$ ____

÷1.19…. ÷1.19….

$P =$ ____

11 $950 = P \times (1 + 0.05)^3$

$950 = P \times$ ____

÷1.15…. ÷1.15….

$P =$ ____

These are just the same, but this time you have to get the numbers from the question.

An investment is compounded yearly for 3 years at 10% p.a. The investment results in an account balance of $945. Find P.

12 $A = P(1 + r)^n$

____ $= P \times (1 +$ ____ $)^{\square}$

____ $= P \times$ ____

$P =$ ____

Round to 2 d.p. and show the $ sign

An investment for 2 years at 6% p.a. is compounded yearly. An investment results in an account balance of $11 000. Find P, the principal invested.

13 $A = P(1 + r)^n$

____ $= P \times (1 +$ ____ $)^{\square}$

____ $= P \times$ ____

$P =$ ____

An investment for 3 years at 8% p.a. is compounded yearly and results in an account balance of $38 200. Find P, the principal invested.

14 $A = P(1 + r)^n$

Investing Money - Compound Interest - Calculating P (Sheet 2)

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Find P, but first perform a calculation. Show the $ sign and round to 2 d.p.



Example:

An investment is compounded annually for 5 years at 7.8% p.a. Find the amount invested (P) when the account balance is $7 300.

$A = P(1 + r)^n$

$7\,300 = P \times (1 + 0.078)^5$

Calc: (1 + 0 • 0 7 8) $x^{\blacksquare}$ 5 =

$7\,300 = P \times 1.455\ldots$ (calc)

÷1.455.. ÷1.455..

Calc: 7 3 0 0 ÷ Ans =

$P = \$5\,014.52$

Round to 2 d.p. and show the $ sign

An investment is compounded annually for 4 years at 5% p.a. Find the principal when the account balance is $4 750.

1 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

A | 5% decimal | years

$\square = P \times 1.\square\square\square\ldots$ (calc)

÷ 1.215 ... ÷ 1.215 ...

$P =$

$ sign and round to 2 d.p.

An investment is compounded annually for 3 years at 7% p.a. Find the principal when the account balance is $850.

2 $A = P(1 + r)^n$

$\square = P \times \square$

$\square = P \times \square$

$P =$

Ben needs $8 000 in 3 years. He chooses an investment which offers 7% p.a. compounded annually for 3 years. Find the amount he needs to invest (P) now to achieve the $8 000 account balance.

3 $A = P(1 + r)^n$

$\square = P \times \square$

$\square = P \times \square$

$P =$

Tilly needs $15 000 to buy a car in three years time. How much does she need to invest now to have a $15 000 account balance? The investment is 5% p.a. compounded annually for three years.

4

Ryan needs $9 000 for a trip he wants to take in two years time. An investment offers interest at 4.2% p.a. compounded annually for 2 years.

5 Express 4.2% as a decimal.

6 Calculate the amount Ryan has to invest for a $9 000 account balance in two years.

Josh needs $32 000 for a new car in 3 years time. An investment offers interest at 9.3% p.a. compounded annually for 3 years.

7 Express 9.3% as a decimal.

8 Calculate the amount Josh has to invest for a $32 000 account balance in three years.

Imogen needs $45 000 for a deposit for a home purchase in 4 years time. An investment offers interest at 5.8% p.a. compounded annually for 4 years.

9 Express 5.8% as a decimal.

10 Calculate the amount Imogen has to invest now to achieve a $45 000 account balance in four years time.

11 How much interest will Imogen earn on the investment?

Investing Money - Compound Interest - Calculating P (Sheet 1)

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Solve these 1-step equations using division.

Example:

$1\,350 = P \times 1.2$

÷1.2 ÷1.2

$P = 1\,1\,25$

1 $448 = P \times 1.6$

÷1.6 ÷1.6

$P =$ 280

2 $823.50 = P \times 1.35$

÷1.35 ÷1.35

$P =$ 610

3 $4\,830 = P \times 1.15$

÷1.15 ÷1.15

$P =$ 4 200

4 $39\,750 = P \times 1.06$

÷1.06 ÷1.06

$P = 37\,500$

5 $20\,886 = P \times 1.77$

÷1.77 ÷1.77

$P = 11\,800$

The next two have to be rounded to 2 d.p.

6 $15\,660 = P \times 1.033$

÷1.033 ÷1.033

$P = 15\,159.73$

Round to 2 d.p.

7 $2\,200 = P \times 1.813$

÷1.813 ÷1.813

$P = 1\,213.46$

Round to 2 d.p.

Find P, but first perform a calculation. Show the $ sign and round to 2 d.p.

Example:

$3\,500 = P \times (1 + 0.1)^4$

Calc: (1+0•1)$x^{\blacksquare}$4=

$3\,500 = P \times 1.4641$

÷1.4641 ÷1.4641

$P =$ $2 390.55

Round to 2 d.p. and show the $ sign

8 $5\,600 = P \times (1 + 0.15)^2$

Calc: (1+0•15)x^2=

$5\,600 = P \times$ 1.3225

÷1.3225 ÷1.3225

$P =$ $ 4 234.40

Round to 2 d.p.

9 $850 = P \times (1 + 0.04)^3$

$850 = P \times$ 1.124864

÷1.12…. ÷1.12….

Calc: 850÷Ans=

$P =$ $755.65

Round to 2 d.p. and show the $ sign

10 $4\,440 = P \times (1 + 0.06)^3$

$4\,440 = P \times$ 1.191016

÷1.19…. ÷1.19….

$P =$ $3 727.91

11 $950 = P \times (1 + 0.05)^3$

$950 = P \times$ 1.157625

÷1.15…. ÷1.15….

$P =$ $820.65

These are just the same, but this time you have to get the numbers from the question.

An investment is compounded yearly for 3 years at 10% p.a. The investment results in an account balance of $945. Find P.

12 $A = P(1 + r)^n$

945 $= P \times (1 +$ 0.1 $)^{3}$

945 $= P \times$ 1.331

÷1.331 ÷1.331

$P =$ $709.99

Round to 2 d.p. and show the $ sign

An investment for 2 years at 6% p.a. is compounded yearly. An investment results in an account balance of $11 000. Find P, the principal invested.

13 $A = P(1 + r)^n$

11 000 $= P \times (1 +$ 0.06 $)^{2}$

11 000 $= P \times$ 1.1236

÷1.1236 ÷1.1236

$P =$ $9 789.96

An investment for 3 years at 8% p.a. is compounded yearly and results in an account balance of $38 200. Find P, the principal invested.

14 $A = P(1 + r)^n$

$38\,200 = P \times (1 + 0.08)^3$

$38\,200 = P \times 1.259712$

÷1.12….. ÷1.12…..

$P =$ $30 324.39

Investing Money - Compound Interest - Calculating P (Sheet 2)

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Find P, but first perform a calculation. Show the $ sign and round to 2 d.p.



Example:

An investment is compounded annually for 5 years at 7.8% p.a. Find the amount invested (P) when the account balance is $7 300.

$A = P(1 + r)^n$

$7\,300 = P \times (1 + 0.078)^5$

Calc: (1 + 0 • 0 7 8) $x^{\blacksquare}$ 5 =

$7\,300 = P \times 1.455\ldots$ (calc)

÷1.455.. ÷1.455..

Calc: 7 3 0 0 ÷ Ans =

$P = \$5\,014.52$

Round to 2 d.p. and show the $ sign

An investment is compounded annually for 4 years at 5% p.a. Find the principal when the account balance is $4 750.

1 $A = P(1 + r)^n$

$4\,750 = P \times (1 + 0.05)^4$

A | 5% decimal | years

$4\,750 = P \times 1.215$ ...(calc)

÷ 1.215 ... ÷ 1.215 ...

$P = \$3\,907.84$

$ sign and round to 2 d.p.

An investment is compounded annually for 3 years at 7% p.a. Find the principal when the account balance is $850.

2 $A = P(1 + r)^n$

$850 = P \times (1 + 0.07)^3$

$850 = P \times 1.225043$

÷1.225 ... ÷1.225 ...

$P = \$693.85$

Ben needs $8 000 in 3 years. He chooses an investment which offers 7% p.a. compounded annually for 3 years. Find the amount he needs to invest (P) now to achieve the $8 000 account balance.

3 $A = P(1 + r)^n$

$8\,000 = P \times (1 + 0.07)^3$

$8\,000 = P \times 1.225043$

÷1.225... ÷1.225...

$P = \$6\,530.38$

Tilly needs $15 000 to buy a car in three years time. How much does she need to invest now to have a $15 000 account balance? The investment is 5% p.a. compounded annually for three years.

4 $A = P(1 + r)^n$

$15\,000 = P \times (1 + 0.05)^3$

$15\,000 = P \times 1.57625$

÷1.576... ÷1.576...

$P = \$9\,516.26$

Ryan needs $9 000 for a trip he wants to take in two years time. An investment offers interest at 4.2% p.a. compounded annually for 2 years.

5 Express 4.2% as a decimal.

$r = 4.2 \div 100 = 0.042$

6 Calculate the amount Ryan has to invest for a $9 000 account balance in two years.

$A = P(1 + r)^n$

$9\,000 = P \times (1 + 0.042)^2$

$9\,000 = P \times 1.085764$

÷1.085... ÷1.085...

$P = \$8\,289.09$

Josh needs $32 000 for a new car in 3 years time. An investment offers interest at 9.3% p.a. compounded annually for 3 years.

7 Express 9.3% as a decimal.

$r = 9.3 \div 100 = 0.093$

8 Calculate the amount Josh has to invest for a $32 000 account balance in three years.

$A = P(1 + r)^n$

$32\,000 = P \times (1 + 0.093)^3$

$32\,000 = P \times 1.305$ ....(calc)

÷1.305... ÷1.305...

$P = \$24\,506.96$

Imogen needs $45 000 for a deposit for a home purchase in 4 years time. An investment offers interest at 5.8% p.a. compounded annually for 4 years.

9 Express 5.8% as a decimal.

$r = 5.8 \div 100 = 0.058$

10 Calculate the amount Imogen has to invest now to achieve a $45 000 account balance in four years time.

$A = P(1 + r)^n$

$45\,000 = P \times (1 + 0.058)^4$

$45\,000 = P \times 1.252$ ....(calc)

÷1.252... ÷1.252...

$P = \$35\,914.50$

11 How much interest will Imogen earn on the investment?

$I = 45\,000 - 35\,914.50$

$I = \$9\,085.50$

Investing Money - Compound Interest - Calculating P (Homework Strip)

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Date Due: _____

An investment for 2 years at 5% p.a. is compounded yearly. If the investment results in an account balance of $3 000, find P, the principal invested.

1 Express 5% as a decimal.

2 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

$\square = P \times \square$

$P =$

Lisa needs $5 000 for a trip she wants to take in three years time. An investment offers interest at 8% p.a. compounded annually for 3 years. Find the principal.

3 Express 8% as a decimal.

4 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

$\square = P \times \square$

$P =$

Date Due: _____

An investment for 2 years at 5% p.a. is compounded yearly. If the investment results in an account balance of $3 000, find P, the principal invested.

1 Express 5% as a decimal.

2 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

$\square = P \times \square$

$P =$

Lisa needs $5 000 for a trip she wants to take in three years time. An investment offers interest at 8% p.a. compounded annually for 3 years. Find the principal.

3 Express 8% as a decimal.

4 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

$\square = P \times \square$

$P =$

Date Due: _____

An investment for 2 years at 5% p.a. is compounded yearly. If the investment results in an account balance of $3 000, find P, the principal invested.

1 Express 5% as a decimal.

2 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

$\square = P \times \square$

$P =$

Lisa needs $5 000 for a trip she wants to take in three years time. An investment offers interest at 8% p.a. compounded annually for 3 years. Find the principal.

3 Express 8% as a decimal.

4 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

$\square = P \times \square$

$P =$

Date Due: _____

An investment for 2 years at 5% p.a. is compounded yearly. If the investment results in an account balance of $3 000, find P, the principal invested.

1 Express 5% as a decimal.

2 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

$\square = P \times \square$

$P =$

Lisa needs $5 000 for a trip she wants to take in three years time. An investment offers interest at 8% p.a. compounded annually for 3 years. Find the principal.

3 Express 8% as a decimal.

4 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

$\square = P \times \square$

$P =$

Investing Money - Compound Interest - Calculating P (Homework Strip)

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Date Due:

An investment for 2 years at 5% p.a. is compounded yearly. If the investment results in an account balance of \$3 000, find P, the principal invested.

1 Express 5% as a decimal.

$r = 5 \div 100 = 0.05$

2 $A = P(1 + r)^n$

$3\,000 = P \times (1 + 0.05)^2$

$3\,000 = P \times 1.1025$

$\div 1.1025$ $\div 1.1025$

$P =$ \$2 721.09

Lisa needs \$5 000 for a trip she wants to take in three years time. An investment offers interest at 8% p.a. compounded annually for 3 years. Find the principal.

3 Express 8% as a decimal.

$r = 8 \div 100 = 0.08$

4 $A = P(1 + r)^n$

$5\,000 = P \times (1 + 0.08)^3$

$5\,000 = P \times 1.259712$

$\div 1.259....$ $\div 1.259....$

$P =$ \$3 969.16

Investing Money - Compound Interest - Guess & Check

Compound Interest investments involve making an investment and getting your money back later with interest. If you are aiming to save for something in the future then you can determine how much to deposit now, or how long you need to invest, or what rate you need to get that future amount. Or in maths speak: Given A, find P (or r or n)?

These questions are solved by substituting guessed values into the compound interest formula, then if the answer is too high, reduced the substituted value …. If the answer is too low, then increase it. It is important that you look at how close the answer is, because if it is 'way off' then the size of the change is larger. The more questions you do, the quicker you will get the answer.

Compound Interest Formula: $A = P(1 + r)^n$

Where:
A = Account balance
P = Principal
r = Interest rate expressed as a decimal
n = Number of compound periods

Example: Geoff wants \$13 230 in two years time. He has \$12 000 to invest, what interest rate is required for him to achieve the \$13 230?

$12\,000 \times (1 + r)^2$ ← Substitute the r values, until you get a number equal to 13 230.

$13\,738.80 = 12\,000 \times (1 + 0.07)^2$ ← USING $r = 0.07$, you are over the 13 230 required

$13\,230 = 12\,000 \times (1 + 0.05)^2$ ← USING $r = 0.05$, you get the 13 230 required

$\times\ 100$

Interest rate needed is 5% ← Multiply 0.05 by 100 to convert it to a percentage and you are done.

Investing Money - Compound Interest - Guess & Check *n* & *P* (Sheet 1)

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Change the power until you get the amount (2 d.p.) in the star. Use the calculator back button.

1 576.24 Number 2 - 9

$430 \times (1+0.05)$ ☐

Calc: 4 3 0 × (1 + 0 • 0 5) $x^{\blacksquare}$? =

2 13 109.11

$11\,450 \times (1+0.07)$ ☐

3 9 326.31

$5\,720 \times (1+0.13)$ ☐

You can find the investment term by guessing, checking and guessing again. Find '*n*'.

4 $784.50 = 620 \times (1+0.04)$ ☐

5 $15\,396.41 = 13\,300 \times (1+0.05)$ ☐

6 $6\,443.20 = 3\,750 \times (1+0.07)$ ☐

7 $1\,258.82 = 800 \times (1+0.12)$ ☐

8 $4\,637.10 = 4\,000 \times (1+0.03)$ ☐

9 $6\,059.45 = 5\,195 \times (1+0.08)$ ☐

10 $14\,363 = 13\,550 \times (1+0.06)$ ☐

These are the same, but you have to build the calculation line yourself. Find '*n*'.

An investment of $720 at 5% p.a. compounded annually, results in an account balance of $1 013.11. How many years was the investment term?

11 $1\,013.11 = 720 \times (1+0.05)$ ☐

The term was for ☐ years.

An investment of $400 at 3% p.a. compounded annually, results in an account balance of $450.20. How many years was the investment term?

12 *Remember use 3% as a decimal

The term was for ☐ years.

An investment of $4 000 at 8% p.a. compounded annually, results in an account balance of $7 403.72. How many years was the investment term?

13

An investment of $7 700 at 5% p.a. compounded annually, results in an account balance of $8 913.71. How many years was the investment term?

14

Find *P*. *P* will end with a '000' or '500'. **Put *P* last** in your calculator as it is easier go back and change.

15 $6\,928.42 = \square \times (1+0.08)^3$

Calc: (1 + 0 • 0 8) $x^{\blacksquare}$ 3 ▶ × ? =

16 $7\,737.38 = \square \times (1+0.12)^7$

17 $9\,363.60 = \square \times (1+0.02)^2$

18 $1\,966.19 = \square \times (1+0.07)^4$

19 $9\,124.90 = \square \times (1+0.04)^5$

Write the calculation line yourself. Find '*P*'. It ends with '000' or '500'.

A 7 year investment results in an account balance of $9 773.60. If the interest rate was 6% p.a. compounded annually, find the amount invested.

20 *Remember use 6% as a decimal

The principal was ☐

Show $ sign

Anna wants $9 680 to buy a car in 2 years. If an investment offers 10% p.a. compounded annually, calculate the amount she needs to invest now to achieve a $9 680 account balance.

21

Investing Money - Compound Interest - Guess & Check *r* (Sheet 2)

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Change the decimal until you get the amount (2 d.p.) in the star. Use calculator back key.

1 257.37

Decimal 0.01 to 0.15

$220 \times (1 + \square)^4$

Calc: 2 2 0 × (1 + 0 • ? ?) $x^{\blacksquare}$ 4 =

2 38 019.20

$32\,000 \times (1 + \square)^2$

3 3 500.63

$1\,900 \times (1 + \square)^5$

4 6 186.75

$5\,500 \times (1 + \square)^3$

Guess and check to find the decimal, '*r*'. The Answer will be between 0.01 and 0.15.

5 $1\,133.74 = 900 \times (1 + \square)^3$

6 $3\,221.02 = 2\,000 \times (1 + \square)^5$

7 $5\,445 = 4\,500 \times (1 + \square)^2$

8 $3\,040.13 = 1\,800 \times (1 + \square)^4$

9 $1\,402.81 = 750 \times (1 + \square)^6$

This time, find the decimal but then multiply it by 100 to express it as a percentage.

10 $795.68 = 750 \times (1 + \square)^2$

Interest rate = ☐ %

Multiply your top answer by 100

11 $4\,799.38 = 3\,400 \times (1 + \square)^4$

Interest rate = ☐ %

12 $9\,076.06 = 5\,150 \times (1 + \square)^5$

Interest rate = ☐

Show % sign

13 $3\,084.88 = 1\,800 \times (1 + \square)^7$

14 $912.53 = 600 \times (1 + \square)^3$

These are the same, but you have to build the calculation line yourself. Find '*r*' as a decimal then convert to a %.

An investment of $7 000 compounded annually for 4 years, results in an account balance of $8 837.34. What was the prevailing interest rate?

15

Interest rate = ☐ %

An investment of $3 850 compounded annually for 2 years, results in an account balance of $4 829.44. What was the prevailing interest rate?

16

Interest rate = ☐ %

An investment of $15 100 compounded annually for 7 years, results in an account balance of $19 870.57. What was the interest rate applied to the investment?

17

With these two, find the required interest rate to the **nearest whole percent**.

James has $40 000 to invest for 3 years. If he wants $50 000 after the 3 years, what annual interest rate is needed. Assume investments are compounded annually.

18

Helen needs $12 000 in 5 years. She is investing $6 000 in a compound investment which compounds annually. What annual interest rate is required for her to achieve the $12 000 account balance?

19

Investing Money - Compound Interest - Guess & Check n & P (Sheet 1)

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Change the power until you get the amount (2 d.p.) in the star. Use the calculator back button.

1 — 576.24 — Number 2 - 9

$430 \times (1+0.05)^{6}$

Calc: 4 3 0 × (1 + 0 • 0 5) $x^{\blacksquare}$? =

2 — 13 109.11

$11\,450 \times (1+0.07)^{2}$

3 — 9 326.31

$5\,720 \times (1+0.13)^{4}$

You can find the investment term by guessing, checking and guessing again. Find 'n'.

4 $784.50 = 620 \times (1+0.04)^{6}$

5 $15\,396.41 = 13\,300 \times (1+0.05)^{3}$

6 $6\,443.20 = 3\,750 \times (1+0.07)^{8}$

7 $1\,258.82 = 800 \times (1+0.12)^{4}$

8 $4\,637.10 = 4\,000 \times (1+0.03)^{5}$

9 $6\,059.45 = 5\,195 \times (1+0.08)^{2}$

10 $14\,363 = 13\,550 \times (1+0.06)^{1}$

These are the same, but you have to build the calculation line yourself. Find 'n'.

An investment of \$720 at 5% p.a. compounded annually, results in an account balance of \$1 013.11. How many years was the investment term?

11 $1\,013.11 = 720 \times (1+0.05)^{7}$

The term was for 7 years.

An investment of \$400 at 3% p.a. compounded annually, results in an account balance of \$450.20. How many years was the investment term?

12 *Remember use 3% as a decimal

$450.20 = 400 \times (1+0.03)^{4}$

The term was for 4 years.

An investment of \$4 000 at 8% p.a. compounded annually, results in an account balance of \$7 403.72. How many years was the investment term?

13 $7\,403.72 = 4\,000 \times (1+0.08)^{8}$

The term was for 8 years.

An investment of \$7 700 at 5% p.a. compounded annually, results in an account balance of \$8 913.71. How many years was the investment term?

14 $8\,913.71 = 7\,700 \times (1+0.05)^{3}$

The term was for 3 years.

Find P. P will end with a '000' or '500'. **Put *P* last** in your calculator as it is easier go back and change.

15 $6\,928.42 = 5\,000 \times (1+0.08)^{3}$

Calc: (1 + 0 • 0 8) $x^{\blacksquare}$ 3 ▶ × ? =

16 $7\,737.38 = 3\,500 \times (1+0.12)^{7}$

17 $9\,363.60 = 9\,000 \times (1+0.02)^{2}$

18 $1\,966.19 = 1\,500 \times (1+0.07)^{4}$

19 $9\,124.90 = 7\,500 \times (1+0.04)^{5}$

Write the calculation line yourself. Find 'P'. It ends with '000' or '500'.

A 7 year investment results in an account balance of \$9 773.60. If the interest rate was 6% p.a. compounded annually, find the amount invested.

20 *Remember use 6% as a decimal

$9\,773.60 = 4\,000 \times (1+0.06)^{7}$

The principal was \$6 500

Show \$ sign

Anna wants \$9 680 to buy a car in 2 years. If an investment offers 10% p.a. compounded annually, calculate the amount she needs to invest now to achieve a \$9 680 account balance.

21 $9\,680 = 4\,000 \times (1+0.10)^{2}$

\$8 000 needs to be invested

Investing Money - Compound Interest - Guess & Check *r* (Sheet 2)

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Change the decimal until you get the amount (2 d.p.) in the star. Use calculator back key.

1 257.37 — Decimal 0.01 to 0.15

$220 \times (1 + \boxed{0.04})^4$

Calc: 2 2 0 × (1 + 0 • ? ?) $x^{\blacksquare}$ 4 =

2 38 019.20

$32\,000 \times (1 + \boxed{0.09})^2$

3 3 500.63

$1\,900 \times (1 + \boxed{0.13})^5$

4 6 186.75

$5\,500 \times (1 + \boxed{0.04})^3$

Guess and check to find the decimal, '*r*'. The Answer will be between 0.01 and 0.15.

5 $1\,133.74 = 900 \times (1 + \boxed{0.08})^3$

6 $3\,221.02 = 2\,000 \times (1 + \boxed{0.1})^5$

7 $5\,445 = 4\,500 \times (1 + \boxed{0.1})^2$

8 $3\,040.13 = 1\,800 \times (1 + \boxed{0.14})^4$

9 $1\,402.81 = 750 \times (1 + \boxed{0.11})^6$

This time, find the decimal but then multiply it by 100 to express it as a percentage.

10 $795.68 = 750 \times (1 + \boxed{0.03})^2$

Interest rate = 3 %

Multiply your top answer by 100

11 $4\,799.38 = 3\,400 \times (1 + \boxed{0.09})^4$

Interest rate = 9 %

12 $9\,076.06 = 5\,150 \times (1 + \boxed{0.12})^5$

Interest rate = 12%

Show % sign

13 $3\,084.88 = 1\,800 \times (1 + \boxed{0.08})^7$

Interest rate = 8%

14 $912.53 = 600 \times (1 + \boxed{0.15})^3$

Interest rate = 15%

These are the same, but you have to build the calculation line yourself. Find '*r*' as a decimal then convert to a %.

An investment of $7 000 compounded annually for 4 years, results in an account balance of $8 837.34. What was the prevailing interest rate?

15 $8\,837.34 = 7\,000 \times (1 + 0.06)^4$

Interest rate = 6 %

An investment of $3 850 compounded annually for 2 years, results in an account balance of $4 829.44. What was the prevailing interest rate?

16 $4\,829.44 = 3\,850 \times (1 + 0.12)^2$

Interest rate = 12 %

An investment of $15 100 compounded annually for 7 years, results in an account balance of $19 870.57. What was the interest rate applied to the investment?

17 $19\,870.57 = 15\,100 \times (1 + 0.04)^7$

Interest rate = 4%

With these two, find the required interest rate to the **nearest whole percent**.

James has $40 000 to invest for 3 years. If he wants $50 000 after the 3 years, what annual interest rate is needed. Assume investments are compounded annually.

18 $50\,388.48 = 40\,000 \times (1 + 0.08)^3$

Interest rate = 8%

Helen needs $12 000 in 5 years. She is investing $6 000 in a compound investment which compounds annually. What annual interest rate is required for her to achieve the $12 000 account balance?

19 $12\,068.14 = 6\,000 \times (1 + 0.15)^5$

Interest rate = 15%

Investing Money - Compound Interest - Guess & Check (Homework Strip)

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Date Due: _____

Find the value of 'n' (in the box) required to get the number in the star.

1. 1 356.78 = $855 \times (1 + 0.08)^{\square}$

Calc: 8 5 5 × (1 + 0 • 0 8) $x^{\square}$? =

2. 7 165.13 = $5\,100 \times (1 + 0.12)^{\square}$

Find the value of 'r' (in the box) required to get the number in the star.

3. 802.99 = $660 \times (1 + 0.\square\square)^{5}$

4. 36 388.80 = $28\,000 \times (1 + \square)^{2}$

5. 6 927.57 = $6\,400 \times (1 + \square)^{4}$

Date Due: _____

Find the value of 'n' (in the box) required to get the number in the star.

1. 1 356.78 = $855 \times (1 + 0.08)^{\square}$

Calc: 8 5 5 × (1 + 0 • 0 8) $x^{\square}$? =

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Date Due: _____

Find the value of 'n' (in the box) required to get the number in the star.

1. 1 356.78 = $855 \times (1 + 0.08)^{\square}$

Calc: 8 5 5 × (1 + 0 • 0 8) $x^{\square}$? =

2. 7 165.13 = $5\,100 \times (1 + 0.12)^{\square}$

Find the value of 'r' (in the box) required to get the number in the star.

3. 802.99 = $660 \times (1 + 0.\square\square)^{5}$

4. 36 388.80 = $28\,000 \times (1 + \square)^{2}$

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Investing Money - Compound Interest - Guess & Check (Homework Strip)

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Date Due: _____

Find the value of '*n*' (in the box) required to get the number in the star.

1 $\dfrac{1\,356.78}{855\times(1+0.08)^{\boxed{6}}}$

Calc: 8 5 5 × (1 + 0 • 0 8) $x^{\blacksquare}$? =

2 $\dfrac{7\,165.13}{5\,100\times(1+0.12)^{\boxed{3}}}$

Find the value of '*r*' (in the box) required to get the number in the star.

3 $\dfrac{802.99}{660\times(1+0.\boxed{0}\boxed{4})^{5}}$

4 $\dfrac{36\,388.80}{28\,000\times(1+\boxed{0.14})^{2}}$

5 $\dfrac{6\,927.57}{6\,400\times(1+\boxed{0.02})^{4}}$

Investing Money - Depreciation - Straight Line Method

When you but a new product from a shop, as soon as you open the packet, it loses its value and this continues through the life of the product. Unless it becomes rare or sought after. This is called depreciation, but the loss in value changes in rate throughout the life of the product.

Straight line depreciation is used for taxation reasons. It allows businesses to deduct a certain amount of a product's worth from their taxable income. So it is a theoretical figure rather than an actual value of a product.

$$S = V_0 - Dn$$

There are 4 variables in the formula, these are:

S = Salvage Value (or book value), which is the amount the item is worth at a given time
V_0 = Initial purchase price, the amount you paid for the item (at $n = 0$)
D = the amount the product depreciates by each year (annual)
n = the term, the number of years the salvage value is required to be based on

Example: A fishing trawler is purchased for $345 000. It is depreciated at 8% p.a. Calculate the:

a) Annual depreciation, and

b) Salvage value after 6 years

STEP 1

FIND DEPRECIATION

8 ÷ 100

a) $D = 0.08 \times 345\,000$

$D = \$27\,600$

Show the $ sign

Calculate the Depreciation

Multiply the percentage by the purchase price of the product. You have to convert the percentage to a decimal first by dividing by 100.

STEP 2

USE $S = V_0 - Dn$

b) $S = V_0 - Dn$

$= 345\,000 - 27\,600 \times 6$

$S = \$179\,400$

Calculate the Salvage value

- Write the formula
- Make the substitution
- Write the answer. Show $ and round to 2 d.p. if cents are involved.

So after 6 years the trawler is worth $179 400. Of course if it has been immaculately kept, or the opposite, if it has been damaged, the salvage value is the same. So the salvage value is a tax-related value, not a genuine value. As real-life factors aren't considered in assigning the salvage value.

Investing Money - Depreciation - Straight Line Method (Sheet 1)

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Convert these percentages to decimals by ÷ 100, or move the d.p. twice to the left.

Example:

16% = 16 ÷ 100 = 0.16

1 6% = ☐ ÷ 100 = ☐

2 8% = ☐ ÷ ☐ = ☐

3 12% = ☐ ÷ ☐ = ☐

Answer only, from now on.

4 19% = ☐

5 12.7% = ☐

6 11.6% = ☐

7 5.05% = ☐

Find a percentage of these quantities. In Mathematics 'of' means times '×'.

Example: Find 19% of $350

0.19 × 350 = $66.50

Convert 19% to a decimal

Show '$' sign and round to 2 d.p.

8 Find 7% of $420

☐ × 420 = $☐☐.☐☐

9 Find 13% of $1 200

☐ × ☐ = $☐

10 Find 3% of $4 090

☐ × ☐ = ☐

Answer with a '$' sign and cents to 2 d.p.

11 Find 6.5% of $11 500

12 Find 9.2% of $30 900

Straight Line Depreciation uses: $S = V_0 - Dn$. Calculate D, then the salvage value S.

James buys a car with a sale price of $46 900. It depreciates by 15% each year. Calculate the:

13 Annual depreciation.

$D =$ 0.15 × 46 900

$D =$ $☐☐☐☐

14 Salvage value after 3 years.

$S = V_0 - Dn$

= 46 900 - ☐ × 3

$S =$ $☐

15 Salvage value after 5 years.

$S = V_0 - Dn$

= 46 900 - ☐ × 5

$S =$ ☐

Catherine buys a delivery van with a sale price of $29 550. It depreciates by 18% each year. Calculate the:

16 Annual depreciation.

$D =$ ☐ × ☐

% as a decimal × purchase price

$D =$ ☐ $ sign!

17 Salvage value after 2 years.

$S = V_0 - Dn$

= ☐ - ☐ × ☐

Purchase price | Depreciation | Years

$S =$ ☐

Jonathan buys a forklift with a sale price of $18 998. It depreciates by 12.5% each year. Calculate the:

18 Annual depreciation.

19 Salvage value after 6 years.

Formula

Substitute values

Show $ and cents to 2 d.p.

Daisy buys a new display cabinet for $1 115. The cabinet depreciates at 16.5% p.a. Calculate the:

20 Annual depreciation.

21 Salvage value after 3 years.

Bethany buys a laptop for $1 335. The laptop has an annual depreciation rate of 22.5% p.a. Calculate the:

22 Annual depreciation.

23 Salvage value after 4 years.

Investing Money - Depreciation - Graphing S/L Depreciation (Sheet 2)

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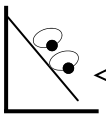


Straight line depreciation forms a straight line when graphed!!!! Calculate D, then find S and complete the table, then plot the values.

Yogi and Boo-Boo is a business that fabricates picnic baskets. A weaving machine costs \$8 000 and is depreciated at 15% p.a.

1 Calculate the annual depreciation.

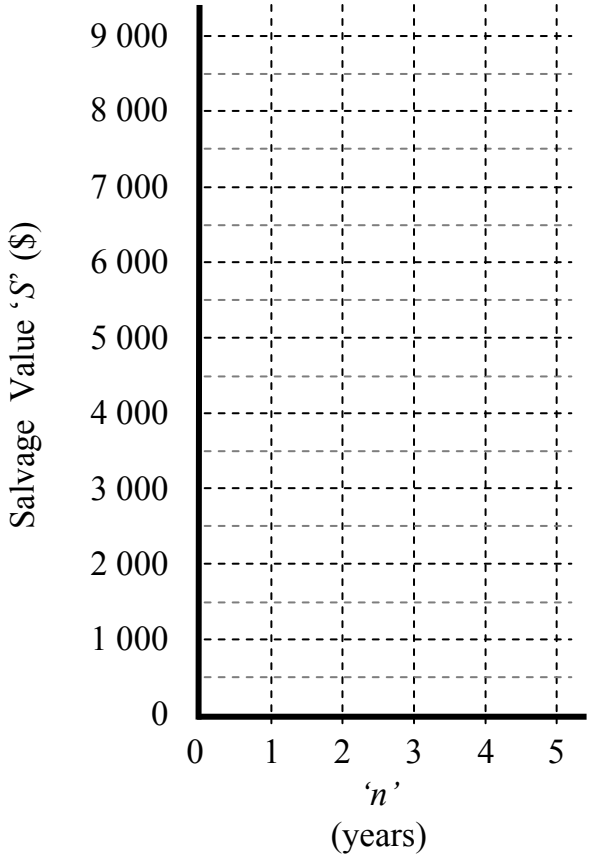
$D =$ [ ] $\times$ [ ] $=$ [ ]

[% as a decimal] [purchase price] [\$ sign]

2 Complete the table

	n Years	Salvage Value S \$
Initial Purchase Price →	0	8 000
Year 0 'S' value - D →	1	
Year 1 'S' value - D →	2	
Year 2 'S' value - D →	3	
Year 3 'S' value - D →	4	
Year 4 'S' value - D →	5	

3 Plot the points then use a ruler to construct a straight line.



A jet-ski hiring company has bought a new jet-ski for \$12 000. It is depreciated at 20% p.a.

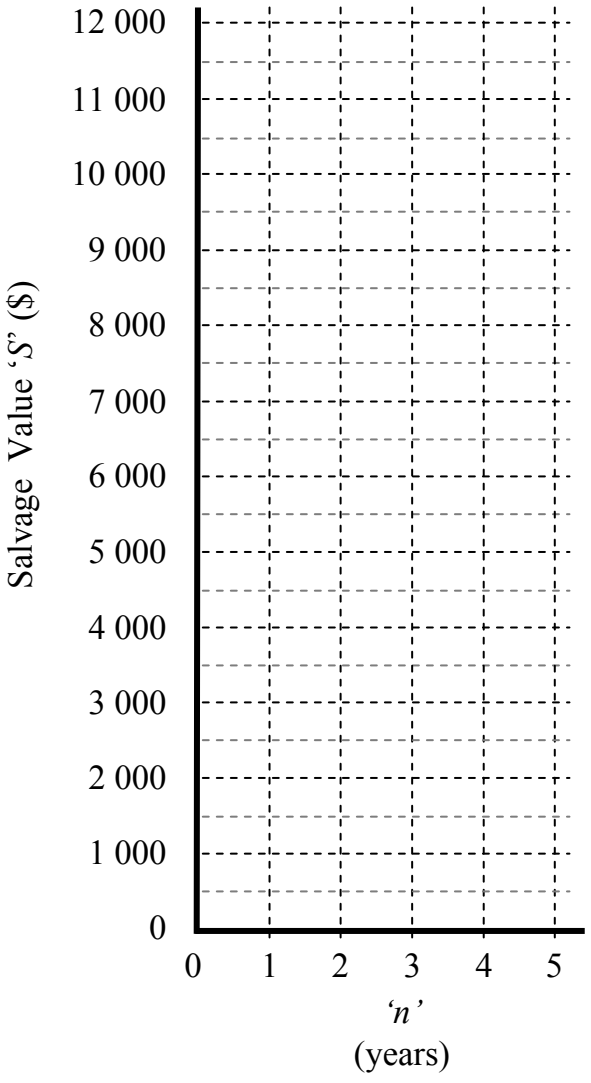
4 Calculate the annual depreciation.

$D =$ [ ] $\times$ [ ] $=$ [ ]

5 Complete the table

Year n	Salvage Value S \$
0	
1	
2	
3	
4	
5	

6 Display the results in graphical form below.



Investing Money - Depreciation - Straight Line Method (Sheet 1)

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Convert these percentages to decimals by ÷ 100, or move the d.p. twice to the left.

Example:

16% = 16 ÷ 100 = 0.16

1 6% = 6 ÷ 100 = 0.06

2 8% = 8 ÷ 100 = 0.08

3 12% = 12 ÷ 100 = 0.12

Answer only, from now on.

4 19% = 0.19

5 12.7% = 0.127

6 11.6% = 0.116

7 5.05% = 0.505

Find a percentage of these quantities. In Mathematics 'of' means times '×'.

Example: Find 19% of $350

0.19 × 350 = $66.50

Convert 19% to a decimal

Show '$' sign and round to 2 d.p.

8 Find 7% of $420

0.07 × 420 = $ 29.40

9 Find 13% of $1 200

0.13 × 1 200 = $ 156

10 Find 3% of $4 090

0.03 × 4 090 = $122.70

Answer with a '$' sign and cents to 2 d.p.

11 Find 6.5% of $11 500

0.065 × 11 500 = $747.50

12 Find 9.2% of $30 900

0.092 × 30 900 = $2 842.80

Straight Line Depreciation uses: $S = V_0 - Dn$. Calculate D, then the salvage value S.

James buys a car with a sale price of $46 900. It depreciates by 15% each year. Calculate the:

13 Annual depreciation.

$D = 0.15 \times 46\,900$

$D = \$7035$

14 Salvage value after 3 years.

$S = V_0 - Dn$

$= 46\,900 - 7\,035 \times 3$

$S = \$25\,795$

15 Salvage value after 5 years.

$S = V_0 - Dn$

$= 46\,900 - 7\,035 \times 5$

$S = \$11\,725$

Catherine buys a delivery van with a sale price of $29 550. It depreciates by 18% each year. Calculate the:

16 Annual depreciation.

$D = 0.18 \times 29\,550$

% as a decimal × purchase price

$D = \$5\,319$ $ sign!

17 Salvage value after 2 years.

$S = V_0 - Dn$

$= 29\,550 - 5\,319 \times 2$

Purchase price; Depreciation; Years

$S = \$18\,912$

Jonathan buys a forklift with a sale price of $18 998. It depreciates by 12.5% each year. Calculate the:

18 Annual depreciation.

$D = 0.125 \times 18\,998$

$D = \$2\,374.75$

19 Salvage value after 6 years.

$S = V_0 - Dn$

Formula

$= 18\,998 - 2\,374.75 \times 6$

Substitute values

$S = \$4\,749.50$

Show $ and cents to 2 d.p.

Daisy buys a new display cabinet for $1 115. The cabinet depreciates at 16.5% p.a. Calculate the:

20 Annual depreciation.

$D = 0.165 \times 1\,115$

$D = \$183.98$

21 Salvage value after 3 years.

$S = V_0 - Dn$

$= 1\,115 - 183.98 \times 3$

$S = \$563.06$

Bethany buys a laptop for $1 335. The laptop has an annual depreciation rate of 22.5% p.a. Calculate the:

22 Annual depreciation.

$D = 0.225 \times 1\,335$

$D = \$300.38$

23 Salvage value after 4 years.

$S = V_0 - Dn$

$= 1\,335 - 300.38 \times 4$

$S = \$133.48$

Investing Money - Depreciation - Graphing S/L Depreciation (Sheet 2)

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Straight line depreciation forms a straight line when graphed!!!! Calculate D, then find S and complete the table, then plot the values.

Yogi and Boo-Boo is a business that fabricates picnic baskets. A weaving machine costs \$8 000 and is depreciated at 15% p.a.

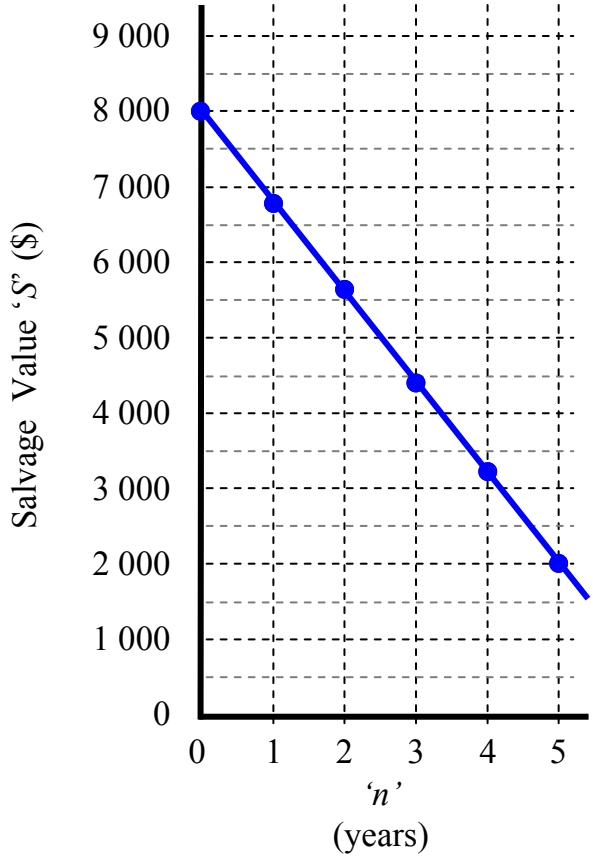
1 Calculate the annual depreciation.

$D =$ 0.15 (% as a decimal) $\times$ 8 000 (purchase price) $=$ \$1 200 (\$ sign)

2 Complete the table

	n Years	Salvage Value S \$
Initial Purchase Price →	0	8 000
Year 0 'S' value - D →	1	6 800
Year 1 'S' value - D →	2	5 600
Year 2 'S' value - D →	3	4 400
Year 3 'S' value - D →	4	3 200
Year 4 'S' value - D →	5	2 000

3 Plot the points then use a ruler to construct a straight line.



A jet-ski hiring company has bought a new jet-ski for \$12 000. It is depreciated at 20% p.a.

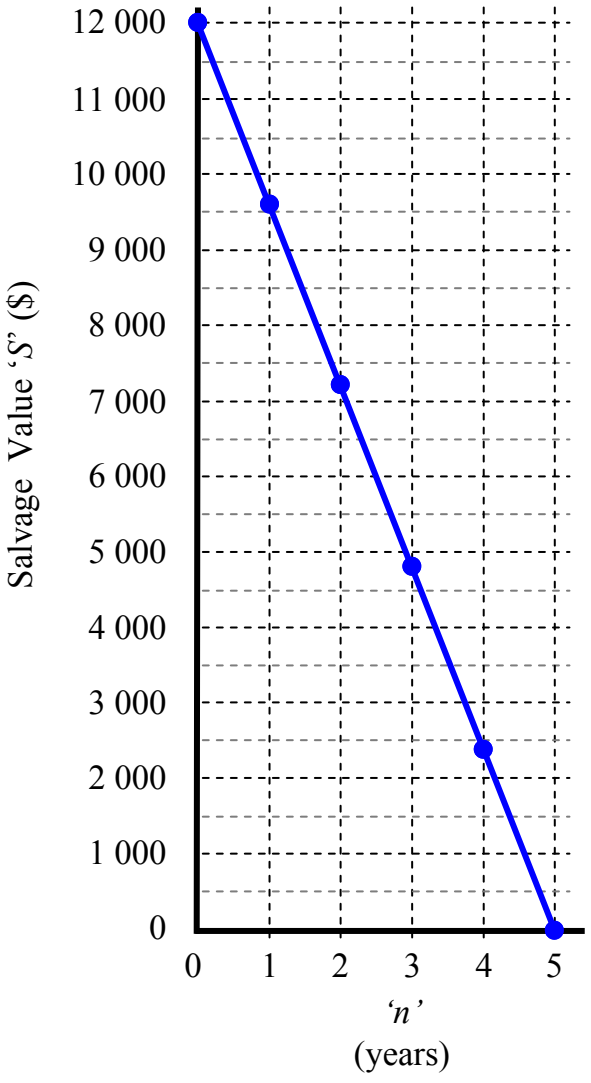
4 Calculate the annual depreciation.

$D =$ 0.2 $\times$ 12 000 $=$ \$2 400

5 Complete the table

Year n	Salvage Value S \$
0	12 000
1	9 600
2	7 200
3	4 800
4	2 400
5	0

6 Display the results in graphical form below.



Investing Money - Depreciation - Straight Line Method (Homework Strip)

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Date Due: _____

A restaurant buys a stove for $4 775. The stove has an annual depreciation rate of 15% p.a. Calculate the:

1 Annual depreciation.

$D = 0.15 \times 4\,775$

$D = \$$ _____

2 Salvage value after 4 years.

$S = V_0 - Dn$

$= 4\,775 -$ _____ $\times 4$

$S = \$$ _____

An electrician buys a new truck for $42 116. The truck depreciates at 12% p.a. Calculate the:

3 Annual depreciation.

$D =$ _____ $\times$ _____

$D = \$$ _____

4 Salvage value after 2 years.

$S = V_0 - Dn$

$=$ _____ $-$ _____ $\times$ _____

$S =$ _____

Date Due: _____

A restaurant buys a stove for $4 775. The stove has an annual depreciation rate of 15% p.a. Calculate the:

1 Annual depreciation.

$D = 0.15 \times 4\,775$

$D = \$$ _____

2 Salvage value after 4 years.

$S = V_0 - Dn$

$= 4\,775 -$ _____ $\times 4$

$S = \$$ _____

An electrician buys a new truck for $42 116. The truck depreciates at 12% p.a. Calculate the:

3 Annual depreciation.

$D =$ _____ $\times$ _____

$D = \$$ _____

4 Salvage value after 2 years.

$S = V_0 - Dn$

$=$ _____ $-$ _____ $\times$ _____

$S =$ _____

Date Due: _____

A restaurant buys a stove for $4 775. The stove has an annual depreciation rate of 15% p.a. Calculate the:

1 Annual depreciation.

$D = 0.15 \times 4\,775$

$D = \$$ _____

2 Salvage value after 4 years.

$S = V_0 - Dn$

$= 4\,775 -$ _____ $\times 4$

$S = \$$ _____

An electrician buys a new truck for $42 116. The truck depreciates at 12% p.a. Calculate the:

3 Annual depreciation.

$D =$ _____ $\times$ _____

$D = \$$ _____

4 Salvage value after 2 years.

$S = V_0 - Dn$

$=$ _____ $-$ _____ $\times$ _____

$S =$ _____

Date Due: _____

A restaurant buys a stove for $4 775. The stove has an annual depreciation rate of 15% p.a. Calculate the:

1 Annual depreciation.

$D = 0.15 \times 4\,775$

$D = \$$ _____

2 Salvage value after 4 years.

$S = V_0 - Dn$

$= 4\,775 -$ _____ $\times 4$

$S = \$$ _____

An electrician buys a new truck for $42 116. The truck depreciates at 12% p.a. Calculate the:

3 Annual depreciation.

$D =$ _____ $\times$ _____

$D = \$$ _____

4 Salvage value after 2 years.

$S = V_0 - Dn$

$=$ _____ $-$ _____ $\times$ _____

$S =$ _____

Investing Money - Depreciation - Straight Line Method (H'work Strip)

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Date Due:

A restaurant buys a stove for $4 775. The stove has an annual depreciation rate of 15% p.a. Calculate the:

1 Annual depreciation.

$D = 0.15 \times 4\,775$

$D = \$$ 716.25

2 Salvage value after 4 years.

$S = V_0 - Dn$

$= 4\,775 -$ 716.25 $\times 4$

$S = \$$ 1 910

An electrician buys a new truck for $42 116. The truck depreciates at 12% p.a. Calculate the:

3 Annual depreciation.

$D =$ 0.12 $\times$ 42 116

$D = \$$ 5 053.92

4 Salvage value after 2 years.

$S = V_0 - Dn$

$=$ 42 116 $-$ 5 053.92 $\times$ 2

$S =$ $32 008.16

Investing Money - Depreciation - Declining Balance Method in Steps

When you depreciate a 'Capital purchase' you can use two methods, the Straight Line Method or the Declining Balance Method. This sheet uses the method learned in the Straight line Method and solves Declining Balance questions using this method. In other words, you can solve declining balance questions using the straight line method.

The method isn't outlined here because this is a stepped sheet with assistance through the entire process, but this is the Decling Balance Formula:

$$S = V_0 (1 - r)^n$$

There are 4 variables in the formula, these are:

S = Salvage Value (or book value), which is the amount the item is worth at a given time
V_0 = Initial purchase price, the amount you paid for the item (at $n = 0$)
r = the rate of depreciation p.a., as a decimal
n = the term, the number of years the salvage value is required to be based on

The decling balance formula is different because:

- Smaller and smaller amounts of depreciation are deducted each year. Not the same amount each year like straight line method.
- The item is never fully depreciated, it continues to be depreciated by smaller and smaller amounts every year, forever.
- The straight line method requires you to calculate the Depreciation amount ***THEN*** the Salvage Value.
- The declining balance method requires you to calculate the Salvage Value ***THEN*** the Depreciation amount (if required).

So the calculation process order is reversed in the declining balance method.

Investing Money - Depreciation - Declining Balance - Stepped (Sheet 1)

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Calculate D, the depreciation then calculate S. Then redo this step again. D declines.

Jacqueline buys a car with a sale price of $27 100. It depreciates by 15% each year. Calculate the:

Year 1

1 Depreciation in the 1st year.

$D = 0.15 \times 27\,100$

% as a decimal × Purchase price

$D = \$$ ☐☐☐☐

2 Salvage value after 1 year.

$S = 27\,100 -$ ☐ (Q.1)

$S =$ ☐

This value is used in the next calculation

Year 2

3 Depreciation in the 2nd year.

$D = 0.15 \times$ ☐

% as a decimal × Q.2 answer

$D = \$$ ☐

4 Salvage value after 2 years.

$S =$ ☐ - ☐

Q.2 answer | Q.3 answer

$S =$ ☐

This value is used in the next calculation

Year 3

5 Depreciation in the 3rd year.

$D = 0.15 \times$ ☐

% as a decimal × Q.4 answer

$D = \$$ ☐

6 Salvage value after 3 years.

$S =$ ☐ - ☐

Q.4 answer | Q.5 answer

$S =$ ☐

This is S after 3 yr, declining balance method

This time a farmer has bought a cool room. Find S after 3 years using the D.B. method

A cool room (for storing apples), costs $17 000 and depreciates by 12% p.a. Calculate the:

Year 1

7 Depreciation in the 1st year.

$D =$ ☐ × ☐

% as a decimal × Purchase price

$D = \$$ ☐

8 Salvage value after 1 year.

$S =$ ☐ - ☐

Purchase price | Q.7 Depreciation

$S =$ ☐

Year 2

9 Depreciation in the 2nd year.

$D =$ ☐ × ☐

% as a decimal × Q.8 answer

$D = \$$ ☐

10 Salvage value after 2 years.

$S =$ ☐ - ☐

Q.8 answer | Q.9 answer

$S =$ ☐

Year 3

11 Depreciation in the 3rd year.

$D =$ ☐ × ☐

$D = \$$ ☐

12 Salvage value after 3 years.

$S =$ ☐ - ☐

$S =$ ☐

This is Salvage value after 3 years, using the declining balance method in 3 applications

Use the brackets keys and power key to evaluate these. Answer to 2 decimal places.

13 $450 \times (1 - 0.08)^4$

$=$ ☐ (Round to 2 decimal places)

Calc: 4 5 0 × (1 - 0 • 0 8) x▪ 4 =

14 $5\,500 \times (1 - 0.11)^5$

$=$ ☐

15 $38\,200 \times (1 - 0.18)^7$

$=$ ☐

16 $26\,000 \times (1 - 0.09)^3$

$=$ ☐

Now answer Columns 1 and 2 in one step. Don't get angry, its called learning! Find 'S' (2 d.p.)

A car with a sale price of $27 100 depreciates by 15% each year. Calculate S after 3 years.

17 $S = V_0 (1 - r)^n$

$=$ ☐ (1 - ☐) ☐

Purchase price | % decimal | years

$S =$ ☐

18 Is your Q.6 answer the same as your Q.17 answer?

○ Yes ○ Arrrrggghhhh!

A cool room costs $17 000 and depreciates by 12% p.a. Calculate the salvage value after 3 years.

19 $S = V_0 (1 - r)^n$

$=$ ☐ (1 - ☐) ☐

$S =$ ☐

20 Is your Q.12 answer the same as your Q.19 answer?

○ Yes ○ I decline to answer

Investing Money - Depreciation - Declining Balance - Stepped (Sheet 1)

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Calculate D, the depreciation then calculate S. Then redo this step again. D declines.

Jacqueline buys a car with a sale price of $27 100. It depreciates by 15% each year. Calculate the:

Year 1

1 Depreciation in the 1st year.

$D =$ 0.15 × 27 100

% as a decimal × Purchase price

$D =$ $ 4 0 6 5

2 Salvage value after 1 year.

$S =$ 27 100 - 4 065

$S =$ $23 035

This value is used in the next calculation

Year 2

3 Depreciation in the 2nd year.

$D =$ 0.15 × 23 035

% as a decimal × Q.2 answer

$D =$ $ 3 455.25

4 Salvage value after 2 years.

$S =$ 23 035 - 3 455.25

Q.2 answer | Q.3 answer

$S =$ $19 579.75

This value is used in the next calculation

Year 3

5 Depreciation in the 3rd year.

$D =$ 0.15 × 19 579.75

% as a decimal × Q.4 answer

$D =$ $ 2 936.96

6 Salvage value after 3 years.

$S =$ 19 579.75 - 2 936.96

Q.4 answer | Q.5 answer

$S =$ $16 642.79

This is S after 3 yr, declining balance method

This time a farmer has bought a cool room. Find S after 3 years using the D.B. method

A cool room (for storing apples), costs $17 000 and depreciates by 12% p.a. Calculate the:

Year 1

7 Depreciation in the 1st year.

$D =$ 0.12 × 17 000

% as a decimal × Purchase price

$D =$ $ 2 040

8 Salvage value after 1 year.

$S =$ 17 000 - 2 040

Purchase price | Q.7 Depreciation

$S =$ $14 960

Year 2

9 Depreciation in the 2nd year.

$D =$ 0.12 × 14 960

% as a decimal × Q.8 answer

$D =$ $ 1 795.20

10 Salvage value after 2 years.

$S =$ 14 960 - 1 795.20

Q.8 answer | Q.9 answer

$S =$ $13 164.80

Year 3

11 Depreciation in the 3rd year.

$D =$ 0.12 × 13 164.80

$D =$ $ 1 579.78

12 Salvage value after 3 years.

$S =$ 13 164.80 - 1 579.78

$S =$ $11 585.02

This is Salvage value after 3 years, using the declining balance method in 3 applications

Use the brackets keys and power key to evaluate these. Answer to 2 decimal places.

13 $450 \times (1 - 0.08)^4$

$=$ 322.38

Round to 2 decimal places

Calc: 4 5 0 × (1 - 0 • 0 8) x▪ 4 =

14 $5\,500 \times (1 - 0.11)^5$

$=$ 3 071.23

15 $38\,200 \times (1 - 0.18)^7$

$=$ 9 522.70

16 $26\,000 \times (1 - 0.09)^3$

$=$ 19 592.85

Now answer Columns 1 and 2 in one step. Don't get angry, its called learning! Find 'S' (2 d.p.)

A car with a sale price of $27 100 depreciates by 15% each year. Calculate S after 3 years.

17 $S = V_0 (1 - r)^n$

$=$ 27 100 (1 - 0.15) 3

Purchase price | % decimal | years

$S =$ $16 642.79

18 Is your Q.6 answer the same as your Q.17 answer?

● Yes ○ Arrrrggghhhh!

A cool room costs $17 000 and depreciates by 12% p.a. Calculate the salvage value after 3 years.

19 $S = V_0 (1 - r)^n$

$=$ 17 000 (1 - 0.12) 3

$S =$ $11 585.02

20 Is your Q.12 answer the same as your Q.19 answer?

● Yes ○ I decline to answer

Investing Money - Depreciation - Declining Balance - Stepped (Homework Strip)

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Date Due:

A $16 550 van is depreciated by 12% p.a. Calculate the salvage value by the declining balance method in steps.

Year 1

1 Depreciation in the 1st year.

$D = 0.12 \times 16\,550$

$D = \$$

2 Salvage value after 1 year.

$S = 16\,550 -$ (Q.1)

$S =$

Year 2

3 Depreciation in the 2nd year.

$D = 0.12 \times$ (Q.2)

$D = \$$

4 Salvage value after 2 years.

$S =$ (Q.2 answer) $-$ (Q.3 answer)

$S =$

5 Calculate: $16\,550\,(1 - 0.12)^2$

$=$

Date Due:

A $16 550 van is depreciated by 12% p.a. Calculate the salvage value by the declining balance method in steps.

Year 1

1 Depreciation in the 1st year.

$D = 0.12 \times 16\,550$

$D = \$$

2 Salvage value after 1 year.

$S = 16\,550 -$ (Q.1)

$S =$

Year 2

3 Depreciation in the 2nd year.

$D = 0.12 \times$ (Q.2)

$D = \$$

4 Salvage value after 2 years.

$S =$ (Q.2 answer) $-$ (Q.3 answer)

$S =$

5 Calculate: $16\,550\,(1 - 0.12)^2$

$=$

Date Due:

A $16 550 van is depreciated by 12% p.a. Calculate the salvage value by the declining balance method in steps.

Year 1

1 Depreciation in the 1st year.

$D = 0.12 \times 16\,550$

$D = \$$

2 Salvage value after 1 year.

$S = 16\,550 -$ (Q.1)

$S =$

Year 2

3 Depreciation in the 2nd year.

$D = 0.12 \times$ (Q.2)

$D = \$$

4 Salvage value after 2 years.

$S =$ (Q.2 answer) $-$ (Q.3 answer)

$S =$

5 Calculate: $16\,550\,(1 - 0.12)^2$

$=$

Date Due:

A $16 550 van is depreciated by 12% p.a. Calculate the salvage value by the declining balance method in steps.

Year 1

1 Depreciation in the 1st year.

$D = 0.12 \times 16\,550$

$D = \$$

2 Salvage value after 1 year.

$S = 16\,550 -$ (Q.1)

$S =$

Year 2

3 Depreciation in the 2nd year.

$D = 0.12 \times$ (Q.2)

$D = \$$

4 Salvage value after 2 years.

$S =$ (Q.2 answer) $-$ (Q.3 answer)

$S =$

5 Calculate: $16\,550\,(1 - 0.12)^2$

$=$

Investing Money - Depreciation - Declining Balance - Stepped (HW Strip)

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Date Due:

A $16 550 van is depreciated by 12% p.a. Calculate the salvage value by the declining balance method in steps.

Year 1

1 Depreciation in the 1st year.

$D = 0.12 \times 16\,550$

$D = \$\ 1\,986$

2 Salvage value after 1 year.

$S = 16\,550 - 1\,986$ (Q.1)

$S = \$14\,564$

Year 2

3 Depreciation in the 2nd year.

$D = 0.12 \times 14\,564$ (Q.2)

$D = \$\ 1\,747.68$

4 Salvage value after 2 years.

$S =$ 14 564 (Q.2 answer) $-$ 1 747.68 (Q.3 answer)

$S = \$12\,816.32$

5 Calculate: $16\,550\,(1 - 0.12)^2$

$= \$12\,816.32$

Investing Money - Depreciation - Declining Balance Method

When you but a new product from a shop, as soon as you open the packet, it loses its value and this continues through the life of the product. Unless it becomes rare or sought after. This is called depreciation, but the loss in value changes in rate throughout the life of the product.

Declining balance depreciation is used for taxation reasons. It allows businesses to deduct a certain amount of a product's worth from their taxable income. So it is a theoretical figure rather than an actual value of a product.

$$S = V_0 (1 - r)^n$$

There are 4 variables in the formula, these are:

S = Salvage Value (or book value), which is the amount the item is worth at a given time
V_0 = Initial purchase price, the amount you paid for the item (at $n = 0$)
r = the rate of depreciation p.a., as a decimal
n = the term, the number of years the salvage value is required to be based on

Example: A fishing trawler is purchased for $345 000. It is depreciated at 8% p.a. Calculate the:

a) Salvage value after 4 years

b) The amount the trawler has depreciated over the 4 years.

STEP 1

FIND SALVAGE VALUE

a) $S = V_0 (1 - r)^n$

$= 345\,000 (1 - 0.08)^4$

Calc: 345000(1-0.08)x^4=

$S = \$247\,155.57$

Show the $ sign

Calculate the Salvage value

- Write the formula
- Make the substitution
- Write the answer. Show $ and round to 2 d.p. if cents are involved.

STEP 2

FIND THE DEPRECIATION

b) $D = 345\,000 - 247\,155.57$

$D = \$97\,844.43$

Show the $ sign

Calculate the Depreciation

Subtract the salvage value from the initial purchase price to calculate the amount the item has depreciated.

So after 6 years the trawler is worth $247 155.57. Of course if it has been immaculately kept, or the opposite, if it has been damaged, the calculated salvage value is the same. So the salvage value is a tax related value, not a genuine value. As real-life factors aren't considered in assigning the salvage value.

Investing Money - Depreciation - Declining Balance Method (Sheet 1)

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Use the brackets and power keys to solve these. Answer to 2 d.p.

1 $42\,700 \times (1 - 0.09)^5$

$=$ ____

4 2 7 0 0 × (1 - 0 • 0 9) $x^{\blacksquare}$ 5 =

2 $19\,804 \times (1 - 0.06)^3$

$=$ ____

3 $1\,447 \times (1 - 0.15)^6$

$=$ ____

4 $20\,550 \times (1 - 0.125)^4$

$=$ ____

Calculate the salvage value for these purchases. Answer to 2 d.p.

A car with a sale price of $30 499 depreciates by 15% each year. Calculate S after 3 years.

5 $S = V_0 (1 - r)^n$

$=$ ____ × (1 - ____) ☐

Purchase price | % decimal | years

$S = \$$ ☐☐ ☐☐☐ . ☐☐

Calculate S after 5 years.

6 $S = V_0 (1 - r)^n$

$=$ ____ × (1 - ____) ☐

$S = \$$ ____

Calculate S after 9 years.

7 $S = V_0 (1 - r)^n$

$=$ ____ × (1 - ____) ☐

$S =$ ____

$ Sign!

A table saw with a sale price of $2 100 depreciates by 20% each year. Calculate S after 3 years.

8 ____

Formula

____

Substitute Values

____

Answer to 2 d.p. showing the $ sign

Calculate S after 7 years.

9 ____

A new trench digger costs $9 030 and depreciates by 15% each year. Calculate S after 2 years.

10 ____

Calculate S after 4 years.

11 ____

A café's coffee machine costs $3 046 and depreciates by 9% each year. Calculate S after 3 years.

12 ____

Calculate S after 5 years.

13 ____

Now calculate the salvage value, then subtract it from the purchase price to calculate D.

A cherry picker costs $48 771 and depreciates by 16% each year. Calculate S after 2 years.

14 $S = V_0 (1 - r)^n$

Find the depreciation after 2 years

15 $D = 48\,771 -$ ____

Q.14

$D =$ ____

A Hair salon chair and wash basin costs $4 330 and is depreciated at 22% p.a. Calculate the salvage value after 5 years.

16 ____

Find the depreciation after 5 years

17 $D = 4\,330 -$ ____

$D =$ ____

A complete shop re-fit costs $18 667 and depreciates by 12.5% each year. Calculate S after 3 years.

18 ____

Find the depreciation after 3 years

19 ____

Investing Money - Depreciation - Graphing D/B Salvage Value (Sheet 2)

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Declining Balance depreciation forms a curve when graphed!!!! Calculate S, complete the table, then plot the values and create a curve.

A Fitness centre has purchased a cable cross-over machine for \$3 900. The machine is depreciated by the declining balance method at 15% p.a.

1 Calculate the salvage values for 4 years.

Yr 0: Value = \$3 900

Yr 1: $S = 3\,900 \times (1 - 0.15)^1 =$

Yr 2: $S = 3\,900 \times (1 - 0.15)^2 =$

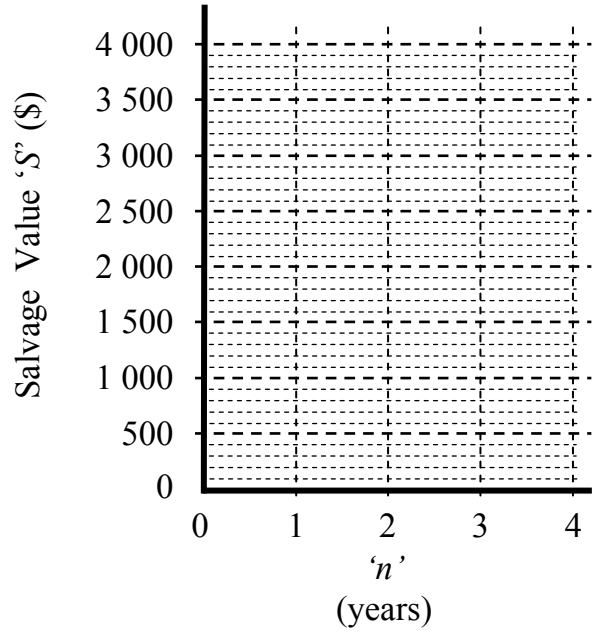
Yr 3: $S =$

Yr 4: $S =$

2 Complete the table

n Years	Salvage Value S \$
0	
1	
2	
3	
4	

3 Plot the points, use a pencil first to construct a smooth curve, then a pen afterwards.

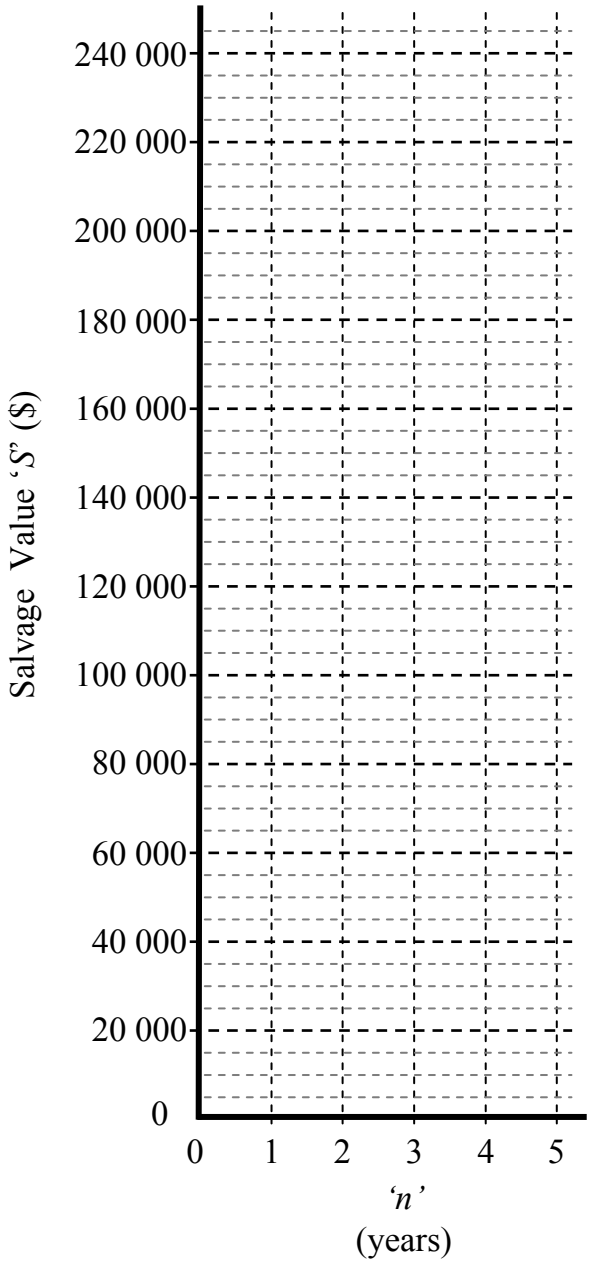


A legal firm has paid \$245 000 for a office fit-out. It is depreciated by the declining balance method at 12.5% p.a.

4 Complete the table

	Year n	Salvage Value S \$
	0	
$245\,000 \times (1 - 0.125)^1$	1	
$245\,000 \times (1 - 0.125)^2$	2	
$245\,000 \times (1 - 0.125)^3$	3	
$245\,000 \times (1 - 0.125)^4$	4	
$245\,000 \times (1 - 0.125)^5$	5	

5 Display the results on the graph below.



Investing Money - Depreciation - Declining Balance Method (Sheet 1)

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Use the brackets and power keys to solve these. Answer to 2 d.p.

1 $42\,700 \times (1 - 0.09)^5$

$= 26\,646.17$

4 2 7 0 0 × (1 - 0 • 0 9) $x^{\blacksquare}$ 5 =

2 $19\,804 \times (1 - 0.06)^3$

$= 16\,448.89$

3 $1\,447 \times (1 - 0.15)^6$

$= 545.74$

4 $20\,550 \times (1 - 0.125)^4$

$= 12\,046.03$

Calculate the salvage value for these purchases. Answer to 2 d.p.

A car with a sale price of $30 499 depreciates by 15% each year. Calculate S after 3 years.

5 $S = V_0 (1 - r)^n$

$= 30\,499 \times (1 - 0.15)^3$

Purchase price; % decimal; years

$S = \$18\,730.20$

Calculate S after 5 years.

6 $S = V_0 (1 - r)^n$

$= 30\,499 \times (1 - 0.15)^5$

$S = \$13\,532.57$

Calculate S after 9 years.

7 $S = V_0 (1 - r)^n$

$= 30\,499 \times (1 - 0.15)^9$

$S =$ $7 064.09

$ Sign!

A table saw with a sale price of $2 100 depreciates by 20% each year. Calculate S after 3 years.

8 $S = V_0 (1 - r)^n$

Formula

$= 2\,100 \times (1 - 0.2)^3$

Substitute Values

$S = \$1\,075.20$

Answer to 2 d.p. showing the $ sign

Calculate S after 7 years.

9 $S = V_0 (1 - r)^n$

$= 2\,100 \times (1 - 0.2)^7$

$S = \$440.40$

A new trench digger costs $9 030 and depreciates by 15% each year. Calculate S after 2 years.

10 $S = V_0 (1 - r)^n$

$= 9\,030 \times (1 - 0.15)^2$

$S = \$6\,524.18$

Calculate S after 4 years.

11 $S = V_0 (1 - r)^n$

$= 9\,030 \times (1 - 0.15)^4$

$S = \$4\,713.72$

A café's coffee machine costs $3 046 and depreciates by 9% each year. Calculate S after 3 years.

12 $S = V_0 (1 - r)^n$

$= 3\,046 \times (1 - 0.09)^3$

$S = \$2\,295.38$

Calculate S after 5 years.

13 $S = V_0 (1 - r)^n$

$= 3\,046 \times (1 - 0.09)^5$

$S = \$1\,900.80$

Now calculate the salvage value, then subtract it from the purchase price to calculate D.

A cherry picker costs $48 771 and depreciates by 16% each year. Calculate S after 2 years.

14 $S = V_0 (1 - r)^n$

$= 48\,771 \times (1 - 0.16)^2$

$S = \$34\,412.82$

Find the depreciation after 2 years

15 $D = 48\,771 -$ 34 412.82

Q.14

$D =$ $14 358.18

A Hair salon chair and wash basin costs $4 330 and is depreciated at 22% p.a. Calculate the salvage value after 5 years.

16 $S = V_0 (1 - r)^n$

$= 4\,330 \times (1 - 0.22)^5$

$S = \$1\,250.15$

Find the depreciation after 5 years

17 $D = 4\,330 -$ 1 250.15

$D =$ $3 079.85

A complete shop re-fit costs $18 667 and depreciates by 12.5% each year. Calculate S after 3 years.

18 $S = V_0 (1 - r)^n$

$= 18\,667 \times (1 - 0.125)^3$

$S = \$12\,505.43$

Find the depreciation after 3 years

19

$D = 18\,667 - 12\,505.43$

$D = \$6\,161.57$

Investing Money - Depreciation - Graphing D/B Salvage Value (Sheet 2)

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Declining Balance depreciation forms a curve when graphed!!!! Calculate S, complete the table, then plot the values and create a curve.

A Fitness centre has purchased a cable cross-over machine for $3 900. The machine is depreciated by the declining balance method at 15% p.a.

1 Calculate the salvage values for 4 years.

Yr 0: Value = $3 900

Yr 1: $S = 3\,900 \times (1 - 0.15)^1 =$ $3 315

Yr 2: $S = 3\,900 \times (1 - 0.15)^2 =$ $2 817.75

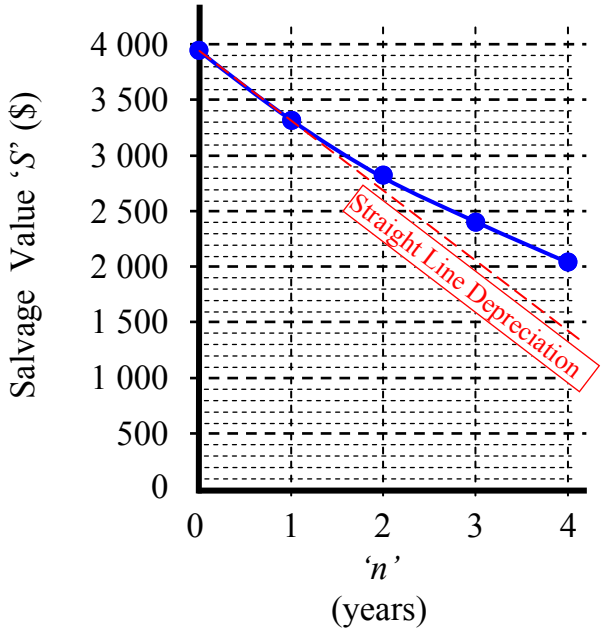
Yr 3: $S = 3\,900 \times (1 - 0.15)^3 = \$2\,395.09$

Yr 4: $S = 3\,900 \times (1 - 0.15)^4 = \$2\,035.82$

2 Complete the table

n Years	Salvage Value S $
0	$3 900
1	$3 315
2	$2 817.75
3	$2 395.09
4	$2 035.82

3 Plot the points, use a pencil first to construct a smooth curve, then a pen afterwards.

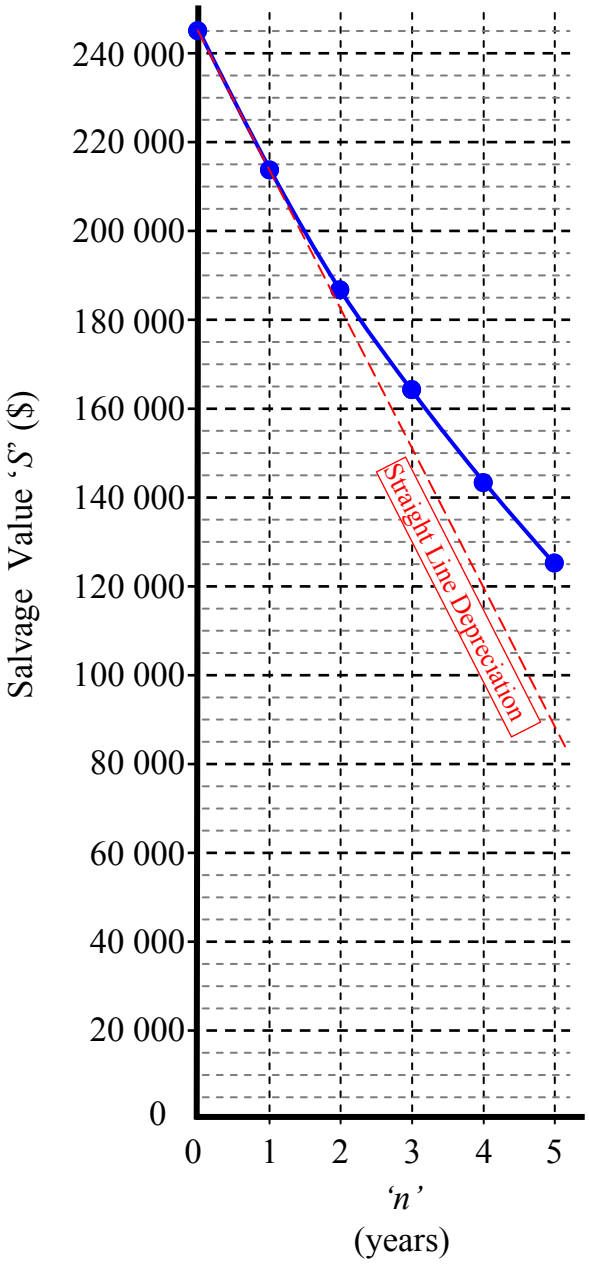


A legal firm has paid $245 000 for a office fit-out. It is depreciated by the declining balance method at 12.5% p.a.

4 Complete the table

	Year n	Salvage Value S $
	0	$245 000
$245\,000 \times (1 - 0.125)^1$	1	$214 375
$245\,000 \times (1 - 0.125)^2$	2	$187 578.13
$245\,000 \times (1 - 0.125)^3$	3	$164 130.86
$245\,000 \times (1 - 0.125)^4$	4	$143 614.50
$245\,000 \times (1 - 0.125)^5$	5	$125 662.69

5 Display the results on the graph below.



Investing Money - Depreciation - Declining Balance Method (Homework Strip)

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Date Due:

A dentist chair costs $8 990 and is depreciated by 8% p.a. Calculate the:

1 Salvage value after 6 years

$S = V_0 (1 - r)^n$

$= \square \times (1 - \square)^{\square}$

$S = \$ \square\square\square\square.\square\square$

2 Depreciation after 6 years

$D = 8\,990 - \square$

$D = \square$

A $2 785 book case in a legal firm is depreciated by 14% p.a. Calculate the:

3 Salvage value after 3 years

4 Depreciation after 3 years

Date Due:

A dentist chair costs $8 990 and is depreciated by 8% p.a. Calculate the:

1 Salvage value after 6 years

$S = V_0 (1 - r)^n$

$= \square \times (1 - \square)^{\square}$

$S = \$ \square\square\square\square.\square\square$

2 Depreciation after 6 years

$D = 8\,990 - \square$

$D = \square$

A $2 785 book case in a legal firm is depreciated by 14% p.a. Calculate the:

3 Salvage value after 3 years

4 Depreciation after 3 years

Date Due:

A dentist chair costs $8 990 and is depreciated by 8% p.a. Calculate the:

1 Salvage value after 6 years

$S = V_0 (1 - r)^n$

$= \square \times (1 - \square)^{\square}$

$S = \$ \square\square\square\square.\square\square$

2 Depreciation after 6 years

$D = 8\,990 - \square$

$D = \square$

A $2 785 book case in a legal firm is depreciated by 14% p.a. Calculate the:

3 Salvage value after 3 years

4 Depreciation after 3 years

Date Due:

A dentist chair costs $8 990 and is depreciated by 8% p.a. Calculate the:

1 Salvage value after 6 years

$S = V_0 (1 - r)^n$

$= \square \times (1 - \square)^{\square}$

$S = \$ \square\square\square\square.\square\square$

2 Depreciation after 6 years

$D = 8\,990 - \square$

$D = \square$

A $2 785 book case in a legal firm is depreciated by 14% p.a. Calculate the:

3 Salvage value after 3 years

4 Depreciation after 3 years

Investing Money - Depreciation - Declining Balance Method (HW Strip)

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Date Due:

A dentist chair costs \$8 990 and is depreciated by 8% p.a. Calculate the:

1 Salvage value after 6 years

$S = V_0 (1 - r)^n$

$= 8\,990 \times (1 - 0.08)^6$

$S = \$5\,451.13$

2 Depreciation after 6 years

$D = 8\,990 - 5\,451.13$

$D = \$3\,538.87$

A \$2 785 book case in a legal firm is depreciated by 14% p.a. Calculate the:

3 Salvage value after 3 years

$S = V_0 (1 - r)^n$

$= 2\,785 (1 - 0.14)^3$

$S = \$1\,771.42$

4 Depreciation after 3 years

$D = 2\,785 - 1\,771.42$

$D = \$1\,013.58$

Investing Money - Reducing Balance Home Loans

When people buy a home, they will usually get a home loan. These loans are based on simple interest and usually every month the bank charges interest and requires a repayment. The repayment is usually larger than the interest payable, so this drops down the amount the person owes the bank each month.

The tables have 6 columns:

1st Column is the Period. Which for these worksheets in measured in months.
2nd Column is the Principal. Which is the amount owed to the bank.
3rd Column is the Interest. This is the interest payable on the amount borrowed
4th column is the Principal plus the interest. So it is Column 2 + Column 3.
5th Column is the repayment. The amount the borrower pays each month.
6th Column is the Principal + Interest - Repayment. So Column 4 - Column 5.

Investing Money - Reducing Balance Home Loans (Sheet 1)

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A home loan charges simple interest on a monthly period. Repayments larger than the interest charged are made which not only pay the interest, but also reduce the principal. So each month the interest is slightly less and the principal reduces. Complete these home loan balance sheets for the first 4 months of the loan.



1 Home loan is for $180 000 at 7.2% p.a., with repayments of $1 750 per month.

Convert 7.2% p.a. to a monthly decimal: $r = 7.2 \div 12 \div 100 =$ ☐.☐☐☐

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	180 000	☐ 180 000 × 0.006	☐ 180 000 + 1 080	1 750	☐ 181 080 - 1 750
2	179 330	☐ 179 330 × 0.006	☐ 179 330 + 1 075.98	1 750	☐ 180 405.98 - 1 750
3	178 655.98	☐ 178 655.98 × 0.006	☐ 178 655.98 +1 071.94	1 750	☐ 179 727.92 - 1 750
4	177 977.92	☐ 177 977.92 × 0.006	☐ 177 977.92+1 067.87	1 750	☐ 179 045.79 - 1 750

2 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = ☐ (Amount borrowed) - ☐ (Period 4 Column 6) = ☐ (Show the $ sign)

3 Home loan is for $350 000 at 5.4% p.a., with repayments of $2 800 per month.

Convert 5.4% p.a. to a monthly decimal: $r = 5.4 \div 12 \div 100 =$ ☐

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	350 000	☐ 350 000 × r	☐ 350 000 + I	2 800	☐
2	☐	☐	☐	2 800	☐
3	☐	☐	☐	2 800	☐
4	☐	☐	☐	2 800	☐

4 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = ☐ - ☐ = ☐

Investing Money - Reducing Balance Home Loans (Sheet 2)

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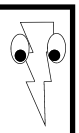
Complete these home loan balance sheets for the first 4 months of these loans. At the end of each row, the last cell has the answer there, except it has two digits 'obscured'. Check your answer at the end of each row, if you don't have a match then you need to back track and find where you have made the mistake.



1 Home loan is for $295 000 at 6% p.a., with repayments of $2 100 per month.

Convert 6% p.a. to a monthly decimal: $r = 6 \div 12 \div 100 =$ ☐.☐☐☐

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1					29X X75
2					29X 74X.88
3					2X3 115.X1
4					292 X81.X9

2 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = ☐ - ☐ = ☐

3 Home loan is for $520 000 at 6.6% p.a., with repayments of $3 800 per month.

Convert 6.6% p.a. to a monthly decimal: $r = 6.6 \div 12 \div 100 =$ ☐

Period	P	I	$P + I$	R	$P + I - R$
1					519 X6X
2					5X8 1X4.83
3					51X 164.X6
4					5X6 20X.86

4 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = ☐ - ☐ = ☐

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A home loan charges simple interest on a monthly period. Repayments larger than the interest charged are made which not only pay the interest, but also reduce the principal. So each month the interest is slightly less and the principal reduces. Complete these home loan balance sheets for the first 4 months of the loan.



1 Home loan is for $180 000 at 7.2% p.a., with repayments of $1 750 per month.

Convert 7.2% p.a. to a monthly decimal: $r = 7.2 \div 12 \div 100 =$ 0.006

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	180 000	1 080 180 000 × 0.006	181 080 180 000 + 1 080	1 750	179 330 181 080 - 1 750
2	179 330	1 075.98 179 330 × 0.006	180 405.98 179 330 + 1 075.98	1 750	178 655.98 180 405.98 - 1 750
3	178 655.98	1 071.94 178 655.98 × 0.006	179 727.92 178 655.98 +1 071.94	1 750	177 977.92 179 727.92 - 1 750
4	177 977.92	1 067.87 177 977.92 × 0.006	179 045.79 177 977.92+1 067.87	1 750	177 295.79 179 045.79 - 1 750

2 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = 180 000 (Amount borrowed) - 177 295.79 (Period 4 Column 6) = $2 704.21 (Show the $ sign)

3 Home loan is for $350 000 at 5.4% p.a., with repayments of $2 800 per month.

Convert 5.4% p.a. to a monthly decimal: $r = 5.4 \div 12 \div 100 =$ 0.0045

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	350 000	1 575 350 000 × r	351 575 350 000 + I	2 800	348 775
2	348 775	1 569.49	350 344.49	2 800	347 544.49
3	347 544.49	1 563.95	349 108.44	2 800	346 308.44
4	346 308.44	1 558.39	347 866.83	2 800	345 066.83

4 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = 350 000 - 345 066.83 = $4 933.17

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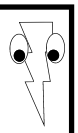
Complete these home loan balance sheets for the first 4 months of these loans. At the end of each row, the last cell has the answer there, except it has two digits 'obscured'. Check your answer at the end of each row, if you don't have a match then you need to back track and find where you have made the mistake.



1 Home loan is for $295 000 at 6% p.a., with repayments of $2 100 per month.

Convert 6% p.a. to a monthly decimal: $r = 6 \div 12 \div 100 =$ 0.005

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	295 000	1 475	296 475	2 100	294 375 29X X75
2	294 375	1 471.88	295 846.88	2 100	293 746.88 29X 74X.88
3	293 746.88	1 468.73	295 215.61	2 100	293 115.61 2X3 115.X1
4	293 115.61	1 465.58	294 581.19	2 100	292 481.19 292 X81.X9

2 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = 295 000 - 292 481.19 = $2 518.81

3 Home loan is for $520 000 at 6.6% p.a., with repayments of $3 800 per month.

Convert 6.6% p.a. to a monthly decimal: $r = 6.6 \div 12 \div 100 =$ 0.0055

Period	P	I	$P + I$	R	$P + I - R$
1	520 000	2 860	522 860	3 800	519 060 519 X6X
2	519 060	2 854.83	521 914.83	3 800	518 114.83 5X8 1X4.83
3	518 114.83	2 849.63	520 964.46	3 800	517 164.46 51X 164.X6
4	517 164.46	2 844.40	520 008.86	3 800	516 208.86 5X6 20X.86

4 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = 520 000 - 516 208.86 = $3 791.14

Investing Money - Reducing Balance Home Loans (Homework Strip)

Date Due:

Complete the table

Period (Months)	P (Principal)	I (Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	50 000	50 000 × 0.004	50 000 + 200	800	50 200 - 800
2	49 400	49 400 × 0.004	49 400 + 197.60	800	49 597.60 - 800
3	48 797.60	48 797.60 × 0.004	48 797.60 +195.19	800	48 992.79 - 800
4	48 192.79	48 192.79 × 0.004	48 192.79 +192.77	800	48 385.56 - 800

Date Due:

Complete the table

Period (Months)	P (Principal)	I (Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	50 000	50 000 × 0.004	50 000 + 200	800	50 200 - 800
2	49 400	49 400 × 0.004	49 400 + 197.60	800	49 597.60 - 800
3	48 797.60	48 797.60 × 0.004	48 797.60 +195.19	800	48 992.79 - 800
4	48 192.79	48 192.79 × 0.004	48 192.79 +192.77	800	48 385.56 - 800

Date Due:

Complete the table

Period (Months)	P (Principal)	I (Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	50 000	50 000 × 0.004	50 000 + 200	800	50 200 - 800
2	49 400	49 400 × 0.004	49 400 + 197.60	800	49 597.60 - 800
3	48 797.60	48 797.60 × 0.004	48 797.60 +195.19	800	48 992.79 - 800
4	48 192.79	48 192.79 × 0.004	48 192.79 +192.77	800	48 385.56 - 800

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Date Due:

Complete the table

Period (Months)	P (Principal)	I (Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	50 000	200 50 000 × 0.004	50 200 50 000 + 200	800	49 400 50 200 - 800
2	49 400	197.60 49 400 × 0.004	49 597.60 49 400 + 197.60	800	48 797.60 49 597.60 - 800
3	48 797.60	195.19 48 797.60 × 0.004	48 992.79 48 797.60 +195.19	800	48 192.79 48 992.79 - 800
4	48 192.79	192.77 48 192.79 × 0.004	48 385.56 48 192.79 +192.77	800	47 585.56 48 385.56 - 800

Investing Money - Introduction to Credit Cards

There are two different 'models' of Credit Cards. There are:

Type 1 - When you buy a product it is placed on a statement with all the other goods you buy. At the end of the month if you pay the statement balance in full, you pay no interest.

Type 2 - When you buy a product it is placed on a statement with all the other goods you buy. You are charged interest from the day you purchased the item, until you pay it back.

Type 1 is an excellent product so long as you pay it back at the end of each month, as there are no costs. These type of cards aren't covered in this course as there is no interest to calculate.

Type 2 results in the owner of the card paying interest on products. And sometimes this can lead a card user who has a 'Buy it Now' approach, being in debt and finding themselves in financial difficulty.

If you can't pay off an 'Interest Free period' credit card, credit is best avoided. Instead you should be patient, save up for the product then buy it with savings.

Credit card interest is 'Simple Interest' calculated monthly and billed usually near the end of the month. The purchase accrues interest from the day of purchase until, and including, the day you pay the bank back.

Example: A coffee machine costs $382. Interest is 17% p.a., for 24 days.

$I = Prn$ ← Simple Interest Formula

$= 382 \times 0.17 \times \frac{24}{365}$ ← Substitute values; Convert r to a decimal; Express days as a fraction over 365

$I = \$4.27$ ← Solve, showing the $ sign and round to the nearest cent

Investing Money - Introduction to Credit Cards (Sheet 1)

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Calculate the interest payable on these credit card purchases.

Example Stove $1 850, 16% p.a. for 22 days.

$I = Prn$ ← Write formula

$= 1\,850 \times 0.16 \times \frac{22}{365}$ Substitute Values

$I = \$17.84$ ← Answer to nearest cent

1 Desk $495, 19% p.a. for 13 days.

$I = Prn$

$= \square \times \square \times \frac{\square}{365}$ (Days)

Item cost | Rate (decimal)

4 9 5 × 0 • 1 9 × 1 3 ÷ 3 6 5 =

$I =$ ____

Show the '$' sign and cents to 2 d.p.

2 Clothes $721, 15% p.a., 24 days.

$I = Prn$

$= \square \times \square \times \frac{\square}{365}$

$I =$ ____

3 Laptop $1 135, 22% p.a., 9 days.

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I =$ ____

4 Phone $408, 18% p.a., 15 days.

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I =$ ____

5 Flight $206, 12.5% p.a., 27 days.

$I = Prn$

$=$

$I =$

6 Petrol $65.80, 17% p.a., 13 days.

$I = Prn$

$=$

$I =$

7 Bill $425.15, 23% p.a., 4 days.

Banks charge interest from the day of purchase, until the day you pay it back. Count the days.

Example Purchased: 9th June
Paid: 26th June

Days = 26 - 9 + 1 = 18 days

8 Purchased: 3rd February
Paid: 27th February

Days = ☐ - ☐ + 1

Days = ☐ days

9 Purchased: 11th March
Paid: 25th March

Days = ☐ - ☐ + ☐

Days = ☐ days

10 Purchased: 6th September
Paid: 23rd September

Days = ☐ - ☐ + ☐

Days = ☐ days

Calculate the interest payable on these credit card purchases.

11 Clothes $178, 14% p.a.
Purchased: 8th April
Paid: 25th April

Days = ☐ - ☐ + ☐

Days = ☐ days

$I = Prn$

$= \square \times \square \times \frac{\square}{365}$

$I =$ ____

12 Holiday $3 840, 17% p.a.
Purchased: 2nd January
Paid: 28th January

Days = ☐ - ☐ + ☐

Days = ☐ days

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I =$ ____

13 Headphones $272, 23% p.a.
Purchased: 24th March
Paid: 28th March

Days = ☐ - ☐ + ☐

Days = ☐ days

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I =$ ____

Investing Money - Introduction to Credit Cards (Sheet 2)

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Calculate the accrued days then calculate the interest payable on these credit card purchases.

1 Insurance $480.35, 23% p.a.
Purchased: 16th July
Paid: 23rd July

Days = [ ] - [ ] + 1

Days = [ ] days

I = *Prn*

= [ ] × [ ] × [ ] / 365

I = [ ]

2 Movie Tickets $58.90, 16% p.a.
Purchased: 3rd October
Paid: 24th October

Days = [ ] - [ ] + [ ]

Days = [ ] days

I = *Prn*

= [ ] × [ ] × [ ] / [ ]

I = [ ]

3 Phone Credit $45.55, 19% p.a.
Purchased: 17th March
Paid: 28th March

Days = [ ] - [ ] + [ ]

Days = [ ] days

I = *Prn*

= [ ] × [ ] × [ ] / [ ]

I = [ ]

4 Refrigerator $1 099, 21% p.a.
Purchased: 4th June
Paid: 23rd June

Days =

Days =

I = *Prn*

=

I =

5 Electricity Bill $597, 21% p.a.
Purchased: 13th June
Paid: 23rd June

Days =

Days =

I = *Prn*

=

I =

6 Shoes $68.70, 21% p.a.
Purchased: 18th June
Paid: 23rd June

Days =

Days =

I = *Prn*

=

I =

7 Add Interest in Q 4, 5 and 6.

I = [ ] + [ ] + [ ]

I = [ ]

8 Add Purchases in Q 4, 5 and 6.

I = [ ] + [ ] + [ ]

I = [ ]

9 Television $485, 14% p.a.
Purchased: 3rd March
Paid: 27th March

Days =

Days =

I = *Prn*

=

I =

10 Kitchenware $93.75, 14% p.a.
Purchased: 14th March
Paid: 27th March

11 Food $143, 14% p.a.
Purchased: 24th March
Paid: 27th March

12 Add Interest in Q 9, 10 and 11.

I = [ ] + [ ] + [ ]

I = [ ]

13 Add Purchases in Q 9, 10 and 11.

I = [ ] + [ ] + [ ]

I = [ ]

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Calculate the interest payable on these credit card purchases.

Example Stove $1 850, 16% p.a. for 22 days.

$I = Prn$ ← Write formula

$= 1\,850 \times 0.16 \times \frac{22}{365}$ Substitute Values

$I = \$17.84$ ← Answer to nearest cent

1 Desk $495, 19% p.a. for 13 days.

$I = Prn$

$= 495 \times 0.19 \times \frac{13}{365}$ (13 Days; 495 Item cost; 0.19 Rate (decimal))

4 9 5 × 0 • 1 9 × ▪ 1 3 ◯ 3 6 5 =

$I = \$3.35$

Show the '$' sign and cents to 2 d.p.

2 Clothes $721, 15% p.a., 24 days.

$I = Prn$

$= 721 \times 0.15 \times \frac{24}{365}$

$I = \$7.11$

3 Laptop $1 135, 22% p.a., 9 days.

$I = Prn$

$= 1\,135 \times 0.22 \times \frac{9}{365}$

$I = \$6.16$

4 Phone $408, 18% p.a., 15 days.

$I = Prn$

$= 408 \times 0.18 \times \frac{15}{365}$

$I = \$3.02$

5 Flight $206, 12.5% p.a., 27 days.

$I = Prn$

$= 206 \times 0.125 \times \frac{27}{365}$

$I = \$1.90$

6 Petrol $65.80, 17% p.a., 13 days.

$I = Prn$

$= 65.80 \times 0.17 \times \frac{13}{365}$

$I = \$0.40$

7 Bill $425.15, 23% p.a., 4 days.

$I = Prn$

$= 425.15 \times 0.23 \times \frac{4}{365}$

$I = \$1.07$

Banks charge interest from the day of purchase, until the day you pay it back. Count the days.

Example Purchased: 9th June
Paid: 26th June

Days = 26 - 9 + 1 = 18 days

8 Purchased: 3rd February
Paid: 27th February

Days = 27 - 3 + 1

Days = 25 days

9 Purchased: 11th March
Paid: 25th March

Days = 25 - 11 + 1

Days = 15 days

10 Purchased: 6th September
Paid: 23rd September

Days = 23 - 6 + 1

Days = 18 days

Calculate the interest payable on these credit card purchases.

11 Clothes $178, 14% p.a.
Purchased: 8th April
Paid: 25th April

Days = 25 - 8 + 1

Days = 18 days

$I = Prn$

$= 178 \times 0.14 \times \frac{18}{365}$

$I = \$1.23$

12 Holiday $3 840, 17% p.a.
Purchased: 2nd January
Paid: 28th January

Days = 28 - 2 + 1

Days = 27 days

$I = Prn$

$= 3\,840 \times 0.17 \times \frac{27}{365}$

$I = \$48.29$

13 Headphones $272, 23% p.a.
Purchased: 24th March
Paid: 28th March

Days = 28 - 24 + 1

Days = 5 days

$I = Prn$

$= 272 \times 0.23 \times \frac{5}{365}$

$I = \$0.86$

Investing Money - Introduction to Credit Cards (Sheet 2)

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Calculate the accrued days then calculate the interest payable on these credit card purchases.

1 Insurance $480.35, 23% p.a.
Purchased: 16th July
Paid: 23rd July

Days = 23 - 16 + 1

Days = 8 days

$I = Prn$

$= 480.35 \times 0.23 \times \frac{8}{365}$

$I =$ $2.42

2 Movie Tickets $58.90, 16% p.a.
Purchased: 3rd October
Paid: 24th October

Days = 24 - 3 + 1

Days = 22 days

$I = Prn$

$= 58.90 \times 0.16 \times \frac{22}{365}$

$I =$ $0.57

3 Phone Credit $45.55, 19% p.a.
Purchased: 17th March
Paid: 28th March

Days = 28 - 17 + 1

Days = 12 days

$I = Prn$

$= 45.55 \times 0.19 \times \frac{12}{365}$

$I =$ $0.28

4 Refrigerator $1 099, 21% p.a.
Purchased: 4th June
Paid: 23rd June

Days = 23 - 4 + 1

Days = 20 days

$I = Prn$

$= 1\,099 \times 0.21 \times \frac{20}{365}$

$I =$ $12.65

5 Electricity Bill $597, 21% p.a.
Purchased: 13th June
Paid: 23rd June

Days = 23 - 13 + 1

Days = 11 days

$I = Prn$

$= 597 \times 0.21 \times \frac{11}{365}$

$I =$ $3.78

6 Shoes $68.70, 21% p.a.
Purchased: 18th June
Paid: 23rd June

Days = 23 - 18 + 1

Days = 6 days

$I = Prn$

$= 68.70 \times 0.21 \times \frac{6}{365}$

$I =$ $0.24

7 Add Interest in Q 4, 5 and 6.

$I = 12.65 + 3.78 + 0.24$

$I =$ $16.67

8 Add Purchases in Q 4, 5 and 6.

$I = 1\,099 + 597 + 68.70$

$I =$ $1 764.70

9 Television $485, 14% p.a.
Purchased: 3rd March
Paid: 27th March

Days = 27 - 3 + 1

Days = 25 days

$I = Prn$

$= 485 \times 0.14 \times \frac{25}{365}$

$I =$ $4.65

10 Kitchenware $93.75, 14% p.a.
Purchased: 14th March
Paid: 27th March

Days = 27 - 14 + 1

Days = 14 days

$I = Prn$

$= 93.75 \times 0.14 \times \frac{14}{365}$

$I =$ $0.50

11 Food $143, 14% p.a.
Purchased: 24th March
Paid: 27th March

Days = 27 - 24 + 1

Days = 4 days

$I = Prn$

$= 143 \times 0.14 \times \frac{4}{365}$

$I =$ $0.22

12 Add Interest in Q 9, 10 and 11.

$I = 4.65 + 0.50 + 0.22$

$I =$ $5.37

13 Add Purchases in Q 9, 10 and 11.

$I = 485 + 93.75 + 143$

$I =$ $721.75

Investing Money - Introduction to Credit Cards (Homework Strip)

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Date Due:

Calculate the interest payable

1 Clothes $280, 16% p.a., 23 days.

$I = Prn$

$= \frac{\square \times 0.16 \times \frac{\square}{365}}{}$ (Item cost; Rate (decimal); Days)

$I =$

Show the '$' sign and cents to 2 d.p.

2 Hotel $119, 22% p.a., 13 days.

$I = Prn$

$= \frac{\square \times \square \times \frac{\square}{\square}}{}$ (Item cost; Rate (decimal))

$I =$

Show the '$' sign and cents to 2 d.p.

3 Gas bill $337, 19% p.a., 8 days.

$I = Prn$

$= \frac{\square \times \square \times \frac{\square}{\square}}{}$

$I =$

Show the '$' sign and cents to 2 d.p.

Date Due:

Calculate the interest payable

1 Clothes $280, 16% p.a., 23 days.

$I = Prn$

$= \frac{\square \times 0.16 \times \frac{\square}{365}}{}$ (Item cost; Rate (decimal); Days)

$I =$

Show the '$' sign and cents to 2 d.p.

2 Hotel $119, 22% p.a., 13 days.

$I = Prn$

$= \frac{\square \times \square \times \frac{\square}{\square}}{}$ (Item cost; Rate (decimal))

$I =$

Show the '$' sign and cents to 2 d.p.

3 Gas bill $337, 19% p.a., 8 days.

$I = Prn$

$= \frac{\square \times \square \times \frac{\square}{\square}}{}$

$I =$

Show the '$' sign and cents to 2 d.p.

Date Due:

Calculate the interest payable

1 Clothes $280, 16% p.a., 23 days.

$I = Prn$

$= \frac{\square \times 0.16 \times \frac{\square}{365}}{}$ (Item cost; Rate (decimal); Days)

$I =$

Show the '$' sign and cents to 2 d.p.

2 Hotel $119, 22% p.a., 13 days.

$I = Prn$

$= \frac{\square \times \square \times \frac{\square}{\square}}{}$ (Item cost; Rate (decimal))

$I =$

Show the '$' sign and cents to 2 d.p.

3 Gas bill $337, 19% p.a., 8 days.

$I = Prn$

$= \frac{\square \times \square \times \frac{\square}{\square}}{}$

$I =$

Show the '$' sign and cents to 2 d.p.

Date Due:

Calculate the interest payable

1 Clothes $280, 16% p.a., 23 days.

$I = Prn$

$= \frac{\square \times 0.16 \times \frac{\square}{365}}{}$ (Item cost; Rate (decimal); Days)

$I =$

Show the '$' sign and cents to 2 d.p.

2 Hotel $119, 22% p.a., 13 days.

$I = Prn$

$= \frac{\square \times \square \times \frac{\square}{\square}}{}$ (Item cost; Rate (decimal))

$I =$

Show the '$' sign and cents to 2 d.p.

3 Gas bill $337, 19% p.a., 8 days.

$I = Prn$

$= \frac{\square \times \square \times \frac{\square}{\square}}{}$

$I =$

Show the '$' sign and cents to 2 d.p.

Investing Money - Introduction to Credit Cards (Homework Strip)

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Date Due:

Calculate the interest payable

1 Clothes $280, 16% p.a., 23 days.

$I = Prn$

$= 280 \times 0.16 \times \frac{23}{365}$ Days

Item cost; Rate (decimal)

$I =$ $2.82

Show the '$' sign and cents to 2 d.p.

2 Hotel $119, 22% p.a., 13 days.

$I = Prn$

$= 119 \times 0.22 \times \frac{13}{365}$

Item cost; Rate (decimal)

$I =$ $0.93

Show the '$' sign and cents to 2 d.p.

3 Gas bill $337, 19% p.a., 8 days.

$I = Prn$

$= 337 \times 0.19 \times \frac{8}{365}$

$I =$ $1.40

Show the '$' sign and cents to 2 d.p.

Investing Money - Credit Cards

Credit cards use Simple Interest to calculate the interest payable. The interest is calculated from the day of the purchase until the payment due date, including the day the payment is made.

The interest calculation is done in two steps:

Step 1 - Calculate the number of days the interest will be calculated on.

Step 2 - Calculate the interest payable.

On these worksheets, both steps are below the statement area. Make sure you notice the 'Due Date' at the top right hand side of the statement.

Example: Interest Rate is 13% p.a. The date Due is 27th November

Date	Description	Amount	Balance
5/11	Clothing	$175	$175
19/11	Electricity	$288	$463
22/11	Groceries	$144	$607

Rewrite 1st purchase

$175 + $288 = $463

$463 + $144 = $607

5/11 Days: $27 - 5 + 1 =$ 23 days

Due date

Date of Purchase

+1 for extra day

5/11 Interest: $I = 175 \times 0.13 \times \frac{23}{365} =$ $1.43

Clothing Cost

13% as a decimal

23 days as a fraction of a year

Interest payable*

*This is the amount payable for the clothing purchase only

Investing Money - Credit Cards (Sheet 1)

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These credit cards charge interest from, and including, the date of purchase to, and including, the payment due date. Find the interest payable on each of the purchases. Note the date due.



1 Rate = 14% p.a. Date Due 30/10

Date	Description	Amount	Balance
12/10	Vet Fee	$90	$90
17/10	CD Sale	$25	$115
22/10	Groceries	$135	$250

12/10 Days: $30 - 12 + 1 =$ ____ days

12/10 Interest: $I = 90 \times 0.14 \times \frac{\square}{365} =$ $

17/10 Days: $30 - 17 + 1 =$ ____ days

17/10 Interest: $I = 25 \times 0.14 \times \frac{\square}{365} =$ ____

22/10 Days: $30 - 22 + 1 =$ ____ days

22/10 Interest: $I = 135 \times 0.14 \times \frac{\square}{365} =$ ____

2 Rate = 18% p.a. Date Due 29/4

Date	Description	Amount	Balance
2/4	Phone Bill	$157	$157
13/4	Insurance	$530	$687
17/4	Groceries	$210	$897

2/4 Days: 29 - ____ + ____ = ____ days

2/4 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

13/4 Days: ____ - ____ + ____ = ____ days

13/4 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

17/4 Days: ____ - ____ + ____ = ____ days

17/4 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____



These are the same except you also have to fill in the balance boxes in the statement as well.

3 Rate = 17% p.a. Date Due 24/6

Date	Description	Amount	Balance
8/6	Clothing	$293 →	
11/6	Ticket	+ $164 →	
20/6	Phone Credit	+ $45 →	

8/6 Days: ____ - ____ + ____ = ____ days

8/6 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

11/6 Days: ____ - ____ + ____ = ____ days

11/6 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

20/6 Days: ____ - ____ + ____ = ____ days

20/6 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

4 Rate = 21% p.a. Date Due 28/3

Date	Description	Amount	Balance
9/3	Groceries	$148	
17/3	Restaurant	$95	
22/3	Theme Park	$65	

9/3 Days: ____ - ____ + ____ = ____ days

9/3 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

17/3 Days: ____ - ____ + ____ = ____ days

17/3 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

22/3 Days: ____ - ____ + ____ = ____ days

22/3 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

Investing Money - Credit Cards (Sheet 2)

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Complete the balance boxes in the statement. Find the interest payable on each purchase. Add all the interest and write total in 'Interest Payable' box, then add to the last balance box.



1 Rate = 18% p.a. Date Due 30/7

Date	Description	Amount	Balance
2/7	Cable TV	$68	
5/7	Clothing	$165	
8/7	Broadband	$72	
14/7	Petrol	$88	
23/7	Electricity	$395	

30/7 Interest Payable

Add all five of the interest calculations below, write in box

30/7 Amount Payable

Add the 'Interest Payable' box to the final box in the statement

2/7 Days: ☐ - ☐ + ☐ = ☐ days

2/7 Int: $I =$ ☐ × ☐ × ☐/☐ = $

5/7 Days: ☐ - ☐ + ☐ = ☐ days

5/7 Int: $I =$ ☐ × ☐ × ☐/☐ =

8/7 Days: ☐ - ☐ + ☐ = ☐ days

8/7 Int: $I =$ ☐ × ☐ × ☐/☐ =

14/7 Days: ☐ - ☐ + ☐ = ☐ days

14/7 Int: $I =$ ☐ × ☐ × ☐/☐ =

23/7 Days: ☐ - ☐ + ☐ = ☐ days

23/7 Int: $I =$ ☐ × ☐ × ☐/☐ =

Same process, except a Cash Advance is made, the interest rate is higher for cash advances.

2 Purchases = 16% p.a. Date Due 28/11

Cash Advance* = 24% p.a.

Date	Description	Amount	Balance
1/11	eTunes	$82	
6/11	Cash Adv.*	$600	
13/11	Groceries	$72	
14/11	JC Hi-Fi	$415	
24/11	Big UU	$209	

28/11 Interest Payable

Add all five of the interest calculations below, write in box

28/11 Amount Payable

Add the 'Interest Payable' box to the final box in the statement

1/11 Days: ☐ - ☐ + ☐ = ☐ days

1/11 Int: $I =$ ☐ × ☐ × ☐/☐ = $

6/11 Days: ☐ - ☐ + ☐ = ☐ days

6/11 Int*: $I =$ ☐ × ☐ × ☐/☐ =

Cash advance* use 24%

13/11 Days: ☐ - ☐ + ☐ = ☐ days

13/11 Int: $I =$ ☐ × ☐ × ☐/☐ =

14/11 Days: ☐ - ☐ + ☐ = ☐ days

14/11 Int: $I =$ ☐ × ☐ × ☐/☐ =

24/11 Days: ☐ - ☐ + ☐ = ☐ days

24/11 Int: $I =$ ☐ × ☐ × ☐/☐ =

Investing Money - Credit Cards (Sheet 1)

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These credit cards charge interest from, and including, the date of purchase to, and including, the payment due date. Find the interest payable on each of the purchases. Note the date due.



1 Rate = 14% p.a. Date Due 30/10

Date	Description	Amount	Balance
12/10	Vet Fee	$90	$90
17/10	CD Sale	$25	$115
22/10	Groceries	$135	$250

12/10 Days: 30 - 12 + 1 = 19 days

12/10 Interest: $I = 90 \times 0.14 \times \frac{19}{365} =$ $0.66

17/10 Days: 30 - 17 + 1 = 14 days

17/10 Interest: $I = 25 \times 0.14 \times \frac{14}{365} =$ $0.13

22/10 Days: 30 - 22 + 1 = 9 days

22/10 Interest: $I = 135 \times 0.14 \times \frac{9}{365} =$ $0.47

2 Rate = 18% p.a. Date Due 29/4

Date	Description	Amount	Balance
2/4	Phone Bill	$157	$157
13/4	Insurance	$530	$687
17/4	Groceries	$210	$897

2/4 Days: 29 - 2 + 1 = 28 days

2/4 Int: $I = 157 \times 0.18 \times \frac{28}{365} =$ $2.17

13/4 Days: 29 - 13 + 1 = 17 days

13/4 Int: $I = 530 \times 0.18 \times \frac{17}{365} =$ $4.44

17/4 Days: 29 - 17 + 1 = 13 days

17/4 Int: $I = 210 \times 0.18 \times \frac{13}{365} =$ $1.35



These are the same except you also have to fill in the balance boxes in the statement as well.

3 Rate = 17% p.a. Date Due 24/6

Date	Description	Amount	Balance
8/6	Clothing	$293 →	$293
11/6	Ticket	+ $164 →	$457
20/6	Phone Credit	+ $45 →	$502

8/6 Days: 24 - 8 + 1 = 17 days

8/6 Int: $I = 293 \times 0.17 \times \frac{17}{365} =$ $2.32

11/6 Days: 24 - 11 + 1 = 14 days

11/6 Int: $I = 164 \times 0.17 \times \frac{14}{365} =$ $1.07

20/6 Days: 24 - 20 + 1 = 5 days

20/6 Int: $I = 45 \times 0.17 \times \frac{5}{365} =$ $0.10

4 Rate = 21% p.a. Date Due 28/3

Date	Description	Amount	Balance
9/3	Groceries	$148	$148
17/3	Restaurant	$95	$243
22/3	Theme Park	$65	$308

9/3 Days: 28 - 9 + 1 = 20 days

9/3 Int: $I = 148 \times 0.21 \times \frac{20}{365} =$ $1.70

17/3 Days: 28 - 17 + 1 = 12 days

17/3 Int: $I = 95 \times 0.21 \times \frac{12}{365} =$ $0.66

22/3 Days: 28 - 22 + 1 = 7 days

22/3 Int: $I = 65 \times 0.21 \times \frac{7}{365} =$ $0.26

Investing Money - Credit Cards (Sheet 2)

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Complete the balance boxes in the statement. Find the interest payable on each purchase. Add all the interest and write total in 'Interest Payable' box, then add to the last balance box.



1 | Rate = 18% p.a. | Date Due 30/7

Date	Description	Amount	Balance
2/7	Cable TV	$68	$68
5/7	Clothing	$165	$233
8/7	Broadband	$72	$305
14/7	Petrol	$88	$393
23/7	Electricity	$395	$788

30/7 Interest Payable: $6.20

Add all five of the interest calculations below, write in box

30/7 Amount Payable: $794.20

Add the 'Interest Payable' box to the final box in the statement

2/7 Days: 30 - 2 + 1 = 29 days

2/7 Int: $I = 68 \times 0.18 \times \frac{29}{365} = \$0.97$

5/7 Days: 30 - 5 + 1 = 26 days

5/7 Int: $I = 165 \times 0.18 \times \frac{26}{365} = \$2.11$

8/7 Days: 30 - 8 + 1 = 23 days

8/7 Int: $I = 72 \times 0.18 \times \frac{23}{365} = \$0.82$

14/7 Days: 30 - 14 + 1 = 17 days

14/7 Int: $I = 88 \times 0.18 \times \frac{17}{365} = \$0.74$

23/7 Days: 30 - 23 + 1 = 8 days

23/7 Int: $I = 395 \times 0.18 \times \frac{8}{365} = \$1.56$



Same process, except a Cash Advance is made, the interest rate is higher for cash advances.

2 | Purchases = 16% p.a. | Date Due 28/11

Cash Advance* = 24% p.a.

Date	Description	Amount	Balance
1/11	eTunes	$82	$82
6/11	Cash Adv.*	$600	$682
13/11	Groceries	$72	$754
14/11	JC Hi-Fi	$415	$1 169
24/11	Big UU	$209	$1 378

28/11 Interest Payable: $13.77

Add all five of the interest calculations below, write in box

28/11 Amount Payable: $1 391.77

Add the 'Interest Payable' box to the final box in the statement

1/11 Days: 28 - 1 + 1 = 28 days

1/11 Int: $I = 82 \times 0.16 \times \frac{28}{365} = \$1.01$

6/11 Days: 28 - 6 + 1 = 23 days

6/11 Int*: $I = 600 \times 0.24 \times \frac{23}{365} = \$9.07$

Cash advance* use 24%

13/11 Days: 28 - 13 + 1 = 16 days

13/11 Int: $I = 72 \times 0.16 \times \frac{16}{365} = \$0.50$

14/11 Days: 28 - 14 + 1 = 15 days

14/11 Int: $I = 415 \times 0.16 \times \frac{15}{365} = \$2.73$

24/11 Days: 28 - 24 + 1 = 5 days

24/11 Int: $I = 209 \times 0.16 \times \frac{5}{365} = \$0.46$

Investing Money - Credit Cards (Homework Strip)

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Date Due: _____

Calculate the Interest on each of these purchases.

1 | Rate = 18% p.a. | Date Due 30/10

Date	Description	Amount	Balance
4/10	Flight	$1 050	$1 050
13/10	Hotel	$985	$2 035
24/10	Clothing	$677	$2 712

4/10 Days: 30 - 4 + 1 = ____ days

4/10 Interest: $I = 1\,050 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

13/10 Days: 30 - 13 + 1 = ____ days

13/10 Interest: $I = 985 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

24/10 Days: 30 - 24 + 1 = ____ days

24/10 Interest: $I = 677 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

30/10 Interest Payable _____

Add all three interest calculations above, write in box

30/10 Amount Payable _____

Add the 'Interest Payable' box to the final statement box

Date Due: _____

Calculate the Interest on each of these purchases.

1 | Rate = 18% p.a. | Date Due 30/10

Date	Description	Amount	Balance
4/10	Flight	$1 050	$1 050
13/10	Hotel	$985	$2 035
24/10	Clothing	$677	$2 712

4/10 Days: 30 - 4 + 1 = ____ days

4/10 Interest: $I = 1\,050 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

13/10 Days: 30 - 13 + 1 = ____ days

13/10 Interest: $I = 985 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

24/10 Days: 30 - 24 + 1 = ____ days

24/10 Interest: $I = 677 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

30/10 Interest Payable _____

Add all three interest calculations above, write in box

30/10 Amount Payable _____

Add the 'Interest Payable' box to the final statement box

Date Due: _____

Calculate the Interest on each of these purchases.

1 | Rate = 18% p.a. | Date Due 30/10

Date	Description	Amount	Balance
4/10	Flight	$1 050	$1 050
13/10	Hotel	$985	$2 035
24/10	Clothing	$677	$2 712

4/10 Days: 30 - 4 + 1 = ____ days

4/10 Interest: $I = 1\,050 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

13/10 Days: 30 - 13 + 1 = ____ days

13/10 Interest: $I = 985 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

24/10 Days: 30 - 24 + 1 = ____ days

24/10 Interest: $I = 677 \times 0.18 \times \frac{\square}{365} =$ $\_\_\_\_$

30/10 Interest Payable _____

Add all three interest calculations above, write in box

30/10 Amount Payable _____

Add the 'Interest Payable' box to the final statement box

Investing Money - Credit Cards (Homework Strip)

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Date Due:

Calculate the Interest on each of these purchases.

1 | Rate = 18% p.a. | Date Due 30/10

Date	Description	Amount	Balance
4/10	Flight	$1 050	$1 050
13/10	Hotel	$985	$2 035
24/10	Clothing	$677	$2 712

4/10 Days: $30 - 4 + 1 =$ 27 days

4/10 Interest: $I = 1\,050 \times 0.18 \times \frac{27}{365} =$ $13.98

13/10 Days: $30 - 13 + 1 =$ 18 days

13/10 Interest: $I = 985 \times 0.18 \times \frac{18}{365} =$ $8.74

24/10 Days: $30 - 24 + 1 =$ 7 days

24/10 Interest: $I = 677 \times 0.18 \times \frac{7}{365} =$ $2.34

30/10 Interest Payable $25.06

Add all three interest calculations above, write in box

30/10 Amount Payable $2 737.06

Add the 'Interest Payable' box to the final statement box

Investing Money Extension Review

This is the review of the Investing Money Extension Topic.

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Convert these times to the specified periods.

1 1 year = ☐ months

2 2 years = ☐ weeks

3 5 years = ☐ months

Eve makes an investment of $9 000 for 4 years compounded monthly at 3% per month.
Use: $A = P(1 + r)^n$

4 Convert 3% to a decimal

5 Calculate the number of compound periods.

6 Calculate Eve's account balance

7 Calculate the interest Eve has earned.

Declan is investing $5 000 for 3 quarters at 10% p.a., compounded quarterly.
Use: $A = P(1 + r)^n$

8 Convert 10% p.a. to a quarterly decimal

9 Calculate Declan's account balance

10 Calculate the interest Declan has earned.

Use guess and check methods to find the term (power) for this compound investment. Answer in the box.

11 $7\,078.09 = 4\,130 \times (1 + 0.08)^{\square}$

An investment of $26 500 at 6% p.a. compounded annually, results in an account balance of $35 462.98. How many years was the investment term? Use G & C.

12

Use guess and check methods to find the rate as a decimal (2 d.p.) for this compound investment. Answer in the box.

13 $3\,587.04 = 2\,200 \times (1 + \square)^4$

An investment of $9 330 compounded annually for 6 years, results in an account balance of $14 001.81. What was the prevailing interest rate?

14

Solve to find the value of P.

15 $3\,780 = P \times 1.08$

16 $1\,190.04 = P \times (1 + 0.05)^3$

An investment for 3 years at 6% p.a. is compounded yearly and results in an account balance of $7 384.30. Find P, the principal invested.

17

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Glen buys a ute for $22 400. It is depreciated at 15% p.a. using the straight line depreciation method.

18 Calculate the yearly depreciation on the ute.

19 Calculate the salvage value after 4 years. Use:
$S = V_0 - Dn$

Kate buys a new laptop for $1 240. It is depreciated at 12% p.a. using the declining balance method.

20 Calculate the salvage value after 3 years. Use:
$S = V_0 (1 - r)^n$

21 Calculate the amount the laptop has depreciated by after the 3 year period.

Caitlyn has recorded her purchases on her credit card below. The card charges interest at 21% p.a. from the day of purchase until and including the day the balance is due.

23 Calculate the values in the balance column

Date	Description	Amount	Balance
4/3	Groceries	$258	
12/3	Restaurant	$105	

24 The balance is due on 28/3. Calculate the number of days interest is payable for:

4/3 purchase:

12/3 purchase:

25 Calculate the simple interest payable on each purchase. Use $I = Prn$, the interest rate is 21% p.a.

4/3 purchase:

12/3 purchase:

26 Calculate the amount due on 28/3.

22 Complete the home loan table below. The home loan is for $130 000 at an interest rate of 0.5% per month, with repayments of $1 400 per month. Use $I = Prn$ to calculate simple interest.

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$
1	130 000			1 600	
2				1 600	
3				1 600	

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Convert these times to the specified periods.

1 1 year = 12 months

2 2 years = 104 weeks

3 5 years = 60 months

Eve makes an investment of $9 000 for 4 years compounded monthly at 3% per month.
Use: $A = P(1 + r)^n$

4 Convert 3% to a decimal

$3 \div 100 = 0.03$

5 Calculate the number of compound periods.

$4 \times 12 = 48$ months

6 Calculate Eve's account balance

$A = P(1 + r)^n$

$= 9\,000 \times (1 + 0.03)^4$

$A = \$10\,129.58$

7 Calculate the interest Eve has earned.

$I = 10\,129.58 - 9\,000 = \$1\,129.58$

Declan is investing $5 000 for 3 quarters at 10% p.a., compounded quarterly.
Use: $A = P(1 + r)^n$

8 Convert 10% p.a. to a quarterly decimal

$10 \div 4 \div 100 = 0.025$

9 Calculate Declan's account balance

$A = P(1 + r)^n$

$= 5\,000 \times (1 + 0.025)^3$

$A = \$5\,384.45$

10 Calculate the interest Declan has earned.

$I = 5\,384.45 - 5\,000 = \$384.45$

Use guess and check methods to find the term (power) for this compound investment. Answer in the box.

11 $7\,078.09 = 4\,130 \times (1 + 0.08)^{\boxed{7}}$

An investment of $26 500 at 6% p.a. compounded annually, results in an account balance of $35 462.98. How many years was the investment term? Use G & C.

12

$35\,462.98 = 26\,500 \times (1 + 0.06)^5$

The investment term was for 5 years.

Use guess and check methods to find the rate as a decimal (2 d.p.) for this compound investment. Answer in the box.

13 $3\,587.04 = 2\,200 \times (1 + \boxed{0.13})^4$

An investment of $9 330 compounded annually for 6 years, results in an account balance of $14 001.81. What was the prevailing interest rate?

14

$14\,001.81 = 9\,330 \times (1 + 0.07)^6$

$0.07 \times 100 = 7\%$ p.a.

Solve to find the value of P.

15 $3\,780 = P \times 1.08$

$\div 1.08$ $\div 1.08$

$P = 3\,500$

16 $1\,190.04 = P \times (1 + 0.05)^3$

$1\,190.04 = P \times 1.57625$

$\div 1.57...$ $\div 1.57...$

$P = 1\,028$

An investment for 3 years at 6% p.a. is compounded yearly and results in an account balance of $7 384.30. Find P, the principal invested.

17

$7\,384.30 = P \times (1 + 0.06)^3$

$7\,384.30 = P \times 1.191016$

$\div 1.19...$ $\div 1.19...$

$P = \$6\,200$

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Glen buys a ute for $22 400. It is depreciated at 15% p.a. using the straight line depreciation method.

18 Calculate the yearly depreciation on the ute.

$D = 0.15 \times 22\,400$

$D = \$3\,360$

19 Calculate the salvage value after 4 years. Use: $S = V_0 - Dn$

$S = V_0 - Dn$

$= 22\,400 - 3\,360 \times 4$

$S = \$8\,960$

Kate buys a new laptop for $1 240. It is depreciated at 12% p.a. using the declining balance method.

20 Calculate the salvage value after 3 years. Use: $S = V_0 (1 - r)^n$

$S = V_0 (1 - r)^n$

$= 1\,240 \times (1 - 0.12)^3$

$S = \$845.03$

21 Calculate the amount the laptop has depreciated by after the 3 year period.

$Dep.\ Amount = 1\,240 - 845.03$

$Dep.\ Amount = \$394.97$

Caitlyn has recorded her purchases on her credit card below. The card charges interest at 21% p.a. from the day of purchase until and including the day the balance is due.

23 Calculate the values in the balance column

Date	Description	Amount	Balance
4/3	Groceries	$258	$258
12/3	Restaurant	$105	$363

24 The balance is due on 28/3. Calculate the number of days interest is payable for:

4/3 purchase: $28 - 4 + 1 = 25$ days

12/3 purchase: $28 - 12 + 1 = 17$ days

25 Calculate the simple interest payable on each purchase. Use $I = Prn$, the interest rate is 21% p.a.

4/3 purchase: $I = 258 \times 0.21 \times \frac{25}{365}$

$I = \$3.71$

12/3 purchase: $I = 105 \times 0.21 \times \frac{17}{365}$

$I = \$1.03$

26 Calculate the amount due on 28/3.

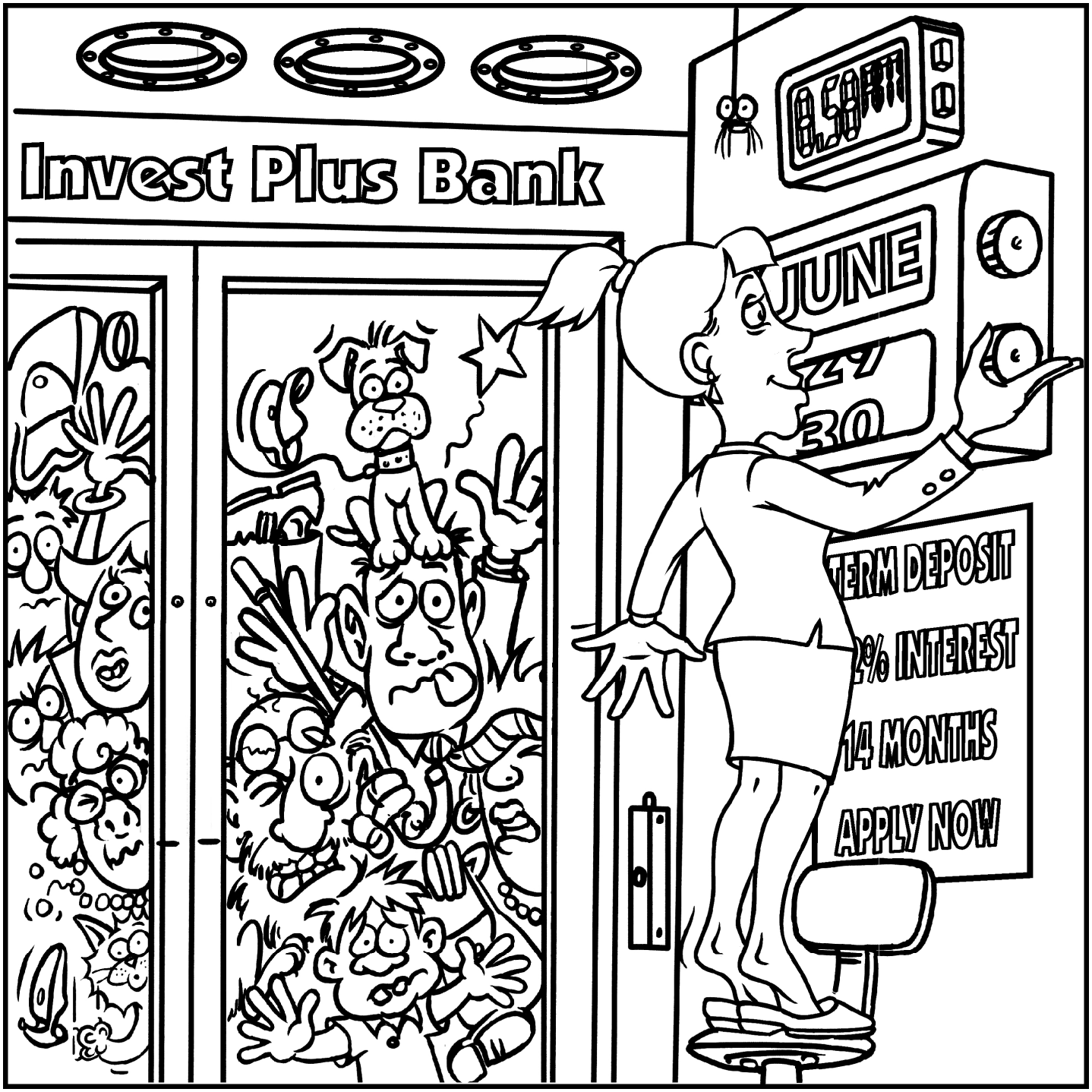
$Amount\ Due = 363 + 3.71 + 1.03$

$Amount\ Due = \$367.74$

22 Complete the home loan table below. The home loan is for $130 000 at an interest rate of 0.5% per month, with repayments of $1 400 per month. Use $I = Prn$ to calculate simple interest.

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$
1	130 000	650	130 650	1 600	129 050
2	129 050	645.25	129 695.25	1 600	128 095.25
3	128 095.25	640.48	128 735.73	1 600	127 135.73

Investing Money Extension



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Investing Money - Compound Interest Changing Periods (Sheet 1)

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Convert these periods to months.

1 1 year = ☐ months

2 2 years = ☐

3 ½ year = ☐

4 1½ years = ☐

Convert these periods to months and use the new number as the power to evaluate these.

$3 000 invested for **1 year compounded monthly** at 1.5% per month.

5 $A = 3\,000 \times (1 + 0.015)^{\square}$

3 0 0 0 × (1 + 0 • 0 1 5) x▪ 1 2 =

$A = \$$ ☐

Round to 2 d.p.

$1 140 invested for **2 years compounded monthly** at 0.3% per month.

6 $A = 1\,140 \times (1 + 0.003)^{\square}$

$A = \$$ ☐

$895 invested for **1½ years compounded monthly** at 0.5% per month.

7 $A = 895 \times (1 + 0.005)^{\square}$

$4 000 borrowed for 4 years compounded monthly at 1.8% per month.

8 $A = 4\,000 \times (1 + 0.018)^{\square}$

$600 borrowed for ½ a year compounded monthly at 2% per month.

9 $A = 600 \times (1 + 0.02)^{\square}$

Convert the periods to months and calculate the account balance.

Formula → Substitution → Answer $ sign

$5 000 invested for **1 year compounded monthly** at 1% per month.

10 $A = P(1 + r)^n$

$= \square \times (\square + \square)^{\square}$

$A =$ ☐

$750 invested for **2 years compounded monthly** at 0.5 % per month.

11 $A = P(1 + r)^n$

$=$

$A =$ ☐

$16 000 borrowed for 4 years compounded monthly at 3% per month.

12 $A = P(1 + r)^n$

$1 200 invested for ½ year compounded monthly at 0.7% per month.

13

$30 500 invested for 1½ years compounded monthly at 0.2% per month.

14

Now it is weekly. There are 52 weeks in a year, convert these terms to weeks, to find A.

$600 invested for **1 year compounded weekly** at 0.1% per week.

15 $A = P(1 + r)^n$

$= \square \times (\square + \square)^{\square}$

$A =$ ☐

$17 400 invested for 2 years compounded weekly at 0.2% per week.

16

$6 130 invested for 3 years compounded weekly at 0.1% per week.

17

Now biannually. That means compounded every 6 months, or, if you prefer, twice a year.

$4 085 invested for 5 years compounded biannually at 3.7% per 6 months.

18

Semiannually is the same as Biannually

$2 330 invested for 2 years compounded semiannually at 5.4% per 6 months.

19

Investing Money - Compound Interest Changing Periods (Sheet 2)

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Convert the periods to quarters and then calculate the account balance and interest earned.

Formula → Substitution → Answer $ sign

$27 500 invested for **1 year compounded quarterly** at 2% per quarter.

1 $A = P(1 + r)^n$

$= 27\,500 \times (1 + 0.02)$ ☐

$A =$ ☐

Now Calculate the Interest Earned

2 $I =$ ☐ $- 27\,500$

$I =$ ☐

$1 700 invested for **3 years compounded quarterly** at 3% per quarter.

3 ☐

Now Calculate the Interest Earned

4 $I =$ ☐ $- 1\,700$

$I =$ ☐

$3 120 invested for **5 years compounded quarterly** at 2.2% per quarter.

5 ☐

Now Calculate the Interest Earned

6 ☐

Find the interest earned and the account balance with these.

Bella is saving for a home deposit. She invests $25 400 for 3 years which is compounded monthly at 0.7% per month.

7 Express 0.7% as a decimal.

$r =$ ☐ $\div$ ☐ $=$ ☐

8 Calculate the number of compound periods in 3 years.

$n =$ ☐ $\times$ ☐ $=$ ☐

9 Find the account balance.

Dave has two investment options which both are 12.48% p.a.:

- 1.04% per month, compounded monthly
- 0.24% per week, compounded weekly

He has $5 000 to invest for 2 years.

10 Convert 1.04% to a decimal.

11 Convert 2 years to monthly compound periods.

12 Find the account balance for the 1.04% per month option.

13 Convert 0.24% to a decimal.

14 Convert 2 years to weekly compound periods.

15 Find the account balance for the 0.24% per week option.

16 Which option results in the best financial outcome?

○ 1.04% monthly ○ 0.24% weekly

17 By how much?

Georgie invests $3 000 for 4 years at 0.5% per month, compounded monthly.

18 Convert 0.5% to a decimal.

19 Find the account balance.

20 Calculate the interest earned

Investing Money - Compound Interest Changing Rates (Sheet 1)

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Change the rates to match the compound period. Then convert them to decimals.

1 10.8% p.a. to monthly

$r =$ ☐ %/month

÷ 10.8% by 12 to get monthly

$r =$ ☐.☐☐☐

÷ 100 to convert to a decimal

2 7.8% p.a. to weekly

$r =$ ☐ %/week

÷ 7.8% by 52 to get weekly

$r =$ ☐.☐☐☐☐

÷ 100 to convert to a decimal

3 6.4% p.a. to quarterly

$r =$ ☐ %/quarter

÷ 6.4% by 4 to get quarterly

$r =$ ☐

÷ 100 to convert to a decimal

4 7.2% p.a. to monthly

$r =$ ☐ %/month

$r =$ ☐

5 9.2% p.a. to quarterly

$r =$ ☐ %/quarter

$r =$ ☐

6 11.44% p.a. to weekly

$r =$ ☐ %/week

$r =$ ☐

7 9.6% p.a. to monthly

$r =$ ☐ %/month

$r =$ ☐

Convert the rates to monthly and calculate the account balance on these investments.

$8 000 is invested for 5 months and is compounded monthly at 9% p.a.

8 Find r as a decimal

$r = 9 \div 12 \div 100$

$r =$ ☐

9 $A = P(1 + r)^n$

$= 8\,000 \times (1 +$ ☐ $)^5$

8 0 0 0 × (1 + 0 • 0 0 7 5) x▪ 5 =

$A = \$$ ☐

Round to 2 d.p.

$3 400 is invested for 8 months and is compounded monthly at 14.4% p.a.

10 Find r as a decimal

$r =$ ☐ ÷ ☐ ÷ 100

$r =$ ☐

11 $A = P(1 + r)^n$

$= 3\,400 \times (1 +$ ☐ $)^8$

$A = \$$ ☐

$23 000 is invested for 15 months and is compounded monthly at 6% p.a.

12 Find r as a decimal

13

Convert the rates to weekly and calculate the account balance.

$2 500 is invested for 14 weeks and is compounded weekly at 7.8% p.a.

14 Find r as a decimal

$r =$ ☐ ÷ 52 ÷ ☐

$r =$ ☐

15

$5 390 is invested for 40 weeks and is compounded weekly at 9.36% p.a.

16

17

$770 is invested for 8 weeks and is compounded weekly at 7.28% p.a.

18

19

Calculate the interest earned on the $770 investment.

20

$I =$ ☐ - 770

$I =$

Investing Money - Compound Interest Changing Rates (Sheet 2)

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Change the rates to a quarterly decimal and then calculate the account balance.



\$650 is invested for 3 quarters and is compounded quarterly at 8% p.a.

1 $r = \square \div 4 \div \square$

$r = \square$

2 $A = P(1 + r)^n$

\$48 000 is invested for 7 quarters and is compounded quarterly at 13.6% p.a.

3

4

Now its daily, so divide the rate by 365. Find the daily rate then calculate the account balance.



\$4 450 is invested for 95 days and is compounded daily at 10.95% p.a.

5

6

\$11 250 is invested for 200 days and is compounded daily at 7.3% p.a.

7

8

Find the rate, the account balance and then the interest earned on these investments.



Eliza invests \$5 000 for 18 weeks at 6.24% p.a., compounded weekly.

9 Convert 6.24% p.a. to weekly, express as a decimal.

10 Find the account balance

11 Calculate the interest earned

$I = \square - 5\,000$

$I = \square$

12 Eliza needs \$8 000 for a car. Calculate the amount she still requires after the investment matures.

Leung invests \$7 200 for 15 months at 5.4% p.a., compounded monthly.

13 Convert 5.4% p.a. to monthly, express as a decimal.

14 Find the account balance

15 Calculate the interest earned

Omar invests \$14 900 for 300 days at 8.76% p.a., compounded daily.

16 Convert 8.76% p.a. to daily, expressing it as a decimal.

17 Find the account balance

18 Calculate the interest earned

Investing Money - Compound Interest - Calculating P (Sheet 1)

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Solve these 1-step equations using division.

Example:

$1\,350 = P \times 1.2$

÷1.2 ÷1.2

$P = 1\,125$

1 $448 = P \times 1.6$

÷1.6 ÷1.6

$P =$ ____

2 $823.50 = P \times 1.35$

$P =$ ____

3 $4\,830 = P \times 1.15$

$P =$ ____

4 $39\,750 = P \times 1.06$

5 $20\,886 = P \times 1.77$

The next two have to be rounded to 2 d.p.

6 $15\,660 = P \times 1.033$

Round to 2 d.p.

7 $2\,200 = P \times 1.813$

Round to 2 d.p.

Find P, but first perform a calculation. Show the $ sign and round to 2 d.p.

Example:

$3\,500 = P \times (1 + 0.1)^4$

Calc: (1 + 0 • 1) $x^{\blacksquare}$ 4 =

$3\,500 = P \times 1.4641$

÷1.4641 ÷1.4641

$P = \$2\,390.55$

Round to 2 d.p. and show the $ sign

8 $5\,600 = P \times (1 + 0.15)^2$

Calc: (1 + 0 • 1 5) x^2 =

$5\,600 = P \times$ ____

÷1.3225 ÷1.3225

$P = \$$ ____

Round to 2 d.p.

9 $850 = P \times (1 + 0.04)^3$

$850 = P \times$ ____

÷1.12…. ÷1.12….

Calc: 8 5 0 ÷ Ans =

$P =$ ____

Round to 2 d.p. and show the $ sign

10 $4\,440 = P \times (1 + 0.06)^3$

$4\,440 = P \times$ ____

÷1.19…. ÷1.19….

$P =$ ____

11 $950 = P \times (1 + 0.05)^3$

$950 = P \times$ ____

÷1.15…. ÷1.15….

$P =$ ____

These are just the same, but this time you have to get the numbers from the question.

An investment is compounded yearly for 3 years at 10% p.a. The investment results in an account balance of $945. Find P.

12 $A = P(1 + r)^n$

____ $= P \times (1 +$ ____ $)^{\square}$

____ $= P \times$ ____

$P =$ ____

Round to 2 d.p. and show the $ sign

An investment for 2 years at 6% p.a. is compounded yearly. An investment results in an account balance of $11 000. Find P, the principal invested.

13 $A = P(1 + r)^n$

____ $= P \times (1 +$ ____ $)^{\square}$

____ $= P \times$ ____

$P =$ ____

An investment for 3 years at 8% p.a. is compounded yearly and results in an account balance of $38 200. Find P, the principal invested.

14 $A = P(1 + r)^n$

Investing Money - Compound Interest - Calculating P (Sheet 2)

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Find P, but first perform a calculation. Show the $ sign and round to 2 d.p.



Example:

An investment is compounded annually for 5 years at 7.8% p.a. Find the amount invested (P) when the account balance is $7 300.

$A = P(1 + r)^n$

$7\,300 = P \times (1 + 0.078)^5$

Calc: (1 + 0 • 0 7 8) $x^{\blacksquare}$ 5 =

$7\,300 = P \times 1.455\ldots$ (calc)

÷1.455.. ÷1.455..

Calc: 7 3 0 0 ÷ Ans =

$P = \$5\,014.52$

Round to 2 d.p. and show the $ sign

An investment is compounded annually for 4 years at 5% p.a. Find the principal when the account balance is $4 750.

1 $A = P(1 + r)^n$

$\square = P \times (1 + \square)^{\square}$

A | 5% decimal | years

$\square = P \times 1.\square\square\square\ldots$ (calc)

÷ 1.215 ... ÷ 1.215 ...

$P =$

$ sign and round to 2 d.p.

An investment is compounded annually for 3 years at 7% p.a. Find the principal when the account balance is $850.

2 $A = P(1 + r)^n$

$\square = P \times \square$

$\square = P \times \square$

$P =$

Ben needs $8 000 in 3 years. He chooses an investment which offers 7% p.a. compounded annually for 3 years. Find the amount he needs to invest (P) now to achieve the $8 000 account balance.

3 $A = P(1 + r)^n$

$\square = P \times \square$

$\square = P \times \square$

$P =$

Tilly needs $15 000 to buy a car in three years time. How much does she need to invest now to have a $15 000 account balance? The investment is 5% p.a. compounded annually for three years.

4

Ryan needs $9 000 for a trip he wants to take in two years time. An investment offers interest at 4.2% p.a. compounded annually for 2 years.

5 Express 4.2% as a decimal.

6 Calculate the amount Ryan has to invest for a $9 000 account balance in two years.

Josh needs $32 000 for a new car in 3 years time. An investment offers interest at 9.3% p.a. compounded annually for 3 years.

7 Express 9.3% as a decimal.

8 Calculate the amount Josh has to invest for a $32 000 account balance in three years.

Imogen needs $45 000 for a deposit for a home purchase in 4 years time. An investment offers interest at 5.8% p.a. compounded annually for 4 years.

9 Express 5.8% as a decimal.

10 Calculate the amount Imogen has to invest now to achieve a $45 000 account balance in four years time.

11 How much interest will Imogen earn on the investment?

Investing Money - Compound Interest - Guess & Check *n* & *P* (Sheet 1)

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Change the power until you get the amount (2 d.p.) in the star. Use the calculator back button.

1 576.24 Number 2 - 9

$430 \times (1+0.05)$ ☐

Calc: 4 3 0 × (1 + 0 • 0 5) $x^{\blacksquare}$? =

2 13 109.11

$11\,450 \times (1+0.07)$ ☐

3 9 326.31

$5\,720 \times (1+0.13)$ ☐

You can find the investment term by guessing, checking and guessing again. Find '*n*'.

4 $784.50 = 620 \times (1+0.04)$ ☐

5 $15\,396.41 = 13\,300 \times (1+0.05)$ ☐

6 $6\,443.20 = 3\,750 \times (1+0.07)$ ☐

7 $1\,258.82 = 800 \times (1+0.12)$ ☐

8 $4\,637.10 = 4\,000 \times (1+0.03)$ ☐

9 $6\,059.45 = 5\,195 \times (1+0.08)$ ☐

10 $14\,363 = 13\,550 \times (1+0.06)$ ☐

These are the same, but you have to build the calculation line yourself. Find '*n*'.

An investment of \$720 at 5% p.a. compounded annually, results in an account balance of \$1 013.11. How many years was the investment term?

11 $1\,013.11 = 720 \times (1+0.05)$ ☐

The term was for ☐ years.

An investment of \$400 at 3% p.a. compounded annually, results in an account balance of \$450.20. How many years was the investment term?

12 *Remember use 3% as a decimal

The term was for ☐ years.

An investment of \$4 000 at 8% p.a. compounded annually, results in an account balance of \$7 403.72. How many years was the investment term?

13

An investment of \$7 700 at 5% p.a. compounded annually, results in an account balance of \$8 913.71. How many years was the investment term?

14

Find *P*. *P* will end with a '000' or '500'. **Put *P* last** in your calculator as it is easier go back and change.

15 $6\,928.42 = \square \times (1+0.08)^3$

Calc: (1 + 0 • 0 8) $x^{\blacksquare}$ 3 ▶ × ? =

16 $7\,737.38 = \square \times (1+0.12)^7$

17 $9\,363.60 = \square \times (1+0.02)^2$

18 $1\,966.19 = \square \times (1+0.07)^4$

19 $9\,124.90 = \square \times (1+0.04)^5$

Write the calculation line yourself. Find '*P*'. It ends with '000' or '500'.

A 7 year investment results in an account balance of \$9 773.60. If the interest rate was 6% p.a. compounded annually, find the amount invested.

20 *Remember use 6% as a decimal

The principal was ☐

Show \$ sign

Anna wants \$9 680 to buy a car in 2 years. If an investment offers 10% p.a. compounded annually, calculate the amount she needs to invest now to achieve a \$9 680 account balance.

21

Investing Money - Compound Interest - Guess & Check *r* (Sheet 2)

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Change the decimal until you get the amount (2 d.p.) in the star. Use calculator back key.

1 257.37

Decimal 0.01 to 0.15

$220 \times (1 + \square)^4$

Calc: 2 2 0 × (1 + 0 • ? ?) $x^{\blacksquare}$ 4 =

2 38 019.20

$32\,000 \times (1 + \square)^2$

3 3 500.63

$1\,900 \times (1 + \square)^5$

4 6 186.75

$5\,500 \times (1 + \square)^3$

Guess and check to find the decimal, '*r*'. The Answer will be between 0.01 and 0.15.

5 $1\,133.74 = 900 \times (1 + \square)^3$

6 $3\,221.02 = 2\,000 \times (1 + \square)^5$

7 $5\,445 = 4\,500 \times (1 + \square)^2$

8 $3\,040.13 = 1\,800 \times (1 + \square)^4$

9 $1\,402.81 = 750 \times (1 + \square)^6$

This time, find the decimal but then multiply it by 100 to express it as a percentage.

10 $795.68 = 750 \times (1 + \square)^2$

Interest rate = ☐ %

Multiply your top answer by 100

11 $4\,799.38 = 3\,400 \times (1 + \square)^4$

Interest rate = ☐ %

12 $9\,076.06 = 5\,150 \times (1 + \square)^5$

Interest rate = ☐

Show % sign

13 $3\,084.88 = 1\,800 \times (1 + \square)^7$

14 $912.53 = 600 \times (1 + \square)^3$

These are the same, but you have to build the calculation line yourself. Find '*r*' as a decimal then convert to a %.

An investment of $7 000 compounded annually for 4 years, results in an account balance of $8 837.34. What was the prevailing interest rate?

15

Interest rate = ☐ %

An investment of $3 850 compounded annually for 2 years, results in an account balance of $4 829.44. What was the prevailing interest rate?

16

Interest rate = ☐ %

An investment of $15 100 compounded annually for 7 years, results in an account balance of $19 870.57. What was the interest rate applied to the investment?

17

With these two, find the required interest rate to the **nearest whole percent**.

James has $40 000 to invest for 3 years. If he wants $50 000 after the 3 years, what annual interest rate is needed. Assume investments are compounded annually.

18

Helen needs $12 000 in 5 years. She is investing $6 000 in a compound investment which compounds annually. What annual interest rate is required for her to achieve the $12 000 account balance?

19

Investing Money - Depreciation - Straight Line Method (Sheet 1)

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Convert these percentages to decimals by ÷ 100, or move the d.p. twice to the left.

Example:

16% = 16 ÷ 100 = 0.16

1 6% = ☐ ÷ 100 = ☐

2 8% = ☐ ÷ ☐ = ☐

3 12% = ☐ ÷ ☐ = ☐

Answer only, from now on.

4 19% = ☐

5 12.7% = ☐

6 11.6% = ☐

7 5.05% = ☐

Find a percentage of these quantities. In Mathematics 'of' means times '×'.

Example: Find 19% of $350

0.19 × 350 = $66.50

Convert 19% to a decimal

Show '$' sign and round to 2 d.p.

8 Find 7% of $420

☐ × 420 = $☐☐.☐☐

9 Find 13% of $1 200

☐ × ☐ = $☐

10 Find 3% of $4 090

☐ × ☐ = ☐

Answer with a '$' sign and cents to 2 d.p.

11 Find 6.5% of $11 500

12 Find 9.2% of $30 900

Straight Line Depreciation uses: $S = V_0 - Dn$. Calculate D, then the salvage value S.

James buys a car with a sale price of $46 900. It depreciates by 15% each year. Calculate the:

13 Annual depreciation.

$D =$ 0.15 × 46 900

$D =$ $☐☐☐☐

14 Salvage value after 3 years.

$S = V_0 - Dn$

= 46 900 - ☐ × 3

$S =$ $☐

15 Salvage value after 5 years.

$S = V_0 - Dn$

= 46 900 - ☐ × 5

$S =$ ☐

Catherine buys a delivery van with a sale price of $29 550. It depreciates by 18% each year. Calculate the:

16 Annual depreciation.

$D =$ ☐ × ☐

% as a decimal × purchase price

$D =$ ☐ $ sign!

17 Salvage value after 2 years.

$S = V_0 - Dn$

= ☐ - ☐ × ☐

Purchase price | Depreciation | Years

$S =$ ☐

Jonathan buys a forklift with a sale price of $18 998. It depreciates by 12.5% each year. Calculate the:

18 Annual depreciation.

19 Salvage value after 6 years.

Formula

Substitute values

Show $ and cents to 2 d.p.

Daisy buys a new display cabinet for $1 115. The cabinet depreciates at 16.5% p.a. Calculate the:

20 Annual depreciation.

21 Salvage value after 3 years.

Bethany buys a laptop for $1 335. The laptop has an annual depreciation rate of 22.5% p.a. Calculate the:

22 Annual depreciation.

23 Salvage value after 4 years.

Investing Money - Depreciation - Graphing S/L Depreciation (Sheet 2)

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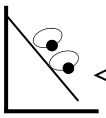


Straight line depreciation forms a straight line when graphed!!!! Calculate D, then find S and complete the table, then plot the values.

Yogi and Boo-Boo is a business that fabricates picnic baskets. A weaving machine costs \$8 000 and is depreciated at 15% p.a.

1 Calculate the annual depreciation.

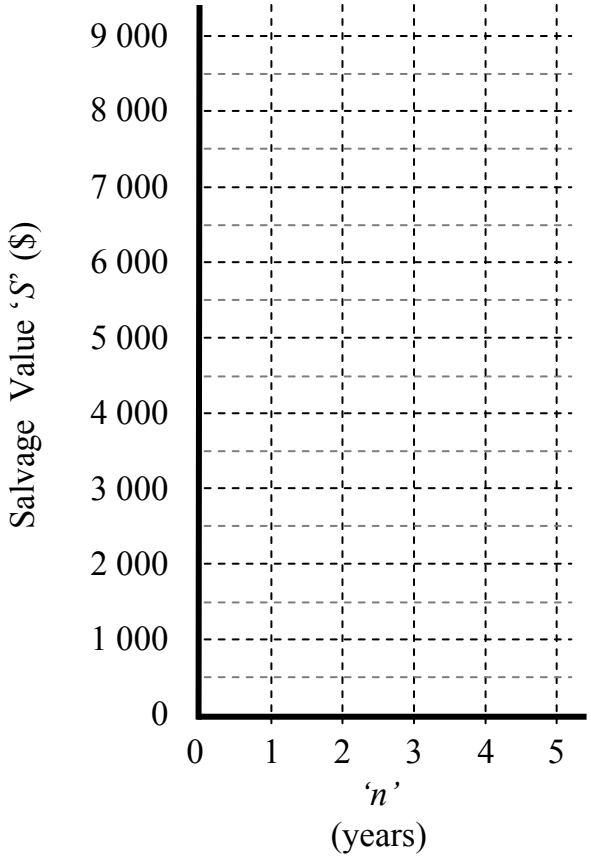
$D =$ [ ] $\times$ [ ] $=$ [ ]

[% as a decimal] [purchase price] [\$ sign]

2 Complete the table

	n Years	Salvage Value S \$
Initial Purchase Price →	0	8 000
Year 0 'S' value - D →	1	
Year 1 'S' value - D →	2	
Year 2 'S' value - D →	3	
Year 3 'S' value - D →	4	
Year 4 'S' value - D →	5	

3 Plot the points then use a ruler to construct a straight line.



A jet-ski hiring company has bought a new jet-ski for \$12 000. It is depreciated at 20% p.a.

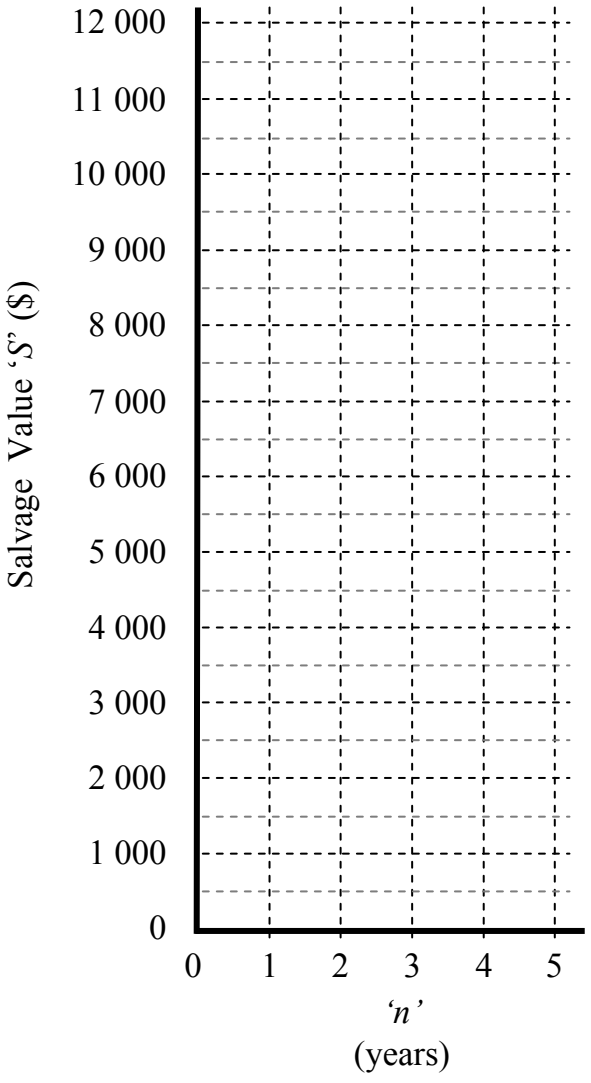
4 Calculate the annual depreciation.

$D =$ [ ] $\times$ [ ] $=$ [ ]

5 Complete the table

Year n	Salvage Value S \$
0	
1	
2	
3	
4	
5	

6 Display the results in graphical form below.



Investing Money - Depreciation - Declining Balance - Stepped (Sheet 1)

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Calculate D, the depreciation then calculate S. Then redo this step again. D declines.

Jacqueline buys a car with a sale price of $27 100. It depreciates by 15% each year. Calculate the:

Year 1

1 Depreciation in the 1st year.

$D = 0.15 \times 27\,100$

% as a decimal × Purchase price

$D = \$$ ☐☐☐☐

2 Salvage value after 1 year.

$S = 27\,100 -$ ☐ (Q.1)

$S =$ ☐

This value is used in the next calculation

Year 2

3 Depreciation in the 2nd year.

$D = 0.15 \times$ ☐

% as a decimal × Q.2 answer

$D = \$$ ☐

4 Salvage value after 2 years.

$S =$ ☐ - ☐

Q.2 answer | Q.3 answer

$S =$ ☐

This value is used in the next calculation

Year 3

5 Depreciation in the 3rd year.

$D = 0.15 \times$ ☐

% as a decimal × Q.4 answer

$D = \$$ ☐

6 Salvage value after 3 years.

$S =$ ☐ - ☐

Q.4 answer | Q.5 answer

$S =$ ☐

This is S after 3 yr, declining balance method

This time a farmer has bought a cool room. Find S after 3 years using the D.B. method

A cool room (for storing apples), costs $17 000 and depreciates by 12% p.a. Calculate the:

Year 1

7 Depreciation in the 1st year.

$D =$ ☐ × ☐

% as a decimal × Purchase price

$D = \$$ ☐

8 Salvage value after 1 year.

$S =$ ☐ - ☐

Purchase price | Q.7 Depreciation

$S =$ ☐

Year 2

9 Depreciation in the 2nd year.

$D =$ ☐ × ☐

% as a decimal × Q.8 answer

$D = \$$ ☐

10 Salvage value after 2 years.

$S =$ ☐ - ☐

Q.8 answer | Q.9 answer

$S =$ ☐

Year 3

11 Depreciation in the 3rd year.

$D =$ ☐ × ☐

$D = \$$ ☐

12 Salvage value after 3 years.

$S =$ ☐ - ☐

$S =$ ☐

This is Salvage value after 3 years, using the declining balance method in 3 applications

Use the brackets keys and power key to evaluate these. Answer to 2 decimal places.

13 $450 \times (1 - 0.08)^4$

$=$ ☐ (Round to 2 decimal places)

Calc: 4 5 0 × (1 - 0 • 0 8) x▪ 4 =

14 $5\,500 \times (1 - 0.11)^5$

$=$ ☐

15 $38\,200 \times (1 - 0.18)^7$

$=$ ☐

16 $26\,000 \times (1 - 0.09)^3$

$=$ ☐

Now answer Columns 1 and 2 in one step. Don't get angry, its called learning! Find 'S' (2 d.p.)

A car with a sale price of $27 100 depreciates by 15% each year. Calculate S after 3 years.

17 $S = V_0 (1 - r)^n$

$=$ ☐ (1 - ☐) ☐

Purchase price | % decimal | years

$S =$ ☐

18 Is your Q.6 answer the same as your Q.17 answer?

○ Yes ○ Arrrrggghhhh!

A cool room costs $17 000 and depreciates by 12% p.a. Calculate the salvage value after 3 years.

19 $S = V_0 (1 - r)^n$

$=$ ☐ (1 - ☐) ☐

$S =$ ☐

20 Is your Q.12 answer the same as your Q.19 answer?

○ Yes ○ I decline to answer

Investing Money - Depreciation - Declining Balance Method (Sheet 1)

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Use the brackets and power keys to solve these. Answer to 2 d.p.

1 $42\,700 \times (1 - 0.09)^5$

$=$ ____

4 2 7 0 0 × (1 - 0 • 0 9) $x^{\blacksquare}$ 5 =

2 $19\,804 \times (1 - 0.06)^3$

$=$ ____

3 $1\,447 \times (1 - 0.15)^6$

$=$ ____

4 $20\,550 \times (1 - 0.125)^4$

$=$ ____

Calculate the salvage value for these purchases. Answer to 2 d.p.

A car with a sale price of $30 499 depreciates by 15% each year. Calculate S after 3 years.

5 $S = V_0 (1 - r)^n$

$=$ ____ × (1 - ____) ☐

Purchase price | % decimal | years

$S = \$$ ☐☐ ☐☐☐ . ☐☐

Calculate S after 5 years.

6 $S = V_0 (1 - r)^n$

$=$ ____ × (1 - ____) ☐

$S = \$$ ____

Calculate S after 9 years.

7 $S = V_0 (1 - r)^n$

$=$ ____ × (1 - ____) ☐

$S =$ ____

$ Sign!

A table saw with a sale price of $2 100 depreciates by 20% each year. Calculate S after 3 years.

8 ____

Formula

____

Substitute Values

____

Answer to 2 d.p. showing the $ sign

Calculate S after 7 years.

9 ____

A new trench digger costs $9 030 and depreciates by 15% each year. Calculate S after 2 years.

10 ____

Calculate S after 4 years.

11 ____

A café's coffee machine costs $3 046 and depreciates by 9% each year. Calculate S after 3 years.

12 ____

Calculate S after 5 years.

13 ____

Now calculate the salvage value, then subtract it from the purchase price to calculate D.

A cherry picker costs $48 771 and depreciates by 16% each year. Calculate S after 2 years.

14 $S = V_0 (1 - r)^n$

Find the depreciation after 2 years

15 $D = 48\,771 -$ ____

Q.14

$D =$ ____

A Hair salon chair and wash basin costs $4 330 and is depreciated at 22% p.a. Calculate the salvage value after 5 years.

16 ____

Find the depreciation after 5 years

17 $D = 4\,330 -$ ____

$D =$ ____

A complete shop re-fit costs $18 667 and depreciates by 12.5% each year. Calculate S after 3 years.

18 ____

Find the depreciation after 3 years

19 ____

Investing Money - Depreciation - Graphing D/B Salvage Value (Sheet 2)

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Declining Balance depreciation forms a curve when graphed!!!! Calculate S, complete the table, then plot the values and create a curve.

A Fitness centre has purchased a cable cross-over machine for \$3 900. The machine is depreciated by the declining balance method at 15% p.a.

1 Calculate the salvage values for 4 years.

Yr 0: Value = \$3 900

Yr 1: $S = 3\,900 \times (1 - 0.15)^1 =$

Yr 2: $S = 3\,900 \times (1 - 0.15)^2 =$

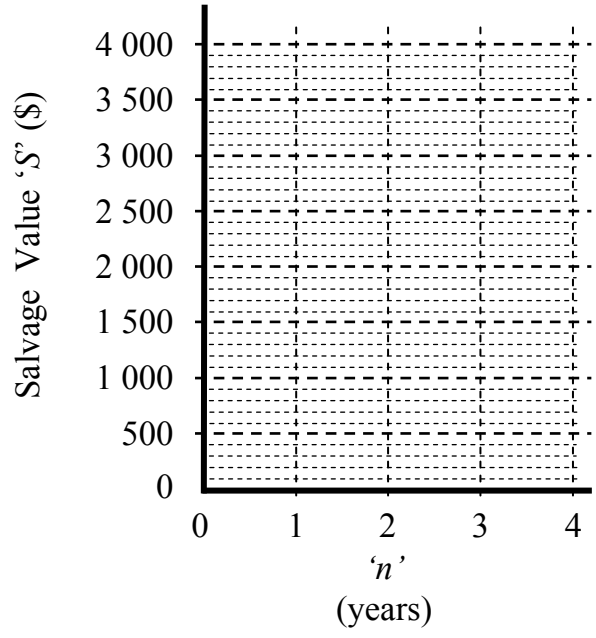
Yr 3: $S =$

Yr 4: $S =$

2 Complete the table

n Years	Salvage Value S \$
0	
1	
2	
3	
4	

3 Plot the points, use a pencil first to construct a smooth curve, then a pen afterwards.

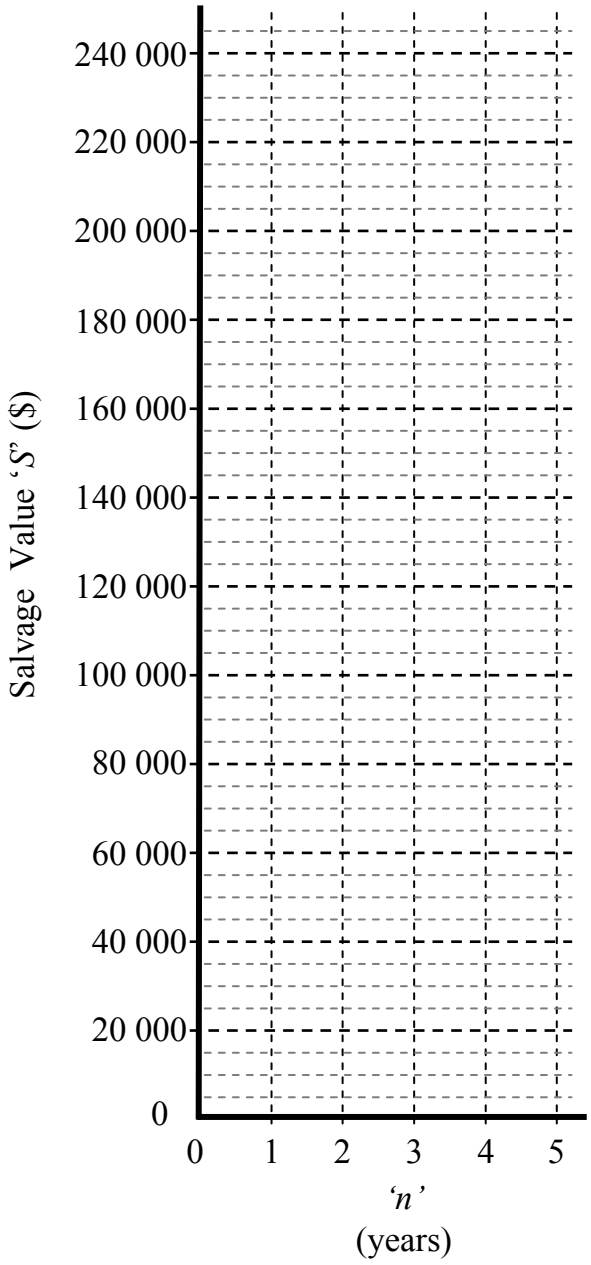


A legal firm has paid \$245 000 for a office fit-out. It is depreciated by the declining balance method at 12.5% p.a.

4 Complete the table

	Year n	Salvage Value S \$
	0	
$245\,000 \times (1 - 0.125)^1$	1	
$245\,000 \times (1 - 0.125)^2$	2	
$245\,000 \times (1 - 0.125)^3$	3	
$245\,000 \times (1 - 0.125)^4$	4	
$245\,000 \times (1 - 0.125)^5$	5	

5 Display the results on the graph below.



Investing Money - Reducing Balance Home Loans (Sheet 1)

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A home loan charges simple interest on a monthly period. Repayments larger than the interest charged are made which not only pay the interest, but also reduce the principal. So each month the interest is slightly less and the principal reduces. Complete these home loan balance sheets for the first 4 months of the loan.



1 Home loan is for $180 000 at 7.2% p.a., with repayments of $1 750 per month.

Convert 7.2% p.a. to a monthly decimal: $r = 7.2 \div 12 \div 100 =$ ☐.☐☐☐

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	180 000	☐ 180 000 × 0.006	☐ 180 000 + 1 080	1 750	☐ 181 080 - 1 750
2	179 330	☐ 179 330 × 0.006	☐ 179 330 + 1 075.98	1 750	☐ 180 405.98 - 1 750
3	178 655.98	☐ 178 655.98 × 0.006	☐ 178 655.98 +1 071.94	1 750	☐ 179 727.92 - 1 750
4	177 977.92	☐ 177 977.92 × 0.006	☐ 177 977.92+1 067.87	1 750	☐ 179 045.79 - 1 750

2 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = ☐ (Amount borrowed) - ☐ (Period 4 Column 6) = ☐ (Show the $ sign)

3 Home loan is for $350 000 at 5.4% p.a., with repayments of $2 800 per month.

Convert 5.4% p.a. to a monthly decimal: $r = 5.4 \div 12 \div 100 =$ ☐

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1	350 000	☐ 350 000 × r	☐ 350 000 + I	2 800	☐
2	☐	☐	☐	2 800	☐
3	☐	☐	☐	2 800	☐
4	☐	☐	☐	2 800	☐

4 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = ☐ - ☐ = ☐

Investing Money - Reducing Balance Home Loans (Sheet 2)

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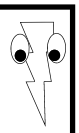
Complete these home loan balance sheets for the first 4 months of these loans. At the end of each row, the last cell has the answer there, except it has two digits 'obscured'. Check your answer at the end of each row, if you don't have a match then you need to back track and find where you have made the mistake.



1 Home loan is for $295 000 at 6% p.a., with repayments of $2 100 per month.

Convert 6% p.a. to a monthly decimal: $r = 6 \div 12 \div 100 =$ ☐.☐☐☐

Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$ (Col. 4 - Col. 5)
1					29X X75
2					29X 74X.88
3					2X3 115.X1
4					292 X81.X9

2 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = ☐ - ☐ = ☐

3 Home loan is for $520 000 at 6.6% p.a., with repayments of $3 800 per month.

Convert 6.6% p.a. to a monthly decimal: $r = 6.6 \div 12 \div 100 =$ ☐

Period	P	I	$P + I$	R	$P + I - R$
1					519 X6X
2					5X8 1X4.83
3					51X 164.X6
4					5X6 20X.86

4 How much principal had been paid off the loan after the 4 month period?

Amount Paid Off = ☐ - ☐ = ☐

Investing Money - Introduction to Credit Cards (Sheet 1)

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Calculate the interest payable on these credit card purchases.

Example Stove $1 850, 16% p.a. for 22 days.

$I = Prn$ ← Write formula

$= 1\,850 \times 0.16 \times \frac{22}{365}$ Substitute Values

$I = \$17.84$ ← Answer to nearest cent

1 Desk $495, 19% p.a. for 13 days.

$I = Prn$

$= \square \times \square \times \frac{\square}{365}$ (Days)

Item cost | Rate (decimal)

4 9 5 × 0 • 1 9 × 1 3 ÷ 3 6 5 =

$I =$ ____

Show the '$' sign and cents to 2 d.p.

2 Clothes $721, 15% p.a., 24 days.

$I = Prn$

$= \square \times \square \times \frac{\square}{365}$

$I =$ ____

3 Laptop $1 135, 22% p.a., 9 days.

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I =$ ____

4 Phone $408, 18% p.a., 15 days.

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I =$ ____

5 Flight $206, 12.5% p.a., 27 days.

$I = Prn$

$=$

$I =$

6 Petrol $65.80, 17% p.a., 13 days.

$I = Prn$

$=$

$I =$

7 Bill $425.15, 23% p.a., 4 days.

Banks charge interest from the day of purchase, until the day you pay it back. Count the days.

Example Purchased: 9th June
Paid: 26th June

Days = 26 - 9 + 1 = 18 days

8 Purchased: 3rd February
Paid: 27th February

Days = ☐ - ☐ + 1

Days = ☐ days

9 Purchased: 11th March
Paid: 25th March

Days = ☐ - ☐ + ☐

Days = ☐ days

10 Purchased: 6th September
Paid: 23rd September

Days = ☐ - ☐ + ☐

Days = ☐ days

Calculate the interest payable on these credit card purchases.

11 Clothes $178, 14% p.a.
Purchased: 8th April
Paid: 25th April

Days = ☐ - ☐ + ☐

Days = ☐ days

$I = Prn$

$= \square \times \square \times \frac{\square}{365}$

$I =$ ____

12 Holiday $3 840, 17% p.a.
Purchased: 2nd January
Paid: 28th January

Days = ☐ - ☐ + ☐

Days = ☐ days

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I =$ ____

13 Headphones $272, 23% p.a.
Purchased: 24th March
Paid: 28th March

Days = ☐ - ☐ + ☐

Days = ☐ days

$I = Prn$

$= \square \times \square \times \frac{\square}{\square}$

$I =$ ____

Investing Money - Introduction to Credit Cards (Sheet 2)

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Calculate the accrued days then calculate the interest payable on these credit card purchases.

1 Insurance $480.35, 23% p.a.
Purchased: 16th July
Paid: 23rd July

Days = [ ] - [ ] + 1

Days = [ ] days

I = *Prn*

= [ ] × [ ] × [ ] / 365

I = [ ]

2 Movie Tickets $58.90, 16% p.a.
Purchased: 3rd October
Paid: 24th October

Days = [ ] - [ ] + [ ]

Days = [ ] days

I = *Prn*

= [ ] × [ ] × [ ] / [ ]

I = [ ]

3 Phone Credit $45.55, 19% p.a.
Purchased: 17th March
Paid: 28th March

Days = [ ] - [ ] + [ ]

Days = [ ] days

I = *Prn*

= [ ] × [ ] × [ ] / [ ]

I = [ ]

4 Refrigerator $1 099, 21% p.a.
Purchased: 4th June
Paid: 23rd June

Days =

Days =

I = *Prn*

=

I =

5 Electricity Bill $597, 21% p.a.
Purchased: 13th June
Paid: 23rd June

Days =

Days =

I = *Prn*

=

I =

6 Shoes $68.70, 21% p.a.
Purchased: 18th June
Paid: 23rd June

Days =

Days =

I = *Prn*

=

I =

7 Add Interest in Q 4, 5 and 6.

I = [ ] + [ ] + [ ]

I = [ ]

8 Add Purchases in Q 4, 5 and 6.

I = [ ] + [ ] + [ ]

I = [ ]

9 Television $485, 14% p.a.
Purchased: 3rd March
Paid: 27th March

Days =

Days =

I = *Prn*

=

I =

10 Kitchenware $93.75, 14% p.a.
Purchased: 14th March
Paid: 27th March

11 Food $143, 14% p.a.
Purchased: 24th March
Paid: 27th March

12 Add Interest in Q 9, 10 and 11.

I = [ ] + [ ] + [ ]

I = [ ]

13 Add Purchases in Q 9, 10 and 11.

I = [ ] + [ ] + [ ]

I = [ ]

Investing Money - Credit Cards (Sheet 1)

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These credit cards charge interest from, and including, the date of purchase to, and including, the payment due date. Find the interest payable on each of the purchases. Note the date due.



1 Rate = 14% p.a. Date Due 30/10

Date	Description	Amount	Balance
12/10	Vet Fee	$90	$90
17/10	CD Sale	$25	$115
22/10	Groceries	$135	$250

12/10 Days: $30 - 12 + 1 =$ ____ days

12/10 Interest: $I = 90 \times 0.14 \times \frac{\square}{365} =$ $

17/10 Days: $30 - 17 + 1 =$ ____ days

17/10 Interest: $I = 25 \times 0.14 \times \frac{\square}{365} =$ ____

22/10 Days: $30 - 22 + 1 =$ ____ days

22/10 Interest: $I = 135 \times 0.14 \times \frac{\square}{365} =$ ____

2 Rate = 18% p.a. Date Due 29/4

Date	Description	Amount	Balance
2/4	Phone Bill	$157	$157
13/4	Insurance	$530	$687
17/4	Groceries	$210	$897

2/4 Days: 29 - ____ + ____ = ____ days

2/4 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

13/4 Days: ____ - ____ + ____ = ____ days

13/4 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

17/4 Days: ____ - ____ + ____ = ____ days

17/4 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____



These are the same except you also have to fill in the balance boxes in the statement as well.

3 Rate = 17% p.a. Date Due 24/6

Date	Description	Amount	Balance
8/6	Clothing	$293 →	
11/6	Ticket	+ $164 →	
20/6	Phone Credit	+ $45 →	

8/6 Days: ____ - ____ + ____ = ____ days

8/6 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

11/6 Days: ____ - ____ + ____ = ____ days

11/6 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

20/6 Days: ____ - ____ + ____ = ____ days

20/6 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

4 Rate = 21% p.a. Date Due 28/3

Date	Description	Amount	Balance
9/3	Groceries	$148	
17/3	Restaurant	$95	
22/3	Theme Park	$65	

9/3 Days: ____ - ____ + ____ = ____ days

9/3 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

17/3 Days: ____ - ____ + ____ = ____ days

17/3 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

22/3 Days: ____ - ____ + ____ = ____ days

22/3 Int: $I =$ ____ $\times$ ____ $\times \frac{\square}{\square} =$ ____

Investing Money - Credit Cards (Sheet 2)

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Complete the balance boxes in the statement. Find the interest payable on each purchase. Add all the interest and write total in 'Interest Payable' box, then add to the last balance box.



1 Rate = 18% p.a. Date Due 30/7

Date	Description	Amount	Balance
2/7	Cable TV	$68	
5/7	Clothing	$165	
8/7	Broadband	$72	
14/7	Petrol	$88	
23/7	Electricity	$395	

30/7 Interest Payable

Add all five of the interest calculations below, write in box

30/7 Amount Payable

Add the 'Interest Payable' box to the final box in the statement

2/7 Days: ☐ - ☐ + ☐ = ☐ days

2/7 Int: $I =$ ☐ × ☐ × ☐/☐ = $

5/7 Days: ☐ - ☐ + ☐ = ☐ days

5/7 Int: $I =$ ☐ × ☐ × ☐/☐ =

8/7 Days: ☐ - ☐ + ☐ = ☐ days

8/7 Int: $I =$ ☐ × ☐ × ☐/☐ =

14/7 Days: ☐ - ☐ + ☐ = ☐ days

14/7 Int: $I =$ ☐ × ☐ × ☐/☐ =

23/7 Days: ☐ - ☐ + ☐ = ☐ days

23/7 Int: $I =$ ☐ × ☐ × ☐/☐ =

Same process, except a Cash Advance is made, the interest rate is higher for cash advances.

2 Purchases = 16% p.a. Date Due 28/11

Cash Advance* = 24% p.a.

Date	Description	Amount	Balance
1/11	eTunes	$82	
6/11	Cash Adv.*	$600	
13/11	Groceries	$72	
14/11	JC Hi-Fi	$415	
24/11	Big UU	$209	

28/11 Interest Payable

Add all five of the interest calculations below, write in box

28/11 Amount Payable

Add the 'Interest Payable' box to the final box in the statement

1/11 Days: ☐ - ☐ + ☐ = ☐ days

1/11 Int: $I =$ ☐ × ☐ × ☐/☐ = $

6/11 Days: ☐ - ☐ + ☐ = ☐ days

6/11 Int*: $I =$ ☐ × ☐ × ☐/☐ =

Cash advance* use 24%

13/11 Days: ☐ - ☐ + ☐ = ☐ days

13/11 Int: $I =$ ☐ × ☐ × ☐/☐ =

14/11 Days: ☐ - ☐ + ☐ = ☐ days

14/11 Int: $I =$ ☐ × ☐ × ☐/☐ =

24/11 Days: ☐ - ☐ + ☐ = ☐ days

24/11 Int: $I =$ ☐ × ☐ × ☐/☐ =

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Convert these times to the specified periods.

1 1 year = ☐ months

2 2 years = ☐ weeks

3 5 years = ☐ months

Eve makes an investment of $9 000 for 4 years compounded monthly at 3% per month.
Use: $A = P(1 + r)^n$

4 Convert 3% to a decimal

5 Calculate the number of compound periods.

6 Calculate Eve's account balance

7 Calculate the interest Eve has earned.

Declan is investing $5 000 for 3 quarters at 10% p.a., compounded quarterly.
Use: $A = P(1 + r)^n$

8 Convert 10% p.a. to a quarterly decimal

9 Calculate Declan's account balance

10 Calculate the interest Declan has earned.

Use guess and check methods to find the term (power) for this compound investment. Answer in the box.

11 $7\,078.09 = 4\,130 \times (1 + 0.08)^{\square}$

An investment of $26 500 at 6% p.a. compounded annually, results in an account balance of $35 462.98. How many years was the investment term? Use G & C.

12

Use guess and check methods to find the rate as a decimal (2 d.p.) for this compound investment. Answer in the box.

13 $3\,587.04 = 2\,200 \times (1 + \square)^4$

An investment of $9 330 compounded annually for 6 years, results in an account balance of $14 001.81. What was the prevailing interest rate?

14

Solve to find the value of P.

15 $3\,780 = P \times 1.08$

16 $1\,190.04 = P \times (1 + 0.05)^3$

An investment for 3 years at 6% p.a. is compounded yearly and results in an account balance of $7 384.30. Find P, the principal invested.

17

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Glen buys a ute for $22 400. It is depreciated at 15% p.a. using the straight line depreciation method.

18 Calculate the yearly depreciation on the ute.

19 Calculate the salvage value after 4 years. Use:
$S = V_0 - Dn$

Kate buys a new laptop for $1 240. It is depreciated at 12% p.a. using the declining balance method.

20 Calculate the salvage value after 3 years. Use:
$S = V_0 (1 - r)^n$

21 Calculate the amount the laptop has depreciated by after the 3 year period.

Caitlyn has recorded her purchases on her credit card below. The card charges interest at 21% p.a. from the day of purchase until and including the day the balance is due.

23 Calculate the values in the balance column

Date	Description	Amount	Balance
4/3	Groceries	$258	
12/3	Restaurant	$105	

24 The balance is due on 28/3. Calculate the number of days interest is payable for:

4/3 purchase:

12/3 purchase:

25 Calculate the simple interest payable on each purchase. Use $I = Prn$, the interest rate is 21% p.a.

4/3 purchase:

12/3 purchase:

26 Calculate the amount due on 28/3.

22 Complete the home loan table below. The home loan is for $130 000 at an interest rate of 0.5% per month, with repayments of $1 400 per month. Use $I = Prn$ to calculate simple interest.

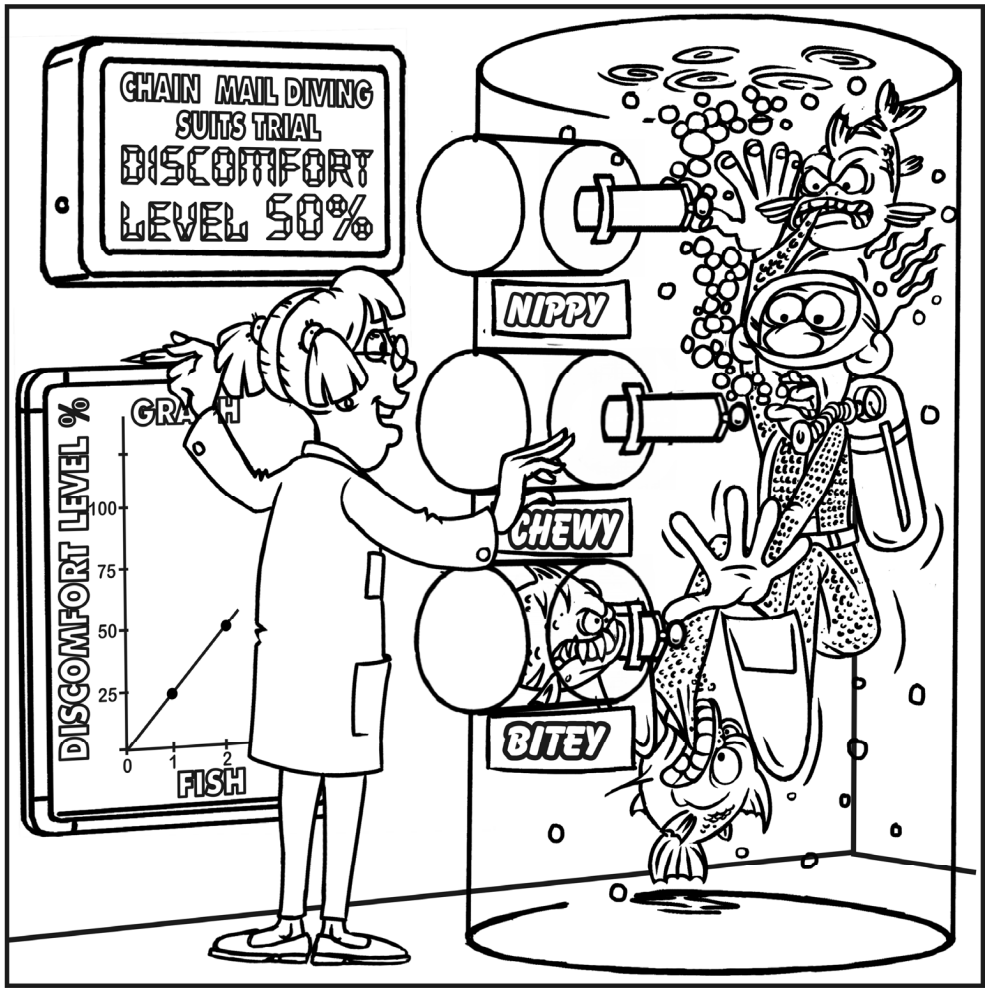
Period (Months)	P (Principal)	I (Month's Interest)	$P + I$ (Principal + Interest)	R (Repayments)	$P + I - R$
1	130 000			1 600	
2				1 600	
3				1 600	

VELOCITY

LINEAR RELATIONSHIPS

THE CARTESIAN PLANE TOPIC SHOULD BE COMPLETED BEFORE THIS TOPIC

Linear Relationships



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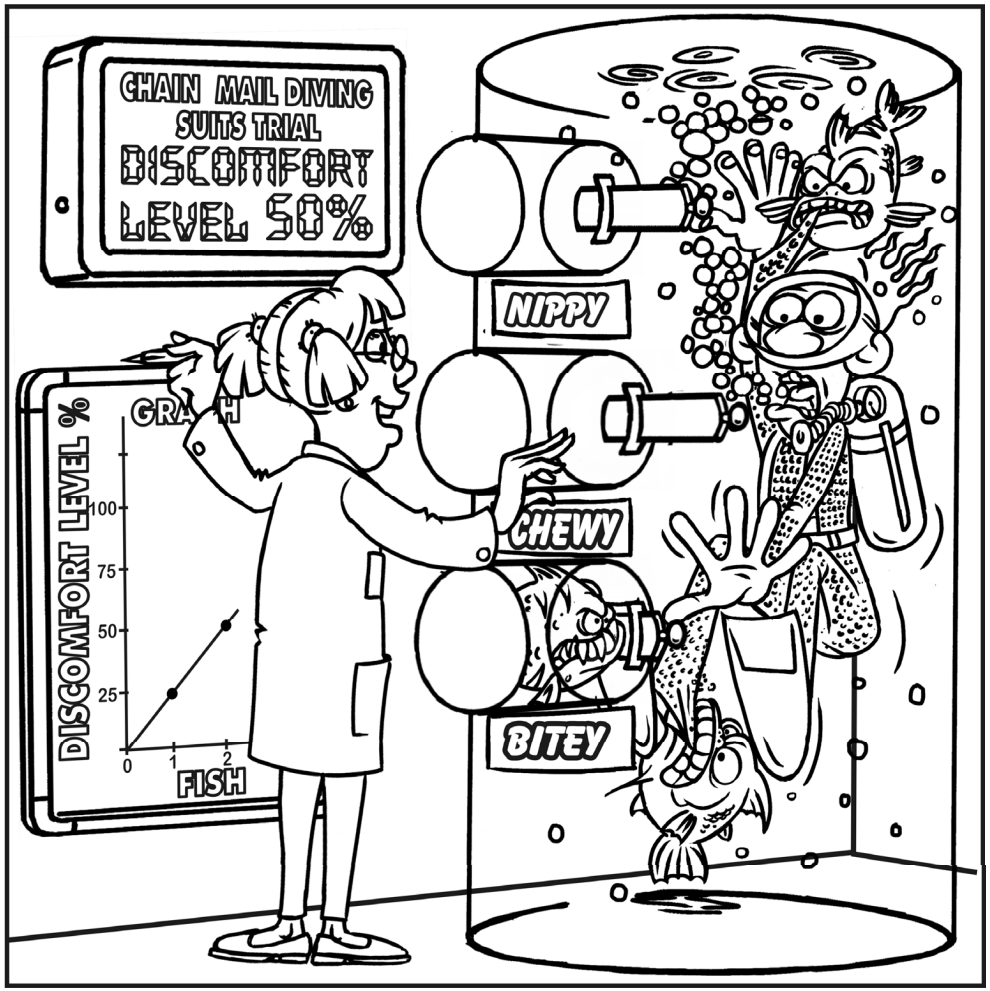
Linear Relationships



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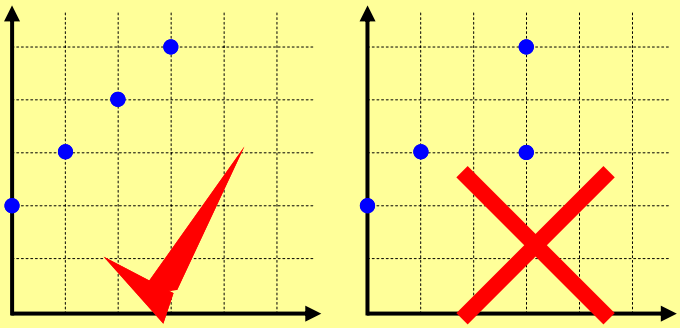
Linear Relationships - Plotting Points & Drawing Straight Lines

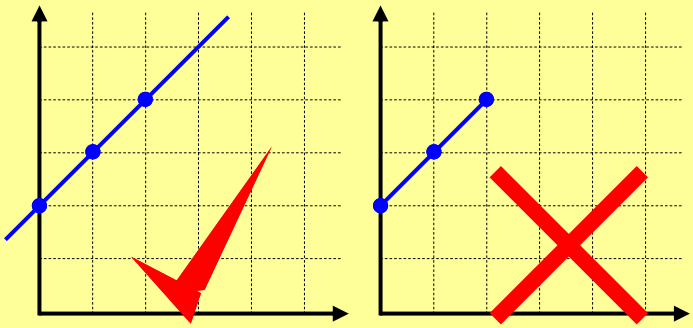
If you can plot points on a grid, drawing straight lines is just drawing a line through them. This sheet removes calculation from the process and just relies on you plotting points correctly and then drawing a line correctly.

There are three key features of drawing a straight line. These are:

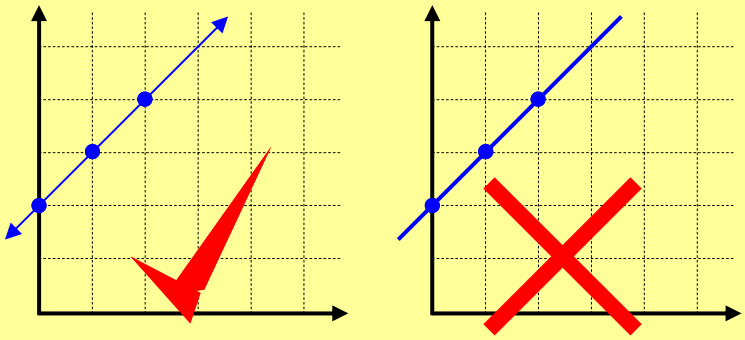
STEP 1 Make sure that your points are in a straight line.



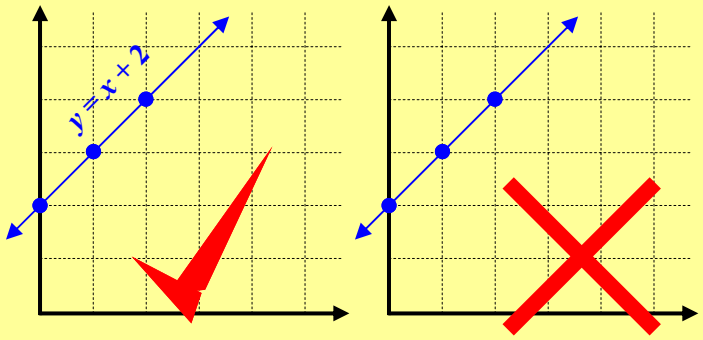
STEP 2 Draw a line the full size of the grid, not just between the first and last dot.



STEP 3 Put arrow heads on each end of the line.



STEP 4 Write the equation for the line on top of the line. Write it in pencil as another line may plot over it later and you will have to move the name.



Column 1 gives you the points to plot as ordered pairs (x co-ord, y co-ord). Plot the points then draw a line through them, keeping the above in mind. The second graph asks you to list all the points where an intersection of lines occurs.

Column 2 is the same except that rather than the points being given as ordered pairs they are in tables. The x value for the point is in the top row, the y value is in the bottom row.

The first worksheet deals with the positive quadrant only, the second worksheet deals with the full plane.

Linear Rel- Plotting Points & Drawing Straight Lines - + Quad (Sheet 1)

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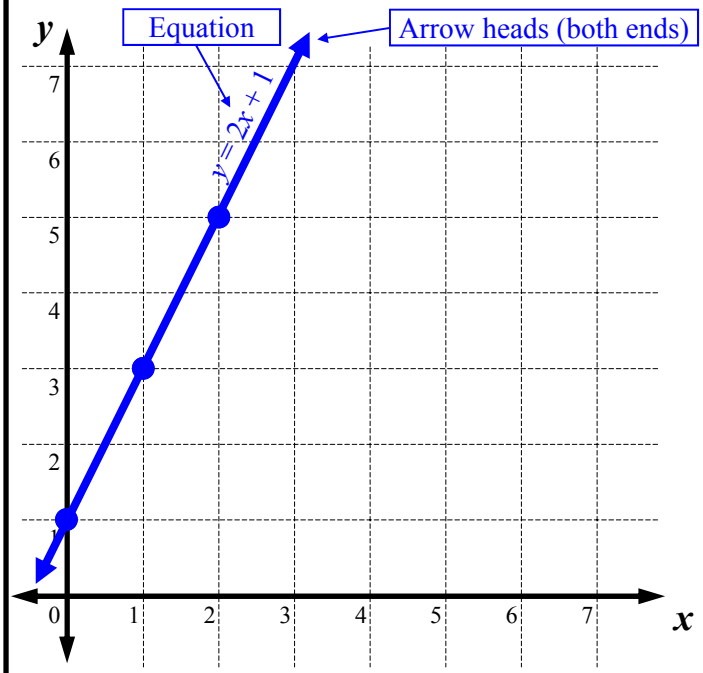
Plot the points then draw a line through them. Use a ruler, show arrow heads and write the equation of the line on the line. **The line should be the full size of the graph**, not stop at the last point.

1 $y = 2x + 1$ Plot Points: (0, 1), (1, 3), (2, 5).

2 $y = -x + 6$ Plot Points: (0, 6), (2, 4), (4, 2).

3 $y = x - 3$ Plot Points: (3, 0), (4, 1), (5, 2).

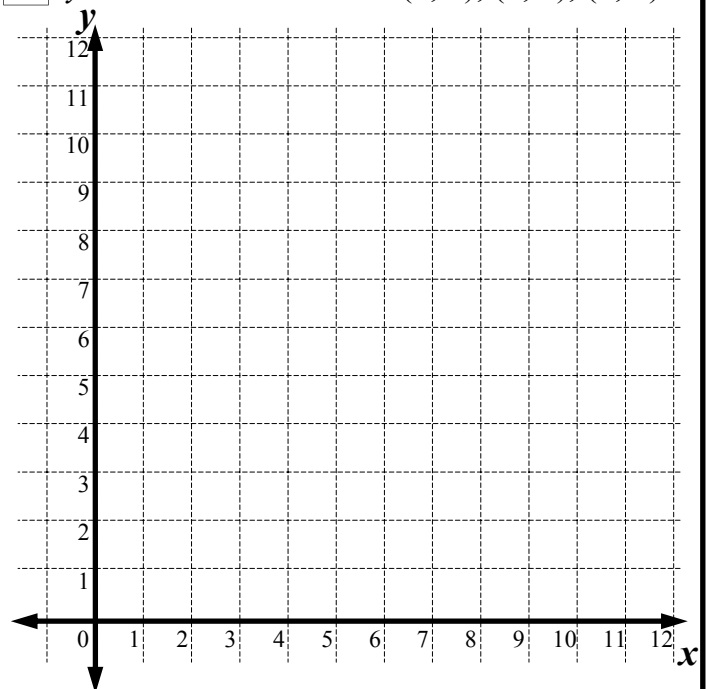


Use the lower grid for these

4 $y = 12 - x$ Plot Points: (0, 12), (2, 10), (4, 8)

5 $y = 3x$ Plot Points: (0, 0), (2, 6), (4, 12)

6 $y = x + 2$ Plot Points: (0, 2), (2, 4), (4, 6)



7 List the 3 points where the lines intersect.

(□, □), (□, □), (□, □).

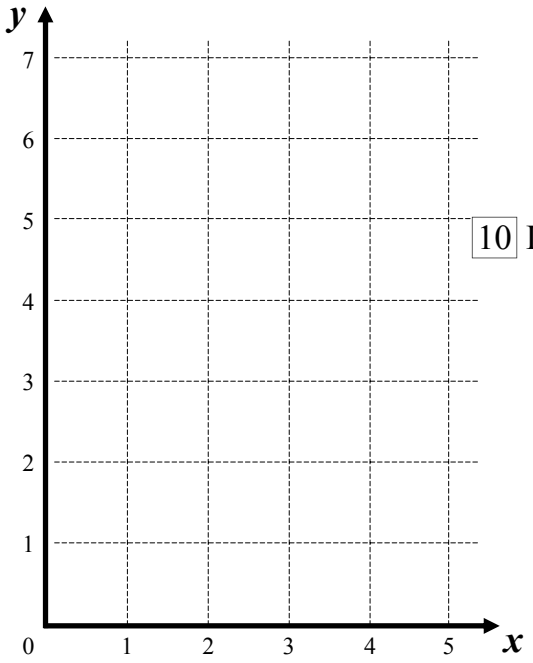
Now use the table of values to plot the lines. Go across 'x' and up 'y'.

8 $y = x$

x	0	1	2	3	4
y	0	1	2	3	4

9 $y = -x + 6$

x	0	1	2	3	4
y	6	5	4	3	2



10 Intersection

(□, □)

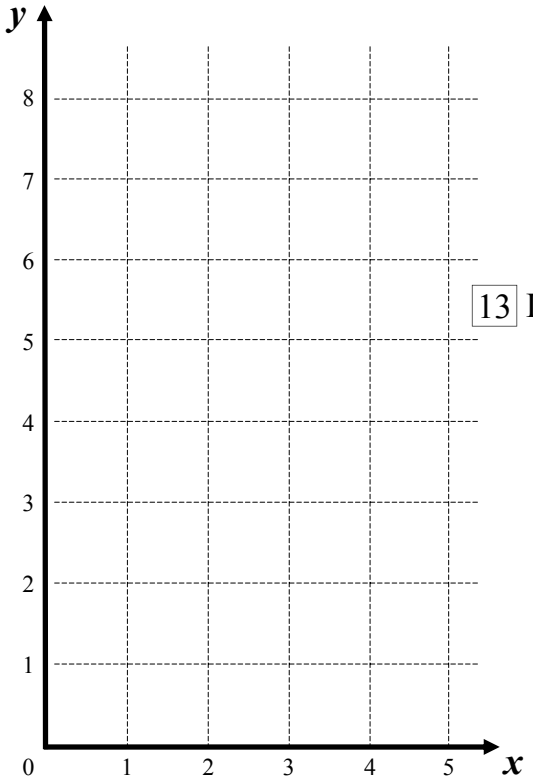
Point

11 $y = 2x$

x	0	1	2	3	4
y	0	2	4	6	8

12 $y = -2x + 8$

x	0	1	2	3	4
y	8	6	4	2	0



13 Intersection

(□, □)

Point

Linear Rel- Plotting Points & Drawing Straight Lines - 4 Quad (Sheet 2)

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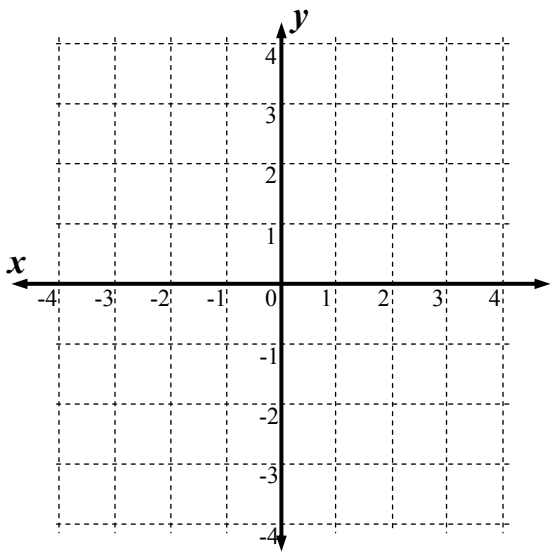
Plot the points then draw a line through them. Use a ruler, show arrow heads and write the equation of the line on the line. The line should be the full size of the graph, not stop at the last point.

1 $y = x + 3$ Plot Points: (-4, -1), (-1, 2), (1, 4).

2 $y = -x + 1$ Plot Points: (-3, 4), (2, -1), (4, -3).

3 $y = -3x - 7$ Plot Points: (1, -4), (2, -1), (3, 2).

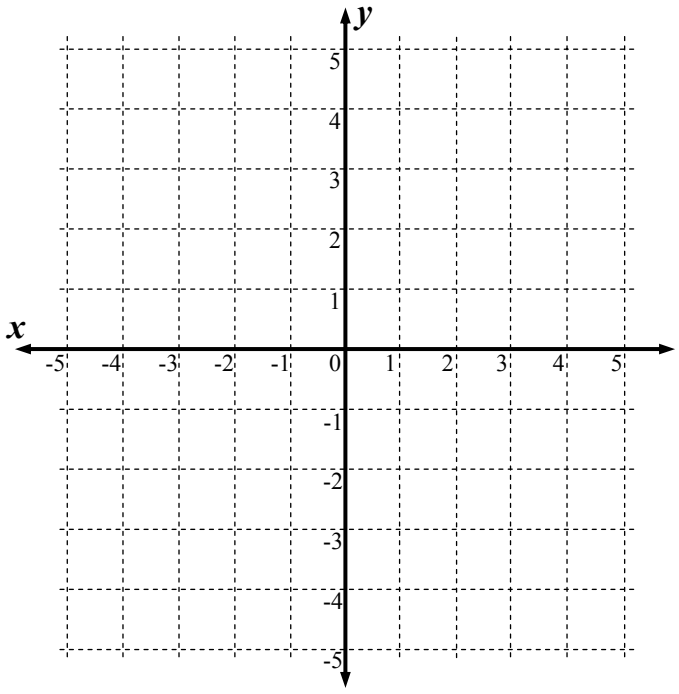


Use the lower grid for Questions 4-6

4 $y = 3x - 2$ Plot Points: (-1, -5), (0, -2), (1, 1)

5 $y = -2x - 2$ Plot Points: (-3, 4), (-1, 0), (1, -4)

6 $y = \frac{1}{2}x + 3$ Plot Points: (-4, 1), (0, 3), (4, 5)



7 List the 3 points where the lines intersect.

(□, □), (□, □), (□, □).

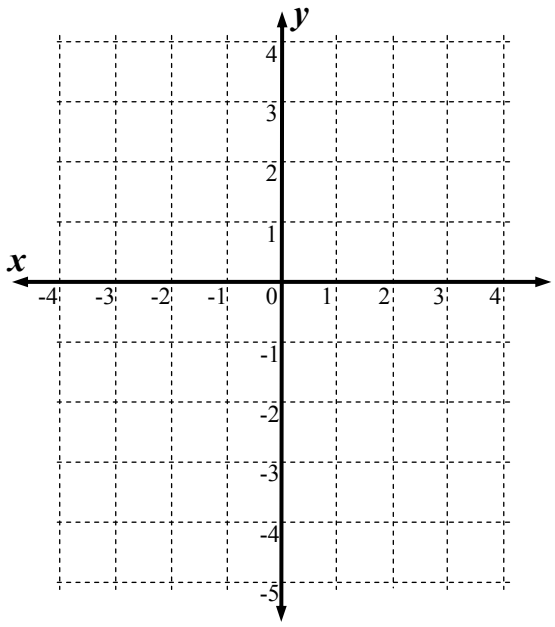
Now use the table of values to plot the lines. Go across the 'x' value and up/down the 'y' value.

8 $y = 2x$

x	-2	-1	0	1	2
y	-4	-2	0	2	4

9 $y = -x - 3$

x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5



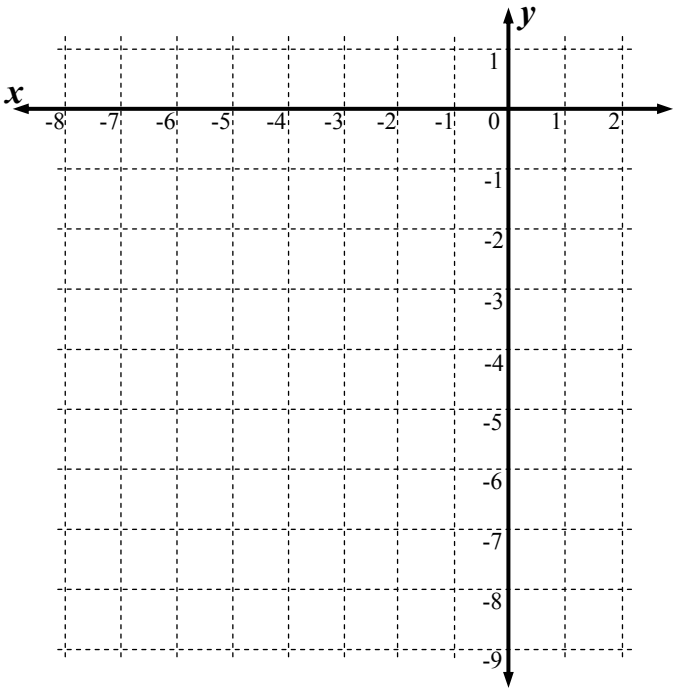
10 Intersection Point (□, □)

11 $y = -2x - 13$

x	-6	-5	-4	-3	-2
y	-1	-3	-5	-7	-9

12 $y = x - 1$

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



13 Intersection Point (□, □)

Linear Rel- Plotting Points & Drawing Straight Lines - + Quad (Sheet 1)

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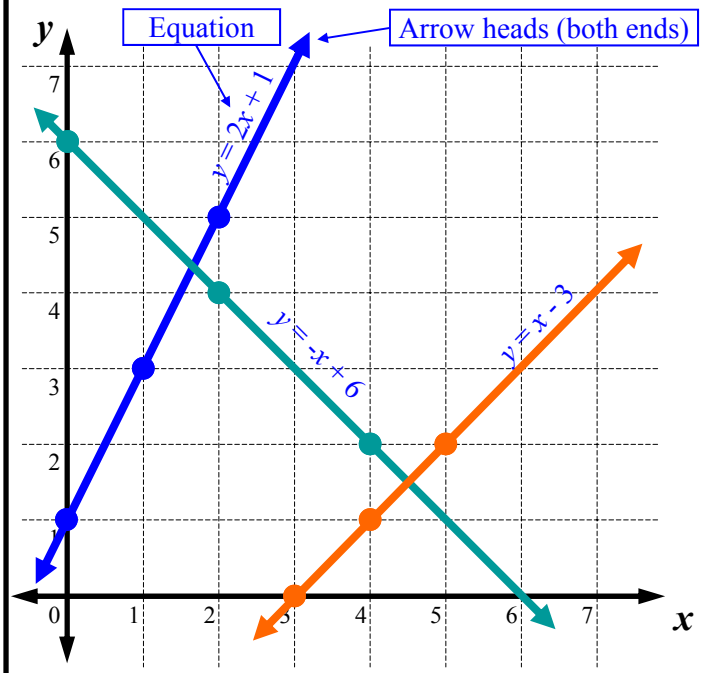
Plot the points then draw a line through them. Use a ruler, show arrow heads and write the equation of the line on the line. **The line should be the full size of the graph**, not stop at the last point.

1 $y = 2x + 1$ Plot Points: (0, 1), (1, 3), (2, 5).

2 $y = -x + 6$ Plot Points: (0, 6), (2, 4), (4, 2).

3 $y = x - 3$ Plot Points: (3, 0), (4, 1), (5, 2).

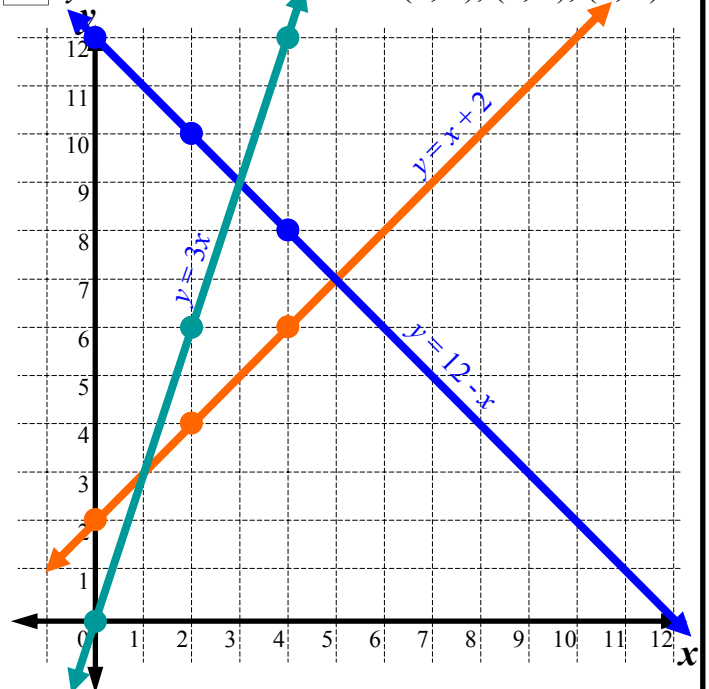


Use the lower grid for these

4 $y = 12 - x$ Plot Points: (0, 12), (2, 10), (4, 8)

5 $y = 3x$ Plot Points: (0, 0), (2, 6), (4, 12)

6 $y = x + 2$ Plot Points: (0, 2), (2, 4), (4, 6)



7 List the 3 points where the lines intersect.

([5], [7]), ([1], [3]), ([3], [9]).

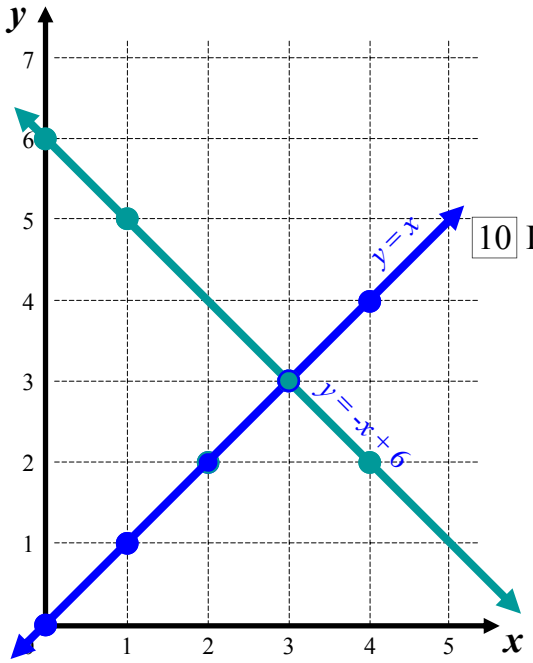
Now use the table of values to plot the lines. Go across 'x' and up 'y'.

8 $y = x$

x	0	1	2	3	4
y	0	1	2	3	4

9 $y = -x + 6$

x	0	1	2	3	4
y	6	5	4	3	2



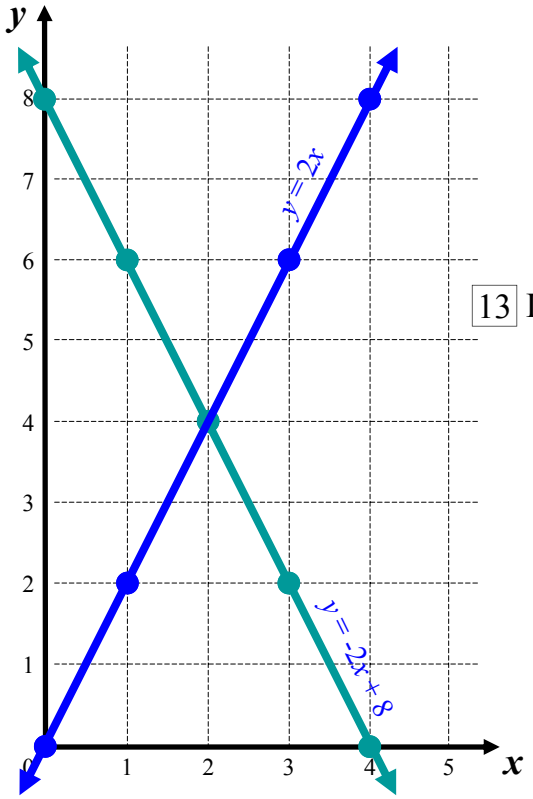
10 Intersection ([3], [3]) Point

11 $y = 2x$

x	0	1	2	3	4
y	0	2	4	6	8

12 $y = -2x + 8$

x	0	1	2	3	4
y	8	6	4	2	0



13 Intersection ([2], [4]) Point

Linear Rel- Plotting Points & Drawing Straight Lines - 4 Quad (Sheet 2)

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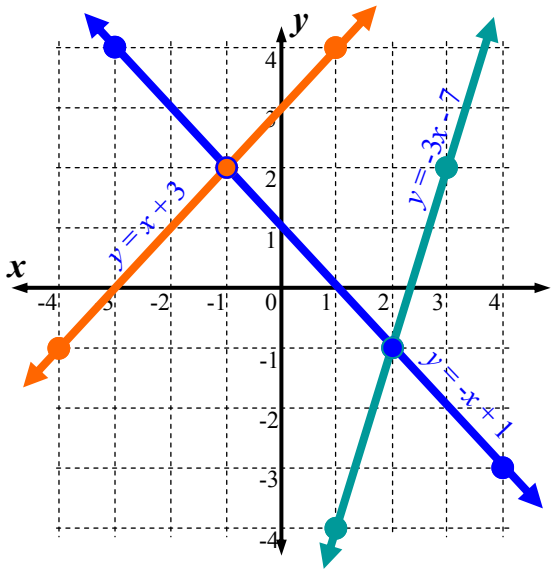
Plot the points then draw a line through them. Use a ruler, show arrow heads and write the equation of the line on the line. The line should be the full size of the graph, not stop at the last point.

1 $y = x + 3$ Plot Points: (-4, -1), (-1, 2), (1, 4).

2 $y = -x + 1$ Plot Points: (-3, 4), (2, -1), (4, -3).

3 $y = -3x - 7$ Plot Points: (1, -4), (2, -1), (3, 2).

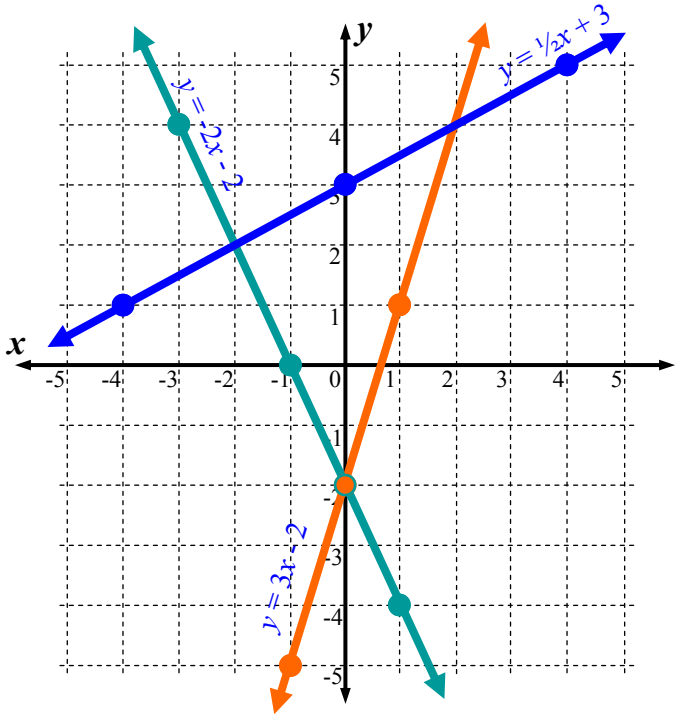


Use the lower grid for Questions 4-6

4 $y = 3x - 2$ Plot Points: (-1, -5), (0, -2), (1, 1)

5 $y = -2x - 2$ Plot Points: (-3, 4), (-1, 0), (1, -4)

6 $y = \frac{1}{2}x + 3$ Plot Points: (-4, 1), (0, 3), (4, 5)



7 List the 3 points where the lines intersect.

(-2, 2), (0, -5), (2, 4).

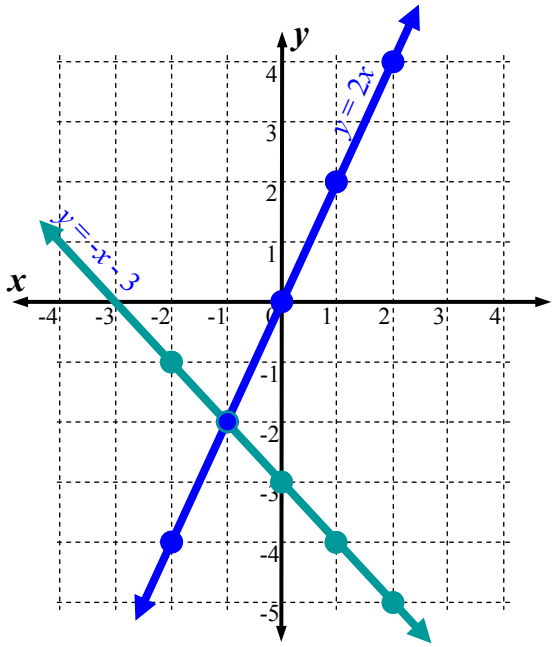
Now use the table of values to plot the lines. Go across the 'x' value and up/down the 'y' value.

8 $y = 2x$

x	-2	-1	0	1	2
y	-4	-2	0	2	4

9 $y = -x - 3$

x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5



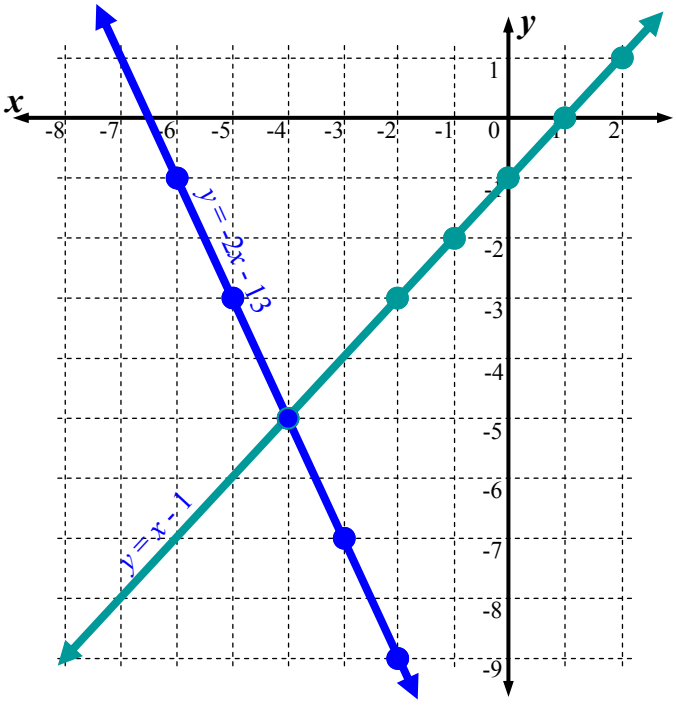
10 Intersection Point (-1, -2)

11 $y = -2x - 13$

x	-6	-5	-4	-3	-2
y	-1	-3	-5	-7	-9

12 $y = x - 1$

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



13 Intersection Point (-4, -5)

Linear Relationships - Plotting Points & Drawing Straight Lines (Homework Strip)

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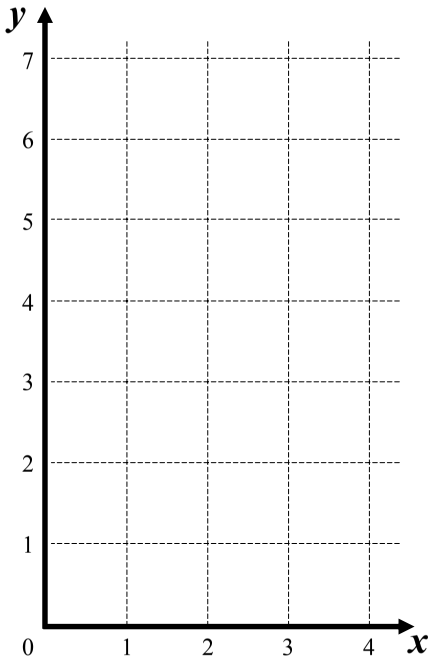
Date Due: _____

Plot the three points for each line then draw the line through the points. Label the line, show arrow heads.

1 $y = x + 2$, Plot Points:
(0, 2), (2, 4), (4, 6).

2 $y = 4 - x$, Plot Points:
(0, 4), (1, 3), (3, 1).



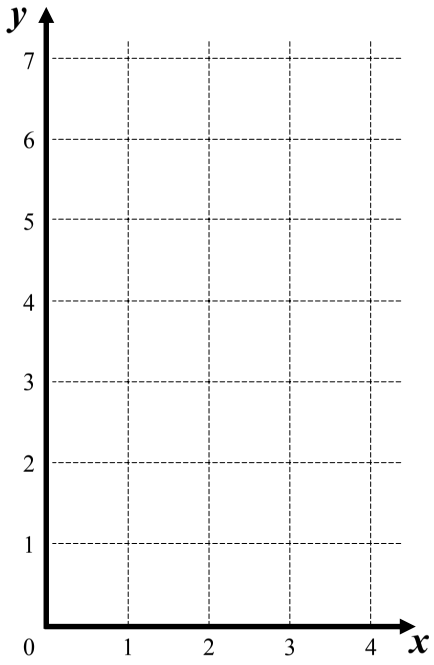
3 Intersection Point (___, ___)

Date Due: _____

Plot the three points for each line then draw the line through the points. Label the line, show arrow heads.

1 $y = x + 2$, Plot Points:
(0, 2), (2, 4), (4, 6).

2 $y = 4 - x$, Plot Points:
(0, 4), (1, 3), (3, 1).



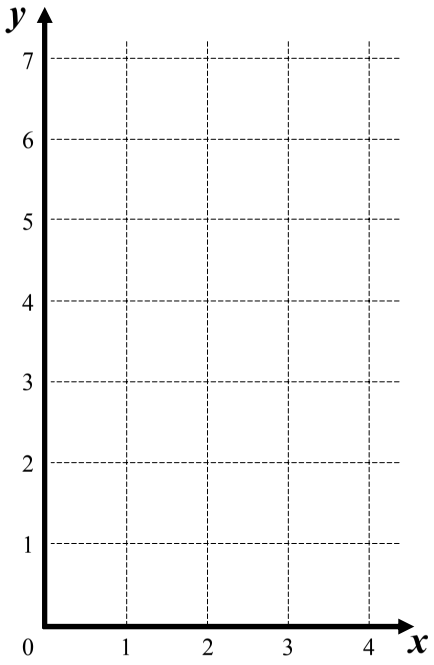
3 Intersection Point (___, ___)

Date Due: _____

Plot the three points for each line then draw the line through the points. Label the line, show arrow heads.

1 $y = x + 2$, Plot Points:
(0, 2), (2, 4), (4, 6).

2 $y = 4 - x$, Plot Points:
(0, 4), (1, 3), (3, 1).



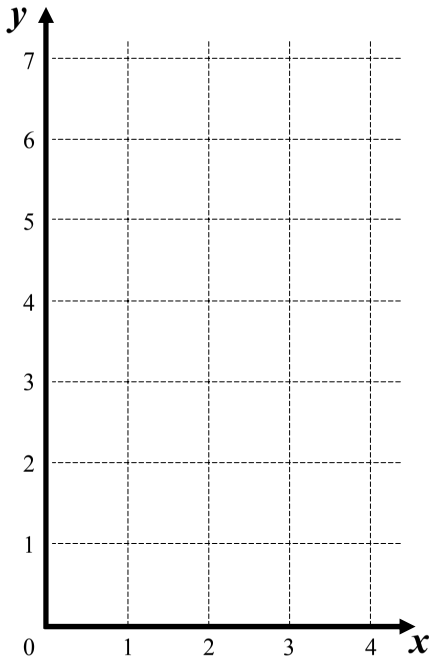
3 Intersection Point (___, ___)

Date Due: _____

Plot the three points for each line then draw the line through the points. Label the line, show arrow heads.

1 $y = x + 2$, Plot Points:
(0, 2), (2, 4), (4, 6).

2 $y = 4 - x$, Plot Points:
(0, 4), (1, 3), (3, 1).



3 Intersection Point (___, ___)

Linear Rel - Plotting Points & Drawing Straight Lines (Homework Strip)

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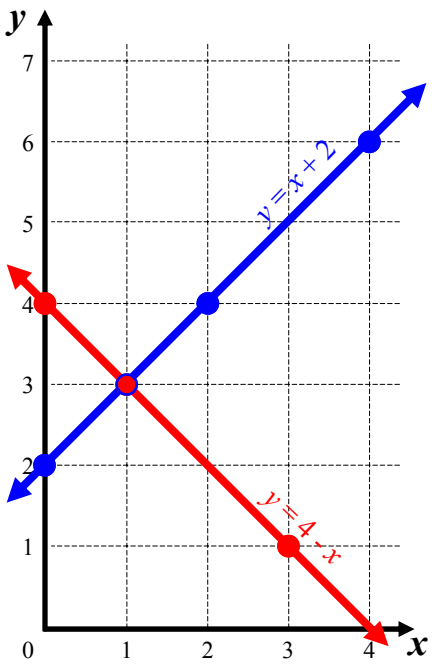
Date Due: _____

Plot the three points for each line then draw the line through the points.
Label the line, show arrow heads.

1 $y = x + 2$, Plot Points:
(0, 2), (2, 4), (4, 6).

2 $y = 4 - x$, Plot Points:
(0, 4), (1, 3), (3, 1).



3 Intersection Point (1 , 3)

Linear Relationships - Plotting Straight Lines

Completing a table of values is just substitution but changing the value in steps. So you replace the x with 0 for the first value. Then replace x with 1 for the second value.

Example - Completing a table of values

$y = 3x + 2$

x	0	1	2	3	4
y	2	5	8	11	14

$y = 3 \times 0 + 2 = 2$

$y = 3 \times 1 + 2 = 5$

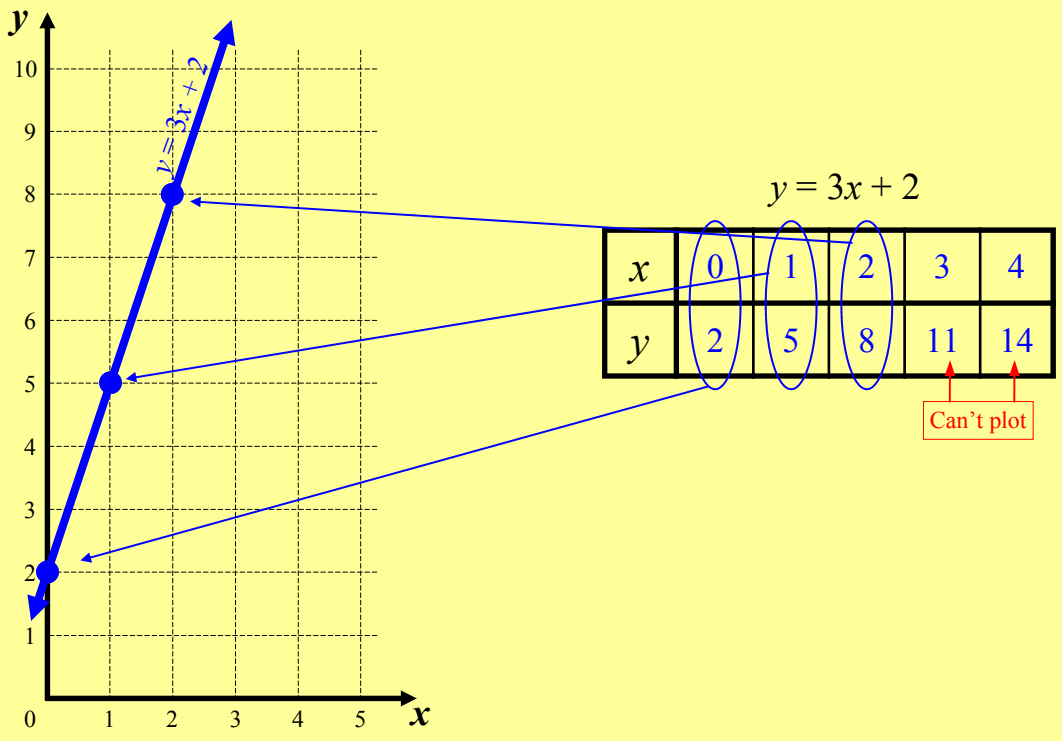
$y = 3 \times 2 + 2 = 8$

$y = 3 \times 3 + 2 = 11$

$y = 3 \times 4 + 2 = 14$

Notice that there is a pattern, the y-values are increasing by three. You should use this as a check, if the numbers aren't increasing or decreasing in a regular pattern, then you have made a mistake.

From the second column you are asked to complete a table and then plot the values, the x value goes across and the y value goes up. Because your number plane is a set size, it is possible that values in a table can't fit, because the numbers are too big. So you can't always plot the values in a table on the number plane.



Remember to always extend the line to the full size of the number plane, put arrowheads on the line and write the equation for the line, on the line. The 2nd worksheet is in all quadrants.

Linear Relationships - Plotting Straight Lines - Pos Quad (Sheet 1)

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Complete these table of values. There are hints below each box.

1 $y = x + 2$

x	0	1	2	3	4
y					

$0 + 2$ $1 + 2$ $2 + 2$ $3 + 2$ $4 + 2$

2 $y = 4x$

x	0	1	2	3	4
y					

4×0 4×1 4×2 4×3 4×4

3 $y = 3x + 6$

x	0	1	2	3	4
y					

$3 \times 0 + 6$ $3 \times 1 + 6$ $3 \times 2 + 6$ $3 \times 3 + 6$ $3 \times 4 + 6$

4 $y = 8 - 2x$

x	0	1	2	3	4
y					

$8 - 2 \times 0$ $8 - 2 \times 1$ $8 - 2 \times 2$ $8 - 2 \times 3$ $8 - 2 \times 4$

The next table will have answers that have ½'s in them

5 $y = \frac{1}{2}x + 7$

x	0	1	2	3	4
y					

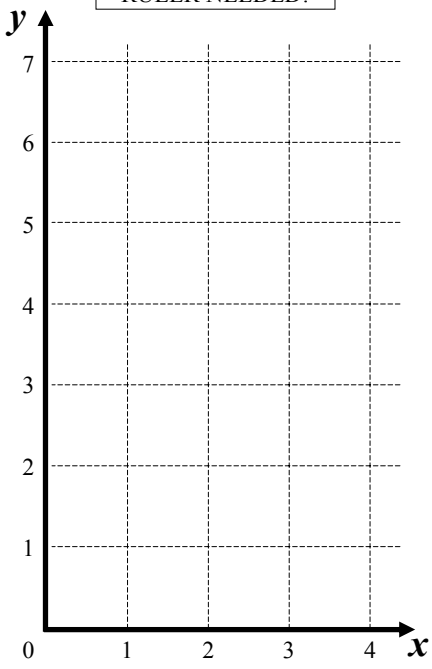
$\frac{1}{2} \times 0 + 7$ $\frac{1}{2} \times 1 + 7$ $\frac{1}{2} \times 2 + 7$ $\frac{1}{2} \times 3 + 7$ $\frac{1}{2} \times 4 + 7$

Complete the tables, then plot the results on the number plane.

6 $y = x + 3$

x	0	1	2	3	4
y					

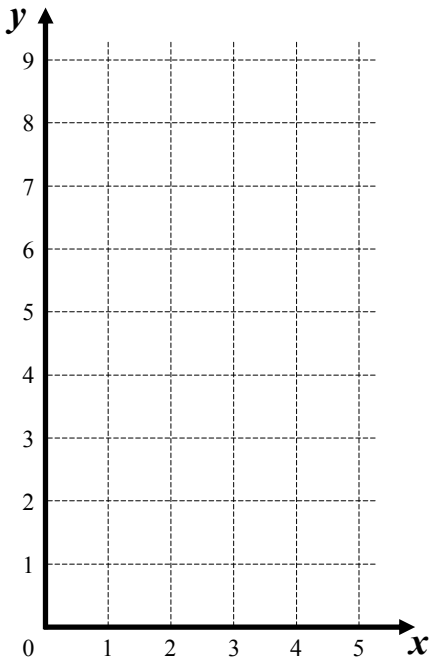
RULER NEEDED!



Check: Equation ✓ Arrowheads ✓ Full size ✓

7 $y = 2x + 1$

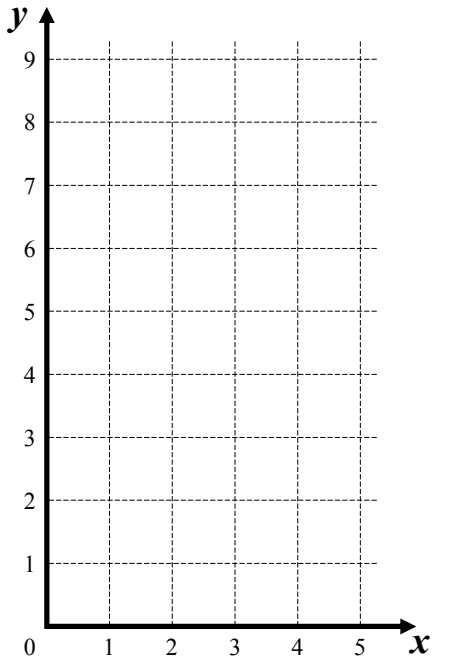
x	0	1	2	3	4
y					



Check: Equation ☐ Arrowheads ☐ Full size ☐

8 $y = 9 - 2x$

x	0	1	2	3	4
y					

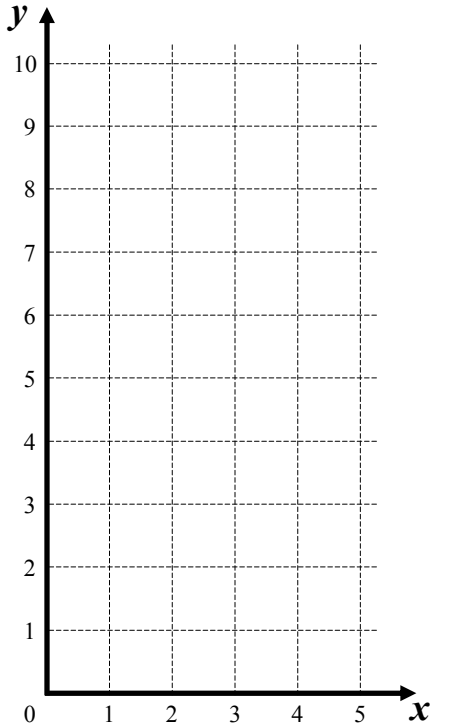


Check: Equation ☐ Arrowheads ☐ Full size ☐

9 $y = 3x - 2$

x	0	1	2	3	4
y					

Can't plot



Linear Relationships - Plotting Straight Lines - All Quads (Sheet 2)

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Complete these table of values.

1 $y = x + 2$

x	-4	-2	0	2	4
y					

2 $y = 4x$

x	-2	-1	0	1	2
y					

3 $y = 2x + 3$

x	-4	-2	0	2	4
y					

4 $y = 10 - x$

x	-4	-2	0	2	4
y					

Calc: 1 0 - - 4 =

5 $y = -3x - 6$

x	-2	-1	0	1	2
y					

Calc: - 3 × - 2 - 6 =

6 $y = ½x + 5$

x	-4	-2	0	2	4
y					

7 $x + y = 2$

x	-4	-2	0	2	4
y					

-4 + 'something' equals 2

Complete the table of values, then plot the two lines on the Cartesian Plane. Record the point of intersection. Write line equations on each line, show arrow heads and extend the lines the full size of the grid.

8 $y = x + 1$

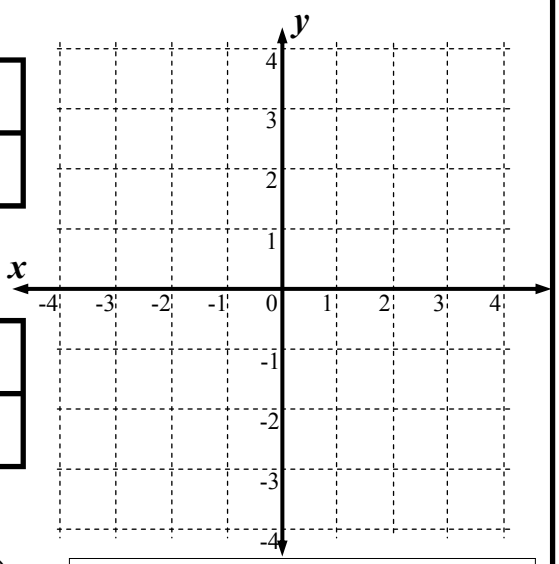
x	-4	-2	0	2	4
y					

Can't plot

9 $y = -2x + 1$

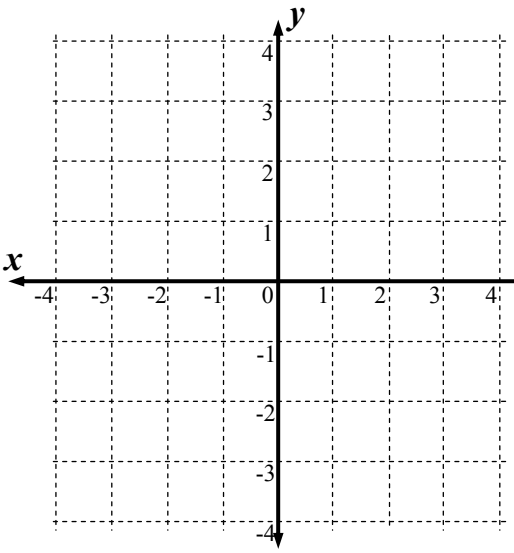
x	-2	-1	0	1	2
y					

Can't plot



Check: Equation ☐ Arrowheads ☐ Full size ☐

10 Intersection Point (☐ , ☐)



11 $y = -x$

x	-4	-2	0	2	4
y					

12 $y = ½x - 3$

x	-4	-2	0	2	4
y					

Can't plot

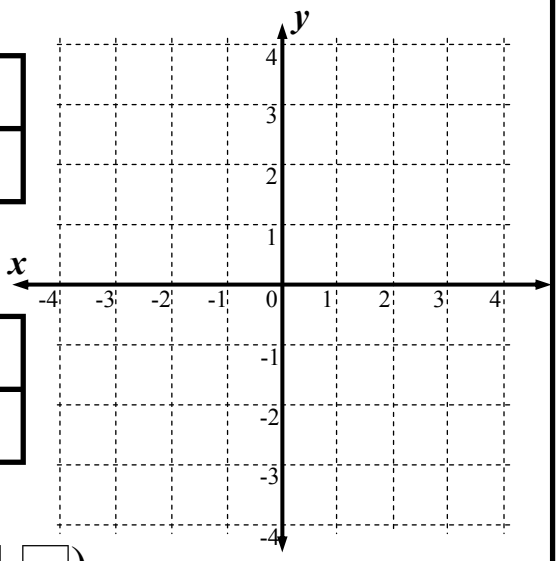
13 Intersection Point (☐ , ☐)

14 $y = -x + 2$

x	-4	-2	0	2	4
y					

15 $y = 3x - 6$

x	-2	-1	0	1	2
y					



16 Intersection Point (☐ , ☐)

Linear Relationships - Plotting Straight Lines - Pos Quad (Sheet 1)

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Complete these table of values. There are hints below each box.

1 $y = x + 2$

x	0	1	2	3	4
y	2	3	4	5	6
	0 + 2	1 + 2	2 + 2	3 + 2	4 + 2

2 $y = 4x$

x	0	1	2	3	4
y	0	4	8	12	16
	4 × 0	4 × 1	4 × 2	4 × 3	4 × 4

3 $y = 3x + 6$

x	0	1	2	3	4
y	6	9	12	15	18
	3 × 0 + 6	3 × 1 + 6	3 × 2 + 6	3 × 3 + 6	3 × 4 + 6

4 $y = 8 - 2x$

x	0	1	2	3	4
y	8	6	4	2	0
	8 - 2 × 0	8 - 2 × 1	8 - 2 × 2	8 - 2 × 3	8 - 2 × 4

The next table will have answers that have ½'s in them

5 $y = ½x + 7$

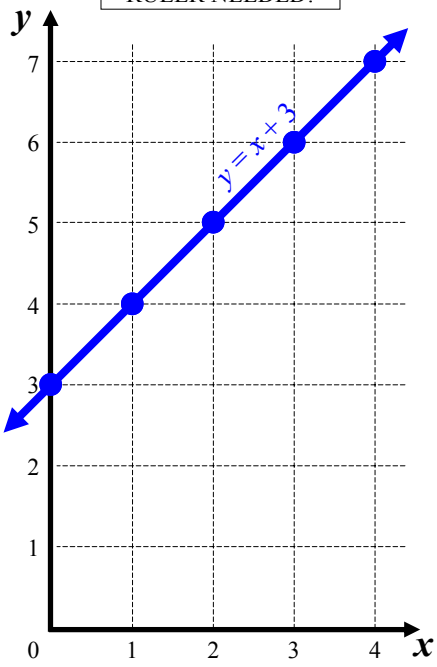
x	0	1	2	3	4
y	7	7½	8	8½	9
	½ × 0 + 7	½ × 1 + 7	½ × 2 + 7	½ × 3 + 7	½ × 4 + 7

Complete the tables, then plot the results on the number plane.

6 $y = x + 3$

x	0	1	2	3	4
y	3	4	5	6	7

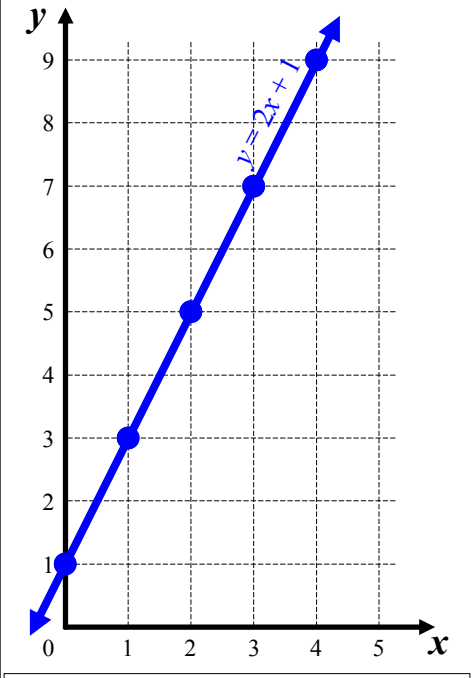
RULER NEEDED!



Check: Equation ✓ Arrowheads ✓ Full size ✓

7 $y = 2x + 1$

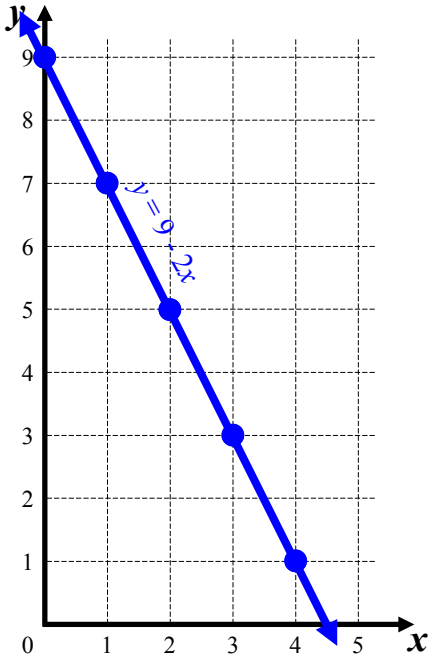
x	0	1	2	3	4
y	1	3	5	7	9



Check: Equation ☑ Arrowheads ☑ Full size ☑

8 $y = 9 - 2x$

x	0	1	2	3	4
y	9	7	5	3	1

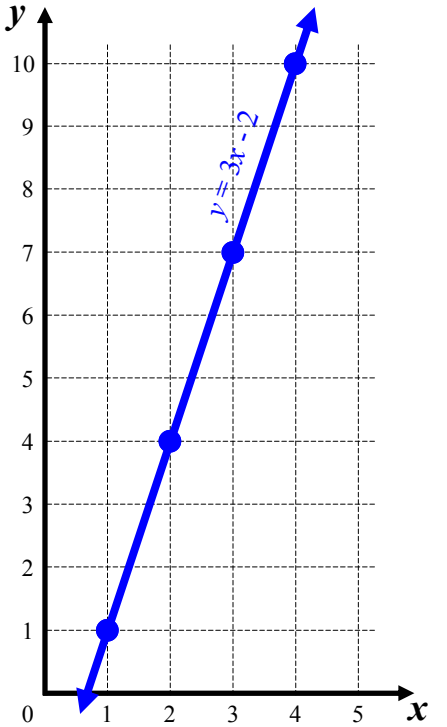


Check: Equation ☑ Arrowheads ☑ Full size ☑

9 $y = 3x - 2$

x	0	1	2	3	4
y	-2	1	4	7	10

Can't plot



Linear Relationships - Plotting Straight Lines - All Quads (Sheet 2)

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Complete these table of values.

1 $y = x + 2$

x	-4	-2	0	2	4
y	-2	0	2	4	6

2 $y = 4x$

x	-2	-1	0	1	2
y	-8	-4	0	4	8

3 $y = 2x + 3$

x	-4	-2	0	2	4
y	-5	-1	3	7	11

4 $y = 10 - x$

x	-4	-2	0	2	4
y	14	12	10	8	6

Calc: 1 0 - - 4 =

5 $y = -3x - 6$

x	-2	-1	0	1	2
y	0	-3	-6	-9	-12

Calc: - 3 × - 2 - 6 =

6 $y = ½x + 5$

x	-4	-2	0	2	4
y	3	4	5	6	7

7 $x + y = 2$

x	-4	-2	0	2	4
y	6	4	2	0	-2

-4 + 'something' equals 2

Complete the table of values, then plot the two lines on the Cartesian Plane. Record the point of intersection. Write line equations on each line, show arrow heads and extend the lines the full size of the grid.

8 $y = x + 1$

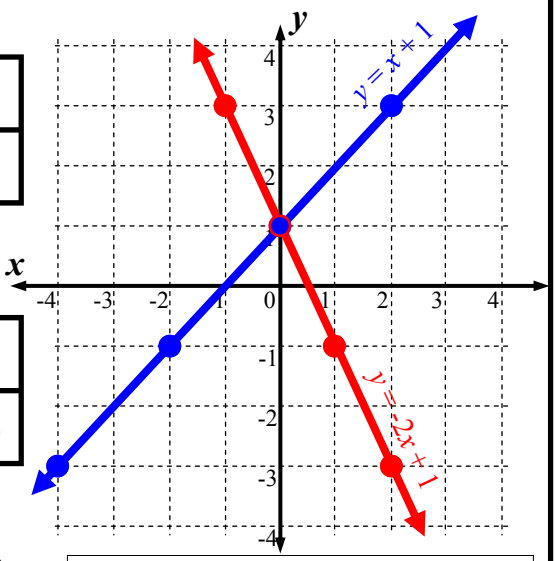
x	-4	-2	0	2	4
y	-3	-1	1	3	5

Can't plot

9 $y = -2x + 1$

x	-2	-1	0	1	2
y	5	3	1	-1	-3

Can't plot



Check: Equation ☑ Arrowheads ☑ Full size ☑

10 Intersection Point (0 , 1)

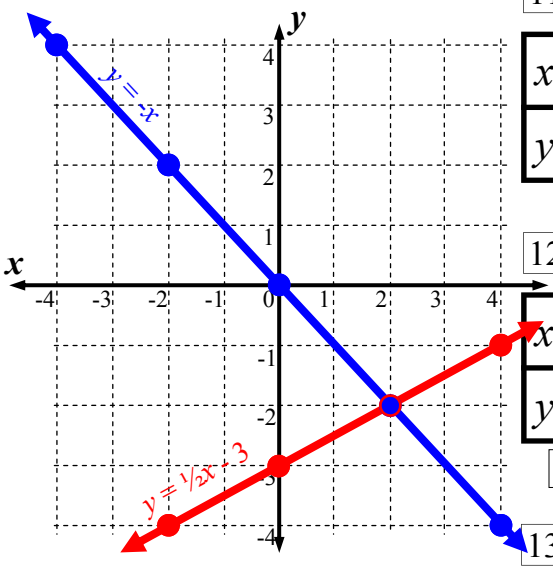
11 $y = -x$

x	-4	-2	0	2	4
y	4	2	0	-2	-4

12 $y = ½x - 3$

x	-4	-2	0	2	4
y	-5	-4	-3	-2	-1

Can't plot



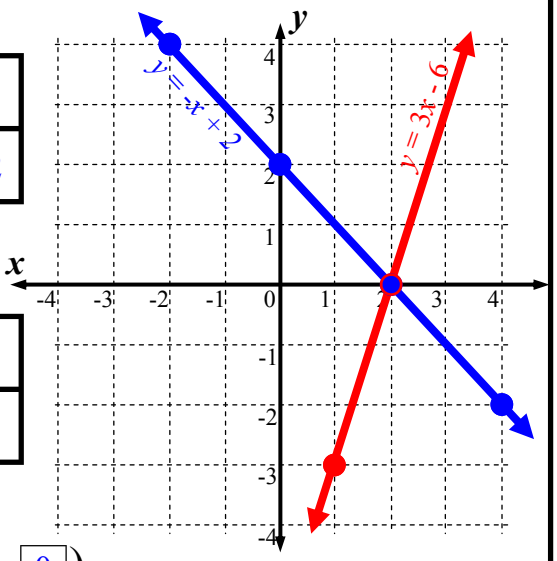
13 Intersection Point (2 , -2)

14 $y = -x + 2$

x	-4	-2	0	2	4
y	6	4	2	0	-2

15 $y = 3x - 6$

x	-2	-1	0	1	2
y	-12	-9	-6	-3	0



16 Intersection Point (2 , 0)

Linear Relationships - Table of Values & Drawing Straight Lines (Homework Strip)

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Date Due: _____

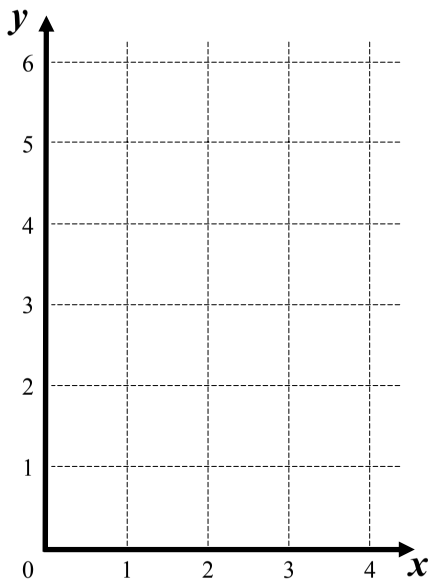
Complete the table of values and then plot the straight lines.

1 $y = x + 2$

x	0	1	2	3	4
y					

2 $y = 6 - x$

x	0	1	2	3	4
y					



3 Intersection Point (___, ___)

Date Due: _____

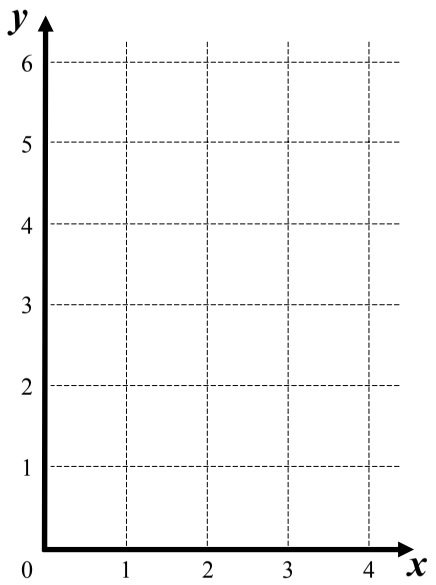
Complete the table of values and then plot the straight lines.

1 $y = x + 2$

x	0	1	2	3	4
y					

2 $y = 6 - x$

x	0	1	2	3	4
y					



3 Intersection Point (___, ___)

Date Due: _____

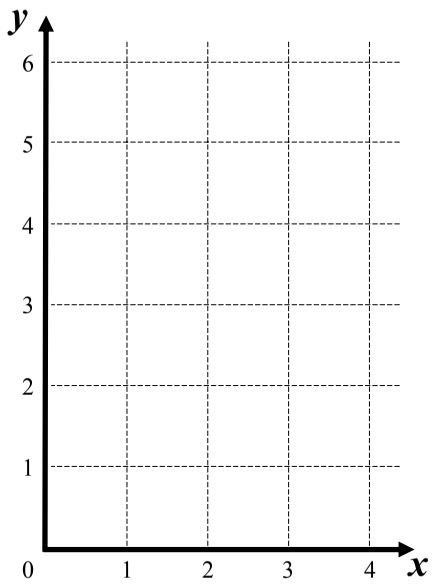
Complete the table of values and then plot the straight lines.

1 $y = x + 2$

x	0	1	2	3	4
y					

2 $y = 6 - x$

x	0	1	2	3	4
y					



3 Intersection Point (___, ___)

Date Due: _____

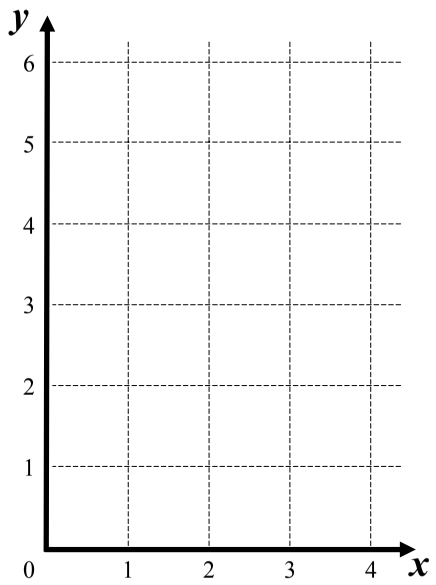
Complete the table of values and then plot the straight lines.

1 $y = x + 2$

x	0	1	2	3	4
y					

2 $y = 6 - x$

x	0	1	2	3	4
y					



3 Intersection Point (___, ___)

Linear Rel - Table of Values & Drawing Straight Lines (Homework Strip)

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Date Due:

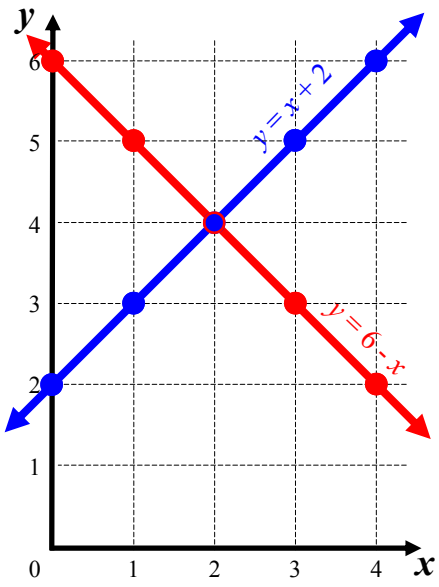
Complete the table of values and then plot the straight lines.

1 $y = x + 2$

x	0	1	2	3	4
y	2	3	4	5	6

2 $y = 6 - x$

x	0	1	2	3	4
y	6	5	4	3	2



3 Intersection Point (2 , 4)

Linear Relationships - Further Straight Lines

This sheet deals with plotting straight lines with points being plotted in all quadrants. The most difficult aspect of this sheet is with the use of negative numbers. You have to remember that when you multiply numbers with a different sign that the answer is negative, or when the signs are both negative then the answer would be positive.

Example

$y = -2x + 3$

x	-4	-2	0	2	4
y	11	7	3	-1	-5

$-2 \times 4 + 3 = -8 + 3 = -5$

$-2 \times 2 + 3 = -4 + 3 = -1$

$-2 \times 0 + 3 = 0 + 3 = 3$

$-2 \times -2 + 3 = 4 + 3 = 7$

$-2 \times -4 + 3 = 8 + 3 = 11$

It is very easy to make a mistake with negative number operations. But remember that the y-values in the Table of Values increase or decrease in a pattern.

$y = -2x + 3$

x	-4	-2	0	2	4
y	11	7	3	-1	-5

-4 -4 -4 -4

The y-values decrease by 4, so if a point wasn't 4 less than the previous number, you know to check for an error.

IMPORTANT - It may be that you use the pattern to complete the table after you have used substitution for two values. If you do this the check system won't work.

Then plot the points and draw a line through them. Remember arrow heads and to label the line. Label the line in pencil so that you can rub it out later if another line gets in the way.

Important features of lines are:

Slope (or gradient) - is the line going up to the left or up to the right? Is the line more or less steep than the other lines?

y-intercept - where is the line cutting the y-axis, is this point visible in the line equation?

Linear Relationships - Further Straight Lines (Sheet 1)

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Complete the table of values then plot the points. Draw the line with arrow heads and the equation on the line. With values in the table that can't be plotted, construct the line with the points that you can plot.

1 $y = -2x$

x	-4	-2	0	2	4
y					

2 $y = x + 1$

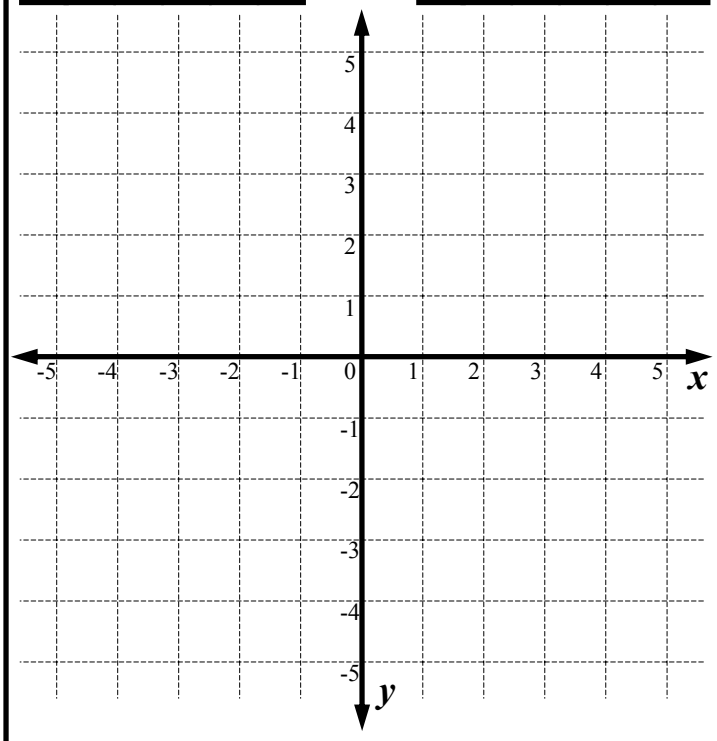
x	-4	-2	0	2	4
y					

3 $y = -\frac{1}{2}x + 1$

x	-4	-2	0	2	4
y					

4 $y = -2x - 5$

x	-4	-2	0	2	4
y					



5 Lines in Q.1 and Q.4 both have '-2' in front of the x. Look at the lines and describe a common feature. _____

6 Lines in Q.2 and Q.3 both have '+1' in the equation. Look at the lines, describe a common feature. _____

Use the top grid, in the next column for Q.7, 8 & 9. below

7 $y = 3x - 6$

x	-4	-2	0	2	4
y					

8 $y = 2x + 1$

x	-4	-2	0	2	4
y					

9 $y = x$

x	-4	-2	0	2	4
y					

Use the lower grid, in the next column for these below

10 $y = -2x + 1$

x	-4	-2	0	2	4
y					

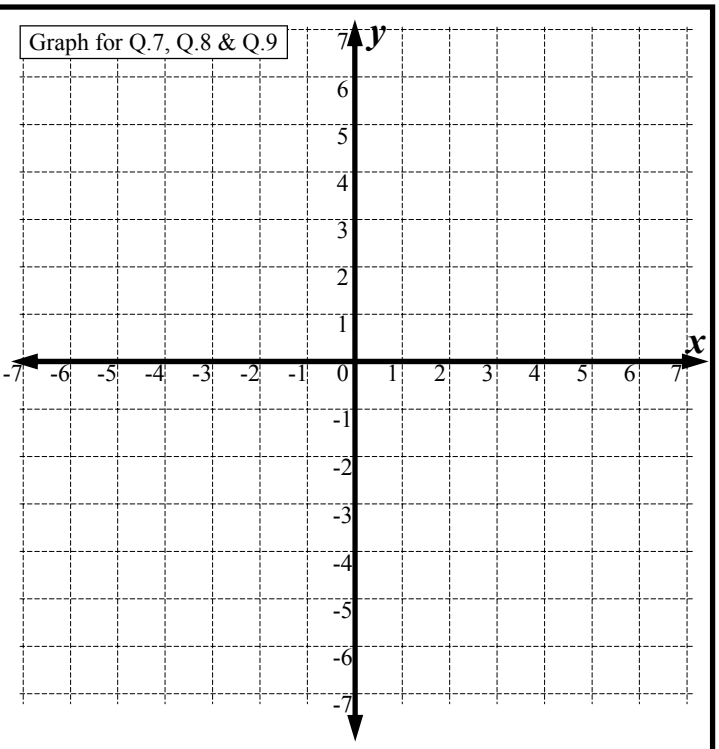
11 $y = -3x - 6$

x	-4	-2	0	2	4
y					

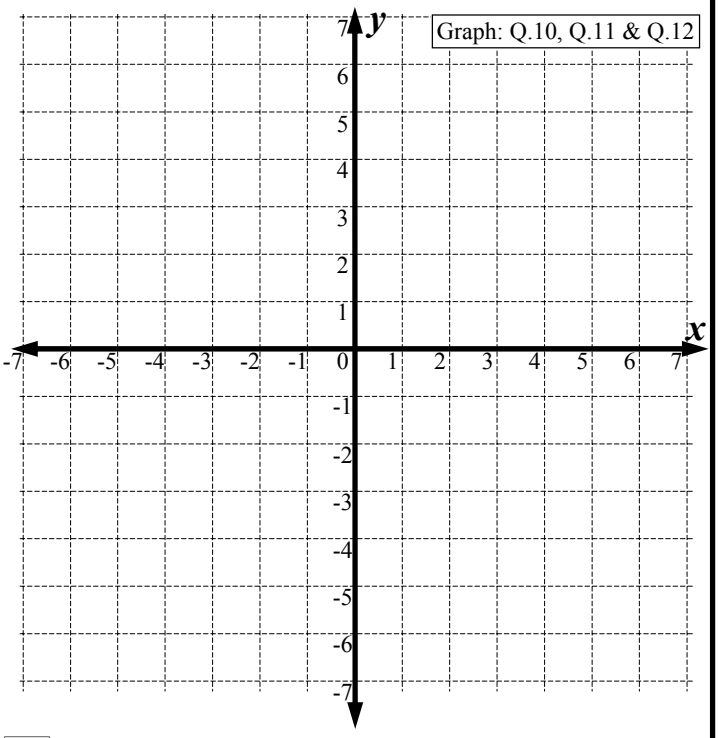
12 $y = -x$

x	-4	-2	0	2	4
y					

Graph for Q.7, Q.8 & Q.9



Graph: Q.10, Q.11 & Q.12



13 What do all the lines in the top grid have in common? _____

14 What do all the lines in the lower grid have in common? _____

15 Complete: A negative number in front of the x means that the line 'goes up' to the _____

16 Complete: A positive number in front of the x means that the line 'goes up' to the _____

17 Some lines are more steep than others. Which is more steep … $y = 4x$ or $y = 3x$? _____

Linear Relationships - Further Straight Lines (Sheet 2)

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Complete the table of values then plot the lines. These include equations involving fractions/denominators.

Colour same as line colour

1 $y = x - 9$

x	-1	0	1	2	3
y					

2 $y = \frac{x}{3} + 2$

x	-6	-3	0	3	6
y					

3 $y = -4x - 6$

x	-3	-2	-1	0	1
y					

4 $y = \frac{x + 10}{2}$

x	-4	-2	0	2	4
y					

5 $y = \frac{-x + 2}{2}$

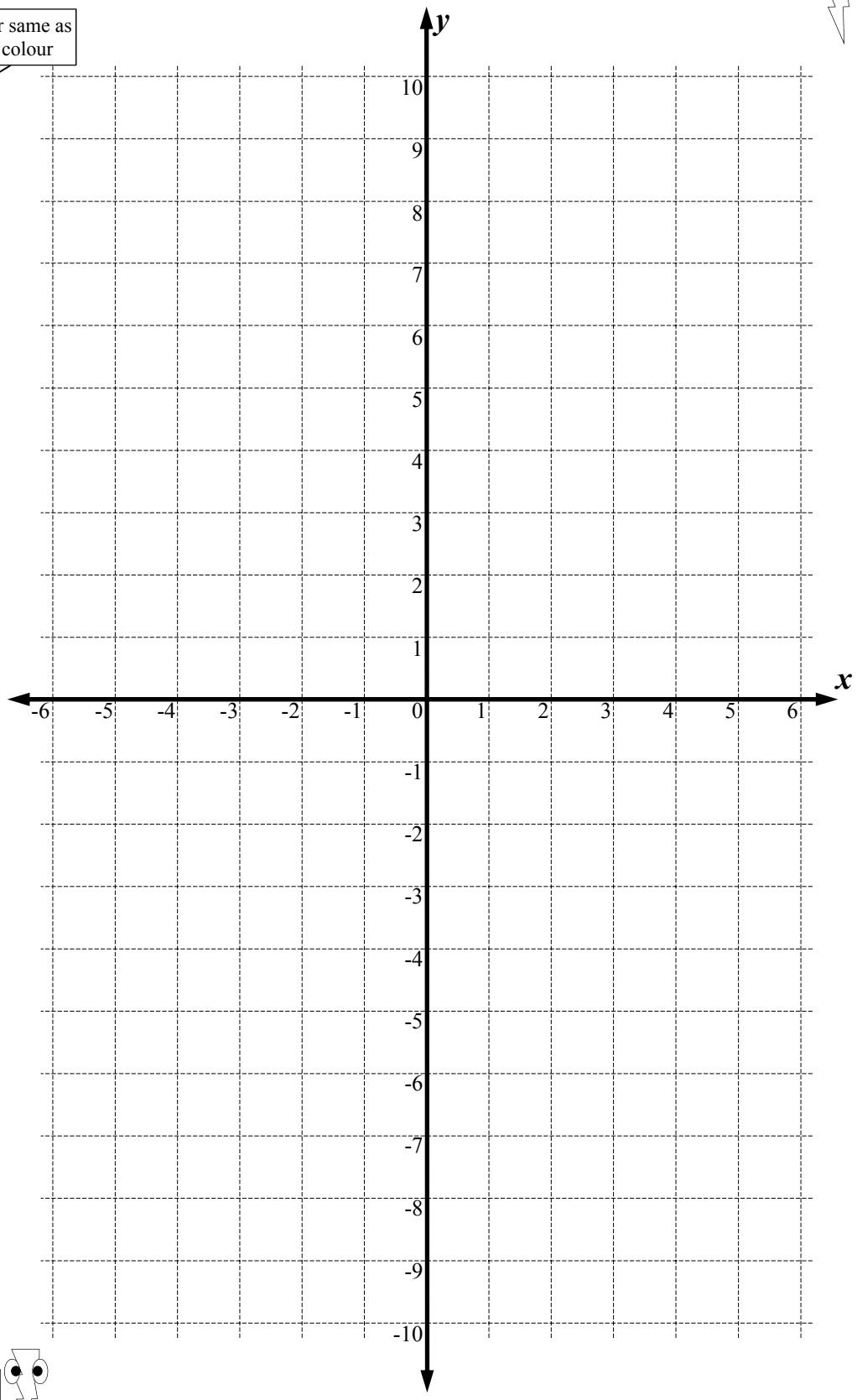
x	-4	-2	0	2	4
y					

6 $y = 3x + 6$

x	-3	-2	-1	0	1
y					

7 $y = \frac{-x - 9}{2}$

x	-5	-3	-1	1	3
y					



Find the points of intersection for these lines.

8 $y = \frac{x + 10}{2}$, $y = \frac{-x + 2}{2}$ (,)

9 $y = \frac{-x - 9}{2}$, $y = 3x + 6$ (,)

10 $y = \frac{-x - 9}{2}$, $y = x - 9$ (,)

Linear Relationships - Further Straight Lines (Sheet 1)

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Complete the table of values then plot the points. Draw the line with arrow heads and the equation on the line. With values in the table that can't be plotted, construct the line with the points that you can plot.

1 $y = -2x$

x	-4	-2	0	2	4
y	8	4	0	-4	-8

2 $y = x + 1$

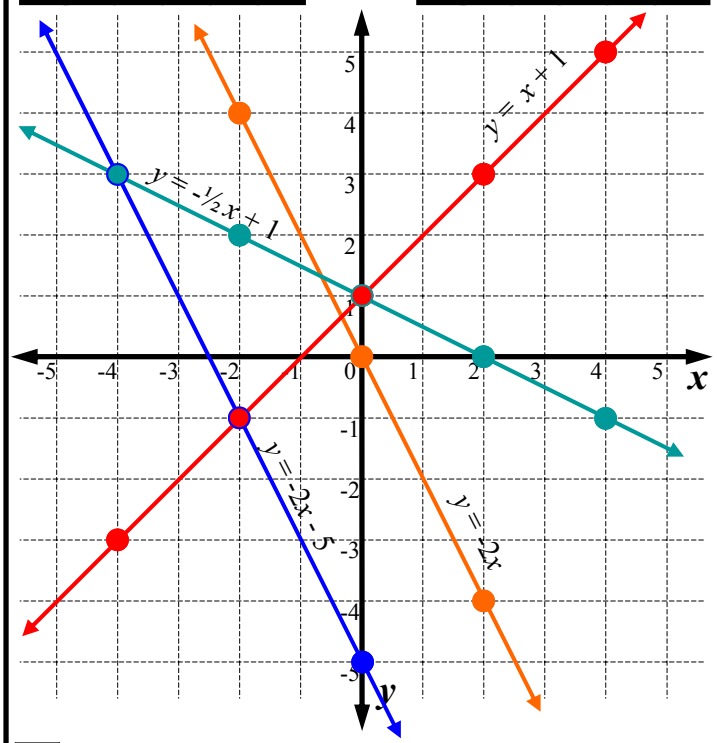
x	-4	-2	0	2	4
y	-3	-1	1	3	5

3 $y = -\frac{1}{2}x + 1$

x	-4	-2	0	2	4
y	3	2	1	0	-1

4 $y = -2x - 5$

x	-4	-2	0	2	4
y	3	-1	-5	-9	-13



5 Lines in Q.1 and Q.4 both have '-2' in front of the x. Look at the lines and describe a common feature. The lines are parallel. (Same gradient)

6 Lines in Q.2 and Q.3 both have '+1' in the equation. Look at the lines, describe a common feature. Both cut y axis at 1. (Same y-intercept)

Use the top grid, in the next column for Q.7, 8 & 9. below

7 $y = 3x - 6$

x	-4	-2	0	2	4
y	-18	-12	-6	0	6

8 $y = 2x + 1$

x	-4	-2	0	2	4
y	-7	-3	1	5	9

9 $y = x$

x	-4	-2	0	2	4
y	-4	-2	0	2	4

Use the lower grid, in the next column for these below

10 $y = -2x + 1$

x	-4	-2	0	2	4
y	9	5	1	-3	-7

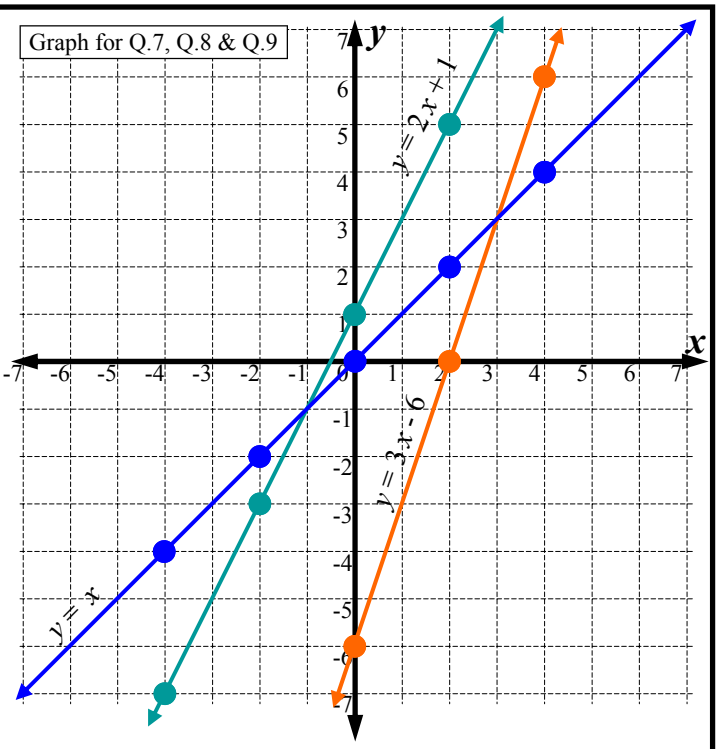
11 $y = -3x - 6$

x	-4	-2	0	2	4
y	6	0	-6	-12	-18

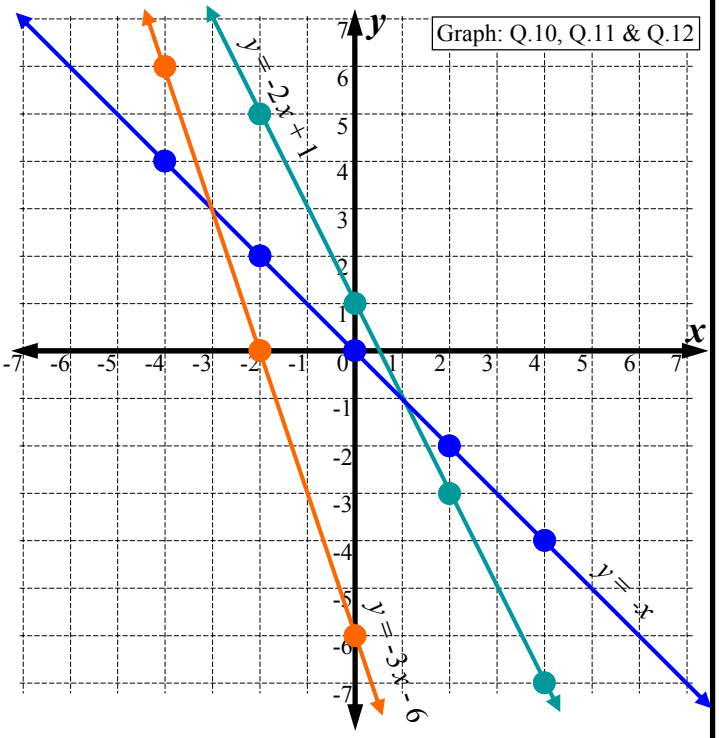
12 $y = -x$

x	-4	-2	0	2	4
y	4	2	0	-2	-4

Graph for Q.7, Q.8 & Q.9



Graph: Q.10, Q.11 & Q.12



13 What do all the lines in the top grid have in common? All go up to the right. (or down to left)

14 What do all the lines in the lower grid have in common? All go up to the left. (or down to right)

15 Complete: A negative number in front of the x means that the line 'goes up' to the left.

16 Complete: A positive number in front of the x means that the line 'goes up' to the right.

17 Some lines are more steep than others. Which is more steep … $y = 4x$ or $y = 3x$? $y = 4x$

Linear Relationships - Further Straight Lines (Sheet 2)

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Complete the table of values then plot the lines. These include equations involving fractions/denominators.

Colour same as line colour

1 $y = x - 9$

x	-1	0	1	2	3
y	-10	-9	-8	-7	-6

2 $y = \frac{x}{3} + 2$

x	-6	-3	0	3	6
y	0	1	2	3	4

3 $y = -4x - 6$

x	-3	-2	-1	0	1
y	6	2	-2	-6	-10

4 $y = \frac{x + 10}{2}$

x	-4	-2	0	2	4
y	3	4	5	6	7

5 $y = \frac{-x + 2}{2}$

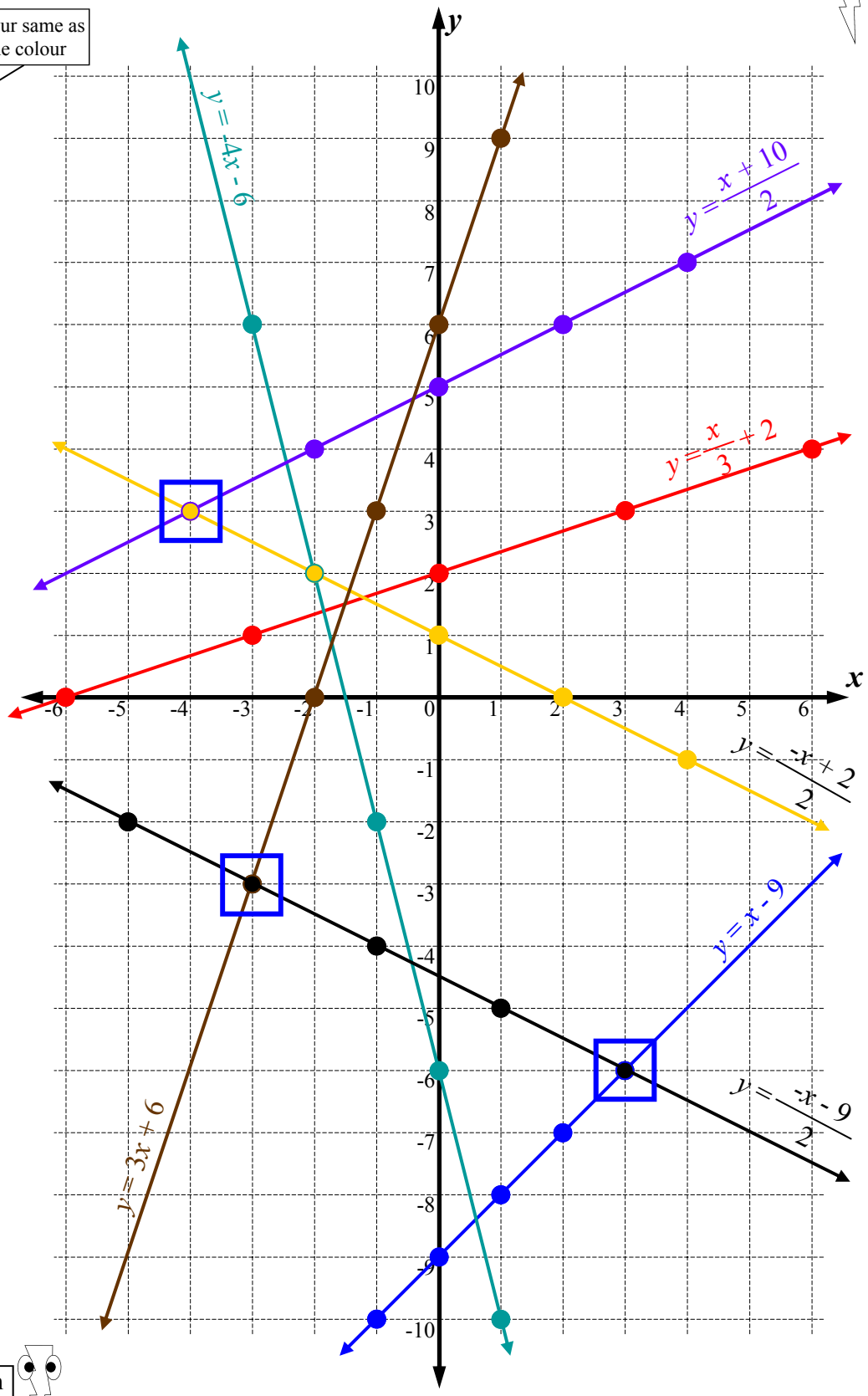
x	-4	-2	0	2	4
y	3	2	1	0	-1

6 $y = 3x + 6$

x	-3	-2	-1	0	1
y	-3	0	3	6	9

7 $y = \frac{-x - 9}{2}$

x	-5	-3	-1	1	3
y	-2	-3	-4	-5	-6



Find the points of intersection for these lines.

8 $y = \frac{x + 10}{2}$, $y = \frac{-x + 2}{2}$ ([-4], [3])

9 $y = \frac{-x - 9}{2}$, $y = 3x + 6$ ([-3], [-3])

10 $y = \frac{-x - 9}{2}$, $y = x - 9$ ([3], [-6])

Linear Relationships - Further Straight Lines (Homework Strip)

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Date Due:

Complete the table of values and then plot the three straight lines.

1 $y = 2x - 3$

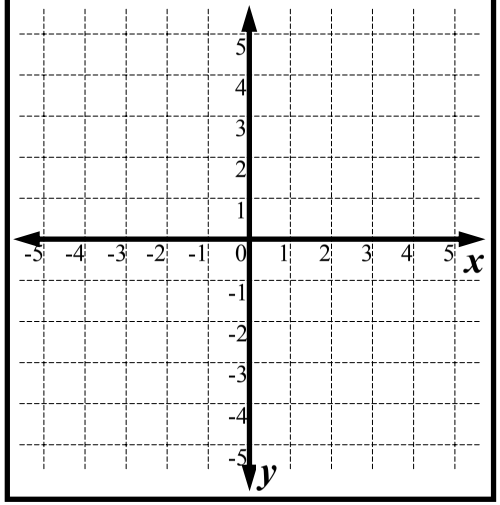
x	-1	0	1	2	3
y					

2 $y = 2 - x$

x	-2	-1	0	1	2
y					

3 $y = -x - 3$

x	-2	-1	0	1	2
y					



Date Due:

Complete the table of values and then plot the three straight lines.

1 $y = 2x - 3$

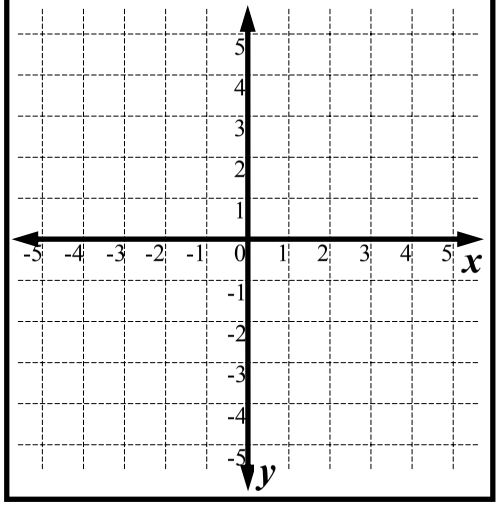
x	-1	0	1	2	3
y					

2 $y = 2 - x$

x	-2	-1	0	1	2
y					

3 $y = -x - 3$

x	-2	-1	0	1	2
y					



Date Due:

Complete the table of values and then plot the three straight lines.

1 $y = 2x - 3$

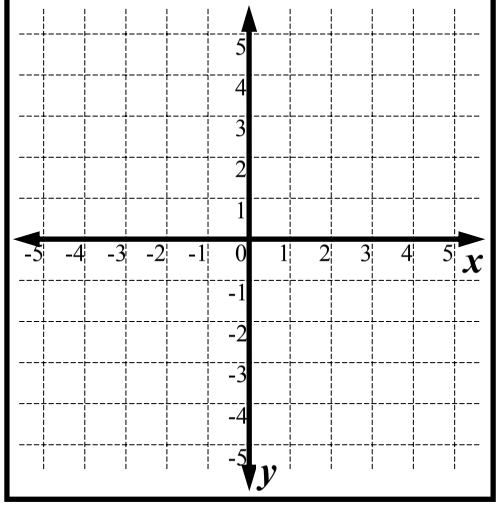
x	-1	0	1	2	3
y					

2 $y = 2 - x$

x	-2	-1	0	1	2
y					

3 $y = -x - 3$

x	-2	-1	0	1	2
y					



Date Due:

Complete the table of values and then plot the three straight lines.

1 $y = 2x - 3$

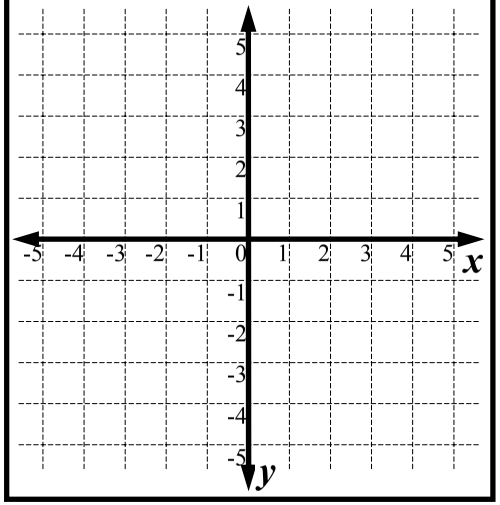
x	-1	0	1	2	3
y					

2 $y = 2 - x$

x	-2	-1	0	1	2
y					

3 $y = -x - 3$

x	-2	-1	0	1	2
y					



Linear Relationships - Further Straight Lines (Homework Strip)

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Date Due:

Complete the table of values and then plot the three straight lines.

1 $y = 2x - 3$

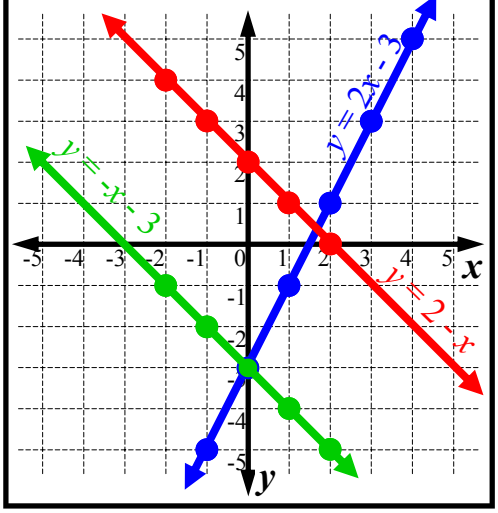
x	-1	0	1	2	3
y	-5	-3	-1	1	3

2 $y = 2 - x$

x	-2	-1	0	1	2
y	4	3	2	1	0

3 $y = -x - 3$

x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5

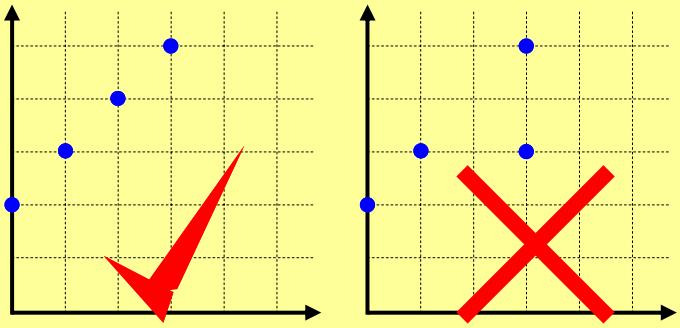


Linear Relationships - Cost Functions

A cost function is a linear relationship that reflects a business model. It might be sales, costs, customer satisfaction …. all can be plotted on a graph.

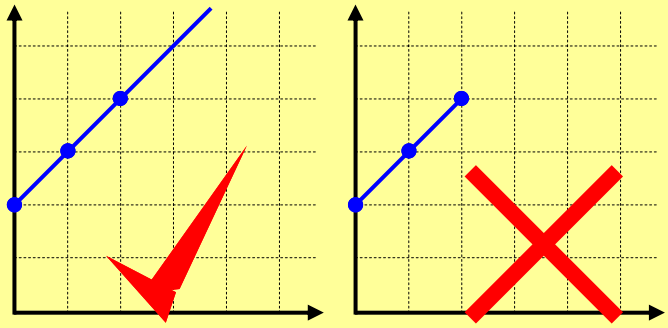
STEP 1

Make sure that your points are in a straight line.

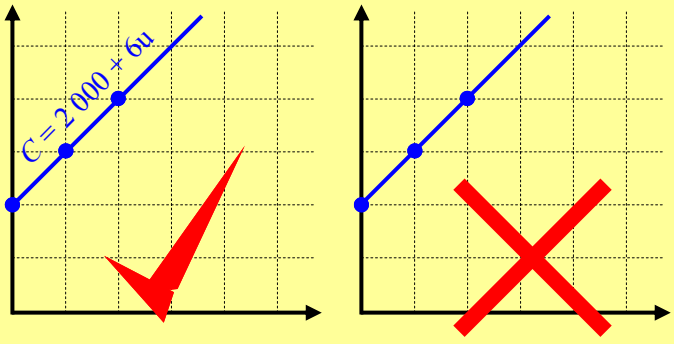


STEP 2

Draw a line the full size of the grid, not just between the first and last dot.



STEP 3



Write the equation for the line on top of the line. Write it in pencil as another line may plot over it later and you will have to move the name.

An important part of a cost function is the break even point. This is the point where one cost function becomes more efficient than the other. It also is the point where a loss, changes to a profit, when costs are plotted against revenue.

Linear Relationships - Cost Functions (Sheet 1)

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Complete the table of values for these Cost Functions.

1 *Consultation Fees:* $C = 60n + 120$

n	1	2	3	4	5
C					

2 *Labour Costs:* $C = 720h + 8\,000$

h	0	10	20	30	40
C					

3 *Unit Rates:* $R = 250e + 320$

e	0	5	10	15	20
R					

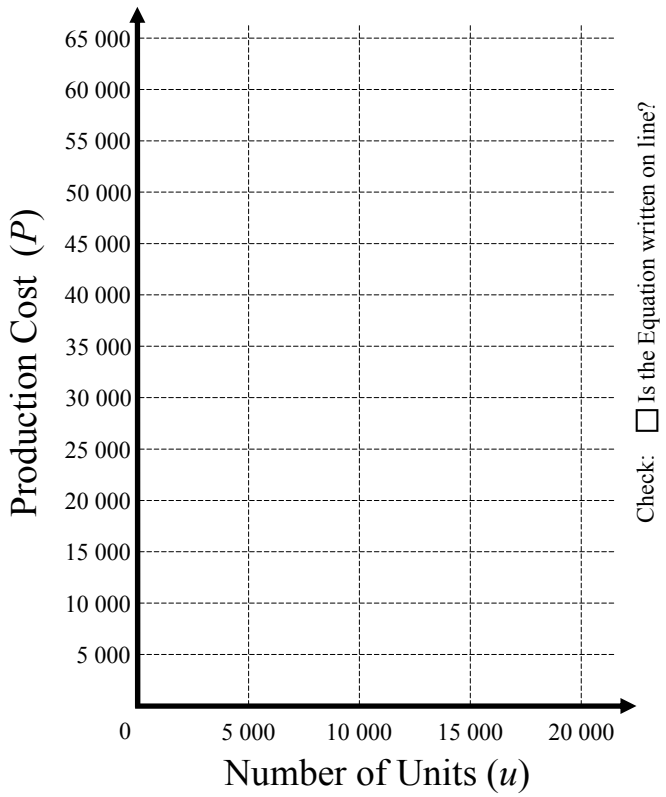
4 *Advertising Outlay :* $A = 180p + 12\,000$

p	0	60	120	180	240
A					

OK it's plotting time. Complete the cost function table then plot the values on the graph and construct a straight line. Write the function on the line.

5 *Production Costs :* $P = 2u + 15\,000$

u	0	5 000	10 000	15 000	20 000
P					



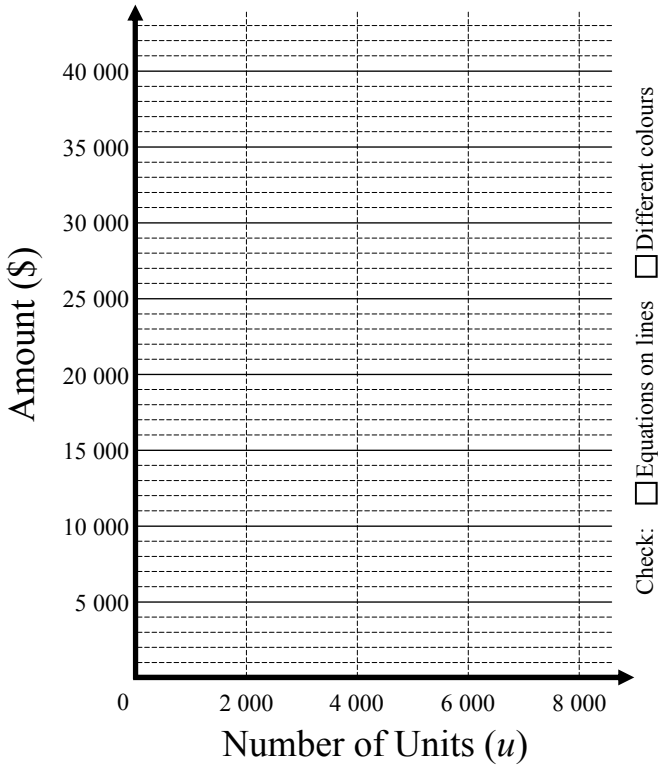
Complete the tables for these Cost Functions, plot them both on the same graph. Use two colours.

6 *Manufacturing Cost :* $C = 3u + 14\,000$

u	0	2 000	4 000	6 000	8 000
P					

7 *Revenue :* $R = 5u$

u	0	2 000	4 000	6 000	8 000
R					



8 Estimate the cost to make 3 000 units. _____ $ sign

9 Estimate the revenue on selling 3 000 units. _____

10 Calculate the loss made on selling 3 000 units.

Loss = _____ (Cost (Q.8)) - _____ (Revenue (Q.9)) = _____

11 Estimate how many units have to be sold for the company to break even. _____

12 Calculate the loss made on selling 6 000 units.

Loss = _____ - _____ = _____

13 Calculate the profit made on selling 8 000 units.

Profit = _____ - _____ = _____

Linear Relationships - Cost Functions (Sheet 2)

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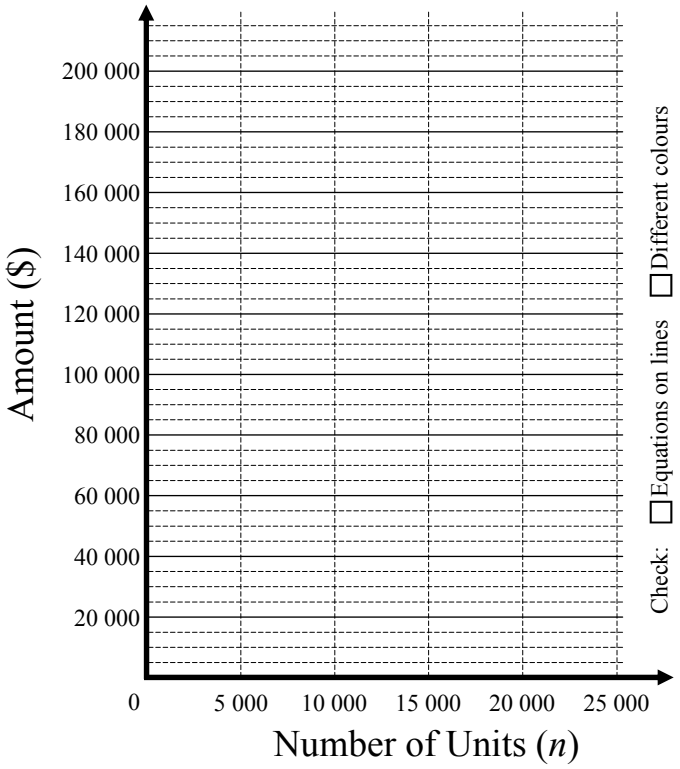
Complete the tables for these Cost Functions, plot them both on the same graph. Use two colours.

1 *Advertising Budget : $A = 5n + 45\,000$*

n	0	5 000	10 000	15 000	20 000
A					

2 *Sales Revenue : $S = 8n$*

n	0	5 000	10 000	15 000	20 000
S					



3 If the advertising budget is \$105 000, estimate the number of units produced. _____

4 If sales revenue is \$140 000, estimate how many units are produced. _____

5 Calculate the loss made on selling 5 000 units.

Loss = _____ - _____ = _____

6 How many units have to be sold for the company to break even? _____

7 Calculate the profit made on selling 20 000 units.

_____

8 Estimate the profit made on selling 25 000 units.

_____

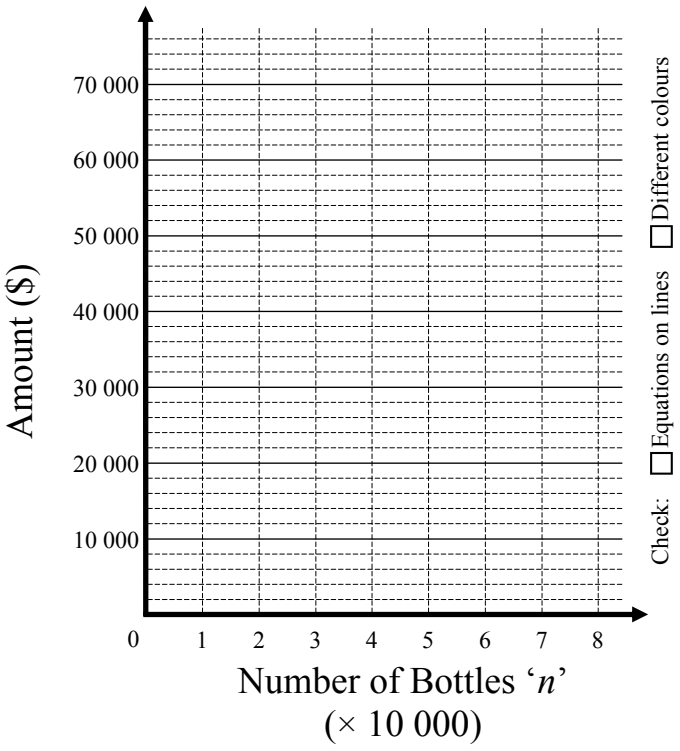
A soft drink manufacturer has cost and revenue functions for daily production. Extend the lines the full extent of the graph to answer the questions.

9 *Daily Production Cost : $P = 0.5n + 20\,000$*

n	0	10 000	20 000	30 000	40 000
P					

10 *Sales Revenue : $S = 0.9n$*

n	0	10 000	20 000	30 000	40 000
S					



11 Estimate the production cost for producing:

_____ 25 000 bottles _____ 60 000 bottles

12 Estimate the bottles produced which gives revenue of:

_____ \$30 000 in sales _____ \$50 000 in sales

13 Estimate how many bottles have to be sold for the company to break even? _____

14 Estimate the profit (or loss) on bottle production of:

15 000 _____

80 000 _____

Linear Relationships - Cost Functions (Sheet 1)

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Complete the table of values for these Cost Functions.

1 *Consultation Fees:* $C = 60n + 120$

n	1	2	3	4	5
C	180	240	300	360	420

2 *Labour Costs:* $C = 720h + 8\,000$

h	0	10	20	30	40
C	8 000	15 200	22 400	29 600	36 800

3 *Unit Rates:* $R = 250e + 320$

e	0	5	10	15	20
R	320	1 570	2 820	4 070	5 320

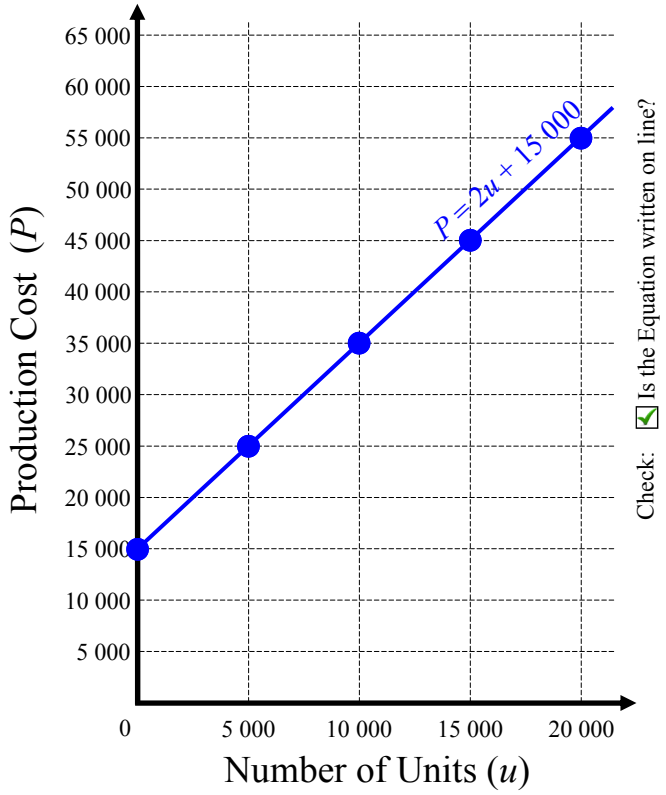
4 *Advertising Outlay :* $A = 180p + 12\,000$

p	0	60	120	180	240
A	12 000	22 800	33 600	44 400	55 200

OK it's plotting time. Complete the cost function table then plot the values on the graph and construct a straight line. Write the function on the line.

5 *Production Costs :* $P = 2u + 15\,000$

u	0	5 000	10 000	15 000	20 000
P	15 000	25 000	35 000	45 000	55 000



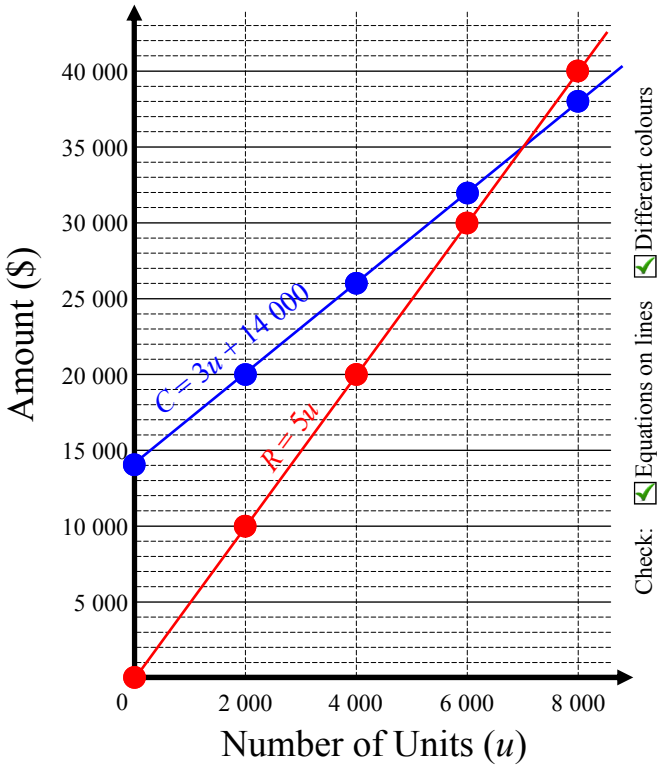
Complete the tables for these Cost Functions, plot them both on the same graph. Use two colours.

6 *Manufacturing Cost :* $C = 3u + 14\,000$

u	0	2 000	4 000	6 000	8 000
P	14 000	20 000	26 000	32 000	38 000

7 *Revenue :* $R = 5u$

u	0	2 000	4 000	6 000	8 000
R	0	10 000	20 000	30 000	40 000



8 Estimate the cost to make 3 000 units. $23 000 ($ sign)

9 Estimate the revenue on selling 3 000 units. $15 000

10 Calculate the loss made on selling 3 000 units.

Loss = 23 000 (Cost (Q.8)) - 15 000 (Revenue (Q.9)) = $8 000

11 Estimate how many units have to be sold for the company to break even. 7 000 units

12 Calculate the loss made on selling 6 000 units.

Loss = 32 000 - 30 000 = $2 000

13 Calculate the profit made on selling 8 000 units.

Profit = 40 000 - 38 000 = $2 000

Linear Relationships - Cost Functions (Sheet 2)

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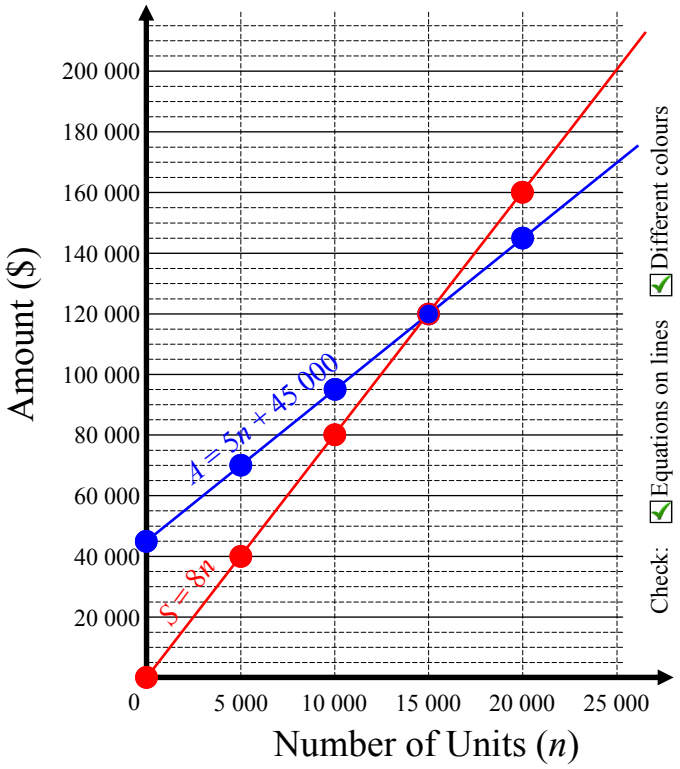
Complete the tables for these Cost Functions, plot them both on the same graph. Use two colours.

1 *Advertising Budget : $A = 5n + 45\,000$*

n	0	5 000	10 000	15 000	20 000
A	45 000	70 000	95 000	120 000	145 000

2 *Sales Revenue : $S = 8n$*

n	0	5 000	10 000	15 000	20 000
S	0	40 000	80 000	120 000	160 000



3 If the advertising budget is \$105 000, estimate the number of units produced. 12 000 units

4 If sales revenue is \$140 000, estimate how many units are produced. 17 500 units

5 Calculate the loss made on selling 5 000 units.

Loss = 70 000 - 40 000 = \$30 000

6 How many units have to be sold for the company to break even? 15 000 units

7 Calculate the profit made on selling 20 000 units.

Profit = 160 000 - 145 000 = \$15 000

8 Estimate the profit made on selling 25 000 units.

Profit = 200 000 - 170 000 = \$30 000

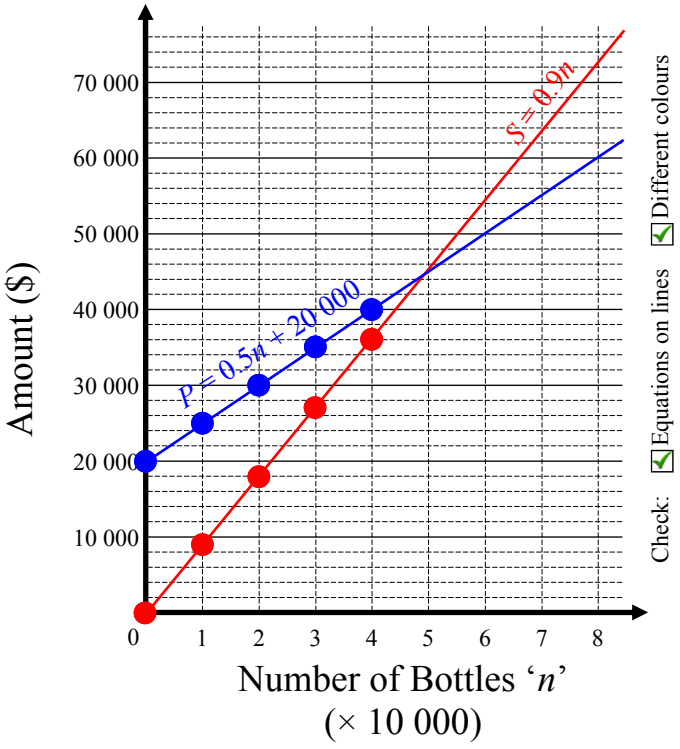
A soft drink manufacturer has cost and revenue functions for daily production. Extend the lines the full extent of the graph to answer the questions.

9 *Daily Production Cost : $P = 0.5n + 20\,000$*

n	0	10 000	20 000	30 000	40 000
P	20 000	25 000	30 000	35 000	40 000

10 *Sales Revenue : $S = 0.9n$*

n	0	10 000	20 000	30 000	40 000
S	0	9 000	18 000	27 000	36 000



11 Estimate the production cost for producing:

\$32 500 for 25 000 bottles

\$5 000 for 60 000 bottles

12 Estimate the bottles produced which gives revenue of:

≈ 3 300 bottles for \$30 000 in sales

≈ 55 000 bottles for \$50 000 in sales

13 Estimate how many bottles have to be sold for the company to break even? 50 000

14 Estimate the profit (or loss) on bottle production of:

15 000 Loss = 28 000 - 14 000 = \$14 000

80 000 Profit = 72 000 - 60 000 = \$12 000

Linear Relationships - Cost Functions (Homework Strip)

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Date Due: _____

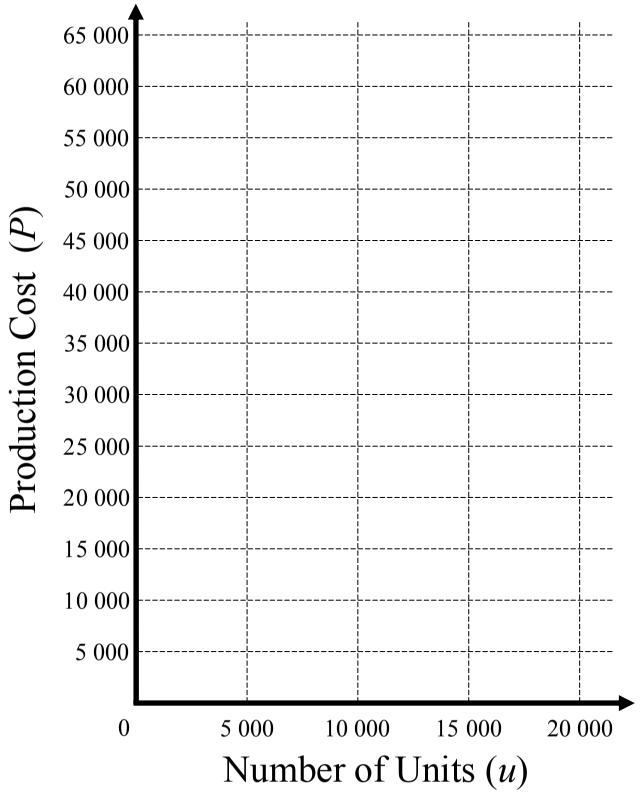
Complete the tables and plot the cost functions

1 *Production Costs : $P = 2u + 20\,000$*

u	0	5 000	10 000	15 000	20 000
P					

2 *Production Costs : $P = 3u + 5\,000$*

u	0	5 000	10 000	15 000	20 000
P					



Date Due: _____

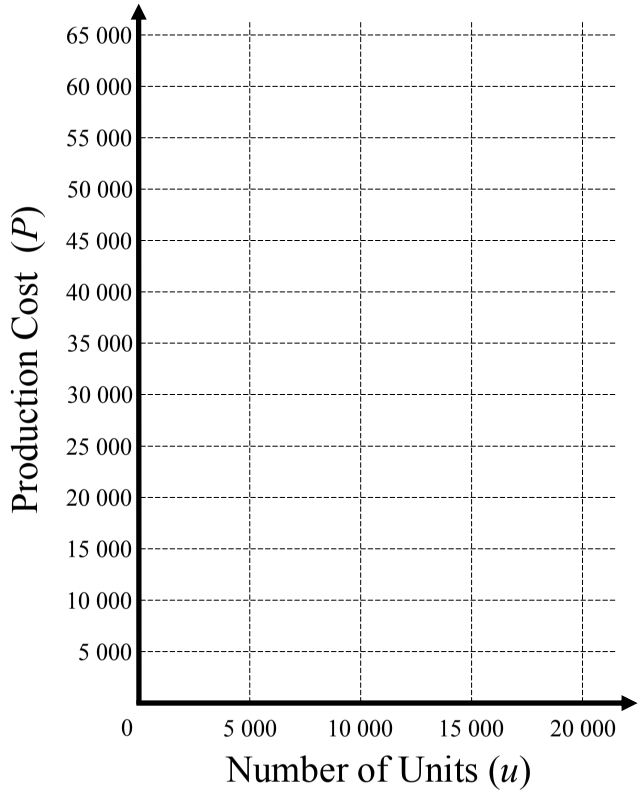
Complete the tables and plot the cost functions

1 *Production Costs : $P = 2u + 20\,000$*

u	0	5 000	10 000	15 000	20 000
P					

2 *Production Costs : $P = 3u + 5\,000$*

u	0	5 000	10 000	15 000	20 000
P					



Date Due: _____

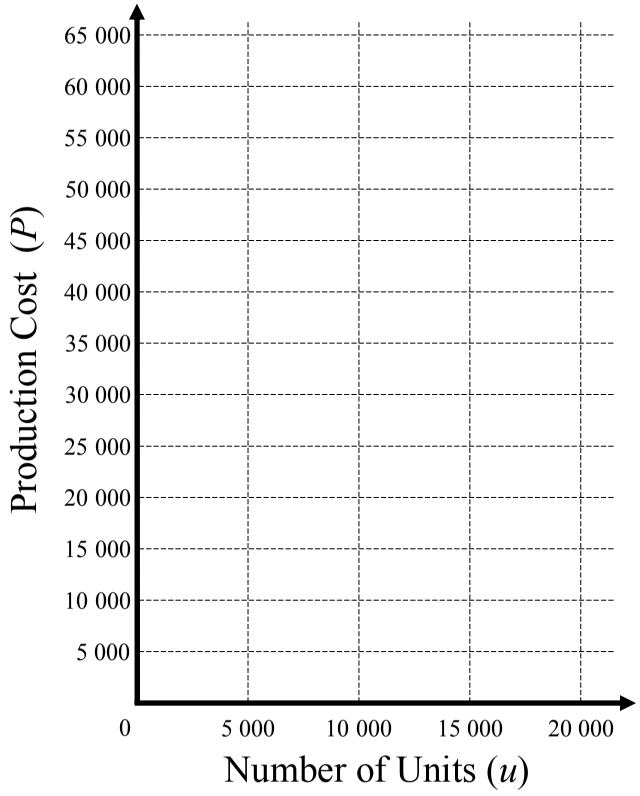
Complete the tables and plot the cost functions

1 *Production Costs : $P = 2u + 20\,000$*

u	0	5 000	10 000	15 000	20 000
P					

2 *Production Costs : $P = 3u + 5\,000$*

u	0	5 000	10 000	15 000	20 000
P					



Linear Relationships - Cost Functions (Homework Strip)

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Date Due:

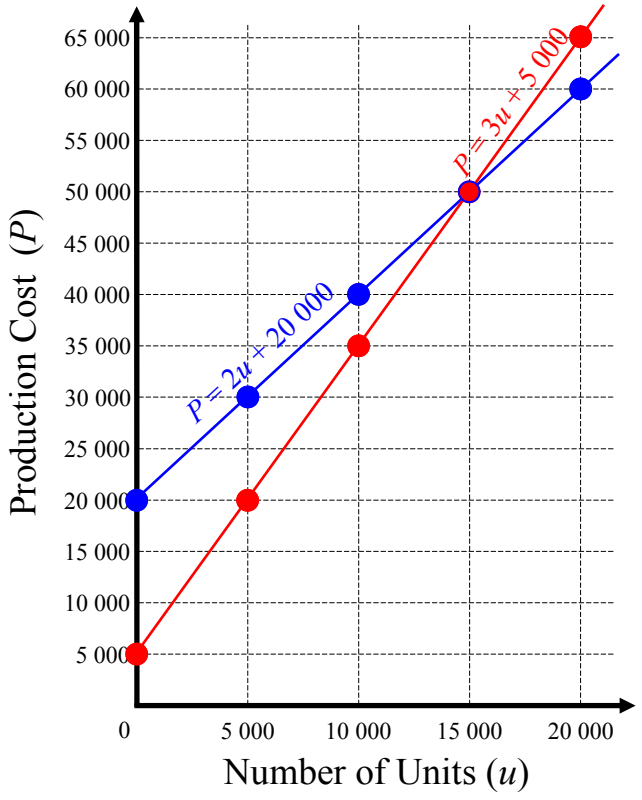
Complete the tables and plot the cost functions

1 *Production Costs : $P = 2u + 20\,000$*

u	0	5 000	10 000	15 000	20 000
P	20 000	30 000	40 000	50 000	60 000

2 *Production Costs : $P = 3u + 5\,000$*

u	0	5 000	10 000	15 000	20 000
P	5 000	20 000	35 000	50 000	65 000



Linear Relationships - Horizontal and Vertical Line Equations

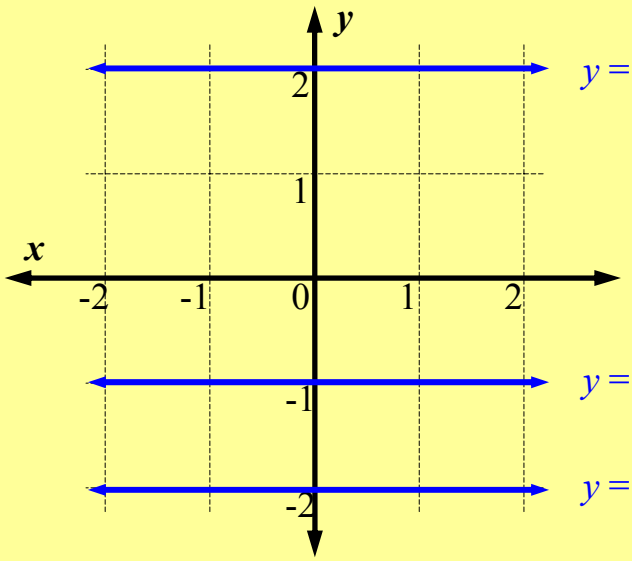
These lines are special because they are horizontal or vertical and only have one letter in them, instead of two. They are also unusual because they seem to be the opposite to what you think.

These lines are in the form:

$y = b$ where b is the number where the line cuts the y-axis

$x = a$ where a is the number where the line cuts the x-axis

Here are some examples, first $y = b$:



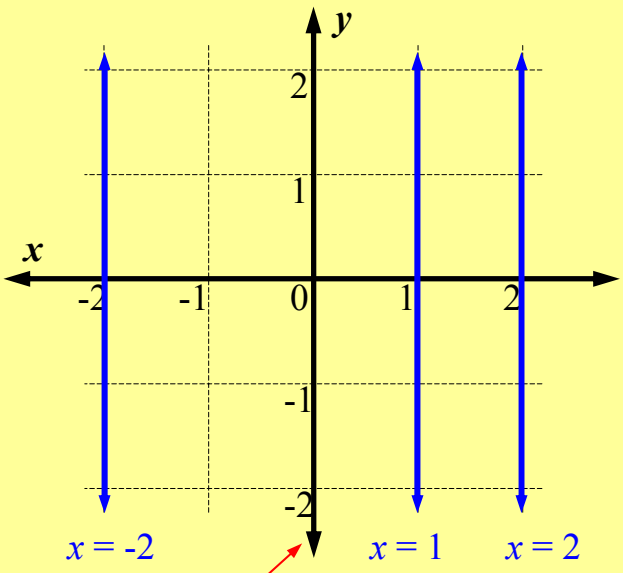
$y = 2$ (cuts the y-axis at 2)

$y = 0$ (cuts the y-axis at 0), so the equation for the x-axis is $y = 0$.

$y = -1$ (cuts the y-axis at -1)

$y = -2$ (cuts the y-axis at -2)

Here are some examples for $x = a$:



$x = -2$ $x = 1$ $x = 2$

$x = 0$ (cuts the x-axis at 0), so the equation for the y-axis is $x = 0$.

So these lines are simple to plot, the only problem is that they seem the opposite to what you would think.

Lines parallel to the y-axis aren't in the format '$y =$' they are '$x =$'. Likewise for the lines parallel to the x-axis, being '$y =$'.

If you can remember this, these are the easiest lines to plot.

Linear Relationships - Horizontal and Vertical Line Equations (Sheet 1)

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Plot these points on the number plane. Draw a line through the points and write the name of the line, on the line. Remember to extend line, arrowheads.

1 Points: (3 , 2), (3 , -4) and (3, 5) $x = 3$

Plot these points on the plane below

Write name on the line

$x = 3$ cuts the x-axis at: ☐

$x = 3$ is: ○ horizontal or ○ vertical

2 Points: (-2 , 1), (-2 , 3) and (-2, -5) $x = -2$

Write name on the line

$x = -2$ cuts the x-axis at: ☐

$x = -2$ is: ○ horizontal or ○ vertical

3 Points: (4 , -4), (0 , -4) and (-3, -4) $y = -4$

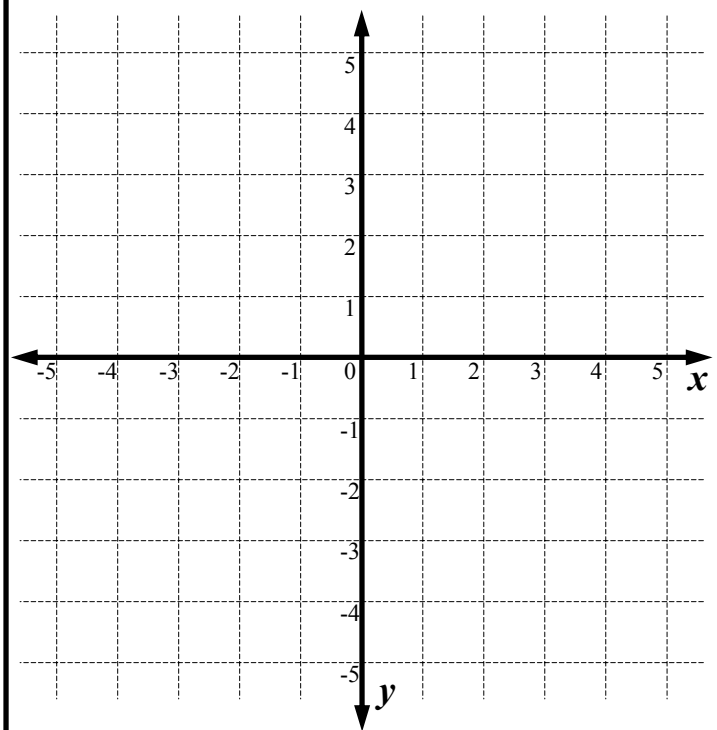
$y = -4$ cuts the y-axis at: ☐

$y = -4$ is: ○ horizontal or ○ vertical

4 Points: (-3 , 2), (-1 , 2) and (2, 2) $y = 2$

$y = 2$ cuts the y-axis at: ☐

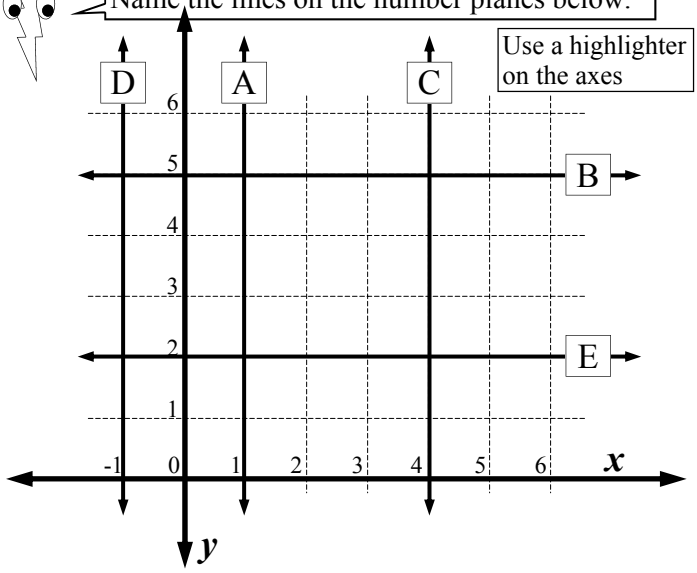
$y = 2$ is: ○ horizontal or ○ vertical



Check:
- ☐ Arrow heads on each line?
- ☐ Is the Equation written on each line?
- ☐ Is each line extended fully on the plane?

Name the lines on the number planes below.

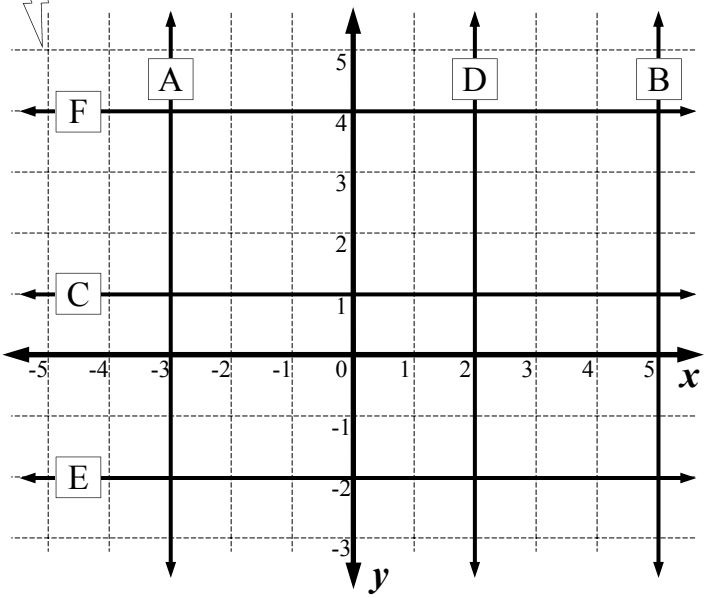
Use a highlighter on the axes



5 A _____ B _____

C _____ D _____ E _____

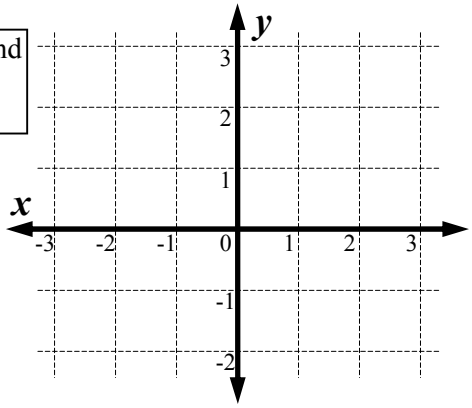
Now try these, use a highlighter on the axes.



6 A _____ B _____ C _____

D _____ E _____ F _____

Plot $y = -2$ and $x = 2$ on the plane.



7

Check:
- ☐ Arrow heads?
- ☐ Equation on lines?
- ☐ Extended fully?

Linear Relationships - Horizontal and Vertical Line Equations (Sheet 2)

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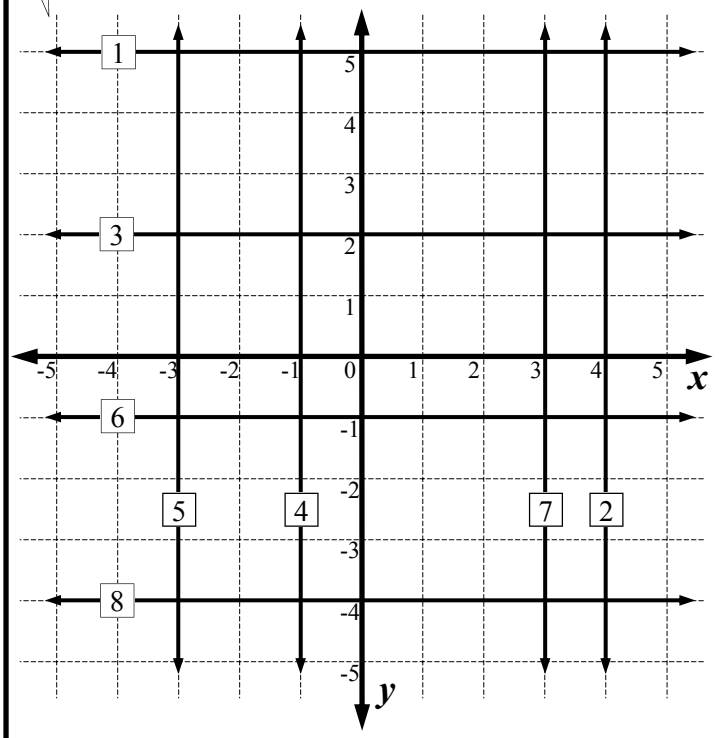
Name the lines shown. Use the spaces below the number plane then label the lines on the plane also.



1 $y =$ ☐ 2 $x =$ ☐ 3 _____ 4 _____

5 _____ 6 _____ 7 _____ 8 _____

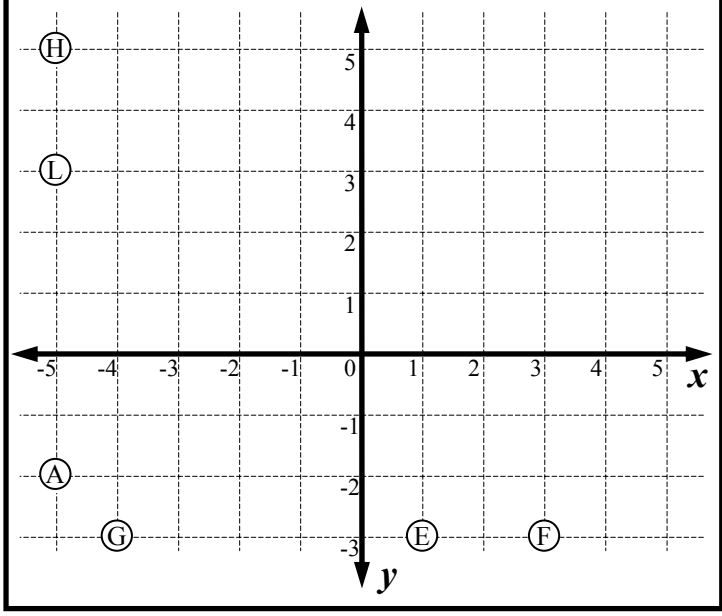
9 What is the equation for the y-axis? _____

10 What is the equation for the x-axis? _____

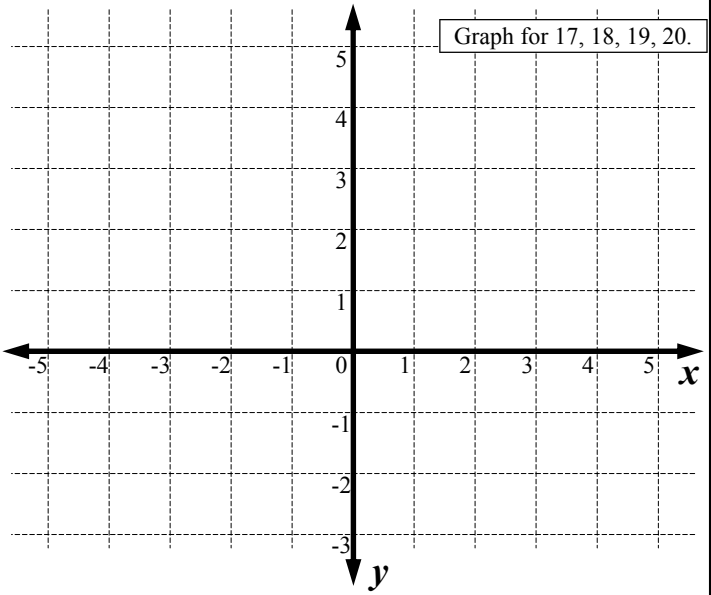
Draw these lines on the Cartesian plane, show arrow heads and labels. After you draw the line it will pass through a letter, write it in the box.

11 $y = -2$ ☐ Letter 12 $x = 3$ ☐ Letter 13 $x = -4$ ☐ Letter

14 $y = 5$ ☐ Letter 15 $x = 1$ ☐ Letter 16 $y = 3$ ☐ Letter



Draw the lines on the grid. Once drawn there will be four intersection points. Give the coordinates for these points as well as the name of shape formed.

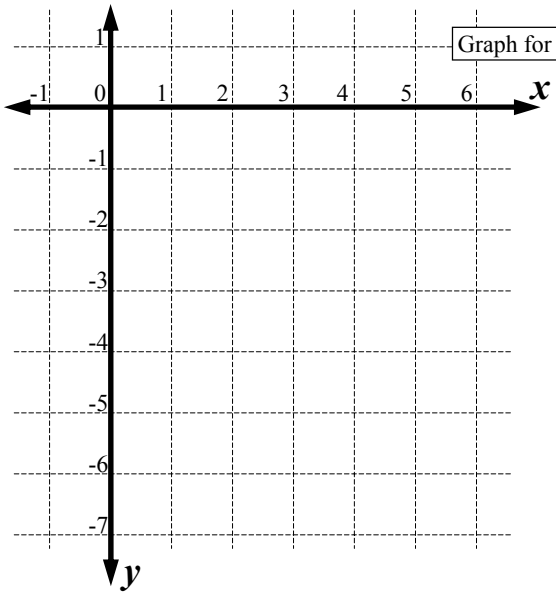


17 $y = 2$ 18 $y = -1$

19 $x = -4$ 20 $x = 1$

21 Points: _____

22 Shape formed: _____



23 $y = -2x + 3$

x	1	2	3	4	5
y					

24 $y = -5$

25 $x = 2$

26 Points: (☐, ☐), (☐, ☐), (☐, ☐).

27 Shape formed: _____

Linear Relationships - Horizontal and Vertical Line Equations (Sheet 1)

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Plot these points on the number plane. Draw a line through the points and write the name of the line, on the line. Remember to extend line, arrowheads.

1 Points: (3 , 2), (3 , -4) and (3, 5) $x = 3$

Plot these points on the plane below

Write name on the line

$x = 3$ cuts the x-axis at: 3

$x = 3$ is: ○ horizontal or ● vertical

2 Points: (-2 , 1), (-2 , 3) and (-2, -5) $x = -2$

Write name on the line

$x = -2$ cuts the x-axis at: -2

$x = -2$ is: ○ horizontal or ● vertical

3 Points: (4 , -4), (0 , -4) and (-3, -4) $y = -4$

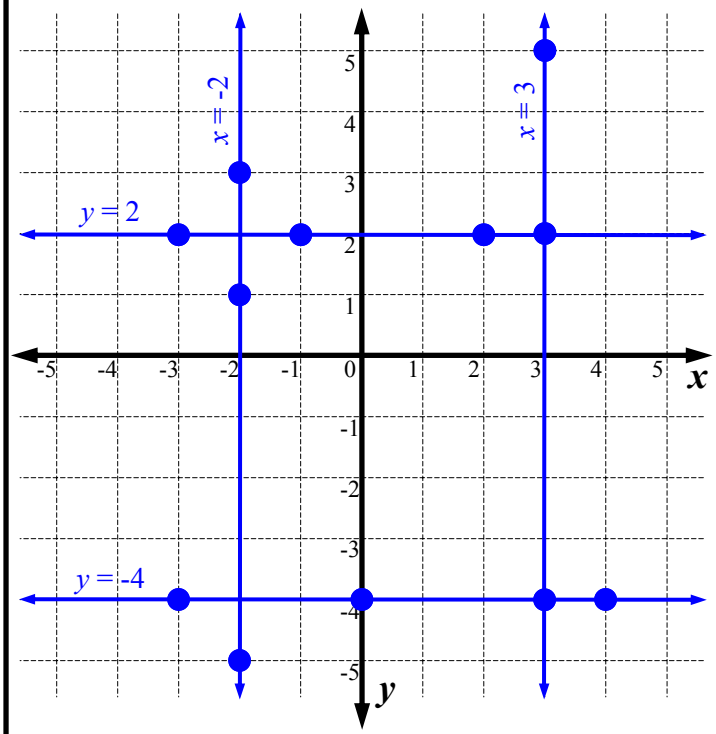
$y = -4$ cuts the y-axis at: -4

$y = -4$ is: ● horizontal or ○ vertical

4 Points: (-3 , 2), (-1 , 2) and (2, 2) $y = 2$

$y = 2$ cuts the y-axis at: 2

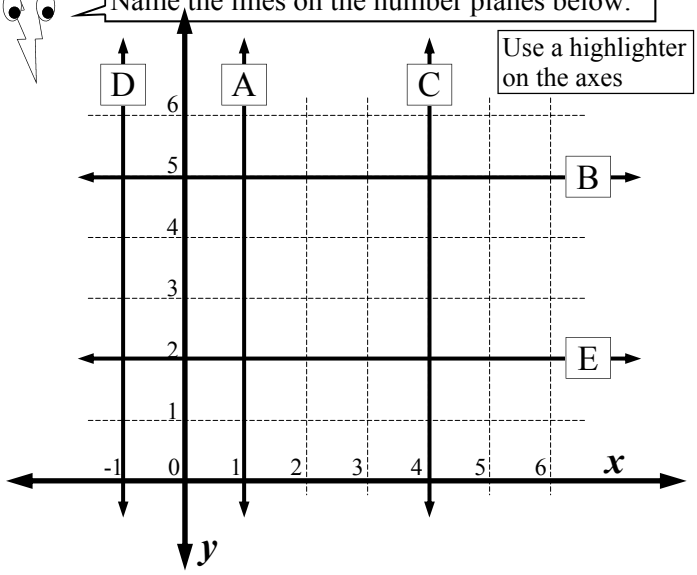
$y = 2$ is: ● horizontal or ○ vertical



Check: ☑ Arrow heads on each line?
☑ Is the Equation written on each line?
☑ Is each line extended fully on the plane?

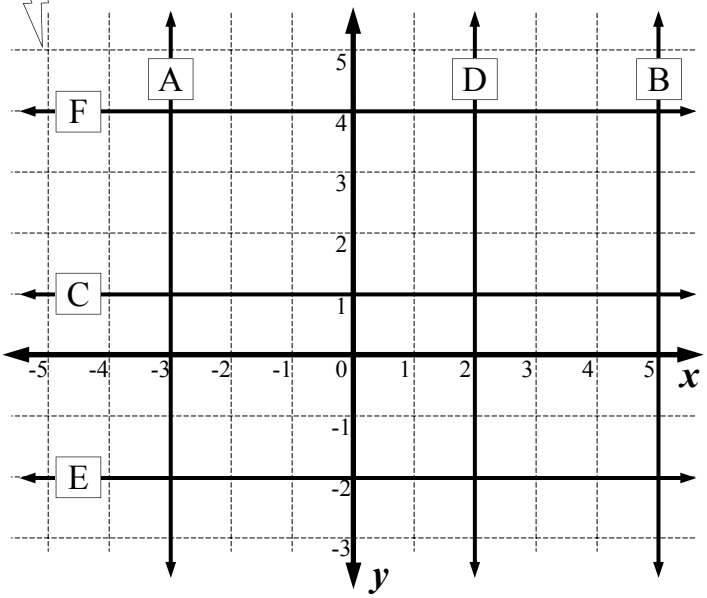
Name the lines on the number planes below.

Use a highlighter on the axes



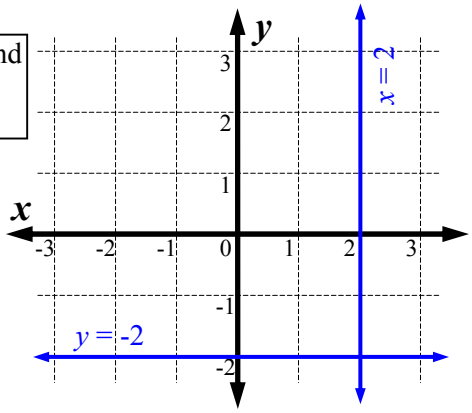
5 A $x = 1$ B $y = 5$ C $x = 4$ D $x = -1$ E $y = 2$

Now try these, use a highlighter on the axes.



6 A $x = -3$ B $x = 5$ C $y = 1$ D $x = 2$ E $y = -2$ F $y = 4$

Plot $y = -2$ and $x = 2$ on the plane.



7

Check:
☑ Arrow heads?
☑ Equation on lines?
☑ Extended fully?

Linear Relationships - Horizontal and Vertical Line Equations (Sheet 2)

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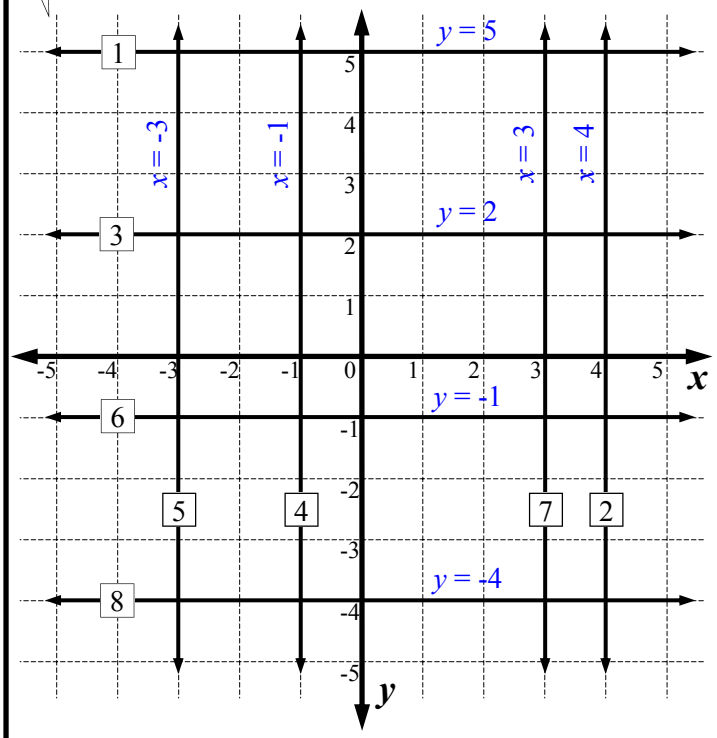
Name the lines shown. Use the spaces below the number plane then label the lines on the plane also.



1 $y =$ 5 2 $x =$ 4 3 $y = 2$ 4 $x = -1$

5 $x = -3$ 6 $y = -1$ 7 $x = 3$ 8 $y = -4$

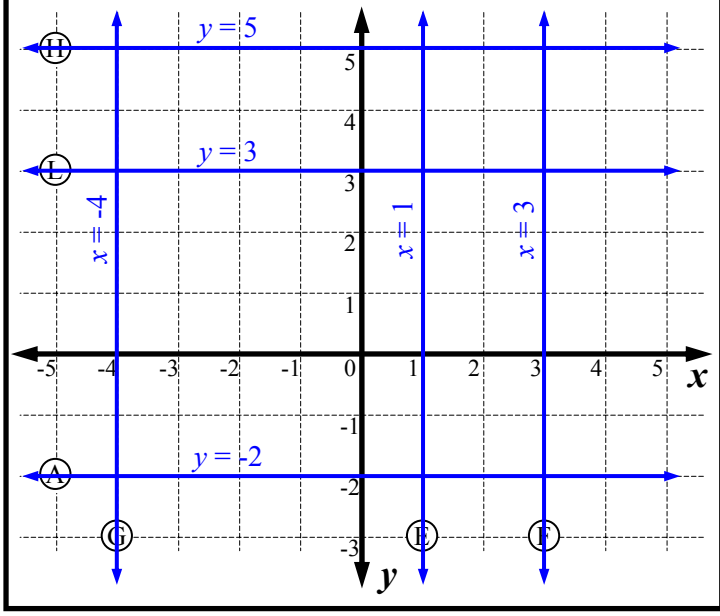
9 What is the equation for the y-axis? $x = 0$

10 What is the equation for the x-axis? $y = 0$

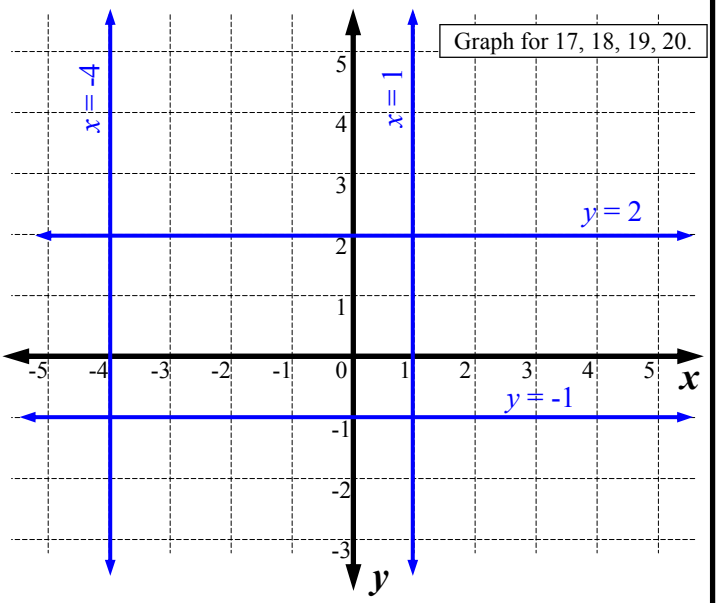
Draw these lines on the Cartesian plane, show arrow heads and labels. After you draw the line it will pass through a letter, write it in the box.

11 $y = -2$ A Letter 12 $x = 3$ F Letter 13 $x = -4$ G Letter

14 $y = 5$ H Letter 15 $x = 1$ E Letter 16 $y = 3$ L Letter



Draw the lines on the grid. Once drawn there will be four intersection points. Give the coordinates for these points as well as the name of shape formed.

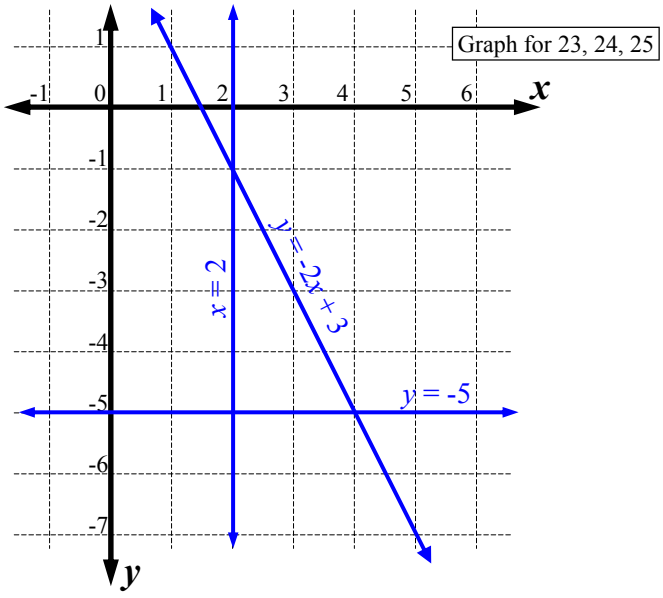


17 $y = 2$ 18 $y = -1$

19 $x = -4$ 20 $x = 1$

21 Points: (-4, 2), (1, 2), (1, -1), (-4, -1).

22 Shape formed: Rectangle



23 $y = -2x + 3$

x	1	2	3	4	5
y	1	-1	-3	-5	-7

24 $y = -5$

25 $x = 2$

26 Points: (2, -1), (2, -5), (4, -5).

27 Shape formed: Right angled triangle

Linear Relationships - Horizontal and Vertical Line Equations (Homework Strip)

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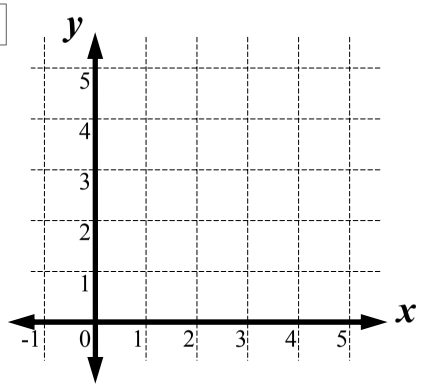
Date Due:

Plot these on the grid:
$x = 2, y = 1, x = 4, y = 3$

1

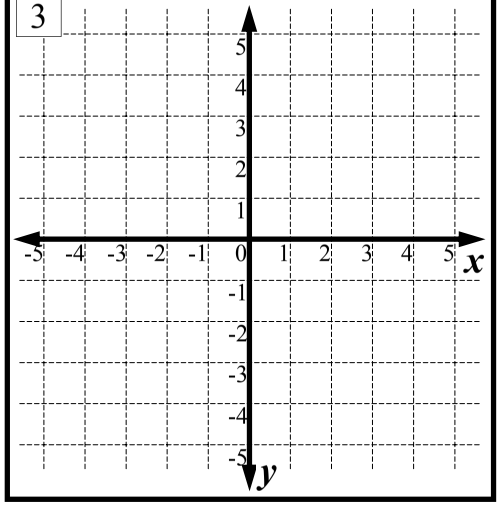


Give the 4 points of intersection

2 Points: $(\square, \square), (\square, \square),$
$(\square, \square), (\square, \square).$

Plot these on the grid below:
$x = -4, y = 3, x = 2, y = -5.$

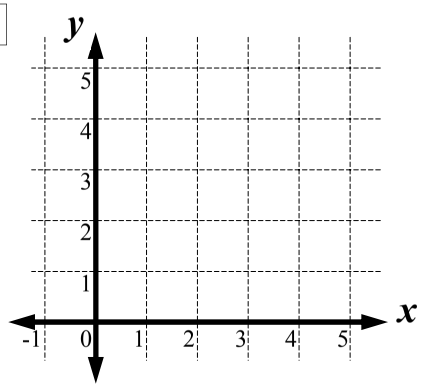
3



Date Due:

Plot these on the grid:
$x = 2, y = 1, x = 4, y = 3$

1

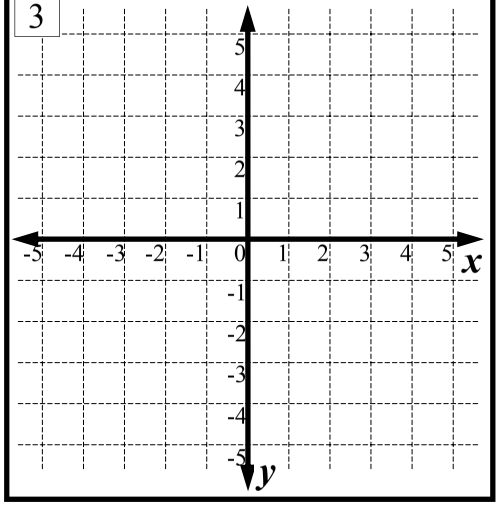


Give the 4 points of intersection

2 Points: $(\square, \square), (\square, \square),$
$(\square, \square), (\square, \square).$

Plot these on the grid below:
$x = -4, y = 3, x = 2, y = -5.$

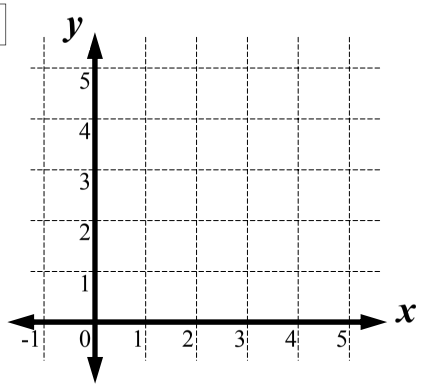
3



Date Due:

Plot these on the grid:
$x = 2, y = 1, x = 4, y = 3$

1

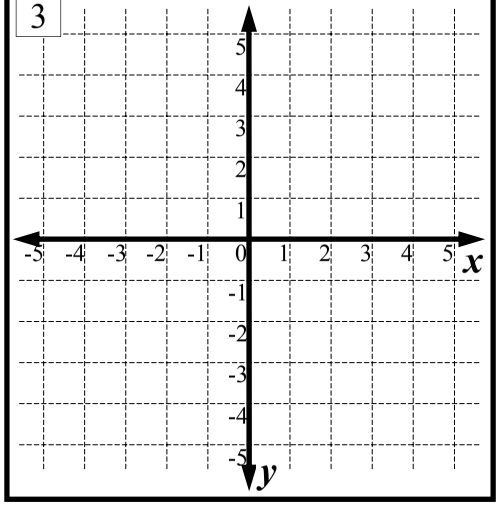


Give the 4 points of intersection

2 Points: $(\square, \square), (\square, \square),$
$(\square, \square), (\square, \square).$

Plot these on the grid below:
$x = -4, y = 3, x = 2, y = -5.$

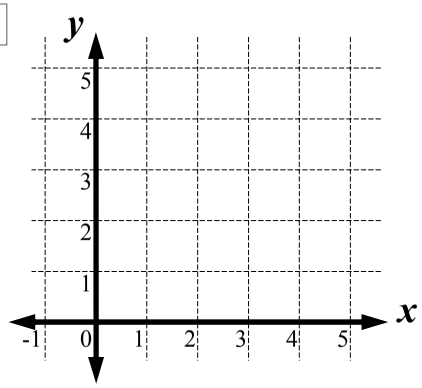
3



Date Due:

Plot these on the grid:
$x = 2, y = 1, x = 4, y = 3$

1

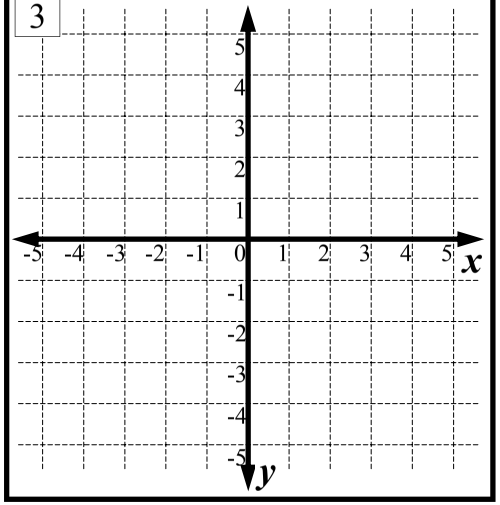


Give the 4 points of intersection

2 Points: $(\square, \square), (\square, \square),$
$(\square, \square), (\square, \square).$

Plot these on the grid below:
$x = -4, y = 3, x = 2, y = -5.$

3



Linear Relationships - Horizontal and Vertical Line Equations (HW Strip)

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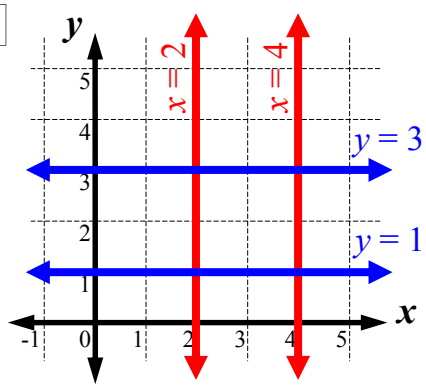
Date Due:

Plot these on the grid:
$x = 2, y = 1, x = 4, y = 3$

1

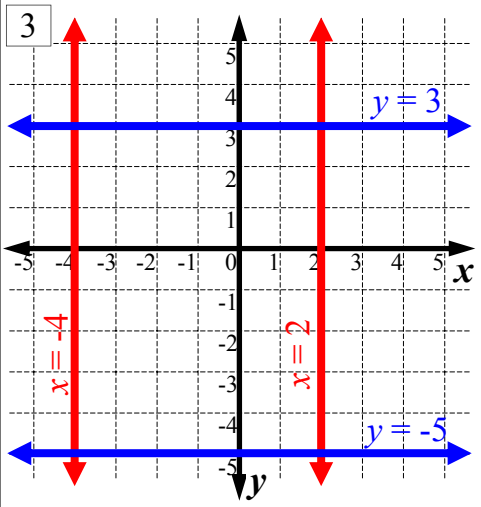


Give the 4 points of intersection

2 Points: (2, 1), (2, 3), (4, 1), (4, 3).

Plot these on the grid below:
$x = -4, y = 3, x = 2, y = -5.$

3



Linear Relationships - Naming Straight Lines

This sheet deals with naming straight lines. The standard equation for a straight line is:

$$y = mx + b$$

Where: m = the gradient or slope of the line, and
b = the y-intercept, which is the point at which the line cuts or intersects the y-axis. (the y-axis is the vertical axis)

Examples

A line has a gradient of 2 and a y-intercept of 4.

$y = 2x + 4$

Replace the m with 2 and the b with 4

A line has a gradient of -4 and a y-intercept of 0.

$y = -4x$

Replace the m with -4 and don't write '+ 0'

A line has a gradient of 1 and a y-intercept of 3.

$y = x + 3$

m is 1 but don't write '1x' leave it as x

This sheet doesn't require the calculation of gradient, it is always given to you.

Linear Relationships - Naming Straight Lines (Sheet 1)

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Name the straight lines, given the gradient and the y-intercept. $y = mx + b$, m = gradient & b = y-int.

1 Gradient = 4, y-int = 1. $y =$ ☐ x + ☐

2 Gradient = -3, y-int = -6. $y =$ ☐ x - ☐

3 Gradient = -2, y-int = 5. $y =$ ☐

4 y-int = 3, Gradient = -4. ☐

5 y-int = 0, Gradient = -2. ☐

When the y-int = 0, you don't include it

6 Gradient = 1, y-int = 2. ☐

Don't include '1' or '-1' just 'x' or '-x'

7 y-int = -3, Gradient = -1. ☐

8 Gradient = ½, y-int = 0. ☐

9 Gradient = 2, y-int = -6. ☐

10 y-int = 0, Gradient = -1. ☐

This time you are given the equation, you need to find the gradient and y-intercept.

11 $y = 3x + 2$ $m =$ ☐ $b =$ ☐

12 $y = x - 7$ $b =$ ☐ $m =$ ☐

13 $y = -4x$ $b =$ ☐ $m =$ ☐

14 $y = ½x + 2$ $m =$ ☐ $b =$ ☐

15 $y = 10 - 2x$ $b =$ ☐ $m =$ ☐

If lines are parallel they have the same gradient. Find the equations of these lines.

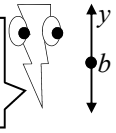
16 Find the equation of the line with a y-int of 6 which is parallel to $y = 3x + 1$. $y =$ ☐ x + ☐

17 Find the equation of the line with a y-int of 4 which is parallel to $y = -2x - 3$. ☐

17 Find the equation of the line with a y-int of -3 which is parallel to $y = -3x$. ☐

18 Find the equation of the line with a y-int of 0 which is parallel to $y = 5x + 2$. ☐

You are given the gradient, read off the y-int and then construct the equation for the line. 'b' is where the line 'cuts' the y-axis. (vertical axis)



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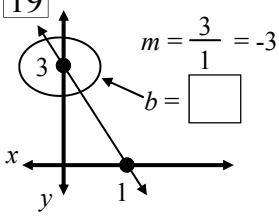
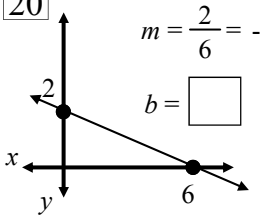


$m = \frac{3}{1} = -3$

$b =$ ☐

$y =$ ☐ x + ☐

20



$m = \frac{2}{6} = -\frac{1}{3}$

$b =$ ☐

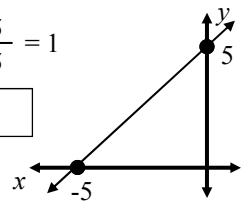
$y =$ ☐ x + ☐

21

$y =$ ☐

$m = \frac{5}{5} = 1$

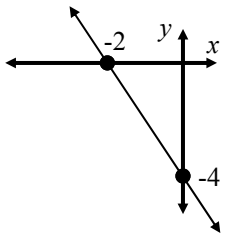
$b =$ ☐



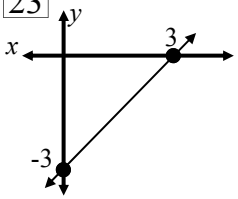
22 ☐

$m = \frac{4}{2} = -2$

$b =$ ☐



23



$m = \frac{3}{3} = 1$

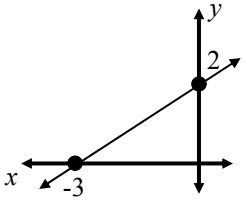
$b =$ ☐

☐

24 ☐

$m = \frac{2}{3}$

$b =$ ☐



Linear Relationships - Naming Straight Lines (Sheet 1)

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Name the straight lines, given the gradient and the y-intercept. $y = mx + b$, m = gradient & b = y-int.

1 Gradient = 4, y-int = 1. $y =$ 4 x + 1

2 Gradient = -3, y-int = -6. $y =$ -3 x - 6

3 Gradient = -2, y-int = 5. $y =$ $y = -2x + 5$

4 y-int = 3, Gradient = -4. $y = -4x + 3$

5 y-int = 0, Gradient = -2. $y = -2x$

When the y-int = 0, you don't include it

6 Gradient = 1, y-int = 2. $y = x + 2$

Don't include '1' or '-1' just 'x' or '$-x$'

7 y-int = -3, Gradient = -1. $y = -x - 3$

8 Gradient = ½, y-int = 0. $y = ½x$

9 Gradient = 2, y-int = -6. $y = 2x - 6$

10 y-int = 0, Gradient = -1. $y = -x$

This time you are given the equation, you need to find the gradient and y-intercept.

11 $y = 3x + 2$ $m =$ 3 $b =$ 2

12 $y = x - 7$ $b =$ -7 $m =$ 1

13 $y = -4x$ $b =$ 0 $m =$ -4

14 $y = ½x + 2$ $m =$ ½ $b =$ 2

15 $y = 10 - 2x$ $b =$ 10 $m =$ -2

If lines are parallel they have the same gradient. Find the equations of these lines.

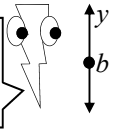
16 Find the equation of the line with a y-int of 6 which is parallel to $y = 3x + 1$. $y =$ 3 x + 6

17 Find the equation of the line with a y-int of 4 which is parallel to $y = -2x - 3$. $y = -2x + 4$

17 Find the equation of the line with a y-int of -3 which is parallel to $y = -3x$. $y = -3x - 3$

18 Find the equation of the line with a y-int of 0 which is parallel to $y = 5x + 2$. $y = 5x$

You are given the gradient, read off the y-int and then construct the equation for the line. 'b' is where the line 'cuts' the y-axis. (vertical axis)



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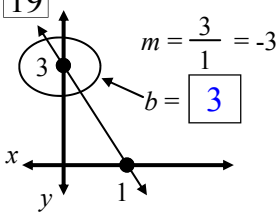
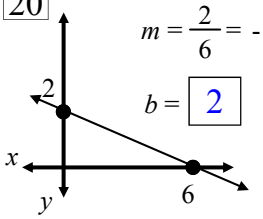


$m = \frac{3}{1} = -3$

$b =$ 3

$y =$ -3 x + 3

20



$m = \frac{2}{6} = -\frac{1}{3}$

$b =$ 2

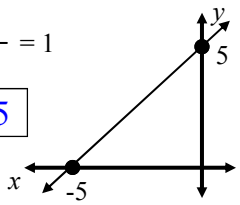
$y =$ $-\frac{1}{3}$ x + 2

21

$y =$ $y = x + 5$

$m = \frac{5}{5} = 1$

$b =$ 5

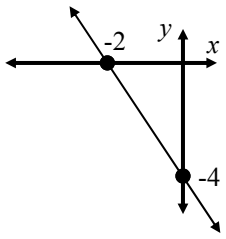


22

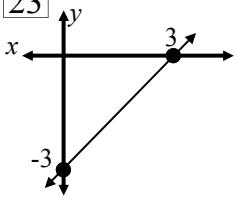
$y = -2x - 4$

$m = \frac{4}{2} = -2$

$b =$ -4



23



$m = \frac{3}{3} = 1$

$b =$ -3

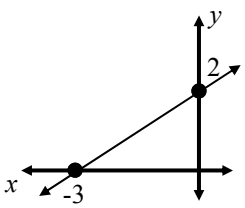
$y = x - 3$

24

$y = \frac{2}{3}x + 2$

$m = \frac{2}{3}$

$b =$ 2



Linear Relationships - Naming Straight Lines (Homework Strip)

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Date Due: _____

Write the equations of these lines, given the gradient and *y*-intercept

1. Gradient = 2, *y*-int = -2

   $y =$ ☐ x - ☐

2. Gradient = -3, *y*-int = 7

   $y =$ ☐ x + ☐

3. *y*-int = -4, Gradient = 1

4. *y*-int = 0, Gradient = -2

Give the gradient and *y*-intercept for these straight line equations

5. $y = 4x - 7$

   $m =$ ☐ $b =$ ☐

6. $y = -x + 3$

   $b =$ ☐ $m =$ ☐

7. $y = 5 - 4x$

   $m =$ ☐ $b =$ ☐

Date Due: _____

Write the equations of these lines, given the gradient and *y*-intercept

1. Gradient = 2, *y*-int = -2

   $y =$ ☐ x - ☐

2. Gradient = -3, *y*-int = 7

   $y =$ ☐ x + ☐

3. *y*-int = -4, Gradient = 1

4. *y*-int = 0, Gradient = -2

Give the gradient and *y*-intercept for these straight line equations

5. $y = 4x - 7$

   $m =$ ☐ $b =$ ☐

6. $y = -x + 3$

   $b =$ ☐ $m =$ ☐

7. $y = 5 - 4x$

   $m =$ ☐ $b =$ ☐

Date Due: _____

Write the equations of these lines, given the gradient and *y*-intercept

1. Gradient = 2, *y*-int = -2

   $y =$ ☐ x - ☐

2. Gradient = -3, *y*-int = 7

   $y =$ ☐ x + ☐

3. *y*-int = -4, Gradient = 1

4. *y*-int = 0, Gradient = -2

Give the gradient and *y*-intercept for these straight line equations

5. $y = 4x - 7$

   $m =$ ☐ $b =$ ☐

6. $y = -x + 3$

   $b =$ ☐ $m =$ ☐

7. $y = 5 - 4x$

   $m =$ ☐ $b =$ ☐

Date Due: _____

Write the equations of these lines, given the gradient and *y*-intercept

1. Gradient = 2, *y*-int = -2

   $y =$ ☐ x - ☐

2. Gradient = -3, *y*-int = 7

   $y =$ ☐ x + ☐

3. *y*-int = -4, Gradient = 1

4. *y*-int = 0, Gradient = -2

Give the gradient and *y*-intercept for these straight line equations

5. $y = 4x - 7$

   $m =$ ☐ $b =$ ☐

6. $y = -x + 3$

   $b =$ ☐ $m =$ ☐

7. $y = 5 - 4x$

   $m =$ ☐ $b =$ ☐

Linear Relationships - Naming Straight Lines (Homework Strip)

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Date Due:

Write the equations of these lines, given the gradient and *y*-intercept

1 Gradient = 2, *y*-int = -2

$y =$ 2 x - 2

2 Gradient = -3, *y*-int = 7

$y =$ -3 x + 7

3 *y*-int = -4, Gradient = 1

$y = x - 4$

4 *y*-int = 0, Gradient = -2

$y = -2x$

Give the gradient and *y*-intercept for these straight line equations

5 $y = 4x - 7$

$m =$ 4 $b =$ -7

6 $y = -x + 3$

$b =$ 3 $m =$ -1

7 $y = 5 - 4x$

$m =$ -4 $b =$ 5

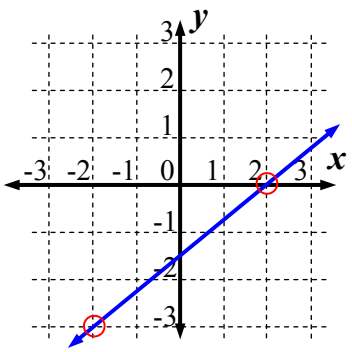
Cartesian Plane - Gradient

Gradient is a measure of slope. So the larger the number, the steeper the line is. A gradient of 5 is much steeper than a gradient of 1. Gradient is found creating a 'gradient triangle' and measuring the rise over the run, the vertical length over the horizontal. Gradient is given the symbol '*m*', it has nothing to do with the midpoint.

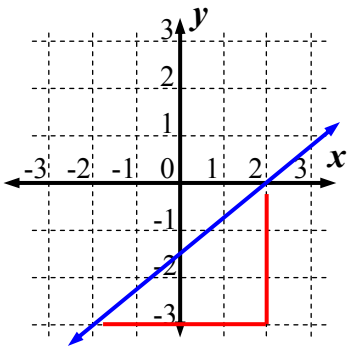
$$m = \frac{Rise}{Run} = \frac{Vertical\ Distance}{Horizontal\ Distance}$$

Here is an example of finding the gradient.

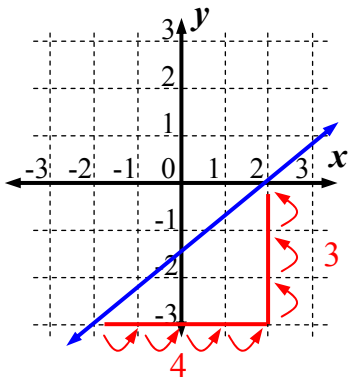
Example: Find the gradient of the line below.



Locate where the line cuts through exact coordinates. (in red)



Construct a gradient triangle, which is a right angle triangle with the line as the hypotenuse



Count and mark the rise and the run on the sides of the triangle.

$$m = \frac{3}{4}$$

Find the gradient by writing the rise over the run. Then finally, check if it is a positive or negative slope.

The process for finding the gradient is straight forward. The only issue is you can have positive and negative gradient. Positive Gradient 'Goes up to the right', Negative Gradient' goes 'up to the left'.

Positive Gradient goes up to the right

Negative Gradient goes up to the left

If the line is flat, it has a gradient of zero ($m = 0$). If the line is vertical (straight up and down), it has a gradient that is undefined (m = undef).

Cartesian Plane - Gradient (Sheet 1)

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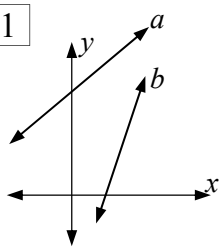
State whether the following have a positive or negative gradient.

Up to the left is -ve

Up to the right is +ve

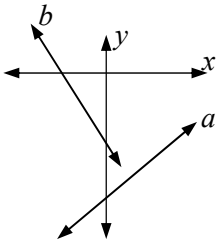
1



a) _____ Positive or Negative

b) _____ Positive or Negative

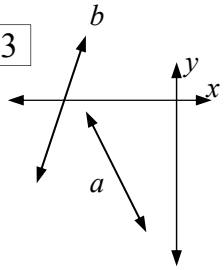
2



a) _____

b) _____

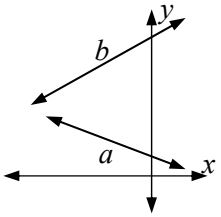
3



a) _____

b) _____

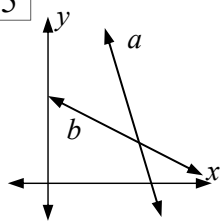
4



a) _____

b) _____

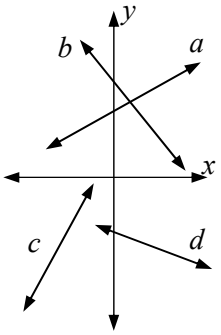
5



a) _____

b) _____

6



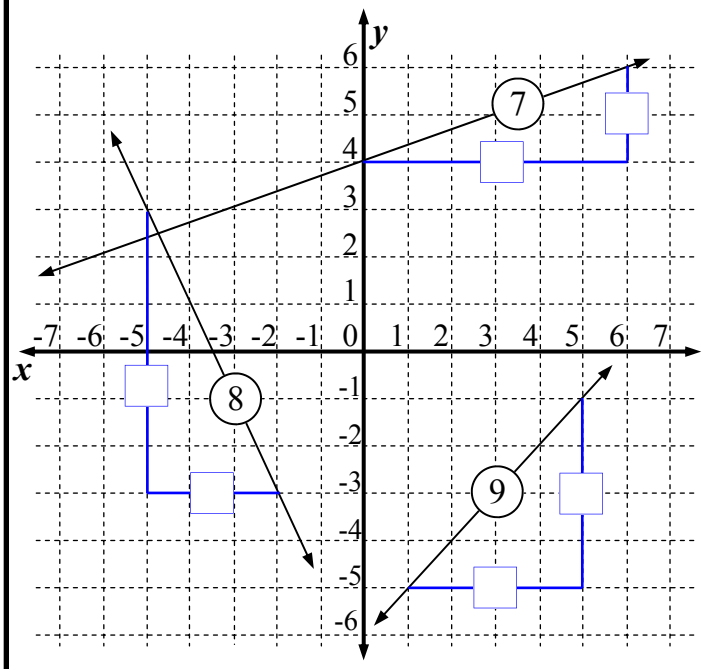
a) _____

b) _____

c) _____

d) _____

These lines have gradient triangles installed! Write the rise and run in the boxes on the triangles sides, then find the gradient. Make sure you remember to check for positive and negative gradient at the end.



7 $m = \frac{\square}{\square} = \frac{\square}{\square}$

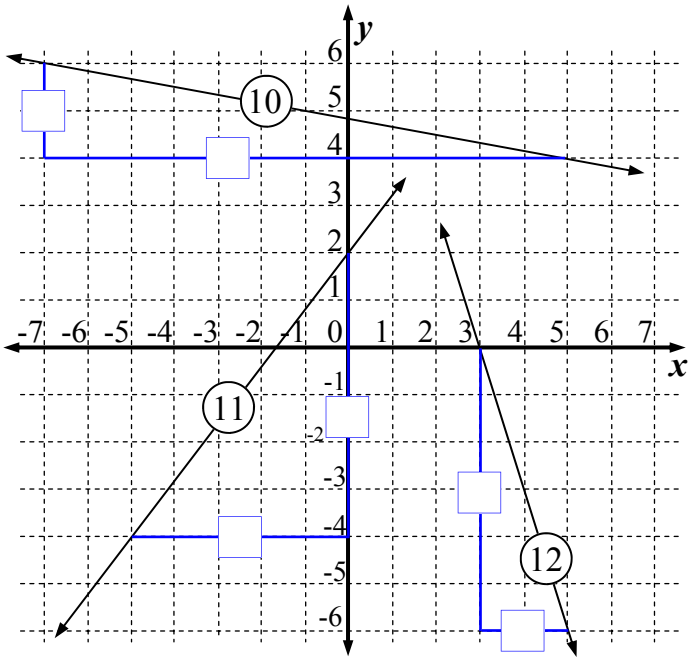
8 $m = \frac{\square}{\square} = \square$

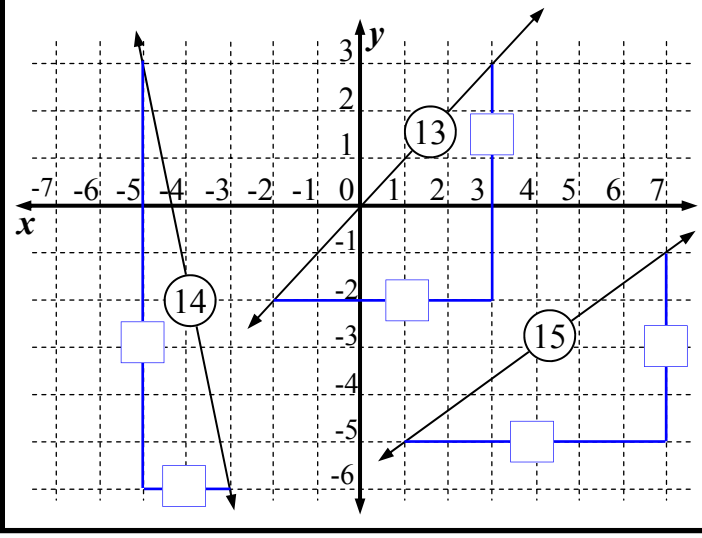
9 $m = \frac{\square}{\square} = \square$

10 $m = \frac{\square}{\square} = \frac{\square}{\square}$

11 $m = \frac{\square}{\square}$

12 $m = \frac{\square}{\square} = \square$





13 $m = \frac{\square}{\square} = \square$

14 $m = \frac{\square}{\square}$

15 $m = \frac{\square}{\square} = \frac{\square}{\square}$

Cartesian Plane - Gradient (Sheet 2)

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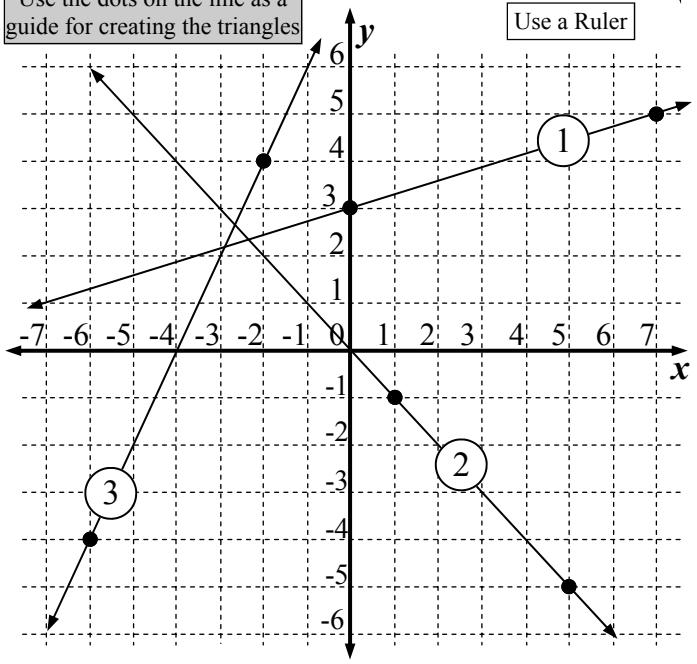
Find the gradient for these lines. Create gradient triangles and remember your positive and negatives.

Use the dots on the line as a guide for creating the triangles

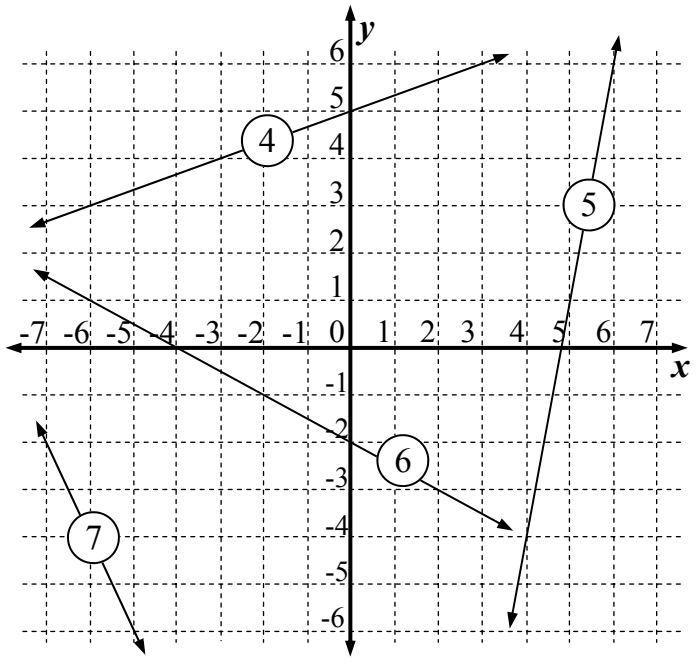
Use a Ruler



1 $m = \frac{\square}{\square}$

2 $m = \frac{\square}{\square} = \square$

3 $m = \frac{\square}{\square} = \square$

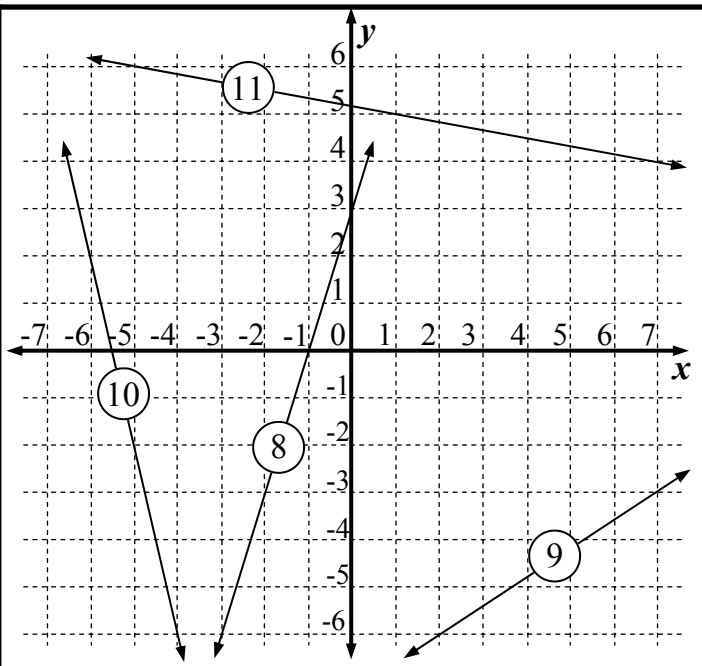


4 $m = \frac{\square}{\square} = \frac{\square}{\square}$

5 $m = \frac{\square}{\square} = \square$

6 $m = \frac{\square}{\square} = \frac{\square}{\square}$

7 $m = \frac{\square}{\square} = \square$

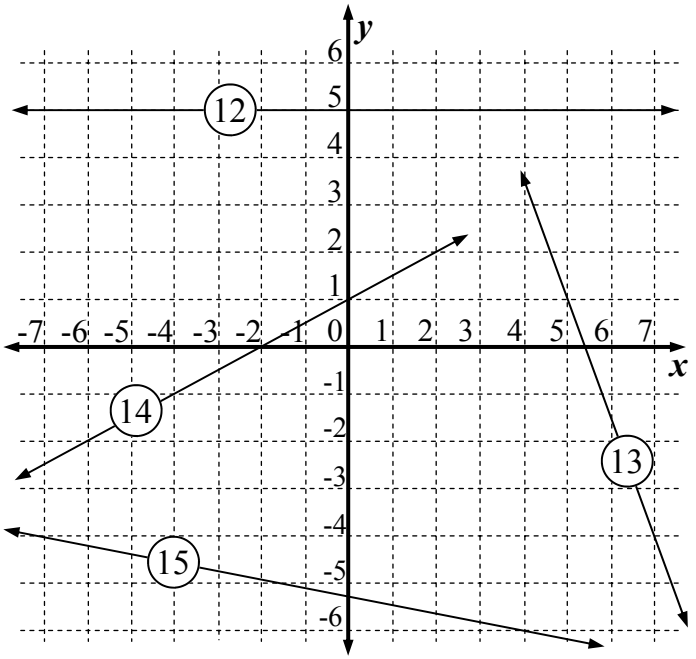


8 $m =$ _____

9 $m =$ _____

10 _____

11 _____



12 _____

13 _____

14 _____

15 _____

Cartesian Plane - Gradient (Sheet 1)

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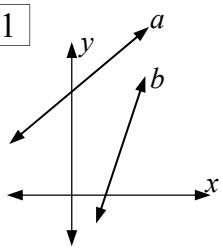
State whether the following have a positive or negative gradient.

Up to the left is -ve

Up to the right is +ve

1



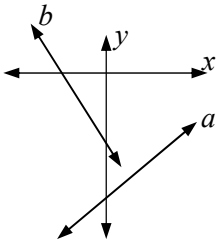
a) Positive

Positive or Negative

b) Positive

Positive or Negative

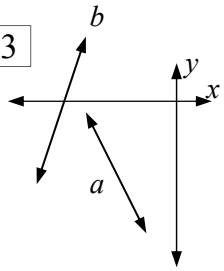
2



a) Positive

b) Negative

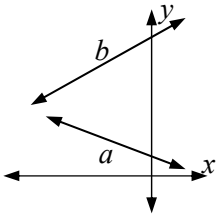
3



a) Negative

b) Positive

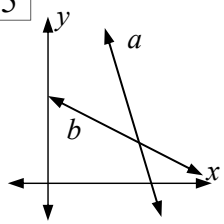
4



a) Negative

b) Positive

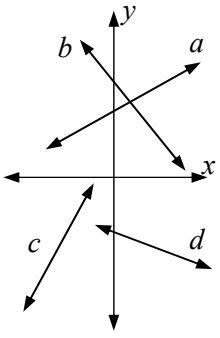
5



a) Negative

b) Negative

6



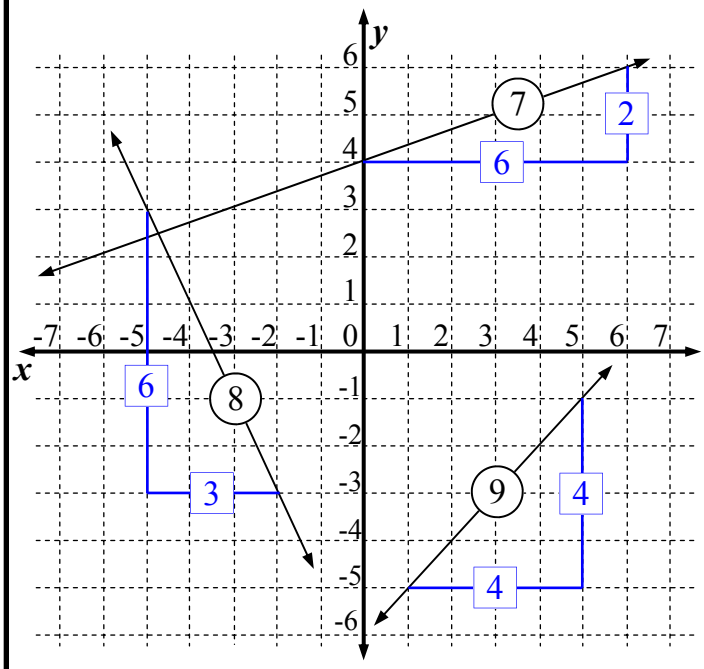
a) Positive

b) Negative

c) Positive

d) Negative

These lines have gradient triangles installed! Write the rise and run in the boxes on the triangles sides, then find the gradient. Make sure you remember to check for positive and negative gradient at the end.



7 $m = \frac{2}{6} = \frac{1}{3}$

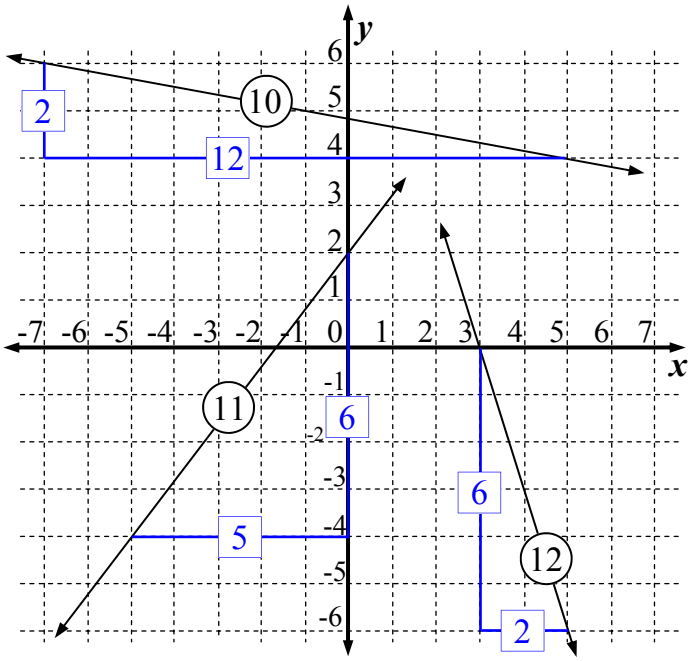
8 $m = \frac{6}{3} = -2$

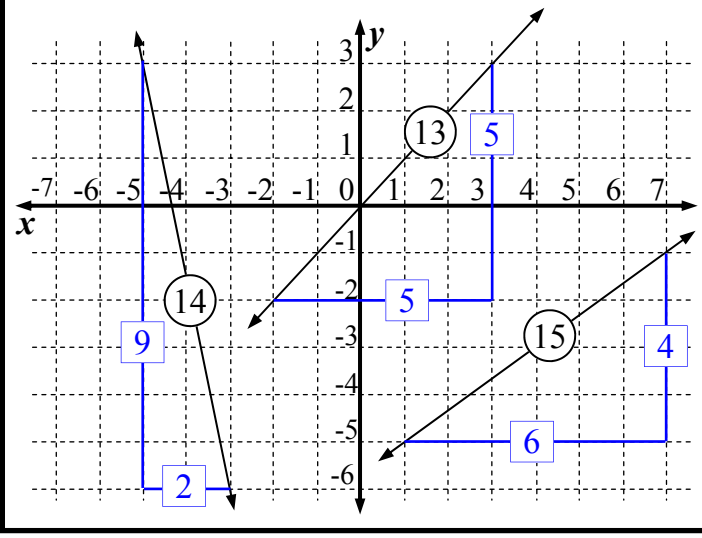
9 $m = \frac{4}{4} = 1$

10 $m = \frac{2}{12} = \frac{-1}{6}$

11 $m = \frac{6}{5}$

12 $m = \frac{6}{2} = -3$





13 $m = \frac{5}{5} = 1$

14 $m = \frac{-9}{2}$

15 $m = \frac{4}{6} = \frac{2}{3}$

Cartesian Plane - Gradient (Sheet 2)

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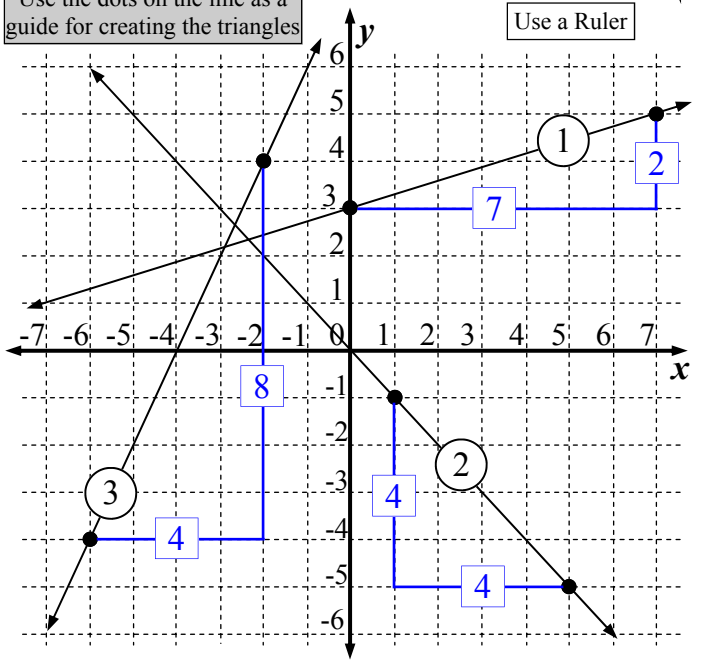
Find the gradient for these lines. Create gradient triangles and remember your positive and negatives.

Use the dots on the line as a guide for creating the triangles

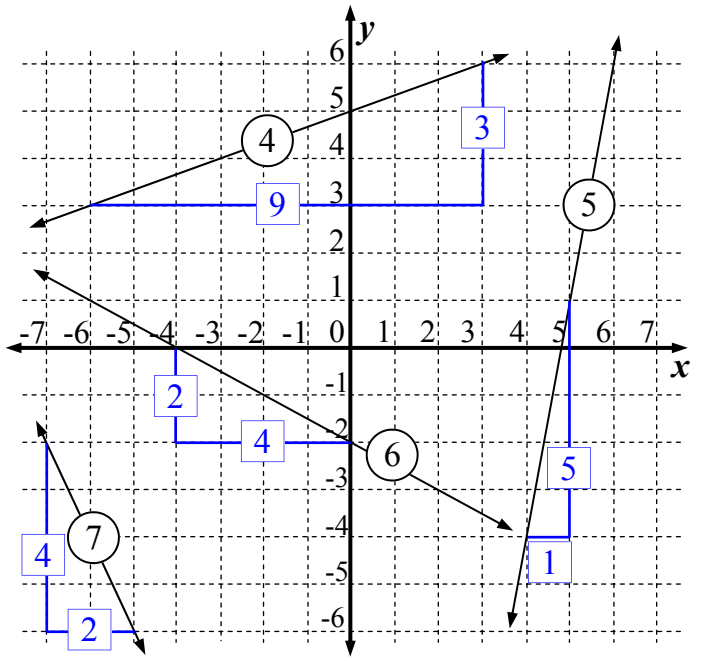
Use a Ruler



1 $m = \frac{2}{7}$

2 $m = \frac{4}{4} = -1$

3 $m = \frac{8}{4} = 2$

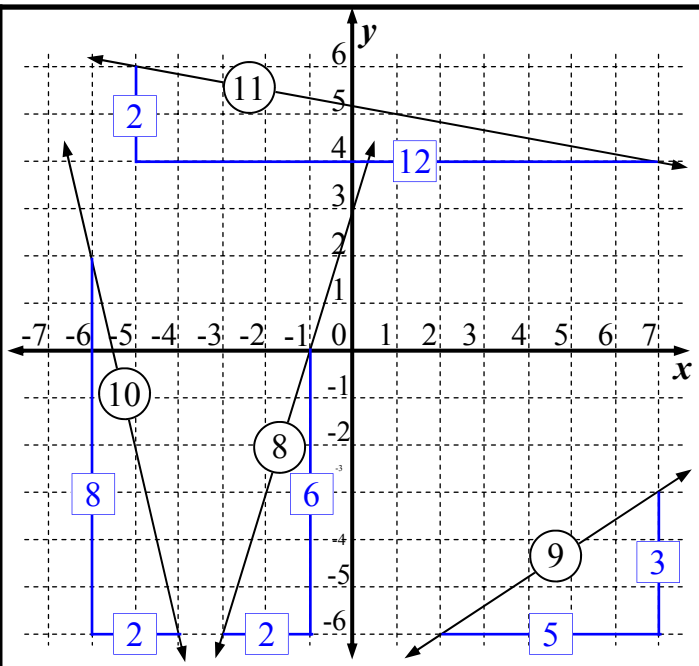


4 $m = \frac{3}{9} = \frac{1}{3}$

5 $m = \frac{5}{1} = 5$

6 $m = \frac{2}{4} = \frac{-1}{2}$

7 $m = \frac{4}{2} = -2$

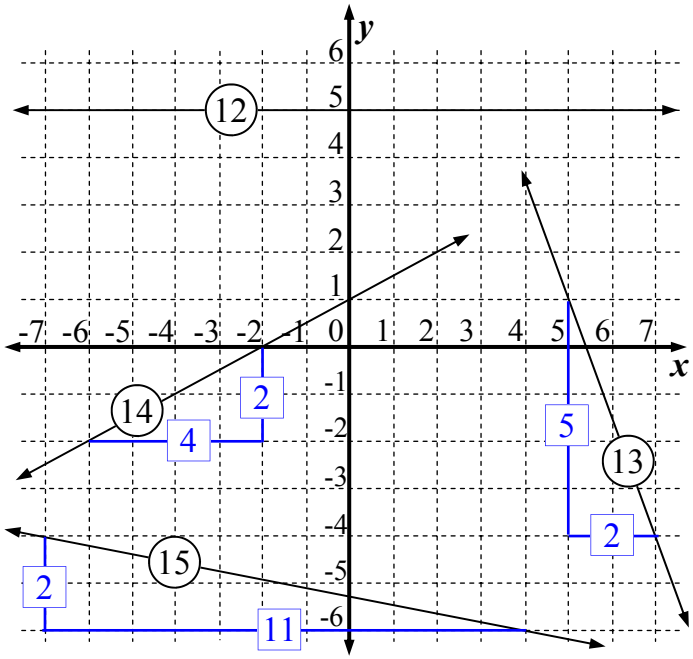


8 $m = \frac{6}{2} = 3$

9 $m = \frac{3}{5}$

10 $m = \frac{8}{2} = -4$

11 $m = \frac{2}{12} = \frac{-1}{6}$



12 $m = 0$

13 $m = \frac{-5}{2}$

14 $m = \frac{2}{4} = \frac{1}{2}$

15 $m = \frac{-2}{11}$

Cartesian Plane - Gradient (Homework Strip)

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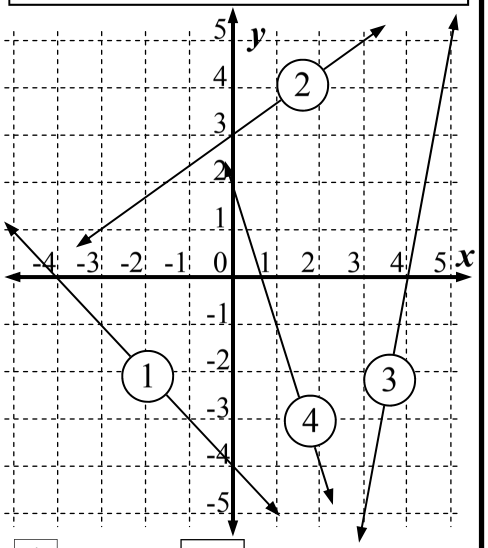
Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{\square}{\square} = \square$ Positive or negative?

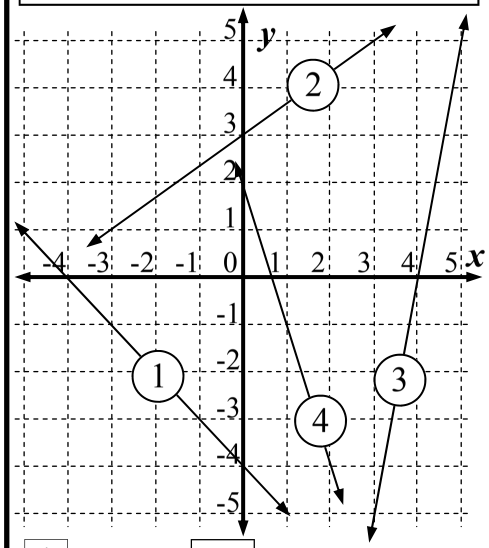
2 $m = \frac{\square}{\square} = \frac{\square}{\square}$ You might not need these boxes

3 $m = \frac{\square}{\square} = \square$

4 $m = \frac{\square}{\square} = \square$

Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{\square}{\square} = \square$ Positive or negative?

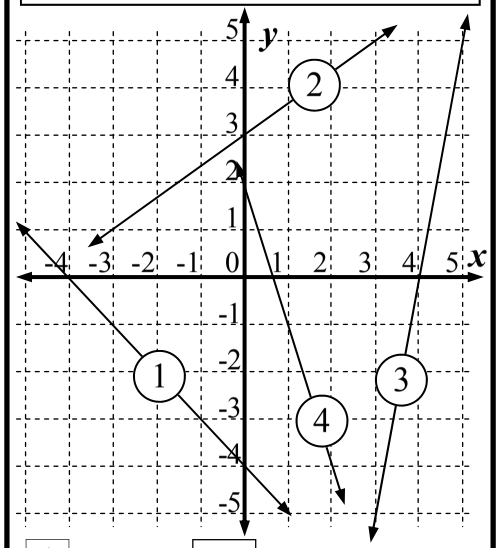
2 $m = \frac{\square}{\square} = \frac{\square}{\square}$ You might not need these boxes

3 $m = \frac{\square}{\square} = \square$

4 $m = \frac{\square}{\square} = \square$

Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{\square}{\square} = \square$ Positive or negative?

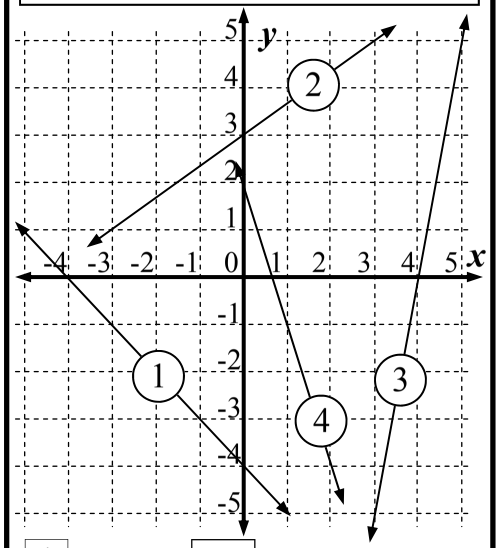
2 $m = \frac{\square}{\square} = \frac{\square}{\square}$ You might not need these boxes

3 $m = \frac{\square}{\square} = \square$

4 $m = \frac{\square}{\square} = \square$

Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{\square}{\square} = \square$ Positive or negative?

2 $m = \frac{\square}{\square} = \frac{\square}{\square}$ You might not need these boxes

3 $m = \frac{\square}{\square} = \square$

4 $m = \frac{\square}{\square} = \square$

Cartesian Plane - Gradient (Homework Strip)

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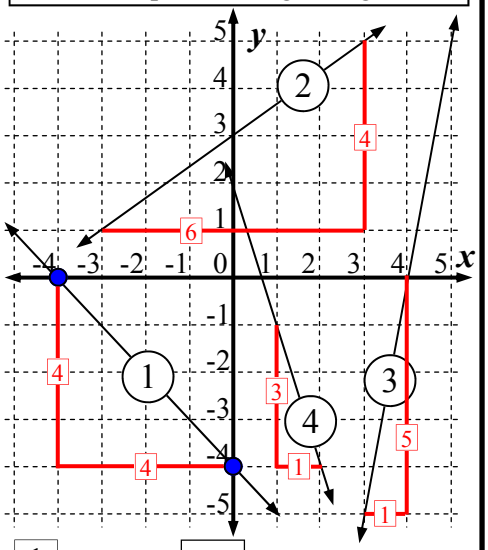
Date Due:

Find the gradient for these four lines.
Remember positive/negative gradient.



1 $m = \frac{4}{4} = -1$ Positive or negative?

2 $m = \frac{4}{6} = \frac{2}{3}$ You might not need these boxes

3 $m = \frac{5}{1} = 5$

4 $m = \frac{3}{1} = -3$

Linear Relationships - Naming Straight Lines - Find *m*

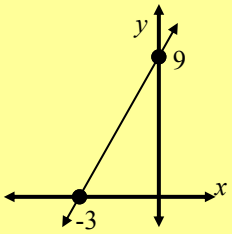
This sheet deals with naming straight lines. The standard equation for a straight line is:

$$y = mx + b$$

Where: m = the gradient or slope of the line, and
b = the y-intercept, which is the point at which the line cuts or intersects the y-axis. (the y-axis is the vertical axis)

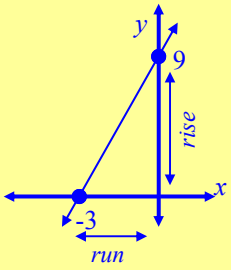
This is a difficult concept for some students, but it is important to realise that there are three steps, with only one of those steps being tricky, the other two are easy. As long as you can find the gradient, finding the y-intercept and getting the line equation is straight-forward.

Example - Find the equation of the straight line shown.



STEP 1

FIND THE GRADIENT



FIND THE GRADIENT

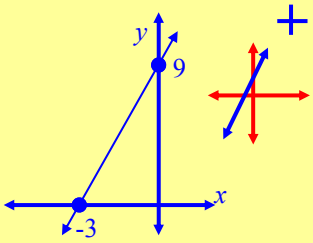
$$m = \frac{\text{rise}}{\text{run}} = \frac{9}{3} = 3$$

Don't use -2

Gradient is rise over the run. So the vertical distance over the horizontal. Remember only use positive numbers, so change -3 to 3.

STEP 2

POSITIVE OR NEGATIVE

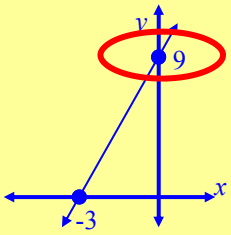


POSITIVE OR NEGATIVE

If the line is going up to the right it is positive. (Which this question is). If it is going up to the left, it is negative. It is vital you ask yourself this question each time you find the gradient. Put a '-' sign in front of the gradient if it is negative. Otherwise leave it.

STEP 3

FIND THE y-INTERCEPT



FIND THE y-INTERCEPT

Where does the line cut the y-axis? It cuts at +9

STEP 4

CONSTRUCT THE EQUATION

$y = 3x + 9$
m b

CONSTRUCT THE EQUATION

Using $y = mx + b$, replace the m with the gradient and b with the y-intercept. You are done.

Linear Relationships - Naming Straight Lines - Find *m* (Sheet 1)

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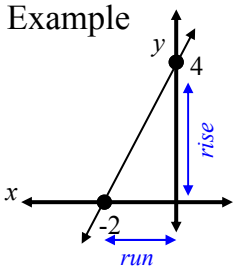
Find the gradient and *y*-intercept, then the equation for these straight lines. All gradients are positive.

Example

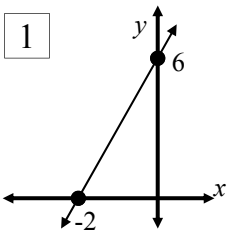


$m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$ — Positive as up to the right; Don't use -2

$b = 4$ — Cuts the *y*-axis at 4

$y = 2x + 4$ (m, b)

1

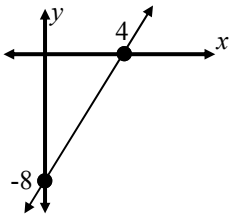


$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square x + \square$

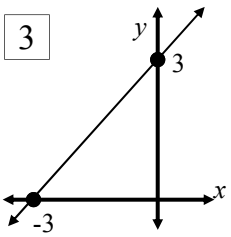
2

$m = \frac{\square}{\square} = \square$ $\quad b = \square$



$y = \square x - \square$

3

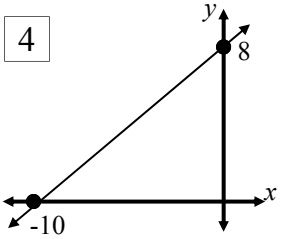


$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square$

These gradients will be fractions, fully simplify them.

4



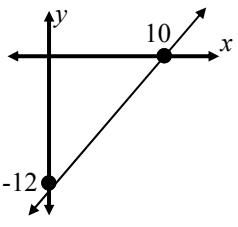
$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$

Calc: 8 ▪ 1 0 =

$y = \frac{\square}{\square} x + \square$

5

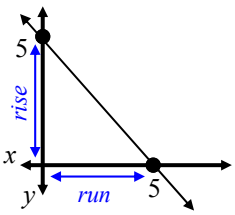
$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$



$\square$

All the gradients are up to the left, so all are negative gradients. Find the equations for these lines.

Example



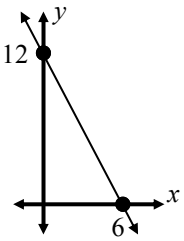
$m = \frac{\text{rise}}{\text{run}} = \frac{5}{5} = -1$ — Negative as up to the left

$b = 5$ — Cuts the *y*-axis at 5

$y = -x + 5$ — Write -*x* not -1*x* (m, b)

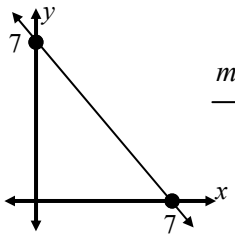
6

$m = \frac{\square}{\square} = \square$ $\quad b = \square$



$y = \square$

7

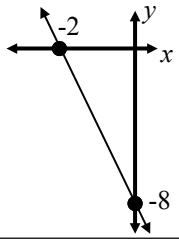


$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$\square$

8

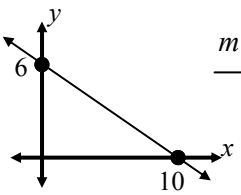
$m = \frac{\square}{\square} = \square$ $\quad b = \square$



$\square$

These gradients will be fractions, fully simplify them.

9

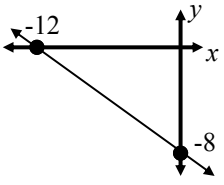


$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$

$y = \frac{\square}{\square} x + \square$

10

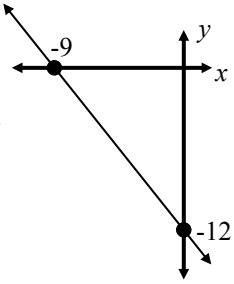
$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$



$\square$

11

$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$



$\square$

Linear Relationships - Naming Straight Lines - Find *m* (Sheet 2)

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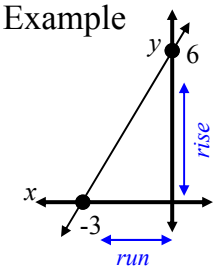
Find the gradient and *y*-intercept, then the equation for these straight lines. Watch for gradient sign.

Example

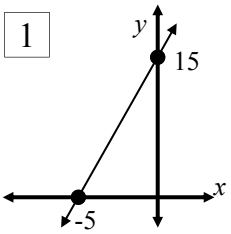


$m = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2$ — Positive as up to the right; Don't use -3

$b = 6$ — Cuts the *y*-axis at 6

$y = 2x + 6$ (m, b)

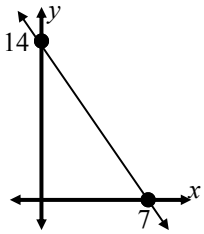
1



$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square x + \square$

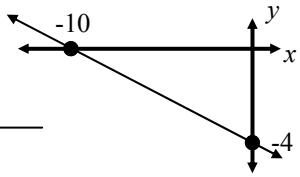
2



$m =$ _____ $\quad b =$ _____

$y =$ _____

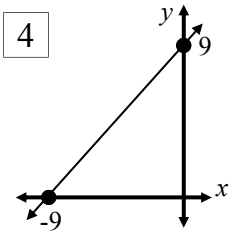
3 Fraction alert!



$m =$ _____ $\quad b =$ _____

$y =$ _____

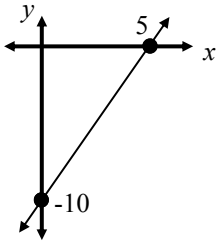
4



_____ _____

_____

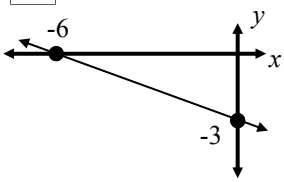
5



_____ _____

_____

6



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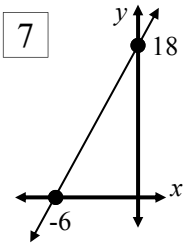
7



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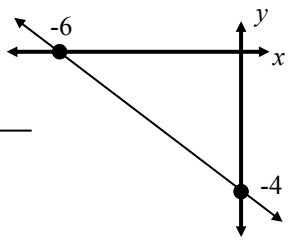
8



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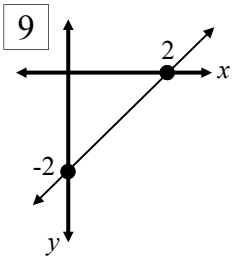
9



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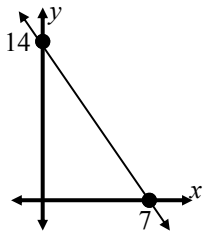
10



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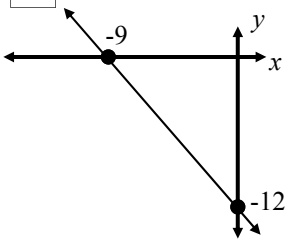
11



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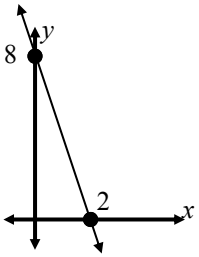
12



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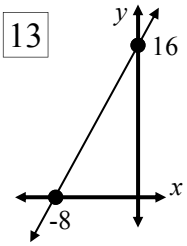
13



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Linear Relationships - Naming Straight Lines - Find *m* (Sheet 1)

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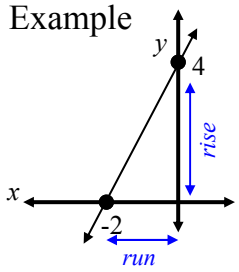
Find the gradient and *y*-intercept, then the equation for these straight lines. All gradients are positive.

Example

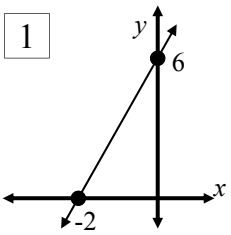


$m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$ — Positive as up to the right; Don't use -2

$b = 4$ — Cuts the *y*-axis at 4

$y = 2x + 4$ (m, b)

1

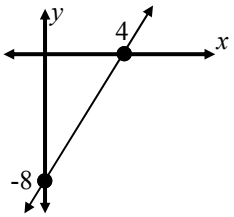


$m = \frac{6}{2} = 3$ $\quad b = 6$

$y = 3x + 6$

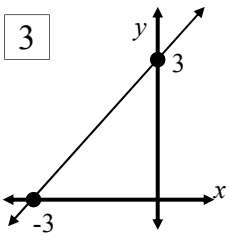
2

$m = \frac{8}{4} = 2$ $\quad b = -8$



$y = 2x - 8$

3

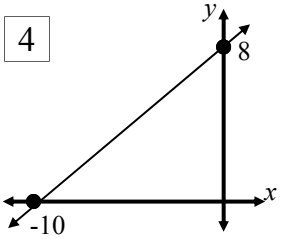


$m = \frac{3}{3} = 1$ $\quad b = 3$

$y = x + 3$

These gradients will be fractions, fully simplify them.

4



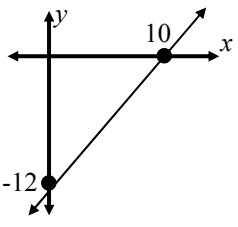
$m = \frac{8}{10} = \frac{4}{5}$ $\quad b = 8$

Calc: 8 ▪ 1 0 =

$y = \frac{4}{5}x + 8$

5

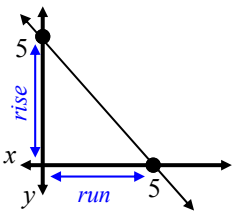
$m = \frac{12}{10} = \frac{6}{5}$ $\quad b = -12$



$y = {}^{6}/_{5}\,x - 12$

All the gradients are up to the left, so all are negative gradients. Find the equations for these lines.

Example



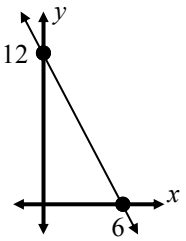
$m = \frac{\text{rise}}{\text{run}} = \frac{5}{5} = -1$ — Negative as up to the left

$b = 5$ — Cuts the *y*-axis at 5

$y = -x + 5$ — Write -*x* not -1*x* (m, b)

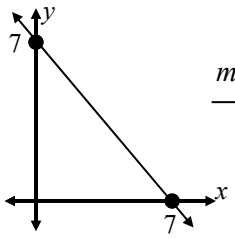
6

$m = \frac{12}{6} = -2$ $\quad b = 12$



$y = -2x + 12$

7

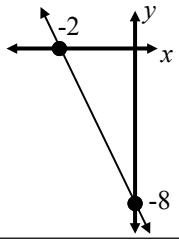


$m = \frac{7}{7} = -1$ $\quad b = 7$

$y = -x + 7$

8

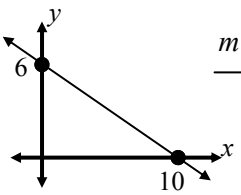
$m = \frac{8}{2} = -4$ $\quad b = -8$



$y = -4x - 8$

These gradients will be fractions, fully simplify them.

9

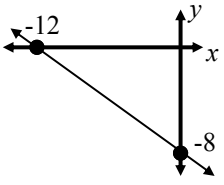


$m = \frac{6}{10} = \frac{-3}{5}$ $\quad b = 6$

$y = \frac{-3}{5}x + 6$

10

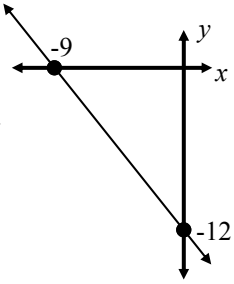
$m = \frac{8}{12} = \frac{-2}{3}$ $\quad b = -8$



$y = {}^{-2}/_{3}\,x - 8$

11

$m = \frac{12}{9} = \frac{-4}{3}$ $\quad b = -12$



$y = {}^{-4}/_{3}\,x - 12$

Linear Relationships - Naming Straight Lines - Find *m* (Sheet 2)

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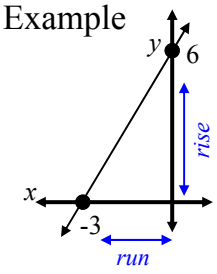
Find the gradient and *y*-intercept, then the equation for these straight lines. Watch for gradient sign.

Example

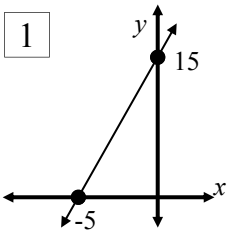


$m = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2$ — Positive as up to the right; Don't use -3

$b = 6$ — Cuts the *y*-axis at 6

$y = 2x + 6$ (*m*, *b*)

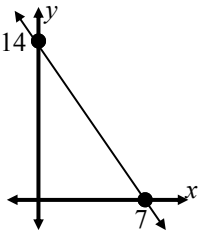
1



$m = \frac{15}{5} = 3$ $\quad b = 15$

$y = 3x + 15$

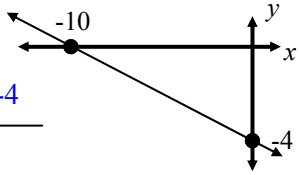
2



$m = \frac{14}{7} = -2$ $\quad b = 14$

$y = -2x + 14$

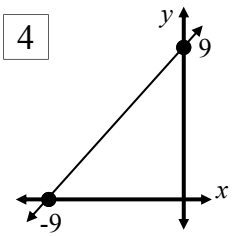
3 Fraction alert!



$m = \frac{4}{10} = \frac{-2}{5}$ $\quad b = -4$

$y = \frac{-2}{5}x - 4$

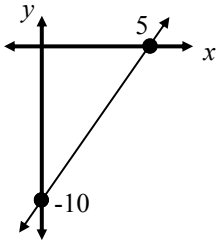
4



$m = \frac{9}{9} = 1$ $\quad b = 9$

$y = x + 9$

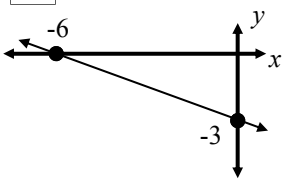
5



$m = \frac{10}{5} = 2$ $\quad b = -10$

$y = 2x - 10$

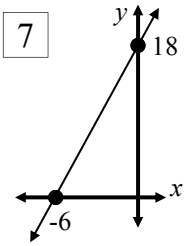
6



$m = \frac{3}{6} = \frac{-1}{2}$ $\quad b = -3$

$y = \frac{-1}{2}x - 3$

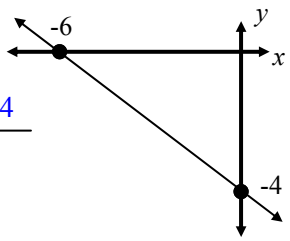
7



$m = \frac{18}{6} = 3$ $\quad b = 18$

$y = 3x + 18$

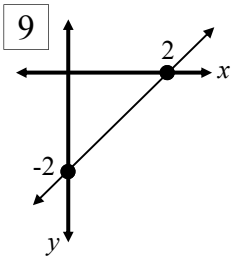
8



$m = \frac{4}{6} = \frac{-2}{3}$ $\quad b = -4$

$y = \frac{-2}{3}x - 4$

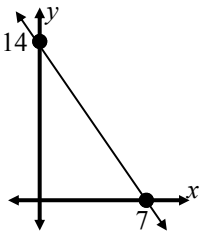
9



$m = \frac{2}{2} = 1$ $\quad b = -2$

$y = x - 2$

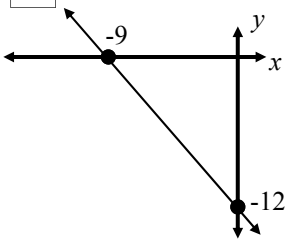
10



$m = \frac{14}{7} = -2$ $\quad b = 14$

$y = -2x + 14$

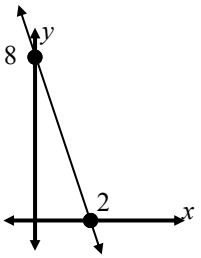
11



$m = \frac{12}{9} = \frac{-4}{3}$ $\quad b = -12$

$y = \frac{-4}{3}x - 12$

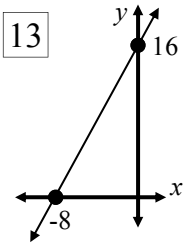
12



$m = \frac{8}{2} = -4$ $\quad b = 8$

$y = -4x + 8$

13



$m = \frac{16}{8} = 2$ $\quad b = 16$

$y = 2x + 16$

Linear Relationships - Naming Straight Lines - Find *m* (Homework Strip)

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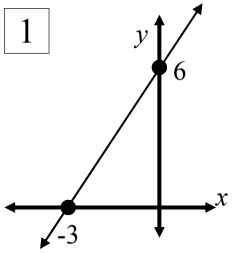
Date Due:

Write the equations for these lines

1

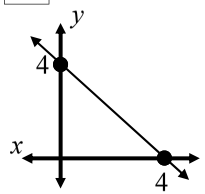


$m = \frac{\square}{\square} = \square$

$b = \square$

$y = \square x + \square$

2

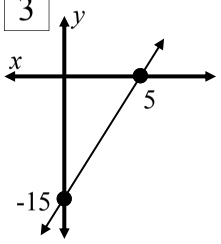


$m = \frac{\square}{\square} = \square$

$b = \square$

$y = \square$

3



$m = \frac{\square}{\square} = \square$

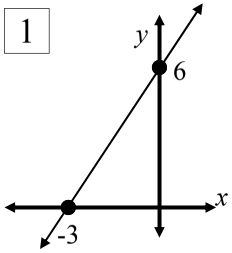
$b = \square$

$\square$

Date Due:

Write the equations for these lines

1

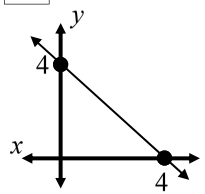


$m = \frac{\square}{\square} = \square$

$b = \square$

$y = \square x + \square$

2

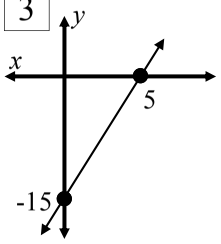


$m = \frac{\square}{\square} = \square$

$b = \square$

$y = \square$

3



$m = \frac{\square}{\square} = \square$

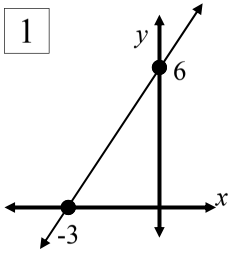
$b = \square$

$\square$

Date Due:

Write the equations for these lines

1

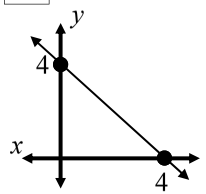


$m = \frac{\square}{\square} = \square$

$b = \square$

$y = \square x + \square$

2

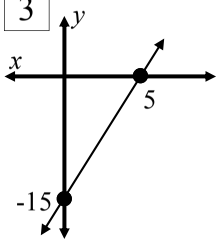


$m = \frac{\square}{\square} = \square$

$b = \square$

$y = \square$

3



$m = \frac{\square}{\square} = \square$

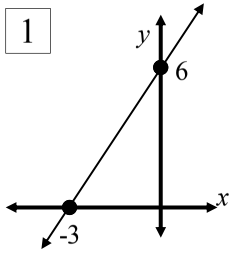
$b = \square$

$\square$

Date Due:

Write the equations for these lines

1

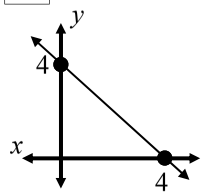


$m = \frac{\square}{\square} = \square$

$b = \square$

$y = \square x + \square$

2

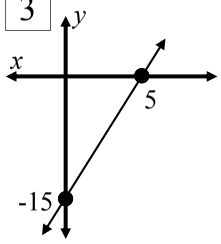


$m = \frac{\square}{\square} = \square$

$b = \square$

$y = \square$

3



$m = \frac{\square}{\square} = \square$

$b = \square$

$\square$

Linear Relationships - Naming Straight Lines - Find *m* (Homework Strip)

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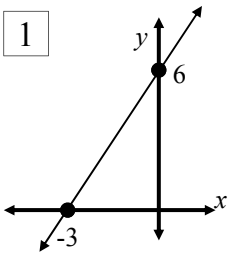
Date Due:

Write the equations for these lines

1

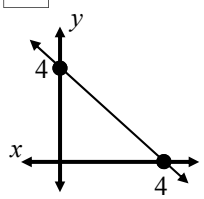


$m = \frac{6}{3} = 2$

$b = 6$

$y = 2x + 6$

2

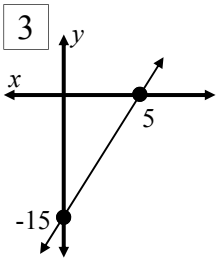


$m = \frac{4}{4} = -1$

$b = 4$

$y = -x + 4$

3



$m = \frac{15}{5} = 3$

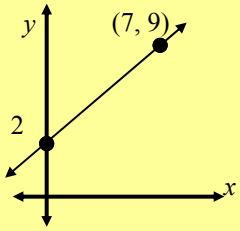
$b = -15$

$y = 3x - 15$

Linear Relationships - Further Naming Straight Lines

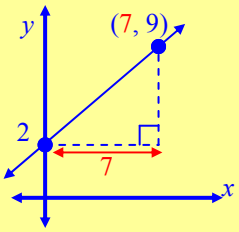
This sheet deals with naming straight lines. This is a more difficult concept for some students, because one point is not on an axis. You are always given the *y*-intercept, but the other point doesn't lie on the *x*-axis.

Example - Find the equation of the straight line shown.



STEP 1

CONSTRUCT GRADIENT TRIANGLE & FIND RUN

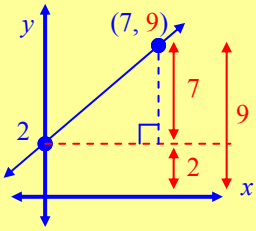


CONSTRUCT TRIANGLE - FIND RUN

Construct a triangle. Draw a horizontal line from the *y*-axis across from the *y*-intercept, then up to the point. Then write the run on the base of the triangle. The horizontal length is 7, because the point is (7, 9), which is 7 across, 9 up.

STEP 2

FIND RISE

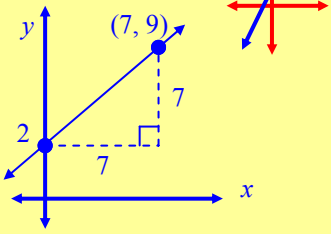


FIND RISE

The rise is found by using the 2nd number in the point, because that is the distance 'up'. If the point is 'up 9', but the triangle is raised 2 above the axis, then the triangle height is 7. (9 - 2 = 7)

STEP 3

CALCULATE THE GRADIENT



CALCULATE THE GRADIENT

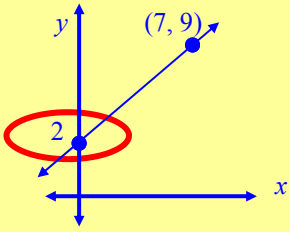
$$m = \frac{\text{rise}}{\text{run}} = \frac{7}{7} = 1$$

Calculate the gradient using the rise and run you found.

Remember to check if it is positive or negative.

STEP 4

FIND THE *y*-INTERCEPT



FIND THE *y*-INTERCEPT

Where does the line cut the *y*-axis? It cuts at +2

STEP 5

CONSTRUCT THE EQUATION

$y = x + 2$

m b

CONSTRUCT THE EQUATION

Using $y = mx + b$, replace the m with the gradient and b with the *y*-intercept. You are done.

Linear Relationships - Further Naming Straight Lines (Sheet 1)

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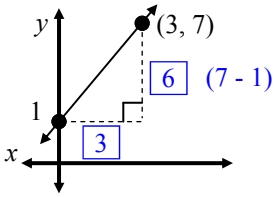
The gradient triangle is constructed. Write in the rise and run. All values are positive, no negatives!

Example



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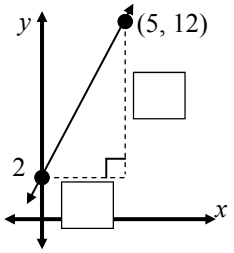

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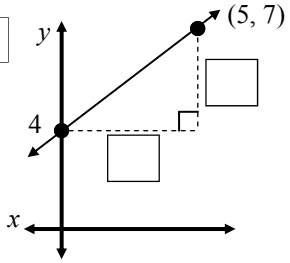

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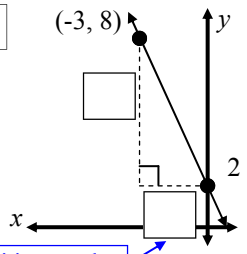
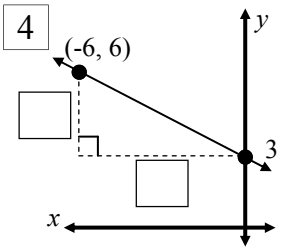


positive number

4



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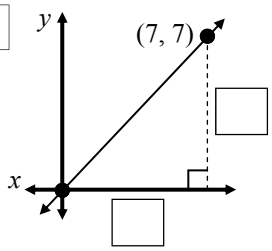
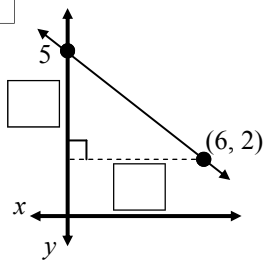


This time the rise is written on the *y*-axis

6



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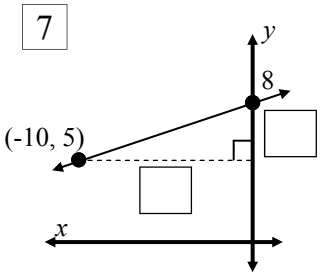

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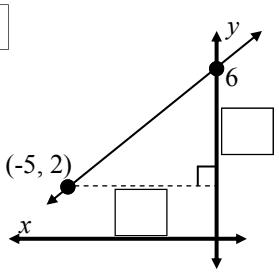

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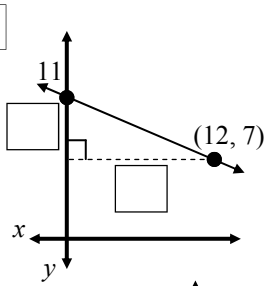

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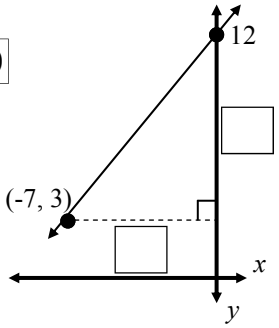

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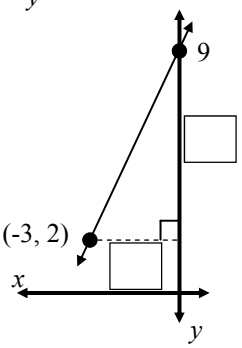
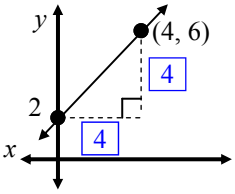


Find the rise, run, gradient, *y*-intercept and equation.

Example

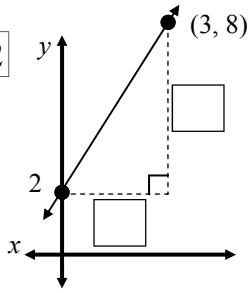
Gradient: ⊕ ⊖



$m = \frac{4}{4} = 1$ $\quad b = 2$

$y = x + 2$

12



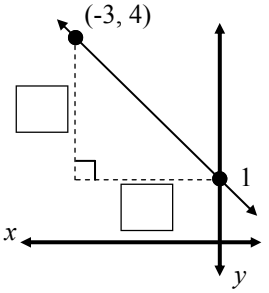
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square x + \square$

13

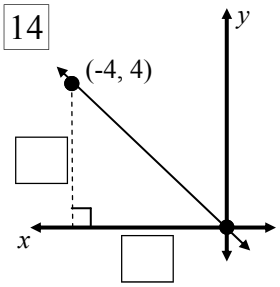
Gradient: ⊕ ⊖



$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square x + \square$

14



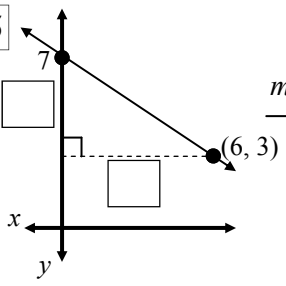
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square$

This time the gradient is a fraction

15



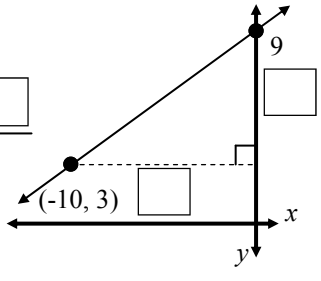
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$

$y = \frac{\square}{\square} x + \square$

16

Gradient: ⊕ ⊖



$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$

$y = \frac{\square}{\square} x + \square$

Linear Relationships - Further Naming Straight Lines (Sheet 2)

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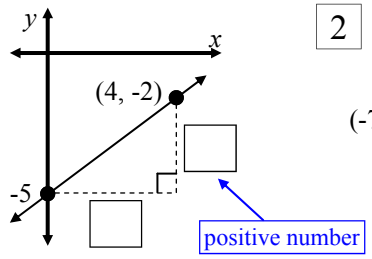
We are below the x-axis this time. Find the rise and run. Keep all your numbers positive.

1



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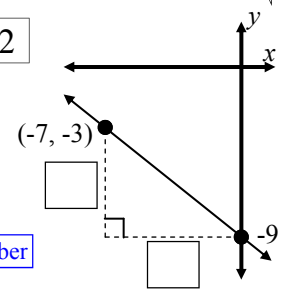

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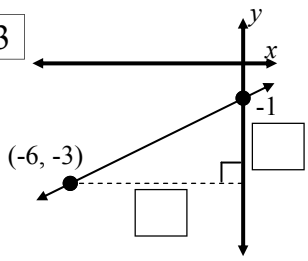

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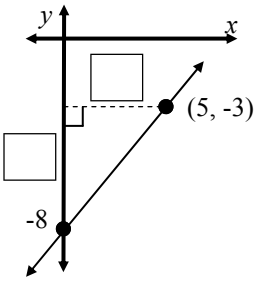

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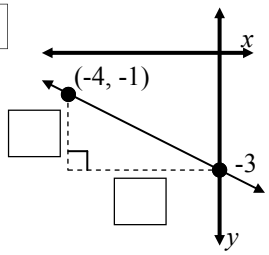

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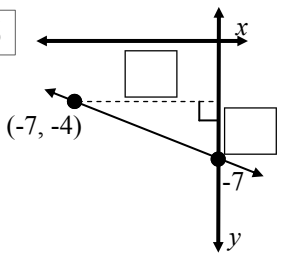
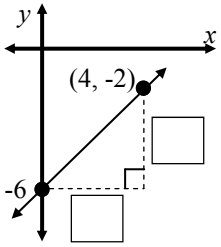


Find the rise, run, gradient, y-intercept and equation.

7



Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

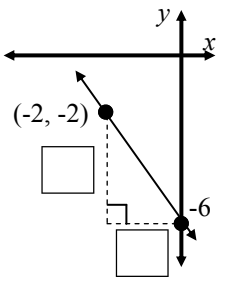
$y = x -$ ☐

8

Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

$y = \square x -$ ☐



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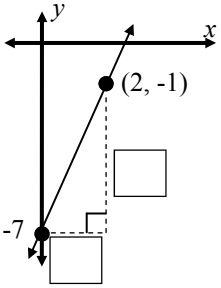


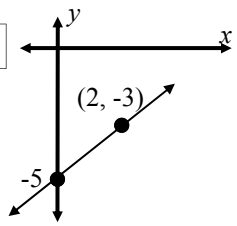
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

$y = \square x -$ ☐

Find the rise, run, gradient, y-intercept and equation. This time you have to construct the triangle yourself.

10



Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

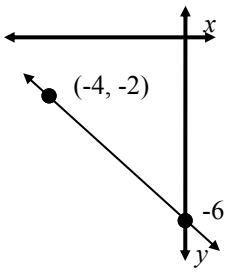
$y =$ ☐

11

Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

$y =$ ☐



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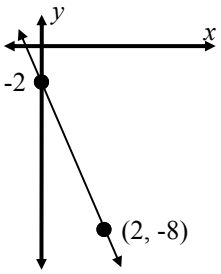


Gradient: ⊕ ⊖

$m =$ _____ $b =$ _____

$y =$ _____

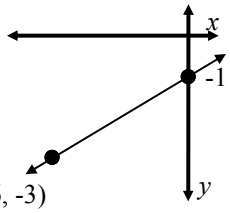
This time the gradient is a fraction

13

Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \frac{\square}{\square}$ $b = \square$

$y = \frac{\square}{\square} x -$ ☐



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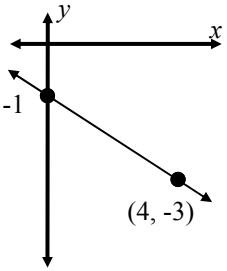


Gradient: ⊕ ⊖

_____ _____

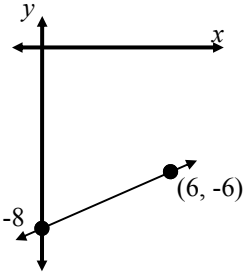
_____

15

Gradient: ⊕ ⊖

_____ _____

_____



Linear Relationships - Further Naming Straight Lines (Sheet 1)

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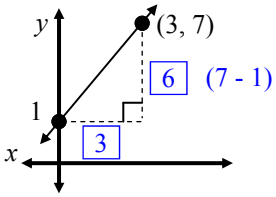
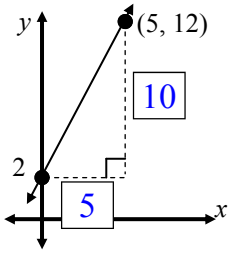
The gradient triangle is constructed. Write in the rise and run. All values are positive, no negatives!

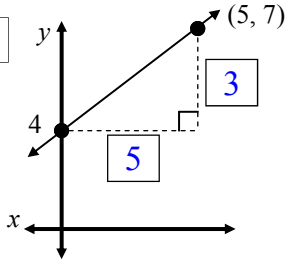
Example



1



2



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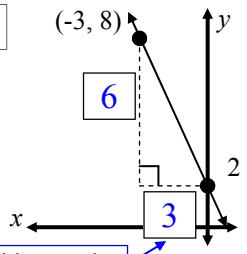
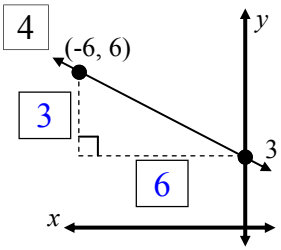


positive number

4



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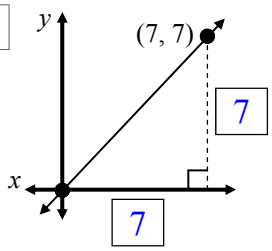
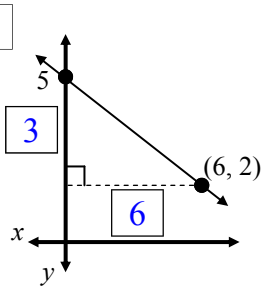


This time the rise is written on the *y*-axis

6



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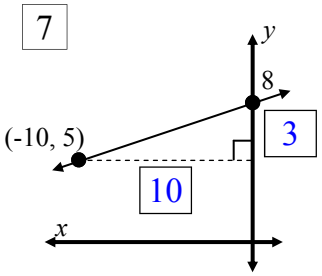

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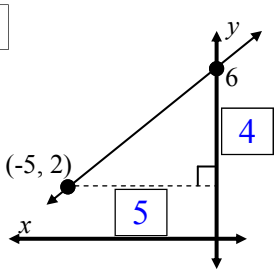

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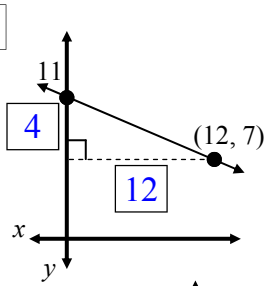

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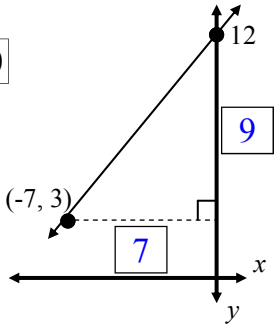

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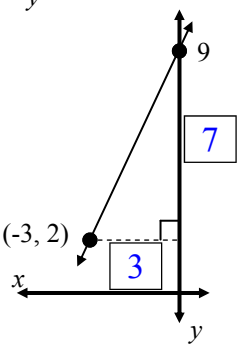
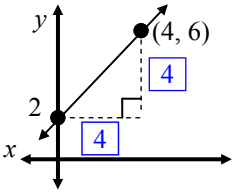


Find the rise, run, gradient, *y*-intercept and equation.

Example

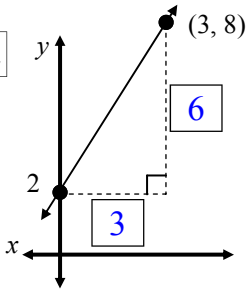
Gradient: ⊕ ⊖



$m = \frac{4}{4} = 1$ $b = 2$

$y = x + 2$

12



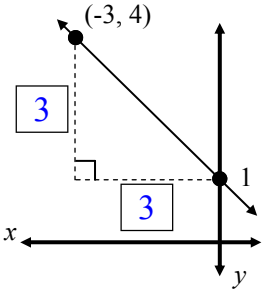
Gradient: ● ⊖

$m = \frac{6}{3} = 2$ $b = 2$

$y = 2x + 2$

13

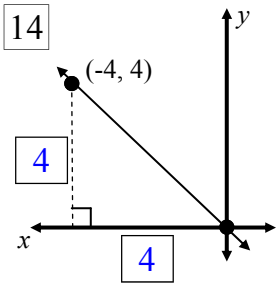
Gradient: ⊕ ●



$m = \frac{3}{3} = -1$ $b = 1$

$y = -x + 1$

14



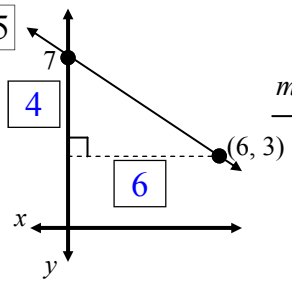
Gradient: ⊕ ●

$m = \frac{4}{4} = -1$ $b = 0$

$y = -x$

This time the gradient is a fraction

15



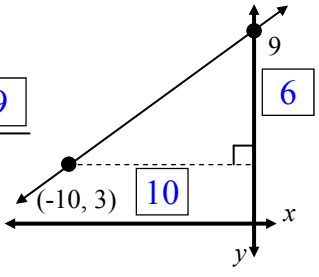
Gradient: ⊕ ●

$m = \frac{4}{6} = \frac{2}{3}$ $b = 7$

$y = \frac{-2}{3}x + 7$

16

Gradient: ● ⊖



$m = \frac{6}{10} = \frac{3}{5}$ $b = 9$

$y = \frac{3}{5}x + 9$

Linear Relationships - Further Naming Straight Lines (Sheet 2)

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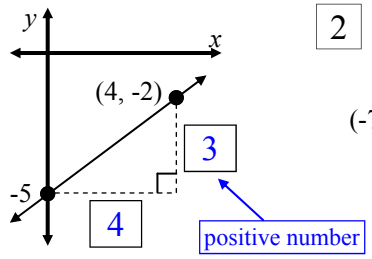
We are below the x-axis this time. Find the rise and run. Keep all your numbers positive.

1



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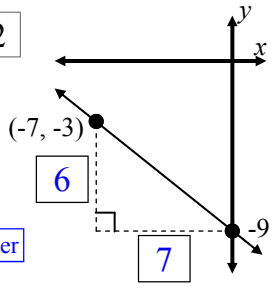

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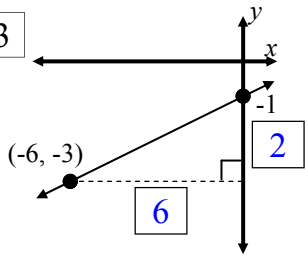

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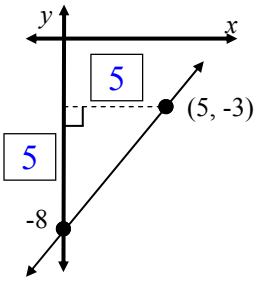

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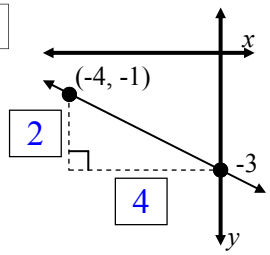

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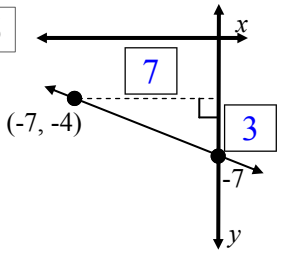
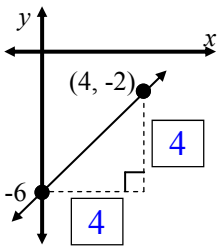


Find the rise, run, gradient, y-intercept and equation.

7



Gradient: ⊕ ⊖

$m = \frac{4}{4} = 1$ $\quad b = -6$

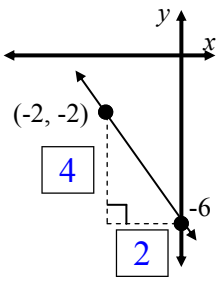
$y = x - 6$

8

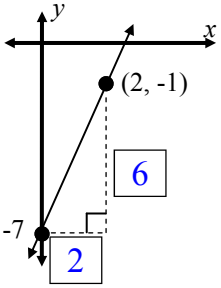
Gradient: ⊕ ⊖

$m = \frac{4}{2} = -2$ $\quad b = -6$

$y = -2x - 6$



9



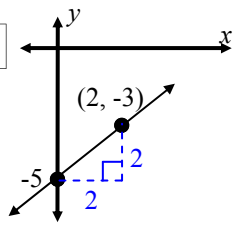
Gradient: ⊕ ⊖

$m = \frac{6}{2} = 3$ $\quad b = -7$

$y = 3x - 7$

Find the rise, run, gradient, y-intercept and equation. This time you have to construct the triangle yourself.

10



Gradient: ⊕ ⊖

$m = \frac{2}{2} = 1$ $\quad b = -5$

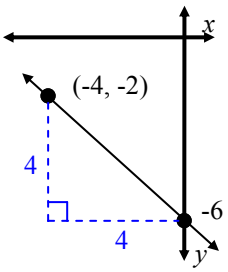
$y = x - 5$

11

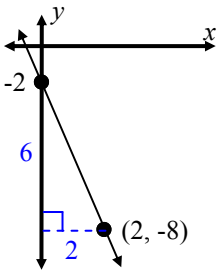
Gradient: ⊕ ⊖

$m = \frac{4}{4} = -1$ $\quad b = -6$

$y = -x - 6$



12



Gradient: ⊕ ⊖

$m = \frac{6}{2} = -3$ $\quad b = -2$

$y = -3x - 2$

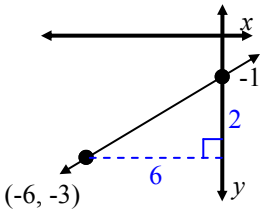
This time the gradient is a fraction

13

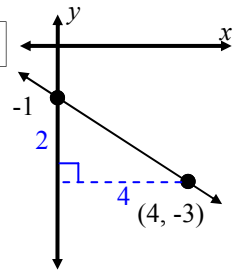
Gradient: ⊕ ⊖

$m = \frac{2}{6} = \frac{1}{3}$ $\quad b = -1$

$y = \frac{1}{3}x - 1$



14



Gradient: ⊕ ⊖

$m = \frac{2}{4} = \frac{-1}{2}$ $\quad b = -1$

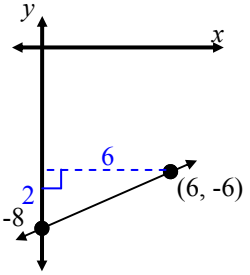
$y = \frac{-1}{2}x - 1$

15

Gradient: ⊕ ⊖

$m = \frac{2}{6} = \frac{1}{3}$ $\quad b = -8$

$y = \frac{1}{3}x - 8$



Linear Relationships - Further Naming Straight Lines (Homework Strip)

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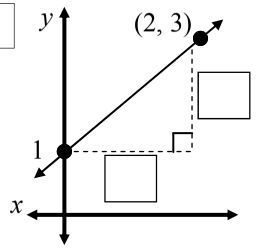
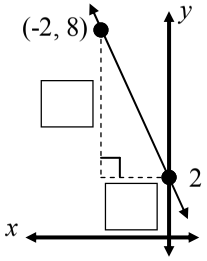
Date Due:

Find the gradient for these lines.

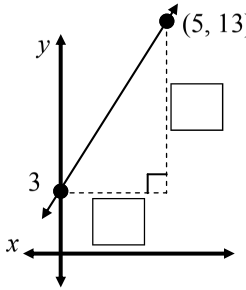
1 

$m = \frac{\square}{\square}$ — $m = \square$

2 

$m = \frac{\square}{\square}$ — $m = \square$

Find the equation for this line.

3 

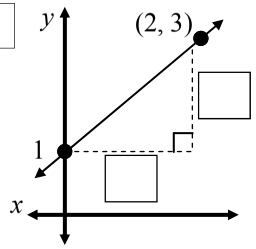
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

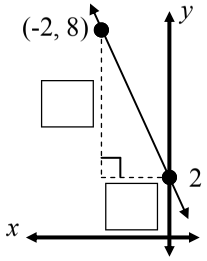
$y = \square x + \square$

Date Due:

Find the gradient for these lines.

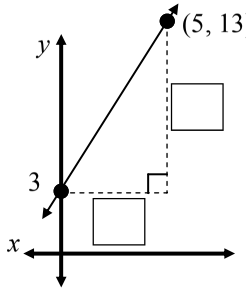
1 

$m = \frac{\square}{\square}$ — $m = \square$

2 

$m = \frac{\square}{\square}$ — $m = \square$

Find the equation for this line.

3 

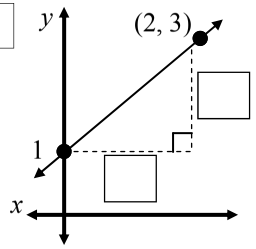
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

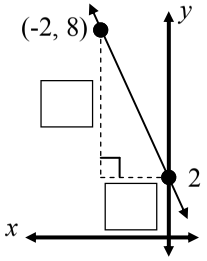
$y = \square x + \square$

Date Due:

Find the gradient for these lines.

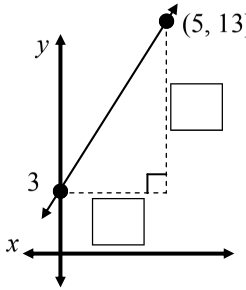
1 

$m = \frac{\square}{\square}$ — $m = \square$

2 

$m = \frac{\square}{\square}$ — $m = \square$

Find the equation for this line.

3 

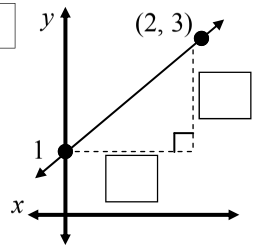
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

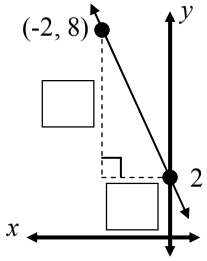
$y = \square x + \square$

Date Due:

Find the gradient for these lines.

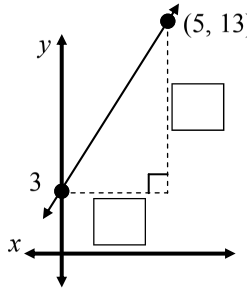
1 

$m = \frac{\square}{\square}$ — $m = \square$

2 

$m = \frac{\square}{\square}$ — $m = \square$

Find the equation for this line.

3 

Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

$y = \square x + \square$

Linear Relationships - Further Naming Straight Lines (Homework Strip)

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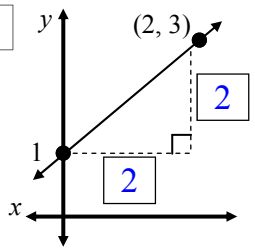
Date Due:

Find the gradient for these lines.

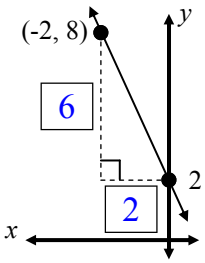
1



$m = \frac{2}{2}$

$m = 1$

2

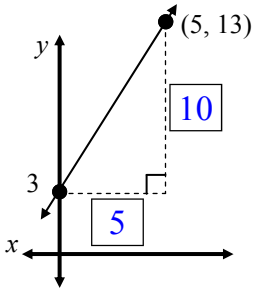


$m = \frac{6}{2}$

$m = -3$

Find the equation for this line.

3



Gradient: ● ⊖

$m = \frac{10}{5} = 2$

$b = 3$

$y = 2x + 3$

Linear Relationships - Proving if a Point is on a Line

If a point is on a line when you substitute the point's co-ordinates into the straight line equation, it satisfies it (which means 'it makes sense'). If the point is substituted into the equation and it doesn't satisfy it (make sense), then it isn't on the line.

The worksheet starts off with basic substitution, this is given an x-value and an equation, find the value of y. Here is an example:

Example: $y = 7x - 3$, find y when $x = 6$.

$y = 7x - 3$ ← Write the equation

$y = 7 \times 6 - 3$ ← Substitute $x = 6$

$y = 39$ ← Calculate the y value

What this is used for, is, for finding a point on the line for a given value of x. So the above found a value, not a point. If you were asked to find the point, you include the x-value in the answer. These are in the second column.

Example: $y = 7x - 3$, find the co-ordinates of the point when $x = 6$.

$y = 7x - 3$ ← Write the equation

$y = 7 \times 6 - 3$ ← Substitute $x = 6$

$y = 39$ ← Calculate the y value

$\therefore (x, y) = (6, 39)$ ← Give the point

The third column deals with finding if a point is on the line. With these you substitute both x and y values into the equation. If the equation is satisfied, then the point is on the line. No satisfaction? Then the point isn't on the line.

Example: Does the point (6, 39) lie on the line $y = 7x - 3$?

x y ← Write x and y here

$y = 7x - 3$ ← Write the equation

$39 = 7 \times 6 - 3$ ← Substitute $x = 6$ and $y = 39$

$39 = 39$ ← Calculate the RHS

RHS = Right hand side (of the equation)

LHS = RHS. $\therefore$ Point is on line. ← Make a statement

Linear Relationships - Proving if a Point is on a Line (Sheet 1)

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Given x, find the value of y.

Example $y = 3x + 2$ $x = 2$

$y = 3x + 2$ ← Write equation

$y = 3 \times 2 + 2$ ← Substitute $x = 2$

$y = 8$ ← Answer

1 $y = 7x - 5$ $x = 3$

$y = 7x - 5$ ← Write equation

$y = 7 \times \square - 5$ ← Substitute $x = 3$

$y = \square$ ← Answer

2 $y = x - 7$ $x = 5$

____ ← Write equation

____ ← Substitute $x = 5$

____ ← Answer

3 $y = -2x + 4$ $x = 6$

____

____

____

4 $y = 12 - 3x$ $x = 0$

____

____

____

5 $y = -x - 9$ $x = 4$

____

____

____

Given an x co-ordinate, give the co-ordinates for a point on a line.

Example $y = 2x + 7$ $(5, y)$

$y = 2x + 7$ ← Write equation

$y = 2 \times 5 + 7$ ← Substitute $x = 5$

$y = 17$ ← y-value

$\therefore (x, y) = (5, 17)$ ← Point

6 $y = 4x - 8$ $(3, y)$

____ ← Write equation

____ ← Substitute $x = 3$

____ ← y-value

$\therefore (x, y) = (3, \square)$ ← Point

7 $y = -5x + 7$ $(-4, y)$

____

____

____

____

8 $y = -x - 11$ $(-1, y)$

____

____

____

____

9 $y = 15 - 5x$ $(-7, y)$

____

____

____

____

Find if the point is on the line.

Example $y = 4x + 5$ $(2, 11)$

x y ← Label x, y.

$y = 4x + 5$ ← Equation

$11 = 4 \times 2 + 5$ ← Substitute $x = 2, y = 11$

$11 \neq 13$ ← Solve RHS

LHS ≠ RHS. ∴ Not on line.

10 $y = 6x + 2$ $(5, 28)$

Label x, y. → __ __

____ ← Write equation

$\square = 6 \times \square + 2$ ← Substitute $x = 5, y = 28$

____ ← Solve Use = or ≠

____

Using LHS/RHS State if point on the line

__ __

11 Does the point (8, 17) lie on the line $y = 3x - 7$?

____ ← Equation

____ ← Substitute

____ ← Solve

____

__ __

12 Does the point (-3, -13) lie on the line $y = -4x - 25$?

____

____

____

____

Linear Relationships - Proving if a Point is on a Line (Sheet 2)

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Find if the point is on the line.

Example $2x + y = 2$ $(-2, 6)$

x y — Label x, y.

$2x + y = 2$ — Equation

$2 \times (-2) + 6 = 2$ — Substitute $x = -2, y = 6$

$2 = 2$ — Solve

LHS = RHS. ∴ Point on line.

1 $5y + 11 = 6x$ $(5, 6)$

Label x, y. → ☐ ☐

Write equation

Substitute $x = 5, y = 6$

Solve Use = or ≠

Using LHS/RHS State if point on the line

☐ ☐

2 Does the point (-2, 4) lie on the line $10y + 3x = 46$?

Equation

Substitute

Solve

Using LHS/RHS State if point on the line

☐ ☐

3 Does the point (-7, 9) lie on the line $13 - 2x = 3y$?

This time you have two lines and you have to check if they intersect at the given point.

4 Do the lines $y = 2x + 6$ and $y = 5x - 15$ intersect at (7, 20)?

☐ ☐

Write $y = 2x + 6$

Substitute $x = 7, y = 20$

Solve Use = or ≠

LHS ☐ RHS ∴

= or ≠ Point is or not on line

Write $y = 5x - 15$

Substitute $x = 7, y = 20$

Solve Use = or ≠

LHS ☐ RHS ∴

= or ≠ Point is or not on line

Yes they, or no they don't, intersect at (7, 20)

5 Do the lines $y = 15 - 3x$ and $y = -2x + 12$ intersect at (3, 6)?

☐ ☐

6 Do the lines $3y - 2x = -5$ and $y = -3x + 8$ intersect at (7, 3)?

☐ ☐

7 Do the lines $y = -x + 12$ and $y = 5x - 12$ intersect at (4, 8)?

☐ ☐

Linear Relationships - Proving if a Point is on a Line (Sheet 1)

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Given x, find the value of y.

Example $y = 3x + 2$ $x = 2$

$y = 3x + 2$ ← Write equation

$y = 3 \times 2 + 2$ ← Substitute $x = 2$

$y = 8$ ← Answer

1 $y = 7x - 5$ $x = 3$

$y = 7x - 5$ Write equation

$y = 7 \times \boxed{3} - 5$ Substitute $x = 3$

$y = \boxed{16}$ Answer

2 $y = x - 7$ $x = 5$

$y = x - 7$ Write equation

$y = 5 - 7$ Substitute $x = 5$

$y = -2$ Answer

3 $y = -2x + 4$ $x = 6$

$y = -2x + 4$

$y = -2 \times 6 + 4$

$y = -8$

4 $y = 12 - 3x$ $x = 0$

$y = 12 - 3x$

$y = 12 - 3 \times 0$

$y = 12$

5 $y = -x - 9$ $x = 4$

$y = -x - 9$

$y = -4 - 9$

$y = -13$

Given an x co-ordinate, give the co-ordinates for a point on a line.

Example $y = 2x + 7$ $(5, y)$

$y = 2x + 7$ ← Write equation

$y = 2 \times 5 + 7$ ← Substitute $x = 5$

$y = 17$ ← y-value

$\therefore (x, y) = (5, 17)$ ← Point

6 $y = 4x - 8$ $(3, y)$

$y = 4x - 8$ Write equation

$y = 4 \times 3 - 8$ Substitute $x = 3$

$y = 4$ y-value

$\therefore (x, y) = (3, \boxed{4})$ Point

7 $y = -5x + 7$ $(-4, y)$

$y = -5x + 7$

$y = -5 \times (-4) + 7$

$y = 27$

$\therefore (x, y) = (-4, 27)$

8 $y = -x - 11$ $(-1, y)$

$y = -x - 11$

$y = -(-1) - 11$

$y = 10$

$\therefore (x, y) = (-1, 10)$

9 $y = 15 - 5x$ $(-7, y)$

$y = 15 - 5x$

$y = 15 - 5 \times (-7)$

$y = 50$

$\therefore (x, y) = (-7, 50)$

Find if the point is on the line.

Example $y = 4x + 5$ $(2, 11)$

x y

$y = 4x + 5$ ← Label x, y. / Equation

$11 = 4 \times 2 + 5$ ← Substitute $x = 2, y = 11$

$11 \neq 13$ ← Solve RHS

LHS ≠ RHS. $\therefore$ Not on line.

10 $y = 6x + 2$ $(5, 28)$

Label x, y. → x y

$y = 6x + 2$ Write equation

$\boxed{28} = 6 \times \boxed{5} + 2$ Substitute $x = 5, y = 28$

$28 \neq 32$ Solve Use = or ≠

LHS ≠ RHS. $\therefore$ Not on line.

Using LHS/RHS State if point on the line

x y

11 Does the point (8, 17) lie on the line $y = 3x - 7$?

$y = 3x - 7$ Equation

$17 = 3 \times 8 - 7$ Substitute

$17 = 17$ Solve

LHS = RHS. $\therefore$ Point is on line.

x y

12 Does the point (-3, -13) lie on the line $y = -4x - 25$?

$y = -4x - 25$

$-13 = -4 \times (-3) - 25$

$-13 = -13$

LHS = RHS. $\therefore$ Point is on line.

Linear Relationships - Proving if a Point is on a Line (Sheet 2)

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Find if the point is on the line.

Example $2x + y = 2$ (-2, 6)

x y

$2x + y = 2$ — Label x, y. / Equation

$2 \times (-2) + 6 = 2$ — Substitute $x = -2, y = 6$

$2 = 2$ — Solve

LHS = RHS. ∴ Point on line.

1 $5y + 11 = 6x$ (5, 6)

Label x, y. → x y

$5y + 11 = 6x$ — Write equation

$5 \times 6 + 11 = 6 \times 5$ — Substitute $x = 5, y = 6$

$41 \neq 30$ — Solve Use = or ≠

LHS ≠ RHS. ∴ Point not on line

Using LHS/RHS State if point on the line

x y

2 Does the point (-2, 4) lie on the line $10y + 3x = 46$?

$10y + 3x = 46$ — Equation

$10 \times 4 + 3 \times (-2) = 46$ — Substitute

$34 \neq 46$ — Solve

LHS ≠ RHS. ∴ Point not on line

Using LHS/RHS State if point on the line

x y

3 Does the point (-7, 9) lie on the line $13 - 2x = 3y$?

$13 - 2x = 3y$

$13 - 2 \times (-7) = 3 \times 9$

$27 = 27$

LHS = RHS. ∴ Point is on line

This time you have two lines and you have to check if they intersect at the given point.

4 Do the lines $y = 2x + 6$ and $y = 5x - 15$ intersect at (7, 20)?

$y = 2x + 6$ — Write $y = 2x + 6$

$20 = 2 \times 7 + 6$ — Substitute $x = 7, y = 20$

$20 = 20$ — Solve Use = or ≠

LHS = RHS ∴ Point is on line

= or ≠ Point is or not on line

$y = 5x - 15$ — Write $y = 5x - 15$

$20 = 5 \times 7 - 15$ — Substitute $x = 7, y = 20$

$20 = 20$ — Solve Use = or ≠

LHS = RHS ∴ Point is on line

= or ≠ Point is or not on line

Yes the lines intersect at (7, 20)

Yes they, or no they don't, intersect at (7, 20)

5 Do the lines $y = 15 - 3x$ and $y = -2x + 12$ intersect at (3, 6)?

x y

$y = 15 - 3x$

$6 = 15 - 3 \times 3$

$6 = 6$

LHS = RHS. ∴ Point is on line

$y = -2x + 12$

$6 = -2 \times 3 + 12$

$6 = 6$

LHS = RHS. ∴ Point is on line

Yes the lines intersect at (3, 6)

6 Do the lines $3y - 2x = -5$ and $y = -3x + 8$ intersect at (7, 3)?

x y

$3y - 2x = -5$

$3 \times 3 - 2 \times 7 = -5$

$-5 = -5$

LHS = RHS. ∴ Point on line.

$y = -3x + 8$

$3 = -3 \times 7 + 8$

$3 \neq -13$

LHS ≠ RHS. ∴ Point not on line.

The lines don't intersect at (7, 3)

7 Do the lines $y = -x + 12$ and $y = 5x - 12$ intersect at (4, 8)?

x y

$y = -x + 12$

$8 = -4 + 12$

$8 = 8$

LHS = RHS. ∴ Point on line.

$y = 5x - 12$

$8 = 5 \times 4 - 12$

$8 = 8$

LHS = RHS. ∴ Point on line.

Yes the lines intersect at (4, 8)

Linear Relationships - Proving if a Point is on a Line (Homework Strip)

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Date Due:

1 Does the point (2, 6) lie on the line $y = 4x - 2$? ☐☐

$y = 4x - 2$ — Write equation

☐ = 4 × ☐ - 2 — Substitute $x = 2, y = 6$

☐ ☐ ☐ — Solve Use = or ≠

= or ≠

LHS ☐ RHS ∴ ☐

= or ≠ Point on/not on line

2 Does the point (10, 6) lie on the line $y = 2x - 12$? ☐☐

☐ — Write equation

☐ — Substitute $x = 10, y = 6$

☐ — Solve Use = or ≠

☐

Using LHS/RHS State if point on the line

Date Due:

1 Does the point (2, 6) lie on the line $y = 4x - 2$? ☐☐

$y = 4x - 2$ — Write equation

☐ = 4 × ☐ - 2 — Substitute $x = 2, y = 6$

☐ ☐ ☐ — Solve Use = or ≠

= or ≠

LHS ☐ RHS ∴ ☐

= or ≠ Point on/not on line

2 Does the point (10, 6) lie on the line $y = 2x - 12$? ☐☐

☐ — Write equation

☐ — Substitute $x = 10, y = 6$

☐ — Solve Use = or ≠

☐

Using LHS/RHS State if point on the line

Date Due:

1 Does the point (2, 6) lie on the line $y = 4x - 2$? ☐☐

$y = 4x - 2$ — Write equation

☐ = 4 × ☐ - 2 — Substitute $x = 2, y = 6$

☐ ☐ ☐ — Solve Use = or ≠

= or ≠

LHS ☐ RHS ∴ ☐

= or ≠ Point on/not on line

2 Does the point (10, 6) lie on the line $y = 2x - 12$? ☐☐

☐ — Write equation

☐ — Substitute $x = 10, y = 6$

☐ — Solve Use = or ≠

☐

Using LHS/RHS State if point on the line

Date Due:

1 Does the point (2, 6) lie on the line $y = 4x - 2$? ☐☐

$y = 4x - 2$ — Write equation

☐ = 4 × ☐ - 2 — Substitute $x = 2, y = 6$

☐ ☐ ☐ — Solve Use = or ≠

= or ≠

LHS ☐ RHS ∴ ☐

= or ≠ Point on/not on line

2 Does the point (10, 6) lie on the line $y = 2x - 12$? ☐☐

☐ — Write equation

☐ — Substitute $x = 10, y = 6$

☐ — Solve Use = or ≠

☐

Using LHS/RHS State if point on the line

Linear Relationships - Proving if a Point is on a Line (Homework Strip)

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Date Due:

1 Does the point (2, 6) lie on the line $y = 4x - 2$? x y

$y = 4x - 2$ — Write equation

6 $= 4 \times$ 2 $- 2$ — Substitute $x = 2, y = 6$

6 = 6 — Solve Use = or ≠

= or ≠

LHS = RHS ∴ Point on line

= or ≠ Point on/not on line

2 Does the point (10, 6) lie on the line $y = 2x - 12$? x y

$y = 2x - 12$ — Write equation

$6 = 2 \times 10 - 12$ — Substitute $x = 10, y = 6$

$6 \neq 8$ — Solve Use = or ≠

LHS ≠ RHS. Point not on line.

Using LHS/RHS State if point on the line

Linear Relationships - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don't colour it in too dark (using a texta for example) so that you can still read it.

Linear Relationships - Find A Word

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Find the words in the puzzle from the wordlist.



O	O	L	V	S	P	E	R	D	W	A	R	R	A	C
L	P	P	E	W	E	T	A	N	I	D	R	O	O	C
T	B	L	R	O	P	U	O	D	R	I	A	S	L	G
P	R	N	T	R	L	T	D	A	A	X	T	E	R	E
L	E	E	I	R	L	I	W	P	E	G	B	R	U	G
T	A	G	C	A	R	T	E	S	I	A	N	L	L	R
E	K	A	A	G	A	S	L	E	L	L	A	R	A	P
Q	E	T	L	E	L	B	A	T	P	V	L	E	T	G
U	V	I	R	A	P	U	P	P	P	L	P	I	N	R
A	E	V	I	T	I	S	O	P	L	O	F	S	O	A
T	N	E	I	D	A	R	G	P	L	O	S	O	Z	E
I	X	N	O	I	T	C	E	S	R	E	T	N	I	N
O	I	N	T	E	R	C	E	P	T	P	P	L	R	I
N	F	U	N	C	T	I	O	N	I	G	I	R	O	L
T	I	V	E	P	O	S	S	T	R	A	I	G	H	T

WORDLIST

HORIZONTAL	DRAW	NEGATIVE	STRAIGHT	ORIGIN
COORDINATE	TABLE	EQUATION	PLOT	PROFIT
SUBSTITUTE	AXES	LINEAR	VERTICAL	VALUE
LABEL	PARALLEL	GRID	FUNCTION	CARTESIAN
BREAK-EVEN	PAIR	COST	SLOPE	ARROWS
No Hyphen in grid	GRADIENT	INTERCEPT	POSITIVE	INTERSECTION

Linear Relationships - Find A Word

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Find the words in the puzzle from the wordlist.



O	O	L	V	S	P	E	R	D	W	A	R	R	A	C
L	P	P	E	W	E	T	A	N	I	D	R	O	O	C
T	B	L	R	O	P	U	O	D	R	I	A	S	L	G
P	R	N	T	R	L	T	D	A	A	X	T	E	R	E
L	E	E	I	R	L	I	W	P	E	G	B	R	U	G
T	A	G	C	A	R	T	E	S	I	A	N	L	L	R
E	K	A	A	G	A	S	L	E	L	L	A	R	A	P
Q	E	T	L	E	L	B	A	T	P	V	L	E	T	G
U	V	I	R	A	P	U	P	P	P	L	P	I	N	R
A	E	V	I	T	I	S	O	P	L	O	F	S	O	A
T	N	E	I	D	A	R	G	P	L	O	S	O	Z	E
I	X	N	O	I	T	C	E	S	R	E	T	N	I	N
O	I	N	T	E	R	C	E	P	T	P	P	L	R	I
N	F	U	N	C	T	I	O	N	I	G	I	R	O	L
T	I	V	E	P	O	S	S	T	R	A	I	G	H	T

WORDLIST

HORIZONTAL
COORDINATE
SUBSTITUTE
LABEL
BREAK-EVEN
No Hyphen in grid

DRAW
TABLE
AXES
PARALLEL
PAIR
GRADIENT

NEGATIVE
EQUATION
LINEAR
GRID
COST
INTERCEPT

STRAIGHT
PLOT
VERTICAL
FUNCTION
SLOPE
POSITIVE

ORIGIN
PROFIT
VALUE
CARTESIAN
ARROWS
INTERSECTION

Linear Relationships Review

This is the review of the Linear Relationships Topic.

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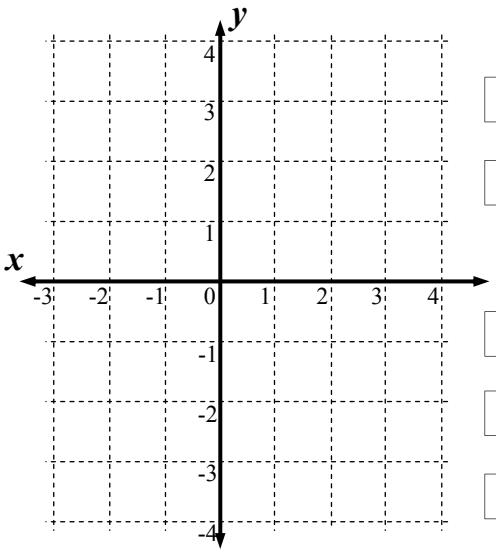
Plot the points on the number plane below.



1 Ⓐ (4, 2)

2 Ⓑ (-1, 4)

3 Ⓒ (-3, 0)

4 Ⓓ (2, -3)

5 Ⓔ (-2, -4)

Complete these table of values

6 $y = x + 5$

x	0	1	2	3	4
y					

7 $y = 3x - 7$

x	-4	-2	0	2	4
y					

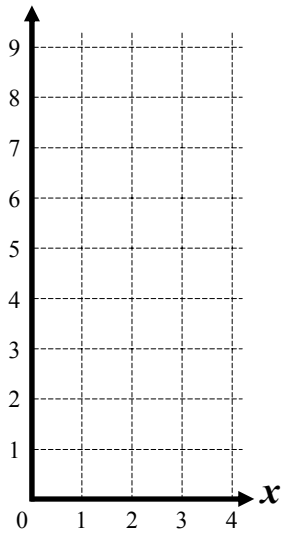
8 $y = -\frac{1}{2}x + 6$

x	-4	-2	0	2	4
y					

Plot this line on the number plane below

9 $y = 2x + 1$

x	0	1	2	3	4
y					



Plot the two lines on the number plane, then give the coordinates for the point of intersection.

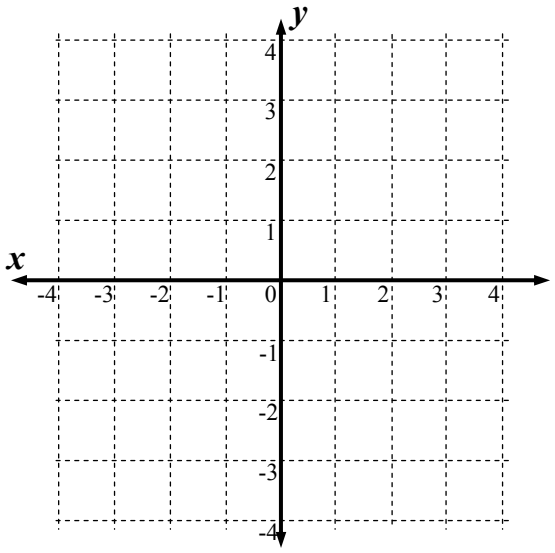
10

$y = x + 4$

x	-4	-2	0	2	4
y					

$y = -2x + 1$

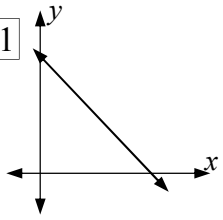
x	-2	-1	0	1	2
y					



Intersection Point (,)

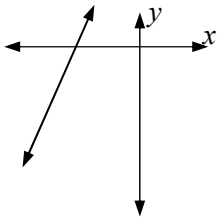
Are these gradients positive or negative?

11



○ Positive ○ Negative

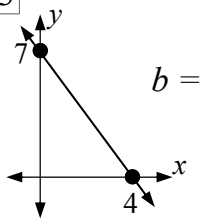
12



○ Positive ○ Negative

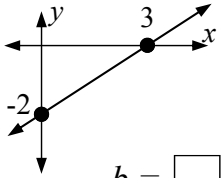
Give the y-intercept for these lines.

13



$b =$ ☐

14



$b =$ ☐

What is the gradient and y-intercept for these lines.

15 $y = -3x + 7$ $m =$ ☐ $b =$ ☐

16 $y = -x$ $m =$ ☐ $b =$ ☐

17 $y = 6 - 2x$ $m =$ ☐ $b =$ ☐

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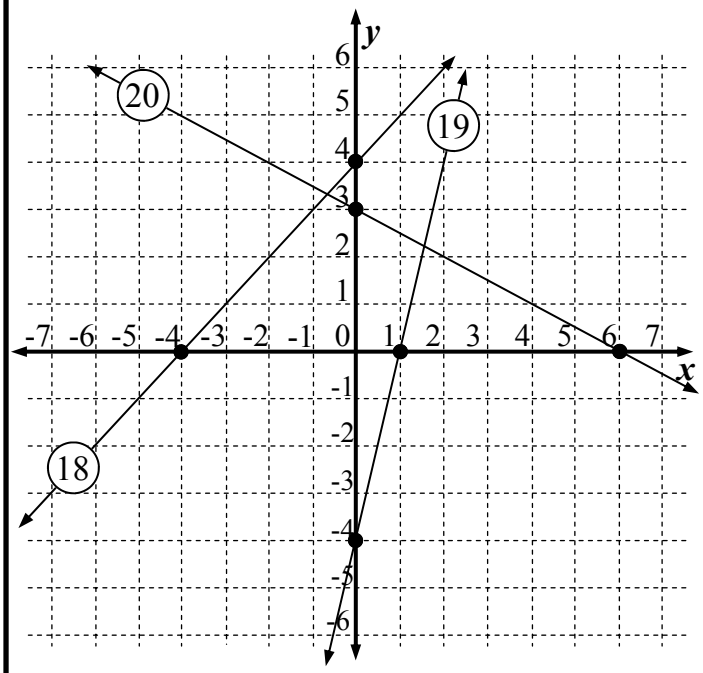
Find the gradient of the lines shown below.



18 _____

19 _____

20 _____

Give the equation for the three lines above

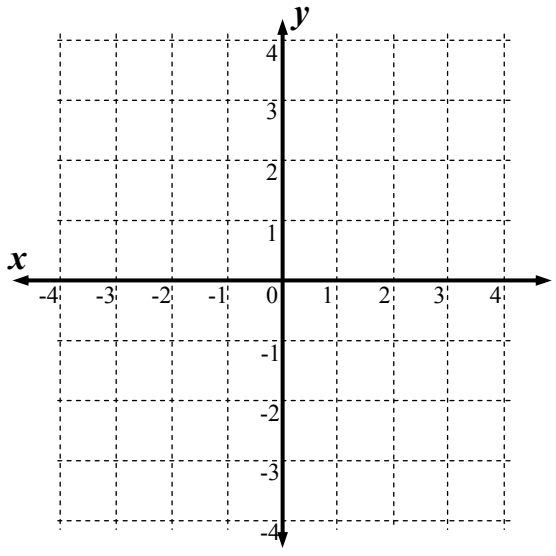
21 Name (18) _____

22 Name (19) _____

23 Name (20) _____

Plot the two lines $x = 2$ and $y = -3$ on the grid below

24



A tennis racquet manufacturer has daily figures below. Complete the tables.

25 *Production Cost : P* = 10*n* + 30 000

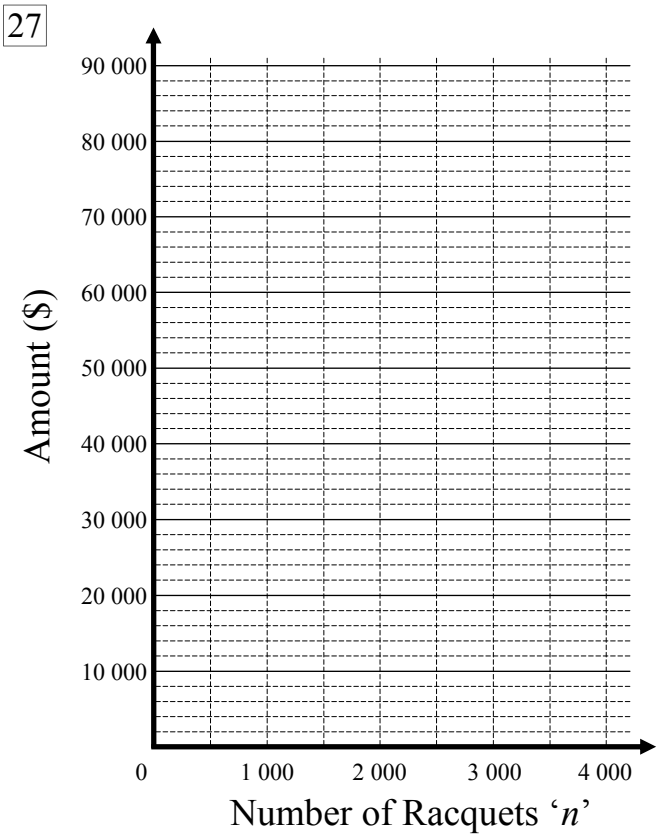
n	0	1 000	2 000	3 000	4 000
P					

26 *Sales Revenue : S* = 22*n*

n	0	1 000	2 000	3 000	4 000
S					

Plot the production and sales cost functions on the grid below.

27



28 Estimate the number of racquets required to be manufactured to break even.

_____

29 Estimate the profit made when 3 500 racquets are manufactured.

_____

Show that the point (3, 13) lies on the line $y = 5x - 2$

30 _____

_____

_____

_____

Linear Relationships - Topic Review

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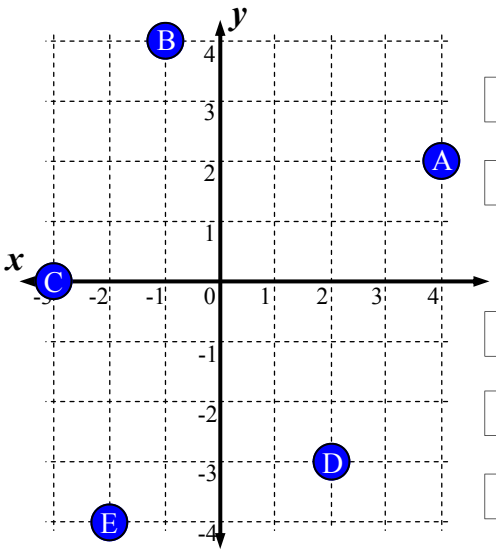
Plot the points on the number plane below.



1 Ⓐ (4, 2)

2 Ⓑ (-1, 4)

3 Ⓒ (-3, 0)

4 Ⓓ (2, -3)

5 Ⓔ (-2, -4)

Complete these table of values

6 $y = x + 5$

x	0	1	2	3	4
y	5	6	7	8	9

7 $y = 3x - 7$

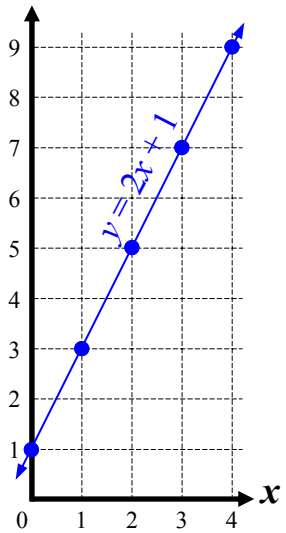
x	-4	-2	0	2	4
y	-19	-13	-7	-1	5

8 $y = -\frac{1}{2}x + 6$

x	-4	-2	0	2	4
y	8	7	6	5	4

Plot this line on the number plane below

9



$y = 2x + 1$

x	0	1	2	3	4
y	1	3	5	7	9

Plot the two lines on the number plane, then give the coordinates for the point of intersection.

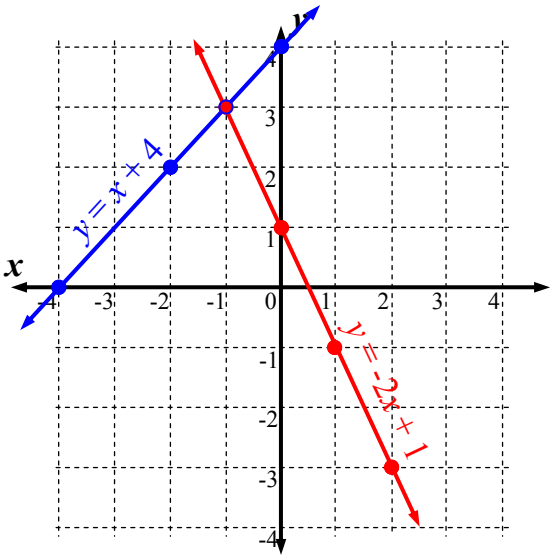
10

$y = x + 4$

x	-4	-2	0	2	4
y	0	2	4	6	8

$y = -2x + 1$

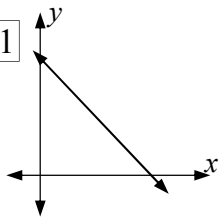
x	-2	-1	0	1	2
y	5	3	1	-1	-3



Intersection Point ([-1] , [3])

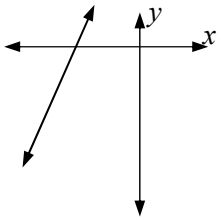
Are these gradients positive or negative?

11



○ Positive ● Negative

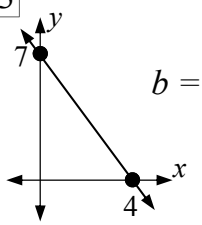
12



● Positive ○ Negative

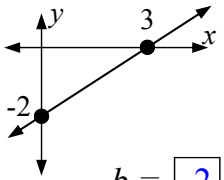
Give the y-intercept for these lines.

13



b = [7]

14



b = [-2]

What is the gradient and y-intercept for these lines.

15 $y = -3x + 7$ m = [-3] b = [7]

16 $y = -x$ m = [-1] b = [0]

17 $y = 6 - 2x$ m = [-2] b = [6]

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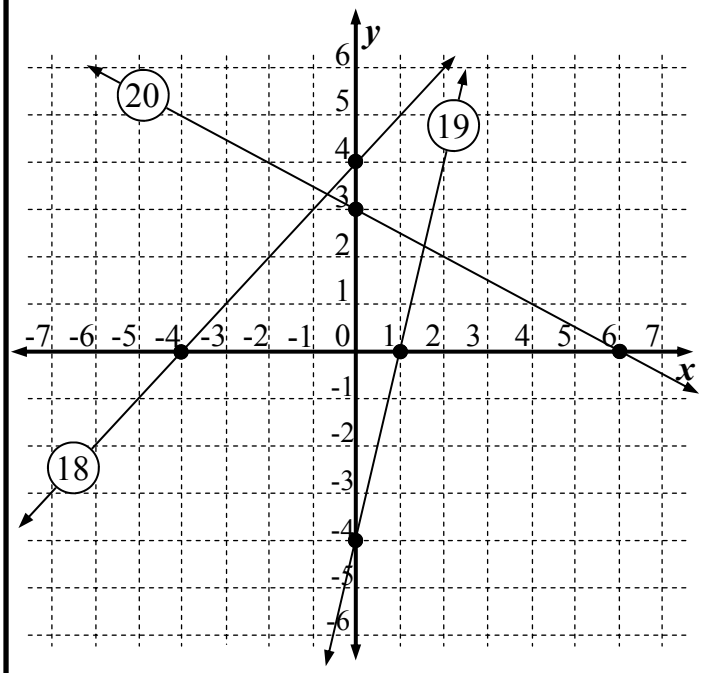
Find the gradient of the lines shown below.



18 $m = \frac{4}{4} = 1$

19 $m = \frac{4}{1} = 4$

20 $m = \frac{3}{6} = \frac{-1}{2}$

Give the equation for the three lines above

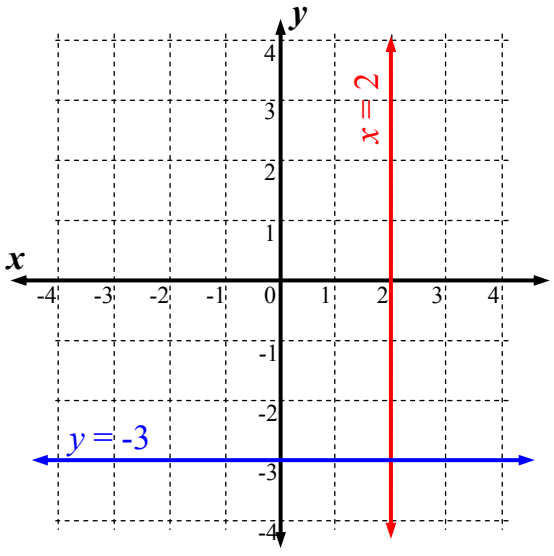
21 Name (18) $y = x + 4$

22 Name (19) $y = 4x - 4$

23 Name (20) $y = \frac{-1}{2}x + 3$

Plot the two lines $x = 2$ and $y = -3$ on the grid below

24



A tennis racquet manufacturer has daily figures below. Complete the tables.

25 *Production Cost* : $P = 10n + 30\,000$

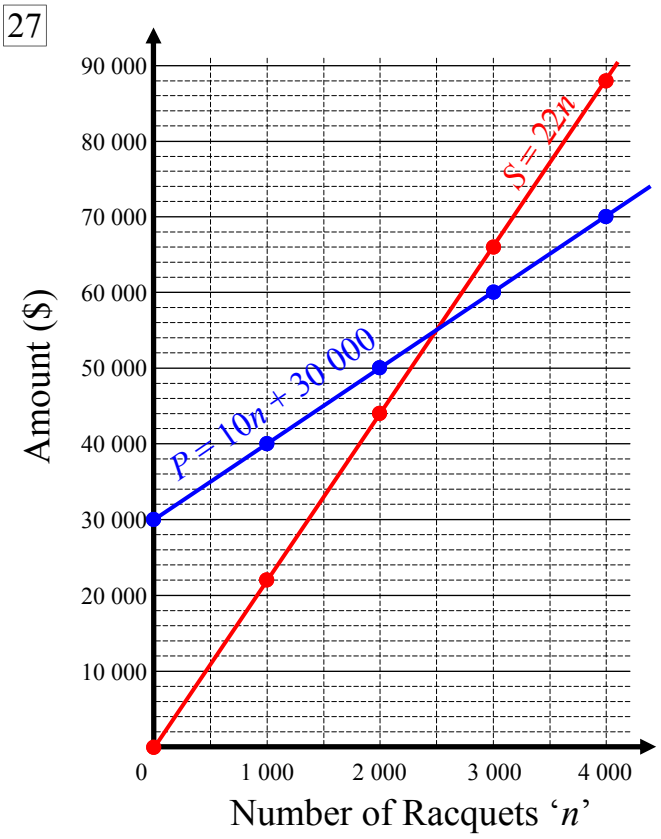
n	0	1 000	2 000	3 000	4 000
P	30 000	40 000	50 000	40 000	70 000

26 *Sales Revenue* : $S = 22n$

n	0	1 000	2 000	3 000	4 000
S	0	22 000	44 000	66 000	88 000

Plot the production and sales cost functions on the grid below.

27



28 Estimate the number of racquets required to be manufactured to break even.

2 500 racquets

29 Estimate the profit made when 3 500 racquets are manufactured.

77 000 - 65 000 = \$12 000

Show that the point (3, 13) lies on the line $y = 5x - 2$

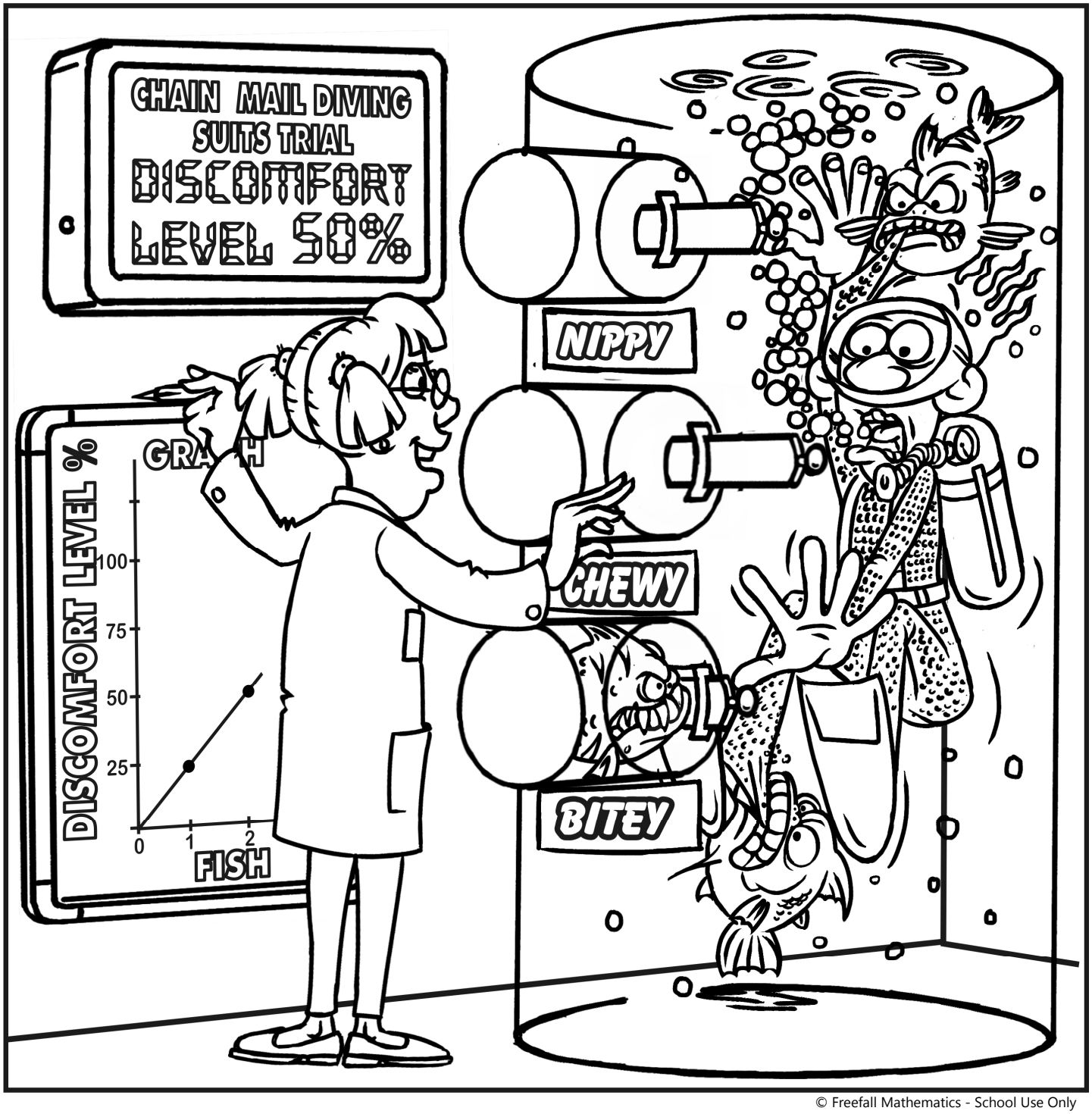
30 $y = 5x - 2$

$13 = 5 \times 3 - 2$

$13 = 13$

LHS = RHS ∴ Point is on line.

Linear Relationships



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Linear Rel- Plotting Points & Drawing Straight Lines - + Quad (Sheet 1)

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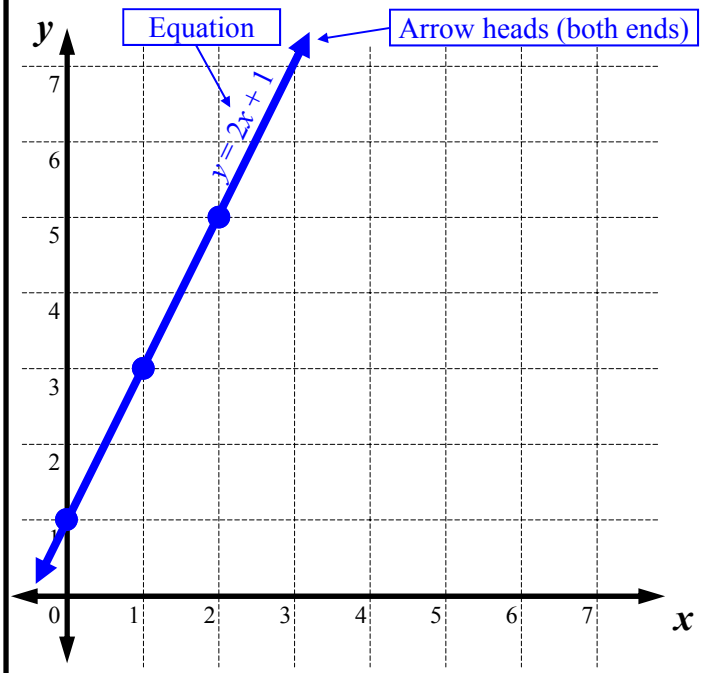
Plot the points then draw a line through them. Use a ruler, show arrow heads and write the equation of the line on the line. **The line should be the full size of the graph**, not stop at the last point.

1 $y = 2x + 1$ Plot Points: (0, 1), (1, 3), (2, 5).

2 $y = -x + 6$ Plot Points: (0, 6), (2, 4), (4, 2).

3 $y = x - 3$ Plot Points: (3, 0), (4, 1), (5, 2).

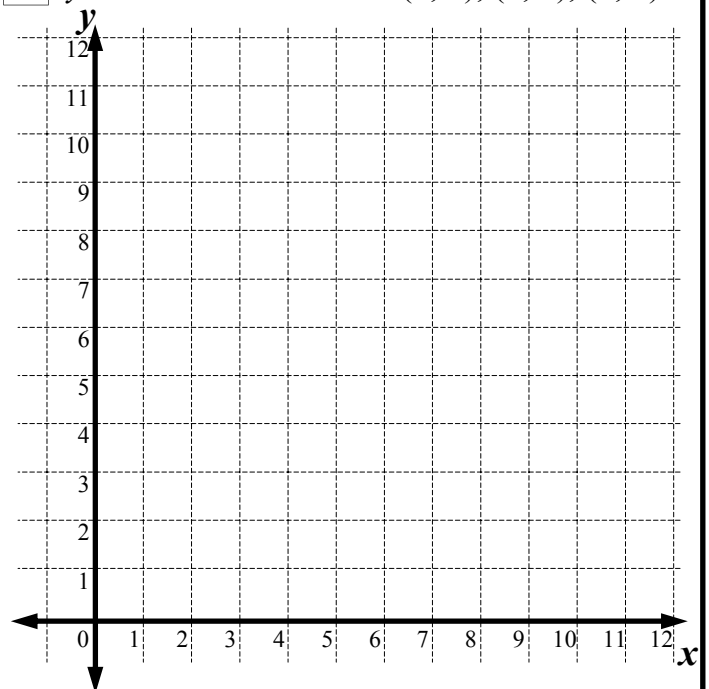


Use the lower grid for these

4 $y = 12 - x$ Plot Points: (0, 12), (2, 10), (4, 8)

5 $y = 3x$ Plot Points: (0, 0), (2, 6), (4, 12)

6 $y = x + 2$ Plot Points: (0, 2), (2, 4), (4, 6)



7 List the 3 points where the lines intersect.

(□, □), (□, □), (□, □).

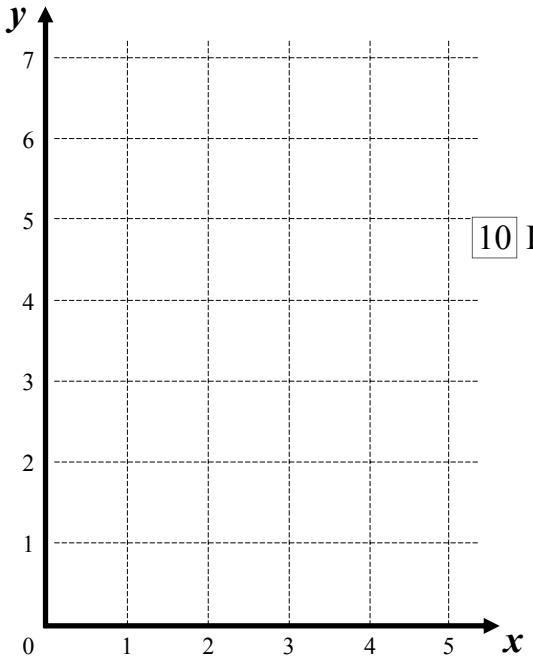
Now use the table of values to plot the lines. Go across 'x' and up 'y'.

8 $y = x$

x	0	1	2	3	4
y	0	1	2	3	4

9 $y = -x + 6$

x	0	1	2	3	4
y	6	5	4	3	2



10 Intersection

(□, □)

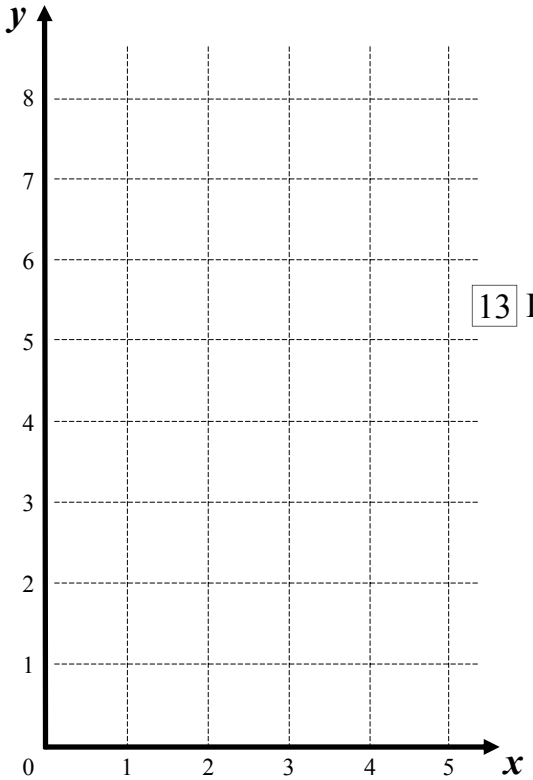
Point

11 $y = 2x$

x	0	1	2	3	4
y	0	2	4	6	8

12 $y = -2x + 8$

x	0	1	2	3	4
y	8	6	4	2	0



13 Intersection

(□, □)

Point

Linear Rel- Plotting Points & Drawing Straight Lines - 4 Quad (Sheet 2)

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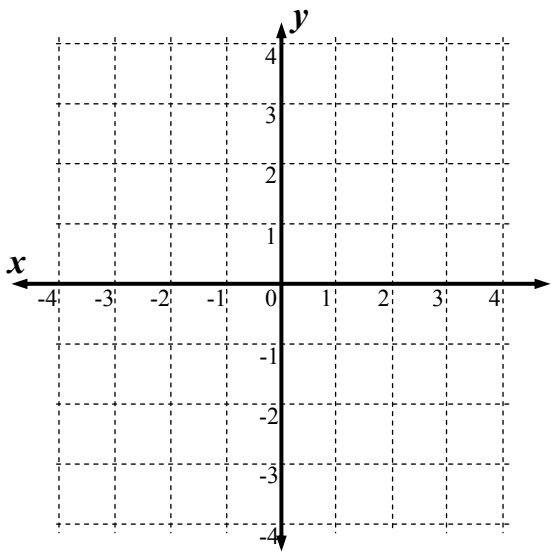
Plot the points then draw a line through them. Use a ruler, show arrow heads and write the equation of the line on the line. The line should be the full size of the graph, not stop at the last point.

1 $y = x + 3$ Plot Points: (-4, -1), (-1, 2), (1, 4).

2 $y = -x + 1$ Plot Points: (-3, 4), (2, -1), (4, -3).

3 $y = -3x - 7$ Plot Points: (1, -4), (2, -1), (3, 2).

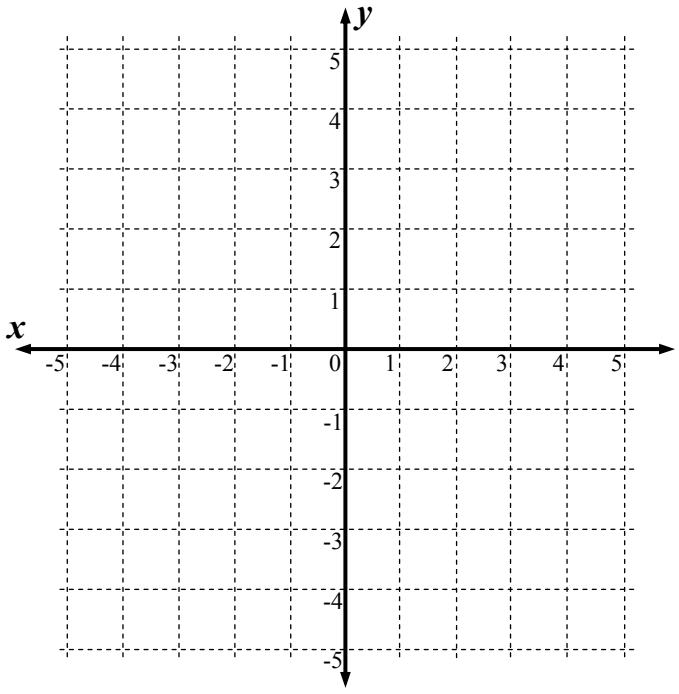


Use the lower grid for Questions 4-6

4 $y = 3x - 2$ Plot Points: (-1, -5), (0, -2), (1, 1)

5 $y = -2x - 2$ Plot Points: (-3, 4), (-1, 0), (1, -4)

6 $y = \frac{1}{2}x + 3$ Plot Points: (-4, 1), (0, 3), (4, 5)



7 List the 3 points where the lines intersect.

(□, □), (□, □), (□, □).

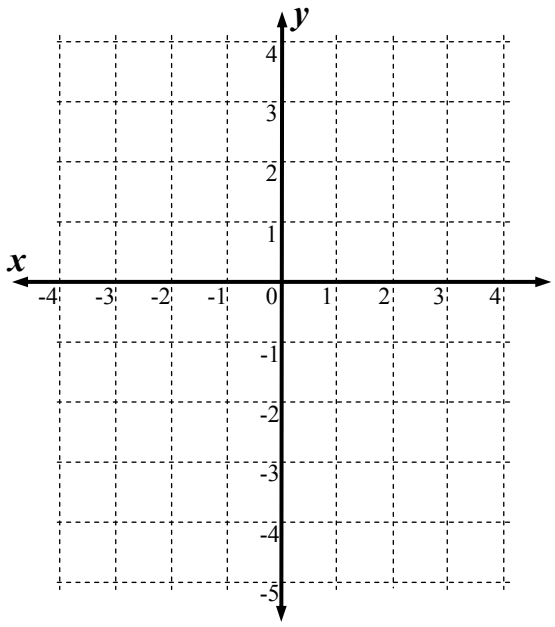
Now use the table of values to plot the lines. Go across the 'x' value and up/down the 'y' value.

8 $y = 2x$

x	-2	-1	0	1	2
y	-4	-2	0	2	4

9 $y = -x - 3$

x	-2	-1	0	1	2
y	-1	-2	-3	-4	-5



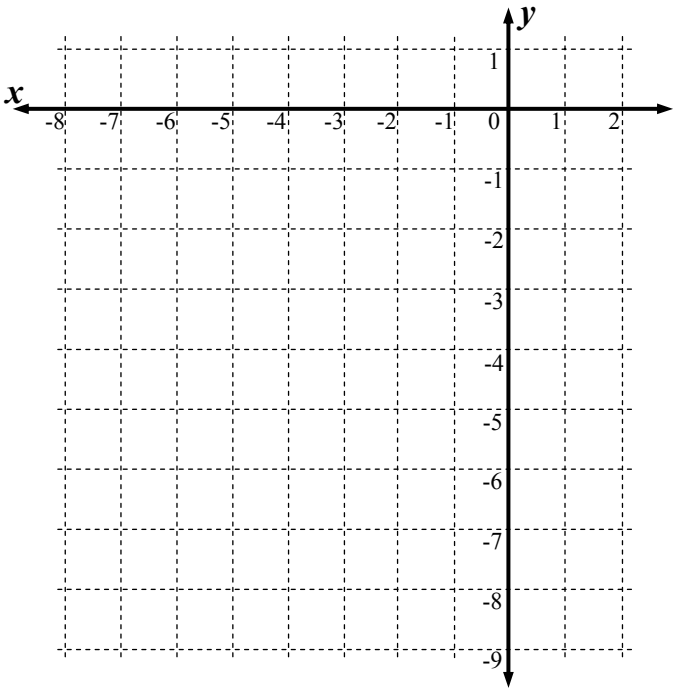
10 Intersection Point (□, □)

11 $y = -2x - 13$

x	-6	-5	-4	-3	-2
y	-1	-3	-5	-7	-9

12 $y = x - 1$

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



13 Intersection Point (□, □)

Linear Relationships - Plotting Straight Lines - Pos Quad (Sheet 1)

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Complete these table of values. There are hints below each box.

1 $y = x + 2$

x	0	1	2	3	4
y					

$0 + 2$ $1 + 2$ $2 + 2$ $3 + 2$ $4 + 2$

2 $y = 4x$

x	0	1	2	3	4
y					

4×0 4×1 4×2 4×3 4×4

3 $y = 3x + 6$

x	0	1	2	3	4
y					

$3 \times 0 + 6$ $3 \times 1 + 6$ $3 \times 2 + 6$ $3 \times 3 + 6$ $3 \times 4 + 6$

4 $y = 8 - 2x$

x	0	1	2	3	4
y					

$8 - 2 \times 0$ $8 - 2 \times 1$ $8 - 2 \times 2$ $8 - 2 \times 3$ $8 - 2 \times 4$

The next table will have answers that have ½'s in them

5 $y = \frac{1}{2}x + 7$

x	0	1	2	3	4
y					

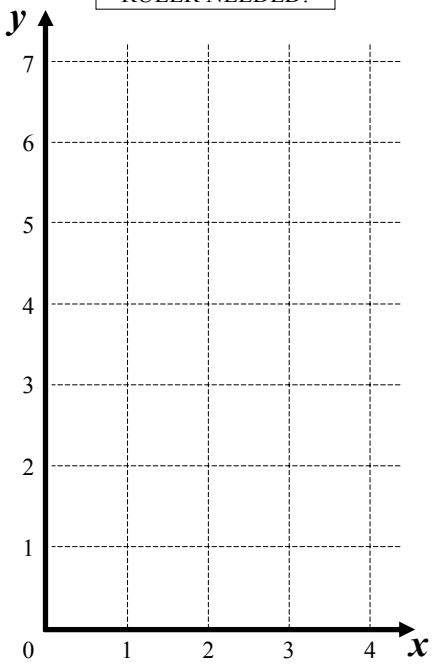
$\frac{1}{2} \times 0 + 7$ $\frac{1}{2} \times 1 + 7$ $\frac{1}{2} \times 2 + 7$ $\frac{1}{2} \times 3 + 7$ $\frac{1}{2} \times 4 + 7$

Complete the tables, then plot the results on the number plane.

6 $y = x + 3$

x	0	1	2	3	4
y					

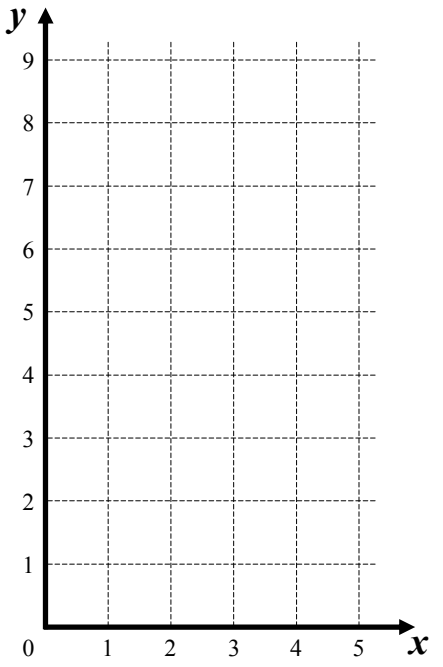
RULER NEEDED!



Check: Equation ✓ Arrowheads ✓ Full size ✓

7 $y = 2x + 1$

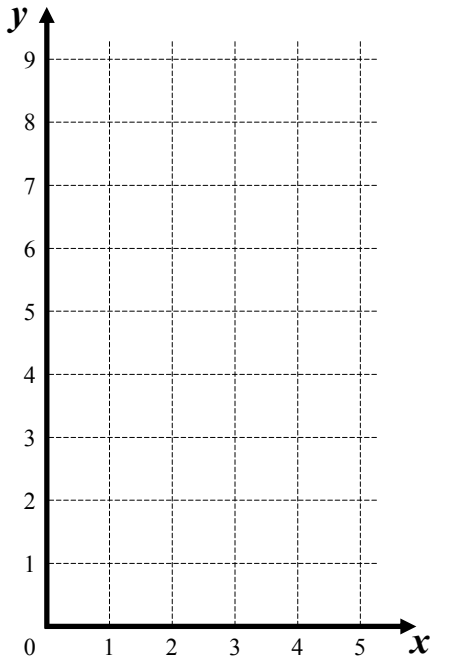
x	0	1	2	3	4
y					



Check: Equation ☐ Arrowheads ☐ Full size ☐

8 $y = 9 - 2x$

x	0	1	2	3	4
y					

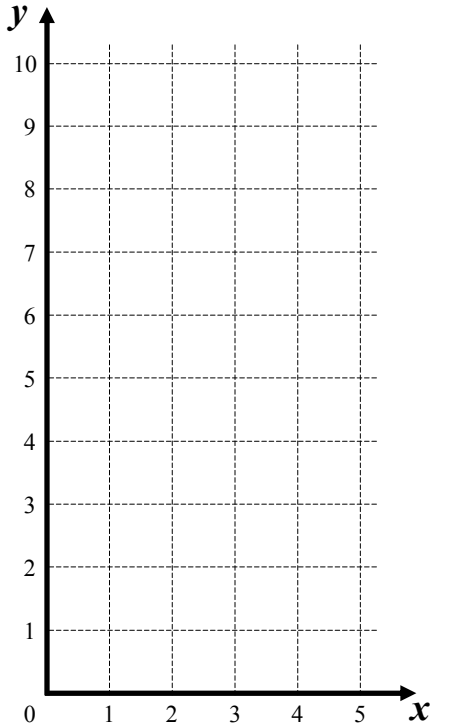


Check: Equation ☐ Arrowheads ☐ Full size ☐

9 $y = 3x - 2$

x	0	1	2	3	4
y					

Can't plot



Linear Relationships - Plotting Straight Lines - All Quads (Sheet 2)

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Complete these table of values.

1 $y = x + 2$

x	-4	-2	0	2	4
y					

2 $y = 4x$

x	-2	-1	0	1	2
y					

3 $y = 2x + 3$

x	-4	-2	0	2	4
y					

4 $y = 10 - x$

x	-4	-2	0	2	4
y					

Calc: 1 0 - - 4 =

5 $y = -3x - 6$

x	-2	-1	0	1	2
y					

Calc: - 3 × - 2 - 6 =

6 $y = ½x + 5$

x	-4	-2	0	2	4
y					

7 $x + y = 2$

x	-4	-2	0	2	4
y					

-4 + 'something' equals 2

Complete the table of values, then plot the two lines on the Cartesian Plane. Record the point of intersection. Write line equations on each line, show arrow heads and extend the lines the full size of the grid.

8 $y = x + 1$

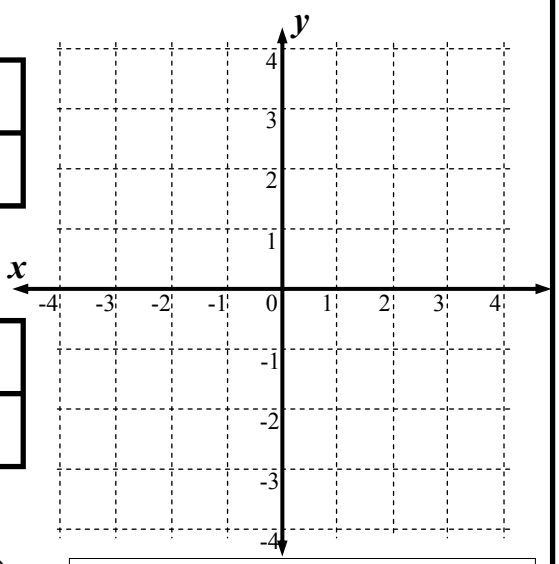
x	-4	-2	0	2	4
y					

Can't plot

9 $y = -2x + 1$

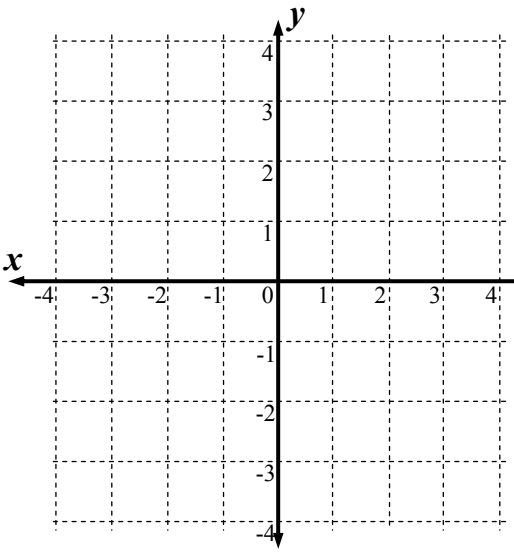
x	-2	-1	0	1	2
y					

Can't plot



Check: Equation ☐ Arrowheads ☐ Full size ☐

10 Intersection Point (☐ , ☐)



11 $y = -x$

x	-4	-2	0	2	4
y					

12 $y = ½x - 3$

x	-4	-2	0	2	4
y					

Can't plot

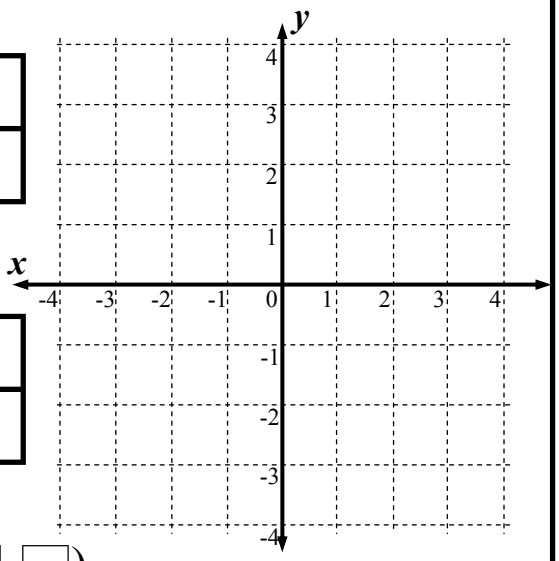
13 Intersection Point (☐ , ☐)

14 $y = -x + 2$

x	-4	-2	0	2	4
y					

15 $y = 3x - 6$

x	-2	-1	0	1	2
y					



16 Intersection Point (☐ , ☐)

Linear Relationships - Further Straight Lines (Sheet 1)

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Complete the table of values then plot the points. Draw the line with arrow heads and the equation on the line. With values in the table that can't be plotted, construct the line with the points that you can plot.

1 $y = -2x$

x	-4	-2	0	2	4
y					

2 $y = x + 1$

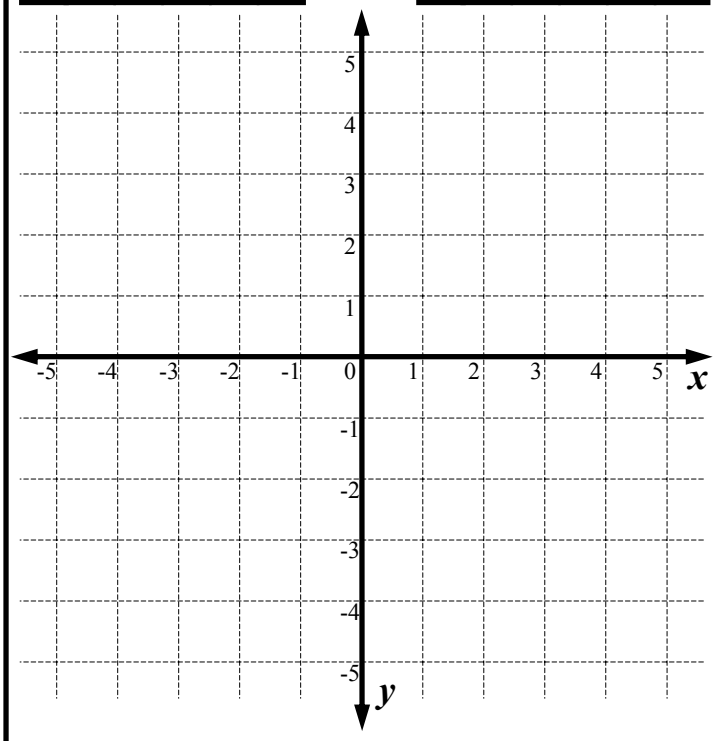
x	-4	-2	0	2	4
y					

3 $y = -\frac{1}{2}x + 1$

x	-4	-2	0	2	4
y					

4 $y = -2x - 5$

x	-4	-2	0	2	4
y					



5 Lines in Q.1 and Q.4 both have '-2' in front of the x. Look at the lines and describe a common feature. _____

6 Lines in Q.2 and Q.3 both have '+1' in the equation. Look at the lines, describe a common feature. _____

Use the top grid, in the next column for Q.7, 8 & 9. below

7 $y = 3x - 6$

x	-4	-2	0	2	4
y					

8 $y = 2x + 1$

x	-4	-2	0	2	4
y					

9 $y = x$

x	-4	-2	0	2	4
y					

Use the lower grid, in the next column for these below

10 $y = -2x + 1$

x	-4	-2	0	2	4
y					

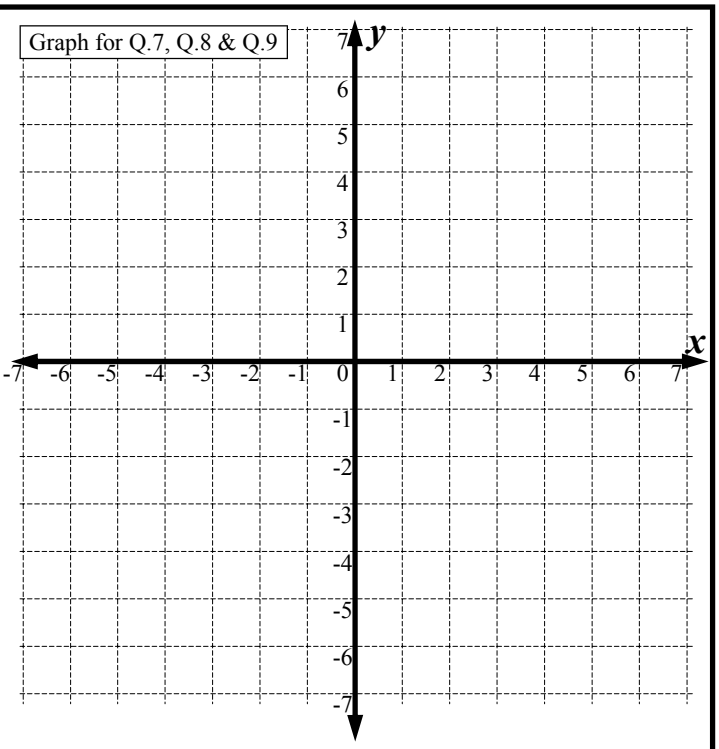
11 $y = -3x - 6$

x	-4	-2	0	2	4
y					

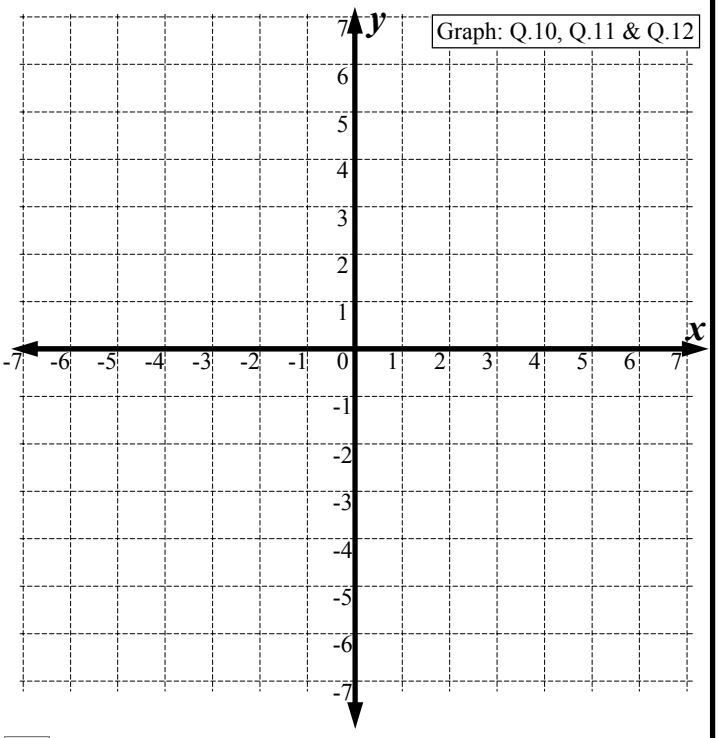
12 $y = -x$

x	-4	-2	0	2	4
y					

Graph for Q.7, Q.8 & Q.9



Graph: Q.10, Q.11 & Q.12



13 What do all the lines in the top grid have in common? _____

14 What do all the lines in the lower grid have in common? _____

15 Complete: A negative number in front of the x means that the line 'goes up' to the _____

16 Complete: A positive number in front of the x means that the line 'goes up' to the _____

17 Some lines are more steep than others. Which is more steep … $y = 4x$ or $y = 3x$? _____

Linear Relationships - Further Straight Lines (Sheet 2)

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Complete the table of values then plot the lines. These include equations involving fractions/denominators.

Colour same as line colour

1 $y = x - 9$

x	-1	0	1	2	3
y					

2 $y = \frac{x}{3} + 2$

x	-6	-3	0	3	6
y					

3 $y = -4x - 6$

x	-3	-2	-1	0	1
y					

4 $y = \frac{x + 10}{2}$

x	-4	-2	0	2	4
y					

5 $y = \frac{-x + 2}{2}$

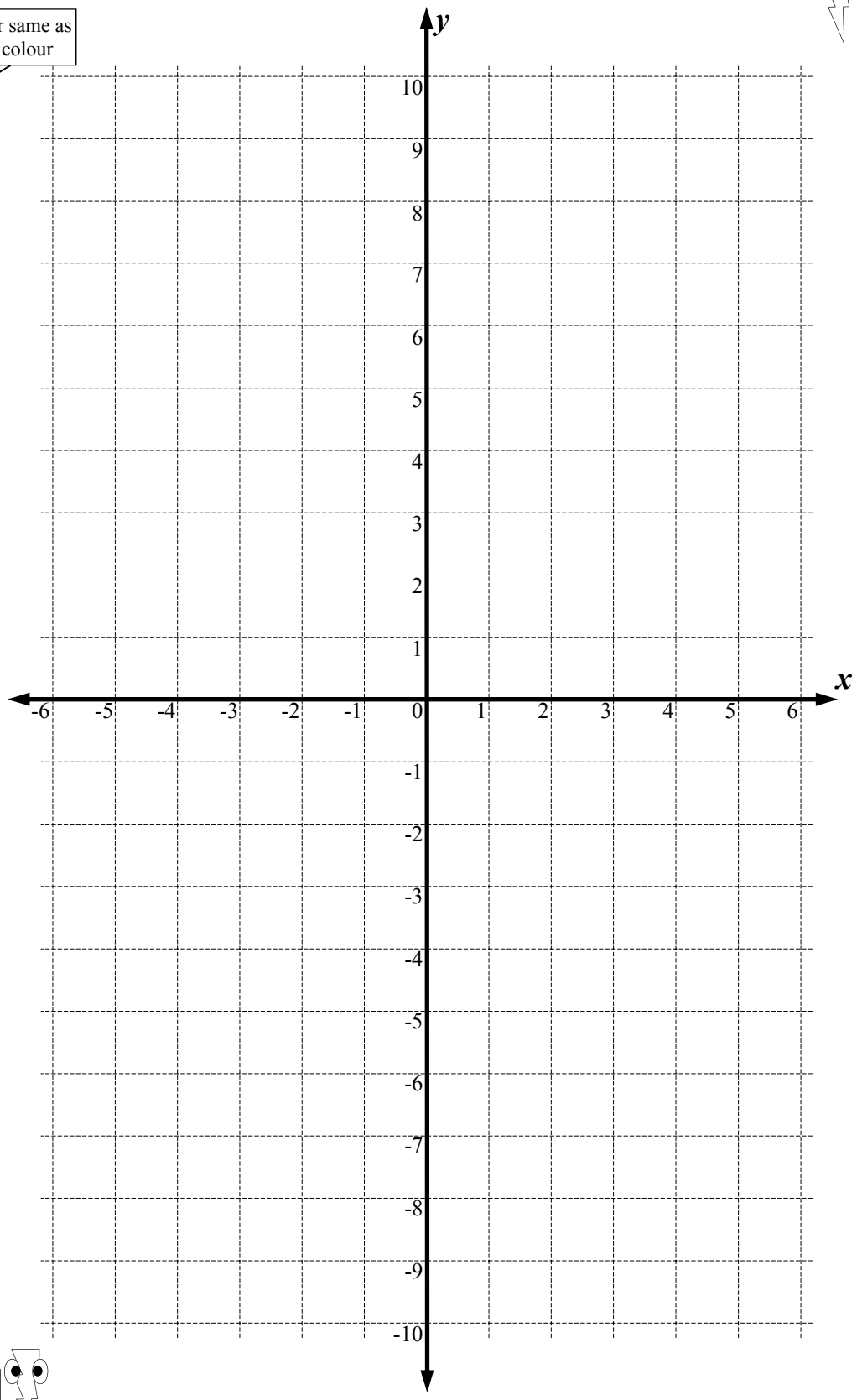
x	-4	-2	0	2	4
y					

6 $y = 3x + 6$

x	-3	-2	-1	0	1
y					

7 $y = \frac{-x - 9}{2}$

x	-5	-3	-1	1	3
y					



Find the points of intersection for these lines.

8 $y = \frac{x + 10}{2}$, $y = \frac{-x + 2}{2}$ (,)

9 $y = \frac{-x - 9}{2}$, $y = 3x + 6$ (,)

10 $y = \frac{-x - 9}{2}$, $y = x - 9$ (,)

Linear Relationships - Cost Functions (Sheet 1)

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Complete the table of values for these Cost Functions.

1 *Consultation Fees:* $C = 60n + 120$

n	1	2	3	4	5
C					

2 *Labour Costs:* $C = 720h + 8\,000$

h	0	10	20	30	40
C					

3 *Unit Rates:* $R = 250e + 320$

e	0	5	10	15	20
R					

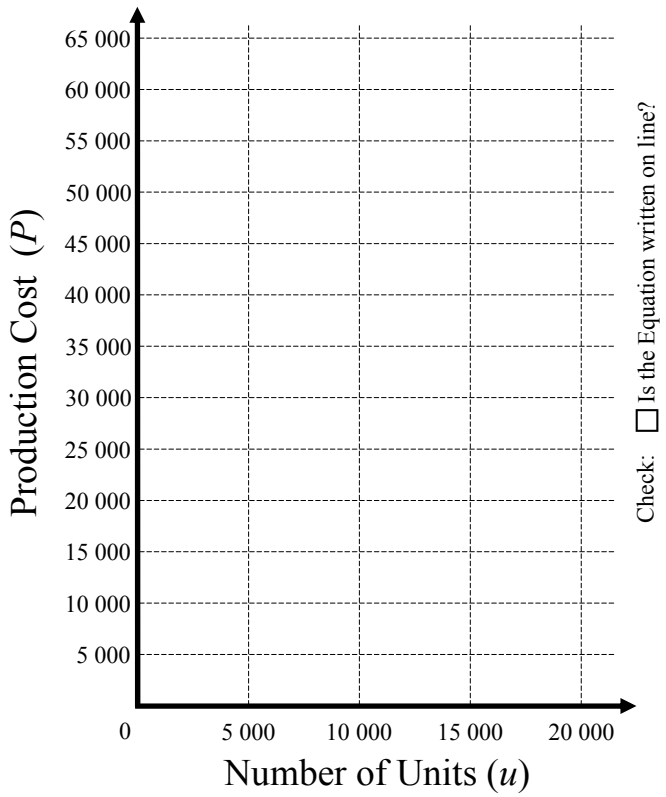
4 *Advertising Outlay :* $A = 180p + 12\,000$

p	0	60	120	180	240
A					

OK it's plotting time. Complete the cost function table then plot the values on the graph and construct a straight line. Write the function on the line.

5 *Production Costs :* $P = 2u + 15\,000$

u	0	5 000	10 000	15 000	20 000
P					



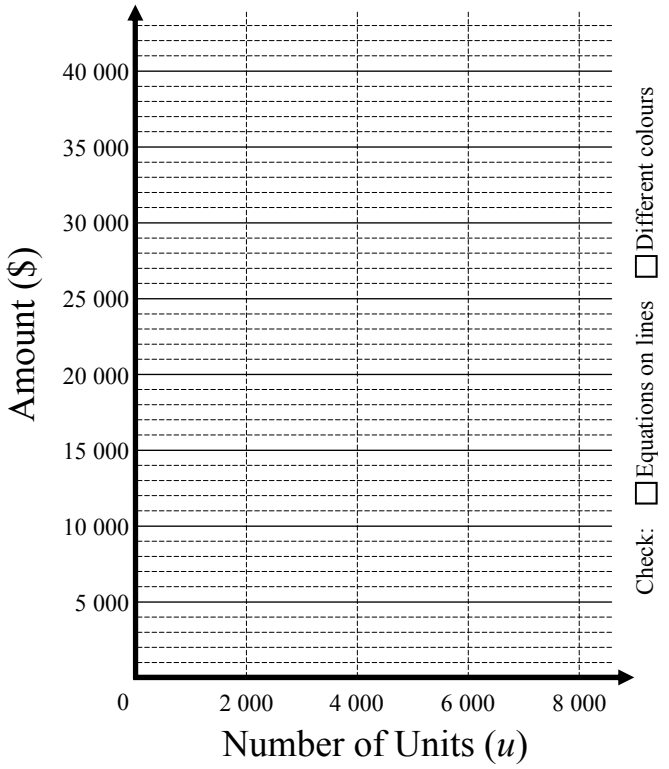
Complete the tables for these Cost Functions, plot them both on the same graph. Use two colours.

6 *Manufacturing Cost :* $C = 3u + 14\,000$

u	0	2 000	4 000	6 000	8 000
P					

7 *Revenue :* $R = 5u$

u	0	2 000	4 000	6 000	8 000
R					



8 Estimate the cost to make 3 000 units. _____ $ sign

9 Estimate the revenue on selling 3 000 units. _____

10 Calculate the loss made on selling 3 000 units.

Loss = _____ (Cost (Q.8)) - _____ (Revenue (Q.9)) = _____

11 Estimate how many units have to be sold for the company to break even. _____

12 Calculate the loss made on selling 6 000 units.

Loss = _____ - _____ = _____

13 Calculate the profit made on selling 8 000 units.

Profit = _____ - _____ = _____

Linear Relationships - Cost Functions (Sheet 2)

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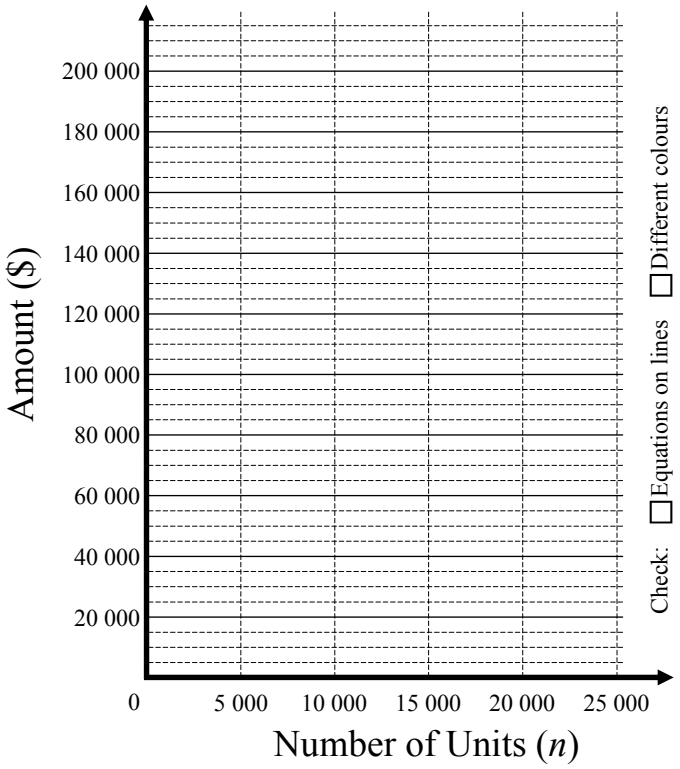
Complete the tables for these Cost Functions, plot them both on the same graph. Use two colours.

1 *Advertising Budget : $A = 5n + 45\,000$*

n	0	5 000	10 000	15 000	20 000
A					

2 *Sales Revenue : $S = 8n$*

n	0	5 000	10 000	15 000	20 000
S					



3 If the advertising budget is \$105 000, estimate the number of units produced. _____

4 If sales revenue is \$140 000, estimate how many units are produced. _____

5 Calculate the loss made on selling 5 000 units.

Loss = _____ - _____ = _____

6 How many units have to be sold for the company to break even? _____

7 Calculate the profit made on selling 20 000 units.

_____

8 Estimate the profit made on selling 25 000 units.

_____

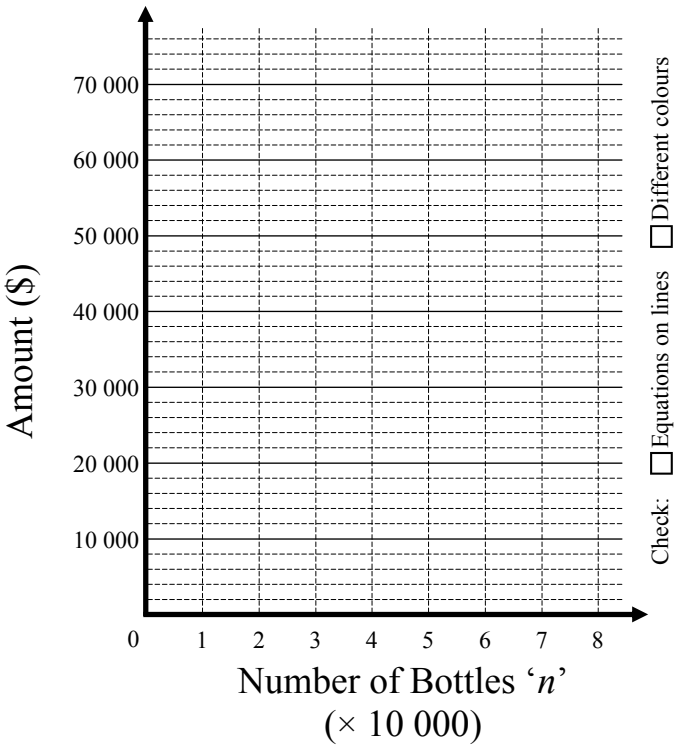
A soft drink manufacturer has cost and revenue functions for daily production. Extend the lines the full extent of the graph to answer the questions.

9 *Daily Production Cost : $P = 0.5n + 20\,000$*

n	0	10 000	20 000	30 000	40 000
P					

10 *Sales Revenue : $S = 0.9n$*

n	0	10 000	20 000	30 000	40 000
S					



11 Estimate the production cost for producing:

_____ 25 000 bottles _____ 60 000 bottles

12 Estimate the bottles produced which gives revenue of:

_____ \$30 000 in sales _____ \$50 000 in sales

13 Estimate how many bottles have to be sold for the company to break even? _____

14 Estimate the profit (or loss) on bottle production of:

15 000 _____

80 000 _____

Linear Relationships - Horizontal and Vertical Line Equations (Sheet 1)

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Plot these points on the number plane. Draw a line through the points and write the name of the line, on the line. Remember to extend line, arrowheads.

1 Points: (3 , 2), (3 , -4) and (3, 5) $x = 3$

Plot these points on the plane below

Write name on the line

$x = 3$ cuts the x-axis at: ☐

$x = 3$ is: ○ horizontal or ○ vertical

2 Points: (-2 , 1), (-2 , 3) and (-2, -5) $x = -2$

Write name on the line

$x = -2$ cuts the x-axis at: ☐

$x = -2$ is: ○ horizontal or ○ vertical

3 Points: (4 , -4), (0 , -4) and (-3, -4) $y = -4$

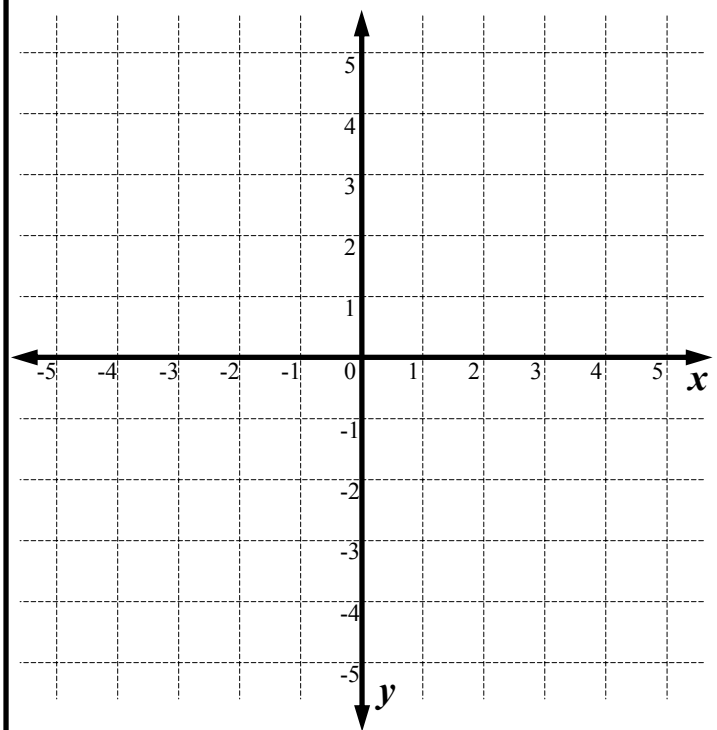
$y = -4$ cuts the y-axis at: ☐

$y = -4$ is: ○ horizontal or ○ vertical

4 Points: (-3 , 2), (-1 , 2) and (2, 2) $y = 2$

$y = 2$ cuts the y-axis at: ☐

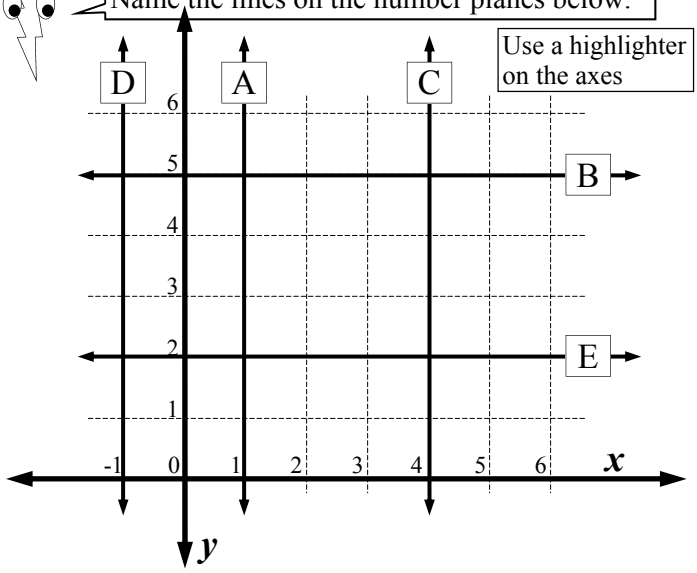
$y = 2$ is: ○ horizontal or ○ vertical



Check:
- ☐ Arrow heads on each line?
- ☐ Is the Equation written on each line?
- ☐ Is each line extended fully on the plane?

Name the lines on the number planes below.

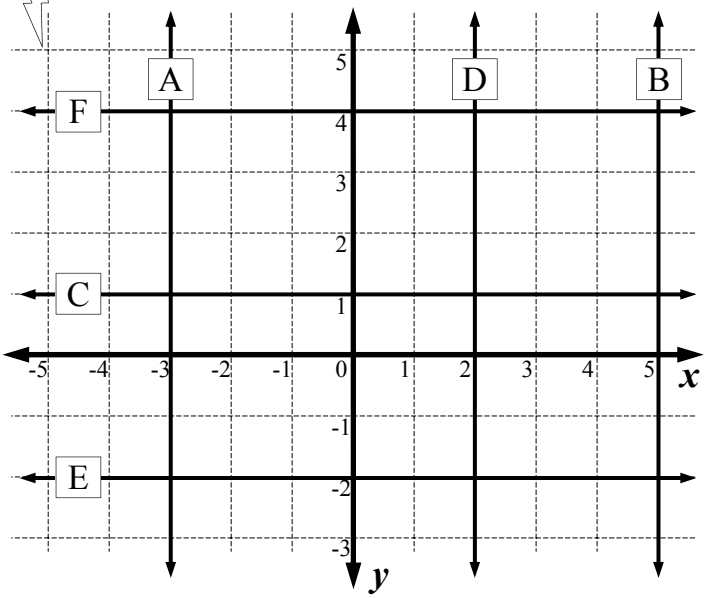
Use a highlighter on the axes



5 A _____ B _____

C _____ D _____ E _____

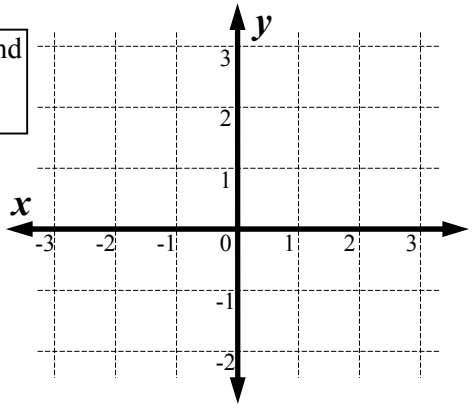
Now try these, use a highlighter on the axes.



6 A _____ B _____ C _____

D _____ E _____ F _____

Plot $y = -2$ and $x = 2$ on the plane.



7

Check:
- ☐ Arrow heads?
- ☐ Equation on lines?
- ☐ Extended fully?

Linear Relationships - Horizontal and Vertical Line Equations (Sheet 2)

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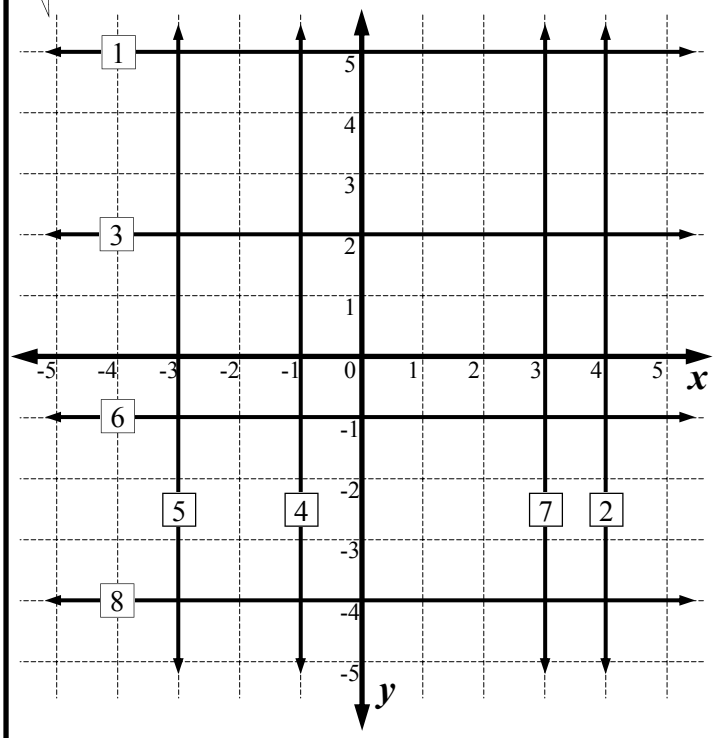
Name the lines shown. Use the spaces below the number plane then label the lines on the plane also.



1 $y =$ ☐ 2 $x =$ ☐ 3 _____ 4 _____

5 _____ 6 _____ 7 _____ 8 _____

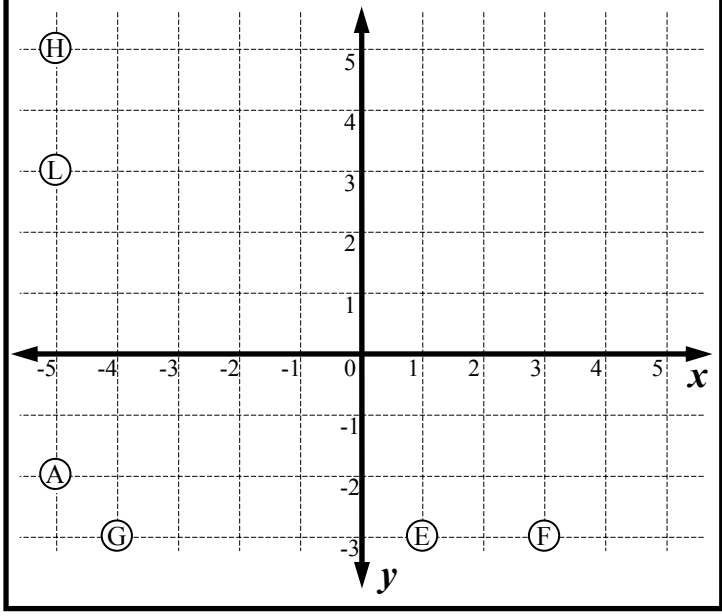
9 What is the equation for the y-axis? _____

10 What is the equation for the x-axis? _____

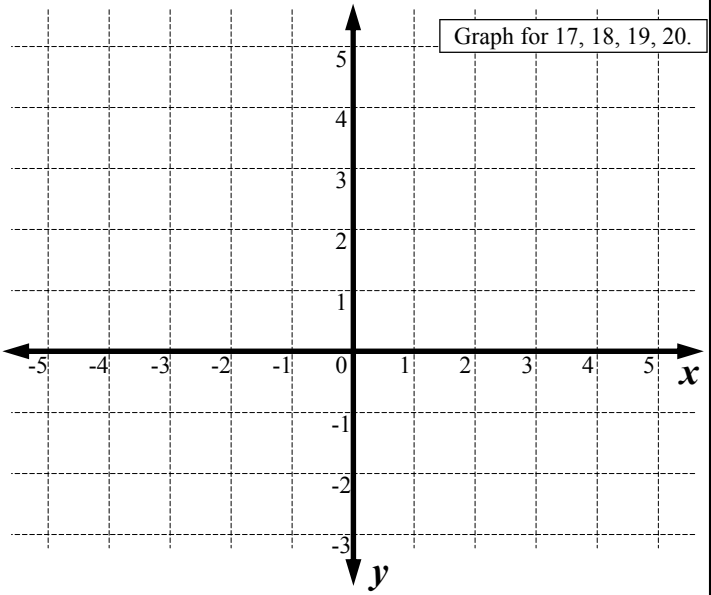
Draw these lines on the Cartesian plane, show arrow heads and labels. After you draw the line it will pass through a letter, write it in the box.

11 $y = -2$ ☐ Letter 12 $x = 3$ ☐ Letter 13 $x = -4$ ☐ Letter

14 $y = 5$ ☐ Letter 15 $x = 1$ ☐ Letter 16 $y = 3$ ☐ Letter



Draw the lines on the grid. Once drawn there will be four intersection points. Give the coordinates for these points as well as the name of shape formed.

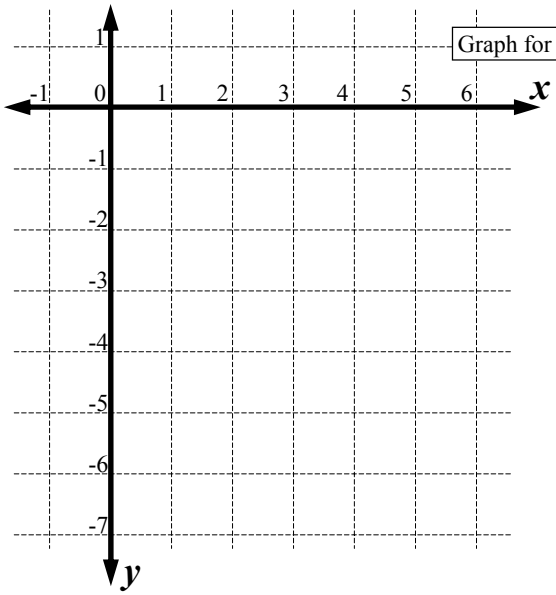


17 $y = 2$ 18 $y = -1$

19 $x = -4$ 20 $x = 1$

21 Points: _____

22 Shape formed: _____



23 $y = -2x + 3$

x	1	2	3	4	5
y					

24 $y = -5$

25 $x = 2$

26 Points: (☐, ☐), (☐, ☐), (☐, ☐).

27 Shape formed: _____

Linear Relationships - Naming Straight Lines (Sheet 1)

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Name the straight lines, given the gradient and the y-intercept. $y = mx + b$, m = gradient & b = y-int.

1 Gradient = 4, y-int = 1. $y =$ ☐ x + ☐

2 Gradient = -3, y-int = -6. $y =$ ☐ x - ☐

3 Gradient = -2, y-int = 5. $y =$ ☐

4 y-int = 3, Gradient = -4. ☐

5 y-int = 0, Gradient = -2. ☐

When the y-int = 0, you don't include it

6 Gradient = 1, y-int = 2. ☐

Don't include '1' or '-1' just 'x' or '-x'

7 y-int = -3, Gradient = -1. ☐

8 Gradient = ½, y-int = 0. ☐

9 Gradient = 2, y-int = -6. ☐

10 y-int = 0, Gradient = -1. ☐

This time you are given the equation, you need to find the gradient and y-intercept.

11 $y = 3x + 2$ $m =$ ☐ $b =$ ☐

12 $y = x - 7$ $b =$ ☐ $m =$ ☐

13 $y = -4x$ $b =$ ☐ $m =$ ☐

14 $y = ½x + 2$ $m =$ ☐ $b =$ ☐

15 $y = 10 - 2x$ $b =$ ☐ $m =$ ☐

If lines are parallel they have the same gradient. Find the equations of these lines.

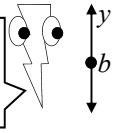
16 Find the equation of the line with a y-int of 6 which is parallel to $y = 3x + 1$. $y =$ ☐ x + ☐

17 Find the equation of the line with a y-int of 4 which is parallel to $y = -2x - 3$. ☐

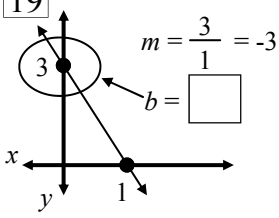
17 Find the equation of the line with a y-int of -3 which is parallel to $y = -3x$. ☐

18 Find the equation of the line with a y-int of 0 which is parallel to $y = 5x + 2$. ☐

You are given the gradient, read off the y-int and then construct the equation for the line. 'b' is where the line 'cuts' the y-axis. (vertical axis)



19

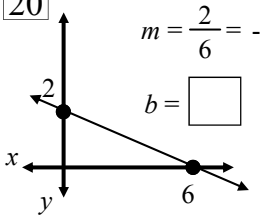


$m = \frac{3}{1} = -3$

$b =$ ☐

$y =$ ☐ x + ☐

20



$m = \frac{2}{6} = -\frac{1}{3}$

$b =$ ☐

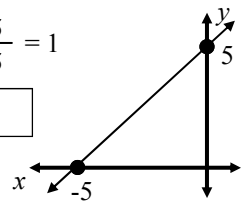
$y =$ ☐ x + ☐

21

$y =$ ☐

$m = \frac{5}{5} = 1$

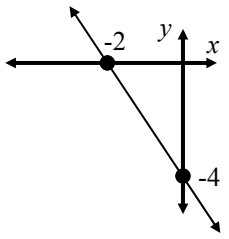
$b =$ ☐



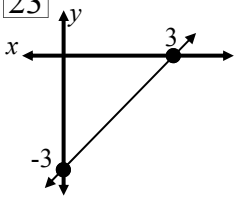
22 ☐

$m = \frac{4}{2} = -2$

$b =$ ☐



23



$m = \frac{3}{3} = 1$

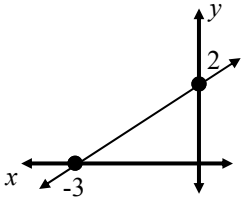
$b =$ ☐

☐

24 ☐

$m = \frac{2}{3}$

$b =$ ☐



Cartesian Plane - Gradient (Sheet 1)

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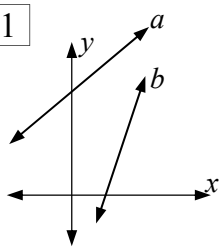
State whether the following have a positive or negative gradient.

Up to the left is -ve

Up to the right is +ve

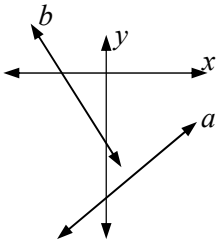
1



a) _____ Positive or Negative

b) _____ Positive or Negative

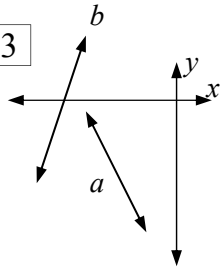
2



a) _____

b) _____

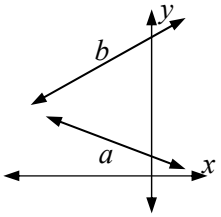
3



a) _____

b) _____

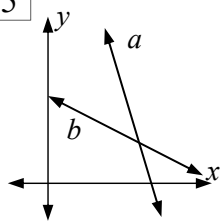
4



a) _____

b) _____

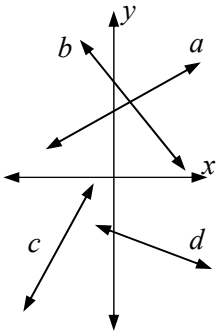
5



a) _____

b) _____

6



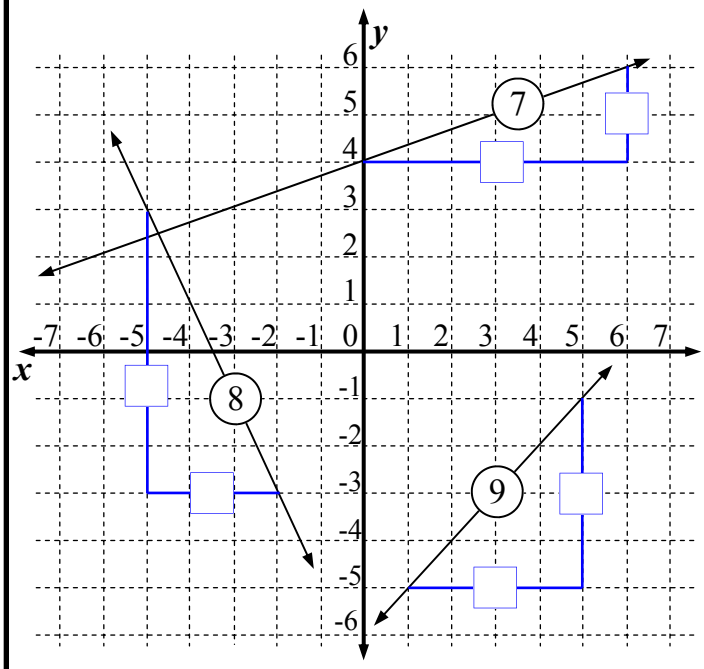
a) _____

b) _____

c) _____

d) _____

These lines have gradient triangles installed! Write the rise and run in the boxes on the triangles sides, then find the gradient. Make sure you remember to check for positive and negative gradient at the end.



7 $m = \frac{\square}{\square} = \frac{\square}{\square}$

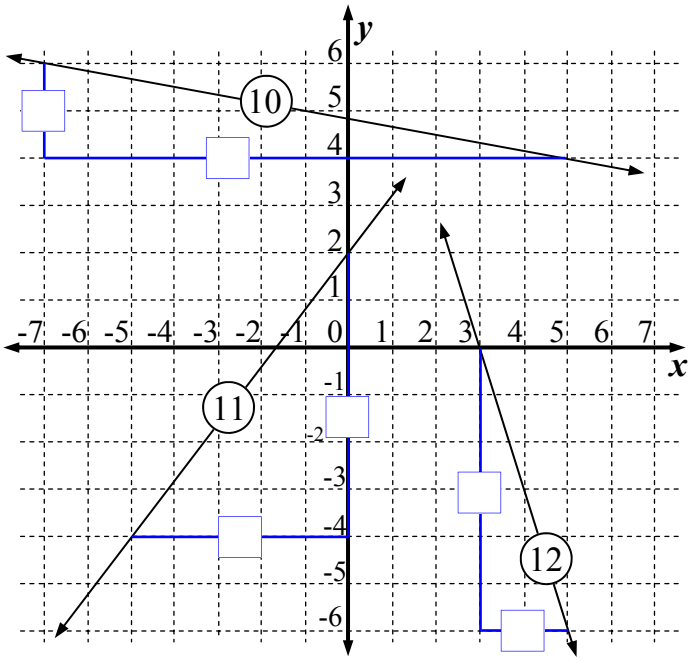
8 $m = \frac{\square}{\square} = \square$

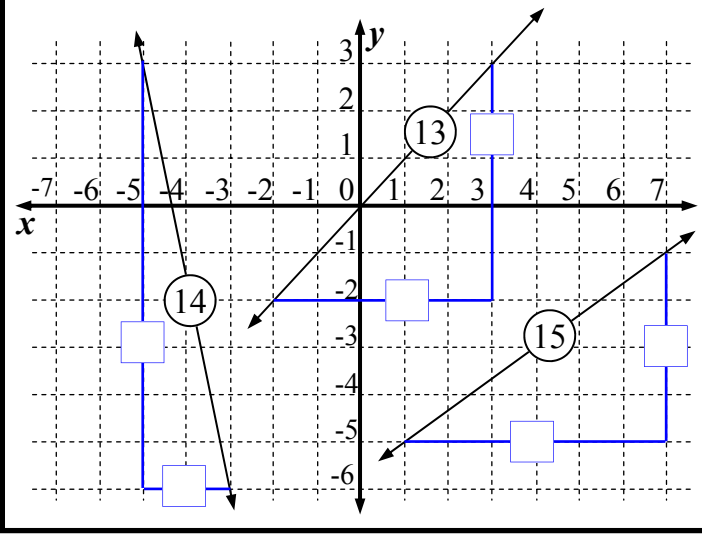
9 $m = \frac{\square}{\square} = \square$

10 $m = \frac{\square}{\square} = \frac{\square}{\square}$

11 $m = \frac{\square}{\square}$

12 $m = \frac{\square}{\square} = \square$





13 $m = \frac{\square}{\square} = \square$

14 $m = \frac{\square}{\square}$

15 $m = \frac{\square}{\square} = \frac{\square}{\square}$

Cartesian Plane - Gradient (Sheet 2)

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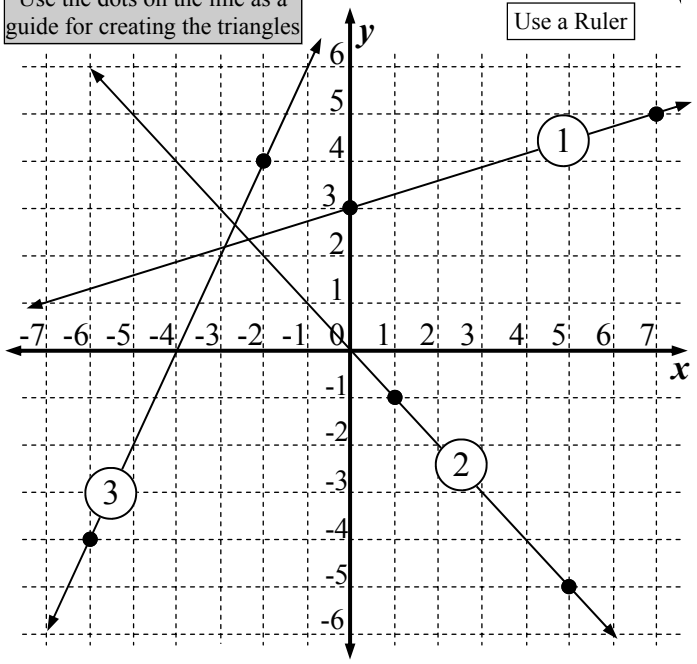
Find the gradient for these lines. Create gradient triangles and remember your positive and negatives.

Use the dots on the line as a guide for creating the triangles

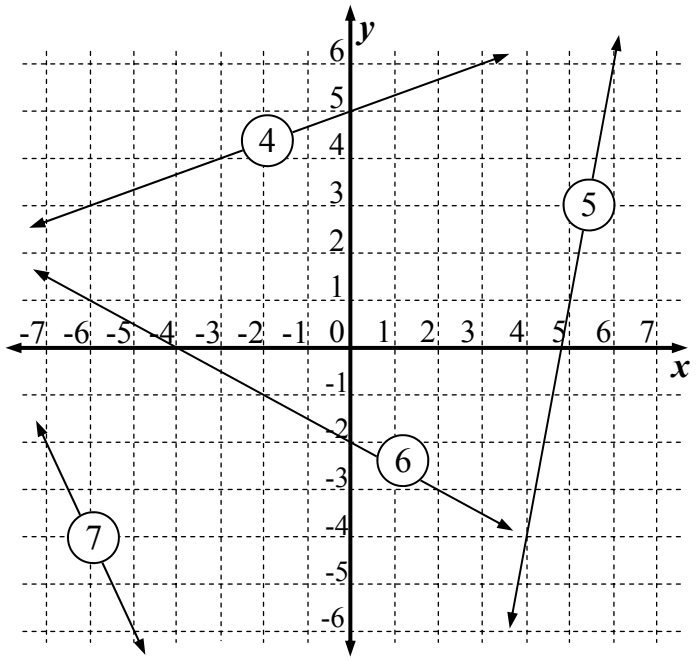
Use a Ruler



1 $m = \frac{\square}{\square}$

2 $m = \frac{\square}{\square} = \square$

3 $m = \frac{\square}{\square} = \square$

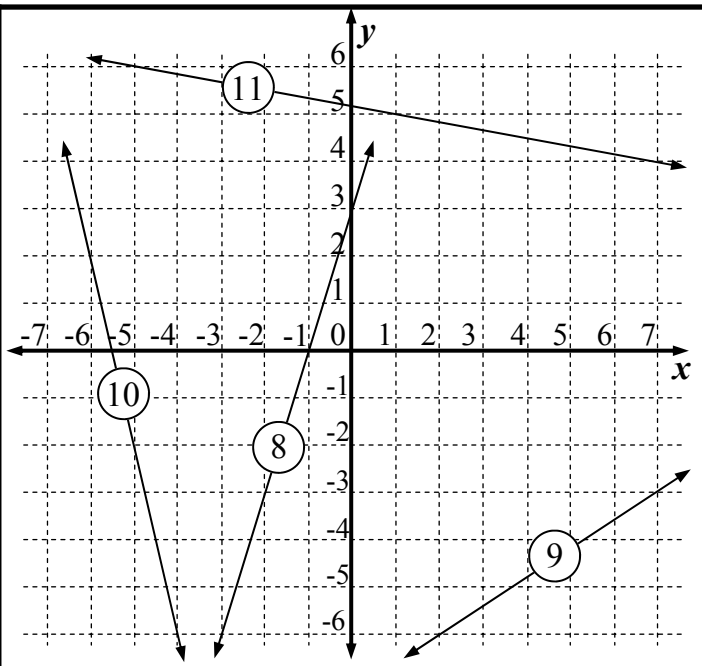


4 $m = \frac{\square}{\square} = \frac{\square}{\square}$

5 $m = \frac{\square}{\square} = \square$

6 $m = \frac{\square}{\square} = \frac{\square}{\square}$

7 $m = \frac{\square}{\square} = \square$

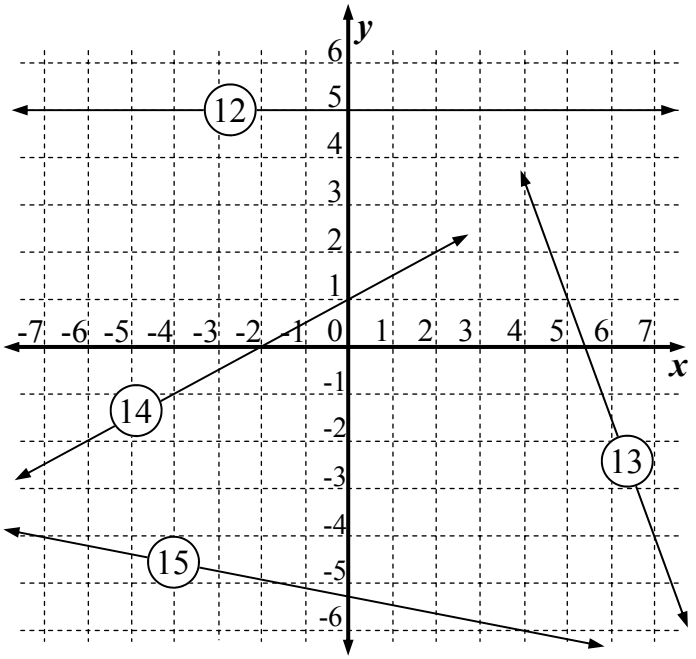


8 $m =$ _____

9 $m =$ _____

10 _____

11 _____



12 _____

13 _____

14 _____

15 _____

Linear Relationships - Naming Straight Lines - Find *m* (Sheet 1)

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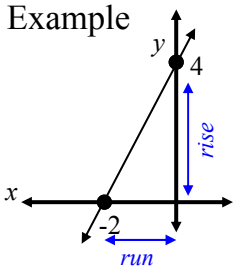
Find the gradient and *y*-intercept, then the equation for these straight lines. All gradients are positive.

Example

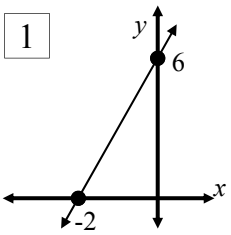


$m = \frac{\text{rise}}{\text{run}} = \frac{4}{2} = 2$ — Positive as up to the right; Don't use -2

$b = 4$ — Cuts the *y*-axis at 4

$y = 2x + 4$ (m, b)

1

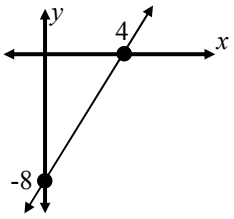


$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square x + \square$

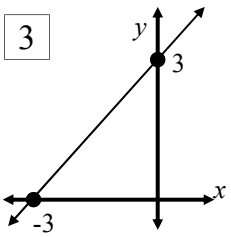
2

$m = \frac{\square}{\square} = \square$ $\quad b = \square$



$y = \square x - \square$

3

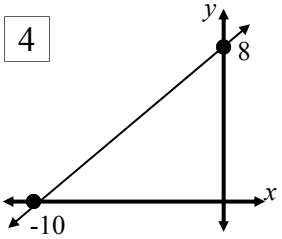


$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square$

These gradients will be fractions, fully simplify them.

4



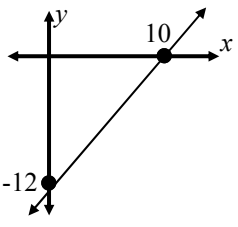
$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$

Calc: 8 ▪ 1 0 =

$y = \frac{\square}{\square} x + \square$

5

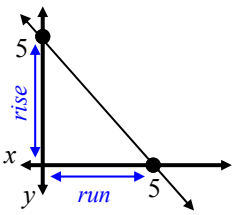
$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$



$\square$

All the gradients are up to the left, so all are negative gradients. Find the equations for these lines.

Example



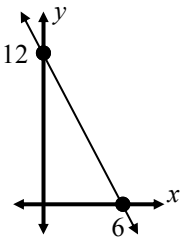
$m = \frac{\text{rise}}{\text{run}} = \frac{5}{5} = -1$ — Negative as up to the left

$b = 5$ — Cuts the *y*-axis at 5

$y = -x + 5$ — Write -*x* not -1*x* (m, b)

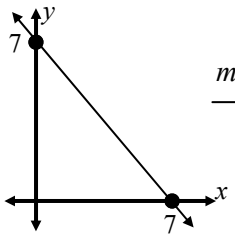
6

$m = \frac{\square}{\square} = \square$ $\quad b = \square$



$y = \square$

7

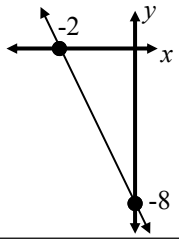


$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$\square$

8

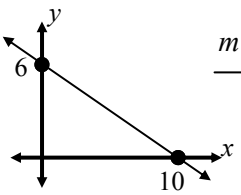
$m = \frac{\square}{\square} = \square$ $\quad b = \square$



$\square$

These gradients will be fractions, fully simplify them.

9

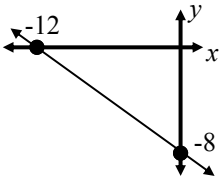


$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$

$y = \frac{\square}{\square} x + \square$

10

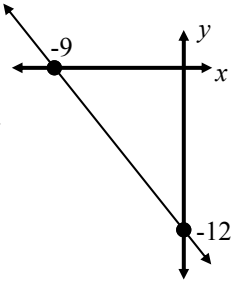
$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$



$\square$

11

$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$



$\square$

Linear Relationships - Naming Straight Lines - Find *m* (Sheet 2)

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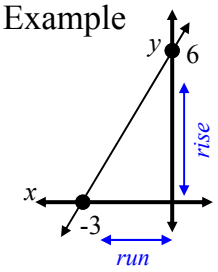
Find the gradient and *y*-intercept, then the equation for these straight lines. Watch for gradient sign.

Example

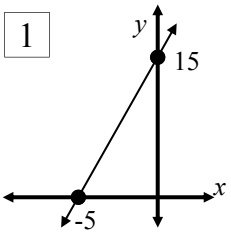


$m = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2$ — Positive as up to the right; Don't use -3

$b = 6$ — Cuts the *y*-axis at 6

$y = 2x + 6$ (m, b)

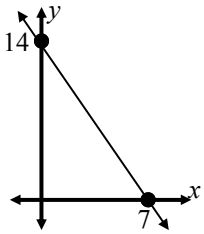
1



$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square x + \square$

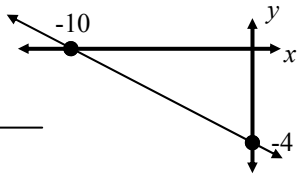
2



$m =$ _____ $\quad b =$ _____

$y =$ _____

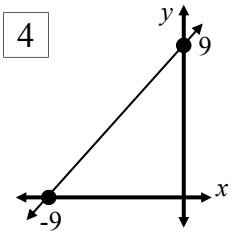
3 Fraction alert!



$m =$ _____ $\quad b =$ _____

$y =$ _____

4



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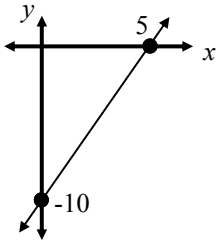
5



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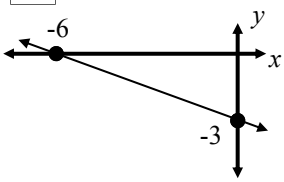
6



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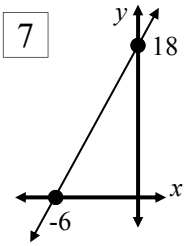
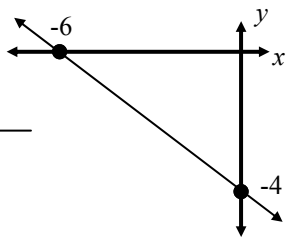
7



_____ _____

_____

8



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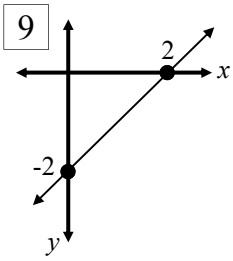
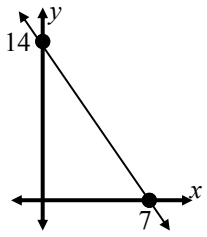
9



_____ _____

_____

10



_____ _____

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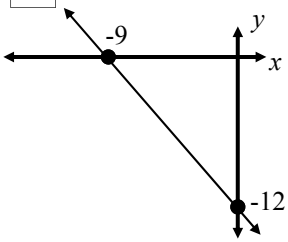
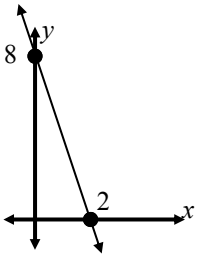
11



_____ _____

_____

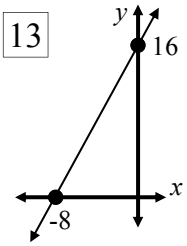
12



_____ _____

_____

13



_____ _____

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Linear Relationships - Further Naming Straight Lines (Sheet 1)

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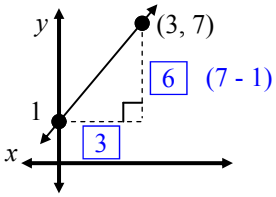
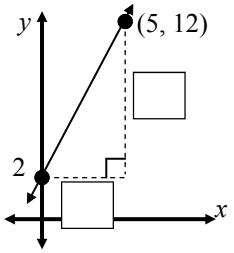
The gradient triangle is constructed. Write in the rise and run. All values are positive, no negatives!

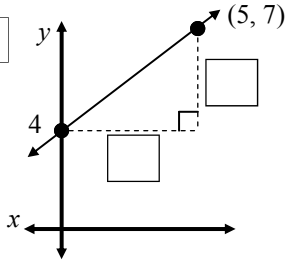
Example



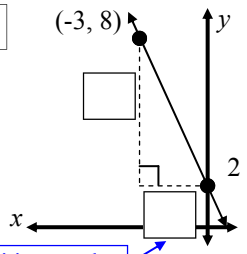
1



2

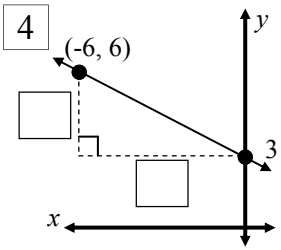


3

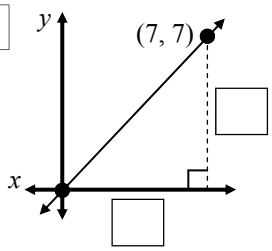


positive number

4

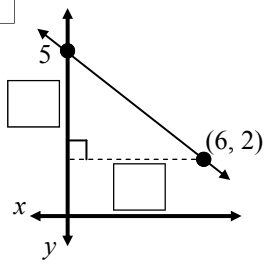


5

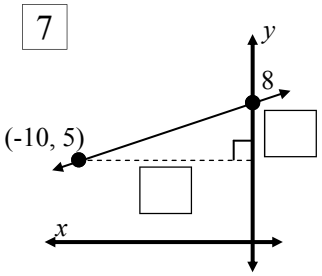


This time the rise is written on the *y*-axis

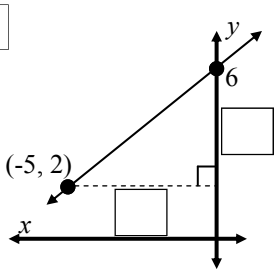
6



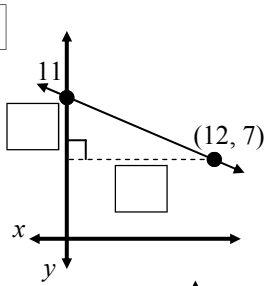
7



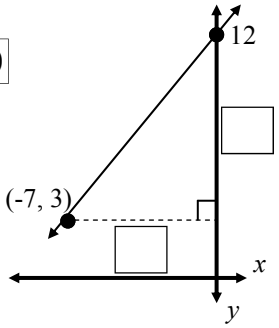
8



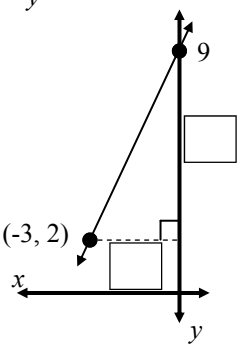
9



10



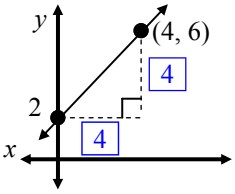
11



Find the rise, run, gradient, *y*-intercept and equation.

Example

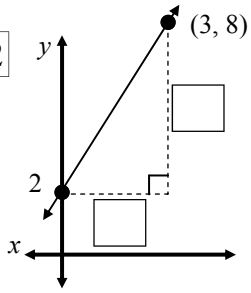
Gradient: ⊕ ⊖



$m = \frac{4}{4} = 1$ $\quad b = 2$

$y = x + 2$

12



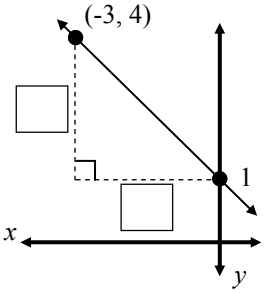
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square x + \square$

13

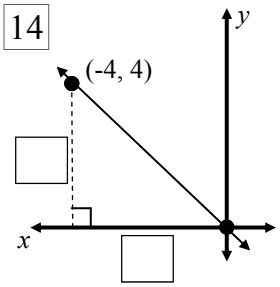
Gradient: ⊕ ⊖



$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square x + \square$

14



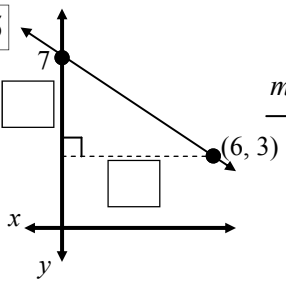
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $\quad b = \square$

$y = \square$

This time the gradient is a fraction

15



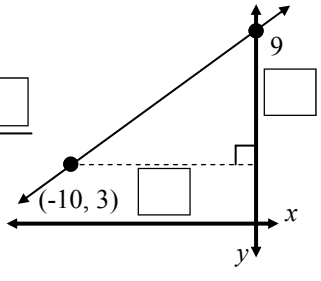
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$

$y = \frac{\square}{\square} x + \square$

16

Gradient: ⊕ ⊖



$m = \frac{\square}{\square} = \frac{\square}{\square}$ $\quad b = \square$

$y = \frac{\square}{\square} x + \square$

Linear Relationships - Further Naming Straight Lines (Sheet 2)

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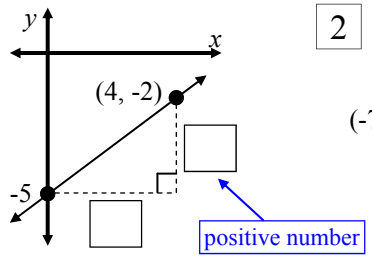
We are below the x-axis this time. Find the rise and run. Keep all your numbers positive.

1



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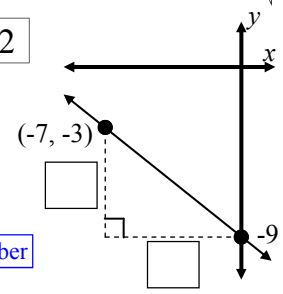

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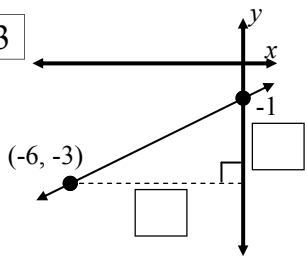

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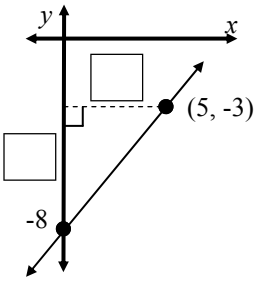

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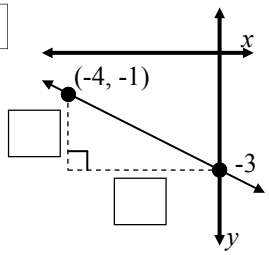

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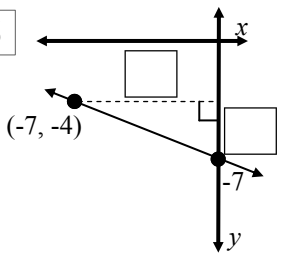
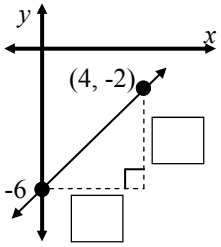


Find the rise, run, gradient, y-intercept and equation.

7



Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

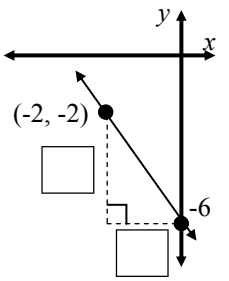
$y = x -$ ☐

8

Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

$y = \square x -$ ☐



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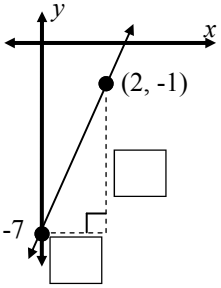


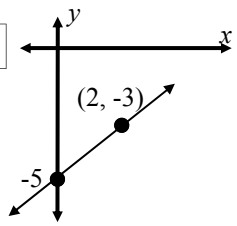
Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

$y = \square x -$ ☐

Find the rise, run, gradient, y-intercept and equation. This time you have to construct the triangle yourself.

10



Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

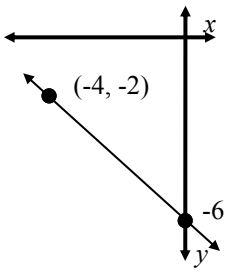
$y =$ ☐

11

Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \square$ $b = \square$

$y =$ ☐



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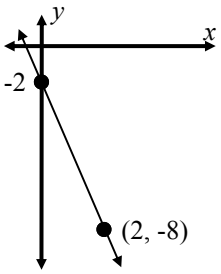


Gradient: ⊕ ⊖

$m =$ _____ $b =$ _____

$y =$ _____

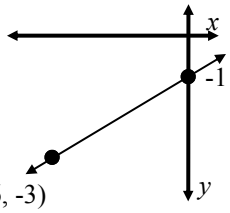
This time the gradient is a fraction

13

Gradient: ⊕ ⊖

$m = \frac{\square}{\square} = \frac{\square}{\square}$ $b = \square$

$y = \frac{\square}{\square} x -$ ☐



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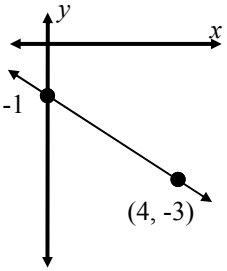


Gradient: ⊕ ⊖

_____ _____

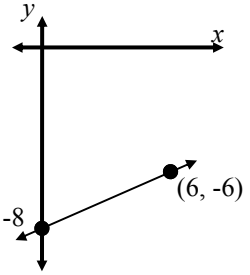
_____

15

Gradient: ⊕ ⊖

_____ _____

_____



Linear Relationships - Proving if a Point is on a Line (Sheet 1)

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Given x, find the value of y.

Example $y = 3x + 2$ $x = 2$

$y = 3x + 2$ ← Write equation

$y = 3 \times 2 + 2$ ← Substitute $x = 2$

$y = 8$ ← Answer

1 $y = 7x - 5$ $x = 3$

$y = 7x - 5$ ← Write equation

$y = 7 \times \square - 5$ ← Substitute $x = 3$

$y = \square$ ← Answer

2 $y = x - 7$ $x = 5$

____ ← Write equation

____ ← Substitute $x = 5$

____ ← Answer

3 $y = -2x + 4$ $x = 6$

____

____

____

4 $y = 12 - 3x$ $x = 0$

____

____

____

5 $y = -x - 9$ $x = 4$

____

____

____

Given an x co-ordinate, give the co-ordinates for a point on a line.

Example $y = 2x + 7$ $(5, y)$

$y = 2x + 7$ ← Write equation

$y = 2 \times 5 + 7$ ← Substitute $x = 5$

$y = 17$ ← y-value

$\therefore (x, y) = (5, 17)$ ← Point

6 $y = 4x - 8$ $(3, y)$

____ ← Write equation

____ ← Substitute $x = 3$

____ ← y-value

$\therefore (x, y) = (3, \square)$ ← Point

7 $y = -5x + 7$ $(-4, y)$

____

____

____

____

8 $y = -x - 11$ $(-1, y)$

____

____

____

____

9 $y = 15 - 5x$ $(-7, y)$

____

____

____

____

Find if the point is on the line.

Example $y = 4x + 5$ $(2, 11)$

x y ← Label x, y.

$y = 4x + 5$ ← Equation

$11 = 4 \times 2 + 5$ ← Substitute $x = 2, y = 11$

$11 \neq 13$ ← Solve RHS

LHS ≠ RHS. ∴ Not on line.

10 $y = 6x + 2$ $(5, 28)$

Label x, y. → __ __

____ ← Write equation

$\square = 6 \times \square + 2$ ← Substitute $x = 5, y = 28$

____ ← Solve Use = or ≠

____

Using LHS/RHS State if point on the line

__ __

11 Does the point (8, 17) lie on the line $y = 3x - 7$?

____ ← Equation

____ ← Substitute

____ ← Solve

____

__ __

12 Does the point (-3, -13) lie on the line $y = -4x - 25$?

____

____

____

____

Linear Relationships - Proving if a Point is on a Line (Sheet 2)

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Find if the point is on the line.

Example $2x + y = 2$ $(-2, 6)$

x y — Label x, y.

$2x + y = 2$ — Equation

$2 \times (-2) + 6 = 2$ — Substitute $x = -2, y = 6$

$2 = 2$ — Solve

LHS = RHS. ∴ Point on line.

1 $5y + 11 = 6x$ $(5, 6)$

Label x, y. → ☐ ☐

Write equation

Substitute $x = 5, y = 6$

Solve Use = or ≠

Using LHS/RHS State if point on the line

☐ ☐

2 Does the point (-2, 4) lie on the line $10y + 3x = 46$?

Equation

Substitute

Solve

Using LHS/RHS State if point on the line

☐ ☐

3 Does the point (-7, 9) lie on the line $13 - 2x = 3y$?

This time you have two lines and you have to check if they intersect at the given point.

4 Do the lines $y = 2x + 6$ and $y = 5x - 15$ intersect at (7, 20)?

☐ ☐

Write $y = 2x + 6$

Substitute $x = 7, y = 20$

Solve Use = or ≠

LHS ☐ RHS ∴

= or ≠ Point is or not on line

Write $y = 5x - 15$

Substitute $x = 7, y = 20$

Solve Use = or ≠

LHS ☐ RHS ∴

= or ≠ Point is or not on line

Yes they, or no they don't, intersect at (7, 20)

5 Do the lines $y = 15 - 3x$ and $y = -2x + 12$ intersect at (3, 6)?

☐ ☐

6 Do the lines $3y - 2x = -5$ and $y = -3x + 8$ intersect at (7, 3)?

☐ ☐

7 Do the lines $y = -x + 12$ and $y = 5x - 12$ intersect at (4, 8)?

☐ ☐

Linear Relationships - Find A Word

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Find the words in the puzzle from the wordlist.



O	O	L	V	S	P	E	R	D	W	A	R	R	A	C
L	P	P	E	W	E	T	A	N	I	D	R	O	O	C
T	B	L	R	O	P	U	O	D	R	I	A	S	L	G
P	R	N	T	R	L	T	D	A	A	X	T	E	R	E
L	E	E	I	R	L	I	W	P	E	G	B	R	U	G
T	A	G	C	A	R	T	E	S	I	A	N	L	L	R
E	K	A	A	G	A	S	L	E	L	L	A	R	A	P
Q	E	T	L	E	L	B	A	T	P	V	L	E	T	G
U	V	I	R	A	P	U	P	P	P	L	P	I	N	R
A	E	V	I	T	I	S	O	P	L	O	F	S	O	A
T	N	E	I	D	A	R	G	P	L	O	S	O	Z	E
I	X	N	O	I	T	C	E	S	R	E	T	N	I	N
O	I	N	T	E	R	C	E	P	T	P	P	L	R	I
N	F	U	N	C	T	I	O	N	I	G	I	R	O	L
T	I	V	E	P	O	S	S	T	R	A	I	G	H	T

WORDLIST

HORIZONTAL	DRAW	NEGATIVE	STRAIGHT	ORIGIN
COORDINATE	TABLE	EQUATION	PLOT	PROFIT
SUBSTITUTE	AXES	LINEAR	VERTICAL	VALUE
LABEL	PARALLEL	GRID	FUNCTION	CARTESIAN
BREAK-EVEN	PAIR	COST	SLOPE	ARROWS
No Hyphen in grid	GRADIENT	INTERCEPT	POSITIVE	INTERSECTION

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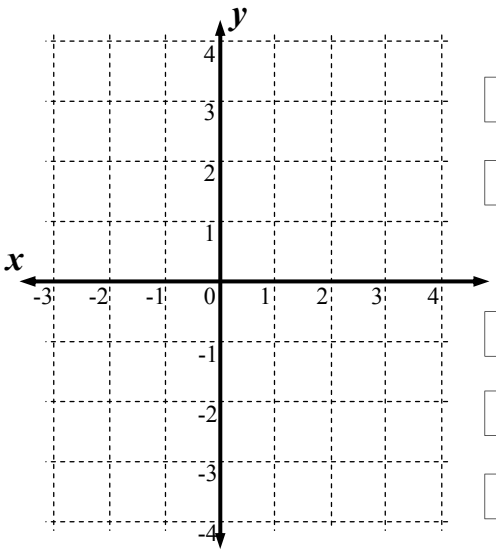
Plot the points on the number plane below.



1 Ⓐ (4, 2)

2 Ⓑ (-1, 4)

3 Ⓒ (-3, 0)

4 Ⓓ (2, -3)

5 Ⓔ (-2, -4)

Complete these table of values

6 $y = x + 5$

x	0	1	2	3	4
y					

7 $y = 3x - 7$

x	-4	-2	0	2	4
y					

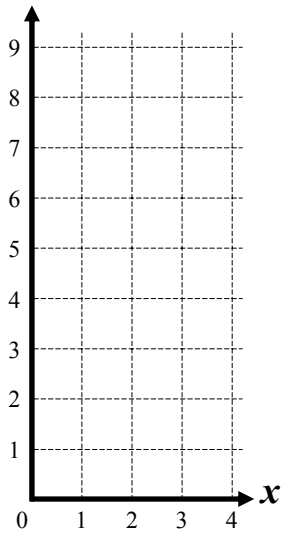
8 $y = -\frac{1}{2}x + 6$

x	-4	-2	0	2	4
y					

Plot this line on the number plane below

9 $y = 2x + 1$

x	0	1	2	3	4
y					



Plot the two lines on the number plane, then give the coordinates for the point of intersection.

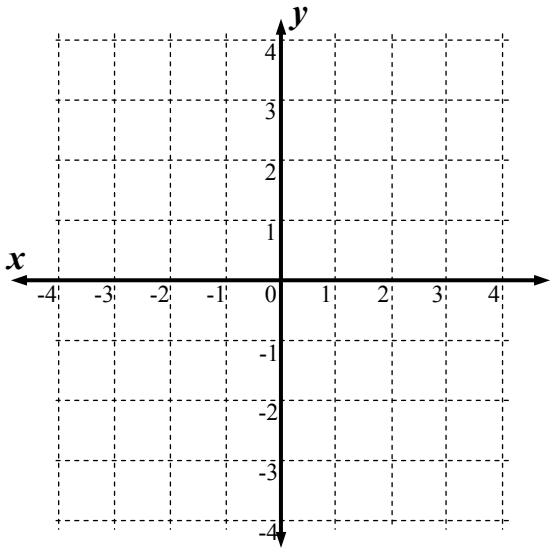
10

$y = x + 4$

x	-4	-2	0	2	4
y					

$y = -2x + 1$

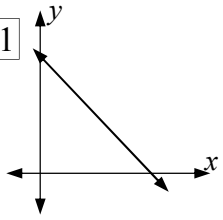
x	-2	-1	0	1	2
y					



Intersection Point (,)

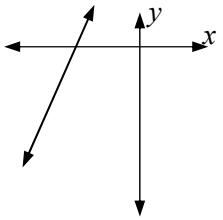
Are these gradients positive or negative?

11



○ Positive ○ Negative

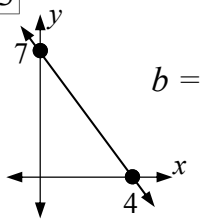
12



○ Positive ○ Negative

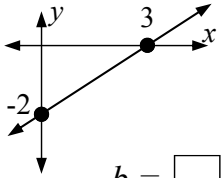
Give the y-intercept for these lines.

13



$b =$ ☐

14



$b =$ ☐

What is the gradient and y-intercept for these lines.

15 $y = -3x + 7$ $m =$ ☐ $b =$ ☐

16 $y = -x$ $m =$ ☐ $b =$ ☐

17 $y = 6 - 2x$ $m =$ ☐ $b =$ ☐

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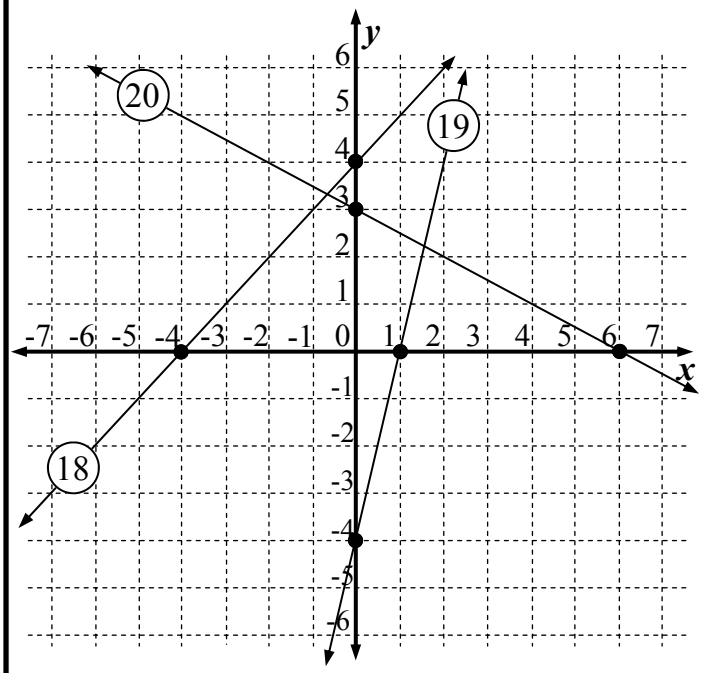
Find the gradient of the lines shown below.



18 _____

19 _____

20 _____

Give the equation for the three lines above

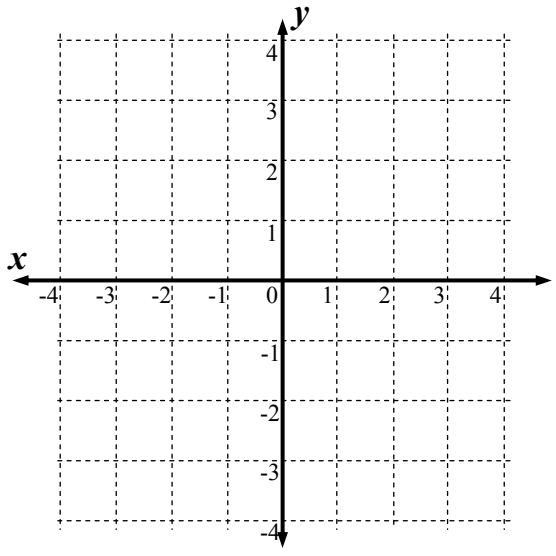
21 Name (18) _____

22 Name (19) _____

23 Name (20) _____

Plot the two lines $x = 2$ and $y = -3$ on the grid below

24



A tennis racquet manufacturer has daily figures below. Complete the tables.

25 *Production Cost : P* = 10*n* + 30 000

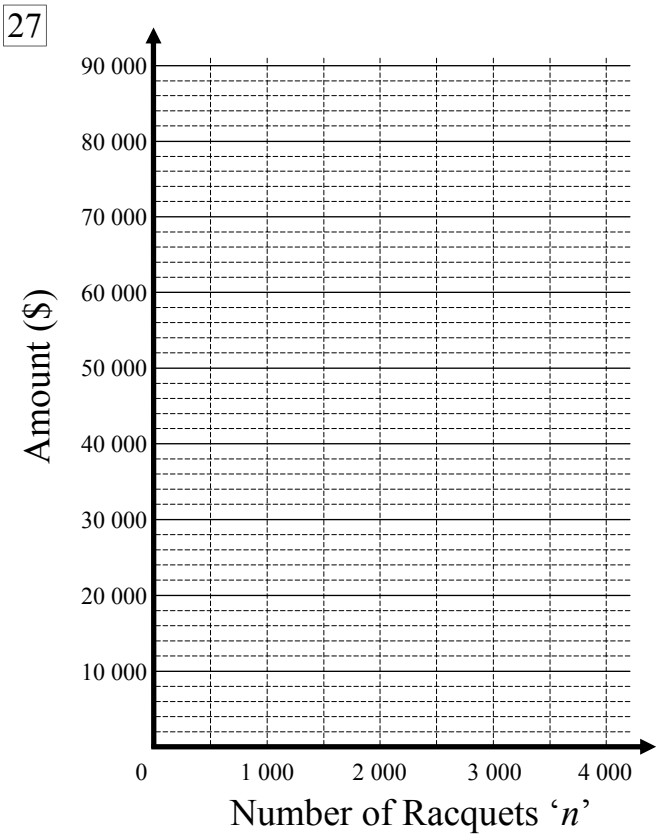
n	0	1 000	2 000	3 000	4 000
P					

26 *Sales Revenue : S* = 22*n*

n	0	1 000	2 000	3 000	4 000
S					

Plot the production and sales cost functions on the grid below.

27



28 Estimate the number of racquets required to be manufactured to break even.

_____

29 Estimate the profit made when 3 500 racquets are manufactured.

_____

Show that the point (3, 13) lies on the line $y = 5x - 2$

30 _____

_____

_____

_____

VELOCITY

SURFACE AREA

Surface Area



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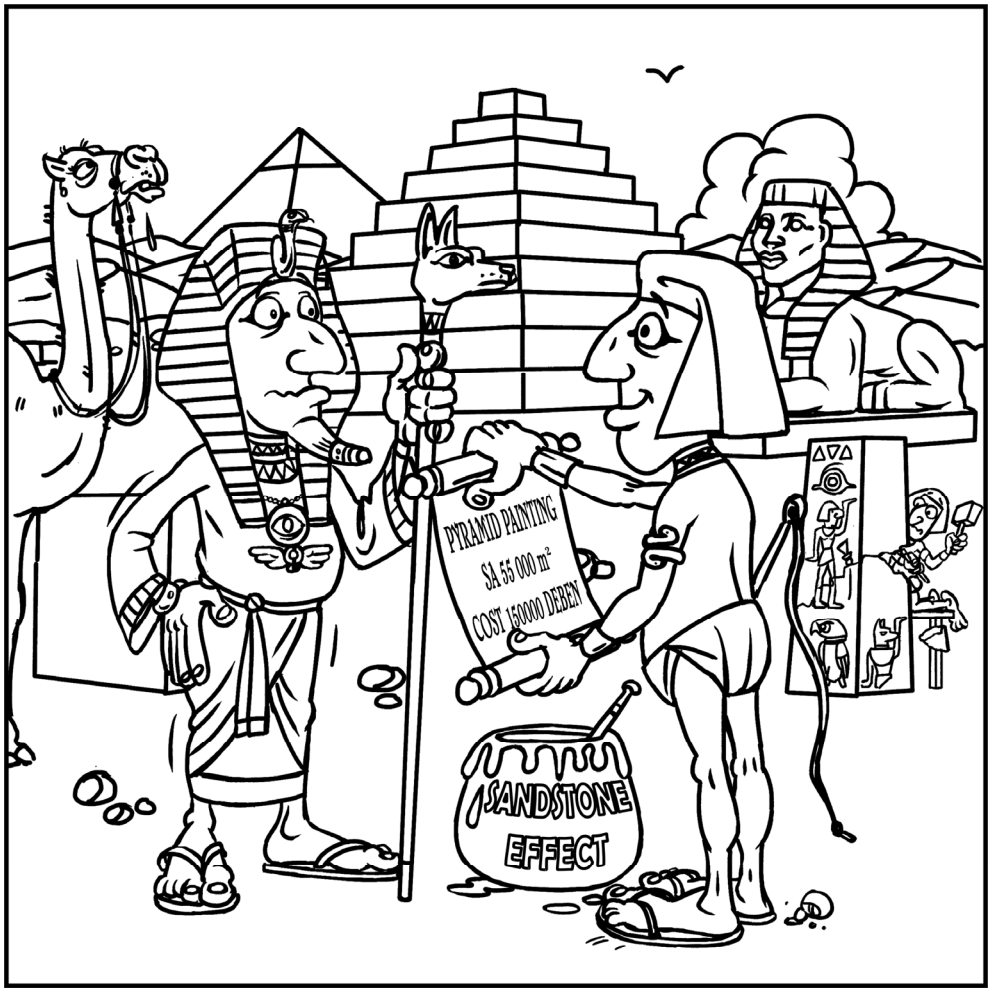
Surface Area



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Surface Area - Templates for Building

These templates are designed to be printed, cut out, coloured-in and either:

Sheets 1 & 2 - stick into books to fold out of page

Sheet 3 - 6 - cut out and assemble to make 3D shapes

Surface Area - Constructing Cubes and Square Pyramids

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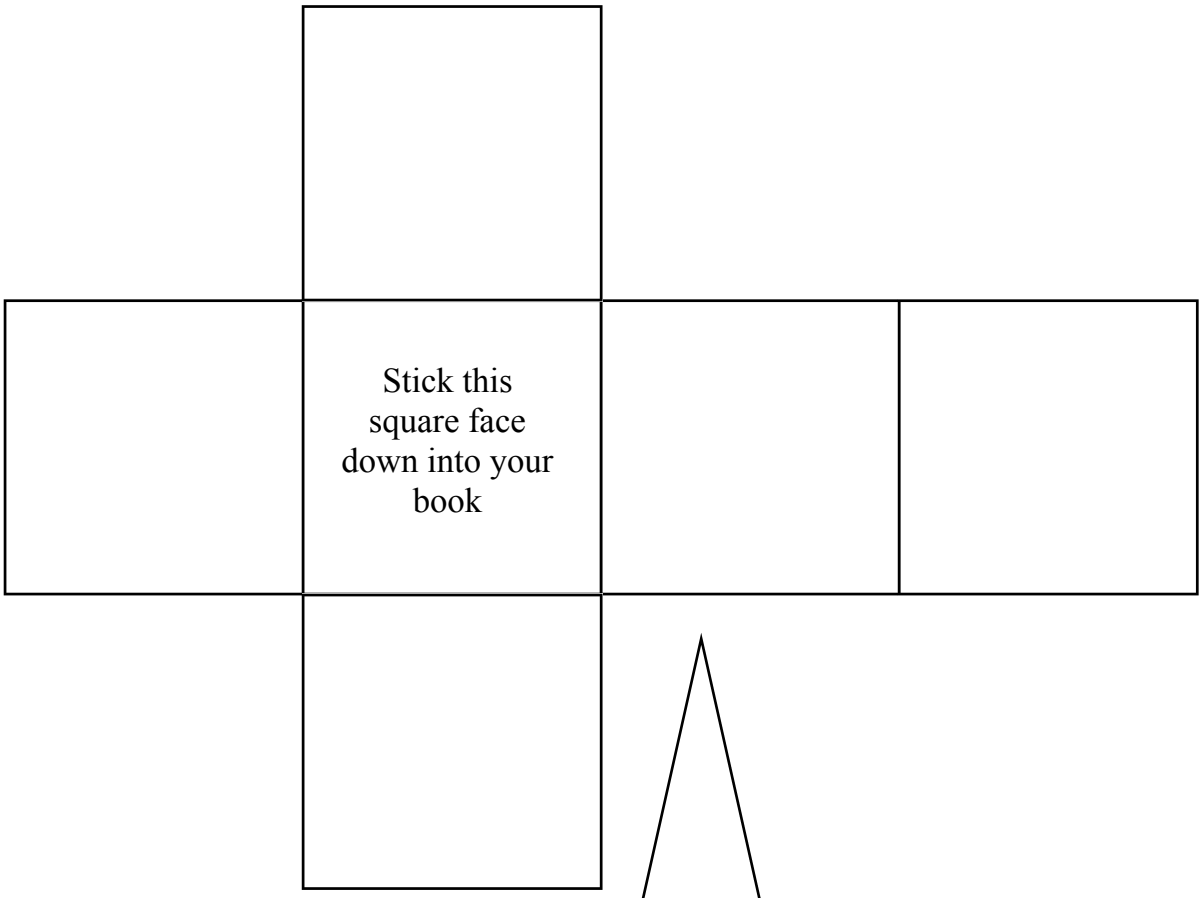

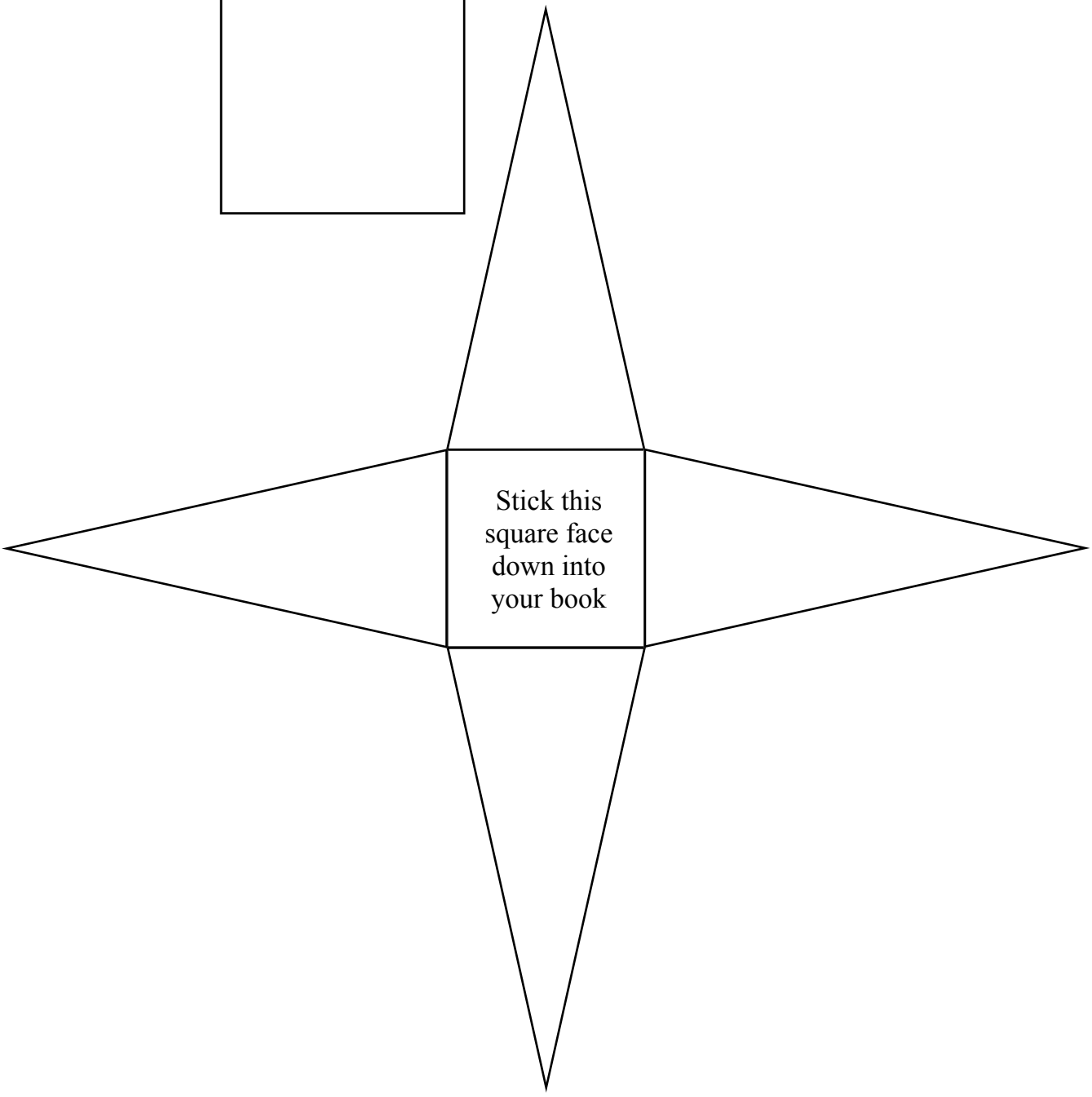

Surface Area - Constructing Rectangular Prisms and Triangular Prisms

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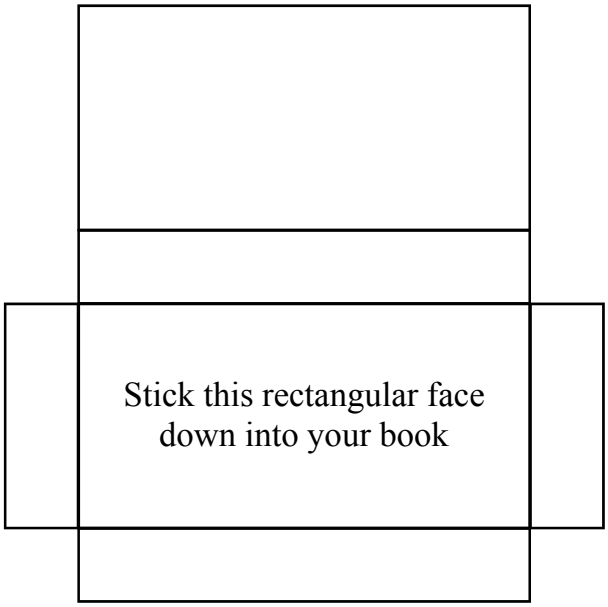

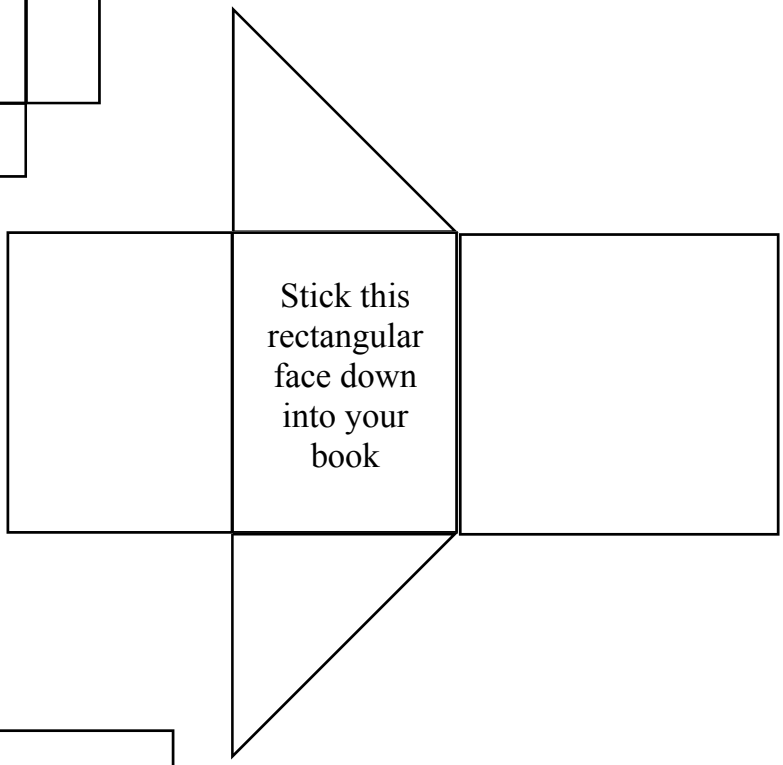

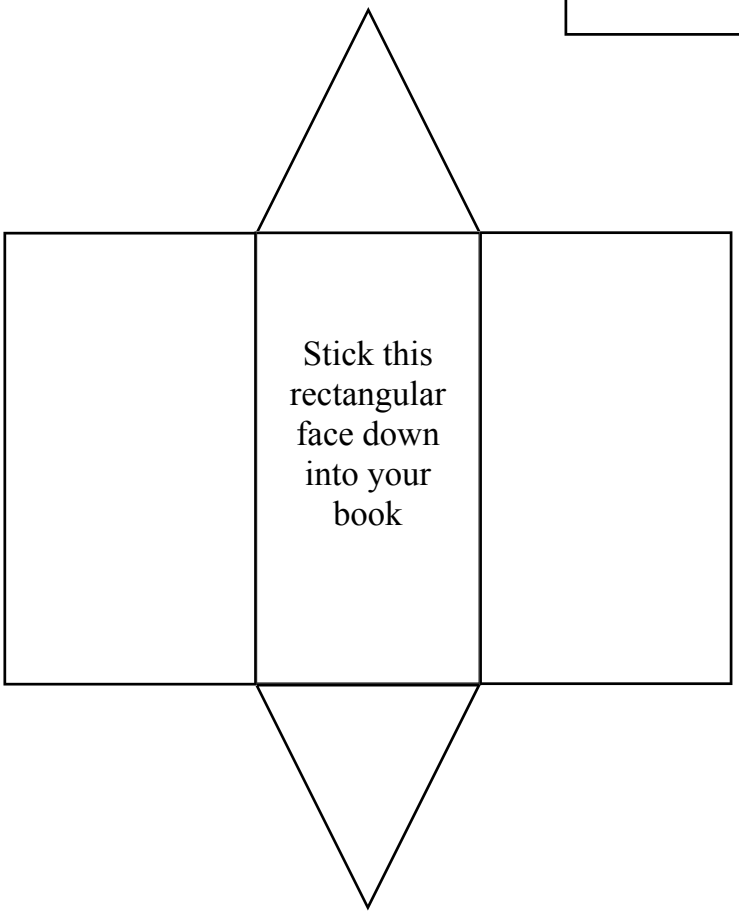

Surface Area - Constructing Cubes and Square Pyramids (with tabs)

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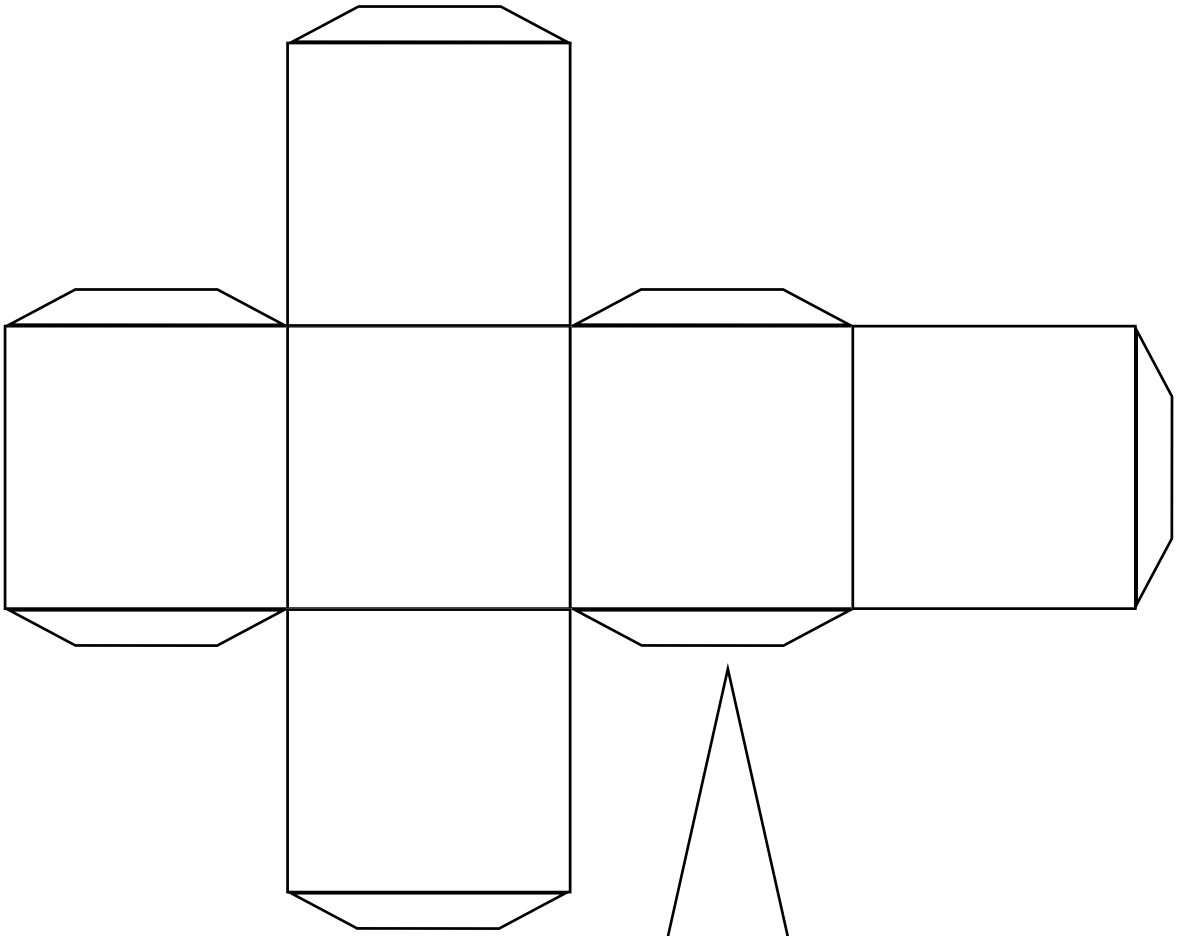

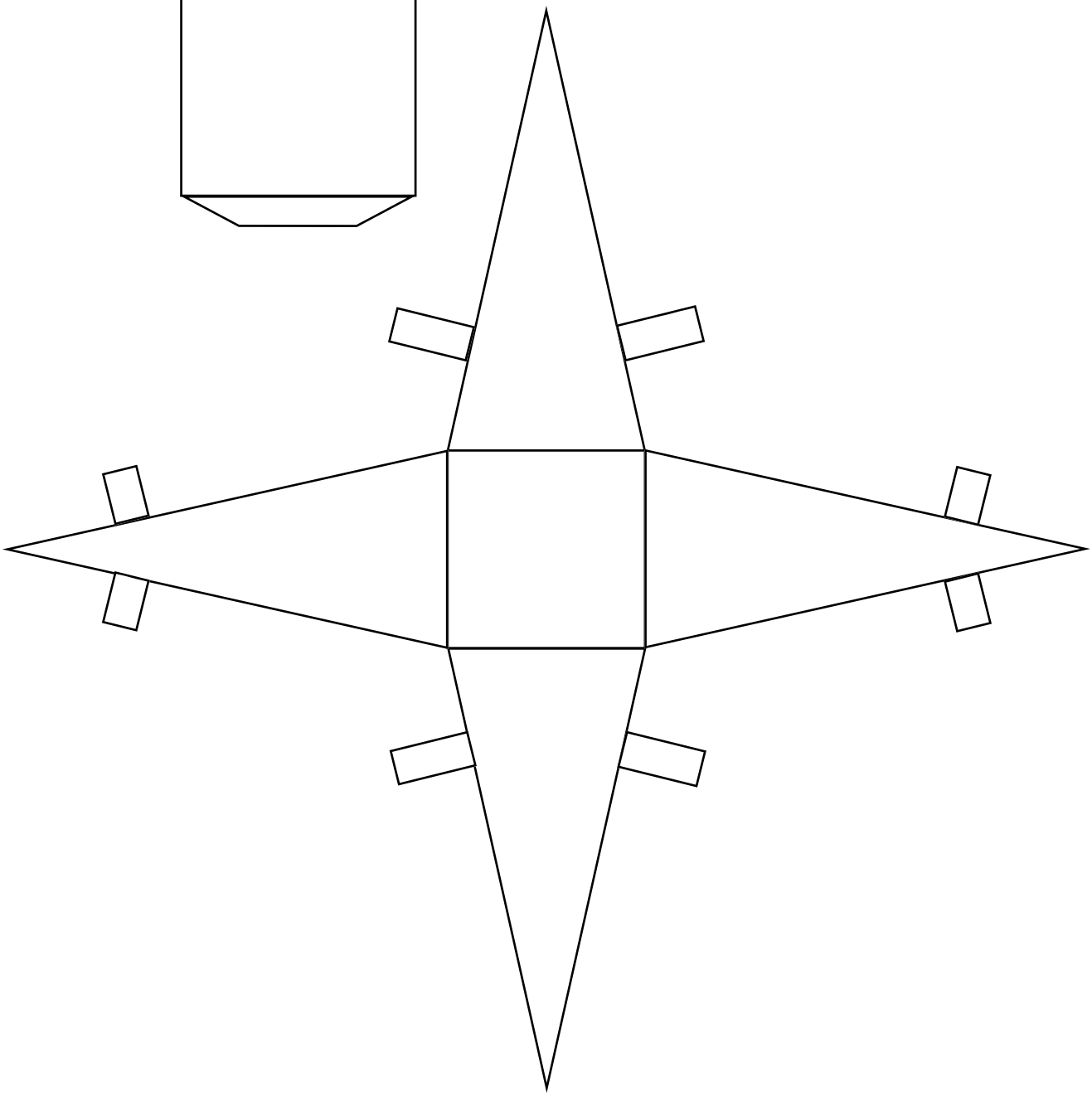

Surface Area - Constructing Rectangular & Triangular Prisms (with tabs)

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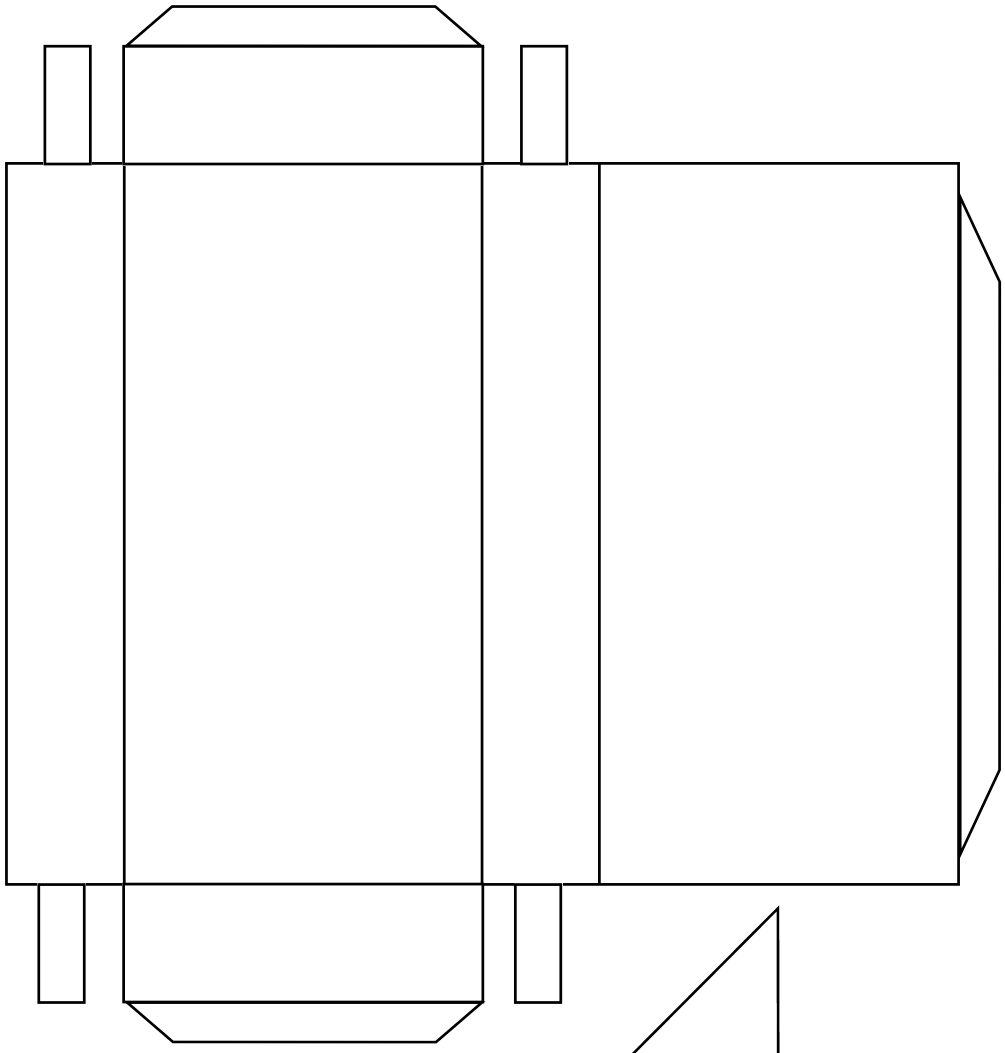

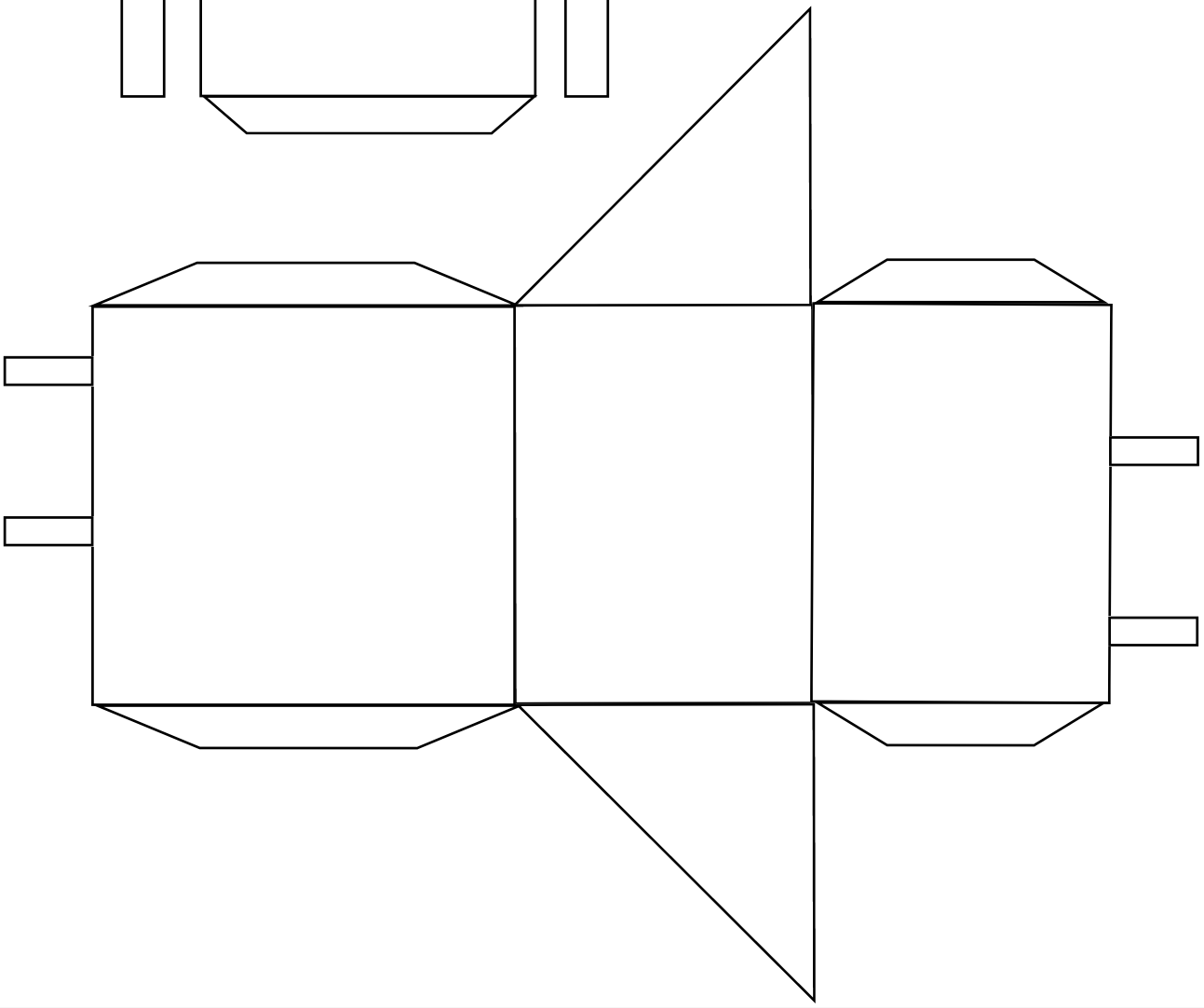

Surface Area - Constructing Cylinders and Triangular Prisms (with tabs)

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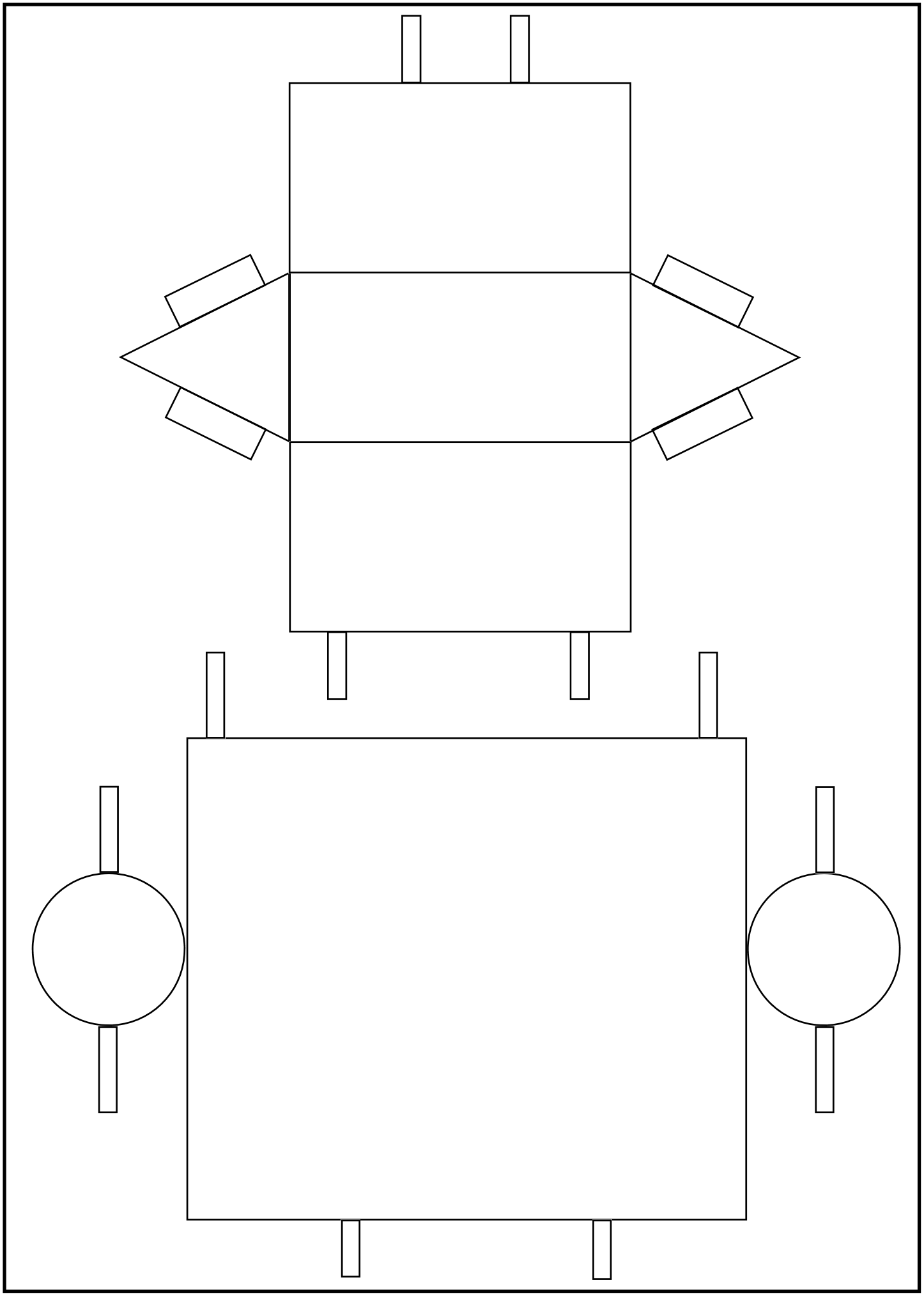

Surface Area - Constructing Cones and Pentagonal Prisms (with tabs)

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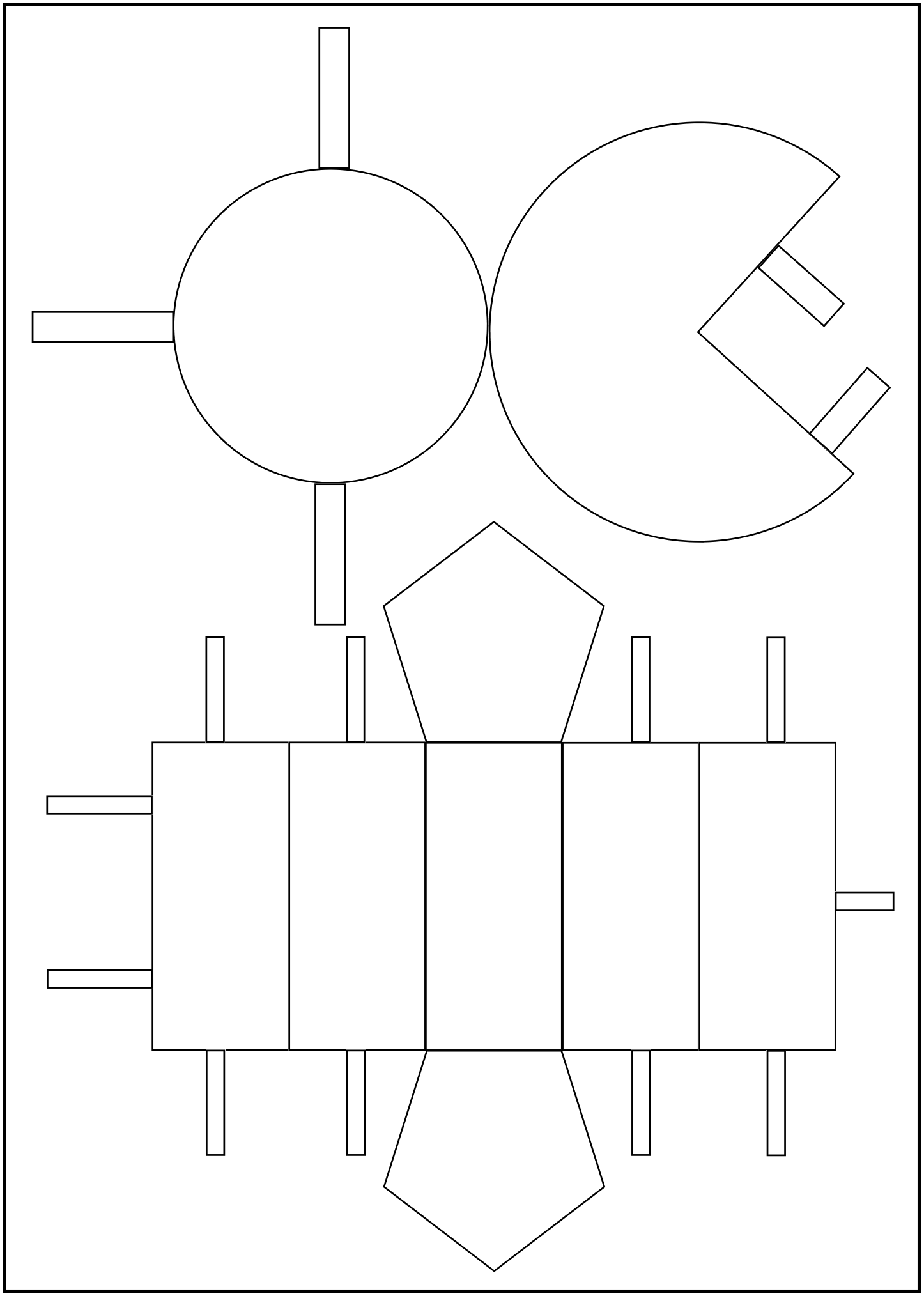

Surface Area - Identifying Solids

When identifying solids first decide whether the solid is a prism or a pyramid. Then look at the base to name the solid.

Use these words:
Triangular (3 sides), Rectangular (4 sides), Pentagonal (5 sides), Hexagonal (6) and Octagonal (8 sides). End these words with either the word 'pyramid' or 'prism'.

The most confusion occurs with triangles, is it a pyramid or a prism? If there are only two triangles in the solid then it is a triangular prism. If there is more than two triangles then the solid is a pyramid, look at its base to decide what type of pyramid it is.

Other shapes to consider are cube, cone, sphere (and hemisphere), and cylinder.

These sheets repeat a few solids so don't think that you have made a mistake if you have used the same name more than once.

A net in mathematics refers to the way a solid would look if it were made from paper and unfolded. So to answer this sheet you have to look at the net and fold it up in your mind and decide what shape it makes when it is folded up.

For most students the difficulty comes when prisms and pyramids are involved. Think about this:

- If a net only has 2 triangles in the net, it will always be a triangular prism.
- If a net has more that 2 triangles in the net it will always be pyramid.
- If a net is of a pyramid then the name of the net will come from the shape in the net that isn't a triangle.
- If the net is of a prism then the name of the prism will come from the shape that isn't a rectangle or square. (except rectangular prisms)

Remember that a shape can't be a pyramid if it has no triangles in its net. A shape can't be a prism if it has more than two triangles in its net. To answer the sheet write the solid's name in the table at the bottom of the page next to the net's number and fill the circle to indicate if it is a prism or not.

Surface Area - Identifying Solids (Sheet 1)

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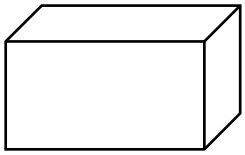
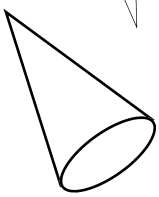
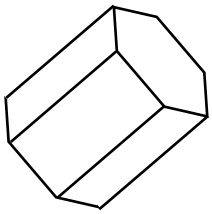
Name the solids and also indicate if they are a prism or not.

1 

O Prism O Non Prism

2 

O Prism
O Non Prism

3 

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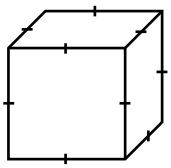
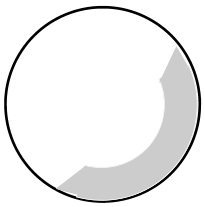
4 

O Prism
O Non Prism

5 

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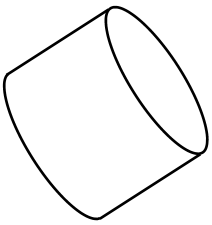
6 

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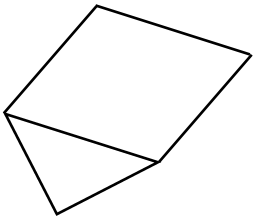
7 

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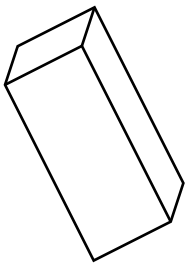
8 

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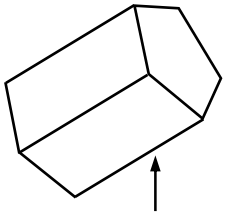
9 

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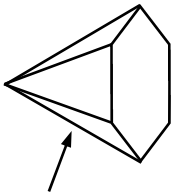
10 

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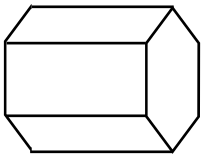
11 

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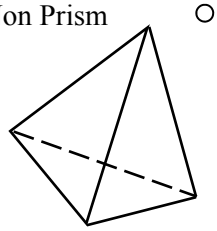
12 

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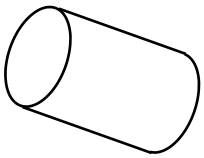
13 

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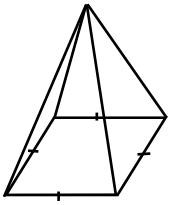
14 

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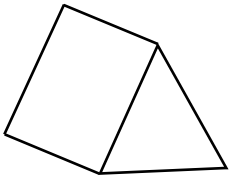
15 

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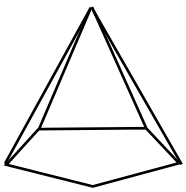
16 

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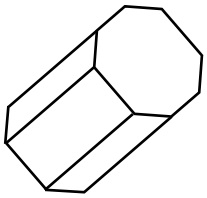
17 

O Prism O Non Prism

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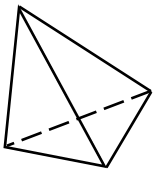
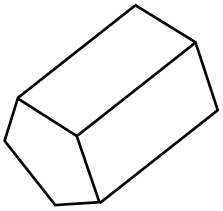
O Prism O Non Prism

19 

O Prism O Non Prism

20 

O Prism
O Non Prism

21 

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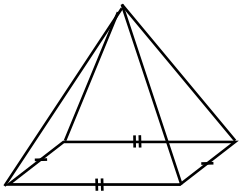
22 

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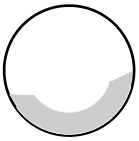
23 

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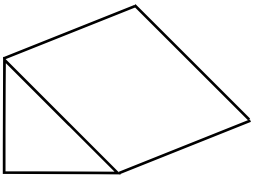
24 

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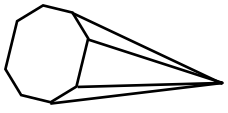
25 

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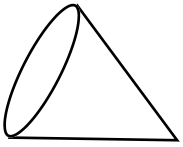
26 

O Prism O Non Prism

Surface Area - Identifying Solids From Their Nets (Sheet 2)

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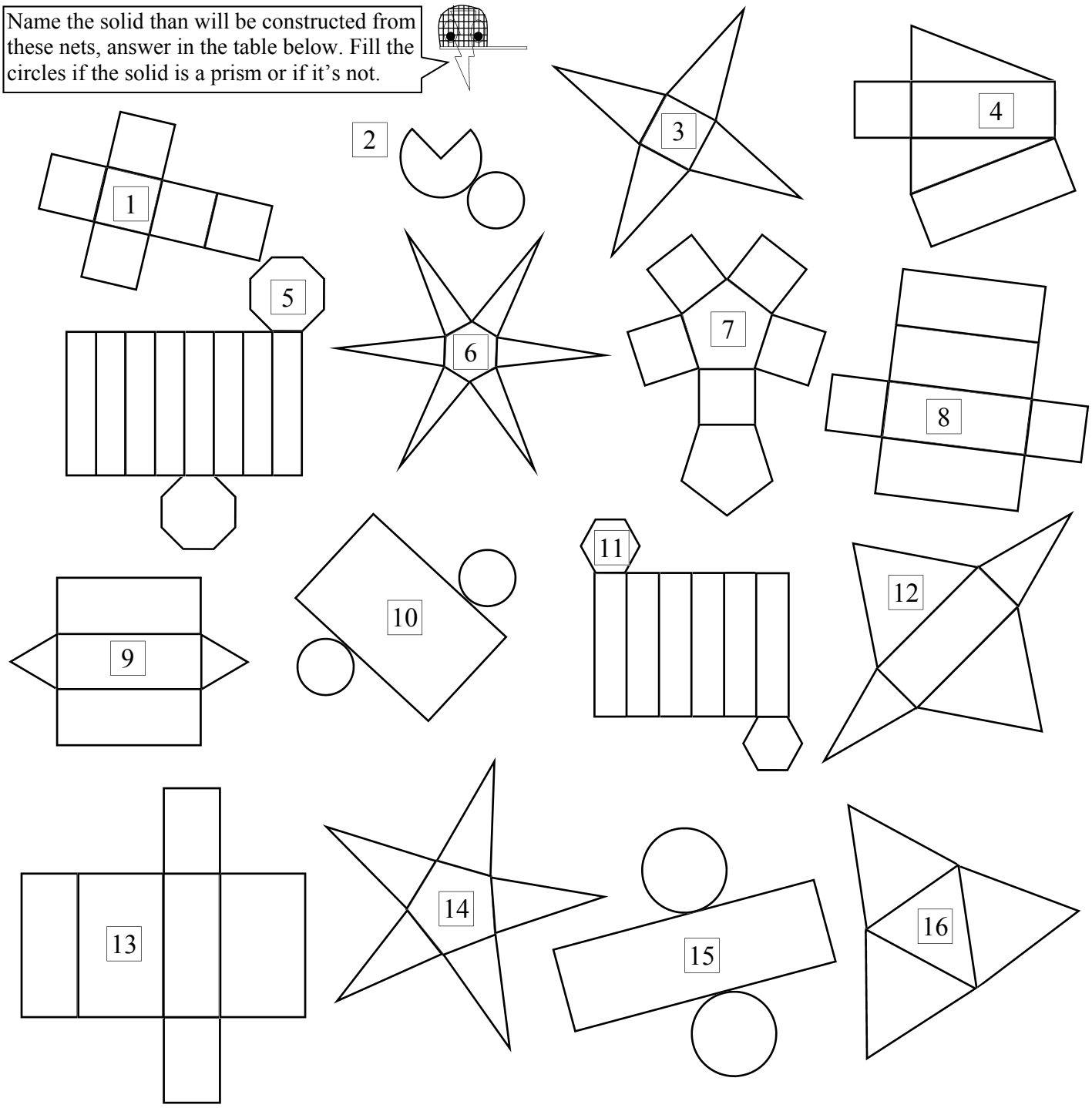
Name the solid than will be constructed from these nets, answer in the table below. Fill the circles if the solid is a prism or if it's not.



1 ____ ○ Prism ○ Non Prism	2 ____ ○ Prism ○ Non Prism	3 ____ ○ Prism ○ Non Prism	4 ____ ○ Prism ○ Non Prism
5 ____ ○ Prism ○ Non Prism	6 ____ ○ Prism ○ Non Prism	7 ____ ○ Prism ○ Non Prism	8 ____ ○ Prism ○ Non Prism
9 ____ ○ Prism ○ Non Prism	10 ____ ○ Prism ○ Non Prism	11 ____ ○ Prism ○ Non Prism	12 ____ ○ Prism ○ Non Prism
13 ____ ○ Prism ○ Non Prism	14 ____ ○ Prism ○ Non Prism	15 ____ ○ Prism ○ Non Prism	16 ____ ○ Prism ○ Non Prism

Surface Area - Drawing the Nets of Solids (Sheet 3)

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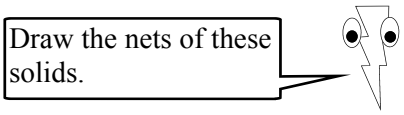

1 Cube

2 Cone

3 Rectangular Prism

4 Triangular Prism

5 Pentagonal Pyramid

6 Hexagonal Prism

7 Octagonal Prism

8 Square Pyramid

9 Pentagonal Prism

10 Triangular Pyramid

11 Rectangular Pyramid

12 Hexagonal Pyramid

Surface Area - Identifying Solids (Sheet 1)

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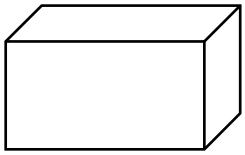
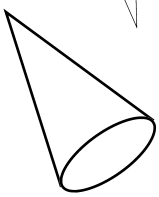
Name the solids and also indicate if they are a prism or not.



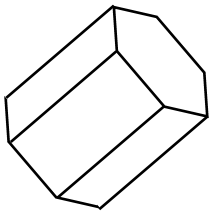
1 Rectangular Prism

● Prism ○ Non Prism



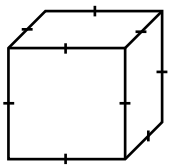
2 Cone

○ Prism
● Non Prism



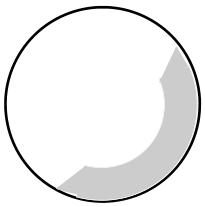
3 Hexagonal Prism

● Prism ○ Non Prism



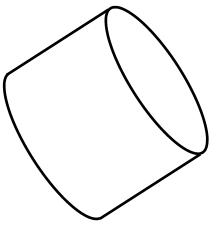
4 Cube

● Prism
○ Non Prism



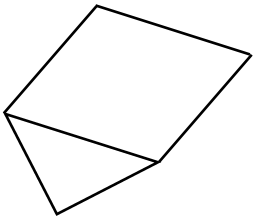
5 Sphere

○ Prism
● Non Prism



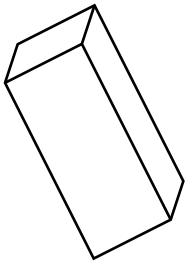
6 Cylinder

● Prism ○ Non Prism



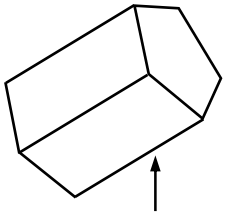
7 Triangular Prism

● Prism ○ Non Prism



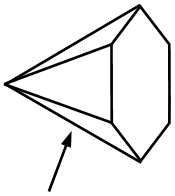
8 Rectangular Prism

● Prism ○ Non Prism



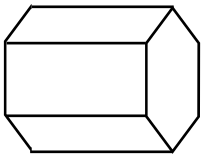
9 Pentagonal Prism

● Prism ○ Non Prism



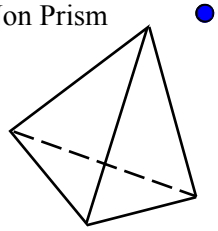
10 Hexagonal Pyramid

○ Prism ● Non Prism



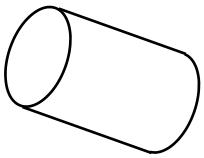
11 Hexagonal Prism

● Prism ○ Non Prism



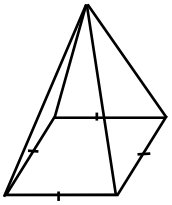
12 Triangular Pyramid

○ Prism ● Non Prism



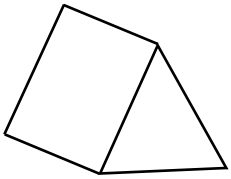
13 Cylinder

● Prism ○ Non Prism



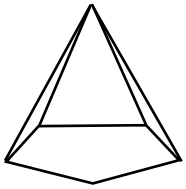
14 Square Pyramid

○ Prism ● Non Prism



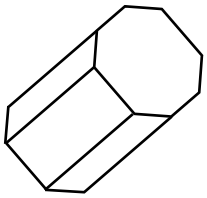
15 Triangular Prism

● Prism ○ Non Prism



16 Pentagonal Pyramid

○ Prism ● Non Prism



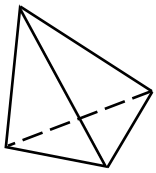
17 Octagonal Prism

● Prism ○ Non Prism



18 Rectangular Prism

● Prism ○ Non Prism



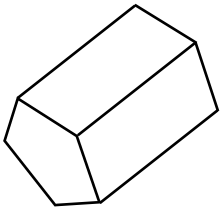
19 Triangular Pyramid

○ Prism ● Non Prism



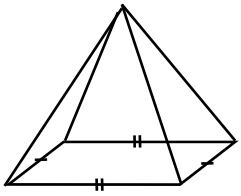
20 Hemisphere

○ Prism
● Non Prism



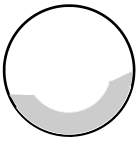
21 Pentagonal Prism

● Prism ○ Non Prism



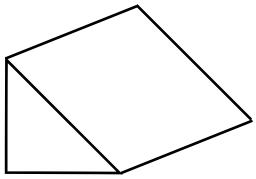
22 Rectangular Pyramid

○ Prism ● Non Prism



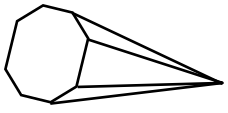
23 Sphere

○ Prism
● Non Prism



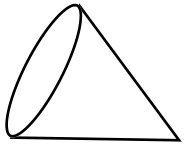
24 Triangular Prism

● Prism ○ Non Prism



25 Octagonal Pyramid

○ Prism ● Non Prism



26 Cone

○ Prism ● Non Prism

Surface Area - Identifying Solids From Their Nets (Sheet 2)

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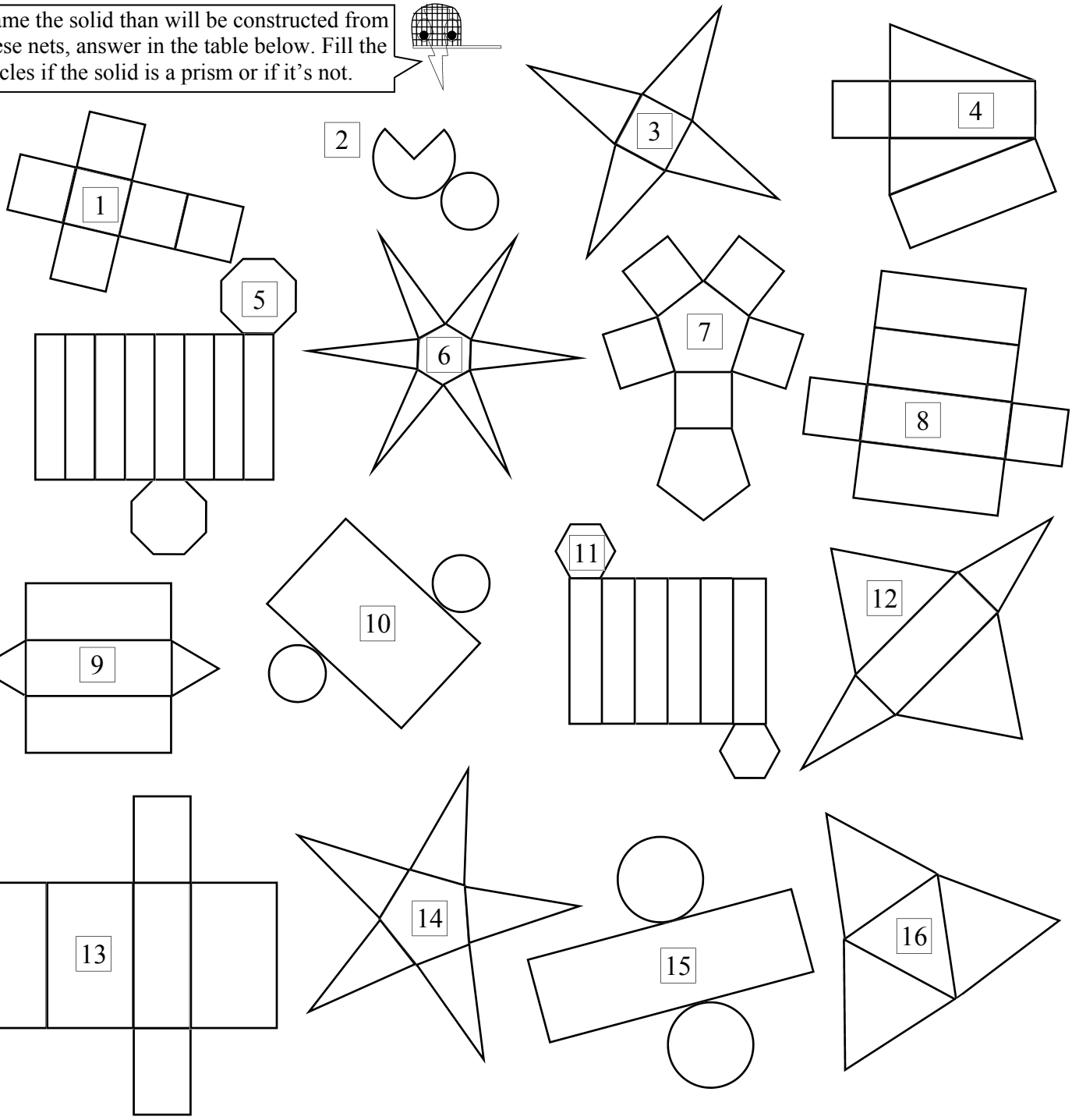
Name the solid than will be constructed from these nets, answer in the table below. Fill the circles if the solid is a prism or if it's not.



1 Cube ● Prism ○ Non Prism	2 Cone ○ Prism ● Non Prism	3 Square Pyramid ○ Prism ● Non Prism	4 Triangular Prism ● Prism ○ Non Prism
5 Octagonal Prism ● Prism ○ Non Prism	6 Hexagonal Pyramid ○ Prism ● Non Prism	7 Pentagonal Prism ● Prism ○ Non Prism	8 Rectangular Prism ● Prism ○ Non Prism
9 Triangular Prism ● Prism ○ Non Prism	10 Cylinder ● Prism ○ Non Prism	11 Hexagonal Prism ● Prism ○ Non Prism	12 Rectangular Pyramid ○ Prism ● Non Prism
13 Rectangular Prism ● Prism ○ Non Prism	14 Pentagonal Pyramid ○ Prism ● Non Prism	15 Cylinder ● Prism ○ Non Prism	16 Triangular Pyramid ○ Prism ● Non Prism

Surface Area - Drawing the Nets of Solids (Sheet 3)

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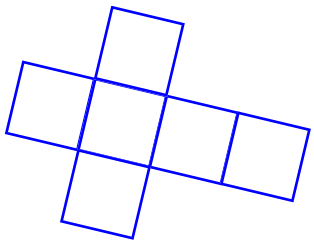
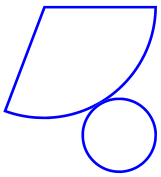
Draw the nets of these solids

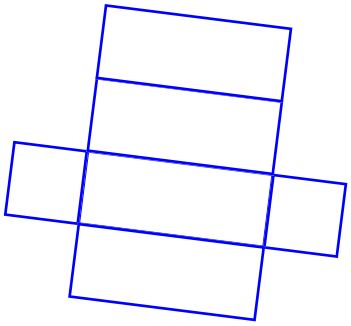
1 Cube



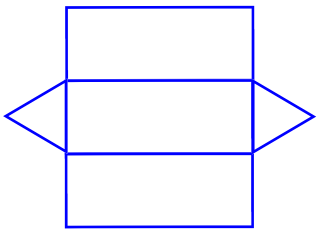
2 Cone



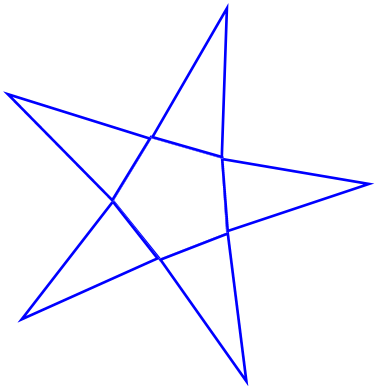
3 Rectangular Prism



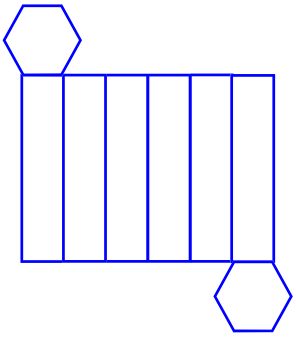
4 Triangular Prism



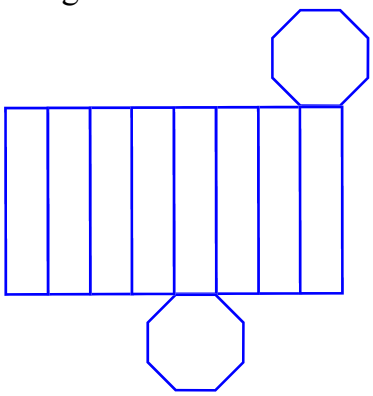
5 Pentagonal Pyramid



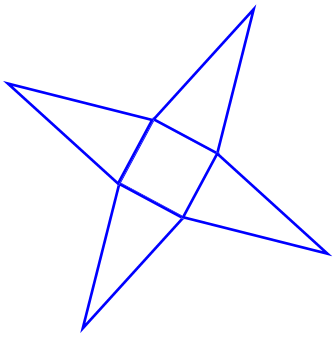
6 Hexagonal Prism



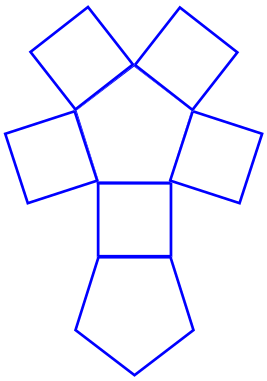
7 Octagonal Prism



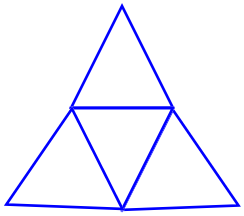
8 Square Pyramid



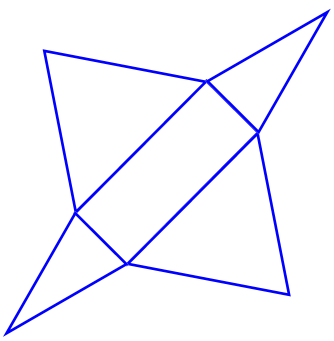
9 Pentagonal Prism



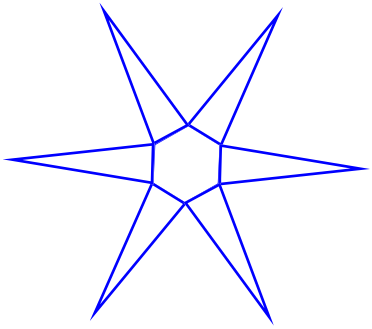
10 Triangular Pyramid



11 Rectangular Pyramid



12 Hexagonal Pyramid



Surface Area - Identifying Solids (Homework Strip)

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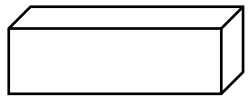
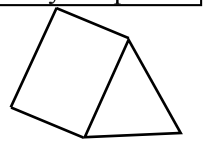
Date Due:

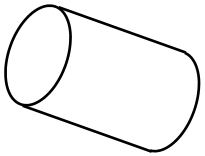
Name the solids shown below. State if they are prisms.

1 

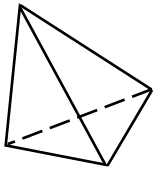
○ Prism ○ Non Prism

2 

○ Prism ○ Non Prism

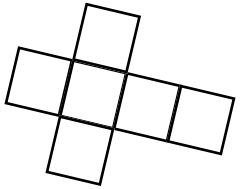
3 

○ Prism ○ Non Prism

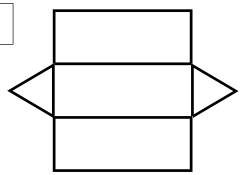
4 

○ Prism ○ Non Prism

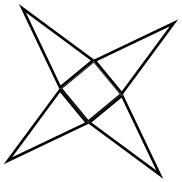
Name the solids shown as nets below.

5 

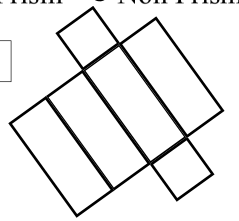
○ Prism ○ Non Prism

6 

○ Prism ○ Non Prism

7 

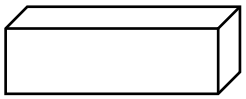
○ Prism ○ Non Prism

8 

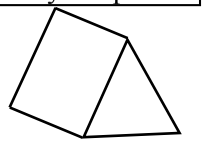
○ Prism ○ Non Prism

Date Due:

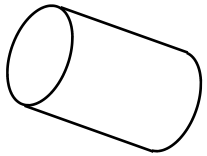
Name the solids shown below. State if they are prisms.

1 

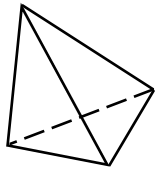
○ Prism ○ Non Prism

2 

○ Prism ○ Non Prism

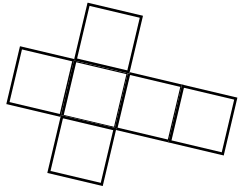
3 

○ Prism ○ Non Prism

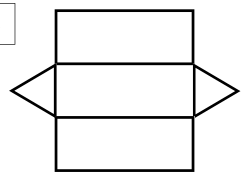
4 

○ Prism ○ Non Prism

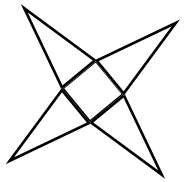
Name the solids shown as nets below.

5 

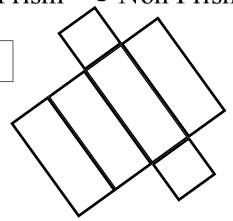
○ Prism ○ Non Prism

6 

○ Prism ○ Non Prism

7 

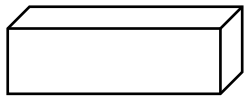
○ Prism ○ Non Prism

8 

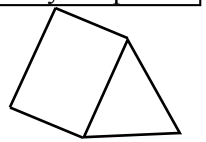
○ Prism ○ Non Prism

Date Due:

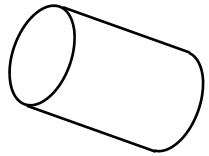
Name the solids shown below. State if they are prisms.

1 

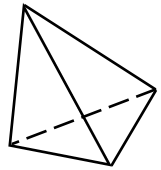
○ Prism ○ Non Prism

2 

○ Prism ○ Non Prism

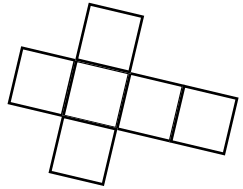
3 

○ Prism ○ Non Prism

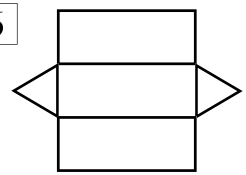
4 

○ Prism ○ Non Prism

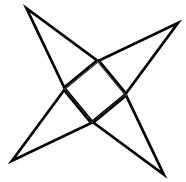
Name the solids shown as nets below.

5 

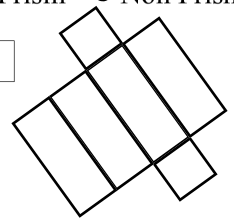
○ Prism ○ Non Prism

6 

○ Prism ○ Non Prism

7 

○ Prism ○ Non Prism

8 

○ Prism ○ Non Prism

Surface Area - Identifying Solids (Homework Strip)

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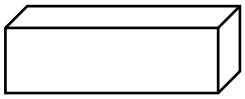
Date Due:

Name the solids shown below. State if they are prisms.

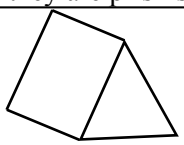
1



Rectangular Prism

● Prism ○ Non Prism

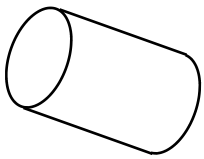
2



Triangular Prism

● Prism ○ Non Prism

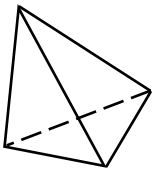
3



Cylinder

● Prism ○ Non Prism

4

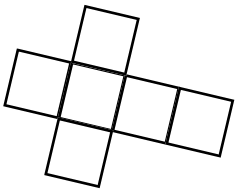


Triangular Pyramid

○ Prism ● Non Prism

Name the solids shown as nets below.

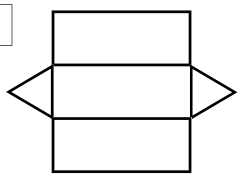
5



Cube

● Prism ○ Non Prism

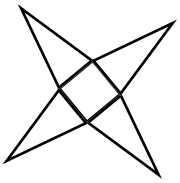
6



Triangular Prism

● Prism ○ Non Prism

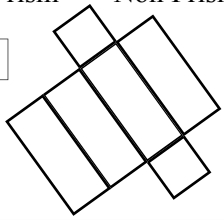
7



Square Pyramid

○ Prism ● Non Prism

8



Rectangular Prism

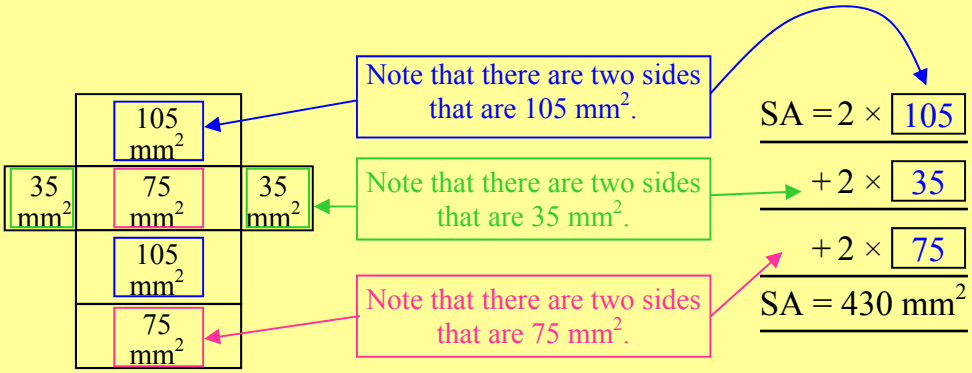
● Prism ○ Non Prism

Surface Area - Face Areas and Edges

Surface area (S.A.) is the outside area of the faces/curved surfaces of a solid. So to calculate the S.A. of a solid you add all the areas of the individual shapes/surfaces that make up the solid. These sheets also show the **nets** of popular solids. A net is like a solid that has been opened up, like it was constructed from paper and unfolded.

The first sheet has three rectangular prisms shown as nets. The SA is found in two ways. Firstly by counting the grid squares within the nets. So count the squares and get the total. This gives you the surface area, but before you move on, work out the surface area another way. Write the side lengths on the side of the nets in the boxes and then find the area of the three Areas, White, Spotted and Grey, by multiplying the sides. Add these three areas to get a total, then double this as there are two matching faces and you have worked the area out for just one face. If the first number matches the last number, great, if not ….. You have to check your working out or your counting.

The second sheet starts with showing nets that have the area for each shape that makes the solid labeled with its area. Note that there are no side lengths … just areas. So the S.A. is found by adding all these areas together. But because some areas are repeated, these are gathered together and multiplication is used to speed things up.



Columns 2 and 3 show solids with the areas labeled on them. But they aren't shown as nets so you can't see all the faces of the solid. But the faces are repeated and you have all the numbers you need, just ask …. How many of each area is there? Then you solve the question using the same method as Column 1.

Surface Area - Face Areas and Edges (Sheet 1)

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The grid below is in 1 cm^2. Measure the edge lengths and write their lengths (in cm) in the boxes. The surface area can be measured in two ways. By counting all the squares in each net or using the edge lengths to find the area, then adding the areas together and doubling the answer. Find the surface area using both methods.

1

Grid squares = ☐ cm^2 ← Count all the squares in the net in Q1 and write it here.

Grey = 1 × 3 = ☐

Spotty = 2×3 = ☐

White = 2×1 = ☐

Total = ☐ cm^2

SA = 2 × ☐ = ☐ cm^2

2

3

2 Grid squares = ☐ cm^2

Grey Area = ☐ × ☐ = ☐ cm^2

Spotty Area = ☐ × ☐ = ☐ cm^2

White Area = ☐ × ☐ = ☐ cm^2

Total = ☐ cm^2

SA = 2 × ☐ = ☐ cm^2

3 Grid squares = ☐ cm^2

Grey Area = ☐ × ☐ = ☐ cm^2

Spotty Area = ☐ × ☐ = ☐ cm^2

White Area = ☐ × ☐ = ☐ cm^2

Total = ☐ cm^2

SA = 2 × ☐ = ☐ cm^2

Surface Area - Face Areas and Edges (Sheet 2)

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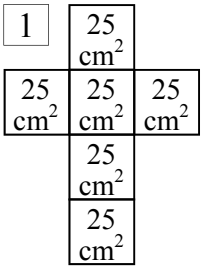
Find the surface area of the nets by adding the face areas. These are areas not side lengths!

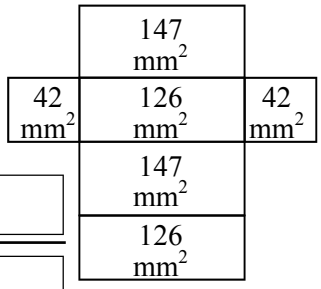
1



SA = 6 × ☐

SA = ☐ cm²

2



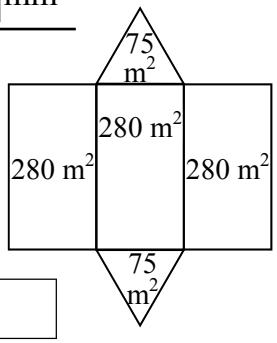
SA = 2 × ☐

+ 2 × ☐

+ 2 × ☐

SA = ☐ mm²

3

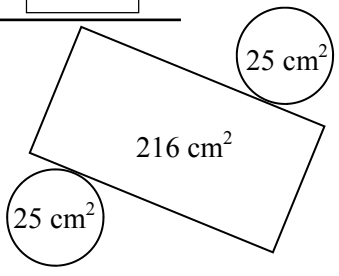


SA = 2 × ☐

+ 3 × ☐

SA = ☐ m²

4



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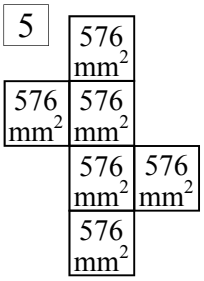
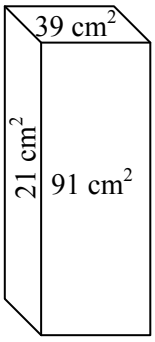


Remember units

Remember the matching sides, find the SA!

6



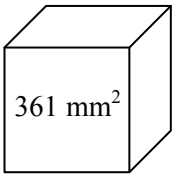
SA = 2 × ☐

+ 2 × ☐

+ 2 × ☐

SA = ☐ cm²

7 Cube



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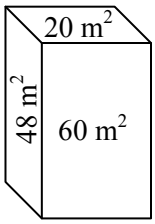
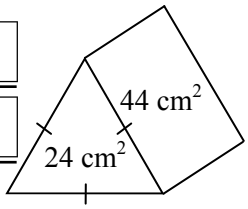


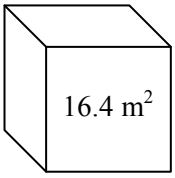
9

SA = 2 × ☐

+ 3 × ☐



10 Cube



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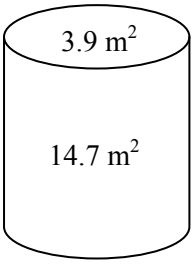

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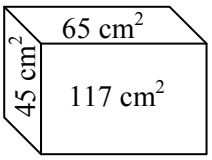

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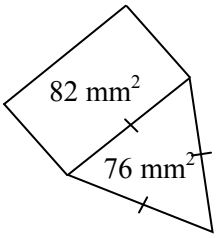

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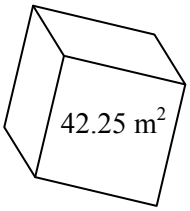

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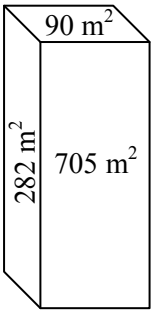

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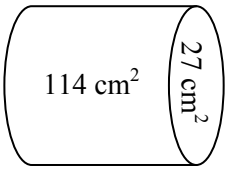

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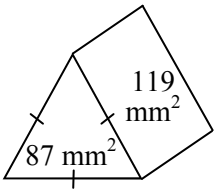
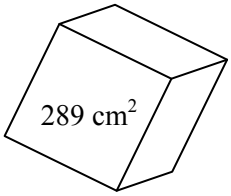


18 Cube



Surface Area - Face Areas and Edges (Sheet 1)

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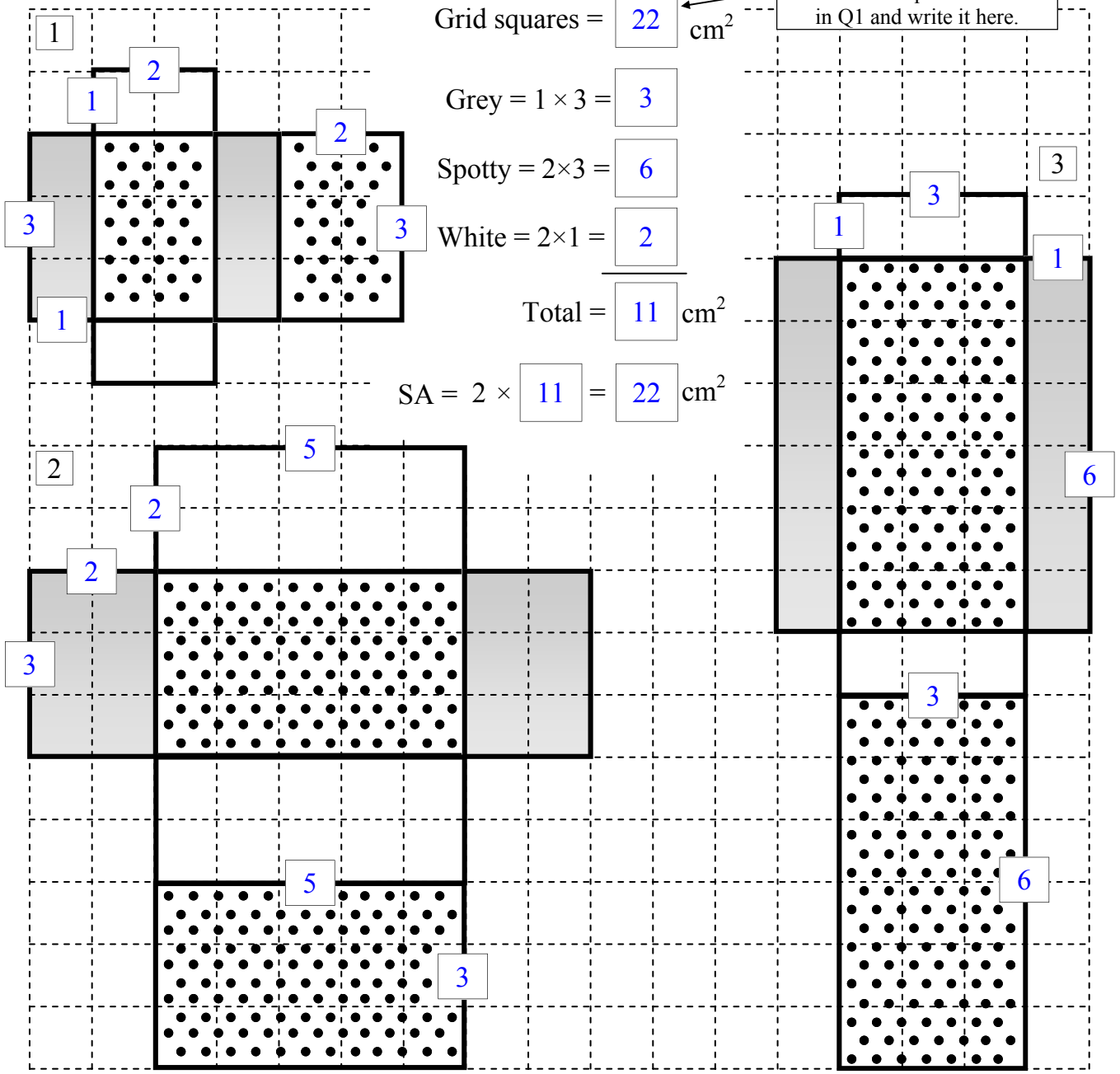
The grid below is in 1 cm^2. Measure the edge lengths and write their lengths (in cm) in the boxes. The surface area can be measured in two ways. By counting all the squares in each net or using the edge lengths to find the area, then adding the areas together and doubling the answer. Find the surface area using both methods.



1

Grid squares = 22 cm^2 ← Count all the squares in the net in Q1 and write it here.

Grey = 1 × 3 = 3

Spotty = 2×3 = 6

White = 2×1 = 2

Total = 11 cm^2

SA = 2 × 11 = 22 cm^2

2 Grid squares = 62 cm^2

Grey Area = 2 × 3 = 6 cm^2

Spotty Area = 5 × 3 = 15 cm^2

White Area = 2 × 5 = 10 cm^2

Total = 31 cm^2

SA = 2 × 31 = 62 cm^2

3 Grid squares = 54 cm^2

Grey Area = 6 × 1 = 6 cm^2

Spotty Area = 6 × 3 = 18 cm^2

White Area = 3 × 1 = 3 cm^2

Total = 27 cm^2

SA = 2 × 27 = 54 cm^2

Surface Area - Face Areas and Edges (Sheet 2)

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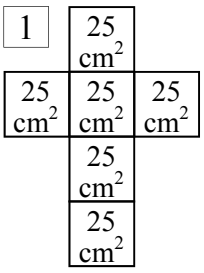
Find the surface area of the nets by adding the face areas. These are areas not side lengths!

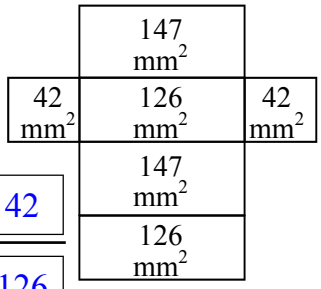
1



$SA = 6 \times 25$

$SA = 150$ cm²

2



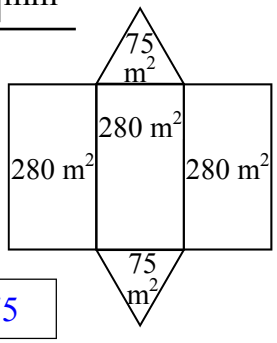
$SA = 2 \times 42$

$+ 2 \times 126$

$+ 2 \times 147$

$SA = 630$ mm²

3

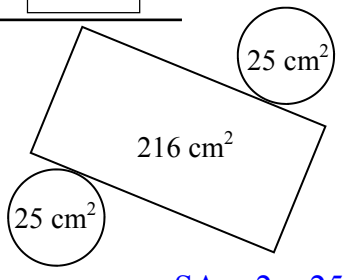


$SA = 2 \times 75$

$+ 3 \times 280$

$SA = 990$ m²

4



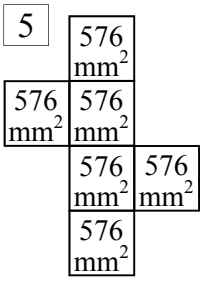
$SA = 2 \times 25$

$+ 216$

$SA = 266$ cm²

Remember units

5

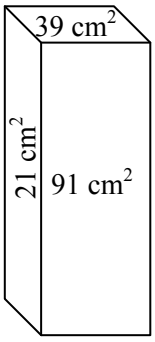


$SA = 6 \times 576$

$SA = 3\,456$ mm²

Remember the matching sides, find the SA!

6



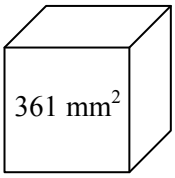
$SA = 2 \times 39$

$+ 2 \times 21$

$+ 2 \times 91$

$SA = 302$ cm²

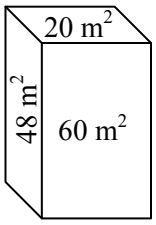
7 Cube



$SA = 6 \times 361$

$SA = 2\,166$ mm²

8



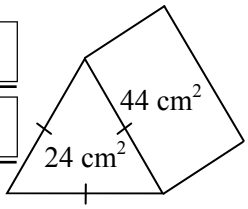
$SA = 2 \times 20$

$+ 2 \times 60$

$+ 2 \times 48$

$SA = 256$ m²

9

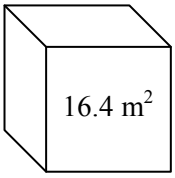


$SA = 2 \times 24$

$+ 3 \times 44$

$SA = 180$ cm²

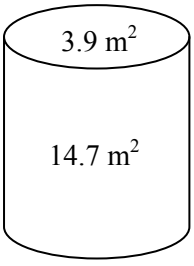
10 Cube



$SA = 6 \times 16.4$

$SA = 98.4$ m²

11

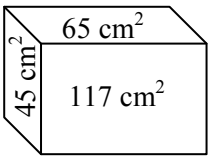


$SA = 2 \times 3.9$

$+ 14.7$

$SA = 22.5$ m²

12



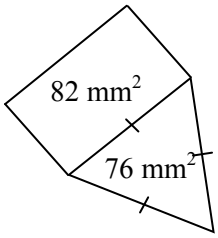
$SA = 2 \times 65$

$+ 2 \times 45$

$+ 2 \times 117$

$SA = 454$ cm²

13

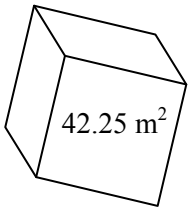


$SA = 2 \times 76$

$+ 3 \times 82$

$SA = 398$ mm²

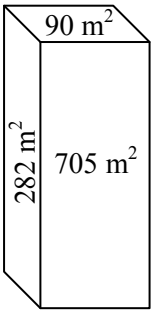
14 Cube



$SA = 6 \times 42.25$

$SA = 253.5$ m²

15



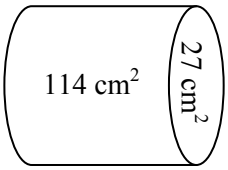
$SA = 2 \times 90$

$+ 2 \times 282$

$+ 2 \times 705$

$SA = 2\,154$ m²

16

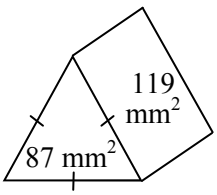


$SA = 2 \times 27$

$+ 114$

$SA = 168$ cm²

17

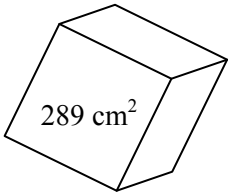


$SA = 2 \times 87$

$+ 3 \times 119$

$SA = 531$ mm²

18 Cube



$SA = 6 \times 289$

$SA = 1\,734$ cm²

Surface Area - Face Areas and Edges (Homework Strip)

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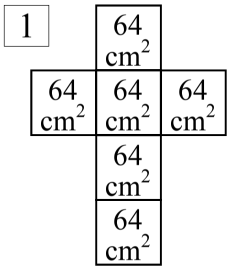
Date Due: _____

Find the SA by adding the face areas.

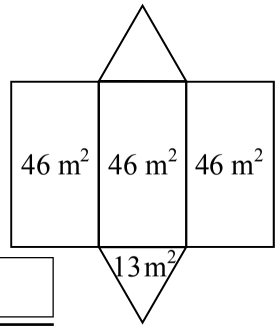
1



SA = 6 × _____

SA = _____ cm²

2

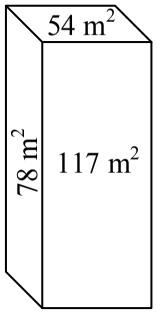


SA = 2 × _____

+ 3 × _____

SA = _____ m²

3



SA = 2× _____

+2 × _____

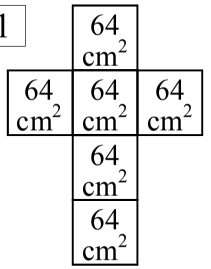
+2 × _____

SA = _____

Date Due: _____

Find the SA by adding the face areas.

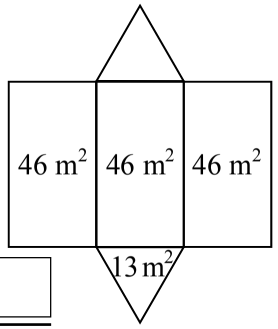
1



SA = 6 × _____

SA = _____ cm²

2

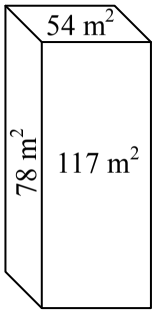


SA = 2 × _____

+ 3 × _____

SA = _____ m²

3



SA = 2× _____

+2 × _____

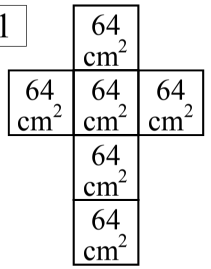
+2 × _____

SA = _____

Date Due: _____

Find the SA by adding the face areas.

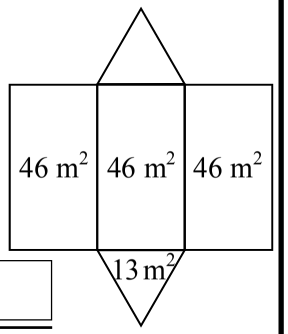
1



SA = 6 × _____

SA = _____ cm²

2

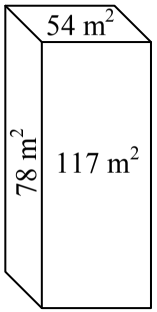


SA = 2 × _____

+ 3 × _____

SA = _____ m²

3



SA = 2× _____

+2 × _____

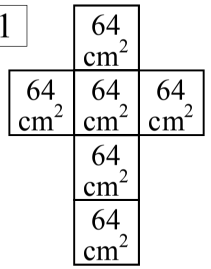
+2 × _____

SA = _____

Date Due: _____

Find the SA by adding the face areas.

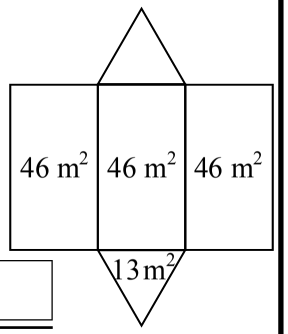
1



SA = 6 × _____

SA = _____ cm²

2

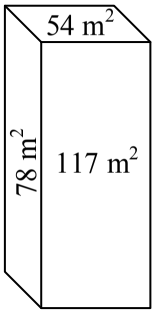


SA = 2 × _____

+ 3 × _____

SA = _____ m²

3



SA = 2× _____

+2 × _____

+2 × _____

SA = _____

Surface Area - Face Areas and Edges (Homework Strip)

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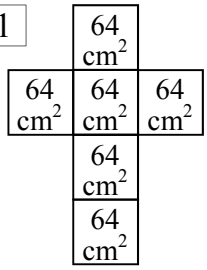
Date Due: _____

Find the SA by adding the face areas.

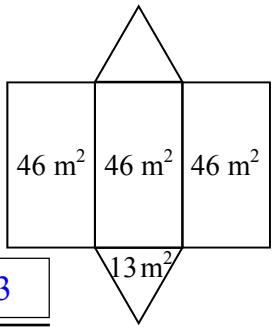
1



SA = 6 × 64

SA = 384 cm^2

2

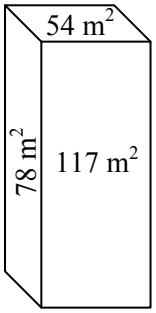


SA = 2 × 13

+ 3 × 46

SA = 164 m^2

3



SA = 2× 54

+2 × 78

+2 × 117

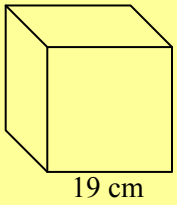
SA = 498 m^2

Surface Area - Cubes and Rectangular Prisms

The previous sheet gave you the areas of each face of a solid. This exercise gives you the length of the edges of the solids. So you have to calculate the areas yourself.

Column 1 asks you to find the S.A. of cubes. A cube has six identical square faces, so calculate the area of one square face, then multiply (times) by 6. But there is a formula as well and it is: S.A. = $6l^2$, where 'l' is the side length of the square face.

Example



$SA = 6l^2$ ← Write out the formula.

$= 6 \times 19^2$ ← Substitute the side length into the formula.

$SA = 2\ 166\ cm^2$ ← Use your calculator and write the answer showing the units, which are square units.

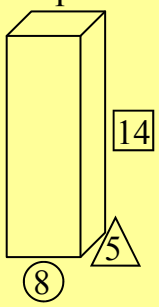
S.A. of a Cube Calculator Steps:

To Find 6×19^2

[6] [×] [1] [9] [x^2] [=]

After a rounding exercise with cubes, column 2 deals with finding the S.A. of rectangular prisms. These are more complicated than cubes because they have 3 pairs of identical face areas. So you have to perform three separate calculations. This column assists you by putting shapes around the edge lengths, this matches the shapes in the working out area …. so just rewrite the number in the correct position and so on. The lengths are all cm, so your final answer should have cm^2 in it.

Example



$SA = 2 \times \boxed{14} \times \triangle 5$ ← The square is 14 and the triangle is 5.

$+ 2 \times \boxed{14} \times (8)$ ← The square is 14 and the circle is 8.

$+ 2 \times (8) \times \triangle 5$ ← The circle is 8 and the triangle is 5.

$SA = 444\ cm^2$ ← Use your calculator and write the answer showing the units, which are all cm^2.

S.A. of a Rectangular Prism Calculator Steps:

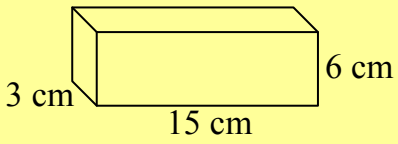
[2] [×] [1] [4] [×] [5] [+] [2] [×] [1] [4] [×] [8] [+] [2] [×] [8] [×] [5] [=]

Column 3 is a mix of cubes and rectangular prisms. But this time the rectangular prisms don't have the shapes around the lengths. So you have to make sure that you use two of each number when you calculate the S.A.

A step-by step process is on the next page.

When you select the sides you are going to use, you can place a stroke through them. Once a number has a cross through it, you don't use it again.

Example



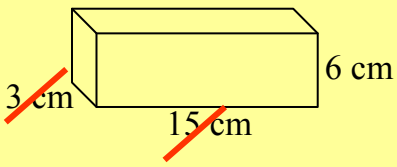
SA = 2 × ×
+ 2 × ×
+ 2 × ×
SA =

STEP 1

Write SA = 2 × ×
+ 2 × ×
+ 2 × ×
SA =

This is always the same, this sets you up to write the side lengths in.

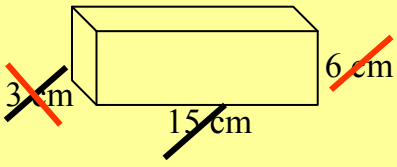
SA = 2 × **3** × **15**
+ 2 × ×
+ 2 × ×
SA =



STEP 2

Pick two side lengths, whichever two you like. Write them in the top line and put a stroke through the numbers on the diagram.

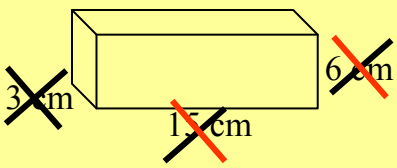
SA = 2 × 3 × 15
+ 2 × **3** × **6**
+ 2 × ×
SA =



STEP 3

Pick the side you haven't used (6 cm) and one side that you have (we used 3 cm). Write them in the second line and put a stroke through the numbers on the solid. Note that the 3 cm side has a cross through it, it is no longer able to be used.

SA = 2 × 3 × 15
+ 2 × 3 × 6
+ 2 × **15** × **6**
SA =



STEP 4

The sides with only one stroke are now used, so use 15 cm and 6 cm. Write these in the third line and you are now ready to calculate the answer.

2	×	3	×	1	5	+	2	×	3	×	6	+	2	×	1	5	×	6	=

SA = 306 cm^2

STEP 5

Calculate the answer and write the square units on the answer. You are done!

Surface Area - Cubes and Rectangular Prisms (Sheet 1)

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Calculate these using your square key on your calculator.

1 $6 \times 14^2 =$ ☐

Calc: 6 × 1 4 x^2 =

2 $6 \times 7^2 =$ ☐

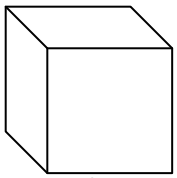
3 $6 \times 17^2 =$ ☐

4 $6 \times 35^2 =$ ☐

5 $6 \times 26^2 =$ ☐

Use SA = $6l^2$ to find the SA of these cubes. Your answer should include square units.

6



$SA = 6l^2$

$= 6 \times$ ☐ ^{2}

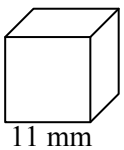
SA = ☐ m^2

7

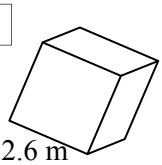
$SA = 6l^2$

=

SA = ☐ mm^2



8



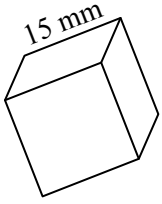
SA =

=

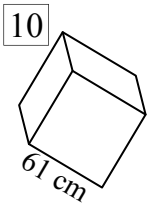
SA =

Square units

9

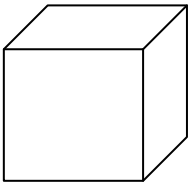


10

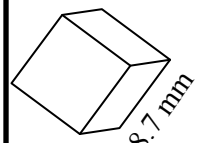


Find the SA of these cubes. Round your answer to 1 d.p.

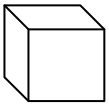
11



12



13



Use the Δ, □, and ○ to help you to find the SA of these rectangular prisms. All side lengths are in cm.

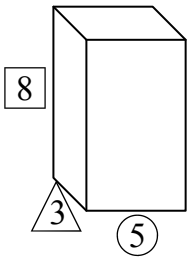
14

$SA = 2 \times$ ☐ $\times$ △

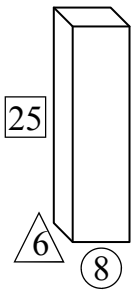
$+ 2 \times$ ☐ $\times$ ○

$+ 2 \times$ ○ $\times$ △

SA = ☐ cm^2



15



$SA = 2 \times$ ☐ $\times$ △

$+ 2 \times$ ☐ $\times$ ○

$+ 2 \times$ ○ $\times$ △

SA = ☐ cm^2

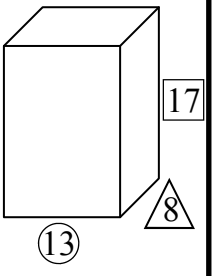
16

$SA = 2 \times$ ☐ $\times$ △

$+ 2 \times$ ☐ $\times$ ○

$+ 2 \times$ ○ $\times$ △

SA = ☐ cm^2



Find the SA of these cubes and rectangular prisms.

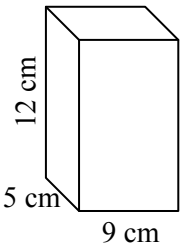
17

$SA = 2 \times$ ☐ $\times$ ☐

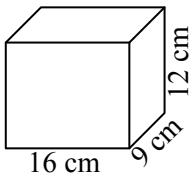
$+ 2 \times$ ☐ $\times$ ☐

$+ 2 \times$ ☐ $\times$ ☐

SA = ☐ cm^2

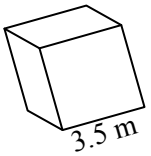


18



19

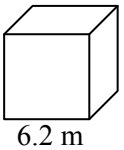
Cube



20 A rectangular prism has sides 2.4 m, 3.8 m and 4.3 m.

21 A rectangular prism has sides 78 cm, 14 cm and 23 cm.

22 Cube



Surface Area - Cubes and Rectangular Prisms (Sheet 2)

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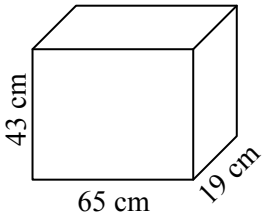
Find the surface area of these rectangular prisms. Remember to show the square units.

1 $SA = 2 \times \square \times \square$
$+ 2 \times \square \times \square$
$+ 2 \times \square \times \square$
$SA = \square\ cm^2$



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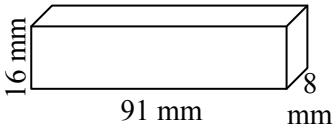

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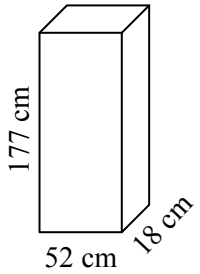

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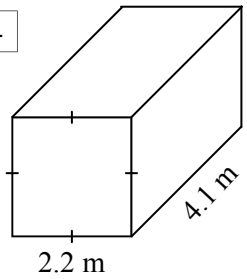

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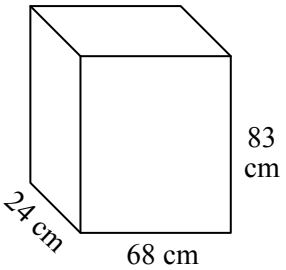

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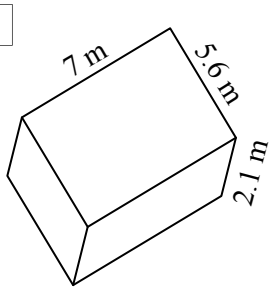
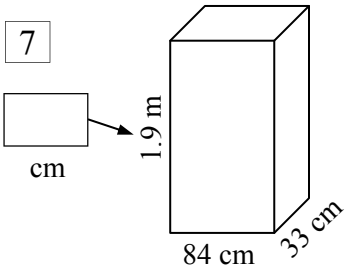


Give the surface area in cm^2. So change all units that aren't in cm, to cm first. Then perform the SA calculation. Round to 1 d.p. if needed.

7



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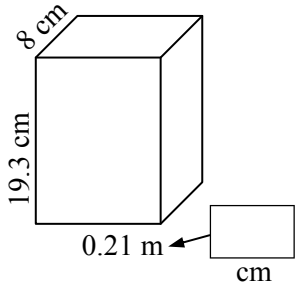

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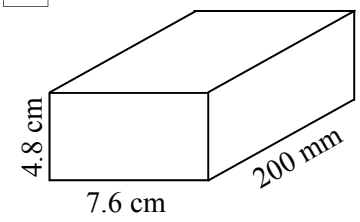

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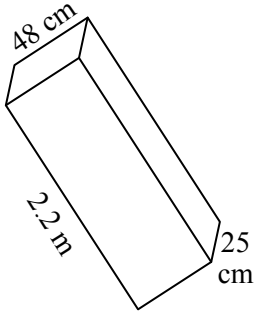

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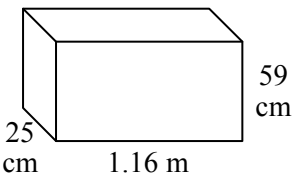

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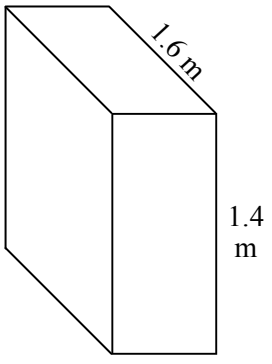

Surface Area - Cubes and Rectangular Prisms (Sheet 1)

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Calculate these using your square key on your calculator.

1 $6 \times 14^2 =$ **1 176**

Calc: 6 × 1 4 x^2 =

2 $6 \times 7^2 =$ **294**

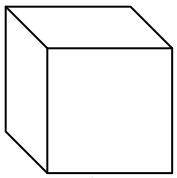
3 $6 \times 17^2 =$ **1 734**

4 $6 \times 35^2 =$ **7 350**

5 $6 \times 26^2 =$ **4 056**

Use $SA = 6l^2$ to find the SA of these cubes. Your answer should include square units.

6



$SA = 6l^2$

$= 6 \times 9^2$

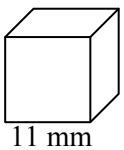
$SA =$ **486** m^2

7

$SA = 6l^2$

$= 6 \times 11^2$

$SA =$ **726** mm^2



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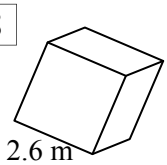


$SA = 6l^2$

$= 6 \times 2.6^2$

$SA =$ **40.56** m^2

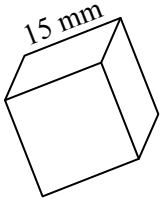
Square units

9

$SA = 6l^2$

$= 6 \times 15^2$

$SA =$ **1 350** mm^2



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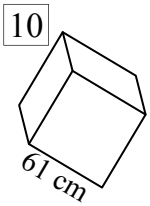


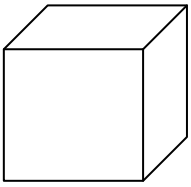
$SA = 6l^2$

$= 6 \times 61^2$

$SA =$ **22 326** cm^2

Find the SA of these cubes. Round your answer to 1 d.p.

11

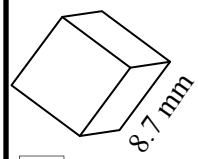


$SA = 6l^2$

$= 6 \times 3.6^2$

$SA = 77.8\ m^2$

12

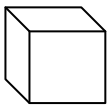


$SA = 6l^2$

$= 6 \times 8.7^2$

$SA = 454.1\ mm^2$

13



$SA = 6l^2$

$= 6 \times 1.25^2$

$SA = 9.4\ m^2$

Use the Δ, □, and ○ to help you to find the SA of these rectangular prisms. All side lengths are in cm.

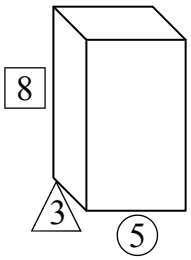
14

$SA = 2 \times$ **8** $\times$ **3**

$+ 2 \times$ **8** $\times$ **5**

$+ 2 \times$ **5** $\times$ **3**

$SA =$ **158** cm^2



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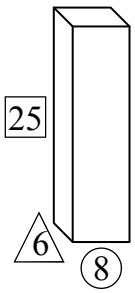


$SA = 2 \times$ **25** $\times$ **6**

$+ 2 \times$ **25** $\times$ **8**

$+ 2 \times$ **8** $\times$ **6**

$SA =$ **796** cm^2

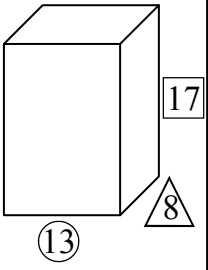
16

$SA = 2 \times$ **17** $\times$ **8**

$+ 2 \times$ **17** $\times$ **13**

$+ 2 \times$ **13** $\times$ **8**

$SA =$ **922** cm^2



Find the SA of these cubes and rectangular prisms.

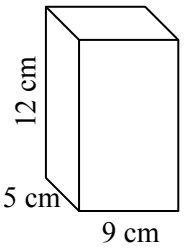
17

$SA = 2 \times$ **5** $\times$ **9**

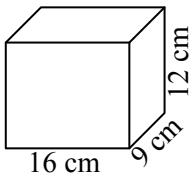
$+ 2 \times$ **5** $\times$ **12**

$+ 2 \times$ **9** $\times$ **12**

$SA =$ **426** cm^2



18



$SA = 2 \times 16 \times 9$

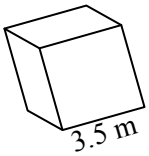
$+ 2 \times 16 \times 12$

$+ 2 \times 9 \times 12$

$SA = 888\ cm^2$

19

Cube



$SA = 6l^2$

$= 6 \times 3.5^2$

$SA = 73.5\ m^2$

20 A rectangular prism has sides 2.4 m, 3.8 m and 4.3 m.

$SA = 2 \times 2.4 \times 3.8$

$+ 2 \times 2.4 \times 4.3$

$+ 2 \times 3.8 \times 4.3$

$SA = 71.56\ m^2$

21 A rectangular prism has sides 78 cm, 14 cm and 23 cm.

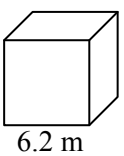
$SA = 2 \times 78 \times 14$

$+ 2 \times 78 \times 23$

$+ 2 \times 14 \times 23$

$SA = 6\,416\ cm^2$

22 Cube



$SA = 6l^2$

$= 6 \times 6.2^2$

$SA = 230.64\ m^2$

Surface Area - Cubes and Rectangular Prisms (Sheet 2)

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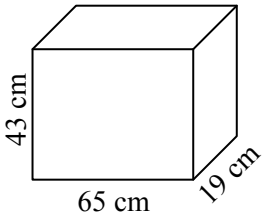
Find the surface area of these rectangular prisms. Remember to show the square units.

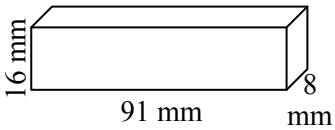
Give the surface area in cm^2. So change all units that aren't in cm, to cm first. Then perform the SA calculation. Round to 1 d.p. if needed.

1

$SA = 2 \times 43 \times 65$
$+ 2 \times 43 \times 19$
$+ 2 \times 65 \times 19$
$SA = 9\,694\ cm^2$



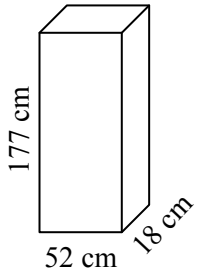
2



$SA = 2 \times 91 \times 8$
$+ 2 \times 91 \times 16$
$+ 2 \times 8 \times 16$
$SA = 4\,624\ mm^2$

3

$SA = 2 \times 177 \times 52$
$+ 2 \times 177 \times 18$
$+ 2 \times 52 \times 18$
$SA = 26\,652\ cm^2$



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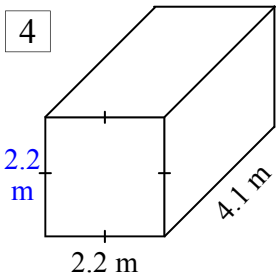
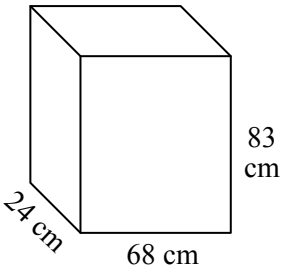


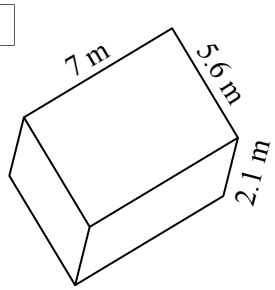
$SA = 2 \times 2.2 \times 4.1$
$+ 2 \times 2.2 \times 2.2$
$+ 2 \times 4.1 \times 2.2$
$SA = 45.76\ m^2$

5

$SA = 2 \times 68 \times 24$
$+ 2 \times 68 \times 83$
$+ 2 \times 24 \times 83$
$SA = 18\,536\ cm^2$



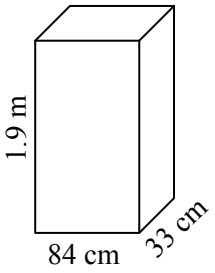
6



$SA = 2 \times 2.1 \times 5.6$
$+ 2 \times 2.1 \times 7$
$+ 2 \times 5.6 \times 7$
$SA = 131.32\ m^2$

7

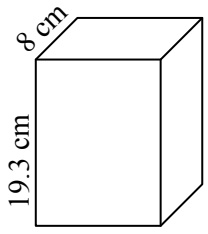
190 cm



$SA = 2 \times 190 \times 84$
$+ 2 \times 190 \times 33$
$+ 2 \times 84 \times 33$
$SA = 50\,0004\ cm^2$

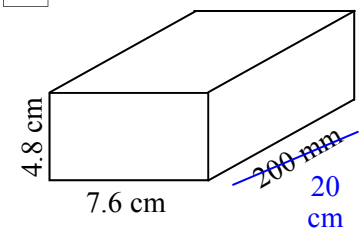
8

$SA = 2 \times 21 \times 8$
$+ 2 \times 21 \times 19.3$
$+ 2 \times 8 \times 19.3$
$SA = 1\,455.4\ cm^2$



21 cm

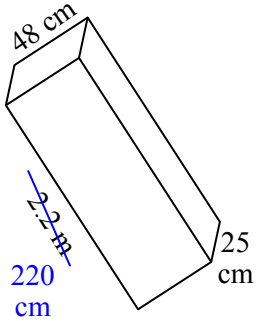
9



$SA = 2 \times 20 \times 7.6$
$+ 2 \times 20 \times 4.8$
$+ 2 \times 7.6 \times 4.8$
$SA = 568.96\ cm^2$

10

$SA = 2 \times 220 \times 48$
$+ 2 \times 220 \times 25$
$+ 2 \times 48 \times 25$
$SA = 34\,520\ cm^2$



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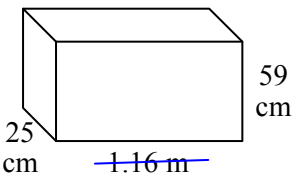
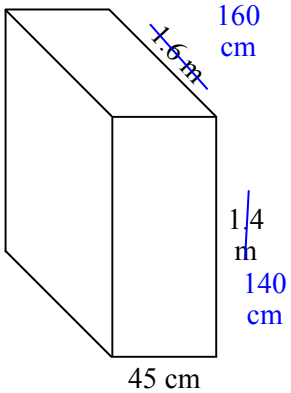


116 cm

$SA = 2 \times 116 \times 59$
$+ 2 \times 116 \times 25$
$+ 2 \times 59 \times 25$
$SA = 22\,438\ cm^2$

12

$SA = 2 \times 160 \times 45$
$+ 2 \times 160 \times 140$
$+ 2 \times 45 \times 140$
$SA = 71\,800\ cm^2$



Surface Area - Cubes and Rectangular Prisms (Homework Strip)

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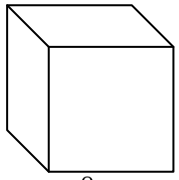
Date Due: _____

Find the SA of these two cubes.

1

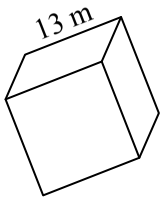


$SA = 6l^2$

$= 6 \times \square^2$

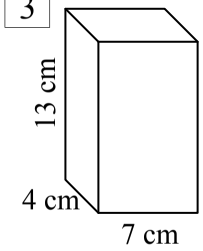
$SA = \square\ cm^2$

2



Find the SA of the rectangular prism.

3



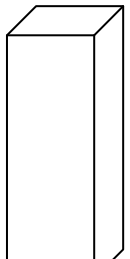
$SA = 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$SA = \square\ cm^2$

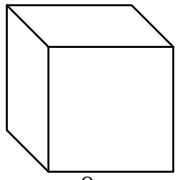
4



Date Due: _____

Find the SA of these two cubes.

1

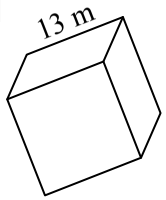


$SA = 6l^2$

$= 6 \times \square^2$

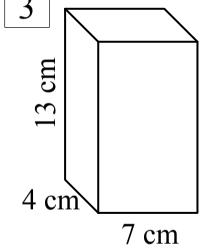
$SA = \square\ cm^2$

2



Find the SA of the rectangular prism.

3



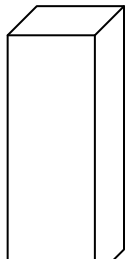
$SA = 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$SA = \square\ cm^2$

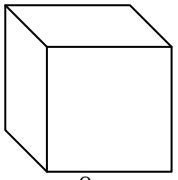
4



Date Due: _____

Find the SA of these two cubes.

1

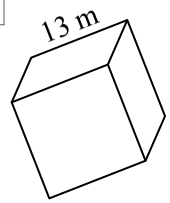


$SA = 6l^2$

$= 6 \times \square^2$

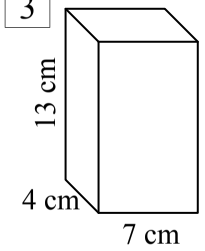
$SA = \square\ cm^2$

2



Find the SA of the rectangular prism.

3



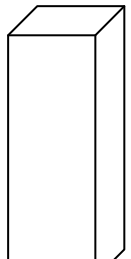
$SA = 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$SA = \square\ cm^2$

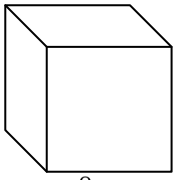
4



Date Due: _____

Find the SA of these two cubes.

1

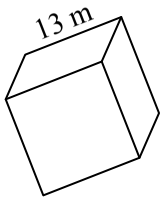


$SA = 6l^2$

$= 6 \times \square^2$

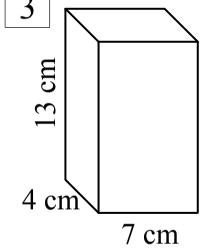
$SA = \square\ cm^2$

2



Find the SA of the rectangular prism.

3



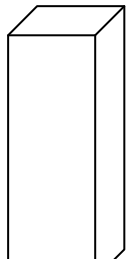
$SA = 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$SA = \square\ cm^2$

4



Surface Area - Cubes and Rectangular Prisms (Homework Strip)

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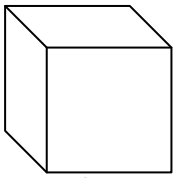
Date Due: _____

Find the SA of these two cubes.

1

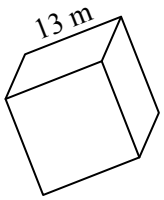


$SA = 6l^2$

$= 6 \times 8^2$

$SA = 384 \text{ cm}^2$

2



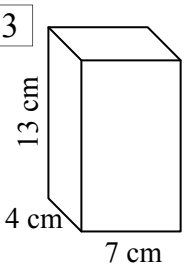
$SA = 6l^2$

$= 6 \times 13^2$

$SA = 1\,014 \text{ m}^2$

Find the SA of the rectangular prism.

3



$SA = 2 \times 7 \times 4$

$+ 2 \times 7 \times 13$

$+ 2 \times 4 \times 13$

$SA = 342 \text{ cm}^2$

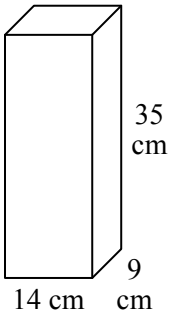
4

$SA = 2 \times 14 \times 9$

$+ 2 \times 14 \times 35$

$+ 2 \times 9 \times 35$

$SA = 1\,862 \text{ cm}^2$



Surface Area - Cubes and Rectangular Prisms Problems.

This worksheet takes your skills with finding the surface area of cubes and rectangular prisms and applies it to different situations.

Column 1 starts with constructing a steel tool box. What happens when you double the size of something, how does that affect the surface area of the object?

Column 2 checks the same theory with cubes, does it also apply? Surface area of a rectangular prism is then applied to re-plastering a room. Calculating how many plaster sheets are required to replace the walls and ceiling.

Column 3 ends with production of a cereal box. Remember that if you need 12.2 m^2 of material, but the material is supplied only in full m^2. You don't round 12.2 m^2 down to 12 m^2. Yes it is true that 12.2 m^2, rounded to the nearest m^2, is 12m^2. but if you need 12.2 m^2, then you won't have enough material for the job.

So rounding up is always required when you buy things like plastering, paint, tiles and anything else that is 'better to have more of', than 'not enough of'.

Surface Area - Cubes and Rectangular Prisms Problems

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Find the SA of these tool boxes!

A steel rectangular shaped toolbox is 35 cm wide, 20 cm high and 25 cm deep, draw a diagram and find the SA

Diagram - Use a Ruler

1

2 SA = 2 × ☐ × ☐

\+ 2 × ☐ × ☐

\+ 2 × ☐ × ☐

SA = ☐ cm^2

Double each of the sides of the tool box.

3 35: 2 × 35 = ☐ cm

20: 2 × 20 = ☐ cm

25: 2 × 25 = ☐ cm

Draw a diagram and then find the SA of the enlarged tool box.

Diagram - Use a Ruler

4

5 SA = 2 × ☐ × ☐

\+ 2 × ☐ × ☐

\+ 2 × ☐ × ☐

SA = ☐ cm^2

Divide (÷) the larger tool box's SA by the smaller tool box's SA.

6 ☐ (Larger Area) / ☐ (Smaller Area) = ☐

Complete this statement

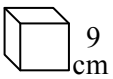
7 When I double the side lengths of a rectangular prism the SA is multiplied by a factor of ☐.

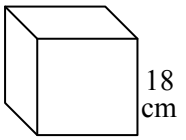
Apply to cubes, find the SA.

8 $SA = 6l^2$

$= 6 \times$ ☐^{2}

SA = ☐ cm^2





Divide the larger cube's SA by the smaller cube's SA.

9 ☐ / ☐ = ☐

Make a statement about the size & SA

10 When I double _____

Jen measures her room, its floor size is 3.7 m by 4.5 m, and the ceiling height is 2.4 m. Calculate the SA of the room.

11 SA = 2 × ☐ × ☐

\+ 2 × ☐ × ☐

\+ 2 × ☐ × ☐

SA = ☐ m^2

Calculate the floor area of the room. Floor size is 3.7 m by 4.5 m.

12 A = ☐ × ☐

A = ☐ m^2

The ceiling and walls are to be re-plastered. Find their area.

13 A = ☐ (Room SA) - ☐ (Floor Area)

A = ☐

Remember units

The plaster sheets are 1.2 m by 2.4 m, calculate the area of one plaster sheet.

14 A = ☐ × ☐

A = ☐

How many **FULL** sheets are required to plaster Jen's ceiling and walls?

15 ☐ (Q13) / ☐ (Q14) = ☐ sheets

A box of 'Oaties' cereal is 0.32 m by 0.23 m by 0.06 m. Find the box's SA.

16 _____

Don't round your answer

The company produces 30 000 boxes in a day, what is the area of cardboard needed per day (ignoring seams)?

17 _____

If cardboard is supplied in rolls of 220 m^2. How many complete rolls are required for one day's production?

18 _____

Surface Area - Cubes and Rectangular Prisms Problems

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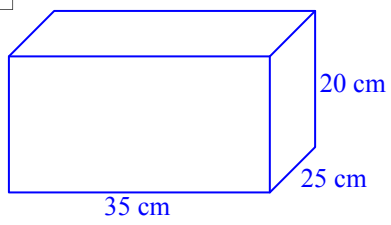
Find the SA of these tool boxes!

A steel rectangular shaped toolbox is 35 cm (L)× 25 cm (W)× 20 (H) cm, draw a diagram and find the SA.

Diagram - Use a Ruler

1



2 SA = 2 × 35 × 25
+ 2 × 35 × 20
+ 2 × 25 × 20
SA = 4 150 cm²

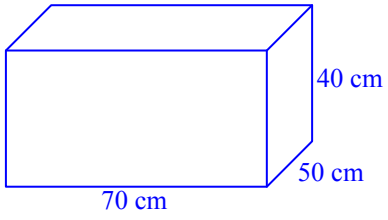
Double each of the sides of the tool box.

3 35: 2 × 35 = 70 cm
20: 2 × 20 = 40 cm
25: 2 × 25 = 50 cm

Draw a diagram and then find the SA of the enlarged tool box.

Diagram - Use a Ruler

4



5 SA = 2 × 70 × 40
+ 2 × 70 × 50
+ 2 × 40 × 50
SA = 16 600 cm²

Divide (÷) the larger tool box's SA by the smaller tool box's SA.

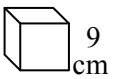
6 16 600 (Larger Area) / 4 150 (Smaller Area) = 4

Complete this statement

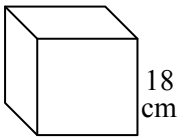
7 When I double the side lengths of a rectangular prism the SA is multiplied by a factor of 4.

Apply to cubes, find the SA.

8 $SA = 6l^2$
$= 6 \times 9^2$
SA = 486 cm²



$SA = 6l^2$
$= 6 \times 18^2$
$SA = 1\,944$ cm²



Divide the larger cube's SA by the smaller cube's SA.

9 1 944 / 486 = 4

Make a statement about the size & SA

10 When I double the side lengths of a cube, the SA is multiplied by a factor of 4.

Jen measures her room, its floor size is 3.7 m by 4.5 m, and the ceiling height is 2.4 m. Calculate the SA of the room.

11 SA = 2 × 3.7 × 4.5
+ 2 × 3.7 × 2.4
+ 2 × 4.5 × 2.4
SA = 72.66 m²

Calculate the floor area of the room. Floor size is 3.7 m by 4.5 m.

12 A = 3.7 × 4.5
A = 16.65 m²

The ceiling and walls are to be re-plastered. Find their area.

13 A = 72.66 (Room SA) - 16.65 (Floor Area)
A = 56.01 m²
Remember units

The plaster sheets are 1.2 m by 2.4 m, calculate the area of one plaster sheet.

14 A = 1.2 × 2.4
A = 2.88 m²

How many **FULL** sheets are required to plaster Jen's ceiling and walls?

15 56.01 (Q13) / 2.88 (Q14) = 20 sheets

A box of 'Oaties' cereal is 0.32 m by 0.23 m by 0.06 m. Find the box's SA.

16 SA = 2 × 0.32 × 0.23
+ 2 × 0.32 × 0.06
+ 2 × 0.23 × 0.06

Don't round your answer

The company produces 30 000 boxes in a day, what is the area of cardboard needed per day (ignoring seams)?

17 A = 0.2132 × 30 000
A = 6 396 m²

If cardboard is supplied in rolls of 220 m². How many complete rolls are required for one day's production?

18 Rolls Req'd = 6 396 ÷ 220
Rolls Req'd = 30 rolls

Surface Area - Cubes and Rectangular Prisms Problems (Homework Strip)

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Date Due: _____

A computer game comes with a special edition sheet metal box, with sides: 15 cm × 9 cm × 11 cm.

1 Calculate the box's Surface Area.

$SA = 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$+ 2 \times \square \times \square$

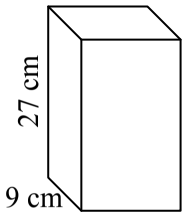
$SA = \square \text{ cm}^2$

Sheet metal weighs 3.4 grams per cm^2.

2 Calculate the box's weight.

A decorative wooden frame is to be covered with rice paper. Find:

3 The amount of paper needed, in cm^2.



Date Due: _____

A computer game comes with a special edition sheet metal box, with sides: 15 cm × 9 cm × 11 cm.

1 Calculate the box's Surface Area.

$SA = 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$+ 2 \times \square \times \square$

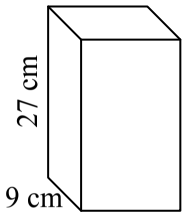
$SA = \square \text{ cm}^2$

Sheet metal weighs 3.4 grams per cm^2.

2 Calculate the box's weight.

A decorative wooden frame is to be covered with rice paper. Find:

3 The amount of paper needed, in cm^2.



Date Due: _____

A computer game comes with a special edition sheet metal box, with sides: 15 cm × 9 cm × 11 cm.

1 Calculate the box's Surface Area.

$SA = 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$+ 2 \times \square \times \square$

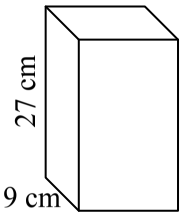
$SA = \square \text{ cm}^2$

Sheet metal weighs 3.4 grams per cm^2.

2 Calculate the box's weight.

A decorative wooden frame is to be covered with rice paper. Find:

3 The amount of paper needed, in cm^2.



Date Due: _____

A computer game comes with a special edition sheet metal box, with sides: 15 cm × 9 cm × 11 cm.

1 Calculate the box's Surface Area.

$SA = 2 \times \square \times \square$

$+ 2 \times \square \times \square$

$+ 2 \times \square \times \square$

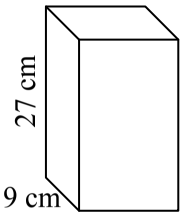
$SA = \square \text{ cm}^2$

Sheet metal weighs 3.4 grams per cm^2.

2 Calculate the box's weight.

A decorative wooden frame is to be covered with rice paper. Find:

3 The amount of paper needed, in cm^2.



Surface Area - Cubes & Rectangular Prisms Problems (Homework Strip)

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Date Due:

A computer game comes with a special edition sheet metal box, with sides: 15 cm × 9 cm × 11 cm.

1 Calculate the box's Surface Area.

$SA = 2 \times 15 \times 9$

$+ 2 \times 15 \times 11$

$+ 2 \times 9 \times 22$

$SA = 996 \text{ cm}^2$

Sheet metal weighs 3.4 grams per cm^2.

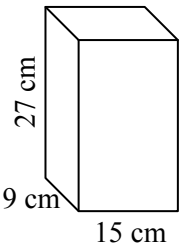
2 Calculate the box's weight.

$\text{Weight} = 996 \times 3.4$

$= 3\,386.4 \text{ g}$

A decorative wooden frame is to be covered with rice paper. Find:

3 The amount of paper needed, in cm^2.



$SA = 2 \times 15 \times 9$

$+ 2 \times 15 \times 27$

$+ 2 \times 9 \times 27$

$SA = 1\,566 \text{ cm}^2$

Surface Area - SA of Nets of Triangular Prisms

When you calculate the Surface Area of a Triangular Prism you are adding 5 areas together. There are the two triangular ends of the solid, then there are the three rectangles that wrap around the solid.

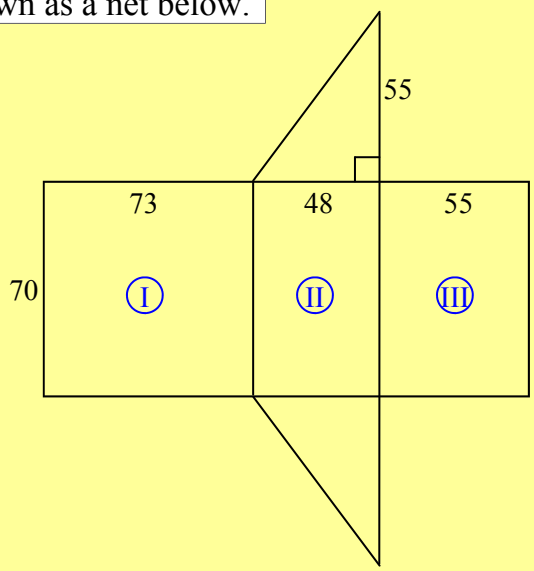
This sheet takes you through the procedure in parts to assist you to understand the steps involved in the process. The first sheet revises finding the area of a triangle and the area of a rectangle, which should be straight-forward revision.

The second sheet deals with nets of triangular prisms. With nets, you can see all the plane shapes that build to make the solid, as they are folded out in front of you. With nets of triangular prisms, there are:

- 2 matching triangular ends
- 3 rectangular areas (sometimes these areas might be the same)

So the surface area is found by finding these areas, then adding them together. An example is below.

Example - Find the surface area of the triangular prism, shown as a net below.



$2\blacktriangle$: $2 \times \frac{1}{2} \times 48 \times 55$

$= 2\,640 \text{ cm}^2$

This is the area of the two triangles

Ⓘ: $73 \times 70 = 5\,110 \text{ cm}^2$

Ⓘ: $48 \times 70 = 3\,360 \text{ cm}^2$

Ⓘ: $55 \times 70 = 3\,850 \text{ cm}^2$

This is the area of the three rectangles

$SA = 2\,640 + 5\,110 + 3\,360 + 3\,850$

$SA = 14\,960 \text{ cm}^2$

Add all the areas to find the surface area (SA)

Surface Area - Area of Triangles and Rectangles (Sheet 1)

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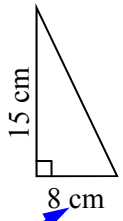
Find the area of these triangles. Use A = ½*bh*, using either 0.5 or ½ on your calculator.

Example

A = ½*bh*

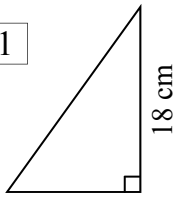
= ½×8×15

A = 60 cm^2



units^2 must match

1

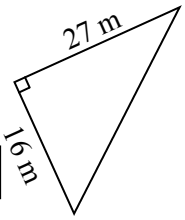


A = ½*bh*

= ½× ☐ × ☐

A = ☐ cm^2

2

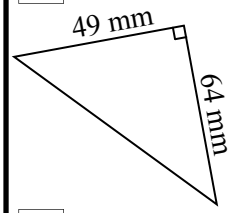


A = ½*bh*

= ½ × ☐ × ☐

A = ☐ m^2

3

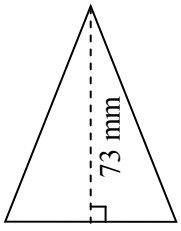


A =

A =

Show square units

4



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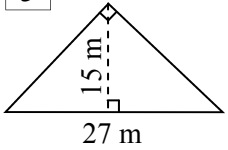

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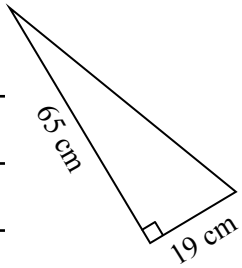
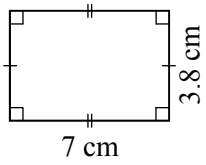


Now rectangles. Use A=*lb* to find the area, round to 1 d.p. when required.

Example



A = *lb*

= 7 × 3.8

A = 26.6 cm^2

Show square units

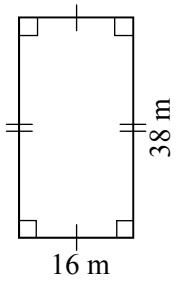
7

A = *lb*

= ☐ × ☐

Multiply, in any order

A = ☐ m^2



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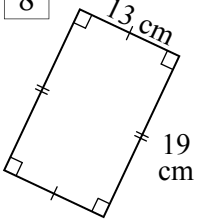


A = *lb*

= ☐ × ☐

A = ☐ cm^2

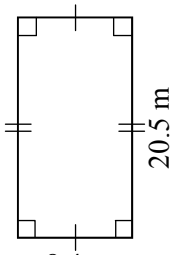
9

A = *lb*

= ☐ × ☐

A = ☐

Show square units



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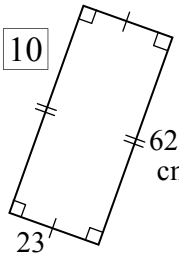

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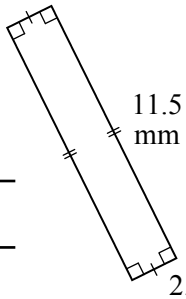

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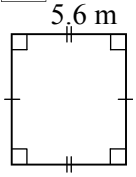
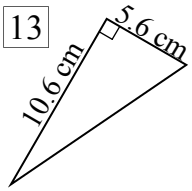


Now a mixture, find the area of these triangles and rectangles, answer to 1 d.p. when required.

13



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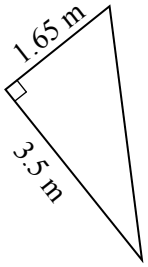

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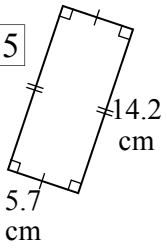

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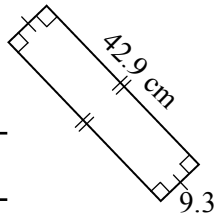

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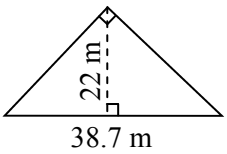

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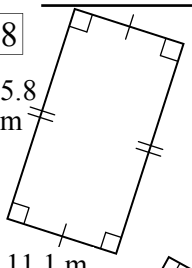

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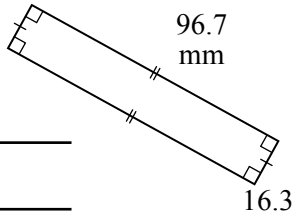

Surface Area - SA of Nets of Triangular Prisms (Sheet 2)

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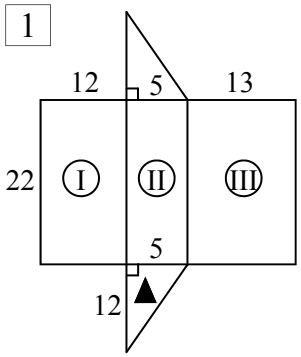
These are nets of triangular prisms. Find the areas individually, then get the total for the surface area of the triangular prism. All measurements are in cm.

1



2▲: $2 \times \frac{1}{2} \times 5 \times 12$

$= \square$ cm^2

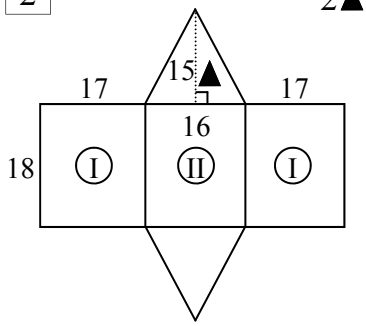
I: $12 \times 22 = \square$ cm^2

II: $5 \times 22 = \square$ cm^2

III: $13 \times 22 = \square$ cm^2

SA = $\square + \square + \square + \square = \square$ cm^2

2



2▲: $2 \times \frac{1}{2} \times \square \times \square$

$= \square$ cm^2

2 I: $2 \times \square \times \square$

$= \square$ cm^2

II: $\square \times \square$

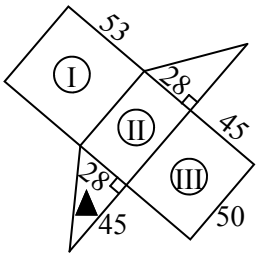
$= \square$ cm^2

SA = $\square + \square + \square = \square$ cm^2

3

2▲: $2 \times \frac{1}{2} \times \square \times \square$

$= \square$ cm^2



I: $\square \times \square = \square$ cm^2

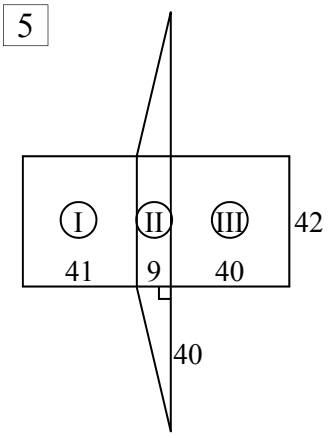
II: $\square \times \square = \square$ cm^2

III: $\square \times \square = \square$ cm^2

SA = $\square + \square + \square + \square$

SA = $\square$ cm^2

5



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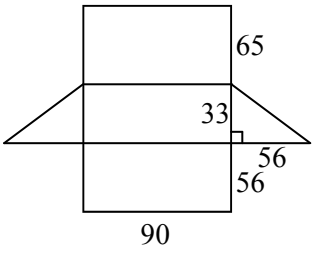

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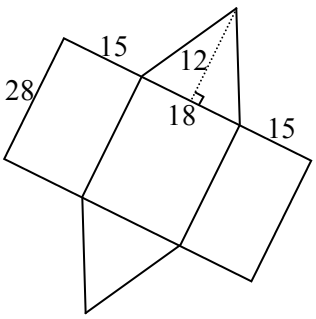

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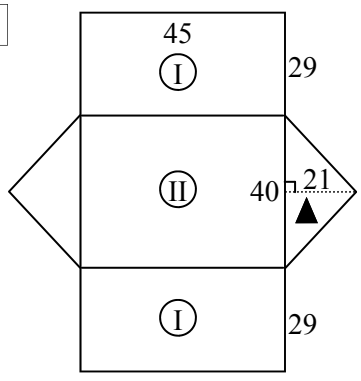

2▲:

2 I:

II:

SA =

Surface Area - Area of Triangles and Rectangles (Sheet 1)

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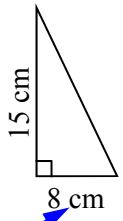
Find the area of these triangles. Use A = ½*bh*, using either 0.5 or ½ on your calculator.

Example

$A = \frac{1}{2}bh$

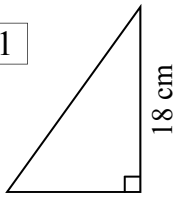
$= \frac{1}{2} \times 8 \times 15$

$A = 60\ cm^2$



units² must match

1

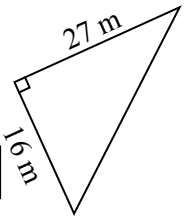


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 11 \times 18$

$A = 99\ cm^2$

2

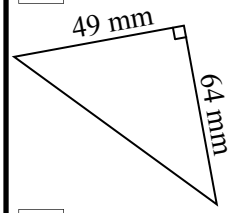


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 16 \times 27$

$A = 216\ m^2$

3



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 49 \times 64$

$A = 1\,568\ mm^2$

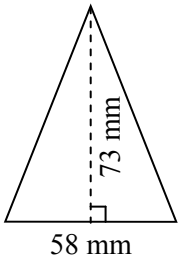
Show square units

4

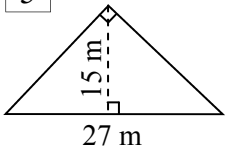
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 58 \times 73$

$A = 2\,044\ mm^2$



5



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 27 \times 15$

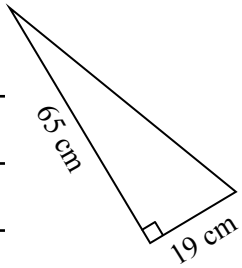
$A = 202.5\ m^2$

6

$A = \frac{1}{2}bh$

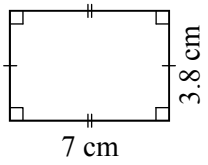
$= \frac{1}{2} \times 19 \times 65$

$A = 617.5\ cm^2$



Now rectangles. Use A=*lb* to find the area, round to 1 d.p. when required.

Example



$A = lb$

$= 7 \times 3.8$

$A = 26.6\ cm^2$

Show square units

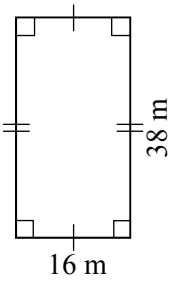
7

$A = lb$

$= 38 \times 16$

Multiply, in any order

$A = 608\ m^2$



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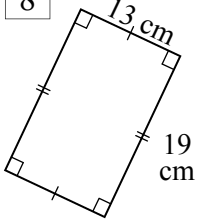


$A = lb$

$= 19 \times 13$

$A = 247\ cm^2$

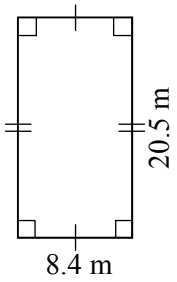
9

$A = lb$

$= 20.5 \times 8.4$

$A = 172.2\ m^2$

Show square units



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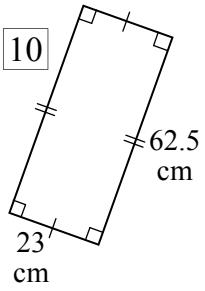


$A = lb$

$= 62.5 \times 23$

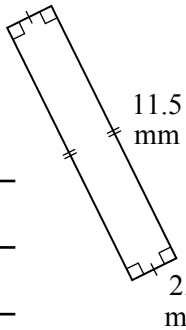
$A = 1\,437.5\ cm^2$

11

$A = lb$

$= 11.5 \times 2.8$

$A = 32.2\ mm^2$



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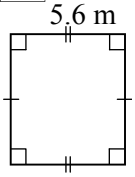


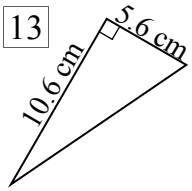
$A = lb$

$= 7.8 \times 5.6$

$A = 43.68\ m^2$

Now a mixture, find the area of these triangles and rectangles, answer to 1 d.p. when required.

13



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5.6 \times 10.6$

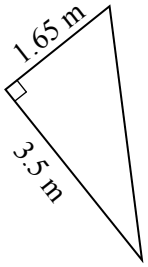
$A = 29.7\ cm^2$

14

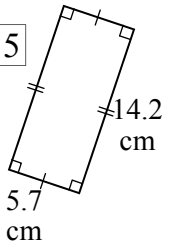
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 1.65 \times 3.5$

$A = 2.9\ m^2$



15



$A = lb$

$= 5.7 \times 14.2$

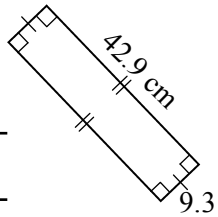
$A = 40.5\ cm^2$

16

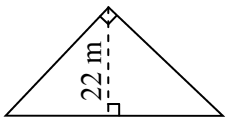
$A = lb$

$= 42.9 \times 9.3$

$A = 399.0\ cm^2$



17

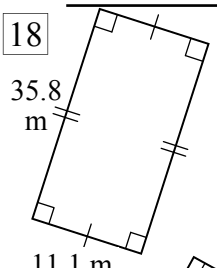


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 38.7 \times 22$

$A = 425.7\ m^2$

18



$A = lb$

$= 35.8 \times 11.1$

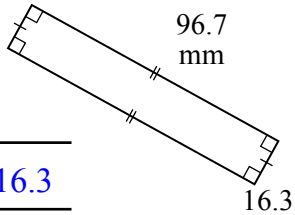
$A = 397.4\ m^2$

19

$A = lb$

$= 96.7 \times 16.3$

$A = 1\,576.2\ mm^2$



Surface Area - SA of Nets of Triangular Prisms (Sheet 2)

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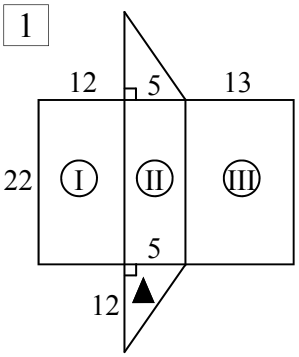
These are nets of triangular prisms. Find the areas individually, then get the total for the surface area of the triangular prism. All measurements are in cm.

1



2▲: $2 \times \frac{1}{2} \times 5 \times 12$

$= 60$ cm^2

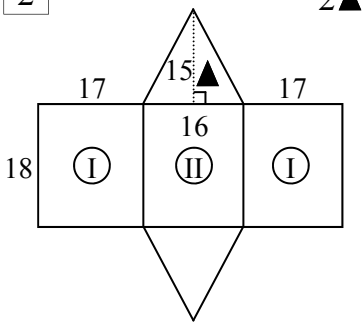
I: $12 \times 22 = 264$ cm^2

II: $5 \times 22 = 110$ cm^2

III: $13 \times 22 = 286$ cm^2

SA = 60 + 264 + 110 + 286 = 720 cm^2

2



2▲: $2 \times \frac{1}{2} \times 16 \times 15$

$= 240$ cm^2

2 I: $2 \times 17 \times 18$

$= 612$ cm^2

II: 16×18

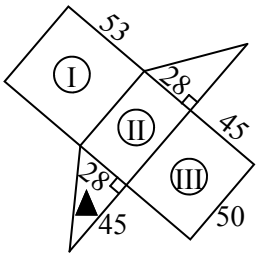
$= 288$ cm^2

SA = 240 + 612 + 288 = 1 140 cm^2

3

2▲: $2 \times \frac{1}{2} \times 28 \times 45$

$= 1\,260$ cm^2



I: $53 \times 50 = 2\,650$ cm^2

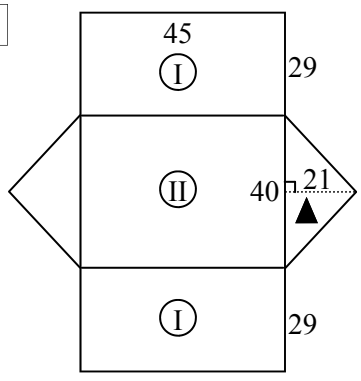
II: $28 \times 50 = 1\,400$ cm^2

III: $45 \times 50 = 2\,250$ cm^2

SA = 1 260 + 2 650 + 1 400 + 2 250

SA = 7 560 cm^2

4



2▲: $2 \times \frac{1}{2} \times 40 \times 21$

$= 840$ cm^2

2 I: $2 \times 29 \times 45$

$= 2\,610$ cm^2

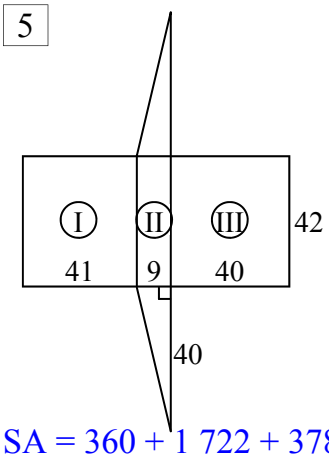
II: 40×45

$= 1\,800$ cm^2

SA = 840 + 2 610 + 1 800

SA = 5 250 cm^2

5



2▲: $2 \times \frac{1}{2} \times 9 \times 40$

$= 360$ cm^2

I: $41 \times 42 = 1\,722$ cm^2

II: $9 \times 42 = 378$ cm^2

III: $40 \times 42 = 1\,680$ cm^2

SA = 360 + 1 722 + 378 + 1 680 = 4 140 cm^2

6

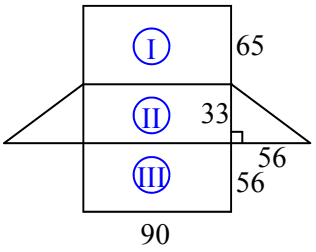
2▲: $2 \times \frac{1}{2} \times 33 \times 56$

$= 1\,848$ cm^2

I: $65 \times 90 = 5\,850$ cm^2

II: $33 \times 90 = 2\,970$ cm^2

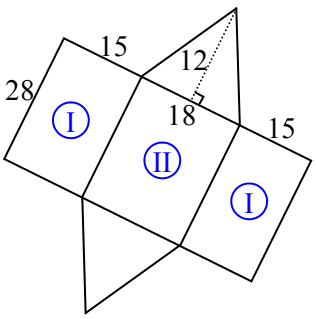
III: $56 \times 90 = 5\,040$ cm^2



SA = 1 848 + 5 850 + 2 970 + 5 040

SA = 15 708 cm^2

7



2▲: $2 \times \frac{1}{2} \times 18 \times 12$

$= 216$ cm^2

2 I: $2 \times 15 \times 28$

$= 840$ cm^2

II: $18 \times 28 = 504$ cm^2

SA = 216 + 840 + 504 = 1 560 cm^2

Surface Area - SA of Nets of Triangular Prisms (Homework Strip)

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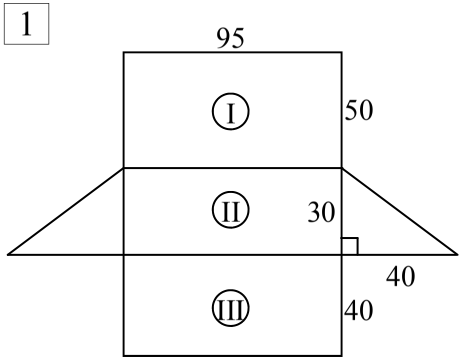
Date Due: _____

Find the SA of the net of a triangular prism shown below.

1



2▲: $2 \times \frac{1}{2} \times \square \times \square$

$= \square$ cm^2

Ⓘ: $\square \times \square = \square$ cm^2

Ⓘⓘ: $\square \times \square = \square$ cm^2

Ⓘⓘⓘ: $\square \times \square = \square$ cm^2

SA = $\square + \square$

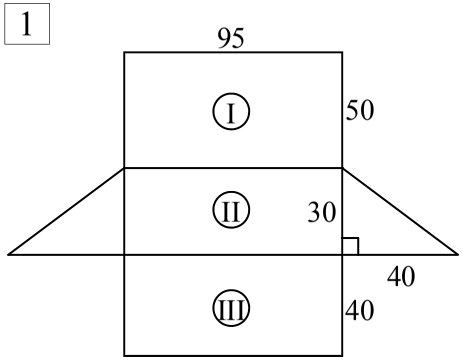
$+ \square + \square$

SA = $\square$ cm^2

Date Due: _____

Find the SA of the net of a triangular prism shown below.

1



2▲: $2 \times \frac{1}{2} \times \square \times \square$

$= \square$ cm^2

Ⓘ: $\square \times \square = \square$ cm^2

Ⓘⓘ: $\square \times \square = \square$ cm^2

Ⓘⓘⓘ: $\square \times \square = \square$ cm^2

SA = $\square + \square$

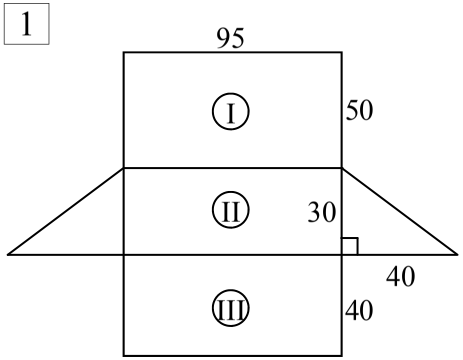
$+ \square + \square$

SA = $\square$ cm^2

Date Due: _____

Find the SA of the net of a triangular prism shown below.

1



2▲: $2 \times \frac{1}{2} \times \square \times \square$

$= \square$ cm^2

Ⓘ: $\square \times \square = \square$ cm^2

Ⓘⓘ: $\square \times \square = \square$ cm^2

Ⓘⓘⓘ: $\square \times \square = \square$ cm^2

SA = $\square + \square$

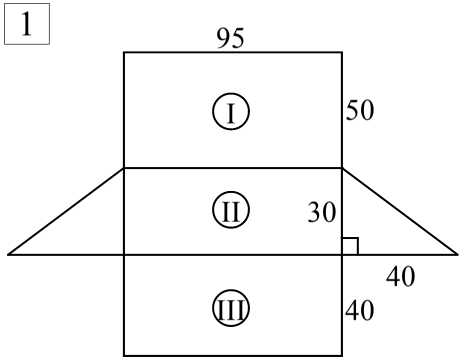
$+ \square + \square$

SA = $\square$ cm^2

Date Due: _____

Find the SA of the net of a triangular prism shown below.

1



2▲: $2 \times \frac{1}{2} \times \square \times \square$

$= \square$ cm^2

Ⓘ: $\square \times \square = \square$ cm^2

Ⓘⓘ: $\square \times \square = \square$ cm^2

Ⓘⓘⓘ: $\square \times \square = \square$ cm^2

SA = $\square + \square$

$+ \square + \square$

SA = $\square$ cm^2

Surface Area - SA of Nets of Triangular Prisms (Homework Strip)

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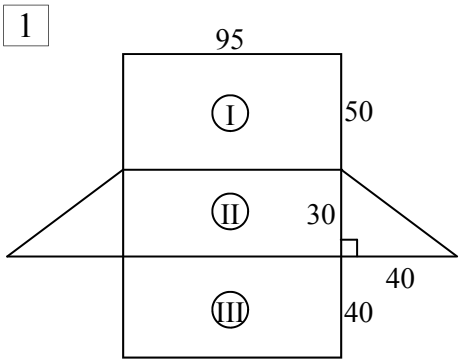
Date Due: _____

Find the SA of the net of a triangular prism shown below.

1



2▲: $2 \times \frac{1}{2} \times 30 \times 40$

$= 1\,200 \text{ cm}^2$

Ⓘ: $50 \times 95 = 4\,750 \text{ cm}^2$

Ⓘⓘ: $30 \times 95 = 2\,850 \text{ cm}^2$

Ⓘⓘⓘ: $40 \times 95 = 3\,800 \text{ cm}^2$

SA = $1\,200 + 4\,750$

$+ 2\,850 + 3\,800$

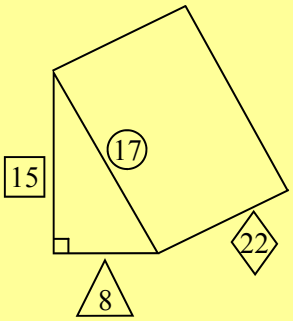
SA = $12\,600 \text{ cm}^2$

Surface Area - Triangular Prisms

The surface area of a triangular prism is found by finding the area of the two identical triangular ends and the three rectangles wrapped around the prism, then adding them together.

Because there are four numbers and it is important which is multiplied by which, the first sheet uses shapes around the numbers that match their correct position in the calculation. Just take the numbers from the shape and place them into the calculation and away you go!

Example - Finding the SA of the triangular prism shown below



The shape has a circle, square, triangle and rhombus around its dimensions. Place each into their correct position in the calculation below.

$SA = 2 \times \frac{1}{2} \times 8 \times 15$

The shape has two identical triangles on the front and back. This line calculates their combined area. Remember that ½ = 0.5, easier on your calculator and maybe you know what 2 × 0.5 is? Is it necessary?

$+ 8 \times 22$

This is calculating the rectangular base of the triangular prism.

$+ 15 \times 22$

This is calculating the rectangular side of the triangular prism. (The vertical face on the left side of the shape).

$+ 17 \times 22$

This is calculating the rectangular side of the triangular prism. (The slanted face on the right side of the shape).

$SA = 1\ 000\ cm^2$

The second sheet doesn't have the shapes around the numbers, but the method to find the SA is the same.

Surface Area - Triangular Prisms Method Sheet (Sheet 1)

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Use the Δ, □, ◊ and ○ to help you to find the SA of these triangular prisms. All measurements are in cm.

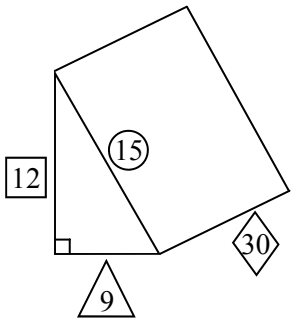
1 SA = 2×½× △ × □

\+ △ × ◊

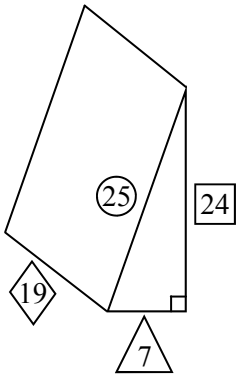
\+ □ × ◊

\+ ○ × ◊

SA = _____ cm^2



2



SA = 2×½× △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____ cm^2

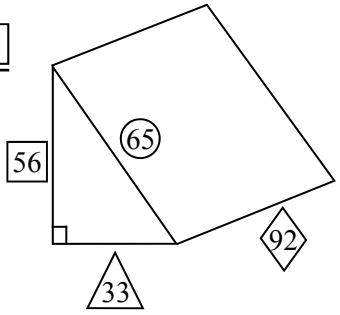
3 SA = 2×½× △ × □

\+ △ × ◊

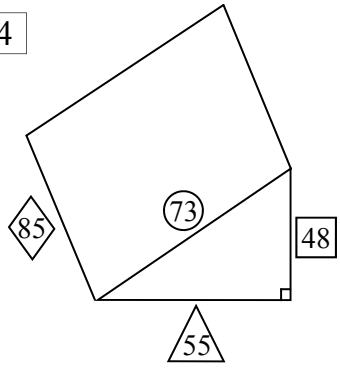
\+ □ × ◊

\+ ○ × ◊

SA = _____



4



SA = × × △ × □

Write the '2' and the '½' above

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____

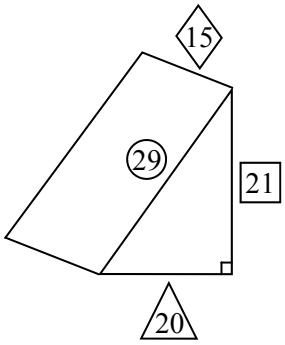
5 SA = × × △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____



6 SA = 2×½× △ × □

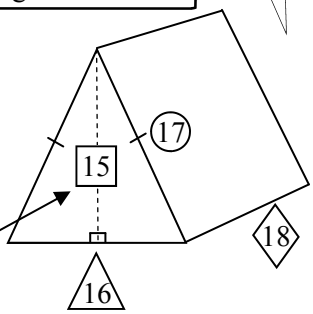
\+ △ × ◊

\+2 × ○ × ◊

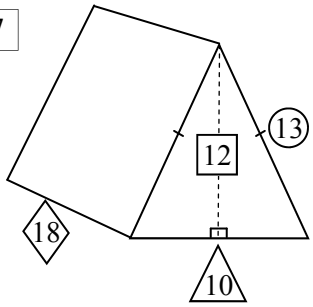
SA = _____ cm^2

15 is the perpendicular height

These are isosceles triangles. Careful!



7



SA = × × △ × □

\+ △ × ◊

\+2 × ○ × ◊

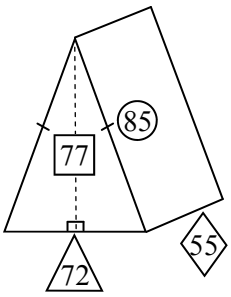
SA = _____

8 SA = × × △ × □

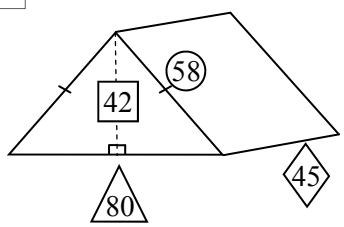
\+ △ × ◊

\+2 × ○ × ◊

SA = _____



9



SA = × × △ × □

\+ △ × ◊

\+2 × ○ × ◊

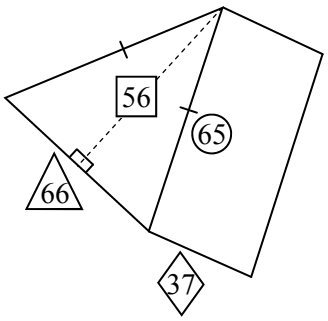
SA = _____

10 SA = × × △ × □

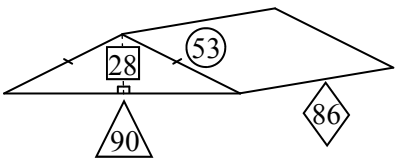
\+ △ × ◊

\+2 × ○ × ◊

SA = _____



11



SA = × × △ × □

\+ △ × ◊

\+2 × ○ × ◊

SA = _____

Surface Area - Triangular Prisms (Sheet 2)

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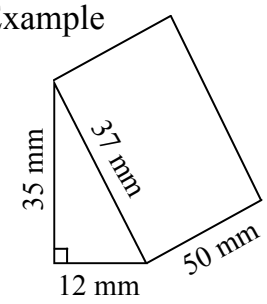
Find the surface area of these triangular prisms.

Example



$SA = 2 \times \frac{1}{2} \times 12 \times 35$

$+ 12 \times 50$

$+ 35 \times 50$

$+ 37 \times 50$

$SA = 4\,620 \text{ mm}^2$

1

base height

$SA = 2 \times \frac{1}{2} \times \square \times \square$

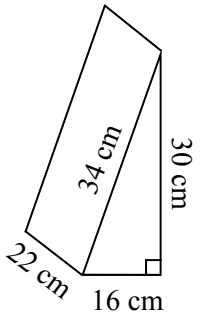
$+ \square \times \square$

$+ \square \times \square$

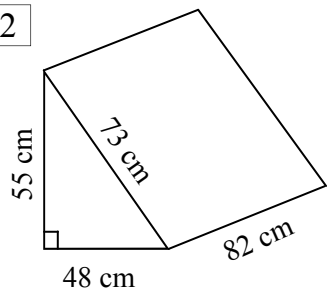
$+ \square \times \square$

Front three edges, all × 22

$SA = \square \text{ cm}^2$



2



base height

$SA = 2 \times \frac{1}{2} \times \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

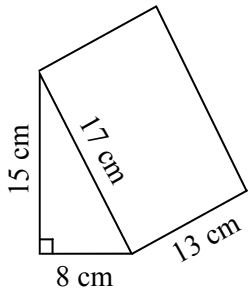
$+ \square \times \square$

Front 3 edges, all × 82

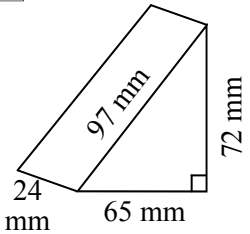
$SA = \square \text{ cm}^2$

3

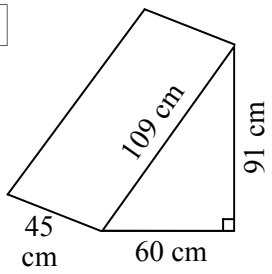
Remember square units



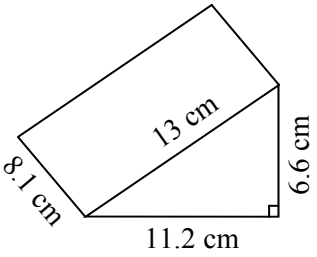
4



5

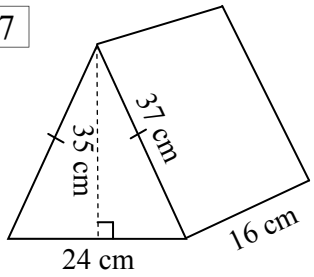


6



These have matching pairs of sides, so you can double the repeated side. Find the surface area.

7



base height

$SA = 2 \times \frac{1}{2} \times \square \times \square$

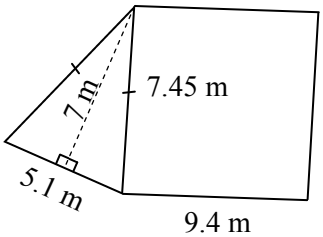
$+ \square \times \square$ base rectangle

$+ 2 \times \square \times \square$

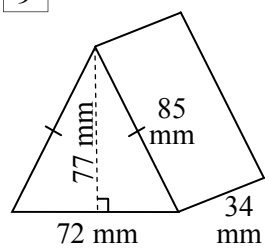
2 × identical (slanted) rectangles

$SA = \square \text{ cm}^2$

8



9

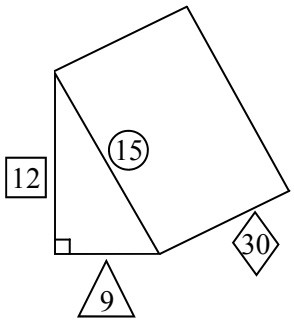


Surface Area - Triangular Prisms Method Sheet (Sheet 1)

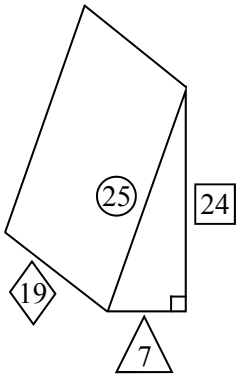
Use the Δ, □, ◊ and ○ to help you to find the SA of these triangular prisms. All measurements are in cm.

1

SA = 2×½× Δ 9 × □ 12
+ Δ 9 × ◊ 30
+ □ 12 × ◊ 30
+ ○ 15 × ◊ 30
SA = 1 188 cm²



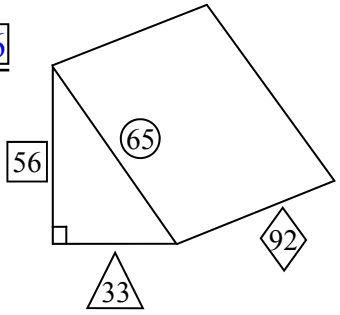
2



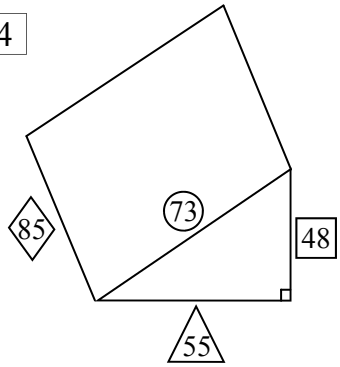
SA = 2×½× Δ 7 × □ 24
+ Δ 7 × ◊ 19
+ □ 24 × ◊ 19
+ ○ 25 × ◊ 19
SA = 1 232 cm²

3

SA = 2×½× Δ 33 × □ 56
+ Δ 33 × ◊ 92
+ □ 56 × ◊ 92
+ ○ 65 × ◊ 92
SA = 16 016 cm²



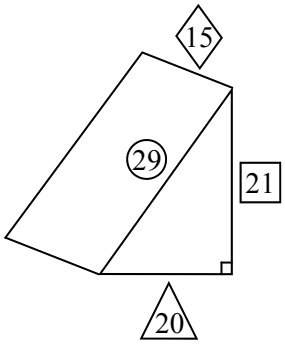
4



SA = 2×½× Δ 55 × □ 48
Write the '2' and the '½' above
+ Δ 55 × ◊ 85
+ □ 48 × ◊ 85
+ ○ 73 × ◊ 85
SA = 17 600 cm²

5

SA = 2×½× Δ 20 × □ 21
+ Δ 20 × ◊ 15
+ □ 21 × ◊ 15
+ ○ 29 × ◊ 15
SA = 1 470 cm²

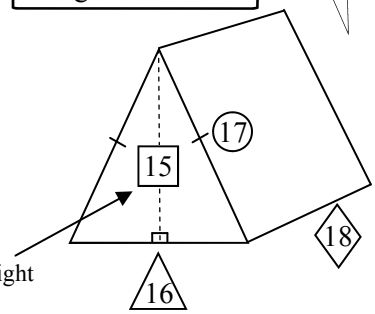


6

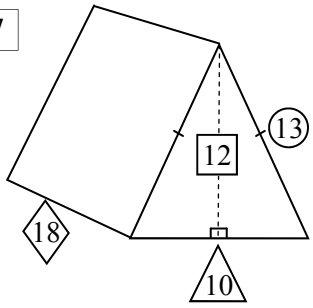
SA = 2×½× Δ 16 × □ 15
+ Δ 16 × ◊ 18
+2 × ○ 17 × ◊ 18
SA = 1 140 cm²

□ 15 is the perpendicular height

These are isosceles triangles. Careful!



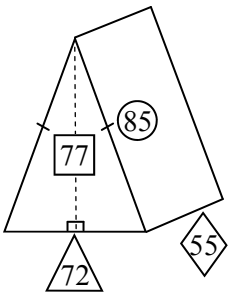
7



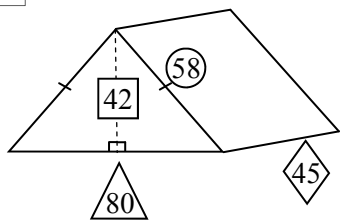
SA = 2×½× Δ 10 × □ 12
+ Δ 10 × ◊ 18
+2 × ○ 13 × ◊ 18
SA = 768 cm²

8

SA = 2×½× Δ 72 × □ 77
+ Δ 72 × ◊ 55
+2 × ○ 85 × ◊ 55
SA = 18 854 cm²



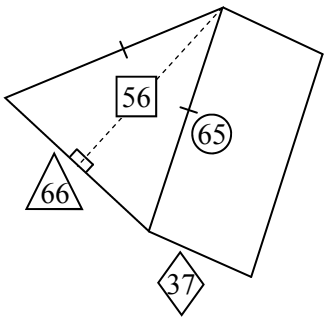
9



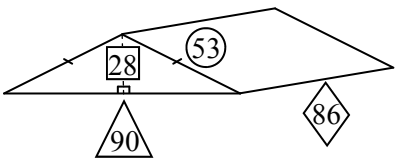
SA = 2×½× Δ 80 × □ 42
+ Δ 80 × ◊ 45
+2 × ○ 58 × ◊ 45
SA = 12 180 cm²

10

SA = 2×½× Δ 66 × □ 56
+ Δ 66 × ◊ 37
+2 × ○ 65 × ◊ 37
SA = 10 948 cm²



11



SA = 2×½× Δ 90 × □ 28
+ Δ 90 × ◊ 86
+2 × ○ 53 × ◊ 86
SA = 19 376 cm²

Surface Area - Triangular Prisms (Sheet 2)

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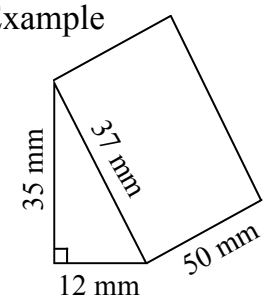
Find the surface area of these triangular prisms.

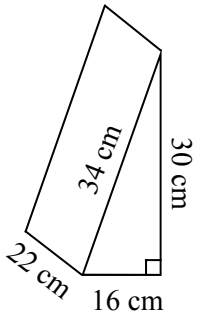
Example



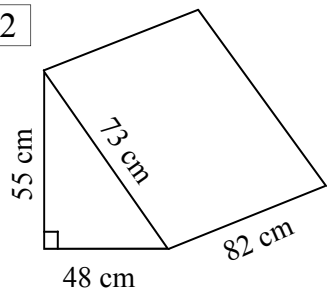
SA = 2 × ½ × 12 × 35
+ 12 × 50
+ 35 × 50
+ 37 × 50
SA = 4 620 mm²

1

base height
SA = 2 × ½ × 16 × 30
+ 30 × 22
+ 34 × 22
+ 16 × 22
Front three edges, all × 22
SA = 2 240 cm²



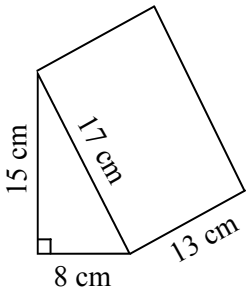
2



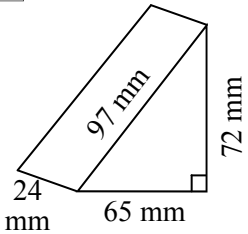
base height
SA = 2 × ½ × 48 × 55
+ 48 × 82
+ 55 × 82
+ 73 × 82
Front 3 edges, all × 82
SA = 17 072 cm²

3

SA = 2 × ½ × 8 × 15
+ 8 × 13
+ 15 × 13
+ 17 × 13
SA = 640 cm²
Remember square units

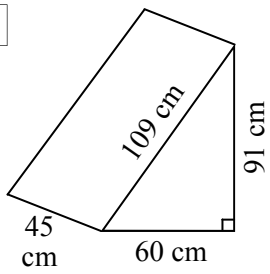


4



SA = 2 × ½ × 65 × 72
+ 65 × 24
+ 72 × 24
+ 97 × 24
SA = 10 296 mm²

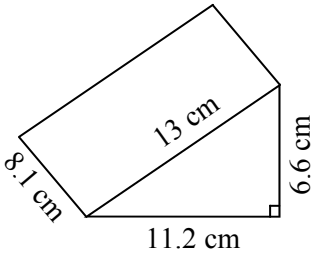
5



SA = 2 × ½ × 60 × 91
+ 60 × 45
+ 91 × 45
+ 109 × 45
SA = 17 160 cm²

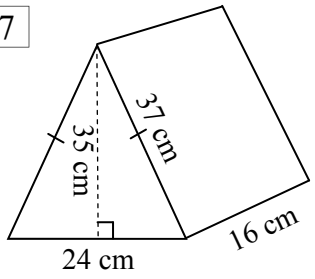
6

SA = 2 × ½ × 11.2 × 6.6
+ 11.2 × 8.1
+ 6.6 × 8.1
+ 13 × 8.1
SA = 323.4 cm²



These have matching pairs of sides, so you can double the repeated side. Find the surface area.

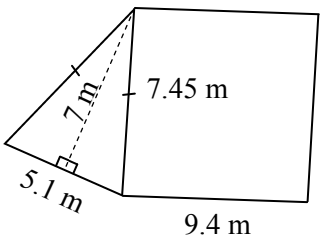
7



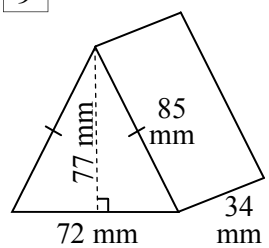
base height
SA = 2 × ½ × 24 × 35
+ 24 × 16 base rectangle
+ 2 × 37 × 16
2 × identical (slanted) rectangles
SA = 2 408 cm²

8

SA = 2 × ½ × 5.1 × 7
+ 5.1 × 9.4
+ 2 × 7.45 × 9.4
SA = 223.7 m²



9



SA = 2 × ½ × 72 × 77
+ 72 × 34
+ 2 × 85 × 34
SA = 13 772 mm²

Surface Area - Triangular Prisms (Homework Strip)

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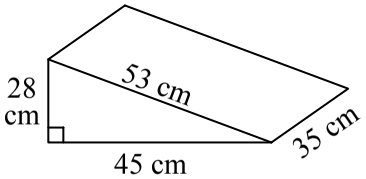
Date Due: _____

Find the SA of the triangular prisms.

1



SA = 2 × ½ × ☐ × ☐

\+ ☐ × ☐

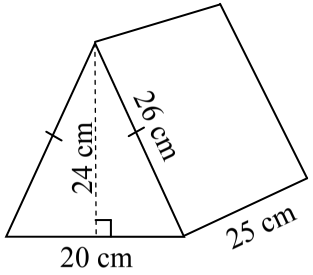
\+ ☐ × ☐

\+ ☐ × ☐

Front three edges, all × 35

SA = ☐ cm²

2



SA = 2 × ½ × ☐ × ☐

\+ ☐ × ☐ ← Base Rectangle

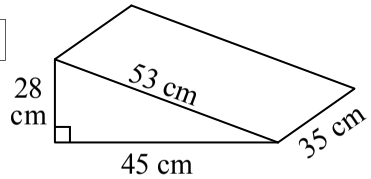
\+ 2 × ☐ × ☐ ← 2×slanted Rectangles

SA = ☐ cm²

Date Due: _____

Find the SA of the triangular prisms.

1



SA = 2 × ½ × ☐ × ☐

\+ ☐ × ☐

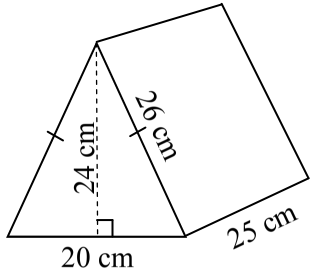
\+ ☐ × ☐

\+ ☐ × ☐

Front three edges, all × 35

SA = ☐ cm²

2



SA = 2 × ½ × ☐ × ☐

\+ ☐ × ☐ ← Base Rectangle

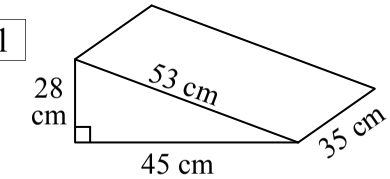
\+ 2 × ☐ × ☐ ← 2×slanted Rectangles

SA = ☐ cm²

Date Due: _____

Find the SA of the triangular prisms.

1



SA = 2 × ½ × ☐ × ☐

\+ ☐ × ☐

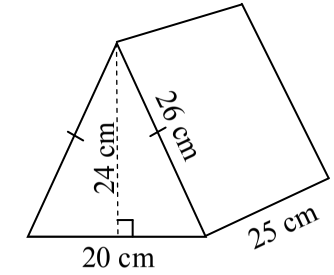
\+ ☐ × ☐

\+ ☐ × ☐

Front three edges, all × 35

SA = ☐ cm²

2



SA = 2 × ½ × ☐ × ☐

\+ ☐ × ☐ ← Base Rectangle

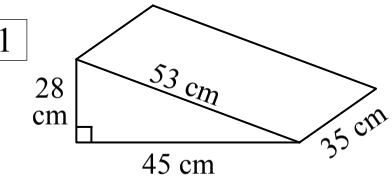
\+ 2 × ☐ × ☐ ← 2×slanted Rectangles

SA = ☐ cm²

Date Due: _____

Find the SA of the triangular prisms.

1



SA = 2 × ½ × ☐ × ☐

\+ ☐ × ☐

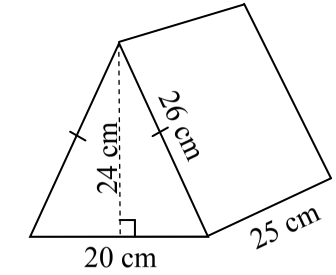
\+ ☐ × ☐

\+ ☐ × ☐

Front three edges, all × 35

SA = ☐ cm²

2



SA = 2 × ½ × ☐ × ☐

\+ ☐ × ☐ ← Base Rectangle

\+ 2 × ☐ × ☐ ← 2×slanted Rectangles

SA = ☐ cm²

Surface Area - Triangular Prisms (Homework Strip)

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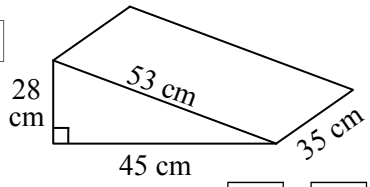
Date Due: _____

Find the SA of the triangular prisms.

1



$SA = 2 \times \frac{1}{2} \times 45 \times 28$

$+ 28 \times 35$

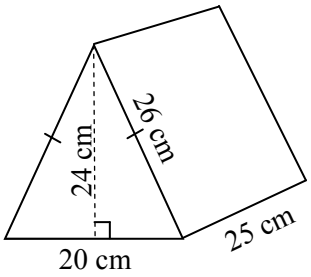
$+ 45 \times 35$

$+ 53 \times 35$

Front three edges, all × 35

$SA = 5\,670 \text{ cm}^2$

2



$SA = 2 \times \frac{1}{2} \times 20 \times 24$

$+ 20 \times 25$ ← Base Rectangle

$+ 2 \times 26 \times 25$ ← 2×slanted Rectangles

$SA = 2\,280 \text{ cm}^2$

Surface Area - Triangular Prisms using Pythagoras Theorem

This worksheet covers finding the surface area of a triangular prism but you need to find a missing side length first.

The first worksheet starts with revising Pythagoras Theorem. All the questions use addition. Given triangles, find the hypotenuse. There is an example at the top of the column.

Column 2 is just the same as the first column except the triangles are the faces of Triangular Prisms. The only trick is remembering to ignore the 'depth' measurement. You don't use all the measurements.

Column 3 asks you to find the hypotenuse just like the previous columns, but then also find the surface area. So these questions are lengthy.

The second worksheet carries on with the same questions, except then in Column 2, you are reminded of the process of finding a short side using Pythagoras Theorem. This is then applied to surface area questions in Column 3.

Surface Area - Triangular Prisms using Pythagoras Theorem (Sheet 1)

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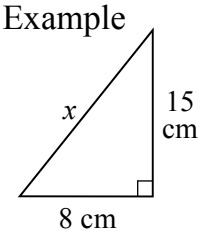
Use Pythagoras Theorem (only addition) for these. Find the length of the hypotenuse. All lengths are whole numbers.

Example



$x^2 = 8^2 + 15^2$

$x^2 = 289$

$x = \sqrt{289}$

$x = 17$ cm

1

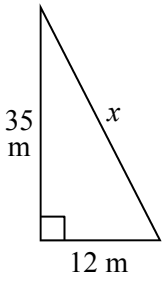
$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ m

Answer is a whole number



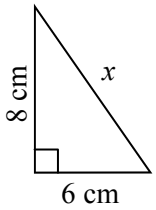
2

$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm



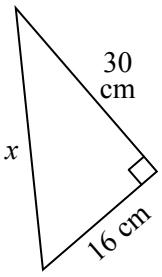
3

$x^2 =$

$x^2 =$

$x =$

$x =$



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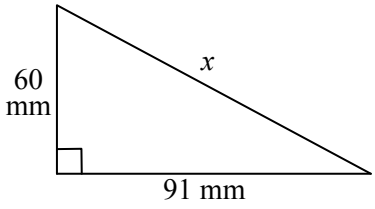
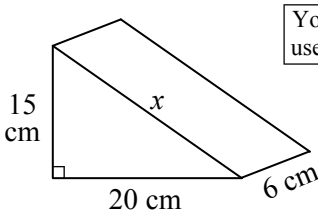


Use Pythagoras Theorem (only addition) to find the length '*x*'.

5

You don't use the '6'



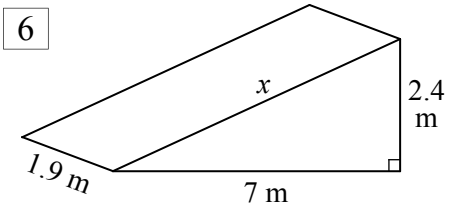
$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$

$x =$

$x =$

6



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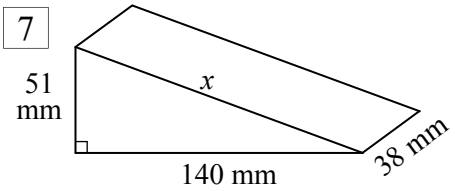

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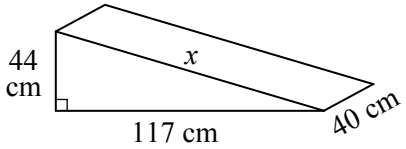
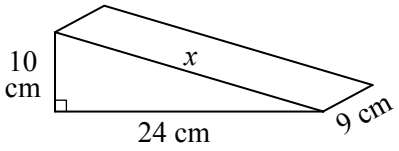


Find the length '*x*' first, then calculate the surface area

9



$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm

Now calculate the surface area

SA = 2 × ½ × ☐ × ☐

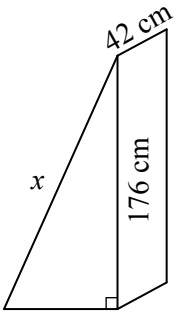
+ ☐ × 9

+ ☐ × 9

+ ☐ × 9

SA = ☐ cm^2

10



$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm

Now calculate the surface area

SA = 2 × ½ × ☐ × ☐

+ ☐ × ☐

+ ☐ × ☐

+ ☐ × ☐

SA = ☐ cm^2

Surface Area - Triangular Prisms using Pythagoras Theorem (Sheet 2)

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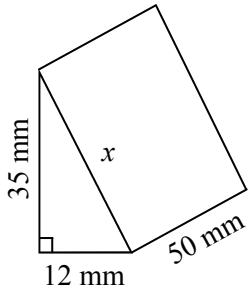
Find the SA of these triangular prisms. Pythagoras Theorem is required as a 1st step.

Example



$x^2 = 12^2 + 35^2$

$x^2 = 1\ 369$

$x = \sqrt{1\ 369}$

$x = 37$ mm

$SA = 2 \times ½ \times 12 \times 35$

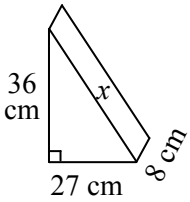
$+ 12 \times 50$

$+ 35 \times 50$

$+ 37 \times 50$

$SA = 4\ 620$ mm^2

1



$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm

$SA = 2 \times ½ \times \square \times \square$

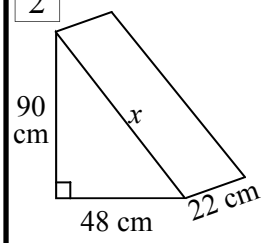
$+ \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

$SA = \square$ cm^2

2



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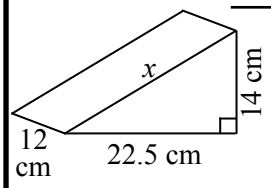

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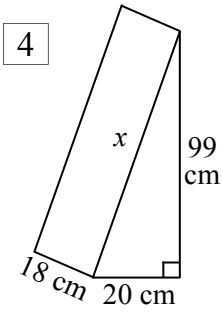
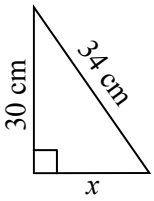


Use Pythagoras Theorem (subtraction). Find the length of the short side.

5



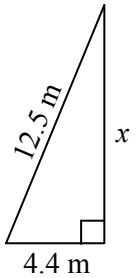
$x^2 = \square^2 - \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

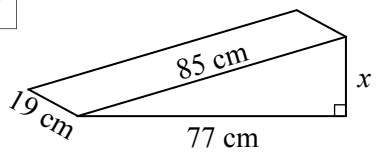
$x = \square$ cm

6



Find the SA of these triangular prisms.

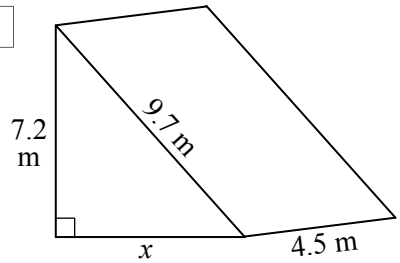
7



Calculate the triangle's height (P.T. minus)

Calculate the triangular prism's surface area

8



Calculate the triangle's base length

Calculate the triangular prism's surface area

Surface Area - Triangular Prisms using Pythagoras Theorem (Sheet 1)

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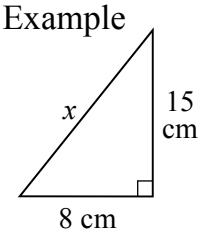
Use Pythagoras Theorem (only addition) for these. Find the length of the hypotenuse. All lengths are whole numbers.

Example



$x^2 = 8^2 + 15^2$

$x^2 = 289$

$x = \sqrt{289}$

$x = 17$ cm

1

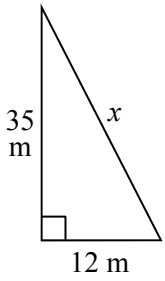
$x^2 =$ 12 $^2 +$ 35 ^{2}

$x^2 =$ 1 369

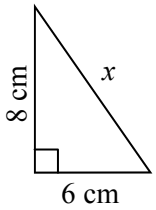
$x =$ $\sqrt{\ 1\ 369\ }$

$x =$ 37 m

Answer is a whole number



2



$x^2 =$ 6 $^2 +$ 8 ^{2}

$x^2 =$ 100

$x =$ $\sqrt{\ 100\ }$

$x =$ 10 cm

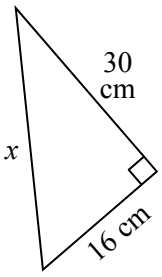
3

$x^2 = 16^2 + 30^2$

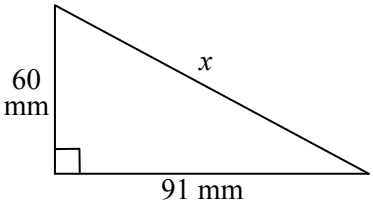
$x^2 = 1\ 156$

$x = \sqrt{1\ 156}$

$x = 34$ cm



4



$x^2 = 60^2 + 91^2$

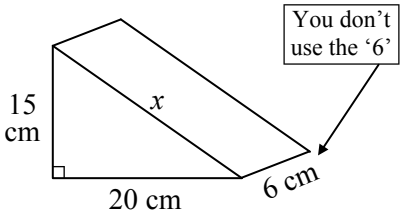
$x^2 = 11\ 881$

$x = \sqrt{11\ 881}$

$x = 109$ mm

Use Pythagoras Theorem (only addition) to find the length 'x'.

5



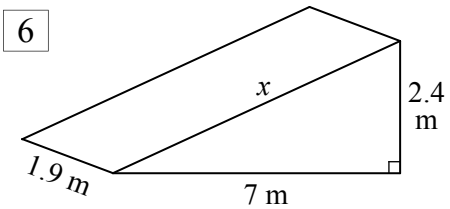
$x^2 =$ 15 $^2 +$ 20 ^{2}

$x^2 = 625$

$x = \sqrt{625}$

$x = 25$ cm

6



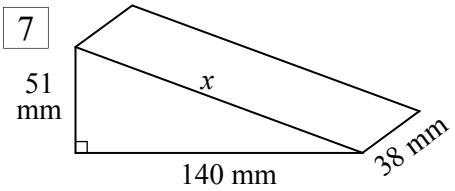
$x^2 = 7^2 + 2.4^2$

$x^2 = 54.76$

$x = \sqrt{54.76}$

$x = 7.4$ m

7



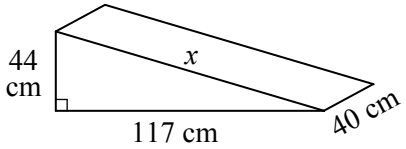
$x^2 = 51^2 + 140^2$

$x^2 = 22\ 201$

$x = \sqrt{22\ 201}$

$x = 149$ mm

8



$x^2 = 44^2 + 117^2$

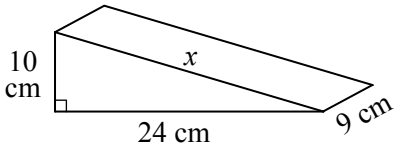
$x^2 = 15\ 625$

$x = \sqrt{15\ 625}$

$x = 125$ cm

Find the length 'x' first, then calculate the surface area

9



$x^2 =$ 10 $^2 +$ 24 ^{2}

$x^2 =$ 676

$x =$ $\sqrt{\ 676\ }$

$x =$ 26 cm

Now calculate the surface area

SA = 2 × ½ × 24 × 10

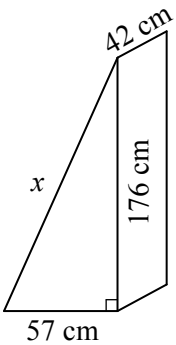
+ 10 × 9

+ 24 × 9

+ 26 × 9

SA = 780 cm^2

10



$x^2 =$ 57 $^2 +$ 176 ^{2}

$x^2 =$ 34 225

$x =$ $\sqrt{\ 34\ 225\ }$

$x =$ 185 cm

Now calculate the surface area

SA = 2 × ½ × 57 × 176

+ 57 × 42

+ 176 × 42

+ 185 × 42

SA = 27 588 cm^2

Surface Area - Triangular Prisms using Pythagoras Theorem (Sheet 2)

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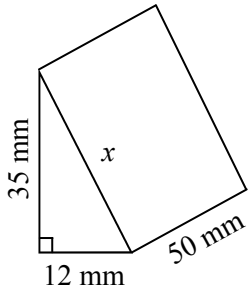
Find the SA of these triangular prisms. Pythagoras Theorem is required as a 1st step.

Example



$x^2 = 12^2 + 35^2$

$x^2 = 1\ 369$

$x = \sqrt{1\ 369}$

$x = 37$ mm

$SA = 2 \times ½ \times 12 \times 35$

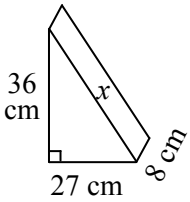
$+ 12 \times 50$

$+ 35 \times 50$

$+ 37 \times 50$

$SA = 4\ 620$ mm^2

1



$x^2 = \boxed{27}^2 + \boxed{36}^2$

$x^2 = \boxed{2\ 025}$

$x = \sqrt{\boxed{2\ 025}}$

$x = \boxed{45}$ cm

$SA = 2 \times ½ \times \boxed{27} \times \boxed{36}$

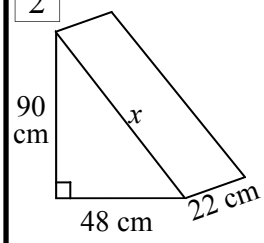
$+ \boxed{27} \times \boxed{8}$

$+ \boxed{36} \times \boxed{8}$

$+ \boxed{45} \times \boxed{8}$

$SA = \boxed{1\ 836}$ cm^2

2



$x^2 = 48^2 + 90^2$

$x^2 = 10\ 404$

$x = \sqrt{10\ 404}$

$x = 102$ cm

$SA = 2 \times ½ \times 48 \times 90$

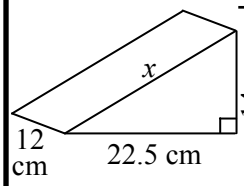
$+ 48 \times 22$

$+ 90 \times 22$

$+ 102 \times 22$

$SA = 9\ 600$ cm^2

3



$x^2 = 14^2 + 22.5^2$

$x^2 = 702.25$

$x = \sqrt{702.25}$

$x = 26.5$ cm

$SA = 2 \times ½ \times 22.5 \times 14$

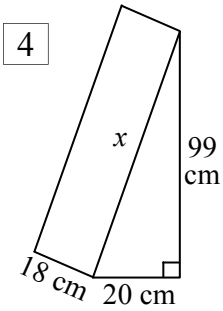
$+ 22.5 \times 12$

$+ 14 \times 12$

$+ 26.5 \times 12$

$SA = 1\ 071$ cm^2

4



$x^2 = 20^2 + 99^2$

$x^2 = 10\ 201$

$x = \sqrt{10\ 201}$

$x = 101$ cm

$SA = 2 \times ½ \times 20 \times 99$

$+ 20 \times 18$

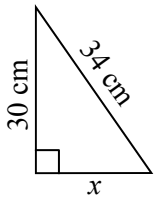
$+ 99 \times 18$

$+ 101 \times 18$

$SA = 5\ 940$ cm^2

Use Pythagoras Theorem (subtraction). Find the length of the short side.

5



$x^2 = \boxed{34}^2 - \boxed{30}^2$

$x^2 = \boxed{256}$

$x = \sqrt{\boxed{256}}$

$x = \boxed{16}$ cm

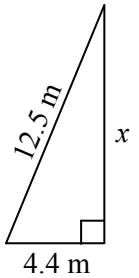
6

$x^2 = 12.5^2 - 4.4^2$

$x^2 = 136.89$

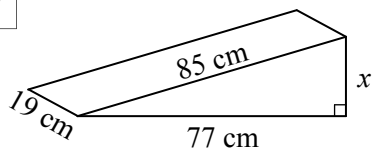
$x = \sqrt{136.89}$

$x = 11.7$ m



Find the SA of these triangular prisms.

7



Calculate the triangle's height (P.T. minus)

$x^2 = 85^2 - 77^2$

$x^2 = 1\ 296$

$x = \sqrt{1\ 296}$

$x = 36$ cm

Calculate the triangular prism's surface area

$SA = 2 \times ½ \times 77 \times 36$

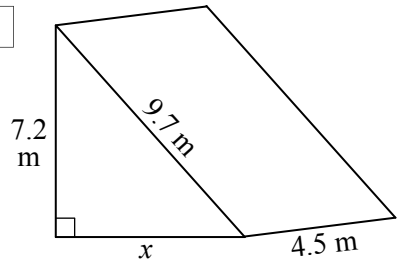
$+ 77 \times 19$

$+ 36 \times 19$

$+ 85 \times 19$

$SA = 6\ 534$ cm^2

8



Calculate the triangle's base length

$x^2 = 9.7^2 - 7.2^2$

$x^2 = 42.25$

$x = \sqrt{42.25}$

$x = 6.5$ m

Calculate the triangular prism's surface area

$SA = 2 \times ½ \times 6.5 \times 7.2$

$+ 6.5 \times 4.5$

$+ 7.2 \times 4.5$

$+ 9.7 \times 4.5$

$SA = 152.1$ m^2

Surface Area - Triangular Prisms using Pythagoras Theorem (Homework Strip)

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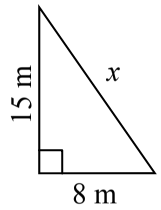
Date Due: _____

Find the length of the side 'x'

1



$x^2 = \square^2 + \square^2$

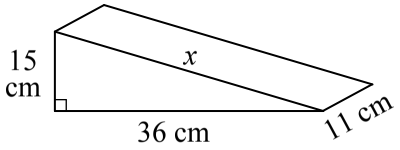
$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ m

Find the length of the side 'x'

2



$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm

Find the surface area.

3

$SA = 2 \times \frac{1}{2} \times \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

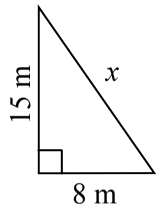
$+ \square \times \square$

$SA = \square$ cm^2

Date Due: _____

Find the length of the side 'x'

1



$x^2 = \square^2 + \square^2$

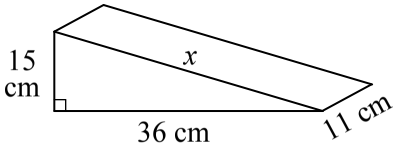
$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ m

Find the length of the side 'x'

2



$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm

Find the surface area.

3

$SA = 2 \times \frac{1}{2} \times \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

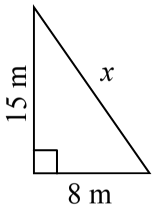
$+ \square \times \square$

$SA = \square$ cm^2

Date Due: _____

Find the length of the side 'x'

1



$x^2 = \square^2 + \square^2$

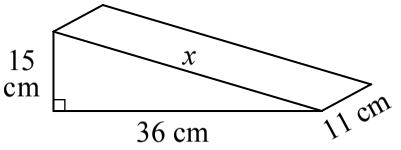
$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ m

Find the length of the side 'x'

2



$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm

Find the surface area.

3

$SA = 2 \times \frac{1}{2} \times \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

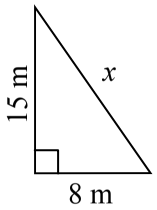
$+ \square \times \square$

$SA = \square$ cm^2

Date Due: _____

Find the length of the side 'x'

1



$x^2 = \square^2 + \square^2$

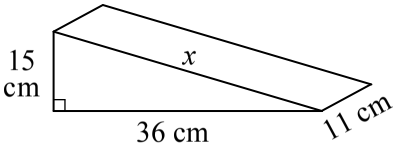
$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ m

Find the length of the side 'x'

2



$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm

Find the surface area.

3

$SA = 2 \times \frac{1}{2} \times \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

$SA = \square$ cm^2

Surface Area - Triangular Prisms using Pythagoras Theorem (HW Strip)

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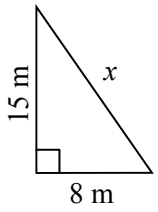
Date Due:

Find the length of the side 'x'

1



$x^2 = 8^2 + 15^2$

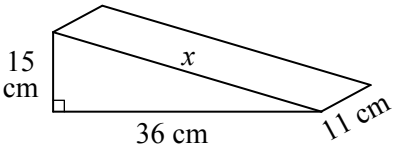
$x^2 = 289$

$x = \sqrt{289}$

$x = 17$ m

Find the length of the side 'x'

2



$x^2 = 15^2 + 36^2$

$x^2 = 1\,521$

$x = \sqrt{1\,521}$

$x = 39$ cm

Find the surface area.

3

$SA = 2 \times \frac{1}{2} \times 36 \times 15$

$+ 15 \times 11$

$+ 36 \times 11$

$+ 39 \times 11$

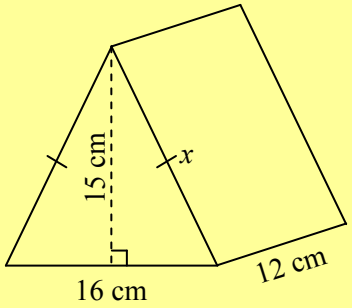
$SA = 1\,530$ cm^2

Surface Area - Further Triangular Prisms using Pythagoras Theorem

This worksheet covers finding the surface area of a triangular prism but you need to find a missing side length first. This sheet is more difficult than the previous sheet because the triangular faces are isosceles not right-angled triangles.

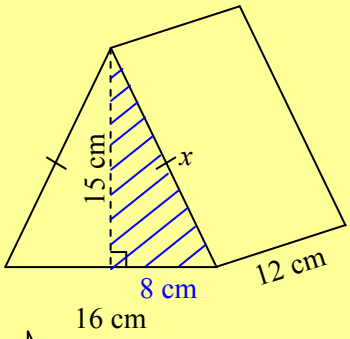
Here is an example of the process.

Calculate the SA of the triangular prism below, answer to 1 decimal place.



STEP 1

HALVE THE BASE

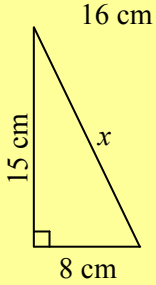


HALVE THE BASE

You need to create a right angled triangle to use Pythagoras Theorem. So divide the base by two. Then deal just with the shaded triangle, for x.

STEP 2

APPLY PYTHAGORAS



$x^2 = 8^2 + 15^2$

$x^2 = 289$

$x^2 = \sqrt{289}$

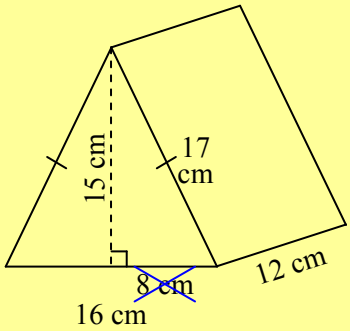
$x^2 = 17$ cm

FIND THE LENGTH

Find the missing length using Pythagoras Theorem.

STEP 3

CALCULATE THE SA



SA = 2 × ½ × 16 × 15

+ 2 × 17 × 12

+ 16 × 12

SA = 840 cm^2

CALCULATE THE SURFACE AREA

The most common mistake to avoid is using half the base instead of the full base. You use the 8 cm (above) in the Pythagoras calculation. Once that is done, you don't use it in the SA calculation.

Surface Area - Further Triangular Prisms & Pythagoras Theorem (Sheet 1)

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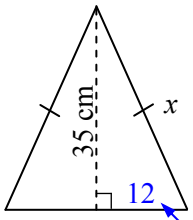
Halve the base first, then use P.T. to find the length 'x'. All lengths are whole numbers.

Example



$x^2 = 12^2 + 35^2$

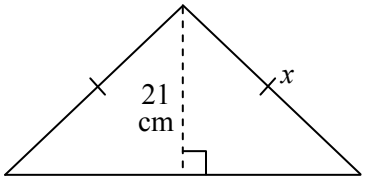
$x^2 = 1\,369$

$x = \sqrt{1\,369}$

$x = 37$ cm

24 ÷ 2

1



$x^2 =$ ☐^{2} + ☐^{2}

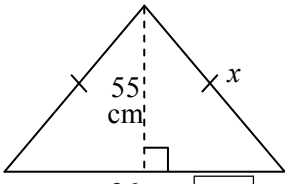
$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm

Answer is a whole number

2



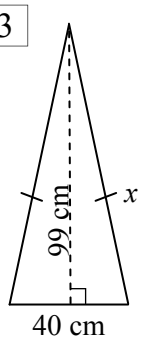
$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

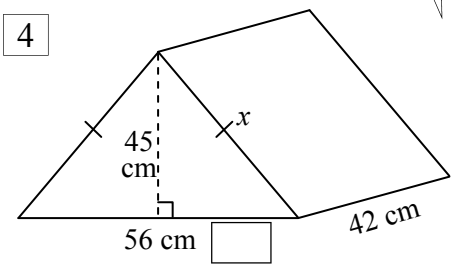
$x =$ ☐ cm

3



Halve the base, find 'x' using P.T., then the SA …. R U OK?

4



$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm

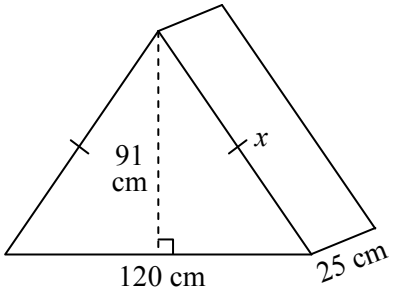
SA = 2 × ½ × 56 × 45

+ 2 × ☐ × 42 (☐ = x)

+ 56 × 42

SA = ☐ cm^2

5



$x^2 =$

$x^2 =$

$x =$

$x =$

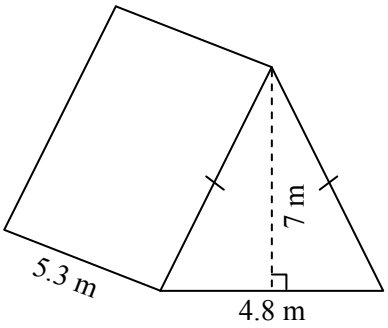
SA = 2 × ½ × ☐ × ☐

+ 2 × ☐ × ☐

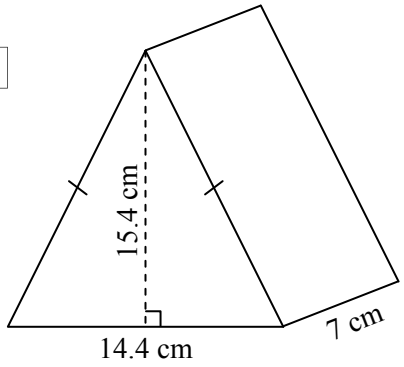
+ ☐ × ☐

SA = ☐ cm^2

6



7



Surface Area - Further Triangular Prisms & Pythagoras Theorem (Sheet 2)

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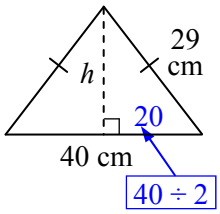
Halve the base, then use P.T. (subtraction) to find the height 'h'. Heights are whole numbers.

Example



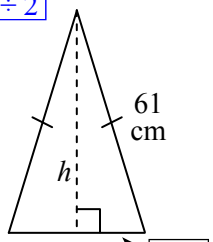
$h^2 = 29^2 - 20^2$

$h^2 = 441$

$h = \sqrt{441}$

$h = 21$ cm

1



$h^2 =$ ☐^{2} - ☐^{2}

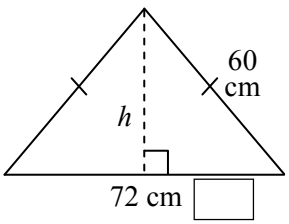
$h^2 =$ ☐

$h = \sqrt{}$☐

$h =$ ☐ cm

Answer is a whole number

2



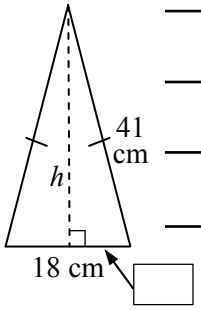
$h^2 =$ ☐^{2} - ☐^{2}

$h^2 =$ ☐

$h = \sqrt{}$☐

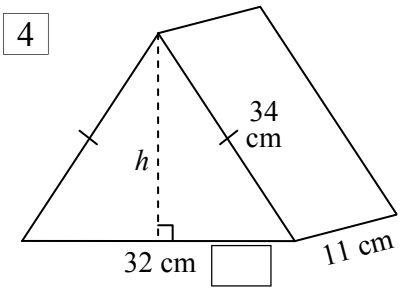
$h =$ ☐ cm

3



Halve the base, find 'h' using P.T., then calculate the SA.

4



$h^2 =$ ☐^{2} - ☐^{2}

$h^2 =$ ☐

$h = \sqrt{}$☐

$h =$ ☐ cm

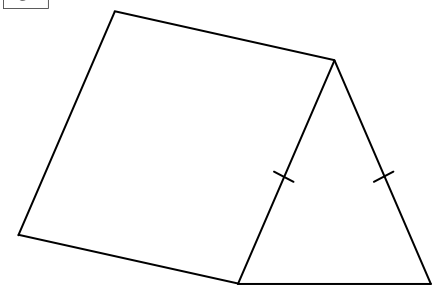
SA = 2 × ½ × 32 × ☐ (h)

+ 2 × 34 × 11

+ 32 × 11

SA = ☐ cm^2

5



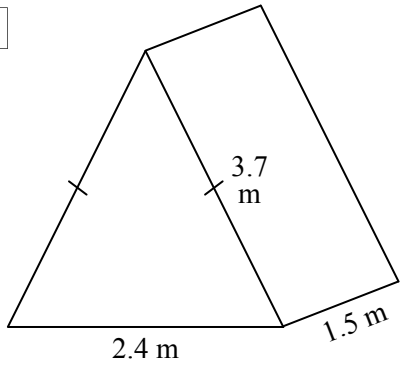
SA = 2 × ½ × ☐ × ☐

+ 2 × ☐ × ☐

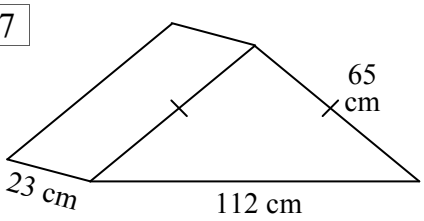
+ ☐ × ☐

SA = ☐ mm^2

6



7



Surface Area - Further Triangular Prisms & Pythagoras Theorem (Sheet 1)

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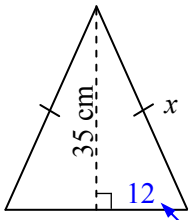
Halve the base first, then use P.T. to find the length 'x'. All lengths are whole numbers.

Example



$x^2 = 12^2 + 35^2$

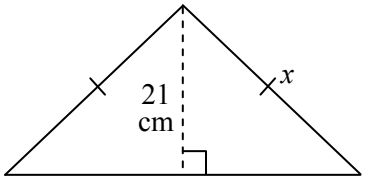
$x^2 = 1\,369$

$x = \sqrt{1\,369}$

$x = 37$ cm

24 ÷ 2

1



20

$x^2 =$ [20]$^2 +$ [21]^2

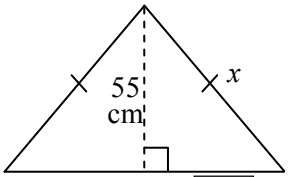
$x^2 =$ 841

$x = \sqrt{841}$

$x =$ 29 cm

Answer is a whole number

2



48

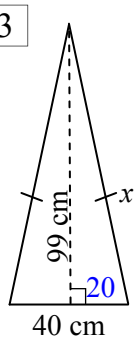
$x^2 =$ [48]$^2 +$ [55]^2

$x^2 =$ 5 329

$x = \sqrt{5\,329}$

$x =$ 73 cm

3



$x^2 = 20^2 + 99^2$

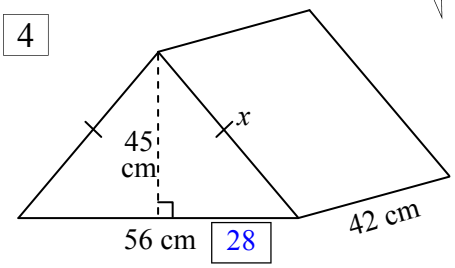
$x^2 = 10\,201$

$x = \sqrt{10\,201}$

$x = 101$ cm

Halve the base, find 'x' using P.T., then the SA …. R U OK?

4



$x^2 =$ [28]$^2 +$ [45]^2

$x^2 =$ 2 809

$x = \sqrt{2\,809}$

$x =$ 53 cm

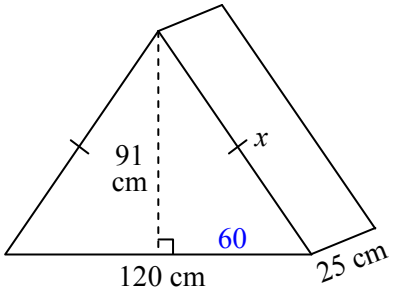
SA = 2 × ½ × 56 × 45

+ 2 × 53 × 42 (53 = x)

+ 56 × 42

SA = 9 324 cm^2

5



$x^2 = 60^2 + 91^2$

$x^2 = 11\,881$

$x = \sqrt{11\,881}$

$x = 109$ cm

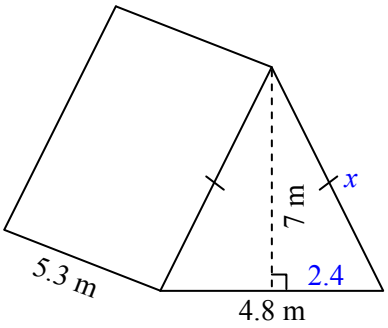
SA = 2 × ½ × 120 × 91

+ 2 × 109 × 25

+ 120 × 25

SA = 19 370 cm^2

6



$x^2 = 2.4^2 + 7^2$

$x^2 = 54.76$

$x = \sqrt{54.76}$

$x = 7.4$ m

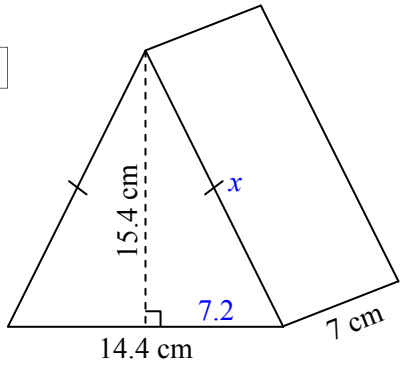
SA = 2 × ½ × 4.8 × 7

+ 2 × 7.4 × 5.3

+ 4.8 × 5.3

SA = 137.48 m^2

7



$x^2 = 7.2^2 + 15.4^2$

$x^2 = 289$

$x = \sqrt{289}$

$x = 17$ cm

SA = 2 × ½ × 14.4 × 15.4

+ 2 × 17 × 7

+ 14.4 × 7

SA = 560.56 cm^2

Surface Area - Further Triangular Prisms & Pythagoras Theorem (Sheet 2)

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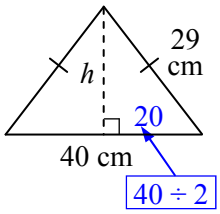
Halve the base, then use P.T. (subtraction) to find the height 'h'. Heights are whole numbers.

Example



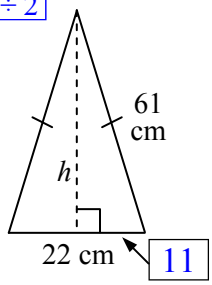
$h^2 = 29^2 - 20^2$

$h^2 = 441$

$h = \sqrt{441}$

$h = 21$ cm

1



$h^2 =$ 61 $^2 -$ 11 ^{2}

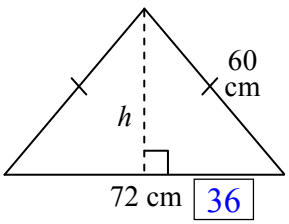
$h^2 =$ 3 600

$h = \sqrt{3\ 600}$

$h =$ 60 cm

Answer is a whole number

2



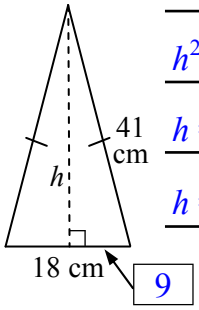
$h^2 =$ 60 $^2 -$ 36 ^{2}

$h^2 =$ 2 304

$h = \sqrt{2\ 304}$

$h =$ 48 cm

3



$h^2 = 41^2 - 9^2$

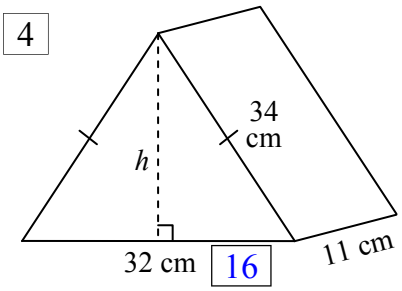
$h^2 = 1\ 600$

$h = \sqrt{1\ 600}$

$h = 40$ cm

Halve the base, find 'h' using P.T., then calculate the SA.

4



$h^2 =$ 34 $^2 -$ 16 ^{2}

$h^2 =$ 900

$h = \sqrt{900}$

$h =$ 30 cm

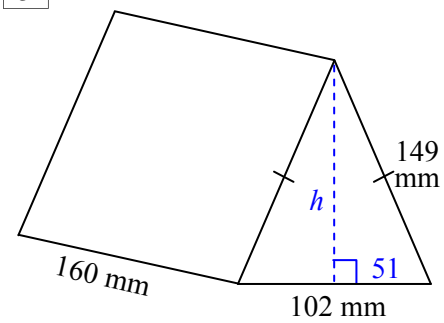
SA = 2 × ½ × 32 × 30 (h)

+ 2 × 34 × 11

+ 32 × 11

SA = 2 060 cm^2

5



$h^2 = 149^2 - 51^2$

$h^2 = 19\ 600$

$h = \sqrt{19\ 600}$

$h = 140$ mm

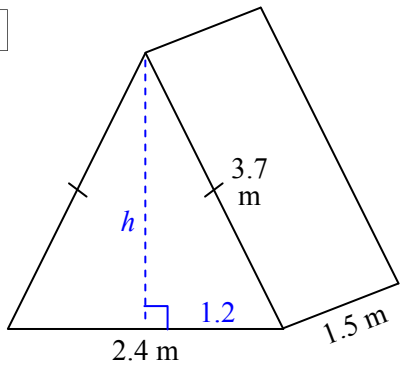
SA = 2 × ½ × 102 × 140

+ 2 × 149 × 160

+ 102 × 160

SA = 78 280 mm^2

6



$h^2 = 3.7^2 - 1.2^2$

$h^2 = 12.25$

$h = \sqrt{12.25}$

$h = 3.5$ m

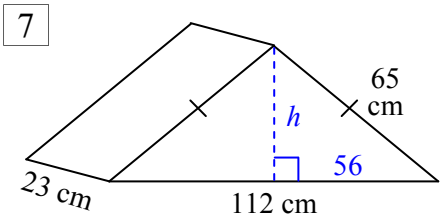
SA = 2 × ½ × 2.4 × 3.5

+ 2 × 3.7 × 1.5

+ 2.4 × 1.5

SA = 23.1 m^2

7



$h^2 = 65^2 - 56^2$

$h^2 = 1\ 089$

$h = \sqrt{1\ 089}$

$h = 33$ cm

SA = 2 × ½ × 112 × 33

+ 2 × 65 × 23

+ 112 × 23

SA = 9 262 cm^2

Surface Area - Further Triangular Prisms using Pythagoras Theorem (Homework Strip)

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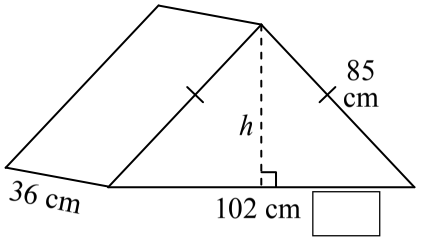
Date Due: _____

Find half the base width and write the answer in the box.

1



Use Pythagoras Theorem to calculate the height of the triangular face.

2

$h^2 =$

$h^2 =$

$h =$

$h =$

Calculate the surface area of the triangular prism.

3

SA = 2 × ½ × ____ × ____

+ 2 × ____ × ____

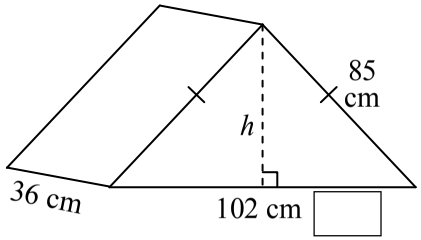
+ ____ × ____

SA = ____ cm^2

Date Due: _____

Find half the base width and write the answer in the box.

1



Use Pythagoras Theorem to calculate the height of the triangular face.

2

$h^2 =$

$h^2 =$

$h =$

$h =$

Calculate the surface area of the triangular prism.

3

SA = 2 × ½ × ____ × ____

+ 2 × ____ × ____

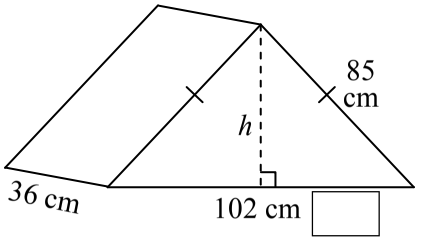
+ ____ × ____

SA = ____ cm^2

Date Due: _____

Find half the base width and write the answer in the box.

1



Use Pythagoras Theorem to calculate the height of the triangular face.

2

$h^2 =$

$h^2 =$

$h =$

$h =$

Calculate the surface area of the triangular prism.

3

SA = 2 × ½ × ____ × ____

+ 2 × ____ × ____

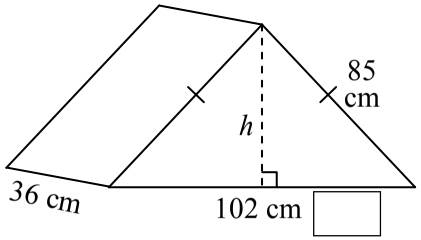
+ ____ × ____

SA = ____ cm^2

Date Due: _____

Find half the base width and write the answer in the box.

1



Use Pythagoras Theorem to calculate the height of the triangular face.

2

$h^2 =$

$h^2 =$

$h =$

$h =$

Calculate the surface area of the triangular prism.

3

SA = 2 × ½ × ____ × ____

+ 2 × ____ × ____

+ ____ × ____

SA = ____ cm^2

Surface Area - Further Triangular Prisms using P.T. (Homework Strip)

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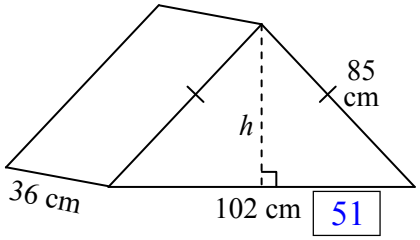
Date Due:

Find half the base width and write the answer in the box.

1



51

Use Pythagoras Theorem to calculate the height of the triangular face.

2

$h^2 = 85^2 - 51^2$

$h^2 = 4\,624$

$h = \sqrt{4\,624}$

$h = 68$ cm

Calculate the surface area of the triangular prism.

3

SA = $2 \times \frac{1}{2} \times$ 102 $\times$ 68

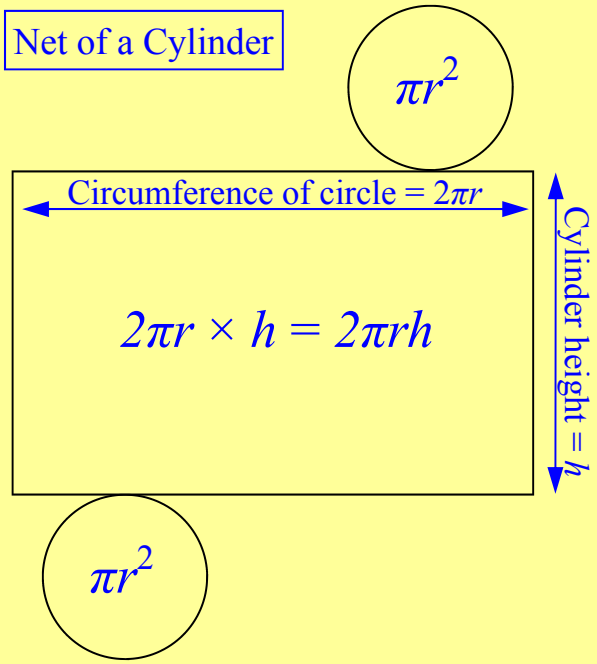
$+ 2 \times$ 85 $\times$ 36

$+$ 102 $\times$ 36

SA = 13 668 cm^2

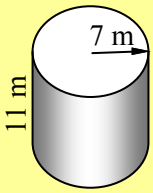
Surface Area - Cylinders

The surface area of a cylinder is the sum of the areas of the shapes that make up the solid. As the net of a cylinder below shows, each circle has an area of πr^2. So the two circles added together gives an area of $2\pi r^2$. The cylinder wall, (also called the curved surface) is the rectangle in the net, it wraps around the circles so one of its sides is the same length as the circumference of the circular ends, which is $2\pi r$. Its height is the height of the cylinder, which is h. As this area is a rectangular area, its area is one side times the other, or $2\pi r \times h = 2\pi rh$.



SA of a Cylinder = $2\pi r^2 + 2\pi rh$

Example - SA of a cylinder



$SA = 2\pi r^2 + 2\pi rh$	Write the formula
$= 2\times\pi\times7^2 + 2\times\pi\times7\times11$	Substitute r and h
$SA = 791.7\ m^2$ (1 d.p.)	Find surface area

Calculator [2] [×] [SHIFT] [×10^x] [×] [7] [x²] [+] [2] [×] [SHIFT] [×10^x] [×] [7] [×] [1] [1] [=]

Surface Area - Cylinders (Sheet 1)

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Use your calculator to answer these to 1 d.p.

1 $2\times\pi\times5^2+2\times\pi\times5\times9=$ ____

2 × SHIFT ×10^x × 5 x² + 2 × SHIFT ×10^x × 5 × 9 =

2 $2\times\pi\times7^2+2\times\pi\times7\times4=$ ____

3 $2\times\pi\times9^2+2\times\pi\times9\times20=$ ____

4 $2\times\pi\times12^2+2\times\pi\times12\times6=$ ____

5 $2\times\pi\times18^2+2\times\pi\times18\times12=$ ____

6 $2\times\pi\times4.6^2+2\times\pi\times4.6\times8=$ ____

7 $2\times\pi\times1.6^2+2\times\pi\times1.6\times3.8=$ ____

8 $2\times\pi\times33^2+2\times\pi\times33\times47=$ ____

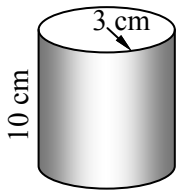
Find the SA of these cylinders. Round your answer to 1 decimal place. Formula, Substitution, Answer.

Example

$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 3^2 + 2 \times \pi \times 3 \times 10$

$SA = 245.0 \text{ cm}^2$ (1 d.p.)

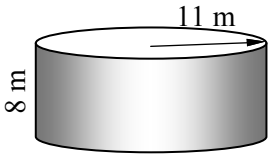


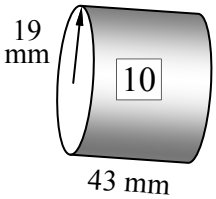
9 $SA = 2\pi r^2 + 2\pi rh$

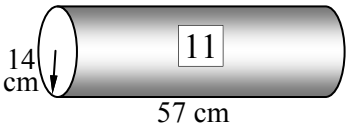
$= 2 \times \pi \times$ ☐ $^2 + 2 \times \pi \times$ ☐ $\times$ ☐

2 × SHIFT ×10^x × 1 1 x² + 2 × SHIFT ×10^x × 1 1 × 8 =

$SA =$ ☐ m^2







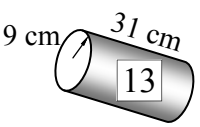
10 $SA = 2\pi r^2 + 2\pi rh$

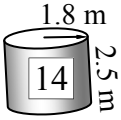
$= 2 \times \pi \times$ ☐ $^2 + 2 \times \pi \times$ ☐ $\times$ ☐

$SA =$ ☐ mm^2

11 $SA = 2\pi r^2 + 2\pi rh$



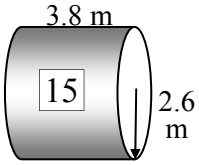


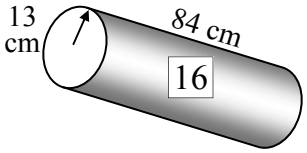


12 ____

13 ____

14 ____





15 ____

16 ____

A tin can of 'Otis' chocolate powder has a radius of 7 cm and a height of 16 cm. Calculate, to 1 decimal place:

17 The area of metal sheet used to fabricate 1 can

18 The weight of the can, if each cm^2 weighs 0.13 g.

Surface Area - Cylinders with Diameters (Sheet 2)

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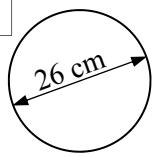
Divide these diameters by 2, to find the radii.
Ignore the height of the cylinders (Q4 and Q5).

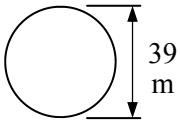
1



$r = 26 \div 2$

$r =$ ☐ cm

2

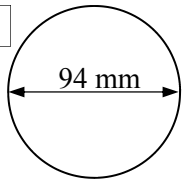


$r =$ ☐ ÷ ☐

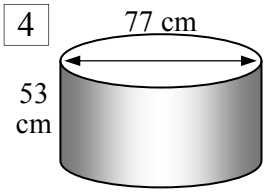
$r =$ ☐

Write the units

3



4

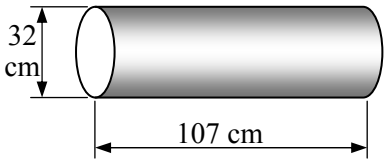


$r = 77 \div 2$

$r =$

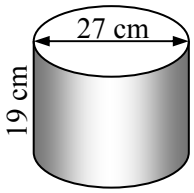
Write the units

5



Calculate the radius first, then find the SA of the cylinder, round your answer to 1 decimal place.

Example



$r = 13.5$ cm

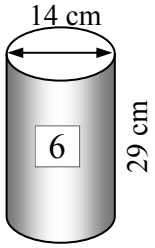
$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 13.5^2 + 2 \times \pi \times 13.5 \times 19$

$SA = 2\,756.7$ cm^2 (1 d.p.)

This is surface area so show square units

6



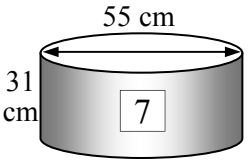
$r =$ ☐ cm

$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times$ ☐ $^2 + 2 \times \pi \times$ ☐ $\times$ ☐

$SA =$ ☐ cm^2

7



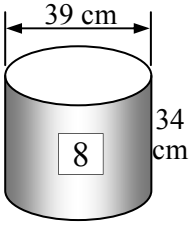
$r =$ ☐

$SA = 2\pi r^2 + 2\pi rh$

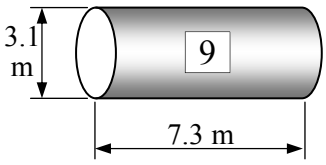
$=$

$SA =$ ☐ cm^2

8



9



Don't round the radius

Find the SA of the water tank to 1 decimal place.

'WeSaveU' cylindrical water tanks are 3.6 m high and 4.3 m in diameter.

10 What is the tank's radius?

Don't round the radius

11 Calculate the surface area of the tank.

12 If powder coating costs \$6.78/m^2, estimate the cost to coat the tank, answer to the nearest \$10.

13 If Teresa pays cash she gets a \$150 discount. The tank (uncoated) costs \$2 990, calculate the price paid with powder coating added and a discount for cash applied.

14 If the sheet metal that is used to build the tank weighs 0.8 kg/m^2. Calculate the weight of the empty tank.

Surface Area - Cylinders (Sheet 1)

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Use your calculator to answer these to 1 d.p.

1 $2\times\pi\times5^2+2\times\pi\times5\times9=$ 439.8

2 × SHIFT ×10^x × 5 x² + 2 × SHIFT ×10^x × 5 × 9 =

2 $2\times\pi\times7^2+2\times\pi\times7\times4 =$ 483.8

3 $2\times\pi\times9^2+2\times\pi\times9\times20 =$ 1 639.9

4 $2\times\pi\times12^2+2\times\pi\times12\times6 =$ 1 357.2

5 $2\times\pi\times18^2+2\times\pi\times18\times12 =$ 3 392.9

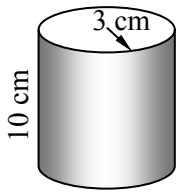
6 $2\times\pi\times4.6^2+2\times\pi\times4.6\times8 =$ 364.2

7 $2\times\pi\times1.6^2+2\times\pi\times1.6\times3.8 =$ 54.3

8 $2\times\pi\times33^2+2\times\pi\times33\times47 =$ 16 587.6

Find the SA of these cylinders. Round your answer to 1 decimal place. Formula, Substitution, Answer.

Example

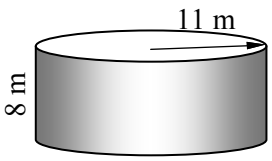


$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 3^2 + 2 \times \pi \times 3 \times 10$

$SA = 245.0 \text{ cm}^2$ (1 d.p.)

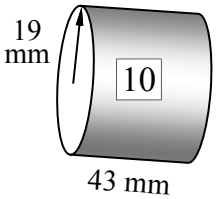
9 $SA = 2\pi r^2 + 2\pi rh$

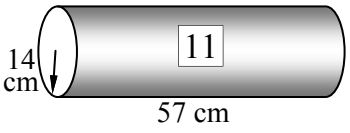


$= 2 \times \pi \times 11^2 + 2 \times \pi \times 11 \times 8$

2 × SHIFT ×10^x × 1 1 x² + 2 × SHIFT ×10^x × 1 1 × 8 =

SA = 1 313.2 m^2





10 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 19^2 + 2 \times \pi \times 19 \times 43$

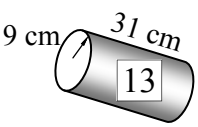
SA = 7 401.6 mm^2

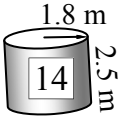
11 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 14^2 + 2 \times \pi \times 14 \times 57$

$SA = 6\,245.5 \text{ cm}^2$







12 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 70^2 + 2 \times \pi \times 70 \times 85$

$SA = 68\,172.6 \text{ mm}^2$

13 $SA = 2\pi r^2 + 2\pi rh$

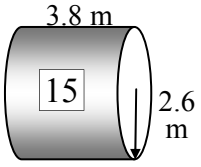
$= 2 \times \pi \times 9^2 + 2 \times \pi \times 9 \times 31$

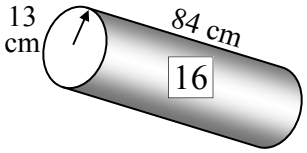
$SA = 2\,261.9 \text{ cm}^2$

14 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 1.8^2 + 2 \times \pi \times 1.8 \times 2.5$

$SA = 48.6 \text{ m}^2$





15 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 2.6^2 + 2 \times \pi \times 2.6 \times 3.8$

$SA = 104.6 \text{ m}^2$

16 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 13^2 + 2 \times \pi \times 13 \times 84$

$SA = 7\,923.1 \text{ cm}^2$

A tin can of 'Otis' chocolate powder has a radius of 7 cm and a height of 16 cm. Calculate, to 1 decimal place:

17 The area of metal sheet used to fabricate 1 can

$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 7^2 + 2 \times \pi \times 7 \times 16$

$SA = 1\,011.6 \text{ cm}^2$

18 The weight of the can, if each cm^2 weighs 0.13 g.

Weight = 1 011.6 × 0.13

Weight = 131.5 g

Surface Area - Cylinders with Diameters (Sheet 2)

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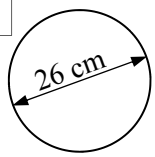
Divide these diameters by 2, to find the radii.
Ignore the height of the cylinders (Q4 and Q5).

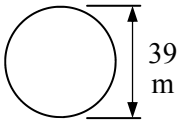
1



$r = 26 \div 2$

$r =$ 13 cm

2

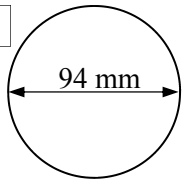


$r =$ 39 $\div$ 2

$r =$ 19.5 m

Write the units

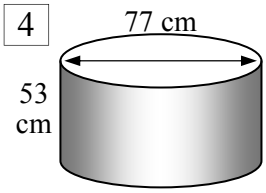
3



$r = 94 \div 2$

$r = 47$ mm

4

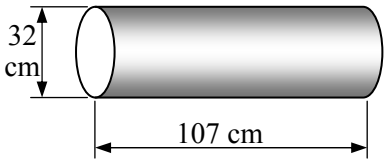


$r = 77 \div 2$

$r =$ 38.5 cm

Write the units

5

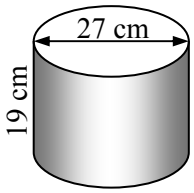


$r = 32 \div 2$

$r = 16$ cm

Calculate the radius first, then find the SA of the cylinder, round your answer to 1 decimal place.

Example



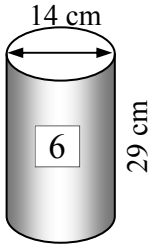
$r = 13.5$ cm

$SA = 2\pi r^2 + 2\pi rh$

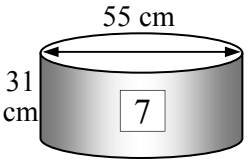
$= 2 \times \pi \times 13.5^2 + 2 \times \pi \times 13.5 \times 19$

$SA = 2\,756.7$ cm^2 (1 d.p.)

This is surface area so show square units



$r =$ 7 cm



$r =$ 27.5 cm

6 $SA = 2\pi r^2 + 2\pi rh$

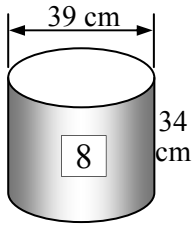
$= 2 \times \pi \times$ 7 $^2 + 2 \times \pi \times$ 7 $\times$ 29

$SA =$ 1 583.4 cm^2

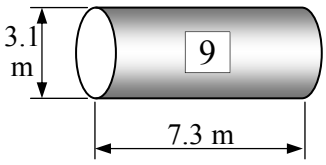
7 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 27.5^2 + 2 \times \pi \times 27.5 \times 31$

$SA =$ 10 108.1 cm^2



$r = 19.5$ cm



$r = 1.55$ m

Don't round the radius

8 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 19.5^2 + 2 \times \pi \times 19.5 \times 34$

$SA = 6\,554.9$ cm^2

9 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 1.55^2 + 2 \times \pi \times 1.55 \times 7.3$

$SA = 86.2$ m^2

Find the SA of the water tank to 1 decimal place.

'WeSaveU' cylindrical water tanks are 3.6 m high and 4.3 m in diameter.

10 What is the tank's radius? $r = 2.15$ m

Don't round the radius

11 Calculate the surface area of the tank.

$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 2.15^2 + 2 \times \pi \times 2.15 \times 3.6$

$SA = 77.7$ m^2

12 If powder coating costs \$6.78/m^2, estimate the cost to coat the tank, answer to the nearest \$10.

Cost = 77.7 × 6.78 = \$526.81 = \$530

13 If Teresa pays cash she gets a \$150 discount. The tank (uncoated) costs \$2 990, calculate the price paid with powder coating added and a discount for cash applied.

Price = 2 990 + 530 –150 = \$3 370 or

Price = 2 990 + 526.81 –150 = \$3 366.81

14 If the sheet metal that is used to build the tank weighs 0.8 kg/m^2. Calculate the weight of the empty tank.

Weight = 77.7 × 0.8 = 62.16 kg (62.2 kg - 1d.p.)

Surface Area - Cylinders (Homework Strip)

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Date Due: _____

Calculate these to 1 decimal place.

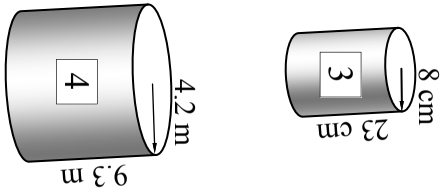
1 $2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 11$

$= \square\square\square.\square$

2 $2 \times \pi \times 12^2 + 2 \times \pi \times 12 \times 25$

$=$ _____

Find the SA to 1 decimal place



3 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times \square^2 + 2 \times \pi \times \square \times \square$

$SA = \square$ cm^2

4

Date Due: _____

Calculate these to 1 decimal place.

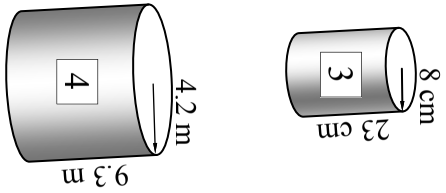
1 $2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 11$

$= \square\square\square.\square$

2 $2 \times \pi \times 12^2 + 2 \times \pi \times 12 \times 25$

$=$ _____

Find the SA to 1 decimal place



3 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times \square^2 + 2 \times \pi \times \square \times \square$

$SA = \square$ cm^2

4

Date Due: _____

Calculate these to 1 decimal place.

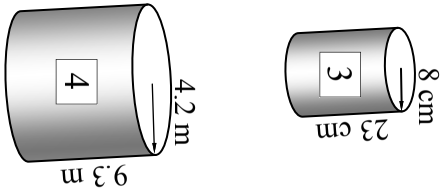
1 $2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 11$

$= \square\square\square.\square$

2 $2 \times \pi \times 12^2 + 2 \times \pi \times 12 \times 25$

$=$ _____

Find the SA to 1 decimal place



3 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times \square^2 + 2 \times \pi \times \square \times \square$

$SA = \square$ cm^2

4

Date Due: _____

Calculate these to 1 decimal place.

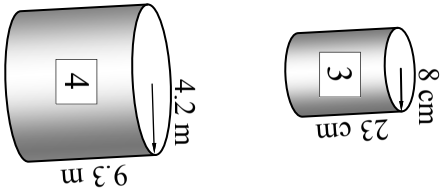
1 $2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 11$

$= \square\square\square.\square$

2 $2 \times \pi \times 12^2 + 2 \times \pi \times 12 \times 25$

$=$ _____

Find the SA to 1 decimal place



3 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times \square^2 + 2 \times \pi \times \square \times \square$

$SA = \square$ cm^2

4

Surface Area - Cylinders (Homework Strip)

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Date Due:

Calculate these to 1 decimal place.

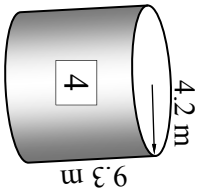
1 $2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 11$

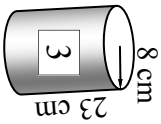
$= 640.9$

2 $2 \times \pi \times 12^2 + 2 \times \pi \times 12 \times 25$

$= 2\,789.7$

Find the SA to 1 decimal place





3 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 8^2 + 2 \times \pi \times 8 \times 23$

$SA = 1\,558.2 \text{ cm}^2$

4 $SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 4.2^2 + 2 \times \pi \times 4.2 \times 9.3$

$SA = 356.3 \text{ m}^2$

Surface Area - Find a Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Surface Area - Find a Word

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Find the words in the puzzle from the wordlist.



R	X	C	A	Q	E	B	S	S	U	R	F	A	C	E
E	E	L	C	R	I	C	C	U	E	R	A	U	Q	S
C	B	B	Q	U	A	R	E	T	I	B	E	U	U	A
T	U	E	R	O	B	L	I	X	R	D	A	A	A	B
A	C	Q	E	S	U	S	U	Q	T	C	A	E	R	A
N	G	E	T	U	O	T	O	G	Y	E	X	R	X	U
G	N	X	E	P	P	X	S	L	N	X	R	N	E	Q
U	I	P	M	X	E	A	I	I	I	A	E	N	E	U
L	H	O	A	X	X	N	I	E	D	D	I	K	A	T
A	C	S	I	E	D	C	U	R	V	E	D	R	N	L
R	T	E	D	E	T	X	N	T	A	M	P	A	T	M
X	A	D	R	T	H	G	I	E	H	U	L	T	A	S
E	M	Q	L	E	N	G	T	H	X	S	Q	T	A	I
D	I	M	A	R	Y	P	S	E	L	L	A	W	X	R
U	Q	R	A	L	U	C	I	D	N	E	P	R	E	P

WORDLIST

COMPOSITE	EXTERNAL	CIRCLE	NET	TRIANGULAR
WALL	HEIGHT	LENGTH	EXPOSED	SURFACE
SQUARE	RADIUS	RECTANGULAR	SLANT	SUM
CURVED	MATCHING	AREA	BASE	PERPENDICULAR
PYRAMID	CUBE	DIAMETER	UNITS	CYLINDER
	OUTSIDE	SOLID	PRISM	

Surface Area - Find a Word

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Find the words in the puzzle from the wordlist.



R	X	C	A	Q	E	B	S	S	U	R	F	A	C	E
E	E	L	C	R	I	C	C	U	E	R	A	U	Q	S
C	B	B	Q	U	A	R	E	T	I	B	E	U	U	A
T	U	E	R	O	B	L	I	X	R	D	A	A	A	B
A	C	Q	E	S	U	S	U	Q	T	C	A	E	R	A
N	G	E	T	U	O	T	O	G	Y	E	X	R	X	U
G	N	X	E	P	P	X	S	L	N	X	R	N	E	Q
U	I	P	M	X	E	A	I	I	I	A	E	N	E	U
L	H	O	A	X	X	N	I	E	D	D	I	K	A	T
A	C	S	I	E	D	C	U	R	V	E	D	R	N	L
R	T	E	D	E	T	X	N	T	A	M	P	A	T	M
X	A	D	R	T	H	G	I	E	H	U	L	T	A	S
E	M	Q	L	E	N	G	T	H	X	S	Q	T	A	I
D	I	M	A	R	Y	P	S	E	L	L	A	W	X	R
U	Q	R	A	L	U	C	I	D	N	E	P	R	E	P

WORDLIST

~~COMPOSITE~~
~~WALL~~
~~SQUARE~~
~~CURVED~~
~~PYRAMID~~
~~EXTERNAL~~
~~HEIGHT~~
~~RADIUS~~
~~MATCHING~~
~~CUBE~~
~~OUTSIDE~~
~~CIRCLE~~
~~LENGTH~~
~~RECTANGULAR~~
~~AREA~~
~~DIAMETER~~
~~SOLID~~
~~NET~~
~~EXPOSED~~
~~SLANT~~
~~BASE~~
~~UNITS~~
~~PRISM~~
~~TRIANGULAR~~
~~SURFACE~~
~~SUM~~
~~PERPENDICULAR~~
~~CYLINDER~~

Surface Area Review

This is the review of the Surface Area Topic.

Surface Area - Topic Review

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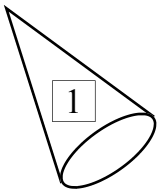
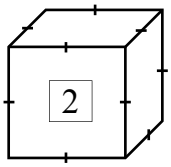
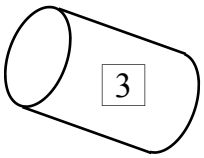
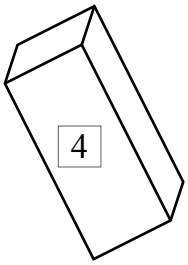
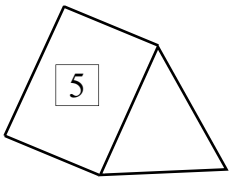
Name these solids.

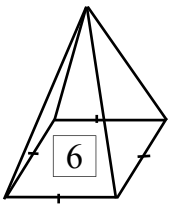
1 

2 

3 

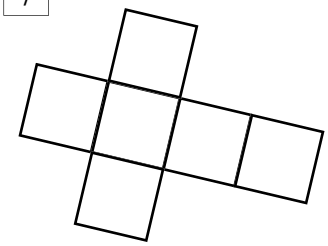
4 

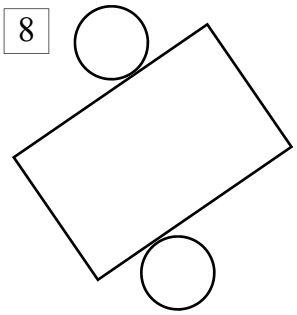
5 

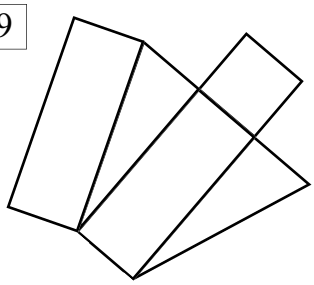
6 

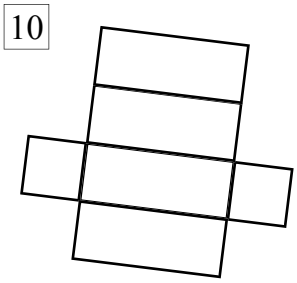
Two Word Description

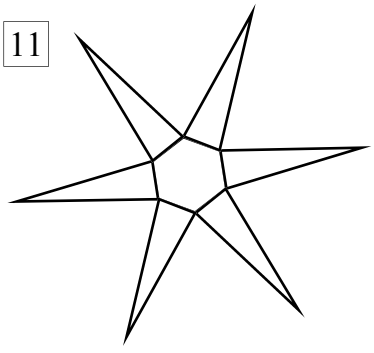
Name the solids shown below as nets.

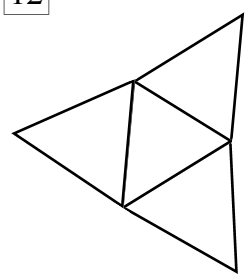
7 

8 

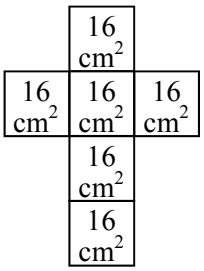
9 

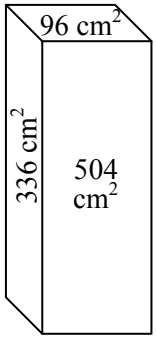
10 

11 

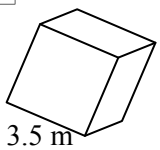
12 

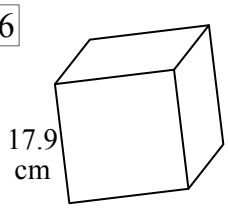
Add the face areas to calculate the total surface area

13 

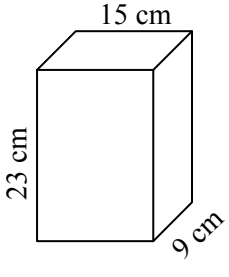
14 

Use SA = $6l^2$ to calculate the surface area of these cubes. Answer to 1 decimal place.

15 

16 

Calculate the surface area of the rectangular prism.

17 

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A rectangular room is 2.4 m × 3.7 m × 4.1 m in size.
Calculate the surface area of the entire room.
Answer to 1 decimal place.

17

A rectangular cereal box has sides 23 cm, 35 cm & 6 cm.
Calculate the total surface area of the cardboard box.

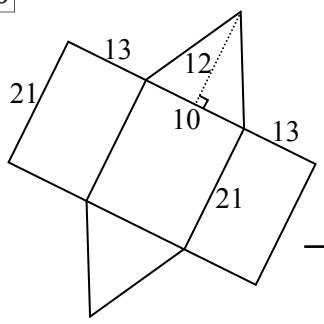
18

An advertisement on the back of the cereal box (above) occupies an area of 358 cm^2. What percentage of the surface area is taken by the advertisement.
Answer to the nearest whole percent.

19

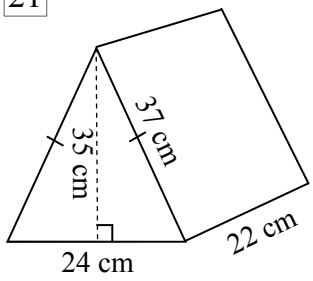
Calculate the total surface area of the triangular prism shown below as a net. All measurements are in cm.

20



Calculate the surface area of these two triangular prisms.

21



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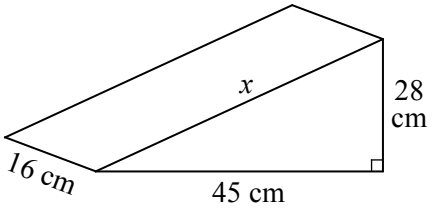
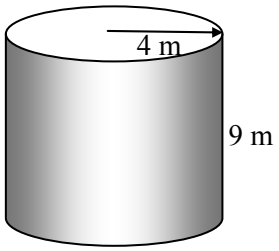


Calculate the surface area of these two cylinders. Use:
$SA = 2\pi r^2 + 2\pi rh$
Round your answer to 2 decimal places.

23



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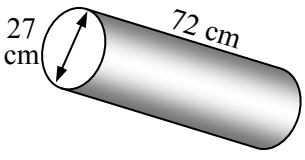

Surface Area - Topic Review

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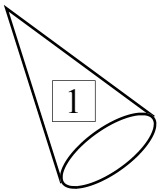
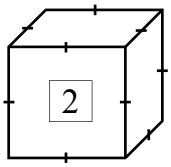
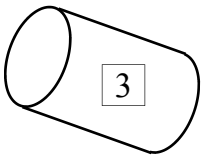
Name these solids.

1 

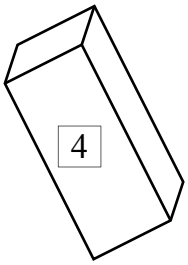
cone

2 

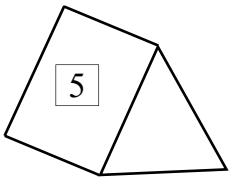
cube

3 

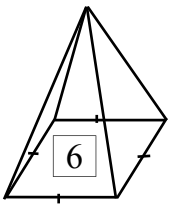
cylinder

4 

Rectangular prism

5 

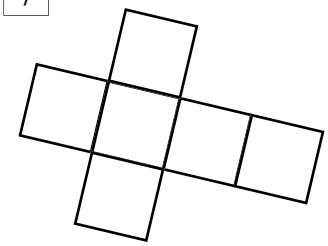
Triangular prism

6 

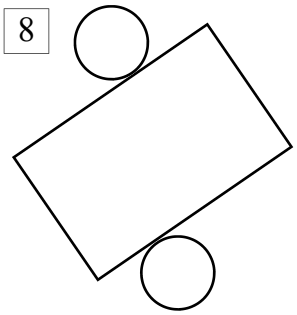
Square pyramid

Two Word Description

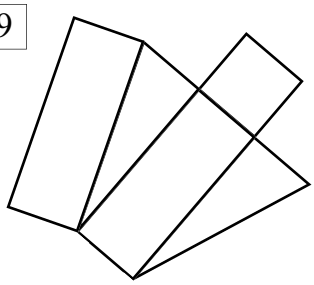
Name the solids shown below as nets.

7 

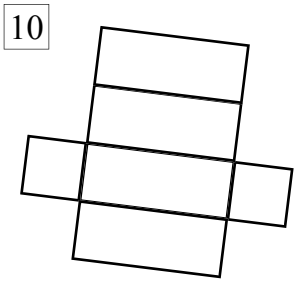
Cube

8 

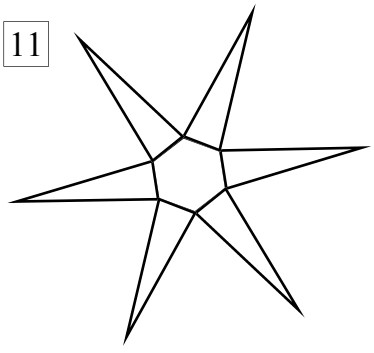
Cylinder

9 

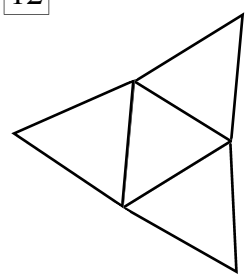
Triangular prism

10 

Rectangular prism

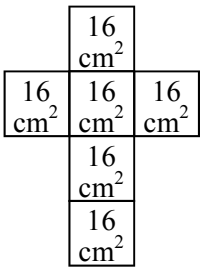
11 

Hexagonal pyramid

12 

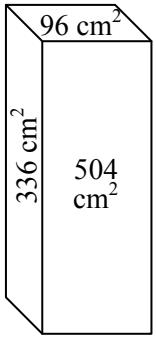
Triangular pyramid (Tetrahedron)

Add the face areas to calculate the total surface area

13 

$SA = 6 \times 16$

$SA = 96 \text{ cm}^2$

14 

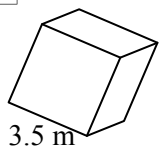
$SA = 2 \times 96$

$+ 2 \times 336$

$+ 2 \times 504$

$SA = 1\,872 \text{ cm}^2$

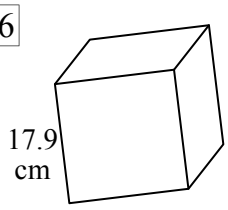
Use $SA = 6l^2$ to calculate the surface area of these cubes. Answer to 1 decimal place.

15 

$SA = 6l^2$

$= 6 \times 3.5^2$

$SA = 73.5 \text{ m}^2$

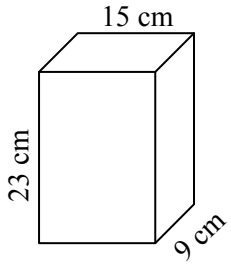
16 

$SA = 6l^2$

$= 6 \times 17.9^2$

$SA = 1\,922.5 \text{ cm}^2$

Calculate the surface area of the rectangular prism.

17 

$SA = 2 \times 23 \times 9$

$+ 2 \times 23 \times 15$

$+ 2 \times 9 \times 15$

$SA = 1\,374 \text{ cm}^2$

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A rectangular room is 2.4 m × 3.7 m × 4.1 m in size. Calculate the surface area of the entire room. Answer to 1 decimal place.

17

$SA = 2 \times 2.4 \times 3.7$

$+ 2 \times 2.4 \times 4.1$

$+ 2 \times 3.7 \times 4.1$

$SA = 67.8 \text{ m}^2$

A rectangular cereal box has sides 23 cm, 35 cm & 6 cm. Calculate the total surface area of the cardboard box.

18

$SA = 2 \times 23 \times 35$

$+ 2 \times 23 \times 6$

$+ 2 \times 35 \times 6$

$SA = 2\,306 \text{ cm}^2$

An advertisement on the back of the cereal box (above) occupies an area of 358 cm^2. What percentage of the surface area is taken by the advertisement. Answer to the nearest whole percent.

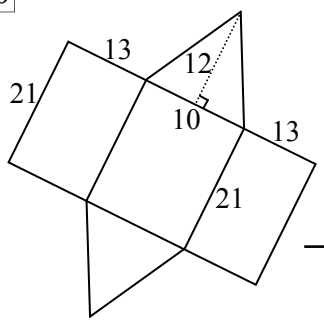
19

$\text{Area \%} = \dfrac{358}{2\,306} \times 100$

Area % = 16% (15.5247….)

Calculate the total surface area of the triangular prism shown below as a net. All measurements are in cm.

20



$SA = 2 \times ½ \times 10 \times 12$

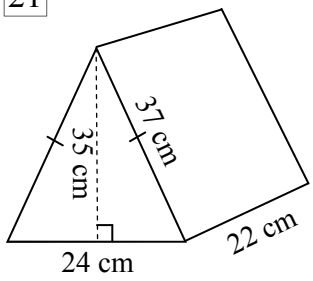
$+ 2 \times 21 \times 13$

$+ 21 \times 10$

$SA = 876 \text{ cm}^2$

Calculate the surface area of these two triangular prisms.

21



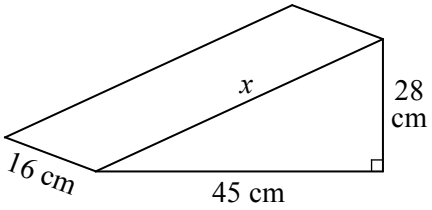
$SA = 2 \times ½ \times 24 \times 35$

$+ 2 \times 37 \times 22$

$+ 24 \times 22$

$SA = 2\,996 \text{ cm}^2$

22



$x^2 = 28^2 + 45^2$

$x^2 = 2\,809$

$x = \sqrt{2\,809}$

$x = 53 \text{ cm}$

$SA = 2 \times ½ \times 45 \times 28$

$+ 28 \times 16$

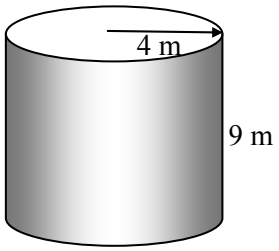
$+ 45 \times 16$

$+ 53 \times 16$

$SA = 3\,276 \text{ cm}^2$

Calculate the surface area of these two cylinders. Use: $SA = 2\pi r^2 + 2\pi rh$ Round your answer to 2 decimal places.

23

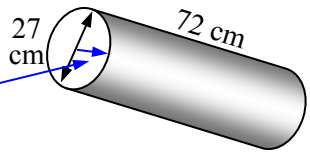


$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 4^2 + 2 \times \pi \times 4 \times 9$

$SA = 326.73 \text{ m}^2$ (326.725636)

24



$r = 13.5 \text{ cm}$

$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 13.5^2 + 2 \times \pi \times 13.5 \times 72$

$SA = 7\,252.37 \text{ cm}^2$ (7 252.366641)

Surface Area



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Surface Area - Constructing Cubes and Square Pyramids

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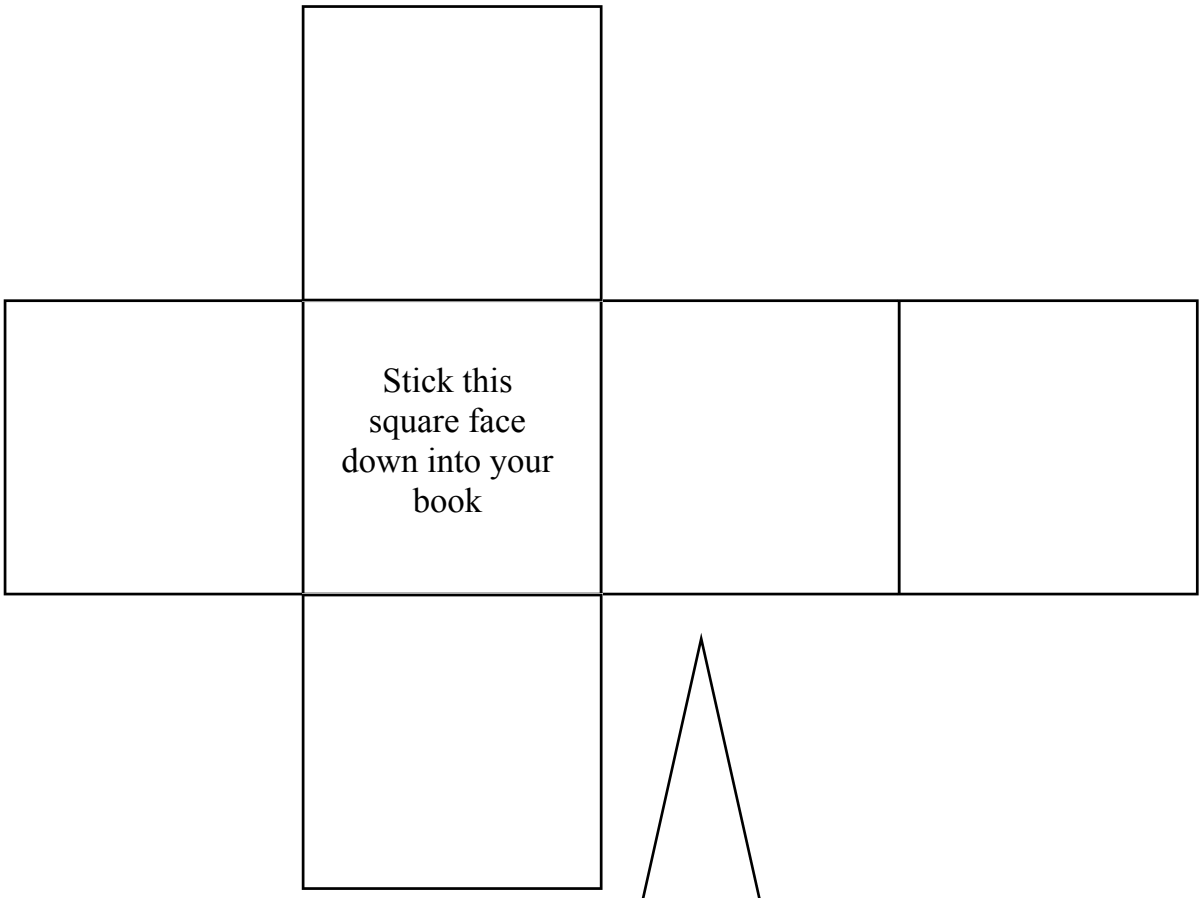

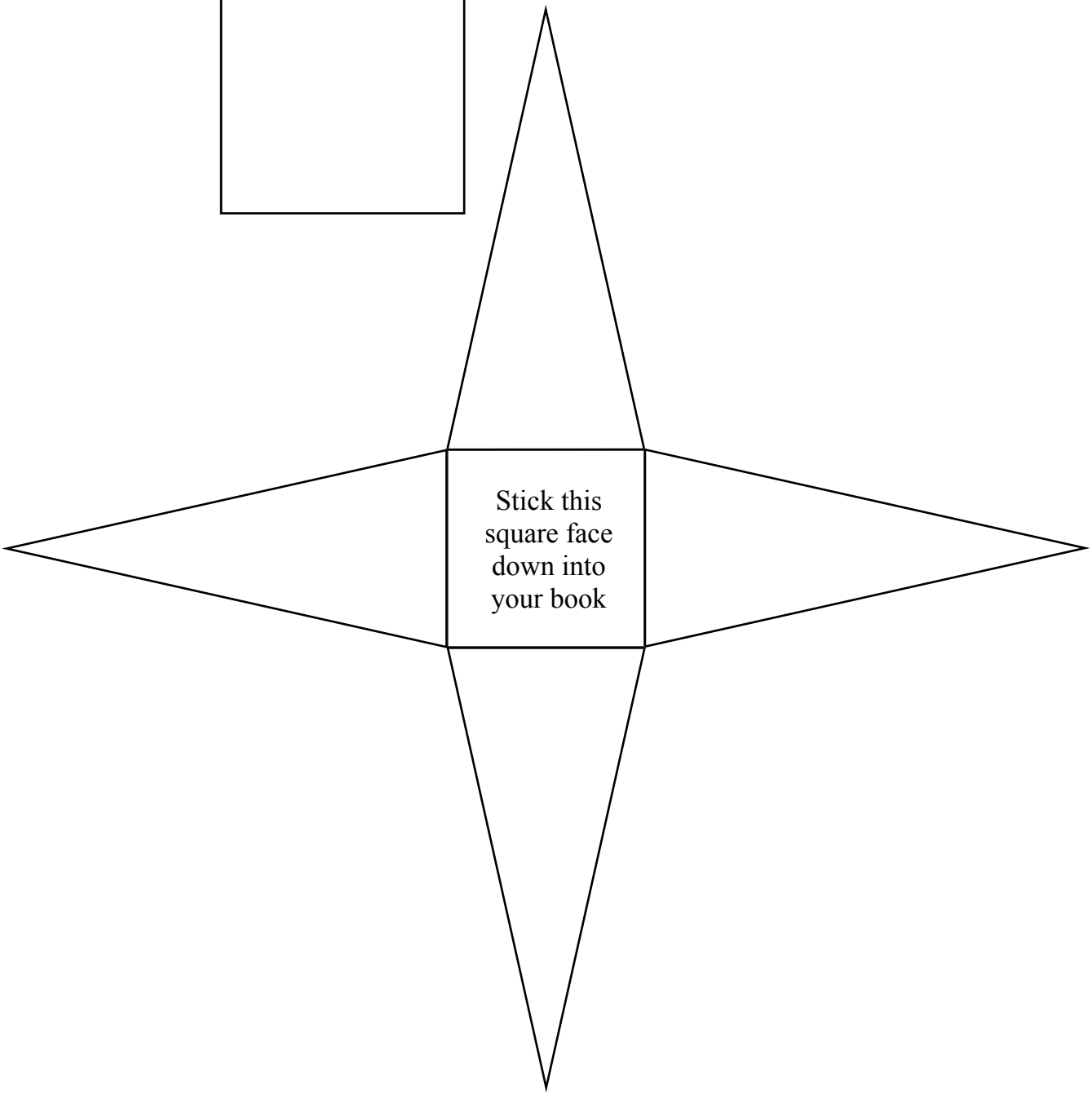

Surface Area - Constructing Rectangular Prisms and Triangular Prisms

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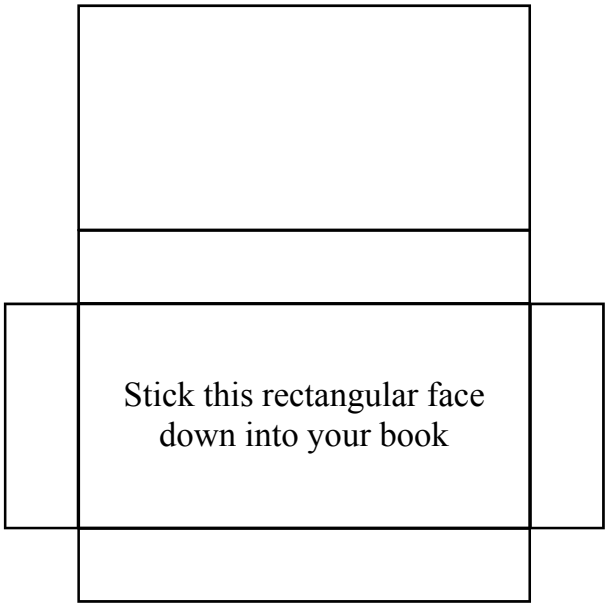

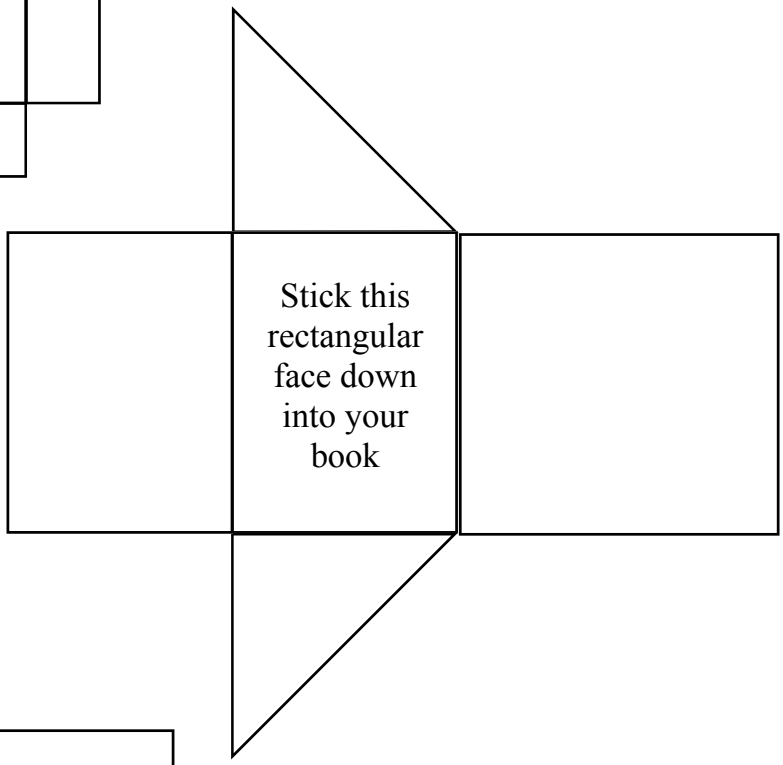

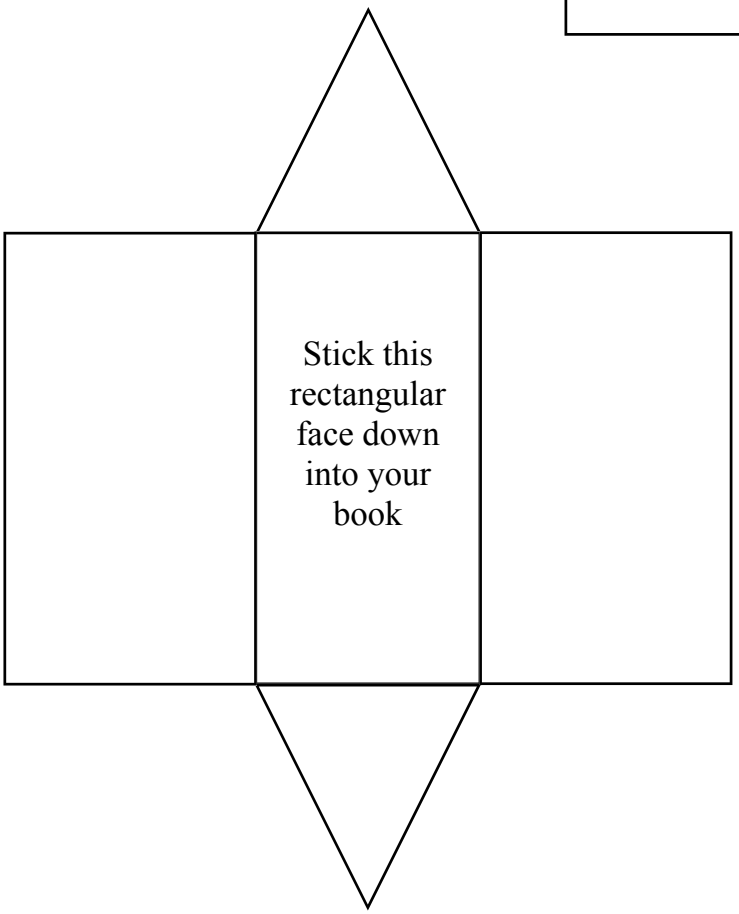

Surface Area - Constructing Cubes and Square Pyramids (with tabs)

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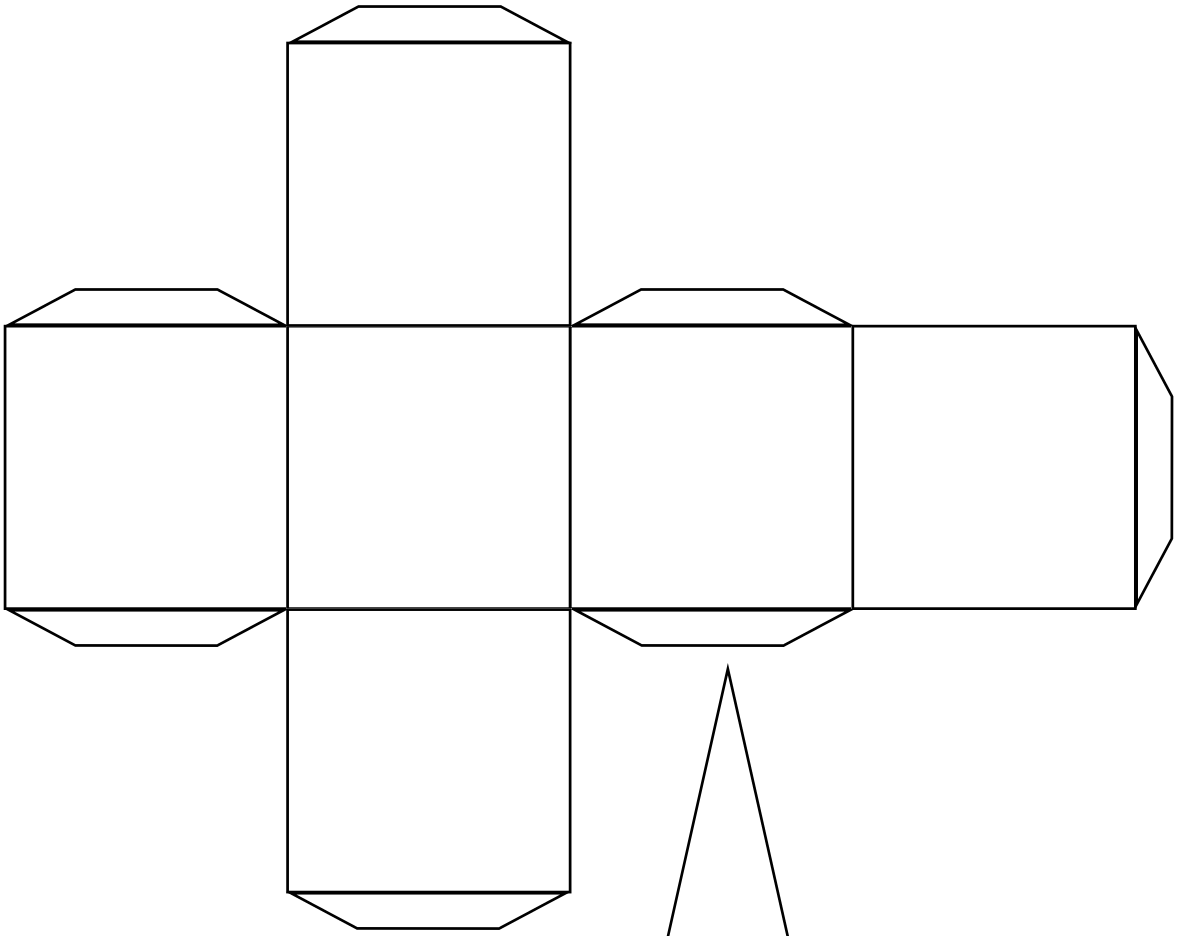

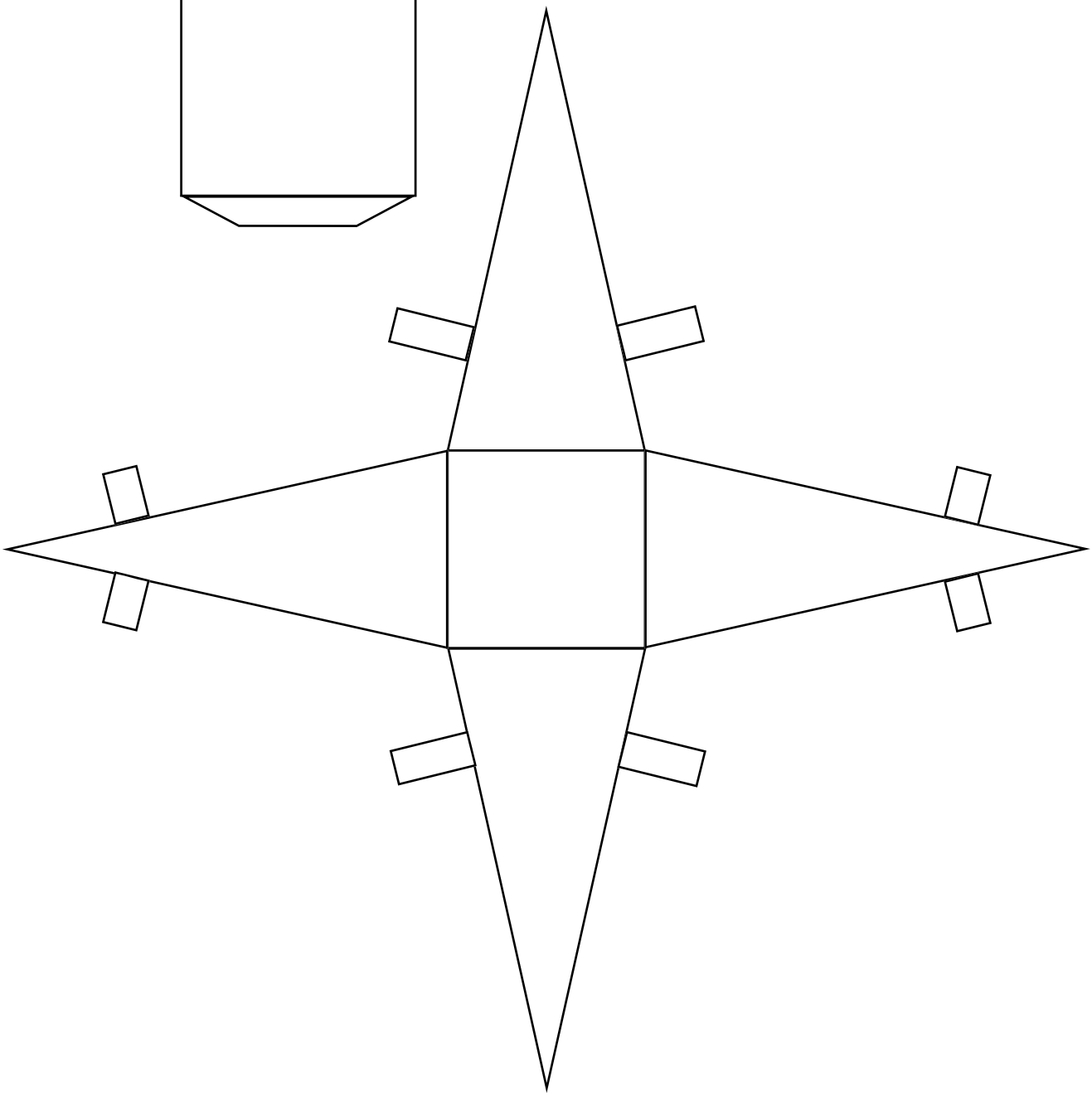

Surface Area - Constructing Rectangular & Triangular Prisms (with tabs)

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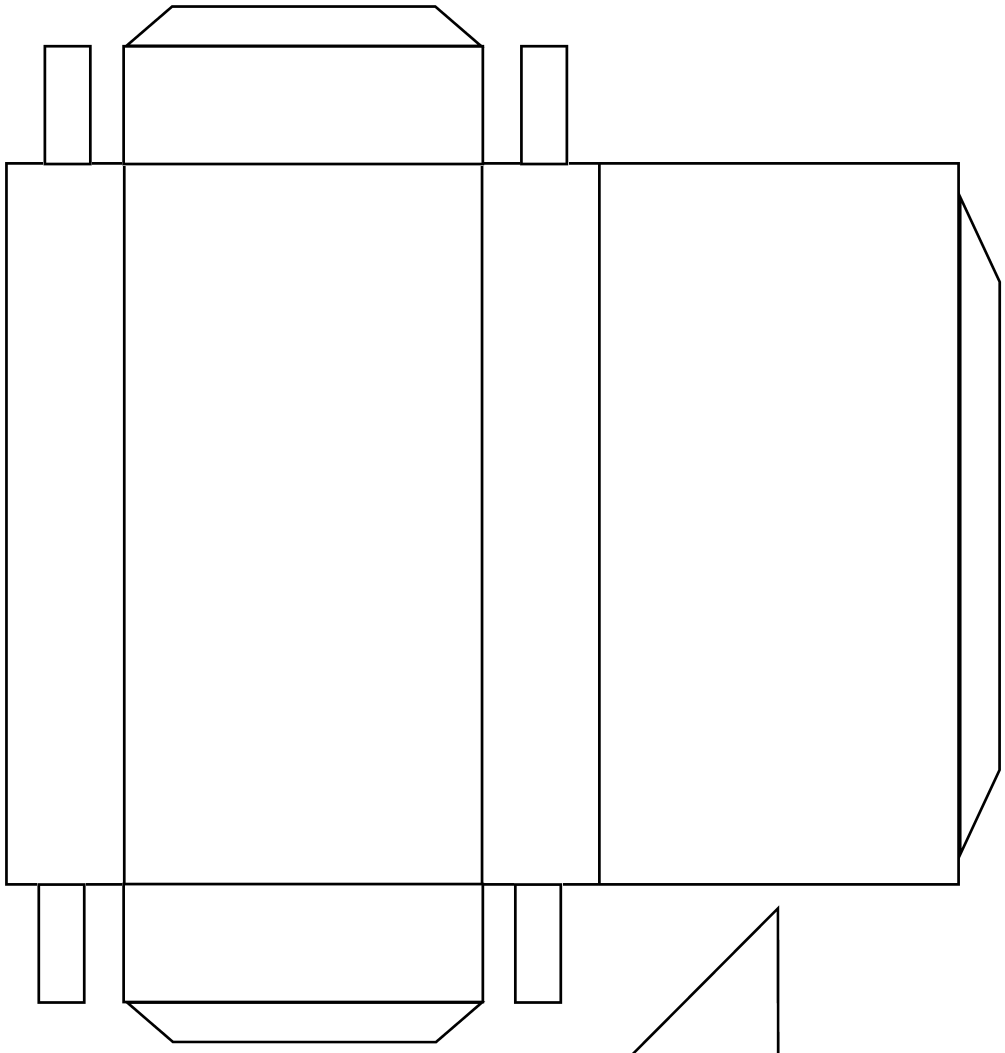

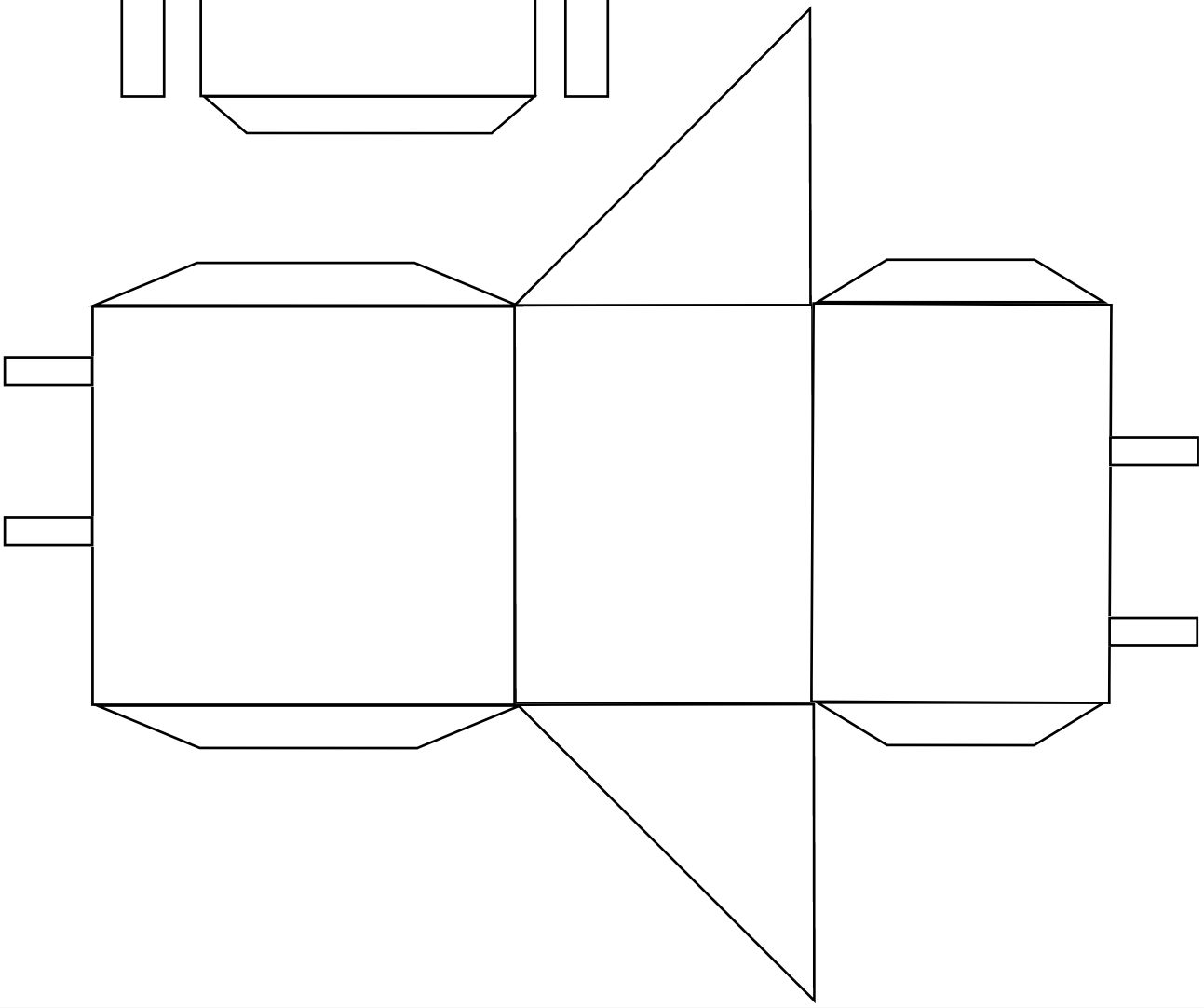

Surface Area - Constructing Cylinders and Triangular Prisms (with tabs)

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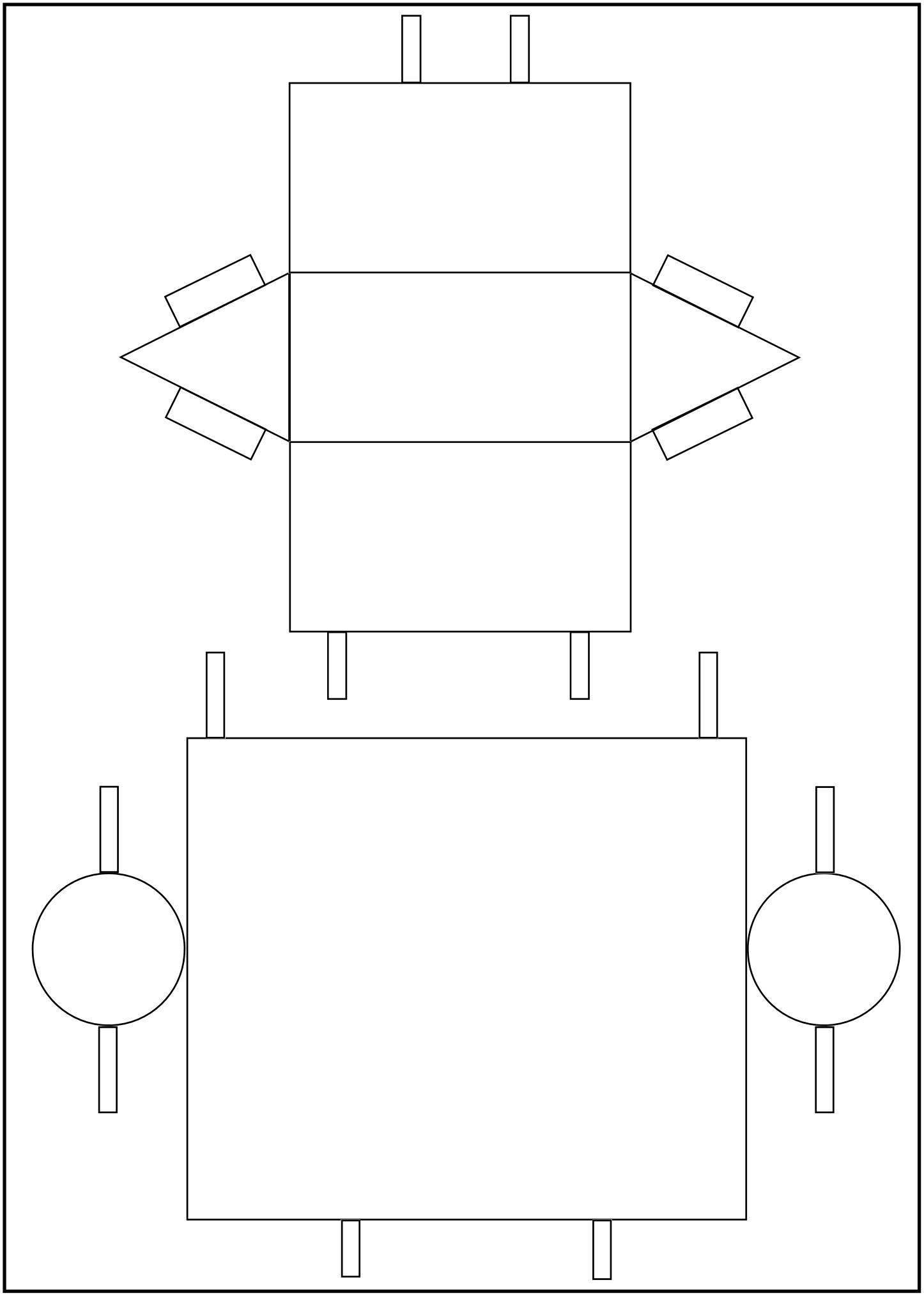

Surface Area - Constructing Cones and Pentagonal Prisms (with tabs)

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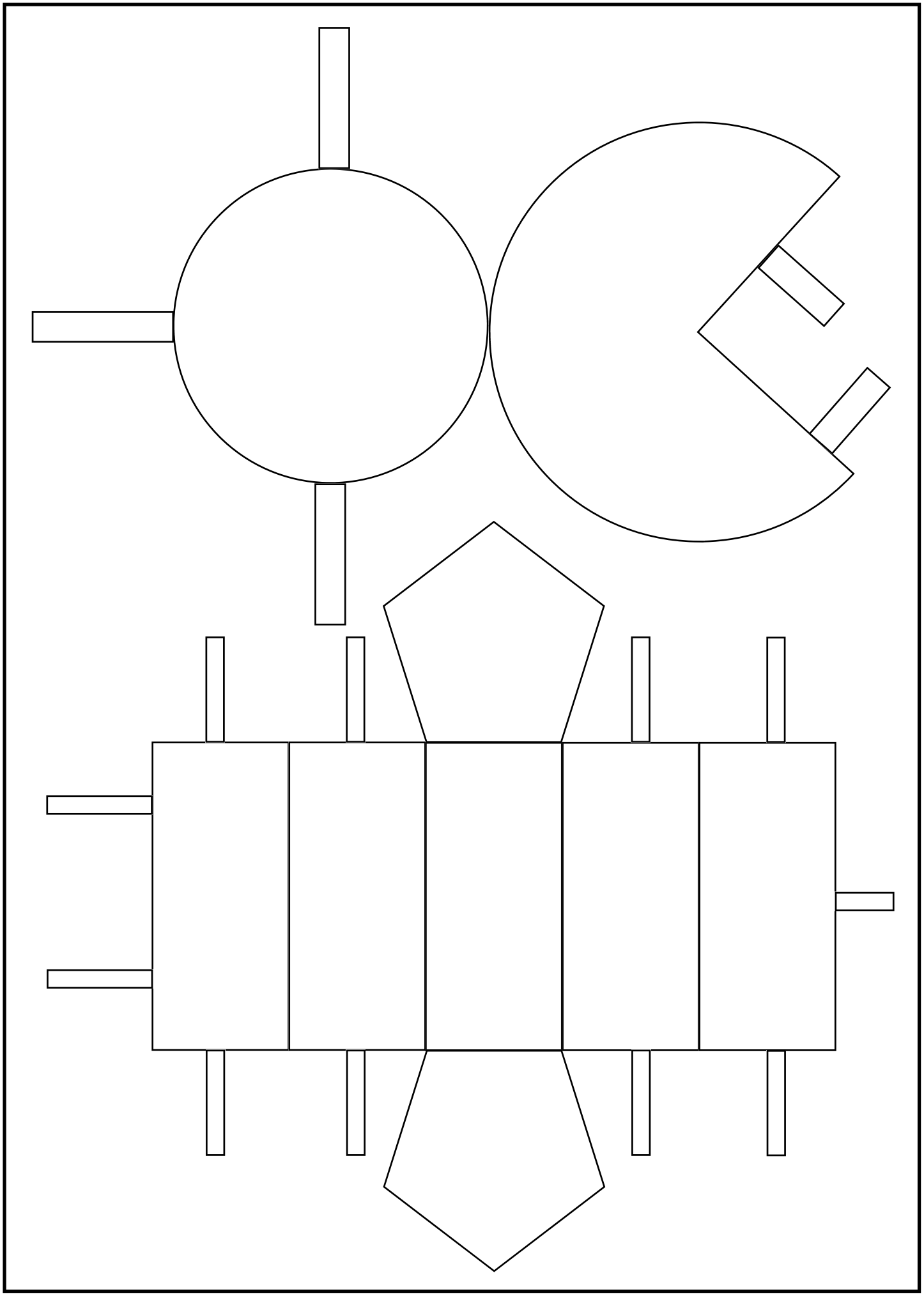

Surface Area - Identifying Solids (Sheet 1)

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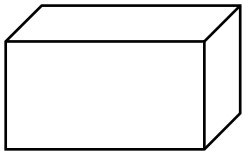
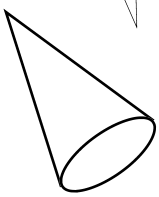
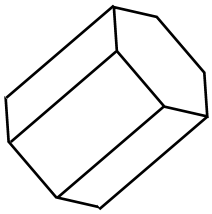
Name the solids and also indicate if they are a prism or not.

1 

O Prism O Non Prism

2 

O Prism
O Non Prism

3 

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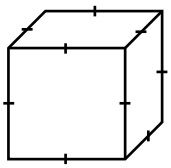
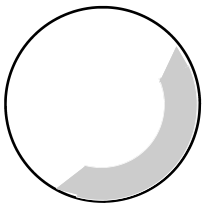
4 

O Prism
O Non Prism

5 

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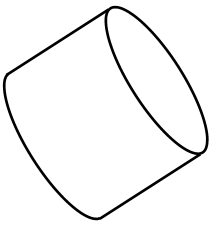
6 

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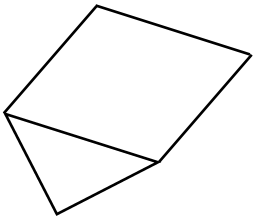
7 

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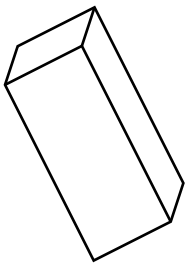
8 

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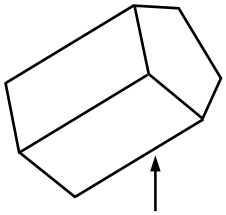
9 

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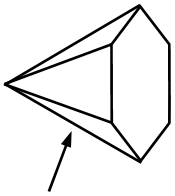
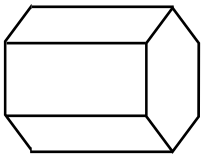
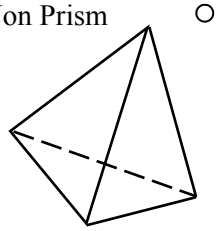
10 

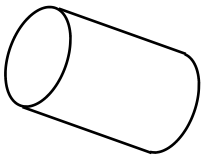
O Prism O Non Prism

11 

O Prism O Non Prism

12 

O Prism O Non Prism

13 

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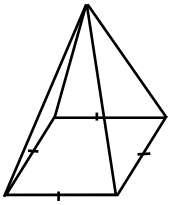
14 

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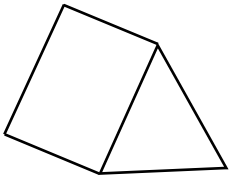
15 

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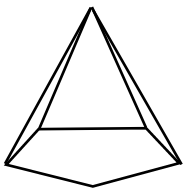
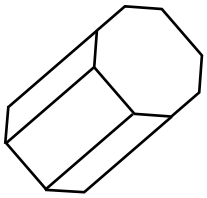
16 

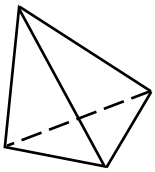
O Prism O Non Prism

17 

O Prism O Non Prism

18 

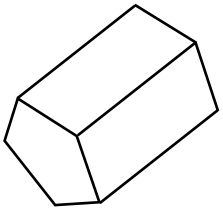
O Prism O Non Prism

19 

O Prism O Non Prism

20 

O Prism
O Non Prism

21 

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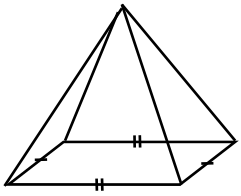
22 

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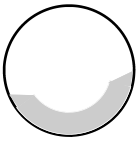
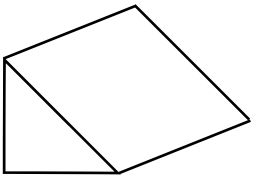
23 

O Prism
O Non Prism

24 

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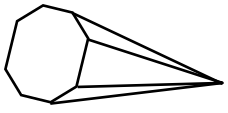
25 

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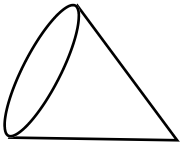
26 

O Prism O Non Prism

Surface Area - Identifying Solids From Their Nets (Sheet 2)

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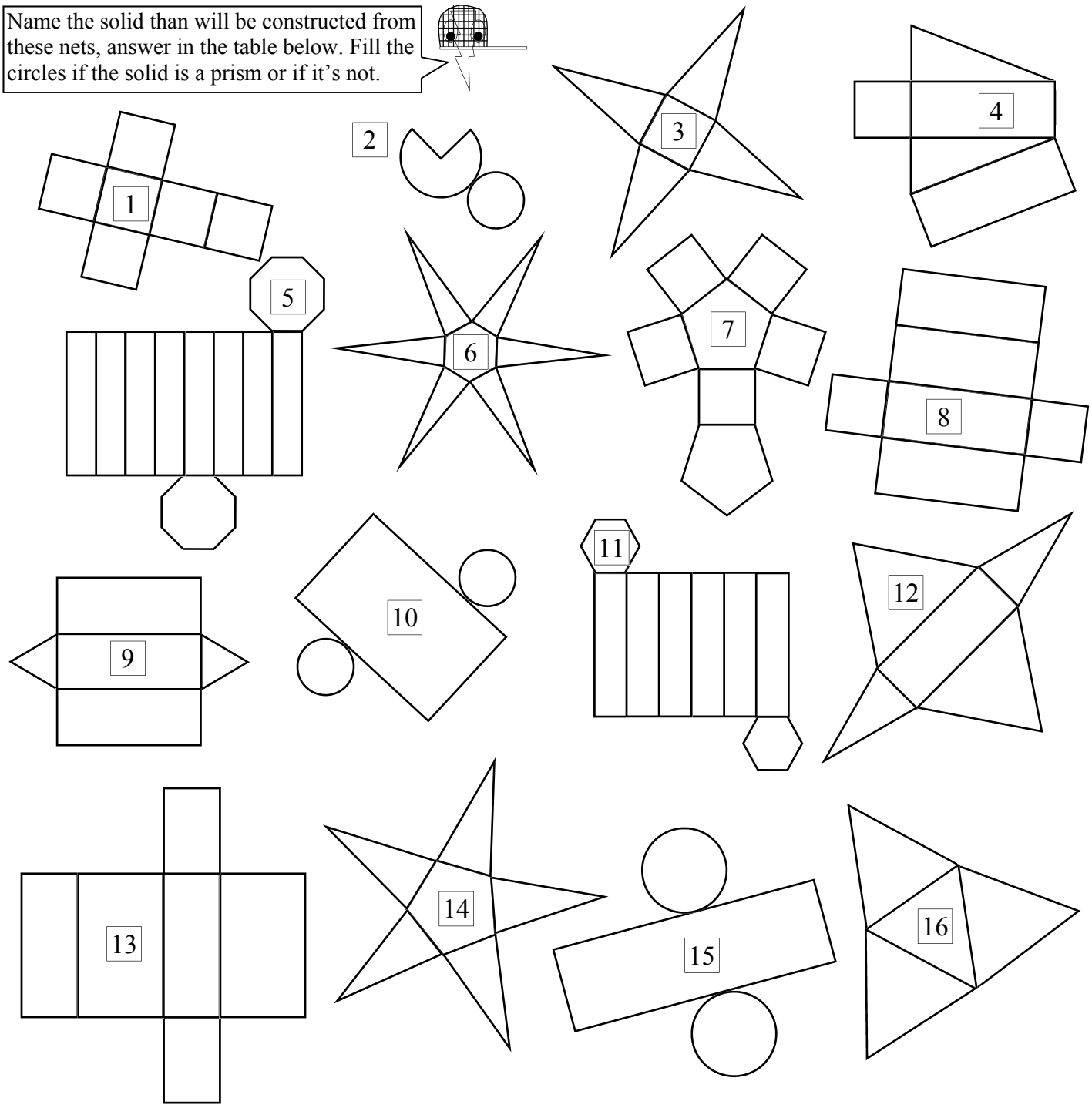
Name the solid than will be constructed from these nets, answer in the table below. Fill the circles if the solid is a prism or if it's not.



1 ____ ○ Prism ○ Non Prism	2 ____ ○ Prism ○ Non Prism	3 ____ ○ Prism ○ Non Prism	4 ____ ○ Prism ○ Non Prism
5 ____ ○ Prism ○ Non Prism	6 ____ ○ Prism ○ Non Prism	7 ____ ○ Prism ○ Non Prism	8 ____ ○ Prism ○ Non Prism
9 ____ ○ Prism ○ Non Prism	10 ____ ○ Prism ○ Non Prism	11 ____ ○ Prism ○ Non Prism	12 ____ ○ Prism ○ Non Prism
13 ____ ○ Prism ○ Non Prism	14 ____ ○ Prism ○ Non Prism	15 ____ ○ Prism ○ Non Prism	16 ____ ○ Prism ○ Non Prism

Surface Area - Drawing the Nets of Solids (Sheet 3)

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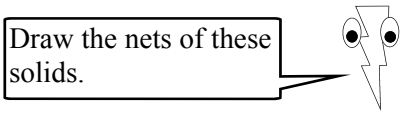

1 Cube

2 Cone

3 Rectangular Prism

4 Triangular Prism

5 Pentagonal Pyramid

6 Hexagonal Prism

7 Octagonal Prism

8 Square Pyramid

9 Pentagonal Prism

10 Triangular Pyramid

11 Rectangular Pyramid

12 Hexagonal Pyramid

Surface Area - Face Areas and Edges (Sheet 1)

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The grid below is in 1 cm^2. Measure the edge lengths and write their lengths (in cm) in the boxes. The surface area can be measured in two ways. By counting all the squares in each net or using the edge lengths to find the area, then adding the areas together and doubling the answer. Find the surface area using both methods.

1

Grid squares = ☐ cm^2 ← Count all the squares in the net in Q1 and write it here.

Grey = 1 × 3 = ☐

Spotty = 2×3 = ☐

White = 2×1 = ☐

Total = ☐ cm^2

SA = 2 × ☐ = ☐ cm^2

2

3

2 Grid squares = ☐ cm^2

Grey Area = ☐ × ☐ = ☐ cm^2

Spotty Area = ☐ × ☐ = ☐ cm^2

White Area = ☐ × ☐ = ☐ cm^2

Total = ☐ cm^2

SA = 2 × ☐ = ☐ cm^2

3 Grid squares = ☐ cm^2

Grey Area = ☐ × ☐ = ☐ cm^2

Spotty Area = ☐ × ☐ = ☐ cm^2

White Area = ☐ × ☐ = ☐ cm^2

Total = ☐ cm^2

SA = 2 × ☐ = ☐ cm^2

Surface Area - Face Areas and Edges (Sheet 2)

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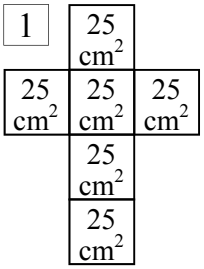
Find the surface area of the nets by adding the face areas. These are areas not side lengths!

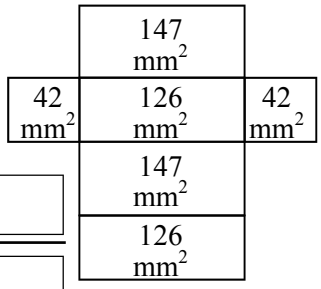
1



SA = 6 × ☐

SA = ☐ cm²

2



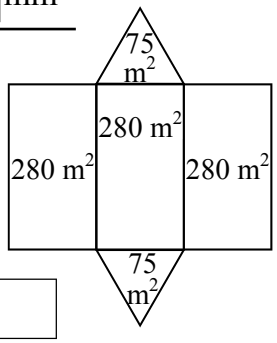
SA = 2 × ☐

+ 2 × ☐

+ 2 × ☐

SA = ☐ mm²

3

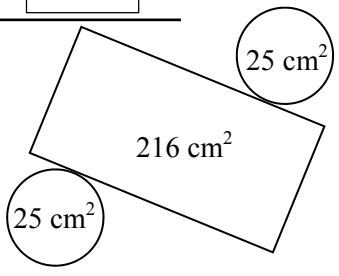


SA = 2 × ☐

+ 3 × ☐

SA = ☐ m²

4



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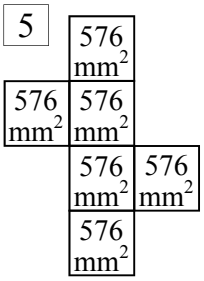
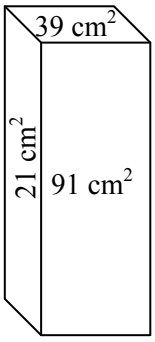


Remember units

Remember the matching sides, find the SA!

6



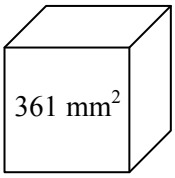
SA = 2 × ☐

+ 2 × ☐

+ 2 × ☐

SA = ☐ cm²

7 Cube



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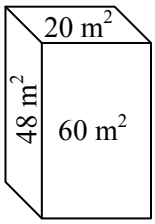
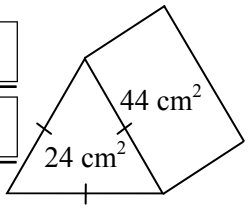


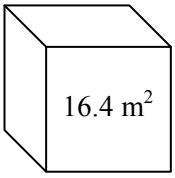
9

SA = 2 × ☐

+ 3 × ☐



10 Cube



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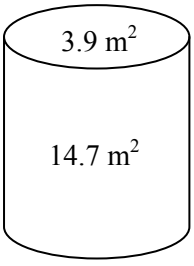

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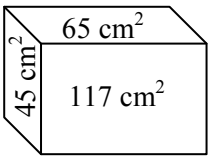

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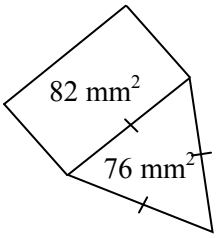
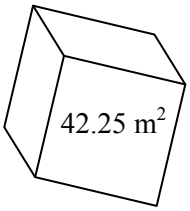


14 Cube



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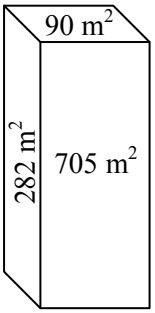

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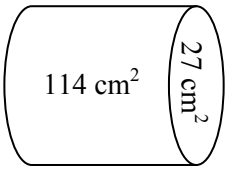

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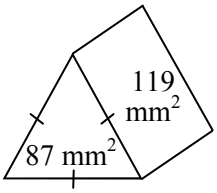
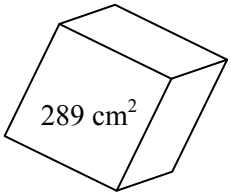


18 Cube



Surface Area - Cubes and Rectangular Prisms (Sheet 1)

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Calculate these using your square key on your calculator.

1 $6 \times 14^2 =$ ☐

Calc: 6 × 1 4 x^2 =

2 $6 \times 7^2 =$ ☐

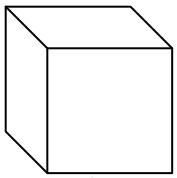
3 $6 \times 17^2 =$ ☐

4 $6 \times 35^2 =$ ☐

5 $6 \times 26^2 =$ ☐

Use SA = $6l^2$ to find the SA of these cubes. Your answer should include square units.

6



$SA = 6l^2$

$= 6 \times$ ☐ ^{2}

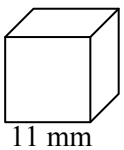
SA = ☐ m^2

7

$SA = 6l^2$

=

SA = ☐ mm^2



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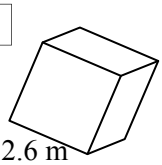


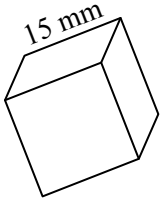
SA =

=

SA =

Square units

9



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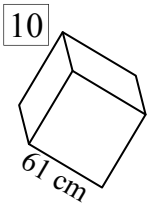
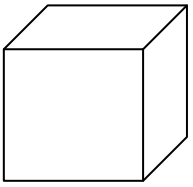


Find the SA of these cubes. Round your answer to 1 d.p.

11



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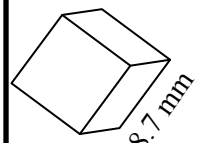

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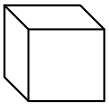


Use the Δ, □, and ○ to help you to find the SA of these rectangular prisms. All side lengths are in cm.

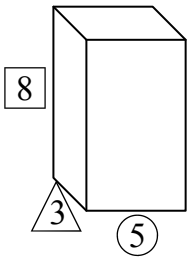
14

$SA = 2 \times$ ☐ $\times$ △

$+ 2 \times$ ☐ $\times$ ○

$+ 2 \times$ ○ $\times$ △

SA = ☐ cm^2



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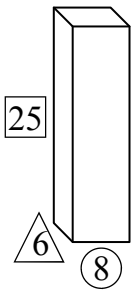


$SA = 2 \times$ ☐ $\times$ △

$+ 2 \times$ ☐ $\times$ ○

$+ 2 \times$ ○ $\times$ △

SA = ☐ cm^2

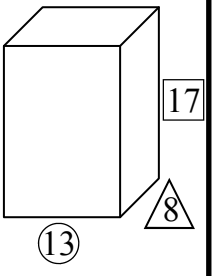
16

$SA = 2 \times$ ☐ $\times$ △

$+ 2 \times$ ☐ $\times$ ○

$+ 2 \times$ ○ $\times$ △

SA = ☐ cm^2



Find the SA of these cubes and rectangular prisms.

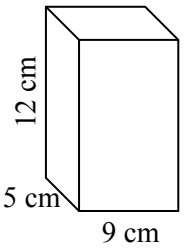
17

$SA = 2 \times$ ☐ $\times$ ☐

$+ 2 \times$ ☐ $\times$ ☐

$+ 2 \times$ ☐ $\times$ ☐

SA = ☐ cm^2



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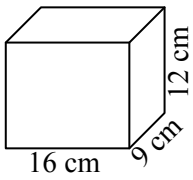
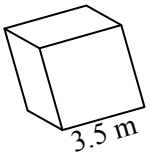


19

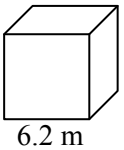
Cube



20 A rectangular prism has sides 2.4 m, 3.8 m and 4.3 m.

21 A rectangular prism has sides 78 cm, 14 cm and 23 cm.

22 Cube



Surface Area - Cubes and Rectangular Prisms (Sheet 2)

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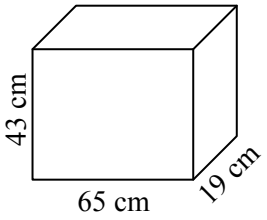
Find the surface area of these rectangular prisms. Remember to show the square units.

1 $SA = 2 \times \square \times \square$
$+ 2 \times \square \times \square$
$+ 2 \times \square \times \square$
$SA = \square\ cm^2$



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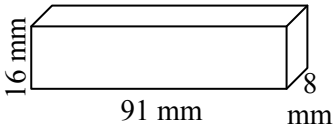

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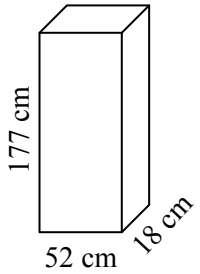

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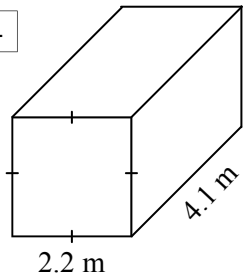

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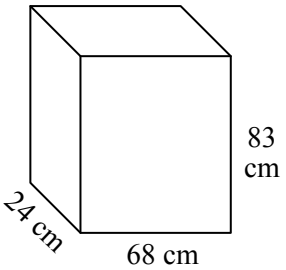

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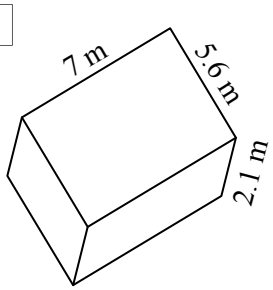
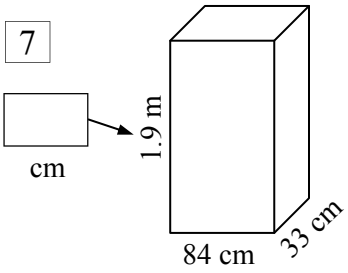


Give the surface area in cm^2. So change all units that aren't in cm, to cm first. Then perform the SA calculation. Round to 1 d.p. if needed.

7



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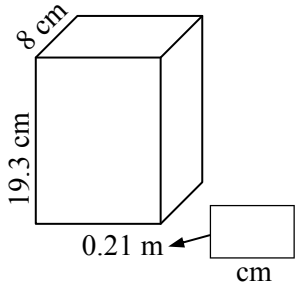

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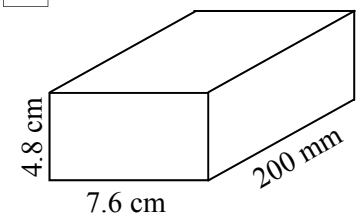

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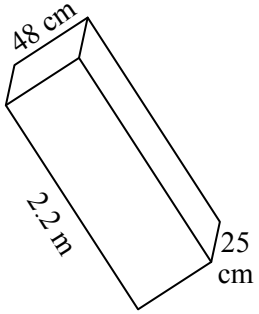

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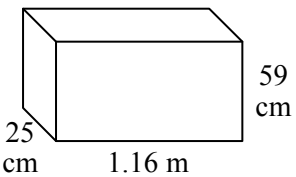

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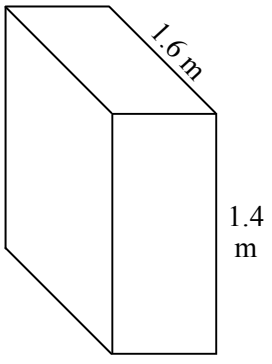

Surface Area - Cubes and Rectangular Prisms Problems

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Find the SA of these tool boxes!

A steel rectangular shaped toolbox is 35 cm wide, 20 cm high and 25 cm deep, draw a diagram and find the SA

Diagram - Use a Ruler

1

2 SA = 2 × ☐ × ☐

\+ 2 × ☐ × ☐

\+ 2 × ☐ × ☐

SA = ☐ cm^2

Double each of the sides of the tool box.

3 35: 2 × 35 = ☐ cm

20: 2 × 20 = ☐ cm

25: 2 × 25 = ☐ cm

Draw a diagram and then find the SA of the enlarged tool box.

Diagram - Use a Ruler

4

5 SA = 2 × ☐ × ☐

\+ 2 × ☐ × ☐

\+ 2 × ☐ × ☐

SA = ☐ cm^2

Divide (÷) the larger tool box's SA by the smaller tool box's SA.

6 ☐ (Larger Area) / ☐ (Smaller Area) = ☐

Complete this statement

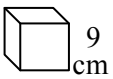
7 When I double the side lengths of a rectangular prism the SA is multiplied by a factor of ☐.

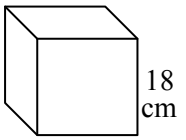
Apply to cubes, find the SA.

8 $SA = 6l^2$

$= 6 \times$ ☐^{2}

SA = ☐ cm^2





Divide the larger cube's SA by the smaller cube's SA.

9 ☐ / ☐ = ☐

Make a statement about the size & SA

10 When I double _____

Jen measures her room, its floor size is 3.7 m by 4.5 m, and the ceiling height is 2.4 m. Calculate the SA of the room.

11 SA = 2 × ☐ × ☐

\+ 2 × ☐ × ☐

\+ 2 × ☐ × ☐

SA = ☐ m^2

Calculate the floor area of the room. Floor size is 3.7 m by 4.5 m.

12 A = ☐ × ☐

A = ☐ m^2

The ceiling and walls are to be re-plastered. Find their area.

13 A = ☐ (Room SA) - ☐ (Floor Area)

A = ☐

Remember units

The plaster sheets are 1.2 m by 2.4 m, calculate the area of one plaster sheet.

14 A = ☐ × ☐

A = ☐

How many **FULL** sheets are required to plaster Jen's ceiling and walls?

15 ☐ (Q13) / ☐ (Q14) = ☐ sheets

A box of 'Oaties' cereal is 0.32 m by 0.23 m by 0.06 m. Find the box's SA.

16 _____

Don't round your answer

The company produces 30 000 boxes in a day, what is the area of cardboard needed per day (ignoring seams)?

17 _____

If cardboard is supplied in rolls of 220 m^2. How many complete rolls are required for one day's production?

18 _____

Surface Area - Area of Triangles and Rectangles (Sheet 1)

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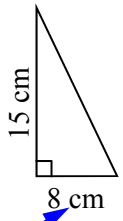
Find the area of these triangles. Use A = ½*bh*, using either 0.5 or ½ on your calculator.

Example

A = ½*bh*

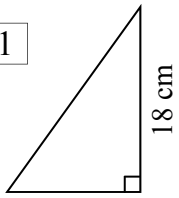
= ½×8×15

A = 60 cm^2



units^2 must match

1

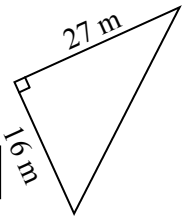


A = ½*bh*

= ½× ☐ × ☐

A = ☐ cm^2

2

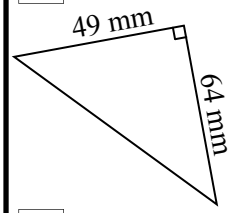


A = ½*bh*

= ½ × ☐ × ☐

A = ☐ m^2

3

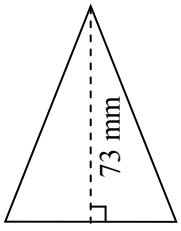


A =

A =

Show square units

4



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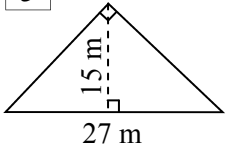

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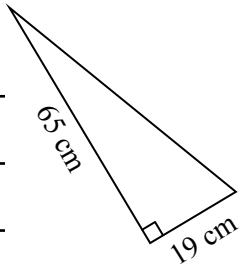
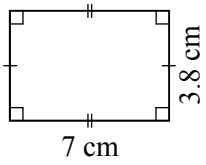


Now rectangles. Use A=*lb* to find the area, round to 1 d.p. when required.

Example



A = *lb*

= 7 × 3.8

A = 26.6 cm^2

Show square units

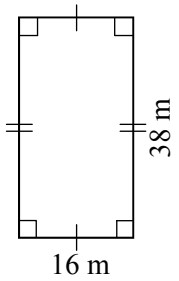
7

A = *lb*

= ☐ × ☐

Multiply, in any order

A = ☐ m^2



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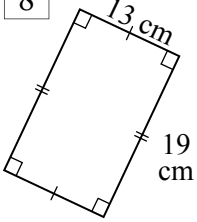


A = *lb*

= ☐ × ☐

A = ☐ cm^2

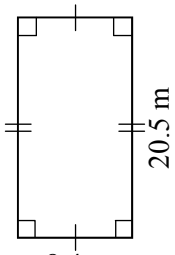
9

A = *lb*

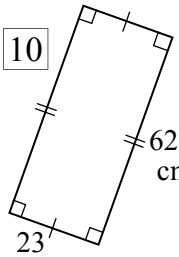
= ☐ × ☐

A = ☐

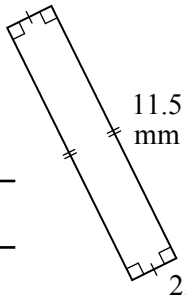
Show square units



10



11



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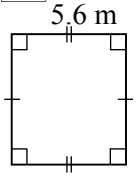
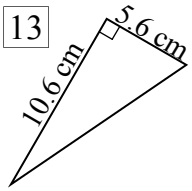


Now a mixture, find the area of these triangles and rectangles, answer to 1 d.p. when required.

13



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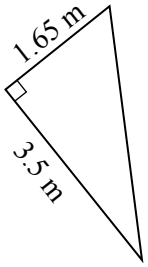

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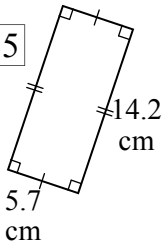

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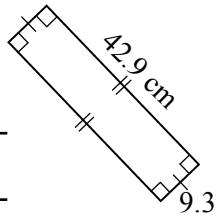

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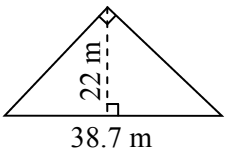

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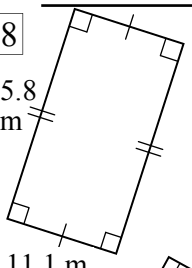

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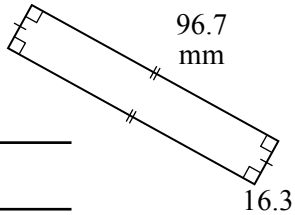

Surface Area - SA of Nets of Triangular Prisms (Sheet 2)

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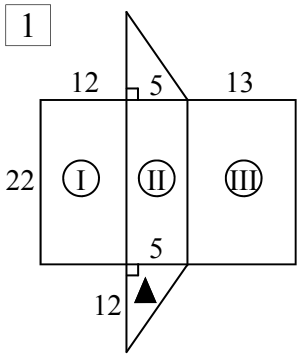
These are nets of triangular prisms. Find the areas individually, then get the total for the surface area of the triangular prism. All measurements are in cm.

1



2▲: $2 \times \frac{1}{2} \times 5 \times 12$

$= \square$ cm^2

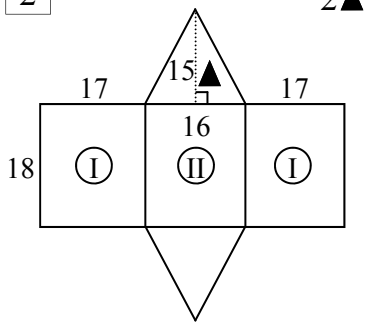
I: $12 \times 22 = \square$ cm^2

II: $5 \times 22 = \square$ cm^2

III: $13 \times 22 = \square$ cm^2

SA = $\square + \square + \square + \square = \square$ cm^2

2



2▲: $2 \times \frac{1}{2} \times \square \times \square$

$= \square$ cm^2

2 I: $2 \times \square \times \square$

$= \square$ cm^2

II: $\square \times \square$

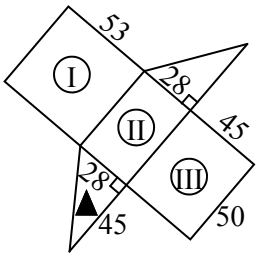
$= \square$ cm^2

SA = $\square + \square + \square = \square$ cm^2

3

2▲: $2 \times \frac{1}{2} \times \square \times \square$

$= \square$ cm^2



I: $\square \times \square = \square$ cm^2

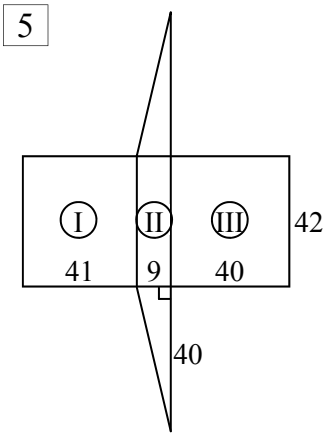
II: $\square \times \square = \square$ cm^2

III: $\square \times \square = \square$ cm^2

SA = $\square + \square + \square + \square$

SA = $\square$ cm^2

5



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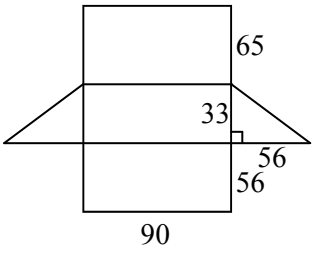

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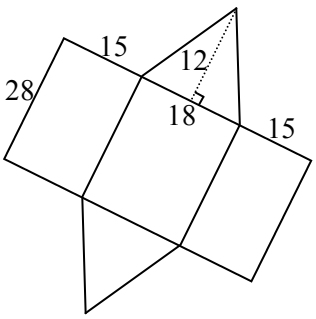

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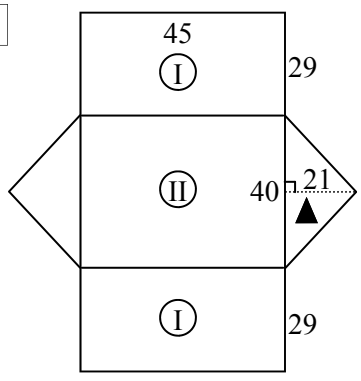

2▲:

2 I:

II:

SA =

Surface Area - Triangular Prisms Method Sheet (Sheet 1)

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Use the Δ, □, ◊ and ○ to help you to find the SA of these triangular prisms. All measurements are in cm.

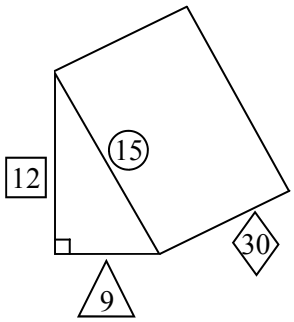
1 SA = 2×½× △ × □

\+ △ × ◊

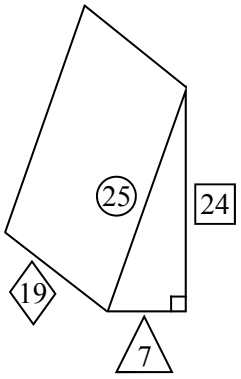
\+ □ × ◊

\+ ○ × ◊

SA = _____ cm^2



2



SA = 2×½× △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____ cm^2

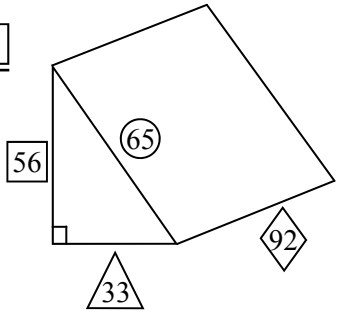
3 SA = 2×½× △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____



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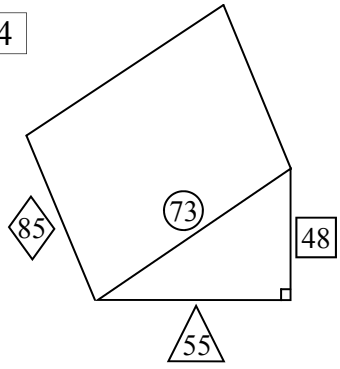


SA = × × △ × □

Write the '2' and the '½' above

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____

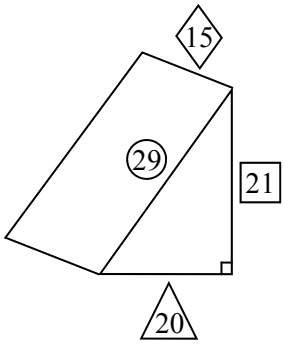
5 SA = × × △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____



6 SA = 2×½× △ × □

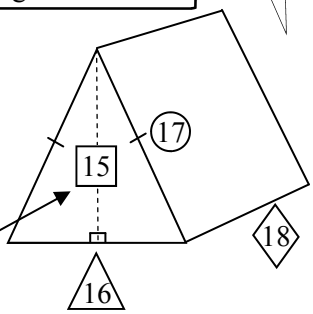
\+ △ × ◊

\+2 × ○ × ◊

SA = _____ cm^2

15 is the perpendicular height

These are isosceles triangles. Careful!



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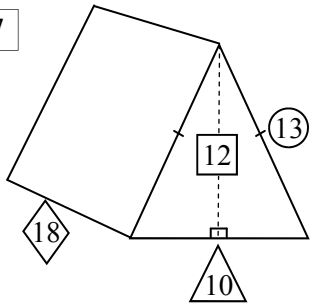


SA = × × △ × □

\+ △ × ◊

\+2 × ○ × ◊

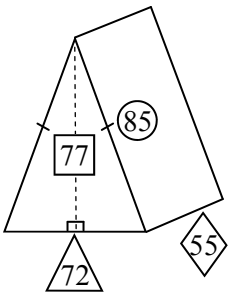
SA = _____

8 SA = × × △ × □

\+ △ × ◊

\+2 × ○ × ◊

SA = _____



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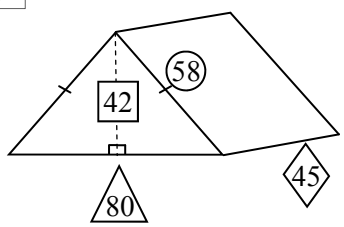


SA = × × △ × □

\+ △ × ◊

\+2 × ○ × ◊

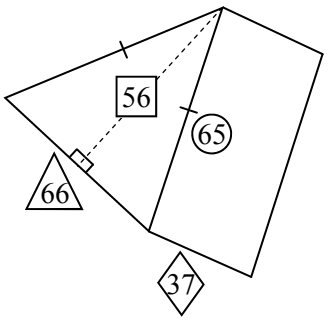
SA = _____

10 SA = × × △ × □

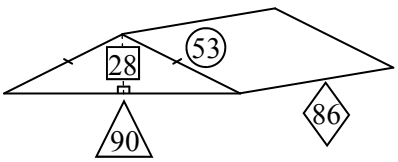
\+ △ × ◊

\+2 × ○ × ◊

SA = _____



11



SA = × × △ × □

\+ △ × ◊

\+2 × ○ × ◊

SA = _____

Surface Area - Triangular Prisms (Sheet 2)

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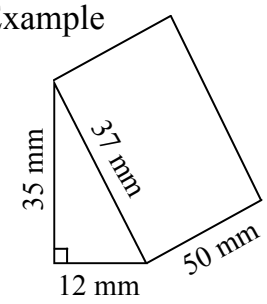
Find the surface area of these triangular prisms.

Example



$SA = 2 \times \frac{1}{2} \times 12 \times 35$

$+ 12 \times 50$

$+ 35 \times 50$

$+ 37 \times 50$

$SA = 4\,620 \text{ mm}^2$

1

base height

$SA = 2 \times \frac{1}{2} \times \square \times \square$

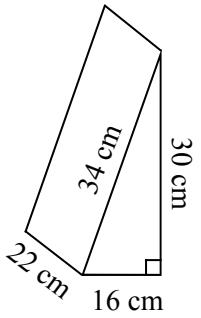
$+ \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

Front three edges, all × 22

$SA = \square \text{ cm}^2$



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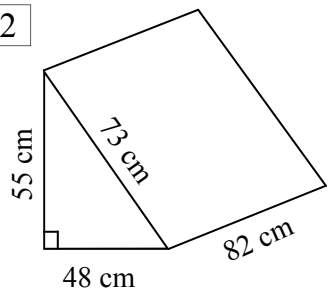


base height

$SA = 2 \times \frac{1}{2} \times \square \times \square$

$+ \square \times \square$

$+ \square \times \square$

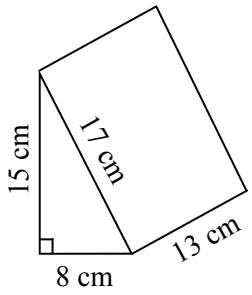
$+ \square \times \square$

Front 3 edges, all × 82

$SA = \square \text{ cm}^2$

3

Remember square units



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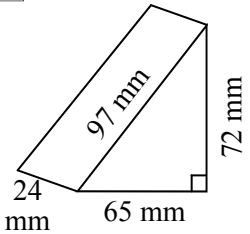

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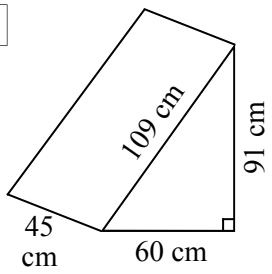

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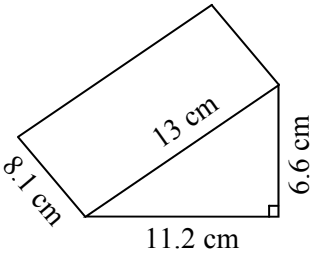
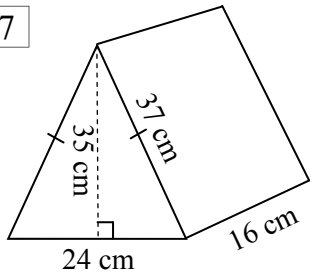


These have matching pairs of sides, so you can double the repeated side. Find the surface area.

7



base height

$SA = 2 \times \frac{1}{2} \times \square \times \square$

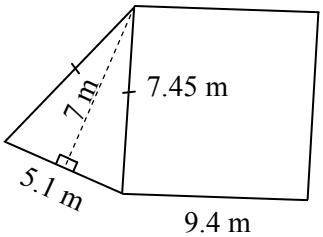
$+ \square \times \square$ base rectangle

$+ 2 \times \square \times \square$

2 × identical (slanted) rectangles

$SA = \square \text{ cm}^2$

8



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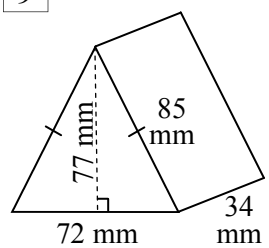

Surface Area - Triangular Prisms using Pythagoras Theorem (Sheet 1)

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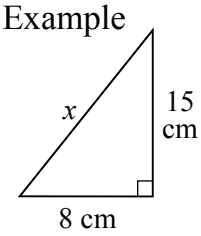
Use Pythagoras Theorem (only addition) for these. Find the length of the hypotenuse. All lengths are whole numbers.

Example



$x^2 = 8^2 + 15^2$

$x^2 = 289$

$x = \sqrt{289}$

$x = 17$ cm

1

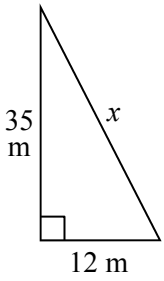
$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ m

Answer is a whole number



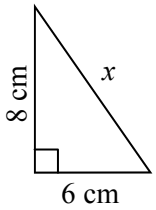
2

$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm



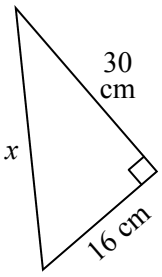
3

$x^2 =$

$x^2 =$

$x =$

$x =$



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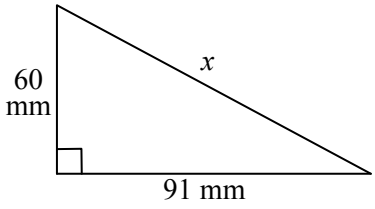
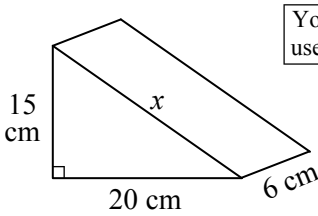


Use Pythagoras Theorem (only addition) to find the length '*x*'.

5

You don't use the '6'



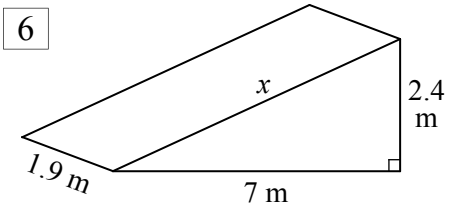
$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$

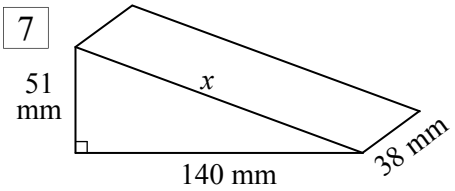
$x =$

$x =$

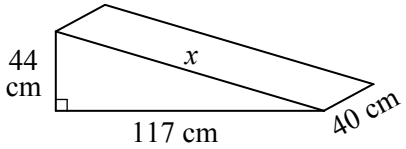
6



7

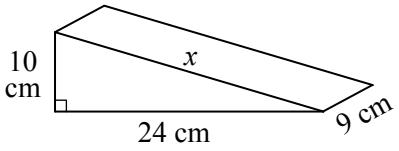


8



Find the length '*x*' first, then calculate the surface area

9



$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm

Now calculate the surface area

SA = 2 × ½ × ☐ × ☐

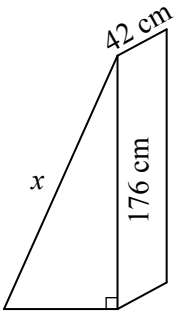
+ ☐ × 9

+ ☐ × 9

+ ☐ × 9

SA = ☐ cm^2

10



$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm

Now calculate the surface area

SA = 2 × ½ × ☐ × ☐

+ ☐ × ☐

+ ☐ × ☐

+ ☐ × ☐

SA = ☐ cm^2

Surface Area - Triangular Prisms using Pythagoras Theorem (Sheet 2)

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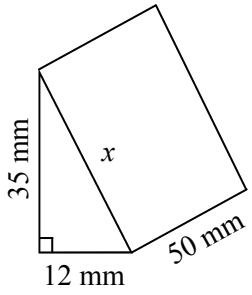
Find the SA of these triangular prisms. Pythagoras Theorem is required as a 1st step.

Example



$x^2 = 12^2 + 35^2$

$x^2 = 1\ 369$

$x = \sqrt{1\ 369}$

$x = 37$ mm

$SA = 2 \times ½ \times 12 \times 35$

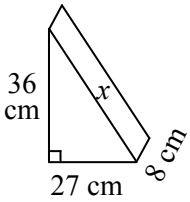
$+ 12 \times 50$

$+ 35 \times 50$

$+ 37 \times 50$

$SA = 4\ 620$ mm^2

1



$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm

$SA = 2 \times ½ \times \square \times \square$

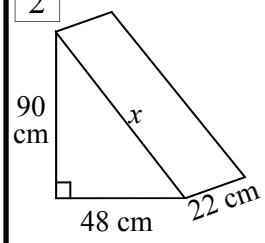
$+ \square \times \square$

$+ \square \times \square$

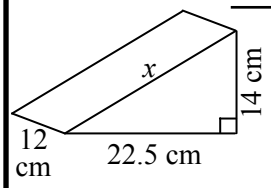
$+ \square \times \square$

$SA = \square$ cm^2

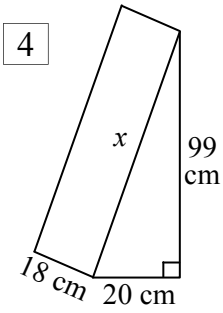
2



3

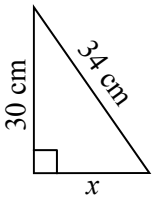


4



Use Pythagoras Theorem (subtraction). Find the length of the short side.

5



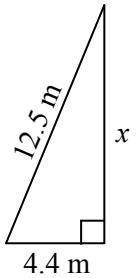
$x^2 = \square^2 - \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

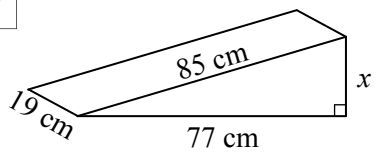
$x = \square$ cm

6



Find the SA of these triangular prisms.

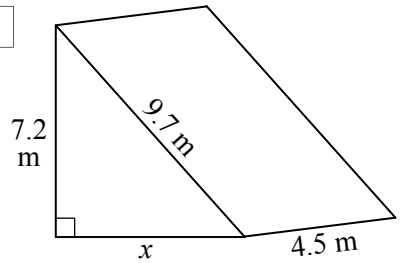
7



Calculate the triangle's height (P.T. minus)

Calculate the triangular prism's surface area

8



Calculate the triangle's base length

Calculate the triangular prism's surface area

Surface Area - Further Triangular Prisms & Pythagoras Theorem (Sheet 1)

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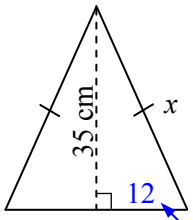
Halve the base first, then use P.T. to find the length 'x'. All lengths are whole numbers.

Example



$x^2 = 12^2 + 35^2$

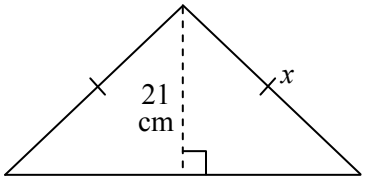
$x^2 = 1\,369$

$x = \sqrt{1\,369}$

$x = 37$ cm

24 ÷ 2

1



$x^2 =$ ☐^{2} + ☐^{2}

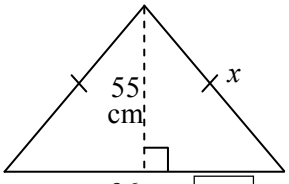
$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm

Answer is a whole number

2



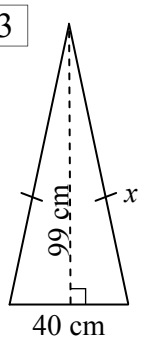
$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

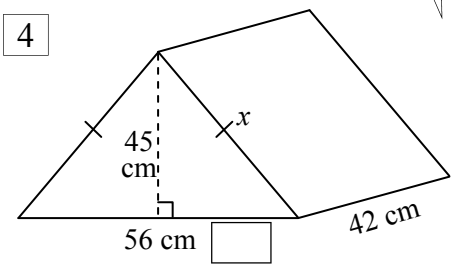
$x =$ ☐ cm

3



Halve the base, find 'x' using P.T., then the SA …. R U OK?

4



$x^2 =$ ☐^{2} + ☐^{2}

$x^2 =$ ☐

$x = \sqrt{}$ ☐

$x =$ ☐ cm

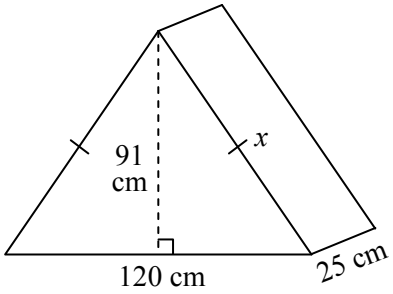
SA = 2 × ½ × 56 × 45

+ 2 × ☐ × 42 (☐ = x)

+ 56 × 42

SA = ☐ cm^2

5



$x^2 =$

$x^2 =$

$x =$

$x =$

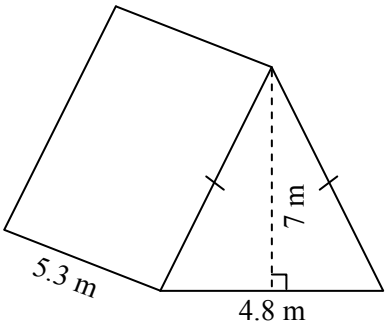
SA = 2 × ½ × ☐ × ☐

+ 2 × ☐ × ☐

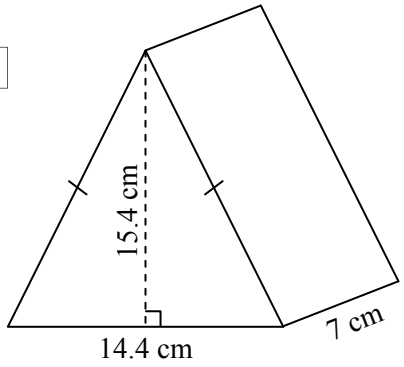
+ ☐ × ☐

SA = ☐ cm^2

6



7



Surface Area - Further Triangular Prisms & Pythagoras Theorem (Sheet 2)

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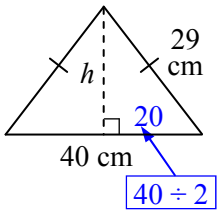
Halve the base, then use P.T. (subtraction) to find the height 'h'. Heights are whole numbers.

Example



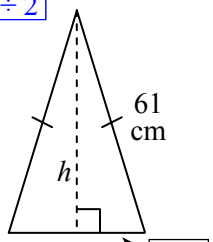
$h^2 = 29^2 - 20^2$

$h^2 = 441$

$h = \sqrt{441}$

$h = 21$ cm

1



$h^2 =$ ☐^{2} - ☐^{2}

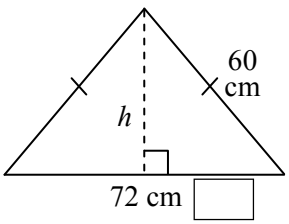
$h^2 =$ ☐

$h = \sqrt{}$☐

$h =$ ☐ cm

Answer is a whole number

2



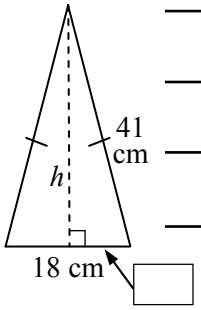
$h^2 =$ ☐^{2} - ☐^{2}

$h^2 =$ ☐

$h = \sqrt{}$☐

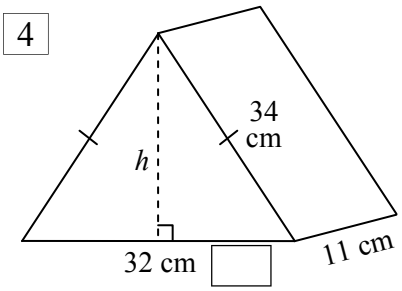
$h =$ ☐ cm

3



Halve the base, find 'h' using P.T., then calculate the SA.

4



$h^2 =$ ☐^{2} - ☐^{2}

$h^2 =$ ☐

$h = \sqrt{}$☐

$h =$ ☐ cm

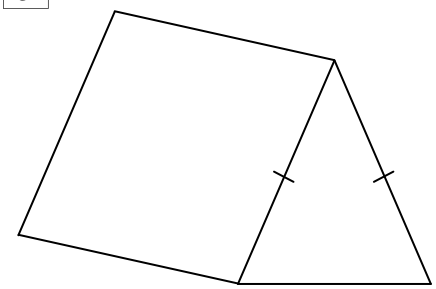
SA = 2 × ½ × 32 × ☐ (h)

+ 2 × 34 × 11

+ 32 × 11

SA = ☐ cm^2

5



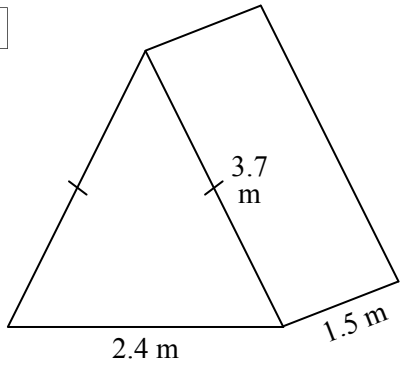
SA = 2 × ½ × ☐ × ☐

+ 2 × ☐ × ☐

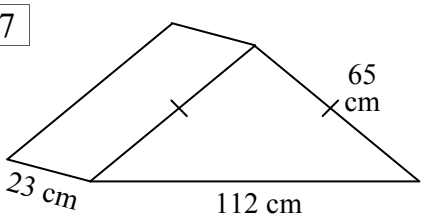
+ ☐ × ☐

SA = ☐ mm^2

6



7



Surface Area - Cylinders (Sheet 1)

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Use your calculator to answer these to 1 d.p.

1 $2\times\pi\times5^2+2\times\pi\times5\times9=$ ____

2 × SHIFT ×10^x × 5 x² + 2 × SHIFT ×10^x × 5 × 9 =

2 $2\times\pi\times7^2+2\times\pi\times7\times4=$ ____

3 $2\times\pi\times9^2+2\times\pi\times9\times20=$ ____

4 $2\times\pi\times12^2+2\times\pi\times12\times6=$ ____

5 $2\times\pi\times18^2+2\times\pi\times18\times12=$ ____

6 $2\times\pi\times4.6^2+2\times\pi\times4.6\times8=$ ____

7 $2\times\pi\times1.6^2+2\times\pi\times1.6\times3.8=$ ____

8 $2\times\pi\times33^2+2\times\pi\times33\times47=$ ____

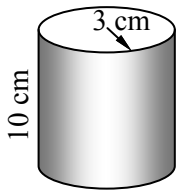
Find the SA of these cylinders. Round your answer to 1 decimal place. Formula, Substitution, Answer.

Example

$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 3^2 + 2 \times \pi \times 3 \times 10$

$SA = 245.0 \text{ cm}^2$ (1 d.p.)

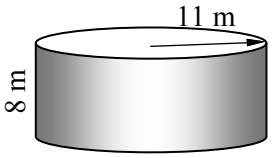


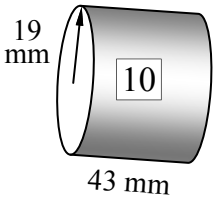
9 $SA = 2\pi r^2 + 2\pi rh$

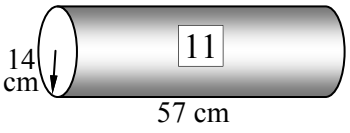
$= 2 \times \pi \times$ ☐ $^2 + 2 \times \pi \times$ ☐ $\times$ ☐

2 × SHIFT ×10^x × 1 1 x² + 2 × SHIFT ×10^x × 1 1 × 8 =

$SA =$ ☐ m^2







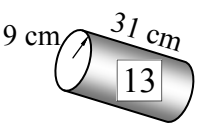
10 $SA = 2\pi r^2 + 2\pi rh$

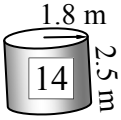
$= 2 \times \pi \times$ ☐ $^2 + 2 \times \pi \times$ ☐ $\times$ ☐

$SA =$ ☐ mm^2

11 $SA = 2\pi r^2 + 2\pi rh$



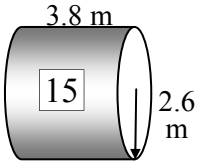


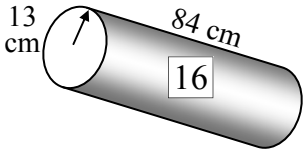


12 ____

13 ____

14 ____





15 ____

16 ____

A tin can of 'Otis' chocolate powder has a radius of 7 cm and a height of 16 cm. Calculate, to 1 decimal place:

17 The area of metal sheet used to fabricate 1 can

18 The weight of the can, if each cm^2 weighs 0.13 g.

Surface Area - Cylinders with Diameters (Sheet 2)

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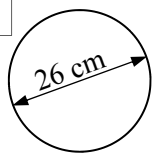
Divide these diameters by 2, to find the radii.
Ignore the height of the cylinders (Q4 and Q5).

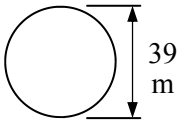
1



$r = 26 \div 2$

$r =$ ☐ cm

2

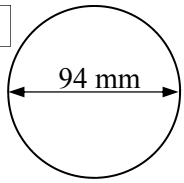


$r =$ ☐ ÷ ☐

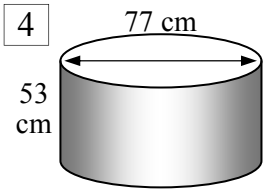
$r =$ ☐

Write the units

3



4

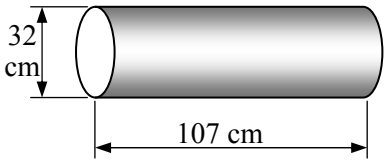


$r = 77 \div 2$

$r =$

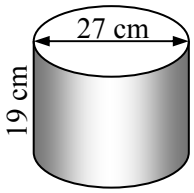
Write the units

5



Calculate the radius first, then find the SA of the cylinder, round your answer to 1 decimal place.

Example



$r = 13.5$ cm

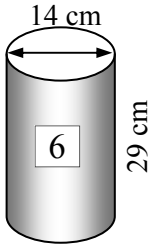
$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 13.5^2 + 2 \times \pi \times 13.5 \times 19$

$SA = 2\,756.7$ cm^2 (1 d.p.)

This is surface area so show square units

6



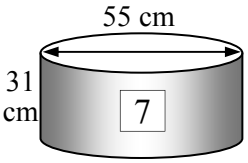
$r =$ ☐ cm

$SA = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times$ ☐ $^2 + 2 \times \pi \times$ ☐ $\times$ ☐

$SA =$ ☐ cm^2

7



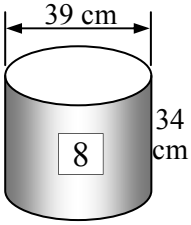
$r =$ ☐

$SA = 2\pi r^2 + 2\pi rh$

$=$

$SA =$ ☐ cm^2

8



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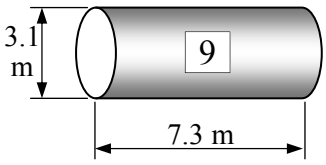

Don't round the radius

Find the SA of the water tank to 1 decimal place.

'WeSaveU' cylindrical water tanks are 3.6 m high and 4.3 m in diameter.

10 What is the tank's radius?

Don't round the radius

11 Calculate the surface area of the tank.

12 If powder coating costs \$6.78/m^2, estimate the cost to coat the tank, answer to the nearest \$10.

13 If Teresa pays cash she gets a \$150 discount. The tank (uncoated) costs \$2 990, calculate the price paid with powder coating added and a discount for cash applied.

14 If the sheet metal that is used to build the tank weighs 0.8 kg/m^2. Calculate the weight of the empty tank.

Surface Area - Find a Word

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Find the words in the puzzle from the wordlist.



R	X	C	A	Q	E	B	S	S	U	R	F	A	C	E
E	E	L	C	R	I	C	C	U	E	R	A	U	Q	S
C	B	B	Q	U	A	R	E	T	I	B	E	U	U	A
T	U	E	R	O	B	L	I	X	R	D	A	A	A	B
A	C	Q	E	S	U	S	U	Q	T	C	A	E	R	A
N	G	E	T	U	O	T	O	G	Y	E	X	R	X	U
G	N	X	E	P	P	X	S	L	N	X	R	N	E	Q
U	I	P	M	X	E	A	I	I	I	A	E	N	E	U
L	H	O	A	X	X	N	I	E	D	D	I	K	A	T
A	C	S	I	E	D	C	U	R	V	E	D	R	N	L
R	T	E	D	E	T	X	N	T	A	M	P	A	T	M
X	A	D	R	T	H	G	I	E	H	U	L	T	A	S
E	M	Q	L	E	N	G	T	H	X	S	Q	T	A	I
D	I	M	A	R	Y	P	S	E	L	L	A	W	X	R
U	Q	R	A	L	U	C	I	D	N	E	P	R	E	P

WORDLIST

COMPOSITE	EXTERNAL	CIRCLE	NET	TRIANGULAR
WALL	HEIGHT	LENGTH	EXPOSED	SURFACE
SQUARE	RADIUS	RECTANGULAR	SLANT	SUM
CURVED	MATCHING	AREA	BASE	PERPENDICULAR
PYRAMID	CUBE	DIAMETER	UNITS	CYLINDER
	OUTSIDE	SOLID	PRISM	

Surface Area - Topic Review

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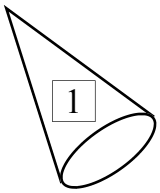
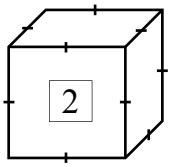
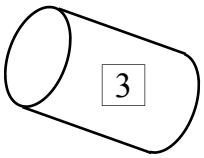
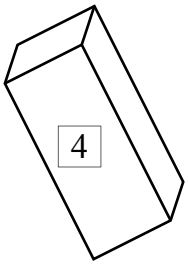
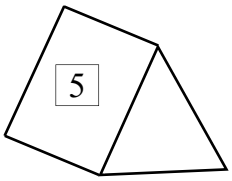
Name these solids.

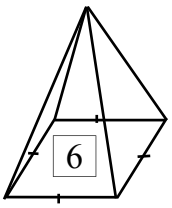
1 

2 

3 

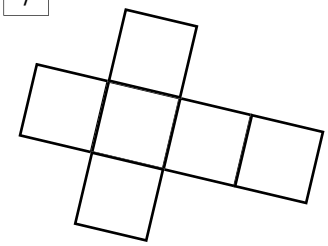
4 

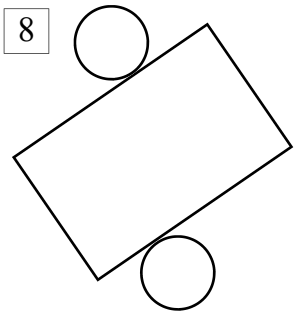
5 

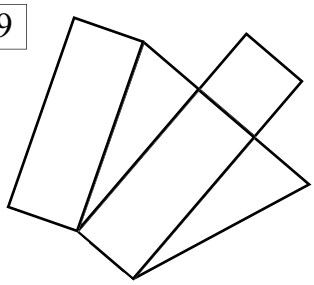
6 

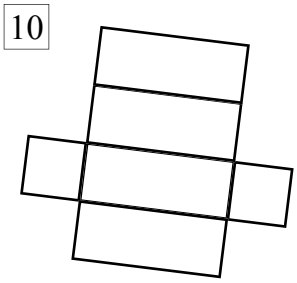
Two Word Description

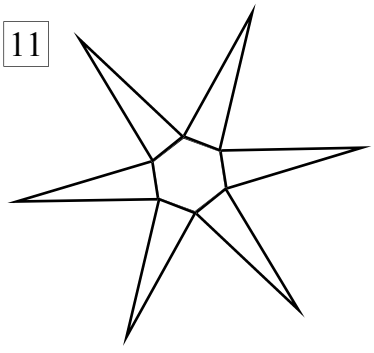
Name the solids shown below as nets.

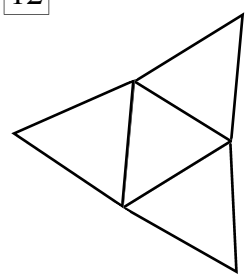
7 

8 

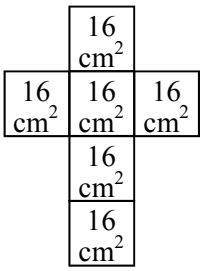
9 

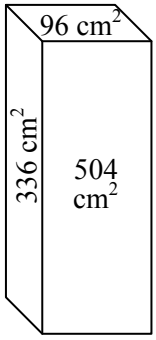
10 

11 

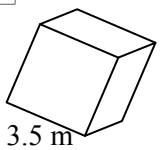
12 

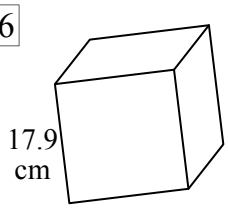
Add the face areas to calculate the total surface area

13 

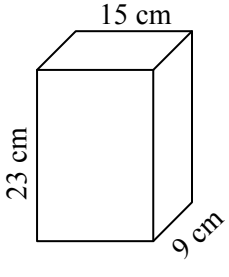
14 

Use SA = $6l^2$ to calculate the surface area of these cubes. Answer to 1 decimal place.

15 

16 

Calculate the surface area of the rectangular prism.

17 

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A rectangular room is 2.4 m × 3.7 m × 4.1 m in size.
Calculate the surface area of the entire room.
Answer to 1 decimal place.

17

A rectangular cereal box has sides 23 cm, 35 cm & 6 cm.
Calculate the total surface area of the cardboard box.

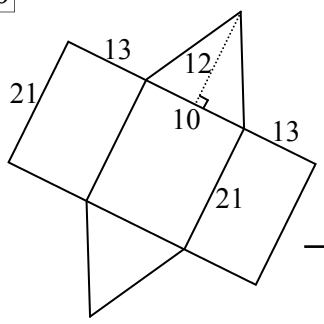
18

An advertisement on the back of the cereal box (above) occupies an area of 358 cm^2. What percentage of the surface area is taken by the advertisement.
Answer to the nearest whole percent.

19

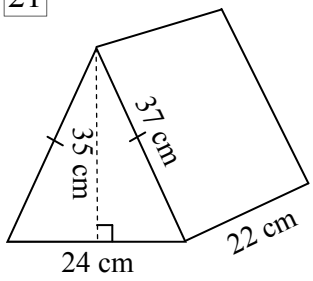
Calculate the total surface area of the triangular prism shown below as a net. All measurements are in cm.

20

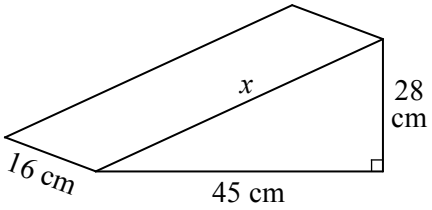


Calculate the surface area of these two triangular prisms.

21

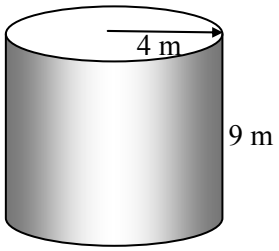


22



Calculate the surface area of these two cylinders. Use:
$SA = 2\pi r^2 + 2\pi rh$
Round your answer to 2 decimal places.

23



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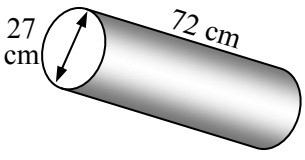
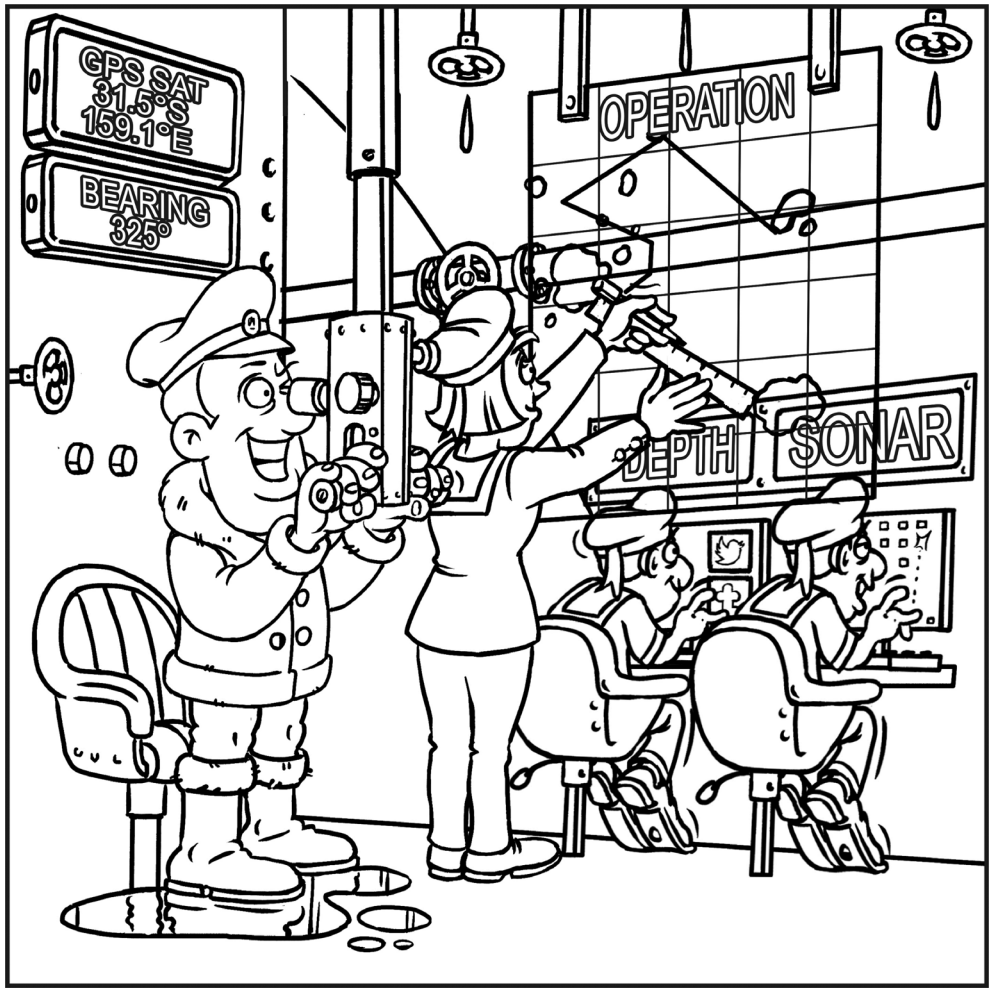


VELOCITY

TRIGONOMETRY

Trigonometry



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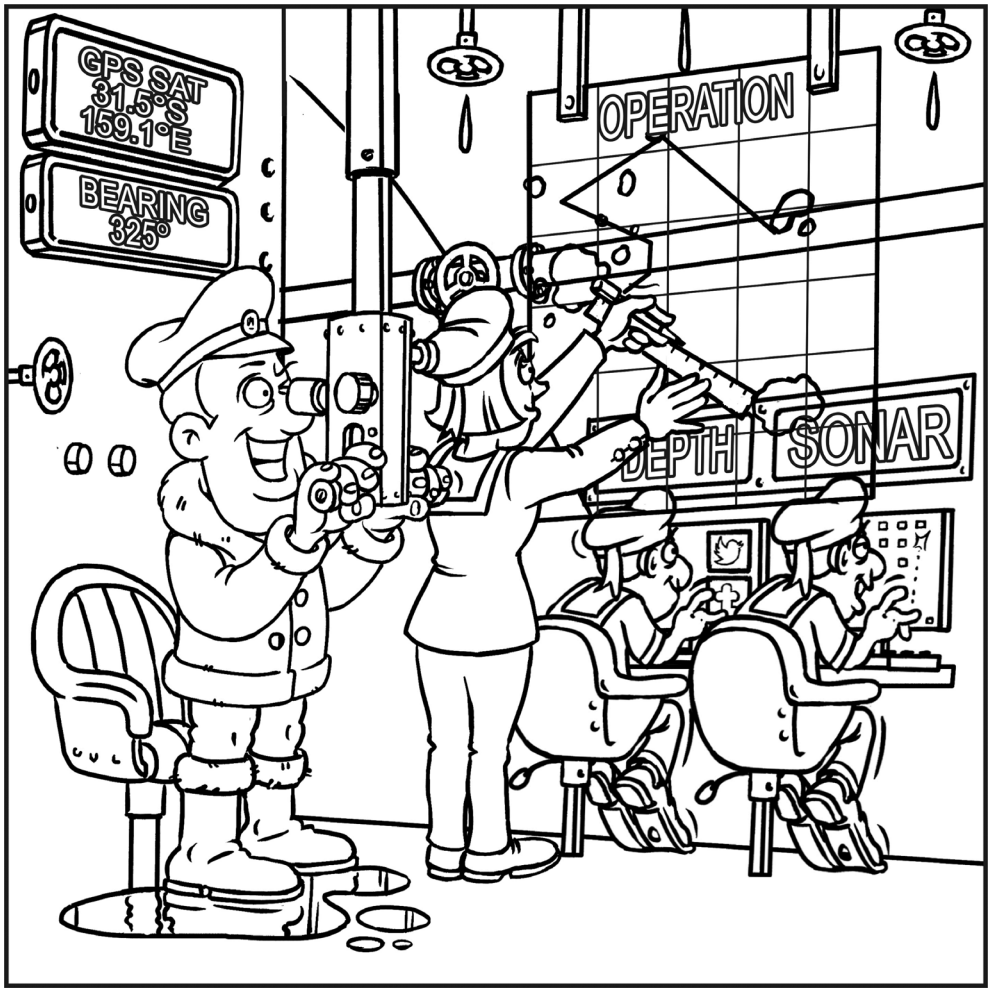
Trigonometry



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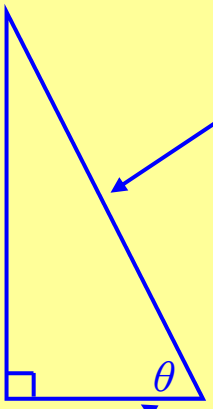
Trigonometry - Naming the Sides

Trigonometry begins with first getting the basics right. The most essential skill to start using trigonometry to solve questions is naming the three sides of a triangle. If you get this wrong the entire process collapses and so it is essential that you are able to name the three sides. The good news is that it is a straight-forward process.

A triangle consists of three sides, they are:



THE HYPOTENUSE (H)

The hypotenuse should be the first side you find. It is the longest side of the triangle so is easy to pick, it also doesn't depend on the position of the angle θ. Note that the hypotenuse is always opposite the right angle.

THE OPPOSITE SIDE (O)

The opposite side should be the last side you find. As you have already found H and A. Note that the opposite side doesn't touch the angle.

THE ADJACENT SIDE (A)

The adjacent side should be the second side you find. The angle θ is always held by the hypotenuse and the adjacent side. So it and the hypotenuse 'touch' or create the angle.

The sheet starts with naming three sides of a triangle. Just like above, just start with H, then A, then O. The second and third column require you to name two sides only. When you use trigonometry to calculate angles in triangles in later exercises you will require two sides only. There is less chance of confusion if you just label two sides as it makes the choice of trig function less confusing.

The second sheet is a variation of the first sheet, it also asks you to measure lengths.

Trigonometry - Naming the Sides (Sheet 1)

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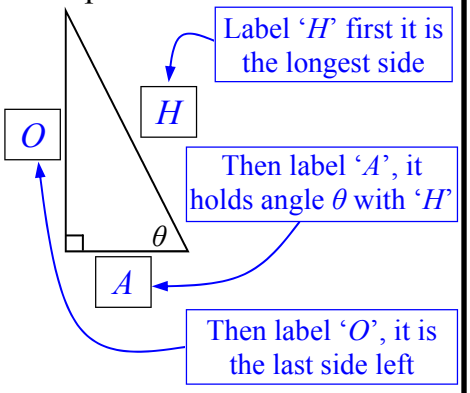
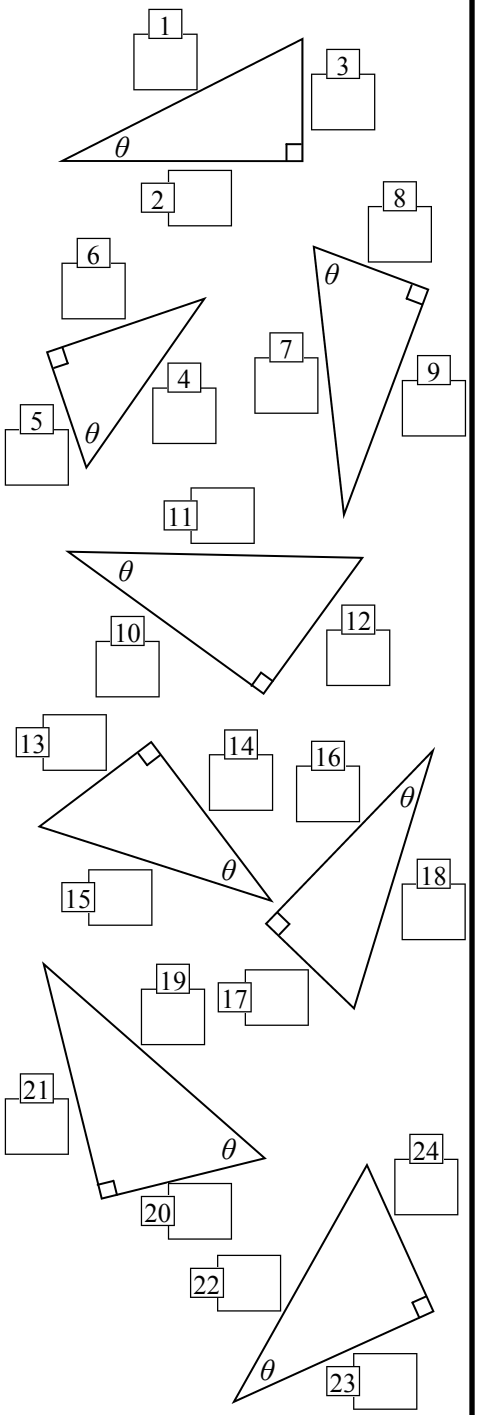
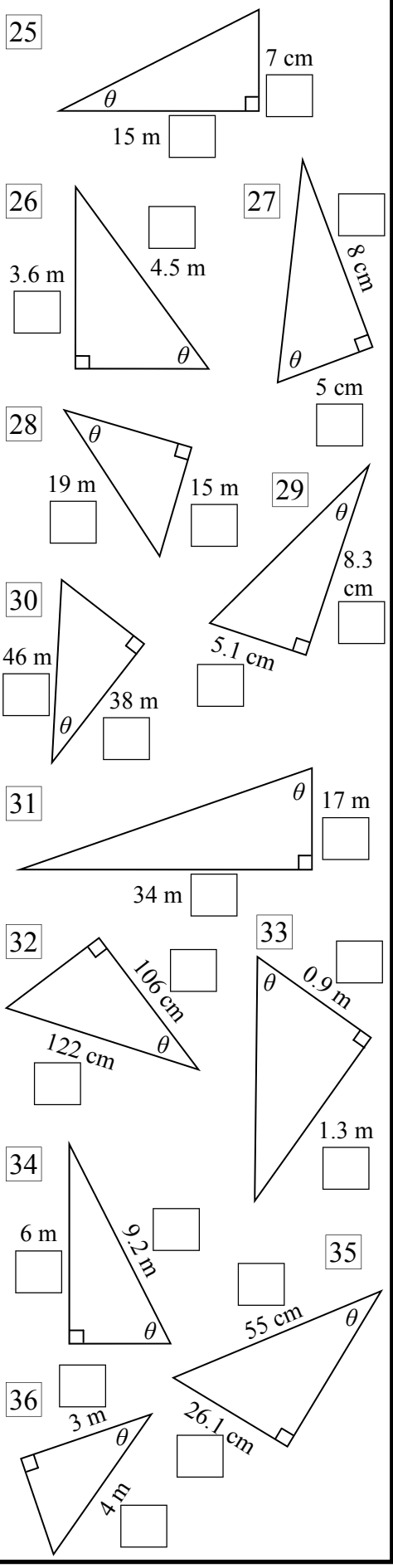
Label the sides as '*O*', '*A*' and '*H*'. Start with '*H*', the angle is always held by '*A*' and '*H*'.

Example:

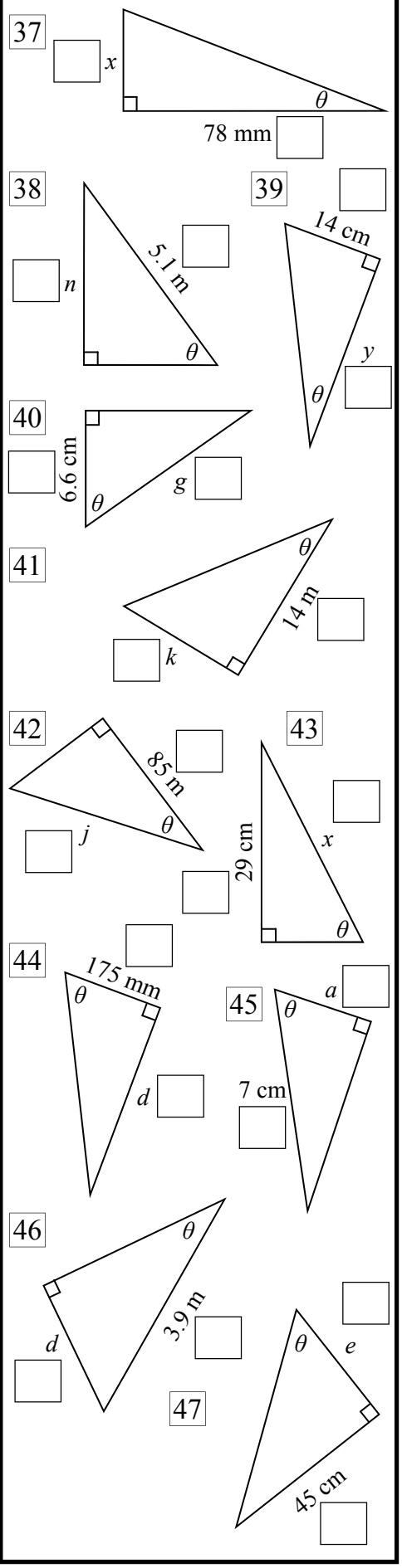




Only label two sides now. If a side has no number, don't label it. Use either '*O*', '*A*' or '*H*'.



These are the same except there is a number and a term. Label two sides '*O*', '*A*' or '*H*'.



Trigonometry - Naming the Sides (Sheet 2)

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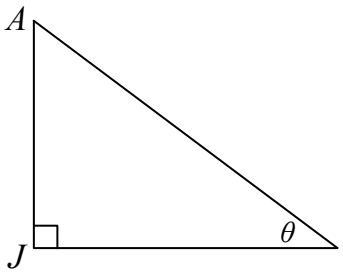
Use a ruler to measure the lengths of each side and write the name of the side and its length (whole cm) in the box.

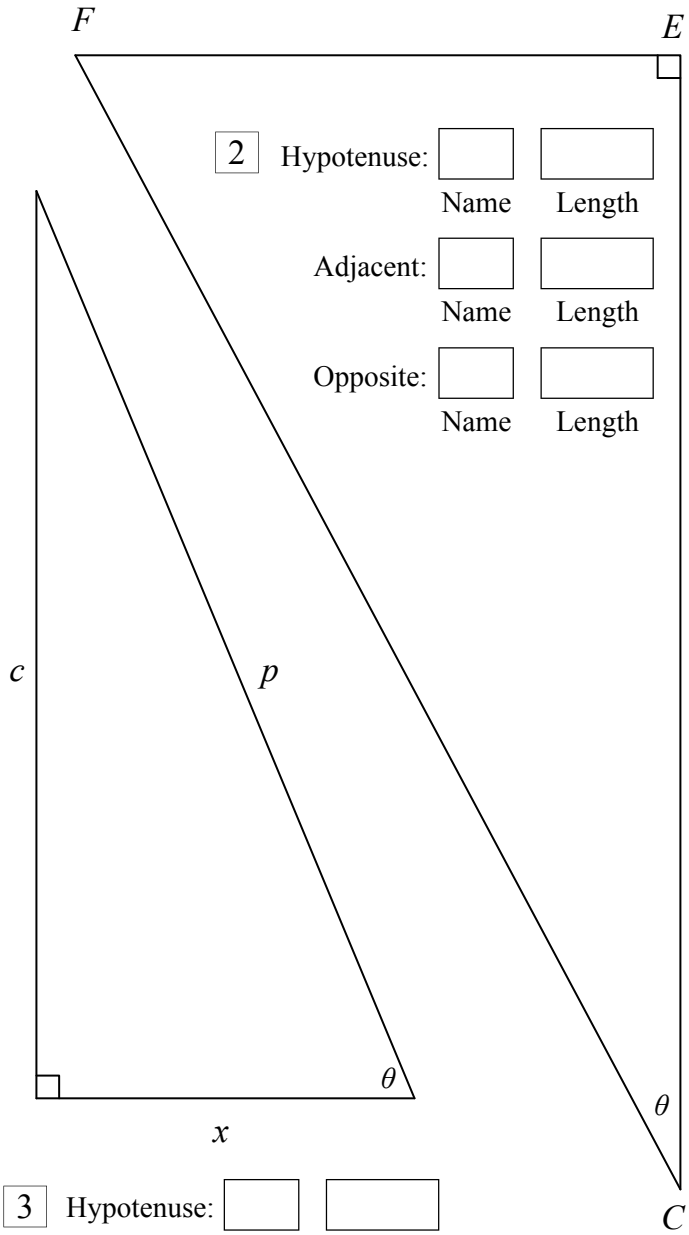
1



Hypotenuse: Name ____ Length ____ *cm*

Adjacent: Name ____ Length ____ *cm*

Opposite: Name ____ Length ____ *cm*



2 Hypotenuse: Name ____ Length ____

Adjacent: Name ____ Length ____

Opposite: Name ____ Length ____

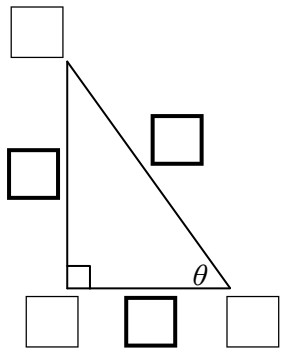
3 Hypotenuse: Name ____ Length ____

Adjacent: Name ____ Length ____

Opposite: Name ____ Length ____

Label the sides '*O*', '*A*' and '*H*' then add the letters to the vertices (corners) of the triangles. Use a pencil in case you need to change letters.

4

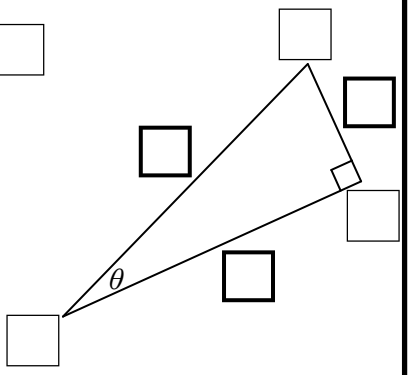


Hypotenuse: *EC*

Adjacent: *QC*

Opposite: ____ Name

5

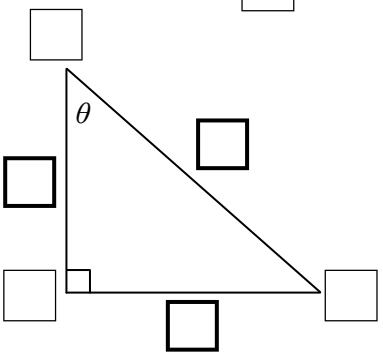


Hypotenuse: *TJ*

Adjacent: *TG*

Opposite: ____

6

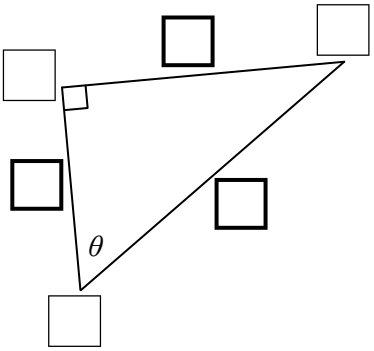


Hypotenuse: *TG*

Opposite: *CT*

Adjacent: ____

7

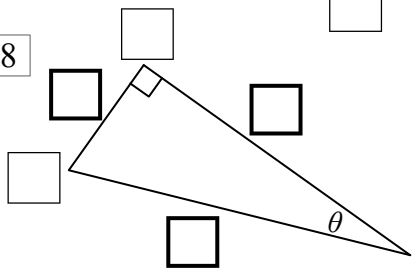


Opposite: *YR*

Adjacent: *YK*

Hypotenuse: ____

8

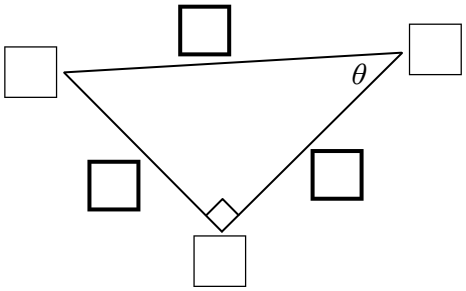


Adjacent: *WB*

Opposite: *BC*

Hypotenuse: ____

9



Hypotenuse: *EB*

Opposite: *XE*

Adjacent: ____

Trigonometry - Naming the Sides (Sheet 1)

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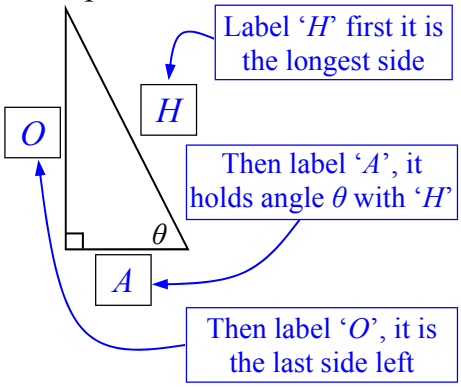
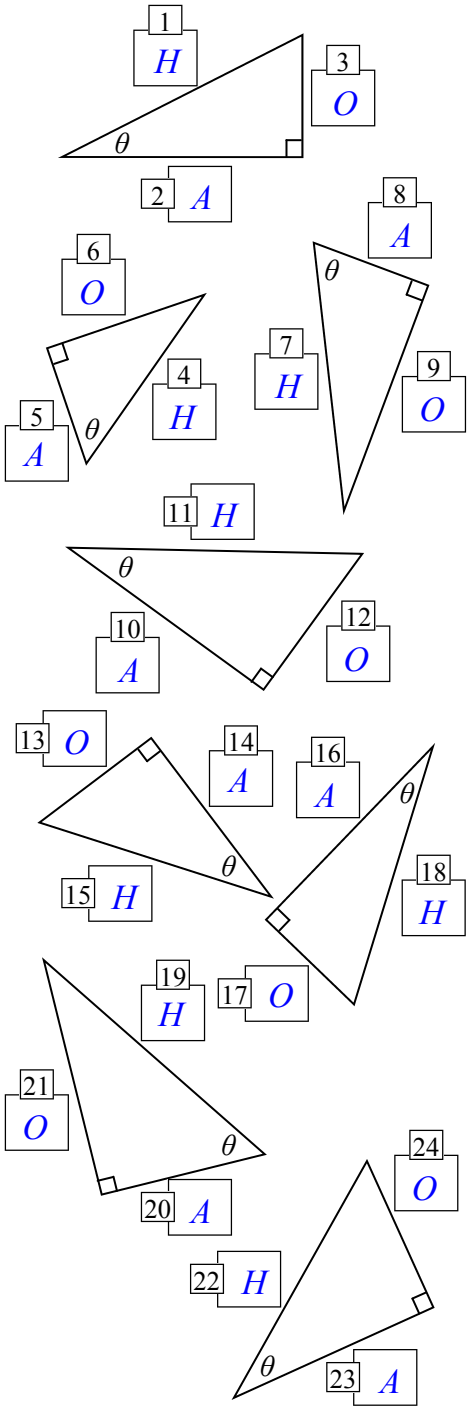
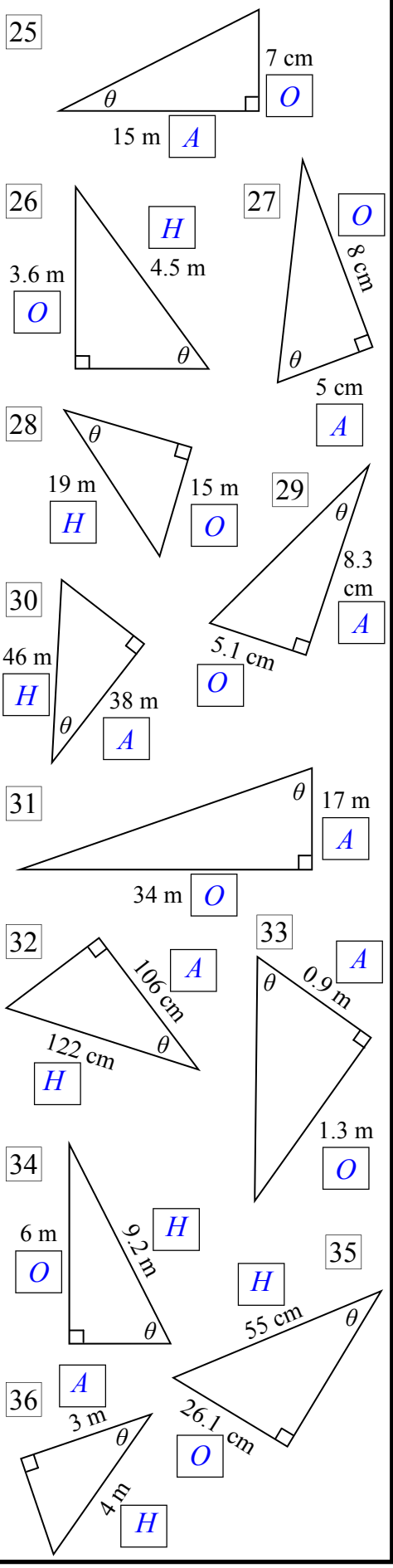
Label the sides as '*O*', '*A*' and '*H*'. Start with '*H*', the angle is always held by '*A*' and '*H*'.

Example:

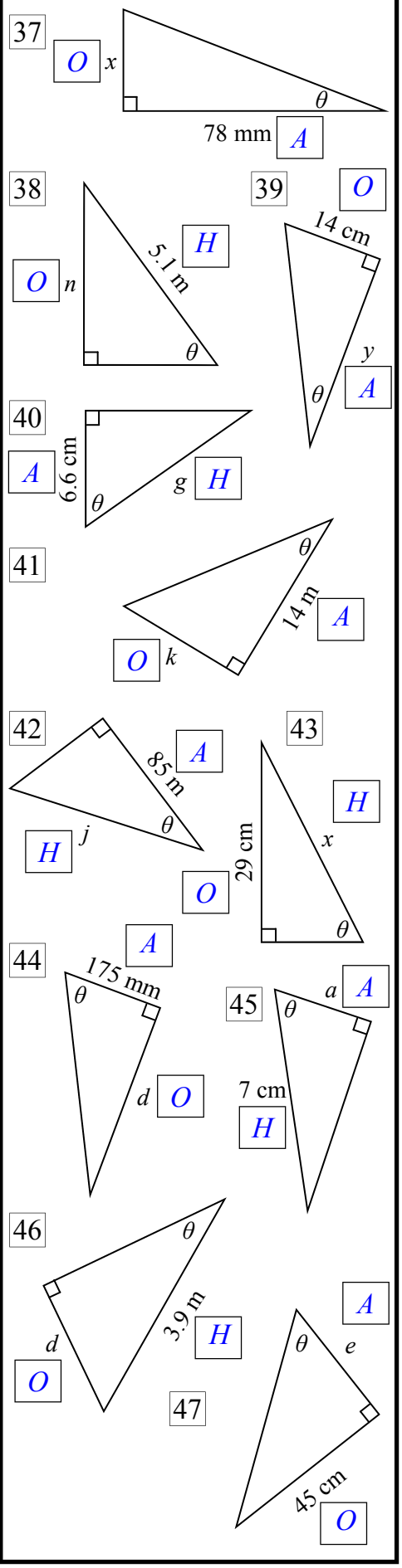




Only label two sides now. If a side has no number, don't label it. Use either '*O*', '*A*' or '*H*'.



These are the same except there is a number and a term. Label two sides '*O*', '*A*' or '*H*'.



Trigonometry - Naming the Sides (Sheet 2)

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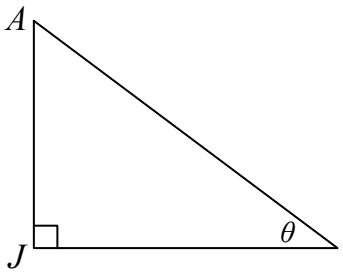
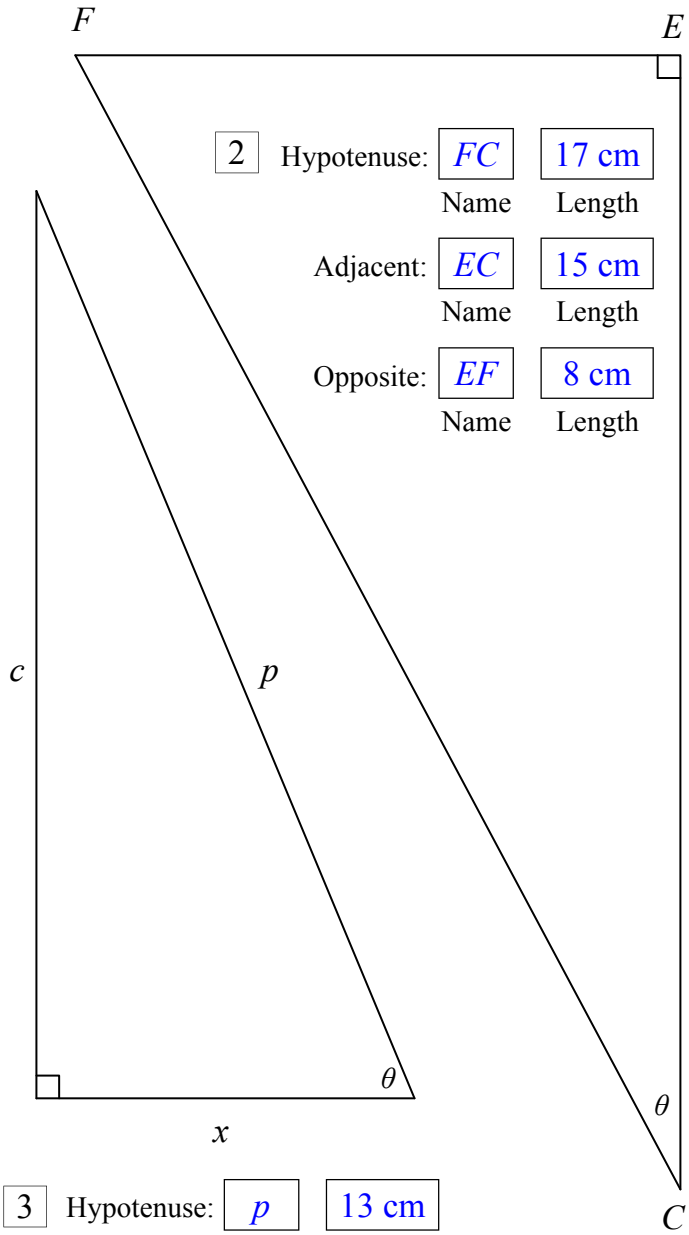
Use a ruler to measure the lengths of each side and write the name of the side and its length (whole cm) in the box.

1



	Name	Length
Hypotenuse:	*AX*	5 cm
Adjacent:	*JX*	4 cm
Opposite:	*AJ*	3 cm

2



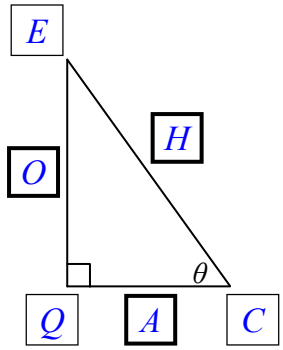
	Name	Length
Hypotenuse:	*FC*	17 cm
Adjacent:	*EC*	15 cm
Opposite:	*EF*	8 cm

3

	Name	Length
Hypotenuse:	*p*	13 cm
Adjacent:	*x*	5 cm
Opposite:	*c*	12 cm

Label the sides '*O*', '*A*' and '*H*' then add the letters to the vertices (corners) of the triangles. Use a pencil in case you need to change letters.

4

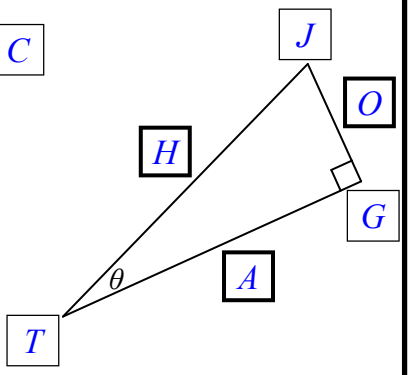


Hypotenuse: *EC*

Adjacent: *QC*

Opposite: *EQ* (Name)

5

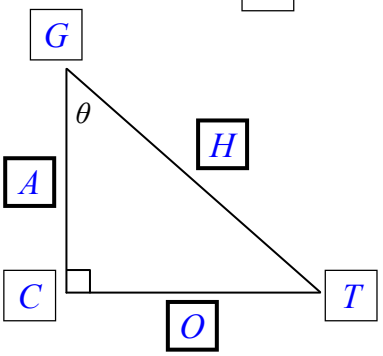


Hypotenuse: *TJ*

Adjacent: *TG*

Opposite: *JG*

6

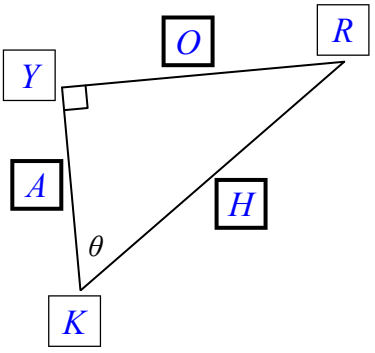


Hypotenuse: *TG*

Opposite: *CT*

Adjacent: *GC*

7

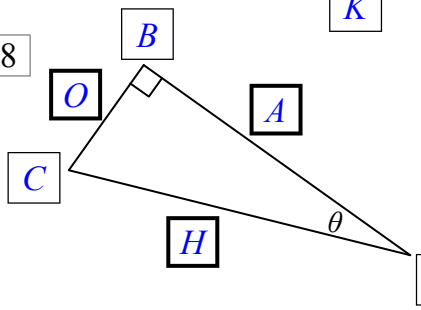


Opposite: *YR*

Adjacent: *YK*

Hypotenuse: *KR*

8

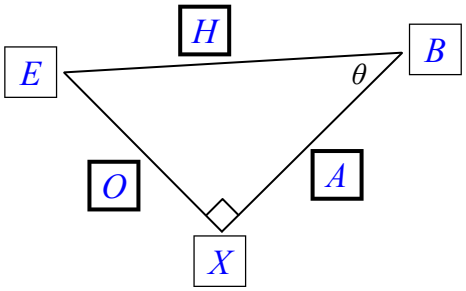


Adjacent: *WB*

Opposite: *BC*

Hypotenuse: *CW*

9



Hypotenuse: *EB*

Opposite: *XE*

Adjacent: *XB*

Trigonometry - Naming the Sides (Homework Strip)

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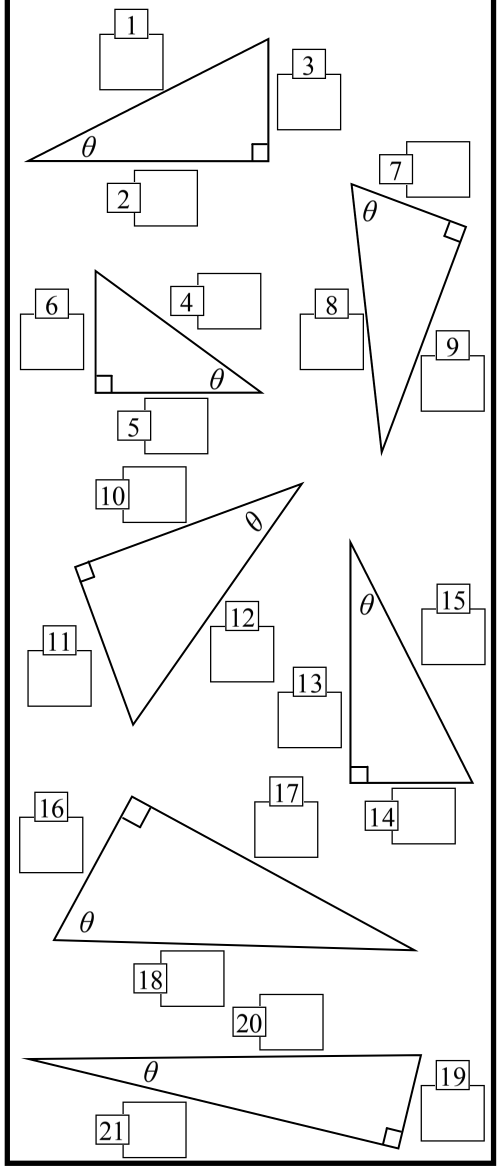
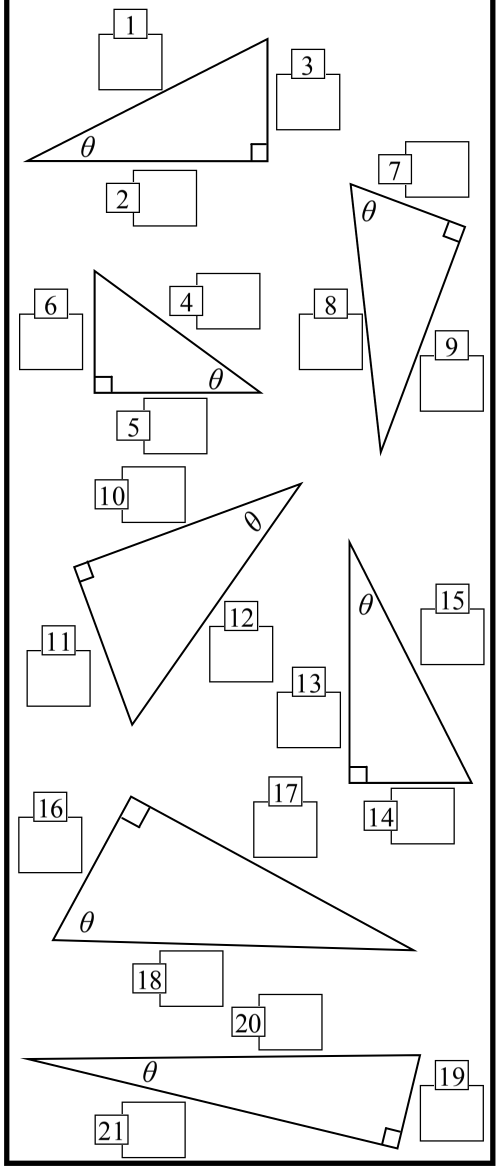
Date Due:

Name the Hypotenuse, Adjacent and Opposite sides of these triangles.



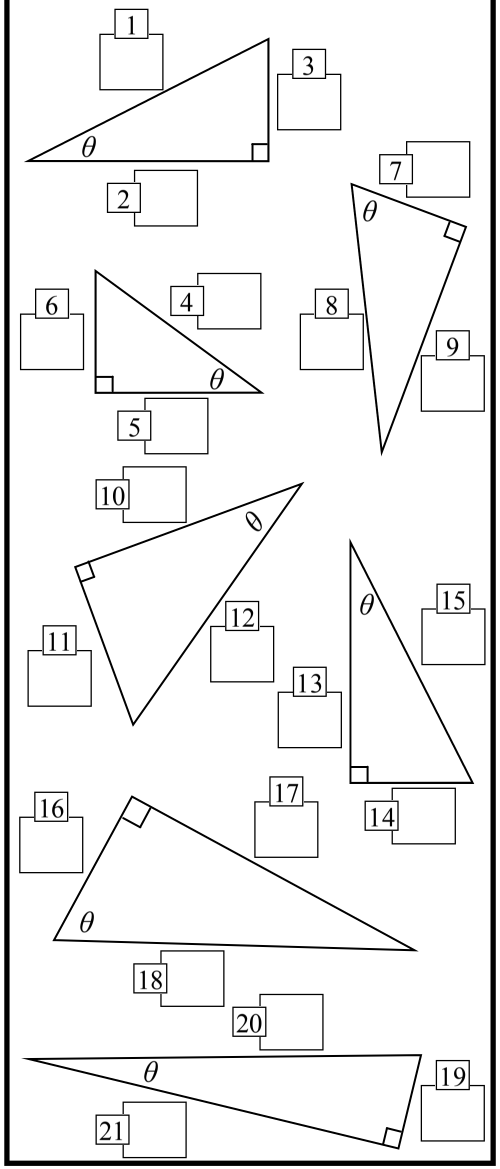
Date Due:

Name the Hypotenuse, Adjacent and Opposite sides of these triangles.



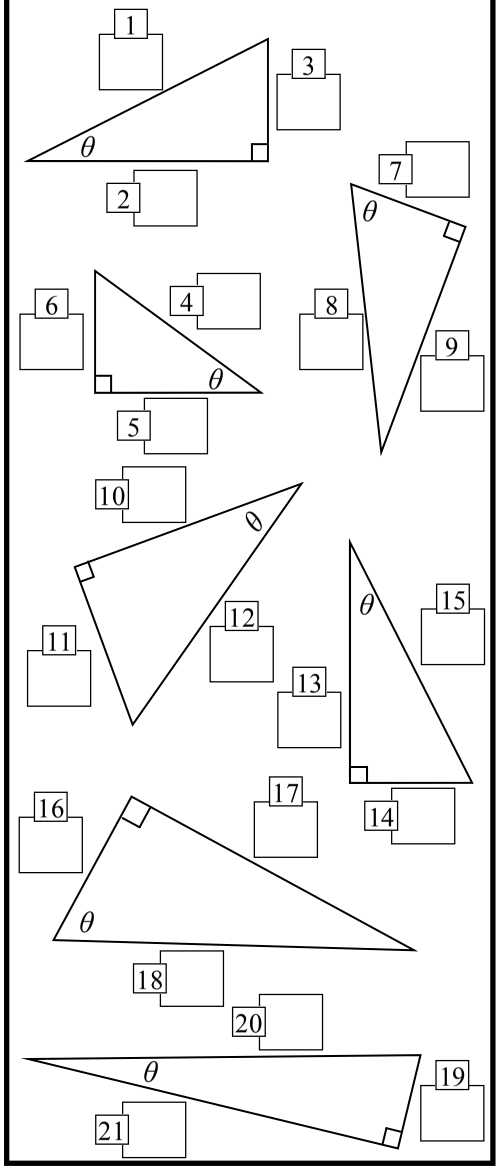
Date Due:

Name the Hypotenuse, Adjacent and Opposite sides of these triangles.



Date Due:

Name the Hypotenuse, Adjacent and Opposite sides of these triangles.



Trigonometry - Naming the Sides (Homework Strip)

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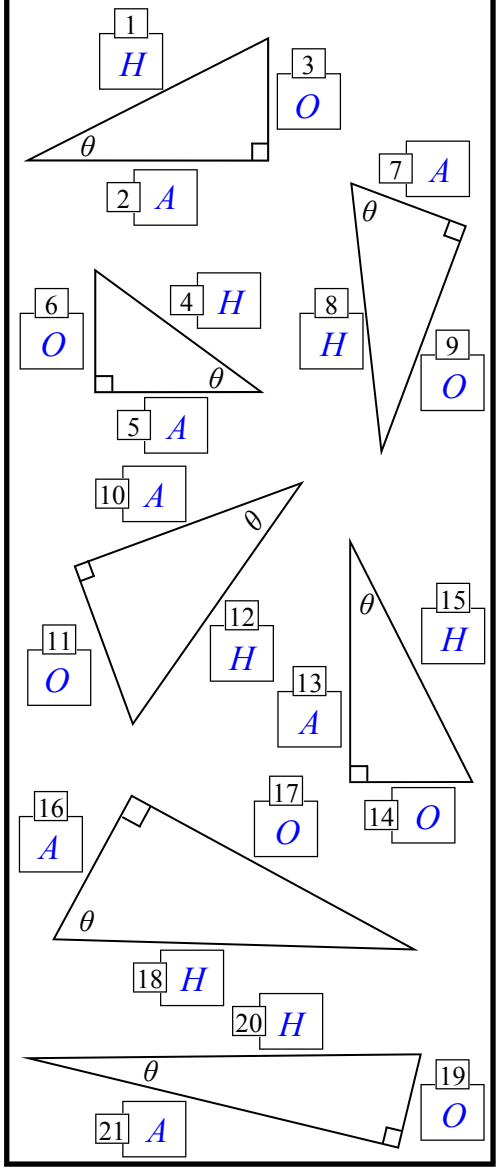
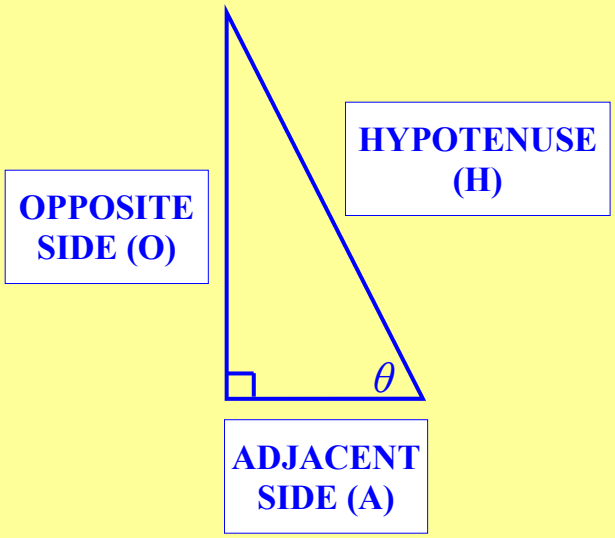
Date Due:

Name the Hypotenuse, Adjacent and Opposite sides of these triangles.



Trigonometry - Trigonometric Ratios

Trigonometric ratios involves creating fractions using the three sides of a triangle. The fractions are created using this rule:

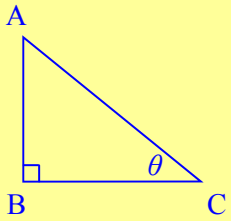


SOH CAH TOA

$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$

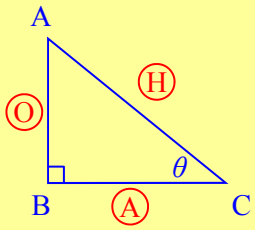
Like normal fractions these can be numbers. But unlike fractions, these can also involve letters, including pairs of letters. Here are some examples.

Example: Find the trigonometric ratios for the triangle shown.



STEP 1

LABEL THE SIDES

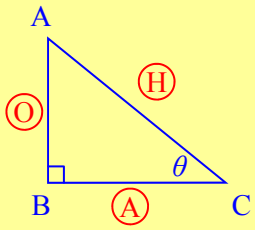


LABEL THE SIDES

Always label the sides O, A, and H. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with letters on the vertices of the triangle.

STEP 2

FIND THE RATIOS

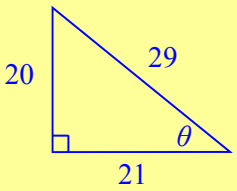


FIND THE TRIG RATIOS

Use SOH CAH TOA to remember the ratios, and construct the ratios. Use two letters to name each side, answers can vary because letters can be backwards. For example the hypotenuse is both 'AC' and 'CA'.

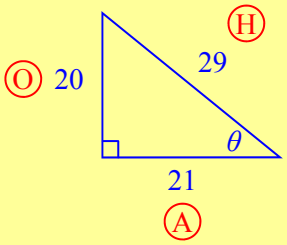
$\sin\theta = \frac{O}{H} = \frac{AB}{AC}$ $\cos\theta = \frac{A}{H} = \frac{BC}{AC}$ $\tan\theta = \frac{O}{A} = \frac{AB}{BC}$

Example: Find the trigonometric ratios for the triangle shown.



STEP 1

LABEL THE SIDES

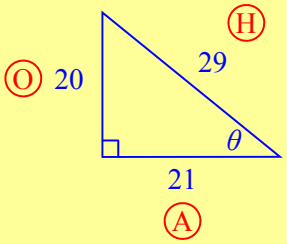


LABEL THE SIDES

Always label the sides O, A, and H.

STEP 2

FIND THE RATIOS

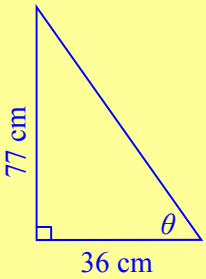


FIND THE TRIG RATIOS

Use SOH CAH TOA to remember the ratios, and construct the ratios. Use the numbers on each side,

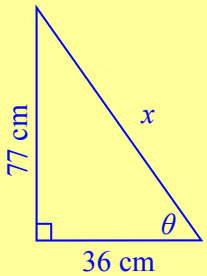
$$\sin\theta = \frac{O}{H} = \frac{20}{29} \qquad \cos\theta = \frac{A}{H} = \frac{21}{29} \qquad \tan\theta = \frac{O}{A} = \frac{20}{21}$$

Example: Find the trigonometric ratios for the triangle shown.



STEP 1

USE PYTHAGORAS



$$x^2 = 36^2 + 77^2$$

$$x^2 = 7\,225$$

$$x = \sqrt{7\,225}$$

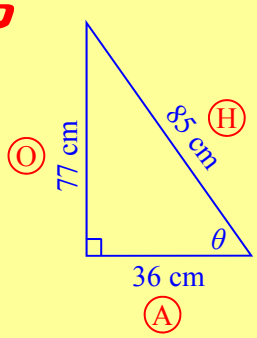
$$x = 85 \text{ cm}$$

USE PYTHAGORAS THEOREM

You need three sides to find all three trig ratios. So before you start the trig, you first need to find the missing side length. Because the hypotenuse is the unknown side, use P.T. addition.

STEP 2

LABEL THE SIDES



STEP 3

FIND THE RATIOS

$$\sin\theta = \frac{O}{H} = \frac{77}{85}$$

$$\cos\theta = \frac{A}{H} = \frac{36}{85}$$

$$\tan\theta = \frac{O}{A} = \frac{77}{36}$$

Trigonometry - Trigonometric Ratios (Sheet 1)

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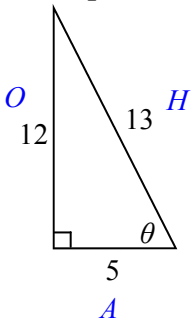
Label the sides as '*O*', '*A*' and '*H*'. Then find sin θ, cos θ and tan θ. Use *SOH CAH TOA!*

Example:

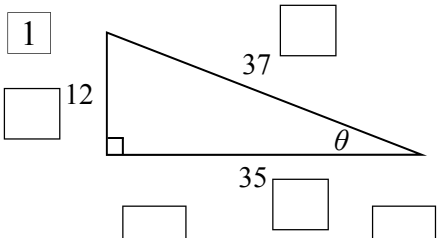


$$\sin\theta = \frac{O}{H} = \frac{12}{13}$$

$$\cos\theta = \frac{A}{H} = \frac{5}{13}$$

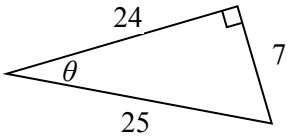
$$\tan\theta = \frac{O}{A} = \frac{12}{5}$$

1



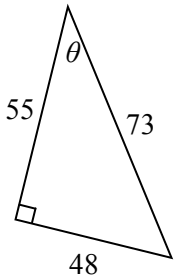
$\sin\theta =$ ☐/☐ $\quad \cos\theta =$ ☐/☐ $\quad \tan\theta =$ ☐/☐

2



$\sin\theta =$ ☐/☐ $\quad \cos\theta =$ ☐/☐ $\quad \tan\theta =$ ☐/☐

3

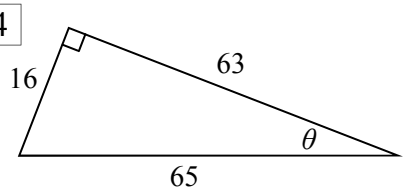


$\tan\theta =$ ☐/☐ $\quad \sin\theta =$ ☐/☐ $\quad \cos\theta =$ ☐/☐

Note: The order of the functions has changed!

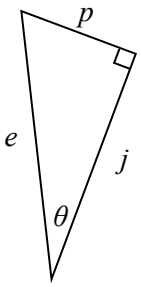
SOH CAH TOA

4



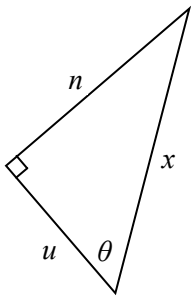
These involve algebra... same process

5



$\cos\theta =$ ☐/☐ $\quad \tan\theta =$ ☐/☐ $\quad \sin\theta =$ ☐/☐

6



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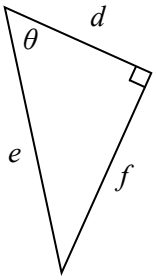
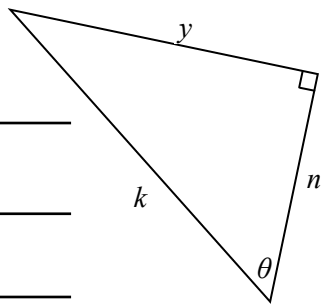
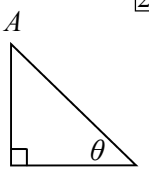


8



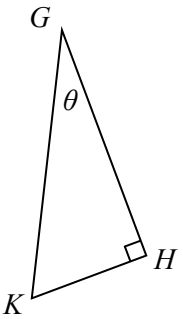
When letters are on the vertices (corners) of the triangle, use two letters to name the sides.

9

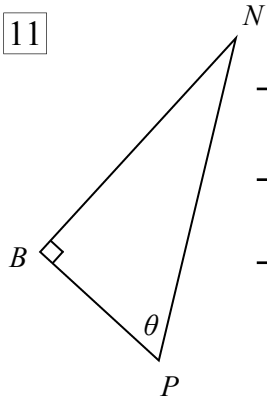


2 letters → ☐ $\quad \cos\theta =$ ☐/☐ 2 letters → ☐ $\quad \tan\theta =$ ☐/☐ $\quad \sin\theta =$ ☐/☐

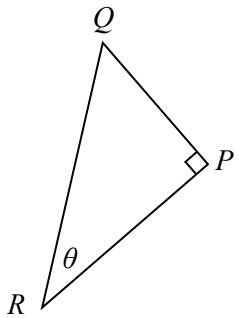
10



11



12



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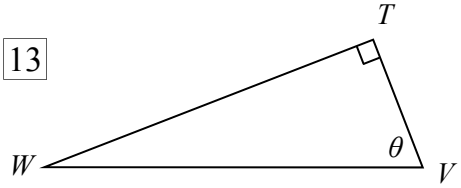

Trigonometry - Trigonometric Ratios (Sheet 2)

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When a side measurement is missing use Pythagoras. Review Pythagoras here first.

Use addition to find the hypotenuse

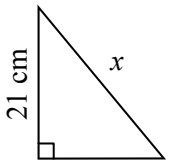
1

$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm



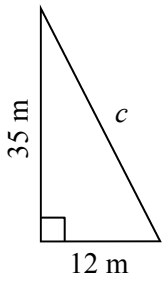
2

$c^2 = \square^2 + \square^2$

$c^2 = \square$

$c = \sqrt{\square}$

$c = \square$ m



Use subtraction to find a short side

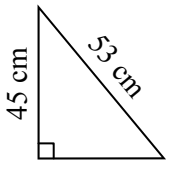
3

$x^2 = 53^2 - \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm



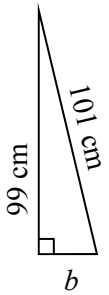
4

$b^2 = \square^2 - \square^2$

$b^2 = \square$

$b = \sqrt{\square}$

$b = \square$ cm



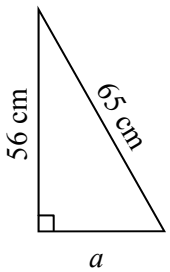
5

$a^2 =$

$a^2 =$

$a =$

$a =$



SOH CAH TOA

Find the missing side then find the trig ratios for sin θ, cos θ and tan θ.

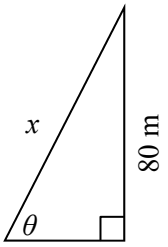
6

$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ m



Label triangle sides *H*, *A* & *O* first!

$\sin\theta = \frac{\square}{\square}$ $\cos\theta = \frac{\square}{\square}$

$\tan\theta = \frac{\square}{\square}$

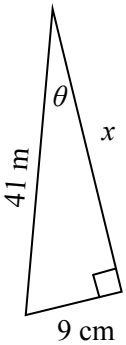
7

$x^2 = \square^2 - \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ m



$\tan\theta = \frac{\square}{\square}$ $\sin\theta = \frac{\square}{\square}$

$\cos\theta = \frac{\square}{\square}$

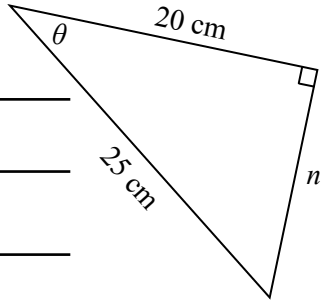
8

$n^2 =$

$n^2 =$

$n =$

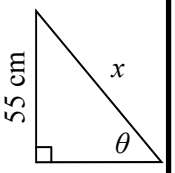
$n =$



$\tan\theta = \frac{\square}{\square}$ $\cos\theta = \frac{\square}{\square}$

$\sin\theta = \frac{\square}{\square}$

9



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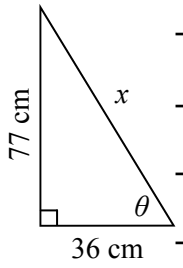

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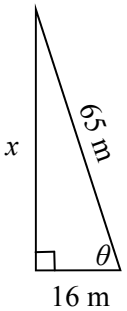

Trigonometry - Trigonometric Ratios (Sheet 1)

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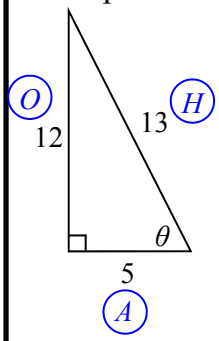
Label the sides as '*O*', '*A*' and '*H*'. Then find sin θ, cos θ and tan θ. Use *SOH CAH TOA!*

Example:

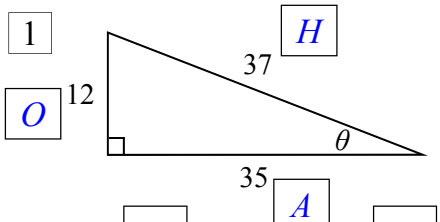


$\sin\theta = \frac{O}{H} = \frac{12}{13}$

$\cos\theta = \frac{A}{H} = \frac{5}{13}$

$\tan\theta = \frac{O}{A} = \frac{12}{5}$

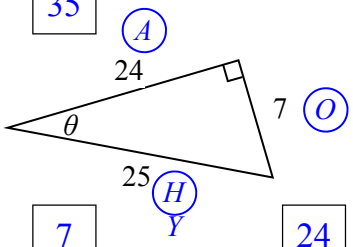
1



$\sin\theta = \frac{12}{35}$ $\quad \cos\theta = \frac{35}{37}$

$\tan\theta = \frac{12}{35}$

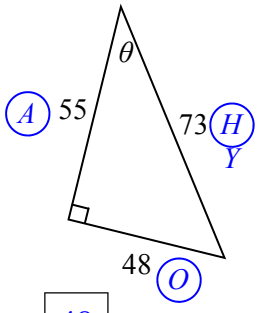
2



$\sin\theta = \frac{7}{25}$ $\quad \cos\theta = \frac{24}{25}$

$\tan\theta = \frac{7}{24}$

3



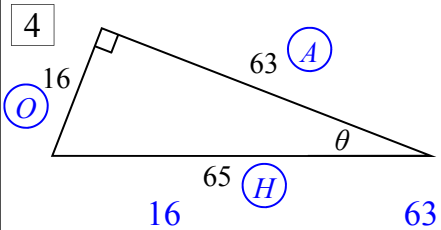
$\tan\theta = \frac{48}{55}$ $\quad \sin\theta = \frac{48}{73}$

$\cos\theta = \frac{55}{73}$

Note: The order of the functions has changed!

SOH CAH TOA

4

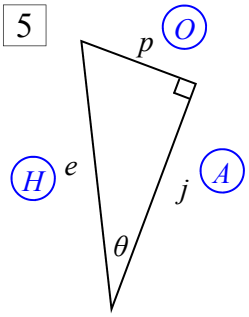


$\sin\theta = \frac{16}{65}$ $\quad \cos\theta = \frac{63}{65}$

$\tan\theta = \frac{16}{63}$

These involve algebra... same process

5



$\cos\theta = \frac{j}{e}$

$\tan\theta = \frac{p}{j}$

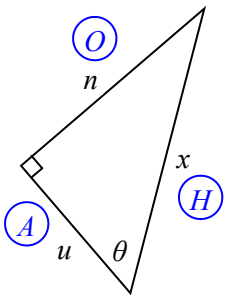
$\sin\theta = \frac{p}{e}$

6

$\sin\theta = \frac{n}{x}$

$\cos\theta = \frac{u}{x}$

$\tan\theta = \frac{n}{u}$



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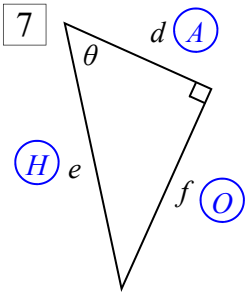
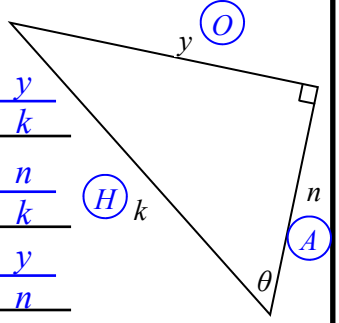


$\sin\theta = \frac{f}{e}$

$\cos\theta = \frac{d}{e}$

$\tan\theta = \frac{f}{d}$

8



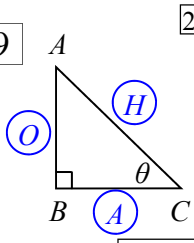
$\sin\theta = \frac{y}{k}$

$\cos\theta = \frac{n}{k}$

$\tan\theta = \frac{y}{n}$

When letters are on the vertices (corners) of the triangle, use two letters to name the sides.

9

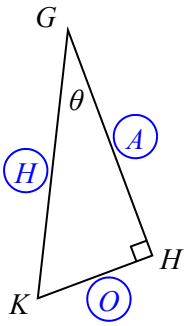


2 letters → $\cos\theta = \frac{BC}{AC}$ (2 letters →)

$\tan\theta = \frac{AB}{BC}$

$\sin\theta = \frac{AB}{AC}$

10

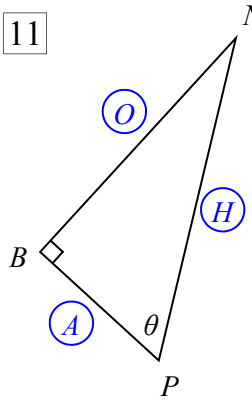


$\sin\theta = \frac{HK}{GK}$

$\cos\theta = \frac{GH}{GK}$

$\tan\theta = \frac{HK}{GH}$

11

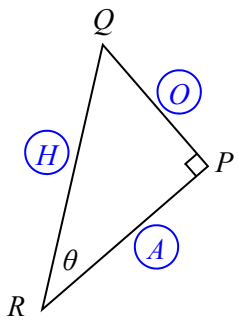


$\sin\theta = \frac{BN}{NP}$

$\cos\theta = \frac{BP}{NP}$

$\tan\theta = \frac{BN}{BP}$

12

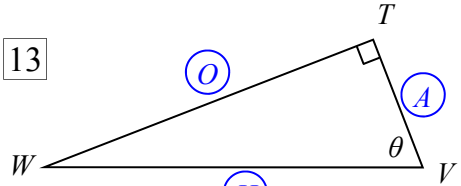


$\sin\theta = \frac{QP}{QR}$

$\cos\theta = \frac{RP}{QR}$

$\tan\theta = \frac{QP}{RP}$

13



$\sin\theta = \frac{WT}{WV}$ $\quad \cos\theta = \frac{TV}{WV}$

$\tan\theta = \frac{WT}{TV}$

Trigonometry - Trigonometric Ratios (Sheet 2)

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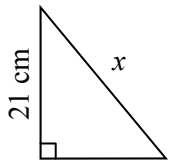
When a side measurement is missing use Pythagoras. Review Pythagoras here first.

Use addition to find the hypotenuse

1



$x^2 = 20^2 + 21^2$

$x^2 = 841$

$x = \sqrt{841}$

$x = 29$ cm

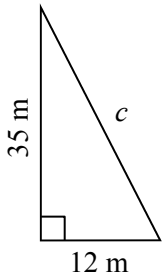
2

$c^2 = 12^2 + 35^2$

$c^2 = 1\,369$

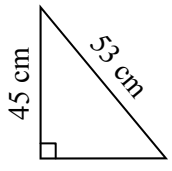
$c = \sqrt{1\,369}$

$c = 37$ m



Use subtraction to find a short side

3



$x^2 = 53^2 - 45^2$

$x^2 = 784$

$x = \sqrt{784}$

$x = 28$ cm

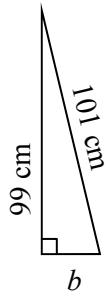
4

$b^2 = 101^2 - 99^2$

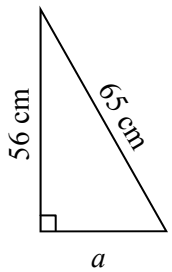
$b^2 = 400$

$b = \sqrt{400}$

$b = 20$ cm



5



$a^2 = 65^2 - 56^2$

$a^2 = 1\,089$

$a = \sqrt{1\,089}$

$a = 33$ cm

SOH CAH TOA

Find the missing side then find the trig ratios for sin θ, cos θ and tan θ.

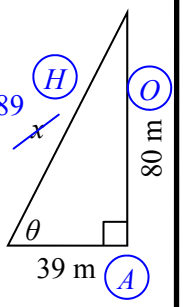
6

$x^2 = 39^2 + 80^2$

$x^2 = 7\,921$

$x = \sqrt{7\,921}$

$x = 89$ m



Label triangle sides *H*, *A* & *O* first!

$\sin\theta = \frac{80}{89}$ $\cos\theta = \frac{39}{89}$

$\tan\theta = \frac{80}{39}$

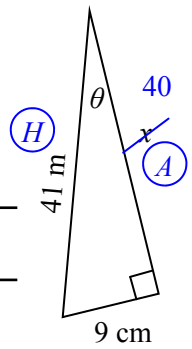
7

$x^2 = 41^2 - 9^2$

$x^2 = 1\,600$

$x = \sqrt{1\,600}$

$x = 40$ m



$\tan\theta = \frac{9}{40}$ $\sin\theta = \frac{9}{41}$

$\cos\theta = \frac{40}{41}$

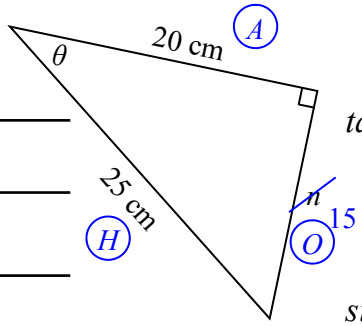
8

$n^2 = 25^2 - 20^2$

$n^2 = 225$

$n = \sqrt{225}$

$n = 15$ cm



$\tan\theta = \frac{15}{20}$ $\cos\theta = \frac{20}{25}$

$\sin\theta = \frac{15}{25}$

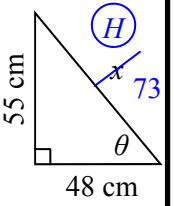
9

$x^2 = 48^2 + 55^2$

$x^2 = 5\,329$

$x = \sqrt{5\,329}$

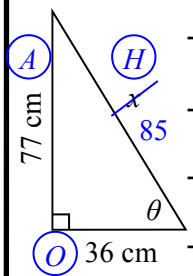
$x = 73$ cm



$\sin\theta = \frac{55}{73}$ $\cos\theta = \frac{48}{73}$ A

$\tan\theta = \frac{55}{48}$

10



$x^2 = 36^2 + 77^2$

$x^2 = 7\,225$

$x = \sqrt{7\,225}$

$x = 85$ cm

$\sin\theta = \frac{36}{85}$ $\cos\theta = \frac{77}{85}$

$\tan\theta = \frac{36}{77}$

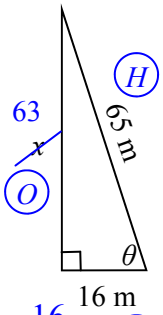
11

$x^2 = 65^2 - 16^2$

$x^2 = 3\,969$

$x = \sqrt{3\,969}$

$x = 63$ m



$\sin\theta = \frac{63}{65}$ $\cos\theta = \frac{16}{65}$ A

$\tan\theta = \frac{63}{16}$

Trigonometry - Trigonometric Ratios (Homework Strip)

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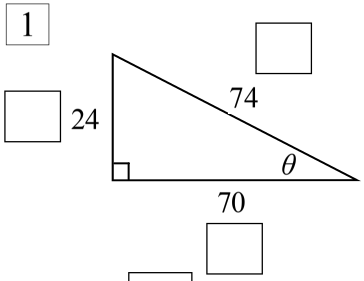
Date Due: _____ SOH CAH TOA

Find the trigonometric ratios for these triangles.

1

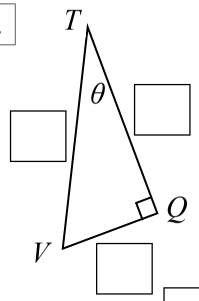


$\sin\theta =$ ☐/☐

$\cos\theta =$ ☐/☐

$\tan\theta =$ ☐/☐

2

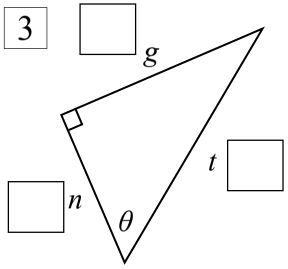


$\tan\theta =$ ☐/☐

$\sin\theta =$ ☐/☐

$\cos\theta =$ ☐/☐

3

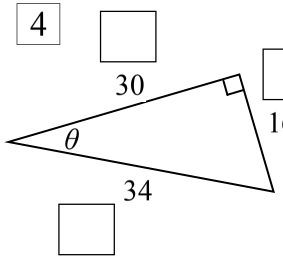


$\sin\theta =$ _____

$\tan\theta =$ _____

$\cos\theta =$ _____

4



$\cos\theta =$ _____

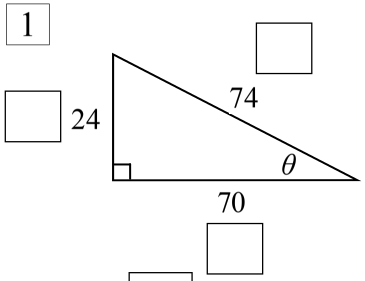
$\sin\theta =$ _____

$\tan\theta =$ _____

Date Due: _____ SOH CAH TOA

Find the trigonometric ratios for these triangles.

1

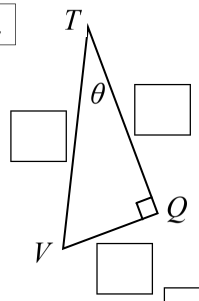


$\sin\theta =$ ☐/☐

$\cos\theta =$ ☐/☐

$\tan\theta =$ ☐/☐

2

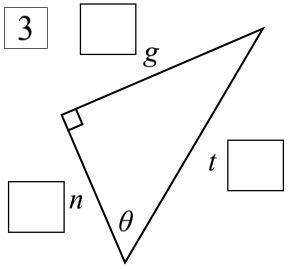


$\tan\theta =$ ☐/☐

$\sin\theta =$ ☐/☐

$\cos\theta =$ ☐/☐

3

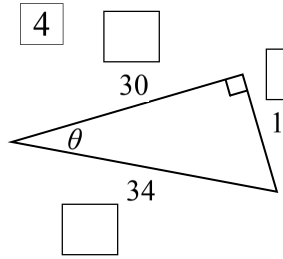


$\sin\theta =$ _____

$\tan\theta =$ _____

$\cos\theta =$ _____

4



$\cos\theta =$ _____

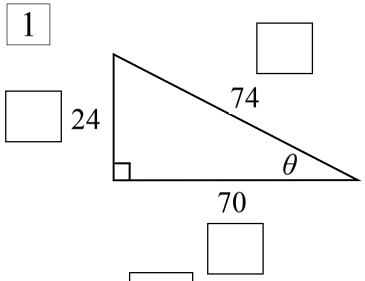
$\sin\theta =$ _____

$\tan\theta =$ _____

Date Due: _____ SOH CAH TOA

Find the trigonometric ratios for these triangles.

1

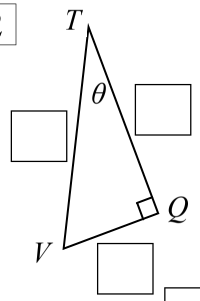


$\sin\theta =$ ☐/☐

$\cos\theta =$ ☐/☐

$\tan\theta =$ ☐/☐

2

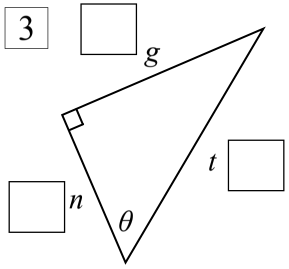


$\tan\theta =$ ☐/☐

$\sin\theta =$ ☐/☐

$\cos\theta =$ ☐/☐

3

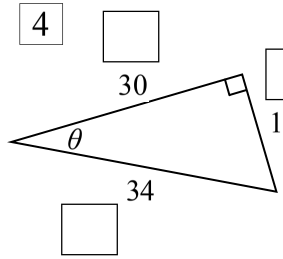


$\sin\theta =$ _____

$\tan\theta =$ _____

$\cos\theta =$ _____

4



$\cos\theta =$ _____

$\sin\theta =$ _____

$\tan\theta =$ _____

Trigonometry - Trigonometric Ratios (Homework Strip)

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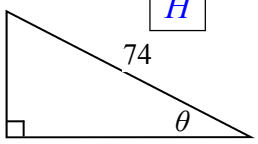
Date Due: _____ SOH CAH TOA

Find the trigonometric ratios for these triangles.

1

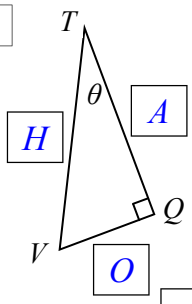


$\sin\theta = \frac{24}{74}$

$\cos\theta = \frac{70}{74}$

$\tan\theta = \frac{24}{70}$

2

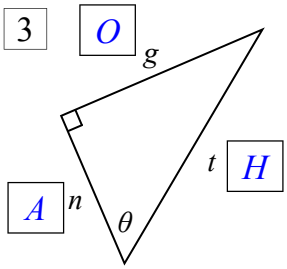


$\tan\theta = \frac{VQ}{TQ}$

$\sin\theta = \frac{VQ}{VT}$

$\cos\theta = \frac{TQ}{VT}$

3

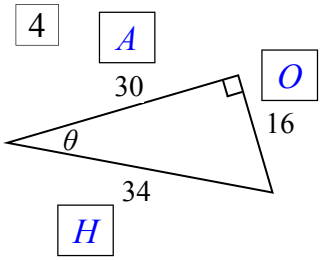


$\sin\theta = \frac{g}{t}$

$\tan\theta = \frac{g}{n}$

$\cos\theta = \frac{n}{t}$

4



$\cos\theta = \frac{30}{34}$

$\sin\theta = \frac{16}{34}$

$\tan\theta = \frac{16}{30}$

Trigonometry - Calculator Use (Angle)

Trigonometry can be used to calculate the size of an angle in a triangle. Angles are measured in degrees, more accuracy involves degrees and minutes, more accuracy still is degrees, minutes and seconds. This course deals with degrees only.

Rounding to the nearest degree can be done in two ways. You can use decimals to guide you, or use minutes to guide you.

Examples: Round these angles to the nearest degree.

1 57.3º = 57º Because 0.3 is less than 0.5, it rounds down

2 51.7º = 52º Because 0.7 is more (or equal to) than 0.5, it rounds up

Now using degrees, minutes and seconds

1 57º 19’ 38.44’’ = 57º Because 19 is less than 30, it rounds down. SECONDS ARE IGNORED

2 51º 40’ 11.72’’ = 52º Because 40 is more than 30, it rounds up. SECONDS ARE IGNORED

Trigonometry uses a calculator’s trig ratios of sin, cos and tan. It is important that you remember that:

TO GET AN ANGLE YOU USE **SHIFT** THEN THE RATIO

The sheet has questions involving fractions and decimals. Below are examples for both.

Examples: Find θ to the nearest degree

DECIMAL

$\sin\theta = 0.317$ $\theta =$ 18°

Calc: SHIFT or 2ndF SIN 0 • 3 1 7 =

Display 18.48159427

Less than 0.5 so round down

FRACTION

$\cos\theta = \frac{8}{17}$ $\theta =$ 62°

Calc: SHIFT or 2ndF COS 8 ▪/□ 1 7 =

Display 61.92751306

More than 0.5 so round up

USING THE DMS KEY

$\cos\theta = \frac{8}{17}$ $\theta =$ 62°

Calc: SHIFT or 2ndF COS 8 ▪/□ 1 7 = º ’ ’’

Display 61º 55’ 39.05’’

More than 30 so round up

Trigonometry - Calculator Use (Angle)

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Round these angles to the nearest degree. No calculator.

Examples

$58.\underline{7}^\circ = 59^\circ$ — More than 0.5 Round UP

$18.\underline{3}^\circ = 18^\circ$ — Less than 0.5 Round DOWN

1 15.8° ____°

2 22.3° ____ Show ° symbol

3 35.7° ____

4 29.8° ____

5 11.73° ____

6 43.2177° ____

7 64.195° ____

8 59.60613° ____

Now complete the calculation and round to the nearest degree.

CALCULATOR IN DEG MODE

Example $\tan\theta = 0.3407$

Calc: SHIFT or 2ndF TAN 0 • 3 4 0 7 =

Display: 18.813……(calc)

Answer: $\theta = 19^\circ$

9 $\tan\theta = 0.556$ $\theta =$ ____°

10 $\sin\theta = 0.12$ $\theta =$ ____°

11 $\cos\theta = 0.5$ $\theta =$ ____ Show ° symbol

12 $\sin\theta = 0.2271$ ____ Write '$\theta =$'

13 $\tan\theta = 1.766$ ____

14 $\sin\theta = 0.4208$ ____

15 $\cos\theta = 0.317$ ____

Complete the calculation to find the angle θ, rounding your answer to the nearest degree.

CALCULATOR IN DEG MODE

16 $\tan\theta = \frac{4}{7}$ $\theta =$ ____°

Calc: SHIFT or 2ndF TAN 4 ▪/□ 7 =

17 $\sin\theta = \frac{7}{9}$ $\theta =$ ____°

18 $\cos\theta = \frac{2}{3}$ $\theta =$ ____°

19 $\tan\theta = \frac{6}{5}$ ____

20 $\cos\theta = \frac{4}{15}$ ____

21 $\sin\theta = \frac{13}{19}$ ____

22 $\sin\theta = \frac{2}{7}$ ____

23 $\tan\theta = \frac{7}{3}$ ____

Round these angles to the nearest degree. No calculator.

Examples

$58^\circ\ \underline{47}' = 59^\circ$ — More than 30' Round UP

$18^\circ\ \underline{27}' = 18^\circ$ — Less than 30' Round DOWN

24 11° 38' ____°

25 55° 16' ____°

26 49° 46' ____

27 79° 28' ____

28 30° 30' ____

From now on seconds are included

29 43° 52' 11.71'' ____ Use Ignore

30 35° 13' 42.05'' ____ Use Ignore

31 42° 59' 28.76'' ____ Use Ignore

32 19° 27' 11.66'' ____

33 76° 49' 23.05'' ____

34 14° 30' 55.17'' ____

Write the full answer in degrees, minutes and seconds, then round the answer to the nearest degree.

35 $\cos\theta = \frac{13}{19}$

Calc: SHIFT or 2ndF TAN 1 3 ▪/□ 1 9 = ° ' ''

Calc: ____

Use ° ' '' key, write answer in D° M' S''

Nearest °: ____

36 $\tan\theta = \frac{23}{38}$

Calc: ____

Nearest °: ____

37 $\sin\theta = 0.5715$

Calc: ____

Nearest °: ____

38 $\tan\theta = 3.2281$

Calc: ____

Nearest °: ____

Trigonometry - Calculator Use (Angle)

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Round these angles to the nearest degree. No calculator.

Examples

$58.\underline{7}^\circ = 59^\circ$ — More than 0.5 Round UP

$18.\underline{3}^\circ = 18^\circ$ — Less than 0.5 Round DOWN

1 15.8° 16°

2 22.3° 22° Show ° symbol

3 35.7° 36°

4 29.8° 30°

5 11.73° 12°

6 43.2177° 43°

7 64.195° 64°

8 59.60613° 60°

Now complete the calculation and round to the nearest degree.

CALCULATOR IN DEG MODE

Example $\tan\theta = 0.3407$

Calc: SHIFT or 2ndF TAN 0 • 3 4 0 7 =

Display: 18.813……(calc)

Answer: $\theta = 19^\circ$

9 $\tan\theta = 0.556$ $\theta =$ 29°

10 $\sin\theta = 0.12$ $\theta =$ 7°

11 $\cos\theta = 0.5$ $\theta =$ 60° Show ° symbol

12 $\sin\theta = 0.2271$ $\theta = 13^\circ$ Write '$\theta =$'

13 $\tan\theta = 1.766$ $\theta = 60^\circ$

14 $\sin\theta = 0.4208$ $\theta = 25^\circ$

15 $\cos\theta = 0.317$ $\theta = 72^\circ$

Complete the calculation to find the angle θ, rounding your answer to the nearest degree.

CALCULATOR IN DEG MODE

16 $\tan\theta = \frac{4}{7}$ $\theta =$ 30°

Calc: SHIFT or 2ndF TAN 4 ▪/□ 7 =

17 $\sin\theta = \frac{7}{9}$ $\theta =$ 51°

18 $\cos\theta = \frac{2}{3}$ $\theta =$ 48°

19 $\tan\theta = \frac{6}{5}$ $\theta = 50^\circ$

20 $\cos\theta = \frac{4}{15}$ $\theta = 75^\circ$

21 $\sin\theta = \frac{13}{19}$ $\theta = 43^\circ$

22 $\sin\theta = \frac{2}{7}$ $\theta = 17^\circ$

23 $\tan\theta = \frac{7}{3}$ $\theta = 67^\circ$

Round these angles to the nearest degree. No calculator.

Examples

$58^\circ\ \underline{47}' = 59^\circ$ — More than 30' Round UP

$18^\circ\ \underline{27}' = 18^\circ$ — Less than 30' Round DOWN

24 11° 38' 12°

25 55° 16' 55°

26 49° 46' 50°

27 79° 28' 79°

28 30° 30' 31°

From now on seconds are included

29 43° 52' 11.71'' (Use / Ignore) 44°

30 35° 13' 42.05'' (Use / Ignore) 35°

31 42° 59' 28.76'' (Use / Ignore) 43°

32 19° 27' 11.66'' 19°

33 76° 49' 23.05'' 77°

34 14° 30' 55.17'' 15°

Write the full answer in degrees, minutes and seconds, then round the answer to the nearest degree.

35 $\cos\theta = \frac{13}{19}$

Calc: SHIFT or 2ndF TAN 1 3 ▪/□ 1 9 = ° ' ''

Calc: 46° 49' 85.22''

Use ° ' '' key, write answer in D° M' S''

Nearest °: 47°

36 $\tan\theta = \frac{23}{38}$

Calc: 31° 11' 5.91''

Nearest °: 31°

37 $\sin\theta = 0.5715$

Calc: 34° 51' 17.61''

Nearest °: 35°

38 $\tan\theta = 3.2281$

Calc: 72° 47' 17.08''

Nearest °: 73°

Trigonometry - Calculator Use - Angle (Homework Strip)

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Date Due: _____

Round these angles to the nearest degree.

1 24.3° _____ °

2 47.6° _____ °

3 59.703° _____ °

4 34° 41’ 23.46’’ _____ ° symbol

5 22° 15’ 47.88’’ _____

6 59° 40’ 13.21’’ _____

Use your calculator to find the value for θ. Round to the nearest degree.

7 $\tan\theta = 0.614$ $\theta =$ _____

8 $\cos\theta = 0.108$ $\theta =$ _____

9 $\sin\theta = 0.882$ $\theta =$ _____

10 $\tan\theta = \frac{8}{5}$ _____ θ = & ° symbol

11 $\cos\theta = \frac{7}{11}$ _____

12 $\sin\theta = \frac{3}{5}$ _____

Date Due: _____

Round these angles to the nearest degree.

1 24.3° _____ °

2 47.6° _____ °

3 59.703° _____ °

4 34° 41’ 23.46’’ _____ ° symbol

5 22° 15’ 47.88’’ _____

6 59° 40’ 13.21’’ _____

Use your calculator to find the value for θ. Round to the nearest degree.

7 $\tan\theta = 0.614$ $\theta =$ _____

8 $\cos\theta = 0.108$ $\theta =$ _____

9 $\sin\theta = 0.882$ $\theta =$ _____

10 $\tan\theta = \frac{8}{5}$ _____ θ = & ° symbol

11 $\cos\theta = \frac{7}{11}$ _____

12 $\sin\theta = \frac{3}{5}$ _____

Date Due: _____

Round these angles to the nearest degree.

1 24.3° _____ °

2 47.6° _____ °

3 59.703° _____ °

4 34° 41’ 23.46’’ _____ ° symbol

5 22° 15’ 47.88’’ _____

6 59° 40’ 13.21’’ _____

Use your calculator to find the value for θ. Round to the nearest degree.

7 $\tan\theta = 0.614$ $\theta =$ _____

8 $\cos\theta = 0.108$ $\theta =$ _____

9 $\sin\theta = 0.882$ $\theta =$ _____

10 $\tan\theta = \frac{8}{5}$ _____ θ = & ° symbol

11 $\cos\theta = \frac{7}{11}$ _____

12 $\sin\theta = \frac{3}{5}$ _____

Date Due: _____

Round these angles to the nearest degree.

1 24.3° _____ °

2 47.6° _____ °

3 59.703° _____ °

4 34° 41’ 23.46’’ _____ ° symbol

5 22° 15’ 47.88’’ _____

6 59° 40’ 13.21’’ _____

Use your calculator to find the value for θ. Round to the nearest degree.

7 $\tan\theta = 0.614$ $\theta =$ _____

8 $\cos\theta = 0.108$ $\theta =$ _____

9 $\sin\theta = 0.882$ $\theta =$ _____

10 $\tan\theta = \frac{8}{5}$ _____ θ = & ° symbol

11 $\cos\theta = \frac{7}{11}$ _____

12 $\sin\theta = \frac{3}{5}$ _____

Trigonometry - Calculator Use - Angle (Homework Strip)

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Date Due:

Round these angles to the nearest degree.

1 24.3° 24 °

2 47.6° 48 °

3 59.703° 60 °

4 34° 41’ 23.46’’ 35º

º symbol

5 22° 15’ 47.88’’ 22º

6 59° 40’ 13.21’’ 60º

Use your calculator to find the value for θ. Round to the nearest degree.

7 $\tan\theta = 0.614$ $\theta =$ 32º

8 $\cos\theta = 0.108$ $\theta =$ 84º

9 $\sin\theta = 0.882$ $\theta =$ 62º

10 $\tan\theta = \frac{8}{5}$ $\theta = 58º$

θ = & º symbol

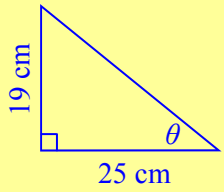
11 $\cos\theta = \frac{7}{11}$ $\theta = 50º$

12 $\sin\theta = \frac{3}{5}$ $\theta = 37º$

Trigonometry - Finding an Angle

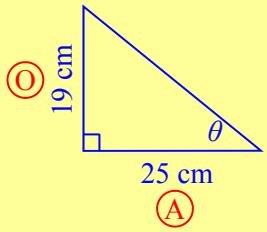
Trigonometry can be used to calculate the size of an angle in a triangle. Below are the steps.

Example: Find the angle θ to the nearest degree



STEP 1

LABEL THE SIDES

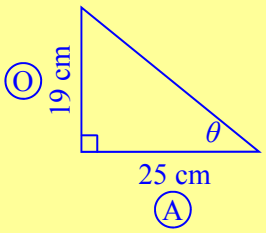


LABEL THE SIDES

Always label the sides O, A, and H. You only have to label the sides that have a measurement on them. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the vertices of the triangle.

STEP 2

PICK THE RATIO

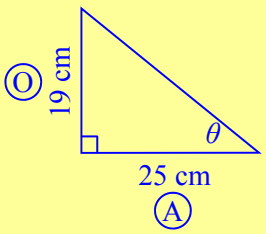


PICK THE RATIO

Write SOH CAH TOA on your page if it isn't there. Then pick the ratio that has the two sides in it. The example has O and A. So this is TOA, which means you use tan to solve this question.

STEP 3

SOLVE



SOLVE

Use the ratio and the two sides to solve, as below.

When you round you can use the decimal as the guide or press the 'DMS' key and use the minutes to guide you whether to round up or down.

$$\tan\theta = \frac{19}{25} \quad \frac{O}{A}$$

Calc: SHIFT or 2ndF TAN 1 9 ■ 2 5 = ° ' '' (optional)

$\theta = 37°$

Actual 37.23483398 — Less than 0.5 so round down

Using DMS 37° 14' 5.4" — Less than 30' so round down

Trigonometry - Finding an Angle (Sheet 1)

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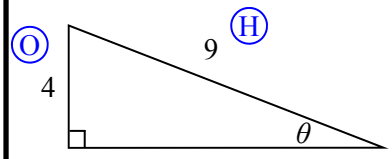
Label two of the sides either '*O*', '*A*' and '*H*'. Then pick which ratio you'd use.

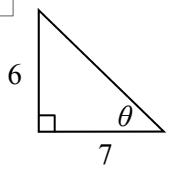
SOH $\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$ CAH $\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$ TOA $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$

Example:



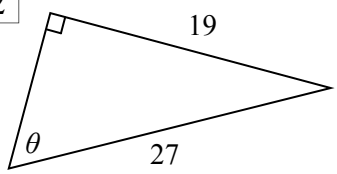
● sin ○ cos ○ tan

1



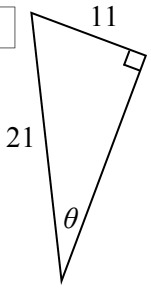
○ cos ○ tan ○ sin

2



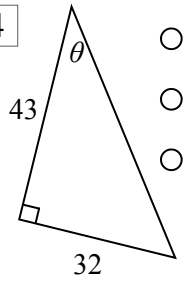
○ sin ○ cos ○ tan

3



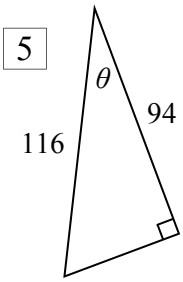
○ cos ○ tan ○ sin

4



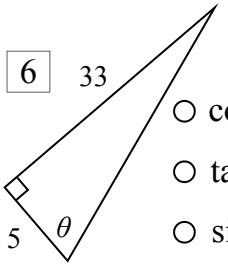
○ tan ○ sin ○ cos

5



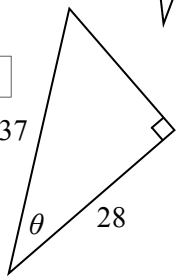
○ tan ○ cos ○ sin

6



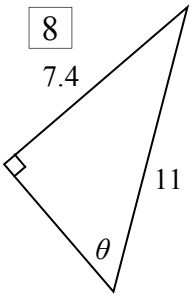
○ cos ○ tan ○ sin

7



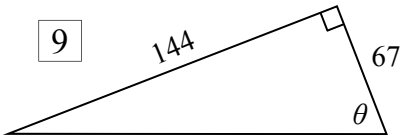
○ tan ○ cos ○ sin

8



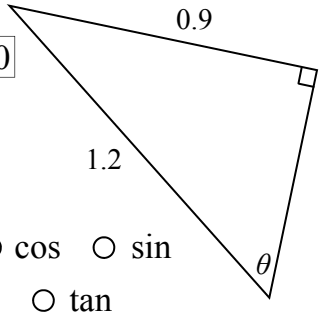
○ tan ○ cos ○ sin

9



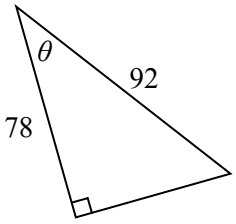
○ cos ○ tan ○ sin

10



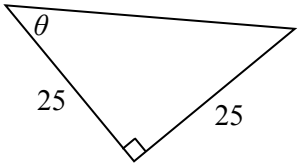
○ cos ○ sin ○ tan

11



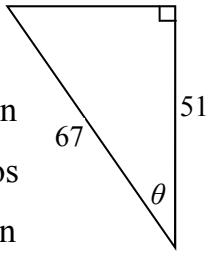
○ cos ○ tan ○ sin

12



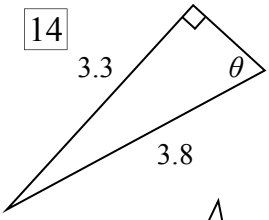
○ cos ○ sin ○ tan

13



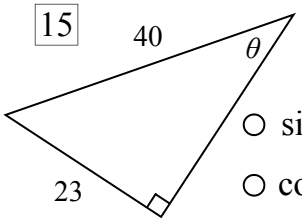
○ tan ○ cos ○ sin

14



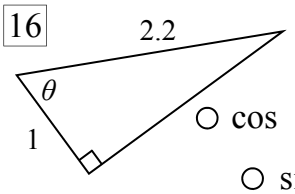
○ cos ○ tan ○ sin

15



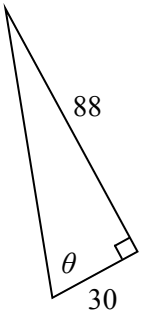
○ sin ○ cos ○ tan

16



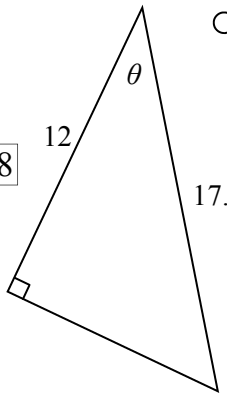
○ cos ○ tan ○ sin

17



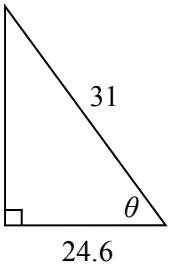
○ cos ○ sin ○ tan

18



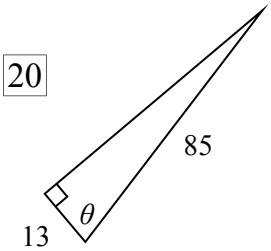
○ tan ○ cos ○ sin

19



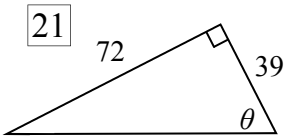
○ tan ○ sin ○ cos

20



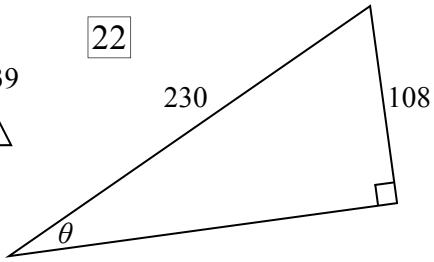
○ sin ○ tan ○ cos

21



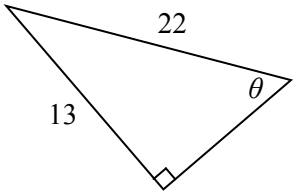
○ tan ○ cos ○ sin

22



○ cos ○ sin ○ tan

23



○ tan ○ sin ○ cos

Trigonometry - Finding an Angle (Sheet 2)

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Complete the calculation to find the angle θ, rounding your answer to the nearest degree.

CALCULATOR IN DEG MODE

1 $\sin\theta = 0.433$

Calc: SHIFT or 2ndF, SIN, 0, •, 4, 3, 3, =, °’’’ (optional)

$\theta =$ ☐ °

2 $\cos\theta = 0.76$

$\theta =$ ☐ °

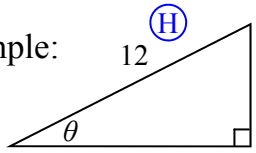
3 $\tan\theta = \frac{4}{7}$

Calc: SHIFT or 2ndF, TAN, 4, ☐, 7, =, °’’’ (optional)

$\theta =$ ☐ °

Calculate the angle θ, rounding to the nearest degree. Steps are: Label sides, pick the ratio and then solve.

Example:

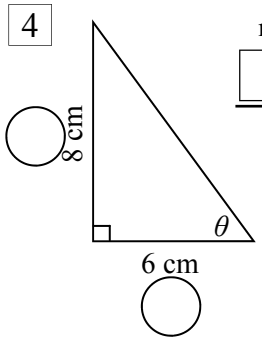


$\cos\theta = \frac{A}{H} = \frac{9}{12}$

Calc: SHIFT or 2ndF, COS, 9, ☐, 1, 2, =, °’’’ (optional)

$\theta = 41°$

4



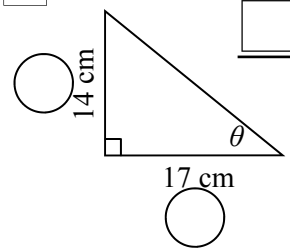
ratio ☐ / ☐ $\theta =$ ☐ / ☐ (O / A)

$\theta =$ ☐ °

Nearest Degree

SOH CAH TOA

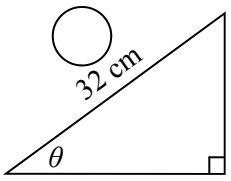
5



☐ $\theta =$ ☐ / ☐

$\theta =$ ☐ °

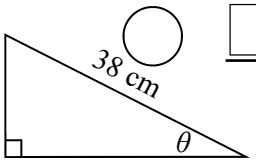
6



☐ $\theta =$ ☐ / ☐

$\theta =$ ☐ °

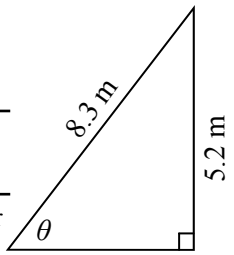
7



☐ $\theta =$ ☐ / ☐

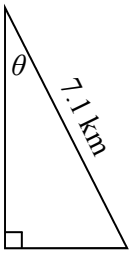
$\theta =$ ☐ °

8



Write ° after answer

9



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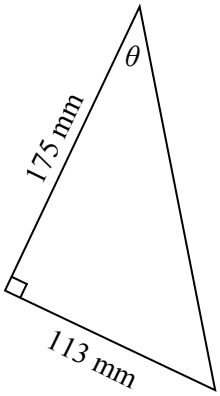

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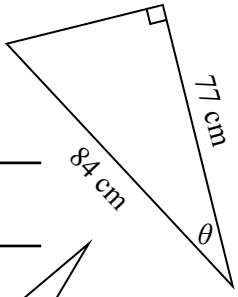

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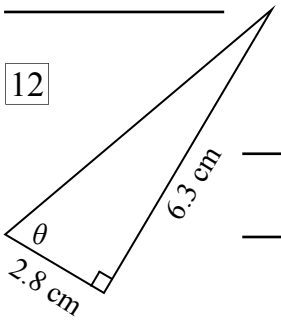
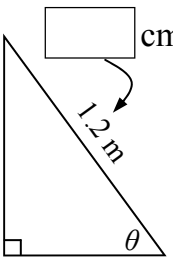
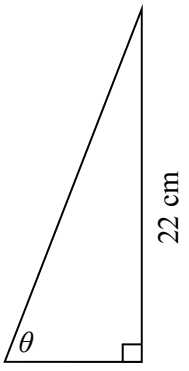


These are the same but first change the units to match each other. Find θ to the nearest °.

13



14



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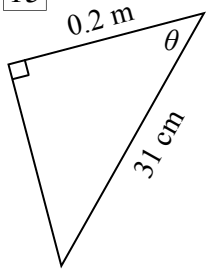

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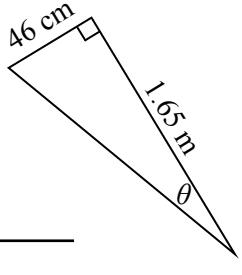

Trigonometry - Finding an Angle (Sheet 3)

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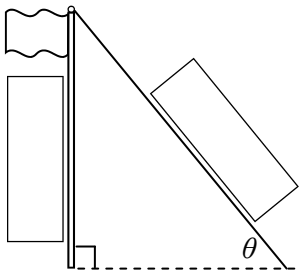
Calculate the angle θ, rounding to the nearest degree.

CALCULATOR IN DEG MODE

A 4.5 m tall flagpole has a 5.6 m guy wire anchored into the ground.

1 Write the two dimensions in the boxes on the diagram below.



2 Calculate the angle θ that the guy wire makes with the ground.

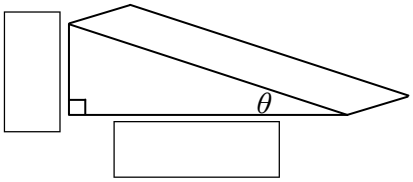
$\theta =$ ☐ / ☐

$\theta =$ ☐ °

A triangular skateboard ramp is 3.5 m long and has a height of 80 cm.

3 Convert 3.5 m to cm

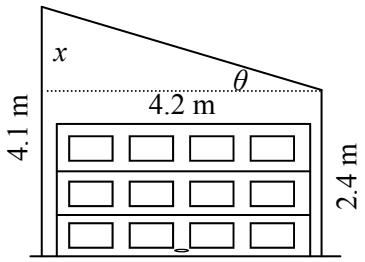
4 Add the two dimensions (in cm) to the diagram below.



5 Calculate the angle θ that the ramp makes with the ground.

SOH CAH TOA

A garage has a skillion roof shown in the diagram below.

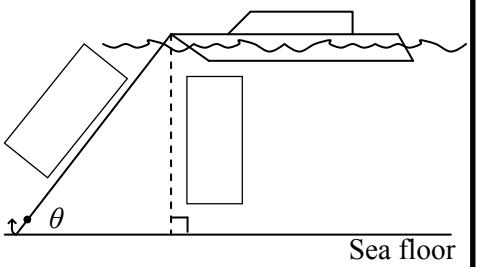


6 Calculate the distance x, being the difference in height between the two walls.

7 Calculate the 'pitch' of the roof, which is the angle θ on the diagram.

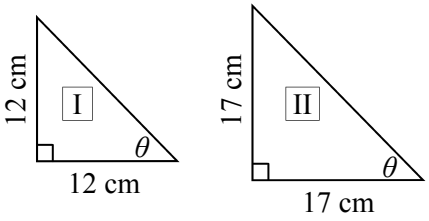
A fishing boat is anchored in 65 m deep water. The anchor rope is 90 m long and is fully extended.

8 Write the two dimensions in the boxes on the diagram below.



9 Calculate the angle θ that the anchor rope makes with the sea floor.

Toby has constructed two isosceles right triangles. He says that when the two short sides of a right triangle are the same, the angle θ is always 45º.



10 Calculate the angle θ in the two triangles.

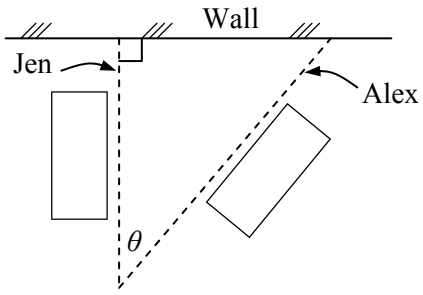
I :

II :

11 Is Toby correct?

Jen measured the straight distance to a wall of 55m. Alex walked to the wall at an angle and measured the distance as 72 m.

PLAN (TOP) VIEW



12 Write the two dimensions in the boxes on the diagram above.

13 Calculate the angle θ that Alex's route took compared to Jen's.

Trigonometry - Finding an Angle (Sheet 1)

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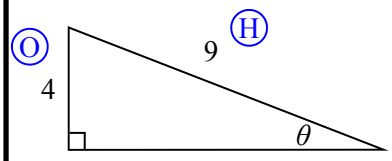
Label two of the sides either '*O*', '*A*' and '*H*'. Then pick which ratio you'd use.

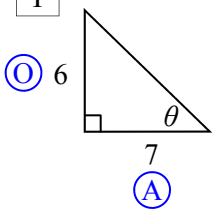
SOH $\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$ CAH $\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$ TOA $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$

Example:



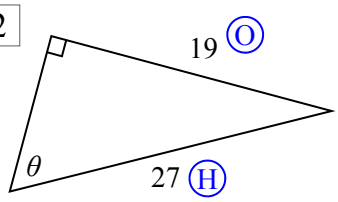
● sin ○ cos ○ tan

1



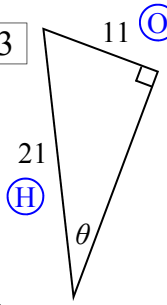
○ cos ● tan ○ sin

2



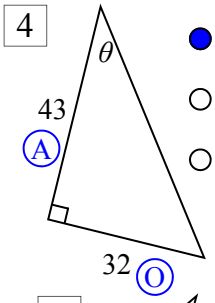
● sin ○ cos ○ tan

3



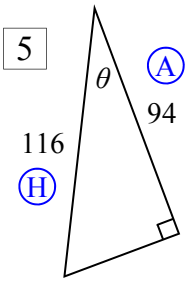
○ cos ○ tan ● sin

4



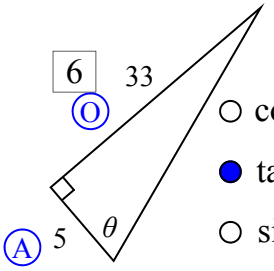
● tan ○ sin ○ cos

5



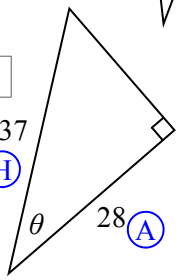
○ tan ● cos ○ sin

6



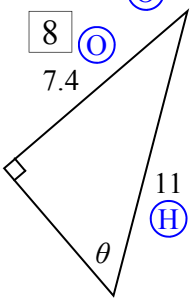
○ cos ● tan ○ sin

7



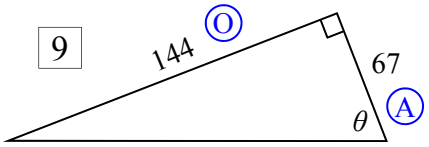
○ tan ● cos ○ sin

8



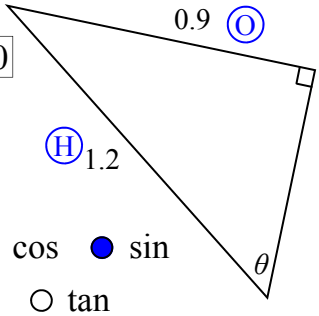
○ tan ○ cos ● sin

9



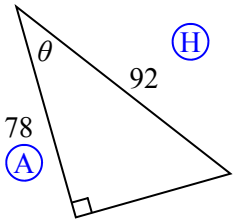
○ cos ● tan ○ sin

10



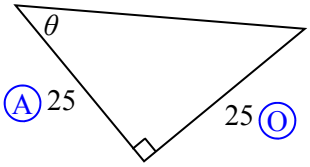
○ cos ● sin ○ tan

11



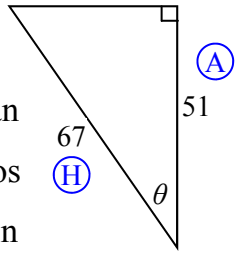
● cos ○ tan ○ sin

12



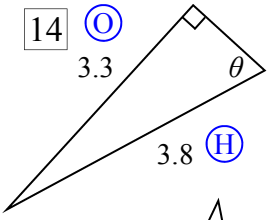
○ cos ○ sin ● tan

13



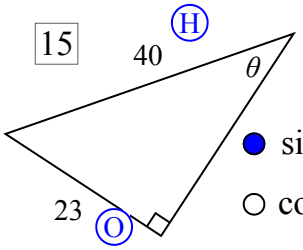
○ tan ● cos ○ sin

14



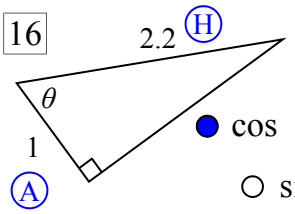
○ cos ○ tan ● sin

15



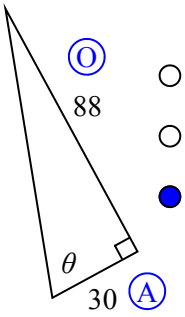
● sin ○ cos ○ tan

16



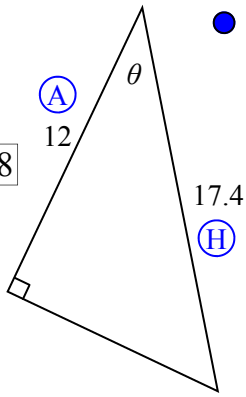
● cos ○ tan ○ sin

17



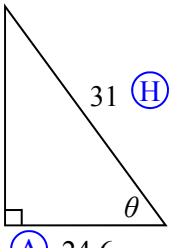
○ cos ○ sin ● tan

18



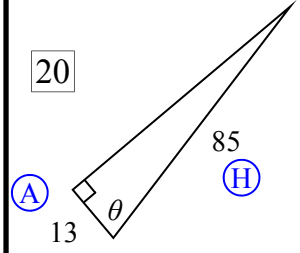
○ tan ● cos ○ sin

19



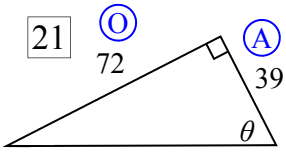
○ tan ○ sin ● cos

20



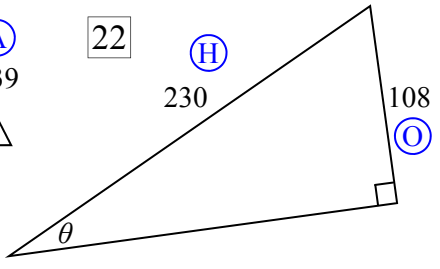
○ sin ○ tan ● cos

21



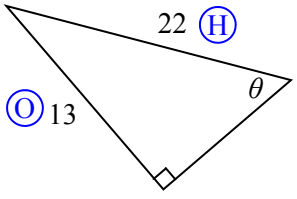
● tan ○ cos ○ sin

22



○ cos ● sin ○ tan

23



○ tan ● sin ○ cos

Trigonometry - Finding an Angle (Sheet 2)

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Complete the calculation to find the angle θ, rounding your answer to the nearest degree.

CALCULATOR IN DEG MODE

1 $\sin\theta = 0.433$

Calc: SHIFT or 2ndF SIN 0 • 4 3 3 = °’’’ (optional)

$\theta = 26°$

2 $\cos\theta = 0.76$

$\theta = 41°$

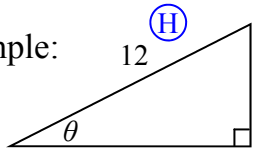
3 $\tan\theta = \frac{4}{7}$

Calc: SHIFT or 2ndF TAN 4 ▪/□ 7 = °’’’ (optional)

$\theta = 30°$

Calculate the angle θ, rounding to the nearest degree. Steps are: Label sides, pick the ratio and then solve.

Example:

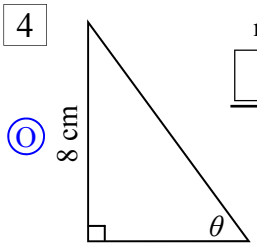


$\cos\theta = \frac{A}{H} = \frac{9}{12}$

Calc: SHIFT or 2ndF COS 9 ▪/□ 1 2 = °’’’ (optional)

$\theta = 41°$

4

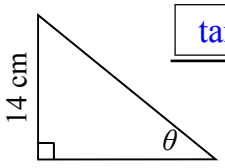


ratio: $\tan\theta = \frac{O}{A} = \frac{8}{6}$

$\theta = 53°$ Nearest Degree

SOH CAH TOA

5

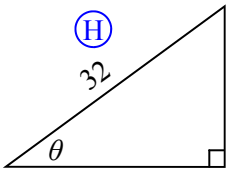


$\tan\theta = \frac{14}{17}$

$\theta = 40°$

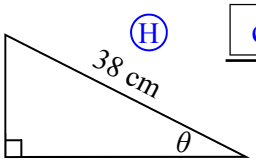
6

$\cos\theta = \frac{24}{32}$



$\theta = 41°$

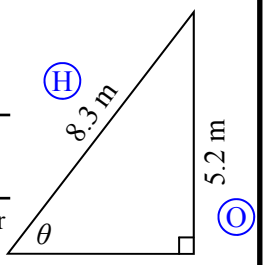
7



$\cos\theta = \frac{31}{38}$

$\theta = 35°$

8

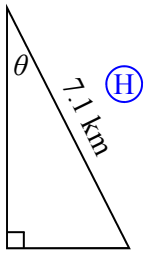


$\sin\theta = \frac{5.2}{8.3}$

$\theta = 39°$

Write ° after answer

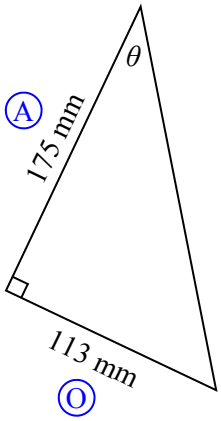
9



$\sin\theta = \frac{3.3}{7.1}$

$\theta = 28°$

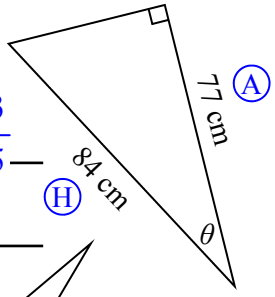
10



$\tan\theta = \frac{113}{175}$

$\theta = 33°$

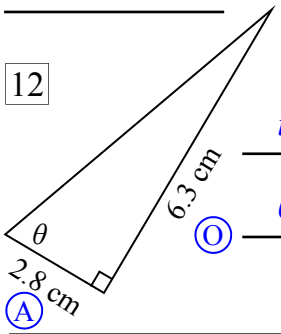
11



$\cos\theta = \frac{113}{175}$

$\theta = 33°$

12

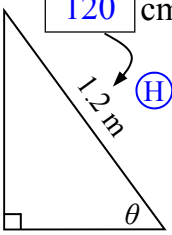


$\tan\theta = \frac{6.3}{2.8}$

$\theta = 66°$ O

These are the same but first change the units to match each other. Find θ to the nearest °.

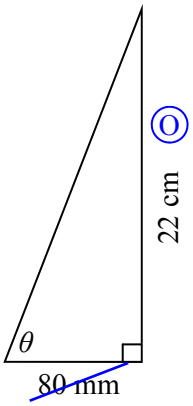
13



$\cos\theta = \frac{56}{120}$

$\theta = 62°$

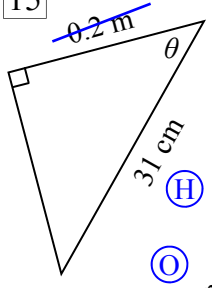
14



$\tan\theta = \frac{22}{8}$

$\theta = 70°$

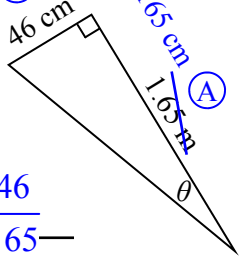
15



$\cos\theta = \frac{20}{31}$

$\theta = 50°$

16



$\tan\theta = \frac{46}{165}$

$\theta = 16°$

Trigonometry - Finding an Angle (Homework Strip)

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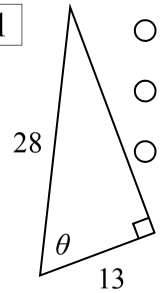
Date Due:

SOH CAH TOA | DEG MODE

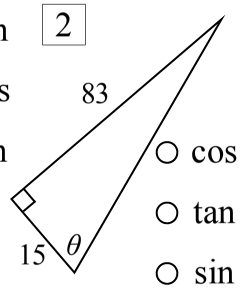
Which Trig ratio would you use to solve these?

1



○ tan
○ cos
○ sin

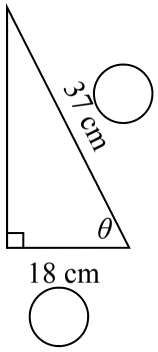
2



○ cos
○ tan
○ sin

Find the size of the angle θ, rounding your answer to the nearest degree.

3

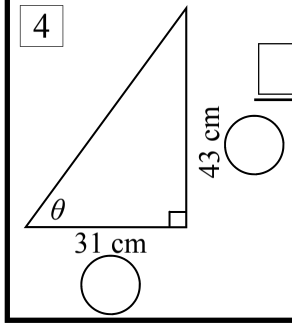


ratio $\theta = \frac{\square}{\square}$

$\theta = \square^{\circ}$

Nearest Degree

4



$\theta = \frac{\square}{\square}$

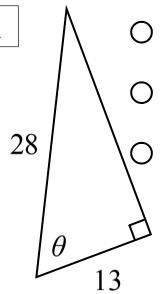
$\theta = \square^{\circ}$

Date Due:

SOH CAH TOA | DEG MODE

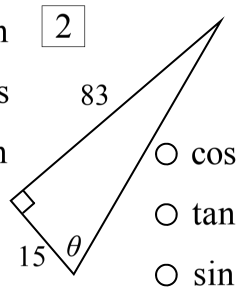
Which Trig ratio would you use to solve these?

1



○ tan
○ cos
○ sin

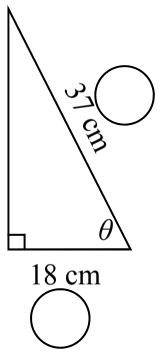
2



○ cos
○ tan
○ sin

Find the size of the angle θ, rounding your answer to the nearest degree.

3

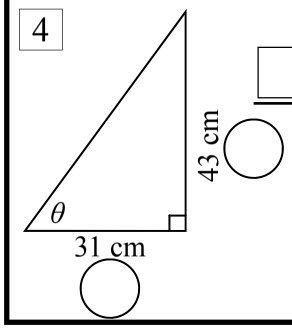


ratio $\theta = \frac{\square}{\square}$

$\theta = \square^{\circ}$

Nearest Degree

4



$\theta = \frac{\square}{\square}$

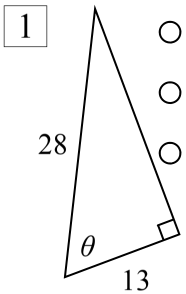
$\theta = \square^{\circ}$

Date Due:

SOH CAH TOA | DEG MODE

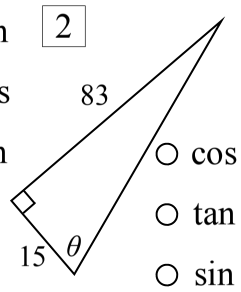
Which Trig ratio would you use to solve these?

1



○ tan
○ cos
○ sin

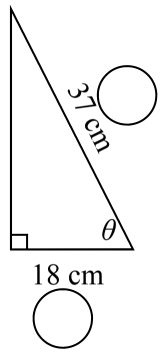
2



○ cos
○ tan
○ sin

Find the size of the angle θ, rounding your answer to the nearest degree.

3

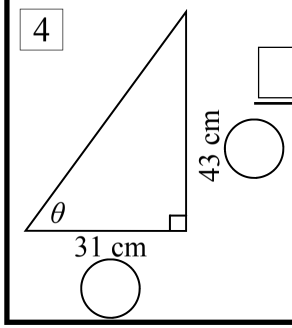


ratio $\theta = \frac{\square}{\square}$

$\theta = \square^{\circ}$

Nearest Degree

4



$\theta = \frac{\square}{\square}$

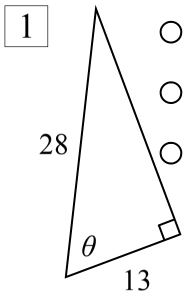
$\theta = \square^{\circ}$

Date Due:

SOH CAH TOA | DEG MODE

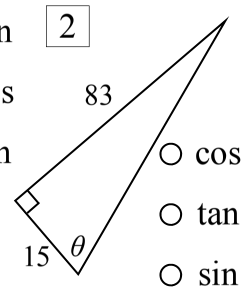
Which Trig ratio would you use to solve these?

1



○ tan
○ cos
○ sin

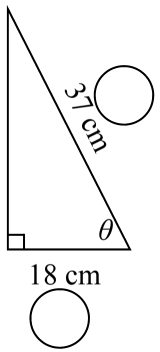
2



○ cos
○ tan
○ sin

Find the size of the angle θ, rounding your answer to the nearest degree.

3

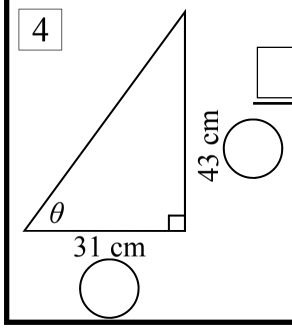


ratio $\theta = \frac{\square}{\square}$

$\theta = \square^{\circ}$

Nearest Degree

4



$\theta = \frac{\square}{\square}$

$\theta = \square^{\circ}$

Trigonometry - Finding an Angle (Homework Strip)

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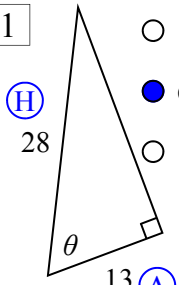
Date Due:

SOH CAH TOA | DEG MODE

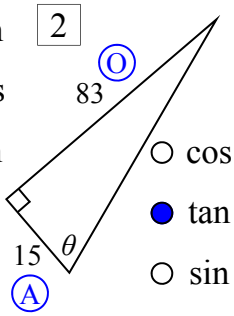
Which Trig ratio would you use to solve these?

1



○ tan
● cos
○ sin

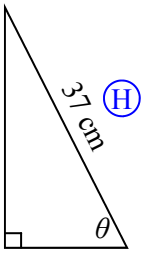
2



○ cos
● tan
○ sin

Find the size of the angle θ, rounding your answer to the nearest degree.

3

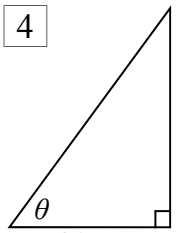


ratio: *cos* $\theta = \frac{18}{37}$

$\theta =$ 61 °

Nearest Degree

4



tan $\theta = \frac{43}{31}$

$\theta =$ 54 °

Trigonometry - Finding an Angle with Protractor Verification

In the previous sheet you learned how to calculate the angle in a triangle. This sheet is just the same except instead of being given side lengths, you have to measure them with a ruler yourself.

STEP 1

MEASURE THE SIDES

Use a ruler to measure the sides that have a box on them. Write the lengths in cm in the box.

STEP 2

LABEL THE SIDES

Label the two sides either ‘O’ or ‘A’ or ‘H’.

STEP 3

CALCULATE ANGLE

Pick the appropriate trig ratio, either ‘sin’, ‘cos’ or ‘tan’ and then solve using your calculator. Write the angle to the nearest degree.

STEP 4

MEASURE ANGLE

Use a protractor to measure the angle with a protractor. Verify that your calculation is correct. Note that you could do this step first if you want to.

Trigonometry - Finding an Angle with Protractor Verification

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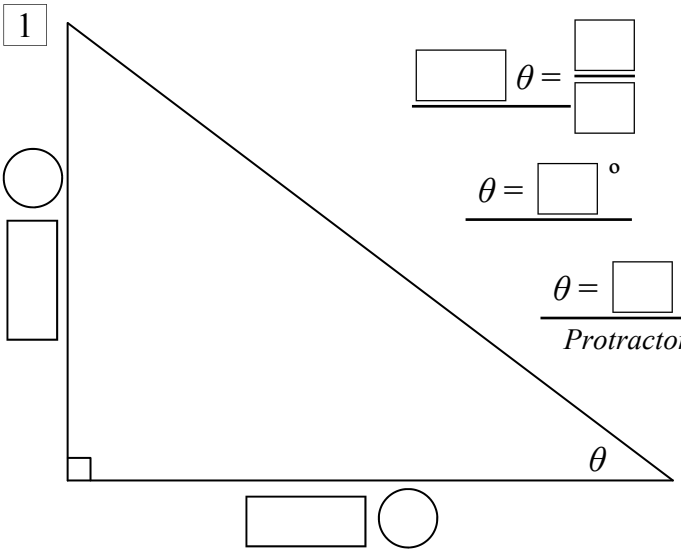
Use a ruler to measure the lengths of the sides with the box, write the side lengths (in whole cm). Then use the circles for '*O*'/'*A*'/'*H*' and use trigonometry to calculate the angle θ, to the nearest degree. Then verify the size of the angle and your calculation by checking it manually with a protractor. SOH CAH TOA

1

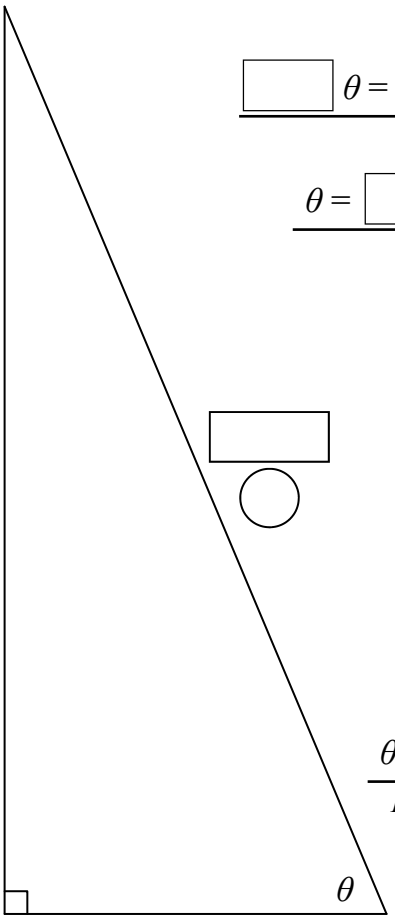


$\square\ \theta = \frac{\square}{\square}$

$\theta = \underline{\square}\ ^{o}$

$\theta = \underline{\square}\ ^{o}$ *Protractor*

2

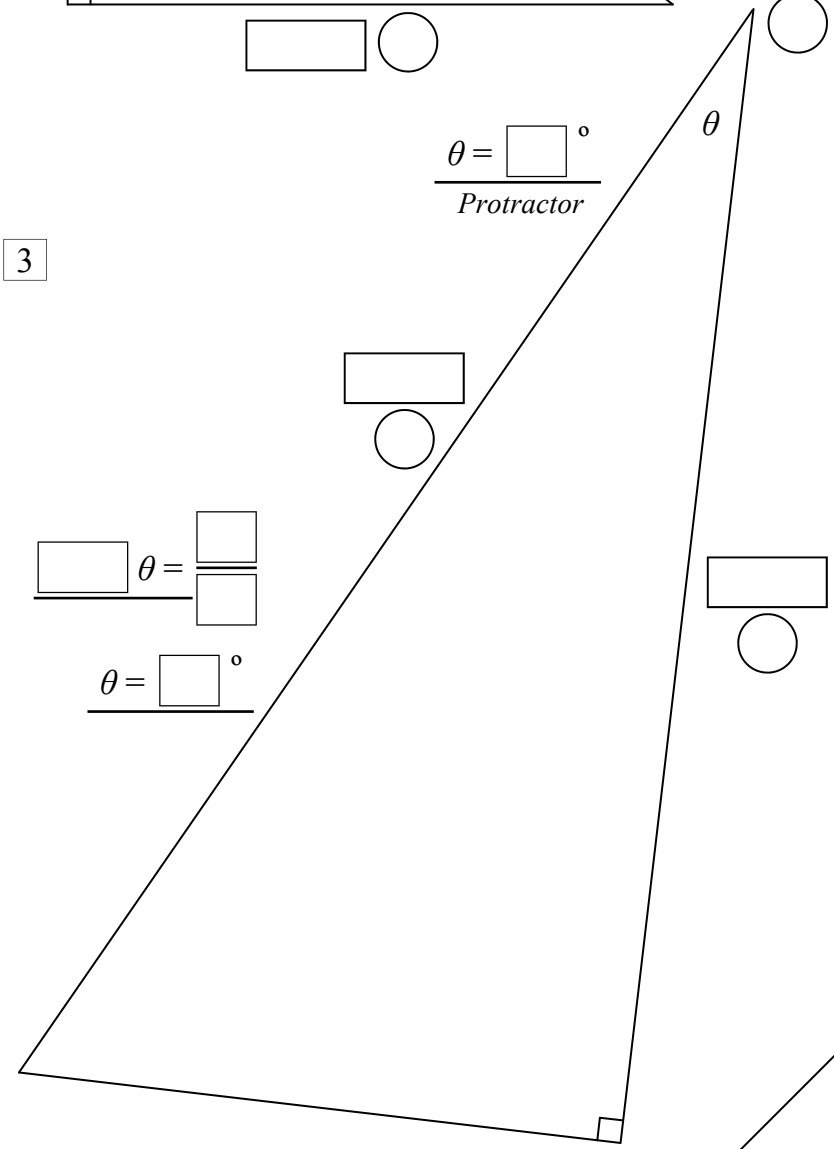


$\square\ \theta = \frac{\square}{\square}$

$\theta = \underline{\square}\ ^{o}$

$\theta = \underline{\square}\ ^{o}$ *Protractor*

3

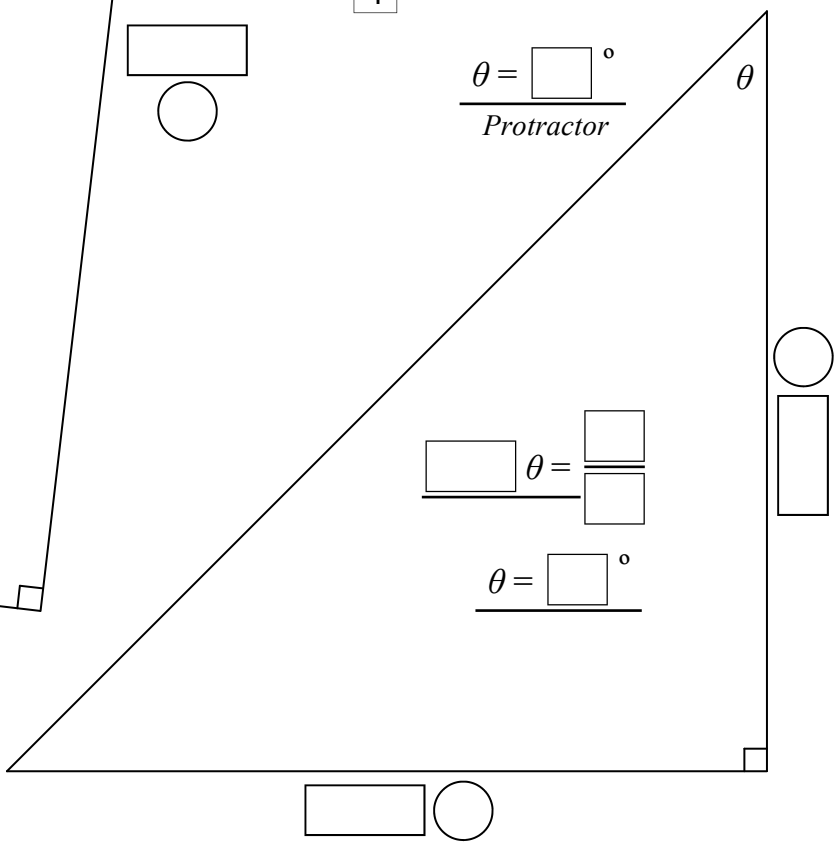


$\square\ \theta = \frac{\square}{\square}$

$\theta = \underline{\square}\ ^{o}$

$\theta = \underline{\square}\ ^{o}$ *Protractor*

4



$\square\ \theta = \frac{\square}{\square}$

$\theta = \underline{\square}\ ^{o}$

$\theta = \underline{\square}\ ^{o}$ *Protractor*

Trigonometry - Finding an Angle with Protractor Verification

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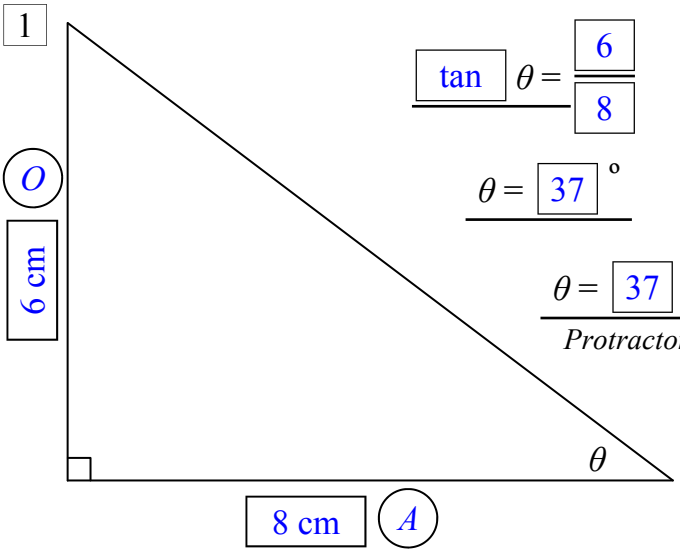
Use a ruler to measure the lengths of the sides with the box, write the side lengths (in whole cm). Then use the circles for '*O*'/'*A*'/'*H*' and use trigonometry to calculate the angle θ, to the nearest degree. Then verify the size of the angle and your calculation by checking it manually with a protractor. SOH CAH TOA

1

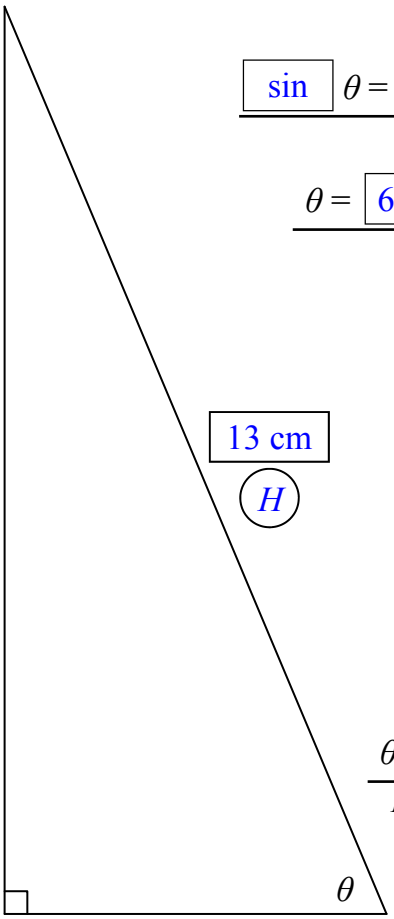


$\tan \theta = \frac{6}{8}$

$\theta = 37^\circ$

$\theta = 37^\circ$ *Protractor*

2

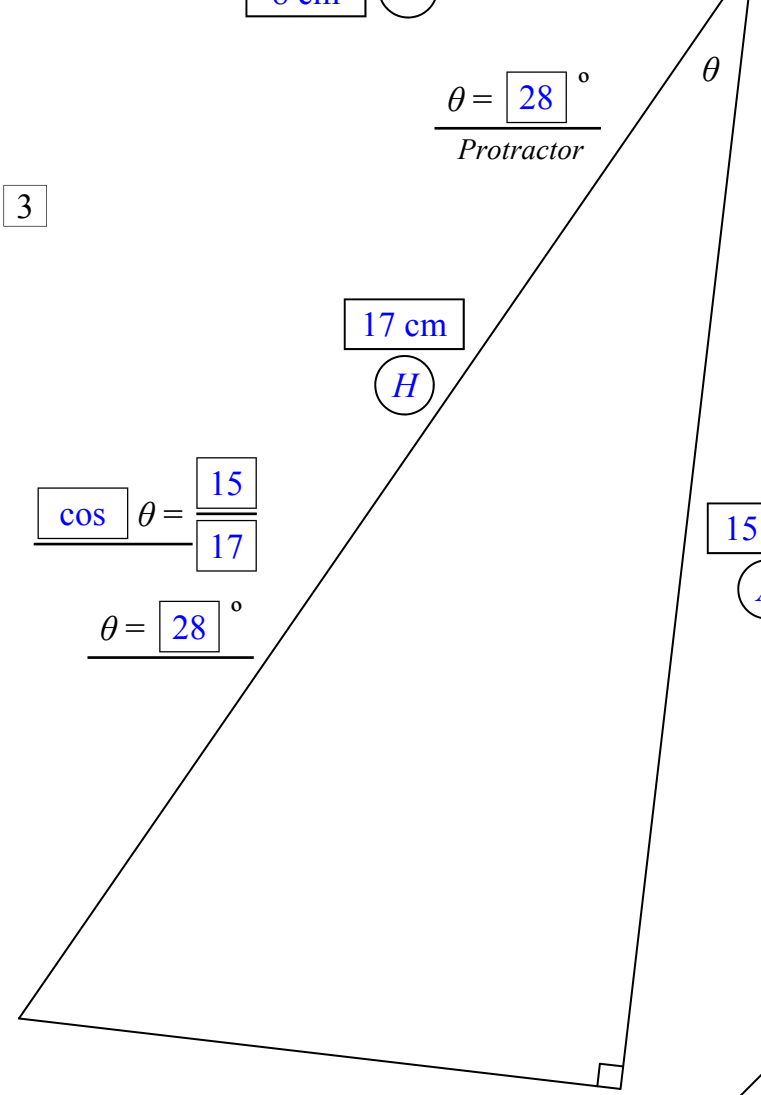


$\sin \theta = \frac{12}{13}$

$\theta = 67^\circ$

$\theta = 67^\circ$ *Protractor*

3

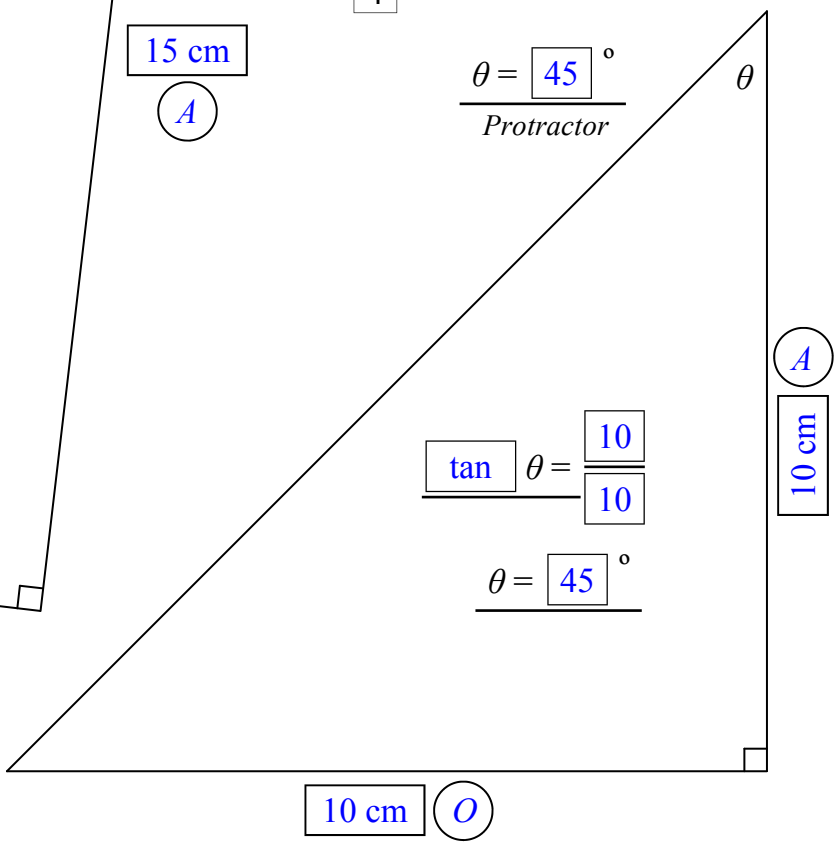


$\cos \theta = \frac{15}{17}$

$\theta = 28^\circ$

$\theta = 28^\circ$ *Protractor*

4



$\theta = 45^\circ$ *Protractor*

$\tan \theta = \frac{10}{10}$

$\theta = 45^\circ$

Trigonometry - Finding an Angle (Using a Ruler) (Homework Strip)

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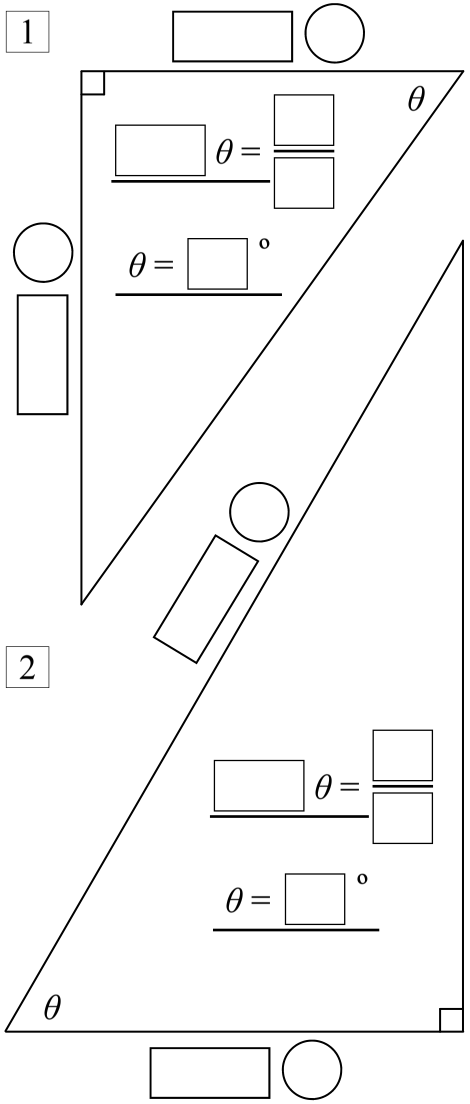
Date Due:

SOH CAH TOA | DEG MODE

Measure the boxed sides and calculate the angle θ to the nearest degree.

1

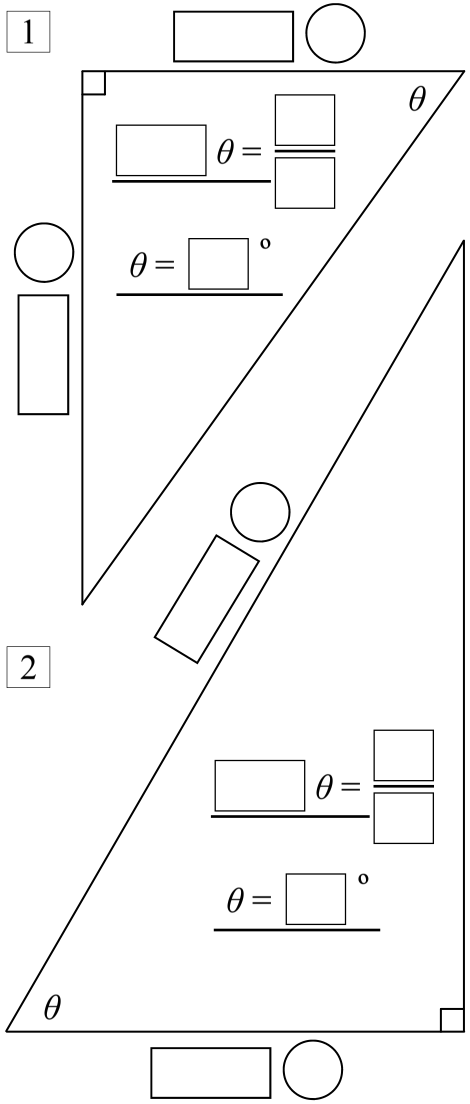


Date Due:

SOH CAH TOA | DEG MODE

Measure the boxed sides and calculate the angle θ to the nearest degree.

1

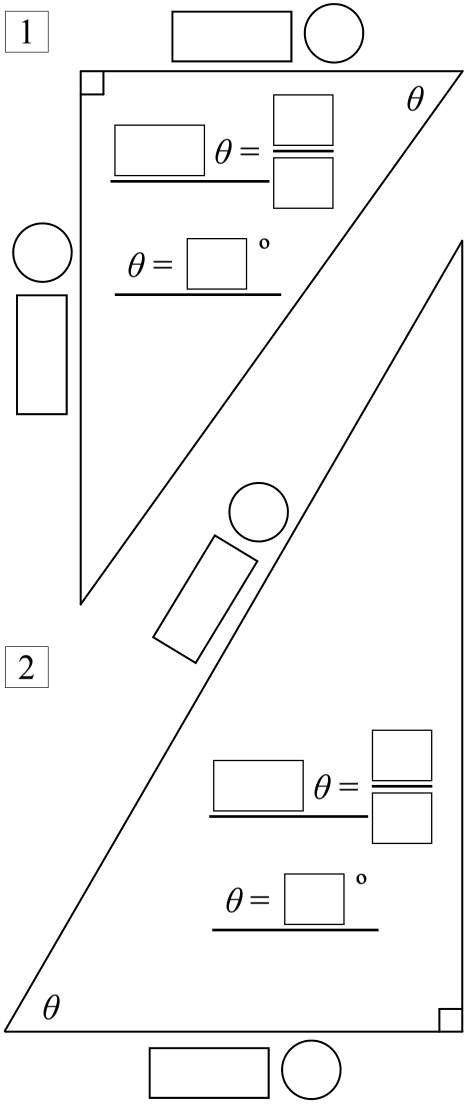


Date Due:

SOH CAH TOA | DEG MODE

Measure the boxed sides and calculate the angle θ to the nearest degree.

1

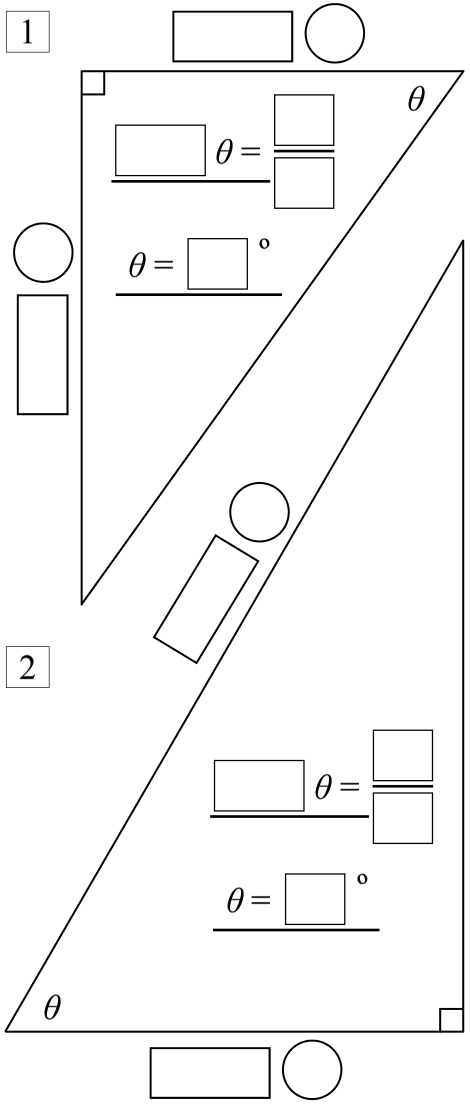


Date Due:

SOH CAH TOA | DEG MODE

Measure the boxed sides and calculate the angle θ to the nearest degree.

1



Trigonometry - Finding an Angle (Using a Ruler) (Homework Strip)

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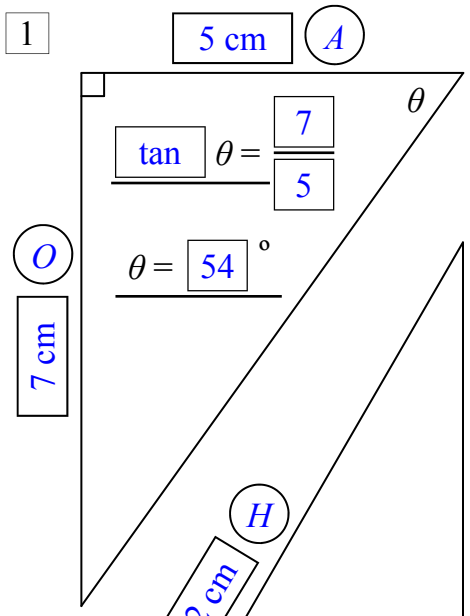
Date Due:

SOH CAH TOA | DEG MODE

Measure the boxed sides and calculate the angle θ to the nearest degree.

1



$\tan \theta = \frac{7}{5}$

$\theta = 54^\circ$

2

$\cos \theta = \frac{6}{12}$

$\theta = 60^\circ$

Trigonometry - Calculator Use (Side)

Trigonometry can be used to calculate the length of side of a triangle. Lengths may have to be rounded and this sheet deals with rounding lengths and using the relevant trig ratio.

Examples: Round these decimals to 1 decimal place.

1 57.362 = 57.4 Because '6' is more than (or equal to) 5, it rounds up

2 51.7206 = 51.7 Because '2' is less than 5, it rounds down

Round these decimals to 2 decimal places.

1 33.362 = 33.36 Because '2' is less than 5, it rounds down

2 10.8963 = 10.90 Because '6' is more than (or equal to) 5, it rounds up

When calculating the side length of a triangle, you could use multiplication or division. Below are examples for both.

Examples: Evaluate the following to 1 decimal place.

MULTIPLICATION

14 tan 32°

Calc: 1 4 TAN 3 2 =

Display 8.748170927

Less than 5 so round down

= 8.7

DIVISION FRACTION KEY

$$\frac{42}{\tan 17^\circ}$$

Calc: 4 2 a/b TAN 1 7) =

Display 137.37581

More than 5 so round up

= 137.4

DIVISION ÷ KEY

$$\frac{135}{\sin 71^\circ}$$

Calc: 1 3 5 ÷ SIN 7 1 =

Display 142.77878792

More than 5 so round up

= 142.8

Trigonometry - Calculator Use (Side)

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Round these decimals to 1 decimal place. No calculator.

Examples

$30.382 = 30.4$ More than 5 Round UP

$73.127 = 73.1$ Less than 5 Round DOWN

1 5.87 ____

2 37.612 ____

3 592.0681 ____

4 136.9716 ____

5 200.6366 ____

Now round to 2 decimal places

6 8.3745 ____

7 95.08863 ____

8 173.9952 ____

9 44.30594 ____

10 599.9953 ____

Perform these calculations and round to 1 decimal place. You don't use the SHIFT key.

11 $5.7 \tan 38^\circ$ = ____

Calc: 5 • 7 TAN 3 8 =

12 $23 \cos 61^\circ$ = ____

13 $82 \tan 18^\circ$ = ____

14 $76.2 \cos 40^\circ$ = ____

15 $114 \tan 48^\circ$ = ____

16 $13.7 \sin 31^\circ$ = ____

17 $57.8 \cos 77^\circ$ = ____

Perform these calculations and round to 2 decimal places. You don't use the SHIFT key.

18 $48 \sin 37^\circ$ = ____

19 $66 \sin 74^\circ$ = ____

20 $14.3 \tan 61^\circ$ = ____

21 $32.7 \cos 19^\circ$ = ____

22 $150.71 \tan 53^\circ$ = ____

23 $0.8 \cos 32^\circ$ = ____

These calculations use the fraction key or division. Answer these to 1 d.p.

24 $\dfrac{9}{\sin 57^\circ}$ = ____

No bracket may give an error

Calc: 9 ▪/□ SIN 5 7) = or

9 ÷ SIN 5 7 =

25 $\dfrac{18.7}{\cos 26^\circ}$ = ____

26 $\dfrac{138}{\tan 71^\circ}$ = ____

27 $\dfrac{54.7}{\tan 20^\circ}$ = ____

28 $\dfrac{3.4}{\sin 66^\circ}$ = ____

29 $\dfrac{277}{\sin 11^\circ}$ = ____

30 $\dfrac{83.5}{\cos 48^\circ}$ = ____

31 $\dfrac{4.2}{\tan 31^\circ}$ = ____

32 $\dfrac{0.64}{\sin 19^\circ}$ = ____

Match the left hand side question with the right hand side answer (to 1 d.p.).

33

$10 \sin 27^\circ$	62.7
$\dfrac{58.5}{\tan 43^\circ}$	4.6
$\dfrac{14.13}{\cos 77^\circ}$	353.6
$11.4 \tan 22^\circ$	4.5
$58 \cos 39^\circ$	45.1
$\dfrac{155}{\sin 26^\circ}$	62.6
$8.8 \tan 82^\circ$	62.8

These include algebraic terms and measurement units. Round to 1 decimal place.

Example

$y = 80.6 \tan 28^\circ$ [cm]

$y = 42.9$ cm

Write letter Show units

34 $x = 38 \sin 52^\circ$ [m]

$x =$ ____ m

35 $y = 52.7 \cos 19^\circ$ [cm]

36 $a = 255 \tan 62^\circ$ [mm]

37 $x = 30.6 \sin 40^\circ$ [m]

38 $y = 450 \tan 45^\circ$ [km]

Trigonometry - Calculator Use (Side)

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Round these decimals to 1 decimal place. No calculator.

Examples

$30.382 = 30.4$ — More than 5 Round UP

$73.127 = 73.1$ — Less than 5 Round DOWN

1. 5.87 — 5.9
2. 37.612 — 37.6
3. 592.0681 — 592.1
4. 136.9716 — 137.0
5. 200.6366 — 200.6

Now round to 2 decimal places

6. 8.3745 — 8.37
7. 95.08863 — 95.09
8. 173.9952 — 174.00
9. 44.30594 — 44.31
10. 599.9953 — 600.00

Perform these calculations and round to 1 decimal place. You don't use the SHIFT key.

11. $5.7 \tan 38^\circ$ = 4.5
    Calc: 5 • 7 TAN 3 8 =
12. $23 \cos 61^\circ$ = 11.2
13. $82 \tan 18^\circ$ = 26.6
14. $76.2 \cos 40^\circ$ = 58.4
15. $114 \tan 48^\circ$ = 126.6
16. $13.7 \sin 31^\circ$ = 7.1
17. $57.8 \cos 77^\circ$ = 13.0

Perform these calculations and round to 2 decimal places. You don't use the SHIFT key.

18. $48 \sin 37^\circ$ = 28.89
19. $66 \sin 74^\circ$ = 63.44
20. $14.3 \tan 61^\circ$ = 25.80
21. $32.7 \cos 19^\circ$ = 30.92
22. $150.71 \tan 53^\circ$ = 200.00
23. $0.8 \cos 32^\circ$ = 0.68

These calculations use the fraction key or division. Answer these to 1 d.p.

24. $\dfrac{9}{\sin 57^\circ}$ = 10.7

No bracket may give an error

Calc: 9 ▪/□ SIN 5 7) = or 9 ÷ SIN 5 7 =

25. $\dfrac{18.7}{\cos 26^\circ}$ = 20.8

26. $\dfrac{138}{\tan 71^\circ}$ = 47.5

27. $\dfrac{54.7}{\tan 20^\circ}$ = 150.3

28. $\dfrac{3.4}{\sin 66^\circ}$ = 3.7

29. $\dfrac{277}{\sin 11^\circ}$ = 1 451.7

30. $\dfrac{83.5}{\cos 48^\circ}$ = 124.8

31. $\dfrac{4.2}{\tan 31^\circ}$ = 7.0

32. $\dfrac{0.64}{\sin 19^\circ}$ = 2.0

Match the left hand side question with the right hand side answer (to 1 d.p.).

33.

$10 \sin 27^\circ$	62.7
$\dfrac{58.5}{\tan 43^\circ}$	4.6
$\dfrac{14.13}{\cos 77^\circ}$	353.6
$11.4 \tan 22^\circ$	4.5
$58 \cos 39^\circ$	45.1
$\dfrac{155}{\sin 26^\circ}$	62.6
$8.8 \tan 82^\circ$	62.8

These include algebraic terms and measurement units. Round to 1 decimal place.

Example

$y = 80.6 \tan 28^\circ$ [cm]

$y = 42.9$ cm

Write letter — Show units

34. $x = 38 \sin 52^\circ$ [m]
    $x =$ 29.9 m
35. $y = 52.7 \cos 19^\circ$ [cm]
    $y = 49.8$ cm
36. $a = 255 \tan 62^\circ$ [mm]
    $a = 479.6$ mm
37. $x = 30.6 \sin 40^\circ$ [m]
    $x = 19.7$ m
38. $y = 450 \tan 45^\circ$ [km]
    $y = 450.0$ km

Trigonometry - Calculator Use (Side) (Homework Strip)

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Date Due: _____

Round these to 1 decimal place

1 45.25 _____

2 13.4927 _____

Round these to 2 decimal places

3 56.4241 _____

4 8.33774 _____

Evaluate these to 1 decimal place. Ensure your calculator is in DEG mode.

5 $13 \sin 71^\circ$ = _____

6 $56 \cos 19^\circ$ = _____

7 $\frac{42}{\tan 38^\circ}$ = _____

8 $\frac{117}{\cos 52^\circ}$ = _____

9 $106 \sin 69^\circ$ = _____

10 $\frac{220}{\cos 59^\circ}$ = _____

11 $\frac{65.4}{\tan 22^\circ}$ = _____

12 $104 \sin 47^\circ$ = _____

Date Due: _____

Round these to 1 decimal place

1 45.25 _____

2 13.4927 _____

Round these to 2 decimal places

3 56.4241 _____

4 8.33774 _____

Evaluate these to 1 decimal place. Ensure your calculator is in DEG mode.

5 $13 \sin 71^\circ$ = _____

6 $56 \cos 19^\circ$ = _____

7 $\frac{42}{\tan 38^\circ}$ = _____

8 $\frac{117}{\cos 52^\circ}$ = _____

9 $106 \sin 69^\circ$ = _____

10 $\frac{220}{\cos 59^\circ}$ = _____

11 $\frac{65.4}{\tan 22^\circ}$ = _____

12 $104 \sin 47^\circ$ = _____

Date Due: _____

Round these to 1 decimal place

1 45.25 _____

2 13.4927 _____

Round these to 2 decimal places

3 56.4241 _____

4 8.33774 _____

Evaluate these to 1 decimal place. Ensure your calculator is in DEG mode.

5 $13 \sin 71^\circ$ = _____

6 $56 \cos 19^\circ$ = _____

7 $\frac{42}{\tan 38^\circ}$ = _____

8 $\frac{117}{\cos 52^\circ}$ = _____

9 $106 \sin 69^\circ$ = _____

10 $\frac{220}{\cos 59^\circ}$ = _____

11 $\frac{65.4}{\tan 22^\circ}$ = _____

12 $104 \sin 47^\circ$ = _____

Date Due: _____

Round these to 1 decimal place

1 45.25 _____

2 13.4927 _____

Round these to 2 decimal places

3 56.4241 _____

4 8.33774 _____

Evaluate these to 1 decimal place. Ensure your calculator is in DEG mode.

5 $13 \sin 71^\circ$ = _____

6 $56 \cos 19^\circ$ = _____

7 $\frac{42}{\tan 38^\circ}$ = _____

8 $\frac{117}{\cos 52^\circ}$ = _____

9 $106 \sin 69^\circ$ = _____

10 $\frac{220}{\cos 59^\circ}$ = _____

11 $\frac{65.4}{\tan 22^\circ}$ = _____

12 $104 \sin 47^\circ$ = _____

Trigonometry - Calculator Use (Side) (Homework Strip)

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Date Due:

Round these to 1 decimal place

1 45.25 → 45.3

2 13.4927 → 13.5

Round these to 2 decimal places

3 56.4241 → 56.42

4 8.33774 → 8.34

Evaluate these to 1 decimal place. Ensure your calculator is in DEG mode.

5 $13 \sin 71^\circ$ = 12.3

6 $56 \cos 19^\circ$ = 52.9

7 $\dfrac{42}{\tan 38^\circ}$ = 53.8

8 $\dfrac{117}{\cos 52^\circ}$ = 190.0

9 $106 \sin 69^\circ$ = 99.0

10 $\dfrac{220}{\cos 59^\circ}$ = 427.2

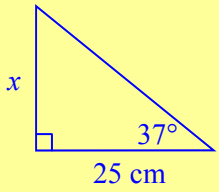
11 $\dfrac{65.4}{\tan 22^\circ}$ = 161.9

12 $104 \sin 47^\circ$ = 76.1

Trigonometry - Finding a Side (tan)

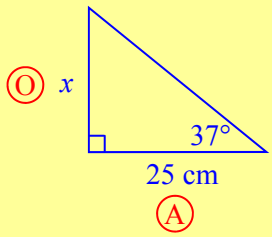
Trigonometry can be used to calculate the length of a side of a triangle. Below are the steps.

Example: Find the length 'x' to 1 decimal place ('x' on top)



STEP 1

LABEL THE SIDES

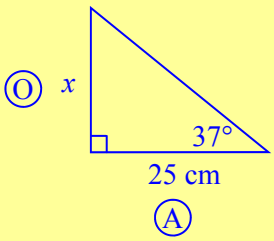


LABEL THE SIDES

Always label the sides O and A. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

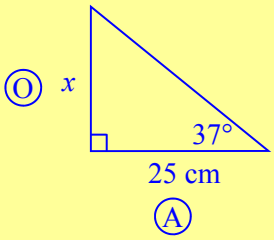


PICK THE RATIO

All the questions on these sheets are TAN, so this step will always result in TAN being used.

STEP 3

WRITE FIRST LINE



WRITE THE FIRST LINE

Write: $\tan 37 = \frac{x}{25}$ — x: OPPOSITE, OVER; 25: ADJACENT

STEP 4

REMOVE THE FRACTION

REMOVE THE FRACTION

$\tan 37 = \frac{x}{25}$

×25 ×25

$25 \tan 37 = x$

$x = 25 \tan 37$

To remove the divided by 25, multiply both sides by 25

This is correct but write it backwards with x on the left hand side

STEP 5

SOLVE

SOLVE USING YOUR CALCULATOR

$x = 25 \tan 37$

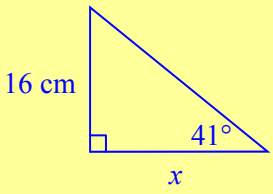
Calc: 2 5 TAN 3 7 =

$x = 18.8$ cm

You will have to round to specified decimal places. Remember the units.

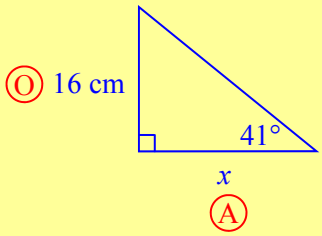
Trigonometry - Finding a Side (tan)

Example: Find the length 'x' to 1 decimal place ('x' on the bottom)



STEP 1

LABEL THE SIDES

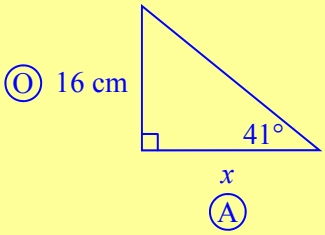


LABEL THE SIDES

Always label the sides O and A. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

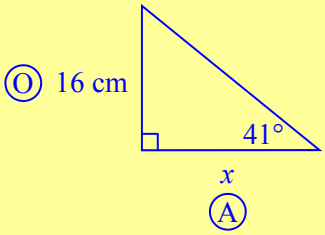


PICK THE RATIO

All the questions on these sheets are TAN, so this step will always result in TAN being used.

STEP 3

WRITE FIRST LINE



WRITE THE FIRST LINE

Write: $\tan 41 = \dfrac{16}{x}$ — 16: OPPOSITE, OVER; x: ADJACENT

STEP 4

SWAP x AND THE TRIG

Look at this example

$3 = \dfrac{6}{2}$ Yes, 3 equals 6 divided by 2

swap

$2 = \dfrac{6}{3}$ Yes, 2 equals 6 divided by 3

SWAP X WITH THE TRIG RATIO

$\tan 41 = \dfrac{16}{x}$ To remove the divided by x, swap the x with the tan 41

swap

$x = \dfrac{16}{\tan 41}$ After the swap, you get this equation

STEP 5

SOLVE

SOLVE USING YOUR CALCULATOR

$x = \dfrac{16}{\tan 41}$

Calc: [1][6][÷][TAN][4][1][)][=]

or

Calc: [1][6][÷][TAN][4][1][=]

$x = 18.4$ cm

The bracket after the 41, will depend on your calculator. You will have to round to specified decimal places. Remember the units.

Trigonometry - Finding a Side (tan) (Sheet 1)

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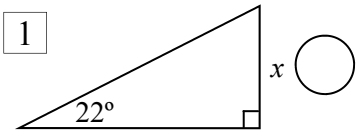
Label these sides '*O*', and '*A*'.

1



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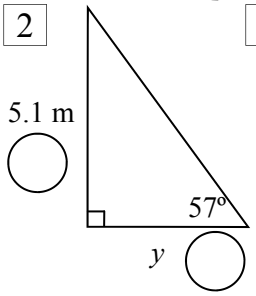

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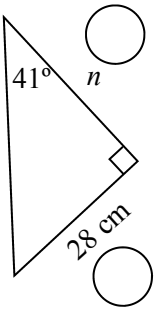

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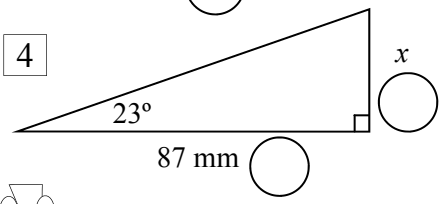


Calculate these, answer to 1 d.p.

5 14 tan 53º = ____

Calc: 1 4 TAN 5 3 =

6 9 tan 35º = ____

7 43.8 tan 32º = ____

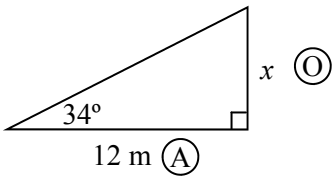
8 $\frac{77}{\tan 63º}$ = ____

9 $\frac{120}{\tan 46º}$ = ____

Calc: 7 7 ÷ TAN 6 3) =

Calculate the side length.
Answering to 1 decimal place.

10



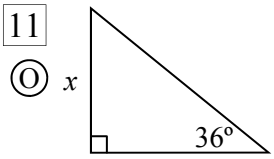
$\tan 34 = \frac{O}{A}$

$\times 12$ $\times 12$

$x = \tan$

$x =$ ____ m 1 d.p.

11



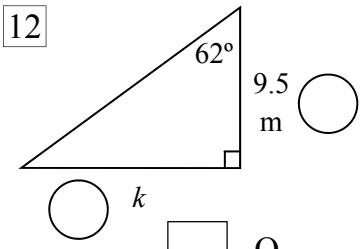
$\tan 36 = \frac{O}{A}$

$\times 55$ $\times 55$

$x = \tan$

$x =$ ____ cm 1 d.p.

12



$\tan 62 = \frac{O}{A}$

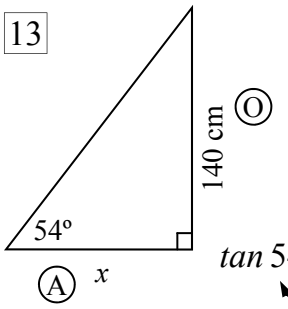
$\times 9.5$ $\times 9.5$

$k = \tan$

$k =$ ____ m 1 d.p.

Now '*x*' (the term) is on the bottom

13



$\tan 54 = \frac{O}{A}$

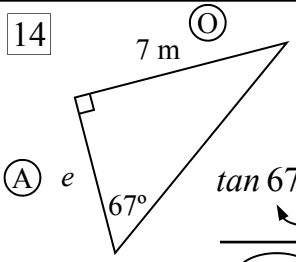
swap swap

$x = \frac{}{\tan}$

$x =$ ____ cm

1 d.p.

14



$\tan 67 = \frac{O}{A}$

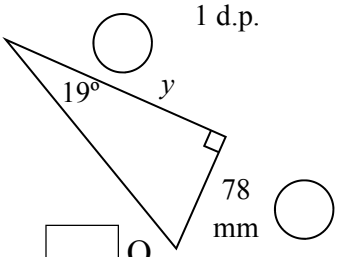
swap swap

$e = \frac{}{\tan}$

$e =$ ____ m

1 d.p.

15



$\tan 19 = \frac{O}{A}$

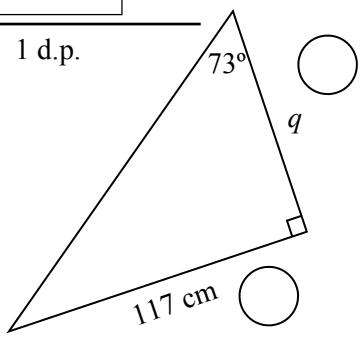
swap swap

$y = \frac{}{\tan}$

$y =$ ____ mm

1 d.p.

16



$\tan 73 = \frac{O}{A}$

swap swap

$q = \frac{}{\tan}$

$q =$ ____ cm

1 d.p.

Trigonometry - Finding a Side (tan) (Sheet 2)

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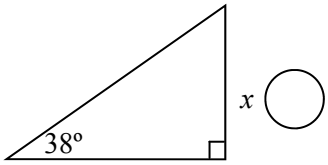
Find the side length (1 d.p.). All are tan, but are a mix of '*x* on top' and '*x* on the bottom'.

1



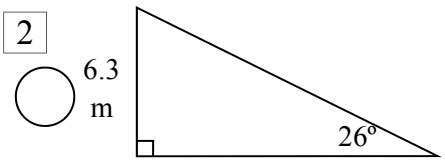
$\tan(\square) = \frac{\square\ \text{O}}{\square\ \text{A}}$

$\times 13$ $\times 13$

$x = \square \tan \square$

$x = \square$ cm 1 d.p.

2



$\tan(\square) = \frac{\square\ \text{O}}{\square\ \text{A}}$

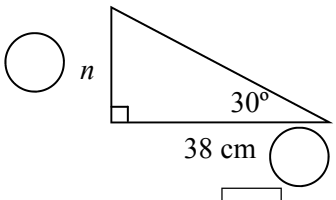
swap swap

$d = \frac{\square}{\tan \square}$

$d = \square$

Remember the units

3

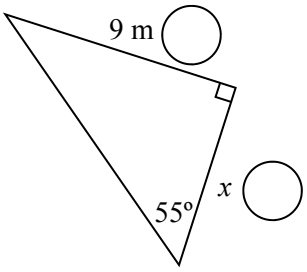


$\tan(\square) = \frac{\square}{\square}$

$n = \square \tan \square$

$n = \square$

4



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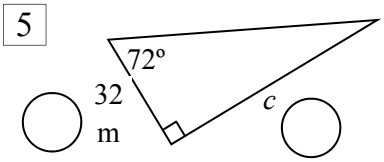

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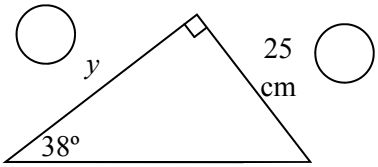

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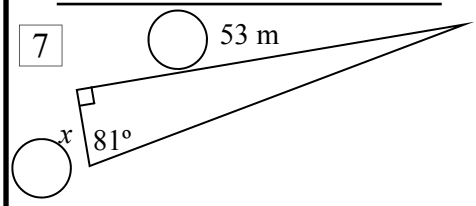
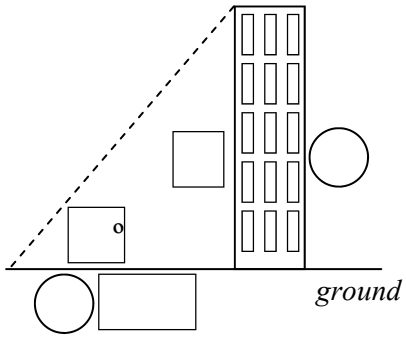


Answer these using your tan trig skills. Find the length to 1 decimal place.

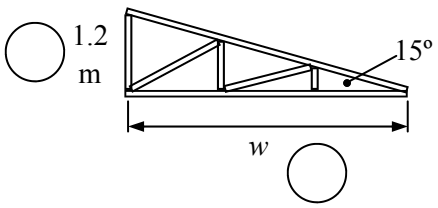
Jess looks at a building that is 72 m away. She looks up at an angle of 54°

8 Write '*x*' as the height of the building, then write the angle and the distance in the boxes on the diagram.



9 Label the distances '*O*' and '*A*' then calculate the building's height.

A roof truss is angled at 15° and has a height of 1.2 m. The width of the roof '*w*' is shown on the diagram.



10 Find '*w*', the width of the roof.

Trigonometry - Finding a Side (tan) (Sheet 1)

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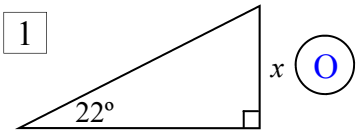
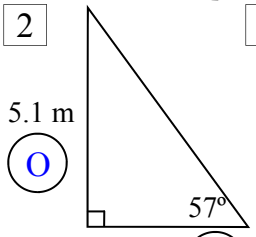
Label these sides '*O*', and '*A*'.

1



2



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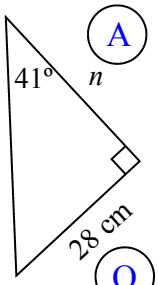
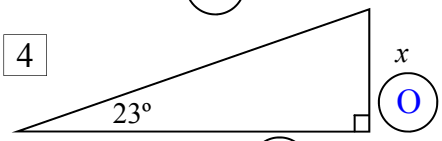


4



Calculate these, answer to 1 d.p.

5 $14 \tan 53° = 18.6$

Calc: 1 4 TAN 5 3 =

6 $9 \tan 35° = 6.3$

7 $43.8 \tan 32° = 27.4$

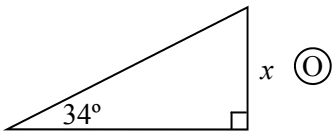
8 $\dfrac{77}{\tan 63°} = 39.2$

9 $\dfrac{120}{\tan 46°} = 115.9$

Calc: 7 7 ▪ TAN 6 3) =

Calculate the side length.
Answering to 1 decimal place.

10

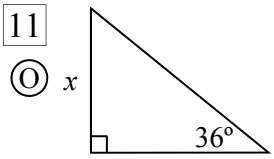


$\tan 34 = \dfrac{x\ \text{O}}{12\ \text{A}}$ (× 12) (× 12)

$x = 12 \tan 34$

$x = 8.1$ m 1 d.p.

11

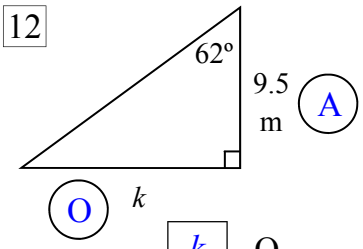


$\tan 36 = \dfrac{x\ \text{O}}{55\ \text{A}}$ (× 55) (× 55)

$x = 55 \tan 36$

$x = 40.0$ cm 1 d.p.

12



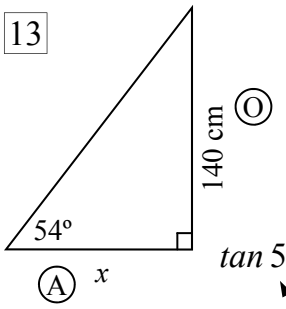
$\tan 62 = \dfrac{k\ \text{O}}{9.5\ \text{A}}$ (× 9.5) (× 9.5)

$k = 9.5 \tan 62$

$k = 17.9$ m 1 d.p.

Now '*x*' (the term) is on the bottom

13

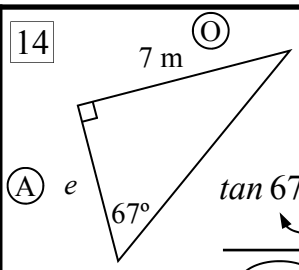


$\tan 54 = \dfrac{140\ \text{O}}{x\ \text{A}}$ swap swap

$x = \dfrac{140}{\tan 54}$

$x = 101.7$ cm 1 d.p.

14

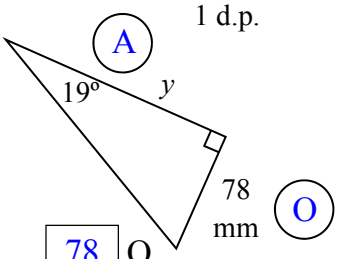


$\tan 67 = \dfrac{7\ \text{O}}{e\ \text{A}}$ swap swap

$e = \dfrac{7}{\tan 67}$

$e = 3.0$ m 1 d.p.

15

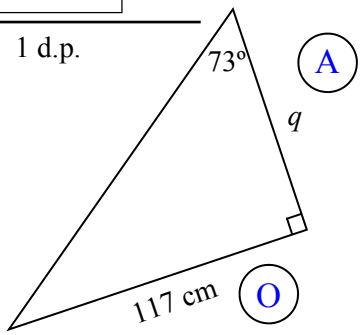


$\tan 19 = \dfrac{78\ \text{O}}{y\ \text{A}}$ swap swap

$y = \dfrac{78}{\tan 19}$

$y = 226.5$ mm 1 d.p.

16



$\tan 73 = \dfrac{117\ \text{O}}{q\ \text{A}}$ swap swap

$q = \dfrac{117}{\tan 73}$

$q = 35.8$ cm 1 d.p.

Trigonometry - Finding a Side (tan) (Sheet 2)

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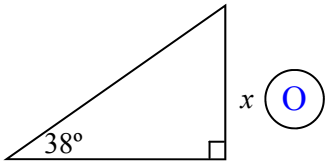
Find the side length (1 d.p.). All are tan, but are a mix of '*x* on top' and '*x* on the bottom'.

1



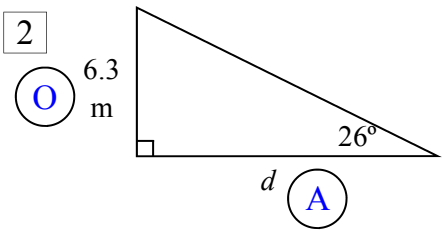
$\tan 38 = \frac{x}{13}$ (O / A)

$\times 13$ $\times 13$

$x = 13 \tan 38$

$x = 10.2$ cm 1 d.p.

2



$\tan 26 = \frac{6.3}{d}$ (O / A)

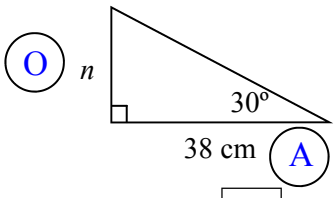
swap swap

$d = \frac{6.3}{\tan 26}$

$d = 12.9$ m

Remember the units

3



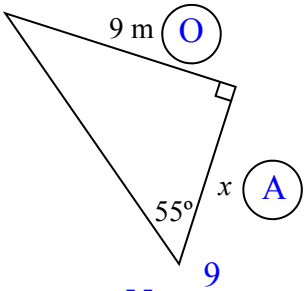
$\tan 30 = \frac{n}{38}$

$\times 38$ $\times 38$

$n = 38 \tan 30$

$n = 21.9$ cm

4



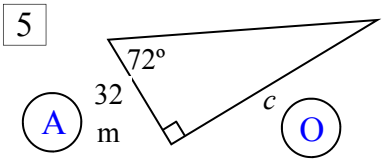
$\tan 55 = \frac{9}{x}$

swap swap

$x = \frac{9}{\tan 55}$

$x = 6.3$ m

5



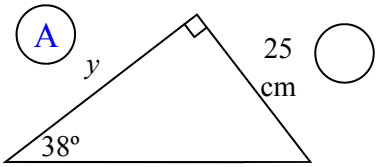
$\tan 72 = \frac{c}{32}$

$\times 32$ $\times 32$

$c = 32 \tan 72$

$c = 98.5$ m

6



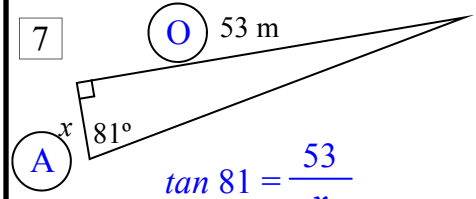
$\tan 38 = \frac{25}{y}$

swap swap

$y = \frac{25}{\tan 38}$

$y = 32.0$ cm

7



$\tan 81 = \frac{53}{x}$

swap swap

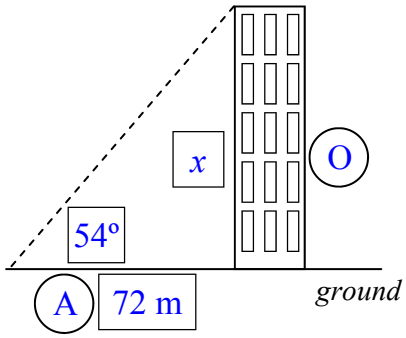
$x = \frac{53}{\tan 81}$

$y = 8.4$ m

Answer these using your tan trig skills. Find the length to 1 decimal place.

Jess looks at a building that is 72 m away. She looks up at an angle of 54°

8 Write '*x*' as the height of the building, then write the angle and the distance in the boxes on the diagram.



9 Label the distances '*O*' and '*A*' then calculate the building's height.

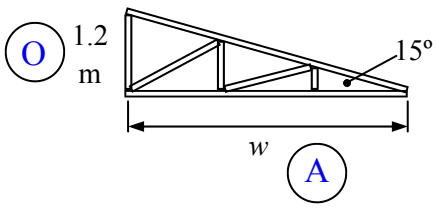
$\tan 54 = \frac{x}{72}$

$\times 72$ $\times 72$

$x = 72 \tan 54$

$x = 99.1$ m

A roof truss is angled at 15° and has a height of 1.2 m. The width of the roof '*w*' is shown on the diagram.



10 Find '*w*', the width of the roof.

$\tan 15 = \frac{1.2}{w}$

swap swap

$w = \frac{1.}{\tan 15}$

$w = 4.5$ m

Trigonometry - Finding a Side (tan) (Homework Strip)

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Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 17 tan 23º = _____

2 60.3 tan 67º = _____

3 $\frac{106}{\tan 44º}$ = _____

4 $\frac{40.7}{\tan 52º}$ = _____

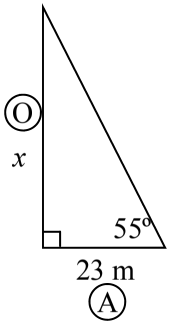
5 $\tan 55 = \frac{\square\ O}{\square\ A}$

× 23 × 23

$x = \square \tan \square$

$x = \square$ m

1 d.p.



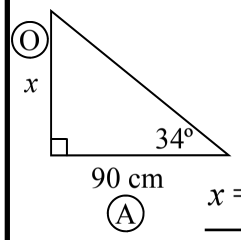
6 $\tan 34 = \frac{\square\ O}{\square\ A}$

× 90 × 90

$x = \square \tan \square$

$x = \square$ cm

1 d.p.



Date Due: _____

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1 17 tan 23º = _____

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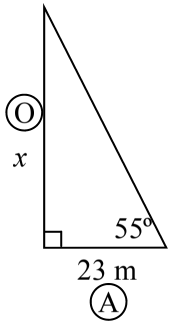
× 23 × 23

$x = \square \tan \square$

$x = \square$ m

1 d.p.



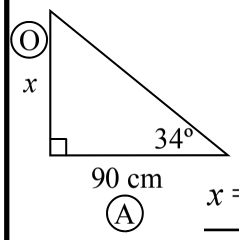
6 $\tan 34 = \frac{\square\ O}{\square\ A}$

× 90 × 90

$x = \square \tan \square$

$x = \square$ cm

1 d.p.



Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 17 tan 23º = _____

2 60.3 tan 67º = _____

3 $\frac{106}{\tan 44º}$ = _____

4 $\frac{40.7}{\tan 52º}$ = _____

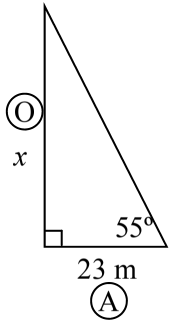
5 $\tan 55 = \frac{\square\ O}{\square\ A}$

× 23 × 23

$x = \square \tan \square$

$x = \square$ m

1 d.p.



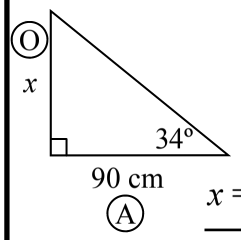
6 $\tan 34 = \frac{\square\ O}{\square\ A}$

× 90 × 90

$x = \square \tan \square$

$x = \square$ cm

1 d.p.



Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 17 tan 23º = _____

2 60.3 tan 67º = _____

3 $\frac{106}{\tan 44º}$ = _____

4 $\frac{40.7}{\tan 52º}$ = _____

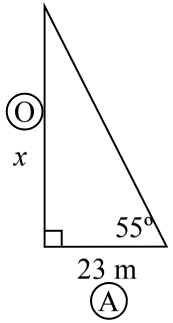
5 $\tan 55 = \frac{\square\ O}{\square\ A}$

× 23 × 23

$x = \square \tan \square$

$x = \square$ m

1 d.p.



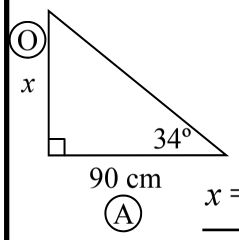
6 $\tan 34 = \frac{\square\ O}{\square\ A}$

× 90 × 90

$x = \square \tan \square$

$x = \square$ cm

1 d.p.



Trigonometry - Finding a Side (tan) (Homework Strip)

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Date Due:

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $17 \tan 23^\circ = 7.2$

2 $60.3 \tan 67^\circ = 142.1$

3 $\dfrac{106}{\tan 44^\circ} = 109.8$

4 $\dfrac{40.7}{\tan 52^\circ} = 31.8$

5

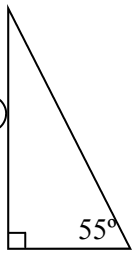
$\tan 55 = \dfrac{x \text{ (O)}}{23 \text{ (A)}}$

$\times 23$ $\times 23$

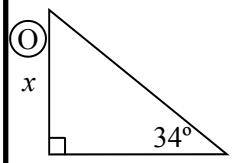
$x = 23 \tan 55$

$x = 32.8$ m

1 d.p.



6



$\tan 34 = \dfrac{x \text{ (O)}}{90 \text{ (A)}}$

$\times 90$ $\times 90$

$x = 90 \tan 34$

$x = 60.7$ cm

1 d.p.

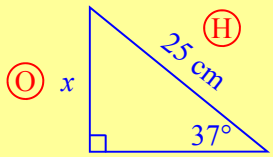
Trigonometry - Finding a Side (sin)

Trigonometry can be used to calculate the length of a side of a triangle. Below are the steps.

Example: Find the length 'x' to 1 decimal place ('x' on top)

STEP 1

LABEL THE SIDES

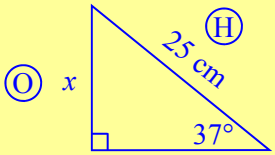


LABEL THE SIDES

Always label the sides O and H. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

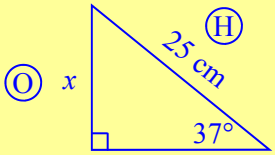


PICK THE RATIO

All the questions on these sheets are SIN, so this step will always result in SIN being used.

STEP 3

WRITE FIRST LINE



WRITE THE FIRST LINE

Write: $\sin 37 = \frac{x}{25}$

x ← OPPOSITE

OVER

25 ← HYPOTENUSE

STEP 4

REMOVE THE FRACTION

REMOVE THE FRACTION

$\sin 37 = \frac{x}{25}$

To remove the divided by 25, multiply both sides by 25

×25 ×25

$25 \sin 37 = x$

$x = 25 \sin 37$

This is correct but write it backwards with x on the left hand side

STEP 5

SOLVE

SOLVE USING YOUR CALCULATOR

$x = 25 \sin 37$

Calc: 2 5 SIN 3 7 =

$x = 15.0$ cm

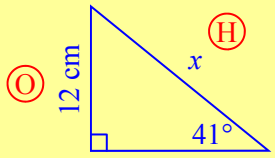
You will have to round to specified decimal places. Remember the units.

Trigonometry - Finding a Side (sin)

Example: Find the length 'x' to 1 decimal place ('x' on the bottom)

STEP 1

LABEL THE SIDES

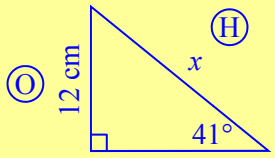


LABEL THE SIDES

Always label the sides O and H. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

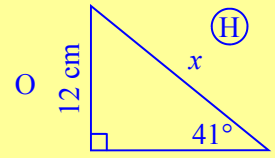


PICK THE RATIO

All the questions on these sheets are SIN, so this step will always result in SIN being used.

STEP 3

WRITE FIRST LINE



WRITE THE FIRST LINE

Write: $\sin 41 = \dfrac{12}{x}$

12 ← OPPOSITE

OVER

x ← HYPOTENUSE

STEP 4

SWAP x AND THE TRIG

Look at this example

$3 = \dfrac{6}{2}$ Yes, 3 equals 6 divided by 2

swap

$2 = \dfrac{6}{3}$ Yes, 2 equals 6 divided by 3

SWAP X WITH THE TRIG RATIO

$\sin 41 = \dfrac{12}{x}$ To remove the divided by x, swap the x with the sin 41

swap

$x = \dfrac{12}{\sin 41}$ After the swap, you get this equation

STEP 5

SOLVE

SOLVE USING YOUR CALCULATOR

$x = \dfrac{12}{\sin 41}$

Calc: 1 2 ■ SIN 4 1) =

or

Calc: 1 2 ÷ SIN 4 1 =

$x = 18.3$ cm

The bracket after the 41, will depend on your calculator.
You will have to round to specified decimal places.
Remember the units.

Trigonometry - Finding a Side (sin) (Sheet 1)

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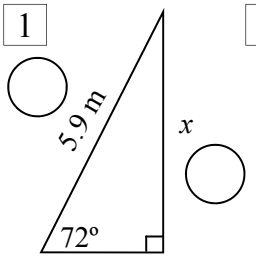
Label these sides '*O*', and '*H*'.

1



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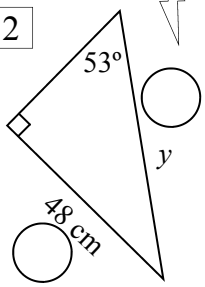
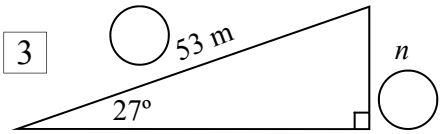


3



Calculate these, answer to 1 d.p.

4 $\dfrac{13}{\sin 21^\circ}$ = ____

5 $\dfrac{54.7}{\sin 77^\circ}$ = ____

Calc: 1 3 ▪/□ SIN 2 1) =

6 $94 \sin 36^\circ$ = ____

Calc: 9 4 SIN 3 6 =

7 $11.2 \sin 62^\circ$ = ____

8 $118 \sin 48^\circ$ = ____

9 $\dfrac{200}{\sin 55^\circ}$ = ____

10 $\dfrac{0.8}{\sin 32^\circ}$ = ____

Calculate the side length.
Answering to 1 decimal place.

11 (O) *x*, (H) 31 cm, 28°

$\sin 28 = \dfrac{O}{H}$

$\times 31$ $\times 31$

$x =$ ____ $\sin$ ____

$x =$ ____ cm 1 d.p.

12 (H) 185 mm, *e*, (O), 38°

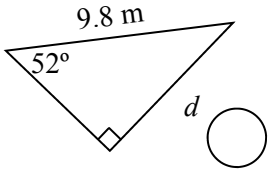
$\sin 38 = \dfrac{O}{H}$

$\times 185$ $\times 185$

$e =$ ____ $\sin$ ____

$e =$ ____ mm 1 d.p.

13



$\sin 52 = \dfrac{\square}{\square}$

$\times 9.8$ $\times 9.8$

$d =$ ____ $\sin$ ____

$d =$ ____ m 1 d.p.

Now '*x*' (the term) is on the bottom

14 (H) *x*, 18°, 3.5 m, (O)

$\sin 18 = \dfrac{O}{H}$

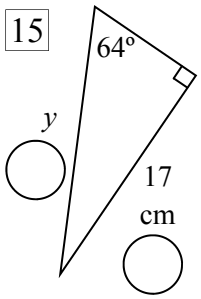
swap swap

$x =$ ____ / $\sin$ ____

$x =$ ____ m

1 d.p.

15



$\sin 64 = \dfrac{O}{H}$

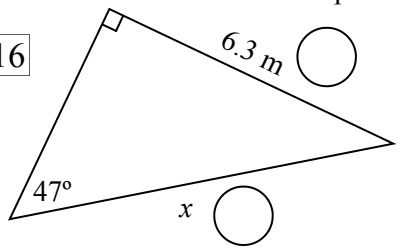
swap swap

$y =$ ____ / $\sin$ ____

$y =$ ____ cm

1 d.p.

16



$\sin 47 = \dfrac{\square}{\square}$

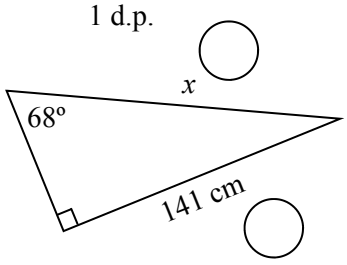
swap swap

$x =$ ____ / $\sin$ ____

$x =$ ____ m

1 d.p.

17



$\sin 68 = \dfrac{\square}{\square}$

swap swap

$x =$ ____ / $\sin$ ____

$x =$ ____ cm

1 d.p.

Trigonometry - Finding a Side (sin) (Sheet 2)

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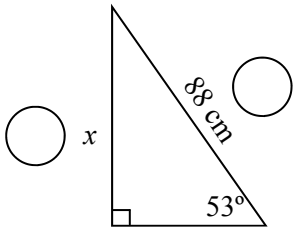
Find the side length (1 d.p.). All are sin, but are a mix of '*x* on top' and '*x* on the bottom'.

1



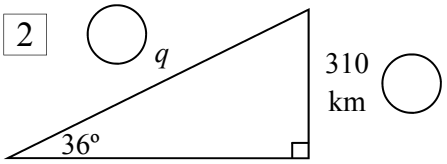
$\sin(\square) = \frac{\square\ \text{O}}{\square\ \text{H}}$

$\times 88$ $\times 88$

$x = \square \sin \square$

$x = \square$ cm 1 d.p.

2



$\sin(\square) = \frac{\square\ \text{O}}{\square\ \text{H}}$

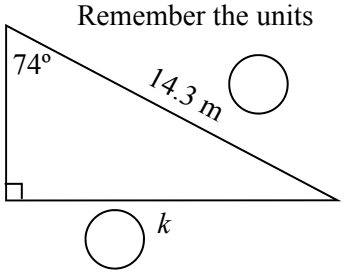
swap swap

$q = \frac{\square}{\sin \square}$

$q = \square$

Remember the units

3

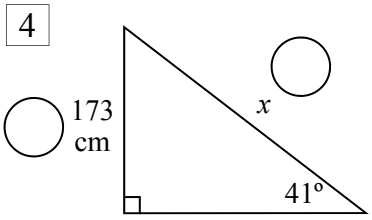


$\sin(\square) = \frac{\square}{\square}$

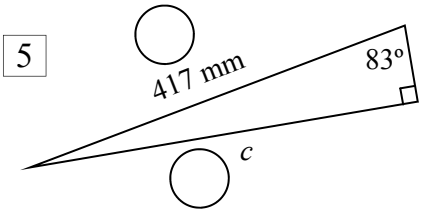
$k = \square \sin \square$

$k = \square$

4



5



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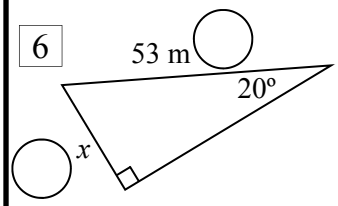

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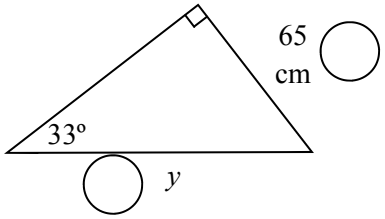
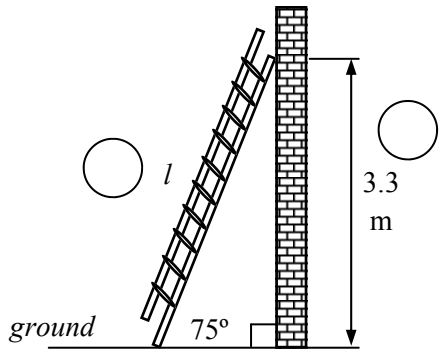


Answer these using your sin trig skills. Find the lengths to 1 decimal place.

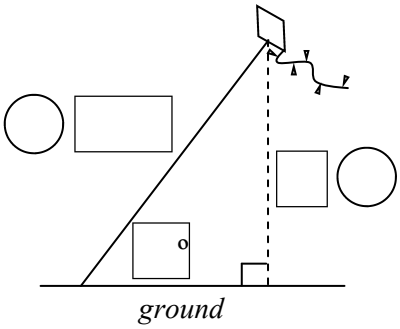
A ladder is leaning against a wall making an angle of 75° with the ground. The ladder reaches 3.3m up the wall.



8 Find '*l*', the length of the ladder.

A kite string is 55 m long and is fully extended. The string makes an angle of 64° with the ground.

9 Write '*x*' as the height of the kite. Then write the string length and angle in the two boxes on the diagram



10 Label the distances 'O' and 'H' then calculate the height of the kite.

Trigonometry - Finding a Side (sin) (Sheet 1)

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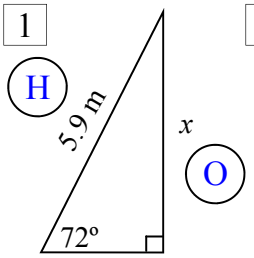
Label these sides '*O*', and '*H*'.

1



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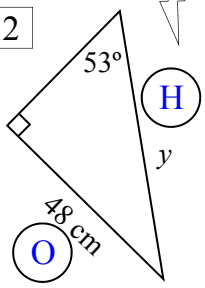
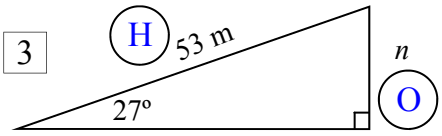


3



Calculate these, answer to 1 d.p.

4 $\dfrac{13}{\sin 21°}$ = 36.3

5 $\dfrac{54.7}{\sin 77°}$ = 56.1

Calc: 1 3 ▪ SIN 2 1) =

6 94 sin 36° = 55.3

Calc: 9 4 SIN 3 6 =

7 11.2 sin 62° = 9.9

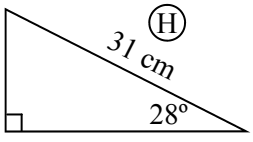
8 118 sin 48° = 87.7

9 $\dfrac{200}{\sin 55°}$ = 244.2

10 $\dfrac{0.8}{\sin 32°}$ = 1.5

Calculate the side length. Answering to 1 decimal place.

11

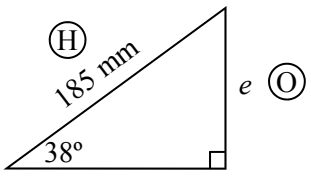


$\sin 28 = \dfrac{x\ \text{O}}{31\ \text{H}}$ (×31) (×31)

$x =$ 31 sin 28

$x =$ 14.6 cm 1 d.p.

12

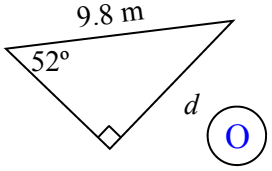


$\sin 38 = \dfrac{e\ \text{O}}{185\ \text{H}}$ (×185) (×185)

$e =$ 185 sin 38

$e =$ 113.9 mm 1 d.p.

13



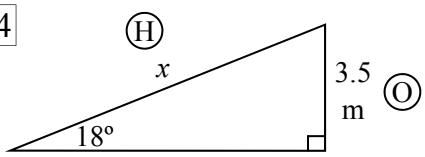
$\sin 52 = \dfrac{d}{9.8}$ (×9.8) (×9.8)

$d =$ 9.8 sin 52

$d =$ 7.7 m 1 d.p.

Now '*x*' (the term) is on the bottom

14



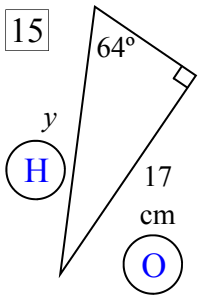
$\sin 18 = \dfrac{3.5\ \text{O}}{x\ \text{H}}$ swap swap

$x = \dfrac{3.5}{\sin 18}$

$x =$ 11.3 m

1 d.p.

15



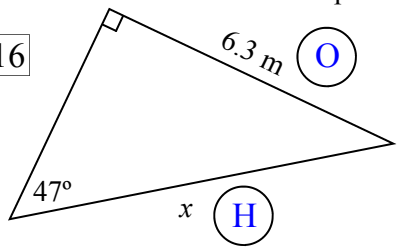
$\sin 64 = \dfrac{17\ \text{O}}{y\ \text{H}}$ swap swap

$y = \dfrac{17}{\sin 64}$

$y =$ 18.9 cm

1 d.p.

16



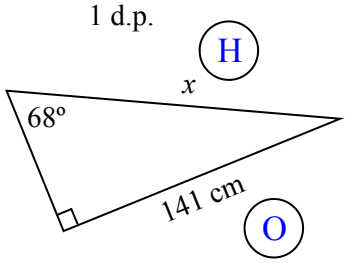
$\sin 47 = \dfrac{6.3}{x}$ swap swap

$x = \dfrac{6.3}{\sin 47}$

$x =$ 8.6 m

1 d.p.

17



$\sin 68 = \dfrac{141}{x}$ swap swap

$x = \dfrac{141}{\sin 68}$

$x =$ 152.1 cm

1 d.p.

Trigonometry - Finding a Side (sin) (Sheet 2)

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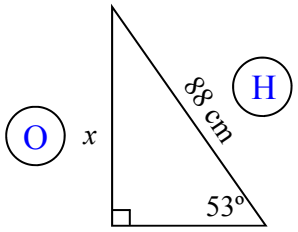
Find the side length (1 d.p.). All are sin, but are a mix of '*x* on top' and '*x* on the bottom'.

1



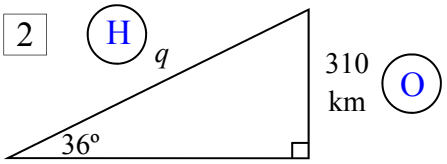
$\sin 53 = \frac{x}{88}$ (O / H)

× 88 × 88

$x = 88 \sin 53$

$x = 70.3$ cm 1 d.p.

2



$\sin 36 = \frac{310}{q}$ (O / H)

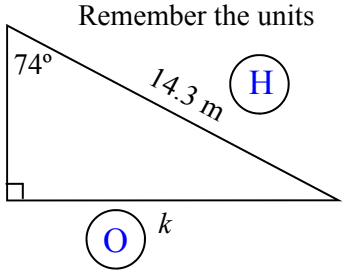
swap swap

$q = \frac{310}{\sin 36}$

$q =$ 527.4 km

Remember the units

3



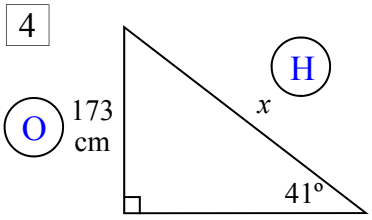
$\sin 74 = \frac{k}{14.3}$

× 14.3 × 14.3

$k = 14.3 \sin 74$

$k =$ 13.8 m

4



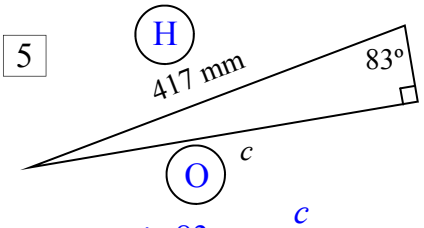
$\sin 41 = \frac{173}{x}$

swap swap

$x = \frac{173}{\sin 41}$

$x = 263.7$ cm

5



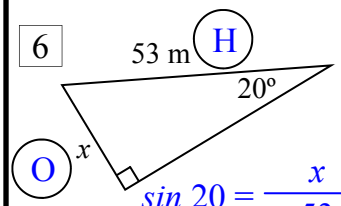
$\sin 83 = \frac{c}{417}$

× 417 × 417

$c = 417 \sin 72$

$c = 396.6$ mm

6



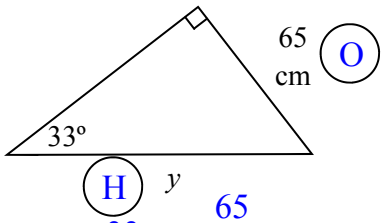
$\sin 20 = \frac{x}{53}$

× 53 × 53

$x = 53 \sin 20$

$x = 18.1$ m

7



$\sin 33 = \frac{65}{y}$

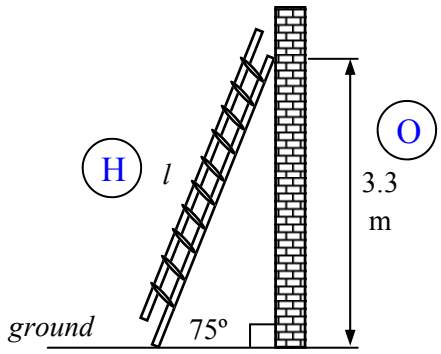
swap swap

$y = \frac{65}{\sin 33}$

$y = 119.3$ cm

Answer these using your sin trig skills. Find the lengths to 1 decimal place.

A ladder is leaning against a wall making an angle of 75° with the ground. The ladder reaches 3.3m up the wall.



8 Find '*l*', the length of the ladder.

$\sin 75 = \frac{3.3}{l}$

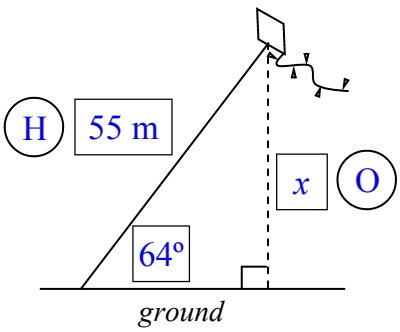
swap swap

$l = \frac{3.3}{\sin 75}$

$l = 3.4$ m

A kite string is 55 m long and is fully extended. The string makes an angle of 64° with the ground.

9 Write '*x*' as the height of the kite. Then write the string length and angle in the two boxes on the diagram



10 Label the distances 'O' and 'H' then calculate the height of the kite.

$\sin 64 = \frac{x}{55}$

× 55 × 55

$x = 55 \sin 64$

$x = 49.4$ m

Trigonometry - Finding a Side (sin) (Homework Strip)

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Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $30 \sin 53^\circ$ = _____

2 $90.6 \sin 41^\circ$ = _____

3 $\dfrac{322}{\sin 67^\circ}$ = _____

4 $\dfrac{8.04}{\sin 76^\circ}$ = _____

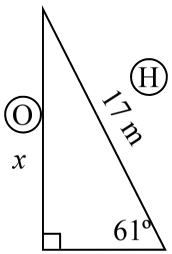
5 $\sin 61 = \dfrac{\square\ O}{\square\ H}$

$\dfrac{}{\times 17 \quad \times 17}$

$x = \square \sin \square$

$x =$ _____ m

1 d.p.



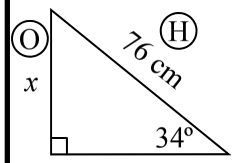
6 $\sin 34 = \dfrac{\square\ O}{\square\ H}$

$\dfrac{}{\times 76 \quad \times 76}$

$x = \square \sin \square$

$x =$ _____ cm

1 d.p.



Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $30 \sin 53^\circ$ = _____

2 $90.6 \sin 41^\circ$ = _____

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4 $\dfrac{8.04}{\sin 76^\circ}$ = _____

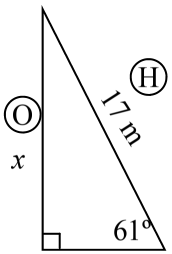
5 $\sin 61 = \dfrac{\square\ O}{\square\ H}$

$\dfrac{}{\times 17 \quad \times 17}$

$x = \square \sin \square$

$x =$ _____ m

1 d.p.



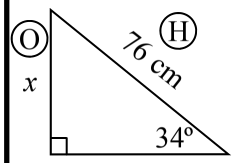
6 $\sin 34 = \dfrac{\square\ O}{\square\ H}$

$\dfrac{}{\times 76 \quad \times 76}$

$x = \square \sin \square$

$x =$ _____ cm

1 d.p.



Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $30 \sin 53^\circ$ = _____

2 $90.6 \sin 41^\circ$ = _____

3 $\dfrac{322}{\sin 67^\circ}$ = _____

4 $\dfrac{8.04}{\sin 76^\circ}$ = _____

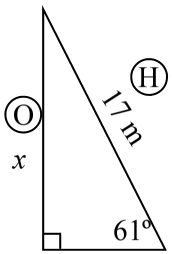
5 $\sin 61 = \dfrac{\square\ O}{\square\ H}$

$\dfrac{}{\times 17 \quad \times 17}$

$x = \square \sin \square$

$x =$ _____ m

1 d.p.



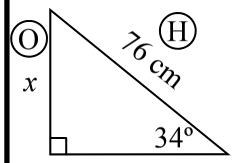
6 $\sin 34 = \dfrac{\square\ O}{\square\ H}$

$\dfrac{}{\times 76 \quad \times 76}$

$x = \square \sin \square$

$x =$ _____ cm

1 d.p.



Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $30 \sin 53^\circ$ = _____

2 $90.6 \sin 41^\circ$ = _____

3 $\dfrac{322}{\sin 67^\circ}$ = _____

4 $\dfrac{8.04}{\sin 76^\circ}$ = _____

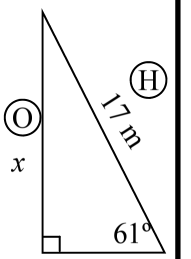
5 $\sin 61 = \dfrac{\square\ O}{\square\ H}$

$\dfrac{}{\times 17 \quad \times 17}$

$x = \square \sin \square$

$x =$ _____ m

1 d.p.



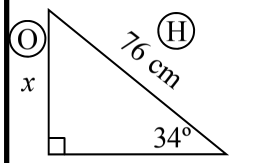
6 $\sin 34 = \dfrac{\square\ O}{\square\ H}$

$\dfrac{}{\times 76 \quad \times 76}$

$x = \square \sin \square$

$x =$ _____ cm

1 d.p.



Trigonometry - Finding a Side (sin) (Homework Strip)

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Date Due:

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $30 \sin 53^\circ$ = 24.0

2 $90.6 \sin 41^\circ$ = 59.4

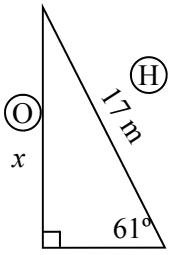
3 $\dfrac{322}{\sin 67^\circ}$ = 349.8

4 $\dfrac{8.04}{\sin 76^\circ}$ = 8.3

5

$\sin 61 = \dfrac{x \text{ (O)}}{17 \text{ (H)}}$

$\times 17$ $\times 17$

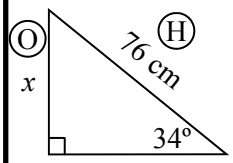


$x =$ 17 $\sin$ 61

$x =$ 14.9 m

1 d.p.

6



$\sin 34 = \dfrac{x \text{ (O)}}{76 \text{ (H)}}$

$\times 76$ $\times 76$

$x =$ 76 $\sin$ 34

$x =$ 42.5 cm

1 d.p.

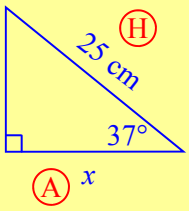
Trigonometry - Finding a Side (cos)

Trigonometry can be used to calculate the length of a side of a triangle. Below are the steps.

Example: Find the length 'x' to 1 decimal place ('x' on top)

STEP 1

LABEL THE SIDES

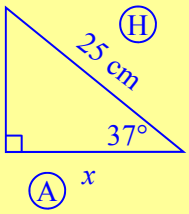


LABEL THE SIDES

Always label the sides A and H. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

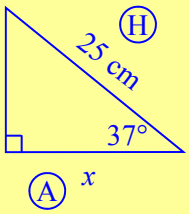


PICK THE RATIO

All the questions on these sheets are COS, so this step will always result in COS being used.

STEP 3

WRITE FIRST LINE



WRITE THE FIRST LINE

Write: $\cos 37 = \frac{x}{25}$

x ← ADJACENT

OVER

25 ← HYPOTENUSE

STEP 4

REMOVE THE FRACTION

REMOVE THE FRACTION

$\cos 37 = \frac{x}{25}$

To get remove the divided by 25, multiply both sides by 25

×25 ×25

$25 \cos 37 = x$

$x = 25 \cos 37$

This is correct but write it backwards with x on the left hand side

STEP 5

SOLVE

SOLVE USING YOUR CALCULATOR

$x = 25 \cos 37$

Calc: 2 5 COS 3 7 =

$x = 20.0$ cm

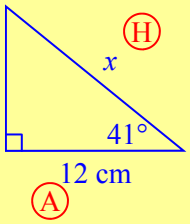
You will have to round to specified decimal places. Remember the units.

Trigonometry - Finding a Side (cos)

Example: Find the length 'x' to 1 decimal place ('x' on the bottom)

STEP 1

LABEL THE SIDES

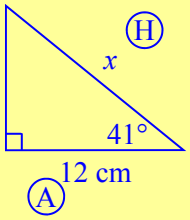


LABEL THE SIDES

Always label the sides A and H. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

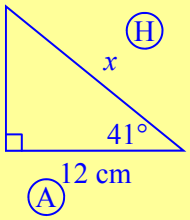


PICK THE RATIO

All the questions on these sheets are COS, so this step will always result in COS being used.

STEP 3

WRITE FIRST LINE



WRITE THE FIRST LINE

Write: $\cos 41 = \frac{12}{x}$ ← ADJACENT, OVER, ← HYPOTENUSE

STEP 4

SWAP x AND THE TRIG

Look at this example

$3 = \frac{6}{2}$ (swap) Yes, 3 equals 6 divided by 2

$2 = \frac{6}{3}$ Yes, 2 equals 6 divided by 3

SWAP X WITH THE TRIG RATIO

$\cos 41 = \frac{12}{x}$ (swap)

To remove the divided by x, swap the x with the sin 41

$x = \frac{12}{\cos 41}$

After the swap, you get this equation

STEP 5

SOLVE

SOLVE USING YOUR CALCULATOR

$x = \frac{12}{\cos 41}$

Calc: 12 ▣ COS 41) =

or

Calc: 12 ÷ COS 41 =

$x = 15.9$ cm

The bracket after the 41, will depend on your calculator.
You will have to round to specified decimal places.
Remember the units.

Trigonometry - Finding a Side (cos) (Sheet 1)

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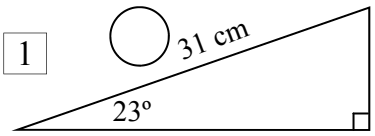
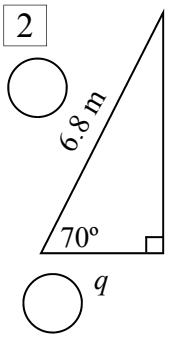
Label these sides '*A*', and '*H*'.

1



2



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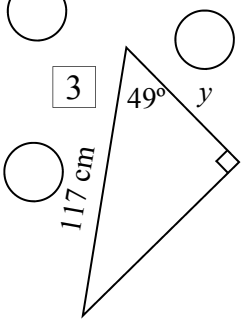


Calculate these, answer to 1 d.p.

4 $\dfrac{57}{\cos 36^\circ}$ = ____

5 $\dfrac{102}{\cos 8^\circ}$ = ____

Calc: 5 7 ▪ COS 3 6) =

6 $94 \cos 36^\circ$ = ____

Calc: 9 4 COS 3 6 =

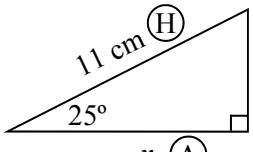
7 $\dfrac{40.4}{\cos 19^\circ}$ = ____

8 $\dfrac{655}{\cos 42^\circ}$ = ____

9 $0.2 \cos 28^\circ$ = ____

Calculate the side length.
Answering to 1 decimal place.

10



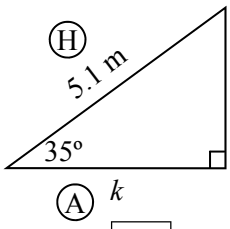
$\cos 25 = \dfrac{\square}{\square}\ \begin{matrix}A\\H\end{matrix}$

(×11) (×11)

$x =$ ____ cos ____

$x =$ ____ cm 1 d.p.

11



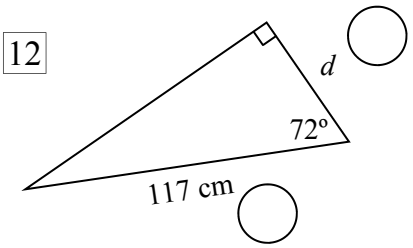
$\cos 35 = \dfrac{\square}{\square}\ \begin{matrix}A\\H\end{matrix}$

(×5.1) (×5.1)

$k =$ ____ cos ____

$k =$ ____ m 1 d.p.

12



$\cos 72 = \dfrac{\square}{\square}$

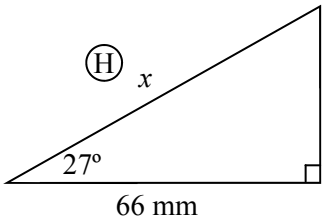
(×117) (×117)

$d =$ ____ cos ____

$d =$ ____ cm 1 d.p.

Now '*x*' (the term) is on the bottom

13



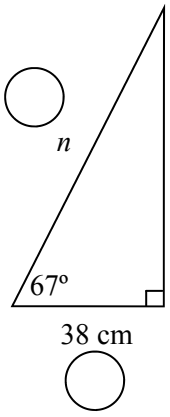
$\cos 27 = \dfrac{\square}{\square}\ \begin{matrix}A\\H\end{matrix}$

swap swap

$x =$ ____ / cos ____

$x =$ ____ mm 1 d.p.

14



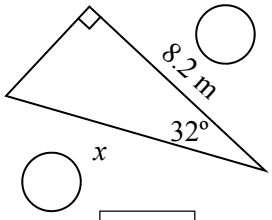
$\cos 67 = \dfrac{\square}{\square}\ \begin{matrix}A\\H\end{matrix}$

swap swap

$n =$ ____ / cos ____

$n =$ ____ cm 1 d.p.

15



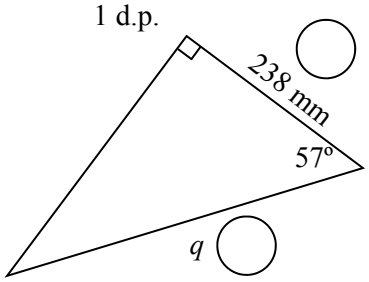
$\cos 32 = \dfrac{\square}{\square}$

swap swap

$x =$ ____ / cos ____

$x =$ ____ m 1 d.p.

16



$\cos 57 = \dfrac{\square}{\square}$

swap swap

$q =$ ____ / cos ____

$q =$ ____ mm 1 d.p.

Trigonometry - Finding a Side (cos) (Sheet 2)

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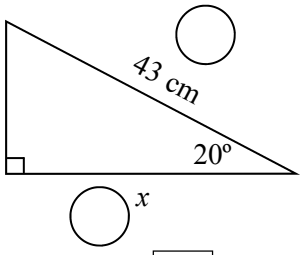
Find the side length (1 d.p.). All are sin, but are a mix of '*x* on top' and '*x* on the bottom'.

1



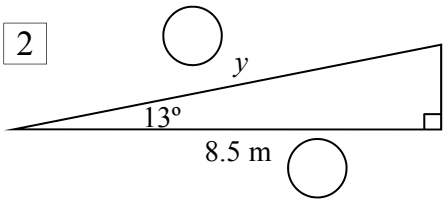
$\cos \square = \frac{\square\ \text{A}}{\square\ \text{H}}$

$\times 43$ $\times 43$

$x = \square \cos \square$

$x = \square$ cm 1 d.p.

2



$\cos \square = \frac{\square\ \text{A}}{\square\ \text{H}}$

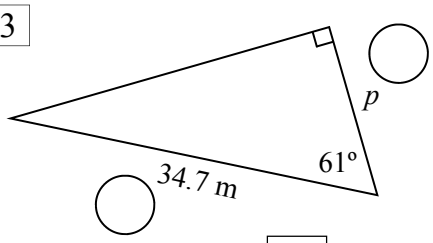
swap swap

$y = \frac{\square}{\cos \square}$

$y = \square$

Remember the units

3

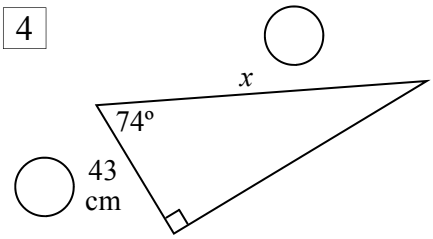


$\cos \square = \frac{\square}{\square}$

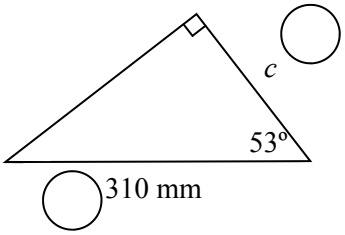
$p = \square \cos \square$

$p = \square$

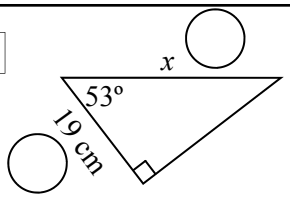
4



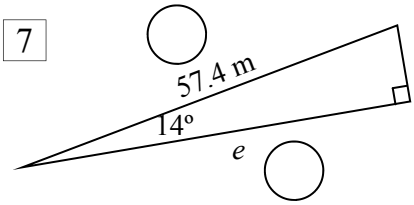
5



6

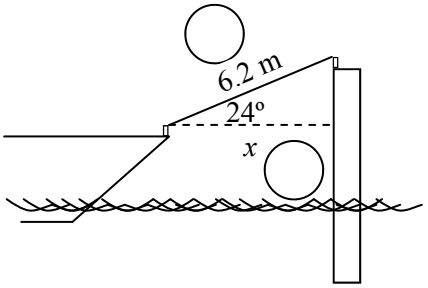


7



Answer these using your cos trig skills. Find the lengths to 1 decimal place.

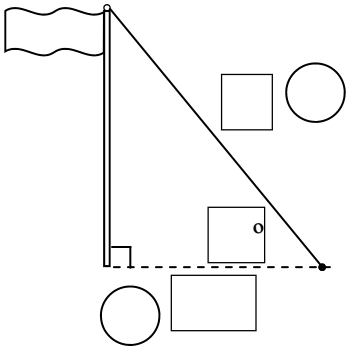
A boat is moored to a jetty. The rope is 6.2 m long and the angle of the rope is 24º. Label sides '*A*' & '*H*'.



8 Calculate the distance '*x*' of the boat from the jetty.

A flag pole has a guy wire attached to it which is secured to the ground 6 m from the pole. The guy wire makes an angle of 51º with the ground.

9 Write '*d*' as the length of the guy wire. Write the distance and angle in the two boxes on the diagram.



10 Label the distances '*A*' and '*H*' then calculate the length of the guy wire.

Trigonometry - Finding a Side (cos) (Sheet 1)

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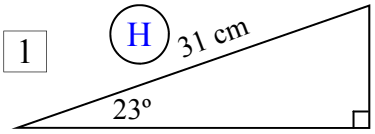
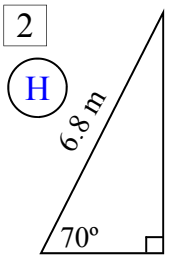
Label these sides '*A*', and '*H*'.

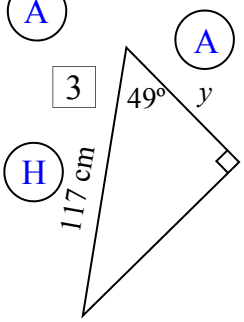
1



2



3



Calculate these, answer to 1 d.p.

4 $\dfrac{57}{\cos 36°}$ = 70.5

5 $\dfrac{102}{\cos 8°}$ = 103.0

Calc: 57 COS 36) =

6 94 cos 36° = 76.0

Calc: 94 COS 36 =

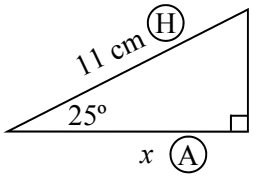
7 $\dfrac{40.4}{\cos 19°}$ = 42.7

8 $\dfrac{655}{\cos 42°}$ = 881.4

9 0.2 cos 28° = 0.2

Calculate the side length. Answering to 1 decimal place.

10

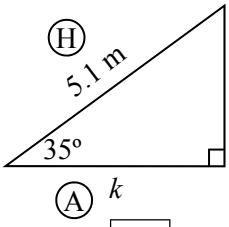


$\cos 25 = \dfrac{x \text{ A}}{11 \text{ H}}$ (×11) (×11)

$x = 11 \cos 25$

$x =$ 10.0 cm 1 d.p.

11

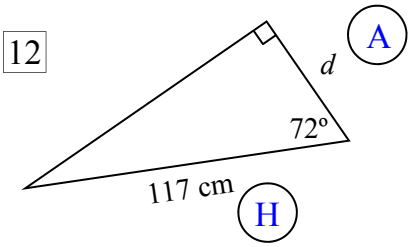


$\cos 35 = \dfrac{k \text{ A}}{5.1 \text{ H}}$ (×5.1) (×5.1)

$k = 5.1 \cos 35$

$k =$ 4.2 m 1 d.p.

12



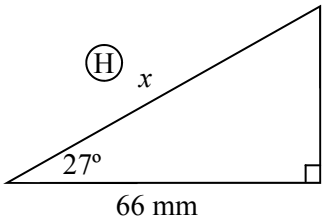
$\cos 72 = \dfrac{d}{117}$ (×117) (×117)

$d = 117 \cos 72$

$d =$ 36.2 cm 1 d.p.

Now '*x*' (the term) is on the bottom

13

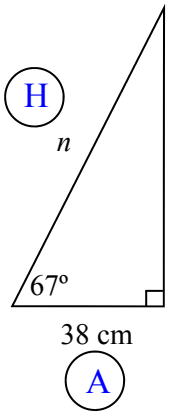


$\cos 27 = \dfrac{66 \text{ A}}{x \text{ H}}$ swap swap

$x = \dfrac{66}{\cos 27}$

$x =$ 74.1 mm 1 d.p.

14

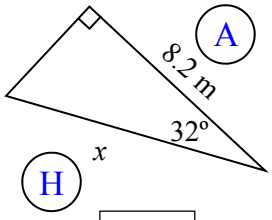


$\cos 67 = \dfrac{38 \text{ A}}{n \text{ H}}$ swap swap

$n = \dfrac{38}{\cos 67}$

$n =$ 97.3 cm 1 d.p.

15

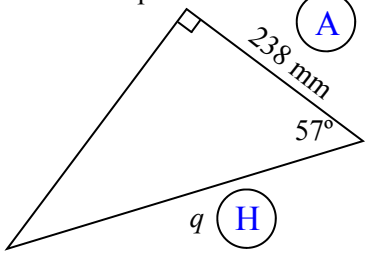


$\cos 32 = \dfrac{8.2}{x}$ swap swap

$x = \dfrac{8.2}{\cos 32}$

$x =$ 9.7 m 1 d.p.

16



$\cos 57 = \dfrac{238}{q}$ swap swap

$q = \dfrac{238}{\cos 57}$

$q =$ 437.0 mm 1 d.p.

Trigonometry - Finding a Side (cos) (Sheet 2)

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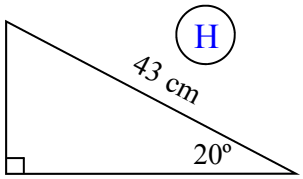
Find the side length (1 d.p.). All are sin, but are a mix of '*x* on top' and '*x* on the bottom'.

1



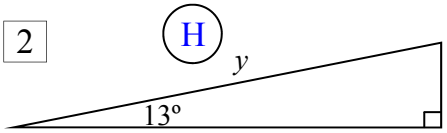
$\cos 20 = \frac{x\ \text{A}}{43\ \text{H}}$

$\times 43$ $\times 43$

$x = 43 \cos 20$

$x = 40.4$ cm 1 d.p.

2



$\cos 13 = \frac{8.5\ \text{A}}{y\ \text{H}}$

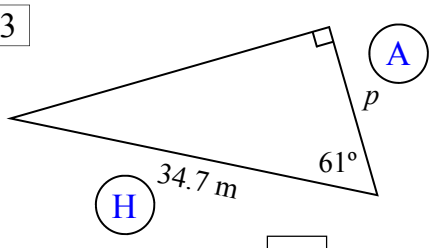
swap swap

$y = \frac{8.5}{\cos 13}$

$y = 8.7$ m

Remember the units

3



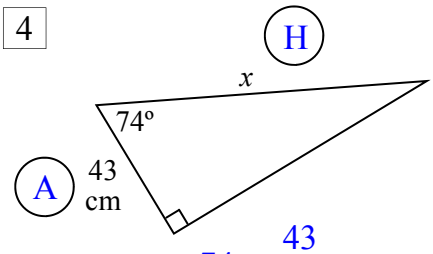
$\cos 61 = \frac{p}{34.7}$

$\times 34.7$ $\times 34.7$

$p = 34.7 \cos 61$

$p = 16.8$ m

4



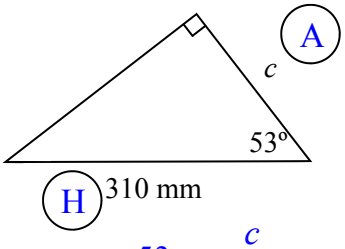
$\cos 74 = \frac{43}{x}$

swap swap

$x = \frac{43}{\cos 74}$

$x = 156.0$ cm

5



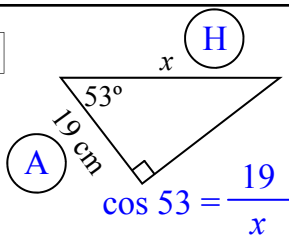
$\cos 53 = \frac{c}{310}$

$\times 310$ $\times 310$

$c = 310 \cos 53$

$c = 186.6$ mm

6



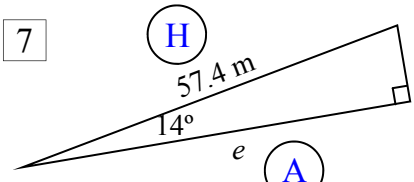
$\cos 53 = \frac{19}{x}$

swap swap

$x = \frac{19}{\cos 53}$

$x = 31.6$ cm

7



$\cos 14 = \frac{e}{57.4}$

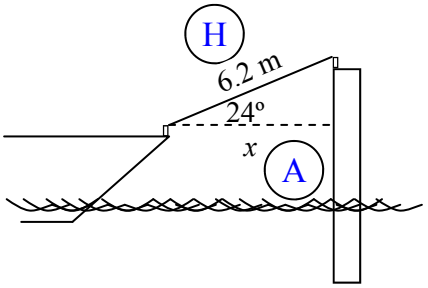
$\times 57.4$ $\times 57.4$

$e = 57.4 \cos 14$

$e = 55.7$ m

Answer these using your cos trig skills. Find the lengths to 1 decimal place.

A boat is moored to a jetty. The rope is 6.2 m long and the angle of the rope is 24°. Label sides '*A*' & '*H*'.



8 Calculate the distance '*x*' of the boat from the jetty.

$\cos 24 = \frac{x}{6.2}$

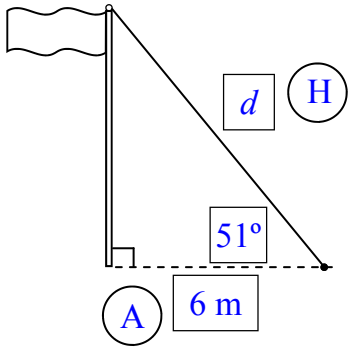
$\times 6.2$ $\times 6.2$

$x = 6.2 \cos 24$

$x = 5.7$ m

A flag pole has a guy wire attached to it which is secured to the ground 6 m from the pole. The guy wire makes an angle of 51° with the ground.

9 Write '*d*' as the length of the guy wire. Write the distance and angle in the two boxes on the diagram.



10 Label the distances '*A*' and '*H*' then calculate the length of the guy wire.

$\cos 51 = \frac{6}{d}$

swap swap

$d = \frac{6}{\cos 51}$

$d = 9.5$ m

Trigonometry - Finding a Side (cos) (Homework Strip)

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Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $29 \cos 64^\circ$ = _____

2 $77.6 \cos 36^\circ$ = _____

3 $\dfrac{19.8}{\cos 51^\circ}$ = _____

4 $\dfrac{560}{\cos 81^\circ}$ = _____

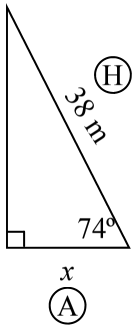
5 $\cos 74 = \dfrac{\square\ (A)}{\square\ (H)}$

$\times 38 \quad \times 38$

$x = \square \cos \square$

$x =$ _____ m

1 d.p.



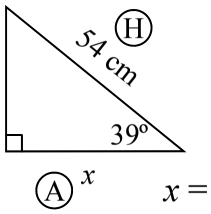
6 $\cos 39 = \dfrac{\square\ (A)}{\square\ (H)}$

$\times 54 \quad \times 54$

$x = \square \cos \square$

$x =$ _____ cm

1 d.p.



Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $29 \cos 64^\circ$ = _____

2 $77.6 \cos 36^\circ$ = _____

3 $\dfrac{19.8}{\cos 51^\circ}$ = _____

4 $\dfrac{560}{\cos 81^\circ}$ = _____

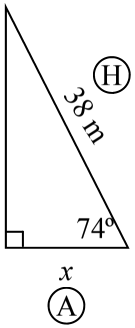
5 $\cos 74 = \dfrac{\square\ (A)}{\square\ (H)}$

$\times 38 \quad \times 38$

$x = \square \cos \square$

$x =$ _____ m

1 d.p.



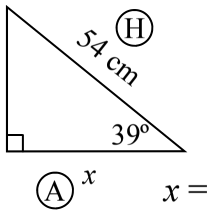
6 $\cos 39 = \dfrac{\square\ (A)}{\square\ (H)}$

$\times 54 \quad \times 54$

$x = \square \cos \square$

$x =$ _____ cm

1 d.p.



Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $29 \cos 64^\circ$ = _____

2 $77.6 \cos 36^\circ$ = _____

3 $\dfrac{19.8}{\cos 51^\circ}$ = _____

4 $\dfrac{560}{\cos 81^\circ}$ = _____

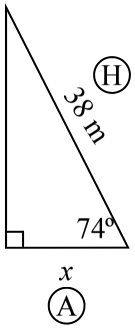
5 $\cos 74 = \dfrac{\square\ (A)}{\square\ (H)}$

$\times 38 \quad \times 38$

$x = \square \cos \square$

$x =$ _____ m

1 d.p.



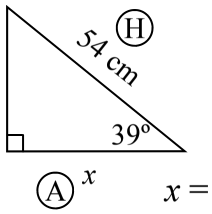
6 $\cos 39 = \dfrac{\square\ (A)}{\square\ (H)}$

$\times 54 \quad \times 54$

$x = \square \cos \square$

$x =$ _____ cm

1 d.p.



Date Due: _____

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $29 \cos 64^\circ$ = _____

2 $77.6 \cos 36^\circ$ = _____

3 $\dfrac{19.8}{\cos 51^\circ}$ = _____

4 $\dfrac{560}{\cos 81^\circ}$ = _____

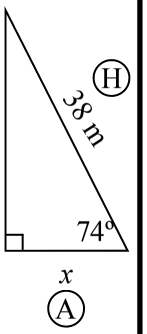
5 $\cos 74 = \dfrac{\square\ (A)}{\square\ (H)}$

$\times 38 \quad \times 38$

$x = \square \cos \square$

$x =$ _____ m

1 d.p.



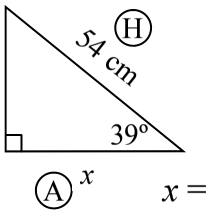
6 $\cos 39 = \dfrac{\square\ (A)}{\square\ (H)}$

$\times 54 \quad \times 54$

$x = \square \cos \square$

$x =$ _____ cm

1 d.p.



Trigonometry - Finding a Side (cos) (Homework Strip)

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Date Due:

CALCULATOR IN DEG MODE

Evaluate these to 1 decimal place

1 $29 \cos 64^\circ = 12.7$

2 $77.6 \cos 36^\circ = 62.5$

3 $\dfrac{19.8}{\cos 51^\circ} = 30.8$

4 $\dfrac{560}{\cos 81^\circ} = 3\,579.8$

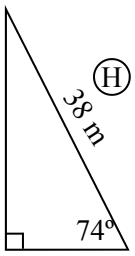
5 $\cos 74 = \dfrac{x\ (A)}{38\ (H)}$

$\times 38$ $\times 38$

$x = 38 \cos 74$

$x = 10.5$ m

1 d.p.



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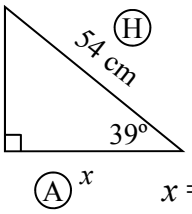


$\cos 39 = \dfrac{x\ (A)}{54\ (H)}$

$\times 54$ $\times 54$

$x = 54 \cos 39$

$x = 42.0$ cm

1 d.p.

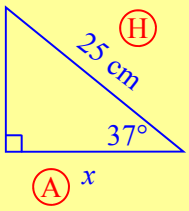
Trigonometry - Finding a Side

Trigonometry can be used to calculate the length of a side of a triangle. Below are the steps.

Example: Find the length 'x' to 1 decimal place ('x' on top)

STEP 1

LABEL THE SIDES

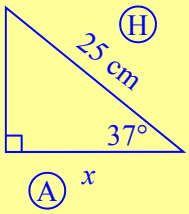


LABEL THE SIDES

Always label the sides, in this case it is A and H. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

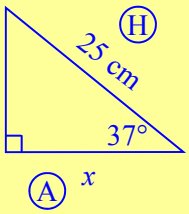


PICK THE RATIO

Use SOH CAH TOA to determine the ratio. This triangle has A and H, that means you use COS.

STEP 3

WRITE FIRST LINE



WRITE THE FIRST LINE

Write: $\cos 37 = \frac{x}{25}$

x ← ADJACENT

OVER

25 ← HYPOTENUSE

STEP 4

REMOVE THE FRACTION

REMOVE THE FRACTION

$\cos 37 = \frac{x}{25}$

To get remove the divided by 25, multiply both sides by 25

×25 ×25

$25 \cos 37 = x$

$x = 25 \cos 37$

This is correct but write it backwards with x on the left hand side

STEP 5

SOLVE

SOLVE USING YOUR CALCULATOR

$x = 25 \cos 37$

Calc: 2 5 COS 3 7 =

$x = 20.0$ cm

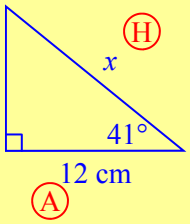
You will have to round to specified decimal places. Remember the units.

Trigonometry - Finding a Side

Example: Find the length 'x' to 1 decimal place ('x' on the bottom)

STEP 1

LABEL THE SIDES

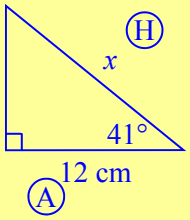


LABEL THE SIDES

Always label the sides, in this case it is A and H. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

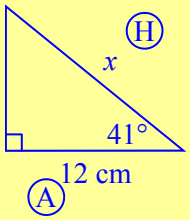


PICK THE RATIO

Use SOH CAH TOA to determine the ratio to use. This triangle has A and H, that means you use COS.

STEP 3

WRITE FIRST LINE



WRITE THE FIRST LINE

Write: $\cos 41 = \dfrac{12}{x}$

12 ← ADJACENT

OVER

x ← HYPOTENUSE

STEP 4

SWAP x AND THE TRIG

Look at this example

$3 = \dfrac{6}{2}$ Yes, 3 equals 6 divided by 2

swap

$2 = \dfrac{6}{3}$ Yes, 2 equals 6 divided by 3

SWAP X WITH THE TRIG RATIO

$\cos 41 = \dfrac{12}{x}$ To remove the divided by x, swap the x with the sin 41

swap

$x = \dfrac{12}{\cos 41}$ After the swap, you get this equation

STEP 5

SOLVE

SOLVE USING YOUR CALCULATOR

$x = \dfrac{12}{\cos 41}$

Calc: 1 2 ■ COS 4 1) =

or

Calc: 1 2 ÷ COS 4 1 =

$x = 15.9$ cm

The bracket after the 41, will depend on your calculator.
You will have to round to specified decimal places.
Remember the units.

Trigonometry - Finding a Side - All Ratios - '*x*' on top (Sheet 1)

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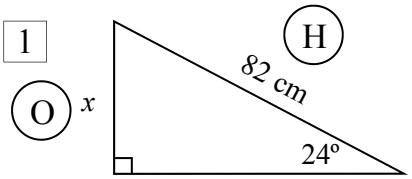
Find the side length to 1 d.p. These are a mix of sin, cos and tan. Use SOH CAH TOA.

1



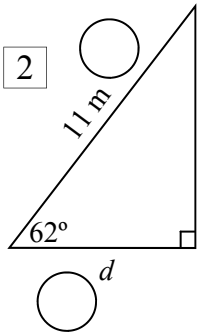
$\sin \square = \frac{\square\ O}{\square\ H}$

$\times 82$ $\times 82$

$x = \square \sin \square$

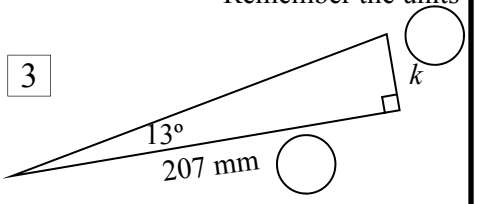
$x = \square$ cm 1 d.p.

2



Remember the units

3



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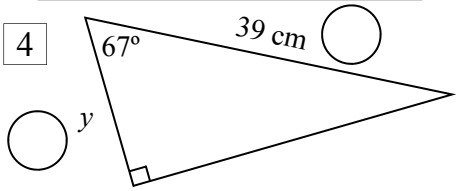
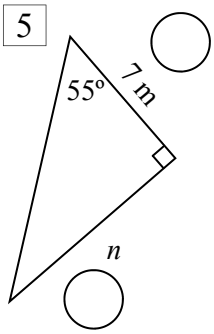


SOH CAH TOA

5



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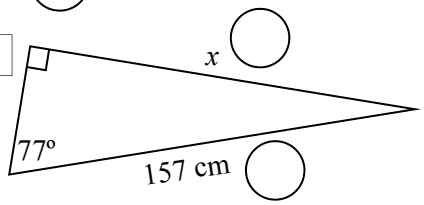

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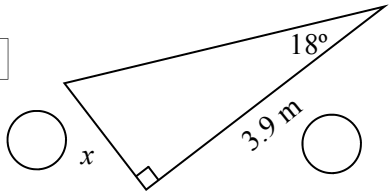

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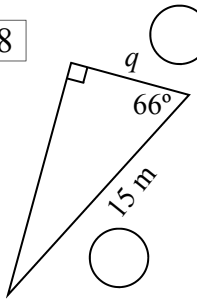

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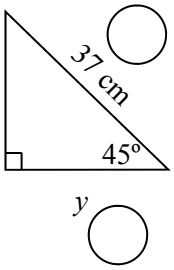

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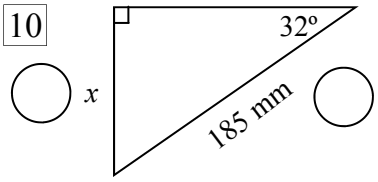

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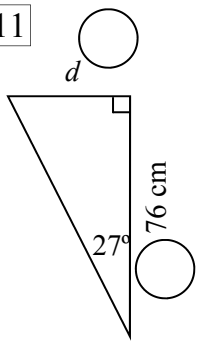

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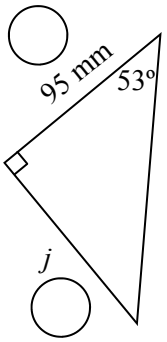

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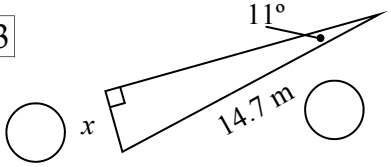

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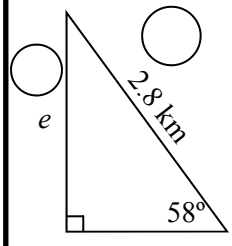

Trigonometry - Finding a Side - All Ratios - '*x*' on bottom (Sheet 2)

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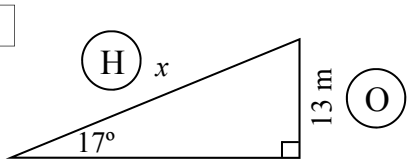
Find the side length to 1 d.p. These are a mix of sin, cos and tan. Use SOH CAH TOA.

SOH CAH TOA

1



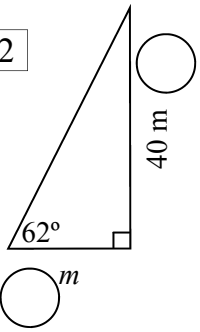
sin ☐ = ☐ / ☐ (O / H)

swap swap

$x =$ ☐ / sin ☐

$x =$ ☐ m 1 d.p.

2



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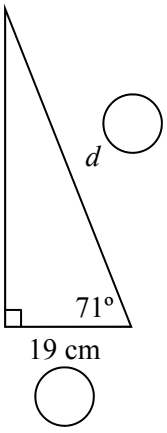

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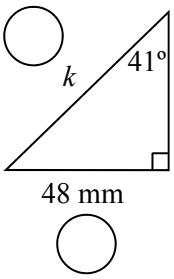

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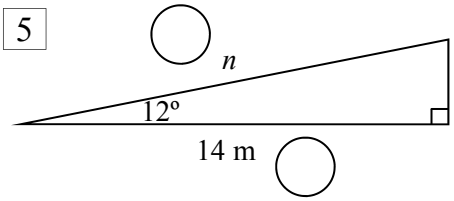

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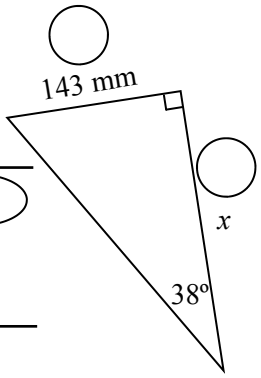

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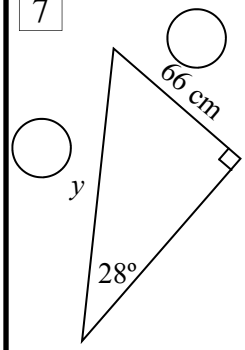

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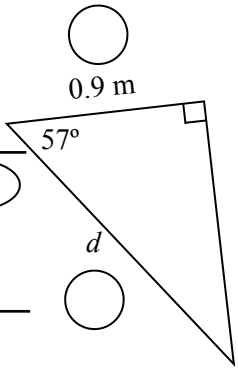

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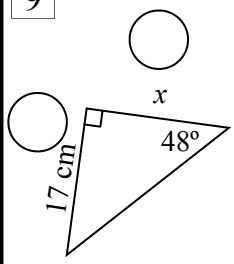

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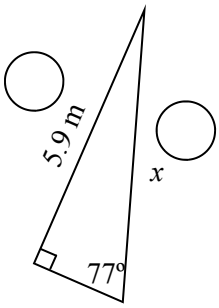

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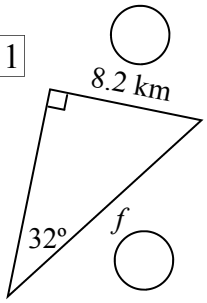

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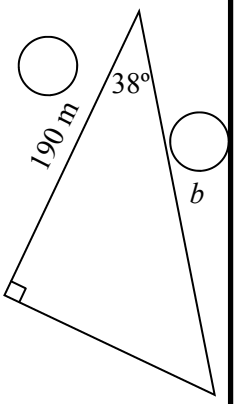

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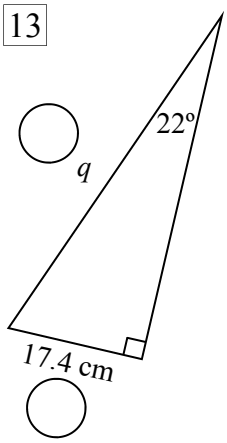

Trigonometry - Finding a Side - All Ratios - Top/bottom Mixed (Sheet 3)

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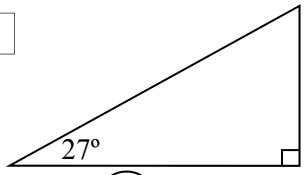
Find the side length to 1 d.p. These are a mix of sin, cos and tan. With '*x*' on top & bottom.

SOH CAH TOA

1



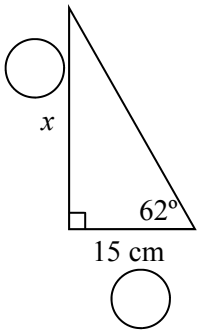
$\tan \square = \frac{\square}{\square}$ (O / A)

swap swap

$x = \frac{\square}{\tan \square}$

$x =$ ____ cm 1 d.p.

2



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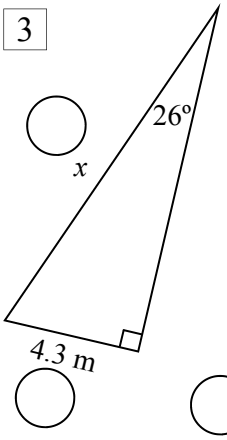

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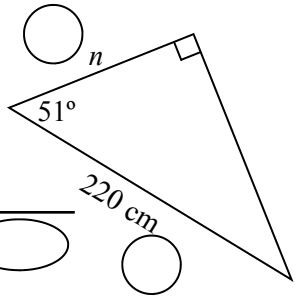

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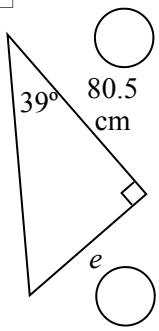

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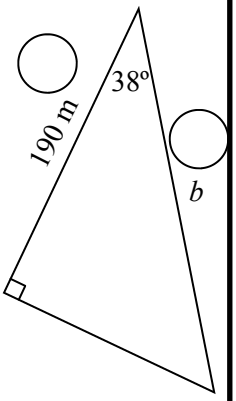

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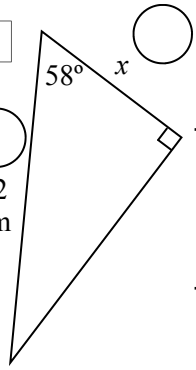

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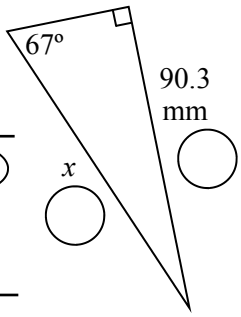

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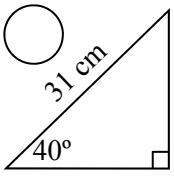

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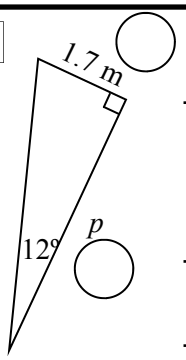

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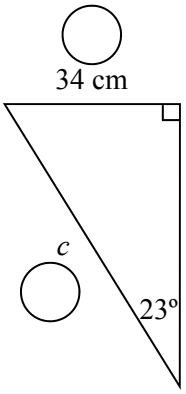

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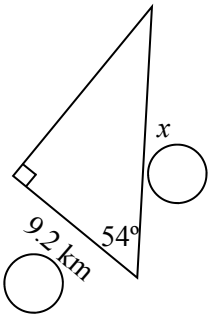

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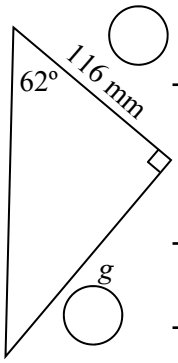

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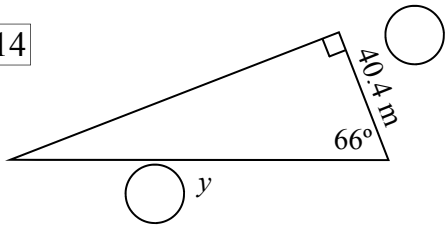

Trigonometry - Finding a Side - All Ratios - '*x*' on top (Sheet 1)

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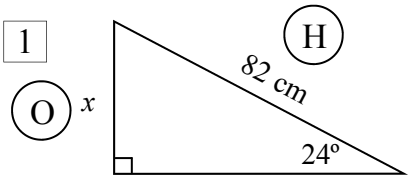
Find the side length to 1 d.p. These are a mix of sin, cos and tan. Use SOH CAH TOA.

1



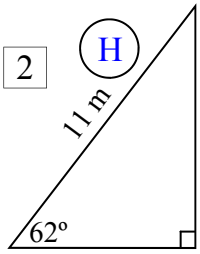
$\sin 24 = \frac{x}{82}$ O / H

$\times 82$ $\times 82$

$x = 82 \sin 24$

$x = 33.4$ cm 1 d.p.

2



$\cos 62 = \frac{d}{11}$

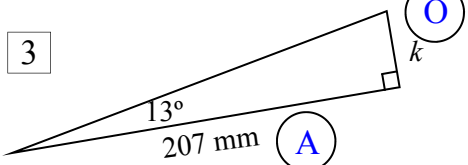
$\times 11$ $\times 11$

$d = 11 \cos 62$

$d = 5.2$ m

Remember the units

3



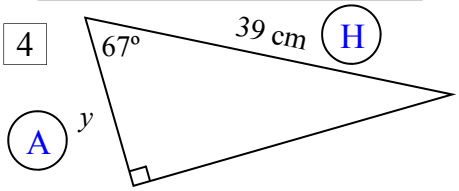
$\tan 13 = \frac{k}{207}$

$\times 207$ $\times 207$

$k = 207 \tan 13$

$k = 47.8$ mm

4



$\cos 67 = \frac{y}{39}$

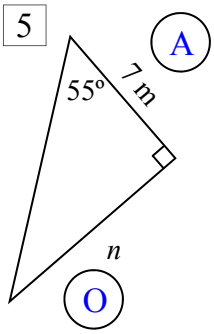
$\times 39$ $\times 39$

$y = 39 \cos 67$

$y = 15.2$ cm

SOH CAH TOA

5



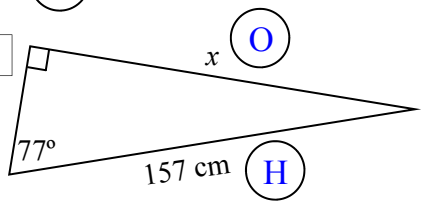
$\tan 55 = \frac{n}{7}$

$\times 7$ $\times 7$

$n = 7 \tan 55$

$n = 10.0$ m

6



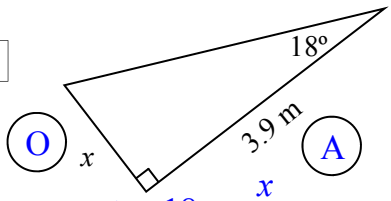
$\sin 77 = \frac{x}{157}$

$\times 157$ $\times 157$

$x = 157 \sin 77$

$x = 131.7$ cm

7



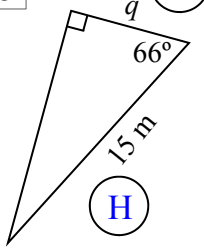
$\tan 18 = \frac{x}{3.9}$

$\times 3.9$ $\times 3.9$

$x = 3.9 \tan 18$

$x = 1.3$ m

8



$\cos 66 = \frac{q}{15}$

$\times 15$ $\times 15$

$q = 15 \cos 66$

$q = 6.1$ m

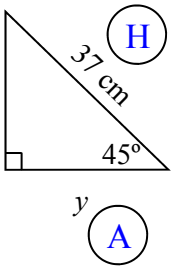
9

$\cos 45 = \frac{y}{37}$

$\times 37$ $\times 37$

$q = 37 \cos 45$

$q = 26.2$ cm



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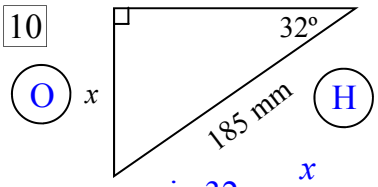


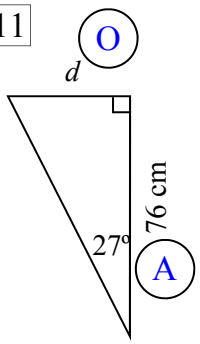
$\sin 32 = \frac{x}{185}$

$\times 185$ $\times 185$

$x = 185 \sin 32$

$x = 98.0$ mm

11



$\tan 27 = \frac{d}{76}$

$\times 76$ $\times 76$

$d = 76 \tan 27$

$d = 38.7$ cm

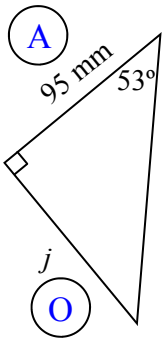
12

$\tan 53 = \frac{j}{95}$

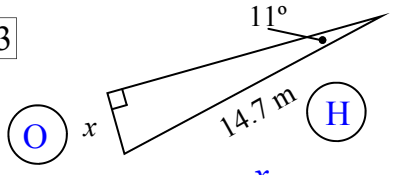
$\times 95$ $\times 95$

$j = 95 \tan 53$

$j = 126.1$ mm



13



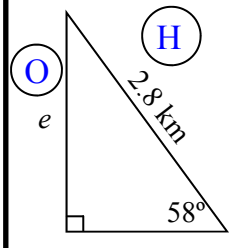
$\sin 11 = \frac{x}{14.7}$

$\times 14.7$ $\times 14.7$

$x = 14.7 \sin 11$

$x = 2.8$ m

14



$\sin 58 = \frac{e}{2.8}$

$\times 2.8$ $\times 2.8$

$e = 2.8 \sin 58$

$e = 2.4$ km

Trigonometry - Finding a Side - All Ratios - '*x*' on bottom (Sheet 2)

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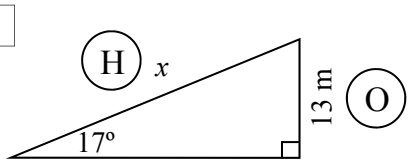
Find the side length to 1 d.p. These are a mix of sin, cos and tan. Use SOH CAH TOA.

SOH CAH TOA

1



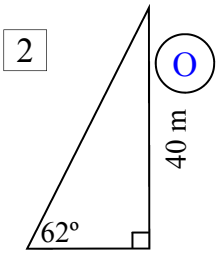
$\sin 17 = \frac{13}{x}$ (13 = O, x = H)

swap swap

$x = \frac{13}{\sin 17}$

$x = 44.5$ m 1 d.p.

2



$\tan 62 = \frac{40}{m}$

swap swap

$m = \frac{40}{\tan 62}$

$m = 21.3$ m

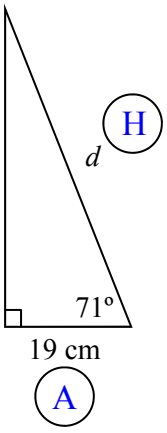
3

$\cos 71 = \frac{19}{d}$

swap swap

$d = \frac{19}{\cos 71}$

$d = 58.4$ cm



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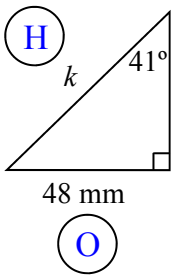


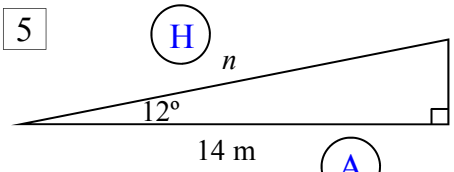
$\sin 41 = \frac{48}{k}$

swap swap

$k = \frac{48}{\sin 41}$

$k = 73.2$ mm

5



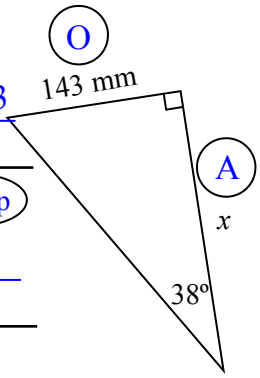
$\cos 12 = \frac{14}{n}$

swap swap

$n = \frac{14}{\cos 12}$

$n = 14.3$ m

6



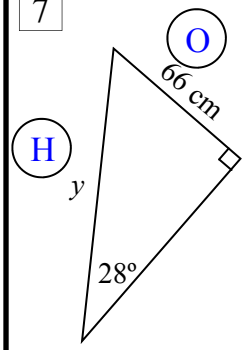
$\tan 38 = \frac{143}{x}$

swap swap

$x = \frac{143}{\tan 38}$

$x = 183.0$ mm

7



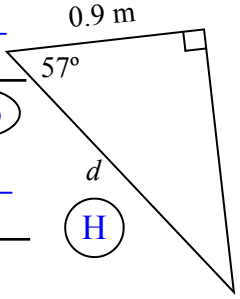
$\sin 28 = \frac{66}{y}$

swap swap

$y = \frac{66}{\sin 28}$

$y = 140.6$ cm

8



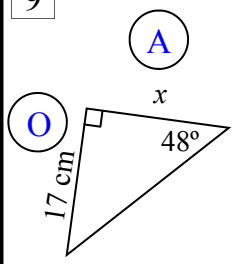
$\cos 57 = \frac{0.9}{d}$

swap swap

$d = \frac{0.9}{\cos 57}$

$d = 1.7$ m

9



$\cos 48 = \frac{17}{x}$

swap swap

$x = \frac{17}{\cos 48}$

$x = 25.4$ cm

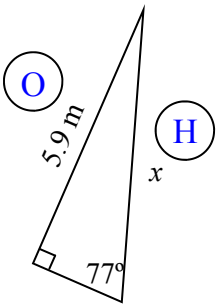
10

$\sin 77 = \frac{5.9}{x}$

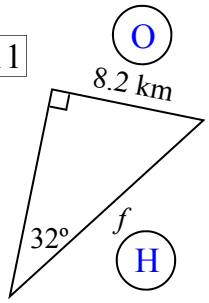
swap swap

$x = \frac{5.9}{\sin 77}$

$x = 6.1$ m



11



$\sin 32 = \frac{8.2}{f}$

swap swap

$f = \frac{8.2}{\sin 32}$

$f = 15.5$ km

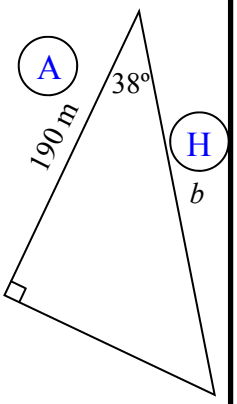
12

$\cos 38 = \frac{190}{b}$

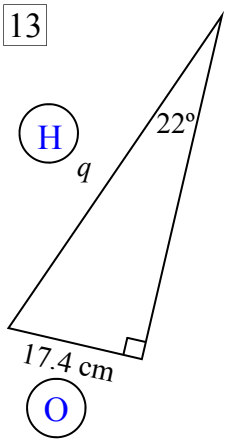
swap swap

$b = \frac{190}{\cos 38}$

$b = 241.1$ m



13



$\sin 22 = \frac{17.4}{q}$

swap swap

$q = \frac{17.4}{\sin 22}$

$q = 46.4$ cm

Trigonometry - Finding a Side - All Ratios - Top/bottom Mixed (Sheet 3)

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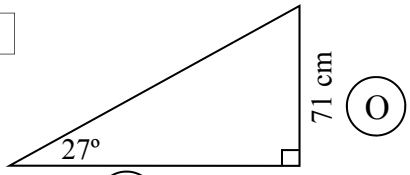
Find the side length to 1 d.p. These are a mix of sin, cos and tan. With 'x' on top & bottom.

SOH CAH TOA

1



O A

$\tan 27 = \frac{71}{x}$ (O / A)

swap swap

$x = \frac{71}{\tan 27}$

$x = 139.3$ cm 1 d.p.

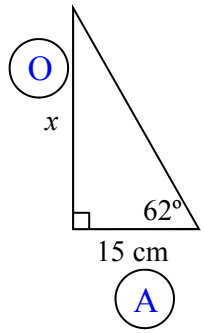
2

$\tan 62 = \frac{x}{15}$

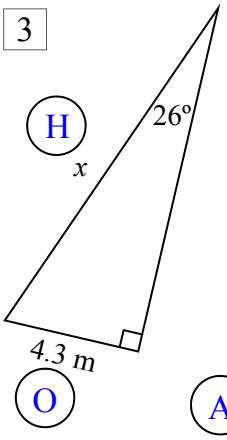
× 15 × 15

$x = 15 \tan 62$

$x = 28.2$ cm



3



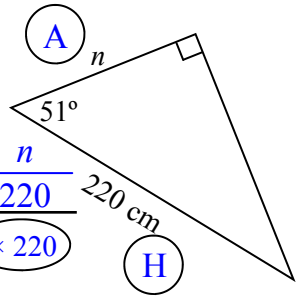
$\sin 26 = \frac{4.3}{x}$

swap swap

$x = \frac{4.3}{\sin 26}$

$x = 9.8$ m

4



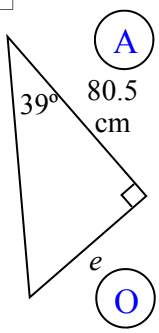
$\cos 51 = \frac{n}{220}$

× 220 × 220

$n = 220 \cos 51$

$n = 138.5$ cm

5



$\tan 39 = \frac{e}{80.5}$

× 80.5 × 80.5

$e = 80.5 \tan 39$

$e = 65.2$ cm

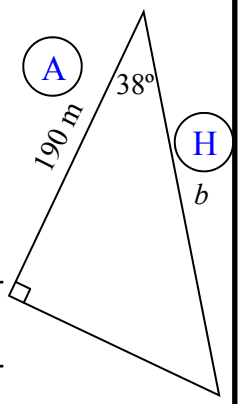
6

$\cos 38 = \frac{190}{b}$

swap swap

$x = \frac{4.3}{\sin 26}$

$x = 9.8$ m



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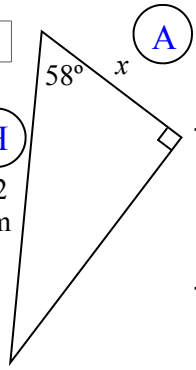


$\cos 58 = \frac{x}{72}$

× 72 × 72

$x = 80.5 \tan 39$

$x = 38.2$ cm

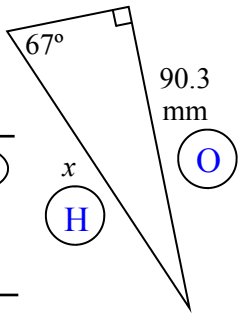
8

$\sin 67 = \frac{90.3}{x}$

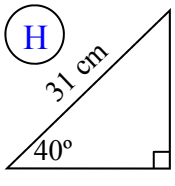
swap swap

$x = \frac{90.3}{\sin 67}$

$x = 98.1$ mm



9



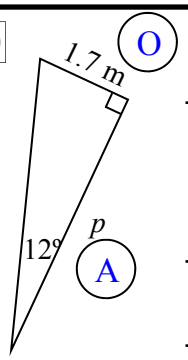
$\cos 40 = \frac{y}{31}$

× 31 × 31

$y = 31 \cos 40$

$y = 23.7$ cm

10



$\tan 12 = \frac{1.7}{p}$

swap swap

$p = \frac{1.7}{\tan 12}$

$p = 8.0$ m

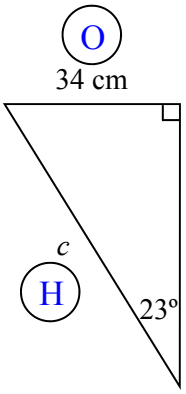
11

$\sin 23 = \frac{34}{c}$

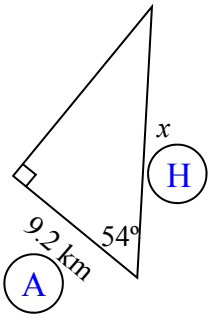
swap swap

$c = \frac{34}{\sin 23}$

$c = 87.0$ cm



12



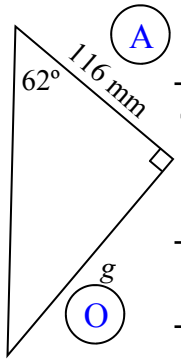
$\cos 54 = \frac{9.2}{x}$

swap swap

$x = \frac{9.2}{\cos 54}$

$x = 15.7$ km

13



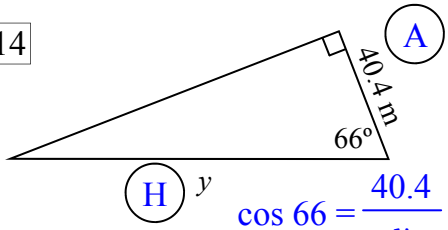
$\tan 62 = \frac{g}{116}$

× 116 × 116

$g = 116 \tan 62$

$g = 218.2$ mm

14



$\cos 66 = \frac{40.4}{y}$

swap swap

$y = \frac{40.4}{\cos 66}$

$y = 99.3$ m

Trigonometry - Finding a Side (All) (Homework Strip)

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Date Due: _____

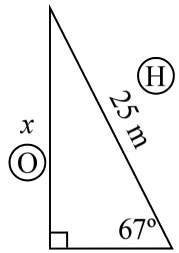
CALCULATOR IN DEG MODE

Find the length to 1 decimal place

1 $\sin 67 = \frac{O}{H}$ $\times 25$ $\times 25$

$x = \sin$

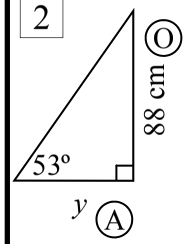
$x =$ m



2 $\tan 53 = \frac{O}{A}$ swap swap

$y = \frac{\quad}{\tan}$

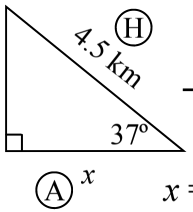
$y =$ cm



3 $\cos 37 = \frac{A}{H}$ $\times 4.5$ $\times 4.5$

$x = \cos$

$x =$ km



Date Due: _____

CALCULATOR IN DEG MODE

Find the length to 1 decimal place

1 $\sin 67 = \frac{O}{H}$ $\times 25$ $\times 25$

$x = \sin$

$x =$ m

2 $\tan 53 = \frac{O}{A}$ swap swap

$y = \frac{\quad}{\tan}$

$y =$ cm

3 $\cos 37 = \frac{A}{H}$ $\times 4.5$ $\times 4.5$

$x = \cos$

$x =$ km

Date Due: _____

CALCULATOR IN DEG MODE

Find the length to 1 decimal place

1 $\sin 67 = \frac{O}{H}$ $\times 25$ $\times 25$

$x = \sin$

$x =$ m

2 $\tan 53 = \frac{O}{A}$ swap swap

$y = \frac{\quad}{\tan}$

$y =$ cm

3 $\cos 37 = \frac{A}{H}$ $\times 4.5$ $\times 4.5$

$x = \cos$

$x =$ km

Date Due: _____

CALCULATOR IN DEG MODE

Find the length to 1 decimal place

1 $\sin 67 = \frac{O}{H}$ $\times 25$ $\times 25$

$x = \sin$

$x =$ m

2 $\tan 53 = \frac{O}{A}$ swap swap

$y = \frac{\quad}{\tan}$

$y =$ cm

3 $\cos 37 = \frac{A}{H}$ $\times 4.5$ $\times 4.5$

$x = \cos$

$x =$ km

Trigonometry - Finding a Side (All) (Homework Strip)

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Date Due:

CALCULATOR IN DEG MODE

Find the length to 1 decimal place

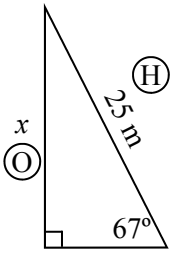
1

$\sin 67 = \frac{x}{25}$ (O / H)

$\times 25$ $\times 25$

$x = 25 \sin 67$

$x =$ 23.0 m



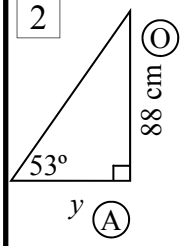
2

$\tan 53 = \frac{88}{y}$ (O / A)

swap swap

$y = \frac{88}{\tan 53}$

$y =$ 66.3 cm



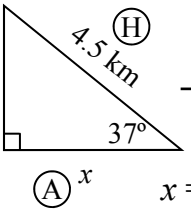
3

$\cos 37 = \frac{x}{4.5}$ (A / H)

$\times 4.5$ $\times 4.5$

$x = 4.5 \cos 37$

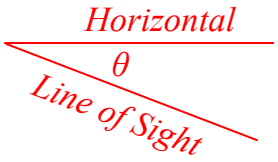
$x =$ 3.6 km



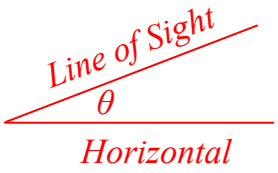
Trigonometry - Angle of Depression and Elevation

Trigonometry can be used to calculate the distance to an object or the angle that you are looking down or up at an object. When a person is depressed they feel down, when they are elevated they are feeling good or they are 'up'. This is much the same in Trigonometry:

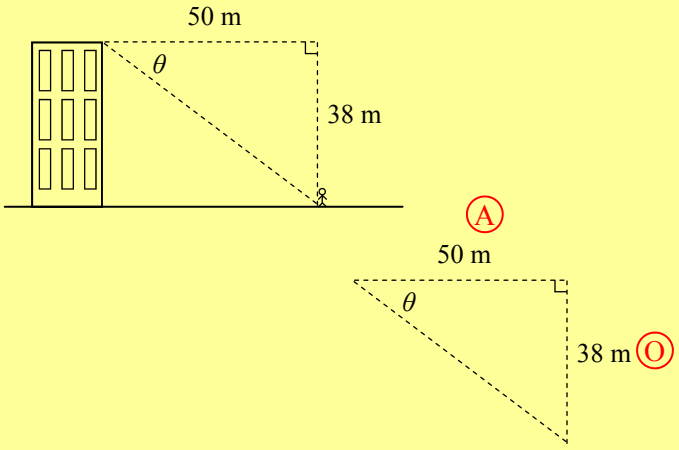
Angle of Depression: When you are looking down, it is the angle made with the 'line of sight' and the horizontal



Angle of Elevation: When you are looking up, it is the angle made with the 'line of sight' and the horizontal



Example: Find the angle of depression 'θ' from the 38 m tall building to a point on the ground 50 m away, where Lucky Phil is standing. Round your answer to the nearest degree.



STEP 1

LABEL THE SIDES

LABEL THE SIDES

Always label the sides, in this case it is A and H. You only have to label the sides that have a measurement on them or have a letter, like 'x'. Most mistakes start with students thinking they don't have to label the sides. Circle the letters that way they aren't confused with any letters that may be on the triangle.

STEP 2

PICK THE RATIO

PICK THE RATIO

Use SOH CAH TOA to determine the ratio. This triangle has O and A, that means you use TAN.

S O H C A H T O A

STEP 3

WRITE FIRST LINE

WRITE THE FIRST LINE

Write: $\tan\theta = \frac{38}{50}$ ← 38 OPPOSITE OVER 50 ← ADJACENT

STEP 4

SOLVE

SOLVE USING YOUR CALCULATOR

Calc: SHIFT TAN (3 8) 5 0 =

Calc: 37.23483398

$\theta = 37°$ *Nearest degree*

Trigonometry - Elevation/Depression - Finding an Angle (Sheet 1)

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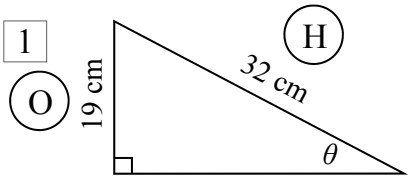
Find the angle 'θ' to the nearest degree.

SOH CAH TOA

1

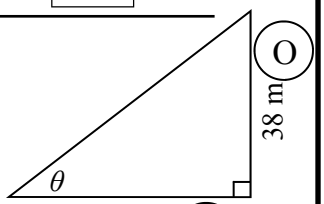


$$\sin\theta = \frac{O}{H}$$

Calc: SHIFT SIN 1 9 ÷ 3 2 =

$\theta =$ ____ °

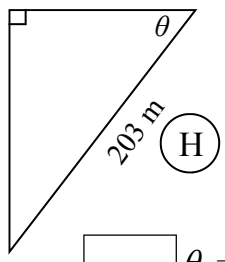
2



$$\theta = \frac{O}{A}$$

$\theta =$ ____ °

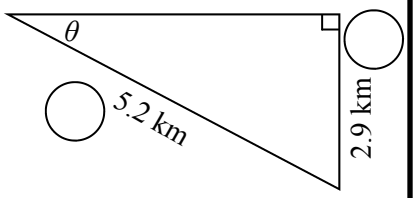
3



$$\theta = \frac{A}{H}$$

$\theta =$ ____ °

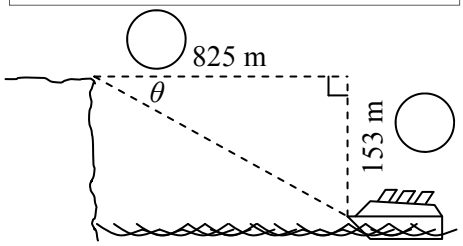
4



$$\theta = \frac{\quad}{\quad}$$

$\theta =$ ____ °

A ship is 825 m from a cliff, which is 153 m tall. Eric is at the top of the cliff, looking down at the ship.

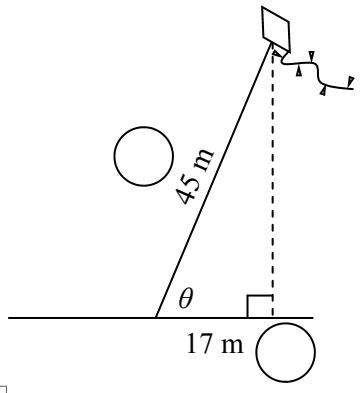


5 Calculate the angle of depression

$$\theta = \frac{\quad}{\quad}$$

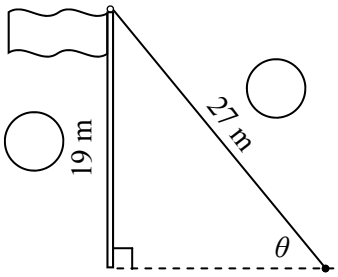
$\theta =$ ____ °

An airborne kite is attached to the ground. Its string is 45 m long and is fully extended. The kite is flying above a point 17 m away.



6 Calculate the angle of elevation from the anchor point up to the kite.

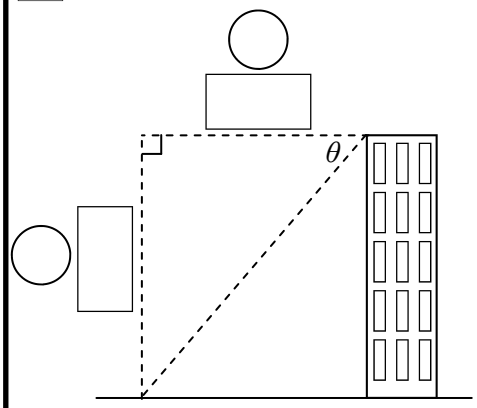
A flag pole has a guy wire attached to reduce vibration. The flagpole is 19 m tall and the guy wire is 27 m long.



7 Calculate the angle of elevation from the anchor point up to the pole.

Tom is on top of a 63 m tall building, looking down at Fred, who is 48 m away from the base of the building.

8 Write the distances in the boxes.



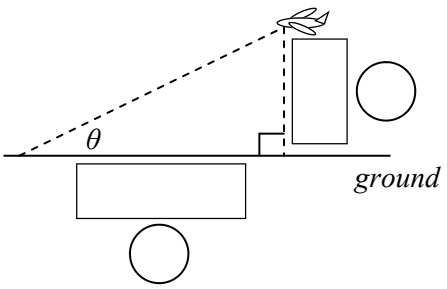
9 Calculate the angle of depression

Yve is looking up at a plane which is 3.7 km away and is flying at a height of 800 m.

10 Convert 3.7 km to metres

3.7 km = ____

11 Write the distances in the boxes.



12 Calculate the angle of elevation

Trigonometry - Elevation/Depression - Finding a Length (Sheet 2)

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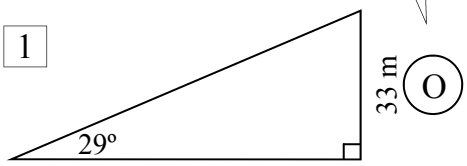
Find the length 'x', rounding to 1 decimal place.

1



$\tan \square = \frac{\square}{\square}$ (O / A)

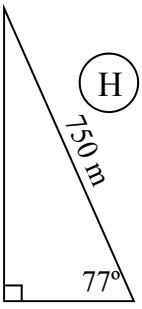
swap swap

$x = \frac{\square}{\tan \square}$

Calc: 3 3 ▪ TAN 2 9) =

$x =$ ____ m 1 d.p.

2



$\cos \square = \frac{\square}{\square}$ (A / H)

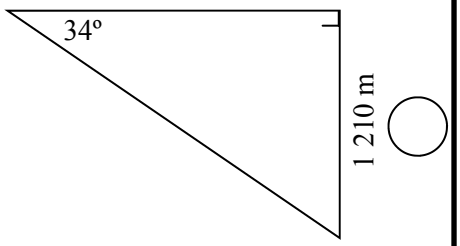
× 750 × 750

$x =$ ____ $\cos$ ____

Calc: 7 5 0 COS 7 7 =

$x =$ ____ m 1 d.p.

3



$\tan \square = \frac{\square}{\square}$ (O / A)

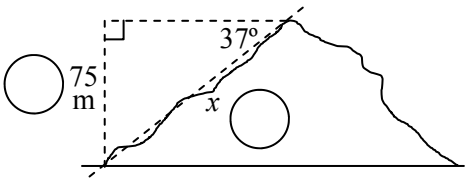
swap swap

$x = \frac{\square}{\tan \square}$

$x =$ ____ m 1 d.p.

SOH CAH TOA

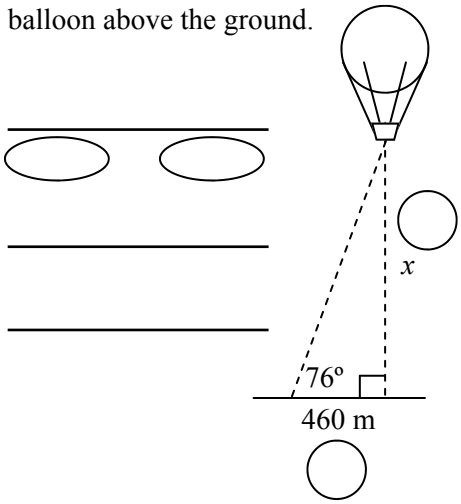
Jill is on top of a hill which has a height of 75 m. The angle of depression from the top to the base of the hill is 37º.



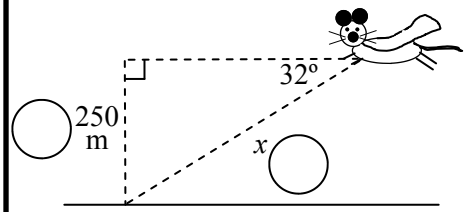
4 Estimate the distance, 'x', Jill travelled to climb the hill.

Ted is looking up at a hot air balloon. The angle of elevation is 76º and the balloon is above a point 460 m away.

5 Calculate 'x', the height of the balloon above the ground.

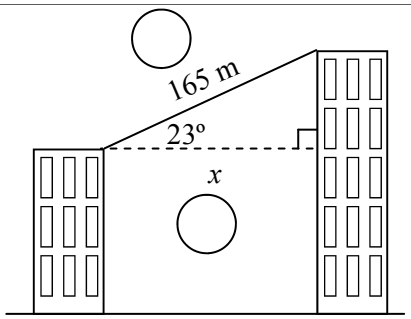


Flighty Mouse, the flying super mouse spies a damsel in distress. The angle of depression to the damsel is 32º and Flighty is flying at a height of 250 m above the ground.



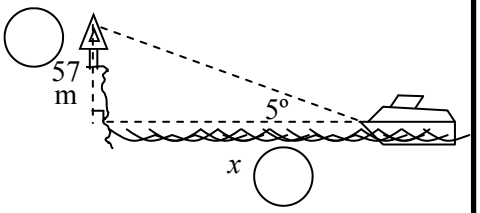
6 Calculate 'x', the flying distance Flighty Mouse has to cover.

Balancing Bec is set to walk a 165 m long tight-rope suspended between two buildings. The angle of elevation from the lower building is 23º.



7 Find 'x', the distance between the two buildings.

Stinky Stew the tug boat captain, spots a cliff beacon at an angle of elevation of 5º. He knows the beacon is 57 m above sea level.



8 Find 'x', the distance to the cliff.

Trigonometry - Elevation/Depression - Finding an Angle (Sheet 1)

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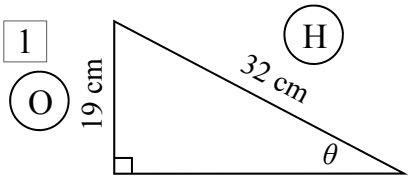
Find the angle 'θ' to the nearest degree.

1

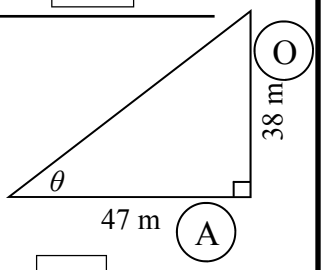


$\sin\theta = \frac{19\ \text{O}}{32\ \text{H}}$

Calc: SHIFT SIN 1 9 ▪ 3 2 =

$\theta = 36^\circ$

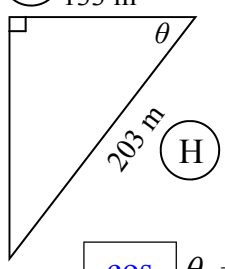
2



$\tan\theta = \frac{38\ \text{O}}{47\ \text{A}}$

$\theta = 39^\circ$

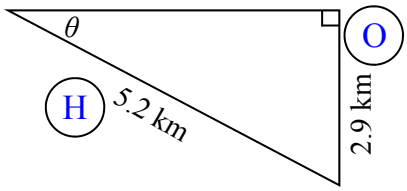
3



$\cos\theta = \frac{133\ \text{A}}{203\ \text{H}}$

$\theta = 49^\circ$

4

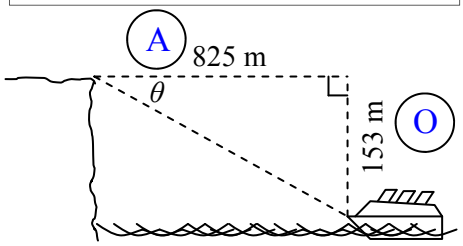


$\sin\theta = \frac{2.9}{5.2}$

$\theta = 34^\circ$

SOH CAH TOA

A ship is 825 m from a cliff, which is 153 m tall. Eric is at the top of the cliff, looking down at the ship.

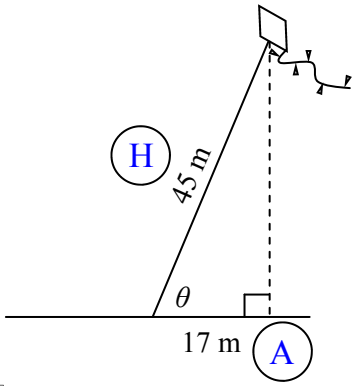


5 Calculate the angle of depression

$\tan\theta = \frac{153}{825}$

$\theta = 11^\circ$

An airborne kite is attached to the ground. Its string is 45 m long and is fully extended. The kite is flying above a point 17 m away.

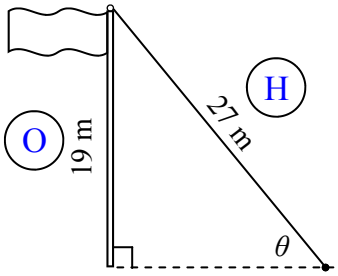


6 Calculate the angle of elevation from the anchor point up to the kite.

$\cos\theta = \frac{17}{45}$

$\theta = 68^\circ$

A flag pole has a guy wire attached to reduce vibration. The flagpole is 19 m tall and the guy wire is 27 m long.



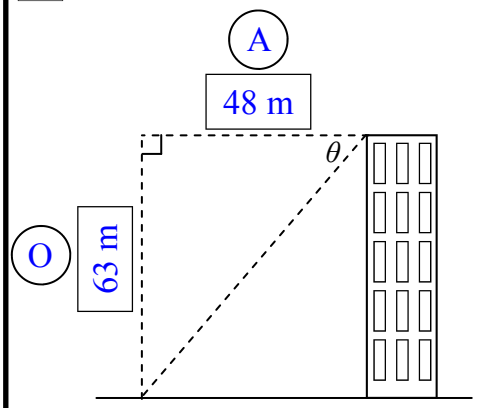
7 Calculate the angle of elevation from the anchor point up to the pole.

$\sin\theta = \frac{19}{27}$

$\theta = 45^\circ$

Tom is on top of a 63 m tall building, looking down at Fred, who is 48 m away from the base of the building.

8 Write the distances in the boxes.



9 Calculate the angle of depression

$\tan\theta = \frac{63}{48}$

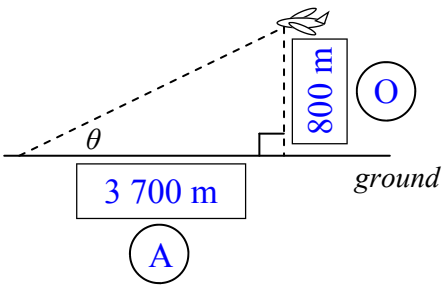
$\theta = 53^\circ$

Yve is looking up at a plane which is 3.7 km away and is flying at a height of 800 m.

10 Convert 3.7 km to metres

3.7 km = 3 700 m

11 Write the distances in the boxes.



12 Calculate the angle of elevation

$\tan\theta = \frac{800}{3\ 700}$

$\theta = 12^\circ$

Trigonometry - Elevation/Depression - Finding a Length (Sheet 2)

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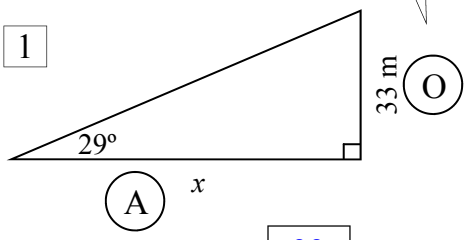
Find the length 'x', rounding to 1 decimal place.

1



$$\tan 29 = \frac{33}{x} \quad \begin{matrix}\text{O}\\ \text{A}\end{matrix}$$

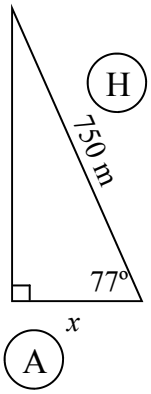
swap swap

$$x = \frac{33}{\tan 29}$$

Calc: 33 ÷ TAN 29) =

$x = 59.5$ m 1 d.p.

2



$$\cos 77 = \frac{x}{750} \quad \begin{matrix}\text{A}\\ \text{H}\end{matrix}$$

× 750 × 750

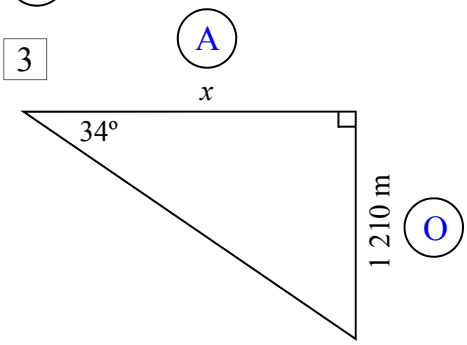
$$x = 750 \cos 77$$

Calc: 750 COS 77 =

$x = 168.7$ m

1 d.p.

3



$$\tan 34 = \frac{1\,210}{x} \quad \begin{matrix}\text{O}\\ \text{A}\end{matrix}$$

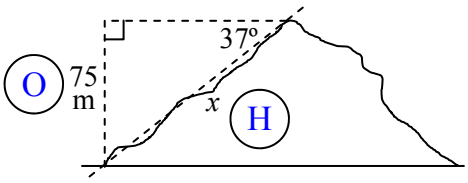
swap swap

$$x = \frac{1\,210}{\tan 34}$$

$x = 1\,793.9$ m 1 d.p.

SOH CAH TOA

Jill is on top of a hill which has a height of 75 m. The angle of depression from the top to the base of the hill is 37°.



4 Estimate the distance, 'x', Jill travelled to climb the hill.

$$\sin 37 = \frac{75}{x}$$

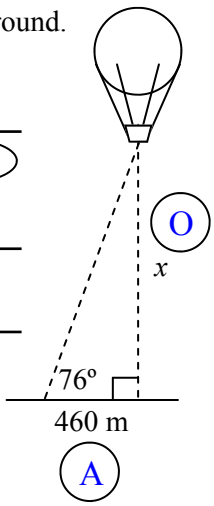
swap swap

$$x = \frac{75}{\sin 37}$$

$$x = 124.6 \text{ m}$$

Ted is looking up at a hot air balloon. The angle of elevation is 76° and the balloon is above a point 460 m away.

5 Calculate 'x', the height of the balloon above the ground.



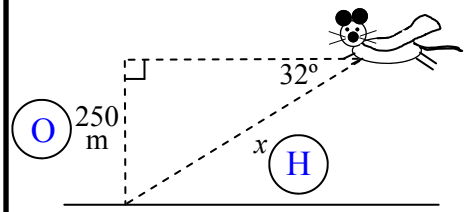
$$\tan 76 = \frac{x}{460}$$

× 460 × 460

$$x = 460 \tan 76$$

$$x = 1\,845.0 \text{ m}$$

Flighty Mouse, the flying super mouse spies a damsel in distress. The angle of depression to the damsel is 32° and Flighty is flying at a height of 250 m above the ground.



6 Calculate 'x', the flying distance Flighty Mouse has to cover.

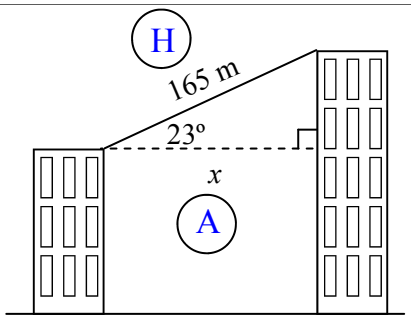
$$\tan 32 = \frac{250}{x}$$

swap swap

$$x = \frac{250}{\tan 32}$$

$$x = 400.1 \text{ m}$$

Balancing Bec is set to walk a 165 m long tight-rope suspended between two buildings. The angle of elevation from the lower building is 23°.



7 Find 'x', the distance between the two buildings.

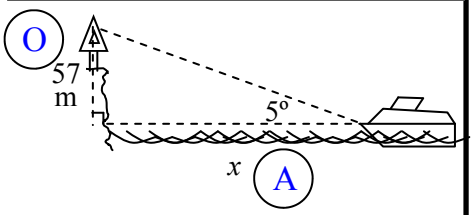
$$\cos 23 = \frac{x}{165}$$

× 165 × 165

$$x = 165 \cos 23$$

$$x = 151.9 \text{ m}$$

Stinky Stew the tug boat captain, spots a cliff beacon at an angle of elevation of 5°. He knows the beacon is 57 m above sea level.



8 Find 'x', the distance to the cliff.

$$\tan 5 = \frac{57}{x}$$

swap swap

$$x = \frac{57}{\tan 5}$$

$$x = 651.5 \text{ m}$$

Trigonometry - Elevation/Depression (Homework Strip)

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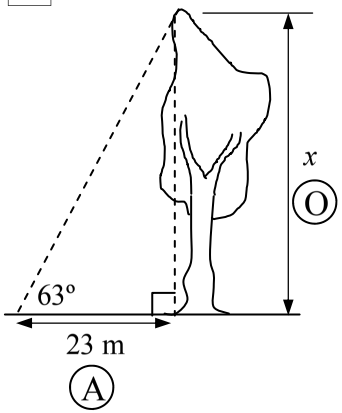
Date Due: _____

Find the height of the tree 'x' given the angle of elevation from a point 23 m away from the tree is 63°. Answer in metres, to 1 decimal point.

1



$\tan$ ☐ $= \frac{\square}{\square}$ O / A

× 23 × 23

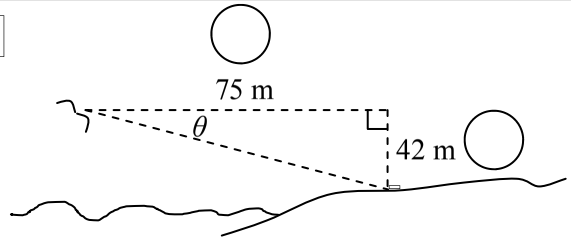
$x =$ ☐ ☐

$x =$ ☐ m

1 d.p.

Jonathan the seagull is flying at 42 m above the beach when he spies a chip on the beach 75m away. Calculate the angle of depression 'θ' for his descent. Answer to the nearest degree.

2



☐ $\theta = \frac{\square}{\square}$

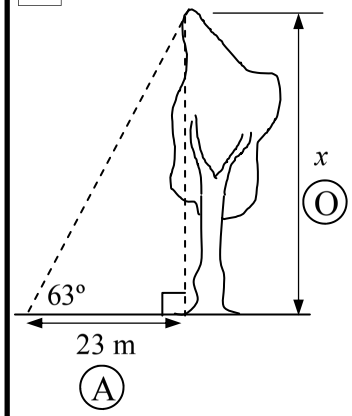
$\theta =$ ☐ °

Use SHIFT to get an angle

Date Due: _____

Find the height of the tree 'x' given the angle of elevation from a point 23 m away from the tree is 63°. Answer in metres, to 1 decimal point.

1



$\tan$ ☐ $= \frac{\square}{\square}$ O / A

× 23 × 23

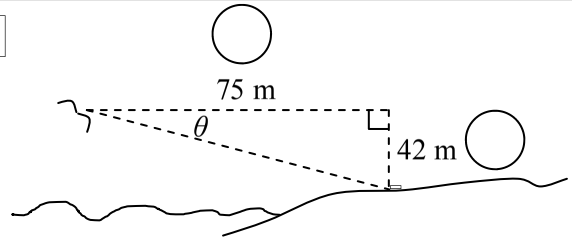
$x =$ ☐ ☐

$x =$ ☐ m

1 d.p.

Jonathan the seagull is flying at 42 m above the beach when he spies a chip on the beach 75m away. Calculate the angle of depression 'θ' for his descent. Answer to the nearest degree.

2



☐ $\theta = \frac{\square}{\square}$

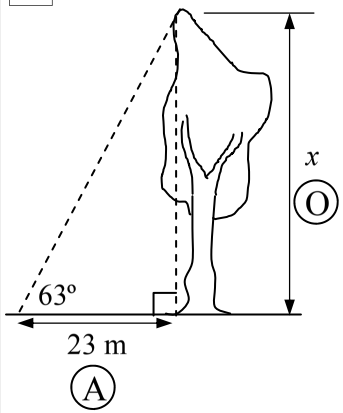
$\theta =$ ☐ °

Use SHIFT to get an angle

Date Due: _____

Find the height of the tree 'x' given the angle of elevation from a point 23 m away from the tree is 63°. Answer in metres, to 1 decimal point.

1



$\tan$ ☐ $= \frac{\square}{\square}$ O / A

× 23 × 23

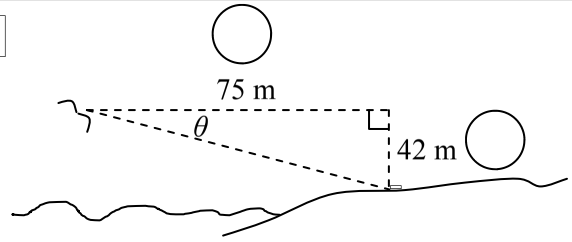
$x =$ ☐ ☐

$x =$ ☐ m

1 d.p.

Jonathan the seagull is flying at 42 m above the beach when he spies a chip on the beach 75m away. Calculate the angle of depression 'θ' for his descent. Answer to the nearest degree.

2



☐ $\theta = \frac{\square}{\square}$

$\theta =$ ☐ °

Use SHIFT to get an angle

Trigonometry - Elevation/Depression (Homework Strip)

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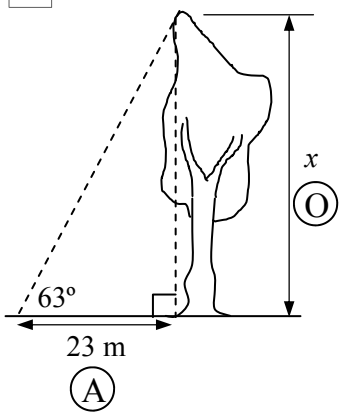
1



$\tan 63 = \frac{x}{23}$ (O / A)

$\times 23$ $\times 23$

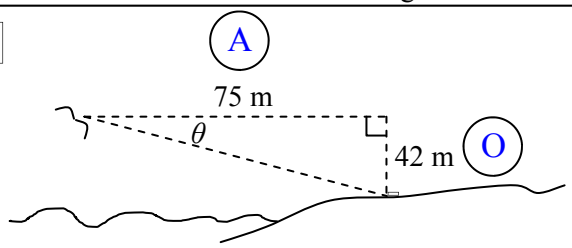
$x = 23 \tan 63$

$x = 45.1$ m

1 d.p.

Jonathan the seagull is flying at 42 m above the beach when he spies a chip on the beach 75m away. Calculate the angle of depression 'θ' for his descent. Answer to the nearest degree.

2



$\tan \theta = \frac{42}{75}$

$\theta = 29°$

Use SHIFT to get an angle

Trigonometry - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Trigonometry - Find A Word

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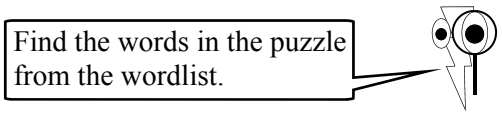

Y	L	U	M	P	A	W	S	N	W	O	N	K	N	U
R	D	L	E	Y	E	L	E	V	A	T	I	O	N	K
O	I	T	A	R	S	U	E	L	G	N	A	I	R	T
U	S	N	S	E	T	I	S	O	P	P	O	H	H	R
N	T	O	U	Z	I	E	D	Q	Q	E	T	Y	N	I
D	A	I	R	R	M	Y	M	E	N	G	P	A	O	G
I	N	S	E	R	A	T	V	I	N	O	P	P	N	O
N	C	S	D	I	T	T	S	E	T	U	N	I	M	N
G	E	E	I	G	E	T	I	E	N	N	L	L	Y	O
G	V	R	V	H	P	E	N	O	E	P	E	P	L	M
Y	L	P	I	T	L	U	M	Q	C	T	H	C	E	E
C	O	E	D	R	S	R	P	R	A	A	H	L	N	T
S	S	D	L	E	B	A	L	V	J	T	G	E	G	R
C	O	Y	S	E	E	R	G	E	D	N	H	E	T	Y
C	C	C	C	A	L	C	U	L	A	T	E	H	H	A

WORDLIST

OPPOSITE	ELEVATION	SINE	ADJACENT	LENGTH
CALCULATE	MEASURE	ANGLE*	DISTANCE	UNKNOWN
DEGREES	SIDE	ROUNDING	DEPRESSION	THETA
COS	DIVIDE	TRIANGLE*	MINUTES	LABEL
MULTIPLY	RATIO	ESTIMATE	RIGHT	TAN
SOLVE	HYPOTENUSE	TRIGONOMETRY	CENTIMETRE	SWAP

* ANGLE AND TRIANGLE CAN BE FOUND IN DIFFERENT LOCATIONS

Trigonometry - Find A Word

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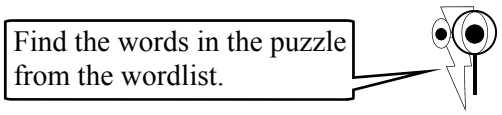

Y	L	U	M	P	A	W	S	N	W	O	N	K	N	U
R	D	L	E	Y	E	L	E	V	A	T	I	O	N	K
O	I	T	A	R	S	U	E	L	G	N	A	I	R	T
U	S	N	S	E	T	I	S	O	P	P	O	H	H	R
N	T	O	U	Z	I	E	D	Q	Q	E	T	Y	N	I
D	A	I	R	R	M	Y	M	E	N	G	P	A	O	G
I	N	S	E	R	A	T	V	I	N	O	P	P	N	O
N	C	S	D	I	T	T	S	E	T	U	N	I	M	N
G	E	E	I	G	E	T	I	E	N	N	L	L	Y	O
G	V	R	V	H	P	E	N	O	E	P	E	P	L	M
Y	L	P	I	T	L	U	M	Q	C	T	H	C	E	E
C	O	E	D	R	S	R	P	R	A	A	H	L	N	T
S	S	D	L	E	B	A	L	V	J	T	G	E	G	R
C	O	Y	S	E	E	R	G	E	D	N	H	E	T	Y
C	C	C	C	A	L	C	U	L	A	T	E	H	H	A

WORDLIST

OPPOSITE	ELEVATION	SINE	ADJACENT	LENGTH
CALCULATE	MEASURE	ANGLE*	DISTANCE	UNKNOWN
DEGREES	SIDE	ROUNDING	DEPRESSION	THETA
COS	DIVIDE	TRIANGLE*	MINUTES	LABEL
MULTIPLY	RATIO	ESTIMATE	RIGHT	TAN
SOLVE	HYPOTENUSE	TRIGONOMETRY	CENTIMETRE	SWAP

* ANGLE AND TRIANGLE CAN BE FOUND IN DIFFERENT LOCATIONS

Trigonometry Review

This is the review of the Trigonometry Topic.

Trigonometry - Topic Review

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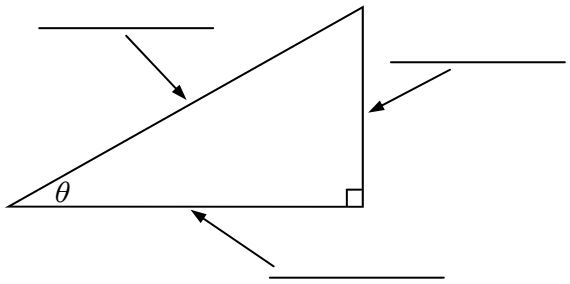
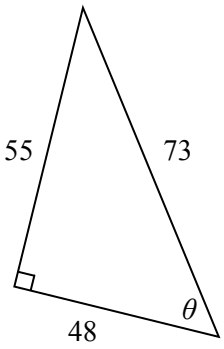
Label the hypotenuse, opposite and adjacent sides.

1

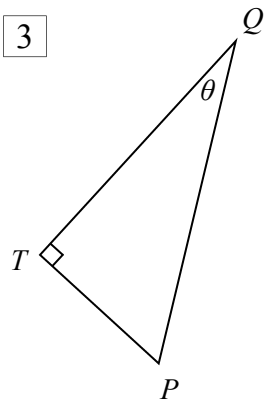


Find the trigonometric ratios for sin θ, cos θ, and tan θ.
NO CALCULATOR REQUIRED.

2



3



Use your calculator to solve these to 1 decimal place.

4 19 tan 38º

5 $\dfrac{57}{\cos 31º}$

Round these angles to the nearest degree.

6 45.36º

7 59.56033º

8 28º 42’

9 55º 13’ 55”

Use your calculator to find θ, round to the nearest degree.

10 $\sin\theta = 0.4228$

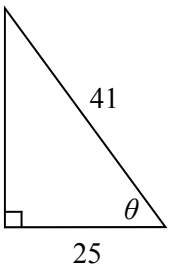
11 $\cos\theta = \dfrac{3}{8}$

12 $\tan\theta = \dfrac{11}{5}$

13 $\tan\theta = 5.4$

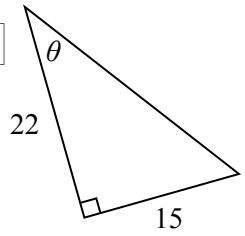
Would you use sin, cos or tan to find these angles?
Colour in the circle to indicate your choice.
DON’T SOLVE THE QUESTIONS.

14



○ sin
○ cos
○ tan

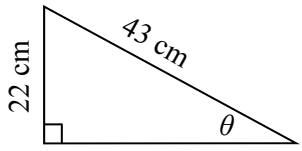
15



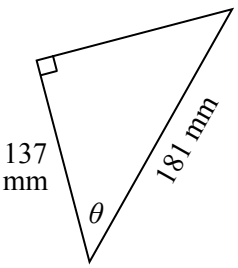
○ sin
○ cos
○ tan

Find the angle θ, round to the nearest degree.

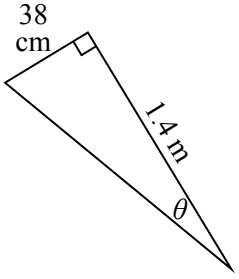
16



17



18



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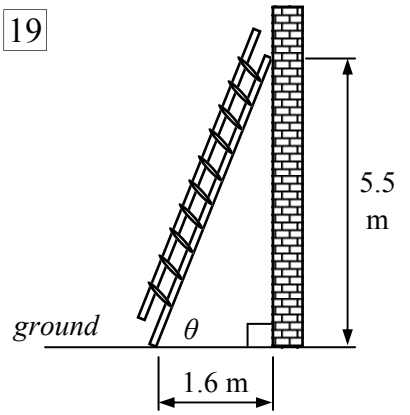
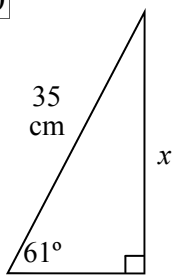
A 5.5 m ladder is leaning against a wall. The toe of the ladder is 1.6 m away from the wall. What angle (shown as θ on the diagram) does the ladder make with the ground? Answer to the nearest degree.

19



Calculate the length 'x' on the triangles below. Answer to 1 decimal place.

20



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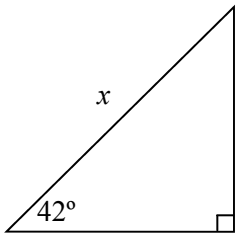

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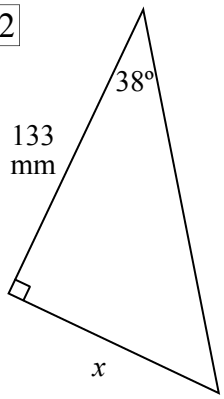
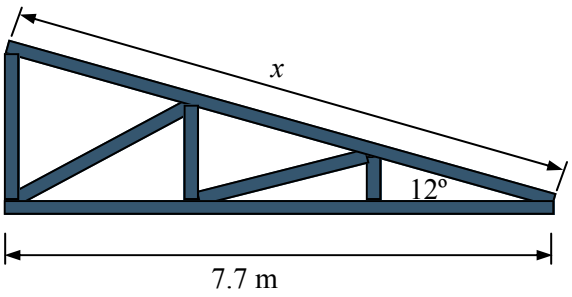


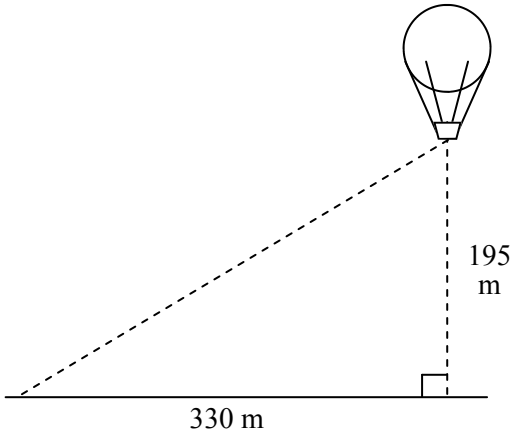
A roof truss is shown below. The roof width is 7.7 m, if the required angle (pitch) of the roof is 12º. Calculate the length of the inclined timber beam, labeled 'x'. Answer in metres, to 1 decimal place.

23



Jessica is watching a hot air balloon which is 330 m away and at a height of 195 m. Calculate the angle of elevation of the balloon from Jessica's position. Answer to the nearest degree.

24



Trigonometry - Topic Review

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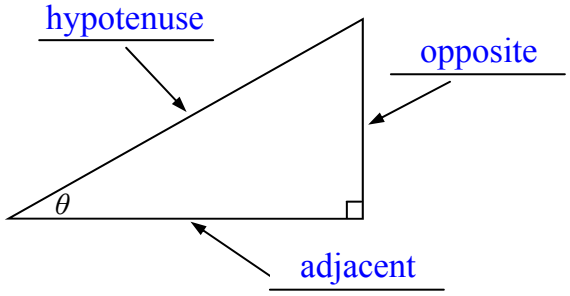
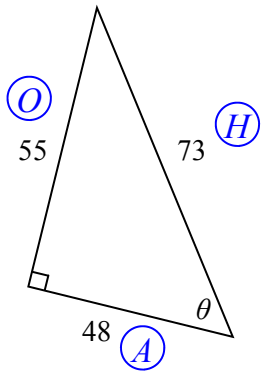
Label the hypotenuse, opposite and adjacent sides.

1



Find the trigonometric ratios for sin θ, cos θ, and tan θ.
NO CALCULATOR REQUIRED.

2

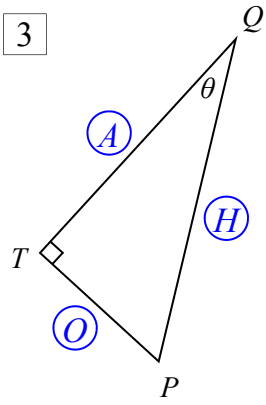


$\sin\theta = \frac{55}{73}$

$\cos\theta = \frac{48}{73}$

$\tan\theta = \frac{55}{48}$

3



$\sin\theta = \frac{TP}{QP}$

$\cos\theta = \frac{TQ}{QP}$

$\tan\theta = \frac{TP}{TQ}$

Use your calculator to solve these to 1 decimal place.

4 19 tan 38°

14.8

5 $\frac{57}{\cos 31°}$

66.5

Round these angles to the nearest degree.

6 45.36°

45°

7 59.56033°

60°

8 28° 42'

29°

9 55° 13' 55"

55°

Use your calculator to find θ, round to the nearest degree.

10 $\sin\theta = 0.4228$

$\theta = 25°$

11 $\cos\theta = \frac{3}{8}$

$\theta = 68°$

12 $\tan\theta = \frac{11}{5}$

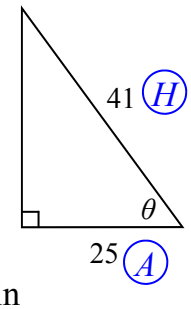
$\theta = 66°$

13 $\tan\theta = 5.4$

$\theta = 80°$

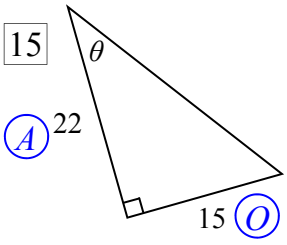
Would you use sin, cos or tan to find these angles?
Colour in the circle to indicate your choice.
DON'T SOLVE THE QUESTIONS.

14



○ sin
● cos
○ tan

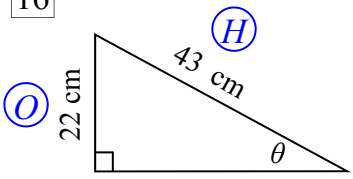
15



○ sin
○ cos
● tan

Find the angle θ, round to the nearest degree.

16



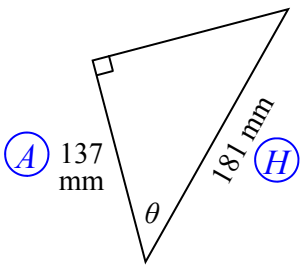
$\sin\theta = \frac{22}{43}$

$\theta = 31°$

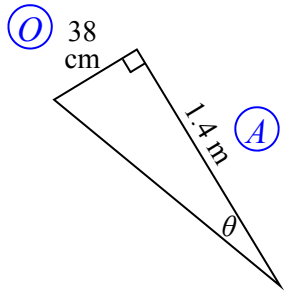
17

$\cos\theta = \frac{137}{181}$

$\theta = 41°$



18



$\tan\theta = \frac{38}{140}$

$\theta = 15°$

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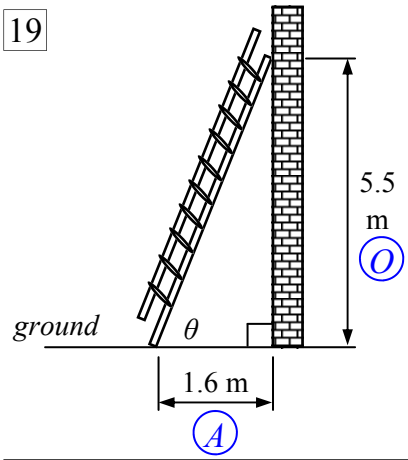
A 5.5 m ladder is leaning against a wall. The toe of the ladder is 1.6 m away from the wall. What angle (shown as θ on the diagram) does the ladder make with the ground? Answer to the nearest degree.

19

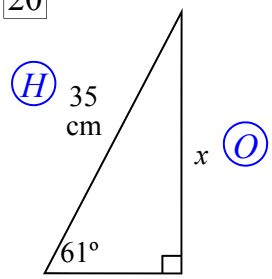


$\tan\theta = \frac{5.5}{1.6}$

$\theta = 74°$

Calculate the length 'x' on the triangles below. Answer to 1 decimal place.

20



$\sin 61 = \frac{x}{35}$

$\times 35$ $\times 35$

$x = 35 \sin 61$

$x = 30.6$ cm

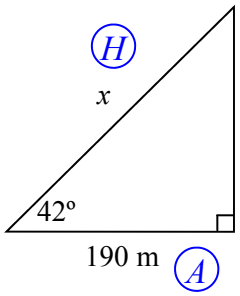
21

$\cos 42 = \frac{190}{x}$

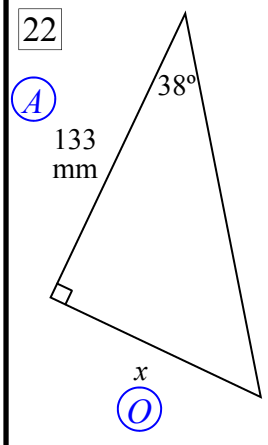
swap swap

$x = \frac{190}{\cos 42}$

$x = 255.7$ m



22



$\tan 38 = \frac{x}{133}$

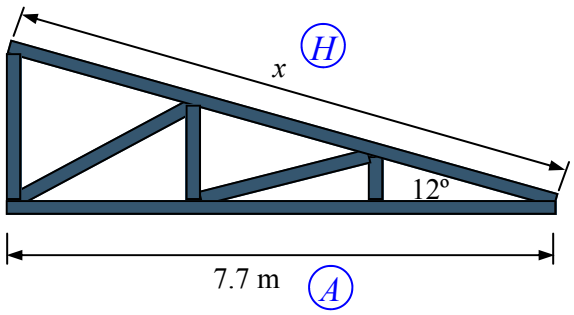
$\times 133$ $\times 133$

$x = 133 \tan 38$

$x = 103.9$ mm

A roof truss is shown below. The roof width is 7.7 m, if the required angle (pitch) of the roof is 12º. Calculate the length of the inclined timber beam, labeled 'x'. Answer in metres, to 1 decimal place.

23



$\cos 12 = \frac{7.7}{x}$

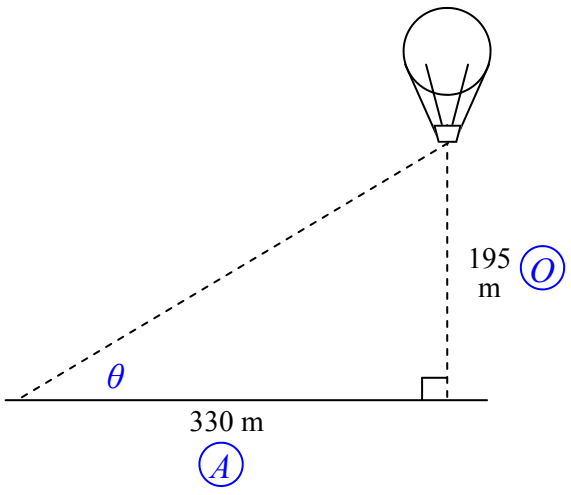
swap swap

$x = \frac{7.7}{\cos 12}$

$x = 7.9$ m

Jessica is watching a hot air balloon which is 330 m away and at a height of 195 m. Calculate the angle of elevation of the balloon from Jessica's position. Answer to the nearest degree.

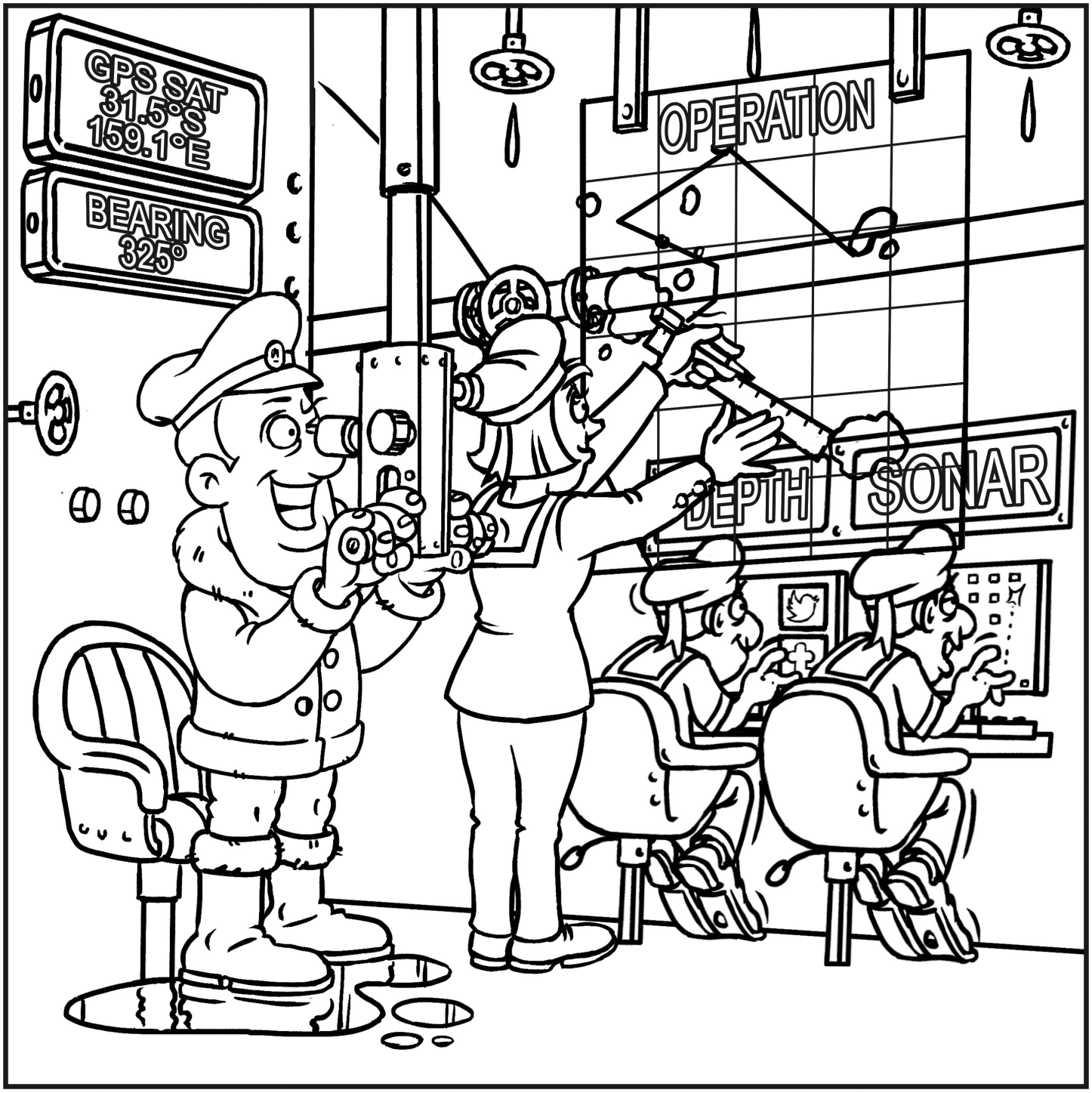
24



$\tan\theta = \frac{195}{330}$

$\theta = 31°$

Trigonometry



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Trigonometry - Naming the Sides (Sheet 1)

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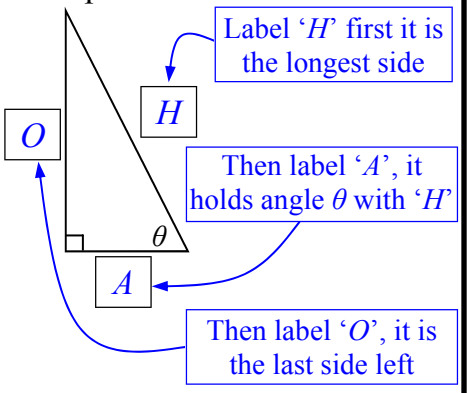
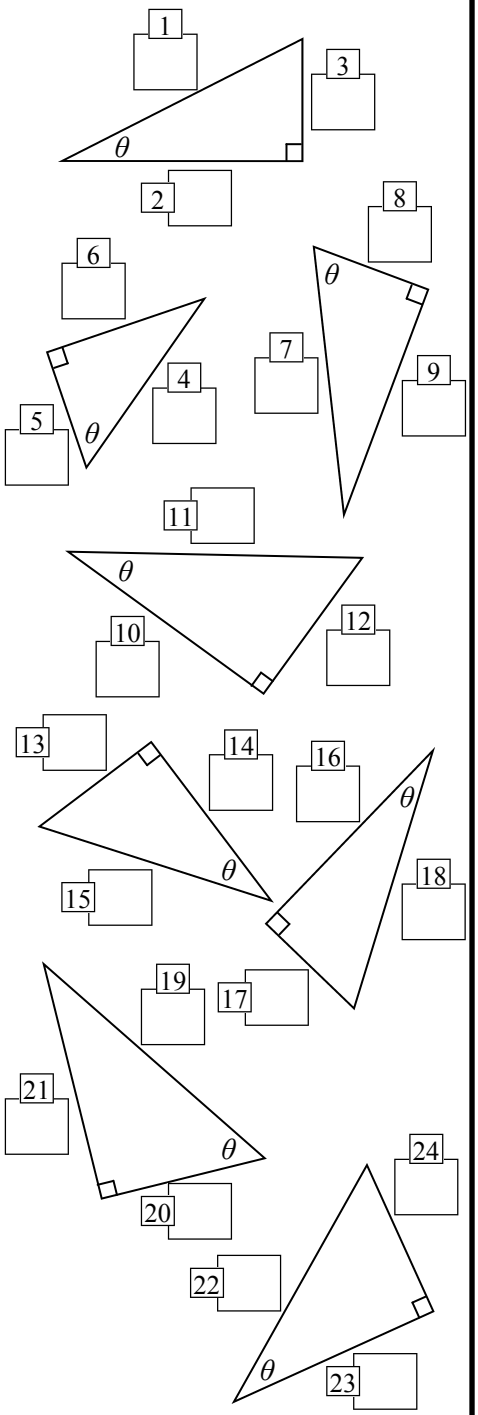
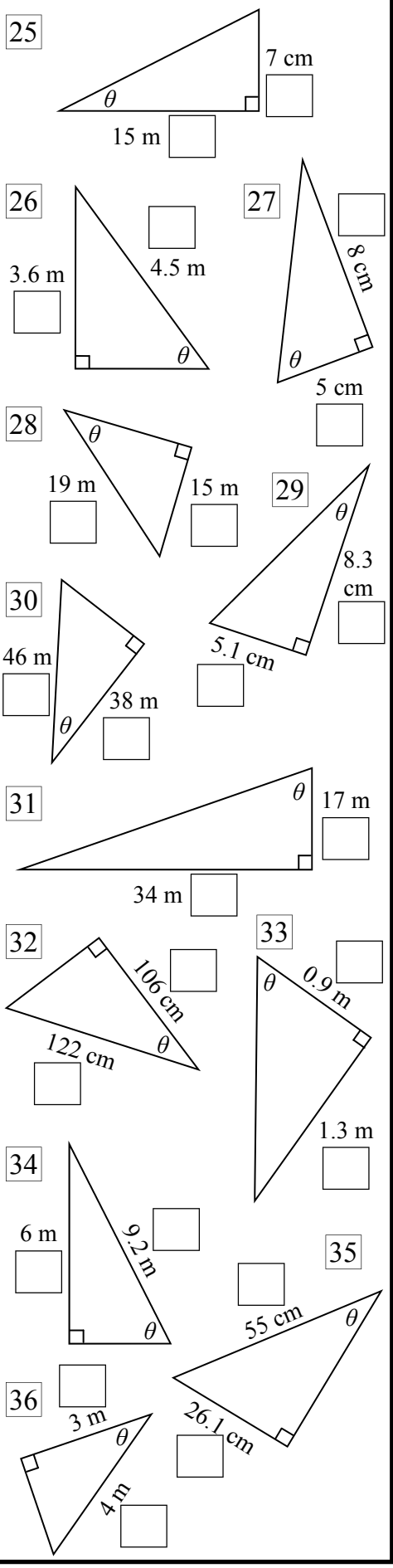
Label the sides as '*O*', '*A*' and '*H*'. Start with '*H*', the angle is always held by '*A*' and '*H*'.

Example:

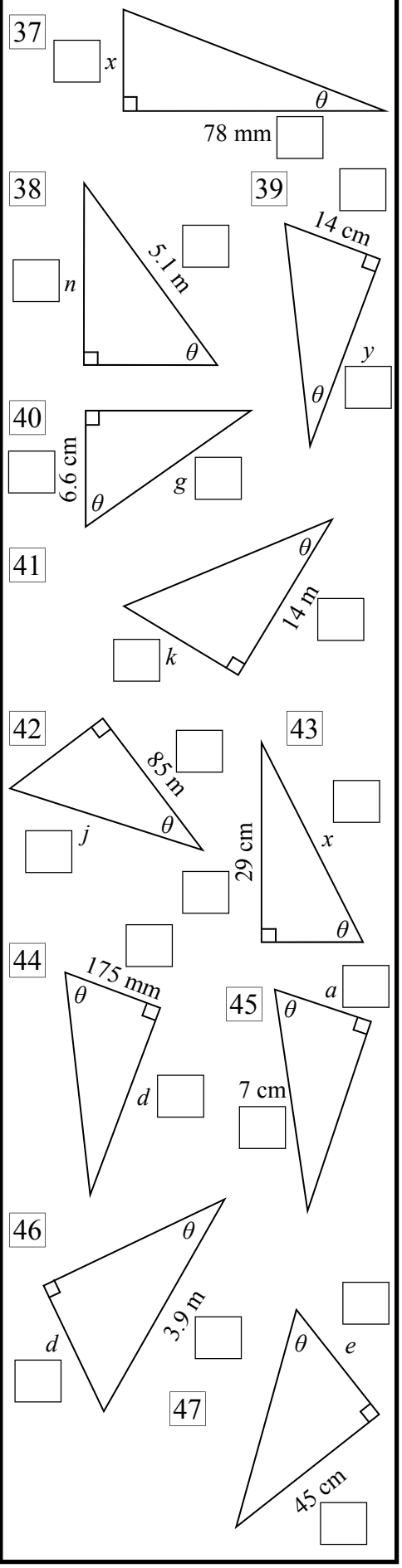




Only label two sides now. If a side has no number, don't label it. Use either '*O*', '*A*' or '*H*'.



These are the same except there is a number and a term. Label two sides '*O*', '*A*' or '*H*'.



Trigonometry - Naming the Sides (Sheet 2)

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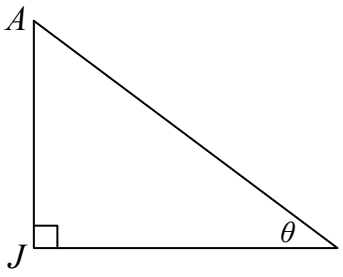
Use a ruler to measure the lengths of each side and write the name of the side and its length (whole cm) in the box.

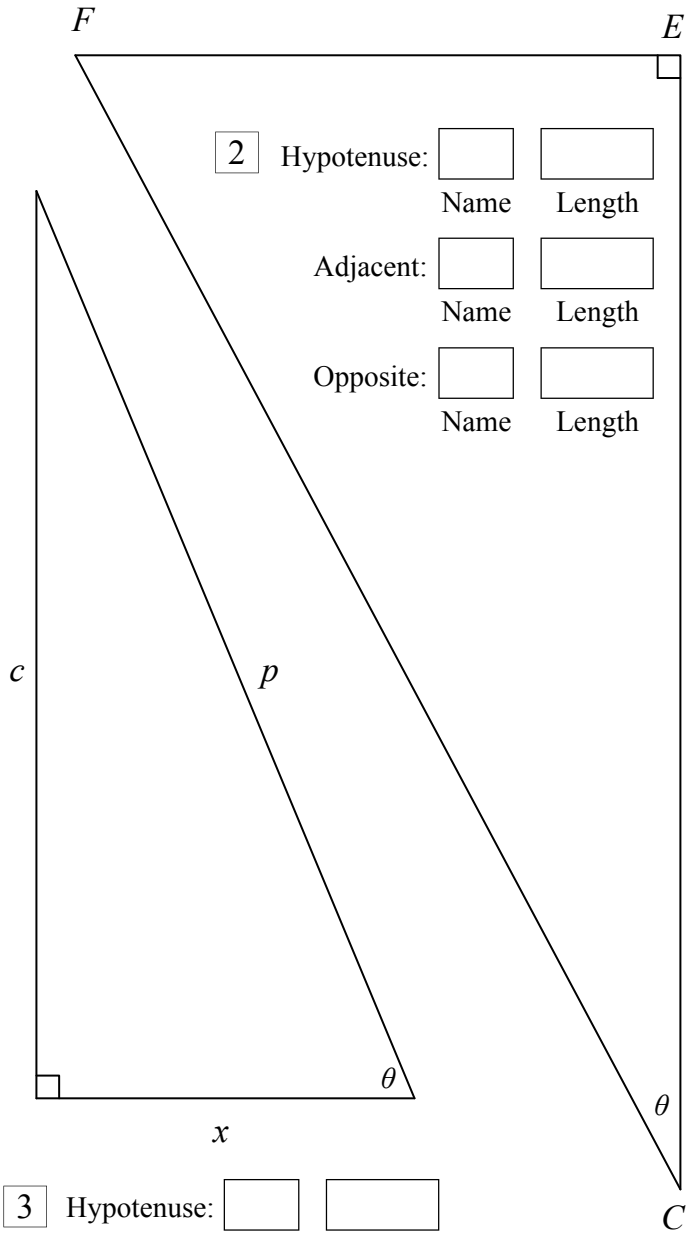
1



Hypotenuse: Name ____ Length ____ *cm*

Adjacent: Name ____ Length ____ *cm*

Opposite: Name ____ Length ____ *cm*



2 Hypotenuse: Name ____ Length ____

Adjacent: Name ____ Length ____

Opposite: Name ____ Length ____

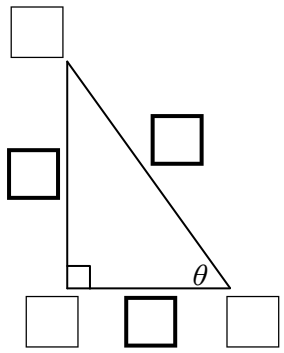
3 Hypotenuse: Name ____ Length ____

Adjacent: Name ____ Length ____

Opposite: Name ____ Length ____

Label the sides '*O*', '*A*' and '*H*' then add the letters to the vertices (corners) of the triangles. Use a pencil in case you need to change letters.

4

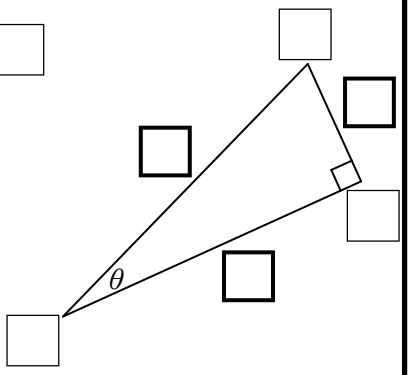


Hypotenuse: *EC*

Adjacent: *QC*

Opposite: ____ Name

5

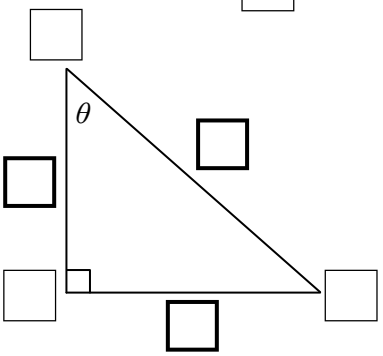


Hypotenuse: *TJ*

Adjacent: *TG*

Opposite: ____

6

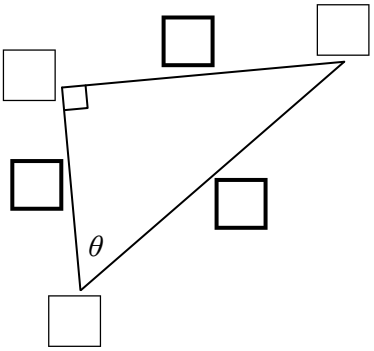


Hypotenuse: *TG*

Opposite: *CT*

Adjacent: ____

7

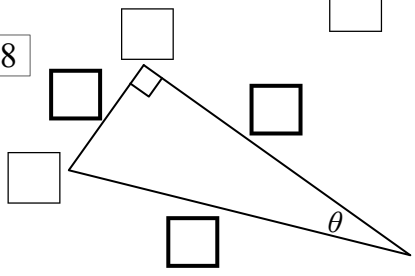


Opposite: *YR*

Adjacent: *YK*

Hypotenuse: ____

8

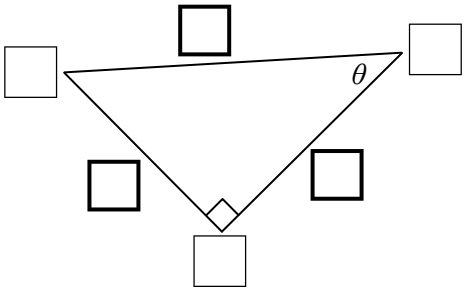


Adjacent: *WB*

Opposite: *BC*

Hypotenuse: ____

9



Hypotenuse: *EB*

Opposite: *XE*

Adjacent: ____

Trigonometry - Trigonometric Ratios (Sheet 1)

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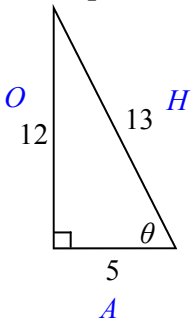
Label the sides as '*O*', '*A*' and '*H*'. Then find sin θ, cos θ and tan θ. Use *SOH CAH TOA!*

Example:

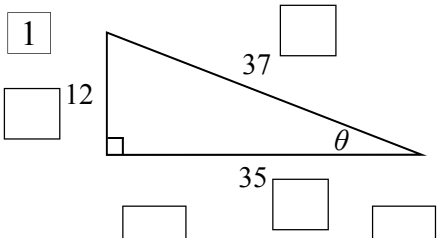


$$\sin\theta = \frac{O}{H} = \frac{12}{13}$$

$$\cos\theta = \frac{A}{H} = \frac{5}{13}$$

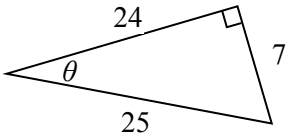
$$\tan\theta = \frac{O}{A} = \frac{12}{5}$$

1



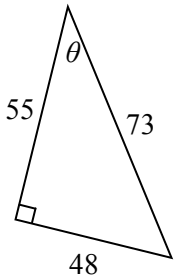
$\sin\theta =$ ☐/☐ $\quad \cos\theta =$ ☐/☐ $\quad \tan\theta =$ ☐/☐

2



$\sin\theta =$ ☐/☐ $\quad \cos\theta =$ ☐/☐ $\quad \tan\theta =$ ☐/☐

3

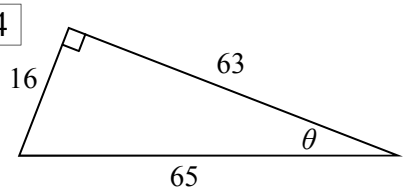


$\tan\theta =$ ☐/☐ $\quad \sin\theta =$ ☐/☐ $\quad \cos\theta =$ ☐/☐

Note: The order of the functions has changed!

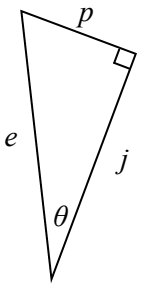
SOH CAH TOA

4



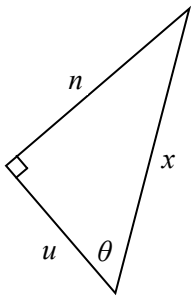
These involve algebra... same process

5

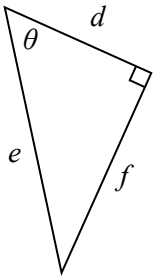


$\cos\theta =$ ☐/☐ $\quad \tan\theta =$ ☐/☐ $\quad \sin\theta =$ ☐/☐

6



7



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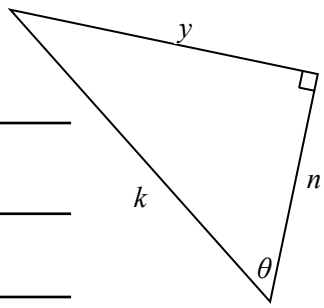
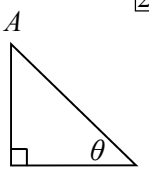


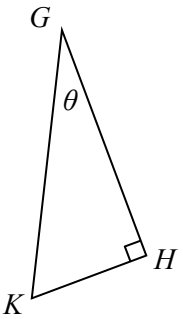
When letters are on the vertices (corners) of the triangle, use two letters to name the sides.

9



2 letters → ☐ $\quad \cos\theta =$ ☐/☐ 2 letters → ☐ $\quad \tan\theta =$ ☐/☐ $\quad \sin\theta =$ ☐/☐

10



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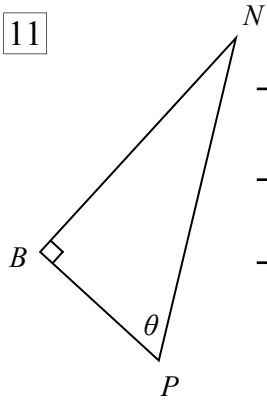

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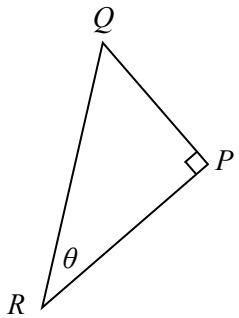

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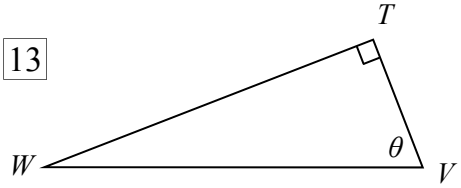

Trigonometry - Trigonometric Ratios (Sheet 2)

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When a side measurement is missing use Pythagoras. Review Pythagoras here first.

Use addition to find the hypotenuse

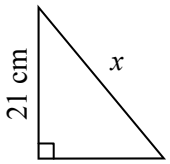
1

$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm



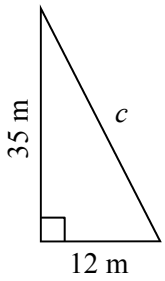
2

$c^2 = \square^2 + \square^2$

$c^2 = \square$

$c = \sqrt{\square}$

$c = \square$ m



Use subtraction to find a short side

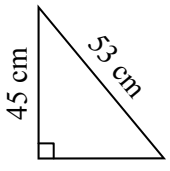
3

$x^2 = 53^2 - \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ cm



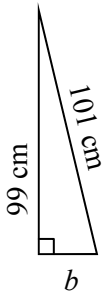
4

$b^2 = \square^2 - \square^2$

$b^2 = \square$

$b = \sqrt{\square}$

$b = \square$ cm



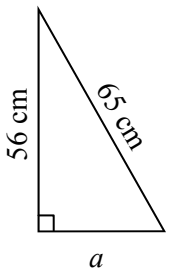
5

$a^2 =$

$a^2 =$

$a =$

$a =$



SOH CAH TOA

Find the missing side then find the trig ratios for sin θ, cos θ and tan θ.

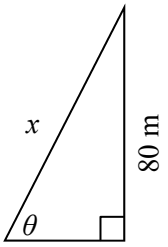
6

$x^2 = \square^2 + \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ m



Label triangle sides *H*, *A* & *O* first!

$\sin\theta = \frac{\square}{\square}$ $\cos\theta = \frac{\square}{\square}$

$\tan\theta = \frac{\square}{\square}$

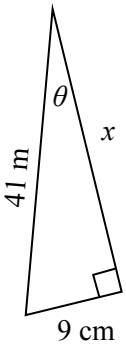
7

$x^2 = \square^2 - \square^2$

$x^2 = \square$

$x = \sqrt{\square}$

$x = \square$ m



$\tan\theta = \frac{\square}{\square}$ $\sin\theta = \frac{\square}{\square}$

$\cos\theta = \frac{\square}{\square}$

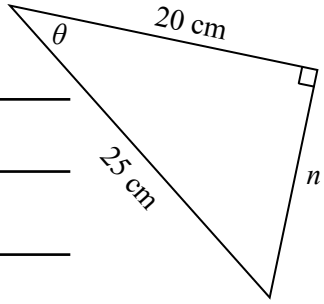
8

$n^2 =$

$n^2 =$

$n =$

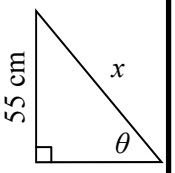
$n =$



$\tan\theta = \frac{\square}{\square}$ $\cos\theta = \frac{\square}{\square}$

$\sin\theta = \frac{\square}{\square}$

9



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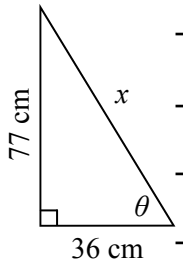

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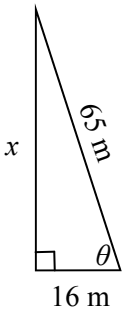

Trigonometry - Calculator Use (Angle)

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Round these angles to the nearest degree. No calculator.

Examples

$58.\underline{7}^\circ = 59^\circ$ — More than 0.5 Round UP

$18.\underline{3}^\circ = 18^\circ$ — Less than 0.5 Round DOWN

1 15.8° ____°

2 22.3° ____ Show ° symbol

3 35.7° ____

4 29.8° ____

5 11.73° ____

6 43.2177° ____

7 64.195° ____

8 59.60613° ____

Now complete the calculation and round to the nearest degree.

CALCULATOR IN DEG MODE

Example $\tan\theta = 0.3407$

Calc: SHIFT or 2ndF TAN 0 • 3 4 0 7 =

Display: 18.813……(calc)

Answer: $\theta = 19^\circ$

9 $\tan\theta = 0.556$ $\theta =$ ____°

10 $\sin\theta = 0.12$ $\theta =$ ____°

11 $\cos\theta = 0.5$ $\theta =$ ____ Show ° symbol

12 $\sin\theta = 0.2271$ ____ Write '$\theta =$'

13 $\tan\theta = 1.766$ ____

14 $\sin\theta = 0.4208$ ____

15 $\cos\theta = 0.317$ ____

Complete the calculation to find the angle θ, rounding your answer to the nearest degree.

CALCULATOR IN DEG MODE

16 $\tan\theta = \frac{4}{7}$ $\theta =$ ____°

Calc: SHIFT or 2ndF TAN 4 ▪/□ 7 =

17 $\sin\theta = \frac{7}{9}$ $\theta =$ ____°

18 $\cos\theta = \frac{2}{3}$ $\theta =$ ____°

19 $\tan\theta = \frac{6}{5}$ ____

20 $\cos\theta = \frac{4}{15}$ ____

21 $\sin\theta = \frac{13}{19}$ ____

22 $\sin\theta = \frac{2}{7}$ ____

23 $\tan\theta = \frac{7}{3}$ ____

Round these angles to the nearest degree. No calculator.

Examples

$58^\circ\ \underline{47}' = 59^\circ$ — More than 30' Round UP

$18^\circ\ \underline{27}' = 18^\circ$ — Less than 30' Round DOWN

24 11° 38' ____°

25 55° 16' ____°

26 49° 46' ____

27 79° 28' ____

28 30° 30' ____

From now on seconds are included

29 43° 52' 11.71'' ____ Use Ignore

30 35° 13' 42.05'' ____ Use Ignore

31 42° 59' 28.76'' ____ Use Ignore

32 19° 27' 11.66'' ____

33 76° 49' 23.05'' ____

34 14° 30' 55.17'' ____

Write the full answer in degrees, minutes and seconds, then round the answer to the nearest degree.

35 $\cos\theta = \frac{13}{19}$

Calc: SHIFT or 2ndF TAN 1 3 ▪/□ 1 9 = ° ' ''

Calc: ____

Use ° ' '' key, write answer in D° M' S''

Nearest °: ____

36 $\tan\theta = \frac{23}{38}$

Calc: ____

Nearest °: ____

37 $\sin\theta = 0.5715$

Calc: ____

Nearest °: ____

38 $\tan\theta = 3.2281$

Calc: ____

Nearest °: ____

Trigonometry - Finding an Angle (Sheet 1)

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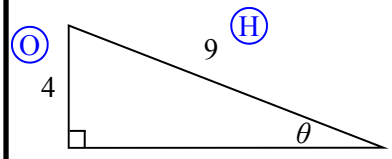
Label two of the sides either '*O*', '*A*' and '*H*'. Then pick which ratio you'd use.

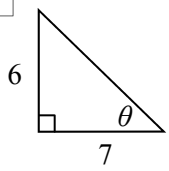
SOH $\sin\theta = \frac{\text{Opp}}{\text{Hyp}}$ CAH $\cos\theta = \frac{\text{Adj}}{\text{Hyp}}$ TOA $\tan\theta = \frac{\text{Opp}}{\text{Adj}}$

Example:



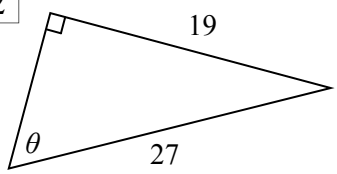
● sin ○ cos ○ tan

1



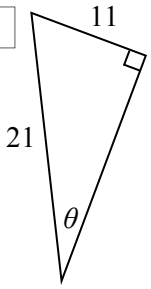
○ cos ○ tan ○ sin

2



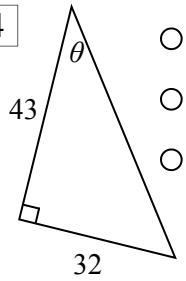
○ sin ○ cos ○ tan

3



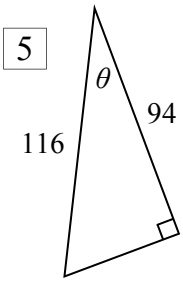
○ cos ○ tan ○ sin

4



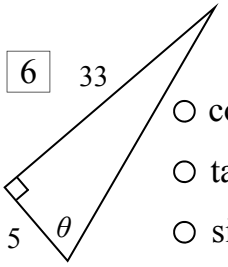
○ tan ○ sin ○ cos

5



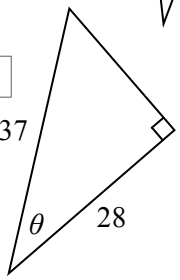
○ tan ○ cos ○ sin

6



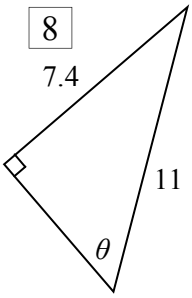
○ cos ○ tan ○ sin

7



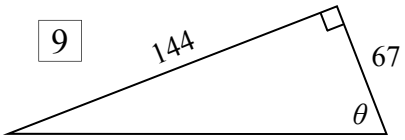
○ tan ○ cos ○ sin

8



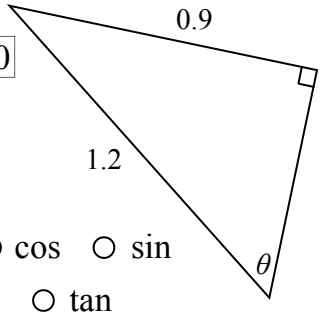
○ tan ○ cos ○ sin

9



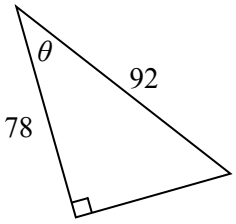
○ cos ○ tan ○ sin

10



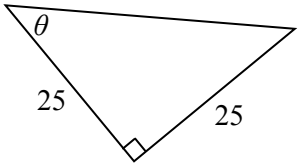
○ cos ○ sin ○ tan

11



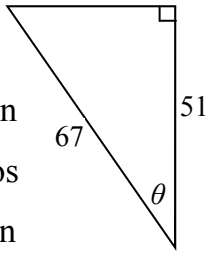
○ cos ○ tan ○ sin

12



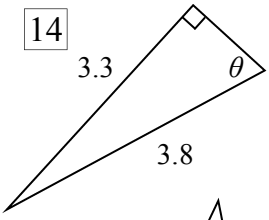
○ cos ○ sin ○ tan

13



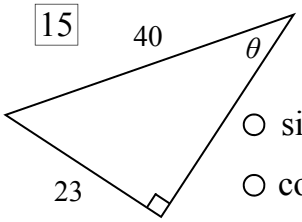
○ tan ○ cos ○ sin

14



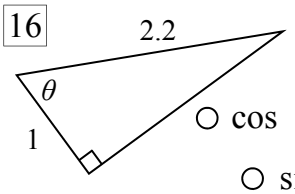
○ cos ○ tan ○ sin

15



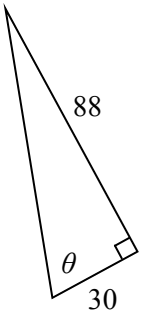
○ sin ○ cos ○ tan

16



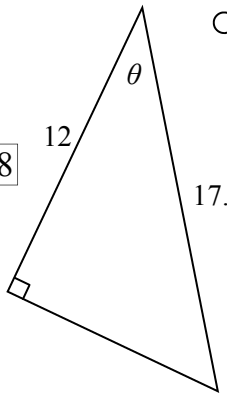
○ cos ○ tan ○ sin

17



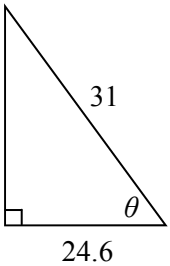
○ cos ○ sin ○ tan

18



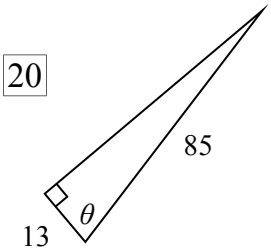
○ tan ○ cos ○ sin

19



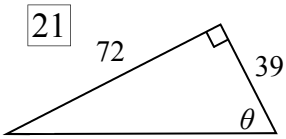
○ tan ○ sin ○ cos

20



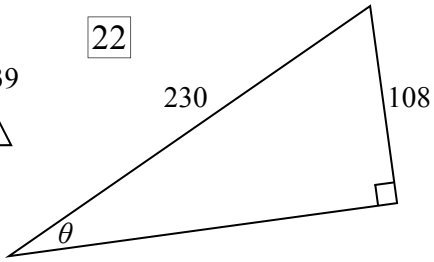
○ sin ○ tan ○ cos

21



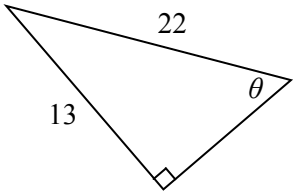
○ tan ○ cos ○ sin

22



○ cos ○ sin ○ tan

23



○ tan ○ sin ○ cos

Trigonometry - Finding an Angle (Sheet 2)

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Complete the calculation to find the angle θ, rounding your answer to the nearest degree.

CALCULATOR IN DEG MODE

1 $\sin\theta = 0.433$

Calc: SHIFT or 2ndF, SIN, 0, •, 4, 3, 3, =, °’’’ (optional)

$\theta =$ ☐ °

2 $\cos\theta = 0.76$

$\theta =$ ☐ °

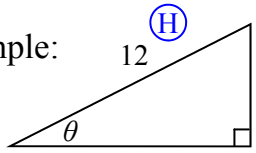
3 $\tan\theta = \frac{4}{7}$

Calc: SHIFT or 2ndF, TAN, 4, ☐, 7, =, °’’’ (optional)

$\theta =$ ☐ °

Calculate the angle θ, rounding to the nearest degree. Steps are: Label sides, pick the ratio and then solve.

Example:

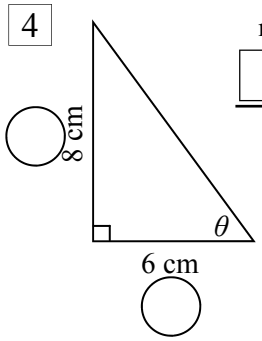


$\cos\theta = \frac{A}{H} = \frac{9}{12}$

Calc: SHIFT or 2ndF, COS, 9, ☐, 1, 2, =, °’’’ (optional)

$\theta = 41°$

4



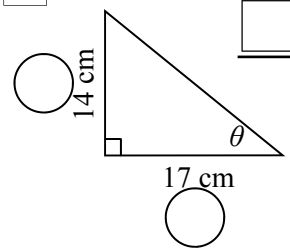
ratio ☐ / ☐ $\theta =$ ☐ / ☐ (O / A)

$\theta =$ ☐ °

Nearest Degree

SOH CAH TOA

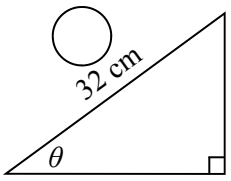
5



☐ $\theta =$ ☐ / ☐

$\theta =$ ☐ °

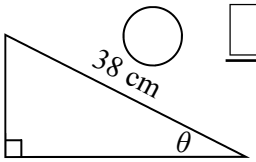
6



☐ $\theta =$ ☐ / ☐

$\theta =$ ☐ °

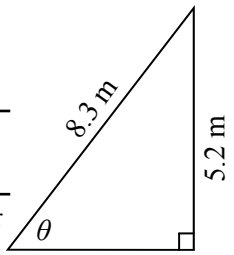
7



☐ $\theta =$ ☐ / ☐

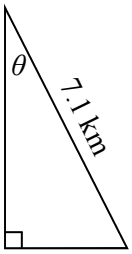
$\theta =$ ☐ °

8



Write ° after answer

9



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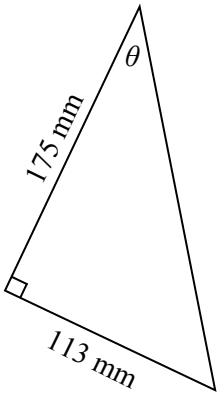

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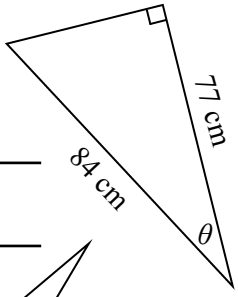

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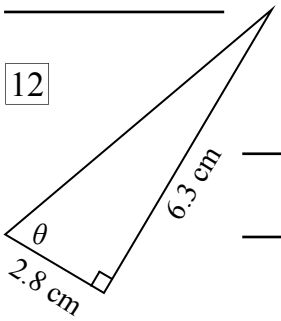
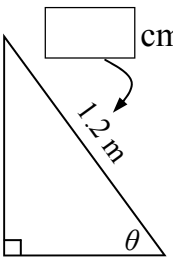


These are the same but first change the units to match each other. Find θ to the nearest °.

13



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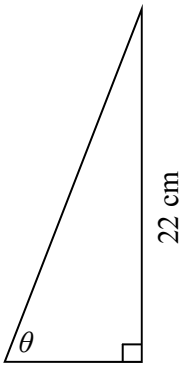

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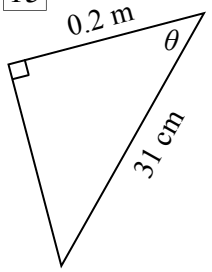
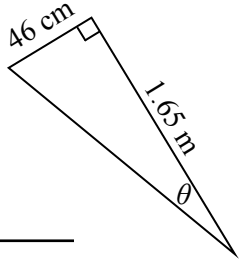


16



Trigonometry - Finding an Angle (Sheet 3)

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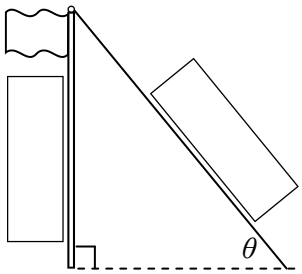
Calculate the angle θ, rounding to the nearest degree.

CALCULATOR IN DEG MODE

A 4.5 m tall flagpole has a 5.6 m guy wire anchored into the ground.

1 Write the two dimensions in the boxes on the diagram below.



2 Calculate the angle θ that the guy wire makes with the ground.

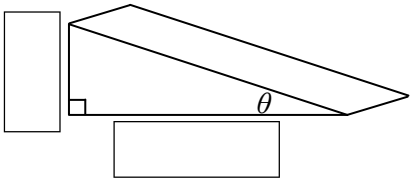
$\theta =$ ☐ / ☐

$\theta =$ ☐ °

A triangular skateboard ramp is 3.5 m long and has a height of 80 cm.

3 Convert 3.5 m to cm

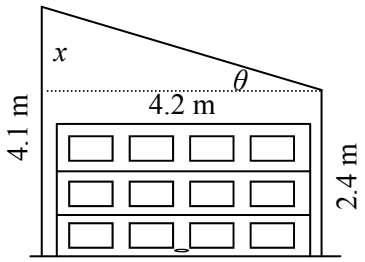
4 Add the two dimensions (in cm) to the diagram below.



5 Calculate the angle θ that the ramp makes with the ground.

SOH CAH TOA

A garage has a skillion roof shown in the diagram below.

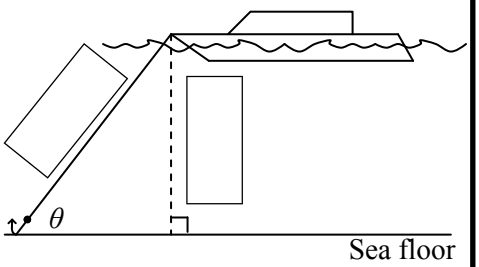


6 Calculate the distance x, being the difference in height between the two walls.

7 Calculate the 'pitch' of the roof, which is the angle θ on the diagram.

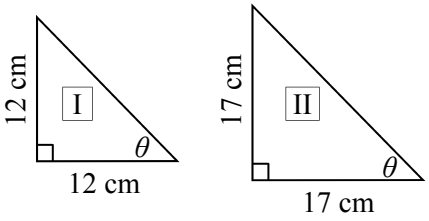
A fishing boat is anchored in 65 m deep water. The anchor rope is 90 m long and is fully extended.

8 Write the two dimensions in the boxes on the diagram below.



9 Calculate the angle θ that the anchor rope makes with the sea floor.

Toby has constructed two isosceles right triangles. He says that when the two short sides of a right triangle are the same, the angle θ is always 45º.



10 Calculate the angle θ in the two triangles.

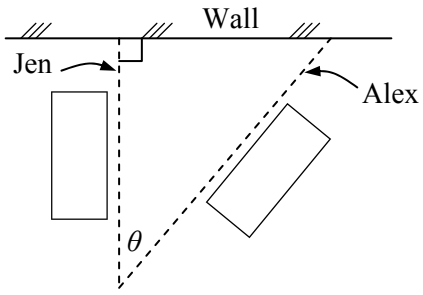
I :

II :

11 Is Toby correct?

Jen measured the straight distance to a wall of 55m. Alex walked to the wall at an angle and measured the distance as 72 m.

PLAN (TOP) VIEW



12 Write the two dimensions in the boxes on the diagram above.

13 Calculate the angle θ that Alex's route took compared to Jen's.

Trigonometry - Finding an Angle with Protractor Verification

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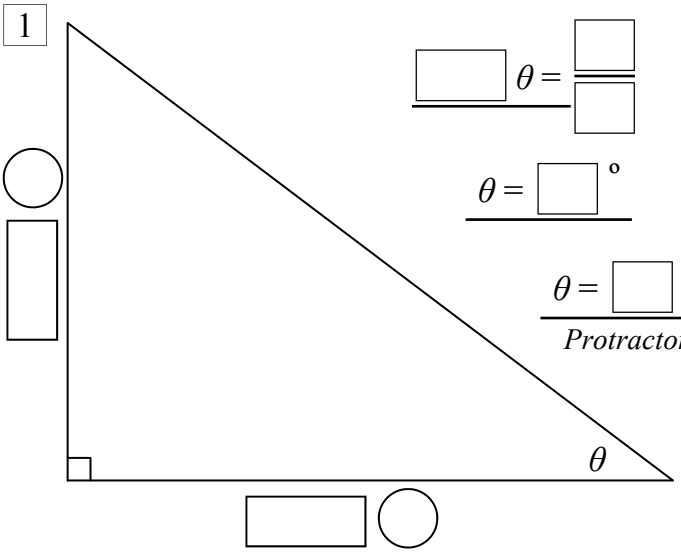
Use a ruler to measure the lengths of the sides with the box, write the side lengths (in whole cm). Then use the circles for '*O*'/'*A*'/'*H*' and use trigonometry to calculate the angle θ, to the nearest degree. Then verify the size of the angle and your calculation by checking it manually with a protractor. SOH CAH TOA

1

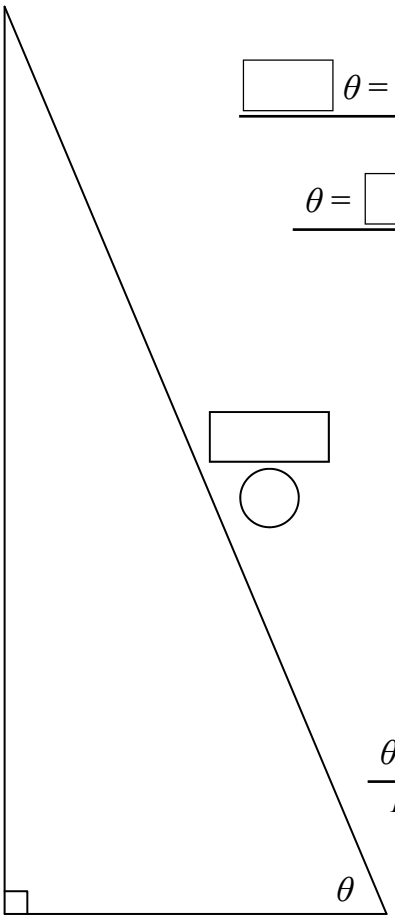


$\square\ \theta = \frac{\square}{\square}$

$\theta = \underline{\square}\ ^{o}$

$\theta = \underline{\square}\ ^{o}$ *Protractor*

2

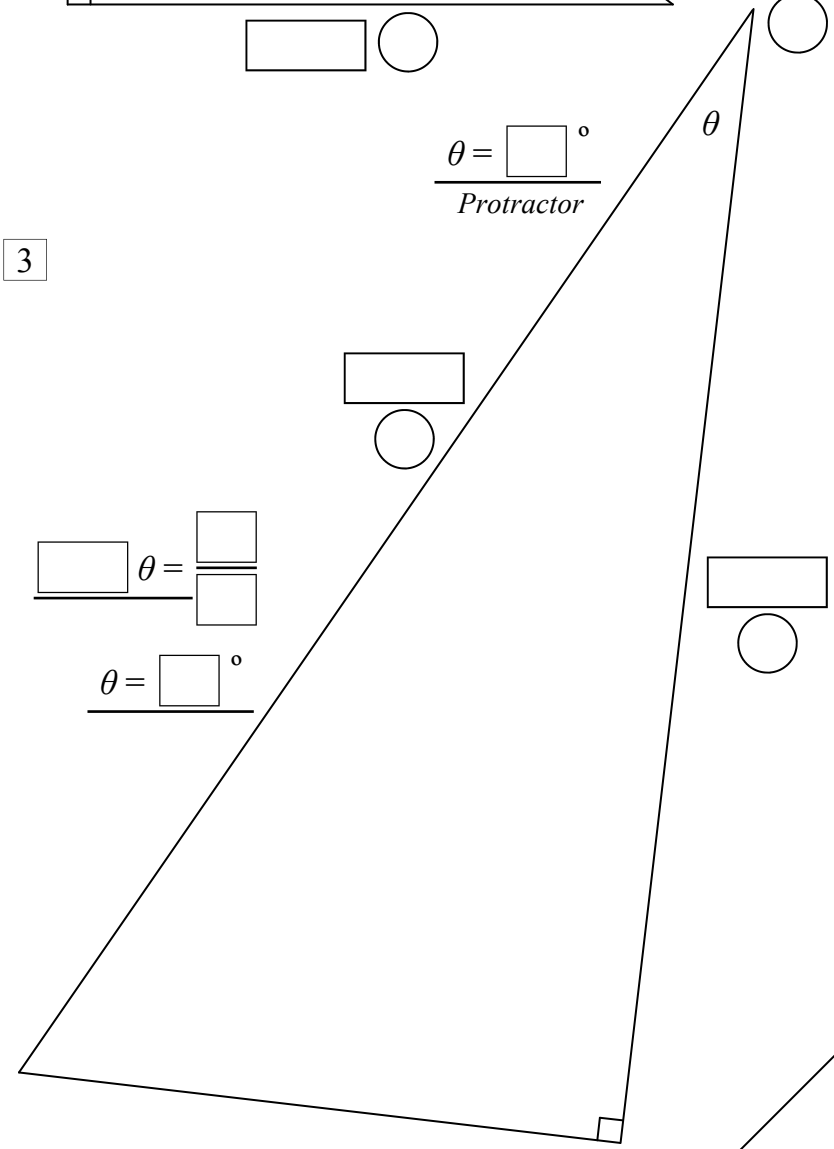


$\square\ \theta = \frac{\square}{\square}$

$\theta = \underline{\square}\ ^{o}$

$\theta = \underline{\square}\ ^{o}$ *Protractor*

3

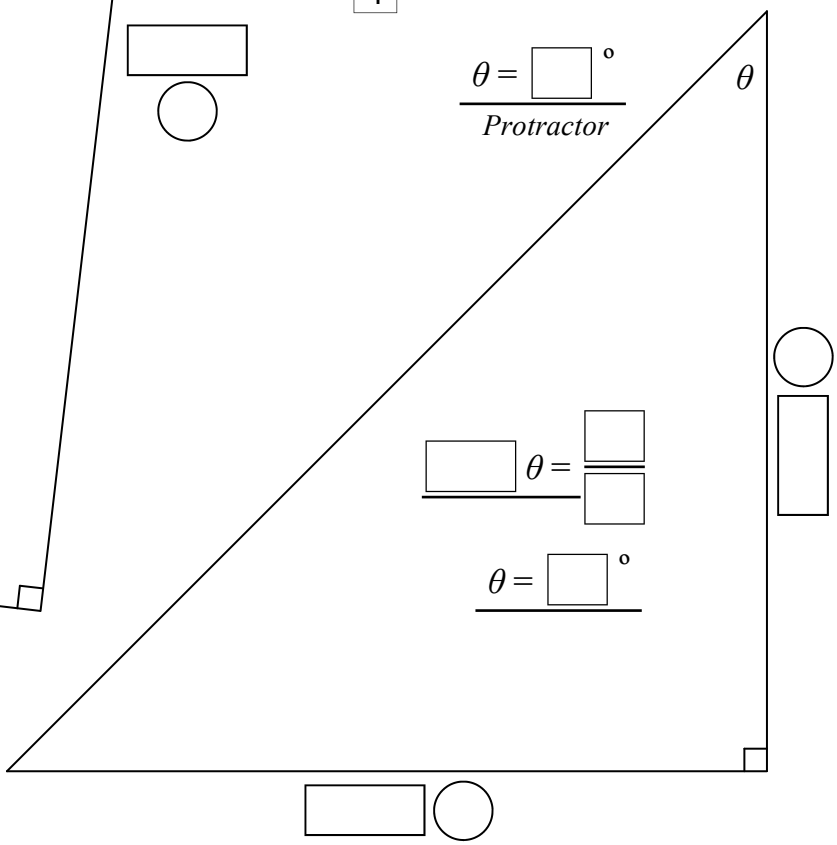


$\square\ \theta = \frac{\square}{\square}$

$\theta = \underline{\square}\ ^{o}$

$\theta = \underline{\square}\ ^{o}$ *Protractor*

4



$\square\ \theta = \frac{\square}{\square}$

$\theta = \underline{\square}\ ^{o}$

$\theta = \underline{\square}\ ^{o}$ *Protractor*

Trigonometry - Calculator Use (Side)

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Round these decimals to 1 decimal place. No calculator.

Examples

$30.382 = 30.4$ More than 5 Round UP

$73.127 = 73.1$ Less than 5 Round DOWN

1 5.87 ____

2 37.612 ____

3 592.0681 ____

4 136.9716 ____

5 200.6366 ____

Now round to 2 decimal places

6 8.3745 ____

7 95.08863 ____

8 173.9952 ____

9 44.30594 ____

10 599.9953 ____

Perform these calculations and round to 1 decimal place. You don't use the SHIFT key.

11 $5.7 \tan 38^\circ$ = ____

Calc: 5 • 7 TAN 3 8 =

12 $23 \cos 61^\circ$ = ____

13 $82 \tan 18^\circ$ = ____

14 $76.2 \cos 40^\circ$ = ____

15 $114 \tan 48^\circ$ = ____

16 $13.7 \sin 31^\circ$ = ____

17 $57.8 \cos 77^\circ$ = ____

Perform these calculations and round to 2 decimal places. You don't use the SHIFT key.

18 $48 \sin 37^\circ$ = ____

19 $66 \sin 74^\circ$ = ____

20 $14.3 \tan 61^\circ$ = ____

21 $32.7 \cos 19^\circ$ = ____

22 $150.71 \tan 53^\circ$ = ____

23 $0.8 \cos 32^\circ$ = ____

These calculations use the fraction key or division. Answer these to 1 d.p.

24 $\dfrac{9}{\sin 57^\circ}$ = ____

No bracket may give an error

Calc: 9 ▪/□ SIN 5 7) = or

9 ÷ SIN 5 7 =

25 $\dfrac{18.7}{\cos 26^\circ}$ = ____

26 $\dfrac{138}{\tan 71^\circ}$ = ____

27 $\dfrac{54.7}{\tan 20^\circ}$ = ____

28 $\dfrac{3.4}{\sin 66^\circ}$ = ____

29 $\dfrac{277}{\sin 11^\circ}$ = ____

30 $\dfrac{83.5}{\cos 48^\circ}$ = ____

31 $\dfrac{4.2}{\tan 31^\circ}$ = ____

32 $\dfrac{0.64}{\sin 19^\circ}$ = ____

Match the left hand side question with the right hand side answer (to 1 d.p.).

33

$10 \sin 27^\circ$	62.7
$\dfrac{58.5}{\tan 43^\circ}$	4.6
$\dfrac{14.13}{\cos 77^\circ}$	353.6
$11.4 \tan 22^\circ$	4.5
$58 \cos 39^\circ$	45.1
$\dfrac{155}{\sin 26^\circ}$	62.6
$8.8 \tan 82^\circ$	62.8

These include algebraic terms and measurement units. Round to 1 decimal place.

Example

$y = 80.6 \tan 28^\circ$ [cm]

$y = 42.9$ cm

Write letter Show units

34 $x = 38 \sin 52^\circ$ [m]

$x =$ ____ m

35 $y = 52.7 \cos 19^\circ$ [cm]

36 $a = 255 \tan 62^\circ$ [mm]

37 $x = 30.6 \sin 40^\circ$ [m]

38 $y = 450 \tan 45^\circ$ [km]

Trigonometry - Finding a Side (tan) (Sheet 1)

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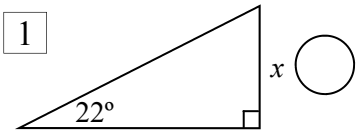
Label these sides '*O*', and '*A*'.

1



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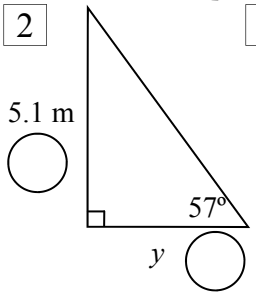

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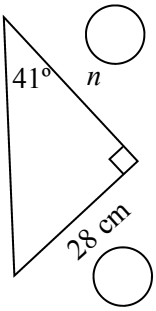

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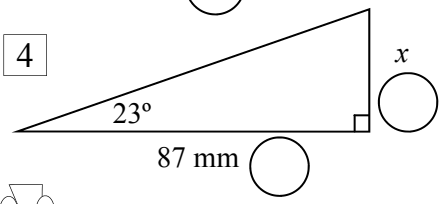


Calculate these, answer to 1 d.p.

5 14 tan 53º = ____

Calc: 1 4 TAN 5 3 =

6 9 tan 35º = ____

7 43.8 tan 32º = ____

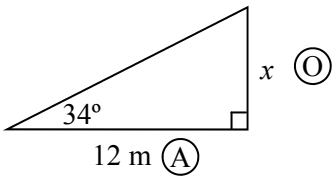
8 $\frac{77}{\tan 63º}$ = ____

9 $\frac{120}{\tan 46º}$ = ____

Calc: 7 7 ÷ TAN 6 3) =

Calculate the side length.
Answering to 1 decimal place.

10



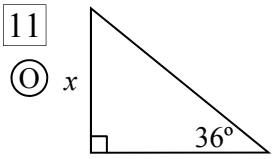
$\tan 34 = \frac{O}{A}$

$\times 12$ $\times 12$

$x = \tan$

$x =$ ____ m 1 d.p.

11



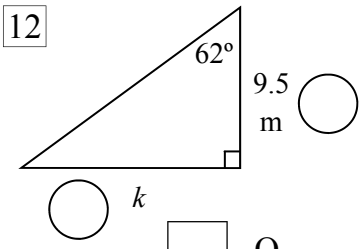
$\tan 36 = \frac{O}{A}$

$\times 55$ $\times 55$

$x = \tan$

$x =$ ____ cm 1 d.p.

12



$\tan 62 = \frac{O}{A}$

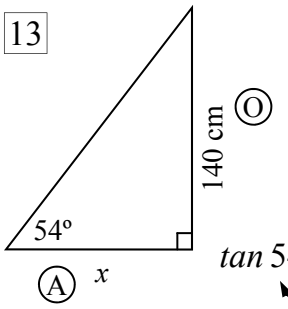
$\times 9.5$ $\times 9.5$

$k = \tan$

$k =$ ____ m 1 d.p.

Now '*x*' (the term) is on the bottom

13



$\tan 54 = \frac{O}{A}$

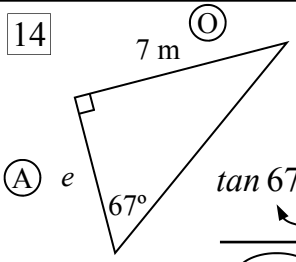
swap swap

$x = \frac{}{\tan}$

$x =$ ____ cm

1 d.p.

14



$\tan 67 = \frac{O}{A}$

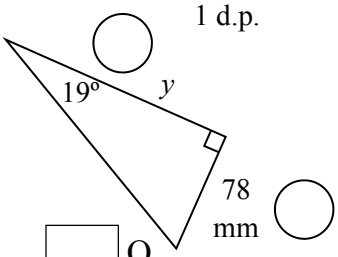
swap swap

$e = \frac{}{\tan}$

$e =$ ____ m

1 d.p.

15



$\tan 19 = \frac{O}{A}$

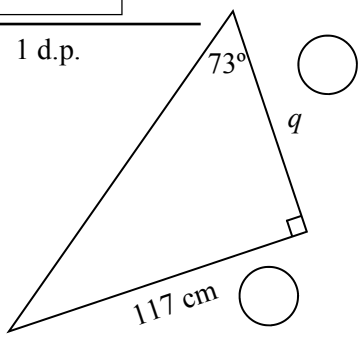
swap swap

$y = \frac{}{\tan}$

$y =$ ____ mm

1 d.p.

16



$\tan 73 = \frac{O}{A}$

swap swap

$q = \frac{}{\tan}$

$q =$ ____ cm

1 d.p.

Trigonometry - Finding a Side (tan) (Sheet 2)

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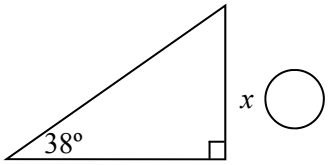
Find the side length (1 d.p.). All are tan, but are a mix of '*x* on top' and '*x* on the bottom'.

1



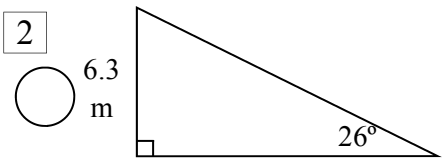
$\tan(\square) = \frac{\square\ \text{O}}{\square\ \text{A}}$

$\times 13$ $\times 13$

$x = \square \tan \square$

$x = \square$ cm 1 d.p.

2



$\tan(\square) = \frac{\square\ \text{O}}{\square\ \text{A}}$

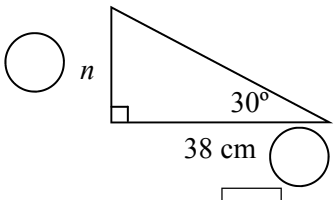
swap swap

$d = \frac{\square}{\tan \square}$

$d = \square$

Remember the units

3

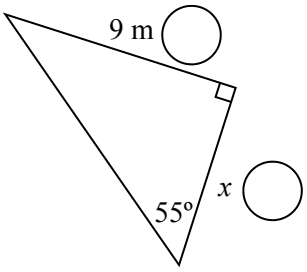


$\tan(\square) = \frac{\square}{\square}$

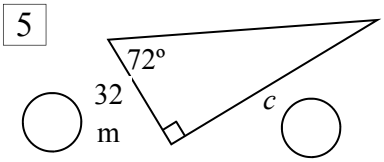
$n = \square \tan \square$

$n = \square$

4



5



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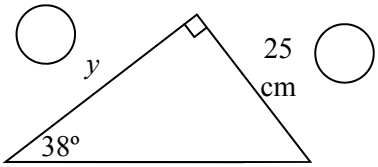
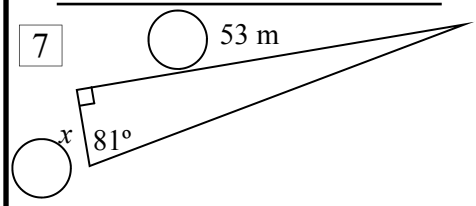


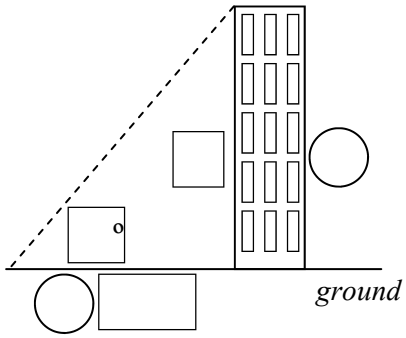
7



Answer these using your tan trig skills. Find the length to 1 decimal place.

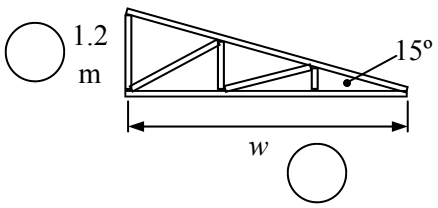
Jess looks at a building that is 72 m away. She looks up at an angle of 54°

8 Write '*x*' as the height of the building, then write the angle and the distance in the boxes on the diagram.



9 Label the distances '*O*' and '*A*' then calculate the building's height.

A roof truss is angled at 15° and has a height of 1.2 m. The width of the roof '*w*' is shown on the diagram.



10 Find '*w*', the width of the roof.

Trigonometry - Finding a Side (sin) (Sheet 1)

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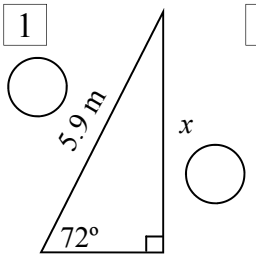
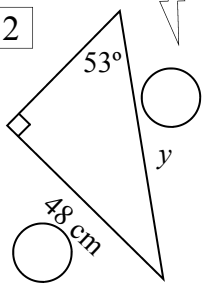
Label these sides '*O*', and '*H*'.

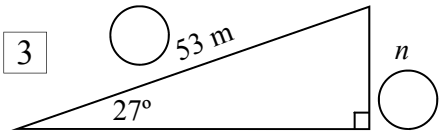
1



2



3



Calculate these, answer to 1 d.p.

4 $\dfrac{13}{\sin 21^\circ}$ = ____

5 $\dfrac{54.7}{\sin 77^\circ}$ = ____

Calc: 1 3 ▪/□ SIN 2 1) =

6 $94 \sin 36^\circ$ = ____

Calc: 9 4 SIN 3 6 =

7 $11.2 \sin 62^\circ$ = ____

8 $118 \sin 48^\circ$ = ____

9 $\dfrac{200}{\sin 55^\circ}$ = ____

10 $\dfrac{0.8}{\sin 32^\circ}$ = ____

Calculate the side length.
Answering to 1 decimal place.

11 (O) *x*, (H) 31 cm, 28°

$\sin 28 = \dfrac{O}{H}$

$\times 31$ $\times 31$

$x =$ ____ $\sin$ ____

$x =$ ____ cm 1 d.p.

12 (H) 185 mm, *e*, (O), 38°

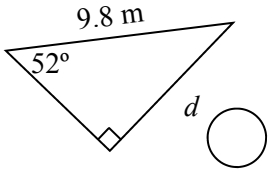
$\sin 38 = \dfrac{O}{H}$

$\times 185$ $\times 185$

$e =$ ____ $\sin$ ____

$e =$ ____ mm 1 d.p.

13



$\sin 52 = \dfrac{\square}{\square}$

$\times 9.8$ $\times 9.8$

$d =$ ____ $\sin$ ____

$d =$ ____ m 1 d.p.

Now '*x*' (the term) is on the bottom

14 (H) *x*, 18°, 3.5 m, (O)

$\sin 18 = \dfrac{O}{H}$

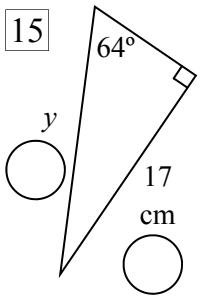
swap swap

$x =$ ____ / $\sin$ ____

$x =$ ____ m

1 d.p.

15



$\sin 64 = \dfrac{O}{H}$

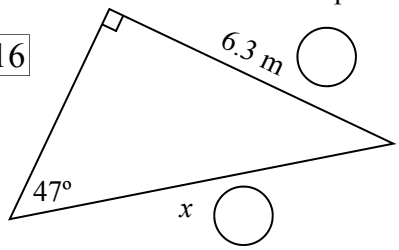
swap swap

$y =$ ____ / $\sin$ ____

$y =$ ____ cm

1 d.p.

16



$\sin 47 = \dfrac{\square}{\square}$

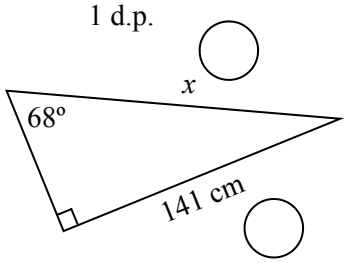
swap swap

$x =$ ____ / $\sin$ ____

$x =$ ____ m

1 d.p.

17



$\sin 68 = \dfrac{\square}{\square}$

swap swap

$x =$ ____ / $\sin$ ____

$x =$ ____ cm

1 d.p.

Trigonometry - Finding a Side (sin) (Sheet 2)

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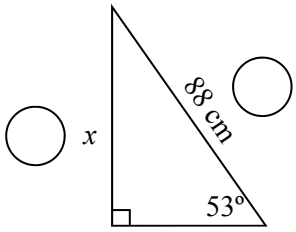
Find the side length (1 d.p.). All are sin, but are a mix of '*x* on top' and '*x* on the bottom'.

1



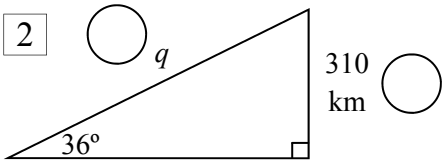
$\sin(\square) = \frac{\square\ \text{O}}{\square\ \text{H}}$

$\times 88$ $\times 88$

$x = \square \sin \square$

$x = \square$ cm 1 d.p.

2



$\sin(\square) = \frac{\square\ \text{O}}{\square\ \text{H}}$

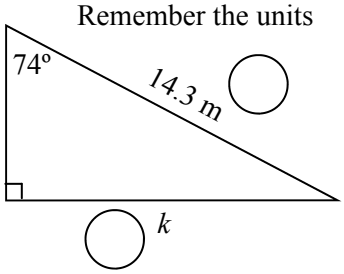
swap swap

$q = \frac{\square}{\sin \square}$

$q = \square$

Remember the units

3

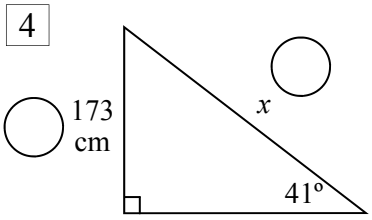


$\sin(\square) = \frac{\square}{\square}$

$k = \square \sin \square$

$k = \square$

4



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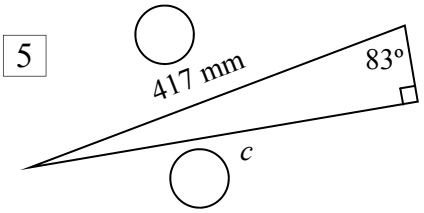

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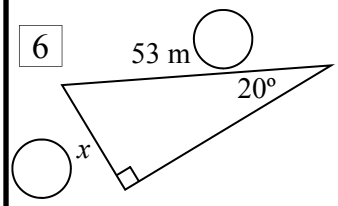

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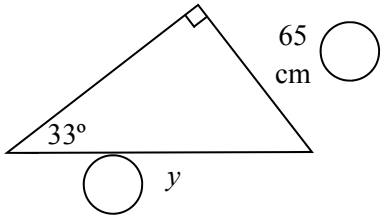
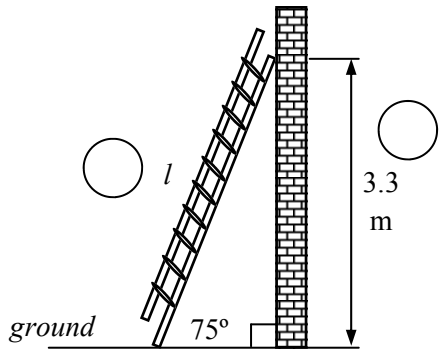


Answer these using your sin trig skills. Find the lengths to 1 decimal place.

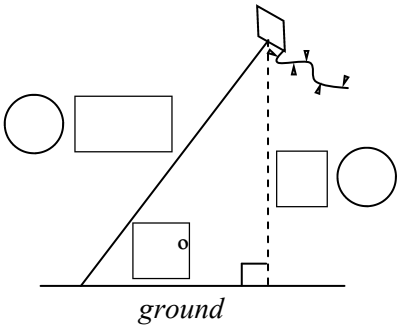
A ladder is leaning against a wall making an angle of 75° with the ground. The ladder reaches 3.3m up the wall.



8 Find '*l*', the length of the ladder.

A kite string is 55 m long and is fully extended. The string makes an angle of 64° with the ground.

9 Write '*x*' as the height of the kite. Then write the string length and angle in the two boxes on the diagram



10 Label the distances 'O' and 'H' then calculate the height of the kite.

Trigonometry - Finding a Side (cos) (Sheet 1)

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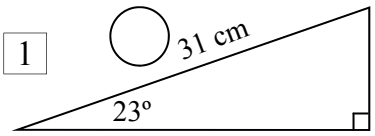
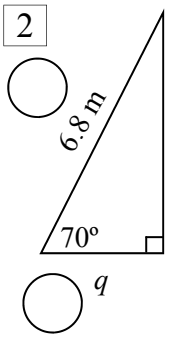
Label these sides '*A*', and '*H*'.

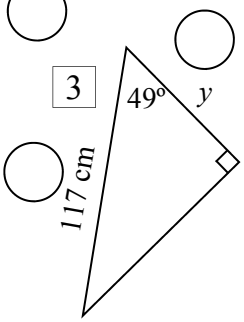
1



2



3



Calculate these, answer to 1 d.p.

4 $\dfrac{57}{\cos 36^\circ}$ = ____

5 $\dfrac{102}{\cos 8^\circ}$ = ____

Calc: 5 7 ▪ COS 3 6) =

6 $94 \cos 36^\circ$ = ____

Calc: 9 4 COS 3 6 =

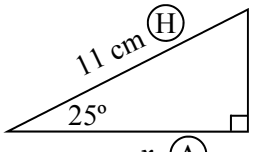
7 $\dfrac{40.4}{\cos 19^\circ}$ = ____

8 $\dfrac{655}{\cos 42^\circ}$ = ____

9 $0.2 \cos 28^\circ$ = ____

Calculate the side length.
Answering to 1 decimal place.

10



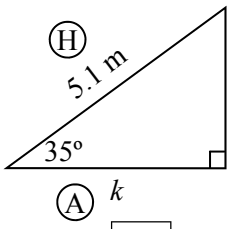
$\cos 25 = \dfrac{\square}{\square}\ \begin{matrix}A\\H\end{matrix}$

(×11) (×11)

$x =$ ____ cos ____

$x =$ ____ cm 1 d.p.

11



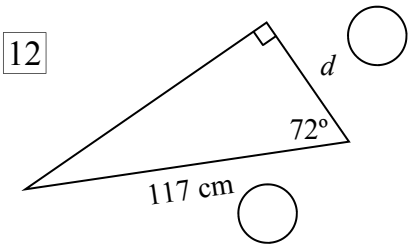
$\cos 35 = \dfrac{\square}{\square}\ \begin{matrix}A\\H\end{matrix}$

(×5.1) (×5.1)

$k =$ ____ cos ____

$k =$ ____ m 1 d.p.

12



$\cos 72 = \dfrac{\square}{\square}$

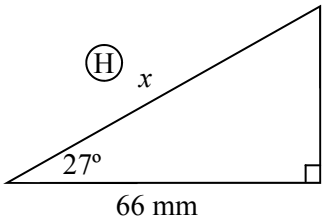
(×117) (×117)

$d =$ ____ cos ____

$d =$ ____ cm 1 d.p.

Now '*x*' (the term) is on the bottom

13



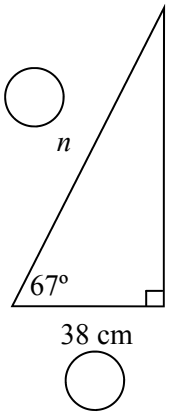
$\cos 27 = \dfrac{\square}{\square}\ \begin{matrix}A\\H\end{matrix}$

swap swap

$x =$ ____ / cos ____

$x =$ ____ mm 1 d.p.

14



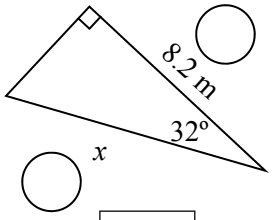
$\cos 67 = \dfrac{\square}{\square}\ \begin{matrix}A\\H\end{matrix}$

swap swap

$n =$ ____ / cos ____

$n =$ ____ cm 1 d.p.

15



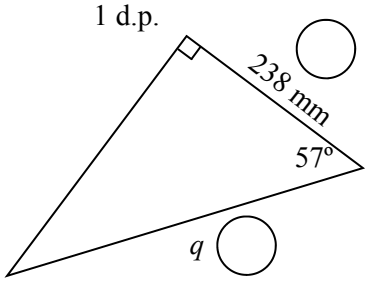
$\cos 32 = \dfrac{\square}{\square}$

swap swap

$x =$ ____ / cos ____

$x =$ ____ m 1 d.p.

16



$\cos 57 = \dfrac{\square}{\square}$

swap swap

$q =$ ____ / cos ____

$q =$ ____ mm 1 d.p.

Trigonometry - Finding a Side (cos) (Sheet 2)

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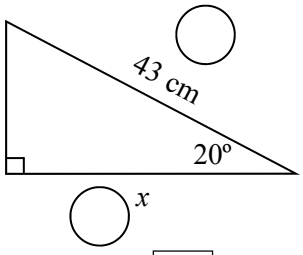
Find the side length (1 d.p.). All are sin, but are a mix of '*x* on top' and '*x* on the bottom'.

1



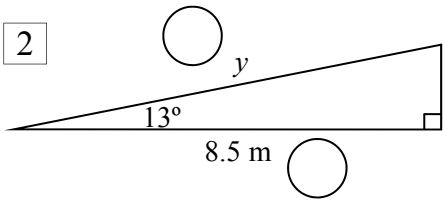
$\cos \square = \frac{\square\ \text{A}}{\square\ \text{H}}$

$\times 43$ $\times 43$

$x = \square \cos \square$

$x = \square$ cm 1 d.p.

2



$\cos \square = \frac{\square\ \text{A}}{\square\ \text{H}}$

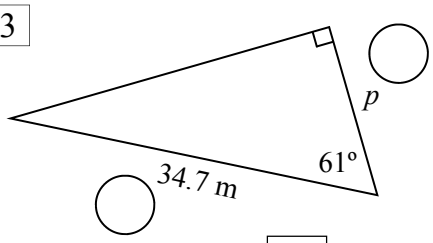
swap swap

$y = \frac{\square}{\cos \square}$

$y = \square$

Remember the units

3

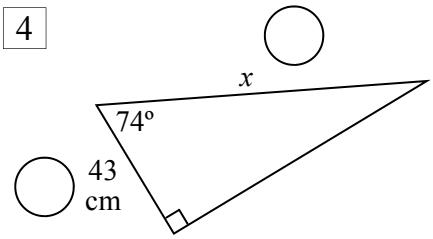


$\cos \square = \frac{\square}{\square}$

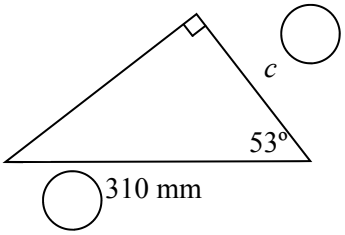
$p = \square \cos \square$

$p = \square$

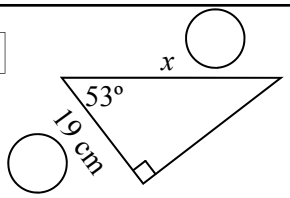
4



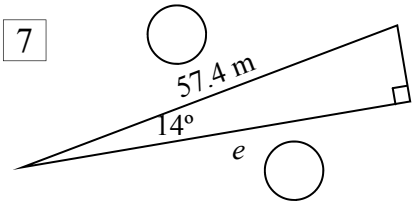
5



6

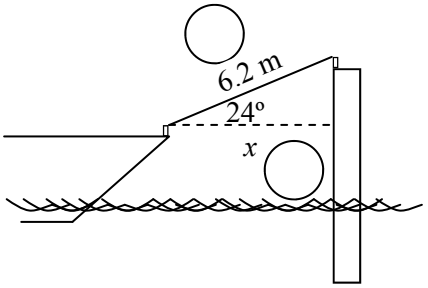


7



Answer these using your cos trig skills. Find the lengths to 1 decimal place.

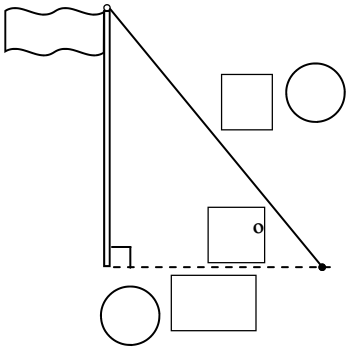
A boat is moored to a jetty. The rope is 6.2 m long and the angle of the rope is 24º. Label sides '*A*' & '*H*'.



8 Calculate the distance '*x*' of the boat from the jetty.

A flag pole has a guy wire attached to it which is secured to the ground 6 m from the pole. The guy wire makes an angle of 51º with the ground.

9 Write '*d*' as the length of the guy wire. Write the distance and angle in the two boxes on the diagram.



10 Label the distances '*A*' and '*H*' then calculate the length of the guy wire.

Trigonometry - Finding a Side - All Ratios - '*x*' on top (Sheet 1)

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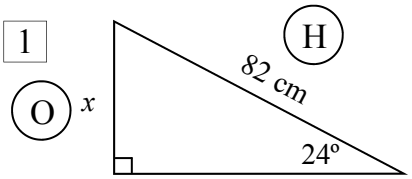
Find the side length to 1 d.p. These are a mix of sin, cos and tan. Use SOH CAH TOA.

1



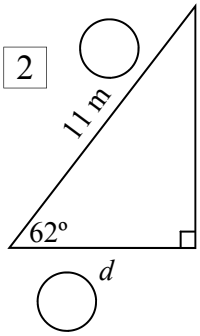
$\sin \square = \frac{\square\ O}{\square\ H}$

$\times 82$ $\times 82$

$x = \square \sin \square$

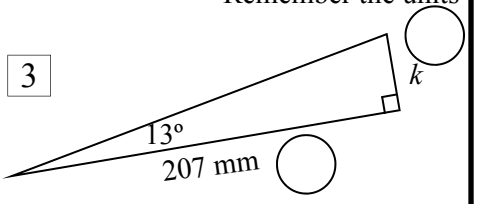
$x = \square$ cm 1 d.p.

2



Remember the units

3



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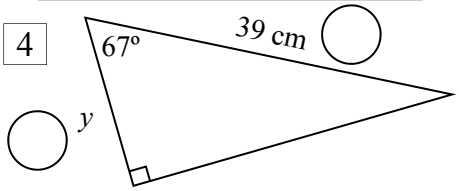
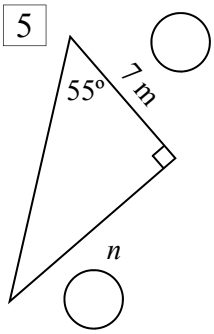


SOH CAH TOA

5



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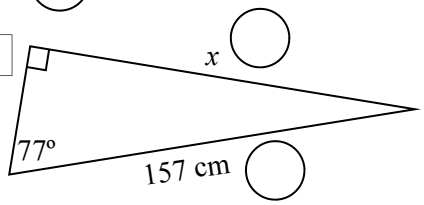

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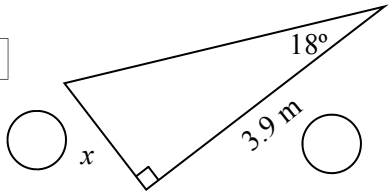

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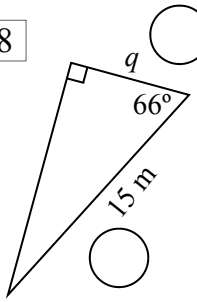

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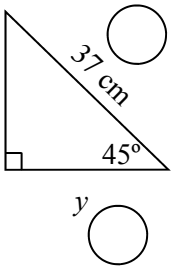

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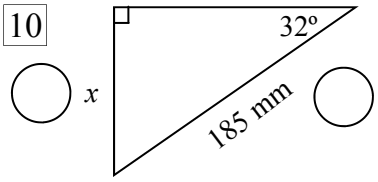

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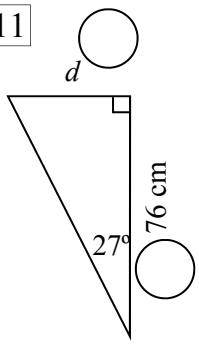

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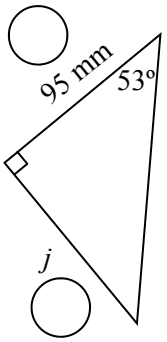

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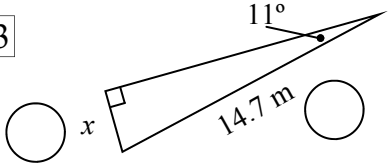

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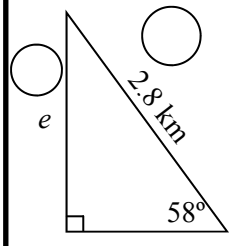

Trigonometry - Finding a Side - All Ratios - '*x*' on bottom (Sheet 2)

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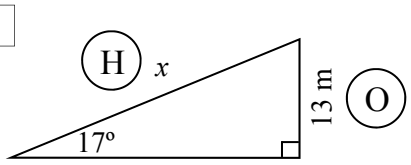
Find the side length to 1 d.p. These are a mix of sin, cos and tan. Use SOH CAH TOA.

SOH CAH TOA

1



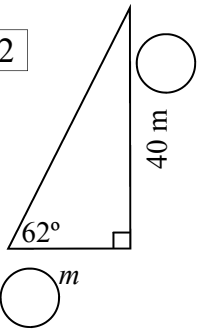
sin ☐ = ☐ / ☐ (O / H)

swap swap

$x =$ ☐ / sin ☐

$x =$ ☐ m 1 d.p.

2



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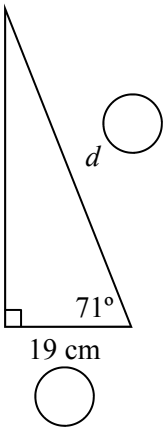

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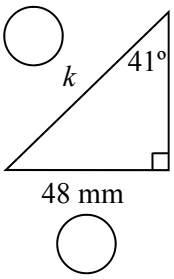

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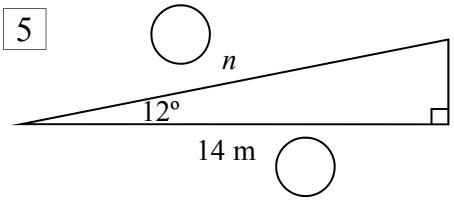

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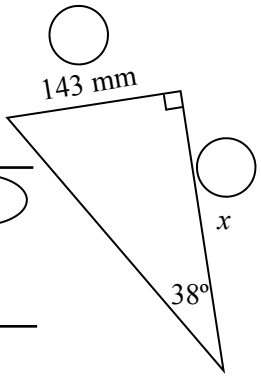

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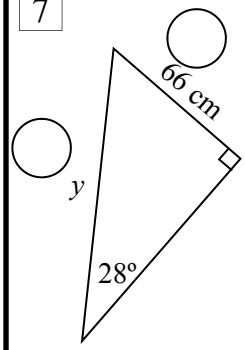

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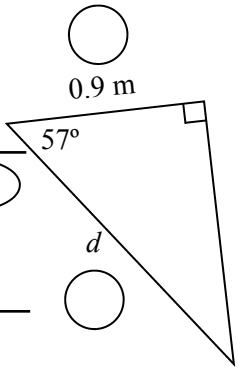

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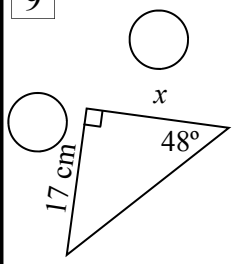

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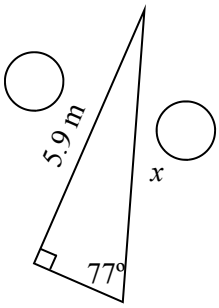

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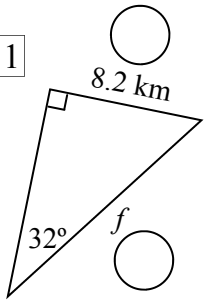

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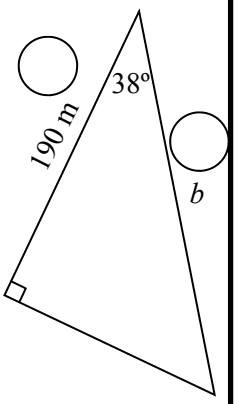

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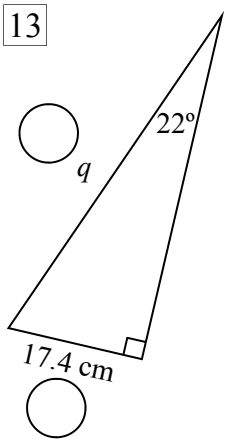

Trigonometry - Finding a Side - All Ratios - Top/bottom Mixed (Sheet 3)

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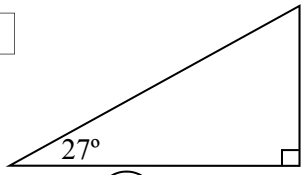
Find the side length to 1 d.p. These are a mix of sin, cos and tan. With '*x*' on top & bottom.

SOH CAH TOA

1



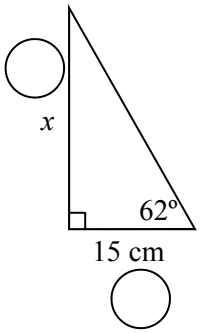
$\tan \square = \frac{\square}{\square}$ (O / A)

swap swap

$x = \frac{\square}{\tan \square}$

$x =$ ____ cm 1 d.p.

2



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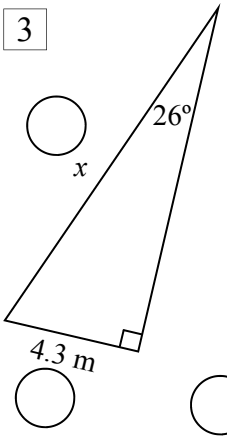

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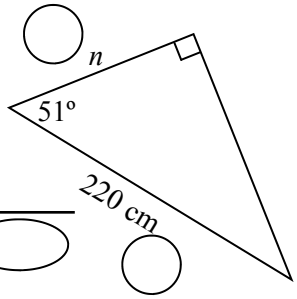

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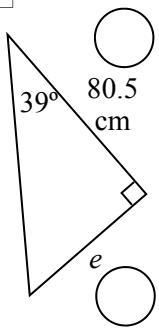

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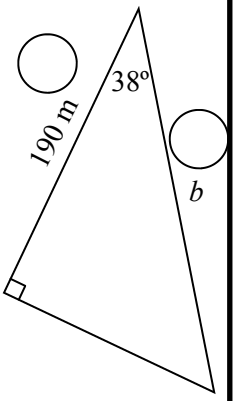

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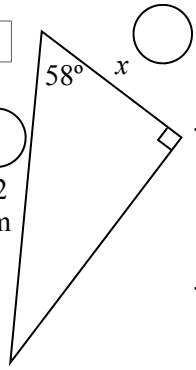

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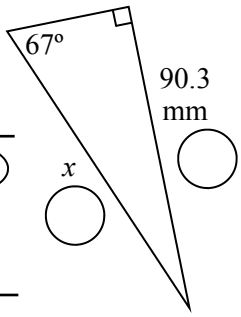

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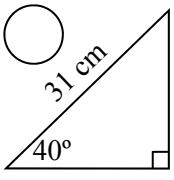

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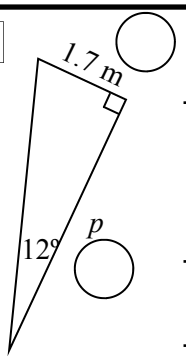

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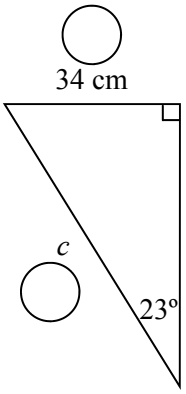

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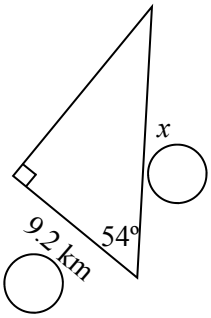

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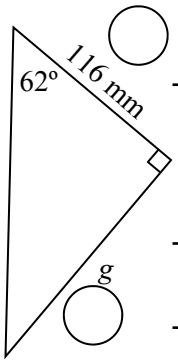

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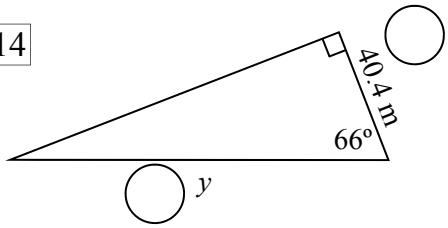

Trigonometry - Elevation/Depression - Finding an Angle (Sheet 1)

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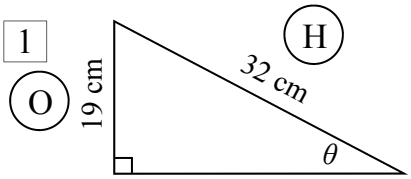
Find the angle 'θ' to the nearest degree.

SOH CAH TOA

1

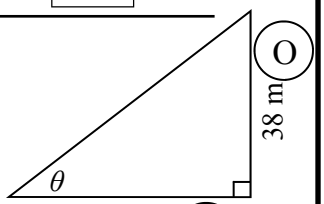


$$\sin\theta = \frac{O}{H}$$

Calc: SHIFT SIN 1 9 ÷ 3 2 =

$\theta =$ ____ °

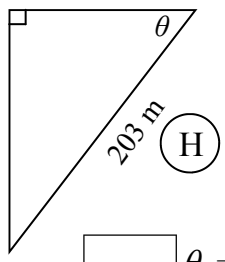
2



$$\theta = \frac{O}{A}$$

$\theta =$ ____ °

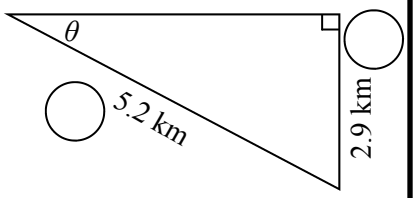
3



$$\theta = \frac{A}{H}$$

$\theta =$ ____ °

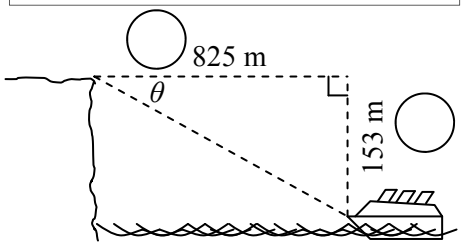
4



$$\theta = \frac{\quad}{\quad}$$

$\theta =$ ____ °

A ship is 825 m from a cliff, which is 153 m tall. Eric is at the top of the cliff, looking down at the ship.

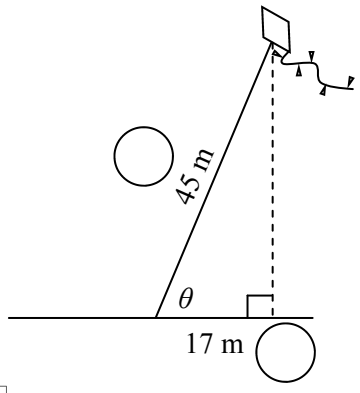


5 Calculate the angle of depression

$$\theta = \frac{\quad}{\quad}$$

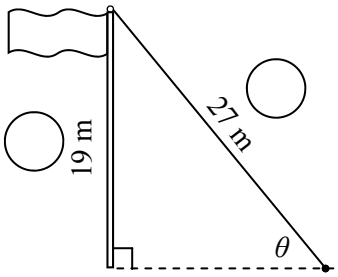
$\theta =$ ____ °

An airborne kite is attached to the ground. Its string is 45 m long and is fully extended. The kite is flying above a point 17 m away.



6 Calculate the angle of elevation from the anchor point up to the kite.

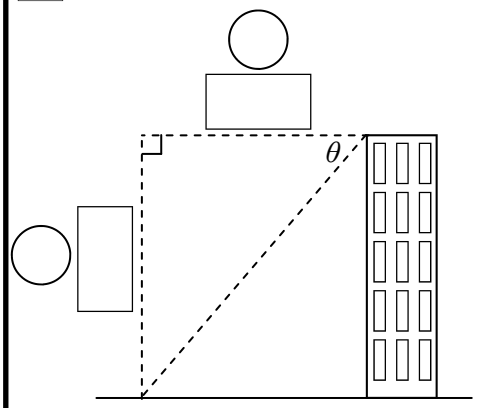
A flag pole has a guy wire attached to reduce vibration. The flagpole is 19 m tall and the guy wire is 27 m long.



7 Calculate the angle of elevation from the anchor point up to the pole.

Tom is on top of a 63 m tall building, looking down at Fred, who is 48 m away from the base of the building.

8 Write the distances in the boxes.



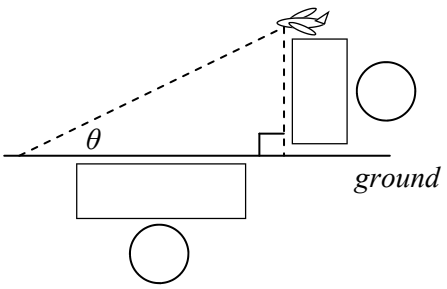
9 Calculate the angle of depression

Yve is looking up at a plane which is 3.7 km away and is flying at a height of 800 m.

10 Convert 3.7 km to metres

3.7 km = ____

11 Write the distances in the boxes.



12 Calculate the angle of elevation

Trigonometry - Elevation/Depression - Finding a Length (Sheet 2)

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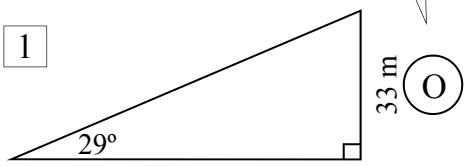
Find the length 'x', rounding to 1 decimal place.

1



$\tan \square = \frac{\square}{\square}$ (O / A)

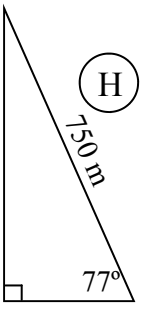
swap swap

$x = \frac{\square}{\tan \square}$

Calc: 3 3 ▪ TAN 2 9) =

$x =$ ____ m 1 d.p.

2



$\cos \square = \frac{\square}{\square}$ (A / H)

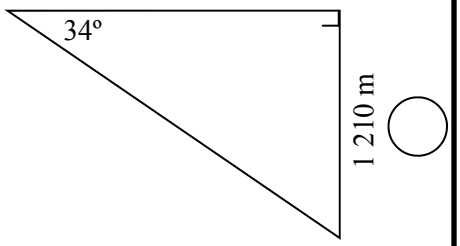
× 750 × 750

$x =$ ____ $\cos$ ____

Calc: 7 5 0 COS 7 7 =

$x =$ ____ m 1 d.p.

3



$\tan \square = \frac{\square}{\square}$ (O / A)

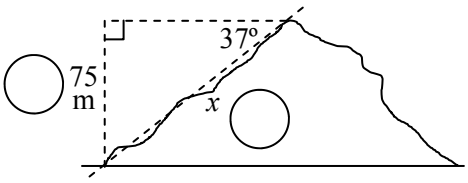
swap swap

$x = \frac{\square}{\tan \square}$

$x =$ ____ m 1 d.p.

SOH CAH TOA

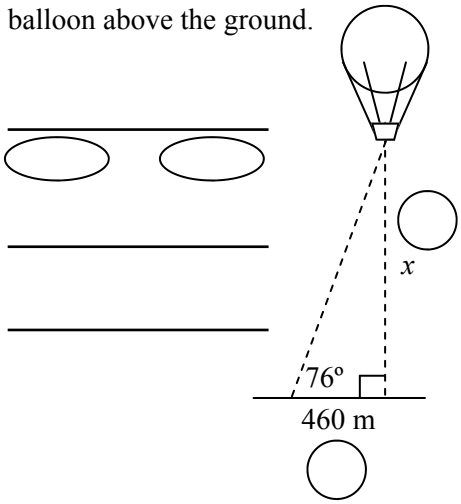
Jill is on top of a hill which has a height of 75 m. The angle of depression from the top to the base of the hill is 37º.



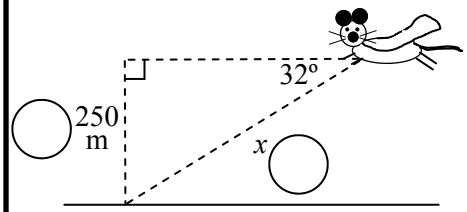
4 Estimate the distance, 'x', Jill travelled to climb the hill.

Ted is looking up at a hot air balloon. The angle of elevation is 76º and the balloon is above a point 460 m away.

5 Calculate 'x', the height of the balloon above the ground.

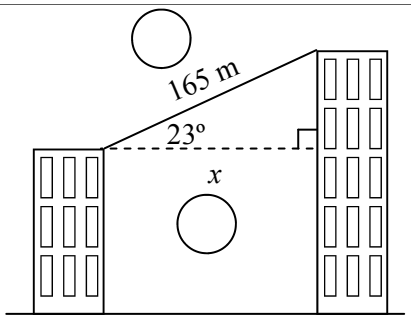


Flighty Mouse, the flying super mouse spies a damsel in distress. The angle of depression to the damsel is 32º and Flighty is flying at a height of 250 m above the ground.



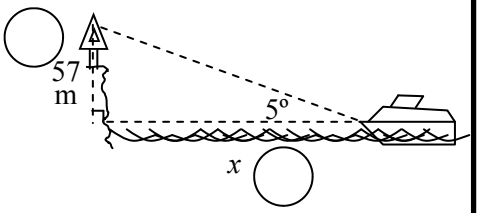
6 Calculate 'x', the flying distance Flighty Mouse has to cover.

Balancing Bec is set to walk a 165 m long tight-rope suspended between two buildings. The angle of elevation from the lower building is 23º.



7 Find 'x', the distance between the two buildings.

Stinky Stew the tug boat captain, spots a cliff beacon at an angle of elevation of 5º. He knows the beacon is 57 m above sea level.



8 Find 'x', the distance to the cliff.

Trigonometry - Find A Word

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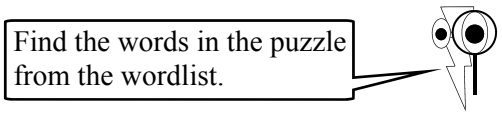

Y	L	U	M	P	A	W	S	N	W	O	N	K	N	U
R	D	L	E	Y	E	L	E	V	A	T	I	O	N	K
O	I	T	A	R	S	U	E	L	G	N	A	I	R	T
U	S	N	S	E	T	I	S	O	P	P	O	H	H	R
N	T	O	U	Z	I	E	D	Q	Q	E	T	Y	N	I
D	A	I	R	R	M	Y	M	E	N	G	P	A	O	G
I	N	S	E	R	A	T	V	I	N	O	P	P	N	O
N	C	S	D	I	T	T	S	E	T	U	N	I	M	N
G	E	E	I	G	E	T	I	E	N	N	L	L	Y	O
G	V	R	V	H	P	E	N	O	E	P	E	P	L	M
Y	L	P	I	T	L	U	M	Q	C	T	H	C	E	E
C	O	E	D	R	S	R	P	R	A	A	H	L	N	T
S	S	D	L	E	B	A	L	V	J	T	G	E	G	R
C	O	Y	S	E	E	R	G	E	D	N	H	E	T	Y
C	C	C	C	A	L	C	U	L	A	T	E	H	H	A

WORDLIST

OPPOSITE	ELEVATION	SINE	ADJACENT	LENGTH
CALCULATE	MEASURE	ANGLE*	DISTANCE	UNKNOWN
DEGREES	SIDE	ROUNDING	DEPRESSION	THETA
COS	DIVIDE	TRIANGLE*	MINUTES	LABEL
MULTIPLY	RATIO	ESTIMATE	RIGHT	TAN
SOLVE	HYPOTENUSE	TRIGONOMETRY	CENTIMETRE	SWAP

* ANGLE AND TRIANGLE CAN BE FOUND IN DIFFERENT LOCATIONS

Trigonometry - Topic Review

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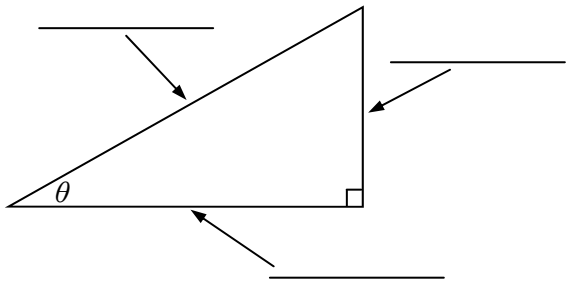
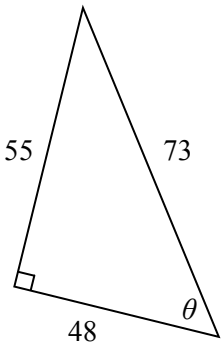
Label the hypotenuse, opposite and adjacent sides.

1

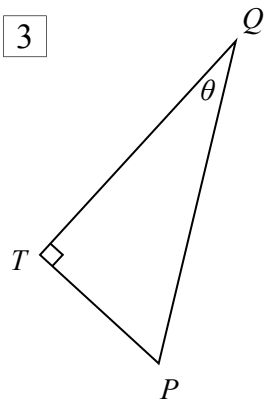


Find the trigonometric ratios for sin θ, cos θ, and tan θ.
NO CALCULATOR REQUIRED.

2



3



Use your calculator to solve these to 1 decimal place.

4 19 tan 38º

5 $\dfrac{57}{\cos 31º}$

Round these angles to the nearest degree.

6 45.36º

7 59.56033º

8 28º 42’

9 55º 13’ 55”

Use your calculator to find θ, round to the nearest degree.

10 $\sin\theta = 0.4228$

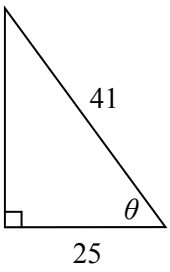
11 $\cos\theta = \dfrac{3}{8}$

12 $\tan\theta = \dfrac{11}{5}$

13 $\tan\theta = 5.4$

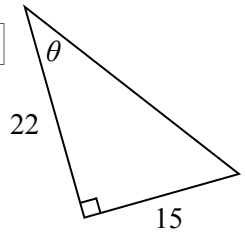
Would you use sin, cos or tan to find these angles?
Colour in the circle to indicate your choice.
DON’T SOLVE THE QUESTIONS.

14



○ sin
○ cos
○ tan

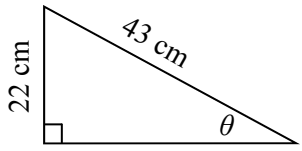
15



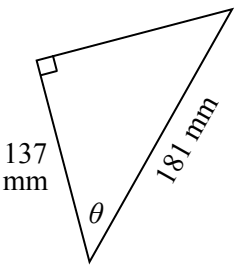
○ sin
○ cos
○ tan

Find the angle θ, round to the nearest degree.

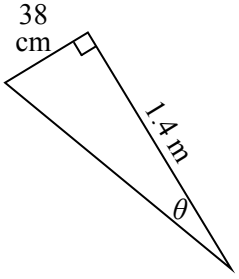
16



17



18



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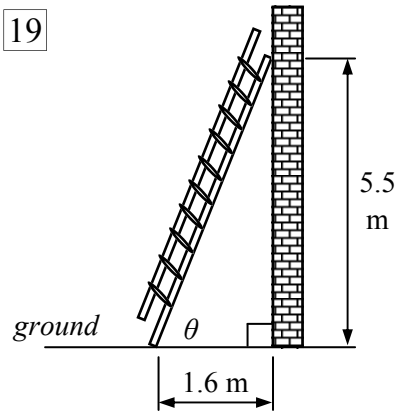
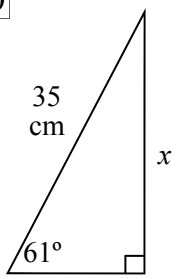
A 5.5 m ladder is leaning against a wall. The toe of the ladder is 1.6 m away from the wall. What angle (shown as θ on the diagram) does the ladder make with the ground? Answer to the nearest degree.

19

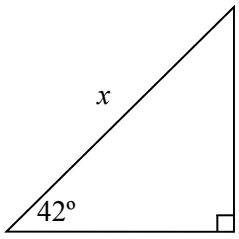


Calculate the length 'x' on the triangles below. Answer to 1 decimal place.

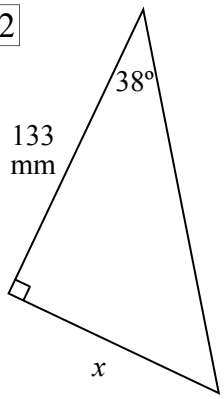
20



21

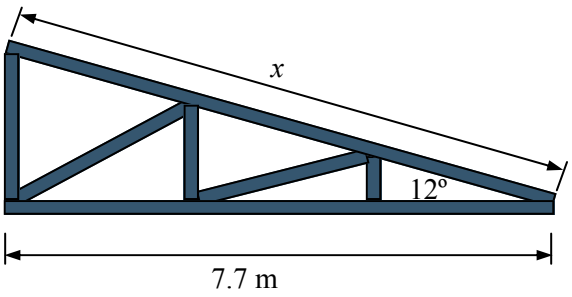


22



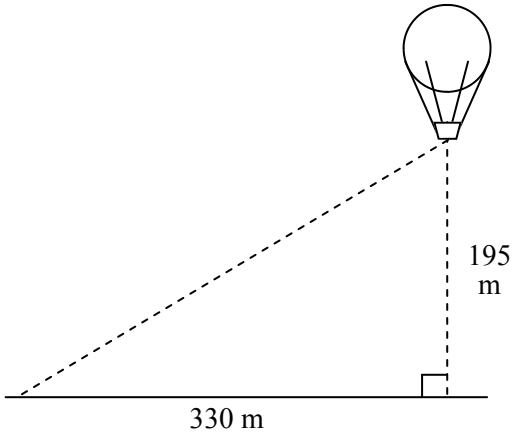
A roof truss is shown below. The roof width is 7.7 m, if the required angle (pitch) of the roof is 12º. Calculate the length of the inclined timber beam, labeled 'x'. Answer in metres, to 1 decimal place.

23



Jessica is watching a hot air balloon which is 330 m away and at a height of 195 m. Calculate the angle of elevation of the balloon from Jessica's position. Answer to the nearest degree.

24



VELOCITY

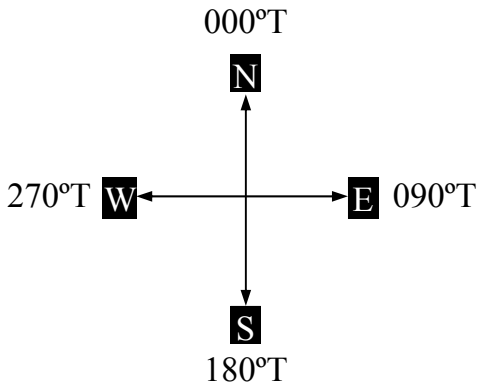
TRIGONOMETRY

EXTENSION

Trigonometry - True Bearings

True bearings are measured from North in a clockwise direction. They are also regularly called 'Whole Circle Bearings'. So bearings start from North 000ºT and increase clockwise, with the largest bearing (in degrees) being 359ºT. Bearings are always shown using 3 digits, so bearings less than 100ºT will have a zero at the front. Example, 073ºT.

North is 000°T,
East is 090°T,
South 180°T and
West 270°T
*North is also considered as 360° for subtraction.

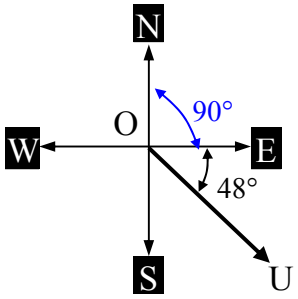


Calculation of bearings uses the values above and either addition or subtraction.

EXAMPLE OF ADDITION

The bearing OU is 48º past East.
The bearing of East is 090ºT, so the bearing of OU is:

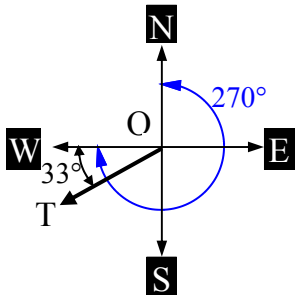
090º + 48º = 138ºT



EXAMPLE OF SUBTRACTION

The bearing OT is 33º before West.
The bearing of West is 270ºT, so the bearing of OT is:

270º - 33º = 237ºT



Trigonometry - True Bearings (Sheet 1)

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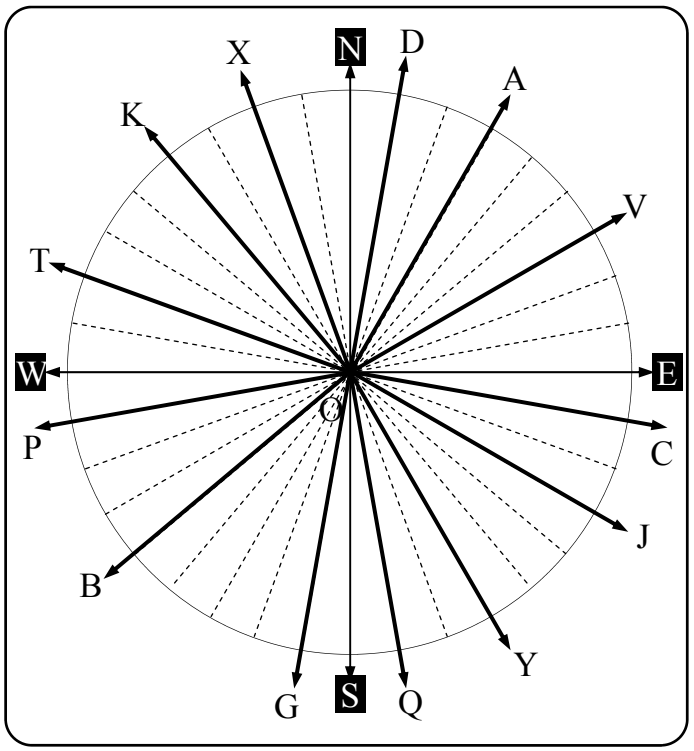
These bearings are on 10° divisions. 'O' is in the centre of the diagram, so bearings are 'OA', 'OB' etc., from the centre out. North is 000° (or 000° T).



Give the True bearing for these. Count the 10º divisions. Remember the bearing for East, South and West, to speed things up.

1 OA ☐☐☐ º T

2 OD ☐☐☐ º T

3 OY ☐ Show º symbol

4 OX ☐

5 OC ☐

6 OB ☐

7 EAST ☐

8 OJ ☐

9 WEST ☐

10 OT ☐

11 OP ☐

12 OK ☐

13 SOUTH ☐

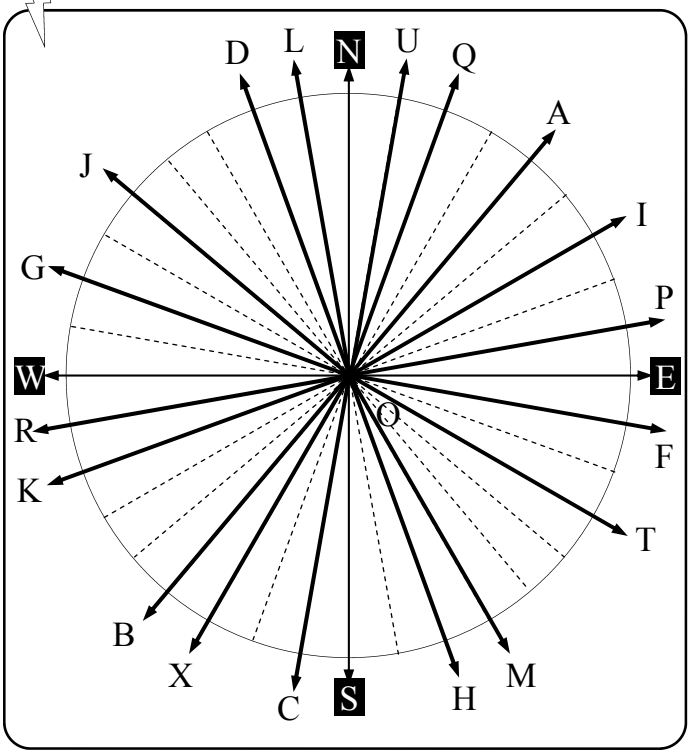
14 OG ☐

Give the bearing that is ….

15 30º clockwise from OQ ☐

16 60º clockwise from OK ☐

Now try this circle. Give the true bearing.



17 OU ☐ Show º symbol

18 OH ☐

19 OJ ☐

20 OP ☐

21 OK ☐

22 OC ☐

23 OA ☐

24 OF ☐

25 OM ☐

26 OD ☐

27 OB ☐

28 OL ☐

29 OG ☐

30 OQ ☐

31 OI ☐

32 OR ☐

33 OT ☐

34 OX ☐

Give the bearing that is ….

35 50º clockwise from OL ☐

36 80º anti-clockwise from OU ☐

37 70º clockwise from OH ☐

Trigonometry - True Bearings (Sheet 2)

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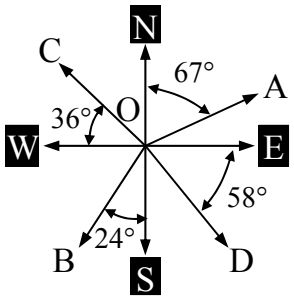
Give the bearing for these. Add to angles for North, East, South and West to get the bearing.

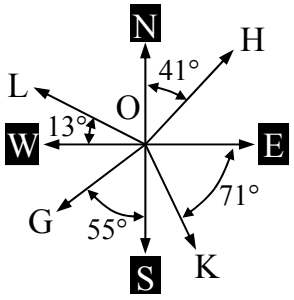


1 OA ___ ° T 000°+ 67°

2 OB ___ ° T 180°+ 24°

3 OC ____ 270°+ 36°

4 OD ____ 090°+ 58°

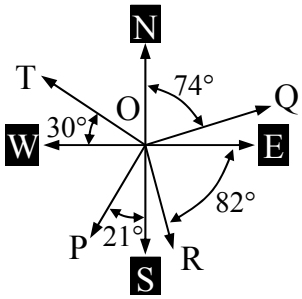


5 OG ____

6 OK ____

7 OH ____

8 OL ____



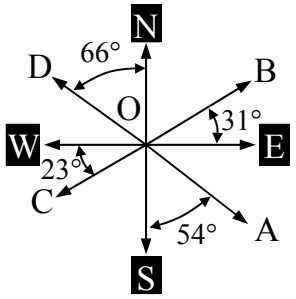
9 ____ OT

10 ____ OQ

11 ____ OP

12 ____ OR

Now use subtraction to find the bearings for these.

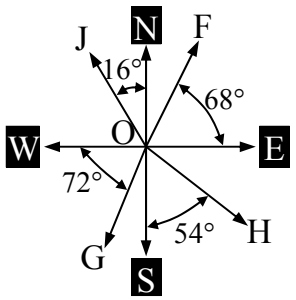


13 OC ____ 270°- 23°

14 OD ____ 360°- 66°

15 OB ____ 090°- 31°

16 OA ____ 180°- 54°

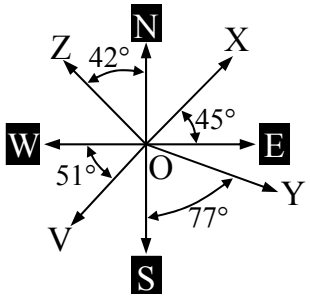


17 ____ OJ

18 ____ OH

19 ____ OF

20 ____ OG



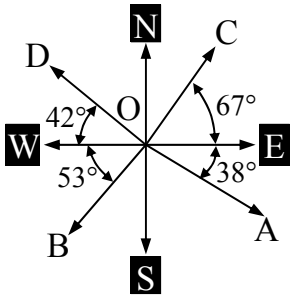
21 OZ ____

22 OX ____

23 OV ____

24 OY ____

These use both subtraction and addition. Find the bearing.

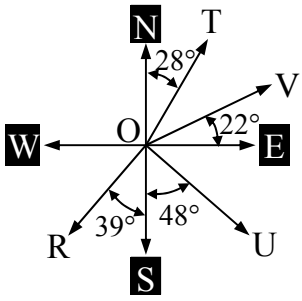


25 OC ____

26 OD ____

27 OB ____

28 OA ____

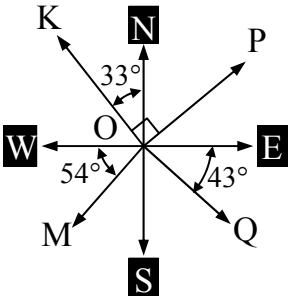


29 OU ____

30 OT ____

31 OR ____

32 OV ____



33 ____ OM

34 ____ OQ

35 ____ OK

36 ____ OP

Trigonometry - True Bearings (Sheet 1)

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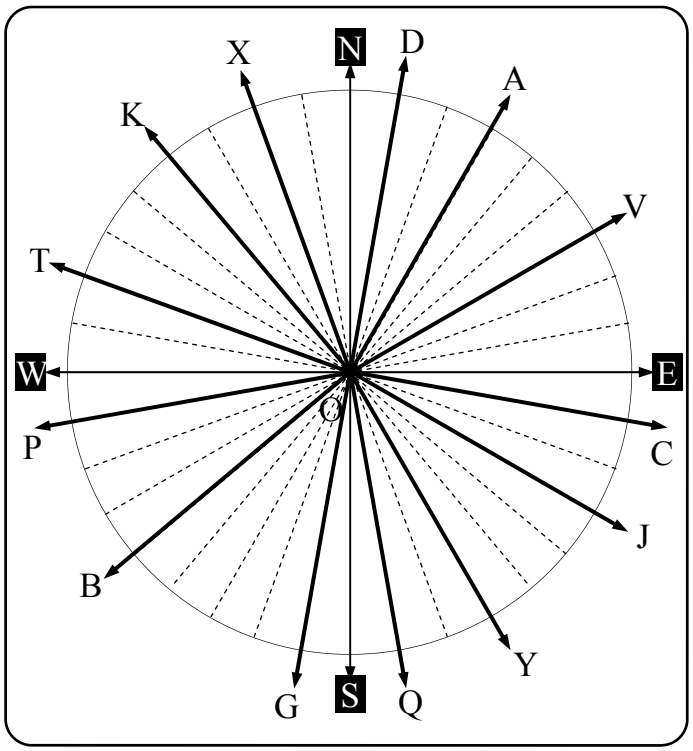
These bearings are on 10° divisions. 'O' is in the centre of the diagram, so bearings are 'OA', 'OB' etc., from the centre out. North is 000° (or 000° T).



Give the True bearing for these. Count the 10° divisions. Remember the bearing for East, South and West, to speed things up.

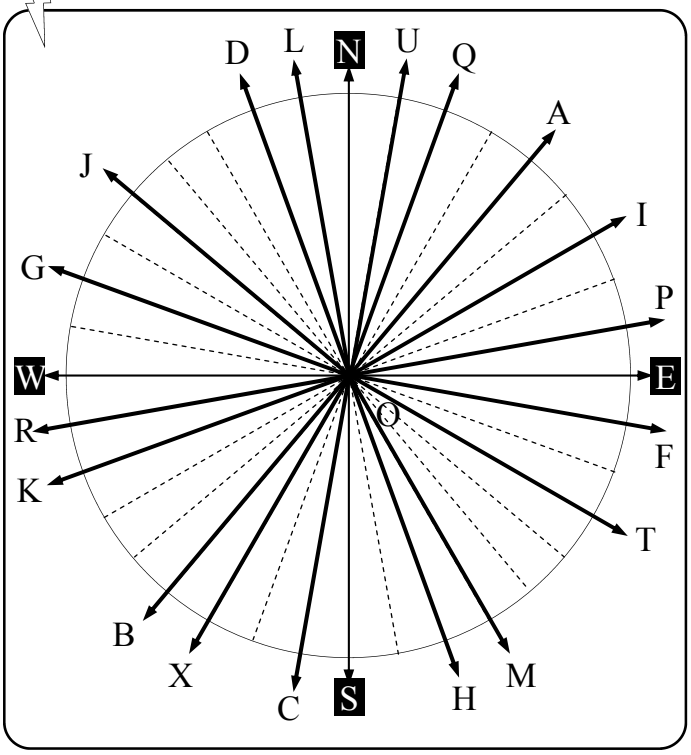
1	OA	030° T		2	OD	010° T	
3	OY	150° T	Show ° symbol	4	OX	340° T	
5	OC	100° T		6	OB	230° T	
7	EAST	090° T		8	OJ	120° T	
9	WEST	270° T		10	OT	290° T	
11	OP	260° T		12	OK	320° T	
13	SOUTH	180° T		14	OG	190° T	

Give the bearing that is ….

15 30° clockwise from OQ — 200° T

16 60° clockwise from OK — 020° T

Now try this circle. Give the true bearing.



17	OU	010° T	Show ° symbol	18	OH	160° T	
19	OJ	310° T		20	OP	080° T	
21	OK	250° T		22	OC	190° T	
23	OA	040° T		24	OF	100° T	
25	OM	150° T		26	OD	340° T	
27	OB	220° T		28	OL	350° T	
29	OG	290° T		30	OQ	020° T	
31	OI	060° T		32	OR	260° T	
33	OT	120° T		34	OX	210° T	

Give the bearing that is ….

35 50° clockwise from OL — 040° T

36 80° anti-clockwise from OU — 290° T

37 70° clockwise from OH — 230° T

Trigonometry - True Bearings (Sheet 2)

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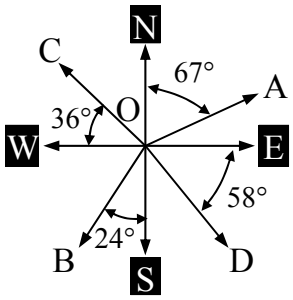
Give the bearing for these. Add to angles for North, East, South and West to get the bearing.

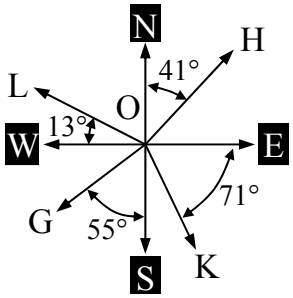


1 OA 067° T 000°+ 67°

2 OB 204° T 180°+ 24°

3 OC 306° T 270°+ 36°

4 OD 148° T 090°+ 58°

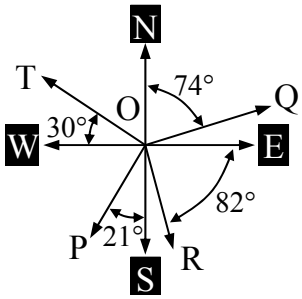


5 OG 235° T

6 OK 161° T

7 OH 041° T

8 OL 283° T



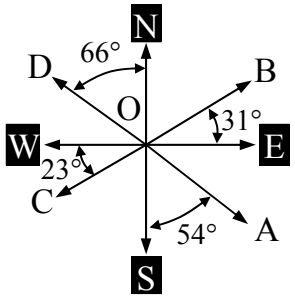
9 300° T OT

10 074° T OQ

11 201° T OP

12 172° T OR

Now use subtraction to find the bearings for these.

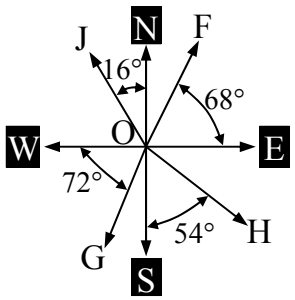


13 OC 247° T 270°- 23°

14 OD 294° T 360°- 66°

15 OB 059° T 090°- 31°

16 OA 126° T 180°- 54°

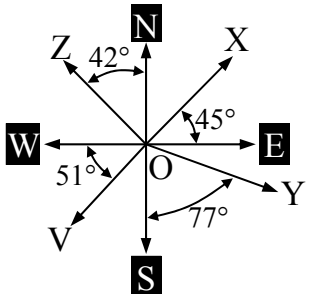


17 344° T OJ

18 126° T OH

19 022° T OF

20 198° T OG



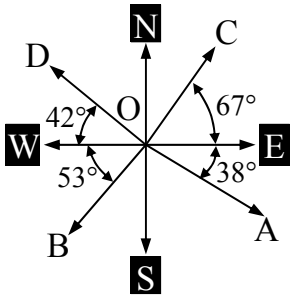
21 OZ 318° T

22 OX 045° T

23 OV 219° T

24 OY 103° T

These use both subtraction and addition. Find the bearing.

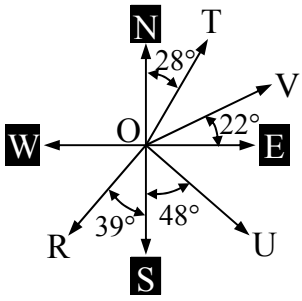


25 OC 023° T

26 OD 312° T

27 OB 217° T

28 OA 128° T

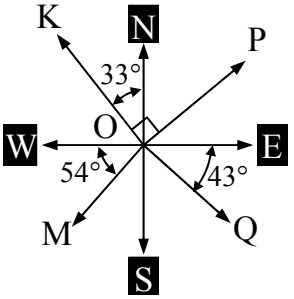


29 OU 132° T

30 OT 028° T

31 OR 219° T

32 OV 068° T



33 216° T OM

34 133° T OQ

35 327° T OK

36 057° T OP

Trigonometry - True Bearings (Homework Strip)

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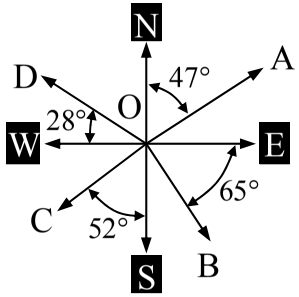
Date Due: _____

Give the true bearing for these.

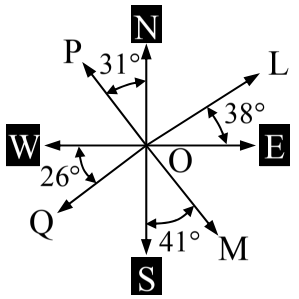


1 _____ OA

2 _____ OC

3 _____ OD

4 _____ OB



5 _____ OM

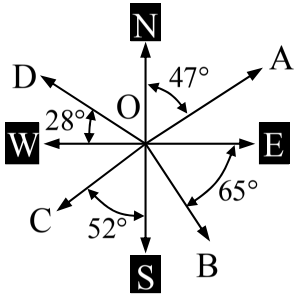
6 _____ OQ

7 _____ OL

8 _____ OP

Date Due: _____

Give the true bearing for these.

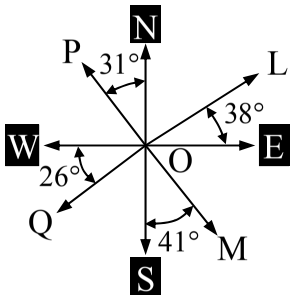


1 _____ OA

2 _____ OC

3 _____ OD

4 _____ OB



5 _____ OM

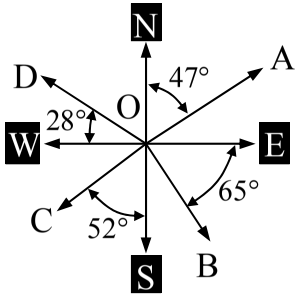
6 _____ OQ

7 _____ OL

8 _____ OP

Date Due: _____

Give the true bearing for these.

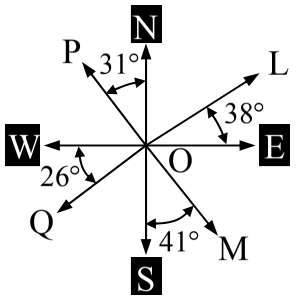


1 _____ OA

2 _____ OC

3 _____ OD

4 _____ OB



5 _____ OM

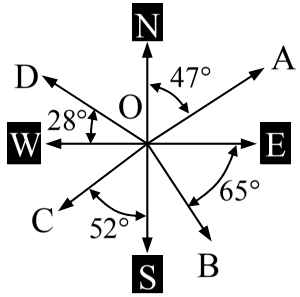
6 _____ OQ

7 _____ OL

8 _____ OP

Date Due: _____

Give the true bearing for these.

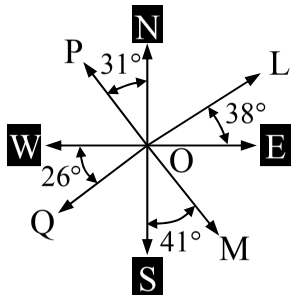


1 _____ OA

2 _____ OC

3 _____ OD

4 _____ OB



5 _____ OM

6 _____ OQ

7 _____ OL

8 _____ OP

Trigonometry - True Bearings (Homework Strip)

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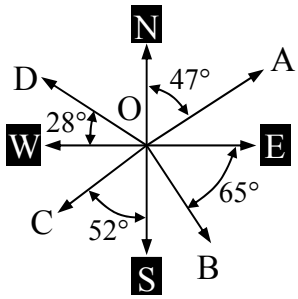
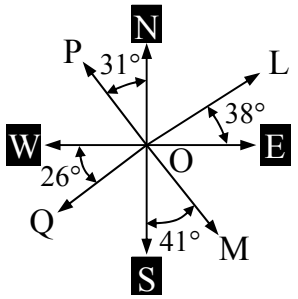
Date Due:

Give the true bearing for these.



1	047°T	2	232°T
	OA		OC
3	298°T	4	155°T
	OD		OB

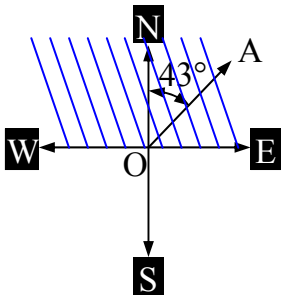


5	139°T	6	244°T
	OM		OQ
7	052°T	8	329°T
	OL		OP

Trigonometry - Compass Bearings

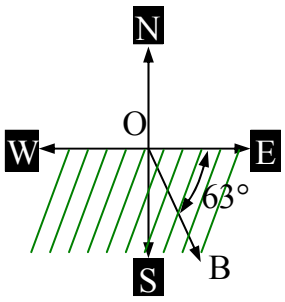
Compass bearings measure the bearing from North, when above the East-West line. Or South, when below the East-West Line.

MEASURED OFF NORTH



OA is above the East-West line.
So it is measured off North.

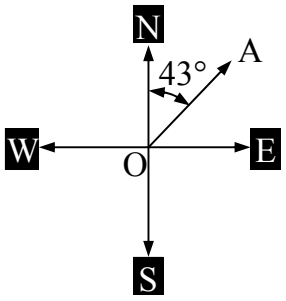
MEASURED OFF SOUTH



OB is below the East-West line.
So it is measured off South.

To give the compass bearing, you use two directions. Firstly North or South, the angle, then the direction the angle is heading, which is either East or West. The angle is always measured from North or from South. If the angle isn't given from North/South, a calculation is performed to change it.

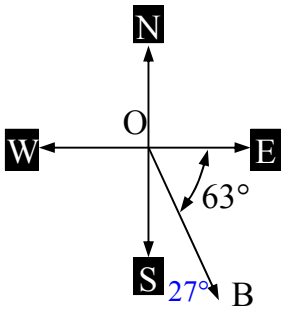
NO CALCULATION REQUIRED



OA is 43° from North in an Easterly direction.

N 43° E

CALCULATION REQUIRED



OB is 27° from South in an Easterly direction. Note 90° - 63° = 27°

S 27° E

Trigonometry - Compass Bearings (Sheet 1)

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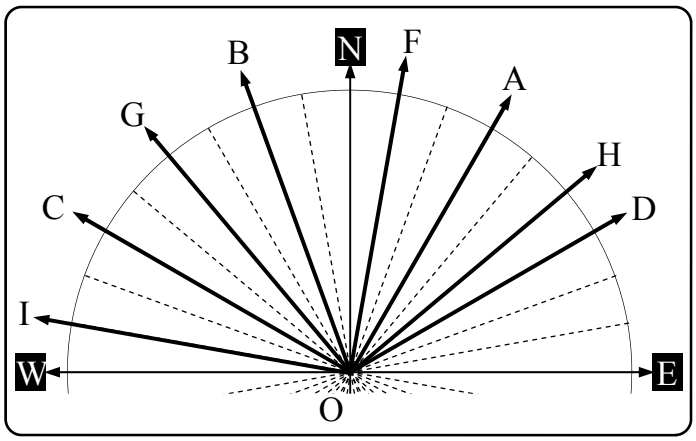
These bearings are on 10° divisions. 'O' is in the centre of the diagram, so bearings are 'OA', 'OB' etc., from the centre out. These are all 'off' North.



Give the compass bearing for these. Count the 10º divisions clockwise or anticlockwise from North, then South (lower diagram).

1 OA N ☐☐° E

2 OC N ☐☐° W

3 OI

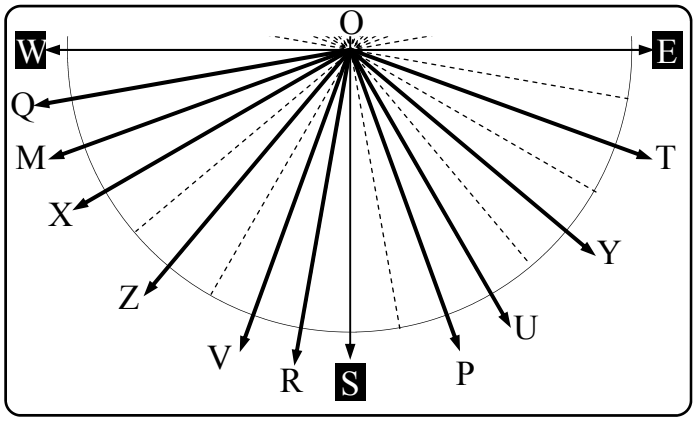
4 OF

5 OG

6 OD

7 OB

8 OH



9 OR S ☐☐° W

10 OT S ☐☐° E

11 OX

12 OP

13 OY

14 OV

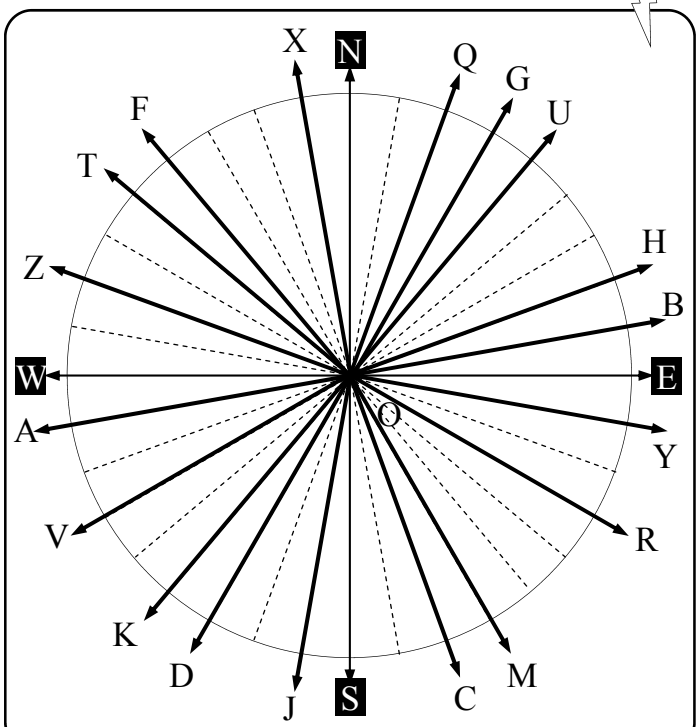
15 OQ

16 OU

17 OM

18 OZ

Now try a full circle. Find the compass bearing.



19 OC

20 OT

21 OV

22 OX

23 OB

24 OY

25 OH

26 OD

27 OZ

28 OQ

29 OR

30 OF

31 OA

32 OM

33 OG

34 OJ

35 OU

36 OK

Give the compass bearing that is ….

37 40º clockwise from OA

38 30º anti-clockwise from OJ

39 80º clockwise from OF

Trigonometry - Compass Bearings (Sheet 2)

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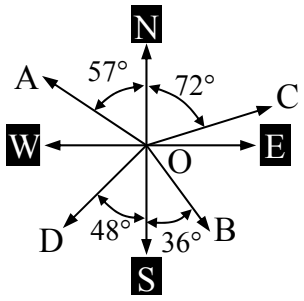
Give the compass bearing for these.

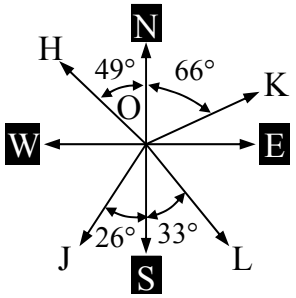


1 OA N ☐☐ º W

2 OB S ☐☐ º E

3 OC ☐

4 OD ☐

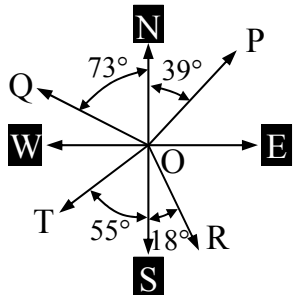


5 OJ ☐

6 OK ☐

7 OH ☐

8 OL ☐



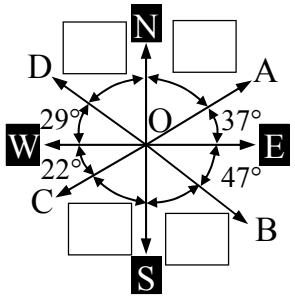
9 ☐ OT

10 ☐ OP

11 ☐ OQ

12 ☐ OR

Use subtraction first, then give the compass bearings for these.

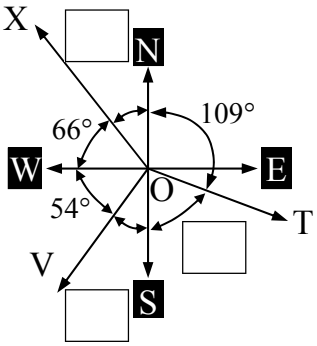


13 OA ☐

14 OD ☐

15 OB ☐

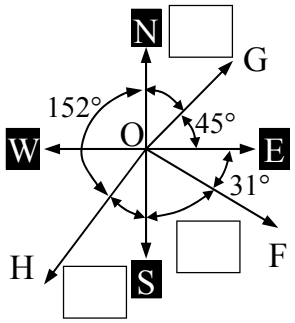
16 OC ☐



17 ☐ OV

18 ☐ OX

19 OT ☐

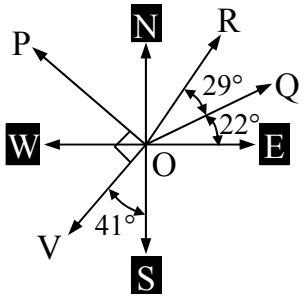


20 OG ☐

21 OF ☐

22 OH ☐

Use subtraction and addition to find the compass bearing.

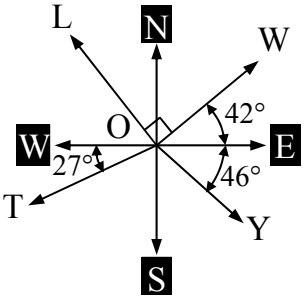


23 OR ☐

24 OQ ☐

25 OV ☐

26 OP ☐

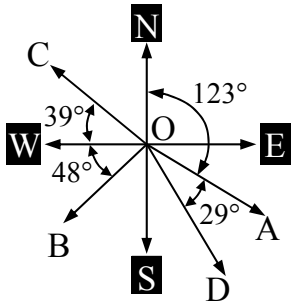


27 ☐ OW

28 ☐ OY

29 ☐ OT

30 ☐ OL



33 ☐ OB

34 ☐ OC

35 ☐ OA

36 ☐ OD

Trigonometry - Compass Bearings (Sheet 1)

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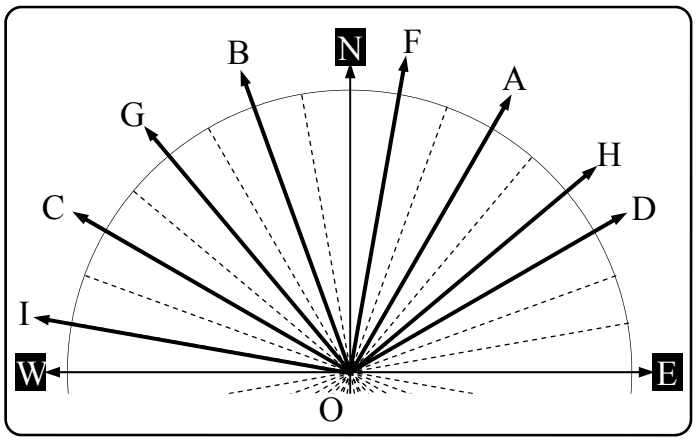
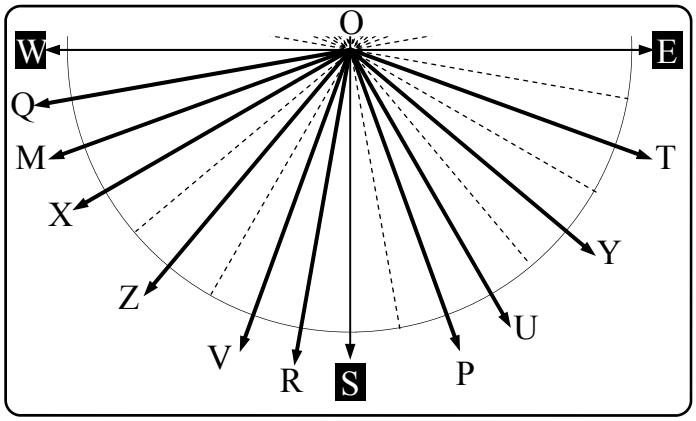
These bearings are on 10° divisions. 'O' is in the centre of the diagram, so bearings are 'OA', 'OB' etc., from the centre out. These are all 'off' North.



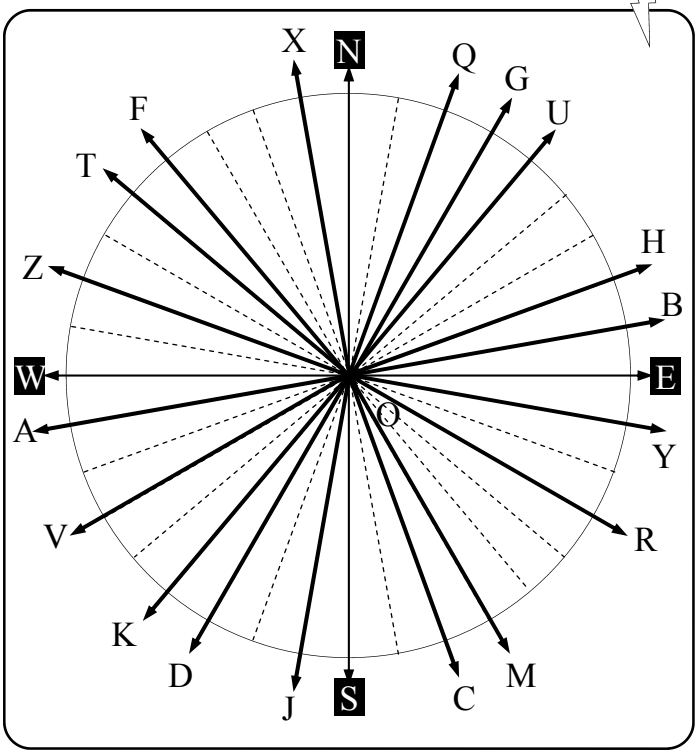
Give the compass bearing for these. Count the 10º divisions clockwise or anticlockwise from North, then South (lower diagram).

1	OA	N 30° E		2	OC	N 60° W	
3	OI	N 80° W		4	OF	N 10° E	
5	OG	N 40° W		6	OD	N 60° E	
7	OB	N 20° W		8	OH	N 50° E	



9	OR	S 10° W		10	OT	S 70° E	
11	OX	S 60° W		12	OP	S 20° E	
13	OY	S 50° E		14	OV	S 20° W	
15	OQ	S 90° W		16	OU	S 30° E	
17	OM	S 70° W		18	OZ	S 40° W	

Now try a full circle. Find the compass bearing.



19	OC	S 20° E		20	OT	N 50° W	
21	OV	S 60° W		22	OX	N 10° W	
23	OB	N 80° E		24	OY	S 80° E	
25	OH	N 70° E		26	OD	S 30° W	
27	OZ	N 70° W		28	OQ	N 20° E	
29	OR	S 60° E		30	OF	N 40° W	
31	OA	S 80° W		32	OM	S 30° E	
33	OG	N 30° E		34	OJ	S 10° W	
35	OU	N 40° E		36	OK	S 40° W	

Give the compass bearing that is ….

37	40º clockwise from OA	N 60° W
38	30º anti-clockwise from OJ	S 20° E
39	80º clockwise from OF	N 40° E

Trigonometry - Compass Bearings (Sheet 2)

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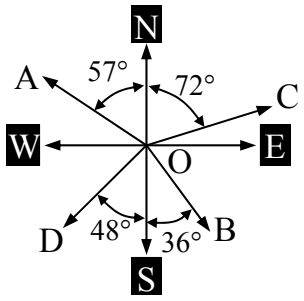
Give the compass bearing for these.

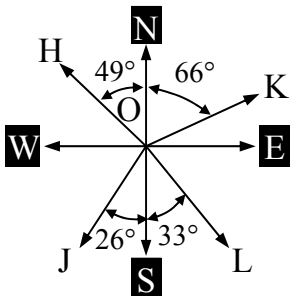


1 OA N 57 ° W

2 OB S 36 ° E

3 OC N 72° E

4 OD S 48° W

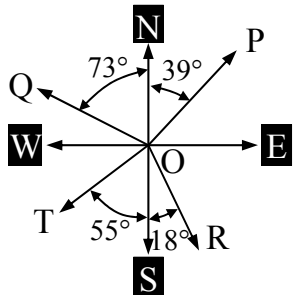


5 OJ S 26° W

6 OK N 66° E

7 OH N 49° W

8 OL S 33° E



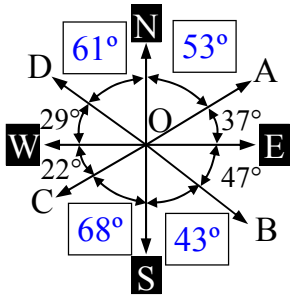
9 OT S 55° W

10 OP N 39° E

11 OQ N 73° W

12 OR S 18° E

Use subtraction first, then give the compass bearings for these.

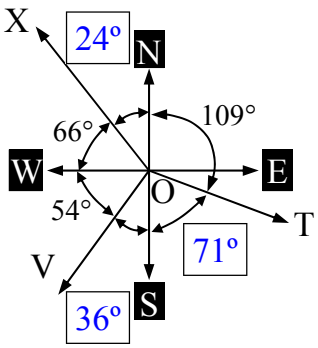


13 OA N 53° E

14 OD N 61° W

15 OB S 43° E

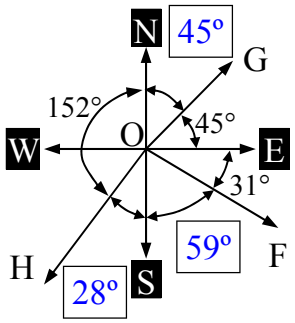
16 OC S 68° W



17 OV S 36° W

18 OX N 24° W

19 OT S 71° E

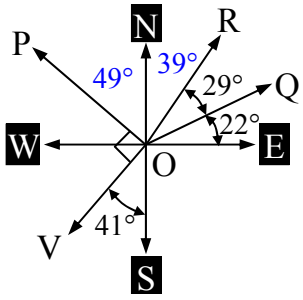


20 OG N 45° E

21 OF S 59° E

22 OH S 28° W

Use subtraction and addition to find the compass bearing.

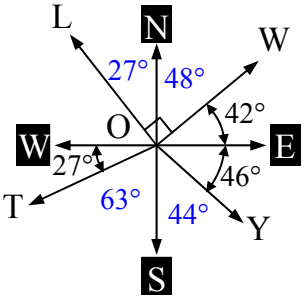


23 OR N 39° E

24 OQ N 68° E

25 OV S 41° W

26 OP N 49° W

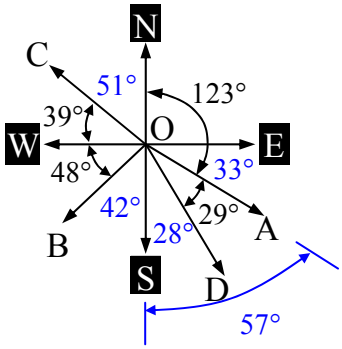


27 OW N 48° W

28 OY S 44° E

29 OT S 63° W

30 OL N 27° W



33 OB S 42° W

34 OC N 51° W

35 OA S 57° E

36 OD S 28° E

Trigonometry - Compass Bearings (Homework Strip)

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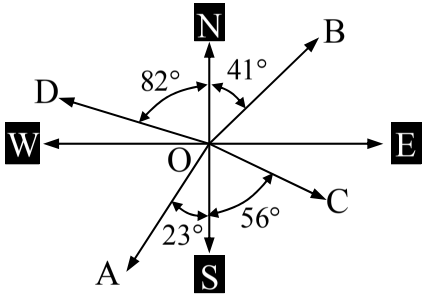
Date Due:

Give the compass bearing for these.

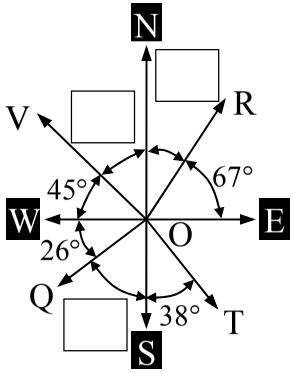


1 OA

2 OD

3 OB

4 OC



5 OR

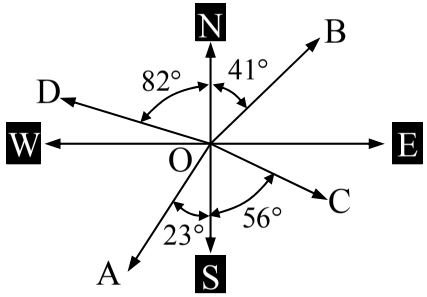
6 OQ

7 OT

8 OV

Date Due:

Give the compass bearing for these.

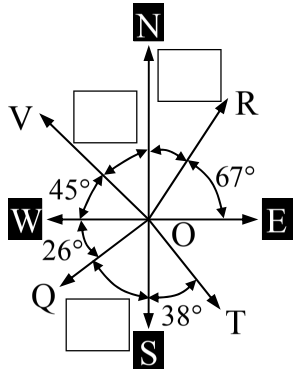


1 OA

2 OD

3 OB

4 OC



5 OR

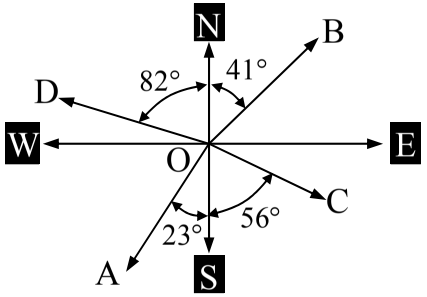
6 OQ

7 OT

8 OV

Date Due:

Give the compass bearing for these.

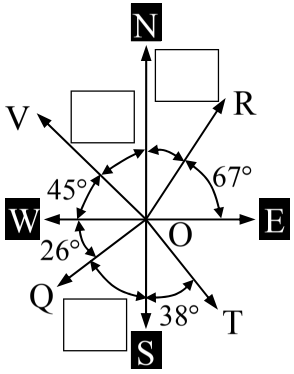


1 OA

2 OD

3 OB

4 OC



5 OR

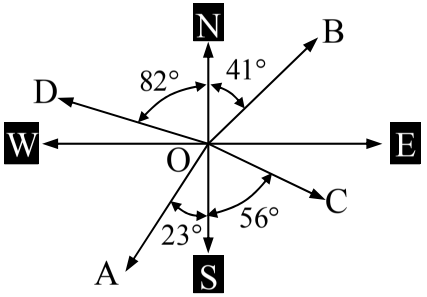
6 OQ

7 OT

8 OV

Date Due:

Give the compass bearing for these.

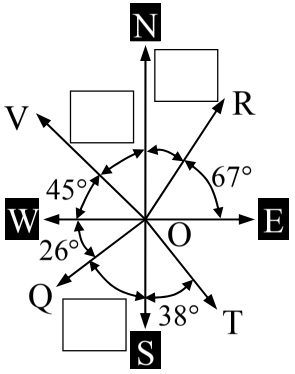


1 OA

2 OD

3 OB

4 OC



5 OR

6 OQ

7 OT

8 OV

Trigonometry - Compass Bearings (Homework Strip)

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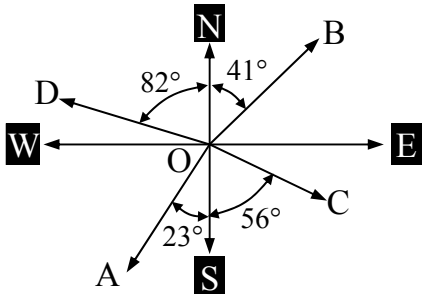
Date Due:

Give the compass bearing for these.

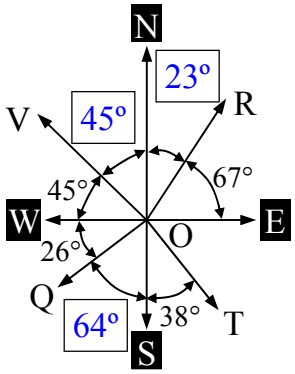


1 S 23° W — OA

2 N 82° W — OD

3 N 41° E — OB

4 S 56° E — OC



5 N 23° E — OR

6 S 64° W — OQ

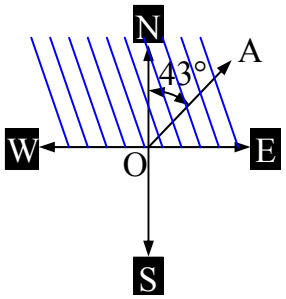
7 S 38° E — OT

8 N 45° W — OV

Trigonometry - Bearing Conversion

Compass bearings measure the bearing from North, when above the East-West line. Or South, when below the East-West Line.

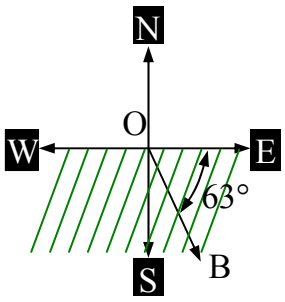
MEASURED OFF NORTH



OA is above the East-West line.
So it is measured off North.

N 43° E

MEASURED OFF SOUTH



OB is below the East-West line.
So it is measured off South.

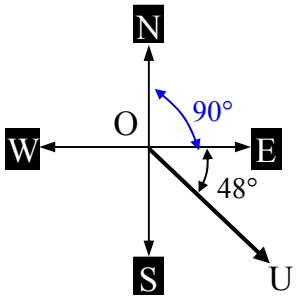
S 27° E

True (or Whole circle) bearings measure the bearing from North, in a clockwise direction using 3 digits. North is 000°T.

EXAMPLE OF ADDITION

The bearing OU is 48° past East.
The bearing of East is 090°T, so the bearing of OU is:

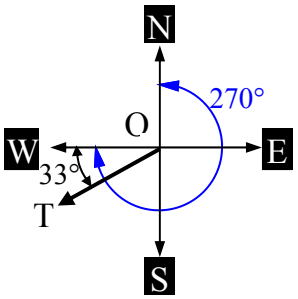
090° + 48° = 138°T



EXAMPLE OF SUBTRACTION

The bearing OT is 33° before West.
The bearing of West is 270°T, so the bearing of OT is:

270° - 33° = 237°T



Trigonometry - Bearing Conversion (Sheet 1)

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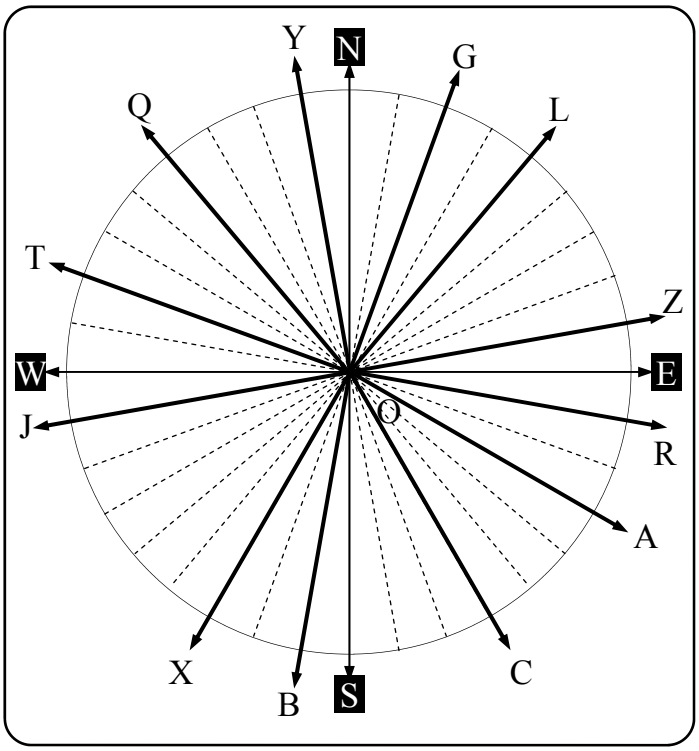
Each division in the circle is 10º. Note that 'O' is in the centre of the circle. Express the bearings in both true bearing and compass bearing format.



1 OC ☐☐☐º T S ☐☐º E

2 OT ☐☐☐º T N ☐☐º W

3 OB

4 OG ☐☐☐º T

5 OZ

6 OJ

7 OQ

8 OX

9 OA

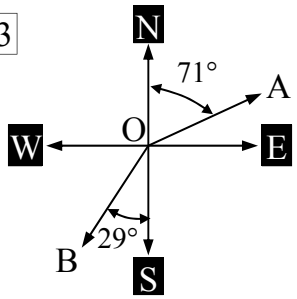
10 OY

11 OL

12 OR

Express these bearings in both true bearing and compass bearing format.

13

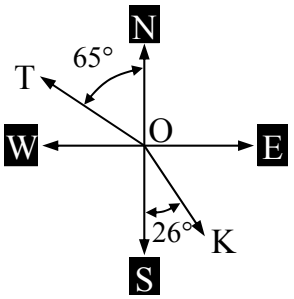


OA ☐☐☐º T N ☐☐º E

OB

14 OT

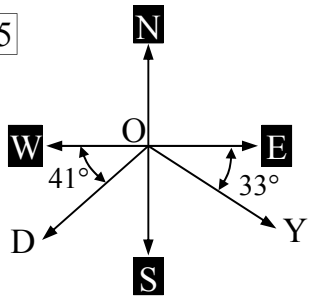
OK



OY

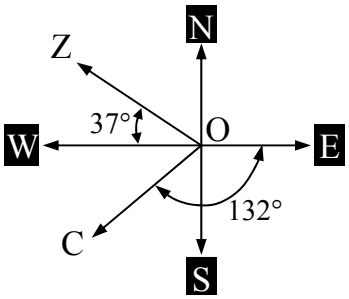
OD

15

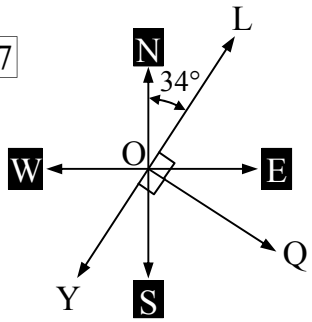


16 OZ

OC



17



OL

OQ

OY

Trigonometry - Bearing Conversion (Sheet 2)

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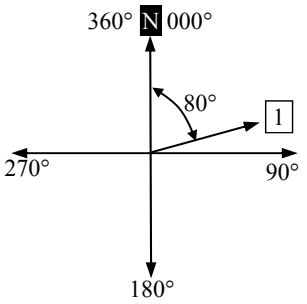
Sketch all the true bearings on the diagram and convert them to compass bearings. Show the angle from North/South on the diagram.

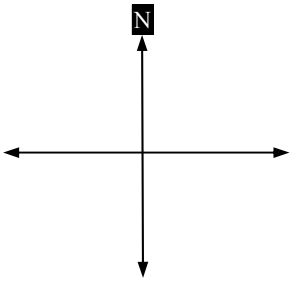
1 080°T N ☐☐° E

2 200°T S ☐☐° W

3 130°T ☐

4 320°T ☐





5 125°T ☐

6 243°T ☐

7 050°T ☐

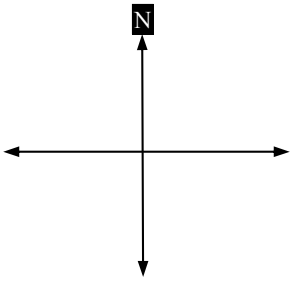
8 294°T ☐

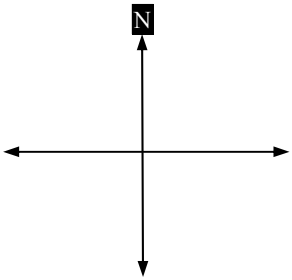
9 305°T ☐

10 193°T ☐

11 110°T ☐

12 044°T ☐





13 255°T ☐

14 072°T ☐

15 330°T ☐

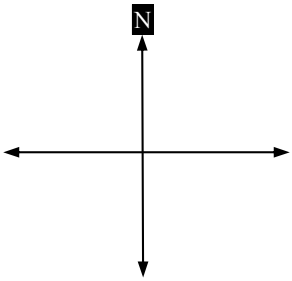
16 165°T ☐

17 110°T ☐

18 010°T ☐

19 310°T ☐

20 210°T ☐



Convert these to true bearings.

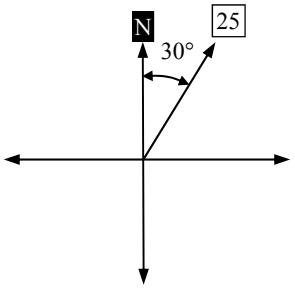
21 North ☐

22 West ☐

23 East ☐

24 South ☐

Convert these to true bearings, show them on the diagram.



25 N 30° E ☐☐☐° T

26 S 67° W ☐

27 S 22° E ☐

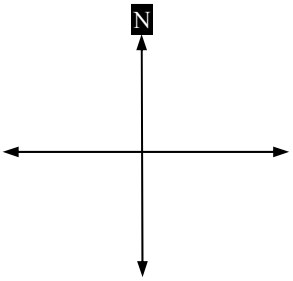
28 N 16° W ☐

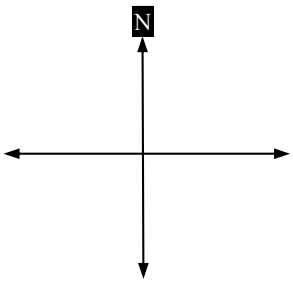
29 N 47° W ☐

30 S 26° W ☐

31 N 60° E ☐

32 S 11° E ☐





33 S 58° W ☐

34 N 23° W ☐

35 N 75° E ☐

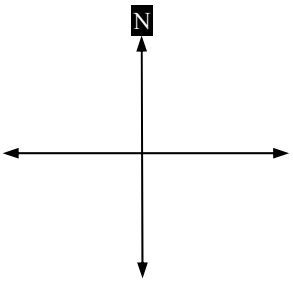
36 S 8° E ☐

37 N 34° W ☐

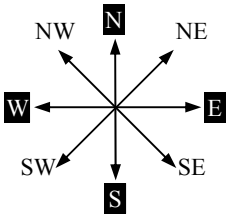
38 S 66° W ☐

39 S 28° E ☐

40 N 55° E ☐



Convert these directions to true bearings.



41 North East ☐

42 South West ☐

43 North West ☐

44 South East ☐

Trigonometry - Bearing Conversion (Sheet 1)

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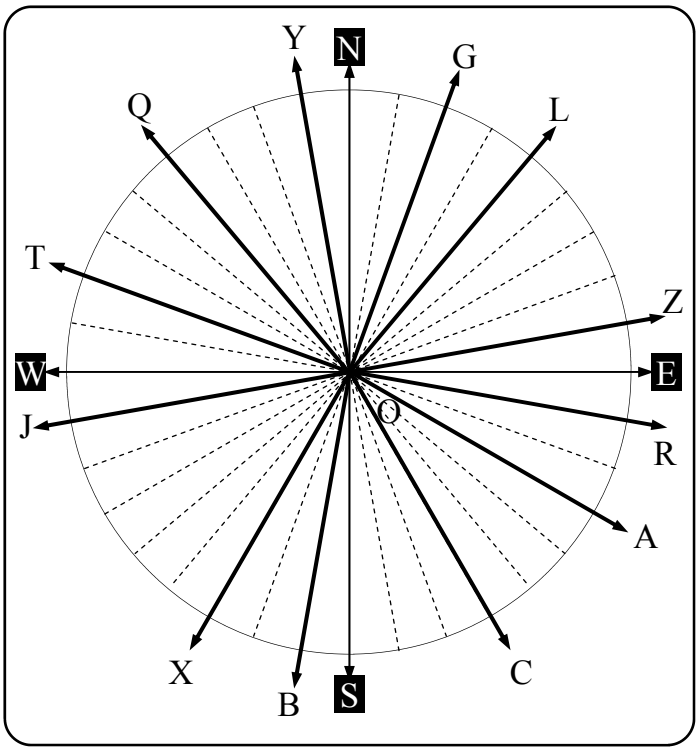
Each division in the circle is 10º. Note that 'O' is in the centre of the circle. Express the bearings in both true bearing and compass bearing format.



1 OC 150º T

S 30º E

2 OT 290º T

N 70º W

3 OB 190ºT

S 10º W

4 OG 020º T

N 20º E

5 OZ 080ºT

N 80º E

6 OJ 260ºT

S 80º W

7 OQ 320ºT

N 40º W

8 OX 210ºT

S 30º W

9 OA 120ºT

S 60º W

10 OY 350ºT

N 10º W

11 OL 040ºT

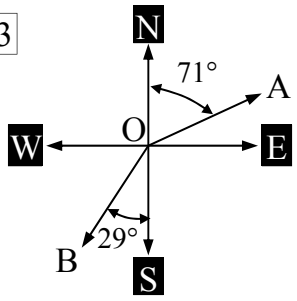
N 40º E

12 OR 100ºT

S 80º E

Express these bearings in both true bearing and compass bearing format.

13



OA 071º T

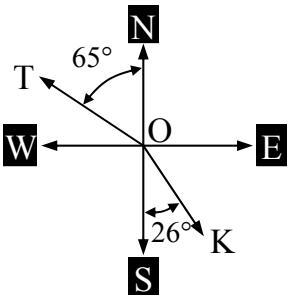
N 71º E

OB 209ºT

S 29º W

14 OT 295ºT

N 29º W



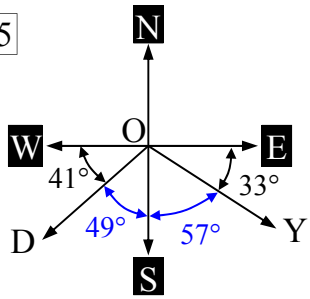
OK 154ºT

S 26º E

OY 123ºT

S 57º E

15

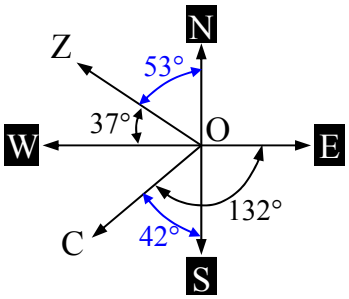


OD 229ºT

S 49º W

16 OZ 307ºT

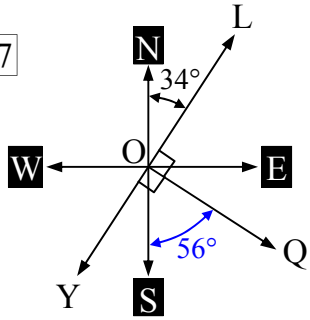
N 53º W



OC 222ºT

S 42º W

17



OL 034ºT

N 34º E

OQ 124ºT

S 56º E

OY 214ºT

S 34º W

Trigonometry - Bearing Conversion (Sheet 2)

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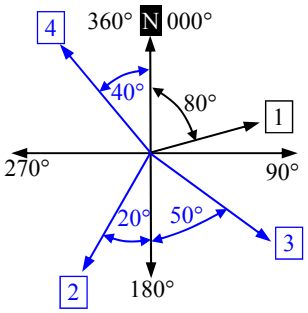
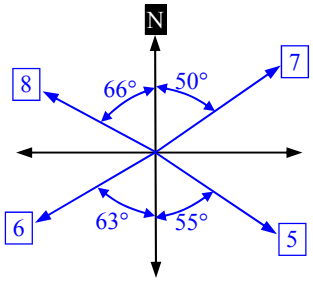
Sketch all the true bearings on the diagram and convert them to compass bearings. Show the angle from North/South on the diagram.

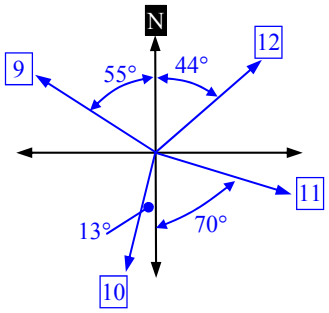
1	080°T	N 80° E
2	200°T	S 20° W
3	130°T	S 50° E
4	320°T	N 40° W

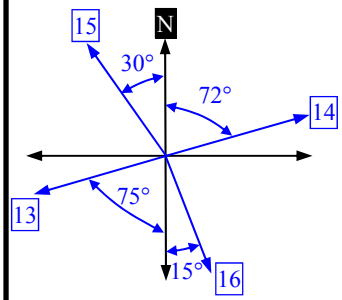




5	125°T	S 55° E
6	243°T	S 63° W
7	050°T	N 50° E
8	294°T	N 66° W

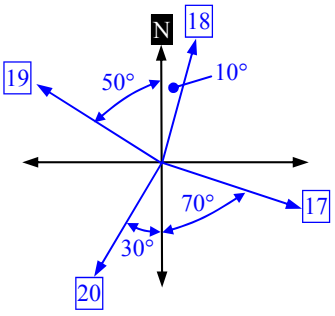
9	305°T	N 55° W
10	193°T	S 13° W
11	110°T	S 70° E
12	044°T	N 44° E





13	255°T	S 75° W
14	072°T	N 72° E
15	330°T	N 30° W
16	165°T	S 15° E

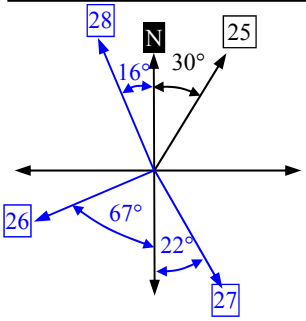
17	110°T	S 70° E
18	010°T	N 10° E
19	310°T	N 50° W
20	210°T	S 30° W



Convert these to true bearings.

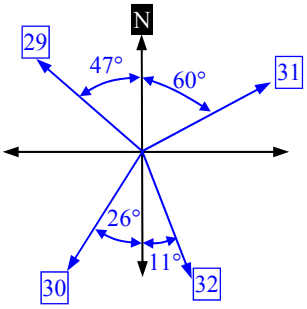
21	North	000°T	22	West	270°T
23	East	090°T	24	South	180°T

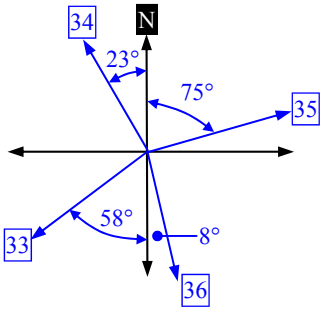
Convert these to true bearings, show them on the diagram.



25	N 30° E	030°T
26	S 67° W	247°T
27	S 22° E	158°T
28	N 16° W	344°T

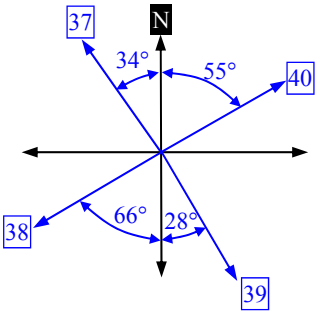
29	N 47° W	313°T
30	S 26° W	206°T
31	N 60° E	060°T
32	S 11° E	169°T



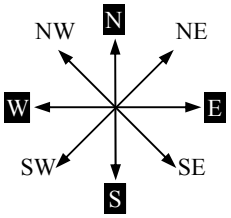


33	S 58° W	238°T
34	N 23° W	337°T
35	N 75° E	075°T
36	S 8° E	172°T

37	N 34° W	326°T
38	S 66° W	246°T
39	S 28° E	152°T
40	N 55° E	055°T



Convert these directions to true bearings.



41	North East	045°T
42	South West	225°T
43	North West	315°T
44	South East	135°T

Trigonometry - Bearing Conversion (Homework Strip)

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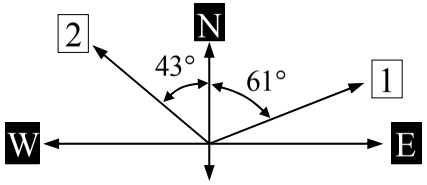
Date Due: _____

Give the compass and true bearings.

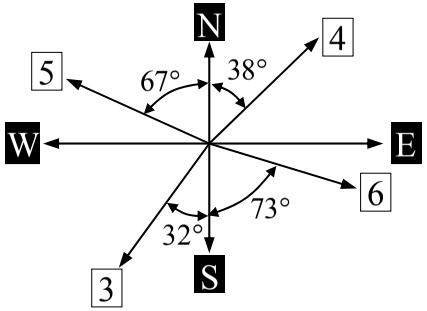


1 ___ ___ ___ ° T — N ___ ___ ° E

True Compass

2 _____ _____

True Compass



3 _____ _____

True Compass

4 _____ _____

True Compass

5 _____ _____

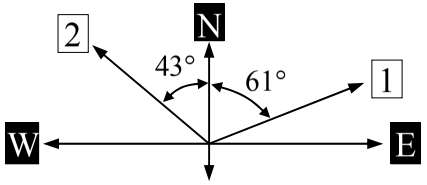
True Compass

6 _____ _____

True Compass

Date Due: _____

Give the compass and true bearings.

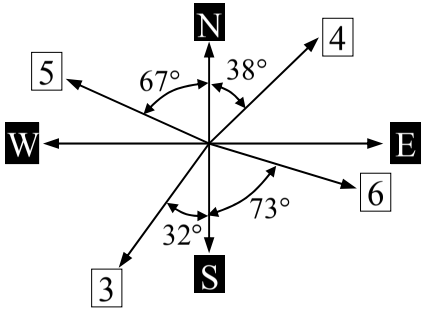


1 ___ ___ ___ ° T — N ___ ___ ° E

True Compass

2 _____ _____

True Compass



3 _____ _____

True Compass

4 _____ _____

True Compass

5 _____ _____

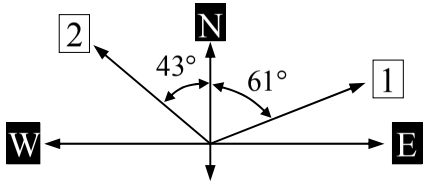
True Compass

6 _____ _____

True Compass

Date Due: _____

Give the compass and true bearings.

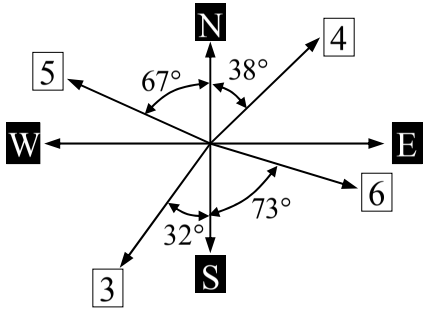


1 ___ ___ ___ ° T — N ___ ___ ° E

True Compass

2 _____ _____

True Compass



3 _____ _____

True Compass

4 _____ _____

True Compass

5 _____ _____

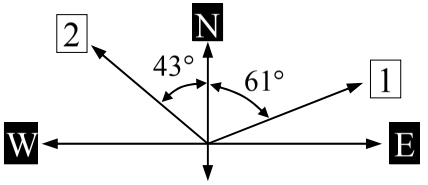
True Compass

6 _____ _____

True Compass

Date Due: _____

Give the compass and true bearings.

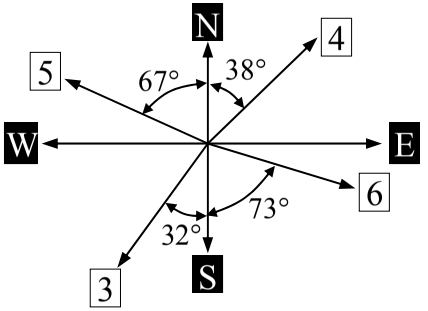


1 ___ ___ ___ ° T — N ___ ___ ° E

True Compass

2 _____ _____

True Compass



3 _____ _____

True Compass

4 _____ _____

True Compass

5 _____ _____

True Compass

6 _____ _____

True Compass

Trigonometry - Bearing Conversion (Homework Strip)

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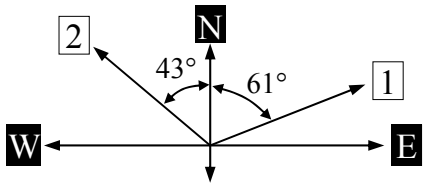
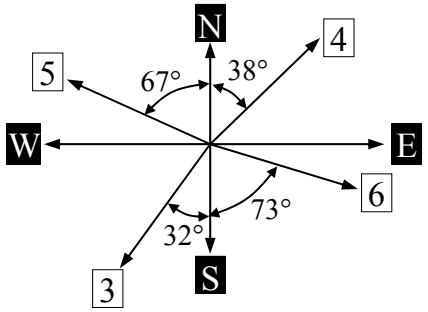
Date Due:

Give the compass and true bearings.



1	061°T	N 61° E
	True	Compass
2	317°T	N 43° W
	True	Compass



3	212°T	S 32° W
	True	Compass
4	038°T	N 38° E
	True	Compass
5	293°T	N 67° W
	True	Compass
6	107°T	S 73° E
	True	Compass

Trigonometry - Bearing Diagrams

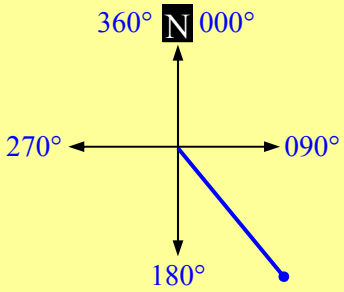
Bearings are often a specific length measurement in a given direction. This can be given in a diagram, or in words. When you apply trigonometry to a question, it is much easier when you have a diagram, but this means you may have to construct one yourself.

These worksheets don't involve scale, so in constructing a diagram you don't have to use a protractor for the angle and apply a scale to rule a line of a set length. They are sketches, not scale diagrams.

Example: Dave drove for 19 km at a bearing of 148ºT.

STEP 1

DRAW THE BEARING



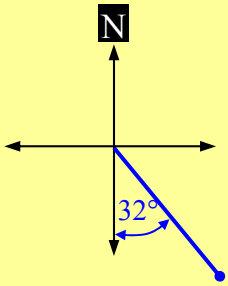
Sketch the bearing.

Think about the angles on the compass points and use that to guide you in where you should draw the bearing.

Unlike a bearing without a distance, you don't use an arrowhead, just a line or put a small circle at the end.

STEP 2

FIND THE ANGLE



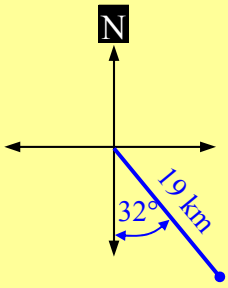
Find the Angle

Because the bearing is below the East-West Line, find the angle to the bearing from South.

Angle = 180 - 148 = 32º

STEP 3

ADD THE DISTANCE



Write the distance

Because the bearing is both direction and length, you also write the distance on the bearing.

Trigonometry - Bearing Diagrams (Sheet 1)

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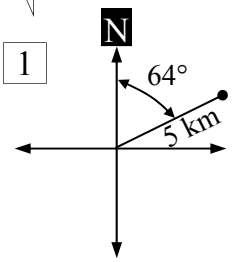
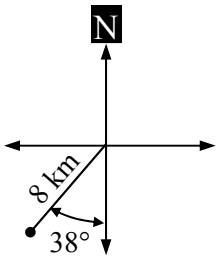
Select which bearing is shown in the diagram.

1



(A) 5 km at N 64° W
(B) 5 km at S 64° E
(C) 5 km at N 64° S
(D) 5 km at N 64° E

2 (A) 8 km at 038°T
(B) 8 km at 218°T
(C) 8 km at 232°T
(D) 8 km at 142°T



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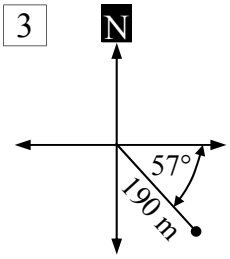
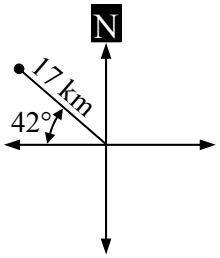
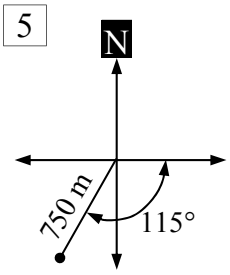


(A) 190 m at 147°T
(B) 190 m at E 57° S
(C) 190 m at 057°T
(D) 190 m at S 57° E

4 (A) 17 km at N 48° W
(B) 17 km at 318°T
(C) 17 km at 222°T
(D) 17 km at N 42° W

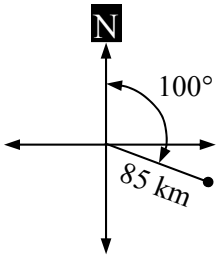


5



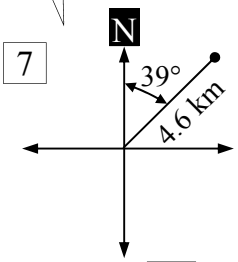
(A) 750 m at 195°T
(B) 750 m at 115°T
(C) 750 m at 205°T
(D) 750 m at S 15° W

6 (A) 85 km at 010°T
(B) 85 km at N 100° E
(C) 85 km at 080°T
(D) 85 km at S 80° E



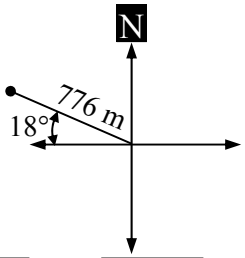
W 20° It is time to fill in the gaps, you may need numbers, you may need letters.

7



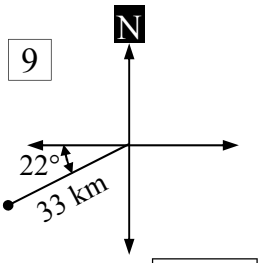
4.6 km at ☐ 39° E

8



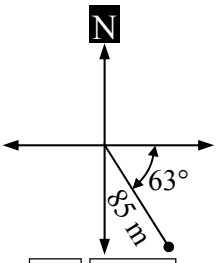
☐ m at ☐ °T

9



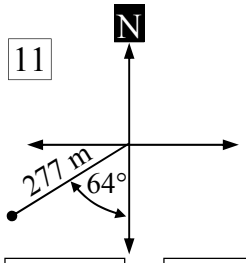
33 km at ☐ °T

10



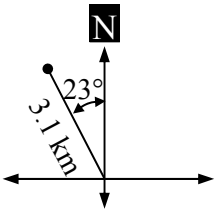
85 m at ☐ ☐ °E

11



☐ at ☐ ° T

12

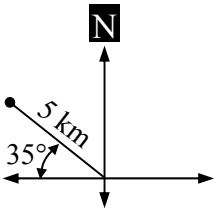


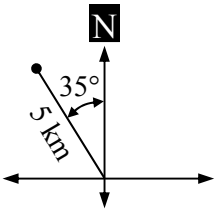
☐ at ☐ °T

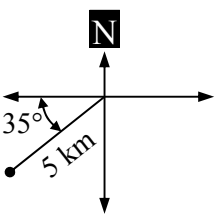
Connect the written description of the bearing to the correct bearing diagram. You will use only three descriptions.

13

5 km at 335° T ●
5 km at S 55° W ●
5 km at N 55° E ●
5 km at 305° T ●
5 km at N 55° W ●
5 km at 325° T ●







Trigonometry - Bearing Diagrams (Sheet 2)

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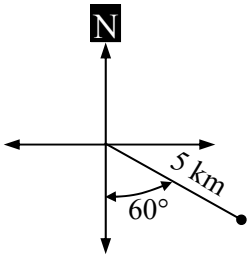
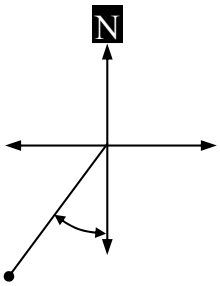
Sketch the bearing and then label the distance and show the angle made with North or South.

Example

John sailed 5 km at a bearing of 120°T.



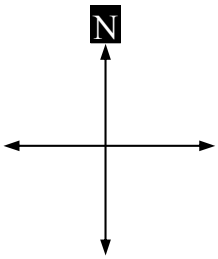
1



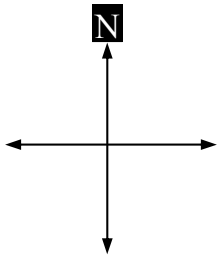
Toni walked for 9 km at a bearing of 210°T.

2

Ben rode for 15 km at a bearing of 320°T.



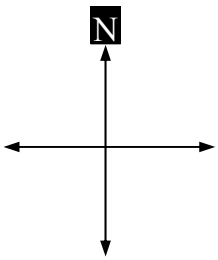
3



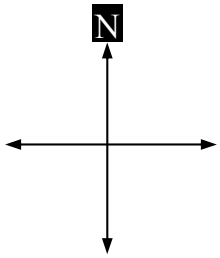
Rohit drove for 46 km at a bearing of 055°T.

4

Keira hiked for 7 km at a bearing of S 30° W.



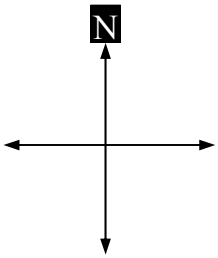
5



Mike sailed for 25 km at a bearing of S 45° E.

6

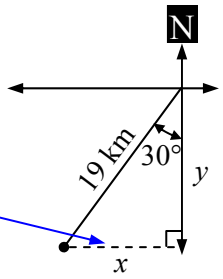
Duncan flew for 300 km at a bearing of N 70° W.



Sketch the bearing, label the distance, show the angle made with North or South. Then label the vertical (y) and horizontal (x) distance travelled.

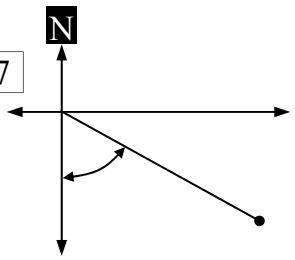
Example

Jade trekked 19 km at a bearing of 210°T.



Draw a dashed line for the x measurement

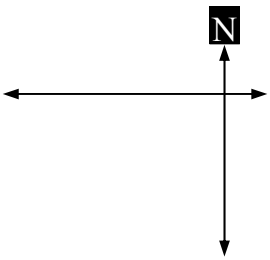
7



Ike walked for 11 km at a bearing of 125°T.

8

Nidal swam for 2.6 km at a bearing of 250°T.



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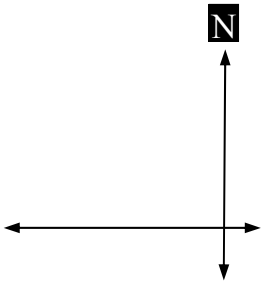
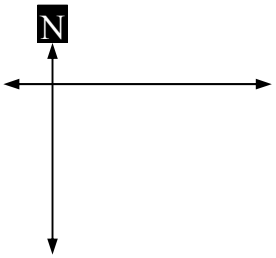


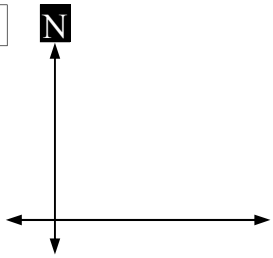
Jiao trekked for 8 km at a bearing of N 25°W.

10



Blake rode for 11 km at a bearing of S 50° E.

11



Aweyes sailed for 25 km at a bearing of N 35° E.

Trigonometry - Bearing Diagrams (Sheet 1)

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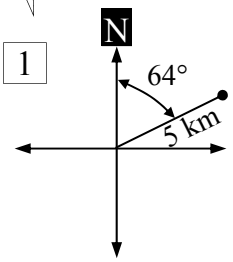
Select which bearing is shown in the diagram.

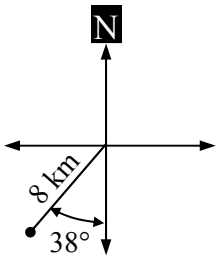
1



(A) 5 km at N 64° W
(B) 5 km at S 64° E
(C) 5 km at N 64° S
(D) 5 km at N 64° E

2

(A) 8 km at 038°T
(B) 8 km at 218°T
(C) 8 km at 232°T
(D) 8 km at 142°T



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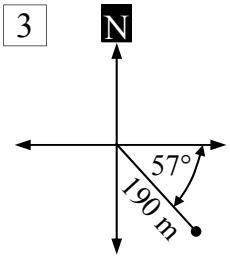
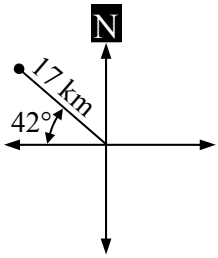


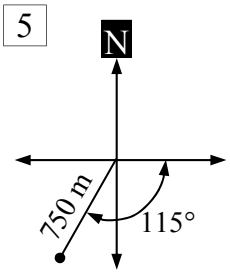
(A) 190 m at 147°T
(B) 190 m at E 57° S
(C) 190 m at 057°T
(D) 190 m at S 57° E

4

(A) 17 km at N 48° W
(B) 17 km at 318°T
(C) 17 km at 222°T
(D) 17 km at N 42° W



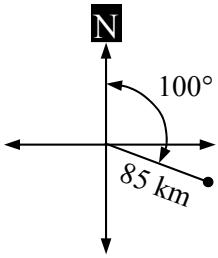
5



(A) 750 m at 195°T
(B) 750 m at 115°T
(C) 750 m at 205°T
(D) 750 m at S 15° W

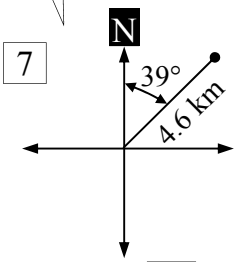
6

(A) 85 km at 010°T
(B) 85 km at N 100° E
(C) 85 km at 080°T
(D) 85 km at S 80° E



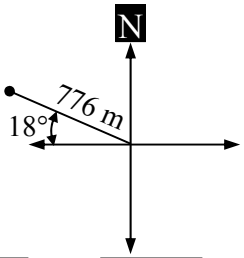
W 20° It is time to fill in the gaps, you may need numbers, you may need letters.

7



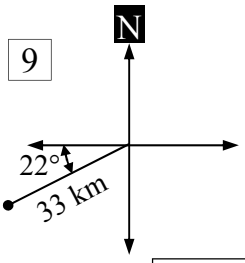
4.6 km at [N] 39° E

8



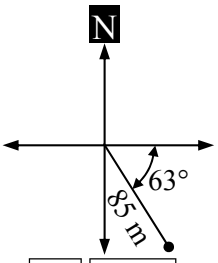
[776] m at [288] °T

9



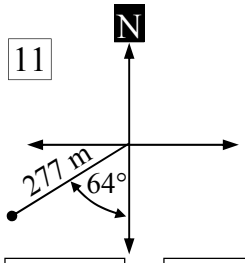
33 km at [248] °T

10



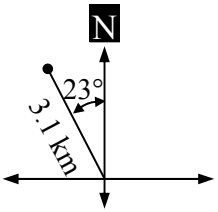
85 m at [S] [27] °E

11



[277 m] at [244] ° T

12

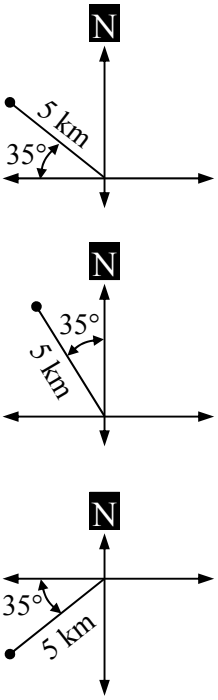


[3.1 km] at [337] °T

Connect the written description of the bearing to the correct bearing diagram. You will use only three descriptions.

13

5 km at 335° T
5 km at S 55° W
5 km at N 55° E
5 km at 305° T
5 km at N 55° W
5 km at 325° T



Trigonometry - Bearing Diagrams (Sheet 2)

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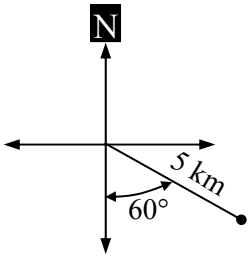
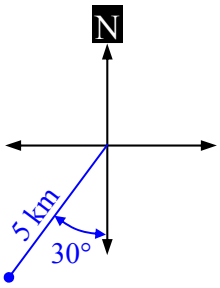
Sketch the bearing and then label the distance and show the angle made with North or South.

Example

John sailed 5 km at a bearing of 120ºT.



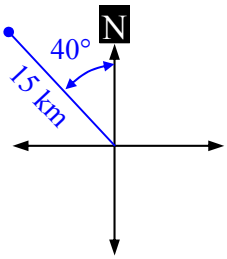
1



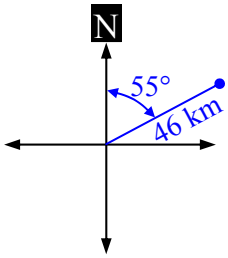
Toni walked for 9 km at a bearing of 210ºT.

2

Ben rode for 15 km at a bearing of 320ºT.



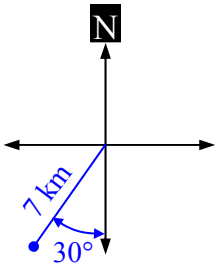
3



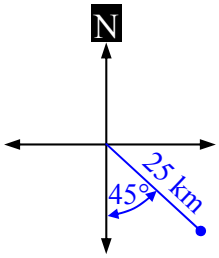
Rohit drove for 46 km at a bearing of 055ºT.

4

Keira hiked for 7 km at a bearing of S 30º W.



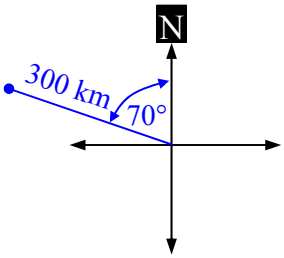
5



Mike sailed for 25 km at a bearing of S 45º E.

6

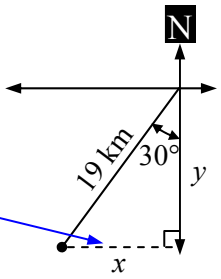
Duncan flew for 300 km at a bearing of N 70º W.



Sketch the bearing, label the distance, show the angle made with North or South. Then label the vertical (y) and horizontal (x) distance travelled.

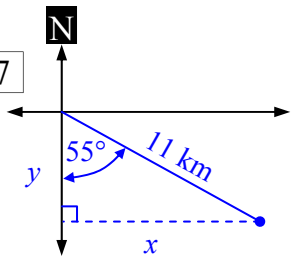
Example

Jade trekked 19 km at a bearing of 210ºT.



Draw a dashed line for the x measurement

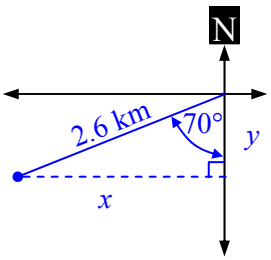
7



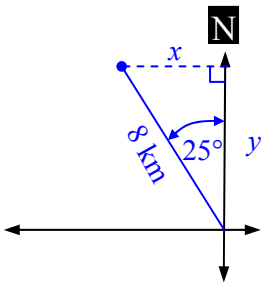
Ike walked for 11 km at a bearing of 125ºT.

8

Nidal swam for 2.6 km at a bearing of 250ºT.

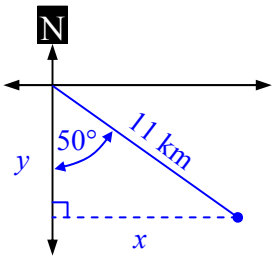


9



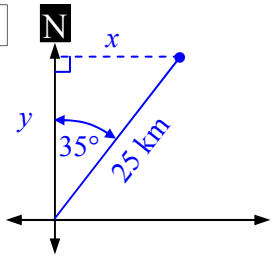
Jiao trekked for 8 km at a bearing of N 25ºW.

10



Blake rode for 11 km at a bearing of S 50º E.

11



Aweyes sailed for 25 km at a bearing of N 35º E.

Trigonometry - Bearing Diagrams (Homework Strip)

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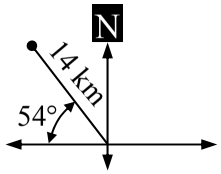
Date Due:

Which bearing is shown below?



1 (A) 14 km at N 54° W

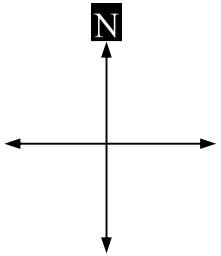
(B) 14 km at 054°T

(C) 14 km at 324°T

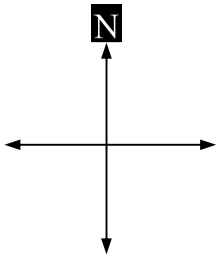
(D) 14 km at W 54° N

Sketch these bearings.

2 9 km at S 30° E

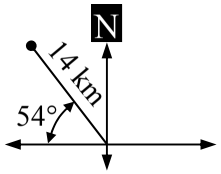


3 850 m at 250°T



Date Due:

Which bearing is shown below?



1 (A) 14 km at N 54° W

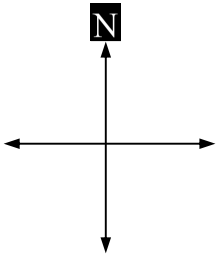
(B) 14 km at 054°T

(C) 14 km at 324°T

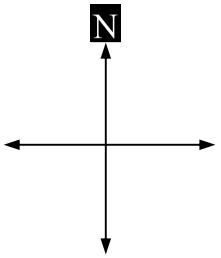
(D) 14 km at W 54° N

Sketch these bearings.

2 9 km at S 30° E

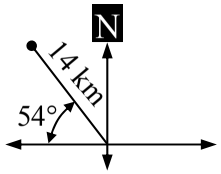


3 850 m at 250°T



Date Due:

Which bearing is shown below?



1 (A) 14 km at N 54° W

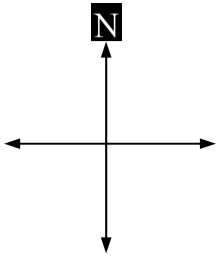
(B) 14 km at 054°T

(C) 14 km at 324°T

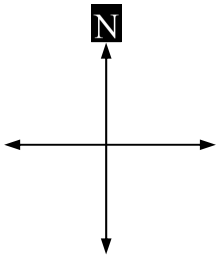
(D) 14 km at W 54° N

Sketch these bearings.

2 9 km at S 30° E

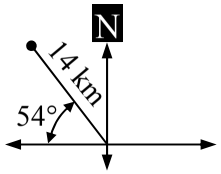


3 850 m at 250°T



Date Due:

Which bearing is shown below?



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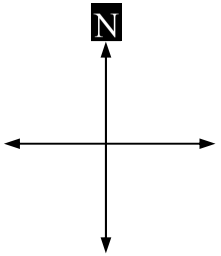
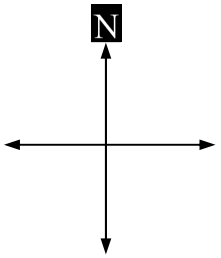
Sketch these bearings.

2 9 km at S 30° E



3 850 m at 250°T



Trigonometry - Bearing Diagrams (Homework Strip)

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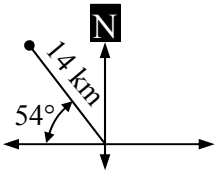
Date Due:

Which bearing is shown below?



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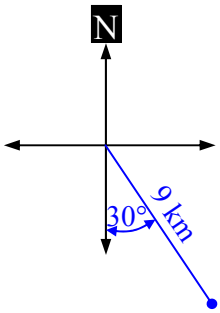
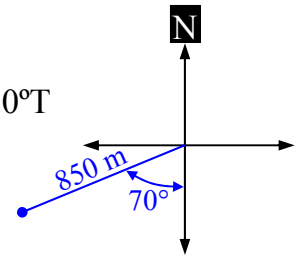


3 850 m at 250°T



Trigonometry - Bearings and Trigonometry Applications

Bearings are often a specific length measurement in a given direction. This can be given in a diagram, or in words. When you apply trigonometry to a question, it is much easier when you have a diagram, but this means you may have to construct one yourself. Combining drawings, interpretation and trigonometry makes this outcome one of the most difficult in the course, so these sheets should be considered very challenging.

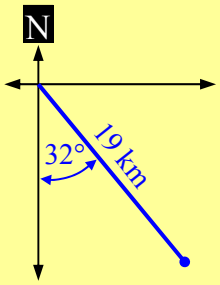
These worksheets don't involve scale, so in constructing a diagram you don't have to use a protractor for the angle and apply a scale to rule a line of a set length. They are sketches, not scale diagrams.

All of the questions on these sheets will be solved using sin or cos, you won't need to use tan.

Example: Dave drove for 19 km at a bearing of 148ºT. How far East of his departure point will he be when he arrives? Answer to 2 decimal places.

STEP 1

DRAW THE BEARING



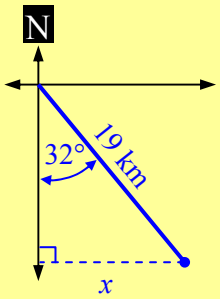
Sketch the bearing.

Position the North-South/East-West lines that favours the quadrant you are going to use …. giving you the most space for your sketch.

Then draw the bearing and label it as well as the angle made with the North-South line.

STEP 2

CONSTRUCT TRIANGLE

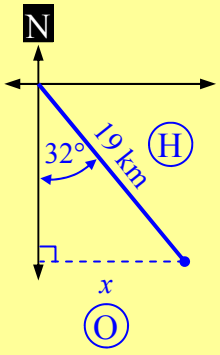


Construct the triangle.

Rule a horizontal line from the end point to North-South line. Include a perpendicular symbol, then write the unknown length on the diagram.

STEP 3

SOLVE



Solve the trig problem.

Label the sides O, A or H. Choose either sin or cos, tan won't be used in these exercises. Also, 'x' will always be on top in these questions, so you will always multiply to solve them.

$$\sin 32 = \frac{x}{19}$$

$\times 19$ $\times 19$

$$x = 19 \sin 32$$

$$x = 10.07 \text{ km} \quad (2 \text{ d.p.})$$

Trigonometry - Bearings and Trigonometry Applications (Sheet 1)

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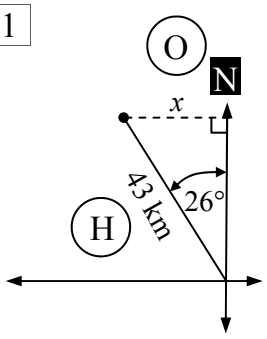
TAN

Label the sides O, A or H and calculate the unknown length to 2 decimal places. Tan free zone.

SOH CAH ~~TOA~~

1



$\sin \square = \frac{\square}{\square}$ O / H

$\times 43$ $\times 43$

$x = \square \sin \square$

$x = \square$ km 2 d.p.

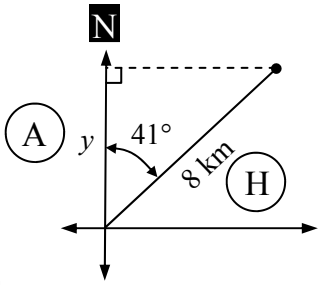
2

$\cos \square = \frac{\square}{\square}$ A / H

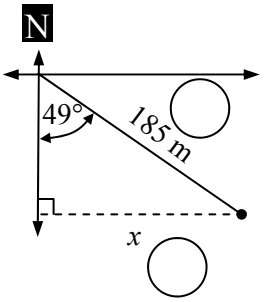
$\times 8$ $\times 8$

$y = \square \cos \square$

$y = \square$ km 2 d.p.



3



$\square = \frac{\square}{\square}$

$\times$ $\times$

$x = \square \sin \square$

$x = \square$ m

2 d.p.

4

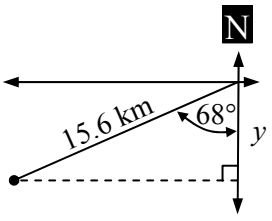
$\square = \frac{\square}{\square}$

$\times$ $\times$

$y =$

$y =$

2 d.p.



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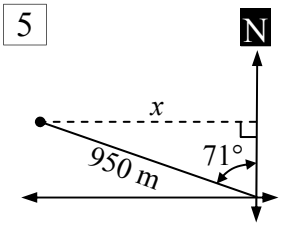


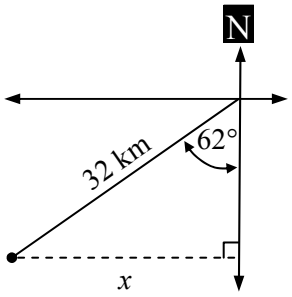
$\square = \frac{\square}{\square}$

$\times$ $\times$

2 d.p.

These questions are the same except they have a worded explanation of the bearing as well. Find the unknown distance to 1 decimal place.

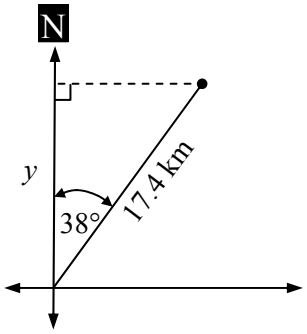
6 A ferry travels 32 km at a bearing of 242ºT. Calculate how far East of its departure point (x on the diagram) the ferry has travelled.



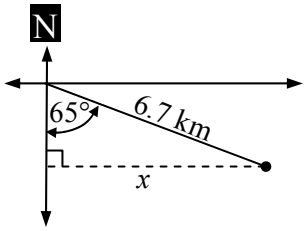
$\times$ $\times$

1 d.p.

7 Murali is trekking for 17.4 km at a bearing of N 38º E. Calculate how far North of his starting point, y on the diagram, he has travelled.



8 Briony is competing in a 6.7 km cross country run at a bearing of 115ºT. Calculate how far East (x) of her starting point she is, when she finishes the run.



Trigonometry - Bearings and Trigonometry Applications (Sheet 2)

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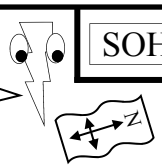
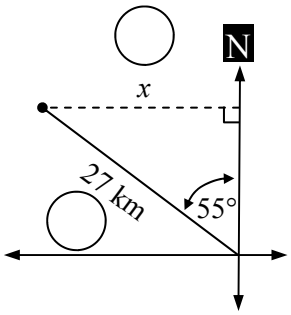
Get ready to construct your own diagrams! Calculate the unknown distance to 1 decimal place.



SOH CAH ~~TOA~~

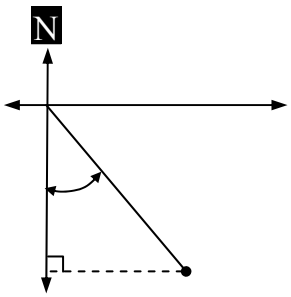
OK you are on your own. Calculate the unknown distance to 1 decimal place.

1 Josh is hiking for 27 km at a bearing of N 55º W. Calculate how far West of his original position, *x* on the diagram, he will be when he finishes the hike.

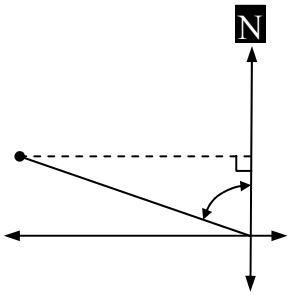


1 d.p.

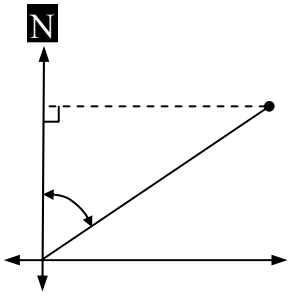
2 Lucy is sailing 33 km at a bearing of 142º T. Add the information to the diagram and calculate how far South of her original position Lucy will be.



3 Darius is bush-walking for 4.2 km at a bearing of 285ºT to a campsite. How far North of his starting point is the campsite?



4 Thien is driving 10.7 km at a bearing of N 41º E. How far East of his starting position will he be when he arrives?



5 Isla is jogging 3.4 km at a bearing of 022ºT. Calculate how far North of her starting point she will be when she finishes.

6 Azmi is trekking at a bearing of S 36º W for 17 km. Calculate how far West Azmi will be from his starting point when he completes the trek.

7 Jodi is looking at a lighthouse which is 5 km away at a bearing of N 58º W. How far North of Jodi is the light-house?

8 Amy is flying at a bearing of 142ºT for 650 km. How far East of her departure point will she be when she lands?

Trigonometry - Bearings and Trigonometry Applications (Sheet 1)

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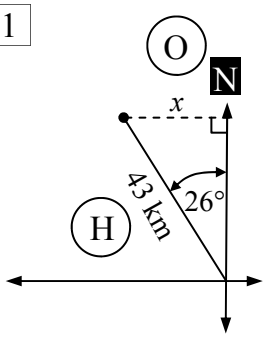
Label the sides O, A or H and calculate the unknown length to 2 decimal places. Tan free zone.

SOH CAH ~~TOA~~

1



$\sin 26 = \frac{x}{43}$ O / H

$\times 43$ $\times 43$

$x = 43 \sin 26$

$x = 18.85$ km 2 d.p.

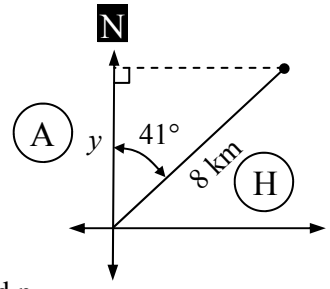
2

$\cos 41 = \frac{y}{8}$ A / H

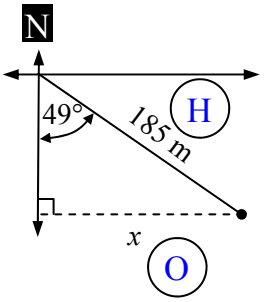
$\times 8$ $\times 8$

$y = 8 \cos 41$

$y = 6.04$ km 2 d.p.



3



$\sin 49 = \frac{x}{185}$

$\times 185$ $\times 185$

$x = 185 \sin 49$

$x = 139.62$ m

2 d.p.

4

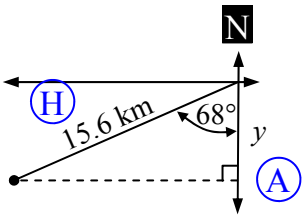
$\cos 68 = \frac{y}{15.6}$

$\times 15.6$ $\times 15.6$

$y = 15.6 \cos 68$

$y = 5.84$ km

2 d.p.



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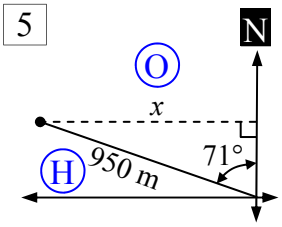


$\sin 71 = \frac{x}{950}$

$\times 950$ $\times 950$

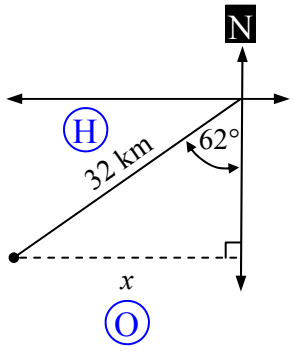
$x = 950 \sin 71$

$x = 898.24$ m

2 d.p.

These questions are the same except they have a worded explanation of the bearing as well. Find the unknown distance to 1 decimal place.

6 A ferry travels 32 km at a bearing of 242°T. Calculate how far East of its departure point (x on the diagram) the ferry has travelled.



$\sin 62 = \frac{x}{32}$

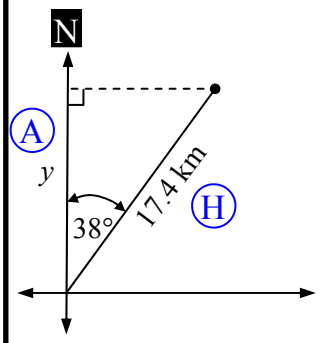
$\times 32$ $\times 32$

$y = 32 \sin 62$

$y = 28.3$ km

1 d.p.

7 Murali is trekking for 17.4 km at a bearing of N 38° E. Calculate how far North of his starting point, y on the diagram, he has travelled.



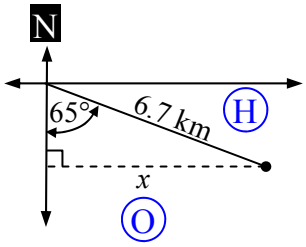
$\cos 38 = \frac{y}{17.4}$

$\times 17.4$ $\times 17.4$

$y = 17.4 \cos 38$

$y = 13.7$ km

8 Briony is competing in a 6.7 km cross country run at a bearing of 115°T. Calculate how far East (x) of her starting point she is, when she finishes the



$\sin 65 = \frac{x}{6.7}$

$\times 6.7$ $\times 6.7$

$x = 6.7 \sin 65$

$x = 6.1$ km

Trigonometry - Bearings and Trigonometry Applications (Sheet 2)

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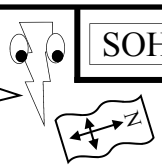

SOH CAH ~~TOA~~

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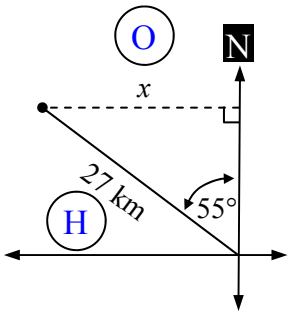


$\sin 55 = \frac{x}{27}$

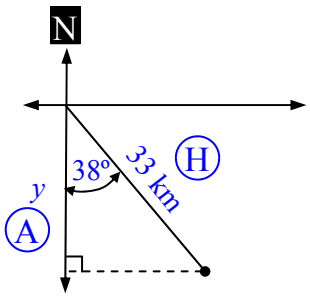
$\times 27 \quad \times 27$

$x = 27 \sin 55$

$x = 22.1$ km

1 d.p.

2 Lucy is sailing 33 km at a bearing of 142º T. Add the information to the diagram and calculate how far South of her original position Lucy will be.



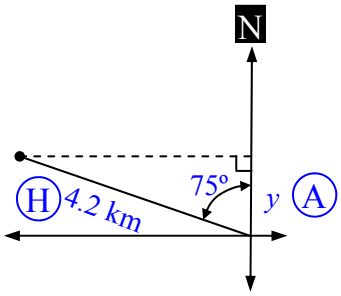
$\cos 38 = \frac{y}{33}$

$\times 33 \quad \times 33$

$y = 33 \cos 38$

$y = 22.1$ km

3 Darius is bush-walking for 4.2 km at a bearing of 285ºT to a campsite. How far North of his starting point is the campsite?



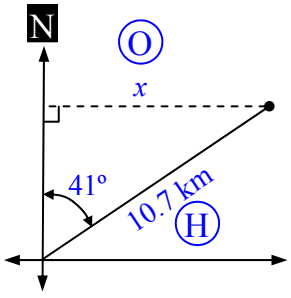
$\cos 75 = \frac{y}{4.2}$

$\times 4.2 \quad \times 4.2$

$y = 4.2 \cos 75$

$y = 1.1$ km

4 Thien is driving 10.7 km at a bearing of N 41º E. How far East of his starting position will he be when he arrives?



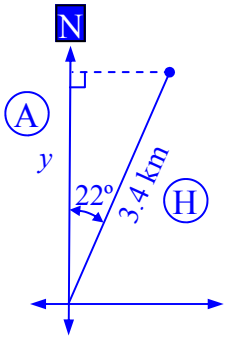
$\sin 41 = \frac{x}{10.7}$

$\times 10.7 \quad \times 10.7$

$x = 10.7 \sin 41$

$x = 7.0$ km

5 Isla is jogging 3.4 km at a bearing of 022ºT. Calculate how far North of her starting point she will be when she finishes.



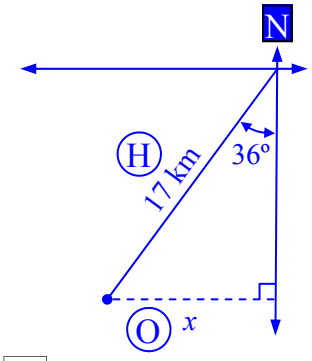
$\cos 22 = \frac{y}{3.4}$

$\times 3.4 \quad \times 3.4$

$y = 3.4 \cos 22$

$y = 3.2$ km

6 Azmi is trekking at a bearing of S 36º W for 17 km. Calculate how far West Azmi will be from his starting point when he completes the trek.



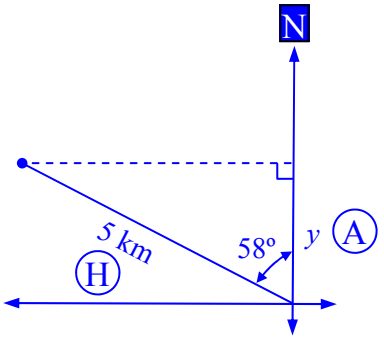
$\sin 36 = \frac{x}{17}$

$\times 17 \quad \times 17$

$x = 17 \sin 36$

$x = 10.0$ km

7 Jodi is looking at a lighthouse which is 5 km away at a bearing of N 58º W. How far North of Jodi is the light-house?



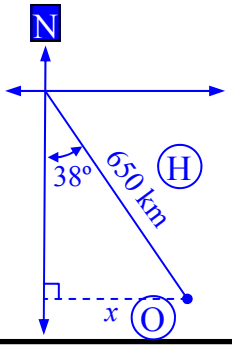
$\cos 59 = \frac{y}{5}$

$\times 5 \quad \times 5$

$y = 5 \cos 59$

$y = 2.6$ km

8 Amy is flying at a bearing of 142ºT for 650 km. How far East of her departure point will she be when she lands?



$\sin 38 = \frac{x}{650}$

$\times 650 \quad \times 650$

$x = 650 \sin 38$

$x = 400.2$ km

Trigonometry - Bearing Diagrams (Sheet 2)

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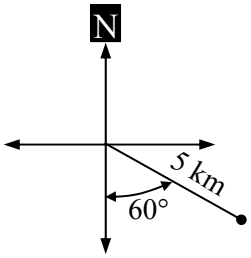
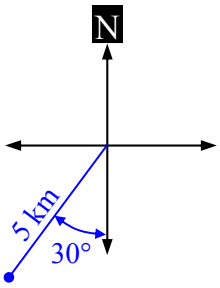
Sketch the bearing and then label the distance and show the angle made with North or South.

Example

John sailed 5 km at a bearing of 120°T.



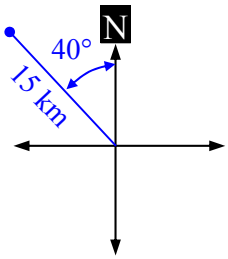
1



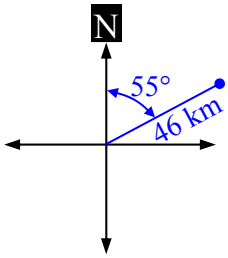
Toni walked for 9 km at a bearing of 210°T.

2

Ben rode for 15 km at a bearing of 320°T.



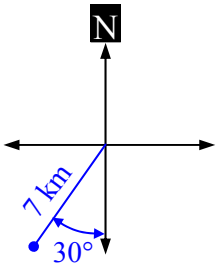
3



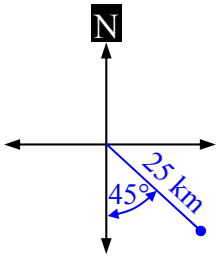
Rohit drove for 46 km at a bearing of 055°T.

4

Keira hiked for 7 km at a bearing of S 30° W.



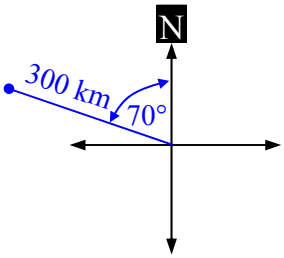
5



Mike sailed for 25 km at a bearing of S 45° E.

6

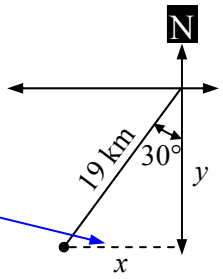
Duncan flew for 300 km at a bearing of N 70° W.



Sketch the bearing, label the distance, show the angle made with North or South. Then label the vertical (y) and horizontal (x) distance travelled.

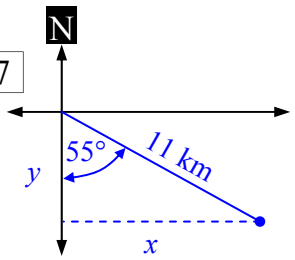
Example

Jade trekked 19 km at a bearing of 210°T.



Draw a dashed line for the x measurement

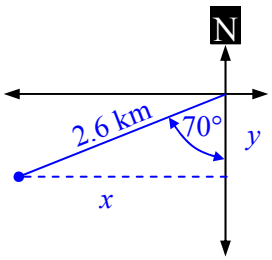
7



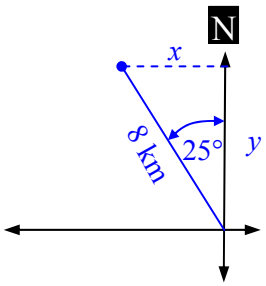
Ike walked for 11 km at a bearing of 125°T.

8

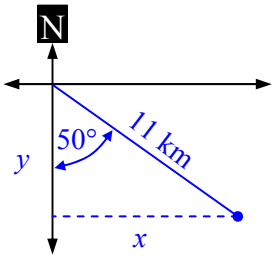
Nidal swam for 2.6 km at a bearing of 250°T.



9

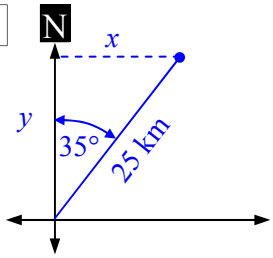


10



Blake rode for 11 km at a bearing of S 50° E.

11



Aweyes sailed for 25 km at a bearing of N 35° E.

Trigonometry - Bearings and Trigonometry Application (Homework Strip)

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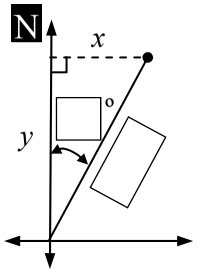
Date Due:

Bree is hiking for 9 km at a bearing of N 27º E. Add the angle and distance to the diagram below (in the boxes).

1



2 Calculate 'x', the distance she will be East of her starting point. (1 d.p.)

3 Calculate 'y', the distance she will be North of her starting point. (1 d.p.)

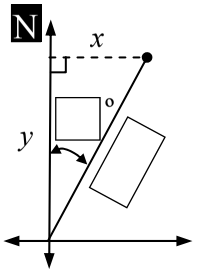
Date Due:

Bree is hiking for 9 km at a bearing of N 27º E. Add the angle and distance to the diagram below (in the boxes).

1

2 Calculate 'x', the distance she will be East of her starting point. (1 d.p.)

3 Calculate 'y', the distance she will be North of her starting point. (1 d.p.)



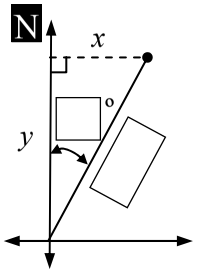
Date Due:

Bree is hiking for 9 km at a bearing of N 27º E. Add the angle and distance to the diagram below (in the boxes).

1

2 Calculate 'x', the distance she will be East of her starting point. (1 d.p.)

3 Calculate 'y', the distance she will be North of her starting point. (1 d.p.)



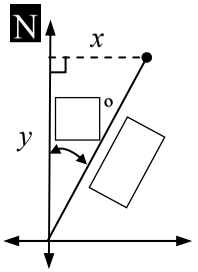
Date Due:

Bree is hiking for 9 km at a bearing of N 27º E. Add the angle and distance to the diagram below (in the boxes).

1

2 Calculate 'x', the distance she will be East of her starting point. (1 d.p.)

3 Calculate 'y', the distance she will be North of her starting point. (1 d.p.)



Trigonometry - Bearings & Trigonometry Application (Homework Strip)

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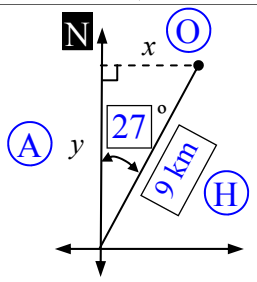
Date Due:

Bree is hiking for 9 km at a bearing of N 27º E. Add the angle and distance to the diagram below (in the boxes).

1



2 Calculate 'x', the distance she will be East of her starting point. (1 d.p.)

$$\sin 27 = \frac{x}{9}$$

$\times 9$ $\times 9$

$$x = 9 \sin 27$$

$$x = 4.1 \text{ km}$$

3 Calculate 'y', the distance she will be North of her starting point. (1 d.p.)

$$\cos 27 = \frac{y}{9}$$

$\times 9$ $\times 9$

$$y = 9 \cos 27$$

$$y = 8.0 \text{ km}$$

Bearings Review

This is the review of the Bearings Extension Topic.

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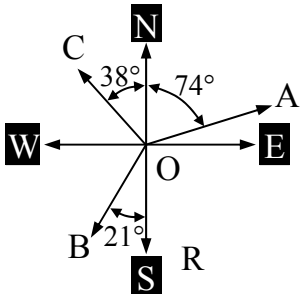
Express the bearings shown as true bearings

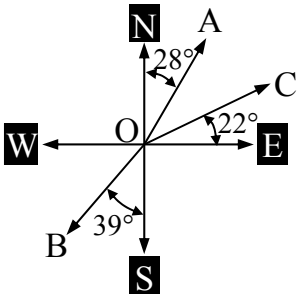


1 OA: _____

2 OB: _____

3 OC: _____

Express the bearings shown as compass bearings



4 OA: _____

5 OB: _____

6 OC: _____

Convert these compass bearings to true bearings

7 N 55° E _____ 8 WEST _____

9 S 13° E _____ 10 N 41° W _____

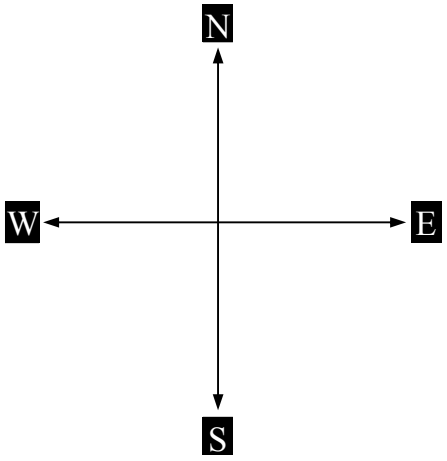
Convert these true bearings to compass bearings

11 058° T _____ 12 267° T _____

13 155° T _____ 14 090° T _____

Given bearings, show them on the diagrams. Include the angle between the North-South line and the bearing, then name each bearing using the letters.

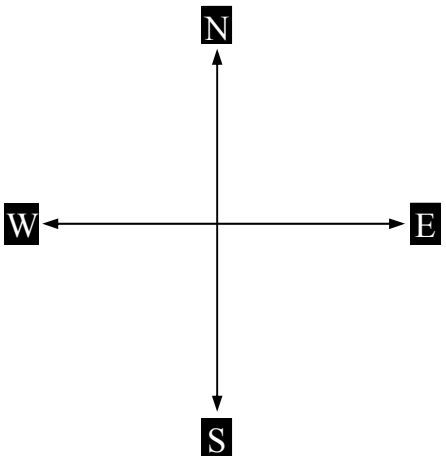
15 OA: 025° T



16 OB: S 45° W

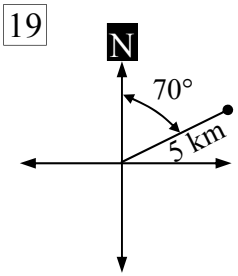
17 OA: 290° T

18 OB: 230° T



Pick which bearing is shown in these diagrams.

19



(A) 5 km at N 70° W

(B) 5 km at S 70° E

(C) 5 km at N 70° S

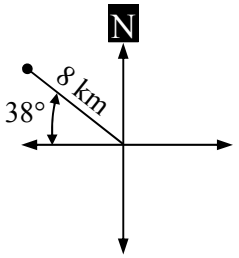
(D) 5 km at N 70° E

20 (A) 8 km at 038°T

(B) 8 km at 308°T

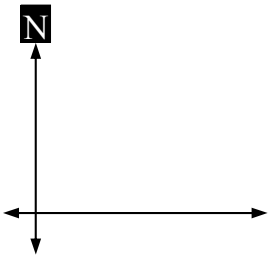
(C) 8 km at 322°T

(D) 8 km at 218°T

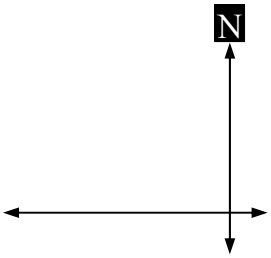


Show these bearings on the diagrams below. Include the distance and the angle made with the North-South line.

21 Jake rode for 11 km at a bearing of 040°T.



22 Joanna walked for 4.8 km at a bearing of N 33° W.



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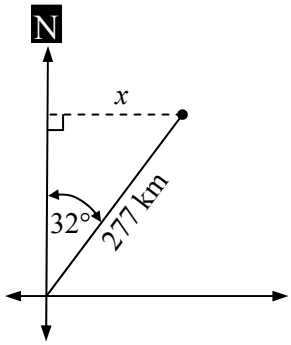
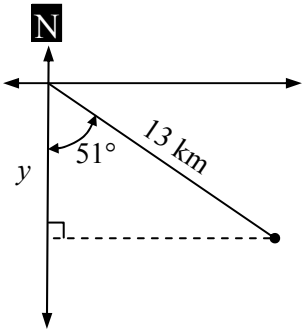
Apply your trigonometry skills to calculate the unknown lengths. Answer to 1 decimal place.

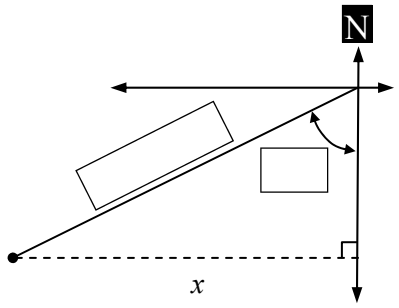
23 A plane travels 277 km at a bearing of 032º T. Calculate how far East of its departure point (x on the diagram) the plane has travelled.



24 Josiah is trekking for 13 km at a bearing of S 51º E. Calculate how far South of his starting point, y on the diagram, he has travelled.



25 Alyssa is bush-walking for 8.2 km at a bearing of 244ºT to a campsite, shown on the diagram below. Write the distance on the bearing and the angle between South and the bearing, in the boxes.



26 Calculate the distance 'x', which is the distance her finishing point is West of her starting point.

27 Brody is sailing at a bearing of 305º T for a distance of 26 km. Construct a diagram showing all relevant data.

28 Calculate how far North of his starting point he will be when he completes the voyage, to 1 decimal place.

29 Kirby is riding at a bearing of S 75º W for a distance of 9.4 km. Construct a diagram showing all relevant data.

30 Calculate how far West of her starting point she will be when she completes riding, to 1 decimal place.

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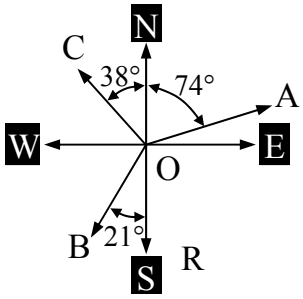
Express the bearings shown as true bearings

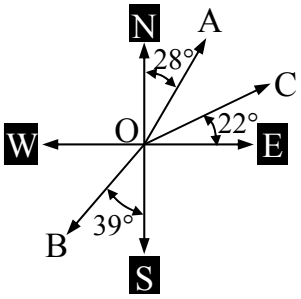


1 OA: 074° T

2 OB: 201° T

3 OC: 322° T

Express the bearings shown as compass bearings



4 OA: N 28° E

5 OB: S 39° W

6 OC: N 68° E

Convert these compass bearings to true bearings

7 N 55° E 055° T

8 WEST 270° T

9 S 13° E 167° T

10 N 41° W 319° T

Convert these true bearings to compass bearings

11 058° T N 58° E

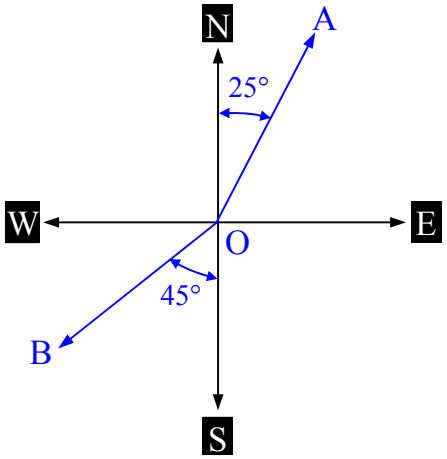
12 267° T S 87° W

13 155° T S 25° E

14 090° T EAST

Given bearings, show them on the diagrams. Include the angle between the North-South line and the bearing, then name each bearing using the letters.

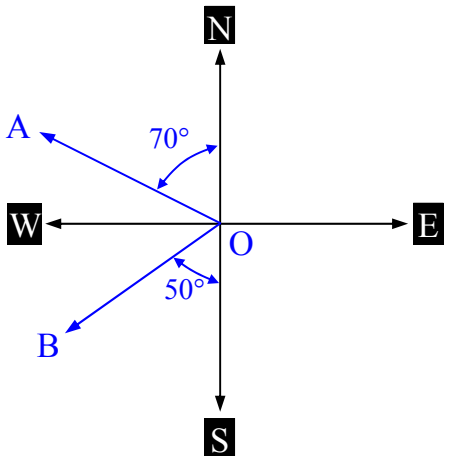
15 OA: 025° T



16 OB: S 45° W

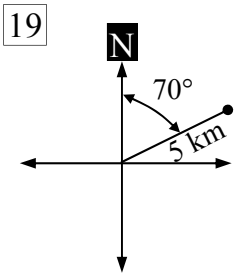
17 OA: 290° T

18 OB: 230° T



Pick which bearing is shown in these diagrams.

19



(A) 5 km at N 70° W

(B) 5 km at S 70° E

(C) 5 km at N 70° S

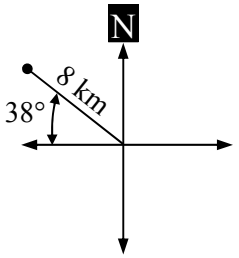
(D) 5 km at N 70° E

20 (A) 8 km at 038°T

(B) 8 km at 308°T

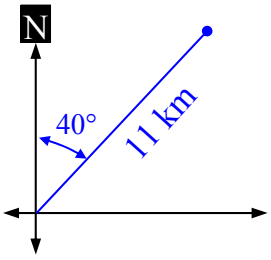
(C) 8 km at 322°T

(D) 8 km at 218°T

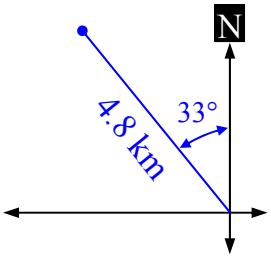


Show these bearings on the diagrams below. Include the distance and the angle made with the North-South line.

21 Jake rode for 11 km at a bearing of 040°T.



22 Joanna walked for 4.8 km at a bearing of N 33° W.



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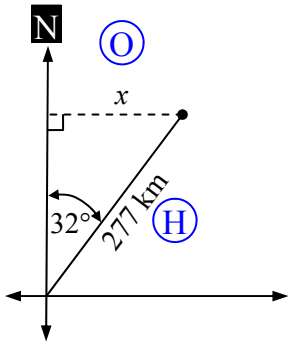
Apply your trigonometry skills to calculate the unknown lengths. Answer to 1 decimal place.

23 A plane travels 277 km at a bearing of 032º T. Calculate how far East of its departure point (x on the diagram) the plane has travelled.



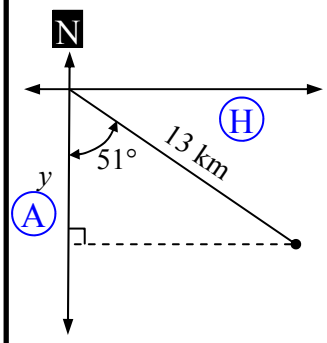
$\sin 32 = \frac{x}{277}$

$\times 277 \quad \times 277$

$x = 277 \sin 32$

$x = 146.8$ km

24 Josiah is trekking for 13 km at a bearing of S 51º E. Calculate how far South of his starting point, y on the diagram, he has travelled.



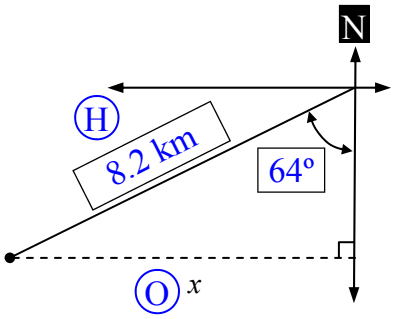
$\cos 51 = \frac{y}{13}$

$\times 13 \quad \times 13$

$y = 13 \cos 51$

$y = 8.2$ km

25 Alyssa is bush-walking for 8.2 km at a bearing of 244ºT to a campsite, shown on the diagram below. Write the distance on the bearing and the angle between South and the bearing, in the boxes.



26 Calculate the distance 'x', which is the distance her finishing point is West of her starting point.

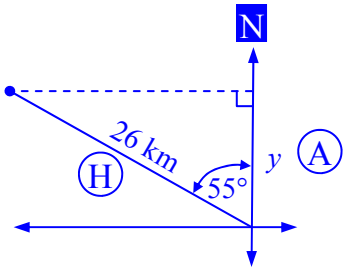
$\sin 64 = \frac{x}{8.2}$

$\times 8.2 \quad \times 8.2$

$x = 8.2 \sin 64$

$x = 7.4$ km

27 Brody is sailing at a bearing of 305º T for a distance of 26 km. Construct a diagram showing all relevant data.



28 Calculate how far North of his starting point he will be when he completes the voyage, to 1 decimal place.

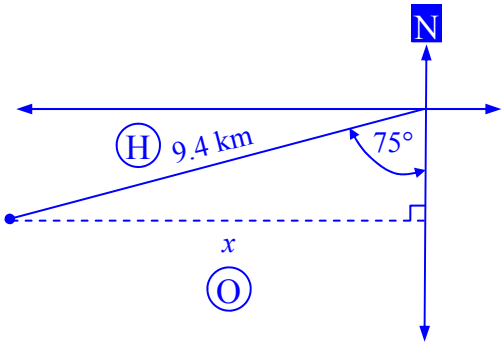
$\cos 55 = \frac{y}{26}$

$\times 26 \quad \times 26$

$y = 26 \cos 55$

$y = 14.9$ km

29 Kirby is riding at a bearing of S 75º W for a distance of 9.4 km. Construct a diagram showing all relevant data.



30 Calculate how far West of her starting point she will be when she completes riding, to 1 decimal place.

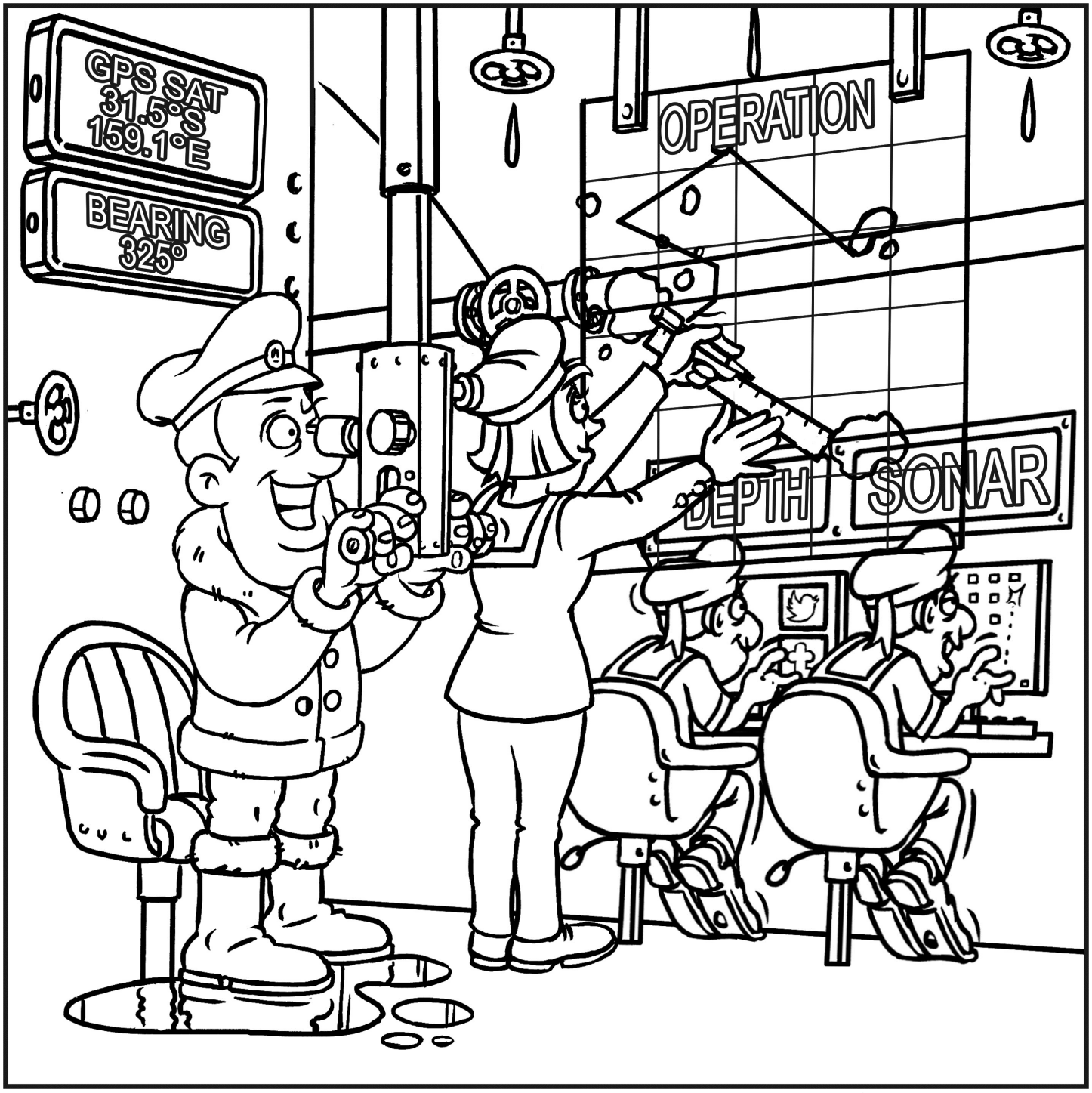
$\sin 75 = \frac{x}{9.4}$

$\times 9.4 \quad \times 9.4$

$x = 9.4 \sin 75$

$x = 9.1$ km

Trigonometry Extension



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Trigonometry - True Bearings (Sheet 1)

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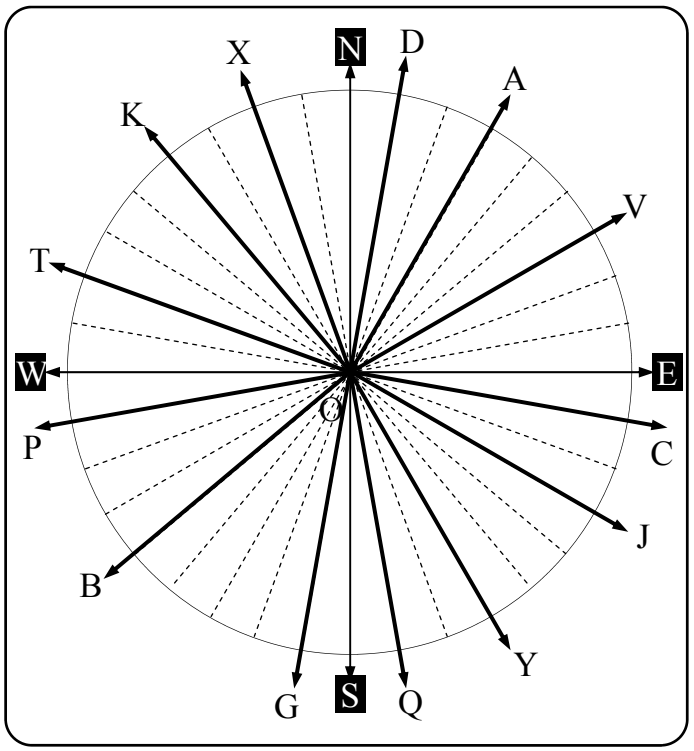
These bearings are on 10° divisions. 'O' is in the centre of the diagram, so bearings are 'OA', 'OB' etc., from the centre out. North is 000° (or 000° T).



Give the True bearing for these. Count the 10º divisions. Remember the bearing for East, South and West, to speed things up.

1 OA ☐☐☐ º T

2 OD ☐☐☐ º T

3 OY ☐ Show º symbol

4 OX ☐

5 OC ☐

6 OB ☐

7 EAST ☐

8 OJ ☐

9 WEST ☐

10 OT ☐

11 OP ☐

12 OK ☐

13 SOUTH ☐

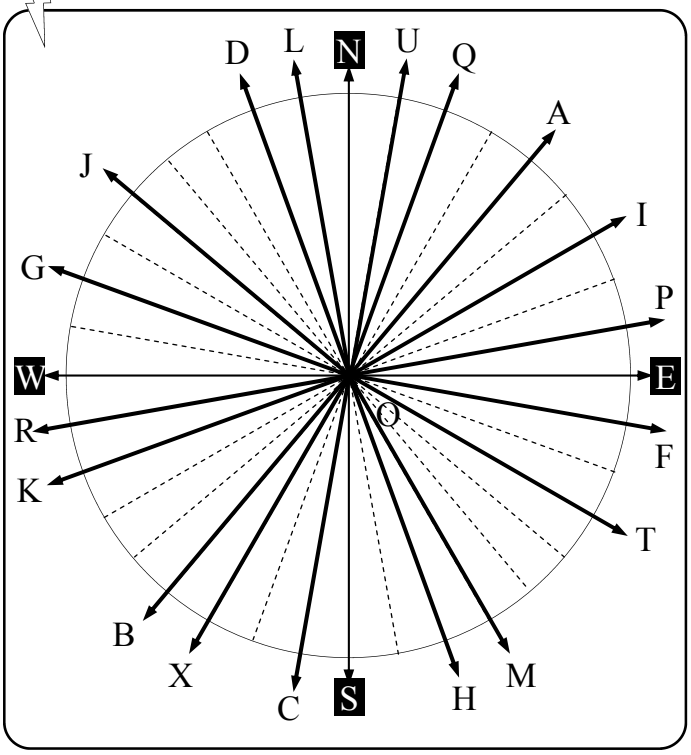
14 OG ☐

Give the bearing that is ….

15 30º clockwise from OQ ☐

16 60º clockwise from OK ☐

Now try this circle. Give the true bearing.



17 OU ☐ Show º symbol

18 OH ☐

19 OJ ☐

20 OP ☐

21 OK ☐

22 OC ☐

23 OA ☐

24 OF ☐

25 OM ☐

26 OD ☐

27 OB ☐

28 OL ☐

29 OG ☐

30 OQ ☐

31 OI ☐

32 OR ☐

33 OT ☐

34 OX ☐

Give the bearing that is ….

35 50º clockwise from OL ☐

36 80º anti-clockwise from OU ☐

37 70º clockwise from OH ☐

Trigonometry - True Bearings (Sheet 2)

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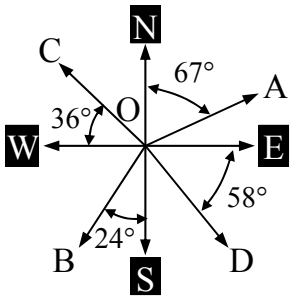
Give the bearing for these. Add to angles for North, East, South and West to get the bearing.

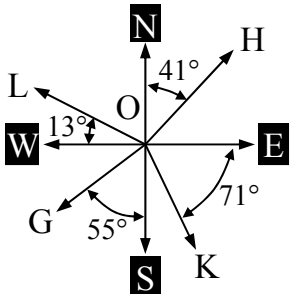


1 OA ___ ° T 000°+ 67°

2 OB ___ ° T 180°+ 24°

3 OC ____ 270°+ 36°

4 OD ____ 090°+ 58°

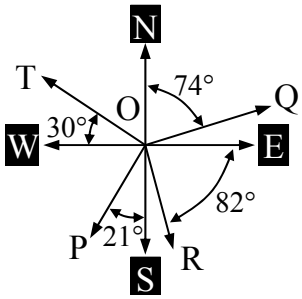


5 OG ____

6 OK ____

7 OH ____

8 OL ____



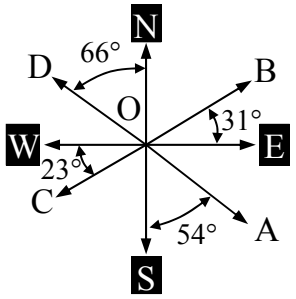
9 ____ OT

10 ____ OQ

11 ____ OP

12 ____ OR

Now use subtraction to find the bearings for these.

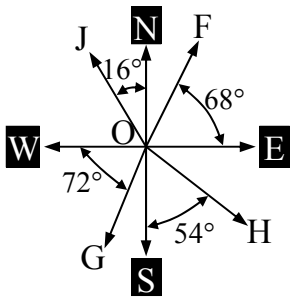


13 OC ____ 270°- 23°

14 OD ____ 360°- 66°

15 OB ____ 090°- 31°

16 OA ____ 180°- 54°

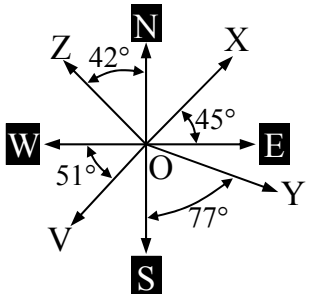


17 ____ OJ

18 ____ OH

19 ____ OF

20 ____ OG



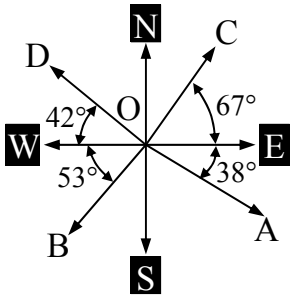
21 OZ ____

22 OX ____

23 OV ____

24 OY ____

These use both subtraction and addition. Find the bearing.

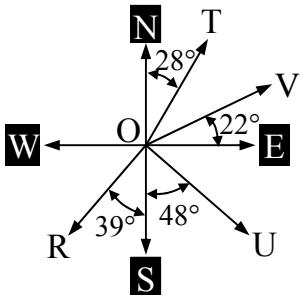


25 OC ____

26 OD ____

27 OB ____

28 OA ____

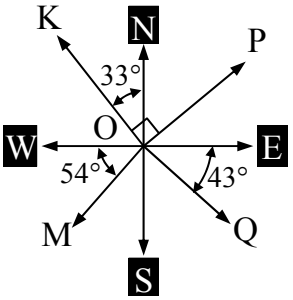


29 OU ____

30 OT ____

31 OR ____

32 OV ____



33 ____ OM

34 ____ OQ

35 ____ OK

36 ____ OP

Trigonometry - Compass Bearings (Sheet 1)

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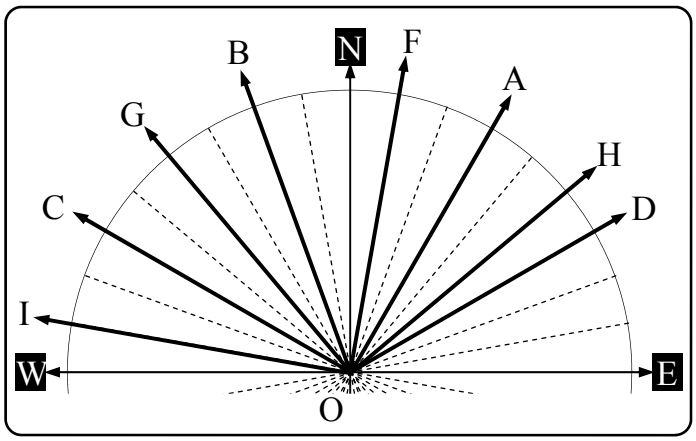
These bearings are on 10° divisions. 'O' is in the centre of the diagram, so bearings are 'OA', 'OB' etc., from the centre out. These are all 'off' North.



Give the compass bearing for these. Count the 10º divisions clockwise or anticlockwise from North, then South (lower diagram).

1 OA N ☐☐° E

2 OC N ☐☐° W

3 OI

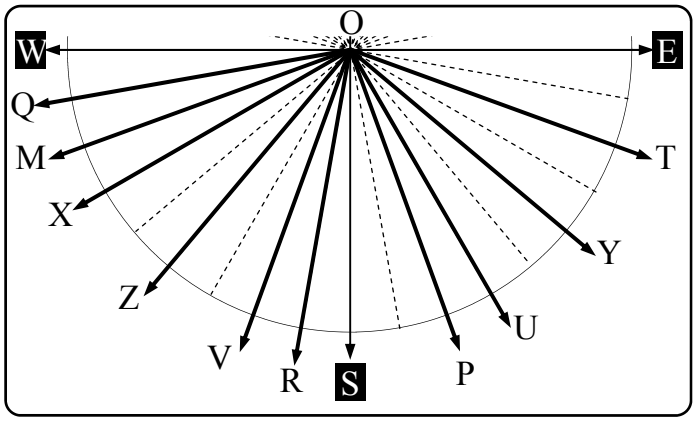
4 OF

5 OG

6 OD

7 OB

8 OH



9 OR S ☐☐° W

10 OT S ☐☐° E

11 OX

12 OP

13 OY

14 OV

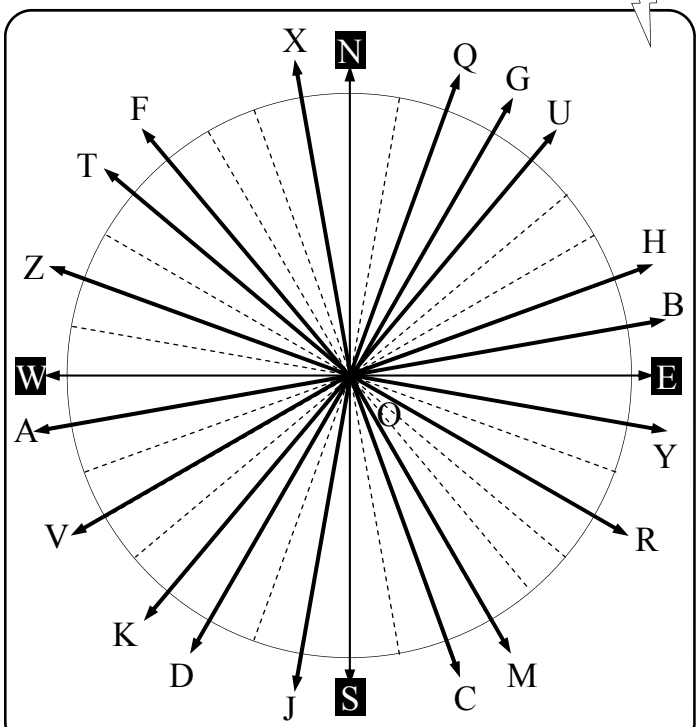
15 OQ

16 OU

17 OM

18 OZ

Now try a full circle. Find the compass bearing.



19 OC

20 OT

21 OV

22 OX

23 OB

24 OY

25 OH

26 OD

27 OZ

28 OQ

29 OR

30 OF

31 OA

32 OM

33 OG

34 OJ

35 OU

36 OK

Give the compass bearing that is ….

37 40º clockwise from OA

38 30º anti-clockwise from OJ

39 80º clockwise from OF

Trigonometry - Compass Bearings (Sheet 2)

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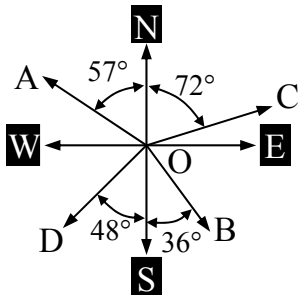
Give the compass bearing for these.

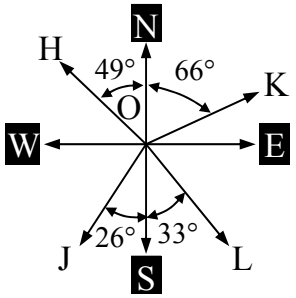


1 OA N ☐☐ º W

2 OB S ☐☐ º E

3 OC ☐

4 OD ☐

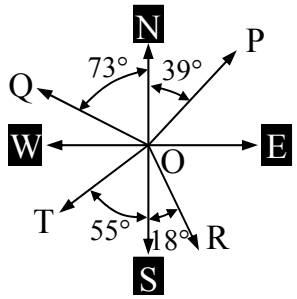


5 OJ ☐

6 OK ☐

7 OH ☐

8 OL ☐



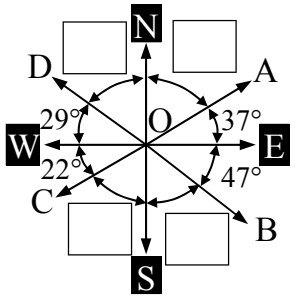
9 ☐ OT

10 ☐ OP

11 ☐ OQ

12 ☐ OR

Use subtraction first, then give the compass bearings for these.

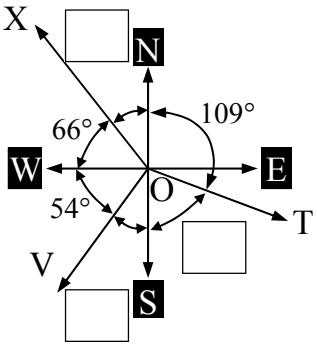


13 OA ☐

14 OD ☐

15 OB ☐

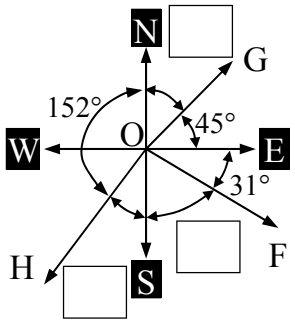
16 OC ☐



17 ☐ OV

18 ☐ OX

19 OT ☐

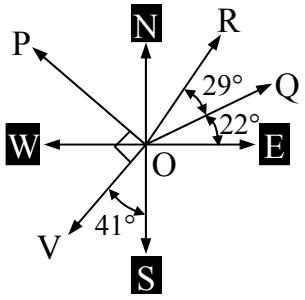


20 OG ☐

21 OF ☐

22 OH ☐

Use subtraction and addition to find the compass bearing.

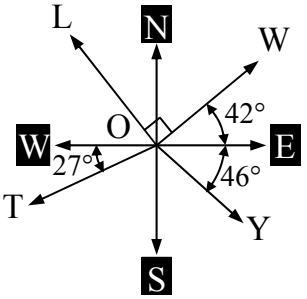


23 OR ☐

24 OQ ☐

25 OV ☐

26 OP ☐

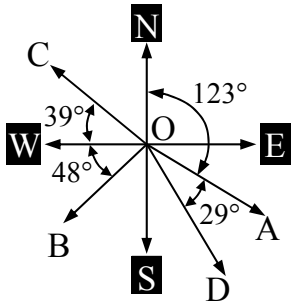


27 ☐ OW

28 ☐ OY

29 ☐ OT

30 ☐ OL



33 ☐ OB

34 ☐ OC

35 ☐ OA

36 ☐ OD

Trigonometry - Bearing Conversion (Sheet 1)

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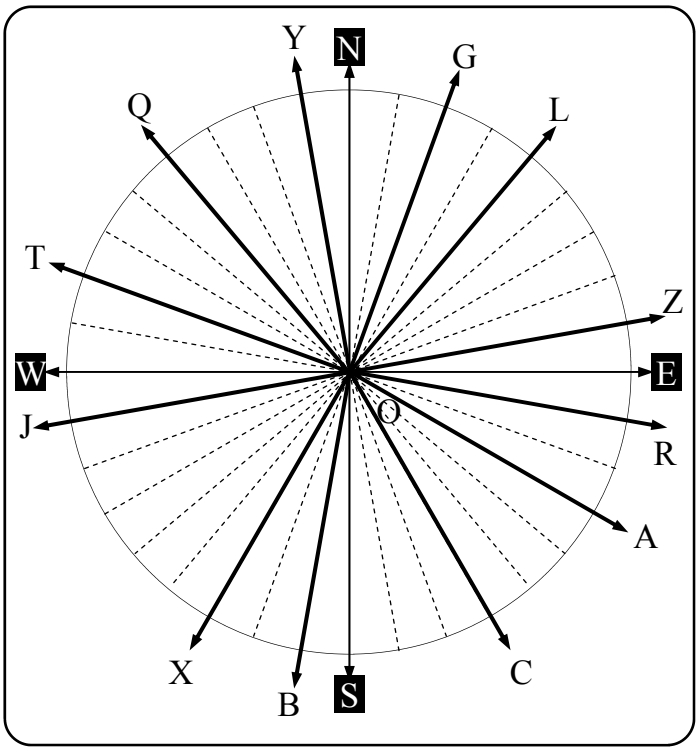
Each division in the circle is 10º. Note that 'O' is in the centre of the circle. Express the bearings in both true bearing and compass bearing format.



1 OC ☐☐☐º T S ☐☐º E

2 OT ☐☐☐º T N ☐☐º W

3 OB

4 OG ☐☐☐º T

5 OZ

6 OJ

7 OQ

8 OX

9 OA

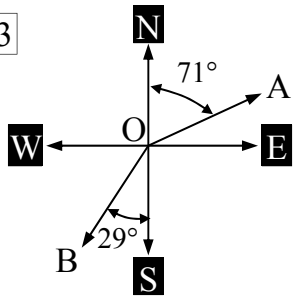
10 OY

11 OL

12 OR

Express these bearings in both true bearing and compass bearing format.

13

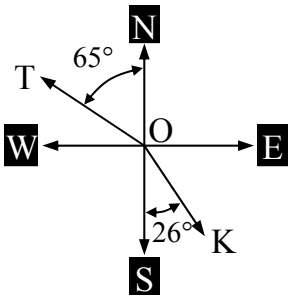


OA ☐☐☐º T N ☐☐º E

OB

14 OT

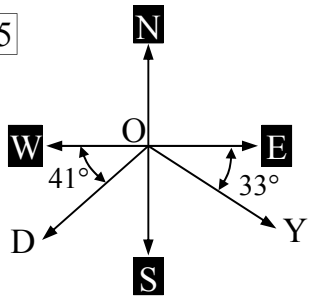
OK



OY

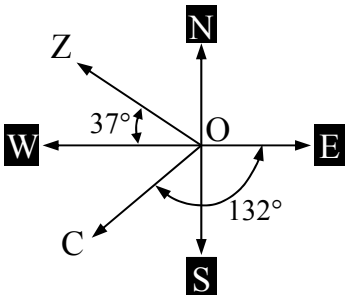
OD

15

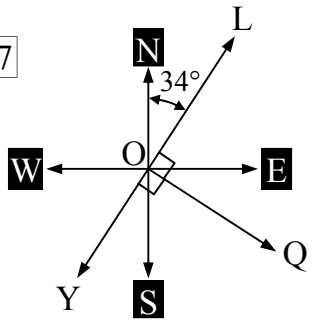


16 OZ

OC



17



OL

OQ

OY

Trigonometry - Bearing Conversion (Sheet 2)

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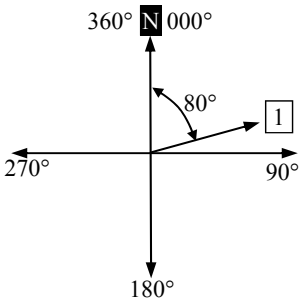
Sketch all the true bearings on the diagram and convert them to compass bearings. Show the angle from North/South on the diagram.

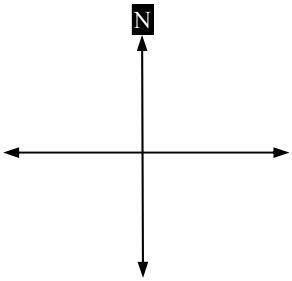
1 080°T N ☐☐° E

2 200°T S ☐☐° W

3 130°T ☐

4 320°T ☐





5 125°T ☐

6 243°T ☐

7 050°T ☐

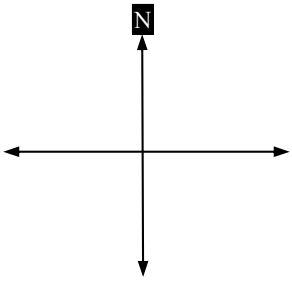
8 294°T ☐

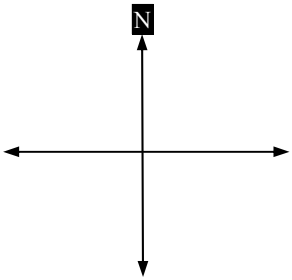
9 305°T ☐

10 193°T ☐

11 110°T ☐

12 044°T ☐





13 255°T ☐

14 072°T ☐

15 330°T ☐

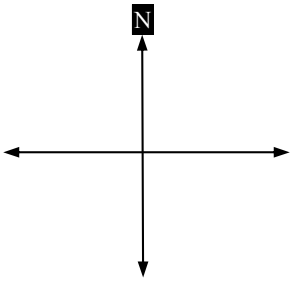
16 165°T ☐

17 110°T ☐

18 010°T ☐

19 310°T ☐

20 210°T ☐



Convert these to true bearings.

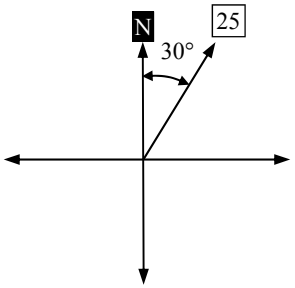
21 North ☐

22 West ☐

23 East ☐

24 South ☐

Convert these to true bearings, show them on the diagram.



25 N 30° E ☐☐☐° T

26 S 67° W ☐

27 S 22° E ☐

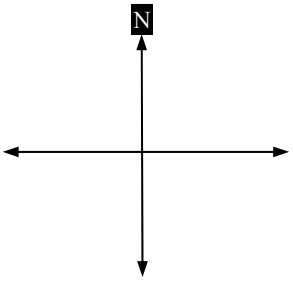
28 N 16° W ☐

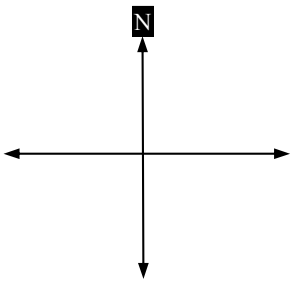
29 N 47° W ☐

30 S 26° W ☐

31 N 60° E ☐

32 S 11° E ☐





33 S 58° W ☐

34 N 23° W ☐

35 N 75° E ☐

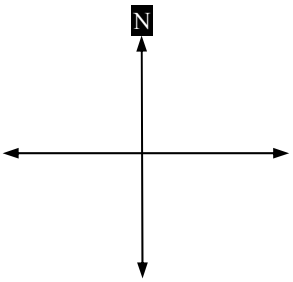
36 S 8° E ☐

37 N 34° W ☐

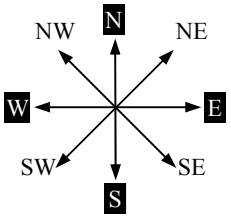
38 S 66° W ☐

39 S 28° E ☐

40 N 55° E ☐



Convert these directions to true bearings.



41 North East ☐

42 South West ☐

43 North West ☐

44 South East ☐

Trigonometry - Bearing Diagrams (Sheet 1)

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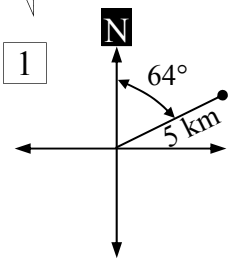
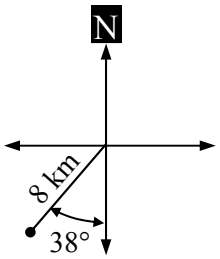
Select which bearing is shown in the diagram.

1

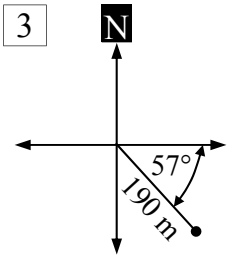


(A) 5 km at N 64° W
(B) 5 km at S 64° E
(C) 5 km at N 64° S
(D) 5 km at N 64° E

2 (A) 8 km at 038°T
(B) 8 km at 218°T
(C) 8 km at 232°T
(D) 8 km at 142°T

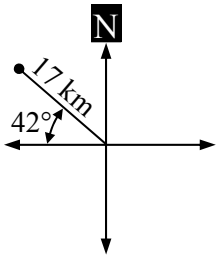


3



(A) 190 m at 147°T
(B) 190 m at E 57° S
(C) 190 m at 057°T
(D) 190 m at S 57° E

4 (A) 17 km at N 48° W
(B) 17 km at 318°T
(C) 17 km at 222°T
(D) 17 km at N 42° W



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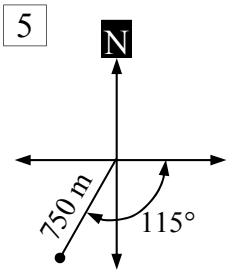
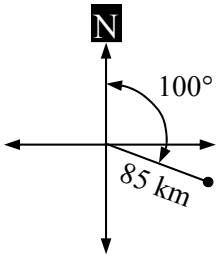


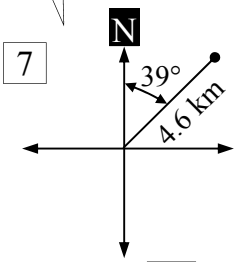
(A) 750 m at 195°T
(B) 750 m at 115°T
(C) 750 m at 205°T
(D) 750 m at S 15° W

6 (A) 85 km at 010°T
(B) 85 km at N 100° E
(C) 85 km at 080°T
(D) 85 km at S 80° E



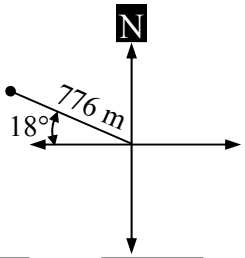
W 20° It is time to fill in the gaps, you may need numbers, you may need letters.

7



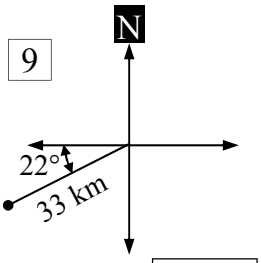
4.6 km at ☐ 39° E

8



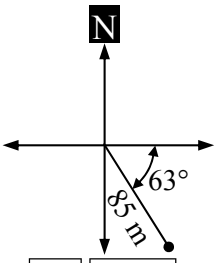
☐ m at ☐ °T

9



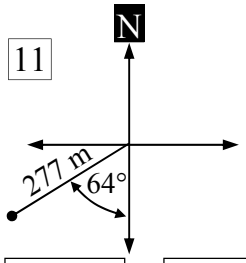
33 km at ☐ °T

10



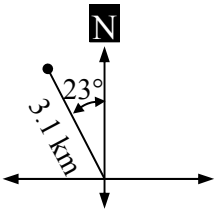
85 m at ☐ ☐ °E

11



☐ at ☐ ° T

12

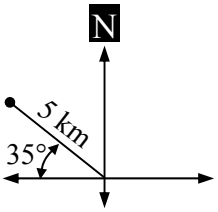


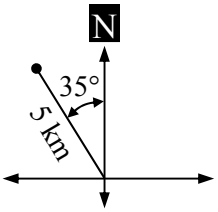
☐ at ☐ °T

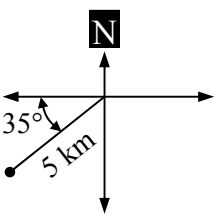
Connect the written description of the bearing to the correct bearing diagram. You will use only three descriptions.

13

5 km at 335° T ●
5 km at S 55° W ●
5 km at N 55° E ●
5 km at 305° T ●
5 km at N 55° W ●
5 km at 325° T ●







Trigonometry - Bearing Diagrams (Sheet 2)

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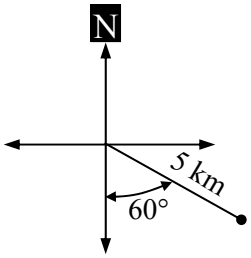
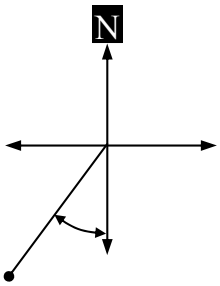
Sketch the bearing and then label the distance and show the angle made with North or South.

Example

John sailed 5 km at a bearing of 120°T.



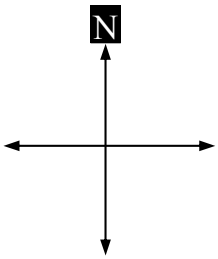
1



Toni walked for 9 km at a bearing of 210°T.

2

Ben rode for 15 km at a bearing of 320°T.



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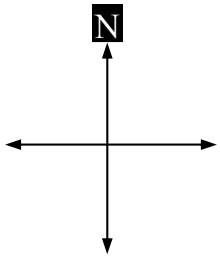
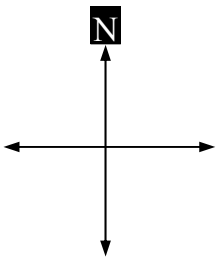


Rohit drove for 46 km at a bearing of 055°T.

4

Keira hiked for 7 km at a bearing of S 30° W.



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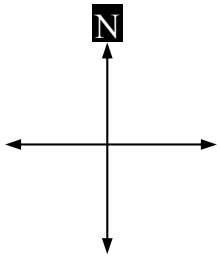
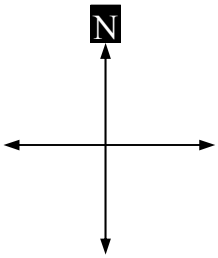


Mike sailed for 25 km at a bearing of S 45° E.

6

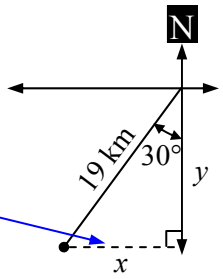
Duncan flew for 300 km at a bearing of N 70° W.



Sketch the bearing, label the distance, show the angle made with North or South. Then label the vertical (y) and horizontal (x) distance travelled.

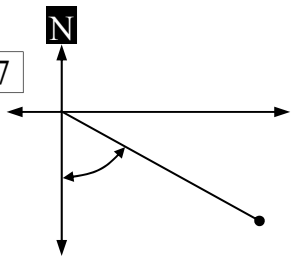
Example

Jade trekked 19 km at a bearing of 210°T.



Draw a dashed line for the x measurement

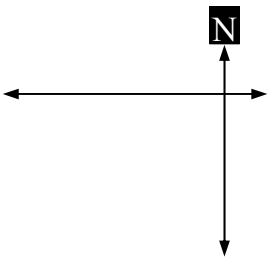
7



Ike walked for 11 km at a bearing of 125°T.

8

Nidal swam for 2.6 km at a bearing of 250°T.



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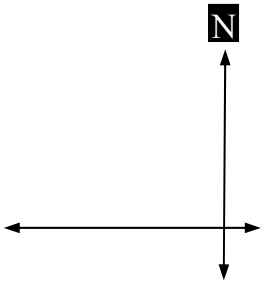
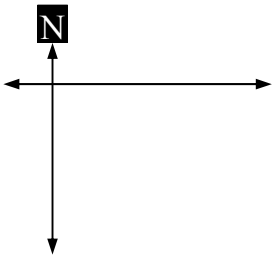


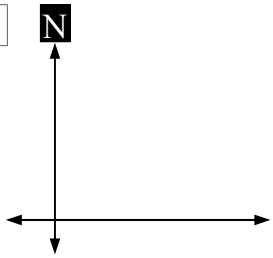
Jiao trekked for 8 km at a bearing of N 25°W.

10



Blake rode for 11 km at a bearing of S 50° E.

11



Aweyes sailed for 25 km at a bearing of N 35° E.

Trigonometry - Bearings and Trigonometry Applications (Sheet 1)

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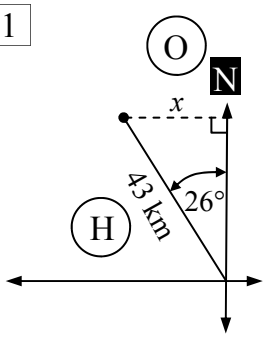
TAN

Label the sides O, A or H and calculate the unknown length to 2 decimal places. Tan free zone.

SOH CAH ~~TOA~~

1



$\sin \square = \frac{\square}{\square}$ O / H

$\times 43$ $\times 43$

$x = \square \sin \square$

$x = \square$ km 2 d.p.

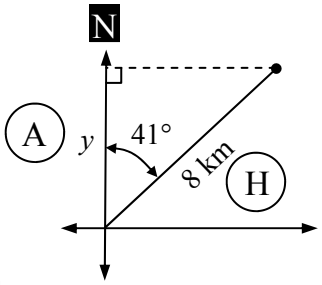
2

$\cos \square = \frac{\square}{\square}$ A / H

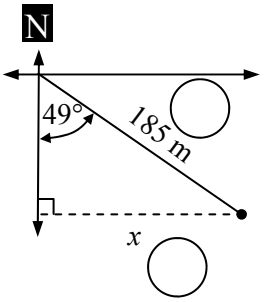
$\times 8$ $\times 8$

$y = \square \cos \square$

$y = \square$ km 2 d.p.



3



$\square = \frac{\square}{\square}$

$\times$ $\times$

$x = \square \sin \square$

$x = \square$ m

2 d.p.

4

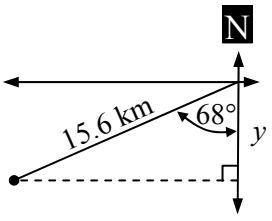
$\square = \frac{\square}{\square}$

$\times$ $\times$

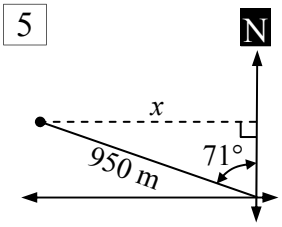
$y =$

$y =$

2 d.p.



5



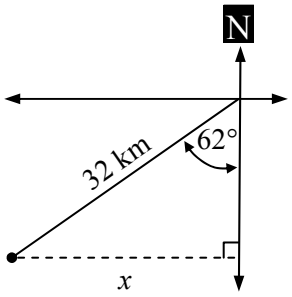
$\square = \frac{\square}{\square}$

$\times$ $\times$

2 d.p.

These questions are the same except they have a worded explanation of the bearing as well. Find the unknown distance to 1 decimal place.

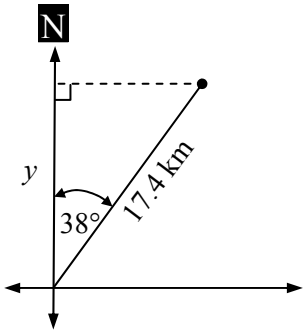
6 A ferry travels 32 km at a bearing of 242ºT. Calculate how far East of its departure point (x on the diagram) the ferry has travelled.



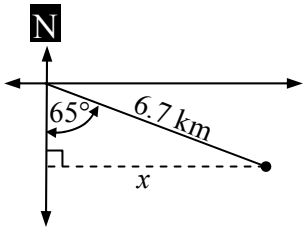
$\times$ $\times$

1 d.p.

7 Murali is trekking for 17.4 km at a bearing of N 38º E. Calculate how far North of his starting point, y on the diagram, he has travelled.



8 Briony is competing in a 6.7 km cross country run at a bearing of 115ºT. Calculate how far East (x) of her starting point she is, when she finishes the run.



Trigonometry - Bearings and Trigonometry Applications (Sheet 2)

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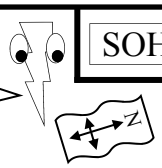
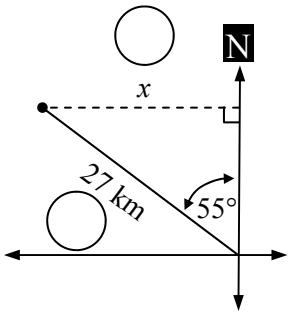
Get ready to construct your own diagrams! Calculate the unknown distance to 1 decimal place.



SOH CAH ~~TOA~~

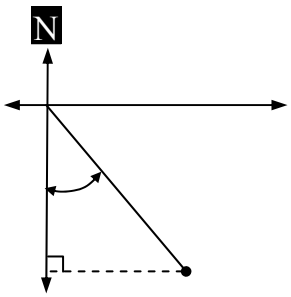
OK you are on your own. Calculate the unknown distance to 1 decimal place.

1 Josh is hiking for 27 km at a bearing of N 55º W. Calculate how far West of his original position, *x* on the diagram, he will be when he finishes the hike.

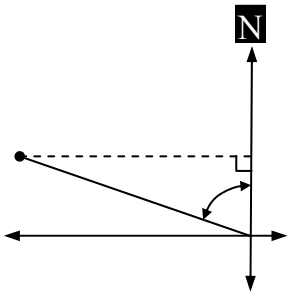


1 d.p.

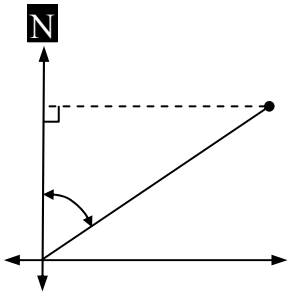
2 Lucy is sailing 33 km at a bearing of 142º T. Add the information to the diagram and calculate how far South of her original position Lucy will be.



3 Darius is bush-walking for 4.2 km at a bearing of 285ºT to a campsite. How far North of his starting point is the campsite?



4 Thien is driving 10.7 km at a bearing of N 41º E. How far East of his starting position will he be when he arrives?



5 Isla is jogging 3.4 km at a bearing of 022ºT. Calculate how far North of her starting point she will be when she finishes.

6 Azmi is trekking at a bearing of S 36º W for 17 km. Calculate how far West Azmi will be from his starting point when he completes the trek.

7 Jodi is looking at a lighthouse which is 5 km away at a bearing of N 58º W. How far North of Jodi is the light-house?

8 Amy is flying at a bearing of 142ºT for 650 km. How far East of her departure point will she be when she lands?

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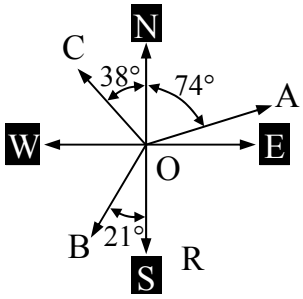
Express the bearings shown as true bearings

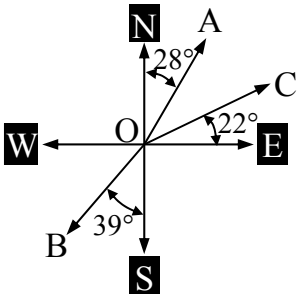


1 OA: _____

2 OB: _____

3 OC: _____

Express the bearings shown as compass bearings



4 OA: _____

5 OB: _____

6 OC: _____

Convert these compass bearings to true bearings

7 N 55° E _____ 8 WEST _____

9 S 13° E _____ 10 N 41° W _____

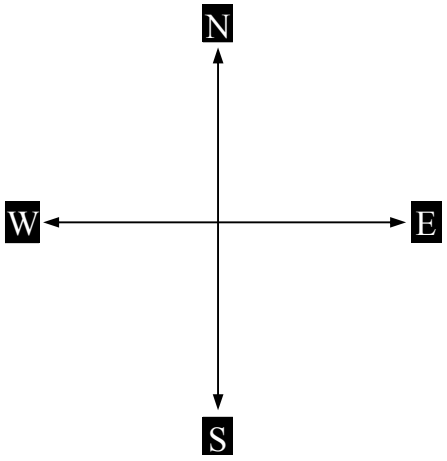
Convert these true bearings to compass bearings

11 058° T _____ 12 267° T _____

13 155° T _____ 14 090° T _____

Given bearings, show them on the diagrams. Include the angle between the North-South line and the bearing, then name each bearing using the letters.

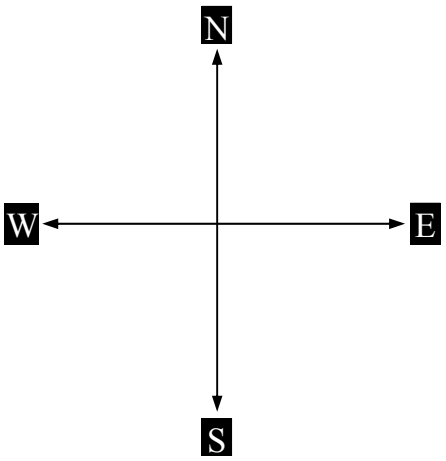
15 OA: 025° T



16 OB: S 45° W

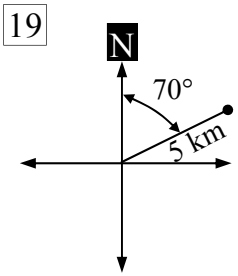
17 OA: 290° T

18 OB: 230° T



Pick which bearing is shown in these diagrams.

19



(A) 5 km at N 70° W

(B) 5 km at S 70° E

(C) 5 km at N 70° S

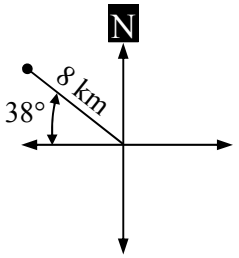
(D) 5 km at N 70° E

20 (A) 8 km at 038°T

(B) 8 km at 308°T

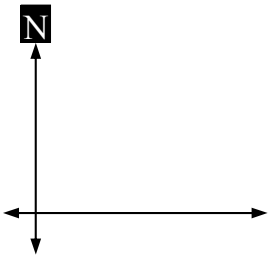
(C) 8 km at 322°T

(D) 8 km at 218°T

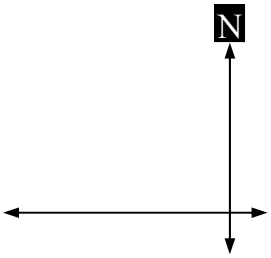


Show these bearings on the diagrams below. Include the distance and the angle made with the North-South line.

21 Jake rode for 11 km at a bearing of 040°T.



22 Joanna walked for 4.8 km at a bearing of N 33° W.



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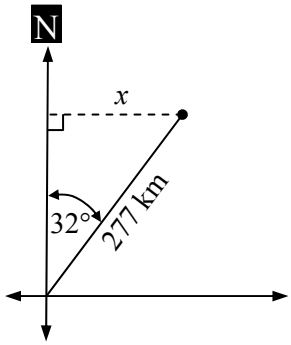
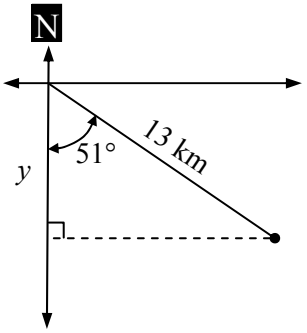
Apply your trigonometry skills to calculate the unknown lengths. Answer to 1 decimal place.

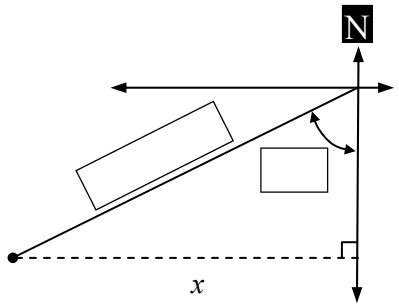
23 A plane travels 277 km at a bearing of 032º T. Calculate how far East of its departure point (x on the diagram) the plane has travelled.



24 Josiah is trekking for 13 km at a bearing of S 51º E. Calculate how far South of his starting point, y on the diagram, he has travelled.



25 Alyssa is bush-walking for 8.2 km at a bearing of 244ºT to a campsite, shown on the diagram below. Write the distance on the bearing and the angle between South and the bearing, in the boxes.



26 Calculate the distance 'x', which is the distance her finishing point is West of her starting point.

27 Brody is sailing at a bearing of 305º T for a distance of 26 km. Construct a diagram showing all relevant data.

28 Calculate how far North of his starting point he will be when he completes the voyage, to 1 decimal place.

29 Kirby is riding at a bearing of S 75º W for a distance of 9.4 km. Construct a diagram showing all relevant data.

30 Calculate how far West of her starting point she will be when she completes riding, to 1 decimal place.

VELOCITY

NON-LINEAR RELATIONSHIPS

Non-Linear Relationships



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Non-Linear Relationships



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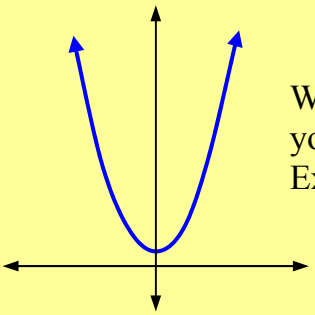
Non-Linear Relationships - Properties of Parabolas

A parabola is a curved line which is like 'U' (or upside-down 'U') shape. In this course the parabola is symmetrical across the *y*-axis, so it is the same either side of the *y*-axis. The equation for a parabola is:

$$y = ax^2 + c$$

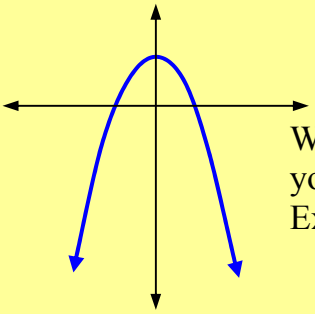
Where: a and c are rational numbers

Positive *a* value



When a is positive the parabola is concave up, with these you have a minimum value which will be on the *y*-axis.
Examples: $y = x^2$, $y = 2x^2$, $y = x^2 + 5$, $y = x^2 - 3$, $y = \frac{1}{2}x^2 + 2$.

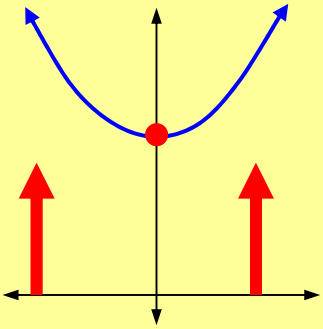
Negative *a* value



When a is negative the parabola is concave down, with these you have a maximum value which will be on the *y*-axis.
Examples: $y = -x^2$, $y = -2x^2$, $y = -x^2 + 5$, $y = -x^2 - 3$, $y = -\frac{1}{2}x^2 + 2$.

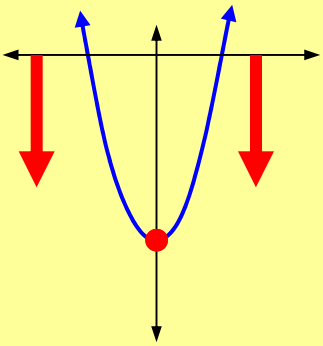
When there is no '*a*' value present '*a*' is 1. So it is positive.

Positive *c* value



When c is positive the parabola cuts the *y*-axis at this value which is ***above*** the *x*-axis.
Examples: $y = x^2 + 5$, $y = -x^2 + 1$, $y = \frac{1}{2}x^2 + 2$.

Negative *c* value

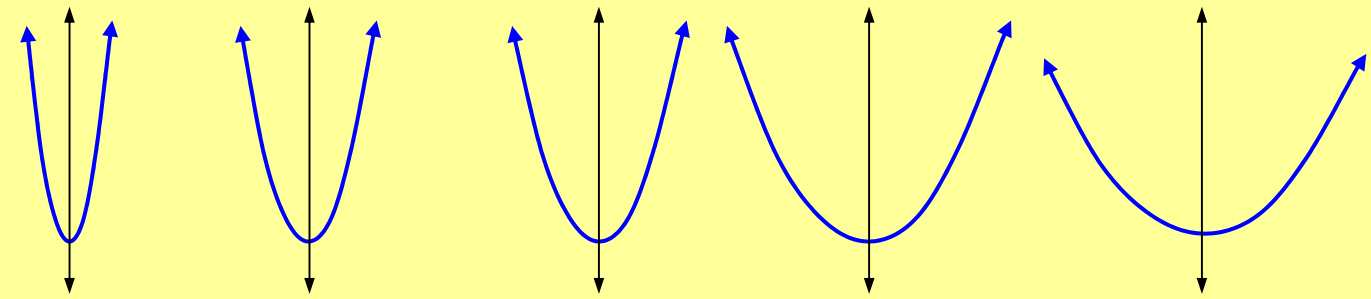


When c is negative the parabola cuts the *y*-axis at this value, which is ***below*** the *x*-axis.
Examples: $y = x^2 - 5$, $y = -x^2 - 1$, $y = \frac{1}{2}x^2 - 2$.

a values

When *a* is larger the parabola is more narrow, or 'hugs' the *y*-axis more closely. Below are sketches, not exact representations

As '*a*' reduces the parabola becomes more wide



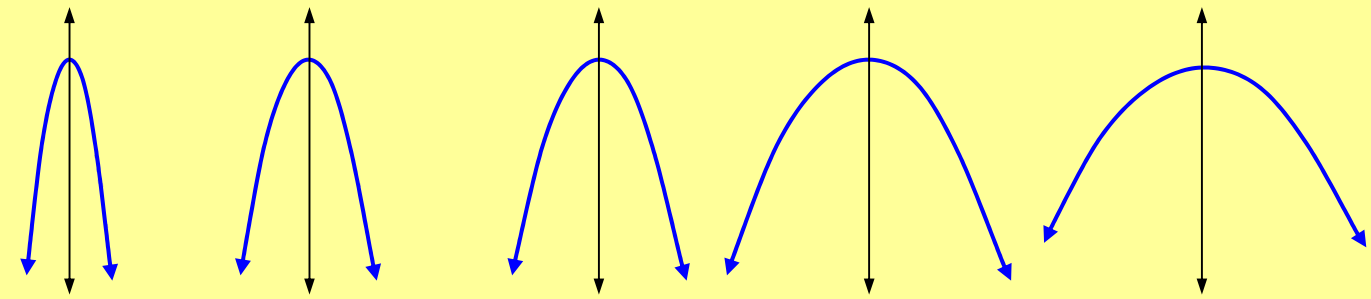
$y = 3x^2$ $y = 2x^2$ $y = x^2$ $y = \frac{1}{2}x^2$ $y = \frac{1}{4}x^2$

($a = 1$)

a values

The same effect occurs with negative values of '*a*'. As the magnitude of the '*a*' values reduce, the parabola flattens.

As the magnitude of '*a*' reduces the parabola becomes more wide



$y = -3x^2$ $y = -2x^2$ $y = -x^2$ $y = -\frac{1}{2}x^2$ $y = -\frac{1}{4}x^2$

($a = -1$)

Non-Linear Relationships - Properties of Parabolas (Sheet 1)

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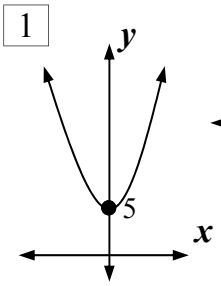
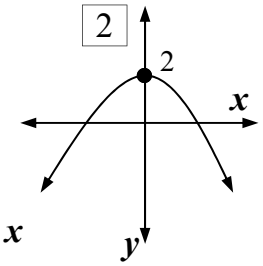
Look at the y-intercept and complete the equations for these parabolas.

1



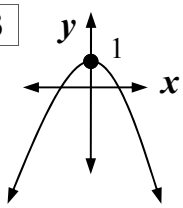
$y = 2x^2 +$ ☐

2



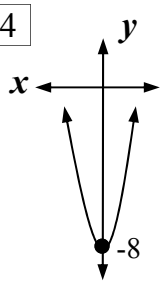
$y = -\frac{1}{2}x^2 +$ ☐

3



$y = -x^2 +$ ☐

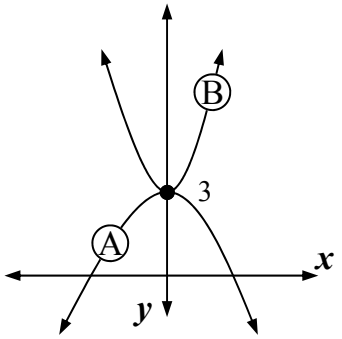
4



$y = 3x^2 -$ ☐

Choose which equation applies to each of the parabolas.

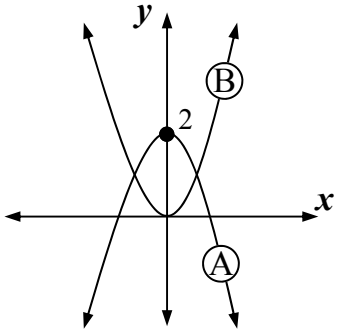
5



$y = x^2 + 3$ Ⓐ Ⓑ

$y = -\frac{1}{2}x^2 + 3$ Ⓐ Ⓑ

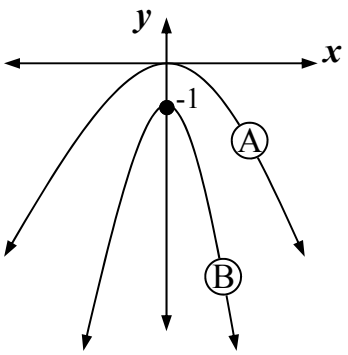
6



$y = x^2$ Ⓐ Ⓑ

$y = -x^2 + 2$ Ⓐ Ⓑ

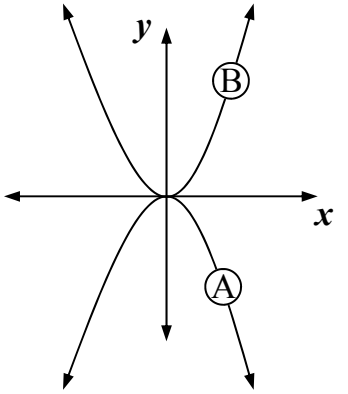
7



$y = -\frac{1}{2}x^2$ Ⓐ Ⓑ

$y = -2x^2 - 1$ Ⓐ Ⓑ

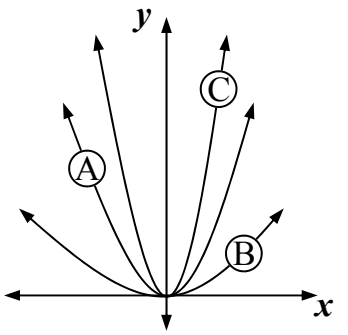
8



$y = -x^2$ Ⓐ Ⓑ

$y = x^2$ Ⓐ Ⓑ

9

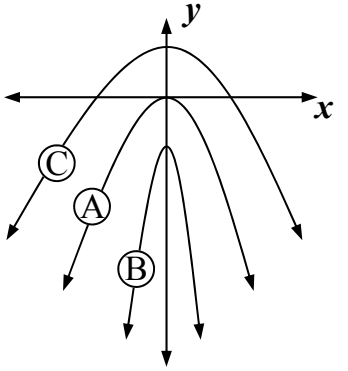


$y = x^2$ ⒶⒷⒸ

$y = 2x^2$ ⒶⒷⒸ

$y = \frac{1}{2}x^2$ ⒶⒷⒸ

10



$y = -\frac{1}{2}x^2 + 3$ ⒶⒷⒸ

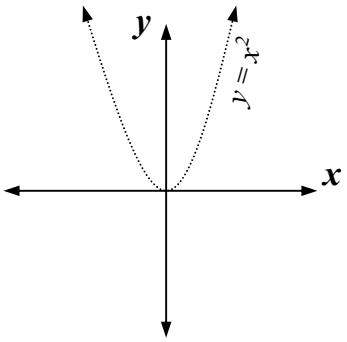
$y = -3x^2 - 3$ ⒶⒷⒸ

$y = -x^2$ ⒶⒷⒸ

Now sketch your own parabolas.

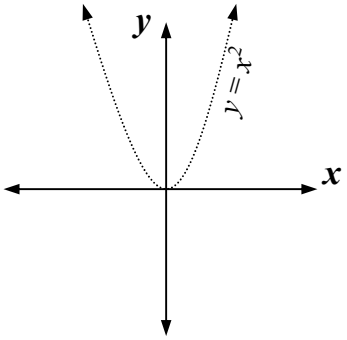
The graph below shows $y = x^2$ as a dotted curve. Sketch $y = -x^2$.

11 Arrowheads, Equation on curve



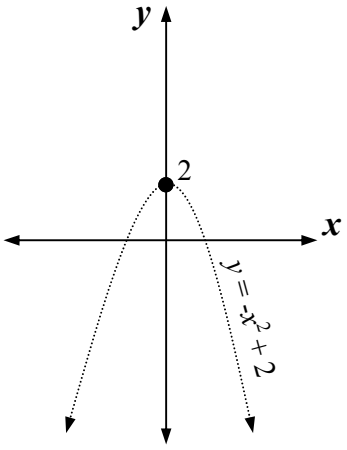
The graph below shows $y = x^2$ as a dotted curve. Sketch $y = x^2 - 3$.

12



The graph below shows $y = -x^2 + 2$ as a dotted curve. Sketch $y = x^2 - 2$.

13



Non-Linear Relationships - Properties of Parabolas (Sheet 2)

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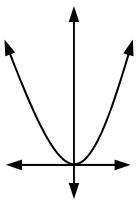
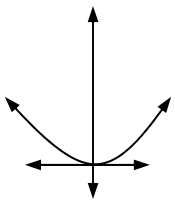
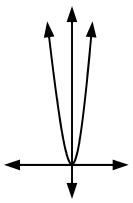
Join the sketch with the equation for each parabola.

1 

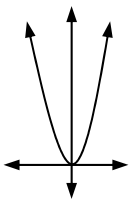
$y = 2x^2$

2 

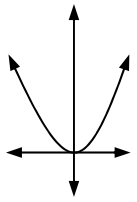
$y = x^2$

3 

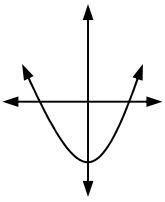
$y = \frac{1}{2}x^2$

4 

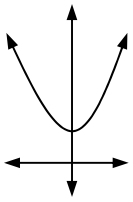
$y = 3x^2$

5 

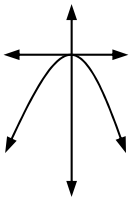
$y = x^2 + 2$

6 

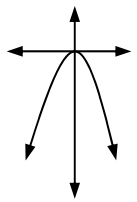
$y = -x^2$

7 

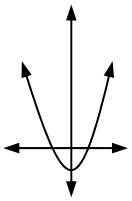
$y = x^2$

8 

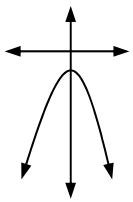
$y = x^2 - 2$

9 

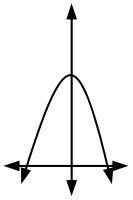
$y = -x^2 - 2$

10 

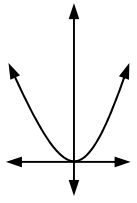
$y = -x^2 + 7$

11 

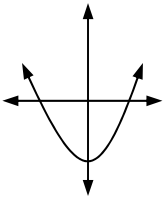
$y = x^2 - 2$

12 

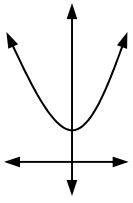
$y = -x^2$

13 

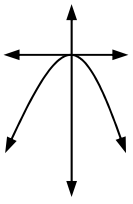
$y = x^2 + 2$

14 

$y = -x^2$

15 

$y = x^2$

16 

$y = x^2 - 2$

Give 2 examples of parabolas that have the following properties.

17 A parabola is concave-up and cuts the y-axis at 4.

$y = x^2 + \square$ $\qquad$ $y = \square x^2 + \square$

18 A parabola is concave-down and cuts the y-axis at the origin.

$y =$ $\qquad$ $y =$

19 A parabola is concave-up and cuts the y-axis at the -3.

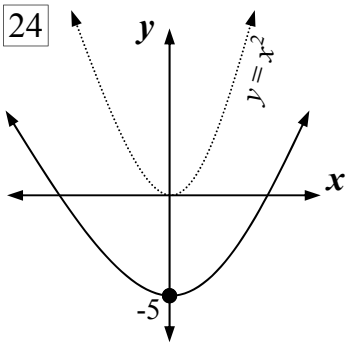
20 A parabola is concave-down and cuts the y-axis at the same intercept as $y = 3x^2 + 5$.

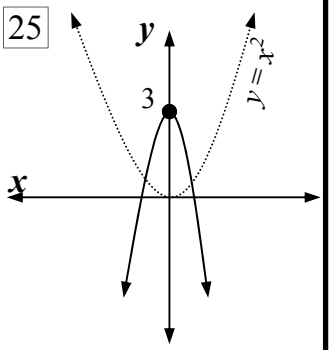
21 A parabola is concave-up and cuts the y-axis at the same intercept as $y = -x^2 - 6$.

22 A parabola is concave-up, cuts the y-axis at the same intercept as $y = -\frac{1}{2}x^2 - 4$ and is more narrow than $y = 2x^2$.

23 A parabola is concave-down, cuts the y-axis at the same intercept as $y = 3x^2 - 6$ and is a more wide than $y = -2x^2$.

Give a possible name for the solid-line unnamed parabolas shown below

24 

25 

Non-Linear Relationships - Properties of Parabolas (Sheet 1)

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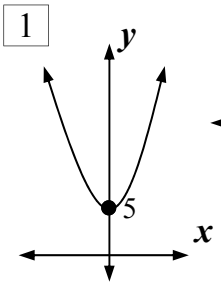
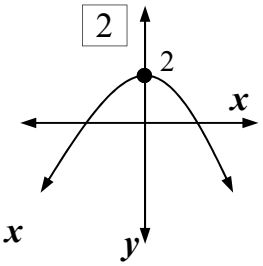
Look at the y-intercept and complete the equations for these parabolas.

1



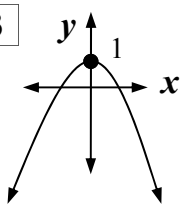
$y = 2x^2 +$ 5

2



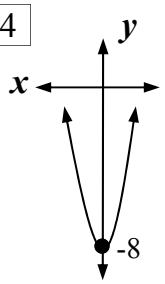
$y = -\frac{1}{2}x^2 +$ 2

3



$y = -x^2 +$ 1

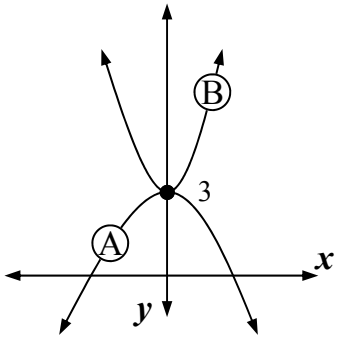
4



$y = 3x^2 -$ 8

Choose which equation applies to each of the parabolas.

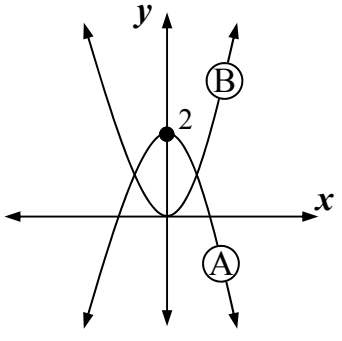
5



$y = x^2 + 3$ $y = -\frac{1}{2}x^2 + 3$

Ⓐ Ⓑ Ⓐ Ⓑ

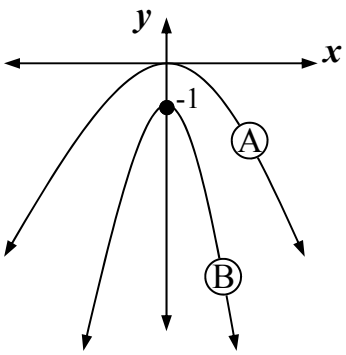
6



$y = x^2$ $y = -x^2 + 2$

Ⓐ Ⓑ Ⓐ Ⓑ

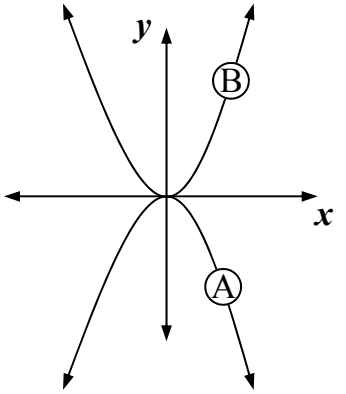
7



$y = -\frac{1}{2}x^2$ $y = -2x^2 - 1$

Ⓐ Ⓑ Ⓐ Ⓑ

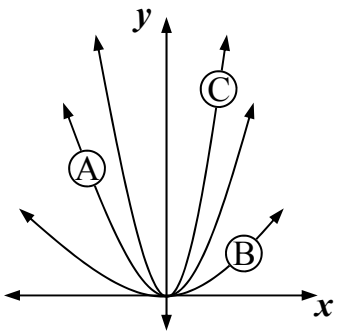
8



$y = -x^2$ $y = x^2$

Ⓐ Ⓑ Ⓐ Ⓑ

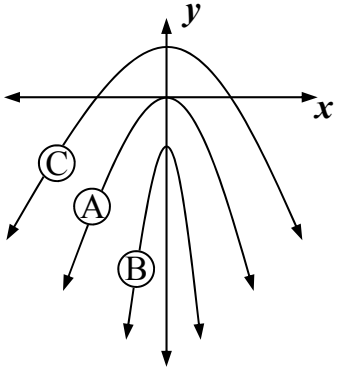
9



$y = x^2$ $y = 2x^2$ $y = \frac{1}{2}x^2$

Ⓐ Ⓑ Ⓒ Ⓐ Ⓑ Ⓒ Ⓐ Ⓑ Ⓒ

10



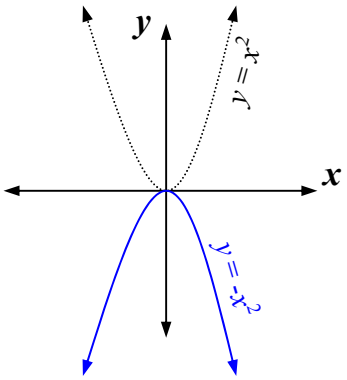
$y = -\frac{1}{2}x^2 + 3$ $y = -3x^2 - 3$ $y = -x^2$

Ⓐ Ⓑ Ⓒ Ⓐ Ⓑ Ⓒ Ⓐ Ⓑ Ⓒ

Now sketch your own parabolas.

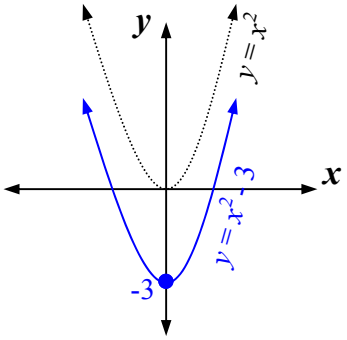
The graph below shows $y = x^2$ as a dotted curve. Sketch $y = -x^2$.

11 Arrowheads, Equation on curve



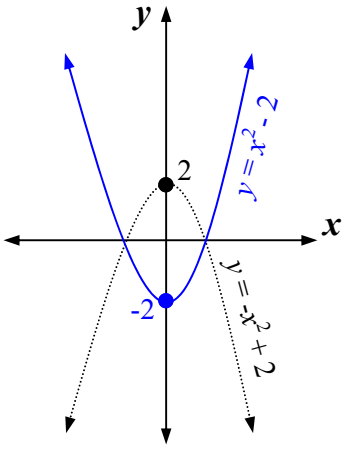
The graph below shows $y = x^2$ as a dotted curve. Sketch $y = x^2 - 3$.

12



The graph below shows $y = -x^2 + 2$ as a dotted curve. Sketch $y = x^2 - 2$.

13



Non-Linear Relationships - Properties of Parabolas (Sheet 2)

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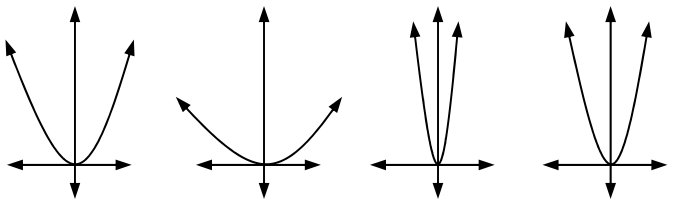
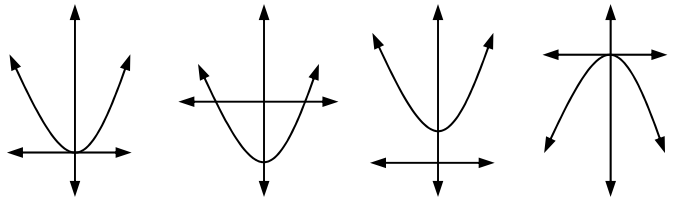
Join the sketch with the equation for each parabola.

1 2 3 4



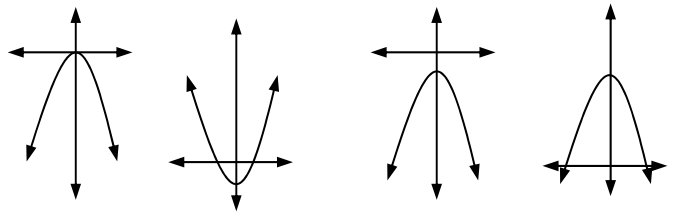
$y = 2x^2$ $y = x^2$ $y = \frac{1}{2}x^2$ $y = 3x^2$

5 6 7 8



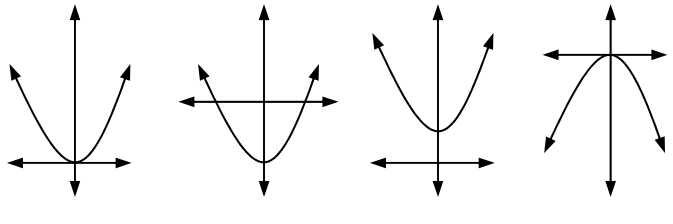
$y = x^2 + 2$ $y = -x^2$ $y = x^2$ $y = x^2 - 2$

9 10 11 12



$y = -x^2 - 2$ $y = -x^2 + 7$ $y = x^2 - 2$ $y = -x^2$

13 14 15 16



$y = x^2 + 2$ $y = -x^2$ $y = x^2$ $y = x^2 - 2$

Give 2 examples of parabolas that have the following properties.

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$y = x^2 - 3$ $y = 2x^2 - 3$

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22 A parabola is concave-up, cuts the y-axis at the same intercept as $y = -\frac{1}{2}x^2 - 4$ and is more narrow than $y = 2x^2$.

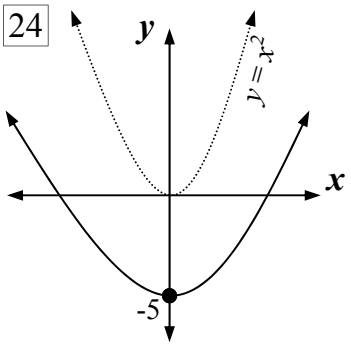
$y = 3x^2 - 4$ $y = 4x^2 - 4$

23 A parabola is concave-down, cuts the y-axis at the same intercept as $y = 3x^2 - 6$ and is a more wide than $y = -2x^2$.

$y = -x^2 - 6$ $y = -\frac{1}{2}x^2 - 6$

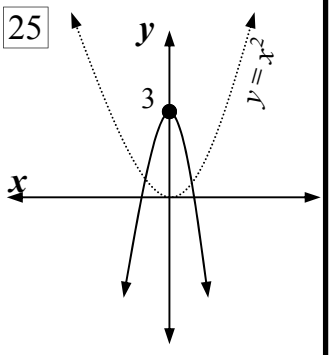
Give a possible name for the solid-line unnamed parabolas shown below

24



$y = \frac{1}{2}x^2 - 5$

25



$y = -2x^2 + 3$

Non-Linear Relationships - Properties of Parabolas (Homework Strip)

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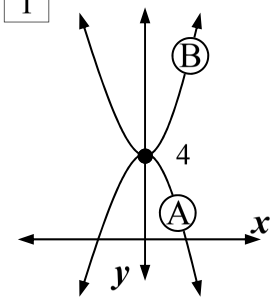
Date Due: _____

Name the parabolas by filling in the letter that matches the sketch.

1



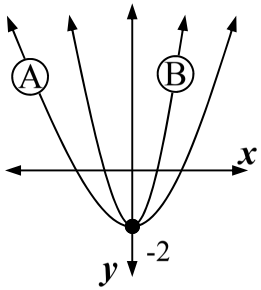
$y = x^2 + 4$ Ⓐ Ⓑ

$y = -x^2 + 4$ Ⓐ Ⓑ

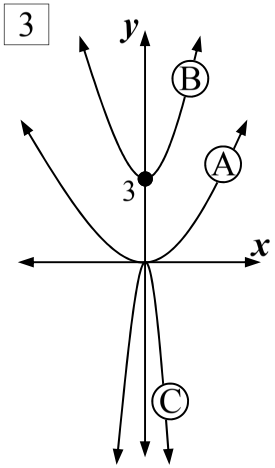
2

$y = x^2 - 2$ Ⓐ Ⓑ

$y = 2x^2 - 2$ Ⓐ Ⓑ



3



$y = \frac{1}{3}x^2$ Ⓐ Ⓑ Ⓒ

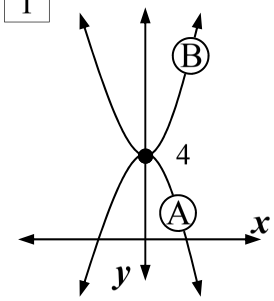
$y = -3x^2$ Ⓐ Ⓑ Ⓒ

$y = x^2 + 3$ Ⓐ Ⓑ Ⓒ

Date Due: _____

Name the parabolas by filling in the letter that matches the sketch.

1



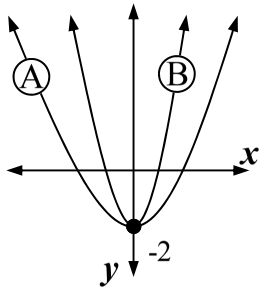
$y = x^2 + 4$ Ⓐ Ⓑ

$y = -x^2 + 4$ Ⓐ Ⓑ

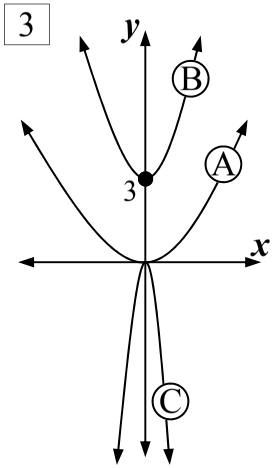
2

$y = x^2 - 2$ Ⓐ Ⓑ

$y = 2x^2 - 2$ Ⓐ Ⓑ



3



$y = \frac{1}{3}x^2$ Ⓐ Ⓑ Ⓒ

$y = -3x^2$ Ⓐ Ⓑ Ⓒ

$y = x^2 + 3$ Ⓐ Ⓑ Ⓒ

Date Due: _____

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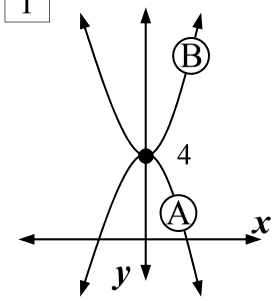
1



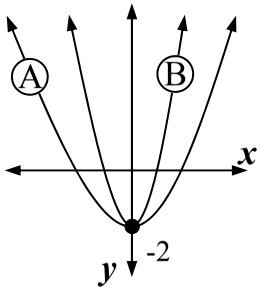
$y = x^2 + 4$ Ⓐ Ⓑ

$y = -x^2 + 4$ Ⓐ Ⓑ

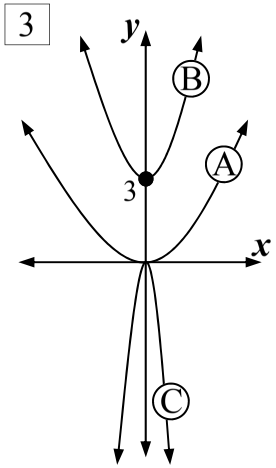
2

$y = x^2 - 2$ Ⓐ Ⓑ

$y = 2x^2 - 2$ Ⓐ Ⓑ



3



$y = \frac{1}{3}x^2$ Ⓐ Ⓑ Ⓒ

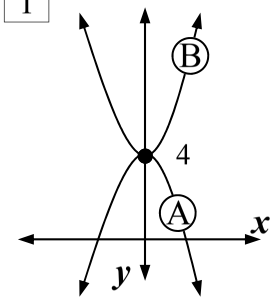
$y = -3x^2$ Ⓐ Ⓑ Ⓒ

$y = x^2 + 3$ Ⓐ Ⓑ Ⓒ

Date Due: _____

Name the parabolas by filling in the letter that matches the sketch.

1



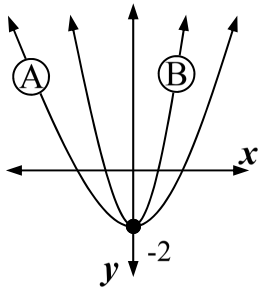
$y = x^2 + 4$ Ⓐ Ⓑ

$y = -x^2 + 4$ Ⓐ Ⓑ

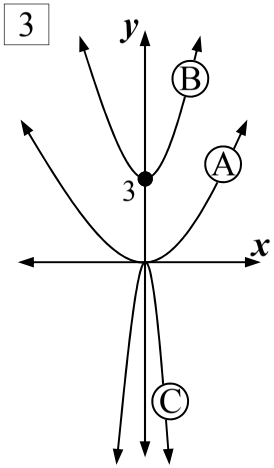
2

$y = x^2 - 2$ Ⓐ Ⓑ

$y = 2x^2 - 2$ Ⓐ Ⓑ



3



$y = \frac{1}{3}x^2$ Ⓐ Ⓑ Ⓒ

$y = -3x^2$ Ⓐ Ⓑ Ⓒ

$y = x^2 + 3$ Ⓐ Ⓑ Ⓒ

Non-Linear Relationships - Properties of Parabolas (Homework Strip)

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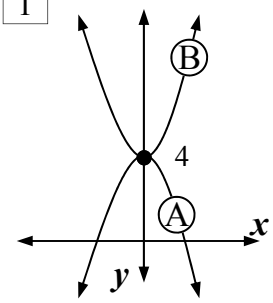
Date Due:

Name the parabolas by filling in the letter that matches the sketch.

1



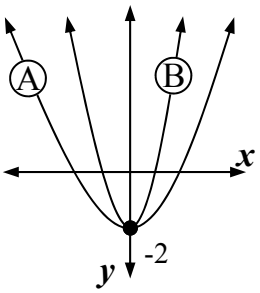
$y = x^2 + 4$ (A) (B)

$y = -x^2 + 4$ (A) (B)

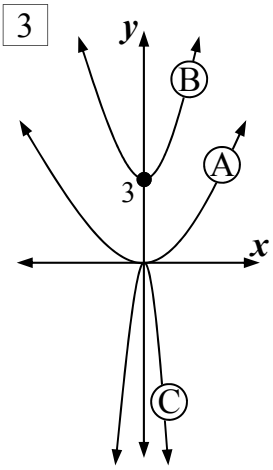
2

$y = x^2 - 2$ (A) (B)

$y = 2x^2 - 2$ (A) (B)



3



$y = \frac{1}{3}x^2$ (A) (B) (C)

$y = -3x^2$ (A) (B) (C)

$y = x^2 + 3$ (A) (B) (C)

Non-Linear Relationships - Plotting Points & Drawing Parabolas

This sheet gives you the points to plot, which when you plot them, creates parabolas. Parabolas are symmetrical curves, which for this course, will be symmetric across the *y*-axis.

Given points to plot, plot them and draw the parabola through the points. Once the curve is drawn, place arrowheads on its ends, then write the equation somewhere on the parabola.

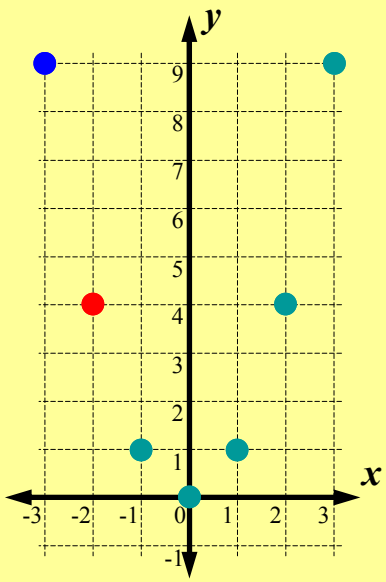
STEP 1 Remember that *x* moves across and *y* moves up and down. Plot the points on the graph.

$y = x^2$

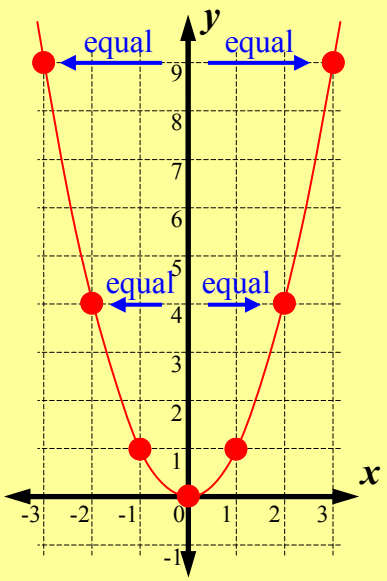
x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

Across to the left -3, then up 9

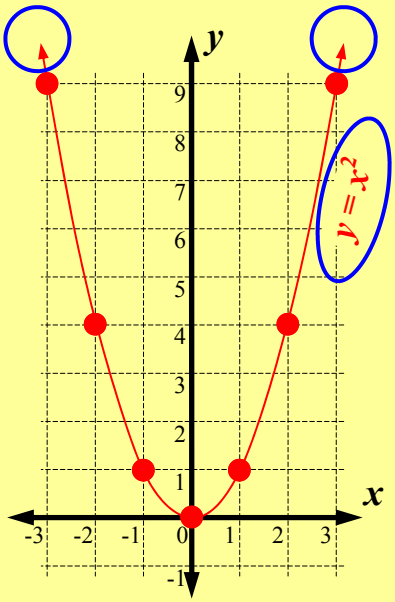
Across to the left -2, then up 4



STEP 2 Draw the curve. Note that the points are symmetrical, check for this before you draw curve.



STEP 3 Add arrowheads and the equation of the parabola. Done.



Non-Linear R'ships - Plotting Points & Drawing Parabolas (Sheet 1)

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Plot the points for each parabola then draw a curve through them. Show arrow heads and write the equation of the parabola on the curve.



1 $y = x^2 + 3$ Plot these points:

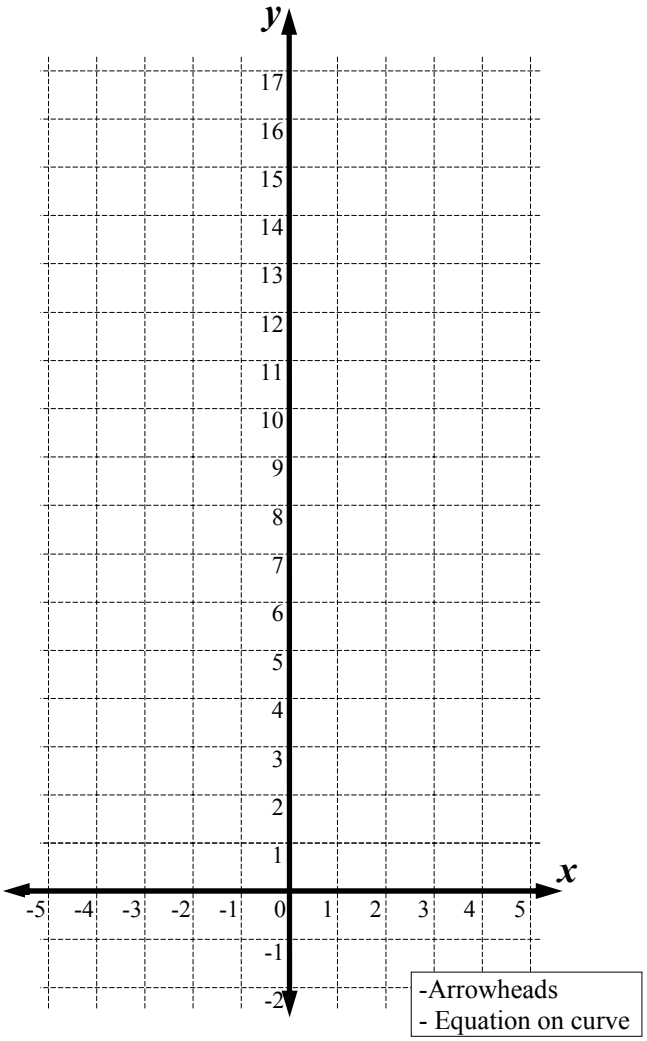
(-3, 12), (0, 3), (2, 7), (-1, 4), (-2, 7), (1, 4), (3, 12).

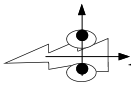
2 $y = x^2$ Plot these points:

(-1, 1), (4, 16), (2, 4), (-3, 9), (-4, 16), (3, 9), (0, 0), (1, 1), (-2, 4).

3 $y = x^2 - 2$ Plot these points:

(2, 2), (4, 14), (3, 7), (-4, 14), (-1, -1), (3, 9), (-2, 2), (1, -1), (-3, 7).





Now use the table of values to plot these parabolas. Go across 'x' and up 'y'.

4 $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18

5 $y = x^2$

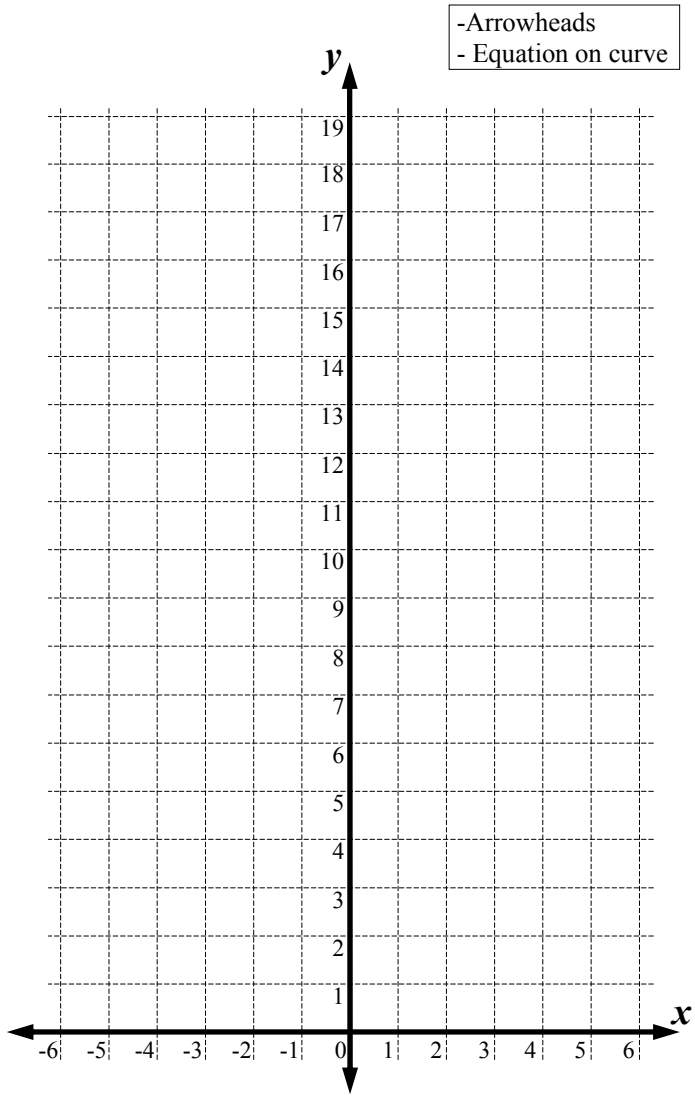
x	-4	-3	-2	1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

6 $y = \frac{1}{2}x^2$

x	-6	-4	-2	-1	0	1	2	4	6
y	18	8	2	½	0	½	2	8	18

7 $y = \frac{1}{4}x^2$

x	-6	-4	-2	0	2	4	6
y	9	4	1	0	1	4	9



8 What effect does the number before the x^2 term have on the parabola?

- ○ A larger coefficient has a more narrow curve.
- ○ A smaller coefficient has a narrower curve.

Non-Linear R'ships - Plotting Points & Drawing Parabolas (Sheet 2)

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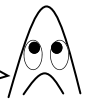
Plot the points for each parabola then draw a curve through them. Show arrow heads and write the equation of the parabola on the curve.



1 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

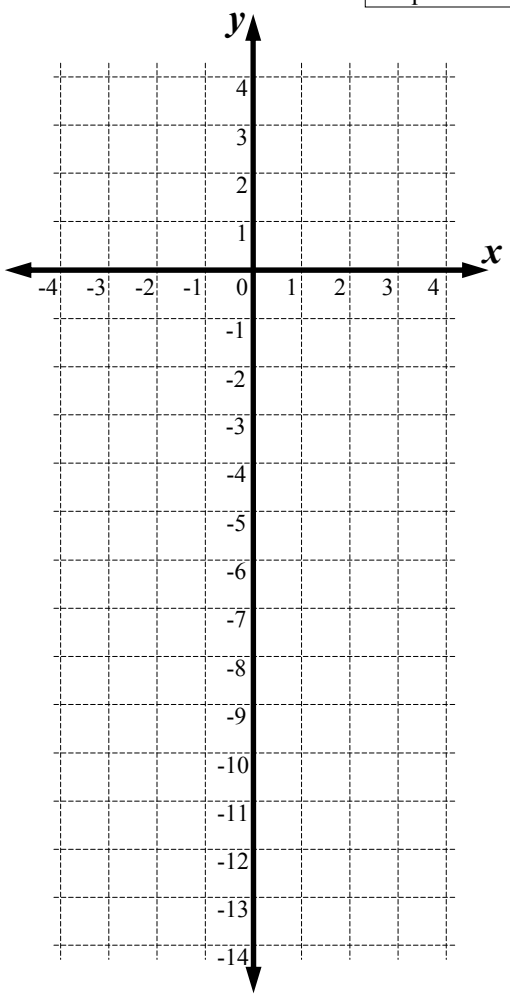
2 $y = -x^2 + 4$

x	-4	-3	-2	1	0	1	2	3	4
y	-12	-5	0	3	4	3	0	-5	-12

3 $y = -x^2 - 3$

x	-3	-2	-1	0	1	2	3
y	-12	-7	-4	-3	-4	-7	-12

-Arrowheads
- Equation on curve

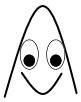


4 What effect does the number after the x^2 term have on the parabola? Choose one …..

○ It shifts the parabola up or down the y-axis.

○ It determines where the parabola cuts the y-axis.

○ Both of the above.



Keep going. This time you have both positive and negative coefficients.

5 $y = -x^2 + 9$

x	-4	-3	-2	1	0	1	2	3	4
y	-7	0	5	8	9	8	5	0	-7

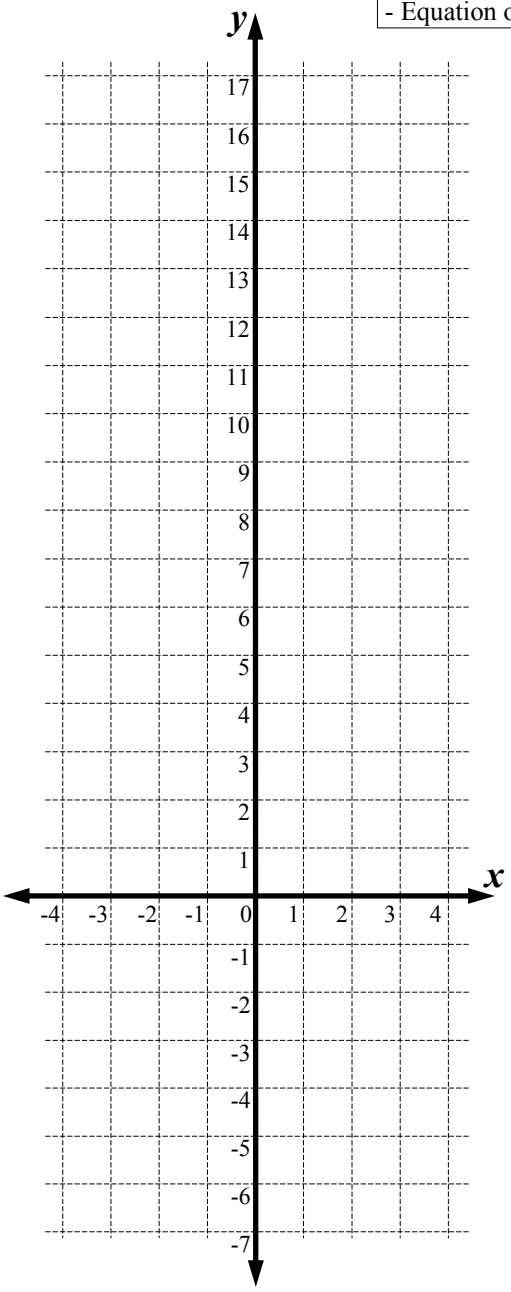
6 $y = x^2 - 6$

x	-4	-3	-2	1	0	1	2	3	4
y	10	3	-2	-5	-6	-5	-2	3	10

7 $y = 2x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14

-Arrowheads
- Equation on curve



Non-Linear R'ships - Plotting Points & Drawing Parabolas (Sheet 1)

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Plot the points for each parabola then draw a curve through them. Show arrow heads and write the equation of the parabola on the curve.

1 $y = x^2 + 3$ Plot these points:

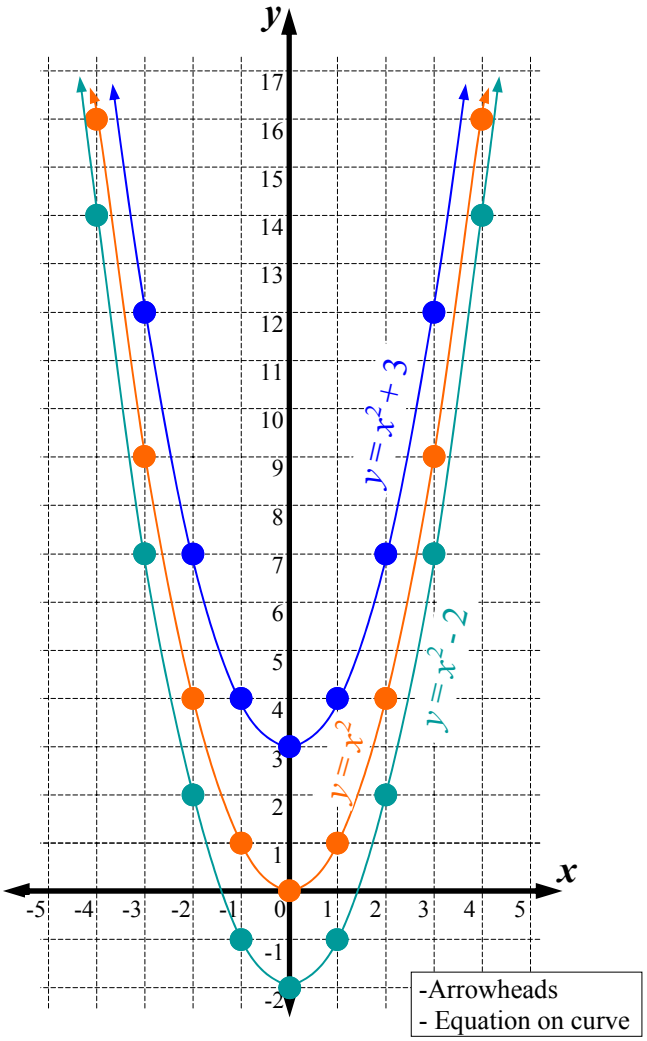
(-3, 12), (0, 3), (2, 7), (-1, 4), (-2, 7), (1, 4), (3, 12).

2 $y = x^2$ Plot these points:

(-1, 1), (4, 16), (2, 4), (-3, 9), (-4, 16), (3, 9), (0, 0), (1, 1), (-2, 4).

3 $y = x^2 - 2$ Plot these points:

(2, 2), (4, 14), (3, 7), (-4, 14), (-1, -1), (3, 9), (-2, 2), (1, -1), (-3, 7).



Now use the table of values to plot these parabolas. Go across 'x' and up 'y'.

4 $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18

5 $y = x^2$

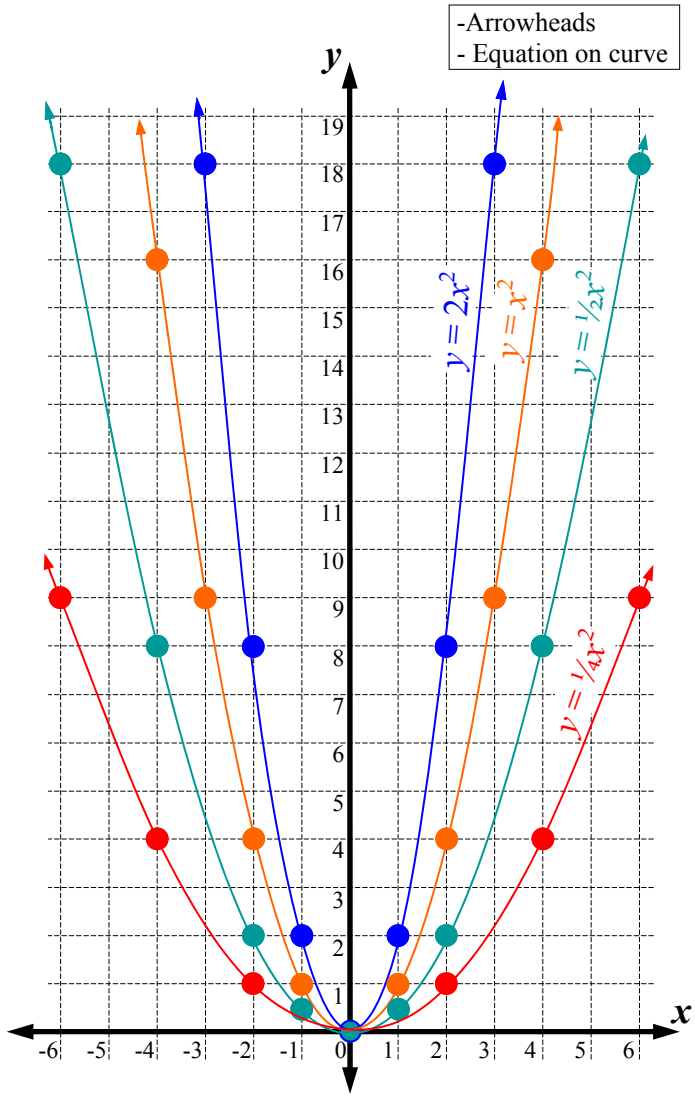
x	-4	-3	-2	1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

6 $y = \frac{1}{2}x^2$

x	-6	-4	-2	-1	0	1	2	4	6
y	18	8	2	½	0	½	2	8	18

7 $y = \frac{1}{4}x^2$

x	-6	-4	-2	0	2	4	6
y	9	4	1	0	1	4	9



8 What effect does the number before the x^2 term have on the parabola?

- ● A larger coefficient has a more narrow curve.
- ○ A smaller coefficient has a narrower curve.

Non-Linear R'ships - Plotting Points & Drawing Parabolas (Sheet 2)

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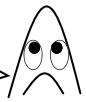
Plot the points for each parabola then draw a curve through them. Show arrow heads and write the equation of the parabola on the curve.



1 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

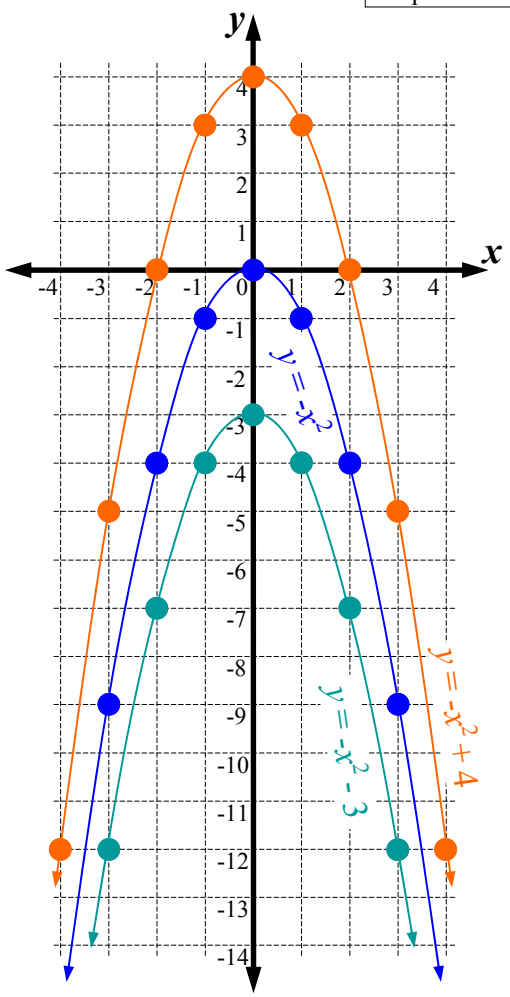
2 $y = -x^2 + 4$

x	-4	-3	-2	1	0	1	2	3	4
y	-12	-5	0	3	4	3	0	-5	-12

3 $y = -x^2 - 3$

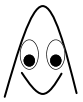
x	-3	-2	-1	0	1	2	3
y	-12	-7	-4	-3	-4	-7	-12

-Arrowheads
- Equation on curve



4 What effect does the number after the x^2 term have on the parabola? Choose one …..

- ○ It shifts the parabola up or down the y-axis.
- ○ It determines where the parabola cuts the y-axis.
- ● Both of the above.



Keep going. This time you have both positive and negative coefficients.

5 $y = -x^2 + 9$

x	-4	-3	-2	1	0	1	2	3	4
y	-7	0	5	8	9	8	5	0	-7

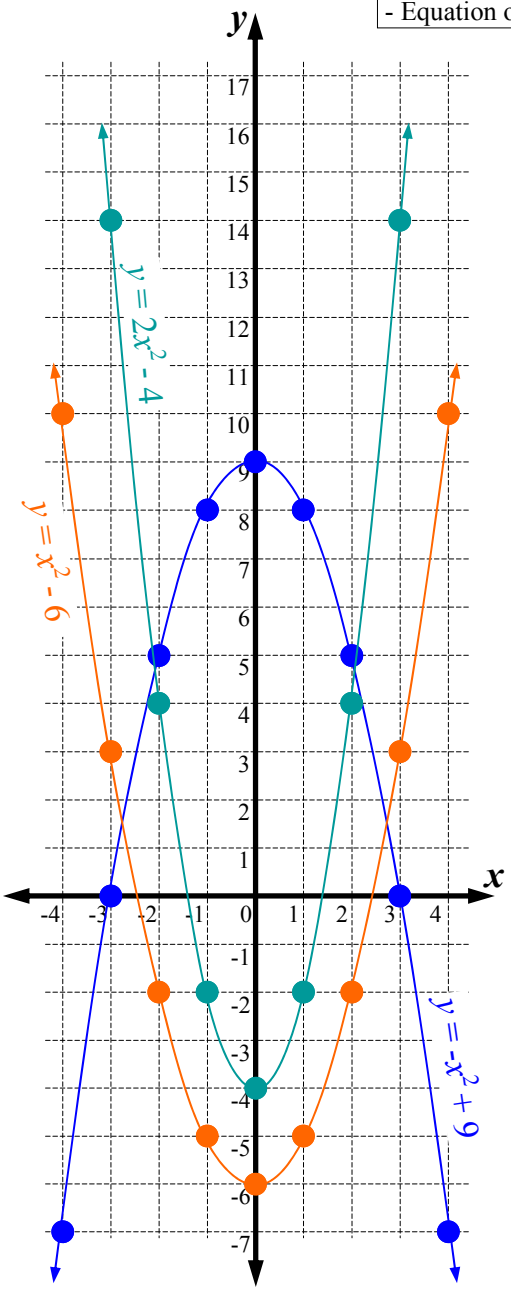
6 $y = x^2 - 6$

x	-4	-3	-2	1	0	1	2	3	4
y	10	3	-2	-5	-6	-5	-2	3	10

7 $y = 2x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14

-Arrowheads
- Equation on curve



Non-Linear Relationships - Plotting Points & Drawing Parabolas (Homework Strip)

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Date Due:

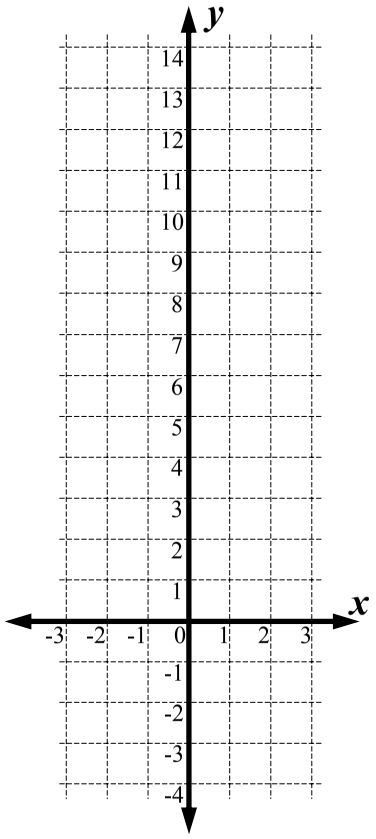
Plot the points in the table of values and create the two parabolas.

1 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12

2 $y = 2x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14



Date Due:

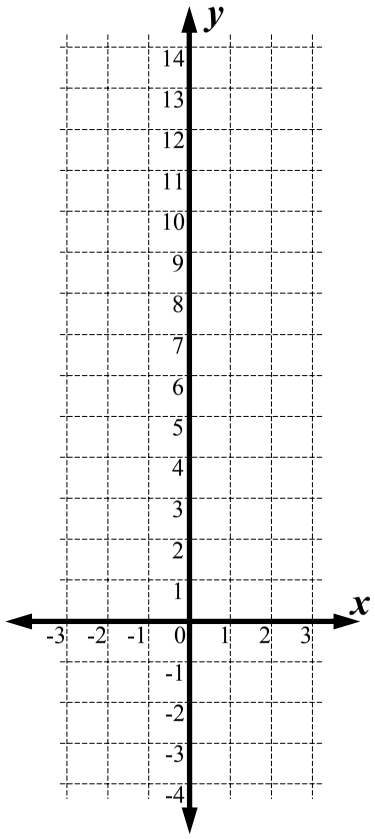
Plot the points in the table of values and create the two parabolas.

1 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12

2 $y = 2x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14



Date Due:

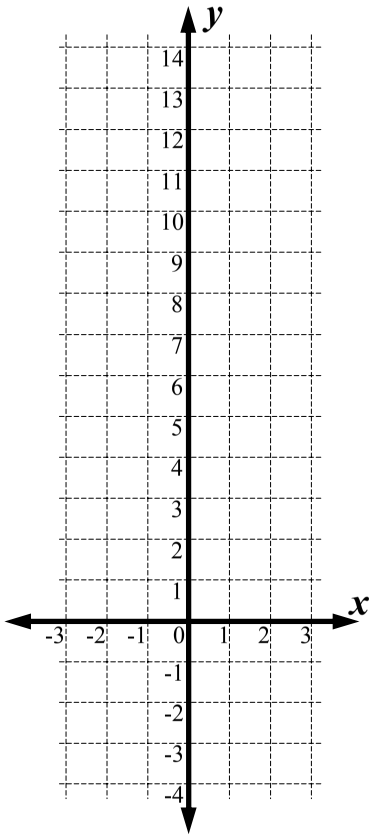
Plot the points in the table of values and create the two parabolas.

1 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12

2 $y = 2x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14



Date Due:

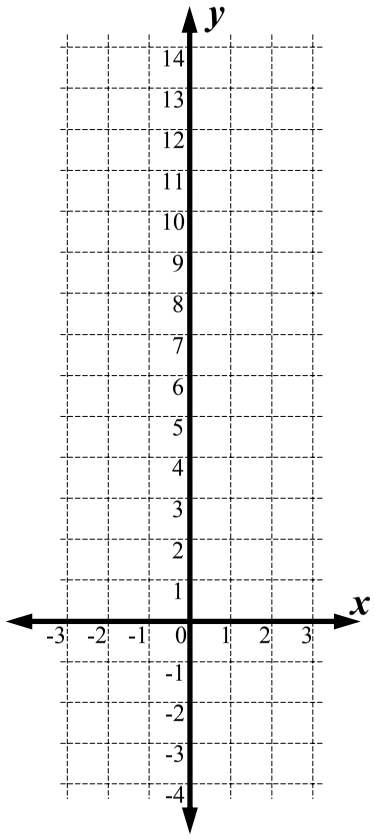
Plot the points in the table of values and create the two parabolas.

1 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12

2 $y = 2x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14



Non-Linear R'ships - Plotting Points & Drawing Parabolas (H'work Strip)

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Date Due:

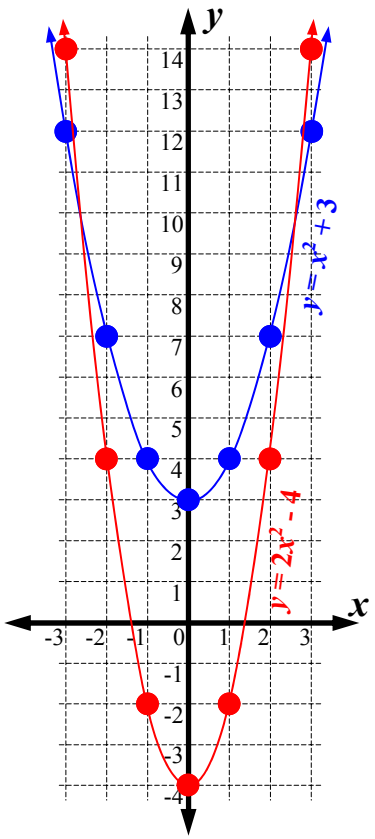
Plot the points in the table of values and create the two parabolas.

1 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12

2 $y = 2x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14



Non-Linear Relationships - Calculator Use

T plot a parabola you need to complete a table of values. These require you to know how to use your calculator, especially when dealing with negative numbers.

The important issue is: You have to place a negative number in brackets to square it.
Example:

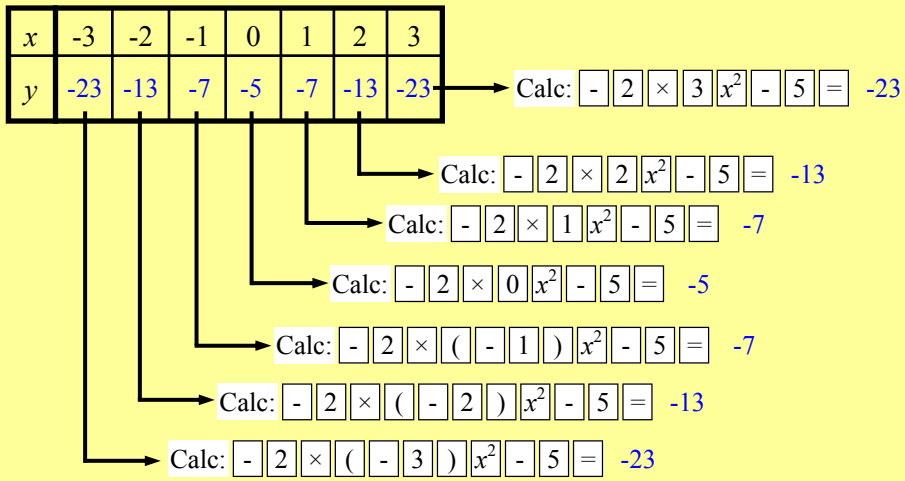
-4^2 is not the same as $(-4)^2$

$-4^2 = -16$ while $(-4)^2$ is 16.

Example

$y = -2x^2 - 5$

x	-3	-2	-1	0	1	2	3
y	-23	-13	-7	-5	-7	-13	-23



Note that the table is symmetrical across the 0 x-value.

$y = -2x^2 - 5$

x	-3	-2	-1	0	1	2	3
y	-23	-13	-7	-5	-7	-13	-23



symmetrical

Non-Linear Relationships - Calculator Use

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Use your calculator to find these.

1 $6^2 - 5 =$ ____

Calc: [6] [x^2] [-] [5] [=]

2 $4^2 + 11 =$ ____

3 $2^2 - 9 =$ ____

4 $3 \times 4^2 + 7 =$ ____

Calc: [3] [×] [4] [x^2] [+] [7] [=]

5 $2 \times 2^2 - 4 =$ ____

6 $-2 \times 3^2 - 8 =$ ____

Calc: [-] [2] [×] [3] [x^2] [-] [8] [=]

Complete the table of values below. Use your back button to speed things up.

7 $y = x^2$

x	0	1	2	3	4	5	6
y							

Calc: [0] [x^2] [=]

8 $y = x^2 + 5$

x	0	1	2	3	4	5	6
y							

Calc: [0] [x^2] [+] [5] [=]

9 $y = -x^2 + 6$

x	0	1	2	3	4	5	6
y							

Calc: [-] [0] [x^2] [+] [6] [=]

10 $y = -2x^2 + 3$

x	0	1	2	3	4	5	6
y							

Calc: [-] [2] [×] [0] [x^2] [+] [3] [=]

These involve negative numbers. To square a negative number you use brackets. Try these.

11 $(-3)^2 =$ ____

Calc: [(] [-] [3] [)] [x^2] [=]

12 $(-6)^2 =$ ____

13 $(-4)^2 + 12 =$ ____

Calc: [(] [-] [4] [)] [x^2] [+] [1] [2] [=]

14 $(-2)^2 - 8 =$ ____

15 $2 \times (-3)^2 - 2 =$ ____

Calc: [2] [×] [(] [-] [3] [)] [x^2] [-] [2] [=]

16 $-3 \times (-1)^2 + 5 =$ ____

Complete these tables which have negative 'x' values.

17 $y = x^2$

x	-6	-5	-4	-3	-2	-1	0
y							

Calc: [(] [-] [6] [)] [x^2] [=]

18 $y = -2x^2 + 4$

x	-6	-5	-4	-3	-2	-1	0
y							

Calc: [-] [2] [×] [(] [-] [6] [)] [x^2] [+] [4] [=]

19 $y = 3x^2 - 10$

x	-6	-5	-4	-3	-2	-1	0
y							

20 $y = -x^2 - 3$

x	-6	-5	-4	-3	-2	-1	0
y							

Match the question on the left with the answer on the right.

21

$8^2 - 11$	152
$2 \times (-2)^2$	32
$3 \times 7^2 + 5$	8
$(-6)^2 - 4$	-20
$-2 \times 5^2 - 4$	25
$-5^2 + 5$	20
$(-5)^2 - 5$	53
$2 \times 4^2 - 7$	-54

Now both positive and negative. Expect to see symmetry, across the '0 value.' Complete these.

22 $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

23 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							

24 $y = -3x^2$

x	-3	-2	-1	0	1	2	3
y							

25 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y							

26 $y = 2x^2 - 9$

x	-3	-2	-1	0	1	2	3
y							

Non-Linear Relationships - Calculator Use

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Use your calculator to find these.

1 $6^2 - 5 =$ **31**

Calc: 6 x^2 - 5 =

2 $4^2 + 11 =$ **27**

3 $2^2 - 9 =$ **-5**

4 $3 \times 4^2 + 7 =$ **55**

Calc: 3 × 4 x^2 + 7 =

5 $2 \times 2^2 - 4 =$ **4**

6 $-2 \times 3^2 - 8 =$ **-26**

Calc: - 2 × 3 x^2 - 8 =

Complete the table of values below. Use your back button to speed things up.

7 $y = x^2$

x	0	1	2	3	4	5	6
y	0	1	4	9	16	25	36

Calc: 0 x^2 =

8 $y = x^2 + 5$

x	0	1	2	3	4	5	6
y	5	6	9	14	21	28	39

Calc: 0 x^2 + 5 =

9 $y = -x^2 + 6$

x	0	1	2	3	4	5	6
y	6	5	2	-3	-10	-19	-30

Calc: - 0 x^2 + 6 =

10 $y = -2x^2 + 3$

x	0	1	2	3	4	5	6
y	3	1	-5	-15	-29	-47	-69

Calc: - 2 × 0 x^2 + 3 =

These involve negative numbers. To square a negative number you use brackets. Try these.

11 $(-3)^2 =$ **9**

Calc: (- 3) x^2 =

12 $(-6)^2 =$ **36**

13 $(-4)^2 + 12 =$ **28**

Calc: (- 4) x^2 + 1 2 =

14 $(-2)^2 - 8 =$ **-4**

15 $2 \times (-3)^2 - 2 =$ **16**

Calc: 2 × (- 3) x^2 - 2 =

16 $-3 \times (-1)^2 + 5 =$ **2**

Complete these tables which have negative 'x' values.

17 $y = x^2$

x	-6	-5	-4	-3	-2	-1	0
y	36	25	16	9	4	1	0

Calc: (- 6) x^2 =

18 $y = -2x^2 + 4$

x	-6	-5	-4	-3	-2	-1	0
y	-68	-46	-28	-14	-4	2	4

Calc: - 2 × (- 6) x^2 + 4 =

19 $y = 3x^2 - 10$

x	-6	-5	-4	-3	-2	-1	0
y	98	65	38	17	2	-7	-10

20 $y = -x^2 - 3$

x	-6	-5	-4	-3	-2	-1	0
y	-39	-28	-19	-12	-7	-4	-3

Match the question on the left with the answer on the right.

21

$8^2 - 11$	152
$2 \times (-2)^2$	32
$3 \times 7^2 + 5$	8
$(-6)^2 - 4$	-20
$-2 \times 5^2 - 4$	25
$-5^2 + 5$	20
$(-5)^2 - 5$	53
$2 \times 4^2 - 7$	-54

Now both positive and negative. Expect to see symmetry, across the '0 value.' Complete these.

22 $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

23 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12

24 $y = -3x^2$

x	-3	-2	-1	0	1	2	3
y	-27	-12	-3	0	-3	-12	-27

25 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

26 $y = 2x^2 - 9$

x	-3	-2	-1	0	1	2	3
y	9	-1	-7	-9	-7	-1	9

Non-Linear Relationships - Plotting Parabolas

This sheet requires you to complete a table of values and then plot points which will form a parabola. All parabolas on these worksheets are symmetrical across the y-axis. So that means each side of the table of values will be the same. Even with negative values.

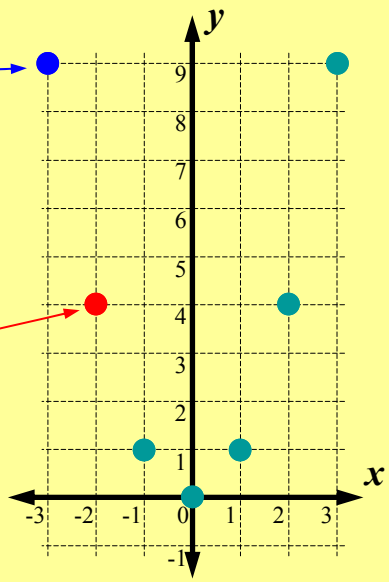
STEP 1 First complete the table. DON'T ANSWER HALF THEN COPY THE VALUES. Otherwise you don't have a check system for your table.

STEP 2 Remember that x moves across and y moves up and down. Plot the points on the graph.

$y = x^2$

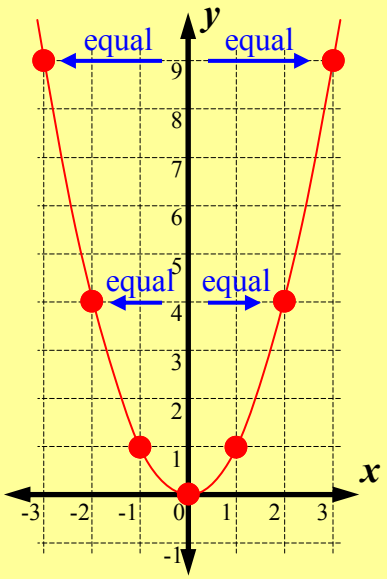
x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

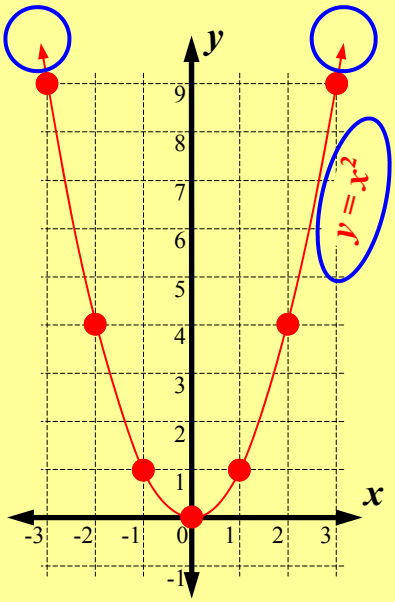
Calc: (- 3) x^2 =



STEP 3 Draw the curve. Note that the points are symmetrical, check for this before you draw curve.

STEP 4 Add arrowheads and the equation of the parabola. Done.





Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 1)

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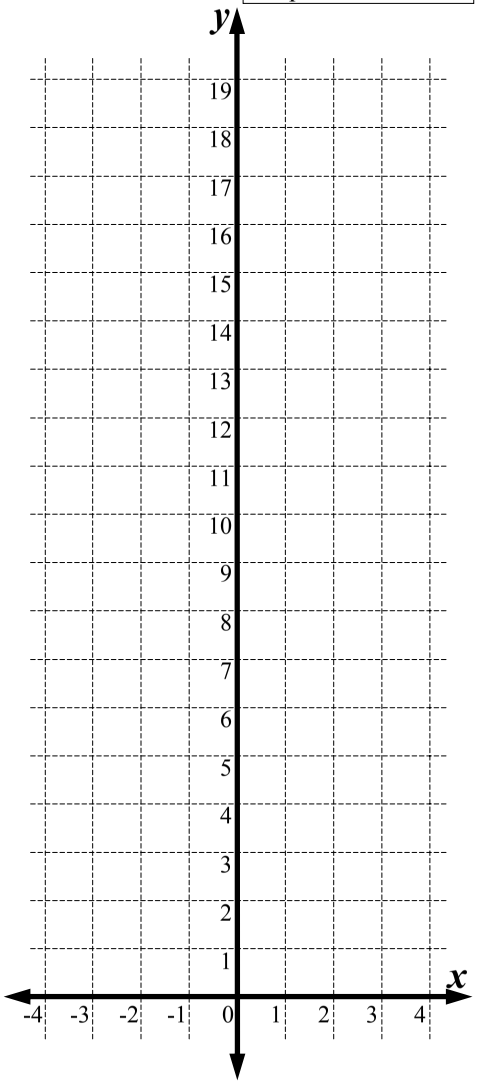
1 $y = \frac{1}{4}x^2$

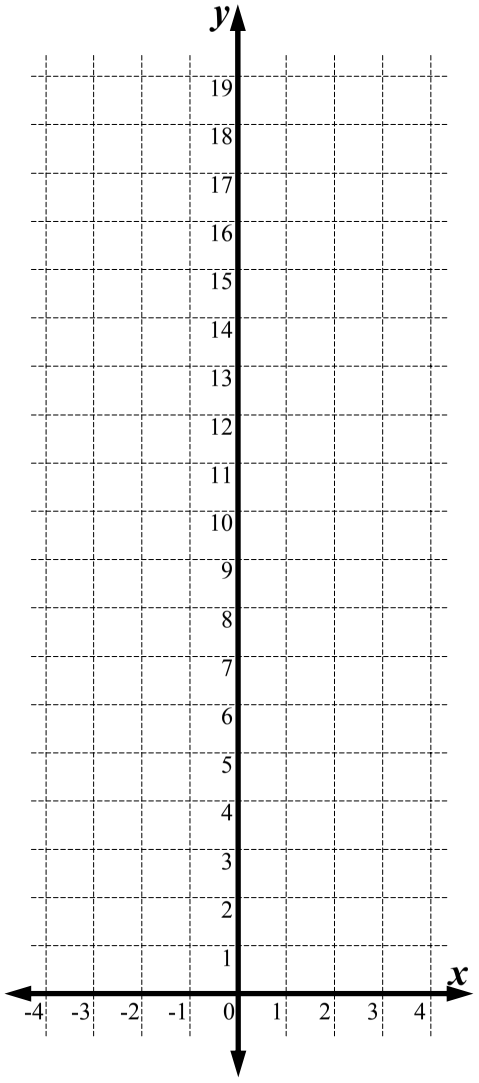
x	-4	-2	-1	0	1	2	4
y							

-Arrowheads
- Equation on curve



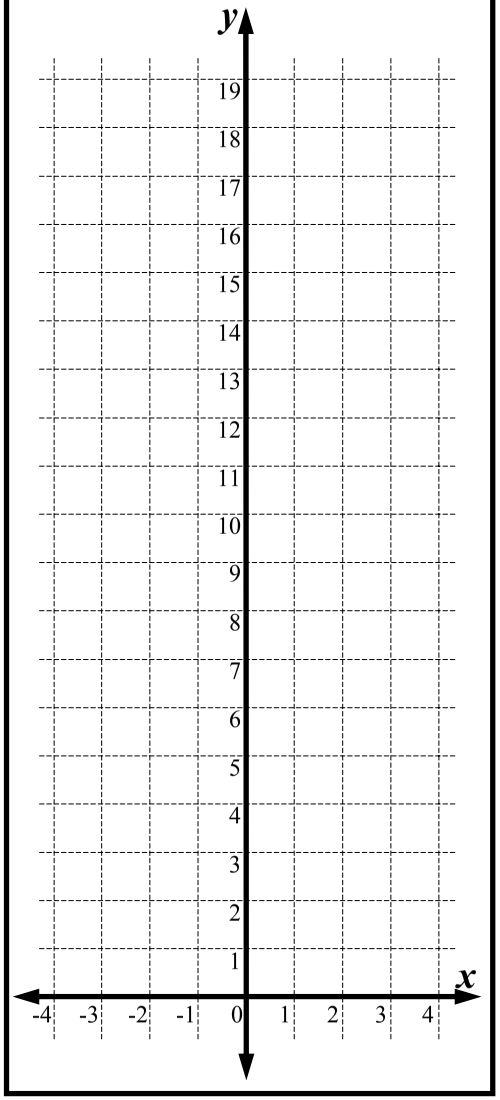
2 $y = \frac{1}{2}x^2$

x	-4	-2	-1	0	1	2	4
y							



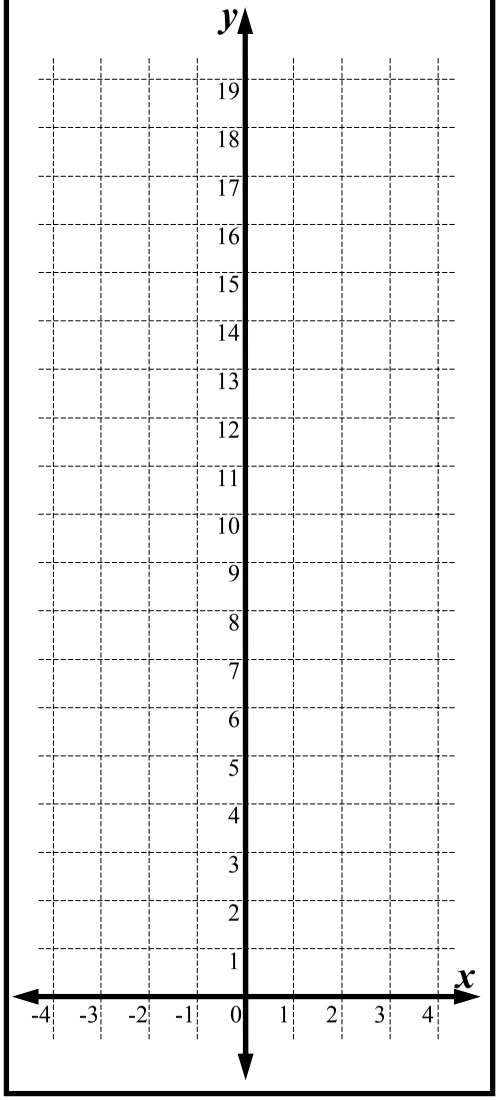
3 $y = x^2$

x	-3	-2	-1	0	1	2	3
y							



4 $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 2)

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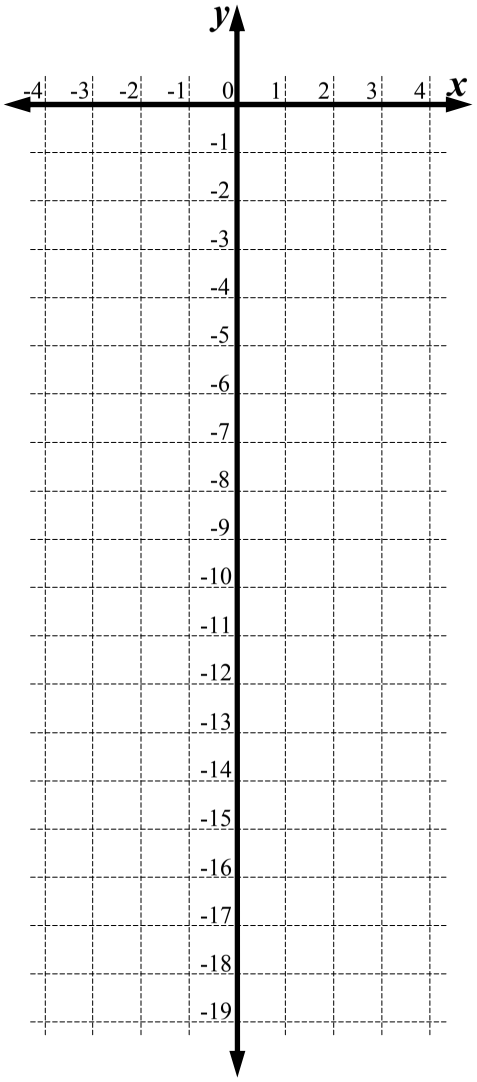
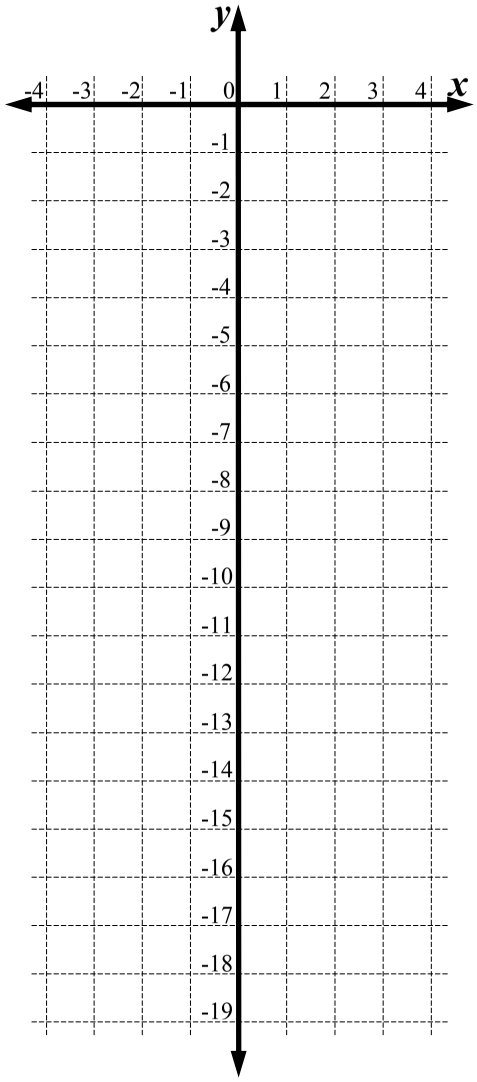
1 $y = -\frac{1}{4}x^2$

x	-4	-2	-1	0	1	2	4
y							



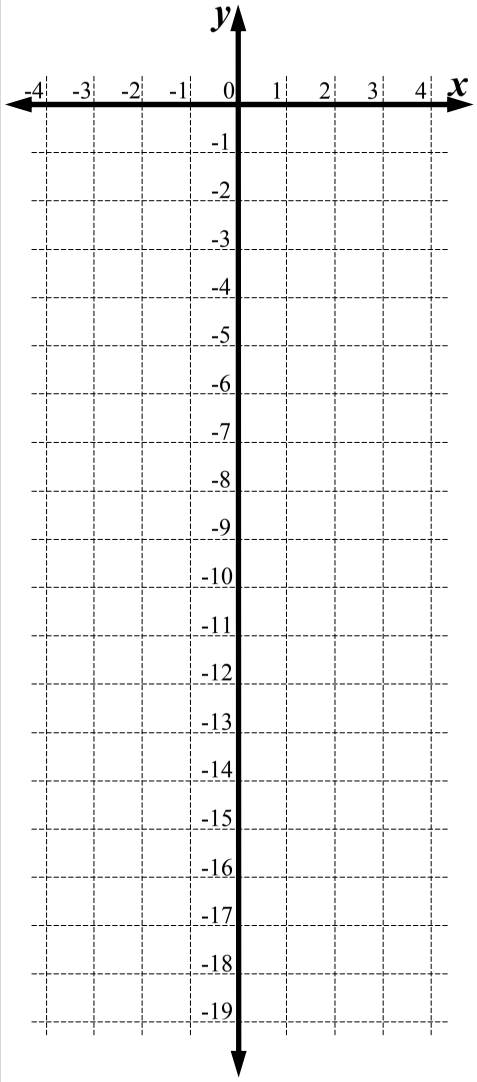
2 $y = -\frac{1}{2}x^2$

x	-4	-2	-1	0	1	2	4
y							



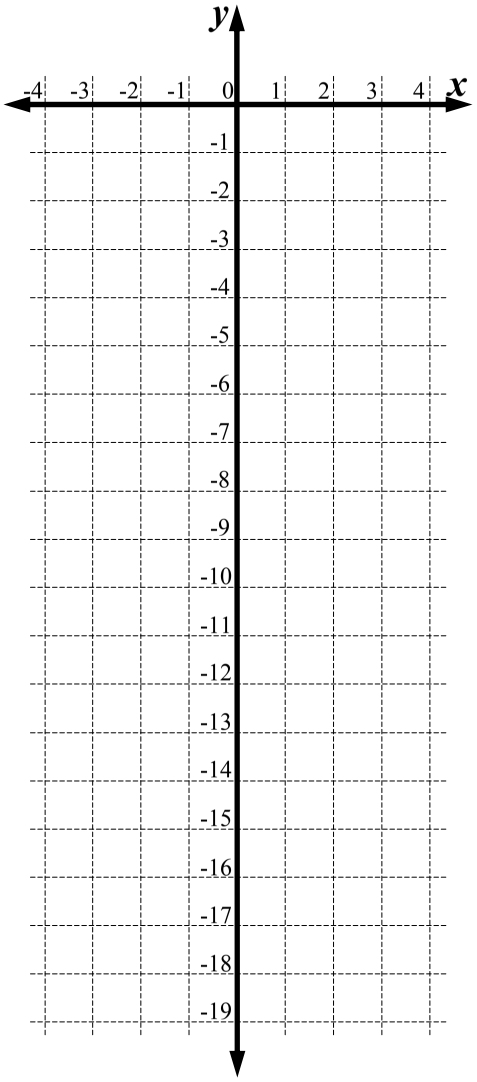
3 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y							



4 $y = -2x^2$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 3)

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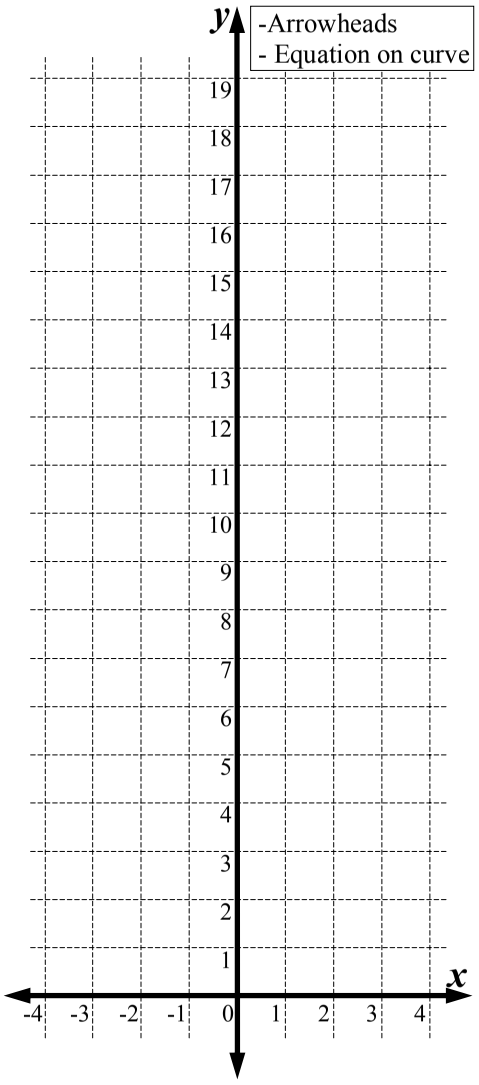
1 $y = x^2 + 5$

x	-3	-2	-1	0	1	2	3
y							

Calc (- 3) x^2 + 5 =

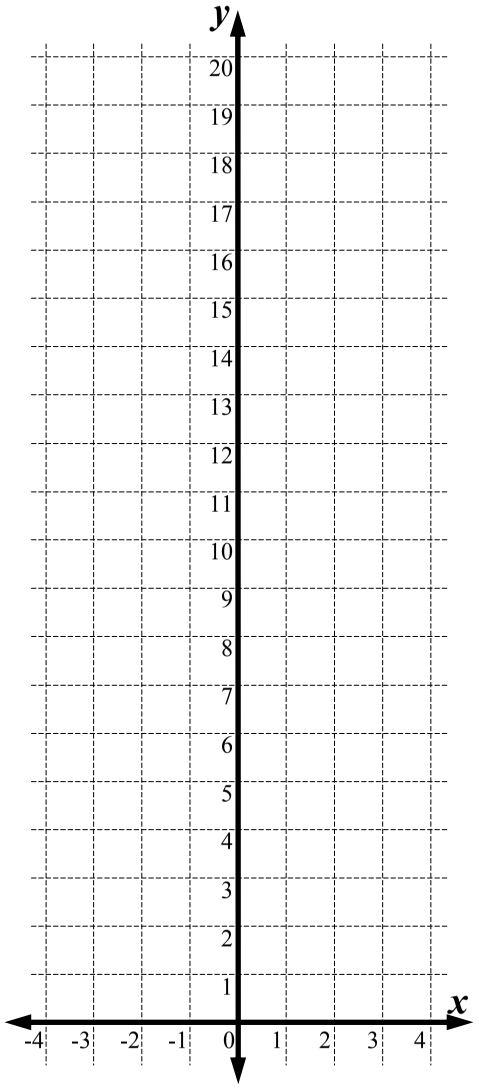
-Arrowheads
- Equation on curve



2 $y = 2x^2 + 2$

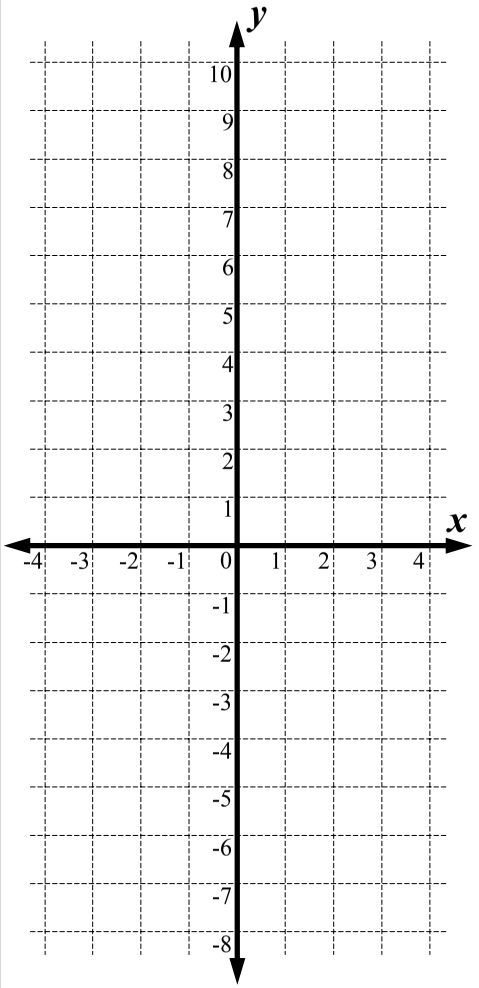
x	-3	-2	-1	0	1	2	3
y							

Calc: 2 × (- 3) x^2 + 2 =



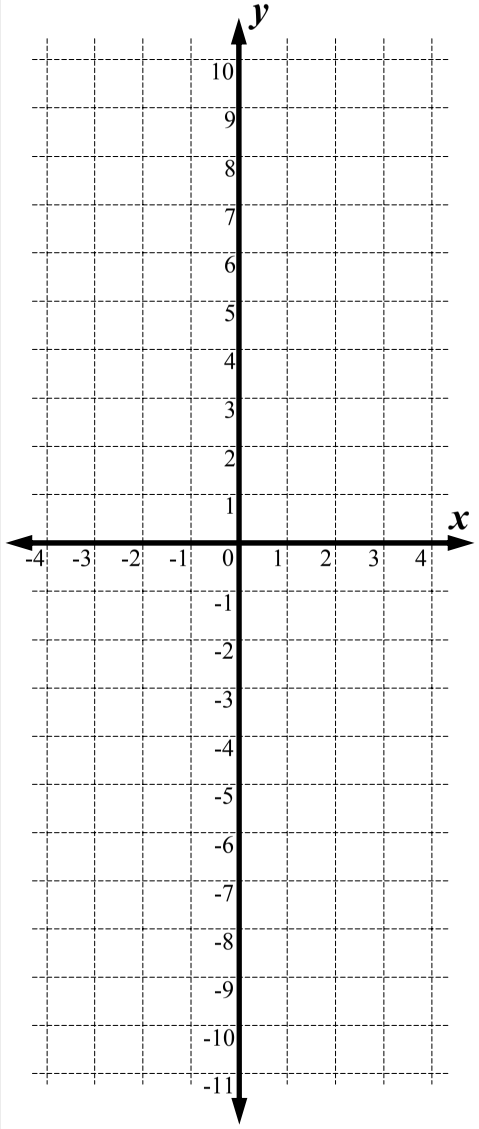
3 $y = x^2 - 7$

x	-4	-3	-2	-1	0	1	2	3	4
y									



4 $y = 2x^2 - 10$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 4)

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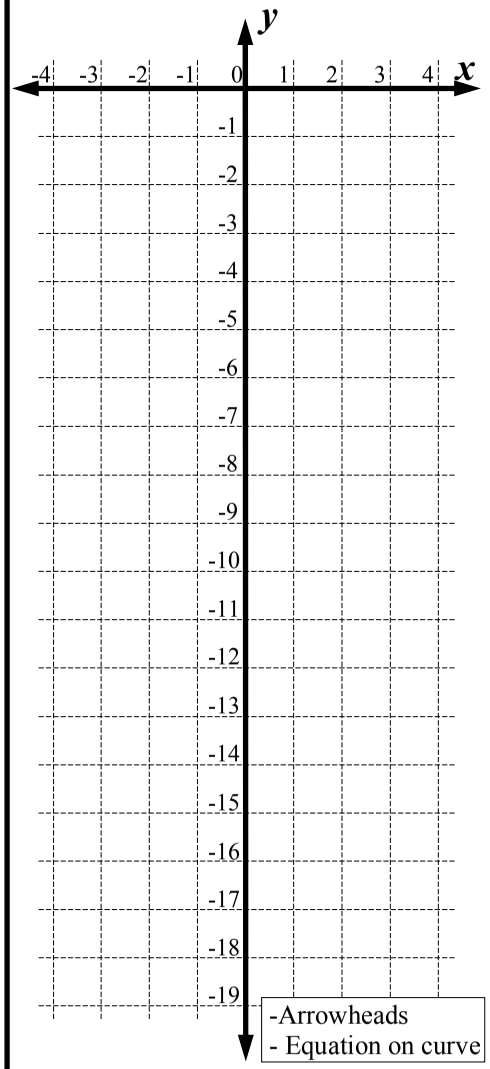
1 $y = -x^2 - 6$

x	-3	-2	-1	0	1	2	3
y							

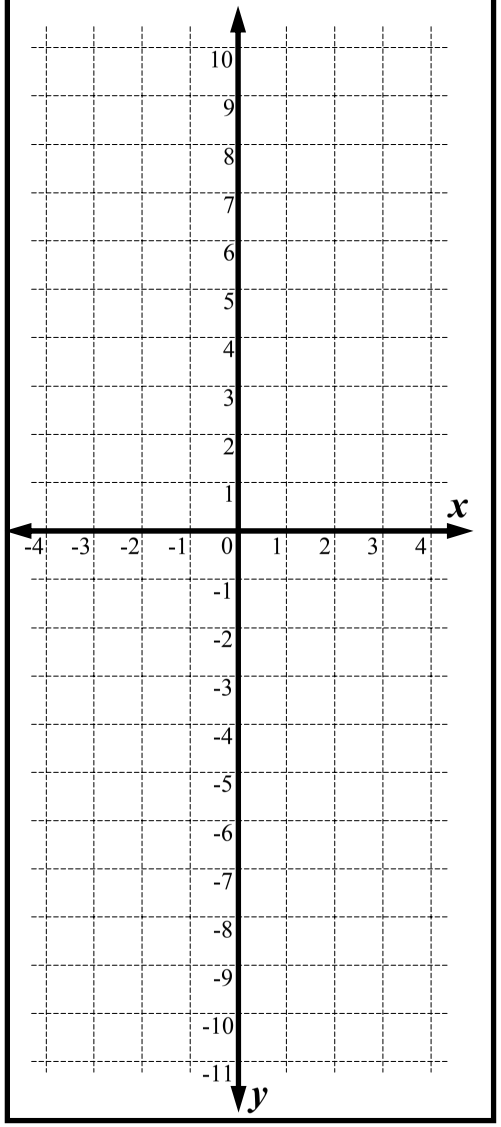
Calc: [-] [(] [-] [3] [)] [x^2] [-] [6] [=]



2 $y = -2x^2 + 7$

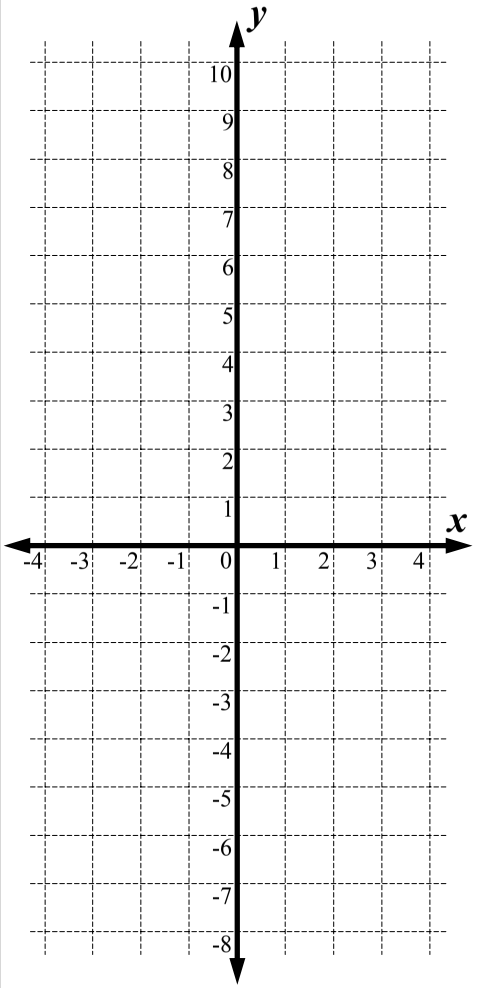
x	-3	-2	-1	0	1	2	3
y							

Calc: [-] [2] [×] [(] [-] [3] [)] [x^2] [+] [7] [=]



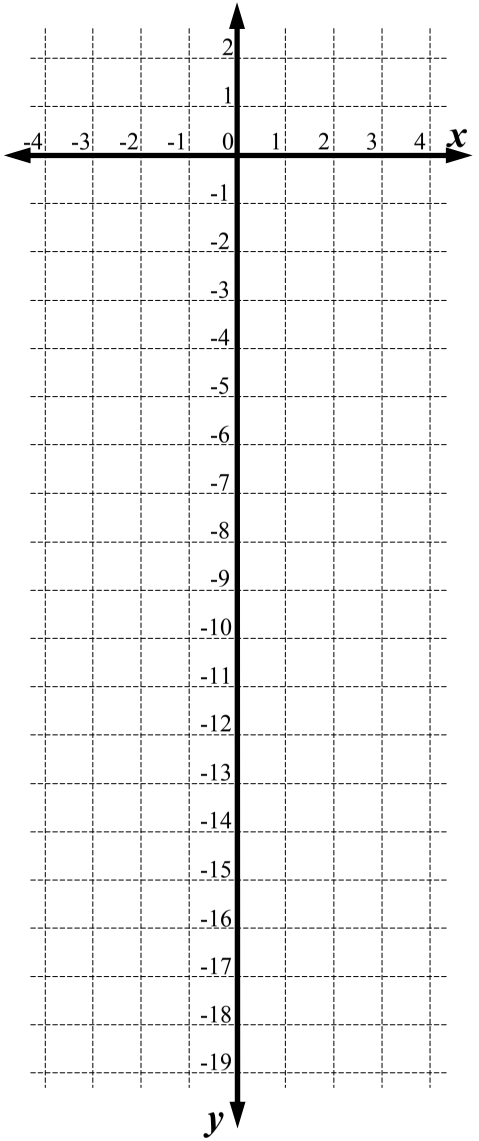
3 $y = -x^2 + 8$

x	-4	-3	-2	-1	0	1	2	3	4
y									



4 $y = -2x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 1)

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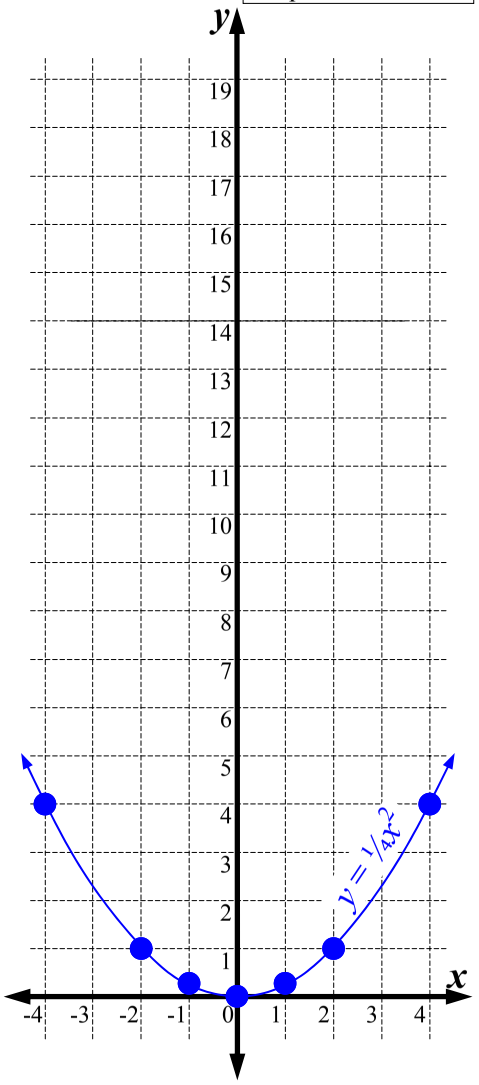
1 $y = \frac{1}{4}x^2$

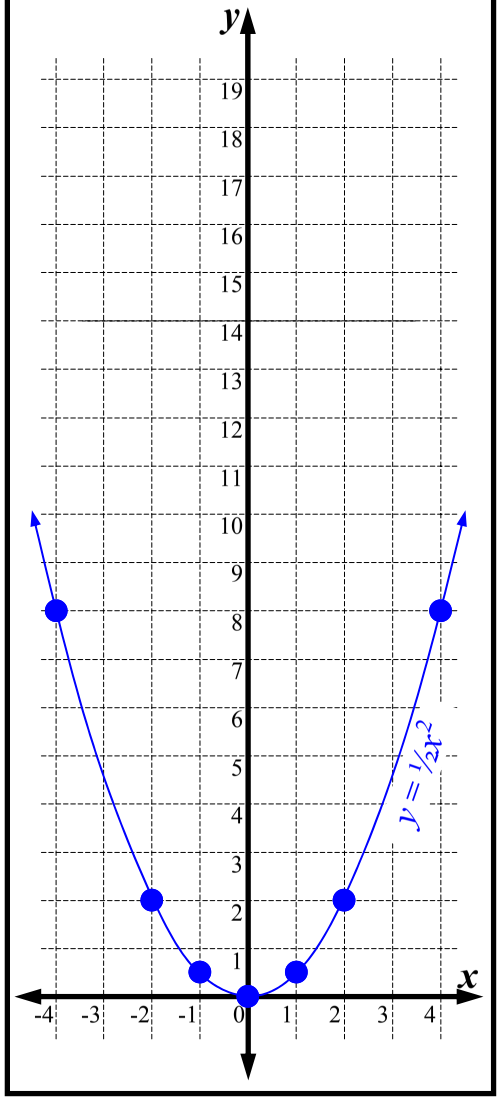
x	-4	-2	-1	0	1	2	4
y	4	1	¼	0	¼	1	4

-Arrowheads
- Equation on curve



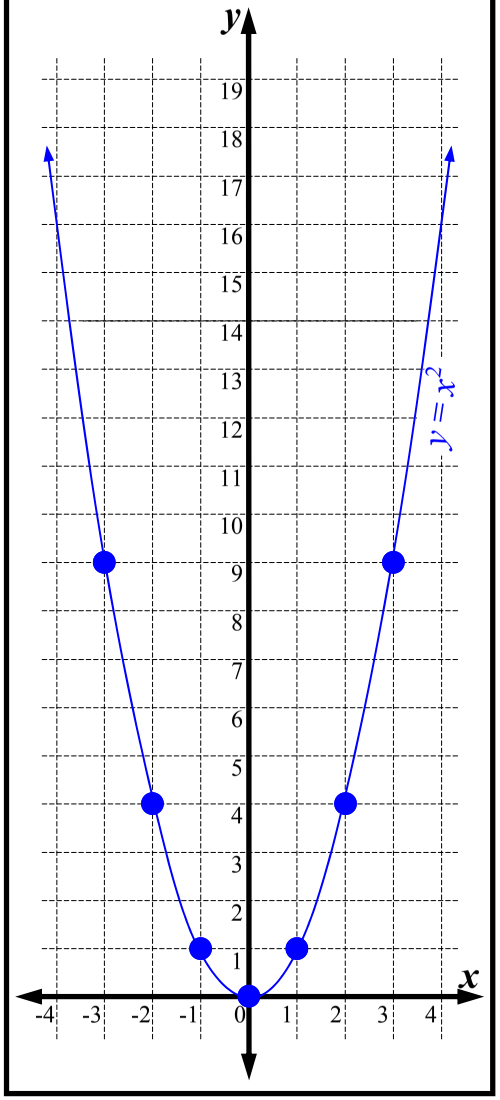
2 $y = \frac{1}{2}x^2$

x	-4	-2	-1	0	1	2	4
y	8	2	½	0	½	2	8



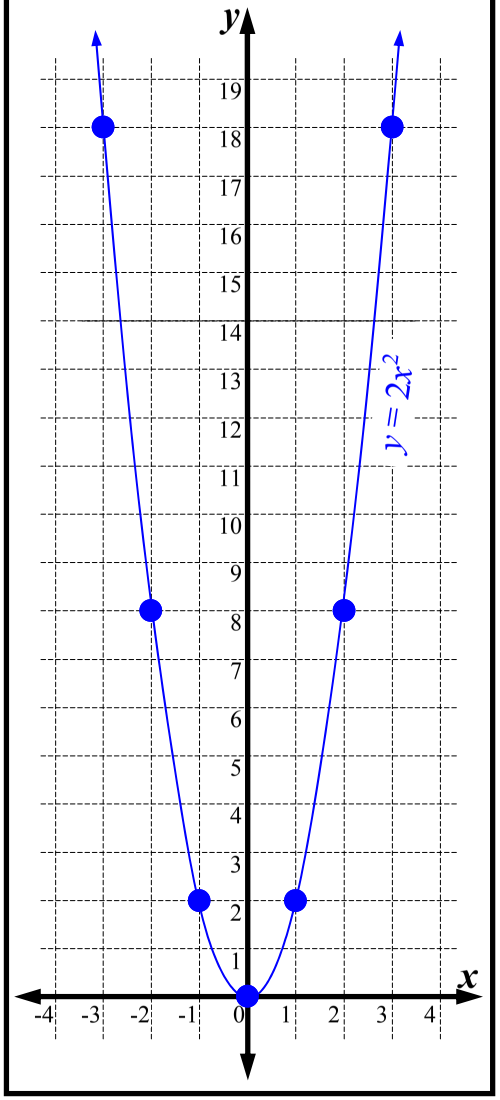
3 $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



4 $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 2)

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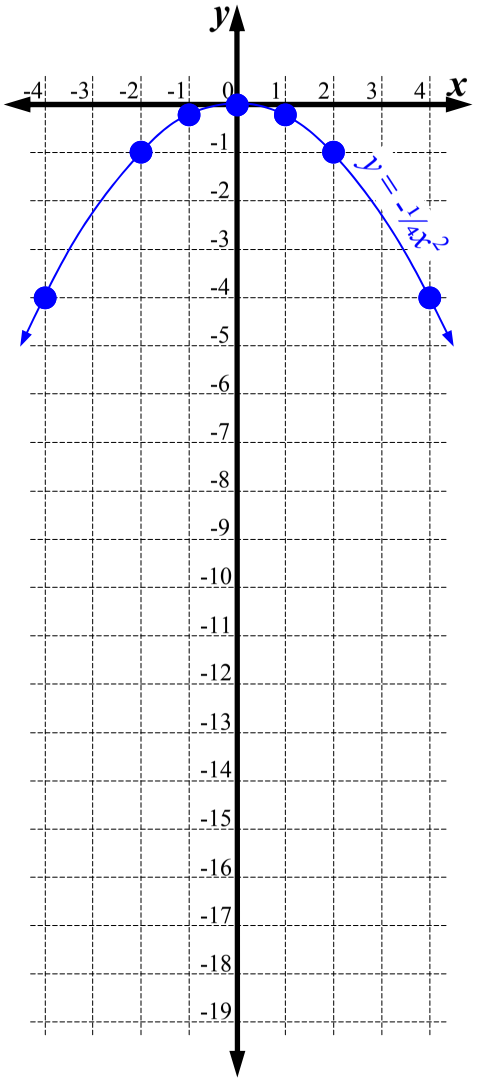
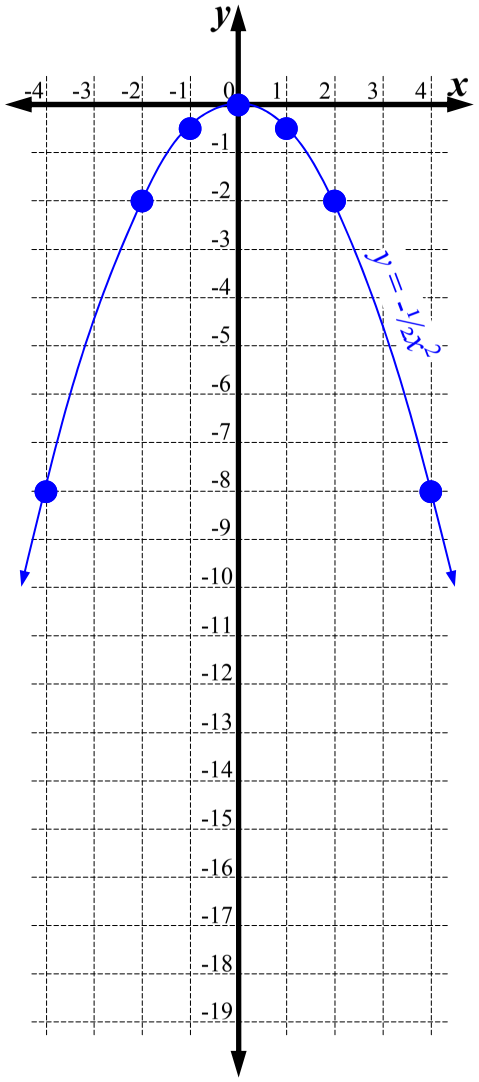
1 $y = -\frac{1}{4}x^2$

x	-4	-2	-1	0	1	2	4
y	-4	-1	-¼	0	-¼	-1	-4



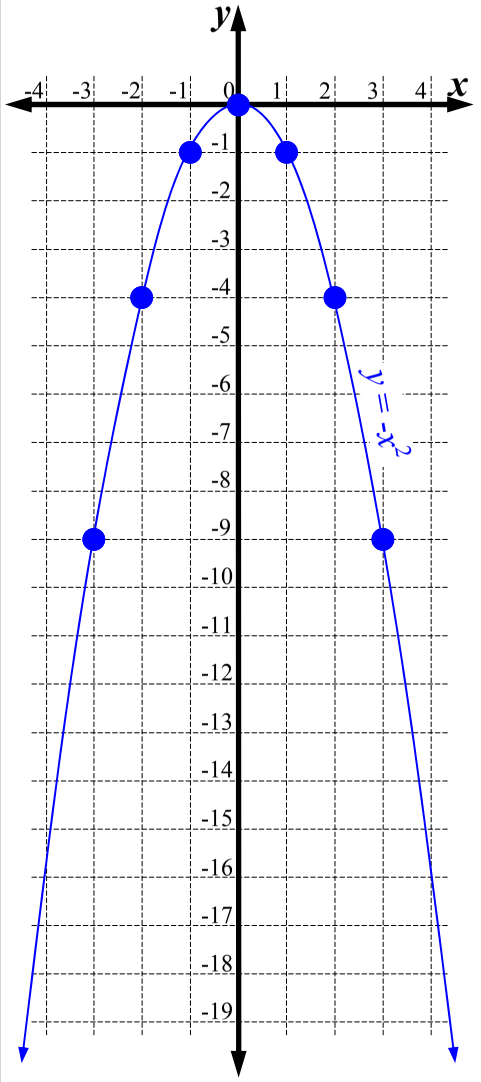
2 $y = -\frac{1}{2}x^2$

x	-4	-2	-1	0	1	2	4
y	-8	-2	-½	0	-½	-2	-8



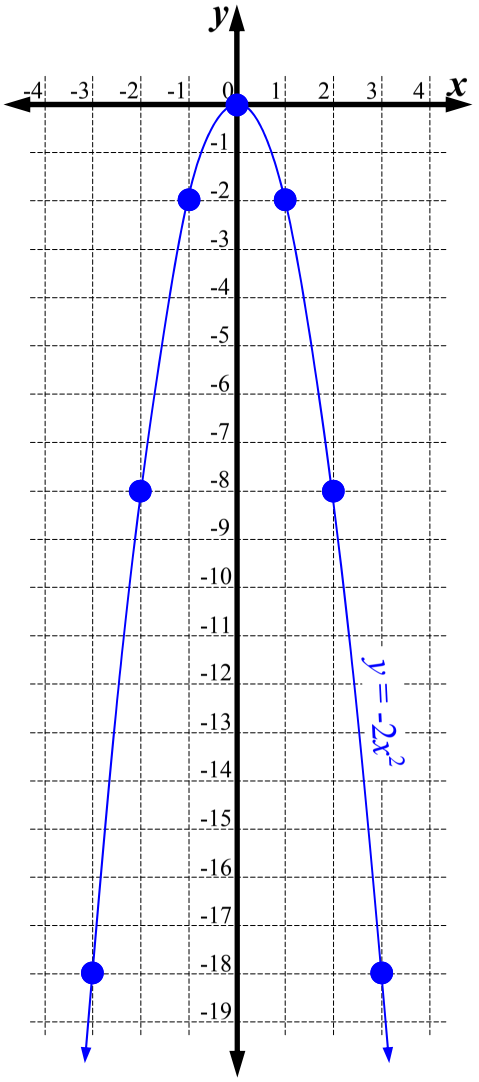
3 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9



4 $y = -2x^2$

x	-3	-2	-1	0	1	2	3
y	-18	-8	-2	0	-2	-8	-18



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 3)

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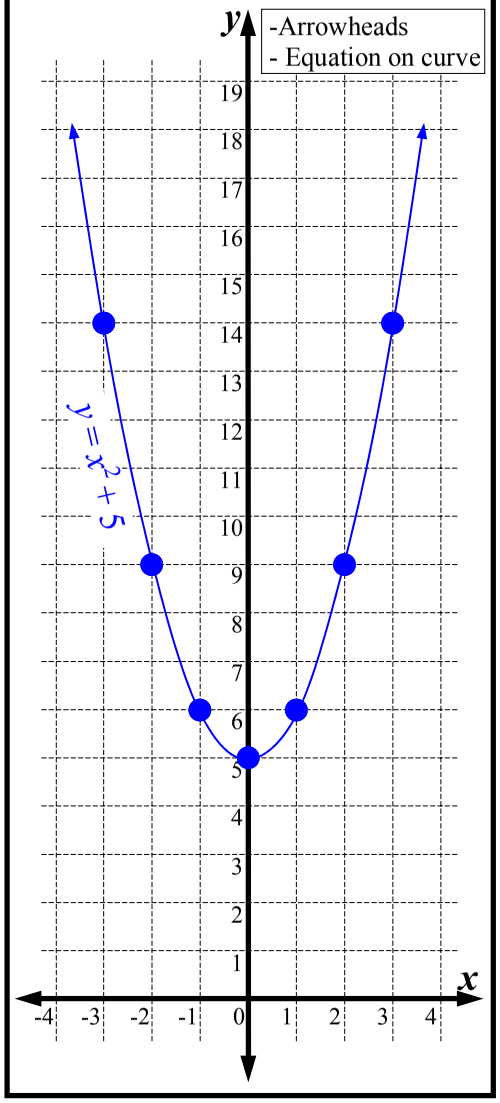
1 $y = x^2 + 5$

x	-3	-2	-1	0	1	2	3
y	14	9	6	5	6	9	14

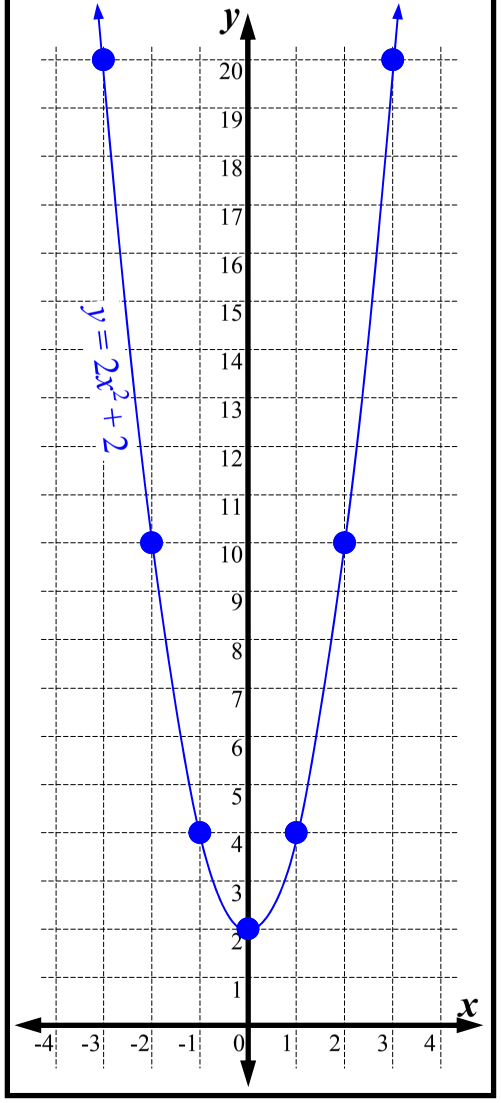
Calc (- 3) x^2 + 5 =



2 $y = 2x^2 + 2$

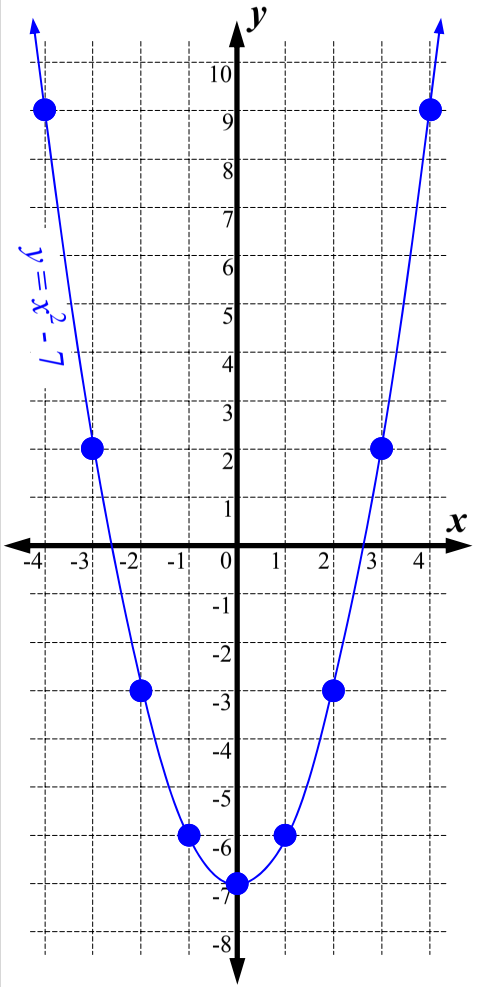
x	-3	-2	-1	0	1	2	3
y	20	10	4	2	4	10	20

Calc: 2 × (- 3) x^2 + 2 =



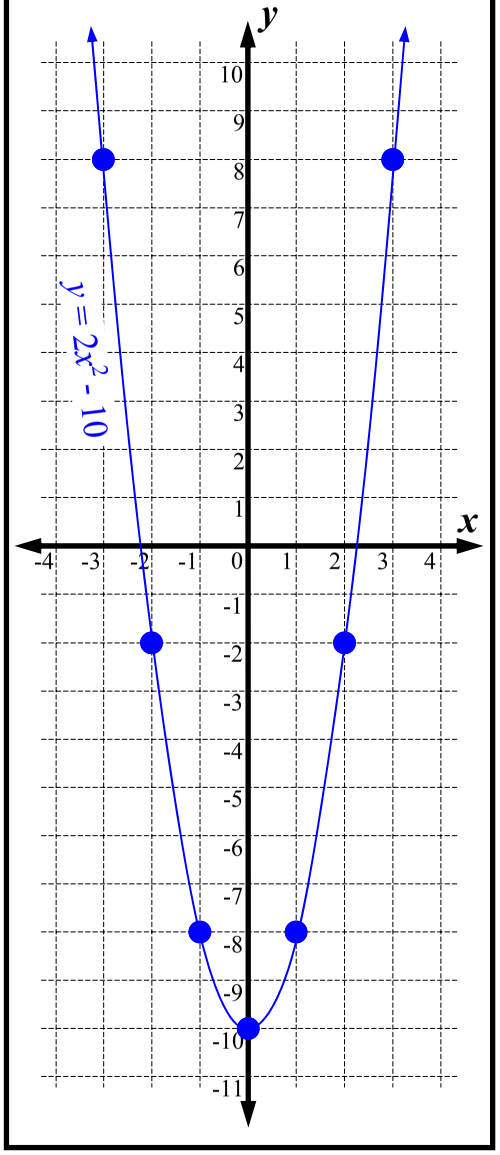
3 $y = x^2 - 7$

x	-4	-3	-2	-1	0	1	2	3	4
y	9	2	-3	-6	-7	-6	-3	2	9



4 $y = 2x^2 - 10$

x	-3	-2	-1	0	1	2	3
y	8	-2	-8	-10	-8	-2	8



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 4)

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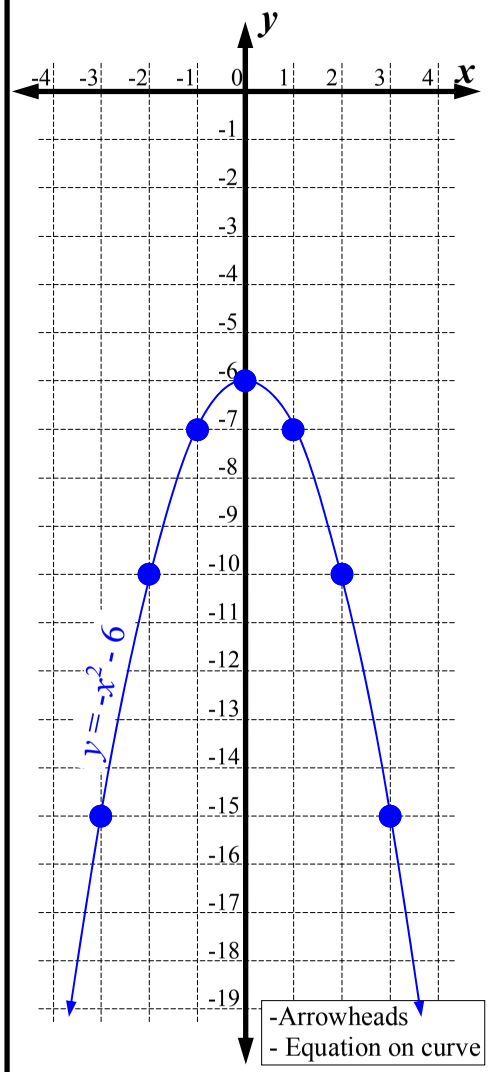
1 $y = -x^2 - 6$

x	-3	-2	-1	0	1	2	3
y	-15	-10	-7	-6	-7	-10	-15

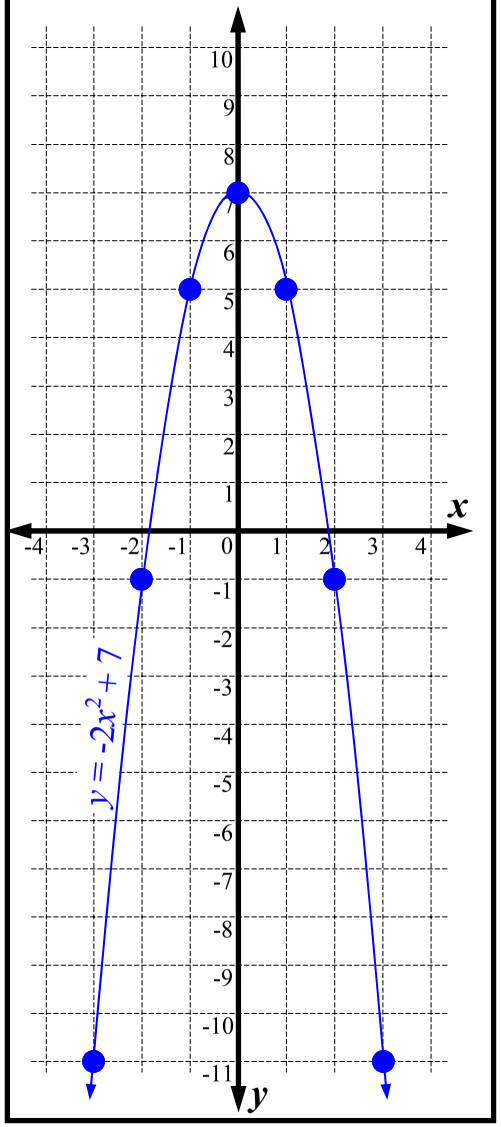
Calc: [-] [(] [-] [3] [)] [x^2] [-] [6] [=]



2 $y = -2x^2 + 7$

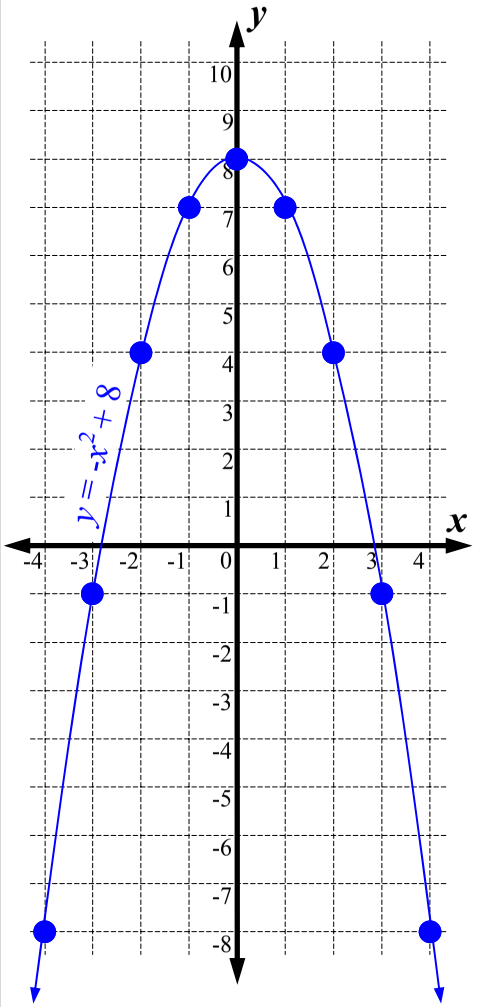
x	-3	-2	-1	0	1	2	3
y	-11	-1	5	7	5	-1	-11

Calc: [-] [2] [×] [(] [-] [3] [)] [x^2] [+] [7] [=]



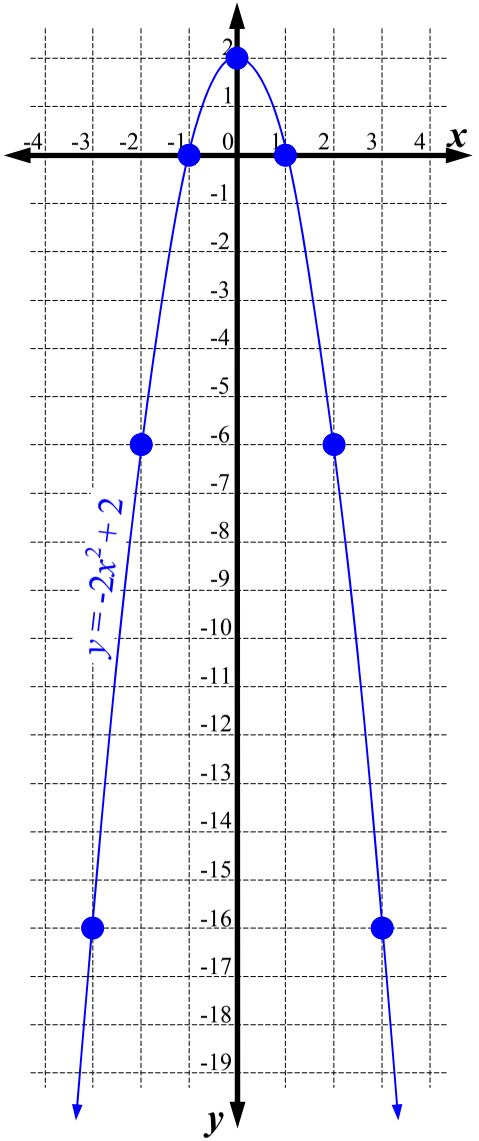
3 $y = -x^2 + 8$

x	-4	-3	-2	-1	0	1	2	3	4
y	-8	-1	4	7	8	7	4	-1	-8



4 $y = -2x^2 + 2$

x	-3	-2	-1	0	1	2	3
y	-16	-6	0	2	0	-6	-16



Non-Linear Relationships - Plotting Parabolas (Homework Strip)

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Date Due:

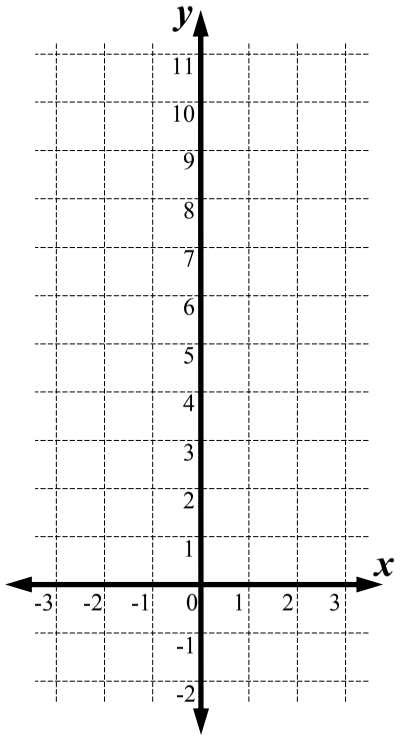
Complete the table of values and create the two parabolas. Use brackets!

1 $y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
y							

2 $y = -x^2 + 10$

x	-3	-2	-1	0	1	2	3
y							



Date Due:

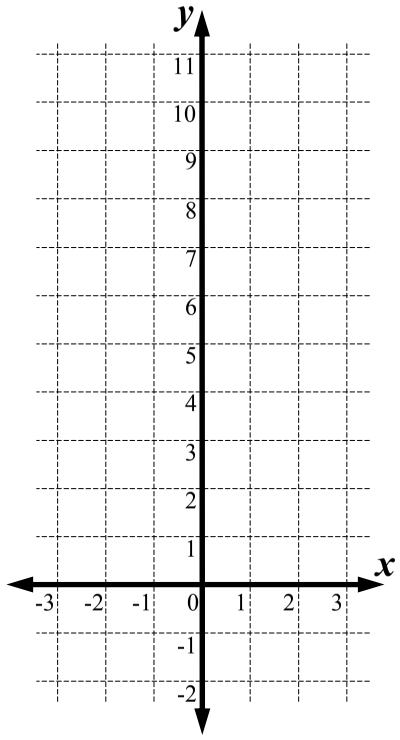
Complete the table of values and create the two parabolas. Use brackets!

1 $y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
y							

2 $y = -x^2 + 10$

x	-3	-2	-1	0	1	2	3
y							



Date Due:

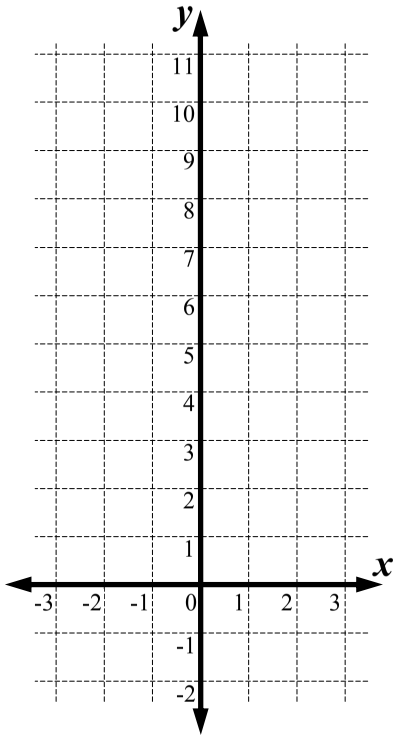
Complete the table of values and create the two parabolas. Use brackets!

1 $y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
y							

2 $y = -x^2 + 10$

x	-3	-2	-1	0	1	2	3
y							



Date Due:

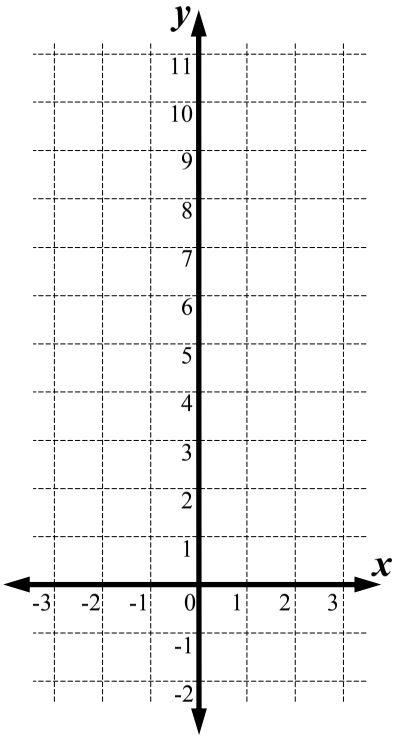
Complete the table of values and create the two parabolas. Use brackets!

1 $y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
y							

2 $y = -x^2 + 10$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Plotting Parabolas (Homework Strip)

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Date Due:

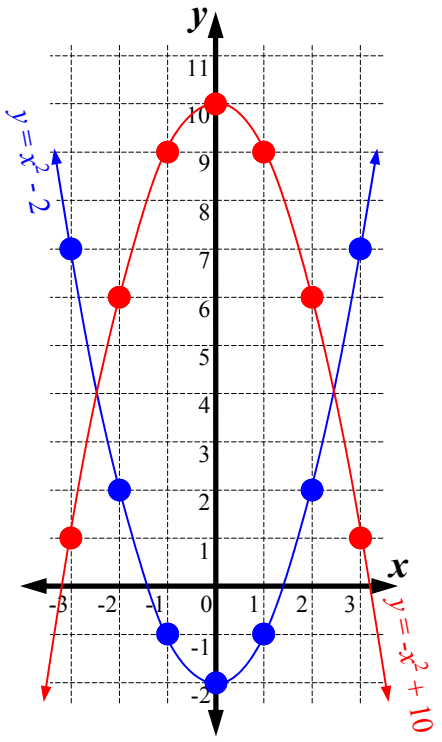
Complete the table of values and create the two parabolas. Use brackets!

1 $y = x^2 - 2$

x	-3	-2	-1	0	1	2	3
y	7	2	-1	-2	-1	2	7

2 $y = -x^2 + 10$

x	-3	-2	-1	0	1	2	3
y	1	6	9	10	9	6	1



Non-Linear Relationships - Further Plotting Parabolas

This sheet requires you to complete a table of values and then plot points which will form a parabola. It is a more difficult sheet than the previous sheet due to the inclusion of fractions. Also each grid has three parabolas on the same grid, so it is more busy. All parabolas on these worksheets are symmetrical across the y-axis. So that means each side of the table of values will be the same. Even with negative values.

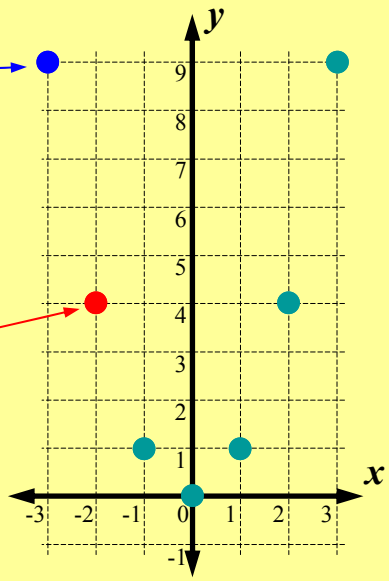
STEP 1 First complete the table. DON'T ANSWER HALF THEN COPY THE VALUES. Otherwise you don't have a check system for your table.

STEP 2 Remember that *x* moves across and *y* moves up and down. Plot the points on the graph.

$y = x^2$

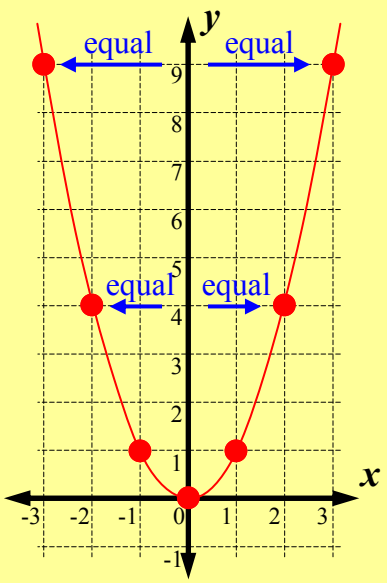
x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

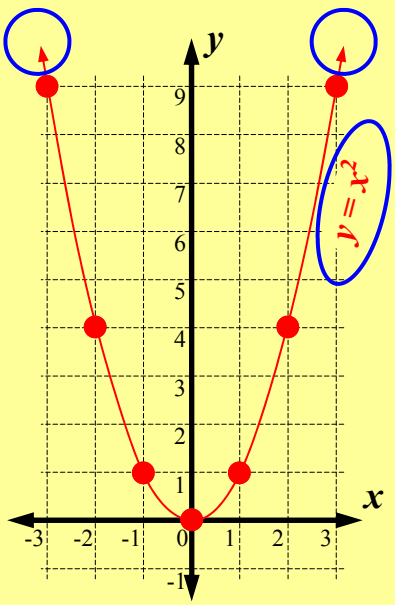
Calc: (- 3) x^2 =



STEP 3 Draw the curve. Note that the points are symmetrical, check for this before you draw curve.

STEP 4 Add arrowheads and the equation of the parabola. Done.





Non-Linear Relationships - Further Plotting Parabolas (Sheet 1)

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Complete the table of values below. Use brackets for negatives & your back button.

1 $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

Calc: (- 3) x^2 =

2 $y = 2x^2 + 5$

x	-3	-2	-1	0	1	2	3
y							

Calc: 2 × (- 3) x^2 + 5 =

3 $y = -x^2 - 1$

x	-3	-2	-1	0	1	2	3
y							

Calc: - (- 3) x^2 - 1 =

4 $y = -2x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							

Calc: - 2 × (- 3) x^2 + 3 =

5 $y = ½x^2$

x	-3	-2	-1	0	1	2	3
y	½						

Calc: 1 a b/c 2 ▶ × (- 3) x^2 = S⇔D

6 $y = -½x^2 + 6$

x	-3	-2	-1	0	1	2	3
y							

Calc: - 1 a b/c 2 ▶ × (- 3) x^2 + 6 = S⇔D

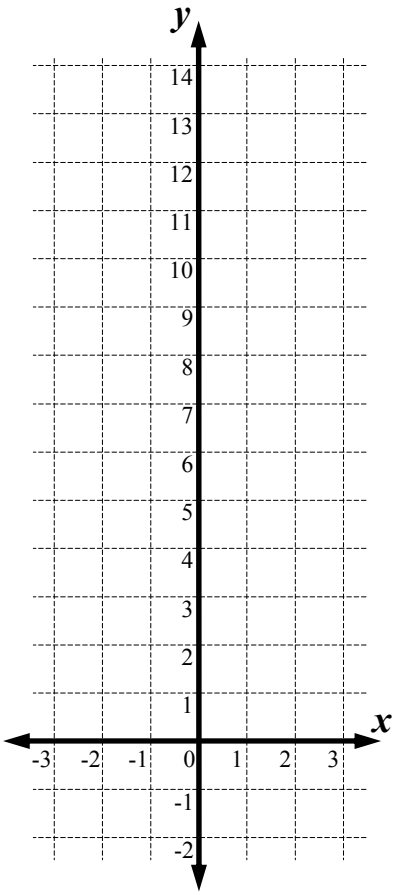
7 $y = -½x^2 - 4$

x	-3	-2	-1	0	1	2	3
y							

Now use the table of values to plot the three parabolas all on the same number plane. Go across 'x' and up 'y'.

8

-Arrowheads
- Equation on curve



$y = x^2$

x	-3	-2	-1	0	1	2	3
y							

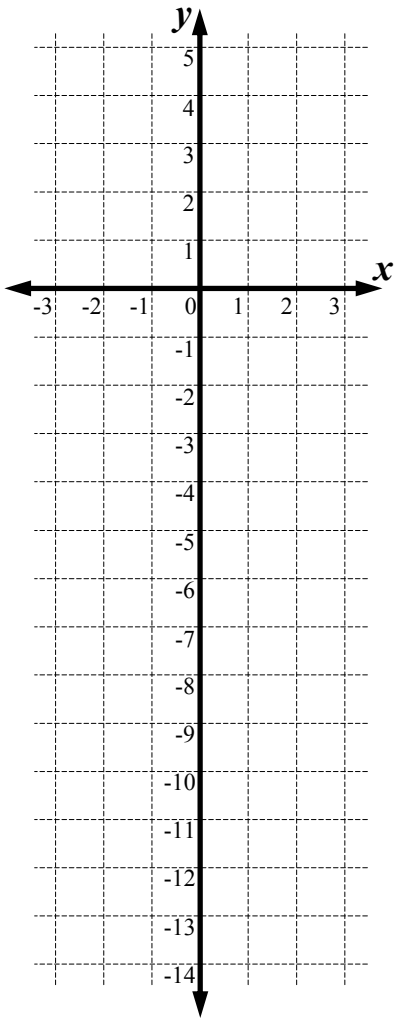
$y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							

$y = -x^2 + 8$

x	-3	-2	-1	0	1	2	3
y							

Calc: - (- 3) x^2 + 8 =



9

$y = -2x^2 + 5$

x	-3	-2	-1	0	1	2	3
y							

$y = -x^2$

x	-3	-2	-1	0	1	2	3
y							

$y = x^2 - 7$

x	-3	-2	-1	0	1	2	3
y							

Arrowheads and Equation on curve

Non-Linear Relationships - Further Plotting Parabolas (Sheet 2)

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Complete the table of values below and plot the parabolas. Fractions in the equations result in flatter parabolas.

1 $y = \frac{1}{2}x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y									

Calc: 1 ▪/□ 2 ▶ × (- 4) x^2 =

Then use the back button

2 $y = -\frac{1}{2}x^2 + 10$

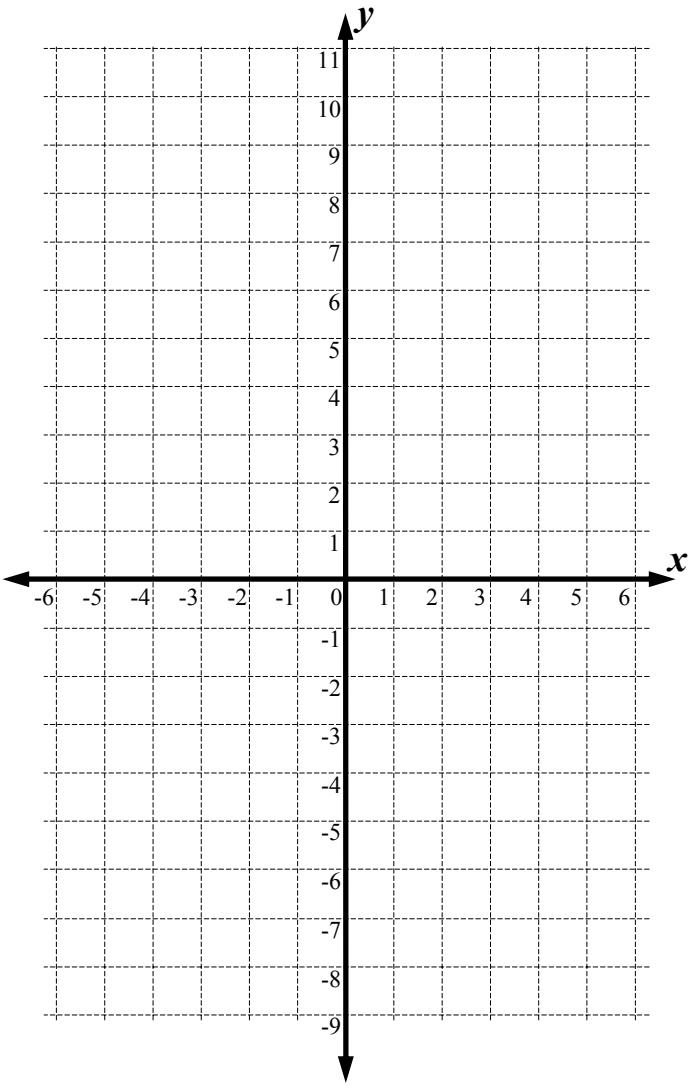
x	-6	-4	-2	-1	0	1	2	4	6
y									

Calc: - 1 ▪/□ 2 ▶ × (- 6) x^2 + 1 0 =

Then use the back button

3 $y = \frac{1}{4}x^2 - 2$

x	-6	-4	-2	0	2	4	6
y							



4 $y = \frac{1}{2}x^2 - 2$

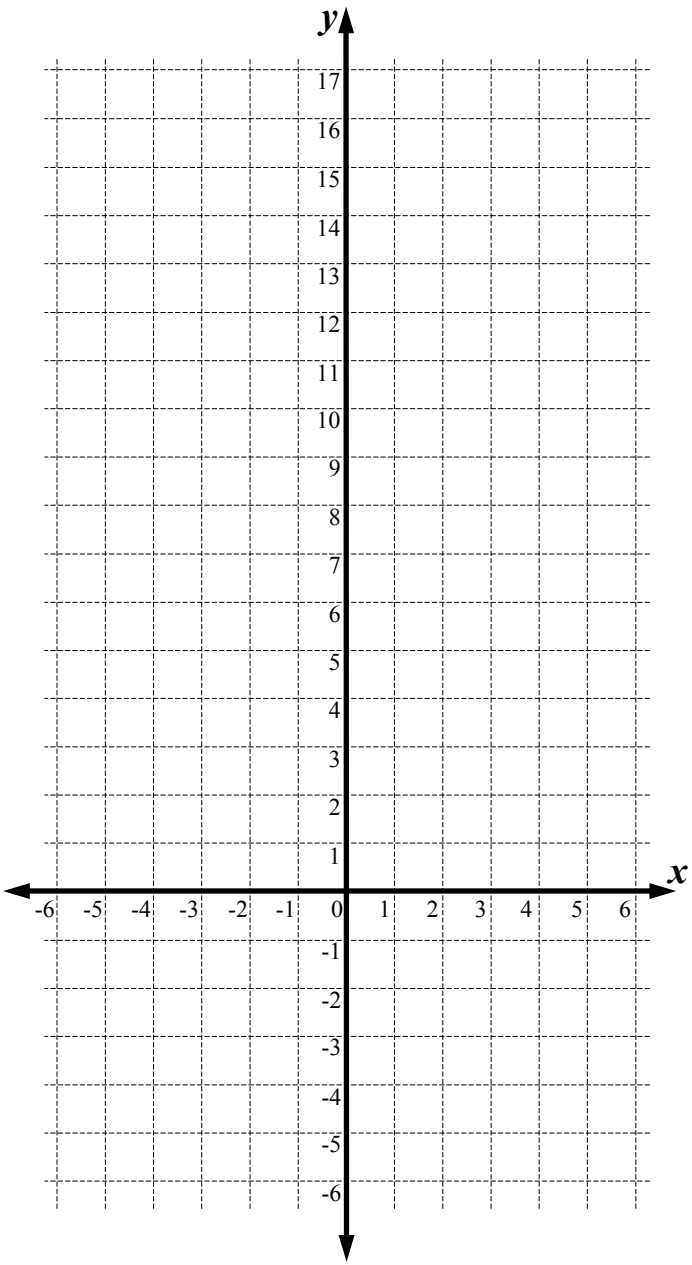
x	-6	-4	-2	-1	0	1	2	4	6
y									

5 $y = x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y									

6 $y = -2x^2 + 12$

x	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
y									



Non-Linear Relationships - Further Plotting Parabolas (Sheet 1)

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Complete the table of values below. Use brackets for negatives & your back button.

1 $y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

Calc: (- 3) x^2 =

2 $y = 2x^2 + 5$

x	-3	-2	-1	0	1	2	3
y	23	13	7	5	7	13	23

Calc: 2 × (- 3) x^2 + 5 =

3 $y = -x^2 - 1$

x	-3	-2	-1	0	1	2	3
y	-10	-5	-2	-1	-2	-5	-10

Calc: - (- 3) x^2 - 1 =

4 $y = -2x^2 + 3$

x	-3	-2	-1	0	1	2	3
y	-15	-5	1	3	1	-5	-15

Calc: - 2 × (- 3) x^2 + 3 =

5 $y = \frac{1}{2}x^2$

x	-3	-2	-1	0	1	2	3
y	4½	2	½	0	½	2	4½

Calc: 1 a/b 2 ▶ × (- 3) x^2 = S⇔D

6 $y = -\frac{1}{2}x^2 + 6$

x	-3	-2	-1	0	1	2	3
y	1½	4	5½	6	5½	4	1½

Calc: - 1 a/b 2 ▶ × (- 3) x^2 + 6 = S⇔D

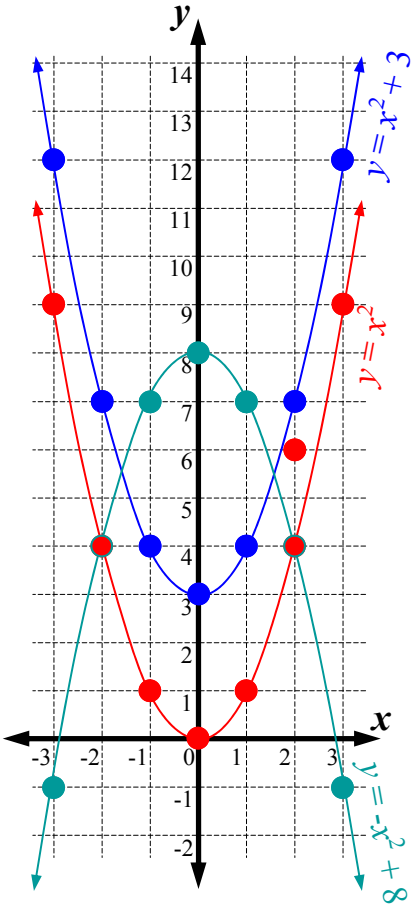
7 $y = -\frac{1}{2}x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	-8½	-6	-4½	-4	-4½	-6	-8½

Now use the table of values to plot the three parabolas all on the same number plane. Go across 'x' and up 'y'.

8

-Arrowheads
- Equation on curve



$y = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

$y = x^2 + 3$

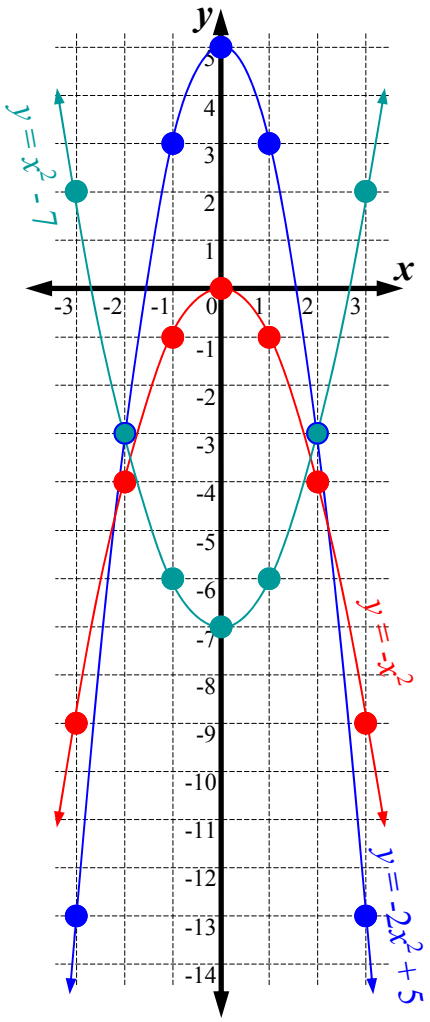
x	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12

$y = -x^2 + 8$

x	-3	-2	-1	0	1	2	3
y	-1	4	7	8	7	4	-1

Calc: - (- 3) x^2 + 8 =

9



$y = -2x^2 + 5$

x	-3	-2	-1	0	1	2	3
y	-13	-3	3	5	3	-3	-13

$y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

$y = x^2 - 7$

x	-3	-2	-1	0	1	2	3
y	2	-3	-6	-7	-6	-3	2

Non-Linear Relationships - Further Plotting Parabolas (Sheet 2)

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Complete the table of values below and plot the parabolas. Fractions in the equations result in flatter parabolas.

1 $y = \frac{1}{2}x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	8	4½	2	½	0	½	2	4½	8

Calc: 1 ▪/□ 2 ▶ × (- 4) x^2 = Then use the back button

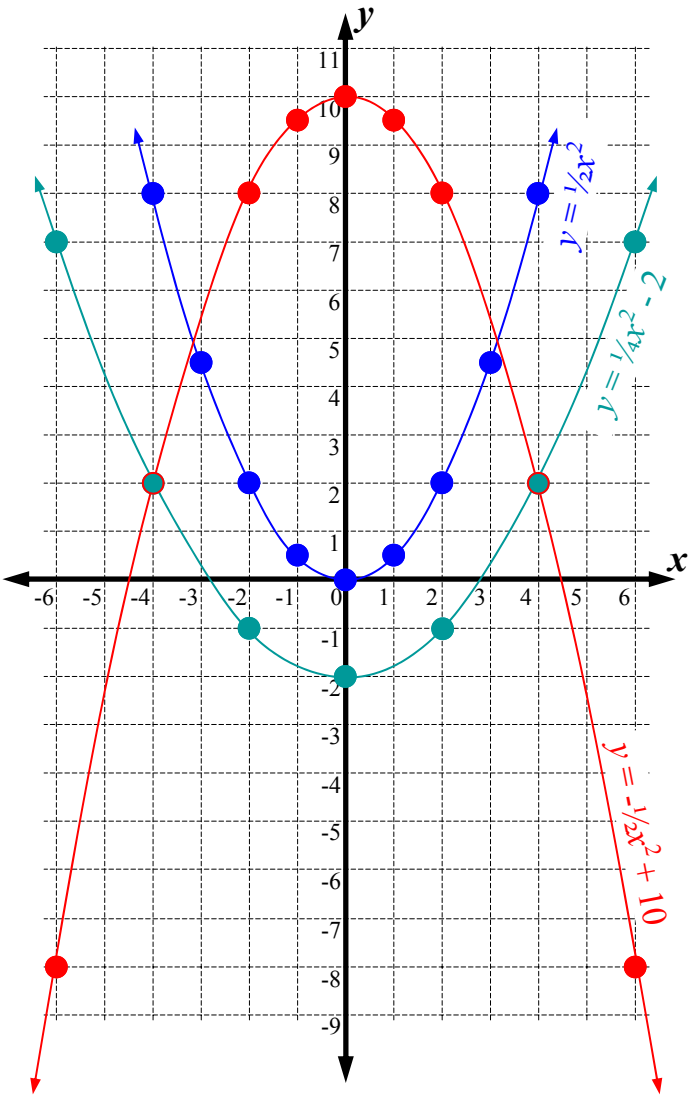
2 $y = -\frac{1}{2}x^2 + 10$

x	-6	-4	-2	-1	0	1	2	4	6
y	-8	2	8	9½	10	9½	8	2	-8

Calc: - 1 ▪/□ 2 ▶ × (- 6) x^2 + 1 0 = Then use the back button

3 $y = \frac{1}{4}x^2 - 2$

x	-6	-4	-2	0	2	4	6
y	7	2	-1	-2	-1	2	7



4 $y = \frac{1}{2}x^2 - 2$

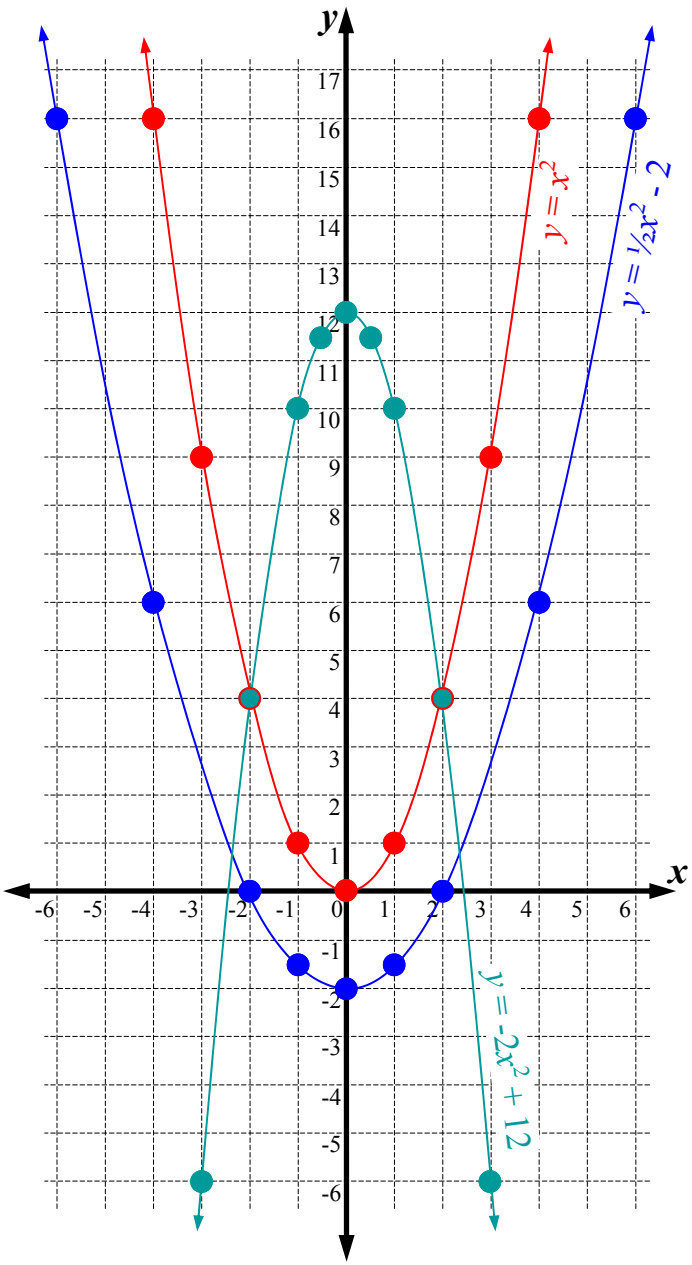
x	-6	-4	-2	-1	0	1	2	4	6
y	16	6	0	-1½	-2	-1½	0	6	16

5 $y = x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

6 $y = -2x^2 + 12$

x	-3	-2	-1	-½	0	½	1	2	3
y	-6	4	10	11½	12	11½	10	4	-6



Non-Linear Relationships - Further Plotting Parabolas (Homework Strip)

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Date Due:

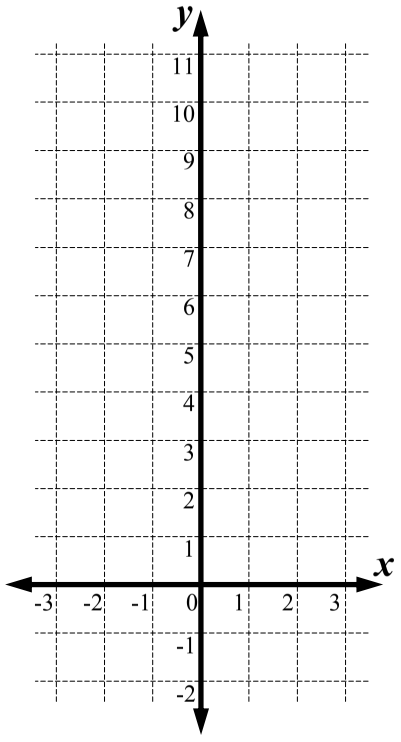
Complete the table of values and create the two parabolas.

1 $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							

2 $y = -x^2 + 7$

x	-3	-2	-1	0	1	2	3
y							



Date Due:

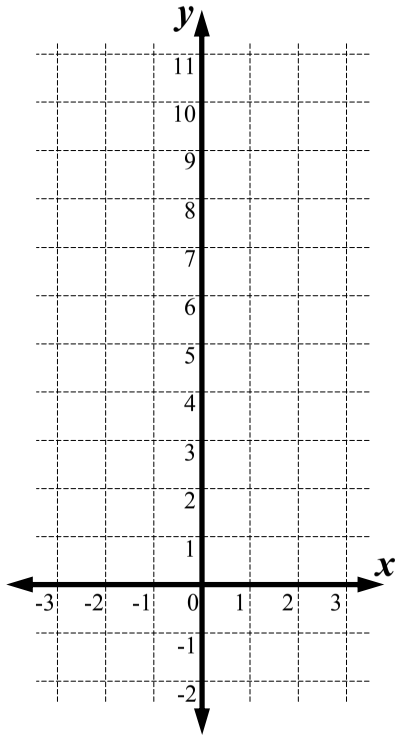
Complete the table of values and create the two parabolas.

1 $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							

2 $y = -x^2 + 7$

x	-3	-2	-1	0	1	2	3
y							



Date Due:

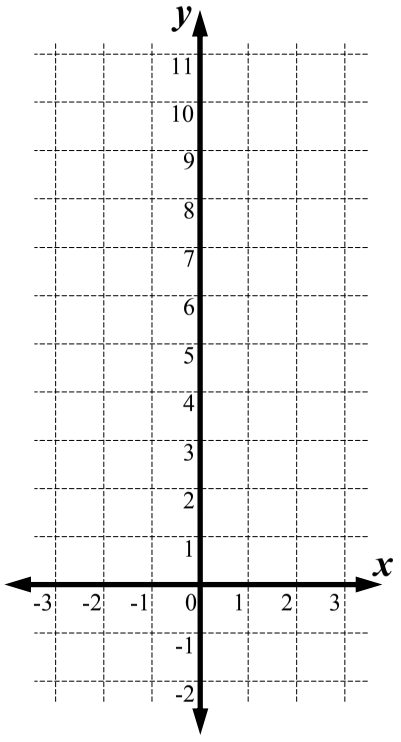
Complete the table of values and create the two parabolas.

1 $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							

2 $y = -x^2 + 7$

x	-3	-2	-1	0	1	2	3
y							



Date Due:

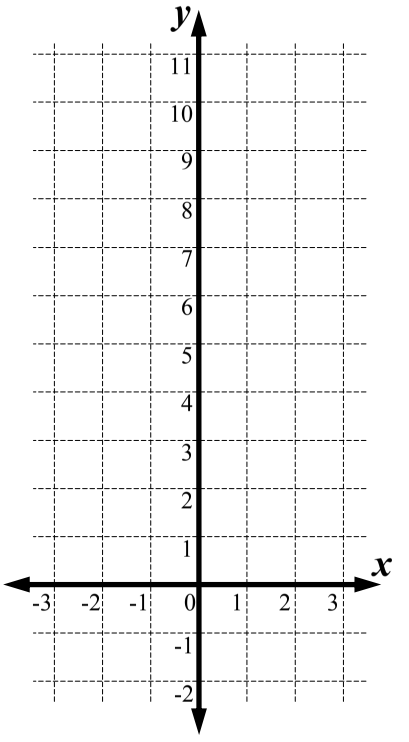
Complete the table of values and create the two parabolas.

1 $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							

2 $y = -x^2 + 7$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Further Plotting Parabolas (Homework Strip)

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Date Due:

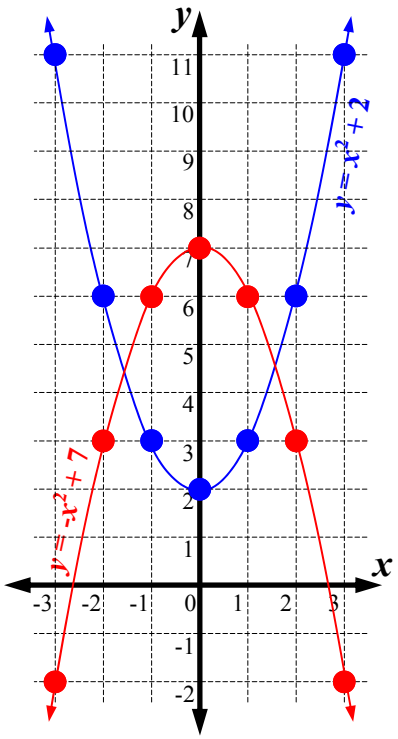
Complete the table of values and create the two parabolas.

1 $y = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y	11	6	3	2	3	6	11

2 $y = -x^2 + 7$

x	-3	-2	-1	0	1	2	3
y	-2	3	6	7	6	3	-2



Non-Linear Relationships - Plotting Exponentials

This sheet requires you to complete a table of values and then plot points which will form an exponential curve. All exponentials on these worksheets are for positive x-values.

Exponentials accelerate quickly and in the space of three x-values, you can have y-values that are thousands of times larger. So when you complete a table of values, you have to be aware that you may not be able to plot all the values from the table on the graph.

STEP 1

First complete the table. You use the power key on your calculator.

STEP 2

Remember that x moves across and y moves up and down. Plot the points on the graph.

STEP 3

Draw the curve. Add an arrowhead and the equation of the curve. Done.

$y = 2^x$

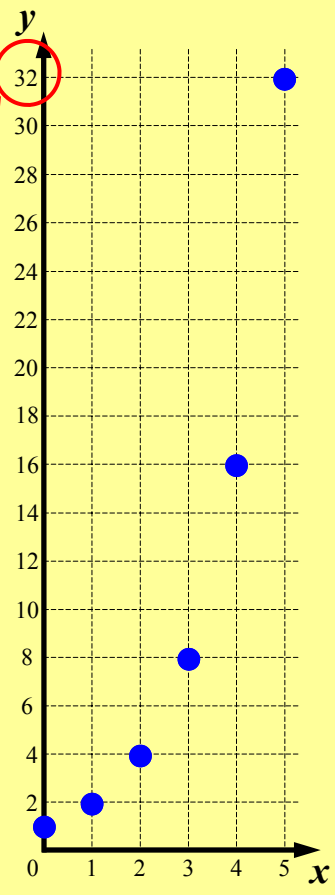
x	0	1	2	3	4	5	6
y	1	2	4	8	16	32	64

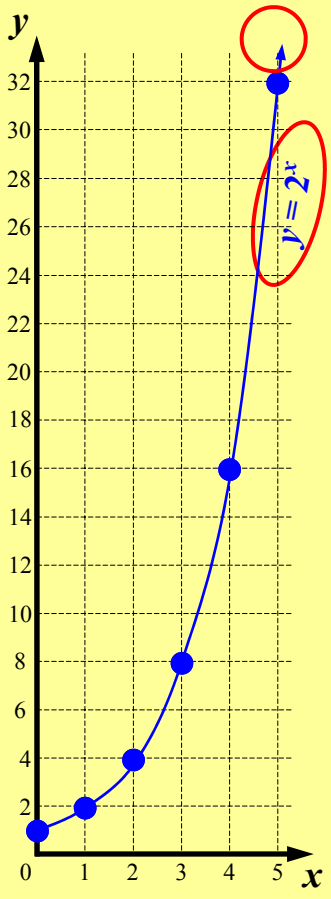
Calc: 2 $x^{\blacksquare}$ 2 =

Calc: 2 $x^{\blacksquare}$ 1 =

Calc: 2 $x^{\blacksquare}$ 0 =

Note that (6, 64) isn't plotted as it won't fit on the graph.





Non-Linear Relationships - Plotting Exponentials (Sheet 1)

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Complete the table of values below. Use your power key.

1 $y = 2^x$

x	0	1	2	3	4	5	6
y							

Calc: 2 $x^{\blacksquare}$ 0 =

2 $y = 3^x$

x	0	1	2	3	4	5
y						

3 $y = 4^x$

x	0	1	2	3	4	5
y						

These negative coefficients will give negative y-values.

4 $y = -2^x$

x	0	1	2	3	4	5	6
y							

Calc: - 2 $x^{\blacksquare}$ 0 =

5 $y = -4^x$

x	0	1	2	3	4	5
y						

6 $y = -5^x$

x	0	1	2	3	4	5
y						

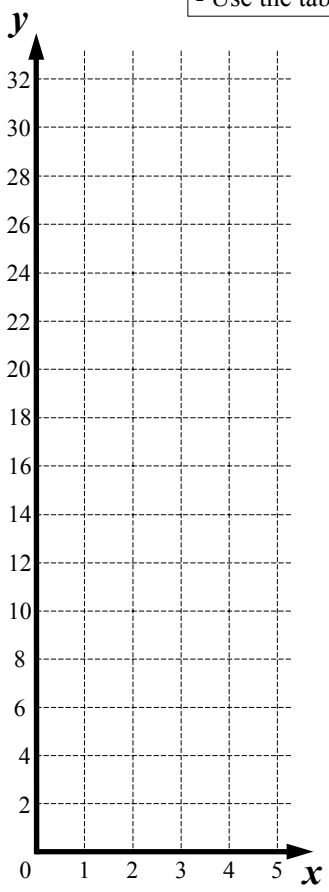
7 $y = -3^x$

x	0	1	2	3	4	5
y						

Use the table of values from your first column to plot these exponentials. Go across 'x' and up 'y'.

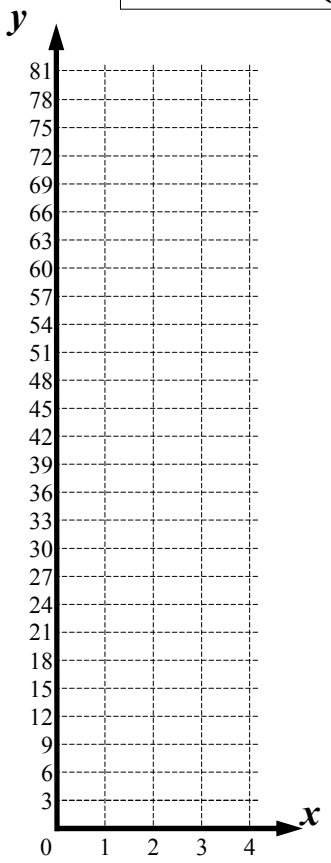
8 $y = 2^x$

- Don't plot (6, 64)
- Equation on curve
- Use the table in Q.1

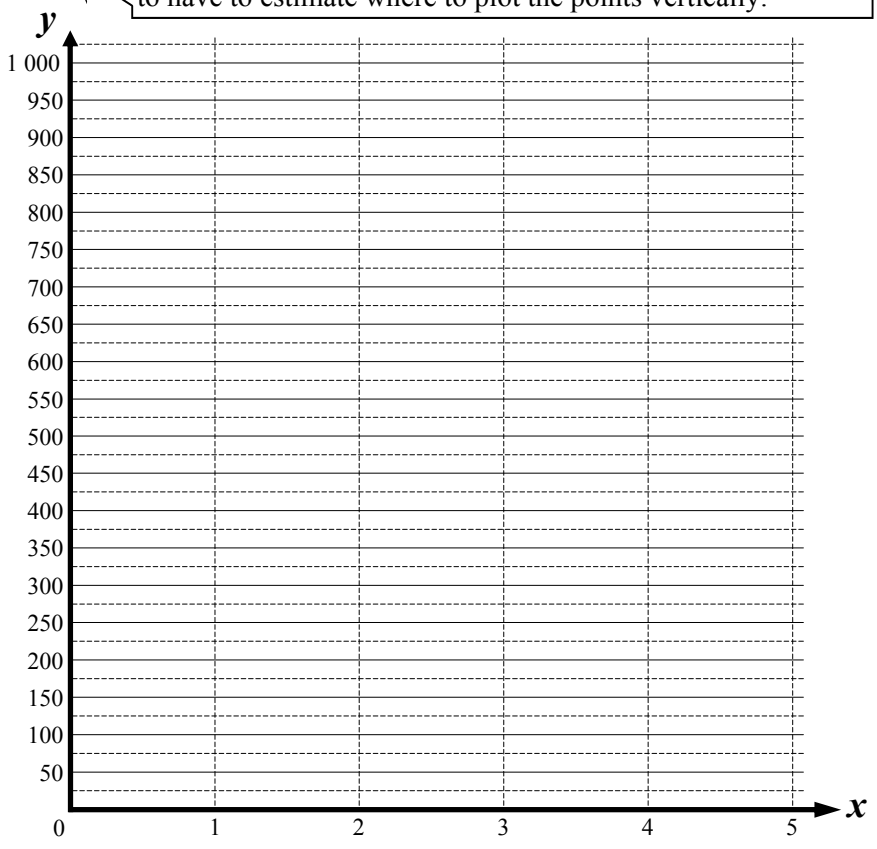


9 $y = 3^x$

- Don't plot (5, 243)
- Equation on curve
- Use the table in Q.2



10 Plot $y = 4^x$ on the grid below. (Table from Q.3). You are going to have to estimate where to plot the points vertically.



Non-Linear Relationships - Plotting Exponentials (Sheet 2)

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Complete the table of values below.
Then plot the curves on the grids.

1 $y = -2^x$

x	0	1	2	3	4	5
y						

- 2 $x^{\blacksquare}$ 0 =

2 $y = -3^x$

x	0	1	2	3	4
y					

3 $y = -4^x$

x	0	1	2	3	4
y					

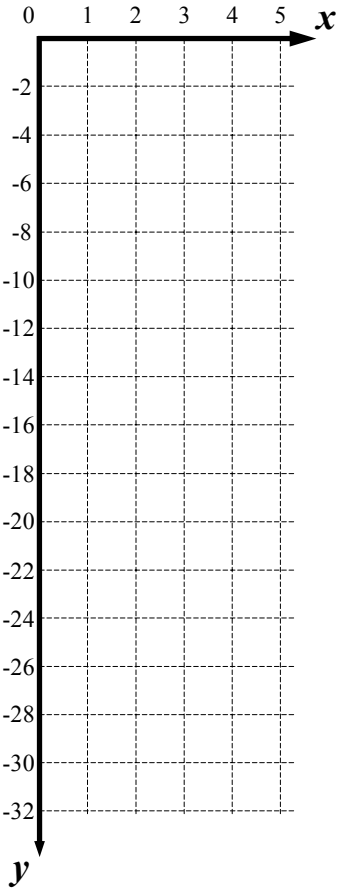
4 $y = 4^x$

x	0	1	2	3	4
y					

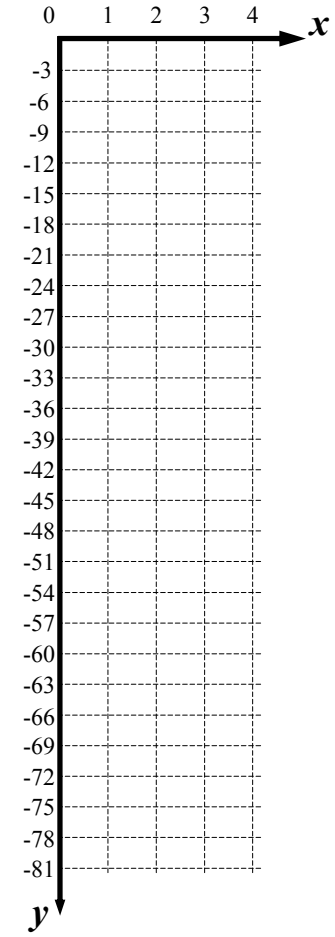
Plot $y = 4^x$ and $y = -4^x$ on the same grid.
(Tables from Q.3 & Q.4)

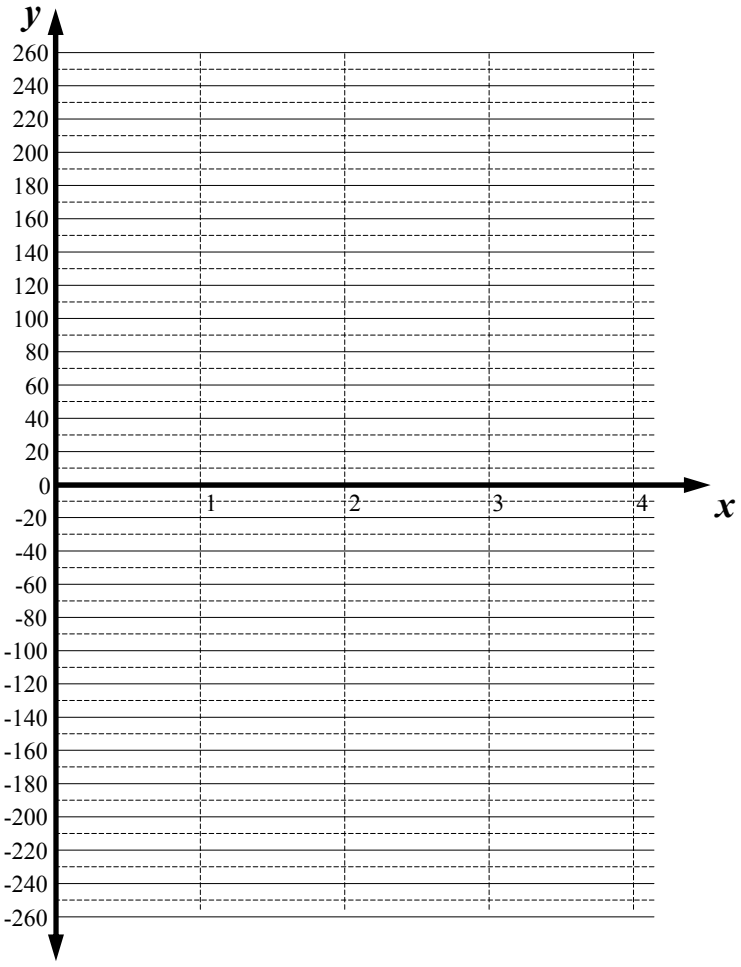
7 $y = -4^x$ & $y = 4^x$

5 $y = -2^x$ (Q.1)



6 $y = -3^x$ (Q.2)





Non-Linear Relationships - Plotting Exponentials (Sheet 1)

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Complete the table of values below. Use your power key.

1 $y = 2^x$

x	0	1	2	3	4	5	6
y	1	2	4	8	16	32	64

Calc: 2 $x^{\blacksquare}$ 0 =

2 $y = 3^x$

x	0	1	2	3	4	5
y	1	3	9	27	81	243

3 $y = 4^x$

x	0	1	2	3	4	5
y	1	4	16	64	256	1 024

These negative coefficients will give negative y-values.

4 $y = -2^x$

x	0	1	2	3	4	5	6
y	-1	-2	-4	-8	-16	-32	-64

Calc: - 2 $x^{\blacksquare}$ 0 =

5 $y = -4^x$

x	0	1	2	3	4	5
y	-1	-4	-16	-64	-256	-1 024

6 $y = -5^x$

x	0	1	2	3	4	5
y	-1	-5	-25	-125	-625	-3 125

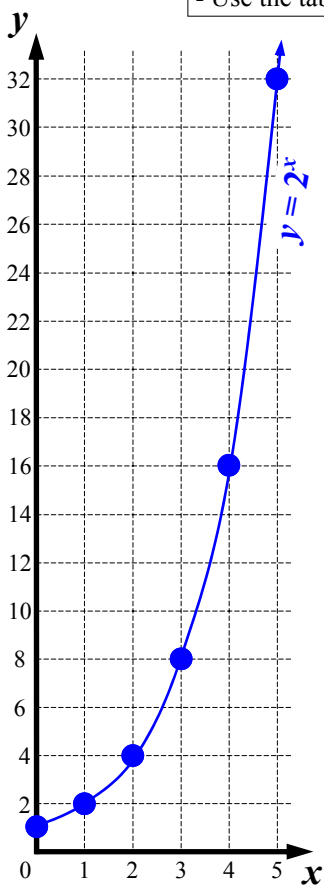
7 $y = -3^x$

x	0	1	2	3	4	5
y	-1	-3	-9	-27	-81	-243

Use the table of values from your first column to plot these exponentials. Go across 'x' and up 'y'.

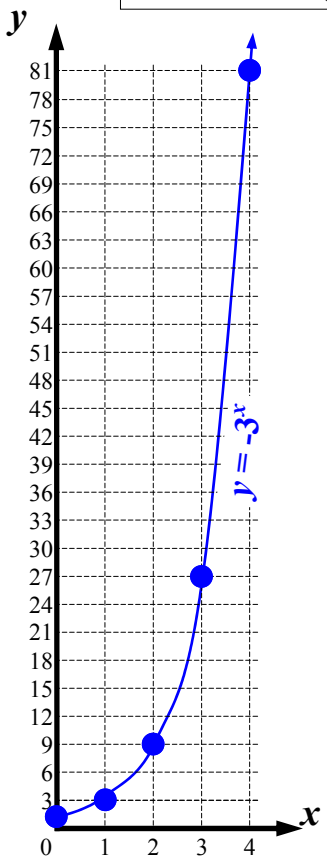
8 $y = 2^x$

- Don't plot (6, 64)
- Equation on curve
- Use the table in Q.1

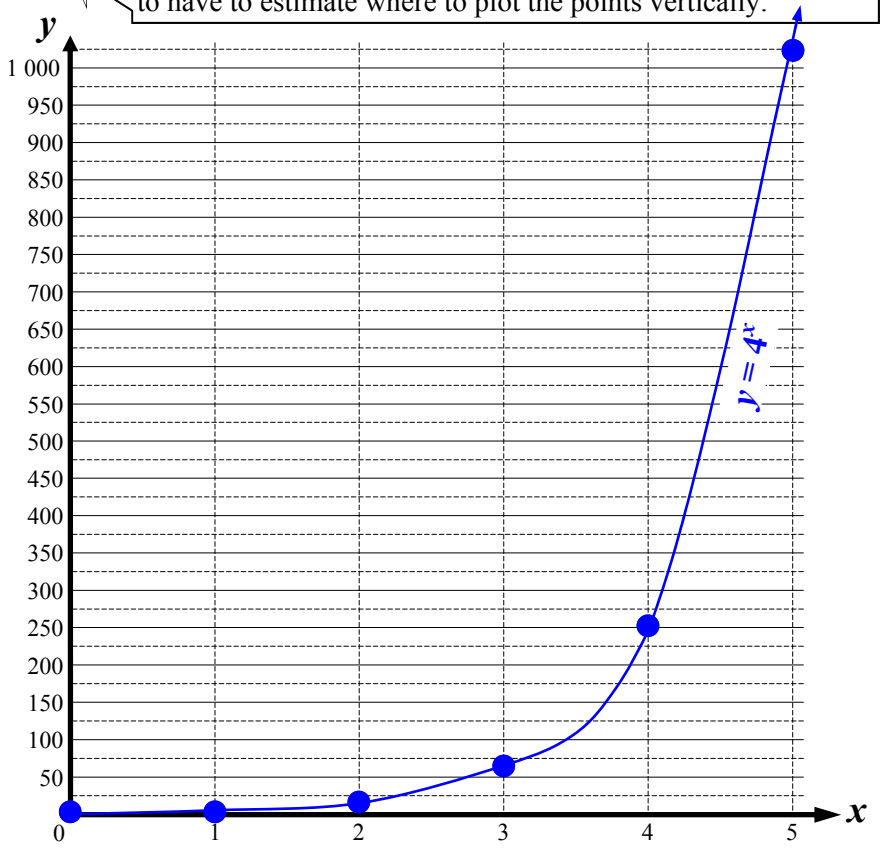


9 $y = 3^x$

- Don't plot (5, 243)
- Equation on curve
- Use the table in Q.2



10 Plot $y = 4^x$ on the grid below. (Table from Q.3). You are going to have to estimate where to plot the points vertically.



Non-Linear Relationships - Plotting Exponentials (Sheet 2)

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Complete the table of values below.
Then plot the curves on the grids.

1 $y = -2^x$

x	0	1	2	3	4	5
y	-1	-2	-4	-8	-16	-32

- 2 $x^{\blacksquare}$ 0 =

2 $y = -3^x$

x	0	1	2	3	4
y	-1	-3	-9	-27	-81

3 $y = -4^x$

x	0	1	2	3	4
y	-1	-4	-16	-64	-256

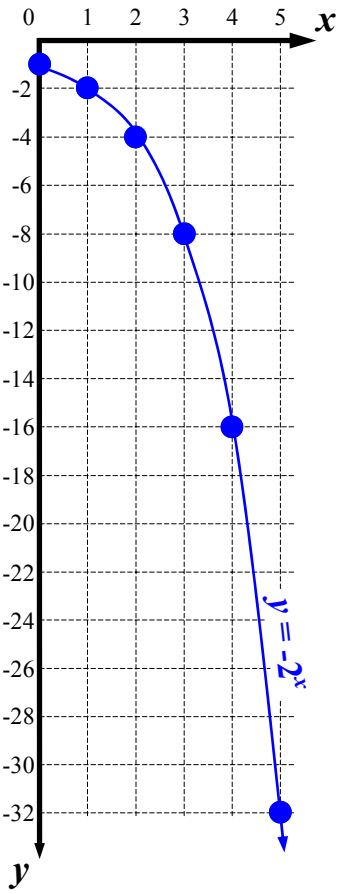
4 $y = 4^x$

x	0	1	2	3	4
y	1	4	16	64	256

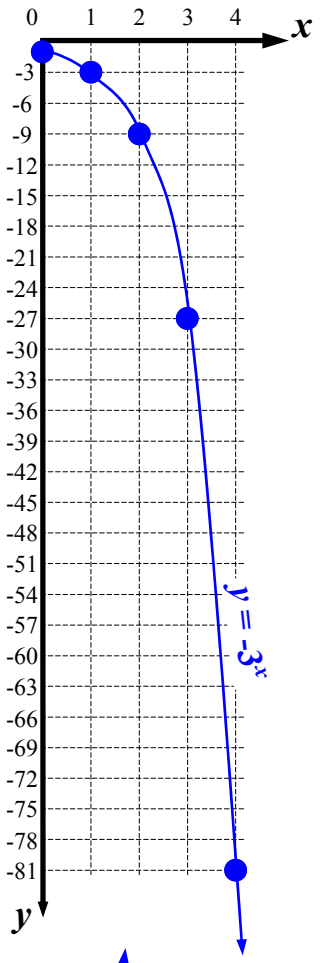
Plot $y = 4^x$ and $y = -4^x$ on the same grid.
(Tables from Q.3 & Q.4)

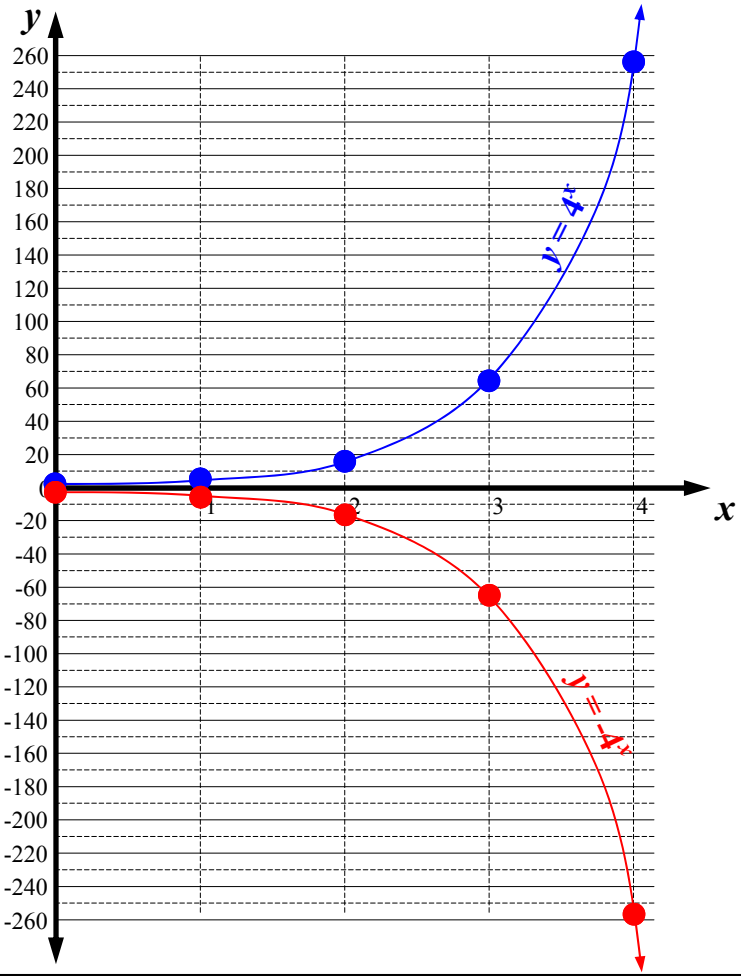
7 $y = -4^x$ & $y = 4^x$

5 $y = -2^x$ (Q.1)



6 $y = -3^x$ (Q.2)





Non-Linear Relationships - Plotting Exponentials (Homework Strip)

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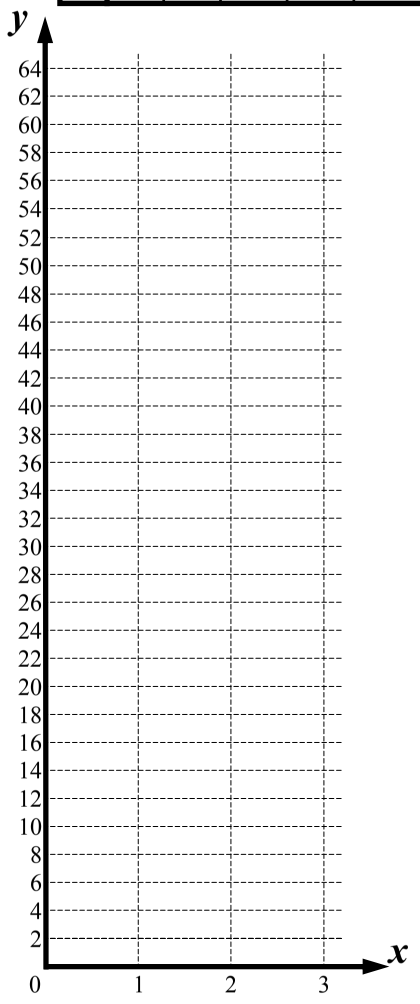
Date Due:

Complete the table of values and plot the exponential.

1 $y = 4^x$

x	0	1	2	3	4
y					

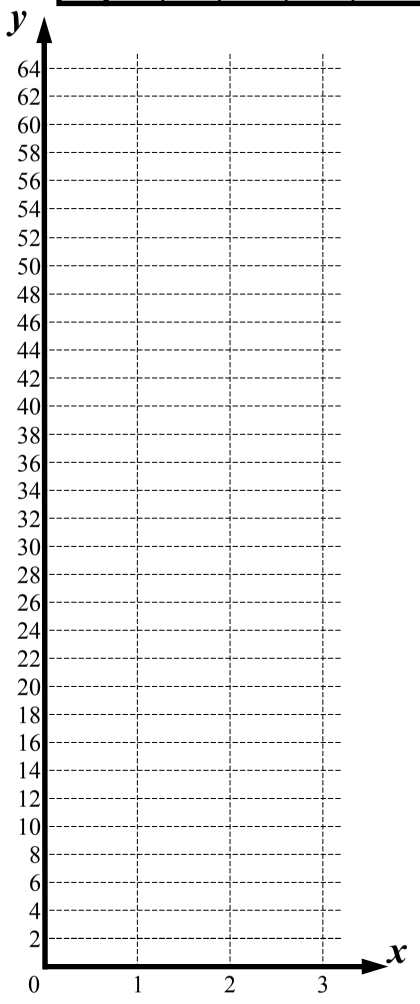


Date Due:

Complete the table of values and plot the exponential.

1 $y = 4^x$

x	0	1	2	3	4
y					

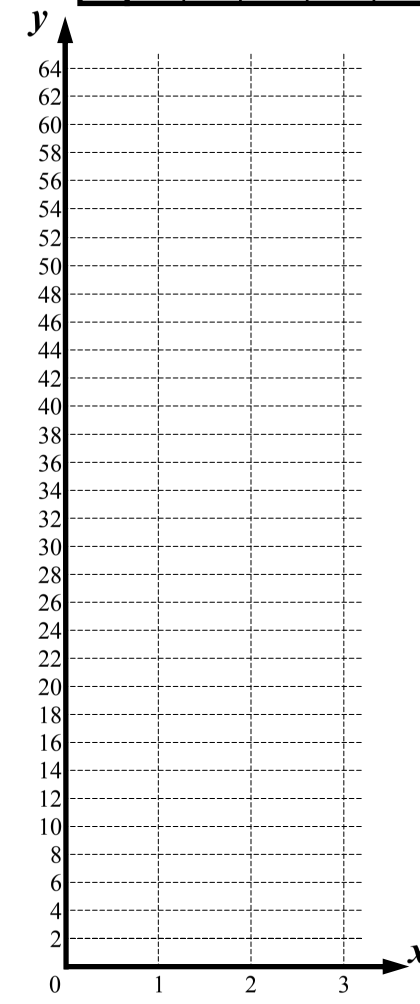


Date Due:

Complete the table of values and plot the exponential.

1 $y = 4^x$

x	0	1	2	3	4
y					

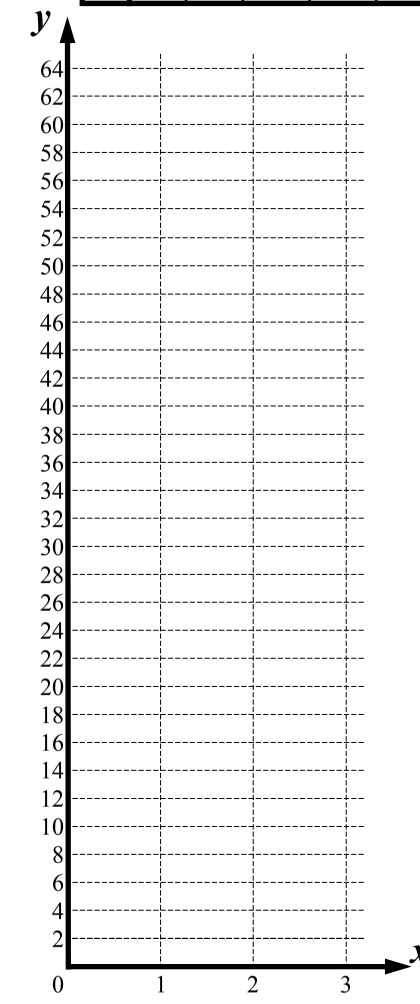


Date Due:

Complete the table of values and plot the exponential.

1 $y = 4^x$

x	0	1	2	3	4
y					



Non-Linear Relationships - Plotting Exponentials (Homework Strip)

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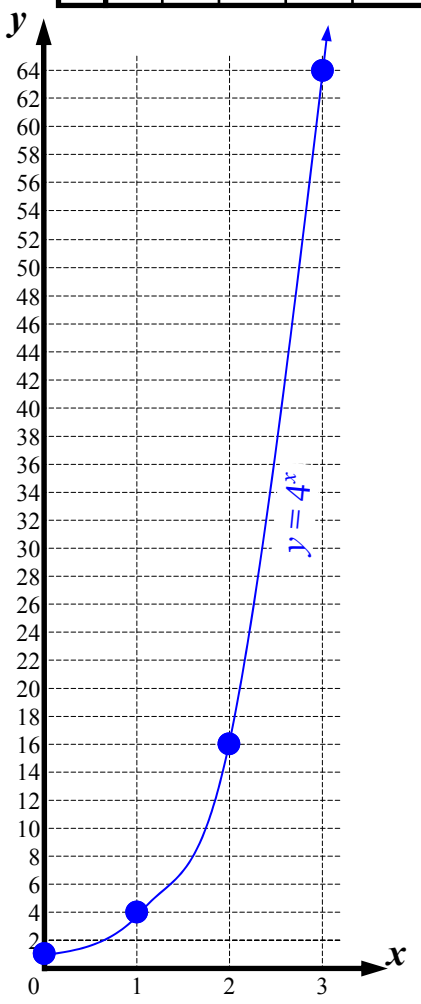
Date Due:

Complete the table of values and plot the exponential.

1

$y = 4^x$

x	0	1	2	3	4
y	1	4	16	64	256



Non-Linear Relationships - Plotting Circles

This sheet requires you to complete a table of values and then plot points which will form a circle.

Circles are different from functions like parabolas and exponentials as for each x-value, there are two y-values. (Except the minimum and maximum x-values). The two y-values are the same except that one is positive and one is negative.

Example Plot the circle: $x^2 + y^2 = 36$.

STEP 1

Rearrange the equation by subtracting x^2 from both sides.

$$x^2 + y^2 = 36 \xrightarrow{\text{Rearrange}} y^2 = 36 - x^2$$

STEP 2

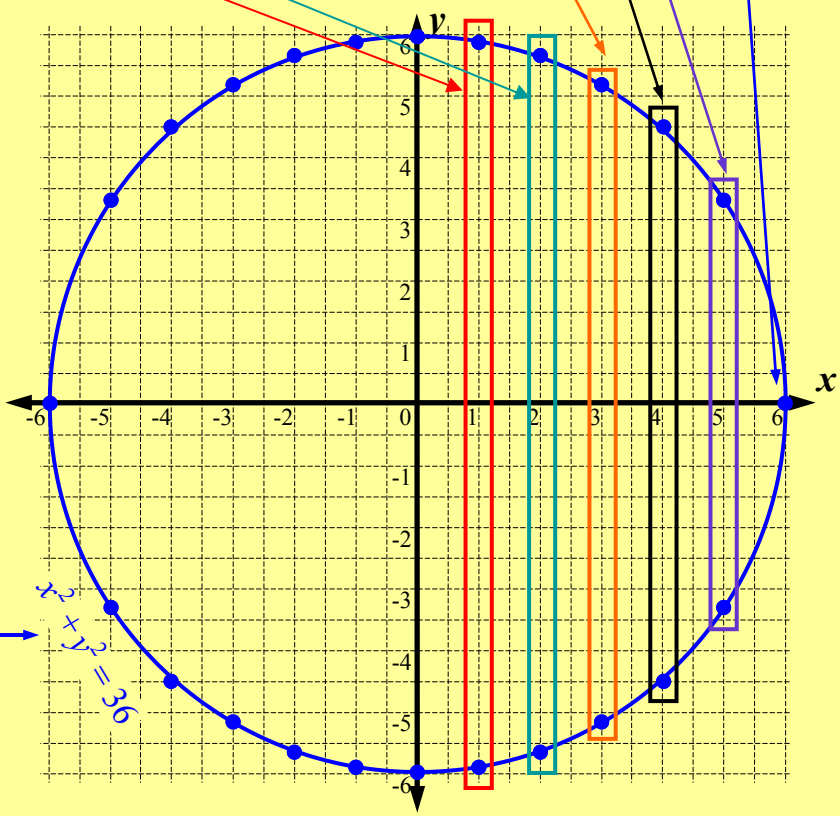
Complete the table, using $y^2 = 36 - x^2$ and substituting the x-values. Round decimal answers to 1 decimal place. Note that the answers are symmetrical across the 0. Use ± for all y-values except the 1st & last.

Symmetrical

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	0	±3.3	±4.5	±5.2	±5.7	±5.9	±6	±5.9	±5.7	±5.2	±4.5	±3.3	0

STEP 3

Plot the points and construct the circle.



STEP 4

Write the original equation on the circle.

Non-Linear Relationships - Plotting Circles (Sheet 1)

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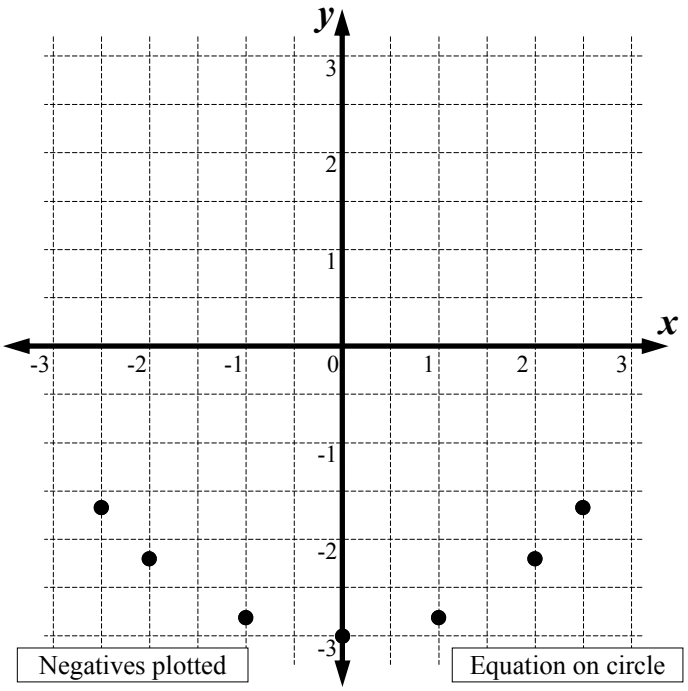
Plot the points for each circle then construct the circle. Write the equation of the circle near it. There are two y-values for each x-value (not ±3).

1 $x^2 + y^2 = 9$

x	-3	-2.5	-2	-1	0	1	2	-2.5	3
y	0	±1.7	±2.2	±2.8	±3	±2.8	±2.2	±1.7	0

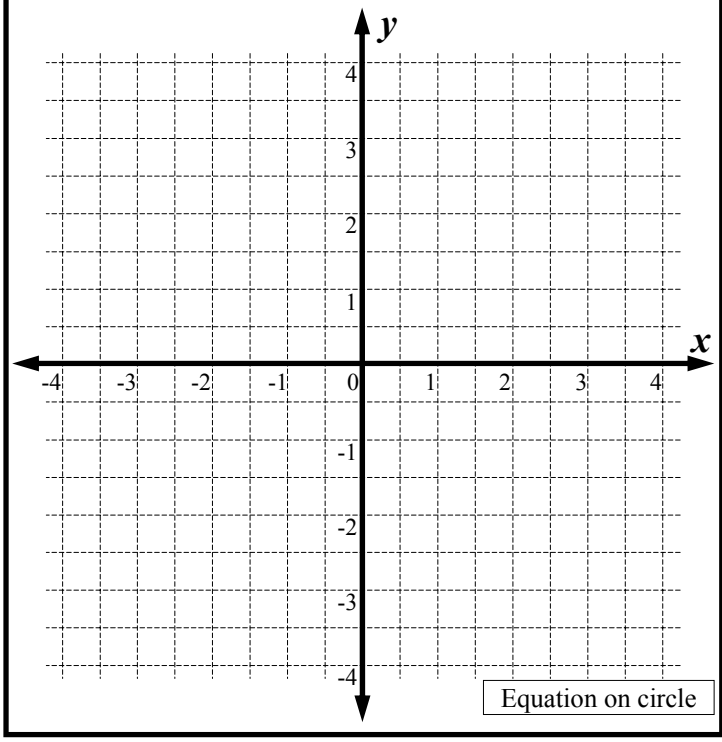


Negatives plotted

Equation on circle

2 $x^2 + y^2 = 16$

x	-4	-3.5	-3	-2	0	2	3	3.5	3
y	0	±1.9	±2.6	±3.5	±4	±3.5	±2.6	±1.9	0



Equation on circle

To complete the table of values yourself, you need to substitute to find y^2 then y. Find y, given these x-values. Round to 1 d.p. if needed (Q. 6).

Example For $y^2 = 20 - x^2$, find y when x is 3.

$y^2 = 20 - x^2$ ← Write equation

$y^2 = 20 - 3^2$ ← Substitute $x = 3$

$y^2 = 11$ ← $20 - 3^2 = 11$

$y = \pm 3.3$ ← $\sqrt{11} = \pm 3.3$ (1 d.p.)

For $y^2 = 25 - x^2$, find y when:

3 $x = 0$

$y^2 =$

$y^2 =$ ← Substitute x-value, $x = 0$

$y^2 =$ ← Calculate y^2

$y = \pm$ ← $\sqrt{\ }$ for y

4 $x = 3$

$y^2 =$ ← Write equation

$y^2 =$ ← Substitute x-value, $x = 3$

$y^2 =$ ← Calculate y^2

$y = \pm$ ← $\sqrt{\ }$ for y

For $y^2 = 50 - x^2$, find y when:

5 $x = 5$

$y^2 =$

$y^2 =$ ← Substitute x-value, $x = 5$

$y^2 =$ ← Calculate y^2

$y =$ ← $\sqrt{\ }$ for y

Write ±

6 $x = 1$

$y^2 =$ ← Write equation

$y^2 =$ ← Substitute x-value, $x = 1$

$y^2 =$ ← Calculate y^2

$y =$ ← $\sqrt{\ }$ for y

Write ±

For $y^2 = 36 - x^2$, find y when:

6 $x = 3$

$y^2 =$

$y^2 =$

$y^2 =$

$y =$

Write ±, to 1 d.p.

6 $x = 5$

$y^2 =$

$y^2 =$

$y^2 =$

$y =$

Write ±, to 1 d.p.

Non-Linear Relationships - Plotting Circles (Sheet 2)

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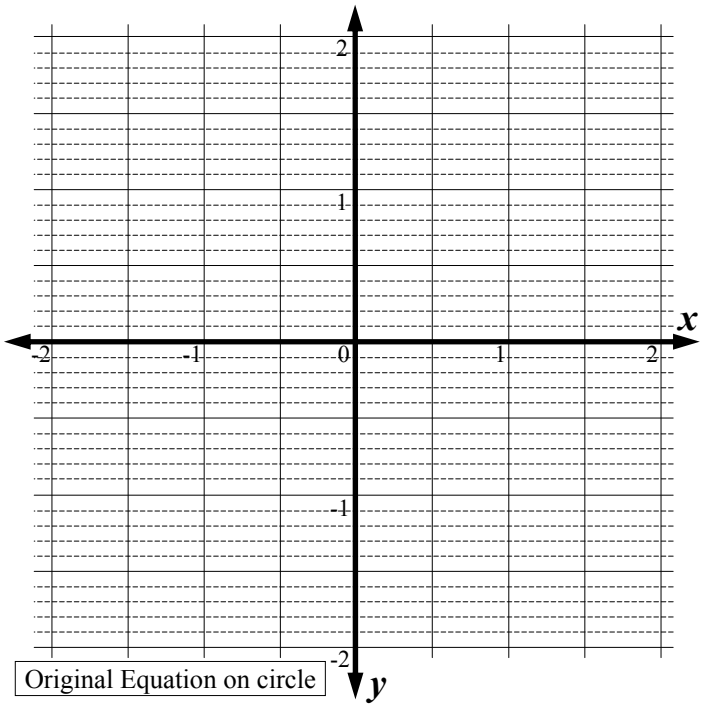
Rewrite the circle equation by subtracting the x^2 term (Q.1. done for you). Complete the table and plot the circle. Round to 1 d.p. when required.

1 $x^2 + y^2 = 4 \xrightarrow{\text{Rearrange}} y^2 = 4 - x^2$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y		$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	

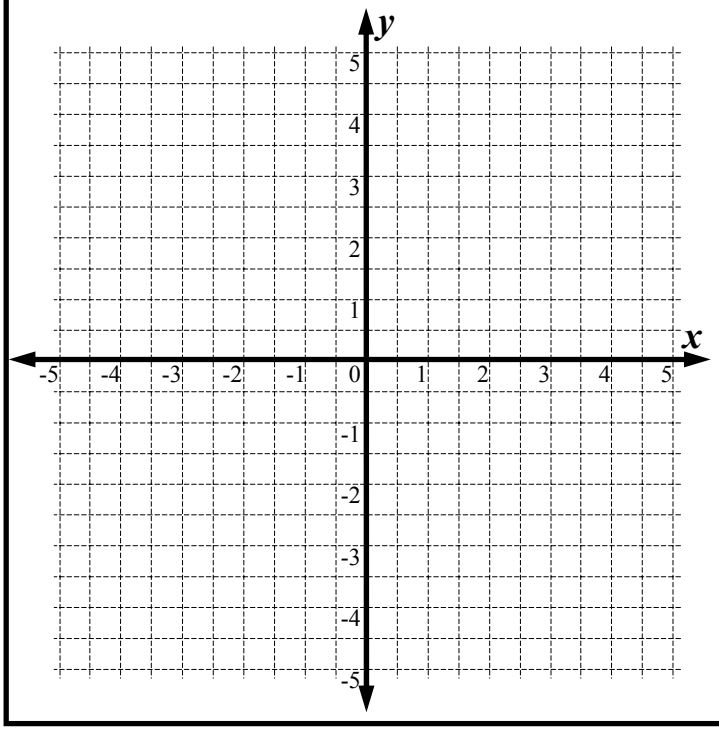
Calc: 4 - (- 2) x^2 = $\sqrt{\blacksquare}$ ANS =



Original Equation on circle

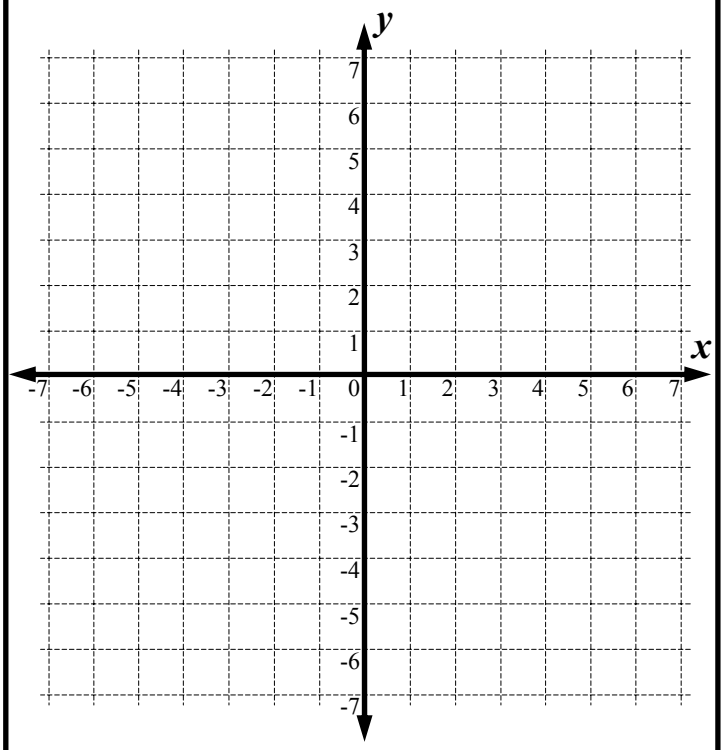
2 $x^2 + y^2 = 25 \xrightarrow{\text{Rearrange}} y^2 =$ ____

x	-5	-4	-3	-2	0	2	3	4	5
y		$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	



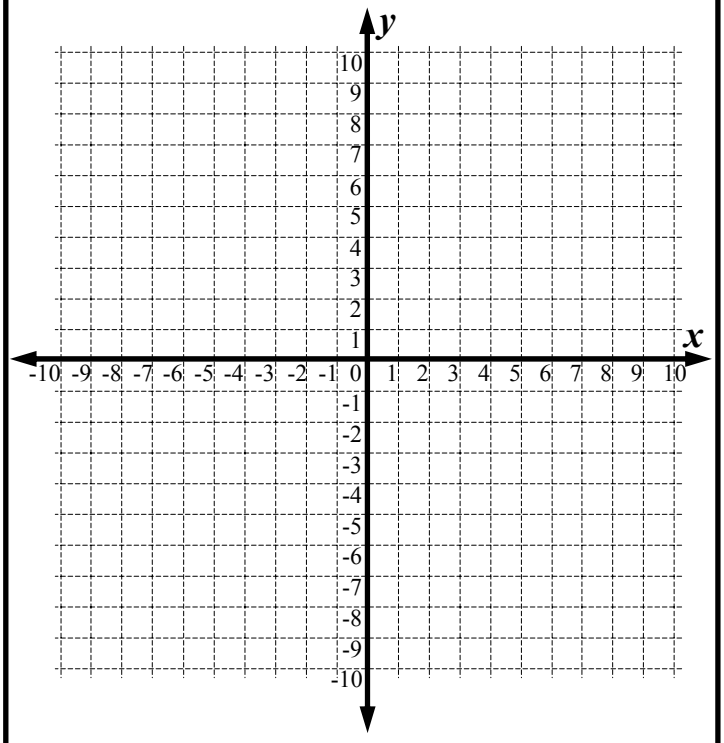
3 $x^2 + y^2 = 49 \Longrightarrow$ ____

x	-7	-6	-4	-2	0	2	4	6	7
y									



4 $x^2 + y^2 = 100 \Longrightarrow$ ____

x	-10	-8	-6	-4	0	-4	6	8	10
y									



Non-Linear Relationships - Plotting Circles (Sheet 1)

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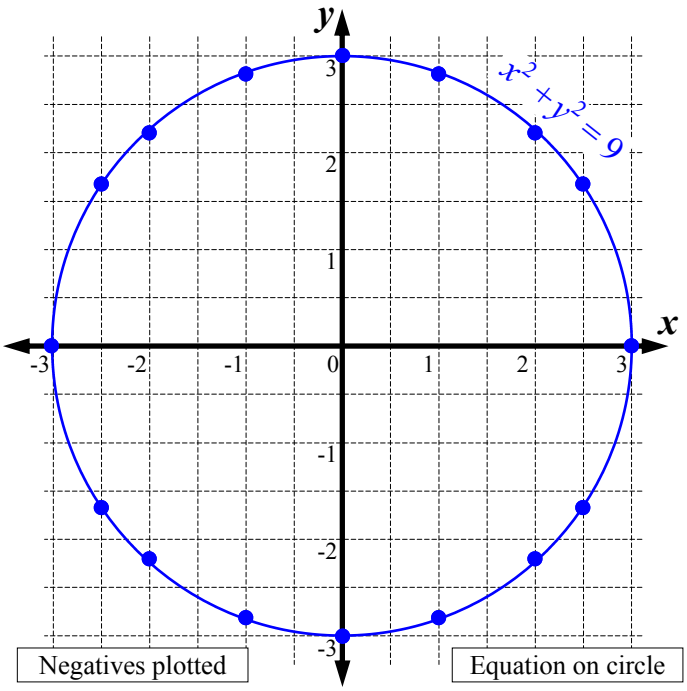
Plot the points for each circle then construct the circle. Write the equation of the circle near it. There are two y-values for each x-value (not ±3).

1 $x^2 + y^2 = 9$

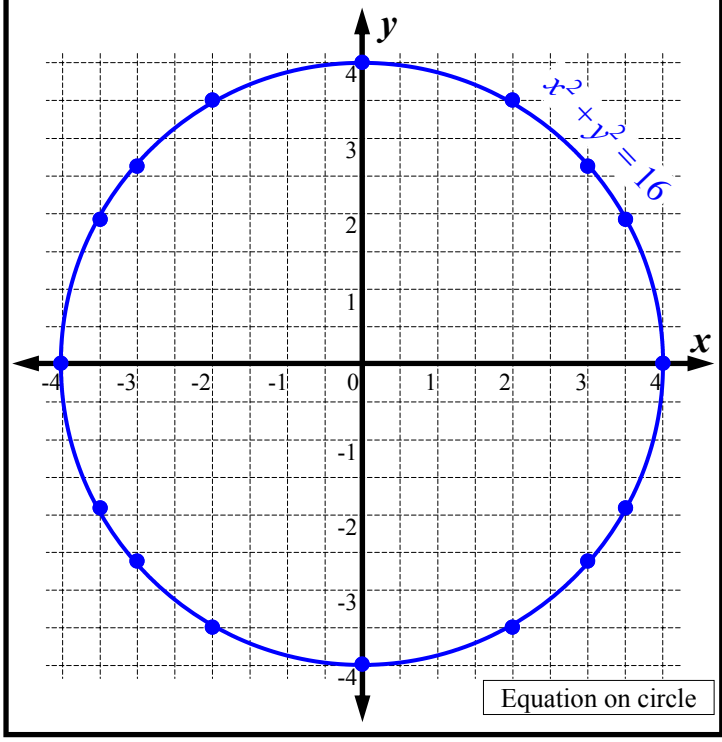
x	-3	-2.5	-2	-1	0	1	2	-2.5	3
y	0	±1.7	±2.2	±2.8	±3	±2.8	±2.2	±1.7	0



Negatives plotted | Equation on circle

2 $x^2 + y^2 = 16$

x	-4	-3.5	-3	-2	0	2	3	3.5	3
y	0	±1.9	±2.6	±3.5	±4	±3.5	±2.6	±1.9	0



Equation on circle

To complete the table of values yourself, you need to substitute to find y^2 then y. Find y, given these x-values. Round to 1 d.p. if needed.

Example For $y^2 = 20 - x^2$, find y when x is 3.

$y^2 = 20 - x^2$ ← Write equation

$y^2 = 20 - 3^2$ ← Substitute $x = 3$

$y^2 = 11$ ← $20 - 3^2 = 11$

$y = \pm 3.3$ ← $\sqrt{11} = \pm 3.3$ (1 d.p.)

For $y^2 = 25 - x^2$, find y when:

3 $x = 0$

$y^2 = 25 - x^2$ ← Write equation

$y^2 = 25 - 0^2$ ← Substitute x-value ($x = 0$)

$y^2 = 25$ ← Calculate y^2

$y = \pm 5$ ← $\sqrt{\ }$ for y

4 $x = 3$

$y^2 = 25 - x^2$ ← Write equation

$y^2 = 25 - 3^2$ ← Substitute x-value ($x = 3$)

$y^2 = 16$ ← Calculate y^2

$y = \pm 4$ ← $\sqrt{\ }$ for y

For $y^2 = 50 - x^2$, find y when:

5 $x = 5$

$y^2 = 50 - x^2$ ← Write equation

$y^2 = 50 - 5^2$ ← Substitute x-value ($x = 5$)

$y^2 = 25$ ← Calculate y^2

$y = \pm 5$ ← $\sqrt{\ }$ for y

Write ±

6 $x = 1$

$y^2 = 50 - x^2$ ← Write equation

$y^2 = 50 - 1^2$ ← Substitute x-value ($x = 1$)

$y^2 = 49$ ← Calculate y^2

$y = \pm 7$ ← $\sqrt{\ }$ for y

Write ±

For $y^2 = 36 - x^2$, find y when:

6 $x = 3$

$y^2 = 36 - x^2$

$y^2 = 36 - 3^2$

$y^2 = 27$

$y = \pm 5.2$

Write ±, to 1 d.p.

6 $x = 5$

$y^2 = 36 - x^2$

$y^2 = 36 - 5^2$

$y^2 = 11$

$y = \pm 3.3$

Write ±, to 1 d.p.

Non-Linear Relationships - Plotting Circles (Sheet 2)

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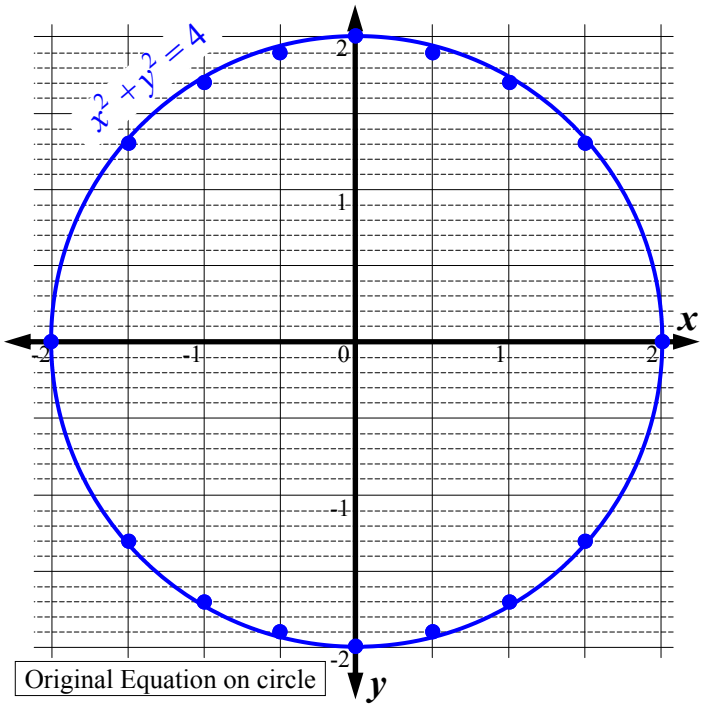
Rewrite the circle equation by subtracting the x^2 term (Q.1. done for you). Complete the table and plot the circle. Round to 1 d.p. when required.

1 $x^2 + y^2 = 4 \Longrightarrow$ Rearrange $\Longrightarrow y^2 = 4 - x^2$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	0	$\pm$1.3	$\pm$1.7	$\pm$1.9	$\pm$2	$\pm$1.9	$\pm$1.7	$\pm$1.3	0

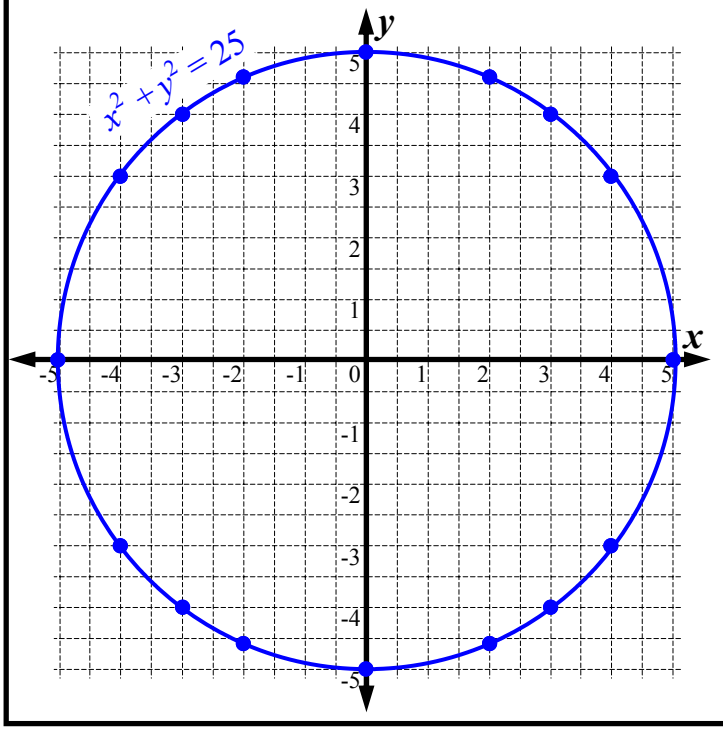
Calc: 4 - (- 2) x^2 = $\int\blacksquare$ ANS =



Original Equation on circle

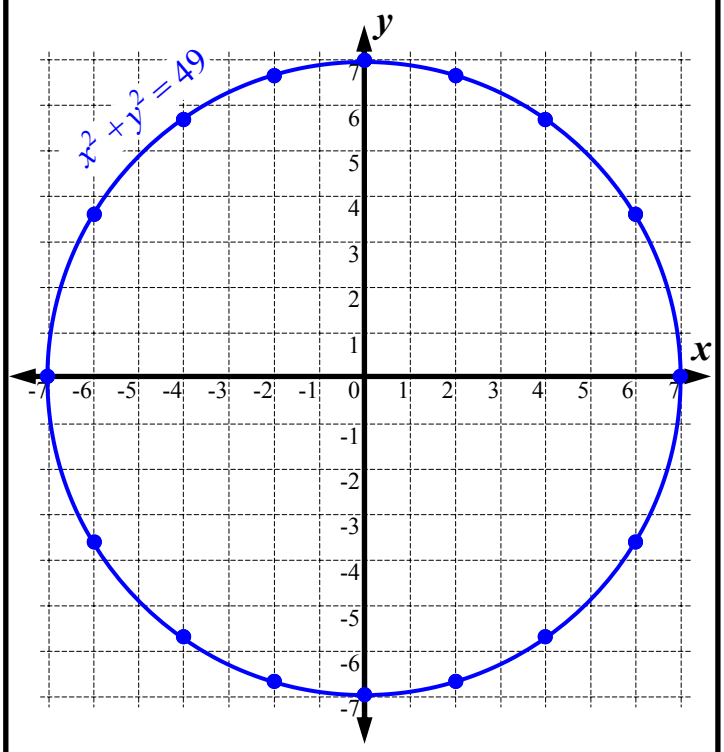
2 $x^2 + y^2 = 25 \Longrightarrow$ Rearrange $\Longrightarrow y^2 =$ $25 - x^2$

x	-5	-4	-3	-2	0	2	3	4	5
y	0	$\pm$ 3	$\pm$ 4	$\pm$4.6	$\pm$ 5	$\pm$4.6	$\pm$ 4	$\pm$ 3	0



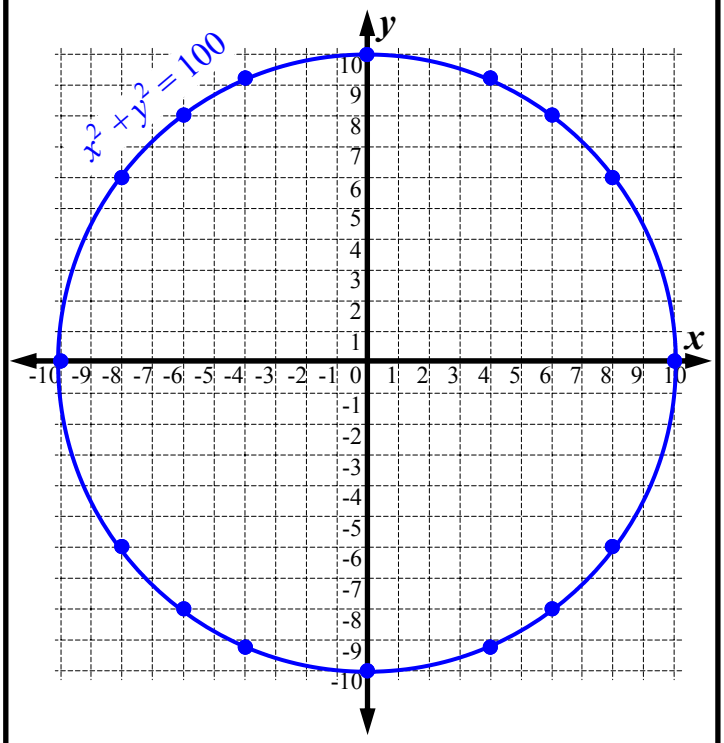
3 $x^2 + y^2 = 49 \Longrightarrow$ $y^2 = 49 - x^2$

x	-7	-6	-4	-2	0	2	4	6	7
y	0	$\pm$3.6	$\pm$5.7	$\pm$6.7	$\pm$ 7	$\pm$6.7	$\pm$5.7	$\pm$3.6	0



4 $x^2 + y^2 = 100 \Longrightarrow$ $y^2 = 100 - x^2$

x	-10	-8	-6	-4	0	-4	6	8	10
y	0	$\pm$6	$\pm$8	$\pm$9.2	$\pm$ 10	$\pm$9.2	$\pm$8	$\pm$6	0



Non-Linear Relationships - Plotting Circles (Homework Strip)

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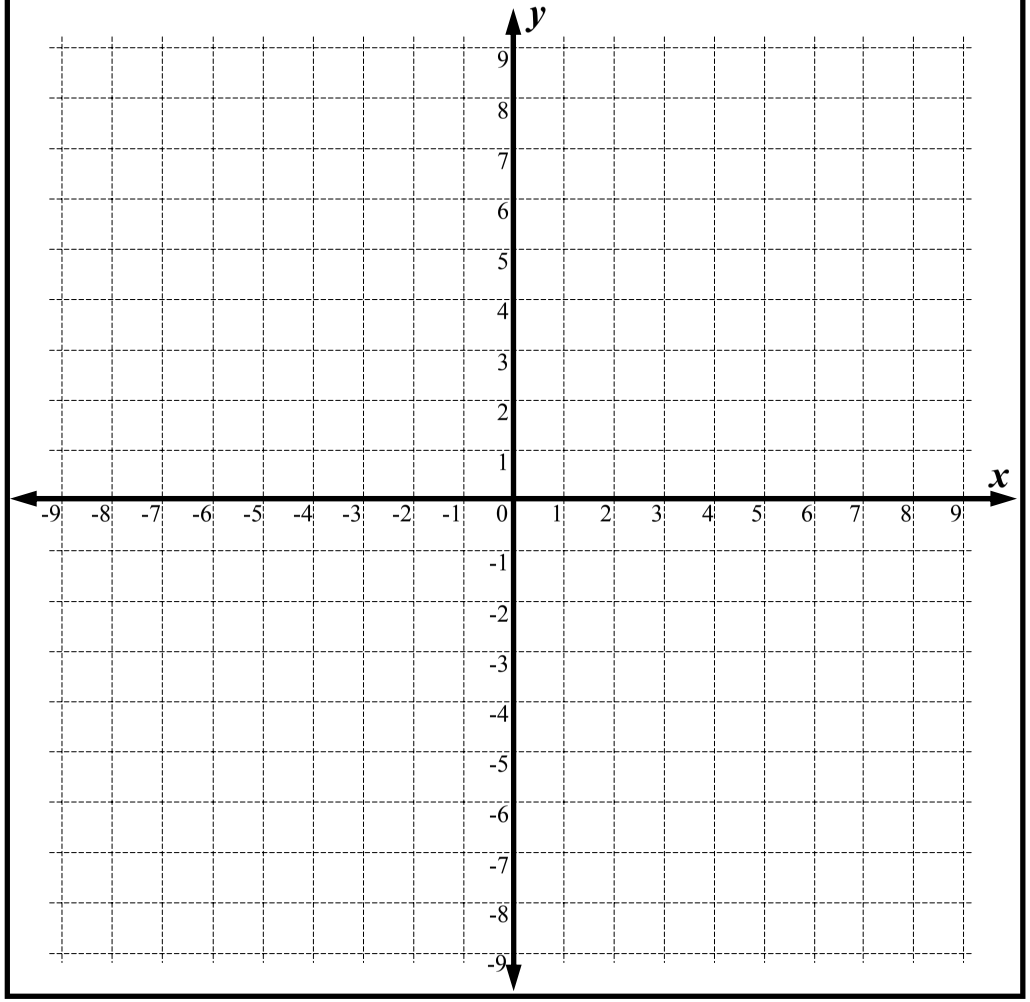
Date Due:

Complete the table of values and plot the circle.

1 $x^2 + y^2 = 81$ $\xrightarrow{\text{Rearrange}}$ $y^2 = 81 - x^2$

x	-9	-8	-7	-6	-4	-2	0	2	4	6	7	8	9
y		$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	

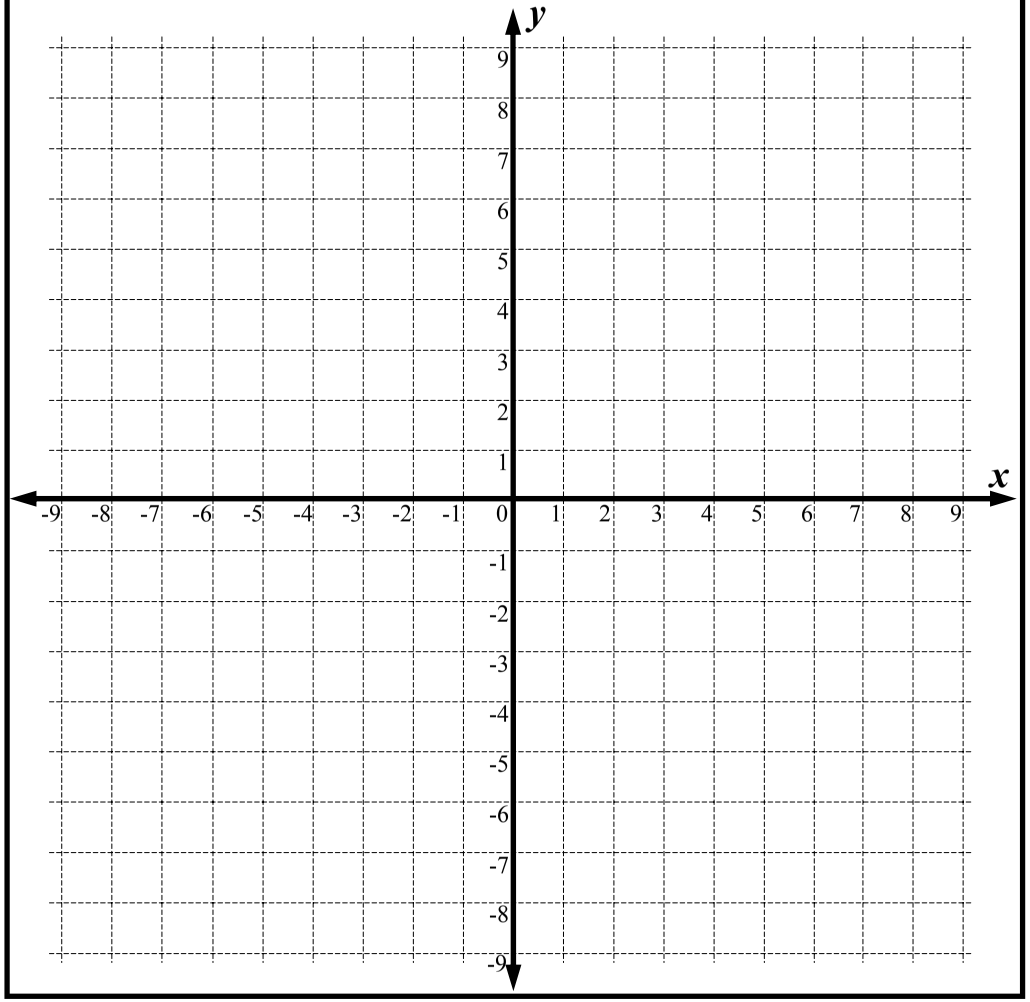


Date Due:

Complete the table of values and plot the circle.

1 $x^2 + y^2 = 81$ $\xrightarrow{\text{Rearrange}}$ $y^2 = 81 - x^2$

x	-9	-8	-7	-6	-4	-2	0	2	4	6	7	8	9
y		$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	



Non-Linear Relationships - Plotting Circles (Homework Strip)

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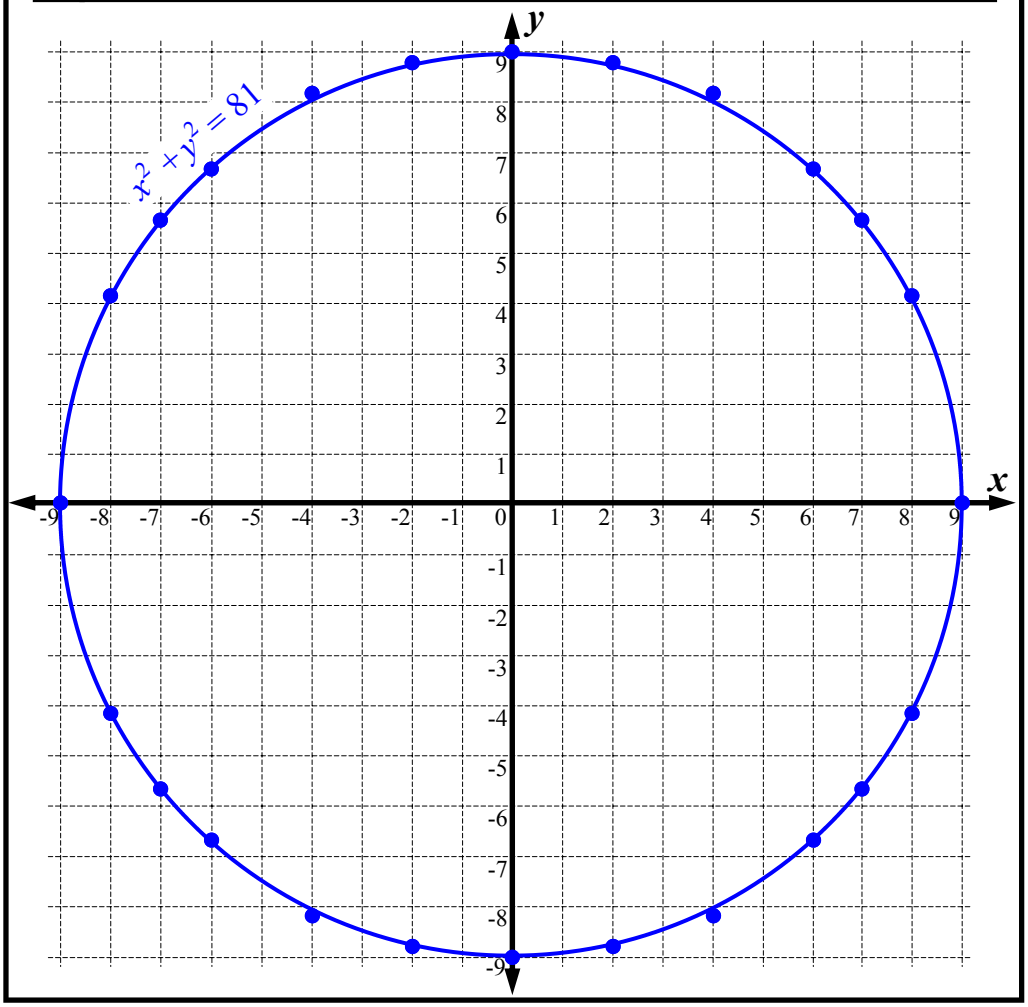
Date Due:

Complete the table of values and plot the circle.

1 $x^2 + y^2 = 81 \xrightarrow{\text{Rearrange}} y^2 = 81 - x^2$

x	-9	-8	-7	-6	-4	-2	0	2	4	6	7	8	9
y	0	$\pm$4.1	$\pm$5.7	$\pm$6.7	$\pm$8.1	$\pm$8.8	$\pm$ 9	$\pm$8.8	$\pm$8.1	$\pm$6.7	$\pm$5.7	$\pm$4.1	0



Non-Linear Relationships - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don't colour it in too dark (using a texta for example) so that you can still read it.

Non-Linear Relationships - Find a Word

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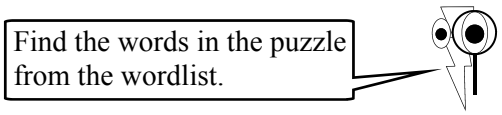

N	I	N	T	E	R	C	E	P	T	P	C	P	E	T
E	E	T	T	N	E	I	C	I	F	F	E	O	C	T
T	S	I	F	E	T	D	A	L	O	B	A	R	A	P
U	N	R	L	L	I	C	C	O	N	C	A	V	E	E
T	V	B	E	R	A	P	M	O	C	C	L	O	T	E
I	A	S	G	W	O	T	I	N	T	E	A	E	T	T
T	L	E	I	I	O	T	T	N	B	P	C	V	N	A
S	U	I	N	X	A	P	L	A	N	E	I	I	E	R
B	E	T	N	U	A	J	L	X	E	X	R	T	G	E
U	S	R	Q	E	X	P	O	N	E	N	T	I	A	L
S	K	E	T	C	H	P	A	R	G	V	E	S	T	E
P	N	P	C	E	L	C	R	I	C	O	M	O	I	C
L	W	O	U	O	W	O	R	R	A	N	M	P	V	C
O	O	R	T	S	U	I	D	A	R	M	Y	S	E	A
W	D	P	S	Y	M	C	A	R	T	E	S	I	A	N

WORDLIST

POWER	UP	PARABOLA	NEGATIVE	PROPERTIES
INTERCEPT	GRAPH	POINTS	TABLE	NARROW
SUBSTITUTE	FLAT	PLANE	EQUATION	CARTESIAN
POSITIVE	SKETCH	CONCAVE	COMPARE	AXIS
VALUES	CUTS	ACCELERATE	GRID	PLOT
SYMMETRICAL	EXPONENTIAL	DOWN	COEFFICIENT	LABEL
	RADIUS		CIRCLE	

Non-Linear Relationships - Find a Word

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Find the words in the puzzle from the wordlist.



N	I	N	T	E	R	C	E	P	T	P	C	P	E	T
E	E	T	T	N	E	I	C	I	F	F	E	O	C	T
T	S	I	F	E	T	D	A	L	O	B	A	R	A	P
U	N	R	L	L	I	C	C	O	N	C	A	V	E	E
T	V	B	E	R	A	P	M	O	C	C	L	O	T	E
I	A	S	G	W	O	T	I	N	T	E	A	E	T	T
T	L	E	I	I	O	T	T	N	B	P	C	V	N	A
S	U	I	N	X	A	P	L	A	N	E	I	I	E	R
B	E	T	N	U	A	J	L	X	E	X	R	T	G	E
U	S	R	Q	E	X	P	O	N	E	N	T	I	A	L
S	K	E	T	C	H	P	A	R	G	V	E	S	T	E
P	N	P	C	E	L	C	R	I	C	O	M	O	I	C
L	W	O	U	O	W	O	R	R	A	N	M	P	V	C
O	O	R	T	S	U	I	D	A	R	M	Y	S	E	A
W	D	P	S	Y	M	C	A	R	T	E	S	I	A	N

WORDLIST

~~POWER~~
~~INTERCEPT~~
~~SUBSTITUTE~~
~~POSITIVE~~
~~VALUES~~
~~SYMMETRICAL~~
~~UP~~
~~GRAPH~~
~~FLAT~~
~~SKETCH~~
~~CUTS~~
~~EXPONENTIAL~~
~~RADIUS~~
~~PARABOLA~~
~~POINTS~~
~~PLANE~~
~~CONCAVE~~
~~ACCELERATE~~
~~DOWN~~
~~NEGATIVE~~
~~TABLE~~
~~EQUATION~~
~~COMPARE~~
~~GRID~~
~~COEFFICIENT~~
~~CIRCLE~~
~~PROPERTIES~~
~~NARROW~~
~~CARTESIAN~~
~~AXIS~~
~~PLOT~~
~~LABEL~~

Non-Linear Relationships Review

This is the review of the Non-Linear Relationships Topic.

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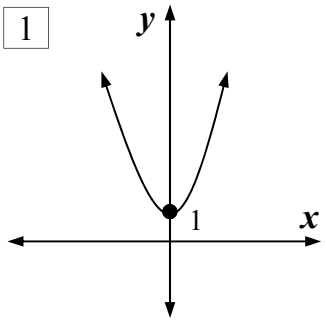
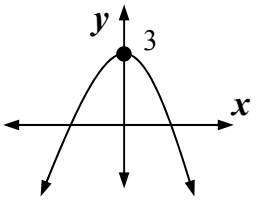
Complete the box to show the parabola's equation

1



$y = x^2 +$ ☐

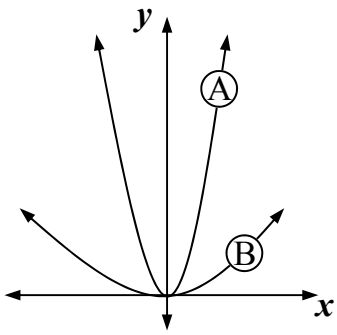
2



$y = -x^2 +$ ☐

Pick the appropriate equation for the parabolas below.

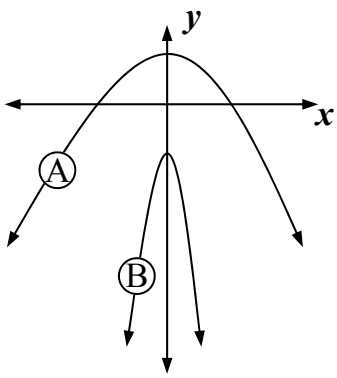
3



$y = 2x^2$ Ⓐ Ⓑ

$y = \frac{1}{2}x^2$ Ⓐ Ⓑ

4



$y = -3x^2 - 2$ Ⓐ Ⓑ

$y = -\frac{1}{2}x^2 + 2$ Ⓐ Ⓑ

Complete the sentence with a number and either up/down.

5 The parabola $y = x^2 + 5$ cuts the y-axis at ☐ and is concave ☐.

6 The parabola $y = -x^2 - 2$ cuts the y-axis at ☐ and is concave ☐.

Complete the sentence with either 'narrow' or 'wide'.

7 The parabola $y = -x^2 - 4$ is more ☐ than $y = -2x^2 - 1$.

8 The parabola $y = 2x^2$ is more ☐ than $y = \frac{1}{2}x^2 + 5$.

Complete these tables.

9 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							

10 $y = -3x^2$

x	-3	-2	-1	0	1	2	3
y							

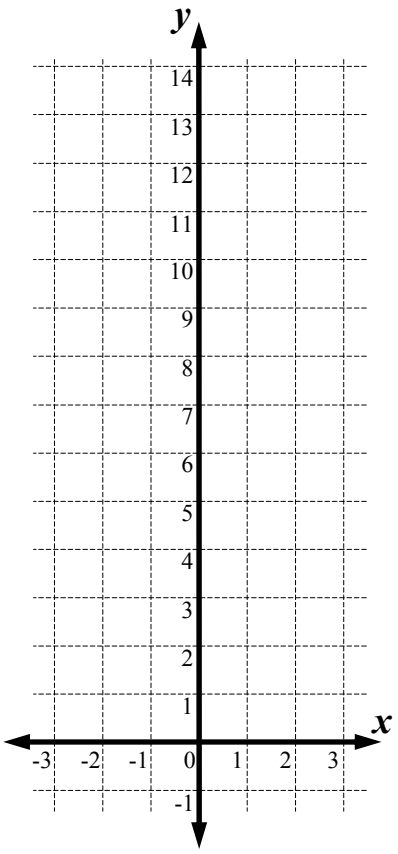
11 $y = \frac{1}{2}x^2 + 4$

x	-6	-4	-2	0	2	4	6
y							

Complete the table then plot the parabola $y = x^2 - 2$ on the number plane.

12 $y = x^2 + 4$

x	-3	-2	-1	0	1	2	3
y							



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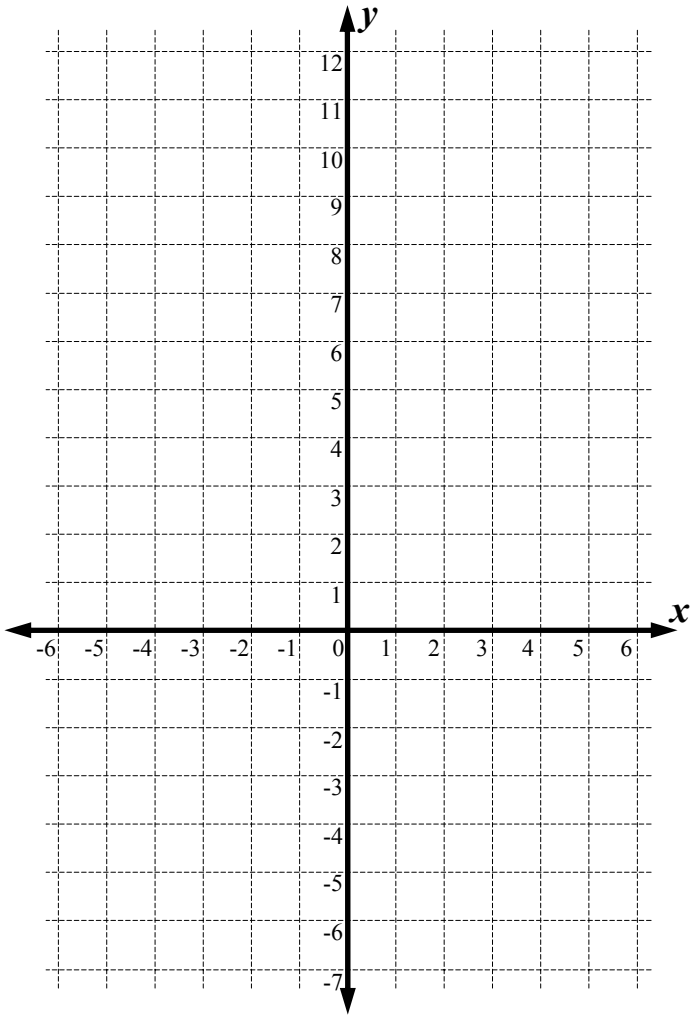
Complete the table, then plot the parabola.

13 $y = \frac{1}{2}x^2 - 6$

x	-6	-4	-2	0	2	4	6
y							



Complete the table of values.

14 $y = 2^x$

x	0	1	2	3	4	5
y						

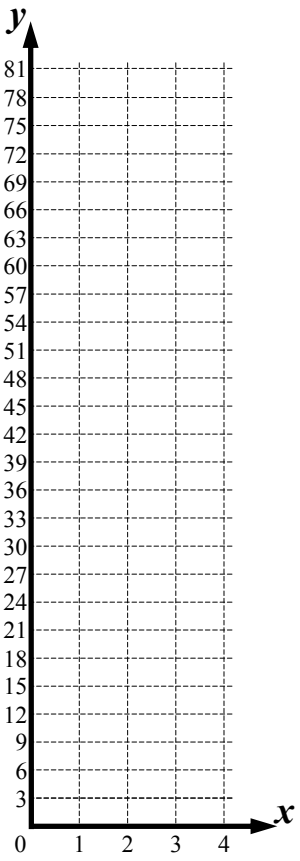
15 $y = 4^x$

x	0	1	2	3	4	5
y						

Complete the table, then plot the exponential.

16 $y = 3^x$

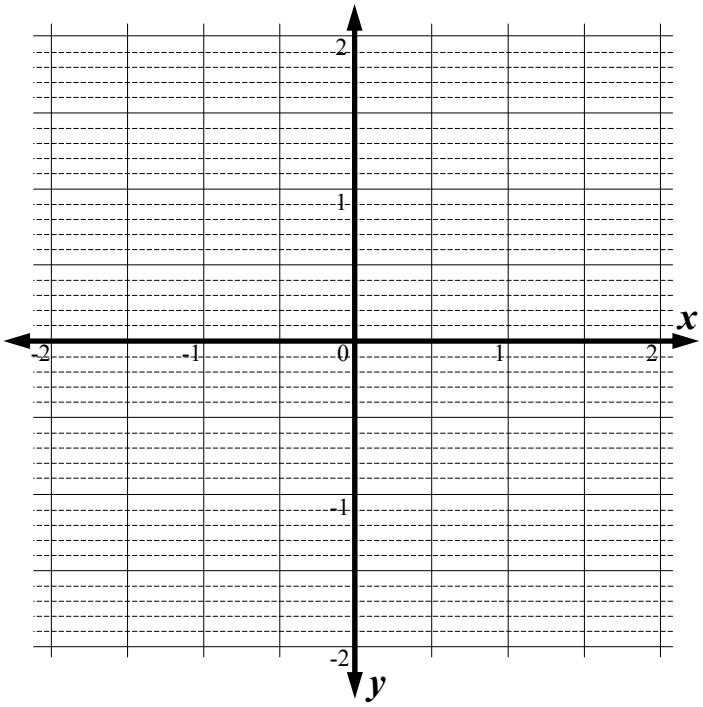
x	0	1	2	3	4
y					



Complete the table to 1 d.p., then plot the circle.

17 $x^2 + y^2 = 4$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y									



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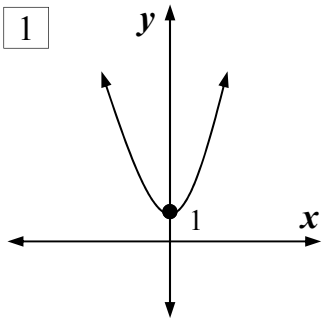
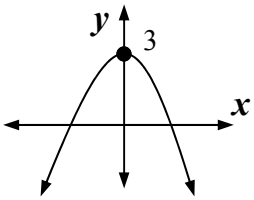
Complete the box to show the parabola's equation

1



$y = x^2 +$ [1]

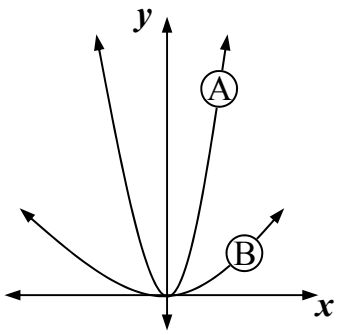
2



$y = -x^2 +$ [3]

Pick the appropriate equation for the parabolas below.

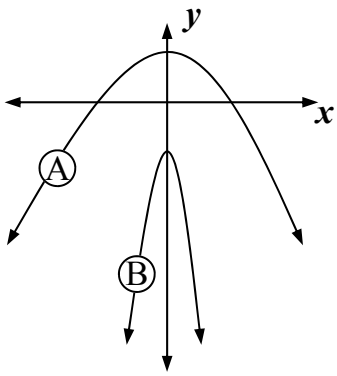
3



$y = 2x^2$ (A) Ⓑ

$y = \frac{1}{2}x^2$ Ⓐ (B)

4



$y = -3x^2 - 2$ (A) Ⓑ

$y = -\frac{1}{2}x^2 + 2$ (A) Ⓑ

Complete the sentence with a number and either up/down.

5 The parabola $y = x^2 + 5$ cuts the y-axis at [5] and is concave [up].

6 The parabola $y = -x^2 - 2$ cuts the y-axis at [-2] and is concave [down].

Complete the sentence with either 'narrow' or 'wide'.

7 The parabola $y = -x^2 - 4$ is more [wide] than $y = -2x^2 - 1$.

8 The parabola $y = 2x^2$ is more [narrow] than $y = \frac{1}{2}x^2 + 5$.

Complete these tables.

9 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y	12	7	4	3	4	7	12

10 $y = -3x^2$

x	-3	-2	-1	0	1	2	3
y	-27	-12	-3	0	-3	-12	-27

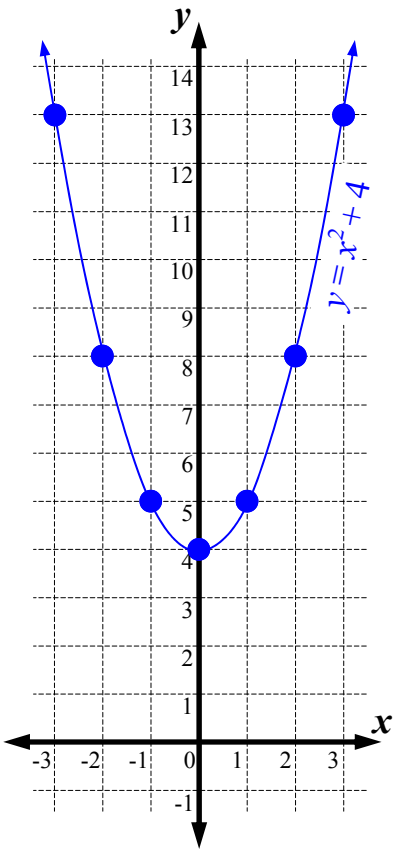
11 $y = \frac{1}{2}x^2 + 4$

x	-6	-4	-2	0	2	4	6
y	22	12	6	4	6	12	22

Complete the table then plot the parabola $y = x^2 - 2$ on the number plane.

12 $y = x^2 + 4$

x	-3	-2	-1	0	1	2	3
y	13	8	5	4	5	8	13



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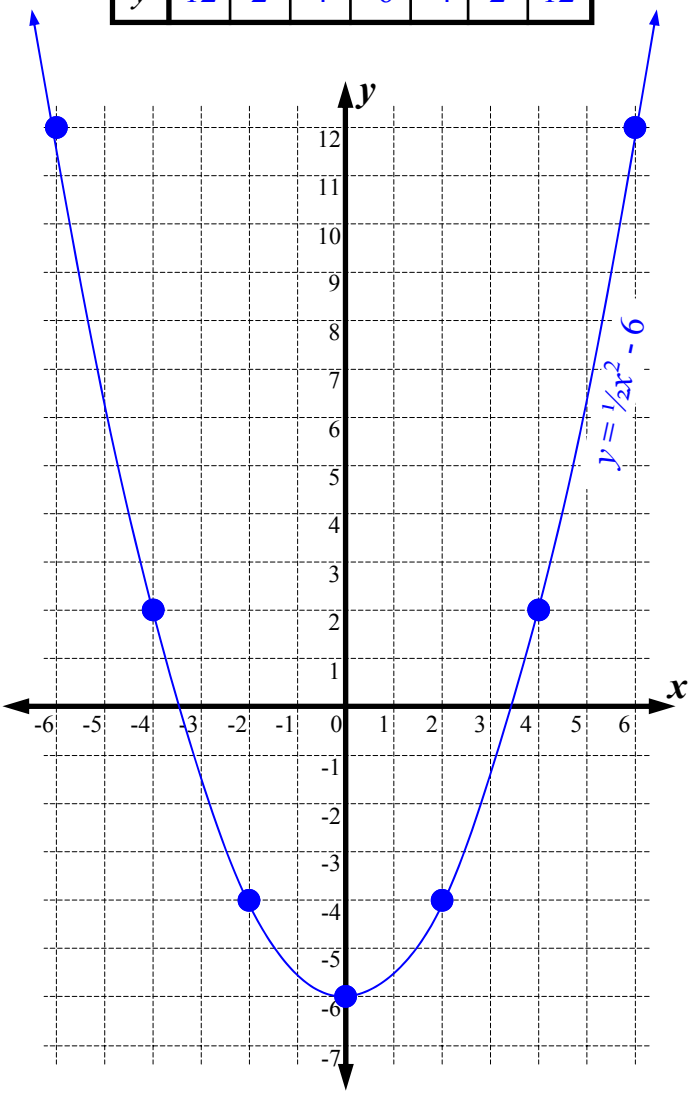
Complete the table, then plot the parabola.

13 $y = \frac{1}{2}x^2 - 6$

x	-6	-4	-2	0	2	4	6
y	12	2	4	-6	-4	2	12



Complete the table of values.

14 $y = 2^x$

x	0	1	2	3	4	5
y	1	2	4	8	16	32

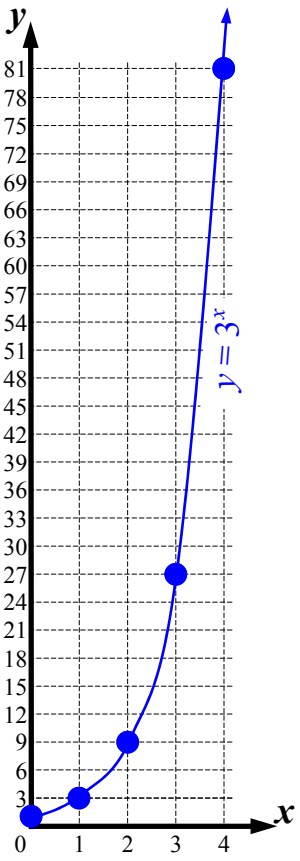
15 $y = 4^x$

x	0	1	2	3	4	5
y	1	4	16	64	256	1 024

Complete the table, then plot the exponential.

16 $y = 3^x$

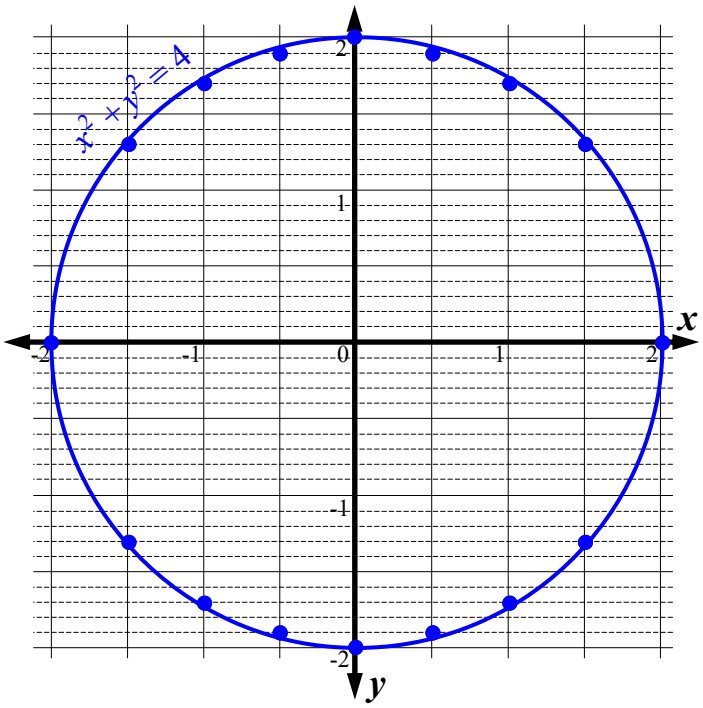
x	0	1	2	3	4
y	1	3	9	27	81



Complete the table to 1 d.p., then plot the circle.

17 $x^2 + y^2 = 4$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	0	±1.3	±1.7	±1.9	±2	±1.9	±1.7	±1.3	0



Non-Linear Relationships



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Non-Linear Relationships - Properties of Parabolas (Sheet 1)

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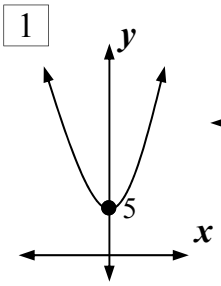
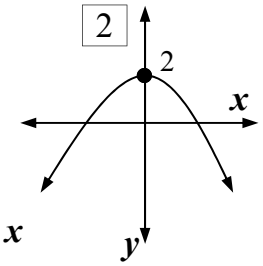
Look at the y-intercept and complete the equations for these parabolas.

1



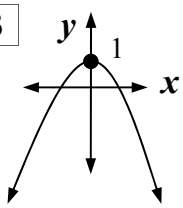
$y = 2x^2 +$ ☐

2



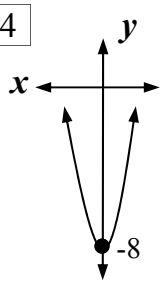
$y = -\frac{1}{2}x^2 +$ ☐

3



$y = -x^2 +$ ☐

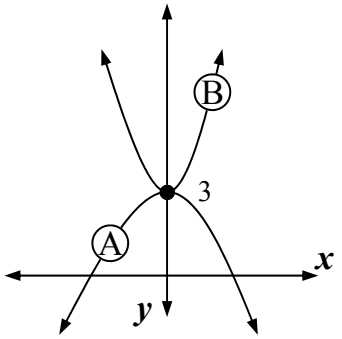
4



$y = 3x^2 -$ ☐

Choose which equation applies to each of the parabolas.

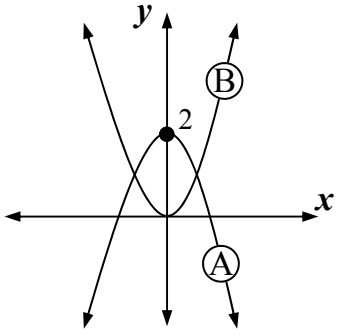
5



$y = x^2 + 3$ Ⓐ Ⓑ

$y = -\frac{1}{2}x^2 + 3$ Ⓐ Ⓑ

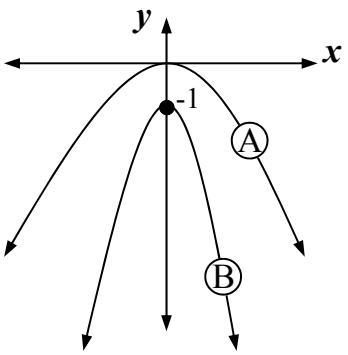
6



$y = x^2$ Ⓐ Ⓑ

$y = -x^2 + 2$ Ⓐ Ⓑ

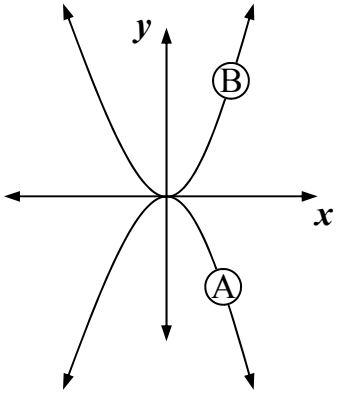
7



$y = -\frac{1}{2}x^2$ Ⓐ Ⓑ

$y = -2x^2 - 1$ Ⓐ Ⓑ

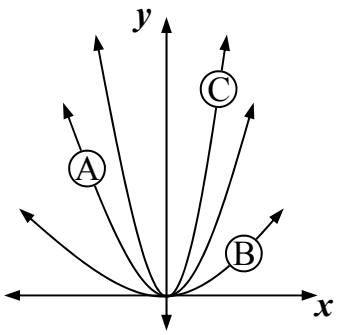
8



$y = -x^2$ Ⓐ Ⓑ

$y = x^2$ Ⓐ Ⓑ

9

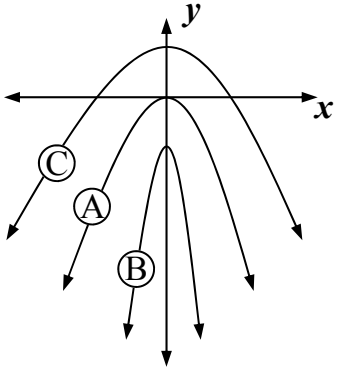


$y = x^2$ ⒶⒷⒸ

$y = 2x^2$ ⒶⒷⒸ

$y = \frac{1}{2}x^2$ ⒶⒷⒸ

10



$y = -\frac{1}{2}x^2 + 3$ ⒶⒷⒸ

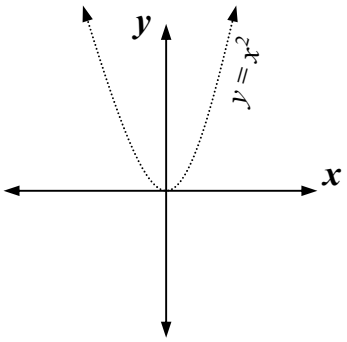
$y = -3x^2 - 3$ ⒶⒷⒸ

$y = -x^2$ ⒶⒷⒸ

Now sketch your own parabolas.

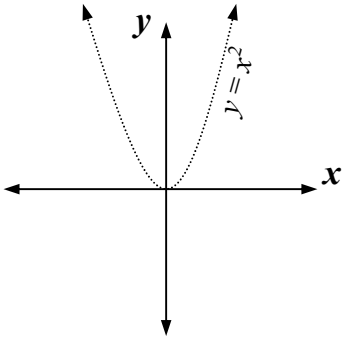
The graph below shows $y = x^2$ as a dotted curve. Sketch $y = -x^2$.

11 Arrowheads, Equation on curve



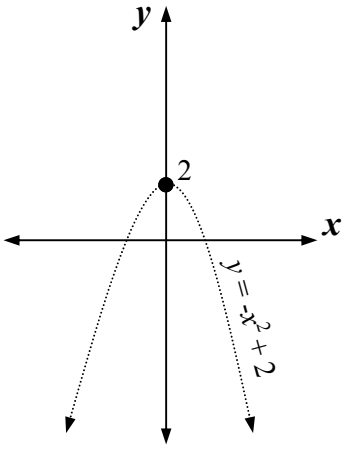
The graph below shows $y = x^2$ as a dotted curve. Sketch $y = x^2 - 3$.

12



The graph below shows $y = -x^2 + 2$ as a dotted curve. Sketch $y = x^2 - 2$.

13



Non-Linear Relationships - Properties of Parabolas (Sheet 2)

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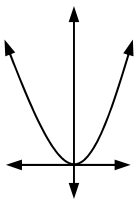
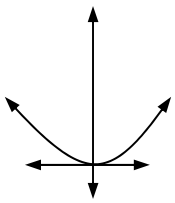
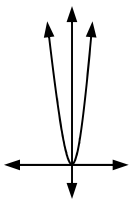
Join the sketch with the equation for each parabola.

1 

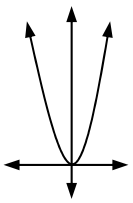
$y = 2x^2$

2 

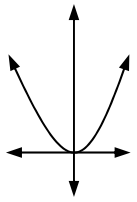
$y = x^2$

3 

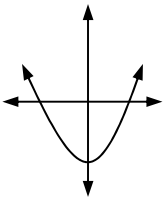
$y = \frac{1}{2}x^2$

4 

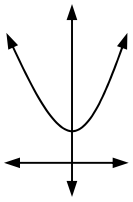
$y = 3x^2$

5 

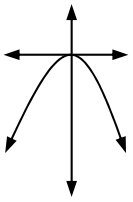
$y = x^2 + 2$

6 

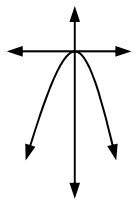
$y = -x^2$

7 

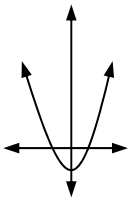
$y = x^2$

8 

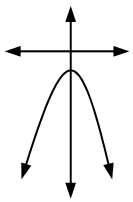
$y = x^2 - 2$

9 

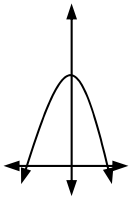
$y = -x^2 - 2$

10 

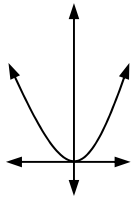
$y = -x^2 + 7$

11 

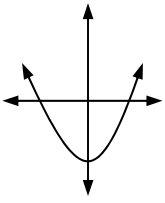
$y = x^2 - 2$

12 

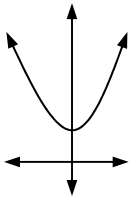
$y = -x^2$

13 

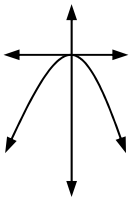
$y = x^2 + 2$

14 

$y = -x^2$

15 

$y = x^2$

16 

$y = x^2 - 2$

Give 2 examples of parabolas that have the following properties.

17 A parabola is concave-up and cuts the y-axis at 4.

$y = x^2 + \square$ $\qquad$ $y = \square x^2 + \square$

18 A parabola is concave-down and cuts the y-axis at the origin.

$y =$ $\qquad$ $y =$

19 A parabola is concave-up and cuts the y-axis at the -3.

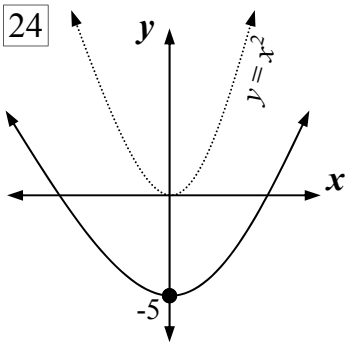
20 A parabola is concave-down and cuts the y-axis at the same intercept as $y = 3x^2 + 5$.

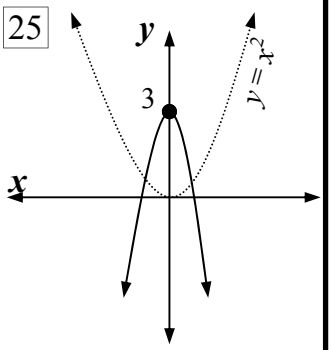
21 A parabola is concave-up and cuts the y-axis at the same intercept as $y = -x^2 - 6$.

22 A parabola is concave-up, cuts the y-axis at the same intercept as $y = -\frac{1}{2}x^2 - 4$ and is more narrow than $y = 2x^2$.

23 A parabola is concave-down, cuts the y-axis at the same intercept as $y = 3x^2 - 6$ and is a more wide than $y = -2x^2$.

Give a possible name for the solid-line unnamed parabolas shown below

24 

25 

Non-Linear R'ships - Plotting Points & Drawing Parabolas (Sheet 1)

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Plot the points for each parabola then draw a curve through them. Show arrow heads and write the equation of the parabola on the curve.



1 $y = x^2 + 3$ Plot these points:

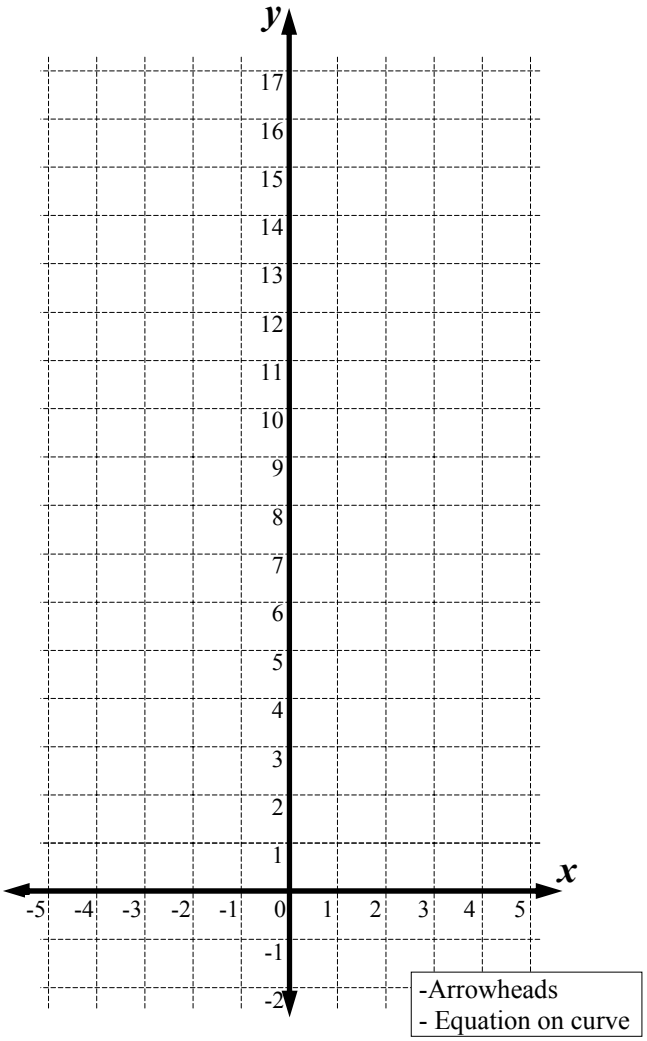
(-3, 12), (0, 3), (2, 7), (-1, 4), (-2, 7), (1, 4), (3, 12).

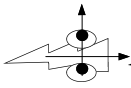
2 $y = x^2$ Plot these points:

(-1, 1), (4, 16), (2, 4), (-3, 9), (-4, 16), (3, 9), (0, 0), (1, 1), (-2, 4).

3 $y = x^2 - 2$ Plot these points:

(2, 2), (4, 14), (3, 7), (-4, 14), (-1, -1), (3, 9), (-2, 2), (1, -1), (-3, 7).





Now use the table of values to plot these parabolas. Go across 'x' and up 'y'.

4 $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y	18	8	2	0	2	8	18

5 $y = x^2$

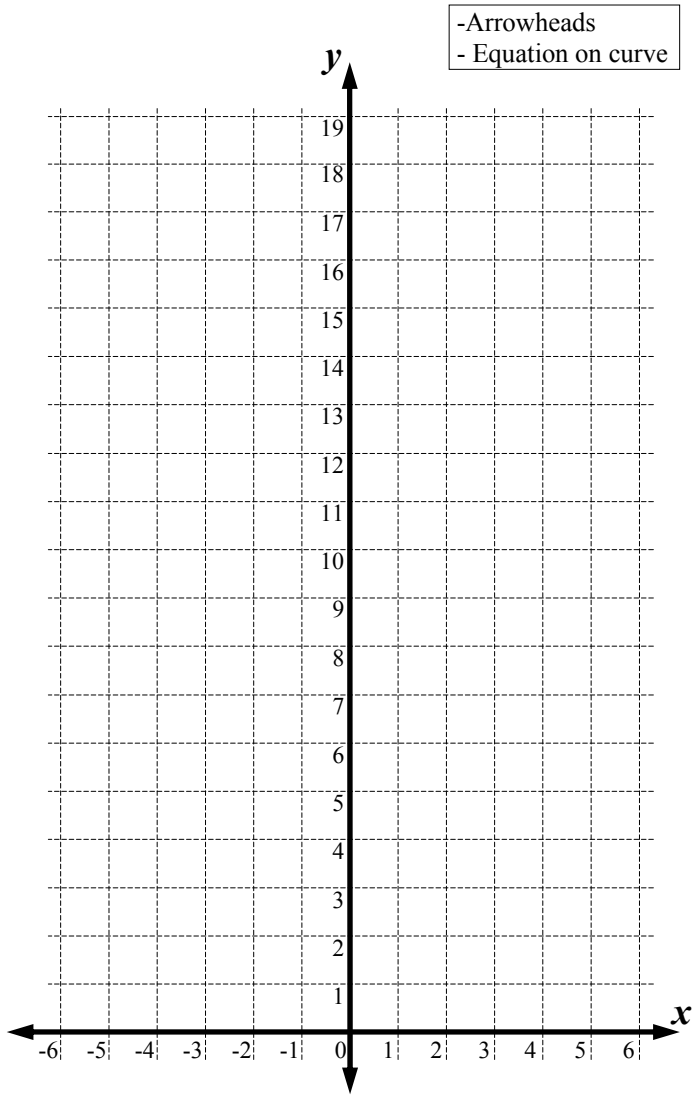
x	-4	-3	-2	1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

6 $y = \frac{1}{2}x^2$

x	-6	-4	-2	-1	0	1	2	4	6
y	18	8	2	½	0	½	2	8	18

7 $y = \frac{1}{4}x^2$

x	-6	-4	-2	0	2	4	6
y	9	4	1	0	1	4	9



8 What effect does the number before the x^2 term have on the parabola?

- ○ A larger coefficient has a more narrow curve.
- ○ A smaller coefficient has a narrower curve.

Non-Linear R'ships - Plotting Points & Drawing Parabolas (Sheet 2)

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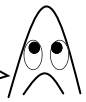
Plot the points for each parabola then draw a curve through them. Show arrow heads and write the equation of the parabola on the curve.



1 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y	-9	-4	-1	0	-1	-4	-9

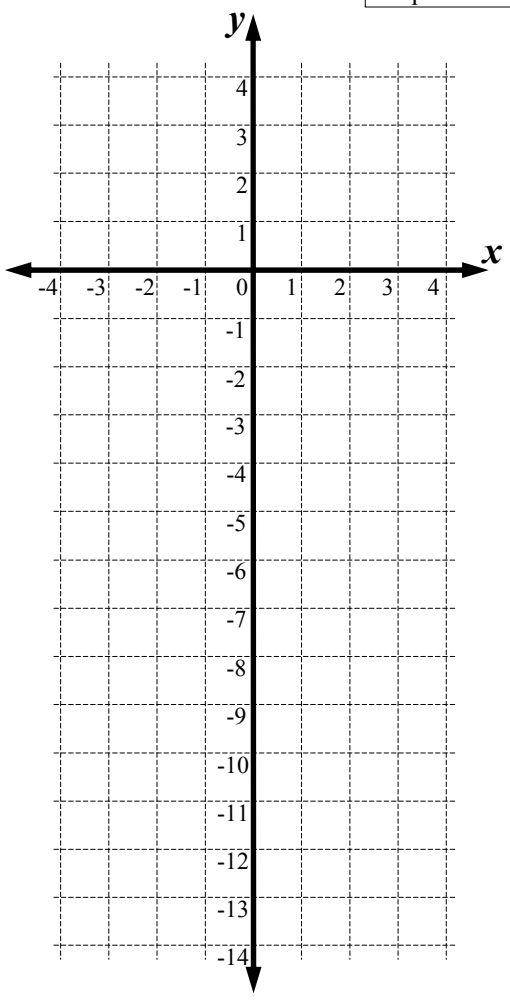
2 $y = -x^2 + 4$

x	-4	-3	-2	1	0	1	2	3	4
y	-12	-5	0	3	4	3	0	-5	-12

3 $y = -x^2 - 3$

x	-3	-2	-1	0	1	2	3
y	-12	-7	-4	-3	-4	-7	-12

-Arrowheads
- Equation on curve

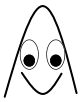


4 What effect does the number after the x^2 term have on the parabola? Choose one …..

○ It shifts the parabola up or down the y-axis.

○ It determines where the parabola cuts the y-axis.

○ Both of the above.



Keep going. This time you have both positive and negative coefficients.

5 $y = -x^2 + 9$

x	-4	-3	-2	1	0	1	2	3	4
y	-7	0	5	8	9	8	5	0	-7

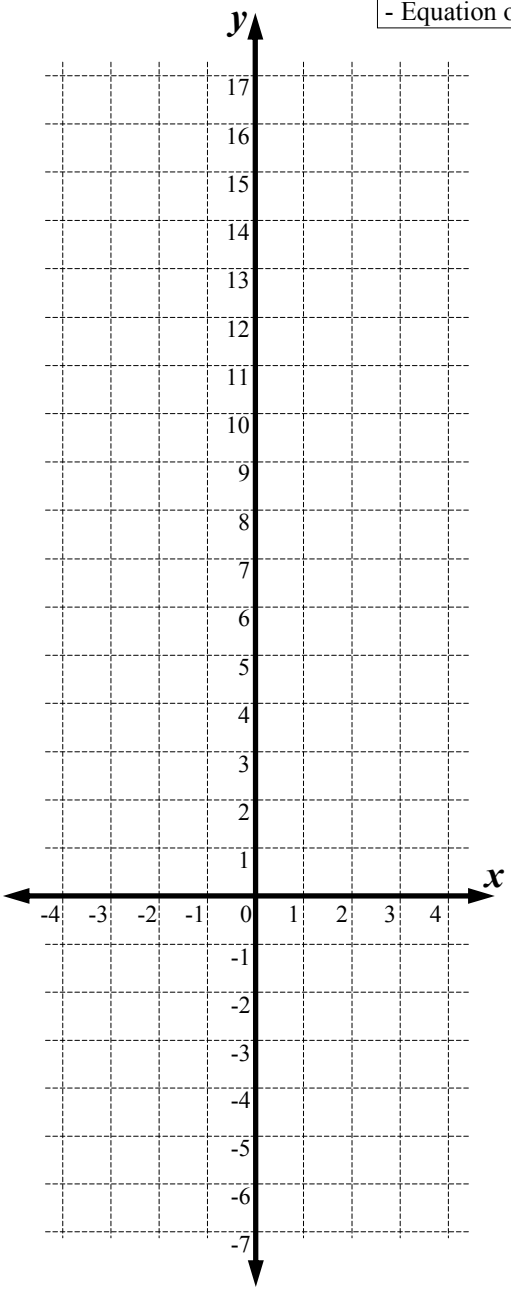
6 $y = x^2 - 6$

x	-4	-3	-2	1	0	1	2	3	4
y	10	3	-2	-5	-6	-5	-2	3	10

7 $y = 2x^2 - 4$

x	-3	-2	-1	0	1	2	3
y	14	4	-2	-4	-2	4	14

-Arrowheads
- Equation on curve



Non-Linear Relationships - Calculator Use

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Use your calculator to find these.

1 $6^2 - 5 =$ ____

Calc: [6] [x^2] [-] [5] [=]

2 $4^2 + 11 =$ ____

3 $2^2 - 9 =$ ____

4 $3 \times 4^2 + 7 =$ ____

Calc: [3] [×] [4] [x^2] [+] [7] [=]

5 $2 \times 2^2 - 4 =$ ____

6 $-2 \times 3^2 - 8 =$ ____

Calc: [-] [2] [×] [3] [x^2] [-] [8] [=]

Complete the table of values below. Use your back button to speed things up.

7 $y = x^2$

x	0	1	2	3	4	5	6
y							

Calc: [0] [x^2] [=]

8 $y = x^2 + 5$

x	0	1	2	3	4	5	6
y							

Calc: [0] [x^2] [+] [5] [=]

9 $y = -x^2 + 6$

x	0	1	2	3	4	5	6
y							

Calc: [-] [0] [x^2] [+] [6] [=]

10 $y = -2x^2 + 3$

x	0	1	2	3	4	5	6
y							

Calc: [-] [2] [×] [0] [x^2] [+] [3] [=]

These involve negative numbers. To square a negative number you use brackets. Try these.

11 $(-3)^2 =$ ____

Calc: [(] [-] [3] [)] [x^2] [=]

12 $(-6)^2 =$ ____

13 $(-4)^2 + 12 =$ ____

Calc: [(] [-] [4] [)] [x^2] [+] [1] [2] [=]

14 $(-2)^2 - 8 =$ ____

15 $2 \times (-3)^2 - 2 =$ ____

Calc: [2] [×] [(] [-] [3] [)] [x^2] [-] [2] [=]

16 $-3 \times (-1)^2 + 5 =$ ____

Complete these tables which have negative 'x' values.

17 $y = x^2$

x	-6	-5	-4	-3	-2	-1	0
y							

Calc: [(] [-] [6] [)] [x^2] [=]

18 $y = -2x^2 + 4$

x	-6	-5	-4	-3	-2	-1	0
y							

Calc: [-] [2] [×] [(] [-] [6] [)] [x^2] [+] [4] [=]

19 $y = 3x^2 - 10$

x	-6	-5	-4	-3	-2	-1	0
y							

20 $y = -x^2 - 3$

x	-6	-5	-4	-3	-2	-1	0
y							

Match the question on the left with the answer on the right.

21

$8^2 - 11$	152
$2 \times (-2)^2$	32
$3 \times 7^2 + 5$	8
$(-6)^2 - 4$	-20
$-2 \times 5^2 - 4$	25
$-5^2 + 5$	20
$(-5)^2 - 5$	53
$2 \times 4^2 - 7$	-54

Now both positive and negative. Expect to see symmetry, across the '0 value.' Complete these.

22 $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

23 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							

24 $y = -3x^2$

x	-3	-2	-1	0	1	2	3
y							

25 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y							

26 $y = 2x^2 - 9$

x	-3	-2	-1	0	1	2	3
y							

Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 1)

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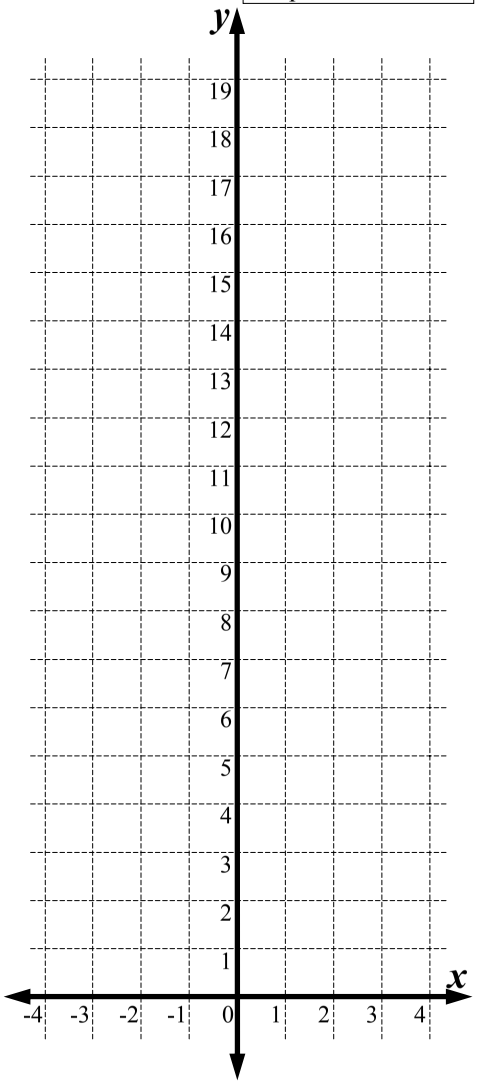
1 $y = \frac{1}{4}x^2$

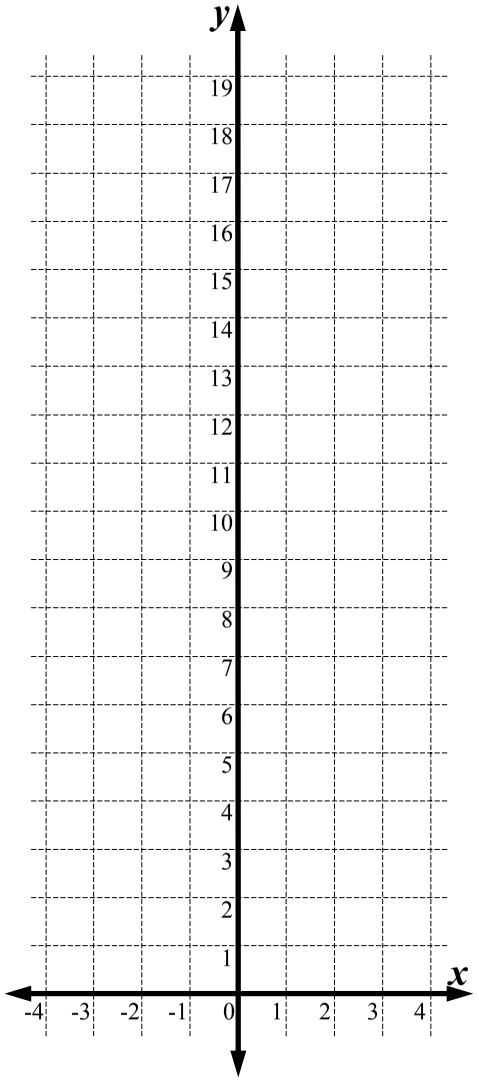
x	-4	-2	-1	0	1	2	4
y							

-Arrowheads
- Equation on curve



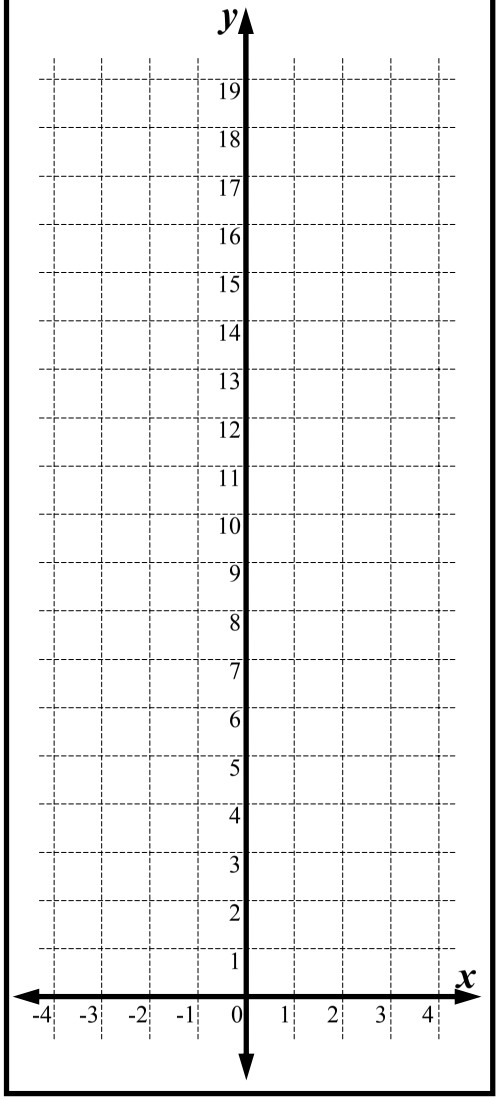
2 $y = \frac{1}{2}x^2$

x	-4	-2	-1	0	1	2	4
y							



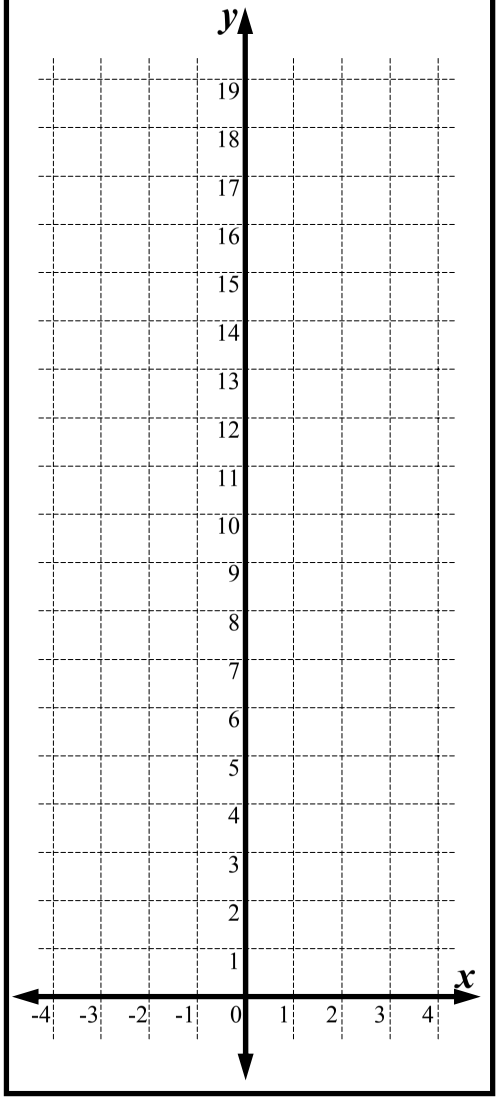
3 $y = x^2$

x	-3	-2	-1	0	1	2	3
y							



4 $y = 2x^2$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 2)

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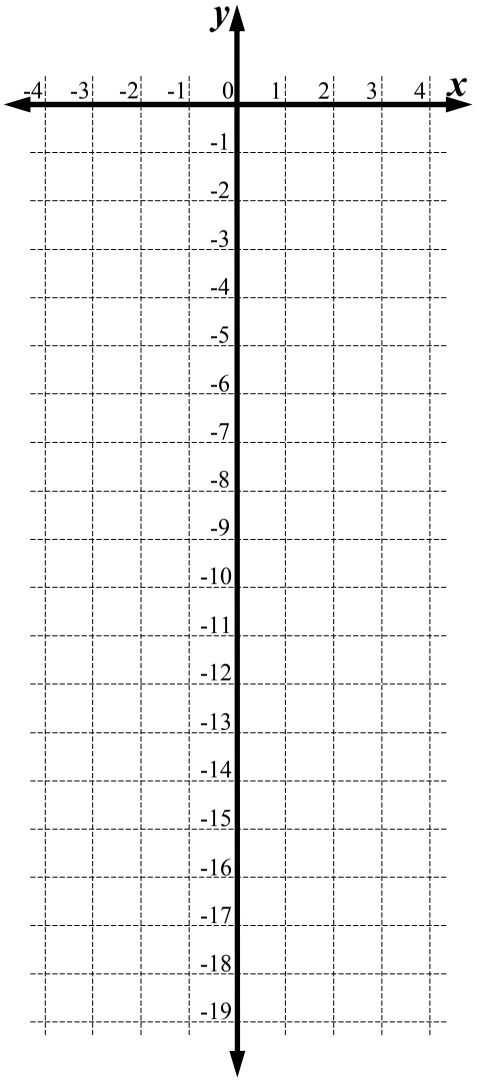
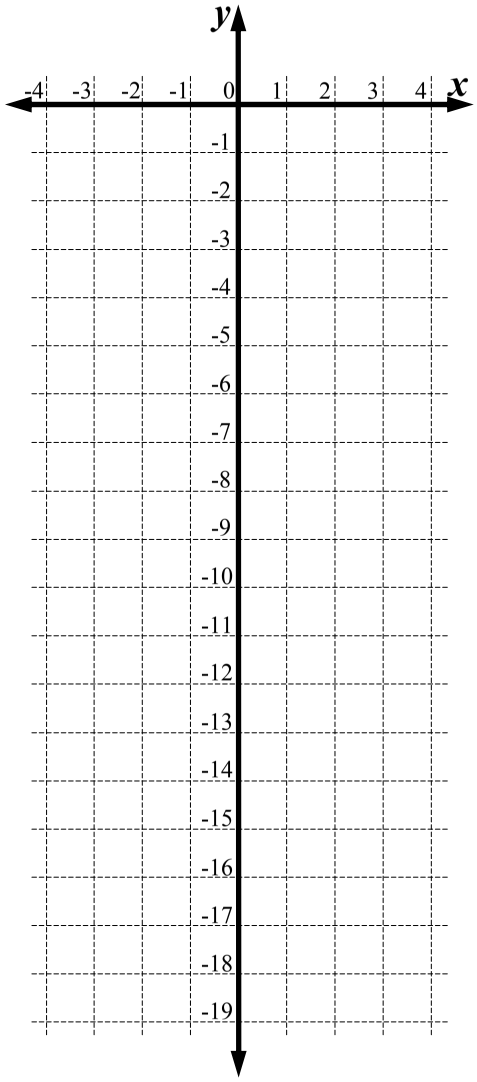
1 $y = -\frac{1}{4}x^2$

x	-4	-2	-1	0	1	2	4
y							



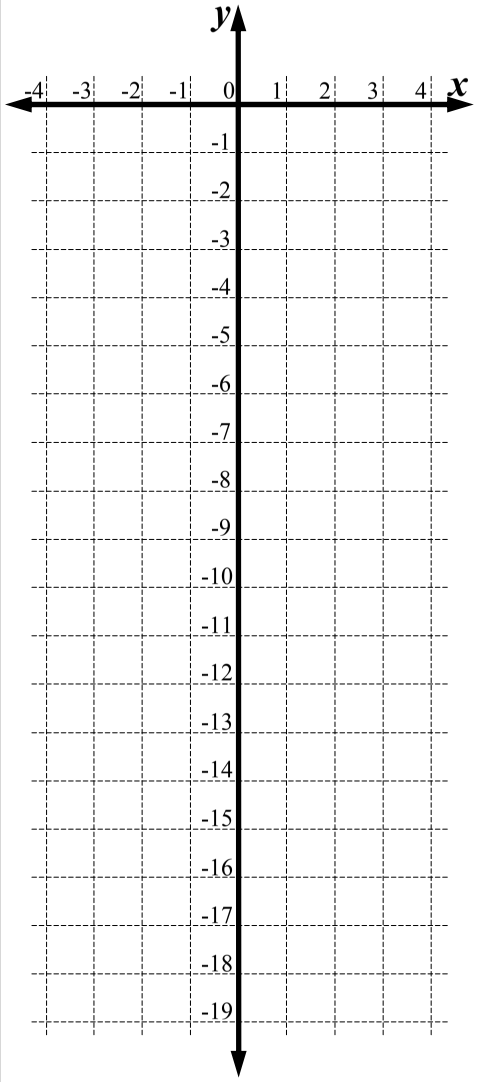
2 $y = -\frac{1}{2}x^2$

x	-4	-2	-1	0	1	2	4
y							



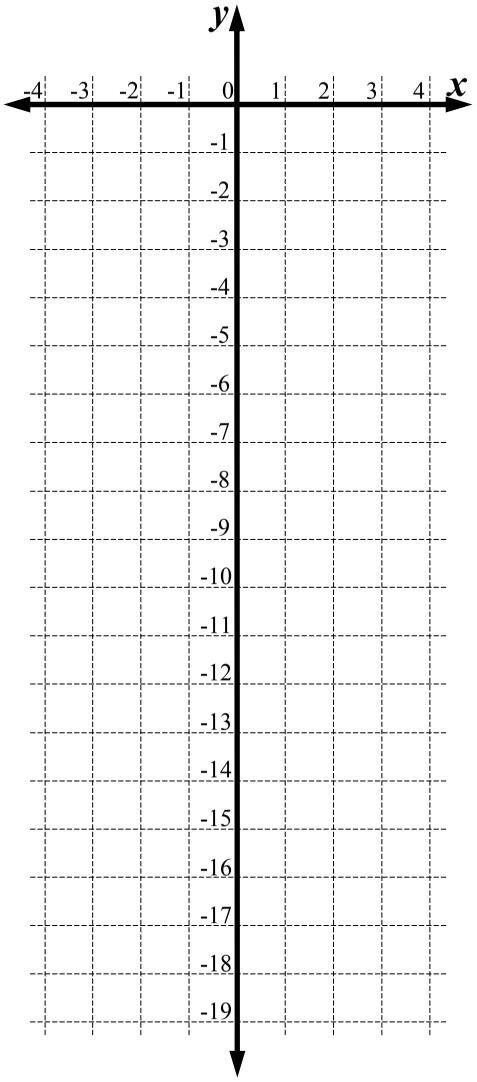
3 $y = -x^2$

x	-3	-2	-1	0	1	2	3
y							



4 $y = -2x^2$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 3)

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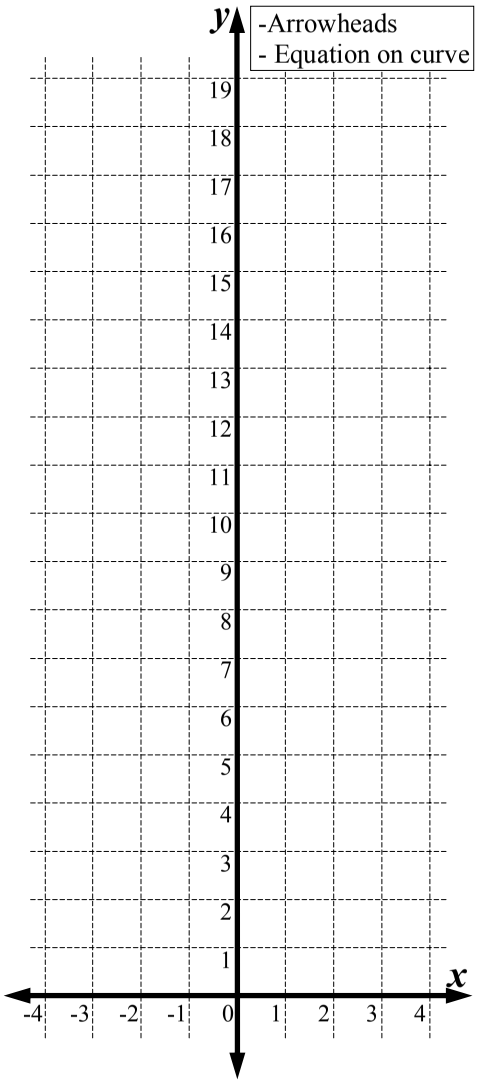
1 $y = x^2 + 5$

x	-3	-2	-1	0	1	2	3
y							

Calc (- 3) x^2 + 5 =

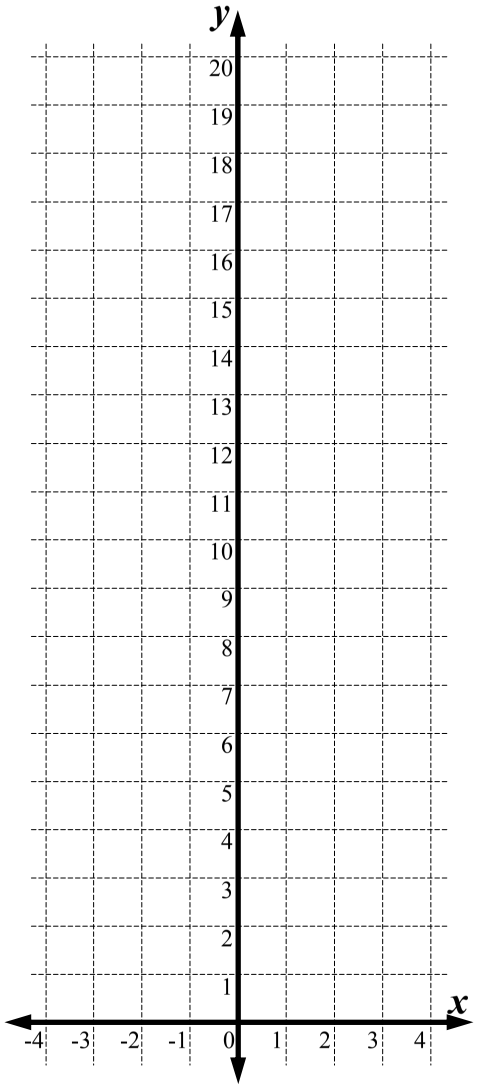
-Arrowheads
- Equation on curve



2 $y = 2x^2 + 2$

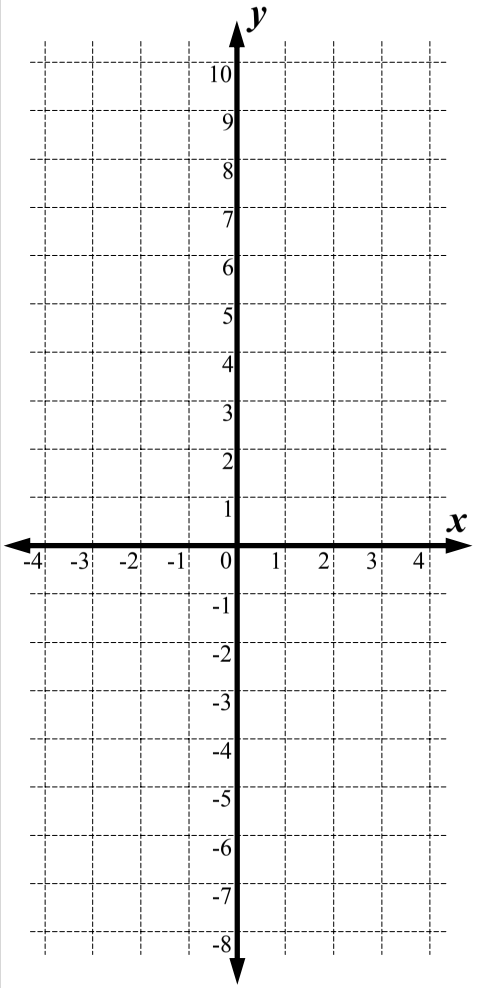
x	-3	-2	-1	0	1	2	3
y							

Calc: 2 × (- 3) x^2 + 2 =



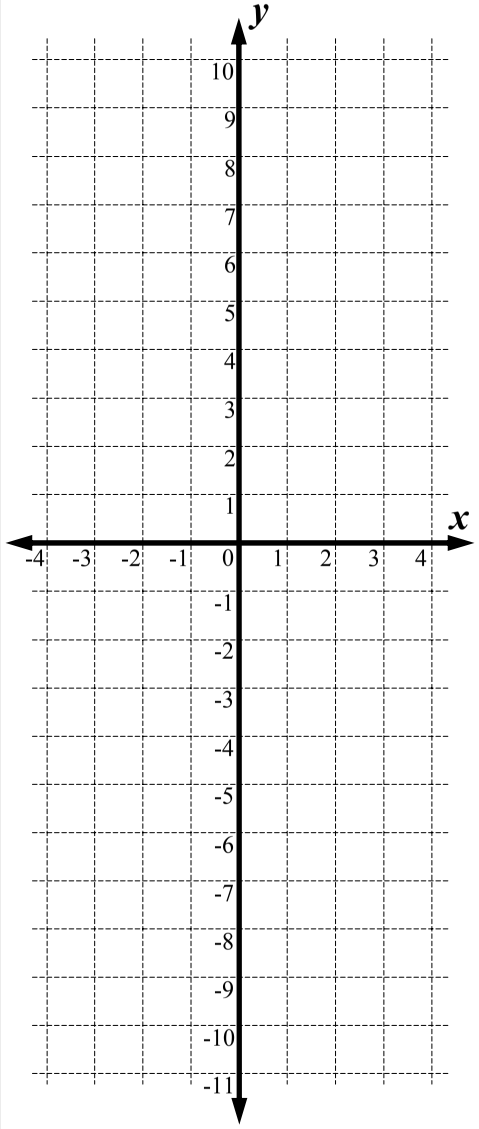
3 $y = x^2 - 7$

x	-4	-3	-2	-1	0	1	2	3	4
y									



4 $y = 2x^2 - 10$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Plotting Parabolas - Individual Grids (Sheet 4)

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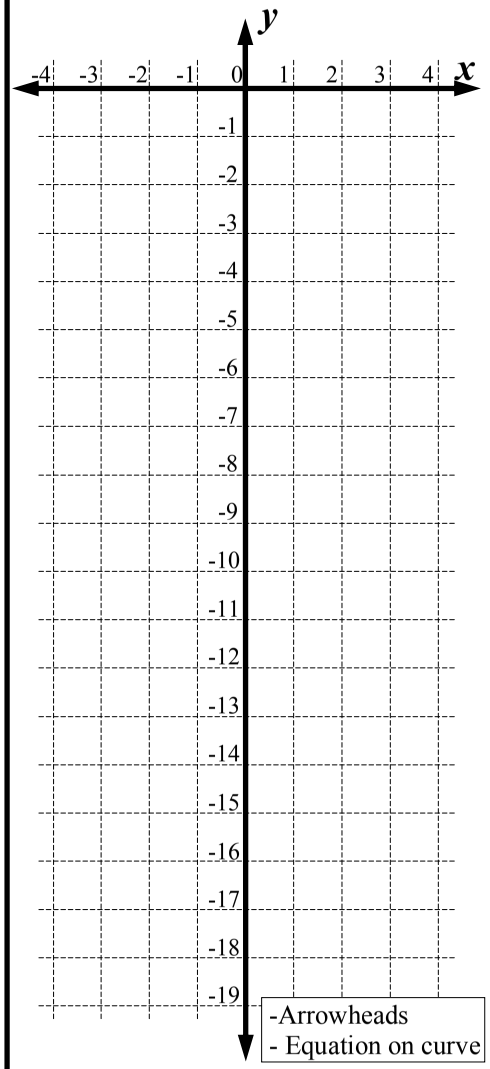
1 $y = -x^2 - 6$

x	-3	-2	-1	0	1	2	3
y							

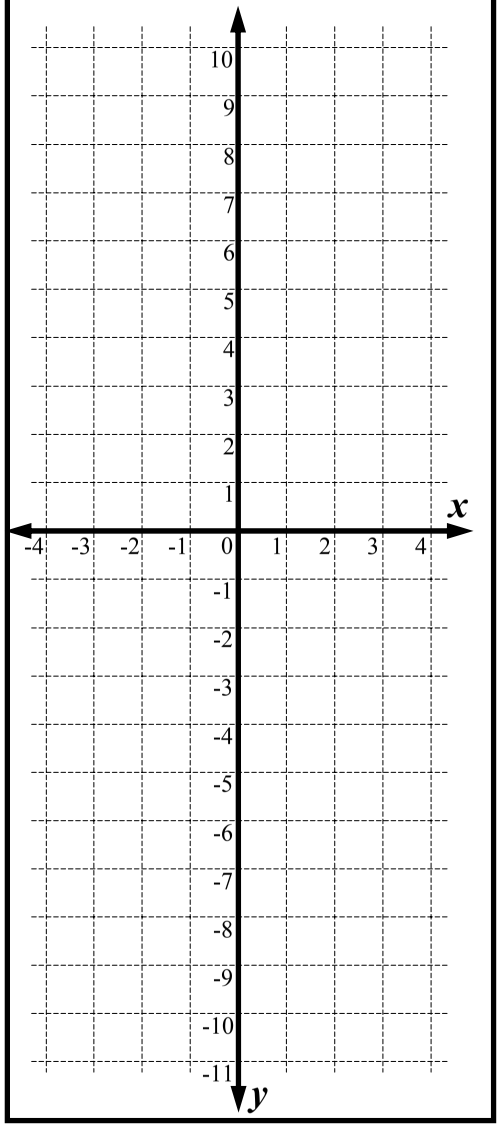
Calc: [-] [(] [-] [3] [)] [x^2] [-] [6] [=]



2 $y = -2x^2 + 7$

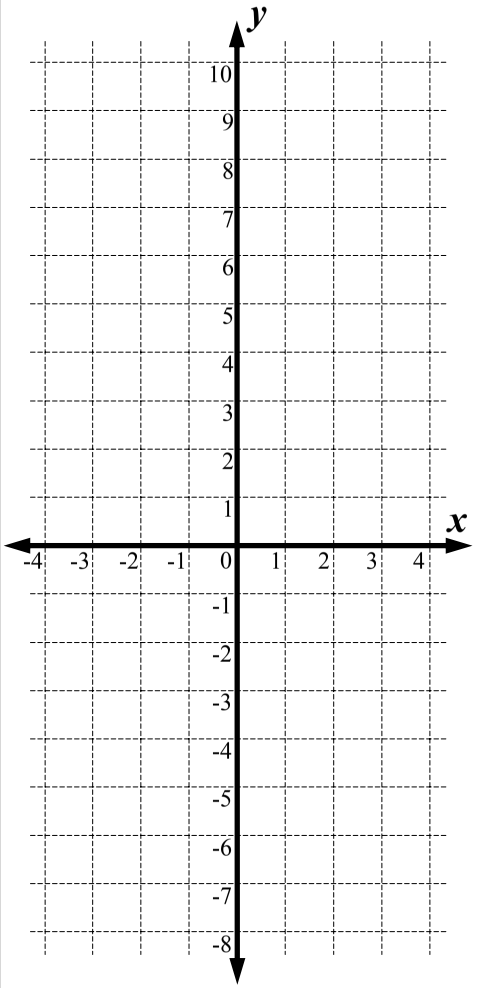
x	-3	-2	-1	0	1	2	3
y							

Calc: [-] [2] [×] [(] [-] [3] [)] [x^2] [+] [7] [=]



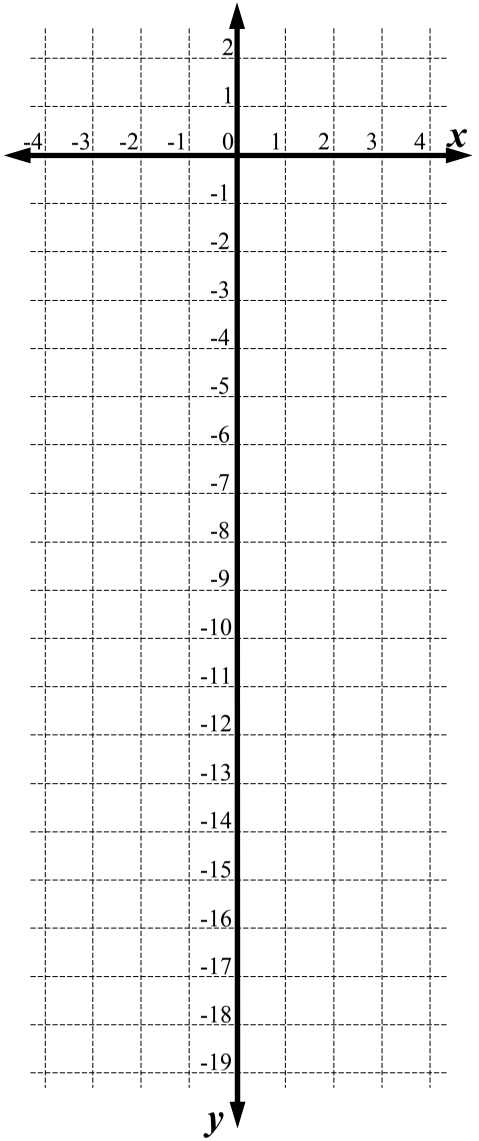
3 $y = -x^2 + 8$

x	-4	-3	-2	-1	0	1	2	3	4
y									



4 $y = -2x^2 + 2$

x	-3	-2	-1	0	1	2	3
y							



Non-Linear Relationships - Further Plotting Parabolas (Sheet 1)

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Complete the table of values below. Use brackets for negatives & your back button.

1 $y = x^2$

x	-3	-2	-1	0	1	2	3
y							

Calc: (- 3) x^2 =

2 $y = 2x^2 + 5$

x	-3	-2	-1	0	1	2	3
y							

Calc: 2 × (- 3) x^2 + 5 =

3 $y = -x^2 - 1$

x	-3	-2	-1	0	1	2	3
y							

Calc: - (- 3) x^2 - 1 =

4 $y = -2x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							

Calc: - 2 × (- 3) x^2 + 3 =

5 $y = ½x^2$

x	-3	-2	-1	0	1	2	3
y	½						

Calc: 1 a b/c 2 ▶ × (- 3) x^2 = S⇔D

6 $y = -½x^2 + 6$

x	-3	-2	-1	0	1	2	3
y							

Calc: - 1 a b/c 2 ▶ × (- 3) x^2 + 6 = S⇔D

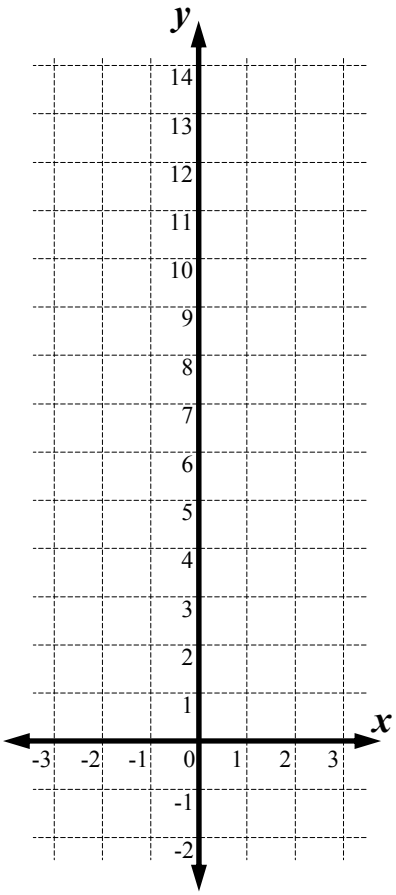
7 $y = -½x^2 - 4$

x	-3	-2	-1	0	1	2	3
y							

Now use the table of values to plot the three parabolas all on the same number plane. Go across 'x' and up 'y'.

8

-Arrowheads
- Equation on curve



$y = x^2$

x	-3	-2	-1	0	1	2	3
y							

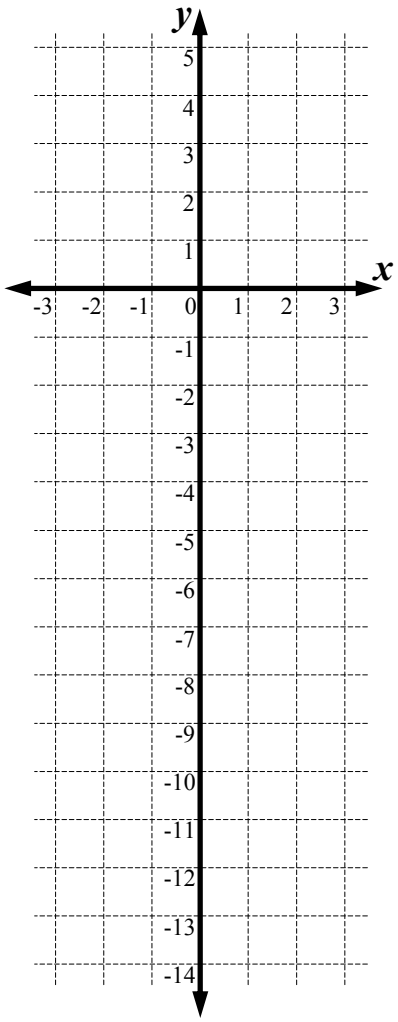
$y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							

$y = -x^2 + 8$

x	-3	-2	-1	0	1	2	3
y							

Calc: - (- 3) x^2 + 8 =



9

$y = -2x^2 + 5$

x	-3	-2	-1	0	1	2	3
y							

$y = -x^2$

x	-3	-2	-1	0	1	2	3
y							

$y = x^2 - 7$

x	-3	-2	-1	0	1	2	3
y							

Arrowheads and Equation on curve

Non-Linear Relationships - Further Plotting Parabolas (Sheet 2)

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Complete the table of values below and plot the parabolas. Fractions in the equations result in flatter parabolas.

1 $y = \frac{1}{2}x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y									

Calc: 1 ▪/□ 2 ▶ × (- 4) x^2 =

Then use the back button

2 $y = -\frac{1}{2}x^2 + 10$

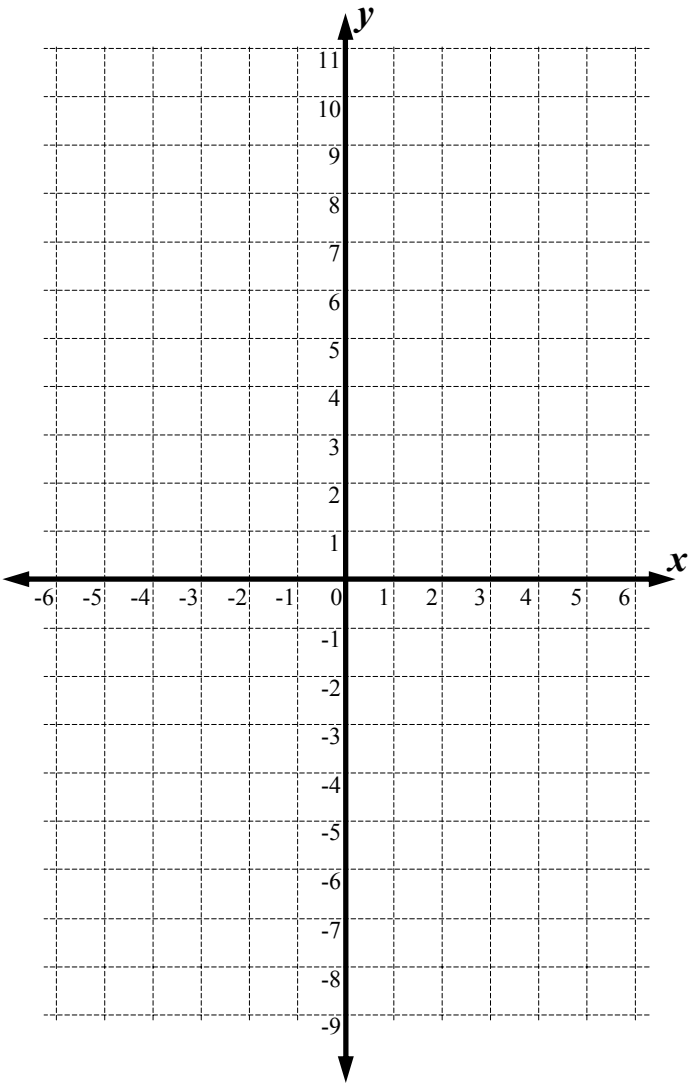
x	-6	-4	-2	-1	0	1	2	4	6
y									

Calc: - 1 ▪/□ 2 ▶ × (- 6) x^2 + 1 0 =

Then use the back button

3 $y = \frac{1}{4}x^2 - 2$

x	-6	-4	-2	0	2	4	6
y							



4 $y = \frac{1}{2}x^2 - 2$

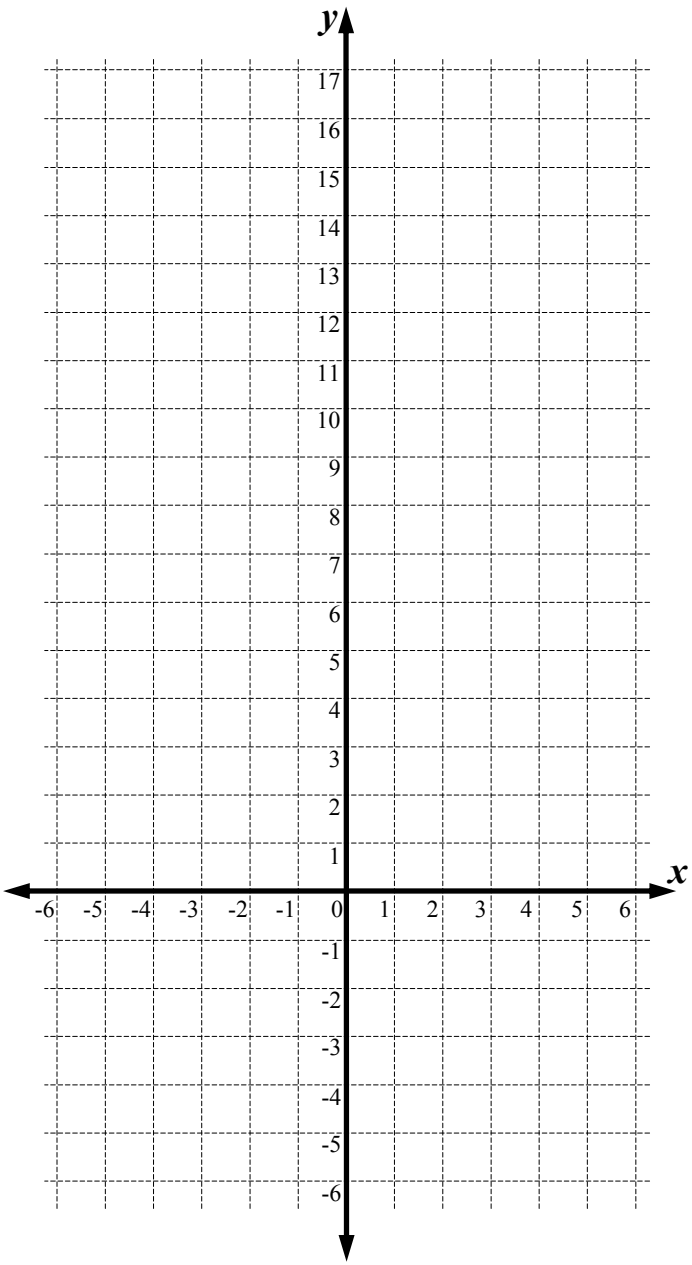
x	-6	-4	-2	-1	0	1	2	4	6
y									

5 $y = x^2$

x	-4	-3	-2	-1	0	1	2	3	4
y									

6 $y = -2x^2 + 12$

x	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
y									



Non-Linear Relationships - Plotting Exponentials (Sheet 1)

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Complete the table of values below. Use your power key.

1 $y = 2^x$

x	0	1	2	3	4	5	6
y							

Calc: 2 $x^{\blacksquare}$ 0 =

2 $y = 3^x$

x	0	1	2	3	4	5
y						

3 $y = 4^x$

x	0	1	2	3	4	5
y						

These negative coefficients will give negative y-values.

4 $y = -2^x$

x	0	1	2	3	4	5	6
y							

Calc: - 2 $x^{\blacksquare}$ 0 =

5 $y = -4^x$

x	0	1	2	3	4	5
y						

6 $y = -5^x$

x	0	1	2	3	4	5
y						

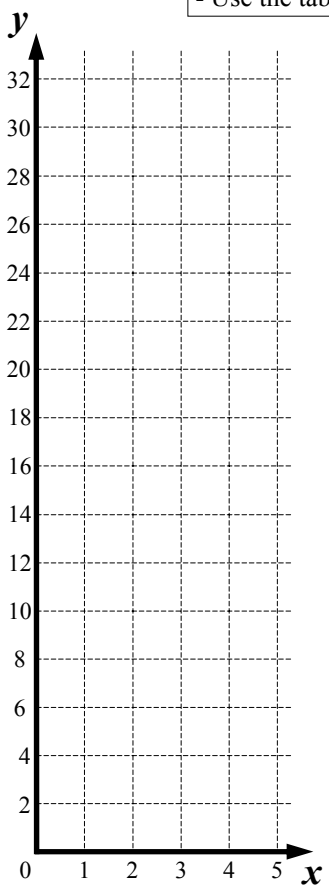
7 $y = -3^x$

x	0	1	2	3	4	5
y						

Use the table of values from your first column to plot these exponentials. Go across 'x' and up 'y'.

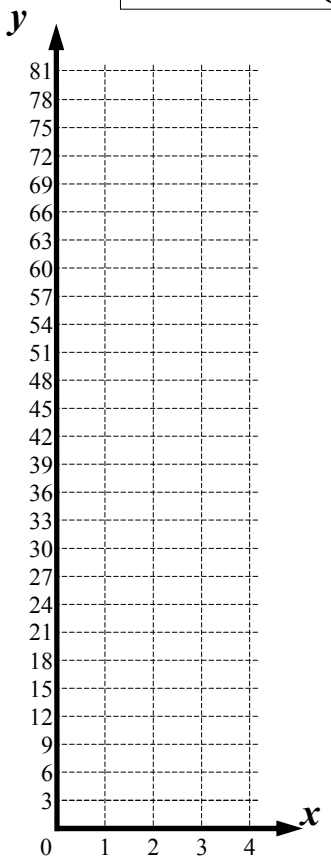
8 $y = 2^x$

- Don't plot (6, 64)
- Equation on curve
- Use the table in Q.1

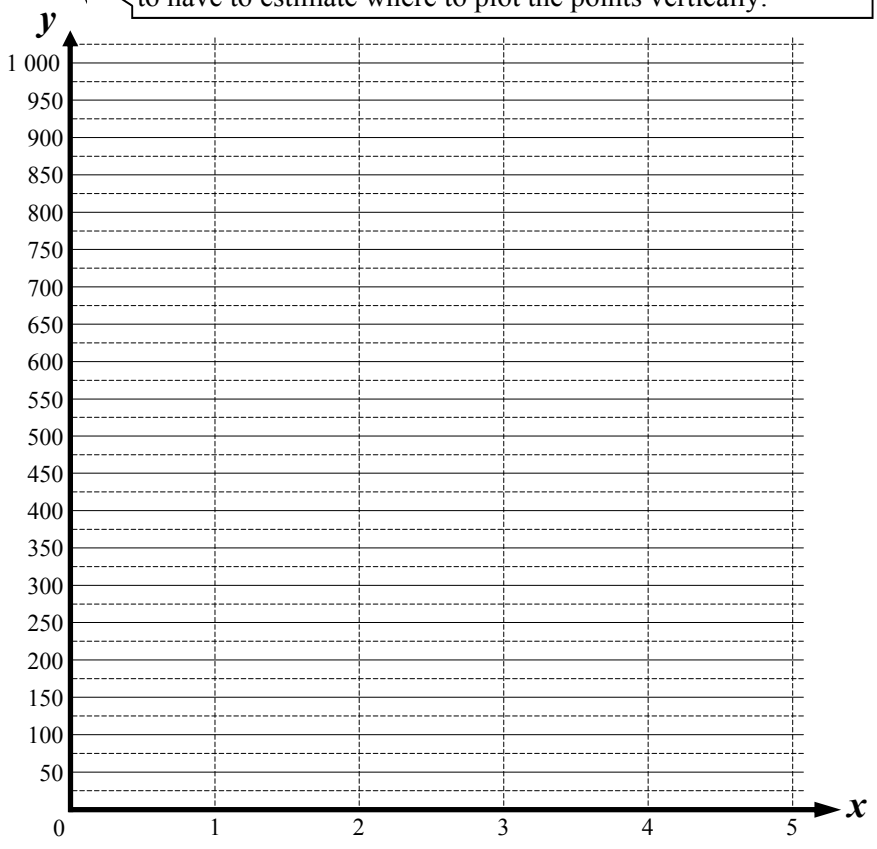


9 $y = 3^x$

- Don't plot (5, 243)
- Equation on curve
- Use the table in Q.2



10 Plot $y = 4^x$ on the grid below. (Table from Q.3). You are going to have to estimate where to plot the points vertically.



Non-Linear Relationships - Plotting Exponentials (Sheet 2)

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Complete the table of values below.
Then plot the curves on the grids.

1 $y = -2^x$

x	0	1	2	3	4	5
y						

- 2 $x^{\blacksquare}$ 0 =

2 $y = -3^x$

x	0	1	2	3	4
y					

3 $y = -4^x$

x	0	1	2	3	4
y					

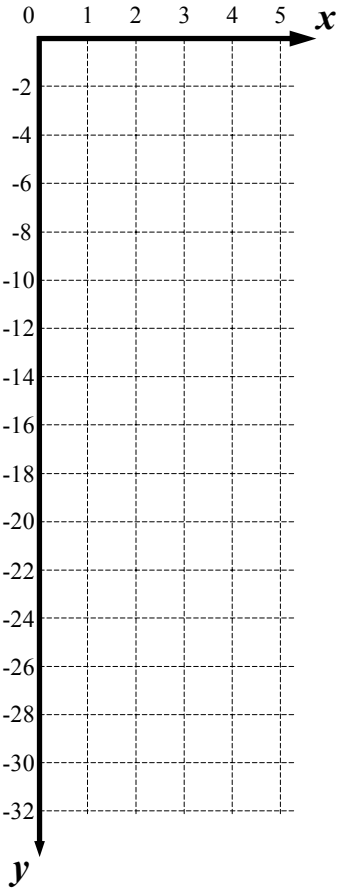
4 $y = 4^x$

x	0	1	2	3	4
y					

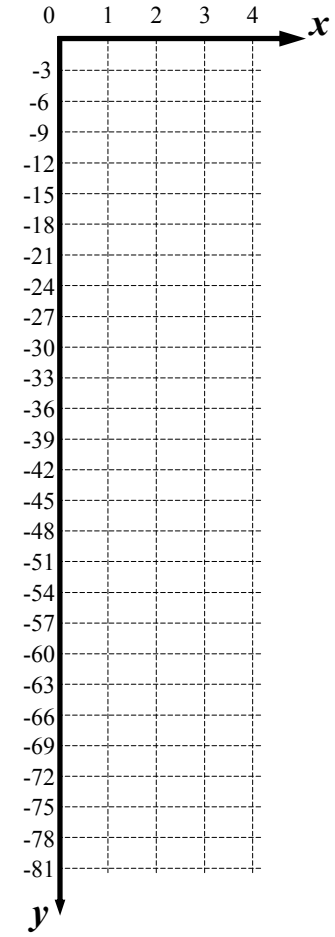
Plot $y = 4^x$ and $y = -4^x$ on the same grid.
(Tables from Q.3 & Q.4)

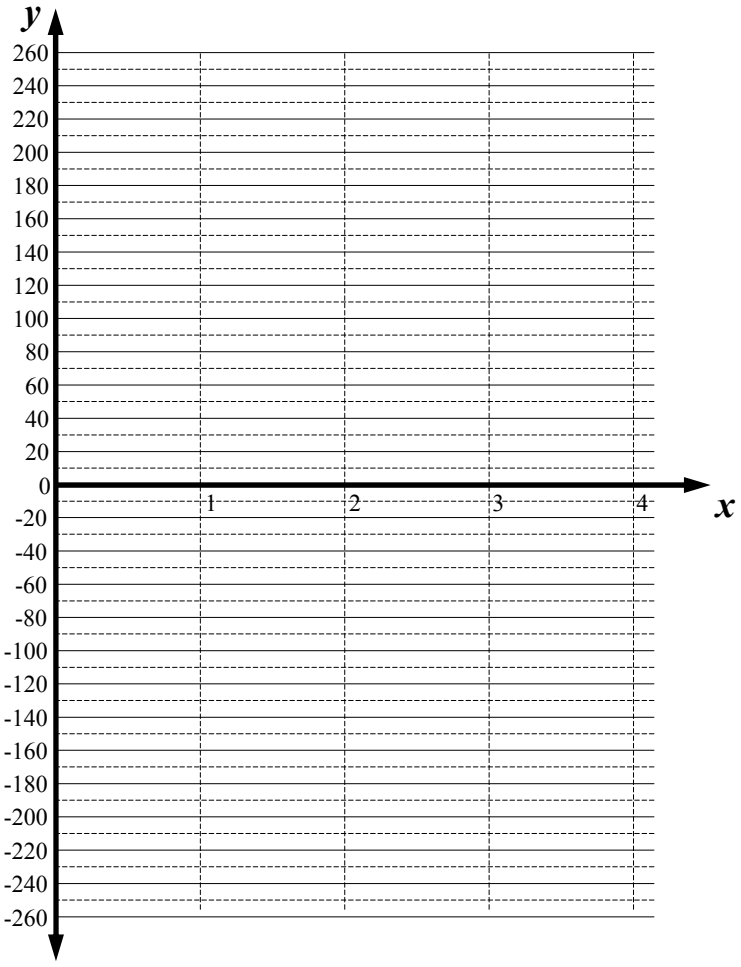
7 $y = -4^x$ & $y = 4^x$

5 $y = -2^x$ (Q.1)



6 $y = -3^x$ (Q.2)





Non-Linear Relationships - Plotting Circles (Sheet 1)

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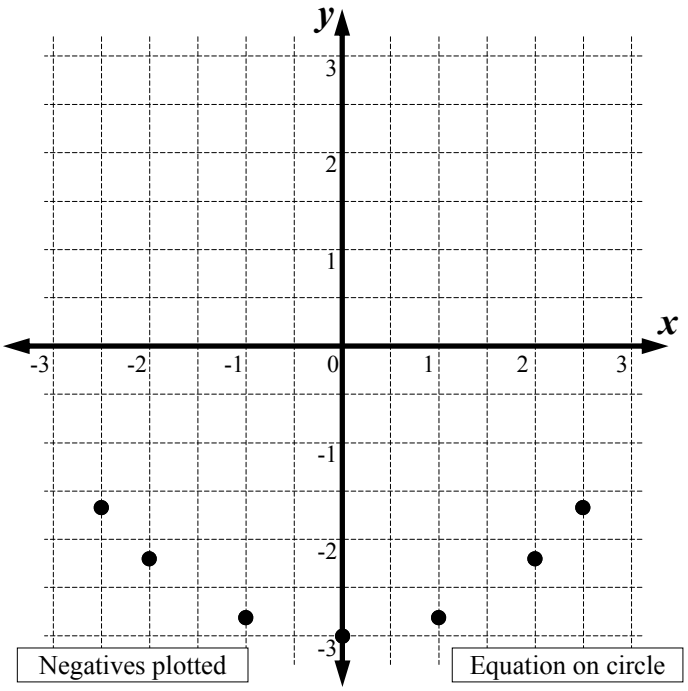
Plot the points for each circle then construct the circle. Write the equation of the circle near it. There are two y-values for each x-value (not ±3).

1 $x^2 + y^2 = 9$

x	-3	-2.5	-2	-1	0	1	2	-2.5	3
y	0	±1.7	±2.2	±2.8	±3	±2.8	±2.2	±1.7	0

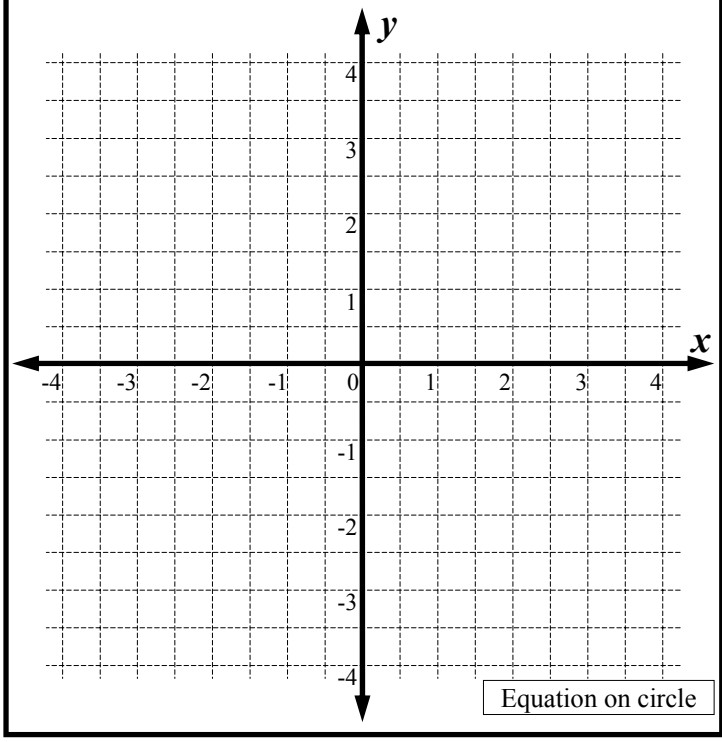


Negatives plotted

Equation on circle

2 $x^2 + y^2 = 16$

x	-4	-3.5	-3	-2	0	2	3	3.5	3
y	0	±1.9	±2.6	±3.5	±4	±3.5	±2.6	±1.9	0



Equation on circle

To complete the table of values yourself, you need to substitute to find y^2 then y. Find y, given these x-values. Round to 1 d.p. if needed (Q. 6).

Example For $y^2 = 20 - x^2$, find y when x is 3.

$y^2 = 20 - x^2$ ← Write equation

$y^2 = 20 - 3^2$ ← Substitute $x = 3$

$y^2 = 11$ ← $20 - 3^2 = 11$

$y = \pm 3.3$ ← $\sqrt{11} = \pm 3.3$ (1 d.p.)

For $y^2 = 25 - x^2$, find y when:

3 $x = 0$

$y^2 =$

$y^2 =$ ← Substitute x-value, $x = 0$

$y^2 =$ ← Calculate y^2

$y = \pm$ ← $\sqrt{\ }$ for y

4 $x = 3$

$y^2 =$ ← Write equation

$y^2 =$ ← Substitute x-value, $x = 3$

$y^2 =$ ← Calculate y^2

$y = \pm$ ← $\sqrt{\ }$ for y

For $y^2 = 50 - x^2$, find y when:

5 $x = 5$

$y^2 =$

$y^2 =$ ← Substitute x-value, $x = 5$

$y^2 =$ ← Calculate y^2

$y =$ ← $\sqrt{\ }$ for y

Write ±

6 $x = 1$

$y^2 =$ ← Write equation

$y^2 =$ ← Substitute x-value, $x = 1$

$y^2 =$ ← Calculate y^2

$y =$ ← $\sqrt{\ }$ for y

Write ±

For $y^2 = 36 - x^2$, find y when:

6 $x = 3$

$y^2 =$

$y^2 =$

$y^2 =$

$y =$

Write ±, to 1 d.p.

6 $x = 5$

$y^2 =$

$y^2 =$

$y^2 =$

$y =$

Write ±, to 1 d.p.

Non-Linear Relationships - Plotting Circles (Sheet 2)

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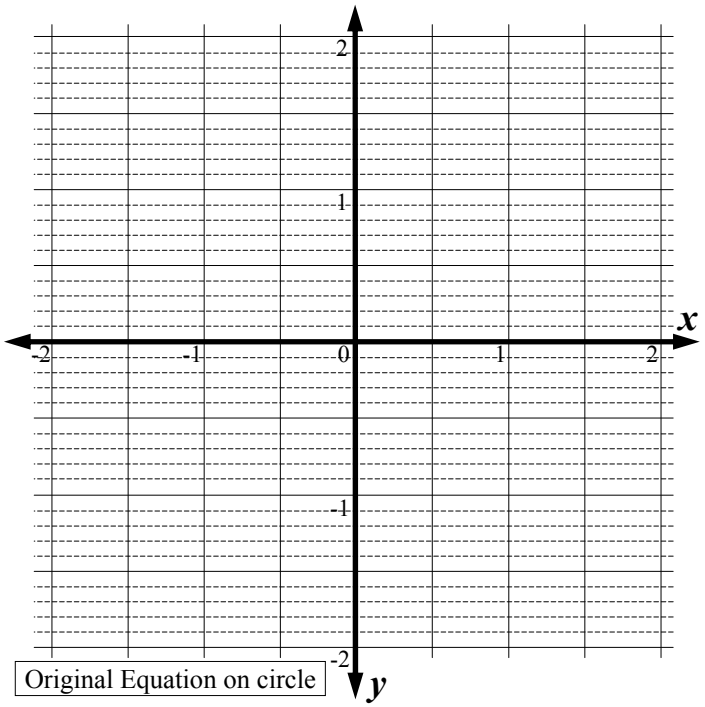
Rewrite the circle equation by subtracting the x^2 term (Q.1. done for you). Complete the table and plot the circle. Round to 1 d.p. when required.

1 $x^2 + y^2 = 4 \xrightarrow{\text{Rearrange}} y^2 = 4 - x^2$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y		$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	

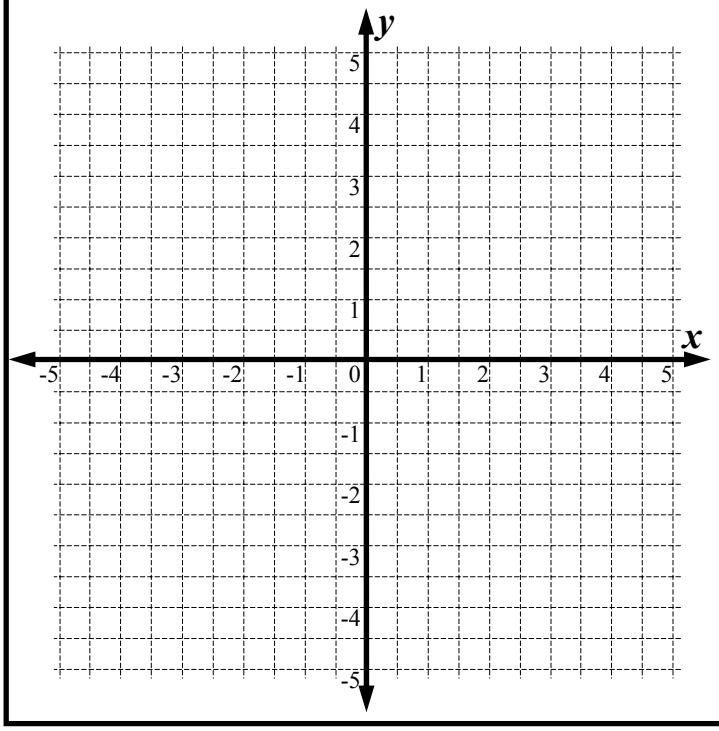
Calc: 4 - (- 2) x^2 = $\sqrt{\blacksquare}$ ANS =



Original Equation on circle

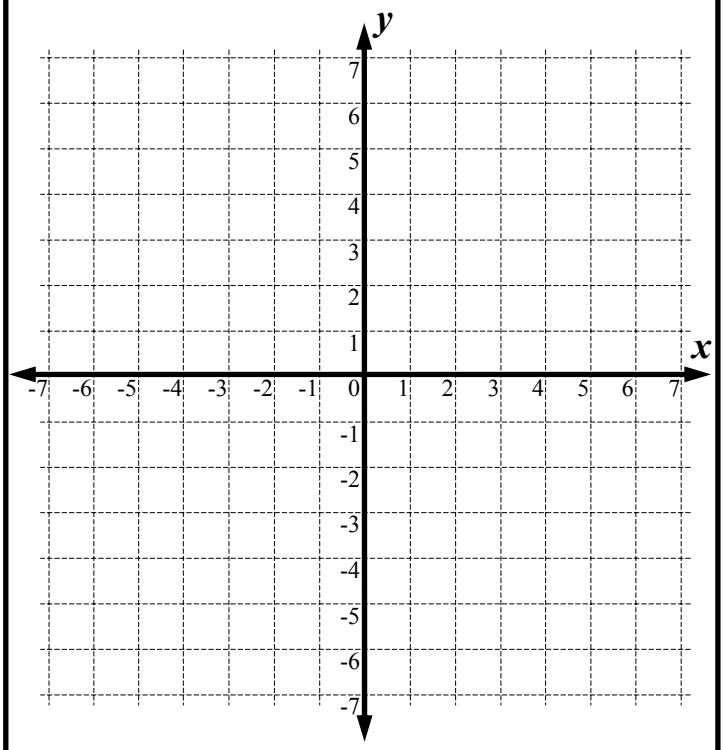
2 $x^2 + y^2 = 25 \xrightarrow{\text{Rearrange}} y^2 =$ ____

x	-5	-4	-3	-2	0	2	3	4	5
y		$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	$\pm$	



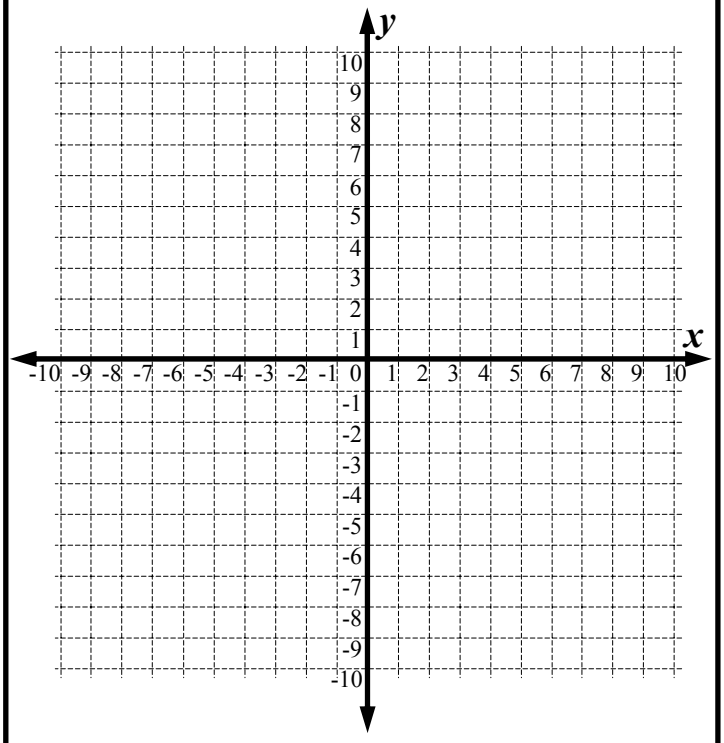
3 $x^2 + y^2 = 49 \Longrightarrow$ ____

x	-7	-6	-4	-2	0	2	4	6	7
y									



4 $x^2 + y^2 = 100 \Longrightarrow$ ____

x	-10	-8	-6	-4	0	-4	6	8	10
y									



Non-Linear Relationships - Find a Word

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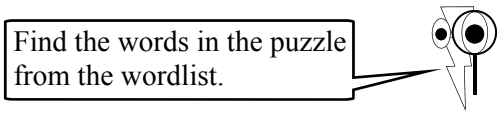

N	I	N	T	E	R	C	E	P	T	P	C	P	E	T
E	E	T	T	N	E	I	C	I	F	F	E	O	C	T
T	S	I	F	E	T	D	A	L	O	B	A	R	A	P
U	N	R	L	L	I	C	C	O	N	C	A	V	E	E
T	V	B	E	R	A	P	M	O	C	C	L	O	T	E
I	A	S	G	W	O	T	I	N	T	E	A	E	T	T
T	L	E	I	I	O	T	T	N	B	P	C	V	N	A
S	U	I	N	X	A	P	L	A	N	E	I	I	E	R
B	E	T	N	U	A	J	L	X	E	X	R	T	G	E
U	S	R	Q	E	X	P	O	N	E	N	T	I	A	L
S	K	E	T	C	H	P	A	R	G	V	E	S	T	E
P	N	P	C	E	L	C	R	I	C	O	M	O	I	C
L	W	O	U	O	W	O	R	R	A	N	M	P	V	C
O	O	R	T	S	U	I	D	A	R	M	Y	S	E	A
W	D	P	S	Y	M	C	A	R	T	E	S	I	A	N

WORDLIST

POWER	UP	PARABOLA	NEGATIVE	PROPERTIES
INTERCEPT	GRAPH	POINTS	TABLE	NARROW
SUBSTITUTE	FLAT	PLANE	EQUATION	CARTESIAN
POSITIVE	SKETCH	CONCAVE	COMPARE	AXIS
VALUES	CUTS	ACCELERATE	GRID	PLOT
SYMMETRICAL	EXPONENTIAL	DOWN	COEFFICIENT	LABEL
	RADIUS		CIRCLE	

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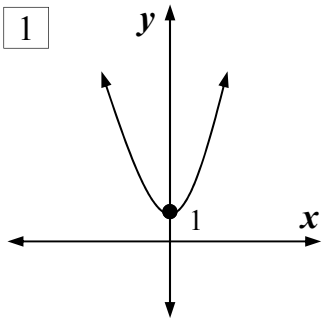
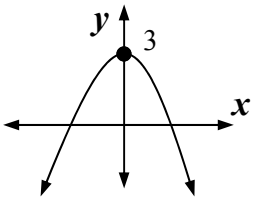
Complete the box to show the parabola's equation

1



$y = x^2 +$ ☐

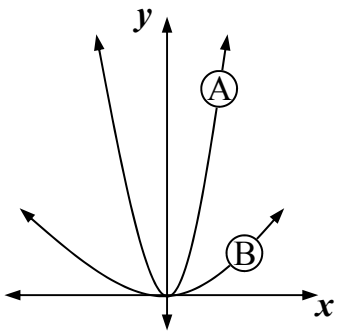
2



$y = -x^2 +$ ☐

Pick the appropriate equation for the parabolas below.

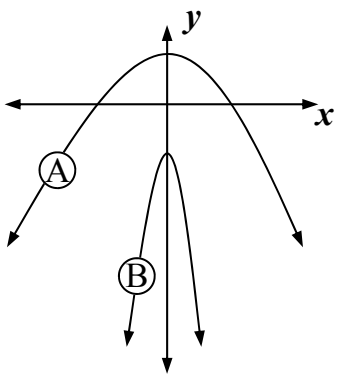
3



$y = 2x^2$ Ⓐ Ⓑ

$y = \frac{1}{2}x^2$ Ⓐ Ⓑ

4



$y = -3x^2 - 2$ Ⓐ Ⓑ

$y = -\frac{1}{2}x^2 + 2$ Ⓐ Ⓑ

Complete the sentence with a number and either up/down.

5 The parabola $y = x^2 + 5$ cuts the y-axis at ☐ and is concave ☐.

6 The parabola $y = -x^2 - 2$ cuts the y-axis at ☐ and is concave ☐.

Complete the sentence with either 'narrow' or 'wide'.

7 The parabola $y = -x^2 - 4$ is more ☐ than $y = -2x^2 - 1$.

8 The parabola $y = 2x^2$ is more ☐ than $y = \frac{1}{2}x^2 + 5$.

Complete these tables.

9 $y = x^2 + 3$

x	-3	-2	-1	0	1	2	3
y							

10 $y = -3x^2$

x	-3	-2	-1	0	1	2	3
y							

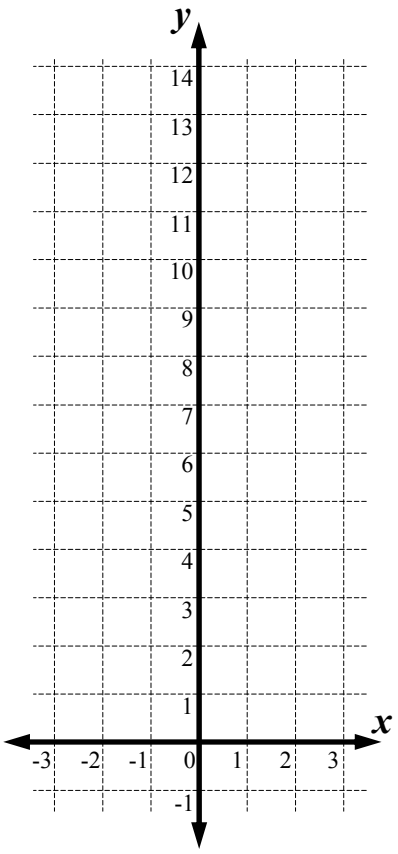
11 $y = \frac{1}{2}x^2 + 4$

x	-6	-4	-2	0	2	4	6
y							

Complete the table then plot the parabola $y = x^2 - 2$ on the number plane.

12 $y = x^2 + 4$

x	-3	-2	-1	0	1	2	3
y							



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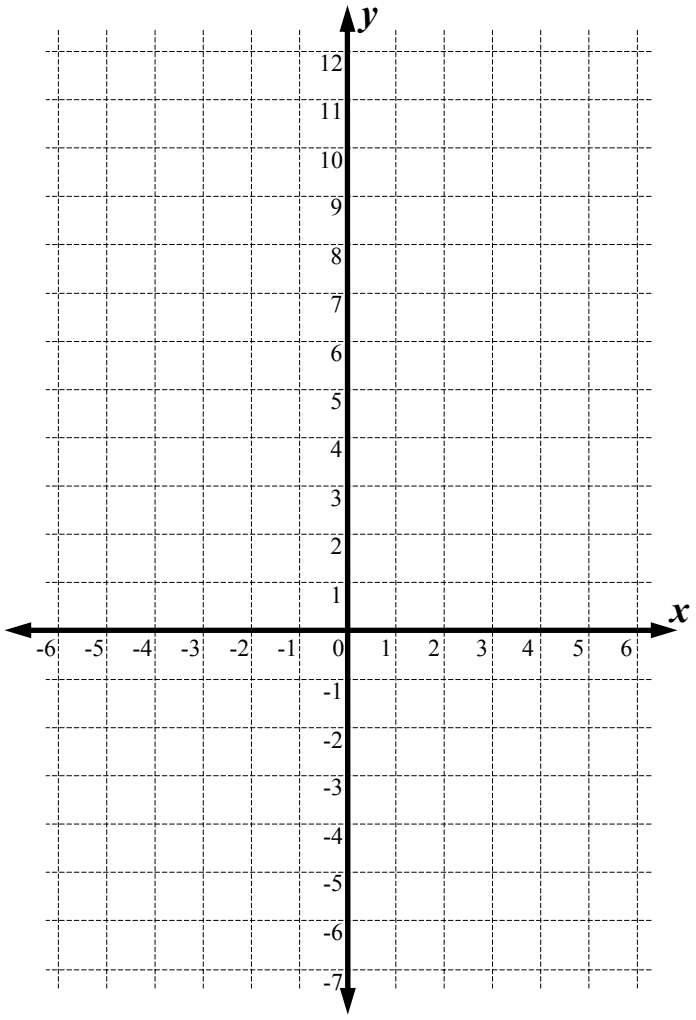
Complete the table, then plot the parabola.

13 $y = \frac{1}{2}x^2 - 6$

x	-6	-4	-2	0	2	4	6
y							



Complete the table of values.

14 $y = 2^x$

x	0	1	2	3	4	5
y						

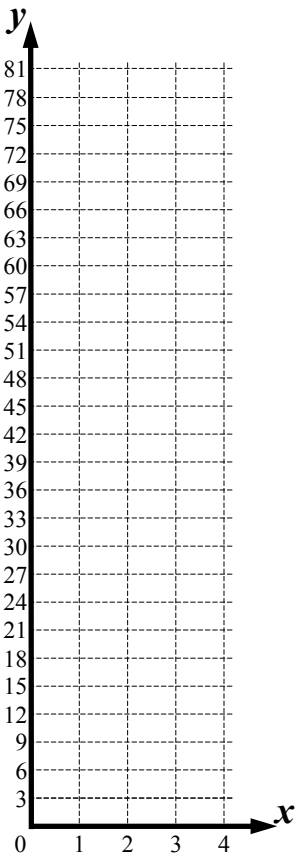
15 $y = 4^x$

x	0	1	2	3	4	5
y						

Complete the table, then plot the exponential.

16 $y = 3^x$

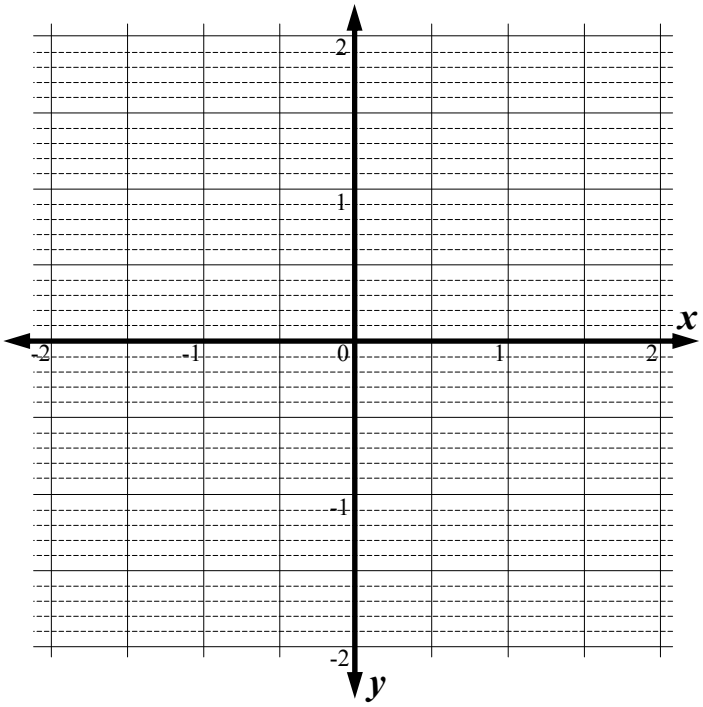
x	0	1	2	3	4
y					



Complete the table to 1 d.p., then plot the circle.

17 $x^2 + y^2 = 4$

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y									



VELOCITY

NON-LINEAR RELATIONSHIPS

EXTENSION

Non-Linear Relationships - Testing Points

If you are given a point and are asked to verify if it is on a parabola, or exponential or circle, the process is the same. Substitute the point into the equation and if it is satisfied, the point is on the equation.

Satisfied means the left hand side of the equation = right hand side (after substitution). Here is an example.

Example: Does the point $\overset{x}{6}, \overset{y}{16}$ lie on the parabola $y = x^2 - 8$?

STEP 1 Label the point in the question with an 'x' and a 'y'.

$\overset{x}{6}, \overset{y}{16}$

STEP 2 Write out the equation of the parabola.

$y = x^2 - 8$

STEP 3 Substitute the values of x and y. Which is the point's coordinates in the question.

$y = x^2 - 8$

$16 = 6^2 - 8$ ← $x = 6, y = 16$.

STEP 4 Calculate the right hand side. If the two sides aren't equal, put a stroke through the 'equals' sign.

$y = x^2 - 8$

$16 = 6^2 - 8$

$16 \neq 28$ Calc: 6 x^2 - 8 =

STEP 5 Make a statement about the left and right hand side of the equation.

If they aren't equal, then the point isn't on the curve. If they are equal the point is on the curve.

$y = x^2 - 8$

$16 = 6^2 - 8$

$16 \neq 28$

LHS ≠ RHS ∴ Point not on Parabola

Non-Linear Relationships - Testing Points and Parabolas (Sheet 1)

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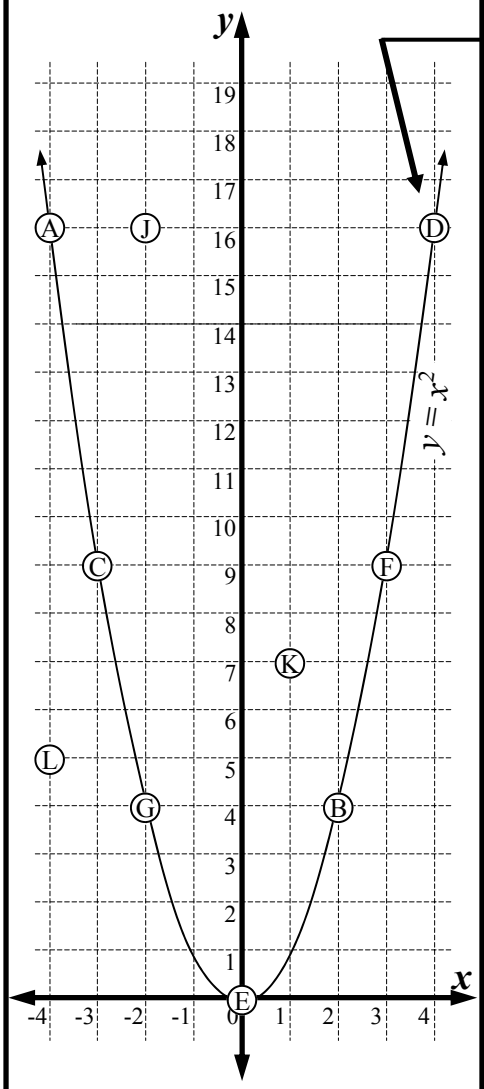
Look at the graph of $y = x^2$ and give the co-ordinates for these points.



Give the co-ordinates for…..

1	A	(,)	These 7 points are on the parabola
2	G	(,)	
3	B	(,)	
4	E	(,)	
5	C	(,)	
6	D	(,)	
7	F	(,)	
8	J	(,)	These 3 points aren't on the parabola
9	K	(,)	
10	L	(,)	

Now show which points are or aren't on the parabola $y = x^2$, algebraically.

Example Point D: (4, 16)
x y

$y = x^2$ ← Write the equation

$16 = 4^2$ ← Substitute x and y

$16 = 16$ ← Calculate 4^2

LHS = RHS ∴ Point on Parabola

12 Point B: (,)
x y

← Write the equation

$\square = \square^2$ ← Substitute x and y

← Calculate

13 Point K: (,)
x y

14 Point A: (,)

← Negative in brackets

15 Point J: (,)

← Negative in brackets

Now the parabolas change each question. Find whether the point is on the parabola or not.

16 Does (2, 7) lie on $y = x^2 + 5$?
x y

← Write equation

$\square = \square^2 + 5$ ← Sub x and y

← Calculation

17 Does (-3, 8) lie on $y = x^2 - 1$?

$\square = \square^2 - \square$

← Brackets

18 Does (5, 85) lie on $y = 2x^2 - 15$?

19 Does (-4, 9) lie on $y = -x^2 - 7$?

20 Does (4, -43) lie on $y = -3x^2 + 5$?

Non-Linear Relationships - Testing Points Circles/Exponentials (Sheet 2)

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Substitute in x and y into these exponential equations to find if the point is on the curve.

Example

Does (3, 143) lie on $y = 4^x$?

x y

$y = 4^x$ ← Write the equation

$143 = 4^3$ ← Substitute x and y

$143 \neq 64$ ← Calculate 4^3

4 $x^{\blacksquare}$ 3 =

LHS ≠ RHS ∴ Point not on curve

1 Does (5, 32) lie on $y = 2^x$?

x y

← Write equation

$\square = 2^{\square}$ ← Substitute x and y

← Calculate 2^5

2 $x^{\blacksquare}$ 5 =

2 Does (7, 2 187) lie on $y = 3^x$?

3 Does (3, 10 000) lie on $y = 10^x$?

4 Does (10, 1 024) lie on $y = 2^x$?

Now it is circles, sub in x and y to show if the point is on the circle.

Example

Does (3, 6) lie on $x^2 + y^2 = 45$?

x y

$x^2 + y^2 = 45$ ← Write the equation

$3^2 + 6^2 = 45$ ← Substitute x and y

$45 = 45$ ← Calculate $3^2 + 6^2$

3 x^2 + 6 x^2 =

LHS = RHS ∴ Point on circle

5 Does (2, 3) lie on $x^2 + y^2 = 15$?

x y

← Write equation

$\square^2 + \square^2 = 15$ ← Sub x and y

← Calculate

6 Does (7, 6) lie on $x^2 + y^2 = 77$?

7 Does (9, 12) lie on $x^2 + y^2 = 225$?

8 Does (1, 8) lie on $x^2 + y^2 = 60$?

If $x^2 + y^2$ is less than the constant the point is inside the circle, more and it's outside. Find if the point is in or out!

LHS < RHS ⟶ Point Inside

LHS > RHS ⟶ Point Outside

Example

x y

Is the point (3, 7) inside or outside of the circle $x^2 + y^2 = 60$?

$x^2 + y^2 = 60$ ← Write the equation

$3^2 + 7^2 = 60$ ← Substitute x and y

$58 < 60$ ← Calculate $3^2 + 7^2$

3 x^2 + 7 x^2 =

LHS < RHS ∴ Point inside circle

9 Is the point (5, 2) inside or outside of the circle $x^2 + y^2 = 20$?

10 Is the point (7, 5) inside or outside of the circle $x^2 + y^2 = 64$?

11 Is the point (3, 8) inside or outside of the circle $x^2 + y^2 = 75$?

Non-Linear Relationships - Testing Points and Parabolas (Sheet 1)

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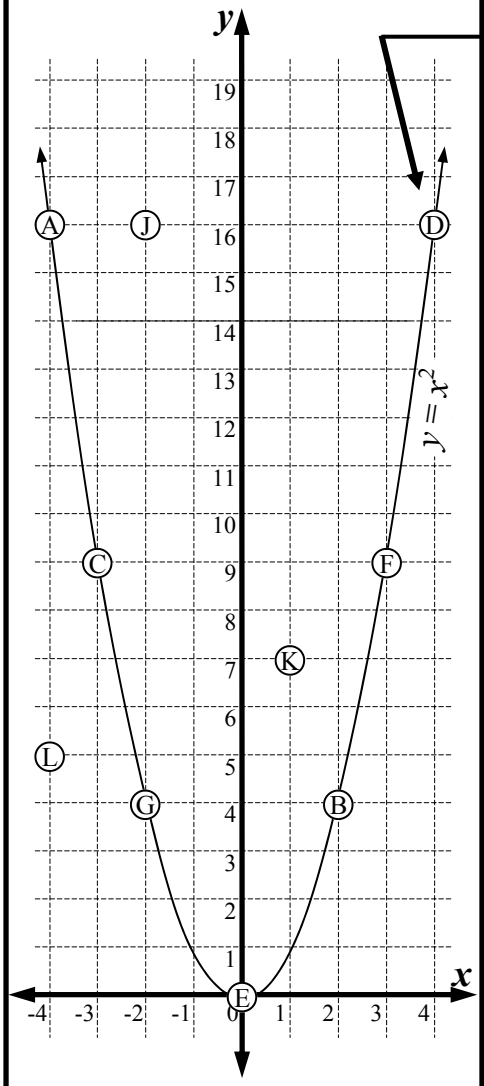
Look at the graph of $y = x^2$ and give the co-ordinates for these points.



Give the co-ordinates for…..

1	A	(-4, 16)	These 7 points are on the parabola
2	G	(-2, 4)	
3	B	(2, 4)	
4	E	(0, 0)	
5	C	(-3, 9)	
6	D	(4, 16)	
7	F	(3, 9)	
8	J	(-2, 16)	These 3 points aren't on the parabola
9	K	(1, 7)	
10	L	(-4, 5)	

Now show which points are or aren't on the parabola $y = x^2$, algebraically.

Example Point D: (4, 16)

$y = x^2$ ← Write the equation

$16 = 4^2$ ← Substitute x and y

$16 = 16$ ← Calculate 4^2

LHS = RHS ∴ Point on Parabola

12 Point B: (2 , 4)

$y = x^2$ ← Write the equation

$4 = 2^2$ ← Substitute x and y

$4 = 4$ ← Calculate

LHS = RHS ∴ Point on Parabola

13 Point K: (1 , 7)

$y = x^2$

$7 = 1^2$

$7 \neq 1$

LHS ≠ RHS ∴ Point not on Parabola

14 Point A: (-4 , 16)

$y = x^2$

$16 = (-4)^2$

$16 = 16$ ← Negative in brackets

LHS = RHS ∴ Point on Parabola

15 Point J: (-2 , 16)

$y = x^2$

$16 = (-2)^2$

$16 \neq 4$ ← Negative in brackets

LHS ≠ RHS ∴ Point not on Parabola

Now the parabolas change each question. Find whether the point is on the parabola or not.

16 Does (2, 7) lie on $y = x^2 + 5$?

$y = x^2 + 5$ ← Write equation

$7 = 2^2 + 5$ ← Sub x and y

$7 \neq 9$ ← Calculation

LHS ≠ RHS ∴ Point not on Parabola

17 Does (-3, 8) lie on $y = x^2 - 1$?

$y = x^2 - 1$

$8 = (-3)^2 - 1$

$8 = 8$ ← Brackets

LHS = RHS ∴ Point on Parabola

18 Does (5, 85) lie on $y = 2x^2 - 15$?

$y = 2x^2 - 15$

$85 = 2 \times 5^2 - 15$

$85 \neq 35$

LHS ≠ RHS ∴ Point not on Parabola

19 Does (-4, 9) lie on $y = -x^2 - 7$?

$y = -x^2 - 7$

$9 = -(-4)^2 - 7$

$9 \neq -23$

LHS ≠ RHS ∴ Point not on Parabola

20 Does (4, -43) lie on $y = -3x^2 + 5$?

$y = -3x^2 + 5$

$-43 = -3 \times 4^2 + 5$

$-43 = -43$

LHS = RHS ∴ Point on Parabola

Non-Linear Relationships - Testing Points Circles/Exponentials (Sheet 2)

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Substitute in x and y into these exponential equations to find if the point is on the curve.

Example

Does (3, 143) lie on $y = 4^x$?

x y

$y = 4^x$ ← Write the equation

$143 = 4^3$ ← Substitute x and y

$143 \neq 64$ ← Calculate 4^3

4 $x^{\blacksquare}$ 3 =

LHS ≠ RHS ∴ Point not on curve

1 Does (5, 32) lie on $y = 2^x$?

x y

$y = 2^x$ ← Write equation

$32 = 2^{5}$ ← Substitute x and y

$32 = 32$ ← Calculate 2^5

2 $x^{\blacksquare}$ 5 =

LHS = RHS ∴ Point is on curve

2 Does (7, 2 187) lie on $y = 3^x$?

x y

$y = 3^x$

$2\,187 = 3^7$

$2\,187 = 2\,187$

LHS = RHS ∴ Point is on curve

3 Does (3, 10 000) lie on $y = 10^x$?

x y

$y = 10^x$

$10\,000 = 10^3$

$10\,000 \neq 1\,000$

LHS ≠ RHS ∴ Point not on curve

4 Does (10, 1 024) lie on $y = 2^x$?

x y

$y = 2^x$

$1\,024 = 2^{10}$

$1\,024 = 1\,024$

LHS = RHS ∴ Point is on curve

Now it is circles, sub in x and y to show if the point is on the circle.

Example

Does (3, 6) lie on $x^2 + y^2 = 45$?

x y

$x^2 + y^2 = 45$ ← Write the equation

$3^2 + 6^2 = 45$ ← Substitute x and y

$45 = 45$ ← Calculate $3^2 + 6^2$

3 x^2 + 6 x^2 =

LHS = RHS ∴ Point on circle

5 Does (2, 3) lie on $x^2 + y^2 = 15$?

x y

$x^2 + y^2 = 15$ ← Write equation

$2^{2} + 3^{2} = 15$ ← Sub x and y

$13 \neq 15$ ← Calculate

LHS ≠ RHS ∴ Point not on circle

6 Does (7, 6) lie on $x^2 + y^2 = 77$?

x y

$x^2 + y^2 = 77$

$7^2 + 6^2 = 77$

$85 \neq 77$

LHS ≠ RHS ∴ Point not on circle

7 Does (9, 12) lie on $x^2 + y^2 = 225$?

x y

$x^2 + y^2 = 225$

$9^2 + 12^2 = 225$

$225 = 225$

LHS = RHS ∴ Point on circle

8 Does (1, 8) lie on $x^2 + y^2 = 60$?

x y

$x^2 + y^2 = 60$

$1^2 + 8^2 = 60$

$65 \neq 60$

LHS ≠ RHS ∴ Point not on circle

If $x^2 + y^2$ is less than the constant the point is inside the circle, more and it's outside. Find if the point is in or out!

LHS < RHS ⟶ Point Inside

LHS > RHS ⟶ Point Outside

Example

Is the point (3, 7) inside or outside of the circle $x^2 + y^2 = 60$?

x y

$x^2 + y^2 = 60$ ← Write the equation

$3^2 + 7^2 = 60$ ← Substitute x and y

$58 < 60$ ← Calculate $3^2 + 7^2$

3 x^2 + 7 x^2 =

LHS < RHS ∴ Point inside circle

9 Is the point (5, 2) inside or outside of the circle $x^2 + y^2 = 20$?

x y

$x^2 + y^2 = 20$

$5^2 + 2^2 = 20$

$29 > 20$

LHS > RHS ∴ Point outside circle

10 Is the point (7, 5) inside or outside of the circle $x^2 + y^2 = 64$?

x y

$x^2 + y^2 = 64$

$7^2 + 5^2 = 64$

$74 > 64$

LHS > RHS ∴ Point outside circle

11 Is the point (3, 8) inside or outside of the circle $x^2 + y^2 = 75$?

x y

$x^2 + y^2 = 75$

$3^2 + 8^2 = 75$

$73 < 75$

LHS < RHS ∴ Point inside circle

Non-Linear Relationships - Testing Points (Homework Strip)

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Date Due: _____

Show whether these points are on the curves or not.

1 Does (3, 15) lie on $y = x^2 + 6$?

x y

← Write equation

$\square = \square^2 + 6$ ← Sub x and y

$\square = \square$ ← Calculation

LHS = RHS ∴

2 Does (4, 1 296) lie on $y = 6^x$?

3 Does (5, 5) lie on $x^2 + y^2 = 55$?

Date Due: _____

Show whether these points are on the curves or not.

1 Does (3, 15) lie on $y = x^2 + 6$?

x y

← Write equation

$\square = \square^2 + 6$ ← Sub x and y

$\square = \square$ ← Calculation

LHS = RHS ∴

2 Does (4, 1 296) lie on $y = 6^x$?

3 Does (5, 5) lie on $x^2 + y^2 = 55$?

Date Due: _____

Show whether these points are on the curves or not.

1 Does (3, 15) lie on $y = x^2 + 6$?

x y

← Write equation

$\square = \square^2 + 6$ ← Sub x and y

$\square = \square$ ← Calculation

LHS = RHS ∴

2 Does (4, 1 296) lie on $y = 6^x$?

3 Does (5, 5) lie on $x^2 + y^2 = 55$?

Date Due: _____

Show whether these points are on the curves or not.

1 Does (3, 15) lie on $y = x^2 + 6$?

x y

← Write equation

$\square = \square^2 + 6$ ← Sub x and y

$\square = \square$ ← Calculation

LHS = RHS ∴

2 Does (4, 1 296) lie on $y = 6^x$?

3 Does (5, 5) lie on $x^2 + y^2 = 55$?

Non-Linear Relationships - Testing Points (Homework Strip)

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Date Due:

Show whether these points are on the curves or not.

1 Does (3, 15) lie on $y = x^2 + 6$?

$x \quad y$

$y = x^2 + 6$ ← Write equation

$15 = 3^2 + 6$ ← Sub x and y

$15 = 15$ ← Calculation

LHS = RHS ∴ Point on Parabola

2 Does (4, 1 296) lie on $y = 6^x$?

$x \quad y$

$y = 6^x$

$1\,296 = 6^4$

$1\,296 = 1\,296$

LHS = RHS ∴ Point is on curve

3 Does (5, 5) lie on $x^2 + y^2 = 55$?

$x \quad y$

$x^2 + y^2 = 55$

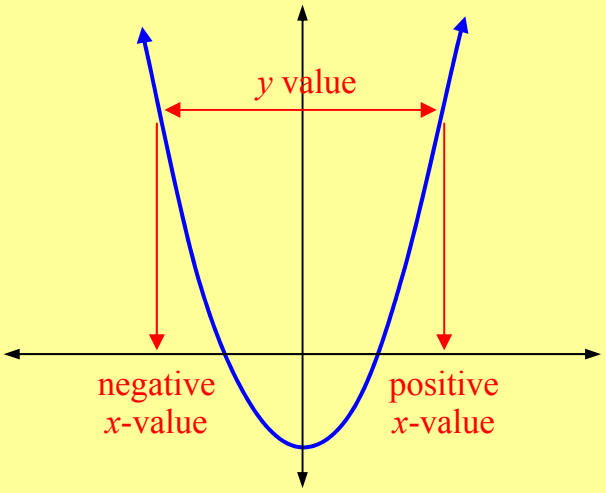
$5^2 + 5^2 = 55$

$50 \neq 55$

LHS ≠ RHS ∴ Point not on circle

Non-Linear Relationships - Finding *x*-values

A parabola is a symmetrical 'U-shaped' curve. Because it is U-shaped, it means that for each *y*-value (except at the minimum or maximum), there are two *x*-values.



This course deals with parabolas that are symmetrical across the *y*-axis. So that means the two *x*-values are the same magnitude, except that one is positive and the other is negative

These questions are solved by substitution and solving the resulting equation. Below is an example.

Example Find the *x*-values when $y = 44$ for the parabola $y = x^2 - 5$.

$y = x^2 - 5$ ← Write the equation

$44 = x^2 - 5$ ← Substitute $y = 44$

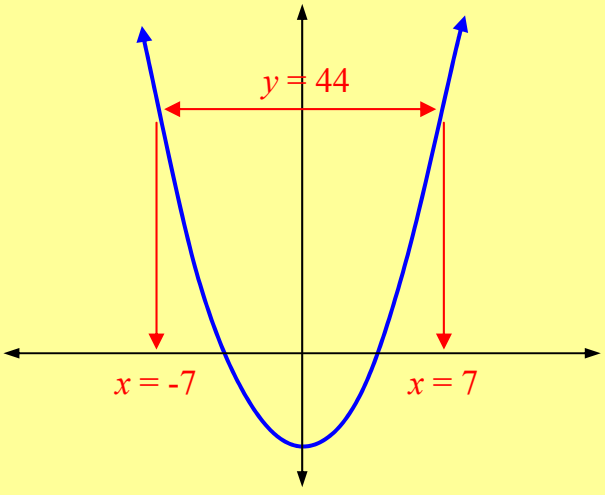
$+5 \quad +5$ ← +5 each side to get the x^2 term by itself

$49 = x^2$

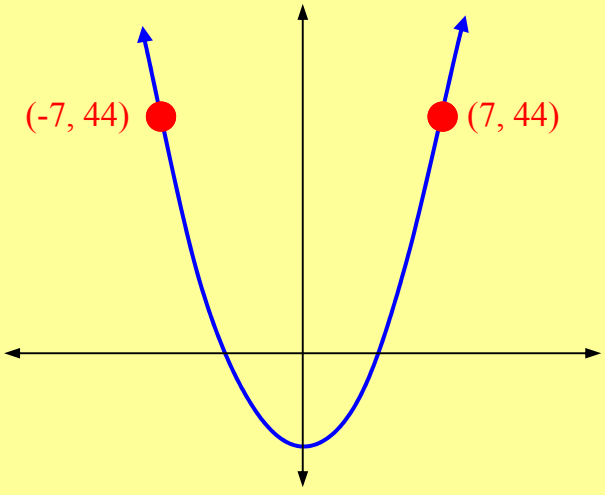
$\sqrt{\ } \quad \sqrt{\ }$ ← $\sqrt{\ }$ each side

$x = \pm 7$

So the *x*-values are -7 and 7.



That means the points on the parabola are (-7, 44) and (7, 44).



Non-Linear Relationships - Finding x-values (Sheet 1)

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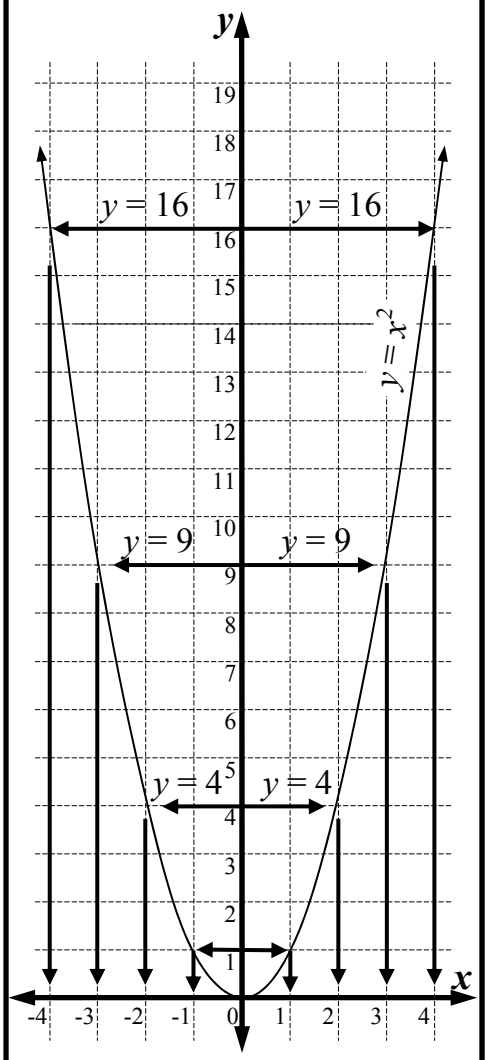
Read off the two x-values for each y-value on $y = x^2$.



1 When $y = 16$: $x =$ ☐ & ☐

2 When $y = 4$: $x =$ ☐ & ☐

3 When $y = 1$: $x =$ ☐ & ☐

4 When $y = 9$: $x =$ ☐ & ☐

The square root of a number results in two answers. Find the two answers for these.

5 $\sqrt{25} = \pm$ ☐ Calc: √ 2 5 =

6 $\sqrt{169} = \pm$ ☐

7 $\sqrt{324} =$ ☐

8 $\sqrt{121} =$ ☐

9 $\sqrt{81} =$ ☐

Solve these equations. The steps are set up for you.

10 $23 = x^2 + 7$

(-7) (-7)

☐ $= x^2$

(√) (√)

$x = \pm$ ☐

11 $39 = x^2 + 3$

(-3) (-3)

☐ $= x^2$

(√) (√)

$x = \pm$ ☐

12 $58 = x^2 + 9$

(-9) (-9)

☐ $= x^2$

(√) (√)

$x = \pm$ ☐

13 $50 = x^2 - 14$

(+14) (+14)

☐ $= x^2$

(√) (√)

$x = \pm$ ☐

Now solve these equations yourself. All answers are whole numbers. Remember ±

14 $9 = x^2 + 8$

○ ○

☐ $= x^2$

○ ○

$x = \pm$ ☐

15 $91 = x^2 - 9$

○ ○

☐ $= x^2$

○ ○

$x = \pm$ ☐

16 $51 = x^2 - 13$

○ ○

☐ $= x^2$

○ ○

$x =$ _____

17 $26 = x^2 + 22$

○ ○

☐ $= x^2$

○ ○

$x =$ _____

Now apply the skill to finding the x-values when given a y-value on a parabola.

18 Find the x-values when $y = 10$ for the parabola $y = x^2 + 6$.

$y = x^2 + 6$ ← Write the equation

$10 = x^2 + 6$ ← Substitute $y = 10$

○ ○ ← -6 each side

☐ $= x^2$

○ ○ ← √ each side

$x = \pm$ ☐

19 Find the x-values when $y = 33$ for the parabola $y = x^2 - 3$.

_____

_____

○ ○

☐ $= x^2$

○ ○

$x = \pm$ ☐

20 Find the x-values when $y = 25$ for the parabola $y = x^2 + 9$.

_____

_____

○ ○

☐ $= x^2$

○ ○

$x = \pm$ ☐

Non-Linear Relationships - Finding x-values (Sheet 2)

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Find the x-values when given a y-value on a parabola. Then give the points on the parabola.

1 Find the x-values when $y = 57$ for the parabola $y = x^2 - 7$.

_____ ← Write eqn

_____ ← Sub $y = 57$

◯ ◯ ← +7 each side

□ $= x^2$

◯ ◯ ← √ each side

$x =$ ±□

2 The two points are:

(□ , 57) & (□ , 57)

3 Find the x-values when $y = 0$ for the parabola $y = x^2 - 4$.

_____

_____

◯ ◯

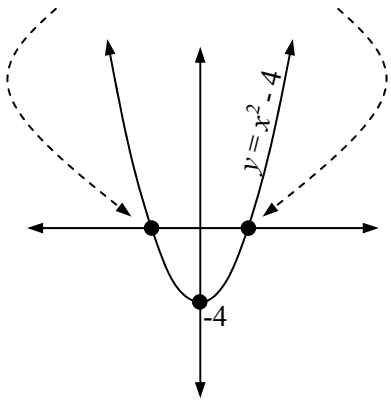
_____

◯ ◯

_____

4 The two points are:

(□ , □) & (□ , □)



With negatives, you switch the sign either side of the equation. Find the x-values for these.

5 Find the x-values when $y = 3$ for the parabola $y = -x^2 + 12$.

_____ ← Write eqn

_____ ← Sub $y = 3$

◯ ◯ ← -12 each side

□ $= -x^2$

swap swap

$x^2 =$ □ ← Change the sign

◯ ◯ ← √ each side

_____

6 The two points are:

(□ , 3) & (□ , 3)

7 Find the x-values when $y = -32$ for the parabola $y = -x^2 - 7$.

_____

◯ ◯

□ $= -x^2$

swap swap

$x^2 =$ □

◯ ◯

_____

8 The two points are:

(□ , □) & (□ , □)

Now there is a co-efficient. This requires division as an extra step. Find the x-values.

9 Find the x-values when $y = 14$ for the parabola $y = 2x^2 + 6$.

_____ ← Write eqn

_____ ← Sub $y = 14$

◯ ◯ ← -6 each side

□ $= 2x^2$

÷2 ÷2

$x^2 =$ □

◯ ◯ ← √ each side

_____

10 The two points are:

(□ , □) & (□ , □)

11 Find the x-values when $y = 46$ for the parabola $y = 3x^2 - 2$.

_____

◯ ◯

_____

◯ ◯

_____

◯ ◯

_____

12 The two points are:

(□ , □) & (□ , □)

Non-Linear Relationships - Finding x-values (Sheet 1)

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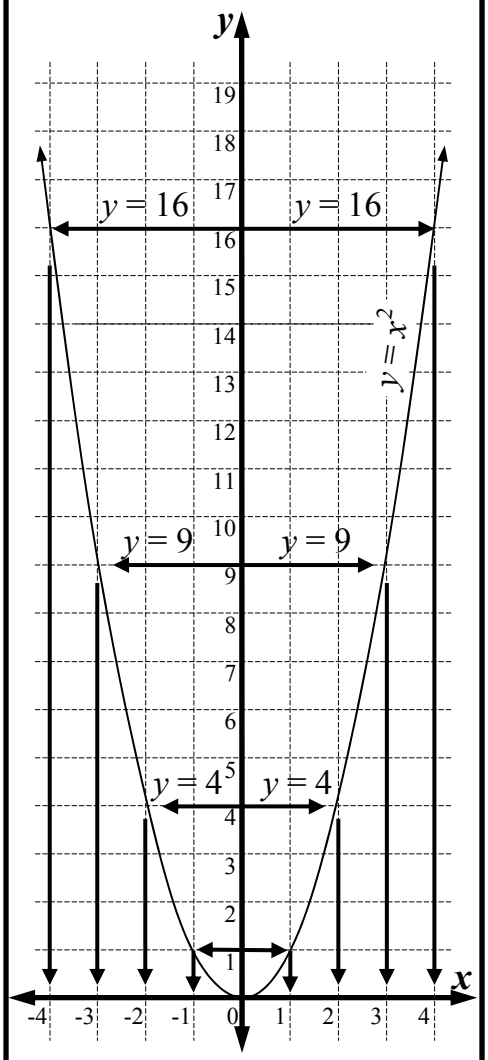
Read off the two x-values for each y-value on $y = x^2$.



1 When $y = 16$: $x =$ -4 & -4

2 When $y = 4$: $x =$ -2 & 2

3 When $y = 1$: $x =$ -1 & 1

4 When $y = 9$: $x =$ -3 & 3

The square root of a number results in two answers. Find the two answers for these.

5 $\sqrt{25} = \pm 5$ Calc: √ 2 5 =

6 $\sqrt{169} = \pm 13$

7 $\sqrt{324} = \pm 18$

8 $\sqrt{121} = \pm 11$

9 $\sqrt{81} = \pm 9$

Solve these equations. The steps are set up for you.

10

$23 = x^2 + 7$

-7 -7

$16 = x^2$

√ √

$x = \pm 4$

11

$39 = x^2 + 3$

-3 -3

$36 = x^2$

√ √

$x = \pm 6$

12

$58 = x^2 + 9$

-9 -9

$49 = x^2$

√ √

$x = \pm 7$

13

$50 = x^2 - 14$

+14 +14

$64 = x^2$

√ √

$x = \pm 8$

Now solve these equations yourself. All answers are whole numbers. Remember ±

14

$9 = x^2 + 8$

-8 -8

$1 = x^2$

√ √

$x = \pm 1$

15

$91 = x^2 - 9$

+9 +9

$100 = x^2$

√ √

$x = \pm 10$

16

$51 = x^2 - 13$

+13 +13

$64 = x^2$

√ √

$x = \pm 8$

17

$26 = x^2 + 22$

-22 -22

$4 = x^2$

√ √

$x = \pm 2$

Now apply the skill to finding the x-values when given a y-value on a parabola.

18 Find the x-values when $y = 10$ for the parabola $y = x^2 + 6$.

$y = x^2 + 6$ ← Write the equation

$10 = x^2 + 6$ ← Substitute $y = 10$

-6 -6 ← -6 each side

$4 = x^2$

√ √ ← √ each side

$x = \pm 2$

19 Find the x-values when $y = 33$ for the parabola $y = x^2 - 3$.

$y = x^2 - 3$

$33 = x^2 - 3$

+3 +3

$36 = x^2$

√ √

$x = \pm 6$

20 Find the x-values when $y = 25$ for the parabola $y = x^2 + 9$.

$y = x^2 + 9$

$25 = x^2 + 9$

-9 -9

$16 = x^2$

√ √

$x = \pm 4$

Non-Linear Relationships - Finding x-values (Sheet 2)

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Find the x-values when given a y-value on a parabola. Then give the points on the parabola.

1 Find the x-values when $y = 57$ for the parabola $y = x^2 - 7$.

$y = x^2 - 7$ ← Write eqn

$57 = x^2 - 7$ ← Sub $y = 57$

$+7$ $+7$ ← $+7$ each side

$64 = x^2$

$\sqrt{}$ $\sqrt{}$ ← $\sqrt{}$ each side

$x = \pm 8$

2 The two points are:

(8 , 57) & (-8 , 57)

3 Find the x-values when $y = 0$ for the parabola $y = x^2 - 4$.

$y = x^2 - 4$

$0 = x^2 - 4$

$+4$ $+4$

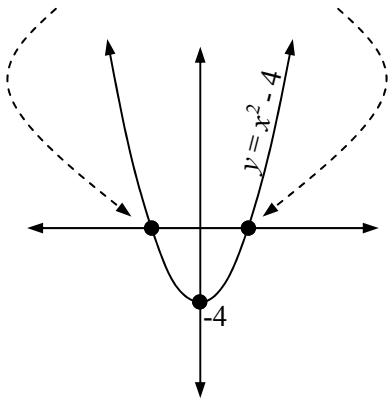
$x^2 = 4$

$\sqrt{}$ $\sqrt{}$

$x = \pm 2$

4 The two points are:

(-2 , 0) & (2 , 0)



With negatives, you switch the sign either side of the equation. Find the x-values for these.

5 Find the x-values when $y = 3$ for the parabola $y = -x^2 + 12$.

$y = -x^2 + 12$ ← Write eqn

$3 = -x^2 + 12$ ← Sub $y = 3$

-12 -12 ← -12 each side

$-9 = -x^2$

swap swap

$x^2 = 9$ ← Change the sign

$\sqrt{}$ $\sqrt{}$ ← $\sqrt{}$ each side

$x = \pm 3$

6 The two points are:

(-3 , 3) & (3 , 3)

7 Find the x-values when $y = -32$ for the parabola $y = -x^2 - 7$.

$y = -x^2 - 7$

$-32 = -x^2 - 7$

$+7$ $+7$

$-25 = -x^2$

swap swap

$x^2 = 25$

$\sqrt{}$ $\sqrt{}$

$x = \pm 5$

8 The two points are:

(-5 , -32) & (5 , -32)

Now there is a co-efficient. This requires division as an extra step. Find the x-values.

9 Find the x-values when $y = 14$ for the parabola $y = 2x^2 + 6$.

$y = 2x^2 + 6$ ← Write eqn

$14 = 2x^2 + 6$ ← Sub $y = 14$

-6 -6 ← -6 each side

$8 = 2x^2$

$\div 2$ $\div 2$

$x^2 = 4$

$\sqrt{}$ $\sqrt{}$ ← $\sqrt{}$ each side

$x = \pm 2$

10 The two points are:

(-2 , 14) & (2 , 14)

11 Find the x-values when $y = 46$ for the parabola $y = 3x^2 - 2$.

$y = 3x^2 - 2$

$46 = 3x^2 - 2$

$+2$ $+2$

$48 = 3x^2$

$\div 3$ $\div 3$

$x^2 = 16$

$\sqrt{}$ $\sqrt{}$

$x = \pm 4$

12 The two points are:

(-4 , 46) & (4 , 46)

Non-Linear Relationships - Finding x-values (Homework Strip)

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Date Due: _____

Find the x-values when $y = 44$ for the parabola $y = x^2 + 8$.

1 _____ ← Write eqn

_____ ← Sub $y = 44$

○ ○ ← -8 each side

□ $= x^2$

○ ○ ← $\sqrt{}$ each side

$x = \pm$ □

Find the x-values when $y = 4$ for the parabola $y = x^2 - 12$.

2 _____ ← Write eqn

_____ ← Sub $y = 4$

○ ○ ← +12 each side

□ $= x^2$

○ ○ ← $\sqrt{}$ each side

$x = \pm$ □

Date Due: _____

Find the x-values when $y = 44$ for the parabola $y = x^2 + 8$.

1 _____ ← Write eqn

_____ ← Sub $y = 44$

○ ○ ← -8 each side

□ $= x^2$

○ ○ ← $\sqrt{}$ each side

$x = \pm$ □

Find the x-values when $y = 4$ for the parabola $y = x^2 - 12$.

2 _____ ← Write eqn

_____ ← Sub $y = 4$

○ ○ ← +12 each side

□ $= x^2$

○ ○ ← $\sqrt{}$ each side

$x = \pm$ □

Date Due: _____

Find the x-values when $y = 44$ for the parabola $y = x^2 + 8$.

1 _____ ← Write eqn

_____ ← Sub $y = 44$

○ ○ ← -8 each side

□ $= x^2$

○ ○ ← $\sqrt{}$ each side

$x = \pm$ □

Find the x-values when $y = 4$ for the parabola $y = x^2 - 12$.

2 _____ ← Write eqn

_____ ← Sub $y = 4$

○ ○ ← +12 each side

□ $= x^2$

○ ○ ← $\sqrt{}$ each side

$x = \pm$ □

Date Due: _____

Find the x-values when $y = 44$ for the parabola $y = x^2 + 8$.

1 _____ ← Write eqn

_____ ← Sub $y = 44$

○ ○ ← -8 each side

□ $= x^2$

○ ○ ← $\sqrt{}$ each side

$x = \pm$ □

Find the x-values when $y = 4$ for the parabola $y = x^2 - 12$.

2 _____ ← Write eqn

_____ ← Sub $y = 4$

○ ○ ← +12 each side

□ $= x^2$

○ ○ ← $\sqrt{}$ each side

$x = \pm$ □

Non-Linear Relationships - Finding x-values (Homework Strip)

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Date Due:

Find the x-values when $y = 44$ for the parabola $y = x^2 + 8$.

1 $y = x^2 + 8$ ← Write eqn

$44 = x^2 + 8$ ← Sub $y = 44$

-8 -8 ← -8 each side

$36 = x^2$

$\sqrt{\ }$ $\sqrt{\ }$ ← $\sqrt{\ }$ each side

$x = \pm 6$

Find the x-values when $y = 4$ for the parabola $y = x^2 - 12$.

2 $y = x^2 - 12$ ← Write eqn

$4 = x^2 - 12$ ← Sub $y = 4$

$+12$ $+12$ ← $+12$ each side

$16 = x^2$

$\sqrt{\ }$ $\sqrt{\ }$ ← $\sqrt{\ }$ each side

$x = \pm 4$

Non-Linear Relationships - Properties of Circles

This sheet deals with finding the equation of a circle given its radius, or, given an equation of a circle, sketch the circle. The circles on this sheet all have their centre at the origin and so are symmetrical across both axes. The general equation for a circle with its centre at the origin is:

$$x^2 + y^2 = r^2$$

Where: r is the radius of the circle

Example Find the equation of a circle with radius 6 units (centre at the origin)

STEP 1 Write the general equation of a circle

$$x^2 + y^2 = r^2$$

STEP 2 Substitute the radius into the equation (radius = 6)

$$x^2 + y^2 = r^2$$
$$x^2 + y^2 = 6^2$$

STEP 3 Calculate 6^2 and you are done.

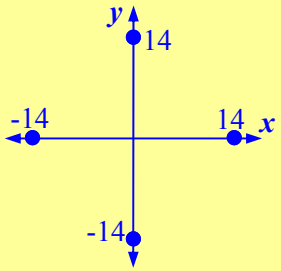
$$x^2 + y^2 = r^2$$
$$x^2 + y^2 = 6^2$$
$$x^2 + y^2 = 36$$

Example Sketch the circle $x^2 + y^2 = 196$

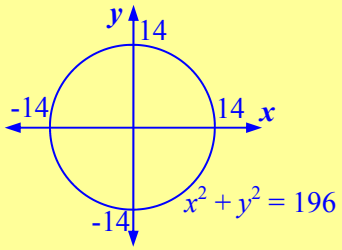
STEP 1 Find the radius of the circle. Square root 196 as it is r^2, and you want r.

$$r = \sqrt{196} = 14 \text{ units}$$

STEP 2 Rule a number plane and plot points at 14 and -14 on both the x and y axes.



STEP 3 Sketch the circle and write the equation next to the circle.



Non-Linear Relationships - Properties of Circles

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The equation of a circle with centre at the origin is $x^2+y^2=r^2$. Find the equation for these circles centred at the origin.

Example

Circle with radius of 5 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = 5^2$ ← Substitute radius

$x^2 + y^2 = 25$ ← Calculate 5^2

1 Circle with radius of 2 units.

← Write equation

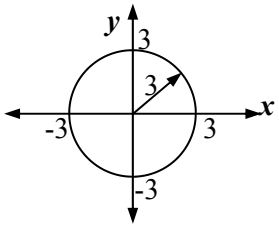
← Substitute radius

← Calculate 2^2

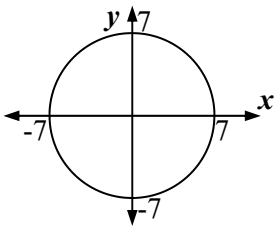
2 Circle with radius of 13 units.

3 Circle with radius of 9 units.

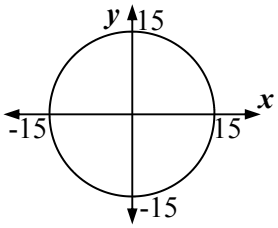
4 The circle shown below.



5 The circle shown below.



6 The circle shown below.



If you are given a diameter, halve it to find the radius. Find the equations for these circles with centres at the origin.

7 Circle with diameter of 16 units.

$r = 16 \div 2 =$ ☐ units

← Write equation

← Substitute **radius**

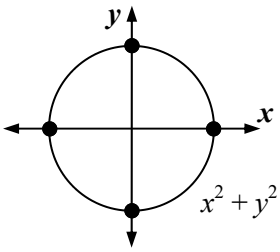
← Calculate

8 Circle with diameter of 34 units.

Sketch these circles on a number plane. Find the radius first by using your $\sqrt{}$ key.

9 $x^2 + y^2 = 144$

$r = \sqrt{144} =$ ☐ units

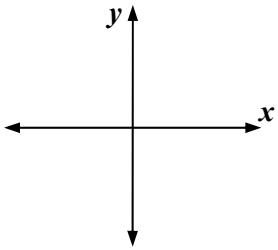


Write x and y intercepts at the 4 dots

$x^2 + y^2 = 144$

10 $x^2 + y^2 = 400$

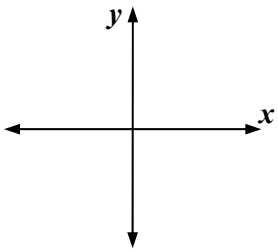
$r = \sqrt{400} =$ ☐ units



Write equation next to circle

11 $x^2 + y^2 = 729$

$r =$



12 $x^2 + y^2 = 361$

Non-Linear Relationships - Properties of Circles

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The equation of a circle with centre at the origin is $x^2+y^2=r^2$. Find the equation for these circles centred at the origin.

Example

Circle with radius of 5 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = 5^2$ ← Substitute radius

$x^2 + y^2 = 25$ ← Calculate 5^2

1 Circle with radius of 2 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = 2^2$ ← Substitute radius

$x^2 + y^2 = 4$ ← Calculate 2^2

2 Circle with radius of 13 units.

$x^2 + y^2 = r^2$

$x^2 + y^2 = 13^2$

$x^2 + y^2 = 169$

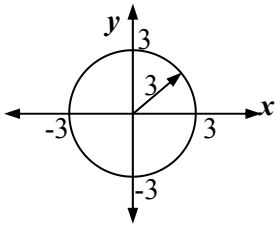
3 Circle with radius of 9 units.

$x^2 + y^2 = r^2$

$x^2 + y^2 = 9^2$

$x^2 + y^2 = 81$

4 The circle shown below.

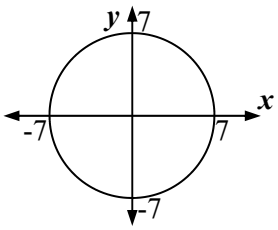


$x^2 + y^2 = r^2$

$x^2 + y^2 = 3^2$

$x^2 + y^2 = 9$

5 The circle shown below.

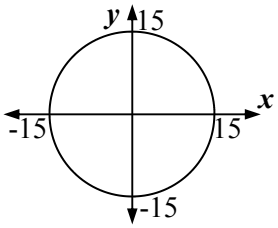


$x^2 + y^2 = r^2$

$x^2 + y^2 = 7^2$

$x^2 + y^2 = 49$

6 The circle shown below.



$x^2 + y^2 = r^2$

$x^2 + y^2 = 15^2$

$x^2 + y^2 = 225$

If you are given a diameter, halve it to find the radius. Find the equations for these circles with centres at the origin.

7 Circle with diameter of 16 units.

$r = 16 \div 2 =$ 8 units

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = 8^2$ ← Substitute **radius**

$x^2 + y^2 = 64$ ← Calculate

8 Circle with diameter of 34 units.

$r = 34 \div 2 = 17$ units

$x^2 + y^2 = r^2$

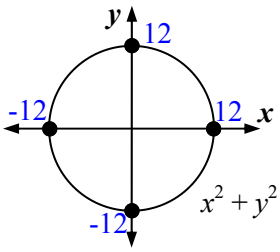
$x^2 + y^2 = 17^2$

$x^2 + y^2 = 289$

Sketch these circles on a number plane. Find the radius first by using your $\sqrt{}$ key.

9 $x^2 + y^2 = 144$

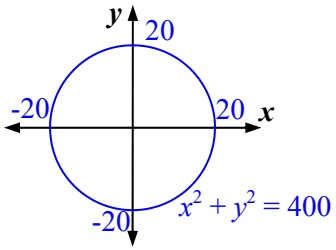
$r = \sqrt{144} =$ 12 units



Write x and y intercepts at the 4 dots

10 $x^2 + y^2 = 400$

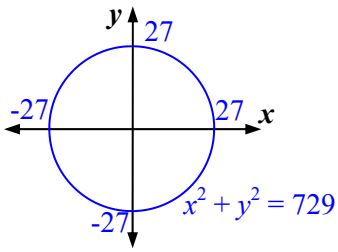
$r = \sqrt{400} =$ 20 units



Write equation next to circle

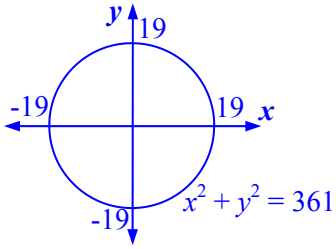
11 $x^2 + y^2 = 729$

$r = \sqrt{729} = 27$ units



12 $x^2 + y^2 = 361$

$r = \sqrt{361} = 19$ units



Non-Linear Relationships - Properties of Circles (Homework Strip)

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Date Due: _____

Find the equation for these circles which have centres at the origin.

1 Circle with radius of 5 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = \square^2$ ← Substitute radius

$x^2 + y^2 = \square$ ← Calculate 5^2

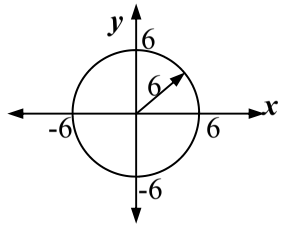
2 Circle with radius of 18 units.

_____ ← Write equation

_____ ← Substitute radius

_____ ← Calculate 18^2

3 The circle shown below.



_____

_____

_____

Date Due: _____

Find the equation for these circles which have centres at the origin.

1 Circle with radius of 5 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = \square^2$ ← Substitute radius

$x^2 + y^2 = \square$ ← Calculate 5^2

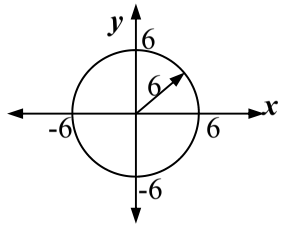
2 Circle with radius of 18 units.

_____ ← Write equation

_____ ← Substitute radius

_____ ← Calculate 18^2

3 The circle shown below.



_____

_____

_____

Date Due: _____

Find the equation for these circles which have centres at the origin.

1 Circle with radius of 5 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = \square^2$ ← Substitute radius

$x^2 + y^2 = \square$ ← Calculate 5^2

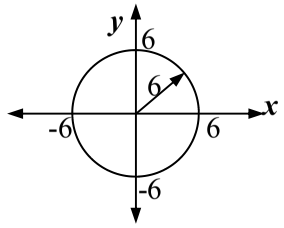
2 Circle with radius of 18 units.

_____ ← Write equation

_____ ← Substitute radius

_____ ← Calculate 18^2

3 The circle shown below.



_____

_____

_____

Date Due: _____

Find the equation for these circles which have centres at the origin.

1 Circle with radius of 5 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = \square^2$ ← Substitute radius

$x^2 + y^2 = \square$ ← Calculate 5^2

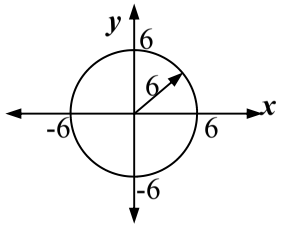
2 Circle with radius of 18 units.

_____ ← Write equation

_____ ← Substitute radius

_____ ← Calculate 18^2

3 The circle shown below.



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Non-Linear Relationships - Properties of Circles (Homework Strip)

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Date Due:

Find the equation for these circles which have centres at the origin.

1 Circle with radius of 5 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = \boxed{5}^2$ ← Substitute radius

$x^2 + y^2 = \boxed{25}$ ← Calculate 5^2

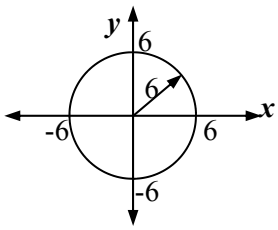
2 Circle with radius of 18 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = 18^2$ ← Substitute radius

$x^2 + y^2 = 324$ ← Calculate 18^2

3 The circle shown below.



$x^2 + y^2 = r^2$

$x^2 + y^2 = 6^2$

$x^2 + y^2 = 36$

Non-Linear Relationships - Point on Circle Find Equation - Pythagoras

If you are given the radius of a circle which has its centre at the origin then you can use the general equation:

$$x^2 + y^2 = r^2$$

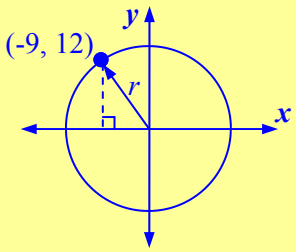
Where: r is the radius of the circle

The equation for the circle is found by substituting the value of r into the above equation.

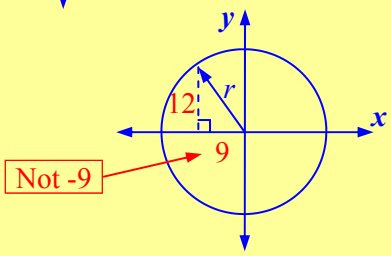
This sheet adds a step to the skill you learned on the previous sheet. Rather than being given r, you have to calculate the radius using a given point on the circle. A right angle triangle is created using the point, then Pythagoras Theorem (Addition) is used. Here is an example.

Example Find the equation of a circle, with centre at the origin, that passes through the point (-9, 12).

STEP 1 This is what the question looks like graphically. All the questions show the diagram.



STEP 2 Write the two short side distances on the triangle. Note that the distances are positive.



STEP 3 Calculate r using Pythagoras Theorem (Addition).

$r^2 = 9^2 + 12^2$

$r^2 = 225$

$r = \sqrt{225}$

$r = 15$ units

STEP 4 Write the general equation for a circle $x^2 + y^2 = r^2$.

$x^2 + y^2 = r^2$

STEP 5 Substitute the radius, that you calculated, into the equation.

$x^2 + y^2 = 15^2$

STEP 6 Calculate the square and you are done.

$x^2 + y^2 = 225$

Non-Linear Relationships - Point on Circle Find Equation - Pythagoras

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Use Pythagoras Theorem (Addition) to calculate the radius of these circles*.

*Only part of the circle is shown

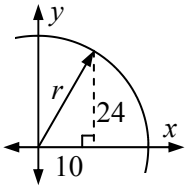
1

$r^2 = 10^2 + 24^2$

$r^2 =$ ☐

$r =$ $\sqrt{}$ ☐

$r =$ ☐ units



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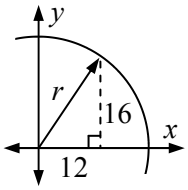


$r^2 =$ ☐^{2} + ☐^{2}

$r^2 =$ ☐

$r =$ $\sqrt{}$ ☐

$r =$ ☐ units

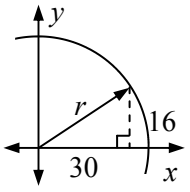
3

$r^2 =$

$r^2 =$

$r =$

$r =$ ☐ units



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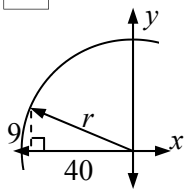
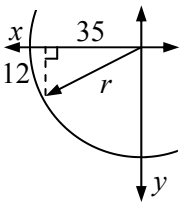
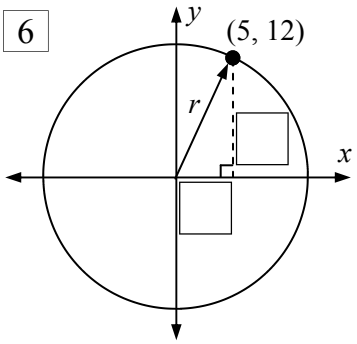


5



Given a point on the circle, write a distance in the boxes, which is always positive, then use Pythagoras to find the radius. After you find the radius, use $x^2 + y^2 = r^2$ to find the equation for the circle.

6



$r^2 =$ ☐^{2} + ☐^{2}

$r^2 =$ ☐

$r =$ $\sqrt{}$ ☐

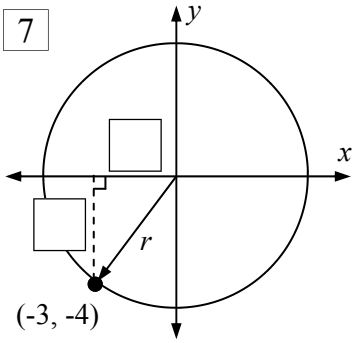
$r =$ ☐ units

$x^2 + y^2 = r^2$

$x^2 + y^2 =$ ☐^{2}

$x^2 + y^2 =$ ☐

7



$r^2 =$

$r^2 =$

$r =$

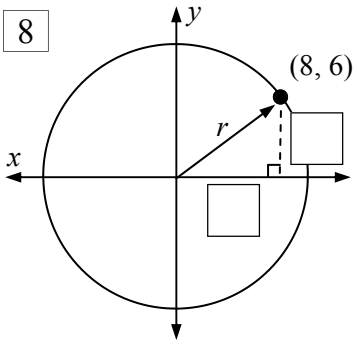
$r =$ ☐ units

$x^2 + y^2 = r^2$

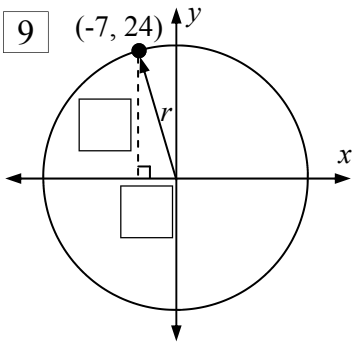
$x^2 + y^2 =$ ☐^{2}

$x^2 + y^2 =$ ☐

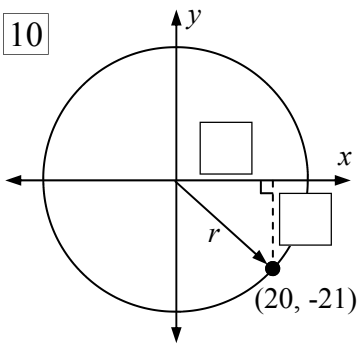
8



9



10



Non-Linear Relationships - Point on Circle Find Equation - Pythagoras

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Use Pythagoras Theorem (Addition) to calculate the radius of these circles*.

*Only part of the circle is shown

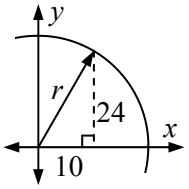
1

$r^2 = 10^2 + 24^2$

$r^2 = 676$

$r = \sqrt{676}$

$r = 26$ units



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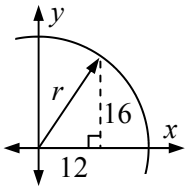


$r^2 = 12^2 + 16^2$

$r^2 = 400$

$r = \sqrt{400}$

$r = 20$ units

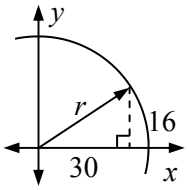
3

$r^2 = 30^2 + 16^2$

$r^2 = 1\,156$

$r = \sqrt{1\,156}$

$r = 34$ units



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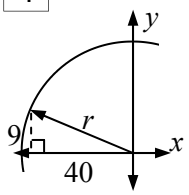


$r^2 = 9^2 + 40^2$

$r^2 = 1\,681$

$r = \sqrt{1\,681}$

$r = 41$ units

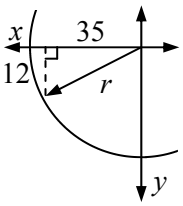
5

$r^2 = 12^2 + 35^2$

$r^2 = 1\,369$

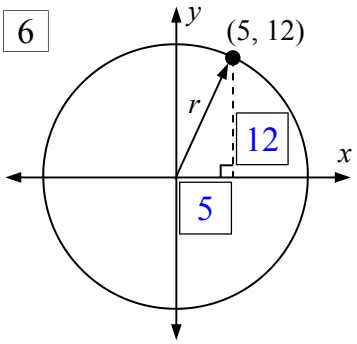
$r = \sqrt{1\,369}$

$r = 37$ units



Given a point on the circle, write a distance in the boxes, which is always positive, then use Pythagoras to find the radius. After you find the radius, use $x^2 + y^2 = r^2$ to find the equation for the circle.

6



$r^2 = 5^2 + 12^2$

$r^2 = 169$

$r = \sqrt{169}$

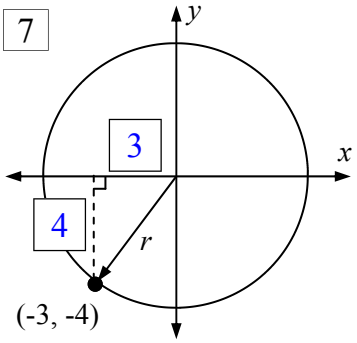
$r = 13$ units

$x^2 + y^2 = r^2$

$x^2 + y^2 = 13^2$

$x^2 + y^2 = 169$

7



$r^2 = 3^2 + 4^2$

$r^2 = 25$

$r = \sqrt{25}$

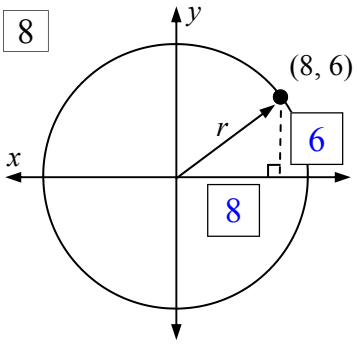
$r = 5$ units

$x^2 + y^2 = r^2$

$x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

8



$r^2 = 6^2 + 8^2$

$r^2 = 100$

$r = \sqrt{100}$

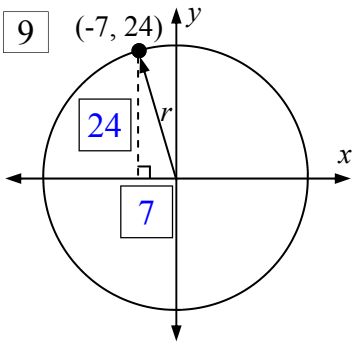
$r = 10$ units

$x^2 + y^2 = r^2$

$x^2 + y^2 = 10^2$

$x^2 + y^2 = 100$

9



$r^2 = 7^2 + 24^2$

$r^2 = 625$

$r = \sqrt{625}$

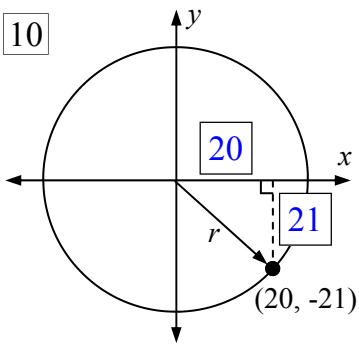
$r = 25$ units

$x^2 + y^2 = r^2$

$x^2 + y^2 = 25^2$

$x^2 + y^2 = 625$

10



$r^2 = 20^2 + 21^2$

$r^2 = 841$

$r = \sqrt{841}$

$r = 29$ units

$x^2 + y^2 = r^2$

$x^2 + y^2 = 29^2$

$x^2 + y^2 = 841$

Non-Linear Relationships - Point on Circle - Pythagoras (Homework Strip)

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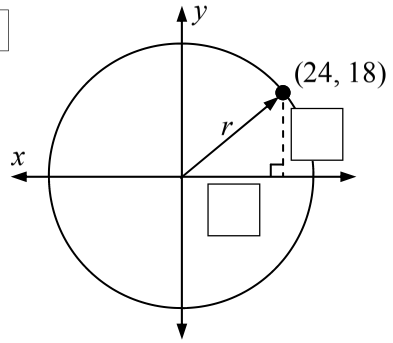
Date Due: _____

Write the short side distances on the triangle, given the point on the circle.

1



Use Pythagoras Theorem to find the radius.

2

$r^2 = \square^2 + \square^2$

$r^2 = \square$

$r = \sqrt{\square}$

$r = \square$ units

Find the equation of the circle above.

3

$x^2 + y^2 = r^2$

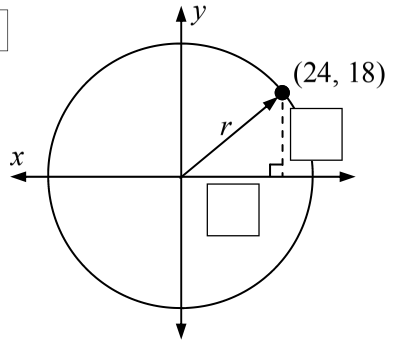
$x^2 + y^2 = \square^2$

$x^2 + y^2 = \square$

Date Due: _____

Write the short side distances on the triangle, given the point on the circle.

1



Use Pythagoras Theorem to find the radius.

2

$r^2 = \square^2 + \square^2$

$r^2 = \square$

$r = \sqrt{\square}$

$r = \square$ units

Find the equation of the circle above.

3

$x^2 + y^2 = r^2$

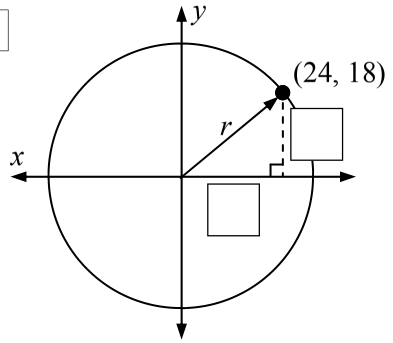
$x^2 + y^2 = \square^2$

$x^2 + y^2 = \square$

Date Due: _____

Write the short side distances on the triangle, given the point on the circle.

1



Use Pythagoras Theorem to find the radius.

2

$r^2 = \square^2 + \square^2$

$r^2 = \square$

$r = \sqrt{\square}$

$r = \square$ units

Find the equation of the circle above.

3

$x^2 + y^2 = r^2$

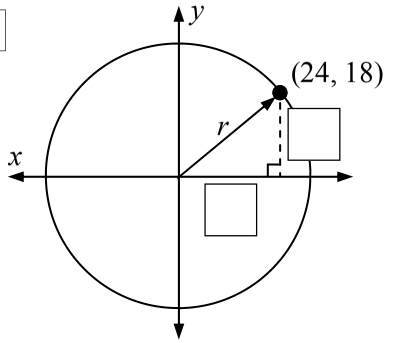
$x^2 + y^2 = \square^2$

$x^2 + y^2 = \square$

Date Due: _____

Write the short side distances on the triangle, given the point on the circle.

1



Use Pythagoras Theorem to find the radius.

2

$r^2 = \square^2 + \square^2$

$r^2 = \square$

$r = \sqrt{\square}$

$r = \square$ units

Find the equation of the circle above.

3

$x^2 + y^2 = r^2$

$x^2 + y^2 = \square^2$

$x^2 + y^2 = \square$

Non-Linear Relationships - Point on Circle - Pythagoras (H'work Strip)

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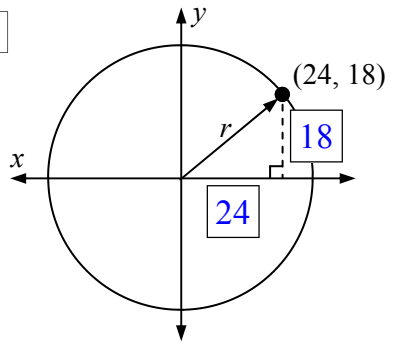
Date Due:

Write the short side distances on the triangle, given the point on the circle.

1



Use Pythagoras Theorem to find the radius.

2

$r^2 = 18^2 + 24^2$

$r^2 = 900$

$r = \sqrt{900}$

$r = 30$ units

Find the equation of the circle above.

3

$x^2 + y^2 = r^2$

$x^2 + y^2 = 30^2$

$x^2 + y^2 = 900$

Non-Linear Relationships - Exponentials - Negative Powers

This sheet requires you to complete a table of values and then plot points which will form an exponential curve. The exponentials on these worksheets feature both positive and negative x-values and also include decimals.

Exponentials accelerate quickly and in the space of three x-values, you can have y-values that are significantly larger.

STEP 1

First complete the table. You use the power key on your calculator.

Round to 1 d.p. for graphing.

$y = 4^x$

x	-2	-1.5	-1	-0.5	0	0.5	1
y	0.1	0.1	0.3	0.5	1	2	4

Calc: [4] [$x^{\blacksquare}$] [-] [2] [=]

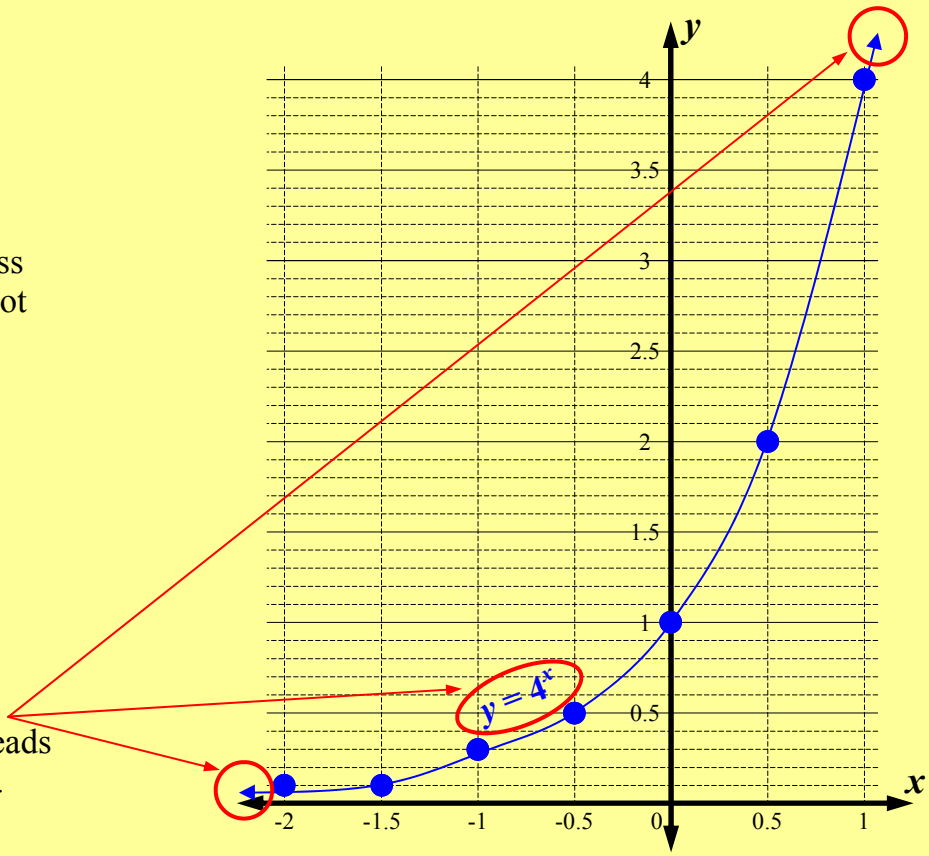
Calc: [4] [$x^{\blacksquare}$] [-] [1] [.] [5] [=]

STEP 2

Remember that x moves across and y moves up and down. Plot the points on the graph.

STEP 3

Draw the curve. Add arrowheads and the equation of the curve. Done.



Non-Linear Relationships - Exponentials - Negative Powers

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Evaluate these, rounding your answer to 1 decimal place.

1 $2^{-1} = 0.\square$

Calc: 2 $x^{\blacksquare}$ - 1 =

2 $2^{-2} = 0.\square$ Round to 1 d.p.

3 $3^{-2} =$ ____

4 $5^{-1} =$ ____

5 $3^{-1} =$ ____

These are decimal powers.

6 $3^{-1.5} =$ ____

7 $2^{-2.5} =$ ____

8 $5^{-0.5} =$ ____

Complete the tables rounding your answers to 1 decimal place when required.

6 $y = 3^x$

x	-2	-1	0	1	2
y					

7 $y = 2^x$

x	-2	-1	0	1	2
y					

8 $y = 5^x$

x	-2	-1	0	1	2
y					

9 $y = 4^x$

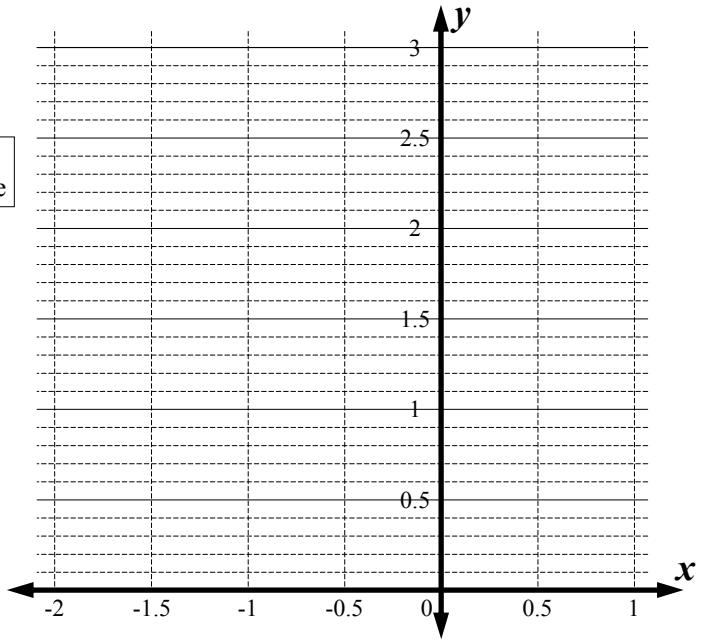
x	-2	-1	0	1	2
y					

Complete the tables round answer to 1 decimal place when required. Then plot the exponential curves, place arrowheads on the curve as well as the equation of the curve.

10 $y = 3^x$

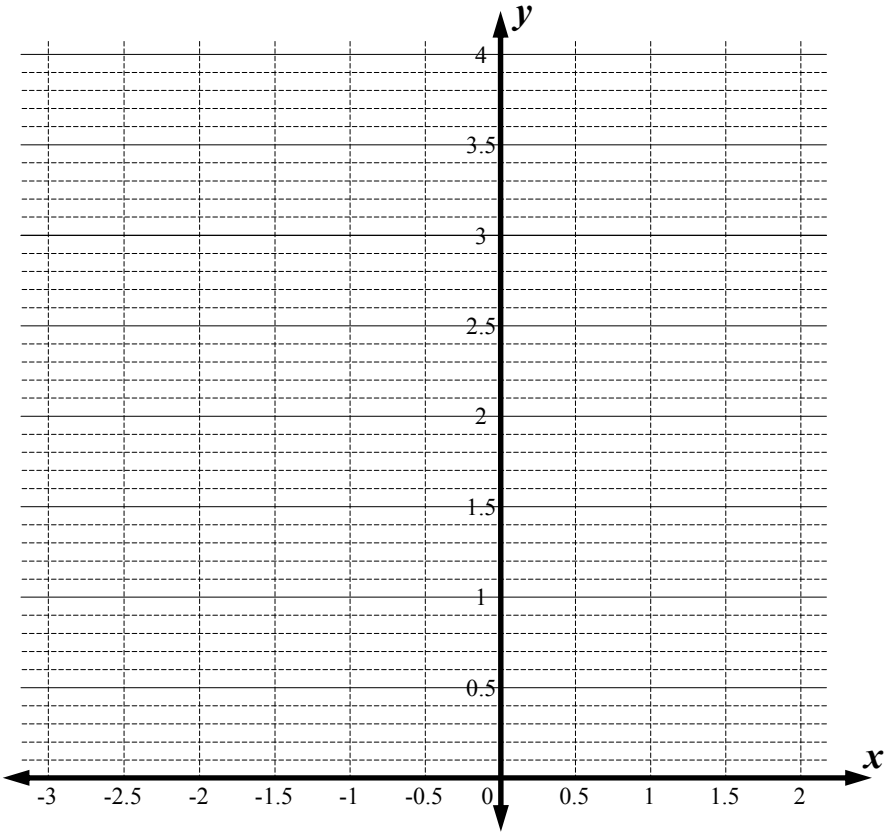
x	-2	-1.5	-1	-0.5	0	0.5	1
y							

- Arrowheads
- Equation on curve



11 $y = 2^x$

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y											



Non-Linear Relationships - Exponentials - Negative Powers

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Evaluate these, rounding your answer to 1 decimal place.

1 $2^{-1} = 0.5$

Calc: 2 $x^{\blacksquare}$ - 1 =

2 $2^{-2} = 0.3$ Round to 1 d.p.

3 $3^{-2} = 0.1$

4 $5^{-1} = 0.2$

5 $3^{-1} = 0.3$

These are decimal powers.

6 $3^{-1.5} = 0.2$

7 $2^{-2.5} = 0.2$

8 $5^{-0.5} = 0.4$

Complete the tables rounding your answers to 1 decimal place when required.

6 $y = 3^x$

x	-2	-1	0	1	2
y	0.1	0.3	1	3	9

7 $y = 2^x$

x	-2	-1	0	1	2
y	0.3	0.5	1	2	4

8 $y = 5^x$

x	-2	-1	0	1	2
y	0.0	0.2	1	5	25

9 $y = 4^x$

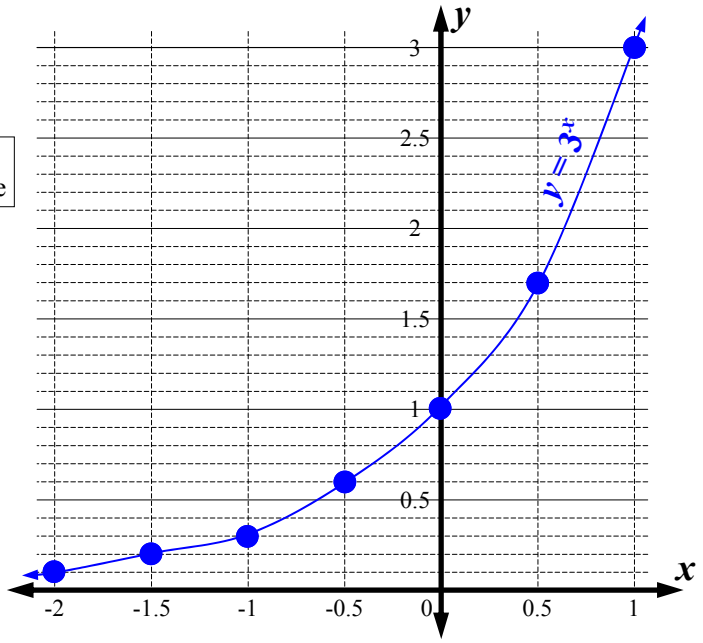
x	-2	-1	0	1	2
y	0.1	0.3	1	4	16

Complete the tables round answer to 1 decimal place when required. Then plot the exponential curves, place arrowheads on the curve as well as the equation of the curve.

10 $y = 3^x$

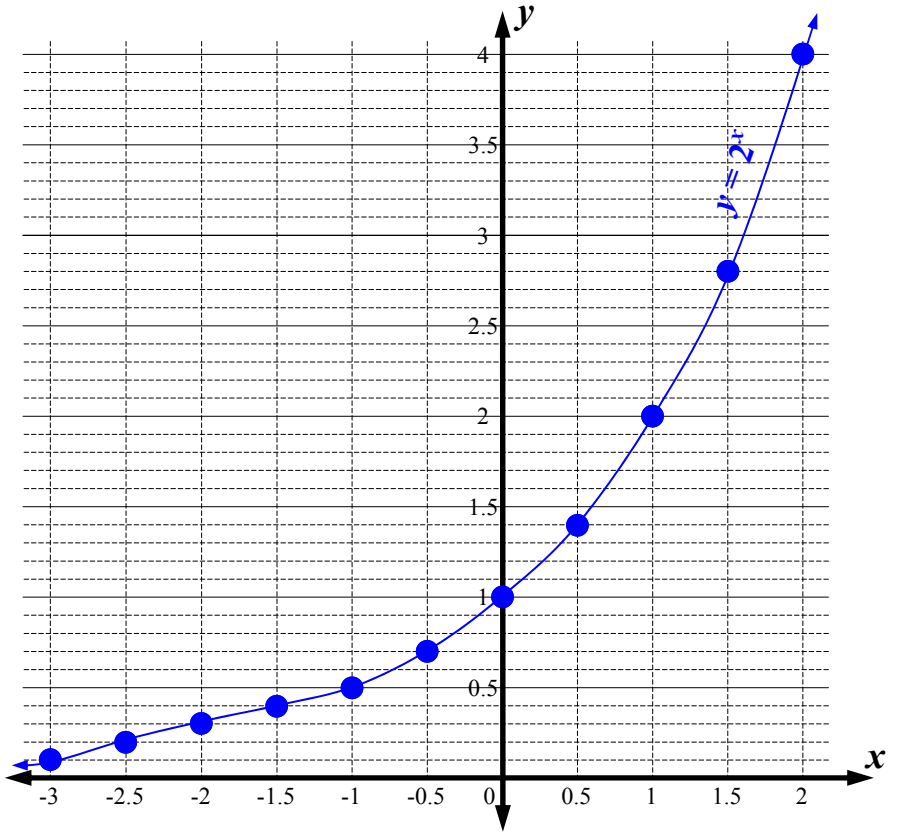
x	-2	-1.5	-1	-0.5	0	0.5	1
y	0.1	0.2	0.3	0.6	1	1.7	3

- Arrowheads
- Equation on curve



11 $y = 2^x$

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	0.1	0.2	0.3	0.4	0.5	0.7	1	1.4	2	2.8	4



Non-Linear Relationships - Exponentials - Negative Powers (Homework Strip)

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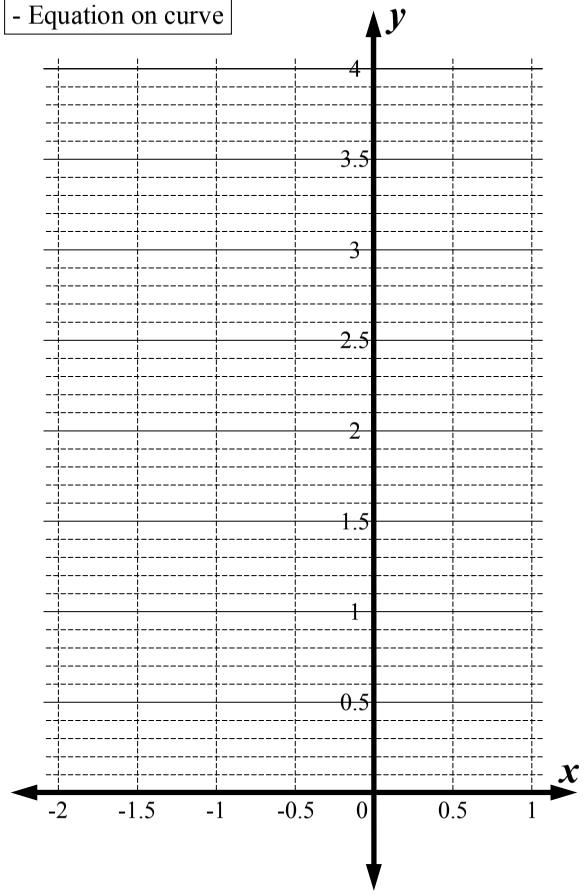
Date Due:

Complete the table and plot the exponential.
Round decimals to 1 decimal place.

$y = 4^x$

x	-2	-1.5	-1	-0.5	0	0.5	1
y							

- Arrowheads
- Equation on curve



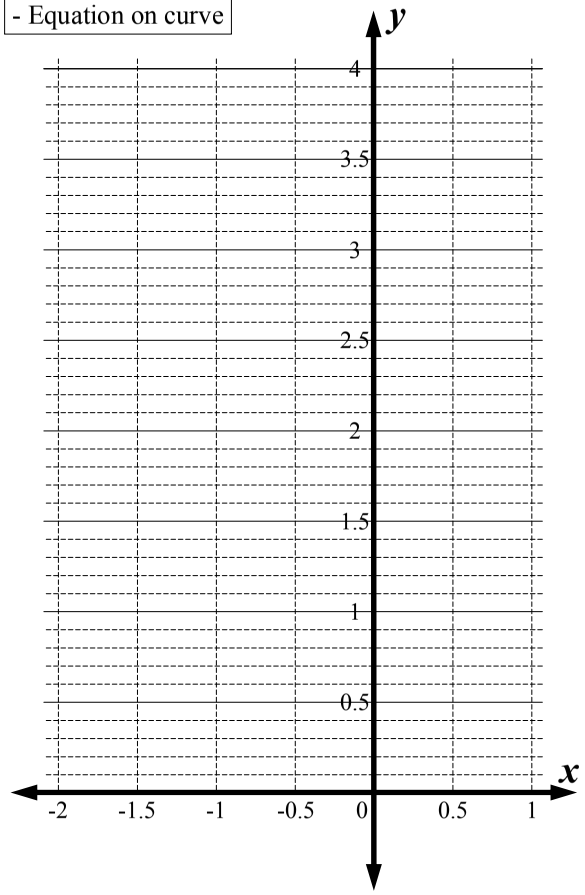
Date Due:

Complete the table and plot the exponential.
Round decimals to 1 decimal place.

$y = 4^x$

x	-2	-1.5	-1	-0.5	0	0.5	1
y							

- Arrowheads
- Equation on curve



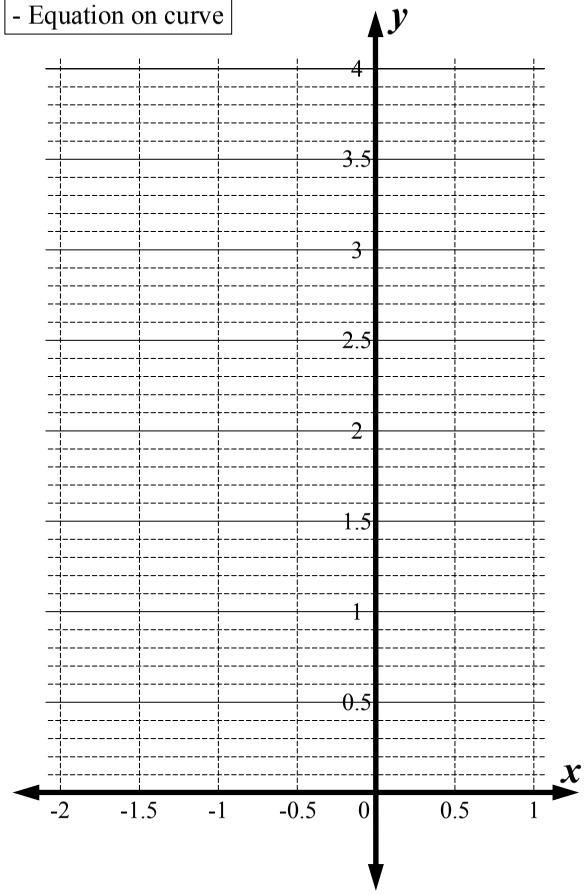
Date Due:

Complete the table and plot the exponential.
Round decimals to 1 decimal place.

$y = 4^x$

x	-2	-1.5	-1	-0.5	0	0.5	1
y							

- Arrowheads
- Equation on curve



Non-Linear Relationships - Exponentials - Negative Powers (HW Strip)

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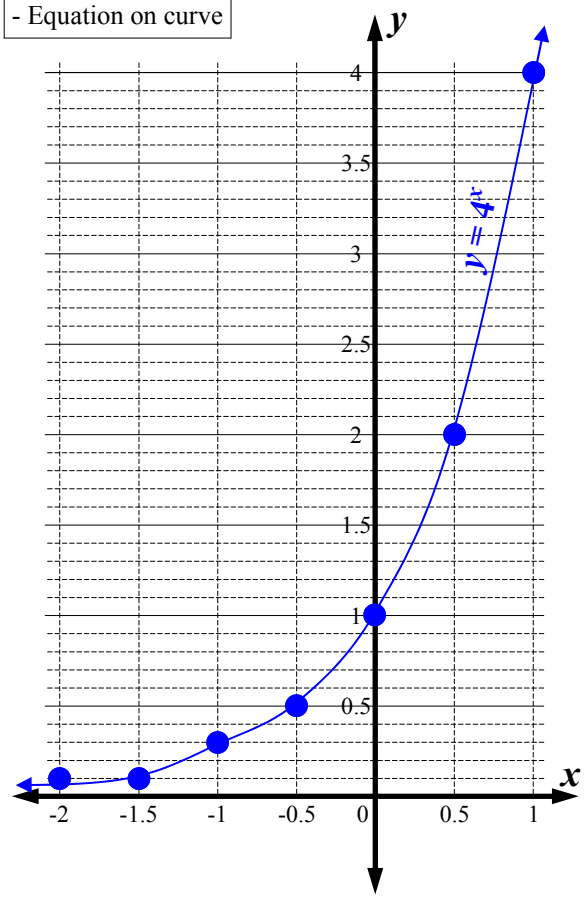
Date Due:

Complete the table and plot the exponential.
Round decimals to 1 decimal place.

$y = 4^x$

x	-2	-1.5	-1	-0.5	0	0.5	1
y	0.1	0.1	0.3	0.5	1	2	4

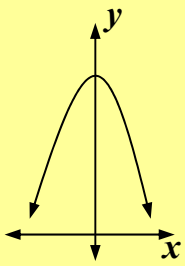
- Arrowheads
- Equation on curve



Non-Linear Relationships - Recognising Equations

When you are given an equation or shown a sketch of an equation, you should be able to state whether the equation is of a parabola, straight line (linear), circle or exponential. Below are the way you pick them out.

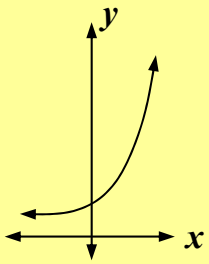
PARABOLAS



$y = 2x^2 + 3$

- Parabolas are U-shaped
- Parabolas are symmetrical across the y-axis. (this course)
- Parabolas have an x^2 term

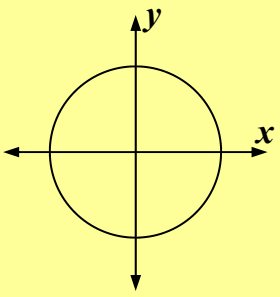
EXPONENTIALS



$y = 2^x$

- Exponentials flatten toward the x-axis
- Exponentials are curves but aren't symmetrical.
- Exponentials have x as a power

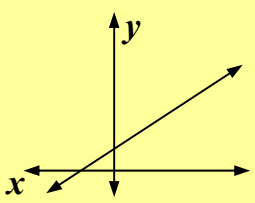
CIRCLES



$x^2 + y^2 = 25$

- Circles look like circles!
- Circles have an x^2 and a y^2 term.

LINEAR



$y = 2x + 4$

- Linear equations are straight lines
- Linear equations have no powers

Non-Linear Relationships - Recognising Equations

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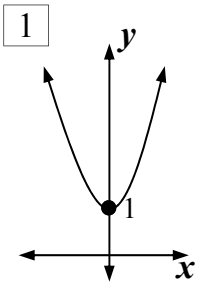
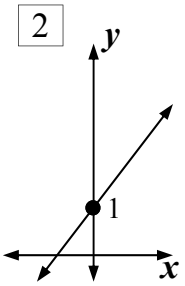
Join the dots between the graph and the equation that matches. You have linear relationships (straight lines), parabolas, circles and exponentials.

1



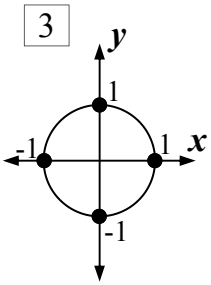
Ⓐ $y = x + 1$

2



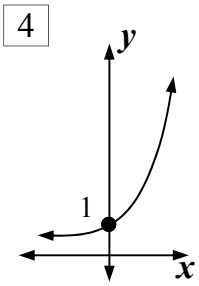
Ⓑ $x^2 + y^2 = 1$

3



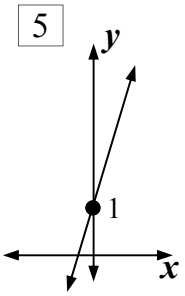
Ⓒ $y = x^2 + 1$

4



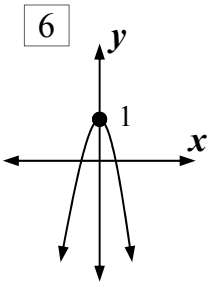
Ⓐ $y = -3x^2 + 1$

5



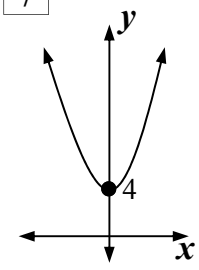
Ⓑ $y = 3x + 1$

6



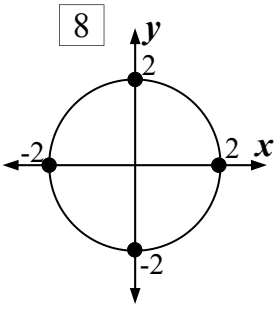
Ⓒ $y = 3^x$

7



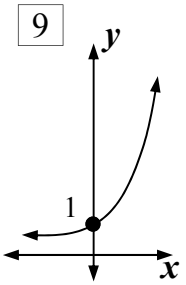
Ⓐ $x^2 + y^2 = 4$

8



Ⓑ $y = x^2 + 4$

9



Ⓒ $y = 4^x$

Write either: 'circle', 'linear', 'parabola' or 'exponential' next to these.

10 $y = 2x^2$ _____

11 $y = 5^x$ _____

12 $y = 4x + 2$ _____

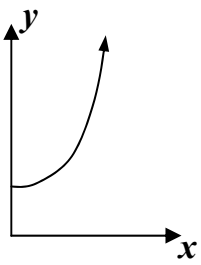
13 $x^2 + y^2 = 16$ _____

14 $y = 3^x$ _____

15 $y = x$ _____

16 $x^2 + y^2 = 5$ _____

17



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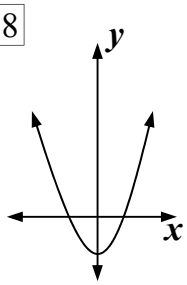

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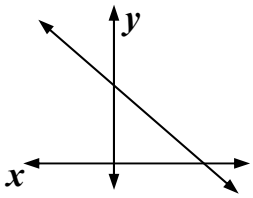

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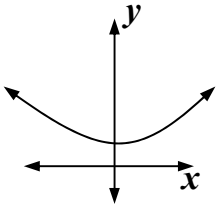

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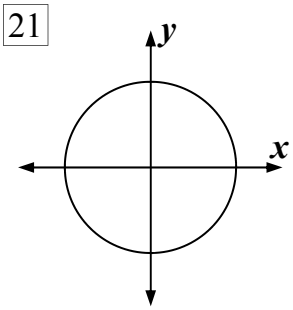

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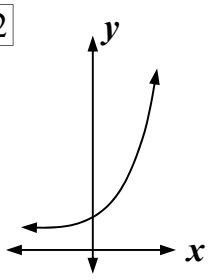

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Non-Linear Relationships - Recognising Equations

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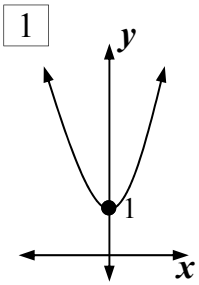
Join the dots between the graph and the equation that matches. You have linear relationships (straight lines), parabolas, circles and exponentials.

1



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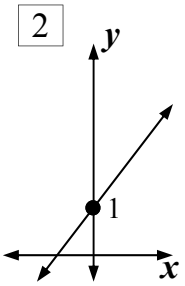

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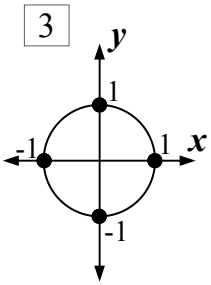
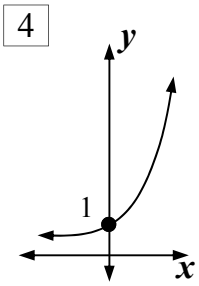


A $y = x + 1$

B $x^2 + y^2 = 1$

C $y = x^2 + 1$

4



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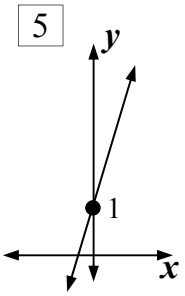

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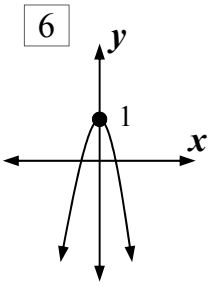
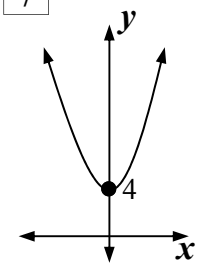


A $y = -3x^2 + 1$

B $y = 3x + 1$

C $y = 3^x$

7



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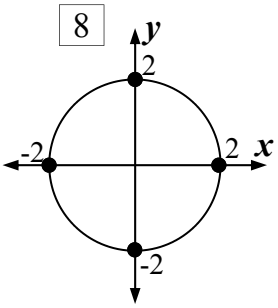

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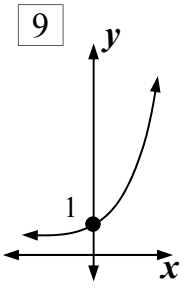


A $x^2 + y^2 = 4$

B $y = x^2 + 4$

C $y = 4^x$

Write either: 'circle', 'linear', 'parabola' or 'exponential' next to these.

10 $y = 2x^2$ — parabola

11 $y = 5^x$ — exponential

12 $y = 4x + 2$ — linear

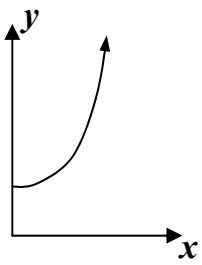
13 $x^2 + y^2 = 16$ — circle

14 $y = 3^x$ — exponential

15 $y = x$ — linear

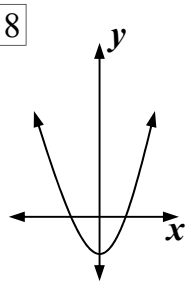
16 $x^2 + y^2 = 5$ — circle

17



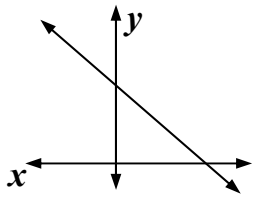
exponential

18



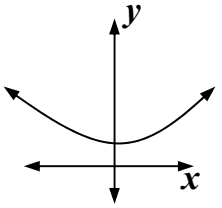
parabola

19



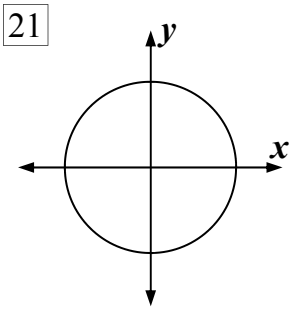
linear

20



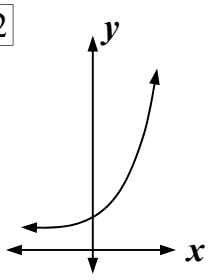
parabola

21



circle

22



exponential

Non-Linear Relationships Extension Review

This is the review of the Non-Linear Relationships Extension Topic.

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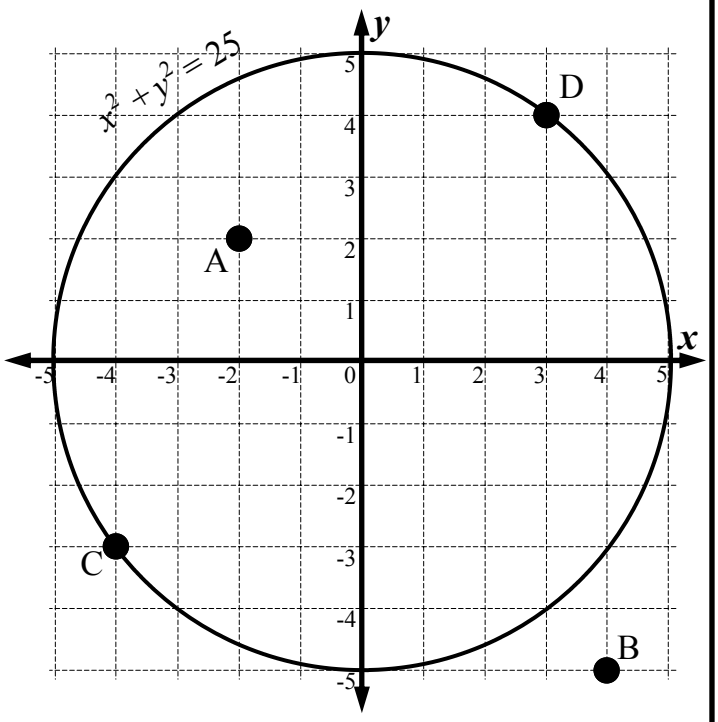
Name and give the coordinates for the points that are on the circle and the points that are inside and outside.



1 On Circle: _____ _____

2 Outside Circle: _____

3 Inside Circle: _____

Show algebraically that the point (4, 21) lies on the parabola $y = x^2 + 5$.

4

Is the point (-2, 7) inside or outside the circle $x^2 + y^2 = 36$? Show working to justify your answer.

5

Complete these and solve the equations.

6

$31 = x^2 + 6$

$\square = x^2$

$x = \pm \square$

7

$1 = x^2 - 8$

$\square = x^2$

$x = \pm \square$

Find the x-values when $y = 21$ for the parabola $y = x^2 + 5$. Show all working.

8

Find the equation for a circle with its centre at the origin that has a radius of 6 units.

9

Find the equation for a circle with its centre at the origin that has a diameter of 16 units.

10

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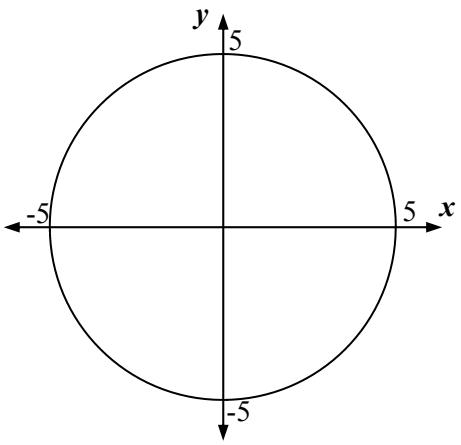
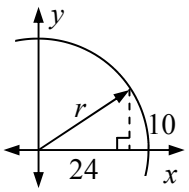
Give the equation for the circle below.

11



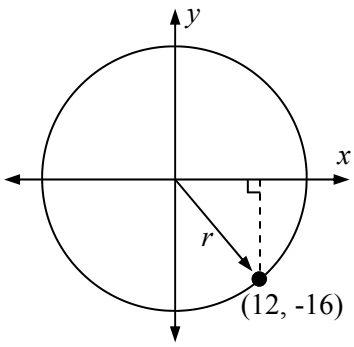
Use Pythagoras Theorem to calculate the radius of the circle below. (Only part of the circle is shown)

12



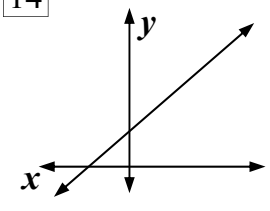
Use Pythagoras Theorem to calculate the radius of the circle below. Then give the equation of the circle.

13



Describe these graphs as either: 'Parabola', 'Circle', 'Linear' or 'Exponential'.

14



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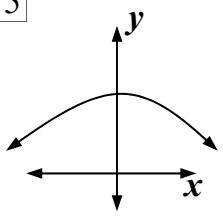
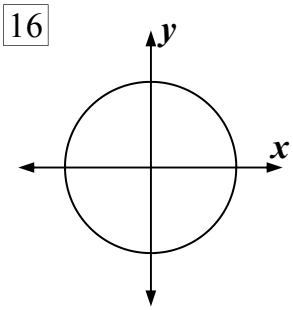
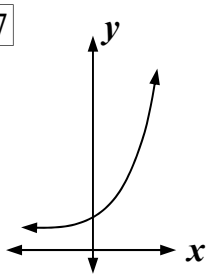


Describe these as either: 'Parabola', 'Circle', 'Linear' or 'Exponential'.

16



17



18 $y = 2x^2$

19 $y = 5^x$

20 $y = 4x + 2$

Complete the table for the exponential equation $y = 3^x$.

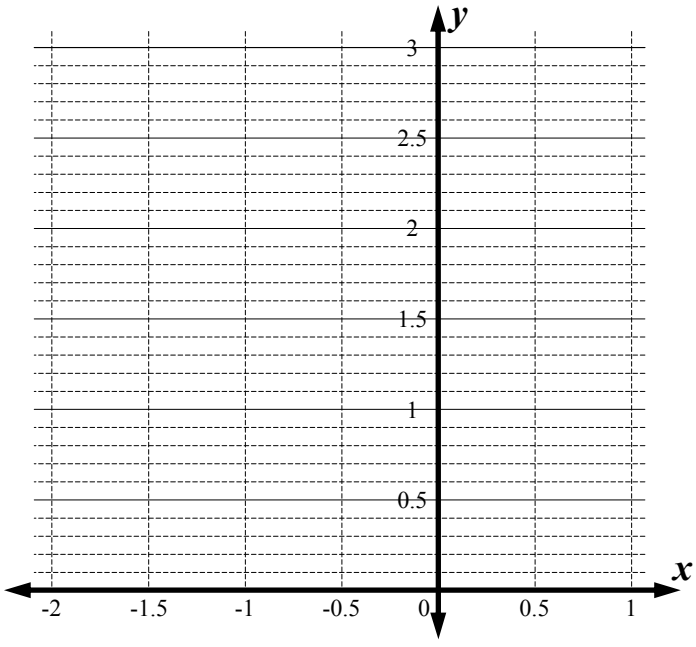
21 $y = 3^x$

x	-2	-1.5	-1	-0.5	0	0.5	1
y							

Round decimals to 1 decimal place

Plot the exponential equation $y = 3^x$ on the graph below.

22



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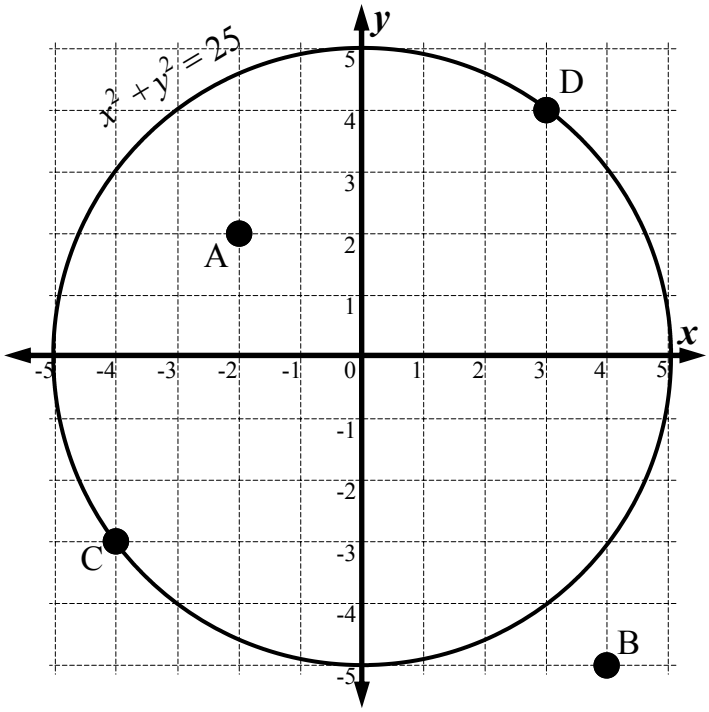
Name and give the coordinates for the points that are on the circle and the points that are inside and outside.



1 On Circle: C (-4, -3) D (3, 4)

2 Outside Circle: B (4, -5)

3 Inside Circle: A (-2, 2)

Show algebraically that the point (4, 21) lies on the parabola $y = x^2 + 5$.

4

$y = x^2 + 5$

$21 = 4^2 + 5$

$21 = 21$

LHS = RHS ∴ Point on parabola

Is the point (-2, 7) inside or outside the circle $x^2 + y^2 = 36$? Show working to justify your answer.

5

$x^2 + y^2 = 36$

$(-2)^2 + 7^2 = 36$

$53 > 36$

LHS > RHS ∴ Point is outside circle

Complete these and solve the equations.

6

$31 = x^2 + 6$

(-6) (-6)

$25 = x^2$

(√) (√)

$x = \pm 5$

7

$1 = x^2 - 8$

(+8) (+8)

$9 = x^2$

(√) (√)

$x = \pm 3$

Find the x-values when $y = 21$ for the parabola $y = x^2 + 5$. Show all working.

8

$y = x^2 + 5$

$21 = x^2 + 5$

(-5) (-5)

$16 = x^2$

$x = \pm 4$

Find the equation for a circle with its centre at the origin that has a radius of 6 units.

9

$x^2 + y^2 = r^2$

$x^2 + y^2 = 6^2$

$x^2 + y^2 = 36$

Find the equation for a circle with its centre at the origin that has a diameter of 16 units.

10

$r = 16 \div 2 = 8$ units

$x^2 + y^2 = r^2$

$x^2 + y^2 = 8^2$

$x^2 + y^2 = 64$

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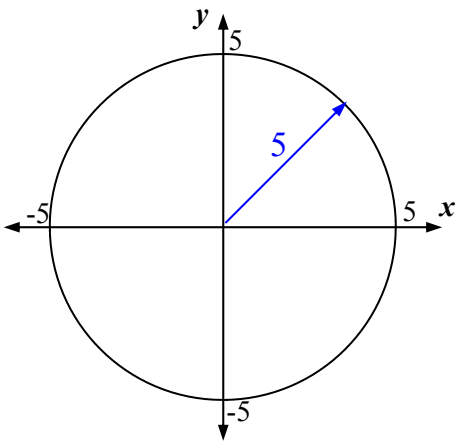
Give the equation for the circle below.

11



$x^2 + y^2 = r^2$

$x^2 + y^2 = 5^2$

$x^2 + y^2 = 25$

Use Pythagoras Theorem to calculate the radius of the circle below. (Only part of the circle is shown)

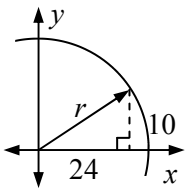
12

$r^2 = 10^2 + 24^2$

$r^2 = 676$

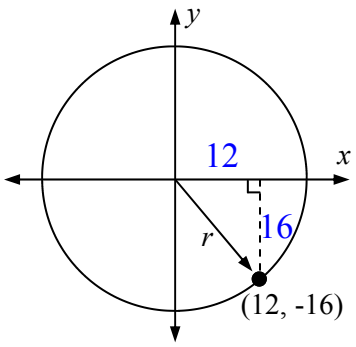
$r = \sqrt{676}$

$r = 26$ units



Use Pythagoras Theorem to calculate the radius of the circle below. Then give the equation of the circle.

13



$r^2 = 12^2 + 16^2$

$r^2 = 400$

$r = \sqrt{400}$

$r = 20$ units

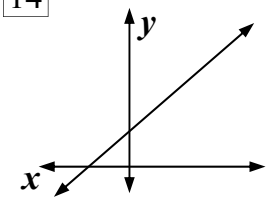
$x^2 + y^2 = r^2$

$x^2 + y^2 = 20^2$

$x^2 + y^2 = 400$

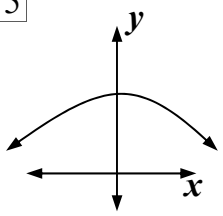
Describe these graphs as either: 'Parabola', 'Circle', 'Linear' or 'Exponential'.

14



linear

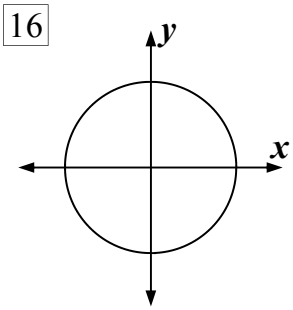
15



parabola

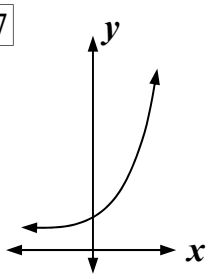
Describe these as either: 'Parabola', 'Circle', 'Linear' or 'Exponential'.

16



circle

17



exponential

18 $y = 2x^2$ parabola

19 $y = 5^x$ exponential

20 $y = 4x + 2$ linear

Complete the table for the exponential equation $y = 3^x$.

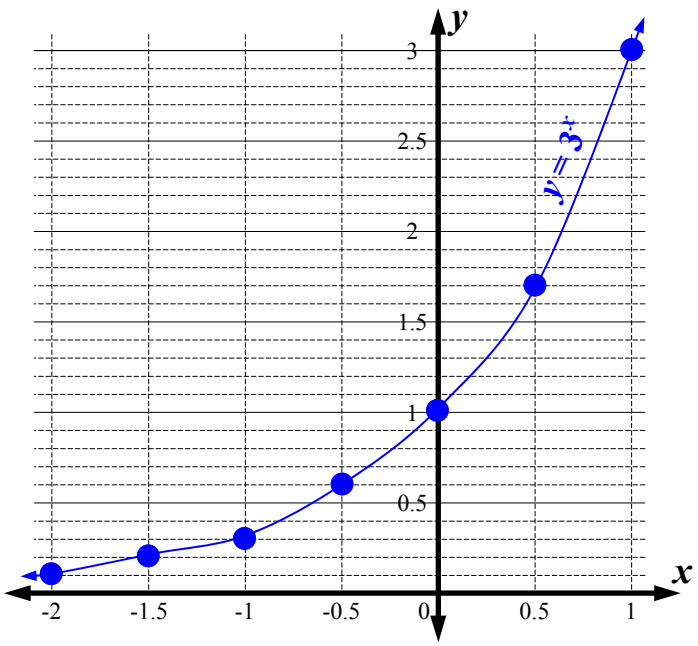
21 $y = 3^x$

x	-2	-1.5	-1	-0.5	0	0.5	1
y	0.1	0.2	0.3	0.6	1	1.7	3

Round decimals to 1 decimal place

Plot the exponential equation $y = 3^x$ on the graph below.

22



Non-Linear Relationships Extension



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Non-Linear Relationships - Testing Points and Parabolas (Sheet 1)

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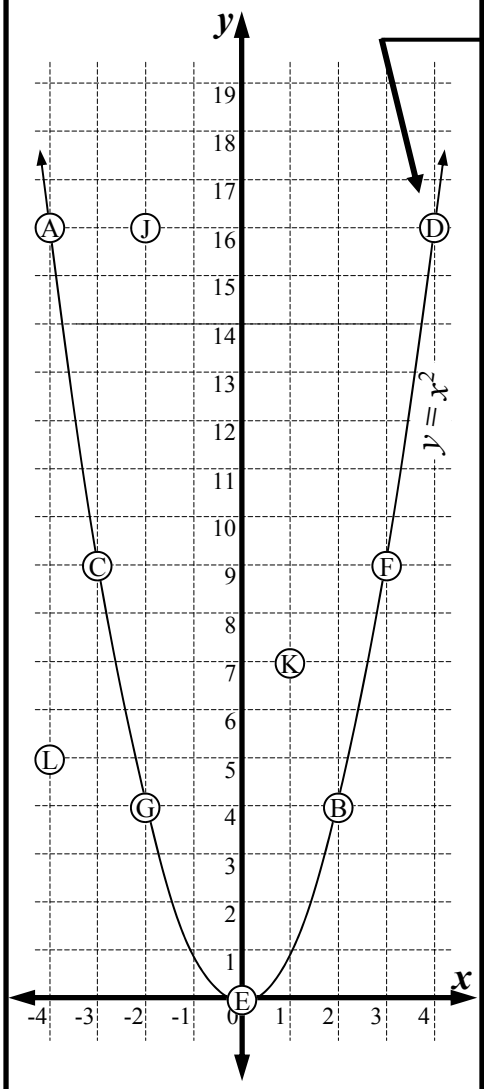
Look at the graph of $y = x^2$ and give the co-ordinates for these points.



Give the co-ordinates for…..

1	A	(,)	These 7 points are on the parabola
2	G	(,)	
3	B	(,)	
4	E	(,)	
5	C	(,)	
6	D	(,)	
7	F	(,)	
8	J	(,)	These 3 points aren't on the parabola
9	K	(,)	
10	L	(,)	

Now show which points are or aren't on the parabola $y = x^2$, algebraically.

Example Point D: (4, 16)
x y

$y = x^2$ ← Write the equation

$16 = 4^2$ ← Substitute x and y

$16 = 16$ ← Calculate 4^2

LHS = RHS ∴ Point on Parabola

12 Point B: (,)
x y

← Write the equation

$\square = \square^2$ ← Substitute x and y

← Calculate

13 Point K: (,)
x y

14 Point A: (,)

← Negative in brackets

15 Point J: (,)

← Negative in brackets

Now the parabolas change each question. Find whether the point is on the parabola or not.

16 Does (2, 7) lie on $y = x^2 + 5$?
x y

← Write equation

$\square = \square^2 + 5$ ← Sub x and y

← Calculation

17 Does (-3, 8) lie on $y = x^2 - 1$?

$\square = \square^2 - \square$

← Brackets

18 Does (5, 85) lie on $y = 2x^2 - 15$?

19 Does (-4, 9) lie on $y = -x^2 - 7$?

20 Does (4, -43) lie on $y = -3x^2 + 5$?

Non-Linear Relationships - Testing Points Circles/Exponentials (Sheet 2)

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Substitute in x and y into these exponential equations to find if the point is on the curve.

Example

Does (3, 143) lie on $y = 4^x$?

x y

$y = 4^x$ ← Write the equation

$143 = 4^3$ ← Substitute x and y

$143 \neq 64$ ← Calculate 4^3

4 $x^{\blacksquare}$ 3 =

LHS ≠ RHS ∴ Point not on curve

1 Does (5, 32) lie on $y = 2^x$?

x y

← Write equation

$\square = 2^{\square}$ ← Substitute x and y

← Calculate 2^5

2 $x^{\blacksquare}$ 5 =

2 Does (7, 2 187) lie on $y = 3^x$?

3 Does (3, 10 000) lie on $y = 10^x$?

4 Does (10, 1 024) lie on $y = 2^x$?

Now it is circles, sub in x and y to show if the point is on the circle.

Example

Does (3, 6) lie on $x^2 + y^2 = 45$?

x y

$x^2 + y^2 = 45$ ← Write the equation

$3^2 + 6^2 = 45$ ← Substitute x and y

$45 = 45$ ← Calculate $3^2 + 6^2$

3 x^2 + 6 x^2 =

LHS = RHS ∴ Point on circle

5 Does (2, 3) lie on $x^2 + y^2 = 15$?

x y

← Write equation

$\square^2 + \square^2 = 15$ ← Sub x and y

← Calculate

6 Does (7, 6) lie on $x^2 + y^2 = 77$?

7 Does (9, 12) lie on $x^2 + y^2 = 225$?

8 Does (1, 8) lie on $x^2 + y^2 = 60$?

If $x^2 + y^2$ is less than the constant the point is inside the circle, more and it's outside. Find if the point is in or out!

LHS < RHS ⟶ Point Inside

LHS > RHS ⟶ Point Outside

Example

x y

Is the point (3, 7) inside or outside of the circle $x^2 + y^2 = 60$?

$x^2 + y^2 = 60$ ← Write the equation

$3^2 + 7^2 = 60$ ← Substitute x and y

$58 < 60$ ← Calculate $3^2 + 7^2$

3 x^2 + 7 x^2 =

LHS < RHS ∴ Point inside circle

9 Is the point (5, 2) inside or outside of the circle $x^2 + y^2 = 20$?

10 Is the point (7, 5) inside or outside of the circle $x^2 + y^2 = 64$?

11 Is the point (3, 8) inside or outside of the circle $x^2 + y^2 = 75$?

Non-Linear Relationships - Finding x-values (Sheet 1)

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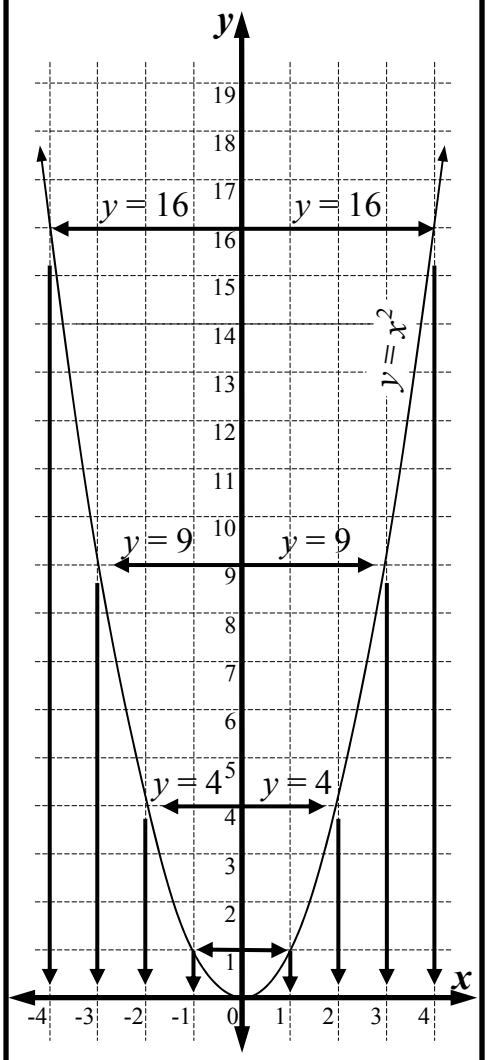
Read off the two x-values for each y-value on $y = x^2$.



1 When $y = 16$: $x =$ ☐ & ☐

2 When $y = 4$: $x =$ ☐ & ☐

3 When $y = 1$: $x =$ ☐ & ☐

4 When $y = 9$: $x =$ ☐ & ☐

The square root of a number results in two answers. Find the two answers for these.

5 $\sqrt{25} = \pm$ ☐ Calc: √ 2 5 =

6 $\sqrt{169} = \pm$ ☐

7 $\sqrt{324} =$ ☐

8 $\sqrt{121} =$ ☐

9 $\sqrt{81} =$ ☐

Solve these equations. The steps are set up for you.

10 $23 = x^2 + 7$

(-7) (-7)

☐ $= x^2$

(√) (√)

$x = \pm$ ☐

11 $39 = x^2 + 3$

(-3) (-3)

☐ $= x^2$

(√) (√)

$x = \pm$ ☐

12 $58 = x^2 + 9$

(-9) (-9)

☐ $= x^2$

(√) (√)

$x = \pm$ ☐

13 $50 = x^2 - 14$

(+14) (+14)

☐ $= x^2$

(√) (√)

$x = \pm$ ☐

Now solve these equations yourself. All answers are whole numbers. Remember ±

14 $9 = x^2 + 8$

○ ○

☐ $= x^2$

○ ○

$x = \pm$ ☐

15 $91 = x^2 - 9$

○ ○

☐ $= x^2$

○ ○

$x = \pm$ ☐

16 $51 = x^2 - 13$

○ ○

☐ $= x^2$

○ ○

$x =$ _____

17 $26 = x^2 + 22$

○ ○

☐ $= x^2$

○ ○

$x =$ _____

Now apply the skill to finding the x-values when given a y-value on a parabola.

18 Find the x-values when $y = 10$ for the parabola $y = x^2 + 6$.

$y = x^2 + 6$ ← Write the equation

$10 = x^2 + 6$ ← Substitute $y = 10$

○ ○ ← -6 each side

☐ $= x^2$

○ ○ ← √ each side

$x = \pm$ ☐

19 Find the x-values when $y = 33$ for the parabola $y = x^2 - 3$.

_____

_____

○ ○

☐ $= x^2$

○ ○

$x = \pm$ ☐

20 Find the x-values when $y = 25$ for the parabola $y = x^2 + 9$.

_____

_____

○ ○

☐ $= x^2$

○ ○

$x = \pm$ ☐

Non-Linear Relationships - Finding x-values (Sheet 2)

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Find the x-values when given a y-value on a parabola. Then give the points on the parabola.

1 Find the x-values when $y = 57$ for the parabola $y = x^2 - 7$.

_____ ← Write eqn

_____ ← Sub $y = 57$

◯ ◯ ← +7 each side

□ $= x^2$

◯ ◯ ← √ each side

$x =$ ±□

2 The two points are:

(□ , 57) & (□ , 57)

3 Find the x-values when $y = 0$ for the parabola $y = x^2 - 4$.

_____

_____

◯ ◯

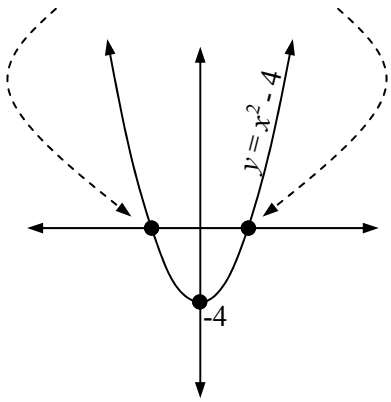
_____

◯ ◯

_____

4 The two points are:

(□ , □) & (□ , □)



With negatives, you switch the sign either side of the equation. Find the x-values for these.

5 Find the x-values when $y = 3$ for the parabola $y = -x^2 + 12$.

_____ ← Write eqn

_____ ← Sub $y = 3$

◯ ◯ ← -12 each side

□ $= -x^2$

swap swap

$x^2 =$ □ ← Change the sign

◯ ◯ ← √ each side

_____

6 The two points are:

(□ , 3) & (□ , 3)

7 Find the x-values when $y = -32$ for the parabola $y = -x^2 - 7$.

_____

◯ ◯

□ $= -x^2$

swap swap

$x^2 =$ □

◯ ◯

_____

8 The two points are:

(□ , □) & (□ , □)

Now there is a co-efficient. This requires division as an extra step. Find the x-values.

9 Find the x-values when $y = 14$ for the parabola $y = 2x^2 + 6$.

_____ ← Write eqn

_____ ← Sub $y = 14$

◯ ◯ ← -6 each side

□ $= 2x^2$

÷2 ÷2

$x^2 =$ □

◯ ◯ ← √ each side

_____

10 The two points are:

(□ , □) & (□ , □)

11 Find the x-values when $y = 46$ for the parabola $y = 3x^2 - 2$.

_____

◯ ◯

_____

◯ ◯

_____

◯ ◯

_____

12 The two points are:

(□ , □) & (□ , □)

Non-Linear Relationships - Properties of Circles

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The equation of a circle with centre at the origin is $x^2+y^2=r^2$. Find the equation for these circles centred at the origin.

Example

Circle with radius of 5 units.

$x^2 + y^2 = r^2$ ← Write equation

$x^2 + y^2 = 5^2$ ← Substitute radius

$x^2 + y^2 = 25$ ← Calculate 5^2

1 Circle with radius of 2 units.

← Write equation

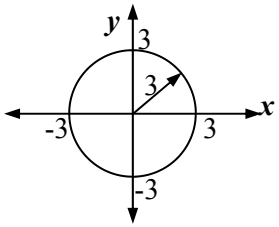
← Substitute radius

← Calculate 2^2

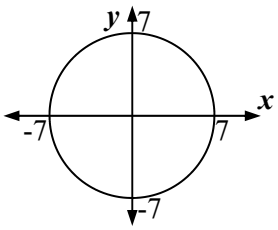
2 Circle with radius of 13 units.

3 Circle with radius of 9 units.

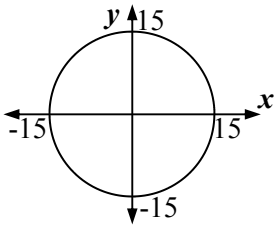
4 The circle shown below.



5 The circle shown below.



6 The circle shown below.



If you are given a diameter, halve it to find the radius. Find the equations for these circles with centres at the origin.

7 Circle with diameter of 16 units.

$r = 16 \div 2 =$ ☐ units

← Write equation

← Substitute **radius**

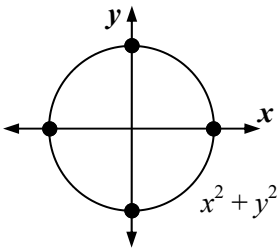
← Calculate

8 Circle with diameter of 34 units.

Sketch these circles on a number plane. Find the radius first by using your $\sqrt{}$ key.

9 $x^2 + y^2 = 144$

$r = \sqrt{144} =$ ☐ units

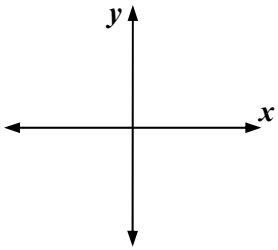


Write x and y intercepts at the 4 dots

$x^2 + y^2 = 144$

10 $x^2 + y^2 = 400$

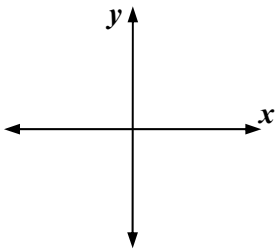
$r = \sqrt{400} =$ ☐ units



Write equation next to circle

11 $x^2 + y^2 = 729$

$r =$



12 $x^2 + y^2 = 361$

Non-Linear Relationships - Point on Circle Find Equation - Pythagoras

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Use Pythagoras Theorem (Addition) to calculate the radius of these circles*.

*Only part of the circle is shown

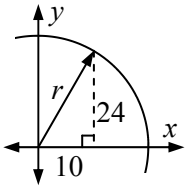
1

$r^2 = 10^2 + 24^2$

$r^2 =$ ☐

$r =$ $\sqrt{}$ ☐

$r =$ ☐ units



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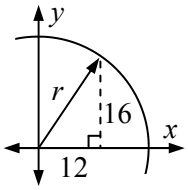


$r^2 =$ ☐^{2} + ☐^{2}

$r^2 =$ ☐

$r =$ $\sqrt{}$ ☐

$r =$ ☐ units

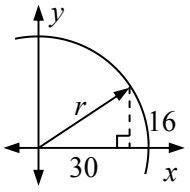
3

$r^2 =$

$r^2 =$

$r =$

$r =$ ☐ units



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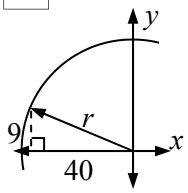

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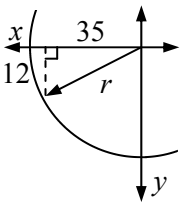
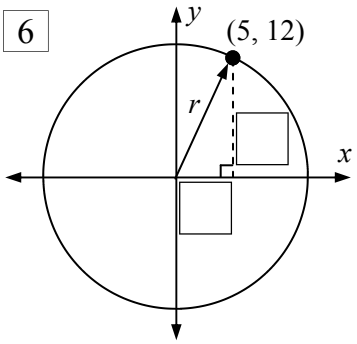


Given a point on the circle, write a distance in the boxes, which is always positive, then use Pythagoras to find the radius. After you find the radius, use $x^2 + y^2 = r^2$ to find the equation for the circle.

6



$r^2 =$ ☐^{2} + ☐^{2}

$r^2 =$ ☐

$r =$ $\sqrt{}$ ☐

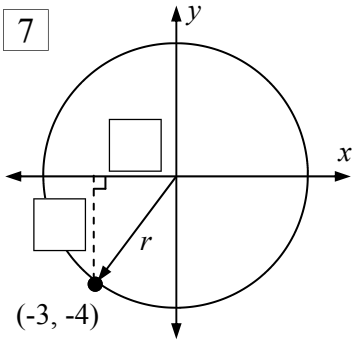
$r =$ ☐ units

$x^2 + y^2 = r^2$

$x^2 + y^2 =$ ☐^{2}

$x^2 + y^2 =$ ☐

7



$r^2 =$

$r^2 =$

$r =$

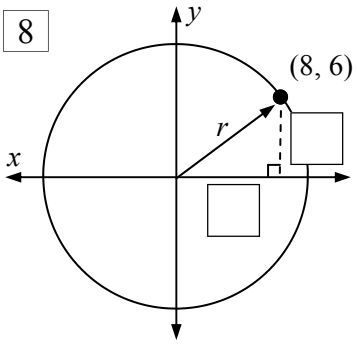
$r =$ ☐ units

$x^2 + y^2 = r^2$

$x^2 + y^2 =$ ☐^{2}

$x^2 + y^2 =$ ☐

8



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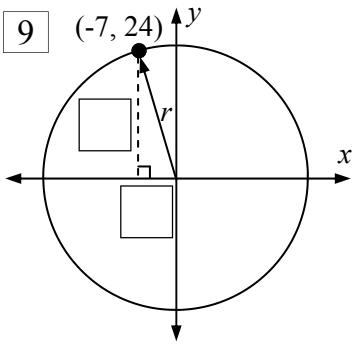

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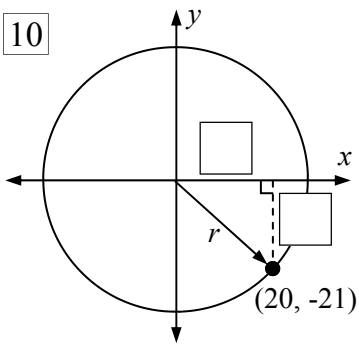

Non-Linear Relationships - Exponentials - Negative Powers

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Evaluate these, rounding your answer to 1 decimal place.

1 $2^{-1} = 0.\square$

Calc: 2 $x^{\blacksquare}$ - 1 =

2 $2^{-2} = 0.\square$ Round to 1 d.p.

3 $3^{-2} =$ ____

4 $5^{-1} =$ ____

5 $3^{-1} =$ ____

These are decimal powers.

6 $3^{-1.5} =$ ____

7 $2^{-2.5} =$ ____

8 $5^{-0.5} =$ ____

Complete the tables rounding your answers to 1 decimal place when required.

6 $y = 3^x$

x	-2	-1	0	1	2
y					

7 $y = 2^x$

x	-2	-1	0	1	2
y					

8 $y = 5^x$

x	-2	-1	0	1	2
y					

9 $y = 4^x$

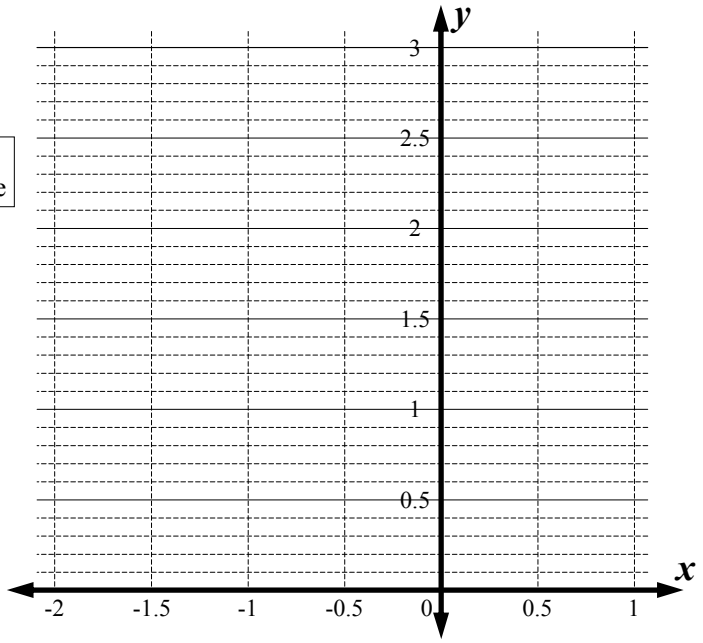
x	-2	-1	0	1	2
y					

Complete the tables round answer to 1 decimal place when required. Then plot the exponential curves, place arrowheads on the curve as well as the equation of the curve.

10 $y = 3^x$

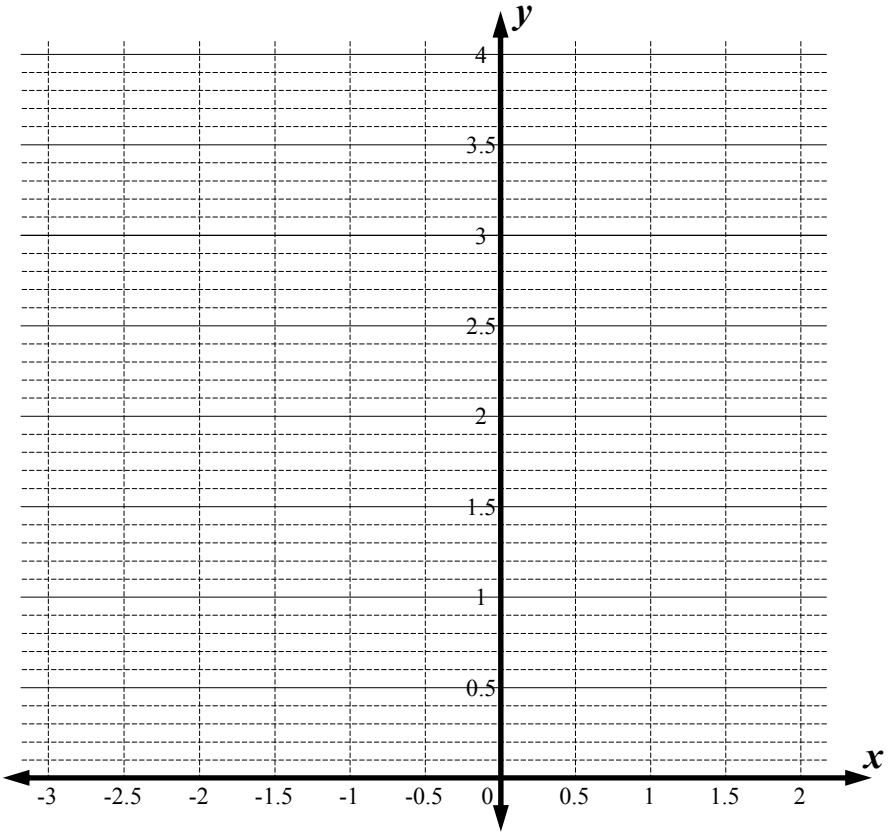
x	-2	-1.5	-1	-0.5	0	0.5	1
y							

- Arrowheads
- Equation on curve



11 $y = 2^x$

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y											



Non-Linear Relationships - Recognising Equations

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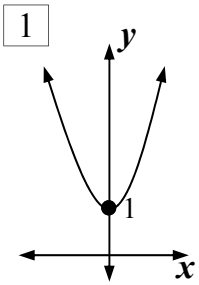
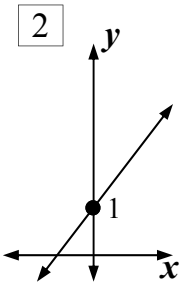
Join the dots between the graph and the equation that matches. You have linear relationships (straight lines), parabolas, circles and exponentials.

1



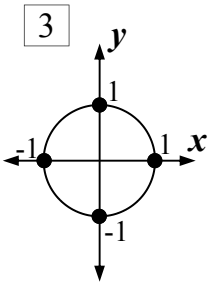
Ⓐ $y = x + 1$

2



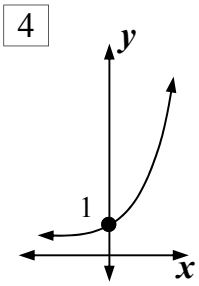
Ⓑ $x^2 + y^2 = 1$

3



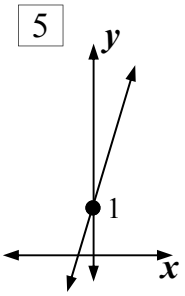
Ⓒ $y = x^2 + 1$

4



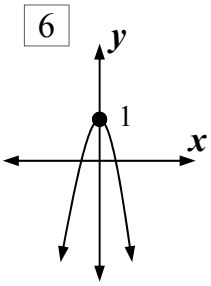
Ⓐ $y = -3x^2 + 1$

5



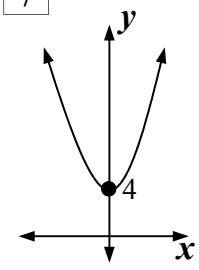
Ⓑ $y = 3x + 1$

6



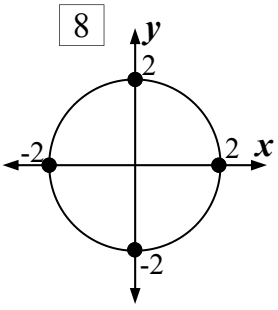
Ⓒ $y = 3^x$

7



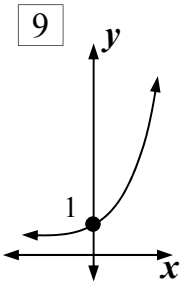
Ⓐ $x^2 + y^2 = 4$

8



Ⓑ $y = x^2 + 4$

9



Ⓒ $y = 4^x$

Write either: 'circle', 'linear', 'parabola' or 'exponential' next to these.

10 $y = 2x^2$ _____

11 $y = 5^x$ _____

12 $y = 4x + 2$ _____

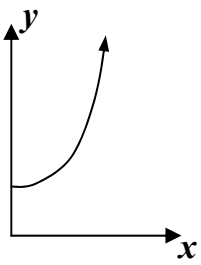
13 $x^2 + y^2 = 16$ _____

14 $y = 3^x$ _____

15 $y = x$ _____

16 $x^2 + y^2 = 5$ _____

17



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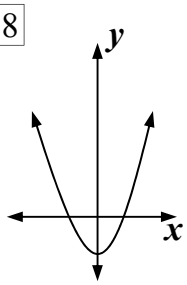
18



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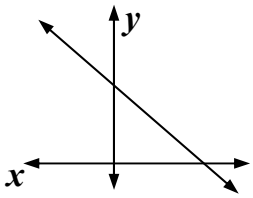
19



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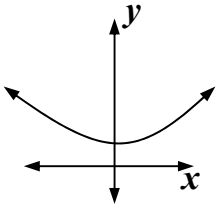
20



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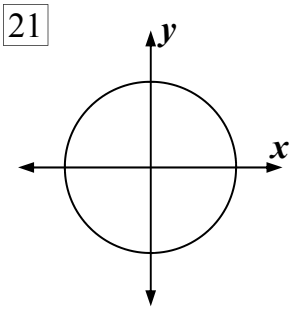
21



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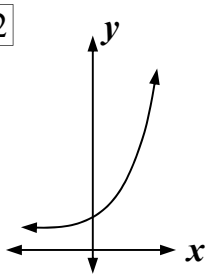
22



_____

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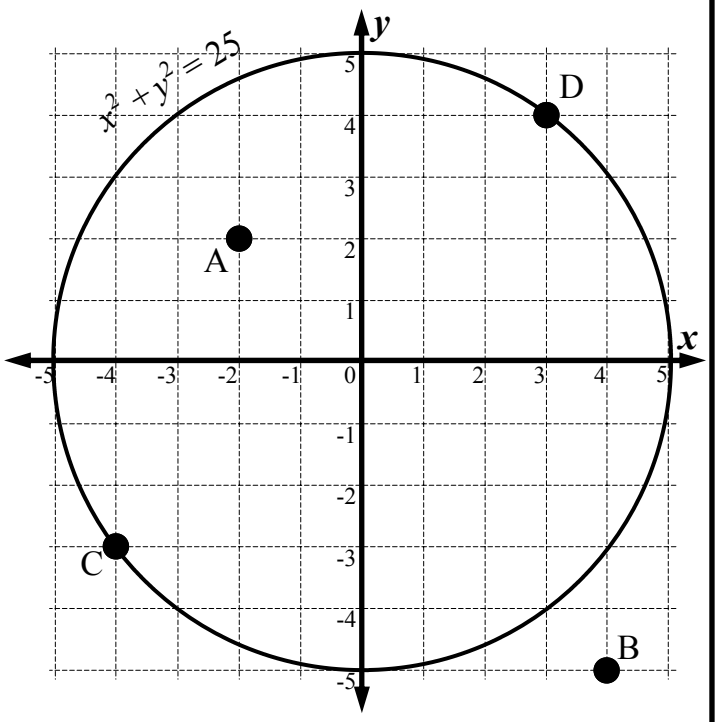
Name and give the coordinates for the points that are on the circle and the points that are inside and outside.



1 On Circle: _____ _____

2 Outside Circle: _____

3 Inside Circle: _____

Show algebraically that the point (4, 21) lies on the parabola $y = x^2 + 5$.

4

Is the point (-2, 7) inside or outside the circle $x^2 + y^2 = 36$? Show working to justify your answer.

5

Complete these and solve the equations.

6

$31 = x^2 + 6$

$\square = x^2$

$x = \pm \square$

7

$1 = x^2 - 8$

$\square = x^2$

$x = \pm \square$

Find the x-values when $y = 21$ for the parabola $y = x^2 + 5$. Show all working.

8

Find the equation for a circle with its centre at the origin that has a radius of 6 units.

9

Find the equation for a circle with its centre at the origin that has a diameter of 16 units.

10

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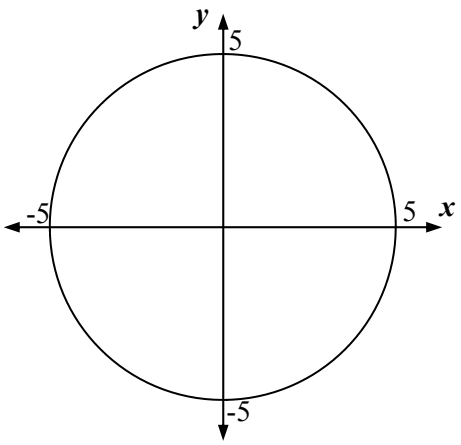
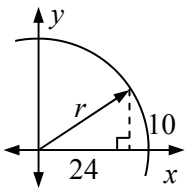
Give the equation for the circle below.

11



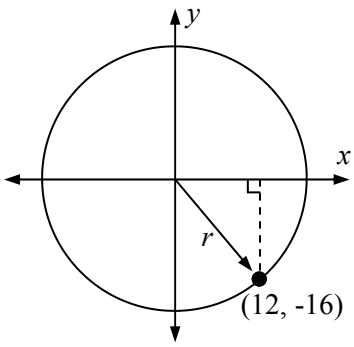
Use Pythagoras Theorem to calculate the radius of the circle below. (Only part of the circle is shown)

12



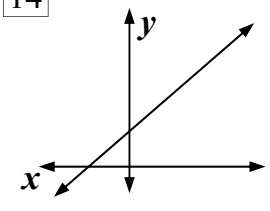
Use Pythagoras Theorem to calculate the radius of the circle below. Then give the equation of the circle.

13

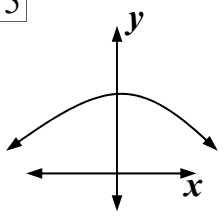


Describe these graphs as either: 'Parabola', 'Circle', 'Linear' or 'Exponential'.

14

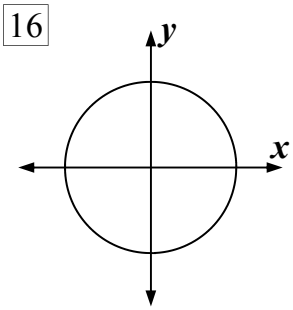


15

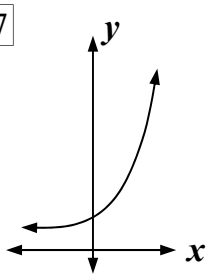


Describe these as either: 'Parabola', 'Circle', 'Linear' or 'Exponential'.

16



17



18 $y = 2x^2$

19 $y = 5^x$

20 $y = 4x + 2$

Complete the table for the exponential equation $y = 3^x$.

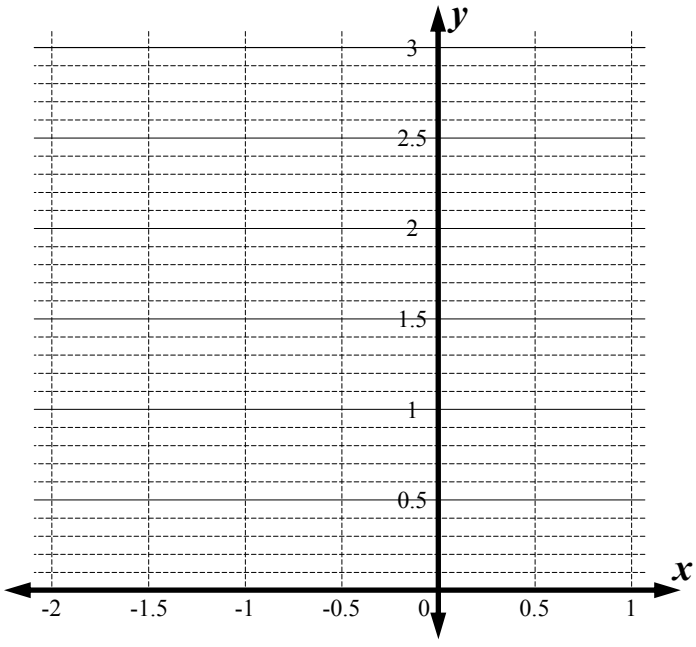
21 $y = 3^x$

x	-2	-1.5	-1	-0.5	0	0.5	1
y							

Round decimals to 1 decimal place

Plot the exponential equation $y = 3^x$ on the graph below.

22



VELOCITY

GEOMETRICAL PROPERTIES

Geometrical Properties



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Geometrical Properties

2



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Geometrical Properties - Enlargements - Drawing Exercises

There are two worksheets for this exercise. The first is a simplified sketch of a ladybird beetle (or ladybug). This is an easier exercise, the second drawing is of King Tutankhamun's Death Mask. This sketch is more complicated.

Geometrical Properties - Enlargements - Drawing Exercise (Sheet 1)

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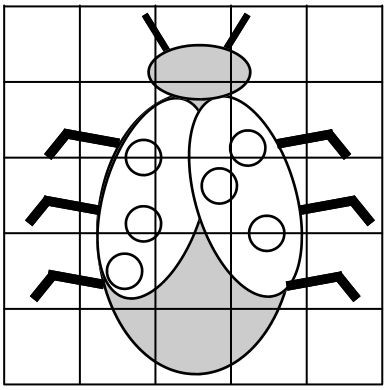

Reproduce the Ladybird beetle in the space below. Once you are done colour the wings red and the body and spots black.



Geometrical Properties - Enlargements - Drawing Exercise (Sheet 2)

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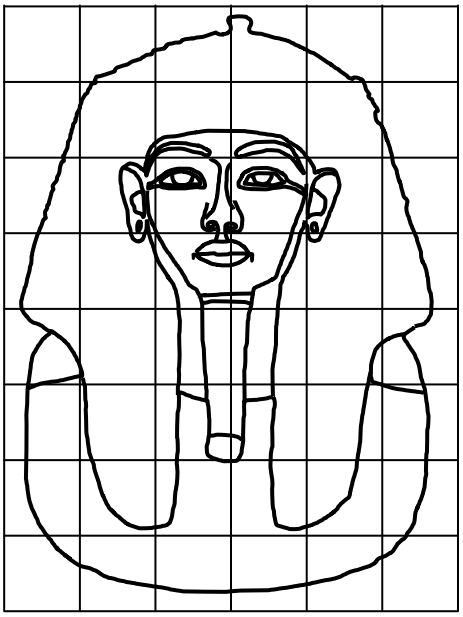

Reproduce the Death Mask of King Tutankhamun in the space below. Once you are done perform an image search on the web to colour it in correctly.



Geometrical Properties - Enlargements - Drawing Exercise (Homework Strip)

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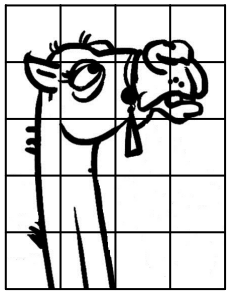
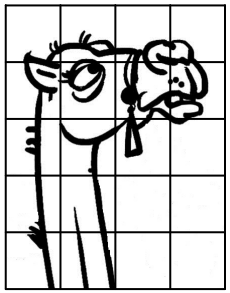
Date Due:

Sketch an enlargement of the camel.



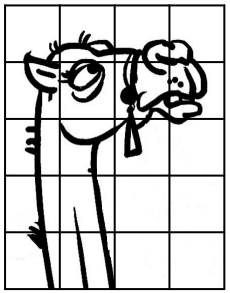
Date Due:

Sketch an enlargement of the camel.



Date Due:

Sketch an enlargement of the camel.



Geometrical Properties - Enlargements - Drawing Exercise (H'work Strip)

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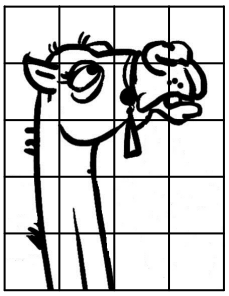
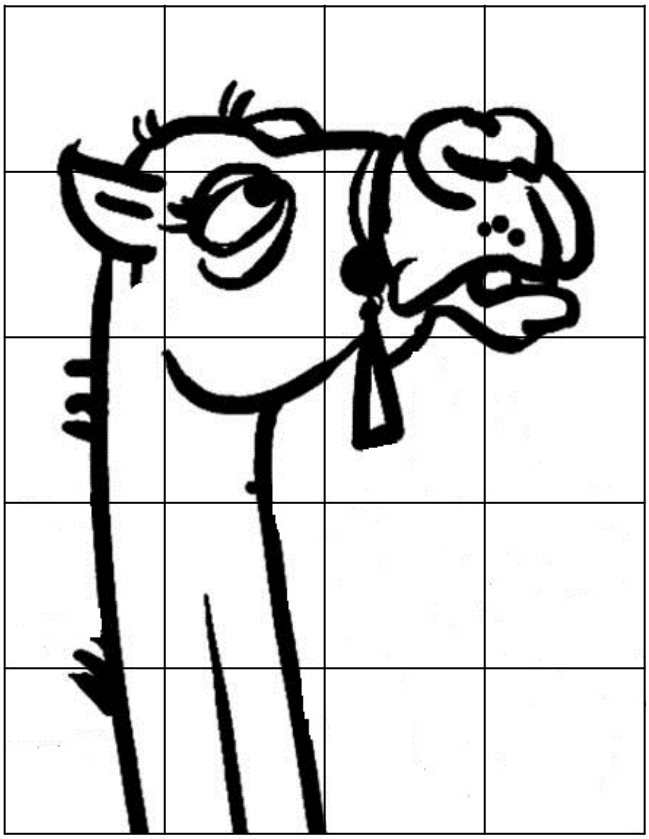
Date Due:

Sketch an enlargement of the camel.





Geometrical Properties - Angles & Similarity - Practical

When you enlarge a shape, the sides of the larger shape are longer, but what about the internal angles? Well unlike the sides, the angles stay the same.

These sheets involve creating an enlargement and then checking internal angles to verify that they remain the same. Two methods are used to enlarge the shapes.

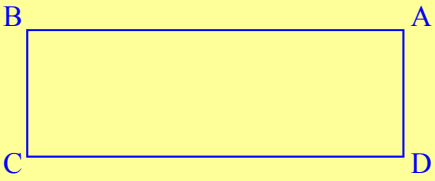
The first worksheet features a grid with a shape inside the grid with vertices on intersecting grid lines. Use the larger grid to copy the shape then measure the internal angles. Note that the original shape is small, if edges are too small for your protractor, remember that you can use a pencil and ruler to extend the lines so that you can then use the protractor.

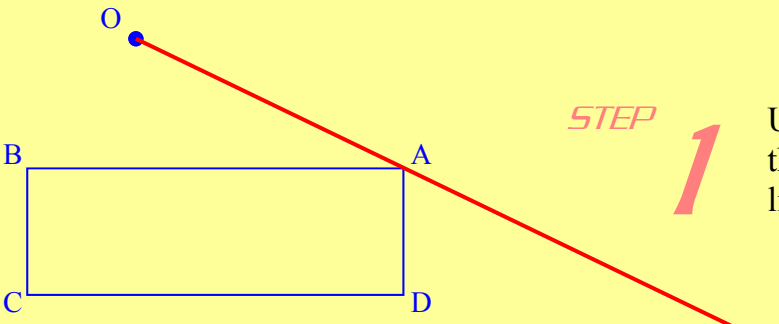
The second worksheet uses a point called a centre of enlargement (CoE). Draw lines from the point marked 'O' through the vertices. Measure the length from 'O' to the corner and triple the distance for Q.1, double the distance for Q.2.

Example

O

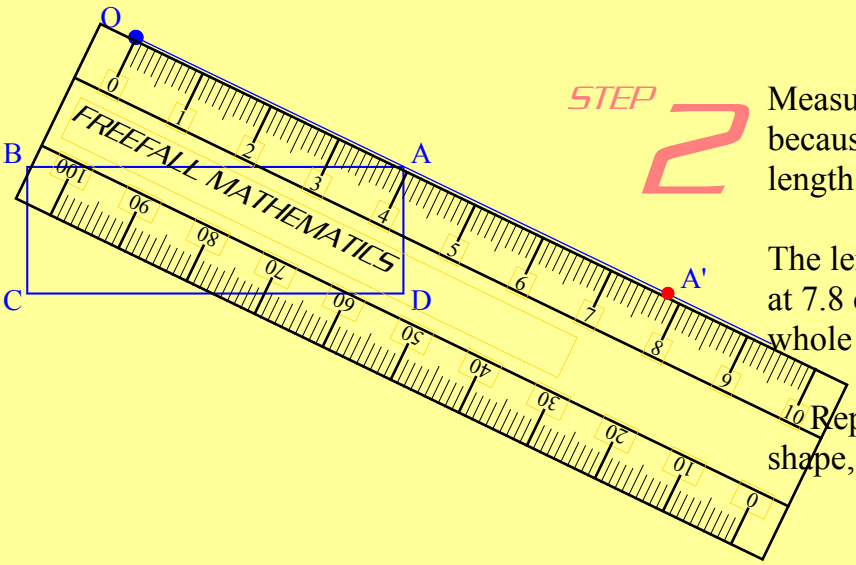
An enlargement of rectangle ABCD with a scale factor of 2 is required.





STEP 1

Use a ruler to draw a line from the CoE through one point. In this case it is the red line from O through A.



STEP 2

Measure the length OA and then double it, because the scale factor is 2. Mark the length on the constructed line.

The length was 3.9 cm, so a mark is placed at 7.8 cm. All lengths on Worksheet 2 are whole centimeters.

Repeat for the 4 vertices, draw the shape, label vertices with a '.

Geometrical Properties - Angles & Similarity - Practical (Sheet 1)

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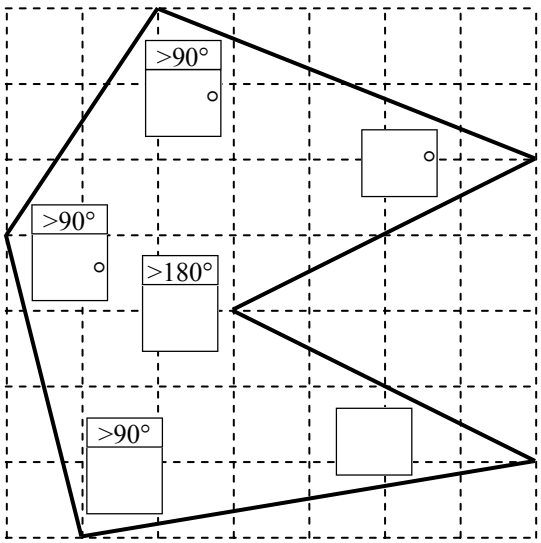
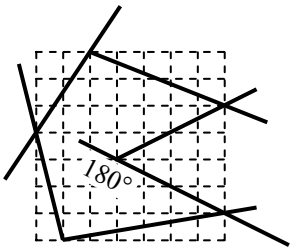
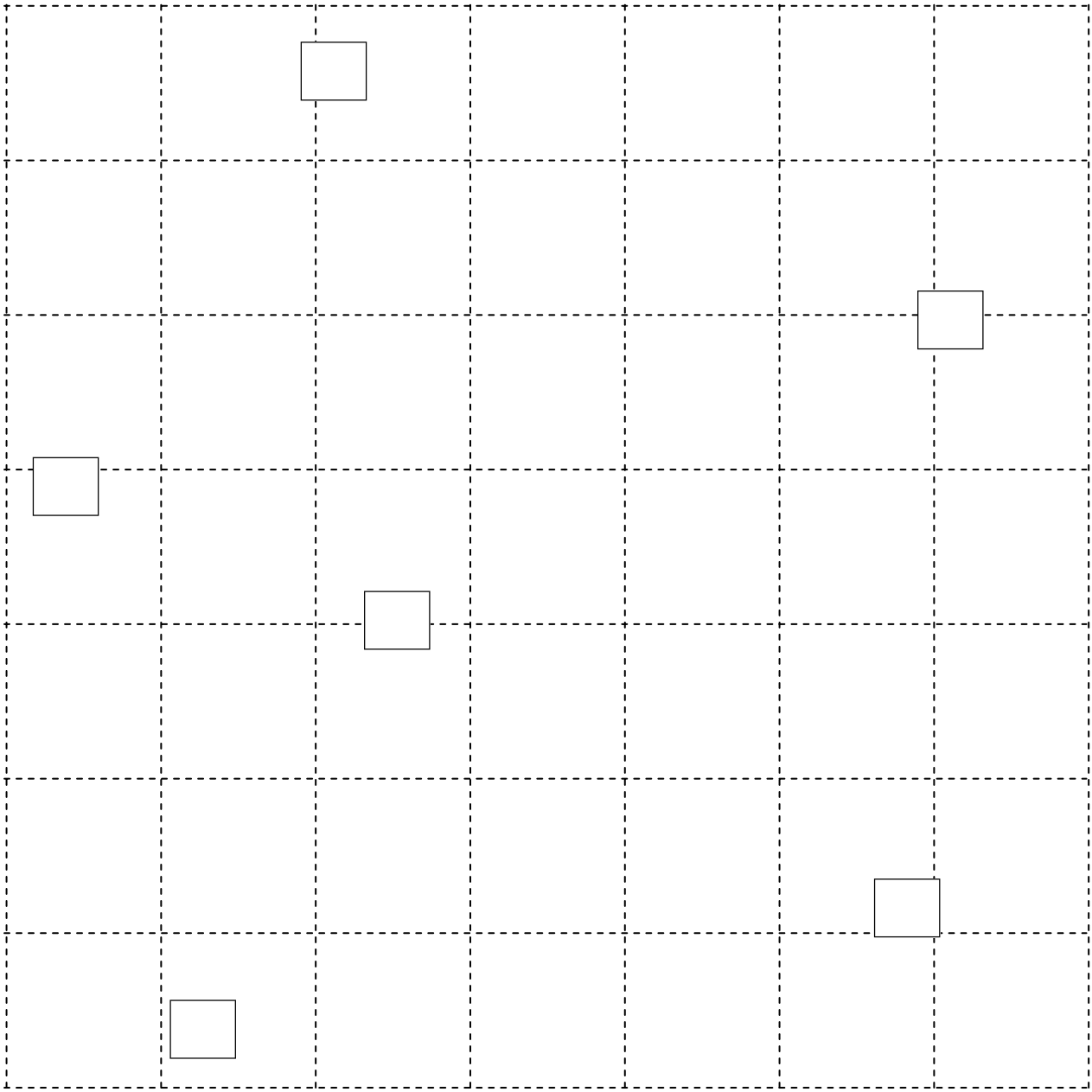
Use a protractor to measure internal angles in the polygon. Write all the angles in the boxes. You can extend lines to use your protractor. Then enlarge this polygon to create a similar figure using the grid. Measure the angles in the similar figure. Do they match?



Extend lines to allow protractor use.



Steps:
- Measure angles in smaller figure
- Construct enlarged (similar) polygon
- Measure angles in enlargement.



Geometrical Properties - Angles & Similarity - Practical (Sheet 2)

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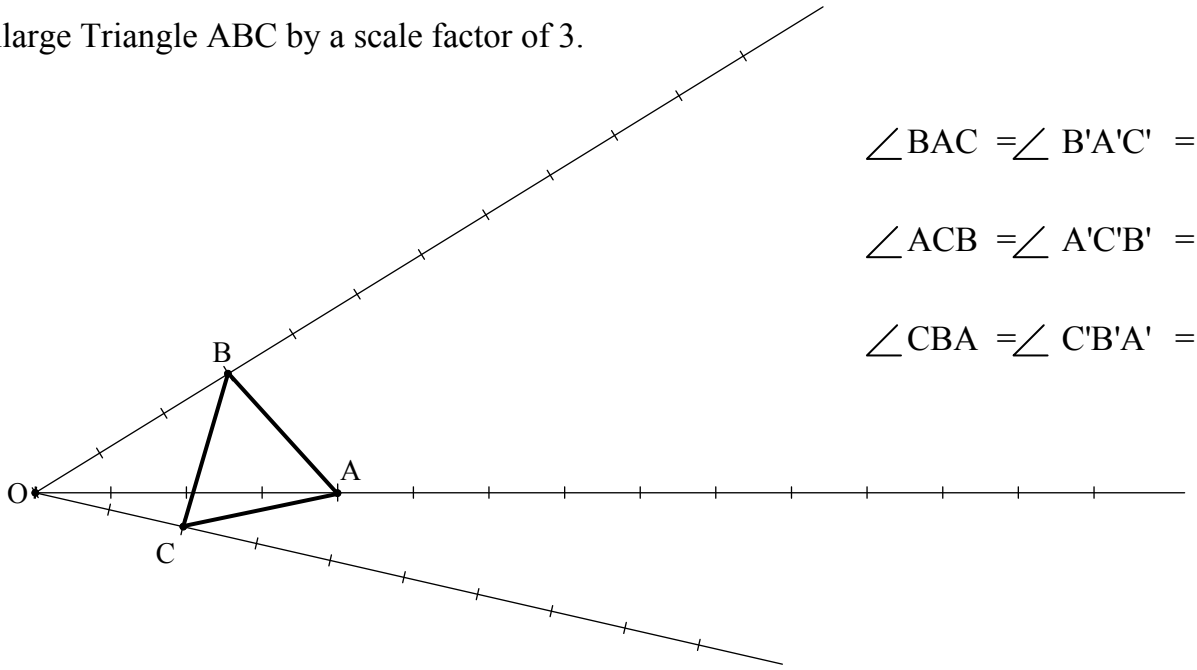
Enlarge the shapes shown using 'O' as the centre of enlargement. Label the similar figure as A'B'C', and so on. Measure the internal angles to confirm that internal angles are the same in similar figures.

1 Enlarge Triangle ABC by a scale factor of 3.

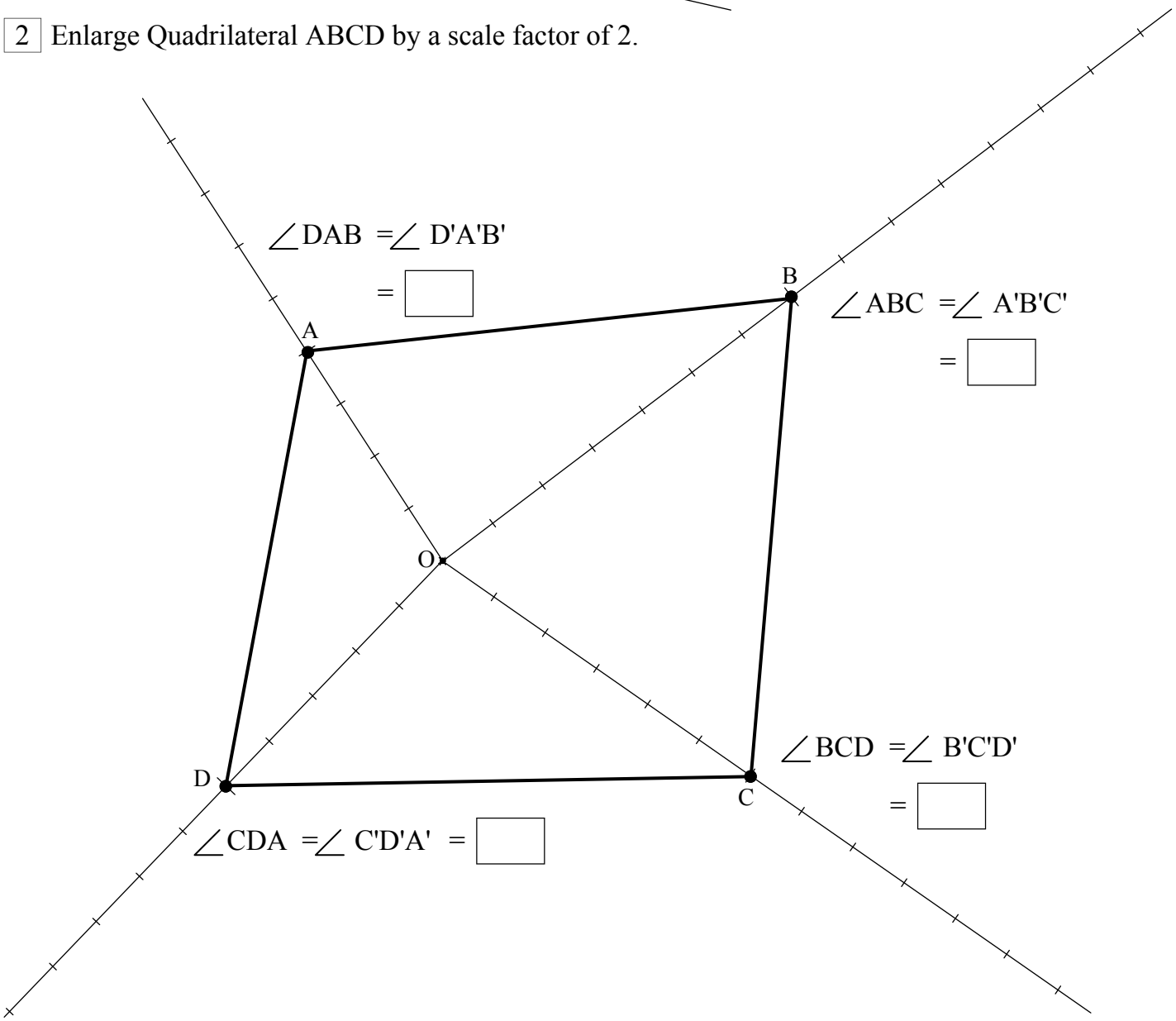


$\angle BAC = \angle B'A'C' =$ ☐°

$\angle ACB = \angle A'C'B' =$ ☐

$\angle CBA = \angle C'B'A' =$ ☐

2 Enlarge Quadrilateral ABCD by a scale factor of 2.



$\angle DAB = \angle D'A'B' =$ ☐

$\angle ABC = \angle A'B'C' =$ ☐

$\angle BCD = \angle B'C'D' =$ ☐

$\angle CDA = \angle C'D'A' =$ ☐

Geometrical Properties - Angles & Similarity - Practical (Sheet 2)

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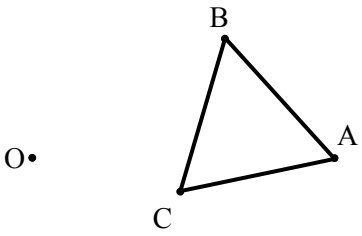
1 Enlarge Triangle ABC by a scale factor of 3.

$\angle BAC = \angle B'A'C' =$ ☐°

$\angle ACB = \angle A'C'B' =$ ☐

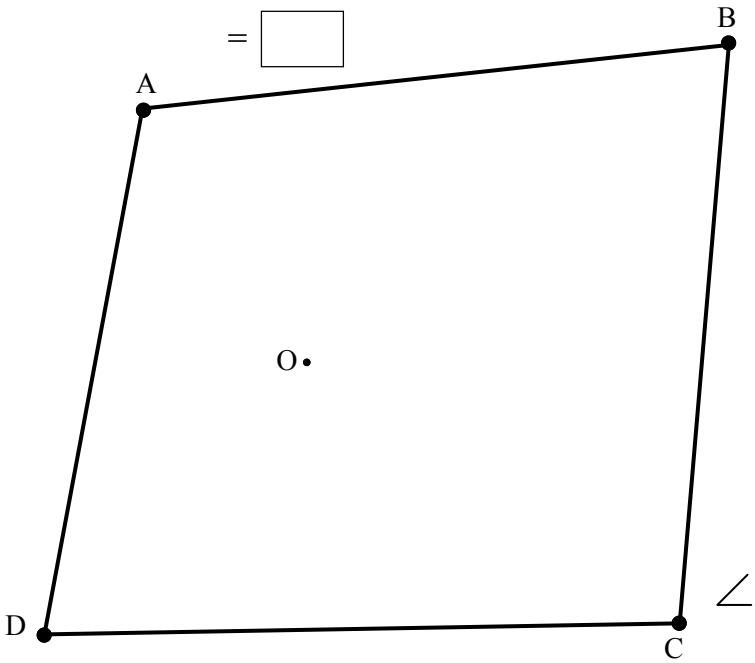
$\angle CBA = \angle C'B'A' =$ ☐



2 Enlarge Quadrilateral ABCD by a scale factor of 2.

$\angle DAB = \angle D'A'B' =$ ☐

$\angle ABC = \angle A'B'C' =$ ☐



$\angle BCD = \angle B'C'D' =$ ☐

$\angle CDA = \angle C'D'A' =$ ☐

Geometrical Properties - Angles & Similarity - Practical (Sheet 1)

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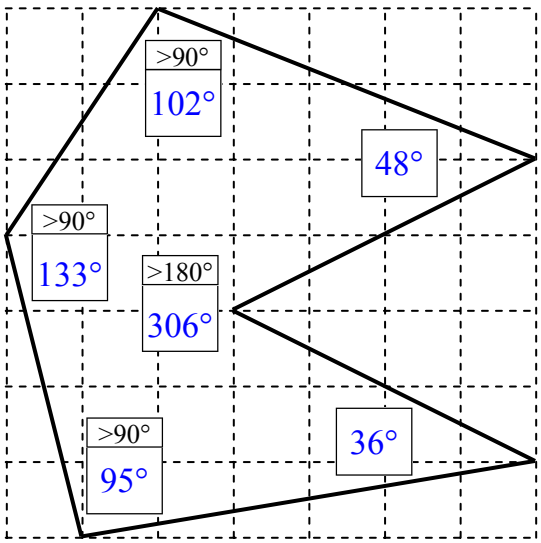
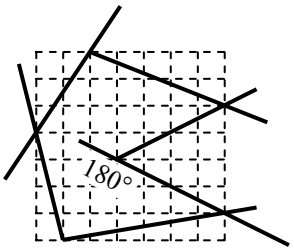
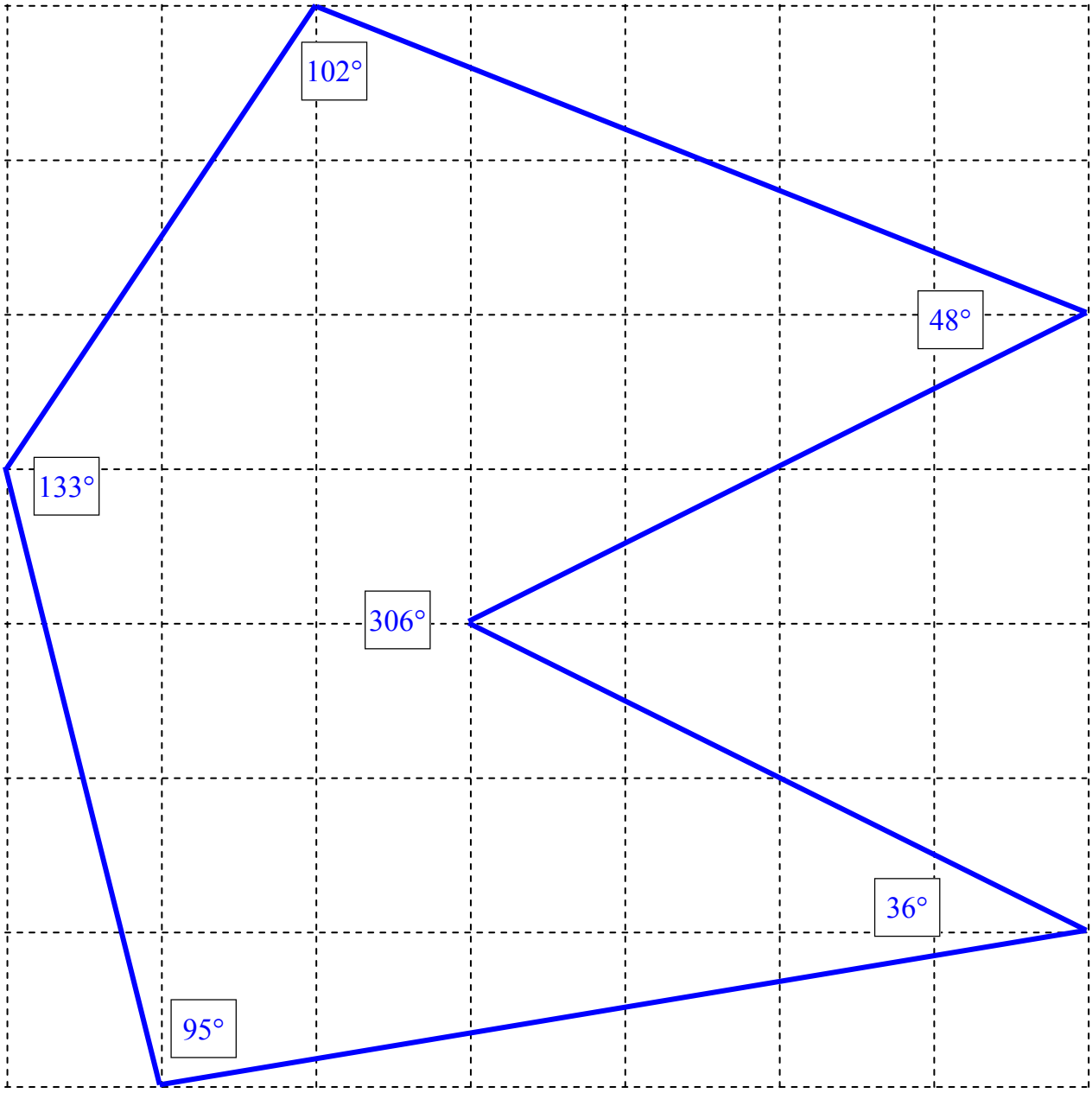
Use a protractor to measure internal angles in the polygon. Write all the angles in the boxes. You can extend lines to use your protractor. Then enlarge this polygon to create a similar figure using the grid. Measure the angles in the similar figure. Do they match?



Extend lines to allow protractor use.



Steps:
- Measure angles in smaller figure
- Construct enlarged (similar) polygon
- Measure angles in enlargement.



Geometrical Properties - Angles & Similarity - Practical (Sheet 2)

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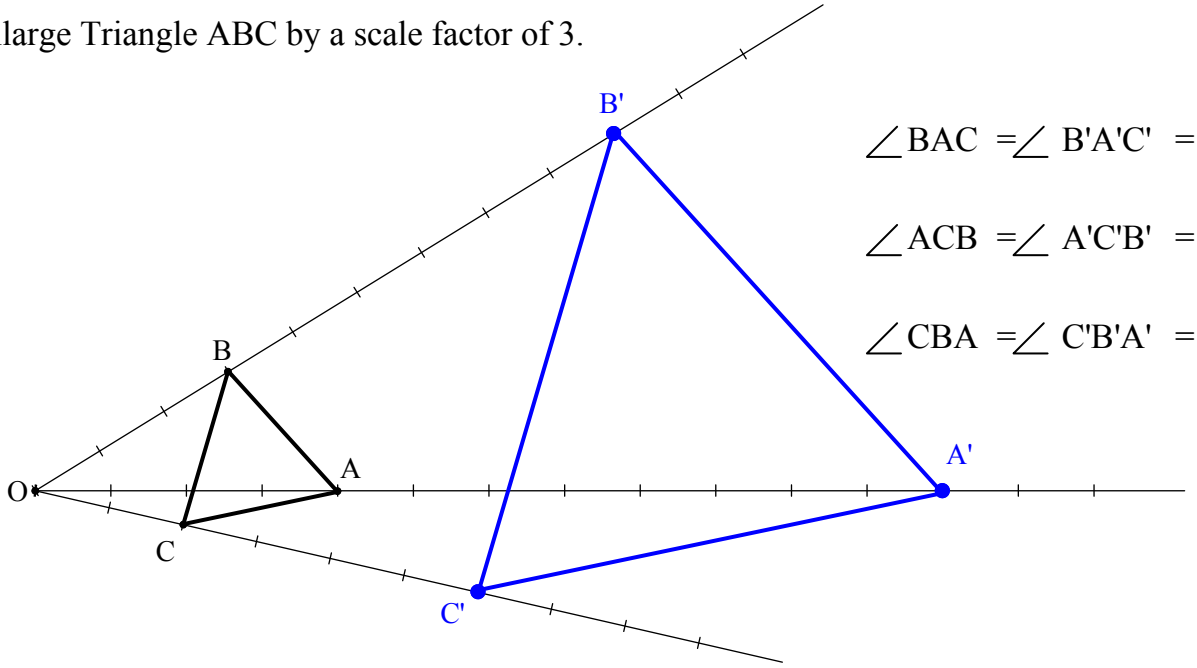
Enlarge the shapes shown using 'O' as the centre of enlargement. Label the similar figure as A'B'C', and so on. Measure the internal angles to confirm that internal angles are the same in similar figures.

1 Enlarge Triangle ABC by a scale factor of 3.

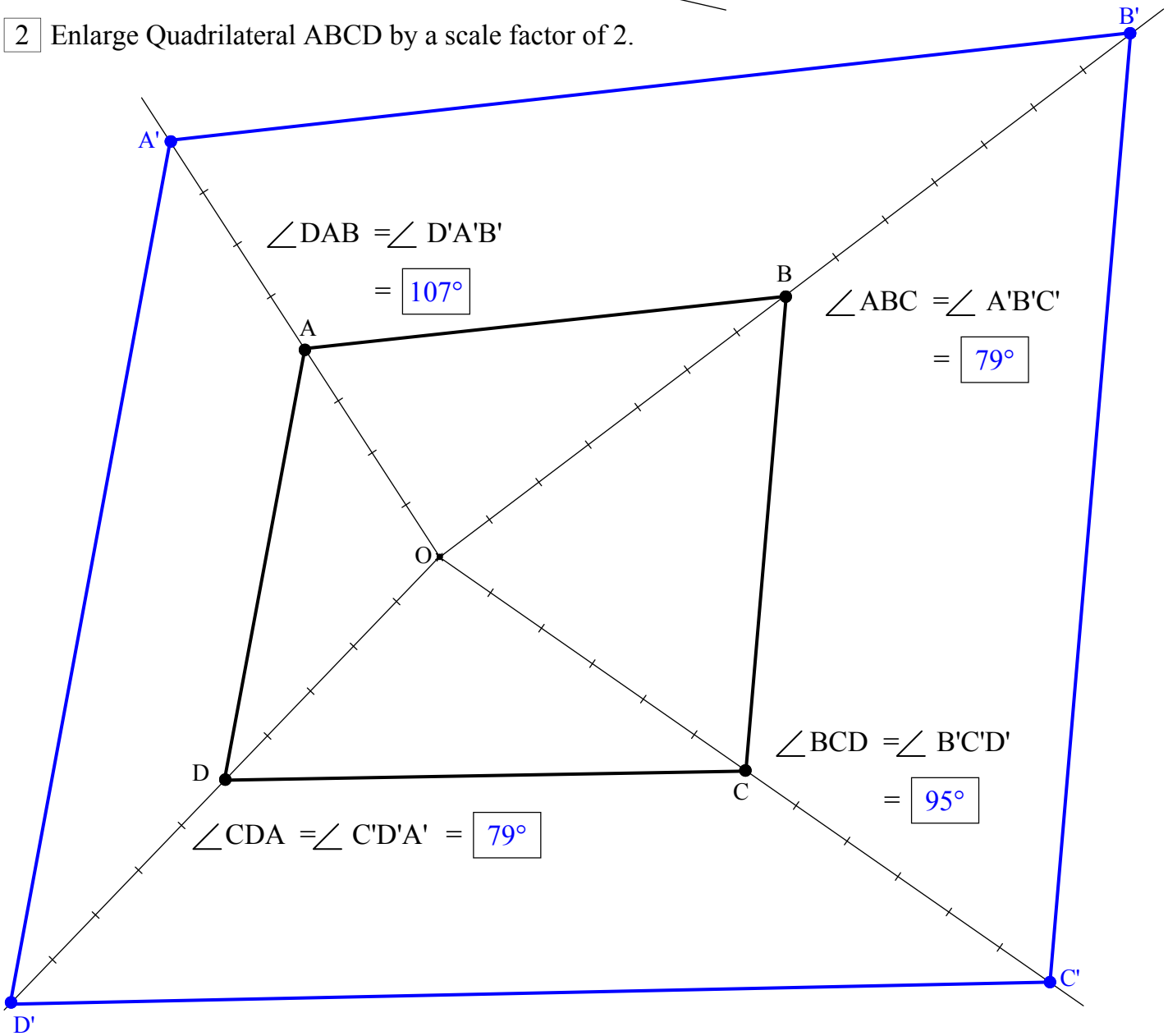


$\angle BAC = \angle B'A'C' = 59°$

$\angle ACB = \angle A'C'B' = 62°$

$\angle CBA = \angle C'B'A' = 59°$

2 Enlarge Quadrilateral ABCD by a scale factor of 2.



$\angle DAB = \angle D'A'B' = 107°$

$\angle ABC = \angle A'B'C' = 79°$

$\angle BCD = \angle B'C'D' = 95°$

$\angle CDA = \angle C'D'A' = 79°$

Geometrical Properties - Angles & Similarity - Practical (Homework Strip)

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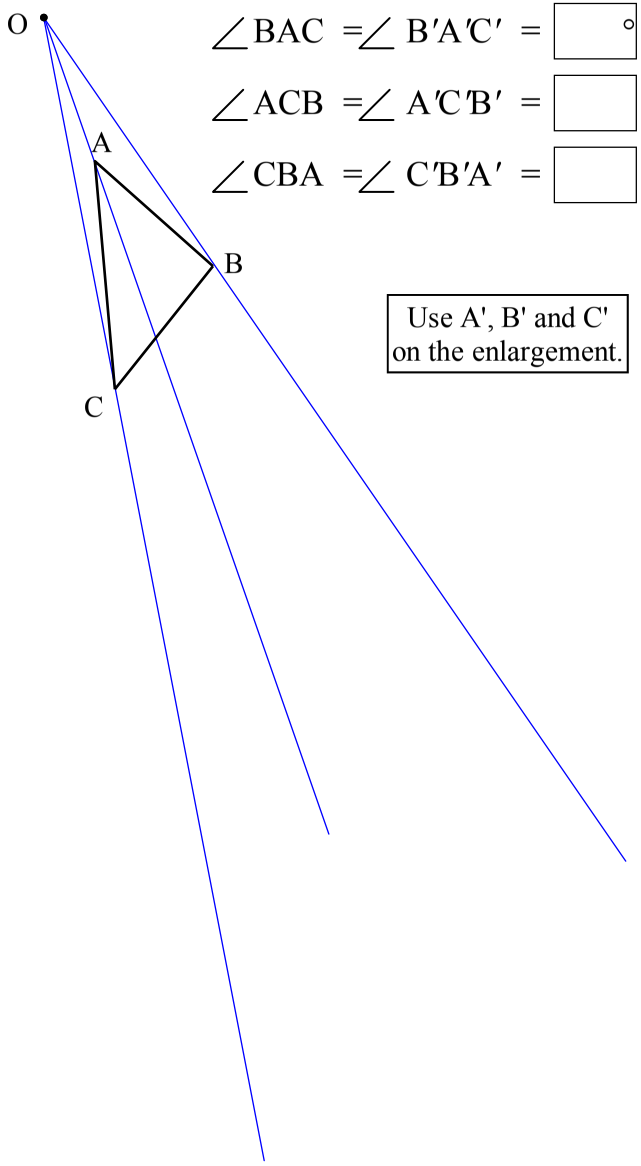
Date Due:

Enlarge the triangle with a scale factor of 2. Measure the angles to see that they match the original.

$\angle BAC = \angle B'A'C' =$ ____°

$\angle ACB = \angle A'C'B' =$ ____

$\angle CBA = \angle C'B'A' =$ ____



Use A', B' and C' on the enlargement.

Date Due:

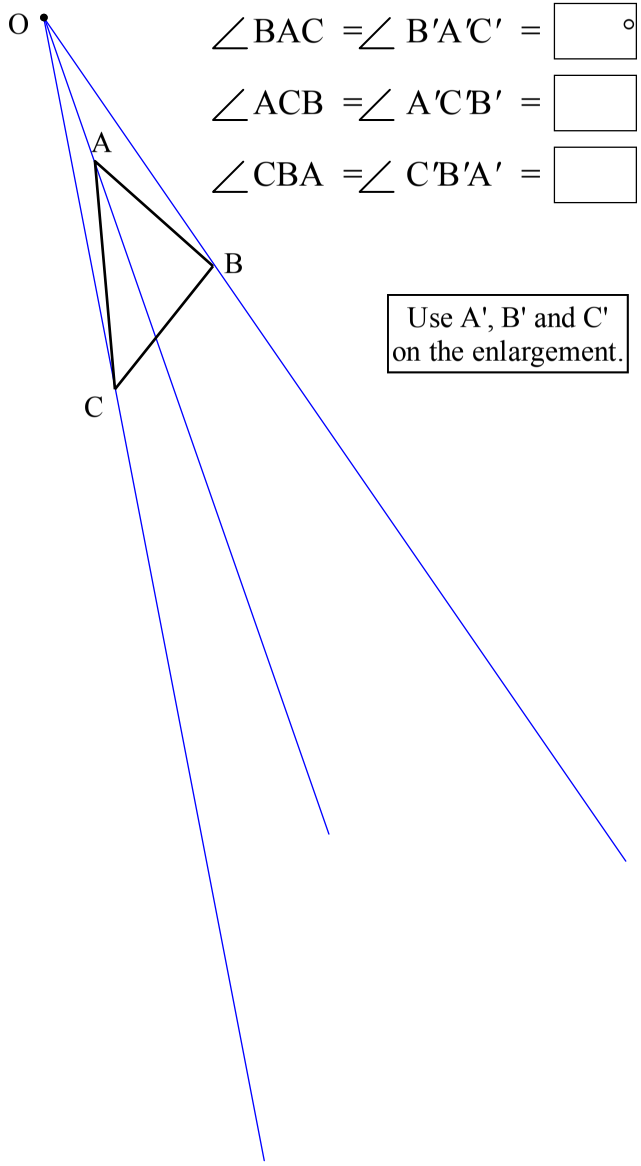
Enlarge the triangle with a scale factor of 2. Measure the angles to see that they match the original.

$\angle BAC = \angle B'A'C' =$ ____°

$\angle ACB = \angle A'C'B' =$ ____

$\angle CBA = \angle C'B'A' =$ ____

Use A', B' and C' on the enlargement.



Date Due:

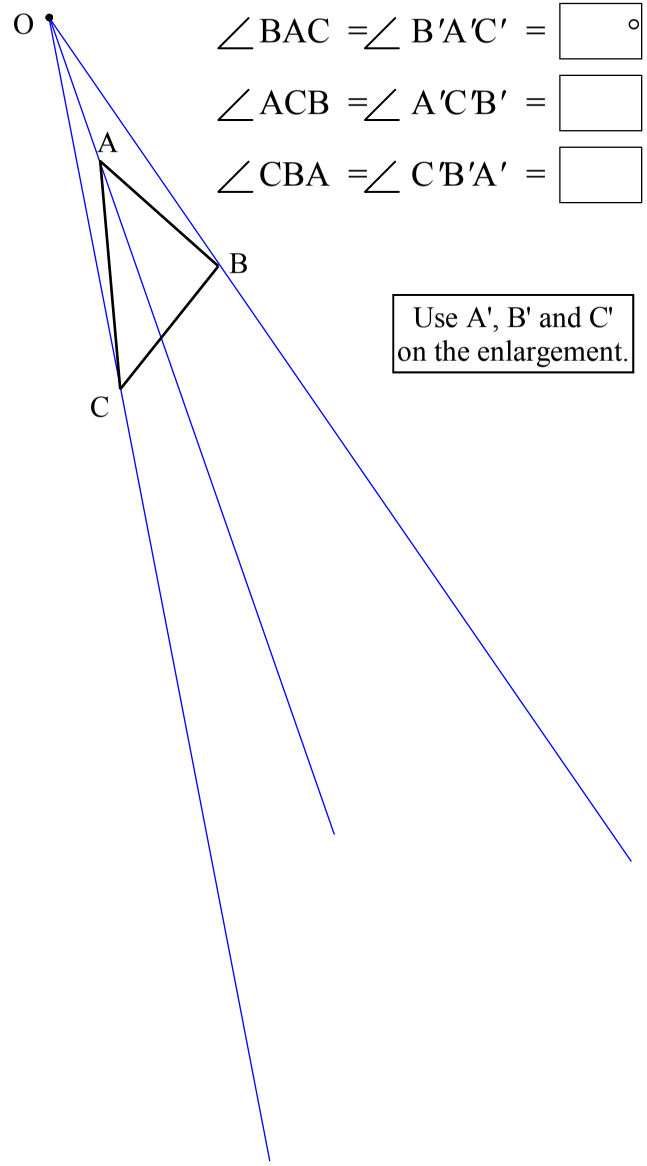
Enlarge the triangle with a scale factor of 2. Measure the angles to see that they match the original.

$\angle BAC = \angle B'A'C' =$ ____°

$\angle ACB = \angle A'C'B' =$ ____

$\angle CBA = \angle C'B'A' =$ ____

Use A', B' and C' on the enlargement.



Geometrical Properties - Angles & Similarity - Practical (Homework Strip)

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Date Due: _____

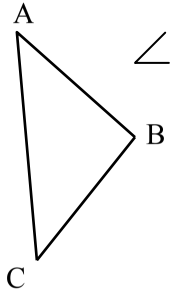
Enlarge the triangle with a scale factor of 2. Measure the angles to see that they match the original.

O •

$\angle BAC = \angle B'A'C' =$ ____°

$\angle ACB = \angle A'C'B' =$ ____

$\angle CBA = \angle C'B'A' =$ ____



Use A', B' and C' on the enlargement.

Date Due: _____

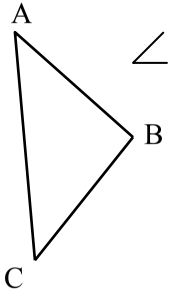
Enlarge the triangle with a scale factor of 2. Measure the angles to see that they match the original.

O •

$\angle BAC = \angle B'A'C' =$ ____°

$\angle ACB = \angle A'C'B' =$ ____

$\angle CBA = \angle C'B'A' =$ ____



Use A', B' and C' on the enlargement.

Date Due: _____

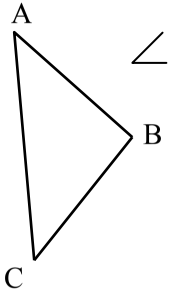
Enlarge the triangle with a scale factor of 2. Measure the angles to see that they match the original.

O •

$\angle BAC = \angle B'A'C' =$ ____°

$\angle ACB = \angle A'C'B' =$ ____

$\angle CBA = \angle C'B'A' =$ ____



Use A', B' and C' on the enlargement.

Geometrical Properties - Angles & Similarity - Practical (H'work Strip)

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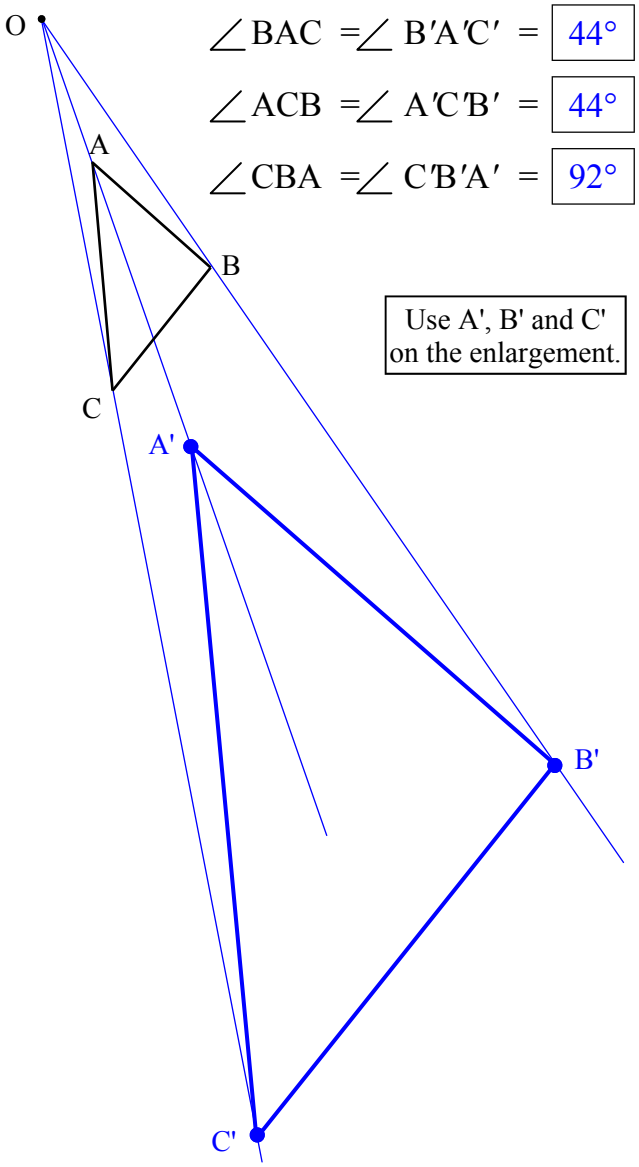
Date Due:

Enlarge the triangle with a scale factor of 2. Measure the angles to see that they match the original.



$\angle BAC = \angle B'A'C' =$ 44°

$\angle ACB = \angle A'C'B' =$ 44°

$\angle CBA = \angle C'B'A' =$ 92°

Use A', B' and C' on the enlargement.

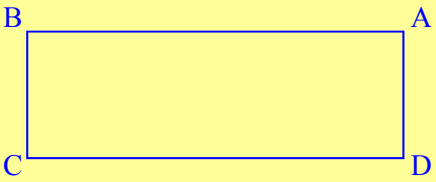
Geometrical Properties - Further Enlargements (Ratios)

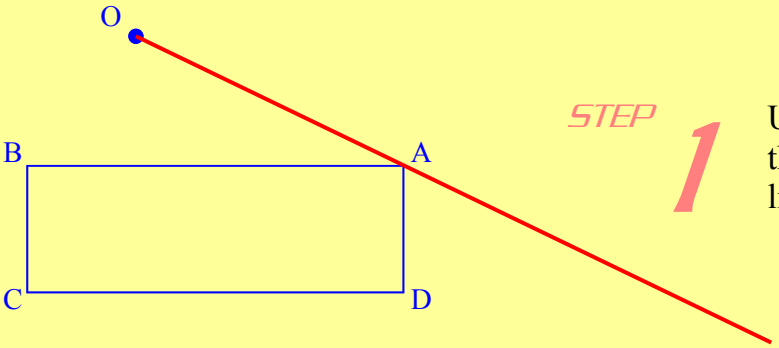
One way to make a similar figure is to create an enlargement of the shape. A point called a centre of enlargement (CoE) is used by drawing lines from it to all the vertices (corners) of the shape. The length from the CoE is measured then multiplied by the scale factor to move that point along the line. The scale factors on this sheet are all more than 1, that means that the enlarged shape will be larger than the original shape.

The CoE can be placed anywhere. It can be inside the shape (Q.2) and outside the shape. It can also be on one of the lines that make the shape. Below are the steps to follow to make an enlargement.

O

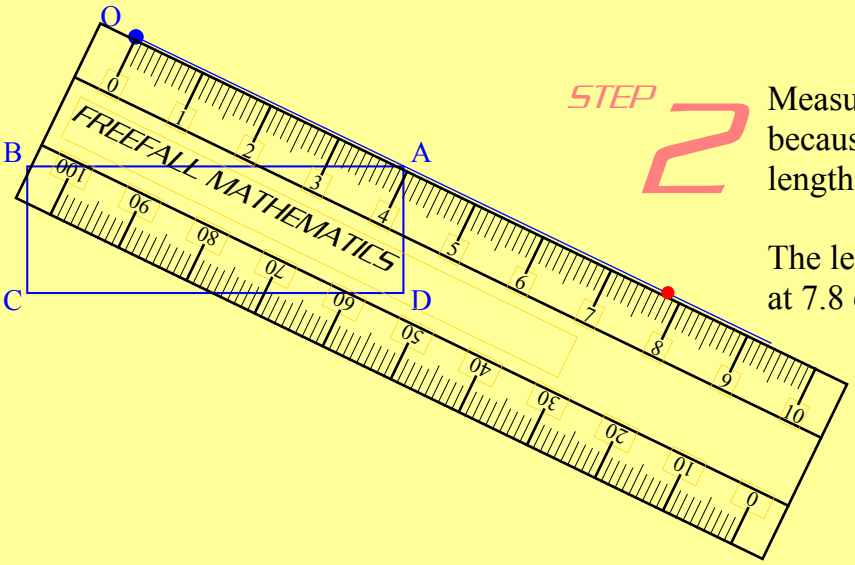
An enlargement of rectangle ABCD with a scale factor of 2 is required.





STEP 1

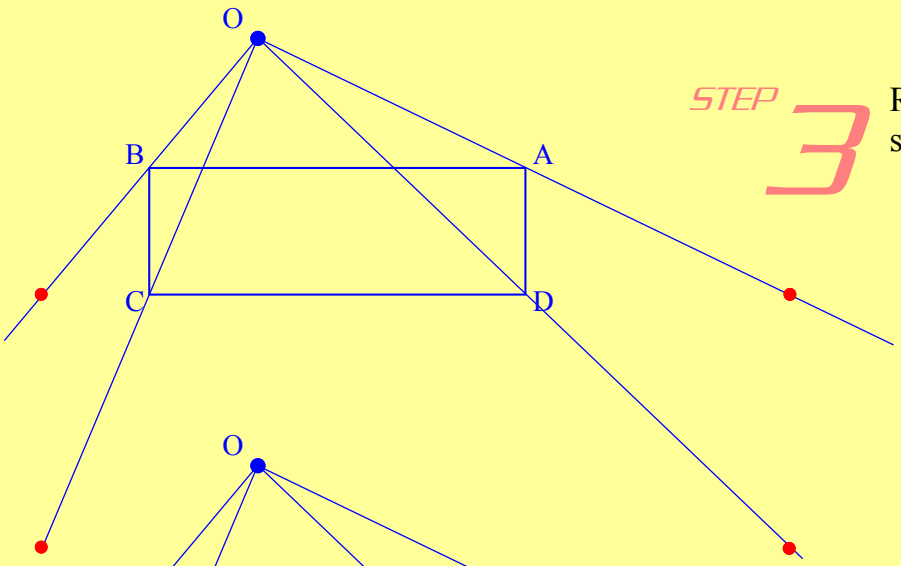
Use a ruler to draw a line from the CoE through one point. In this case it is the red line from O through A.



STEP 2

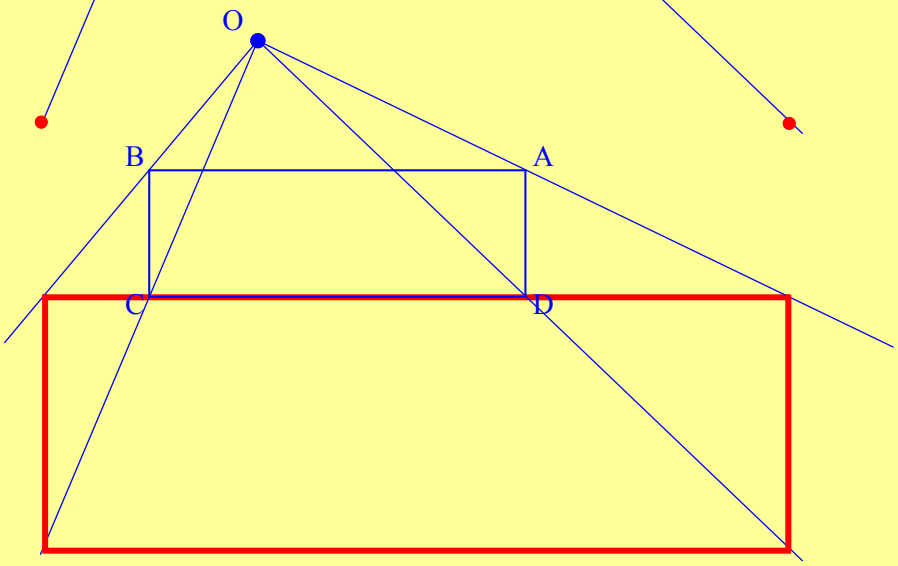
Measure the length OA and then double it, because the scale factor is 2. Mark the length on the constructed line.

The length was 3.9 cm, so a mark is placed at 7.8 cm.



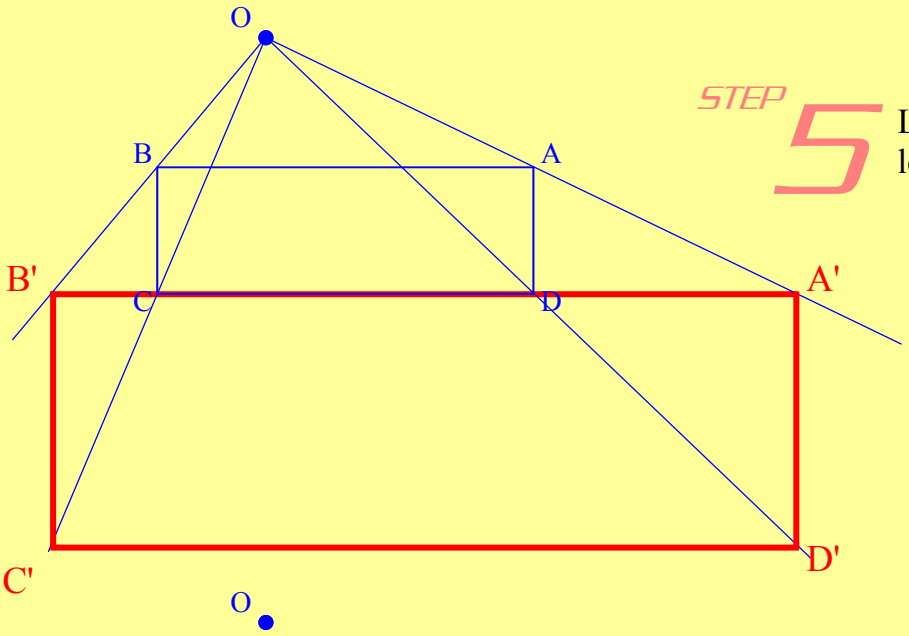
STEP 3

Repeat for the three other corners of the shape.



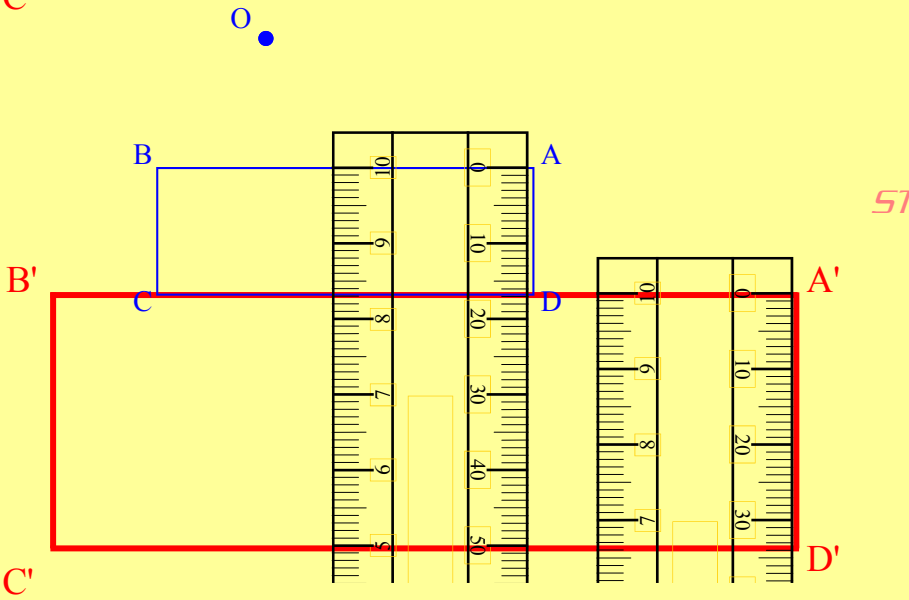
STEP 4

Draw the lines to make the shape.



STEP 5

Label the sides of the shape, use the same letters and add a dash. You are all done!



STEP 6

Measure matching sides, one should be double the other.

AD = 1.7 cm

A'D' = 3.4 cm Looks good.

Geometrical Properties - Further Enlargements (Ratios)

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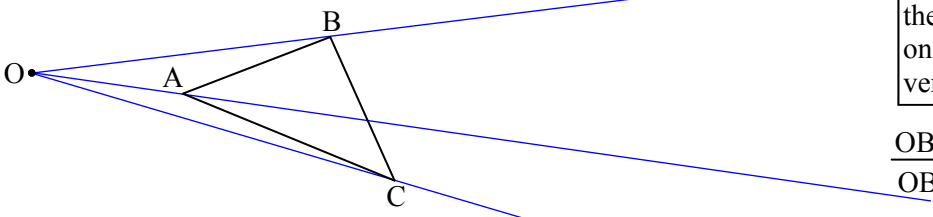
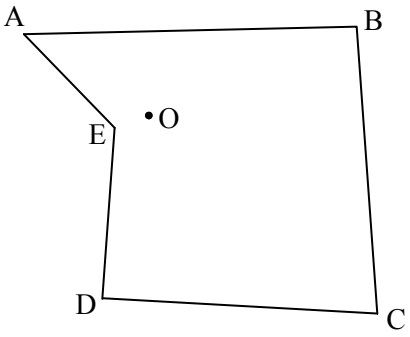
1 Enlarge Triangle ABC by a scale factor of 2.



Enlarge the shapes shown using 'O' as the centre of enlargement. Label the enlarged shape as A'B'C', and so on. Then complete the ratios to verify scale factor properties.

$\frac{OB'}{OB} = \frac{\square}{\square} = \square$ $\frac{A'B'}{AB} = \frac{\square}{\square} = \square$

2 Enlarge Pentagon ABCDE by a scale factor of 3.

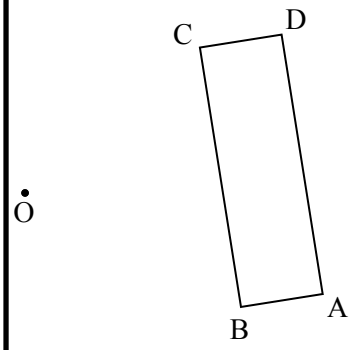


$\frac{OA'}{OA} = \frac{\square}{\square} = \square$

$\frac{B'C'}{BC} = \frac{\square}{\square} = \square$

$\frac{C'D'}{CD} = \frac{\square}{\square} = \square$

3 Enlarge Rectangle ABCD by a scale factor of 2.5.



$\frac{OD'}{OD} = \frac{\square}{\square} = \square$

$\frac{A'D'}{AD} = \frac{\square}{\square} = \square$

$\frac{B'D'}{BD} = \frac{\square}{\square} = \square$

Geometrical Properties - Further Enlargements (Ratios)

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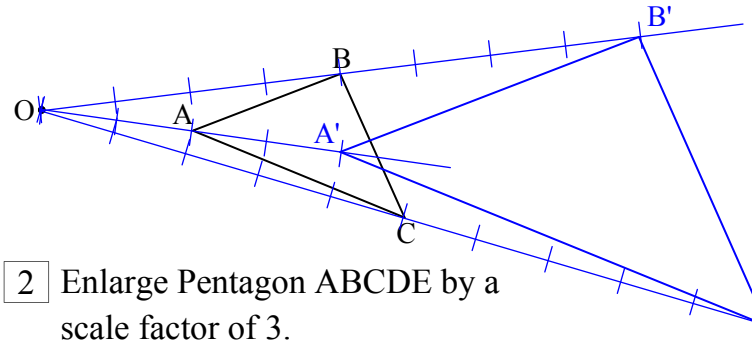
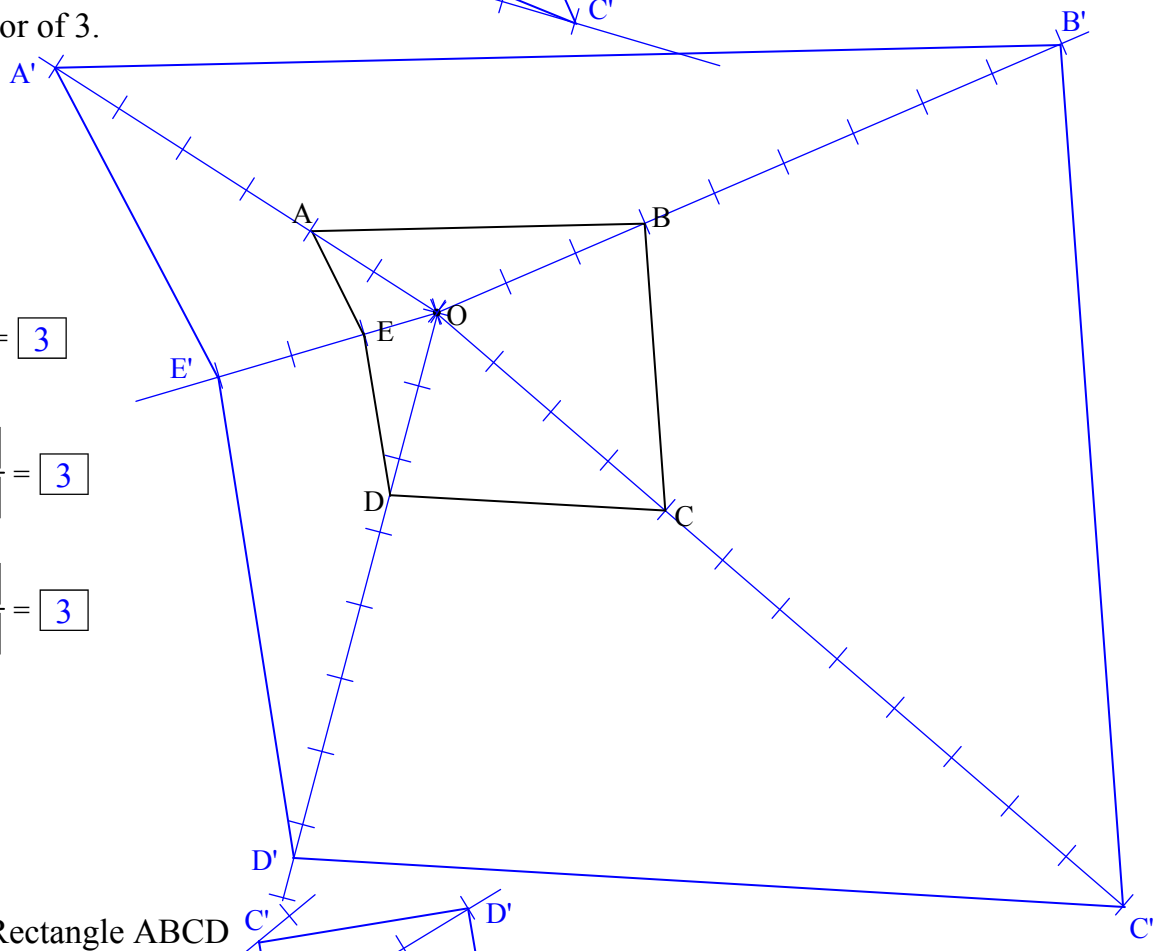
1 Enlarge Triangle ABC by a scale factor of 2.



Enlarge the shapes shown using 'O' as the centre of enlargement. Label the enlarged shape as A'B'C', and so on. Then complete the ratios to verify scale factor properties.

$\frac{OB'}{OB} = \frac{80}{40} = 2$ $\quad \frac{A'B'}{AB} = \frac{42}{21} = 2$

2 Enlarge Pentagon ABCDE by a scale factor of 3.

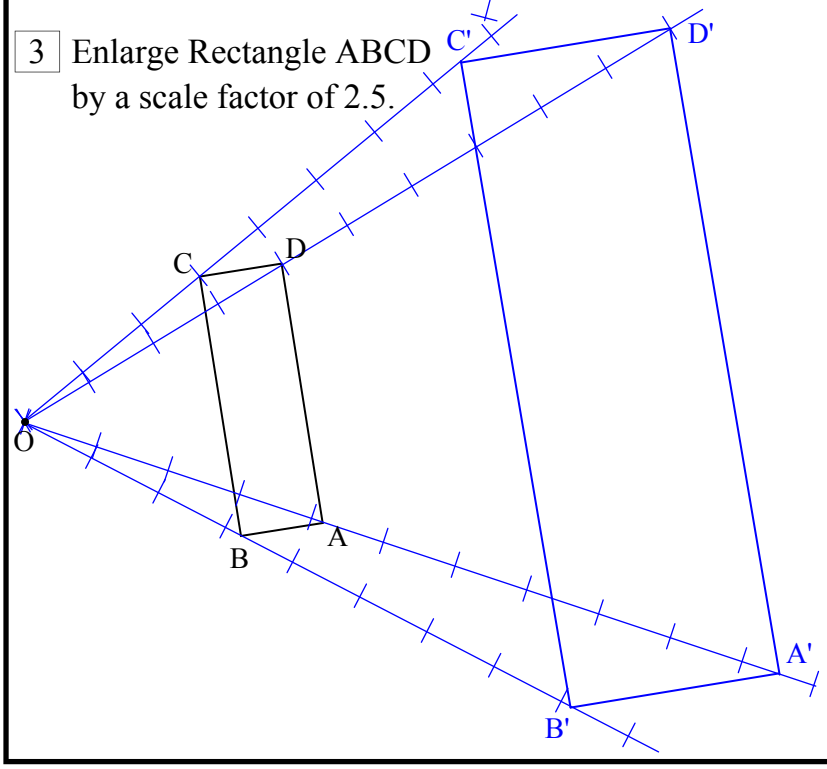


$\frac{OA'}{OA} = \frac{6}{2} = 3$

$\frac{B'C'}{BC} = \frac{114}{38} = 3$

$\frac{C'D'}{CD} = \frac{110}{36.5} = 3$

3 Enlarge Rectangle ABCD by a scale factor of 2.5.



$\frac{OD'}{OD} = \frac{100}{40} = 2.5$

$\frac{A'D'}{AD} = \frac{86}{34} = 2.5$

$\frac{B'D'}{BD} = \frac{90}{36} = 2.5$

Geometrical Properties - Further Enlargements (Homework Strip)

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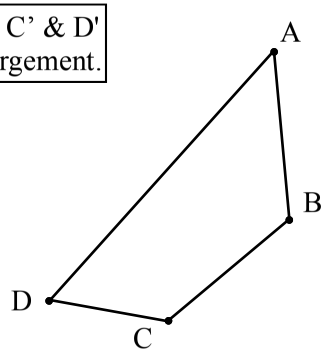
Date Due: _____

Enlarge the trapezium with a scale factor of 2. Complete the ratios to show ratios of side lengths are equal with similar figures and result in the scale factor.

O •

Use A', B', C' & D' on the enlargement.



$\frac{A'D'}{AD} = \frac{\text{in mm}}{\text{in mm}} = \square$ $\quad \frac{B'C'}{BC} = \frac{\square}{\square} = \square$

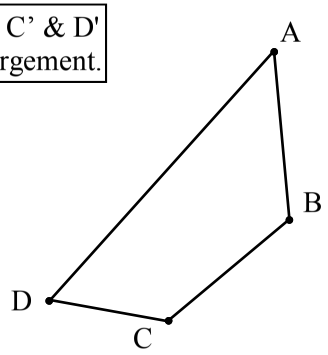
$\frac{A'B'}{AB} = \frac{\square}{\square} = \square$ $\quad \frac{C'D'}{CD} = \frac{\square}{\square} = \square$

Date Due: _____

Enlarge the trapezium with a scale factor of 2. Complete the ratios to show ratios of side lengths are equal with similar figures and result in the scale factor.

O •

Use A', B', C' & D' on the enlargement.



$\frac{A'D'}{AD} = \frac{\text{in mm}}{\text{in mm}} = \square$ $\quad \frac{B'C'}{BC} = \frac{\square}{\square} = \square$

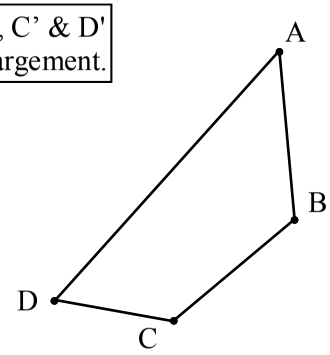
$\frac{A'B'}{AB} = \frac{\square}{\square} = \square$ $\quad \frac{C'D'}{CD} = \frac{\square}{\square} = \square$

Date Due: _____

Enlarge the trapezium with a scale factor of 2. Complete the ratios to show ratios of side lengths are equal with similar figures and result in the scale factor.

O •

Use A', B', C' & D' on the enlargement.



$\frac{A'D'}{AD} = \frac{\text{in mm}}{\text{in mm}} = \square$ $\quad \frac{B'C'}{BC} = \frac{\square}{\square} = \square$

$\frac{A'B'}{AB} = \frac{\square}{\square} = \square$ $\quad \frac{C'D'}{CD} = \frac{\square}{\square} = \square$

Geometrical Properties - Further Enlargements (Homework Strip)

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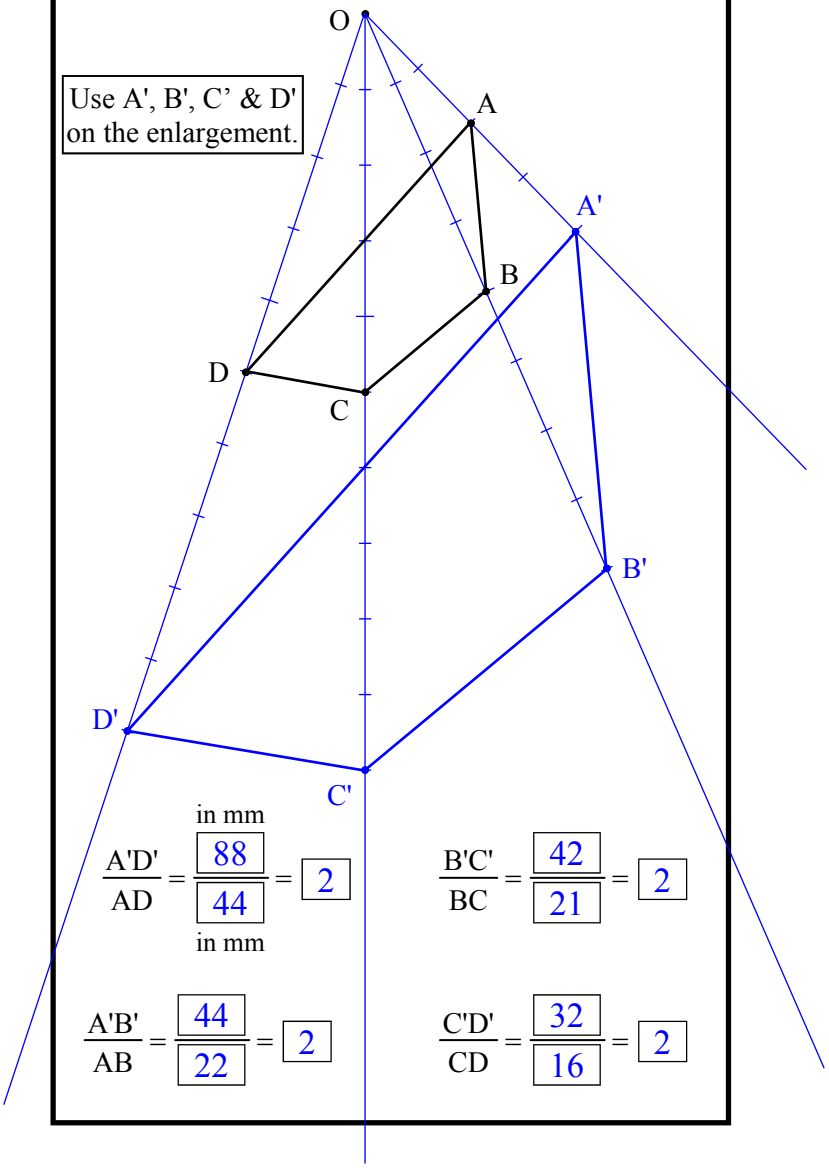
Date Due:

Enlarge the trapezium with a scale factor of 2. Complete the ratios to show ratios of side lengths are equal with similar figures and result in the scale factor.

Use A', B', C' & D' on the enlargement.



$$\frac{A'D'}{AD} = \frac{88 \text{ in mm}}{44 \text{ in mm}} = 2$$

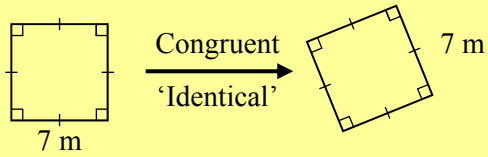
$$\frac{B'C'}{BC} = \frac{42}{21} = 2$$

$$\frac{A'B'}{AB} = \frac{44}{22} = 2$$

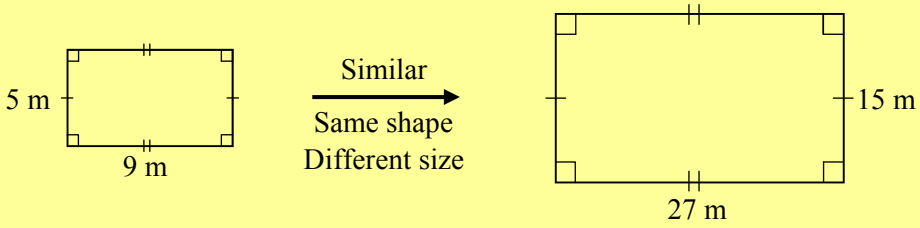
$$\frac{C'D'}{CD} = \frac{32}{16} = 2$$

Geometrical Properties - Similarity and Congruence

Congruence means identical. So internal angles are the same on both shapes and the length of sides (or radius) is the same. Note that shapes can be in different orientation (eg. rotated).



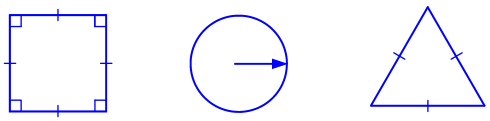
Similar means same shape but different size. So internal angles are the same, but the side lengths are larger or smaller, multiplied by a scale factor.



In the above example the internal angles are all 90°. But the right rectangle is 3 times larger than the left rectangle. Because both sides are multiplied by three, the rectangles are similar.

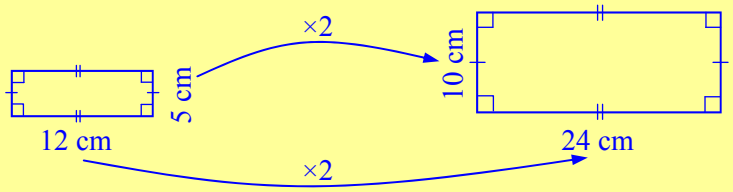
When a shape features only one length, other shapes of its type are always either congruent or similar. Squares, equilateral triangles and circles are examples of shapes that must be either similar or congruent.

These shapes must be either similar or congruent



With shapes like rectangles and isosceles triangles, the shapes are congruent if the sides are the same length. If matching sides on the two shapes have lengths that have a common scale factor, then those shapes are similar. So find the scale factor to show similarity.

Example



● Similar
○ Not similar
because the scale factor
● is uniform
○ is inconsistent

If one side was ×2 and the other was ×3 then the shapes aren't similar, because the pairs of matching sides don't have the same scale factor.

Geometrical Properties - Similarity and Congruence (Sheet 1)

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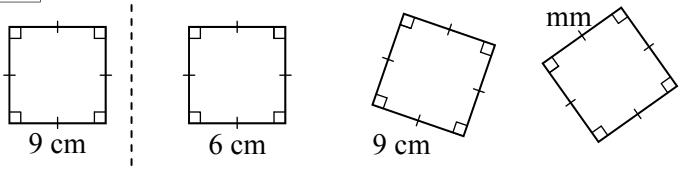
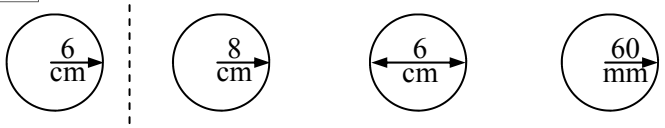
All squares, circles and equilateral triangles are either similar or congruent. Decide if these are congruent or similar to the original shape.

1



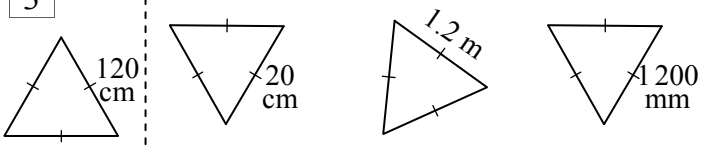
○ similar ○ congruent | ○ similar ○ congruent | ○ similar ○ congruent

2



○ similar ○ congruent | ○ similar ○ congruent | ○ similar ○ congruent

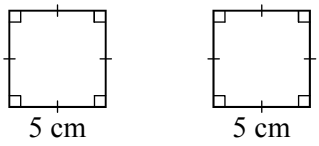
3



○ similar ○ congruent | ○ similar ○ congruent | ○ similar ○ congruent

State why these are similar or congruent. Use:
Congruent - all sides & angles are the same, or
Similarity - all sides enlarged by same scale factor.

4

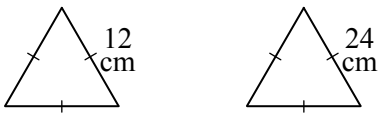


○ similar
○ congruent

Reason: _____

_____

5

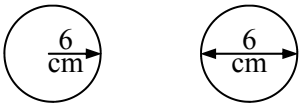


○ similar
○ congruent

Reason: _____

_____

6



Use the word 'radius' not 'side'

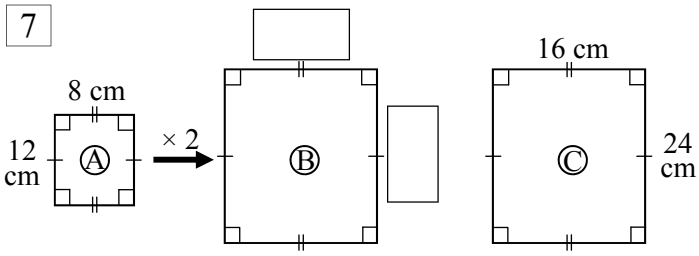
○ similar
○ congruent

Reason: _____

_____

Enlarge the first shape 'A' given a scale factor. If the enlarged shape 'B' is congruent with the third shape 'C', then you have similarity between A & C.

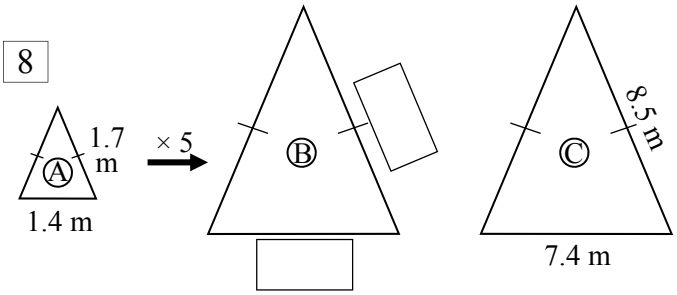
7



Because rectangle B ○is ○isn't congruent to rectangle C,

rectangle A ○is ○isn't similar to rectangle C.

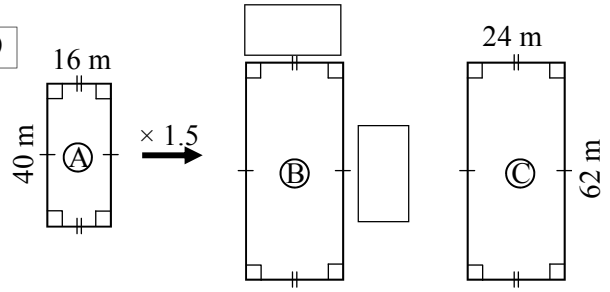
8



Because triangle B ○is ○isn't congruent to triangle C,

triangle A ○is ○isn't similar to triangle C.

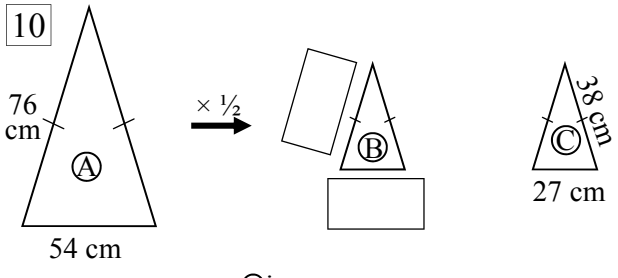
9



Because rectangle B ○is ○isn't congruent to rectangle C,

rectangle A ○is ○isn't similar to rectangle C.

10



Because triangle B ○is ○isn't congruent to triangle C,

triangle A ○is ○isn't similar to triangle C.

Geometrical Prop - Similarity and Congruence - Scale Factors (Sheet 2)

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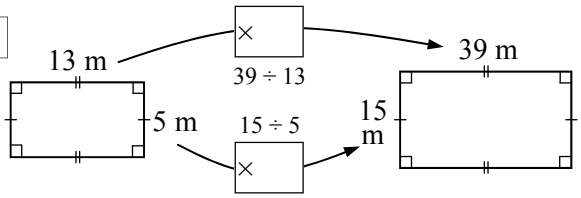
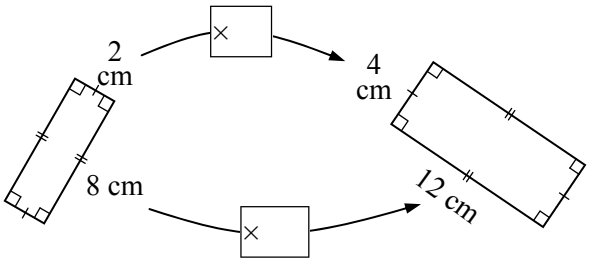
With rectangles, if the scale factor for both sides is the same, you have similarity. Test these rectangles for similarity.

1



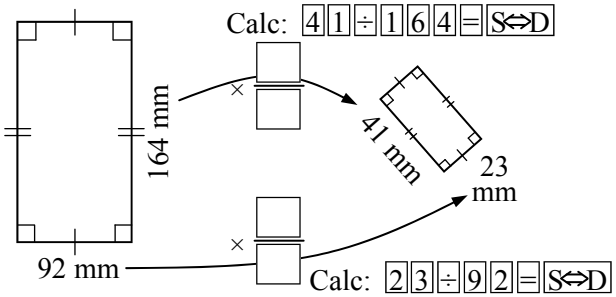
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

2



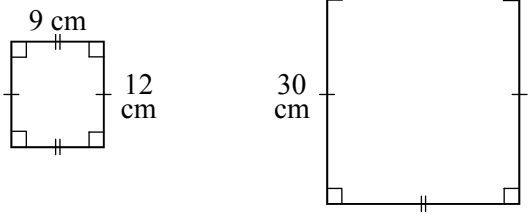
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

3



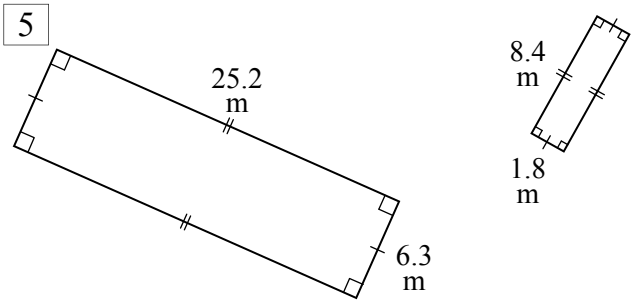
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

4



○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

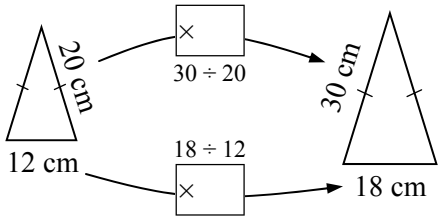
5



○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

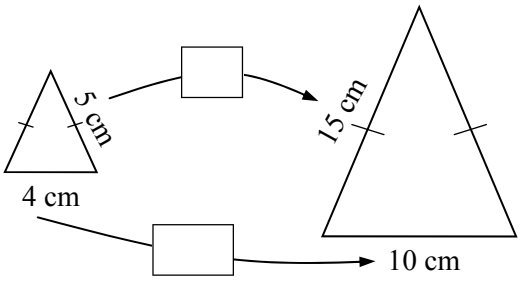
Like rectangles, triangles can be tested for similarity using the scale factor. Test these triangles for similarity.

6



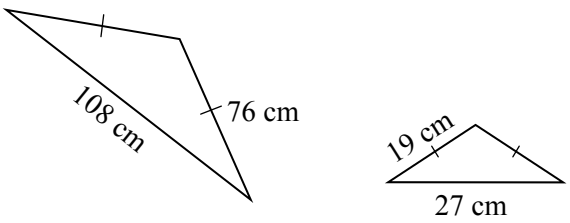
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

7



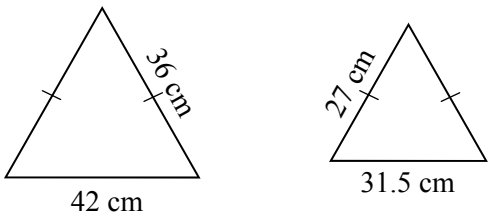
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

8



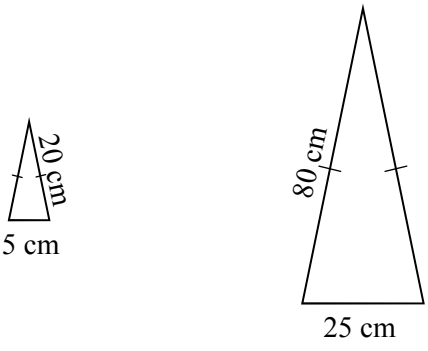
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

9



○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

10



○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

Geometrical Properties - Similarity and Congruence (Sheet 1)

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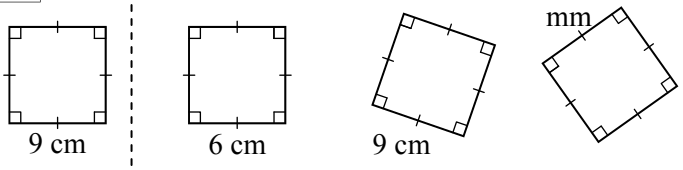
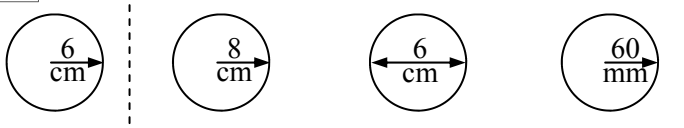
All squares, circles and equilateral triangles are either similar or congruent. Decide if these are congruent or similar to the original shape.

1



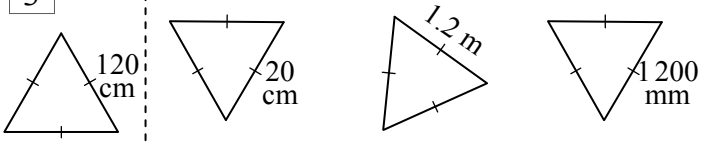
● similar ○ congruent	○ similar ● congruent	○ similar ● congruent

2



● similar ○ congruent	● similar ○ congruent	○ similar ● congruent

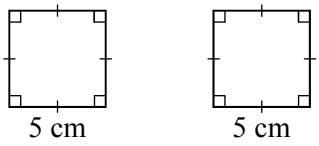
3



● similar ○ congruent	○ similar ● congruent	○ similar ● congruent

State why these are similar or congruent. Use:
Congruent - all sides & angles are the same, or
Similarity - all sides enlarged by same scale factor.

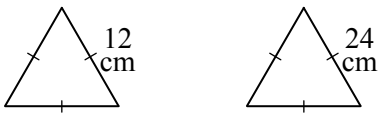
4



○ similar
● congruent

Reason: All sides and internal angles are the same.

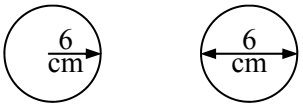
5



● similar
○ congruent

Reason: All sides enlarged by same scale factor (×2).

6



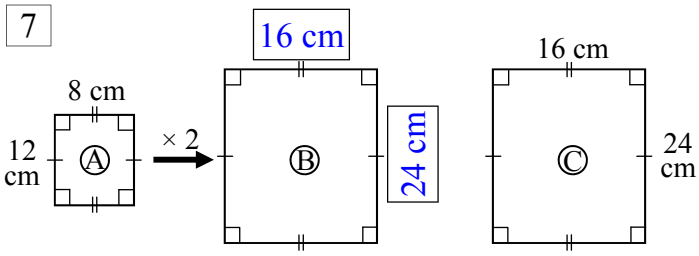
Use the word 'radius' not side

● similar
○ congruent

Reason: Radius enlarged by same scale factor (×½).

Enlarge the first shape 'A' given a scale factor. If the enlarged shape 'B' is congruent with the third shape 'C', then you have similarity between A & C.

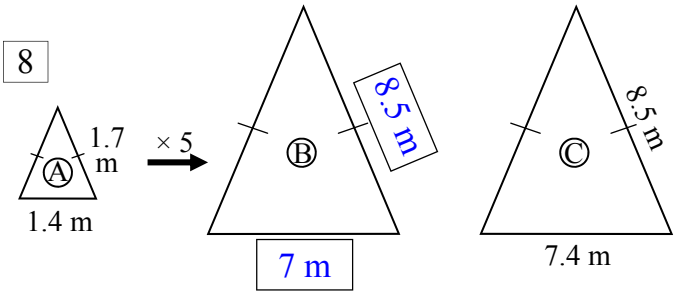
7



Because rectangle B ● is ○ isn't congruent to rectangle C,

rectangle A ● is ○ isn't similar to rectangle C.

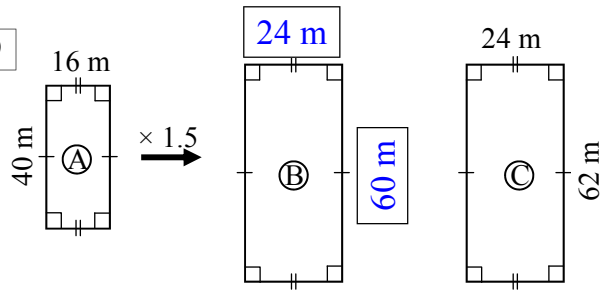
8



Because triangle B ○ is ● isn't congruent to triangle C,

triangle A ○ is ● isn't similar to triangle C.

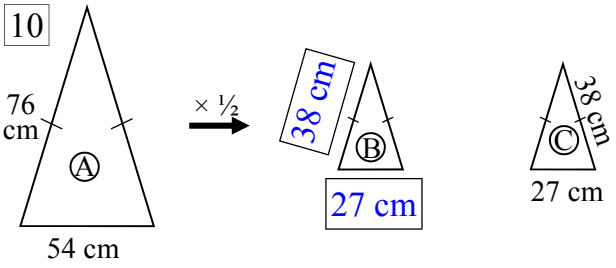
9



Because rectangle B ○ is ● isn't congruent to rectangle C,

rectangle A ○ is ● isn't similar to rectangle C.

10



Because triangle B ● is ○ isn't congruent to triangle C,

triangle A ● is ○ isn't similar to triangle C.

Geometrical Prop - Similarity and Congruence - Scale Factors (Sheet 2)

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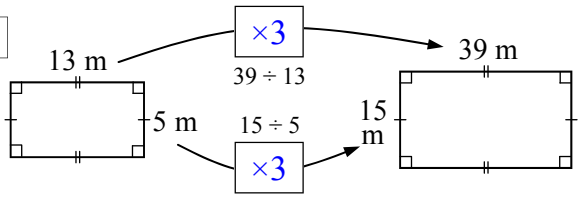
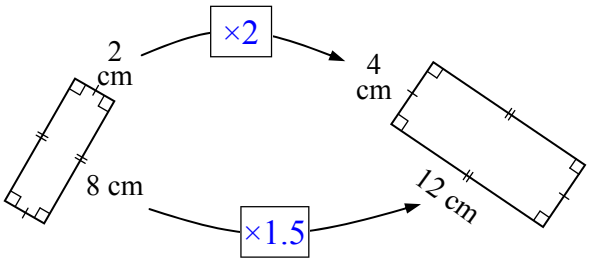
With rectangles, if the scale factor for both sides is the same, you have similarity. Test these rectangles for similarity.

1



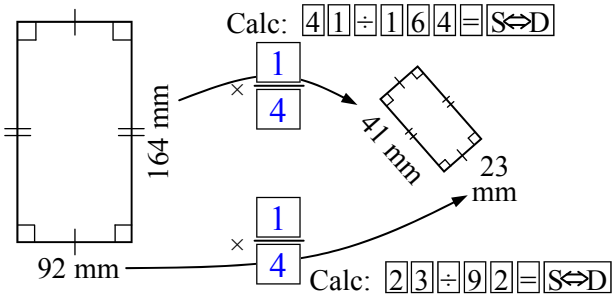
● Similar ○ Not similar because the scale factor ● is uniform ○ is inconsistent

2



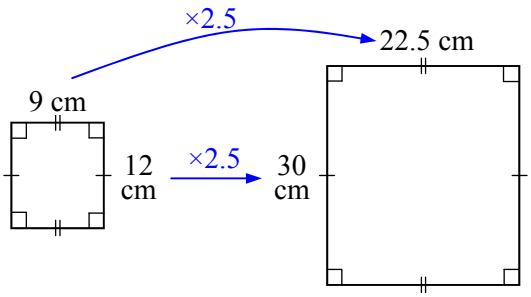
○ Similar ● Not similar because the scale factor ○ is uniform ● is inconsistent

3



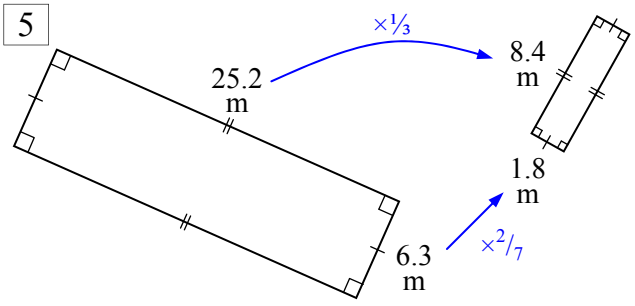
● Similar ○ Not similar because the scale factor ● is uniform ○ is inconsistent

4



● Similar ○ Not similar because the scale factor ● is uniform ○ is inconsistent

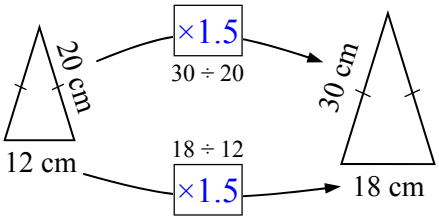
5



○ Similar ● Not similar because the scale factor ○ is uniform ● is inconsistent

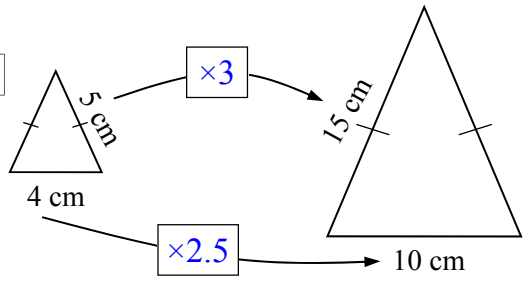
Like rectangles, triangles can be tested for similarity using the scale factor. Test these triangles for similarity.

6



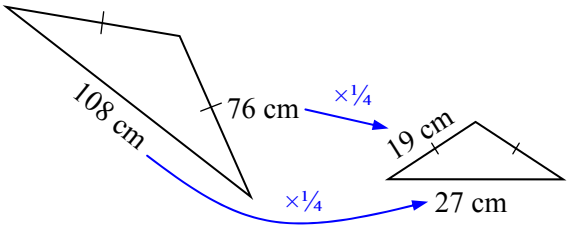
● Similar ○ Not similar because the scale factor ● is uniform ○ is inconsistent

7



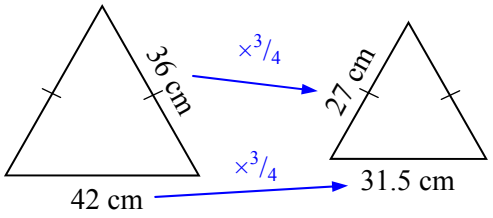
○ Similar ● Not similar because the scale factor ○ is uniform ● is inconsistent

8



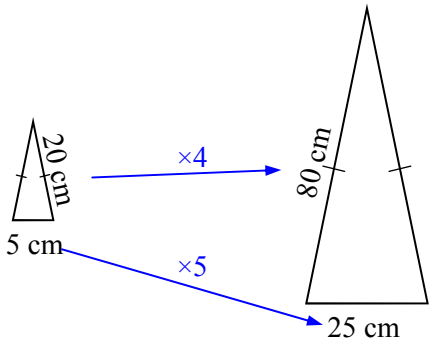
● Similar ○ Not similar because the scale factor ● is uniform ○ is inconsistent

9



● Similar ○ Not similar because the scale factor ● is uniform ○ is inconsistent

10



○ Similar ● Not similar because the scale factor ○ is uniform ● is inconsistent

Geometrical Properties - Similarity and Congruence (Homework Strip)

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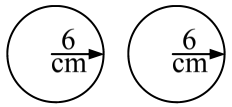
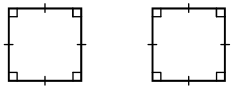
Date Due: _____

Are these shapes similar or congruent?

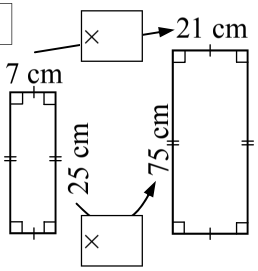
1 

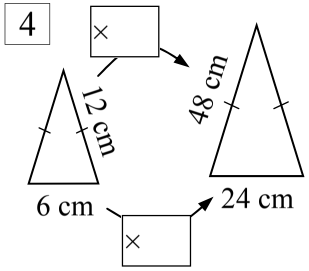
○ similar ○ congruent

2 

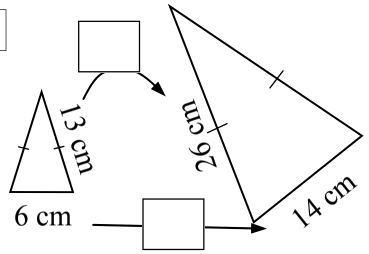
○ similar ○ congruent

Find the scale factor for these similar figures.

3 

4 

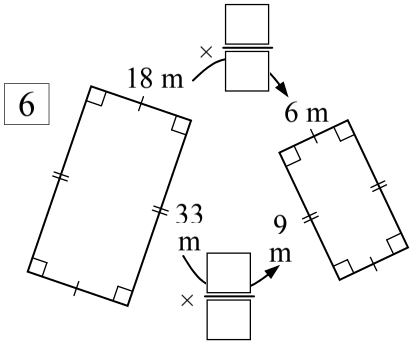
Are these similar figures?

5 

○ Similar
○ Not similar

because the scale factor

○ is uniform
○ is inconsistent

6 

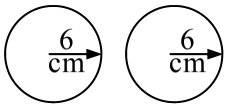
○ Similar
○ Not similar

because the scale factor

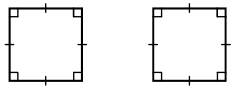
○ is uniform
○ is inconsistent

Date Due: _____

Are these shapes similar or congruent?

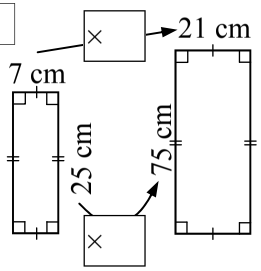
1 

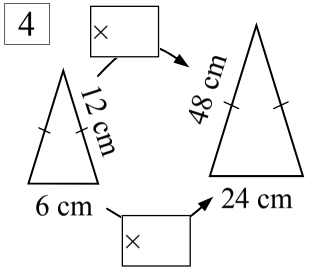
○ similar ○ congruent

2 

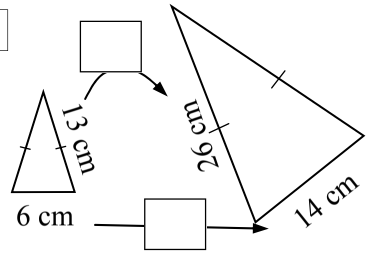
○ similar ○ congruent

Find the scale factor for these similar figures.

3 

4 

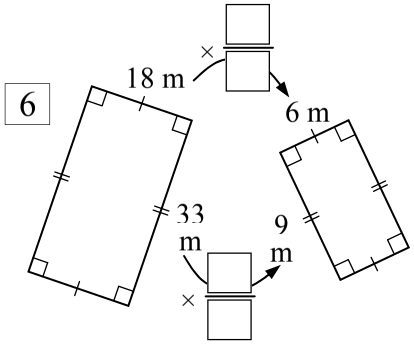
Are these similar figures?

5 

○ Similar
○ Not similar

because the scale factor

○ is uniform
○ is inconsistent

6 

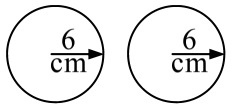
○ Similar
○ Not similar

because the scale factor

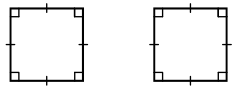
○ is uniform
○ is inconsistent

Date Due: _____

Are these shapes similar or congruent?

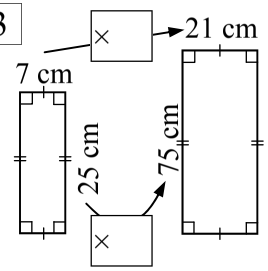
1 

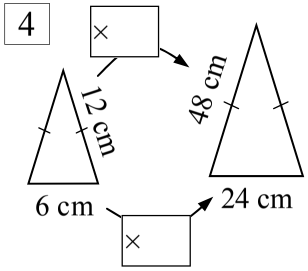
○ similar ○ congruent

2 

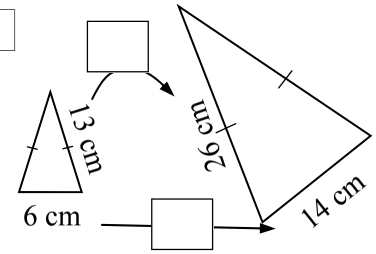
○ similar ○ congruent

Find the scale factor for these similar figures.

3 

4 

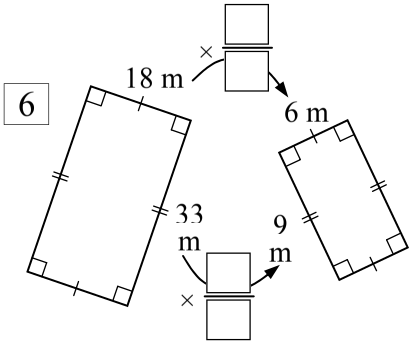
Are these similar figures?

5 

○ Similar
○ Not similar

because the scale factor

○ is uniform
○ is inconsistent

6 

○ Similar
○ Not similar

because the scale factor

○ is uniform
○ is inconsistent

Geometrical Properties - Similarity and Congruence (Homework Strip)

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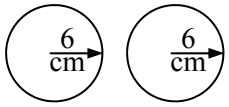
Date Due:

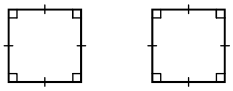
Are these shapes similar or congruent?

1



○ similar ● congruent

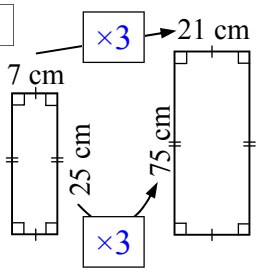
2



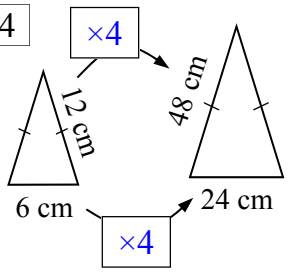
● similar ○ congruent

Find the scale factor for these similar figures.

3

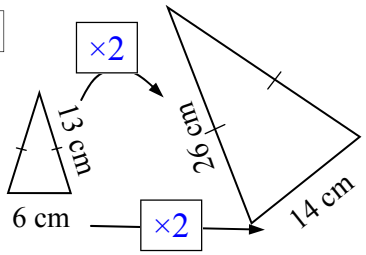


4



Are these similar figures?

5

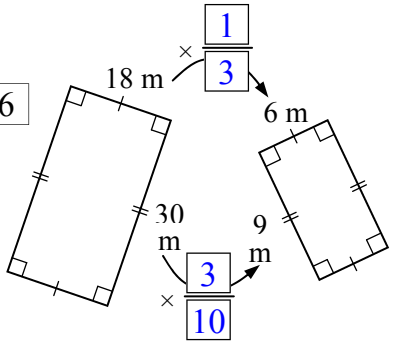


● Similar
○ Not similar

because the scale factor

● is uniform
○ is inconsistent

6



○ Similar
● Not similar

because the scale factor

○ is uniform
● is inconsistent

Geometrical Properties - Similar Figures & Similarity Statements

Similar figures are shapes that are the same shape only a scale factor has been applied to enlarge one of the shapes. Angles inside the shape are identical, but the side lengths are different, having been multiplied by a scale factor. Similarity is stated by using three vertical strokes, $|||$.

The first worksheet starts with pairs of similar shapes scattered around the column. Find the similar pairs, that is, identical shapes just one is larger than the other. When you have found them, write the answer in the boxes at the base of the column. Strike each shape out with a lead pencil that you can rub out later, because you can colour the matching pairs the same colour. There is one shape that doesn't have a match, write its letter/number in the box at the bottom of the column.

The next column asks you to write similarity statements. When you name a shape you start at one vertex (corner) of the shape and go around it either in a clockwise or anti-clockwise direction. With similarity (and later congruence) statements you must start at matching vertices and move around the shape always matching vertices. The first column gives you the order with either symbols or numbers below the boxes. Just write the letters in the boxes that match the angle/symbol.

Because angles match in similar figures, you can check that the angles are the same when identifying matching vertices. So check that the internal angles are the same to make sure you are listing the letters the correct way.

Geometrical Props - Similar Figures & Similarity Statements (Sheet 1)

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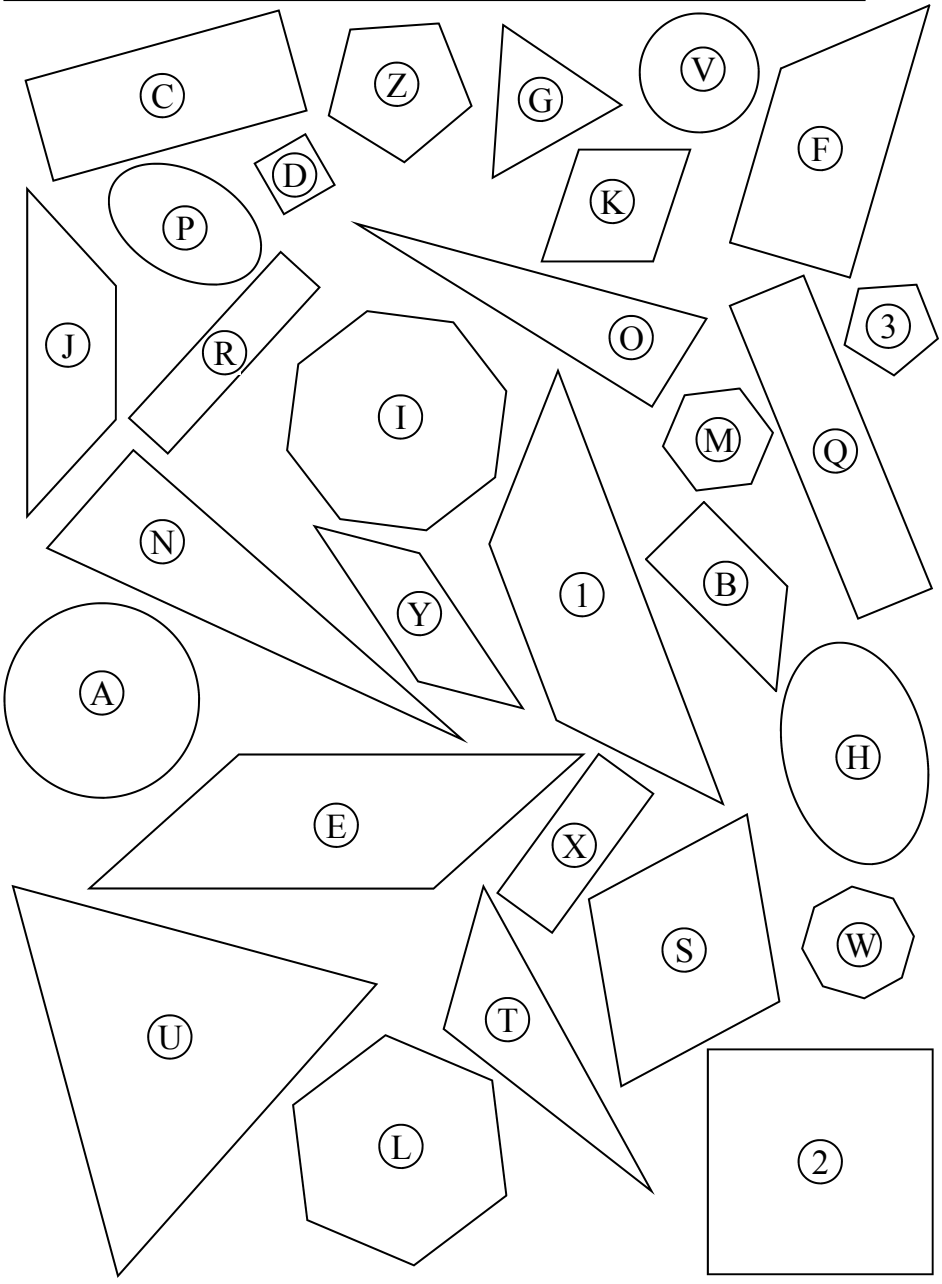
Similar figures have matching internal angles but have a scale factor applied to their side lengths. In other words, one is an enlargement of the other. Find all the similar pairs below, there is one non-paired shape.



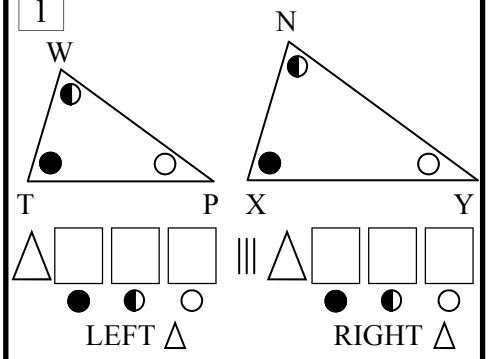
Write your matches below, once you are finished colour the similar shapes the same colour.

☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
	☐ & ☐	☐ & ☐	

There is one unmatched shape, it is: ☐

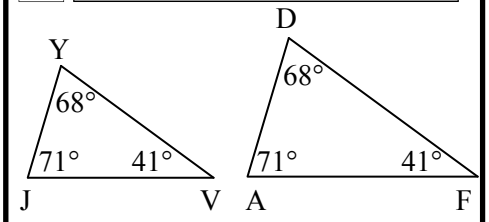
These triangles are similar. To state that, you have to start at matching vertices and go around the same direction.

1



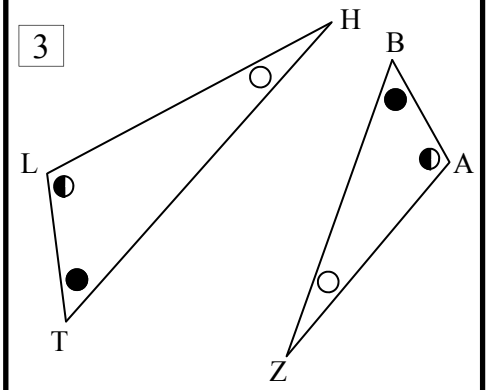
△ ☐ ☐ ☐ LEFT △ | ||| △ ☐ ☐ ☐ RIGHT △

2 Now use the matching angles



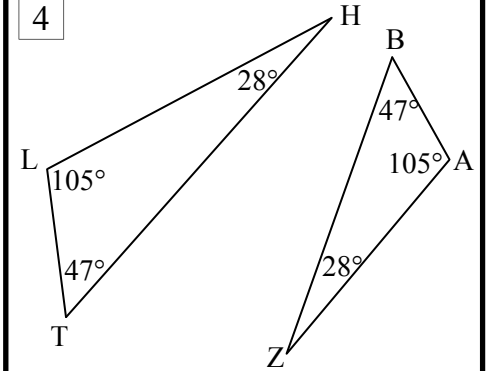
△ ☐ ☐ ☐ 71° 68° 41° LEFT △ | ||| △ ☐ ☐ ☐ 71° 68° 41° RIGHT △

3



△ ☐ ☐ ☐ | ||| △ ☐ ☐ ☐

4



△ ☐ ☐ ☐ 47° 105° 28° | ||| △ ☐ ☐ ☐ 47° 105° 28°

Geometrical Props - Similar Figures & Similarity Statements (Sheet 2)

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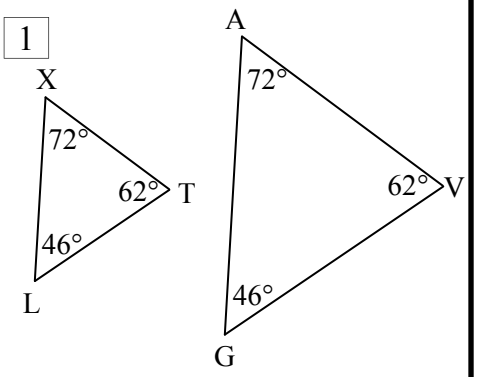
Write the similarity statements for these. Start at matching vertices and travel one way.

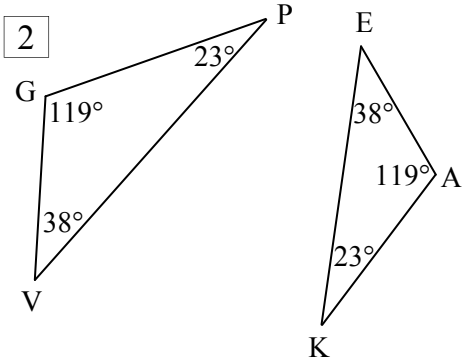
1



△ ☐☐☐ ||| △ ☐☐☐

46° 72° 62° 46° 72° 62°

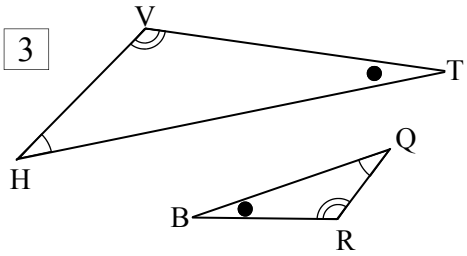
2



△ ☐☐☐ ||| △ ☐☐☐

38° ← Start at 38°, it is easier to mark

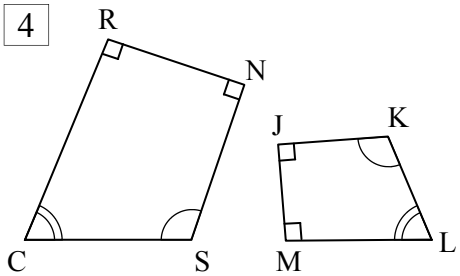
3



△ ☐☐☐ ||| △ ☐☐☐

Start at H

4

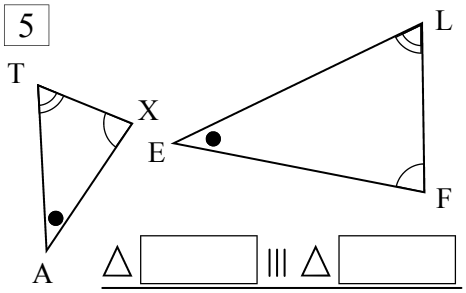


☐☐☐☐ ||| ☐☐☐☐

Start at C - Four letters required

Write similarity statements for these similar shapes.

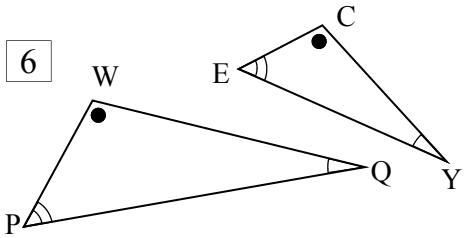
5



△ ☐☐☐ ||| △ ☐☐☐

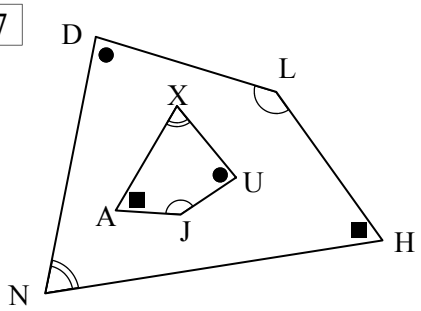
Start at A

6



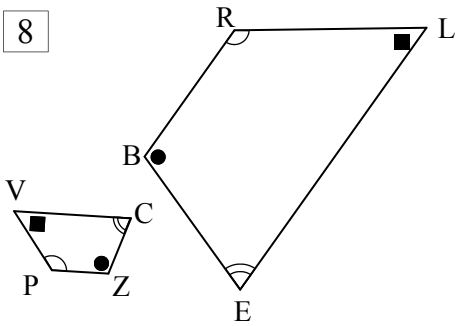
Start at P

7



Start at N

8



Start at P

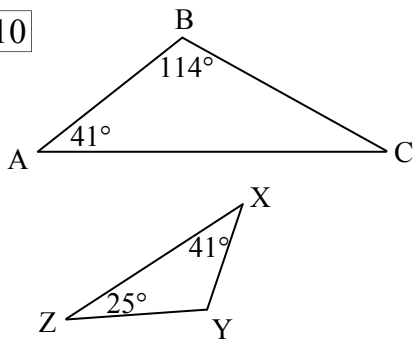
9 List the matching sides from Q8:

PV & ☐☐ & ☐☐

☐☐ & ☐☐ & ☐☐

Write similarity statements for these similar shapes. Then list matching sides and angles.

10



Start at A

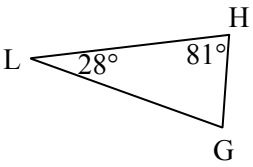
11 Angles: ∠A = ∠☐

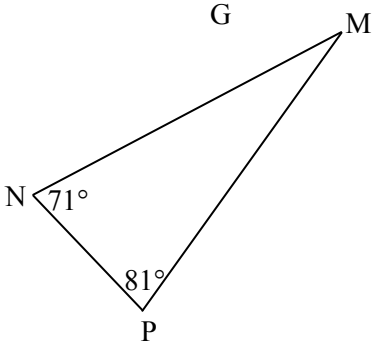
∠☐ = ∠☐ ∠☐ = ∠☐

12 Sides: AB & ☐☐

☐☐ & ☐☐ ☐☐ & ☐☐

13





Start at L

14 Angles: _____

_____

15 Sides: _____

_____

Geometrical Props - Similar Figures & Similarity Statements (Sheet 1)

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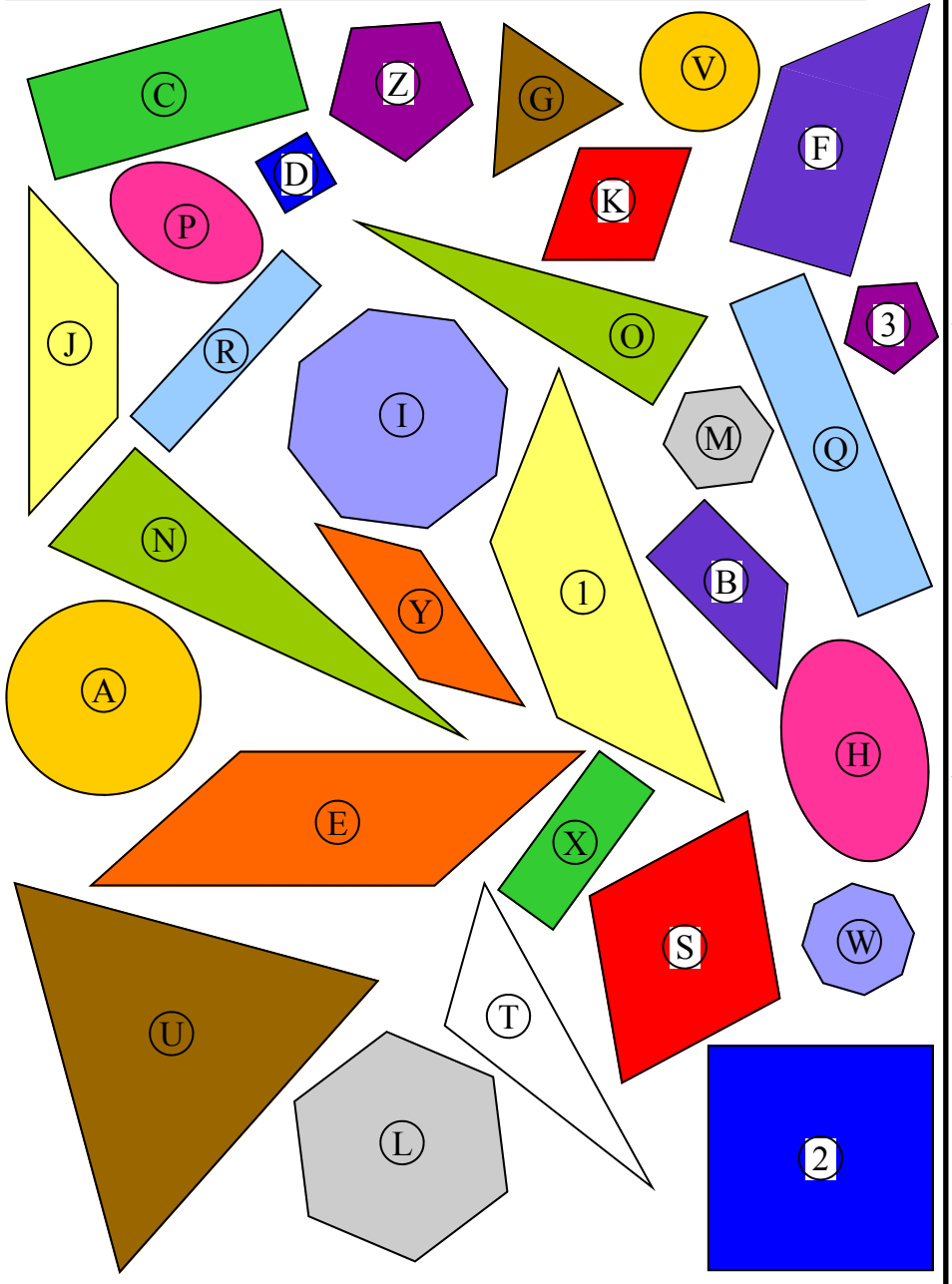
Similar figures have matching internal angles but have a scale factor applied to their side lengths. In other words, one is an enlargement of the other. Find all the similar pairs below, there is one non-paired shape.



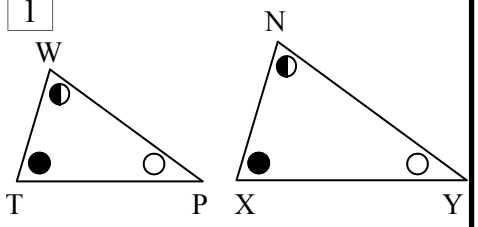
Write your matches below, once you are finished colour the similar shapes the same colour.

E & Y	K & S	J & 1	L & M
C & X	D & 2	A & V	O & N
Q & R	P & H	F & B	W & I
	U & G	Z & 3	

There is one unmatched shape, it is: T

These triangles are similar. To state that, you have to start at matching vertices and go around the same direction.

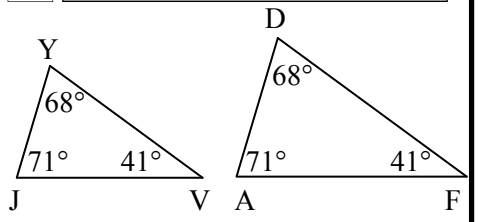
1



△ T W P ||| △ X N Y

LEFT △ RIGHT △

2 Now use the matching angles

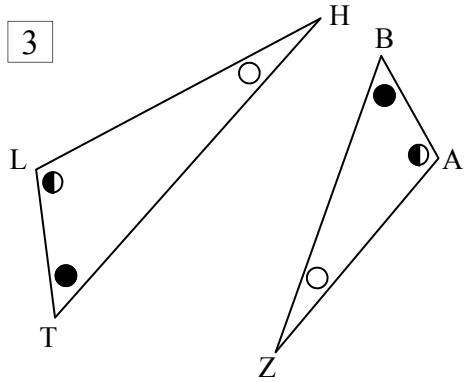


△ J Y V ||| △ A D F

71° 68° 41° | 71° 68° 41°

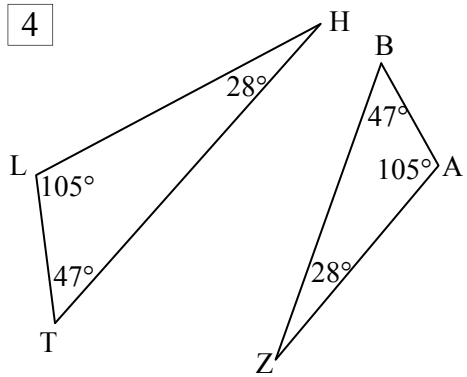
LEFT △ RIGHT △

3



△ T L H ||| △ B A Z

4



△ T L H ||| △ B A Z

47° 105° 28° | 47° 105° 28°

Geometrical Props - Similar Figures & Similarity Statements (Sheet 2)

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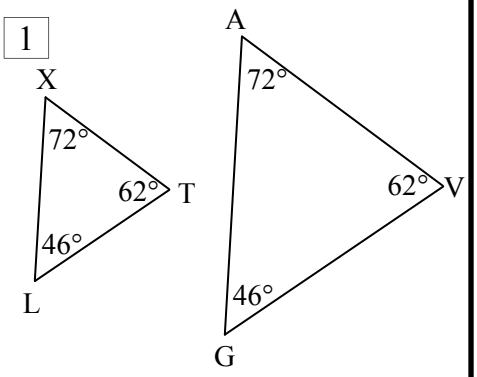
Write the similarity statements for these. Start at matching vertices and travel one way.

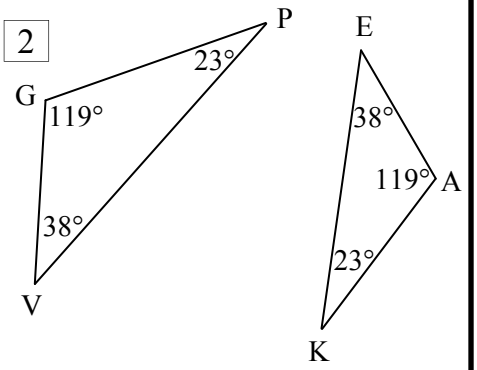
1



△ L X T ||| △ G A V

46° 72° 62° 46° 72° 62°

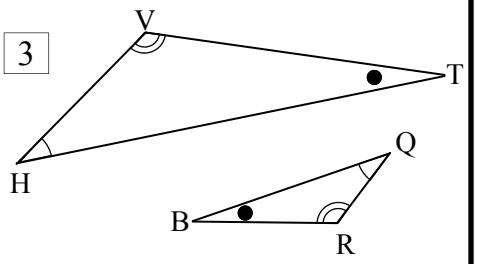
2



△ V G P ||| △ E A K

OR △ VPG ||| △ EKA

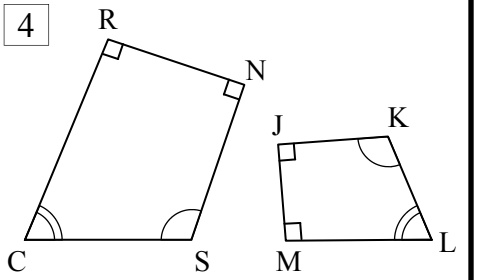
3



△ H V T ||| △ Q R B

OR △ HTV ||| △ QBR

4

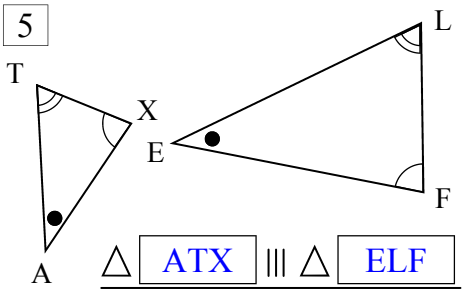


C R N S ||| L M J K

OR CSNR ||| LKJM

Write similarity statements for these similar shapes.

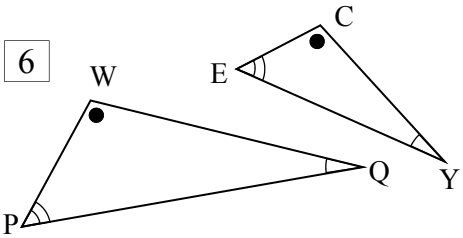
5



△ ATX ||| △ ELF

OR △ AXT ||| △ EFL

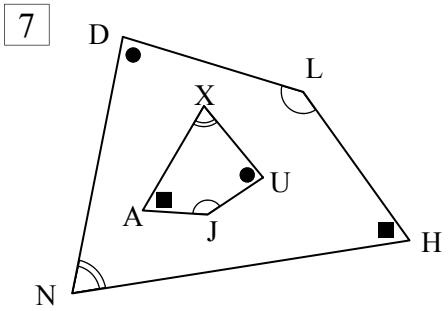
6



△ PWQ ||| △ ECY

OR △ PQW ||| △ EYC

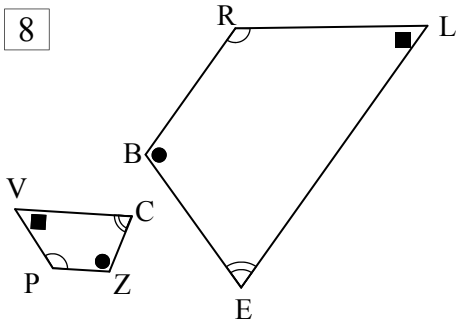
7



NDLH ||| XUJA

OR NHLD ||| XAJU

8



PVCZ ||| RLEB

OR PZCV ||| RBEL

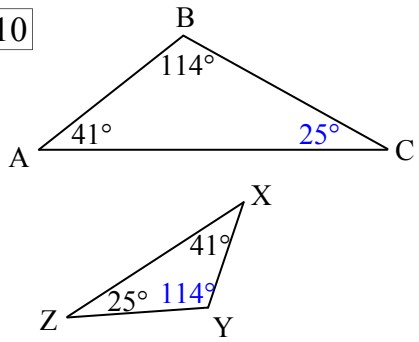
9 List the matching sides from Q8:

PV & RL VC & LE

CZ & EB ZP & BR

Write similarity statements for these similar shapes. Then list matching sides and angles.

10



△ ABC ||| △ XYZ

OR △ ACB ||| △ XZY

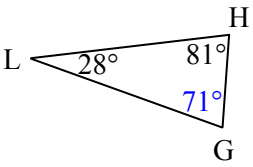
11 Angles: ∠A = ∠X

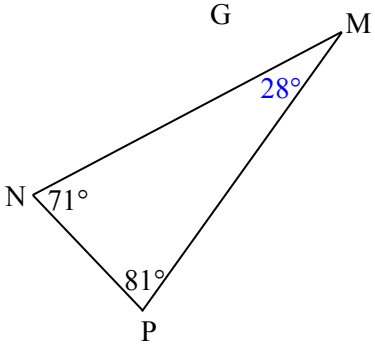
∠B = ∠Y ∠C = ∠Z

12 Sides: AB & XY

BC & YZ CA & ZX

13





△ LHG ||| △ MPN

OR △ LGH ||| △ MNP

14 Angles: ∠L = ∠M

∠H = ∠P ∠G = ∠N

15 Sides: LH & MP

HG & PN GL & NM

Geometrical Props - Similar Figures & Similarity Statements (Homework Strip)

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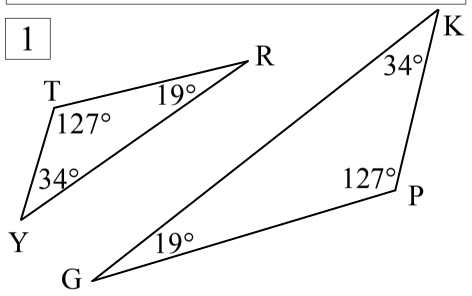
Date Due:

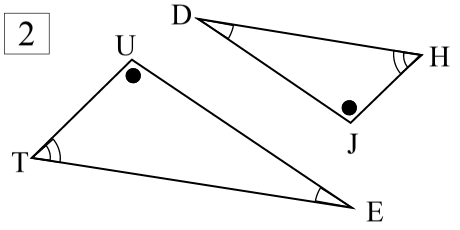
Write similarity statements for these shapes.

1



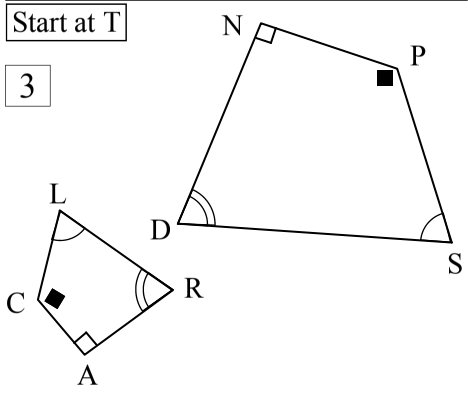
Y

2



Start at T

3

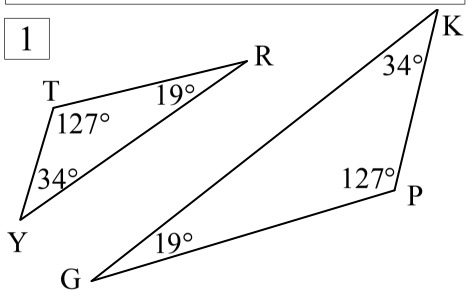


Start at A

Date Due:

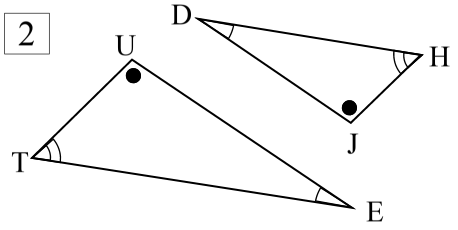
Write similarity statements for these shapes.

1



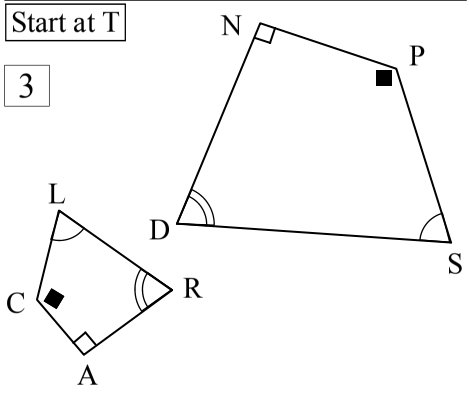
Y

2



Start at T

3

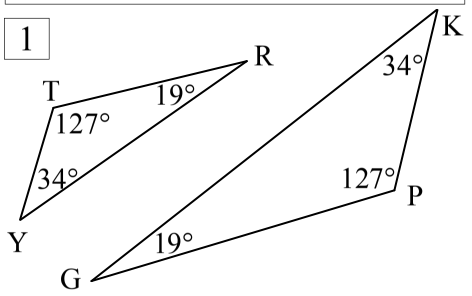


Start at A

Date Due:

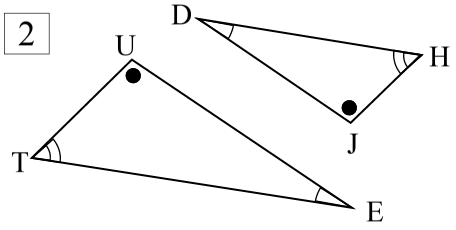
Write similarity statements for these shapes.

1



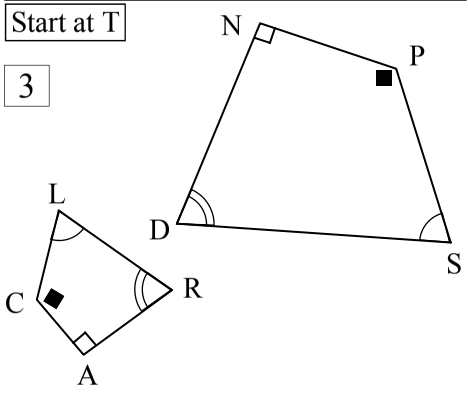
Y

2



Start at T

3

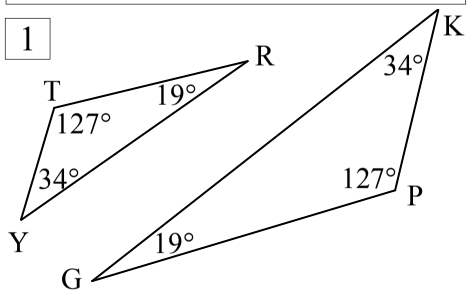


Start at A

Date Due:

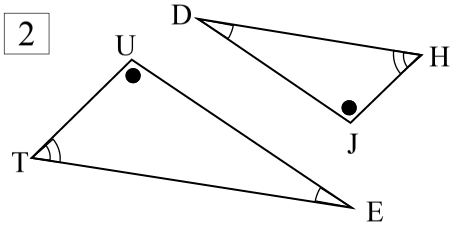
Write similarity statements for these shapes.

1



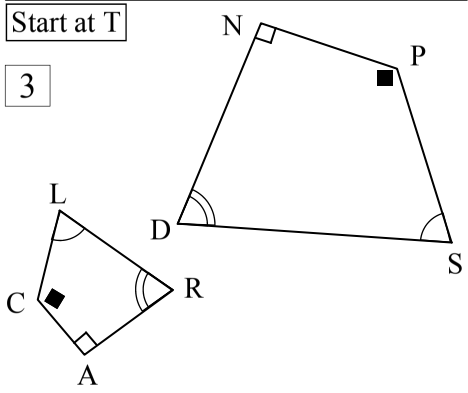
Y

2



Start at T

3



Start at A

Geometrical Props - Similar Figures & Similarity Statements (HW Strip)

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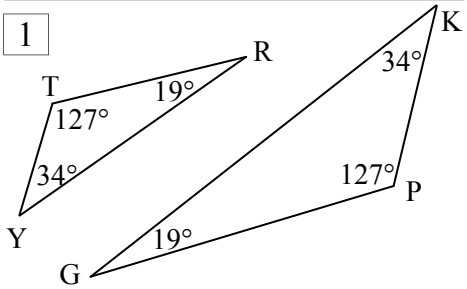
Date Due:

Write similarity statements for these shapes.

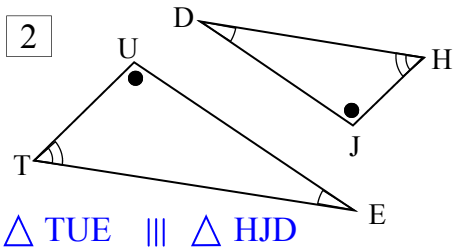
1



△ Y T R ||| △ K P G

Y OR △ YRT ||| △ KGP

2

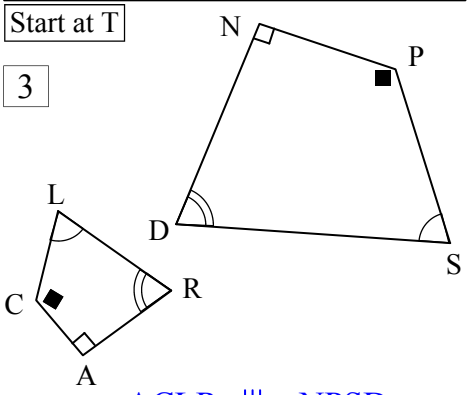


△ TUE ||| △ HJD

OR △ TEU ||| △ HDJ

Start at T

3



ACLR ||| NPSD

Start at A OR ARLC ||| NDSP

Geometrical Properties - Congruent Shapes & Congruence Statements

This sheet compares shapes and asks you if they are congruent. When shapes are congruent they are identical, same internal angles and same side lengths. Note that a congruent shape can be formed by reflection.

Column 1 starts with three questions and asks you to use a ruler and write down all the side lengths. Once you do this if all the sides match, and are in the same position, then the shapes are congruent. Fill in the yes circle if they are.

Column 2 has a number of shapes labeled, A to Z and 1 - 9. So there are 35 shapes. There are 17 pairs of matching shapes and one ‘non-matching shape’. Some shapes are easy as there is only one possible match, but others aren’t so straight forward. With slight variations and rotation as well it is difficult to tell. You have three options:

- Reach for your ruler.
- Use a piece of tracing paper to trace the more difficult shapes, then move the trace over the possible matching shapes.
- Or your teacher may give you a transparency which has all the shapes on it. Use the transparency to check for congruence.

Once you have found your pairs write them in the boxes below the shapes. After you have found all the matches there will be one shape left, write that at the base of the page. Your last step is to colour the shapes that match the same colour.

Geometrical Properties - Matching Congruent Shapes (Sheet 1)

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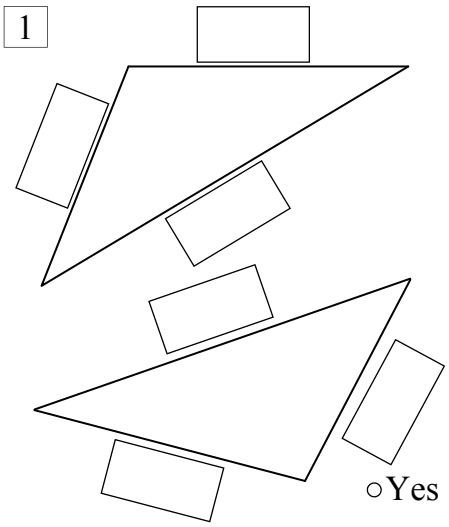
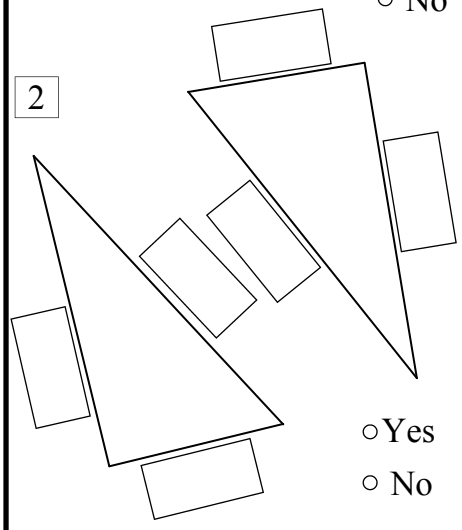
Using a ruler, measure and then write the side lengths on the triangles. Are they congruent?

1



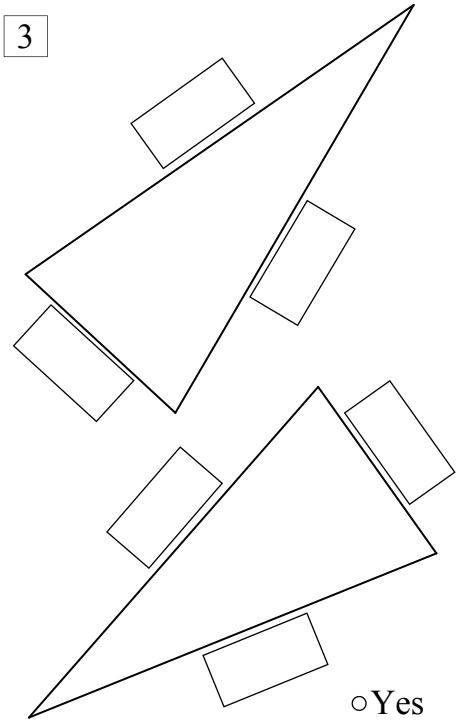
○ Yes
○ No

2



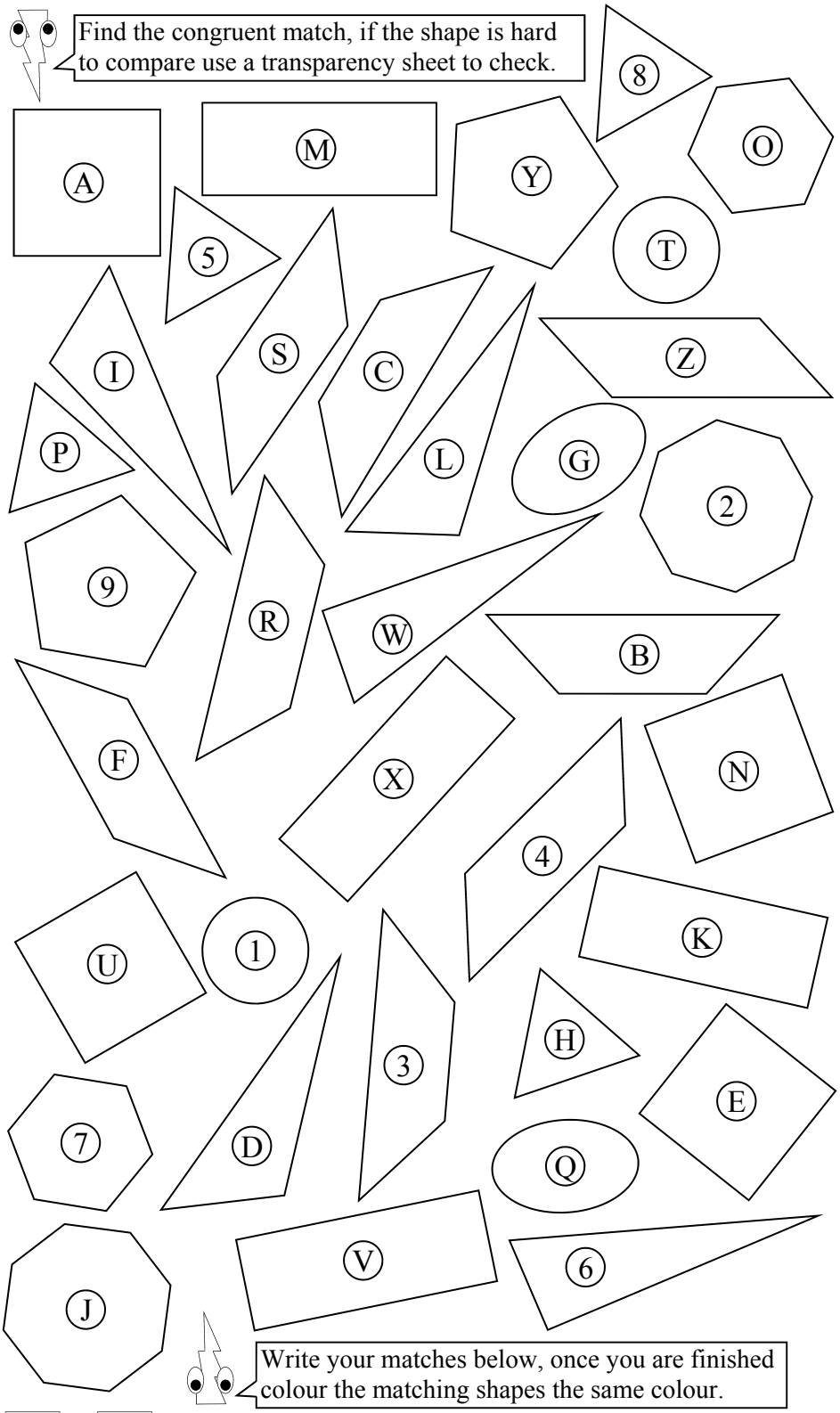
○ Yes
○ No

3



○ Yes
○ No

Find the congruent match, if the shape is hard to compare use a transparency sheet to check.



Write your matches below, once you are finished colour the matching shapes the same colour.

☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐			

There is one unmatched shape, it is: ☐

Geometrical Properties - Congruence Statements (Sheet 2)

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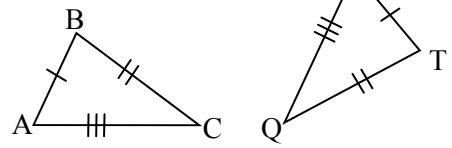
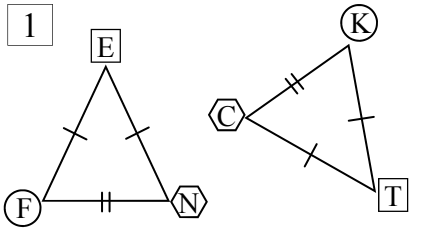
A congruence statement has the matching vertices named in the same order. Write a congruence statement for these.

Example



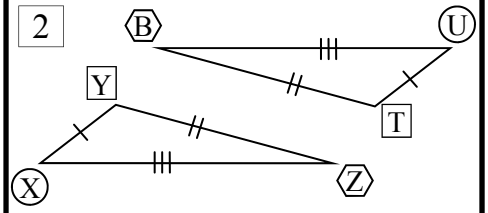
$\triangle ABC \equiv \triangle MTQ$

1



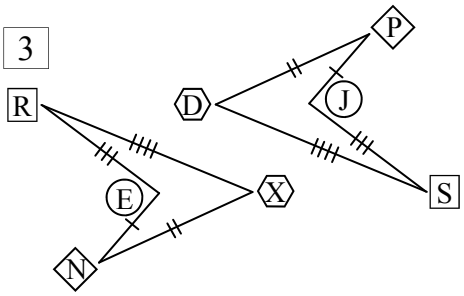
$\triangle\bigcirc\square\hexagon \equiv \triangle\bigcirc\square\hexagon$

2



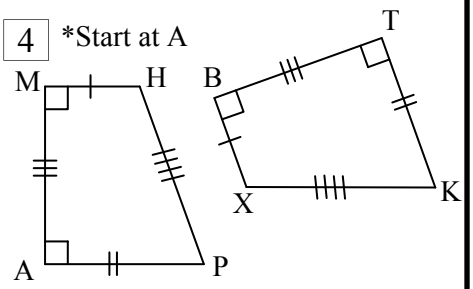
$\triangle\bigcirc\square\hexagon \equiv \triangle\bigcirc\square\hexagon$

3

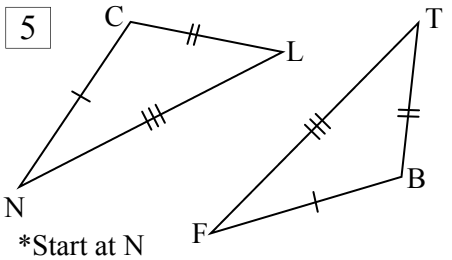


$\bigcirc\square\hexagon\diamond \equiv \bigcirc\square\hexagon\diamond$

4 *Start at A



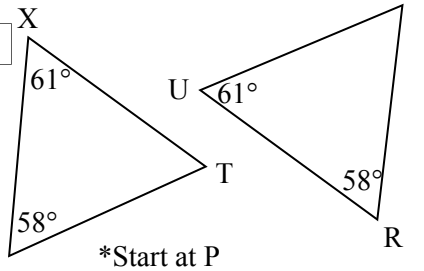
5



*Start at N

State which angles and sides are equal & complete congruence statements for these congruent shapes.

6



*Start at P

$\angle P = \angle R$ $\angle X = \angle\square$

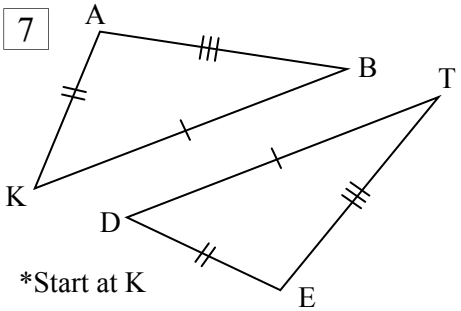
$\angle T = \angle\square$ $XP = \square$

$PT = \square$ $XT = \square$

$\triangle\square \equiv \triangle\square$

Remember same starting point & order.

7



*Start at K

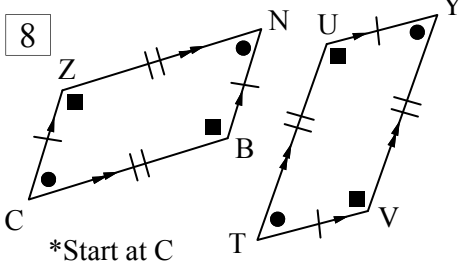
$\angle K = \angle\square$ $\angle\square = \angle\square$

$\angle\square = \angle\square$ $AK = \square$

$\square = \square$ $\square = \square$

$\triangle\square \equiv \triangle\square$

8



*Start at C

$\angle\square = \angle\square = \angle\square = \angle\square$

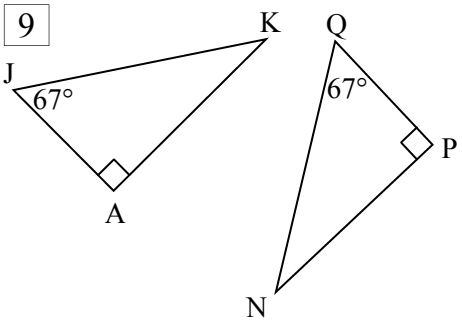
$\angle\square = \angle\square = \angle\square = \angle\square$

$\square = \square = \square = \square$

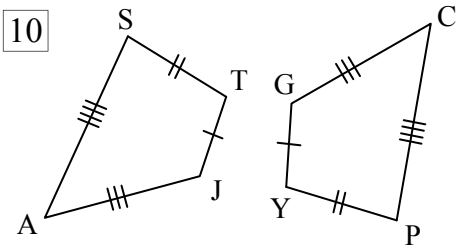
$\square = \square = \square = \square$

$\square \equiv \square$

9

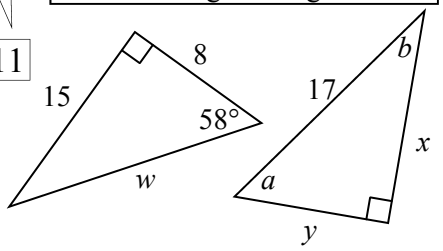


10



Find the value of all unknowns, for these congruent figures.

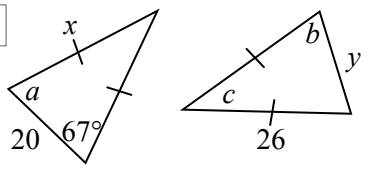
11



$a =$ $b =$ $w =$

$x =$ $y =$

12



$a =$ $b =$ $c =$

$x =$ $y =$

Geometrical Properties - Matching Congruent Shapes (Sheet 1)

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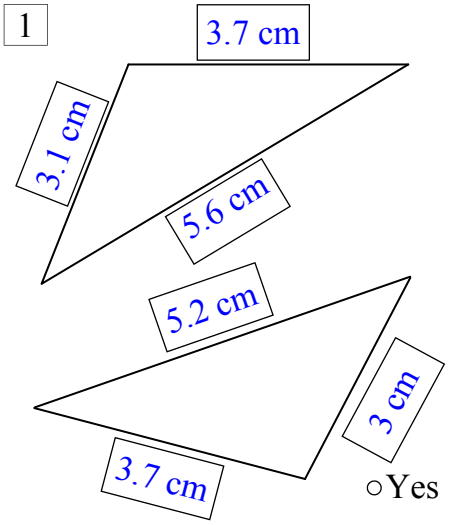
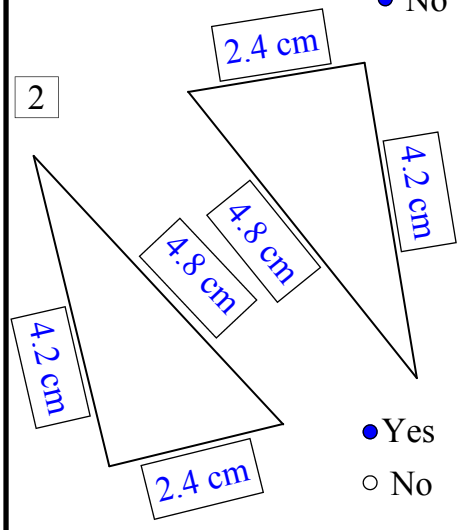
Using a ruler, measure and then write the side lengths on the triangles. Are they congruent?

1



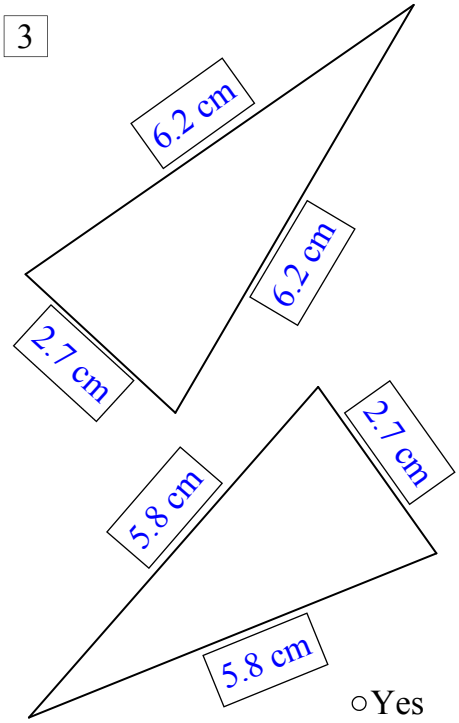
○ Yes
● No

2



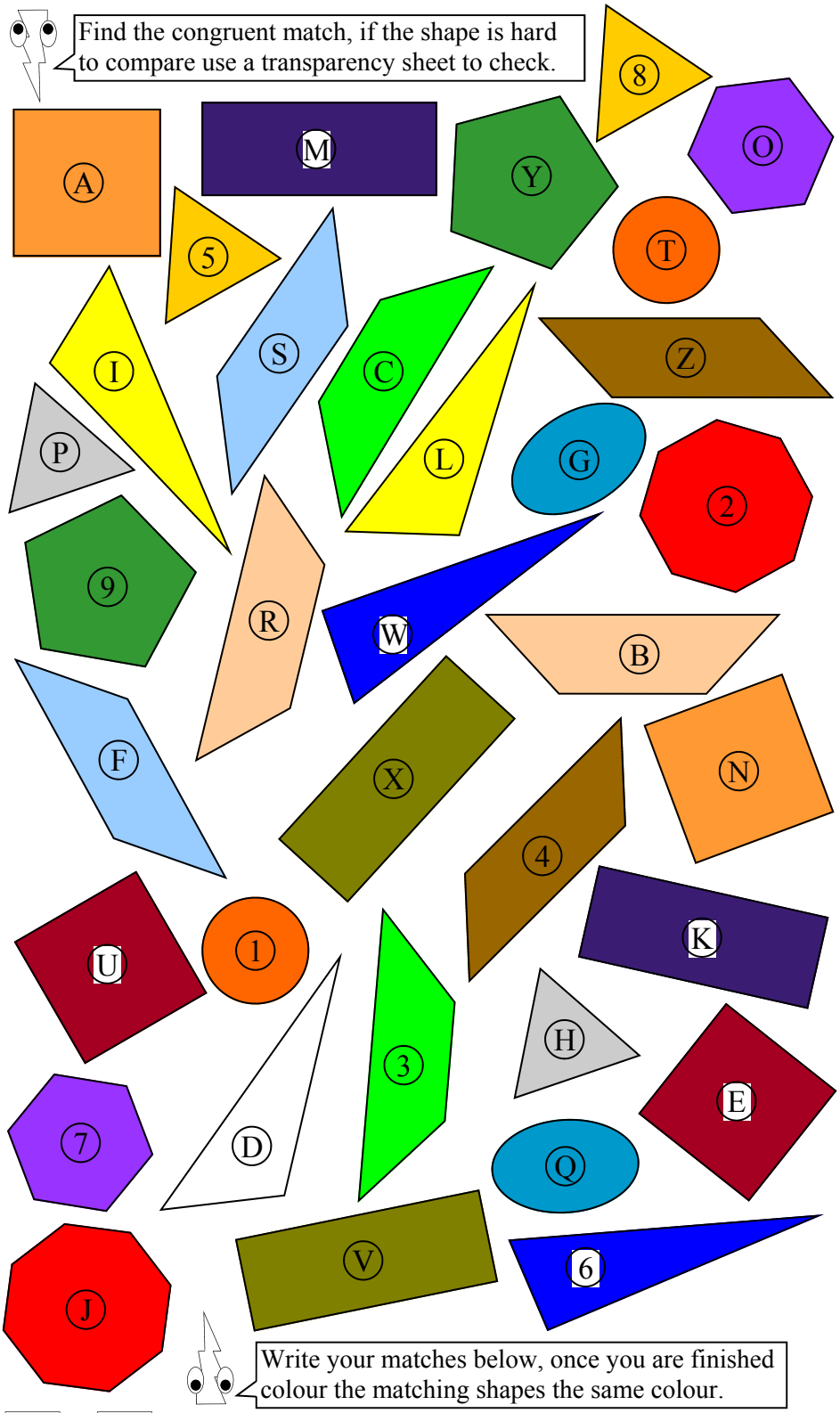
● Yes
○ No

3



○ Yes
● No

Find the congruent match, if the shape is hard to compare use a transparency sheet to check.



Write your matches below, once you are finished colour the matching shapes the same colour.

T & 1	H & P	E & U	M & K
G & Q	A & N	C & 3	W & 6
J & 2	R & B	X & V	5 & 8
W & Y	F & S	I & L	Z & 4
7 & O			

There is one unmatched shape, it is: D

Geometrical Properties - Congruence Statements (Sheet 2)

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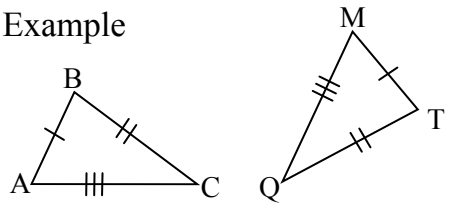
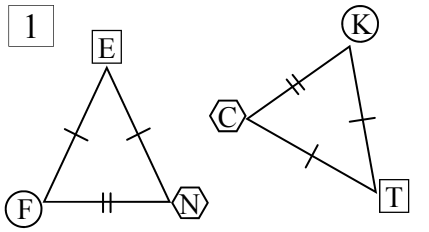
A congruence statement has the matching vertices named in the same order. Write a congruence statement for these.

Example



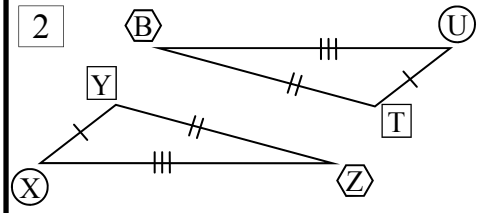
$\triangle ABC \equiv \triangle MTQ$

1



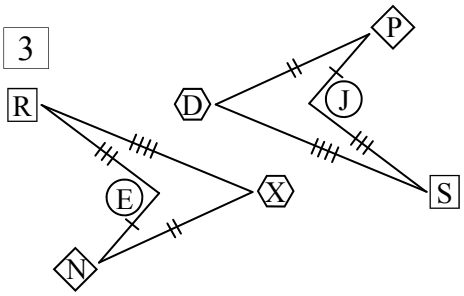
$\triangle FEN \equiv \triangle KTC$

2



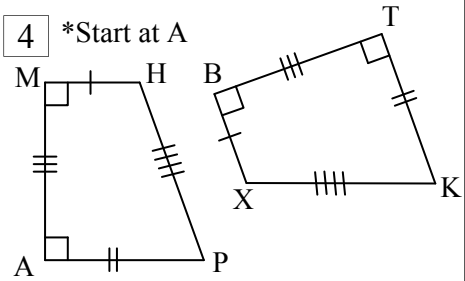
$\triangle XYZ \equiv \triangle UTB$

3



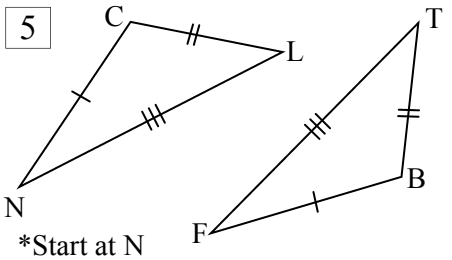
$ERXN \equiv JSDP$

4 *Start at A



$AMHP \equiv TBXK$

5

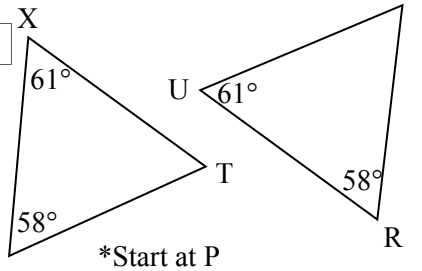


*Start at N

$\triangle CLN \equiv \triangle BTF$

State which angles and sides are equal & complete congruence statements for these congruent shapes.

6



*Start at P

$\angle P = \angle R$ $\angle X = \angle U$

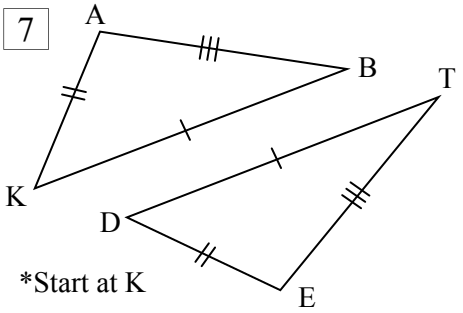
$\angle T = \angle E$ $XP = UR$

$PT = RE$ $XT = UE$

$\triangle PXT \equiv \triangle RUE$

Remember same starting point & order.

7



*Start at K

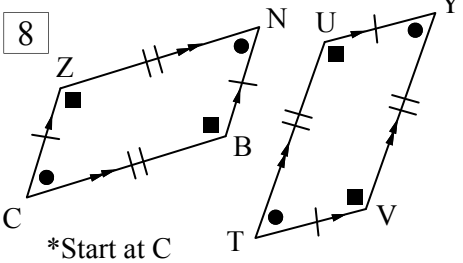
$\angle K = \angle D$ $\angle A = \angle E$

$\angle B = \angle T$ $AK = ED$

$AB = ET$ $TD = BK$

$\triangle ABK \equiv \triangle ETD$

8



*Start at C

$\angle C = \angle N = \angle T = \angle Y$

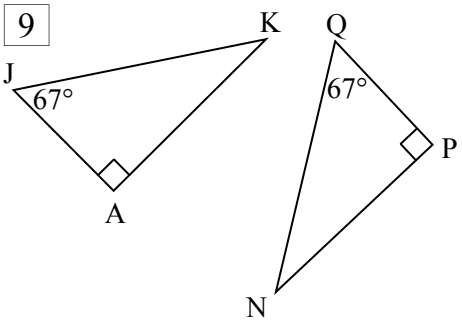
$\angle Z = \angle B = \angle U = \angle V$

$ZC = NB = YU = VT$

$ZN = CB = YV = UT$

$CZNB \equiv TUYV$

9



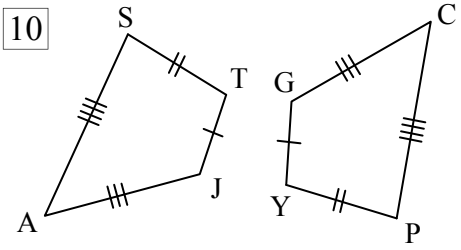
$\angle A = \angle P$ $\angle N = \angle K$

$\angle J = \angle Q$ $JA = QP$

$AK = QP$ $QN = JK$

$\triangle AJK \equiv \triangle PQN$

10



$\angle A = \angle C$ $\angle J = \angle G$

$\angle T = \angle Y$ $\angle S = \angle P$

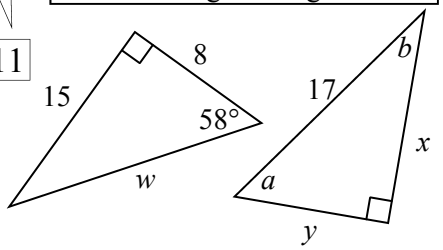
$AS = CP$ $ST = PY$

$TJ = YG$ $JA = GC$

$ASTJ \equiv CPYG$

Find the value of all unknowns, for these congruent figures.

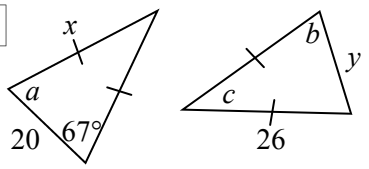
11



$a = 58°$ $b = 32°$ $w = 17$

$x = 15$ $y = 8$

12



$a = 67°$ $b = 67°$ $c = 46°$

$x = 26$ $y = 20$

Geom Props - Matching Congruent Shapes - Transparency Printout Sheet

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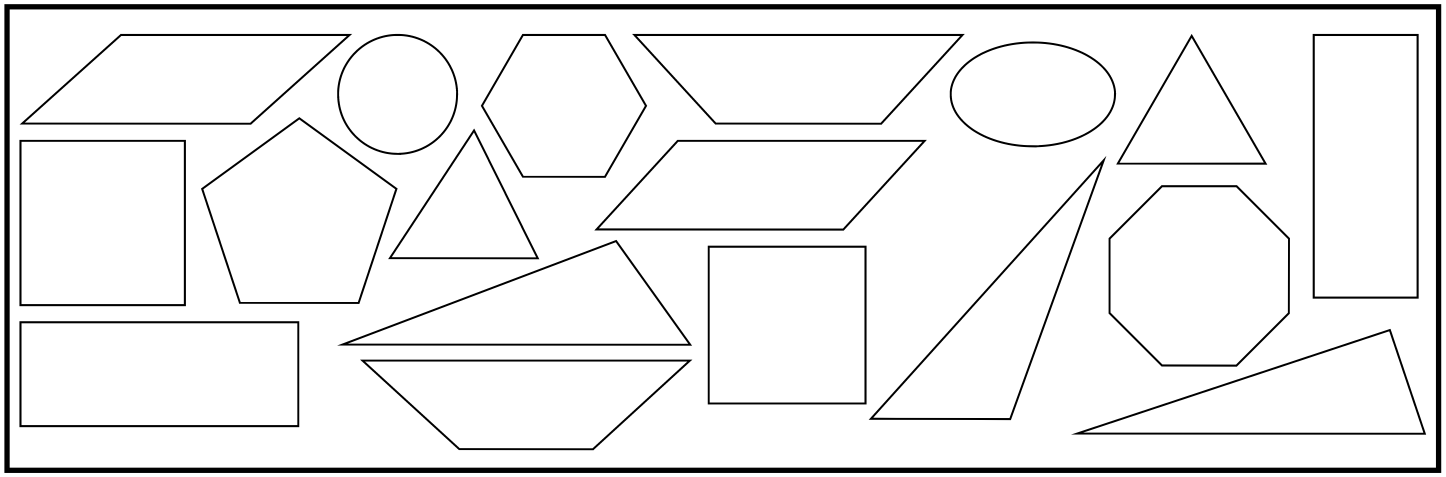

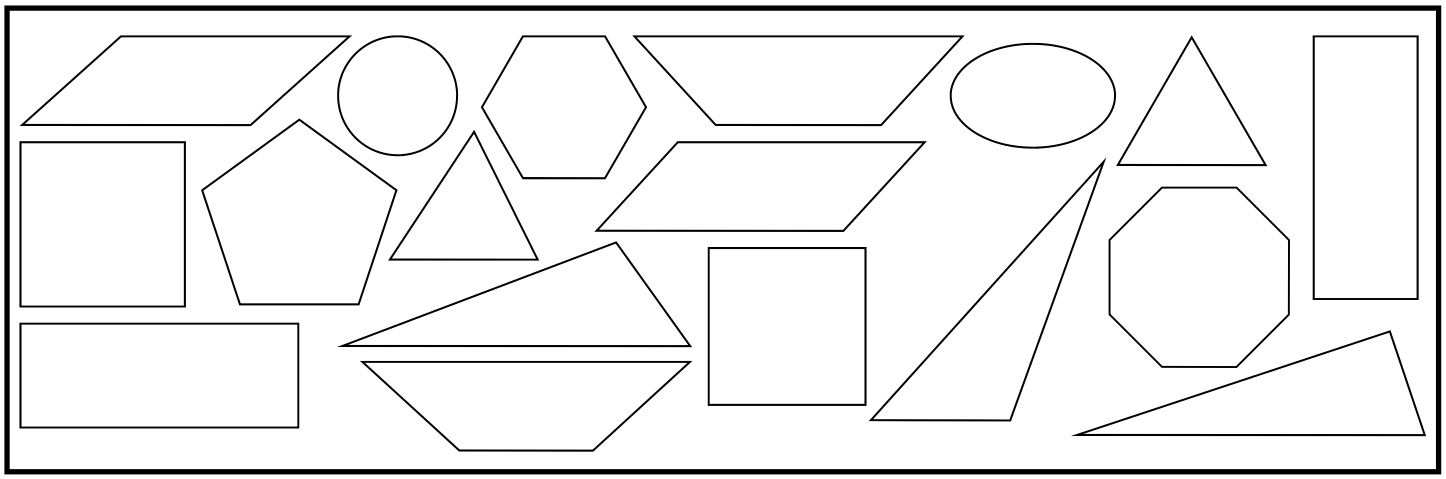

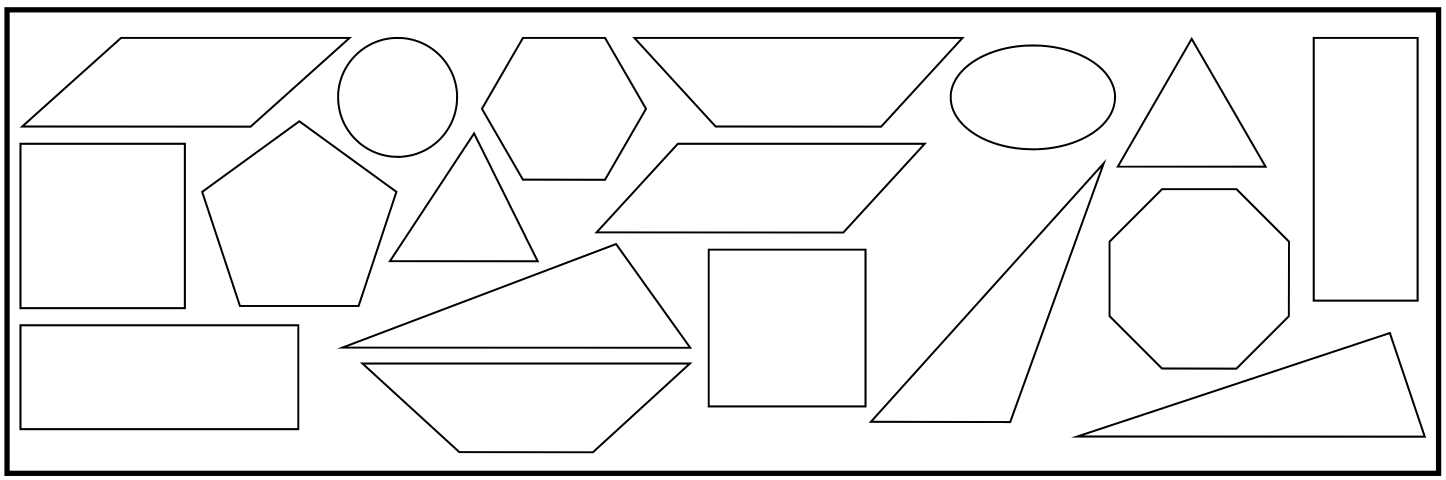

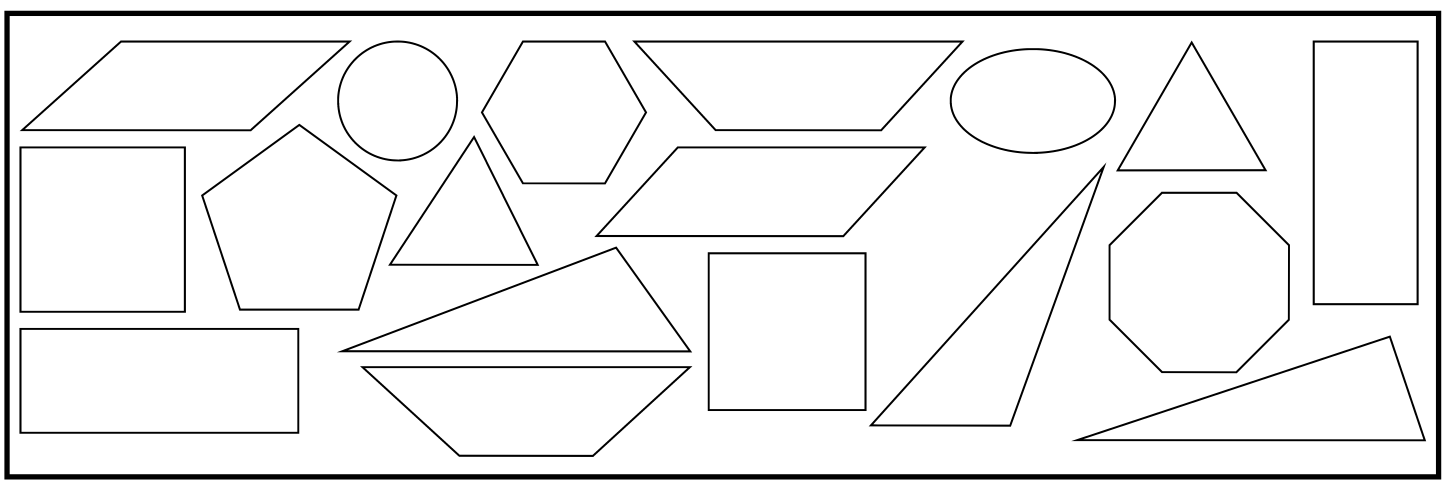

Geometrical Properties - Congruence Statements (Homework Strip)

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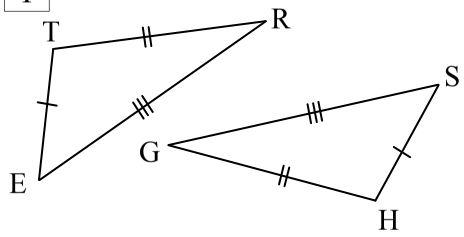
Date Due: _____

Write congruence statements for these congruent shapes.

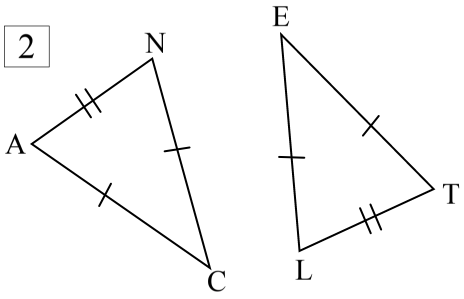
1



△ ☐ ☐ ☐ ≡ △ ☐ ☐ ☐

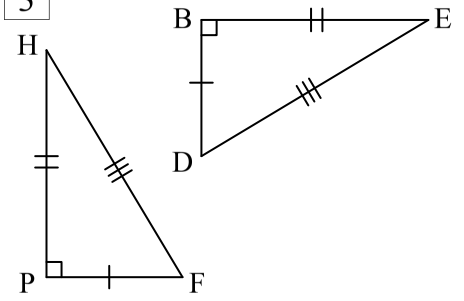
E

2



Start at A _____

3

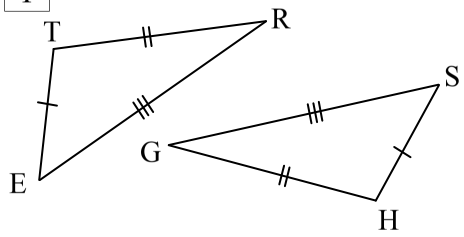


Start at P _____

Date Due: _____

Write congruence statements for these congruent shapes.

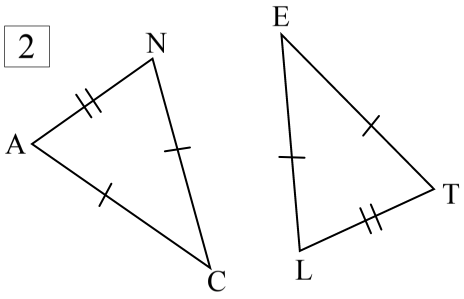
1



△ ☐ ☐ ☐ ≡ △ ☐ ☐ ☐

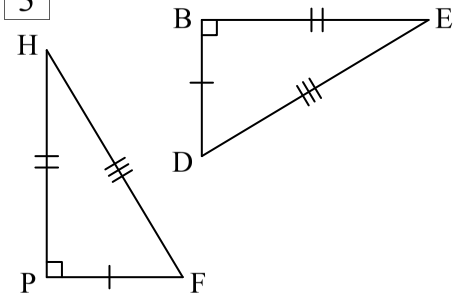
E

2



Start at A _____

3

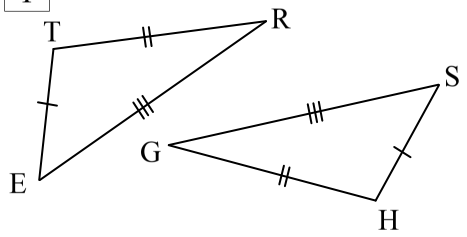


Start at P _____

Date Due: _____

Write congruence statements for these congruent shapes.

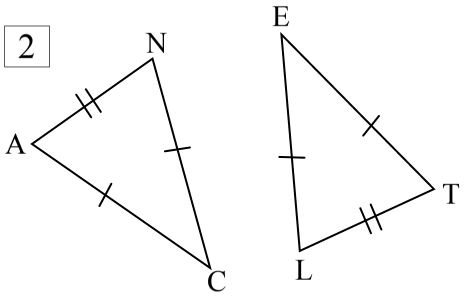
1



△ ☐ ☐ ☐ ≡ △ ☐ ☐ ☐

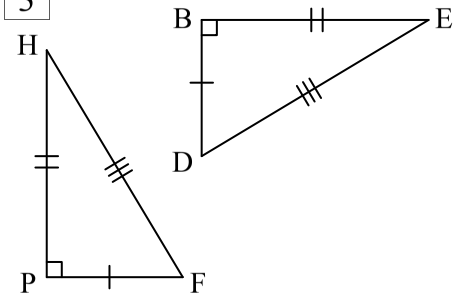
E

2



Start at A _____

3

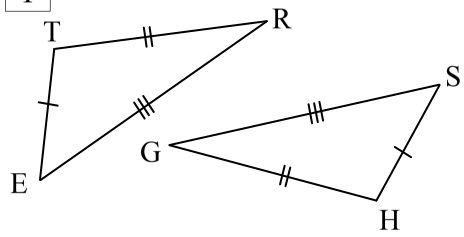


Start at P _____

Date Due: _____

Write congruence statements for these congruent shapes.

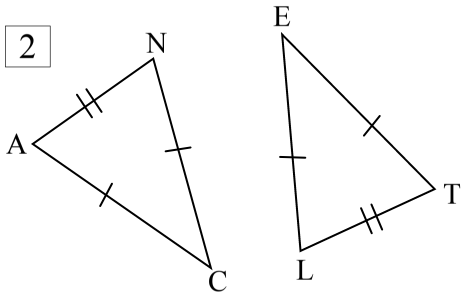
1



△ ☐ ☐ ☐ ≡ △ ☐ ☐ ☐

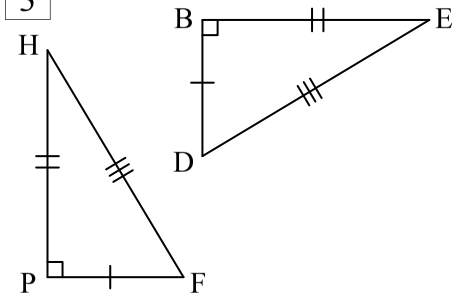
E

2



Start at A _____

3



Start at P _____

Geometrical Properties - Congruence Statements (Homework Strip)

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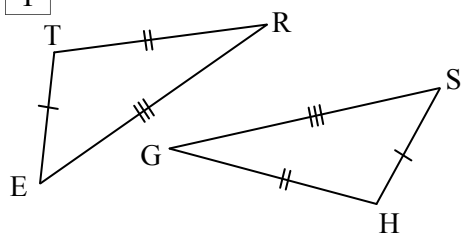
Date Due:

Write congruence statements for these congruent shapes.

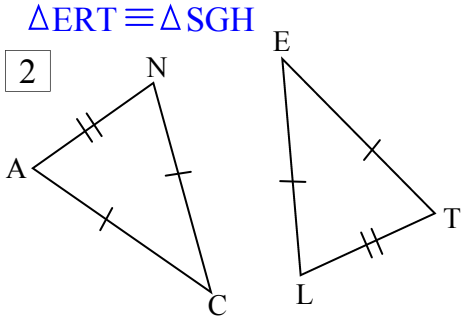
1



△ E T R ≡ △ S H G

△ERT ≡ △SGH

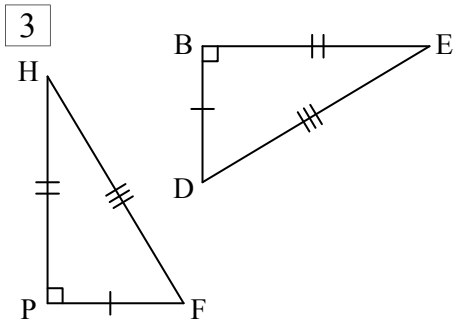
2



△ANC ≡ △TLE

Start at A △ACN ≡ △TEL

3



△PHF ≡ △BED

Start at P △PFH ≡ △BDE

Geometrical Properties - Similarity - Finding Sides

If shapes are similar then the sides of one shape can be found by multiplying (×) the sides of the other by a number, called the scale factor.

So given a matching side (in location on the shape) you can work out the scale factor. Then apply this to work out other sides. These sheets focus on triangles. Similar triangles have an important place in history, because they were used to estimate heights of objects that you couldn't physically measure, but due to similarity they could be. Similarity has been used to help construct some of the world's most amazing structures from ancient times.

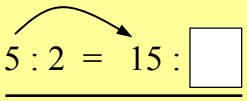
Column 1 starts with equivalent ratios. Sides are found using equivalent ratios, so this is a refresher on calculating them. Use multiplication to find the value of the box.

This method is then applied to triangles. Using ratios and the scale factor to find missing sides. If you can handle the equivalent ratios in the first column, you can handle similarity.

Example - Complete to find the equivalent ratio $3 : 7 = 15 : \square$

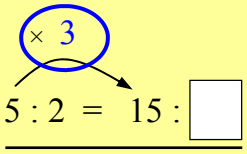
STEP 1

Match the ratios, so Position 1 in the first ratio matches Position 1 in the 2nd ratio.



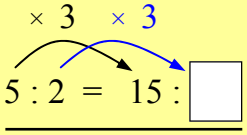
STEP 2

Ask your self "What do I multiply 5 by to get 15?" The answer is 3. Write '×3' above the arrow.



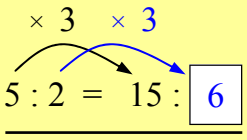
STEP 3

Draw another arrow matching the numbers in Position 2, and write the same '×3'.



STEP 4

Ask yourself "What is 2 times 3 (2 × 3)?" It is 6. Guess what …. you are done!



Geometrical Properties - Similarity - Finding Sides (Sheet 1)

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Let's start with a quick review of equivalent ratios. Find the value of the box in the ratios.

1 $5 : 2 = 30 : \square$ (×6 ×6)

2 $4 : 3 = 16 : \square$ (×4 ×4)

3 $2 : 3 = 22 : \square$ (×11 ×11)

Now find the scale factor (multiplier) yourself.

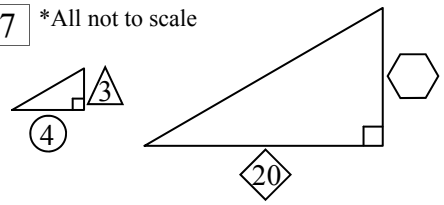
4 $5 : 2 = 15 : \square$ (× □ × □)

5 $1 : 6 = 7 : \square$ (× □ × □)

6 $5 : 12 = 10 : \square$ (× □ × □)

Build the ratios from these similar triangles*

7 *All not to scale



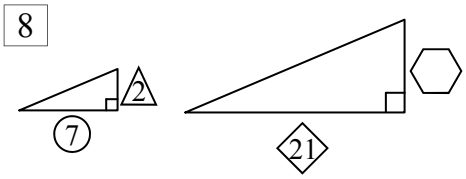
× □ × □

□ : △ = ◇ : ⬡

Base Height Base Height

Left △ Right △

8



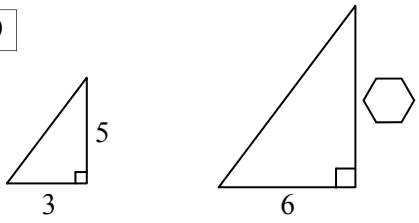
× □ × □

□ : △ = ◇ : ⬡

Base Height Base Height

Left △ Right △

9



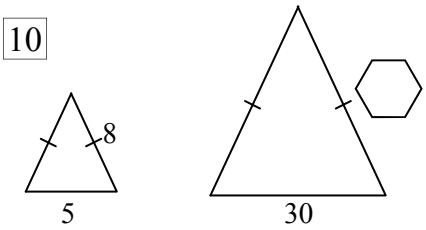
× □ × □

□ : □ = □ : ⬡

Base Height Base Height

Left △ Right △

10

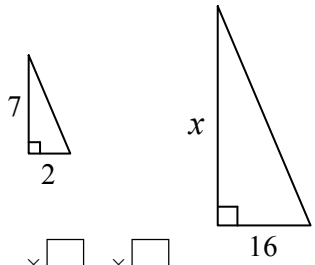


× □ × □

□ : □ = □ : ⬡

From now on the unknown side is 'x'

11

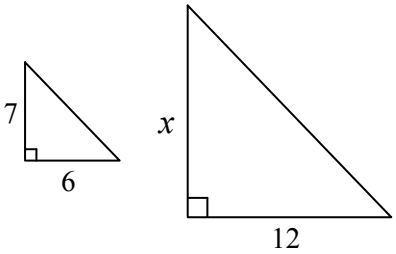


× □ × □

□ : □ = □ : x

$x = \square$

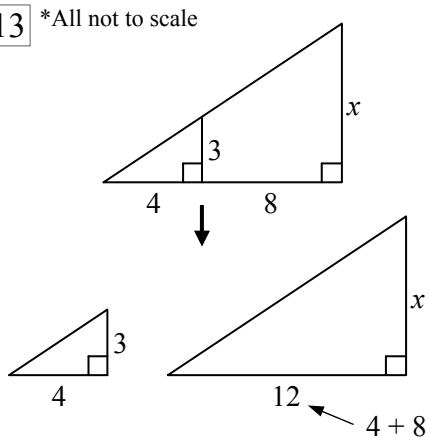
12



$x = \square$

Deconstruct these and solve like before. Change the diagram to two triangles. All are similar.

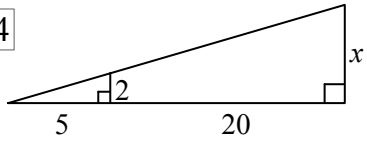
13 *All not to scale



□ : □ = □ : x

$x = \square$

14

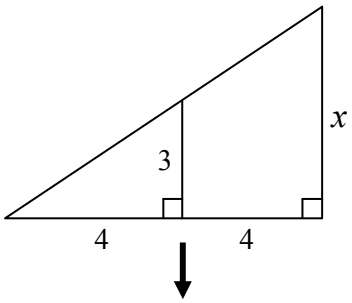


Redraw 2 triangles

□ : □ = □ : x

$x = \square$

15



Geometrical Properties - Similarity - Finding Sides (Sheet 2)

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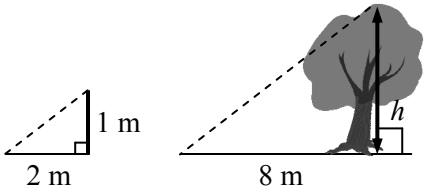
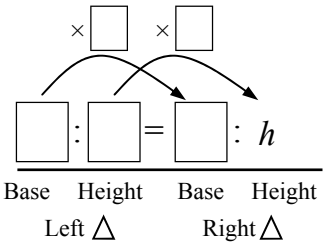
Find the height of these objects, by using similarity to calculate the unknown height.

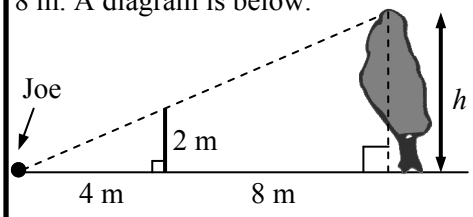
1 A 1 m high post casts a 2 m long shadow. How tall is a tree that casts an 8 m long shadow?



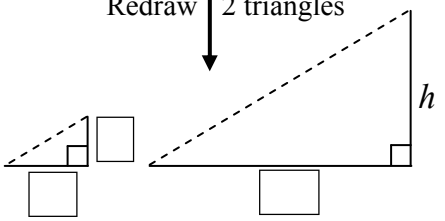


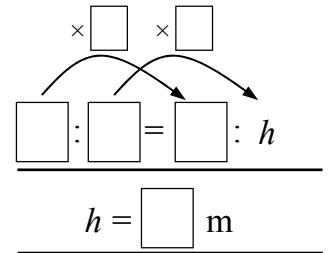
$h =$ ☐ m

2 Joe looks at the top of a tree and moves until a 2 m post's top is in line with the top of the tree. The post is 4 m away. He then measures the distance from the post to the tree, it is 8 m. A diagram is below.



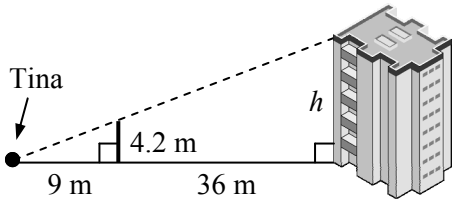
Redraw | 2 triangles





$h =$ ☐ m

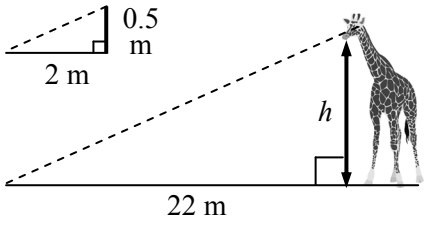
3 Tina looks at the top of a building and she moves until the top of a 4.2 m post is in line with the building's top. The post is 9 m away, the building is 36 m from the post. Diagram below.



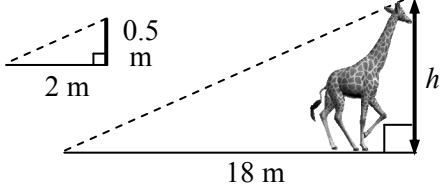
Redraw | 2 triangles

Find the height of the building.

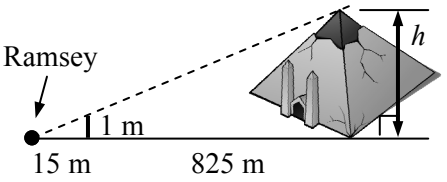
4 A 0.5 m high post casts a 2 m long shadow. How tall is 'Spotty' the giraffe who casts an 22 m shadow?



5 'Shorty' the giraffe is standing next to Spotty. He casts an 18 m long shadow. Calculate Shorty's height.



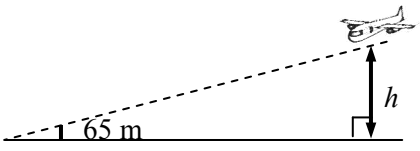
6 Ramsey is estimating the height of a pyramid using a 1 m vertical staff. The staff is 15 m away and its top is in the line of sight of the top of the pyramid, which is 825 m away.



Redraw | 2 triangles

Find the height of the pyramid.

6 Brie is looking at a plane coming in to land. The plane is 6 km away from the flight control tower. Brie has the top of the tower in the same line of sight as the plane. If the tower is 1.2 km away and is 65 m tall, how high is the plane?



Redraw | 2 triangles

Find the height of the plane.

Geometrical Properties - Similarity - Finding Sides (Sheet 1)

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Let's start with a quick review of equivalent ratios. Find the value of the box in the ratios.

1 $5 : 2 = 30 : \boxed{12}$ (×6, ×6)

2 $4 : 3 = 16 : \boxed{12}$ (×4, ×4)

3 $2 : 3 = 22 : \boxed{33}$ (×11, ×11)

Now find the scale factor (multiplier) yourself.

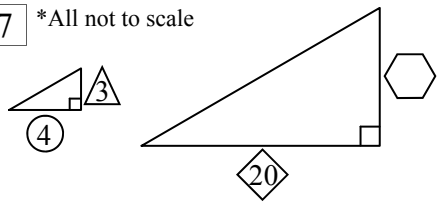
4 $5 : 2 = 15 : \boxed{6}$ (×3, ×3)

5 $1 : 6 = 7 : \boxed{42}$ (×7, ×7)

6 $5 : 12 = 10 : \boxed{24}$ (×2, ×2)

Build the ratios from these similar triangles*

7 *All not to scale

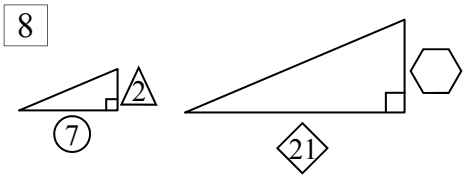


$4 : 3 = 20 : 15$ (×5, ×5)

Base Height Base Height

Left △ Right △

8

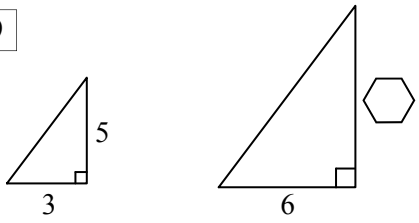


$7 : 2 = 21 : 6$ (×3, ×3)

Base Height Base Height

Left △ Right △

9

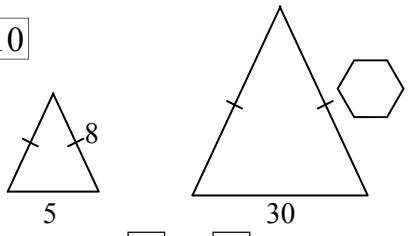


$3 : 5 = 6 : 10$ (×2, ×2)

Base Height Base Height

Left △ Right △

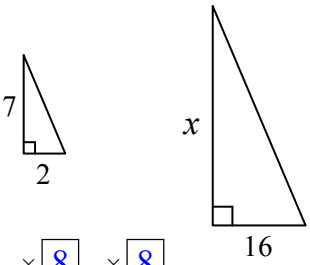
10



$5 : 8 = 30 : 48$ (×6, ×6)

From now on the unknown side is 'x'

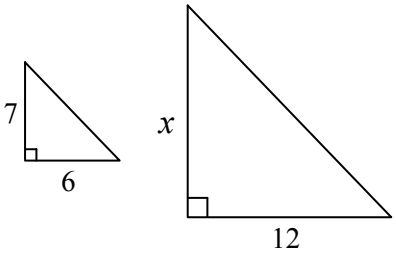
11



$2 : 7 = 16 : x$ (×8, ×8)

$x = 16$

12

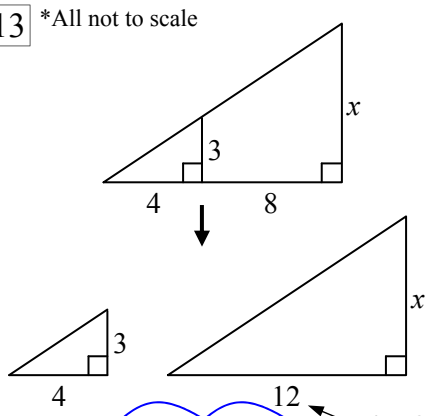


$6 : 7 = 12 : x$ (×2, ×2)

$x = 14$

Deconstruct these and solve like before. Change the diagram to two triangles. All are similar.

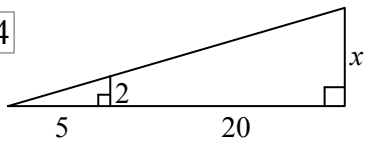
13 *All not to scale



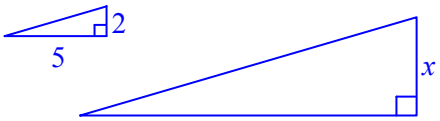
$4 : 3 = 12 : x$ (×3, ×3) 4 + 8

$x = 9$

14



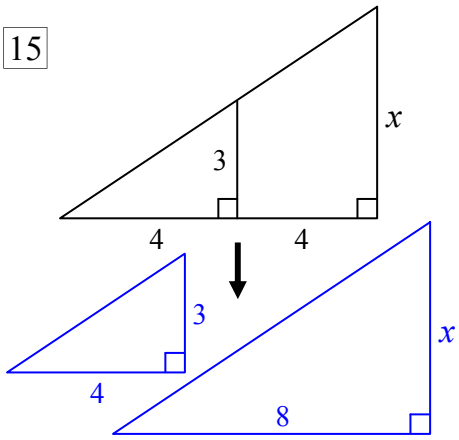
Redraw 2 triangles



$5 : 2 = 25 : x$ (×5, ×5)

$x = 10$

15



$4 : 3 = 8 : x$ (×2, ×2)

$x = 6$

Geometrical Properties - Similarity - Finding Sides (Sheet 2)

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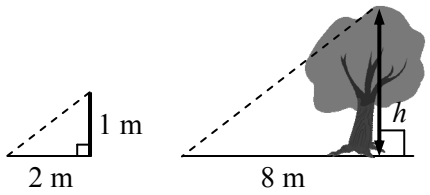
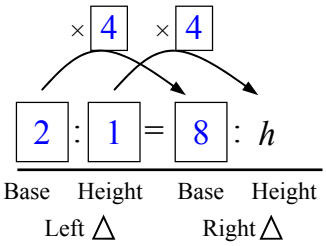
Find the height of these objects, by using similarity to calculate the unknown height.

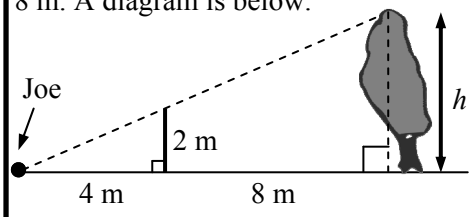
1 A 1 m high post casts a 2 m long shadow. How tall is a tree that casts an 8 m long shadow?



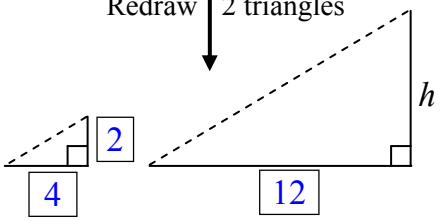


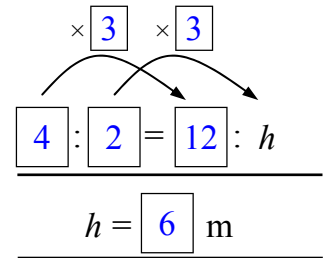
$h =$ 4 m

2 Joe looks at the top of a tree and moves until a 2 m post's top is in line with the top of the tree. The post is 4 m away. He then measures the distance from the post to the tree, it is 8 m. A diagram is below.



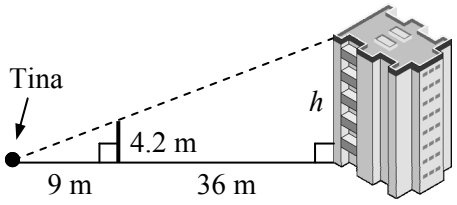
Redraw 2 triangles



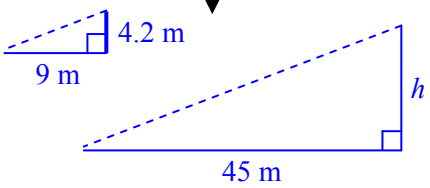


$h =$ 6 m

3 Tina looks at the top of a building and she moves until the top of a 4.2 m post is in line with the building's top. The post is 9 m away, the building is 36 m from the post. Diagram below.



Redraw 2 triangles



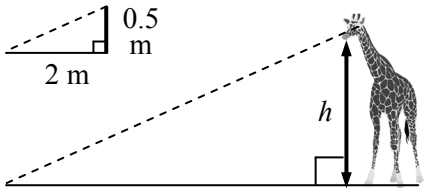
Find the height of the building.

× 5 × 5

9 : 4.2 = 45 : h

$h = 21$ m

4 A 0.5 m high post casts a 2 m long shadow. How tall is 'Spotty' the giraffe who casts an 22 m shadow?

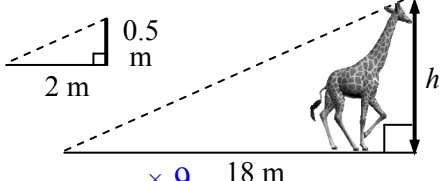


× 11

2 : 0.5 = 22 : h

$h = 5.5$ m

5 'Shorty' the giraffe is standing next to Spotty. He casts an 18 m long shadow. Calculate Shorty's height.

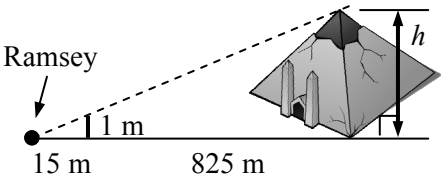


× 9

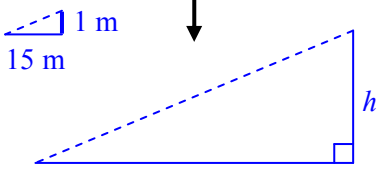
2 : 0.5 = 18 : h

$h = 4.5$ m

6 Ramsey is estimating the height of a pyramid using a 1 m vertical staff. The staff is 15 m away and its top is in the line of sight of the top of the pyramid, which is 825 m away.



Redraw 2 triangles



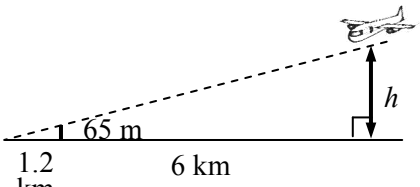
Find the height of the pyramid.

× 56

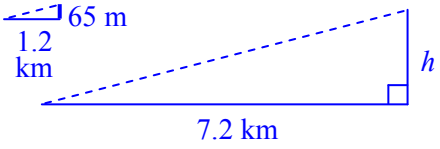
15 : 1 = 840 : h

$h = 56$ m

6 Brie is looking at a plane coming in to land. The plane is 6 km away from the flight control tower. Brie has the top of the tower in the same line of sight as the plane. If the tower is 1.2 km away and is 65 m tall, how high is the plane?



Redraw 2 triangles



Find the height of the plane.

× 6

1.2 : 65 = 7.2 : h

$h = 390$ m

Geometrical Properties - Similarity - Finding Sides (Homework Strip)

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Date Due: _____

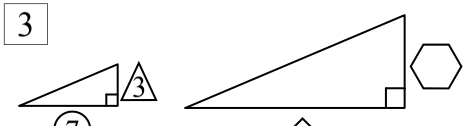
Find the box for these equivalent ratios.

1 $4 : 7 = 16 : \square$ (×4, ×4)

2 $8 : 3 = 24 : \square$ (× □, × □)

Find the missing side for these similar Δ

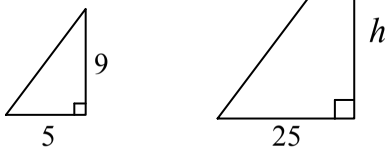
3



Base Height Base Height

Left Δ Right Δ

4



$\square : \square = \square : h$

$h = \square$

Date Due: _____

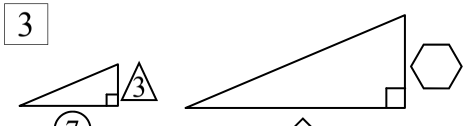
Find the box for these equivalent ratios.

1 $4 : 7 = 16 : \square$ (×4, ×4)

2 $8 : 3 = 24 : \square$ (× □, × □)

Find the missing side for these similar Δ

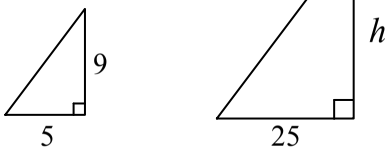
3



Base Height Base Height

Left Δ Right Δ

4



$\square : \square = \square : h$

$h = \square$

Date Due: _____

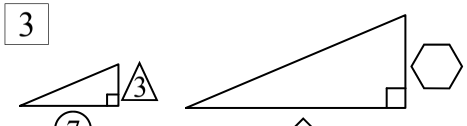
Find the box for these equivalent ratios.

1 $4 : 7 = 16 : \square$ (×4, ×4)

2 $8 : 3 = 24 : \square$ (× □, × □)

Find the missing side for these similar Δ

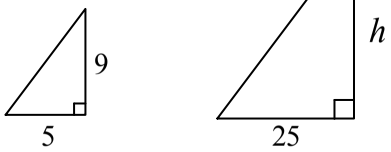
3



Base Height Base Height

Left Δ Right Δ

4



$\square : \square = \square : h$

$h = \square$

Date Due: _____

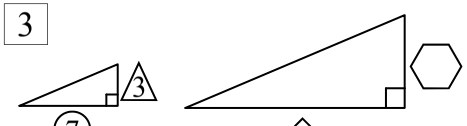
Find the box for these equivalent ratios.

1 $4 : 7 = 16 : \square$ (×4, ×4)

2 $8 : 3 = 24 : \square$ (× □, × □)

Find the missing side for these similar Δ

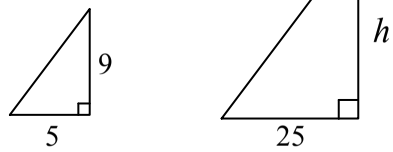
3



Base Height Base Height

Left Δ Right Δ

4



$\square : \square = \square : h$

$h = \square$

Geometrical Properties - Similarity - Finding Sides (Homework Strip)

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Date Due: _____

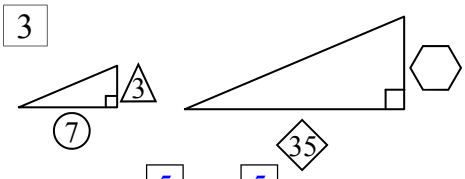
Find the box for these equivalent ratios.

1 $4 : 7 = 16 : \boxed{28}$ (×4, ×4)

2 $8 : 3 = 24 : \boxed{9}$ (× 3, × 3)

Find the missing side for these similar Δ

3



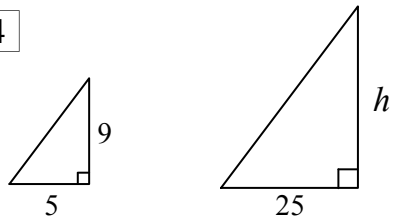
× 5, × 5

$7 : 3 = 35 : 15$

Base Height Base Height

Left Δ Right Δ

4



× 5, × 5

$5 : 9 = 25 : h$

$h = 45$

Geometrical Properties - Scale Diagrams

Two common applications of scale diagrams is maps and construction drawings. Maps are perhaps less relevant with GPS now used, but many walkers have to know how to use a map and compass in the case technology fails them. Some prefer maps also.

When a building is designed it is shown in two dimensional views. It can be shown from the sides (elevation and end elevation) or from above, which is called the plan view. This sheet deals with a scale drawing diagram in plan view.

So the roof has been ripped off and you are like a giant bent over the room looking down. The kitchen shown on the worksheet contains a pantry cupboard, a sink (wet area), a cooking area and other cupboards. While you can't see how tall they are, you can measure them and find their length and width.

The scale of this diagram is 1 : 50. So every measurement you make has to be multiplied by 50. Once you do that, change the length from cm to m. All the lengths on the sheet are either exact cm lengths, or 0.5 cm lengths.

So measure the lengths, make the conversion using the scale and answer the questions. You should notice an easier way to convert than using your calculator all the time. Nothing that a giant couldn't manage, as long as they have a ruler of course.

With maps you use the same skill as above. The scale is in a different format, rather than being a ratio it tells you that 1cm = 4 km. So multiply all lengths by 4 (×4), and change the cm to km. Measure from the centre of the dots, record the length in cm, then make the conversion.

Geometrical Props - Scale Diagrams - Construction Plan Views (Sheet 1)

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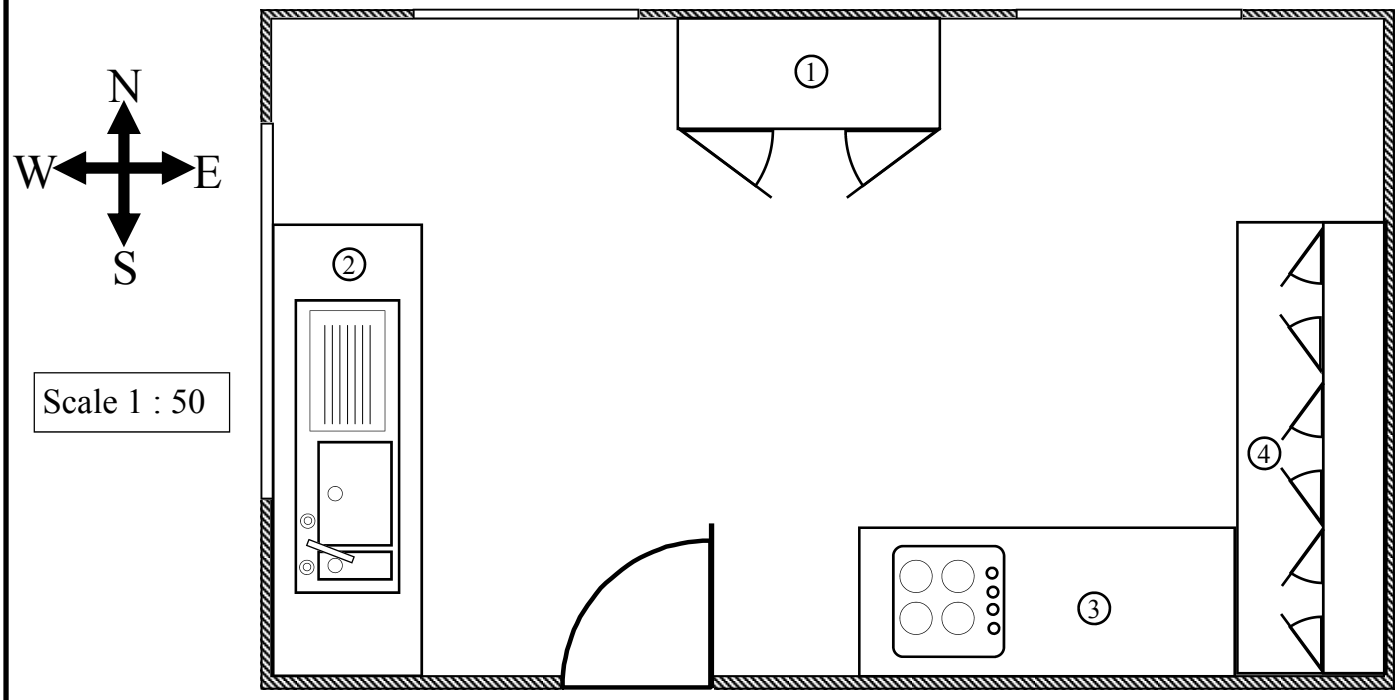

Scale 1 : 50

Get out your ruler and measure lengths in cm, to the nearest 0.5 cm. Use the scale to find the actual lengths. Round answers to 2 d.p where required.

1 The length of the eastern wall (in m).

Length = ____ cm = ____ cm = ____ m
(× 50 →) (÷ 100 →)

2 The length of the northern wall (in m).

Length = ____ cm = ____ cm = ____ m
(× 50 →) (÷ 100 →)

3 The floor area (empty) of the room (m^2).

Q.1 × Q.2

$A = lb$

$= \square \times \square$

$A = \square$ m^2

2 d.p.

4 The length, breadth and floor area of the pantry. ① on diagram.

Length = ____ m

Breadth = ____ m

Round to 2 d.p., in m^2.

5 The length, breadth and floor area of the sink bench. ② on diagram.

Length = ____ m

Breadth = ____ m

6 The length, breadth and floor area of the cook top. ③ on diagram.

Length = ____ m

Breadth = ____ m

7 The length, breadth and floor area of the cupboards. ④ on diagram.

Length = ____ m

Breadth = ____ m

8 Find the total covered floor area. = ① + ② + ③ + ④

Area = ____ + ____ + ____ + ____

Area = ____

9 Find the total exposed floor area. Q.3 - Q.8

Area =

10 Estimate the cost of tiling the exposed floor area when tiles cost \$35/m^2. Available in whole m^2 only.

Exposed floor area rounded up to whole m^2 = ____

Cost = ____ × ____ = ____

\$ sign

Geometrical Properties - Scale Diagrams - Using Maps (Sheet 2)

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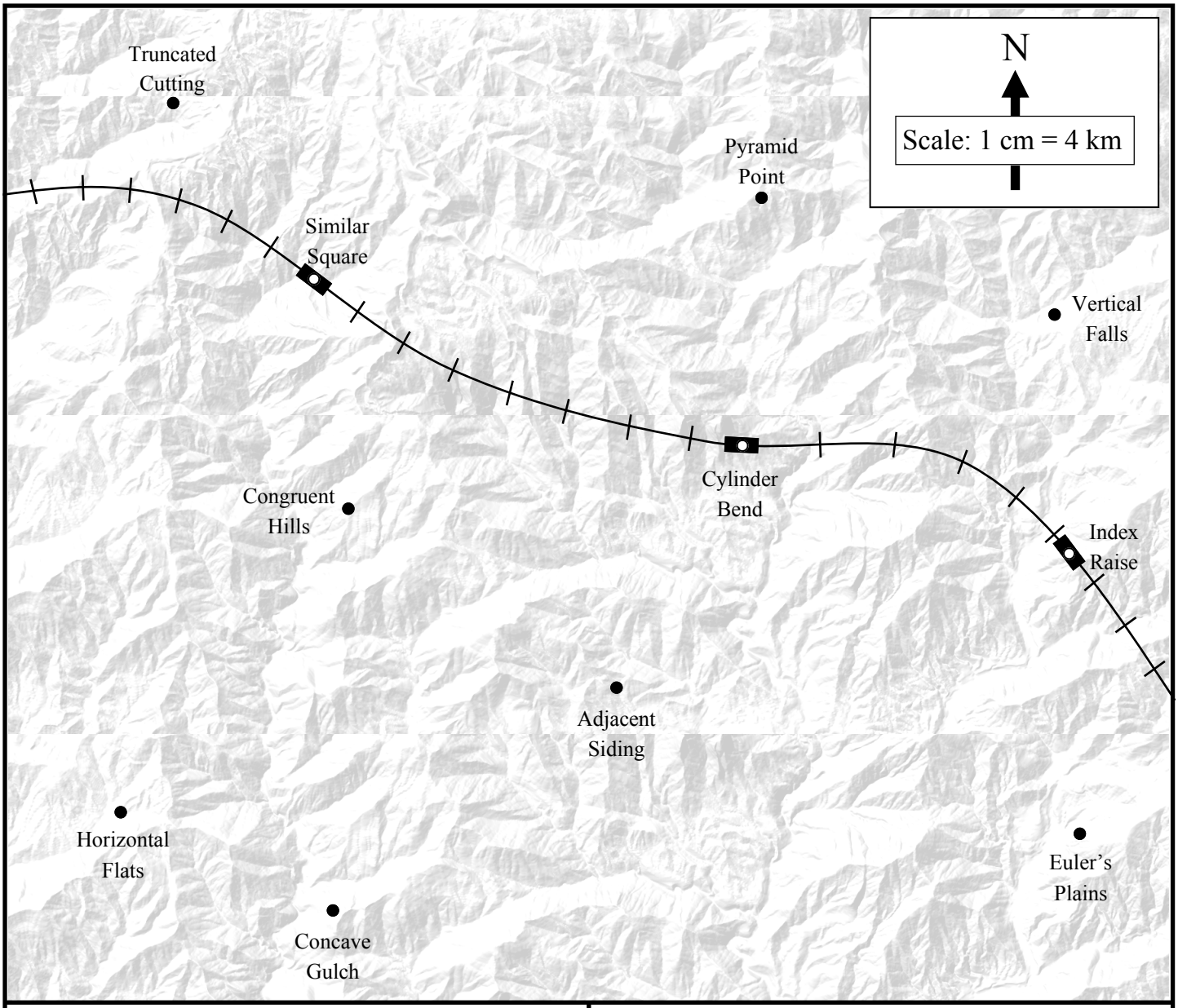

Get out your ruler and measure lengths in cm, to the nearest 0.1 cm. Use the scale (top right) to find the actual distances. Round answers to 1 d.p if required.

1 Index Raise station to Euler's Plains.

Distance = ☐.☐ cm = ☐ km

× 4

2 Similar Square station to Horizontal Flats.

Distance = ☐.☐ cm = ☐ km

READ THE NEXT QUESTION CAREFULLY!

3 Pyramid Point to Index Raise, **via Vertical Falls**.

Distance = ☐ cm + ☐ cm = ☐ cm = ☐ km

4 Concave Gulch to Cylinder Bend, via Adjacent Siding.

Distance = ☐ cm + ☐ cm = ☐ cm = ☐ km

Hansel travels from Truncated Cutting to Adjacent Siding via Congruent Hills. While **Gretel** travels from Truncated Cutting to Adjacent Siding via Pyramid Point.

5 Calculate the distance Hansel travels

6 Calculate the distance Gretel travels

7 Who travels the furthest and by how much?

Which station is the closest to **Adjacent Siding** and by how much? Similar Square Station or Index Raise Station?

8

Geometrical Props - Scale Diagrams - Construction Plan Views (Sheet 1)

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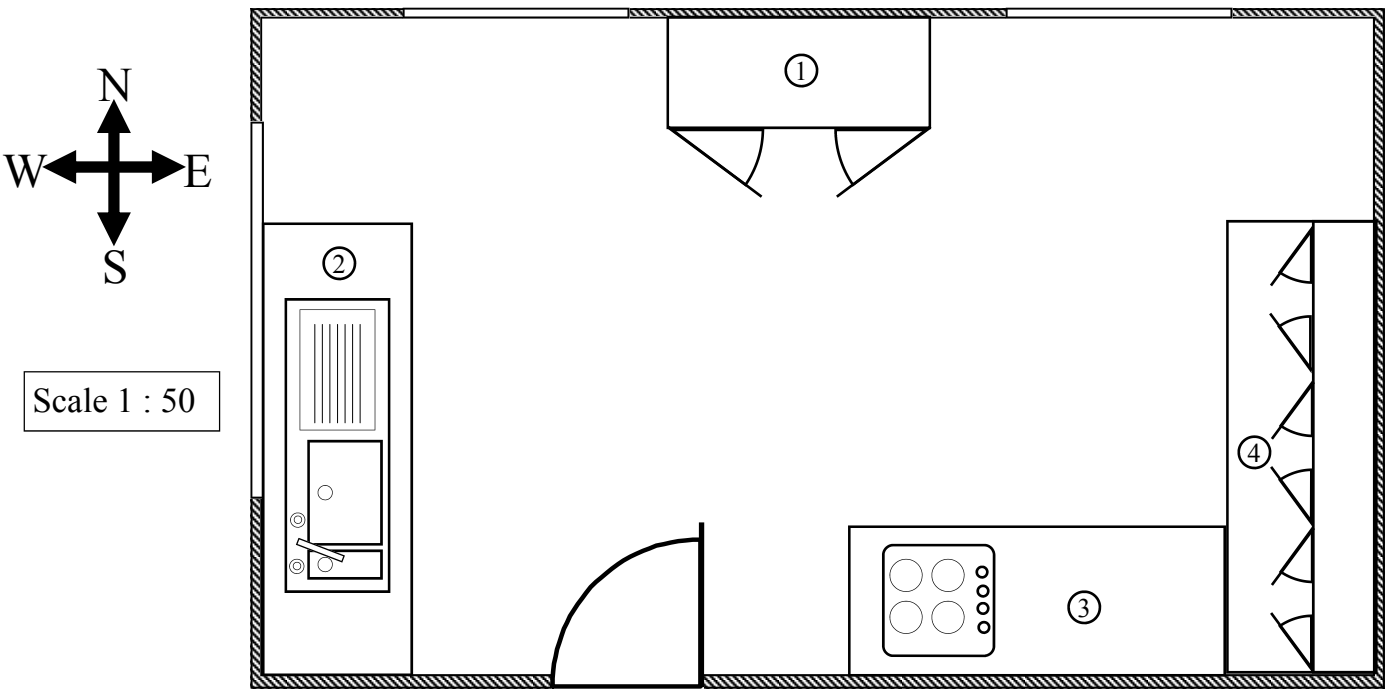

Scale 1 : 50

Measure lengths to the nearest whole or 0.5 cm.

Get out your ruler and measure lengths in cm, to the nearest 0.5 cm. Use the scale to find the actual lengths. Round answers to 2 d.p where required.

1 The length of the eastern wall (in m).

Length = 9 cm = 450 cm = 4.5 m
(× 50 → ; ÷ 100 →)

2 The length of the northern wall (in m).

Length = 15 cm = 750 cm = 7.5 m
(× 50 → ; ÷ 100 →)

3 The floor area (empty) of the room (m^2).

Q.1 × Q.2

$A = lb$

$= 7.5 \times 4.5$

$A = 33.75$ m^2

2 d.p.

4 The length, breadth and floor area of the pantry. ① on diagram.

Length = 1.75 m

Breadth = 0.75 m

$A = lb$

$= 1.75 \times 0.75$

$A = 1.31$ m^2

Round to 2 d.p., in m^2.

5 The length, breadth and floor area of the sink bench. ② on diagram.

Length = 3 m

Breadth = 1 m

$A = lb$

$= 3 \times 1$

$A = 3$ m^2

6 The length, breadth and floor area of the cook top. ③ on diagram.

Length = 2.5 m

Breadth = 1 m

$A = lb$

$= 2.5 \times 1$

$A = 2.5$ m^2

7 The length, breadth and floor area of the cupboards. ④ on diagram.

Length = 3 m

Breadth = 1 m

$A = lb$

$= 3 \times 1$

$A = 3$ m^2

8 Find the total covered floor area. = ① + ② + ③ + ④

Area = 1.31 + 3 + 2.5 + 3

Area = 9.81 m^2

9 Find the total exposed floor area. Q.3 - Q.8

Area = 33.75 - 9.81 = 23.94 m^2

10 Estimate the cost of tiling the exposed floor area when tiles cost \$35/m^2. Available in whole m^2 only.

Exposed floor area rounded up to whole m^2 = 24 m^2

Cost = 24 × 35 = \$840

\$ sign

Geometrical Properties - Scale Diagrams - Using Maps (Sheet 2)

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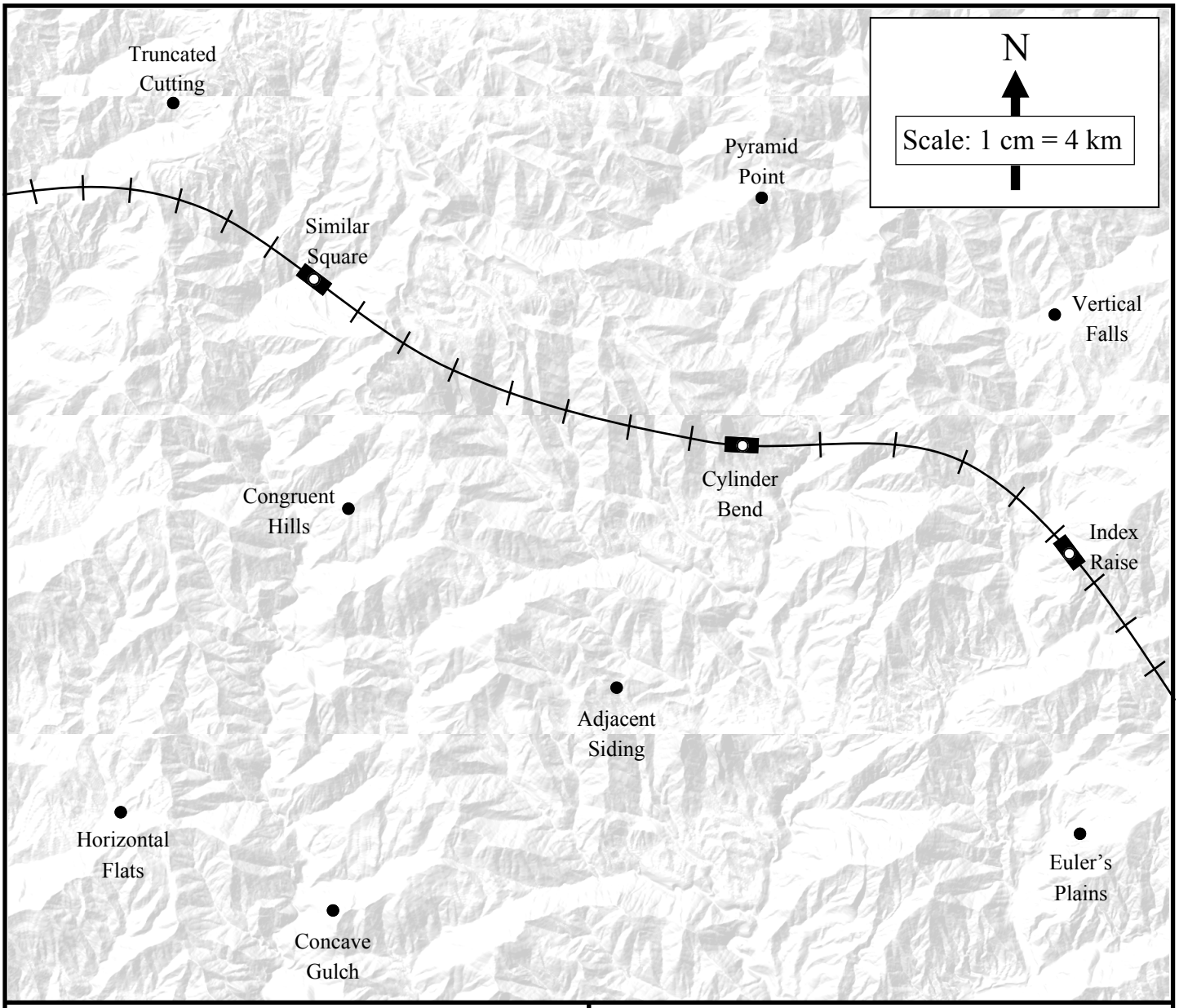

Get out your ruler and measure lengths in cm, to the nearest 0.1 cm. Use the scale (top right) to find the actual distances. Round answers to 1 d.p if required.

1 Index Raise station to Euler's Plains.

Distance = 4.5 cm = 18 km

× 4

2 Similar Square station to Horizontal Flats.

Distance = 9.2 cm = 36.8 km

READ THE NEXT QUESTION CAREFULLY!

3 Pyramid Point to Index Raise, **via Vertical Falls**.

Distance = 5.1 cm + 3.9 cm = 9 cm = 36 km

4 Concave Gulch to Cylinder Bend, via Adjacent Siding.

Distance = 5.8 cm + 4.4 cm = 10.2 cm = 40.8 km

Hansel travels from Truncated Cutting to Adjacent Siding via Congruent Hills. While **Gretel** travels from Truncated Cutting to Adjacent Siding via Pyramid Point.

5 Calculate the distance Hansel travels

Distance = 7.1 + 5.2 = 12.3 cm = 49.2 km

6 Calculate the distance Gretel travels

Distance = 9.6 + 8.3 = 17.9 cm = 71.6 km

7 Who travels the furthest and by how much?

Gretel. Difference = 71.6 - 49.2 = 22.4 km

Which station is the closest to **Adjacent Siding** and by how much? Similar Square Station or Index Raise Station?

8 Similar Square: Distance = 8.2 cm = 32.8 km

Index Raise: Distance = 7.6 cm = 30.4 km

Difference = 32.8 - 30.4 = 2.4 km

Geometrical Properties - Scale Diagrams (Homework Strip)

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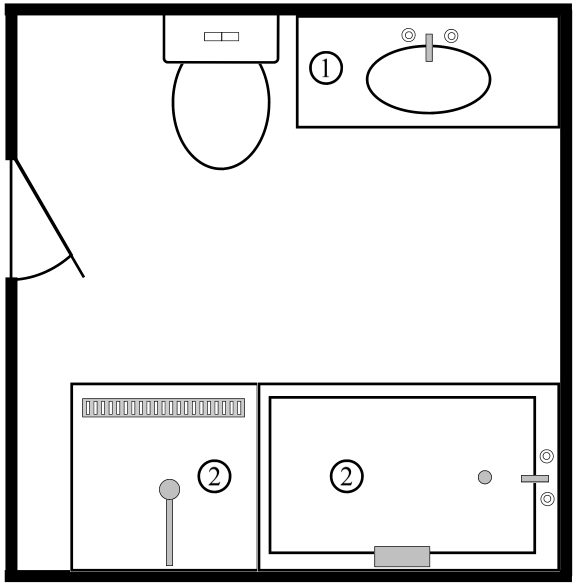
Date Due: _____

PLAN VIEW - ENSUITE BATHROOM



Scale is 1 : 40

Calculate the length and breadth (nearest 0.5 cm) of:

1 The vanity unit ①

Length = ON PLAN _____ cm × 40 = ACTUAL _____ cm

Breadth = ON PLAN _____ cm = ACTUAL _____ cm

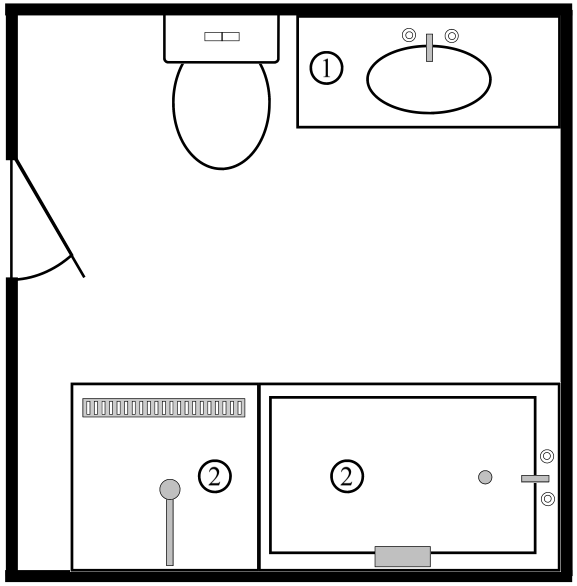
2 The bath and shower unit (combined) ②

Length = ON PLAN _____ cm = ACTUAL _____ cm

Breadth = ON PLAN _____ cm = ACTUAL _____ cm

Date Due: _____

PLAN VIEW - ENSUITE BATHROOM



Scale is 1 : 40

Calculate the length and breadth (nearest 0.5 cm) of:

1 The vanity unit ①

Length = ON PLAN _____ cm × 40 = ACTUAL _____ cm

Breadth = ON PLAN _____ cm = ACTUAL _____ cm

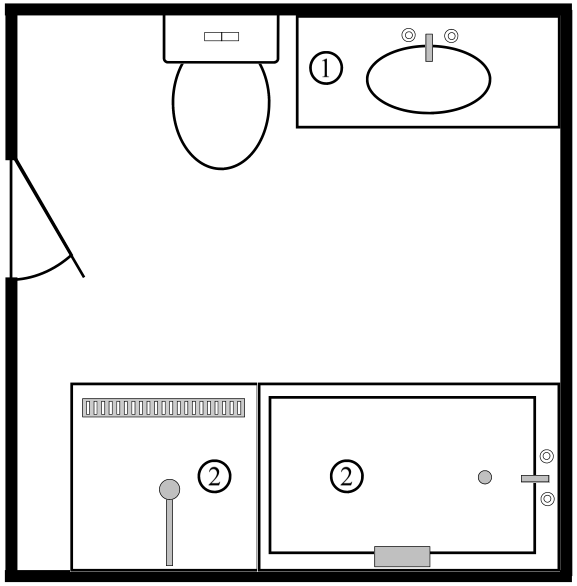
2 The bath and shower unit (combined) ②

Length = ON PLAN _____ cm = ACTUAL _____ cm

Breadth = ON PLAN _____ cm = ACTUAL _____ cm

Date Due: _____

PLAN VIEW - ENSUITE BATHROOM



Scale is 1 : 40

Calculate the length and breadth (nearest 0.5 cm) of:

1 The vanity unit ①

Length = ON PLAN _____ cm × 40 = ACTUAL _____ cm

Breadth = ON PLAN _____ cm = ACTUAL _____ cm

2 The bath and shower unit (combined) ②

Length = ON PLAN _____ cm = ACTUAL _____ cm

Breadth = ON PLAN _____ cm = ACTUAL _____ cm

Geometrical Properties - Scale Diagrams (Homework Strip)

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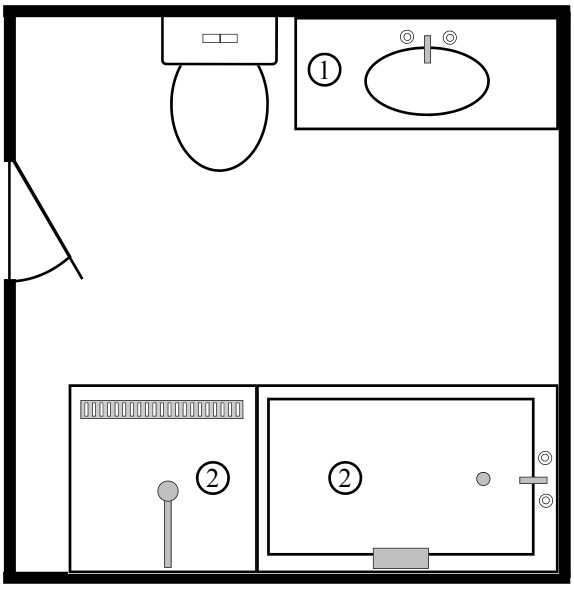
Date Due:

PLAN VIEW - ENSUITE BATHROOM



Scale is 1 : 40

Calculate the length and breadth (nearest 0.5 cm) of:

1 The vanity unit ①

Length = 2.5 cm → × 40 → = 100 cm

Breadth = 1.5 cm → × 40 → = 60 cm

2 The bath and shower unit (combined) ②

Length = 6.5 cm → × 40 → = 260 cm

Breadth = 2.5 cm → × 40 → = 100 cm

Geometrical Properties - Further Scale Drawing

Scale drawings allow us to use images to measure distances. By multiplying by the scale any length on a scale diagram can be used to determine the actual length.

The first worksheet asks you to enlarge triangles and rectangles using a scale. The rule with using a scale is that you multiply by the second number and divide by the first.

Example

If a drawing of an image is 55 mm long what is the actual length, given a scale factor of 5 : 4.

$$55 \times \frac{4}{5} = 44 \text{ mm}$$

Once you convert lengths, draw the shapes on the grids provided. Use dashes on the vertice's letters to indicate it is an image. For example A becomes A′. (An image is the drawing after the scale factor has been applied).

The second worksheet features diagrams and images of animals/insects in nature and man-made objects, as well as the scale ratio applied to the image. How do you find the actual length? Measure the length in mm, then multiply by the second number in the ratio, then divide by the first number.

Write the measured length in the table at the base of the column, using mm. Apply the scale and write the actual length in the RHS of the table. Again use mm, but if the column asks you to convert this to cm or m, then divide by 10 or 1 000, and rewrite in the units requested.

Column 2 involves travel between two wharves. One person drives on land, the other uses a ferry. Use the scale to measure the distance between the two points requested in km. Then using the speed of the ferry complete a timetable for its operation.

Further down Column 2 is a map with roads connecting towns. Measure the straight line distances between the points, then apply the scale. You should notice a quick way to apply the scale rather than multiplying and dividing by the large numbers. The lengths aren't exact in mm, sometimes they are slightly out, so answers will vary by small amounts. Write the distances in the boxes.

Geometrical Properties - Further Scale Drawing (Sheet 1)

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Measure the sides of these shapes and then apply the scale factor to construct an enlargement. The construction will be larger than the original.

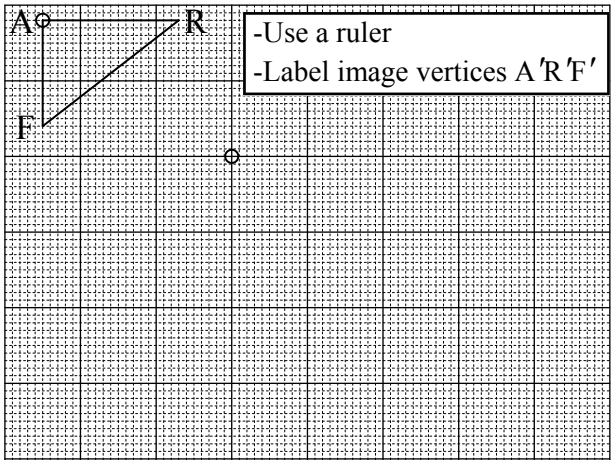
1 Scale = 2 : 5

A′R′: ☐ × $\frac{5}{2}$ = ☐ mm

Length in mm

A′F′: ☐ × $\frac{5}{2}$ = ☐ mm

Construct △A′R′F′ by drawing sides A′R′ and A′F′ first.

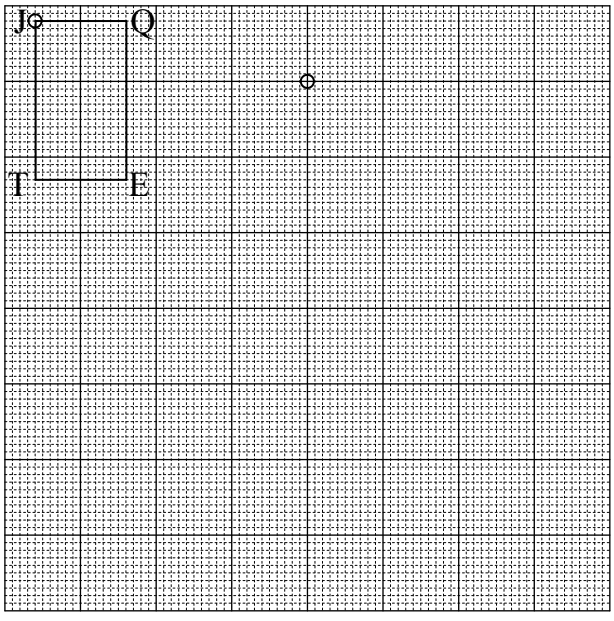


2 Scale = 3 : 8

J′Q′: ☐ × $\frac{\square}{\square}$ = ☐

J′T′: ☐ × $\frac{\square}{\square}$ = ☐

units!

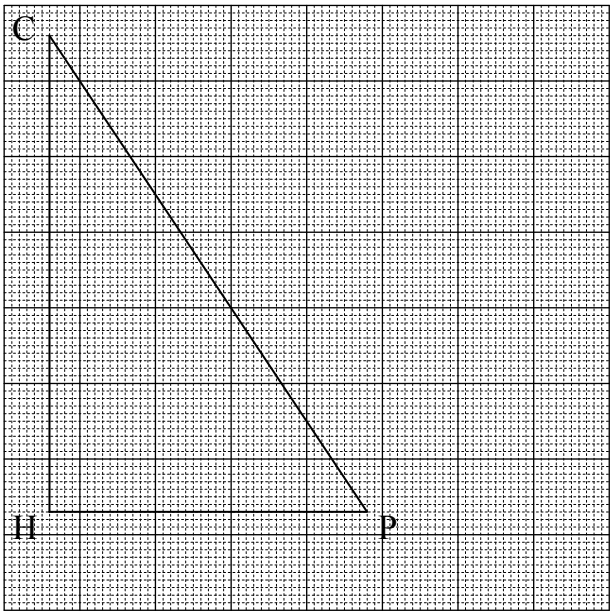


These are enlargements … but they are smaller!

3 Scale = 7 : 2

C′H′: ☐ × $\frac{\square}{\square}$ = ☐

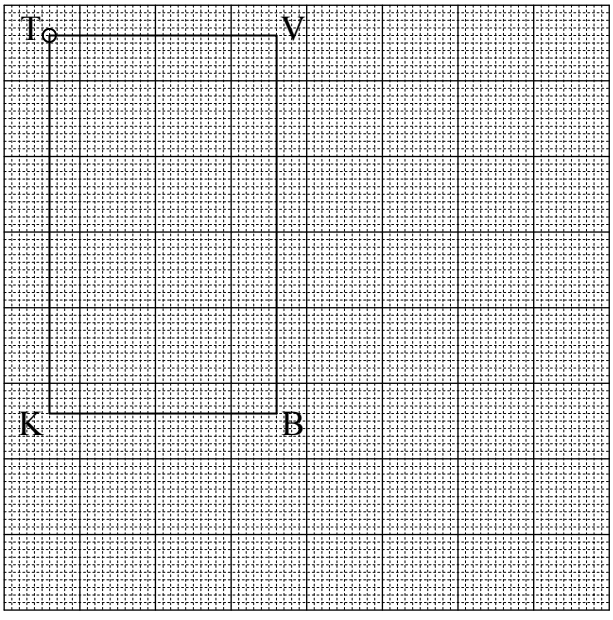
H′P′: ☐ × $\frac{\square}{\square}$ = ☐



4 Scale = 10 : 3

T′V′: ☐ × $\frac{\square}{\square}$ = ☐

T′K′: ☐ × $\frac{\square}{\square}$ = ☐



Geometrical Properties - Further Scale Drawing (Sheet 2)

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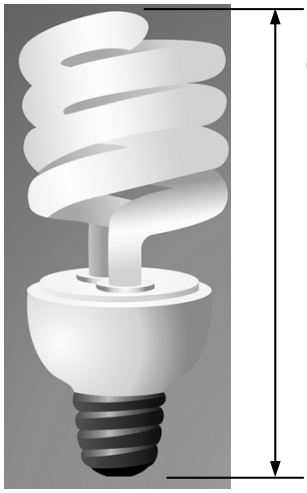
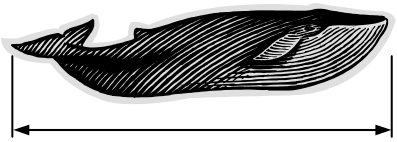
Measure the lengths in mm, using the arrowed length. Write the length in the 'image' box below. Apply the scale to find the actual length. Multiply by the second number in the ratio, divide by the first.

1 Scale = 1 : 2



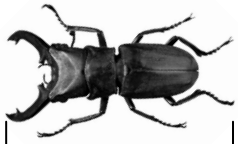
2 Scale = 1 : 660



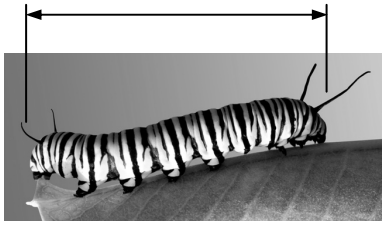
3 Scale = 9 : 1



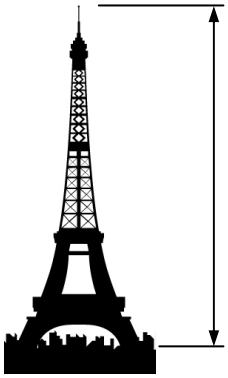
4 Scale = 3 : 2



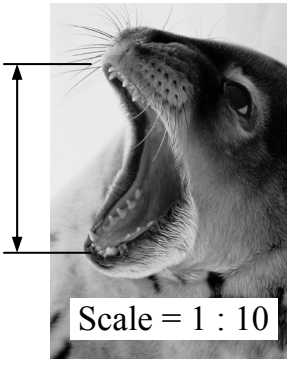
5 Scale = 5 : 3



6 Scale = 1 : 7 200

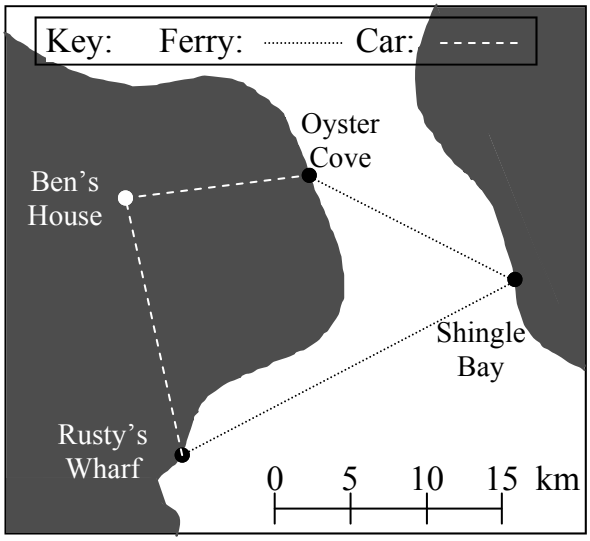


7



Q	Image Length (mm)	Actual Length (mm)
1		
2		mm = m
3		
4		
5		mm = cm
6		mm = m
7		mm = cm

Ben drove from Rusty's Wharf to Oyster Cove, while Tim caught the ferry. Use the scale to answer these questions.



Find the distance in km between:

8 Rusty's Wharf and Shingle Bay _____

9 Shingle Bay and Oyster Cove _____

10 Rusty's Wharf and Ben's House _____

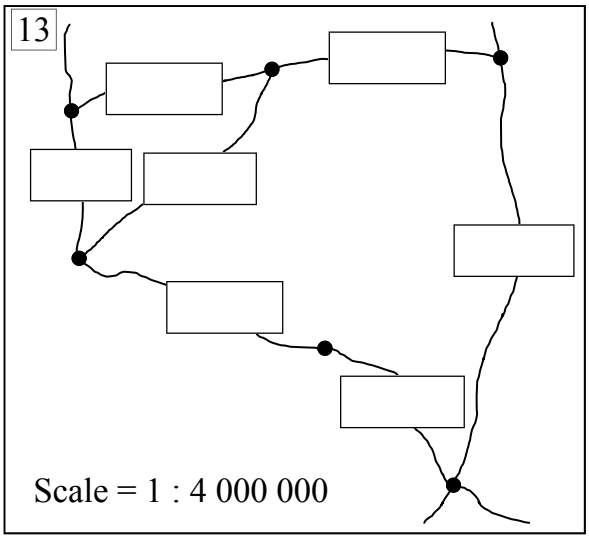
11 Ben's house and Oyster Cove _____

12 The ferry averages 20 km/h (5 km in 15 min), and it stops at all wharves for 10 min. Complete the time table below.

Location	Arrives	Departs
Rusty's Wharf	12.25 pm.	
Shingle Bay		
Oyster Cove		

Measure straight lengths in mm (ignore curves). Use the scale, then write the distances (km) in the boxes.

13



mm → m → km = ÷ 1 000 ÷ 1 000

Geometrical Properties - Further Scale Drawing (Sheet 1)

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Measure the sides of these shapes and then apply the scale factor to construct an enlargement. The construction will be larger than the original.

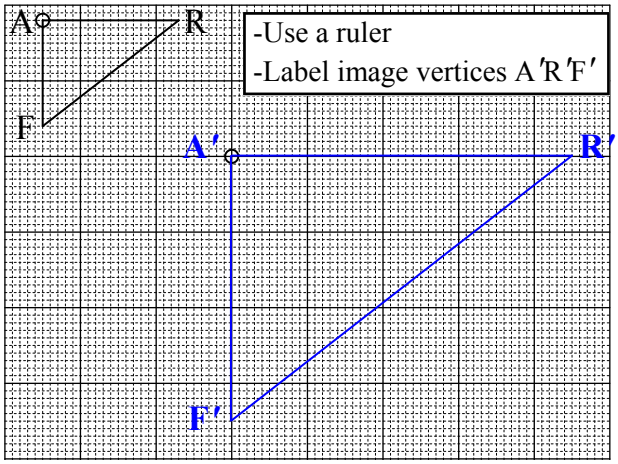
1 Scale = 2 : 5

A′R′: 18 × $\frac{5}{2}$ = 45 mm

Length in mm

A′F′: 14 × $\frac{5}{2}$ = 35 mm

Construct △A′R′F′ by drawing sides A′R′ and A′F′ first.



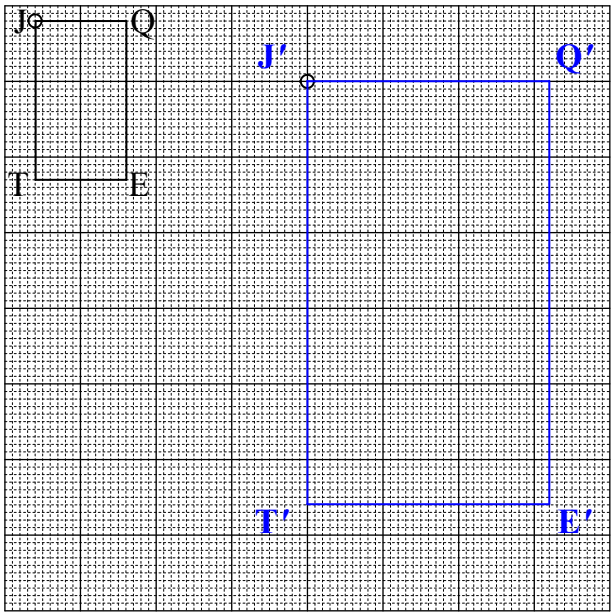
-Use a ruler
-Label image vertices A′R′F′

2 Scale = 3 : 8

J′Q′: 12 × $\frac{8}{3}$ = 32 mm

J′T′: 21 × $\frac{8}{3}$ = 56 mm

units!

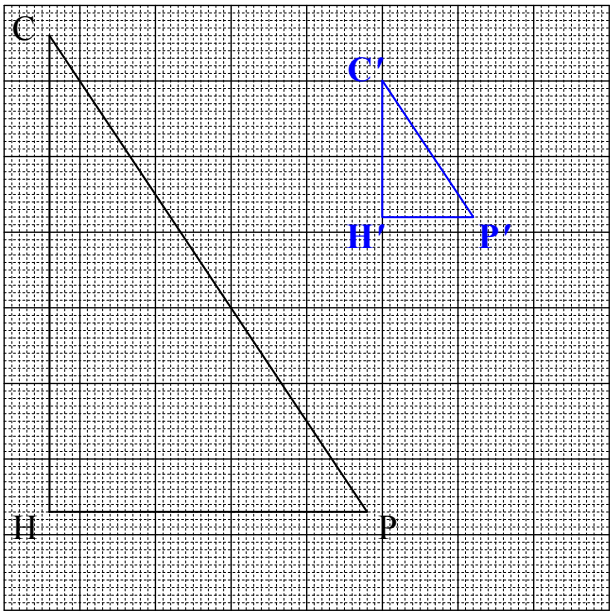


These are enlargements … but they are smaller!

3 Scale = 7 : 2

C′H′: 63 × $\frac{2}{7}$ = 18 mm

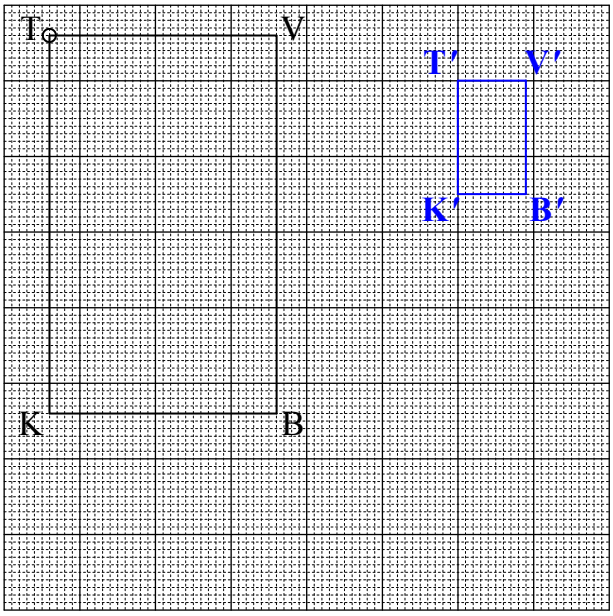
H′P′: 42 × $\frac{2}{7}$ = 12 mm



4 Scale = 10 : 3

T′V′: 30 × $\frac{3}{10}$ = 9 mm

T′K′: 50 × $\frac{3}{10}$ = 15 mm



Geometrical Properties - Further Scale Drawing (Sheet 2)

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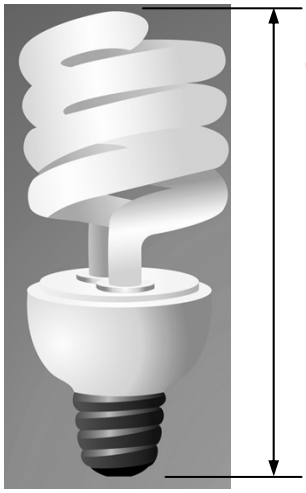
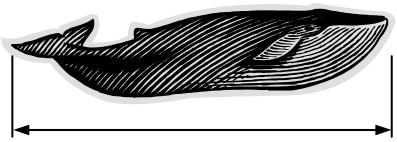
Measure the lengths in mm, using the arrowed length. Write the length in the 'image' box below. Apply the scale to find the actual length. Multiply by the second number in the ratio, divide by the first.

1 Scale = 1 : 2



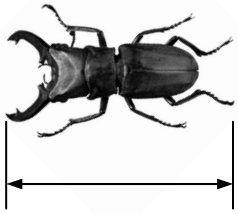
2 Scale = 1 : 660



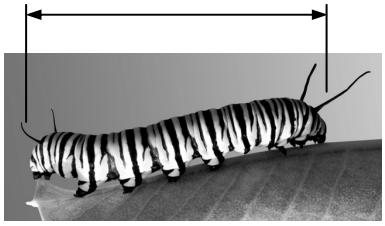
3 Scale = 9 : 1



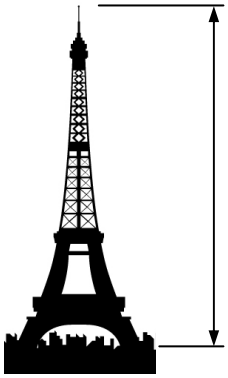
4 Scale = 3 : 2



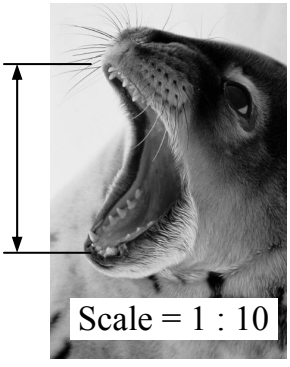
5 Scale = 5 : 3



6 Scale = 1 : 7 200

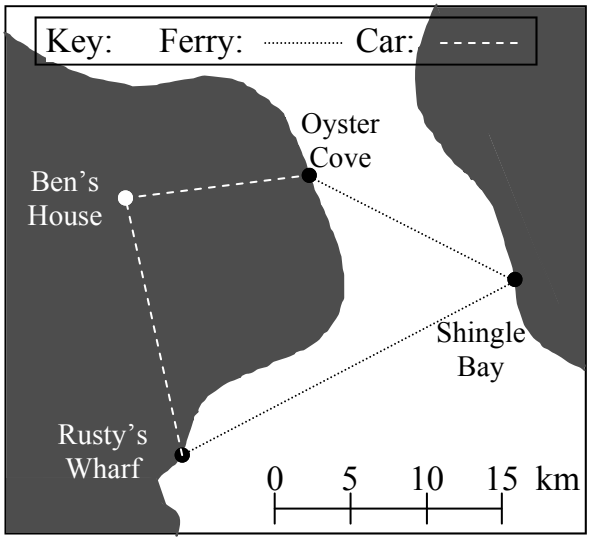


7



Q	Image Length (mm)	Actual Length (mm)
1	62 mm	124 mm
2	50 mm	33 000 mm = 33 m
3	36 mm	4 mm
4	30 mm	20 mm
5	40 mm	24 mm = 2.4 cm
6	45 mm	324 000 mm = 324 m
7	25 mm	250 mm mm = 25 cm

Ben drove from Rusty's Wharf to Oyster Cove, while Tim caught the ferry. Use the scale to answer these questions.



Find the distance in km between:

8 Rusty's Wharf and Shingle Bay 25 km

9 Shingle Bay and Oyster Cove 15 km

10 Rusty's Wharf and Ben's House 17.5 km

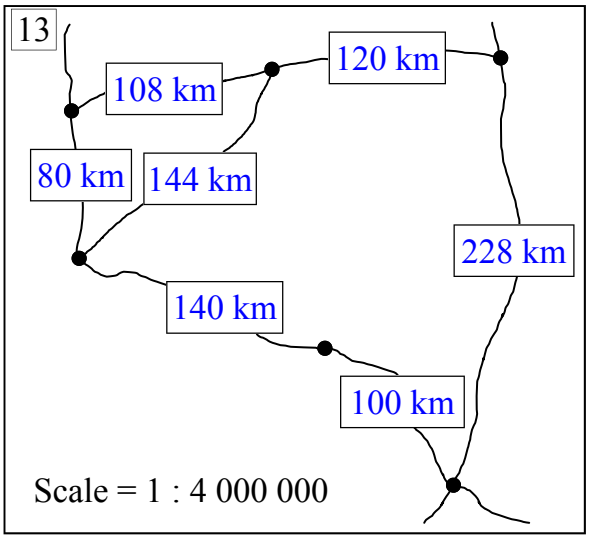
11 Ben's house and Oyster Cove 12.5 km

12 The ferry averages 20 km/h (5 km in 15 min), and it stops at all wharves for 10 min. Complete the time table below.

Location	Arrives	Departs
Rusty's Wharf	12.25 pm.	12.35 pm.
Shingle Bay	1.50 pm.	2.00 pm.
Oyster Cove	2.45 pm.	2.55 pm.

Measure straight lengths in mm (ignore curves). Use the scale, then write the distances (km) in the boxes.

13



mm → m → km = ÷ 1 000 ÷ 1 000

Geometrical Properties - Further Scale Diagrams (Homework Strip)

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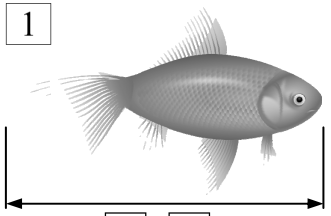
Date Due: _____

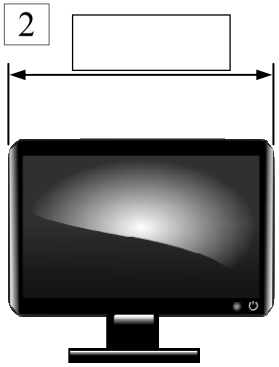
Measure the lengths on these images in cm. (1 d.p.)

1

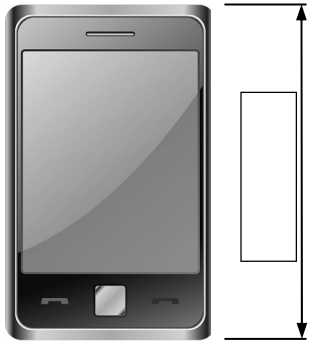


_____ . _____ cm

2



3



Apply the scale factor to calculate the actual lengths in cm for these. Round to 1 d.p.

5 Fish, Scale 2 : 5 _____ × $\frac{\square}{\square}$ = _____

6 TV, Scale 1 : 24 _____ × $\frac{\square}{\square}$ = _____

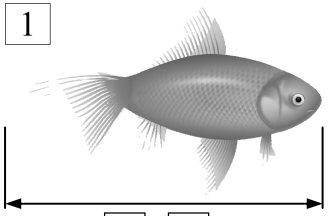
7 Phone, Scale 4 : 9

_____ × $\frac{\square}{\square}$ = _____

Date Due: _____

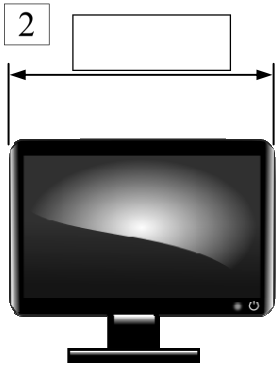
Measure the lengths on these images in cm. (1 d.p.)

1

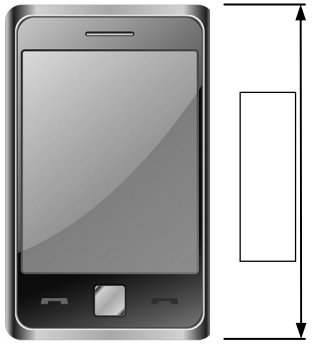


_____ . _____ cm

2



3



Apply the scale factor to calculate the actual lengths in cm for these. Round to 1 d.p.

5 Fish, Scale 2 : 5 _____ × $\frac{\square}{\square}$ = _____

6 TV, Scale 1 : 24 _____ × $\frac{\square}{\square}$ = _____

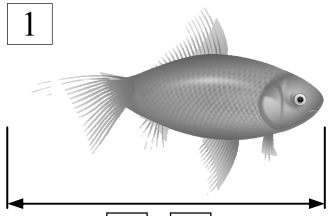
7 Phone, Scale 4 : 9

_____ × $\frac{\square}{\square}$ = _____

Date Due: _____

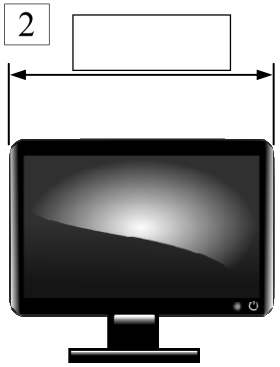
Measure the lengths on these images in cm. (1 d.p.)

1

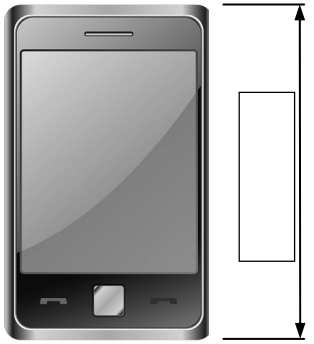


_____ . _____ cm

2



3



Apply the scale factor to calculate the actual lengths in cm for these. Round to 1 d.p.

5 Fish, Scale 2 : 5 _____ × $\frac{\square}{\square}$ = _____

6 TV, Scale 1 : 24 _____ × $\frac{\square}{\square}$ = _____

7 Phone, Scale 4 : 9

_____ × $\frac{\square}{\square}$ = _____

Geometrical Properties - Further Scale Diagrams (Homework Strip)

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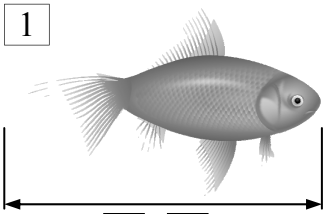
Date Due: _____

Measure the lengths on these images in cm. (1 d.p.)

1



4.2 cm

2

3.5 cm



3



4.4 cm

Apply the scale factor to calculate the actual lengths in cm for these. Round to 1 d.p.

5 Fish, Scale 2 : 5 $4.2 \times \frac{5}{2} = 10.5$ cm

6 TV, Scale 1 : 24 $3.5 \times \frac{24}{1} = 84$ cm

7 Phone, Scale 4 : 9

$4.4 \times \frac{9}{4} = 9.9$ cm

Geometrical Properties - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Geometrical Properties - Find A Word

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Find the words in the puzzle from the wordlist.



M	D	M	K	S	T	A	T	E	M	E	N	T	C	N
U	I	I	X	E	H	E	N	O	I	T	A	T	O	R
L	X	X	V	C	I	H	E	I	G	H	T	I	N	E
T	T	I	R	I	G	H	R	T	R	T	T	P	G	C
I	N	X	E	T	D	Q	U	L	O	A	R	P	R	T
P	E	U	L	R	Q	E	S	Q	T	U	I	Y	U	A
L	M	N	U	E	A	Q	A	N	C	A	A	T	E	N
Y	E	K	R	V	U	P	E	U	A	Q	N	I	N	G
K	G	N	N	K	K	I	M	O	F	U	G	R	C	L
P	R	O	P	O	R	T	I	O	N	A	L	A	E	E
M	A	W	E	O	E	T	U	Q	C	E	E	L	L	S
A	L	N	X	L	A	U	Q	E	A	L	S	I	C	C
T	N	X	G	R	A	X	U	A	Q	U	X	M	R	I
C	E	J	I	L	A	C	I	T	N	E	D	I	I	C
H	H	T	G	N	E	L	S	S	X	Q	E	S	C	L

WORDLIST

TRIANGLES	FACTOR	IDENTICAL	LENGTH	MULTIPLY
ENLARGEMENT	RECTANGLES	DIVIDE	UNKNOWN	VERTICES
PROPORTIONAL	RULER	COMPARE	SCALE	ROTATION
ORIENTATION	ANGLE	SIMILARITY	RATIO	MATCH
CONGRUENCE	STATEMENT	EQUAL	MEASURE	AXIS
	HEIGHT		CIRCLE	

Geometrical Properties - Find A Word

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Find the words in the puzzle from the wordlist.



M	D	M	K	S	T	A	T	E	M	E	N	T	C	N
U	I	I	X	E	H	E	N	O	I	T	A	T	O	R
L	X	X	V	C	I	H	E	I	G	H	T	I	N	E
T	T	I	R	I	G	H	R	T	R	T	T	P	G	C
I	N	X	E	T	D	Q	U	L	O	A	R	P	R	T
P	E	U	L	R	Q	E	S	Q	T	U	I	Y	U	A
L	M	N	U	E	A	Q	A	N	C	A	A	T	E	N
Y	E	K	R	V	U	P	E	U	A	Q	N	I	N	G
K	G	N	N	K	K	I	M	O	F	U	G	R	C	L
P	R	O	P	O	R	T	I	O	N	A	L	A	E	E
M	A	W	E	O	E	T	U	Q	C	E	E	L	L	S
A	L	N	X	L	A	U	Q	E	A	L	S	I	C	C
T	N	X	G	R	A	X	U	A	Q	U	X	M	R	I
C	E	J	I	L	A	C	I	T	N	E	D	I	I	C
H	H	T	G	N	E	L	S	S	X	Q	E	S	C	L

WORDLIST

TRIANGLES
ENLARGEMENT
PROPORTIONAL
ORIENTATION
CONGRUENCE
FACTOR
RECTANGLES
RULER
ANGLE
STATEMENT
HEIGHT
IDENTICAL
DIVIDE
COMPARE
SIMILARITY
EQUAL
LENGTH
UNKNOWN
SCALE
RATIO
MEASURE
CIRCLE
MULTIPLY
VERTICES
ROTATION
MATCH
AXIS

Geometrical Properties Review

This is the review of the Geometrical Properties Topic.

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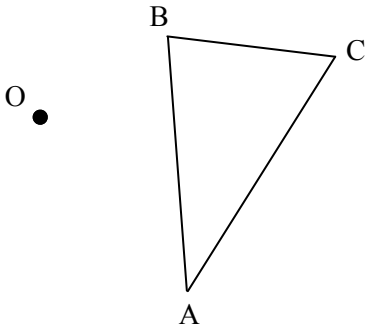
Enlarge triangle ABC by a scale factor of 2, using the centre of enlargement labeled 'O'.

1



The car below is drawn to a scale of 1 : 35. Measure the length of the wheelbase on the image then use the scale factor to give the actual car's wheelbase length.

2

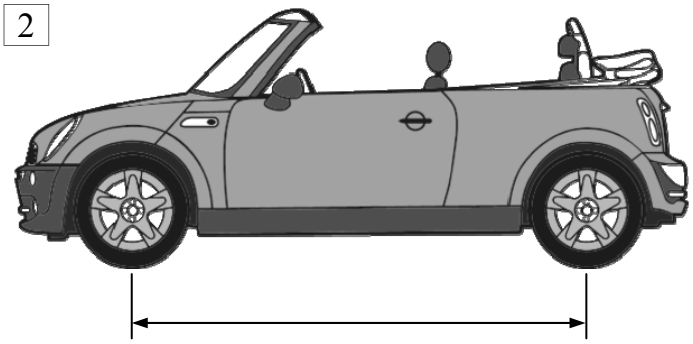
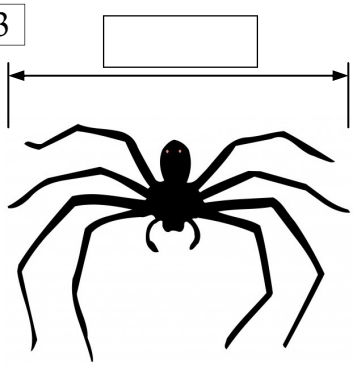


Image length (cm): ____

Actual length (cm): ____

The spider below is drawn to a scale of 3 : 2. Use the scale to calculate the width (cm) of this spider in real life.

3

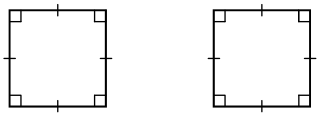


Actual length (cm): _____

_____

Is the first square below similar, congruent or neither when compared to the second square? Give a reason.

4



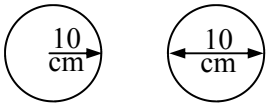
○similar
○congruent
○neither

Reason: _____

_____

Is the first circle below similar, congruent or neither when compared to the second circle? Give a reason.

5 Not to scale



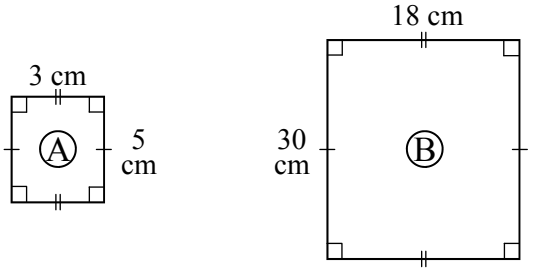
○similar
○congruent
○neither

Reason: _____

_____

Calculate the scale factor used to get Rectangle B from Rectangle A.

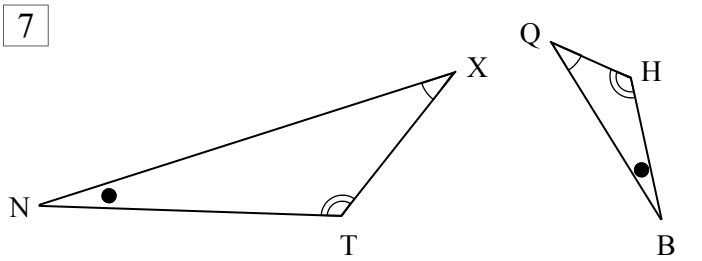
6



Scale factor: _____

The two triangles below are similar. Write a similarity statement for the two triangles.

7



_____

List 3 pairs of matching (similar) sides for the triangles.

8

____ & ____, ____ & ____, ____ & ____.

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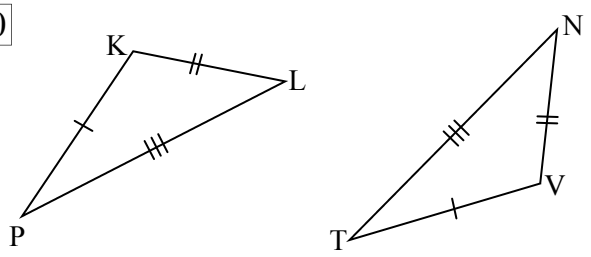
The triangles below are congruent. Write a congruence statement for the two triangles.

10



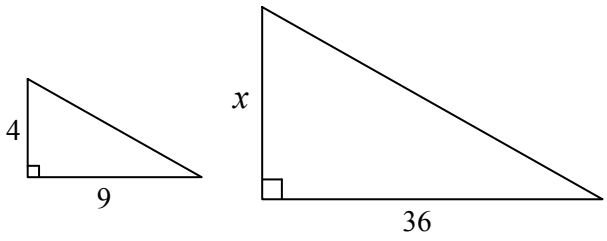
Complete the equivalent ratios below.

11 $5 : 6 = 30 : \square$

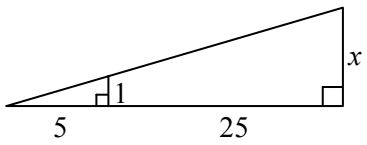
12 $10 : 4 = 20 : \square$

Find the unknown length on the triangles. (Not to scale). The two triangles are similar.

13



14



A plan view of Lucy's room is shown below. The scale is 1 : 30. Find the actual lengths of these objects in cm.

15 Desk width (w) _____

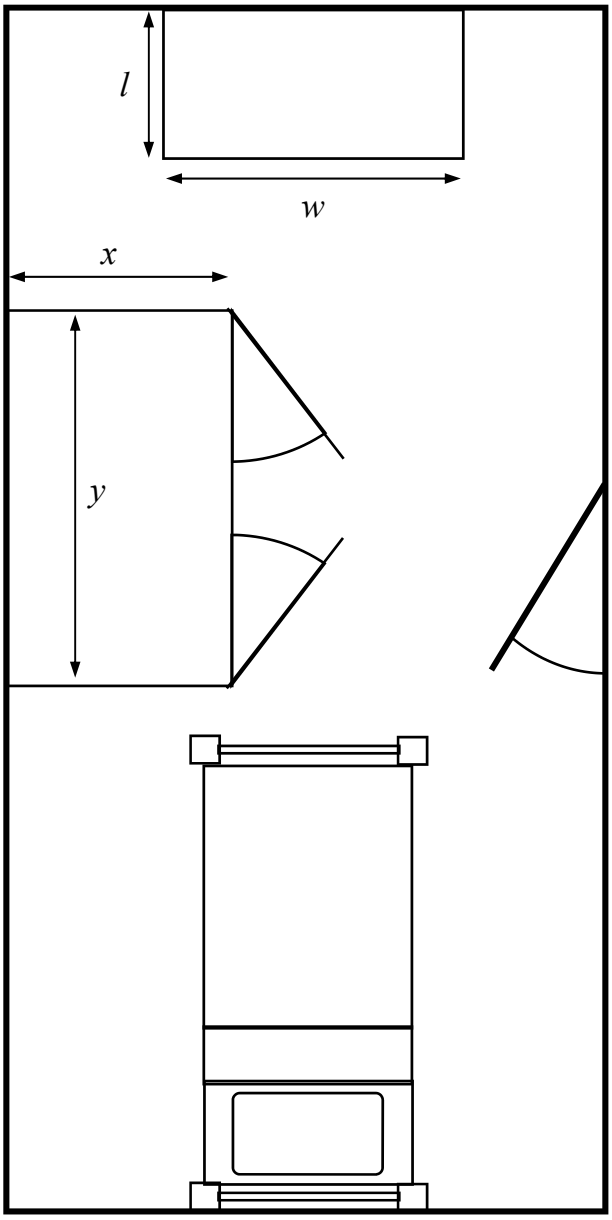
Actual object width, not diagram width

16 Desk depth (l) _____

17 Wardrobe depth (x) _____

18 Wardrobe width (y) _____

Scale 1 : 30



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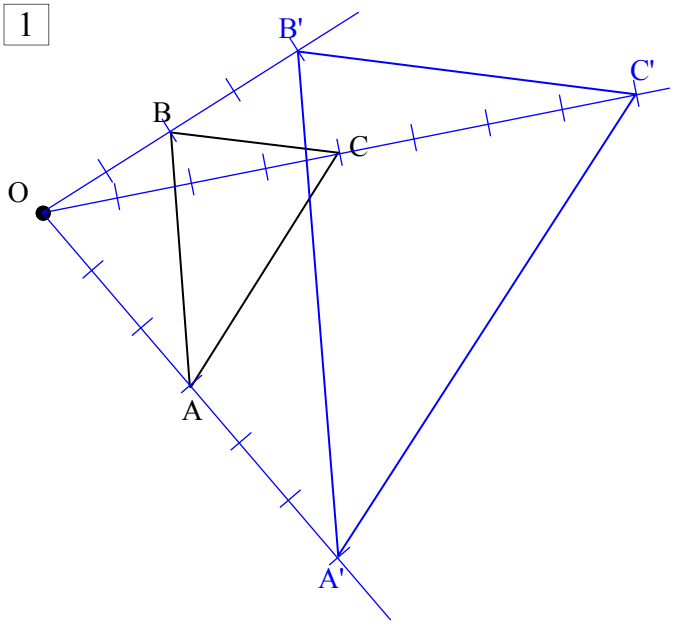
Enlarge triangle ABC by a scale factor of 2, using the centre of enlargement labeled 'O'.

1



The car below is drawn to a scale of 1 : 35. Measure the length of the wheelbase on the image then use the scale factor to give the actual car's wheelbase length.

2

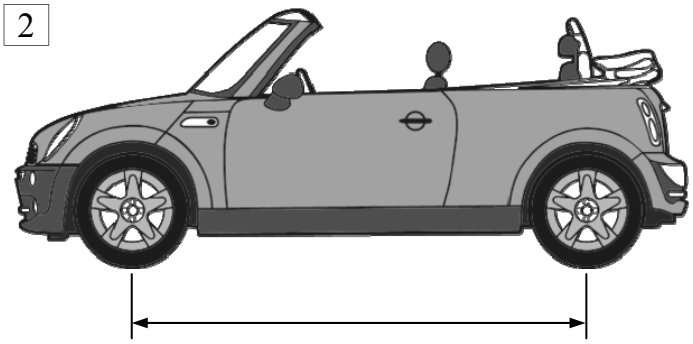
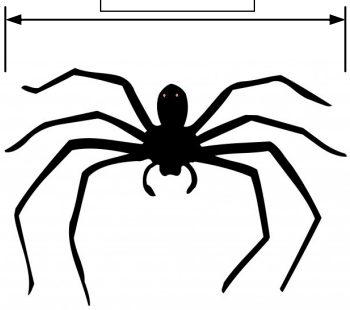


Image length (cm): 6 cm

Actual length (cm): 210 cm

The spider below is drawn to a scale of 3 : 2. Use the scale to calculate the width (cm) of this spider in real life.

3



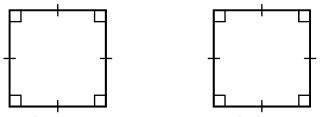
Actual length (cm):

Length = $4.5 \times \frac{2}{3}$

Length = 3 cm

Is the first square below similar, congruent or neither when compared to the second square? Give a reason.

4



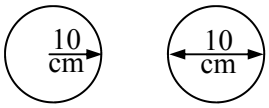
○similar
●congruent
○neither

Reason: Same side lengths and internal angles.

Is the first circle below similar, congruent or neither when compared to the second circle? Give a reason.

5

Not to scale



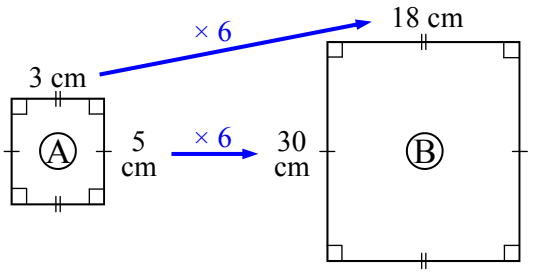
●similar
○congruent
○neither

Reason: Circles are either congruent or similar.

One has radius of 5 cm, the other 10 cm. Similar.

Calculate the scale factor used to get Rectangle B from Rectangle A.

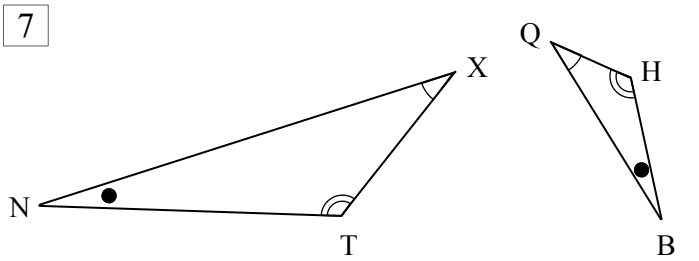
6



Scale factor: 6 (scale of 1 : 6)

The two triangles below are similar. Write a similarity statement for the two triangles.

7



△ NXT ||| △ BQH

List 3 pairs of matching (similar) sides for the triangles.

8

NX & BQ, XT & QH, TN & HB.

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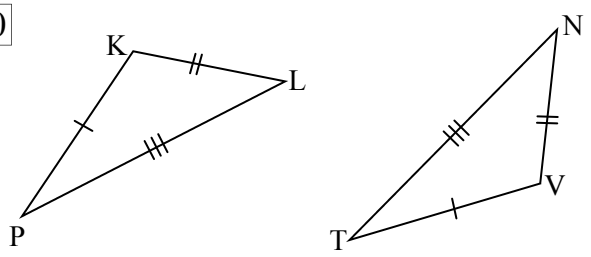
The triangles below are congruent. Write a congruence statement for the two triangles.

10



△PKL ≡ △TVN

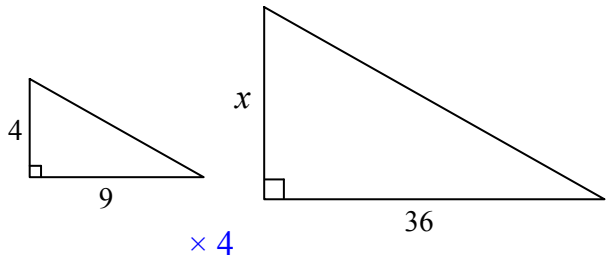
Complete the equivalent ratios below.

11 5 : 6 = 30 : 36

12 10 : 4 = 20 : 8

Find the unknown length on the triangles. (Not to scale). The two triangles are similar.

13

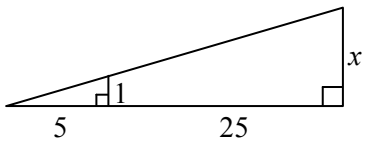


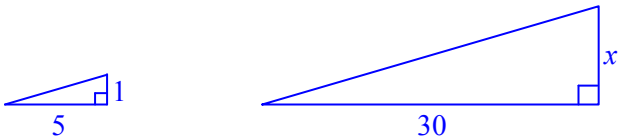
× 4 →

9 : 4 = 36 : x

x = 16

14





× 6 →

5 : 1 = 30 : x

x = 6

A plan view of Lucy's room is shown below. The scale is 1 : 30. Find the actual lengths of these objects in cm.

15 Desk width (w) 2 cm = 60 cm

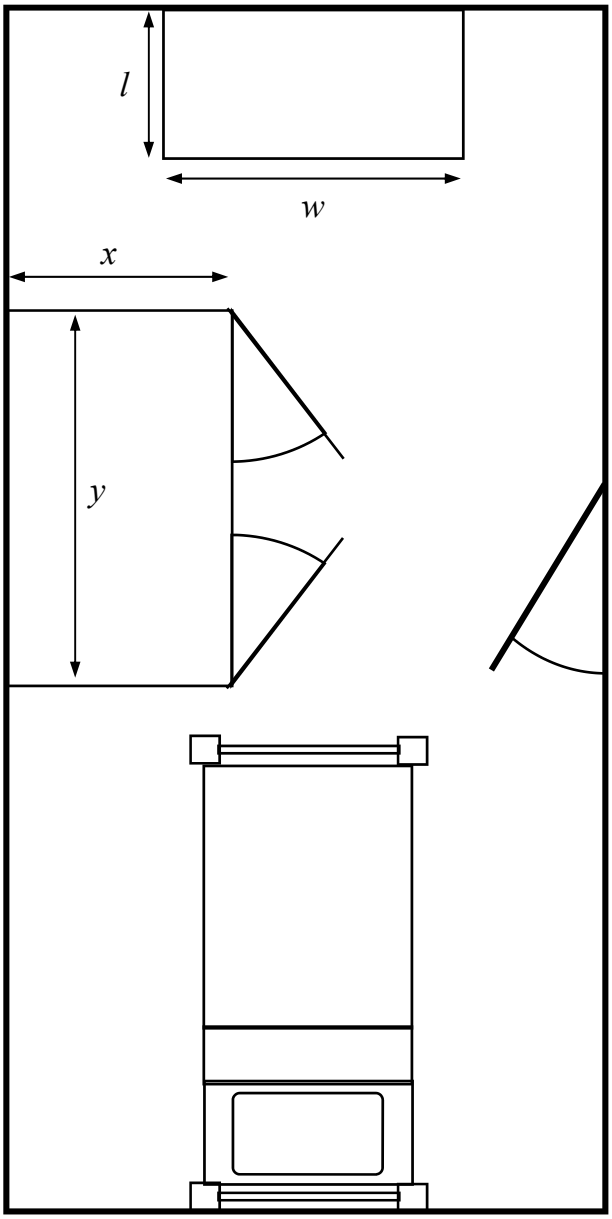
Actual object width, not diagram width

16 Desk depth (l) 4 cm = 120 cm

17 Wardrobe depth (x) 3 cm = 90 cm

18 Wardrobe width (y) 5 cm = 150 cm

Scale 1 : 30



Geometrical Properties



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Geometrical Properties - Enlargements - Drawing Exercise (Sheet 1)

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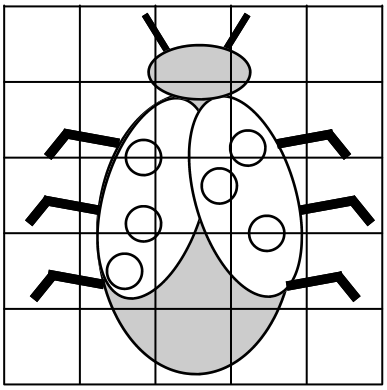

Reproduce the Ladybird beetle in the space below. Once you are done colour the wings red and the body and spots black.



Geometrical Properties - Enlargements - Drawing Exercise (Sheet 2)

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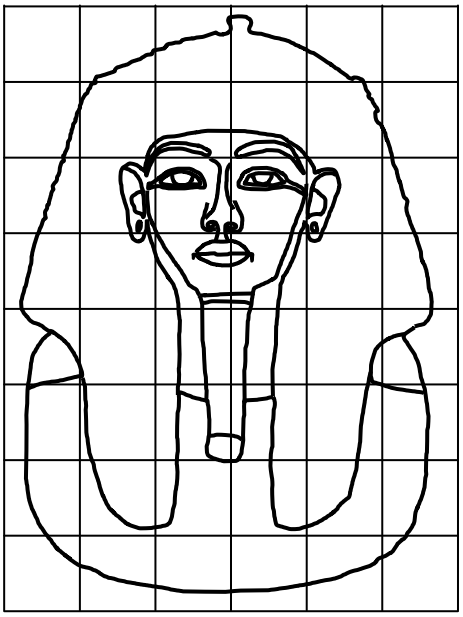

Reproduce the Death Mask of King Tutankhamun in the space below. Once you are done perform an image search on the web to colour it in correctly.



Geometrical Properties - Angles & Similarity - Practical (Sheet 1)

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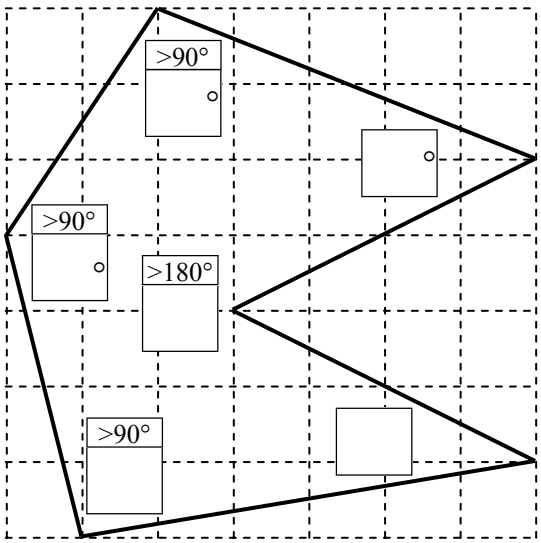
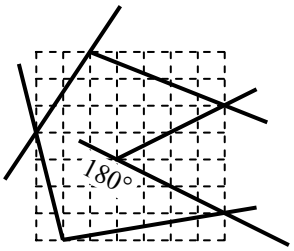
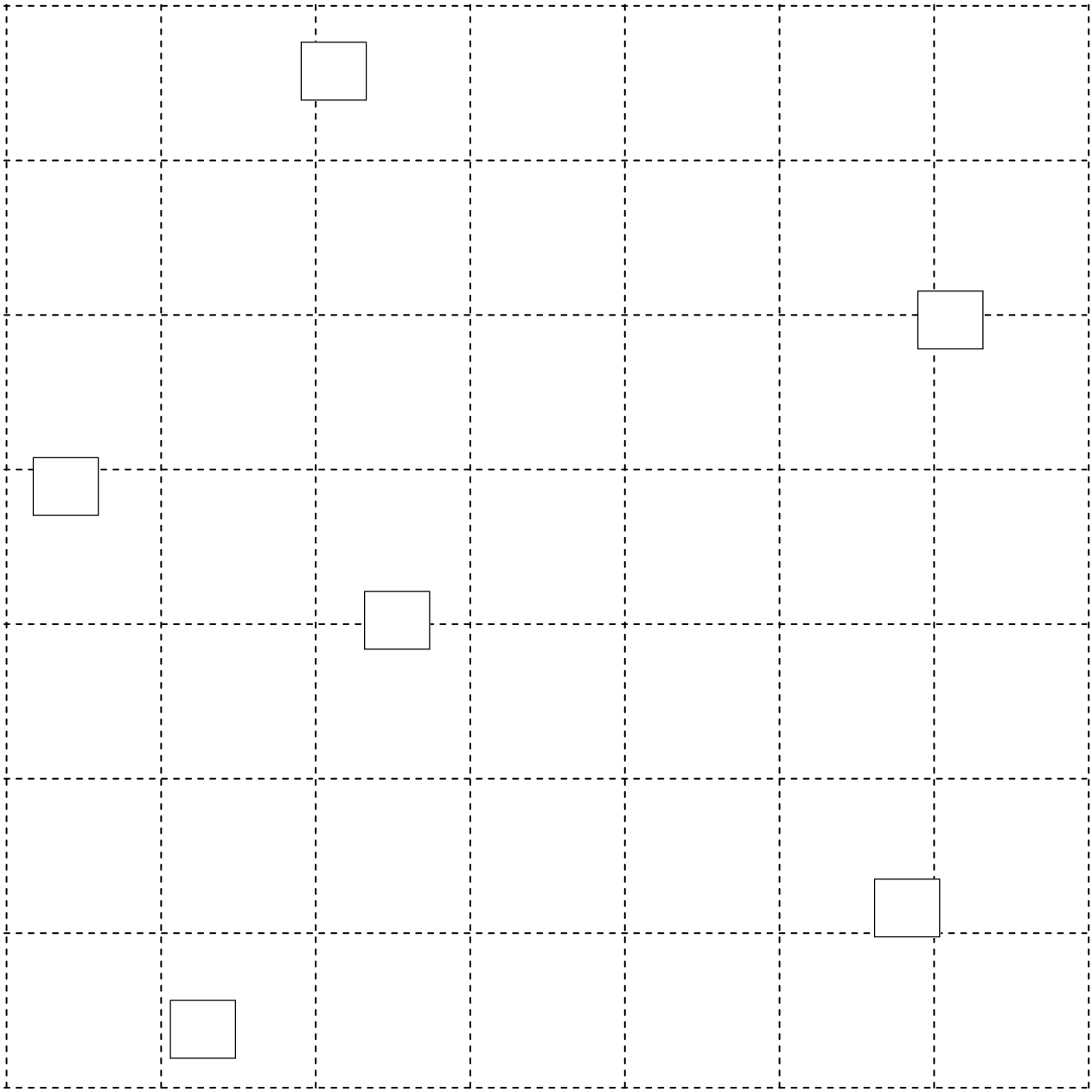
Use a protractor to measure internal angles in the polygon. Write all the angles in the boxes. You can extend lines to use your protractor. Then enlarge this polygon to create a similar figure using the grid. Measure the angles in the similar figure. Do they match?



Extend lines to allow protractor use.



Steps:
- Measure angles in smaller figure
- Construct enlarged (similar) polygon
- Measure angles in enlargement.



Geometrical Properties - Angles & Similarity - Practical (Sheet 2)

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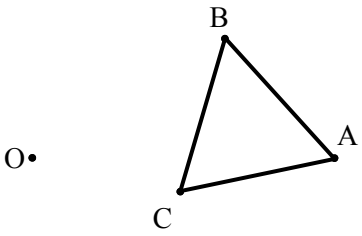
Enlarge the shapes shown using 'O' as the centre of enlargement. Label the similar figure as A'B'C', and so on. Measure the internal angles to confirm that internal angles are the same in similar figures.

1 Enlarge Triangle ABC by a scale factor of 3.

$\angle BAC = \angle B'A'C' =$ ☐°

$\angle ACB = \angle A'C'B' =$ ☐

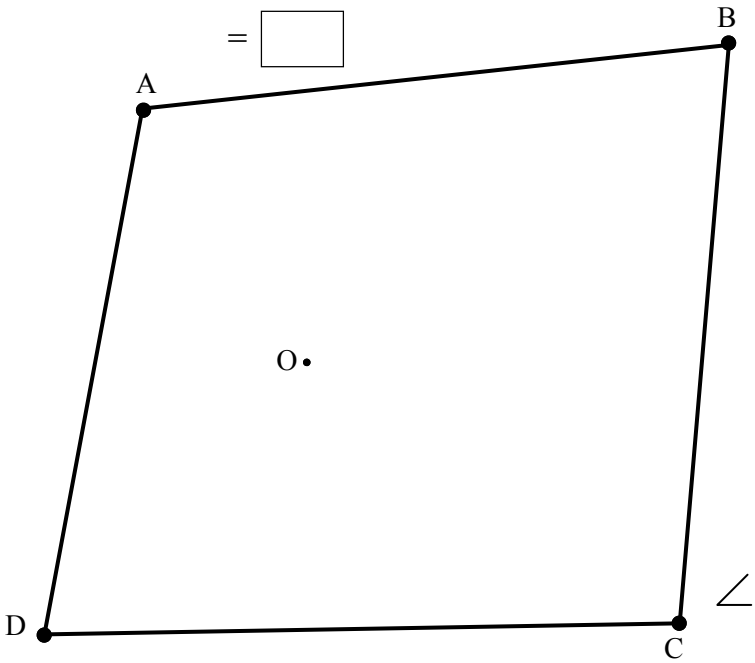
$\angle CBA = \angle C'B'A' =$ ☐



2 Enlarge Quadrilateral ABCD by a scale factor of 2.

$\angle DAB = \angle D'A'B' =$ ☐

$\angle ABC = \angle A'B'C' =$ ☐



$\angle BCD = \angle B'C'D' =$ ☐

$\angle CDA = \angle C'D'A' =$ ☐

Geometrical Properties - Further Enlargements (Ratios)

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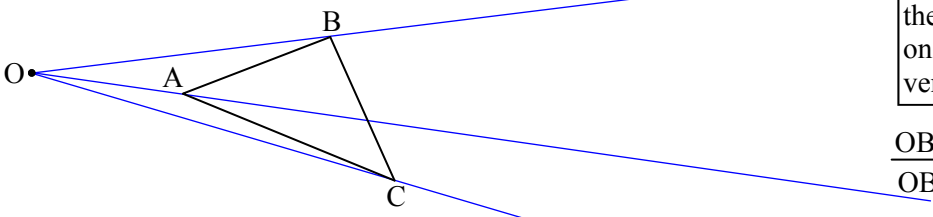
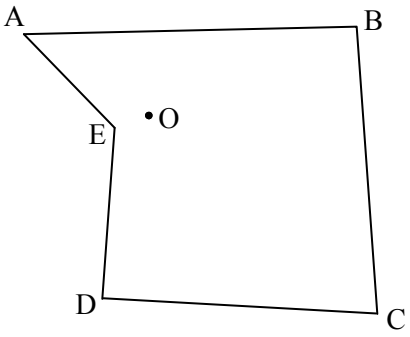
1 Enlarge Triangle ABC by a scale factor of 2.



Enlarge the shapes shown using 'O' as the centre of enlargement. Label the enlarged shape as A'B'C', and so on. Then complete the ratios to verify scale factor properties.

$\frac{OB'}{OB} = \frac{\square}{\square} = \square$ $\frac{A'B'}{AB} = \frac{\square}{\square} = \square$

2 Enlarge Pentagon ABCDE by a scale factor of 3.

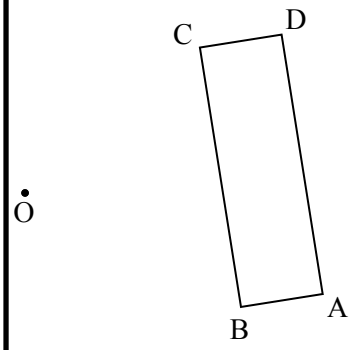


$\frac{OA'}{OA} = \frac{\square}{\square} = \square$

$\frac{B'C'}{BC} = \frac{\square}{\square} = \square$

$\frac{C'D'}{CD} = \frac{\square}{\square} = \square$

3 Enlarge Rectangle ABCD by a scale factor of 2.5.



$\frac{OD'}{OD} = \frac{\square}{\square} = \square$

$\frac{A'D'}{AD} = \frac{\square}{\square} = \square$

$\frac{B'D'}{BD} = \frac{\square}{\square} = \square$

Geometrical Properties - Similarity and Congruence (Sheet 1)

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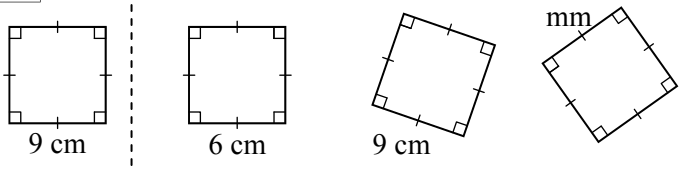
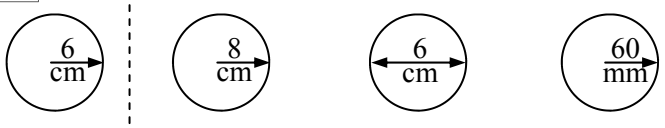
All squares, circles and equilateral triangles are either similar or congruent. Decide if these are congruent or similar to the original shape.

1



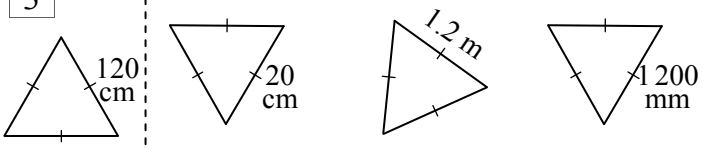
○ similar ○ congruent | ○ similar ○ congruent | ○ similar ○ congruent

2



○ similar ○ congruent | ○ similar ○ congruent | ○ similar ○ congruent

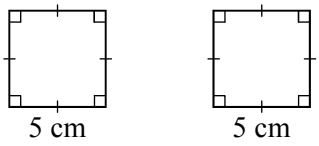
3



○ similar ○ congruent | ○ similar ○ congruent | ○ similar ○ congruent

State why these are similar or congruent. Use:
Congruent - all sides & angles are the same, or
Similarity - all sides enlarged by same scale factor.

4

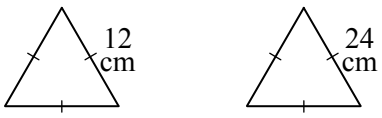


○ similar
○ congruent

Reason: _____

_____

5

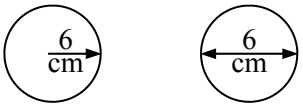


○ similar
○ congruent

Reason: _____

_____

6



Use the word 'radius' not 'side'

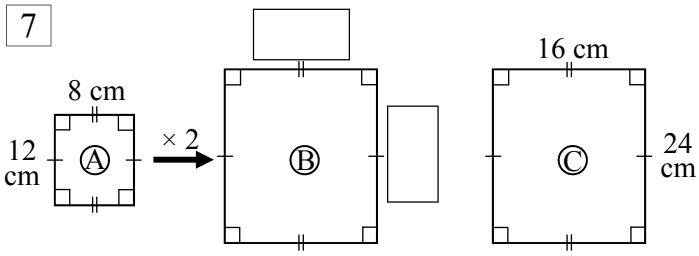
○ similar
○ congruent

Reason: _____

_____

Enlarge the first shape 'A' given a scale factor. If the enlarged shape 'B' is congruent with the third shape 'C', then you have similarity between A & C.

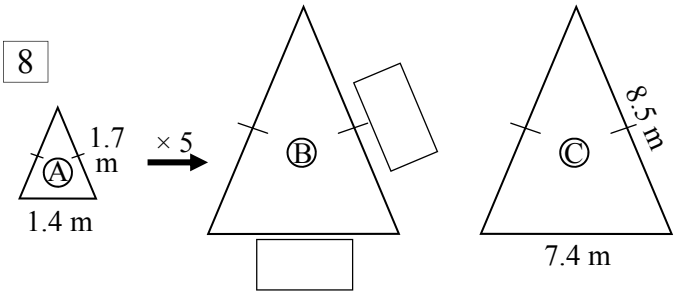
7



Because rectangle B ○is ○isn't congruent to rectangle C,

rectangle A ○is ○isn't similar to rectangle C.

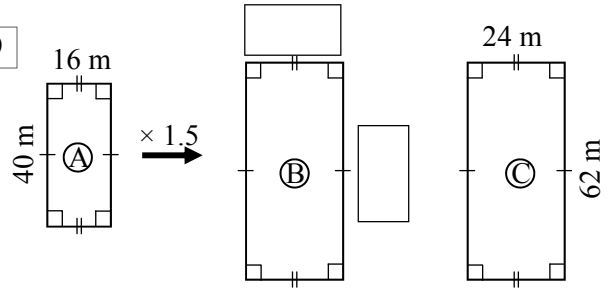
8



Because triangle B ○is ○isn't congruent to triangle C,

triangle A ○is ○isn't similar to triangle C.

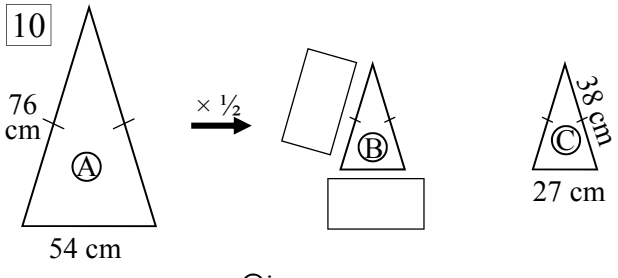
9



Because rectangle B ○is ○isn't congruent to rectangle C,

rectangle A ○is ○isn't similar to rectangle C.

10



Because triangle B ○is ○isn't congruent to triangle C,

triangle A ○is ○isn't similar to triangle C.

Geometrical Prop - Similarity and Congruence - Scale Factors (Sheet 2)

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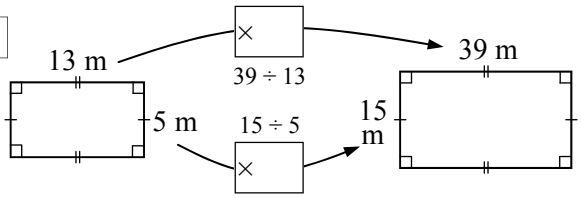
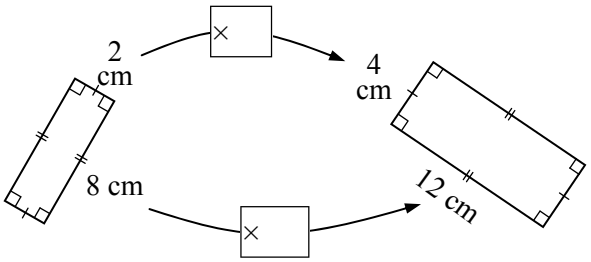
With rectangles, if the scale factor for both sides is the same, you have similarity. Test these rectangles for similarity.

1



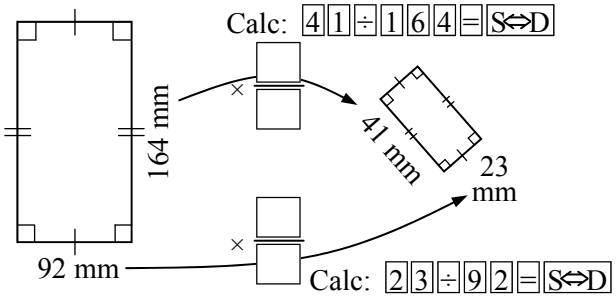
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

2



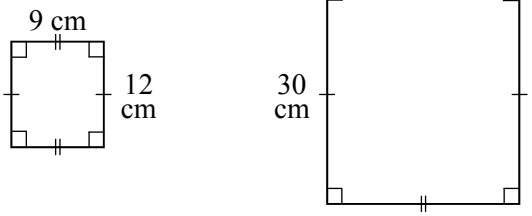
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

3



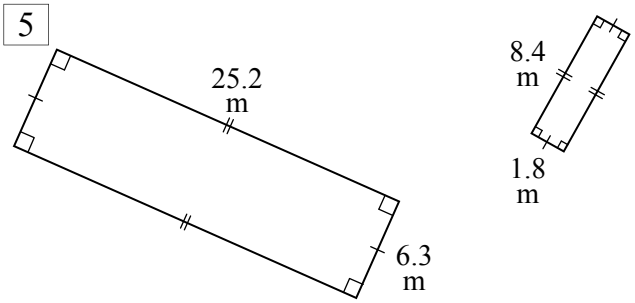
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

4



○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

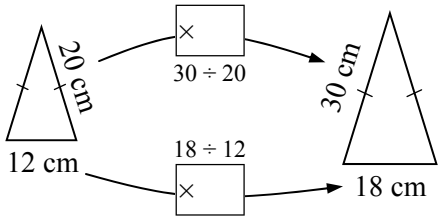
5



○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

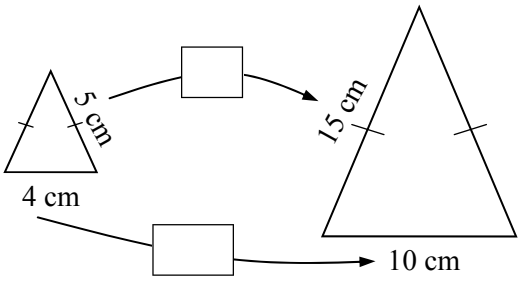
Like rectangles, triangles can be tested for similarity using the scale factor. Test these triangles for similarity.

6



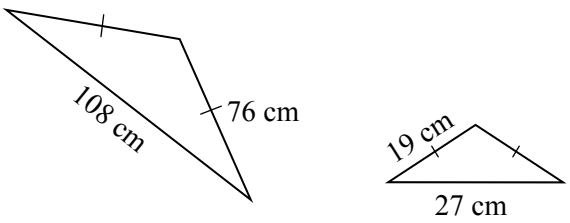
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

7



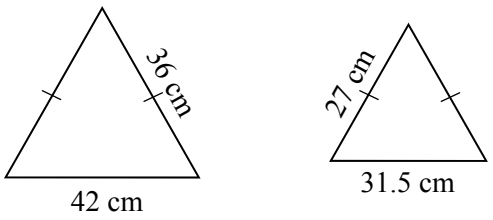
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

8



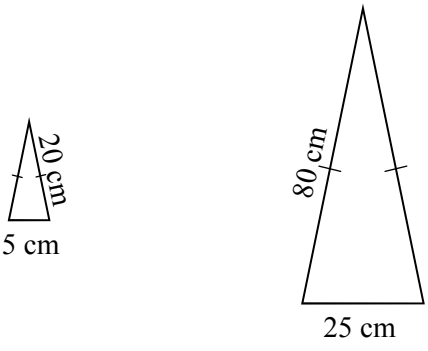
○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

9



○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

10



○ Similar ○ Not similar because the scale factor ○ is uniform ○ is inconsistent

Geometrical Props - Similar Figures & Similarity Statements (Sheet 1)

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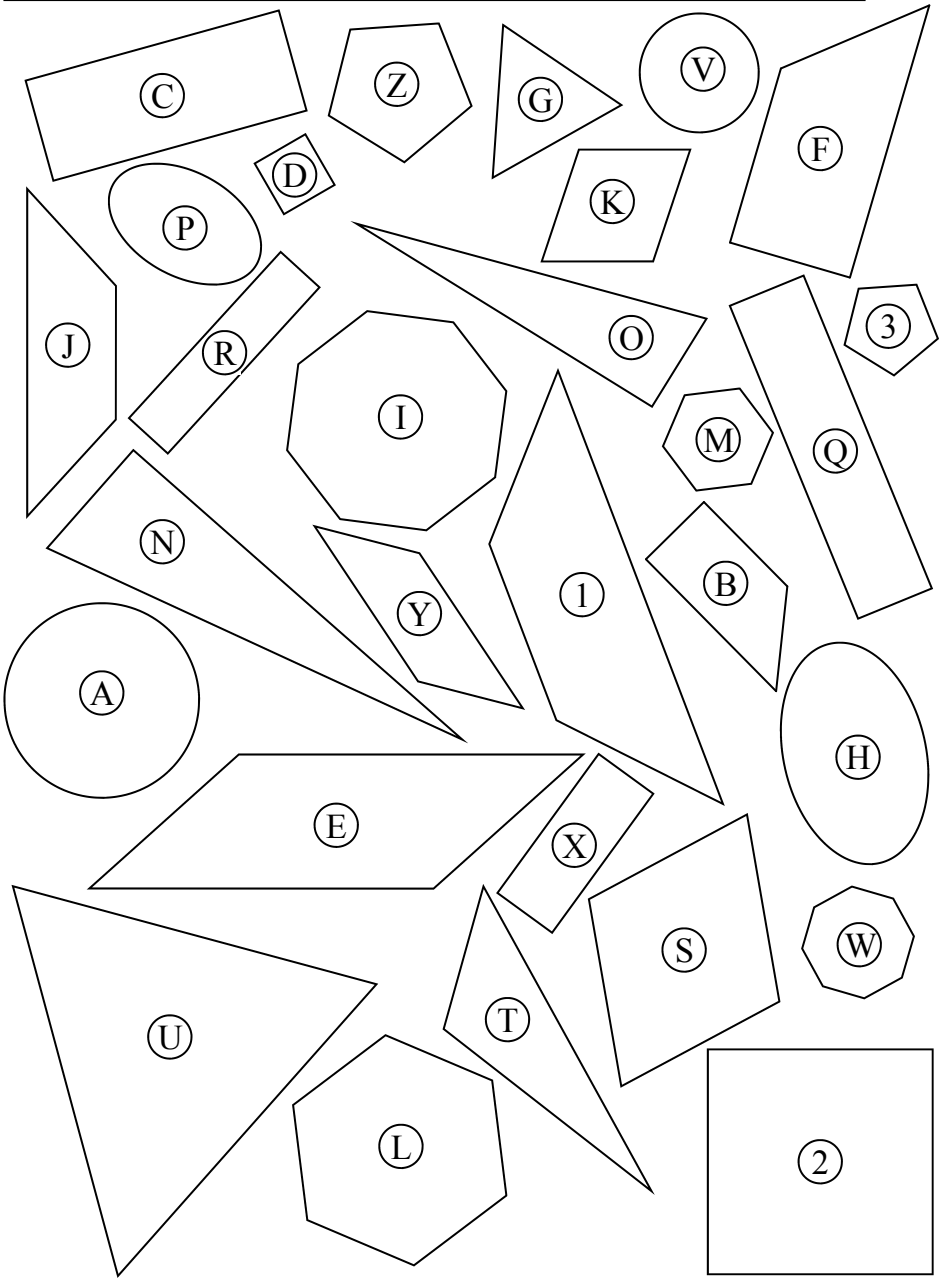
Similar figures have matching internal angles but have a scale factor applied to their side lengths. In other words, one is an enlargement of the other. Find all the similar pairs below, there is one non-paired shape.



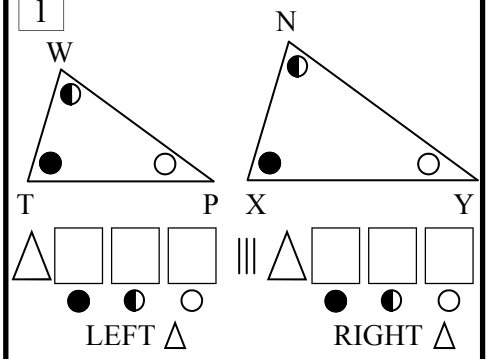
Write your matches below, once you are finished colour the similar shapes the same colour.

☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
	☐ & ☐	☐ & ☐	

There is one unmatched shape, it is: ☐

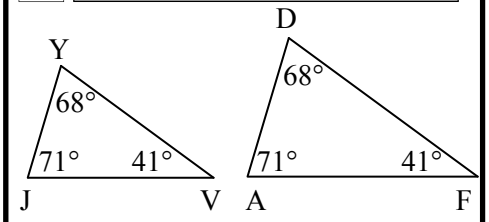
These triangles are similar. To state that, you have to start at matching vertices and go around the same direction.

1



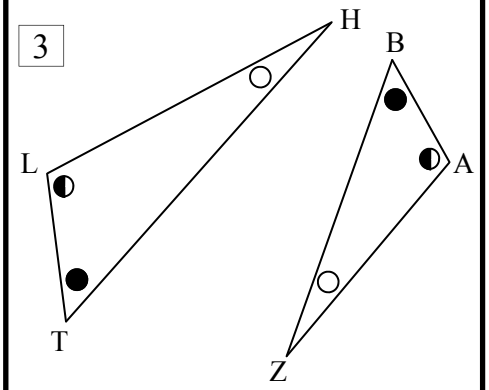
△ ☐ ☐ ☐ LEFT △ | ||| △ ☐ ☐ ☐ RIGHT △

2 Now use the matching angles



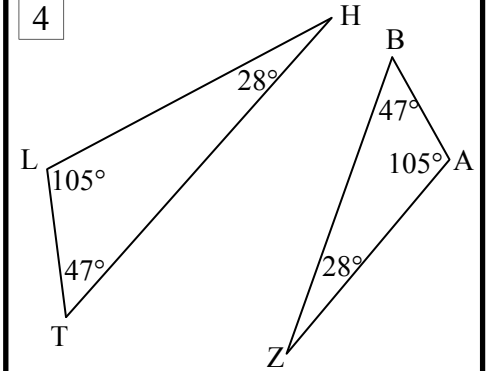
△ ☐ ☐ ☐ 71° 68° 41° LEFT △ | ||| △ ☐ ☐ ☐ 71° 68° 41° RIGHT △

3



△ ☐ ☐ ☐ | ||| △ ☐ ☐ ☐

4



△ ☐ ☐ ☐ 47° 105° 28° | ||| △ ☐ ☐ ☐ 47° 105° 28°

Geometrical Props - Similar Figures & Similarity Statements (Sheet 2)

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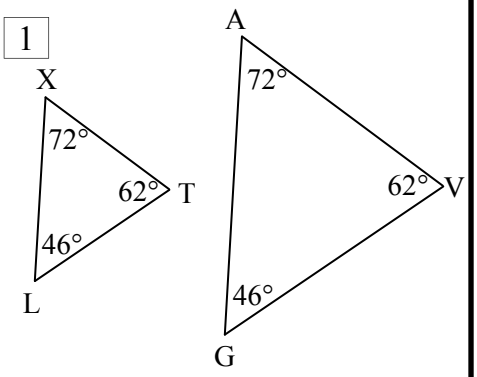
Write the similarity statements for these. Start at matching vertices and travel one way.

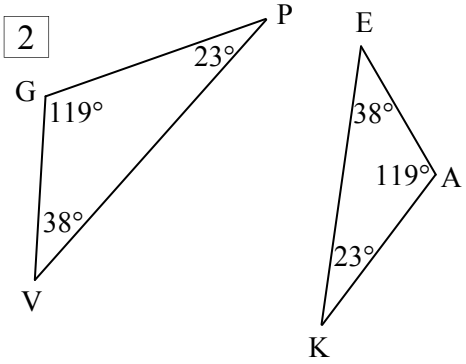
1



△ ☐☐☐ ||| △ ☐☐☐

46° 72° 62° 46° 72° 62°

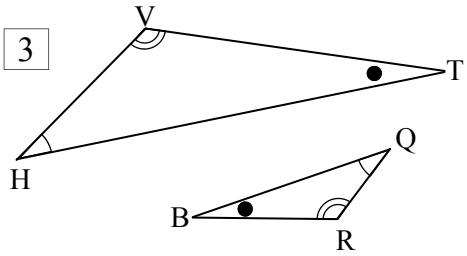
2



△ ☐☐☐ ||| △ ☐☐☐

38° ← Start at 38°, it is easier to mark

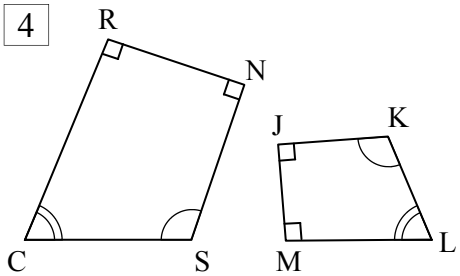
3



△ ☐☐☐ ||| △ ☐☐☐

Start at H

4

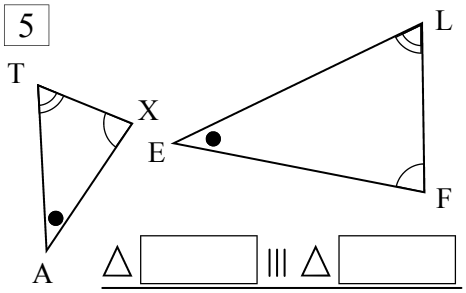


☐☐☐☐ ||| ☐☐☐☐

Start at C - Four letters required

Write similarity statements for these similar shapes.

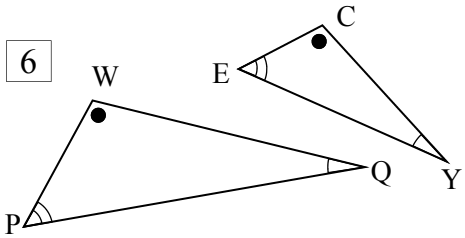
5



△ ☐☐☐ ||| △ ☐☐☐

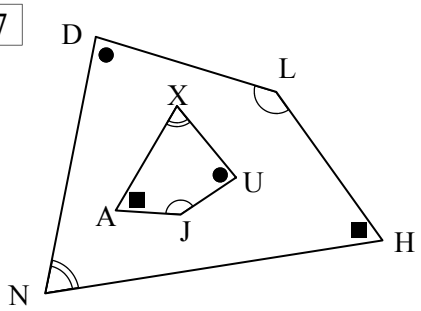
Start at A

6



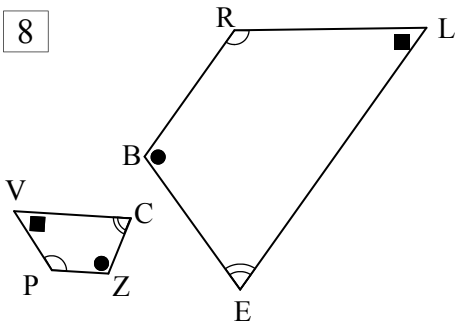
Start at P

7



Start at N

8



Start at P

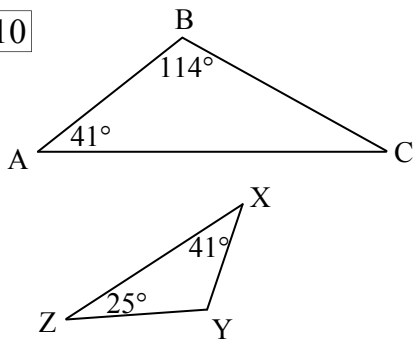
9 List the matching sides from Q8:

PV & ☐☐ & ☐☐

☐☐ & ☐☐ & ☐☐

Write similarity statements for these similar shapes. Then list matching sides and angles.

10



Start at A

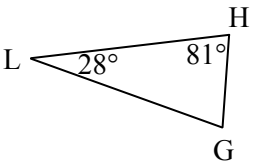
11 Angles: ∠A = ∠☐

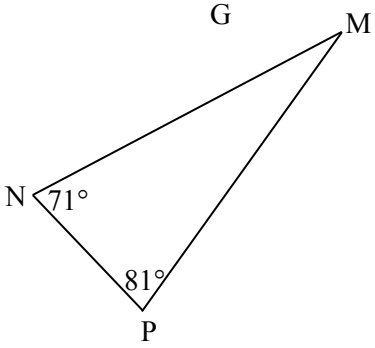
∠☐ = ∠☐ ∠☐ = ∠☐

12 Sides: AB & ☐☐

☐☐ & ☐☐ ☐☐ & ☐☐

13





Start at L

14 Angles: _____

_____

15 Sides: _____

_____

Geometrical Properties - Matching Congruent Shapes (Sheet 1)

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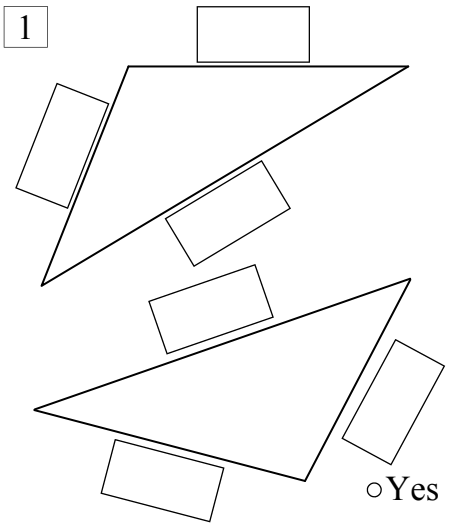
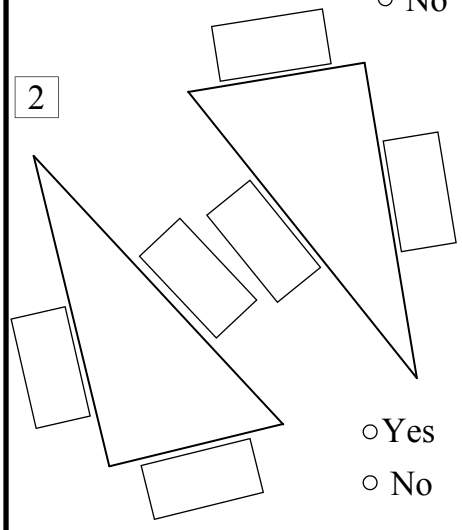
Using a ruler, measure and then write the side lengths on the triangles. Are they congruent?

1



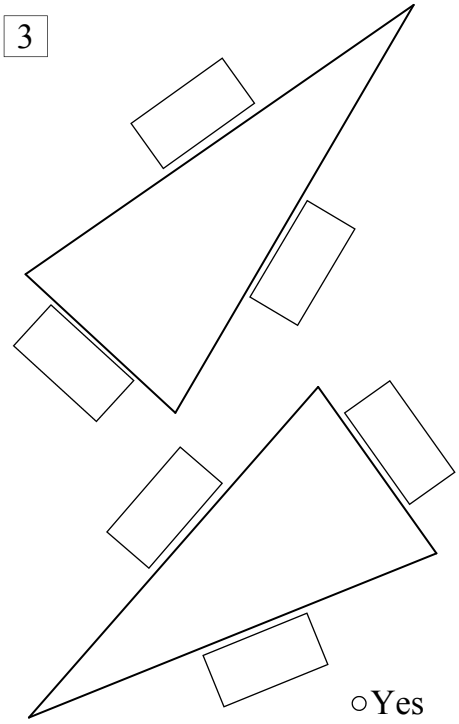
○ Yes
○ No

2



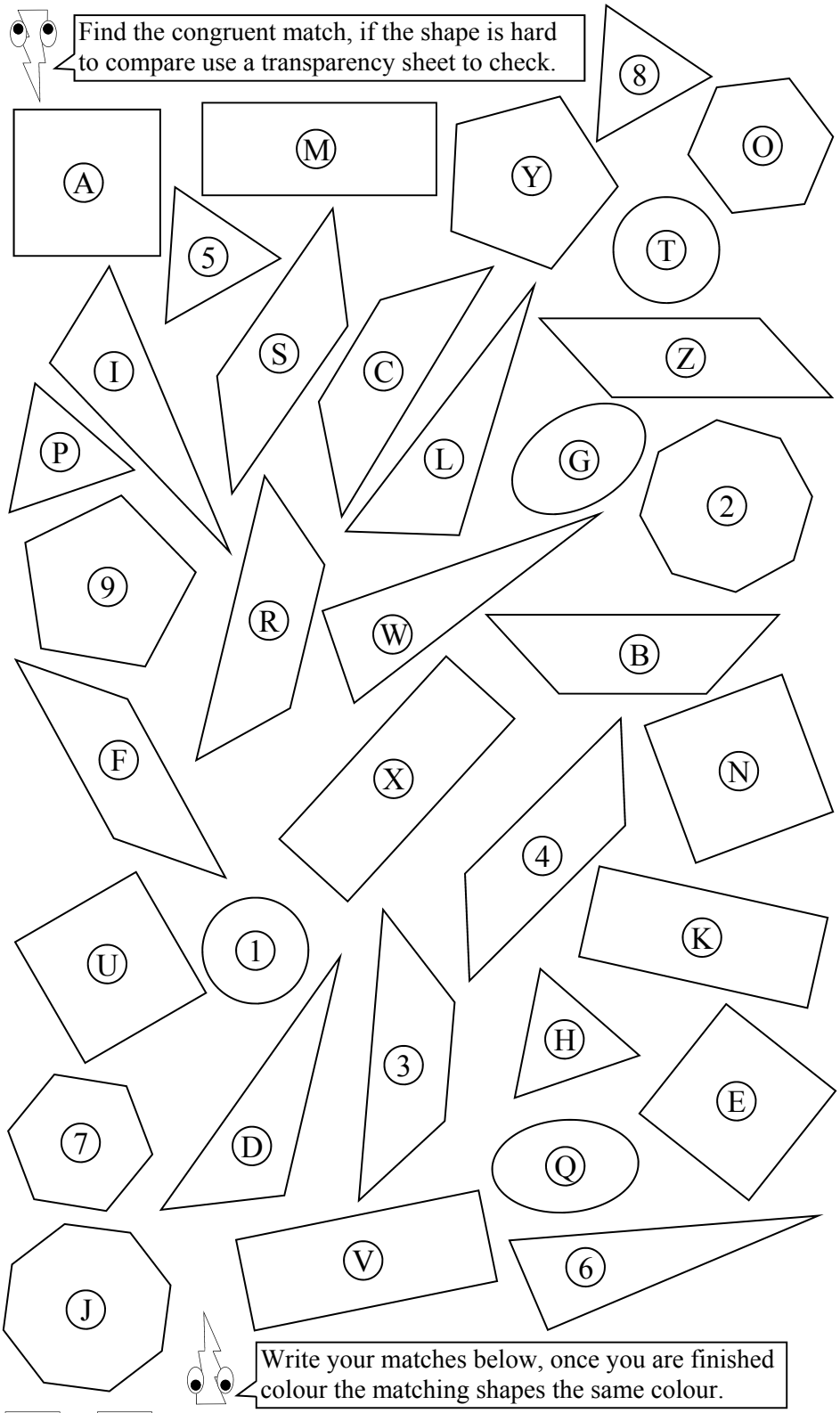
○ Yes
○ No

3



○ Yes
○ No

Find the congruent match, if the shape is hard to compare use a transparency sheet to check.



Write your matches below, once you are finished colour the matching shapes the same colour.

☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐			

There is one unmatched shape, it is: ☐

Geometrical Properties - Congruence Statements (Sheet 2)

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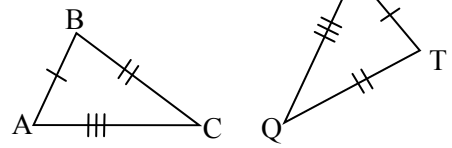
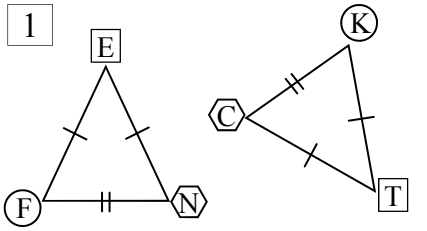
A congruence statement has the matching vertices named in the same order. Write a congruence statement for these.

Example



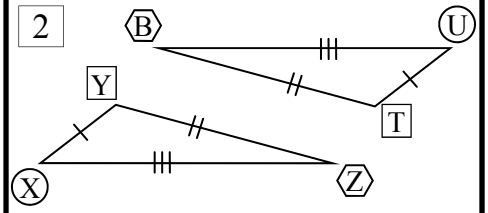
$\triangle ABC \equiv \triangle MTQ$

1



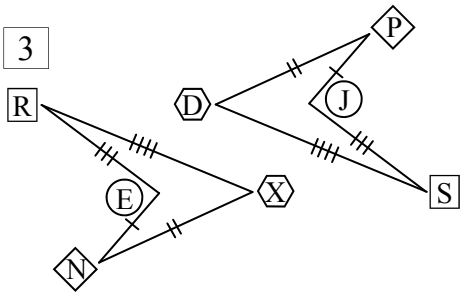
$\triangle\bigcirc\square\hexagon \equiv \triangle\bigcirc\square\hexagon$

2



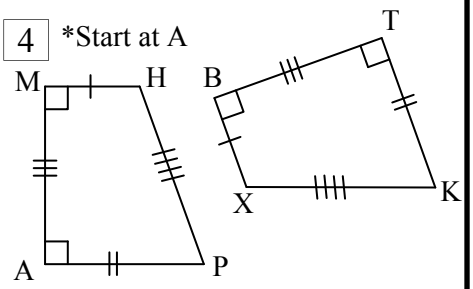
$\triangle\bigcirc\square\hexagon \equiv \triangle\bigcirc\square\hexagon$

3

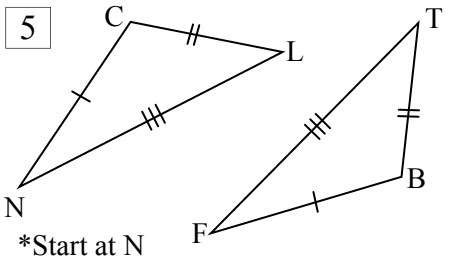


$\bigcirc\square\hexagon\diamond \equiv \bigcirc\square\hexagon\diamond$

4 *Start at A



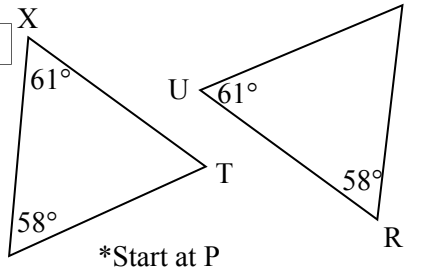
5



*Start at N

State which angles and sides are equal & complete congruence statements for these congruent shapes.

6



*Start at P

$\angle P = \angle R$ $\angle X = \angle\square$

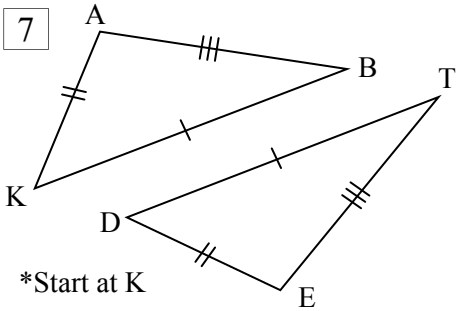
$\angle T = \angle\square$ $XP = \square$

$PT = \square$ $XT = \square$

$\triangle\square \equiv \triangle\square$

Remember same starting point & order.

7



*Start at K

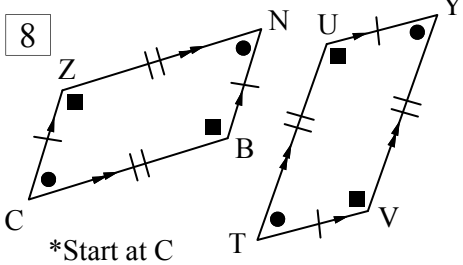
$\angle K = \angle\square$ $\angle\square = \angle\square$

$\angle\square = \angle\square$ $AK = \square$

$\square = \square$ $\square = \square$

$\triangle\square \equiv \triangle\square$

8



*Start at C

$\angle\square = \angle\square = \angle\square = \angle\square$

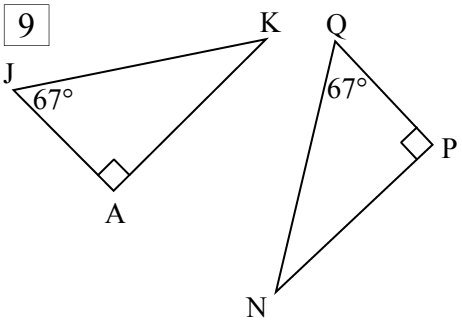
$\angle\square = \angle\square = \angle\square = \angle\square$

$\square = \square = \square = \square$

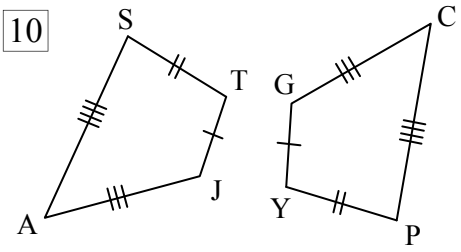
$\square = \square = \square = \square$

$\square \equiv \square$

9

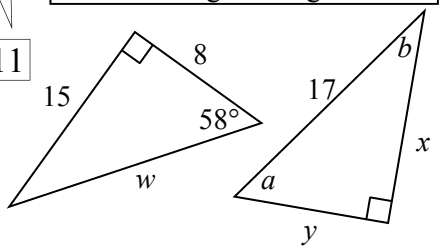


10



Find the value of all unknowns, for these congruent figures.

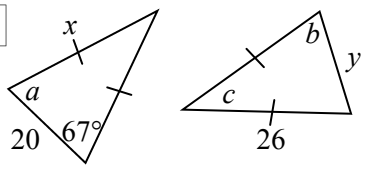
11



$a =$ $b =$ $w =$

$x =$ $y =$

12



$a =$ $b =$ $c =$

$x =$ $y =$

Geometrical Properties - Similarity - Finding Sides (Sheet 1)

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Let's start with a quick review of equivalent ratios. Find the value of the box in the ratios.

1 $5 : 2 = 30 : \square$ (×6 ×6)

2 $4 : 3 = 16 : \square$ (×4 ×4)

3 $2 : 3 = 22 : \square$ (×11 ×11)

Now find the scale factor (multiplier) yourself.

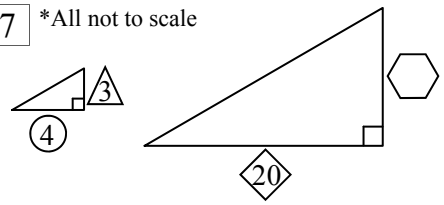
4 $5 : 2 = 15 : \square$ (× □ × □)

5 $1 : 6 = 7 : \square$ (× □ × □)

6 $5 : 12 = 10 : \square$ (× □ × □)

Build the ratios from these similar triangles*

7 *All not to scale



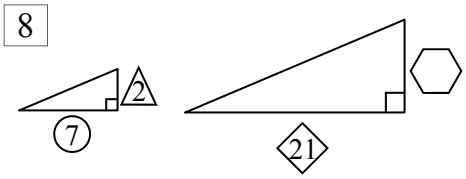
× □ × □

□ : △ = ◇ : ⬡

Base Height Base Height

Left △ Right △

8



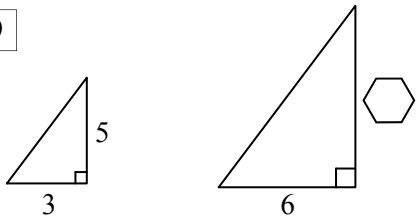
× □ × □

□ : △ = ◇ : ⬡

Base Height Base Height

Left △ Right △

9



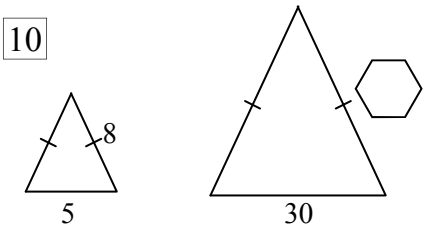
× □ × □

□ : □ = □ : ⬡

Base Height Base Height

Left △ Right △

10

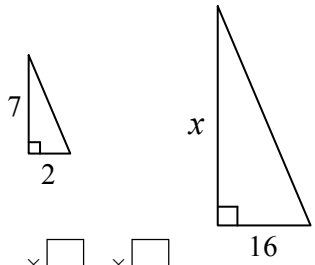


× □ × □

□ : □ = □ : ⬡

From now on the unknown side is 'x'

11

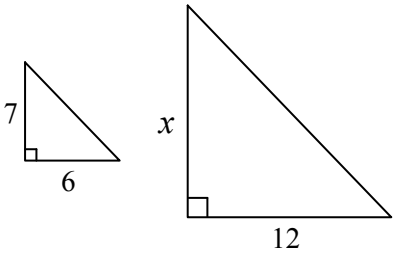


× □ × □

□ : □ = □ : x

$x = \square$

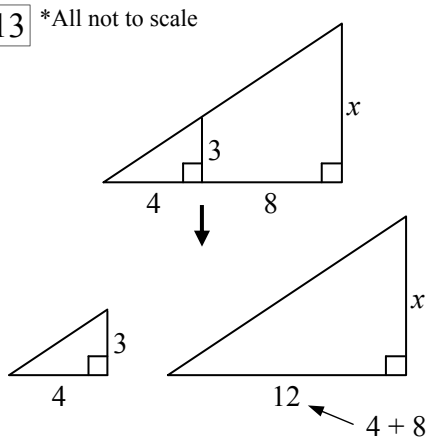
12



$x = \square$

Deconstruct these and solve like before. Change the diagram to two triangles. All are similar.

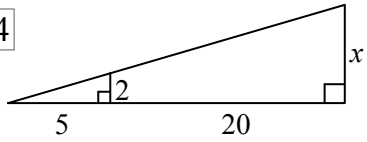
13 *All not to scale



□ : □ = □ : x

$x = \square$

14

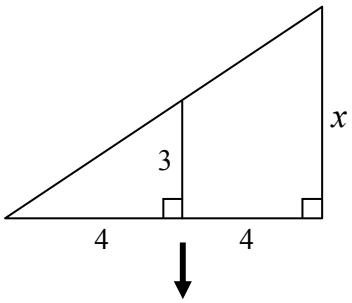


Redraw 2 triangles

□ : □ = □ : x

$x = \square$

15



Geometrical Properties - Similarity - Finding Sides (Sheet 2)

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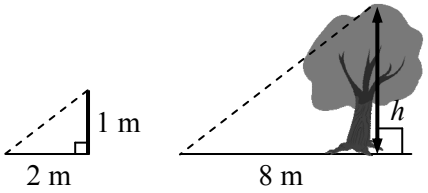
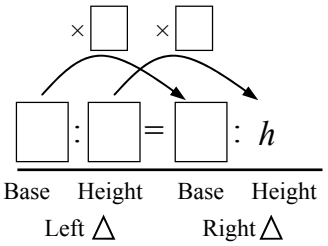
Find the height of these objects, by using similarity to calculate the unknown height.

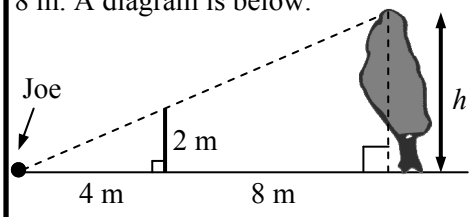
1 A 1 m high post casts a 2 m long shadow. How tall is a tree that casts an 8 m long shadow?



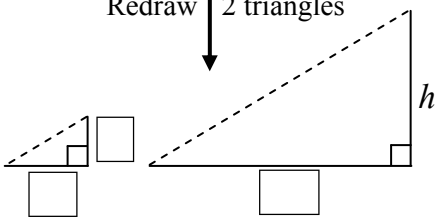


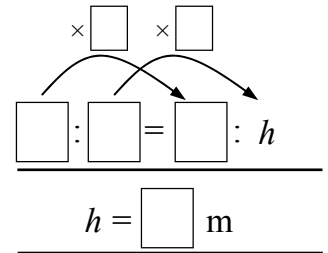
$h =$ ☐ m

2 Joe looks at the top of a tree and moves until a 2 m post's top is in line with the top of the tree. The post is 4 m away. He then measures the distance from the post to the tree, it is 8 m. A diagram is below.



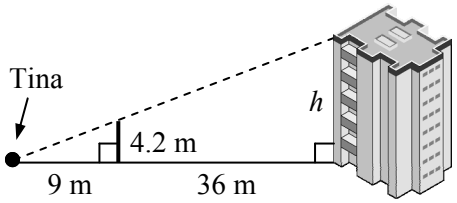
Redraw | 2 triangles





$h =$ ☐ m

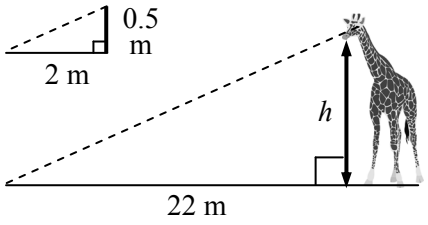
3 Tina looks at the top of a building and she moves until the top of a 4.2 m post is in line with the building's top. The post is 9 m away, the building is 36 m from the post. Diagram below.



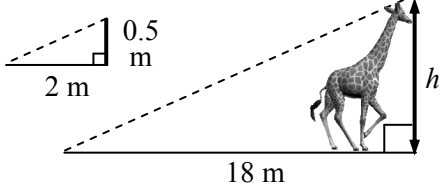
Redraw | 2 triangles

Find the height of the building.

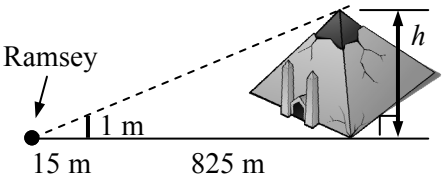
4 A 0.5 m high post casts a 2 m long shadow. How tall is 'Spotty' the giraffe who casts an 22 m shadow?



5 'Shorty' the giraffe is standing next to Spotty. He casts an 18 m long shadow. Calculate Shorty's height.



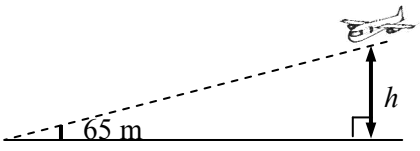
6 Ramsey is estimating the height of a pyramid using a 1 m vertical staff. The staff is 15 m away and its top is in the line of sight of the top of the pyramid, which is 825 m away.



Redraw | 2 triangles

Find the height of the pyramid.

6 Brie is looking at a plane coming in to land. The plane is 6 km away from the flight control tower. Brie has the top of the tower in the same line of sight as the plane. If the tower is 1.2 km away and is 65 m tall, how high is the plane?



Redraw | 2 triangles

Find the height of the plane.

Geometrical Props - Scale Diagrams - Construction Plan Views (Sheet 1)

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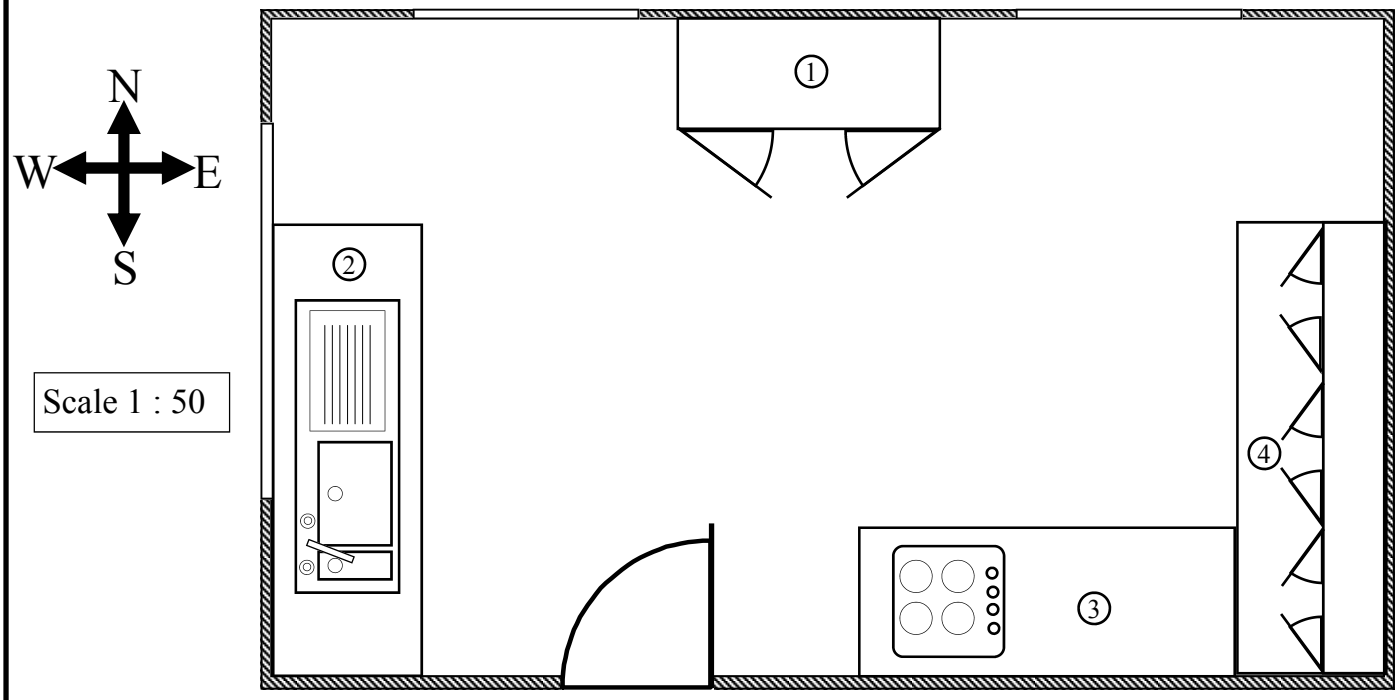

Scale 1 : 50

Get out your ruler and measure lengths in cm, to the nearest 0.5 cm. Use the scale to find the actual lengths. Round answers to 2 d.p where required.

1 The length of the eastern wall (in m).

Length = ____ cm = ____ cm = ____ m
(× 50 →) (÷ 100 →)

2 The length of the northern wall (in m).

Length = ____ cm = ____ cm = ____ m
(× 50 →) (÷ 100 →)

3 The floor area (empty) of the room (m^2).

Q.1 × Q.2

$A = lb$

$= \square \times \square$

$A = \square$ m^2

2 d.p.

4 The length, breadth and floor area of the pantry. ① on diagram.

Length = ____ m

Breadth = ____ m

Round to 2 d.p., in m^2.

5 The length, breadth and floor area of the sink bench. ② on diagram.

Length = ____ m

Breadth = ____ m

6 The length, breadth and floor area of the cook top. ③ on diagram.

Length = ____ m

Breadth = ____ m

7 The length, breadth and floor area of the cupboards. ④ on diagram.

Length = ____ m

Breadth = ____ m

8 Find the total covered floor area. = ① + ② + ③ + ④

Area = ____ + ____ + ____ + ____

Area = ____

9 Find the total exposed floor area. Q.3 - Q.8

Area =

10 Estimate the cost of tiling the exposed floor area when tiles cost \$35/m^2. Available in whole m^2 only.

Exposed floor area rounded up to whole m^2 = ____

Cost = ____ × ____ = ____

\$ sign

Geometrical Properties - Scale Diagrams - Using Maps (Sheet 2)

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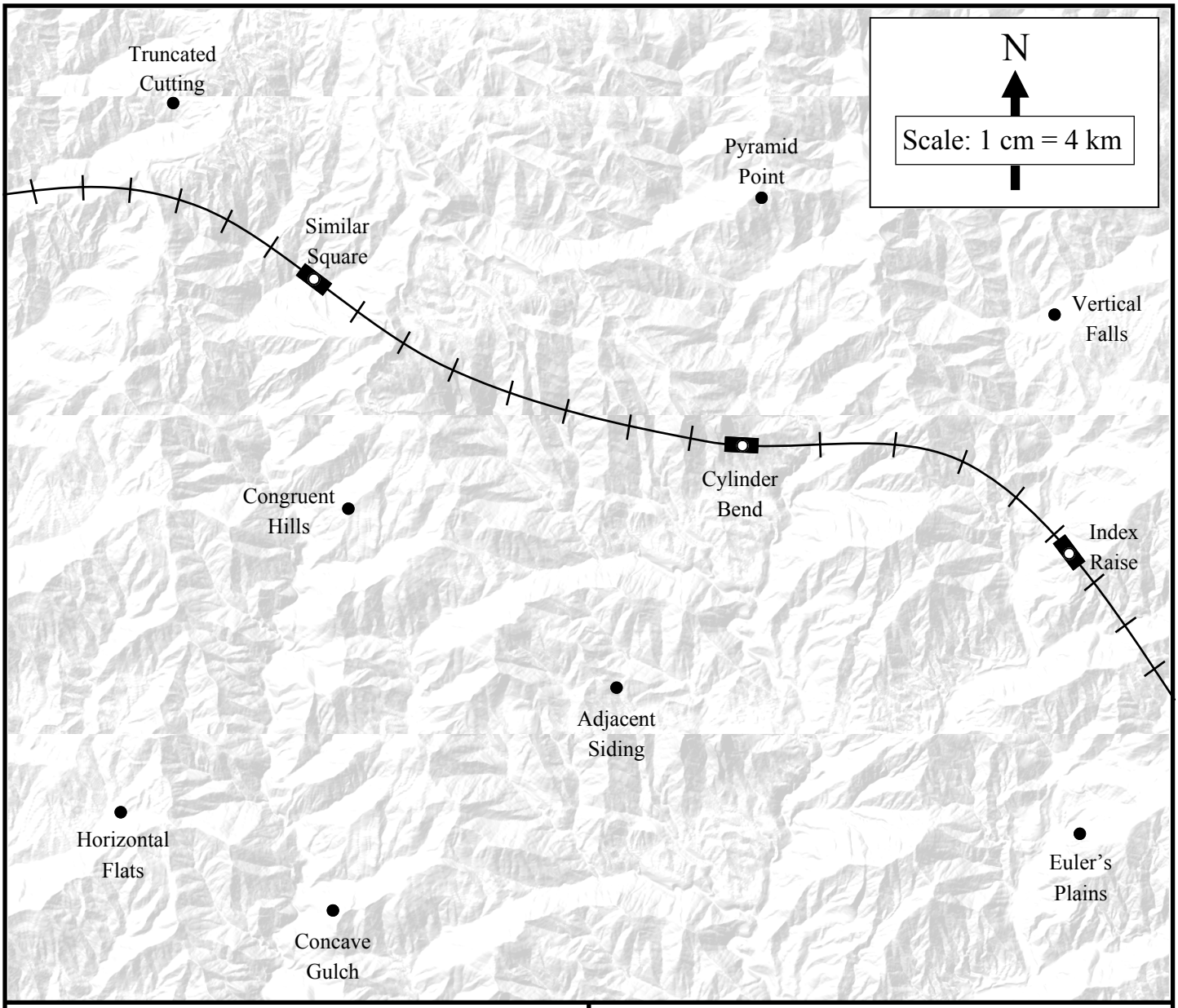

Get out your ruler and measure lengths in cm, to the nearest 0.1 cm. Use the scale (top right) to find the actual distances. Round answers to 1 d.p if required.

1 Index Raise station to Euler's Plains.

Distance = ☐.☐ cm = ☐ km

× 4

2 Similar Square station to Horizontal Flats.

Distance = ☐.☐ cm = ☐ km

READ THE NEXT QUESTION CAREFULLY!

3 Pyramid Point to Index Raise, **via Vertical Falls**.

Distance = ☐ cm + ☐ cm = ☐ cm = ☐ km

4 Concave Gulch to Cylinder Bend, via Adjacent Siding.

Distance = ☐ cm + ☐ cm = ☐ cm = ☐ km

Hansel travels from Truncated Cutting to Adjacent Siding via Congruent Hills. While **Gretel** travels from Truncated Cutting to Adjacent Siding via Pyramid Point.

5 Calculate the distance Hansel travels

6 Calculate the distance Gretel travels

7 Who travels the furthest and by how much?

Which station is the closest to **Adjacent Siding** and by how much? Similar Square Station or Index Raise Station?

8

Geometrical Properties - Further Scale Drawing (Sheet 1)

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Measure the sides of these shapes and then apply the scale factor to construct an enlargement. The construction will be larger than the original.

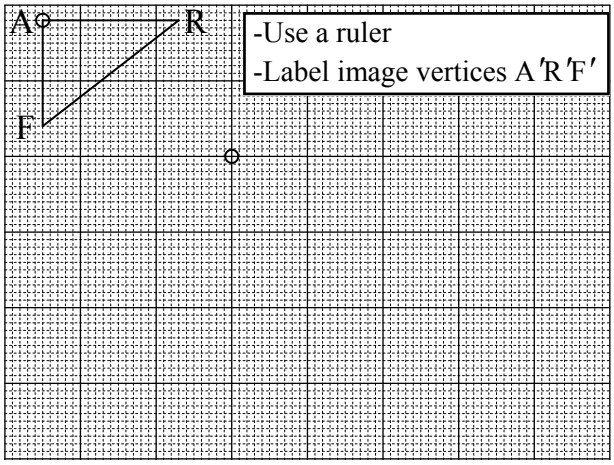
1 Scale = 2 : 5

A′R′: ☐ × $\frac{5}{2}$ = ☐ mm

Length in mm

A′F′: ☐ × $\frac{5}{2}$ = ☐ mm

Construct △A′R′F′ by drawing sides A′R′ and A′F′ first.

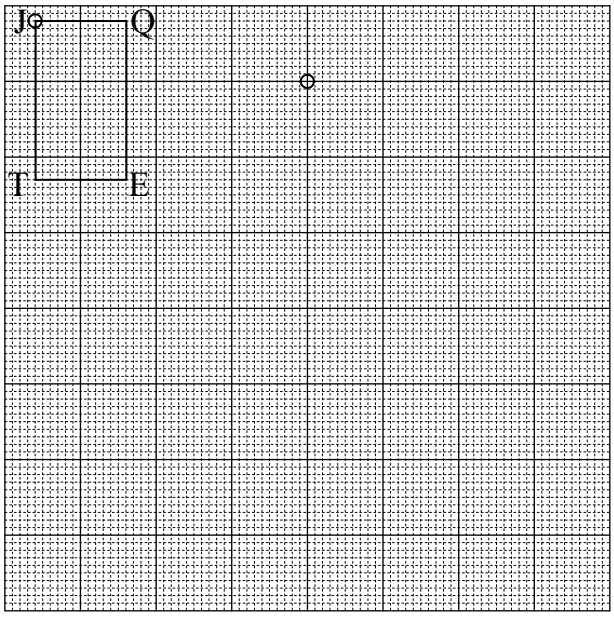


2 Scale = 3 : 8

J′Q′: ☐ × $\frac{\square}{\square}$ = ☐

J′T′: ☐ × $\frac{\square}{\square}$ = ☐

units!

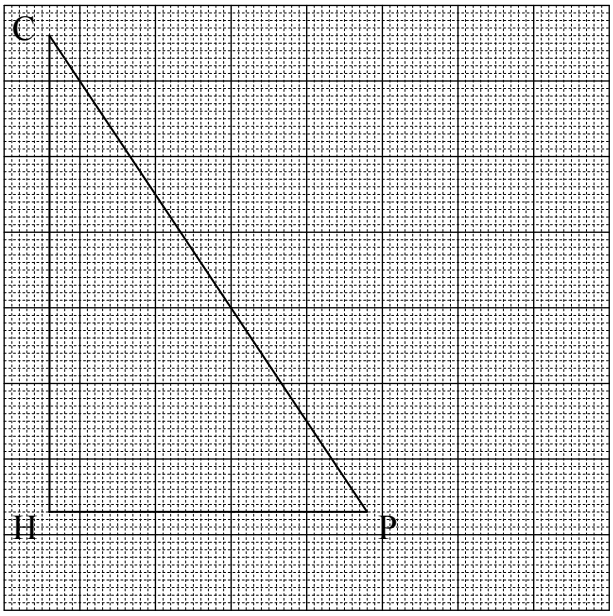


These are enlargements … but they are smaller!

3 Scale = 7 : 2

C′H′: ☐ × $\frac{\square}{\square}$ = ☐

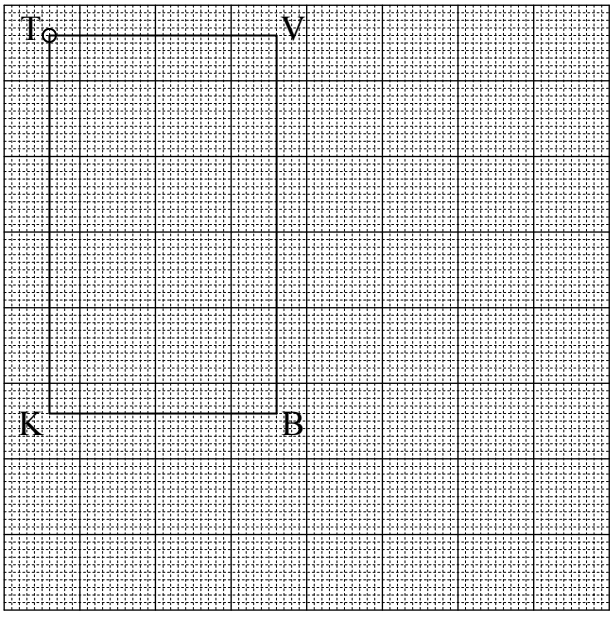
H′P′: ☐ × $\frac{\square}{\square}$ = ☐



4 Scale = 10 : 3

T′V′: ☐ × $\frac{\square}{\square}$ = ☐

T′K′: ☐ × $\frac{\square}{\square}$ = ☐



Geometrical Properties - Further Scale Drawing (Sheet 2)

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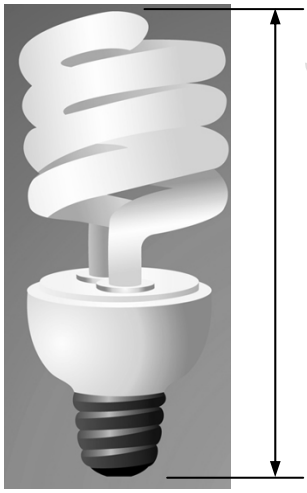
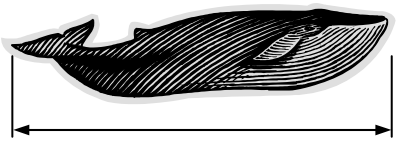
Measure the lengths in mm, using the arrowed length. Write the length in the 'image' box below. Apply the scale to find the actual length. Multiply by the second number in the ratio, divide by the first.

1 Scale = 1 : 2



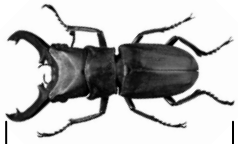
2 Scale = 1 : 660



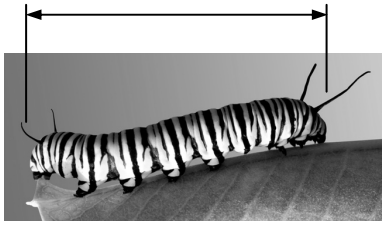
3 Scale = 9 : 1



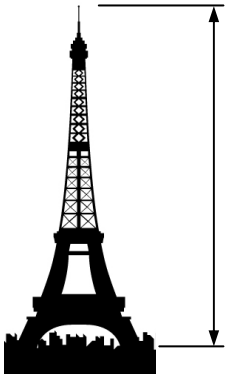
4 Scale = 3 : 2



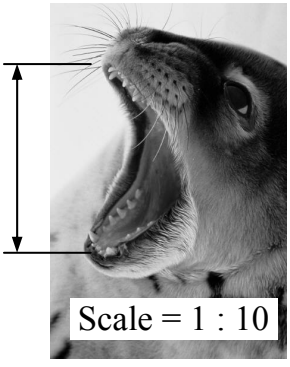
5 Scale = 5 : 3



6 Scale = 1 : 7 200

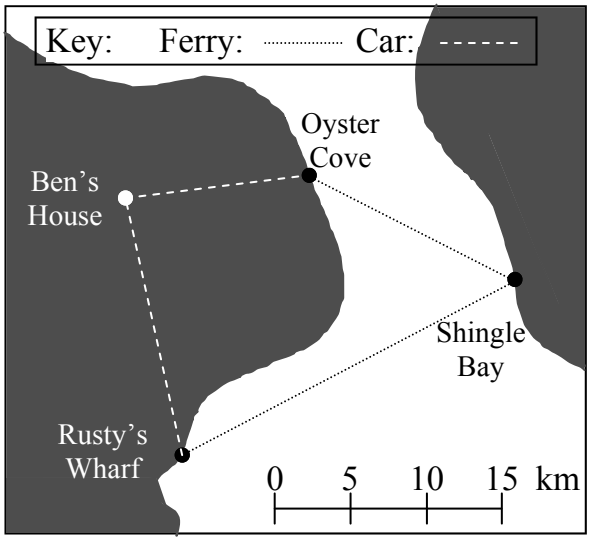


7



Q	Image Length (mm)	Actual Length (mm)
1		
2		mm = m
3		
4		
5		mm = cm
6		mm = m
7		mm = cm

Ben drove from Rusty's Wharf to Oyster Cove, while Tim caught the ferry. Use the scale to answer these questions.



Find the distance in km between:

8 Rusty's Wharf and Shingle Bay _____

9 Shingle Bay and Oyster Cove _____

10 Rusty's Wharf and Ben's House _____

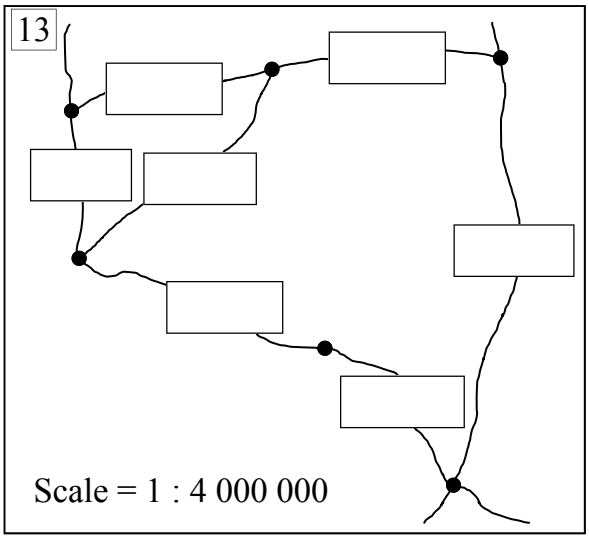
11 Ben's house and Oyster Cove _____

12 The ferry averages 20 km/h (5 km in 15 min), and it stops at all wharves for 10 min. Complete the time table below.

Location	Arrives	Departs
Rusty's Wharf	12.25 pm.	
Shingle Bay		
Oyster Cove		

Measure straight lengths in mm (ignore curves). Use the scale, then write the distances (km) in the boxes.

13



mm → m → km = ÷ 1 000 ÷ 1 000

Geometrical Properties - Find A Word

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Find the words in the puzzle from the wordlist.



M	D	M	K	S	T	A	T	E	M	E	N	T	C	N
U	I	I	X	E	H	E	N	O	I	T	A	T	O	R
L	X	X	V	C	I	H	E	I	G	H	T	I	N	E
T	T	I	R	I	G	H	R	T	R	T	T	P	G	C
I	N	X	E	T	D	Q	U	L	O	A	R	P	R	T
P	E	U	L	R	Q	E	S	Q	T	U	I	Y	U	A
L	M	N	U	E	A	Q	A	N	C	A	A	T	E	N
Y	E	K	R	V	U	P	E	U	A	Q	N	I	N	G
K	G	N	N	K	K	I	M	O	F	U	G	R	C	L
P	R	O	P	O	R	T	I	O	N	A	L	A	E	E
M	A	W	E	O	E	T	U	Q	C	E	E	L	L	S
A	L	N	X	L	A	U	Q	E	A	L	S	I	C	C
T	N	X	G	R	A	X	U	A	Q	U	X	M	R	I
C	E	J	I	L	A	C	I	T	N	E	D	I	I	C
H	H	T	G	N	E	L	S	S	X	Q	E	S	C	L

WORDLIST

TRIANGLES	FACTOR	IDENTICAL	LENGTH	MULTIPLY
ENLARGEMENT	RECTANGLES	DIVIDE	UNKNOWN	VERTICES
PROPORTIONAL	RULER	COMPARE	SCALE	ROTATION
ORIENTATION	ANGLE	SIMILARITY	RATIO	MATCH
CONGRUENCE	STATEMENT	EQUAL	MEASURE	AXIS
	HEIGHT		CIRCLE	

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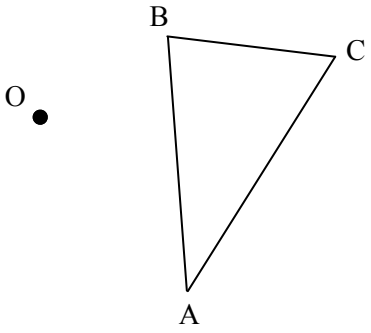
Enlarge triangle ABC by a scale factor of 2, using the centre of enlargement labeled 'O'.

1



The car below is drawn to a scale of 1 : 35. Measure the length of the wheelbase on the image then use the scale factor to give the actual car's wheelbase length.

2

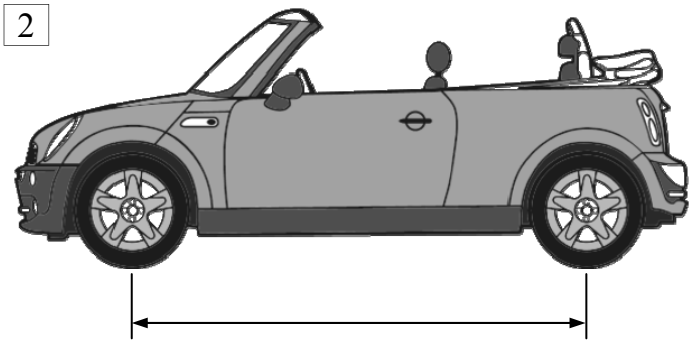
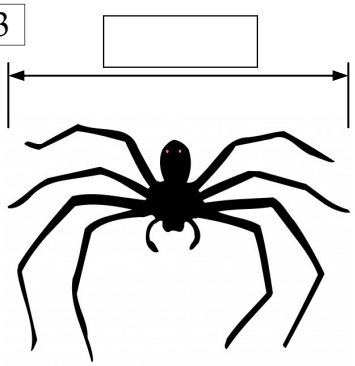


Image length (cm): ____

Actual length (cm): ____

The spider below is drawn to a scale of 3 : 2. Use the scale to calculate the width (cm) of this spider in real life.

3

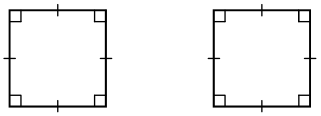


Actual length (cm): _____

_____

Is the first square below similar, congruent or neither when compared to the second square? Give a reason.

4



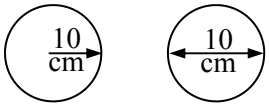
○similar
○congruent
○neither

Reason: _____

_____

Is the first circle below similar, congruent or neither when compared to the second circle? Give a reason.

5 Not to scale



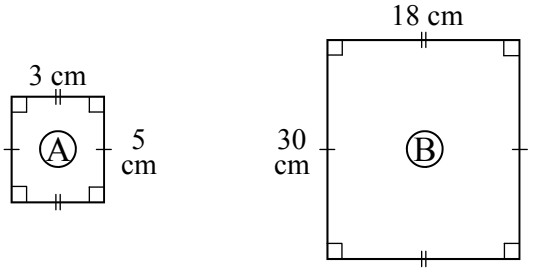
○similar
○congruent
○neither

Reason: _____

_____

Calculate the scale factor used to get Rectangle B from Rectangle A.

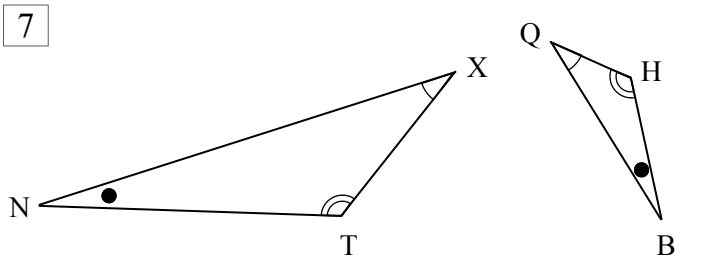
6



Scale factor: _____

The two triangles below are similar. Write a similarity statement for the two triangles.

7



_____

List 3 pairs of matching (similar) sides for the triangles.

8

____ & ____, ____ & ____, ____ & ____.

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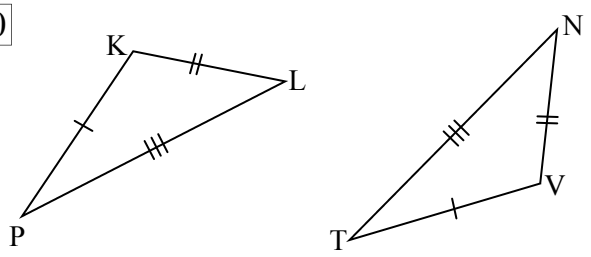
The triangles below are congruent. Write a congruence statement for the two triangles.

10



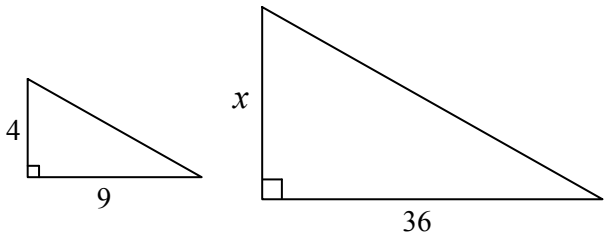
Complete the equivalent ratios below.

11 $5 : 6 = 30 : \square$

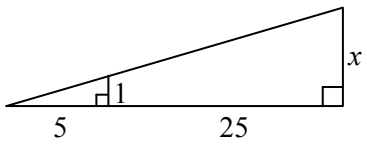
12 $10 : 4 = 20 : \square$

Find the unknown length on the triangles. (Not to scale). The two triangles are similar.

13



14



A plan view of Lucy's room is shown below. The scale is 1 : 30. Find the actual lengths of these objects in cm.

15 Desk width (w) _____

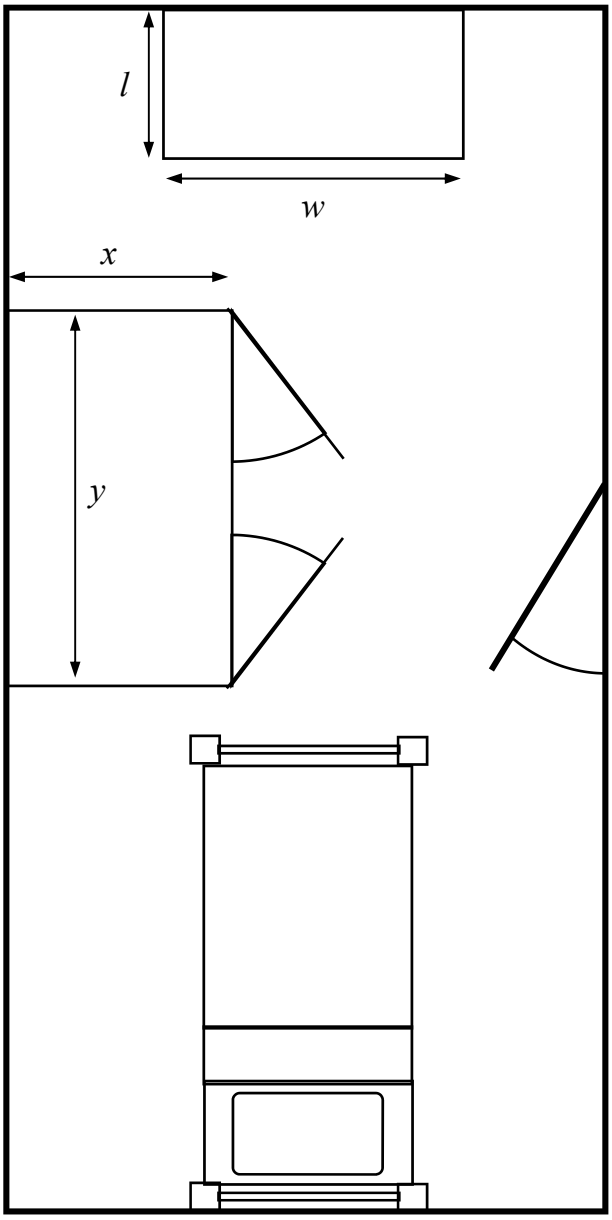
Actual object width, not diagram width

16 Desk depth (l) _____

17 Wardrobe depth (x) _____

18 Wardrobe width (y) _____

Scale 1 : 30



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VELOCITY

Michael Farlam



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ALTITUDE

BOOK ONE

FREEFALL MATHEMATICS

ALTITUDE

NUMBER

Operations Tables - Whole Numbers

This sheet is about completing tables. The tables are addition, multiplication and subtraction. With subtraction ensure the number in the column (up and down) is taken away from the number in the row, otherwise negative answers will occur.

The question mark starts with the number 13 and then the operation in blue is completed, until the last square is filled. All answers are whole numbers so fractions or decimal answers mean that a mistake has been made.

The small tables at the bottom of the page are division. Divide the larger number by the smaller number and then write the answer in the box below.

Operations Tables - Whole Numbers

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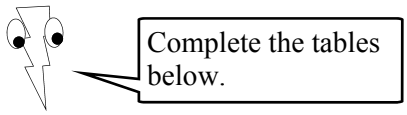
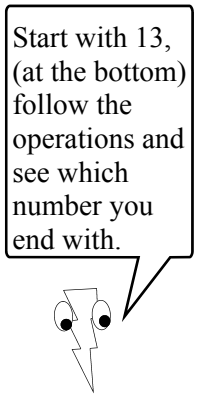
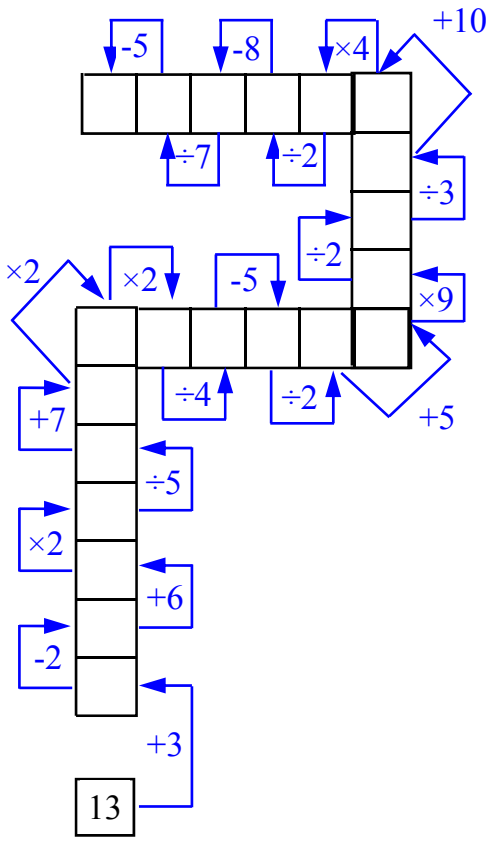


+	5	2	1	6	10	7	8	3	4	9
10										
5										
6										
8										
1										
3										
2										
4										
7										
9										

×	5	2	1	6	10	7	8	3	4	9
10										
5										
6										
8										
1										
3										
2										
4										
7										
9										

-	15	12	11	16	10	17	18	13	14	19
10										
5										
6										
8										
1										
3										
2										
4										
7										
9										





8 ÷	2	4	8

12 ÷	4	3	6

30 ÷	6	5	15

40 ÷	5	8	10

16 ÷	4	8	2

20 ÷	4	10	5

60 ÷	6	3	12

100 ÷	5	25	10

Operations Tables - Whole Numbers

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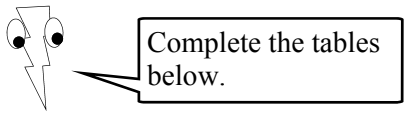
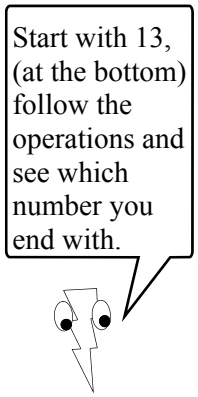
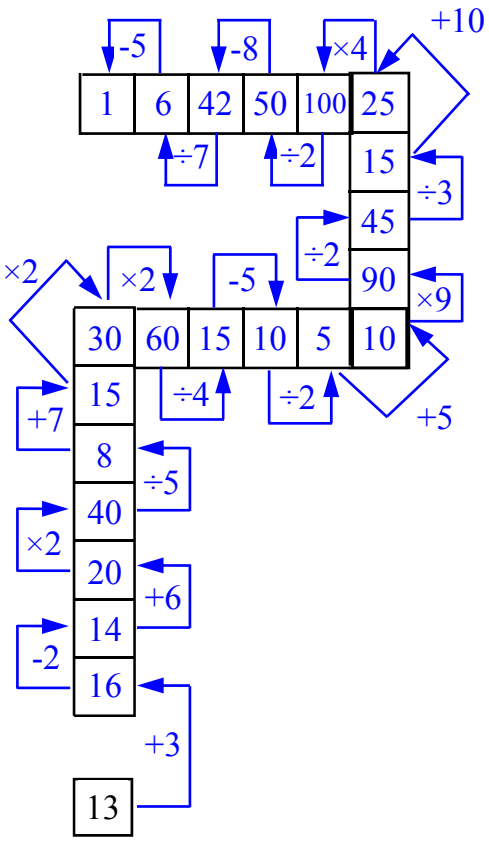


+	5	2	1	6	10	7	8	3	4	9
10	15	12	11	16	20	17	18	13	14	19
5	10	7	6	11	15	12	13	8	9	14
6	11	8	7	12	16	13	14	9	10	15
8	13	10	9	14	18	15	16	11	12	17
1	6	3	2	7	11	8	9	4	5	10
3	8	5	4	9	13	10	11	6	7	12
2	7	4	3	8	12	9	10	5	6	11
4	9	6	5	10	14	11	12	7	8	13
7	12	9	8	13	17	14	15	10	11	16
9	14	11	10	15	19	16	17	12	13	18

×	5	2	1	6	10	7	8	3	4	9
10	50	20	10	60	100	70	80	30	40	90
5	25	10	5	30	50	35	40	15	20	45
6	30	12	6	36	60	42	48	18	24	54
8	40	16	8	48	80	56	64	24	32	72
1	5	2	1	6	10	7	8	3	4	9
3	15	6	3	18	30	21	24	9	12	27
2	10	4	2	12	20	14	16	6	8	18
4	20	8	4	24	40	28	32	12	16	13
7	35	14	7	42	70	49	56	21	28	63
9	45	18	9	54	90	63	72	27	36	81

-	15	12	11	16	10	17	18	13	14	19
10	5	2	1	6	0	7	8	3	4	9
5	10	7	6	11	5	12	13	8	9	14
6	9	6	5	10	4	11	12	7	8	13
8	7	4	3	8	2	9	10	5	6	11
1	14	11	10	15	9	16	17	12	13	18
3	12	9	8	13	7	14	15	10	11	16
2	13	10	9	14	8	15	16	11	12	17
4	11	8	7	12	6	13	14	9	10	15
7	8	5	4	9	3	10	11	6	7	12
9	6	3	2	7	1	8	9	4	5	10





8 ÷	2	4	8
	4	2	1

12 ÷	4	3	6
	3	4	2

30 ÷	6	5	15
	5	6	2

40 ÷	5	8	10
	8	5	4

16 ÷	4	8	2
	4	2	8

20 ÷	4	10	5
	5	2	4

60 ÷	6	3	12
	10	20	5

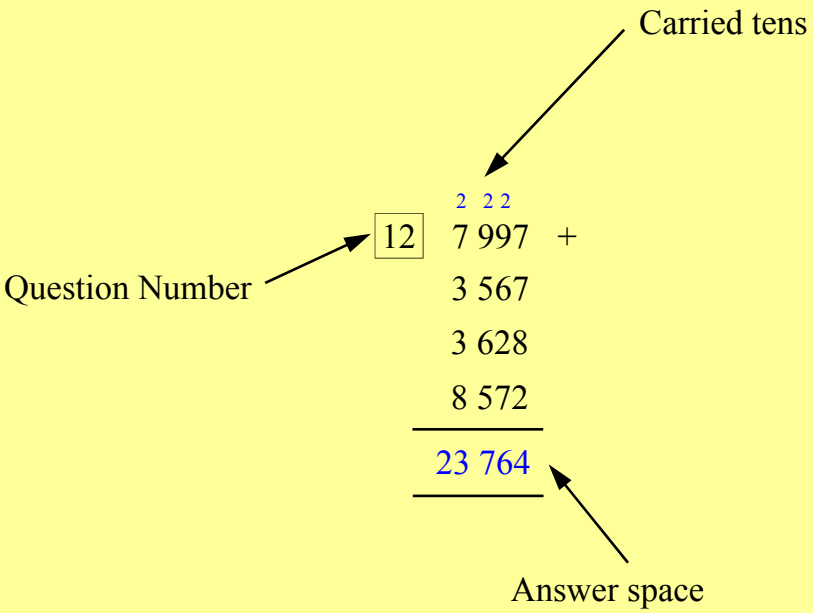
100 ÷	5	25	10
	20	4	10

Addition - Whole Numbers

To perform additions move from right to left and “carry the tens” when the total exceeds 9. The example below shows how to carry the tens.

First answer the table, then complete the exercises, follow this method:

- total down the right hand column and if the sum is less than 10 write it in then move to the next column.
- If the total is ten or more write the last digit and write the tens digit above the next column, sometimes said as carry the 1, 2 or so on.
- Then add the next column including the figure above in your calculation, repeat the process if the total is 10 or more, and so on, until you are finished.



Addition - Whole Numbers

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Complete the table below, then try the exercises



+	5	2	10	3	7	9	6	1	8	4
3										
10										
9										
4										
1										
8										
5										
7										
2										
6										

1 $22 + 34$ =

2 $32 + 67$ =

3 $76 + 53$ =

4 $92 + 75$ =

5 $187 + 39$ =

6 $387 + 64$ =

7 $874 + 232$ =

8 $807 + 696$ =

9 $383 + 86 + 27$ =

10 $140 + 855 + 21$ =

11 $705 + 493 + 287$ =

12 $783 + 821 + 906$ =

13 $378 + 674 + 197$ =

14 $1\,382 + 765 + 43$ =

15 $4\,352 + 894 + 130$ =

16 $5\,030 + 1\,349 + 211$ =

17 $8\,434 + 2\,159 + 3\,550$ =

18 $1\,923 + 2\,002 + 3\,641$ =

19 $3\,658 + 1\,291 + 1\,006 + 844$ =

20 $5\,337 + 3\,724 + 1\,467 + 2\,440$ =

21 $2\,241 + 5\,423 + 8\,711 + 2\,009$ =

22 $8\,907 + 5\,024 + 7\,255 + 3\,029$ =

23 $7\,066 + 3\,879 + 9\,054 + 8\,777$ =

Addition - Whole Numbers

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Complete the table below, then try the exercises



+	5	2	10	3	7	9	6	1	8	4
3	8	5	13	6	10	12	9	4	11	7
10	15	12	20	13	17	19	16	11	18	14
9	14	11	19	12	16	18	15	10	17	13
4	9	6	14	7	11	13	10	5	12	8
1	6	3	11	4	8	10	7	2	9	5
8	13	10	18	11	15	17	14	9	16	12
5	10	7	15	8	12	45	11	6	13	9
7	12	9	17	10	14	16	13	8	15	11
2	7	4	12	5	9	11	8	3	10	6
6	11	8	16	9	13	15	12	7	14	10

1
22 +
34
56

2
32 +
67
99

3
76 +
53
129

4
92 +
75
167

5
1 1
187 +
39
226

6
1 1
387 +
64
451

7
1
874 +
232
1 106

8
1 1
807 +
696
1 503

9
1 1
383 +
86
27
496

10
1
140 +
855
21
1 016

11
1 1
705 +
493
287
1 485

12
1 1
783 +
821
906
2 510

13
2 1
378 +
674
197
1 249

14
1 1 1
1 382 +
765
43
2 190

15
1 1
4 352 +
894
130
5 376

16
1
5 030 +
1 349
211
6 590

17
1 1 1
8 434 +
2 159
3 550
14 143

18
1
1 923 +
2 002
3 641
7 566

19
1 1 1
3 658 +
1 291
1 006
844
6 799

20
1 1 1
5 337 +
3 724
1 467
2 440
12 968

21
1 1
2 241 +
5 423
8 711
2 009
18 384

22
1 1 2
8 907 +
5 024
7 255
3 029
24 215

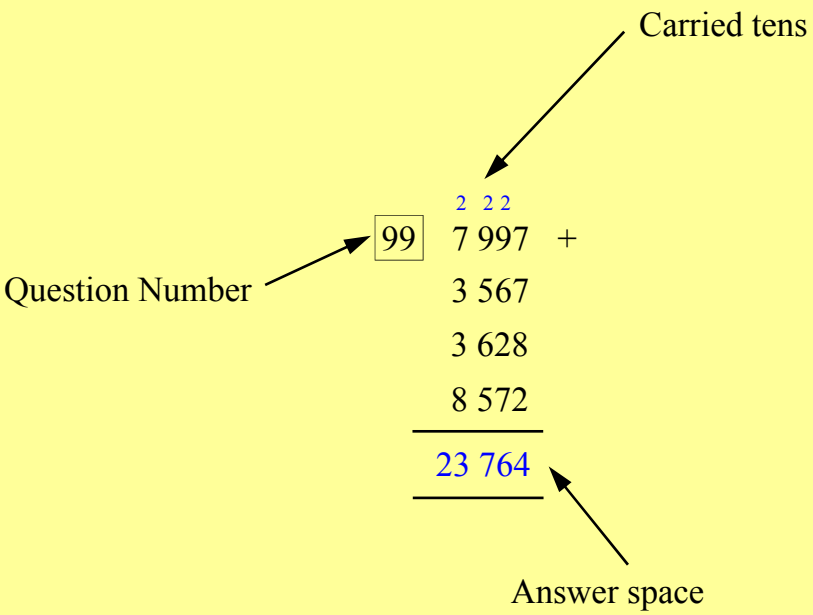
23
1 2 2
7 066 +
3 879
9 054
8 777
28 776

Further Addition - Whole Numbers

To perform additions move from right to left and “carry then tens” when the total exceeds 9. The example below shows how to carry the tens.

Follow this method:

- total down the right hand column and if the sum is less than 10 write it in then move to the next column.
- If the sum is ten or more write the last digit and write the tens digit above the next column, sometimes said as carry the 1, 2 or so on.
- Then add the next column including the figure above in your calculation, repeat the process if the total is 10 or more, and so on, until you are finished.



Further Addition - Whole Numbers

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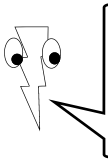

Put a space or comma between the hundreds and thousands column

1 7 397 + 8 935 =

2 1 875 + 9 660 =

3 4 699 + 7 065 =

4 7 331 + 8 059 =

5 7 429 + 6 578 + 8 995 =

6 8 226 + 7 168 + 8 490 =

7 9 676 + 3 923 + 8 511 =

8 8 220 + 5 707 + 9 888 =

9 7 499 + 2 728 + 7 493 =

10 5 663 + 2 880 + 7 395 + 286 =

11 2 760 + 9 833 + 5 809 + 661 =

12 9 448 + 8 615 + 4 833 + 49 =

13 2 986 + 4 443 + 9 277 + 764 =

14 6 905 + 7 884 + 6 593 + 139 =

15 7 339 + 1 674 + 3 770 + 5 993 =

16 9 334 + 5 665 + 7 320 + 9 758 =

17 6 775 + 9 034 + 6 311 + 4 843 =

18 1 006 + 9 914 + 4 788 + 4 729 =

19 8 655 + 9 272 + 1 843 + 8 702 =

20 26 951 + 18 373 + 11 694 + 52 205 + 4 951 =

21 85 388 + 29 744 + 36 401 + 2 945 + 44 989 =

22 70 563 + 26 906 + 7 824 + 67 224 + 63 990 =

23 19 335 + 86 777 + 41 177 + 7 355 + 93 228 =

24 73 002 + 9 677 + 39 405 + 53 547 + 89 430 =

25 74 359 + 36 933 + 72 450 + 24 302 + 52 888 =

26 11 963 + 43 811 + 19 330 + 96 075 + 62 797 =

27 82 471 + 93 402 + 11 639 + 72 865 + 90 958 =

28 20 773 + 58 836 + 43 729 + 71 090 + 33 677 =

29 98 765 + 87 654 + 76 543 + 65 432 + 54 321 =

Further Addition - Whole Numbers

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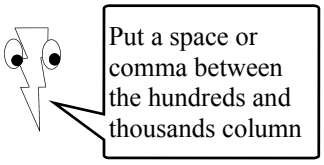


1 1 11
7 397 +
8 935
16 332

2 1 1
1 875 +
9 660
11 535

3 11
4 699 +
7 065
11 764

4 1
7 331 +
8 059
15 390

5 2 22
7 429 +
6 578
8 995
23 002

6 11
8 226 +
7 168
8 490
23 884

7 2 11
9 676 +
3 923
8 511
22 110

8 1 11
8 220 +
5 707
9 888
23 815

9 1 22
7 499 +
2 728
7 493
17 720

10 2 31
5 663 +
2 880
7 395
286
16 224

11 3 11
2 760 +
9 833
5 809
661
19 063

12 1 12
9 448 +
8 615
4 833
49
22 945

13 2 22
2 986 +
4 443
9 277
764
17 470

14 2 22
6 905 +
7 884
6 593
139
21 521

15 2 21
7 339 +
1 674
3 770
5 993
18 776

16 2 11
9 334 +
5 665
7 320
9 758
32 077

17 1 11
6 775 +
9 034
6 311
4 843
26 963

18 2 12
1 006 +
9 914
4 788
4 729
20 437

19 2 11
8 655 +
9 272
1 843
8 702
28 472

20 2 3 21
26 951 +
18 373
11 694
52 205
4 951
114 174

21 2 3 22
85 388 +
29 744
36 401
2 945
44 989
199 467

22 2 3 21
70 563 +
26 906
7 824
67 224
63 990
236 507

23 2 1 23
19 335 +
86 777
41 177
7 355
93 228
247 872

24 3 2 12
73 002 +
9 677
39 405
53 547
89 430
265 061

25 2 2 22
74 359 +
36 933
72 450
24 302
52 888
260 932

26 2 2 21
11 963 +
43 811
19 330
96 075
62 797
233 976

27 1 3 22
82 471 +
93 402
11 639
72 865
90 958
351 335

28 1 3 32
20 773 +
58 836
43 729
71 090
33 677
228 105

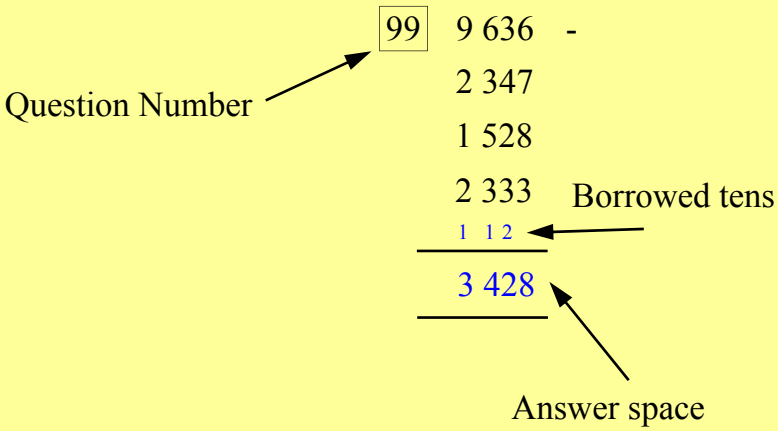
29 3 2 21
98 765 +
87 654
76 543
65 432
54 321
382 715

Subtraction - Whole Numbers

To perform subtractions move from right to left and “borrow tens” when the top number is smaller than the bottom number. The example below outlines how to show your working.

Firstly answer the table by subtracting the numbers in the column (up and down) from the numbers in the row. Then start the exercises, follow this method:

- subtract down the right hand column and if the top number is greater than the bottom number, or the total of the numbers below it, subtract.
- If the top number is less than the total of the bottom numbers 'borrow' 1 or more from the next column, these are 10’s. Write the amount borrowed underneath the column you borrowed from, below the numbers.
- Then subtract the next column including the figure you borrowed in your calculation, repeat the process until you are finished.



Subtraction - Whole Numbers

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Complete the table below by subtracting the column from the row, then try the exercises.



-	10	15	11	20	18	14	19	13	17	12
3										
10										
9										
4										
1										
8										
5										
7										
2										
6										

1 67 - 24 = _____

2 89 - 45 = _____

3 76 - 47 = _____

4 56 - 29 = _____

5 157 - 68 = _____

6 253 - 157 = _____

7 439 - 263 = _____

8 736 - 578 = _____

9 357 - 134 - 84 = _____

10 578 - 207 - 76 = _____

11 627 - 274 - 170 = _____

12 923 - 467 - 266 = _____

13 873 - 394 - 387 = _____

14 1 274 - 490 - 377 = _____

15 4 880 - 967 - 479 = _____

16 6 550 - 3 542 - 876 = _____

17 4 255 - 1 633 - 2 150 = _____

18 7 638 - 2 952 - 2 008 = _____

19 6 557 - 1 264 - 2 765 - 304 = _____

20 5 670 - 3 622 - 1 109 - 576 = _____

21 8 990 - 2 378 - 3 482 - 1 676 = _____

22 7 553 - 1 865 - 1 777 - 2 039 = _____

23 9 741 - 2 503 - 3 780 - 1 986 = _____

Subtraction - Whole Numbers

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Complete the table below by subtracting the column from the row, then try the exercises.



-	10	15	11	20	18	14	19	13	17	12
3	7	12	8	17	15	11	16	10	14	9
10	0	5	1	10	8	4	9	3	7	2
9	1	6	2	11	9	5	10	4	8	3
4	6	11	7	16	14	10	15	9	13	8
1	9	14	10	19	17	13	18	12	16	11
8	2	7	3	12	10	6	11	5	9	4
5	5	10	6	15	13	9	14	8	12	7
7	3	8	4	13	11	7	12	6	10	5
2	8	13	9	18	16	12	17	11	15	10
6	4	9	5	14	12	8	13	7	11	6

1 67 − 24 = 43

2 89 − 45 = 44

3 76 − 47 = 29 (1)

4 56 − 29 = 27 (1)

5 157 − 68 = 89 (1 1)

6 253 − 157 = 96 (1 1)

7 439 − 263 = 176 (1)

8 736 − 578 = 158 (1 1)

9 357 − 134 − 84 = 139 (1 1)

10 578 − 207 − 76 = 295 (1 1)

11 627 − 274 − 170 = 183 (2)

12 923 − 467 − 266 = 190 (2 1)

13 873 − 394 − 387 = 92 (2 1)

14 1 274 − 490 − 377 = 407 (1 1 1)

15 4 880 − 967 − 479 = 3 434 (1 1 2)

16 6 550 − 3 542 − 876 = 2 132 (1 1 1)

17 4 255 − 1 633 − 2 150 = 472 (1 1)

18 7 638 − 2 952 − 2 008 = 2 678 (1 1 1)

19 6 557 − 1 264 − 2 765 − 304 = 2 224 (1 1 1)

20 5 670 − 3 622 − 1 109 − 576 = 363 (1 1 2)

21 8 990 − 2 378 − 3 482 − 1 676 = 1 454 (1 2 2)

22 7 553 − 1 865 − 1 777 − 2 039 = 1 872 (2 2 2)

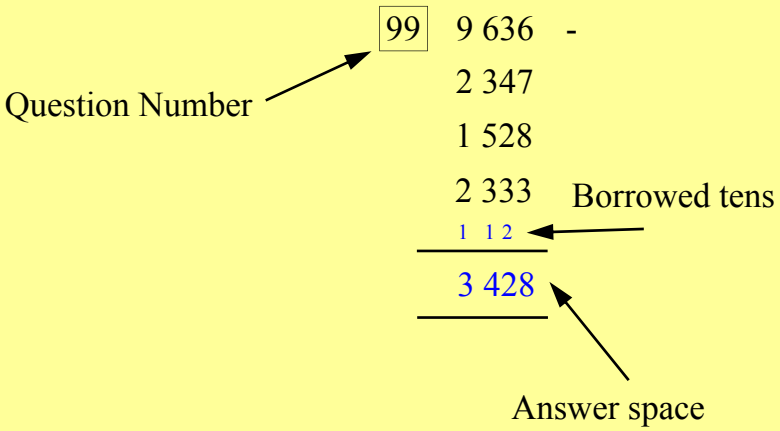
23 9 741 − 2 503 − 3 780 − 1 986 = 1 472 (2 2 1)

Further Subtraction - Whole Numbers

To perform subtractions move from right to left and “borrow tens” when the top number is smaller than the bottom number. The example below outlines how to show your working.

Follow this method:

- subtract down the right hand column and if the top number is greater than the bottom number, or the total of the numbers below it, subtract.
- If the top number is less than the total of the bottom numbers 'borrow' 1 or more from the next column, these are 10’s. Write the amount borrowed underneath the column you borrowed from, below the numbers.
- Then subtract the next column including the figure you borrowed in your calculation, repeat the process until you are finished.



Further Subtraction - Whole Numbers

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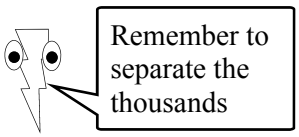

1) 73 − 27	2) 62 − 28	3) 81 − 39	4) 95 − 57	
5) 167 − 76	6) 275 − 128	7) 334 − 169	8) 633 − 458	9) 927 − 309
10) 588 − 394	11) 674 − 169	12) 702 − 555	13) 871 − 219	14) 931 − 133
15) 877 − 156 − 273	16) 900 − 256 − 75	17) 734 − 183 − 228	18) 927 − 336 − 481	19) 853 − 94 − 637
20) 2 345 − 755 − 430	21) 6 557 − 1 820 − 429	22) 4 350 − 1 564 − 2 319	23) 8 005 − 1 621 − 599	24) 7 256 − 3 711 − 1 888
25) 5 288 − 1 969 − 2 145 − 1 010	26) 7 450 − 2 322 − 2 069 − 1 442	27) 4 329 − 1 247 − 1 603 − 956	28) 8 731 − 2 933 − 1 830 − 3 555	29) 7 601 − 1 663 − 2 937 − 2 450
30) 56 730 − 4 300 − 11 275 − 38 795	31) 31 890 − 8 635 − 4 777 − 11 000	32) 79 342 − 12 309 − 14 117 − 34 818	33) 48 067 − 8 094 − 6 207 − 21 955	34) 82 311 − 8 538 − 17 653 − 37 202

Further Subtraction - Whole Numbers

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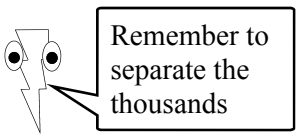


1
73 -
27
1
46

2
62 -
28
1
34

3
81 -
39
1
42

4
95 -
57
1
38

5
167 -
76
1
91

6
275 -
128
1
147

7
334 -
169
1 1
165

8
633 -
458
1 1
175

9
927 -
309
1
618

10
588 -
394
1
194

11
674 -
169
1
505

12
702 -
555
1 1
147

13
871 -
219
1
652

14
931 -
133
1 1
798

15
877 -
156
273
1 1
448

16
900 -
256
75
2 2
569

17
734 -
183
228
1 1
323

18
927 -
336
481
1
110

19
853 -
94
637
1 1
122

20
2 345 -
755
430
1 1
1 160

21
6 557 -
1 820
429
1 1
4 308

22
4 350 -
1 564
2 319
1 1 2
467

23
8 005 -
1 621
599
2 2 1
5 785

24
7 256 -
3 711
1 888
2 1 1
1 657

25
5 288 -
1 969
2 145
1 010
1 1 1
164

26
7 450 -
2 322
2 069
1 442
1 1 2
1 617

27
4 329 -
1 247
1 603
956
2 1 1
523

28
8 731 -
2 933
1 830
3 555
2 1 1
413

29
7 601 -
1 663
2 937
2 450
2 2 1
551

30
56 730 -
4 300
11 275
38 795
1 1 2 1
2 360

31
31 890 -
8 635
4 777
11 000
2 1 1 2
7 478

32
79 342 -
12 309
14 117
34 818
1 1 1 3
18 098

33
48 067 -
8 094
6 207
21 955
1 2 1 1
11 811

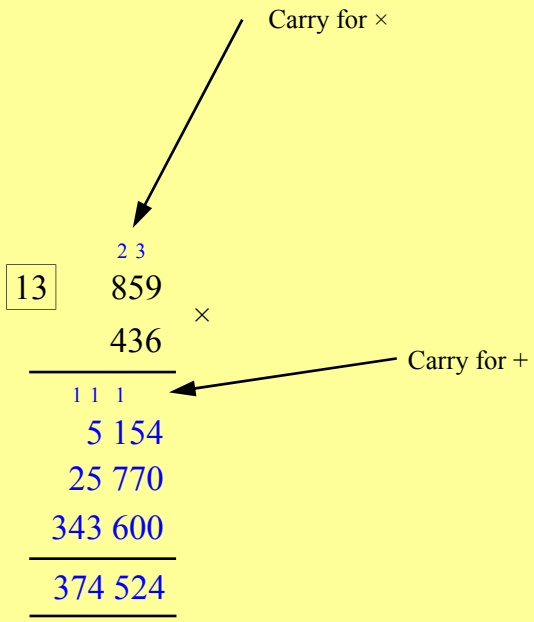
34
82 311 -
8 538
17 653
37 202
3 2 1 2
18 918

Multiplication - Whole Numbers

To perform multiplication move from right to left and carry the tens when the answer is greater than 9. The example below will show you how to do so.

Firstly answer the table, then when you are ready to start the exercises, follow this method:

- multiply the first numbers together and write the answer.
- If the answer is greater than 9 write the "units" digit then the tens digit above the numbers in the column to the left.
- Then repeat the multiplication but then add the figure above in your calculation, repeat the process until you are finished.
- From question 7 on, two rows are used. The first row is for the units in the bottom number, the second row is used when you multiply the tens digit. Write a 0 (zero) in the second row before you start to multiply the tens digit.
- When the two rows are completed you will have to add them. So carry the tens above the two rows as shown in the example below.



Multiplication - Whole Numbers

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Complete the table below, then try the exercises



×	5	2	4	8	10	6	3	9	1	7
2										
10										
6										
3										
9										
5										
7										
1										
4										
8										

1 12×4

2 71×3

3 46×5

4 92×8

5 312×7

6 716×4

7 34×13

8 62×46

9 22×14

10 71×53

11 86×34

12 73×96

13 67×57

14 311×27

15 177×43

16 466×27

17 822×92

18 760×63

19 $8\,230 \times 27$

20 $6\,758 \times 53$

21 2962×47

22 $7\,738 \times 69$

23 $8\,965 \times 82$

Multiplication - Whole Numbers

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Complete the table below, then try the exercises



×	5	2	4	8	10	6	3	9	1	7
2	10	4	8	16	20	12	6	18	2	14
10	50	20	40	80	100	60	30	90	10	70
6	30	12	24	48	60	36	18	54	6	42
3	15	6	12	24	30	18	9	27	3	21
9	45	18	36	72	90	54	27	81	9	63
5	25	10	20	40	50	30	15	45	5	35
7	35	14	28	56	70	42	21	63	7	49
1	5	2	4	8	10	6	3	9	1	7
4	20	8	16	32	40	24	12	36	4	28
8	40	16	32	64	80	48	24	72	8	56

1 12 × 4 = 48

2 71 × 3 = 213

3 46 × 5 = 230

4 92 × 8 = 736

5 312 × 7 = 2 184

6 716 × 4 = 2 864

7 34 × 13 = 102 + 340 = 442

8 62 × 46 = 372 + 2 480 = 2 852

9 22 × 14 = 88 + 220 = 308

10 71 × 53 = 213 + 3 550 = 3 763

11 86 × 34 = 344 + 2 580 = 2 924

12 73 × 96 = 438 + 6 570 = 7 008

13 67 × 57 = 469 + 3 350 = 3 819

14 311 × 27 = 2 177 + 6 220 = 8 397

15 177 × 43 = 531 + 7 080 = 7 611

16 466 × 27 = 3 262 + 9 320 = 12 582

17 822 × 92 = 1 644 + 73 980 = 75 624

18 760 × 63 = 2 280 + 45 600 = 47 880

19 8 230 × 27 = 57 610 + 164 600 = 222 210

20 6 758 × 53 = 20 274 + 337 900 = 358 174

21 2962 × 47 = 20 734 + 118 480 = 139 214

22 7 738 × 69 = 69 642 + 464 280 = 533 922

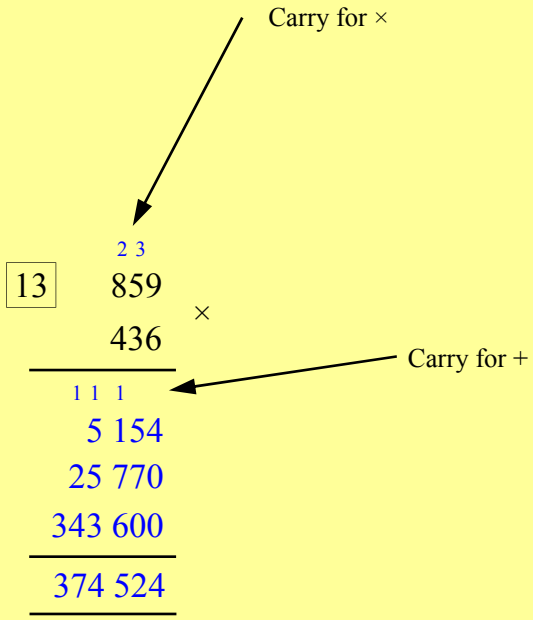
23 8 965 × 82 = 17 930 + 717 200 = 735 130

Further Multiplication - Whole Numbers

To perform multiplication move from right to left and carry the tens when the result is greater than 9. The example below will show you how to do so.

Follow this method:

- multiply the first numbers together and write the answer.
- If the answer is greater than 9 write the "units" digit then the tens digit above the numbers in the column to the left.
- Then repeat the multiplication but then add the figure above in your calculation, repeat the process until you are finished.
- From questions 5 to 9 two rows are used. The first row is for the units in the bottom number, the second row is used when you multiply the tens digit. Write a 0 (zero) in the second row before you start to multiply the tens digit. From Q.10 add two zeros when using a third row.
- When the rows are completed you will have to add them. So carry the tens above the rows as shown in the example below.



Further Multiplication - Whole Numbers

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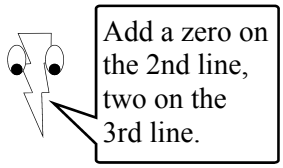

1 1 379 × 4

2 8 758 × 5

3 4 568 × 3

4 7 022 × 8

5 3 699 × 13

6 4 668 × 37

7 5 433 × 64

8 6 880 × 82

9 7 889 × 76

10 222 × 198

11 308 × 234

12 711 × 507

13 459 × 276

14 787 × 527

15 832 × 146

16 371 × 141

17 539 × 411

18 620 × 373

19 874 × 453

20 743 × 605

21 687 × 272

22 866 × 357

23 830 × 462

24 999 × 999

Further Multiplication - Whole Numbers

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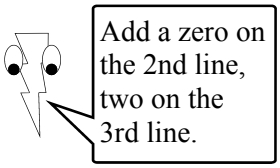


1 1 379 × 4 = 5 516

2 8 758 × 5 = 43 790

3 4 568 × 3 = 13 704

4 7 022 × 8 = 56 176

5 3 699 × 13 = 11 097 + 36 990 = 48 087

6 4 668 × 37 = 32 676 + 140 040 = 172 716

7 5 433 × 64 = 21 732 + 325 980 = 347 712

8 6 880 × 82 = 13 760 + 550 400 = 564 160

9 7 889 × 76 = 47 334 + 552 230 = 599 564

10 222 × 198 = 1 776 + 19 980 + 22 200 = 43 956

11 308 × 234 = 1 232 + 9 240 + 61 600 = 72 072

12 711 × 507 = 4 977 + 0 + 355 500 = 360 477

13 459 × 276 = 2 754 + 32 130 + 91 800 = 126 684

14 787 × 527 = 5 509 + 15 740 + 393 500 = 414 749

15 832 × 146 = 4 992 + 33 280 + 83 200 = 121 472

16 371 × 141 = 371 + 14 840 + 37 100 = 52 311

17 539 × 411 = 539 + 5 390 + 215 600 = 221 529

18 620 × 373 = 1 860 + 43 400 + 186 000 = 231 260

19 874 × 453 = 2 622 + 43 700 + 349 600 = 395 922

20 743 × 605 = 3 715 + 0 + 445 800 = 449 515

21 687 × 272 = 1 374 + 48 090 + 137 400 = 186 864

22 866 × 357 = 6 062 + 43 300 + 259 800 = 309 162

23 830 × 462 = 1 660 + 49 800 + 332 000 = 383 460

24 999 × 999 = 8 991 + 89 910 + 899 100 = 998 001

Division - Whole Numbers

Division is calculating how many times a number ‘goes into’ another. The answer after the division is called the quotient.

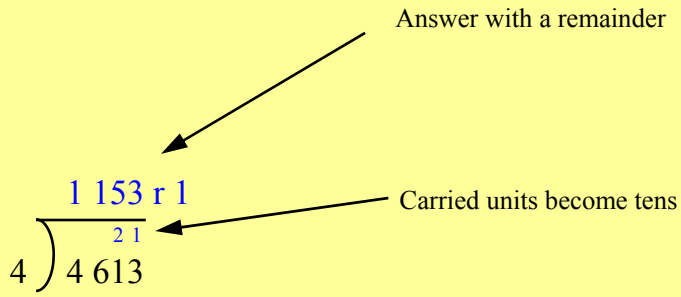
The first 8 questions involve completing tables. A number is given, the large number at the left side of the box, and this is to be divided by the numbers in the top row. Write the answer below each number.

Questions 9 through 23 involve divisions that don’t have a remainder. So the quotient will be a whole number only. While there isn’t a remainder at the end there will be remainders during the calculation which are carried through. This won’t be necessary in the first 6 questions as there are no remainders in the calculation stage.

Divide the outside number into the first number on the inside. If the number can divide into the other number write how many times it can do so, then write the remainder as a small number above the next number to its right. The remainder becomes the ‘tens’ digit and the number below it the units (ones) digit, then repeat the process.

Questions 24 through to 41 will have a remainder. Express this as in the example below, with the answer, a space, a ‘r’, a space, then the remainder.

Questions 42 through 45 are ‘long division’ questions. These all have remainders.



Division - Whole Numbers

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Try the tables first then the exercises, the first 23 won't have remainders. Position your answer on top.

1	16 ÷	2	8	4	16
2	20 ÷	4	5	2	10
3	30 ÷	5	15	2	6
4	45 ÷	15	5	3	9
5	50 ÷	25	10	2	5
6	42 ÷	6	14	21	7
7	64 ÷	8	4	2	16
8	80 ÷	10	20	40	8

9 $3\overline{)369}$ 10 $4\overline{)844}$ 11 $2\overline{)4\,862}$

12 $5\overline{)5\,055}$ 13 $4\overline{)8\,480}$ 14 $6\overline{)6\,060}$

15 $6\overline{)2\,412}$ 16 $7\overline{)3\,577}$ 17 $9\overline{)6\,318}$

18 $4\overline{)3\,604}$ 19 $7\overline{)5\,642}$ 20 $5\overline{)4\,535}$

21 $3\overline{)2\,463}$ 22 $8\overline{)5\,616}$ 23 $2\overline{)1\,206}$



These have remainders. Look at the example.

1 153 r 1

2 1

$4\overline{)4\,613}$ 24 $3\overline{)7\,984}$ 25 $5\overline{)8\,266}$ 26 $8\overline{)9\,742}$

27 $4\overline{)6\,830}$ 28 $6\overline{)2\,467}$ 29 $7\overline{)3\,823}$ 30 $5\overline{)8\,788}$ 31 $8\overline{)9\,115}$

32 $7\overline{)4\,668}$ 33 $3\overline{)7\,133}$ 34 $4\overline{)5\,297}$ 35 $9\overline{)6\,670}$ 36 $5\overline{)7\,239}$

37 $6\overline{)2\,775}$ 38 $4\overline{)7\,882}$ 39 $2\overline{)1\,875}$ 40 $8\overline{)8\,140}$ 41 $7\overline{)4\,222}$

42 $13\overline{)2\,895}$ 43 $18\overline{)7\,939}$ 44 $16\overline{)3\,924}$ 45 $12\overline{)5\,563}$

Division - Whole Numbers

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Try the tables first then the exercises, the first 23 won't have remainders. Position your answer on top.

1

16 ÷	2	8	4	16
	8	2	4	1

2

20 ÷	4	5	2	10
	5	4	10	2

3

30 ÷	5	15	2	6
	6	2	15	5

4

45 ÷	15	5	3	9
	3	9	15	5

5

50 ÷	25	10	2	5
	2	5	25	10

6

42 ÷	6	14	21	7
	7	3	2	6

7

64 ÷	8	4	2	16
	8	16	32	4

8

80 ÷	10	20	40	8
	8	4	2	10

9 $3\overline{)369} = 123$

10 $4\overline{)844} = 211$

11 $2\overline{)4\,862} = 2\,431$

12 $5\overline{)5\,055} = 1\,011$

13 $4\overline{)8\,480} = 2\,120$

14 $6\overline{)6\,060} = 1\,010$

15 $6\overline{)2\,412} = 402$

16 $7\overline{)3\,577} = 511$

17 $9\overline{)6\,318} = 702$

18 $4\overline{)3\,604} = 901$

19 $7\overline{)5\,642} = 806$

20 $5\overline{)4\,535} = 907$

21 $3\overline{)2\,463} = 821$

22 $8\overline{)5\,616} = 702$

23 $2\overline{)1\,206} = 603$



These have remainders. Look at the example.

$4\overline{)4\,613} = 1\,153$ r 1 (2 1)

24 $3\overline{)7\,984} = 2\,661$ r 1 (1 1)

25 $5\overline{)8\,266} = 1\,653$ r 1 (3 2 1)

26 $8\overline{)9\,742} = 1\,217$ r 6 (1 1 6)

27 $4\overline{)6\,830} = 1\,707$ r 2 (2)

28 $6\overline{)2\,467} = 411$ r 1

29 $7\overline{)3\,823} = 546$ r 1 (3 4)

30 $5\overline{)8\,788} = 1\,757$ r 3 (3 2 3)

31 $8\overline{)9\,115} = 1\,139$ r 3 (1 3 7)

32 $7\overline{)4\,668} = 666$ r 6 (4 4)

33 $3\overline{)7\,133} = 2\,377$ r 2 (1 2 2)

34 $4\overline{)5\,297} = 1\,324$ r 1 (1 1)

35 $9\overline{)6\,670} = 741$ r 1 (3 1)

36 $5\overline{)7\,239} = 1\,447$ r 4 (2 2 3)

37 $6\overline{)2\,775} = 462$ r 3 (3 1)

38 $4\overline{)7\,882} = 1970$ r 2 (3 2)

39 $2\overline{)1\,875} = 937$ r 1 (1)

40 $8\overline{)8\,140} = 1\,017$ r 4 (1 6)

41 $7\overline{)4\,222} = 603$ r 1 (2)

42 $13\overline{)2\,895} = 222$ r 9

26
29
26
35
26
1
9

43 $18\overline{)7\,939} = 441$ r 1

72
73
72
19
18
1

44 $16\overline{)3\,924} = 245$ r 4

32
72
64
1
84
80
4

45 $12\overline{)5\,563} = 463$ r 7

48
1
76
72
43
36
1
7

Further Division - Whole Numbers

Division is calculating how many times a number ‘goes into’ another. The answer after the division is called the quotient.

Column 1 starts first with dividing by 10, 100 and 1 000. With these, the division process involves moving the decimal point. When dividing by 10 move the decimal point 1 place to the left, dividing by 100 moves it 2 places to the left and with 1 000, move the decimal point 3 places to the left. Question 17 is an introduction to writing divisions with remainders as fractions. Answer the question then select the fraction by circling it.

Column 2 are division exercises which will have a remainder, write the remainder on the top line of the fraction (the numerator) and write the number you were dividing by as the bottom number, (the denominator). The example above question 20 shows this.

Division by 10 and Fraction Remainders

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When you divide by 10 move the decimal point 1 place to the left. 100 = 2 places and 1000 = 3 places.

÷ 10 — 1 place left

÷ 100 — 2 places left

÷ 1 000 — 3 places left

Example

$10 \overline{)\,34\,698} = 3\,469.8$

1 $10 \overline{)\,5\,450}$

2 $10 \overline{)\,7\,330}$

3 $100 \overline{)\,27\,880}$

4 $100 \overline{)\,14\,050}$

5 $1\,000 \overline{)\,53\,456}$

6 $1\,000 \overline{)\,73\,992}$

7 $\frac{180}{10} = $ ____

8 $\frac{270}{10} = $ ____

9 $\frac{573}{100} = $ ____

10 $\frac{692}{100} = $ ____

11 $\frac{1\,370}{100} = $ ____

12 $\frac{9\,600}{1\,000} = $ ____

13 $\frac{5\,687}{1\,000} = $ ____

14 $\frac{7\,808}{1\,000} = $ ____

15 $450 \div 10 = $ ____

16 $309 \div 10 = $ ____

Read the problem below and solve in the space provided.

17 If 77 jelly beans are shared equally between 2 children, how many is each child given and how many are left? Solve, then complete the statement.

$2 \overline{)\,77}$

Each child gets ☐ jelly beans and there is ☐ extra.

If the remainder was cut up equally, circle the fraction of a jelly bean received by each child.

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{3}$

Instead of writing 'r' then the remainder write it as a fraction as shown

$6 \overline{)\,2\,777}$ = 462 r 5 = $462\frac{5}{6}$

18 $3 \overline{)\,5\,386}$

19 $2 \overline{)\,1\,657}$

20 $5 \overline{)\,8\,791}$

21 $4 \overline{)\,3\,563}$

22 $8 \overline{)\,2\,441}$

23 $2 \overline{)\,7\,265}$

24 $5 \overline{)\,4\,678}$

25 $6 \overline{)\,9\,005}$

26 $9 \overline{)\,9\,121}$

27 $7 \overline{)\,76\,595}$

28 $9 \overline{)\,31\,217}$

29 $3 \overline{)\,70\,303}$

30 $7 \overline{)\,56\,729}$

31 $4 \overline{)\,34\,251}$

32 $6 \overline{)\,92\,335}$

33 $8 \overline{)\,43\,729}$

34 $7 \overline{)\,96\,839}$

35 $6 \overline{)\,70\,207}$

36 $4 \overline{)\,35\,447}$

37 $8 \overline{)\,82\,635}$

38 $5 \overline{)\,78\,118}$

39 $11 \overline{)\,4\,655}$

40 $16 \overline{)\,5\,795}$

41 $13 \overline{)\,4\,642}$

Division by 10 and Fraction Remainders

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When you divide by 10 move the decimal point 1 place to the left. 100 = 2 places and 1000 = 3 places.

÷ 10 — 1 place left

÷ 100 — 2 places left

÷ 1 000 — 3 places left

Example

$10 \overline{)\,34\,698}$ = 3 469.8

1 $10 \overline{)\,5\,450}$ = 545

2 $10 \overline{)\,7\,330}$ = 733

3 $100 \overline{)\,27\,880}$ = 278.8

4 $100 \overline{)\,14\,050}$ = 140.5

5 $1\,000 \overline{)\,53\,456}$ = 53.456

6 $1\,000 \overline{)\,73\,992}$ = 73.992

7 $\frac{180}{10}$ = 18

8 $\frac{270}{10}$ = 27

9 $\frac{573}{100}$ = 5.73

10 $\frac{692}{100}$ = 6.92

11 $\frac{1\,370}{100}$ = 13.7

12 $\frac{9\,600}{1\,000}$ = 9.6

13 $\frac{5\,687}{1\,000}$ = 5.687

14 $\frac{7\,808}{1\,000}$ = 7.808

15 450 ÷ 10 = 45

16 309 ÷ 10 = 3.09

Read the problem below and solve in the space provided.

17 If 77 jelly beans are shared equally between 2 children, how many is each child given and how many are left? Solve, then complete the statement.

$2 \overline{)\,77}$ = 38 r 1 (1)

Each child gets [38] jelly beans and there is [1] extra.

If the remainder was cut up equally, circle the fraction of a jelly bean received by each child.

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{3}$

Instead of writing 'r' then the remainder write it as a fraction as shown

$6 \overline{)\,2\,777}$ = 462 r 5 (3 1) → 462 $\frac{5}{6}$

18 $3 \overline{)\,5\,386}$ = $1\,795\frac{1}{3}$ (2 2 1)

19 $2 \overline{)\,1\,657}$ = $828\frac{1}{2}$ (1 1)

20 $5 \overline{)\,8\,791}$ = $1\,758\frac{1}{2}$ (3 2 4)

21 $4 \overline{)\,3\,563}$ = $890\frac{3}{4}$ (3 3)

22 $8 \overline{)\,2\,441}$ = $305\frac{1}{8}$ (2 4)

23 $2 \overline{)\,7\,265}$ = $3\,632\frac{1}{2}$ (1)

24 $5 \overline{)\,4\,678}$ = $935\frac{3}{5}$ (4 1 2)

25 $6 \overline{)\,9\,005}$ = $1\,500\frac{5}{6}$ (3)

26 $9 \overline{)\,9\,121}$ = $1\,013\frac{4}{9}$ (1 3)

27 $7 \overline{)\,76\,595}$ = $10\,942\frac{1}{7}$ (6 2 1)

28 $9 \overline{)\,31\,217}$ = $3\,468\frac{5}{9}$ (3 4 6 7)

29 $3 \overline{)\,70\,303}$ = $23\,434\frac{1}{3}$ (1 1 1 1)

30 $7 \overline{)\,56\,729}$ = $8\,104\frac{1}{7}$

31 $4 \overline{)\,34\,251}$ = $8\,562\frac{3}{4}$ (3 2 2 1)

32 $6 \overline{)\,92\,335}$ = $15\,389\frac{1}{6}$ (3 2 5 5)

33 $8 \overline{)\,43\,729}$ = $5\,466\frac{1}{8}$ (4 3 5 4)

34 $7 \overline{)\,96\,839}$ = $13\,834\frac{1}{7}$ (2 5 2 2)

35 $6 \overline{)\,70\,207}$ = $11\,701\frac{1}{6}$ (1 4)

36 $4 \overline{)\,35\,447}$ = $8\,861\frac{3}{4}$ (3 3 2)

37 $8 \overline{)\,82\,635}$ = $10\,329\frac{3}{8}$ (2 2 7)

38 $5 \overline{)\,78\,118}$ = $15\,623\frac{3}{5}$ (2 3 1 1)

39 $11 \overline{)\,4\,655}$ = $423\frac{2}{11}$

44 / 25 / 22 / 35 / 33 / 2

40 $16 \overline{)\,5\,795}$ = $362\frac{3}{16}$

48 / 99 / 96 / 35 / 32 / 3

41 $13 \overline{)\,4\,642}$ = $357\frac{1}{13}$

39 / 74 / 65 / 92 / 91 / 1

Mental Strategies with Larger Numbers

When you add or multiply or subtract numbers that are larger than you are comfortable with, you can use a strategy to tackle the problem. Break the operation up so that you can evaluate the operation in two steps rather than one.

The process breaks the number up into tens and units and uses the first step to deals with the ‘tens’ part and the second step to deal with the units. There are examples at the top of each Column that show you how to approach the questions.

Mental Strategies with Larger Numbers

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Separate the units from one of the numbers to make the process of addition more manageable.

Example

Break up 39 into 30 and 9

27 + 39 = 27 + 30 + 9

= 57 + 9 (Add the tens first)

= 66 (Then add the units)

1

28 + 47 = 28 + 40 + 7

= ☐ + ☐

= ☐

2

57 + 19 = ☐ + ☐ + ☐

= ☐ + ☐

= ☐

3

88 + 36 = ☐ + ☐ + ☐

= ☐ + ☐

= ☐

4

117 + 67 = ☐ + ☐ + ☐

= ☐ + ☐

= ☐

5

127 + 28 = ☐ + ☐ + ☐

= ☐ + ☐

= ☐

6

77 + 77 = ☐ + ☐ + ☐

= ☐ + ☐

= ☐

Break the numbers up to perform these multiplications.

Example

Break up 14 into 10 and 4

14 × 8 = 10 × 8 + 4 × 8

= 80 + 32 (Calculate the 2 parts)

= 112 (Then add the 2 parts)

7

16 × 7 = ☐ × 7 + ☐ × 7

= ☐ + ☐

= ☐

8

12 × 9 = ☐ × ☐ + ☐ × ☐

= ☐ + ☐

= ☐

9

17 × 8 = ☐ × ☐ + ☐ × ☐

= ☐ + ☐

= ☐

10

27 × 5 = 20 × 5 + ☐ × ☐

= ☐ + ☐

= ☐

11

19 × 9 = ☐ × ☐ + ☐ × ☐

= ☐ + ☐

= ☐

12

26 × 7 = ☐ × ☐ + ☐ × ☐

= ☐ + ☐

= ☐

Now try some subtracting. Use two steps to evaluate.

Example

Break up 2nd number

54 - 39 = 54 - 30 - 9

= 24 - 9 (Subtract the 'tens' first)

= 15 (Then subtract the units for the answer)

13 46 - 28 = 46 - 20 - 8

= ☐ - ☐

= ☐

14 72 - 59 = ☐ - ☐ - ☐

= ☐ - ☐

= ☐

15 81 - 24 = ☐ - ☐ - ☐

= ☐ - ☐

= ☐

16 56 - 18 = ☐ - ☐ - ☐

= ☐ - ☐

= ☐

17 92 - 67 = ☐ - ☐ - ☐

= ☐ - ☐

= ☐

18 74 - 57 = ☐ - ☐ - ☐

= ☐ - ☐

= ☐

19 88 - 79 = ☐ - ☐ - ☐

= ☐ - ☐

= ☐

Mental Strategies with Larger Numbers

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Separate the units from one of the numbers to make the process of addition more manageable.

Example

Break up 39 into 30 and 9

27 + 39 = 27 + 30 + 9

= 57 + 9 (Add the tens first)

= 66 (Then add the units)

1

28 + 47 = 28 + 40 + 7

= 68 + 7

= 75

2

57 + 19 = 57 + 10 + 9

= 67 + 9

= 76

3

88 + 36 = 88 + 30 + 6

= 118 + 6

= 124

4

117 + 67 = 117 + 60 + 7

= 177 + 7

= 184

5

127 + 28 = 127 + 20 + 8

= 147 + 8

= 155

6

77 + 77 = 77 + 70 + 7

= 147 + 7

= 154

Break the numbers up to perform these multiplications.

Example

Break up 14 into 10 and 4

14 × 8 = 10 × 8 + 4 × 8

= 80 + 32 (Calculate the 2 parts)

= 112 (Then add the 2 parts)

7

16 × 7 = 10 × 7 + 6 × 7

= 70 + 42

= 112

8

12 × 9 = 10 × 9 + 2 × 9

= 90 + 18

= 108

9

17 × 8 = 10 × 8 + 7 × 8

= 80 + 56

= 136

10

27 × 5 = 20 × 5 + 7 × 5

= 100 + 35

= 135

11

19 × 9 = 10 × 9 + 9 × 9

= 90 + 81

= 171

12

26 × 7 = 20 × 7 + 6 × 7

= 140 + 42

= 182

Now try some subtracting. Use two steps to evaluate.

Example

Break up 2nd number

54 - 39 = 54 - 30 - 9

= 24 - 9 (Subtract the 'tens' first)

= 15 (Then subtract the units for the answer)

13 46 - 28 = 46 - 20 - 8

= 26 - 8

= 18

14 72 - 59 = 72 - 50 - 9

= 22 - 9

= 13

15 81 - 24 = 81 - 20 - 4

= 61 - 4

= 57

16 56 - 18 = 56 - 10 - 8

= 46 - 8

= 38

17 92 - 67 = 92 - 60 - 7

= 32 - 7

= 25

18 74 - 57 = 74 - 50 - 7

= 24 - 7

= 17

19 88 - 79 = 88 - 70 - 9

= 18 - 9

= 9

Magic Squares

Magic squares are small puzzles that use your addition and subtraction ability. The total after the sum of the numbers in a line of a magic square must all be the same, there are 8 lines in all, 3 horizontal (flat), 3 vertical (up and down) and 2 diagonal.

To solve magic squares:

- One line of numbers will be complete, add the numbers along this line and write the total in the magic number box, called the 'Magic Number' below the magic square.
- There is one other number not in the line. Use this number together with one of the other numbers to form another line. Add the two numbers, then subtract the total from the Magic Number. Write the answer in the square that completes the line, then move to the next line.
- Continue this process until all the squares are filled.

Magic Squares

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Find the magic number and put in in the box, then fill the missing squares.

1

		8
9	5	1

The Magic Number is ☐

2

7	8	
	6	
	4	

The Magic Number is ☐

3

4		
9	7	
		10

The Magic Number is ☐

4

	13	6
	9	
12		

The Magic Number is ☐

5

10	8	6
		7

The Magic Number is ☐

6

13		
	10	12
		7

The Magic Number is ☐

7

		11
		16
	10	15

The Magic Number is ☐

8

	9	
	11	
8	13	

The Magic Number is ☐

9

		8
	10	6
12		

The Magic Number is ☐

10

21		
24		
9	30	

The Magic Number is ☐

11

16		
36		20
32		

The Magic Number is ☐

12

	60	
	40	
45	20	

The Magic Number is ☐

Magic Squares

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Find the magic number and put in in the box, then fill the missing squares.

1

4	3	8
9	5	1
2	7	6

The Magic Number is 15

2

7	8	3
2	6	10
9	4	5

The Magic Number is 18

3

4	11	6
9	7	5
8	3	10

The Magic Number is 21

4

8	13	6
7	9	11
12	5	10

The Magic Number is 27

5

9	4	11
10	8	6
5	12	7

The Magic Number is 24

6

13	6	11
8	10	12
9	14	7

The Magic Number is 30

7

13	18	11
12	14	16
17	10	15

The Magic Number is 42

8

10	9	14
15	11	7
8	13	12

The Magic Number is 33

9

4	18	8
14	10	6
12	2	16

The Magic Number is 30

10

21	6	27
24	18	12
9	30	15

The Magic Number is 54

11

16	44	24
36	28	20
32	12	40

The Magic Number is 84

12

25	60	35
50	40	30
45	20	55

The Magic Number is 120

Mixed Operations - Whole Numbers

This sheet involves using division, multiplication, addition and subtraction. As there are several operations in each question there are two methods you can use to answer the questions. The first is to calculate the entire question mentally. If you can do this great, but if you have difficulty you should use this method.

- Split the question up into chunks, in the example at the top of the first column you have 50 - 13 + 4 - 11 = ?
- Find 50 - 13 first, that's 37, you then write that answer above the 13 then strike out those first 2 numbers.
- Then using 37 find 37 + 4, that's 41, write that number above the 4 and strike out the 4
- Then using the 41 find 41 - 11, that is 30, so write the answer in the square.

Column 3 involves reading a sentence and translating it to numbers and operation signs. There are 3 answers supplied, only one of them is correct. Read the sentence and select the matching mathematical statement. To select your choice fill the circle. Then solve A, B and C not just the correct one, answer all three. Use the same method to solve as in the earlier columns.

Some terms you should be familiar with:

- Sum, total, increase, add - all mean Addition
- Reduce, decrease, find the difference, less, minus and subtract - all mean Subtraction
- Divide and 'find the quotient of' - mean Division
- Product, times and 'lots of' - mean Multiplication

Mixed Operations - Whole Numbers

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Strike out the numbers and write the new total as you go across

Example

50 - 13 + 4 - 11 = 30 (37, 41)

1 $3 + 7 - 5 + 6 =$ ☐

2 $12 - 4 + 20 - 3 =$ ☐

3 $20 + 15 - 8 + 3 =$ ☐

4 $16 - 10 - 3 + 18 =$ ☐

5 $3 + 30 - 6 + 11 =$ ☐

6 $6 + 3 - 7 + 50 =$ ☐

7 $29 - 11 + 6 - 9 =$ ☐

8 $14 + 40 - 7 + 8 =$ ☐

9 $14 + 7 - 2 +$ ☐ $= 30$

10 $15 + 22 + 18 -$ ☐ $= 40$

11 $13 + 8 - 6 +$ ☐ $= 27$

12 $16 + 5 + 13 +$ ☐ $= 52$

13 $14 + 17 - 8 -$ ☐ $= 8$

14 $21 - 9 - 7 -$ ☐ $= 0$

15 $42 + 22 - 12 -$ ☐ $= 35$

16 $53 - 9 + 11 +$ ☐ $= 80$

17 $17 + 8 + 20 -$ ☐ $= 33$

18 $20 - 8 + 30 -$ ☐ $= 36$

19 $15 + 20 - 17 -$ ☐ $= 8$

20 $5 \times 4 \times 2 =$ ☐

21 $8 \times 2 \div 4 =$ ☐

22 $6 \times 4 \div 8 =$ ☐

23 $10 \div 5 \times 15 =$ ☐

24 $6 \times 5 \div 10 =$ ☐

25 $20 \times 3 \times 2 \div 40 =$ ☐

26 $6 \div 2 \times 5 \times 3 =$ ☐

27 $10 \div 2 \times 10 \div 2 =$ ☐

28 $20 \times 2 \div 8 \times 3 =$ ☐

29 $42 \div 7 \times 3 \div 9 =$ ☐

30 $8 \times 9 \div 12 \times 4 =$ ☐

31 $3 \times 12 \div 4 \times$ ☐ $= 63$

32 $8 \times 3 \div 4 \times$ ☐ $= 54$

33 $9 \times 5 \div 3 \times$ ☐ $= 30$

34 $10 \div 2 \times 15 \times$ ☐ $= 0$

35 $2 \times 2 \times 2 \times$ ☐ $= 64$

36 $5 \times 4 \times 4 \div$ ☐ $= 4$

37 $6 \times 8 \div 12 \times$ ☐ $= 32$

38 $9 \times 4 \div 18 \times$ ☐ $= 100$

39 $50 \div 5 \div 2 \div$ ☐ $= 1$

40 $6 \times 2 \times 5 \div$ ☐ $= 10$

Read the sentence, select the matching operation but solve all three.

41 The difference of 12 and 7 plus 25.

A $12 + 7 + 25 =$ ☐

B $12 - 7 + 25 =$ ☐

C $12 \times 7 + 25 =$ ☐

42 Increase the product of 7 and 8 by 15.

A $7 + 8 + 15 =$ ☐

B $8 - 7 + 15 =$ ☐

C $7 \times 8 + 15 =$ ☐

43 Reduce the quotient of 24 and 6 by 4.

A $24 - 6 - 4 =$ ☐

B $24 \div 6 - 4 =$ ☐

C $24 + 6 - 4 =$ ☐

44 From the product of 2, 3 and 4 subtract 9.

A $2 \times 3 \times 4 + 9 =$ ☐

B $2 + 3 + 4 - 9 =$ ☐

C $2 \times 3 \times 4 - 9 =$ ☐

45 Find the total of 22, 33 and 17 and reduce the result by 9.

A $22 + 33 + 17 - 9 =$ ☐

B $22 + 33 - 17 + 9 =$ ☐

C $22 + 33 + 17 + 9 =$ ☐

46 Reduce the sum of 14 and 21 by 7 then add 22 to the result.

A $21 - 14 - 7 + 22 =$ ☐

B $21 - 14 + 7 + 22 =$ ☐

C $21 + 14 - 7 + 22 =$ ☐

Mixed Operations - Whole Numbers

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Strike out the numbers and write the new total as you go across

Example

50 - 13 + 4 - 11 = **30**

1. 3 + 7 - 5 + 6 = **11**
2. 12 - 4 + 20 - 3 = **25**
3. 20 + 15 - 8 + 3 = **30**
4. 16 - 10 - 3 + 18 = **21**
5. 3 + 30 - 6 + 11 = **38**
6. 6 + 3 - 7 + 50 = **52**
7. 29 - 11 + 6 - 9 = **15**
8. 14 + 40 - 7 + 8 = **55**
9. 14 + 7 - 2 + **11** = 30
10. 15 + 22 + 18 - **15** = 40
11. 13 + 8 - 6 + **12** = 27
12. 16 + 5 + 13 + **18** = 52
13. 14 + 17 - 8 - **15** = 8
14. 21 - 9 - 7 - **5** = 0
15. 42 + 22 - 12 - **17** = 35
16. 53 - 9 + 11 + **25** = 80
17. 17 + 8 + 20 - **8** = 33
18. 20 - 8 + 30 - **6** = 36
19. 15 + 20 - 17 - **10** = 8
20. 5 × 4 × 2 = **40**
21. 8 × 2 ÷ 4 = **4**
22. 6 × 4 ÷ 8 = **3**
23. 10 ÷ 5 × 15 = **30**
24. 6 × 5 ÷ 10 = **3**
25. 20 × 3 × 2 ÷ 40 = **3**
26. 6 ÷ 2 × 5 × 3 = **45**
27. 10 ÷ 2 × 10 ÷ 2 = **25**
28. 20 × 2 ÷ 8 × 3 = **15**
29. 42 ÷ 7 × 3 ÷ 9 = **2**
30. 8 × 9 ÷ 12 × 4 = **24**
31. 3 × 12 ÷ 4 × **7** = 63
32. 8 × 3 ÷ 4 × **9** = 54
33. 9 × 5 ÷ 3 × **2** = 30
34. 10 ÷ 2 × 15 × **0** = 0
35. 2 × 2 × 2 × **8** = 64
36. 5 × 4 × 4 ÷ **20** = 4
37. 6 × 8 ÷ 12 × **8** = 32
38. 9 × 4 ÷ 18 × **50** = 100
39. 50 ÷ 5 ÷ 2 ÷ **5** = 1
40. 6 × 2 × 5 ÷ **6** = 10

Read the sentence, select the matching operation but solve all three.

41. The difference of 12 and 7 plus 25.
    - A) 12 + 7 + 25 = **44**
    - ● 12 - 7 + 25 = **30**
    - C) 12 × 7 + 25 = **109**
42. Increase the product of 7 and 8 by 15.
    - A) 7 + 8 + 15 = **30**
    - B) 8 - 7 + 15 = **16**
    - ● 7 × 8 + 15 = **71**
43. Reduce the quotient of 24 and 6 by 4.
    - A) 24 - 6 - 4 = **14**
    - ● 24 ÷ 6 - 4 = **0**
    - C) 24 + 6 - 4 = **26**
44. From the product of 2, 3 and 4 subtract 9.
    - A) 2 × 3 × 4 + 9 = **33**
    - ● 2 + 3 + 4 - 9 = **0**
    - C) 2 × 3 × 4 - 9 = **11**
45. Find the total of 22, 33 and 17 and reduce the result by 9.
    - ● 22 + 33 + 17 - 9 = **63**
    - B) 22 + 33 - 17 + 9 = **47**
    - C) 22 + 33 + 17 + 9 = **81**
46. Reduce the sum of 14 and 21 by 7 then add 22 to the result.
    - A) 21 - 14 - 7 + 22 = **22**
    - B) 21 - 14 + 7 + 22 = **36**
    - ● 21 + 14 - 7 + 22 = **50**

Egyptian Numerals

Egyptian people used symbols to represent numbers. These symbols were of common objects that the people were familiar with. The table below has the names of these objects, the symbols themselves and the modern day numbers that these represent.

The system works by having up to 9 of each symbol, so 15 would be 1 heel bone and 5 vertical staffs, 20 would be 2 heel bones and 6 750 would be 6 Lotus flowers, 7 coiled ropes and 5 heel bones. The symbols are grouped together in any way you like and in any order you like. But for our purpose, let the order be from the largest down to the lowest value symbol.

Column 1 has groups of symbols, rewrite these in today's modern numerals, often referred to as the Hindu-Arabic number system. The table below will have to used so have it on screen or copy it in your book. The column ends with a pair of road signs, change the distances from modern numerals to Egyptian numerals, and write the symbols on the signs.

Column 2 is similar to the road sign questions giving numbers which are to be converted to Egyptian numerals.

Column 3 has questions that require an operation to be performed with modern numbers, find the answer, then rewrite that answer in Egyptian numerals.

1		Vertical staff
10		Heel bone
100		Coiled rope
1 000		Lotus flower
10 000		Bent reed
100 000		Fish
1 000 000		Amazed man

Egyptian Numerals

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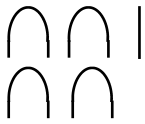
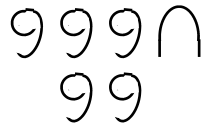
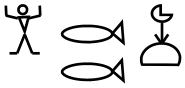
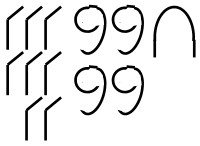
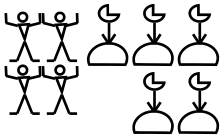
Change these Egyptian numerals to our modern (Hindu-Arabic) numbers

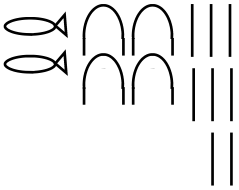
1 

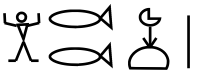
2 

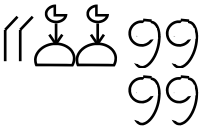
3 

4 

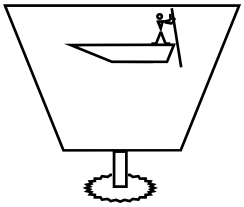
5 

6 

7 

8 

Complete the numerical distances in Egyptian numerals

9 

Nile Ferry
1 100

10 Cairo

Cairo
402

Convert these modern numerals to Egyptian numerals

11 32

12 3040

13 15 200

14 420 031

15 4 202

16 1 032 000

17 3 040

18 93 002

19 4 270 035

Answer these questions in Egyptian numerals

20 Write this year in Egyptian numerals.

21 Ali is offered a sculpture for  he haggles and is offered a price of:  How much did he save? Write the answer in Egyptian numerals.

-

22 Slimy Snake World sells asps in boxes of 12. If they have 28 boxes how many asps are there? Write your answer in Egyptian numerals.

X

23 Ali Baba and his forty thieves each need new socks. If they buy 8 pair each, how many socks are bought in all. Answer in Egyptian numerals.

X

Egyptian Numerals

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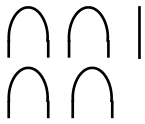
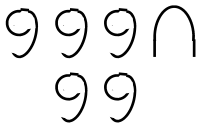
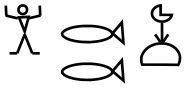
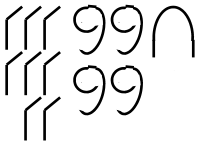
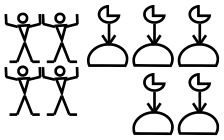
Change these Egyptian numerals to our modern (Hindu-Arabic) numbers

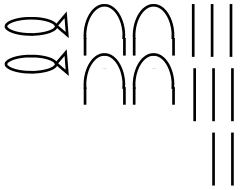
1  41

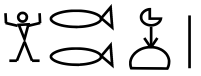
2  510

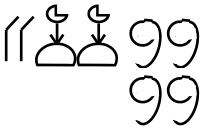
3  1 201 000

4  80 410

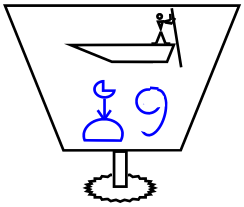
5  4 005 000

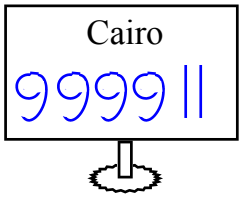
6  200 048

7  1 201 001

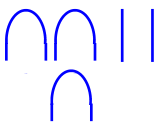
8  22 400

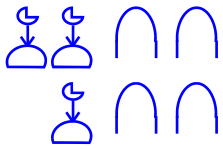
Complete the numerical distances in Egyptian numerals

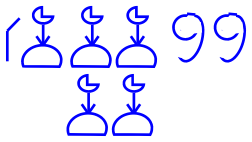
9 

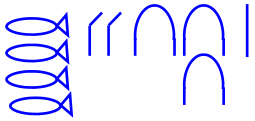
10 

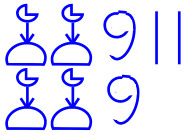
Convert these modern numerals to Egyptian numerals

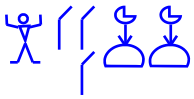
11 32 

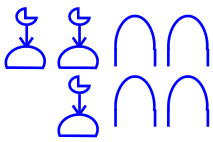
12 3040 

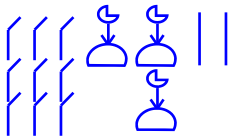
13 15 200 

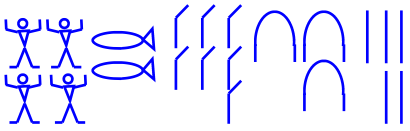
14 420 031 

15 4 202 

16 1 032 000 

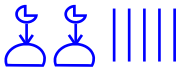
17 3 040 

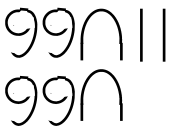
18 93 002 

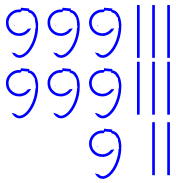
19 4 270 035 

Answer these questions in Egyptian numerals

20 Write this year in Egyptian numerals.



21 Ali is offered a sculpture for  he haggles and is offered a price of:  How much did he save? Write the answer in Egyptian numerals.



1 130 -
422
1 1
708

22 Slimy Snake World sells asps in boxes of 12. If they have 28 boxes how many asps are there? Write your answer in Egyptian numerals.



1
28 X
12
1
56
280
336

23 Ali Baba and his forty thieves each need new socks. If they buy 8 pair each, how many socks are bought in all. Answer in Egyptian numerals.



41 X
16
246
410
656

Roman Numerals

Much like the Egyptian system, Roman numerals build up numbers by combining several symbols together. The differences are:

- Instead of pictures, letters are used to represent the numbers, (see the table below)
- The letters are written in a single row starting from largest to smallest
- A maximum of 3 of the same letter can be used, instead of 4 letters you subtract the letter from the next largest letter. Subtraction is the only time that a smaller value letter comes before a letter of larger value.

Column 1 starts with converting Roman numbers to our modern day (Hindu-Arabic) system. Use the table below for reference if you need it. Complete the tables first then continue down the column. There are no 'subtraction' style questions in this column.

Column 2 asks you to answer questions in the same way, except this time there are subtraction style questions present. Remember there is no single Roman numeral for the following: 4, 9, 40, 90, 400 and 900. You have to 'subtract' numerals to get these numbers.
Eg. 4 = IV, 9 = IX, 40 = XL, 90 = XC, 400 = CD and 900 = CM.

You can take 1 from 10, 10 from 50 and 10 from 100, 100 from 500 and 100 from 1 000. But You can't take 1 from 500 to make 499, or 10 from 500 to make 490. So, 490 = CDXC and 499 = CDXCIX.

Column 3 asks you to write the given numbers in Roman numerals.

1	I
5	V
10	X
50	L
100	C
500	D
1 000	M

Roman Numerals

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Complete the tables below then answer the questions, use capital letters.

I	
	5
X	
L	
	100
	500
M	

III	
VI	
XV	
LII	
CX	
DV	
MV	



Change these Roman numerals to our modern numbers.

1 XXV _____

2 LXVI _____

3 CCXV _____

4 CVI _____

5 LXXXVI _____

6 CCLI _____

7 DCL _____

8 MDI _____

9 MMDL _____

10 MDCLX _____

11 MMMIII _____

12 DCLXI _____

13 DXXI _____

14 MMXX _____

15 MDXV _____



Change these to our modern numbers, these will have 'subtraction' questions among them.

II	
III	
IV	
VII	
VIII	
IX	
XI	

L	
LX	
XL	
C	
CX	
XC	
CD	

16 XXVIII _____

17 XXIX _____

18 XCI _____

19 XCIV _____

20 XCIX _____

21 CMX _____

22 CMXL _____

23 MMXC _____

24 MCMXL _____

25 XLIV _____

26 CXIX _____

27 MMCMI _____

28 DCXCIV _____

29 CXXIV _____

30 MCXLII _____

31 CMXLIII _____

32 CDXCIV _____



Write the Roman numerals for the modern numbers shown

6	
12	
20	
53	
110	
150	
140	

190	
502	
160	
540	
901	
19	
62	

33 56 _____

34 456 _____

35 39 _____

36 74 _____

37 594 _____

38 381 _____

39 490 _____

40 3100 _____

41 366 _____

42 199 _____

43 673 _____

44 1 241 _____

45 895 _____

46 394 _____

47 1 999 _____

48 2 440 _____

49 1 599 _____

Roman Numerals

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Complete the tables below then answer the questions, use capital letters.

I	1
V	5
X	10
L	50
C	100
D	500
M	1 000

III	3
VI	6
XV	15
LII	52
CX	110
DV	505
MV	1 005

Change these Roman numerals to our modern numbers.

1 XXV = 25

2 LXVI = 66

3 CCXV = 215

4 CVI = 106

5 LXXXVI = 86

6 CCLI = 251

7 DCL = 650

8 MDI = 1 501

9 MMDL = 2 550

10 MDCLX = 1 660

11 MMMIII = 3 003

12 DCLXI = 661

13 DXXI = 521

14 MMXX = 2 020

15 MDXV = 1 515

Change these to our modern numbers, these will have 'subtraction' questions among them.

II	2
III	3
IV	4
VII	7
VIII	8
IX	9
XI	11

L	50
LX	60
XL	40
C	100
CX	110
XC	90
CD	400

16 XXVIII = 28

17 XXIX = 29

18 XCI = 91

19 XCIV = 94

20 XCIX = 99

21 CMX = 910

22 CMXL = 940

23 MMXC = 2 090

24 MCMXL = 1 940

25 XLIV = 44

26 CXIX = 119

27 MMCMI = 2 901

28 DCXCIV = 694

29 CXXIV = 124

30 MCXLII = 1 142

31 CMXLIII = 943

32 CDXCIV = 494

Write the Roman numerals for the modern numbers shown

6	VI
12	XII
20	XX
53	LIII
110	CX
150	CL
140	CXL

190	CXC
502	DII
160	CLX
540	DXL
901	CMI
19	XIX
62	LXII

33 56 = LVI

34 456 = CDLVI

35 39 = XXXIX

36 74 = LXXIV

37 594 = DXCIV

38 381 = CCCLXXXI

39 490 = CDXC

40 3100 = MMMC

41 366 = CCCLXVI

42 199 = CXCIX

43 673 = DCLXXIII

44 1 241 = MCCXLI

45 895 = DCCCXCV

46 394 = CCCXCIV

47 1 999 = MCMXCIX

48 2 440 = MMCDXL

49 1 599 = MDXCIX

Inequalities, Ascending and Descending Order

Inequalities in mathematics refer to the use of < (less than) and > (greater than) signs. This sheet asks you to look at two numbers, determine which number is the smallest one and put a < or > in the box that separates them. If you have trouble remembering the 'less than' sign (<) and the 'greater than' sign (>) then imagine that they are arrows. Then if you point the arrow to the smallest number you will always be correct. Just to shake things up a little some require = signs not just < or >.

Column 1 asks to place < or > between two numbers, then later, instead of two numbers, each side has a small operation to perform. Find the answer and write it below the operation and then 'point the arrows'. The example at the top of the column shows the use of canceling strokes and finding the total as you move across.

Column 2 is similar to column 1 except that this time there are 3 numbers (or operations) rather than 2. The other difference is that the signs are given, you write the numbers (and operations) in the boxes rather than the signs. Fizzle is giving you a warning at question 22 because the signs have changed direction, so watch them! Again look at the example for the method to be used.

Column 3 involves placing numbers in ascending and descending order, this is actually what you have already been doing in column 2. How do you remember that ascending goes up (from lowest to highest) and descending goes down (from highest to lowest). Descending starts with a D, just like Down. When you go down you go from high to low. I always say: "What about ascending?...WHO CARES!!" Because you only have to remember one of them, the other one is just the opposite, so just remember descending.

So for the first part of the column write the smallest number in the row below it, then cancel it out on the top line with a stroke (so you know you have used it), then look for the next highest number and so on. The second part of the column is to be written in descending order which starts with the largest number and goes down, cancel them as before.

Inequalities, Ascending and Descending Order

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Put <, > or = in the square to make true

Example

$\cancel{30} + \cancel{9}^{39} - 8$ [<] $22 + 17$

31 39

1 23 [ ] 37

2 110 [ ] 101

3 373 [ ] 337

4 10 010 [ ] 10 005

5 5 + 23 [ ] 28

6 23 - 6 [ ] 9 + 6

7 6 + 22 - 11 [ ] 22 - 5

8 9 × 4 + 14 [ ] 70 - 15

9 3 × 5 × 2 [ ] 5 × 6

10 32 ÷ 8 × 20 [ ] 9 × 10

11 37 + 15 - 9 [ ] 7 × 4

12 40 - 12 - 11 [ ] 10 + 5

13 33 - 15 [ ] 3 × 5 + 8

14 4 × 5 × 3 [ ] 90 - 20

15 18 + 11 + 15 [ ] 5 × 9

16 90 ÷ 10 × 4 [ ] 3 × 5 × 2

Put the numbers given into the correct position, you may have to do some calculating first.

Example

30, 18 + 9, 7 × 4

30 27 28

[30] > [7 × 4] > [18 + 9]

17 46, 23, 17

[ ] < [ ] < [ ]

18 153, 135, 351

[ ] < [ ] < [ ]

19 247, 89, 196

[ ] > [ ] > [ ]

20 111, 101, 110

[ ] < [ ] < [ ]

21 12 + 4, 6 +3, 14

[ ] > [ ] > [ ]

22 14 + 12, 5 × 7, 50 - 19

[ ] > [ ] > [ ]

23 37 + 25, 7 × 8, 9 × 6

[ ] > [ ] > [ ]

24 35 ÷ 7, 45 ÷ 5, 56 ÷ 8

[ ] > [ ] > [ ]

25 200 + 150, 500 - 120, 70 × 3

[ ] > [ ] > [ ]

Rewrite the numbers in ascending order

26 123, 281, 142, 412, 336

→ _____

27 1 232, 898, 1 199, 959

→ _____

28 9 857, 8 667, 10 023, 9 156

→ _____

29 43, 103, 79, 110, 130, 101

→ _____

30 1 076, 10 760, 1 067, 7 601

→ _____

31 564, 692, 288, 1 206, 1 089

→ _____

Rewrite the numbers in descending order

32 65, 75, 12, 33, 90, 29, 50

→ _____

33 130, 105, 103, 150, 203

→ _____

34 297, 369, 403, 1 003, 676

→ _____

35 8 766, 3 452, 10 299, 5 677

→ _____

36 221, 390, 667, 767, 903, 93

→ _____

37 11, 23, 8, 15, 31, 0, 12, 5, 7

→ _____

Inequalities, Ascending and Descending Order

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Put <, > or = in the square to make true

Example

$30 + 9 - 8$ (39) $\boxed{<}$ $22 + 17$ (39)

1 $23 \boxed{<} 37$ (31 < 37)

2 $110 \boxed{>} 101$

3 $373 \boxed{>} 337$

4 $10\,010 \boxed{>} 10\,005$

5 $5 + 23 \boxed{=} 28$ (28 = 28)

6 $23 - 6 \boxed{>} 9 + 6$ (17 > 15)

7 $6 + 22 - 11 \boxed{=} 22 - 5$ (28 − 11 = 17 = 22 − 5)

8 $9 \times 4 + 14 \boxed{<} 70 - 15$ (36 + 14 = 50 < 55)

9 $3 \times 5 \times 2 \boxed{=} 5 \times 6$ (15 × 2 = 30 = 30)

10 $32 \div 8 \times 20 \boxed{<} 9 \times 10$ (4 × 20 = 80 < 90)

11 $37 + 15 - 9 \boxed{>} 7 \times 4$ (52 − 9 = 43 > 28)

12 $40 - 12 - 11 \boxed{>} 10 + 5$ (28 − 11 = 17 > 15)

13 $33 - 15 \boxed{<} 3 \times 5 + 8$ (18 < 15 + 8 = 23)

14 $4 \times 5 \times 3 \boxed{<} 90 - 20$ (20 × 3 = 60 < 70)

15 $18 + 11 + 15 \boxed{<} 5 \times 9$ (29 + 15 = 44 < 45)

16 $90 \div 10 \times 4 \boxed{>} 3 \times 5 \times 2$ (9 × 4 = 36 > 30)

Put the numbers given into the correct position, you may have to do some calculating first.

Example

$30,\ 18 + 9,\ 7 \times 4$ (30, 27, 28)

$\boxed{30} > \boxed{7 \times 4} > \boxed{18 + 9}$

17 46, 23, 17

$\boxed{17} < \boxed{23} < \boxed{46}$

18 153, 135, 351

$\boxed{135} < \boxed{153} < \boxed{351}$

19 247, 89, 196

$\boxed{247} > \boxed{196} > \boxed{89}$

20 111, 101, 110

$\boxed{101} < \boxed{110} < \boxed{111}$

21 $12 + 4,\ 6 + 3,\ 14$ (16, 9, 14)

$\boxed{12 + 4} > \boxed{14} > \boxed{6 + 3}$

22 $14 + 12,\ 5 \times 7,\ 50 - 19$ (26, 35, 31)

$\boxed{5 \times 7} > \boxed{50 - 19} > \boxed{14 + 12}$

23 $37 + 25,\ 7 \times 8,\ 9 \times 6$ (62, 56, 54)

$\boxed{37 + 25} > \boxed{7 \times 8} > \boxed{9 \times 6}$

24 $35 \div 7,\ 45 \div 5,\ 56 \div 8$ (5, 9, 7)

$\boxed{45 \div 5} > \boxed{56 \div 8} > \boxed{35 \div 7}$

25 $200 + 150,\ 500 - 120,\ 70 \times 3$ (350, 380, 210)

$\boxed{500 - 120} > \boxed{200 + 150} > \boxed{70 \times 3}$

Rewrite the numbers in ascending order

26 ~~123~~, ~~281~~, ~~142~~, ~~412~~, ~~336~~
→ 123, 142, 281, 336, 412

27 ~~1 232~~, ~~898~~, ~~1 199~~, ~~959~~
→ 898, 959, 1 199, 1 232

28 ~~9 857~~, ~~8 667~~, ~~10 023~~, ~~9 156~~
→ 8 667, 9 156, 9 857, 10 023

29 ~~43~~, ~~103~~, ~~79~~, ~~110~~, ~~130~~, ~~101~~
→ 43, 79, 101, 103, 110, 130

30 ~~1 076~~, ~~10 760~~, ~~1 067~~, ~~7 601~~
→ 1 067, 1 076, 7 601, 10 760

31 ~~564~~, ~~692~~, ~~288~~, ~~1 206~~, ~~1 089~~
→ 288, 564, 692, 1 089, 1 206

Rewrite the numbers in descending order

32 ~~65~~, ~~75~~, ~~12~~, ~~33~~, ~~90~~, ~~29~~, ~~50~~
→ 90, 75, 65, 50, 33, 29, 12

33 ~~130~~, ~~105~~, ~~103~~, ~~150~~, ~~203~~
→ 203, 150, 130, 105, 103

34 ~~297~~, ~~369~~, ~~403~~, ~~1 003~~, ~~676~~
→ 1 003, 676, 403, 369, 297

35 ~~8 766~~, ~~3 452~~, ~~10 299~~, ~~5 677~~
→ 10 299, 8 766, 5 677, 3 452

36 ~~221~~, ~~390~~, ~~667~~, ~~767~~, ~~903~~, ~~93~~
→ 903, 767, 667, 390, 221, 93

37 ~~11~~, ~~23~~, ~~8~~, ~~15~~, ~~31~~, ~~0~~, ~~12~~, ~~5~~, ~~7~~
→ 31, 23, 15, 12, 11, 8, 7, 5, 0

Order of Operations

With a mathematical operation you normally move left to right, with order of operations this can change. Order of operations requires you to evaluate the question in this order:

- Evaluate × or ÷ first, then the + and - section of the question.
- Note × and ÷ have no priority over each other, it's simply the first one of the two as you move (to the right) through the question
- Likewise with + and - they have no priority, it's the first one you encounter.

Column 1 asks for the answer to each question considering order of operations. Look at each question as if it is in 'sections', the sections will be made by the × and ÷ signs, find totals for these then carry out the addition or subtraction between the sections. Look at the examples below.

Column 2 asks to evaluate the exercise each side of a box, then write either <, > or = to make them true. These are easier as there are 2 small operations rather than large one, so don't let the size of the question daunt you. Use the same method as with the first column except:

- Once you finish evaluating one side write the total in the box below each part of the question.
- Repeat for the other side, then using the 2 lower numbers write <, > or = in the box. Remember think of the < and > as arrows and point the arrow at the smallest number.

The bottom of column 2 has some questions which have a number missing. One side is complete, find the total for this side then write the answer in both lower boxes. Then find the value for the box that will give the total.

Column 3 are worded questions, colour the circle in front of the expression that matches the sentence, but once you have done that, solve all three.

This question doesn't require order of operations consideration because the × is first in the exercise.

$4 \times 3 + 2 =$ [14] (12)

This does as the × is after the + so calculate the × section first.

$4 + 3 \times 2 =$ [10] (6)

This requires the ÷ to be done first, so evaluate it first then work through the question from left to right.

$40 - 9 \div 3 + 2$ (3)

$= 40 - 3 + 2$ (37) $=$ [39]

This requires the × and ÷ to be done first, but because the × is first you do it first, get your total then do the ÷.

$80 - 4 \times 9 \div 2$ (36)

$= 80 - 36 \div 2$ (18) $=$ [62]

Order of Operations

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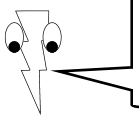

Try these, remember to do the × and ÷ before you do the + and –.

Example

$12 + 3 \times 4 = 24$ (12)

1 $4 + 6 \times 3 =$ ☐

2 $4 \times 6 + 3 =$ ☐

3 $10 + 15 \div 5 =$ ☐

4 $36 - 27 \div 9 =$ ☐

5 $28 - 12 \div 2 =$ ☐

6 $4 \times 20 \div 10 =$ ☐

7 $4 \times 5 + 3 \times 9$

$= \_\_\_\_\_ =$ ☐

8 $10 \times 6 - 5 \times 5$

$= \_\_\_\_\_ =$ ☐

9 $30 \div 6 + 9 \times 5$

$= \_\_\_\_\_ =$ ☐

10 $11 \times 6 - 24 \div 6$

$= \_\_\_\_\_ =$ ☐

11 $60 - 20 + 7 \times 3$

$= \_\_\_\_\_ =$ ☐

12 $9 \times 6 - 4 \times 6$

$= \_\_\_\_\_ =$ ☐

13 $72 \div 6 + 36 \div 3$

$= \_\_\_\_\_ =$ ☐

14 $12 + 16 \div 4 - 16$

$= \_\_\_\_\_ =$ ☐

15 $30 - 4 \times 5 + 17$

$= \_\_\_\_\_ =$ ☐



Fill the box with either <, >, or =. Remember point 'the arrow' to the smallest number.

Example

$9 + 2 \times 4$ (8) ☐ $30 - 8 \times 2$ (16)

17 > Total each side < 14

16 $10 + 15 \times 2$ ☐ $40 + 10$

17 $86 - 6 \times 11$ ☐ $5 + 3 \times 3$

18 $10 \div 2 \times 5$ ☐ $15 + 5 \times 2$

19 $6 + 3 \times 11$ ☐ $9 + 24 \div 3$

20 $99 - 81 \div 9$ ☐ $48 \div 6 \times 2$

21 $12 + 64 \div 8$ ☐ $80 \div 5 + 5$

22 $15 + 55 \div 5$ ☐ $7 + 39 \div 3$



Find the missing number that makes these true

23 $10 + 5 \times$ ☐ $= 60 - 5$

24 $30 - 12 \times 2 = 10 -$ ☐

25 $20 + 16 \div 4 = 9 + 5 \times$ ☐

26 $35 - 3 \times$ ☐ $= 12 \div 4 + 2$



Read the sentence, select the matching operation but solve all three.

27 James is carrying 23 pair of shoes at once, if he drops three shoes, how many shoes is he carrying?

(A) $23 - 3 \times 2 =$ ☐

(B) $23 \times 2 - 3 \times 2$

$= \_\_\_\_\_ =$ ☐

(C) $23 \times 2 - 3 =$ ☐

28 Jane has a sheep farm. If she has 14 sheep in the barn and three fields with 15 sheep in each field, how many sheep does she have?

(A) $15 + 14 \times 3 =$ ☐

(B) $14 + 3 \times 15 =$ ☐

(C) $15 \times 3 - 14 =$ ☐

29 An ex-soldier wishes to give all of his 14 service ribbons and half of his 32 medal collection to his grand-daughter. How many awards does the child receive?

(A) $32 + 14 \div 2 =$ ☐

(B) $14 + 32 \div 2 =$ ☐

(C) $32 \div 2 - 14 =$ ☐

30 A school music production has several acts by different groups. There will be 3 quartets, 4 duos and one trio. How many students are performing?

(A) $3 \times 4 \times 4 \times 2 + 1$

$= \_\_\_\_\_ =$ ☐

(B) $3 \times 4 + 4 \times 2 + 3$

$= \_\_\_\_\_ =$ ☐

(C) $3 \times 4 + 4 \times 2 + 1$

$= \_\_\_\_\_ =$ ☐

Order of Operations

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Try these, remember to do the × and ÷ before you do the + and –.

Example

$12 + 3 \times 4 = 24$ (12)

1 $4 + 6 \times 3 = 22$ (18)

2 $4 \times 6 + 3 = 27$ (24)

3 $10 + 15 \div 5 = 13$ (3)

4 $36 - 27 \div 9 = 33$ (3)

5 $28 - 12 \div 2 = 22$ (6)

6 $4 \times 20 \div 10 = 8$ (80)

7 $4 \times 5 + 3 \times 9$
$= 20 + 27 = 47$

8 $10 \times 6 - 5 \times 5$
$= 60 - 25 = 35$

9 $30 \div 6 + 9 \times 5$
$= 5 + 45 = 50$

10 $11 \times 6 - 24 \div 6$
$= 66 - 4 = 62$

11 $60 - 20 + 7 \times 3$ (21)
$= 40 + 21 = 61$

12 $9 \times 6 - 4 \times 6$
$= 54 - 24 = 30$

13 $72 \div 6 + 36 \div 3$
$= 12 + 12 = 24$

14 $12 + 16 \div 4 - 16$ (4)
$= 12 + 4 - 16 = 0$ (16)

15 $30 - 4 \times 5 + 17$ (20)
$= 30 - 20 + 17 = 27$ (10)

Fill the box with either <, >, or =. Remember point 'the arrow' to the smallest number.

Example

$9 + 2 \times 4$ (8) $\boxed{>}$ $30 - 8 \times 2$ (16)

17 Total each side 14

16 $10 + 15 \times 2$ (30) $\boxed{<}$ $40 + 10$
40 50

17 $86 - 6 \times 11$ (66) $\boxed{>}$ $5 + 3 \times 3$ (9)
20 14

18 $10 \div 2 \times 5$ (5) $\boxed{=}$ $15 + 5 \times 2$ (10)
25 25

19 $6 + 3 \times 11$ (33) $\boxed{>}$ $9 + 24 \div 3$ (8)
39 17

20 $99 - 81 \div 9$ (9) $\boxed{>}$ $48 \div 6 \times 2$ (8)
90 16

21 $12 + 64 \div 8$ (8) $\boxed{<}$ $80 \div 5 + 5$ (16)
20 21

22 $15 + 55 \div 5$ (11) $\boxed{>}$ $7 + 39 \div 3$ (13)
26 20

Find the missing number that makes these true

23 $10 + 5 \times \boxed{9} = 60 - 5$
55 55

24 $30 - 12 \times 2$ (24) $= 10 - \boxed{4}$
6 6

25 $20 + 16 \div 4$ (4) $= 9 + 5 \times \boxed{3}$
24 24

26 $35 - 3 \times \boxed{10} = 12 \div 4$ (3) $+ 2$
5 5

Read the sentence, select the matching operation but solve all three.

27 James is carrying 23 pair of shoes at once, if he drops three shoes, how many shoes is he carrying?

A $23 - 3 \times 2$ (6) $= 17$

B $23 \times 2 - 3 \times 2$
$= 46 - 6 = 40$ (46)

● $23 \times 2 - 3 = 43$

28 Jane has a sheep farm. If she has 14 sheep in the barn and three fields with 15 sheep in each field, how many sheep does she have?

A $15 + 14 \times 3 = 57$ (42)

● $14 + 3 \times 15 = 59$ (45)

C $15 \times 3 - 14 = 31$ (45)

29 An ex-soldier wishes to give all of his 14 service ribbons and half of his 32 medal collection to his grand-daughter. How many awards does the child receive?

A $32 + 14 \div 2 = 39$ (7)

● $14 + 32 \div 2 = 30$ (16)

C $32 \div 2 - 14 = 2$ (16)

30 A school music production has several acts by different groups. There will be 3 quartets, 4 duos and one trio. How many students are performing?

A $3 \times 4 \times 4 \times 2 + 1$ (12) (8)
$= 12 \times 8 + 1 = 97$ (96)

● $3 \times 4 + 4 \times 2 + 3$ (12) (8)
$= 12 + 8 + 3 = 23$ (20)

C $3 \times 4 + 4 \times 2 + 1$ (12) (8)
$= 12 + 8 + 1 = 21$ (20)

Using Brackets

Before attempting this sheet order of operations should be attempted first. This sheet involves using Brackets and their priority over the 4 operations. Square roots and powers are occasionally referred to as Orders, these come after brackets. Division and/or Multiplication comes before Addition and/or Subtraction, so….BODMAS is what you have.

Students forget what O refers to, it also implies that Division comes before Multiplication, which isn't correct, they have the same priority. As does Addition and Subtraction, so you can use BODMAS to help you remember, but it has its problems.

Column 1 starts with questions which have brackets. As brackets come first, evaluate the operation in the brackets then complete the rest of the question. The first 8 questions have no order of operation rules to apply, other than brackets. Like the example below.

$$100 \div (10 + 15) \div 2$$

$$= \underline{100 \div 25 \div 2} = \boxed{2}$$

Note that the example at the top of the column on the worksheet is an order of operations question, Q9 onwards are this style of question, testing your knowledge of order of operations.

Column 2 require brackets to be placed if they are required for the operation to equal the answer given. Questions 18 to 31 require one set of brackets or no brackets, if you put brackets in a question where they aren't required, usually due to order of operations, you may get the question wrong in a test. These questions are challenging. The last seven questions can either have 1, 2, or no pairs of brackets, so these could also be a challenge. Realise that you can place the brackets, test and see if it works, and if it doesn't, try placing them in a different position. This is a logical way of solving the question.

Column 3 starts with finding the number in the brackets for a given answer. The method is to find the total that the brackets must equal first, write the answer above the bracketed numbers, then write the number in the square inside the brackets that will get you the total.

Square roots and division operations are then briefly looked at. With square roots you find the total in the square root sign, then square root it. When there are operations above and below the division bar, total the top and total the bottom and then complete the division. If you aren't familiar with square roots: a square root is the number which when multiplied by itself equals the number in the square root sign.

Using Brackets

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Brackets are done before other operations. Evaluate these (find the answer).

Example

$10 + (\cancel{5} + \cancel{3}^{8}) \times 4$

$= 10 + \cancel{8}^{32} \times \cancel{4}$ $= \boxed{42}$

1 $10 \times (12 - 7) =$ ☐

2 $16 \div (5 + 3) =$ ☐

3 $(20 + 40) \div 30 =$ ☐

4 $(13 + 12) \times 3 =$ ☐

5 $80 \div (25 + 6 - 11)$

$=$ _____ $=$ ☐

6 $12 \times (17 + 10 - 23)$

$=$ _____ $=$ ☐

7 $(65 - 13 + 4) \div 7$

$=$ _____ $=$ ☐

8 $4 \times (4 + 5) \div 12$

$=$ _____ $=$ ☐

Remember brackets first, but then look for order of operation rules to apply.

9 $6 + (9 - 4) \times 9$

$=$ _____ $=$ ☐

10 $50 - (12 + 3) \times 3$

$=$ _____ $=$ ☐

11 $(34 - 16) \div 6 + 3$

$=$ _____ $=$ ☐

12 $120 \div (13 + 17) + 10$

$=$ _____ $=$ ☐

Put in <u>one</u> set of brackets to make these true, but only if they are required.

Example

$(\cancel{3} + \cancel{2}^{5}) \times 4 = 20$

13 $4 \times 3 + 2 = 14$

14 $6 + 5 \times 7 = 41$

15 $10 + 2 \div 2 = 6$

16 $19 - 4 \div 3 = 5$

17 $6 + 4 \times 5 = 26$

18 $8 - 3 \times 2 = 10$

19 $12 - 8 \div 4 = 10$

20 $25 - 18 \div 3 \times 2 = 22$

21 $9 + 2 \times 5 - 2 \times 4 = 47$

22 $80 \div 2 - 3 \times 7 - 6 = 25$

23 $48 + 3 \times 12 \div 4 = 57$

24 $12 + 7 \times 2 + 3 = 47$

25 $30 - 4 \times 8 - 3 = 1$

26 $24 \div 6 + 3 - 7 \times 2 = 24$

These <u>may</u> require up to 2 sets of brackets

27 $18 + 24 \div 2 + 4 = 7$

28 $9 - 6 \times 2 \times 4 - 3 = 6$

29 $30 \div 6 - 3 \times 5 - 1 = 40$

30 $60 \div 20 + 30 - 6 \div 2 = 30$

31 $3 + 3 \times 3 - 3 \div 3 = 3$

32 $10 + 2 \times 10 \div 1 + 4 = 38$

33 $50 - 24 \div 3 + 1 \times 6 = 48$

Find the missing value to make these correct.

34 $(3 + \square) \times 6 = 36$

35 $(2 + \square) \div 3 = 6$

36 $3 \times (8 - \square) = 9$

37 $35 \div (10 - \square) = 5$

38 $40 \div (3 + \square) = 8$

39 $(12 - \square) \times 5 = 25$

The division bar and the square root sign group like brackets. Try these.

Example (1) $\sqrt{12 - 3} = \sqrt{9} = 3$

Example (2) $\dfrac{29 + 13}{22 - 15} = \dfrac{42}{7} = 6$

Remember this means ÷

40 $\sqrt{65 - 40} = \sqrt{\square} = \square$

41 $\sqrt{4 \times 9} = \sqrt{\square} = \square$

42 $\dfrac{15 + 45}{3 \times 4} = \dfrac{\square}{\square} = \square$

43 $\sqrt{35 + 2 \times 7} = \sqrt{\square} = \square$

44 $\dfrac{100 - 8 \times 8}{32 \div 4 + 1} = \dfrac{\square}{\square} = \square$

45 $\dfrac{3 + 54 \div 9}{6 \times 6 \div 4} = \dfrac{\square}{\square} = \square$

46 $\sqrt{7 + 21 \times 2} = \sqrt{\square} = \square$

Using Brackets

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Brackets are done before other operations. Evaluate these (find the answer).

Example

$10 + (5 + 3) \times 4$

$= 10 + 8 \times 4 = 42$

1 $10 \times (12 - 7) = 50$

2 $16 \div (5 + 3) = 2$

3 $(20 + 40) \div 30 = 2$

4 $(13 + 12) \times 3 = 75$

5 $80 \div (25 + 6 - 11)$

$= 80 \div 20 = 4$

6 $12 \times (17 + 10 - 23)$

$= 12 \times 4 = 48$

7 $(65 - 13 + 4) \div 7$

$= 56 \div 7 = 8$

8 $4 \times (4 + 5) \div 12$

$= 4 \times 9 \div 12 = 3$

Remember brackets first, but then look for order of operation rules to apply.

9 $6 + (9 - 4) \times 9$

$= 6 + 5 \times 9 = 51$

10 $50 - (12 + 3) \times 3$

$= 50 - 15 \times 3 = 5$

11 $(34 - 16) \div 6 + 3$

$= 18 \div 6 + 3 = 6$

12 $120 \div (13 + 17) + 10$

$= 120 \div 30 + 10 = 14$

Put in one set of brackets to make these true, but only if they are required.

Example

$(3 + 2) \times 4 = 20$

13 $4 \times 3 + 2 = 14$

14 $6 + 5 \times 7 = 41$

15 $(10 + 2) \div 2 = 6$

16 $(19 - 4) \div 3 = 5$

17 $6 + 4 \times 5 = 26$

18 $(8 - 3) \times 2 = 10$

19 $12 - 8 \div 4 = 10$

20 $25 - 18 \div (3 \times 2) = 22$

21 $(9 + 2) \times 5 - 2 \times 4 = 47$

22 $80 \div 2 - (3 \times 7 - 6) = 25$

23 $48 + 3 \times 12 \div 4 = 57$

24 $12 + 7 \times (2 + 3) = 47$

25 $30 - (4 \times 8 - 3) = 1$

26 $24 \div (6 + 3 - 7) \times 2 = 24$

These may require up to 2 sets of brackets

27 $(18 + 24) \div (2 + 4) = 7$

28 $(9 - 6) \times 2 \times (4 - 3) = 6$

29 $30 \div (6 - 3) \times (5 - 1) = 40$

30 $60 \div 20 + 30 - 6 \div 2 = 30$

31 $3 + 3 \times (3 - 3) \div 3 = 3$

32 $10 + 2 \times (10 \div 1 + 4) = 38$

33 $(50 - 24 \div 3) + 1 \times 6 = 48$

Find the missing value to make these correct.

34 $(3 + \boxed{3}) \times 6 = 36$

35 $(2 + \boxed{16}) \div 3 = 6$

36 $3 \times (8 - \boxed{5}) = 9$

37 $35 \div (10 - \boxed{3}) = 5$

38 $40 \div (3 + \boxed{2}) = 8$

39 $(12 - \boxed{7}) \times 5 = 25$

The division bar and the square root sign group like brackets. Try these.

Example (1) $\sqrt{12 - 3} = \sqrt{9} = 3$

Example (2) $\dfrac{29 + 13}{22 - 15} = \dfrac{42}{7} = 6$

Remember this means ÷

40 $\sqrt{65 - 40} = \sqrt{25} = 5$

41 $\sqrt{4 \times 9} = \sqrt{36} = 6$

42 $\dfrac{15 + 45}{3 \times 4} = \dfrac{60}{12} = 5$

43 $\sqrt{35 + 2 \times 7} = \sqrt{49} = 7$

44 $\dfrac{100 - 8 \times 8}{32 \div 4 + 1} = \dfrac{36}{9} = 4$

45 $\dfrac{3 + 54 \div 9}{6 \times 6 \div 4} = \dfrac{9}{9} = 1$

46 $\sqrt{7 + 21 \times 2} = \sqrt{49} = 7$

Place Value

Place value is the value of a number in a set position. For example the value of the digit 3 in the following would be:

- 30 in 1 234
- 300 in 2 346.04
- 30 000 in 1 236 790
- 0.03 in 12.439.

If the number is in front of the decimal point count how many digits are behind it until the decimal point. Then write the number and count off the zeros behind it.

1 234

Example. For the place value of 5 in 457 609.7, write '5' then count the digits behind the 5 back to the decimal point, there are 4 digits so write 4 zeros, 50 000. If the number is behind the decimal point write '0.' then count the digits between the decimal point and the number.

12

Example. For the place value of 9 in 344.379, write '0.' then count the digits between the decimal point and the 9. There are 2 digits so write 2 zeros then the 9, 0.009. (which is zero point zero zero nine or just point zero zero nine).

Column 1 has 10 numbers, 5 written in words, the other 5 in numerals. Start by placing all of the numbers in the table at the top of the column then find the place value of both the 5 and the 7 in each number.

Column 2 starts with 10 questions on the place value of a number which is given in brackets. This is the same as the first column except that you are asked to give your answer in words. There are two acceptable ways as the example shows at the top of the column. The first way is as used in the first column, but in words. The other way is to write the value of the position. In the example it is ten thousands.

The last part of the column has three numbers in each question, you are asked to write the number which gives the largest place value for the number in brackets. Once you have done this you are asked to write your answer in words. Note that the commas in these questions separate the numbers, if you write 1 000 as 1,000 don't get confused with the commas, the actual numbers don't have commas in them.

Place Value

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Put the numbers in the questions below, in the table. Then give the place value of both the 5 and of the 7 in each number. The first one has been done for you.

Table 1 : Place Values

	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units (or ones)	Decimal Point	Tenths	Hundredths	Thousandths
	2	4	5	0	1	1	9	.	7		
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											

Example (see table also)

Two million, four hundred and fifty thousand, one hundred and nineteen and seven tenths. 5: 50 000 7: 0.7

1 Five hundred and twenty-seven thousand and three, point zero four six. 5: _____ 7: _____

2 Eight million, seventy-six thousand, three hundred and ten, point two five. 5: _____ 7: _____

3 Two hundred and thirty-six thousand, seven hundred and twenty-three, point eight nine five. 5: _____ 7: _____

4 Five million, four hundred and thirty-seven thousand, one hundred and two. 5: _____ 7: _____

5 One thousand, five hundred and thirty-nine, point zero two seven 5: _____ 7: _____

6 117 002.5 5: _____ 7: _____

7 2 750 032 5: _____ 7: _____

8 67 025.43 5: _____ 7: _____

9 29 536.07 5: _____ 7: _____

10 315 019.7 5: _____ 7: _____

Give the place value of the bracketed number in words, either of the two formats shown is acceptable

Example

364 257.1 [6] Six ten thousands or Sixty thousand

11 3 583.452 [4] _____

12 2 877 643 [8] _____

13 9 227.023 [3] _____

14 345 002 [4] _____

15 311.129 [9] _____

16 768 909 [6] _____

17 9 076 223 [9] _____

18 7.008 [7] _____

19 329 777.3 [2] _____

20 555 712.6 [1] _____

Using the number in brackets, write the number (and in words) which has the largest corresponding place value.

Example

14 887, 2 300 403, 99 994 [4] 14 887

400 → 4 000 is largest place value of 4

Fourteen thousand eight hundred and eighty-seven

21 8 455 237, 322.095, 3 668 [3] _____

22 2 347.8, 554 237, 20.009 [2] _____

23 790, 567 352.96, 378 002 [7] _____

24 9 065, 336 950.8, 93.007 [9] _____

Place Value

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Put the numbers in the questions below, in the table. Then give the place value of both the 5 and of the 7 in each number. The first one has been done for you.



Table 1 : Place Values

	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Units (or ones)	Decimal Point	Tenths	Hundredths	Thousandths
	2	4	5	0	1	1	9	.	7		
1		5	2	7	0	0	3	.	0	4	6
2	8	0	7	6	3	1	0	.	2	5	
3		2	3	6	7	2	3	.	8	9	5
4	5	4	3	7	1	0	2				
5				1	5	3	9	.	0	2	7
6		1	1	7	0	0	2	.	5		
7	2	7	5	0	0	3	2				
8			6	7	0	2	5	.	4	3	
9			2	9	5	3	6	.	0	7	
10		3	1	5	0	1	9	.	7		

Example (see table also)

Two million, four hundred and fifty thousand, one hundred and nineteen and seven tenths. 5: 50 000 7: 0.7

1 Five hundred and twenty-seven thousand and three, point zero four six. 5: 500 000 7: 7 000

2 Eight million, seventy-six thousand, three hundred and ten, point two five. 5: 0.05 7: 70 000

3 Two hundred and thirty-six thousand, seven hundred and twenty-three, point eight nine five. 5: 0.005 7: 700

4 Five million, four hundred and thirty-seven thousand, one hundred and two. 5: 5 000 000 7: 7 000

5 One thousand, five hundred and thirty-nine, point zero two seven 5: 500 7: 0.007

6 117 002.5 5: 0.5 7: 7 000

7 2 750 032 5: 50 000 7: 700 000

8 67 025.43 5: 5 7: 7 000

9 29 536.07 5: 500 7: 0.07

10 315 019.7 5: 5 000 7: 0.7



Give the place value of the bracketed number in words, either of the two formats shown is acceptable

Example

364 257.1 [6] Six ten thousands or Sixty thousand

11 3 583.452 [4] Four tenths

12 2 877 643 [8] Eight hundred thousands

13 9 227.023 [3] Three thousandths

14 345 002 [4] Four ten thousands

15 311.129 [9] Nine thousandths

16 768 909 [6] Six ten thousands

17 9 076 223 [9] Nine millions

18 7.008 [7] Seven units (or ones)

19 329 777.3 [2] Two ten thousands

20 555 712.6 [1] One tens



Using the number in brackets, write the number (and in words) which has the largest corresponding place value.

Example

400 4

14 887, 2 300 403, 99 994 [4] 14 887

4 000 is largest place value of 4

Fourteen thousand eight hundred and eighty-seven

21 8 455 237, 322.095, 3 668 [3] 3 668

Three thousand, six hundred and sixty-eight

22 2 347.8, 554 237, 20.009 [2] 2 347.8

Two thousand three hundred and forty-seven point eight

23 790, 567 352.96, 378 002 [7] 378 002

Three hundred and seventy-eight thousand and two

24 9 065, 336 950.8, 93.007 [9] 9 065

Nine thousand and sixty-five

Expanded Notation

This sheet has some questions written sideways. To read these questions click on the "View" menu, select the rotate option then "counter clockwise".

Numbers written in expanded notation are broken up into their place values. Multiplying the number by the place value of its position.

In column 1 a number has been expressed in expanded notation, this is to be rewritten in basic numerals. The best way to attempt these problems is to look at them digit by digit, for example : $7 \times 100\,000 + 4 \times 1\,000 + 8 \times 100 + 6$, follow these steps:

- The largest unit is 100 000 write the number 7
- After 100 000's comes 10 000's, how many are there? None, so write a zero
- How many thousands? Write the number 4
- How many hundreds? Write the number 8
- How many tens? None, so write a zero
- How many units (or ones)? Write the number 6
- You now have 704 806.

After the first 9 questions the sheet is written side ways, so follow the instruction in red above.

Column 2 works in the reverse, remember if there is a zero in the number write nothing for it.

Expanded Notation

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Change from expanded notation to basic numerals

Example

$7 \times 100\,000 + 4 \times 1\,000 + 3 = 704\,003$

1. $5 \times 10\,000 + 4 \times 1\,000 + 2 \times 10 =$ _____
2. $3 \times 1\,000 + 6 \times 100 + 3 \times 10 =$ _____
3. $9 \times 1\,000 + 7 \times 10 + 8 =$ _____
4. $1 \times 1\,000\,000 + 7 \times 10\,000 + 7 =$ _____
5. $4 \times 10\,000 + 8 \times 1\,000 + 5 \times 10 =$ _____
6. $2 \times 100\,000 + 5 \times 10\,000 + 9 =$ _____
7. $6 \times 1\,000 + 7 \times 100 + 2 \times 10 =$ _____
8. $8 \times 1\,000\,000 + 4 \times 10\,000 + 1 =$ _____
9. $1 \times 100\,000 + 6 \times 100 + 5 =$ _____
10. $5 \times 100\,000 + 2 \times 10\,000 + 3 \times 100 + 7 =$ _____
11. $9 \times 1\,000\,000 + 4 \times 1\,000 + 5 \times 100 + 3 \times 10 + 4 =$ _____
12. $8 \times 1\,000\,000 + 3 \times 10\,000 + 7 \times 1\,000 + 9 \times 10 =$ _____
13. $2 \times 100\,000 + 3 \times 10\,000 + 6 \times 100 + 4 \times 10 =$ _____
14. $7 \times 1\,000\,000 + 1 \times 100\,000 + 6 \times 100 =$ _____
15. $8 \times 100\,000 + 4 \times 1\,000 + 4 \times 10 + 7 =$ _____
16. $2 \times 1\,000\,000 + 6 \times 10\,000 + 5 \times 1\,000 + 9 \times 100 + 6 =$ _____
17. $9 \times 10\,000 + 3 \times 1\,000 + 8 \times 100 + 8 =$ _____
18. $4 \times 100\,000 + 5 \times 1\,000 + 9 \times 100 + 3 \times 10 + 3 =$ _____
19. $5 \times 100\,000 + 9 \times 10\,000 + 6 \times 1\,000 =$ _____
20. $8 \times 1\,000\,000 + 3 \times 100\,000 + 5 \times 100 + 8 =$ _____

Now the reverse, write these numbers in expanded notation



Example

$102\,700 = 1 \times 100\,000 + 2 \times 1\,000 + 7 \times 100$

21. $593 =$ _____
22. $1\,270 =$ _____
23. $50\,035 =$ _____
24. $9\,010\,002 =$ _____
25. $150 =$ _____
26. $70\,330 =$ _____
27. $8\,070 =$ _____
28. $3\,410 =$ _____
29. $7\,000\,900 =$ _____
30. $17\,431 =$ _____
31. $105\,309 =$ _____
32. $85\,317 =$ _____
33. $5\,608\,020 =$ _____
34. $4\,841 =$ _____
35. $6\,040\,902 =$ _____
36. $110\,882 =$ _____
37. $7\,720\,000 =$ _____
38. $15\,936 =$ _____
39. $276\,410 =$ _____
40. $5\,059\,107 =$ _____

Expanded Notation

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Change from expanded notation to basic numerals

Example

$7 \times 100\,000 + 4 \times 1\,000 + 3 = 704\,003$

1. $5 \times 10\,000 + 4 \times 1\,000 + 2 \times 10 = 54\,020$
2. $3 \times 1\,000 + 6 \times 100 + 3 \times 10 = 3\,630$
3. $9 \times 1\,000 + 7 \times 10 + 8 = 9\,078$
4. $1 \times 1\,000\,000 + 7 \times 10\,000 + 7 = 1\,070\,007$
5. $4 \times 10\,000 + 8 \times 1\,000 + 5 \times 10 = 48\,050$
6. $2 \times 100\,000 + 5 \times 10\,000 + 9 = 250\,009$
7. $6 \times 1\,000 + 7 \times 100 + 2 \times 10 = 6\,720$
8. $8 \times 1\,000\,000 + 4 \times 10\,000 + 1 = 8\,040\,001$
9. $1 \times 100\,000 + 6 \times 100 + 5 = 100\,605$
10. $5 \times 100\,000 + 2 \times 10\,000 + 3 \times 100 + 7 = 520\,307$
11. $9 \times 1\,000\,000 + 4 \times 1\,000 + 5 \times 100 + 3 \times 10 + 4 = 9\,004\,534$
12. $8 \times 1\,000\,000 + 3 \times 10\,000 + 7 \times 1\,000 + 9 \times 10 = 8\,037\,090$
13. $2 \times 100\,000 + 3 \times 10\,000 + 6 \times 100 + 4 \times 10 = 237\,090$
14. $7 \times 1\,000\,000 + 1 \times 100\,000 + 6 \times 100 = 7\,100\,600$
15. $8 \times 100\,000 + 4 \times 1\,000 + 4 \times 10 + 7 = 804\,047$
16. $2 \times 1\,000\,000 + 6 \times 10\,000 + 5 \times 1\,000 + 9 \times 100 + 6 = 2\,065\,906$
17. $9 \times 10\,000 + 3 \times 1\,000 + 8 \times 100 + 8 = 93\,808$
18. $4 \times 100\,000 + 5 \times 1\,000 + 9 \times 100 + 3 \times 10 + 3 = 405\,933$
19. $5 \times 100\,000 + 9 \times 10\,000 + 6 \times 1\,000 = 596\,000$
20. $8 \times 1\,000\,000 + 3 \times 100\,000 + 5 \times 100 + 8 = 8\,300\,508$



Now the reverse, write these numbers in expanded notation

Example

$102\,700 = 1 \times 100\,000 + 2 \times 1\,000 + 7 \times 100$

21. $593 = 5 \times 100 + 9 \times 10 + 3$
22. $1\,270 = 1 \times 1\,000 + 2 \times 100 + 7 \times 10$
23. $50\,035 = 5 \times 10\,000 + 3 \times 10 + 5$
24. $9\,010\,002 = 9 \times 1\,000\,000 + 1 \times 10\,000 + 2$
25. $150 = 1 \times 100 + 5 \times 10$
26. $70\,330 = 7 \times 10\,000 + 3 \times 100 + 3 \times 10$
27. $8\,070 = 8 \times 1\,000 + 7 \times 10$
28. $3\,410 = 3 \times 1\,000 + 4 \times 100 + 1 \times 10$
29. $7\,000\,900 = 7 \times 1\,000\,000 + 9 \times 100$
30. $17\,431 = 1 \times 10\,000 + 7 \times 1\,000 + 4 \times 100 + 3 \times 10 + 1$
31. $105\,309 = 1 \times 100\,000 + 5 \times 1\,000 + 3 \times 100 + 9$
32. $85\,317 = 8 \times 10\,000 + 5 \times 1\,000 + 3 \times 100 + 1 \times 10 + 7$
33. $5\,608\,020 = 5 \times 1\,000\,000 + 6 \times 100\,000 + 8 \times 1\,000 + 2 \times 10$
34. $4\,841 = 4 \times 1\,000 + 8 \times 100 + 4 \times 10 + 1$
35. $6\,040\,902 = 6 \times 1\,000\,000 + 4 \times 10\,000 + 9 \times 100 + 2$
36. $110\,882 = 1 \times 100\,000 + 1 \times 10\,000 + 8 \times 100 + 8 \times 10 + 2$
37. $7\,720\,000 = 7 \times 1\,000\,000 + 7 \times 100\,000 + 2 \times 10\,000$
38. $15\,936 = 1 \times 10\,000 + 5 \times 1\,000 + 9 \times 100 + 3 \times 10 + 6$
39. $276\,410 = 2 \times 100\,000 + 7 \times 10\,000 + 6 \times 1\,000 + 4 \times 100 + 1 \times 10$
40. $5\,059\,107 = 5 \times 1\,000\,000 + 5 \times 10\,000 + 9 \times 1\,000 + 1 \times 100 + 7$

Rounding Numbers

Numbers are often rounded when it is considered the smaller digits are not needed. An example would be a company's profits being rounded to the nearest $1 000, or even to the nearest $1 000 000 for large corporations, as the smaller amounts have little significance in overall profit.

Column 1 requires numbers to be rounded to the nearest 5. This means the answer will either end with a 5 or a 0. Note that 2.5 is halfway between 0 and 5 and 7.5 is halfway between 5 and 10. So if the last digit is less than 2.5 (1 or 2) it is changed to a 0. If it is between 2.5 and 7.5 (3, 4, 5, 6, 7) it is changed to a 5. If it is an 8 or 9 it is changed to a zero and 1 is added to the tens column.

To round to the nearest 10 the halfway point is 5. If the last digit is less than 5, ie (1, 2, 3 or 4) it is changed to a zero. If the last digit is 5 or more (5, 6, 7, 8 , 9) it is also changed to a zero but 1 is added to the tens column also.

Column 2 involves rounding to the nearest 100 and then 1 000. The method is the much the same as with 10's. For 100 the halfway point is 50 and with 1 000 the halfway point is 500. This means that all answers will end in '00' for hundreds and '000' when rounded to the nearest thousand.

An important point is that when a number is rounded it is possible for a number with value to become 0. For example, 430 when rounded to the nearest 1 000 becomes 0, as it is less than 500.

Column 3 is comparing answers if rounding is done at different stages of an addition. Given several numbers, add the numbers then round the answer to the nearest 100. Then perform the addition again except this time round the numbers before the addition.

Rounding Numbers

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Round these numbers to the nearest 5

Examples

i) 52 50 — In between 50 and 55 and is closer to 50 (< 52.5)

ii) 98 100 — In between 95 and 100 and is closer to 100 (≥ 97.5)

1	7	2	33
3	9	4	71
5	99	6	203
7	2	8	91
9	497	10	777
11	106	12	382

13 1 048

14 9 497

15 28 498

16 73 444



Round these numbers to the nearest 10

Example 167 170 — In between 160 and 170 and is closer to 170 (≥ 165)

17	12	18	56
19	104	20	255
21	4	22	86
23	193	24	464
25	298	26	99
27	11	28	67

29 1 011

30 9 889

31 10 573

32 19 996



Round these to the nearest 100

Example 1 371 1 400 — In between 1 300 and 1 400 and is closer to 1 400 (≥ 1 350)

33	149	34	670
35	29	36	250

37 1 809

38 5 037

39 3 099

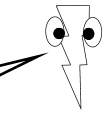
40 14 636

41 7 591

42 28 370

43 39 889

44 71 656



Round these to the nearest 1 000

Example 1 442 1 000

45 9 027

46 3 641

47 22 555

48 396

49 11 099

50 34 495

51 99 199

52 274 902

53 600 513

54 599 714

55 99 899



Add the numbers and round the answer to the nearest 100, then round the numbers to the nearest 100 and add them afterwards.

Example : 776, 345, 881 and 252

	1 1	
776 +		800 +
345		300
881		900
252		300
2 184 →	2 200	2 300

56 839, 242, 1 487, 3 510

\+ \+

57 112, 5 861, 8 170, 923, 756

\+ \+

57 2 859, 933, 5 211, 43, 1 169

\+ \+

Rounding Numbers

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Round these numbers to the nearest 5

Examples

i) 52 50 — In between 50 and 55 and is closer to 50 (< 52.5)

ii) 98 100 — In between 95 and 100 and is closer to 100 (≥ 97.5)

1	7	5		2	33	35	
3	9	10		4	71	70	
5	99	100		6	203	205	
7	2	0		8	91	90	
9	497	495		10	777	775	
11	106	105		12	382	380	
13	1 048	1 050					
14	9 497	9 495					
15	28 498	28 500					
16	73 444	73 445					



Round these numbers to the nearest 10

Example 167 170 — In between 160 and 170 and is closer to 170 (≥ 165)

17	12	10		18	56	60	
19	104	100		20	255	260	
21	4	0		22	86	90	
23	193	190		24	464	460	
25	298	300		26	99	100	
27	11	10		28	67	70	
29	1 011	1 010					
30	9 889	9 890					
31	10 573	10 570					
32	19 996	20 000					



Round these to the nearest 100

Example 1 371 1 400 — In between 1 300 and 1 400 and is closer to 1 400 (≥ 1 350)

33	149	100		34	670	700	
35	29	0		36	250	300	
37	1 809	1 800					
38	5 037	5 000					
39	3 099	3 100					
40	14 636	14 600					
41	7 591	7 600					
42	28 370	28 400					
43	39 889	39 900					
44	71 656	71 700					



Round these to the nearest 1 000

Example 1 442 1 000

45	9 027	9 000					
46	3 641	4 000					
47	22 555	23 000					
48	396	0					
49	11 099	11 000					
50	34 495	34 000					
51	99 199	99 000					
52	274 902	275 000					
53	600 513	601 000					
54	599 714	600 000					
55	99 899	100 000					



Add the numbers and round the answer to the nearest 100, then round the numbers to the nearest 100 and add them afterwards.

Example : 776, 345, 881 and 252

1 1			
776 +		800 +	
345		300	
881		900	
252		300	
2 184 →	2 200	2 300	

56 839, 242, 1 487, 3 510

2 1 1		2	
839 +		800 +	
242		200	
1 487		1 500	
3 510		3 500	
6 079 →	6 100	6 000	

57 112, 5 861, 8 170, 923, 756

2 2 1		2	
112 +		100 +	
5 861		5 900	
2 170		2 200	
923		900	
756		800	
9 822 →	9 800	9 900	

57 2 859, 933, 5 211, 43, 1 169

1 2 2		1	
2 859 +		2 900 +	
633		600	
4 211		4 200	
43		0	
1 169		1 200	
8 915 →	8 900	8 900	

Number - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Number

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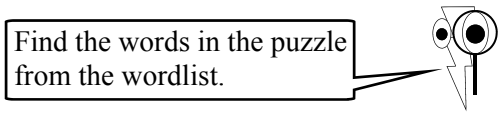

N	J	B	N	R	E	D	U	C	E	S	S	Y	F	G
G	B	Y	C	P	W	Y	A	D	D	I	T	I	O	N
R	T	T	D	P	R	L	N	U	M	E	R	A	L	K
H	C	I	G	O	Z	A	M	I	N	U	S	Z	U	X
B	J	L	M	Q	E	T	N	E	U	O	O	A	N	N
G	H	A	M	N	S	O	O	C	S	D	D	S	N	C
S	N	U	U	O	A	T	I	N	T	D	P	U	L	S
S	J	Q	L	I	E	E	T	E	E	A	R	B	E	U
E	M	E	T	S	R	C	A	R	K	D	O	T	S	L
L	U	N	I	I	C	A	R	E	C	B	D	R	A	P
G	S	I	P	V	N	L	E	F	A	L	U	A	E	R
O	D	Y	L	I	I	P	P	F	R	H	C	C	R	E
C	Y	S	Y	D	B	W	O	I	B	E	T	T	C	D
X	T	I	M	E	S	D	E	D	N	A	P	X	E	R
V	Z	Q	U	O	T	I	E	N	T	B	G	D	D	O

WORDLIST

ADD	QUOTIENT	MINUS	DIVISION	TIMES
DECREASE	SUBTRACT	OPERATION	INEQUALITY	ROMAN
EXPANDED	TOTAL	PLUS	MULTIPLY	BRACKETS
LESS	ADDITION	REDUCE	ORDER	INCREASE
NUMERAL	DIFFERENCE	SUM	PRODUCT	PLACE

Number

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Find the words in the puzzle from the wordlist.



N	J	B	N	R	E	D	U	C	E	S	S	Y	F	G
G	B	Y	C	P	W	Y	A	D	D	I	T	I	O	N
R	T	T	D	P	R	L	N	U	M	E	R	A	L	K
H	C	I	G	O	Z	A	M	I	N	U	S	Z	U	X
B	J	L	M	Q	E	T	N	E	U	O	O	A	N	N
G	H	A	M	N	S	O	O	C	S	D	D	S	N	C
S	N	U	U	O	A	T	I	N	T	D	P	U	L	S
S	J	Q	L	I	E	E	T	E	E	A	R	B	E	U
E	M	E	T	S	R	C	A	R	K	D	O	T	S	L
L	U	N	I	I	C	A	R	E	C	B	D	R	A	P
G	S	I	P	V	N	L	E	F	A	L	U	A	E	R
O	D	Y	L	I	I	P	P	F	R	H	C	C	R	E
C	Y	S	Y	D	B	W	O	I	B	E	T	T	C	D
X	T	I	M	E	S	D	E	D	N	A	P	X	E	R
V	Z	Q	U	O	T	I	E	N	T	B	G	D	D	O

WORDLIST

~~ADD~~
~~DECREASE~~
~~EXPANDED~~
~~LESS~~
~~NUMERAL~~
~~QUOTIENT~~
~~SUBTRACT~~
~~TOTAL~~
~~ADDITION~~
~~DIFFERENCE~~
~~MINUS~~
~~OPERATION~~
~~PLUS~~
~~REDUCE~~
~~SUM~~
~~DIVISION~~
~~INEQUALITY~~
~~MULTIPLY~~
~~ORDER~~
~~PRODUCT~~
~~TIMES~~
~~ROMAN~~
~~BRACKETS~~
~~INCREASE~~
~~PLACE~~

FREEFALL MATHEMATICS

ALTITUDE

SHAPES & SOLIDS

Naming Plane Shapes

With naming shapes it is important to first count the sides and after that examine the properties like:

- parallel sides
- equal side lengths
- internal angles

Some descriptions used:
Triangles - right (has a right angle), scalene (unequal sides), equilateral (all sides and angles equal) and isosceles (two sides and two angles are equal).

Quadrilaterals - square, rectangle, rhombus, kite, parallelogram and trapezium. If a 4-sided shape doesn't match these names then label it a 'quadrilateral'.

Circle - semi-circle (half circle), oval or ellipse (stretched)

Shapes with more than 4 sides - pentagon (5), hexagon (6), heptagon (or septagon) (7), octagon (8), nonagon (9), and decagon (10).

If a shape with 5 sides or more has unequal sides or internal angles then the word 'irregular' is used to describe it. For example a five sided shape that has sides of different length is called an irregular pentagon.

Note that some shapes appear more than once.

Naming Plane Shapes

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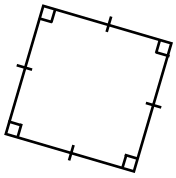
Name these shapes.
Hey! I'm irregular!





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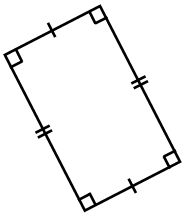

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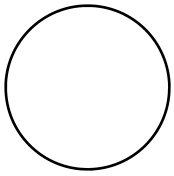

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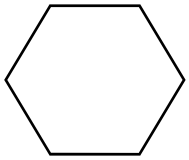

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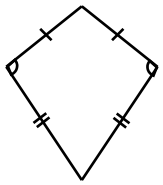

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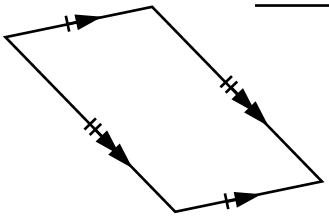

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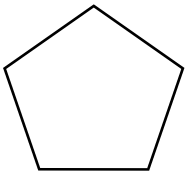

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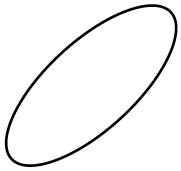

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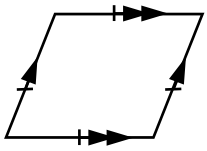

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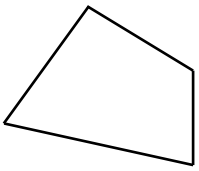

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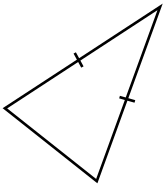

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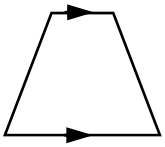

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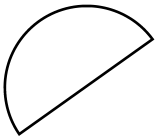

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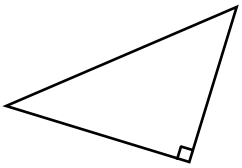

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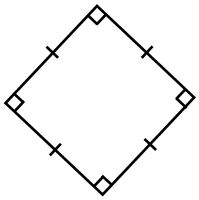

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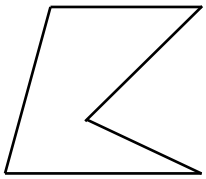

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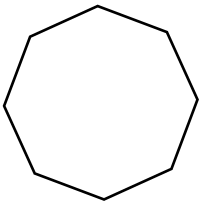

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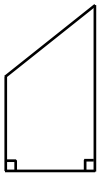

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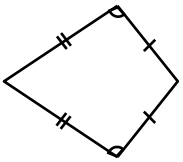

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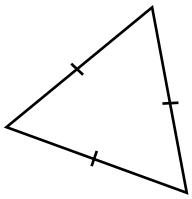

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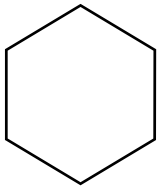

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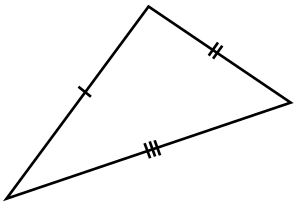

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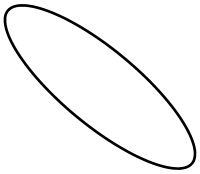

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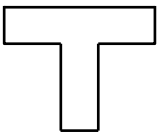

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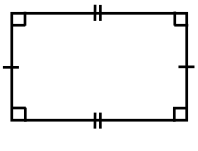
(2 word description)



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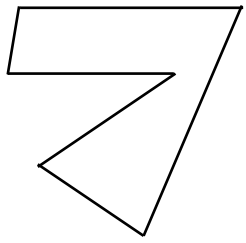

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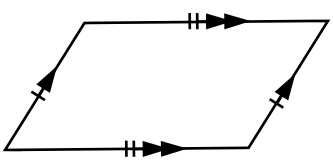

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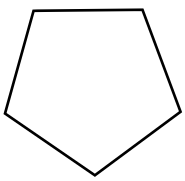

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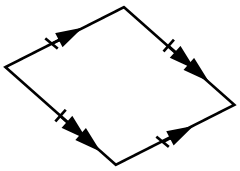

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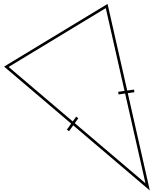

30 _____
(2 word description)

Naming Plane Shapes

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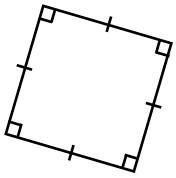
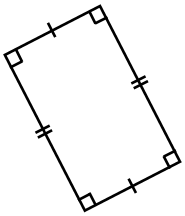
Name these shapes.
Hey! I'm irregular!

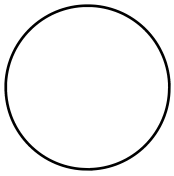




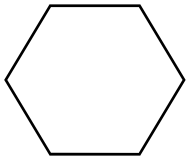
1 square



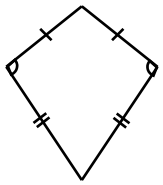
2 rectangle



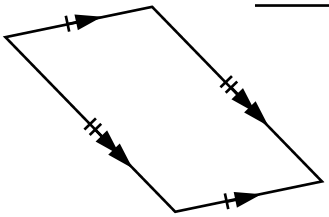
3 circle



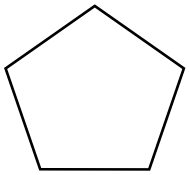
4 hexagon



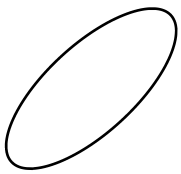
5 kite



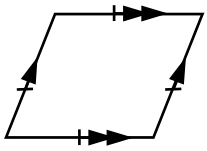
6 parallelogram



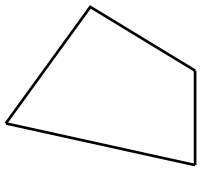
7 pentagon



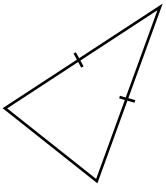
8 oval



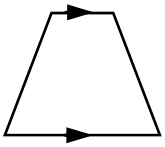
9 rhombus



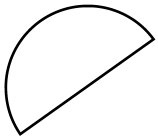
10 quadrilateral



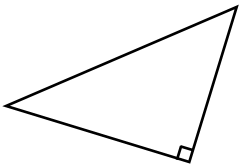
11 Isosceles triangle
(2 word description)



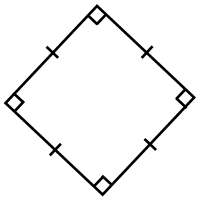
12 trapezium



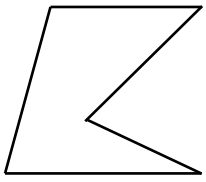
13 Semi-circle



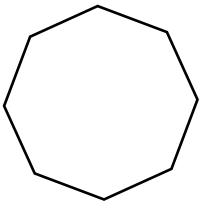
14 Right angled triangle
(2 or 3 word description)



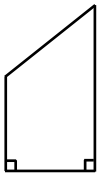
15 square



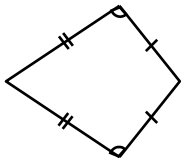
16 Irregular pentagon
(2 word description)



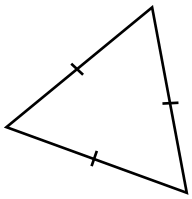
17 octagon



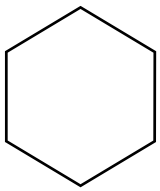
18 trapezium



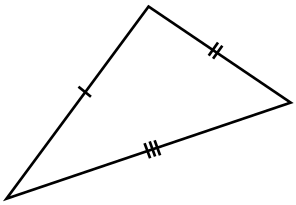
19 kite



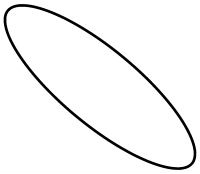
20 Equilateral triangle
(2 word description)



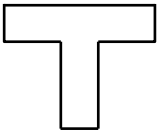
21 hexagon



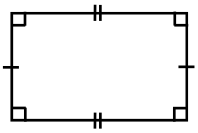
22 Scalene triangle
(2 word description)



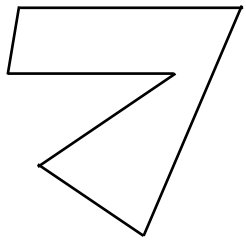
23 oval



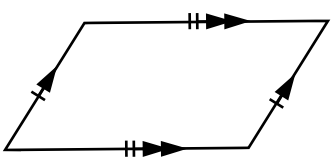
24 Irregular octagon
(2 word description)



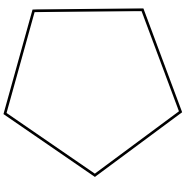
25 rectangle



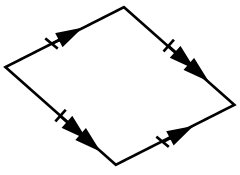
26 Irregular hexagon
(2 word description)



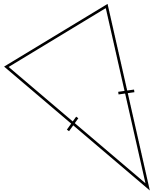
27 parallelogram



28 pentagon



29 rhombus



30 Isosceles triangle
(2 word description)

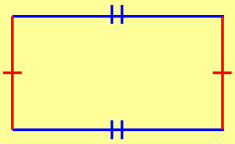
Drawing Plane Shapes

This sheet involves the drawing of 2D shapes, referred to as Plane shapes. In mathematics a drawn shape gives information about the shape. So that a person can look at the drawing and not confuse it with another similar shape. For example a square and a rectangle.

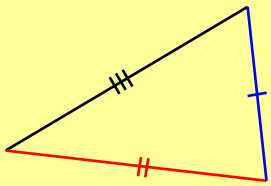
There are 3 features to include in all your drawings:

- Equal or unequal side lengths

Make sure that if a side is the same length as another side you mark it as so in your diagram, this is done by placing a dash on the sides that are the same length. If there are more than one pair of equal length sides then use two dashes on the next pair and so on. If a shape has unequal side lengths then mark the sides as unequal, see below.



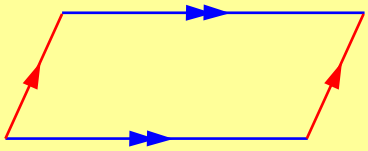
Red sides are the same length, blue sides are the same length. Because the blue sides use 2 dashes and the red sides have only 1 dash, then it is known that they are different lengths



The sides of this triangle have a different numbers of dashes, so all the side lengths are unequal.

- Parallel sides

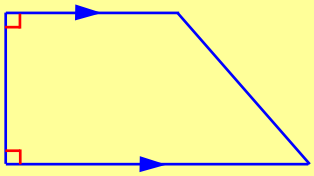
If pairs of sides are parallel then this should be shown on the diagram, this is done by placing arrow heads on sides that are parallel. Generally sides aren't labeled as parallel on shapes with more than 4 sides, a hexagon for example.



Red sides are parallel.
Blue sides are parallel.

- Right Angles

If a shape has one or more right angles then show this on the diagram.



It may be that you are expected to show matching angle pairs on diagrams also, check with your teacher.

Drawing Plane Shapes

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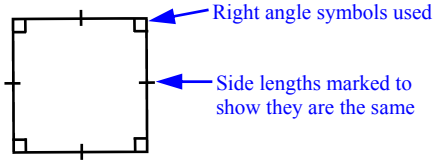
Draw these shapes, show parallel lines ►► ,same side lengths ⊣ ⊣⊢ and right angles. ⊾



Example: Draw a square



1 Draw a rectangle showing same side lengths and right angles.

2 Draw a scalene triangle, label the sides as unequal.

3 Draw an equilateral triangle and label the sides as equal.

4 Draw a parallelogram and label each pair of sides as being equal and parallel.

5 Draw an oval which has a width greater than its height.

6 Draw an octagon and label all the sides as equal, (think of a stop sign).

7 Draw a rhombus, label the sides as equal and each pair of sides as parallel.

8 Draw a trapezium and label two sides as parallel.

9 Draw a pentagon and label the sides as equal.

10 Draw a isosceles triangle and label two sides as equal.

11 Draw a hexagon and label the sides as equal.

Drawing Plane Shapes

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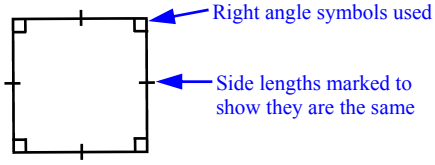
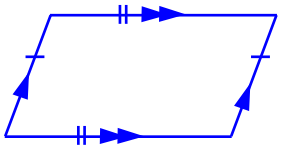
Draw these shapes, show parallel lines ►► ,same side lengths ─┼─ ─┼┼─ and right angles. ⊾



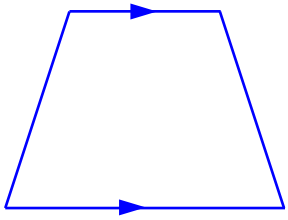
Example: Draw a square



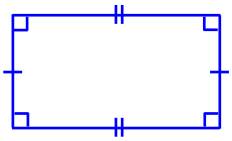
4 Draw a parallelogram and label each pair of sides as being equal and parallel.



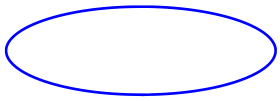
8 Draw a trapezium and label two sides as parallel.



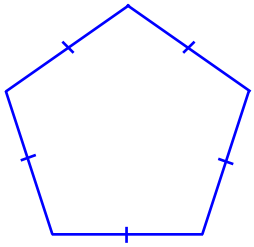
1 Draw a rectangle showing same side lengths and right angles.



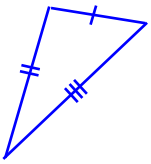
5 Draw an oval which has a width greater than its height.



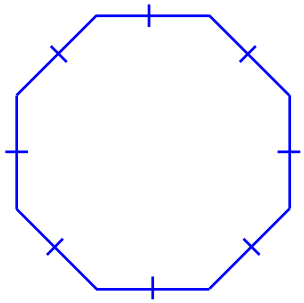
9 Draw a pentagon and label the sides as equal.



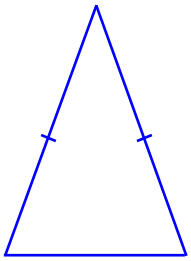
2 Draw a scalene triangle, label the sides as unequal.



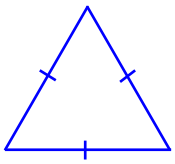
6 Draw an octagon and label all the sides as equal, (think of a stop sign).



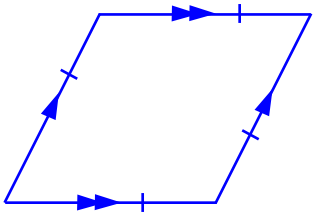
10 Draw a isosceles triangle and label two sides as equal.



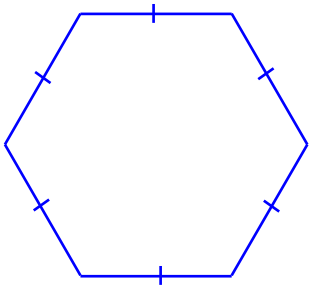
3 Draw an equilateral triangle and label the sides as equal.



7 Draw a rhombus, label the sides as equal and each pair of sides as parallel.



11 Draw a hexagon and label the sides as equal.



Naming Solids

When naming solids first decide whether it is a prism or a pyramid. Then look at the base to name the solid.

Use these words:
Triangular (3 sides), Rectangular (4 sides), Pentagonal (5 sides), Hexagonal (6) and Octagonal (8 sides). End these words with either the word ‘pyramid’ or ‘prism’.

The most confusion occurs with triangles, is it a pyramid or a prism? If there are only two triangles in the solid then it is a triangular prism. If there is more than two triangles then the solid is a pyramid, look at its base to decide what type of pyramid it is.

Other shapes to consider are cube, cone, sphere (and hemisphere), and cylinder.

This sheet repeats a few solids so don’t think that you have made a mistake if you have used the same name more than once.

Naming Solids

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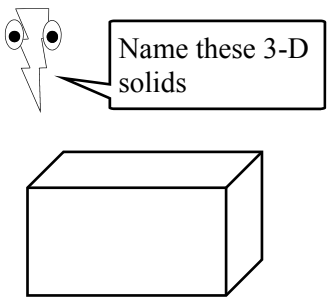

1 _____ 2 _____ 3 _____ 4 _____ 5 _____

6 _____ 7 _____ 8 _____ 9 _____ 10 _____

11 _____ 12 _____ 13 _____ 14 _____ 15 _____

16 _____ 17 _____ 18 _____ 19 _____ 20 _____

21 _____ 22 _____ 23 _____ 24 _____ 25 _____

26 _____ 27 _____ 28 _____ 29 _____ 30 _____

Naming Solids

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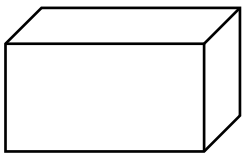
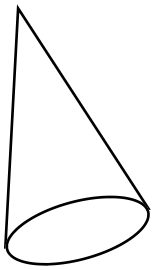
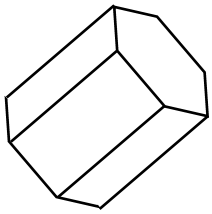
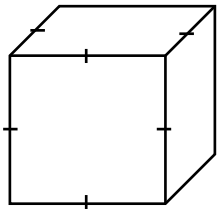
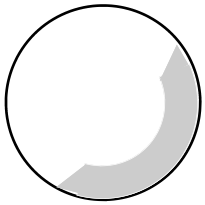
Name these 3-D solids



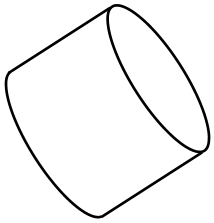


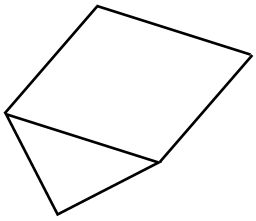


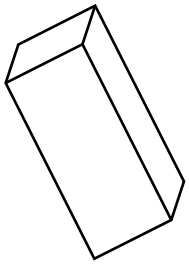


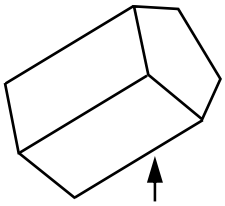


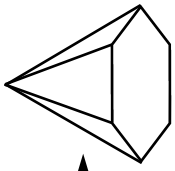
1 Rectangular prism 2 Cone 3 Hexagonal prism 4 Cube 5 Sphere



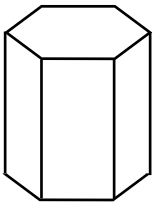


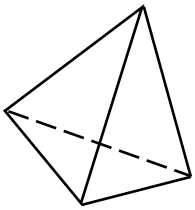


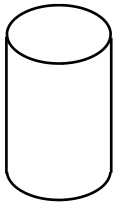


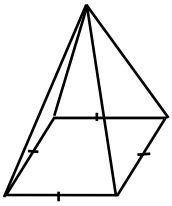


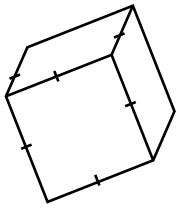
6 Cylinder 7 Triangular prism 8 Rectangular prism 9 Pentagonal prism 10 Hexagonal pyramid



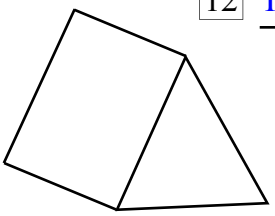


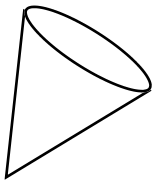


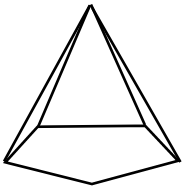


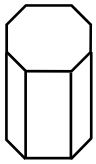


11 Hexagonal prism 12 Triangular pyramid 13 Cylinder 14 Square pyramid 15 Cube



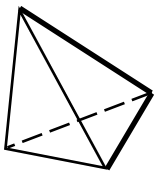




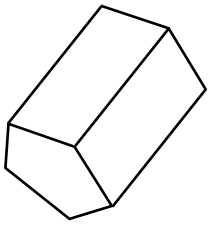


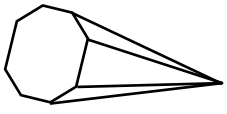


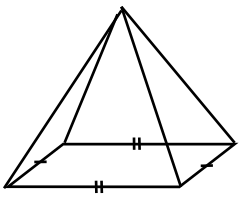
16 Triangular prism 17 Cone 18 Pentagonal pyramid 19 Octagonal prism 20 Rectangular prism



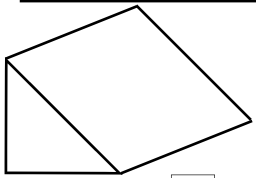


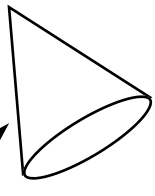


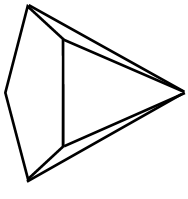


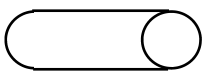


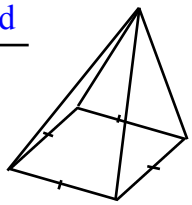
21 Triangular Pyramid 22 Hemisphere 23 Pentagonal Prism 24 Octagonal pyramid 25 Rectangular pyramid









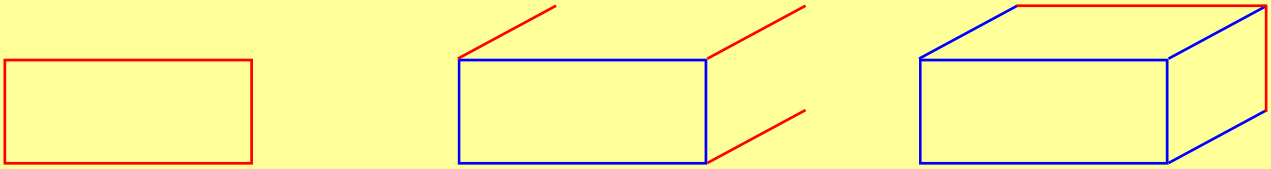


26 Triangular prism 27 Cone 28 Pentagonal pyramid 29 Cylinder 30 Square pyramid

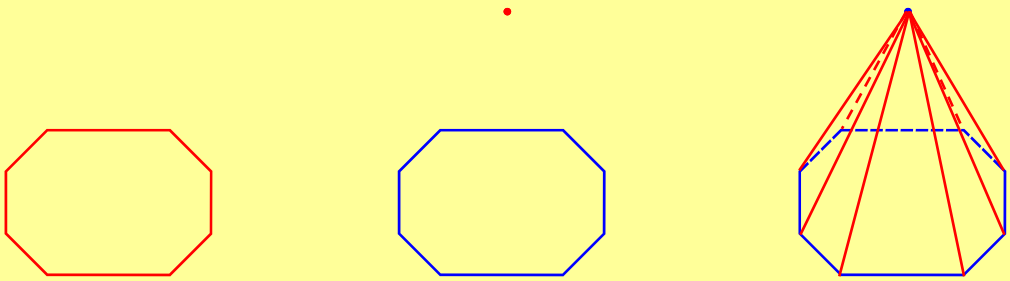
Drawing Solids

Drawing solids can be challenging as showing a 3D solid as a drawing in 2D is not always an easy task.

The way to approach drawing solids is to complete one face first, for example with a rectangular prism, draw the rectangle first. Then take each corner back, then join the ends.



With solids that have pentagonal, hexagonal or octagonal faces try to ‘squash’ the faces when you draw them, for example a octagonal pyramid. Draw the octagon, but squash it. Then pick a point above the centre of the octagon and connect to its vertices.



If you want to produce a more complicated diagram try to show hidden lines as broken lines. As shown in the example above and also at the top of the first column of the worksheet.

Drawing Solids

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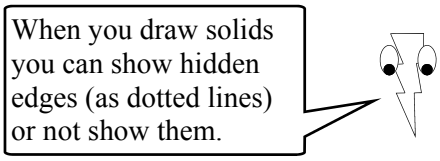
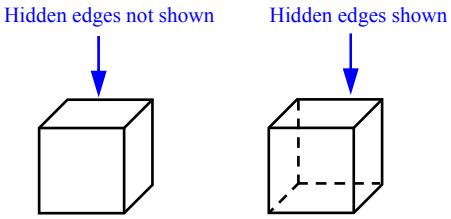


Example: Drawing a cube



1 Draw a cube

2 Draw a rectangular prism

3 Draw a cylinder

4 Draw a triangular prism

5 Draw a square pyramid

6 Draw a pentagonal prism

7 Draw a pentagonal pyramid

8 Draw a hexagonal prism

9 Draw a hexagonal pyramid

10 Draw a triangular pyramid

11 Draw an octagonal prism

Drawing Solids

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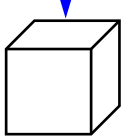
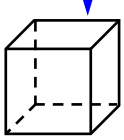
When you draw solids you can show hidden edges (as dotted lines) or not show them.

Example: Drawing a cube

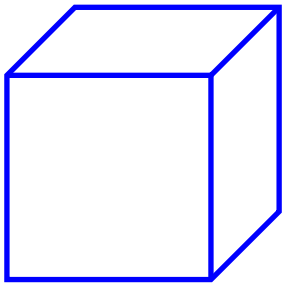
Hidden edges not shown



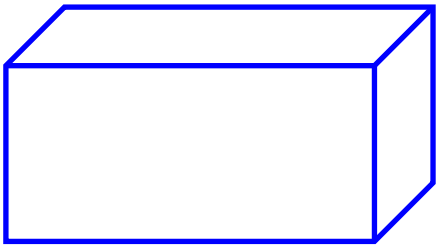
Hidden edges shown



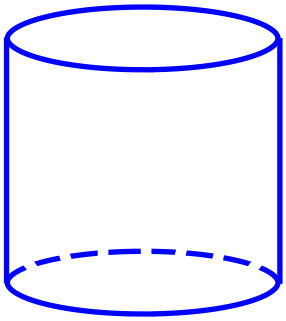
1 Draw a cube



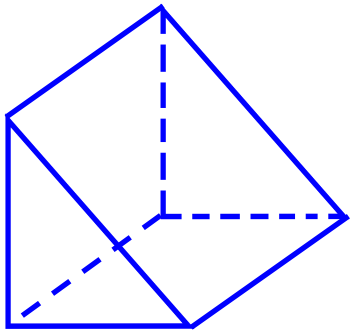
2 Draw a rectangular prism



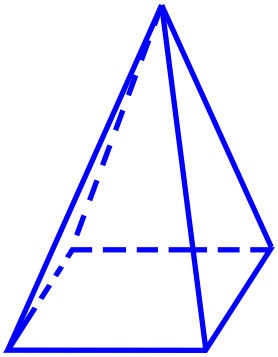
3 Draw a cylinder



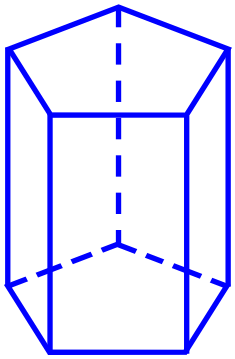
4 Draw a triangular prism



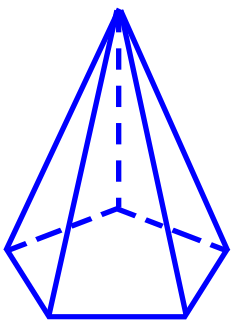
5 Draw a square pyramid



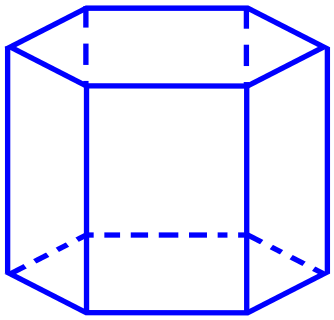
6 Draw a pentagonal prism



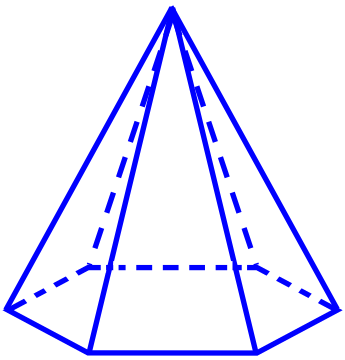
7 Draw a pentagonal pyramid



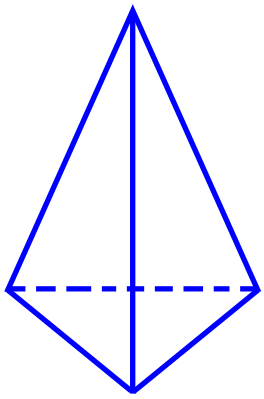
8 Draw a hexagonal prism



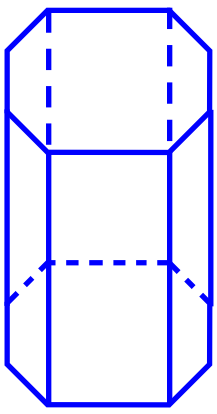
9 Draw a hexagonal pyramid



10 Draw a triangular pyramid



11 Draw an octagonal prism



Shapes in the Environment

This sheet deals with the different shapes that you have learnt and where you may find them in your world.

Column 1 starts with diagrams of objects and asks you to name the shapes/solids that make up the object. Some objects have more than one shape included in them, there will be two or more answer lines supplied when this is the case. The shapes can be plane shapes (2D) such as squares, rectangles, trapeziums, parallelograms etc, as well as solids (3D) such as cones, spheres and prisms.

Column 2 starts with questions on naming objects that may include the given shape/solid. Try and name objects that differ from those given in the previous column. List as many as you can in the space provided.

The bridge at the bottom of column 2 has steel beams which outline many different shapes, see if you can list 5 different shapes.

Column 3 starts with looking at bricks. Name the different shapes present in each brick then circle the letter of the brick with the most shapes. Question 16 shows an artists model and again you are asked to name all the shapes that you can see. Once you have done this, colour-in the shapes.

Shapes in the Environment

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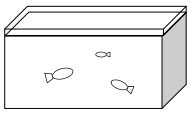
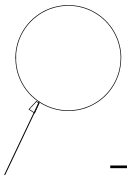
Look at the objects and suggest a shape or solid.

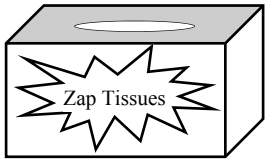
1 Fish tank



2 Balloon



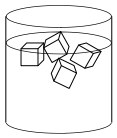
3 Box of tissues - 2



a) Opening:

b) Box:

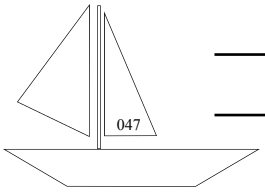
4 Drink with ice - 2



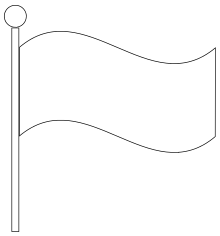
a) Ice:

b) Glass:

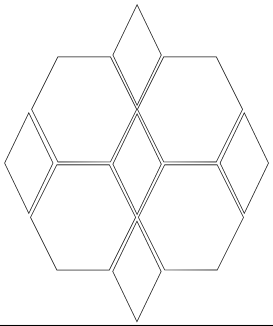
5 Sail boat - 4



6 Flag on a pole - 3



7 Floor tiles - 2



This time think of objects that look like the given shape/solid.

8 Cylinder

9 Sphere

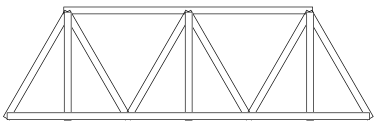
10 Rectangle

11 Cube

12 Cone

13 Rectangular prism

Name the shapes you can see in the steel bridge drawn below.



14

a)

b)

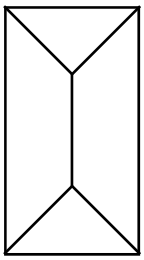
c)

d)

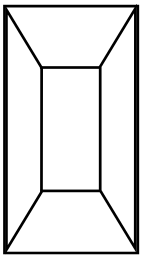
e)

Name the 2-D shapes in each brick, then name the brick with the most shapes

15 Brick A



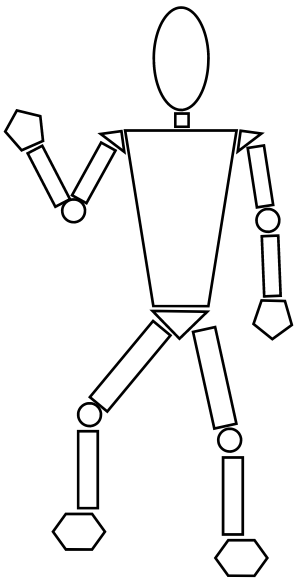
Brick B



Circle A / B

Name the shapes in the artist model below, then colour it in.

16



Shapes in the Environment

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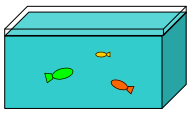
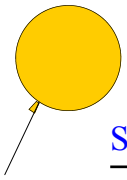
Look at the objects and suggest a shape or solid.

1 Fish tank



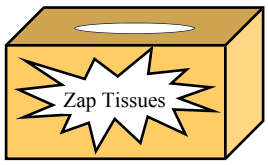
Rectangular prism

2 Balloon



Sphere

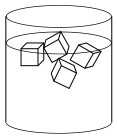
3 Box of tissues - 2



a) Opening: Oval

b) Box: Rectangular prism

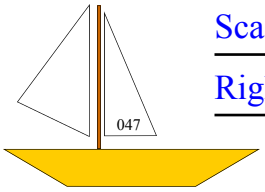
4 Drink with ice - 2



a) Ice: cube

b) Glass: cylinder

5 Sail boat - 4



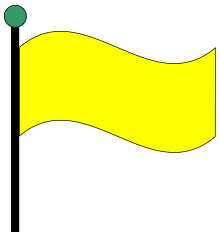
Scalene triangle

Right triangle

cylinder

trapezium

6 Flag on a pole - 3

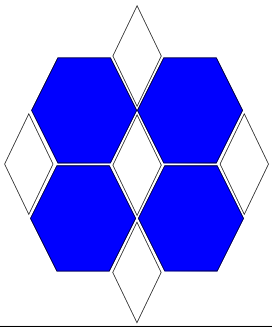


Rectangle

cylinder

sphere

7 Floor tiles - 2



Hexagon

Rhombus

This time think of objects that look like the given shape/solid.

8 Cylinder Cans, tubes, poles, pipes, cakes, glasses

9 Sphere Planets, balls, scoop of ice-cream, oranges

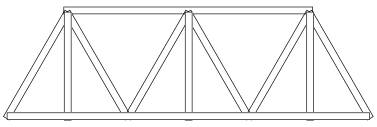
10 Rectangle Sheet of paper, towels, windows, playground

11 Cube Ice cubes, centi-cubes, dice

12 Cone Ice cream cone, party hat, funnel, pile of dirt

13 Rectangular prism Boxes, buildings, books, doors, suit cases, rooms and buses

Name the shapes you can see in the steel bridge drawn below.



14

a) Right triangle

b) Isosceles or Equilateral Δ

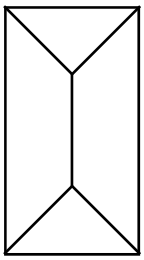
c) parallelogram & rhombus

d) rectangle

e) trapezium

Name the 2-D shapes in each brick, then name the brick with the most shapes

15 Brick A

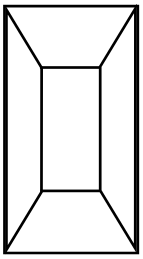


Isosceles Triangle

Trapezium

Rectangle

Brick B



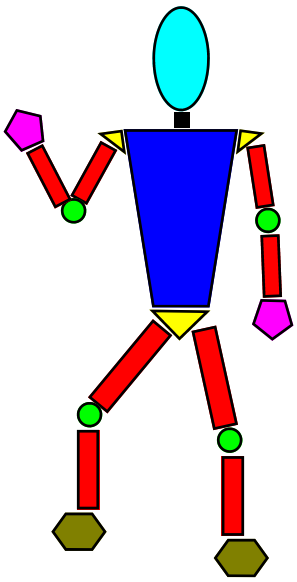
rectangle

trapezium

Circle (A) / B

Name the shapes in the artist model below, then colour it in.

16



Oval

Circle

Pentagon

Hexagon

Rectangle

Triangle

Square

Trapezium

Tessellations

Tessellations are patterns made by the repeated use of a shape, without leaving gaps.

Question 1 in the first column features tessellated triangles. Colour the adjacent triangles (that share a common side) to build the shapes shown. Then colour the square in front of the shape's name the same colour. Like a key or legend in a map.

Column 1 then has three tessellated diagrams, complete the patterns. If you want to use different colours then print the second worksheet, which is unpainted.

Column 2 consists of irregular tessellations. This is when two or more different shapes are used. Colour them to form patterns.

The final tessellation requires you to complete it by using the dots as guides. Once you have drawn it colour in the stars that have been formed.

Tessellations

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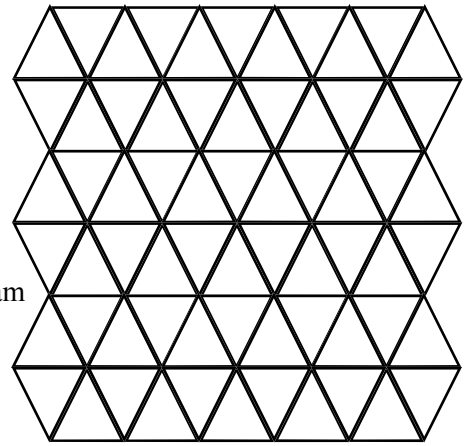
Create the shapes below on the grid by colouring the triangles. Then match that colour in one of the key squares before the shape name.

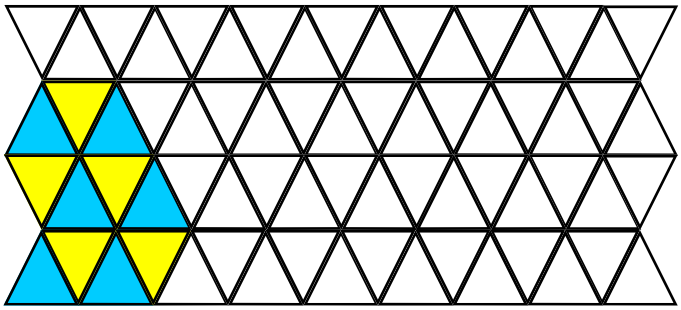
1



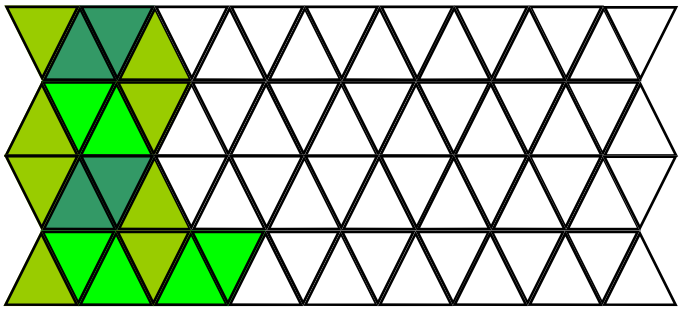
- ☐ Hexagon
- ☐ Rhombus
- ☐ Parallelogram
- ☐ Trapezium

Complete these regular tessellations

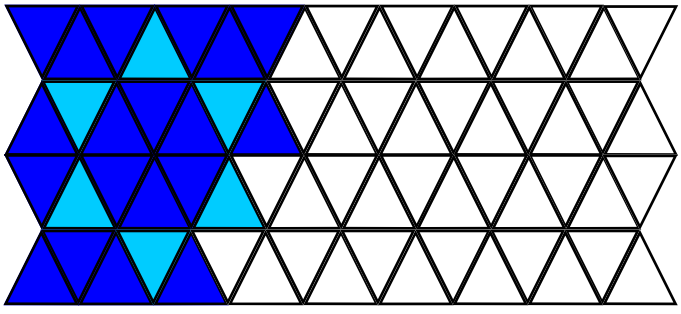
2 Triangle



3 Rhombus

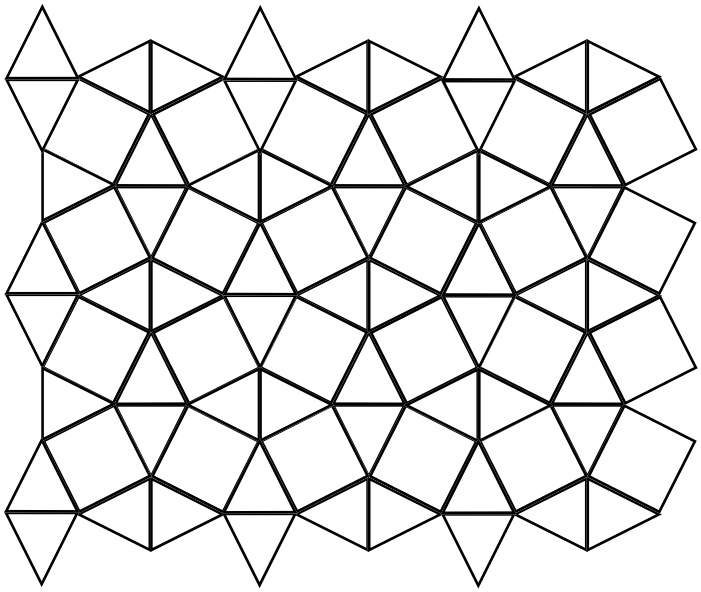


4 Hexagon

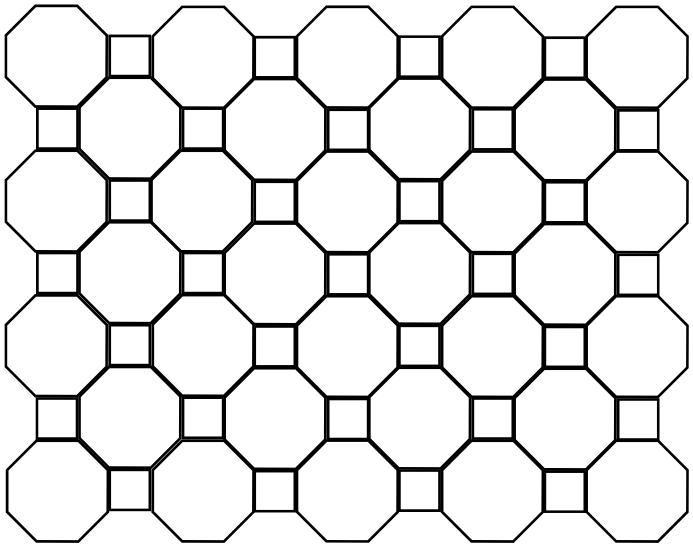


Colour-in these irregular tessellations.

5 Triangles and Squares

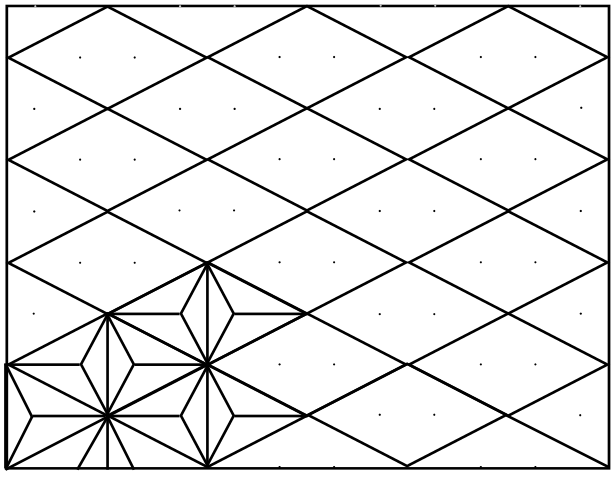


5 Octagons and Squares



Complete this irregular tessellation, then colour-in the stars

7



Tessellations

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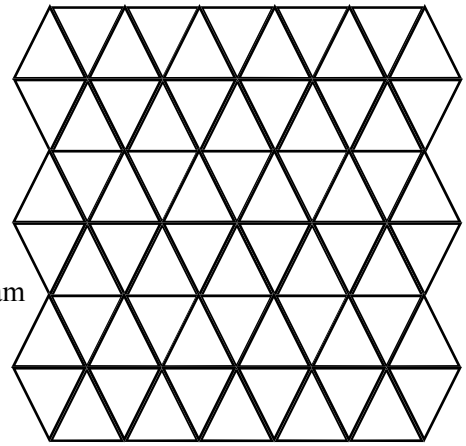
Create the shapes below on the grid by colouring the triangles. Then match that colour in one of the key squares before the shape name.

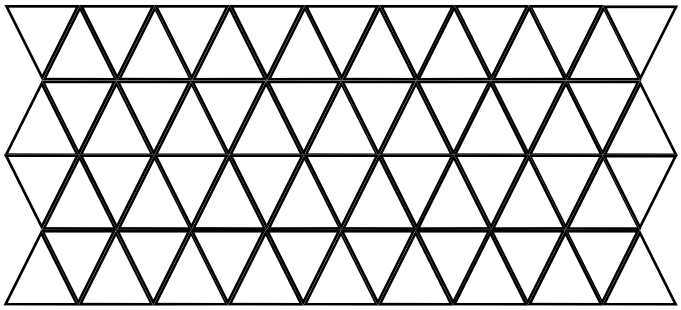
1



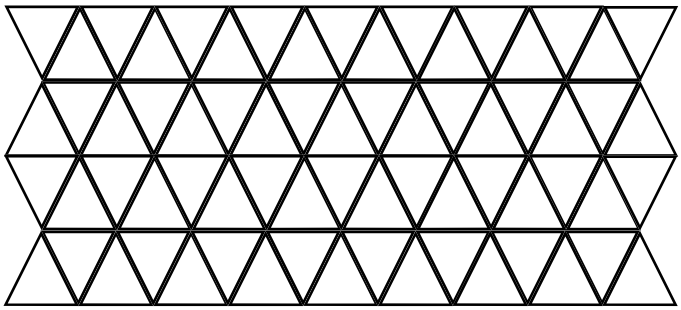
- Hexagon
- Rhombus
- Parallelogram
- Trapezium

Complete these regular tessellations

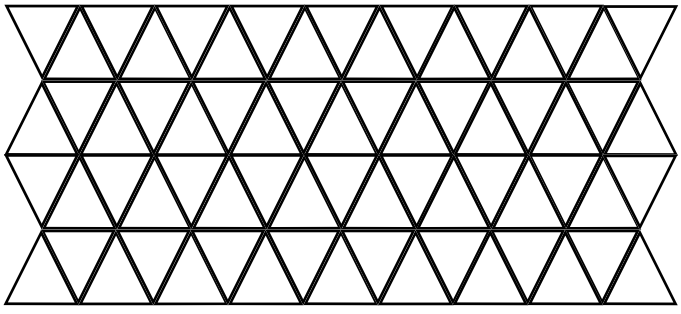
2 Triangle



3 Rhombus

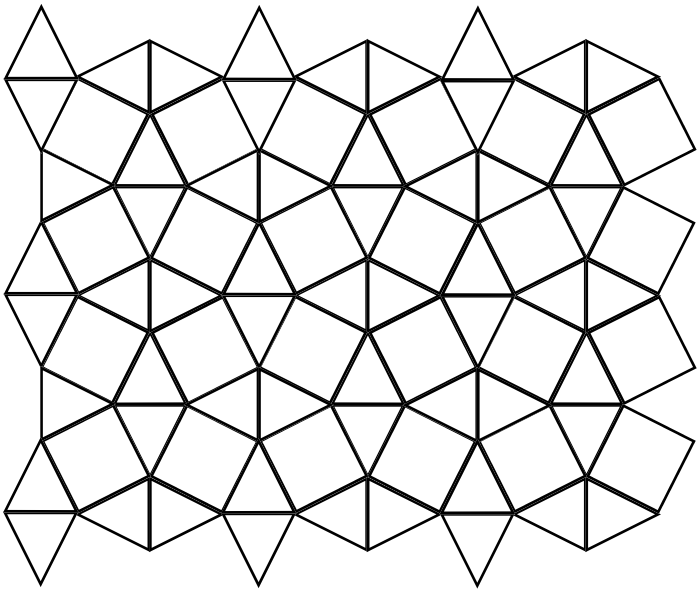


4 Hexagon

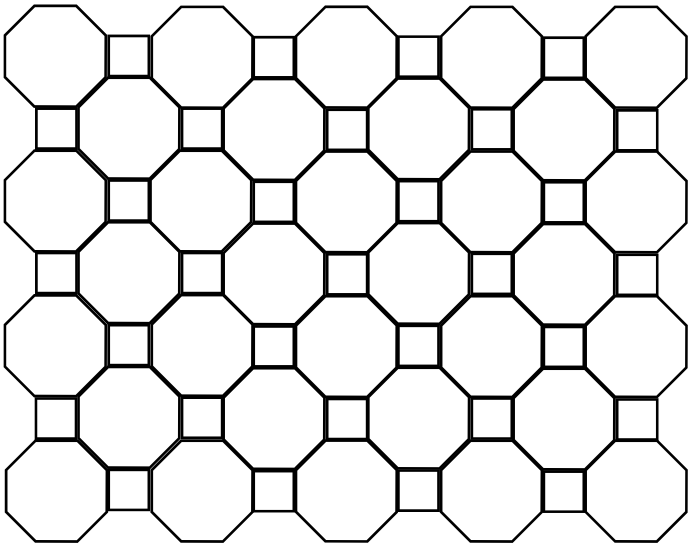


Colour-in these irregular tessellations.

5 Triangles and Squares

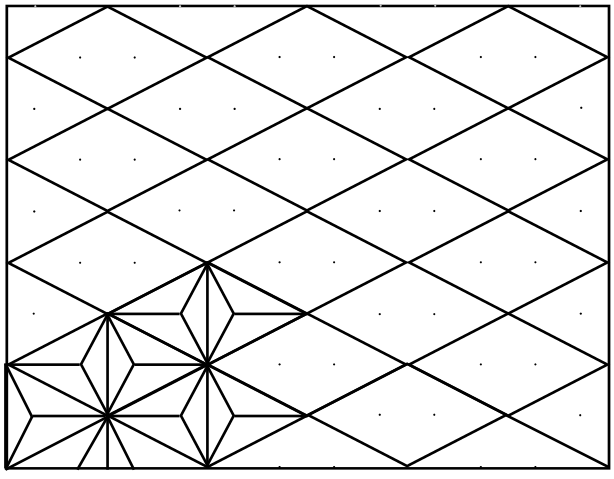


5 Octagons and Squares



Complete this irregular tessellation, then colour-in the stars

7



Tessellations

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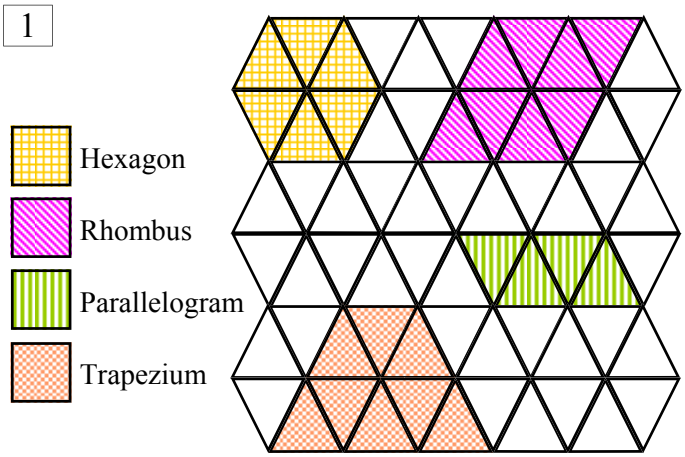
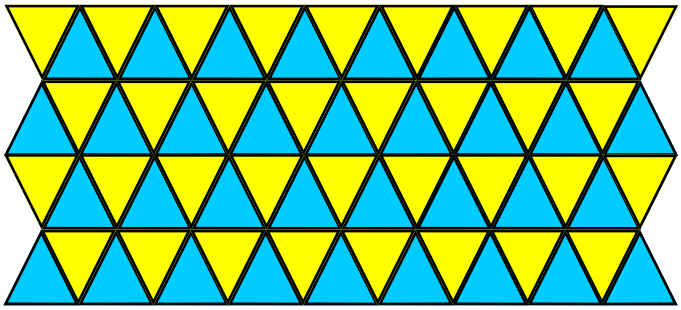
Create the shapes below on the grid by colouring the triangles. Then match that colour in one of the key squares before the shape name.

1

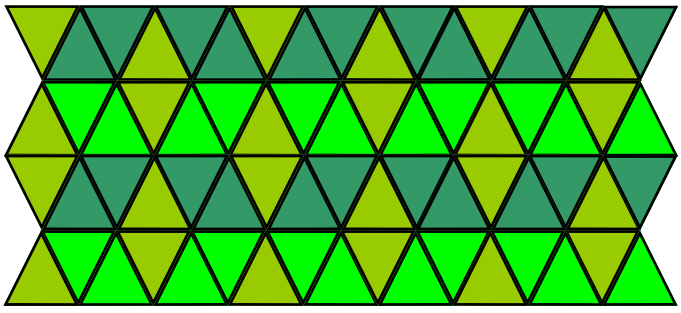


Complete these regular tessellations

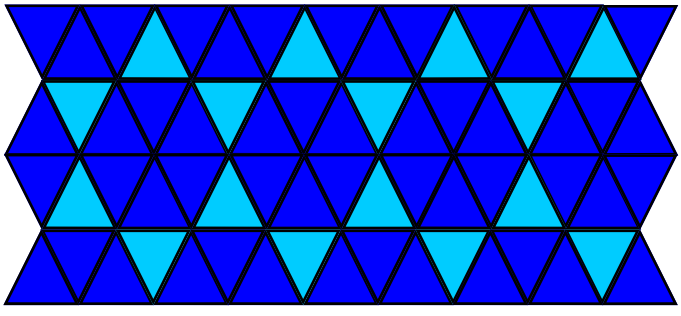
2 Triangle



3 Rhombus

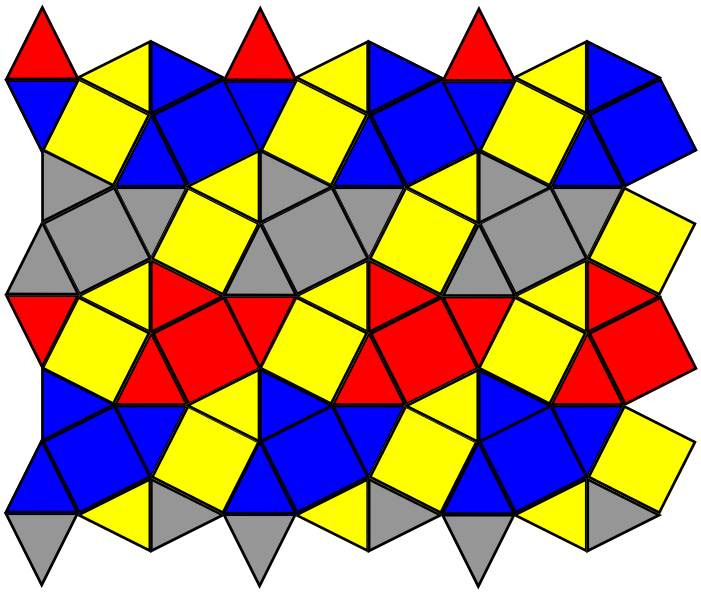


4 Hexagon

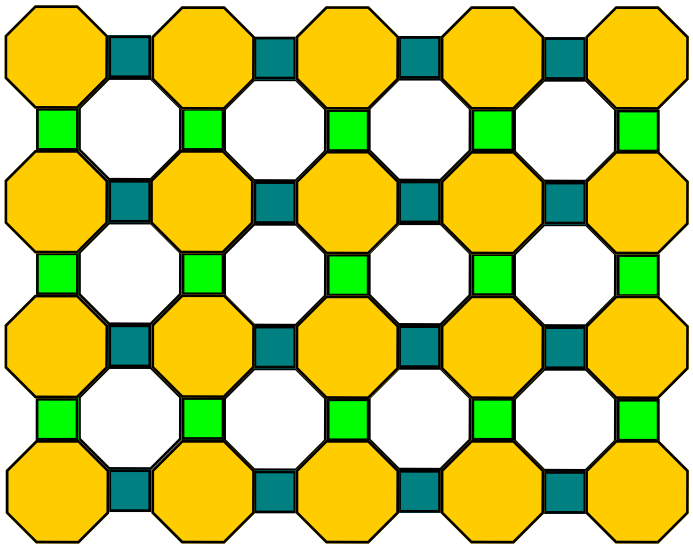


Colour-in these irregular tessellations.

5 Triangles and Squares

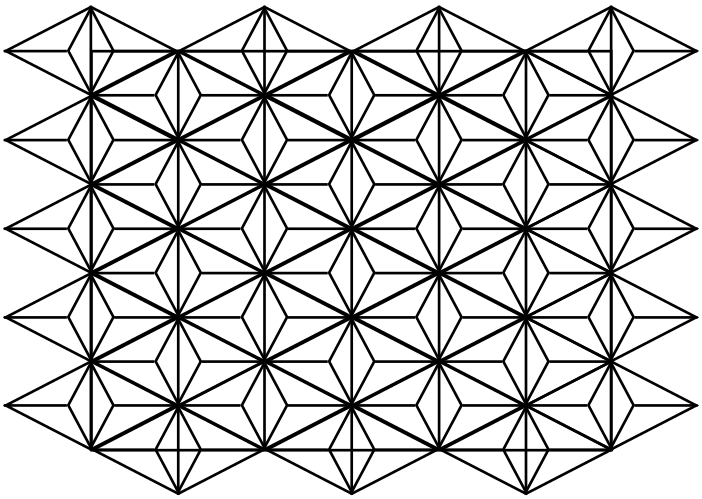


5 Octagons and Squares



Complete this irregular tessellation, then colour-in the stars

7



Axis of Symmetry 1

An axis of symmetry is a line which divides a shape into two identical shapes. The shapes have to be a mirror image of each other. To complete this sheet on paper cut the shapes out and fold the shape on top of itself, the crease made by the fold is the axis of symmetry, count the creases and write the number in the box inside the shape.

If working on screen, imagine a line cutting the shape and check whether the shape either side of the imaginary line is an identical image.

Axis of Symmetry 1

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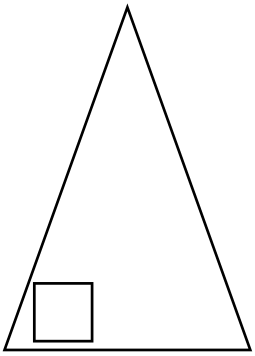
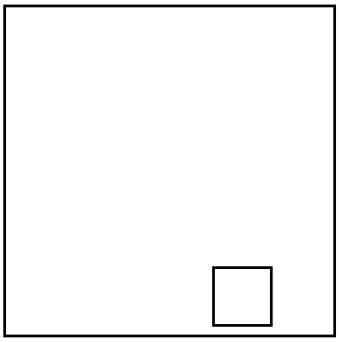
Cut the shapes out and fold them. If the shape doubles back on itself so that the fold mark creates two shapes that are the same, then that is an axis of symmetry. Repeat until all are found. Then write the number found in the square of the shape and stick the shape and its name in your book.

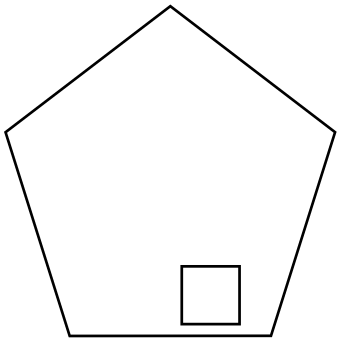
1 Isosceles triangle



2 Square



3 Pentagon



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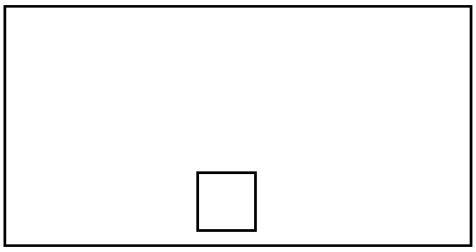
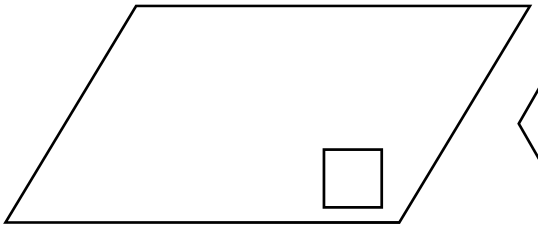
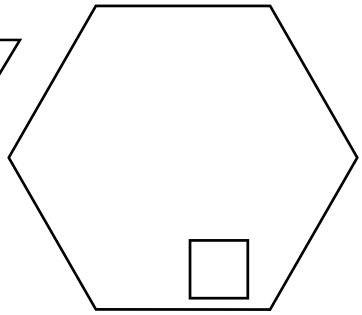


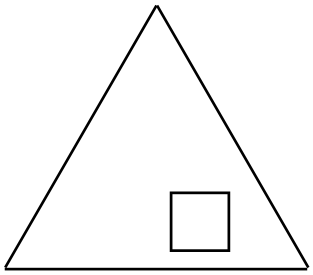
5 Parallelogram



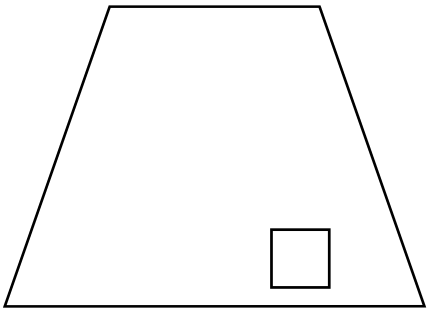
6 Hexagon



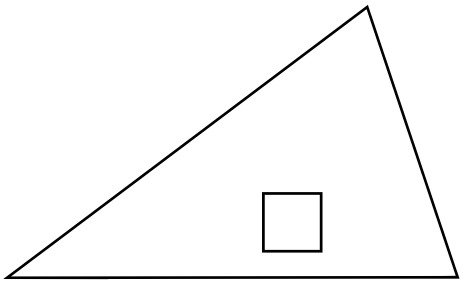
7 Equilateral triangle



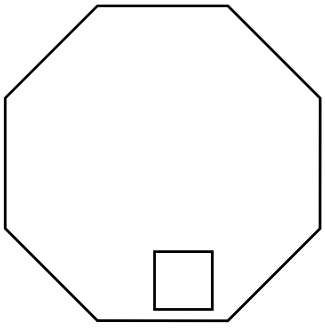
8 Trapezium



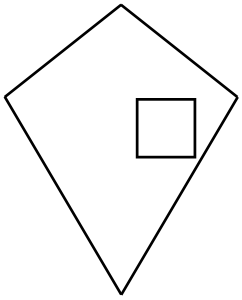
9 Scalene triangle



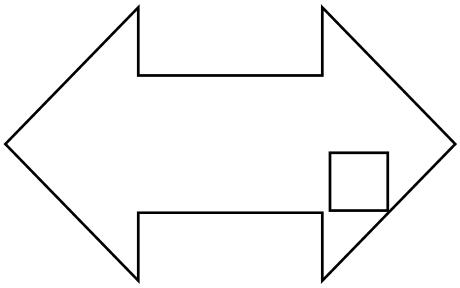
10 Octagon



11 Kite



12 Irregular Decagon



Axis of Symmetry 1

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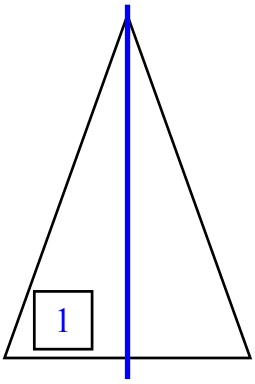
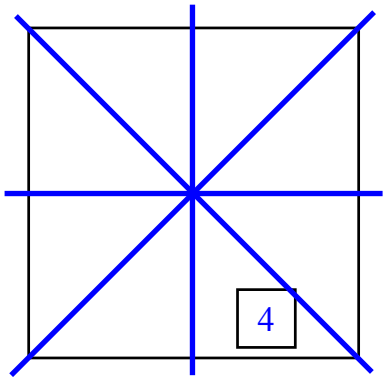
Cut the shapes out and fold them. If the shape doubles back on itself so that the fold mark creates two shapes that are the same, then that is an axis of symmetry. Repeat until all are found. Then write the number found in the square of the shape and stick the shape and its name in your book.

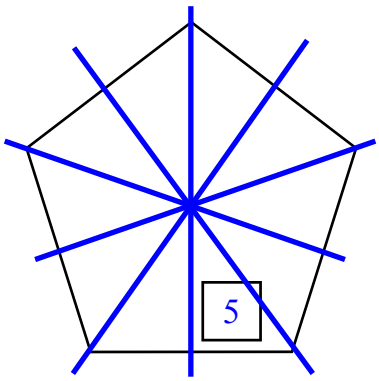
1 Isosceles triangle



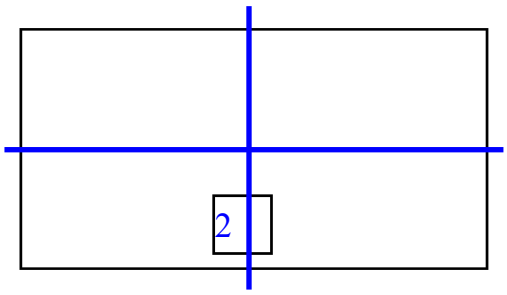
2 Square



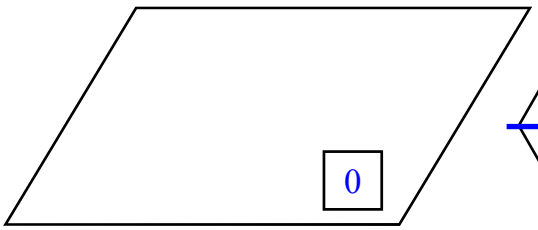
3 Pentagon



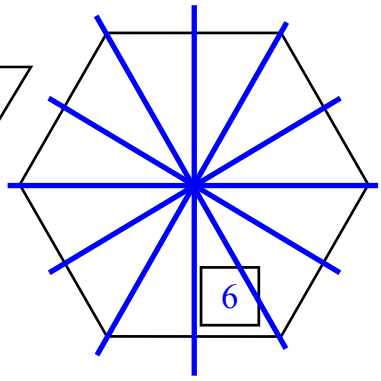
4 Rectangle



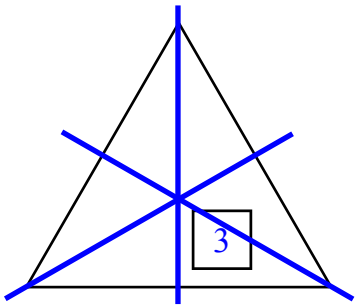
5 Parallelogram



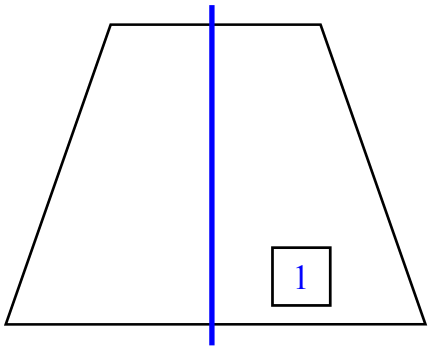
6 Hexagon



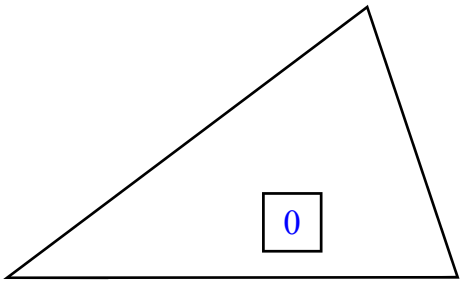
7 Equilateral triangle



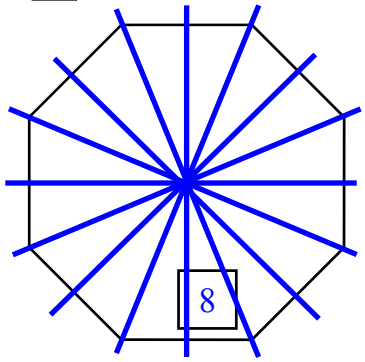
8 Trapezium



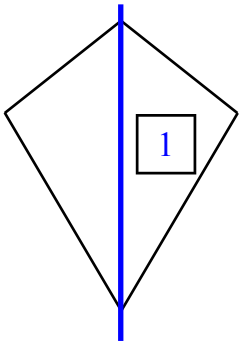
9 Scalene triangle



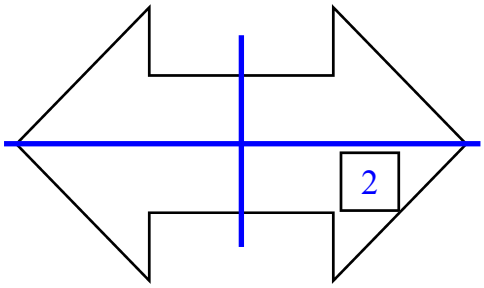
10 Octagon



11 Kite



12 Irregular Decagon



Axis of Symmetry 2

The axis of symmetry is just like a fold line where the image on one side of the axis is reproduced on the other side. Remember that the position of the shape is the same distance away from the line on the reflected image, so be careful with questions where the shape isn't touching the axis.

To complete the first column reflect the shape across to the right hand side of the axis of symmetry. Using the grid, colour in the squares (or triangles) to make a reflection. Use a pencil so that you can remove any mistakes easily.

Column 2 has two axes, so reflect to the right then reflect down, then it is your choice how you get the bottom right image, either by reflecting across or reflecting down.

The shapes made are in black but your reflection can use all the different colours you want, this isn't an exact reflection but it is more fun. You can even recolour the original shape different colours then reflect an exact copy.

Column 3 has drawings that you complete with one or several reflected images. Reflect them and then colour them in!

Axis of Symmetry 2

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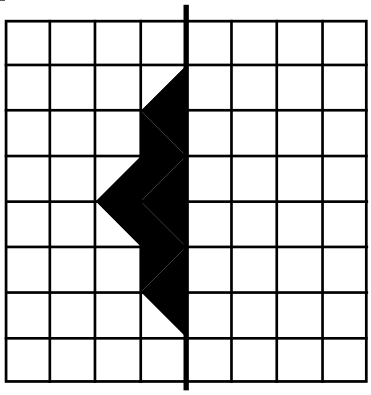
Reflect the shape to the right side of the axis of symmetry (of the final shape).

1



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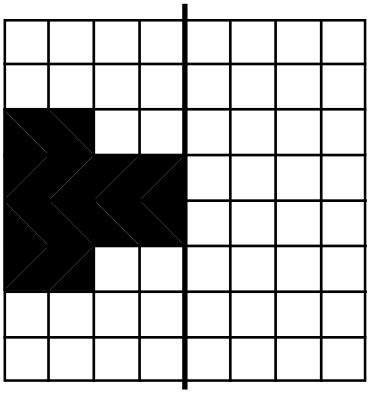

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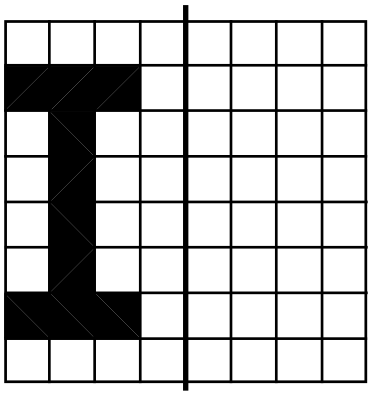

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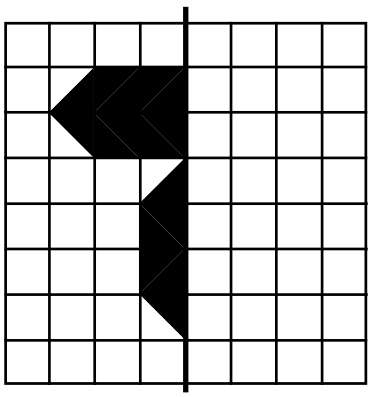
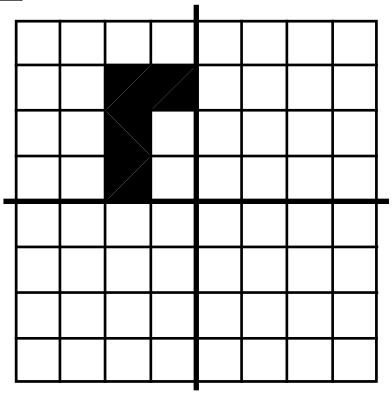




Reflect the shape both vertically and horizontally.

5



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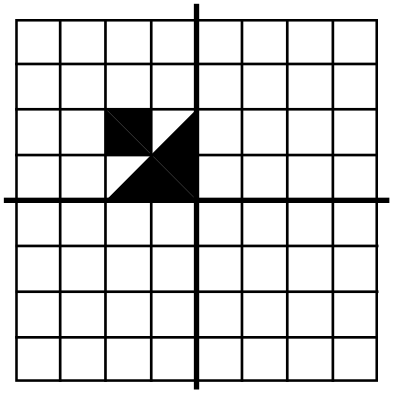

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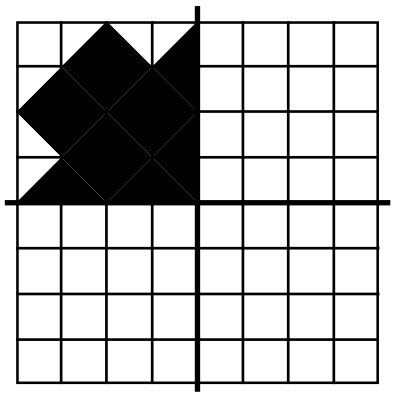

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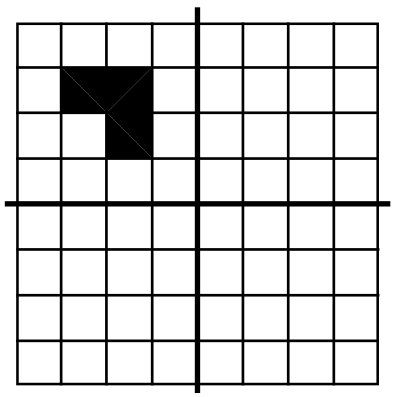
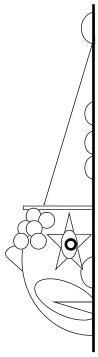




Reflect the first pair once and the second pair twice then colour them in! Are you seeing double?

9



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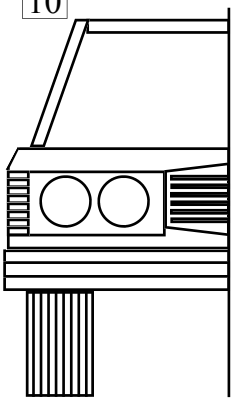

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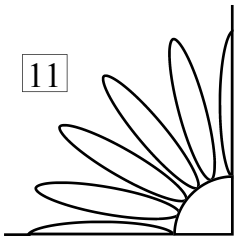

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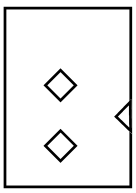

Axis of Symmetry 2

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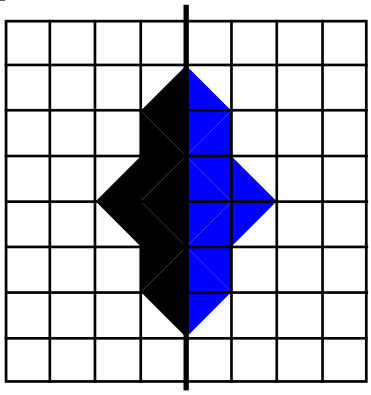
Reflect the shape to the right side of the axis of symmetry (of the final shape).

1



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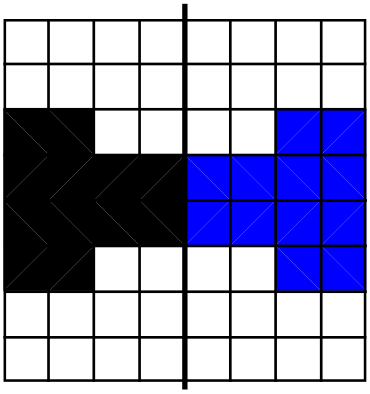

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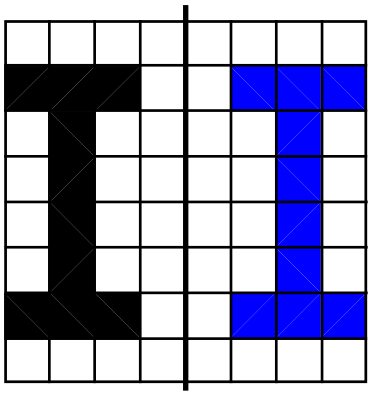

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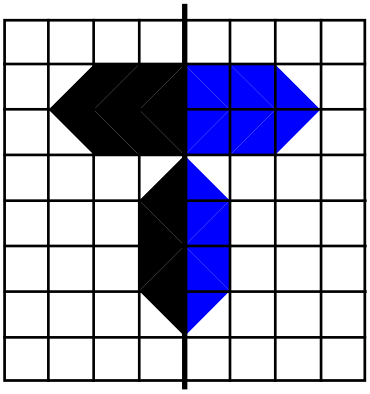
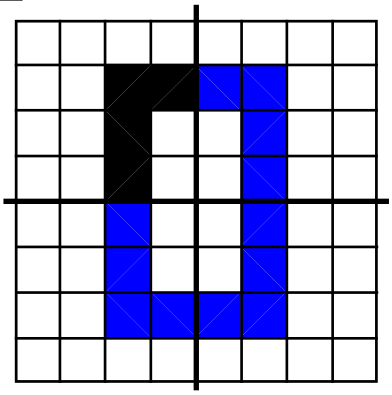


Reflect the shape both vertically and horizontally.

5



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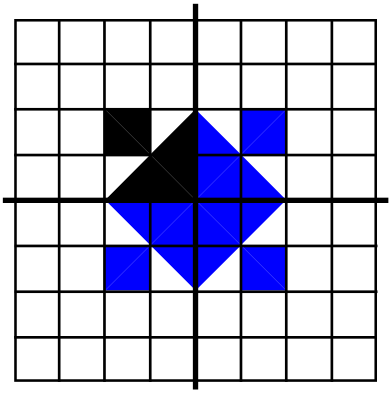

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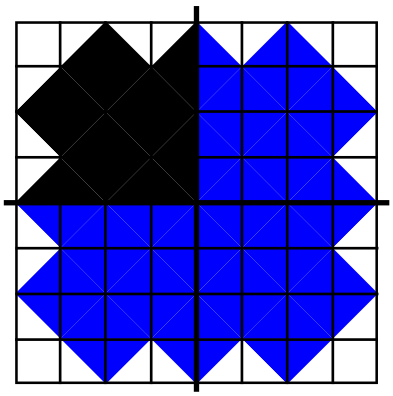

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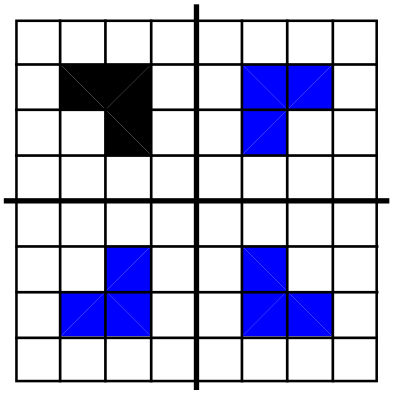
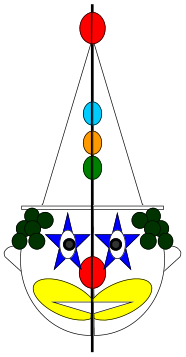


Reflect the first pair once and the second pair twice then colour them in! Are you seeing double?

9



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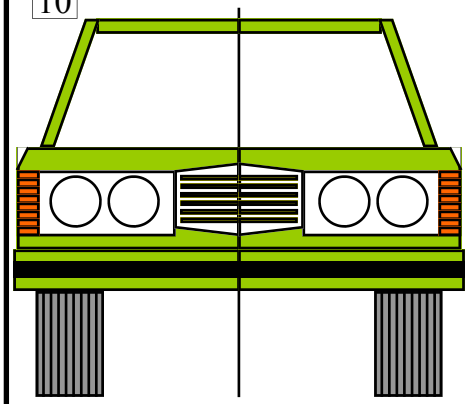

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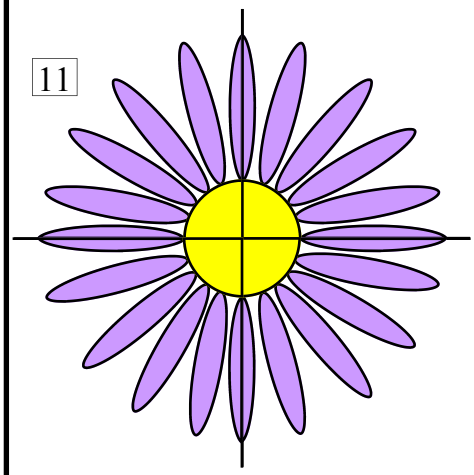

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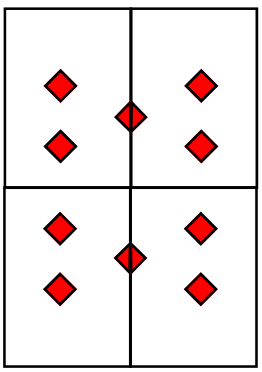

Rotational and Point Symmetry

Rotational symmetry is shown by rotating a shape and having it appear as the same shape during the rotation, before completing a revolution. A shape isn't rotationally symmetrical if it matches only after full rotation.

Column 1 could be answered by tracing the shape and spinning the trace over the original and counting the number of times it matches the original shape, including when it returns to its original position. But this can also be attempted mentally by imagining the process.

Column 2 involves identifying point symmetry. Point symmetry is when a shape is identical after a 180° rotation (half a rotation). These questions can be answered by observation or you can choose to turn the page upside-down. If the upside-down image is identical to the original then the object is point symmetric.

To flip the sheet upside-down, select the "View" menu, then the rotate option then "counter clockwise" or "clockwise" but repeat a second time.

Column 3 involves identifying point symmetry, line symmetry and rotational symmetry. If a line can be drawn anywhere through the object creating a mirror image on each side then the object has line symmetry. Point symmetry and rotational symmetry can then be found using the previous methods. If the type of symmetry exists answer 'yes'.

Rotational and Point Symmetry

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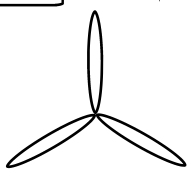
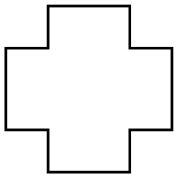
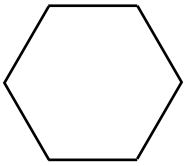
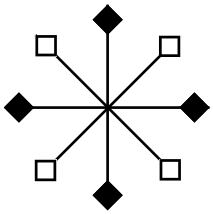
These objects all have rotational symmetry state their order.

1 Order = 

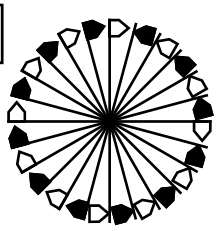
2 Order = 

3 Order = 

4 Order = 

5 Order = 

5 Order = 

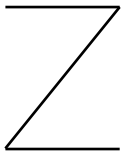
6 Order = 

7 Name 2 more objects that have rotational symmetry.

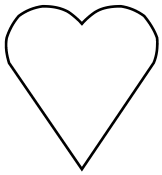
a) _____

b) _____

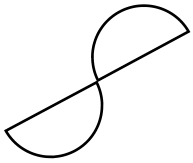
Do these objects have point symmetry? Answer yes or no.

8 

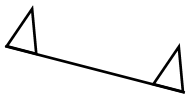
Yes ○ No ○

9 

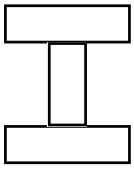
Yes ○ No ○

10 

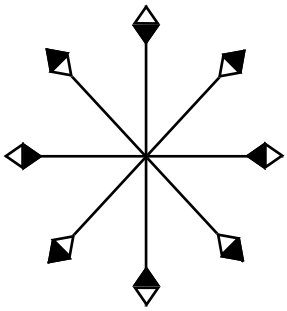
Yes ○ No ○

11 

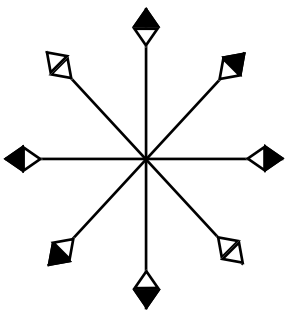
Yes ○ No ○

12 

Yes ○ No ○

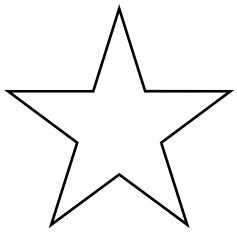
13 

Yes ○ No ○

14 

Yes ○ No ○

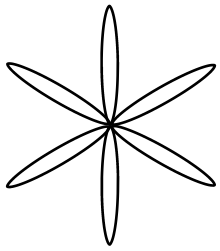
Do these objects have:
- Line symmetry,
- Point symmetry and
- Rotational symmetry.

15 

Line Symmetry Ⓨ Ⓝ

Point Symmetry Ⓨ Ⓝ

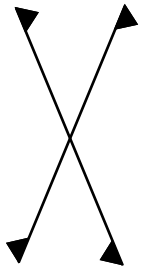
Rotational Symmetry Ⓨ Ⓝ

16 

Line Symmetry Ⓨ Ⓝ

Point Symmetry Ⓨ Ⓝ

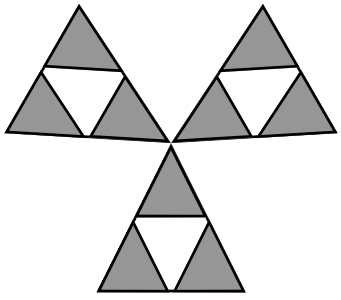
Rotational Symmetry Ⓨ Ⓝ

17 

Line Symmetry Ⓨ Ⓝ

Point Symmetry Ⓨ Ⓝ

Rotational Symmetry Ⓨ Ⓝ

18 

Line Symmetry Ⓨ Ⓝ

Point Symmetry Ⓨ Ⓝ

Rotational Symmetry Ⓨ Ⓝ

Rotational and Point Symmetry

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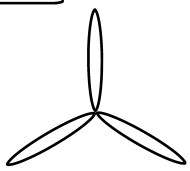
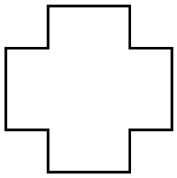
These objects all have rotational symmetry state their order.

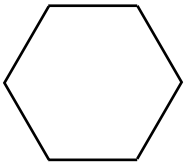
1 Order = 3



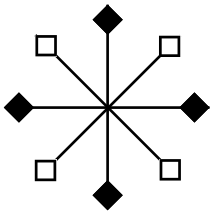
2 Order = 4



3 Order = 6



4 Order = 4



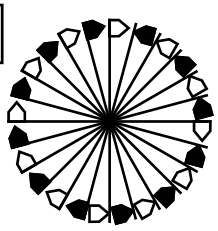
5 Order = 5



5 Order = 8



6 Order = 12



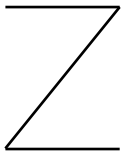
7 Name 2 more objects that have rotational symmetry.

a) Fan or Propeller

b) Certain flowers

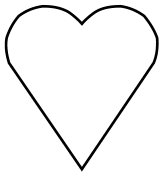
Do these objects have point symmetry? Answer yes or no.

8



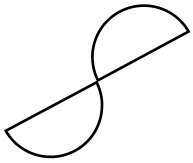
Yes ● No ○

9



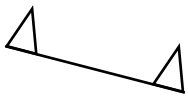
Yes ○ No ●

10



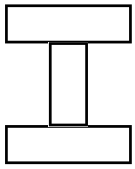
Yes ● No ○

11



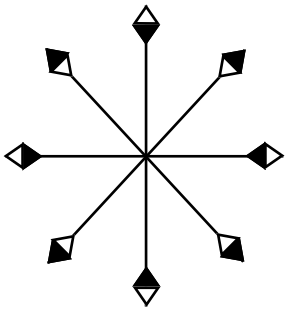
Yes ○ No ●

12



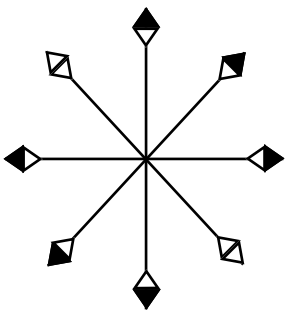
Yes ● No ○

13



Yes ● No ○

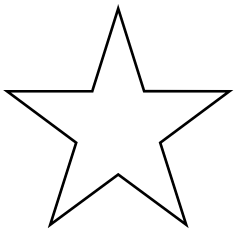
14



Yes ● No ○

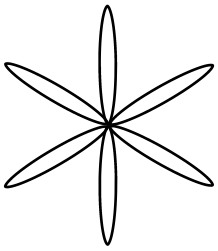
Do these objects have:
- Line symmetry,
- Point symmetry and
- Rotational symmetry.

15



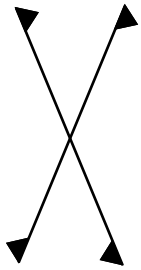
Line Symmetry ● (N)
Point Symmetry (Y) ●
Rotational Symmetry ● (N)

16



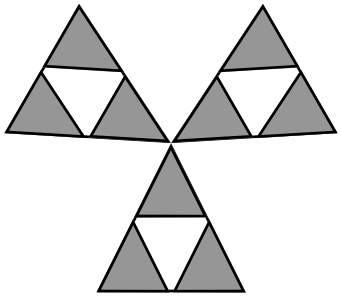
Line Symmetry ● (N)
Point Symmetry ● (N)
Rotational Symmetry ● (N)

17



Line Symmetry (Y) ●
Point Symmetry ● (N)
Rotational Symmetry ● (N)

18



Line Symmetry ● (N)
Point Symmetry (Y) ●
Rotational Symmetry ● (N)

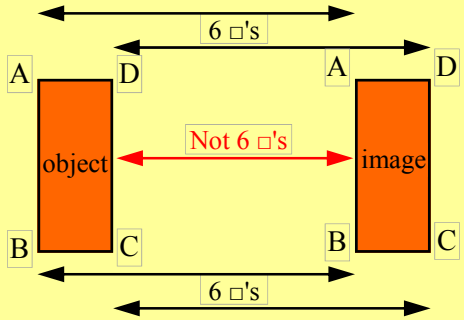
Transformations : Translation, Reflection and Rotation

When something is transformed it is changed in some way. In mathematics there are 3 different ways to transform a shape.

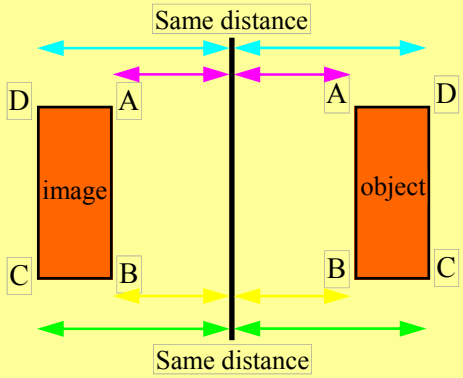
- Translation - this simply means to move, on the worksheet the shapes are on a grid and you are asked to move the object a certain number of squares on the grid.
- Refection - this is like a mirror image, the shape is reflected across a plane (line). The new shape will be the same distance away from the plane as the original, only upside-down or back-to-front.
- Rotation means to spin around a point, the point could be on the shape or away from the shape, this type of transformation requires the most thought.

With positioning the letters on the shape, the letters stay in the same position with translation, reversed in reflection and will be rotated in rotation, look at the examples below.

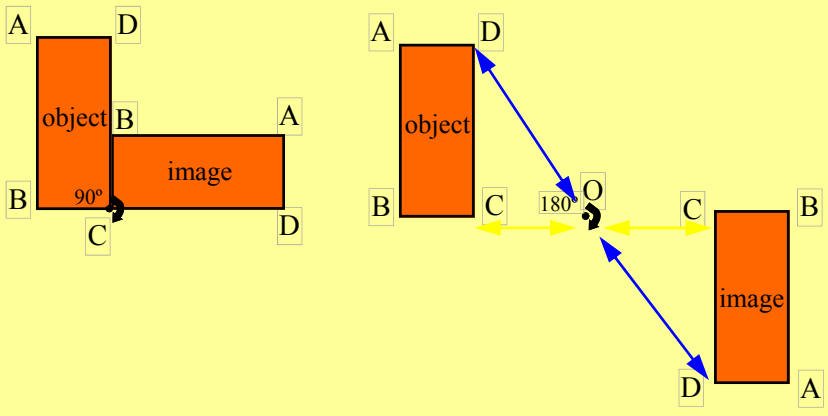
Translation means to move, the letters stay in the same position on the image. If you move the shape 6 □'s to the left then the distance between the same points is 6 □'s. The gap between the object and its image isn't 6 □'s, the most frequent mistake made.



Reflection means to copy back-to-front across a plane, the letters stay the same distance from the plane which means they appear to be reversed.



Rotation means to spin around a point, the point can be on the object or away from the object. The direction of rotation is given as either clockwise or anticlockwise. The distance of a point on the object from the rotation point remains the same once rotated.



Transformations : Translation, Reflection and Rotation

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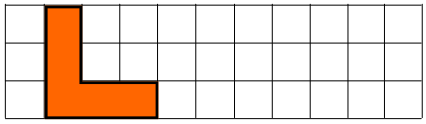
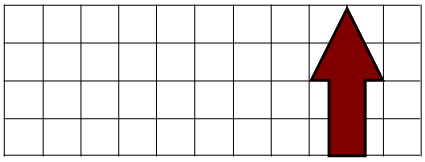
Translation means to move, redraw the shape in its final position.

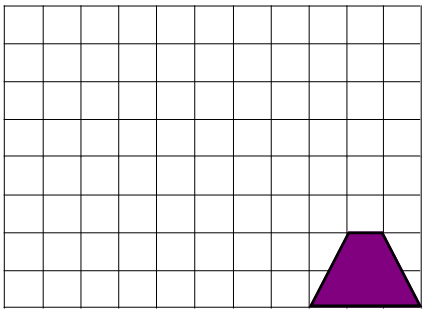
1 Translate 6 ❑'s right



2 Translate 8 ❑'s left

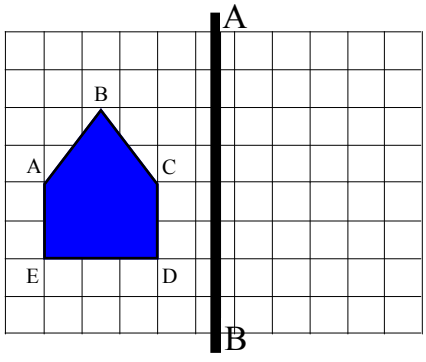


3 Translate 6 ❑'s left and then 4 ❑'s up

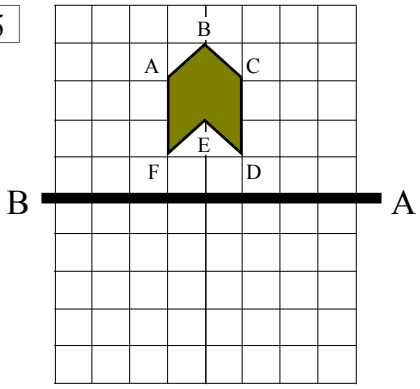


Reflection is 'flipping the shape over' across a line of reflection.

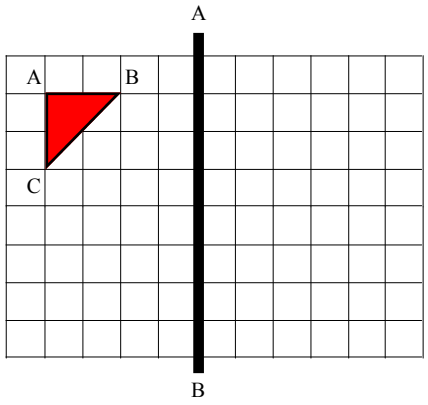
4 Reflect the shape in line AB show letters on new shape



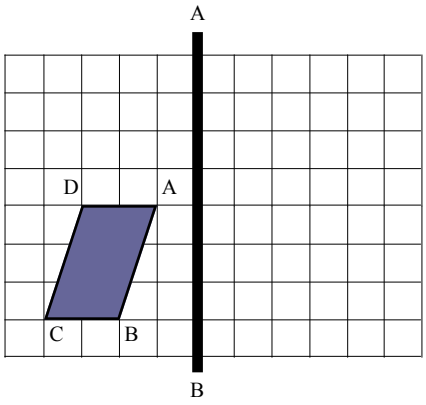
5



6 Translate 4 ❑'s down and reflect in line AB.

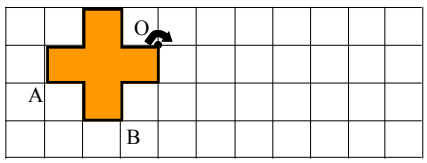


7 Reflect in line AB then translate 4 ❑'s up then 2 ❑'s right

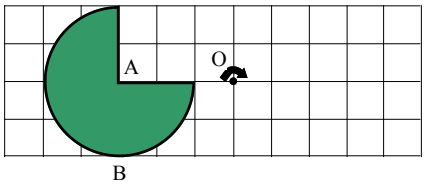


Rotation means to spin around a point, redraw the shape in its final position showing two letters.

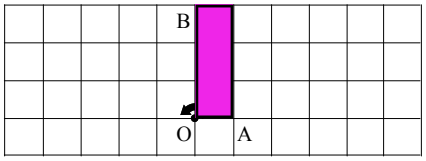
8 Rotate 180º clockwise at O



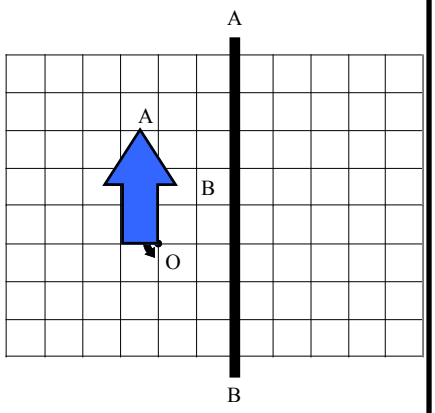
9 Rotate 180º clockwise at O



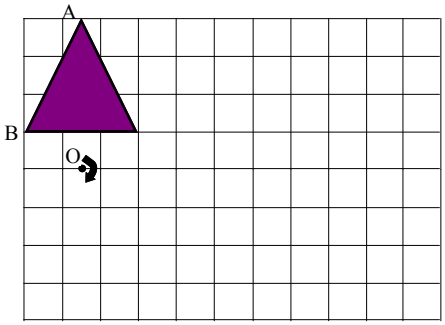
10 Rotate 90º anticlockwise at O



11 Rotate anticlockwise 90º at O then reflect in line AB

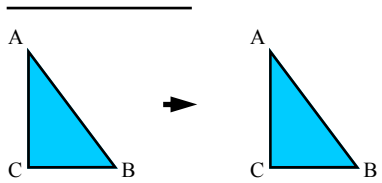


12 Rotate clockwise 180º at O then translate 6 ❑'s right

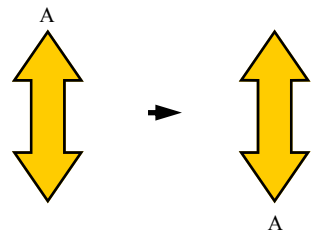


Which transformation would have occurred to create the image shown.

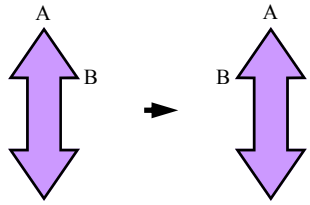
13



14



15



Transformations : Translation, Reflection and Rotation

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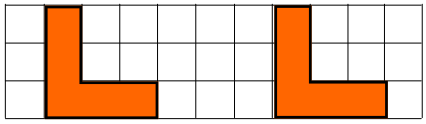
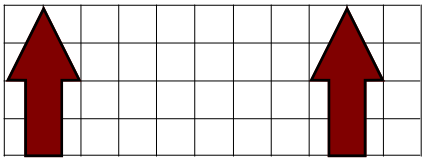
Translation means to move, redraw the shape in its final position.

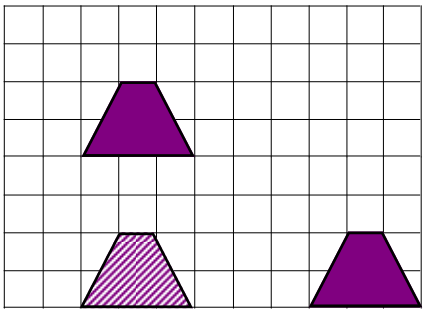
1 Translate 6 ❑'s right



2 Translate 8 ❑'s left

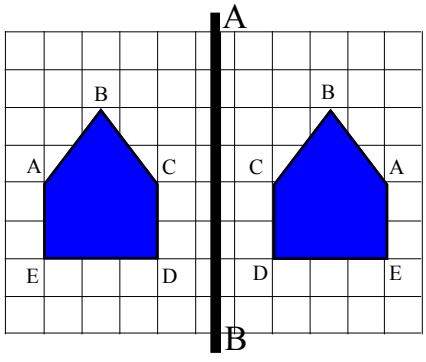


3 Translate 6 ❑'s left and then 4 ❑'s up

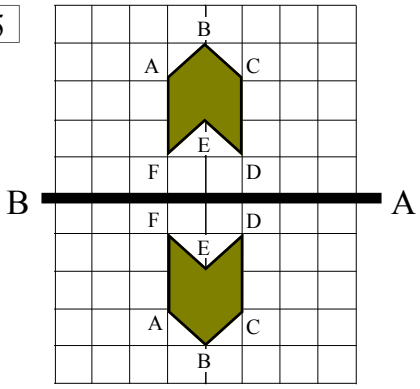


Reflection is 'flipping the shape over' across a line of reflection.

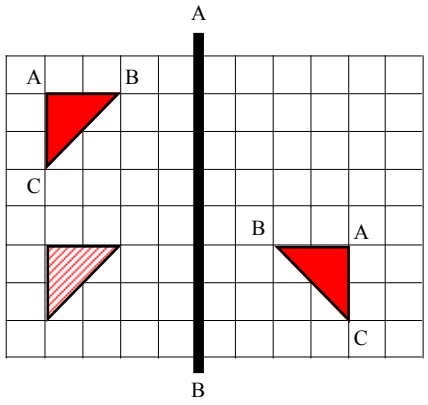
4 Reflect the shape in line AB show letters on new shape



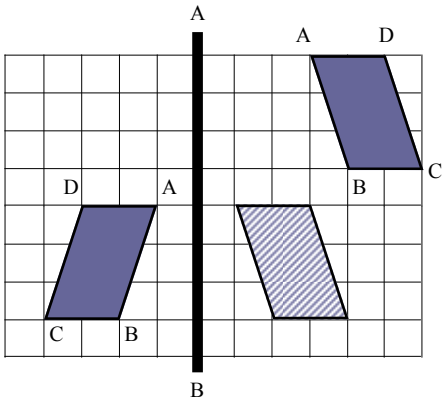
5



6 Translate 4 ❑'s down and reflect in line AB.

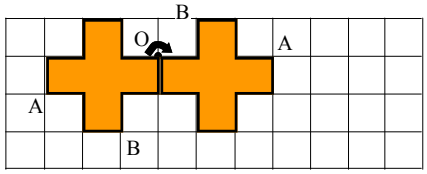


7 Reflect in line AB then translate 4 ❑'s up then 2 ❑'s right

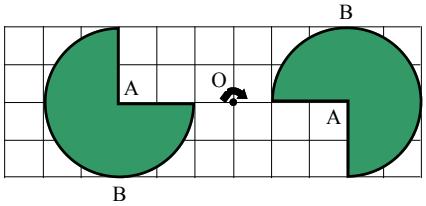


Rotation means to spin around a point, redraw the shape in its final position showing two letters.

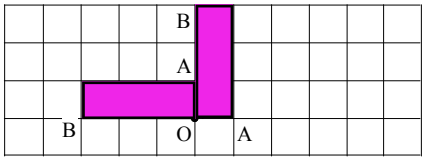
8 Rotate 180º clockwise at O



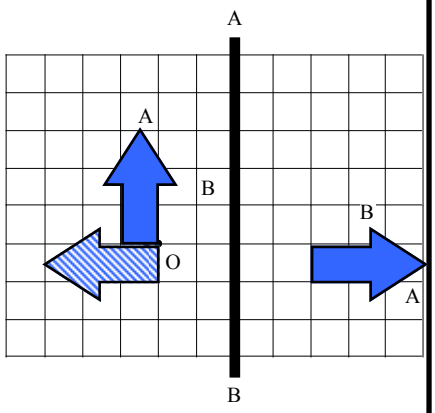
9 Rotate 180º clockwise at O



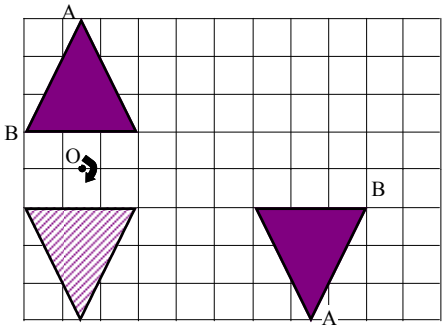
10 Rotate 90º anticlockwise at O



11 Rotate anticlockwise 90º at O then reflect in line AB

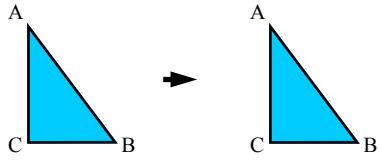


12 Rotate clockwise 180º at O then translate 6 ❑'s right

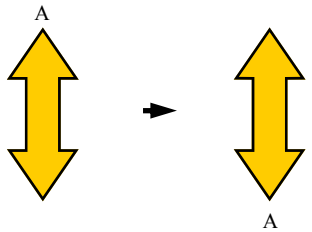


Which transformation would have occurred to create the image shown.

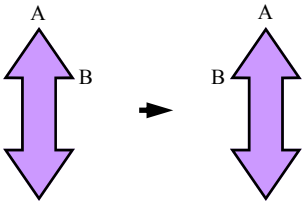
13 translation



14 rotation



15 reflection



Templates for Building

These templates are designed to be printed, cut out, coloured-in and either:

Sheets 1 & 2 - stick into books to fold out of page

Sheet 3 - 6 - cut out and assemble to make 3D shapes

Constructing Cubes and Square Pyramids

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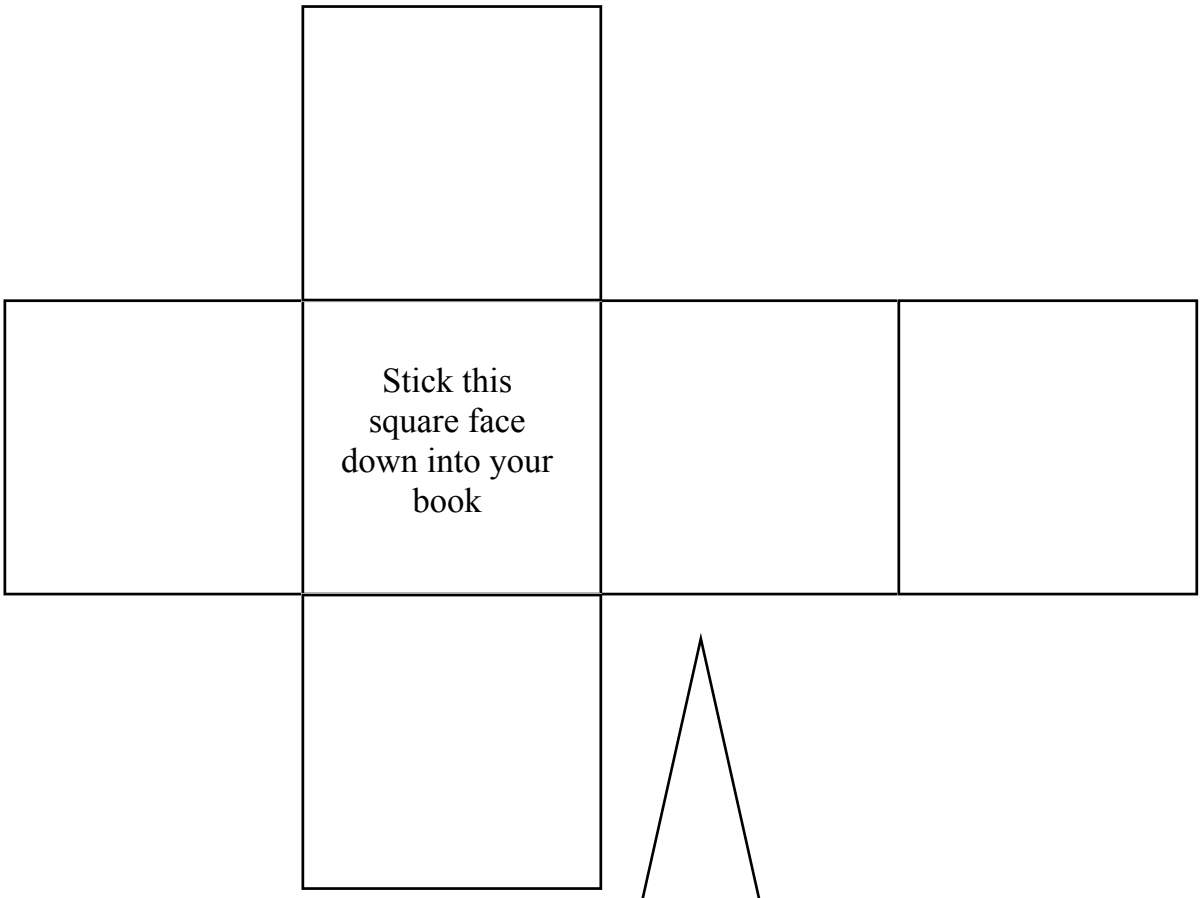

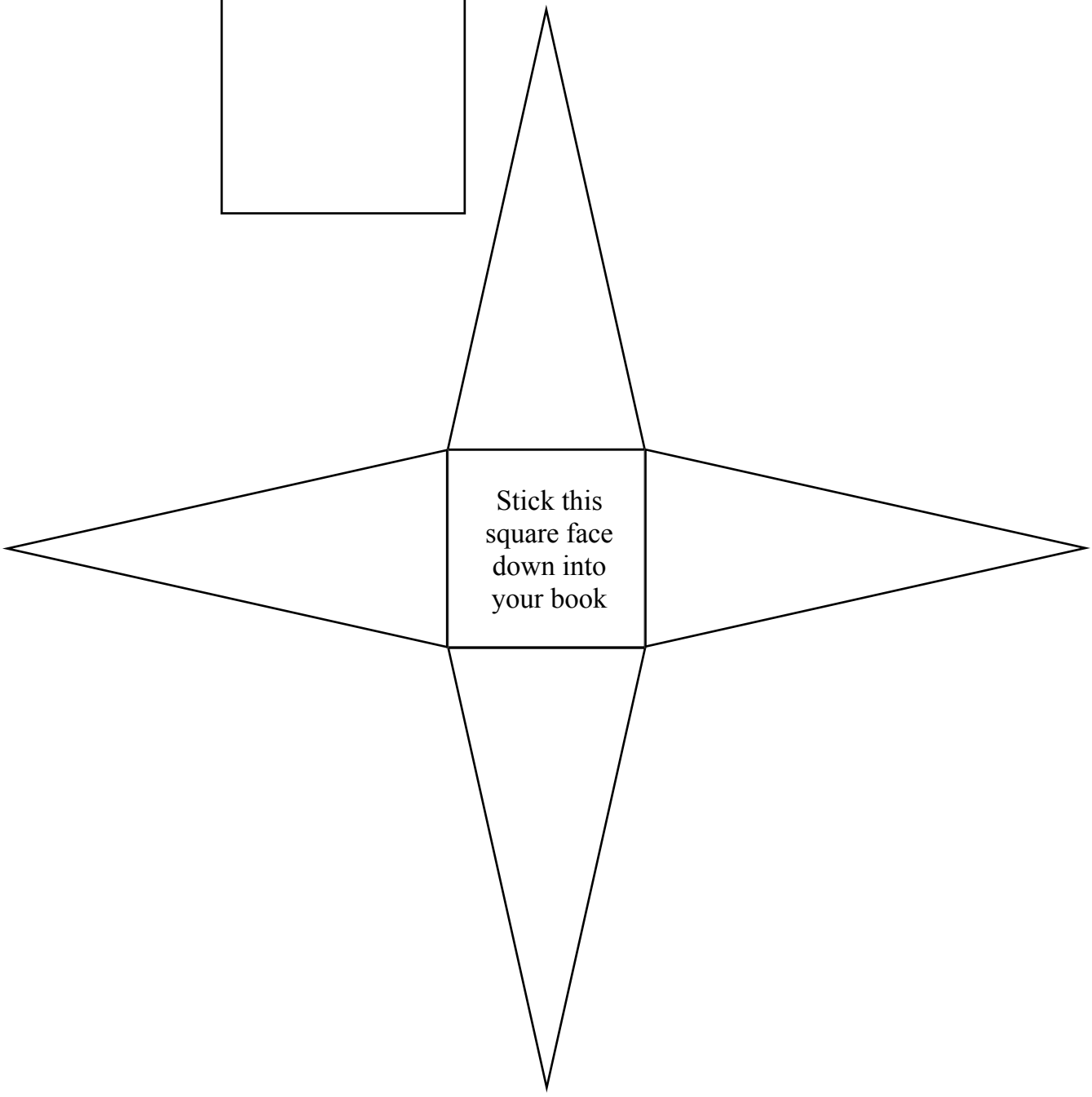

Constructing Rectangular Prisms and Triangular Prisms

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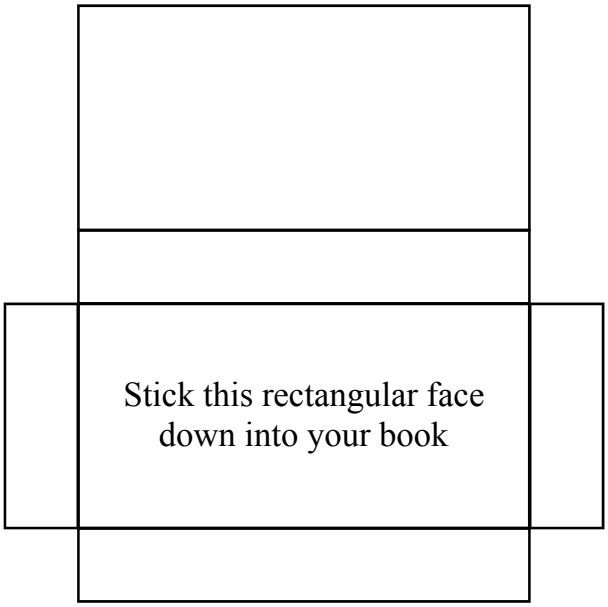

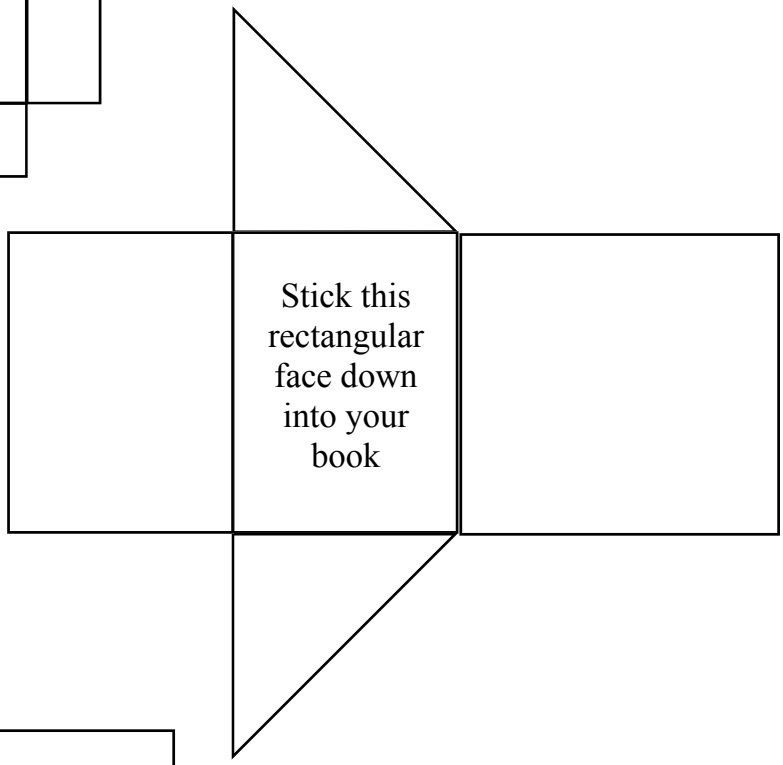

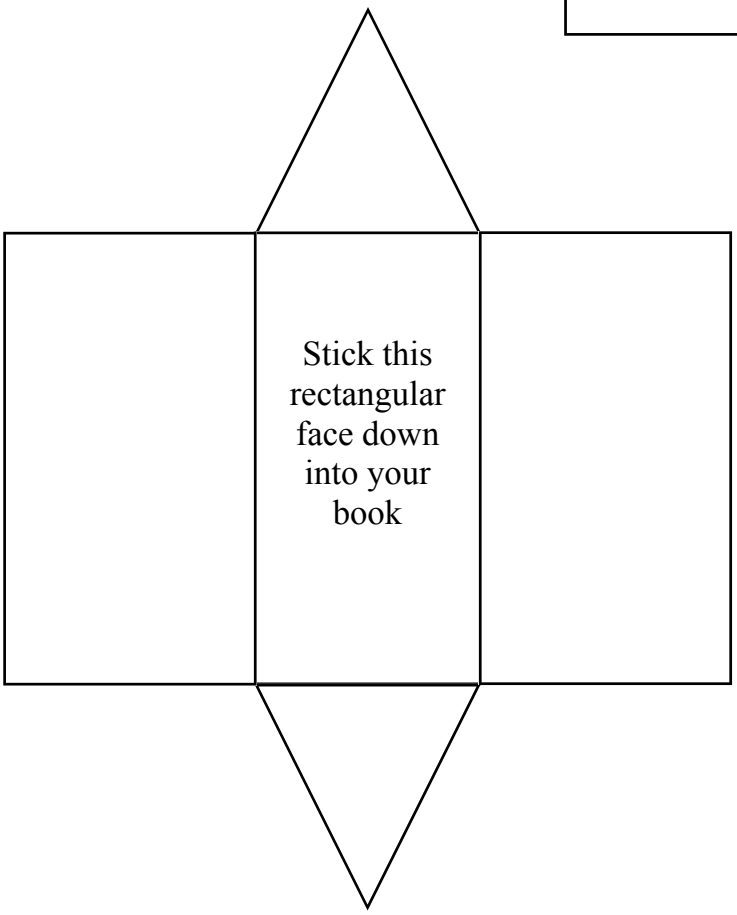

Constructing Cubes and Square Pyramids (with tabs)

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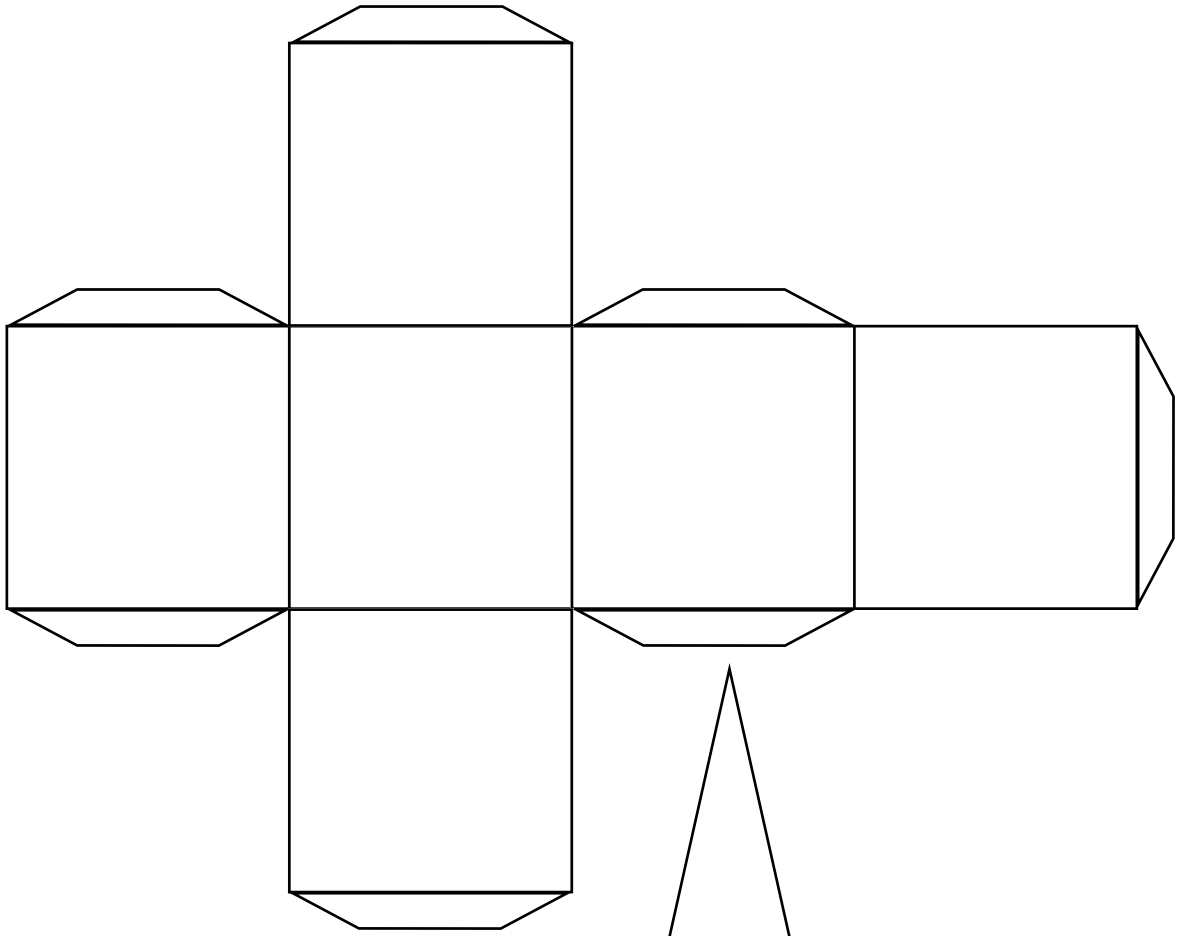

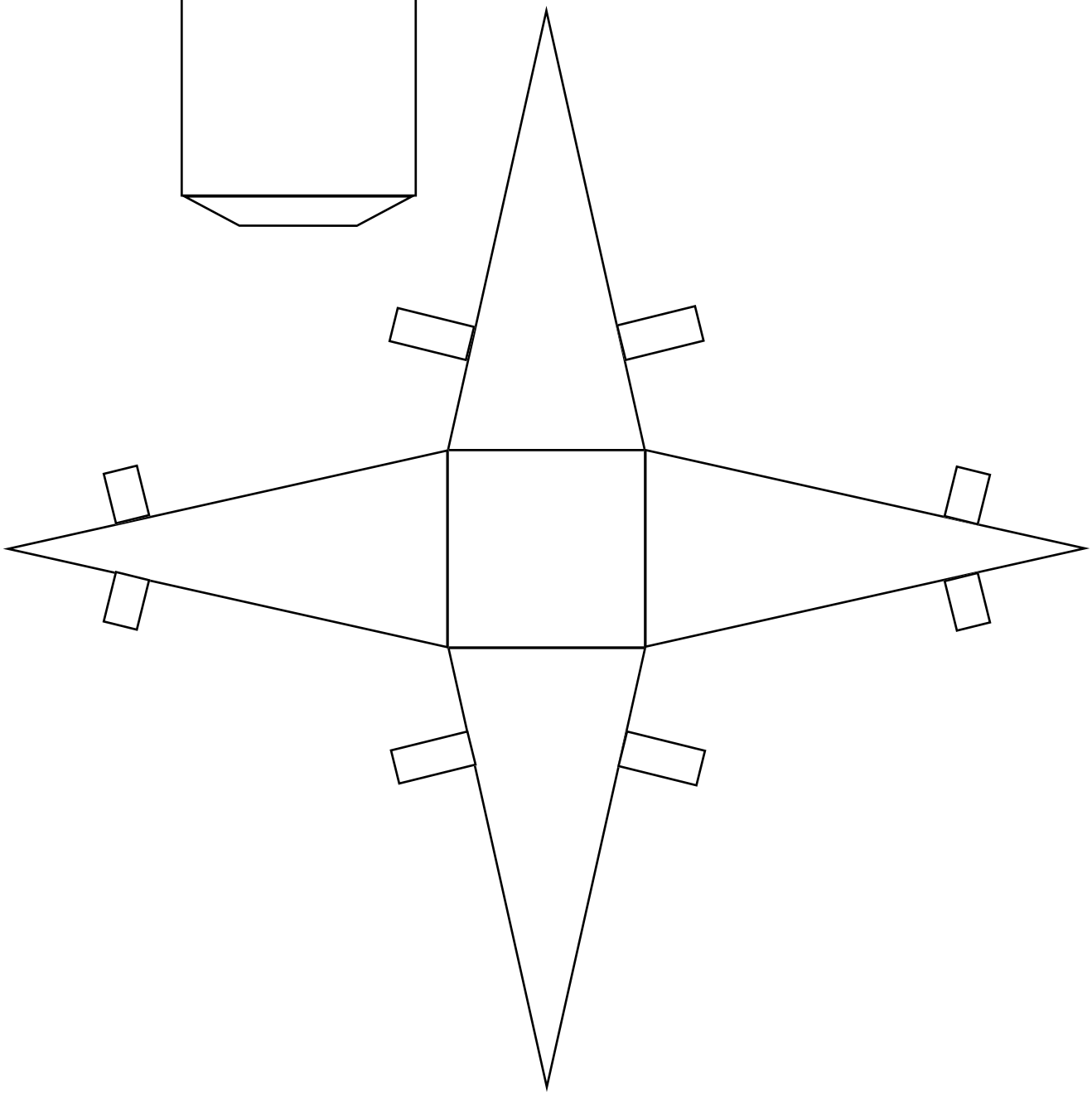

Constructing Rectangular Prisms and Triangular Prisms (with tabs)

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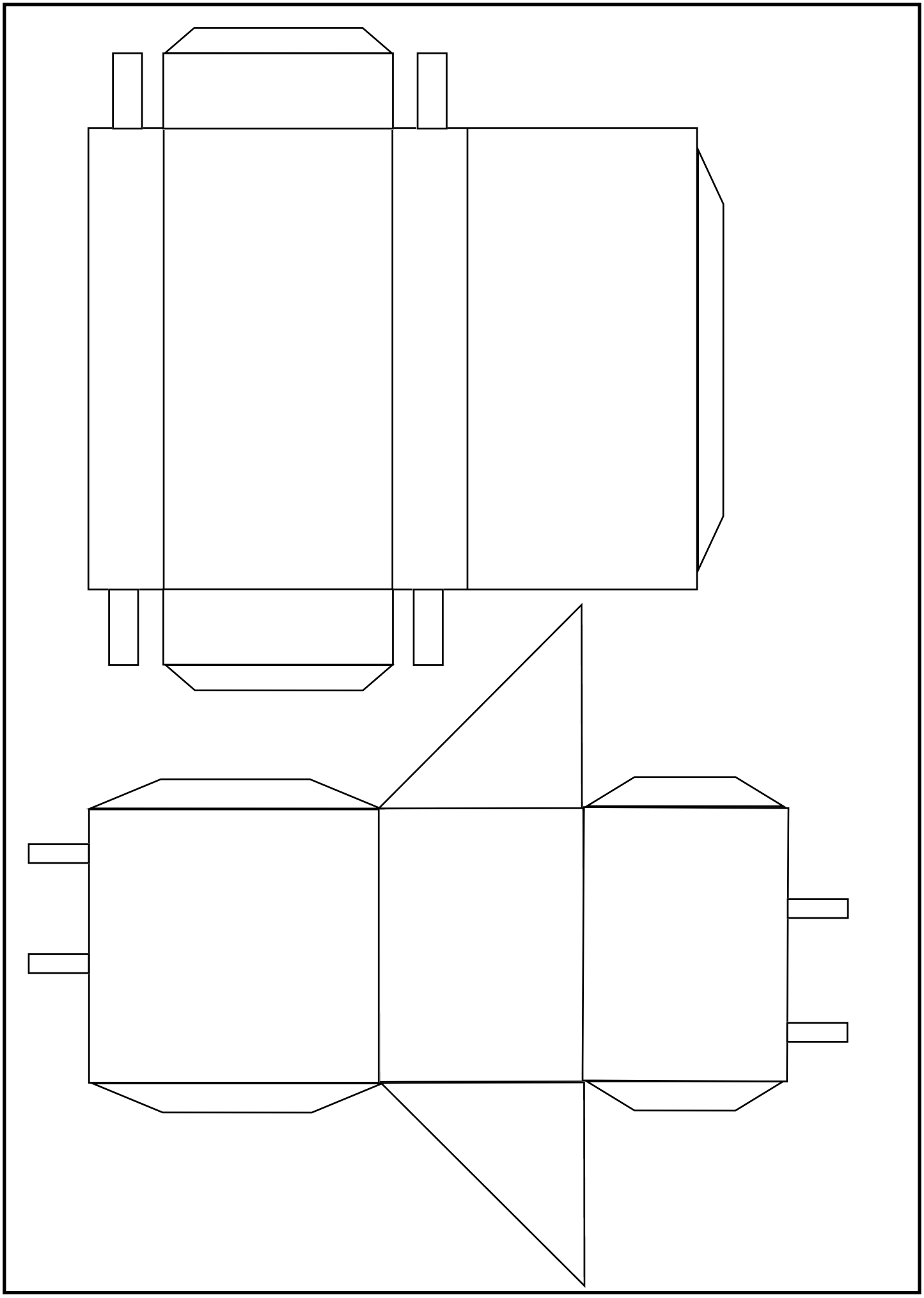

Constructing Cylinders and Triangular Prisms (with tabs)

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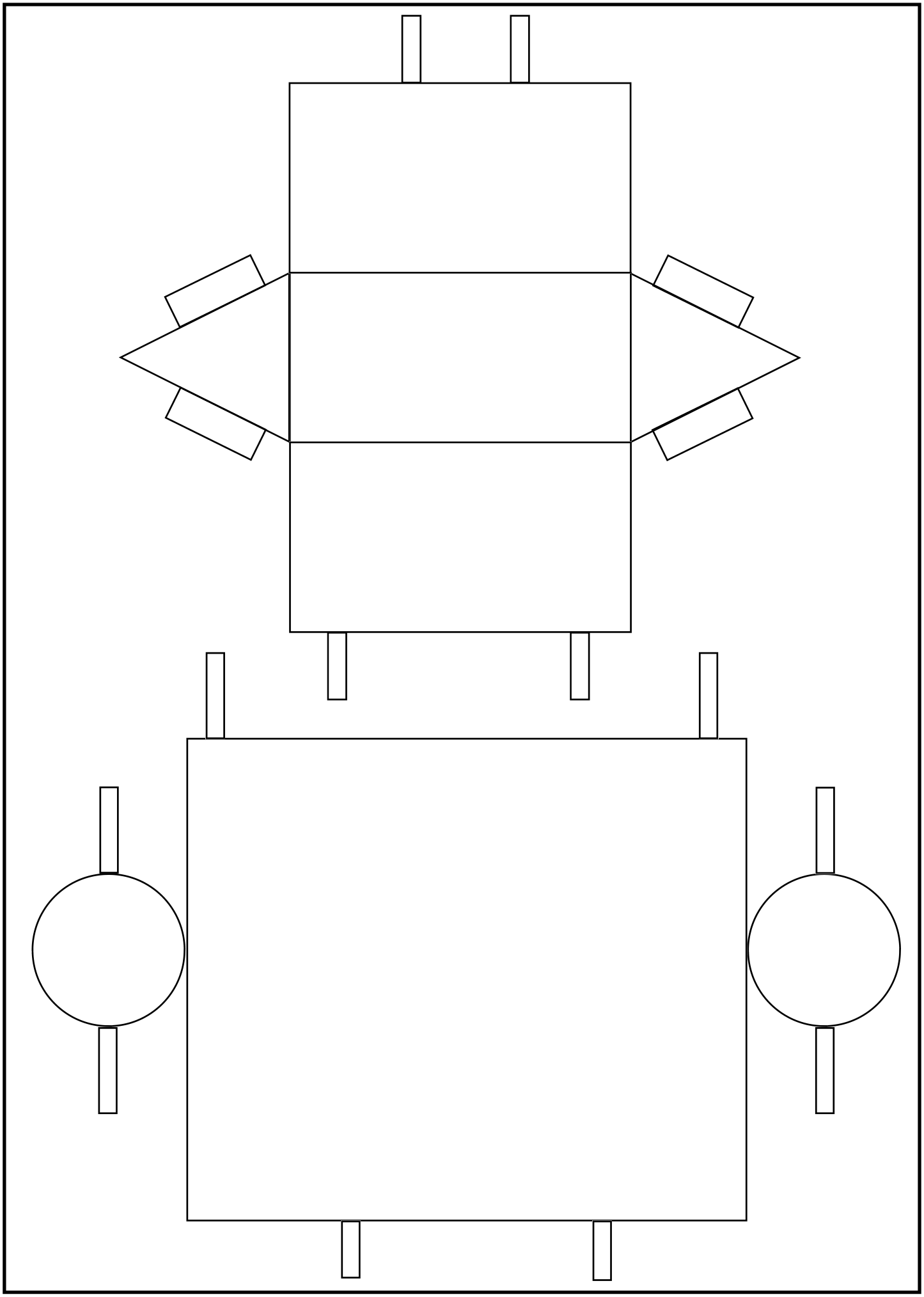

Constructing Cones and Pentagonal Prisms (with tabs)

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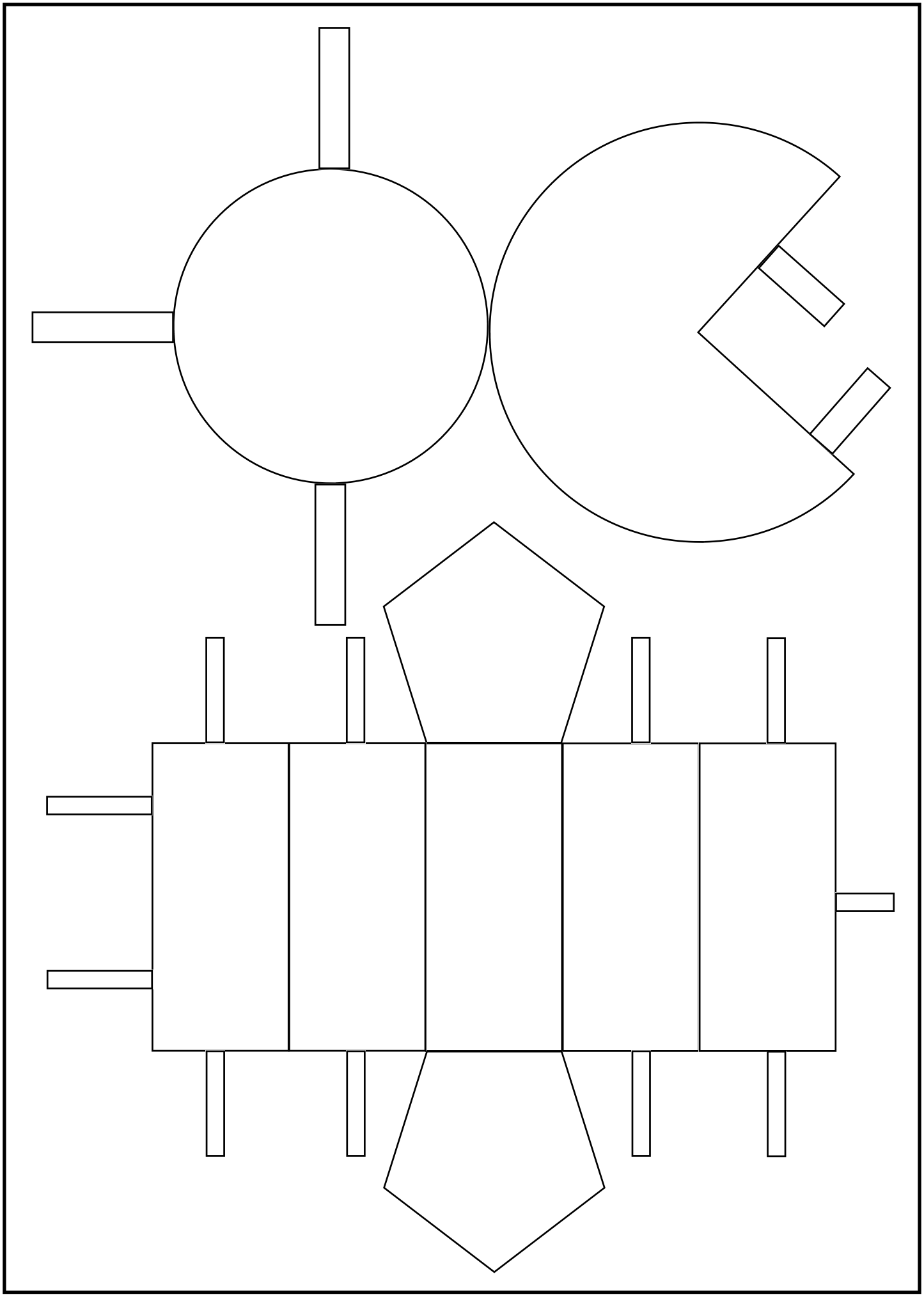

Nets of Solids 1

A net in mathematics refers to the way a solid would look if it were made from paper and unfolded. So to answer this sheet you have to look at the net and fold it up in your mind and decide what shape it makes when it is folded up.

For most students the difficulty comes when prisms and pyramids are involved. Think about this:

- If a net only has 2 triangles in the net, it will always be a triangular prism.
- If a net has more that 2 triangles in the net it will always be pyramid.
- If a net is of a pyramid then the name of the net will come from the shape in the net that isn't a triangle.
- If the net is of a prism then the name of the prism will come from the shape that isn't a rectangle or square. (except rectangular prisms)

Remember that a shape can't be a pyramid if it has no triangles in its net. A shape can't be a prism if it has more than two triangles in its net. To answer the sheet write the solid's name in the table at the bottom of the page next to the net's number.

Nets of Solids 1

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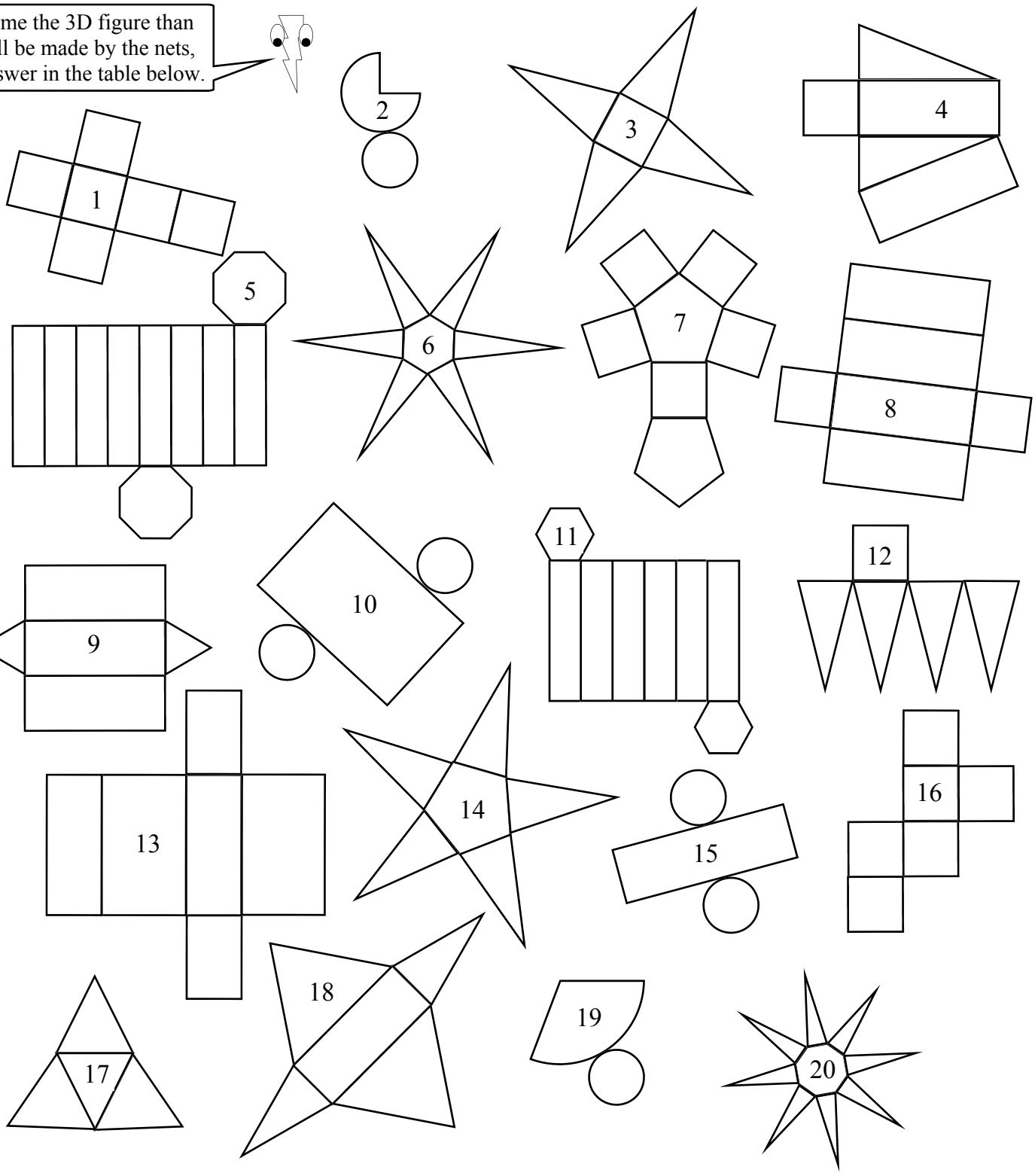
Name the 3D figure than will be made by the nets, answer in the table below.



1 _____	2 _____	3 _____	4 _____
5 _____	6 _____	7 _____	8 _____
9 _____	10 _____	11 _____	12 _____
13 _____	14 _____	15 _____	16 _____
17 _____	18 _____	19 _____	20 _____

Nets of Solids 1

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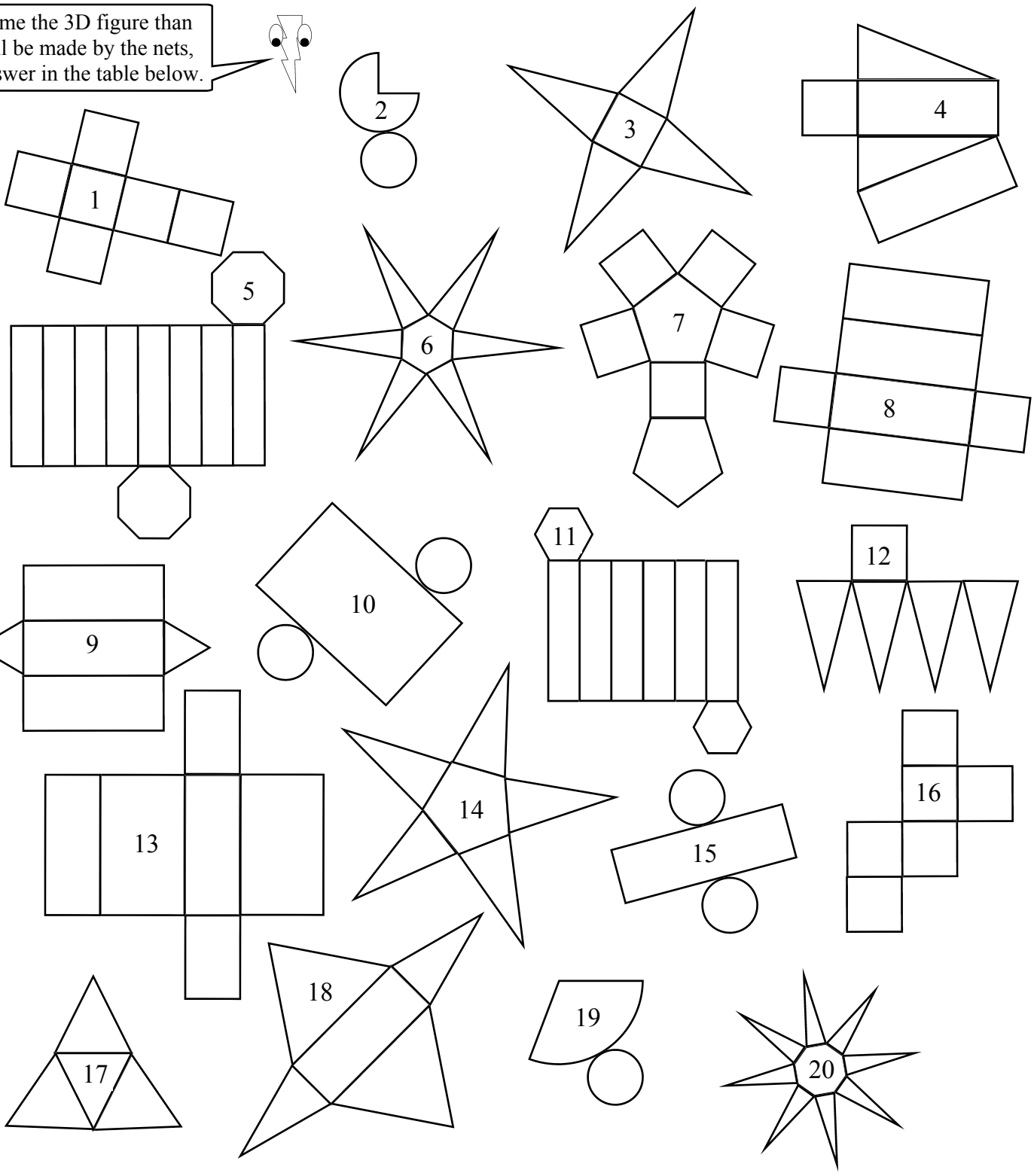
Name the 3D figure than will be made by the nets, answer in the table below.



1 Cube	2 Cone	3 Square pyramid	4 Triangular prism
5 Octagonal prism	6 Hexagonal pyramid	7 Pentagonal prism	8 Rectangular prism
9 Triangular prism	10 Cylinder	11 Hexagonal prism	12 Square pyramid
13 Rectangular prism	14 Pentagonal pyramid	15 Cylinder	16 Cube
17 Triangular pyramid	18 Rectangular pyramid	19 Cone	20 Octagonal pyramid

Nets of Solids 2

Nets of solids are drawings of solids as though they are made from paper and unfolded. So to draw a net think of the solid unfolded. The previous sheet “Nets of Solids 1” can be referred to if you need help.

A prism will always have rectangles in its net and two other shapes as its base shape. So a pentagonal prism would have two pentagons and five rectangles, an octagonal prism would have two octagons and eight rectangles in its net.

A pyramid will always have triangles in its net and a different shape as its base shape (except triangular pyramids. So a hexagonal pyramid would have a single hexagon with six triangles, while a triangular pyramid would have a triangle with three triangles off it.

Use a pencil so that you can correct mistakes easily.

Nets of Solids 2

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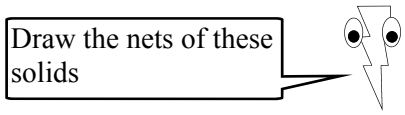

1 Cube

2 Cone

3 Rectangular Prism

4 Triangular Prism

5 Pentagonal Pyramid

6 Hexagonal Prism

7 Octagonal Prism

8 Square Pyramid

9 Pentagonal Prism

10 Triangular Pyramid

11 Rectangular Pyramid

12 Hexagonal Pyramid

Nets of Solids 2

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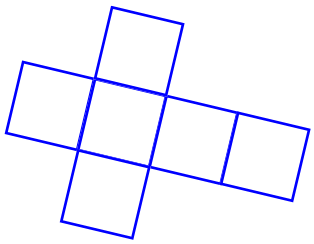
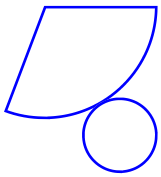
Draw the nets of these solids

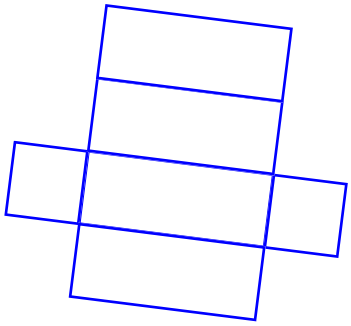
1 Cube



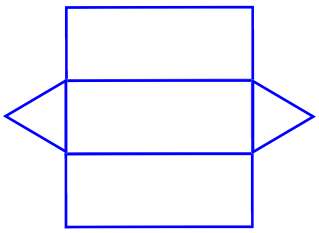
2 Cone



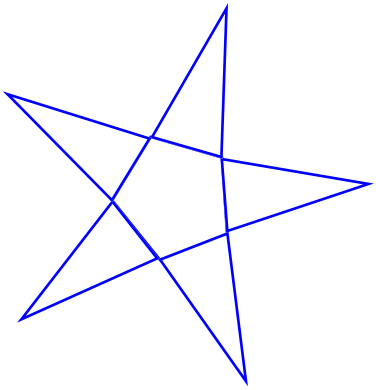
3 Rectangular Prism



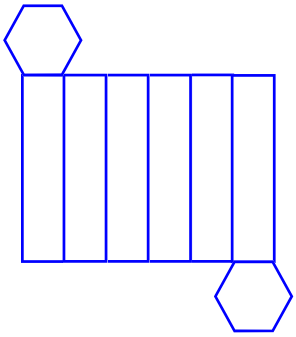
4 Triangular Prism



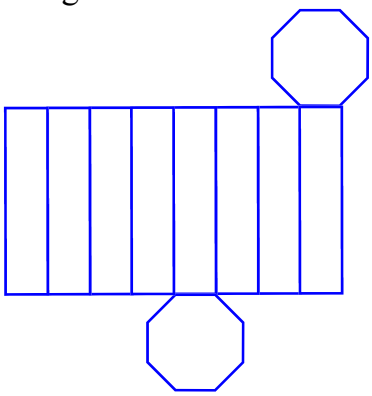
5 Pentagonal Pyramid



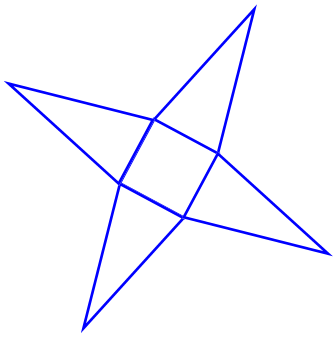
6 Hexagonal Prism



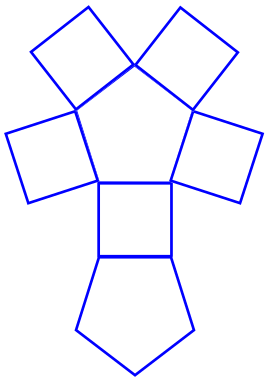
7 Octagonal Prism



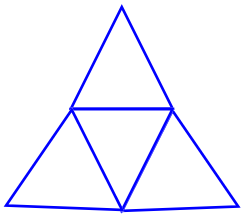
8 Square Pyramid



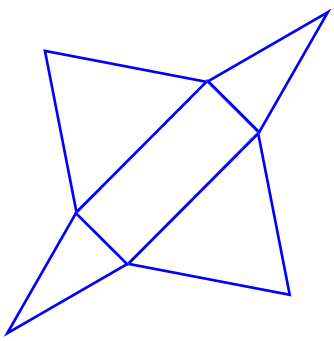
9 Pentagonal Prism



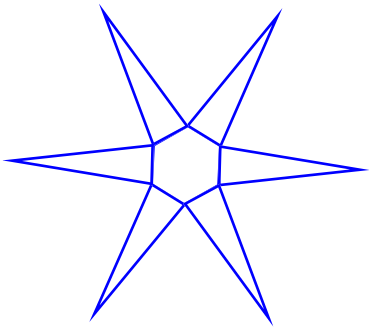
10 Triangular Pyramid



11 Rectangular Pyramid



12 Hexagonal Pyramid



Views of Solids

When you look at a solid from the side, front or top you will see a plane (2D) shape. Imagine looking at a box from the top, side and front. You would have 3 views that would be either rectangles or squares.

Column 1 starts by giving you 3 views of a solid and asks you to name it. This exercise tests your visual skills and there aren't any helpful tips except that pyramids will have more than one view that includes a triangle, with that triangle being either isosceles or equilateral, not right or scalene.

Column 2 has diagrams of 5 solids and asks what shape you would see if you were looking from the direction of the arrow. The answers are all in the box at the bottom of the column, write in the letter (from A → I) that matches the object in the space provided. Note that the same letter can be used more than once.

Column 3 asks you to build you own shapes to show the view from the direction that is arrowed.

Views of Solids

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Name the 3-D solids that would have these top and side views.

1 _____

Top view

Front view Side view

2 _____

Top view

Front view Side view

3 _____

Top view

Front view Side view

4 _____

Top view

Front view Side view

5 _____

Top view

Front view Side view

6 _____

Top view

Front view Side view

7 _____

Top view

Front view Side view

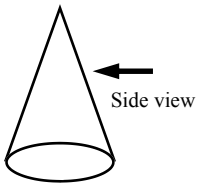
8 _____

Top view

Front view Side view

Match the views shown with the shapes below, write the letter.

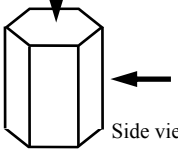
9 Top view



Top ___

Side ___

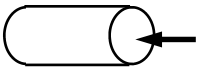
10 Top view



Top ___

Side ___

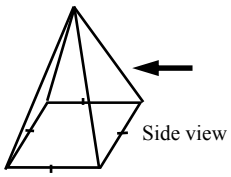
11 Top view



Top ___

Side ___

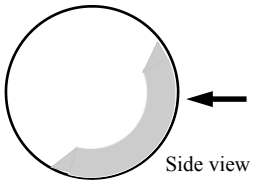
12 Top view



Top ___

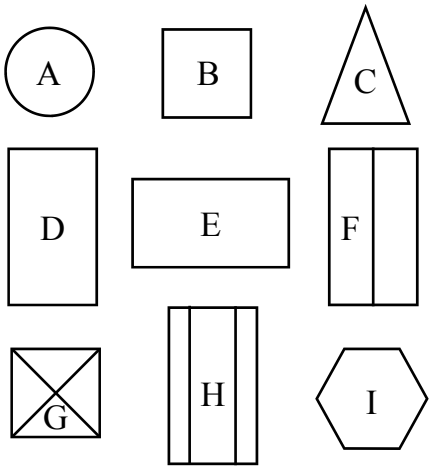
Side ___

13 Top view



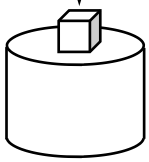
Top ___

Side ___



Sketch the view of the 3-D solids indicated by the arrow.

14 Top view



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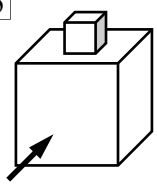

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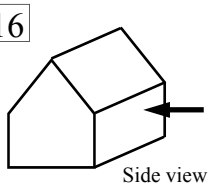
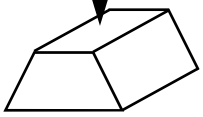


17 Top view



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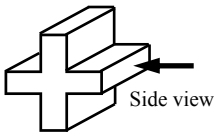
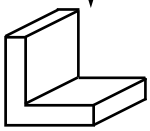
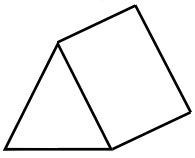


19 Top view



20 Top view



Views of Solids

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Name the 3-D solids that would have these top and side views.

1 Rectangular Prism

Top view 

Front view  Side view 

2 Cube

Top view 

Front view  Side view 

3 Triangular prism

Top view 

Front view  Side view 

4 Cylinder

Top view 

Front view  Side view 

5 Cone

Top view 

Front view  Side view 

6 Sphere

Top view 

Front view  Side view 

7 Square pyramid

Top view 

Front view  Side view 

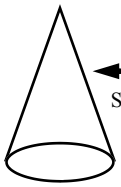
8 Hexagonal pyramid

Top view 

Front view  Side view 

Match the views shown with the shapes below, write the letter.

9

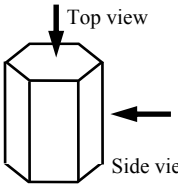
Top view 

Side view

Top A

Side C

10

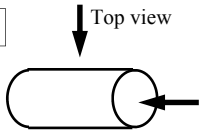
Top view 

Side view

Top I

Side F

11

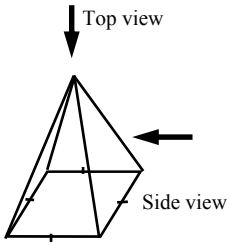
Top view 

Side view

Top E

Side A

12

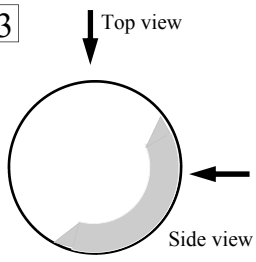
Top view 

Side view

Top G

Side C

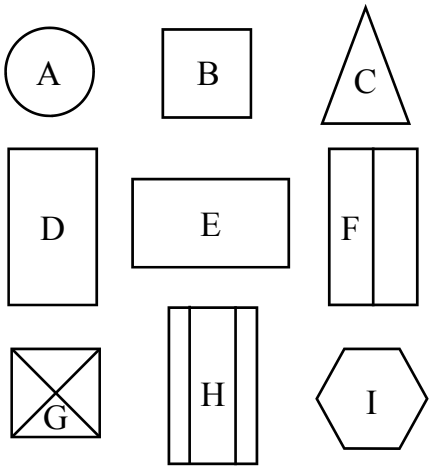
13

Top view 

Side view

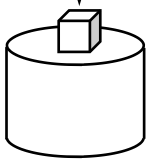
Top A

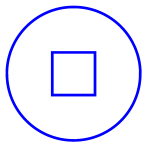
Side A



Sketch the view of the 3-D solids indicated by the arrow.

14

Top view 



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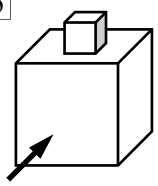
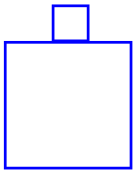


Front view



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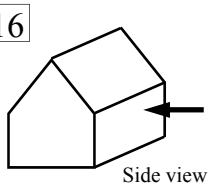
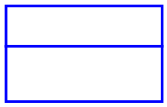


Side view



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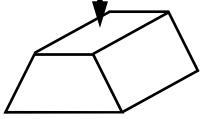
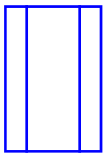
Top view 



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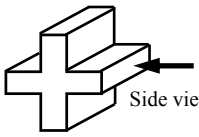


Side view



19

Top view

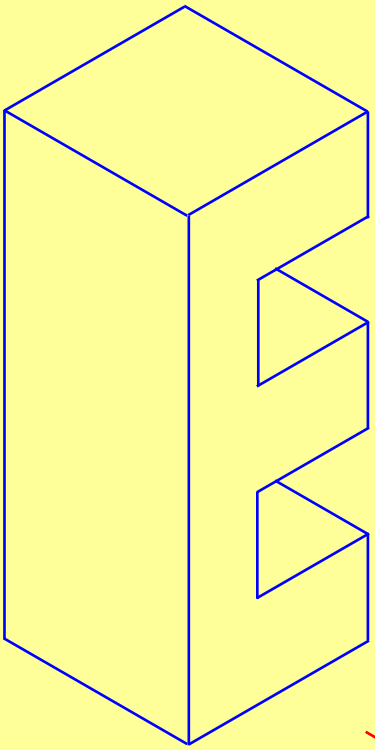
20

Top view

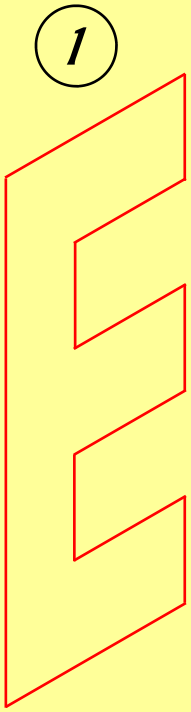
Isometric Drawing

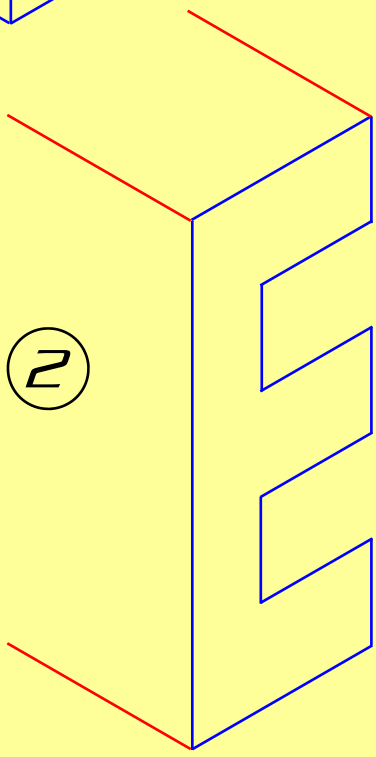
Isometric drawing is a method of showing a shape in a way that looks 3 dimensional. The bottom corner of the drawn shape has a BLUE COLOURED DOT, this is the point that you should start at and it is shown at the bottom of the page.

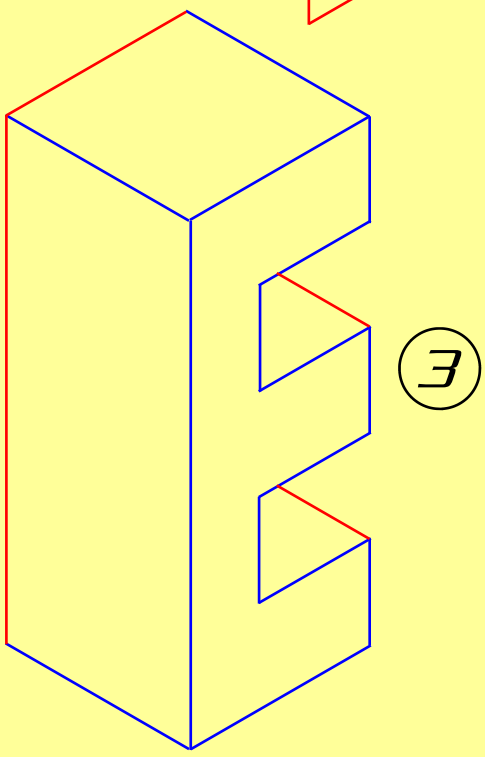
There is no 'best way' as it depends on how you visualise the solid, but one way is to try to complete one face first and then move on



So to draw the solid to the left, draw the front face first (right). Then take each corner back the required depth (bottom left). Join the edges and don't forget to take the two edges back as shown (bottom right)







Isometric Drawing - Sheet 1

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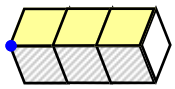
Copy the shapes in each question. Make sure that you start your drawing at the dot, otherwise you may run out of space. Colour them in when you are finished.

1



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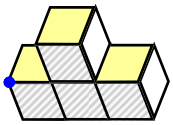

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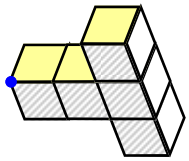

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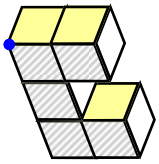

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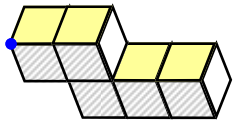

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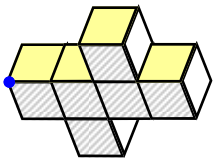

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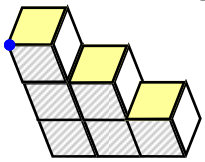

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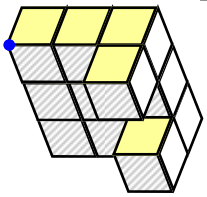

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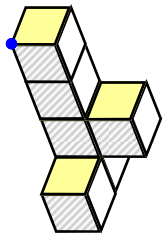

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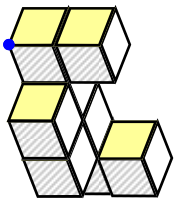

Isometric Drawing - Sheet 2

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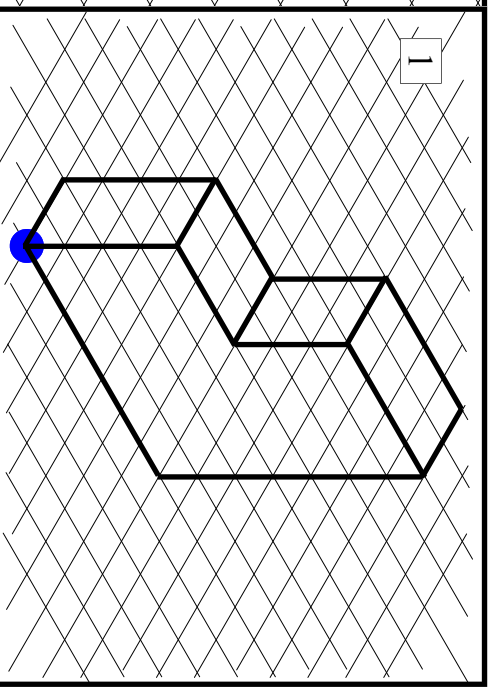
Copy the shapes in each question. Make sure that you start your drawing at the dot, otherwise you may run out of space. Colour them in when you are finished.

1



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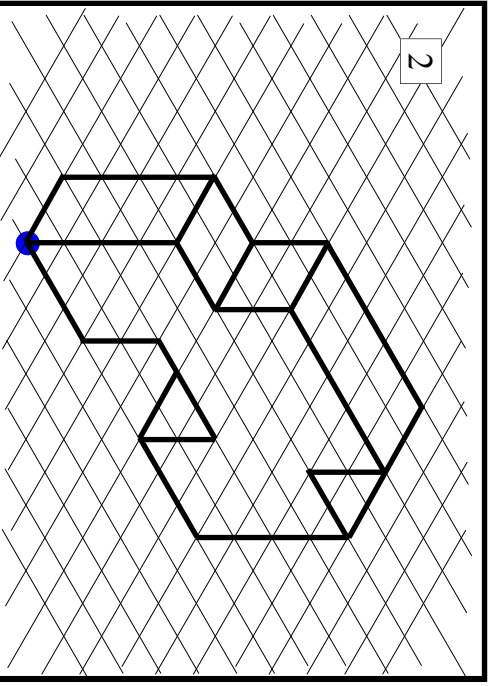

Isometric Drawing - Sheet 1

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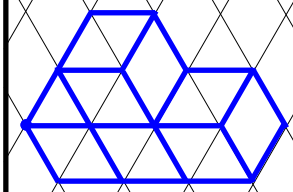

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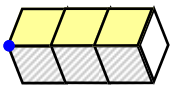

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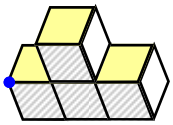

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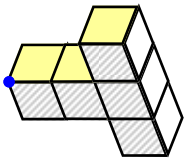

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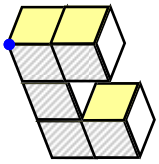

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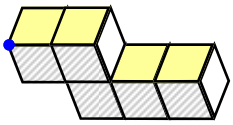

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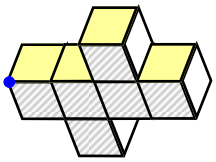

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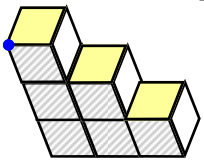

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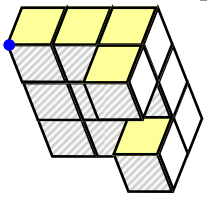

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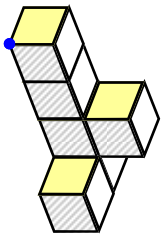

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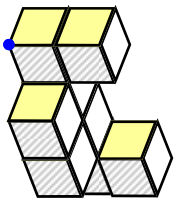

Isometric Drawing - Sheet 2

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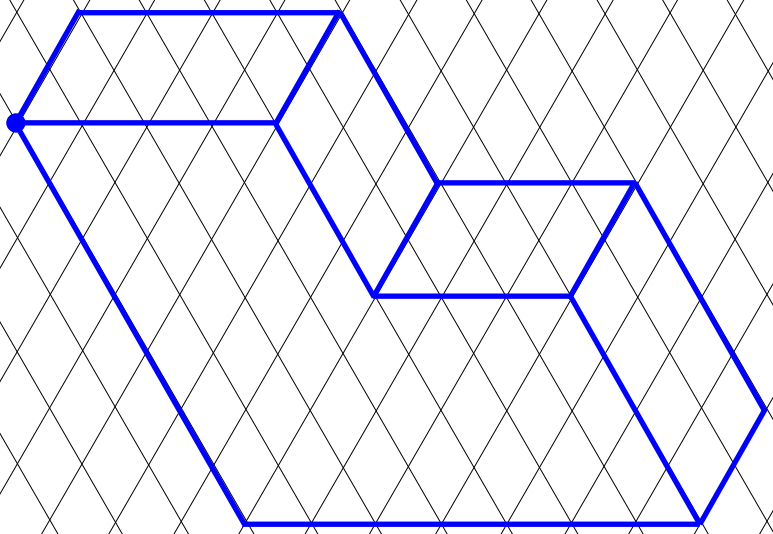

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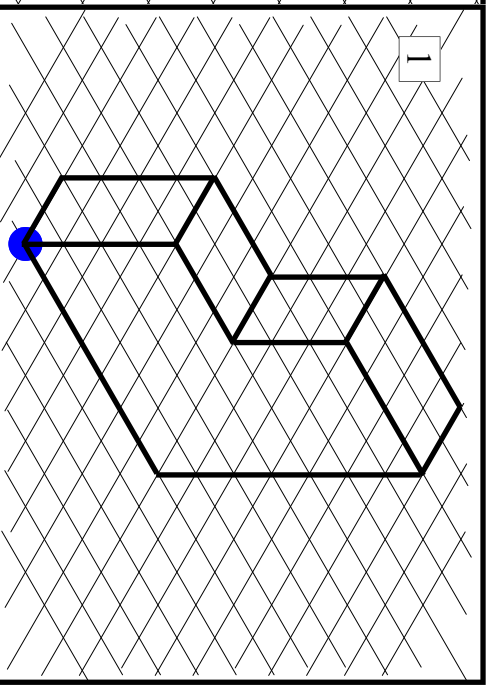

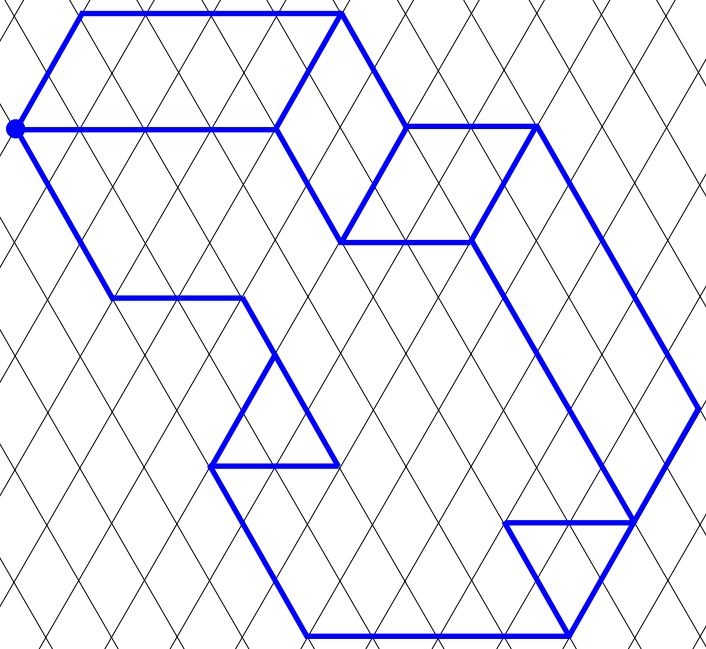

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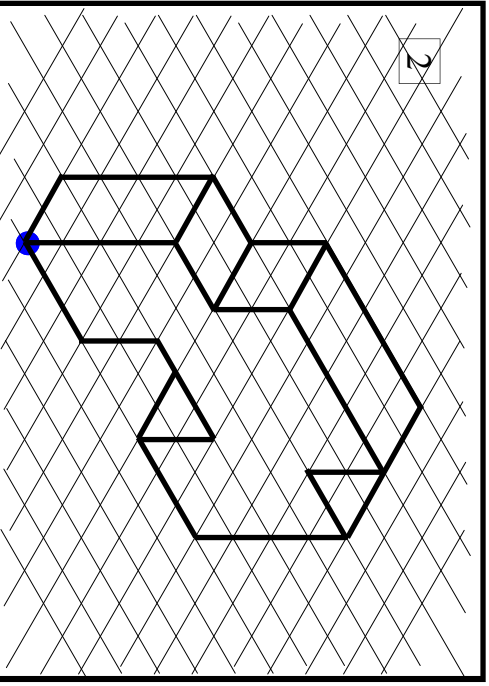

Drawing Solids from Views

Isometric drawing is a method of drawing a solid in a way that it looks 3 dimensional. The bottom part of the sheet has a blue dot, this is the point that you should start from. The ‘front view’ arrow should be pointing at the front view, the object must be drawn in this way or it may not fit in the page.

Question 1 should be attempted first, Question 2 should be considered challenging.

“15 Isometric drawing” should have been completed before this Worksheet is attempted.

Drawing Solids from Views

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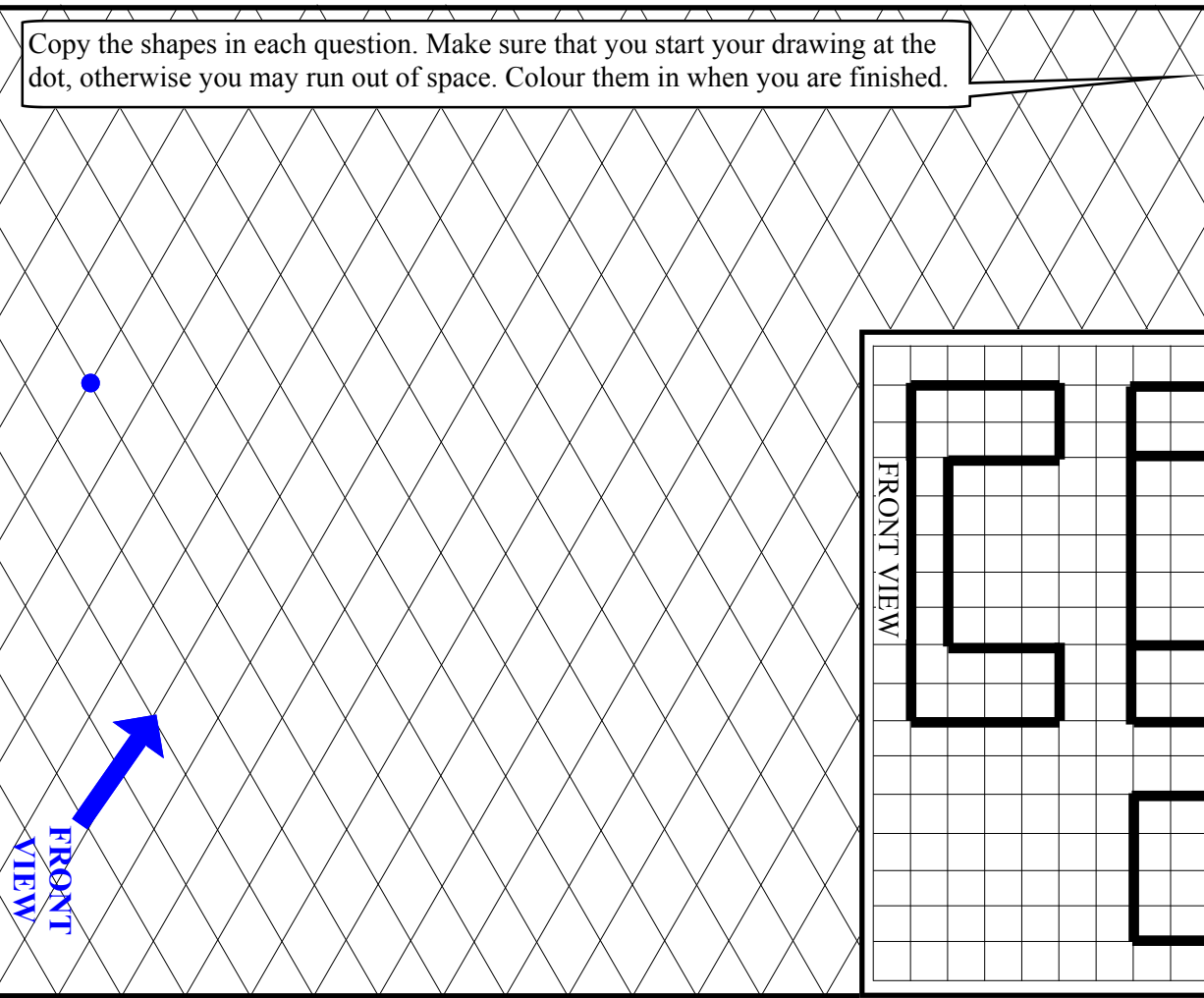
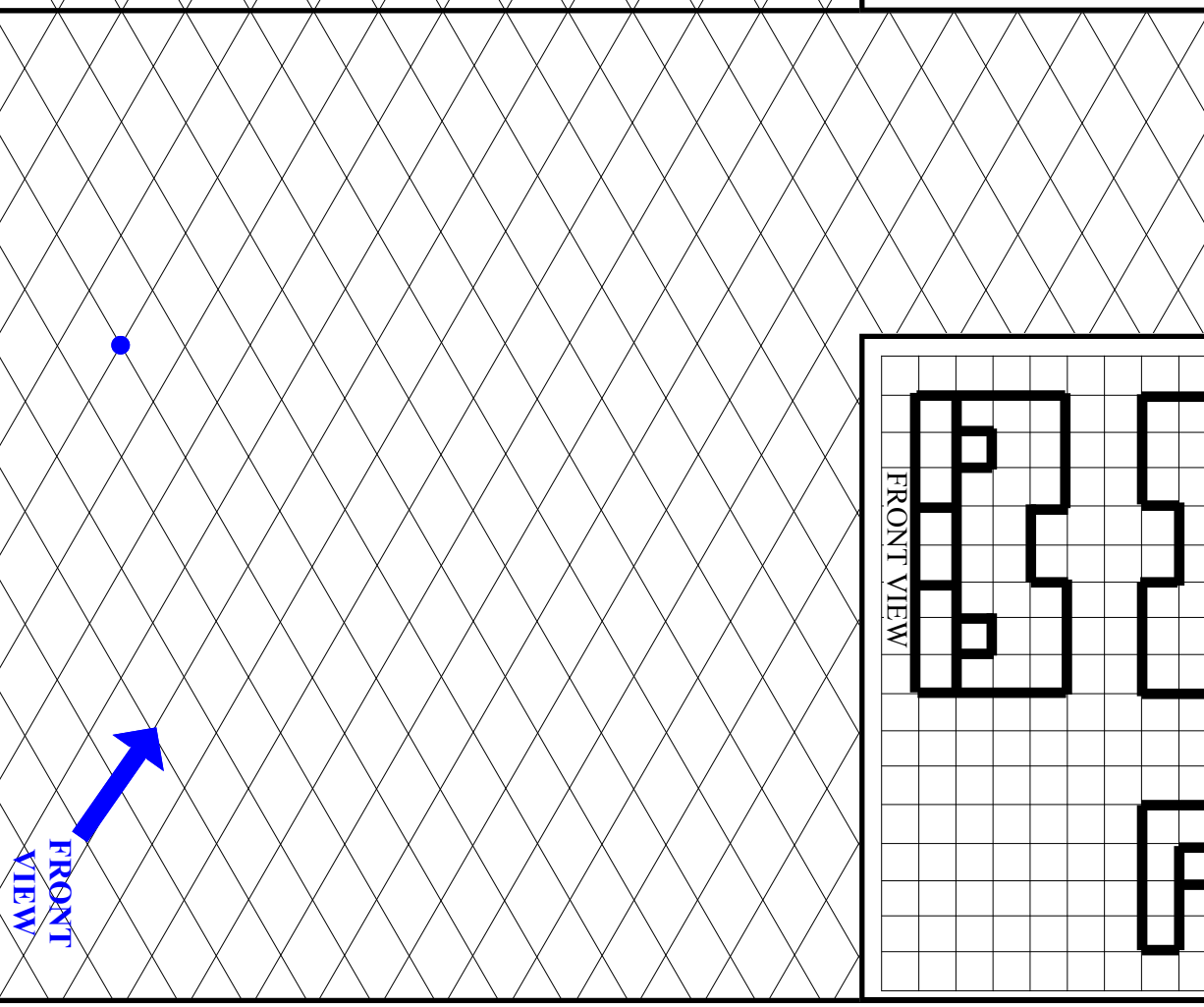
Copy the shapes in each question. Make sure that you start your drawing at the dot, otherwise you may run out of space. Colour them in when you are finished.





Drawing Solids from Views

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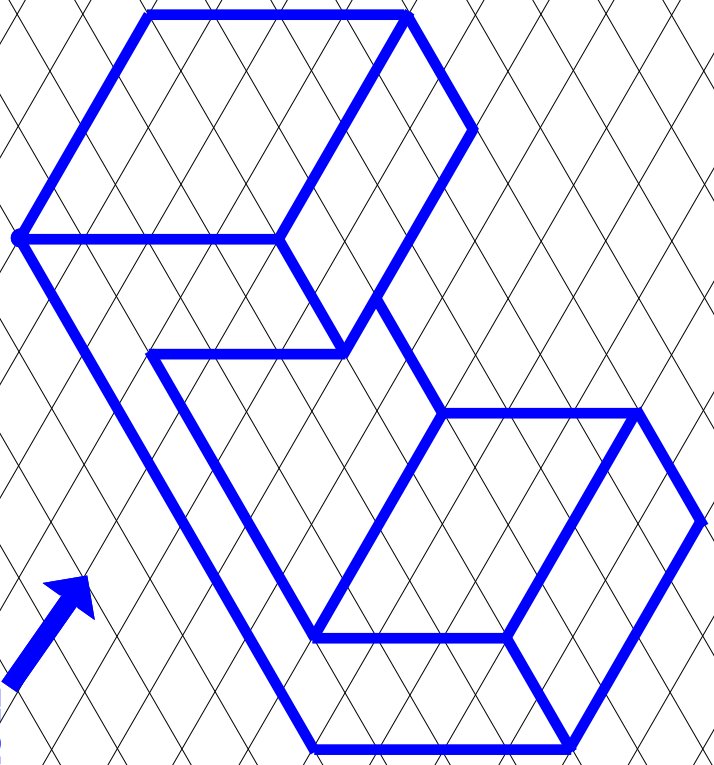
Copy the shapes in each question. Make sure that you start your drawing at the dot, otherwise you may run out of space. Colour them in when you are finished.

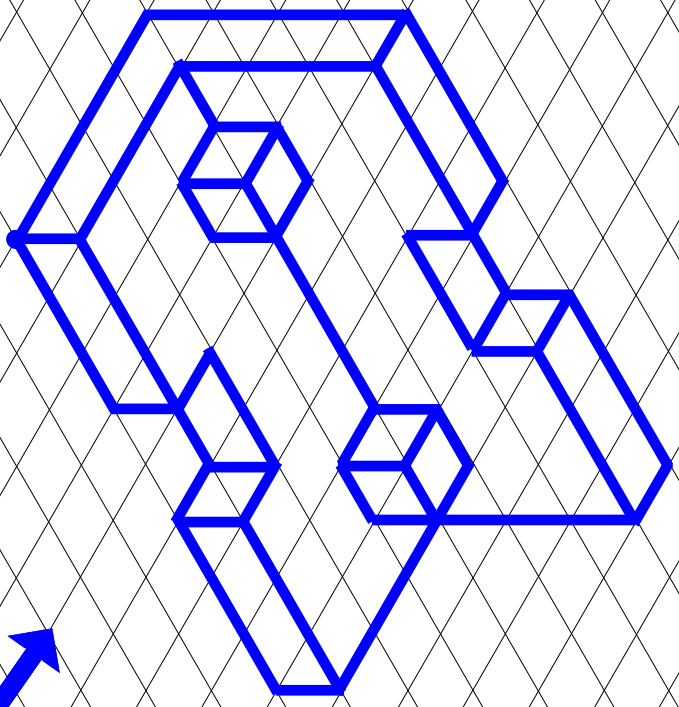


1

FRONT VIEW

TOP VIEW

SIDE VIEW



2

FRONT VIEW

TOP VIEW

SIDE VIEW

Euler's Formula

This formula relates the properties of a solid. The properties are the:

F - number of faces on the shape (the sides of the solid)
V - number of vertices on the shape (corners where edges meet)
E - number of edges on the shape (the lines that separate faces)

The Euler formula is: F + V - E = 2

To answer the questions look at each solid, the solids aren't numbered so start where you wish. Then:

- Write the name in the first column
- Count the faces and write the number in the next column
- Count the vertices and again, write it in the next column
- Count the edges and write in the next column
- Solve for F + V - E and it should equal 2.

This formula is a good check to see that you can accurately identify the properties of a shape as you know that if the answer isn't 2 you have to go back and find your mistake.

Euler's Formula

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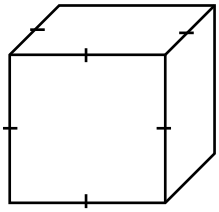
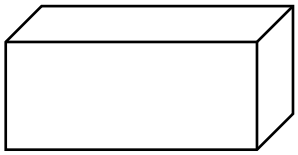
Apply the Euler formula to these 3-D solids : F + V - E = 2, by completing the table below.

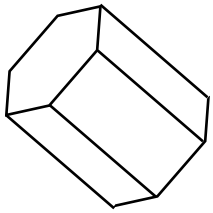
1 Cube



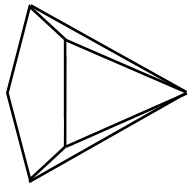
2 Rectangular Prism



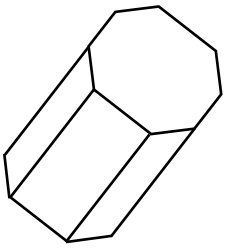
3 Hexagonal Prism



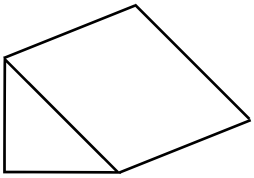
4 Pentagonal Pyramid



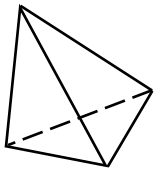
5 Octagonal Prism



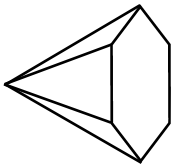
6 Triangular Prism



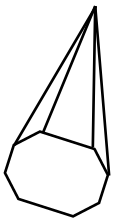
7 Triangular Pyramid



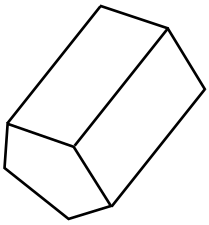
8 Hexagonal Pyramid



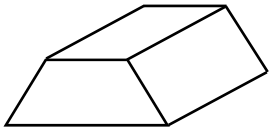
9 Octagonal Pyramid



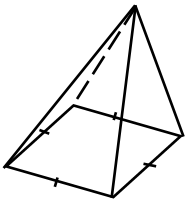
10 Pentagonal Prism



11 Trapezoidal Prism



12 Square Pyramid



	NAME OF SOLID	FACES (F)	VERTICES (V)	EDGES (E)	F + V - E =
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Euler's Formula

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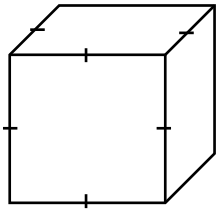
Apply the Euler formula to these 3-D solids : F + V - E = 2, by completing the table below.

1



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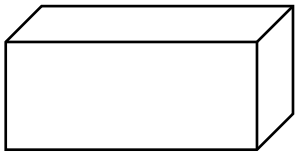

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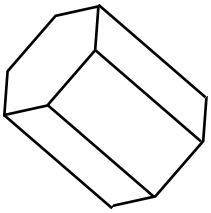

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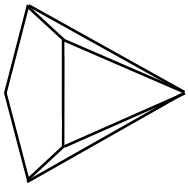

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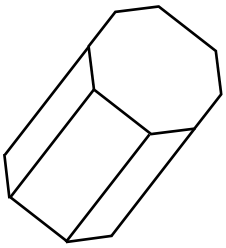

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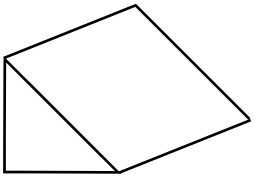

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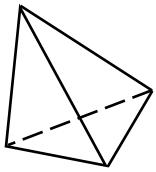

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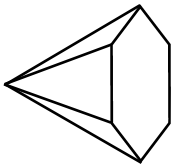

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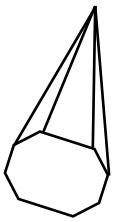

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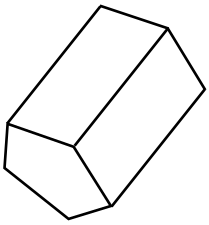

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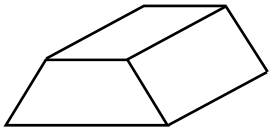

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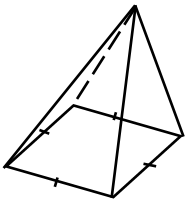

	NAME OF SOLID	FACES (F)	VERTICES (V)	EDGES (E)	F + V - E =
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					

Euler's Formula

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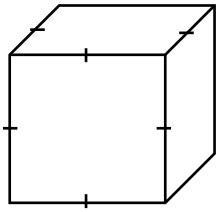
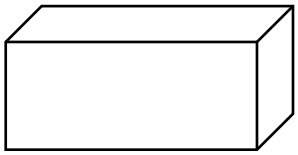
Apply the Euler formula to these 3-D solids : F + V - E = 2, by completing the table below.

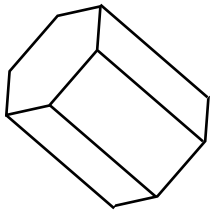
1 Cube



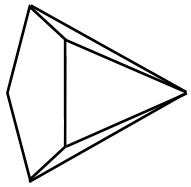
2 Rectangular Prism



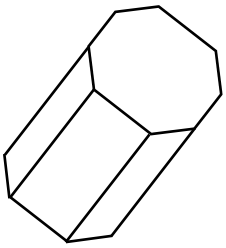
3 Hexagonal Prism



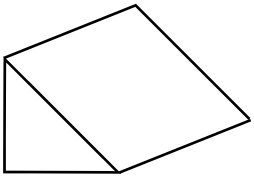
4 Pentagonal Pyramid



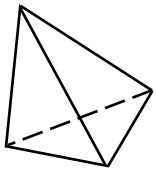
5 Octagonal Prism



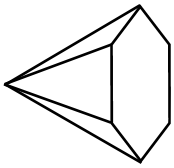
6 Triangular Prism



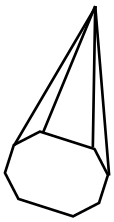
7 Triangular Pyramid



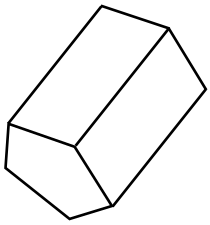
8 Hexagonal Pyramid



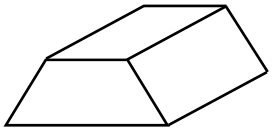
9 Octagonal Pyramid



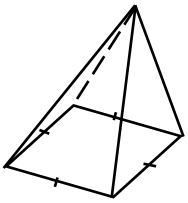
10 Pentagonal Prism



11 Trapezoidal Prism



12 Square Pyramid



	NAME OF SOLID	FACES (F)	VERTICES (V)	EDGES (E)	F + V - E =
1	Cube	6	8	12	2
2	Rectangular Prism	6	8	12	2
3	Hexagonal Prism	8	12	18	2
4	Pentagonal Pyramid	6	6	10	2
5	Octagonal Prism	10	16	24	2
6	Triangular Prism	5	6	9	2
7	Triangular Pyramid	4	4	6	2
8	Hexagonal Pyramid	7	7	12	2
9	Octagonal Pyramid	9	9	16	2
10	Pentagonal Prism	7	10	15	2
11	Trapezoidal Prism	6	8	12	2
12	Square Pyramid	5	5	8	2

Cross-sections of Shapes

A cross-sectional view of a solid is like you are slicing through the solid and then looking at the surface that you have just cut. Imagine slicing a loaf of bread (rectangular prism) and looking at the slice (a square?). The important rule is that if you slice a prism you get a uniform cross-section, that is, the cross section is identical wherever you choose to cut through the solid. Just like slices of bread. But what if the cross sections aren't the same. That means the shape isn't a prism. It doesn't automatically mean that it is a pyramid though, as you will find out.

Columns 1 and 2 start with giving three cross sections, these sections are all cut in the same direction, like sliced bread, not any direction through the solid. You are asked to write the name of the solid that could have these cross sections. Question 1 has two possible answers.

Important - don't confuse cross sections with front, end and top views of a solid, all the sections are views from the one direction.

Column 3 shows a cross section and asks you to give some examples of solids that would have that section, the last part of the column asks you to name objects that would have cross sections that are rectangular, circular or triangular.

Cross-sections of Shapes

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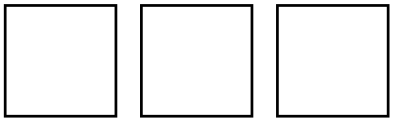
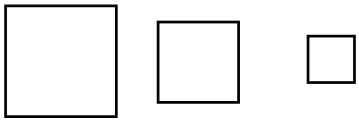
Name the 3-D shape(s) that would have these cross-sections.

1 1st section 2nd section 3rd section

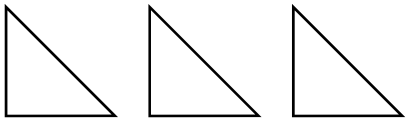


2 solids possible for these sections

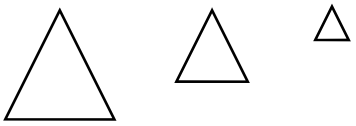
2 1st section 2nd section 3rd section



3 1st section 2nd section 3rd section



4 1st section 2nd section 3rd section



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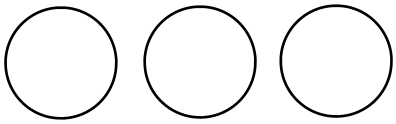

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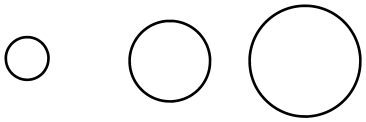

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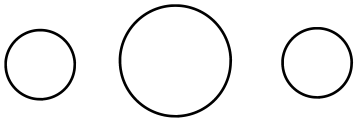

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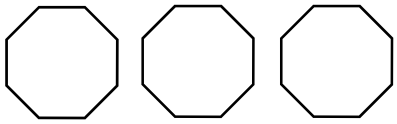

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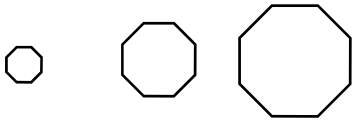

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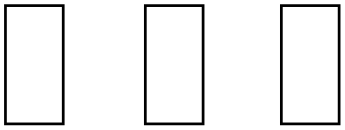

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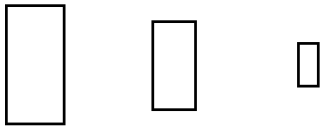

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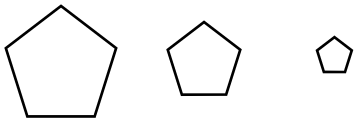
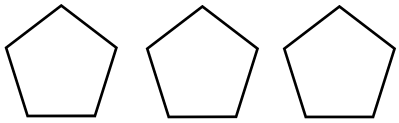
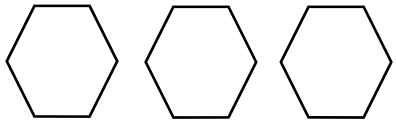


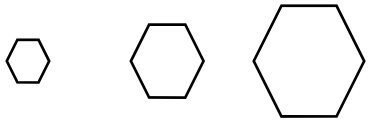
13 1st section 2nd section 3rd section



14 1st section 2nd section 3rd section

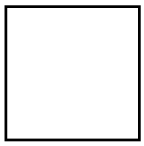


15 1st section 2nd section 3rd section



Name some solids that could have the cross sections shown below.

16



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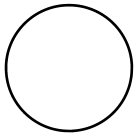

18



Name objects in your environment that when sliced have cross-sections of….

19 Circles

20 Rectangles

21 Triangles

Cross-sections of Shapes

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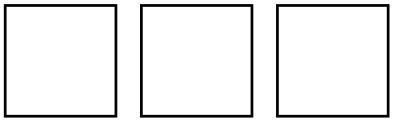
Name the 3-D shape(s) that would have these cross-sections.

1 1st section 2nd section 3rd section

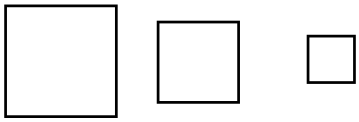


Cube

2 solids possible for these sections

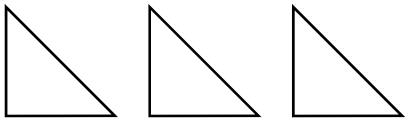
Rectangular prism

2 1st section 2nd section 3rd section



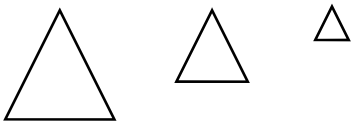
Square pyramid

3 1st section 2nd section 3rd section



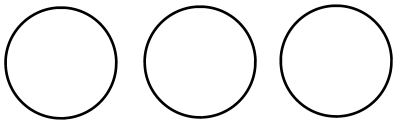
Triangular prism

4 1st section 2nd section 3rd section



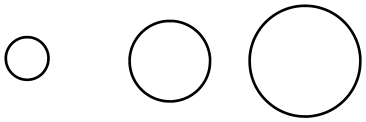
Triangular pyramid

5 1st section 2nd section 3rd section



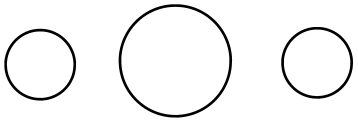
Cylinder

6 1st section 2nd section 3rd section



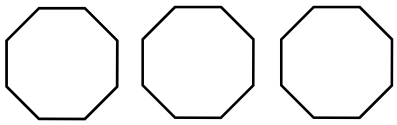
Cone

7 1st section 2nd section 3rd section



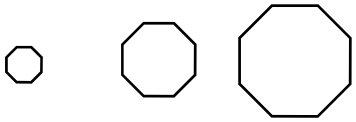
Sphere

8 1st section 2nd section 3rd section



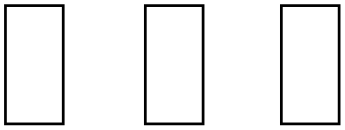
Octagonal prism

9 1st section 2nd section 3rd section



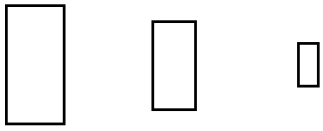
Octagonal pyramid

10 1st section 2nd section 3rd section



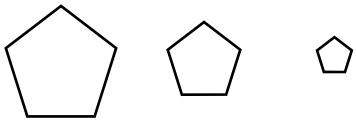
Rectangular prism

11 1st section 2nd section 3rd section



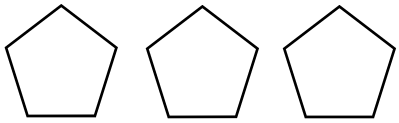
Rectangular pyramid

12 1st section 2nd section 3rd section



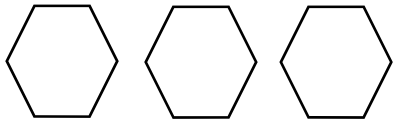
Pentagonal pyramid

13 1st section 2nd section 3rd section



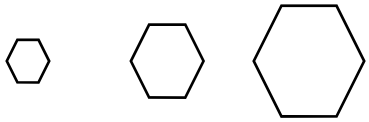
Pentagonal prism

14 1st section 2nd section 3rd section



Hexagonal prism

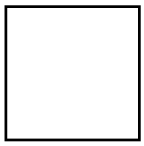
15 1st section 2nd section 3rd section



Hexagonal Pyramid

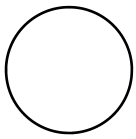
Name some solids that could have the cross sections shown below.

16



Cube, square pyramid, cylinder, rectangular prism, triangular prism (horizontal section)

17



Cone , cylinder, sphere, ellipsoid (vertical section in one direction)

18



Any prism could have this cross-section. Rectangular Pyramid.

Name objects in your environment that when sliced have cross-sections of….

19 Circles

Pipes, tree trunks, cans, sausages, tooth paste, drink in a glass

20 Rectangles

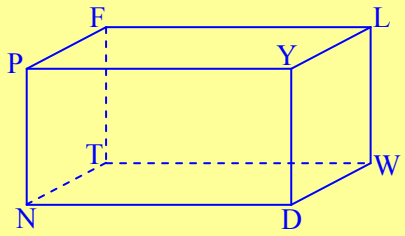
Bread, butter, block of cheese, any box unless it is a cube

21 Triangles

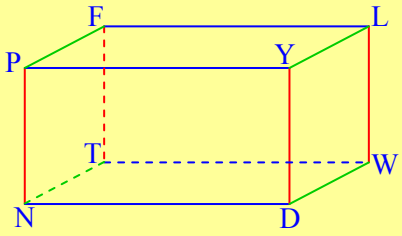
Toblerone, pyramids

Parallel, Perpendicular and Skew

Parallel means the same distance apart and not touching. This sheet deals with parallel faces and parallel edges. The rectangular prism shown below has three pairs of parallel faces.



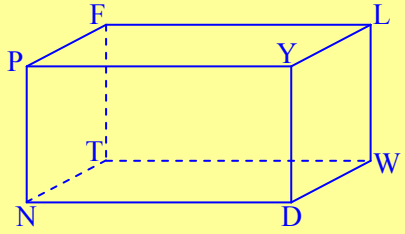
The top face PFLY is parallel to the bottom face NTWD.
The front face PYDN is parallel to the back face FLWT.
The LHS face PFTN is parallel to the RHS face YLWD.



There are parallel edges as well. On the diagram to the right, the edges that are the same colour are all parallel. You can show this using // :

PN // FT // LW // YD ND // PY // FL // TW PF // YL // DW // NT

Perpendicular means ‘at right angles to’ and touching. Again you can have perpendicular faces and also perpendicular edges. Faces first:

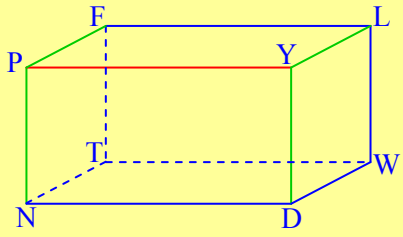


The top face PFLY and the bottom face NTWD are perpendicular to FLWT, YLWD, PYDN and PFTN.
The front face PYDN and the back face FLWT are perpendicular to PFLY, YLWD, WDNT and PFTN.
The side faces PFTN and YLWD are perpendicular to PFLY, FLWT, NTWD and PYDN.

Edges can also be parallel. With rectangular prisms and cubes there will be two perpendicular edges and the end of each edge.

So the edge PY (in red) has perpendicular edges (in green) YL, YD, PF and PN.

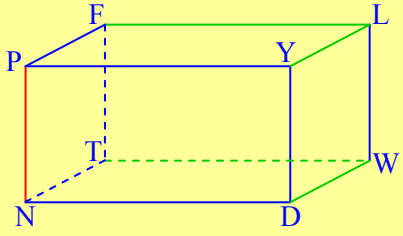
Not the perpendicular symbol is ⊥ .



Skew edges are edges that aren’t parallel and aren’t perpendicular. So if an edge is skew it won’t be parallel and it isn’t going to touch the given edge.

Name the skew edges to PN (shown in red).
The skew edges are shown in green and are FL, TW, YL and DW.

Note that none touch PN, and none are parallel to PN.



Parallel, Perpendicular and Skew

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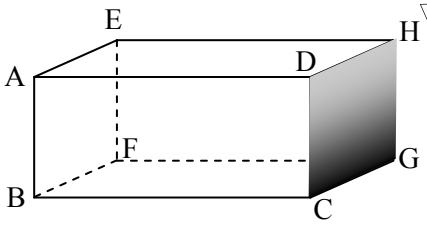
Name the face(s) that ….



1 Is shaded. _____

2 Is parallel to BFGC.

BFGC // _____

3 Is parallel to the shaded face.

_____ // _____

4 Is parallel to ABCD.

_____

5 Is parallel to AEFB.

_____

6 Is perpendicular to ABCD & has an edge EH.

_____ ⊥ _____

7 Are perpendicular to EHGF & have an edge BF, (2 faces).

_____ ⊥ _____ ⊥ _____

8 Are perpendicular to BFGC & perpendicular to AEFB.

[ ] , [ ]

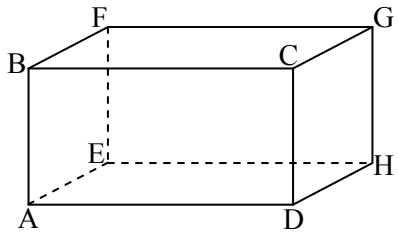
9 Are perpendicular to the shaded face and have edge GF.

10 Is perpendicular to ABCD & has vertices C & F.

11 Is perpendicular to DAEH & has edge BC.

_____ ⊥ _____

Name the edges that are parallel to, & perpendicular to, the edges on the rectangular prisms below.



12 3 edges parallel to AB are:

AB // [ ] // [ ] // [ ]

13 3 edges parallel to GC are:

GC // [ ] // [ ] // [ ]

14 3 edges parallel to EH are:

_____

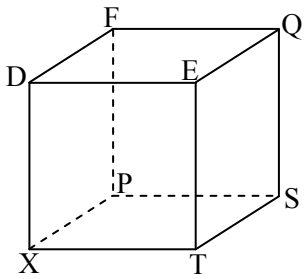
15 4 edges perpendicular to AB

[ ] , [ ] , [ ] , [ ]

16 4 edges perpendicular to GC

_____

Now use the cube shown below.



17 Name the face parallel to QFPS. _____

18 Name the edges parallel to FP. _____

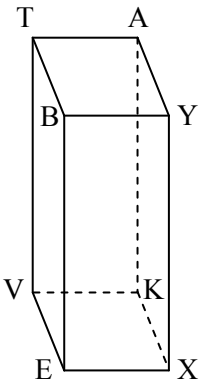
19 Name the horizontal faces perpendicular to DFPX.

_____

20 Name the vertical faces perpendicular to PFQS.

_____

What's skew with you? Name the skew edges in these questions.



21 Name 4 edges that are skew to YB: [ ], [ ], [ ], [ ].

22 Name 4 edges that are skew to VT: _____

23 Name 4 edges that are skew to KX: _____

24 Name two edges that are skew to EX and parallel to EB.

_____

25 Name two edges that are skew to AK and parallel to AT.

_____

26 Name an edge that is skew to VT and parallel to BY. _____

27 Name an edge that is skew to KV and parallel to YX. _____

28 Name an edge that is skew to EX and is on the face TBEV.

_____

29 Name two edges that are skew to AK and perpendicular to VT. _____

30 Name two edges that are skew to KV and perpendicular to EX. _____

31 Name two edges that are skew to XY and are on the face ATVK. _____

Parallel, Perpendicular and Skew

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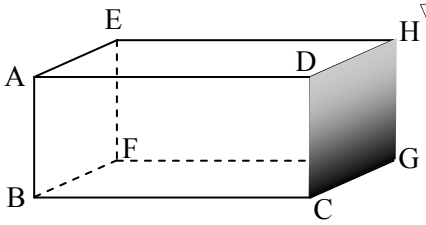
Name the face(s) that ….



1 Is shaded. GCDH

2 Is parallel to BFGC.

BFGC // AEHD

3 Is parallel to the shaded face.

GCDH // FBAE

4 Is parallel to ABCD.

ABCD // EFGH

5 Is parallel to AEFB.

AEFB // DHGC

6 Is perpendicular to ABCD & has an edge EH.

ABCD ⊥ AEHD

7 Are perpendicular to EHGF & have an edge BF, (2 faces).

EHGF ⊥ AEFB ⊥ BFGC

8 Are perpendicular to BFGC & perpendicular to AEFB.

ABCD, EHGF

9 Are perpendicular to the shaded face and have edge GF.

CDHG ⊥ CBFG ⊥ GFEH

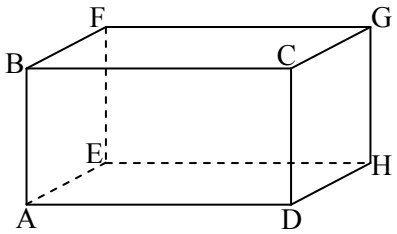
10 Is perpendicular to ABCD & has vertices C & F.

ABCD ⊥ CBFG

11 Is perpendicular to DAEH & has edge BC.

DAEH ⊥ ABCD

Name the edges that are parallel to, & perpendicular to, the edges on the rectangular prisms below.



12 3 edges parallel to AB are:

AB // EF // HG // DC

13 3 edges parallel to GC are:

GC // HD // EA // FB

14 3 edges parallel to EH are:

EH // AD // BC // FG

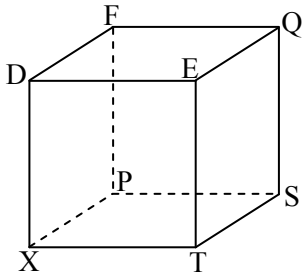
15 4 edges perpendicular to AB

AD , AE , BC , BF

16 4 edges perpendicular to GC

CB , CD , GH , GF

Now use the cube shown below.



17 Name the face parallel to QFPS. EDXT

18 Name the edges parallel to FP. FP // DX // ET // QS

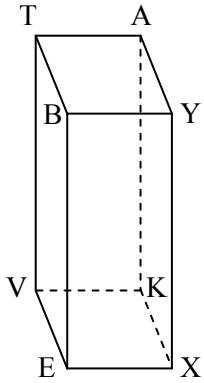
19 Name the horizontal faces perpendicular to DFPX.

DFQE, XPST

20 Name the vertical faces perpendicular to PFQS.

EQST, DFPX

What's skew with you? Name the skew edges in these questions.



21 Name 4 edges that are skew to YB: TV , AK , VE , KX .

22 Name 4 edges that are skew to VT: BY, EX, AY, KX.

23 Name 4 edges that are skew to KX: BY, TA, TV, BE.

24 Name two edges that are skew to EX and parallel to EB.

VT, KA.

25 Name two edges that are skew to AK and parallel to AT.

BY, EX.

26 Name an edge that is skew to VT and parallel to BY. EX

27 Name an edge that is skew to KV and parallel to YX. BE

28 Name an edge that is skew to EX and is on the face TBEV.

TB or TV

29 Name two edges that are skew to AK and perpendicular to VT. TB, VE.

30 Name two edges that are skew to KV and perpendicular to EX. BE, YX.

31 Name two edges that are skew to XY and are on the face ATVK. VK, TA.

Shapes - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Shapes Find-A-Word

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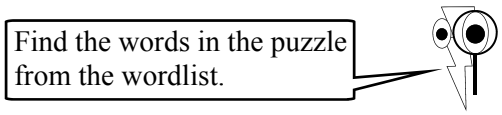

X	Q	S	Z	T	R	A	N	S	L	A	T	I	O	N
V	V	E	E	P	E	N	T	A	G	O	N	Z	M	R
E	T	C	Y	L	I	N	D	E	R	V	E	X	A	E
R	A	L	U	G	E	R	R	I	R	R	T	U	R	F
T	T	B	I	H	E	C	O	N	E	Q	S	E	G	L
E	E	R	D	T	T	O	S	V	M	K	H	P	O	E
X	M	L	I	H	A	C	Q	O	E	P	H	R	L	C
T	P	K	M	A	Z	T	D	W	S	Q	H	I	E	T
E	L	M	A	H	N	A	O	O	C	I	E	S	L	I
L	A	H	R	C	Q	G	P	R	A	E	X	M	L	O
C	T	O	Y	U	V	O	L	S	L	R	A	A	A	N
R	E	R	P	B	B	N	L	E	E	A	G	M	R	P
I	S	O	M	E	T	R	I	C	N	U	O	T	A	N
C	R	H	O	M	B	U	S	V	E	Q	N	X	P	E
V	N	O	I	T	A	M	R	O	F	S	N	A	R	T

WORDLIST

AXIS	NET	PYRAMID	SQUARE	REFLECTION
CUBE	CIRCLE	VERTEX	TRIANGLE	TRANSLATION
ISOMETRIC	CYLINDER	CONE	SPHERE	PRISM
OCTAGON	KITE	HEXAGON	TEMPLATE	IRREGULAR
SCALENE	ROTATE	PENTAGON	ISOSCELES	RHOMBUS
PARALLELOGRAM		TRANSFORMATION		SKEW

Shapes Find-A-Word

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Find the words in the puzzle from the wordlist.



X	Q	S	Z	T	R	A	N	S	L	A	T	I	O	N
V	V	E	E	P	E	N	T	A	G	O	N	Z	M	R
E	T	C	Y	L	I	N	D	E	R	V	E	X	A	E
R	A	L	U	G	E	R	R	I	R	R	T	U	R	F
T	T	B	E	H	E	C	O	N	E	Q	S	E	G	L
E	E	R	D	T	T	O	S	V	M	K	H	P	O	E
X	M	L	I	H	A	C	Q	O	E	P	H	R	L	C
T	P	K	M	A	Z	T	D	W	S	Q	H	I	E	T
E	L	M	A	H	N	A	O	O	C	I	E	S	L	I
L	A	H	R	C	Q	G	P	R	A	E	X	M	L	O
C	T	O	Y	U	V	O	L	S	L	R	A	A	A	N
R	E	R	P	B	B	N	L	E	E	A	G	M	R	P
I	S	O	M	E	T	R	I	C	N	U	O	T	A	N
C	R	H	O	M	B	U	S	V	E	Q	N	X	P	E
V	N	O	I	T	A	M	R	O	F	S	N	A	R	T

WORDLIST

~~AXIS~~ ~~NET~~ ~~PYRAMID~~ ~~SQUARE~~ ~~REFLECTION~~

~~CUBE~~ ~~CIRCLE~~ ~~VERTEX~~ ~~TRIANGLE~~ ~~TRANSLATION~~

~~ISOMETRIC~~ ~~CYLINDER~~ ~~CONE~~ ~~SPHERE~~ ~~PRISM~~

~~OCTAGON~~ ~~KITE~~ ~~HEXAGON~~ ~~TEMPLATE~~ ~~IRREGULAR~~

~~SCALENE~~ ~~ROTATE~~ ~~PENTAGON~~ ~~ISOSCELES~~ ~~RHOMBUS~~

~~PARALLELOGRAM~~ ~~TRANSFORMATION~~ ~~SKEW~~

FREEFALL MATHEMATICS

ALTITUDE

NUMBER THEORY

Odd and Even Numbers

Odd and even numbers separate each other and can be quickly identified, as:

- Odd numbers end in either a 1, 3, 5, 7 or 9
- Even numbers end in either a 0, 2, 4, 6, 8
- 0 is usually considered neither odd or even

Column 1 starts with 18 questions on classifying numbers as odd or even. The last digit of the number determines if it is odd or even, so ignore all the digits except the last one. The next four questions ask you to write the odd number before and after the number given. If the number is an even number then subtract 1 from it for the odd number before, add one to it for the odd number after. If the number is odd itself, subtract two from the number for the odd number before it, add two for the odd number after it. The last 4 questions involve even numbers before and after. Use the same method, if the number is odd subtract/add one to get the even numbers before and after and if it is even, subtract/add two.

Column 2 starts with listing all the odd numbers between two given numbers. One of the most asked questions from students is “does between include the numbers in the question?”, no it doesn’t. So look at the first number if it is even, the first odd number is one more than it, if it is odd, the first odd number is found by adding two to it. Then add two to get the next odd number and so on until the last odd number written is less than the larger number in the question. Repeat the same process for even numbers. Separate the numbers with a comma.

The last part of column 2 is circling the odd numbers and placing a square around the even numbers.

Column 3 starts with three questions on completing the pattern, if the numbers are even write in the next three even numbers, if they are odd put in the next three odd numbers. The rest of the column is devoted to showing properties of odd and even numbers. These are:

- Multiply (×) an even number with an even number you get an even number as the answer
- Multiply (×) an odd number with odd number you get an odd number as the answer
- Multiply (×) and even number with an odd number (or the reverse) you get an even number as the answer
- The sum (+) of an even number with an even number gives an even numbered answer
- The sum (+) of an odd number with an odd number gives an even numbered answer
- The sum (+) of an odd number with an even number (or the reverse) will give an odd number as the answer

When students are asked “What is an odd number times an even number" in a test and don't answer it they usually say "I forgot the rules", you don't have to remember them, just use 2 as your even number and 3 as your odd number and answer the question in your head. You get the answer 6 so its “even”.

Odd and Even Numbers

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State whether the numbers below are odd or even.

1 3 ____ 2 11 ____

3 8 ____ 4 17 ____

5 30 ____ 6 57 ____

7 43 ____ 8 102 ____

9 95 ____ 10 230 ____

11 333 ____ 12 507 ____

13 896 ____ 14 439 ____

15 771 ____ 16 603 ____

17 813 ____ 18 400 ____

Give the odd number before and after the number given below

19 234 Before ____ After ____

20 1 050 Before ____ After ____

21 499 Before ____ After ____

22 5 000 Before ____ After ____

Now give the even number immediately before and after these

23 711 Before ____ After ____

24 1 677 Before ____ After ____

25 3 894 Before ____ After ____

26 9 999 Before ____ After ____

Write all the odd numbers between these 2 numbers

27 768 and 776

28 809 and 816

29 495 and 506

30 444 and 455

31 291 and 301

Give the even numbers between these 2 numbers

32 611 and 622

33 838 and 852

34 992 and 1 003

35 9 895 and 9 903

Circle the odd numbers and put a square around the even numbers

36

		181	800	39
	198			
			776	70
555		459		
11	221		663	
		302		
152	107	308	930	

Complete the patterns below

37 76, 78, 80, ___, ___, ___

38 185, 187, 189, ___, ___, ___

39 293, 295, 297, ___, ___, ___

Carry out the operations then state if each answer is odd or even

Example

5 (odd) + 9 (odd) = 14 even

40 11 (odd) × 5 (odd) = ____ ____

41 33 (odd) + 17 (odd) = ____ ____

From the above you can say:

42 an odd + an odd = ____

43 an odd × an odd = ____

44 6 (even) × 12 (even) = ____ ____

45 66 (even) + 14 (even) = ____ ____

From the above you can say:

46 an even + an even = ____

47 an even × an even = ____

48 7 (odd) × 6 (even) = ____ ____

49 8 (even) × 5 (odd) = ____ ____

50 17 (odd) + 8 (even) = ____ ____

51 24 (even) + 13 (odd) = ____ ____

From the above you can say:

52 an odd × an even =
an even × an odd = ____

53 an odd + an even =
an even + an odd = ____

Odd and Even Numbers

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State whether the numbers below are odd or even.

1	3	odd	2	11	odd
3	8	even	4	17	odd
5	30	even	6	57	odd
7	43	odd	8	102	even
9	95	odd	10	230	even
11	333	odd	12	507	odd
13	896	even	14	439	odd
15	771	odd	16	603	odd
17	813	odd	18	400	even

Give the odd number before and after the number given below

		Before	After
19	234	233	235
20	1 050	1 049	1 051
21	499	497	501
22	5 000	4 999	5 001

Now give the even number immediately before and after these

		Before	After
23	711	710	712
24	1 677	1 676	1 678
25	3 894	3 892	3 896
26	9 999	9 998	10 000

Write all the odd numbers between these 2 numbers

27 768 and 776
769, 771, 773, 775

28 809 and 816
811, 813, 815

29 495 and 506
497, 499, 501, 503, 505

30 444 and 455
445, 447, 449, 451, 453

31 291 and 301
293, 295, 297, 299

Give the even numbers between these 2 numbers

32 611 and 622
612, 614, 616, 618, 620

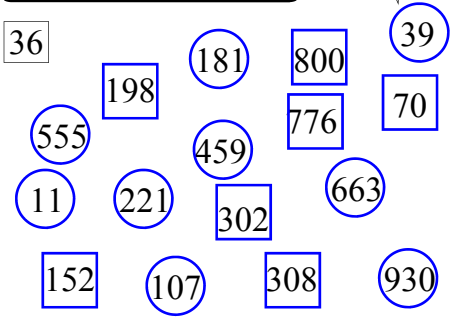
33 838 and 852
840, 842, 844, 846, 848, 850

34 992 and 1 003
994, 996, 998, 1 000, 1 002

35 9 895 and 9 903
9 896, 9 898, 9 900, 9 902

Circle the odd numbers and put a square around the even numbers

36



Complete the patterns below

37 76, 78, 80, 82 , 84 , 86

38 185, 187, 189, 191 , 193 , 195

39 293, 295, 297, 299 , 301 , 303

Carry out the operations then state if each answer is odd or even

Example

5 (odd) + 9 (odd) = 14 even

40 11 (odd) × 5 (odd) = 55 odd

41 33 (odd) + 17 (odd) = 40 even

From the above you can say:

42 an odd + an odd = even

43 an odd × an odd = odd

44 6 (even) × 12 (even) = 72 even

45 66 (even) + 14 (even) = 80 even

From the above you can say:

46 an even + an even = even

47 an even × an even = even

48 7 (odd) × 6 (even) = 42 even

49 8 (even) × 5 (odd) = 40 even

50 17 (odd) + 8 (even) = 25 odd

51 24 (even) + 13 (odd) = 37 odd

From the above you can say:

52 an odd × an even =
an even × an odd = even

53 an odd + an even =
an even + an odd = odd

Triangular Numbers, Square Numbers and Square Roots

Part of understanding number theory is understanding number patterns. Triangular numbers are the result of a pattern that can be easily represented graphically by dots which form triangles (look at the top of the first column). The first line is 1 dot, increase the next line by a dot so on the next line has 2 dots, the third line increases it by another dot and you get 3 dots, and so on, adding another dot to each new line. Square numbers could be seen as adding a dot to its width and adding a dot to its height. See half way down the column, before Q. 4.

In column 1 the first 3 questions deal with triangular numbers. You can work out the number mentally then 'do the dots' or the reverse, its your choice. Just add a row that is 1 greater than the previous row. With square numbers you can use a similar method, just add a dot to the width and a dot to the height to make the next largest square.

Column 2 involves exercises to make sure you understand how you get a square number. Some students forget and incorrectly think that when you square a number you multiply it by 2, like $4^2 = 4 \times 2 = 8$. It is important that you remember that squared means "times itself" ie. $4^2 = 4 \times 4 = 16$. You should know your 10 times table and so these should be just a mental exercise. Others know higher squares but many don't, so use the working spaces to calculate the next 10 square numbers.

Column 3 from Q. 28 involves square roots. A square root is the opposite to squaring a number. It is asking "what number multiplied by itself equals the number in the square root sign". You know that $2 \times 2 = 4$, that would mean that the $\sqrt{4}$ is 2. Or what number multiplied by itself equals 4? Well $2 \times 2 = 4$, so its 2. All the square roots asked for in these questions are within the first 20 square numbers. That means you have all the answers in the previous questions, just look at the previous answers and work backwards. Note that you can take square roots of non-square numbers, the answers will be in decimals however, a job for a calculator.

Triangular Numbers, Square Numbers and Square Roots

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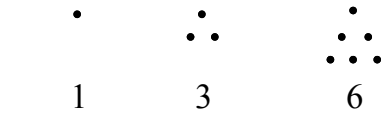
Find the next 3 triangular numbers and draw in the dots to represent them

Add a row, each row increases by 1 dot



1 3 6

1 ____ 4th number →

2 ____ 5th number →

3 ____ 6th number →

Find the next 4 square numbers and draw them with dots.



1 4 9

4 ____ 4th number →

5 ____ 5th number →

6 ____ 6th number →

7 ____ 7th number →

List the first 10 square numbers then calculate the next 10

8 $1^2 = 1 \times 1 =$ ____

9 $2^2 = 2 \times 2 =$ ____

10 $3^2 =$ __ × __ = ____

11 $4^2 =$ ____

12 $5^2 =$ ____

13 $6^2 =$ ____

14 $7^2 =$ ____

15 $8^2 =$ ____

16 $9^2 =$ ____

17 $10^2 =$ ____

18 $11^2 =$ ____

11 ×
11

0

19 $12^2 =$ ____

12 ×
12

20 $13^2 =$ ____

13 ×
13

21 $14^2 =$ ____

14 ×
14

22 $15^2 =$ ____

15 ×
15

23 $16^2 =$ ____

16 ×
16

24 $17^2 =$ ____

17 ×
17

25 $18^2 =$ ____

18 ×
18

26 $19^2 =$ ____

19 ×
19

27 $20^2 =$ ____

20 ×
20

Using your previous answers find the square root of these numbers

28 $\sqrt{81} =$ ____

29 $\sqrt{49} =$ ____

30 $\sqrt{100} =$ ____

31 $\sqrt{16} =$ ____

32 $\sqrt{121} =$ ____

33 $\sqrt{225} =$ ____

34 $\sqrt{400} =$ ____

35 $\sqrt{256} =$ ____

36 $\sqrt{324} =$ ____

37 $\sqrt{81} =$ ____

38 $\sqrt{361} =$ ____

39 $\sqrt{289} =$ ____

Triangular Numbers, Square Numbers and Square Roots

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Find the next 3 triangular numbers and draw in the dots to represent them

Add a row, each row increases by 1 dot

1 3 6

1 10 (4th number) →

2 15 (5th number) →

3 21 (6th number) →

Find the next 4 square numbers and draw them with dots.

1 4 9

4 16 (4th number) →

5 25 (5th number) →

6 36 (6th number) →

7 49 (7th number) →

List the first 10 square numbers then calculate the next 10

8 $1^2 = 1 \times 1 = 1$

9 $2^2 = 2 \times 2 = 4$

10 $3^2 = 3 \times 3 = 9$

11 $4^2 = 4 \times 4 = 16$

12 $5^2 = 5 \times 5 = 25$

13 $6^2 = 6 \times 6 = 36$

14 $7^2 = 7 \times 7 = 49$

15 $8^2 = 8 \times 8 = 64$

16 $9^2 = 9 \times 9 = 81$

17 $10^2 = 10 \times 10 = 100$

18 $11^2 = 121$

11 ×
11
11
110
121

19 $12^2 = 144$

12 ×
12
24
120
144

20 $13^2 = 169$

13 ×
13
39
130
169

21 $14^2 = 196$

14 ×
14
56
140
196

22 $15^2 = 225$

15 ×
15
1
75
150
225

23 $16^2 = 256$

16 ×
16
1
96
160
256

24 $17^2 = 289$

17 ×
17
119
170
289

25 $18^2 = 324$

18 ×
18
1
144
180
324

26 $19^2 = 361$

19 ×
19
1
171
190
361

27 $20^2 = 400$

20 ×
20
400

Using your previous answers find the square root of these numbers

28 $\sqrt{81} = 9$

29 $\sqrt{49} = 7$

30 $\sqrt{100} = 10$

31 $\sqrt{16} = 4$

32 $\sqrt{121} = 11$

33 $\sqrt{225} = 15$

34 $\sqrt{400} = 20$

35 $\sqrt{256} = 16$

36 $\sqrt{324} = 18$

37 $\sqrt{81} = 9$

38 $\sqrt{361} = 19$

39 $\sqrt{289} = 17$

Palindromic, Fibonacci Numbers and Number Patterns

Palindromic numbers can be read the same way forwards or backwards. For example some word palindromes are: rotor, radar, mum and dad. The same applies with numbers, examples are: 121, 54 145, 11, 444 and 7 227.

Column 1 asks you to circle the palindromic numbers. To check them look at the first number and the last number, if they are different the number isn't palindromic. If they are the same look at the second number and the second last number and compare. Do this until you reach the middle number(s). If you have matching pairs of numbers all the way through, (if there is a middle number alone it is as if it has a matching pair) then the number is palindromic or a palindrome.

Questions 2 through 17 have numbers missing that you need to fill in to make the numbers palindromes. Compare the position of the box with the same position from the other end, if it's the first number compare it with the last number, if it's the second number compare it with the second last position or the reverse. The last question asks you to find the next 11 palindromic years after 2002, the first one being given to you (2112).

Column 2 introduces Fibonacci numbers. This is an unusual number pattern because it has the number 1 twice! The numbers are 0, 1, 1, 2, 3, 5, 8, 13, … each number is found by adding the previous two numbers, this is shown at the top of the second column. So 0 and the first 1 don't fit this rule (they don't have 2 numbers in front of them) but the second 1 is part of the method and from then on the rule applies. (0 + 1 = 1), (1 + 1 = 2) etc. This column asks you to find the next 10 numbers. From question 32 working spaces are supplied to help with the harder addition.

Column 3 is about number patterns. The first section deals with addition or subtraction. Look at the first two numbers. If they are increasing in size its an addition. To solve, subtract the first number from the second and that's the amount that you need to add to get the next number. If the numbers are reducing in size you know it's a subtraction, subtract the second number from the first number to find the amount to subtract.

The next group is multiplication and division, if they increase its multiplication, if they decrease its division. Find the number you multiply the first number by to get the second. Then, apply this to find the missing number. Division will have the numbers decreasing in size and so find the number that the first number was divided by to get the second number. Note that if the numbers are too large or difficult use the 2nd and 3rd numbers.

The last section is more difficult. This time you apply two operations (× or ÷ with + or -). Instead of just using the first 2 numbers make sure that the solution applies to all the given numbers. So investigate further into each question when you have a mixture of different types of patterns.

Palindromic, Fibonacci and Number Patterns

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Circle (or oval?) all the palindromic numbers

1

1 991 112 233 88 11 16 016 6 565 45 545 644 2 002 100 333 1 212 121

Find the missing number that would make these palindromes

2 5 9 ☐ 5

3 ☐ 7

4 ☐ 114

5 ☐ 33

6 ☐ 1

7 21 ☐

8 225 ☐ 22

9 6 ☐ 106

10 1 0 ☐ 1

11 902 ☐ 09

12 ☐ 43 345

13 6 ☐ 86

Now there's two missing

14 2 3 ☐ ☐

15 ☐ 9 3 ☐ 1

16 ☐ 8 ☐ 7

17 ☐ 34 ☐ 33

Give the dates of the next 10 palindromic years that follow 2112.

2002

2112

18 ____

19 ____

20 ____

21 ____

22 ____

23 ____

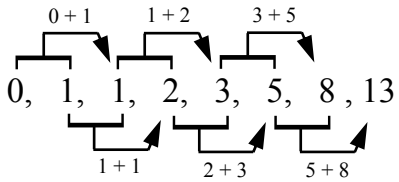
24 ____

25 ____

26 ____

27 ____

Find the next 10 numbers in the Fibonacci pattern



28 8 + 13 = ___

29 13 + ___ = ___

30 _____ = ___

31 _____ = ___

32 _____ + _____ = ___

33 _____ + _____ = ___

34 _____ + _____ = ___

35 _____ + _____ = ___

36 _____ + _____ = ___

37 _____ + _____ = ___

Use addition or subtraction to find the next 3 numbers in these number patterns

38 1, 3, 5, ___, ___, ___

39 2, 4, 6, ___, ___, ___

40 1, 5, 9, 13, ___, ___, ___

41 15, 12, 9, ___, ___, ___

42 50, 75, 100 ___, ___, ___

43 36, 29, 22, ___, ___, ___

44 93, 82, 71, ___, ___, ___

45 0, 80, 160, ___, ___, ___

Multiply or divide to find the next 3 numbers in these number patterns

Example

×3 ×3 ×3 ×3 ×3

1, 3, 9, 27, 81, 243

46 2, 4, 8, ___, ___, ___

47 800, 400, 200, ___, ___, ___

48 15, 30, 60, ___, ___, ___

49 1 024, 256, 64, ___, ___, ___

50 ¼, 1, 4, ___, ___, ___

These use a combination and are more difficult.

51 2, 5, 11, 23, ___, ___, ___

52 4, 7, 13, 25, ___, ___, ___

53 10, 15, 25, 45, ___, ___, ___

54 5, 20, 50, 110, ___, ___

Palindromic, Fibonacci and Number Patterns

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Circle (or oval?) all the palindromic numbers

1

1 991 112 233

88 11 6 565

16 016

644 2 002

45 545

100 333

1 212 121

Find the missing number that would make these palindromes

2 5 9 $\boxed{9}$ 5

3 $\boxed{7}$ 7

4 $\boxed{4}$ 114

5 $\boxed{3}$ 33

6 $\boxed{1}$ 1

7 21 $\boxed{2}$

8 225 $\boxed{5}$ 22

9 6 $\boxed{0}$ 106

10 1 0 $\boxed{0}$ 1

11 902 $\boxed{2}$ 09

12 $\boxed{5}$ 43 345

13 6 $\boxed{8}$ 86

Now there's two missing

14 2 3 $\boxed{3}$ $\boxed{2}$

15 $\boxed{1}$ 9 3 $\boxed{9}$ 1

16 $\boxed{7}$ 8 $\boxed{8}$ 7

17 $\boxed{3}$ 34 $\boxed{4}$ 33

Give the dates of the next 10 palindromic years that follow 2112.

	2002	22	2662
	2112	23	2772
18	2222	24	2882
19	2332	25	2992
20	2442	26	3003
21	2552	27	3113

Find the next 10 numbers in the Fibonacci pattern

0 + 1 1 + 2 3 + 5

0, 1, 1, 2, 3, 5, 8 , 13

1 + 1 2 + 3 5 + 8

28 8 + 13 = 21

29 13 + 21 = 34

30 21 + 34 = 55

31 34 + 55 = 89

32 55 + 89 = 144

$$\begin{array}{r} {}^{1} \\ 89 \\ +\ 55 \\ \hline 144 \end{array}$$

33 89 + 144 = 233

$$\begin{array}{r} {}^{1\,1} \\ 144 \\ +\ 89 \\ \hline 233 \end{array}$$

34 144 + 233 = 377

$$\begin{array}{r} 233 \\ +\ 144 \\ \hline 377 \end{array}$$

35 233 + 377 = 610

$$\begin{array}{r} {}^{1\,1} \\ 377 \\ +\ 233 \\ \hline 610 \end{array}$$

36 377 + 610 = 987

$$\begin{array}{r} 610 \\ +\ 377 \\ \hline 987 \end{array}$$

37 610 + 987 = 1 597

$$\begin{array}{r} 987 \\ +\ 610 \\ \hline 1\,597 \end{array}$$

Use addition or subtraction to find the next 3 numbers in these number patterns

38 1, 3, 5, 7 , 9 , 11

39 2, 4, 6, 8 , 10 , 12

40 1, 5, 9, 13, 17 , 21 , 25

41 15, 12, 9, 6 , 3 , 0

42 50, 75, 100 125 , 150 , 175

43 36, 29, 22, 15 , 8 , 1

44 93, 82, 71, 60 , 49 , 38

45 0, 80, 160, 240 , 320 , 400

Multiply or divide to find the next 3 numbers in these number patterns

Example

×3 ×3 ×3 ×3 ×3

1, 3, 9, 27 , 81 , 243

46 2, 4, 8, 16 , 32 , 64

47 800, 400, 200, 100 , 50 , 25

48 15, 30, 60, 120 , 240 , 480

49 1 024, 256, 64, 16 , 4 , 1

50 ¼, 1, 4, 16 , 64 , 256

These use a combination and are more difficult.

51 2, 5, 11, 23, 47 , 95 , 191

52 4, 7, 13, 25, 49 , 97 , 193

53 10, 15, 25, 45, 85 , 165 , 325

54 5, 20, 50, 110, 230 , 470

Factors - Finding the HCF

Factors are numbers that divide into another number without a remainder. All numbers have factors, the number itself and 1. If a number has only these two factors then it is classified as being prime. If additional factors are found then the number is called composite.

Column 1 asks you to list the factors of the numbers. Follow this method:

- Write 1 straight away then see if 2 is a factor and test each number 3, 4, 5, etc
- Once you get to half of the number you can stop and write the number itself, there are no factors greater than half the number except itself
- Perform a check, the first number × the last number = the number, the second number × the second last number = the number and work your way into the middle. The middle two numbers when multiplied = the number. Note that if the number is a square number then the middle number is the square root.

A common mistake is students finding the smaller factors and missing the larger factors. If you perform the check you won't miss them. When you list the numbers separate them with a comma.

Column 2 asks you to find the HCF (Highest Common Factor). When given 2 numbers (or more) the largest factor that is a factor of both numbers is the HCF. The process is the same as column 1 only you perform it twice, for the 2 numbers. The bottom of the column asks you to state whether the numbers are prime or composite. Note that 0 and 1 are usually considered neither prime or composite and 2 is the only even prime number.

Column 3 is an introduction to factor trees. Factor trees are used to break a number down into its prime factors. The first nine questions give a number at the top of the 'tree', then another number is given below it, divide the top number by the lower number and write the answer in the box. Or ask yourself what number times the lower number equals the top number? The next four questions use the same process only it is extended a further branch. Find the missing number as with the earlier questions, then break these numbers down to the two (prime) factors that multiply together to equal it. If the number is prime it doesn't have any factors so a single branch and circle are below it, just rewrite the same number in the circle.

To list the prime factors write the numbers that are in the bottom row of the tree, you only write each number once. That means that if there are, for example, three 2's and a five you only write 2 and 5, there isn't space for you to write all the numbers.

Factors - Finding the HCF

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List the factors for the given numbers

Example

8 1, 2, 4, 8

1 6 _____

2 10 _____

3 12 _____

4 20 _____

5 40 _____

6 24 _____

7 14 _____

8 44 _____

9 48 _____

_____

10 56 _____

11 100 _____

_____

12 60 _____

_____

13 80 _____

_____

14 120 _____

_____

15 150 _____

_____

16 27 _____

List the factors for each number then circle the HCF of the numbers

17 12 _____

42 _____

The HCF is _____

18 24 _____

56 _____

The HCF is _____

19 45 _____

18 _____

The HCF is _____

20 28 _____

36 _____

The HCF is _____

21 16 _____

42 _____

The HCF is _____

22 24 _____

12 _____

9 _____

The HCF is _____

Are the numbers below prime or composite? Circle the correct answer

23 29 Prime Composite

24 35 Prime Composite

25 33 Prime Composite

26 47 Prime Composite

27 87 Prime Composite

Find the missing factor and write it in the empty circle.

28 (24) (6) 29 (18) (2) 30 (15) (3)

31 (30) (10) 32 (56) (8) 33 (60) (4)

34 (36) (6) 35 (12) (4) 36 (80) (5)

Use the factor trees to find the prime factors of the following numbers

37 30 (5)

Prime factors: _ _ _

38 18 (9)

Prime factors: _ _

39 100 (10)

Prime factors: _ _

40 24 (6)

Prime factors: _ _

Factors - Finding the HCF

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List the factors for the given numbers

Example

8 1, 2, 4, 8

1 6 1, 2, 3, 6

2 10 1, 2, 5, 10

3 12 1, 2, 3, 4, 6, 12

4 20 1, 2, 4, 5, 10, 20

5 40 1, 2, 4, 5, 8, 10, 20, 40

6 24 1, 2, 3, 4, 6, 8, 12, 24

7 14 1, 2, 7, 14

8 44 1, 2, 4, 11, 22, 44

9 48 1, 2 , 3, 4, 6, 8, 12, 16, 24, 48

10 56 1, 2, 4, 7, 8, 14, 28, 56

11 100 1, 2, 4, 5, 10, 20, 25, 50, 100

12 60 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

13 80 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

14 120 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120

15 150 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150

16 27 1, 3, 9, 27

List the factors for each number then circle the HCF of the numbers

17 12 1, 2, 3, 4, (6), 12

42 1, 2, 3, (6), 7, 14, 21, 42

The HCF is 6

18 24 1, 2, 3, 4, 6, (8), 12, 24

56 1, 2, 4, 7, (8), 14, 28, 56

The HCF is 8

19 45 1, 3, 5, (9), 15, 45

18 1, 2, 3, 6, (9), 18

The HCF is 9

20 28 1, 2, (4), 7, 14, 28

36 1, 2, 3, (4), 6, 9, 12, 18, 36

The HCF is 4

21 16 1, (2), 4, 8, 16

42 1, (2), 3, 6, 7, 14, 21, 42

The HCF is 2

22 24 1, 2, (3), 4, 6, 8, 12, 24

12 1, 2, (3), 4, 6, 12

9 1, (3), 9

The HCF is 3

Are the numbers below prime or composite? Circle the correct answer

23 29 (Prime) ~~Composite~~

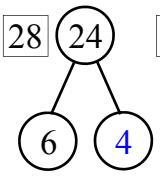
24 35 ~~Prime~~ (Composite)

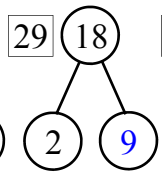
25 33 ~~Prime~~ (Composite)

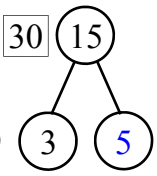
26 47 (Prime) ~~Composite~~

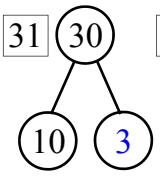
27 87 ~~Prime~~ (Composite)

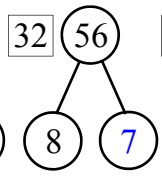
Find the missing factor and write it in the empty circle.

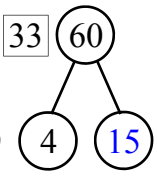
28 

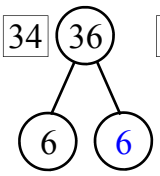
29 

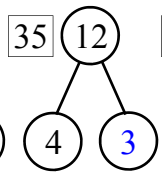
30 

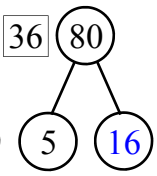
31 

32 

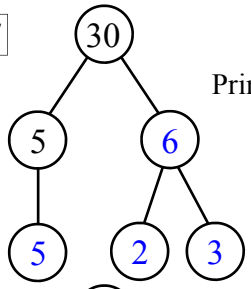
33 

34 

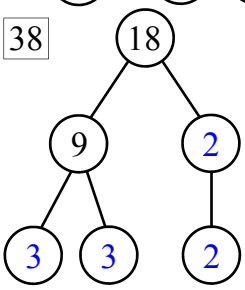
35 

36 

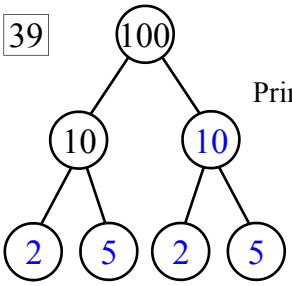
Use the factor trees to find the prime factors of the following numbers

37 

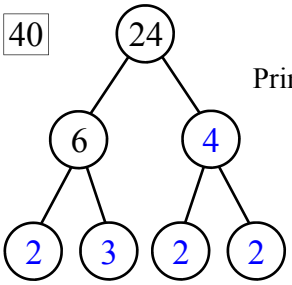
Prime factors: 5 2 3

38 

Prime factors: 3 2

39 

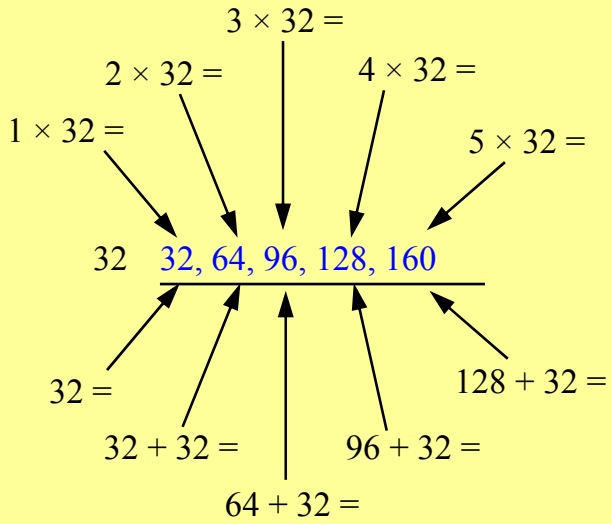
Prime factors: 2 5

40 

Prime factors: 2 3

Multiples - Finding the LCM

When you think of multiples think of being given a number and multiplying it by 1, 2, 3,.etc. So the first multiple of a number is itself, as 1 × a number doesn't change it. But while it is multiplication it is often easier to add. Look at the example below finding the multiples of 32.



By Multiplying

By Adding

Column 1 deals with finding the first five multiples of a number. Write the number then add the number to itself and find the total, then add the number again and find the new total and so on. Or multiply by 1, 2, 3, 4 and 5, the addition is usually easier.

Column 2 asks for multiples of a number between 25 and 45. Remember between doesn't include the numbers, if a multiple is 25 or 45 they aren't between. You should be able to complete the additions mentally. Eg. the number 7...would start with 7 then 14 then 21 then 28 ….that's above 25 so that's my first number, write in 28 then add 7's….28 + 7 = 35, write 35….35 + 7 = 42, write 42….42 + 7 = 49 that's above 45 so we don't include it, so you're done. The next 5 questions ask for the next 3 multiples above 100. The method would be to count mentally in your head the numbers given until they pass 100, then write the next 3 multiples. Rather than counting from the start you should be able to multiply the numbers by a number of your choice to get it close to 100. For example if the number was 12...I know that 7 × 12 = 84….. so start from there 84 + 12 = 96…. so 96 + 12 = 108 that is my first number. Then away you go!

What is the LCM? LCM (Lowest Common Multiple) is the smallest multiple that is shared by two or more numbers. Column 2, Q. 28 through to the end of the sheet all use the same method. You are given 2 (or 3) numbers, list the multiples until you think you have the LCM covered. When you list the multiples for the second number stop once you spot the match. Then circle the matching LCM's and write the answer. An example is below.

5 5, 10, 15, (20) 25, 30 ← List the multiples of 5

4 4, 8, 12, 16, (20) ← List the multiples of 4, stop when you get a match (20). Circle the 2 numbers, then write your answer on the line.

The LCM is 20

Multiples - Finding the LCM

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List the first 5 multiples of the numbers below

Example

4 4, 8, 12, 16, 20

1 10 _____

2 3 _____

3 5 _____

4 7 _____

5 12 _____

6 9 _____

7 15 _____

8 20 _____

9 11 _____

10 25 _____

11 8 _____

12 100 _____

_____

13 150 _____

_____

14 135 _____

_____

15 160 _____

_____

16 202 _____

_____

17 250 _____

_____

List the multiples of the numbers below that are between 25 and 45

18 3 _____

19 5 _____

20 4 _____

21 6 _____

22 12 _____

List the next 3 multiples of the given numbers that are greater than 100

23 5 _____

24 10 _____

25 8 _____

26 9 _____

27 22 _____

List the multiples for each number then circle the LCM of each.

28 4 _____

10 _____

The LCM is _____

29 8 _____

6 _____

The LCM is _____

30 11 _____

33 _____

The LCM is _____

31 20 _____

15 _____

The LCM is _____

32 9 _____

15 _____

The LCM is _____

33 35 _____

21 _____

The LCM is _____

34 12 _____

20 _____

The LCM is _____

35 14 _____

8 _____

The LCM is _____

36 12 _____

15 _____

The LCM is _____

37 24 _____

18 _____

The LCM is _____

38 30 _____

15 _____

45 _____

The LCM is _____

39 9 _____

6 _____

12 _____

The LCM is _____

Multiples - Finding the LCM

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List the first 5 multiples of the numbers below

Example

4 4, 8, 12, 16, 20

1 10 10, 20, 30, 40, 50

2 3 3, 6, 9, 12, 15

3 5 5, 10, 15, 20, 25

4 7 7, 14, 21, 28, 35

5 12 12, 24, 36, 48, 60

6 9 9, 18, 27, 36, 45

7 15 15, 30, 45, 60, 75

8 20 20, 40, 60, 80, 100

9 11 11, 22, 33, 44, 55

10 25 25, 50, 75, 100, 125

11 8 8, 16, 24, 32, 40

12 100 100, 200, 300, 400, 500

13 150 150, 300, 450, 600, 750

14 135 135, 270, 405, 540, 675

15 160 160, 320, 480, 640, 800

16 202 202, 404, 606, 808, 1 010

17 250 250, 500, 750, 1 000, 1 250

List the multiples of the numbers below that are between 25 and 45

18 3 27, 30, 33, 36, 39, 42

19 5 30, 35, 40

20 4 28, 32, 36, 40, 44

21 6 30, 36, 42

22 12 36

List the next 3 multiples of the given numbers that are greater than 100

23 5 105, 110, 115

24 10 110, 120, 130

25 8 104, 112, 120

26 9 108, 117, 126

27 22 110, 132, 154

List the multiples for each number then circle the LCM of each.

28 4 4, 8, 12, 16, (20) 24

10 (10), 20, 30, 40

The LCM is 20

29 8 8, 16, (24) 32, 40

6 6, 12, 18, (24) 30

The LCM is 24

30 11 11, 22, (33) 44

33 (33) 66, 99

The LCM is 33

31 20 20, 40, (60) 80, 100

15 15, 30, 45, (60) 75

The LCM is 60

32 9 9, 18, 27, 36, (45)

15 15, 30, (45) 60, 75

The LCM is 45

33 35 35, 70, (105) 140, 175

21 21, 42, 63, 84, (105)

The LCM is 105

34 12 12, 24, 36, 48, (60)

20 20, 40, (60) 80, 100

The LCM is 60

35 14 14, 28, 42, (56) 70

8 8, 16, 24, 32, 40, 48, (56)

The LCM is 56

36 12 12, 24, 36, 48, (60) 72

15 15, 30, 45, (60) 75

The LCM is 60

37 24 24, 48, (72) 96, 120

18 18, 36, 54, (72) 90

The LCM is 72

38 30 30, 60, (90) 120

15 15, 30, 45, 60, 75, (90)

45 45, (90) 135

The LCM is 90

39 9 9, 18, 27, (36) 45, 54

6 6, 12, 18, 24, 30, (36)

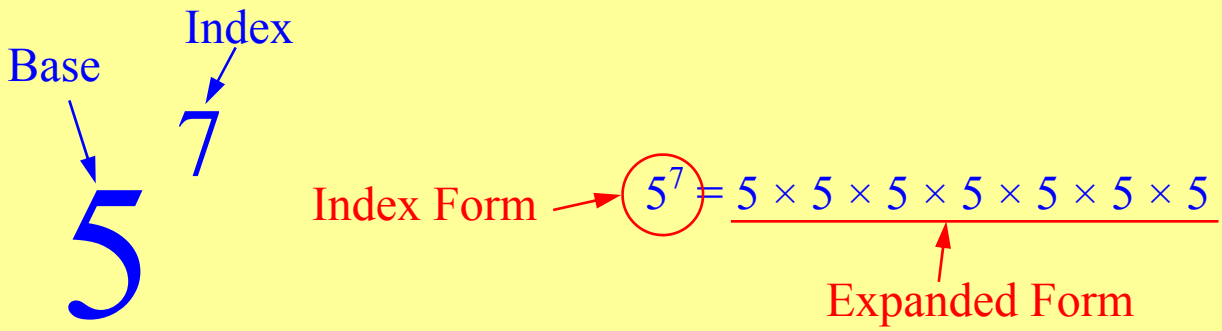
12 12, 24, (36)

The LCM is 36

Indexed Numbers

This sheet has some questions written sideways. To read these questions click on the "View" menu, select the rotate option then "counter clockwise".

Indexed numbers have a base and an index. The index tells you how many times the base is multiplied by itself. So 5^2 is $= 5 \times 5$ and $5^4 = 5 \times 5 \times 5 \times 5$. When a number is shown with an index it is in 'index form', when it is shown with the '×' between the numbers it is said to be in 'expanded' form (that means pulled apart).



Column 1 involves changing numbers from expanded form to index form. Look at the base and write this normal size, then count how many of the numbers there are and write that number next to the base number, as a smaller raised number.

Column 2 is an extension of this, with finding the expansion and then solving for the answer. An example is at the top of the column. Strokes can be used to total along the line (as in the example) or answer it mentally. The next section of column 2 deals with indices with a base of 10. With these you don't need to expand, the answer is just 1 with the index number of zeros after it. For example 10^5 would be a 1 with 5 zeros or 100 000. That means $10^1 = 10$ (a 1 with 1 zero after it). Numbers are then introduced, for example 6×10^5 would be a 6 with 5 zeros or 600 000, and $9 \times 10^1 = 90$.

Column 3 from Q. 43 is the reverse process, look at the number and write it as the first digit in the answer then a '× 10', then the power. Count the number of zeros after the first digit and that's your power!

Indexed Numbers

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Write these numbers in index form don't multiply to get the answer

Example

$8 \times 8 \times 8 \times 8 \times 8 = 8^5$ (1 2 3 4 5)

1 $6 \times 6 \times 6$ = _____

2 $4 \times 4 \times 4 \times 4 \times 4$ = _____

3 5×5 = _____

4 $7 \times 7 \times 7 \times 7$ = _____

5 $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$ = _____

6 $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$ = _____

7 $9 \times 9 \times 9 \times 9 \times 9$ = _____

8 $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$ = _____

Now the reverse, expand these !

16 10^7 =

15 1^{11} =

14 19^6 =

13 8^8 =

12 2^{13} =

11 123^7 =

10 10^9 =

9 5^{12} =

Calculate the numbers that these indexed numbers represent, you may wish to expand them first.

Example

$2^5 = \not{2} \times \not{2} \times \not{2} \times \not{2} \times 2 = 32$ (4 8 16)

17 3^3 = _____ = _____

18 10^3 = _____ = _____

19 8^2 = _____ = _____

20 1^4 = _____ = _____

21 4^3 = _____ = _____

22 5^3 = _____ = _____

23 2^4 = _____ = _____

24 2^6 = _____ = _____

Write these indexed base 10 numbers as numbers without working out

25 10^3 = _____

26 10^2 = _____

27 10^6 = _____

28 10^7 = _____

29 1×10^4 = _____

30 2×10^4 = _____

31 6×10^4 = _____

32 2×10^2 = _____

33 9×10^2 = _____

34 7×10^3 = _____

35 5×10^2 = _____

36 8×10^5 = _____

37 9×10^6 = _____

38 5×10^1 = _____

39 2×10^7 = _____

40 4×10^5 = _____

41 2×10^1 = _____

42 7×10^2 = _____

Write these as a single digit number × a base 10 indexed number

Example

$60\,000 = 6 \times 10^4$

43 400 = _____

44 9 000 = _____

45 20 000 = _____

46 4 000 000 = _____

47 700 000 = _____

48 3 000 = _____

49 90 000 000 = _____

50 70 = _____

51 6 000 = _____

52 500 000 = _____

53 200 = _____

54 8 000 000 = _____

55 30 = _____

Indexed Numbers

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Write these numbers in index form don't multiply to get the answer

Example

$8 \times 8 \times 8 \times 8 \times 8 = 8^5$ (1 2 3 4 5)

1 $6 \times 6 \times 6 = 6^3$

2 $4 \times 4 \times 4 \times 4 \times 4 = 4^5$

3 $5 \times 5 = 5^2$

4 $7 \times 7 \times 7 \times 7 = 7^4$

5 $10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^7$

6 $1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 = 1^9$

7 $9 \times 9 \times 9 \times 9 \times 9 = 9^5$

8 $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^8$

Now the reverse, expand these !

9 $5^{12} = 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

10 $10^9 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

11 $123^7 = 123 \times 123 \times 123 \times 123 \times 123 \times 123 \times 123$

12 $2^{13} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

13 $8^8 = 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$

14 $19^6 = 19 \times 19 \times 19 \times 19 \times 19 \times 19$

15 $1^{11} = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

16 $10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$

Calculate the numbers that these indexed numbers represent, you may wish to expand them first.

Example

$2^5 = \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 = 32$ (4 8 16)

17 $3^3 = \cancel{3} \times \cancel{3} \times 3 = 27$ (9)

18 $10^3 = \cancel{10} \times \cancel{10} \times 10 = 1\,000$ (100)

19 $8^2 = 8 \times 8 = 64$

20 $1^4 = 1 \times 1 \times 1 \times 1 = 1$

21 $4^3 = \cancel{4} \times \cancel{4} \times 4 = 64$ (16)

22 $5^3 = \cancel{5} \times \cancel{5} \times 5 = 125$ (25)

23 $2^4 = \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 = 16$ (4 8)

24 $2^6 = \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times 2 = 64$ (4 8 16 32)

Write these indexed base 10 numbers as numbers without working out

25 $10^3 = 1\,000$

26 $10^2 = 100$

27 $10^6 = 1\,000\,000$

28 $10^7 = 10\,000\,000$

29 $1 \times 10^4 = 10\,000$

30 $2 \times 10^4 = 20\,000$

31 $6 \times 10^4 = 60\,000$

32 $2 \times 10^2 = 200$

33 $9 \times 10^2 = 900$

34 $7 \times 10^3 = 7\,000$

35 $5 \times 10^2 = 500$

36 $8 \times 10^5 = 800\,000$

37 $9 \times 10^6 = 9\,000\,000$

38 $5 \times 10^1 = 50$

39 $2 \times 10^7 = 20\,000\,000$

40 $4 \times 10^5 = 400\,000$

41 $2 \times 10^1 = 20$

42 $7 \times 10^2 = 700$

Write these as a single digit number × a base 10 indexed number

Example

$60\,000 = 6 \times 10^4$

43 $400 = 4 \times 10^2$

44 $9\,000 = 9 \times 10^3$

45 $20\,000 = 2 \times 10^4$

46 $4\,000\,000 = 4 \times 10^6$

47 $700\,000 = 7 \times 10^5$

48 $3\,000 = 7 \times 10^3$

49 $90\,000\,000 = 9 \times 10^7$

50 $70 = 7 \times 10^1$

51 $6\,000 = 6 \times 10^3$

52 $500\,000 = 5 \times 10^5$

53 $200 = 2 \times 10^2$

54 $8\,000\,000 = 8 \times 10^6$

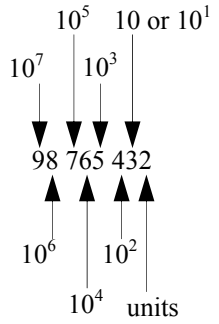
55 $30 = 3 \times 10^1$

Expanded Notation with Indices

This sheet has some questions written sideways. To read these questions click on the "View" menu, select the rotate option then "counter clockwise".

When numbers are written in expanded notation the number is broken up into its place values. Instead of using numbers to represent the millions, hundred thousands etc. indices are used, a list of which is below. You should have attempted 'Indexed Numbers' prior to this worksheet.

The Top 8 Indices Chart



So this number would be:

$9 \times 10^7 + 8 \times 10^6 + 7 \times 10^5 + 6 \times 10^4 + 5 \times 10^3 + 4 \times 10^2 + 3 \times 10 + 2$

In column 1 a number has been expressed in expanded notation (in index form). You are asked to write the number in basic numerals. The best way to attempt these problems is to look at them digit by digit. In the example at the top of the column you have :

$7 \times 10^5 + 4 \times 10^3 + 8 \times 10^2 + 6$, follow these steps:

- Write the 7 straight away, note that its power is 10^5
- Is the next number $\times 10^4$, no it isn't so write a zero
- Is the next number $\times 10^3$, yes it is, so write a 4
- Is the next number $\times 10^2$, yes it is, so write an 8
- Is the next number $\times 10$ (or $\times 10^1$), no it isn't so write a zero
- Are there any single units (or $\times 1$), yes there is, so write a 6
- You now have 704 806.

If you have trouble remembering the powers, remember that with base 10 indices the power is the number of zeros behind the one, i.e. $10^3 = 1\,000$, a 1 with three zeros.

Column 2 works in the reverse, just start at the first digit and work through, remember if it's a zero, write nothing. Count the number of digits behind the first number to find the first base 10 index.

Expanded Notation with Indices

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Change from expanded notation to basic numerals

Example

10^5 10^4 10^3 10^2 10 units

$7 \times 10^5 + 4 \times 10^3 + 8 \times 10^2 + 6 = 704\,806$

1. $2 \times 10^3 + 9 \times 10^2 + 5 \times 10 + 6 =$ _____
2. $3 \times 10^4 + 1 \times 10^3 + 6 \times 10^2 + 3 =$ _____
3. $5 \times 10^5 + 2 \times 10^4 + 7 \times 10 + 1 =$ _____
4. $6 \times 10^5 + 2 \times 10^4 + 6 \times 10^2 =$ _____
5. $7 \times 10^4 + 3 \times 10^2 + 2 \times 10 + 8 =$ _____
6. $8 \times 10^5 + 5 \times 10^4 + 3 \times 10^2 + 5 =$ _____
7. $9 \times 10^7 + 4 \times 10^3 =$ _____
8. $6 \times 10^4 + 2 \times 10^3 + 6 \times 10^2 + 2 =$ _____
9. $7 \times 10^5 + 7 \times 10^4 + 7 \times 10 + 7 =$ _____
10. $3 \times 10^6 + 9 \times 10^5 + 6 \times 10^3 + 8 \times 10^2 + 8 \times 10 + 6 =$ _____
11. $1 \times 10^7 + 4 \times 10^5 + 8 \times 10^4 + 6 \times 10^3 + 3 \times 10^2 + 3 =$ _____
12. $8 \times 10^5 + 6 \times 10^4 + 9 \times 10^3 + 7 \times 10 + 5 =$ _____
13. $2 \times 10^6 + 7 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 9 \times 10 + 6 =$ _____
14. $9 \times 10^7 + 3 \times 10^6 + 9 \times 10^5 + 3 \times 10^4 + 6 \times 10^2 + 4 =$ _____
15. $1 \times 10^6 + 3 \times 10^5 + 1 \times 10^4 + 2 \times 10^2 + 2 \times 10 + 7 =$ _____
16. $2 \times 10^7 + 1 \times 10^6 + 5 \times 10^3 + 1 \times 10^2 + 2 \times 10 + 8 =$ _____
17. $8 \times 10^6 + 5 \times 10^5 + 4 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 =$ _____
18. $3 \times 10^7 + 7 \times 10^6 + 2 \times 10^4 + 1 \times 10^3 + 6 \times 10^2 + 9 =$ _____
19. $4 \times 10^6 + 5 \times 10^4 + 1 \times 10^2 + 5 \times 10 + 3 =$ _____
20. $9 \times 10^7 + 6 \times 10^4 + 6 \times 10^2 + 5 \times 10 + 4 =$ _____

Now the reverse, write these numbers in expanded notation



Example

$59\,003\,007 = 5 \times 10^7 + 9 \times 10^6 + 3 \times 10^3 + 6$

21. $6\,610 =$ _____
22. $271\,004 =$ _____
23. $45\,096 =$ _____
24. $5\,010\,072 =$ _____
25. $86\,090 =$ _____
26. $4\,700\,000 =$ _____
27. $396 =$ _____
28. $107\,902 =$ _____
29. $7\,430\,007 =$ _____
30. $2\,934\,049 =$ _____
31. $5\,773\,088 =$ _____
32. $653\,310 =$ _____
33. $52\,400\,032 =$ _____
34. $3\,756\,407 =$ _____
35. $4\,986\,020 =$ _____
36. $911\,690 =$ _____
37. $1\,804\,125 =$ _____
38. $558\,027 =$ _____
39. $8\,023\,962 =$ _____
40. $45\,770\,000 =$ _____

Expanded Notation with Indices

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Change from expanded notation to basic numerals

Example

10^5 10^4 10^3 10^2 10 units

$7 \times 10^5 + 4 \times 10^3 + 8 \times 10^2 + 6 = 704\,806$

1. $2 \times 10^3 + 9 \times 10^2 + 5 \times 10 + 6 = 2\,956$
2. $3 \times 10^4 + 1 \times 10^3 + 6 \times 10^2 + 3 = 31\,603$
3. $5 \times 10^5 + 2 \times 10^4 + 7 \times 10 + 1 = 520\,071$
4. $6 \times 10^5 + 2 \times 10^4 + 6 \times 10^2 = 620\,600$
5. $7 \times 10^4 + 3 \times 10^2 + 2 \times 10 + 8 = 70\,328$
6. $8 \times 10^5 + 5 \times 10^4 + 3 \times 10^2 + 5 = 850\,305$
7. $9 \times 10^7 + 4 \times 10^3 = 90\,004\,000$
8. $6 \times 10^4 + 2 \times 10^3 + 6 \times 10^2 + 2 = 62\,602$
9. $7 \times 10^5 + 7 \times 10^4 + 7 \times 10 + 7 = 770\,077$
10. $3 \times 10^6 + 9 \times 10^5 + 6 \times 10^3 + 8 \times 10^2 + 8 \times 10 + 6 = 3\,906\,886$
11. $1 \times 10^7 + 4 \times 10^5 + 8 \times 10^4 + 6 \times 10^3 + 3 \times 10^2 + 3 = 10\,486\,303$
12. $8 \times 10^5 + 6 \times 10^4 + 9 \times 10^3 + 7 \times 10 + 5 = 869\,075$
13. $2 \times 10^6 + 7 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 9 \times 10 + 6 = 2\,077\,696$
14. $9 \times 10^7 + 3 \times 10^6 + 9 \times 10^5 + 3 \times 10^4 + 6 \times 10^2 + 4 = 93\,930\,604$
15. $1 \times 10^6 + 3 \times 10^5 + 1 \times 10^4 + 2 \times 10^2 + 2 \times 10 + 7 = 1\,310\,227$
16. $2 \times 10^7 + 1 \times 10^6 + 5 \times 10^3 + 1 \times 10^2 + 2 \times 10 + 8 = 21\,005\,128$
17. $8 \times 10^6 + 5 \times 10^5 + 4 \times 10^4 + 6 \times 10^3 + 5 \times 10^2 = 8\,546\,500$
18. $3 \times 10^7 + 7 \times 10^6 + 2 \times 10^4 + 1 \times 10^3 + 6 \times 10^2 + 9 = 37\,021\,609$
19. $4 \times 10^6 + 5 \times 10^4 + 1 \times 10^2 + 5 \times 10 + 3 = 4\,050\,153$
20. $9 \times 10^7 + 6 \times 10^4 + 6 \times 10^2 + 5 \times 10 + 4 = 90\,060\,654$

Now the reverse, write these numbers in expanded notation



Example

$59\,003\,007 = 5 \times 10^7 + 9 \times 10^6 + 3 \times 10^3 + 6$

21. $6\,610 = 6 \times 10^3 + 6 \times 10^2 + 1 \times 10$
22. $271\,004 = 2 \times 10^5 + 7 \times 10^4 + 1 \times 10^3 + 4$
23. $45\,096 = 4 \times 10^4 + 5 \times 10^3 + 9 \times 10 + 6$
24. $5\,010\,072 = 5 \times 10^6 + 1 \times 10^4 + 7 \times 10 + 2$
25. $86\,090 = 8 \times 10^4 + 6 \times 10^3 + 9 \times 10$
26. $4\,700\,000 = 4 \times 10^6 + 7 \times 10^5$
27. $396 = 3 \times 10^2 + 9 \times 10 + 6$
28. $107\,902 = 1 \times 10^5 + 7 \times 10^3 + 9 \times 10^2 + 2$
29. $7\,430\,007 = 7 \times 10^6 + 4 \times 10^5 + 3 \times 10^4 + 7$
30. $2\,934\,049 = 2 \times 10^6 + 9 \times 10^5 + 3 \times 10^4 + 4 \times 10^3 + 4 \times 10 + 9$
31. $5\,773\,088 = 5 \times 10^6 + 7 \times 10^5 + 7 \times 10^4 + 3 \times 10^3 + 8 \times 10 + 8$
32. $653\,310 = 6 \times 10^5 + 5 \times 10^4 + 3 \times 10^3 + 3 \times 10^2 + 1 \times 10$
33. $52\,400\,032 = 5 \times 10^7 + 2 \times 10^6 + 4 \times 10^5 + 3 \times 10 + 2$
34. $3\,756\,407 = 3 \times 10^6 + 7 \times 10^5 + 5 \times 10^4 + 6 \times 10^3 + 4 \times 10^2 + 7$
35. $4\,986\,020 = 4 \times 10^6 + 9 \times 10^5 + 8 \times 10^4 + 6 \times 10^3 + 2 \times 10$
36. $911\,690 = 9 \times 10^5 + 1 \times 10^4 + 1 \times 10^3 + 6 \times 10^2 + 9 \times 10$
37. $1\,804\,125 = 1 \times 10^6 + 8 \times 10^5 + 4 \times 10^3 + 1 \times 10^2 + 2 \times 10 + 5$
38. $558\,027 = 5 \times 10^5 + 5 \times 10^4 + 8 \times 10^3 + 2 \times 10 + 7$
39. $8\,023\,962 = 8 \times 10^6 + 2 \times 10^4 + 3 \times 10^3 + 9 \times 10^2 + 6 \times 10 + 2$
40. $45\,770\,000 = 4 \times 10^7 + 5 \times 10^6 + 7 \times 10^5 + 7 \times 10^4$

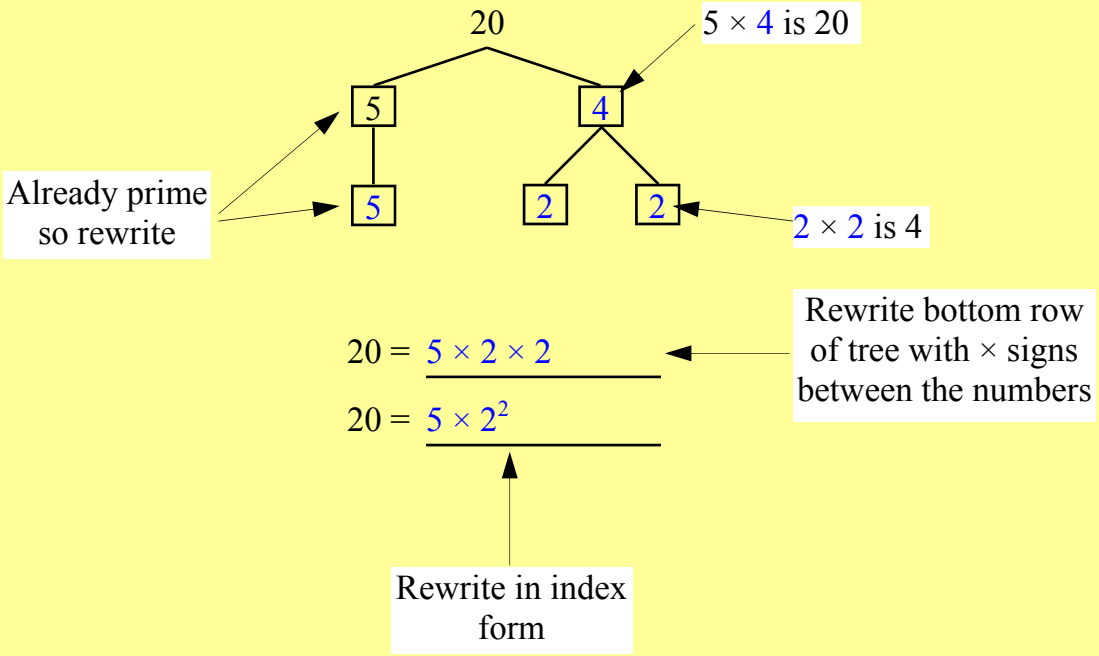
Factor Trees

Factor trees are used to reduce a number down to a multiplication of its prime factors. These factors can be used to find the LCM and HCF of two numbers. This sheet asks you to find the answer as a multiplication of the primes and to also write the answer in index form. Factor trees have the number to be reduced at the top, this is split into two branches and from then on is split again into 2 branches or extended with a single branch if it is a prime number, until all the numbers are prime (don't have factors other than 1 and itself).

Column 1 has factor trees already constructed with a number given on the first tier (first split branch) to help you get started. Here is the method:

- Look at the number to be reduced and divide it by the number on the first tier, that will give you the answer for the other first tier box.
- Then look at the numbers you now have and break them down, if a single branch is provided that means you can't reduce the number so just rewrite it in the box below, if you have another pair of branches then select 2 numbers that multiply together to give the number. Note that you never use 1 in a factor tree.
- Column 1 only has one solution method, where there is a choice, a number is supplied so that there is only one answer.
- Once you have the boxes completed look at the numbers across the bottom row and multiply them to get the answer, it should equal the number at the top of the tree. Then write the numbers in the space provided, then rewrite them in index form, which you should know from the previous sheet.

Column 2 and 3 require you to build the tree without hints. Pick two numbers that multiply to get the top number, you should pick two numbers that are close together to reduce the number in the minimum number of steps. So if the number was say 100 you would use 10 and 10 not 25 and 4.



Factor Trees

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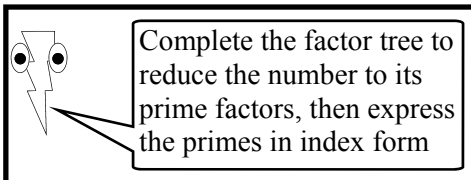
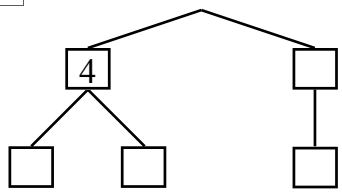


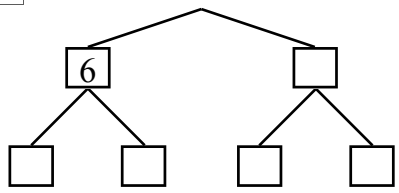
1 12



12 = ___ × ___ × ___

12 = ___^{2} × ___

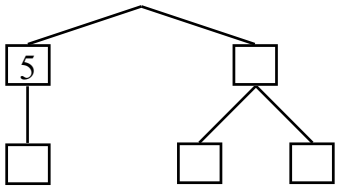
2 36



36 = ___ × ___ × ___ × ___

36 = ___^{2} × ___^{2}

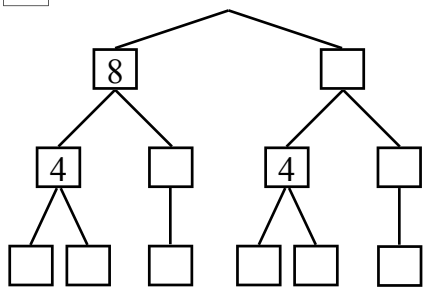
3 50



50 = _____

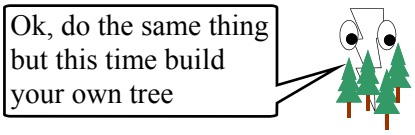
50 = _____

4 64



64 = _____

64 = _____



5 8

8 = _____

8 = _____

6 81

81 = _____

81 = _____

7 48

48 = _____

48 = _____

8 120

120 = _____

120 = _____

9 500

500 = _____

500 = _____

10 180

180 = _____

180 = _____

11 450

450 = _____

450 = _____

12 1 000

1 000 = _____

1 000 = _____

Factor Trees

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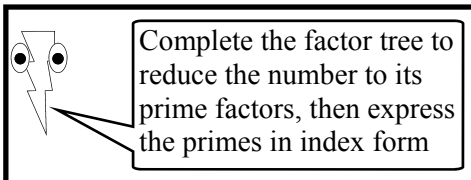
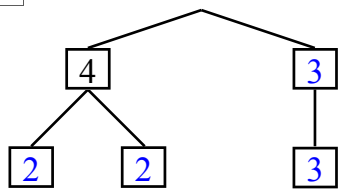


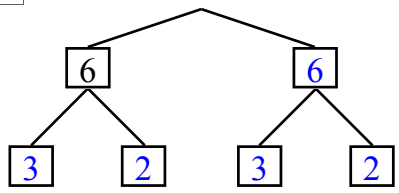
1 12



$12 = 2 \times 2 \times 3$

$12 = 2^2 \times 3$

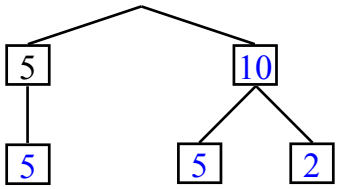
2 36



$36 = 3 \times 2 \times 3 \times 2$

$36 = 3^2 \times 2^2$

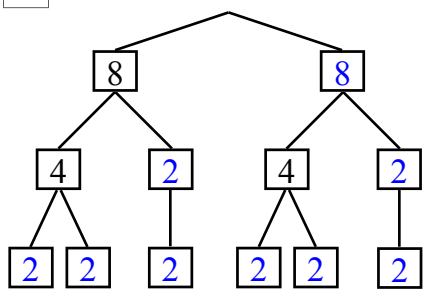
3 50



$50 = 5 \times 5 \times 2$

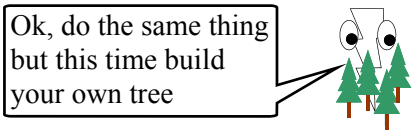
$50 = 5^2 \times 2$

4 64

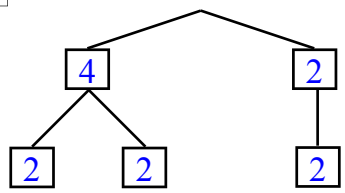


$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$64 = 2^6$



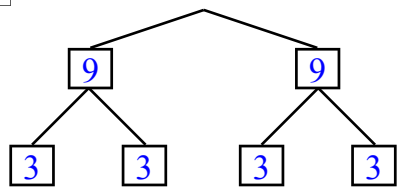
5 8



$8 = 2 \times 2 \times 2$

$8 = 2^3$

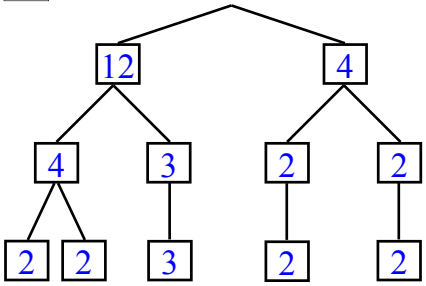
6 81



$81 = 3 \times 3 \times 3 \times 3$

$81 = 3^4$

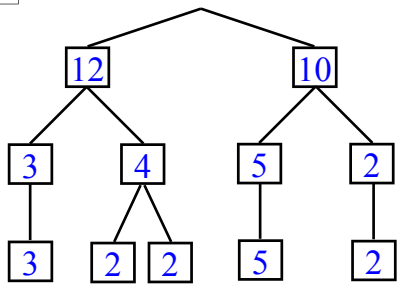
7 48



$48 = 2 \times 2 \times 3 \times 2 \times 2$

$48 = 2^4 \times 3$

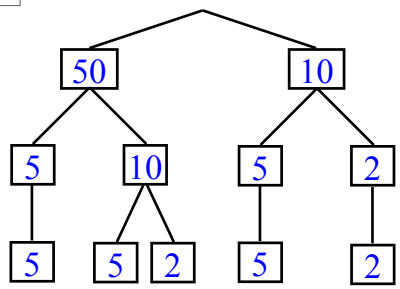
8 120



$120 = 3 \times 2 \times 2 \times 5 \times 2$

$120 = 3 \times 2^3 \times 5$

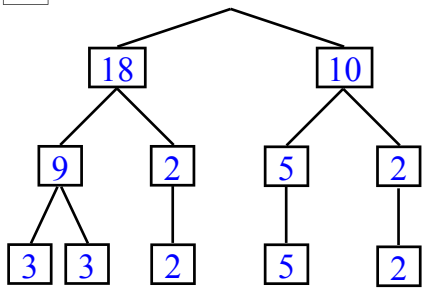
9 500



$500 = 5 \times 5 \times 2 \times 5 \times 2$

$500 = 5^3 \times 2^2$

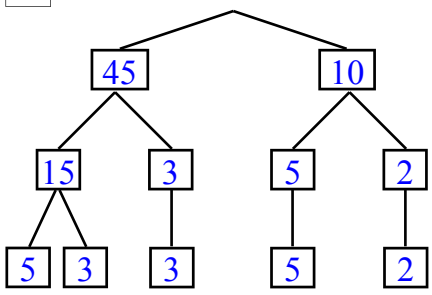
10 180



$180 = 3 \times 3 \times 2 \times 5 \times 2$

$180 = 3^2 \times 2^2 \times 5$

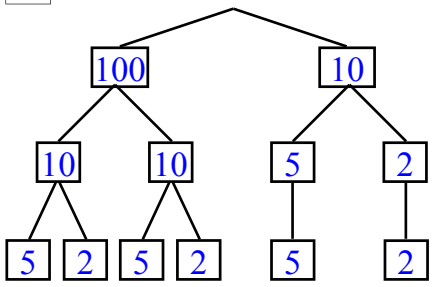
11 450



$450 = 5 \times 3 \times 3 \times 5 \times 2$

$450 = 5^2 \times 3^2 \times 2$

12 1 000



$1\,000 = 5 \times 2 \times 5 \times 2 \times 5 \times 2$

$1\,000 = 5^3 \times 2^3$

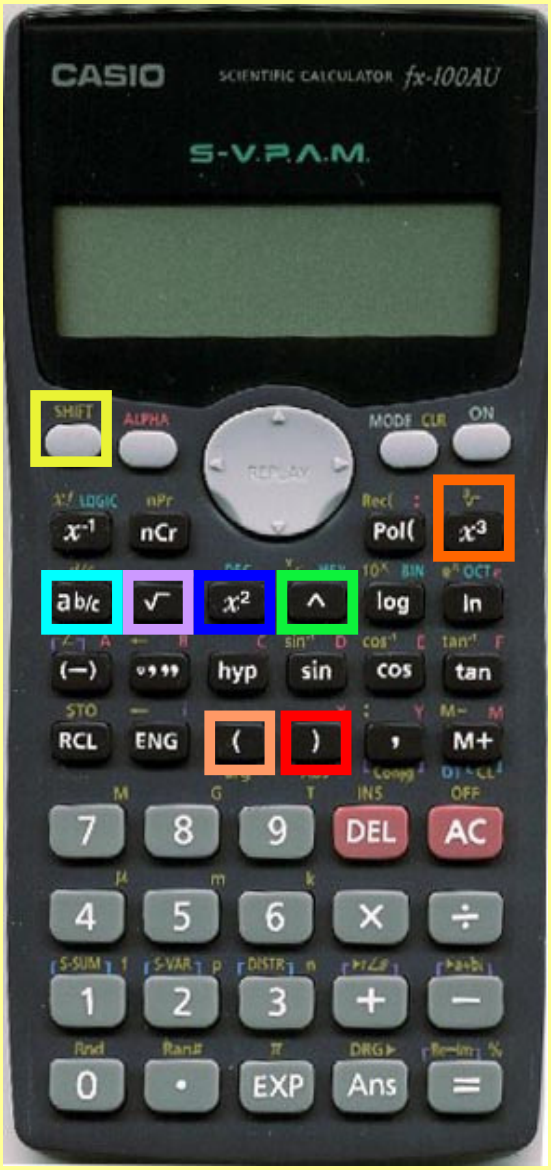
Powers, Roots and the Power Key - Calculator

This sheet is about the use of powers and roots on a calculator. There are examples on the next sheet with a calculator image showing you the buttons. This calculator may be different from your model but the symbols used on the keys should be the same, just look for the symbols.

Column 1 is all square roots, some calculators require you to type the number then the square root, the steps for these calculators are different than those shown, try your calculator on the examples on the next page and see the method you use to get the correct answer. The first part of the column doesn't require brackets, the second part does. Your calculator needs brackets to force order of operations on it, otherwise it will take the square root of the first number only.

Column 2 deals with squares (power of 2) and cubes (power of 3). Some calculators may not have a cube key, if this is your case then use the power key with a power of 3. Some calculators experience difficulty with fractions and powers, with the power being considered on the last number entered only (the denominator), so you may have to put fractions in brackets before entering the power. The second part of the column deals with using brackets. Your teacher may ask you to avoid using brackets, instead using the equals key at selected times, this is just as valid. When dividing, bracket the top operation then divide it by the bracketed bottom operation then press =.

Column 3 starts with using the power key, look at the examples. Single number cube roots are then covered briefly, leading to the more complicated cube roots with brackets. Teachers will ask you to ignore the cube root sign and evaluate the inside first, then cube root it. This simplifies the operation and should be considered as an easier alternative, done in 2 steps. You don't have to solve these calculations in one step, and the use of 3 sets of brackets may lead to mistakes being made.



Examples of steps in calculating square roots

$\sqrt{85}$ = √ 8 5 = = 9.22

$\sqrt{5\ ^2/_7}$ = √ 5 a b/c 2 a b/c 7 = a b/c = 2.30

$\sqrt{13.4 + 22 \times 3.6}$ These answers to 2 d.p.

= √ (1 3 . 4 + 2 2 × 3 . 6) = = 9.62

$\sqrt{\dfrac{34}{45 + 19.7}}$

= √ (3 4 ÷ (4 5 + 1 9 . 7)) = = 0.72

Use the same method with the cube root key only press Shift first

Examples of steps in calculating squares and cubes

5.6^2 = 5 . 6 x^2 = = 31.36

$(5\ ^2/_7)^2$ = (5 a b/c 2 a b/c 7) x^2 = a b/c = 27.94

Use the same method with the cube key x^3

These answers to 2 d.p.

Examples of steps using the power key

5.6^8 = 5 . 6 ^ 8 = = 967 173.12

These answers to 2 d.p.

$(5\ ^2/_7)^9$ = (5 a b/c 2 a b/c 7) ^ 9 = a b/c = 3 220 573.07

Powers and Roots - Calculator Applications

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Use the square root key on your calculate to answer these.

Calc: [√] [4] [4] [1] [=]

1 $\sqrt{441}$ = ____ 2 $\sqrt{361}$ = ____

3 $\sqrt{256}$ = ____ 4 $\sqrt{484}$ = ____

5 $\sqrt{289}$ = ____ 6 $\sqrt{729}$ = ____

7 $\sqrt{0.04}$ = ____ 8 $\sqrt{1\ 369}$

9 $\sqrt{0.3136}$ = ____ = ____

Now round your answers to 1 decimal place

10 $\sqrt{32}$ = ____ 11 $\sqrt{77}$ = ____

12 $\sqrt{205}$ = ____ 13 $\sqrt{888}$ = ____

Use the brackets key to answer these, round to 1 d.p when necessary.

Calc: [√] [(] [2] [3] [×] [1] [6] [)] [=]

14 $\sqrt{(23 \times 16)}$ = ____

15 $\sqrt{(27.2 + 48.9)}$ = ____

16 $\sqrt{(15 \div 3.9)}$ = ____

17 $\sqrt{(106 + 45 \times 19)}$ = ____

The brackets aren't in these questions but you still need to use them on your calculator.

18 $\sqrt{19 \times 42 + 102}$ = ____

19 $\sqrt{\frac{133.6}{8.35}}$ = ____

20 $\sqrt{509 - 21 \times 18.6}$ = ____

21 $\sqrt{17.7 \times 30 \times 1.8}$ = ____

22 $\sqrt{49.3 + 112 + 95}$ = ____

23 $\sqrt{\frac{2\ 293.2}{5.2}}$ = ____

Find these squares and cubes, answer to 2 d.p. where required.

24 35^2 = ____

25 9.7^2 = ____

26 20.6^2 = ____

27 $(3\frac{1}{2})^3$ = ____

28 10.5^3 = ____

29 65^2 = ____

30 12^3 = ____

31 $(4\frac{3}{5})^3$ = ____

32 8.09^3 = ____

33 $19.4^2 + 10.9^3$ = ____

34 18.1×11^3 = ____

35 $\frac{52.5^2}{14^3}$ = ____

36 $25^2 - 12^3$ = ____

37 $34 + 6.2^3 \times 13^2$ = ____

38 $\sqrt{77} + 25^2$ = ____

39 $(\sqrt{66})^2$ = ____

40 $(11 - 26.2)^2$ = ____

41 $11 - 26.2^2$ = ____

42 $11^2 - 26.2^2$ = ____

43 $18^3 - 18^2 - 18$ = ____

44 $18^3 - 18^2 \times 18$ = ____

45 $100^3 \div 10^3$ = ____

46 $100^3 \div 10^3 \div 10^3$ = ____

Use the power key [^] for these. Answer to 2 d.p. if required.

47 2^4 = ____

48 5^5 = ____

49 1^7 = ____

50 10^6 = ____

51 5.2^3 = ____

52 2^{12} = ____

53 $(1^2/_7)^6$ = ____

54 $8^9 - 8^8$ = ____

55 $2^{15} \div 2^{13}$ = ____

Use the cube root key [$\sqrt[3]{}$] to answer these. Express your answer to 2 d.p. when required

56 $\sqrt[3]{216}$ = ____

57 $\sqrt[3]{343}$ = ____

58 $\sqrt[3]{1\ 000}$ = ____

Brackets now required

59 $\sqrt[3]{\frac{5\ 375}{43}}$ = ____

60 $\sqrt[3]{452 \times 17.3}$ = ____

61 $\sqrt[3]{88^3 + 88^3}$ = ____

62 $\sqrt[3]{5\ 833 - 18^3}$ = ____

Careful with this one

63 $\sqrt[3]{\frac{5^3 + 2^7}{11^2}}$ = ____

Powers and Roots - Calculator Applications

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Use the square root key on your calculate to answer these.

Calc: [√] [4] [4] [1] [=]

1 $\sqrt{441} = 21$ 2 $\sqrt{361} = 19$

3 $\sqrt{256} = 16$ 4 $\sqrt{484} = 22$

5 $\sqrt{289} = 17$ 6 $\sqrt{729} = 27$

7 $\sqrt{0.04} = 0.2$ 8 $\sqrt{1\ 369} = 37$

9 $\sqrt{0.3136} = 0.56$

Now round your answers to 1 decimal place

10 $\sqrt{32} = 5.7$ 11 $\sqrt{77} = 8.8$

12 $\sqrt{205} = 14.3$ 13 $888 = 29.8$

Use the brackets key to answer these, round to 1 d.p when necessary.

Calc: [√] [(] [2] [3] [×] [1] [6] [)] [=]

14 $\sqrt{(23 \times 16)} = 19.2$

15 $\sqrt{(27.2 + 48.9)} = 8.7$

16 $\sqrt{(15 \div 3.9)} = 2$

17 $\sqrt{(106 + 45 \times 19)} = 31$

The brackets aren't in these questions but you still need to use them on your calculator.

18 $\sqrt{19 \times 42 + 102} = 30$

19 $\sqrt{\frac{133.6}{8.35}} = 4$

20 $\sqrt{509 - 21 \times 18.6} = 10.9$

21 $\sqrt{17.7 \times 30 \times 1.8} = 9.8$

22 $\sqrt{49.3 + 112 + 95} = 16$

23 $\sqrt{\frac{2\ 293.2}{5.2}} = 21$

Find these squares and cubes, answer to 2 d.p. where required.

24 $35^2 = 1\ 225$

25 $9.7^2 = 94.09$

26 $20.6^2 = 424.36$

27 $(3\frac{1}{2})^3 = 42.88$

28 $10.5^3 = 1\ 157.63$

29 $65^2 = 4\ 225$

30 $12^3 = 1\ 728$

31 $(4^3/_5)^3 = 97.34$

32 $8.09^3 = 529.48$

33 $19.4^2 + 10.9^3 = 1\ 671.39$

34 $18.1 \times 11^3 = 24\ 091.1$

35 $\frac{52.5^2}{14^3} = 1$

36 $25^2 - 12^3 = -1\ 103$

37 $34 + 6.2^3 \times 13^2 = 31\ 493.3$

38 $\sqrt{77} + 25^2 = 633.77$

39 $(\sqrt{66})^2 = 66$

40 $(11 - 26.2)^2 = 231.04$

41 $11 - 26.2^2 = -675.44$

42 $11^2 - 26.2^2 = -565.44$

43 $18^3 - 18^2 - 18 = 5\ 490$

44 $18^3 - 18^2 \times 18 = 0$

45 $100^3 \div 10^3 = 1\ 000$

46 $100^3 \div 10^3 \div 10^3 = 1$

Use the power key [^] for these. Answer to 2 d.p. if required.

47 $2^4 = 16$

48 $5^5 = 3\ 125$

49 $1^7 = 1$

50 $10^6 = 1\ 000\ 000$

51 $5.2^3 = 140.61$

52 $2^{12} = 4\ 096$

53 $(1^2/_7)^6 = 4.52$

54 $8^9 - 8^8 = 117\ 440\ 512$

55 $2^{15} \div 2^{13} = 4$

Use the cube root key [$\sqrt[3]{}$] to answer these. Express your answer to 2 d.p. when required

56 $\sqrt[3]{216} = 6$

57 $\sqrt[3]{343} = 7$

58 $\sqrt[3]{1\ 000} = 10$

Brackets now required

59 $\sqrt[3]{\frac{5\ 375}{43}} = 5$

60 $\sqrt[3]{452 \times 17.3} = 19.85$

61 $\sqrt[3]{88^3 + 88^3} = 110.87$

62 $\sqrt[3]{5\ 833 - 18^3} = 1$

Careful with this one

63 $\sqrt[3]{\frac{5^3 + 2^7}{11^2}} = 1.45$

Further Factor Trees - Finding the HCF

This sheet and the one that follows it are challenging.

Finding the Highest Common Factors of larger numbers can be found using factor trees. The method is as follows:

- Build a factor tree, you should have completed the previous sheet on factor trees and so the process of finding prime factors won't be readdressed.
- Write the number and its prime factors in the table then match off pairs of numbers against each other, one for one, by circling them.
- Write one set of the matching prime factors on the first line separated by × signs
- Evaluate the multiplication and write your answer in the space, this is the HCF.

An example is at the top of the worksheet.

Further Factor Trees - Finding the HCF (or GCF)

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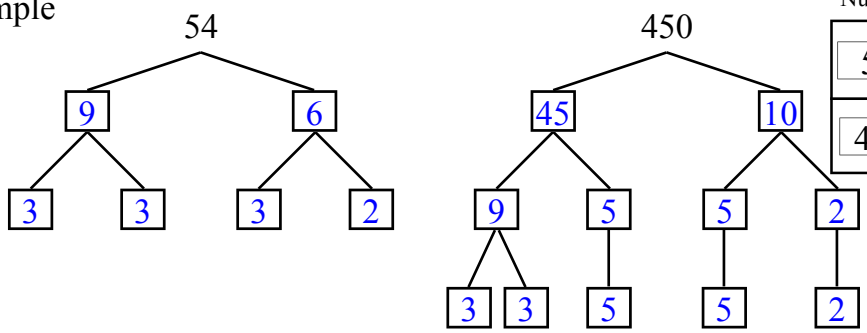
Complete the trees to find the prime factors, put the prime factors in the table, circle factor pairs in both lists, multiply these factors to get the HCF (or GCF)

Example



Number	List of Prime Factors
54	3 3 3 2
450	3 3 5 5 2

HCF = $3 \times 3 \times 2$

HCF = 18

1



Number	List of Prime Factors

HCF = _____

HCF = _____

2



Number	List of Prime Factors

HCF = _____

HCF = _____

3



Number	List of Prime Factors

HCF = _____

HCF = _____

4



Number	List of Prime Factors

HCF = _____

HCF = _____

Further Factor Trees - Finding the HCF (or GCF)

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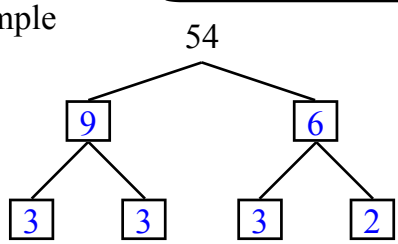
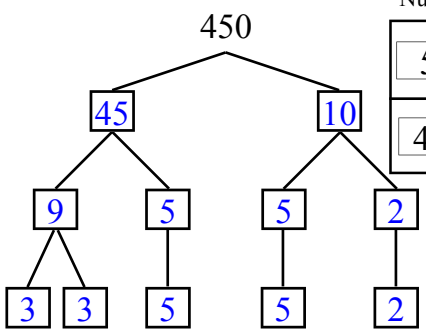
Complete the trees to find the prime factors, put the prime factors in the table, circle factor pairs in both lists, multiply these factors to get the HCF (or GCF)

Example



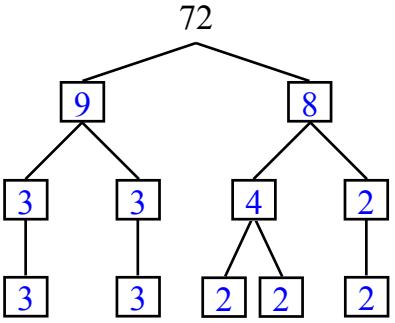


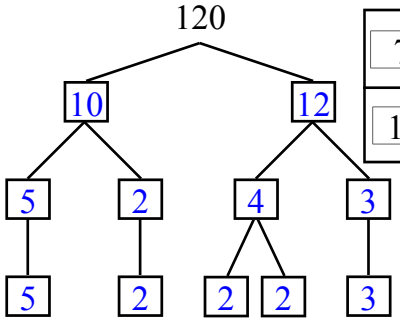
Number	List of Prime Factors
54	3 3 3 2
450	3 3 5 5 2

HCF = $3 \times 3 \times 2$

HCF = 18

1



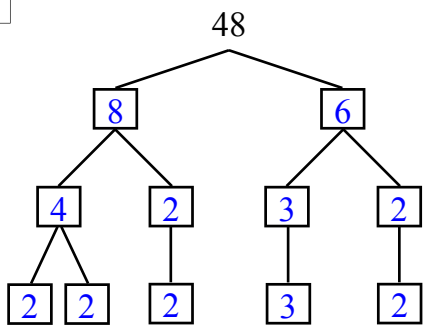


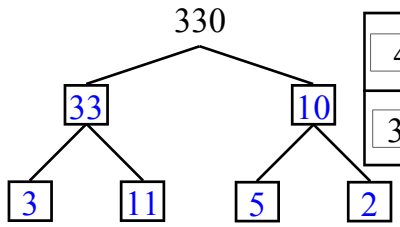
Number	List of Prime Factors
72	3 3 2 2 2
120	5 2 2 2 3

HCF = $3 \times 2 \times 2$

HCF = 12

2



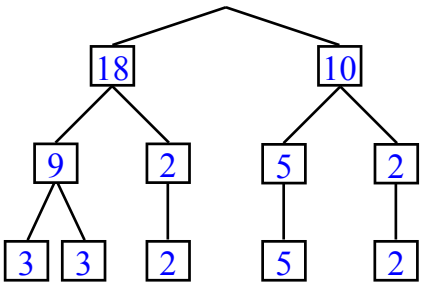


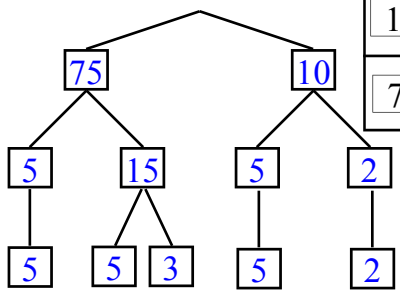
Number	List of Prime Factors
48	2 2 2 3 2
330	3 11 5 2

HCF = 3×2

HCF = 6

3



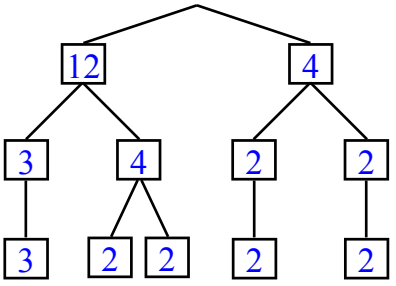


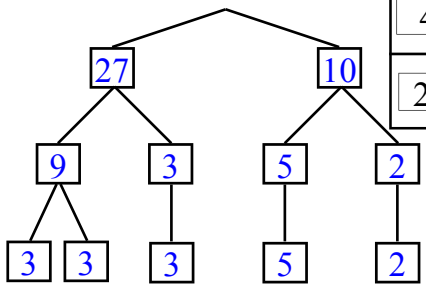
Number	List of Prime Factors
180	3 3 2 5 2
750	5 5 3 5 2

HCF = $3 \times 2 \times 5$

HCF = 30

4





Number	List of Prime Factors
48	3 2 2 2 2
270	3 3 3 5 2

HCF = 3×2

HCF = 6

Further Factor Trees - Finding the LCM

This sheet is a challenging sheet.

Finding the Lowest Common Multiple of larger numbers can be done by using factor trees. The method is as follows:

- Build two factor trees, you should have completed the Factor Tree sheet and so the process of finding prime factors won't be readdressed.
- Write the number and its prime factors in the table for both trees then if a number is in the top list and also in the bottom list strike out the number in the bottom list only.
- Note that this doesn’t mean if you have a 2 in the top list you strike out all the 2's in the bottom list, you just strike out one number for one number. If there are 2 threes in the top list and 4 threes in the bottom then you only strike out 2 threes in the bottom list.
- Write all the non-struck out numbers from both lists, separated by × signs
- Evaluate the multiplication and write your answer in the space, this is the LCM.

The multiplication is performed by trying to multiply numbers together to come up with two large numbers. These larger numbers can then be multiplied. In the example the 3's and the 2's are multiplied separately, leaving 9 × 72. If the numbers are too large then you may have to complete a few multiplication steps.

Further Factor Trees - Finding the LCM

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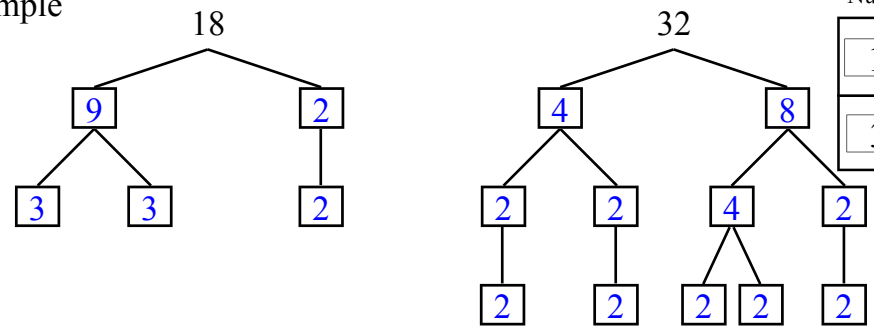
Find the prime factors, put the factors in the table, then strike out all the numbers in the bottom list that are in the top list, multiply all non-stroked numbers

Example



Number	List of Prime Factors
18	3 3 2
32	2 2 2 2 2

LCM = ~~9~~ × ~~3~~ × ~~4~~ × ~~2~~ × ~~8~~ × ~~2~~ × ~~16~~ × ~~2~~ × ~~32~~

LCM = 288

1



Number	List of Prime Factors

LCM = _____

LCM = _____

2



Number	List of Prime Factors

LCM = _____

LCM = _____

3



Number	List of Prime Factors

LCM = _____

LCM = _____

4



Number	List of Prime Factors

LCM = _____

LCM = _____

Further Factor Trees - Finding the LCM

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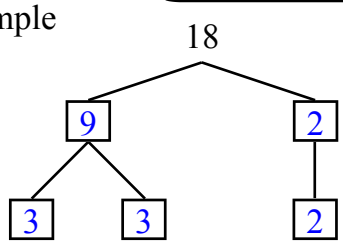
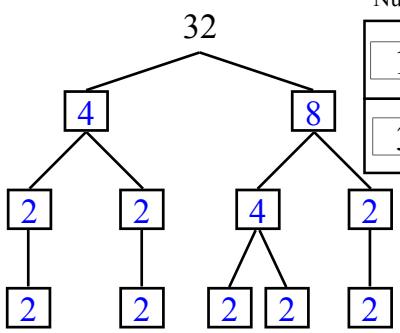
Find the prime factors, put the factors in the table, then strike out all the numbers in the bottom list that are in the top list, multiply all non-stroked numbers

Example



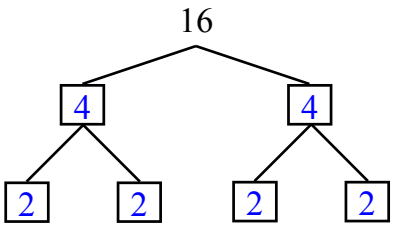


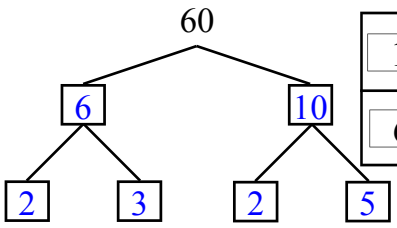
Number	List of Prime Factors
18	3 3 2
32	2 2 2 2 2

LCM = $\cancel{3} \times \cancel{3}^{9} \times \cancel{2} \times \cancel{2}^{4} \times \cancel{2}^{8} \times \cancel{2}^{16} \times \cancel{2}^{32}$

LCM = 288

1



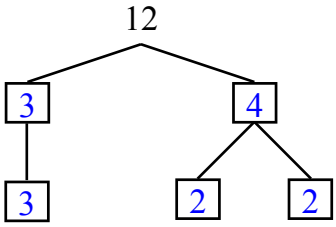


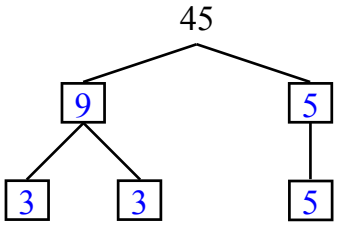
Number	List of Prime Factors
16	2 2 2 2
60	2 3 2 5

LCM = $\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}^{16} \times \cancel{3} \times \cancel{5}^{15}$

LCM = 240

2



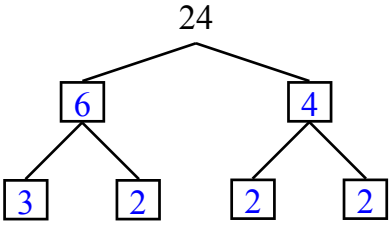


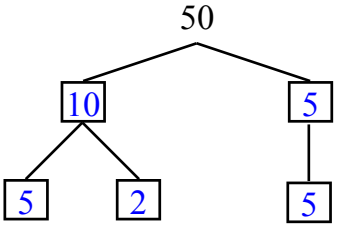
Number	List of Prime Factors
12	3 2 2
45	3 3 5

LCM = $\cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{3}^{36} \times 5$

LCM = 180

3



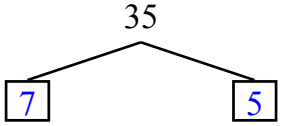


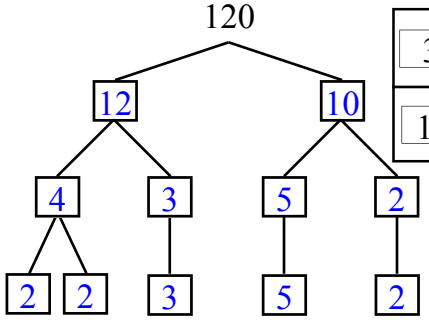
Number	List of Prime Factors
24	3 2 2 2
50	5 2 5

LCM = $\cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{2}^{24} \times \cancel{5} \times \cancel{5}^{25}$

LCM = 600

4





Number	List of Prime Factors
35	5 7
120	2 2 3 5 2

LCM = $\cancel{5} \times \cancel{7}^{35} \times \cancel{2} \times \cancel{2} \times \cancel{3} \times \cancel{2}^{24}$

LCM = 840

Divisibility Tests - 2, 4, 5 and 10

Divisibility tests are rules that check if a number can be divided by another number, without attempting the division to see if there is a remainder. The rules for this sheet are below.

- A number is divisible by 2 if its last digit is a 0, 2, 4, 6, or 8
- A number is divisible by 4 if its last two digits are divisible by 4, this doesn't mean each digit separately, it is the number formed by the 2 digits. For example 32 is divisible by 4, so 532, 1 032 and 77 732 would be also.
- A number is divisible by 5 if its last digit is a 0 or a 5
- A number is divisible by 10 if its last digit is a 0

So the tests all work on the last digit only, with the exception of the 4 test which requires the last 2 digits.

These questions have different ways of being answered. You may have to write the answer. If there is a yes or no answer required, either colour in the circle or strike out the wrong answer.

Divisibility Rules

2 : last digit is 0, 2, 4, 6, 8

4 : last 2 digits form a number divisible by 4

5 : last digit is either 0 or 5

10 : last digit must be 0

Divisibility Tests - 2, 4, 5 and 10

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Test these numbers for divisibility by 2, fill in yes or no

		Yes	No
1	28	○	○
2	66	○	○
3	1 112	○	○
4	1 003	○	○
5	59 691	○	○
6	80 000	○	○
7	27	○	○
8	185	○	○
9	77 772	○	○
10	13 574	○	○

Are the following divisible by 4? Circle your answer.

11	42	Yes / No
12	50	Yes / No
13	68	Yes / No
14	86	Yes / No
15	532	Yes / No
16	102	Yes / No
17	5 794	Yes / No
18	44 448	Yes / No
19	13 252	Yes / No
20	30 006	Yes / No
21	91 428	Yes / No
22	10 100	Yes / No

Circle the numbers that are divisible by both 2 and 4

23

24 654 490 122 6 678 772 88 886 4 930 550 454 9 968 92

The numbers below are divisible by 5. Give the 2 possible numbers it could be due to the unreadable last digit.

24	3▒	____	____
25	9▒	____	____
26	15▒	____	____
27	21▒	____	____
28	1 00▒	____	____
29	86▒	____	____
30	9 07▒	____	____
31	44 05▒	____	____
32	2 11▒	____	____
33	1 00▒	____	____

Test these numbers for divisibility by 10, fill in yes or no

		Yes	No
34	90	○	○
35	643	○	○
36	10 101	○	○
37	3 804	○	○
38	13 330	○	○
39	8 080	○	○

Test for divisibility of 2, 4, 5 and 10. Fill the oval if divisibility exists.

		2	4	5	10
40	30	○	○	○	○
41	60	○	○	○	○
42	110	○	○	○	○
43	205	○	○	○	○
44	776	○	○	○	○
45	490	○	○	○	○
46	2 080	○	○	○	○
47	32 070	○	○	○	○
48	924	○	○	○	○
49	74 650	○	○	○	○

Answer the following questions

Find the next 2 numbers that are greater than these numbers that are divisible by 2, 4, 5 and 10:

50	45	____	____
51	1 00	____	____
52	1 963	____	____
53	594	____	____

54 Claudia believes that only when a number is a multiple of 20, is it divisible by 2, 4, 5 and 10. What do you think?

I stopped listening to her years ago ○

I knew her before she was President ○

Divisibility Tests - 2, 4, 5 and 10

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Test these numbers for divisibility by 2, fill in yes or no

		Yes	No
1	28	●	○
2	66	●	○
3	1 112	●	○
4	1 003	○	●
5	59 691	○	●
6	80 000	●	○
7	27	○	●
8	185	○	●
9	77 772	●	○
10	13 574	●	○

Are the following divisible by 4? Circle your answer.

11	42	~~Yes~~ / No
12	50	~~Yes~~ / No
13	68	Yes / ~~No~~
14	86	~~Yes~~ / No
15	532	Yes / ~~No~~
16	102	~~Yes~~ / No
17	5 794	~~Yes~~ / No
18	44 448	Yes / ~~No~~
19	13 252	Yes / ~~No~~
20	30 006	~~Yes~~ / No
21	91 428	Yes / ~~No~~
22	10 100	Yes / ~~No~~

Circle the numbers that are divisible by both 2 and 4

23

24 654 490 122 6 678 772 88 886 4 930 550 454 9 968 92

The numbers below are divisible by 5. Give the 2 possible numbers it could be due to the unreadable last digit.

24	3▒	30	35
25	9▒	90	95
26	15▒	150	155
27	21▒	210	215
28	1 00▒	1 000	1 005
29	86▒	860	865
30	9 07▒	9 070	9 075
31	44 05▒	44 050	44 055
32	2 11▒	2 110	2 115
33	1 00▒	1 000	1 005

Test these numbers for divisibility by 10, fill in yes or no

		Yes	No
34	90	●	○
35	643	○	●
36	10 101	○	●
37	3 804	○	●
38	13 330	●	○
39	8 080	●	○

Test for divisibility of 2, 4, 5 and 10. Fill the oval if divisibility exists.

		2	4	5	10
40	30	●	○	●	●
41	60	●	●	●	●
42	110	●	○	●	●
43	205	○	○	●	○
44	776	●	●	○	○
45	490	●	○	●	●
46	2 080	●	●	●	●
47	32 070	●	○	●	●
48	924	●	●	○	○
49	74 650	●	○	●	●

Answer the following questions

Find the next 2 numbers that are greater than these numbers that are divisible by 2, 4, 5 and 10:

50	45	60	80
51	1 00	120	140
52	1 963	1 980	2 000
53	594	600	620

54 Claudia believes that only when a number is a multiple of 20, is it divisible by 2, 4, 5 and 10. What do you think?

I stopped listening to her years ago ○

I knew her before she was President ●

Divisibility Tests - 3, 6 and 9

Divisibility tests for numbers can be used rather than division. The tests for 3, 6 and 9 are:

- 3: Find the sum of the digits (add them up) that form the number and if the sum is a multiple of 3 then the number in the question is divisible by 3.
- 6: The number must be divisible by both 2 and 3. The test for 2 is the last digit is even (0, 2, 4, 6 or 8). So use the same method as with 3 but the number given in the question must also be even (not the sum).
- 9: Find the sum of the digits and if the number is a multiple of 9 then the number in the question is divisible by 9.

A card with the rules of divisibility is also below

Divisibility Rules

3 : sum of digits is a multiple of 3

6 : number is divisible by both 3 and 2 (must be even & digit sum is multiple of 3)

9 : sum of digits is a multiple of 9

Divisibility Tests - 3, 6 and 9

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Test these numbers for divisibility by 3, fill in yes or no

Example

8 + 0 + 2 + 2

		Sum digits	Yes	No
	8 022	12	●	○
1	78	____	○	○
2	93	____	○	○
3	112	____	○	○
4	108	____	○	○
5	3 419	____	○	○
6	77 322	____	○	○
7	8 301	____	○	○
8	10 020	____	○	○
9	84 355	____	○	○



Now test for divisibility by 6. Circle Yes or No.

		Sum digits	
10	2 103	____	Yes / No
11	2 104	____	Yes / No
12	5 514	____	Yes / No
13	80 940	____	Yes / No
14	30 003	____	Yes / No
15	11 424	____	Yes / No
16	10 800	____	Yes / No
17	60 682	____	Yes / No
18	99 996	____	Yes / No
19	17 361	____	Yes / No
20	44 004	____	Yes / No
21	88 884	____	Yes / No



Circle the numbers that are divisible by 3, put a box around them if they are divisible by 3 and 6.

22

578 14 053 96 35 730 1 809 8 931 38 690



The numbers below are said to be divisible by 9. Answer true or false

		Sum digits	
23	54	____	____
24	72	____	____
25	105	____	____
26	228	____	____
27	315	____	____
28	882	____	____
29	15 597	____	____
30	88 884	____	____
31	47 333	____	____
32	6 959	____	____
33	87 532	____	____
34	54 738	____	____
35	38 007	____	____
36	7 893	____	____



Test for divisibility of 3, 6 and 9. Fill the oval if divisibility exists.

		Sum digits	3	6	9
37	453	____	○	○	○
38	6 003	____	○	○	○
39	8 934	____	○	○	○
40	2 073	____	○	○	○
41	1 452	____	○	○	○
42	71 712	____	○	○	○
43	4 221	____	○	○	○
44	8 964	____	○	○	○
45	72 159	____	○	○	○
46	5 232	____	○	○	○

Divisibility Tests - 3, 6 and 9

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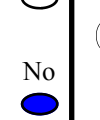
Test these numbers for divisibility by 3, fill in yes or no

Example

		Sum digits	Yes	No
	8 022	12 (8 + 0 + 2 + 2)	●	○
1	78	15	●	○
2	93	12	●	○
3	112	4	○	●
4	108	9	●	○
5	3 419	17	○	●
6	77 322	21	●	○
7	8 301	12	●	○
8	10 020	3	●	○
9	84 355	25	○	●



Circle the numbers that are divisible by 3, put a box around them if they are divisible by 3 and 6.

22 14 053 578 96 35 730 1 809 8 931 38 690



The numbers below are said to be divisible by 9. Answer true or false

		Sum digits	
23	54	9	True
24	72	9	True
25	105	6	False
26	228	12	False
27	315	9	True
28	882	18	True
29	15 597	27	True



Now test for divisibility by 6. Circle Yes or No.

		Sum digits	
10	2 103	6	~~Yes~~ / No
11	2 104	7	~~Yes~~ / No
12	5 514	15	Yes / ~~No~~
13	80 940	21	Yes / ~~No~~
14	30 003	6	~~Yes~~ / No
15	11 424	12	Yes / ~~No~~
16	10 800	9	Yes / ~~No~~
17	60 682	22	~~Yes~~ / No
18	99 996	42	Yes / ~~No~~
19	17 361	18	~~Yes~~ / No
20	44 004	12	Yes / ~~No~~
21	88 884	36	Yes / ~~No~~

		Sum digits	
30	88 884	36	True
31	47 333	20	False
32	7 893	27	True
33	87 532	25	False
34	54 738	27	True
35	38 007	18	True
36	6 959	29	False



Test for divisibility of 3, 6 and 9. Fill the oval if divisibility exists.

		Sum digits	3	6	9
37	453	12	●	○	○
38	6 003	9	●	○	●
39	8 934	24	●	●	○
40	2 073	12	●	○	○
41	1 452	12	●	●	○
42	71 712	18	●	●	●
43	4 221	9	●	○	●
44	8 964	27	●	●	●
45	72 159	24	●	○	○
46	5 232	12	●	●	○

Further Divisibility Tests - 7, 11 and 13

These tests are more difficult than the previous tests. Follow the steps outlined below.
In column 1, to test for divisibility of 7:

- Take the last digit off the number and double it (×2).
- Subtract the number (found above) from the original number in the question without its last digit.
- The above subtraction can be reversed to avoid subtracting a larger number from a smaller number (this occurs in question 1).
- If the subtraction results in a number that is 0 or a multiple of 7 then the number is divisible by 7.
- There is an example at top of the column.

Test for divisibility of 11:

- Counting from left to right add all the odd digits together, that is, the 1st, 3rd, 5th, 7th …. (write the value in the box below the words 'odd digit')
- Then add all the even digits together, that is, 2nd, 4th, 6th, 8th ….(write the value in the box below the words 'even digit')
- Subtract one from the other, space for subtraction is provided though you probably won't need it, if the answer is 0 or a multiple of 11 then it is divisible by 11.
- There is an example at top of the column.

Test for divisibility of 13:

- Complete the multiplications to give the first 9 multiples of 13, you can use these to help you identify the multiples
- This test is similar to the 7 test in that the last digit is removed, only it is multiplied by 9.
- This number is then subtracted from the number in the question without its last digit, this subtraction can be reversed if required.
- If the subtraction results in a number that is 0 or a multiple of 13 then the number is divisible by 13.
- There is an example before question 23.

The card to the right is there if you need it.

Divisibility Rules

7 : Remove the last digit off the number, from it subtract 2 × the last digit. Divisible if the answer is 0 or a multiple of 7.

11 : Add all the odd numbered digits and then the even numbered digits. Subtract them, if answer is 0 or a multiple of 11, it is divisible by 11.

13 : Remove the last digit off the number and from it subtract 9 times the last digit. Divisible if the answer is 0 or a multiple of 13.

Further Divisibility Tests - 7, 11 and 13

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Test these numbers for divisibility by 7, fill in yes or no

Example 476 — 47 - 12 — 35

2 × 6 = 12

Yes ● No ○

1 119 -

Yes ○ No ○

2 181 -

Yes ○ No ○

3 576 -

Yes ○ No ○

4 826 -

Yes ○ No ○

5 957 -

Yes ○ No ○

6 743 -

Yes ○ No ○

7 882 -

Yes ○ No ○

Test these numbers for divisibility by 11, fill in yes or no

Example 88 957 — 24 - 13 — 11

odd digit 24 — even digit 13

Yes ● No ○

8 639 -

odd digit ___ — even digit ___

Yes ○ No ○

9 275 -

odd digit ___ — even digit ___

Yes ○ No ○

10 13 178 -

odd digit ___ — even digit ___

Yes ○ No ○

11 184 796 -

odd digit ___ — even digit ___

Yes ○ No ○

12 557 193 -

odd digit ___ — even digit ___

Yes ○ No ○

13 438 592 -

odd digit ___ — even digit ___

Yes ○ No ○

Find the first 9 multiples of 13, use the answers to help test for divisibility by 13, fill in yes or no

14 1 × 13 = ___ 15 2 × 13 = ___ 16 3 × 13 = ___

17 4 × 13 = ___ 18 5 × 13 = ___ 19 6 × 13 = ___

20 7 × 13 = ___ 21 8 × 13 = ___ 22 9 × 13 = ___

Example 975 — 97 - 45 — 52

9 × last digit 45

Yes ● No ○

23 624 -

9 × last digit ___

Yes ○ No ○

24 663 -

9 × last digit ___

Yes ○ No ○

25 1 765 -

9 × last digit ___

Yes ○ No ○

26 1 898 -

9 × last digit ___

Yes ○ No ○

27 1 846 -

9 × last digit ___

Yes ○ No ○

Further Divisibility Tests - 7, 11 and 13

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Test these numbers for divisibility by 7, fill in yes or no

Example 476

$2 \times 6 = 12$

47 -
12
35

Yes ● No ○

1 119

18 -
11
7

Yes ● No ○

2 181

18 -
2
16

Yes ○ No ●

3 576

57 -
12
45

Yes ● No ○

4 826

82 -
12
70

Yes ● No ○

5 957

95 -
14
81

Yes ○ No ●

6 743

74 -
6
68

Yes ○ No ●

7 882

88 -
4
84

Yes ● No ○

Test these numbers for divisibility by 11, fill in yes or no

Example 88 957

24 -
13
11

odd digit 24 even digit 13

Yes ● No ○

8 639

15 -
3
12

odd digit 15 even digit 3

Yes ○ No ●

9 275

7 -
7
0

odd digit 7 even digit 7

Yes ● No ○

10 13 178

10 -
10
0

odd digit 10 even digit 10

Yes ● No ○

11 184 796

21 -
14
7

odd digit 14 even digit 21

Yes ○ No ●

12 557 193

21 -
9
12

odd digit 21 even digit 9

Yes ○ No ●

13 438 592

21 -
10
11

odd digit 21 even digit 10

Yes ● No ○

Find the first 9 multiples of 13, use the answers to help test for divisibility by 13, fill in yes or no

14 $1 \times 13 = 13$ | 15 $2 \times 13 = 26$ | 16 $3 \times 13 = 39$

17 $4 \times 13 = 52$ | 18 $5 \times 13 = 65$ | 19 $6 \times 13 = 78$

20 $7 \times 13 = 91$ | 21 $8 \times 13 = 104$ | 22 $9 \times 13 = 117$

Example 975

97 -
45
52

9 × last digit 45

Yes ● No ○

23 624

62 -
36
26

9 × last digit 36

Yes ● No ○

24 663

66 -
27
39

9 × last digit 27

Yes ● No ○

25 1 765

176 -
45
131

9 × last digit 45

Yes ○ No ●

26 1 898

189 -
72
117

9 × last digit 72

Yes ● No ○

27 1 846

184 -
54
130

9 × last digit 54

Yes ● No ○

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ALTITUDE

CALCULATOR

THESE SHEETS ARE IN THIS FOLDER AND THEIR CHAPTER FOLDER

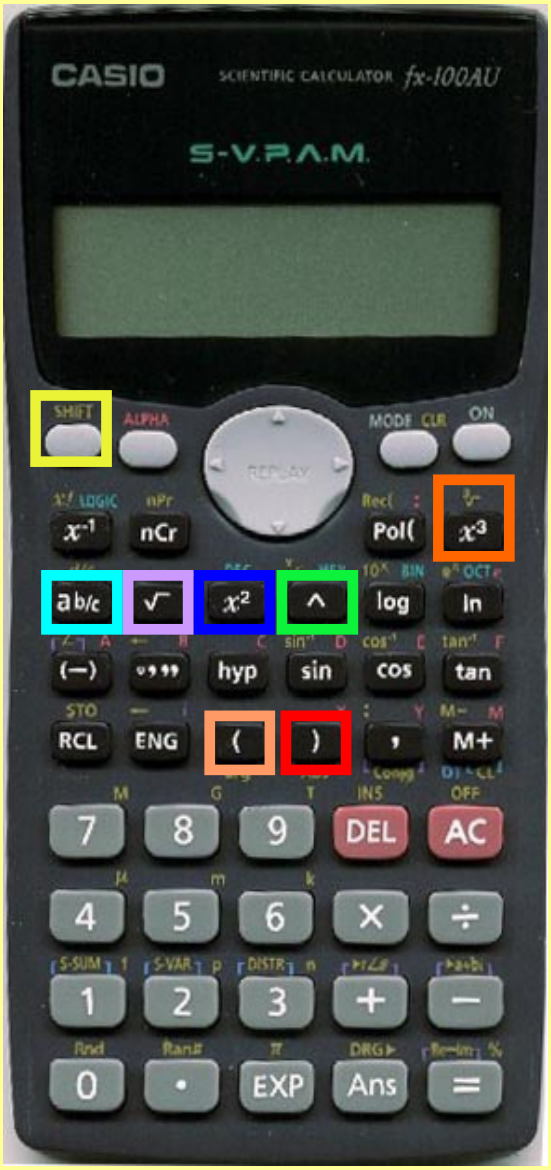
Powers, Roots and the Power Key - Calculator

This sheet is about the use of powers and roots on a calculator. There are examples on the next sheet with a calculator image showing you the buttons. This calculator may be different from your model but the symbols used on the keys should be the same, just look for the symbols.

Column 1 is all square roots, some calculators require you to type the number then the square root, the steps for these calculators are different than those shown, try your calculator on the examples on the next page and see the method you use to get the correct answer. The first part of the column doesn't require brackets, the second part does. Your calculator needs brackets to force order of operations on it, otherwise it will take the square root of the first number only.

Column 2 deals with squares (power of 2) and cubes (power of 3). Some calculators may not have a cube key, if this is your case then use the power key with a power of 3. Some calculators experience difficulty with fractions and powers, with the power being considered on the last number entered only (the denominator), so you may have to put fractions in brackets before entering the power. The second part of the column deals with using brackets. Your teacher may ask you to avoid using brackets, instead using the equals key at selected times, this is just as valid. When dividing, bracket the top operation then divide it by the bracketed bottom operation then press =.

Column 3 starts with using the power key, look at the examples. Single number cube roots are then covered briefly, leading to the more complicated cube roots with brackets. Teachers will ask you to ignore the cube root sign and evaluate the inside first, then cube root it. This simplifies the operation and should be considered as an easier alternative, done in 2 steps. You don't have to solve these calculations in one step, and the use of 3 sets of brackets may lead to mistakes being made.



Examples of steps in calculating square roots

$\sqrt{85}$ = √ 8 5 = = 9.22

$\sqrt{5\ ^2/_7}$ = √ 5 a b/c 2 a b/c 7 = a b/c = 2.30

$\sqrt{13.4 + 22 \times 3.6}$ These answers to 2 d.p.

= √ (1 3 . 4 + 2 2 × 3 . 6) = = 9.62

$\sqrt{\dfrac{34}{45 + 19.7}}$

= √ (3 4 ÷ (4 5 + 1 9 . 7)) = = 0.72

Use the same method with the cube root key only press Shift first

Examples of steps in calculating squares and cubes

5.6^2 = 5 . 6 x^2 = = 31.36

$(5\ ^2/_7)^2$ = (5 a b/c 2 a b/c 7) x^2 = a b/c = 27.94

Use the same method with the cube key x^3

These answers to 2 d.p.

Examples of steps using the power key

5.6^8 = 5 . 6 ^ 8 = = 967 173.12

These answers to 2 d.p.

$(5\ ^2/_7)^9$ = (5 a b/c 2 a b/c 7) ^ 9 = a b/c = 3 220 573.07

Powers and Roots - Calculator Applications

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Use the square root key on your calculate to answer these.

Calc: [√] [4] [4] [1] [=]

1 $\sqrt{441}$ = ____ 2 $\sqrt{361}$ = ____

3 $\sqrt{256}$ = ____ 4 $\sqrt{484}$ = ____

5 $\sqrt{289}$ = ____ 6 $\sqrt{729}$ = ____

7 $\sqrt{0.04}$ = ____ 8 $\sqrt{1\ 369}$

9 $\sqrt{0.3136}$ = ____ = ____

Now round your answers to 1 decimal place

10 $\sqrt{32}$ = ____ 11 $\sqrt{77}$ = ____

12 $\sqrt{205}$ = ____ 13 $\sqrt{888}$ = ____

Use the brackets key to answer these, round to 1 d.p when necessary.

Calc: [√] [(] [2] [3] [×] [1] [6] [)] [=]

14 $\sqrt{(23 \times 16)}$ = ____

15 $\sqrt{(27.2 + 48.9)}$ = ____

16 $\sqrt{(15 \div 3.9)}$ = ____

17 $\sqrt{(106 + 45 \times 19)}$ = ____

The brackets aren't in these questions but you still need to use them on your calculator.

18 $\sqrt{19 \times 42 + 102}$ = ____

19 $\sqrt{\frac{133.6}{8.35}}$ = ____

20 $\sqrt{509 - 21 \times 18.6}$ = ____

21 $\sqrt{17.7 \times 30 \times 1.8}$ = ____

22 $\sqrt{49.3 + 112 + 95}$ = ____

23 $\sqrt{\frac{2\ 293.2}{5.2}}$ = ____

Find these squares and cubes, answer to 2 d.p. where required.

24 35^2 = ____

25 9.7^2 = ____

26 20.6^2 = ____

27 $(3\frac{1}{2})^3$ = ____

28 10.5^3 = ____

29 65^2 = ____

30 12^3 = ____

31 $(4\frac{3}{5})^3$ = ____

32 8.09^3 = ____

33 $19.4^2 + 10.9^3$ = ____

34 18.1×11^3 = ____

35 $\frac{52.5^2}{14^3}$ = ____

36 $25^2 - 12^3$ = ____

37 $34 + 6.2^3 \times 13^2$ = ____

38 $\sqrt{77} + 25^2$ = ____

39 $(\sqrt{66})^2$ = ____

40 $(11 - 26.2)^2$ = ____

41 $11 - 26.2^2$ = ____

42 $11^2 - 26.2^2$ = ____

43 $18^3 - 18^2 - 18$ = ____

44 $18^3 - 18^2 \times 18$ = ____

45 $100^3 \div 10^3$ = ____

46 $100^3 \div 10^3 \div 10^3$ = ____

Use the power key [^] for these. Answer to 2 d.p. if required.

47 2^4 = ____

48 5^5 = ____

49 1^7 = ____

50 10^6 = ____

51 5.2^3 = ____

52 2^{12} = ____

53 $(1^2/_7)^6$ = ____

54 $8^9 - 8^8$ = ____

55 $2^{15} \div 2^{13}$ = ____

Use the cube root key [$\sqrt[3]{}$] to answer these. Express your answer to 2 d.p. when required

56 $\sqrt[3]{216}$ = ____

57 $\sqrt[3]{343}$ = ____

58 $\sqrt[3]{1\ 000}$ = ____

Brackets now required

59 $\sqrt[3]{\frac{5\ 375}{43}}$ = ____

60 $\sqrt[3]{452 \times 17.3}$ = ____

61 $\sqrt[3]{88^3 + 88^3}$ = ____

62 $\sqrt[3]{5\ 833 - 18^3}$ = ____

Careful with this one

63 $\sqrt[3]{\frac{5^3 + 2^7}{11^2}}$ = ____

Powers and Roots - Calculator Applications

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Use the square root key on your calculate to answer these.

Calc: [√] [4] [4] [1] [=]

1 $\sqrt{441} = 21$ 2 $\sqrt{361} = 19$

3 $\sqrt{256} = 16$ 4 $\sqrt{484} = 22$

5 $\sqrt{289} = 17$ 6 $\sqrt{729} = 27$

7 $\sqrt{0.04} = 0.2$ 8 $\sqrt{1\ 369} = 37$

9 $\sqrt{0.3136} = 0.56$

Now round your answers to 1 decimal place

10 $\sqrt{32} = 5.7$ 11 $\sqrt{77} = 8.8$

12 $\sqrt{205} = 14.3$ 13 $888 = 29.8$

Use the brackets key to answer these, round to 1 d.p when necessary.

Calc: [√] [(] [2] [3] [×] [1] [6] [)] [=]

14 $\sqrt{(23 \times 16)} = 19.2$

15 $\sqrt{(27.2 + 48.9)} = 8.7$

16 $\sqrt{(15 \div 3.9)} = 2$

17 $\sqrt{(106 + 45 \times 19)} = 31$

The brackets aren't in these questions but you still need to use them on your calculator.

18 $\sqrt{19 \times 42 + 102} = 30$

19 $\sqrt{\frac{133.6}{8.35}} = 4$

20 $\sqrt{509 - 21 \times 18.6} = 10.9$

21 $\sqrt{17.7 \times 30 \times 1.8} = 9.8$

22 $\sqrt{49.3 + 112 + 95} = 16$

23 $\sqrt{\frac{2\ 293.2}{5.2}} = 21$

Find these squares and cubes, answer to 2 d.p. where required.

24 $35^2 = 1\ 225$

25 $9.7^2 = 94.09$

26 $20.6^2 = 424.36$

27 $(3\frac{1}{2})^3 = 42.88$

28 $10.5^3 = 1\ 157.63$

29 $65^2 = 4\ 225$

30 $12^3 = 1\ 728$

31 $(4^3/_5)^3 = 97.34$

32 $8.09^3 = 529.48$

33 $19.4^2 + 10.9^3 = 1\ 671.39$

34 $18.1 \times 11^3 = 24\ 091.1$

35 $\frac{52.5^2}{14^3} = 1$

36 $25^2 - 12^3 = -1\ 103$

37 $34 + 6.2^3 \times 13^2 = 31\ 493.3$

38 $\sqrt{77} + 25^2 = 633.77$

39 $(\sqrt{66})^2 = 66$

40 $(11 - 26.2)^2 = 231.04$

41 $11 - 26.2^2 = -675.44$

42 $11^2 - 26.2^2 = -565.44$

43 $18^3 - 18^2 - 18 = 5\ 490$

44 $18^3 - 18^2 \times 18 = 0$

45 $100^3 \div 10^3 = 1\ 000$

46 $100^3 \div 10^3 \div 10^3 = 1$

Use the power key [^] for these. Answer to 2 d.p. if required.

47 $2^4 = 16$

48 $5^5 = 3\ 125$

49 $1^7 = 1$

50 $10^6 = 1\ 000\ 000$

51 $5.2^3 = 140.61$

52 $2^{12} = 4\ 096$

53 $(1^2/_7)^6 = 4.52$

54 $8^9 - 8^8 = 117\ 440\ 512$

55 $2^{15} \div 2^{13} = 4$

Use the cube root key [$\sqrt[3]{}$] to answer these. Express your answer to 2 d.p. when required

56 $\sqrt[3]{216} = 6$

57 $\sqrt[3]{343} = 7$

58 $\sqrt[3]{1\ 000} = 10$

Brackets now required

59 $\sqrt[3]{\frac{5\ 375}{43}} = 5$

60 $\sqrt[3]{452 \times 17.3} = 19.85$

61 $\sqrt[3]{88^3 + 88^3} = 110.87$

62 $\sqrt[3]{5\ 833 - 18^3} = 1$

Careful with this one

63 $\sqrt[3]{\frac{5^3 + 2^7}{11^2}} = 1.45$

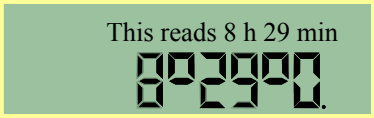
Time Calculations (Calculator)

Time calculations are performed every day, …..how long until lunch?, ….the bus arrives? When you become a wage earner it is important to be able to check that your hours worked are correct, but because minutes and hours are in groups of 60 min this is not always straight forward. But a calculator makes it easy.

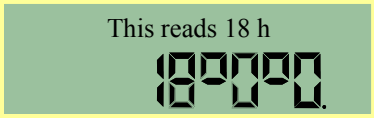
The calculator image on the next page shows the DMS key or the 'bubble button' (the key has an orange border). This key allows you work with hours and minutes.

IMPORTANT YOU MUST ENTER A 0 (THEN DMS KEY) IF DEALING ONLY WITH MINUTES (NO HOURS).

The calculator will always show the hours, minutes and seconds (we won't be using seconds) separated by a degree sign (a small raised o) →°.







In column 1 you are asked to convert the times given in minutes to hours and minutes. This is done by pressing 0 then the DMS key, then the minutes in the question then the DMS key and then press = and the answer will be displayed. Example, change 338 min to hours and minutes.

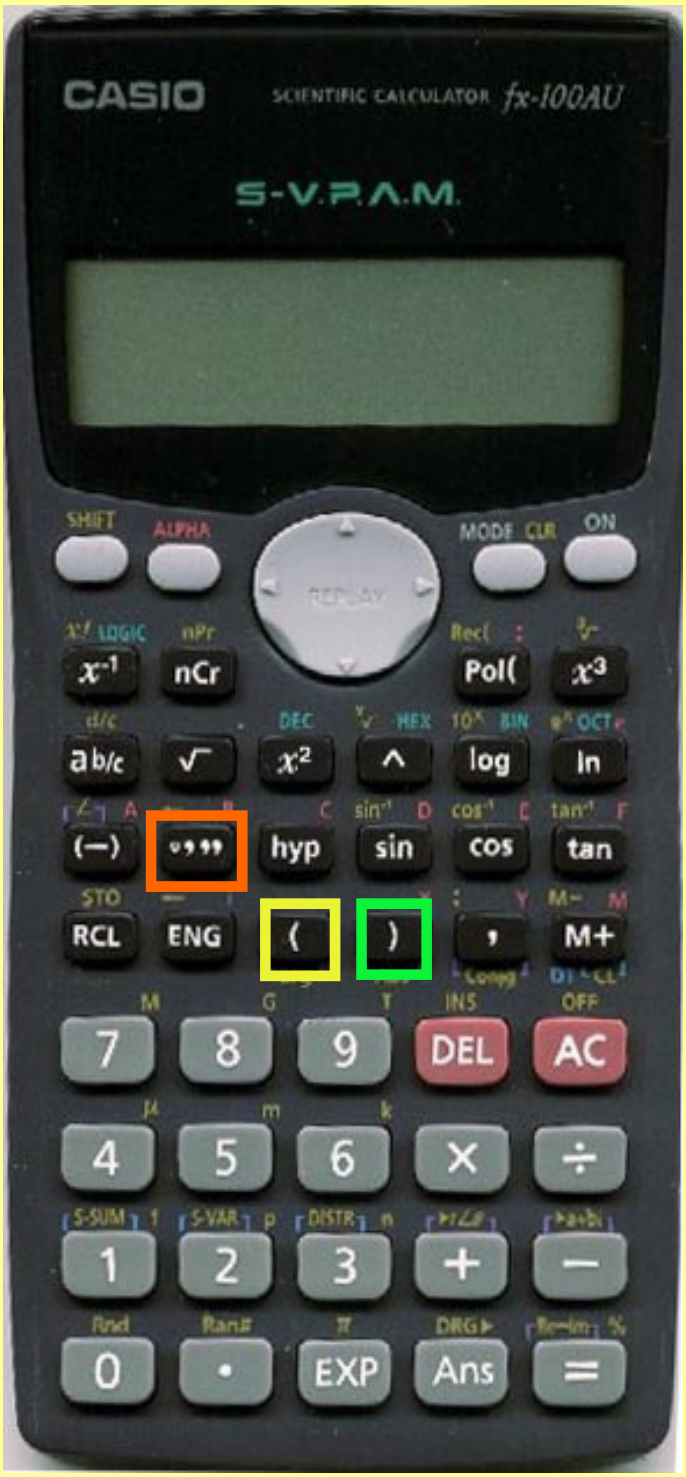




In the 2nd column times are to be added together, put the first time into the calculator then a + then the second time in, press =, done. Finding the difference between two times is done by subtraction, using the same method as above, with one exception YOU MUST CONVERT P.M. TIME TO 24 H TIME. If they are both a.m. times or both p.m. times no conversion is necessary, as soon as a question involves both a p.m. and an a.m. time convert the p.m. time to 24 h format. See the example before Q 18.

The time sheet at the bottom of the column 2 requires you to find the time difference for each day between start and finish times, put them in the spaces provided then add them. Monday is already done for you, check you get the same answer and don't forget to include it in your addition.

The 3rd column is using × and ÷ these are done no differently just remember that you are multiplying by a number not a time and so only use the DMS on the time. The calculation of an average time (Q 31) can be done in two ways refer to the text box below the calculator image for the full key stroke method on the next page.



Example of steps in calculating average time
of 11h 12 min, 8 h 57 min and 10 h 9 min

Using Brackets

(1 1 °’’’ 1 2 °’’’ + 8 °’’’ 5 7 °’’’ + 1 0 °’’’ 9 °’’’) ÷ 3 =

Using = to avoid order of operations

1 1 °’’’ 1 2 °’’’ + 8 °’’’ 5 7 °’’’ + 1 0 °’’’ 9 °’’’ = ÷ 3 =

if you get 10 h 6 min for the question above you are correct

Time Calculations (Calculator)

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Rewrite the calculator displays in h and min.

1 

2 

3 

4 

Convert the following times in min to h and min, remember to put a zero in for the hours first.

5 211 min

6 173 min

7 245 min

8 727 min

9 341 min

10 360 min

11 568 min

12 540 min

13 1331 min

14 1080 min

Add these times together using your calculator

15 8 h 27 min + 11 h 36 min

16 8 h 19 min + 4 h 48 min + 2 h 37 min

17 4 h 39 min + 5 h 25 min + 11 h 56 min

Find the time difference between these times that are on the same day. Use 24 hour time format

Example

1853 h in 24 h

11.17 a.m. and 6.53 p.m.

1 8 ''' 5 3 ''' - 1 1 ''' 1 7 ''' = 7 h 36 min

18 10.30 a.m. and 3.56 p.m.

19 3.52 a.m. and 11.27 a.m.

20 4.23 a.m. and 6.17 p.m.

21 1.09 p.m. and 10.57 p.m.

Complete the time sheet below, then find the total hours worked.

22 A. Jolie Time Sheet

Day	Start	Finish	Hours
Mon	9.30 a.m.	4.45 p.m.	7 h 15 min
Tues	10.45 a.m.	2.30 p.m.	h min
Wed	9.30 a.m.	5.00 p.m.	h min
Thur	9.30 a.m.	3.45 p.m.	h min
Fri	1.15 p.m.	6.00 p.m.	h min
		Total hours for the week	h min

Now use × and ÷ with these times

23 3 × 17 min

24 8 × 23 min

25 7 × 1 h 46 min

26 1 h 55 min ÷ 5

27 19 h 57 min ÷ 9

28 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min

29 8 h 45 min ÷ 5 + 5 min × 17 + 1 h 31 min

Try these time problems

30 Sean watches 3 movies at a 6 h movie marathon, if two of the films were 1h 46 min and 1h 50 min, find the duration of the other.

31 Julie's time for return travel to school for 3 days were: 1h 15 min, 56 min and 1h 22 min. Find the average time for her return trip.

32 It takes Jemma 43 min to wash a car, and 11 min to vacuum a car. How long does she take to wash 5 cars and vacuum 3 of them? Jemma has 40 min for lunch, if she started at 10.25 a.m. estimate her finish time.

Time taken _____ Estimated finish time _____

Time Calculations (Calculator)

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Rewrite the calculator displays in h and min.

1 

52 min

2 

1 h 17 min

3 

3 h

4 

31 h 34 min

Convert the following times in min to h and min, remember to put a 0° in for the hours first.

5 211 min — 3 h 31 min

6 173 min — 2 h 53 min

7 245 min — 4 h 5 min

8 727 min — 12 h 7 min

9 341 min — 5 h 41 min

10 360 min — 6 h

11 568 min — 9 h 28 min

12 540 min — 9 h

13 1331 min — 22 h 11 min

14 1080 min — 18 h

Add these times together using your calculator

15 8 h 27 min + 11 h 36 min

20 h 3 min

16 8 h 19 min + 4 h 48 min + 2 h 37 min — 15 h 44 min

17 4 h 39 min + 5 h 25 min + 11 h 56 min — 22 h

Find the time difference between these times that are on the same day. Use 24 hour time format

Example

1853 in 24 h time

11.17 a.m. and 6.53 p.m.

1 8 °'" 5 3 °'" - 1 1 °'" 1 7 °'" =

7 h 36 min

18 10.30 a.m. and 3.56 p.m.

5 h 26 min

19 3.52 a.m. and 11.27 a.m.

7 h 35 min

20 4.23 a.m. and 6.17 p.m.

13 h 54 min

21 1.09 p.m. and 10.57 p.m.

9 h 48 min

Complete the time sheet below, then find the total hours worked.

22 A. Jolie Time Sheet

Day	Start	Finish	Hours
Mon	9.30 a.m.	4.45 p.m.	7 h 15 min
Tues	10.45 a.m.	2.30 p.m.	3 h 45 min
Wed	9.30 a.m.	5.00 p.m.	7 h 30 min
Thur	9.30 a.m.	3.45 p.m.	6 h 15 min
Fri	1.15 p.m.	6.00 p.m.	4 h 45 min

Total hours for the week 29 h 30 min

Now use × and ÷ with these times

23 3 × 17 min — 51 min

24 8 × 23 min — 3 h 4 min

25 7 × 1 h 46 min — 12h 22 min

26 1 h 55 min ÷ 5 — 23 min

27 19 h 57 min ÷ 9 — 2 h 13 min

28 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min — 7 h 8 min

29 8 h 45 min ÷ 5 + 5 min × 17 + 1 h 31 min — 4 h 41 min

Try these time problems

30 Sean watches 3 movies at a 6 h movie marathon, if two of the films were 1h 46 min and 1h 50 min, find the duration of the other.

2 h 24 min

31 Julie's time for return travel to school for 3 days were: 1h 15 min, 56 min and 1h 22 min. Find the average time for her return trip.

1 h 11 min

32 It takes Jemma 43 min to wash a car, and 11 min to vacuum a car. How long does she take to wash 5 cars and vacuum 3 of them? Jemma has 40 min for lunch, if she started at 10.25 a.m. estimate her finish time.

Time taken: 4 h 8 min

Estimated finish time: 3.13 p.m.

Comparing Fractions (Calculator)

Using a calculator to compare fractions is done by changing the fractions to decimals and comparing the decimals instead. If all rounded to the same decimal place, the decimals can be easily compared, e.g. 0.308, 0.471, 0.235. It is easy to see the largest and smallest decimal, or if they are equal, if you look at the numbers behind the decimal point.

Column 1 asks you to convert the fractions to decimals, the calculator image on the next page shows you how to do this. If you have a decimal 0.100 then write it as 0.100 don't change it to 0.1 as this may lead you to make an error when comparing other numbers that are 3 d.p. (You may look at it as 1 instead of 100).

The rest of the sheet is attempted the same way, by finding the decimal for the fraction then comparing decimals. In the example below, two fractions are being compared, the method is:

- Find the decimals for both
- Compare decimals and put a < or > or = sign in between the decimals
- If this sign matches the one above it then write true, if not, write false.

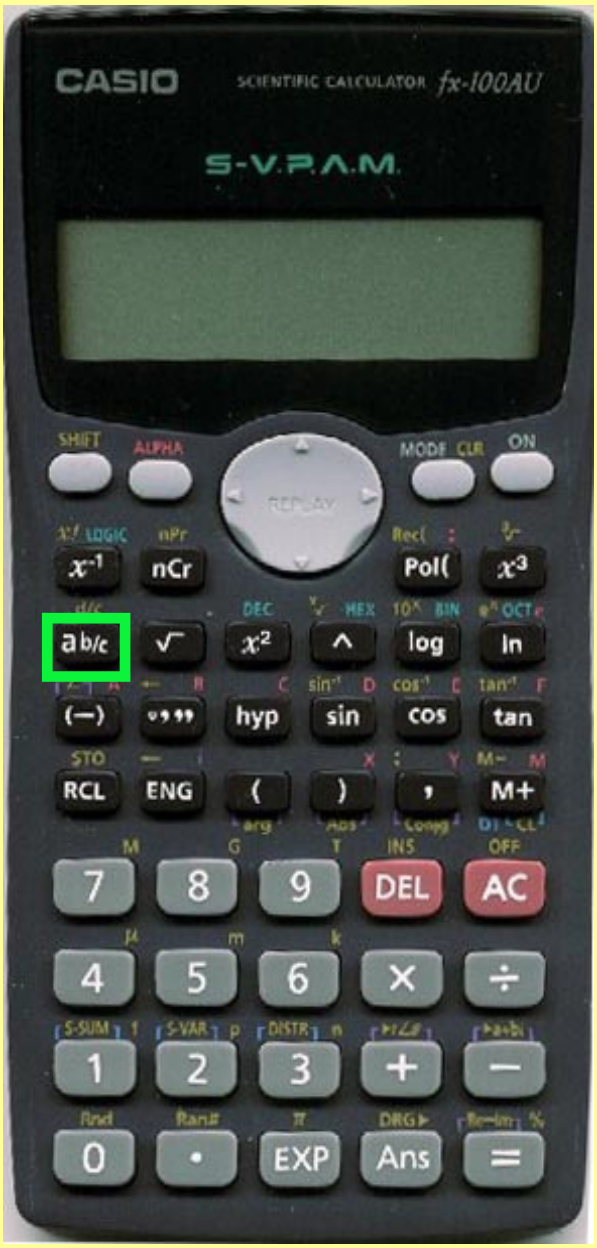
$$\frac{4}{5} > \frac{7}{8} \quad \text{False}$$
$$0.800 < 0.875$$

In the 2nd column, from Q. 31 on, use the same method only this time just write the sign into the box rather than stating true/false.

The third column asks you to compare fractions and arrange them in descending order, use the decimal system then write the fractions again in descending order. In a test the most common mistake is rewriting the decimals in descending order, the question asks for the fractions to be written, not the decimals.

How do you remember the difference between < and >? Imagine they are arrowheads and point them at the smallest number. E.g. $5 < 6$ and $15 > 10$, the arrow points to the smallest one.

Descending order? Remember going down.



Example of how to convert fractions to decimals

$\frac{6}{7} = \_\_\_\_ ?$

Don't forget the 'equals' sign

Method 1: Using Fraction Key [6] [a b/c] [7] [=] [a b/c]

Method 2: Using ÷ Key [6] [÷] [7] [=]

if you get 0.857 (rounded) you have answered it correctly

Comparing Fractions (Calculator)

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Convert these to decimals round to 3 d.p.

1 $\frac{1}{10}$ ____ 2 $\frac{1}{4}$ ____

3 $\frac{2}{3}$ ____ 4 $\frac{3}{8}$ ____

5 $\frac{3}{4}$ ____ 6 $\frac{5}{6}$ ____

7 $\frac{6}{13}$ ____ 8 $\frac{8}{19}$ ____

9 $\frac{3}{7}$ ____ 10 $\frac{24}{27}$ ____

11 $\frac{17}{29}$ ____ 12 $\frac{7}{12}$ ____

13 $\frac{73}{80}$ ____ 14 $\frac{33}{43}$ ____

15 $\frac{17}{42}$ ____ 16 $\frac{96}{99}$ ____

17 $\frac{175}{305}$ ____ 18 $\frac{212}{636}$ ____

19 $\frac{572}{911}$ ____ 20 $\frac{101}{909}$ ____

Write the decimal (3 d.p) under the fraction and then answer true or false

21 $\frac{2}{3}$ < $\frac{3}{4}$ ____

22 $\frac{2}{5}$ > $\frac{3}{7}$ ____

23 $\frac{8}{9}$ < $\frac{9}{10}$ ____

24 $\frac{6}{7}$ > $\frac{8}{10}$ ____

25 $\frac{5}{8}$ < $\frac{7}{14}$ ____

26 $\frac{8}{11}$ > $\frac{5}{7}$ ____

27 $\frac{13}{17}$ < $\frac{14}{19}$ ____

28 $\frac{6}{81}$ < $\frac{4}{63}$ ____

29 $\frac{35}{39}$ > $\frac{60}{70}$ ____

30 $\frac{88}{93}$ < $\frac{17}{19}$ ____

Use the same method only fill in <, > or =

31 $\frac{2}{3}$ ☐ $\frac{3}{5}$

32 $\frac{3}{6}$ ☐ $\frac{18}{36}$

33 $\frac{4}{10}$ ☐ $\frac{27}{60}$

34 $\frac{3}{7}$ ☐ $\frac{4}{9}$

35 $\frac{8}{12}$ ☐ $\frac{2}{3}$

36 $\frac{4}{9}$ ☐ $\frac{17}{45}$

37 $\frac{4}{19}$ ☐ $\frac{14}{60}$

38 $\frac{11}{18}$ ☐ $\frac{26}{37}$

39 $\frac{3}{8}$ ☐ $\frac{21}{56}$

40 $\frac{12}{13}$ ☐ $\frac{16}{19}$

Use decimals to arrange these in descending order

41 $\frac{3}{7}$, $\frac{2}{3}$, $\frac{10}{13}$, $\frac{17}{40}$

Rewrite in Descending order ☐ , ☐ , ☐ , ☐

42 $\frac{39}{95}$, $\frac{23}{60}$, $\frac{13}{40}$, $\frac{1}{4}$

Rewrite in Descending order ☐ , ☐ , ☐ , ☐

43 $\frac{13}{15}$, $\frac{9}{10}$, $\frac{29}{34}$, $\frac{18}{21}$

Rewrite in Descending order ☐ , ☐ , ☐ , ☐

Comparing Fractions (Calculator)

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Convert these to decimals round to 3 d.p.

1. $\frac{1}{10}$ 0.100
2. $\frac{1}{4}$ 0.250
3. $\frac{2}{3}$ 0.667
4. $\frac{3}{8}$ 0.375
5. $\frac{3}{4}$ 0.750
6. $\frac{5}{6}$ 0.833
7. $\frac{6}{13}$ 0.462
8. $\frac{8}{19}$ 0.421
9. $\frac{3}{7}$ 0.429
10. $\frac{24}{27}$ 0.889
11. $\frac{17}{29}$ 0.586
12. $\frac{7}{12}$ 0.583
13. $\frac{73}{80}$ 0.913
14. $\frac{33}{43}$ 0.767
15. $\frac{17}{42}$ 0.405
16. $\frac{96}{99}$ 0.970
17. $\frac{175}{305}$ 0.574
18. $\frac{212}{636}$ 0.333
19. $\frac{572}{911}$ 0.628
20. $\frac{101}{909}$ 0.111

Write the decimal (3 d.p) under the fraction and then answer true or false

21. $\frac{2}{3} < \frac{3}{4}$ True
$0.667 < 0.750$
22. $\frac{2}{5} > \frac{3}{7}$ False
$0.400 < 0.429$
23. $\frac{8}{9} < \frac{9}{10}$ True
$0.889 < 0.900$
24. $\frac{6}{7} > \frac{8}{10}$ True
$0.857 > 0.800$
25. $\frac{5}{8} < \frac{7}{14}$ False
$0.625 > 0.500$
26. $\frac{8}{11} > \frac{5}{7}$ True
$0.727 > 0.714$
27. $\frac{13}{17} < \frac{14}{19}$ False
$0.765 > 0.737$
28. $\frac{6}{81} < \frac{4}{63}$ False
$0.074 > 0.063$
29. $\frac{35}{39} > \frac{60}{70}$ True
$0.897 > 0.857$
30. $\frac{88}{93} < \frac{17}{19}$ False
$0.946 > 0.895$

Use the same method only fill in <, > or =

31. $\frac{2}{3} > \frac{3}{5}$
$0.667 > 0.600$
32. $\frac{3}{6} = \frac{18}{36}$
$0.500 = 0.500$
33. $\frac{4}{10} < \frac{27}{60}$
$0.400 < 0.450$
34. $\frac{3}{7} < \frac{4}{9}$
$0.429 < 0.444$
35. $\frac{8}{12} = \frac{2}{3}$
$0.667 = 0.667$
36. $\frac{4}{9} > \frac{17}{45}$
$0.444 > 0.378$
37. $\frac{4}{19} < \frac{14}{60}$
$0.211 < 0.233$
38. $\frac{11}{18} < \frac{26}{37}$
$0.611 < 0.703$
39. $\frac{3}{8} = \frac{21}{56}$
$0.375 = 0.375$
40. $\frac{12}{13} > \frac{16}{19}$
$0.923 > 0.842$

Use decimals to arrange these in descending order

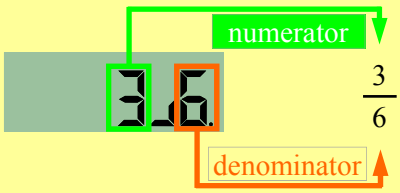
41. $\frac{3}{7}, \frac{2}{3}, \frac{10}{13}, \frac{17}{40}$
0.429 0.667 0.769 0.425
Rewrite in Descending order → $\frac{10}{13}, \frac{2}{3}, \frac{3}{7}, \frac{17}{40}$

42. $\frac{39}{95}, \frac{23}{60}, \frac{13}{40}, \frac{1}{4}$
0.411 0.383 0.325 0.250
Rewrite in Descending order → $\frac{39}{95}, \frac{23}{60}, \frac{13}{40}, \frac{1}{4}$

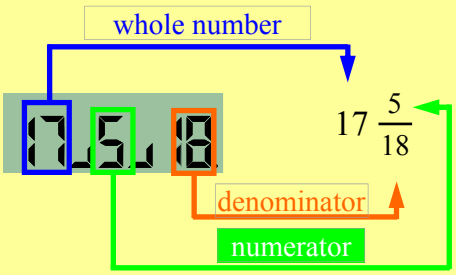
43. $\frac{13}{15}, \frac{9}{10}, \frac{29}{34}, \frac{18}{21}$
0.867 0.900 0.853 0.857
Rewrite in Descending order → $\frac{9}{10}, \frac{13}{15}, \frac{18}{21}, \frac{29}{34}$

Changing Between Mixed Numerals and Improper Fractions (Calculator)

This sheet is designed for you to learn how to change between mixed numerals and improper fractions using your calculator. A calculator separates the numerator and the denominator by a reversed 'L' (or sometimes an ‘r’), for mixed numerals the whole number is also separated by the 'L' (or ‘r’). See example below.



$\frac{3}{6}$



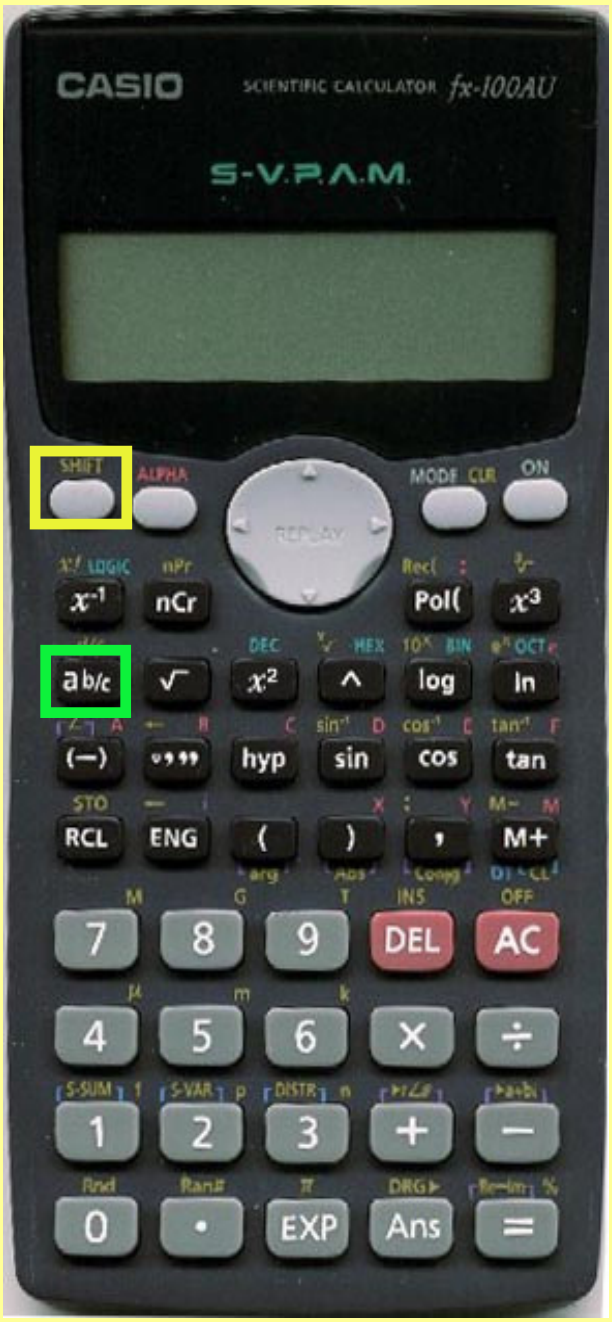
$17\frac{5}{18}$

The first column starts with the way your calculator displays fractions. If there are 2 numbers the first number is the numerator (number on top) the second number is the denominator (number on the bottom). These numbers are separated by the "reversed L". Don’t write fractions the calculator way, write the fractions separated by a line (as above). From Q. 6 on, you are asked to change mixed numerals to improper fractions. The method required is described on the next sheet, in words - type the fraction into the calculator, press =, then press shift and the fraction button.

Column 2 reverses the process and asks you to change improper fractions to mixed numerals, to answer these put the fraction in the calculator and press =, the mixed numeral will be displayed. Question 46 through 53 asks you to repeat the same process, if you use the ÷ sign then you will get a decimal answer, press the fraction key and it will change it to a fraction. Or a quicker step is to use the fraction key instead of the ÷ sign this will give the answer as a fraction straight away.

Note: These are all the same: $8\overline{)3}$ $\frac{3}{8}$ $3 \div 8$

The last column asks you to compare the improper fractions with the mixed numerals. Put the improper fraction into your calculator and press the ‘equals’ key. If the fraction displayed on the screen is the same as the answer supplied write 'true', if it isn't, write ‘false’. At the bottom of the column you are asked to show the pairs that ‘match’ (an improper and a mixed number) by using pairs of shapes (a smaller shape for the improper fractions, larger one for mixed).



Example of how to convert Mixed numerals to Improper Fractions

$3\ \frac{6}{7} = \frac{\quad}{\quad}$?

Don't forget the 'equals' sign

SHIFT

$\boxed{3}\ \boxed{a\,b/c}\ \boxed{6}\ \boxed{a\,b/c}\ \boxed{7}\ \boxed{=}\ \boxed{}\ \boxed{a\,b/c}$

if you get $\frac{27}{7}$ as your answer you have done it correctly

Changing Between Mixed Numerals and Improper Fractions (Calculator)

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Write the fraction represented by the calculator screens

1 

2 

3 

4 

5 

Change these mixed numbers to improper fractions

6 $3\frac{1}{2} = \frac{\square}{\square}$ 7 $1\frac{1}{3} = \frac{\square}{\square}$

8 $2\frac{1}{4} = \frac{\square}{\square}$ 9 $1\frac{1}{2} = \frac{\square}{\square}$

10 $1\frac{2}{5} = \frac{\square}{\square}$ 11 $1\frac{5}{12} = \frac{\square}{\square}$

12 $2\frac{2}{3} = \frac{\square}{\square}$ 13 $2\frac{4}{7} = \frac{\square}{\square}$

14 $2\frac{5}{6} = \frac{\square}{\square}$ 15 $3\frac{1}{4} = \frac{\square}{\square}$

16 $1\frac{3}{4} = \frac{\square}{\square}$ 17 $12\frac{1}{7} = \frac{\square}{\square}$

18 $7\frac{1}{3} = \frac{\square}{\square}$ 19 $3\frac{5}{11} = \frac{\square}{\square}$

20 $3\frac{1}{8} = \frac{\square}{\square}$ 21 $9\frac{2}{3} = \frac{\square}{\square}$

22 $4\frac{4}{5} = \frac{\square}{\square}$ 23 $1\frac{1}{10} = \frac{\square}{\square}$

24 $10\frac{2}{9} = \frac{\square}{\square}$ 25 $11\frac{3}{4} = \frac{\square}{\square}$

Change these improper fractions to mixed numerals

26 $\frac{5}{2} = \square\frac{\square}{\square}$ 27 $\frac{7}{4} = \square\frac{\square}{\square}$

28 $\frac{3}{2} = \square\frac{\square}{\square}$ 29 $\frac{8}{5} = \square\frac{\square}{\square}$

30 $\frac{16}{7} = \square\frac{\square}{\square}$ 31 $\frac{24}{7} = \square\frac{\square}{\square}$

32 $\frac{23}{5} = \square\frac{\square}{\square}$ 33 $\frac{16}{3} = \square\frac{\square}{\square}$

34 $\frac{13}{2} = \square\frac{\square}{\square}$ 35 $\frac{19}{7} = \square\frac{\square}{\square}$

36 $\frac{13}{2} = \square\frac{\square}{\square}$ 37 $\frac{28}{3} = \square\frac{\square}{\square}$

38 $\frac{15}{4} = \square\frac{\square}{\square}$ 39 $\frac{17}{10} = \square\frac{\square}{\square}$

40 $\frac{13}{6} = \square\frac{\square}{\square}$ 41 $\frac{53}{5} = \square\frac{\square}{\square}$

Write the answers as mixed numerals

42 $11 \div 7 = \square\frac{\square}{\square}$

43 $109 \div 10 = \square\frac{\square}{\square}$

44 $3\overline{)125} = \square\frac{\square}{\square}$

45 $20 \div 3 = \square\frac{\square}{\square}$

46 $7\overline{)321} = \square\frac{\square}{\square}$

47 $47 \div 5 = \square\frac{\square}{\square}$

Change improper to mixed to answer true of false to these

48 $\frac{8}{2} = 4\frac{1}{2}$ _____

49 $4\frac{3}{8} = \frac{43}{8}$ _____

50 $7\frac{2}{7} = \frac{51}{7}$ _____

51 $8\frac{5}{9} = \frac{75}{9}$ _____

52 $\frac{11}{6} = 1\frac{5}{6}$ _____

53 $\frac{24}{7} = 3\frac{4}{7}$ _____

54 $1\frac{7}{8} = \frac{15}{8}$ _____

55 $\frac{17}{7} = 2\frac{3}{7}$ _____

56 $\frac{13}{11} = 1\frac{3}{11}$ _____

57 $4\frac{1}{3} = \frac{15}{3}$ _____

Using circles, squares, triangles or other shapes draw the same shape around matching pairs

58

$4\frac{5}{7}$ $\frac{16}{3}$ $5\frac{1}{3}$ $4\frac{3}{7}$ $\frac{33}{7}$ $\frac{22}{7}$ $\frac{17}{3}$ $5\frac{2}{3}$ $6\frac{2}{3}$ $\frac{22}{3}$ $\frac{20}{3}$ $\frac{31}{7}$ $3\frac{1}{7}$ $7\frac{1}{3}$

Changing Between Mixed Numerals and Improper Fractions (Calculator)

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Write the fraction represented by the calculator screens

1 

$\frac{2}{5}$

2 

$\frac{3}{2}$

3 

4 $\frac{8}{9}$

4 

11 $\frac{3}{20}$

5 

10 $\frac{1}{10}$

Change these mixed numbers to improper fractions

6 $3\frac{1}{2} = \frac{7}{2}$

7 $1\frac{1}{3} = \frac{4}{3}$

8 $2\frac{1}{4} = \frac{9}{4}$

9 $1\frac{1}{2} = \frac{3}{2}$

10 $1\frac{2}{5} = \frac{7}{5}$

11 $1\frac{5}{12} = \frac{17}{12}$

12 $2\frac{2}{3} = \frac{8}{3}$

13 $2\frac{4}{7} = \frac{18}{7}$

14 $2\frac{5}{6} = \frac{17}{6}$

15 $3\frac{1}{4} = \frac{13}{4}$

16 $1\frac{3}{4} = \frac{7}{4}$

17 $12\frac{1}{7} = \frac{85}{7}$

18 $7\frac{1}{3} = \frac{22}{3}$

19 $3\frac{5}{11} = \frac{38}{11}$

20 $3\frac{1}{8} = \frac{25}{8}$

21 $9\frac{2}{3} = \frac{29}{3}$

22 $4\frac{4}{5} = \frac{24}{5}$

23 $1\frac{1}{10} = \frac{11}{10}$

24 $10\frac{2}{9} = \frac{92}{10}$

25 $11\frac{3}{4} = \frac{47}{4}$

Change these improper fractions to mixed numerals

26 $\frac{5}{2} = 2\frac{1}{2}$

27 $\frac{7}{4} = 1\frac{3}{4}$

28 $\frac{3}{2} = 1\frac{1}{2}$

29 $\frac{8}{5} = 1\frac{3}{5}$

30 $\frac{16}{7} = 2\frac{2}{7}$

31 $\frac{24}{7} = 3\frac{3}{7}$

32 $\frac{23}{5} = 4\frac{3}{5}$

33 $\frac{16}{3} = 5\frac{1}{3}$

34 $\frac{13}{2} = 6\frac{1}{2}$

35 $\frac{19}{7} = 2\frac{5}{7}$

36 $\frac{25}{9} = 2\frac{7}{9}$

37 $\frac{28}{3} = 9\frac{1}{3}$

38 $\frac{15}{4} = 3\frac{3}{4}$

39 $\frac{17}{10} = 1\frac{7}{10}$

40 $\frac{13}{6} = 2\frac{1}{6}$

41 $\frac{53}{5} = 10\frac{3}{5}$

Write the answers as mixed numerals

42 $11 \div 7 = 1\frac{4}{7}$

43 $109 \div 10 = 10\frac{9}{10}$

44 $3\overline{)125} = 41\frac{2}{3}$

45 $20 \div 3 = 6\frac{2}{3}$

46 $7\overline{)321} = 45\frac{6}{7}$

47 $47 \div 5 = 9\frac{2}{5}$

Change improper to mixed to answer true of false to these

48 $\frac{8}{2} = 4\frac{1}{2}$ False

49 $4\frac{3}{8} = \frac{43}{8}$ False

50 $7\frac{2}{7} = \frac{51}{7}$ True

51 $8\frac{5}{9} = \frac{75}{9}$ False

52 $\frac{11}{6} = 1\frac{5}{6}$ True

53 $\frac{24}{7} = 3\frac{4}{7}$ False

54 $1\frac{7}{8} = \frac{15}{8}$ True

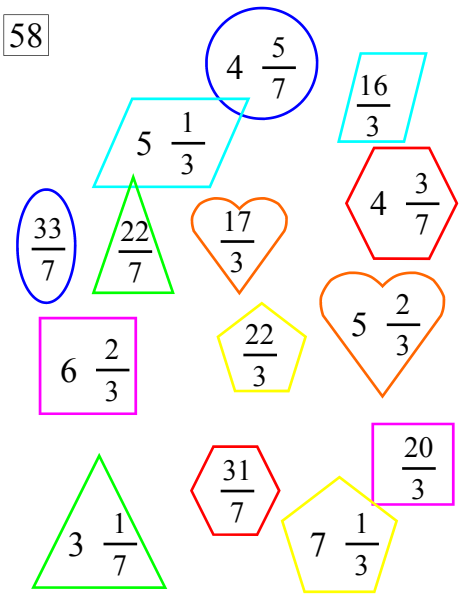
55 $\frac{17}{7} = 2\frac{3}{7}$ True

56 $\frac{13}{11} = 1\frac{3}{11}$ False

57 $4\frac{1}{3} = \frac{15}{3}$ False

Using circles, squares, triangles or other shapes draw the same shape around matching pairs

58



Mixed Operations - Calculator

This sheet is for calculator use so there are no working spaces provided.

The method is simple for this sheet, use your calculator. You don't have to worry about reciprocals or simplifying, the calculator does it all for you. Mistakes made using the calculator can occur anywhere due to keystroke error, there is one more common mistake made though, that is the entering of mixed numbers. Remember that you have to press the fraction key twice to enter a mixed number.

Your calculator should obey order of operations rules in the exercises in column 3, just don't press '=' until you key in the entire question otherwise the calculator may give the wrong answer.

Mixed Operations - Calculator

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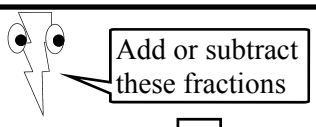


1 $\frac{1}{2} + \frac{1}{3} = \square$

2 $\frac{4}{5} - \frac{3}{7} = \square$

3 $\frac{4}{5} - \frac{3}{10} = \square$

4 $\frac{3}{11} + \frac{3}{5} = \square$

5 $\frac{3}{4} - \frac{1}{6} + \frac{4}{5} = \square$

6 $\frac{4}{5} + \frac{3}{8} - \frac{2}{15} = \square$

7 $\frac{9}{12} + \frac{5}{7} - \frac{1}{4} = \square$

8 $\frac{17}{20} - \frac{3}{10} + \frac{8}{15} = \square$

9 $\frac{4}{5} + 2\frac{2}{3} - \frac{7}{10} = \square$

10 $4\frac{6}{7} - \frac{2}{3} - \frac{4}{9} = \square$

11 $2\frac{4}{11} - 1\frac{1}{2} + \frac{5}{6} = \square$

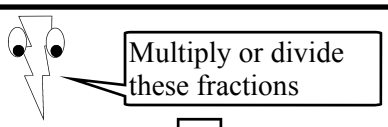
12 $4\frac{3}{5} + \frac{2}{3} - 1\frac{7}{10} = \square$

13 $3\frac{5}{6} - 1\frac{1}{4} + 4\frac{7}{12} = \square$

14 $3\frac{2}{5} + 4\frac{7}{10} + 2\frac{5}{12} = \square$

15 $5\frac{1}{4} - 2\frac{2}{3} - 1\frac{1}{2} = \square$

16 $9\frac{2}{3} - 3\frac{6}{7} + 1\frac{5}{6} = \square$



17 $\frac{3}{5} \times \frac{1}{4} = \square$

18 $\frac{7}{12} \div \frac{2}{3} = \square$

19 $\frac{2}{3} \div \frac{8}{9} = \square$

20 $\frac{3}{5} \times \frac{7}{6} = \square$

21 $\frac{4}{5} \div \frac{1}{3} \times \frac{8}{9} = \square$

22 $\frac{1}{7} \div \frac{3}{10} \times \frac{18}{5} = \square$

23 $\frac{3}{4} \times \frac{17}{10} \div \frac{1}{8} = \square$

24 $\frac{4}{7} \div \frac{1}{10} \div \frac{2}{5} = \square$

25 $\frac{3}{8} \times 4\frac{1}{6} \times \frac{2}{3} = \square$

26 $5\frac{11}{12} \div \frac{3}{4} \times \frac{1}{2} = \square$

27 $3\frac{1}{2} \times 6 \times \frac{4}{5} = \square$

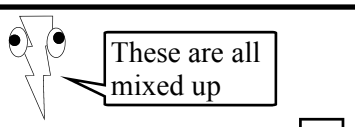
28 $7\frac{3}{4} \times \frac{2}{5} \div 2\frac{2}{3} = \square$

29 $7 \div 1\frac{1}{4} \div 2\frac{3}{5} = \square$

30 $4\frac{5}{6} \times 3\frac{3}{10} \div 2\frac{2}{5} = \square$

31 $5\frac{3}{14} \div 8 \times 4\frac{4}{5} = \square$

32 $2\frac{3}{7} \div 1\frac{2}{3} \times 3 = \square$



33 $\frac{1}{3} + \frac{3}{4} \times \frac{9}{5} = \square$

34 $\frac{1}{5} \div \frac{2}{15} + \frac{2}{3} = \square$

35 $5\frac{1}{6} \times 3 - 4\frac{6}{7} = \square$

36 $7\frac{3}{8} \div 9\frac{1}{4} + \frac{3}{2} = \square$

37 $\frac{5}{4} + 3\frac{1}{2} \div \frac{5}{6} = \square$

38 $5 \div \frac{7}{10} \times \frac{8}{9} = \square$

39 $\frac{1}{2} \times \frac{7}{11} + \frac{3}{4} = \square$

40 $7\frac{5}{6} - \frac{11}{12} \times 1\frac{2}{3} = \square$

41 $3\frac{3}{4} \div \frac{5}{9} - \frac{5}{12} = \square$

42 $2\frac{5}{8} \times 3\frac{1}{6} \div 5 = \square$

43 $9\frac{1}{2} - 3\frac{2}{3} \div 1\frac{4}{5} = \square$

44 $3\frac{8}{9} \times 1\frac{2}{5} \times 3\frac{1}{2} = \square$

45 $\frac{2}{5} \times \frac{8}{3} + \frac{1}{2} = \square$

46 $4\frac{7}{10} \div \frac{1}{4} \div 1\frac{1}{5} = \square$

47 $2\frac{2}{5} + \frac{3}{4} \div \frac{1}{2} = \square$

48 $5\frac{7}{8} \div 2\frac{1}{3} - 1\frac{1}{4} = \square$

Mixed Operations - Calculator

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Add or subtract these fractions

1 $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

2 $\frac{4}{5} - \frac{3}{7} = \frac{13}{35}$

3 $\frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

4 $\frac{3}{11} + \frac{3}{5} = \frac{48}{55}$

5 $\frac{3}{4} - \frac{1}{6} + \frac{4}{5} = 1\frac{23}{60}$

6 $\frac{4}{5} + \frac{3}{8} - \frac{2}{15} = 1\frac{1}{24}$

7 $\frac{9}{12} + \frac{5}{7} - \frac{1}{4} = 1\frac{3}{14}$

8 $\frac{17}{20} - \frac{3}{10} + \frac{8}{15} = 1\frac{1}{12}$

9 $\frac{4}{5} + 2\frac{2}{3} - \frac{7}{10} = 2\frac{23}{30}$

10 $4\frac{6}{7} - \frac{2}{3} - \frac{4}{9} = 3\frac{47}{63}$

11 $2\frac{4}{11} - 1\frac{1}{2} + \frac{5}{6} = 1\frac{23}{33}$

12 $4\frac{3}{5} + \frac{2}{3} - 1\frac{7}{10} = 3\frac{17}{30}$

13 $3\frac{5}{6} - 1\frac{1}{4} + 4\frac{7}{12} = 7\frac{1}{6}$

14 $3\frac{2}{5} + 4\frac{7}{10} + 2\frac{5}{12} = 10\frac{31}{60}$

15 $5\frac{1}{4} - 2\frac{2}{3} - 1\frac{1}{2} = 1\frac{1}{12}$

16 $9\frac{2}{3} - 3\frac{6}{7} + 1\frac{5}{6} = 7\frac{9}{14}$



Multiply or divide these fractions

17 $\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$

18 $\frac{7}{12} \div \frac{2}{3} = \frac{7}{8}$

19 $\frac{2}{3} \div \frac{8}{9} = \frac{3}{4}$

20 $\frac{3}{5} \times \frac{7}{6} = \frac{7}{10}$

21 $\frac{4}{5} \div \frac{1}{3} \times \frac{8}{9} = 2\frac{2}{15}$

22 $\frac{1}{7} \div \frac{3}{10} \times \frac{18}{5} = 1\frac{5}{7}$

23 $\frac{3}{4} \times \frac{17}{10} \div \frac{1}{8} = 10\frac{1}{5}$

24 $\frac{4}{7} \div \frac{1}{10} \div \frac{2}{5} = 14\frac{2}{7}$

25 $\frac{3}{8} \times 4\frac{1}{6} \times \frac{2}{3} = 1\frac{1}{24}$

26 $5\frac{11}{12} \div \frac{3}{4} \times \frac{1}{2} = 3\frac{17}{18}$

27 $3\frac{1}{2} \times 6 \times \frac{4}{5} = 16\frac{4}{5}$

28 $7\frac{3}{4} \times \frac{2}{5} \div 2\frac{2}{3} = 1\frac{13}{80}$

29 $7 \div 1\frac{1}{4} \div 2\frac{3}{5} = 2\frac{2}{13}$

30 $4\frac{5}{6} \times 3\frac{3}{10} \div 2\frac{2}{5} = 6\frac{31}{48}$

31 $5\frac{3}{14} \div 8 \times 4\frac{4}{5} = 3\frac{9}{70}$

32 $2\frac{3}{7} \div 1\frac{2}{3} \times 3 = 4\frac{13}{35}$



These are all mixed up

33 $\frac{1}{3} + \frac{3}{4} \times \frac{9}{5} = 1\frac{41}{60}$

34 $\frac{1}{5} \div \frac{2}{15} + \frac{2}{3} = 2\frac{1}{6}$

35 $5\frac{1}{6} \times 3 - 4\frac{6}{7} = 10\frac{9}{14}$

36 $7\frac{3}{8} \div 9\frac{1}{4} + \frac{3}{2} = 2\frac{11}{37}$

37 $\frac{5}{4} + 3\frac{1}{2} \div \frac{5}{6} = 5\frac{9}{20}$

38 $5 \div \frac{7}{10} \times \frac{8}{9} = 6\frac{22}{63}$

39 $\frac{1}{2} \times \frac{7}{11} + \frac{3}{4} = 1\frac{3}{44}$

40 $7\frac{5}{6} - \frac{11}{12} \times 1\frac{2}{3} = 6\frac{11}{36}$

41 $3\frac{3}{4} \div \frac{5}{9} - \frac{5}{12} = 6\frac{1}{3}$

42 $2\frac{5}{8} \times 3\frac{1}{6} \div 5 = 1\frac{53}{80}$

43 $9\frac{1}{2} - 3\frac{2}{3} \div 1\frac{4}{5} = 7\frac{25}{54}$

44 $3\frac{8}{9} \times 1\frac{2}{5} \times 3\frac{1}{2} = 19\frac{1}{18}$

45 $\frac{2}{5} \times \frac{8}{3} + \frac{1}{2} = 1\frac{17}{30}$

46 $4\frac{7}{10} \div \frac{1}{4} \div 1\frac{1}{5} = 15\frac{2}{3}$

47 $2\frac{2}{5} + \frac{3}{4} \div \frac{1}{2} = 3\frac{9}{10}$

48 $5\frac{7}{8} \div 2\frac{1}{3} - 1\frac{1}{4} = 1\frac{15}{56}$

Calculating Percentages - Calculator

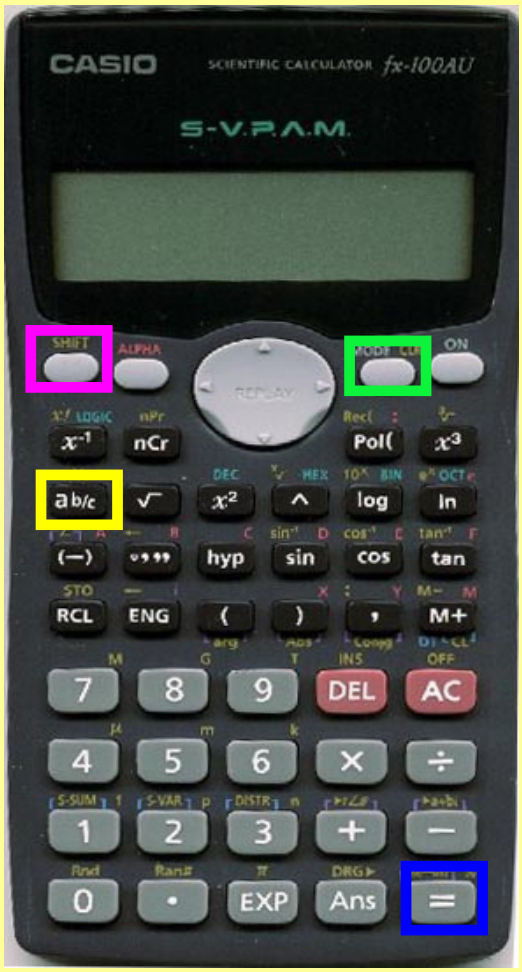
The method of calculating a percentage from a decimal or fraction is to multiply the decimal or fraction by 100. To change a percentage to a decimal or fraction you divide by 100.

A picture of a calculator is shown with below with keys highlighted. The methods used for each column are also listed on the following pages. Where there is a alternative method it is shown, so you can select the method that makes the most sense to you.

Column 1 asks you to convert fractions to percentages to 2 decimal places, then to a specified number of decimal places (in brackets). Multiply the fractions by 100 for the answer. Note that if writing a whole number, such as 45%, when asked to give to 2 d.p. then the answer can be 45% or 45.00%.

Column 2 asks you to convert the decimals to percentages. Again multiply the question by 100.

Column 3 deals with changing percentages back to fractions or decimals. In this case divide by 100.



Column 2

Example of calculating a decimal percentage from a decimal

Convert 0.206 to a percentage

Using × 100

0 . 2 0 6 × 1 0 0 =

if you get 20.6 for the question above you are correct

Column 1

Example of calculating a decimal percentage from a fraction

Convert $^{7}/_{8}$ to a percentage

Using % key (not recommended)

7 ÷ 8 Shift %

Using × 100

7 ÷ 8 × 1 0 0 =

Using fraction key

7 $a^b/_c$ 8 × 1 0 0 = $a^b/_c$

if you get 87.5 (no fix) for the question above you are correct

Example of calculating a fraction percentage from a fraction

Convert $^{7}/_{8}$ to a percentage

Using fraction key

7 $a^b/_c$ 8 × 1 0 0 =

Using × 100

7 ÷ 8 × 1 0 0 = $a^b/_c$

if you get 87 ½ for the question above you are correct
note that this style of question isn't on the sheet

Example of calculating a decimal percentage from a mixed numeral

Convert $2\ ^{7}/_{8}$ to a percentage

Using fraction key

2 $a^b/_c$ 7 $a^b/_c$ 8 × 1 0 0 = $a^b/_c$

Using × 100

2 + 7 ÷ 8 = × 1 0 0 =

if you get 287.5 for the question above you are correct

Column 3

Example of converting a decimal percentage to a decimal

Convert 56.32% to a decimal

Using ÷ 100

5 6 . 3 2 ÷ 1 0 0 =

if you get 0.5632 (no fix) for the question above you are correct

Example of converting a fraction percentage to a decimal

Convert 56 ¼% to a decimal

Using the fraction key

5 6 $a^b/_c$ 1 $a^b/_c$ 4 ÷ 1 0 0 = $a^b/_c$

if you get 0.5625 (no fix) for the question above you are correct

Example of converting a decimal percentage to a fraction

Convert 56.25% to a fraction

Using the fraction key Note that the featured calculator can convert decimals to fractions, this may not be a feature on your calculator

5 6 . 2 5 ÷ 1 0 0 = $a^b/_c$

If your calculator has a fraction key but can't convert decimals to fractions then use this method (note you need some mental skills)

5 6 $a^b/_c$ 2 5 $a^b/_c$ 1 0 0 ÷ 1 0 0 =

if you get $^9/_{16}$ for the question above you are correct

Example of converting a fraction percentage to a fraction

Convert 56 ¼% to a fraction

Using the fraction key

5 6 $a^b/_c$ 1 $a^b/_c$ 4 ÷ 1 0 0 =

if you get $^9/_{16}$ for the question above you are correct

Calculating Percentages - Calculator

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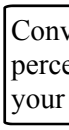
Convert these fractions to percentages (by × 100). Round answer to 2 d.p.

1	$^{2}/_{3}$	=	%
2	$^{5}/_{7}$	=	
3	$^{1}/_{8}$	=	
4	$^{1}/_{12}$	=	
5	$^{4}/_{13}$	=	
6	$1\ ^{56}/_{83}$	=	
7	$^{124}/_{171}$	=	
8	$^{85}/_{302}$	=	
9	$^{206}/_{118}$	=	
10	$4\ ^{357}/_{502}$	=	



Continue converting to percentages but round your answer to the d.p. in the brackets.

11	$^{51}/_{75}$ [1]	=	%
12	$^{131}/_{165}$ [2]	=	
13	$3\ ^{81}/_{97}$ [2]	=	
14	$^{1}/_{3}$ [3]	=	
15	$1\ ^{144}/_{758}$ [1]	=	
16	$^{41}/_{85}$ [3]	=	
17	$^{288}/_{650}$ [3]	=	
18	$^{65}/_{91}$ [1]	=	
19	$2\ ^{32}/_{55}$ [0]	=	
20	$^{193}/_{210}$ [2]	=	
21	$^{5}/_{8}$ [2]	=	
22	$^{249}/_{250}$ [0]	=	



Convert these decimals to percentages (× 100). Give your answer to 1 d.p.

23	0.15	=	%
24	0.439	=	
25	1.266	=	
26	0.78841	=	
27	0.0606	=	
28	3.0295	=	
29	0.0074	=	
30	0.4989	=	
31	1.0907	=	
32	0.9	=	



Round the percentage answer to the d.p. asked

33	0.021141 [3]	=	%
34	1.30055 [2]	=	
35	1.08 [1]	=	
36	0.29992 [2]	=	
37	4.00681 [2]	=	
38	0.0096 [1]	=	
39	11.0016 [1]	=	
40	0.701061 [3]	=	
41	0.349211 [0]	=	
42	0.59097 [2]	=	
43	0.087026 [3]	=	
44	0.389001 [0]	=	
45	1.999802 [1]	=	



Convert these percentages to decimals (÷ 100). Give your answer to 3 d.p.

46	95%	=	
47	102.7%	=	
48	0.13%	=	
49	11.63%	=	
50	225.46%	=	
51	0.3%	=	
52	½%	=	
53	33⅓%	=	
54	1.03%	=	
55	67 ¾%	=	



Write the percentages below as fractions

56	15%	=	$^{\square}/_{\square}$
57	68.4%	=	$^{\square}/_{\square}$
58	12.5%	=	$^{\square}/_{\square}$
59	75.4%	=	$^{\square}/_{\square}$
60	117.5%	=	$\square\ ^{\square}/_{\square}$
61	42.36%	=	$^{\square}/_{\square}$
62	½%	=	$^{\square}/_{\square}$
63	18.8%	=	$^{\square}/_{\square}$

Calculating Percentages - Calculator

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Convert these fractions to percentages (by × 100). Round answer to 2 d.p.

1	$^{2}/_{3}$	= 66.67 %
2	$^{5}/_{7}$	= 71.43%
3	$^{1}/_{8}$	= 12.50%
4	$^{1}/_{12}$	= 8.33%
5	$^{4}/_{13}$	= 30.77%
6	$1\ ^{56}/_{83}$	= 167.47%
7	$^{124}/_{171}$	= 72.51%
8	$^{85}/_{302}$	= 28.15%
9	$^{206}/_{118}$	= 174.58%
10	$4\ ^{357}/_{502}$	= 471.12%

Convert these decimals to percentages (× 100). Give your answer to 1 d.p.

23	0.15	= 15.0 %
24	0.439	= 43.9%
25	1.266	= 126.6%
26	0.78841	= 78.8%
27	0.0606	= 6.1%
28	3.0295	= 303.0%
29	0.0074	= 0.7%
30	0.4989	= 49.9%
31	1.0907	= 109.1%
32	0.9	= 90.0%

Convert these percentages to decimals (÷ 100). Give your answer to 3 d.p.

46	95%	= 0.950
47	102.7%	= 1.027
48	0.13%	= 0.001
49	11.63%	= 0.116
50	225.46%	= 2.255
51	0.3%	= 0.003
52	½%	= 0.005
53	33⅓%	= 0.333
54	1.03%	= 0.010
55	67 ¾%	= 0.678

Continue converting to percentages but round your answer to the d.p. in the brackets.

11	$^{51}/_{75}$ [1]	= 68.0 %
12	$^{131}/_{165}$ [2]	= 79.39%
13	$3\ ^{81}/_{97}$ [2]	= 383.51%
14	$^{1}/_{3}$ [3]	= 33.333%
15	$1\ ^{144}/_{758}$ [1]	= 119.0%
16	$^{41}/_{85}$ [3]	= 48.235%
17	$^{288}/_{650}$ [3]	= 44.308%
18	$^{65}/_{91}$ [1]	= 71.4%
19	$2\ ^{32}/_{55}$ [0]	= 258%
20	$^{193}/_{210}$ [2]	= 91.90%
21	$^{5}/_{8}$ [2]	= 62.50%
22	$^{249}/_{250}$ [0]	= 100%

Round the percentage answer to the d.p. asked

33	0.021141 [3]	= 2.114 %
34	1.30055 [2]	= 130.06%
35	1.08 [1]	= 108.0%
36	0.29992 [2]	= 29.99%
37	4.00681 [2]	= 400.68%
38	0.0096 [1]	= 1.0%
39	11.0016 [1]	= 1 100.2%
40	0.701061 [3]	= 70.106%
41	0.349211 [0]	= 35%
42	0.59097 [2]	= 59.10%
43	0.087026 [3]	= 8.703%
44	0.389001 [0]	= 39%
45	1.999802 [1]	= 200.0%

Write the percentages below as fractions

56	15%	= $^{3}/_{20}$
57	68.4%	= $^{171}/_{250}$
58	12.5%	= $^{1}/_{8}$
59	75.4%	= $^{377}/_{500}$
60	117.5%	= $1\ ^{7}/_{40}$
61	42.36%	= $^{1\,059}/_{2\,500}$
62	½%	= $^{1}/_{200}$
63	18.8%	= $^{47}/_{250}$

FREEFALL MATHEMATICS

ALTITUDE

MEASUREMENT & PERIMETER

Using Scales

One method of measuring weight is to use scales. The faces of these scales are shown in 100 g intervals with larger divisions every 500 g. The kilogram readings have a number assigned. Starting at zero through to 5 kg, the scale travels past 5 kg but doesn't reach 6 kg, the maximum reading being 5.5 kg.

Column 1 asks you to read the weight displayed on the scales. The pointers are on an exact division. So answer in kg, but to one decimal place, ie: readings such as 3.2 kg or 4.1 kg.

Column 2 is the reverse, position the pointers so that they represent the given weights. Note that you are required to approximate the position of the needle as it may have to be positioned in between divisions, (Q.7 and Q.10). It also asks questions in grams.

Column 3 is similar to Column 2 except that an addition is required first. The table describes a certain number of 100 g, 250 g and 1 kg weights that are on a particular scale. Add them across the row of the table and then write the total next to the question number, then point that pointer.

Using Scales

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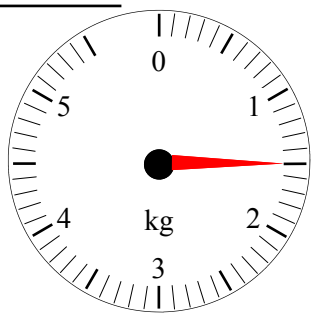
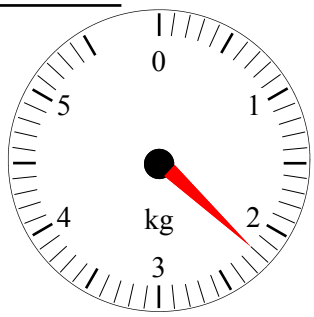
Write the weight shown on the scales below.

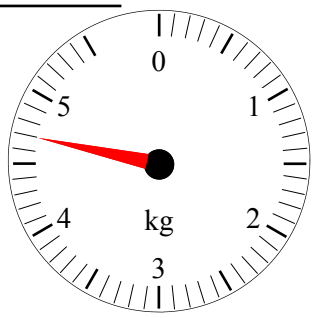
1 _____ kg



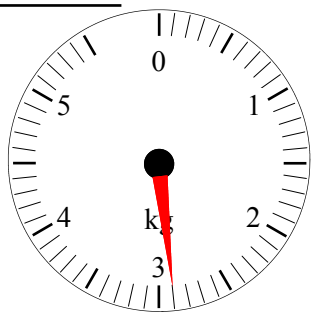
2 _____



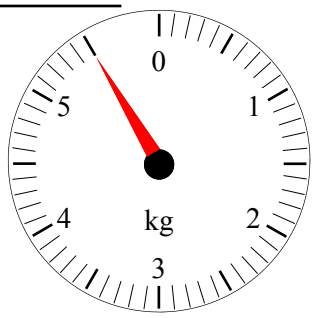
3 _____



4 _____

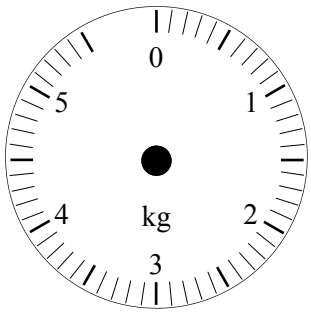


5 _____

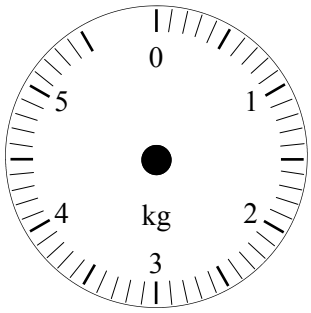


For the weights given draw the pointer in the correct position

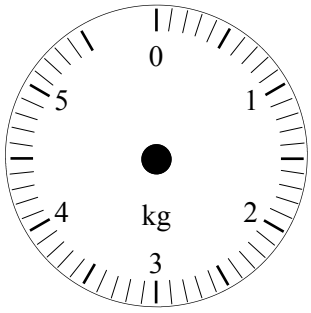
6 4.4 kg



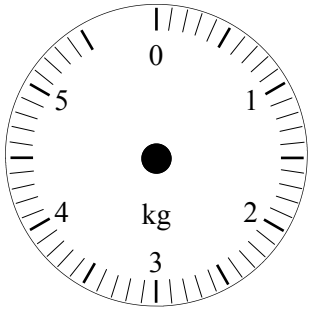
7 2.55 kg



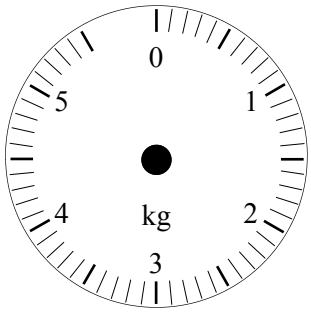
8 900 g



9 3 200 g



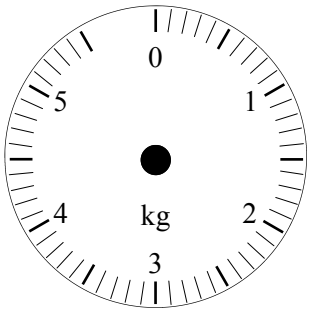
10 5.05 kg



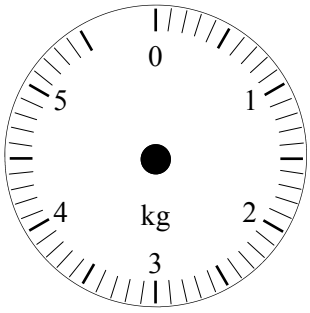
Add the weights in the table below, put the total in the space then point the pointer to the total weight

Scale	100 g	250 g	1 kg
11	3	-	2
12	1	2	3
13	2	-	2
14	1	1	4

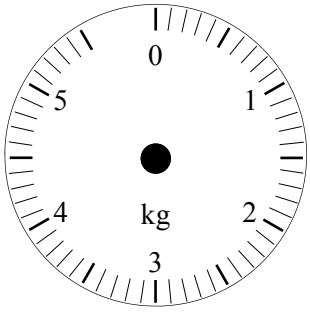
11 _____



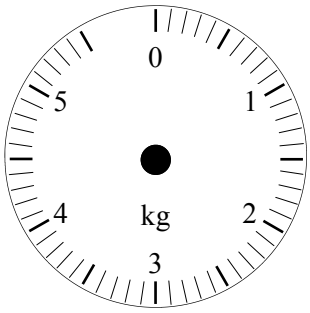
12 _____



13 _____



14 _____



Using Scales

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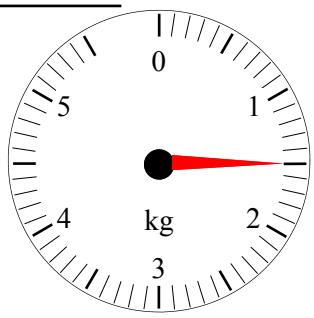
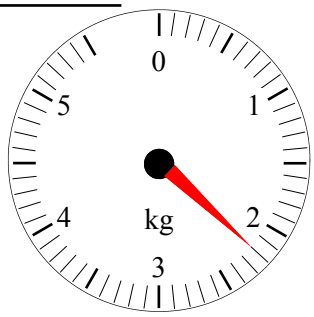
Write the weight shown on the scales below.

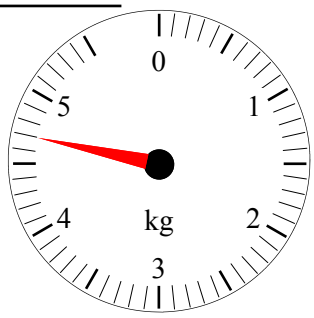
1 1.5 kg



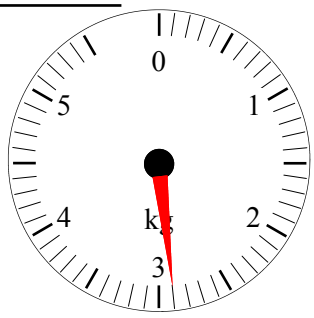
2 2.2 kg



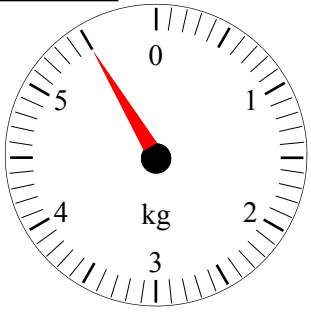
3 4.7 kg



4 2.9 kg

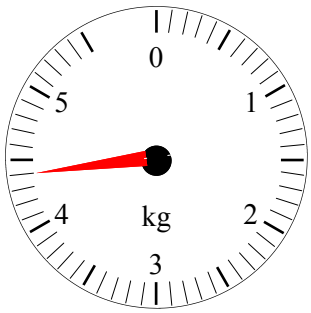


5 5.5 kg

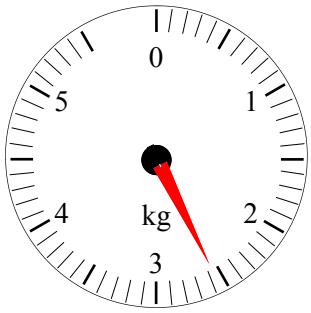


For the weights given draw the pointer in the correct position

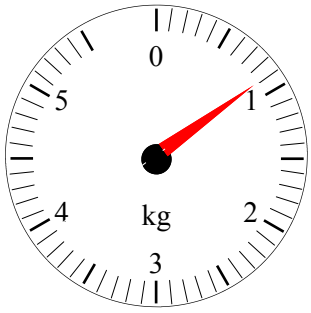
6 4.4 kg



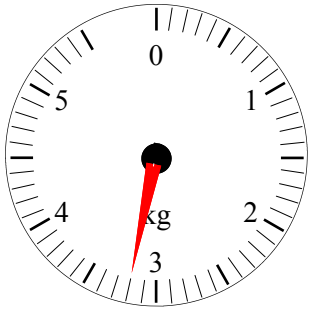
7 2.55 kg



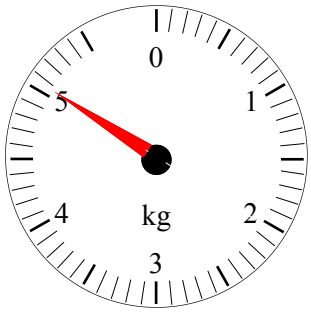
8 900 g



9 3 200 g



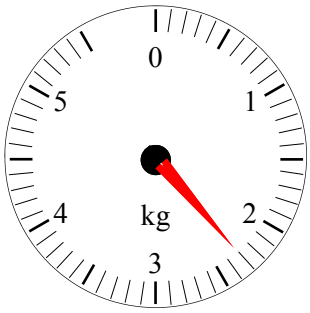
10 5.05 kg



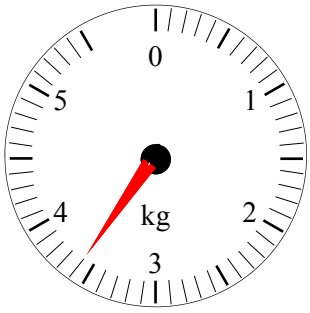
Add the weights in the table below, put the total in the space then point the pointer to the total weight

Scale	100 g	250 g	1 kg
11	3	-	2
12	1	2	3
13	2	-	2
14	1	1	4

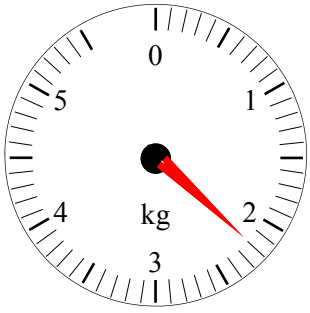
11 2.3 kg



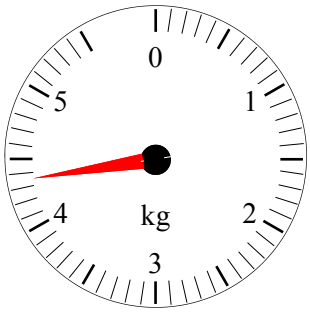
12 3.6 kg



13 2.2 kg



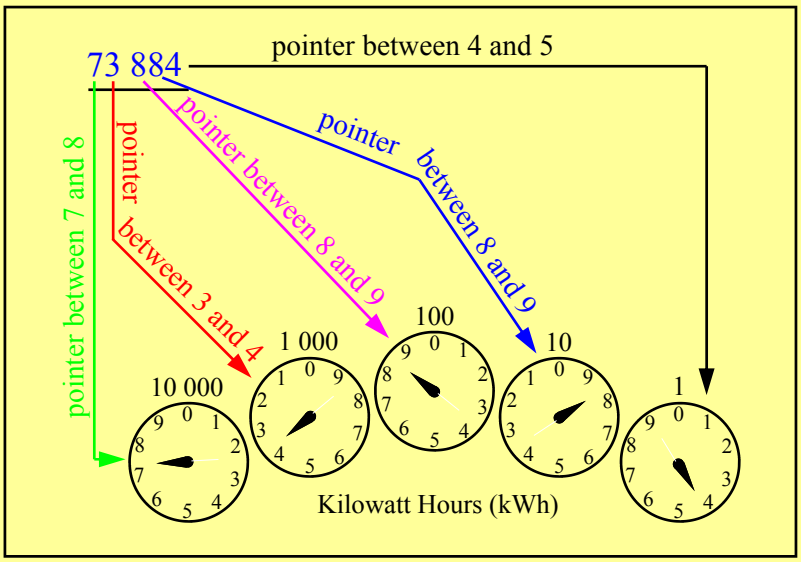
14 4.35 kg



Reading Water and Electricity Meters

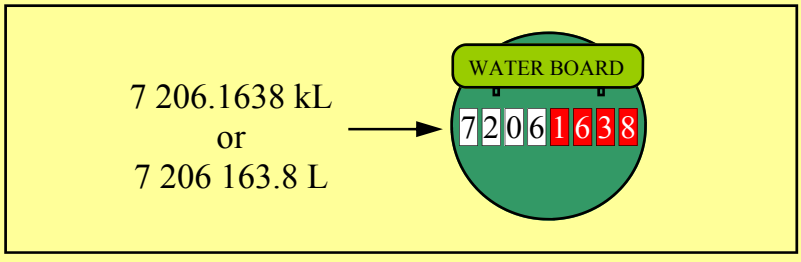
The water and electricity supplied to your home passes through meters which record the amount used. Electricity meters are gear-driven, the scales changing between clockwise and anti-clockwise rotation. So the adjacent pointers spin in opposite directions. There is an easy way to read the scales, just look where the needle is and the number it represents is the smallest of the two numbers the needle is between. Note that zero is both 10 and zero. If the pointer is between the 9 and 0 the smallest number is 9, (zero = 10). If the pointer is between 0 and 1 the smallest number is 0, as zero = 0.

Column 1 starts with 4 questions reading the meters shown. The meters give a 5 digit reading from 10 thousands down to units (1's), the units are in kWh (kilowatt-hours). Read each from left to right and build the five digit number. The next 4 questions ask you to put the pointers in the correct position. Start with the 10 000's first and work your way left to the right.



Column 2 are water meters, these are more standard in layout. The meters measure water usage in kL (kilolitres). The end 4 digits are inversed (white on red). These end digits represent 4 decimal places in kL. If measuring in L, the last digit is tenths of a L (1 d.p). The first 3 questions compare a 'before' and 'after' reading and then require subtraction to calculate the amount of water used. Note that these meters deal with large units, some numbers are not required in the calculation due to them not changing, a short cut - but be careful!

The last 3 questions give the 'before reading' and then require this reading to be added to the usage to get the 'current reading'. After completing the sum, write the answer in the spaces on the meter.



Reading Water and Electricity Meters

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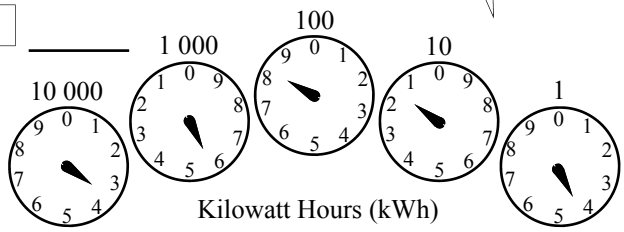
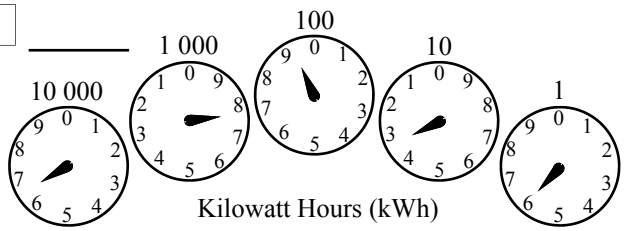
Read the dials below and write the number they represent

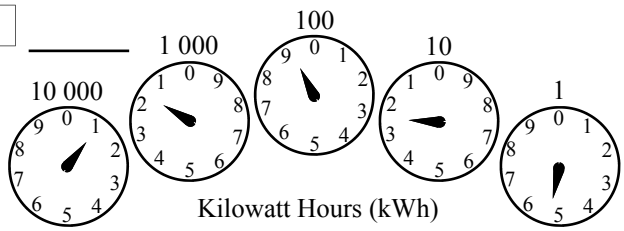
1 _____



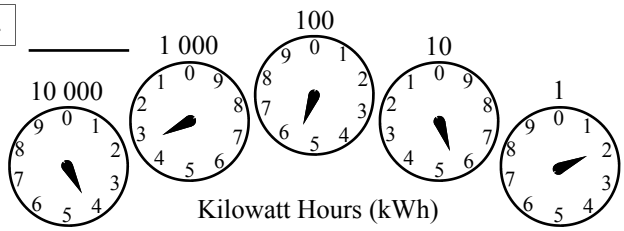
2 _____



3 _____

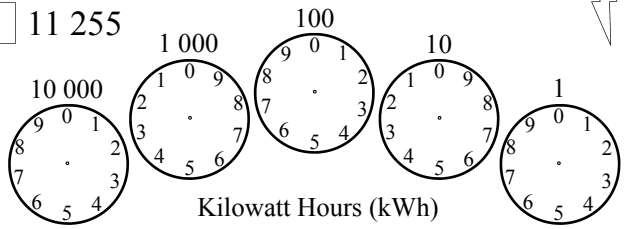


4 _____

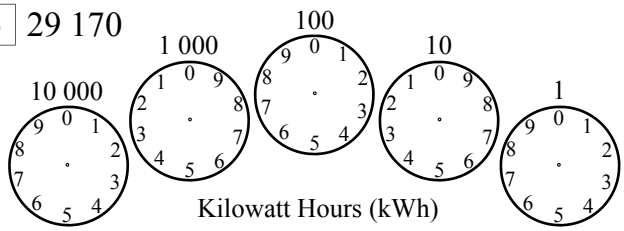


Now the reverse, put in the pointers!

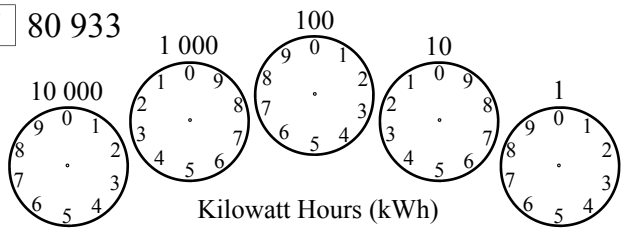
5 11 255



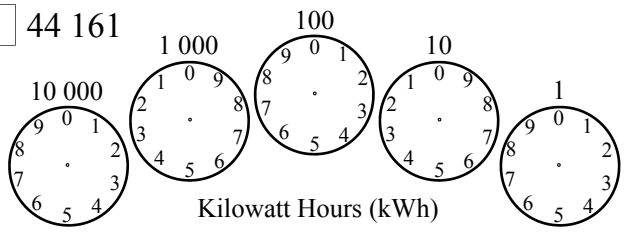
6 29 170



7 80 933

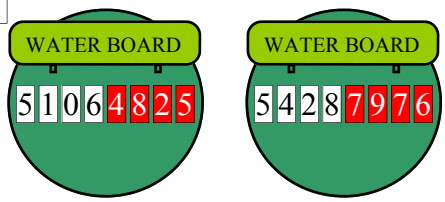


8 44 161



Read the 'last bill' reading then the 'current reading' and calculate the water usage since the last reading.

9



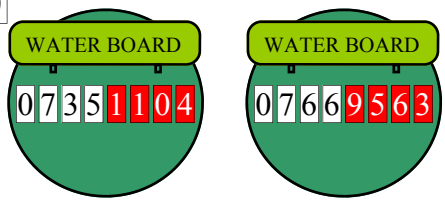
Last Bill Reading Current Reading

5 428.7976 -

_____

kL

10



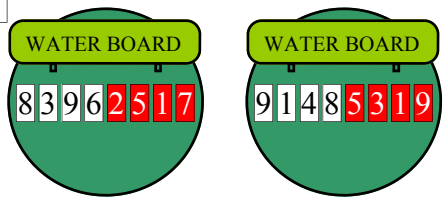
Last Bill Reading Current Reading

-

_____

kL

11



Last Bill Reading Current Reading

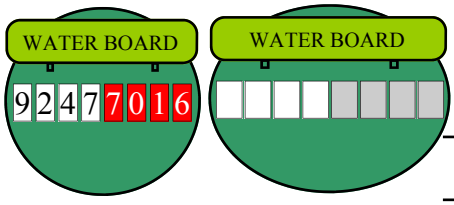
-

_____

kL

Given the water used and the last reading add them and write in the current reading that should be on the meter.

12 306.5908 kL



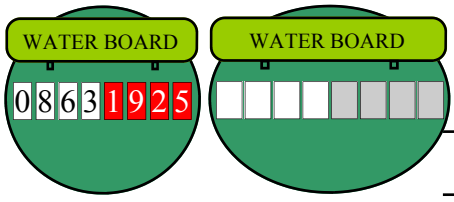
Last Bill Reading Current Reading

+

_____

kL

13 175.6907 kL



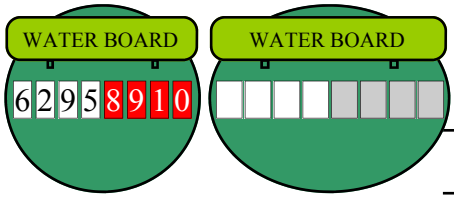
Last Bill Reading Current Reading

+

_____

kL

14 785.9707 kL



Last Bill Reading Current Reading

+

_____

kL

Reading Water and Electricity Meters

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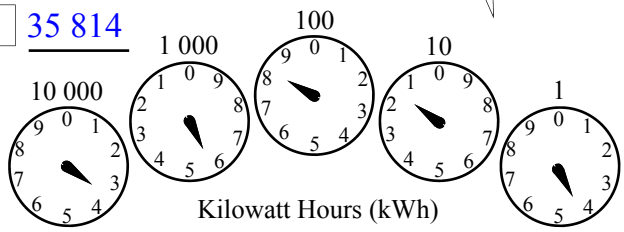
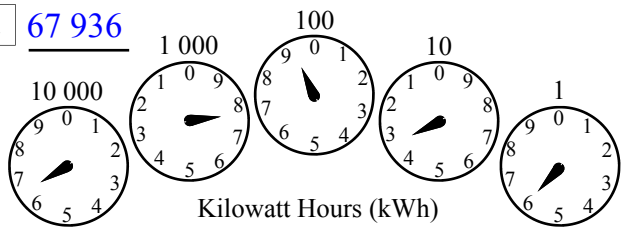
Read the dials below and write the number they represent

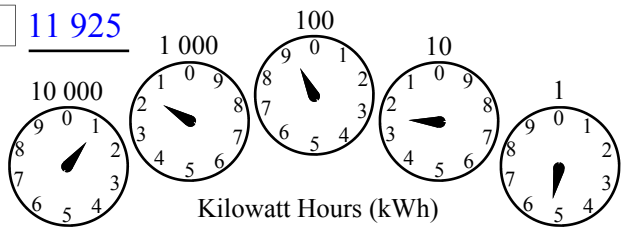
1 35 814



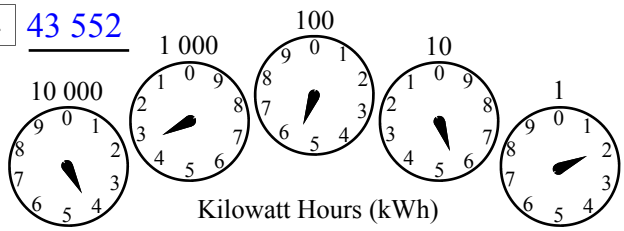
2 67 936



3 11 925

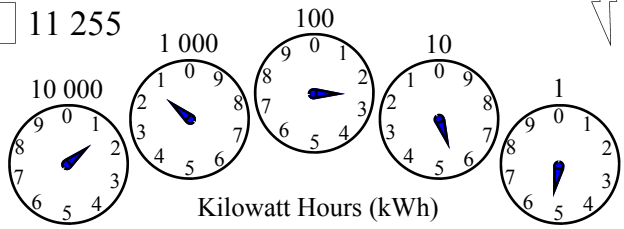


4 43 552

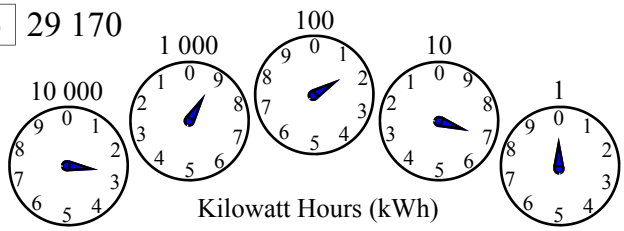


Now the reverse, put in the pointers!

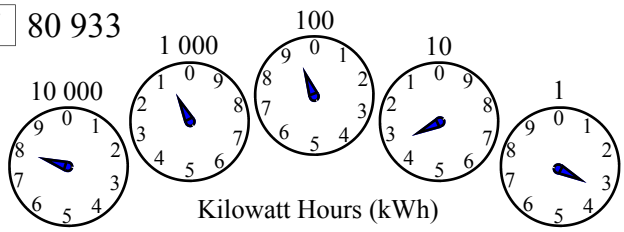
5 11 255



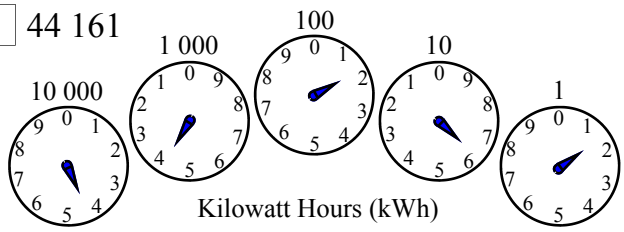
6 29 170



7 80 933



8 44 161



Read the 'last bill' reading then the 'current reading' and calculate the water usage since the last reading.

9



Last Bill Reading



Current Reading

5 428.7976 -
5 106.4825
322.3151 kL

10



Last Bill Reading



Current Reading

766.9563 -
735.1104
31.8459 kL

11



Last Bill Reading



Current Reading

9 148.5319 -
8 396.2517
752.2802 kL

Given the water used and the last reading add them and write in the current reading that should be on the meter.

12 306.5908 kL



Last Bill Reading



Current Reading

9 247.7016 +
306.5908
9 554.2924 kL

13 175.6907 kL



Last Bill Reading



Current Reading

863.1925 +
175.6907
1 038.8832 kL

14 785.9707 kL



Last Bill Reading



Current Reading

6 295.8910 +
785.9707
7 081.8617 kL

Fuel Gauges

Fuel gauges are an indication of how much fuel is in a car's petrol tank. Most commonly fuel gauges have divisions at empty, ¼ of a tank, ½ a tank, ¾ of a tank and full. In the city, gauges aren't that critical, with 24 hr petrol stations in great numbers. In the country away from the conveniences of the city, the understanding of a fuel gauge is more important.

In column 1 the fuel gauges are shown, put the fraction that the gauge's pointer indicates into the spaces provided. Note that the pointers are meant to be pointing exactly at a division or exactly half way between a division. When the pointer is halfway between divisions the reading will be in 'eighths'. Look at the gauge below, it has the eighths marked.

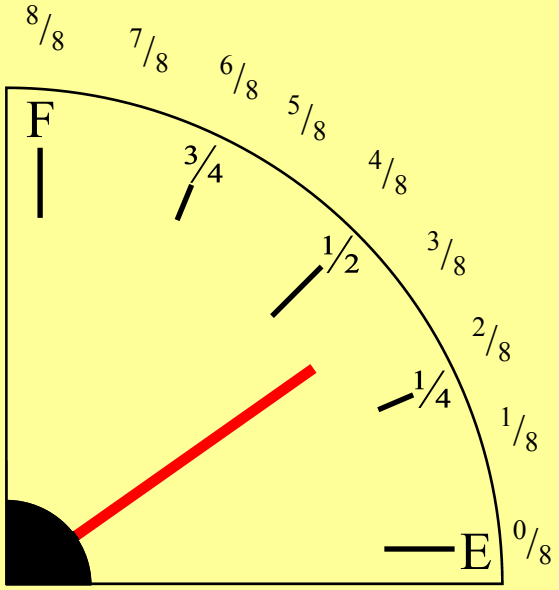
Column 2 is the reverse, given the fraction draw the pointers to the correct position.

Column 3 asks you to use a fuel gauge to estimate the fuel that remains in the tank as well as the distance that the car can continue until it will run out of fuel. This requires some multiplication of fractions skills, the method is as follows:

- To calculate the litres that remain divide 32 L (tank capacity) by the denominator and multiply by the numerator. This means divide 32 by the bottom number in the fraction, then multiply this number by the top number.
- The same method is used for the distance that the car can still travel. Divide 400 (max km) by the denominator (bottom number in fraction) and then multiply the answer by the numerator (top number in the fraction).

For the gauge below the reading is $^{3}/_{8}$. This means that if a car has a 32 L tank and a maximum travel distance on the tank of 400 km (city), then:

Fuel left is $32 \div 8 \times 3 = 4 \times 3 = 12$ L
Distance until empty is $400 \div 8 \times 3 = 50 \times 3 = 150$ km



Fuel Gauges

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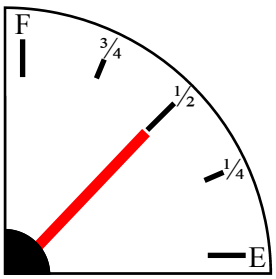
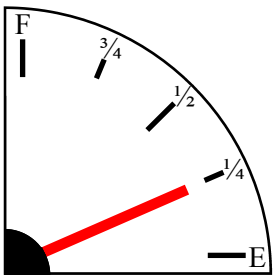
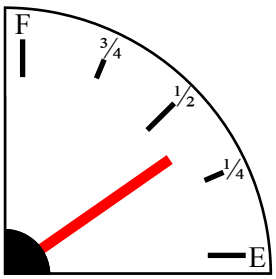
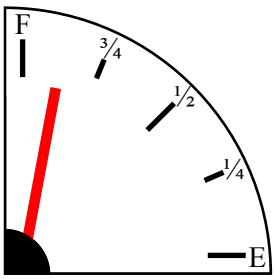
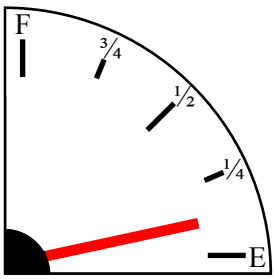
Estimate the fraction of a fuel tank that remains on the gauges below

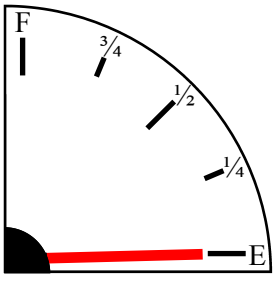
1 

2 

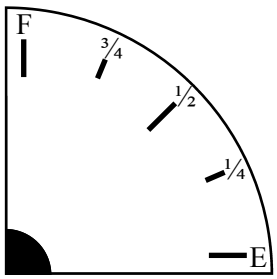
3 

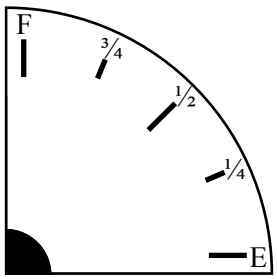
4 

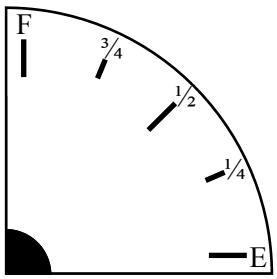
5 

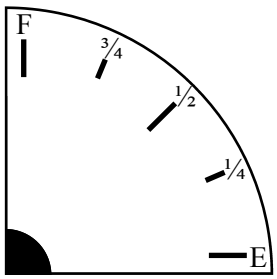
6 

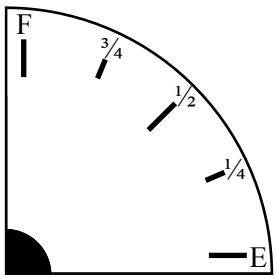
This time the fraction is given, place the needle in the correct position.

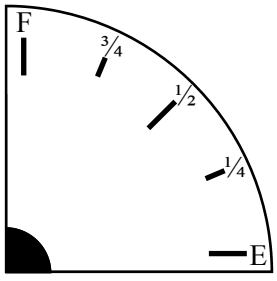
7 $\frac{3}{4}$ 

8 $\frac{7}{8}$ 

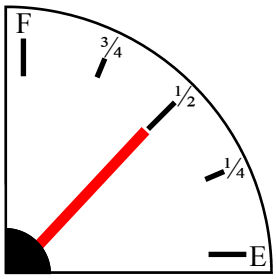
9 $\frac{1}{4}$ 

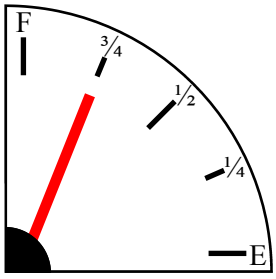
10 $\frac{3}{8}$ 

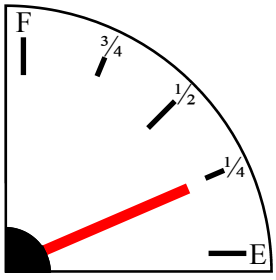
11 $\frac{1}{2}$ 

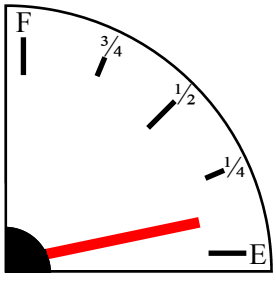
12 $\frac{5}{8}$ 

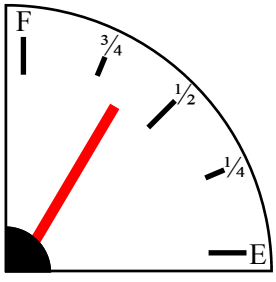
A full tank holds 32 L and travels 400 km (city). Use the gauges to estimate the unused fuel and km that can still be travelled.

13 L: _____ km: _____ 

14 L: _____ km: _____ 

15 L: _____ km: _____ 

16 L: _____ km: _____ 

17 L: _____ km: _____ 

Fuel Gauges

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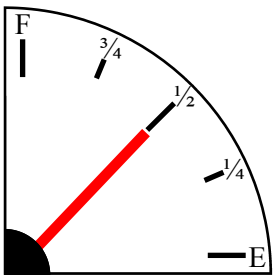
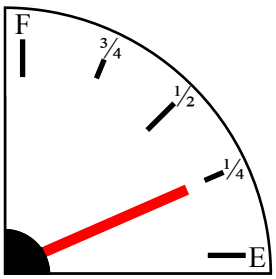
Estimate the fraction of a fuel tank that remains on the gauges below

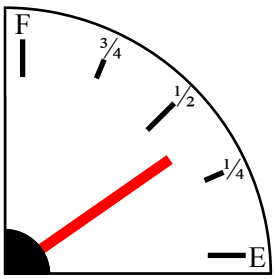
1 $\frac{1}{2}$



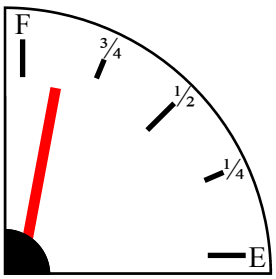
2 $\frac{1}{4}$



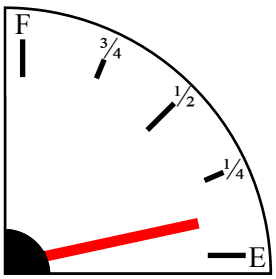
3 $\frac{3}{8}$



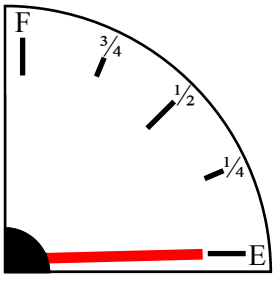
4 $\frac{7}{8}$



5 $\frac{1}{8}$

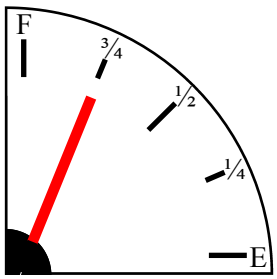


6 $\frac{0}{4}$

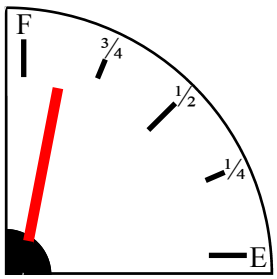


This time the fraction is given, place the needle in the correct position.

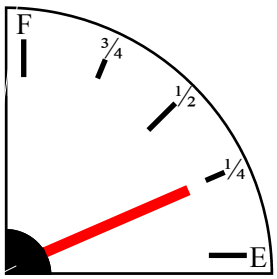
7 $\frac{3}{4}$



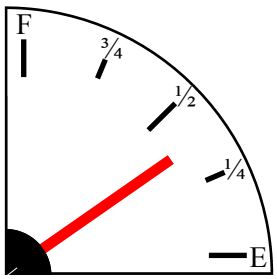
8 $\frac{7}{8}$



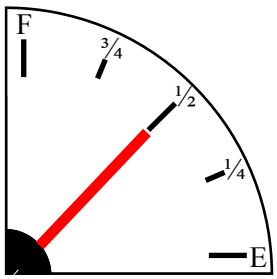
9 $\frac{1}{4}$



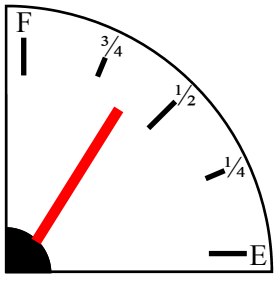
10 $\frac{3}{8}$



11 $\frac{1}{2}$

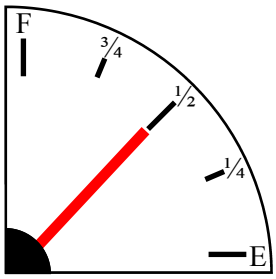


12 $\frac{5}{8}$

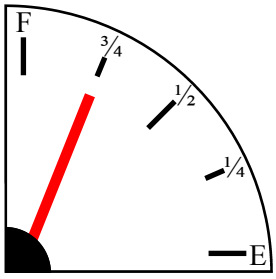


A full tank holds 32 L and travels 400 km (city). Use the gauges to estimate the unused fuel and km that can still be travelled.

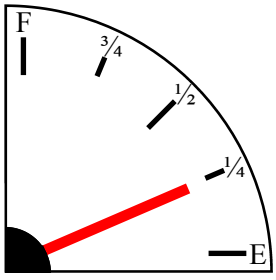
13 L: 16 km: 200



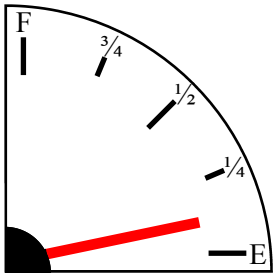
14 L: 24 km: 300



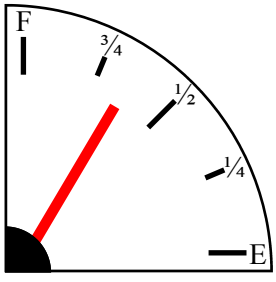
15 L: 8 km: 100



16 L: 4 km: 50



17 L: 20 km: 250



Using a Ruler - Line Measurement

This sheet has lines of a set length. When you print this sheet you must select 'No Scaling' when you print it, otherwise your answers will be wrong.

When a line is measured using a ruler the answer can be expressed in mm or cm. If the ruler has both units on it you can use the side needed for the answer. However you should also be able to convert between the two units. You should know that:

1 cm = 10 mm, so 2 cm = 20 mm, and decimals 3.8 cm = 38 mm etc.

Column 1 has horizontal lines and vertical lines. It is the thick (blue) lines you are measuring, don't confuse them with the thin black lines for writing your answer on. The first column asks for answers in cm (to 1 d.p.), that means 3.7 cm, 11.2 cm and so on. The screw and nut at the bottom of the page can be measured either across the object or use the lines with the double-ended arrow between them. The second column is much the same except the lines are angled, note that measurement for this column are in mm.

Use the same method with column 3 only write the size of each side and then add all the numbers together for the perimeter, the distance around the outside.

Using a Ruler - Line Measurement

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Use a ruler to measure these, express the answer in cm (to 1 d. p.)

1 _____

2 _____

3 _____

4 _____

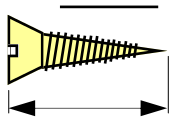
5 _____

6 _____

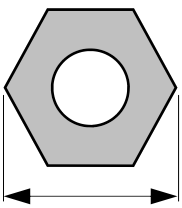
7 _____

13 12 11 10 9 8

14 _____



15 _____



Measure these angled lines to the nearest mm, express your answer in mm

16 _____

17 _____

18 _____

19 _____

20 _____

21 _____

22 _____

Measure the perimeter of these shapes to the nearest mm, write the side length on each side, then total answer in mm.

23 _____

24 _____

25 _____

26 _____

Using a Ruler - Line Measurement

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Use a ruler to measure these, express the answer in cm (to 1 d. p.)

1 1.1 cm

2 0.4 cm

3 1.6 cm

4 0.8 cm

5 2.7 cm

6 2.5 cm

7 3.9 cm

13 14.8 cm

12 13.4 cm

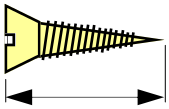
11 16.2 cm

10 7.7 cm

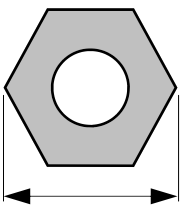
9 9.0 cm

8 5.8 cm

14 2.1 cm



15 2.3 cm



Measure these angled lines to the nearest mm, express your answer in mm

16 32 mm

17 19 mm

18 54 mm

19 46 mm

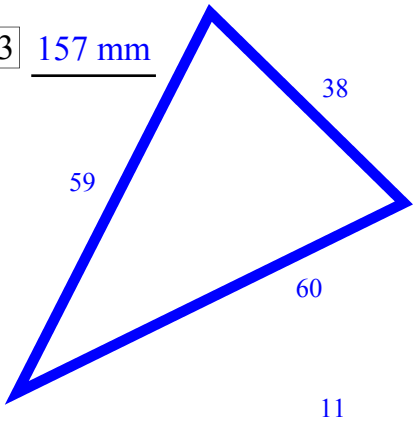
20 98 mm

21 66 mm

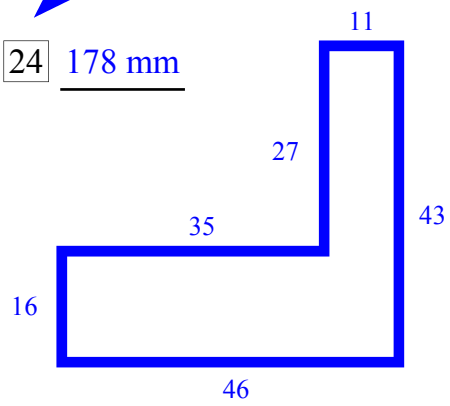
22 53 mm

Measure the perimeter of these shapes to the nearest mm, write the side length on each side, then total answer in mm.

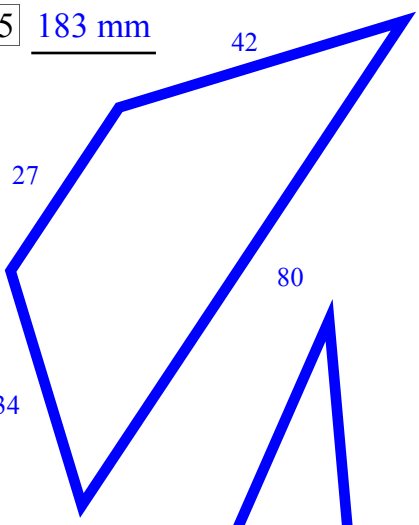
23 157 mm



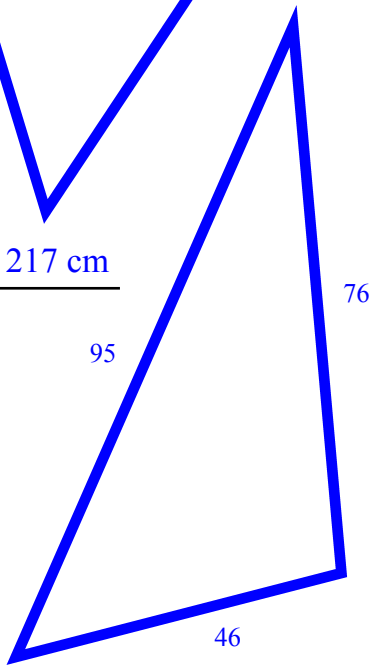
24 178 mm



25 183 mm



26 217 cm

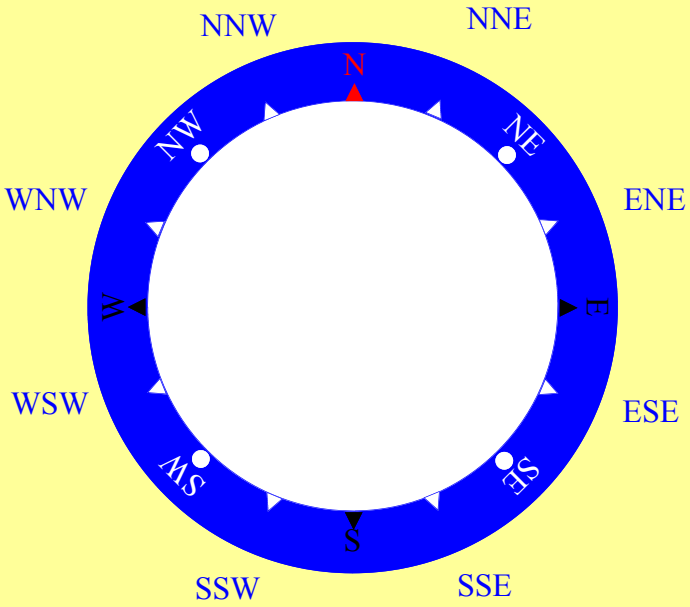


Using a Compass

A compass is used as a measure of direction. A compass needle always points to Magnetic North, so on this sheet the red section of the needle is pointing North and the white and red striped end is pointing South. The view is as if you are holding the compass against your chest looking down at it.

The compasses all have the blue ring with N (North) straight up. To find the direction you are facing count how many divisions (triangles and circles) that the red pointer is from North. Then count back the same number of divisions the opposite way from North and that is the direction you are facing.

The readings on this sheet will be one of the readings on the dial below. If you are not familiar with the 16 directions of the compass take the time to look at the dial, as the dials on the sheet don't have all the directions (shown in blue letters) that you need to know.



Using a Compass

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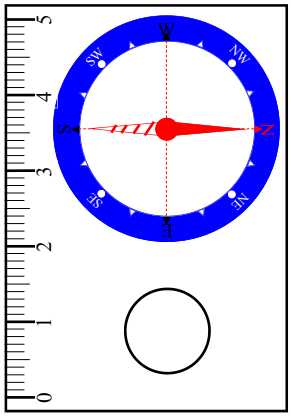
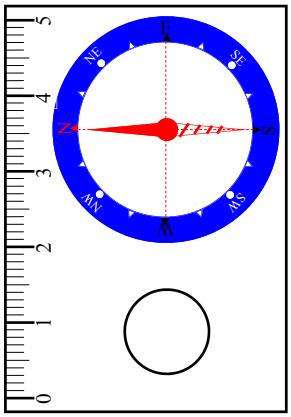
If the compass was held in front of you with the needles as shown which direction are you facing?

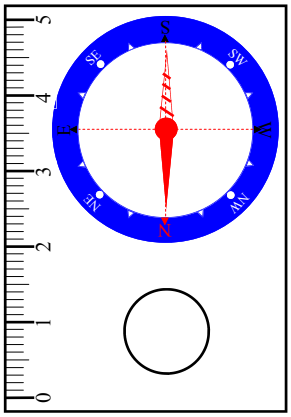
1 _____



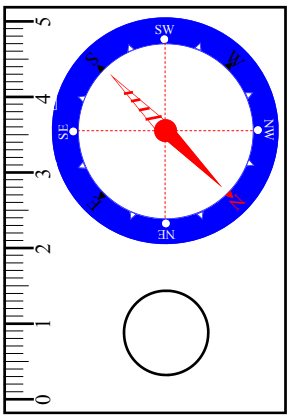
2 _____



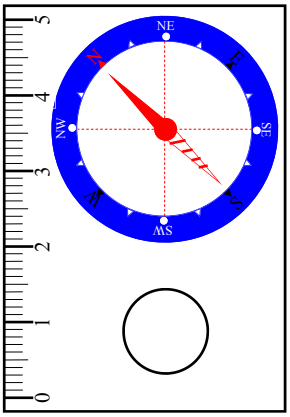
3 _____



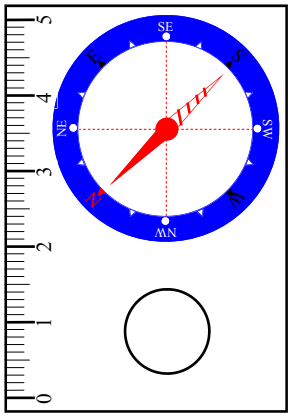
4 _____



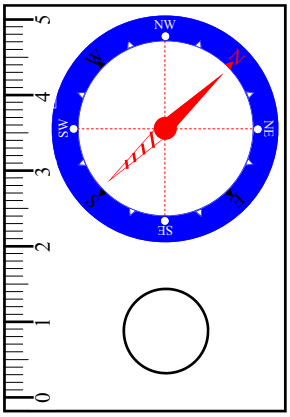
5 _____



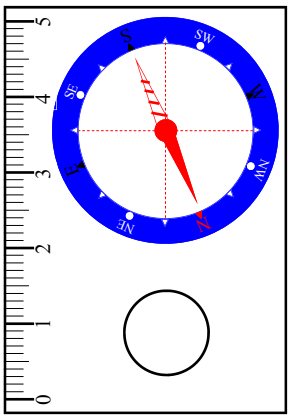
6 _____



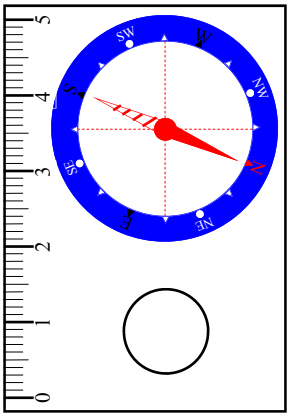
7 _____



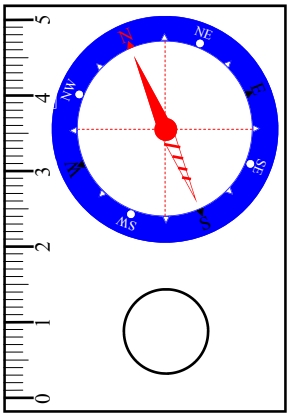
8 _____



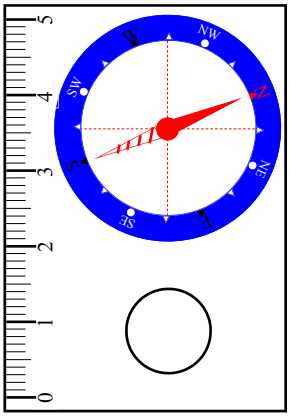
9 _____



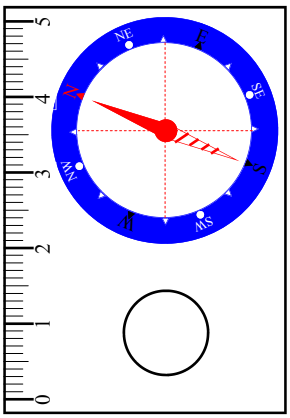
10 _____



11 _____



12 _____



Using a Compass

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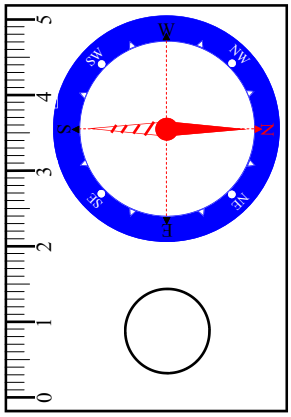
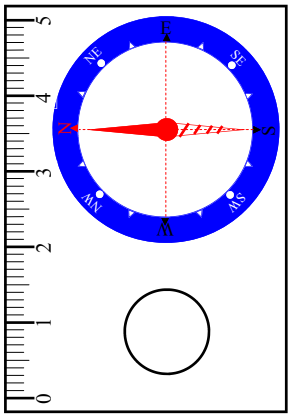
If the compass was held in front of you with the needles as shown which direction are you facing?

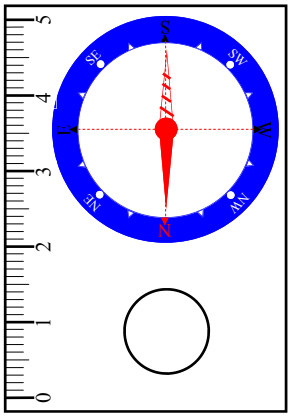
1 W



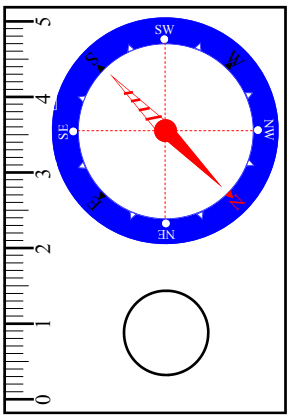
2 E



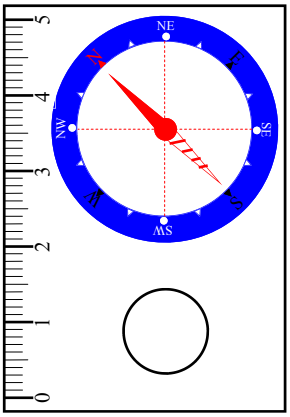
3 S



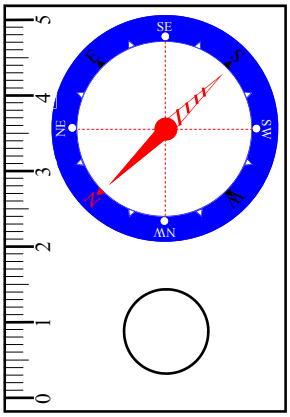
4 SW



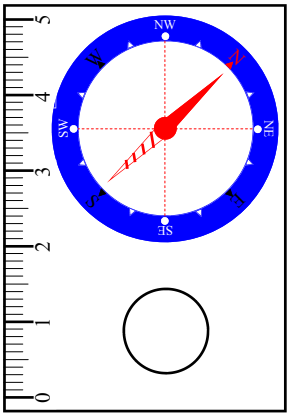
5 NE



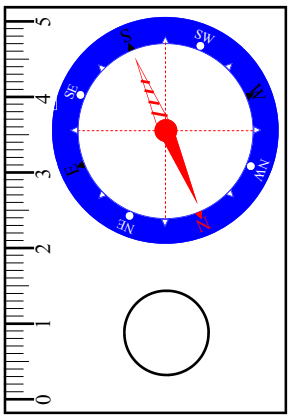
6 SE



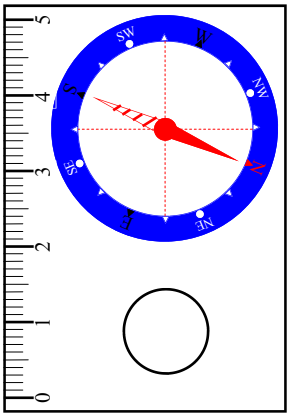
7 NW



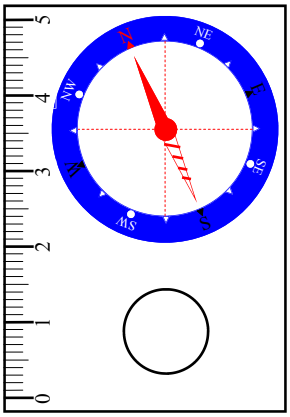
8 SSW



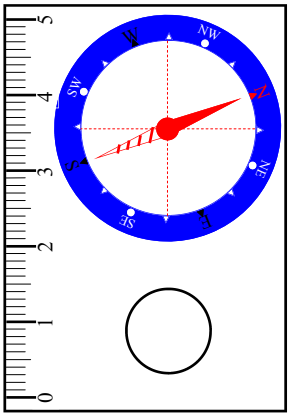
9 WSW



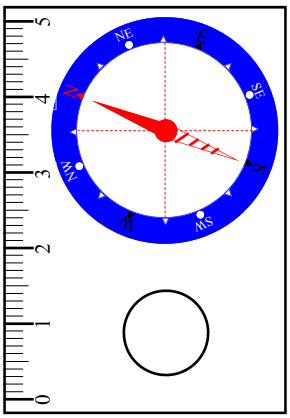
10 NNE



11 WNW



12 ENE



Car Gauges

This sheet asks you to put information on the gauges, a representation of a car's instrument cluster.

This is the method:

- The fuel gauge is on the left hand side of the instrument cluster. Draw a needle to the amount of fuel specified. Remember that between ¼'s are $^{1}/_{8}$'s of a tank, see the fuel gauge sheet if you need to.
- The speedometer is numbered 0 - 200 km/h. The speed increases by 10 km/h divisions with numbers at every 20 km/h. The odometer is the top set of boxes in the speedometer, while the tripmeter is the bottom set.
- The tachometer is the next large faced gauge. It displays engine speed with divisions at every 500 rpm with numbers at every 1 000 rpm. Note that car manufacturers use '× 1 000' so that single numbers are used. So 1 on the dial is 1 000 rpm. 2.5 would be 2 500 rpm. Note there is an LCD readout to show what gear the car is in.
- The temperature gauge is very simple, it has a green section to show the engine is cold, a blue section to indicate normal temperature and a red section to show if the engine is hot.
- The odometer is the record of the distance traveled during the life of your car it is the 6-digit number in your speedometer. So its largest reading is '999 999' km, then it resets to '000 000'. Write the numbers in the spaces.
- The tripmeter also records the distance traveled but it can be reset. It is often reset when the petrol tank is filled to calculate fuel economy or the distance covered on holidays. It has 4 digits but the last digit is tenths of a km. So its maximum reading is 999.9 km before it resets. Write the numbers in the spaces, note that the last digit is often in inverse (white on black) to show it is tenths of a kilometre.
- Some cars have an LCD readout to show the gear you are in, so that you don't have to look down at the gear shift. Cars usually have 5 or 6 gears. Colour the LCD bars.
- When turning at an intersection or overtaking you should use your indicator, colour the arrow, they are usually green or orange.

Car Gauges

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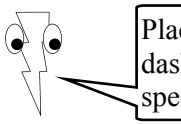
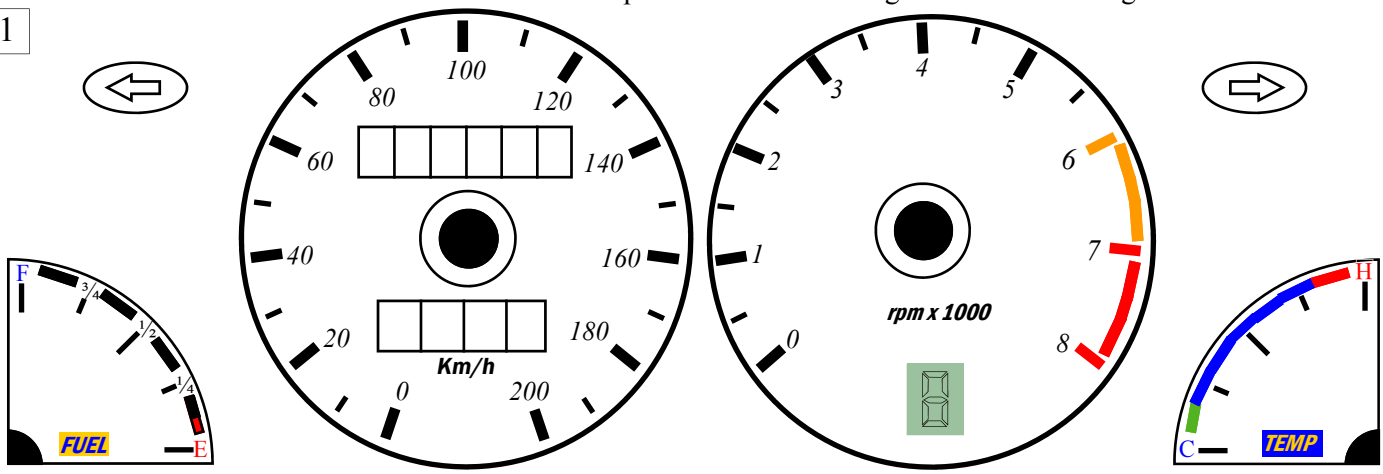


1

Fuel : ¾ tank
Speed : 80 km/h
RPM : 3 000
Temp : normal

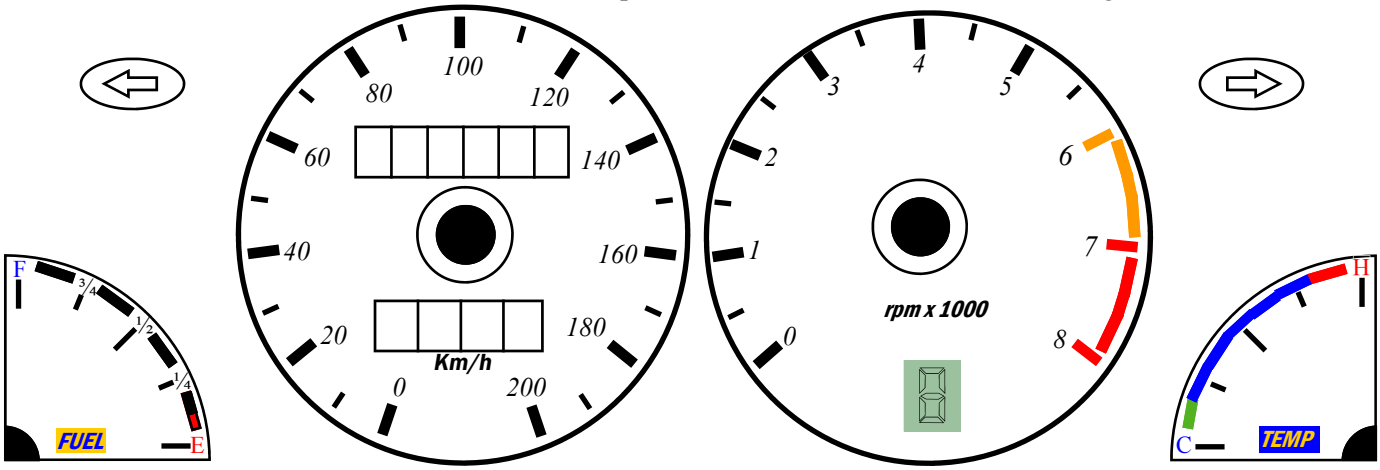
Odo : 56 277 km
Trip : 193.4 km
4th Gear
Right indicator flashing



2

Fuel : ¼ tank
Speed : 70 km/h
RPM : 4 500
Temp : cold

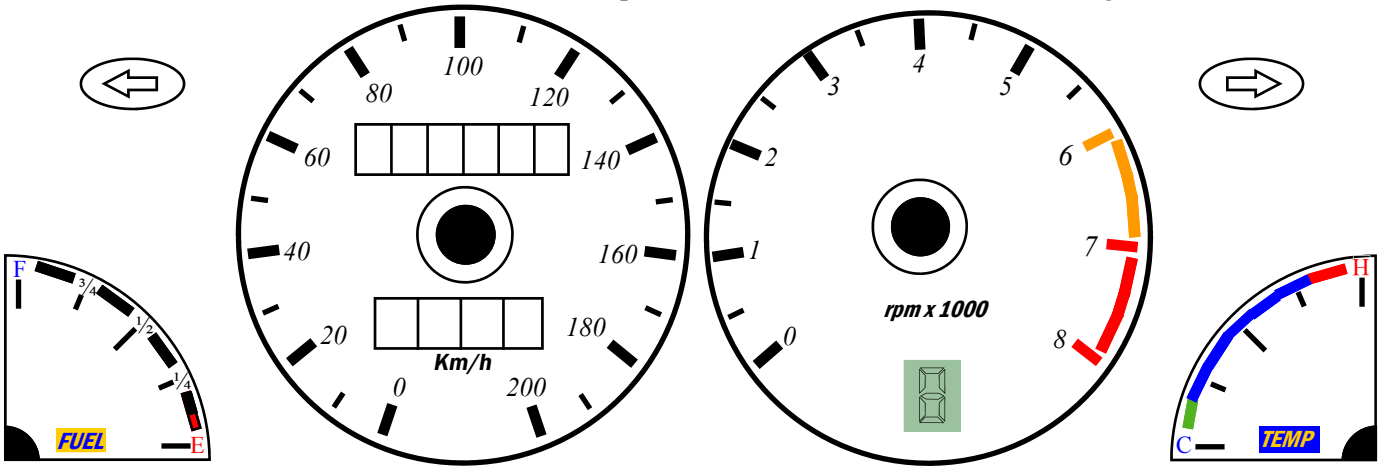
Odo : 79 003 km
Trip : 290.4 km
3rd Gear
Left indicator flashing



3

Fuel : $^{1}/_{8}$ tank
Speed : 105 km/h
RPM : 3 750
Temp : normal

Odo :135 198 km
Trip : 78.2 km
5th Gear
Left indicator flashing



Car Gauges

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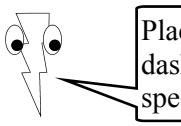
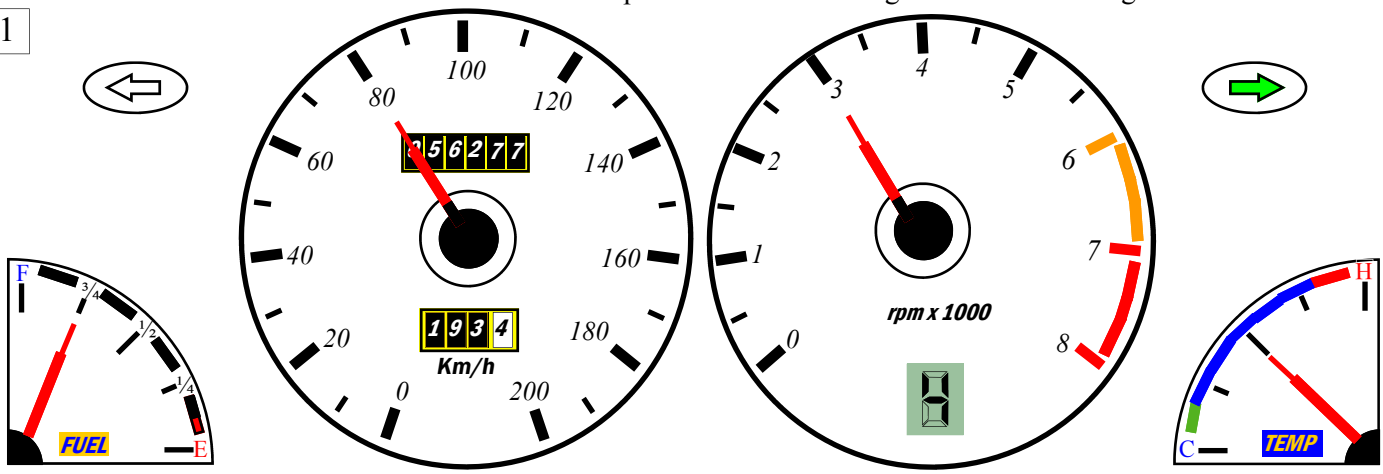


1

Fuel : ¾ tank
Speed : 80 km/h
RPM : 3 000
Temp : normal

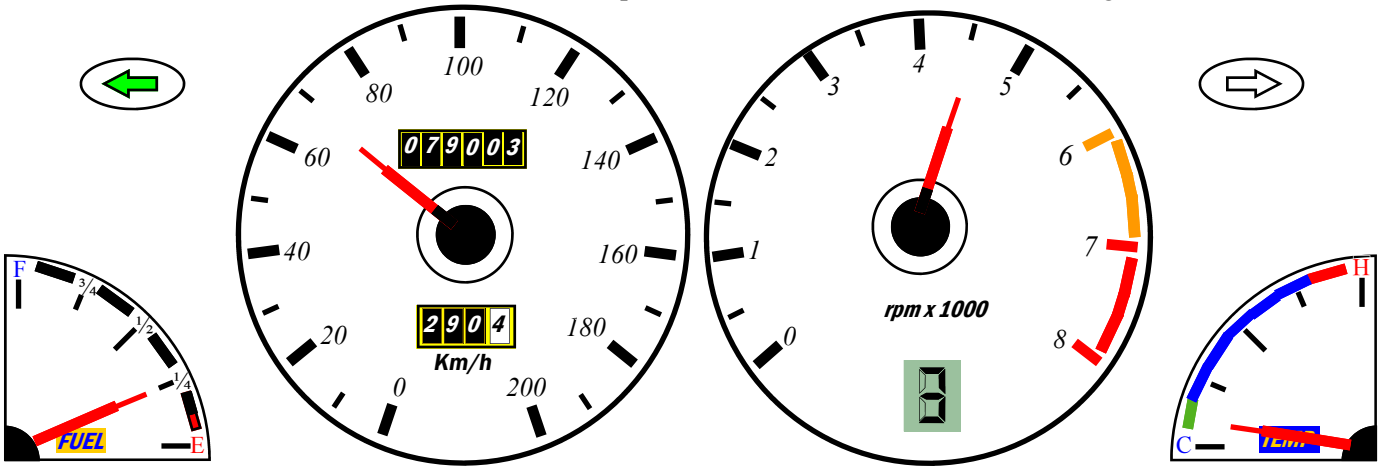
Odo : 56 277 km
Trip : 193.4 km
4th Gear
Right indicator flashing



2

Fuel : ¼ tank
Speed : 70 km/h
RPM : 4 500
Temp : cold

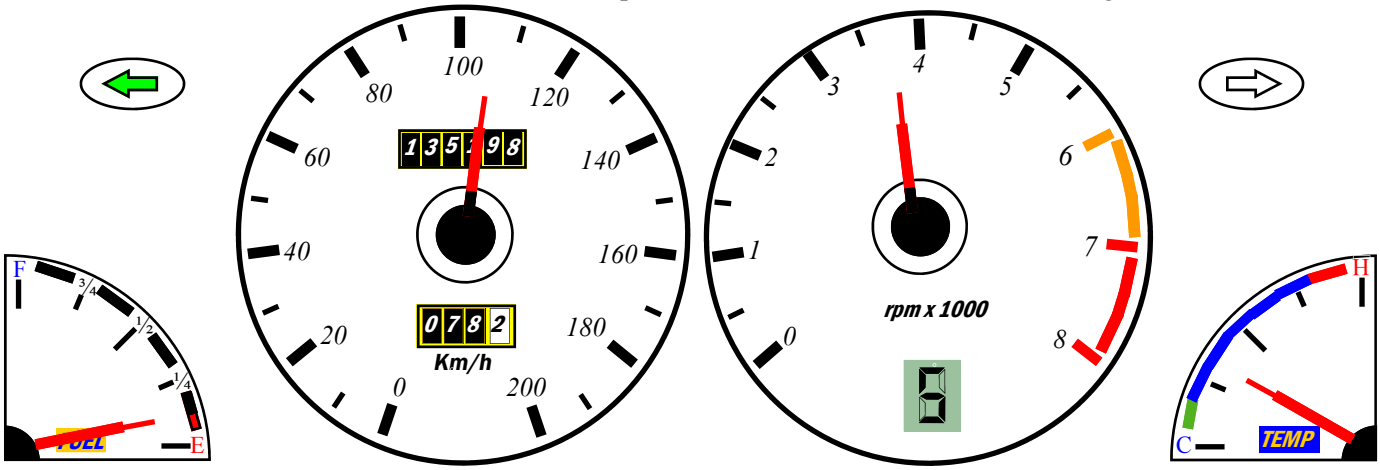
Odo : 79 003 km
Trip : 290.4 km
3rd Gear
Left indicator flashing



3

Fuel : $^{1}/_{8}$ tank
Speed : 105 km/h
RPM : 3 750
Temp : normal

Odo :135 198 km
Trip : 78.2 km
5th Gear
Left indicator flashing



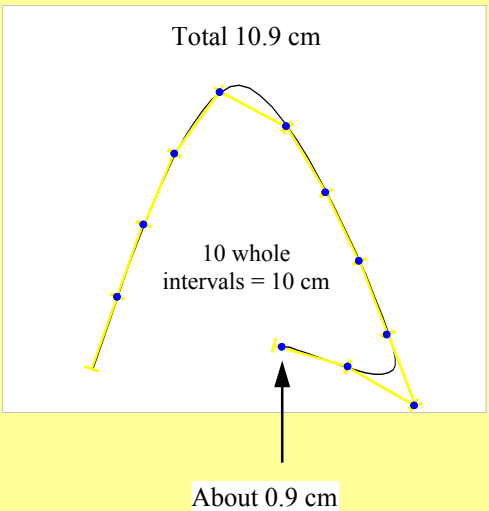
Curved Line Measurement

This sheet outlines the measurement of curved lines. Follow this method:

- Cut a length of string 30 cm long
- Use a felt tip pen to mark off 1 cm intervals along the string by lying the string alongside a ruler.
- Using a glue stick spread a thin film of glue over the first of the printed lines, the idea being not to stick the string to the page but to make it tacky enough to hold the string in place.
- Count the number of 1 cm intervals to get the number of cm then add the portion of the last interval, this will be tenths of a cm at the end, i. e: halfway into the last segment = 0.5 etc.
- Write the answer in the space provided

If you have no glue or string use a compass or set of dividers opened to 1 cm and 'step off' the curve. Or measure straight line lengths of 1 cm with a ruler.

Note that you can leave the line to make up for a corner you have previously cut (see example below). Decide if you are going to reduce the measurement at the corner or increase the measurement at the corner. You can alternate to reduce the error. The answer sheet for this sheet cuts all corners to give the minimum distance you should get for each line.



Using String or Intervals - Curved Line Measurement

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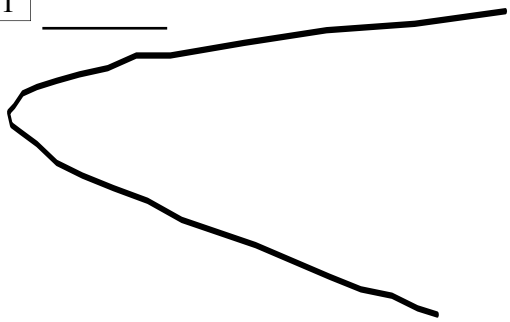
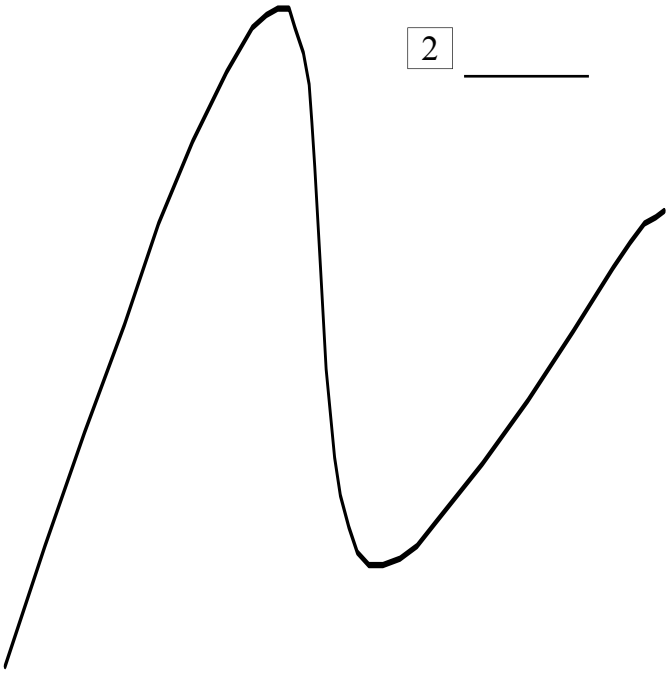
Use a piece of string with 1 cm intervals marked on it with a small amount of glue on the page so that the string doesn't wander about too much. No string? Use a compass and mark off 1 cm intervals. No compass? Use a ruler and mark off 1 cm intervals. Find the length and write it in the space next to the question. Express your answer in cm to 1 d. p.



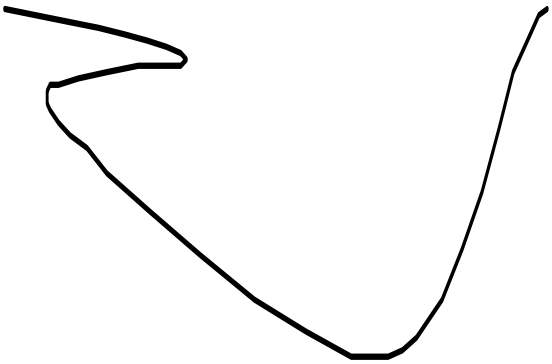
1 _____



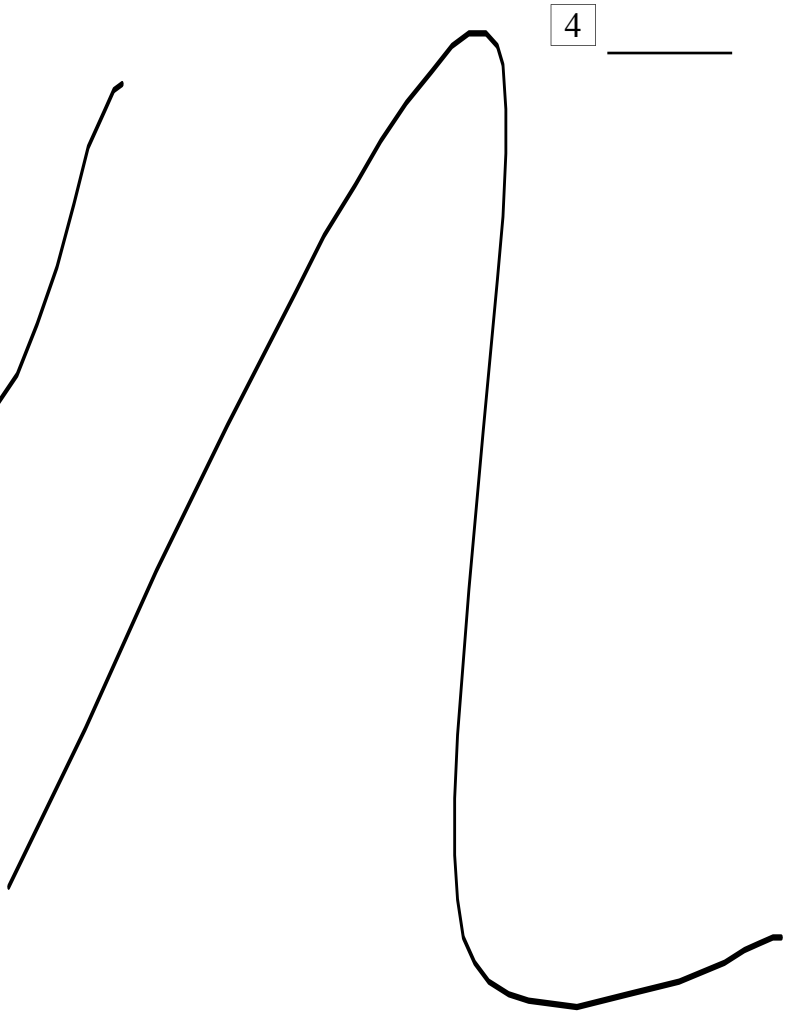
2 _____



3 _____



4 _____



Using String or Intervals - Curved Line Measurement

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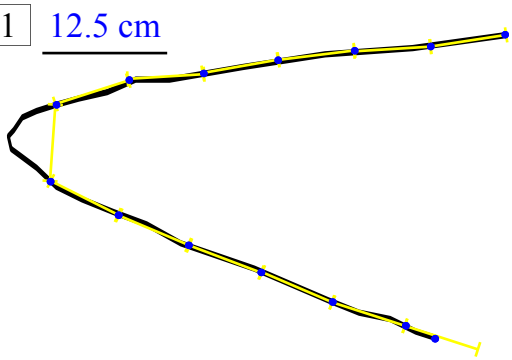
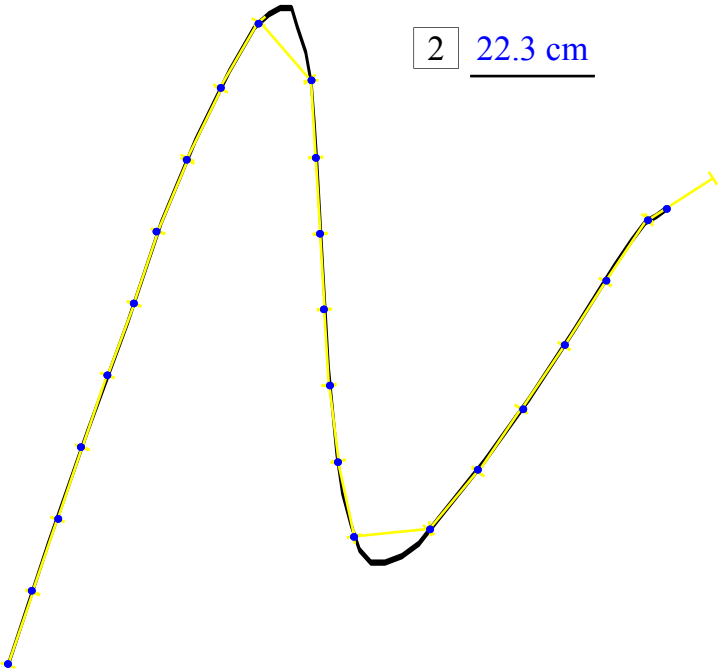
Use a piece of string with 1 cm intervals marked on it with a small amount of glue on the page so that the string doesn't wander about too much. No string? Use a compass and mark off 1 cm intervals. No compass? Use a ruler and mark off 1 cm intervals. Find the length and write it in the space next to the question. Express your answer in cm to 1 d. p.



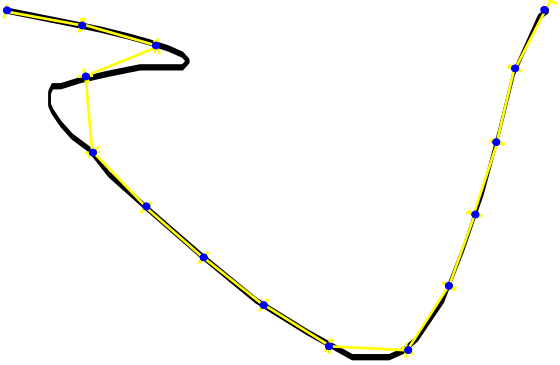
1 12.5 cm



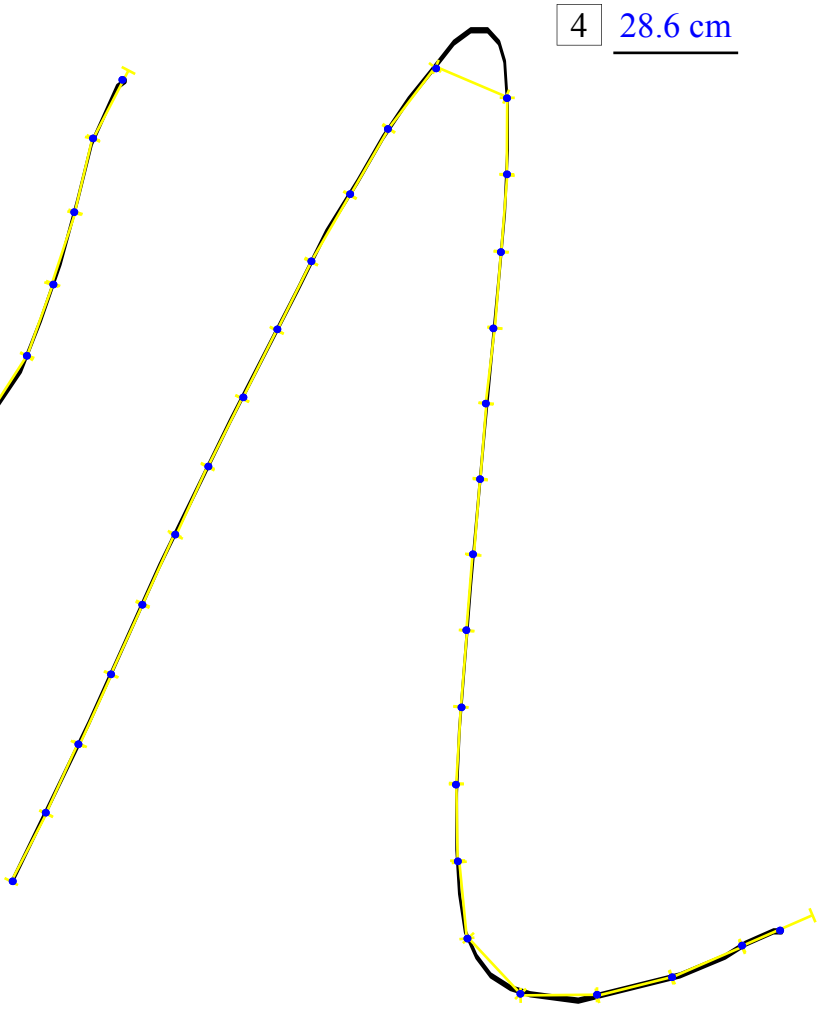
2 22.3 cm



3 13.8 cm



4 28.6 cm



Units of Measurement

When units of measurement are changed, multiply or divide by 10, 100, 1 000 or more. To change to a smaller unit multiply, to convert to a larger unit divide. Some students have difficulty with this, they think that if the unit is larger you must multiply. This isn't the case. Imagine you have 1 m or 100 cm, both are the same distance, there are more of the smaller unit, 1→100, so:

- changing to a smaller unit means - multiply
- changing to a larger unit means - divide

The number used to multiply or divide is the number of small units in the larger unit. For example 2 m to cm, there are 100 cm in a metre, changing to a smaller unit so multiply…so multiply by 100. As only 10, 100 or 1 000 are used to get the answer … move the decimal point. Multiply by 10 moves the decimal place 1 position to the right, 100 move 2 positions and 1 000 move the decimal place 3 positions to the right.

Division is the same number of moves, only this time it's to the left. Divide by 10 and move the decimal place 1 position to the left, 100 move it 2 positions and 1 000 moves 3 positions to the left. A reminder is at the top of each column.

Column 1 starts with changing mm to cm. You should be able to talk it through to your-self….mm to cm is changing to a larger unit...that means division….there are 10 mm in a cm that means I divide by 10….divide by 10 means I move the decimal point 1 place to the left. That means 18 mm = 1.8 cm 113 mm = 11.3 cm and 0.9 mm = 0.09 cm. The next part of the column is the reverse, multiply by 10 means move the decimal place 1 position to the right. So 54 cm = 540 mm, 1.3 cm = 13 mm and 123 cm = 1 230 mm.

Column 2 deals with converting between cm and m. This time the change is by dividing or multiplying by 100. That means a 2 decimal place movement. For cm to m (divide by 100) 12 cm = 0.12 m, 106 cm = 1.06 m and 5.78 cm = 0.0578 m. Then the reverse, m to cm means multiplying by 100 or a 2 decimal place shift to the right: 3 m = 300 cm, 0.7 m = 70 cm and 10.05 m = 1 005 cm.

Column 3 is m to km, this involves multiplying or dividing by 1 000, or a 3 decimal place movement. That would mean 300 m = 0.3 km, 2 305 m = 2.305 km and 4.7 m = 0.0047 km. Then the reverse, changing km to m. That's 3 decimal places to the right: 1.2 km = 1 200m, 0.85 km = 850 m and 0.02 km = 20 m.

Units of Measurement

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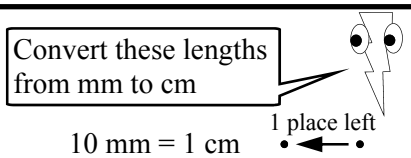
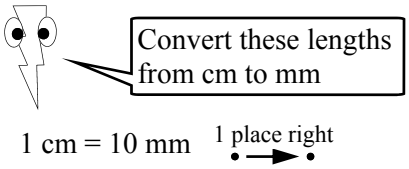


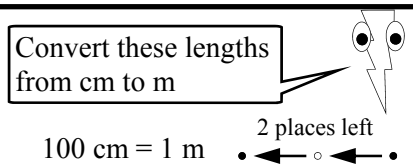
10 mm = 1 cm

1	20 mm	
2	50 mm	
3	130 mm	
4	55 mm	
5	92 mm	
6	200 mm	
7	18.6 mm	
8	2 020 mm	
9	435 mm	
10	0.95 mm	
11	5.47 mm	
12	906.2 mm	



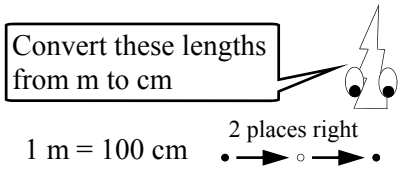
1 cm = 10 mm

13	5 cm	
14	18 cm	
15	6.3 cm	
16	74.8 cm	
17	356.7 cm	
18	8.12 cm	
19	0.4 cm	
20	0.566 cm	
21	155 cm	
22	829.3 cm	
23	1.002 cm	
24	19.91 cm	



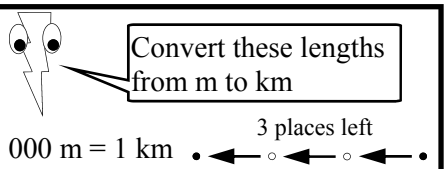
100 cm = 1 m

25	200 cm	
26	600 cm	
27	150 cm	
28	370 cm	
29	12 cm	
30	86.5 cm	
31	7 300 cm	
32	56.05 cm	
33	3 cm	
34	6.7 cm	
35	8 002 cm	
36	27.08 cm	



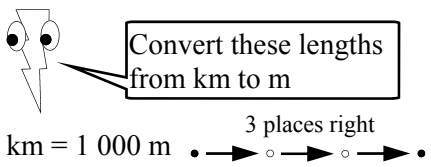
1 m = 100 cm

37	3 m	
38	7 m	
39	2.2 m	
40	5.9 m	
41	0.3 m	
42	0.75 m	
43	0.363 m	
44	5.02 m	
45	10.7 m	
46	0.006 m	
47	5.071 m	
48	30.262 m	



1 000 m = 1 km

49	1 500 m	
50	1 220 m	
51	4 930 m	
52	110 m	
53	550.6 m	
54	1 003 m	
55	85 m	
56	4.7 m	
57	309.7 m	
58	12 032 m	
59	8 771.3 m	
60	763.2 m	



1 km = 1 000 m

61	3 km	
62	8 km	
63	3.2 km	
64	14.7 km	
65	10.6 km	
66	0.9 km	
67	0.673 km	
68	5.12 km	
69	1.007 km	
70	0.4503 km	
71	4.019 km	
72	0.06 km	

Units of Measurement

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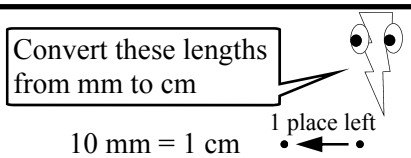
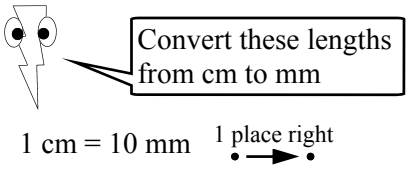
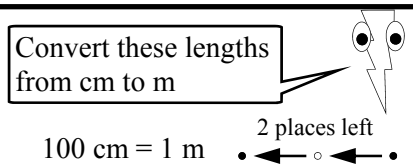


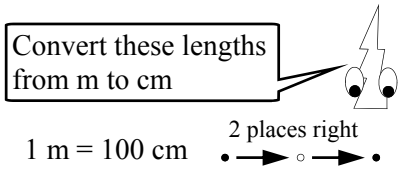
1	20 mm	2 cm
2	50 mm	5 cm
3	130 mm	13 cm
4	55 mm	5.5 cm
5	92 mm	9.2 cm
6	200 mm	20 cm
7	18.6 mm	1.86 cm
8	2 020 mm	202 cm
9	435 mm	43.5 cm
10	0.95 mm	0.095 cm
11	5.47 mm	0.547 cm
12	906.2 mm	90.62 cm



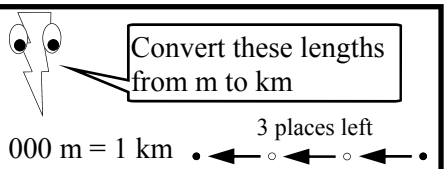
13	5 cm	50 mm
14	18 cm	180 mm
15	6.3 cm	63 mm
16	74.8 cm	748 mm
17	356.7 cm	3567 mm
18	8.12 cm	81.2 mm
19	0.4 cm	4 mm
20	0.566 cm	5.66 mm
21	155 cm	1 550 mm
22	829.3 cm	8 293 mm
23	1.002 cm	10.02 mm
24	19.91 cm	199.1 mm



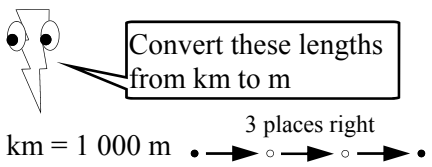
25	200 cm	2 m
26	600 cm	6 m
27	150 cm	1.5 m
28	370 cm	3.7 m
29	12 cm	0.12 m
30	86.5 cm	0.865 m
31	7 300 cm	73 m
32	56.05 cm	0.5605 m
33	3 cm	0.03 m
34	6.7 cm	0.067 m
35	8 002 cm	80.02 m
36	27.08 cm	0.2708 m



37	3 m	300 cm
38	7 m	700 cm
39	2.2 m	220 cm
40	5.9 m	590 cm
41	0.3 m	30 cm
42	0.75 m	75 cm
43	0.363 m	36.3 cm
44	5.02 m	502 cm
45	10.7 m	1 070 cm
46	0.006 m	0.6 cm
47	5.071 m	507.1 cm
48	30.262 m	3 026.2 cm



49	1 500 m	1.5 km
50	1 220 m	1.22 km
51	4 930 m	4.93 km
52	110 m	0.11 km
53	550.6 m	0.5506 km
54	1 003 m	1.003 km
55	85 m	0.085 km
56	4.7 m	0.0047 km
57	309.7 m	0.3097 km
58	12 032 m	12.032 km
59	8 771.3 m	8.7713 km
60	763.2 m	0.7632 km



61	3 km	3 000 m
62	8 km	8 000 m
63	3.2 km	3 200 m
64	14.7 km	14 700 m
65	10.6 km	10 600 m
66	0.9 km	900 m
67	0.673 km	673 m
68	5.12 km	5 120 m
69	1.007 km	1 007 m
70	0.4503 km	450.3 m
71	4.019 km	4 019 m
72	0.06 km	60 m

Further Units of Measurement

The processes are the same as on the previous sheet 'Units of Measurement' except that this time the conversions aren't in groups of units and multiplication/division. Its all jumbled so you need to know whether you divide or multiply and how many decimal places you shift the decimal point. There is also the inclusion of mm to m and the reverse. This is a 3 decimal place shift (× or ÷ by 1 000).

Further Units of Measurement

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Use the guide below to help you

mm → cm	1 place left
mm → m	3 places left
cm → mm	1 place right
cm → m	2 places left
m → mm	3 places right
m → cm	2 places right
m → km	3 places left
km → m	3 places right

Convert these to the units in brackets. They are all mixed up

1. 3.2 m [cm] _____
2. 4.3 cm [mm] _____
3. 0.98 m [cm] _____
4. 200 mm [m] _____
5. 860 m [km] _____
6. 118 mm [cm] _____
7. 1 033 m [km] _____
8. 27.2 cm [mm] _____
9. 185 mm [m] _____
10. 206.5 mm [m] _____
11. 0.35 cm [mm] _____
12. 686 cm [m] _____
13. 0.496 m [mm] _____
14. 4.03 km [m] _____
15. 65.7 cm [m] _____
16. 825 mm [cm] _____
17. 11.3 m [cm] _____
18. 5.062 km [m] _____
19. 0.838 km [m] _____
20. 3.07 cm [mm] _____
21. 121 mm [m] _____
22. 7 777 m [km] _____
23. 918 cm [m] _____
24. 4.2 cm [m] _____
25. 0.5 cm [m] _____
26. 0.89 m [cm] _____
27. 765 mm [cm] _____
28. 19.6 cm [mm] _____
29. 16 010 m [km] _____
30. 78 mm [m] _____
31. 2.1 m [mm] _____
32. 1.93 m [cm] _____
33. 82.6 cm [mm] _____
34. 16 m [cm] _____
35. 1.8 km [m] _____
36. 3.8 mm [cm] _____
37. 9.021 km [m] _____
38. 56.5 cm [m] _____
39. 28.8 m [km] _____
40. 0.83 cm [mm] _____
41. 0.6 m [mm] _____
42. 0.03 m [mm] _____
43. 41 m [cm] _____
44. 850 mm [m] _____
45. 182.5 cm [m] _____
46. 1 405 mm [m] _____
47. 0.95 m [mm] _____
48. 1 727.4 m [km] _____
49. 0.052 cm [mm] _____
50. 39 cm [m] _____
51. 0.2004 km [m] _____
52. 4.52 m [cm] _____
53. 80.05 cm [mm] _____
54. 293 m [km] _____
55. 0.302 m [mm] _____
56. 13 mm [m] _____
57. 43.9 mm [cm] _____
58. 0.67 m [cm] _____
59. 850 mm [m] _____
60. 2 211 mm [m] _____
61. 549.3 m [km] _____
62. 400.3 cm [m] _____
63. 0.74 km [m] _____
64. 4.96 cm [mm] _____
65. 113.2 cm [mm] _____
66. 333.7 cm [m] _____
67. 17 mm [cm] _____
68. 0.005 m [cm] _____
69. 17 mm [cm] _____
70. 0.02 km [m] _____
71. 680 mm [m] _____
72. 82.4 cm [m] _____
73. 0.65 mm [cm] _____
74. 32.6 mm [m] _____
75. 0.006 km [m] _____

Further Units of Measurement

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Use the guide below to help you

mm → cm	1 place left
mm → m	3 places left
cm → mm	1 place right
cm → m	2 places left
m → mm	3 places right
m → cm	2 places right
m → km	3 places left
km → m	3 places right

Convert these to the units in brackets. They are all mixed up

1	3.2 m	[cm]	320 cm
2	4.3 cm	[mm]	43 mm
3	0.98 m	[cm]	98 cm
4	200 mm	[m]	0.2 m
5	860 m	[km]	0.86 km
6	118 mm	[cm]	11.8 cm
7	1 033 m	[km]	1.033 km
8	27.2 cm	[mm]	272 mm
9	185 mm	[m]	0.185 m
10	206.5 mm	[m]	0.2065 m
11	0.35 cm	[mm]	3.5 mm
12	686 cm	[m]	6.86 m
13	0.496 m	[mm]	496 mm
14	4.03 km	[m]	4 030 m
15	65.7 cm	[m]	0.657 m
16	825 mm	[cm]	82.5 cm
17	11.3 m	[cm]	1 130 cm
18	5.062 km	[m]	5 062 m
19	0.838 km	[m]	838 m
20	3.07 cm	[mm]	30.7 mm
21	121 mm	[m]	0.121 m
22	7 777 m	[km]	7.777 km
23	918 cm	[m]	9.18 m
24	4.2 cm	[m]	0.042 m
25	0.5 cm	[m]	0.005 m
26	0.89 m	[cm]	89 cm
27	765 mm	[cm]	76.5 cm
28	19.6 cm	[mm]	196 mm
29	16 010 m	[km]	16.01 km
30	78 mm	[m]	0.078 m
31	2.1 m	[mm]	2 100 mm
32	1.93 m	[cm]	193 cm
33	82.6 cm	[mm]	826 mm
34	16 m	[cm]	1 600 cm
35	1.8 km	[m]	1 800 m
36	3.8 mm	[cm]	0.38 cm
37	9.021 km	[m]	9 021 m
38	56.5 cm	[m]	0.565 m
39	28.8 m	[km]	0.0288 km
40	0.83 cm	[mm]	8.3 mm
41	0.6 m	[mm]	600 mm
42	0.03 m	[mm]	30 mm
43	41 m	[cm]	4 100 cm
44	850 mm	[m]	0.85 m
45	182.5 cm	[m]	1.825 m
46	1 405 mm	[m]	1.405 m
47	0.95 m	[mm]	950 mm
48	1 727.4 m	[km]	1.7274 km
49	0.052 cm	[mm]	0.52 mm
50	39 cm	[m]	0.39 m
51	0.2004 km	[m]	200.4 m
52	4.52 m	[cm]	452 cm
53	80.05 cm	[mm]	800.5 mm
54	293 m	[km]	0.293 km
55	0.302 m	[mm]	302 mm
56	13 mm	[m]	0.013 m
57	43.9 mm	[cm]	4.39 cm
58	0.67 m	[cm]	67 cm
59	850 mm	[m]	0.85 m
60	2 211 mm	[m]	2.211 m
61	549.3 m	[km]	0.5493 km
62	400.3 cm	[m]	4.003 m
63	0.74 km	[m]	740 m
64	4.96 cm	[mm]	49.6 mm
65	113.2 cm	[mm]	1 132 mm
66	333.7 cm	[m]	3.337 m
67	17 mm	[cm]	1.7 cm
68	0.005 m	[cm]	0.5 cm
69	17 mm	[cm]	1.7 cm
70	0.02 km	[m]	20 m
71	680 mm	[m]	0.68 m
72	82.4 cm	[m]	0.824 m
73	0.65 mm	[cm]	0.065 cm
74	32.6 mm	[m]	0.0326 m
75	0.006 km	[m]	6 m

Perimeter of Shapes

The perimeter of a shape is the distance around the outside of the shape. The units of measurement are mm, cm, m and km. Shapes often have side lengths that are the same, rather than writing this distance out again a mark is put on the identical sides to show they are the same length. If there are a number of different pairs of sides that are the same length, 2 marks together will be used and so on, such as with a rectangle.

To answer columns 1 and 2 use the same method, this is:

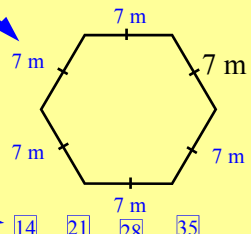
- Look at the side markings and write side lengths on every side. It is up to you if you write the units as well.
- The first line of working is the sum of the sides of the shape, write 'P =' then the lengths separated by + signs. Note that to avoid missing sides don't jump around in the addition. Start on a side then move around the shape in a clockwise or anti-clockwise direction.
- If the sum is challenging to complete mentally, total the sum as you move through, look at the example below.
- The second line is the answer line, again write 'P =' then the answer, then the units, either mm, cm or m.

Column 3 introduces formulae used for squares and rectangles. These are:

- Squares: $P = 4l$ (which is 4 times the side length)
- Rectangles: $P = 2l + 2b$ (which is 2 times the length + 2 times the breadth)

The questions are in tables. With squares, multiply the side length by 4 and write the answer with units. The rectangles are completed in stages, l is given and you double it to get the value of $2l$, b is given and you double it to get the value of $2b$. Then add the values for $2l$ and $2b$ together to get the answer, write in the units (all of which are cm.)

Fill in all the side lengths to avoid missing them



If you have trouble solving the sum mentally use canceling strokes and total as you go through

14 21 28 35

P = 7 + 7 + 7 + 7 + 7 + 7

P = 42 m

Perimeter of Shapes

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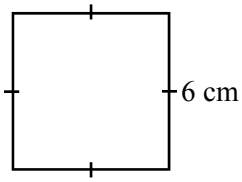
Put measurements on all sides then add them to find the perimeter.

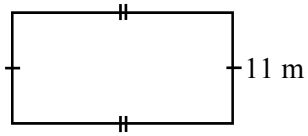
1



P = 6 + + +

P = cm

2



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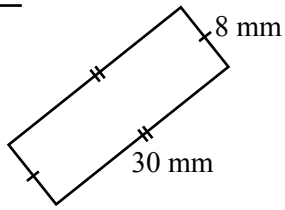

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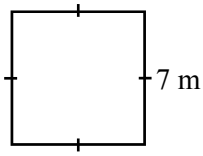

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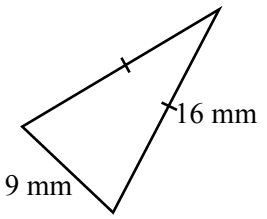

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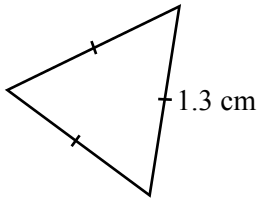

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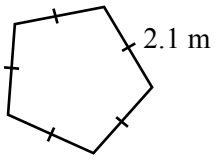

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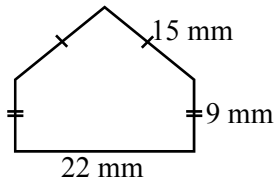

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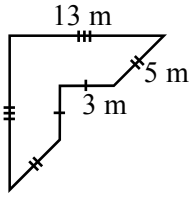

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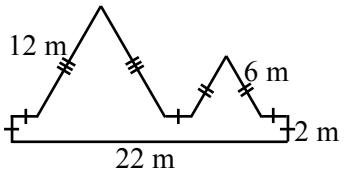

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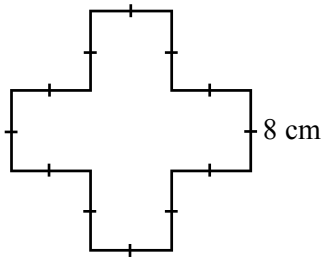
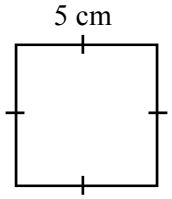


P = ×

Complete the tables, all measurements are in cm.

Perimeter Formula for Squares :

$P = 4l$ where l = side length



$P = 4l$

$= 4 \times 5$

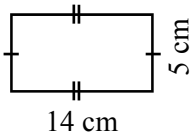
$P = 20$ cm

12 to 26

Length (l)	Perimeter ($4l$)	Length (l)	Perimeter ($4l$)
5	20 cm	15	
8		11	
10		14	
9		30	
3		50	
4		80	
12		55	
20		42	

Perimeter Formula for Rectangles :

$P = 2l + 2b$ where : l = length and b = breadth



$P = 2l + 2b$

$= 2 \times 14 + 2 \times 5$

$P = 38$ cm

27 to 33

Length (l)	$2l$	Breadth (b)	$2b$	Perimeter ($2l + 2b$)
14	28	5	10	38 cm
8		11		
12		5		
9		6		
15		25		
20		16		
7		19		
13		17		

Perimeter of Shapes

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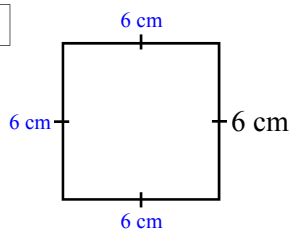
Put measurements on all sides then add them to find the perimeter.

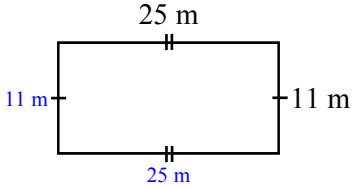
1



P = 6 + 6 + 6 + 6

P = 24 cm

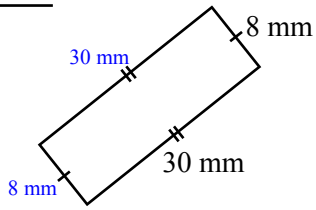
2



P = 25 + 25 + 11 + 11

P = 72 m

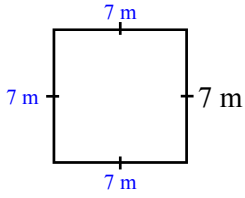
3



P = 30 + 30 + 8 + 8

P = 76 mm

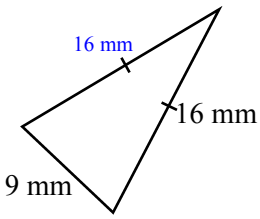
4



P = 7 + 7 + 7 + 7

P = 28 m

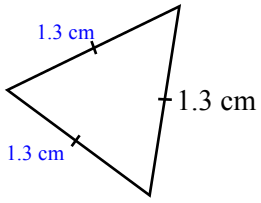
5



P = 16 + 16 + 9

P = 41 mm

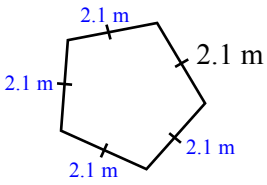
6



P = 1.3 + 1.3 + 1.3

P = 3.9 cm

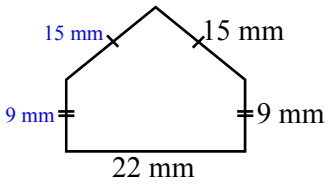
7



P = 2.1 + 2.1 + 2.1 + 2.1 + 2.1

P = 10.5 m

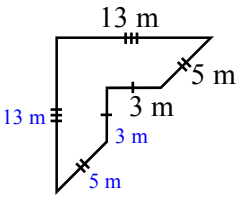
8



P = 22 + 9 + 15 + 15 + 9

P = 70 mm

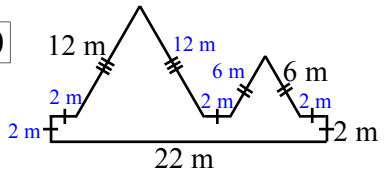
9



P = 5 + 13 + 13 + 5 + 3 + 3

P = 42 m

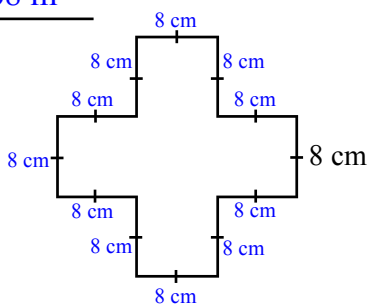
10



P = 22 + 2 + 2 + 12 + 12 + 2 + 6 + 6 + 2 + 2

P = 68 m

11



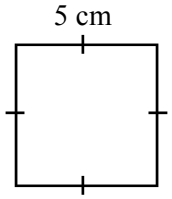
P = 8 × 12

P = 96 cm

Complete the tables, all measurements are in cm.

Perimeter Formula for Squares :

$P = 4l$ where l = side length



$P = 4l$

$= 4 \times 5$

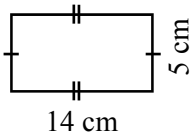
$P = 20$ cm

12 to 26

Length (l)	Perimeter ($4l$)	Length (l)	Perimeter ($4l$)
5	20 cm	15	60 cm
8	32 cm	11	44 cm
10	40 cm	14	56 cm
9	36 cm	30	120 cm
3	12 cm	50	200 cm
4	16 cm	80	320 cm
12	48 cm	55	220 cm
20	80 cm	42	168 cm

Perimeter Formula for Rectangles :

$P = 2l + 2b$ where : l = length and b = breadth



$P = 2l + 2b$

$= 2 \times 14 + 2 \times 5$

$P = 38$ cm

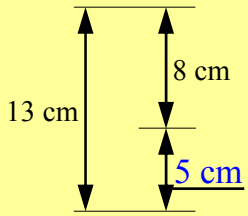
27 to 33

Length (l)	$2l$	Breadth (b)	$2b$	Perimeter ($2l + 2b$)
14	28	5	10	38 cm
8	16	11	22	38 cm
12	24	5	10	34 cm
9	18	6	12	30 cm
15	30	25	50	80 cm
20	40	16	32	72 cm
7	14	19	38	52 cm
13	26	17	34	60 cm

Perimeter - Finding a Missing Side

Sometimes before the perimeter of a shape can be calculated the missing sides have to be found. The missing sides will either have side markings that indicate that they are the same size as another side or they will be found by adding or subtracting sides that you know the value for. Note that this applies due to all the angles being right angles, the corners are not marked as right angles but you can assume that they are all 90º.

In column 1 Q. 1 - 5, two different ways are used to measure the same distance. This means that they must be equal. Use addition or subtraction to find the value. In the example here a distance is measured as 13 cm. The same distance is then broken up into to parts, one of those parts is 8 cm. Ask yourself, 'What number adds to 8 to give 13?'
The answer is 5, so the missing length is 5 cm.



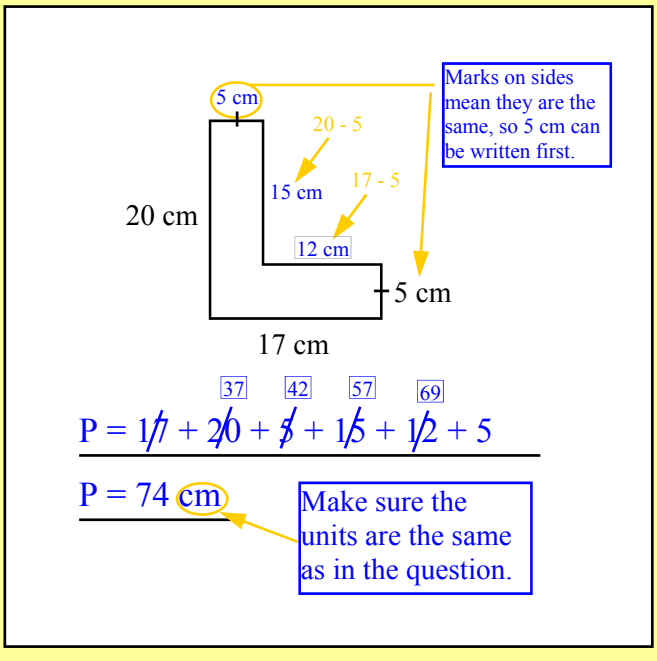
From Q. 6, through the column, shapes are given and you use the same method to find the missing sides. The method is:

- If the shape has side markings write in all the sides that you know.
- Use the given lengths to then calculate the other side lengths

Columns 2 and 3 add one step to column 1, the perimeter is calculated. Use 2 lines of working, the first line being the addition of all the sides and the second line being the answer with the units (mm, cm, m etc). Writing the side lengths on all sides of the shape will help avoid mistakes. Some students like to miss this step because they know the value of the side and don't need to write it, the problem is that when they add the sides to get the perimeter they often forget to include them. A method to avoid this problem is to label every side with its measurement then look at the addition and check off the numbers against the numbers on the sides of the shape. If there are different numbers or there are too many or too few you have to find the problem.

Note that if you add around the sides of the shape in the same way, clockwise or an anti-clockwise direction, it makes it less confusing.

Be careful with question 19 it is different and may trick you.



Perimeter - Finding a Missing Side

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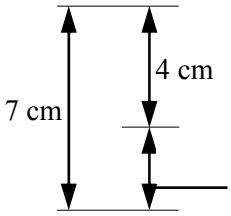
Calculate the missing lengths, all angles are right angles

1



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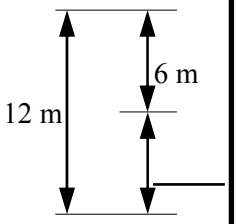

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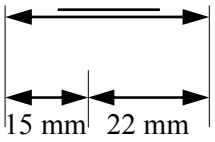

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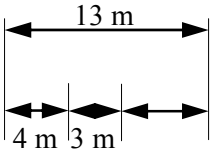

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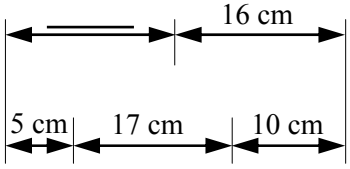

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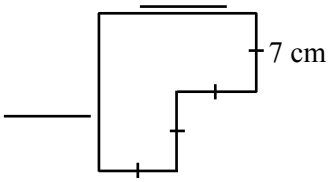

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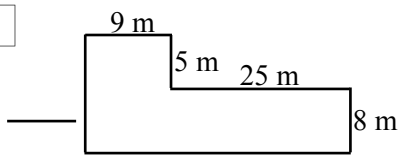

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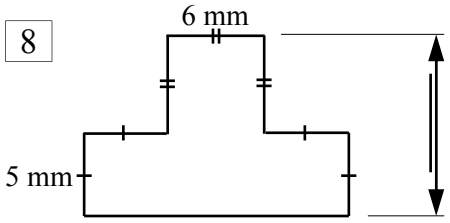

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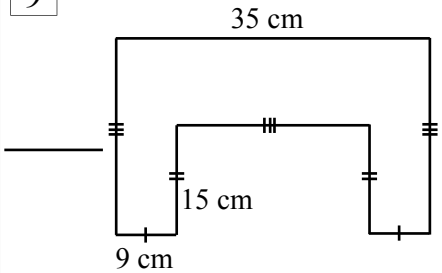
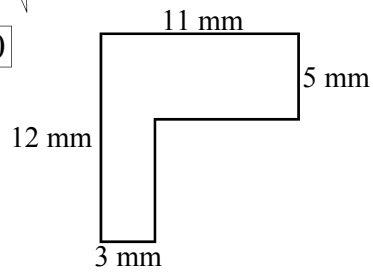


Find the missing sides then find the perimeter

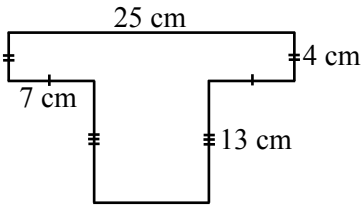
10



P = 3 + 12 + 11 + 5 + + +

P = mm

11



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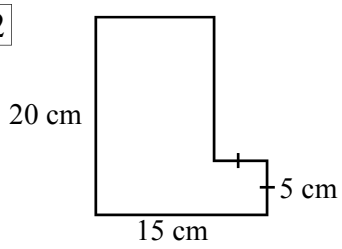

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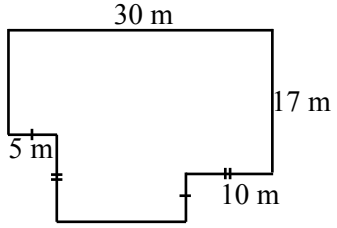

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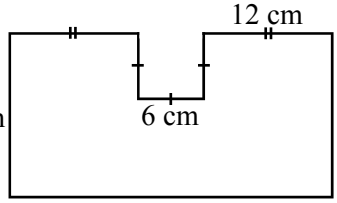

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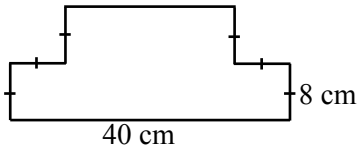

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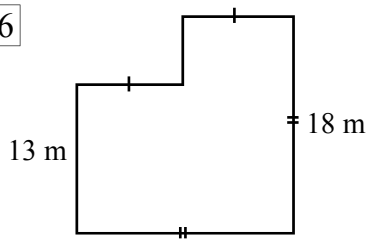

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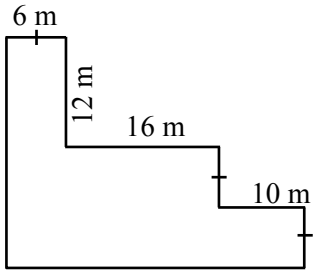

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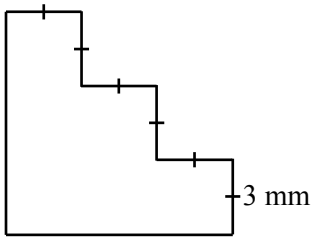

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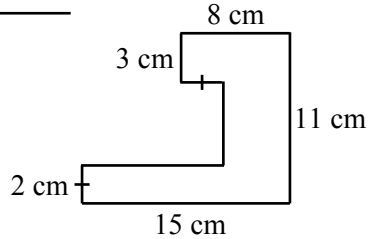

Perimeter - Finding a Missing Side

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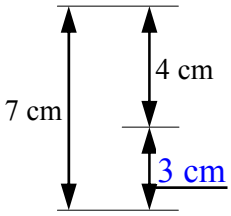
Calculate the missing lengths, all angles are right angles

1



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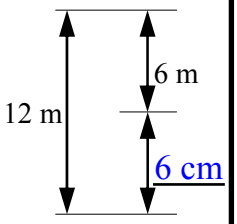

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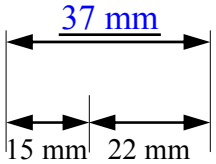

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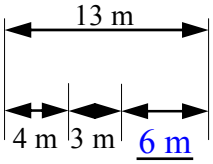

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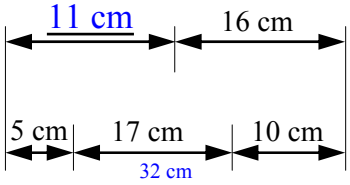

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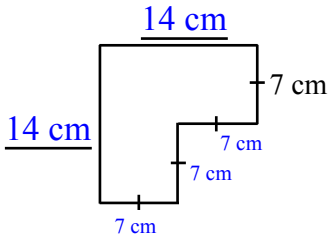

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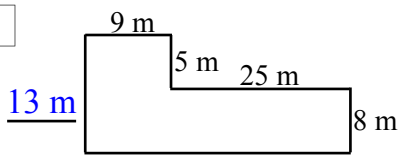
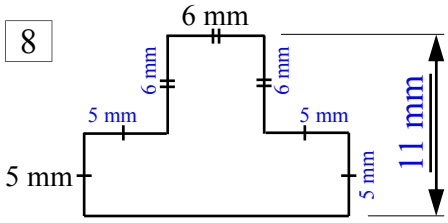


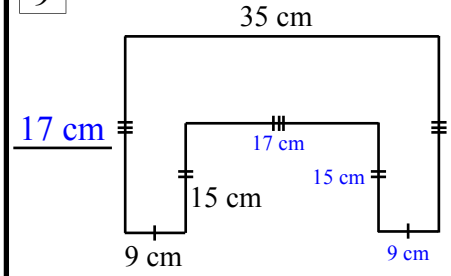
34 m

8



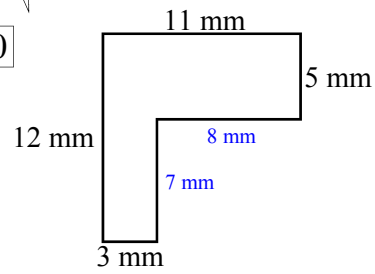
16 mm

9



Find the missing sides then find the perimeter

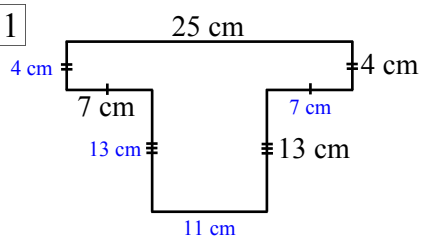
10



P = 3 + 12 + 11 + 5 + 8 + 7

P = 46 mm

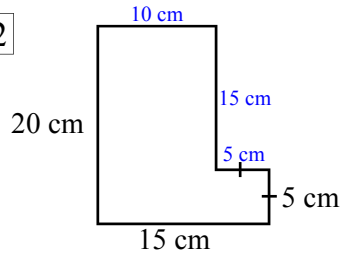
11



P = 11 + 13 + 7 + 4 + 25 + 4 + 7 + 13

P = 84 cm

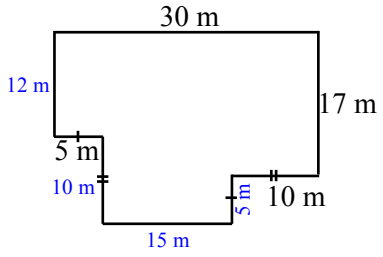
12



P = 15 + 20 + 10 + 15 + 5 + 5

P = 70 cm

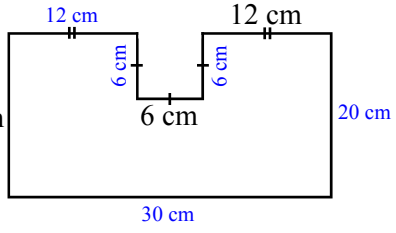
13



P = 15 + 10 + 5 + 12 + 30 + 17 + 10 + 5

P = 104 m

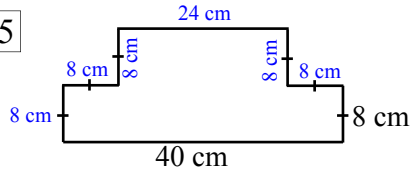
14



P = 30 + 20 + 12 + 6 + 6 + 6 + 12 + 20

P = 112 cm

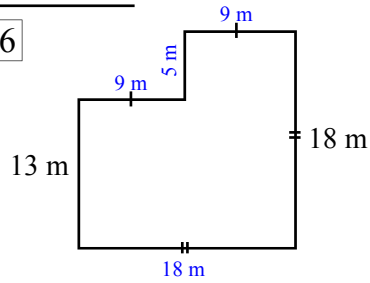
15



P = 40 + 8 + 8 + 8 + 24 + 8 + 8 + 8

P = 112 cm

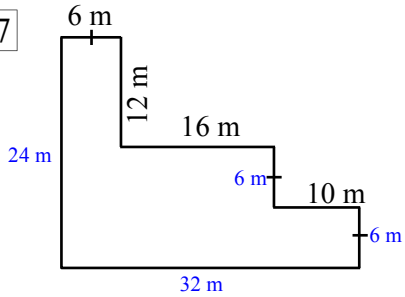
16



P = 18 + 13 + 9 + 5 + 9 + 18

P = 72 m

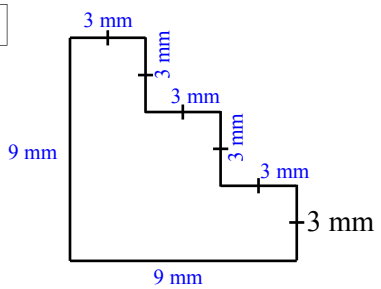
17



P = 32 + 24 + 6 + 12 + 16 + 6 + 10 + 6

P = 112 m

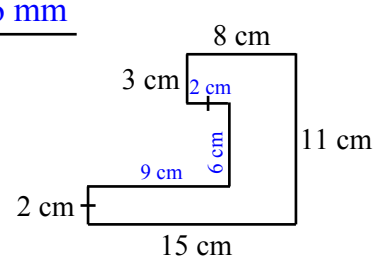
18



P = 9 + 9 + 3 + 3 + 3 + 3 + 3 + 3

P = 36 mm

19



P = 15 + 2 + 9 + 6 + 2 + 3 + 8 + 11

P = 56 cm

Perimeter Problems

These problems are an extension of the exercises covered earlier with an inclusion of cost. A common use of perimeter is in the calculation of cost of fences and walls. To calculate this cost calculate the perimeter first, then multiply the answer by the cost of the fence/wall by its unit cost, (the cost per metre).

In all the problems two lines of working are expected. The first line being the sum of the side lengths:
P = _ + _ + _ + etc

the second line being the total length and the units
P = _____ m (or cm).

Perimeter Problems

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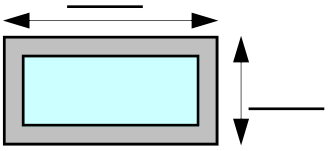
Answer the perimeter problems below.

1 A rectangular swimming pool has sides with lengths 12 m and 4 m. Find the distance around the pool.

2 A path 1 m wide surrounds the same pool (Q1) calculate the length and breadth of the pool and the path combined, put the dimensions on the diagram below and find the perimeter



3 If the pool above has a safety fence around the outside of the path find the cost of fencing the pool when the fence costs $27 per metre. ×

4 In a science experiment Travis noted that on average an ant stopped every 5 cm. If the viewing tray was 30 cm by 20 cm, find its perimeter.

How many times would the ant stop:

i) in 1 lap

ii) in 5 laps

iii) over a 9 m distance

5 Three hobby farmers are each given small garden areas. The plots all occupy the same land area (16 m²) but have different side lengths, find the perimeter of each.

Plot 1 : 16 m by 1 m

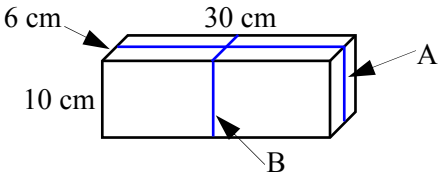
Plot 2 : 8 m by 2 m

Plot 3 : 4 m by 4 m

6 If you were offered a plot and had to pay to fence the area, would it be cheaper to have a square area or a rectangular area?

Circle : square or rectangle

7 A parcel has 2 strings around it as shown below. Calculate the length of the string A and string B, then add them to find the total length.



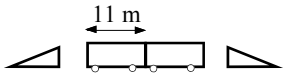
A:

B:

8 Scott says that when you double the side length of a square you get 4 times the perimeter, is he right?

For Not a

9 Dare-devil Dan is about to jump 2 buses, lined up end to end (see below). Dan is superstitious and walks 1 lap around the buses before the jump, how far does he walk?



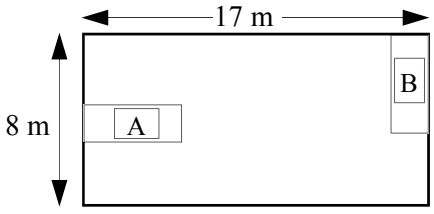
The buses are 11 m long and 3 m wide

10 An inventor created a synthetic material that will stretch to 18 times its length before breaking. If the original length is 7 m, find its maximum length. Will it stretch around a rectangular building with sides 25 m and 35 m? (Find P then circle answer)

Max length ×

Yes / No

11 Leon's dog 'Oblong' always runs around the perimeter of the backyard when he sees Leon arrive home. Find the distance he runs around the empty yard. Then calculate the distances he runs when Leon's car is in position A or in position B, (not both together). Leon's car is 2 m wide and 5 m long.



Position A is in the centre of the yard

No car:

Car A :

Car B :

Perimeter Problems

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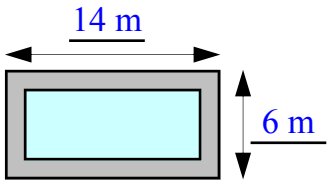
Answer the perimeter problems below.

1 A rectangular swimming pool has sides with lengths 12 m and 4 m. Find the distance around the pool.

$P = 12 + 12 + 4 + 4$

$P = 32$ m

2 A path 1 m wide surrounds the same pool (Q1) calculate the length and breadth of the pool and the path combined, put the dimensions on the diagram below and find the perimeter



$P = 14 + 6 + 14 + 6$

$P = 40$ m

3 If the pool above has a safety fence around the outside of the path find the cost of fencing the pool when the fence costs $27 per metre.

$\text{Cost} = 40 \times 27$

$\text{Cost} = \$1\,080$

$$\begin{array}{r} {\scriptstyle 2} \\ 27 \\ \times\ 40 \\ \hline 1\,080 \end{array}$$

4 In a science experiment Travis noted that on average an ant stopped every 5 cm. If the viewing tray was 30 cm by 20 cm, find its perimeter.

$P = 30 + 30 + 20 + 20$

$P = 100$ cm

How many times would the ant stop:

i) in 1 lap — 20 times

ii) in 5 laps — 100 times

iii) over a 9 m distance — 180 times

5 Three hobby farmers are each given small garden areas. The plots all occupy the same land area (16 m²) but have different side lengths, find the perimeter of each.

Plot 1 : 16 m by 1 m

$P = 16 + 16 + 1 + 1$

$P = 34$ m

Plot 2 : 8 m by 2 m

$P = 8 + 8 + 2 + 2$

$P = 20$ m

Plot 3 : 4 m by 4 m

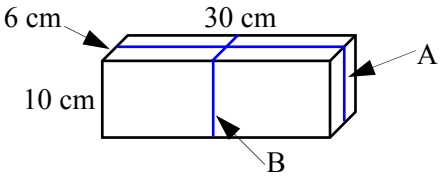
$P = 4 + 4 + 4 + 4$

$P = 16$ m

6 If you were offered a plot and had to pay to fence the area, would it be cheaper to have a square area or a rectangular area?

Circle : (square) or rectangle

7 A parcel has 2 strings around it as shown below. Calculate the length of the string A and string B, then add them to find the total length.



A: $P = 30 + 30 + 10 + 10$

$P = 80$ cm

B: $P = 10 + 10 + 6 + 6$

$P = 32$ cm

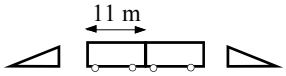
$P = 80 + 32$

$P = 112$ cm

8 Scott says that when you double the side length of a square you get 4 times the perimeter, is he right?

For ○ Not a ●

9 Dare-devil Dan is about to jump 2 buses, lined up end to end (see below). Dan is superstitious and walks 1 lap around the buses before the jump, how far does he walk?



The buses are 11 m long and 3 m wide

$P = 11 + 11 + 3 + 11 + 11 + 3$

$P = 50$ m

10 An inventor created a synthetic material that will stretch to 18 times its length before breaking. If the original length is 7 m, find its maximum length. Will it stretch around a rectangular building with sides 25 m and 35 m? (Find P then circle answer)

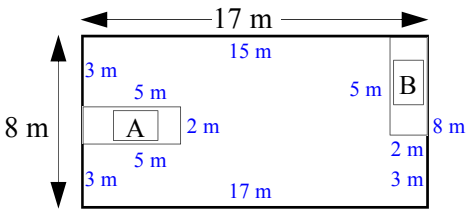
Max length 126 m

$P = 35 + 35 + 25 + 25$

$P = 120$ m (Yes) / No

$$\begin{array}{r} {\scriptstyle 5} \\ 18 \\ \times\ 7 \\ \hline 126 \end{array}$$

11 Leon's dog 'Oblong' always runs around the perimeter of the backyard when he sees Leon arrive home. Find the distance he runs around the empty yard. Then calculate the distances he runs when Leon's car is in position A or in position B, (not both together). Leon's car is 2 m wide and 5 m long.



Position A is in the centre of the yard

No car: $P = 17 + 17 + 8 + 8$

$P = 50$ m

Car A :

$P = 17 + 3 + 5 + 2 + 5 + 3 + 17 + 8$

$P = 60$ m

Car B :

$P = 17 + 8 + 15 + 5 + 2 + 3$

$P = 50$ m

FREEFALL MATHEMATICS

ALTITUDE

AREA

Grid Area of Shapes

This sheet uses areas of 1 cm^2. When you print this sheet you must select 'No Scaling' when you print it, otherwise the grid areas will be scaled and incorrect.

One method of determining area within a shape is to use a grid system in which the space inside a shape is broken up into squares. This sheet uses a 1 cm grid system. The shapes either fit the grid exactly or cross the grid diagonally using ½ a grid space.

The top section of the sheet (Q 1 - 9) requires you to calculate the areas of the nine shapes shown. This is done by counting the number of 1 cm² grid squares inside the shape and writing the answer. Some of the shapes have diagonals that cut the squares in half. Remember that ½ + ½ = 1, an even number of ½'s will give a whole number and an odd number of ½ squares will have the answer being a mixed number with a ½ as the proper fraction part. Or use decimals 0.5 cm² is half a square. Remember to give your answer in cm².

With Q 10 - 13 there are no grid lines on the shapes. This is overcome by either of these two methods:

- Use a grid overlay on a transparent sheet (such as an overhead projector sheet), or
- Rule lines 1 cm apart and create the grid on the paper

A worksheet is supplied that has the grids drawn for you if you can't print a transparent sheet.

The 2nd column asks you to use the same method with the grid overlay to calculate the area. But this time write in the side lengths. Then multiply the two side lengths together and see if this number relates to the area.

The third column then uses the side multiplication method to calculate the area of the shapes in the questions.

Grids for Transparency Printing

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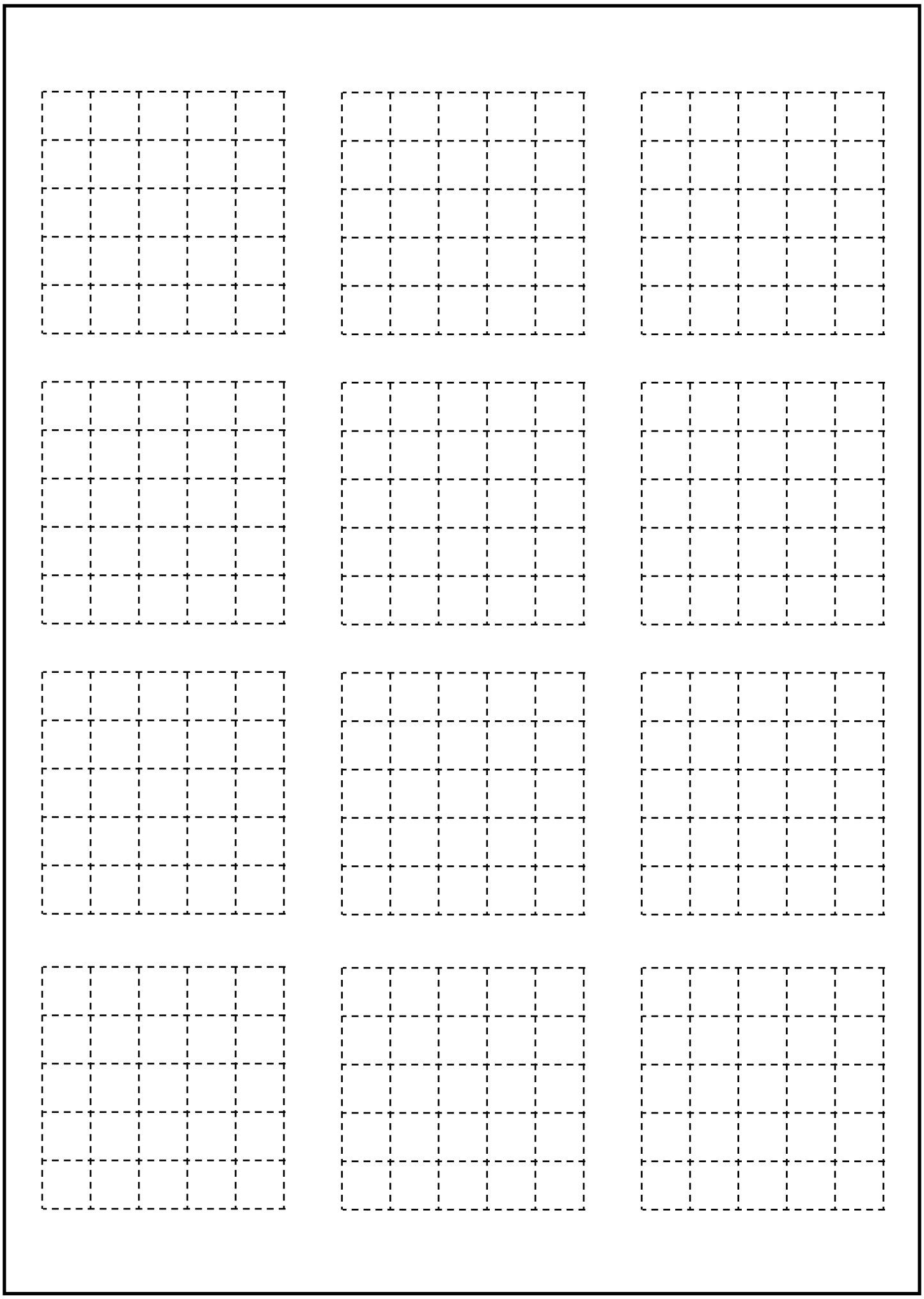

Grid Area of Shapes

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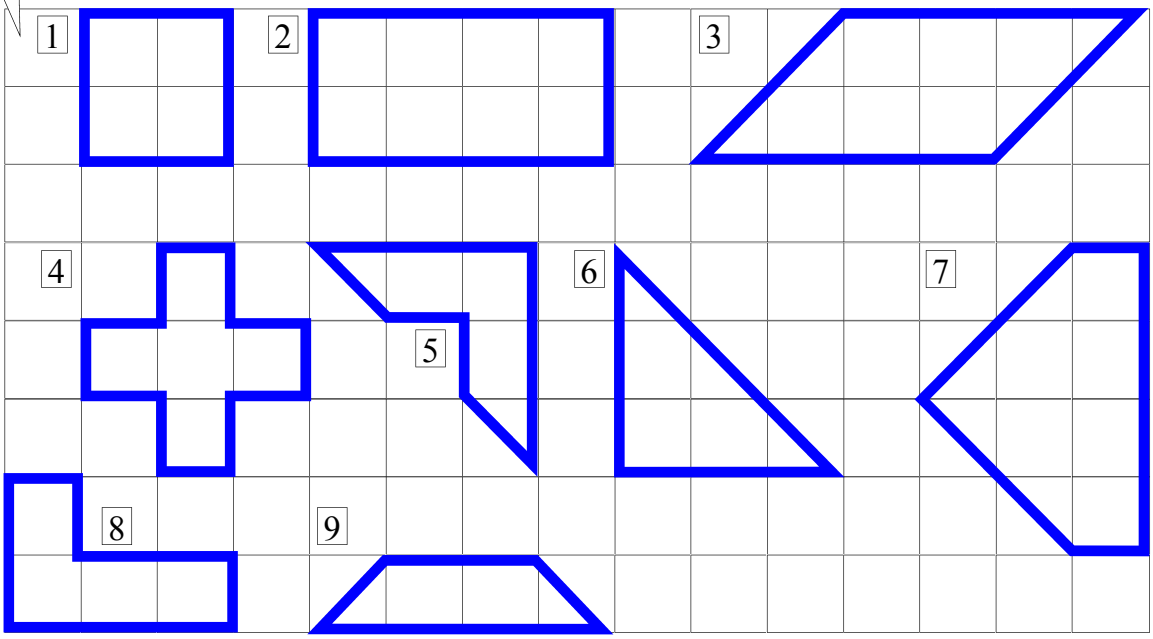
Find the area for the shapes on 1 cm grid



1 _____ cm²

2 _____

3 _____

4 _____

5 _____

6 _____

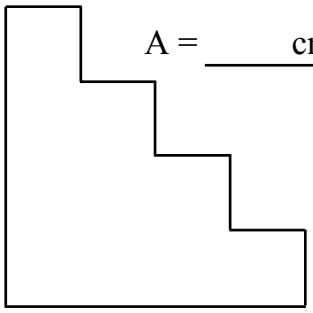
7 _____

8 _____

9 _____

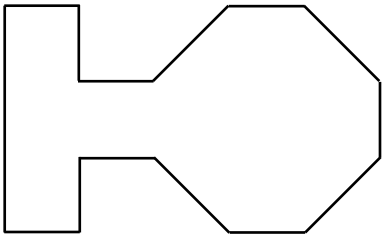
Use grid overlay to find the area of these, or use a rule to make 1 cm spaced lines

10



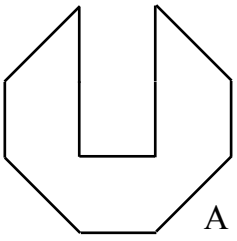
A = _____ cm²

11



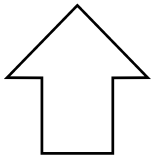
A = _____

12



A = _____

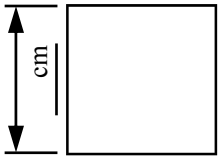
13



A = _____

Use 1 cm grid to find the area of these, then measure the side lengths.

14



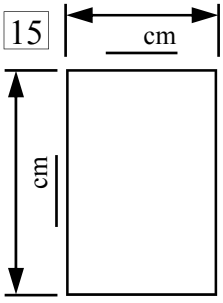
A = _____ cm²

Side = _____ cm

Side = _____ cm

Multiply sides ↓ ☐

15



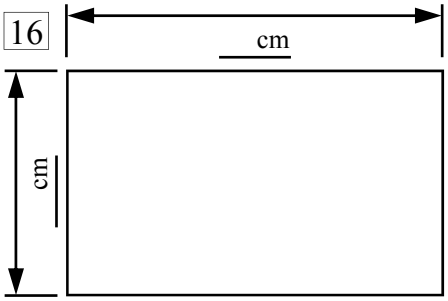
A = _____ cm²

Side = _____ cm

Side = _____ cm

Multiply sides ↓ ☐

16



A = _____ cm²

Side = _____ cm

Side = _____ cm

Multiply sides ☐

Answer the questions without drawing the shapes. Calculate the area of…..

17 A rectangle with side lengths 8 cm and 6 cm.

Area = ___ cm × ___ cm

Area = ___ cm²

18 A rectangle with side lengths 7 cm and 9 cm.

Area = ___ cm × ___ cm

Area = ___ cm²

19 A square with side lengths of 9 m.

Area = ___ m × ___ m

Area = ___ m²

20 A square with side lengths of 7 cm.

Area = ___ cm × ___ cm

Area = ___ cm²

21 A rectangle with side lengths 19 m and 3 m.

Area = ___ m × ___ m

Area = ___ m²

Grid Area of Shapes

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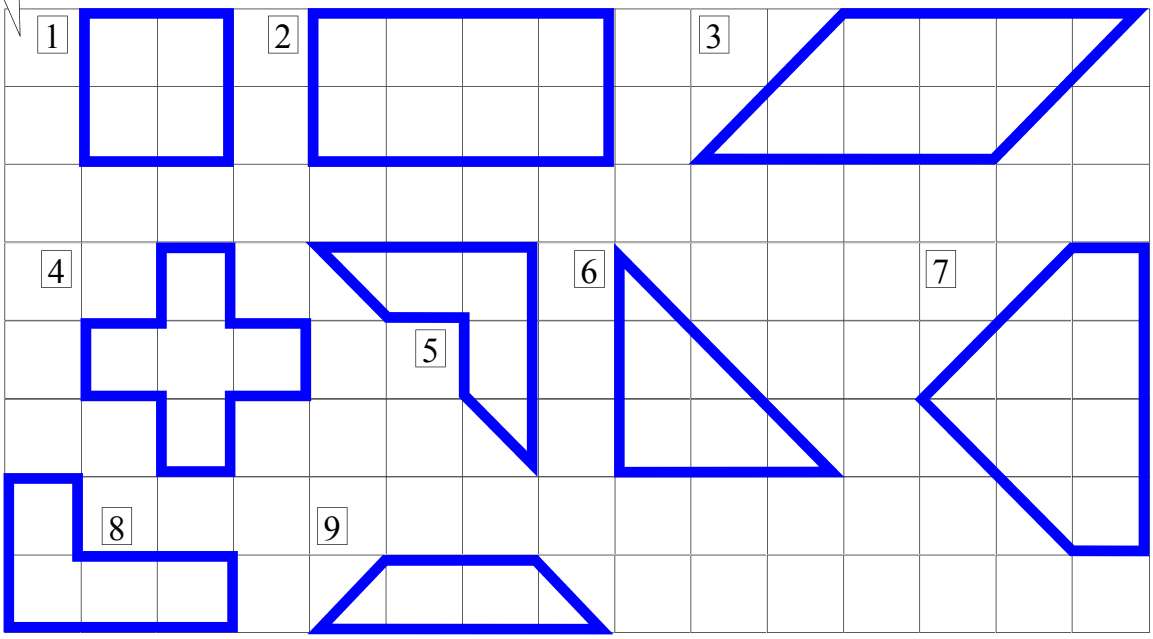
Find the area for the shapes on 1 cm grid



1 _____ cm²

2 _____

3 _____

4 _____

5 _____

6 _____

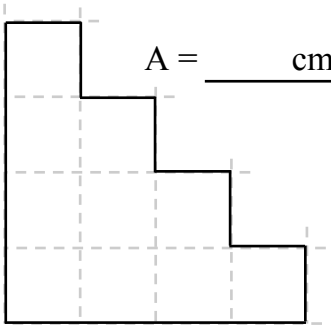
7 _____

8 _____

9 _____

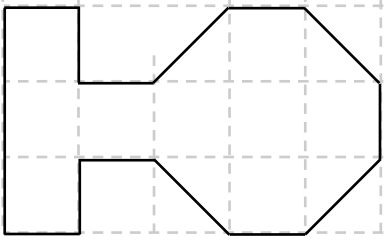
Use grid overlay to find the area of these, or use a rule to make 1 cm spaced lines

10



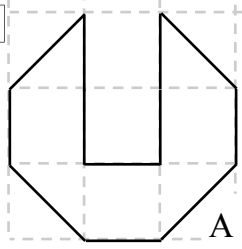
A = _____ cm²

11



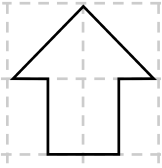
A = _____

12



A = _____

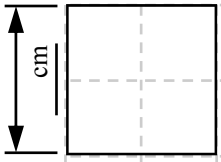
13



A = _____

Use 1 cm grid to find the area of these, then measure the side lengths.

14



A = _____ cm²

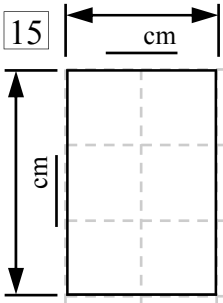
Side = _____ cm

Side = _____ cm

↓ Multiply sides

☐

15



A = _____ cm²

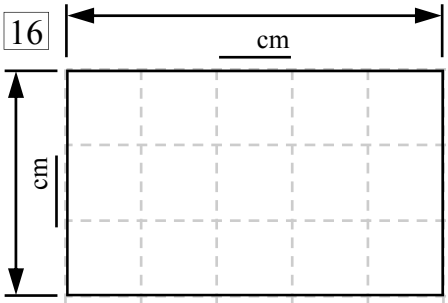
Side = _____ cm

Side = _____ cm

↓ Multiply sides

☐

16



A = _____ cm²

Side = _____ cm

Side = _____ cm

Multiply sides ↘ ↗ ☐

Answer the questions without drawing the shapes. Calculate the area of…..

17 A rectangle with side lengths 8 cm and 6 cm.

Area = ___ cm × ___ cm

Area = ___ cm²

18 A rectangle with side lengths 7 cm and 9 cm.

Area = ___ cm × ___ cm

Area = ___ cm²

19 A square with side lengths of 9 m.

Area = ___ m × ___ m

Area = ___ m²

20 A square with side lengths of 7 cm.

Area = ___ cm × ___ cm

Area = ___ cm²

21 A rectangle with side lengths 19 m and 3 m.

Area = ___ m × ___ m

Area = ___ m²

Grid Area of Shapes

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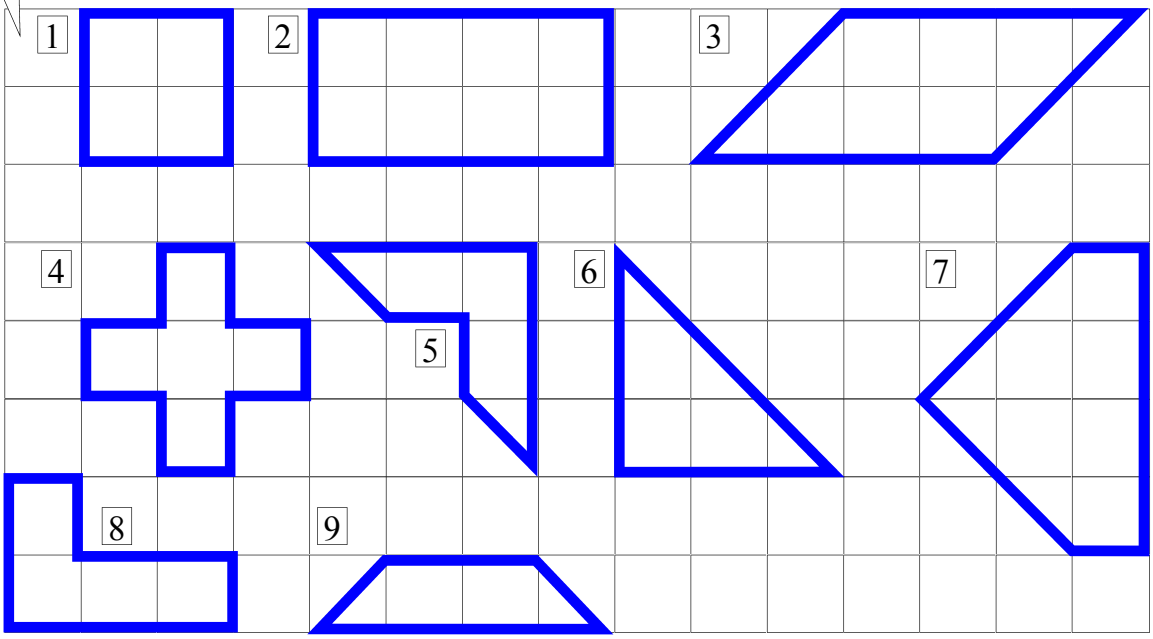
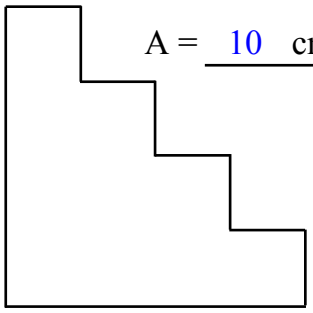
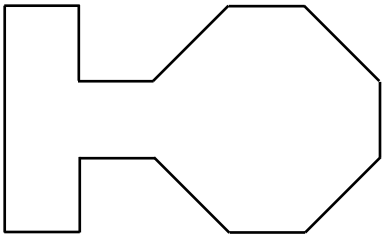
Find the area for the shapes on 1 cm grid

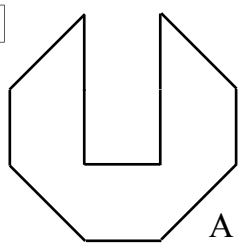


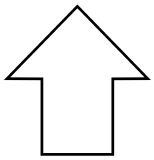
1. 4 cm²
2. 8 cm²
3. 8 cm²
4. 5 cm²
5. 4 cm²
6. 4½ cm²
7. 8 cm²
8. 4 cm²
9. 3 cm²

Use grid overlay to find the area of these, or use a rule to make 1 cm spaced lines

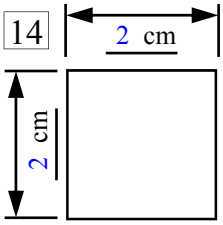
10. 
A = 10 cm²

11. 
A = 11 cm²

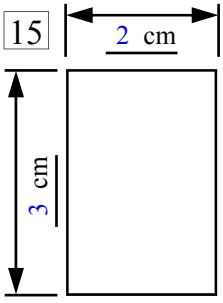
12. 
A = 5 cm²

13. 
A = 2 cm²

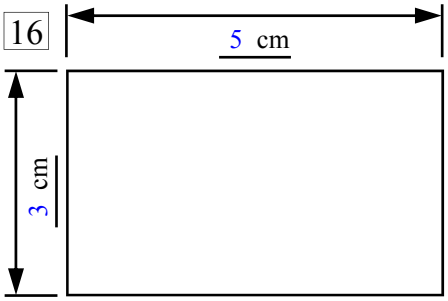
Use 1 cm grid to find the area of these, then measure the side lengths.

14. 

A = 4 cm²
Side = 2 cm
Side = 2 cm
Multiply sides
4

15. 

A = 6 cm²
Side = 2 cm
Side = 3 cm
Multiply sides
6

16. 

A = 15 cm²
Side = 3 cm
Side = 5 cm
Multiply sides
15

Answer the questions without drawing the shapes. Calculate the area of…..

17. A rectangle with side lengths 8 cm and 6 cm.
Area = 8 cm × 6 cm
Area = 48 cm²

18. A rectangle with side lengths 7 cm and 9 cm.
Area = 7 cm × 9 cm
Area = 63 cm²

19. A square with side lengths of 9 m.
Area = 9 m × 9 m
Area = 9 m²

20. A square with side lengths of 7 cm.
Area = 7 cm × 7 cm
Area = 49 cm²

21. A rectangle with side lengths 19 m and 3 m.
Area = 19 m × 3 m
Area = 57 m²

Designing Flooring on a House Plan

This sheet deals with area in a practical use, the floor coverings of a house. The depth to which you complete this sheet depends on your time and creativity. The grid on the third page is used to represent the floor of your house, looking down through a roof that has vanished.

The method of the sheet is as follows:

- Wall off the rooms.
- Try to create: 3 bedrooms, bathroom, kitchen, laundry, lounge room, dining room and a hallway.
- Colour the squares in each room a particular colour to represent a different floor covering, then the basics, fill the house with furniture. This can either be drawn in or the pieces on the next page can be coloured, cut out and stuck on.
- Once you have finished the house complete the last page which is used to find the total cost for flooring. Multiply the length and breadth of the room to get the area, if rooms are rectangular or square, otherwise just count the squares. Multiply this by the cost of the floor covering used, to find the cost of each room. Add the costs to get the total.

There are 3 bedroom spaces on the calculation sheet, you may have a 2 bedroom house though, so leave one blank, or create another room that you may have. There is also one additional space at the bottom of the page if you wish to put in another room. You may like to include a garage and park your car in it.

House Pieces

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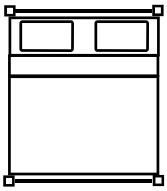
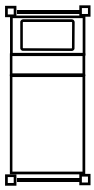
BEDROOMS

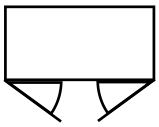
Double Bed



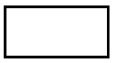
Single Bed

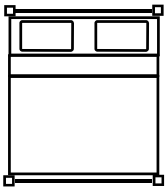


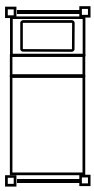
Wardrobe

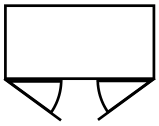


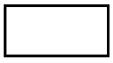
Chest of drawers

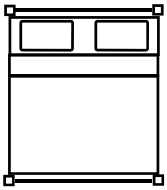


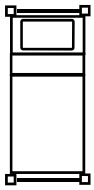


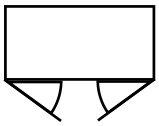


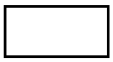










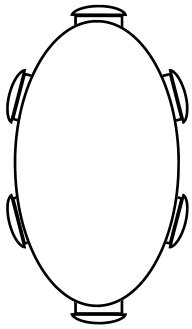


Toilet

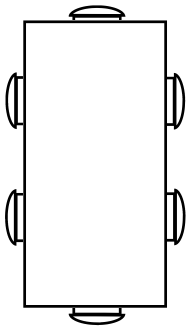


DINING ROOM

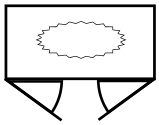
Round Dining Table + chairs



Rectangular Dining Table + chairs

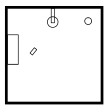


China Cupboard

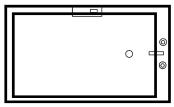


BATH ROOM

Shower with soap left on floor



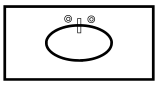
Bath



Toilet

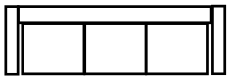


Basin



LOUNGE ROOM

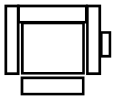
Lounge



Arm Chairs



Chair with TV remote tray and footrest

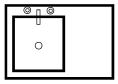


Coffee Table

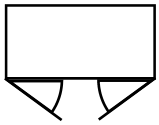


LAUNDRY

Wash Tub



Cupboard

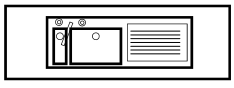


Washing Machine

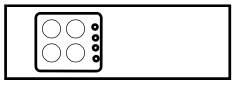


KITCHEN

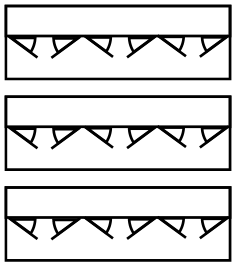
Bench with Sink



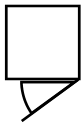
Bench with Cook-top and Oven



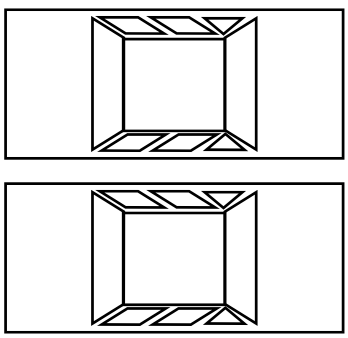
Bench with wall cupboards



Refrigerator



GARAGE/CARPORT



Designing Flooring on a House Plan

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Below is a blank floor Plan for a house that is still in design. Consider it your job to put in the walls to make rooms, naming and fitting out each room (adding furniture). Then using the next page calculate the cost of flooring for each room and then total. The rooms you need to include are:

Up to 3 Bedrooms (MBR, BR2 and BR3) Dining Room (DR) Lounge Room (LR)
Bathroom (Bath) Laundry (LDY) Hallway (HALL)
Kitchen (KN) [+ Pantry (P) optional]

Make sure you use a pencil. The separating walls should be on the grid lines, but thick enough to see. If you have extra room to fill in include rooms like: extra toilet, photographic darkroom, car garage or an additional bedroom. The scale used is 1 cm = 1 m.

Floor Coverings for the Plan

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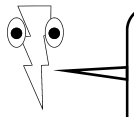

Measure the sides (length and breadth) of each room and multiply to get the area. If the room is not square or rectangular just count the squares. Select the type of flooring, ensure that it matches the type of room you are dealing with. Then multiply the area by the cost to get the cost for the room. Then add all the rooms to get total cost. Remember that 1 square = 1 m². Coverings to choose from:

1. Wool carpet	$95 per m²	2. Hard wear carpet	$32 per m²
3. Tiles	$27 per m²	4. Cork tiles	$48 per m²
5. Polished boards	$38 per m²	6. Vinyl	$12 per m²

Master Bedroom Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Bedroom 2 Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Bedroom 3 Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Dining Room Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Lounge Room Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Kitchen Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Laundry Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Hallway Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Bathroom Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

Length _____ × Breadth _____ = _____ m²

Number of flooring type __ Cost per m² _____ Cost for room _____

TOTAL COST: _____

Designing Flooring on a House Plan

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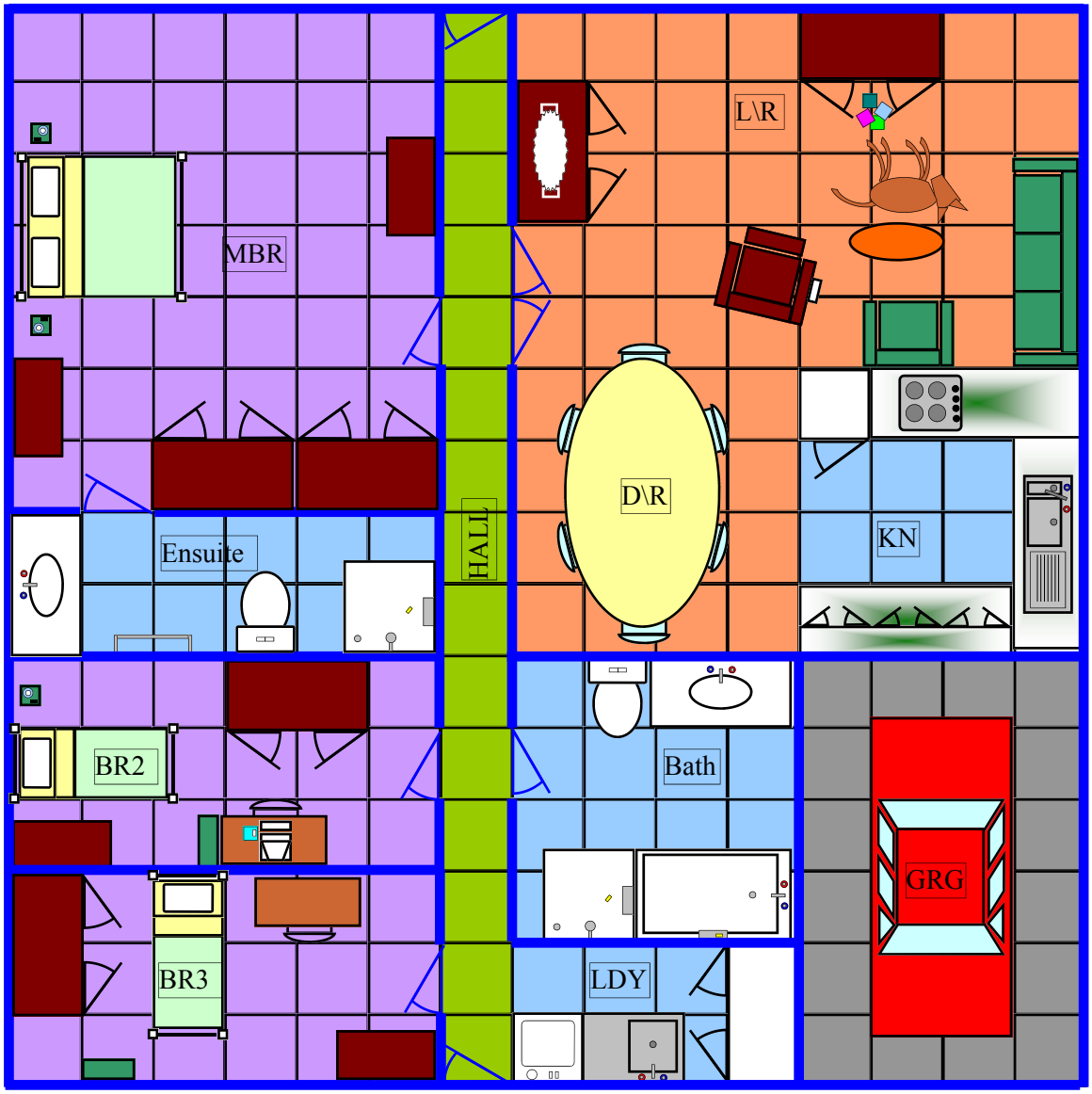
Bathroom (Bath)
Kitchen (KN) [+ Pantry (P) optional]
Dining Room (DR)
Laundry (LDY)
Lounge Room (LR)
Hallway (HALL)

Make sure you use a pencil. The separating walls should be on the grid lines, but thick enough to see. If you have extra room to fill in include rooms like: extra toilet, photographic darkroom, car garage or an additional bedroom. The scale used is 1 cm = 1 m.



Floor Coverings for the Plan

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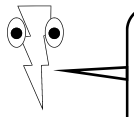

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3. Tiles	$27 per m²	4. Cork tiles	$48 per m²
5. Polished boards	$38 per m²	6. Vinyl	$12 per m²

Master Bedroom Length 6 × Breadth 7 = 42 m²

Number of flooring type 1 Cost per m² 95 Cost for room $3 990

Bedroom 2 Length 3 × Breadth 6 = 18 m²

Number of flooring type 1 Cost per m² 95 Cost for room $1 710

Bedroom 3 Length 3 × Breadth 6 = 18 m²

Number of flooring type 1 Cost per m² 95 Cost for room $1 710

Dining Room Length 4 × Breadth 4 = 16 m²

Number of flooring type 5 Cost per m² 38 Cost for room $608

Lounge Room Length 5 × Breadth 8 = 40 m²

Number of flooring type 5 Cost per m² 38 Cost for room $1 520

Kitchen Length 4 × Breadth 4 = 16 m²

Number of flooring type 3 Cost per m² 27 Cost for room $432

Laundry Length 2 × Breadth 4 = 8 m²

Number of flooring type 3 Cost per m² 27 Cost for room $216

Hallway Length 15 × Breadth 1 = 15 m²

Number of flooring type 2 Cost per m² 32 Cost for room $480

Bathroom Length 4 × Breadth 4 = 16 m²

Number of flooring type 3 Cost per m² 27 Cost for room $432

Ensuite Length 2 × Breadth 6 = 12 m²

Number of flooring type 3 Cost per m² 27 Cost for room $324

TOTAL COST: $11 422

Area of Squares

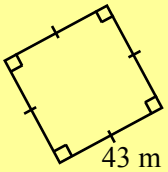
Area is the measurement of space within a shape. The units of measurement are square units, which can be mm², cm², m² and so on. This sheet deals solely with squares. The formula used is $A = l^2$. The l referring to the side length of the square.

Some students have a difficulty with squaring numbers (²). They forget that it means the number times itself and instead think it is the number times 2. You can use the formula $A = l \times l$ if this makes it easier to remember. Note that there are 3 lines of working. The first being the formula, the second being the substitution of values and the third being the answer with square units.

Column 1 questions have sides with 2 digit numbers. Column 2 has sides with 1 or 2 digits plus 1 decimal place. Use the same method as with Column 1, just remember that you will have an answer with 2 decimal places. Column 3 are written problems, solve these in the same way just without the diagram.

Make sure you place the ² on the units. Some students forget and place the ² on the number in the answer instead of the units, eg. instead of writing 14 m² they write 14² m, which is incorrect.

Show 3 lines of working : formula, substitution and answer



$A = l^2$

$= 43^2$

$A = 1\ 849\ m^2$

$$\begin{array}{r} {\scriptstyle 1} \\ 43 \times \\ 43 \\ \hline 129 \\ 1\ 720 \\ \hline 1\ 849 \end{array}$$

Area of Squares

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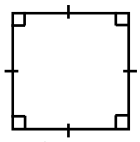
Find the areas of the squares below

1

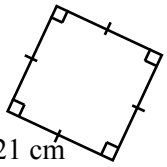


$A = l^2$

$=$ 0

A = m²

2



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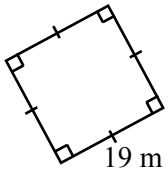

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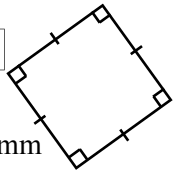

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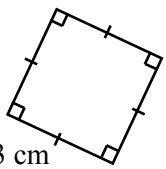
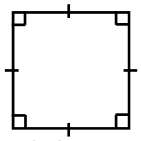


Find the area of these using decimals

6



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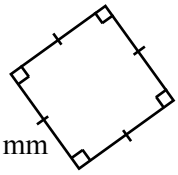

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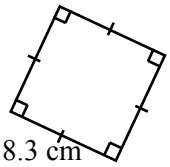

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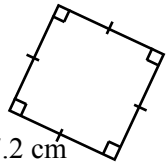

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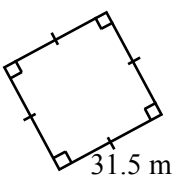

Now instead of a diagram read the problems

11 A quilt is to be made of 17 cm side length square patches. Calculate the area of one of these patches.

12 A garden shed has a square roof with 2.3 m side lengths. Calculate the area of the roof in square metres.

13 A courtyard has a square grassed section with 27 m long sides. Find the area for this section.

14 If turf costs \$4/m² what would be the cost of relaying the grass surface above?

Cost = area × unit cost

Area of Squares

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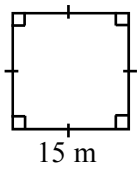
Find the areas of the squares below

1



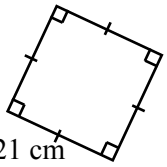
$A = l^2$

$= 15^2$

$A = 225\ m^2$

15 ×
15
1
75
150
225

2



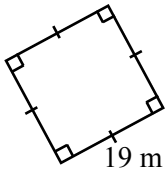
$A = l^2$

$= 21^2$

$A = 441\ cm^2$

21 ×
21
21
420
441

3



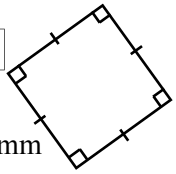
$A = l^2$

$= 19^2$

$A = 361\ m^2$

19 ×
19
1
171
190
361

4



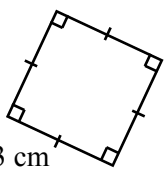
$A = l^2$

$= 37^2$

$A = 1\ 369\ mm^2$

2
37 ×
37
259
1 110
1 369

5



$A = l^2$

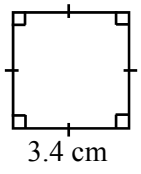
$= 53^2$

$A = 2\ 809\ cm^2$

53 ×
53
1
159
2 650
2 809

Find the area of these using decimals

6



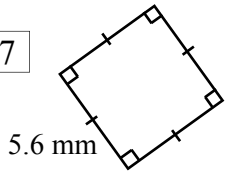
$A = l^2$

$= 3.4^2$

$A = 11.56\ cm^2$

34 ×
34
136
1 020
11.56

7



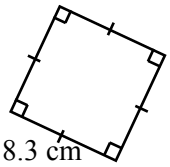
$A = l^2$

$= 5.6^2$

$A = 31.36\ mm^2$

56 ×
56
1
336
2 800
31.36

8



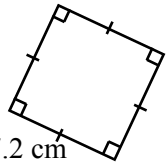
$A = l^2$

$= 8.3^2$

$A = 68.89\ cm^2$

2
83 ×
83
249
6 640
68.89

9



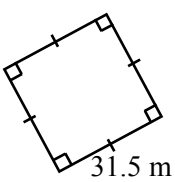
$A = l^2$

$= 17.2^2$

$A = 295.84\ cm^2$

172 ×
172
344
12 040
17 200
295.84

10



$A = l^2$

$= 31.5^2$

$A = 992.25\ m^2$

1
315 ×
315
1 1
1 575
3 150
94 500
992.25

Now instead of a diagram read the problems

11 A quilt is to be made of 17 cm side length square patches. Calculate the area of one of these patches.

$A = l^2$

$= 17^2$

$A = 289\ cm^2$

17 ×
17
119
170
289

12 A garden shed has a square roof with 2.3 m side lengths. Calculate the area of the roof in square metres.

$A = l^2$

$= 2.3^2$

$A = 5.29\ m^2$

23 ×
23
1
69
460
5.29

13 A courtyard has a square grassed section with 27 m long sides. Find the area for this section.

$A = l^2$

$= 27^2$

$A = 729\ m^2$

1
27 ×
27
1
189
540
729

14 If turf costs \$4/m² what would be the cost of relaying the grass surface above?

Cost = area × unit cost

$= 729 \times 4$

Cost = \$2 916

1 3
729 ×
4
2 916

Area of Rectangles

Area is the measurement of space within the shape. The units of measurement are square units, which can be mm², cm², m² and so on. This sheet deals solely with rectangles. The equation used is A = *lb*. The *l* is the side length, *b* is the breadth. It doesn't matter which you choose as *l* and which as *b*, the answer will still be the same, but generally *l* is the longest length.

Note that there are 3 lines of working. The first being the formula, the second being the substitution of values and the third being the answer with units.

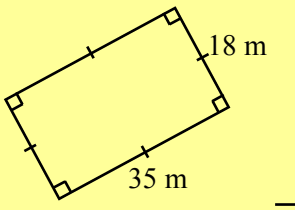
Column 1 questions have sides with 2 digit numbers. Write the first 2 lines of working then use the multiplication working space. Then answer using the third line.

Column 2 has sides with 1 or 2 digits plus 1 decimal place. Use the same method as with Column 1 just remember that you will have an answer with 2 decimal places. Multiply the numbers without the decimal point, then put it back in at the end.

Column 3 has written problems solve these in the same way, just without the diagram.

The most common mistake made is misusing the ², some students forget and put the ² on the number in the answer instead of the units, instead of 14 m² they incorrectly write 14² m.

Show 3 lines of working: formula, substitution and answer.



A = *lb*

= 35 × 18

A = 630 m²

35 ×
18
1
280
350
630

Area of Rectangles

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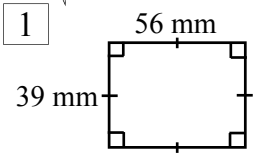
Find the areas of the rectangles below

1

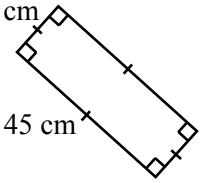


$A = lb$

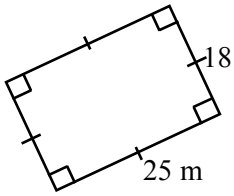
$= \times$ 0

$A =$ mm²

2



3



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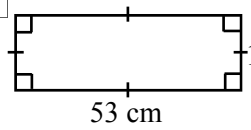

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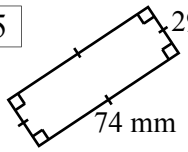
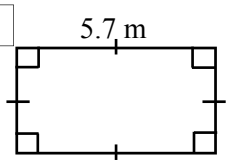


Find the area now using decimals

6



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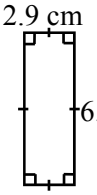

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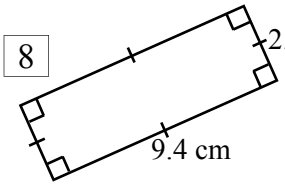

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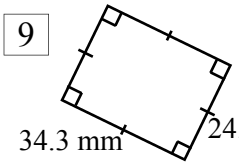

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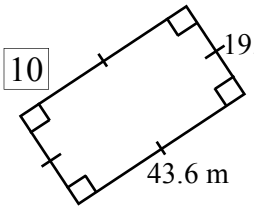

Now find the area in the following problems

11 A rectangular coffee table has side lengths 22 cm and 63 cm. Calculate the area of the table

12 Find the area of an air hockey table which has a rectangular playing area of side lengths 1.1 m and 2.6 m.

13 A foyer of a building is to be tiled. If the area is rectangular with sides 25 m and 48 m, find the area.

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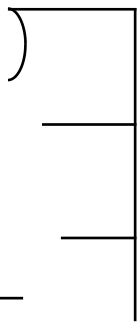

Area of Rectangles

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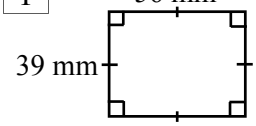
Find the areas of the rectangles below

1



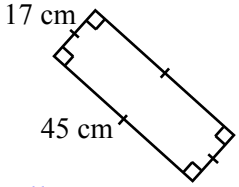
$A = lb$

$= 56 \times 39$

$A = 2\,184 \text{ mm}^2$

$$\begin{array}{r} {\scriptstyle 1} \\ 56 \times \\ 39 \\ \hline {\scriptstyle 1} \\ 504 \\ 1\,680 \\ \hline 2\,184 \end{array}$$

2



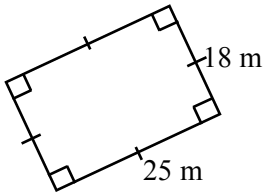
$A = lb$

$= 45 \times 17$

$A = 765 \text{ cm}^2$

$$\begin{array}{r} 45 \times \\ 17 \\ \hline 315 \\ 450 \\ \hline 765 \end{array}$$

3



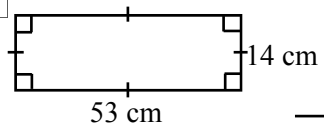
$A = lb$

$= 25 \times 18$

$A = 450 \text{ m}^2$

$$\begin{array}{r} 25 \times \\ 18 \\ \hline 200 \\ 250 \\ \hline 450 \end{array}$$

4



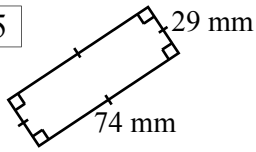
$A = lb$

$= 53 \times 14$

$A = 742 \text{ m}^2$

$$\begin{array}{r} 53 \times \\ 14 \\ \hline 212 \\ 530 \\ \hline 742 \end{array}$$

5



$A = lb$

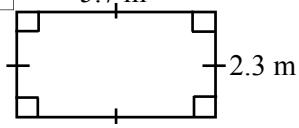
$= 74 \times 29$

$A = 2\,146 \text{ mm}^2$

$$\begin{array}{r} 74 \times \\ 29 \\ \hline {\scriptstyle 1\ 1} \\ 666 \\ 1\,480 \\ \hline 2\,146 \end{array}$$

Find the area now using decimals

6



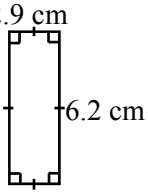
$A = lb$

$= 5.7 \times 2.3$

$A = 13.11 \text{ m}^2$

$$\begin{array}{r} {\scriptstyle 1} \\ 57 \times \\ 23 \\ \hline {\scriptstyle 1} \\ 171 \\ 1\,140 \\ \hline 13.11 \end{array}$$

7



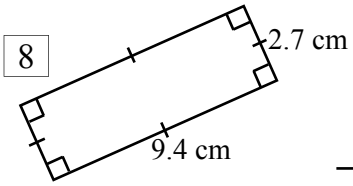
$A = lb$

$= 2.9 \times 6.2$

$A = 17.98 \text{ cm}^2$

$$\begin{array}{r} 62 \times \\ 29 \\ \hline 558 \\ 1\,240 \\ \hline 17.98 \end{array}$$

8



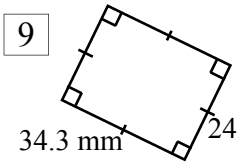
$A = lb$

$= 9.4 \times 2.7$

$A = 25.38 \text{ cm}^2$

$$\begin{array}{r} 94 \times \\ 27 \\ \hline {\scriptstyle 1\ 1} \\ 658 \\ 1\,880 \\ \hline 25.38 \end{array}$$

9



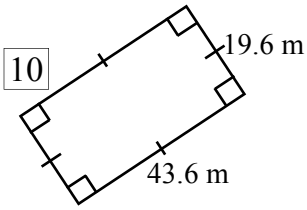
$A = lb$

$= 34.3 \times 24.6$

$A = 843.78 \text{ mm}^2$

$$\begin{array}{r} 343 \times \\ 246 \\ \hline {\scriptstyle 1\ 1} \\ 2\,058 \\ 13\,720 \\ 68\,600 \\ \hline 843.78 \end{array}$$

10



$A = lb$

$= 43.6 \times 19.6$

$A = 854.56 \text{ m}^2$

$$\begin{array}{r} 436 \times \\ 196 \\ \hline {\scriptstyle 1\ 1} \\ 2\,616 \\ 39\,240 \\ 43\,600 \\ \hline 854.56 \end{array}$$

Now find the area in the following problems

11 A rectangular coffee table has side lengths 22 cm and 63 cm. Calculate the area of the table

$A = lb$

$= 22 \times 63$

$A = 1\,386 \text{ cm}^2$

$$\begin{array}{r} 63 \times \\ 22 \\ \hline 126 \\ 1\,260 \\ \hline 1\,386 \end{array}$$

12 Find the area of an air hockey table which has a rectangular playing area of side lengths 1.1 m and 2.6 m.

$A = lb$

$= 1.1 \times 2.6$

$A = 2.86 \text{ m}^2$

$$\begin{array}{r} 26 \times \\ 11 \\ \hline 26 \\ 260 \\ \hline 2.86 \end{array}$$

13 A foyer of a building is to be tiled. If the area is rectangular with sides 25 m and 48 m, find the area.

$A = lb$

$= 25 \times 48$

$A = 1\,200 \text{ m}^2$

$$\begin{array}{r} {\scriptstyle 1} \\ 48 \times \\ 25 \\ \hline {\scriptstyle 1} \\ 240 \\ 960 \\ \hline 1\,200 \end{array}$$

14 If 2 tilers can lay 80 m² per day, calculate the time taken to complete the job

$$\begin{array}{r} 15 \\ 80 \overline{)\ 1\,200} \\ 80 \\ \hline 400 \\ 400 \\ \hline 0 \end{array}$$

It will take 15 days.

Area of Triangles

Area is the measurement of space within a shape. The units of measurement are square units, which can be mm², cm², m² and so on. This sheet deals solely with triangles. The formula used is A = ½*bh*. The *b* referring to triangle's base length, the *h* referring to the triangle's perpendicular height.

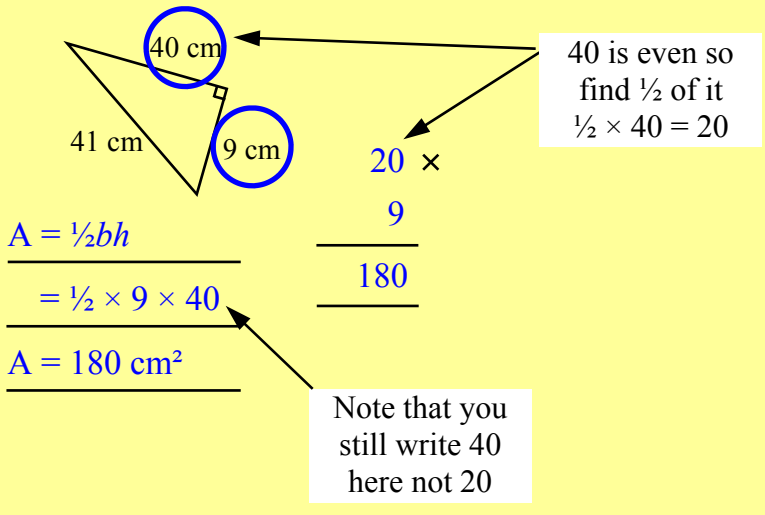
Triangles confuse some students due to there being 3 sides but only 2 measurements being used in the formula. If given 3 sides some students multiply ½ by all 3 sides, this is incorrect. Other students use the horizontal measurement always as the base, this isn't the case with rotated triangles. The critical thing to remember is that the measurements MUST be at right angles (perpendicular) to each other ALWAYS. So long as you never use sides that don't have a right angle between them you will be correct.

With Column 1 the triangles have 3 or more dimensions (measurements) on them. Remember that you only need 2 of the lengths. Circle the sides to be used in the formula. Then find the area using those 2 sides. Note that there are 3 lines of working, the first being the formula, the second being the substitution of values and the third being the answer with units.

Column 1 features triangles with sides having 2 digit numbers. Write the first 2 lines of working then use the multiplication working space. Then write the answer in the third line. With triangles save yourself time by dividing one of the sides by 2 and using it in the multiplication. All the questions have at least one even side which can be divided by 2. See the example below. When you divide a decimal imagine the point isn't there. E.g. 3.4 as 34 half of 34 is 17 so it is 1.7 and 8.8 as 88, half of 88 is 44 so it is 4.4.

Column 2 questions have decimal sides with 1 digit plus 1 decimal place. Use the same method as with column 1 just remember that you will have an answer with 2 decimal places.

Column 3 are written problems solve these in the same way, just without the diagram.



Area of Triangles

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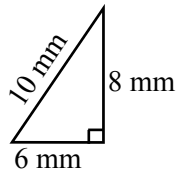
Select the 2 measurements to use, circle them, then find the triangle's area.

1

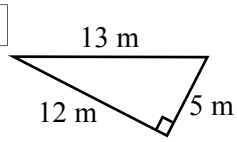


$A = \frac{1}{2}bh$

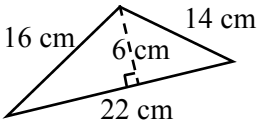
$= \frac{1}{2} \times \quad \times$

$A = \quad mm^2$

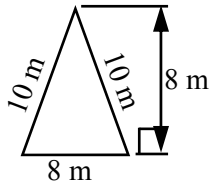
2



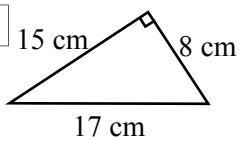
3



4

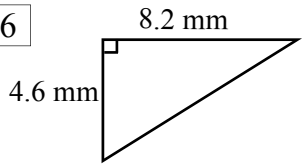


5

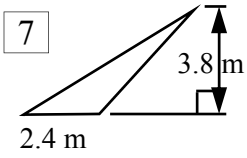


Find the area now using decimals

6



7



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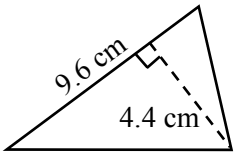

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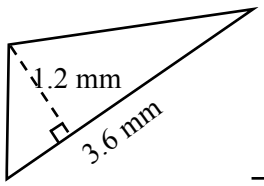

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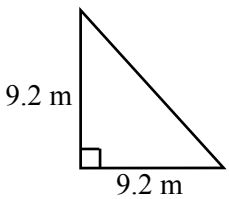

Now find the area in the following problems

11 A triangular sun shade has perpendicular sides of 3.8 m and 6.2 m. Calculate the area

12 A triangular section of glass has a base of 4.8 m and a perpendicular height of 7.6 m. Calculate the area.

13 A triangular sail has vertical height of 5.2 m and a horizontal base length of 3 m. Calculate its area.

14 A triangular garden bed has perpendicular sides of 86 cm and 94 cm. Calculate the bed's area.

Area of Triangles

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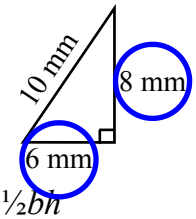
Select the 2 measurements to use, circle them, then find the triangle's area.

1

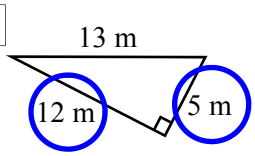


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 8$

$A = 24$ mm²

2



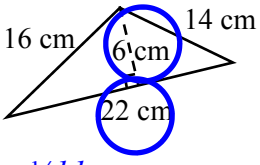
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 12$

$A = 30$ m²

6 × 5 = 30

3



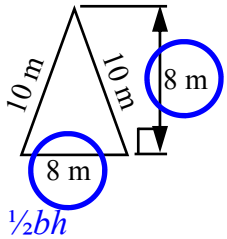
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 22 \times 6$

$A = 66$ cm²

11 × 6 = 66

4

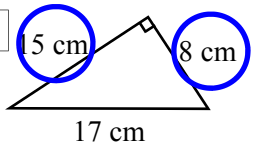


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 8$

$A = 32$ m²

5



$A = \frac{1}{2}bh$

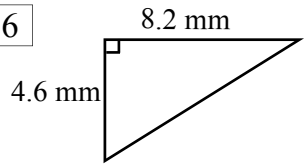
$= \frac{1}{2} \times 8 \times 15$

$A = 60$ cm²

^{2}15 × 4 = 60

Find the area now using decimals

6



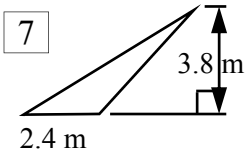
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4.6 \times 8.2$

$A = 18.86$ mm²

82 × 23 = 246 + 1 640 = 18.86

7



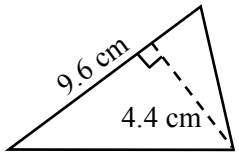
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 2.4 \times 3.8$

$A = 4.56$ m²

38 × 12 = 76 + 380 = 4.56

8



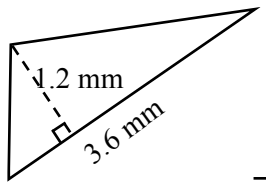
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 9.6 \times 4.4$

$A = 21.12$ cm²

48 × 44 = 192 + 1 920 = 21.12

9



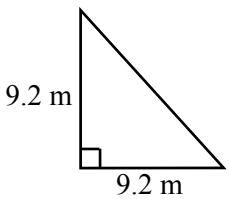
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3.6 \times 1.2$

$A = 2.16$ cm²

18 × 12 = 36 + 180 = 2.16

10



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 9.2 \times 9.2$

$A = 42.32$ m²

92 × 46 = 552 + 3 680 = 42.32

Now find the area in the following problems

11 A triangular sun shade has perpendicular sides of 3.8 m and 6.2 m. Calculate the area

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6.2 \times 3.8$

$A = 11.78$ m²

^{2}38 × 31 = 38 + 1 140 = 11.78

12 A triangular section of glass has a base of 4.8 m and a perpendicular height of 7.6 m. Calculate the area.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4.8 \times 7.6$

$A = 9.12$ m²

^{1}76 × 24 = 304 + 1 520 = 18.24

13 A triangular sail has vertical height of 5.2 m and a horizontal base length of 3 m. Calculate its area.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3 \times 5.2$

$A = 7.8$ m²

52 × 15 = 260 + 520 = 7.80

14 A triangular garden bed has perpendicular sides of 86 cm and 94 cm. Calculate the bed's area.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 86 \times 94$

$A = 4\,042$ cm²

^{1}94 × 43 = ^{1}282 + 3 760 = 4 042

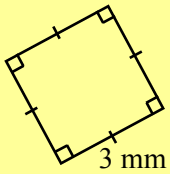
Areas of Squares, Rectangles and Triangles

Area is the measurement of space within a shape. The units of measurement are square units, which can be mm², cm², m² and so on. Note that if you are ever given a question without units, then express your answer in units². The last questions in each column require you to do this.

Column 1 deals solely with squares. The formula used is $A = l^2$. The l referring to the side length. Some students have a difficulty with squaring numbers (²). They forget that it means the number times itself and instead think it is the number times 2. You can use the formula $A = l \times l$ if it easier for you to remember. The answer is obtained by multiplying the side length by itself. Note that there are 3 lines of working. The first being the formula, the second being the substitution of values and the third being the answer with units.

Column 2 is all rectangles. The equation is $A = lb$, the length is normally the longest distance, but it doesn't actually matter which you choose because the answer will be the same. So multiply the two different sides together in any order.

Column 3 are all triangles. The equation is $A = \frac{1}{2}bh$, or 'half times the base times the height'. When you multiply these there is an easier way. The more difficult way is to multiply the two numbers then divide by 2, only use this method if both numbers are odd. The easier way, if at least one side is even, is divide one of the numbers by 2 (usually pick the largest even number) get the answer and then multiply it by the other side. This sheet has been designed to be done mentally, so no working spaces are provided.

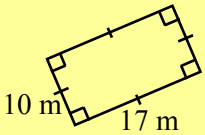


$A = l^2$

$= 3^2$

$A = 9$ mm²

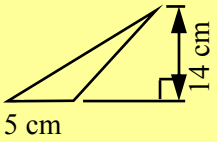
Don't forget to put a 'squared' (²) in the answer



$A = lb$

$= 17 \times 10$

$A = 170$ m²



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 14$

$A = 35$ cm²

Calculate like this:
"a half of 14 is 7, 7 times 5 is 35

Areas of Squares, Rectangles and Triangles

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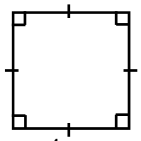
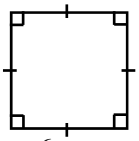
Find the areas of the squares below

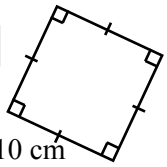
1 

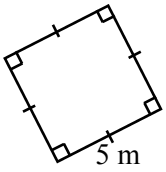
A = l^2

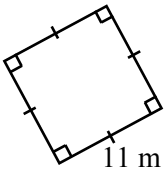
=

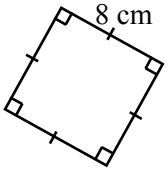
A = m²

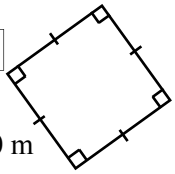
2 

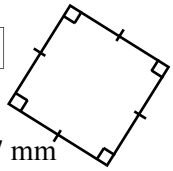
3 

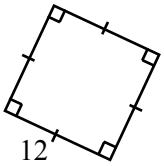
4 

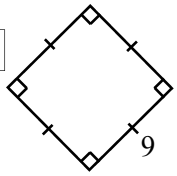
5 

6 

7 

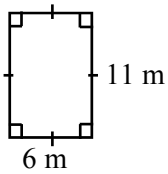
8 

9 

10 

A = units²

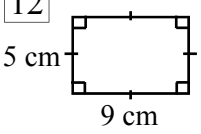
Find the areas of the rectangles below

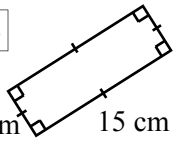
11 

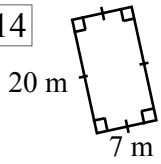
A = lb

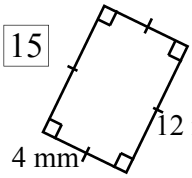
=

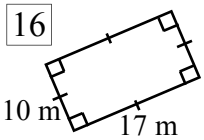
A = m²

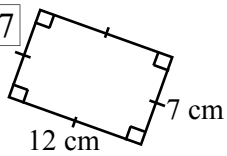
12 

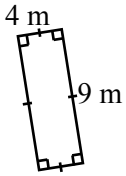
13 

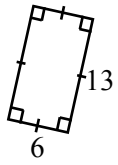
14 

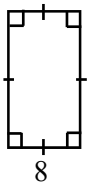
15 

16 

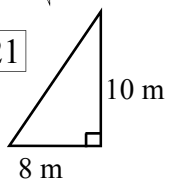
17 

18 

19 

20 

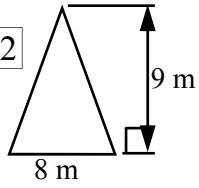
Find the areas of the triangles below

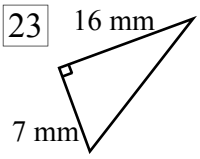
21 

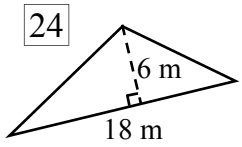
A = ½bh

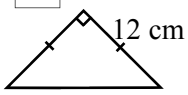
= ½× ×

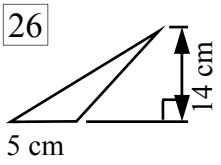
A = m²

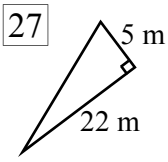
22 

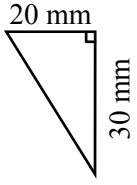
23 

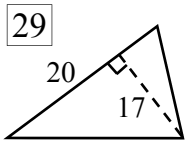
24 

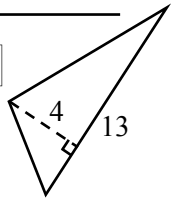
25 

26 

27 

28 

29 

30 

Areas of Squares, Rectangles and Triangles

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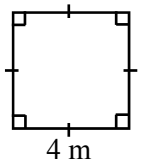
Find the areas of the squares below

1

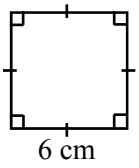


$A = l^2$

$= 4^2$

$A = 16 \text{ m}^2$

2

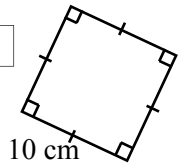


$A = l^2$

$= 6^2$

$A = 36 \text{ cm}^2$

3

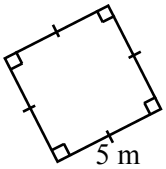


$A = l^2$

$= 10^2$

$A = 100 \text{ cm}^2$

4

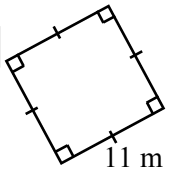


$A = l^2$

$= 5^2$

$A = 25 \text{ m}^2$

5

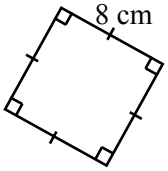


$A = l^2$

$= 11^2$

$A = 121 \text{ m}^2$

6

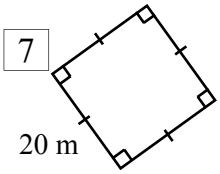


$A = l^2$

$= 8^2$

$A = 64 \text{ cm}^2$

7

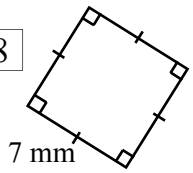


$A = l^2$

$= 20^2$

$A = 400 \text{ m}^2$

8

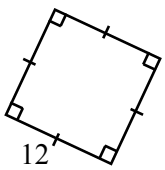


$A = l^2$

$= 7^2$

$A = 49 \text{ mm}^2$

9

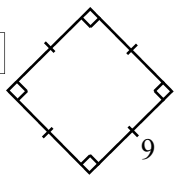


$A = l^2$

$= 12^2$

$A = 144 \text{ units}^2$

10



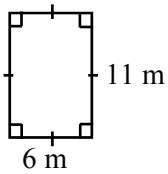
$A = l^2$

$= 9^2$

$A = 81 \text{ units}^2$

Find the areas of the rectangles below

11

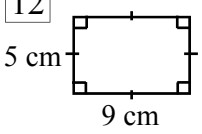


$A = lb$

$= 6 \times 11$

$A = 66 \text{ m}^2$

12

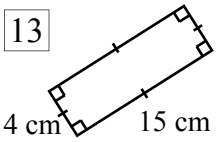


$A = lb$

$= 9 \times 5$

$A = 45 \text{ cm}^2$

13

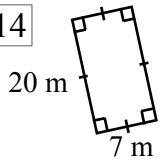


$A = lb$

$= 4 \times 15$

$A = 60 \text{ cm}^2$

14

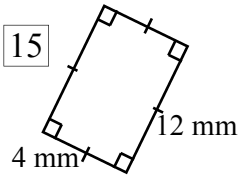


$A = lb$

$= 7 \times 20$

$A = 140 \text{ m}^2$

15

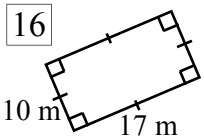


$A = lb$

$= 4 \times 12$

$A = 48 \text{ mm}^2$

16

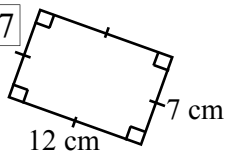


$A = lb$

$= 10 \times 17$

$A = 170 \text{ m}^2$

17

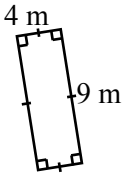


$A = lb$

$= 12 \times 7$

$A = 84 \text{ cm}^2$

18

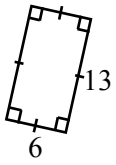


$A = lb$

$= 4 \times 9$

$A = 36 \text{ m}^2$

19

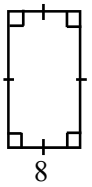


$A = lb$

$= 6 \times 13$

$A = 78 \text{ units}^2$

20



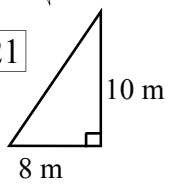
$A = lb$

$= 8 \times 21$

$A = 168 \text{ units}^2$

Find the areas of the triangles below

21

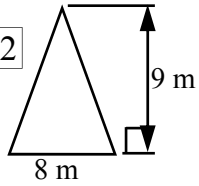


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 10$

$A = 40 \text{ m}^2$

22

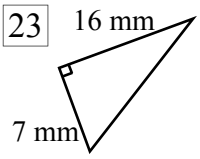


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 9$

$A = 36 \text{ m}^2$

23

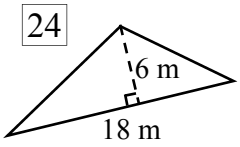


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 7 \times 16$

$A = 56 \text{ mm}^2$

24

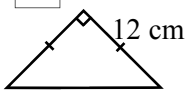


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 6$

$A = 54 \text{ m}^2$

25

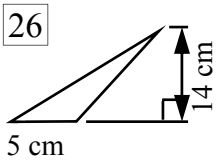


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 12$

$A = 72 \text{ cm}^2$

26

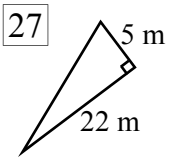


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5 \times 14$

$A = 35 \text{ cm}^2$

27

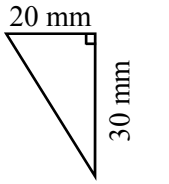


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 22 \times 5$

$A = 55 \text{ m}^2$

28

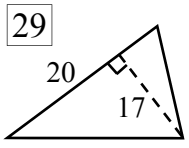


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 20 \times 30$

$A = 300 \text{ mm}^2$

29

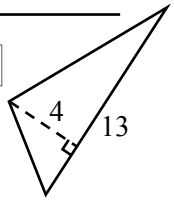


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 20 \times 17$

$A = 170 \text{ units}^2$

30



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 13 \times 4$

$A = 26 \text{ units}^2$

Finding Area with Different Units

This sheet deals with shapes that have measurements in different units on their sides, for example one side in metres and the other in cm. With these the method is to change the larger units to match the smaller units. To convert, multiply the measurement by:

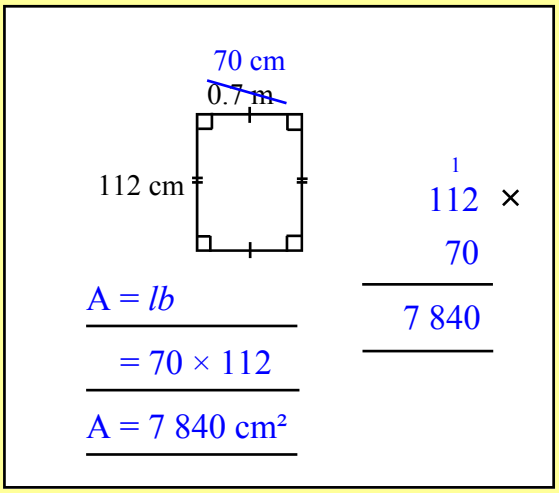
- 10 if converting from cm → mm, e.g. 3 cm = 30 mm, 5.9 cm = 59 mm
- 100 if converting from m → cm, e.g. 3 m = 300 cm, 1.9 m = 190 cm

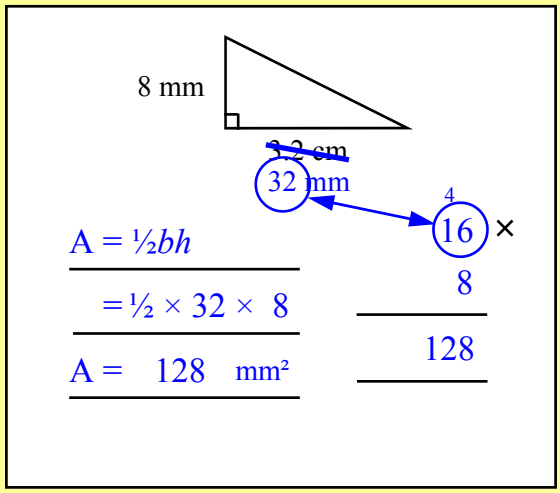
The reverse applies changing from a smaller unit to a larger unit, convert by dividing the measurement by:

- 10 if converting from mm → cm, e.g. 56 mm = 5.6 cm, 134 mm = 13.4 cm
- 100 if converting from cm → m, e.g. 35 cm = 0.35 m, 128 cm = 1.28 m

Column 1 asks you first to convert units, note that some ask for a conversion that jumps a unit m → mm, you may like to do these in 2 stages convert to cm mentally then to mm. The rest of the column and column 2 are rectangle problems. Change the larger unit to the smaller unit and write the conversion next to the measurement, then strike out the old measurement. Show 3 lines of working: formula, substitution and answer with units, follow the method shown in Q. 13. There is a working space provided for your multiplication.

Column 3 deals with triangles. The method is the same except that there is a ½ in the formula. All the numbers in the questions have at least 1 even number, divide it by 2 and then multiply.





Finding Area with Different Units

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Change the measurements given on the left to the units on the right hand side

1 0.8 m = cm

2 7.5 cm = mm

3 1.6 m = cm

4 30 mm = cm

5 1.9 cm = mm

6 43 cm = m

7 0.11 m = cm

8 0.4 m = mm

9 0.37 m = mm

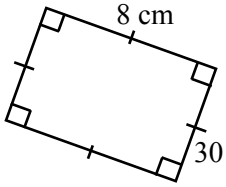
10 273 mm = m

11 0.673 m = cm

12 2.03 m = cm

Change the larger units to the smaller units of the other side. Find the area.

13

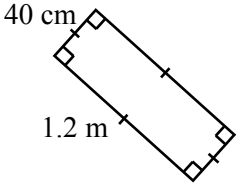


A = *lb*

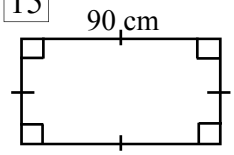
= ×

A = mm²

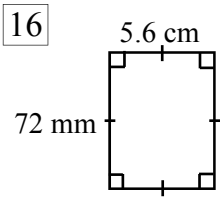
14



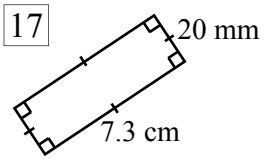
15



16



17



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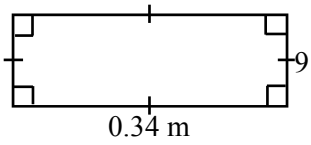

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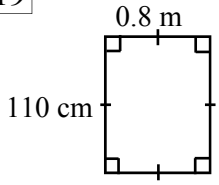
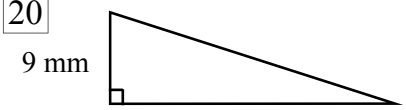


Now lets try triangles

20

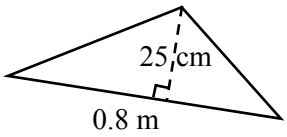


A = ½*bh*

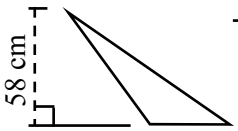
= ½ × ×

A = mm²

21



22



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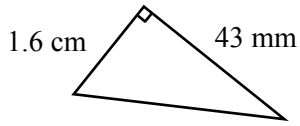
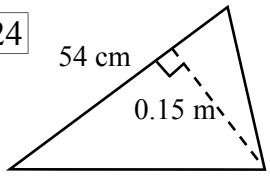


24



Finding Area with Different Units

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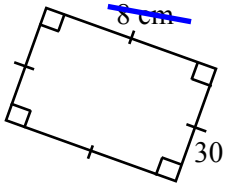
Change the measurements given on the left to the units on the right hand side

1	0.8 m	=	80	cm
2	7.5 cm	=	75	mm
3	1.6 m	=	160	cm
4	30 mm	=	3	cm
5	1.9 cm	=	19	mm
6	43 cm	=	0.43	m
7	0.11 m	=	11	cm
8	0.4 m	=	400	mm
9	0.37 m	=	370	mm
10	273 mm	=	0.273	m
11	0.673 m	=	67.3	cm
12	2.03 m	=	203	cm

Change the larger units to the smaller units of the other side. Find the area.

13



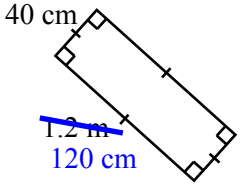
$A = lb$

$= 80 \times 30$

$A = 2\,400$ mm²

80 × 30 = 2 400

14



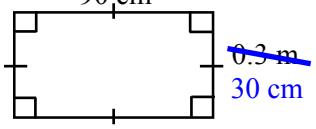
$A = lb$

$= 120 \times 40$

$A = 4\,800$ cm²

120 × 40 = 4 800

15



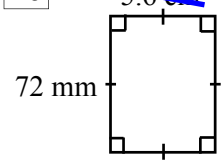
$A = lb$

$= 90 \times 30$

$A = 2\,700$ cm²

90 × 30 = 2 700

16



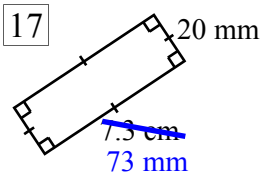
$A = lb$

$= 56 \times 72$

$A = 4\,032$ mm²

72 × 56 = 432 + 3 600 = 4 032

17



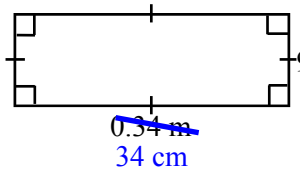
$A = lb$

$= 73 \times 20$

$A = 1\,460$ mm²

73 × 20 = 1 460

18



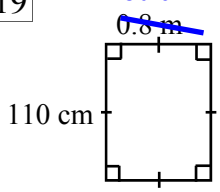
$A = lb$

$= 34 \times 9$

$A = 306$ mm²

34 × 9 = 306

19



$A = lb$

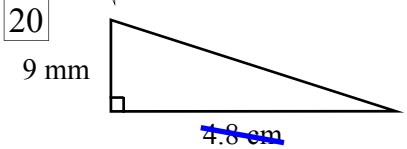
$= 80 \times 110$

$A = 8\,800$ cm²

110 × 80 = 8 800

Now lets try triangles

20



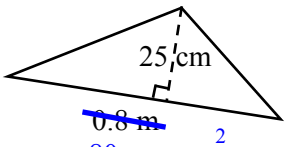
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 48 \times 9$

$A = 216$ mm²

24 × 9 = 216

21



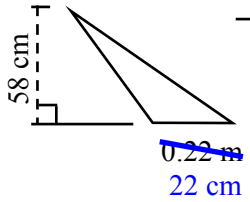
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 80 \times 25$

$A = 1\,000$ cm²

25 × 40 = 1 000

22



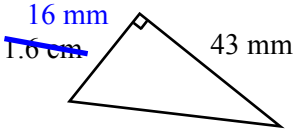
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 58 \times 22$

$A = 638$ cm²

58 × 11 = 58 + 580 = 638

23



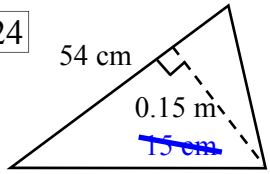
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 16 \times 43$

$A = 344$ cm²

43 × 8 = 344

24



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 54 \times 15$

$A = 405$ cm²

27 × 15 = 135 + 270 = 405

Area Problems

These problems calculate the area of squares, rectangles and triangles and then use the area to involve money, time or a quantity (such as litres). As with areas of shapes, use three lines of working: the formula, substitute the values and the answer including units. Don't forget the ² on the units.

In Column 2, Q. 9 to 16 introduce hectares (ha). Unlike the other units ha doesn't have a ² on it as it isn't a distance unit it is just an area unit. The unit is used for large areas (usually land) such as with farms and parkland. The conversion is:

1 hectare (ha) = 10 000 m².

This means that 20 000 m² = 2 ha, 50 000 m² = 5 ha and so on. These questions ask you to convert the m² units to ha. When you divide by 10 000 you move the decimal point 4 places to the left.

The third column deals with area problems using measurements in metres. Answer the questions in m² then convert the answer to ha using the 10 000 division.

Area Problems

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Find the area of the shapes described below

1 A square with 4 cm sides is cut from paper, a square with 8 cm is then cut out. Find the areas of each of these squares.

$A = l^2$ $A = l^2$

2 The side lengths were multiplied by 2 (4 cm → 8 cm), what number would you multiply the 4 cm square area by to get the 8 cm square area?

3 Now use 5 cm and 10 cm side length squares. Does the same area relationship apply as above?

Circle: Yes / No

4 David wants to paint his team emblem on the football field. The Raiders emblem is a black triangle of base 5m and height 6 m inside a yellow square with sides 8m. Find the shape's area then the yellow area.

Yellow painted area = ❑ - Δ

= - = m²

5 A jar of grass paint covers 10 m² how many jars will be needed and find the total cost. (1 jar costs $3.75)

Jars needed:

yellow:

black: ×

Total: Cost:

6 Karen's room is rectangular with sides 4.5 m and 4 m. Find the floor area and the cost of re-carpeting it if the carpet costs $53 per square metre.

× ×

Cost:

7 Find the area of a field that is 120 m long and 80 m wide. If a lawn mower covers 1 000 m² in a minute. How long will it take to mow the field

×

Time (min): (h:min):

8 Calculate the area of the water surface (in m²) of a swimming pool 50m long and 20 m wide

Area:

A hectare (ha) is 10 000 m² Convert these to ha.

9 30 000 m²

10 40 000 m²

11 35 000 m²

12 100 000 m²

13 120 000 m²

14 125 300 m²

15 4 000 m²

16 300 m²

Answer these in m² then convert to hectares

17 A rural property is rectangular in shape with sides 350 m and 500 m. Find its area

×

Land area:

m²: ha:

18 The owner wants to plant 8 trees per ha for desalination reasons, how many trees should be planted?

Trees = ×

×

19 A triangular section of paddock has perpendicular sides of 80 m and 50 m. Calculate its area.

×

Land area:

m²:

ha:

20 A 16 ha property has a square free range chicken barn with sides of 200 m. Calculate the area of the barn

×

Barn area:

m²: ha:

21 What percentage of the land is taken up by the barn?

Area Problems

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Find the area of the shapes described below

1 A square with 4 cm sides is cut from paper, a square with 8 cm is then cut out. Find the areas of each of these squares.

$A = l^2$ $= 4^2$ $A = 16$ cm²

$A = l^2$ $= 8^2$ $A = 64$ cm²

2 The side lengths were multiplied by 2 (4 cm → 8 cm), what number would you multiply the 4 cm square area by to get the 8 cm square area?

4

3 Now use 5 cm and 10 cm side length squares. Does the same area relationship apply as above?

$A = l^2$ $= 5^2$ $A = 25$ cm²

$A = l^2$ $= 10^2$ $A = 100$ cm²

Circle: Yes / No

4 David wants to paint his team emblem on the football field. The Raiders emblem is a black triangle of base 5m and height 6 m inside a yellow square with sides 8m. Find the shape's area then the yellow area.

$A = l^2$ $= 8^2$ $A = 64$ m²

$A = ½bh$ $= ½\times5\times6$ $A = 15$ m²

Yellow painted area = ❑ - Δ

= 64 - 15 = 49 m²

5 A jar of grass paint covers 10 m² how many jars will be needed and find the total cost. (1 jar costs $3.75)

Jars needed:
yellow: 5
black: 2
Total: 7

5 3
375 ×
7
26.25

Cost: $26.25

6 Karen's room is rectangular with sides 4.5 m and 4 m. Find the floor area and the cost of re-carpeting it if the carpet costs $53 per square metre.

$A = lb$ $= 4.5 \times 4$ $A = 18$ m²

Cost: $954

2
45 ×
4
18.0

53 ×
18
424
530
954

7 Find the area of a field that is 120 m long and 80 m wide. If a lawn mower covers 1 000 m² in a minute. How long will it take to mow the field

$A = lb$ $= 120 \times 80$ $A = 9\,600$ m²

1
120 ×
80
9 600

Time (min): 96 min (h:min): 1 h 36 min

8 Calculate the area of the water surface (in m²) of a swimming pool 50m long and 20 m wide

$A = lb$ $= 50 \times 20$ $A = 1\,000$ m²

Area: 1 000 m²

A hectare (ha) is 10 000 m²
Convert these to ha.

9	30 000 m²	3 ha
10	40 000 m²	4 ha
11	35 000 m²	3.5 ha
12	100 000 m²	10 ha
13	120 000 m²	12 ha
14	125 300 m²	12.53 ha
15	4 000 m²	0.4 ha
16	300 m²	0.03 ha

Answer these in m² then convert to hectares

17 A rural property is rectangular in shape with sides 350 m and 500 m. Find its area

$A = lb$ $=350\times500$ $A = 175\,000$ m²

2
350 ×
500
175 000

Land area:
m²: 175 000 m² ha: 17.5 ha

18 The owner wants to plant 8 trees per ha for desalination reasons, how many trees should be planted?

Trees = 17.5 × 8
= 140 trees

6 4
175 ×
8
140.0

19 A triangular section of paddock has perpendicular sides of 80 m and 50 m. Calculate its area.

$A = ½bh$ $=½\times80\times50$ $A = 2\,000$ m²

50 ×
40
2 000

Land area:
m²: 2 000 m²
ha: 0.2 ha

20 A 16 ha property has a square free range chicken barn with sides of 200 m. Calculate the area of the barn

$A = l^2$ $=200\times200$ $A = 40\,000$ m²

2
200 ×
200
40 000

Barn area:
m²: 40 000 m² ha: 4 ha

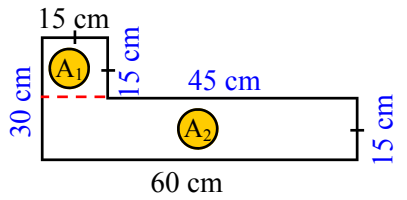
21 What percentage of the land is taken up by the barn? 25%

Composite Area - Addition

Composite areas are shapes that are made from two or more shapes. To calculate the area the shapes are broken up into geometric shapes, in this case squares, rectangles and triangles. The shapes have their areas calculated separately then they are added together to get the total area. Questions 1 to 7 can be broken into 2 shapes, questions 8 and 9 into three shapes. Note that all angles that appear to be right angles are right angles.

The method for the sheet is the same throughout, follow this method:

- Find all the missing side lengths for the shapes and write them on the diagrams
- Draw a line to cut the shape up into basic shapes. Label the separate areas A_1, A_2 … to show which area is which.
- Answer the questions showing full working out, follow the method shown in question 1 and the example below.



$A_1 = l^2$

$= 15^2$

$A_1 = 225 \text{ cm}^2$

$A_2 = lb$

$= 60 \times 15$

$A_2 = 900 \text{ cm}^2$

$A = A_1 + A_2 = 225 + 900$

$A = 1\,125 \text{ cm}^2$

Composite Area - Addition

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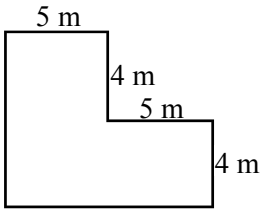
Find the areas of the shapes below

1



$A_1 = lb$ $A_2 = lb$

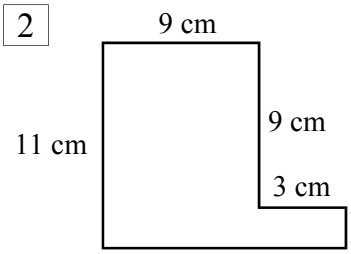
$=$ $=$

$A_1 =$ m² $A_2 =$ m²

$A = A_1 + A_2 =$ +

$A =$

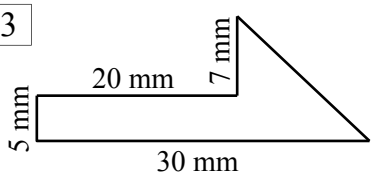
2



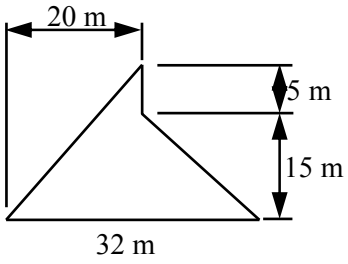
$A = A_1 + A_2 =$ +

$A =$

3



4



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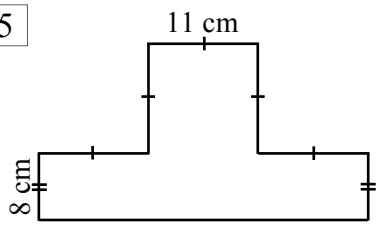

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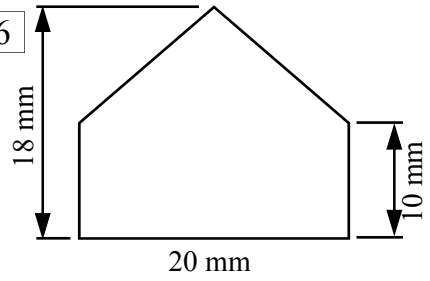

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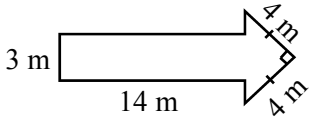
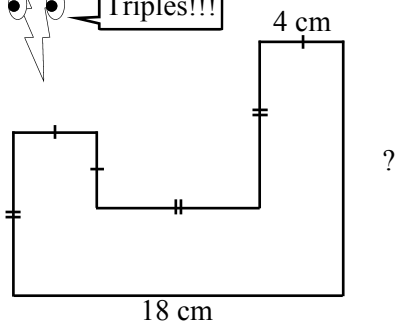


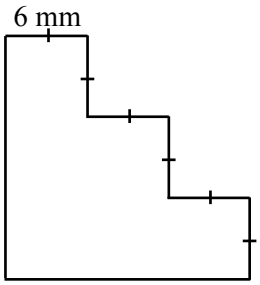
Triples!!!

8



$A = A_1 + A_2 + A_3$

9



Composite Area - Addition

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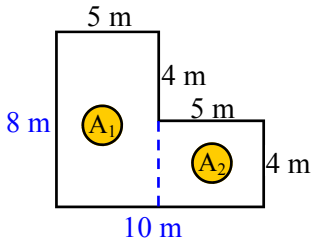
Find the areas of the shapes below

1



$A_1 = lb$ $A_2 = lb$

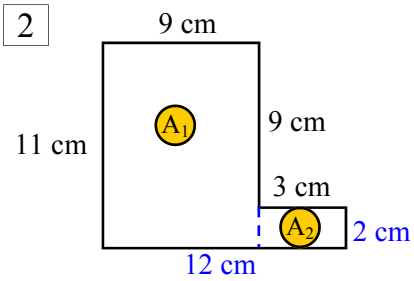
$= 5 \times 8$ $= 5 \times 4$

$A_1 = 40$ m² $A_2 = 20$ m²

$A = A_1 + A_2 = 40 + 20$

$A = 60$ m²

2



$A_1 = lb$ $A_2 = lb$

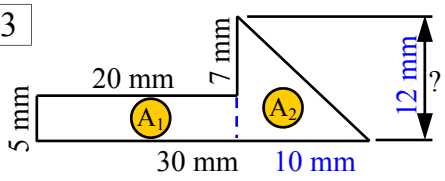
$= 9 \times 11$ $= 3 \times 2$

$A_1 = 99$ cm² $A_2 = 6$ cm²

$A = A_1 + A_2 = 99 + 6$

$A = 105$ cm²

3



$A_1 = lb$ $A_2 = \frac{1}{2}bh$

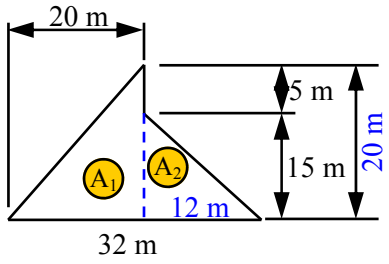
$= 5 \times 20$ $= \frac{1}{2} \times 10 \times 12$

$A_1 = 100$ mm² $A_2 = 60$ mm²

$A = A_1 + A_2 = 100 + 60$

$A = 160$ mm²

4



$A_1 = \frac{1}{2}bh$ $A_2 = \frac{1}{2}bh$

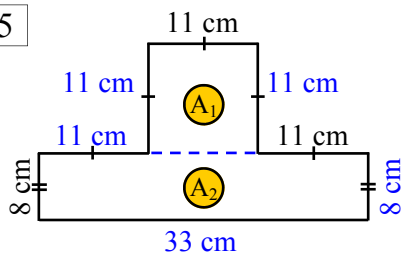
$= \frac{1}{2} \times 20 \times 20$ $= \frac{1}{2} \times 12 \times 15$

$A_1 = 200$ m² $A_2 = 90$ m²

$A = A_1 + A_2 = 200 + 90$

$A = 290$ m²

5



$A_1 = l^2$ $A_2 = lb$

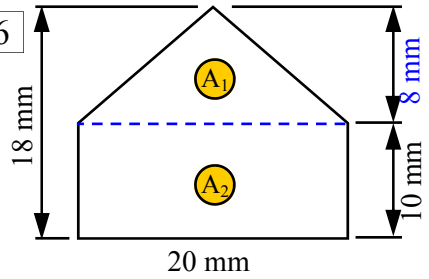
$= 11^2$ $= 33 \times 8$

$A_1 = 121$ cm² $A_2 = 264$ cm²

$A = A_1 + A_2 = 121 + 264$

$A = 385$ cm²

6



$A_1 = \frac{1}{2}bh$ $A_2 = lb$

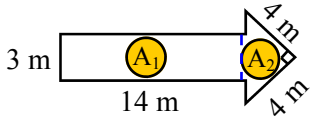
$= \frac{1}{2} \times 20 \times 8$ $= 20 \times 10$

$A_1 = 80$ mm² $A_2 = 200$ mm²

$A = A_1 + A_2 = 80 + 200$

$A = 280$ mm²

7



$A_1 = lb$ $A_2 = \frac{1}{2}bh$

$= 14 \times 3$ $= \frac{1}{2} \times 4 \times 4$

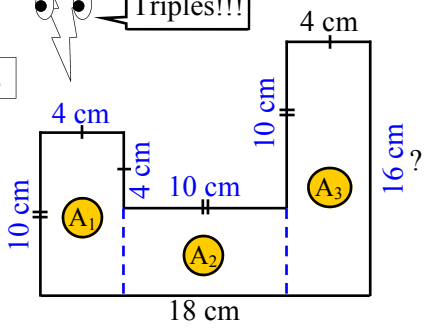
$A_1 = 42$ m² $A_2 = 8$ m²

$A = A_1 + A_2 = 42 + 8$

$A = 50$ m²

8

Triples!!!



$A_1 = lb$ $A_2 = lb$

$= 4 \times 10$ $= 10 \times 6$

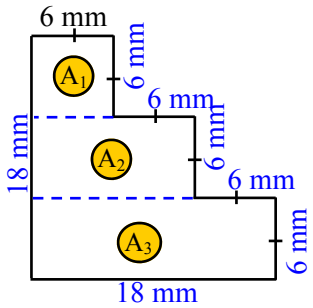
$A_1 = 40$ cm² $A_2 = 60$ cm²

$A_3 = lb$ $A = A_1 + A_2 + A_3$

$= 4 \times 16$ $= 40 + 60 + 64$

$A_1 = 64$ cm² $A = 164$ cm²

9



$A_1 = l^2$ $A_2 = lb$

$= 6^2$ $= 12 \times 6$

$A_1 = 36$ mm² $A_2 = 72$ mm²

$A_3 = lb$ $A = A_1 + A_2 + A_3$

$= 18 \times 6$ $= 36 + 72 + 108$

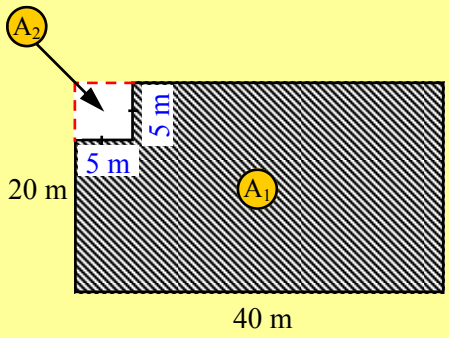
$A_1 = 108$ mm² $A = 216$ mm²

Composite Area - Subtraction

Composite areas are shapes that are composed (made up) of two or more shapes. These shapes can be broken up into geometric shapes, in this case squares, rectangles and triangles to help calculation of area. The shapes have their areas calculated separately then the smaller shapes are subtracted from the larger shape. Questions 1 to 7 have one shape subtracted from the other, Q 8 and 9 have 2 shapes subtracted.

The method for the sheet is the same throughout, follow this method:

- Find all the missing side lengths for the shapes and write them on the diagrams
- If the inside shape touches the perimeter to make an opening draw a broken line across the opening to close it off. Question 3 will need 2 lines.
- Label the areas A_1, A_2 (and A_3 for questions 8 & 9) to show which area is which. Let A_1 be the outside or largest shape area then use A_2 and A_3 for the other areas.
- Answer the questions showing full working out, follow the method shown in question 1 or the example below.



$A_1 = lb$

$= 40 \times 20$

$A_1 = 800\ m^2$

$A_2 = l^2$

$= 5^2$

$A_2 = 25\ m^2$

$A = A_1 - A_2$

$= 800 - 25$

$A = 775\ m^2$

Composite Area - Subtraction

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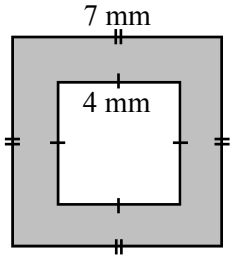
Find the shaded areas of the following shapes

1



$A_1 = l^2$ $A_2 = l^2$

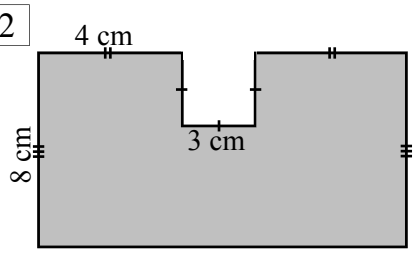
= =

A_1 = mm² A_1 = mm²

$A = A_1 - A_2$ = -

A =

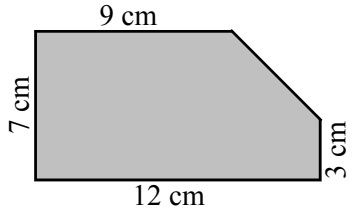
2



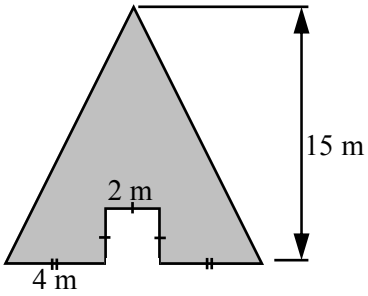
$A = A_1 - A_2$ = -

A =

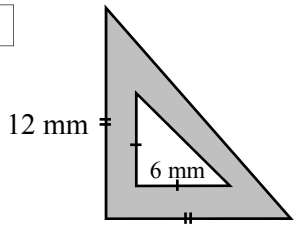
3



4



5



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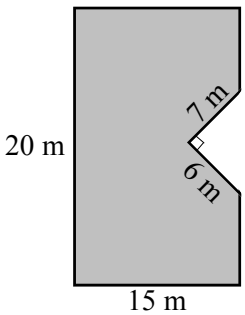
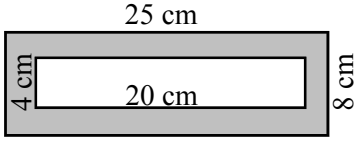
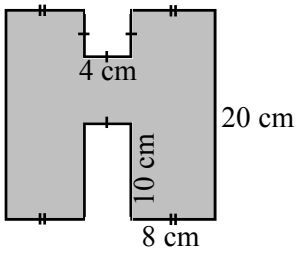


7



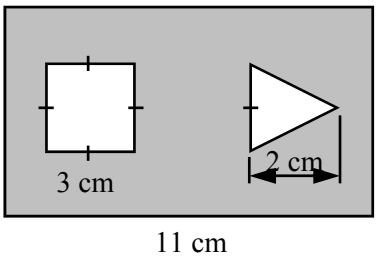
Now subtract two areas

8



$A = A_1 - A_2 - A_3$

9



Composite Area - Subtraction

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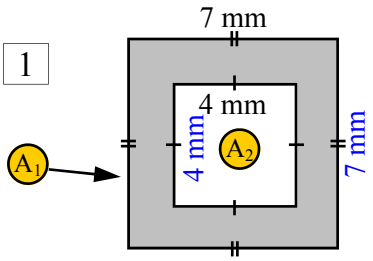
Find the shaded areas of the following shapes

1



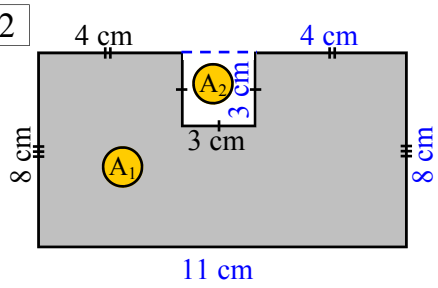
$A_1 = l^2$ $= 7^2$ $A_1 = 49$ mm²

$A_2 = l^2$ $= 4^2$ $A_1 = 16$ mm²

$A = A_1 - A_2 = 49 - 16$

$A = 33$ mm²

2



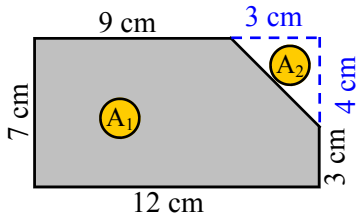
$A_1 = lb$ $= 11 \times 8$ $A_1 = 88$ cm²

$A_2 = l^2$ $= 3^2$ $A_2 = 9$ cm²

$A = A_1 - A_2 = 88 - 9$

$A = 79$ cm²

3



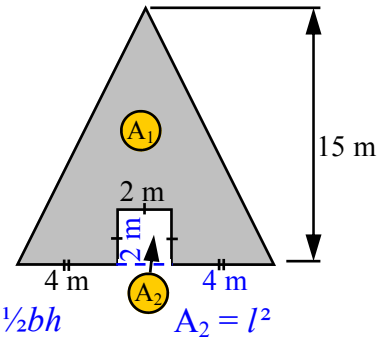
$A_1 = lb$ $= 12 \times 7$ $A_1 = 84$ cm²

$A_2 = \frac{1}{2}bh$ $= \frac{1}{2} \times 4 \times 3$ $A_2 = 6$ cm²

$A = A_1 - A_2 = 84 - 6$

$A = 78$ cm²

4



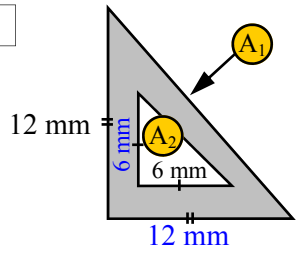
$A_1 = \frac{1}{2}bh$ $= \frac{1}{2} \times 10 \times 15$ $A_1 = 75$ m²

$A_2 = l^2$ $= 2^2$ $A_2 = 4$ m²

$A = A_1 - A_2 = 75 - 4$

$A = 71$ m²

5



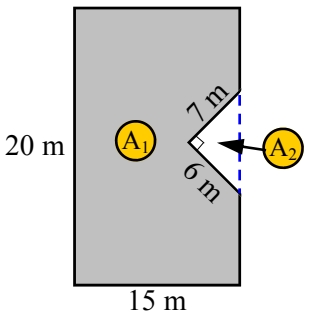
$A_1 = \frac{1}{2}bh$ $= \frac{1}{2} \times 12 \times 12$ $A_1 = 72$ mm²

$A_2 = \frac{1}{2}bh$ $= \frac{1}{2} \times 6 \times 6$ $A_2 = 18$ mm²

$A = A_1 - A_2 = 72 - 18$

$A = 54$ mm²

6



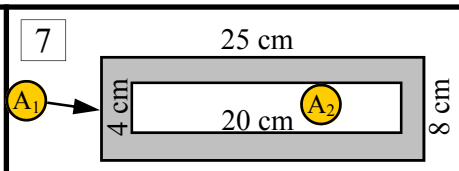
$A_1 = lb$ $= 15 \times 20$ $A_1 = 300$ m²

$A_2 = \frac{1}{2}bh$ $= \frac{1}{2} \times 6 \times 7$ $A_2 = 21$ m²

$A = A_1 - A_2 = 300 - 21$

$A = 279$ m²

7



$A_1 = lb$ $= 25 \times 8$ $A_1 = 200$ cm²

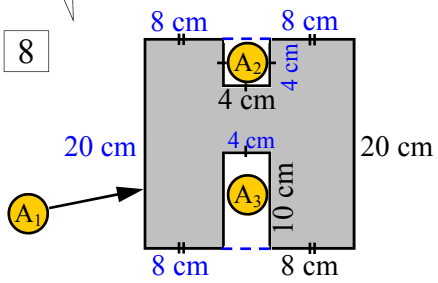
$A_2 = lb$ $= 20 \times 4$ $A_2 = 80$ cm²

$A = A_1 - A_2 = 200 - 80$

$A = 120$ cm²

Now subtract two areas

8



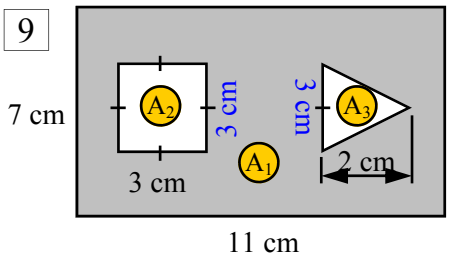
$A_1 = l^2$ $= 20^2$ $A_1 = 400$ cm²

$A_2 = l^2$ $= 4^2$ $A_2 = 16$ cm²

$A_3 = lb$ $= 4 \times 10$ $A_3 = 40$ cm²

$A = A_1 - A_2 - A_3$ $= 400 - 16 - 40$ $A = 344$ cm²

9



$A_1 = lb$ $= 11 \times 7$ $A_1 = 77$ cm²

$A_2 = l^2$ $= 3^2$ $A_2 = 9$ cm²

$A_3 = \frac{1}{2}bh$ $= \frac{1}{2} \times 3 \times 2$ $A_3 = 3$ cm²

$A = A_1 - A_2 - A_3$ $= 77 - 9 - 3$ $A = 65$ cm²

Changing Units of Area

This is perhaps the most difficult part of area calculations, with students often forgetting this work. The reason is that because there are 10 mm in 1 cm, students automatically use this to convert mm^2 to cm^2, but this isn't the case. There is 100 mm^2 in 1 cm^2. The same with converting cm^2 to m^2, there is 10 000 cm^2 in a m^2, not 100.

Column 1 first converts areas from cm^2 to mm^2. Note the diagram at the top of the column. A 1 cm square is redrawn with grid lines at each millimetre. The square has 10 mm sides and as you know the area of a square is its side length squared. As $10^2 = 100$ there are 100 1 mm^2 squares in a square cm. So to convert the measurements multiply each number by 100. This means move the decimal point 2 places to the right. So 5 cm^2 would be 500 mm^2, 5.5 cm^2 would be 550 mm^2 and 5.05 cm^2 would be 505 mm^2. Questions 11- 20 are the reverse, move the decimal point 2 places to the left. So 8 mm^2 is 0.08 cm^2, 130 mm^2 is 1.3 cm^2 and 130.02 mm^2 would be 1.3002 cm^2.

Column 2 then deals with converting between m^2 and cm^2. Look at the diagram and note that there are (100×100) 10 000 cm^2 in 1 m^2. So to change cm^2 to m^2 multiply the number by 10 000 or move the decimal point 4 places to the right. So 8 m^2 would be 80 000 cm^2 and 8.02 m^2 would be 80 200 cm^2. Then the method is reversed for the second part of the column. When you divide by 10 000 move the decimal place 4 places to the left. So 9 cm^2 would be 0.0009 m^2 and 136 cm^2 would be 0.0136 m^2.

Column 3 are problems that you solve using the units as given in the question. Then after you answer the question express the answer in the other unit that is requested. Working spaces are supplied.

Changing Units of Area

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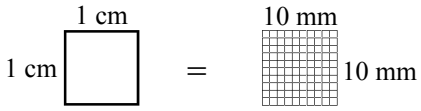
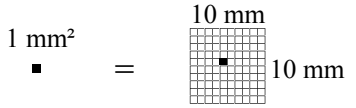
Convert these areas from cm² to mm²



1 cm² = 100 mm²

1 3 cm² _____
2 8 cm² _____
3 7.5 cm² _____
4 18 cm² _____
5 29 cm² _____
6 45.6 cm² _____
7 65.32 cm² _____
8 82.09 cm² _____
9 100 cm² _____
10 100.1 cm² _____

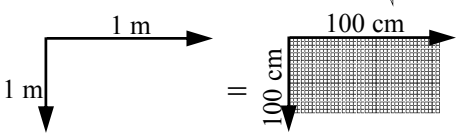
Convert these areas from mm² to cm²



1 mm² = 0.01 cm²

11 5 mm² _____
12 10 mm² _____
13 25 mm² _____
14 125 mm² _____
15 170.1 mm² _____
16 205.4 mm² _____
17 30.03 mm² _____
18 450 mm² _____
19 500 mm² _____
20 500.6 mm² _____

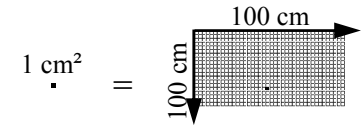
Convert these areas from m² to cm²



1 m² = 10 000 cm²

21 2 m² _____
22 3 m² _____
23 0.3 m² _____
24 0.03 m² _____
25 3.3 m² _____
26 0.725 m² _____
27 0.203 m² _____
28 0.8963 m² _____
29 0.902 m² _____
30 0.0009 m² _____

Convert these areas from cm² to m²



1 cm² = 0.0001 m²

31 7 cm² _____
32 11 cm² _____
33 54 cm² _____
34 154 cm² _____
35 296 cm² _____
36 875 cm² _____
37 1 500 cm² _____
38 8 700 cm² _____
39 12 000 cm² _____
40 15 796 cm² _____

Answer these problems using the methods from the first two columns

41 A rectangular piece of fabric is size 1.5 m × 2.7 m. Find its area in both m² and cm².

A = *lb* _____ ×

_____ _____

_____

Fabric area:

m²: _____ _____

cm²: _____ _____

42 A square steel plate with sides 8 cm has a rectangular slot cut into it with sides 35 mm and 15 mm. Find:
i) the size of the plate without the slot in mm² (A_1)
ii) the size of the slot in mm² (A_2)
iii) the cut plate area in mm² and cm²

i) $A_1 = l^2$ _____

_____

_____

Plate area = _____

ii) ×

_____ _____

_____

_____ _____

_____ _____

iii)

_____

_____ -

_____

Cut Plate area: _____

mm²: _____ _____

cm²: _____

Changing Units of Area

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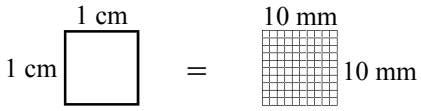
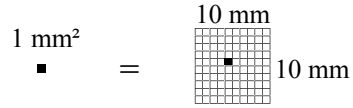
Convert these areas from cm² to mm²



1 cm² = 100 mm²

1	3 cm²	300 mm²
2	8 cm²	800 mm²
3	7.5 cm²	750 mm²
4	18 cm²	1 800 mm²
5	29 cm²	2 900 mm²
6	45.6 cm²	4 560 mm²
7	65.32 cm²	6 532 mm²
8	82.09 cm²	8 209 mm²
9	100 cm²	10 000 mm²
10	100.1 cm²	10 010 mm²

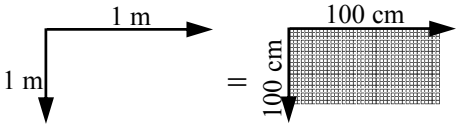
Convert these areas from mm² to cm²



1 mm² = 0.01 cm²

11	5 mm²	0.05 cm²
12	10 mm²	0.1 cm²
13	25 mm²	0.25 cm²
14	125 mm²	1.25 cm²
15	170.1 mm²	1.701 cm²
16	205.4 mm²	2.054 cm²
17	30.03 mm²	0.3003 cm²
18	450 mm²	4.5 cm²
19	500 mm²	5 cm²
20	500.6 mm²	5.006 cm²

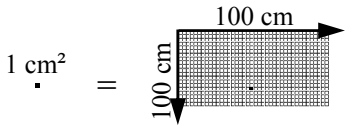
Convert these areas from m² to cm²



1 m² = 10 000 cm²

21	2 m²	20 000 cm²
22	3 m²	30 000 cm²
23	0.3 m²	3 000 cm²
24	0.03 m²	300 cm²
25	3.3 m²	33 000 cm²
26	0.725 m²	7 250 cm²
27	0.203 m²	2 030 cm²
28	0.8963 m²	8 963 cm²
29	0.902 m²	9 020 cm²
30	0.0009 m²	9 cm²

Convert these areas from cm² to m²



1 cm² = 0.0001 m²

31	7 cm²	0.0007 m²
32	11 cm²	0.0011 m²
33	54 cm²	0.0054 m²
34	154 cm²	0.0154 m²
35	296 cm²	0.0296 m²
36	875 cm²	0.0875 m²
37	1 500 cm²	0.15 m²
38	8 700 cm²	0.87 m²
39	12 000 cm²	1.2 m²
40	15 796 cm²	1.5796 m²

Answer these problems using the methods from the first two columns

41 A rectangular piece of fabric is size 1.5 m × 2.7 m. Find its area in both m² and cm².

$A = lb$

$= 1.5 \times 2.7$

$A = 4.05$ m²

Fabric area:

m²: 4.05 m²

cm²: 40 500 cm²

$$\begin{array}{r} 27 \times \\ 15 \\ \hline {}^{1}135 \\ 270 \\ \hline 4.05 \end{array}$$

42 A square steel plate with sides 8 cm has a rectangular slot cut into it with sides 35 mm and 15 mm. Find:
i) the size of the plate without the slot in mm² (A_1)
ii) the size of the slot in mm² (A_2)
iii) the cut plate area in mm² and cm²

i) $A_1 = l^2$

$= 8^2$

$A_1 = 64$ cm²

Plate area = 6 400 mm²

ii)

$A_2 = lb$

$= 35 \times 15$

$A_2 = 525$ mm²

Slot area = 525 mm²

$$\begin{array}{r} 35 \times \\ 15 \\ \hline {}^{1}175 \\ 350 \\ \hline 525 \end{array}$$

iii)

$A = A_1 - A_2$

$= 6\,400 - 525$

$A = 5\,875$ mm²

Cut Plate area:

mm²: 5 875 mm²

cm²: 58.75 cm²

$$\begin{array}{r} 6\,400 \; - \\ 525 \\ {}^{1\;1\;1} \\ \hline 5\,875 \end{array}$$

Finding a Side Given Area

This sheet is difficult as it uses skills that are learnt in the Algebra 2 folder. It is essentially working in reverse, given an area and a side (except with squares) you are asked to find the unknown side. Note that there is up to 5 lines of working, when you may be able to complete the problem mentally, so why all the working? These exercises use whole numbers throughout, but the method can be used for decimals also and the use of decimals would make the exercises too difficult to solve mentally.

Column 1 deals with calculating a square's side length given its area. The first 5 questions test that you understand what a square root is. Eg. $10^2 = 100$ so $\sqrt{100} = 10$. To find the length of a side given the area you square root the area. So if an area was 4 cm² then its side length is 2 cm. As the $\sqrt{4} = 2$. The 3 lines used are again the formula, the substitution then the solution with units matching the area units only without the ².

Column 2 deals with the area of a rectangle, this time given an area and a side. The first line is the formula for a rectangle written backwards $lb = A$. Then the second line has the values substituted in, in the example at the top of the column: $A = 20$ mm² and $b = 5$ mm. The third line divides each side by b (5) to get l by itself (from Algebra 2 folder).

Column 3 involves triangles, it is an extension of column 2 in that a ½ is used. But the method is just the same, look at the example. The formula is written backwards, then the substitution is made, the ½ × the side is calculated then the method is the same as above.

Finding a Side Given Area

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Find the squares of the following and then complete the exercises

1 $4^2 =$ 16 so, $\sqrt{16} = 4$

2 $5^2 =$ ☐ so, $\sqrt{} = 5$

3 $6^2 =$ ☐ so, $\sqrt{} = 6$

4 $7^2 =$ ☐ so, $\sqrt{} = 7$

5 $8^2 =$ ☐ so, $\sqrt{} = 8$

Find the side length of the square with the area given

6

Area = 16 cm²

$l = \sqrt{A}$

$l = \sqrt{}$

$l =$ cm

7

Area = 64 m²

$l = \sqrt{}$

$l = \sqrt{}$

$l =$

8

Area = 100 cm²

$\sqrt{}$

$\sqrt{}$

9

Area = 25 mm²

$\sqrt{}$

$\sqrt{}$

10

Area = 81 m²

$\sqrt{}$

$\sqrt{}$

11

Area = 36 mm²

$\sqrt{}$

$\sqrt{}$

12

Area = 121 cm²

$\sqrt{}$

$\sqrt{}$

13

Area = 144 m²

$\sqrt{}$

$\sqrt{}$

Find the unknown side of the rectangles given their area and one side

Example

A = 20 mm²

$b = 5$ mm

$lb = A$

$l \times 5 = 20$

$l = 20 \div 5$

$l = 4$ mm

14

A = 35 m²

$b = 7$ m

15

A = 60 mm²

$b = 4$ mm

16

A = 120 m²

$b = 6$ m

17

A = 63 cm²

$l = 9$ cm

18

A = 72 cm²

$l = 6$ cm

19

A = 88 mm²

$l = 8$ mm

20

A = 140 cm²

$b = 7$ cm

Find the height (or base) of the triangles below given their area and base (height)

Example

A = 40 mm²

$b = 10$ mm

$\frac{1}{2}bh = A$

$\frac{1}{2} \times 10 \times h = 40$

$5 \times h = 40$

$h = 40 \div 5$

$h = 8$ mm

21

A = 48 cm²

$b = 6$ cm

22

A = 60 m²

$b = 6$ m

23

A = 100 mm²

$b = 20$ mm

24

A = 80 cm²

$h = 10$ cm

25

A = 120 m²

$h = 6$ m

26

A = 56 mm²

$h = 8$ mm

Finding a Side Given Area

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Find the squares of the following and then complete the exercises

1 $4^2 =$ 16 so, $\sqrt{16} = 4$

2 $5^2 =$ 25 so, $\sqrt{25} = 5$

3 $6^2 =$ 36 so, $\sqrt{36} = 6$

4 $7^2 =$ 49 so, $\sqrt{49} = 7$

5 $8^2 =$ 64 so, $\sqrt{64} = 8$

Find the side length of the square with the area given

6

Area = 16 cm²

$l = \sqrt{A}$

$l = \sqrt{16}$

$l =$ 4 cm

7

Area = 64 m²

$l = \sqrt{A}$

$l = \sqrt{64}$

$l =$ 8 m

8

Area = 100 cm²

$l = \sqrt{A}$

$l = \sqrt{100}$

$l = 10$ cm

9

Area = 25 mm²

$l = \sqrt{A}$

$l = \sqrt{25}$

$l = 5$ mm

10

Area = 81 m²

$l = \sqrt{A}$

$l = \sqrt{81}$

$l = 9$ m

11

Area = 36 mm²

$l = \sqrt{A}$

$l = \sqrt{36}$

$l = 6$ mm

12

Area = 121 cm²

$l = \sqrt{A}$

$l = \sqrt{121}$

$l = 11$ cm

13

Area = 144 m²

$l = \sqrt{A}$

$l = \sqrt{144}$

$l = 12$ m

Find the unknown side of the rectangles given their area and one side

Example

A = 20 mm²

$b = 5$ mm

$lb = A$

$l \times 5 = 20$

$l = 20 \div 5$

$l = 4$ mm

14

A = 35 m²

$b = 7$ m

$lb = A$

$l \times 7 = 35$

$l = 35 \div 7$

$l = 5$ m

15

A = 60 mm²

$b = 4$ mm

$lb = A$

$l \times 4 = 60$

$l = 60 \div 4$

$l = 15$ mm

16

A = 120 m²

$b = 6$ m

$lb = A$

$l \times 6 = 120$

$l = 120 \div 6$

$l = 20$ m

17

A = 63 cm²

$l = 9$ cm

$lb = A$

$9 \times b = 63$

$b = 63 \div 9$

$b = 7$ cm

18

A = 72 cm²

$l = 6$ cm

$lb = A$

$6 \times b = 72$

$b = 72 \div 6$

$b = 12$ cm

19

A = 88 mm²

$l = 8$ mm

$lb = A$

$8 \times b = 88$

$b = 88 \div 8$

$b = 11$ mm

20

A = 140 cm²

$b = 7$ cm

$lb = A$

$l \times 7 = 140$

$l = 140 \div 7$

$l = 20$ cm

Find the height (or base) of the triangles below given their area and base (height)

Example

A = 40 mm²

$b = 10$ mm

$\frac{1}{2}bh = A$

$\frac{1}{2} \times 10 \times h = 40$

$5 \times h = 40$

$h = 40 \div 5$

$h = 8$ mm

21

A = 48 cm²

$b = 6$ cm

$\frac{1}{2}bh = A$

$\frac{1}{2} \times 6 \times h = 48$

$3 \times h = 48$

$h = 48 \div 3$

$h = 16$ cm

22

A = 60 m²

$b = 12$ m

$\frac{1}{2}bh = A$

$\frac{1}{2} \times 12 \times h = 60$

$6 \times h = 60$

$h = 60 \div 6$

$h = 10$ m

23

A = 100 mm²

$b = 20$ mm

$\frac{1}{2}bh = A$

$\frac{1}{2} \times 20 \times h = 100$

$10 \times h = 100$

$h = 100 \div 10$

$h = 10$ mm

24

A = 80 cm²

$h = 10$ cm

$\frac{1}{2}bh = A$

$\frac{1}{2} \times b \times 10 = 80$

$5 \times b = 80$

$b = 80 \div 5$

$b = 16$ cm

25

A = 120 m²

$h = 6$ m

$\frac{1}{2}bh = A$

$\frac{1}{2} \times b \times 6 = 120$

$3 \times b = 120$

$b = 120 \div 3$

$b = 40$ m

26

A = 56 mm²

$h = 8$ mm

$\frac{1}{2}bh = A$

$\frac{1}{2} \times b \times 8 = 56$

$4 \times b = 56$

$b = 56 \div 4$

$b = 14$ mm

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TIME

Writing and Reading Time

Writing the time is usually done in one of two ways. The first is the hour then the minutes, such as seven thirty-eight (7.38) or six fifteen (6.15). The other method is to treat the two halves of the clock differently.

- There is a 'past half' (0 - 29 minutes) such as twelve minutes past six (6.12), twenty-five minutes past nine (9.25).
- There is a 'to half' (31 - 59 minutes) such ten to eleven (6.50), twenty-five to three (2.35)
- There is also half past (30 minutes), quarter past (15 minutes) and a quarter to (45 minutes). Such as a quarter past three (3.15), half past eleven (11.30), quarter to one (12.45).

Column 1 gives you the time in words and asks for the time in number form as well as placing and positioning of the hands on the analogue clocks. Then there is the time of day it is, is it morning, night, afternoon, day, evening etc. These words all have a.m./ p.m. meanings and you are asked to write in the a.m. or p.m. as well.

Column 2 asks for the time to be written in words. Also give the time of day as morning, night, afternoon etc.

The 3rd column is using dates, writing the dates in words and then also in numerical form as the example at the top of the column shows you. The calendars below may be of assistance.

JULY 2013

S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

AUGUST 2013

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

SEPTEMBER 2013

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30					

Writing and Reading Time

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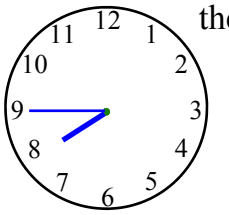
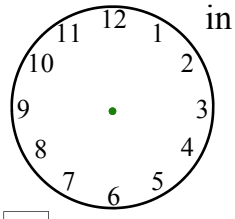
Put the hands on the clock and write these times

Example a quarter to eight in the morning

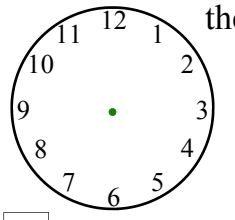


7.45 a.m.

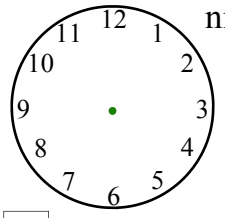
1 twenty-five past one in the day



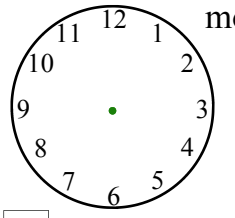
2 four thirty-five in the afternoon



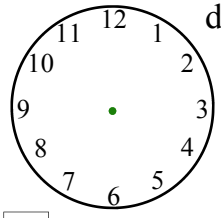
3 twelve to twelve at night



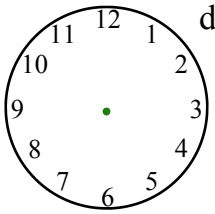
4 half past three in the morning



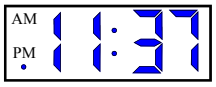
5 twenty past six at dusk



6 a quarter to five at dawn

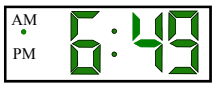


Write two ways of telling the time from the clocks shown. E.g. 7.35 is seven thirty-five or twenty-five to eight. Write morning, afternoon or night as well.

7 

a)

b)

8 

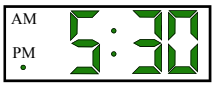
a)

b)

9 

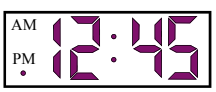
a)

b)

10 

a)

b)

11 

a)

b)

Write these days and their dates on the calendar in words then the date in numerical form.

AUGUST 2013

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Example □

Thursday 29th August, 2013

29 / 08 / 13

12 ♡

13 ⬡

14 ○

15 The date 9 days before ○

16 The date 15 days after ♡

17 The date 10 days after □

18 The date 20 days before ⬡

Writing and Reading Time

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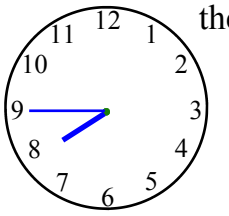
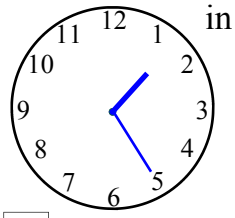
Put the hands on the clock and write these times

Example a quarter to eight in the morning



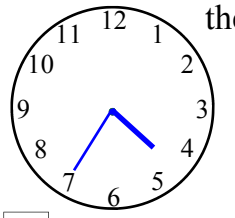
7.45 a.m.

1 twenty-five past one in the day



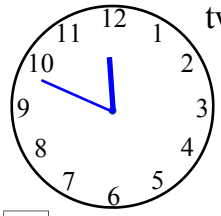
1.25 p.m.

2 four thirty-five in the afternoon



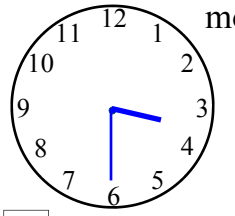
4.35 p.m.

3 twelve minutes to twelve at night



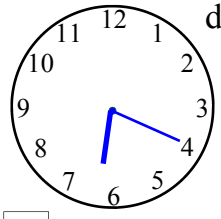
11.48 p.m.

4 half past three in the morning



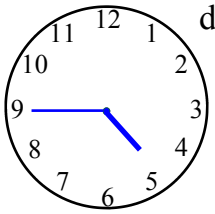
3.30 a.m.

5 twenty past six at dusk



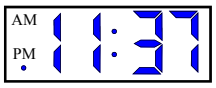
6.20 p.m.

6 a quarter to five at dawn



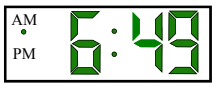
4.45 a.m.

Write two ways of telling the time from the clocks shown. E.g. 7.35 is seven thirty-five or twenty-five to eight. Write morning, afternoon or night as well.

7 

a) eleven thirty-seven at night

b) twenty-three minutes to midnight

8 

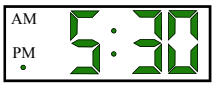
a) six forty-nine in the morning

b) Eleven minutes to seven in the morning

9 

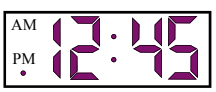
a) twelve fifty-three in the morning

b) seven minutes to one in the morning

10 

a) Five thirty in the afternoon

b) half past five in the afternoon

11 

a) twelve forty-five in the day

b) a quarter to one in the day

Write these days and their dates on the calendar in words then the date in numerical form.

AUGUST 2013

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Example Thursday 29th August, 2013

29/08/13

12 Wednesday 21st August, 2013

21/08/13

13 Sunday 11th August, 2013

11/08/13

14 Friday 9th August, 2013

09/08/13

15 The date 9 days before Wednesday 31st July, 2013

31/07/13

16 The date 15 days after Thursday 5th September, 2013

05/09/13

17 The date 10 days after Sunday 8th September, 2013

08/09/13

18 The date 20 days before Monday 22nd July, 2013

02/07/13

Converting to 24 Hour Time

This time format is used mainly in the armed forces and the travel industry. If you buy computerised tickets at railway stations the time will be usually written in 24 hour time format. It reduces confusion particularly with electronic machinery.

The first 2 digits of 24 hour time are the hours, the second 2 digits are the minutes.
The time starts at 0000 hours (midnight) and ends at 2359 (1 minute before midnight). Note that there is no full stop separating hours and minutes. So:

- The time is 0000 hours at 12 a.m. (midnight), 12.30 a.m. is 0030 hours
- Then each hour adds 100 so 1 a.m. is 0100 hours, 2 a.m. is 0200 hours etc
- Noon is 1200 hours and 1 p.m. is 1300 hours

So once the time reaches 1 p.m. add 12 hours to the normal time to get your answer in 24 hr time format, e.g. 4.45 p.m. = 1645 hours (hours = 4 + 12 = 16, minutes stay the same)

Note that the word 'hours' is written after the time. When the time is spoken there is no reference to minutes. E.g. 0400 is "O four hundred hours, Sir!" and the time 1640 is "sixteen hundred and forty hours, Sir!" (The Sir! is optional)

Time of day in the 3rd column refers to morning, afternoon, evening, night or any other words that describe the time of day

Converting to 24 Hour Time

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Change these times to 24 hour time format, (from 1 p.m. add 12 hours)

Example

4.45 p.m. 1645 hours

1 3 a.m.

2 7 a.m.

3 5 p.m.

4 11.25 a.m.

5 3.52 p.m.

6 5.17 p.m.

7 8.47 p.m.

8 noon

9 4.56 a.m.

10 12.19 a.m.

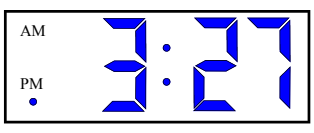
11 10.25 p.m.

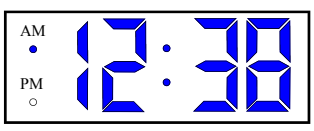
12 midnight

13 8.20 p.m.

14 9.55 a.m.

15 11.39 p.m.

16 

17 

Change the times to a.m./ p.m. format. If the time starts with 13 or more, subtract 12 and write p.m.

Example

1548 hours 3.48 p.m.

18 0400 hours

19 0945 hours

20 1320 hours

21 1717 hours

22 2002 hours

23 2211 hours

24 1539 hours

25 2347 hours

26 0016 hours

27 0303 hours

28 1956 hours

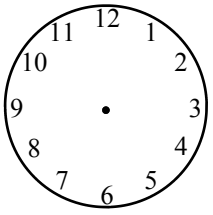
29 2200 hours

30 0000 hours

31 0222 hours

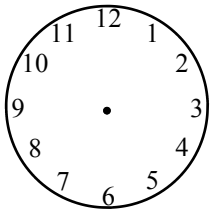
32 1833 hours

33 2008 hours



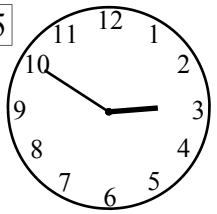
Circle: am pm

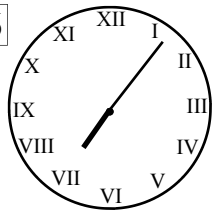
34 0505 hours



Circle: am pm

Write the times below in 24 hour format

35  afternoon

36  evening

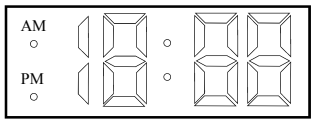
37 A quarter to ten at night

38 Eleven forty-seven (day)

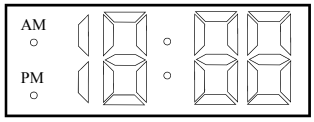
39 Half past twelve at night

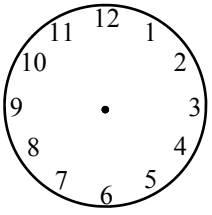
Write/colour the times in a.m./ p.m. format

40 1556 hours



41 0147 hours



42  2218 hours

time of day (words)

43 0855 hours

write answer in words & the time of the day
e.g. afternoon, evening, morning etc

Converting to 24 Hour Time

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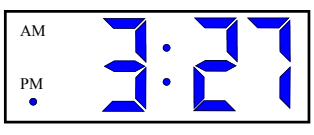
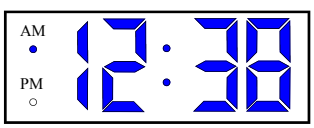
Change these times to 24 hour time format, (from 1 p.m. add 12 hours)

Example

4.45 p.m. — 1645 hours

1. 3 a.m. — 0300 hours
2. 7 a.m. — 0700 hours
3. 5 p.m. — 1700 hours
4. 11.25 a.m. — 1125 hours
5. 3.52 p.m. — 1552 hours
6. 5.17 p.m. — 1717 hours
7. 8.47 p.m. — 2047 hours
8. noon — 1200 hours
9. 4.56 a.m. — 0456 hours
10. 12.19 a.m. — 0019 hours
11. 10.25 p.m. — 2225 hours
12. midnight — 0000 hours
13. 8.20 p.m. — 2020 hours
14. 9.55 a.m. — 0955 hours
15. 11.39 p.m. — 2339 hours
16. 

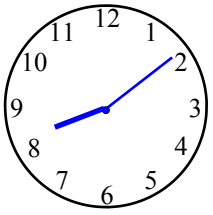
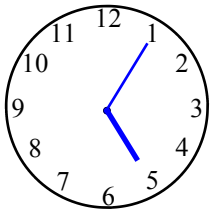
1527 hours
17. 

0038 hours

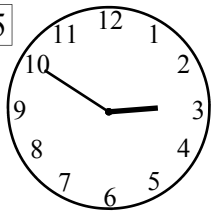
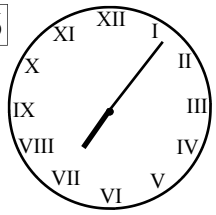
Change the times to a.m./ p.m. format. If the time starts with 13 or more, subtract 12 and write p.m.

Example

1548 hours — 3.48 p.m.

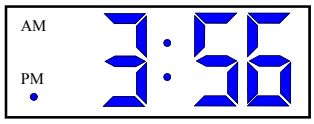
18. 0400 hours — 4 a.m.
19. 0945 hours — 9.45 a.m.
20. 1320 hours — 1.20 p.m.
21. 1717 hours — 5.17 p.m.
22. 2002 hours — 8.02 p.m.
23. 2211 hours — 10.11 p.m.
24. 1539 hours — 3.39 p.m.
25. 2347 hours — 11.47 p.m.
26. 0016 hours — 12.16 a.m.
27. 0303 hours — 3.03 a.m.
28. 1956 hours — 7.56 p.m.
29. 2200 hours — 10 p.m.
30. 0000 hours — 12 a.m.
31. 0222 hours — 2.22 a.m.
32. 1833 hours — 6.33 p.m.
33. 2008 hours

Circle: am (pm)
34. 0505 hours

Circle: (am) pm

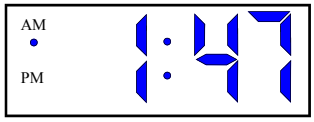
Write the times below in 24 hour format

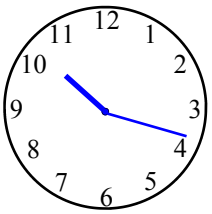
35. 
afternoon
1450 hours
36. 

evening
1907 hours
37. A quarter to ten at night
2145 hours
38. Eleven forty-seven (day)
1147 hours
39. Half past twelve at night
0030 hours

Write/colour the times in a.m./ p.m. format

40. 1556 hours


41. 0147 hours


42. 
2218 hours
night
time of day (words)
43. 0855 hours
Five to nine in the morning
write answer in words & the time of the day
e.g. afternoon, evening, morning etc

Units of Time

This sheet deals with the changing from one unit of time to another. Time is measured in: seconds (s), minutes (min), hours (h), days, weeks, years, and so on. In business and especially in the construction industry units such as years have to be broken into smaller units like weeks or even days, so that accurate scheduling can occur.

Column 1 outlines common time units and asks you to convert these to the larger unit above it, or the smaller unit below it. These should be done mentally, no working spaces have been provided. Some answers will need to be given in decimal or fraction form.

Column 2 requires the conversion of larger time periods to smaller time periods. These questions involve larger numbers, so working spaces are supplied for each question. The method is to convert one of the larger unit to smaller units, then multiply this by the number of larger units. So if converting 6 years to weeks for example, follow these steps:

- State the number of weeks in one year …. 52
- Multiply 52 by 6 and you get your answer …. 312 weeks (a working space is provided with each of these questions)
- State your answer… "There are 312 weeks in 6 years", and you are done.

You may think that the working spaces aren't adequate, Q 23 for example, remember that when you multiply by 60 it is easier to multiply by 6 and just add a zero, just don't forget to add it.

Column 3 adds a step, instead of moving to the next unit down, jump 2 units down. The questions are the same style as Column 2 only you need to make two conversions and multiply them.

Units of Time

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Convert these times to the unit of time on the right.

1 4 weeks = ____ days

2 1 year = ____ weeks

3 3 days = ____ h

4 ¼ h = ____ min

5 26 weeks = ____ years

6 6 min = ____ s

7 2 h = ____ min

8 1 leap year = ____ days

9 6 h = ____ days

10 3 decades = ____ years

11 4 min = ____ s

12 24 months = ____ years

13 4 centuries = ____ decades

14 ¼ year = ____ months

15 ¾ min = ____ s

16 2 fortnights = ____ weeks

17 300 s = ____ min

18 156 weeks = ____ years

19 September = ____ days

20 2 millennia = ____ years

21 March + May = ____ days

Convert to different units. These are harder, break them into parts.

22 Change 7 days → hours

1 day = ____ h

State your answer in words

There are ____ hours in seven days

23 Change 17 min → seconds

1 min = ____ s

State your answer in words

24 Change 18 days → hours

1 day = ____ h

State your answer in words

25 Change 9 years → weeks

1 year = ____ weeks

26 Change 15 years → months

1 year = ____ months

State your answer in words

Jumping from unit to unit requires thought. Change these units, in steps.

27 Change 1 day → min

1 day = ____ h

1 h = ____ min

28 Change 8 weeks → hours

1 week = ____ days

8 weeks = ____ days

1 day = ____ h

29 Change 1 hour → seconds

1 h = ____ min

1 min = ____ s

30 Change 2½ years → days

1 year = ____ weeks +

½ year = ____ weeks

So 2½ years = ____ weeks

1 week = ____ days

Units of Time

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Convert these times to the unit of time on the right.



1 4 weeks = 28 days

2 1 year = 52 weeks

3 3 days = 72 h

4 ¼ h = 15 min

5 26 weeks = ½ years

6 6 min = 360 s

7 2 h = 120 min

8 1 leap year = 366 days

9 6 h = ¼ days

10 3 decades = 30 years

11 4 min = 240 s

12 24 months = 2 years

13 4 centuries = 40 decades

14 ¼ year = 3 months

15 ¾ min = 45 s

16 2 fortnights = 4 weeks

17 300 s = 5 min

18 156 weeks = 3 years

19 September = 30 days

20 2 millennia = 2000 years

21 March + May = 62 days



Convert to different units. These are harder, break them into parts.

22 Change 7 days to hours

1 day = 24 h

State your answer in words

There are 168 hours in seven days

$$\begin{array}{r} {}^{2} \\ 24 \\ \times\ 7 \\ \hline 168 \end{array}$$

23 Change 17 min to seconds

1 min = 60 s

State your answer in words

There are 1 020 s in 17 minutes

$$\begin{array}{r} {}^{4} \\ 17 \\ \times\ 60 \\ \hline 1\,020 \end{array}$$

24 Change 18 days to hours

1 day = 24 h

State your answer in words

There are 432 h in 18 days.

$$\begin{array}{r} 24 \\ \times\ 18 \\ \hline {}^{1} \\ 192 \\ 240 \\ \hline 432 \end{array}$$

25 Change 9 years to weeks

1 year = 52 weeks

There are 468 weeks in 9 years

$$\begin{array}{r} {}^{1} \\ 52 \\ \times\ 9 \\ \hline 468 \end{array}$$

26 Change 15 years to months

1 year = 12 months

State your answer in words

There are 180 months in 15 years

$$\begin{array}{r} 15 \\ \times\ 12 \\ \hline 30 \\ 150 \\ \hline 180 \end{array}$$



Jumping from unit to unit requires thought. Change these units, in steps.

27 Change 1 day to min

1 day = 24 h

1 h = 60 min

There are 1 440 min in 1 day

$$\begin{array}{r} {}^{2} \\ 24 \\ \times\ 60 \\ \hline 1\,440 \end{array}$$

28 Change 8 weeks to hours

1 week = 7 days

8 weeks = 56 days

1 day = 24 h

There are 1 344 h in 8 weeks

$$\begin{array}{r} {}^{1} \\ 56 \\ \times\ 24 \\ \hline 224 \\ 1\,120 \\ \hline 1\,344 \end{array}$$

29 Change 1 hour to seconds

1 h = 60 min

1 min = 60 s

There are 3 600 s in 1 hour

$$\begin{array}{r} 60 \\ \times\ 60 \\ \hline 3\,600 \end{array}$$

30 Change 2½ years to days

1 year = 52 weeks

½ year = 26 weeks

So 2½ years = 130 weeks

1 week = 7 days

There are 910 days in 2.5 years

$$\begin{array}{r} {}^{1} \\ 52 \\ +\ 52 \\ 26 \\ \hline 130 \end{array}$$

$$\begin{array}{r} {}^{2} \\ 130 \\ \times\ 7 \\ \hline 910 \end{array}$$

Using Timetables

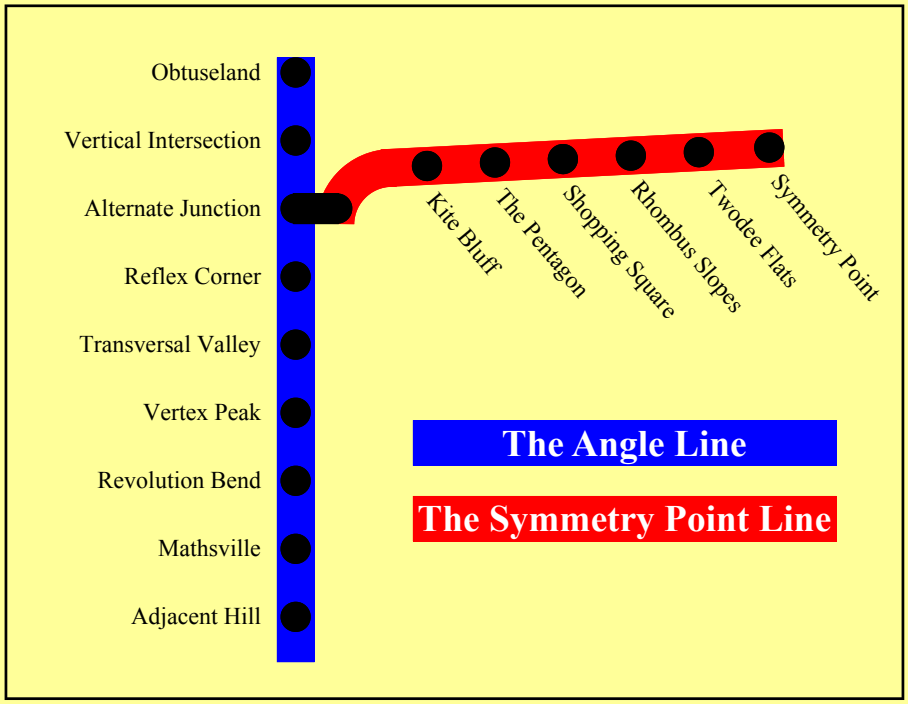
Reading timetables not only helps you to catch your train but it also avoids wasting time. Public transport runs whether you use it or not. By using it your travel is environmentally sound plus it helps maintain the service. The problem with public transport is that it is not flexible with time. But by using a timetable you can keep wasted time to a minimum. In these problems it is outlined that the latest time you can arrive at a station is when the train departs. This is true but not practical, it is always best to arrive early to have spare time in case the train is early, there is a queue for a ticket or a difference in the driver's watch and your own.

In Column 1 the Angle Line timetable is used, read the times from the timetable to answer the questions. Alternate Junction and Mathsville are major stations with branch lines and so the train stops on the platform and waits for 1 minute before departing.

In Column 2 the Symmetry Point Line joins the Angle Line at Alternate Junction station. The train terminates (stops) at Alternate Junction and returns back to Symmetry Point. It doesn't continue along the Angle Line. The questions for this column involve travel on the Symmetry Point Line then changing trains at Alternate Junction to meet the next Angle Line service. So both timetables will be needed! You are asked to complete missing entries on the timetable, enter the times (note you don't need to put a.m. or p.m. as they are at the top of the column).

Column 3 deals just with the Symmetry Point Line. This time a return timetable is used for a return trip. This is the Saturday timetable for the small branch line and only one train operates the service. While answering these questions keep in mind how the trip could have been made more enjoyable by using a timetable and using your skills at measuring time.

There is a schematic map of the two lines below.



Using Timetables

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Use the timetable below for this column.

The Angle Line

Station		a.m.	a.m.	a.m.	a.m.
Obtuseland		7.45	8.05	8.25	8.45
Vertical Intersection		7.49	8.09	8.29	8.49
Alternate Junction	arr.	7.53	8.13	8.33	8.53
	dep.	7.54	8.14	8.34	8.54
Reflex Corner		7.56	8.16	8.36	8.56
Transversal Valley		8.02	8.22	8.42	9.02
Vertex Peak		8.07	8.27	8.47	9.07
Revolution Bend		8.10	8.30	8.50	9.10
Mathsville	arr.	8.15	8.35	8.55	9.15
	dep.	8.16	8.36	8.56	9.16
Adjacent Hill		8.18	8.38	8.58	9.18

1 Will leaves home at 8.00 a.m. and arrives at Obtuseland Station at 8.08 a.m., find the time:

a) the first train arrives _____

b) he could have left home to catch the same train _____

c) he arrives at Mathsville _____

d) he arrives at school, a 5 min walk from Mathsville Stn. _____

e) the train travel takes _____

f) the entire trip takes _____

2 At 8 a.m. Will tries for the earlier train. He can run to Obtuseland Stn. in 6 min or to Vertical Intersection in 9 min, which is successful? _____

3 At what time would he arrive at school now? _____

4 Find the duration of the total trip to school. _____

5 Will wants to see his friend Jerome. If Jerome catches the same train as Will at 8.27 a.m., at what station was he waiting? _____

6 Will wants to buy a hot chocolate at the Alternate Junction platform vending machine. If it is 9 s away how much time does the machine have to fill the cup? _____

The Symmetry Point Line connects to the Angle Line at Alternate Junction so both timetables will be used for this column

The Symmetry Point Line

Station	a.m.	a.m.	a.m.	a.m.
Symmetry Point	7.25	7.45	7.55	8.25
Twodee Flats	7.28	...	7.58	
Rhombus Slopes	7.31	...		
Shopping Square	7.36	7.52	8.06	
The Pentagon	7.40	7.56	8.10	
Kite Bluff	7.44	...		
Alternate Junction	7.50x	8.02x	8.20x	8.50x

... denotes doesn't stop at this station

x denotes terminates here, change here for Angle Line

7 Alicia's timetable was unreadable from rain damage. Complete the table for the Symmetry Point 7.55 and 8.25 a.m. services.

8 Alicia lives at Symmetry Point and wants to catch the 8.14 a.m. Angle Line service which train should she catch? _____

9 Alicia attends the same school as Will. At what time would she arrive at Mathsville? _____

10 If it takes Alicia 5 min to walk to school at what time does she arrive at school? _____

11 If Alicia misses the 7.45 a.m. train will she be late if school starts at 9 a.m.? Circle: Yes / No

12 Calculate Alicia's total travel time for both the 7.45 and 7.55 a.m. services if her house is 4 min away from Symmetry Point Stn. (assume no waiting at Symm Pt.)

7.45 a.m. train _____ 7.55 a.m. train _____

13 Ann lives at Isosceles Hill, an 8 min drive from Kite Bluff. If she wants to catch the 8.14 a.m. Angle Line train find the latest time she can arrive at Kite Bluff and the time she must leave home.

Latest Arrival _____ Time left home _____

On Saturdays one train operates the Symmetry Pt line. A return timetable is included.

Symmetry Pt. → Alternate Jnct

Station	a.m.	a.m.	p.m.	p.m.
Symmetry Point	10.30	11.30	12.30	1.30
Twodee Flats	10.33	11.33	12.33	1.33
Rhombus Slopes	10.36	11.36	12.36	1.36
Shopping Square	10.41	11.41	12.41	1.41
The Pentagon	10.45	11.45	12.45	1.45
Kite Bluff	10.49	11.49	12.49	1.49
Alternate Junction	10.55x	11.55x	12.55x	1.55x

Alternate Jnct → Symmetry Pt.

Station	a.m.	p.m.	p.m.	p.m.
Alternate Junction	11.00	12.00	1.00	2.00
Kite Bluff	11.06	12.06	1.06	2.06
The Pentagon	11.10	12.10	1.10	2.10
Shopping Square	11.14	12.14	1.14	2.14
Rhombus Slopes	11.19	12.19	1.19	2.19
Twodee Flats	11.22	12.22	1.22	2.22
Symmetry Point	11.25x	12.25x	1.25x	2.25x

x denotes terminates here

14 Ian wants to catch a train from Twodee Flats to Shopping Square. Reaching the station at 10.50 a.m. how long does he wait? _____

15 Does Ian see his train travelling in the opposite direction? If so, at what time? Circle: Yes / No _____

16 If he spends 2 h at the shops (with walking time included), when does the next train arrive to return home and how long does he wait?

Time of train _____ Waiting time _____

17 How long is the train journey home? _____

18 If it takes Ian 15 min to walk between home and the station, at what time did he leave home and arrive back? How much time has elapsed during his trip?

Left home _____ Arrived home _____

Elapsed time _____

Using Timetables

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Use the timetable below for this column.

The Angle Line

Station	a.m.	a.m.	a.m.	a.m.
Obtuseland	7.45	8.05	8.25	8.45
Vertical Intersection	7.49	8.09	8.29	8.49
Alternate Junction arr.	7.53	8.13	8.33	8.53
dep.	7.54	8.14	8.34	8.54
Reflex Corner	7.56	8.16	8.36	8.56
Transversal Valley	8.02	8.22	8.42	9.02
Vertex Peak	8.07	8.27	8.47	9.07
Revolution Bend	8.10	8.30	8.50	9.10
Mathsville arr.	8.15	8.35	8.55	9.15
dep.	8.16	8.36	8.56	9.16
Adjacent Hill	8.18	8.38	8.58	9.18

1 Will leaves home at 8.00 a.m. and arrives at Obtuseland Station at 8.08 a.m., find the time:

a) the first train arrives 8.25 a.m.

b) he could have left home to catch the same train 8.17 a.m.

c) he arrives at Mathsville 8.55 a.m.

d) he arrives at school, a 5 min walk from Mathsville Stn. 9.00 a.m.

e) the train travel takes 30 min

f) the entire trip takes 1 h

2 At 8 a.m. Will tries for the earlier train. He can run to Obtuseland Stn. in 6 min or to Vertical Intersection in 9 min, which is successful? Vertical Intersection

3 At what time would he arrive at school now? 8.40 a.m.

4 Find the duration of the total trip to school. 40 min

5 Will wants to see his friend Jerome. If Jerome catches the same train as Will at 8.27 a.m., at what station was he waiting? Vertex Peak

6 Will wants to buy a hot chocolate at the Alternate Junction platform vending machine. If it is 9 s away how much time does the machine have to fill the cup? 42 s

The Symmetry Point Line connects to the Angle Line at Alternate Junction so both timetables will be used for this column

The Symmetry Point Line

Station	a.m.	a.m.	a.m.	a.m.
Symmetry Point	7.25	7.45	7.55	8.25
Twodee Flats	7.28	...	7.58	8.28
Rhombus Slopes	7.31	...	8.01	8.31
Shopping Square	7.36	7.52	8.06	8.36
The Pentagon	7.40	7.56	8.10	8.40
Kite Bluff	7.44	...	8.14	8.44
Alternate Junction	7.50x	8.02x	8.20x	8.50x

... denotes doesn't stop at this station

x denotes terminates here, change here for Angle Line

7 Alicia's timetable was unreadable from rain damage. Complete the table for the Symmetry Point 7.55 and 8.25 a.m. services.

8 Alicia lives at Symmetry Point and wants to catch the 8.14 a.m. Angle Line service which train should she catch? 7.45 a.m.

9 Alicia attends the same school as Will. At what time would she arrive at Mathsville? 8.35 a.m.

10 If it takes Alicia 5 min to walk to school at what time does she arrive at school? 8.40 a.m.

11 If Alicia misses the 7.45 a.m. train will she be late if school starts at 9 a.m.? Circle: Yes / (No)

12 Calculate Alicia's total travel time for both the 7.45 and 7.55 a.m. services if her house is 4 min away from Symmetry Point Stn. (assume no waiting at Symm Pt.)

7.45 a.m. train: 59 min

7.55 a.m. train: 1 h 9 min

13 Ann lives at Isosceles Hill, an 8 min drive from Kite Bluff. If she wants to catch the 8.14 a.m. Angle Line train find the latest time she can arrive at Kite Bluff and the time she must leave home.

Latest Arrival: 7.44 a.m.

Time left home: 7.36 a.m.

On Saturdays one train operates the Symmetry Pt line. A return timetable is included.

Symmetry Pt. → Alternate Jnct

Station	a.m.	a.m.	p.m.	p.m.
Symmetry Point	10.30	11.30	12.30	1.30
Twodee Flats	10.33	11.33	12.33	1.33
Rhombus Slopes	10.36	11.36	12.36	1.36
Shopping Square	10.41	11.41	12.41	1.41
The Pentagon	10.45	11.45	12.45	1.45
Kite Bluff	10.49	11.49	12.49	1.49
Alternate Junction	10.55x	11.55x	12.55x	1.55x

Alternate Jnct → Symmetry Pt.

Station	a.m.	p.m.	p.m.	p.m.
Alternate Junction	11.00	12.00	1.00	2.00
Kite Bluff	11.06	12.06	1.06	2.06
The Pentagon	11.10	12.10	1.10	2.10
Shopping Square	11.14	12.14	1.14	2.14
Rhombus Slopes	11.19	12.19	1.19	2.19
Twodee Flats	11.22	12.22	1.22	2.22
Symmetry Point	11.25x	12.25x	1.25x	2.25x

x denotes terminates here

14 Ian wants to catch a train from Twodee Flats to Shopping Square. Reaching the station at 10.50 a.m. how long does he wait? 43 min

15 Does Ian see his train travelling in the opposite direction? If so, at what time? Circle: (Yes) / No

11.22 a.m.

16 If he spends 2 h at the shops (with walking time included), when does the next train arrive to return home and how long does he wait?

Time of train: 2.14 p.m.

Waiting time: 33 min

17 How long is the train journey home? 8 min

18 If it takes Ian 15 min to walk between home and the station, at what time did he leave home and arrive back? How much time has elapsed during his trip?

Left home: 10.35 a.m.

Arrived home: 2.37 a.m.

Elapsed time: 4 h 2 min

Time Calculations 1

Often times are given in a way that you find difficult to use. One example is a movie on video with its duration given in minutes. If a movie is 173 minutes long, how do you convert it to hours and minutes?

In Column 1 the exercises involves changing min to h and min. Multiples of 60 are required, so $1 \times 60 = 60$ and so on, from then on add 60 to the previous answer to get the next.

To answer Q 2 - 7 find the largest multiple of 60 that is below the number of min in the question and then put the remainder, the amount left over, in the 2nd box. Then change the multiple of 60 to hours and then write the remainder in min. Look at the example above Q.2.

Column 2 requires times to be added. Using the addition space add the two times together, ignoring the fact that the min may exceed 60 min. Then rewrite your answer again in the smaller outlined box. If the minutes exceed 60 convert the min to hours by deducting 60 min off the min total and adding 1 h to the hour total. Of course if the min total is greater than 120 min you have to deduct 120 min off and add 2 hours, and so on for each multiple of 60. See the example at the top of the column.

The 3rd Column requires the method to be reversed. Before subtracting there must be enough min to perform the subtraction (the first time must have more min than the second time). If it doesn't, take an hour off the first time and add 60 min to the minutes. For example 4 h 11 min would become 3 h 71 min, 1 h 35 min would become 95 min etc. There is an example at the top of the column.

Time Calculations 1

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List the first 12 multiples of 60, this will help you convert min to h and min.

1

$1 \times 60 =$ ____ $7 \times 60 =$ ____

$2 \times 60 =$ ____ $8 \times 60 =$ ____

$3 \times 60 =$ ____ $9 \times 60 =$ ____

$4 \times 60 =$ ____ $10 \times 60 =$ ____

$5 \times 60 =$ ____ $11 \times 60 =$ ____

$6 \times 60 =$ ____ $12 \times 60 =$ ____

Using the answers in question 1, convert these times in min to h and min

Example

435 min = 420 + 15 min

= 7 h 15 min

2 157 min = ____ + ____ min

= ____

3 203 min = ____ + ____ min

= ____

4 347 min = ____ + ____ min

= ____

5 577 min = ____ + ____ min

= ____

6 756 min = ____ + ____ min

= ____

7 686 min = ____ + ____ min

= ____

Add these times together. If the min exceed 60 change them to h.

Example

3 h 42 min and 7 h 39 min

1 1
3 42 +
7 39
10 81

10 h 81 min = 11 h 21 min

The total is 11 h 21 min

8 3 h 27 min and 8 h 46 min

+ ____ ____

The total is

9 11 h 52 min and 7 h 35 min

+ ____ ____

10 6 h 49 min and 8 h 44 min

+ ____ ____

11 7 h 56 min and 2 h 48 min

+ ____ ____

12 3 h 43 min, 3 h 37 min and 7 h 51 min

+ ____ ____

Subtract these times, you may have to convert an h to 60 min first (or more).

Example

7 h 15 min - 4 h 57 min

6 75 -
4 57
1
2 18

7 h 15 min = 6 h 75 min

Difference is 2 h 18 min

13 8 h 47 min - 5 h 11 min

- ____ ____

Difference is

14 17 h 25 min - 6 h 52 min

- ____ ____

15 26 h 37 min - 13 h 48 min

- ____ ____

16 16 h 43 min - 9 h 56 min

- ____ ____

17 7 h - 3 h 41 min - 1 h 57 min

- ____ ____

Time Calculations 1

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List the first 12 multiples of 60, this will help you convert min to h and min.

1

1 × 60 =	60	7 × 60 =	420
2 × 60 =	120	8 × 60 =	480
3 × 60 =	180	9 × 60 =	540
4 × 60 =	240	10 × 60 =	600
5 × 60 =	300	11 × 60 =	660
6 × 60 =	360	12 × 60 =	720



Using the answers in question 1, convert these times in min to h and min

Example

435 min = 420 + 15 min

= 7 h 15 min

2 157 min = 120 + 37 min

= 2 h 37 min

3 203 min = 180 + 23 min

= 3 h 23 min

4 347 min = 300 + 47 min

= 5 h 47 min

5 577 min = 540 + 37 min

= 9 h 37 min

6 756 min = 720 + 36 min

= 12 h 36 min

7 686 min = 660 + 26 min

= 11 h 26 min



Add these times together. If the min exceed 60 change them to h.

Example

3 h 42 min and 7 h 39 min

1 1			
3 42	+	10 h 81 min	= 11 h 21 min
7 39			
10 81			

The total is 11 h 21 min

8 3 h 27 min and 8 h 46 min

1 1			
3 27	+	11 h 73 min	= 12 h 13 min
8 46			
11 73			

The total is 12 h 13 min

9 11 h 52 min and 7 h 35 min

11 52	+	18 h 87 min	= 19 h 27 min
7 35			
18 87			

The total is 19 h 27 min

10 6 h 49 min and 8 h 44 min

1 1			
6 49	+	14 h 93 min	= 15 h 33 min
8 44			
14 93			

The total is 15 h 33 min

11 7 h 56 min and 2 h 48 min

1 1			
7 56	+	9 h 104 min	= 10 h 44 min
2 48			
9 104			

The total is 10 h 44 min

12 3 h 43 min, 3 h 37 min and 7 h 51 min

1 1 1			
3 43	+		
3 37		13 h 131 min	= 15 h 11 min
7 51			
13 131			

The total is 15 h 11 min



Subtract these times, you may have to convert an h to 60 min first (or more).

Example

7 h 15 min - 4 h 57 min

6 75	-	7 h 15 min	= 6 h 75 min
4 57			
1			
2 18			

Difference is 2 h 18 min

13 8 h 47 min - 5 h 11 min

8 47	-		
5 11			
3 36			

Difference is 3h 36 min

14 17 h 25 min - 6 h 52 min

16 85	-	17 h 25 min	= 16 h 85 min
6 52			
10 33			

Difference is 10 h 33 min

15 26 h 37 min - 13 h 48 min

25 97	-	26 h 37 min	= 25 h 97 min
13 48			
1			
12 49			

Difference is 12 h 49 min

16 16 h 43 min - 9 h 56 min

15 103	-	16 h 43 min	= 15 h 103 min
9 56			
1 1 1			
6 47			

Difference is 6 h 47 min

17 7 h - 3 h 41 min - 1 h 57 min

5 120	-	7 h	= 5 h 120 min
3 41			
1 57			
1 1			
1 22			

Difference is 1 h 22 min

Time Calculations 2

Often the time until an event occurs is required to be calculated for planning or scheduling.

In column 1 the time in days between two dates is required, count these off to answer them. With Q 4 and 5 add the separate amounts, the number of days to: get to the end of the month, the complete month in-between and then the days in the 3rd month.

Column 2, Q 13 to 16, again use the same counting system. But from Q 17 on a new method is required. The steps are as follows:

- convert the times to 24 h format (add 12 to hours to times after 1 p.m.)
- the later time must have more minutes that the earlier time to be able to subtract, so if the minutes are less take an hour off and add 60 to the minutes column
- Carry out the subtraction and then write the answer in words.

For example 3.35 p.m. = 1535 h to move an hour across take an hour off the 15 h (15 - 1 = 14) and add 60 to the minutes (35 + 60 = 95), thus 1495 h. Look at the example before Q 17.

In the third column these require a further step, use the same method but because the times are on the following day add 24 h for each day that is advanced. For example 5.16 p.m. on the next day would be 1716 + 24 = 4116 h, then perform the subtraction. See the example at the top of the column.

With Q 22 and 23 find the sum (addition), then if the minutes are greater than 60 take 60 min off and add 1 to the hours. For example,

3 h 43 min + 5 h 23 min = 8 h 66 min = 9 h 6 min.

Time Calculations 2

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Find the time difference in days between the following dates.

1 8th June and the 23rd June

2 17th September and the 3rd September

3 18th March and the 9th April

4 16th July and the 28th August

5 15/12/02 and 02/02/03

☐ + ☐ + ☐ =

6 02/02/02 and 05/04/02

☐ + ☐ + ☐ =

Find the time difference in weeks between the following dates.

7 3rd March and 31st March

8 20th November and Christmas day

9 New Years day and 26th February

Find the time in years represented by the following words.

10 Millennium

11 Century

12 Decade

Find the time difference in hours between the following times.

13 3 a.m. Tuesday and 11 p.m. Tuesday

14 12 p.m. Thursday and 6 a.m. Friday

15 1 a.m. Monday and 6 p.m. Tuesday

16 6.15 a.m. Sunday and 4.15 a.m. Monday

Find the time difference. Take an hour off and change it to minutes.

Example

11.27 a.m. and 4.17 p.m.

15 77 - 4.17 p.m. = 1617 h (24h)

11 27 1617 h = 1577 h (1h →min)

The difference is

4 50 4 h 50 min.

17 12.45 p.m. and 3.16 p.m.

3.16 p.m.= ☐ (24h)

\- ☐ (h →min)

18 7.37 a.m. and 5.28 p.m.

☐ (24h)

\- ☐ (h →min)

19 4.19 a.m. and 7.53 p.m.

☐ (24h)

\-

Add 24 hours for every day that you advance. These are harder!

Example 7.26 a.m. Friday and 3.52 p.m. Saturday

39 52 - 3.52 p.m. = 1552 h (24h)

07 26 1552 = 3952 h (+ 24h)

1

32 26 The difference is 32 h 26 min.

20 7.17 p.m. Monday and 4.53 a.m. Tuesday

\- ☐

☐

21 10.08 p.m. Wednesday and 1.37 a.m. Friday

\- ☐

☐

Add these times, if the minutes exceed 60 change them to hours

22 8 h 47 min and 5 h 39 min

☐ = ☐

\+

The total is

23 6 h 57 min, 3 h 43 min, 2 h 16 min and 4 h 28 min

\+

☐ = ☐

Time Calculations 2

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Find the time difference in days between the following dates.

1 8th June and the 23rd June — 15 days

2 17th September and the 3rd September — 14 days

3 18th March and the 9th April — 22 days

4 16th July and the 28th August — 43 days

5 15/12/02 and 02/02/03

16 + 31 + 2 = 49 days

6 02/02/02 and 05/04/02

26 + 31 + 5 = 62 days

Find the time difference in weeks between the following dates.

7 3rd March and 31st March — 4 weeks

8 20th November and Christmas day — 5 weeks

9 New Years day and 26th February — 8 weeks

Find the time in years represented by the following words.

10 Millennium — 1000 years

11 Century — 100 years

12 Decade — 10 years

Find the time difference in hours between the following times.

13 3 a.m. Tuesday and 11 p.m. Tuesday — 20 h

14 12 p.m. Thursday and 6 a.m. Friday — 18 h

15 1 a.m. Monday and 6 p.m. Tuesday — 41 h

16 6.15 a.m. Sunday and 4.15 a.m. Monday — 22 h

Find the time difference. Take an hour off and change it to minutes.

Example

11.27 a.m. and 4.17 p.m.

15 77 - (4.17 p.m. = 1617 h (24h); 1617 h = 1577 (1h →min))

11 27

4 50

The difference is 4 h 50 min.

17 12.45 p.m. and 3.16 p.m.

14 76 - (3.16 p.m.= 1516 h (24h); 1516 = 1476 h (h →min))

12 45

2 31

The difference is 2 h 31 min.

18 7.37 a.m. and 5.28 p.m.

16 88 - (5.28 p.m.= 1728 h (24h); 1728 = 1688 h (h →min))

07 37

1

9 51

The difference is 9 h 51 min.

19 4.19 a.m. and 7.53 p.m.

19 53 - (7.53 p.m.= 1953 h (24h))

04 19

1

15 34

The difference is 15 h 34 min.

Add 24 hours for every day that you advance. These are harder!

Example 7.26 a.m. Friday and 3.52 p.m. Saturday

39 52 - (3.52 p.m. = 1552 h (24h); 1552 = 3952 h (+ 24h))

07 26

1

32 26

The difference is 32 h 26 min.

20 7.17 p.m. Monday and 4.53 a.m. Tuesday

28 53 - (4.53 am = 0453 h (24 h); 0453 = 2853 (+24 h))

19 17

1 1

9 36

The difference is 9 h 36 min.

21 10.08 p.m. Wednesday and 1.37 a.m. Friday

49 37 - (1.37 am = 0137 h (24h); 0137 = 4937 h (+ 48h))

22 08

1

27 29

The difference is 27 h 29 min.

Add these times, if the minutes exceed 60 change them to hours

22 8 h 47 min and 5 h 39 min

1

8 47 +

5 39

13 86 (13 h 86 min = 14 h 26 min)

The total is 14 h 26 min

23 6 h 57 min, 3 h 43 min, 2 h 16 min and 4 h 28 min

2

6 57 +

3 43

2 16

4 28

15 144 (15 h 144 min = 17 h 24 min)

The total is 17 h 24 min

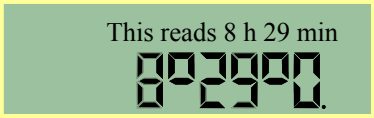
Time Calculations (Calculator)

Time calculations are performed every day, …..how long until lunch?, ….the bus arrives? When you become a wage earner it is important to be able to check that your hours worked are correct, but because minutes and hours are in groups of 60 min this is not always straight forward. But a calculator makes it easy.

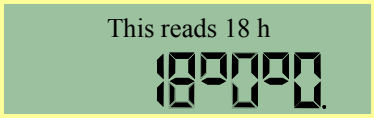
The calculator image on the next page shows the DMS key or the 'bubble button' (the key has an orange border). This key allows you work with hours and minutes.

IMPORTANT YOU MUST ENTER A 0 (THEN DMS KEY) IF DEALING ONLY WITH MINUTES (NO HOURS).

The calculator will always show the hours, minutes and seconds (we won't be using seconds) separated by a degree sign (a small raised o) →°.







In column 1 you are asked to convert the times given in minutes to hours and minutes. This is done by pressing 0 then the DMS key, then the minutes in the question then the DMS key and then press = and the answer will be displayed. Example, change 338 min to hours and minutes.

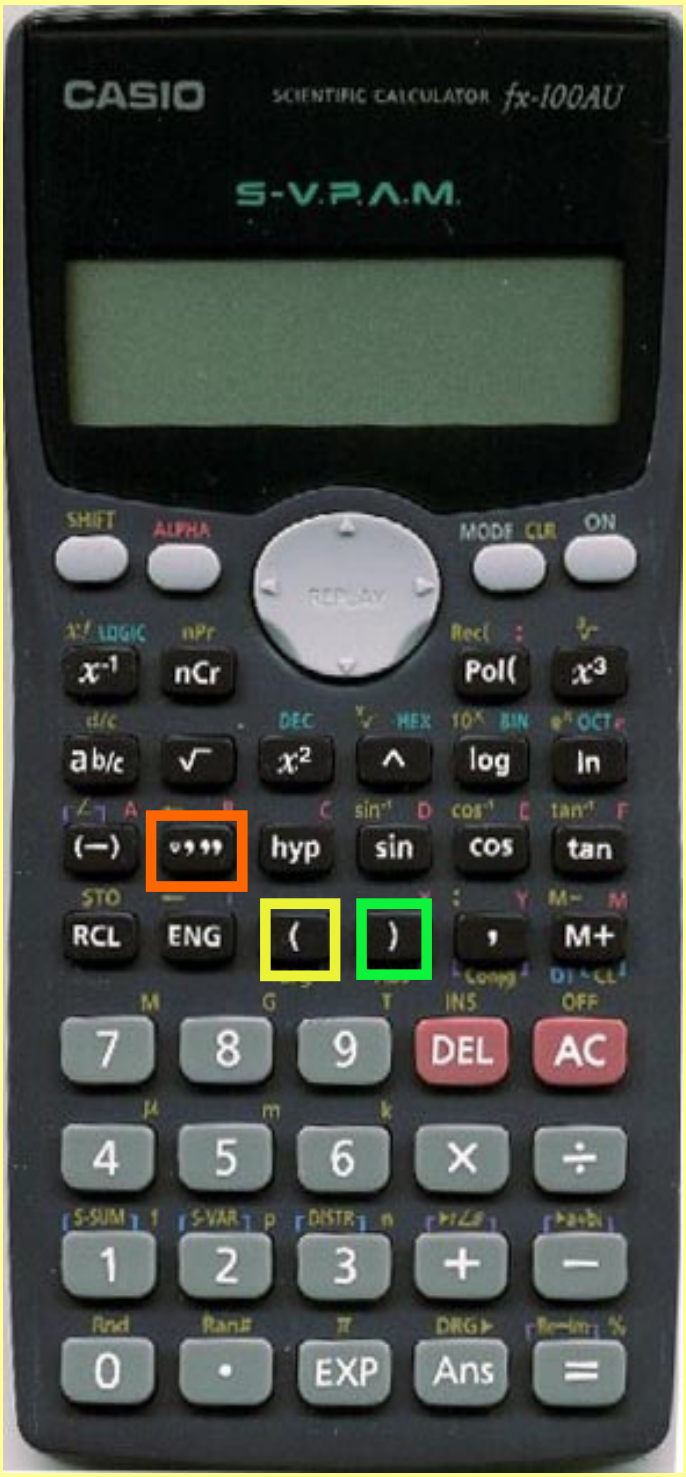




In the 2nd column times are to be added together, put the first time into the calculator then a + then the second time in, press =, done. Finding the difference between two times is done by subtraction, using the same method as above, with one exception YOU MUST CONVERT P.M. TIME TO 24 H TIME. If they are both a.m. times or both p.m. times no conversion is necessary, as soon as a question involves both a p.m. and an a.m. time convert the p.m. time to 24 h format. See the example before Q 18.

The time sheet at the bottom of the column 2 requires you to find the time difference for each day between start and finish times, put them in the spaces provided then add them. Monday is already done for you, check you get the same answer and don't forget to include it in your addition.

The 3rd column is using × and ÷ these are done no differently just remember that you are multiplying by a number not a time and so only use the DMS on the time. The calculation of an average time (Q 31) can be done in two ways refer to the text box below the calculator image for the full key stroke method on the next page.



Example of steps in calculating average time
of 11h 12 min, 8 h 57 min and 10 h 9 min

Using Brackets

(1 1 °’’’ 1 2 °’’’ + 8 °’’’ 5 7 °’’’ + 1 0 °’’’ 9 °’’’) ÷ 3 =

Using = to avoid order of operations

1 1 °’’’ 1 2 °’’’ + 8 °’’’ 5 7 °’’’ + 1 0 °’’’ 9 °’’’ = ÷ 3 =

if you get 10 h 6 min for the question above you are correct

Time Calculations (Calculator)

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Rewrite the calculator displays in h and min.

1 

2 

3 

4 

Convert the following times in min to h and min, remember to put a zero in for the hours first.

5 211 min

6 173 min

7 245 min

8 727 min

9 341 min

10 360 min

11 568 min

12 540 min

13 1331 min

14 1080 min

Add these times together using your calculator

15 8 h 27 min + 11 h 36 min

16 8 h 19 min + 4 h 48 min + 2 h 37 min

17 4 h 39 min + 5 h 25 min + 11 h 56 min

Find the time difference between these times that are on the same day. Use 24 hour time format

Example

1853 h in 24 h

11.17 a.m. and 6.53 p.m.

1 8 ''' 5 3 ''' - 1 1 ''' 1 7 ''' = 7 h 36 min

18 10.30 a.m. and 3.56 p.m.

19 3.52 a.m. and 11.27 a.m.

20 4.23 a.m. and 6.17 p.m.

21 1.09 p.m. and 10.57 p.m.

Complete the time sheet below, then find the total hours worked.

22 A. Jolie Time Sheet

Day	Start	Finish	Hours
Mon	9.30 a.m.	4.45 p.m.	7 h 15 min
Tues	10.45 a.m.	2.30 p.m.	h min
Wed	9.30 a.m.	5.00 p.m.	h min
Thur	9.30 a.m.	3.45 p.m.	h min
Fri	1.15 p.m.	6.00 p.m.	h min
		Total hours for the week	h min

Now use × and ÷ with these times

23 3 × 17 min

24 8 × 23 min

25 7 × 1 h 46 min

26 1 h 55 min ÷ 5

27 19 h 57 min ÷ 9

28 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min

29 8 h 45 min ÷ 5 + 5 min × 17 + 1 h 31 min

Try these time problems

30 Sean watches 3 movies at a 6 h movie marathon, if two of the films were 1h 46 min and 1h 50 min, find the duration of the other.

31 Julie's time for return travel to school for 3 days were: 1h 15 min, 56 min and 1h 22 min. Find the average time for her return trip.

32 It takes Jemma 43 min to wash a car, and 11 min to vacuum a car. How long does she take to wash 5 cars and vacuum 3 of them? Jemma has 40 min for lunch, if she started at 10.25 a.m. estimate her finish time.

Time taken _____ Estimated finish time _____

Time Calculations (Calculator)

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Rewrite the calculator displays in h and min.

1 

52 min

2 

1 h 17 min

3 

3 h

4 

31 h 34 min

Convert the following times in min to h and min, remember to put a 0° in for the hours first.

5 211 min — 3 h 31 min

6 173 min — 2 h 53 min

7 245 min — 4 h 5 min

8 727 min — 12 h 7 min

9 341 min — 5 h 41 min

10 360 min — 6 h

11 568 min — 9 h 28 min

12 540 min — 9 h

13 1331 min — 22 h 11 min

14 1080 min — 18 h

Add these times together using your calculator

15 8 h 27 min + 11 h 36 min

20 h 3 min

16 8 h 19 min + 4 h 48 min + 2 h 37 min — 15 h 44 min

17 4 h 39 min + 5 h 25 min + 11 h 56 min — 22 h

Find the time difference between these times that are on the same day. Use 24 hour time format

Example

1853 in 24 h time

11.17 a.m. and 6.53 p.m.

1 8 °'" 5 3 °'" - 1 1 °'" 1 7 °'" =

7 h 36 min

18 10.30 a.m. and 3.56 p.m.

5 h 26 min

19 3.52 a.m. and 11.27 a.m.

7 h 35 min

20 4.23 a.m. and 6.17 p.m.

13 h 54 min

21 1.09 p.m. and 10.57 p.m.

9 h 48 min

Complete the time sheet below, then find the total hours worked.

22 A. Jolie Time Sheet

Day	Start	Finish	Hours
Mon	9.30 a.m.	4.45 p.m.	7 h 15 min
Tues	10.45 a.m.	2.30 p.m.	3 h 45 min
Wed	9.30 a.m.	5.00 p.m.	7 h 30 min
Thur	9.30 a.m.	3.45 p.m.	6 h 15 min
Fri	1.15 p.m.	6.00 p.m.	4 h 45 min

Total hours for the week 29 h 30 min

Now use × and ÷ with these times

23 3 × 17 min — 51 min

24 8 × 23 min — 3 h 4 min

25 7 × 1 h 46 min — 12h 22 min

26 1 h 55 min ÷ 5 — 23 min

27 19 h 57 min ÷ 9 — 2 h 13 min

28 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min — 7 h 8 min

29 8 h 45 min ÷ 5 + 5 min × 17 + 1 h 31 min — 4 h 41 min

Try these time problems

30 Sean watches 3 movies at a 6 h movie marathon, if two of the films were 1h 46 min and 1h 50 min, find the duration of the other.

2 h 24 min

31 Julie's time for return travel to school for 3 days were: 1h 15 min, 56 min and 1h 22 min. Find the average time for her return trip.

1 h 11 min

32 It takes Jemma 43 min to wash a car, and 11 min to vacuum a car. How long does she take to wash 5 cars and vacuum 3 of them? Jemma has 40 min for lunch, if she started at 10.25 a.m. estimate her finish time.

Time taken: 4 h 8 min

Estimated finish time: 3.13 p.m.

Regional Time Difference

Australia has 3 time zones:

Eastern Standard Time (EST):	Queensland, New South Wales, Australian Capital Territory, Victoria and Tasmania.
Central Standard Time (CST):	Northern Territory and South Australia
Western Standard Time (WST):	Western Australia

New Zealand mainland has a single time zone, but the Chatham Islands are +45 min ahead of NZ mainland time. The Chatham Island's population are the first in the world to see the sunrise each new day.

Time is subtracted when moving west, (so it is earlier) and time is added when moving west (it is later). The amount added/subtracted varies. The arrows at the top and bottom of the map, tell you how much to add or subtract when you step between adjacent time zones.

Column 1 gives you a time in Perth and asks for the current time in other centres.
Example : Perth local time is 4.45 a.m., the time in Canberra is?

To get from Perth to Canberra add a ½ hr to get to CST then 1½ hr to get from CST to EST, a total of 2 hours to be added. So the time in Canberra is 4.45 a.m. + 2 hr = 6.45 a.m.

Column 2 selects Wellington (NZ) as the comparison, this time movement west (deduct time) is involved. Crossing the Tasman Sea to Australia. Remember that the arrows at the top of the map help you to answer these questions.

The last question in the first 2 columns asks you to write in your location (city or town) and calculate the local time.

With the 3rd column the time is required to be written and also placed on the analogue clocks.

Remember that a time requires a.m. or p.m. written after it. (unless in 24 hour time format) and that 12 noon is 12 p.m. and 12 midnight is 12 a.m.

Regional Time Difference

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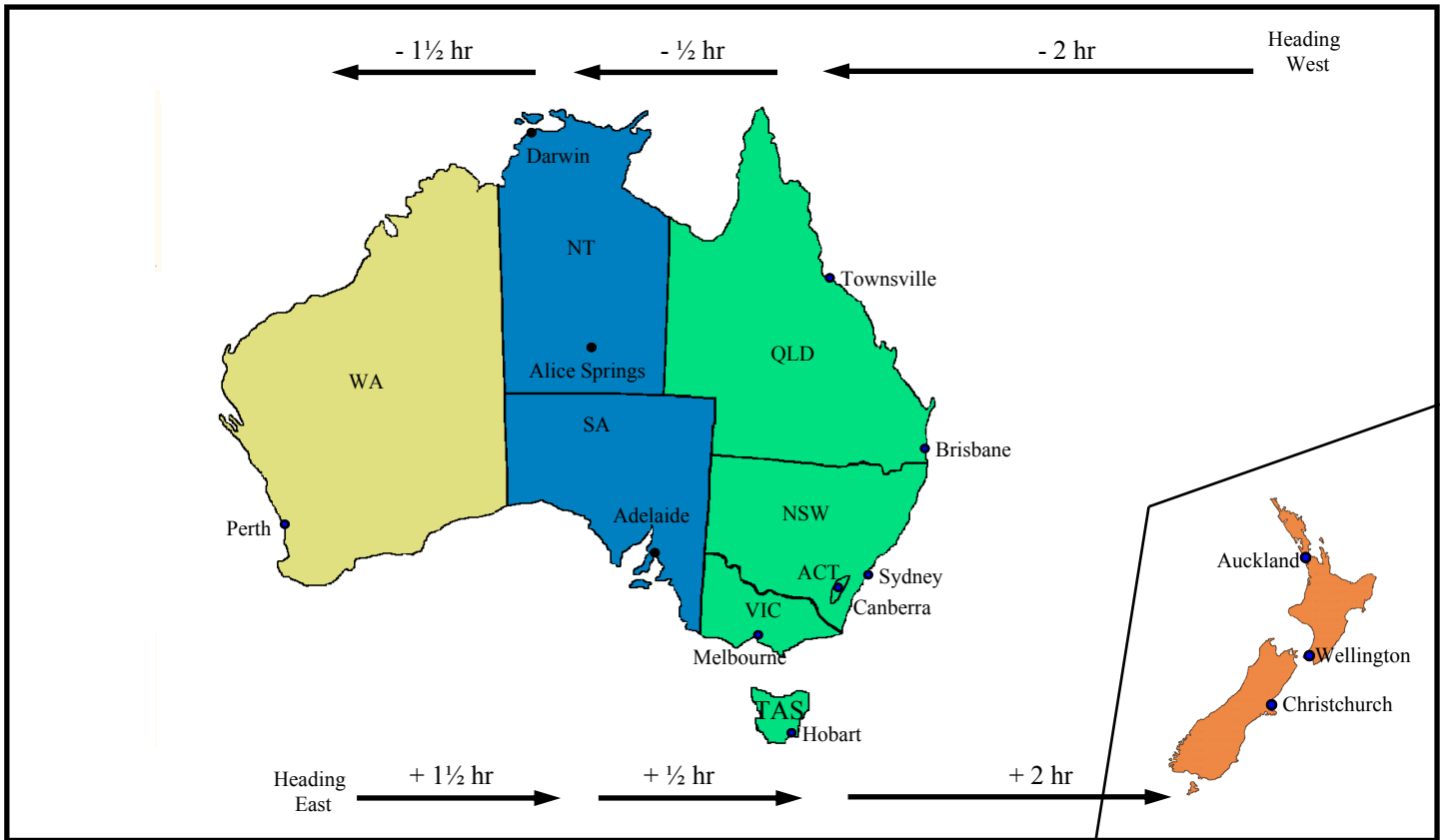


If it is 10 a.m. in Perth calculate the local time in the cities below (am/pm)

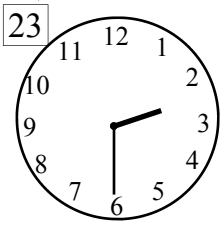
1	Sydney	
2	Hobart	
3	Brisbane	
4	Auckland	
5	Darwin	
6	Melbourne	
7	Wellington	
8	Adelaide	
9	Alice Springs	
10	Townsville	
11	Your location	

If it is 3 a.m. in Wellington calculate the local time in the cities below (am/pm).

12	Auckland	
13	Sydney	
14	Perth	
15	Adelaide	
16	Brisbane	
17	Hobart	
18	Darwin	
19	Christchurch	
20	Canberra	
21	Alice Springs	
22	Your location	

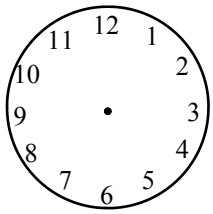
Show and write the local time for the city given that the current time is….

23



Melbourne

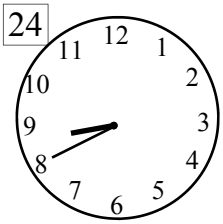
2.30 p.m.



Darwin

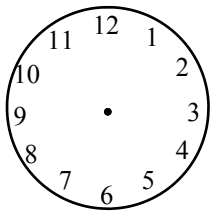
_____

24



Perth

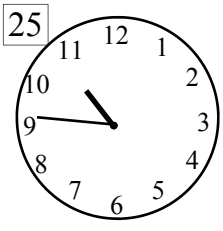
8.40 p.m.



Sydney

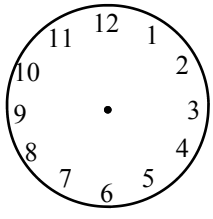
_____

25



Brisbane

10.47 p.m.



Auckland

_____

Regional Time Difference

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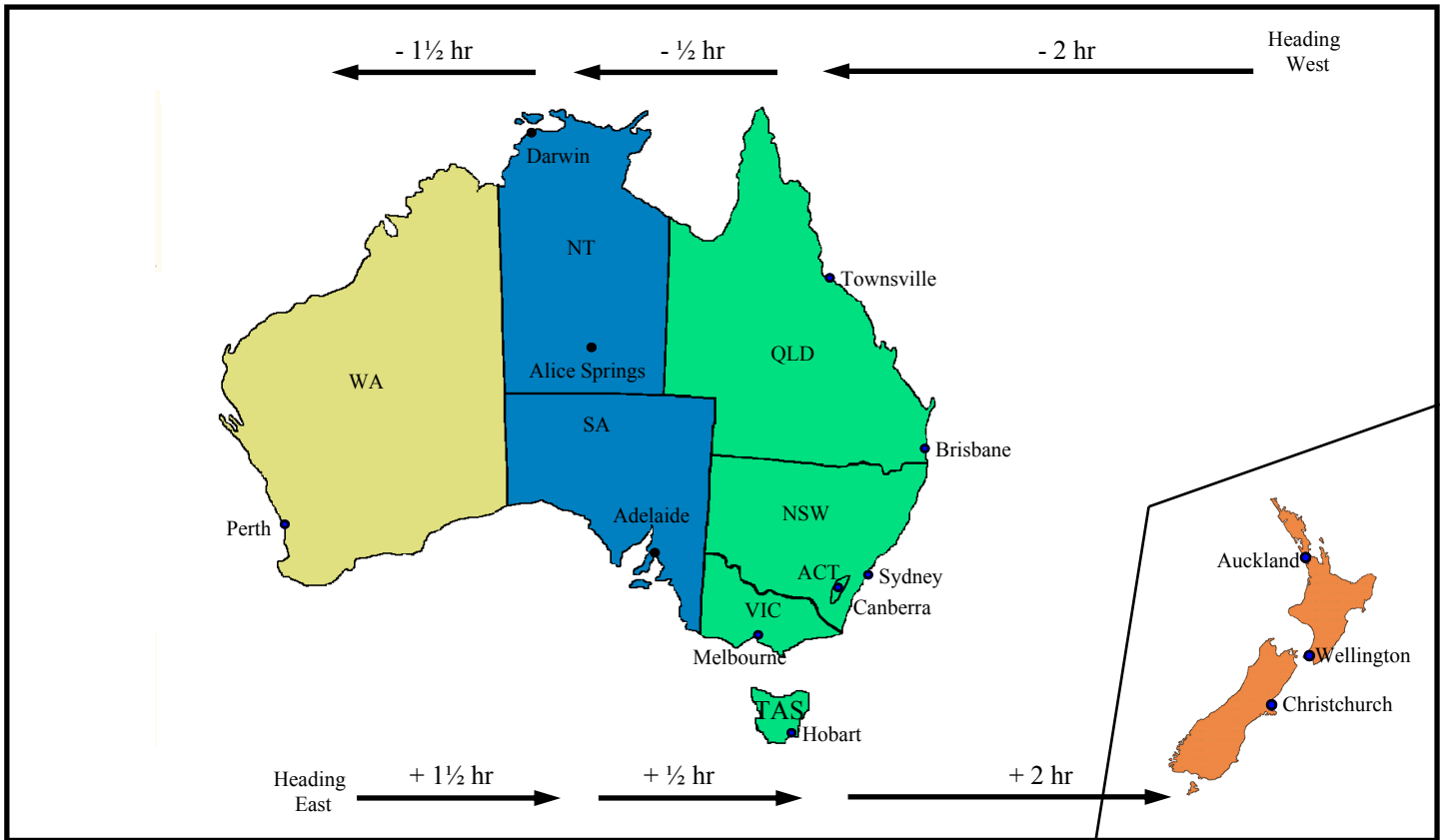


If it is 10 a.m. in Perth calculate the local time in the cities below (am/pm)

1	Sydney	12 p.m.
2	Hobart	12 p.m.
3	Brisbane	12 p.m.
4	Auckland	2 p.m.
5	Darwin	11.30 a.m.
6	Melbourne	12 p.m.
7	Wellington	2 p.m.
8	Adelaide	11.30 a.m.
9	Alice Springs	11.30 a.m.
10	Townsville	12 p.m.
11		

Your location

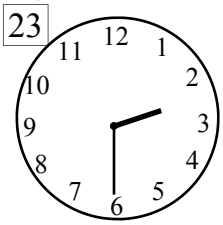
If it is 3 a.m. in Wellington calculate the local time in the cities below (am/pm).

12	Auckland	3 a.m.
13	Sydney	1 a.m.
14	Perth	11 p.m.
15	Adelaide	12.30 a.m.
16	Brisbane	1 a.m.
17	Hobart	1 a.m.
18	Darwin	12.30 a.m.
19	Christchurch	3 a.m.
20	Canberra	1 a.m.
21	Alice Springs	12.30 a.m.
22		

Your location

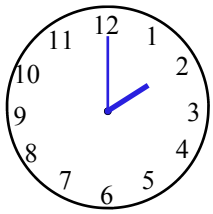
Show and write the local time for the city given the current time is….

23



Melbourne

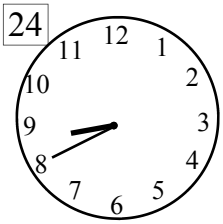
2.30 p.m.



Darwin

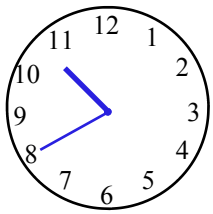
2.00 p.m.

24



Perth

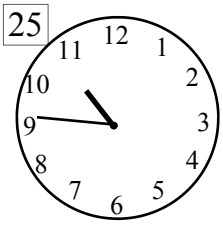
8.40 p.m.



Sydney

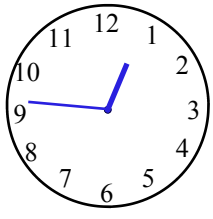
10.40 p.m.

25



Brisbane

10.47 p.m.



Auckland

12.47 a.m.

World Time Difference

Time difference in the world is based on GMT, Greenwich Meridian Time. With locations that are west of GMT time is subtracted. (This means it is earlier than in the U.K.). With locations east time is added (this means it is later than in the U.K.). GMT is now also referred to as UTC (Coordinated Universal Time) but this is not as well known.

Column 1 requires use of the map. Cities have been shown with the time ahead (+) or behind (-) GMT. So to calculate the time in a city in column 1, deduct the time (if -) or add the time (if +) in hours from/to the GMT time. Note that with New Delhi you add 5½ hours.

With column 2 instead of U.K. local time it is now New York time. If it is 10.00 am in New York what is the GMT? Once you know the GMT time you can use the same method as the first column!

Remember that due to cities being up to 12 hours ahead of the U.K. this means than the date in Auckland (and other cities to the east) could actually be different (ahead) of the U.K. and so could be the next day. The opposite could be said for cities to the west, the date could be behind the U.K. and a day earlier.

With column 3, again convert the time in Japan to GMT and then apply the same method as the previous columns. Remember a time requires a.m. or p.m. written after it, unless in 24 hour time format and that 12 noon is 12 p.m. and 12 midnight is 12 a.m.

World Time Difference

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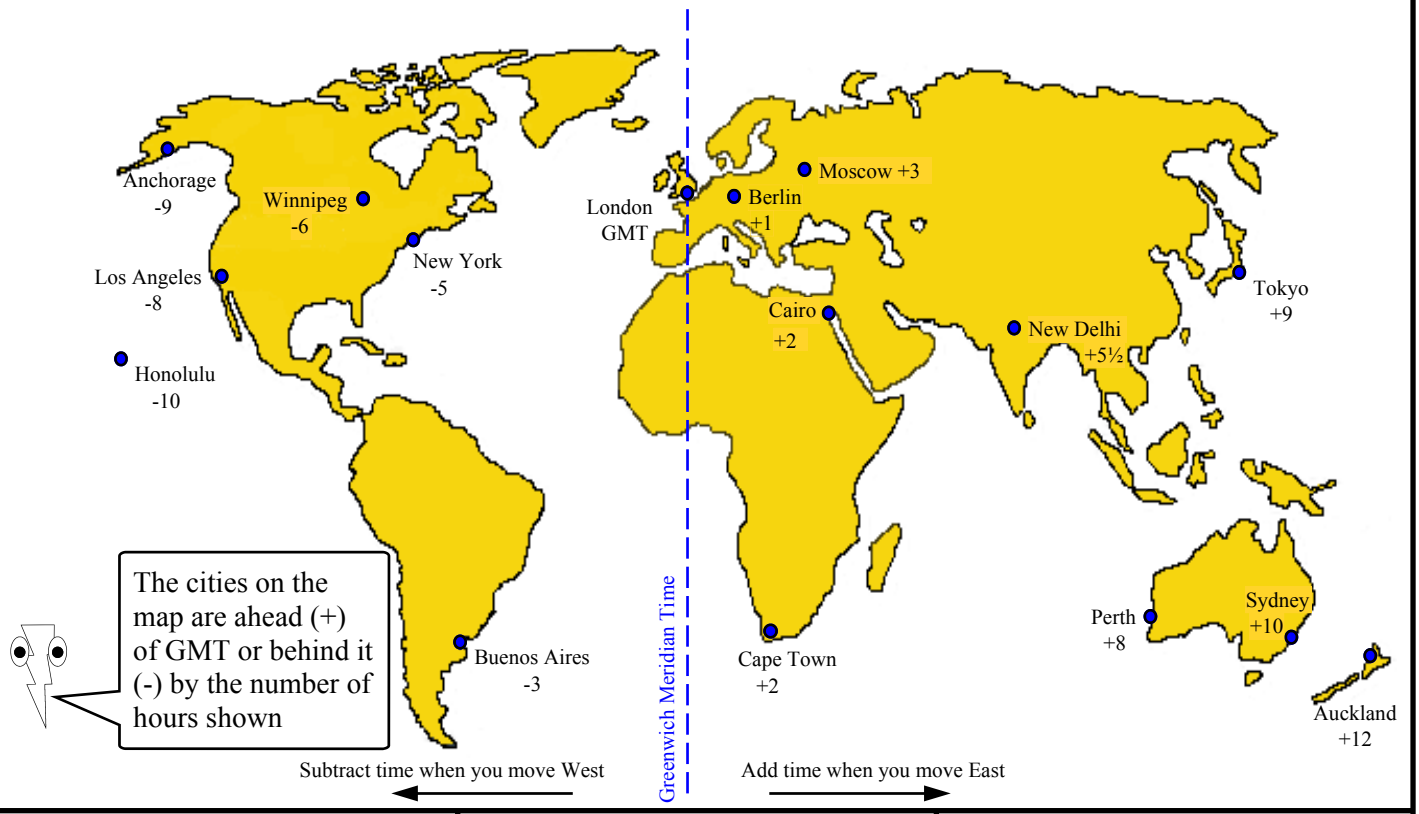


If it is 10 a.m. in London calculate the local time in the cities below (am/pm)

1 Cairo _____

2 Sydney _____

3 New York _____

4 Honolulu _____

5 Buenos Aires _____

6 Moscow _____

7 New Delhi _____

8 Perth _____

9 Los Angeles _____

10 Winnipeg _____

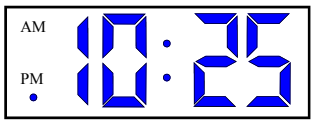
11 Cape Town _____

If it is 10 a.m. on Friday 30/04/02 in New York calculate the local time (am/pm), day and date in the cities below

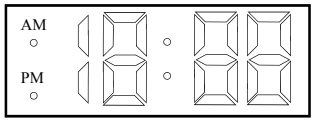
Example		
Anchorage	6.00 a.m.	
Friday	30/04/02	
12 New Delhi		
13 Honolulu		
14 Berlin		
15 Sydney		
16 Auckland		

A company in Tokyo displays times of cities on digital clocks, show these times on the clocks below.

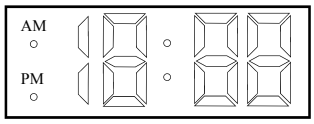
Tokyo local time:



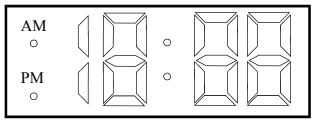
17 Sydney Local Time



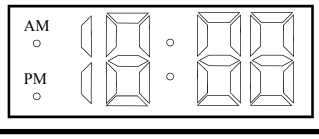
18 London (GMT) Local Time



19 New Delhi Local Time



20 Los Angeles Local Time



World Time Difference

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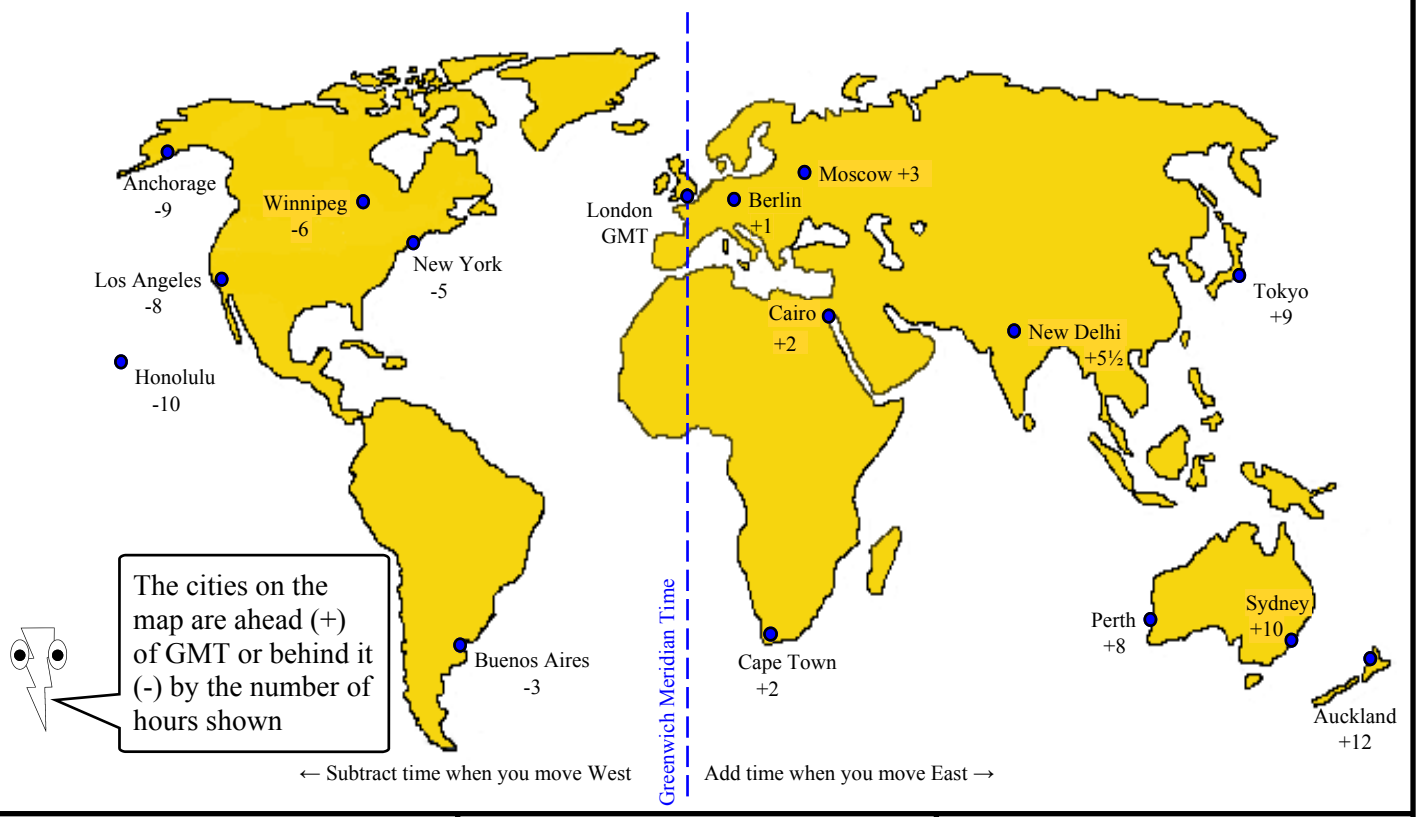


If it is 10 a.m. in London calculate the local time in the cities below (am/pm)

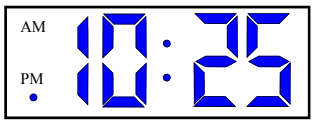
1	Cairo	12 p.m.
2	Sydney	8 p.m.
3	New York	5 a.m.
4	Honolulu	12 a.m.
5	Buenos Aires	7 a.m.
6	Moscow	1 p.m.
7	New Delhi	3.30 p.m.
8	Perth	6 p.m.
9	Los Angeles	2 a.m.
10	Winnipeg	4 a.m.
11	Cape Town	12 p.m.

If it is 10 a.m. on Friday 30/04/02 in New York calculate the local time (am/pm), day and date in the cities below

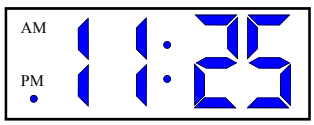
Example			
	Anchorage	9 a.m.	
		Friday	30/04/02
12	New Delhi	8.30 p.m.	
		Friday	30/04/02
13	Honolulu	5 a.m.	
		Friday	30/04/02
14	Berlin	4 p.m.	
		Friday	30/04/02
15	Sydney	1 a.m.	
		Saturday	01/05/02
16	Auckland	3 a.m.	
		Saturday	01/05/02

A company in Tokyo displays times of cities on digital clocks, show these times on the clocks below.

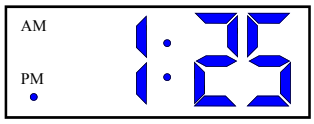
Tokyo local time:



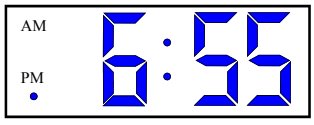
17 Sydney Local Time



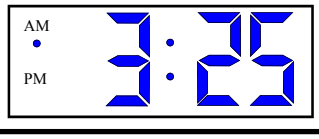
18 London (GMT) Local Time



19 New Delhi Local Time



20 Los Angeles Local Time



Time - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Time Find A Word

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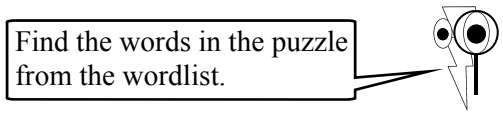

E	L	B	A	T	E	M	I	T	G	D	D	H	A	M
N	S	N	S	O	C	S	G	N	I	N	E	V	E	D
E	C	N	A	V	D	A	D	G	S	M	P	N	C	S
U	H	T	N	O	M	N	I	U	O	U	A	O	A	T
G	E	N	N	T	D	T	P	R	D	U	R	S	D	L
O	D	S	S	E	A	J	N	Q	A	L	T	E	I	I
L	U	E	E	L	M	I	N	U	T	E	S	V	F	N
A	L	Y	T	E	N	T	E	A	E	N	R	I	F	O
N	E	E	B	G	E	U	N	E	O	M	T	R	E	O
A	T	A	S	R	C	A	R	I	K	D	O	R	R	N
T	S	R	U	O	H	S	T	L	O	L	A	A	E	R
U	N	I	I	E	A	A	R	I	E	P	C	B	N	E
C	E	N	T	U	R	Y	D	R	M	A	P	D	C	T
P	G	A	I	U	P	V	N	O	L	E	E	A	E	F
F	L	L	D	U	D	E	C	A	D	E	A	Y	R	A

WORDLIST

TIME	DURATION	DIFFERENCE	DAY	EVENING
HOURS	DIGITAL	CENTURY	APPOINTMENT	TIMETABLE
ADVANCE	DATE	AFTERNOON	COMPARE	DECADE
ARRIVES	YEAR	ANALOGUE	MINUTES	SCHEDULE
SECOND	MORNING	DEPARTS	MONTH	LATE

Time Find A Word

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Find the words in the puzzle from the wordlist.



E	L	B	A	T	E	M	I	T	G	D	D	H	A	M
N	S	N	S	O	C	S	G	N	I	N	E	V	E	D
E	C	N	A	V	D	A	D	G	O	M	P	N	C	S
U	H	T	N	O	M	N	I	C	O	U	A	O	A	T
G	E	N	N	T	D	T	E	R	D	U	R	S	D	L
O	D	S	S	E	A	S	N	Q	A	L	T	E	I	I
L	U	E	E	L	M	I	N	U	T	E	S	V	F	N
A	L	Y	T	E	N	T	E	A	E	N	R	I	F	O
N	E	E	B	G	E	U	N	E	O	M	T	R	E	O
A	T	A	S	R	C	A	R	I	K	D	O	R	R	N
T	S	R	U	O	H	S	T	L	O	L	A	A	E	R
U	N	I	I	E	A	A	R	I	E	P	C	B	N	E
C	E	N	T	U	R	Y	D	R	M	A	P	D	C	T
P	G	A	I	U	P	V	N	O	L	E	E	A	E	F
F	L	L	D	U	D	E	C	A	D	E	A	Y	R	A

WORDLIST

~~TIME~~
~~HOURS~~
~~ADVANCE~~
~~ARRIVES~~
~~SECOND~~

~~DURATION~~
~~DIGITAL~~
~~DATE~~
~~YEAR~~
~~MORNING~~

~~DIFFERENCE~~
~~CENTURY~~
~~AFTERNOON~~
~~ANALOGUE~~
~~DEPARTS~~

~~DAY~~
~~APPOINTMENT~~
~~COMPARE~~
~~MINUTES~~
~~MONTH~~

~~EVENING~~
~~TIMETABLE~~
~~DECADE~~
~~SCHEDULE~~
~~LATE~~

FREEFALL MATHEMATICS

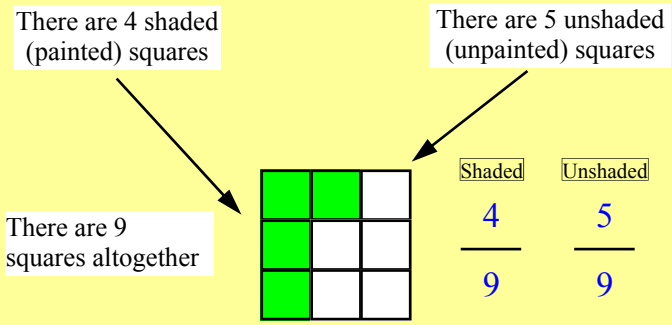
ALTITUDE

FRACTIONS

Fractions of Shapes

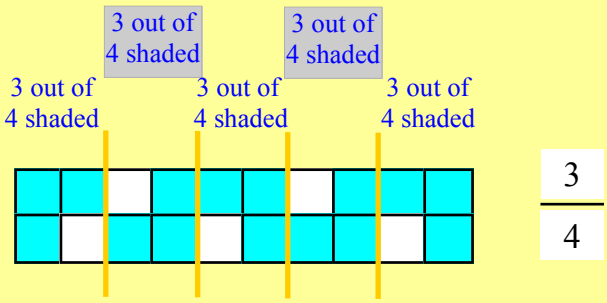
Fractions deal with parts of a number or item. The most important thing to remember is: 'take ____ parts from _____' meaning that for the fraction ¾ you take 3 parts from 4.

In column 1 you are asked to find the fraction of the shape that is both shaded and unshaded. Count the shaded (darkened) sections of the diagram and write this in the top box, then count the total number of sections (both shaded and unshaded) and put this number in the bottom box. The same process is done for the unshaded fraction for the question, count the unshaded sections and put that number in the top box then count the total number of sections (which you have already done) and put that in the bottom box. An example is below.



Column 2 reverses the process, now you are given the fraction and you are asked to shade the fraction on the diagram. The denominators all match the number of parts in the shape, so that means you shade the number of parts that match the top number of the fraction.

In Column 3 is the bottom number of the fraction doesn't match the number of sections in the shape. What you do now is remember to 'take ____ parts from _____". In the example below there are 20 squares and you are asked to find ¾ of them, this means 'take 3 parts from 4' which translates to: count off 4 squares and shade 3 of them. Then count off another 4 squares and shade 3 more and keep going until the you count off the last 4. How many 4's are there in 20? There are 5 lots of 4, and as you are shading 3 out of every 4 that means there is 5 lots of 3 sections to be shaded, so expect to shade 15 squares.



Fractions of Shapes

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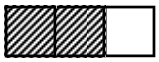
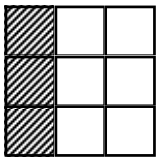
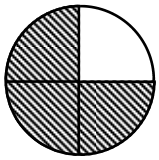
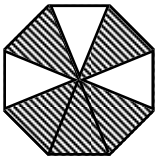
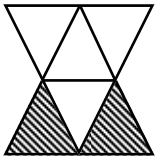
Write the fraction for both the shaded and unshaded parts of the shapes below

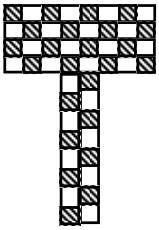
1  Shaded ☐ Unshaded ☐

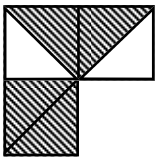
2  Shaded ☐ Unshaded ☐

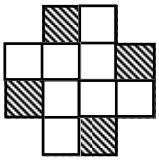
3  Shaded ☐ Unshaded ☐

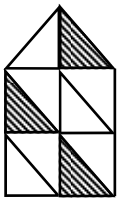
4  Shaded ☐ Unshaded ☐

5  Shaded ☐ Unshaded ☐

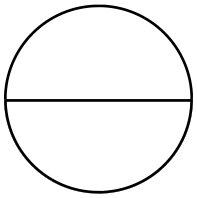
6  Shaded ☐ Unshaded ☐

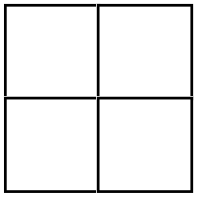
7  Shaded ☐ Unshaded ☐

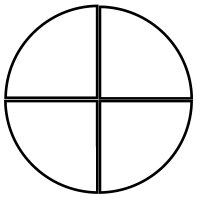
8  Shaded ☐ Unshaded ☐

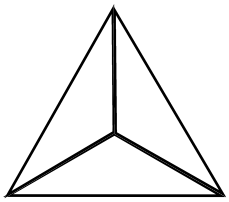
9  Shaded ☐ Unshaded ☐

Shade the following shapes according to the fraction given

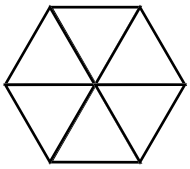
10  $\frac{1}{2}$

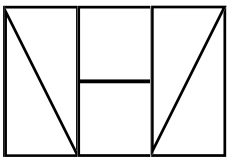
11  $\frac{1}{4}$

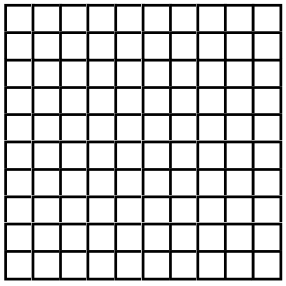
12  $\frac{3}{4}$

13  $\frac{2}{3}$

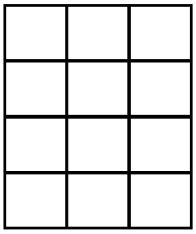
14  $\frac{4}{6}$

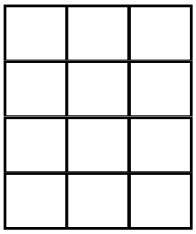
15  $\frac{5}{6}$

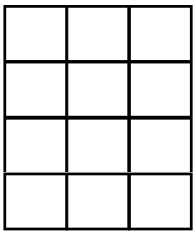
16  $\frac{5}{6}$

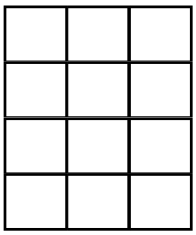
17  $\frac{17}{100}$

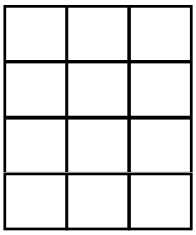
Shade the following shapes according to the fraction given

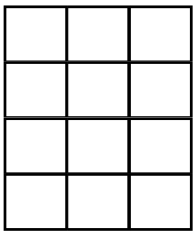
18 $\frac{1}{3}$ 

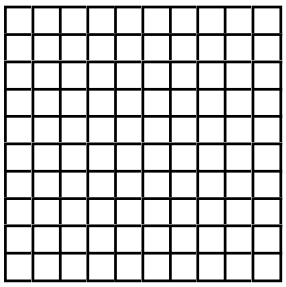
19 $\frac{3}{4}$ 

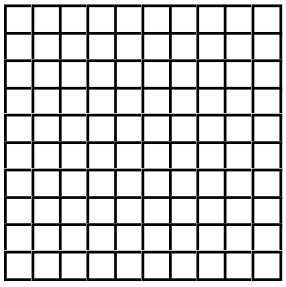
20 $\frac{1}{6}$ 

21 $\frac{2}{3}$ 

22 $\frac{6}{6}$ 

23 $\frac{5}{6}$ 

24  $\frac{5}{20}$

25  $\frac{5}{25}$

Fractions of Shapes

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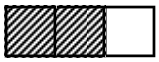
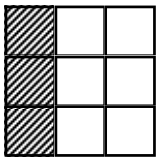
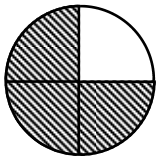
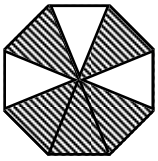
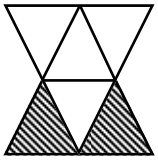
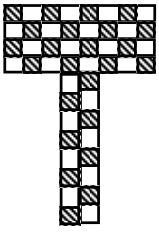
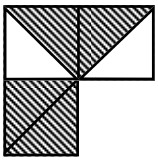
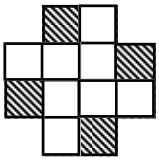
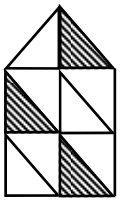
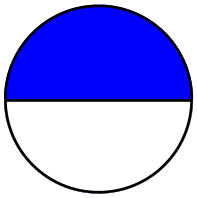
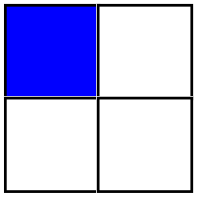
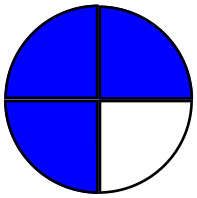
Write the fraction for both the shaded and unshaded parts of the shapes below

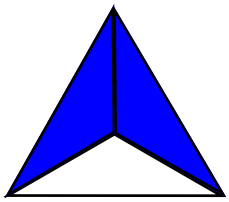
		Shaded	Unshaded
1		2/3	1/3
2		3/9	6/9
3		3/4	1/4
4		5/8	3/8
5		2/6	4/6
6		24/48	24/48
7		4/6	2/6
8		4/12	8/12
9		3/10	7/10

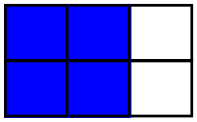
Shade the following shapes according to the fraction given

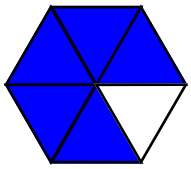
10  1/2

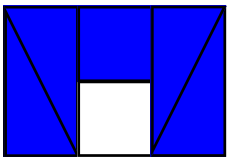
11  1/4

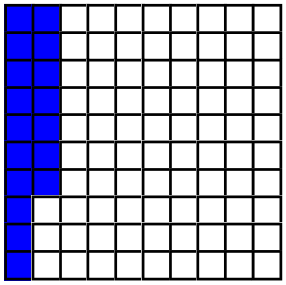
12  3/4

13  2/3

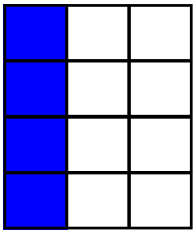
14  4/6

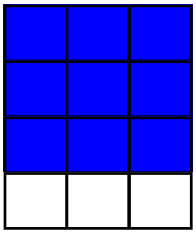
15  5/6

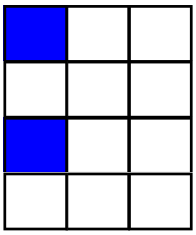
16  5/6

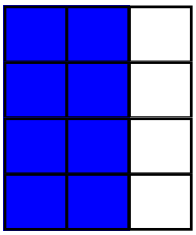
17  17/100

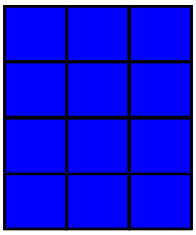
Shade the following shapes according to the fraction given

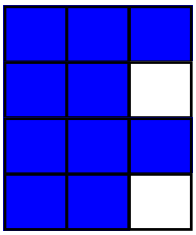
18 1/3 

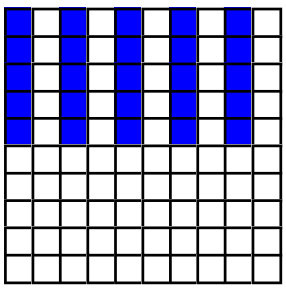
19 3/4 

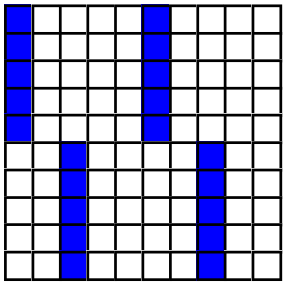
20 1/6 

21 2/3 

22 6/6 

23 5/6 

24  5/20

25  5/25

Graphing and Comparing Fractions

This sheet divides a unit length (between 0 and 1) into fractions and asks you to compare the fractions using the same length interval as a guide. As the fractions are placed directly below each other, the fractions can be compared more easily.

Column 1 asks you to fill in the missing spaces, the first interval is in two halves, the next in three thirds and so on. The denominator (the bottom number) stays the same on each line, the numerator (the top line) counts off starting at 1. Once you have filled the spaces answer the exercises. If a fraction is to the left of another it is less than it (<), if it is to the right it is greater than it (>) and if it is in the same position it is equal (=) to it. For some fractions that are close it may be difficult to tell with your eye, a ruler may need to be used.

With column 2 you are given <, > and = signs. You have to place in the numbers that make the statement true. Sometimes more than one answer will be possible, if that is the case use the first fraction that is less than or greater than. Questions 25 through 32 ask you to answer true or false, again use a ruler if the fractions are difficult to separate with your eye.

Column 3 asks you to arrange the fractions in descending and later ascending order. Remember that descending starts with a 'd' just like the word 'down'. When you go down you go from a high position to a lower position, that means that descending order starts with the highest (largest) fraction and ends with the lowest (smallest) fraction. How do you remember ascending? "Who cares!" Because if you know descending goes from highest to lowest then ascending must be the opposite, lowest to highest. It may make it easier if you strike out the fractions on the top line as you put them in order in the line below.

Remember :

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7}$$

Graphing and Comparing Fractions

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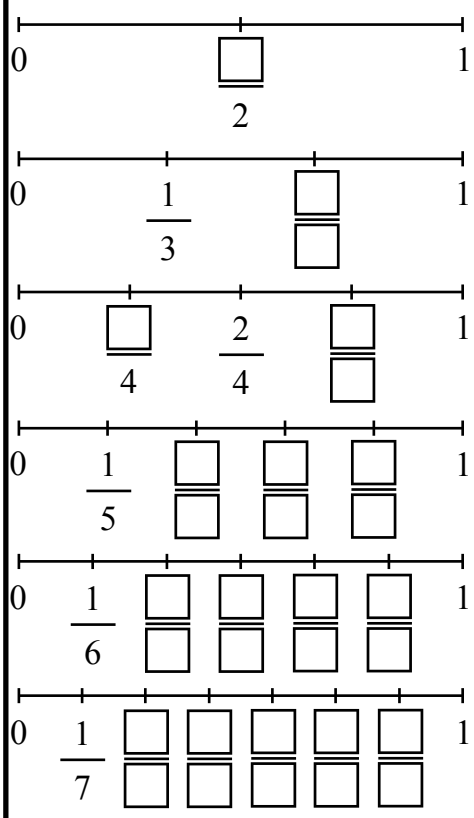
Complete the fractions which divide the intervals below.



Now using the intervals above (and a ruler?), answer the questions.

Fill the box with <, >, or = to make these true.

1 $\frac{2}{3} \square \frac{1}{3}$ 2 $\frac{5}{7} \square \frac{6}{7}$

3 $\frac{3}{4} \square \frac{1}{4}$ 4 $\frac{2}{4} \square \frac{2}{3}$

5 $\frac{1}{3} \square \frac{2}{6}$ 6 $\frac{3}{5} \square \frac{4}{6}$

7 $\frac{4}{7} \square \frac{1}{2}$ 8 $\frac{6}{7} \square \frac{5}{6}$

9 $\frac{2}{3} \square \frac{4}{6}$ 10 $\frac{3}{6} \square \frac{1}{2}$

11 $\frac{2}{5} \square \frac{3}{7}$ 12 $\frac{1}{2} \square \frac{2}{4}$

Find the closest number that makes these true.

13 $\frac{\square}{3} < \frac{2}{5}$ 14 $\frac{\square}{4} > \frac{5}{7}$

15 $\frac{2}{5} < \frac{\square}{3}$ 16 $\frac{2}{3} = \frac{4}{\square}$

17 $\frac{3}{4} < \frac{\square}{6}$ 18 $\frac{3}{\square} > \frac{1}{2}$

19 $\frac{1}{2} > \frac{\square}{7}$ 20 $\frac{3}{5} < \frac{2}{\square}$

21 $\frac{\square}{6} = \frac{\square}{2}$ 22 $\frac{1}{\square} = \frac{\square}{4}$

23 $\frac{\square}{5} > \frac{5}{7}$ 24 $\frac{3}{\square} = \frac{\square}{4}$

State if the following are true or false.

25 $\frac{1}{4} < \frac{1}{7}$ _____

26 $\frac{1}{2} > \frac{3}{7}$ _____

27 $\frac{3}{4} = \frac{4}{6}$ _____

28 $\frac{2}{7} > \frac{1}{4}$ _____

29 $\frac{4}{4} > \frac{3}{3}$ _____

30 $\frac{2}{4} = \frac{3}{6}$ _____

31 $\frac{4}{5} < \frac{4}{7}$ _____

32 $\frac{1}{3} > \frac{2}{6}$ _____

Arrange these fractions in descending order.

33 $\frac{2}{7}, \frac{5}{7}, \frac{3}{7}, \frac{6}{7}$

Rewrite in order → $\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$

34 $\frac{1}{5}, \frac{3}{6}, \frac{4}{5}, \frac{4}{6}$

Rewrite in order → $\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$

35 $\frac{1}{3}, \frac{1}{6}, \frac{3}{5}, \frac{2}{4}$

Rewrite in order → $\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$

36 $\frac{5}{7}, \frac{1}{2}, \frac{2}{3}, \frac{1}{4}$

Rewrite in order → $\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$

Arrange these fractions in ascending order.

37 $\frac{3}{4}, \frac{3}{5}, \frac{6}{7}, \frac{1}{2}$

Rewrite in order → $\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$

38 $\frac{4}{6}, \frac{1}{2}, \frac{3}{7}, \frac{1}{5}$

Rewrite in order → $\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$

39 $\frac{6}{7}, \frac{1}{3}, \frac{3}{5}, \frac{2}{4}$

Rewrite in order → $\frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}, \frac{\square}{\square}$

Graphing and Comparing Fractions

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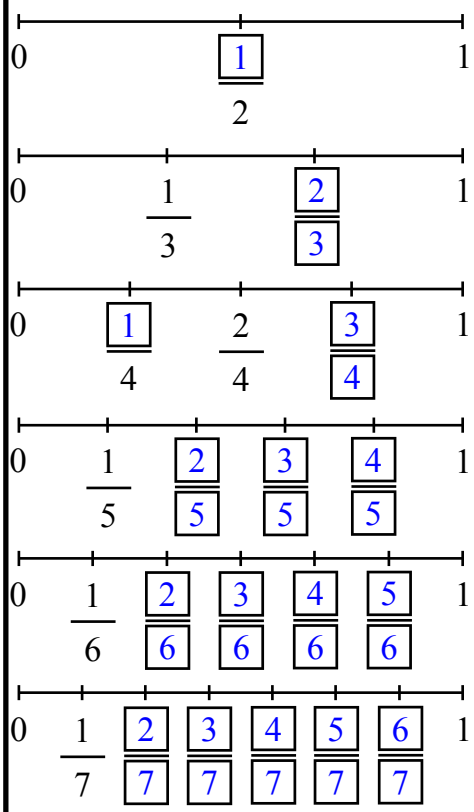
Complete the fractions which divide the intervals below.



Now using the intervals above (and a ruler?), answer the questions.

Fill the box with <, >, or = to make these true.

1. $\frac{2}{3} > \frac{1}{3}$
2. $\frac{5}{7} < \frac{6}{7}$
3. $\frac{3}{4} > \frac{1}{4}$
4. $\frac{2}{4} < \frac{2}{3}$
5. $\frac{1}{3} = \frac{2}{6}$
6. $\frac{3}{5} < \frac{4}{6}$
7. $\frac{4}{7} > \frac{1}{2}$
8. $\frac{6}{7} > \frac{5}{6}$
9. $\frac{2}{3} = \frac{4}{6}$
10. $\frac{3}{6} = \frac{1}{2}$
11. $\frac{2}{5} < \frac{3}{7}$
12. $\frac{1}{2} = \frac{2}{4}$

Find the closest number that makes these true.

13. $\frac{1}{3} < \frac{2}{5}$
14. $\frac{3}{4} > \frac{5}{7}$
15. $\frac{2}{5} < \frac{2}{3}$
16. $\frac{2}{3} = \frac{4}{6}$
17. $\frac{3}{4} < \frac{5}{6}$
18. $\frac{3}{5} > \frac{1}{2}$
19. $\frac{1}{2} > \frac{3}{7}$
20. $\frac{3}{5} < \frac{2}{3}$
21. $\frac{3}{6} = \frac{1}{2}$
22. $\frac{1}{2} = \frac{2}{4}$
23. $\frac{4}{5} > \frac{5}{7}$
24. $\frac{3}{6} = \frac{2}{4}$

State if the following are true or false.

25. $\frac{1}{4} < \frac{1}{7}$ False
26. $\frac{1}{2} > \frac{3}{7}$ True
27. $\frac{3}{4} = \frac{4}{6}$ False
28. $\frac{2}{7} > \frac{1}{4}$ True
29. $\frac{4}{4} > \frac{3}{3}$ False
30. $\frac{2}{4} = \frac{3}{6}$ True
31. $\frac{4}{5} < \frac{4}{7}$ False
32. $\frac{1}{3} > \frac{2}{6}$ False

Arrange these fractions in descending order.

33. $\frac{2}{7}, \frac{5}{7}, \frac{3}{7}, \frac{6}{7}$
Rewrite in order → $\frac{6}{7}, \frac{5}{7}, \frac{3}{7}, \frac{2}{7}$
34. $\frac{1}{5}, \frac{3}{6}, \frac{4}{5}, \frac{4}{6}$
Rewrite in order → $\frac{4}{5}, \frac{4}{6}, \frac{3}{6}, \frac{1}{5}$
35. $\frac{1}{3}, \frac{1}{6}, \frac{3}{5}, \frac{2}{4}$
Rewrite in order → $\frac{3}{5}, \frac{2}{4}, \frac{1}{3}, \frac{1}{6}$
36. $\frac{5}{7}, \frac{1}{2}, \frac{2}{3}, \frac{1}{4}$
Rewrite in order → $\frac{5}{7}, \frac{2}{3}, \frac{1}{2}, \frac{1}{4}$

Arrange these fractions in ascending order.

37. $\frac{3}{4}, \frac{3}{5}, \frac{6}{7}, \frac{1}{2}$
Rewrite in order → $\frac{1}{2}, \frac{3}{5}, \frac{3}{4}, \frac{6}{7}$
38. $\frac{4}{6}, \frac{1}{2}, \frac{3}{7}, \frac{1}{5}$
Rewrite in order → $\frac{1}{5}, \frac{3}{7}, \frac{1}{2}, \frac{4}{6}$
39. $\frac{6}{7}, \frac{1}{3}, \frac{3}{5}, \frac{2}{4}$
Rewrite in order → $\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{6}{7}$

Types of Fractions

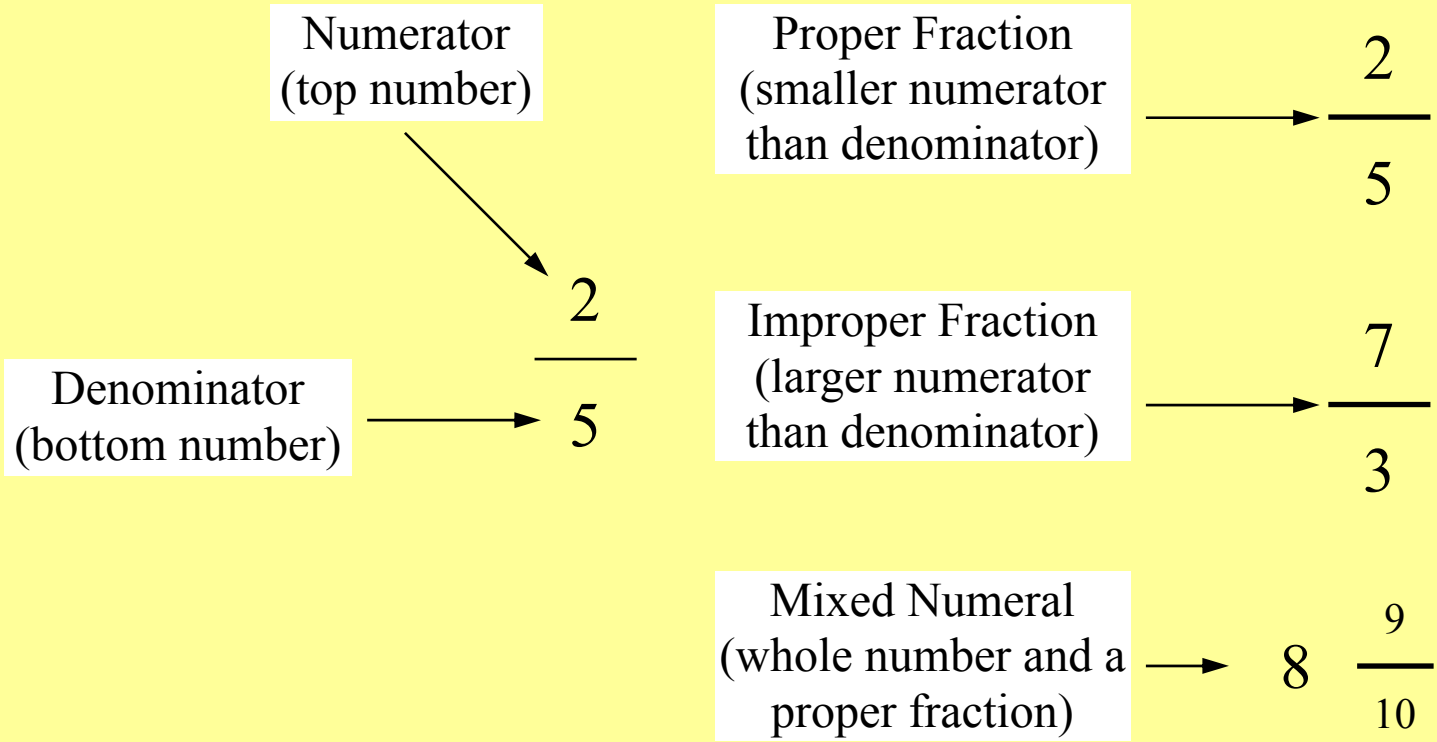
A fraction is composed of one number ‘over’ or ‘on top of’ another. The number on top is called the numerator, the number on the bottom is called the denominator. How do you remember these? Remember that denominator starts with a 'd' as does the word 'down'. So you should remember that the denominator is down below and the numerator is up on top.

Column 1 asks you to write the fraction that has been spelt out then pick the numerator and the denominator and put these in the boxes. Note that the boxes change their position and so don't write the numerator in the denominator box. Questions 9 through 13 are the reverse, this tests your spelling. You can look at the spelling in the questions above to assist you.

Column 2 introduces proper fractions, improper fractions and mixed numerals (or mixed numbers). A proper fraction is 'the way you would expect a fraction to be' with the numerator smaller than the denominator, (the number on top smaller than the number below it). An improper fraction is different because the numerator is larger than the denominator, and a mixed numeral has a whole number and a proper fraction together, so it's mixed up!

Note that because improper fractions have a larger top number they will have a value that is greater than 1. Questions 24, 25 and 26 ask you to build fractions using the numbers 2, 4 and 7. The idea being that with the mixed numbers you use 3 different numbers not one number twice.

The third column has some worded questions about fractions that test your knowledge of them, then the last 3 questions ask you to sort through the fractions at the bottom of the column and rewrite them below the type of fraction that they are.



Types of Fractions

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Write the fraction given in words as numbers and put the numerator and the denominator in the spaces

Example

six sevenths $\frac{6}{7}$ (num 6, denom 7)

1 a half — (denom, num)

2 two quarters — (num, denom)

3 three eighths — (num, denom)

4 seven tenths — (denom, num)

5 ten twelfths — (denom, num)

6 five sixths — (num, denom)

7 ten thirds — (denom, num)

8 six fourths — (num, denom)

Now write the fractions given below in words

9 $\frac{5}{7}$ _____

10 $\frac{3}{12}$ _____

11 $\frac{5}{20}$ _____

12 A fraction has a numerator of 5 and a denominator of 8

_____

13 A fraction has a numerator of 9 and a denominator of 4

_____

Describe the fractions below as either proper, improper or mixed

14 $\frac{3}{11}$ _____

15 $4\frac{5}{6}$ _____

16 $7\frac{1}{2}$ _____

17 $\frac{88}{89}$ _____

18 $\frac{43}{34}$ _____

19 $6\frac{8}{10}$ _____

20 $\frac{11}{7}$ _____

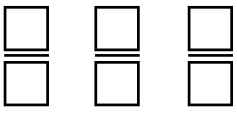
21 $12\frac{5}{6}$ _____

22 $\frac{12}{13}$ _____

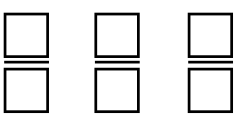
23 $\frac{56}{13}$ _____

Use the numbers 2, 4, or 7 to construct:

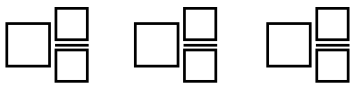
24 3 proper fractions



25 3 improper fractions



26 3 mixed numerals



Name the type or part of a fraction that is being described below.

27 This fraction has a denominator larger than its numerator

_____

28 This fraction can have a value greater than 1

_____

29 This has a proper fraction and a whole number.

_____

30 An answer of 1 occurs when the denominator equals this.

_____

From the fractions below select the:

31 Proper fractions

32 Improper fractions

33 Mixed numerals

$\frac{3}{5}$ $2\frac{3}{7}$ $\frac{67}{63}$ $\frac{11}{3}$ $6\frac{2}{5}$ $\frac{55}{56}$ $\frac{12}{5}$ $8\frac{9}{10}$ $\frac{13}{19}$ $\frac{17}{12}$

Types of Fractions

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Write the fraction given in words as numbers and put the numerator and the denominator in the spaces

Example

six sevenths $\frac{6}{7}$ (num 6, denom 7)

1 a half $\frac{1}{2}$ (denom 2, num 1)

2 two quarters $\frac{2}{4}$ (num 2, denom 4)

3 three eighths $\frac{3}{8}$ (num 3, denom 8)

4 seven tenths $\frac{7}{10}$ (denom 10, num 7)

5 ten twelfths $\frac{10}{12}$ (denom 12, num 10)

6 five sixths $\frac{5}{6}$ (num 5, denom 6)

7 ten thirds $\frac{10}{3}$ (denom 3, num 10)

8 six fourths $\frac{6}{4}$ (num 6, denom 4)

Now write the fractions given below in words

9 $\frac{5}{7}$ five sevenths

10 $\frac{3}{12}$ three twelfths

11 $\frac{5}{20}$ five twentieths

12 A fraction has a numerator of 5 and a denominator of 8

five eighths

13 A fraction has a numerator of 9 and a denominator of 4

nine quarters (or fourths)

Describe the fractions below as either proper, improper or mixed

14 $\frac{3}{11}$ Proper

15 $4\frac{5}{6}$ Mixed

16 $7\frac{1}{2}$ Mixed

17 $\frac{88}{89}$ Proper

18 $\frac{43}{34}$ Improper

19 $6\frac{8}{10}$ Mixed

20 $\frac{11}{7}$ Improper

21 $12\frac{5}{6}$ Mixed

22 $\frac{12}{13}$ Proper

23 $\frac{56}{13}$ Improper

Use the numbers 2, 4, or 7 to construct:

24 3 proper fractions

$\frac{2}{4}$ $\frac{2}{7}$ $\frac{4}{7}$

25 3 improper fractions

$\frac{7}{2}$ $\frac{7}{4}$ $\frac{4}{2}$

26 3 mixed numerals

$7\frac{2}{4}$ $4\frac{2}{7}$ $2\frac{4}{7}$

Name the type or part of a fraction that is being described below.

27 This fraction has a denominator larger than its numerator

Proper fraction

28 This fraction can have a value greater than 1

Improper fraction

29 This has a proper fraction and a whole number.

Mixed Numeral

30 An answer of 1 occurs when the denominator equals this.

numerator

From the fractions below select the:

31 Proper fractions

$\frac{3}{5}$ $\frac{55}{56}$ $\frac{13}{19}$

32 Improper fractions

$\frac{12}{5}$ $\frac{67}{63}$ $\frac{17}{12}$ $\frac{11}{3}$

33 Mixed numerals

$8\frac{9}{10}$ $6\frac{2}{5}$ $2\frac{3}{7}$

Converting between Mixed Numerals and Improper Fractions

Changing between mixed numerals and improper fractions is a skill that must be developed for operations with fractions (+, -, ×, ÷). The first column is an introduction to the process of changing mixed numerals to improper fractions. Questions 1 to 3 are connected and then Questions 4 to 6 are connected. This is to help show you how the action of converting fractions is done. It is important that you realise that $1 = {}^{2}/_{2} = {}^{3}/_{3} = {}^{4}/_{4} = {}^{5}/_{5} = {}^{6}/_{6}$ etc.

Questions 7 through 9 are a graphical representation of mixed numerals, with each separate shape being equal to 1. This means that if the shape is split into 3 parts then it is in ${}^{1}/_{3}$'s. In the example below there are two shapes fully shaded that means 2, then there are 5 triangles shaded in the third shape which has 6 parts, that means ${}^{5}/_{6}$. So we have $2{}^{5}/_{6}$ as the mixed numeral. To write it as an improper fraction look at the shape, it is in 6 parts, so in all we have ${}^{?}/_{6}$. Count all the parts up, the first shape has 6 parts, the next another 6, that's 12, then the 5 makes 17. So the improper fraction would be ${}^{17}/_{6}$.

$\frac{6}{6} = \boxed{1}$ $\frac{6}{6} = \boxed{1}$ $\frac{5}{6}$



Mixed Numeral = $2\frac{5}{6}$ Improper Fraction = $\frac{17}{6}$

The entire 2nd column is about converting mixed numerals to improper fractions. The top of the column shows you how this is done. In words the process is as follows:

- write the denominator for the mixed numeral the same as the denominator in the improper fraction
- multiply the denominator of the mixed numeral by the whole number then add it to the numerator
- write this number as the numerator, and you're done

Converting improper to mixed is just like dividing, column 3 shows the method with an example, but in words:

- write the same denominator in the improper fraction as given in the mixed number
- divide the numerator by the denominator (it must 'go into it' as it is an improper fraction) the answer is the whole number in the mixed numeral, the remainder becomes the numerator.

Converting between Mixed Numerals and Improper Fractions

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Try these, remember that fractions are equal parts of a whole number

1 How many ½ 's are there in 1?

2 How many ½ 's are there in a ½?

3 So how many ½ 's are there in 1½?

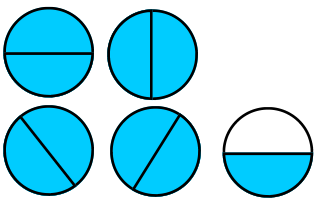
4 How many ¼ 's are there in 1?

5 How many ¼ 's are there in ¾ ?

6 So how many ¼ 's are there in 1¾?

Write the shaded area as a mixed numeral and an improper fraction

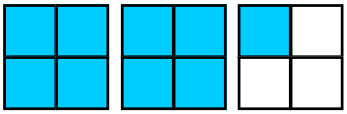
Example



$4\frac{1}{2}$

$\frac{9}{2}$

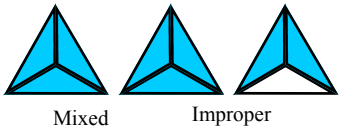
7



Mixed

Improper

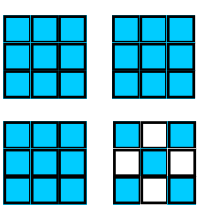
8



Mixed

Improper

9

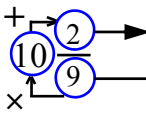


Mixed

Improper

Fill in the missing spaces then find the improper fraction for these.

Example



$\frac{9 \times 10 + 2}{9} = \frac{92}{9}$

10 $2\frac{2}{3} = \frac{\square \times \square + \square}{\square} = \frac{\square}{\square}$

11 $4\frac{3}{5} = \frac{\square \times \square + \square}{\square} = \frac{\square}{\square}$

12 $5\frac{5}{7} = \frac{\square \times \square + \square}{\square} = \frac{\square}{\square}$

13 $6\frac{7}{9} = \frac{\square \times \square + \square}{\square} = \frac{\square}{\square}$

14 $10\frac{3}{8} = \frac{\square \times \square + \square}{\square} = \frac{\square}{\square}$

Now change these to improper fractions without working out spaces

15 $2\frac{3}{4} = \frac{\square}{\square}$

16 $3\frac{3}{5} = \frac{\square}{\square}$

17 $1\frac{6}{7} = \frac{\square}{\square}$

18 $4\frac{3}{4} = \frac{\square}{\square}$

19 $8\frac{2}{3} = \frac{\square}{\square}$

20 $6\frac{2}{5} = \frac{\square}{\square}$

21 $4\frac{9}{10} = \frac{\square}{\square}$

22 $8\frac{2}{7} = \frac{\square}{\square}$

23 $3\frac{5}{6} = \frac{\square}{\square}$

24 $6\frac{3}{8} = \frac{\square}{\square}$

25 $2\frac{6}{13} = \frac{\square}{\square}$

26 $4\frac{3}{7} = \frac{\square}{\square}$

27 $2\frac{5}{16} = \frac{\square}{\square}$

28 $3\frac{7}{20} = \frac{\square}{\square}$

Follow the arrows and put your answer in the square

29 How many 3 's go into 8?

30 What is the remainder?

$\frac{8}{3} = \square\ 3$

31 How many 2 's go into 9?

32 What is the remainder?

$\frac{9}{2} = \square$

33 How many 4 's go into 19?

34 What is the remainder?

$\frac{19}{4} = \square$

Now try converting improper fractions to mixed numerals.

35 $\frac{5}{3} = \square\frac{\square}{\square}$

36 $\frac{7}{2} = \square\frac{\square}{\square}$

37 $\frac{11}{4} = \square\frac{\square}{\square}$

38 $\frac{22}{5} = \square\frac{\square}{\square}$

39 $\frac{17}{2} = \square\frac{\square}{\square}$

40 $\frac{21}{4} = \square\frac{\square}{\square}$

41 $\frac{19}{6} = \square\frac{\square}{\square}$

42 $\frac{25}{8} = \square\frac{\square}{\square}$

43 $\frac{33}{10} = \square\frac{\square}{\square}$

44 $\frac{38}{9} = \square\frac{\square}{\square}$

45 $\frac{41}{4} = \square\frac{\square}{\square}$

46 $\frac{37}{8} = \square\frac{\square}{\square}$

47 $\frac{47}{12} = \square\frac{\square}{\square}$

48 $\frac{60}{11} = \square\frac{\square}{\square}$

49 $\frac{55}{9} = \square\frac{\square}{\square}$

50 $\frac{60}{19} = \square\frac{\square}{\square}$

Converting between Mixed Numerals and Improper Fractions

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Try these, remember that fractions are equal parts of a whole number

1 How many ½ 's are there in 1? 2

2 How many ½ 's are there in a ½? 1

3 So how many ½ 's are there in 1½? 3

4 How many ¼ 's are there in 1? 4

5 How many ¼ 's are there in ¾ ? 3

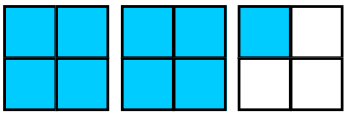
6 So how many ¼ 's are there in 1¾? 7

Write the shaded area as a mixed numeral and an improper fraction

Example

$4\frac{1}{2}$ $\frac{9}{2}$

7



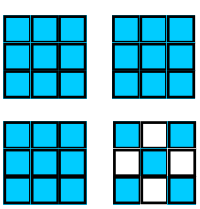
Mixed $2\frac{1}{4}$ Improper $\frac{9}{4}$

8



Mixed $2\frac{2}{3}$ Improper $\frac{8}{3}$

9



Mixed $3\frac{5}{9}$ Improper $\frac{32}{9}$

Fill in the missing spaces then find the improper fraction for these.

Example

$+$ 2, 10, $\times$ 9 → $\frac{9 \times 10 + 2}{9} = \frac{92}{9}$

10 $2\frac{2}{3} = \frac{3 \times 2 + 2}{3} = \frac{8}{3}$

11 $4\frac{3}{5} = \frac{5 \times 4 + 3}{5} = \frac{23}{5}$

12 $5\frac{5}{7} = \frac{7 \times 5 + 5}{7} = \frac{40}{7}$

13 $6\frac{7}{9} = \frac{9 \times 6 + 7}{9} = \frac{61}{9}$

14 $10\frac{3}{8} = \frac{8 \times 10 + 3}{8} = \frac{83}{8}$

Now change these to improper fractions without working out spaces

15 $2\frac{3}{4} = \frac{11}{4}$ 16 $3\frac{3}{5} = \frac{18}{5}$

17 $1\frac{6}{7} = \frac{13}{7}$ 18 $4\frac{3}{4} = \frac{19}{4}$

19 $8\frac{2}{3} = \frac{26}{3}$ 20 $6\frac{2}{5} = \frac{32}{5}$

21 $4\frac{9}{10} = \frac{49}{10}$ 22 $8\frac{2}{7} = \frac{58}{7}$

23 $3\frac{5}{6} = \frac{23}{6}$ 24 $6\frac{3}{8} = \frac{51}{8}$

25 $2\frac{6}{13} = \frac{32}{13}$ 26 $4\frac{3}{7} = \frac{31}{7}$

27 $2\frac{5}{16} = \frac{37}{16}$ 28 $3\frac{7}{20} = \frac{67}{20}$

Follow the arrows and put your answer in the square

29 How many 3 's go into 8?

30 What is the remainder?

$\frac{8}{3} = 2\frac{2}{3}$

31 How many 2 's go into 9?

32 What is the remainder?

$\frac{9}{2} = 4\frac{1}{2}$

33 How many 4 's go into 19?

34 What is the remainder?

$\frac{19}{4} = 4\frac{3}{4}$

Now try converting improper fractions to mixed numerals.

35 $\frac{5}{3} = 1\frac{2}{3}$ 36 $\frac{7}{2} = 3\frac{1}{2}$

37 $\frac{11}{4} = 2\frac{3}{4}$ 38 $\frac{22}{5} = 4\frac{2}{5}$

39 $\frac{17}{2} = 8\frac{1}{2}$ 40 $\frac{21}{4} = 5\frac{1}{4}$

41 $\frac{19}{6} = 3\frac{1}{6}$ 42 $\frac{25}{8} = 3\frac{1}{8}$

43 $\frac{33}{10} = 3\frac{3}{10}$ 44 $\frac{38}{9} = 4\frac{2}{9}$

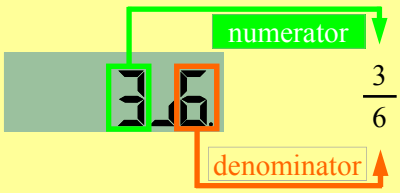
45 $\frac{41}{4} = 10\frac{1}{4}$ 46 $\frac{37}{8} = 4\frac{5}{8}$

47 $\frac{47}{12} = 3\frac{11}{12}$ 48 $\frac{60}{11} = 5\frac{5}{11}$

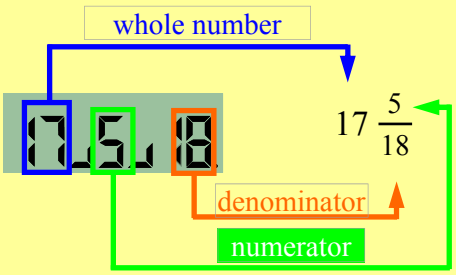
49 $\frac{55}{9} = 6\frac{1}{9}$ 50 $\frac{60}{19} = 3\frac{3}{19}$

Changing Between Mixed Numerals and Improper Fractions (Calculator)

This sheet is designed for you to learn how to change between mixed numerals and improper fractions using your calculator. A calculator separates the numerator and the denominator by a reversed 'L' (or sometimes an ‘r’), for mixed numerals the whole number is also separated by the 'L' (or ‘r’). See example below.



$\frac{3}{6}$



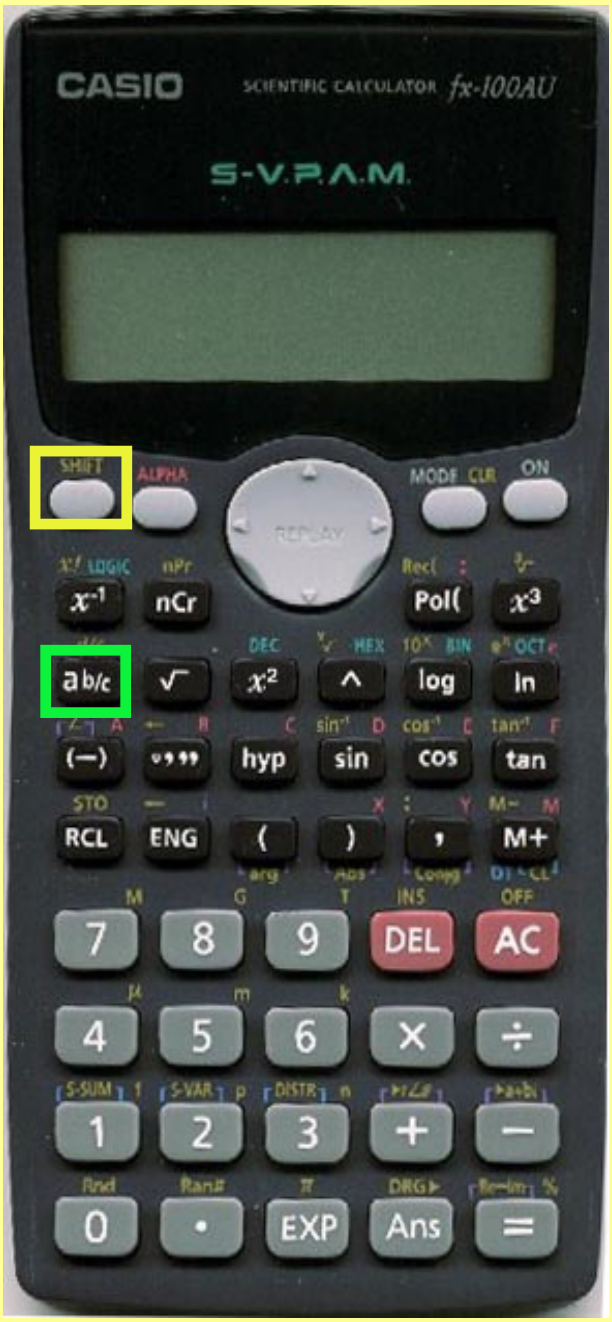
$17\frac{5}{18}$

The first column starts with the way your calculator displays fractions. If there are 2 numbers the first number is the numerator (number on top) the second number is the denominator (number on the bottom). These numbers are separated by the "reversed L". Don’t write fractions the calculator way, write the fractions separated by a line (as above). From Q. 6 on, you are asked to change mixed numerals to improper fractions. The method required is described on the next sheet, in words - type the fraction into the calculator, press =, then press shift and the fraction button.

Column 2 reverses the process and asks you to change improper fractions to mixed numerals, to answer these put the fraction in the calculator and press =, the mixed numeral will be displayed. Question 46 through 53 asks you to repeat the same process, if you use the ÷ sign then you will get a decimal answer, press the fraction key and it will change it to a fraction. Or a quicker step is to use the fraction key instead of the ÷ sign this will give the answer as a fraction straight away.

Note: These are all the same: $8\overline{)3}$ $\frac{3}{8}$ $3 \div 8$

The last column asks you to compare the improper fractions with the mixed numerals. Put the improper fraction into your calculator and press the ‘equals’ key. If the fraction displayed on the screen is the same as the answer supplied write 'true', if it isn't, write ‘false’. At the bottom of the column you are asked to show the pairs that ‘match’ (an improper and a mixed number) by using pairs of shapes (a smaller shape for the improper fractions, larger one for mixed).



Example of how to convert Mixed numerals to Improper Fractions

$3\ \frac{6}{7} = \frac{\quad}{\quad}$?

Don't forget the 'equals' sign

SHIFT

$\boxed{3}\ \boxed{a\,b/c}\ \boxed{6}\ \boxed{a\,b/c}\ \boxed{7}\ \boxed{=}\ \boxed{}\ \boxed{a\,b/c}$

if you get $\frac{27}{7}$ as your answer you have done it correctly

Changing Between Mixed Numerals and Improper Fractions (Calculator)

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Write the fraction represented by the calculator screens

1 

2 

3 

4 

5 

Change these mixed numbers to improper fractions

6 $3\frac{1}{2} = \frac{\square}{\square}$ 7 $1\frac{1}{3} = \frac{\square}{\square}$

8 $2\frac{1}{4} = \frac{\square}{\square}$ 9 $1\frac{1}{2} = \frac{\square}{\square}$

10 $1\frac{2}{5} = \frac{\square}{\square}$ 11 $1\frac{5}{12} = \frac{\square}{\square}$

12 $2\frac{2}{3} = \frac{\square}{\square}$ 13 $2\frac{4}{7} = \frac{\square}{\square}$

14 $2\frac{5}{6} = \frac{\square}{\square}$ 15 $3\frac{1}{4} = \frac{\square}{\square}$

16 $1\frac{3}{4} = \frac{\square}{\square}$ 17 $12\frac{1}{7} = \frac{\square}{\square}$

18 $7\frac{1}{3} = \frac{\square}{\square}$ 19 $3\frac{5}{11} = \frac{\square}{\square}$

20 $3\frac{1}{8} = \frac{\square}{\square}$ 21 $9\frac{2}{3} = \frac{\square}{\square}$

22 $4\frac{4}{5} = \frac{\square}{\square}$ 23 $1\frac{1}{10} = \frac{\square}{\square}$

24 $10\frac{2}{9} = \frac{\square}{\square}$ 25 $11\frac{3}{4} = \frac{\square}{\square}$

Change these improper fractions to mixed numerals

26 $\frac{5}{2} = \square\frac{\square}{\square}$ 27 $\frac{7}{4} = \square\frac{\square}{\square}$

28 $\frac{3}{2} = \square\frac{\square}{\square}$ 29 $\frac{8}{5} = \square\frac{\square}{\square}$

30 $\frac{16}{7} = \square\frac{\square}{\square}$ 31 $\frac{24}{7} = \square\frac{\square}{\square}$

32 $\frac{23}{5} = \square\frac{\square}{\square}$ 33 $\frac{16}{3} = \square\frac{\square}{\square}$

34 $\frac{13}{2} = \square\frac{\square}{\square}$ 35 $\frac{19}{7} = \square\frac{\square}{\square}$

36 $\frac{13}{2} = \square\frac{\square}{\square}$ 37 $\frac{28}{3} = \square\frac{\square}{\square}$

38 $\frac{15}{4} = \square\frac{\square}{\square}$ 39 $\frac{17}{10} = \square\frac{\square}{\square}$

40 $\frac{13}{6} = \square\frac{\square}{\square}$ 41 $\frac{53}{5} = \square\frac{\square}{\square}$

Write the answers as mixed numerals

42 $11 \div 7 = \square\frac{\square}{\square}$

43 $109 \div 10 = \square\frac{\square}{\square}$

44 $3\overline{)125} = \square\frac{\square}{\square}$

45 $20 \div 3 = \square\frac{\square}{\square}$

46 $7\overline{)321} = \square\frac{\square}{\square}$

47 $47 \div 5 = \square\frac{\square}{\square}$

Change improper to mixed to answer true of false to these

48 $\frac{8}{2} = 4\frac{1}{2}$ _____

49 $4\frac{3}{8} = \frac{43}{8}$ _____

50 $7\frac{2}{7} = \frac{51}{7}$ _____

51 $8\frac{5}{9} = \frac{75}{9}$ _____

52 $\frac{11}{6} = 1\frac{5}{6}$ _____

53 $\frac{24}{7} = 3\frac{4}{7}$ _____

54 $1\frac{7}{8} = \frac{15}{8}$ _____

55 $\frac{17}{7} = 2\frac{3}{7}$ _____

56 $\frac{13}{11} = 1\frac{3}{11}$ _____

57 $4\frac{1}{3} = \frac{15}{3}$ _____

Using circles, squares, triangles or other shapes draw the same shape around matching pairs

58

$4\frac{5}{7}$ $\frac{16}{3}$ $5\frac{1}{3}$ $4\frac{3}{7}$ $\frac{33}{7}$ $\frac{22}{7}$ $\frac{17}{3}$ $5\frac{2}{3}$ $6\frac{2}{3}$ $\frac{22}{3}$ $\frac{20}{3}$ $\frac{31}{7}$ $3\frac{1}{7}$ $7\frac{1}{3}$

Changing Between Mixed Numerals and Improper Fractions (Calculator)

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Write the fraction represented by the calculator screens

1 

$\frac{2}{5}$

2 

$\frac{3}{2}$

3 

4 $\frac{8}{9}$

4 

11 $\frac{3}{20}$

5 

10 $\frac{1}{10}$

Change these mixed numbers to improper fractions

6 $3\frac{1}{2} = \frac{7}{2}$

7 $1\frac{1}{3} = \frac{4}{3}$

8 $2\frac{1}{4} = \frac{9}{4}$

9 $1\frac{1}{2} = \frac{3}{2}$

10 $1\frac{2}{5} = \frac{7}{5}$

11 $1\frac{5}{12} = \frac{17}{12}$

12 $2\frac{2}{3} = \frac{8}{3}$

13 $2\frac{4}{7} = \frac{18}{7}$

14 $2\frac{5}{6} = \frac{17}{6}$

15 $3\frac{1}{4} = \frac{13}{4}$

16 $1\frac{3}{4} = \frac{7}{4}$

17 $12\frac{1}{7} = \frac{85}{7}$

18 $7\frac{1}{3} = \frac{22}{3}$

19 $3\frac{5}{11} = \frac{38}{11}$

20 $3\frac{1}{8} = \frac{25}{8}$

21 $9\frac{2}{3} = \frac{29}{3}$

22 $4\frac{4}{5} = \frac{24}{5}$

23 $1\frac{1}{10} = \frac{11}{10}$

24 $10\frac{2}{9} = \frac{92}{10}$

25 $11\frac{3}{4} = \frac{47}{4}$

Change these improper fractions to mixed numerals

26 $\frac{5}{2} = 2\frac{1}{2}$

27 $\frac{7}{4} = 1\frac{3}{4}$

28 $\frac{3}{2} = 1\frac{1}{2}$

29 $\frac{8}{5} = 1\frac{3}{5}$

30 $\frac{16}{7} = 2\frac{2}{7}$

31 $\frac{24}{7} = 3\frac{3}{7}$

32 $\frac{23}{5} = 4\frac{3}{5}$

33 $\frac{16}{3} = 5\frac{1}{3}$

34 $\frac{13}{2} = 6\frac{1}{2}$

35 $\frac{19}{7} = 2\frac{5}{7}$

36 $\frac{25}{9} = 2\frac{7}{9}$

37 $\frac{28}{3} = 9\frac{1}{3}$

38 $\frac{15}{4} = 3\frac{3}{4}$

39 $\frac{17}{10} = 1\frac{7}{10}$

40 $\frac{13}{6} = 2\frac{1}{6}$

41 $\frac{53}{5} = 10\frac{3}{5}$

Write the answers as mixed numerals

42 $11 \div 7 = 1\frac{4}{7}$

43 $109 \div 10 = 10\frac{9}{10}$

44 $3\overline{)125} = 41\frac{2}{3}$

45 $20 \div 3 = 6\frac{2}{3}$

46 $7\overline{)321} = 45\frac{6}{7}$

47 $47 \div 5 = 9\frac{2}{5}$

Change improper to mixed to answer true of false to these

48 $\frac{8}{2} = 4\frac{1}{2}$ False

49 $4\frac{3}{8} = \frac{43}{8}$ False

50 $7\frac{2}{7} = \frac{51}{7}$ True

51 $8\frac{5}{9} = \frac{75}{9}$ False

52 $\frac{11}{6} = 1\frac{5}{6}$ True

53 $\frac{24}{7} = 3\frac{4}{7}$ False

54 $1\frac{7}{8} = \frac{15}{8}$ True

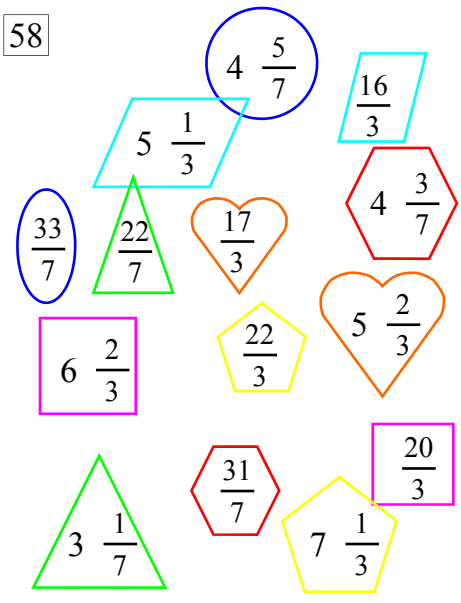
55 $\frac{17}{7} = 2\frac{3}{7}$ True

56 $\frac{13}{11} = 1\frac{3}{11}$ False

57 $4\frac{1}{3} = \frac{15}{3}$ False

Using circles, squares, triangles or other shapes draw the same shape around matching pairs

58



Simplifying Fractions

Simplifying a fraction is the same as expressing a fraction in its simplest form, the fraction you are being given is expressed with smaller numbers. The process of simplifying is finding the largest number that divides into the numerator and the denominator and dividing them both by it. The largest number is the HCF the Highest Common Factor.

In column 1 questions 1 through 20 ask you to find the HCF of two numbers. Look at the two numbers and find the largest number that goes into both of them. Questions 21 through 26 use the HCF to simplify the fraction. In the example (above question 21) the fraction $^{3}/_{12}$ is to be simplified. The HCF is found to be 3, the numerator and denominator are then divided by 3 (the HCF) and the answer placed in the fraction boxes.

Column 2 is all simplification exercises. Find the HCF of the numerator and denominator and divide through. When you complete each question look at your answer and see whether it can be further simplified, if it can be, simplify it again or look at the question again to get the HCF that you have missed and try again from the start.

Column 3 is the same process except the numbers used are larger, make sure that you check your answer to see whether it can be further simplified. Question 79 through 86 asks you to draw a line between the fraction and the simplified fraction. After you calculate the simplified fraction draw a line between it and the question, joining the dots.

Simplifying Fractions

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Find the HCF, the largest number that goes into both of these 2 numbers.

1	3, 9	2	6, 9
3	2, 4	4	8, 10
5	5, 10	6	10, 25
7	8, 16	8	6, 16
9	9, 30	10	12, 30
11	12, 24	12	20, 12
13	4, 14	14	7, 14
15	22, 8	16	50, 15
17	32, 16	18	30, 18
19	14, 42	20	18, 42

Now lets apply this skill to simplifying fractions

Example

$\frac{3}{12}$ HCF 3

$3 \div 3 = 1$

$12 \div 3 = 4$

21 $\frac{3}{9}$ HCF ☐

$3 \div$ ☐ = ☐

$9 \div$ ☐ = ☐

22 $\frac{6}{18}$ HCF ☐

$6 \div$ ☐ = ☐

$18 \div$ ☐ = ☐

23 $\frac{8}{20}$ HCF ☐

$8 \div$ ☐ = ☐

$20 \div$ ☐ = ☐

24 $\frac{10}{25}$ HCF ☐

$10 \div$ ☐ = ☐

$25 \div$ ☐ = ☐

25 $\frac{12}{30}$ HCF ☐

$12 \div$ ☐ = ☐

$30 \div$ ☐ = ☐

26 $\frac{24}{32}$ HCF ☐

$24 \div$ ☐ = ☐

$32 \div$ ☐ = ☐

Now simplify these. Look at your answer, if you can further simplify it you missed the HCF.

27	$\frac{5}{15} = \frac{\square}{\square}$	28	$\frac{4}{8} = \frac{\square}{\square}$
29	$\frac{6}{9} = \frac{\square}{\square}$	30	$\frac{5}{20} = \frac{\square}{\square}$
31	$\frac{9}{15} = \frac{\square}{\square}$	32	$\frac{12}{18} = \frac{\square}{\square}$
33	$\frac{15}{20} = \frac{\square}{\square}$	34	$\frac{18}{30} = \frac{\square}{\square}$
35	$\frac{21}{35} = \frac{\square}{\square}$	36	$\frac{12}{15} = \frac{\square}{\square}$
37	$\frac{16}{20} = \frac{\square}{\square}$	38	$\frac{15}{40} = \frac{\square}{\square}$
39	$\frac{16}{28} = \frac{\square}{\square}$	40	$\frac{30}{50} = \frac{\square}{\square}$
41	$\frac{9}{12} = \frac{\square}{\square}$	42	$\frac{20}{65} = \frac{\square}{\square}$
43	$\frac{19}{38} = \frac{\square}{\square}$	44	$\frac{18}{54} = \frac{\square}{\square}$
45	$\frac{16}{40} = \frac{\square}{\square}$	46	$\frac{45}{60} = \frac{\square}{\square}$
47	$\frac{35}{42} = \frac{\square}{\square}$	48	$\frac{8}{12} = \frac{\square}{\square}$
49	$\frac{2}{50} = \frac{\square}{\square}$	50	$\frac{12}{26} = \frac{\square}{\square}$
51	$\frac{15}{60} = \frac{\square}{\square}$	52	$\frac{18}{27} = \frac{\square}{\square}$
53	$\frac{63}{81} = \frac{\square}{\square}$	54	$\frac{42}{50} = \frac{\square}{\square}$
55	$\frac{56}{64} = \frac{\square}{\square}$	56	$\frac{12}{60} = \frac{\square}{\square}$

Now try larger numbers

57	$\frac{100}{200} = \frac{\square}{\square}$	58	$\frac{350}{500} = \frac{\square}{\square}$
59	$\frac{120}{300} = \frac{\square}{\square}$	60	$\frac{180}{200} = \frac{\square}{\square}$
61	$\frac{90}{210} = \frac{\square}{\square}$	62	$\frac{140}{210} = \frac{\square}{\square}$
63	$\frac{75}{150} = \frac{\square}{\square}$	64	$\frac{400}{500} = \frac{\square}{\square}$
65	$\frac{110}{440} = \frac{\square}{\square}$	66	$\frac{204}{612} = \frac{\square}{\square}$
67	$\frac{80}{150} = \frac{\square}{\square}$	68	$\frac{125}{500} = \frac{\square}{\square}$
69	$\frac{35}{120} = \frac{\square}{\square}$	70	$\frac{195}{200} = \frac{\square}{\square}$
71	$\frac{180}{240} = \frac{\square}{\square}$	72	$\frac{150}{360} = \frac{\square}{\square}$

Draw lines between fractions and their simplified match

73	$\frac{35}{40}$	A	$\frac{1}{2}$
74	$\frac{10}{12}$	B	$\frac{4}{5}$
75	$\frac{12}{24}$	C	$\frac{5}{6}$
76	$\frac{15}{45}$	D	$\frac{3}{4}$
77	$\frac{20}{28}$	E	$\frac{7}{8}$
78	$\frac{9}{12}$	F	$\frac{5}{7}$
79	$\frac{8}{12}$	G	$\frac{2}{3}$
80	$\frac{60}{75}$	H	$\frac{1}{3}$

Simplifying Fractions

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Find the HCF, the largest number that goes into both of these 2 numbers.

1	3, 9	3	2	6, 9	3
3	2, 4	2	4	8, 10	2
5	5, 10	5	6	10, 25	5
7	8, 16	8	8	6, 16	2
9	9, 30	3	10	12, 30	6
11	12, 24	12	12	20, 12	4
13	4, 14	2	14	7, 14	7
15	22, 8	2	16	50, 15	5
17	32, 16	16	18	30, 18	6
19	14, 42	14	20	18, 42	6

Now lets apply this skill to simplifying fractions

Example

$\frac{3}{12}$ HCF 3

$3 \div 3 = 1$

$12 \div 3 = 4$

21 $\frac{3}{9}$ HCF 3

$3 \div 3 = 1$

$9 \div 3 = 3$

22 $\frac{6}{18}$ HCF 6

$6 \div 6 = 1$

$18 \div 6 = 3$

23 $\frac{8}{20}$ HCF 4

$8 \div 4 = 2$

$20 \div 4 = 5$

24 $\frac{10}{25}$ HCF 5

$10 \div 5 = 2$

$25 \div 5 = 5$

25 $\frac{12}{30}$ HCF 6

$12 \div 6 = 2$

$30 \div 6 = 5$

26 $\frac{24}{32}$ HCF 8

$24 \div 8 = 3$

$32 \div 8 = 4$

Now simplify these. Look at your answer, if you can further simplify it you missed the HCF.

27	$\frac{5}{15}$	$\frac{1}{3}$	28	$\frac{4}{8}$	$\frac{1}{2}$
29	$\frac{6}{9}$	$\frac{2}{3}$	30	$\frac{5}{20}$	$\frac{1}{4}$
31	$\frac{9}{15}$	$\frac{3}{5}$	32	$\frac{12}{18}$	$\frac{2}{3}$
33	$\frac{15}{20}$	$\frac{3}{4}$	34	$\frac{18}{30}$	$\frac{3}{5}$
35	$\frac{21}{35}$	$\frac{3}{5}$	36	$\frac{12}{15}$	$\frac{4}{5}$
37	$\frac{16}{20}$	$\frac{4}{5}$	38	$\frac{15}{40}$	$\frac{3}{8}$
39	$\frac{16}{28}$	$\frac{4}{7}$	40	$\frac{30}{50}$	$\frac{3}{5}$
41	$\frac{9}{12}$	$\frac{3}{4}$	42	$\frac{20}{65}$	$\frac{4}{13}$
43	$\frac{19}{38}$	$\frac{1}{2}$	44	$\frac{18}{54}$	$\frac{1}{3}$
45	$\frac{16}{40}$	$\frac{2}{5}$	46	$\frac{45}{60}$	$\frac{3}{4}$
47	$\frac{35}{42}$	$\frac{5}{6}$	48	$\frac{8}{12}$	$\frac{2}{3}$
49	$\frac{2}{50}$	$\frac{1}{26}$	50	$\frac{12}{26}$	$\frac{6}{13}$
51	$\frac{15}{60}$	$\frac{1}{4}$	52	$\frac{18}{27}$	$\frac{2}{3}$
53	$\frac{63}{81}$	$\frac{7}{9}$	54	$\frac{42}{50}$	$\frac{21}{25}$
55	$\frac{56}{64}$	$\frac{7}{8}$	56	$\frac{12}{60}$	$\frac{1}{5}$

Now try larger numbers

57	$\frac{100}{200}$	$\frac{1}{2}$	58	$\frac{350}{500}$	$\frac{7}{10}$
59	$\frac{120}{300}$	$\frac{2}{5}$	60	$\frac{180}{200}$	$\frac{9}{10}$
61	$\frac{90}{210}$	$\frac{3}{7}$	62	$\frac{140}{210}$	$\frac{2}{3}$
63	$\frac{75}{150}$	$\frac{1}{2}$	64	$\frac{400}{500}$	$\frac{4}{5}$
65	$\frac{110}{440}$	$\frac{1}{4}$	66	$\frac{204}{612}$	$\frac{1}{3}$
67	$\frac{80}{150}$	$\frac{8}{15}$	68	$\frac{125}{500}$	$\frac{1}{4}$
69	$\frac{35}{120}$	$\frac{7}{24}$	70	$\frac{195}{200}$	$\frac{39}{40}$
71	$\frac{180}{240}$	$\frac{3}{4}$	72	$\frac{150}{360}$	$\frac{5}{12}$

Draw lines between fractions and their simplified match

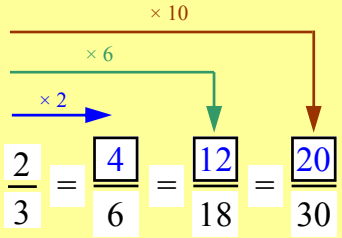
73	$\frac{35}{40}$	$\frac{1}{2}$	A
74	$\frac{10}{12}$	$\frac{4}{5}$	B
75	$\frac{12}{24}$	$\frac{5}{6}$	C
76	$\frac{15}{45}$	$\frac{3}{4}$	D
77	$\frac{20}{28}$	$\frac{7}{8}$	E
78	$\frac{9}{12}$	$\frac{5}{7}$	F
79	$\frac{8}{12}$	$\frac{2}{3}$	G
80	$\frac{60}{75}$	$\frac{1}{3}$	H

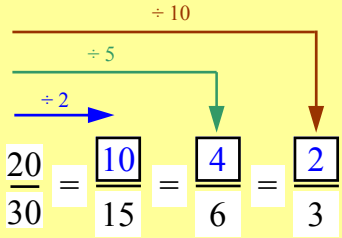
Equivalent and Comparing Fractions

An equivalent fraction is a fraction that is equal to another fraction but has a different denominator and numerator. For example if you scored $^{9}/_{10}$ in a test it would be equivalent to scoring $^{18}/_{20}$ or $^{90}/_{100}$. Equivalent fractions are calculated by multiplying (or dividing) the numerator and the denominator by the same number. To change $^{9}/_{10}$ to $^{18}/_{20}$ we multiplied the top and bottom numbers in the fraction by 2. To change $^{9}/_{10}$ to $^{90}/_{100}$ we multiplied the top and bottom numbers by 10.

Column 1 starts by asking you to find two fractions that represent the shaded area. The first fraction will be found by counting the shaded parts and making that the numerator, and counting the total number of parts and making that number the denominator. Then either multiply both of these numbers by any other number, or simplify to find an equivalent fraction. A hint, do you notice that the shading goes all the way down the columns of the shape? Then if you cover the rows of the shape except for top row you will get an equivalent fraction. Note that if the shading is scattered stick to the multiply or divide method.

The next part of column 1 asks you to find equivalent fractions to the one given. Look at the example below to the left, we are given $^{2}/_{3}$ so the denominator is 3. The first question has a denominator of 6, what do you multiply 3 by to get 6? The answer is 2, so multiply the numerator by 2 and you get $2 \times 2 = 4$ so the first answer is 4. The 2nd question has a denominator of 18, what do you multiply 3 by to get 18? The answer is 6, so multiply the numerator by 6 and you get $2 \times 6 = 12$, so the 2nd answer is 12. The same method again will give you × 10 and thus 20 for the last fraction.





Column 2 uses the same theory but with division. To make smaller equivalent fractions divide the top and bottom numbers in the fraction by another number. Of course when you divide you must find a number than can divide into both numbers exactly without a remainder. Questions 16 to 22 give you a fraction and a different denominator to make an equivalent fraction. Find the number to divide by across the bottom of the fraction and then divide across the top by the same number. You are then given the numerator of equivalent fractions and asked to find the denominator. Exactly the same method is used. YOU NEVER ADD OR SUBTRACT TO GET EQUIVALENT FRACTIONS.

Column 3 asks you to compare fractions, look at the fraction at the right of the empty box, look at its denominator and change the fraction on the left to match that denominator, then compare and fill in the box with < , > or =. Remember point the arrow at the smallest fraction (number). See the example at the top of column 3. Questions 41 to 48 ask you to check if the fractions are equivalent, check the denominators first and find the number that has been used to divide or multiply to get the other fraction, then apply the same number to the top and see if they equal each other. Yes? ……..True, No? …….False.

Equivalent and Comparing Fractions

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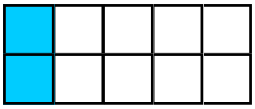
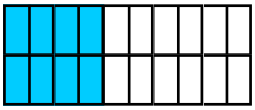
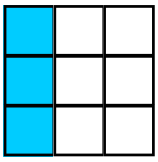
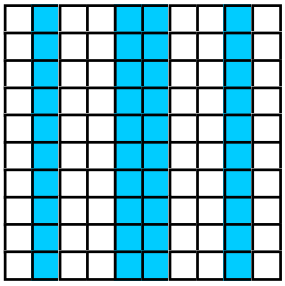
Give two fractions that represent the shaded part of each shape

1 

2 

3 

4 

Find the equivalent fraction with the given denominator

5 $\frac{1}{3} = \frac{\square}{12}$ 6 $\frac{2}{5} = \frac{\square}{10}$

7 $\frac{4}{10} = \frac{\square}{50}$ 8 $\frac{3}{6} = \frac{\square}{18}$

9 $\frac{9}{10} = \frac{\square}{90}$ 10 $\frac{3}{20} = \frac{\square}{40}$

11 $\frac{4}{5} = \frac{\square}{30}$ 12 $\frac{1}{2} = \frac{\square}{50}$

13 $\frac{1}{2} = \frac{\square}{12} = \frac{\square}{20} = \frac{\square}{18} = \frac{\square}{48}$

14 $\frac{2}{3} = \frac{\square}{12} = \frac{\square}{30} = \frac{\square}{9} = \frac{\square}{15}$

15 $\frac{3}{4} = \frac{\square}{20} = \frac{\square}{12} = \frac{\square}{24} = \frac{\square}{32}$

Now use division to find the equivalent fraction

16 $\frac{12}{20} = \frac{\square}{10}$ 17 $\frac{6}{18} = \frac{\square}{6}$

18 $\frac{20}{50} = \frac{\square}{10}$ 19 $\frac{18}{42} = \frac{\square}{7}$

20 $\frac{20}{60} = \frac{\square}{6} = \frac{\square}{30} = \frac{\square}{15} = \frac{\square}{12}$

21 $\frac{12}{24} = \frac{\square}{12} = \frac{\square}{6} = \frac{\square}{2} = \frac{\square}{4}$

22 $\frac{12}{36} = \frac{\square}{9} = \frac{\square}{6} = \frac{\square}{18} = \frac{\square}{3}$

Now find the denominator given the numerator

23 $\frac{2}{3} = \frac{6}{\square}$ 24 $\frac{3}{5} = \frac{15}{\square}$

25 $\frac{4}{12} = \frac{2}{\square}$ 26 $\frac{5}{6} = \frac{20}{\square}$

27 $\frac{12}{18} = \frac{24}{\square} = \frac{6}{\square} = \frac{2}{\square} = \frac{4}{\square}$

28 $\frac{4}{6} = \frac{8}{\square} = \frac{2}{\square} = \frac{40}{\square} = \frac{12}{\square}$

29 $\frac{8}{12} = \frac{24}{\square} = \frac{4}{\square} = \frac{16}{\square} = \frac{2}{\square}$

Now they are all mixed up complete for the fractions

30 $\frac{6}{10} = \frac{\square}{20} = \frac{3}{\square} = \frac{30}{\square} = \frac{\square}{30}$

31 $\frac{6}{9} = \frac{\square}{90} = \frac{2}{\square} = \frac{\square}{18} = \frac{18}{\square}$

32 $\frac{8}{24} = \frac{16}{\square} = \frac{2}{\square} = \frac{\square}{12} = \frac{24}{\square}$

33 $\frac{15}{45} = \frac{1}{\square} = \frac{\square}{15} = \frac{\square}{9} = \frac{30}{\square}$

Use the equivalent fraction to compare the fractions. Use <, > or = to make true

Example

$\frac{4}{9} = \frac{12}{27} \square \frac{10}{27}$ (>)

34 $\frac{1}{2} = \frac{\square}{4} \square \frac{3}{4}$

35 $\frac{3}{5} = \frac{\square}{10} \square \frac{6}{10}$

36 $\frac{8}{90} = \frac{\square}{45} \square \frac{4}{45}$

37 $\frac{8}{12} = \frac{\square}{\square} \square \frac{30}{48}$

38 $\frac{3}{7} = \frac{\square}{\square} \square \frac{12}{28}$

39 $\frac{3}{4} = \frac{\square}{\square} \square \frac{70}{80}$

40 $\frac{63}{70} = \frac{\square}{\square} \square \frac{8}{10}$

Test these to see if these are equivalent. Answer true or false

41 $\frac{1}{2} = \frac{6}{12}$ _____

42 $\frac{3}{5} = \frac{6}{15}$ _____

43 $\frac{9}{30} = \frac{3}{10}$ _____

44 $\frac{2}{3} = \frac{14}{21}$ _____

45 $\frac{9}{12} = \frac{3}{4}$ _____

46 $\frac{15}{40} = \frac{5}{8}$ _____

47 $\frac{2}{5} = \frac{10}{25}$ _____

48 $\frac{5}{9} = \frac{45}{90}$ _____

Equivalent and Comparing Fractions

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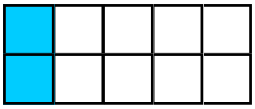
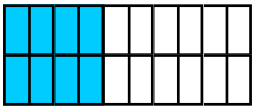
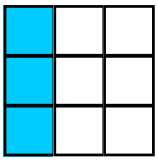
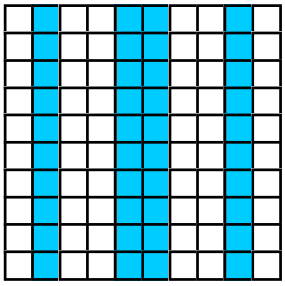
Give two fractions that represent the shaded part of each shape

1  $\frac{2}{10}$ $\frac{1}{5}$

2  $\frac{8}{20}$ $\frac{4}{10}$

3  $\frac{3}{9}$ $\frac{1}{3}$

4  $\frac{40}{100}$ $\frac{4}{10}$

Find the equivalent fraction with the given denominator

5 $\frac{1}{3} = \frac{4}{12}$ 6 $\frac{2}{5} = \frac{4}{10}$

7 $\frac{4}{10} = \frac{20}{50}$ 8 $\frac{3}{6} = \frac{9}{18}$

9 $\frac{9}{10} = \frac{81}{90}$ 10 $\frac{3}{20} = \frac{6}{40}$

11 $\frac{4}{5} = \frac{24}{30}$ 12 $\frac{1}{2} = \frac{25}{50}$

13 $\frac{1}{2} = \frac{6}{12} = \frac{10}{20} = \frac{9}{18} = \frac{24}{48}$

14 $\frac{2}{3} = \frac{8}{12} = \frac{20}{30} = \frac{6}{9} = \frac{10}{15}$

15 $\frac{3}{4} = \frac{15}{20} = \frac{9}{12} = \frac{18}{24} = \frac{24}{32}$

Now use division to find the equivalent fraction

16 $\frac{12}{20} = \frac{6}{10}$ 17 $\frac{6}{18} = \frac{2}{6}$

18 $\frac{20}{50} = \frac{4}{10}$ 19 $\frac{18}{42} = \frac{3}{7}$

20 $\frac{20}{60} = \frac{2}{6} = \frac{10}{30} = \frac{5}{15} = \frac{4}{12}$

21 $\frac{12}{24} = \frac{6}{12} = \frac{3}{6} = \frac{1}{2} = \frac{2}{4}$

22 $\frac{12}{36} = \frac{3}{9} = \frac{2}{6} = \frac{6}{18} = \frac{1}{3}$

Now find the denominator given the numerator

23 $\frac{2}{3} = \frac{6}{9}$ 24 $\frac{3}{5} = \frac{15}{25}$

25 $\frac{4}{12} = \frac{2}{6}$ 26 $\frac{5}{6} = \frac{20}{24}$

27 $\frac{12}{18} = \frac{24}{36} = \frac{6}{9} = \frac{2}{3} = \frac{4}{6}$

28 $\frac{4}{6} = \frac{8}{12} = \frac{2}{3} = \frac{40}{60} = \frac{12}{18}$

29 $\frac{8}{12} = \frac{24}{36} = \frac{4}{6} = \frac{16}{24} = \frac{2}{3}$

Now they are all mixed up complete for the fractions

30 $\frac{6}{10} = \frac{12}{20} = \frac{3}{5} = \frac{30}{50} = \frac{18}{30}$

31 $\frac{6}{9} = \frac{60}{90} = \frac{2}{3} = \frac{12}{18} = \frac{18}{27}$

32 $\frac{8}{24} = \frac{16}{48} = \frac{2}{6} = \frac{4}{12} = \frac{24}{72}$

33 $\frac{15}{45} = \frac{1}{3} = \frac{5}{15} = \frac{3}{9} = \frac{30}{90}$

Use the equivalent fraction to compare the fractions. Use <, > or = to make true

Example $\frac{4}{9} = \frac{12}{27} > \frac{10}{27}$

34 $\frac{1}{2} = \frac{2}{4} < \frac{3}{4}$

35 $\frac{3}{5} = \frac{6}{10} = \frac{6}{10}$

36 $\frac{8}{90} = \frac{4}{45} = \frac{4}{45}$

37 $\frac{8}{12} = \frac{32}{48} > \frac{30}{48}$

38 $\frac{3}{7} = \frac{12}{28} = \frac{12}{28}$

39 $\frac{3}{4} = \frac{60}{80} < \frac{70}{80}$

40 $\frac{63}{70} = \frac{9}{10} > \frac{8}{10}$

Test these to see if these are equivalent. Answer true or false

41 $\frac{1}{2} = \frac{6}{12}$ True

42 $\frac{3}{5} = \frac{6}{15}$ False

43 $\frac{9}{30} = \frac{3}{10}$ True

44 $\frac{2}{3} = \frac{14}{21}$ True

45 $\frac{9}{12} = \frac{3}{4}$ True

46 $\frac{15}{40} = \frac{5}{8}$ False

47 $\frac{2}{5} = \frac{10}{25}$ True

48 $\frac{5}{9} = \frac{45}{90}$ False

Comparing Fractions (Calculator)

Using a calculator to compare fractions is done by changing the fractions to decimals and comparing the decimals instead. If all rounded to the same decimal place, the decimals can be easily compared, e.g. 0.308, 0.471, 0.235. It is easy to see the largest and smallest decimal, or if they are equal, if you look at the numbers behind the decimal point.

Column 1 asks you to convert the fractions to decimals, the calculator image on the next page shows you how to do this. If you have a decimal 0.100 then write it as 0.100 don't change it to 0.1 as this may lead you to make an error when comparing other numbers that are 3 d.p. (You may look at it as 1 instead of 100).

The rest of the sheet is attempted the same way, by finding the decimal for the fraction then comparing decimals. In the example below, two fractions are being compared, the method is:

- Find the decimals for both
- Compare decimals and put a < or > or = sign in between the decimals
- If this sign matches the one above it then write true, if not, write false.

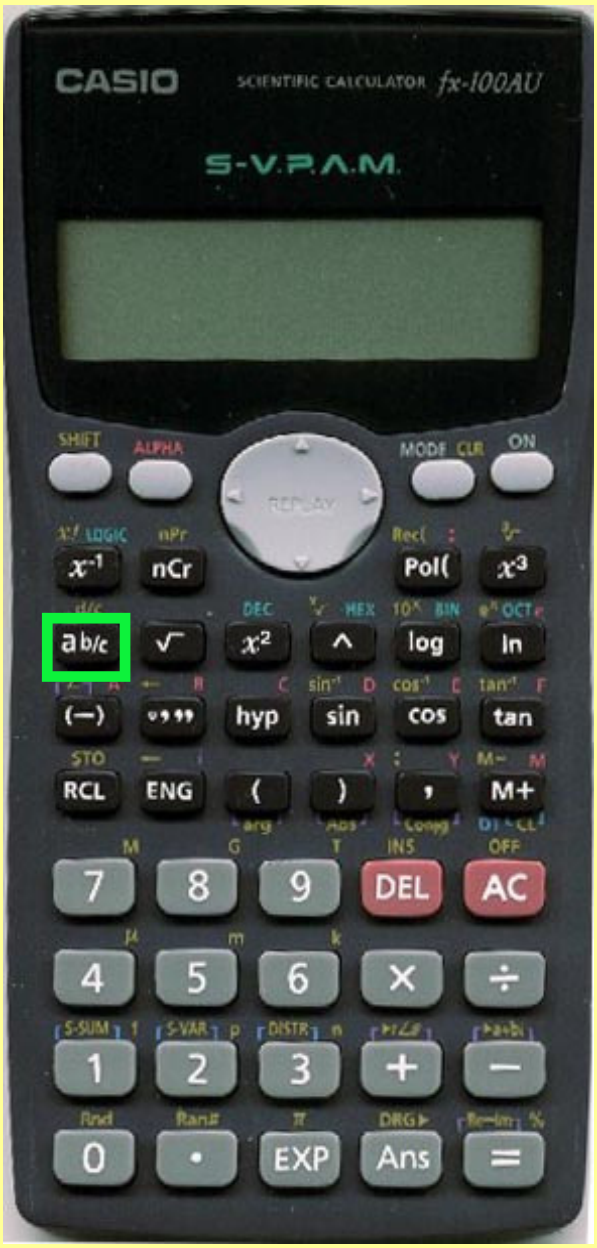
$$\frac{4}{5} > \frac{7}{8} \quad \text{False}$$
$$0.800 < 0.875$$

In the 2nd column, from Q. 31 on, use the same method only this time just write the sign into the box rather than stating true/false.

The third column asks you to compare fractions and arrange them in descending order, use the decimal system then write the fractions again in descending order. In a test the most common mistake is rewriting the decimals in descending order, the question asks for the fractions to be written, not the decimals.

How do you remember the difference between < and >? Imagine they are arrowheads and point them at the smallest number. E.g. $5 < 6$ and $15 > 10$, the arrow points to the smallest one.

Descending order? Remember going down.



Example of how to convert fractions to decimals

$\frac{6}{7} = \_\_\_\_ ?$

Don't forget the 'equals' sign

Method 1: Using Fraction Key [6] [a b/c] [7] [=] [a b/c]

Method 2: Using ÷ Key [6] [÷] [7] [=]

if you get 0.857 (rounded) you have answered it correctly

Comparing Fractions (Calculator)

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Convert these to decimals round to 3 d.p.

1 $\frac{1}{10}$ ____ 2 $\frac{1}{4}$ ____

3 $\frac{2}{3}$ ____ 4 $\frac{3}{8}$ ____

5 $\frac{3}{4}$ ____ 6 $\frac{5}{6}$ ____

7 $\frac{6}{13}$ ____ 8 $\frac{8}{19}$ ____

9 $\frac{3}{7}$ ____ 10 $\frac{24}{27}$ ____

11 $\frac{17}{29}$ ____ 12 $\frac{7}{12}$ ____

13 $\frac{73}{80}$ ____ 14 $\frac{33}{43}$ ____

15 $\frac{17}{42}$ ____ 16 $\frac{96}{99}$ ____

17 $\frac{175}{305}$ ____ 18 $\frac{212}{636}$ ____

19 $\frac{572}{911}$ ____ 20 $\frac{101}{909}$ ____

Write the decimal (3 d.p) under the fraction and then answer true or false

21 $\frac{2}{3}$ < $\frac{3}{4}$ ____

22 $\frac{2}{5}$ > $\frac{3}{7}$ ____

23 $\frac{8}{9}$ < $\frac{9}{10}$ ____

24 $\frac{6}{7}$ > $\frac{8}{10}$ ____

25 $\frac{5}{8}$ < $\frac{7}{14}$ ____

26 $\frac{8}{11}$ > $\frac{5}{7}$ ____

27 $\frac{13}{17}$ < $\frac{14}{19}$ ____

28 $\frac{6}{81}$ < $\frac{4}{63}$ ____

29 $\frac{35}{39}$ > $\frac{60}{70}$ ____

30 $\frac{88}{93}$ < $\frac{17}{19}$ ____

Use the same method only fill in <, > or =

31 $\frac{2}{3}$ ☐ $\frac{3}{5}$

32 $\frac{3}{6}$ ☐ $\frac{18}{36}$

33 $\frac{4}{10}$ ☐ $\frac{27}{60}$

34 $\frac{3}{7}$ ☐ $\frac{4}{9}$

35 $\frac{8}{12}$ ☐ $\frac{2}{3}$

36 $\frac{4}{9}$ ☐ $\frac{17}{45}$

37 $\frac{4}{19}$ ☐ $\frac{14}{60}$

38 $\frac{11}{18}$ ☐ $\frac{26}{37}$

39 $\frac{3}{8}$ ☐ $\frac{21}{56}$

40 $\frac{12}{13}$ ☐ $\frac{16}{19}$

Use decimals to arrange these in descending order

41 $\frac{3}{7}$, $\frac{2}{3}$, $\frac{10}{13}$, $\frac{17}{40}$

Rewrite in Descending order ☐ , ☐ , ☐ , ☐

42 $\frac{39}{95}$, $\frac{23}{60}$, $\frac{13}{40}$, $\frac{1}{4}$

Rewrite in Descending order ☐ , ☐ , ☐ , ☐

43 $\frac{13}{15}$, $\frac{9}{10}$, $\frac{29}{34}$, $\frac{18}{21}$

Rewrite in Descending order ☐ , ☐ , ☐ , ☐

Comparing Fractions (Calculator)

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Convert these to decimals round to 3 d.p.

1. $\frac{1}{10}$ 0.100
2. $\frac{1}{4}$ 0.250
3. $\frac{2}{3}$ 0.667
4. $\frac{3}{8}$ 0.375
5. $\frac{3}{4}$ 0.750
6. $\frac{5}{6}$ 0.833
7. $\frac{6}{13}$ 0.462
8. $\frac{8}{19}$ 0.421
9. $\frac{3}{7}$ 0.429
10. $\frac{24}{27}$ 0.889
11. $\frac{17}{29}$ 0.586
12. $\frac{7}{12}$ 0.583
13. $\frac{73}{80}$ 0.913
14. $\frac{33}{43}$ 0.767
15. $\frac{17}{42}$ 0.405
16. $\frac{96}{99}$ 0.970
17. $\frac{175}{305}$ 0.574
18. $\frac{212}{636}$ 0.333
19. $\frac{572}{911}$ 0.628
20. $\frac{101}{909}$ 0.111

Write the decimal (3 d.p) under the fraction and then answer true or false

21. $\frac{2}{3} < \frac{3}{4}$ True
$0.667 < 0.750$
22. $\frac{2}{5} > \frac{3}{7}$ False
$0.400 < 0.429$
23. $\frac{8}{9} < \frac{9}{10}$ True
$0.889 < 0.900$
24. $\frac{6}{7} > \frac{8}{10}$ True
$0.857 > 0.800$
25. $\frac{5}{8} < \frac{7}{14}$ False
$0.625 > 0.500$
26. $\frac{8}{11} > \frac{5}{7}$ True
$0.727 > 0.714$
27. $\frac{13}{17} < \frac{14}{19}$ False
$0.765 > 0.737$
28. $\frac{6}{81} < \frac{4}{63}$ False
$0.074 > 0.063$
29. $\frac{35}{39} > \frac{60}{70}$ True
$0.897 > 0.857$
30. $\frac{88}{93} < \frac{17}{19}$ False
$0.946 > 0.895$

Use the same method only fill in <, > or =

31. $\frac{2}{3} > \frac{3}{5}$
$0.667 > 0.600$
32. $\frac{3}{6} = \frac{18}{36}$
$0.500 = 0.500$
33. $\frac{4}{10} < \frac{27}{60}$
$0.400 < 0.450$
34. $\frac{3}{7} < \frac{4}{9}$
$0.429 < 0.444$
35. $\frac{8}{12} = \frac{2}{3}$
$0.667 = 0.667$
36. $\frac{4}{9} > \frac{17}{45}$
$0.444 > 0.378$
37. $\frac{4}{19} < \frac{14}{60}$
$0.211 < 0.233$
38. $\frac{11}{18} < \frac{26}{37}$
$0.611 < 0.703$
39. $\frac{3}{8} = \frac{21}{56}$
$0.375 = 0.375$
40. $\frac{12}{13} > \frac{16}{19}$
$0.923 > 0.842$

Use decimals to arrange these in descending order

41. $\frac{3}{7}, \frac{2}{3}, \frac{10}{13}, \frac{17}{40}$
0.429 0.667 0.769 0.425
Rewrite in Descending order → $\frac{10}{13}, \frac{2}{3}, \frac{3}{7}, \frac{17}{40}$

42. $\frac{39}{95}, \frac{23}{60}, \frac{13}{40}, \frac{1}{4}$
0.411 0.383 0.325 0.250
Rewrite in Descending order → $\frac{39}{95}, \frac{23}{60}, \frac{13}{40}, \frac{1}{4}$

43. $\frac{13}{15}, \frac{9}{10}, \frac{29}{34}, \frac{18}{21}$
0.867 0.900 0.853 0.857
Rewrite in Descending order → $\frac{9}{10}, \frac{13}{15}, \frac{18}{21}, \frac{29}{34}$

Adding and Subtracting Fractions (Same Denominator)

Fractions like whole numbers can be added and subtracted. The fractions on this sheet all have the same denominator so they can be added without finding equivalent fractions.

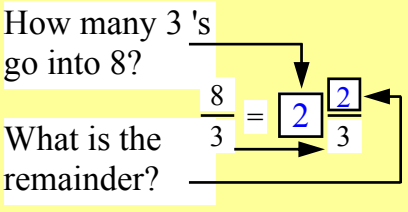
With column 1 the fractions are added and subtracted to give answers that can't be simplified. When you add fractions you add the numerator only, that is, the top numbers. The same with subtraction, you subtract the top numbers not the denominators (bottom numbers). In the example below an extra working step has been included. The top numbers are added but the bottom number is unchanged.

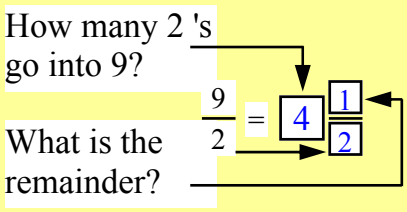
$$\frac{9}{21} + \frac{7}{21} = \frac{9+7}{21} = \frac{16}{21}$$

Column 2 involves the same style of exercise except this time the fractions can be simplified. Complete the addition or subtraction then look at your answer. Find the HCF, that is, the largest number that divides into both the top and bottom numbers. Then divide through by that number. Use canceling if you wish, or otherwise just write the simplified answer in the next fraction boxes. Remember that once you have simplified the answer, look at it again to ensure you can't reduce the fractions any further by dividing again. If you can further simplify that means that you missed the Highest Common Factor, look back and see whether you can find the HCF.

$$\frac{7}{15} - \frac{4}{15} = \frac{3}{15} = \frac{1}{5}$$

Column 3 is one extra step more, like the second column you are given exercises that will result in answers that can be simplified. But this time the answers will be improper fractions. The question asks for mixed numeral answers so you then have to convert them to mixed numbers. You should remember how to do this, if not, look at the example of converting to mixed numbers below. Note that you might like to convert to mixed numbers then simplify afterwards, both methods will result in the same answer.





Adding/Subtracting Fractions (Same Denominator)

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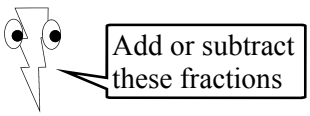


1 $\frac{1}{10} + \frac{2}{10} = \frac{\square}{\square}$

2 $\frac{5}{8} - \frac{2}{8} = \frac{\square}{\square}$

3 $\frac{1}{5} + \frac{2}{5} = \frac{\square}{\square}$

4 $\frac{4}{7} - \frac{3}{7} = \frac{\square}{\square}$

5 $\frac{2}{11} + \frac{7}{11} = \frac{\square}{\square}$

6 $\frac{1}{9} + \frac{4}{9} = \frac{\square}{\square}$

7 $\frac{7}{5} - \frac{4}{5} = \frac{\square}{\square}$

8 $\frac{8}{6} - \frac{3}{6} = \frac{\square}{\square}$

9 $\frac{17}{12} - \frac{6}{12} = \frac{\square}{\square}$

10 $\frac{2}{13} + \frac{7}{13} = \frac{\square}{\square}$

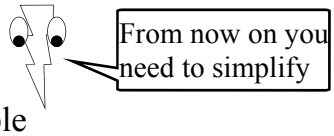
11 $\frac{5}{10} + \frac{4}{10} = \frac{\square}{\square}$

12 $\frac{14}{6} - \frac{9}{6} = \frac{\square}{\square}$

13 $\frac{15}{32} + \frac{12}{32} = \frac{\square}{\square}$

14 $\frac{13}{8} - \frac{6}{8} = \frac{\square}{\square}$

15 $\frac{9}{20} + \frac{4}{20} = \frac{\square}{\square}$



Example

$\frac{9}{10} - \frac{3}{10} = \frac{{}^{3}\not{6}}{{}^{5}\not{10}} = \frac{3}{5}$

16 $\frac{3}{4} - \frac{1}{4} = \frac{\square}{\square} = \frac{\square}{\square}$

17 $\frac{1}{6} + \frac{3}{6} = \frac{\square}{\square} = \frac{\square}{\square}$

18 $\frac{5}{10} + \frac{1}{10} = \frac{\square}{\square} = \frac{\square}{\square}$

19 $\frac{10}{9} - \frac{7}{9} = \frac{\square}{\square} = \frac{\square}{\square}$

20 $\frac{7}{12} - \frac{5}{12} = \frac{\square}{\square} = \frac{\square}{\square}$

21 $\frac{3}{8} + \frac{5}{8} = \frac{\square}{\square} = \square$

22 $\frac{13}{20} - \frac{7}{20} = \frac{\square}{\square} = \frac{\square}{\square}$

23 $\frac{4}{14} + \frac{3}{14} = \frac{\square}{\square} = \frac{\square}{\square}$

24 $\frac{7}{9} - \frac{1}{9} = \frac{\square}{\square} = \frac{\square}{\square}$

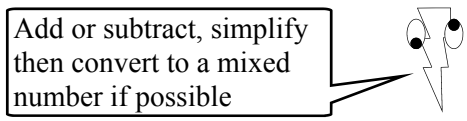
25 $\frac{8}{10} - \frac{3}{10} = \frac{\square}{\square} = \frac{\square}{\square}$

26 $\frac{13}{24} + \frac{7}{24} = \frac{\square}{\square} = \frac{\square}{\square}$

27 $\frac{5}{18} + \frac{4}{18} = \frac{\square}{\square} = \frac{\square}{\square}$

28 $\frac{13}{15} - \frac{3}{15} = \frac{\square}{\square} = \frac{\square}{\square}$

29 $\frac{7}{30} - \frac{4}{30} = \frac{\square}{\square} = \frac{\square}{\square}$



Example

Add: $\frac{9}{10} + \frac{9}{10} = \frac{{}^{9}\not{18}}{{}^{5}\not{10}}$ Simplify: $= \frac{9}{5}$

Change to mixed numeral $= 1\frac{4}{5}$

30 $\frac{9}{6} + \frac{5}{6} = \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

31 $\frac{9}{12} + \frac{7}{12} = \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

32 $\frac{29}{12} - \frac{5}{12} = \frac{\square}{\square} = \square$

33 $\frac{17}{10} + \frac{18}{10} = \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

34 $\frac{44}{9} - \frac{11}{9} = \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

35 $\frac{50}{18} - \frac{11}{18} = \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

36 $\frac{50}{8} + \frac{10}{8} = \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

Adding/Subtracting Fractions (Same Denominator)

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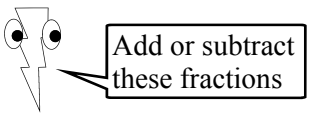


1 $\frac{1}{10} + \frac{2}{10} = \frac{3}{10}$

2 $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

3 $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

4 $\frac{4}{7} - \frac{3}{7} = \frac{1}{7}$

5 $\frac{2}{11} + \frac{7}{11} = \frac{9}{11}$

6 $\frac{1}{9} + \frac{4}{9} = \frac{5}{9}$

7 $\frac{7}{5} - \frac{4}{5} = \frac{3}{5}$

8 $\frac{8}{6} - \frac{3}{6} = \frac{5}{6}$

9 $\frac{17}{12} - \frac{6}{12} = \frac{11}{12}$

10 $\frac{2}{13} + \frac{7}{13} = \frac{9}{13}$

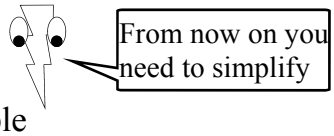
11 $\frac{5}{10} + \frac{4}{10} = \frac{9}{10}$

12 $\frac{14}{6} - \frac{9}{6} = \frac{5}{6}$

13 $\frac{15}{32} + \frac{12}{32} = \frac{27}{32}$

14 $\frac{13}{8} - \frac{6}{8} = \frac{7}{8}$

15 $\frac{9}{20} + \frac{4}{20} = \frac{13}{20}$



Example

$\frac{9}{10} - \frac{3}{10} = \frac{{}^{3}\cancel{6}}{{}^{5}10} = \frac{3}{5}$

16 $\frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$

17 $\frac{1}{6} + \frac{3}{6} = \frac{4}{6} = \frac{2}{3}$

18 $\frac{5}{10} + \frac{1}{10} = \frac{6}{10} = \frac{3}{5}$

19 $\frac{10}{9} - \frac{7}{9} = \frac{3}{9} = \frac{1}{3}$

20 $\frac{7}{12} - \frac{5}{12} = \frac{2}{12} = \frac{1}{6}$

21 $\frac{3}{8} + \frac{5}{8} = \frac{8}{8} = 1$

22 $\frac{13}{20} - \frac{7}{20} = \frac{6}{20} = \frac{3}{10}$

23 $\frac{4}{14} + \frac{3}{14} = \frac{7}{14} = \frac{1}{2}$

24 $\frac{7}{9} - \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$

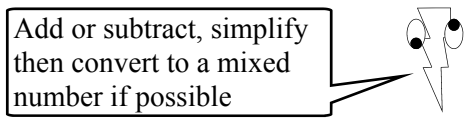
25 $\frac{8}{10} - \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$

26 $\frac{13}{24} + \frac{7}{24} = \frac{20}{24} = \frac{5}{6}$

27 $\frac{5}{18} + \frac{4}{18} = \frac{9}{18} = \frac{1}{2}$

28 $\frac{13}{15} - \frac{3}{15} = \frac{10}{15} = \frac{2}{3}$

29 $\frac{7}{30} - \frac{4}{30} = \frac{3}{30} = \frac{1}{10}$



Example

Add: $\frac{9}{10} + \frac{9}{10} = \frac{{}^{9}18}{{}^{5}10}$ Simplify: $= \frac{9}{5}$

Change to mixed numeral $= 1\frac{4}{5}$

30 $\frac{9}{6} + \frac{5}{6} = \frac{14}{6} = \frac{7}{3} = 2\frac{1}{3}$

31 $\frac{9}{12} + \frac{7}{12} = \frac{16}{12} = \frac{4}{3} = 1\frac{1}{3}$

32 $\frac{29}{12} - \frac{5}{12} = \frac{24}{12} = 2$

33 $\frac{17}{10} + \frac{18}{10} = \frac{35}{10} = \frac{7}{2} = 3\frac{1}{2}$

34 $\frac{44}{9} - \frac{11}{9} = \frac{33}{9} = \frac{11}{3} = 3\frac{2}{3}$

35 $\frac{50}{18} - \frac{11}{18} = \frac{39}{18} = \frac{13}{6} = 2\frac{1}{6}$

36 $\frac{50}{8} + \frac{10}{8} = \frac{60}{8} = \frac{15}{2} = 7\frac{1}{2}$

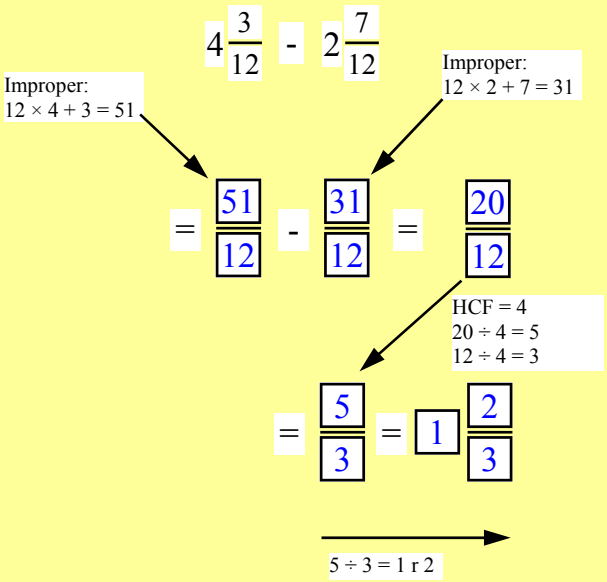
Adding and Subtracting Mixed Numerals
(Same denominator)

Adding and subtracting mixed numerals will be challenging for some students, follow the examples below, then try the exercises.

The entire sheet is done by the one method, you are given 2 mixed numerals and you have to add or subtract them. The method is:

- change the first mixed number into an improper fraction and rewrite it in the boxes below
- Repeat for the 2nd mixed numeral
- Add the numerators (the top numbers) and write the answer in the top box then write the denominator straight into the bottom box, don't add or subtract denominators
- Find the HCF, the largest number that divides into both the top and bottom numbers
- Divide through by this number and write the answer in the boxes on the 3rd line, in other words simplify the fraction
- Look at your answer, can it be further simplified? If it can try and simplify it and try to get the HCF that you missed in the first simplification.
- Change your answer to a mixed number

You are done! It's a long process and a mistake can be made at many of the stages so be careful and take your time. Note that you can drop a step, the simplifying, if you use canceling.



Adding/Subtracting Mixed Numerals (same denominator)

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Solve these, write your answer as a mixed numeral

The whole sheet uses the same method. Convert to an improper fraction , add or subtract, simplify then change back to mixed numbers.

1 $3\frac{1}{4} - 1\frac{3}{4}$

Improper Improper Answer

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

Simplify Rewrite as mixed

2 $4\frac{3}{10} - 1\frac{7}{10}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

3 $3\frac{3}{8} + 2\frac{7}{8}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

4 $2\frac{5}{6} + 3\frac{5}{6}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

5 $6\frac{1}{4} - 2\frac{3}{4}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

6 $3\frac{4}{9} - 1\frac{1}{9}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

7 $6\frac{1}{6} - 1\frac{5}{6}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

8 $3\frac{3}{4} + 2\frac{3}{4}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

9 $3\frac{5}{12} - \frac{8}{12}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

10 $5\frac{3}{8} - 1\frac{7}{8}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

11 $1\frac{7}{10} + 2\frac{7}{10}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

12 $2\frac{7}{12} + 3\frac{9}{12}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

13 $5\frac{3}{4} + 3\frac{3}{4}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

14 $3\frac{3}{20} - 1\frac{8}{20}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

15 $2\frac{5}{16} - \frac{11}{16}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

16 $1\frac{6}{10} + 3\frac{9}{10}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

17 $3\frac{4}{9} + 1\frac{8}{9}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

Adding/Subtracting Mixed Numerals (same denominator)

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Solve these, write your answer as a mixed numeral

The whole sheet uses the same method. Convert to an improper fraction , add or subtract, simplify then change back to mixed numbers.

1 $3\frac{1}{4} - 1\frac{3}{4}$

$= \frac{13}{4} - \frac{7}{4} = \frac{6}{4}$ (Improper, Improper, Answer)

$= \frac{3}{2} = 1\frac{1}{2}$ (Simplify, Rewrite as mixed)

2 $4\frac{3}{10} - 1\frac{7}{10}$

$= \frac{43}{10} - \frac{17}{10} = \frac{26}{10}$

$= \frac{13}{5} = 2\frac{3}{5}$

3 $3\frac{3}{8} + 2\frac{7}{8}$

$= \frac{27}{8} + \frac{23}{8} = \frac{50}{8}$

$= \frac{25}{4} = 6\frac{1}{4}$

4 $2\frac{5}{6} + 3\frac{5}{6}$

$= \frac{17}{6} + \frac{23}{6} = \frac{40}{6}$

$= \frac{20}{3} = 6\frac{2}{3}$

5 $6\frac{1}{4} - 2\frac{3}{4}$

$= \frac{25}{4} - \frac{11}{4} = \frac{14}{4}$

$= \frac{7}{2} = 3\frac{1}{2}$

6 $3\frac{4}{9} - 1\frac{1}{9}$

$= \frac{31}{9} - \frac{10}{9} = \frac{21}{9}$

$= \frac{7}{3} = 2\frac{1}{3}$

7 $6\frac{1}{6} - 1\frac{5}{6}$

$= \frac{37}{6} - \frac{11}{6} = \frac{26}{6}$

$= \frac{13}{3} = 4\frac{1}{3}$

8 $3\frac{3}{4} + 2\frac{3}{4}$

$= \frac{15}{4} + \frac{11}{4} = \frac{26}{4}$

$= \frac{13}{2} = 6\frac{1}{2}$

9 $3\frac{5}{12} - \frac{8}{12}$

$= \frac{41}{12} - \frac{8}{12} = \frac{33}{12}$

$= \frac{11}{4} = 2\frac{3}{4}$

10 $5\frac{3}{8} - 1\frac{7}{8}$

$= \frac{43}{8} - \frac{15}{8} = \frac{28}{8}$

$= \frac{7}{2} = 3\frac{1}{2}$

11 $1\frac{7}{10} + 2\frac{7}{10}$

$= \frac{17}{10} + \frac{27}{10} = \frac{44}{10}$

$= \frac{22}{5} = 4\frac{2}{5}$

12 $2\frac{7}{12} + 3\frac{9}{12}$

$= \frac{31}{12} + \frac{45}{12} = \frac{76}{12}$

$= \frac{19}{3} = 6\frac{1}{3}$

13 $5\frac{3}{4} + 3\frac{3}{4}$

$= \frac{23}{4} + \frac{15}{4} = \frac{38}{4}$

$= \frac{19}{2} = 9\frac{1}{2}$

14 $3\frac{3}{20} - 1\frac{8}{20}$

$= \frac{63}{20} - \frac{28}{20} = \frac{35}{20}$

$= \frac{7}{4} = 1\frac{3}{4}$

15 $2\frac{5}{16} - \frac{11}{16}$

$= \frac{37}{16} - \frac{11}{16} = \frac{26}{16}$

$= \frac{13}{8} = 1\frac{5}{8}$

16 $1\frac{6}{10} + 3\frac{9}{10}$

$= \frac{16}{10} + \frac{39}{10} = \frac{55}{10}$

$= \frac{11}{2} = 5\frac{1}{2}$

17 $3\frac{4}{9} + 1\frac{8}{9}$

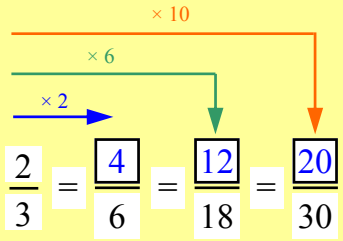
$= \frac{31}{9} + \frac{17}{9} = \frac{48}{9}$

$= \frac{16}{3} = 5\frac{1}{3}$

Adding/Subtracting Fractions
(Different Denominator)

The main requirement for adding fractions with different denominators is that you know how to get an equivalent fraction, this means multiplying the fraction's numerator and denominator (top and bottom number) by a number.

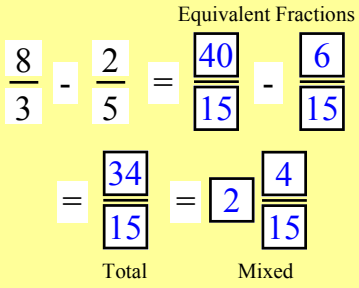
In column 1 you are asked in questions 1 through 7 to find equivalent fractions to the one given. Look at the example below, we are given $^2/_3$ so the denominator is 3. The first question has a denominator of 6, what do you multiply 3 by to get 6? The answer is 2, so multiply the numerator by 2 and you get $2 \times 2 = 4$ so the first answer is 4. The 2nd question has a denominator of 18, what do you multiply 3 by to get 18? The answer is 6, so multiply the numerator by 6 and you get $2 \times 6 = 12$, so the 2nd answer is 12. The same method again will give you 20 for the last question.



Questions 8 through 10 are additions with different denominators, you have to find the LCM (Lowest Common Multiple), in other words the smallest number that both numbers divide into. These three have the LCM found for you with the denominator typed in. In question 8 only one fraction needs to be changed, (because 2 and 8 both divide into 8), questions 9 and 10 require both fractions to be changed. Note that you don't just multiply the denominators together to get the LCM, an example would be 6 & 10 the LCM is 30 not $6 \times 10 = 60$.

The 2nd column are additions which will give a proper fraction answer, the last two questions will need to be simplified. In Column 3 the answer will be an improper fraction and so an extra step is required to change it to a mixed number. An example is below.

Example



Adding/Subtracting Fractions (Different Denominator)

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Find the numerator for these fractions to make them equivalent fractions.

1 $\frac{1}{2} = \frac{\square}{4} = \frac{\square}{8} = \frac{\square}{20}$

2 $\frac{1}{3} = \frac{\square}{6} = \frac{\square}{12} = \frac{\square}{18}$

3 $\frac{2}{3} = \frac{\square}{9} = \frac{\square}{12} = \frac{\square}{21}$

4 $\frac{3}{5} = \frac{\square}{10} = \frac{\square}{15} = \frac{\square}{25}$

5 $\frac{4}{6} = \frac{\square}{3} = \frac{\square}{30} = \frac{\square}{24}$

6 Change these to eighths

$\frac{1}{2} = \frac{\square}{\square}$, $\frac{1}{4} = \frac{\square}{\square}$, $\frac{3}{4} = \frac{\square}{\square}$

7 Change these to twelfths

$\frac{1}{2} = \frac{\square}{\square}$, $\frac{2}{3} = \frac{\square}{\square}$, $\frac{3}{4} = \frac{\square}{\square}$

Add/subtract these. The denominator is given for the equivalent fractions

8 $\frac{1}{2} + \frac{3}{8}$

$= \frac{\square}{8} + \frac{\square}{8} = \frac{\square}{\square}$

9 $\frac{1}{2} + \frac{1}{3}$

$= \frac{\square}{6} + \frac{\square}{6} = \frac{\square}{\square}$

10 $\frac{3}{5} - \frac{1}{3}$

$= \frac{\square}{15} - \frac{\square}{15} = \frac{\square}{\square}$

Where possible simplify

11 $\frac{1}{2} + \frac{3}{10}$

Equivalent Fractions, Total, Simplify

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

12 $\frac{1}{2} - \frac{1}{3}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

13 $\frac{2}{3} + \frac{1}{4}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

14 $\frac{7}{10} - \frac{1}{4}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

15 $\frac{3}{5} + \frac{1}{6}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

16 $\frac{1}{4} - \frac{2}{9}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

17 $\frac{2}{5} + \frac{7}{20}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

18 $\frac{7}{10} - \frac{1}{6}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

The answer to these will be a mixed numeral

19 $\frac{3}{5} + \frac{1}{2} = \frac{\square}{\square} + \frac{\square}{\square}$

Equivalent Fractions

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

Total, Mixed

20 $\frac{4}{10} + \frac{5}{6} = \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

21 $\frac{10}{3} - \frac{4}{5} = \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

22 $\frac{12}{7} - \frac{2}{3} = \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

23 $\frac{7}{9} + \frac{1}{2} = \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

24 $\frac{22}{15} - \frac{1}{4} = \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

25 $\frac{2}{3} + \frac{8}{11} = \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

Adding/Subtracting Fractions (Different Denominator)

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Find the numerator for these fractions to make them equivalent fractions.

1 $\frac{1}{2} = \frac{\boxed{2}}{4} = \frac{\boxed{4}}{8} = \frac{\boxed{10}}{20}$

2 $\frac{1}{3} = \frac{\boxed{2}}{6} = \frac{\boxed{4}}{12} = \frac{\boxed{6}}{18}$

3 $\frac{2}{3} = \frac{\boxed{6}}{9} = \frac{\boxed{8}}{12} = \frac{\boxed{14}}{21}$

4 $\frac{3}{5} = \frac{\boxed{6}}{10} = \frac{\boxed{9}}{15} = \frac{\boxed{15}}{25}$

5 $\frac{4}{6} = \frac{\boxed{2}}{3} = \frac{\boxed{20}}{30} = \frac{\boxed{16}}{24}$

6 Change these to eighths

$\frac{1}{2} = \frac{\boxed{4}}{\boxed{8}}$, $\frac{1}{4} = \frac{\boxed{2}}{\boxed{8}}$, $\frac{3}{4} = \frac{\boxed{6}}{\boxed{8}}$

7 Change these to twelfths

$\frac{1}{2} = \frac{\boxed{6}}{\boxed{12}}$, $\frac{2}{3} = \frac{\boxed{8}}{\boxed{12}}$, $\frac{3}{4} = \frac{\boxed{9}}{\boxed{12}}$

Add/subtract these. The denominator is given for the equivalent fractions

8 $\frac{1}{2} + \frac{3}{8}$

$= \frac{\boxed{4}}{\boxed{8}} + \frac{\boxed{3}}{\boxed{8}} = \frac{\boxed{7}}{\boxed{8}}$

9 $\frac{1}{2} + \frac{1}{3}$

$= \frac{\boxed{3}}{\boxed{6}} + \frac{\boxed{2}}{\boxed{6}} = \frac{\boxed{5}}{\boxed{6}}$

10 $\frac{3}{5} - \frac{1}{3}$

$= \frac{\boxed{9}}{\boxed{15}} - \frac{\boxed{5}}{\boxed{15}} = \frac{\boxed{4}}{\boxed{15}}$

Where possible simplify

11 $\frac{1}{2} + \frac{3}{10}$

Equivalent Fractions, Total, Simplify

$= \frac{\boxed{5}}{\boxed{10}} + \frac{\boxed{3}}{\boxed{10}} = \frac{\boxed{8}}{\boxed{10}} = \frac{\boxed{4}}{\boxed{5}}$

12 $\frac{1}{2} - \frac{1}{3}$

$= \frac{\boxed{3}}{\boxed{6}} - \frac{\boxed{2}}{\boxed{6}} = \frac{\boxed{1}}{\boxed{6}}$

13 $\frac{2}{3} + \frac{1}{4}$

$= \frac{\boxed{8}}{\boxed{12}} + \frac{\boxed{3}}{\boxed{12}} = \frac{\boxed{11}}{\boxed{12}}$

14 $\frac{7}{10} - \frac{1}{4}$

$= \frac{\boxed{14}}{\boxed{20}} - \frac{\boxed{5}}{\boxed{20}} = \frac{\boxed{9}}{\boxed{20}}$

15 $\frac{3}{5} + \frac{1}{6}$

$= \frac{\boxed{18}}{\boxed{30}} + \frac{\boxed{5}}{\boxed{30}} = \frac{\boxed{23}}{\boxed{30}}$

16 $\frac{1}{4} - \frac{2}{9}$

$= \frac{\boxed{9}}{\boxed{36}} - \frac{\boxed{8}}{\boxed{36}} = \frac{\boxed{1}}{\boxed{36}}$

17 $\frac{2}{5} + \frac{7}{20}$

$= \frac{\boxed{8}}{\boxed{20}} + \frac{\boxed{7}}{\boxed{20}} = \frac{\boxed{15}}{\boxed{20}} = \frac{\boxed{3}}{\boxed{4}}$

18 $\frac{7}{10} - \frac{1}{6}$

$= \frac{\boxed{21}}{\boxed{30}} - \frac{\boxed{5}}{\boxed{30}} = \frac{\boxed{16}}{\boxed{30}} = \frac{\boxed{8}}{\boxed{15}}$

The answer to these will be a mixed numeral

19 $\frac{3}{5} + \frac{1}{2}$

Equivalent Fractions

$= \frac{\boxed{6}}{\boxed{10}} + \frac{\boxed{5}}{\boxed{10}}$

$= \frac{\boxed{11}}{\boxed{10}} = \boxed{1}\frac{\boxed{1}}{\boxed{10}}$

Total, Mixed

20 $\frac{4}{10} + \frac{5}{6}$

$= \frac{\boxed{12}}{\boxed{30}} + \frac{\boxed{25}}{\boxed{30}}$

$= \frac{\boxed{37}}{\boxed{30}} = \boxed{1}\frac{\boxed{7}}{\boxed{30}}$

21 $\frac{10}{3} - \frac{4}{5}$

$= \frac{\boxed{50}}{\boxed{15}} - \frac{\boxed{12}}{\boxed{15}}$

$= \frac{\boxed{38}}{\boxed{15}} = \boxed{2}\frac{\boxed{8}}{\boxed{15}}$

22 $\frac{12}{7} - \frac{2}{3}$

$= \frac{\boxed{36}}{\boxed{21}} - \frac{\boxed{14}}{\boxed{21}}$

$= \frac{\boxed{22}}{\boxed{21}} = \boxed{1}\frac{\boxed{1}}{\boxed{21}}$

23 $\frac{7}{9} + \frac{1}{2}$

$= \frac{\boxed{14}}{\boxed{18}} + \frac{\boxed{9}}{\boxed{18}}$

$= \frac{\boxed{23}}{\boxed{18}} = \boxed{1}\frac{\boxed{5}}{\boxed{18}}$

24 $\frac{22}{15} - \frac{1}{4}$

$= \frac{\boxed{88}}{\boxed{60}} - \frac{\boxed{15}}{\boxed{60}}$

$= \frac{\boxed{73}}{\boxed{60}} = \boxed{1}\frac{\boxed{13}}{\boxed{60}}$

25 $\frac{2}{3} + \frac{8}{11}$

$= \frac{\boxed{22}}{\boxed{33}} + \frac{\boxed{24}}{\boxed{33}}$

$= \frac{\boxed{46}}{\boxed{33}} = \boxed{1}\frac{\boxed{13}}{\boxed{33}}$

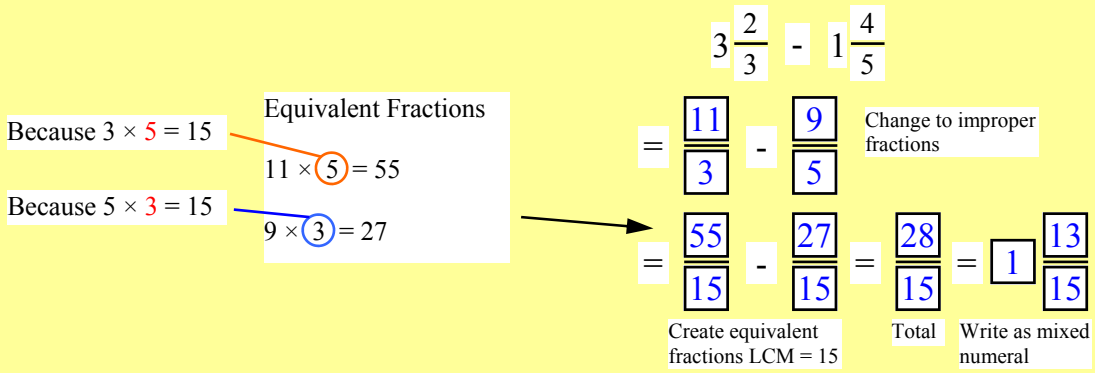
Adding and Subtracting Mixed Numerals (Different Denominators)

This is a challenging sheet and some students may experience difficulty with it. The method for each question is the same and is:

- Change both fractions to improper fractions
- Find the LCM (Lowest Common Multiple) of the 2 denominators
- Create equivalent fractions with the matching denominators
- Add or subtract, you will get an improper fraction as the answer
- Convert the improper fraction to a mixed numeral, and you are done.

Look at the steps below in the example, the first question has the denominators filled in for you to get started.

Example



Adding and Subtracting Mixed Numerals (Different Denominators)

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Add or subtract these fractions, give your answer as a mixed number.



The whole sheet uses the same method. Convert to an improper fraction , create equivalent fractions, add or subtract, simplify if required then change back to mixed numbers.

1 $2\frac{2}{3} + 2\frac{3}{4}$

$= \frac{\square}{3} + \frac{\square}{4}$ Change to improper fractions

$= \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{12} = \square\frac{\square}{12}$

Create equivalent fractions — Total — Write as mixed numeral

2 $2\frac{4}{5} + 3\frac{1}{2}$

$= \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

3 $2\frac{3}{4} - 1\frac{2}{5}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

4 $3\frac{7}{10} - 1\frac{4}{5}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

5 $5\frac{4}{9} - 2\frac{2}{3}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

6 $1\frac{1}{5} + 1\frac{2}{7}$

$= \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

7 $2\frac{3}{4} + 3\frac{2}{12}$

$= \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

8 $4\frac{3}{4} - 2\frac{7}{10}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

9 $5\frac{5}{6} + 2\frac{1}{4}$

$= \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

10 $4\frac{2}{5} - 2\frac{1}{2}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

11 $3\frac{3}{4} - 1\frac{4}{5}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

12 $5\frac{1}{3} - 3\frac{2}{5}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

13 $4\frac{3}{10} + 2\frac{4}{5}$

$= \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

14 $6\frac{3}{7} - 4\frac{1}{2}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

15 $4\frac{1}{4} - 2\frac{5}{6}$

$= \frac{\square}{\square} - \frac{\square}{\square}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

16 $3\frac{2}{7} + 1\frac{1}{3}$

$= \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

17 $5\frac{1}{2} + 2\frac{2}{3}$

$= \frac{\square}{\square} + \frac{\square}{\square}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

Adding and Subtracting Mixed Numerals (Different Denominators)

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Add or subtract these fractions, give your answer as a mixed number.

The whole sheet uses the same method. Convert to an improper fraction , create equivalent fractions, add or subtract, simplify if required then change back to mixed numbers.

1 $2\frac{2}{3} + 2\frac{3}{4}$

$= \frac{8}{3} + \frac{11}{4}$ Change to improper fractions

$= \frac{32}{12} + \frac{33}{12} = \frac{65}{12} = 5\frac{5}{12}$

Create equivalent fractions — Total — Write as mixed numeral

2 $2\frac{4}{5} + 3\frac{1}{2}$

$= \frac{14}{5} + \frac{7}{2}$

$= \frac{28}{10} + \frac{35}{10} = \frac{63}{10} = 6\frac{3}{10}$

3 $2\frac{3}{4} - 1\frac{2}{5}$

$= \frac{11}{4} - \frac{7}{5}$

$= \frac{55}{20} - \frac{28}{20} = \frac{27}{20} = 1\frac{7}{20}$

4 $3\frac{7}{10} - 1\frac{4}{5}$

$= \frac{37}{10} - \frac{9}{5}$

$= \frac{37}{10} - \frac{18}{10} = \frac{19}{10} = 1\frac{9}{10}$

5 $5\frac{4}{9} - 2\frac{2}{3}$

$= \frac{49}{9} - \frac{8}{3}$

$= \frac{49}{9} - \frac{24}{9} = \frac{25}{9} = 2\frac{7}{9}$

6 $1\frac{1}{5} + 1\frac{2}{7}$

$= \frac{6}{5} + \frac{9}{7}$

$= \frac{42}{35} + \frac{45}{35} = \frac{87}{35} = 2\frac{17}{35}$

7 $2\frac{3}{4} + 3\frac{2}{12}$

$= \frac{11}{4} + \frac{38}{12}$

$= \frac{33}{12} + \frac{38}{12} = \frac{71}{12} = 5\frac{11}{12}$

8 $4\frac{3}{4} - 2\frac{7}{10}$

$= \frac{19}{4} - \frac{27}{10}$

$= \frac{95}{20} - \frac{54}{20} = \frac{41}{20} = 2\frac{1}{20}$

9 $5\frac{5}{6} + 2\frac{1}{4}$

$= \frac{35}{6} + \frac{9}{4}$

$= \frac{70}{12} + \frac{27}{12} = \frac{97}{12} = 8\frac{1}{12}$

10 $4\frac{2}{5} - 2\frac{1}{2}$

$= \frac{22}{5} - \frac{5}{2}$

$= \frac{44}{10} - \frac{25}{10} = \frac{19}{10} = 1\frac{9}{10}$

11 $3\frac{3}{4} - 1\frac{4}{5}$

$= \frac{15}{4} - \frac{9}{5}$

$= \frac{75}{20} - \frac{36}{20} = \frac{39}{20} = 1\frac{19}{20}$

12 $5\frac{1}{3} - 3\frac{2}{5}$

$= \frac{16}{3} - \frac{17}{5}$

$= \frac{80}{15} - \frac{51}{15} = \frac{29}{15} = 1\frac{14}{15}$

13 $4\frac{3}{10} + 2\frac{4}{5}$

$= \frac{43}{10} + \frac{14}{5}$

$= \frac{43}{10} + \frac{28}{10} = \frac{71}{10} = 7\frac{1}{10}$

14 $6\frac{3}{7} - 4\frac{1}{2}$

$= \frac{45}{7} - \frac{9}{2}$

$= \frac{90}{14} - \frac{63}{14} = \frac{27}{14} = 1\frac{13}{14}$

15 $4\frac{1}{4} - 2\frac{5}{6}$

$= \frac{17}{4} - \frac{17}{6}$

$= \frac{51}{12} - \frac{34}{12} = \frac{17}{12} = 1\frac{5}{12}$

16 $3\frac{2}{7} + 1\frac{1}{3}$

$= \frac{23}{7} + \frac{4}{3}$

$= \frac{69}{21} + \frac{28}{21} = \frac{97}{21} = 4\frac{13}{21}$

17 $5\frac{1}{2} + 2\frac{2}{3}$

$= \frac{11}{2} + \frac{8}{3}$

$= \frac{33}{6} + \frac{16}{6} = \frac{49}{6} = 8\frac{1}{6}$

Multiplying Fractions

When fractions are multiplied, multiply straight across the top (the numerators) and straight across the bottom (the denominators). Unlike addition and subtraction the denominators are included in the operation.

Column 1 asks you first to multiply a whole number and a fraction. A whole number has no denominator so just multiply the number and the numerator (top number) of the fraction. A whole number written as a fraction is just the number over 1. For example $3 = {}^{3}/_{1}$ and $136 = {}^{136}/_{1}$. An example is below.

$$\frac{3}{19} \times 3 = \frac{9}{19} \quad \leftarrow 3 \times 3 = 9$$

From question 4 on, the column is all done the same way. The top numbers multiplied together and the denominators multiplied together. It doesn't matter if there are two fractions or six fractions multiplied, the process will still be the same. There will also be whole numbers mixed in, make sure you multiply the whole number only with the numerators (top numbers), not the denominator. Note that none of these questions can be simplified.

Column 2 is the same as column 1 only the fraction answer can be simplified.

Column 3 is a further extension, simplify the fractions and convert them to mixed or whole numbers.

$$\frac{2}{3} \times \frac{4}{7} = \frac{8}{21}$$

$2 \times 4 = 8$

$3 \times 7 = 21$

$$\frac{5}{6} \times \frac{3}{4} = \frac{15}{24} = \frac{5}{8}$$

HCF = 3
so ÷ top and bottom by 3

$$\frac{4}{5} \times \frac{10}{3} = \frac{40}{15} = \frac{8}{3} = 2\frac{2}{3}$$

Total — Simplify — Mixed Numeral

Multiplying Fractions

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Multiply these

1 $\frac{1}{3} \times 2 = \frac{\square}{\square}$

2 $\frac{2}{7} \times 3 = \frac{\square}{\square}$

3 $7 \times \frac{2}{15} = \frac{\square}{\square}$

4 $\frac{2}{3} \times \frac{1}{5} = \frac{\square}{\square}$

5 $\frac{1}{2} \times \frac{3}{4} = \frac{\square}{\square}$

6 $\frac{5}{7} \times \frac{2}{3} = \frac{\square}{\square}$

7 $\frac{4}{9} \times \frac{1}{4} = \frac{\square}{\square}$

8 $\frac{4}{11} \times \frac{1}{6} = \frac{\square}{\square}$

9 $\frac{3}{5} \times \frac{1}{4} = \frac{\square}{\square}$

10 $\frac{5}{6} \times \frac{5}{7} = \frac{\square}{\square}$

11 $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{5} = \frac{\square}{\square}$

12 $\frac{3}{5} \times \frac{1}{4} \times \frac{1}{2} = \frac{\square}{\square}$

13 $\frac{5}{9} \times \frac{2}{3} \times 2 = \frac{\square}{\square}$

14 $\frac{3}{11} \times 4 \times \frac{4}{5} = \frac{\square}{\square}$

15 $\frac{2}{5} \times \frac{2}{3} \times 2 = \frac{\square}{\square}$

16 $5 \times \frac{3}{47} \times 2 = \frac{\square}{\square}$

Answer these, they will simplify.

17 $\frac{3}{8} \times 2 = \frac{\square}{\square} = \frac{\square}{\square}$

18 $\frac{4}{15} \times 3 = \frac{\square}{\square} = \frac{\square}{\square}$

19 $4 \times \frac{3}{20} = \frac{\square}{\square} = \frac{\square}{\square}$

20 $\frac{2}{3} \times \frac{3}{5} = \frac{\square}{\square} = \frac{\square}{\square}$

21 $\frac{5}{6} \times \frac{4}{7} = \frac{\square}{\square} = \frac{\square}{\square}$

22 $\frac{3}{10} \times \frac{5}{8} = \frac{\square}{\square} = \frac{\square}{\square}$

23 $\frac{5}{9} \times \frac{3}{4} = \frac{\square}{\square} = \frac{\square}{\square}$

24 $\frac{3}{8} \times \frac{4}{6} = \frac{\square}{\square} = \frac{\square}{\square}$

25 $\frac{7}{12} \times \frac{2}{5} = \frac{\square}{\square} = \frac{\square}{\square}$

26 $\frac{13}{14} \times \frac{2}{3} = \frac{\square}{\square} = \frac{\square}{\square}$

27 $\frac{7}{10} \times \frac{5}{9} = \frac{\square}{\square} = \frac{\square}{\square}$

28 $\frac{2}{3} \times \frac{4}{5} \times \frac{3}{4} = \frac{\square}{\square} = \frac{\square}{\square}$

29 $\frac{4}{5} \times \frac{5}{6} \times \frac{1}{3} = \frac{\square}{\square} = \frac{\square}{\square}$

30 $\frac{4}{10} \times \frac{3}{4} \times \frac{1}{2} = \frac{\square}{\square} = \frac{\square}{\square}$

31 $\frac{4}{7} \times 2 \times \frac{5}{8} = \frac{\square}{\square} = \frac{\square}{\square}$

32 $\frac{3}{10} \times \frac{5}{6} \times 3 = \frac{\square}{\square} = \frac{\square}{\square}$

These will result in mixed or whole numbers.

Total Simplify

33 $\frac{4}{5} \times \frac{10}{3} = \frac{\square}{\square} = \frac{\square}{\square} = \square\ \frac{\square}{\square}$

Mixed Numeral

34 $\frac{4}{6} \times \frac{5}{2} = \frac{\square}{\square} = \frac{\square}{\square} = \square\ \frac{\square}{\square}$

35 $\frac{8}{5} \times \frac{15}{6} = \frac{\square}{\square} = \frac{\square}{\square} = \square$

36 $\frac{10}{7} \times \frac{13}{5} = \frac{\square}{\square} = \frac{\square}{\square} = \square\ \frac{\square}{\square}$

37 $\frac{20}{3} \times \frac{5}{4} = \frac{\square}{\square} = \frac{\square}{\square} = \square\ \frac{\square}{\square}$

38 $\frac{11}{5} \times \frac{10}{11} = \frac{\square}{\square} = \frac{\square}{\square} = \square$

39 $\frac{9}{4} \times \frac{6}{5} = \frac{\square}{\square} = \frac{\square}{\square} = \square\ \frac{\square}{\square}$

Multiplying Fractions

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Multiply these

1 $\frac{1}{3} \times 2 = \frac{2}{3}$

2 $\frac{2}{7} \times 3 = \frac{6}{7}$

3 $7 \times \frac{2}{15} = \frac{14}{15}$

4 $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$

5 $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

6 $\frac{5}{7} \times \frac{2}{3} = \frac{10}{21}$

7 $\frac{4}{9} \times \frac{1}{4} = \frac{4}{36}$

8 $\frac{4}{11} \times \frac{1}{6} = \frac{4}{66}$

9 $\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$

10 $\frac{5}{6} \times \frac{5}{7} = \frac{25}{42}$

11 $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{5} = \frac{1}{30}$

12 $\frac{3}{5} \times \frac{1}{4} \times \frac{1}{2} = \frac{3}{40}$

13 $\frac{5}{9} \times \frac{2}{3} \times 2 = \frac{20}{27}$

14 $\frac{3}{11} \times 4 \times \frac{4}{5} = \frac{48}{55}$

15 $\frac{2}{5} \times \frac{2}{3} \times 2 = \frac{8}{15}$

16 $5 \times \frac{3}{47} \times 2 = \frac{30}{47}$

Answer these, they will simplify.

17 $\frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$

18 $\frac{4}{15} \times 3 = \frac{12}{15} = \frac{4}{5}$

19 $4 \times \frac{3}{20} = \frac{12}{20} = \frac{3}{5}$

20 $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

21 $\frac{5}{6} \times \frac{4}{7} = \frac{20}{42} = \frac{10}{21}$

22 $\frac{3}{10} \times \frac{5}{8} = \frac{15}{80} = \frac{3}{16}$

23 $\frac{5}{9} \times \frac{3}{4} = \frac{15}{36} = \frac{5}{12}$

24 $\frac{3}{8} \times \frac{4}{6} = \frac{12}{48} = \frac{1}{4}$

25 $\frac{7}{12} \times \frac{2}{5} = \frac{14}{60} = \frac{7}{30}$

26 $\frac{13}{14} \times \frac{2}{3} = \frac{26}{42} = \frac{13}{21}$

27 $\frac{7}{10} \times \frac{5}{9} = \frac{35}{90} = \frac{7}{18}$

28 $\frac{2}{3} \times \frac{4}{5} \times \frac{3}{4} = \frac{24}{60} = \frac{2}{5}$

29 $\frac{4}{5} \times \frac{5}{6} \times \frac{1}{3} = \frac{20}{90} = \frac{2}{9}$

30 $\frac{4}{10} \times \frac{3}{4} \times \frac{1}{2} = \frac{12}{80} = \frac{3}{20}$

31 $\frac{4}{7} \times 2 \times \frac{5}{8} = \frac{40}{56} = \frac{5}{7}$

32 $\frac{3}{10} \times \frac{5}{6} \times 3 = \frac{45}{60} = \frac{3}{4}$

These will result in mixed or whole numbers.

Total / Simplify

33 $\frac{4}{5} \times \frac{10}{3} = \frac{40}{15} = \frac{8}{3} = 2\frac{2}{3}$

34 $\frac{4}{6} \times \frac{5}{2} = \frac{20}{12} = \frac{5}{3} = 1\frac{2}{3}$

35 $\frac{8}{5} \times \frac{15}{6} = \frac{120}{30} = \frac{4}{1} = 4$

36 $\frac{10}{7} \times \frac{13}{5} = \frac{130}{35} = \frac{26}{7} = 3\frac{5}{7}$

37 $\frac{20}{3} \times \frac{5}{4} = \frac{100}{12} = \frac{25}{3} = 8\frac{1}{3}$

38 $\frac{11}{5} \times \frac{10}{11} = \frac{110}{55} = \frac{2}{1} = 2$

39 $\frac{9}{4} \times \frac{6}{5} = \frac{54}{20} = \frac{27}{10} = 2\frac{7}{10}$

Mixed Numeral

Fraction of a Quantity

Quantities can be litres, kg, even $, they are a measurement of anything, not just a volume. In column 1 two quantities are given and the fraction is required. Treat these problems as just simplification exercises. Create a fraction with the first value 'over' the second value, then simplify. Your answer will be a fraction with no units (see below left). Question 7 through 12 require a conversion to be made first so that both quantities are in the same units. Find the conversion, then follow the same method, form a fraction and simplify. (See below right)

HCF = 8
8 ÷ 8 = 1
24 ÷ 8 = 3

8 hr of 24 hr $= \frac{8}{24} = \frac{1}{3}$

900 mL of 2.4 L

2.4 L = 2 400 mL $= \frac{900}{2\,400} = \frac{3}{8}$

HCF = 300
900 ÷ 300 = 3
2 400 ÷ 300 = 8

In column 2 a fraction of a quantity is required. These problems are the same as multiplying fractions with whole numbers. Multiply the numerator (top number) by the whole number, the denominator stays the same. The denominator will divide into the numerator and you will be left with a whole number answer, after you write the answer put the units on the end.

3 × 52 = 156

$\frac{3}{4} \times 52\text{ m} = \frac{156}{4} = 39\text{ m}$

156 ÷ 4 = 39

Question 18 through to the end of the sheet uses the same method, only a conversion is to take place first. Given a quantity, change that quantity to a lower unit (to get a whole number answer). Then multiply by the numerator and divide by the denominator to get the final answer. The example below is 1.2 t, change this to kg (in brackets) = 1 200 kg, multiply by the fraction then simplify.

$\frac{3}{4}$ of 1.2 t [kg]

$= \frac{3}{4} \times$ 1 200 kg ← 1.2 t = 1 200 kg

$= \frac{3\,600}{4} =$ 900 kg

3 600 ÷ 4 = 900 kg

Fraction of a Quantity

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Express these parts of a quantity as fractions

1 3 cm of 12 cm = ☐/☐ = ☐/☐

2 6 L of 20 L = ☐/☐ = ☐/☐

3 $40 of $65 = ☐/☐ = ☐/☐

4 18 m of 36 m = ☐/☐ = ☐/☐

5 66 kg of 90 kg = ☐/☐ = ☐/☐

6 16 hr of 24 hr = ☐/☐ = ☐/☐

Change the larger units to the smaller units to solve

7 3 sec of 2 min

2 min = ☐ sec = ☐/☐ = ☐/☐

8 750 kg of 1 t

1 t = ☐ kg = ☐/☐ = ☐/☐

9 60 cm of 2 m

2 m = ☐ cm = ☐/☐ = ☐/☐

10 18 hr of a day

1 day = ☐ hr = ☐/☐ = ☐/☐

11 50 mm of 1.5 m

1.5 m = ☐ mm = ☐/☐ = ☐/☐

12 700 mL of 4.2 L

4.2 L = ☐ mL = ☐/☐ = ☐/☐

Find the new quantity make sure you show units

Example

$\frac{8}{10} \times 5\text{ L} = \frac{40}{10} = 4\text{ L}$

13 $\frac{2}{3} \times \$120 = \frac{☐}{☐} = $ ____

14 $\frac{3}{4} \times 60\text{ cm} = \frac{☐}{☐} = $ ____

15 $\frac{2}{5} \times 75\text{ kg} = \frac{☐}{☐} = $ ____

16 $\frac{3}{8} \times 16\text{ g} = \frac{☐}{☐} = $ ____

17 $\frac{3}{10} \times 90\text{ m} = \frac{☐}{☐} = $ ____

Change to the smaller units in brackets to solve

18 $\frac{4}{5}$ of 8 cm [mm]

$= \frac{4}{5} \times 80\text{ mm}$

$= \frac{☐}{☐} = $ ____

19 $\frac{2}{3}$ of 2 min [s]

$= \frac{☐}{☐} \times ☐$

$= \frac{☐}{☐} = $ ____

20 $\frac{3}{4}$ of 1.2 t [kg]

$= \frac{☐}{☐} \times ☐$

$= \frac{☐}{☐} = $ ____

21 $\frac{5}{8}$ of 1 day [hr]

$= \frac{☐}{☐} \times ☐$

$= \frac{☐}{☐} = $ ____

22 $\frac{3}{10}$ of $1.30 [c]

$= \frac{☐}{☐} \times ☐$

$= \frac{☐}{☐} = $ ____

23 $\frac{2}{5}$ of 2 m [cm]

$= \frac{☐}{☐} \times ☐$

$= \frac{☐}{☐} = $ ____

24 $\frac{1}{4}$ of 3 L [mL]

$= \frac{☐}{☐} \times ☐$

$= \frac{☐}{☐} = $ ____

25 $\frac{7}{10}$ of $1.10 [c]

$= \frac{☐}{☐} \times ☐$

$= \frac{☐}{☐} = $ ____

26 $\frac{2}{3}$ of 0.6m [cm]

$= \frac{☐}{☐} \times ☐$

$= \frac{☐}{☐} = $ ____

Fraction of a Quantity

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Express these parts of a quantity as fractions

1 3 cm of 12 cm = $\frac{3}{12} = \frac{1}{4}$

2 6 L of 20 L = $\frac{6}{20} = \frac{3}{10}$

3 \$40 of \$65 = $\frac{40}{65} = \frac{8}{13}$

4 18 m of 36 m = $\frac{18}{36} = \frac{1}{2}$

5 66 kg of 90 kg = $\frac{66}{90} = \frac{11}{15}$

6 16 hr of 24 hr = $\frac{16}{24} = \frac{2}{3}$

Change the larger units to the smaller units to solve

7 3 sec of 2 min

2 min = 120 sec = $\frac{3}{120} = \frac{1}{40}$

8 750 kg of 1 t

1 t = 1 000 kg = $\frac{750}{1\,000} = \frac{3}{4}$

9 60 cm of 2 m

2 m = 200 cm = $\frac{60}{200} = \frac{3}{10}$

10 18 hr of a day

1 day = 24 hr = $\frac{18}{24} = \frac{3}{4}$

11 50 mm of 1.5 m

1.5 m = 1 500 mm = $\frac{50}{1\,500} = \frac{1}{30}$

12 700 mL of 4.2 L

4.2 L = 4 200 mL = $\frac{700}{4\,200} = \frac{1}{6}$

Find the new quantity make sure you show units

Example

$\frac{8}{10} \times 5\text{ L} = \frac{40}{10} = 4\text{ L}$

13 $\frac{2}{3} \times \$120 = \frac{240}{3} = \$80$

14 $\frac{3}{4} \times 60\text{ cm} = \frac{180}{4} = 45\text{ cm}$

15 $\frac{2}{5} \times 75\text{ kg} = \frac{150}{5} = 30\text{ kg}$

16 $\frac{3}{8} \times 16\text{ g} = \frac{48}{8} = 6\text{ g}$

17 $\frac{3}{10} \times 90\text{ m} = \frac{270}{10} = 27\text{ m}$

Change to the smaller units in brackets to solve

18 $\frac{4}{5}$ of 8 cm [mm]

$= \frac{4}{5} \times 80\text{ mm}$

$= \frac{320}{5} = 64\text{ mm}$

19 $\frac{2}{3}$ of 2 min [s]

$= \frac{2}{3} \times 120\text{ min}$

$= \frac{240}{3} = 80\text{ s}$

20 $\frac{3}{4}$ of 1.2 t [kg]

$= \frac{3}{4} \times 1\,200\text{ kg}$

$= \frac{3\,600}{4} = 900\text{ kg}$

21 $\frac{5}{8}$ of 1 day [hr]

$= \frac{5}{8} \times 24\text{ hr}$

$= \frac{120}{8} = 15\text{ hr}$

22 $\frac{3}{10}$ of \$1.30 [c]

$= \frac{3}{10} \times 130\text{ c}$

$= \frac{390}{10} = 39\text{ c}$

23 $\frac{2}{5}$ of 2 m [cm]

$= \frac{2}{5} \times 200\text{ cm}$

$= \frac{400}{5} = 80\text{ cm}$

24 $\frac{1}{4}$ of 3 L [mL]

$= \frac{1}{4} \times 3\,000\text{ mL}$

$= \frac{3\,000}{4} = 750\text{ mL}$

25 $\frac{7}{10}$ of \$1.10 [c]

$= \frac{7}{10} \times 110\text{ c}$

$= \frac{770}{10} = 77\text{ c}$

26 $\frac{2}{3}$ of 0.6 m [cm]

$= \frac{2}{3} \times 60\text{ cm}$

$= \frac{120}{3} = 40\text{ cm}$

Multiplying Mixed Numerals

When fractions are multiplied, multiply across the top numbers and across the bottom numbers. As mixed numerals have whole numbers in them you first must change them to improper fractions. Once this has been done the same multiplication method is used.

In column 1 a mixed numeral is multiplied by a proper fraction. The method used is:

- change the mixed numeral to an improper fraction and leave the proper fraction as it is.
- multiply the numerators and write the answer then multiply the denominators and write the answer
- simplify by finding the HCF (Highest Common Factor), which is the largest number that goes into both numbers and divide through
- then rewrite as a mixed numeral or a whole number.

An example is at the top of column 1.

Columns 2 and 3 are done in exactly the same way except there are two mixed numerals so you have to convert both to improper fractions not just one of them. There is an example at the top of column 2.

Multiplying Mixed Numerals

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Multiply these fractions and mixed numbers.

Example

$4\frac{2}{3} \times \frac{5}{6}$ Simplified: ÷ by 2

$= \frac{14}{3} \times \frac{5}{6} = \frac{70}{18} = \frac{35}{9}$

Convert to Improper

$= 3\frac{8}{9}$

1 $\frac{2}{3} \times 2\frac{1}{2}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

2 $4\frac{3}{4} \times \frac{2}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

3 $3\frac{1}{2} \times \frac{4}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

4 $\frac{5}{12} \times 2\frac{3}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

5 $6\frac{1}{2} \times \frac{2}{7}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

Try these, they will result in mixed or whole numbers.

6 $2\frac{3}{4} \times 1\frac{3}{5}$

Improper Fractions Total Simplify

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

Mixed Numeral

7 $5\frac{1}{3} \times 1\frac{1}{2}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square$

8 $1\frac{2}{3} \times 2\frac{1}{4}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

9 $2\frac{2}{5} \times 2\frac{1}{2}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square$

10 $3\frac{1}{2} \times 1\frac{3}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

11 $1\frac{3}{10} \times 3\frac{1}{3}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

12 $7\frac{1}{2} \times 1\frac{1}{7}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

13 $1\frac{1}{4} \times 2\frac{2}{9}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

14 $6\frac{1}{4} \times 1\frac{1}{3}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

15 $1\frac{3}{7} \times 1\frac{4}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

16 $2\frac{2}{3} \times 1\frac{1}{4}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

17 $3\frac{1}{3} \times 1\frac{2}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

18 $1\frac{3}{4} \times 1\frac{3}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

Multiplying Mixed Numerals

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Multiply these fractions and mixed numbers.

Example

$4\frac{2}{3} \times \frac{5}{6}$

Simplified: ÷ by 2

$= \frac{14}{3} \times \frac{5}{6} = \frac{70}{18} = \frac{35}{9}$

Convert to Improper

$= 3\frac{8}{9}$

1 $\frac{2}{3} \times 2\frac{1}{2}$

$= \frac{2}{3} \times \frac{5}{2} = \frac{10}{6} = \frac{5}{3}$

$= 1\frac{2}{3}$

2 $4\frac{3}{4} \times \frac{2}{5}$

$= \frac{19}{4} \times \frac{2}{5} = \frac{38}{20} = \frac{19}{10}$

$= 1\frac{9}{10}$

3 $3\frac{1}{2} \times \frac{4}{5}$

$= \frac{7}{2} \times \frac{4}{5} = \frac{28}{10} = \frac{14}{5}$

$= 2\frac{4}{5}$

4 $\frac{5}{12} \times 2\frac{3}{5}$

$= \frac{5}{12} \times \frac{13}{5} = \frac{65}{60} = \frac{13}{12}$

$= 1\frac{1}{12}$

5 $6\frac{1}{2} \times \frac{2}{7}$

$= \frac{13}{2} \times \frac{2}{7} = \frac{26}{14} = \frac{13}{7}$

$= 1\frac{6}{7}$

Try these, they will result in mixed or whole numbers.

6 $2\frac{3}{4} \times 1\frac{3}{5}$

Improper Fractions — Total — Simplify

$= \frac{11}{4} \times \frac{8}{5} = \frac{88}{20} = \frac{22}{5}$

$= 4\frac{2}{5}$

Mixed Numeral

7 $5\frac{1}{3} \times 1\frac{1}{2}$

$= \frac{16}{3} \times \frac{3}{2} = \frac{48}{6} = \frac{8}{1}$

$= 8$

8 $1\frac{2}{3} \times 2\frac{1}{4}$

$= \frac{5}{3} \times \frac{9}{4} = \frac{45}{12} = \frac{15}{4}$

$= 3\frac{3}{4}$

9 $2\frac{2}{5} \times 2\frac{1}{2}$

$= \frac{12}{5} \times \frac{5}{2} = \frac{60}{10} = \frac{6}{1}$

$= 6$

10 $3\frac{1}{2} \times 1\frac{3}{5}$

$= \frac{7}{2} \times \frac{8}{5} = \frac{56}{10} = \frac{28}{5}$

$= 5\frac{3}{5}$

11 $1\frac{3}{10} \times 3\frac{1}{3}$

$= \frac{13}{10} \times \frac{10}{3} = \frac{130}{30} = \frac{13}{3}$

$= 4\frac{1}{3}$

12 $7\frac{1}{2} \times 1\frac{1}{7}$

$= \frac{15}{2} \times \frac{8}{7} = \frac{120}{14} = \frac{60}{7}$

$= 8\frac{4}{7}$

13 $1\frac{1}{4} \times 2\frac{2}{9}$

$= \frac{5}{4} \times \frac{20}{9} = \frac{100}{36} = \frac{25}{9}$

$= 2\frac{7}{9}$

14 $4\frac{1}{4} \times 1\frac{1}{3}$

$= \frac{25}{4} \times \frac{4}{3} = \frac{100}{12} = \frac{25}{3}$

$= 8\frac{1}{3}$

15 $1\frac{3}{7} \times 1\frac{4}{5}$

$= \frac{10}{7} \times \frac{9}{5} = \frac{90}{35} = \frac{18}{7}$

$= 2\frac{4}{7}$

16 $2\frac{2}{3} \times 1\frac{1}{4}$

$= \frac{8}{3} \times \frac{5}{4} = \frac{40}{12} = \frac{10}{3}$

$= 3\frac{1}{3}$

17 $3\frac{1}{3} \times 1\frac{2}{5}$

$= \frac{10}{3} \times \frac{7}{5} = \frac{70}{15} = \frac{14}{3}$

$= 4\frac{2}{3}$

18 $1\frac{3}{4} \times 1\frac{3}{5}$

$= \frac{7}{4} \times \frac{8}{5} = \frac{56}{20} = \frac{14}{5}$

$= 2\frac{4}{5}$

Multiplying Fractions Using Cancellation

When fractions are canceled they are simplified before multiplication, but instead of simplifying the fraction's numerator and denominator you simplify diagonally. This means that the numerator with one fraction is cancelled against the denominator of the other and vice versa.

In Column 1 the fractions are either proper or improper, you have to cancel across diagonals by finding the HCF, the largest number that divides into both numbers and dividing through. Once you divide, strike out the old number and write its replacement, then multiply and change to a mixed or whole number.

Questions 1 - 5 can only be cancelled across one diagonal, questions 6 - 12 can be cancelled across both diagonals.

Column 2 involves proper fractions and mixed numerals, on the line below change the mixed numeral to an improper fraction, rewrite the proper fraction below itself then start canceling. The first 4 questions can be cancelled on one diagonal only, then for the rest of the column canceling will be possible on both diagonals.

Column 3 is a further step again with two mixed numerals having to be multiplied. Convert both this time to improper, cancel, multiply and change back to mixed numerals. There is an example below.

Diagonals 6 and 9 have a HCF of 3:
6 ÷ 3 = 2
9 ÷ 3 = 3

Diagonals 5 and 25 have a HCF of 5:
5 ÷ 5 = 1
25 ÷ 5 = 5

$$\frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{3}{\cancel{9}}} = \frac{10}{3} = 3\frac{1}{3}$$

Convert improper fraction to a mixed numeral

$$1\frac{7}{11} \times 4\frac{1}{8}$$

Diagonals 18 and 8 have a HCF of 2:
18 ÷ 2 = 9
8 ÷ 2 = 4

Diagonals 11 and 33 have a HCF of 11:
11 ÷ 11 = 1
33 ÷ 11 = 3

$$= \frac{\overset{9}{\cancel{18}}}{\underset{1}{\cancel{11}}} \times \frac{\overset{3}{\cancel{33}}}{\underset{4}{\cancel{8}}} = \frac{27}{4} = 6\frac{3}{4}$$

Convert to mixed numerals

Convert improper fraction to a mixed numeral

Multiplying Fractions Using Cancellation

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Use cancelling on one diagonal, then multiply and change from improper to mixed

Example

$\frac{\cancel{16}^{4}}{3} \times \frac{1}{\cancel{4}_{1}} = \frac{4}{3} = 1\frac{1}{3}$

1 $\frac{3}{2} \times \frac{11}{6} = \frac{\square}{\square} = \square\frac{\square}{\square}$

2 $\frac{21}{5} \times \frac{4}{7} = \frac{\square}{\square} = \square\frac{\square}{\square}$

3 $\frac{7}{6} \times \frac{18}{5} = \frac{\square}{\square} = \square\frac{\square}{\square}$

4 $\frac{5}{2} \times \frac{11}{15} = \frac{\square}{\square} = \square\frac{\square}{\square}$

5 $\frac{9}{14} \times \frac{7}{2} = \frac{\square}{\square} = \square\frac{\square}{\square}$



This time cancel across both diagonals

6 $\frac{5}{2} \times \frac{8}{15} = \frac{\square}{\square} = \square\frac{\square}{\square}$

7 $\frac{10}{3} \times \frac{9}{20} = \frac{\square}{\square} = \square\frac{\square}{\square}$

8 $\frac{12}{5} \times \frac{25}{6} = \frac{\square}{\square} = \square$

9 $\frac{3}{4} \times \frac{16}{9} = \frac{\square}{\square} = \square\frac{\square}{\square}$

10 $\frac{50}{7} \times \frac{7}{30} = \frac{\square}{\square} = \square\frac{\square}{\square}$

11 $\frac{55}{9} \times \frac{12}{35} = \frac{\square}{\square} = \square\frac{\square}{\square}$

12 $\frac{17}{20} \times \frac{40}{34} = \frac{\square}{\square} = \square$



Change to improper fractions, then answer as a mixed numeral.

13 $3\frac{1}{2} \times \frac{4}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

14 $\frac{7}{9} \times 1\frac{4}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

15 $2\frac{1}{3} \times \frac{11}{14}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

16 $\frac{8}{11} \times 3\frac{1}{4}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

17 $4\frac{4}{9} \times \frac{3}{10}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

18 $3\frac{3}{4} \times \frac{12}{25}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

19 $\frac{4}{5} \times 1\frac{7}{8}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

20 $5\frac{5}{6} \times \frac{3}{7}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

21 $\frac{20}{27} \times 3\frac{3}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$



Both are mixed numerals, solve these.

22 $1\frac{19}{26} \times 1\frac{3}{10}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

23 $1\frac{9}{35} \times 4\frac{1}{11}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

24 $2\frac{4}{5} \times 1\frac{3}{7}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square$

25 $1\frac{1}{12} \times 1\frac{5}{13}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

26 $2\frac{4}{5} \times 4\frac{1}{6}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

27 $1\frac{5}{7} \times 2\frac{5}{8}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

28 $2\frac{2}{5} \times 2\frac{7}{9}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

29 $3\frac{8}{9} \times 2\frac{7}{10}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

30 $2\frac{11}{12} \times 1\frac{11}{21}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

Multiplying Fractions Using Cancellation

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Use cancelling on one diagonal, then multiply and change from improper to mixed

Example

$\frac{\cancel{16}^{4}}{3} \times \frac{1}{\cancel{4}_{1}} = \frac{4}{3} = 1\frac{1}{3}$

1 $\frac{\cancel{3}^{1}}{2} \times \frac{11}{\cancel{6}_{2}} = \frac{11}{4} = 2\frac{3}{4}$

2 $\frac{\cancel{21}^{3}}{5} \times \frac{4}{\cancel{7}_{1}} = \frac{12}{5} = 2\frac{2}{5}$

3 $\frac{7}{\cancel{6}_{1}} \times \frac{\cancel{18}^{3}}{5} = \frac{21}{5} = 4\frac{1}{5}$

4 $\frac{\cancel{5}^{1}}{2} \times \frac{11}{\cancel{15}_{3}} = \frac{11}{6} = 1\frac{5}{6}$

5 $\frac{9}{\cancel{14}_{2}} \times \frac{\cancel{7}^{1}}{2} = \frac{9}{4} = 2\frac{1}{4}$

This time cancel across both diagonals

6 $\frac{\cancel{5}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{8}^{4}}{\cancel{15}_{3}} = \frac{4}{3} = 1\frac{1}{3}$

7 $\frac{\cancel{10}^{1}}{\cancel{3}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{20}_{2}} = \frac{3}{2} = 1\frac{1}{2}$

8 $\frac{\cancel{12}^{2}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{6}_{1}} = \frac{10}{1} = 10$

9 $\frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{16}^{4}}{\cancel{9}_{3}} = \frac{4}{3} = 1\frac{1}{3}$

10 $\frac{\cancel{50}^{5}}{\cancel{7}_{1}} \times \frac{\cancel{7}^{1}}{\cancel{30}_{3}} = \frac{5}{3} = 1\frac{2}{3}$

11 $\frac{\cancel{55}^{11}}{\cancel{9}_{3}} \times \frac{\cancel{12}^{4}}{\cancel{35}_{7}} = \frac{44}{21} = 2\frac{2}{21}$

12 $\frac{\cancel{17}^{1}}{\cancel{20}_{1}} \times \frac{\cancel{40}^{2}}{\cancel{34}_{2}} = \frac{2}{2} = 1$

Change to improper fractions, then answer as a mixed numeral.

13 $3\frac{1}{2} \times \frac{4}{5}$

$= \frac{\cancel{7}^{1}}{\cancel{2}_{1}} \times \frac{\cancel{4}^{2}}{5} = \frac{14}{5} = 2\frac{4}{5}$

14 $\frac{7}{9} \times 1\frac{4}{5}$

$= \frac{7}{\cancel{9}_{1}} \times \frac{\cancel{9}^{1}}{5} = \frac{7}{5} = 1\frac{2}{5}$

15 $2\frac{1}{3} \times \frac{11}{14}$

$= \frac{\cancel{7}^{1}}{3} \times \frac{11}{\cancel{14}_{2}} = \frac{11}{6} = 1\frac{5}{6}$

16 $\frac{8}{11} \times 3\frac{1}{4}$

$= \frac{\cancel{8}^{2}}{11} \times \frac{13}{\cancel{4}_{1}} = \frac{26}{11} = 2\frac{4}{11}$

17 $4\frac{4}{9} \times \frac{3}{10}$

$= \frac{\cancel{40}^{4}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{\cancel{10}_{1}} = \frac{4}{3} = 1\frac{1}{3}$

18 $3\frac{3}{4} \times \frac{12}{25}$

$= \frac{\cancel{15}^{3}}{\cancel{4}_{1}} \times \frac{\cancel{12}^{3}}{\cancel{25}_{5}} = \frac{9}{5} = 1\frac{4}{5}$

19 $\frac{4}{5} \times 1\frac{7}{8}$

$= \frac{\cancel{4}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{15}^{3}}{\cancel{8}_{2}} = \frac{3}{2} = 1\frac{1}{2}$

20 $5\frac{5}{6} \times \frac{3}{7}$

$= \frac{\cancel{35}^{5}}{\cancel{6}_{2}} \times \frac{\cancel{3}^{1}}{\cancel{7}_{1}} = \frac{5}{2} = 2\frac{1}{2}$

21 $\frac{20}{27} \times 3\frac{3}{5}$

$= \frac{\cancel{20}^{4}}{\cancel{27}_{3}} \times \frac{\cancel{18}^{2}}{\cancel{5}_{1}} = \frac{8}{3} = 2\frac{2}{3}$

Both are mixed numerals, solve these.

22 $1\frac{19}{26} \times 1\frac{3}{10}$

$= \frac{\cancel{45}^{9}}{\cancel{26}_{2}} \times \frac{\cancel{13}^{1}}{\cancel{10}_{2}} = \frac{9}{4} = 2\frac{1}{4}$

23 $1\frac{9}{35} \times 4\frac{1}{11}$

$= \frac{\cancel{44}^{4}}{\cancel{35}_{7}} \times \frac{\cancel{45}^{9}}{\cancel{11}_{1}} = \frac{36}{7} = 5\frac{1}{7}$

24 $2\frac{4}{5} \times 1\frac{3}{7}$

$= \frac{\cancel{14}^{2}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{\cancel{7}_{1}} = \frac{4}{1} = 4$

25 $1\frac{1}{12} \times 1\frac{5}{13}$

$= \frac{\cancel{13}^{1}}{\cancel{12}_{2}} \times \frac{\cancel{18}^{3}}{\cancel{13}_{1}} = \frac{3}{2} = 1\frac{1}{2}$

26 $2\frac{4}{5} \times 4\frac{1}{6}$

$= \frac{\cancel{14}^{7}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{6}_{3}} = \frac{35}{3} = 11\frac{2}{3}$

27 $1\frac{5}{7} \times 2\frac{5}{8}$

$= \frac{\cancel{12}^{3}}{\cancel{7}_{1}} \times \frac{\cancel{21}^{3}}{\cancel{8}_{2}} = \frac{9}{2} = 4\frac{1}{2}$

28 $2\frac{2}{5} \times 2\frac{7}{9}$

$= \frac{\cancel{12}^{4}}{\cancel{5}_{1}} \times \frac{\cancel{25}^{5}}{\cancel{9}_{3}} = \frac{20}{3} = 6\frac{2}{3}$

29 $3\frac{8}{9} \times 2\frac{7}{10}$

$= \frac{\cancel{35}^{7}}{\cancel{9}_{1}} \times \frac{\cancel{27}^{3}}{\cancel{10}_{2}} = \frac{21}{2} = 10\frac{1}{2}$

30 $2\frac{11}{12} \times 1\frac{11}{21}$

$= \frac{\cancel{35}^{5}}{\cancel{12}_{3}} \times \frac{\cancel{32}^{8}}{\cancel{21}_{3}} = \frac{40}{9} = 4\frac{4}{9}$

Dividing Fractions And Reciprocal

The reciprocal of a fraction is the fraction turned upside-down. The reciprocal is used to convert the division process to a multiplication process. When a fraction has a division sign before it turn the fraction upside and change the ÷ sign to a × sign.

Column 1 requires the reciprocal to be found. This is achieved by turning the fraction upside-down. In the case of improper fractions they become proper fractions, and proper fractions become improper fractions. So for example $^{10}/_{11}$ would have a reciprocal of $^{11}/_{10}$ which becomes $1^{1}/_{10}$. Write it as a mixed number for this column because it fully answers the question, but in the division process we would leave it as an improper fraction for calculations. Questions 9 through 15 require you to change mixed and whole numbers to improper fractions then find the reciprocal. A whole number is written as the number over 1, e.g. 23 would be $^{23}/_{1}$, then turn it upside-down and you get $^{1}/_{23}$. Look at the example at the top of the column.

Column 2 uses the reciprocal in the division process. Turn the fraction that is behind the division sign upside-down, then multiply. So there is no division process with fractions calculations, you always use multiplication. Questions 23 through 25 are an extension of this, turn the fractions that have a ÷ sign before them upside-down, make sure you don't turn fractions with a × sign before them upside-down! An example is below, it is done twice, the first time by simplifying, the second by canceling, if you can cancel then use that method.

Column 3 adds a further step in the process as the answer will be a mixed numeral. There is an example at the top of the column

Note that this sheet can be done by canceling or by simplifying after multiplication, the choice is yours. If you do use canceling you will find that the simplifying stage won't be required and you will have unused spaces.

$$\frac{6}{25} \div \frac{3}{10} = \frac{6}{25} \times \frac{10}{3} = \frac{60}{75} = \frac{4}{5}$$

$$\frac{6}{25} \div \frac{3}{10} = \frac{\overset{2}{\cancel{6}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{2}{\cancel{10}}}{\underset{1}{\cancel{3}}} = \frac{4}{5} = \frac{\square}{\square}$$

Dividing Fractions And Reciprocal

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Find the reciprocal, answer as whole or mixed numbers where possible

Example 1 $\frac{55}{9} \rightarrow \frac{9}{55}$ Example 2 $\frac{7}{19} \rightarrow \frac{19}{7} = 2\frac{5}{7}$

1 $\frac{4}{3} \rightarrow$ 2 $\frac{9}{2} \rightarrow$

3 $\frac{4}{5} \rightarrow =$

4 $\frac{10}{3} \rightarrow$ 5 $\frac{73}{8} \rightarrow$

6 $\frac{1}{3} \rightarrow =$

7 $\frac{4}{13} \rightarrow =$

8 $\frac{1}{23} \rightarrow =$

Change to improper fractions then find the reciprocal.

9 $5 = \rightarrow$

10 $2\frac{2}{3} = \rightarrow$

11 $12 = \rightarrow$

12 $5\frac{3}{4} = \rightarrow$

13 $5\frac{3}{10} = \rightarrow$

14 $7 = \rightarrow$

15 $9\frac{1}{2} = \rightarrow$

Find the reciprocal of the fraction behind the ÷, then ×. Simplify if possible.

16 $\frac{1}{4} \div \frac{3}{5} = \times =$

17 $\frac{3}{8} \div \frac{4}{7} = \times =$

18 $\frac{1}{2} \div \frac{5}{9} = \times =$

19 $\frac{2}{5} \div \frac{7}{5} = \times = =$

20 $\frac{7}{11} \div \frac{3}{4} = \times =$

21 $\frac{4}{9} \div \frac{10}{15} = \times = =$

22 $\frac{2}{3} \div \frac{16}{21} = \times = =$

23 $\frac{1}{2} \div \frac{3}{4} \div \frac{5}{6} = \times \times = = =$

24 $\frac{2}{3} \times \frac{3}{10} \div \frac{3}{4} = \times \times = = =$

25 $\frac{5}{12} \div \frac{2}{7} \div 3 = \times \times = = =$

This time the answers will need to be changed to mixed or whole numbers

Example (with canceling) $\frac{3}{4} \div \frac{7}{12} = \frac{3}{\cancel{4}_{1}} \times \frac{\cancel{12}^{3}}{7} = \frac{9}{7}$

Simplify here if you have difficulty with canceling $= = 1\frac{2}{7}$

26 $\frac{3}{4} \div \frac{6}{11} = \times = = =$

27 $\frac{8}{5} \div \frac{8}{11} = \times = = =$

28 $\frac{15}{4} \div \frac{20}{7} = \times = = =$

29 $\frac{9}{8} \div \frac{1}{2} = \times = = =$

30 $\frac{5}{4} \div \frac{5}{7} \div \frac{7}{8} = \times \times = = =$

31 $\frac{2}{3} \times \frac{11}{4} \div \frac{22}{15} = \times \times = = =$

Dividing Fractions And Reciprocal

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Find the reciprocal, answer as whole or mixed numbers where possible

Example 1 Example 2

$\frac{55}{9} \rightarrow \frac{9}{55}$ $\frac{7}{19} \rightarrow \frac{19}{7} = 2\frac{5}{7}$

1 $\frac{4}{3} \rightarrow \frac{3}{4}$ 2 $\frac{9}{2} \rightarrow \frac{2}{9}$

3 $\frac{4}{5} \rightarrow \frac{5}{4} = 1\frac{1}{4}$

4 $\frac{10}{3} \rightarrow \frac{3}{10}$ 5 $\frac{73}{8} \rightarrow \frac{8}{73}$

6 $\frac{1}{3} \rightarrow \frac{3}{1} = 3$

7 $\frac{4}{13} \rightarrow \frac{13}{4} = 3\frac{1}{4}$

8 $\frac{1}{23} \rightarrow \frac{23}{1} = 23$

Change to improper fractions then find the reciprocal.

9 $5 = \frac{5}{1} \rightarrow \frac{1}{5}$

10 $2\frac{2}{3} = \frac{8}{3} \rightarrow \frac{3}{8}$

11 $12 = \frac{12}{1} \rightarrow \frac{1}{12}$

12 $5\frac{3}{4} = \frac{23}{4} \rightarrow \frac{4}{23}$

13 $5\frac{3}{10} = \frac{53}{10} \rightarrow \frac{10}{53}$

14 $7 = \frac{7}{1} \rightarrow \frac{1}{7}$

15 $9\frac{1}{2} = \frac{19}{2} \rightarrow \frac{2}{19}$

Find the reciprocal of the fraction behind the ÷, then ×. Simplify if possible.

16 $\frac{1}{4} \div \frac{3}{5} = \frac{1}{4} \times \frac{5}{3} = \frac{5}{12}$

17 $\frac{3}{8} \div \frac{4}{7} = \frac{3}{8} \times \frac{7}{4} = \frac{21}{32}$

18 $\frac{1}{2} \div \frac{5}{9} = \frac{1}{2} \times \frac{9}{5} = \frac{9}{10}$

19 $\frac{2}{5} \div \frac{7}{5} = \frac{2}{\cancel{5}_1} \times \frac{\cancel{5}^1}{7} = \frac{2}{7} = \frac{\square}{\square}$

20 $\frac{7}{11} \div \frac{3}{4} = \frac{7}{11} \times \frac{4}{3} = \frac{28}{33}$

21 $\frac{4}{9} \div \frac{10}{15} = \frac{\cancel{4}^2}{\cancel{9}_3} \times \frac{\cancel{15}^5}{\cancel{10}_5} = \frac{10}{15} = \frac{2}{3}$

22 $\frac{2}{3} \div \frac{16}{21} = \frac{\cancel{2}^1}{\cancel{3}_1} \times \frac{\cancel{21}^7}{\cancel{16}_8} = \frac{7}{8} = \frac{\square}{\square}$

23 $\frac{1}{2} \div \frac{3}{4} \div \frac{5}{6}$

$= \frac{1}{\cancel{2}_1} \times \frac{\cancel{4}^2}{\cancel{3}_1} \times \frac{\cancel{6}^2}{5} = \frac{4}{5} = \frac{\square}{\square}$

24 $\frac{2}{3} \times \frac{3}{10} \div \frac{3}{4}$

$= \frac{2}{\cancel{3}_1} \times \frac{\cancel{3}^1}{\cancel{10}_5} \times \frac{\cancel{4}^2}{3} = \frac{4}{15} = \frac{\square}{\square}$

25 $\frac{5}{12} \div \frac{2}{7} \div 3$

$= \frac{5}{12} \times \frac{7}{2} \times \frac{1}{3} = \frac{35}{72}$

This time the answers will need to be changed to mixed or whole numbers

Example (with canceling)

$\frac{3}{4} \div \frac{7}{12} = \frac{3}{\cancel{4}_1} \times \frac{\cancel{12}^3}{7} = \frac{9}{7}$

Simplify here if you have difficulty with canceling $= \frac{\square}{\square} = 1\frac{2}{7}$

26 $\frac{3}{4} \div \frac{6}{11} = \frac{\cancel{3}^1}{4} \times \frac{11}{\cancel{6}_2} = \frac{11}{8} = \frac{\square}{\square} = 1\frac{3}{8}$

27 $\frac{8}{5} \div \frac{8}{11} = \frac{\cancel{8}^1}{5} \times \frac{11}{\cancel{8}_1} = \frac{11}{5} = \frac{\square}{\square} = 2\frac{1}{5}$

28 $\frac{15}{4} \div \frac{20}{7} = \frac{\cancel{15}^3}{4} \times \frac{7}{\cancel{20}_4} = \frac{21}{16} = \frac{\square}{\square} = 1\frac{5}{16}$

29 $\frac{9}{8} \div \frac{1}{2} = \frac{9}{\cancel{8}_4} \times \frac{\cancel{2}^1}{1} = \frac{9}{4} = \frac{\square}{\square} = 2\frac{1}{4}$

30 $\frac{5}{4} \div \frac{5}{7} \div \frac{7}{8}$

$= \frac{\cancel{5}^1}{\cancel{4}_1} \times \frac{\cancel{7}^1}{\cancel{5}_1} \times \frac{\cancel{8}^2}{\cancel{7}_1} = \frac{2}{1} = \frac{\square}{\square} = 2$

31 $\frac{2}{3} \times \frac{11}{4} \div \frac{22}{15}$

$= \frac{\cancel{2}^1}{\cancel{3}_1} \times \frac{\cancel{11}^1}{\cancel{4}_2} \times \frac{\cancel{15}^5}{\cancel{22}_2} = \frac{5}{4} = \frac{\square}{\square} = 1\frac{1}{4}$

Dividing Mixed Numerals

Dividing mixed numerals is an extension on dividing fractions, the extra step required is to convert the mixed numeral to an improper fraction.

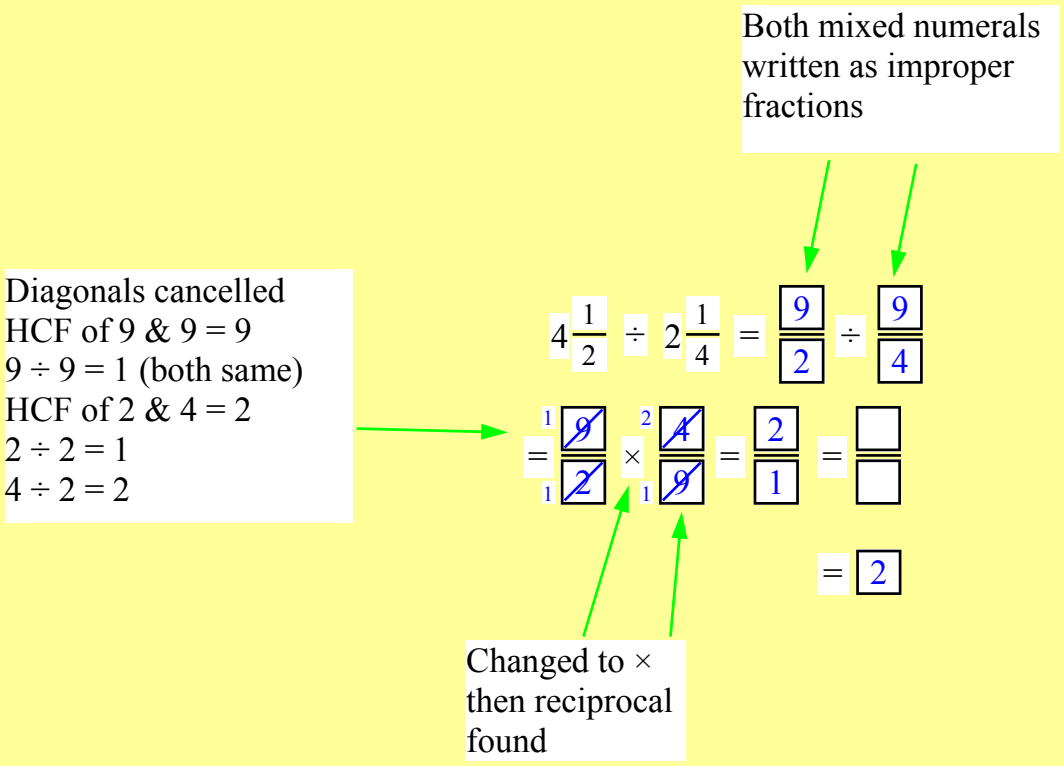
In column 1 a mixed numeral is divided by a proper fraction. To answer these questions follow these steps:

- Change the mixed numeral to an improper fraction
- Turn the proper fraction upside-down and write the reciprocal underneath the fraction
- Multiply across the top (numerators) and the bottom (denominators) to get the answer, it will be an improper fraction
- These questions can't be simplified so change the improper fraction answer to a mixed numeral and you are done.

An example is at the top of the column, note that you only invert (turn upside-down) the fractions that are behind a ÷ sign. In the lower questions multiplication is included to test that you are aware of this. Don't find the reciprocal of fractions that are behind a × sign.

Columns 2 and 3 are the same as column 1 except both are mixed numerals not just one of the numbers. An example is below. The steps are the same as above except that:

- Both must be converted from mixed numbers to improper fractions
- The answers will often need to be simplified
- As with previous sheets if you have difficultly with the concept of canceling there are additional spaces for simplifying your answer, if you are canceling you mostly won't need to use these spaces.
- If you don't use canceling you may find questions 16 and 17 difficult to simplify



Dividing Mixed Numerals

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Divide these fractions and whole numbers.

Example

$3\frac{1}{2} \div \frac{2}{3}$ Reciprocal

$= \frac{7}{2} \times \frac{3}{2} = \frac{21}{4} = 5\frac{1}{4}$

Convert to Improper

1 $2\frac{3}{4} \div \frac{4}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

2 $1\frac{1}{4} \div \frac{2}{3}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

3 $2\frac{1}{2} \div \frac{7}{9}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

4 $5\frac{1}{3} \div \frac{3}{4}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

5 $1\frac{2}{5} \div \frac{3}{4} \times \frac{2}{3}$

$= \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

6 $3\frac{1}{2} \div \frac{1}{3} \times \frac{1}{5}$

$= \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

7 $3\frac{1}{3} \times \frac{2}{3} \div \frac{1}{4}$

$= \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \square\frac{\square}{\square}$

Try these, they will result in mixed or whole numbers.

8 $4\frac{1}{2} \div 1\frac{2}{3} = \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

9 $2\frac{1}{4} \div 1\frac{3}{8} = \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

10 $3\frac{3}{4} \div 2\frac{1}{4} = \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

11 $1\frac{5}{7} \div 1\frac{3}{7} = \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

12 $9\frac{1}{2} \div 3\frac{1}{3} = \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

13 $1\frac{4}{5} \div 1\frac{3}{10} = \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

14 $2\frac{3}{7} \div 1\frac{7}{10} = \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

15 $4\frac{1}{3} \div 1\frac{2}{3} = \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

$= \square\frac{\square}{\square}$

16 $2\frac{1}{3} \div 1\frac{3}{4} \div \frac{2}{5}$

$= \frac{\square}{\square} \div \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

17 $1\frac{1}{2} \div 1\frac{5}{6} \div 1\frac{1}{11}$

$= \frac{\square}{\square} \div \frac{\square}{\square} \div \frac{\square}{\square}$

$= \frac{\square}{\square} \times \frac{\square}{\square} \times \frac{\square}{\square} = \frac{\square}{\square}$

$= \frac{\square}{\square} = \square\frac{\square}{\square}$

Dividing Mixed Numerals

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Divide these fractions and whole numbers.

Example

$3\frac{1}{2} \div \frac{2}{3}$ Reciprocal

$= \frac{7}{2} \times \frac{3}{2} = \frac{21}{4} = 5\frac{1}{4}$

Convert to Improper

1 $2\frac{3}{4} \div \frac{4}{5}$

$= \frac{11}{4} \times \frac{5}{4} = \frac{55}{16} = 3\frac{7}{16}$

2 $1\frac{1}{4} \div \frac{2}{3}$

$= \frac{5}{4} \times \frac{3}{2} = \frac{15}{8} = 1\frac{7}{8}$

3 $2\frac{1}{2} \div \frac{7}{9}$

$= \frac{5}{2} \times \frac{9}{7} = \frac{45}{14} = 3\frac{3}{14}$

4 $5\frac{1}{3} \div \frac{3}{4}$

$= \frac{16}{3} \times \frac{4}{3} = \frac{64}{9} = 7\frac{1}{9}$

5 $1\frac{2}{5} \div \frac{3}{4} \times \frac{2}{3}$

$= \frac{7}{5} \times \frac{4}{3} \times \frac{2}{3} = \frac{56}{45} = 1\frac{11}{45}$

6 $3\frac{1}{2} \div \frac{1}{3} \times \frac{1}{5}$

$= \frac{7}{2} \times \frac{3}{1} \times \frac{1}{5} = \frac{21}{10} = 2\frac{1}{10}$

7 $3\frac{1}{3} \times \frac{2}{3} \div \frac{1}{4}$

$= \frac{10}{3} \times \frac{2}{3} \times \frac{4}{1} = \frac{80}{9} = 8\frac{8}{9}$

Try these, they will result in mixed or whole numbers.

8 $4\frac{1}{2} \div 1\frac{2}{3} = \frac{9}{2} \div \frac{5}{3}$

$= \frac{9}{2} \times \frac{3}{5} = \frac{27}{10} = \square\square$

$= 2\frac{7}{10}$

9 $2\frac{1}{4} \div 1\frac{3}{8} = \frac{9}{4} \div \frac{11}{8}$

$= \frac{\cancel{9}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{11} = \frac{18}{11} = \square\square$

$= 1\frac{7}{11}$

10 $3\frac{3}{4} \div 2\frac{1}{4} = \frac{15}{4} \div \frac{9}{4}$

$= \frac{\cancel{15}^{5}}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{\cancel{9}_{3}} = \frac{5}{3} = \square\square$

$= 1\frac{2}{3}$

11 $1\frac{5}{7} \div 1\frac{3}{7} = \frac{12}{7} \div \frac{10}{7}$

$= \frac{\cancel{12}^{6}}{\cancel{7}_{1}} \times \frac{\cancel{7}^{1}}{\cancel{10}_{5}} = \frac{6}{5} = \square\square$

$= 1\frac{1}{5}$

12 $9\frac{1}{2} \div 3\frac{1}{3} = \frac{19}{2} \div \frac{10}{3}$

$= \frac{19}{2} \times \frac{3}{10} = \frac{57}{20} = \square\square$

$= 2\frac{17}{20}$

13 $1\frac{4}{5} \div 1\frac{3}{10} = \frac{9}{5} \div \frac{13}{10}$

$= \frac{\cancel{9}^{1}}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{13} = \frac{18}{13} = \square\square$

$= 1\frac{5}{13}$

14 $2\frac{3}{7} \div 1\frac{7}{10} = \frac{17}{7} \div \frac{17}{10}$

$= \frac{\cancel{17}^{1}}{7} \times \frac{10}{\cancel{17}_{1}} = \frac{10}{7} = \square\square$

$= 1\frac{3}{7}$

15 $4\frac{1}{3} \div 1\frac{2}{3} = \frac{13}{3} \div \frac{5}{3}$

$= \frac{13}{\cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{5} = \frac{13}{5} = \square\square$

$= 2\frac{3}{5}$

16 $2\frac{1}{3} \div 1\frac{3}{4} \div \frac{2}{5}$

$= \frac{7}{3} \div \frac{7}{4} \div \frac{2}{5}$

$= \frac{\cancel{7}^{1}}{3} \times \frac{\cancel{4}^{2}}{\cancel{7}_{1}} \times \frac{5}{\cancel{2}_{1}} = \frac{10}{3} = \square\square = 3\frac{1}{3}$

17 $2\frac{1}{2} \div 1\frac{5}{6} \div 1\frac{1}{11}$

$= \frac{5}{2} \div \frac{11}{6} \div \frac{12}{11}$

$= \frac{5}{2} \times \frac{\cancel{6}^{1}}{\cancel{11}_{1}} \times \frac{\cancel{11}^{1}}{\cancel{12}_{2}} = \frac{5}{4} = \square\square = 1\frac{1}{4}$

Mixed Operations - Calculator

This sheet is for calculator use so there are no working spaces provided.

The method is simple for this sheet, use your calculator. You don't have to worry about reciprocals or simplifying, the calculator does it all for you. Mistakes made using the calculator can occur anywhere due to keystroke error, there is one more common mistake made though, that is the entering of mixed numbers. Remember that you have to press the fraction key twice to enter a mixed number.

Your calculator should obey order of operations rules in the exercises in column 3, just don't press '=' until you key in the entire question otherwise the calculator may give the wrong answer.

Mixed Operations - Calculator

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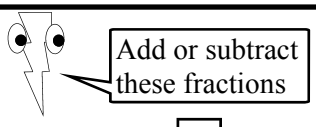


1 $\frac{1}{2} + \frac{1}{3} = \square$

2 $\frac{4}{5} - \frac{3}{7} = \square$

3 $\frac{4}{5} - \frac{3}{10} = \square$

4 $\frac{3}{11} + \frac{3}{5} = \square$

5 $\frac{3}{4} - \frac{1}{6} + \frac{4}{5} = \square$

6 $\frac{4}{5} + \frac{3}{8} - \frac{2}{15} = \square$

7 $\frac{9}{12} + \frac{5}{7} - \frac{1}{4} = \square$

8 $\frac{17}{20} - \frac{3}{10} + \frac{8}{15} = \square$

9 $\frac{4}{5} + 2\frac{2}{3} - \frac{7}{10} = \square$

10 $4\frac{6}{7} - \frac{2}{3} - \frac{4}{9} = \square$

11 $2\frac{4}{11} - 1\frac{1}{2} + \frac{5}{6} = \square$

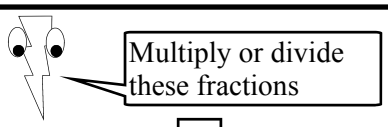
12 $4\frac{3}{5} + \frac{2}{3} - 1\frac{7}{10} = \square$

13 $3\frac{5}{6} - 1\frac{1}{4} + 4\frac{7}{12} = \square$

14 $3\frac{2}{5} + 4\frac{7}{10} + 2\frac{5}{12} = \square$

15 $5\frac{1}{4} - 2\frac{2}{3} - 1\frac{1}{2} = \square$

16 $9\frac{2}{3} - 3\frac{6}{7} + 1\frac{5}{6} = \square$



17 $\frac{3}{5} \times \frac{1}{4} = \square$

18 $\frac{7}{12} \div \frac{2}{3} = \square$

19 $\frac{2}{3} \div \frac{8}{9} = \square$

20 $\frac{3}{5} \times \frac{7}{6} = \square$

21 $\frac{4}{5} \div \frac{1}{3} \times \frac{8}{9} = \square$

22 $\frac{1}{7} \div \frac{3}{10} \times \frac{18}{5} = \square$

23 $\frac{3}{4} \times \frac{17}{10} \div \frac{1}{8} = \square$

24 $\frac{4}{7} \div \frac{1}{10} \div \frac{2}{5} = \square$

25 $\frac{3}{8} \times 4\frac{1}{6} \times \frac{2}{3} = \square$

26 $5\frac{11}{12} \div \frac{3}{4} \times \frac{1}{2} = \square$

27 $3\frac{1}{2} \times 6 \times \frac{4}{5} = \square$

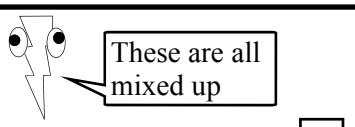
28 $7\frac{3}{4} \times \frac{2}{5} \div 2\frac{2}{3} = \square$

29 $7 \div 1\frac{1}{4} \div 2\frac{3}{5} = \square$

30 $4\frac{5}{6} \times 3\frac{3}{10} \div 2\frac{2}{5} = \square$

31 $5\frac{3}{14} \div 8 \times 4\frac{4}{5} = \square$

32 $2\frac{3}{7} \div 1\frac{2}{3} \times 3 = \square$



33 $\frac{1}{3} + \frac{3}{4} \times \frac{9}{5} = \square$

34 $\frac{1}{5} \div \frac{2}{15} + \frac{2}{3} = \square$

35 $5\frac{1}{6} \times 3 - 4\frac{6}{7} = \square$

36 $7\frac{3}{8} \div 9\frac{1}{4} + \frac{3}{2} = \square$

37 $\frac{5}{4} + 3\frac{1}{2} \div \frac{5}{6} = \square$

38 $5 \div \frac{7}{10} \times \frac{8}{9} = \square$

39 $\frac{1}{2} \times \frac{7}{11} + \frac{3}{4} = \square$

40 $7\frac{5}{6} - \frac{11}{12} \times 1\frac{2}{3} = \square$

41 $3\frac{3}{4} \div \frac{5}{9} - \frac{5}{12} = \square$

42 $2\frac{5}{8} \times 3\frac{1}{6} \div 5 = \square$

43 $9\frac{1}{2} - 3\frac{2}{3} \div 1\frac{4}{5} = \square$

44 $3\frac{8}{9} \times 1\frac{2}{5} \times 3\frac{1}{2} = \square$

45 $\frac{2}{5} \times \frac{8}{3} + \frac{1}{2} = \square$

46 $4\frac{7}{10} \div \frac{1}{4} \div 1\frac{1}{5} = \square$

47 $2\frac{2}{5} + \frac{3}{4} \div \frac{1}{2} = \square$

48 $5\frac{7}{8} \div 2\frac{1}{3} - 1\frac{1}{4} = \square$

Mixed Operations - Calculator

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Add or subtract these fractions

1 $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

2 $\frac{4}{5} - \frac{3}{7} = \frac{13}{35}$

3 $\frac{4}{5} - \frac{3}{10} = \frac{1}{2}$

4 $\frac{3}{11} + \frac{3}{5} = \frac{48}{55}$

5 $\frac{3}{4} - \frac{1}{6} + \frac{4}{5} = 1\frac{23}{60}$

6 $\frac{4}{5} + \frac{3}{8} - \frac{2}{15} = 1\frac{1}{24}$

7 $\frac{9}{12} + \frac{5}{7} - \frac{1}{4} = 1\frac{3}{14}$

8 $\frac{17}{20} - \frac{3}{10} + \frac{8}{15} = 1\frac{1}{12}$

9 $\frac{4}{5} + 2\frac{2}{3} - \frac{7}{10} = 2\frac{23}{30}$

10 $4\frac{6}{7} - \frac{2}{3} - \frac{4}{9} = 3\frac{47}{63}$

11 $2\frac{4}{11} - 1\frac{1}{2} + \frac{5}{6} = 1\frac{23}{33}$

12 $4\frac{3}{5} + \frac{2}{3} - 1\frac{7}{10} = 3\frac{17}{30}$

13 $3\frac{5}{6} - 1\frac{1}{4} + 4\frac{7}{12} = 7\frac{1}{6}$

14 $3\frac{2}{5} + 4\frac{7}{10} + 2\frac{5}{12} = 10\frac{31}{60}$

15 $5\frac{1}{4} - 2\frac{2}{3} - 1\frac{1}{2} = 1\frac{1}{12}$

16 $9\frac{2}{3} - 3\frac{6}{7} + 1\frac{5}{6} = 7\frac{9}{14}$



Multiply or divide these fractions

17 $\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$

18 $\frac{7}{12} \div \frac{2}{3} = \frac{7}{8}$

19 $\frac{2}{3} \div \frac{8}{9} = \frac{3}{4}$

20 $\frac{3}{5} \times \frac{7}{6} = \frac{7}{10}$

21 $\frac{4}{5} \div \frac{1}{3} \times \frac{8}{9} = 2\frac{2}{15}$

22 $\frac{1}{7} \div \frac{3}{10} \times \frac{18}{5} = 1\frac{5}{7}$

23 $\frac{3}{4} \times \frac{17}{10} \div \frac{1}{8} = 10\frac{1}{5}$

24 $\frac{4}{7} \div \frac{1}{10} \div \frac{2}{5} = 14\frac{2}{7}$

25 $\frac{3}{8} \times 4\frac{1}{6} \times \frac{2}{3} = 1\frac{1}{24}$

26 $5\frac{11}{12} \div \frac{3}{4} \times \frac{1}{2} = 3\frac{17}{18}$

27 $3\frac{1}{2} \times 6 \times \frac{4}{5} = 16\frac{4}{5}$

28 $7\frac{3}{4} \times \frac{2}{5} \div 2\frac{2}{3} = 1\frac{13}{80}$

29 $7 \div 1\frac{1}{4} \div 2\frac{3}{5} = 2\frac{2}{13}$

30 $4\frac{5}{6} \times 3\frac{3}{10} \div 2\frac{2}{5} = 6\frac{31}{48}$

31 $5\frac{3}{14} \div 8 \times 4\frac{4}{5} = 3\frac{9}{70}$

32 $2\frac{3}{7} \div 1\frac{2}{3} \times 3 = 4\frac{13}{35}$



These are all mixed up

33 $\frac{1}{3} + \frac{3}{4} \times \frac{9}{5} = 1\frac{41}{60}$

34 $\frac{1}{5} \div \frac{2}{15} + \frac{2}{3} = 2\frac{1}{6}$

35 $5\frac{1}{6} \times 3 - 4\frac{6}{7} = 10\frac{9}{14}$

36 $7\frac{3}{8} \div 9\frac{1}{4} + \frac{3}{2} = 2\frac{11}{37}$

37 $\frac{5}{4} + 3\frac{1}{2} \div \frac{5}{6} = 5\frac{9}{20}$

38 $5 \div \frac{7}{10} \times \frac{8}{9} = 6\frac{22}{63}$

39 $\frac{1}{2} \times \frac{7}{11} + \frac{3}{4} = 1\frac{3}{44}$

40 $7\frac{5}{6} - \frac{11}{12} \times 1\frac{2}{3} = 6\frac{11}{36}$

41 $3\frac{3}{4} \div \frac{5}{9} - \frac{5}{12} = 6\frac{1}{3}$

42 $2\frac{5}{8} \times 3\frac{1}{6} \div 5 = 1\frac{53}{80}$

43 $9\frac{1}{2} - 3\frac{2}{3} \div 1\frac{4}{5} = 7\frac{25}{54}$

44 $3\frac{8}{9} \times 1\frac{2}{5} \times 3\frac{1}{2} = 19\frac{1}{18}$

45 $\frac{2}{5} \times \frac{8}{3} + \frac{1}{2} = 1\frac{17}{30}$

46 $4\frac{7}{10} \div \frac{1}{4} \div 1\frac{1}{5} = 15\frac{2}{3}$

47 $2\frac{2}{5} + \frac{3}{4} \div \frac{1}{2} = 3\frac{9}{10}$

48 $5\frac{7}{8} \div 2\frac{1}{3} - 1\frac{1}{4} = 1\frac{15}{56}$

Fractions - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Fractions Find-A-Word

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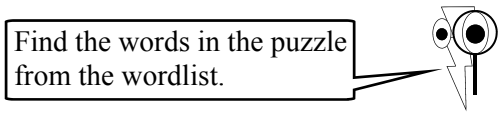

M	D	V	A	R	E	D	N	I	A	M	E	R	N	B
D	Q	U	X	D	L	O	W	E	S	T	R	N	O	C
E	V	Q	T	Z	D	R	T	T	M	I	O	Q	I	I
N	N	V	S	T	L	I	D	E	X	I	M	U	T	M
O	N	U	I	N	R	W	T	D	S	C	P	A	A	P
M	P	D	M	L	E	E	H	I	P	P	M	N	C	R
I	N	T	P	E	Q	R	V	O	O	A	D	T	I	O
N	D	R	L	C	R	I	A	N	L	N	R	I	L	P
A	S	E	I	N	D	A	C	P	I	E	E	T	P	E
T	E	V	F	A	N	V	T	N	M	N	D	Y	I	R
O	P	N	Y	C	X	T	D	O	L	O	U	D	T	B
R	A	O	G	R	E	C	I	P	R	O	C	A	L	P
V	H	C	O	M	M	O	N	N	U	Q	I	G	U	M
M	S	U	B	T	R	A	C	T	I	O	N	V	M	B
B	T	N	E	L	A	V	I	U	Q	E	G	L	L	M

WORDLIST

SUBTRACTION	RECIPROCAL	ADDITION	CANCEL	DIVISION
MULTIPLICATION	CONVERT	INVERT	IMPROPER	REDUCING
DENOMINATOR	REMAINDER	NUMERATOR	SIMPLIFY	MIXED
EQUIVALENT	COMPARE	QUANTITY	SHAPES	WHOLE
	PART	COMMON	LOWEST	

Fractions Find-A-Word

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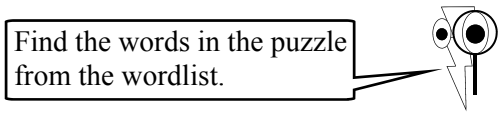

M	D	V	A	R	E	D	N	I	A	M	E	R	N	B
D	Q	U	X	D	L	O	W	E	S	T	R	N	O	C
E	V	Q	T	Z	D	R	T	T	M	I	O	Q	I	I
N	N	V	S	T	L	I	D	E	X	I	M	U	T	M
O	N	U	I	N	R	W	T	D	S	C	P	A	A	P
M	P	D	M	L	E	E	H	I	P	P	M	N	C	R
I	N	T	P	E	Q	R	V	O	O	A	D	T	I	O
N	D	R	L	C	R	I	A	N	L	N	R	I	L	P
A	S	E	I	N	D	A	C	P	I	E	E	T	P	E
T	E	V	F	A	N	V	T	N	M	N	D	Y	I	R
O	P	N	Y	C	X	T	D	O	L	O	U	D	T	B
R	A	O	G	R	E	C	I	P	R	O	C	A	L	P
V	H	C	O	M	M	O	N	N	U	Q	I	G	U	M
M	S	U	B	T	R	A	C	T	I	O	N	V	M	B
B	T	N	E	L	A	V	I	U	Q	E	G	L	L	M

WORDLIST

~~SUBTRACTION~~
~~MULTIPLICATION~~
~~DENOMINATOR~~
~~EQUIVALENT~~
~~RECIPROCAL~~
~~CONVERT~~
~~REMAINDER~~
~~COMPARE~~
~~PART~~
~~ADDITION~~
~~INVERT~~
~~NUMERATOR~~
~~QUANTITY~~
~~COMMON~~
~~CANCEL~~
~~IMPROPER~~
~~SIMPLIFY~~
~~SHAPES~~
~~LOWEST~~
~~DIVISION~~
~~REDUCING~~
~~MIXED~~
~~WHOLE~~

FREEFALL MATHEMATICS

ALTITUDE

DECIMALS

Place Value of Decimals

The position of a digit in a decimal could be to the left of the decimal point: units (ones), tens, hundreds and upwards. Or to the right of the decimal point: tenths, hundredths, thousandths and so on. This location is its place value.

Column 1 asks you to place decimals that have been broken up into parts into the table. Read the question and place the digits in the table according to their place value. Once the digits are in, fill all the spaces between the digits with zeros, except in the decimal point column, just write in a '**.**' .

Column 2 asks you to give the place value of the '4' in all the numbers. Look at its location with the decimal point, the amount of digits either side of it doesn't matter, it is its location to the decimal point that determines its place value. Use the table if you have difficulty with the positions or the spelling. Once you have spelt the place value (just write '4' don't spell it) write the fraction or number that matches the answer.

The next part of column 2 asks you to write the number in decimal form and then expand it. The first decimal place will be a fraction 'over' 10, the second decimal place will be 'over' 100, the third 'over' 1 000. If there is a zero in the decimal then you don't include it in the expansion. So watch for zeros!

Column 3 asks you to state the number of decimal places that each number has. Again don't let the numbers trick you, look for the decimal point and count the number of digits behind (to the right of) it. If you had a number such as 4.200, while you could write it as 4.2 (which has 1 decimal place) the answer would still be that the decimal has 3 decimal places. Note that d.p. = decimal places.

The rest of Column 3 uses comparisons of decimals. Look at the numbers and add zeros to the number with less decimal places, until they have the same number of d.p. (match the number of decimal places). Then compare them, just imagine that there is no decimal point and they are whole numbers. This takes the error factor out of comparing. Remember with < and > you point the arrow to the smallest number (decimal).

A very common mistake when rewriting decimals in order is that you write the altered decimals in order, not the decimals in the question. So remember write the decimals in the question in order, not your working out decimals.

Place Value of Decimals

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Complete the table. Fill blank spaces between the numbers with zeros. All questions must have a zero or whole number in the units column.

Example

4 hundreds, 3 units, 2 tenths and 7 thousandths

1. 3 units and 7 thousandths
2. 8 tens and 7 tenths
3. 7 hundreds, 8 units and 4 hundredths
4. 8 tens, 4 hundredths and 1 thousandth
5. 9 hundreds, 4 tens, 6 units, 5 tenths and 4 thousandths
6. 2 hundreds, 7 units, 3 tenths and 9 hundredths
7. 7 hundreds, 6 units and 8 thousandths
8. 3 hundreds, 9 tens and 5 tenths

	Hundreds 100's	Tens 10's	Units 1's	Decimal Point .	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1\,000}$
Ex.	4	0	3	.	2	0	7
1							
2							
3							
4							
5							
6							
7							
8							

Give the value of the 4 in these. Express both in words and numbers

Example

5.742 = 4 hundredths $\frac{4}{100}$

9. 14.36 = _____
10. 0.034 = _____
11. 127.4 = _____
12. 15.94 = _____
13. 1.423 = _____
14. 7.0542 = _____
15. 12.471 = _____

Write these as decimals, then expand. Place only a single digit in the top (numerator) boxes.

Example

$5\frac{37}{100} = 5.37 = \boxed{5} + \frac{\boxed{3}}{10} + \frac{\boxed{7}}{100}$

16. $\frac{45}{100} = 0.\_\_ = \frac{\square}{10} + \frac{\square}{100}$

17. $\frac{572}{1\,000} = \_\_\_\_ = \frac{\square}{10} + \frac{\square}{100} + \frac{\square}{1\,000}$

18. $7\frac{17}{100} = \_\_\_\_ = \square + \frac{\square}{\square} + \frac{\square}{\square}$

19. $4\frac{703}{1\,000} = \_\_\_\_ = \square + \frac{\square}{\square} + \frac{\square}{\square}$

20. $\frac{68}{100} = \_\_\_\_ = \frac{\square}{\square} + \frac{\square}{\square}$

21. $\frac{39}{1\,000} = \_\_\_\_ = \frac{\square}{\square} + \frac{\square}{\square}$

Give the decimal places of the following

Example 25.742 3

22	850.65	23	0.4529
24	3.7055	25	56.244
26	371.9	27	24.01
28	12.408	29	100.05
30	903.2	31	363.63

Add zeros to give all the decimals the same number of decimal places then use < or > to compare them

Example 0.04 < 0.2

0.04 0.20

32. 0.7 ☐ 0.584
33. 0.72 ☐ 0.8
34. 1.25 ☐ 2.1
35. 3.56 ☐ 3.068
36. 1.963 ☐ 1.9

Rewrite decimals with the same decimal places, then arrange in descending order

37. 0.72 , 0.3 , 1.004 , 1.3

38. 0.555 , 0.5 , 5.05 , 0.55

Place Value of Decimals

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Complete the table. Fill blank spaces between the numbers with zeros. All questions must have a zero or whole number in the units column.

Example

4 hundreds, 3 units, 2 tenths and 7 thousandths

1. 3 units and 7 thousandths
2. 8 tens and 7 tenths
3. 7 hundreds, 8 units and 4 hundredths
4. 8 tens, 4 hundredths and 1 thousandth
5. 9 hundreds, 4 tens, 6 units, 5 tenths and 4 thousandths
6. 2 hundreds, 7 units, 3 tenths and 9 hundredths
7. 7 hundreds, 6 units and 8 thousandths
8. 3 hundreds, 9 tens and 5 tenths

	Hundreds 100's	Tens 10's	Units 1's	Decimal Point .	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1\,000}$
Ex.	4	0	3	.	2	0	7
1			3	.	0	0	7
2		8	0	.	7		
3	7	0	8	.	0	4	
4		8	0	.	0	4	1
5	9	4	6	.	5	0	4
6	2	0	7	.	3	0	9
7	7	0	6	.	0	0	8
8	3	9	0	.	5		

Give the value of the 4 in these. Express both in words and numbers

Example

$5.742 =$ 4 hundredths $\frac{4}{100}$

9. $14.36 =$ 4 units 4
10. $0.034 =$ 4 thousandths $\frac{4}{1\,000}$
11. $127.4 =$ 4 tenths $\frac{4}{10}$
12. $15.94 =$ 4 hundredths $\frac{4}{100}$
13. $1.423 =$ 4 tenths $\frac{4}{10}$
14. $7.0542 =$ 4 thousandths $\frac{4}{1\,000}$
15. $12.471 =$ 4 tenths $\frac{4}{10}$

Write these as decimals, then expand. Place only a single digit in the top (numerator) boxes.

Example

$5\frac{37}{100} = 5.37 = \boxed{5} + \frac{\boxed{3}}{10} + \frac{\boxed{7}}{100}$

16. $\frac{45}{100} = 0.45 = \frac{\boxed{4}}{10} + \frac{\boxed{5}}{100}$

17. $\frac{572}{1\,000} = 0.572 = \frac{\boxed{5}}{10} + \frac{\boxed{7}}{100} + \frac{\boxed{2}}{1\,000}$

18. $7\frac{17}{100} = 7.17 = \boxed{7} + \frac{\boxed{1}}{10} + \frac{\boxed{7}}{100}$

19. $4\frac{703}{1\,000} = 4.703 = \boxed{4} + \frac{\boxed{7}}{10} + \frac{\boxed{3}}{1\,000}$

20. $\frac{68}{100} = 0.68 = \frac{\boxed{6}}{10} + \frac{\boxed{8}}{100}$

21. $\frac{39}{1\,000} = 0.039 = \frac{\boxed{3}}{100} + \frac{\boxed{9}}{1\,000}$

Give the decimal places of the following

Example $25.\underline{742}$ 3

22. 850.65 2
23. 0.4529 4
24. 3.7055 4
25. 56.244 3
26. 371.9 1
27. 24.01 2
28. 12.408 3
29. 100.05 2
30. 903.2 1
31. 363.63 2

Add zeros to give all the decimals the same number of decimal places then use < or > to compare them

Example 0.04 < 0.2

0.040 0.200

32. 0.7 > 0.584

0.700 0.584

33. 0.72 < 0.8

0.72 0.80

34. 1.25 < 2.1

1.25 2.10

35. 3.56 > 3.068

3.560 3.068

36. 1.963 > 1.9

1.963 1.900

Rewrite decimals with the same decimal places, then arrange in descending order

37. 0.72 , 0.3 , 1.004 , 1.3

0.720 0.300 1.004 1.300

1.3, 1.004, 0.72, 0.3

38. 0.555 , 0.5 , 5.05 , 0.55

0.555 0.500 5.050 0.550

5.05, 0.555, 0.55, 0.5

Rounding Decimals

Rounding is used to shorten decimal answers to a level of accuracy required. Often when answering questions, you are asked to give your answer to a specific number of decimal places. This isn't done by just chopping the decimal off at the number of places required, this method is followed:

- Look at the decimal behind the decimal place asked for. For example if rounding to 2 d.p. (2 decimal places) look at the digit in the 3rd decimal place.
- If this digit is 5 or more then add 1 to the digit in front of it, this means that when increasing a 3 for example, it would become a 4. But it also means that if it is a 9 it will become a 0 and will add 1 to the digit in front of it.
- If the digit is less than 5 the digit is unchanged and the number is just 'chopped' at the required decimal place.
- If there isn't a number in the decimal place asked for, then zeros are added to the number until the decimal place has a zero in the position.

Column 1 asks firstly to round the given decimals and whole numbers to 1 d.p. To do this (an example is at the top of the column), look at the number in the 2nd decimal place position. If that number is 5 or more add 1 to the 1 d.p. number then remove all the digits behind it. If the 2 d.p. digit is less than 5 just rewrite the 1 d.p. number unchanged with the digits removed after it. The second part of column 1 asks you to round to 2 d.p. Look at the 3rd digit and use the same method, the answer will have 2 digits after the decimal point.

Column 2 asks you to round to 3 d.p. Look at he 4th decimal place digit and apply the same method. The next part of the column asks you to round to 0 d.p. This could also be asked in the form : 'round to the nearest unit' or 'round to the nearest whole number'. Look at the digit after the decimal point and apply the same method then write the whole number without a decimal point. Note that if a decimal is less than 0.5 then when it is rounded to the nearest whole number it will become 0.

Column 3 is a combination of the previous 2 columns. The first part asks you to round to the number of decimal places specified in the brackets. So instead of the column being all the same rounding, it jumps from one to the next. The second part of the column asks you to round the one number to 1, 2 and 3 d.p.

Examples of rounding
0.54862
To 1 d.p. : 0.54862 → 0.5 (as 4 < 5)
To 2 d.p. : 0.54862 → 0.55 (as 8 > 5)
To 3 d.p. : 0.54862 → 0.549 (as 6 > 5)
To nearest whole number:
0.54862 → 1 (as 5 = 5)

Example: Whole numbers
65
To 1 d.p. : 65.0
To 2 d.p. : 65.00
To 3 d.p. : 65.000

Example:
71.5
To 1 d.p. : 71.5
To 2 d.p. : 71.50
To 3 d.p. : 71.500

Rounding Decimals

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Round these numbers to the nearest tenth (1 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.2

1. 0.87
2. 0.43
3. 0.90347
4. 0.05956
5. 18.374
6. 22.128
7. 67
8. 56.942
9. 129.988
10. 99.953

Round these numbers to the nearest thousandth (3 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.163

22. 2.7674
23. 0.8436
24. 8.0302
25. 6.9969
26. 6.9996
27. 0.7
28. 0.70
29. 0.707
30. 4
31. 9.9999

Round these numbers to the decimal place given in the brackets

43. 29.6751 [2]
44. 1 361.54 [1]
45. 0.4296 [0]
46. 9.1 [3]
47. 19.9971 [2]
48. 6.0606 [3]
49. 86 [2]
50. 56.0438 [1]
51. 11.4099 [0]
52. 8.8997 [3]
53. 5.9949 [2]

Round these numbers to the nearest hundredth (2 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1.16

11. 0.278
12. 0.805
13. 0.4246
14. 0.9793
15. 2.2183
16. 73.2
17. 4.95829
18. 9.99811
19. 67.9942
20. 83
21. 99.9961

Round these numbers to the nearest unit (0 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round it up, if < 5 the digit is unchanged

1.1627 → 1

32. 4.7
33. 9.3
34. 18.78
35. 249.499
36. 7 101.835
37. 83.5004
38. 999.278
39. 999.872
40. 0.808
41. 0.080
42. 9 999.5

Round each decimal to 1, 2 and 3 decimal places

54. 0.1919 [1] _____ [2] _____ [3] _____
55. 3.8225 [1] _____ [2] _____ [3] _____
56. 26.9527 [1] _____ [2] _____ [3] _____
57. 0.3235 [1] _____ [2] _____ [3] _____
58. 9.9994 [1] _____ [2] _____ [3] _____

Rounding Decimals

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Round these numbers to the nearest tenth (1 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round up, if < 5 the digit is unchanged

	1.1627	1.2
1	0.87	0.9
2	0.43	0.4
3	0.90347	0.9
4	0.05956	0.1
5	18.374	18.4
6	22.128	22.1
7	67	67.0
8	56.942	56.9
9	129.988	130.0
10	99.953	100.0

Round these numbers to the nearest thousandth (3 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round up, if < 5 the digit is unchanged

	1.1627	1.163
22	2.7674	2.767
23	0.8436	0.844
24	8.0302	8.030
25	6.9969	6.997
26	6.9996	7.000
27	0.7	0.700
28	0.70	0.700
29	0.707	0.707
30	4	4.000
31	9.9999	10.000

Round these numbers to the decimal place given in the brackets

43	29.6751	[2]	29.68
44	1 361.54	[1]	1 361.5
45	0.4296	[0]	0
46	9.1	[3]	9.100
47	19.9971	[2]	20.00
48	6.0606	[3]	6.061
49	86	[2]	86.00
50	56.0438	[1]	56.0
51	11.4099	[0]	11
52	8.8997	[3]	8.900
53	5.9949	[2]	5.99

Round these numbers to the nearest hundredth (2 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round up, if < 5 the digit is unchanged

	1.1627	1.16
11	0.278	0.28
12	0.805	0.81
13	0.4246	0.42
14	0.9793	0.98
15	2.2183	2.22
16	73.2	73.20
17	4.95829	4.96
18	9.99811	10.00
19	67.9942	67.99
20	83	83.00
21	99.9961	100.00

Round these numbers to the nearest unit (0 d.p.)

Example

Look at the digit to the right of the one to be rounded. If it is 5 or more round up, if < 5 the digit is unchanged

	1.1627	1
32	4.7	5
33	9.3	9
34	18.78	19
35	249.499	249
36	7 101.835	7 102
37	83.5004	84
38	999.278	999
39	999.872	1 000
40	0.808	1
41	0.080	0
42	9 999.5	10 000

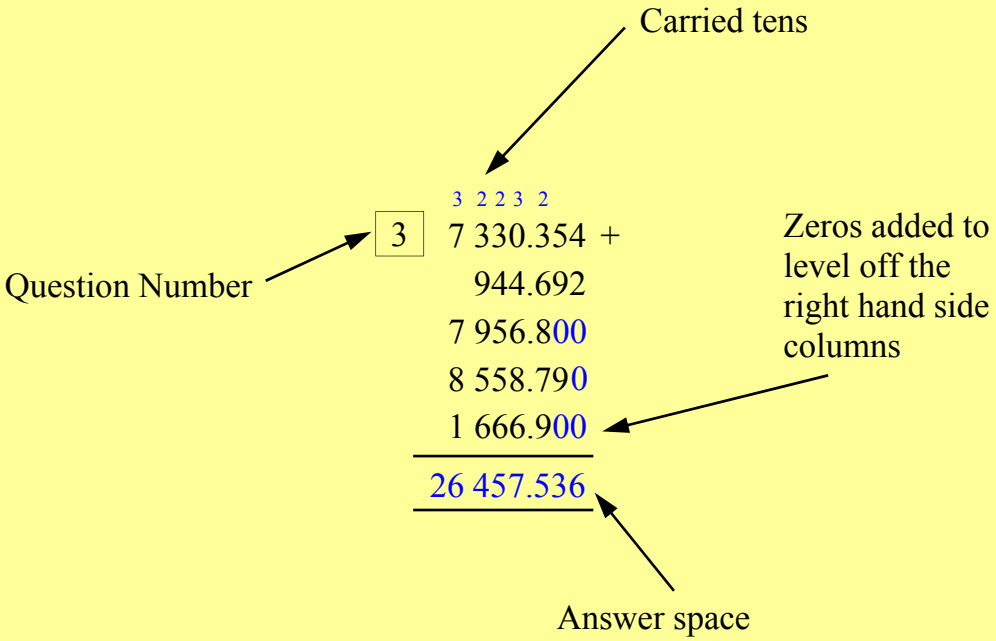
Round each decimal to 1, 2 and 3 decimal places

54	0.1919	[1]	0.2	[2]	0.19	[3]	0.192
55	3.8225	[1]	3.8	[2]	3.82	[3]	3.823
56	26.9527	[1]	27.0	[2]	26.95	[3]	26.953
57	0.3235	[1]	0.3	[2]	0.32	[3]	0.324
58	9.9994	[1]	10.0	[2]	10.00	[3]	9.999

Addition - Decimals

To perform addition move from right to left and carry tens when the total exceeds 9. The example below will show the method:

- If you want to fill the shorter decimal numbers with zeros to make the all the decimals level on the right hand side you can, if a number is a whole number then write the decimal point then the zeros.
- Total down the right hand column and if the total is less than 10 write the answer on the bottom line.
- If the number is ten or more write in the units or last digit and write the first digit (tens) above the next number to the left (carried tens).
- Add the next column including the carried figure (if any) in your calculation, repeat the process until you are finished



Addition - Decimals

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Try these additions, place zeros at the ends to fill 'shorter' decimal numbers if it makes it easier.



1
2.8 +
0.7
=

2
1.4 +
2.9
=

3
6.6 +
1.6
=

4
3.7 +
4.4
=

5
5.8 +
1.5
=

6
47.63 +
25.79
=

7
11.49 +
7.6
=

8
17.5 +
39.64
=

9
26.74 +
58.3
=

10
74.94 +
8.37
=

11
47.33 +
17.6
=

12
50.76 +
14.33
12.58
=

13
8.9 +
30.25
15
=

14
72.8 +
2.06
5.4
=

15
36.1 +
48
13.97
=

16
17.39 +
25.5
40.08
=

17
63.8 +
20.71
8
=

18
98.73 +
143.46
288.12
=

19
447.83 +
106.4
222.6
=

20
507.9 +
200.85
141.06
=

21
75.48 +
312.7
458
=

22
110.79 +
755.63
92.4
=

23
330 +
115.62
176
=

24
900.267 +
12.552
0.708
647.320
=

25
329.8 +
588.139
141.7
827.53
=

26
56.8 +
702.426
879
950.42
=

27
437.166 +
692.4
240.1
855.83
=

28
717.009 +
422.6
196.5
803.72
=

29
6 720.363 +
9 345.869
823.441
2 995.752
=

30
3 209.1 +
9 022.803
4 693
8 974.46
=

31
1 095.736 +
4 577.91
7 211.1
3 820.97
=

32
4 633 +
2 921.506
5 228.35
9 407
=

33
5 338.291 +
2 891.537
3 522.4
7 100.22
=

34
9 352.071 +
1 133.059
6 402.774
3 867.856
5 002.795
=

35
9 222.5 +
6 731.059
8 605
7 322.41
8 511.977
=

36
3 476.093 +
767.811
9 491.2
6 333.79
1 874.9
=

37
5 070.396 +
4 226.5
9 063.9
7 911.26
3 493.12
=

38
7 336.288 +
8 017.5
2 435.851
499.87
6 513
=

Addition - Decimals

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Try these additions, place zeros at the ends to fill 'shorter' decimal numbers if it makes it easier.



1	2	3	4	5
1	1	1	1	1
2.8 +	1.4 +	6.6 +	3.7 +	5.8 +
0.7	2.9	1.6	4.4	1.5
3.5	4.3	8.2	8.1	7.3

6	7	8	9	10	11
1 1 1	1	1 1	1 1	1 1	1
47.63 +	11.49 +	17.50 +	26.74 +	74.94 +	47.33 +
25.79	7.60	39.64	58.30	8.37	17.60
73.42	19.09	57.14	85.04	83.31	64.93

12	13	14	15	16	17
1 1	1 1	1 1	1 1	1 1	1 1
50.76 +	8.90 +	72.80 +	36.10 +	17.39 +	63.80 +
14.33	30.25	2.06	48.00	25.50	20.71
12.58	15.00	5.40	13.97	40.08	8.00
77.67	54.15	80.26	98.07	82.97	92.51

18	19	20	21	22	23
2 2 1 1	1 1	1 1	1 1 1	1 1	1 1
98.73 +	447.83 +	507.90 +	75.48 +	110.79 +	330.00 +
143.46	106.40	200.85	312.70	755.63	115.62
288.12	222.60	141.06	458.00	92.40	176.00
530.31	776.83	849.81	846.18	958.82	621.62

24	25	26	27	28
1 1 1 1	1 2 2	1 1 1	2 1 1	1 1 1
900.267 +	329.800 +	56.800 +	437.166 +	717.009 +
12.552	588.139	702.426	692.400	422.600
0.708	141.700	879.000	240.100	196.500
647.320	827.530	950.420	855.830	803.720
1 560.847	1 887.169	2 588.646	2 225.496	2 139.829

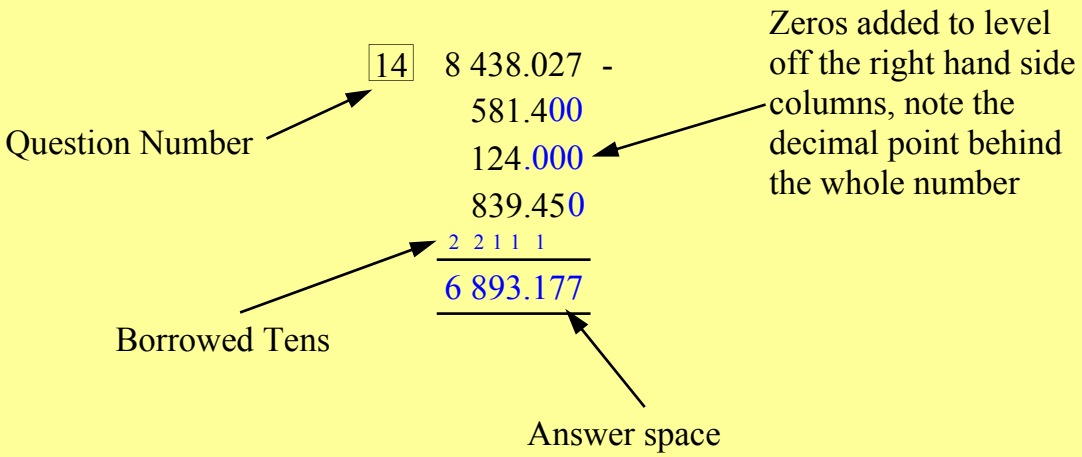
29	30	31	32	33
2 1 1 2 2 1	1 1 1 1	1 2 1 2 1	2 1	1 1 1 1 1
6 720.363 +	3 209.100 +	1 095.736 +	4 633.000 +	5 338.291 +
9 345.869	9 022.803	4 577.910	2 921.506	2 891.537
823.441	4 693.000	7 211.100	5 228.350	3 522.400
2 995.752	8 974.460	3 820.970	9 407.000	7 100.220
19 885.425	25 899.363	16 705.716	22 189.856	18 852.448

34	35	36	37	38
1 1 1 2 3 2	2 1 1 1 1	2 3 2 2 1	1 2 1 2 1	1 2 3 2 2
9 352.071 +	9 222.500 +	3 476.093 +	5 070.396 +	7 336.288 +
1 133.059	6 731.059	767.811	4 226.500	8 017.500
6 402.774	8 605.000	9 491.200	9 063.900	2 435.851
3 867.856	7 322.410	6 333.790	7 911.260	499.870
5 002.795	8 511.977	1 874.900	3 493.120	6 513.000
25 758.555	40 392.946	21 943.794	29 765.176	24 802.509

Subtraction - Decimals

To perform subtractions move from right to left and 'borrow' when the sum of the numbers below the top digit exceeds 9. The example below outlines the method:

- If you want to fill the shorter decimal numbers with zeros to make the numbers level on the right hand side you can, if a number is a whole number then write the decimal point then the zeros. If there is a decimal point already just write in the required number of zeros.
- Subtract down the right hand column if the top digit is large enough to subtract all the digits below it. Write the answer then move to the next column.
- If the top digit isn't large enough, add the digits below it and then borrow the required number of 'tens' from the column to its left, then put the number of tens borrowed below the column from which it was borrowed.
- Then subtract the next column, including the figure that you borrowed in your calculation, repeat the process until you are finished.



Subtraction - Decimals

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Try these subtractions, place zeros at the ends to fill 'shorter' decimal numbers if it makes it easier.



1	8.6	-	2	5.7	-	3	7.4	-	4	4.3	-	5	9.7	-
	5.2			3.9			2.1			0.9			3.5	

6	35.63	-	7	59.06	-	8	31.42	-	9	62.9	-	10	82.48	-	11	24.1	-
	18.87			33.4			26.7			48.05			57.9			9.56	

12	50.76	-	13	73.69	-	14	50	-	15	92.03	-	16	28.27	-	17	62	-
	14.33			47.5			7.8			46.5			17.19			37.6	
	12.58			21.61			25.44			27			9.8			18.71	

18	825.02	-	19	213.55	-	20	495	-	21	629.4	-	22	398.45	-	23	572.93	-
	146.47			96.7			289.7			438			16.7			105.2	
	482.94			104			96.76			177.85			145.9			359	

24	3 290.351	-	25	4 883.027	-	26	1 412.096	-	27	4 408	-	28	7 559.824	-
	462.783			177.4			364.839			950.2			683.5	
	816.447			502			217.6			703.152			297	
	658.215			938.77			92.51			444.06			781.976	

29	7 538.293	-	30	8 356.248	-	31	5 963	-	32	9 814.763	-	33	7 333	-
	1 089.476			986.51			1 286.9			4 011.6			2 463.05	
	2 774.502			3 267.382			542.075			2 452			1 779.841	
	1 946.229			1 344.8			1 766.2			1 835.66			856.22	

34	8 777.292	-	35	7 428	-	36	8 059.214	-	37	4 919.5	-	38	9 423.67	-
	2 856.319			2 441.8			327.162			361.607			3 502.164	
	683.147			855.561			1 588.3			1 053.436			54.34	
	1 936.495			1 587.04			3 762			1 500.61			1 823	
	2 351.866			2 168.7			2 070.11			1 785			2 538.2	

Subtraction - Decimals

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Try these subtractions, place zeros at the ends to fill 'shorter' decimal numbers if it makes it easier.



1	8.6	-	2	5.7	-	3	7.4	-	4	4.3	-	5	9.7	-
	5.2			3.9			2.1			0.9			3.5	
				1						1				
	3.4			1.8			5.3			3.4			6.2	

6	35.63	-	7	59.06	-	8	31.42	-	9	62.90	-	10	82.48	-	11	24.10	-
	18.87			33.40			26.70			48.05			57.90			9.56	
	1 1 1			1			1 1			1 1			1 1			1 1 1	
	16.76			25.66			4.72			14.85			24.58			14.54	

12	50.76	-	13	73.69	-	14	50.00	-	15	92.03	-	16	28.27	-	17	62.00	-
	14.33			47.50			7.80			46.50			17.19			37.60	
	12.58			21.61			25.44			27.00			9.80			18.71	
	1 1 1			1 1			2 2 1			2 1			1 1 1			2 2 1	
	23.85			4.58			16.76			18.53			1.28			5.69	

18	825.02	-	19	213.55	-	20	495.00	-	21	629.40	-	22	398.45	-	23	572.93	-
	146.47			96.70			289.70			438.00			16.70			105.20	
	482.94			104.00			96.76			177.85			145.90			359.00	
	2 1 2 1			1 1 1			1 2 2 1			1 1 1 1			1 2			2	
	195.61			12.85			108.54			13.55			235.85			108.73	

24	3 290.351	-	25	4 883.027	-	26	1 412.096	-	27	4 408.000	-	28	7 559.824	-			
	462.783			177.400			364.839			950.200			683.500				
	816.447			502.000			217.600			703.152			297.000				
	658.215			938.770			92.510			444.060			781.976				
	2 1 2 2 1 2			1 1 2 2 1			1 2 2 2 1			2 1 1 2 1			2 3 1 1 1 1				
	1 352.906			3 264.857			737.147			2 310.588			5 797.348				

29	7 538.293	-	30	8 356.248	-	31	5 963.000	-	32	9 814.763	-	33	7 333.000	-			
	1 089.476			986.510			1 286.900			4 011.600			2 463.050				
	2 774.502			3 267.382			542.075			2 452.000			1 779.841				
	1 946.229			1 344.800			1 766.200			1 835.660			856.220				
	2 2 2 1 1 2			2 2 2 2 1			1 2 2 2 1 1			1 1 1 1			2 2 2 2 2 1				
	1 728.086			2 757.556			2 367.825			1 515.503			2 233.889				

34	8 777.292	-	35	7 428.000	-	36	8 059.214	-	37	4 919.500	-	38	9 423.670	-			
	2 856.319			2 441.800			327.162			361.607			3 502.164				
	683.147			855.561			1 588.300			1 053.436			54.340				
	1 936.495			1 587.040			3 762.000			1 500.610			1 823.000				
	2 351.866			2 168.700			2 070.110			1 785.000			2 538.200				
	3 2 2 2 2 3			2 3 2 3 2 1			2 2 1 1 1			1 2 1 2 1 2			2 1 2 1 1 1				
	949.465			374.899			311.642			218.847			1 505.966				

Multiply and Divide by 10, 100 and 1 000

When you multiply or divide by 10, 100, 1 000, etc a rule applies which allows the calculation to be done without the usual amount of working. The rule is that when you divide by:

- 10 : move the decimal place one place to the left
- 100 : move the decimal place two places to the left
- 1 000 : move the decimal place three places to the left

The opposite applies with multiplication, when you multiply by:

- 10 : move the decimal place one place to the right
- 100 : move the decimal place two places to the right
- 1 000 : move the decimal place three places to the right

Column 1 are all multiplications, just move the decimal point to the right the same number of places as there are zeros in the number : 10 = 1 place, 100 = 2 places and 1 000 = 3 places. Examples are at the top of the column. Remember if a number doesn't have a decimal point then it is as if there is a decimal point behind the last digit, for example: think of 120 = 120**.** and 2 039 = 2 039**.** and so on.

Column 2 is all division. This time move the decimal point to the left, the same number of places as there are zeros in the number, 10 (1), 100 (2) and 1 000 (3).

Column 3 are problem-style which use the same approach as the previous 2 columns. The only difference being an additional calculation is required in most questions.

Multiply and Divide by 10, 100 and 1 000

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When you multiply by 10, 100 or 1 000 you move the decimal point to the right

× 10 — 1 place right — 5.784 × 10 = 57.84

× 100 — 2 places right — 5.78 × 100 = 578

× 1 000 — 3 places right

1 $2.6 \times 10 =$ ____

2 $19.7 \times 10 =$ ____

3 $100 \times 100 =$ ____

4 $5.373 \times 1\,000 =$ ____

5 $933.05 \times 100 =$ ____

6 $7\,707.7 \times 10 =$ ____

7 $0.002 \times 100 =$ ____

8 $2.0202 \times 10 =$ ____

9 $2.0202 \times 100 =$ ____

10 $2.0202 \times 1\,000 =$ ____

11 $17.38 \times 10 =$ ____

12 $0.046 \times 1\,000 =$ ____

13 $90.33 \times 1\,000 =$ ____

14 $12.549 \times 100 =$ ____

15 $528.64 \times 10 =$ ____

16 $0.0003 \times 100 =$ ____

17 $530.2 \times 100 =$ ____

18 $0.07 \times 1\,000 =$ ____

19 $0.8223 \times 10 =$ ____

20 $116.8 \times 1\,000 =$ ____

21 $0.00318 \times 10 =$ ____

When you divide by 10, 100 or 1 000 you move the decimal point to the left

÷ 10 — 1 place left — 578.4 ÷ 10 = 57.84

÷ 100 — 2 places left — $\frac{578.4}{100} = 5.784$

÷ 1 000 — 3 places left

22 $10\overline{)3\,82}$ = ____

23 $1\,549 \div 1\,000 =$ ____

24 $4.63 \div 10 =$ ____

25 $\frac{2\,573.4}{100} =$ ____

26 $100\overline{)63.47}$ = ____

27 $\frac{3.4}{100} =$ ____

28 $25 \div 1\,000 =$ ____

29 $\frac{9.04}{1\,000} =$ ____

30 $10\overline{)13.06}$ = ____

31 $403.02 \div 100 =$ ____

32 $10.7 \div 1\,000 =$ ____

33 $10\overline{)6\,059.8}$ = ____

34 $1\,000\overline{)80.5}$ = ____

35 $\frac{14.07}{100} =$ ____

36 $702.6 \div 10 =$ ____

37 $\frac{999}{10} =$ ____

38 $999 \div 100 =$ ____

39 $999 \div 1\,000 =$ ____

Now try these, write the question mathematically then solve.

40 The airport has a departure levy of $18.30 on each person. If 1 000 people depart in an hour on average, calculate the amount (A) collected in an hour. Then give an estimate on the amount taken for a 16 h day.

A = ____ × ____

A = ____ ____

Amount after 16 hours: ____ ____

41 A small stream feeds 19.45 kL of water into a dam daily. The next day after rain it delivers 10 times this amount. Find the amount (A) for the wet day, then calculate the amount of water delivered for the 2 days.

A = ____

A = ____ + ____

____ ____

42 A tennis ball launcher fires a ball every 6 sec. Find the number of balls (B) that are fired in a minute, then find the ball capacity of the machine if it runs out of balls after 12.5 min.

B = ____

B = ____

43 A grain silo holds 9 625 kg of grain. If it is filled to one hundredth of its capacity, find the mass of the grain inside.

Multiply and Divide by 10, 100 and 1 000

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When you multiply by 10, 100 or 1 000 you move the decimal point to the right

× 10 — 1 place right — 5.784 × 10 = 57.84

× 100 — 2 places right — 5.78 × 100 = 578

× 1 000 — 3 places right

1. 2.6 × 10 = 26
2. 19.7 × 10 = 197
3. 100 × 100 = 10 000
4. 5.373 × 1 000 = 5 373
5. 933.05 × 100 = 93 305
6. 7 707.7 × 10 = 77 077
7. 0.002 × 100 = 0.2
8. 2.0202 × 10 = 20.202
9. 2.0202 × 100 = 202.02
10. 2.0202 × 1 000 = 2 020.2
11. 17.38 ×10 = 173.8
12. 0.046 × 1 000 = 46
13. 90.33 × 1 000 = 90 330
14. 12.549 × 100 = 1 254.9
15. 528.64 × 10 = 5 286.4
16. 0.0003 × 100 = 0.03
17. 530.2 × 100 = 53 020
18. 0.07 × 1 000 = 70
19. 0.8223 × 10 = 8.223
20. 116.8 × 1 000 = 116 800
21. 0.00318 × 10 = 0.0318

When you divide by 10, 100 or 1 000 you move the decimal point to the left

÷ 10 — 1 place left — 578.4 ÷ 10 = 57.84

÷ 100 — 2 places left — $\frac{578.4}{100} = 5.784$

÷ 1 000 — 3 places left

22. $10\overline{)3\,82}$ = 38.2
23. 1 549 ÷ 1 000 = 1.549
24. 4.63 ÷ 10 = 0.463
25. $\frac{2\,573.4}{100}$ = 25.734
26. $100\overline{)63.47}$ = 0.6347
27. $\frac{3.4}{100}$ = 0.034
28. 25 ÷ 1 000 = 0.025
29. $\frac{9.04}{1\,000}$ = 0.00904
30. $10\overline{)13.06}$ = 1.306
31. 403.02 ÷ 100 = 4.0302
32. 10.7 ÷ 1 000 = 0.0107
33. $10\overline{)6\,059.8}$ = 605.98
34. $1\,000\overline{)80.5}$ = 0.0805
35. $\frac{14.07}{100}$ = 0.1407
36. 702.6 ÷ 10 = 70.26
37. $\frac{999}{10}$ = 99.9
38. 999 ÷ 100 = 9.99
39. 999 ÷ 1 000 = 0.999

Now try these, write the question mathematically then solve

40. The airport has a departure levy of $18.30 on each person. If 1 000 people depart in an hour on average, calculate the amount (A) collected in an hour. Then give an estimate on the amount taken for a 16 h day.

A = 18.30 × 1 000

A = $18 300

Amount after 16 hours: $292 800

18 300 × 16

1

109 800

183 000

292 800

41. A small stream feeds 19.45 kL of water into a dam daily. The next day after rain it delivers 10 times this amount. Find the amount (A) for the wet day, then calculate the amount of water delivered for the 2 days.

A = 19.45 × 10

A = 194.5 kL

After 2 days the total is 213.95 kL

1 1

194.50 +

19.45

213.95

42. A tennis ball launcher fires a ball every 6 sec. Find the number of balls that are fired in a minute. Find the ball capacity of the machine if it runs out of balls after 12.5 min.

B = 60 ÷ 6

B = 10 balls

Capacity = 12.5 × 10

Capacity = 125 balls

43. A full grain silo holds 9 625 kg of grain. If it is currently filled to one hundredth of its capacity, find the mass of the grain inside.

Mass = 9 625 ÷ 100

Mass = 96.25 kg

Multiplication of Decimals

When decimals are multiplied, it is easier to take the decimal point out, that way it is just like multiplying two whole numbers. Then replace the decimal point in the answer, but where? Here's where:

- count the number of digits behind the decimal point in the first decimal
- then do the same for the second decimal
- add the two together and that is how many digits will be behind the decimal point in the answer, so put the decimal point in at the end.

In the example in the top row the first decimal has 2 decimal places, the second decimal has 1 decimal place. Add 2 and 1 and you get 3. So the answer will have 3 decimal places. Write that in the box provided below the question number. Then carry out the multiplication, get your answer, then put a decimal point between the 3rd and 4th digit (from the right).

The questions become longer as you work through. Remember to add a zero on the 2nd line and 2 zeros on the third line (if there is one).

Multiplication of Decimals

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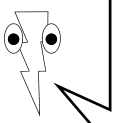

Multiply without the decimal point and put it in at the end. Write the number of d.p.'s in the box.

Example $25.79_{2} \times 0.7_{1}$

3

$$\begin{array}{r} {\scriptstyle 4\ 5\ 6} \\ 2\,579 \\ \times\ \ 7 \\ \hline 18.053 \end{array}$$

1 6.92×0.03

4

$$\begin{array}{r} 692 \\ \times\ \ 3 \\ \hline \\ \hline \end{array}$$

2 183.7×0.04

$$\begin{array}{r} \square \\ \times \\ \hline \\ \hline \end{array}$$

3 7×19.25

$$\begin{array}{r} \square \\ \times \\ \hline \\ \hline \end{array}$$

4 254.6×0.9

$$\begin{array}{r} \square \\ \times \\ \hline \\ \hline \end{array}$$

5 $8\,257 \times 0.05$

$$\begin{array}{r} \square \\ \times \\ \hline \\ \hline \end{array}$$

6 9.003×0.8

$$\begin{array}{r} \square \\ \times \\ \hline \\ \hline \end{array}$$

7 3.411×0.9

$$\begin{array}{r} \square \\ \times \\ \hline \\ \hline \end{array}$$

8 $7\,644 \times 0.009$

$$\begin{array}{r} \square \\ \times \\ \hline \\ \hline \end{array}$$

9 535.3×0.71

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

10 0.1632×59

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

11 2.1×37.16

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

12 5.113×0.35

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

13 112.4×0.061

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

14 37×5.036

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

15 21.55×0.49

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

16 0.5663×4.3

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

17 9.5×2.907

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

18 6.081×0.75

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

19 7.67×8.45

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

20 118×0.522

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

21 41.9×0.138

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

22 8.65×24.5

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

23 94.2×2.56

$$\begin{array}{r} \square \\ \times \\ \hline \\ \\ \hline \end{array}$$

Multiplication of Decimals

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Multiply without the decimal point and put it in at the end. Write the number of d.p.'s in the box.

Example $25.79_{2} \times 0.7_{1}$

3	4 5 6 2 579 × 7 18.053	1	6.92 × 0.03 2 692 × 3 0.2076	2	183.7 × 0.04 3 1 2 1 837 × 4 7.348	3	7 × 19.25 6 1 3 1 925 × 7 134.75
4	254.6 × 0.9 4 4 5 2 546 × 9 229.14	5	8 257 × 0.05 1 2 3 8 257 × 5 412.85	6	9.003 × 0.8 2 9 003 × 8 7.2024	7	3.411 × 0.9 3 3 411 × 9 3.0699	8	7 644 × 0.009 5 3 3 7 644 × 9 68.796
9	535.3 × 0.71 2 3 2 5 353 × 71 1 1 5 353 374 710 380.063	10	0.1632 × 59 3 1 1 1 632 × 59 1 14 788 81 500 9.6288	11	2.1 × 37.16 1 1 3 716 × 21 1 3 716 74 320 78.036	12	5.113 × 0.35 5 113 × 35 1 25 565 153 390 1.78955	13	112.4 × 0.061 1 2 1 124 × 61 1 124 67 440 6.8564
14	37 × 5.036 1 1 5 036 × 37 1 35 252 151 080 186.332	15	21.55 × 0.49 2 2 2 155 × 49 1 19 395 86 200 10.5595	16	0.5663 × 4.3 2 2 1 5 663 × 43 1 1 1 16 989 226 520 2.43509	17	9.5 × 2.907 8 6 2 907 × 95 1 14 535 261 630 27.6165	18	6.081 × 0.75 5 6 081 × 75 1 30 405 425 670 4.56075
19	7.67 × 8.45 5 5 767 × 845 2 1 3 835 30 680 613 600 64.8115	20	118 × 0.522 1 1 522 × 118 1 4 176 5 220 52 200 61.596	21	41.9 × 0.138 1 2 419 × 138 1 1 3 352 12 570 41 900 5.7822	22	8.65 × 24.5 1 1 865 × 245 1 1 4 325 34 600 173 000 211.925	23	94.2 × 2.56 942 × 256 1 2 1 5 652 47 100 188 400 241.152

Division of Decimals

Dividing decimals is performed in the same way as with whole numbers except that when you divide past the decimal point, place a decimal point in the quotient (answer).

Column 1 involves dividing decimals by a whole number. If the number 'won't divide' into the other number then write a zero and carry the digit to the next number (as tens) and then attempt the division again.

Column 2 involves longer questions, these involve adding zeros to enable the division to be completed. The zeros are added until you can remove the remainder. From question 18 answer the divisions using 4 decimal places. This means that you need a 5 decimal place answer which can then be rounded to the 4 d.p answer required.

Column 3 involves dividing decimals by other decimals. This column requires you to divide or multiply both numbers by either 10 or 100 before the division takes place. This is done to change the divisor (the outside number) to a single digit number, for example:

- 46.4 ÷ 0.8 ….multiply both numbers by 10 to get ...464 ÷ 8
- 29.6 ÷ 60 ….. Divide each number by 10 to get …. 2.96 ÷ 6
- 0.597 ÷ 0.04 … multiply each number by 100 to get 59.7 ÷ 4

Although you are changing the numbers you are using, because you are changing both of them in the same way the answer stays the same and doesn't need to be changed.

Division of Decimals

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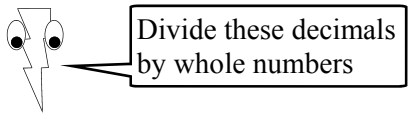


Example $1.56 \div 4$

$$4 \overline{)1.56} = 0.39 \quad (1\ 3)$$

1 $2.56 \div 8$

2 $9.45 \div 3$

3 $2.28 \div 6$

4 $0.572 \div 4$

5 $9.275 \div 5$

6 $45.87 \div 3$

7 $0.092 \div 4$

8 $105.6 \div 8$

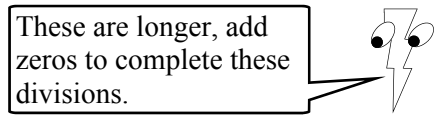
9 $4.752 \div 9$

10 $19.06 \div 2$

11 $3.584 \div 7$

12 $59.36 \div 8$

13 $862.5 \div 5$



Example $3.45 \div 8$

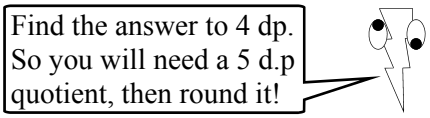
$$8 \overline{)3.45000} = 0.43125 \quad (3\ 2\ 1\ 2\ 4)$$

14 $2.63 \div 5$

15 $19.3 \div 4$

16 $15.5 \div 8$

17 $19.81 \div 2$



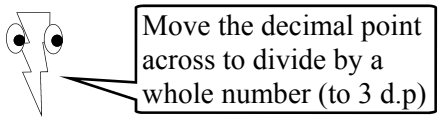
18 $6.5 \div 7$ = _____ (4 d.p.)

19 $14.8 \div 3$ = _____ (4 d.p.)

20 $7.26 \div 9$ = _____ (4 d.p.)

21 $22 \div 7$ = _____ (4 d.p.)

22 $0.41 \div 8$ = _____ (4 d.p.)



Example $52.1 \div 0.8$

$$8 \overline{)521.000} = 65.625 \quad (5\ 4\ 1\ 2\ 4)$$

$52.1 \times 10 = 521$
$0.8 \times 10 = 8$

23 $17 \div 0.4$ = _____ (3 d.p.)

24 $0.72 \div 0.5$ = _____ (3 d.p.)

25 $0.431 \div 0.03$ = _____ (3 d.p.)

26 $1.7 \div 30$ = _____ (3 d.p.)

27 $8.53 \div 0.7$ = _____ (3 d.p.)

28 $1.27 \div 0.4$ = _____ (3 d.p.)

29 $4.32 \div 90$ = _____ (3 d.p.)

30 $0.5 \div 0.06$ = _____ (3 d.p.)

31 $9.2 \div 40$ = _____ (3 d.p.)

Division of Decimals

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Divide these decimals by whole numbers

Example

1.56 ÷ 4 — 4) 1.56 (1 3) = 0.39

1 2.56 ÷ 8 — 8) 2.56 (1) = 0.32

2 9.45 ÷ 3 — 3) 9.45 (1) = 3.15

3 2.28 ÷ 6 — 6) 2.28 (4) = 0.38

4 0.572 ÷ 4 — 4) 0.572 (1 1) = 0.143

5 9.275 ÷ 5 — 5) 9.275 (4 2 2) = 1.855

6 45.87 ÷ 3 — 3) 45.87 (1 2) = 15.29

7 0.092 ÷ 4 — 4) 0.092 (1) = 0.023

8 105.6 ÷ 8 — 8) 105.6 (2 1) = 13.2

9 4.752 ÷ 9 — 9) 4.752 (2 7) = 0.528

10 19.06 ÷ 2 — 2) 19.06 (1) = 9.53

11 3.584 ÷ 7 — 7) 3.584 (1) = 0.512

12 59.36 ÷ 8 — 8) 59.36 (3 1) = 7.42

13 862.5 ÷ 5 — 5) 862.5 (3 1 2) = 172.5

These are longer, add zeros to complete these divisions.

Example

3.45 ÷ 8 — 8) 3.45000 (3 2 1 2 4) = 0.43125

14 2.63 ÷ 5 — 5) 2.630 (1 3) = 0.126

15 19.3 ÷ 4 — 4) 19.3000 (3 1 2) = 4.825

16 15.5 ÷ 8 — 8) 15.5000 (7 3 6 4) = 1.9375

17 19.81 ÷ 2 — 2) 19.810 (1 1) = 9.9405

Find the answer to 4 dp. So you will need a 5 d.p quotient, then round it!

18 6.5 ÷ 7 — 7) 6.50000 (2 6 4 5) = 0.62857

= 0.6286 (4 d.p.)

19 14.8 ÷ 3 — 3) 14.80000 (2 1 1 1 1) = 4.93333

= 4.9333 (4 d.p.)

20 7.26 ÷ 9 — 9) 7.26000 (6 6 6) = 0.80666

= 0.8067 (4 d.p.)

21 22 ÷ 7 — 7) 22.00000 (1 3 2 6 4) = 3.14285

= 3.1429 (4 d.p.)

22 0.41 ÷ 8 — 8) 0.41000 (4 1 2 4) = 0.05125

= 0.0513 (4 d.p.)

Move the decimal point across to divide by a whole number (to 3 d.p)

Example

52.1 ÷ 0.8 — 8) 521.000 (5 4 1 2 4) = 65.625

{52.1 × 10 = 521
{0.8 × 10 = 8

23 17 ÷ 0.4 — 4) 170.0 (1 2) = 42.5

= 42.500 (3 d.p.)

24 0.72 ÷ 0.5 — 5) 7.20 (2 2) = 1.44

= 1.440 (3 d.p.)

25 0.431 ÷ 0.03 — 3) 43.1000 (1 1 2 2 2) = 14.3666

= 14.367 (3 d.p.)

26 1.7 ÷ 30 — 3) 0.17000 (1 2 2 2) = 0.05666

= 0.057 (3 d.p.)

27 8.53 ÷ 0.7 — 7) 85.3000 (1 1 6 4 5) = 12.1857

= 12.186 (3 d.p.)

28 1.27 ÷ 0.4 — 4) 12.700 (3 2) = 3.175

= 3.175 (3 d.p.)

29 4.32 ÷ 90 — 9) 0.432 (4 7) = 0.048

= 0.048 (3 d.p.)

30 0.5 ÷ 0.06 — 6) 50.0000 (2 2 2 2) = 8.3333

= 8.333 (3 d.p.)

31 9.2 ÷ 40 — 4) 0.92 (1) = 0.23

= 0.920 (3 d.p.)

Changing Fractions to Decimals

When you look at a fraction you can consider it as the top number divided by the bottom number. To change a fraction to a decimal you carry out the division and the answer after the division (the quotient) will be in decimal form.

Column 1 starts with changing fractions to decimals that have a denominator of either 10, 100 or 1 000. Remember that this is just like division, when you divide by these numbers move the decimal point to the left. So for division by 10, move the decimal point 1 position to the left, 100 - 2 positions to the left and 1 000 moves 3 positions left. So the answer will be the numerator (top number) with the decimal point moved. There are 9 examples at the top of the column. Remember that when you move the decimal point, zeros that had meaning on the end of whole numbers are not required when you convert them to decimals.

Column 2 gives you fractions that require division to change to decimals. First write out the division with a ÷ sign, this is done for you already in some questions. Then place the numbers in the working space, add a decimal point and zeros to the number in the division, there is room to add up to 7 zeros, you won't need that many though. Then divide. When you divide a decimal by a whole number add a decimal point in the answer when you cross over it in the division. There is an example at the top of column 2. Note that questions 29 and 30 are long division and question 31 asks you to convert a mixed number to an improper fraction then solve.

Column 3 introduces recurring (or repeating) decimals. These decimals have digits or a sequence of digits that constantly repeat themselves in the decimal. Because these go on forever you need to show that this is occurring in your answer. To do this put a dot on the number or group of numbers that are recurring, for example

- 0.76666666... would be written as $0.7\dot{6}$
- 0.7676767676…. would be written as $0.\dot{7}\dot{6}$, because both numbers are recurring
- 0.765876587658…. would be written as $0.\dot{7}65\dot{8}$ (the first and last of the recurring Numbers have dots above them.

Changing Fractions to Decimals

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Change these fractions to decimals, make sure there are no 'end zeros'

Examples

$\frac{1}{10} = 0.1$ | $\frac{1}{100} = 0.01$ | $\frac{1}{1\,000} = 0.001$

$\frac{10}{10} = 1$ | $\frac{10}{100} = 0.1$ | $\frac{10}{1\,000} = 0.01$

$\frac{41}{10} = 4.1$ | $\frac{352}{100} = 3.52$ | $\frac{100}{1\,000} = 0.1$

1 $\frac{7}{10} =$ _____ 2 $\frac{8}{100} =$ _____

3 $\frac{5}{1\,000} =$ _____

4 $\frac{16}{10} =$ _____ 5 $\frac{37}{100} =$ _____

6 $\frac{87}{100} =$ _____ 7 $\frac{63}{10} =$ _____

8 $\frac{430}{1\,000} =$ _____

9 $\frac{906}{1\,000} =$ _____

10 $\frac{408}{100} =$ _____ 11 $\frac{110}{100} =$ _____

12 $\frac{4\,076}{1\,000} =$ _____

13 $\frac{8\,008}{100} =$ _____

14 $\frac{1\,501}{1\,000} =$ _____

15 $\frac{45}{10} =$ _____ 16 $\frac{60}{10} =$ _____

17 $\frac{550}{100} =$ _____ 18 $\frac{120}{100} =$ _____

Now the reverse, change these decimals to fractions

19 0.47 = $\frac{\square}{\square}$ 20 1.1 = $\frac{\square}{\square}$

21 2.13 = $\frac{\square}{\square}$ 22 35.7 = $\frac{\square}{\square}$

Use division to change these fractions to decimals

Example

$\frac{3}{4} = 3 \div 4$ $4\overline{)3.00}$ $= 0.75$ (3 2)

23 $\frac{1}{2} = 1 \div 2$ $2\overline{)1}$

24 $\frac{1}{4} = 1 \div 4$ $4\overline{)1}$

25 $\frac{2}{5} = 2 \div 5$ $\overline{)\ \ }$

26 $\frac{3}{8} = 3 \div 8$ $\overline{)\ \ }$

27 $\frac{4}{5} =$ _____ $\overline{)\ \ }$

28 $\frac{7}{8} =$ _____ $\overline{)\ \ }$

29 $\frac{3}{20} =$ _____ 30 $\frac{7}{40} =$ _____

$\overline{)\ \ }$ $\overline{)\ \ }$

Convert to improper fraction

31 $4\frac{4}{5} = \frac{\square}{\square} =$ _____

$\overline{)\ \ }$

Convert these fractions, which will be recurring decimals

Example

$\frac{1}{6} = 1 \div 6$ $6\overline{)1.0000}$ $= 0.1\dot{6}$ (4 4 4)

32 $\frac{2}{3} =$ _____ $\overline{)\ \ }$

$=$ _____

33 $\frac{1}{9} =$ _____ $\overline{)\ \ }$

$=$ _____

34 $\frac{5}{6} =$ _____ $\overline{)\ \ }$

$=$ _____

35 $\frac{7}{9} =$ _____ $\overline{)\ \ }$

$=$ _____

36 $\frac{5}{11} =$ _____ $\overline{)\ \ }$

$=$ _____

37 $\frac{6}{11} =$ _____ $\overline{)\ \ }$

$=$ _____

38 $\frac{8}{15} =$ _____ $\overline{)\ \ }$

$=$ _____

39 $\frac{5}{12} =$ _____ $\overline{)\ \ }$

$=$ _____

Changing Fractions to Decimals

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Change these fractions to decimals, make sure there are no 'end zeros'

Examples

$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1\,000} = 0.001$
$\frac{10}{10} = 1$	$\frac{10}{100} = 0.1$	$\frac{10}{1\,000} = 0.01$
$\frac{41}{10} = 4.1$	$\frac{352}{100} = 3.52$	$\frac{100}{1\,000} = 0.1$

1 $\frac{7}{10} = 0.7$

2 $\frac{8}{100} = 0.08$

3 $\frac{5}{1\,000} = 0.005$

4 $\frac{16}{10} = 1.6$

5 $\frac{37}{100} = 0.37$

6 $\frac{87}{100} = 0.87$

7 $\frac{63}{10} = 6.3$

8 $\frac{430}{1\,000} = 0.43$

9 $\frac{906}{1\,000} = 0.906$

10 $\frac{408}{100} = 4.08$

11 $\frac{110}{100} = 1.1$

12 $\frac{4\,076}{1\,000} = 4.076$

13 $\frac{8\,008}{100} = 80.08$

14 $\frac{1\,501}{1\,000} = 1.501$

15 $\frac{45}{10} = 4.5$

16 $\frac{60}{10} = 6$

17 $\frac{550}{100} = 5.5$

18 $\frac{120}{100} = 1.2$

Now the reverse, change these decimals to fractions

19 $0.47 = \frac{47}{100}$

20 $1.1 = \frac{11}{10}$

21 $2.13 = \frac{213}{100}$

22 $35.7 = \frac{357}{10}$

Use division to change these fractions to decimals

Example

$\frac{3}{4} = 3 \div 4$ $\quad 4\overline{)3.00}$ = 0.75 (3 2)

23 $\frac{1}{2} = 1 \div 2$ $\quad 2\overline{)1.0}$ = 0.5

24 $\frac{1}{4} = 1 \div 4$ $\quad 4\overline{)1.00}$ = 0.25 (2)

25 $\frac{2}{5} = 2 \div 5$ $\quad 5\overline{)2.0}$ = 0.4

26 $\frac{3}{8} = 3 \div 8$ $\quad 8\overline{)3.000}$ = 0.375 (6 4)

27 $\frac{4}{5} = 4 \div 5$ $\quad 5\overline{)4.0}$ = 0.8

28 $\frac{7}{8} = 7 \div 8$ $\quad 8\overline{)7.000}$ = 0.875 (6 4)

29 $\frac{3}{20} = 3 \div 20$

$20\overline{)3.00}$ = 0.15
20
100
100
0

30 $\frac{7}{40} = 7 \div 40$

$40\overline{)7.000}$ = 0.175
40
300
280
200
200
0

Convert to improper fraction

31 $4\frac{4}{5} = \frac{24}{5} = 24 \div 5$

$5\overline{)24.0}$ = 4.8 (4)

Convert these fractions, which will be recurring decimals

Example

$\frac{1}{6} = 1 \div 6$ $\quad 6\overline{)1.0000}$ = 0.1666 (4 4 4)

$= 0.1\dot{6}$

32 $\frac{2}{3} = 2 \div 3$ $\quad 3\overline{)2.000000}$ = 0.666666 (2 2 2 2 2)

$= 0.\dot{6}$

33 $\frac{1}{9} = 1 \div 9$ $\quad 9\overline{)1.000000}$ = 0.111111 (1 1 1 1 1)

$= 0.\dot{1}$

34 $\frac{5}{6} = 5 \div 6$ $\quad 6\overline{)5.000000}$ = 0.833333 (2 2 2 2 2)

$= 0.\dot{8}\dot{3}$

35 $\frac{7}{9} = 7 \div 9$ $\quad 9\overline{)7.000000}$ = 0.777777 (7 7 7 7 7)

$= 0.\dot{7}$

36 $\frac{5}{11} = 5 \div 11$ $\quad 11\overline{)5.000000}$ = 0.454545 (6 5 6 5 6)

$= 0.\dot{4}\dot{5}$

37 $\frac{6}{11} = 6 \div 11$ $\quad 11\overline{)6.000000}$ = 0.545454 (5 6 5 6 5)

$= 0.\dot{5}\dot{4}$

38 $\frac{8}{15} = 8 \div 15$ $\quad 15\overline{)8.000000}$ = 0.533333 (5 5 5 5 5)

$= 0.5\dot{3}$

39 $\frac{5}{12} = 5 \div 12$ $\quad 12\overline{)5.000000}$ = 0.416666 (2 8 8 8 8)

$= 0.41\dot{6}$

FREEFALL MATHEMATICS

ALTITUDE

ALGEBRA 1

Completing Number Patterns

To complete number patterns either a "rule" is required which tells you how the number pattern is created, or, enough numbers (usually 4) are needed to determine the rule yourself. Once the rule has been found the last number is used to find the next one and so on.

In column 1 the rule is given, use the last number given and put it in the rule to get the first missing number. Then using the new number, put it in the rule and get the next missing value, and so on.

In column 2 you are asked to create the next pattern by building it with matchsticks. Then find the missing numbers to extend the pattern. Then write the rule in words as you were given in the first column.

Column 3 has number patterns, note how the numbers are changing and carry on the process for the unknown numbers in the boxes. The last few questions are more difficult and require two operations, for example multiplying by two an subtracting one.

Completing Number Patterns

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Complete the patterns by finding the missing numbers

1 Start with 2, "add" 3

→ 2, 5, 8, ☐, ☐

2 Start with 21, "subtract" 3

→ 21, 18, 15, ☐, ☐

3 Start with 5, "add" 5

→ 5, 10, ☐, 20, ☐

4 Start with 5, "multiply by" 2

→ 5, 10, ☐, 40, ☐, ☐

5 Start with 80, "divide by" 2

→ 80, 40, ☐, ☐, ☐

6 Start with 25, "subtract" 5

→ 25, 20, ☐, ☐, ☐

7 Start with 2, "multiply by" 2 then add 1

→ 2, 5, 11, ☐, ☐, ☐

8 Start with 5, "multiply by" 3 then subtract 5

→ 5, 10, ☐, 70, ☐, ☐

9 Start with 6, "multiply by" 2 then subtract 4

→ 6, 8, ☐, ☐, ☐, ☐

10 Start with 3, "multiply by" 2 then add 1

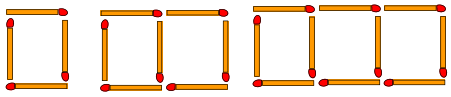
→ 3, 7, ☐, ☐, ☐, ☐

11 Start with 100, "divide by" 2 then add 2

→ 100, 52, ☐, ☐, ☐

Draw the next match-stick pattern then complete the numbers and pattern in words below it.

12

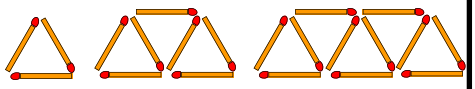


4th → pattern

4, 7, 10, ☐, ☐, ☐, ☐

Start with ☐ "add" ☐

13

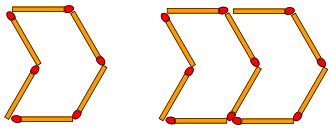


4th → pattern

3, 7, ☐, ☐, ☐, ☐

Start with ☐ ___ ☐

14

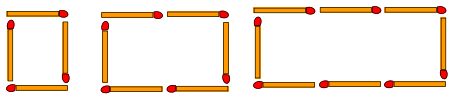


3rd → pattern

6, 10, ☐, ☐, ☐, ☐

___ ☐ ___ ☐

15



4th → pattern

☐, ☐, ☐, ☐, ☐

___ ☐ ___ ☐

Complete the patterns you will at times get answers which include decimals.

16 4, 9, 14, 19, ☐, ☐, ☐

17 20, 35, 50, 65, ☐, ☐

18 1, 2, 4, 8, ☐, ☐, ☐

19 320, 160, 80, 40, ☐, ☐

20 $\frac{1}{5}$, 1, 5, 25, ☐, ☐

21 88, 99, 110, 121, ☐, ☐

22 5, 14, 23, 32, ☐, ☐

23 87, 91, 95, 99, ☐, ☐

(decimal answers)

24 1 000, 100, 10, 1, ☐, ☐

25 1, 7, 13, 19, ☐, ☐, ☐

26 15, 10, 5, 0, ☐, ☐, ☐

27 197, 157, 117, 77, ☐, ☐

28 54, 63, 72, 81, ☐, ☐

These are harder, you will have to multiply by a number then add/subtract.

29 1, 4, 10, 22, ☐, ☐

30 2, 4, 10, 28, ☐, ☐

31 1, 4, 13, 40, ☐, ☐

32 2½, 10, 25, 55, ☐, ☐

33 1, 2, 5, 14, ☐, ☐

34 1, 6, 16, 36, ☐, ☐

Completing Number Patterns

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Complete the patterns by finding the missing numbers

1 Start with 2, "add" 3

→ 2, 5, 8, 11, 14

2 Start with 21, "subtract" 3

→ 21, 18, 15, 12, 9

3 Start with 5, "add" 5

→ 5, 10, 15, 20, 25

4 Start with 5, "multiply by" 2

→ 5, 10, 20, 40, 80, 160

5 Start with 80, "divide by" 2

→ 80, 40, 20, 10, 5

6 Start with 25, "subtract" 5

→ 25, 20, 15, 10, 5

7 Start with 2, "multiply by" 2 then add 1

→ 2, 5, 11, 23, 47, 95

8 Start with 5, "multiply by" 3 then subtract 5

→ 5, 10, 25, 70, 205, 610

9 Start with 6, "multiply by" 2 then subtract 4

→ 6, 8, 12, 20, 36, 68

10 Start with 3, "multiply by" 2 then add 1

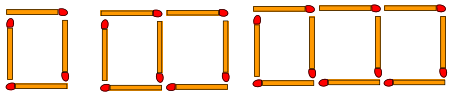
→ 3, 7, 15, 31, 63, 127

11 Start with 100, "divide by" 2 then add 2

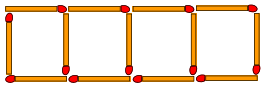
→ 100, 52, 28, 16, 10

Draw the next match-stick pattern then complete the numbers and pattern in words below it.

12



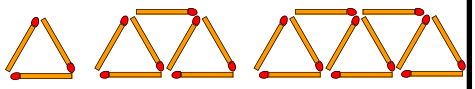
4th pattern →



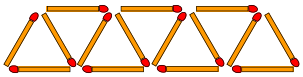
4, 7, 10, 13, 16, 19, 22

Start with 4 "add" 3

13



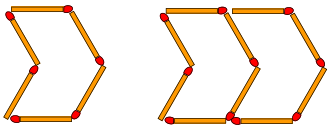
4th pattern →



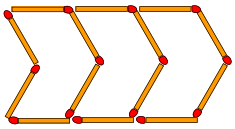
3, 7, 11, 15, 19, 23

Start with 3 "add" 4

14



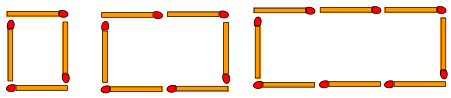
3rd pattern →



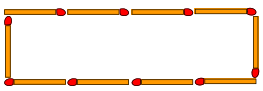
6, 10, 14, 18, 22, 26

Start with 6 "add" 4

15



4th pattern →



4, 6, 8, 10, 12

Start with 4 "add" 2

Complete the patterns you will at times get answers which include decimals.

16 4, 9, 14, 19, 24, 29, 34

17 20, 35, 50, 65, 80, 95

18 1, 2, 4, 8, 16, 32, 64

19 320, 160, 80, 40, 20, 10

20 $\frac{1}{5}$, 1, 5, 25, 125, 625

21 88, 99, 110, 121, 132, 143

22 5, 14, 23, 32, 41, 50

23 87, 91, 95, 99, 103, 107

(decimal answers)

24 1 000, 100, 10, 1, 0.1, 0.01

25 1, 7, 13, 19, 25, 31, 37

26 15, 10, 5, 0, -5, -10, -15

27 197, 157, 117, 77, 37, -3

28 54, 63, 72, 81, 90, 99

These are harder, you will have to multiply by a number then add/subtract.

29 1, 4, 10, 22, 46, 94

30 2, 4, 10, 28, 82, 244

31 1, 4, 13, 40, 121, 364

32 2½, 10, 25, 55, 115, 235

33 1, 2, 5, 14, 41, 122

34 1, 6, 16, 36, 76, 156

Writing Algebraic Expressions

An algebraic expression involves pronumerals (which are letters), like a, b, etc. Why use letters in mathematics? A letter can be used to represent different numbers, so it is a quick way to use the same 'formula' over and over.

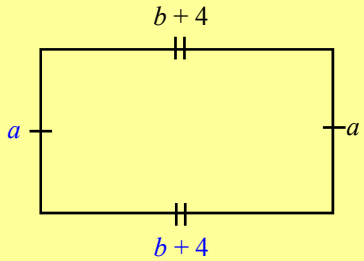
To write expressions in words there are several words that should be understood:
addition - increase, raise, sum of, plus, and add.
subtraction - decrease, reduce, subtract, minus and take away.
division - divide, over, find the quotient (answer after division).
multiplication - times, product of, multiply, times, lots of
squared - square of, multiplied by itself

In Column 1 write the algebraic expression that the sentence describes, this is sometimes not straight-forward. E.g. Subtract g^2 from the product of 9 and $m \rightarrow 9 \times m - g^2$
While g^2 is written first in the sentence, it is the last mathematical operation to be completed so it is written last in the expression.

Column 2 is the reverse of column 1, construct the sentences given the expression. Some will require order of operations to be considered so be careful.

Column 3 requires an expression for the perimeter of the shapes, write each side separated by a '+' sign, then add the numbers together to get a total but leave the letters as they are. Different letters are left with plus signs between them, you are about to learn how to add pronumerals, but for this exercise this is not required.

Example.



$P = b + 4 + a + b + 4 + a$
$= b + b + a + a + 8$ (leave your answer like this or simplify as below)
$= 2b + 2a + 8$ (you will learn this on the next sheet)

Writing Algebraic Expressions

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Read the sentences and write algebraic expressions to describe them.

Example

Increase 15 by t squared

$15 + t^2$

1 The sum of g and 20

2 Decrease w by 29

3 Multiply 5 by d

4 From 15 subtract u

5 The sum of a, b and c

6 Raise 16 by r squared

7 Reduce b by 870

8 Subtract 38 from the sum of w and a

9 Reduce the quotient of 20 and q by 8

10 From the product of m and 9 subtract 6

This time write a sentence that matches the algebraic expressions below

Use words like: squared, times, increase, decrease, raise, reduce, sum of, subtract, product, plus, divide, multiply, minus and add.

11 $2 \times m$

12 $k + 5 - d$

* order of operations

13 $a + 2 \times b$*

14 $854 \div 3 \times f$

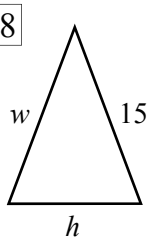
15 $9 + h \times c$

16 $a^2 + 25 - q$

17 $x^2 - 3 \times a$

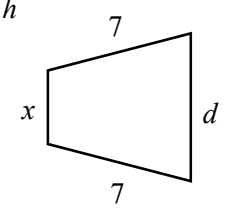
Construct an algebraic expression for the perimeter of the shapes below.

18

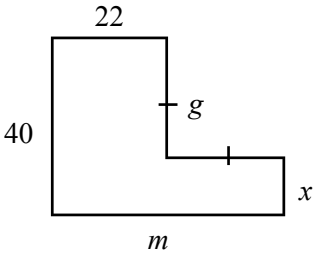


P =

19

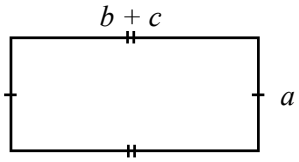


20



These two are harder

21



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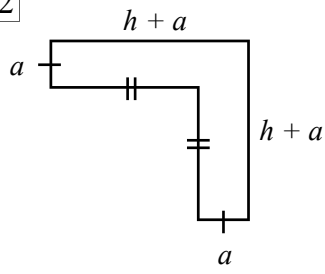

Writing Algebraic Expressions

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Read the sentences and write algebraic expressions to describe them.

Example

Increase 15 by t squared

$15 + t^2$

1 The sum of g and 20

$g + 20$

2 Decrease w by 29

$w - 29$

3 Multiply 5 by d

$5 \times d$

4 From 15 subtract u

$15 - u$

5 The sum of a, b and c

$a + b + c$

6 Raise 16 by r squared

$16 + r^2$

7 Reduce b by 870

$b - 870$

8 Subtract 38 from the sum of w and a

$w + a - 38$

9 Reduce the quotient of 20 and q by 8

$20 \div q - 8$

10 From the product of m and 9 subtract 6

$m \times 9 - 6$

This time write a sentence that matches the algebraic expressions below

Use words like: squared, times, increase, decrease, raise, reduce, sum of, subtract, product, plus, divide, multiply, minus and add.

11 $2 \times m$

The product of 2 and m

12 $k + 5 - d$

From the sum of k and 5

subtract d

* order of operations

13 $a + 2 \times b$*

To the product of 2 and b add a

14 $854 \div 3 \times f$

Divide 854 by 3 then multiply

the quotient by f

15 $9 + h \times c$

To the product of h and c add 9

16 $a^2 + 25 - q$

From the sum of a squared and

25, subtract q

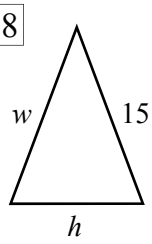
17 $x^2 - 3 \times a$

From x squared subtract the

product of 3 and a

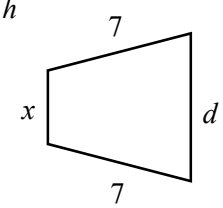
Construct an algebraic expression for the perimeter of the shapes below.

18



$P = w + h + 15$

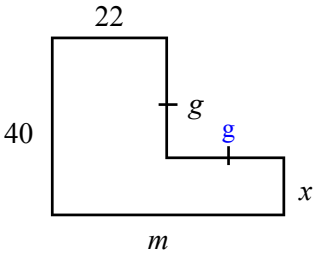
19



$P = x + d + 7 + 7$

$P = x + d + 14$

20

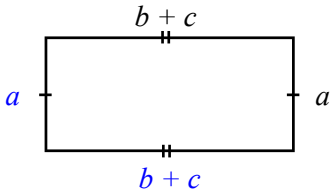


$P = 40 + 22 + g + g + x + m$

$P = 62 + 2g + x + m$

These two are harder

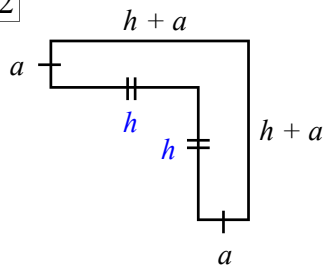
21



$P = a + b + c + a + b + c$

$P = 2a + 2b + 2c$

22



$P = a + h + a + h + a + a + h + h$

$P = 4a + 4h$

Formulae and Substitution

Substitution means to replace a pronumeral 'letter' with a number. This is just like using a formula and putting the values in. Just remember that when a number is before a letter it is as if there is a multiplication sign between them.

Eg. G = 4f, find G when f = 12 min

$G = 4f$

$= 4 \times 12$

$G = 48$ min

Note that G is written on the bottom row and includes the units, whatever they may be, in this case min (minutes).

In the 2nd column you are required to construct a formula that matches the sentence, this may be challenging. Use the underlined letters to make the formula.

Eg. Chris is 4 times older than Ian, if Ian is 9 years old calculate Chris' age.

$C = 4I$

$= 4 \times 9$

$C = 36$ years

The 3rd column requires substitution to be built up in stages, if x^2 has been found then to calculate $2x^2$ just double the answer for x^2. Note that $2x$ isn't squared. Calculate x^2 then multiply it by 2. Write your answers going down the column and the 3rd row will be some small combination of the first two rows.

Formulae and Substitution

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Use the formulae below to substitute then solve

1 Find the perimeter of a square using $P = 4l$ where $l = 12$ cm.

$P = 4l$

use = signs $= 4 \times 12$ show units

$P =$ _____

rewrite letter

2 Find the average age of two girls using: $A = \frac{x + y}{2}$

where $x = 6$ years and $y = 16$ years.

$A = \frac{x + y}{2}$

$=$ _____

_____

3 Change minutes to seconds using $T = 60m$ where $m = 4$ min.

_____

_____

_____

4 Find the area of a triangle using $A = \frac{1}{2}bh$ where $b = 16$ mm and $h = 5$ mm

_____

_____

_____

5 Now use $b = 12$ m and $h = 10$ m to find the Δ area.

_____

_____

_____

Use the underlined letters to construct a formula for the following. Then substitute and solve.

6 Julian is twice as old as Ann. If Ann is 17 years old how old is Julian?

_____

_____

_____

7 There are 12 pieces in one family pizza how many pieces are there in 7?

_____

_____

_____

8 Britney bought a pair of shoes at a price of $76.85. Find her change from $100

_____

_____

_____

9 Sean has half the money that he had when he left home. How much money has he now if he started with $8.70.

_____

_____

_____

10 If a car has 5 wheels and a motorcycle has 2. How many wheels do 3 cars and 6 motorcycles have?

_____

_____

_____

Answer the top row then the 2nd row then add the two rows to help answer the 3rd row.

11

Use $k =$	2	3	4
k^2			
$7k$			
$k^2 + 7k$			

12

Use $g =$	2	4	6
g^2			
$5g$			
$2g^2 + 5g$			

13

Use $m =$	4	7	10
m^2			
$2m$			
$m^2 + 2m - 3$			

14

Use $e =$	5	10	20
e^2			
$4e$			
$2e^2 + 8e$			

Formulae and Substitution

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Use the formulae below to substitute then solve

1 Find the perimeter of a square using $P = 4l$ where $l = 12$ cm.

$P = 4l$

use = signs $= 4 \times 12$ show units

$P = 48$ cm

rewrite letter

2 Find the average age of two girls using: $A = \frac{x + y}{2}$ where $x = 6$ years and $y = 16$ years.

$$A = \frac{x + y}{2}$$

$$= \frac{6 + 16}{2}$$

$A = 11$ years

3 Change minutes to seconds using $T = 60m$ where $m = 4$ min.

$T = 60m$

$= 60 \times 4$

$T = 240$ min

4 Find the area of a triangle using $A = \frac{1}{2}bh$ where $b = 16$ mm and $h = 5$ mm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 16 \times 5$

$A = 40$ mm²

5 Now use $b = 12$ m and $h = 10$ m to find the Δ area.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 10$

$A = 60$ m²

Use the underlined letters to construct a formula for the following. Then substitute and solve.

6 Julian is twice as old as Ann. If Ann is 17 years old how old is Julian?

$J = 2A$

$= 2 \times 17$

$J = 34$ years

7 There are 12 pieces in one family pizza how many pieces are there in 7?

$p = 12f$

$= 12 \times 7$

$p = 84$ pieces

8 Britney bought a pair of shoes at a price of $76.85. Find her change from $100

$c = 100 - p$

$= 100 - 76.85$

$c = \$23.15$

9 Sean has half the money that he had when he left home. How much money has he now if he started with $8.70.

$n = \frac{1}{2}m$

$= \frac{1}{2} \times 8.70$

$n = \$4.35$

10 If a car has 5 wheels and a motorcycle has 2. How many wheels do 3 cars and 6 motorcycles have?

$w = 5c + 2m$

$= 5 \times 3 + 2 \times 6$

$w = 27$ wheels

Answer the top row then the 2nd row then add the two rows to help answer the 3rd row.

11

Use k =	2	3	4
k^2	4	9	16
$7k$	14	21	28
$k^2 + 7k$	18	30	44

12

Use g =	2	4	6
g^2	4	16	36
$5g$	10	20	30
$2g^2 + 5g$	18	52	102

13

Use m =	4	7	10
m^2	16	49	100
$2m$	8	14	20
$m^2 + 2m - 3$	21	60	117

14

Use e =	5	10	20
e^2	25	100	400
$4e$	20	40	80
$2e^2 + 8e$	90	180	960

Substitution

This sheet doesn't feature Negative numbers. Sheet 08 in the Algebra 2 folder does.

Substitution means to replace the pronumeral (letter) with a number, then solve the question just like any other operations exercise. There is only one difference, when the letter is after a number put a × between them.

E.g. If $a = 5$ then $7a = 7 \times 5 = 35$, don't just change the a for a 5, i.e. not $7a = 75$. Another mistake is when a number is squared.

E.g. If $a = 9$ $a^2 = 9 \times 9 = 81$, it is not $a^2 = 9 \times 2 = 18$.

Column 1 asks you to use $a = 4$ in all the equations. Substitute a into each equation then solve the equation (get an answer). Remember to watch for order of operations, an example is at the top of the column.

Column 2 involves squares (^{2}) and there are two letters used, k and e. Use the same process as with the previous column but remember that a squared number is that number times itself. When there is a number before a squared term, such as $5m^2$, then only the m part is squared ($= 5 \times m^2$) not the whole line $(5m)^2$.

Column 3 involves using brackets, remember that brackets are done first. Otherwise these are the same style of question as in the previous column.

Substitution

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Use $a = 4$ to find the value of the other pronumeral

Example

$q = 10a - 11$ ← question

$= 10 \times 4 - 11$ ← substitution

$q = 29$ ← solution (answer)

1 $b = a + 3$

2 $h = a + 8$

3 $m = 2a - 2$

4 $w = 6a - 5$

5 $q = 5a + 7$

6 $u = 11a - 23$

7 $k = 11a - 10$

This time use $k = 5$ and $e = 7$. Remember that $k^2 = 5^2$ or 5×5 not 5×2



Example

$d = e^2 + 5$

$= 7 \times 7 + 5$

$d = 54$

or,

$d = e^2 + 5$

$= 7^2 + 5$

$d = 54$

8 $t = k^2 + 3$

9 $g = e^2 - 5$

10 $m = k^2 - k$

11 $n = \dfrac{e^2 - 4}{9}$

$=$

12 $d = \dfrac{8k^2}{4}$

$=$

13 $b = 100 - e^2$

14 $r = 2k^2$

With brackets solve the inside of the brackets first, (order of operations). Let $m = 10$ and $f = 4$



Example

$t = 5(m + f)$

$= 5 \times (10 + 4)$

$t = 70$

15 $v = 2(m + 7)$

16 $c = 9(f + 5)$

17 $x = f(m - 3)$

18 $g = f(m - f)$

19 $n = 7(20 - m)$

20 $q = m(m - 2f)$

21 $t = 2m \div (f + 1)$

Substitution

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Use $a = 4$ to find the value of the other pronumeral

Example

$q = 10a - 11$ ← question

$= 10 \times 4 - 11$ ← substitution

$q = 29$ ← solution (answer)

1 $b = a + 3$

$= 4 + 3$

$b = 7$

2 $h = a + 8$

$= 4 + 8$

$h = 12$

3 $m = 2a - 2$

$= 2 \times 4 - 2$

$m = 6$

4 $w = 6a - 5$

$= 6 \times 4 - 5$

$w = 19$

5 $q = 5a + 7$

$= 5 \times 4 + 7$

$q = 27$

6 $u = {}^{a}/_{2} + 23$

$= 4 \div 2 + 23$

$u = 25$

7 $k = 11a - 10$

$= 11 \times 4 - 10$

$k = 34$

This time use $k = 5$ and $e = 7$. Remember that $k^2 = 5^2$ or 5×5 not 5×2

Example

$d = e^2 + 5$

$= 7 \times 7 + 5$

$d = 54$

or,

$d = e^2 + 5$

$= 7^2 + 5$

$d = 54$

8 $t = k^2 + 3$

$= 5^2 + 3$

$t = 28$

9 $g = e^2 - 5$

$= 7^2 - 5$

$g = 44$

10 $m = k^2 - k$

$= 5^2 - 5$

$m = 20$

11 $n = \dfrac{e^2 - 4}{9}$

$= \dfrac{7^2 - 4}{9}$

$= \dfrac{45}{9}$

$n = 5$

12 $d = \dfrac{8k^2}{4}$

$= \dfrac{8 \times 5^2}{4}$

$= \dfrac{200}{4}$

$d = 50$

13 $b = 100 - e^2$

$= 100 - 7^2$

$b = 51$

14 $r = 2k^2$

$= 2 \times 5^2$

$r = 50$

With brackets solve the inside of the brackets first, (order of operations). Let $m = 10$ and $f = 4$

Example

$t = 5(m + f)$

$= 5 \times (10 + 4)$

$t = 70$

15 $v = 2(m + 7)$

$= 2 \times (10 + 7)$

$v = 34$

16 $c = 9(f + 5)$

$= 9 \times (4 + 5)$

$c = 81$

17 $x = f(m - 3)$

$= 4 \times (10 - 3)$

$x = 28$

18 $g = f(m - f)$

$= 4 \times (10 - 4)$

$g = 24$

19 $n = 7(20 - m)$

$= 7 \times (20 - 10)$

$n = 70$

20 $q = m(m - 2f)$

$= 10 \times (10 - 2 \times 4)$

$= 10 \times 2$

$q = 20$

21 $t = 2m \div (f + 1)$

$= 2 \times 10 \div (4 + 1)$

$= 20 \div 5$

$t = 4$

Substitution - Table of Values

Substitution is replacing a 'letter' with a number. A number before a letter means multiply the number and the substituted number together. So if the equation is $n = 2a$, *and* $a = 3$. Then, $2 \times 3 = 6$ not $2a = 23$. Similarly, when you have x^2 and $x = 5$ this means $5^2 = 5 \times 5 = 25$ not $5 \times 2 = 10$.

In column 1 an equation or 'rule' is given and you are asked to substitute the number given in the top box for the letter on the right hand side of the equation. Calculate the answer and put the value for the letter in the box below the number used. An example is at the top of the column.

Column 2 requires you to state the rule given a completed table, use this method.

- In a rule: $y = \blacktriangle x + \blacksquare$ the value $\blacksquare$ is found when $x = 0$, (the first value in the bottom row). In the example below $\blacksquare$ is 3, so that means $y = \blacktriangle x + 3$

x	0	1	2	3	4
y	3	7	11	15	19

x	0	1	2	3	4
y	3	7	11	15	19

- Look at the next value, for $x = 1$ we get $y = 7$. So $7 = \blacktriangle \times 1 + 3$. Which gives $y = 4x + 3$.

Question 13 and 14 are different, the numbers are decreasing in size, a rule similar to that used in question 4 applies in this situation.

In the 3rd column complete the table, then to plot the points go across the number in the top box and up the number in the bottom box. For the circled numbers above, for example, this would mean that you go across '0' (which means don't move) then go up 3. Then plot a point. Then plot the 2nd point, by moving across 1 and up 7, and so on. The points will be in a straight line, if they aren't you have made a mistake. Then draw a line through the points. Extend the line the full size of the graph and place arrow heads on each end.

Substitution - Table of Values

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Use the rule and the value in the top row, to find the bottom row value.

Example Rule : $m = 2a + 1$

a	0	1	2	3	4
m	1	3	5	7	9

$= 2 \times 0 + 1 = 1$
$= 2 \times 1 + 1 = 3$
$= 2 \times 2 + 1 = 5$
$= 2 \times 3 + 1 = 7$
$= 2 \times 4 + 1 = 9$

1 Rule : $c = 2k$

k	0	1	2	3	4
c					

2 Rule : $m = 3t + 1$

t	0	5	10	15	20
m					

3 Rule : $d = 3e - 2$

e	2	4	6	8	10
d					

4 Rule : $w = 10 - r$

r	2	4	6	8	10
w					

5 Rule : $d = 3g - 3$

g	50	150	200	250	300
e					

6 Rule : $y = 3x - 2$

x	10	20	30	40	50
y					

7 Rule : $h = c^2$

c	0	1	2	3	4
h					

8 Rule : $b = 2x^2 + 3$

x	0	1	2	3	4
b					

This time find the rule

9 _____

c	0	1	2	3	4
h	0	2	4	6	8

10 _____

x	0	1	2	3	4
b	2	3	4	5	6

11 _____

t	0	1	2	3	4
p	0	5	10	15	20

12 _____

c	0	5	10	15	20
h	5	20	35	50	65

13 _____

a	0	2	4	6	8
k	20	18	16	14	12

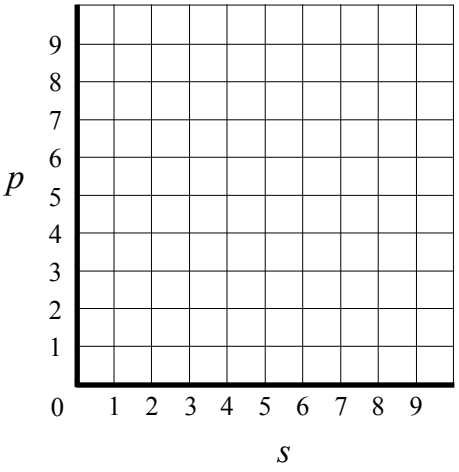
14 _____

i	0	1	2	3	4
r	10	8	6	4	2

Complete the table below then plot the points on the graph, the numbers in the top row go across and the bottom row go up. Then draw a line through them.

15 Rule : $p = 2s + 1$

s	0	1	2	3	4
p					

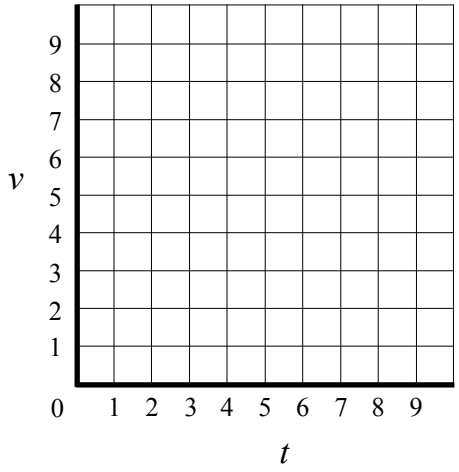


16 Graphs can be used to find values that aren't in the table. Look at the graph in Q15. and give the value of p when:

$s = 2\frac{1}{2}$ $p =$ ☐

17 Rule : $v = 8 - t$

t	0	1	2	3	4
v					



Substitution - Table of Values

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Use the rule and the value in the top row, to find the bottom row value.

Example Rule : $m = 2a + 1$

a	0	1	2	3	4
m	1	3	5	7	9

$= 2 \times 0 + 1 = 1$ $= 2 \times 1 + 1 = 3$ $= 2 \times 2 + 1 = 5$ $= 2 \times 3 + 1 = 7$ $= 2 \times 4 + 1 = 9$

1 Rule : $c = 2k$

k	0	1	2	3	4
c	0	2	4	6	8

2 Rule : $m = 3t + 1$

t	0	5	10	15	20
m	1	16	31	46	61

3 Rule : $d = 3e - 2$

e	2	4	6	8	10
d	4	10	16	22	28

4 Rule : $w = 10 - r$

r	2	4	6	8	10
w	8	6	4	2	0

5 Rule : $d = 3g - 3$

g	50	150	200	250	300
e	147	447	597	747	897

6 Rule : $y = 3x - 2$

x	10	20	30	40	50
y	28	58	88	118	148

7 Rule : $h = c^2$

c	0	1	2	3	4
h	0	1	4	9	16

8 Rule : $b = 2x^2 + 3$

x	0	1	2	3	4
b	3	5	11	21	35

This time find the rule

9 Rule: $h = 2c$

c	0	1	2	3	4
h	0	2	4	6	8

10 Rule: $b = x + 2$

x	0	1	2	3	4
b	2	3	4	5	6

11 Rule: $p = 5t$

t	0	1	2	3	4
p	0	5	10	15	20

12 Rule: $h = 3c + 5$

c	0	5	10	15	20
h	5	20	35	50	65

13 Rule: $k = 20 - a$

a	0	2	4	6	8
k	20	18	16	14	12

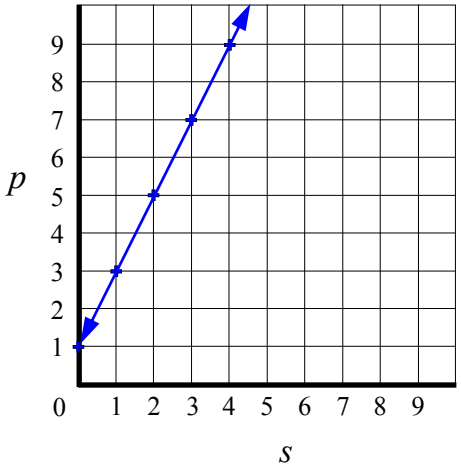
14 Rule: $r = 10 - 2i$

i	0	1	2	3	4
r	10	8	6	4	2

Complete the table below then plot the points on the graph, the numbers in the top row go across and the bottom row go up. Then draw a line through them.

15 Rule : $p = 2s + 1$

s	0	1	2	3	4
p	1	3	5	7	9

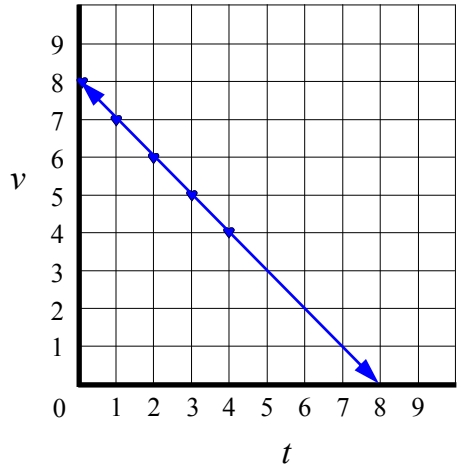


16 Graphs can be used to find values that aren't in the table. Look at the graph in Q15. and give the value of p when:

$s = 2½$ $p =$ 6

17 Rule : $v = 8 - t$

t	0	1	2	3	4
v	8	7	6	5	4



Adding and Subtracting Pronumerals

Think of algebra as being items. Standing at a street corner you may see trucks, cars and buses. If you see 3 trucks ($3t$) and 2 buses ($2b$) you can't add these together, so: $3t + 2b$ remains as $3t + 2b$. If you see 3 cars then 2 buses then 4 more cars, you have seen a total of 7 cars and 2 buses, or $3c + 2b + 4c = 7c + 2b$.

So 'like' terms (the same letter) can be added and subtracted but not different letters.

The same with numbers and terms. Numbers can be collected together and the letters can be collected together but letters can't be added/subtracted with numbers.
For example,
$3 + 4c + 4 + 5c - 2$
$= 9c + 5$

A common mistake is to write a '1' in front of a single letter, this isn't giving your answer in a fully simplified way.
For example,
$5b - 7 + 2b - 6b$
$= b - 7$ (don't write this as $1b - 7$).

Just like numbers, when the same term is subtracted from itself it will give zero.
Example,
$5f + 2 - 3f - 2f$
$= 2$ (as $5f - 5f = 0$)

Try to write the 'letters' part first when the answer is written, ie: write $5t + 6$ rather than $6 + 5t$.

Adding and Subtracting Pronumerals

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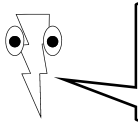
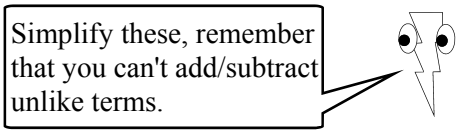


Simplify these, remember that you don't write a 1 in front of a letter, $1m = m$

1. $t + t$
2. $k + 2k$
3. $6m - 4m$
4. $12i + 3i$
5. $2j + 5j$
6. $16e - 9e$
7. $9x + 5x + 3x$
8. $12b + 9b + 4b$
9. $6n + 2n + n$
10. $t + 6t - 2t$
11. $3e + 5e - 4e$
12. $12r - 7r + 2r$
13. $11k - 7k - 4k$
14. $24h - 9h - 14h$
15. $2c + 5c + 7c$
16. $u + 5u - 2u$
17. $6d - 3d - 2d$
18. $14q - 11q - 3q$
19. $8v - 2v - 3v$
20. $12s + 3s - 14s$



Simplify these, remember that you can't add/subtract unlike terms.

21. $a + g + a$
22. $2e + c + e$
23. $4h + 3h + 2d$
24. $10m - 3n - m$
25. $2c - 7q + 3c$
26. $9k + 3k + n$
27. $3e + 2f - f$
28. $m + p - m$
29. $4d - 2m + d$
30. $6m + 3b - 5m$
31. $2d + 3d - h$
32. $t + 5e - t$
33. $5e - 2e + q$
34. $3a - 2d - 2a$
35. $4c + 2b - b$
36. $6w + n - 6w$
37. $6n - 3n + 2d$
38. $2f + 3f + f - 2d$
39. $a + b + a + b$
40. $d + 3d + 3e$



Simplify these, remember that you can't add/subtract numbers and terms.

41. $2a + 3 + 2a$
42. $5e + 4 - 4e$
43. $10d - 3d - 6$
44. $3 + 4q + 2 - q$
45. $4y + 12 - 3y$
46. $4k + 2 + 5 - k$
47. $a + 2 + 2a - 1$
48. $2d - 5 + 4d$
49. $5 + 3e - 2 + e$
50. $t + 4 - t - 2$
51. $3v - 6 - 2v$
52. $10 + 2b - 5$
53. $a + 8 - a + 8$
54. $6x - 10 + 4x$
55. $9m + 6 - 8m$
56. $5 + b - 4 + b$
57. $2f - 10 - f$
58. $10 + k + 15$
59. $h + 3 + h + 1$
60. $y + 2y - 3 + y$

Adding and Subtracting Pronumerals

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Simplify these, remember that you don't write a 1 in front of a letter, $1m = m$

1	$t + t$	$= 2t$
2	$k + 2k$	$= 3k$
3	$6m - 4m$	$= 2m$
4	$12i + 3i$	$= 15i$
5	$2j + 5j$	$= 7j$
6	$16e - 9e$	$= 7e$
7	$9x + 5x + 3x$	$= 17x$
8	$12b + 9b + 4b$	$= 25b$
9	$6n + 2n + n$	$= 9n$
10	$t + 6t - 2t$	$= 5t$
11	$3e + 5e - 4e$	$= 4e$
12	$12r - 7r + 2r$	$= 7r$
13	$11k - 7k - 4k$	$= 0$
14	$24h - 9h - 14h$	$= h$
15	$2c + 5c + 7c$	$= 14c$
16	$u + 5u - 2u$	$= 4u$
17	$6d - 3d - 2d$	$= d$
18	$14q - 11q - 3q$	$= 0$
19	$8v - 2v - 3v$	$= 3v$
20	$12s + 3s - 14s$	$= s$

Simplify these, remember that you can't add/subtract unlike terms.

21	$a + g + a$	$= 2a + g$
22	$2e + c + e$	$= 3e + c$
23	$4h + 3h + 2d$	$= 7h + 2d$
24	$10m - 3n - m$	$= 9m - 3n$
25	$2c - 7q + 3c$	$= 5c - 7q$
26	$9k + 3k + n$	$= 12k + n$
27	$3e + 2f - f$	$= 3e + f$
28	$m + p - m$	$= p$
29	$4d - 2m + d$	$= 5d - 2m$
30	$6m + 3b - 5m$	$= m + 3b$
31	$2d + 3d - h$	$= 5d - h$
32	$t + 5e - t$	$= 5e$
33	$5e - 2e + q$	$= 3e + q$
34	$3a - 2d - 2a$	$= a - 2d$
35	$4c + 2b - b$	$= 4c + b$
36	$6w + n - 6w$	$= n$
37	$6n - 3n + 2d$	$= 3n + 2d$
38	$2f + 3f + f - 2d$	$= 6f - 2d$
39	$a + b + a + b$	$= 2a + 2b$
40	$d + 3d + 3e$	$= 4d + 3e$

Simplify these, remember that you can't add/subtract numbers and terms.

41	$2a + 3 + 2a$	$= 4a + 3$
42	$5e + 4 - 4e$	$= e + 4$
43	$10d - 3d - 6$	$= 7d - 6$
44	$3 + 4q + 2 - q$	$= 3q + 5$
45	$4y + 12 - 3y$	$= y + 12$
46	$4k + 2 + 5 - k$	$= 3k + 7$
47	$a + 2 + 2a - 1$	$= 3a + 1$
48	$2d - 5 + 4d$	$= 6d - 5$
49	$5 + 3e - 2 + e$	$= 4e + 3$
50	$t + 4 - t - 2$	$= 2$
51	$3v - 6 - 2v$	$= v - 6$
52	$10 + 2b - 5$	$= 2b + 5$
53	$a + 8 - a + 8$	$= 16$
54	$6x - 10 + 4x$	$= 10x - 10$
55	$9m + 6 - 8m$	$= m + 6$
56	$5 + b - 4 + b$	$= 2b + 1$
57	$2f - 10 - f$	$= f - 10$
58	$10 + k + 15$	$= k + 25$
59	$h + 3 + h + 1$	$= 2h + 4$
60	$y + 2y - 3 + y$	$= 4y - 3$

Further Adding and Subtracting Pronumerals

To add and subtract letters they must be the same. For example:
$5a + 2a = 7a$,
$7bc - 6bc = bc$ and
$3ab + 2ba = 5ba$ (or $5ab$).

However unlike terms can't be added together:
$4c + 3a = 4c + 3a$,
$7qm - 3q = 7qm - 3q$ and
$3ab - a - 3b = 3ab - a - 3b$.

The order of the letters doesn't matter: $4bc = 4cb$, $5xqm = 5qxm = 5mqx$ so terms can be added when the letters are the same but in a different order.

The same applies to when terms have a power, such as $3b^2$, $5d^2f$ and $2x^2y^2$. Terms with different powers can't be added or subtracted. But you can add/subtract these:
$a^2 + 3a^2 = 4a^2$ and
$5x^2y - 2yx^2 = 3x^2y$,

but not thse :
$a^2 + 2a = a^2 + 2a$,
$3e^2 - 3ed = 3e^2 - 3ed$.

Remember that $3k^2mw^2 = 3mk^2w^2 = 3w^2mk^2$, the order doesn't matter, so long as the powers are on the same letters.

The final column requires the missing sides to be found and then the sides added together to get the perimeter. Follow the example at the top of the column.

Further Adding and Subtracting Pronumerals

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Collect like terms and simplify.

Example

$bx + b + 5 + 2xb + 3b + 7$

$= 3xb + 4b + 12$

1 $3a + 5t + 3 - a + 6 + 4t$

2 $12d + 2a + 16 - 5d - 7 + 3a$

3 $4ux + 3xa - 3xu + 2ax$

4 $5 + 5m + 21 - 4m + 3ma$

5 $12kd - 5dk - 5dk - 8 - kd$

6 $5 + a + b + 6 + 2ab + 3b$

7 $5tu + 17 + 12ut - 12t - 12$

8 $20a - 20 - a + 5ta + 11at$

9 $30ue + 15e - 5e - u$

10 $at + ta + at + ta + a - 4t$

11 $17 + 20as - 3 - 15sa - 3as$

Collect like terms, this time using powers $(^2)$

Example

$50 + a^2 + 3a + 17 - 2a + 3a^2$

$= 4a^2 - a + 67$

12 $17 + 2m^2 - m + m^2 + 14$

13 $7h^2 + 3h - 3h^2 + 4 - 4h^2$

14 $20k^2 + 14 - 12k - 8k^2 - 7$

15 $9r^2 + 5tr + 7 - 3r^2 + 16 - 2rt$

16 $13e + 5e^2 - 8e - 2e^2 - 5 - 4e$

These are harder, note which term has the ^{2}

17 $20cw + 37 - c^2 - 18wc - 19$

18 $9fg^2 + 3f^2g + 3g^2f$

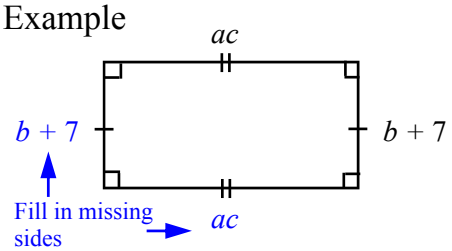
19 $10yt^2 + 3y^2t + 5yt^2$

20 $5a^2 + 3a^2t - 3a^2 - 2ta^2$

21 $y^2 + 2xy^2 + 3y^2 - y^2x^2$

Use your algebra skills to give an expression for the perimeter of the shapes below. Collect like terms.

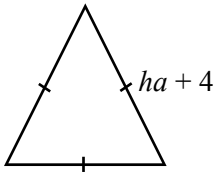
Example



$P = ac + ac + b + 7 + b + 7$

$P = 2ac + 2b + 14$

22



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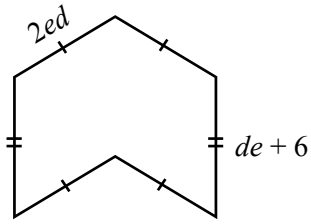

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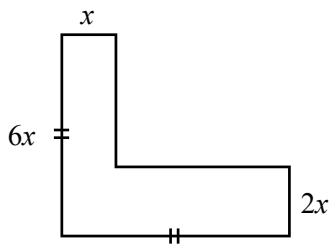

Further Adding and Subtracting Pronumerals

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Collect like terms and simplify.

Example

$bx + b + 5 + 2xb + 3b + 7$

$= 3xb + 4b + 12$

1 $3a + 5t + 3 - a + 6 + 4t$

$= 2a + 9t + 9$

2 $12d + 2a + 16 - 5d - 7 + 3a$

$= 7d + 5a + 9$

3 $4ux + 3xa - 3xu + 2ax$

$= ux + 5ax$

4 $5 + 5m + 21 - 4m + 3ma$

$= 3ma + m + 26$

5 $12kd - 5dk - 5dk - 8 - kd$

$= kd - 8$

6 $5 + a + b + 6 + 2ab + 3b$

$= 2ab + 4b + a + 11$

7 $5tu + 17 + 12ut - 12t - 12$

$= 17ut - 12t + 5$

8 $20a - 20 - a + 5ta + 11at$

$= 16at + 19a - 20$

9 $30ue + 15e - 5e - u$

$= 30ue - 10e - u$

10 $at + ta + at + ta + a - 4t$

$= 4at + a - 4t$

11 $17 + 20as - 3 - 15sa - 3as$

$= 2as + 14$

Collect like terms, this time using powers $(^2)$

Example

$50 + a^2 + 3a + 17 - 2a + 3a^2$

$= 4a^2 - a + 67$

12 $17 + 2m^2 - m + m^2 + 14$

$= 3m^2 - m + 31$

13 $7h^2 + 3h - 3h^2 + 4 - 4h^2$

$= 3h + 4$

14 $20k^2 + 14 - 12k - 8k^2 - 7$

$= 12k^2 - 12k + 7$

15 $9r^2 + 5tr + 7 - 3r^2 + 16 - 2rt$

$= 6r^2 + 3rt + 23$

16 $13e + 5e^2 - 8e - 2e^2 - 5 - 4e$

$= 3e^2 - e - 5$

17 $20cw + 37 - c^2 - 18wc - 19$

$= 2cw - c^2 + 18$

These are harder, note which term has the ^{2}

18 $9fg^2 + 3f^2g + 3g^2f$

$= 12fg^2 + 3f^2g$

19 $10yt^2 + 3y^2t + 5yt^2$

$= 15yt^2 + 3y^2t$

20 $5a^2 + 3a^2t - 3a^2 - 2ta^2$

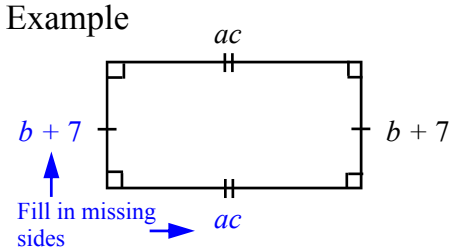
$= 2a^2 - ta^2$

21 $y^2 + 2xy^2 + 3y^2 - y^2x^2$

$= 4y^2 + 2xy^2 - y^2x^2$

Use your algebra skills to give an expression for the perimeter of the shapes below. Collect like terms.

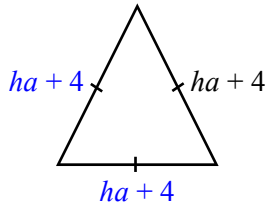
Example



$P = ac + ac + b + 7 + b + 7$

$P = 2ac + 2b + 14$

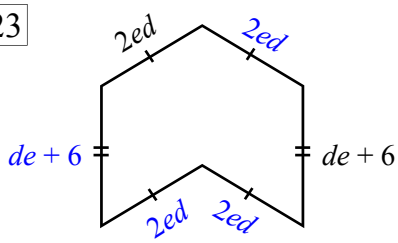
22



$P = ha + 4 + ha + 4 + ha + 4$

$P = 3ha + 12$

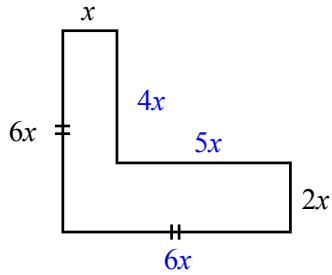
23



$P = 2ed + 2ed + 2ed + 2ed + de + 6 + de + 6$

$P = 10de + 12$

24



$P = 6x + 6x + x + 4x + 5x + 2x$

$P = 24x$

Multiplying, Dividing and Using Brackets with Pronumerals

We show multiplication between numbers and pronumerals as the number first then the letter. For example, $2 \times a = 2a$, $5 \times d = 5d$ and $4 \times a \times d = 4ad$ (or $4da$).

When several numbers and letters are mixed up, calculate the product of the numbers first, then the letters. For example, $5 \times e \times 2 \times g = 10eg$. Note that when the same letter is featured twice a squared term will result. E.g. $3 \times q \times 5 \times q = 15q^2$. A letter times itself is that letter squared. ($a \times a = a^2$, $c \times c = c^2$ etc)

When a pronumeral is divided by a number, write the letter 'over' the number.

So, $x \div 7 = \frac{x}{7}$ $\quad r \div 6 = \frac{r}{6}$ $\quad 3 \times m \div 4 = \frac{3m}{4}$

Sometimes these can be simplified. Like fractions, if both the top and bottom numbers can be divided by another number this will simplify the answer.

$12e \div 8 = \frac{\overset{3}{\cancel{12}e}}{\underset{2}{\cancel{8}}} = \frac{3e}{2}$ (divide by 4)

$12e \div 4 = \frac{\overset{3}{\cancel{12}e}}{\underset{1}{\cancel{4}}} = 3e$ (divide by 4) (you don't write a number over 1)

Remember that the largest number (HCF) that divides into both the top and the bottom numbers (not just any number) is to be found to fully simplify the answer.

Expanding, (removing brackets) is multiplication in steps. The number outside the brackets is multiplied by the first term inside the brackets and then the second term. These are added together if the sign between them is a +, subtracted if there is a negative sign - .

For example,

$5(y + 4) = 5 \times y + 5 \times 4$
$= 5y + 20$

$e(m - e) = e \times m - e \times e$
$= em - e^2$

Multiplying, Dividing and Using Brackets with Pronumerals

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Multiply these remember numbers first then letters. $d \times d = d^2$ not $2d$

1 $2 \times a$

2 $c \times 3$

3 $4 \times e \times 2$

4 $2 \times f \times 5$

5 $3 \times 4 \times u$

6 $8 \times h \times 2$

7 $3 \times x \times y$

8 $7 \times t \times k$

9 $g \times 2 \times g$

10 $a \times b \times c$

11 $q \times u \times q \times 7$

12 $n \times 5 \times 3 \times n$

13 $x \times 4 \times t$

14 $10 \times z \times q$

15 $h \times 3 \times b \times 4$

16 $6 \times n \times n \times b$

17 $e \times e \times t \times t$

18 $d \times a \times d \times 3$

19 $9 \times 6 \times t \times t$

20 $v \times v \times 3 \times y$

Divide these, remember that ÷ is the same as $\frac{\quad}{\quad}$ Simplify numbers when you can

Example

$9 \times m \div 3 = \frac{3}{1}\frac{\cancel{9}m}{\cancel{3}} = 3m$

21 $c \div 7$

22 $2 \times g \div 3$

23 $\frac{15 \times k}{3}$

24 $22 \times e \div 4$

25 $7 \times b \div 7$

26 $4 \times f \div 8$

27 $18 \times a \div 6$

28 $\frac{45 \times y}{9}$

29 $50d \div 10$

30 $9e \div 6$

31 $\frac{35\,y}{10}$

32 $48a \div 3$

Multiply the outside letter or number by the inside parts separately. This is called 'expanding'.

Example 1

$5\,(w + 10)$ $= 5w + 50$

Example 2

$3a\,(a - 5)$ $= 3a^2 - 15a$

33 $4\,(c + 6)$

34 $6\,(m + 3)$

35 $5\,(c - 8)$

36 $7\,(3d + 2)$

37 $h\,(h + 2)$

38 $e\,(k + e)$

39 $5n\,(n - 11)$

40 $3x\,(x - 3)$

41 $k\,(m + d)$

42 $3w\,(a + 2)$

43 $2q\,(3q - 2)$

44 $8e\,(7e + 3)$

45 $2m\,(e + 2m)$

46 $9w\,(w - 4)$

47 $a^2\,(b + c)$

48 $x^2\,(d^2 + t^2)$

49 $3u\,(2 - t^2)$

Multiplying, Dividing and Using Brackets with Pronumerals

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Multiply these remember numbers first then letters. $d \times d = d^2$ not $2d$

1 $2 \times a$ $= 2a$

2 $c \times 3$ $= 3c$

3 $4 \times e \times 2$ $= 8e$

4 $2 \times f \times 5$ $= 10f$

5 $3 \times 4 \times u$ $= 12u$

6 $8 \times h \times 2$ $= 16h$

7 $3 \times x \times y$ $= 3xy$

8 $7 \times t \times k$ $= 7tk$

9 $g \times 2 \times g$ $= 2g^2$

10 $a \times b \times c$ $= abc$

11 $q \times u \times q \times 7$ $= 7uq^2$

12 $n \times 5 \times 3 \times n$ $= 15n^2$

13 $x \times 4 \times t$ $= 4xt$

14 $10 \times z \times q$ $= 10zq$

15 $h \times 3 \times b \times 4$ $= 12hb$

16 $6 \times n \times n \times b$ $= 6n^2b$

17 $e \times e \times t \times t$ $= e^2t^2$

18 $d \times a \times d \times 3$ $= 3d^2a$

19 $9 \times 6 \times t \times t$ $= 54t^2$

20 $v \times v \times 3 \times y$ $= 3v^2y$

Divide these, remember that ÷ is the same as $\frac{\ }{\ }$ Simplify numbers when you can

Example

$9 \times m \div 3 = \frac{{}^{3}\cancel{9}m}{{}_{1}\cancel{3}} = 3m$

21 $c \div 7 = \frac{c}{7}$

22 $2 \times g \div 3 = \frac{2g}{3}$

23 $\frac{{}^{5}\cancel{15} \times k}{{}_{1}\cancel{3}} = 5k$

24 $22 \times e \div 4 = \frac{{}^{11}\cancel{22}e}{{}_{2}\cancel{4}} = \frac{11e}{2}$

25 $7 \times b \div 7 = \frac{{}^{1}\cancel{7}b}{{}_{1}\cancel{7}} = b$

26 $4 \times f \div 8 = \frac{{}^{1}\cancel{4}f}{{}_{2}\cancel{8}} = \frac{f}{2}$

27 $18 \times a \div 6 = \frac{{}^{3}\cancel{18}a}{{}_{1}\cancel{6}} = 3a$

28 $\frac{{}^{5}\cancel{45} \times y}{{}_{1}\cancel{9}} = 5y$

29 $50d \div 10 = \frac{{}^{5}\cancel{50}d}{{}_{1}\cancel{10}} = 5d$

30 $9e \div 6 = \frac{{}^{3}\cancel{9}e}{{}_{2}\cancel{6}} = \frac{3e}{2}$

31 $\frac{{}^{7}\cancel{35}\,y}{{}_{2}\cancel{10}} = \frac{7y}{2}$

32 $48a \div 3 = \frac{{}^{16}\cancel{48}a}{{}_{1}\cancel{3}} = 16a$

Multiply the outside letter or number by the inside parts separately. This is called 'expanding'.

Example 1

$5(w + 10) = 5w + 50$

Example 2

$3a(a - 5) = 3a^2 - 15a$

33 $4(c + 6) = 4c + 24$

34 $6(m + 3) = 6m + 18$

35 $5(c - 8) = 5c - 40$

36 $7(3d + 2) = 21d + 14$

37 $h(h + 2) = h^2 + 2h$

38 $e(k + e) = ek + e^2$

39 $5n(n - 11) = 5n^2 - 55n$

40 $3x(x - 3) = 3x^2 - 9x$

41 $k(m + d) = km + kd$

42 $3w(a + 2) = 3wa + 6w$

43 $2q(3q - 2) = 6q^2 - 4q$

44 $8e(7e + 3) = 56e^2 + 24e$

45 $2m(e + 2m) = 2me + 4m^2$

46 $9w(w - 4) = 9w^2 - 36w$

47 $a^2(b + c) = a^2b + a^2c$

48 $x^2(d^2 + t^2) = x^2d^2 + x^2t^2$

49 $3u(2 - t^2) = 6u - 3ut^2$

Further Multiplying, Dividing and Using Brackets with Pronumerals

Column 1 is multiplication between numbers and pronumerals. Always write the number first then the letter (or letters). For example, $2 \times a = 2a$, $5 \times 3d = 15d$ and $4 \times a \times d \times 3 = 12ad$ (or $12da$).

Note that when the same letter is featured twice a squared term will result. A letter times itself is that letter squared. ($a \times a = a^2$, $c \times c = c^2$ etc) For example, $3q \times 5q = 15q^2$, $2c \times 3c = 6c^2$ and $abc \times bc = ab^2c^2$.

The 2nd column are exercises on division. Like fractions, if both the top and bottom numbers can be divided by another number this will simplify the number part of the answer. Remember that the largest number that divides into both the top and the bottom numbers (not just any number) is required to fully simplify.

$12e \div 8 = \overset{3}{\underset{2}{\frac{12e}{8}}} = \frac{3e}{2}$

(divide top and bottom by 4)

$12e \div 4 = \overset{3}{\underset{1}{\frac{12e}{4}}} = 3e$

(divide by 4)

With division of terms the following applies. A term divided by itself equals 1, so it cancels out.

$m \div m = \frac{m}{m} = 1$

$8m \div 6m = \overset{4}{\underset{3}{\frac{8m}{6m}}} = \frac{4}{3}$

$8mb \div 6m = \overset{4}{\underset{3}{\frac{8mb}{6m}}} = \frac{4b}{3}$

A squared term divided by itself equals the term. ($c^2 \div c = c$)

$d^2 \div d = \frac{dd}{d} = d$

$6d^2 \div 4d = \overset{3}{\underset{2}{\frac{6dd}{4d}}} = \frac{3d}{2}$

$8d^2b \div 6d = \overset{4}{\underset{3}{\frac{8ddb}{6d}}} = \frac{4db}{3}$

The 3rd column uses terms instead of numbers in area formulae. This is the same as multiplication (in the first column). The only difference is that triangles have a ½ in the formula. To allow for this, divide one of the numbers on one of the sides by 2 (hopefully its even). For example:

$\frac{1}{2} \times 6m \times t$ ($= 3m \times t$)

$= 3mt$ units^2

$\frac{1}{2} \times 6m \times 2t$ ($= 3m \times 2t$)

$= 6mt$ units^2

Because no units of measurement are mentioned in the question write units^2 in the answer.

Further Multiplying, Dividing and Using Brackets with Pronumerals

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Multiply these, remember numbers first then letters. These are challenging!

1 $2h \times 3u$

2 $4r \times 3f$

3 $10k \times 3k$

4 $4c \times 2c$

5 $7md \times 2d$

6 $3e \times 2d \times 5$

7 $8y \times 2x \times 3y$

8 $4de \times 3ed \times 2$

9 $5b \times 3c \times bc$

10 $ac \times a \times c \times b$

11 $2t \times utx \times u$

12 $d \times 2d \times u$

13 $cn \times nc \times ab$

14 $4v \times 2w \times 5u$

15 $abc \times abc$

16 $2abc \times 2cba$

17 $9k \times 2ek \times u$

18 $2y \times 3d \times dy$

19 $5fn \times 3 \times fe$

20 $abc \times bcd$

Divide these, remember that ÷ is the same as ___ Simplify the numbers if you can, then the terms.

Example

$8e^2 \div 2e = \frac{\overset{4}{\cancel{8}}\cancel{e}e}{\underset{1}{\cancel{2}}\cancel{e}} = 4e$

21 $3u \div 2$

22 $\frac{15x}{3}$

23 $20k \div 6$

24 $8bc \div 3b$

25 $9ch \div 3h$

26 $14ed \div 4d$

27 $4g^2 \div 2g$

28 $\frac{15b^2}{5b^2}$

29 $6ck^2 \div 2k$

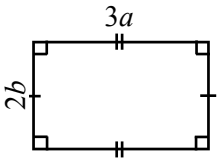
30 $16d^2 \div 6d$

31 $\frac{9ab^2}{6ab}$

32 $a^2b^2 \div ab^2$

Find the areas, here are the formulae. $A = l^2$ (square) $A = lb$ (rectangle) and $A = \frac{1}{2}bh$ (triangle).

Example 1

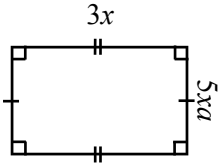


$A = lb$

$= 3a \times 2b$

$A = 6ab$ units²

Example 2

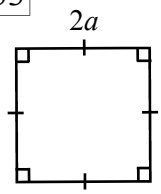


$A = lb$

$= 3x \times 5xa$

$A = 15x^2a$ units²

33



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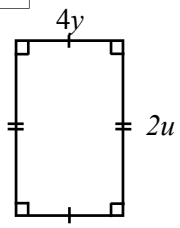

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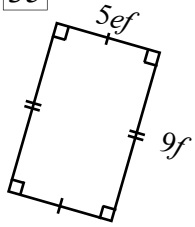
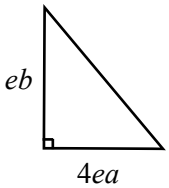
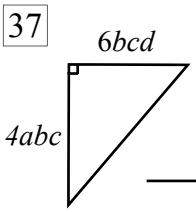


36



$A = \frac{1}{2}bh$

37



Further Multiplying, Dividing and Using Brackets with Pronumerals

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Multiply these remember numbers first then letters. These are challenging!

1 $2h \times 3u$ = $6hu$

2 $4r \times 3f$ = $12rf$

3 $10k \times 3k$ = $30k^2$

4 $4c \times 2c$ = $8c^2$

5 $7md \times 2d$ = $14md^2$

6 $3e \times 2d \times 5$ = $30ed$

7 $8y \times 2x \times 3y$ = $48xy^2$

8 $4de \times 3ed \times 2$ = $24e^2d^2$

9 $5b \times 3c \times bc$ = $15cb^2$

10 $ac \times a \times c \times b$ = a^2c^2b

11 $2t \times utx \times u$ = $2u^2t^2x$

12 $u \times 2d \times ud$ = $2u^2d^2$

13 $cn \times nc \times ab$ = abn^2c^2

14 $4v \times 2w \times 5u$ = $40vwu$

15 $abc \times abc$ = $a^2b^2c^2$

16 $2abc \times 2cba$ = $4a^2b^2c^2$

17 $9k \times 2ek \times u$ = $18k^2eu$

18 $2y \times 3d \times dy$ = $6y^2d^2$

19 $5fn \times 3 \times fe$ = $15f^2ne$

20 $abc \times bcd$ = ab^2c^2d

Divide these, remember that ÷ is the same as $\frac{\ }{\ }$ Simplify the numbers if you can, then the terms.

Example $8e^2 \div 2e = \frac{\overset{4}{\cancel{8}\cancel{e}^2}}{\underset{1}{\cancel{2}\cancel{e}}} = 4e$

21 $3u \div 2 = \frac{3u}{2}$

22 $\frac{\overset{5}{\cancel{15}x}}{\underset{1}{\cancel{3}}} = 5x$

23 $20k \div 6 = \frac{\overset{10}{\cancel{20}k}}{\underset{3}{\cancel{6}}} = \frac{10k}{3}$

24 $8bc \div 3b = \frac{8\cancel{b}c}{3\cancel{b}} = \frac{8c}{3}$

25 $9ch \div 3h = \frac{\overset{3}{\cancel{9}c}\cancel{h}}{\underset{1}{\cancel{3}}\cancel{h}} = 3c$

26 $14ed \div 4d = \frac{\overset{7}{\cancel{14}e}\cancel{d}}{\underset{2}{\cancel{4}}\cancel{d}} = \frac{7e}{2}$

27 $4g^2 \div 2g = \frac{\overset{2}{\cancel{4}g}\cancel{g}}{\underset{1}{\cancel{2}}\cancel{g}} = 2g$

28 $\frac{\overset{3}{\cancel{15}b^2}}{\underset{1}{\cancel{5}b^2}} = 3$

29 $6ck^2 \div 2k = \frac{\overset{3}{\cancel{6}}ck\cancel{k}}{\underset{1}{\cancel{2}}\cancel{k}} = 3ck$

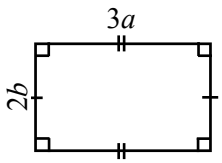
30 $16d^2 \div 6d = \frac{\overset{8}{\cancel{16}}d\cancel{d}}{\underset{3}{\cancel{6}}\cancel{d}} = \frac{8d}{3}$

31 $\frac{\overset{3}{\cancel{9}}d\cancel{b}^2}{\underset{2}{\cancel{6}}d\cancel{b}} = \frac{3b}{2}$

32 $a^2b^2 \div ab^2 = \frac{a\cancel{b}^2}{d\cancel{b}^2} = a$

Find the areas, here are the formulae. $A = l^2$ (square) $A = lb$ (rectangle) and $A = \frac{1}{2}bh$ (triangle).

Example 1

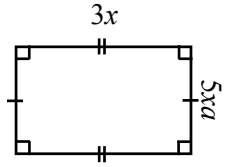


$A = lb$

$= 3a \times 2b$

$A = 6ab$ units²

Example 2

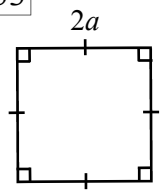


$A = lb$

$= 3x \times 5xa$

$A = 15x^2a$ units²

33

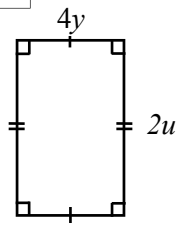


$A = l^2$

$= 2a \times 2a$

$A = 4a^2$ units²

34

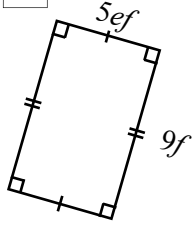


$A = lb$

$= 4y \times 2u$

$A = 8uy$ units²

35

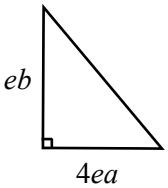


$A = lb$

$= 5ef \times 9f$

$A = 45ef^2$ units²

36

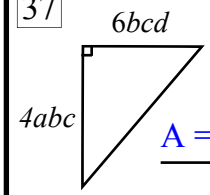


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4ea \times eb$

$A = 2e^2ab$ units²

37



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6bcd \times 4abc$

$A = 12b^2c^2ad$ units²

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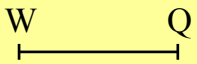
ANGLES

Naming Angles

Naming angles allows you to explain which angle or part of a shape you're dealing with.

The first column introduces intervals, lines and rays. With intervals and lines you can name them in either direction. The interval below could be WQ or QW. You are asked to give both the names in these questions.



With rays, name them in the direction of the arrow only. The ray below is called HG only, not GH.



In column 2 you are asked to name the rays and also the vertex, this is the meeting point of the two rays, the 'elbow' if you like. The vertex is named by the letter at the point alone. To show it is a vertex we put a hat on it, like an upside-down 'v'.

To name an angle we generally use 3 letters, moving from ray-to vertex-to ray. The vertex is always in the middle. To show that it is an angle you put an angle sign in front of it or write the letters placing a hat over the vertex.

$\angle ABC$ or, $\hat{A}BC$

When you name a shape (in column 3) you start at any point then go around the shape the one way. If the letters are in alphabetical order you might like to start at the earliest letter. No symbol is put before the name of a shape, though sometimes this is done with triangles. (e.g. ΔABC)

Naming Angles

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Give the 2 possible names for these intervals.

1 A B or _____

2 K Z or _____

3 D L or _____

4 W U or _____

Give the 2 possible names for these lines.

5 T Q or _____

6 S C or _____

7 H I or _____

8 K E V R i) ii)

i) _____ or _____ ii) _____ or _____

Name these rays

9 X F _____

10 Y N _____

11 P M _____

12 J H _____

Name the 2 rays and the vertex for these angles

13 Y K V ^

ray _____ ray _____ vertex _____

14 P T A _____ ray _____ ray _____ vertex

15 S A E L Z ★ + _____ ray _____ ray _____ vertex

+ _____ ray _____ ray _____ vertex

Name these angles, with the vertex as the middle letter. Use either ^ or ∠

16 A X B ∠ _____

17 L Z F T ★ _____ + _____

18 K A M O Y ♦ _____ ★ _____ + _____

Name these shapes using their letters. Then name the angle with the symbol.

19 D C A B + _____ + _____

20 T H R K V _____ + _____

21 F E A D B C _____ + _____ ★ _____

22 E J N P K T _____ + _____ ★ _____ ♦ _____

Place the given symbol in the following angles

23 D F C E G B A I H

Note E is the point of contact

▲ ∠ BCD ♥ ∠ AED

★ ∠ FED + ∠ AEI

♦ ∠ IHG × ∠ ABC

Naming Angles

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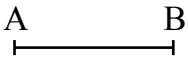
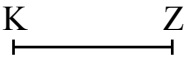
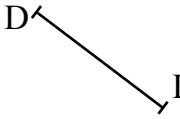
Give the 2 possible names for these intervals.

1 

AB or BA

2 

KZ or ZK

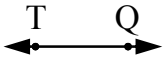
3 

DL or LD

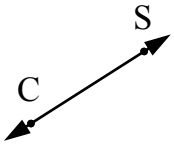
4 

UW or WU

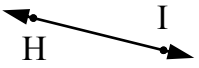
Give the 2 possible names for these lines.

5 

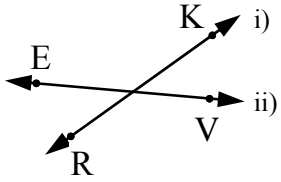
TQ or QT

6 

CS or SC

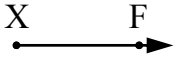
7 

HI or IH

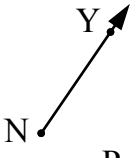
8 

i) RK or KR ii) EV or VE

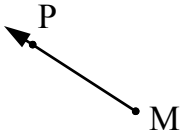
Name these rays

9 

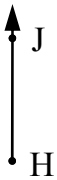
XF

10 

NY

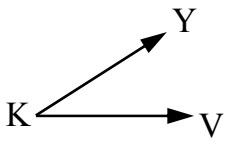
11 

MP

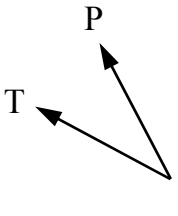
12 

HJ

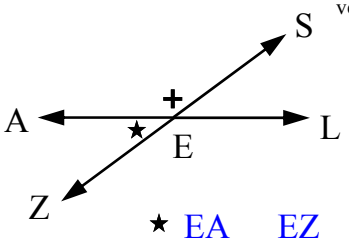
Name the 2 rays and the vertex for these angles

13 

KV ray KY ray ^K vertex

14 

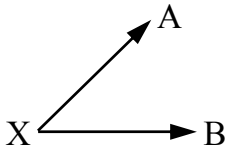
AT ray AP ray ^A vertex

15 

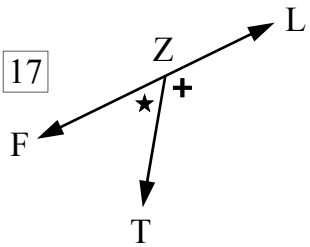
★ EA ray EZ ray ^E vertex

+ EA ray ES ray ^E vertex

Name these angles, with the vertex as the middle letter. Use either ^ or ∠

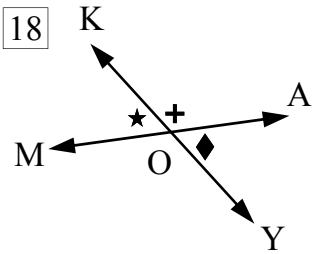
16 

∠ AXB

17 

★ ∠ FZT

+ ∠ TZL

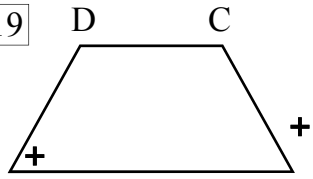
18 

♦ ∠ AOY

★ ∠ KOM

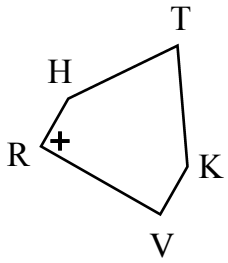
+ ∠ AOK

Name these shapes using their letters. Then name the angle with the symbol.

19 

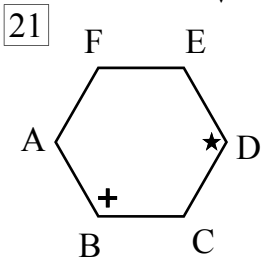
ABCD

+ ∠ DAB

20 

TKVRH

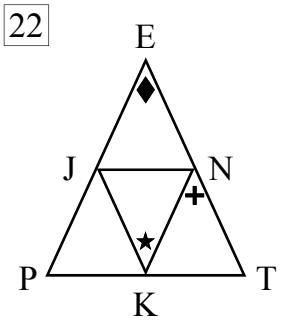
+ ∠ HRV

21 

ABCDEF

+ ∠ ABC

★ ∠ CDE

22 

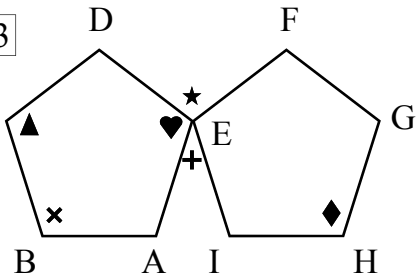
ENTKPJ

+ ∠ TNK

★ ∠ JKN

♦ ∠ JEN

Place the given symbol in the following angles

23 

Note E is the point of contact

▲ ∠ BCD ♥ ∠ AED

★ ∠ FED + ∠ AEI

♦ ∠ IHG × ∠ ABC

Classifying Angles

Classification means to group into categories. There are six classifications in all:

- acute angles are between 0° and 90°
- a right angle is exactly 90°
- obtuse angles are between 90° and 180°
- a straight angle is exactly 180°
- reflex angles are between 180° and 360°
- and a revolution is 360°

How do you remember all these? You should be able to remember the straight angle, right angle and revolution, it is the others you could mix up. Remember that as the angle increases you move through the alphabet. A (acute) is before O(Obtuse) which is before R(reflex). There is a guide at the top of the page to refer to. With the angles, the measured side is the side marked with the arc (part circle) or the right angle symbol.

When asked to name angles remember that you can use either the hat or the angle sign method, as below.

$\angle ABC$ or, $\hat{ABC}$

In column 3 the word internal is used. What does internal mean? It means the inside (interior), so the internal angles are the angles on the inside of a shape.

Classifying Angles

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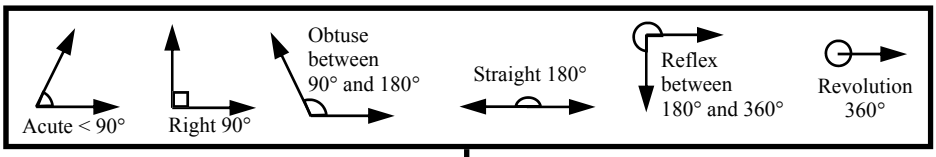


Classify the following angles

1 _____

2 _____

3 _____

4 _____

5 _____

6 _____

7 _____

8 _____

9 _____

10 _____

11 _____

12 _____

13 _____

14 _____

15 _____

16 _____

17 _____

18 _____

19 _____

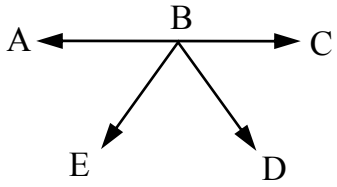
Complete the table below

20

Angle	Classification
115°	
333°	
73°	
165°	
180°	
200°	
360°	

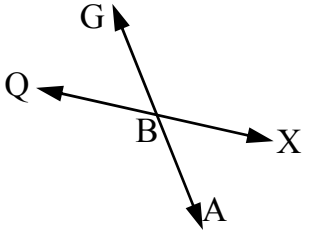
Using the ray AB in all angles name an acute, obtuse and straight angle

21



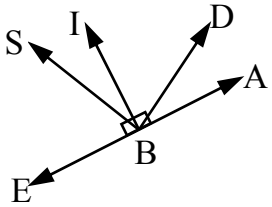
acute _____ obtuse _____ straight _____

22



acute _____ obtuse _____ straight _____

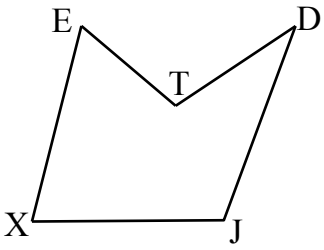
23



acute _____ obtuse _____ straight _____

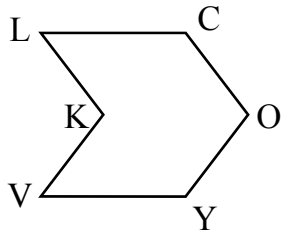
For the two shapes below name internal angles that are acute, obtuse or reflex.

24



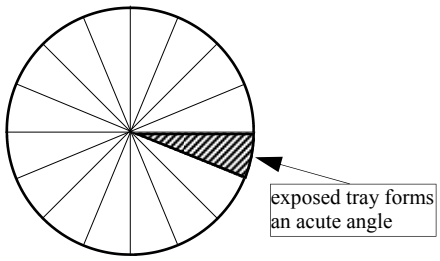
acute _____ obtuse _____ reflex _____

25



acute _____ obtuse _____ reflex _____

26 A family size pizza has the first piece eaten forming an acute angle. Each piece is the same size and when removed increases the size of the angle. Find the number of:



i) acute angles that can be made _____

ii) pieces eaten to make a right angle _____

iii) pieces eaten to make the largest obtuse angle _____

iv) reflex angles that can be made _____

Classifying Angles

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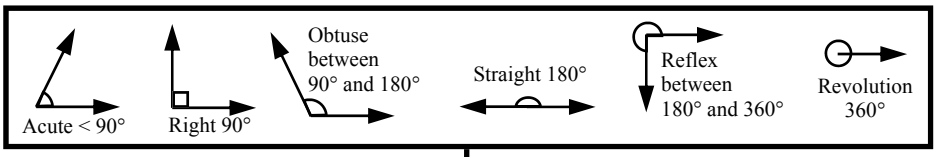


Classify the following angles

1 acute

2 obtuse

3 revolution

4 right

5 reflex

6 straight

7 acute

8 reflex

9 revolution

10 obtuse

11 right

12 obtuse

13 acute

14 acute

15 reflex

16 obtuse

17 straight

18 obtuse

19 reflex

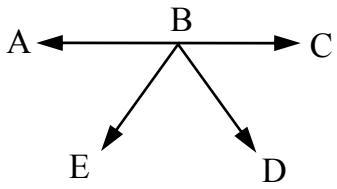
Complete the table below

20

Angle	Classification
115°	obtuse
333°	reflex
73°	acute
165°	obtuse
180°	straight
200°	reflex
360°	revolution

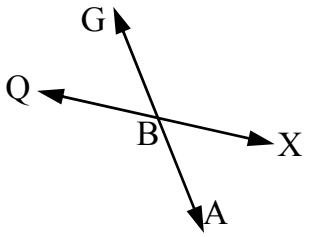
Using the ray AB in all angles name an acute, obtuse and straight angle

21



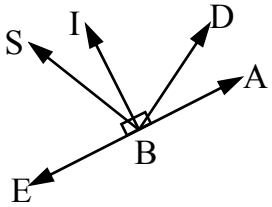
∠ ABE acute ∠ ABD obtuse ∠ ABC straight

22



∠ ABX acute ∠ ABQ obtuse ∠ ABG straight

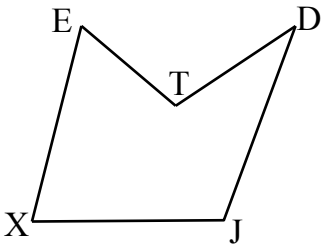
23



∠ ABD acute ∠ ABS obtuse ∠ ABE straight

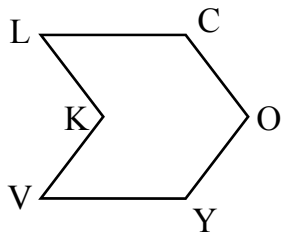
For the two shapes below name internal angles that are acute, obtuse or reflex.

24



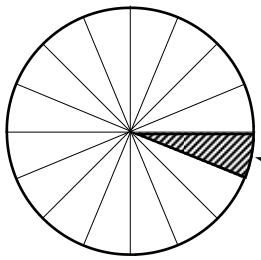
∠ XET acute ∠ DJX obtuse ∠ ETD reflex

25



∠ KLC acute ∠ LCO obtuse ∠ LKV reflex

26 A family size pizza has the first piece eaten forming an acute angle. Each piece is the same size and when removed increases the size of the angle. Find the number of:



i) acute angles that can be made 3

ii) pieces eaten to make a right angle 4

iii) pieces eaten to make the largest obtuse angle 7

iv) reflex angles that can be made 7

Estimating and Measuring Angles

An angle is used to measure rotation, a protractor is required for this worksheet.

To answer this sheet look at each angle and estimate its size, estimate meaning a skilled guess, so it is unlikely to guess the exact answer, but within say 15° would be very good. Don't use a protractor until you have estimated all of the angles, use the guide at the top right of the sheet if you need the help.

When using a standard protractor ensure you use the correct scale (inside or outside, make sure the scale used starts at 0°) and if the angle is a reflex (greater than 180°), measure the outside of the angle then subtract the angle from 360°. If an angle is measured and it is above 90°, then it must be larger than a right angle, if it isn't you have made a mistake. Use your estimation as a guide as well, and then see how close you were.

Estimating and Measuring Angles

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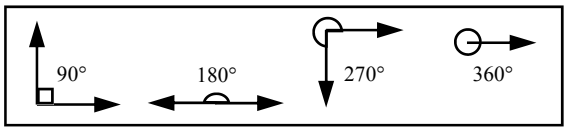
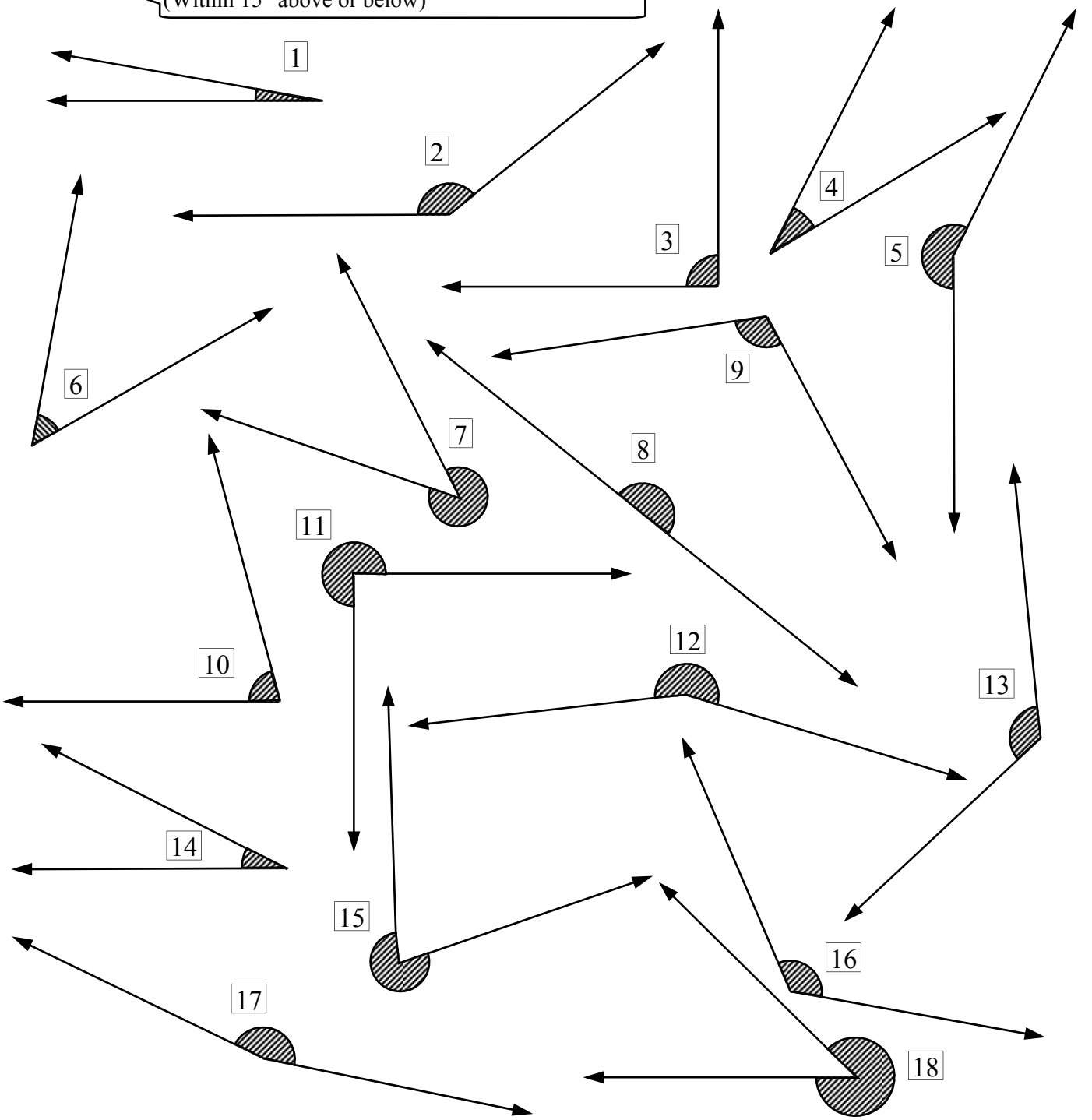
Estimate the size of all the shaded angles and place your answers in the table. Then using a protractor measure all the angles. See how well you estimated. (Within 15° above or below)





No	Estimate	Measured	No	Estimate	Measured	No	Estimate	Measured
1			7			13		
2			8			14		
3			9			15		
4			10			16		
5			11			17		
6			12			18		

Estimating and Measuring Angles

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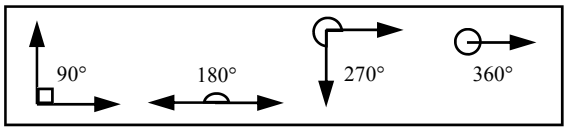
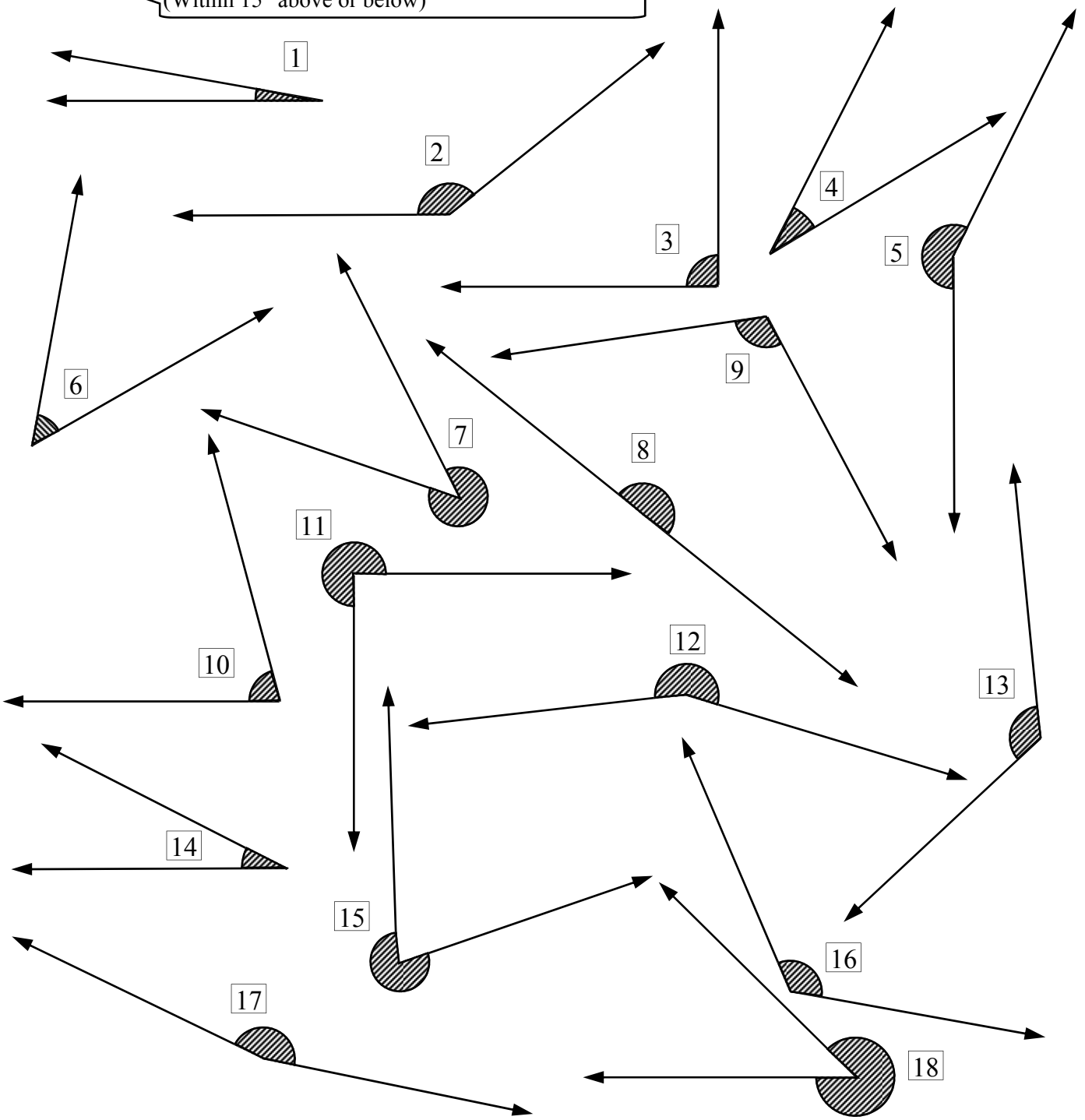
Estimate the size of all the shaded angles and place your answers in the table. Then using a protractor measure all the angles. See how well you estimated. (Within 15° above or below)





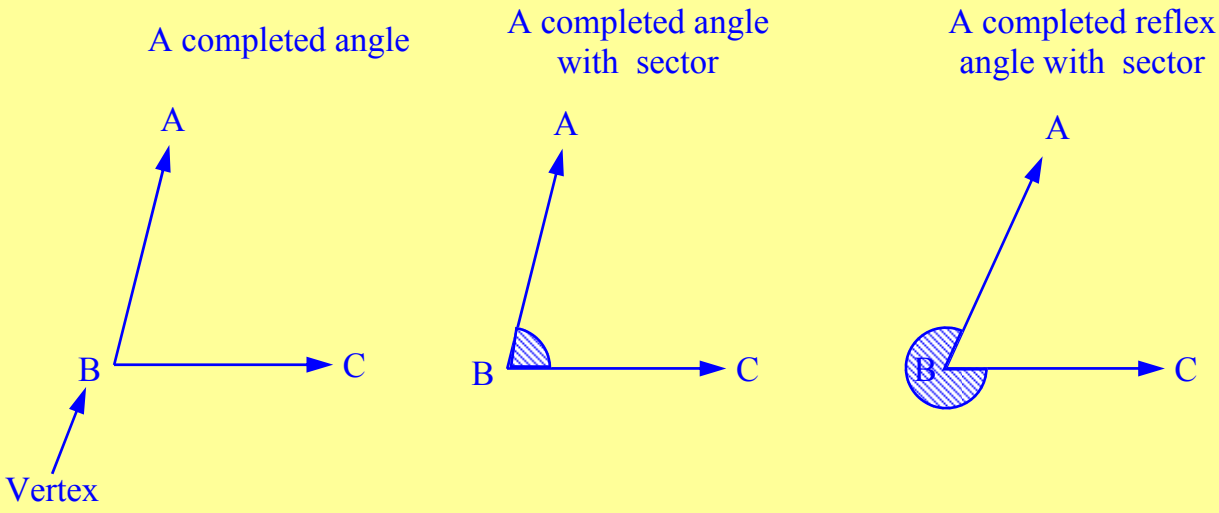
No	Estimate	Measured	No	Estimate	Measured	No	Estimate	Measured
1	5 ~ 25°	10°	7	300 ~ 330°	316°	13	115 ~ 140°	127°
2	125 ~ 155°	141°	8	180°	180°	14	20 ~ 35°	27°
3	85 ~ 95°	90°	9	95 ~ 125°	110°	15	270 ~ 300°	287°
4	20 ~ 40°	32°	10	65 ~ 85°	75°	16	110 ~ 135°	123°
5	190 ~ 220°	206°	11	265 ~ 275°	270°	17	150 ~ 170°	166°
6	40 ~ 60°	50°	12	190 ~ 220°	203°	18	300 ~ 330°	316°

Constructing Angles

The previous sheet involved measuring angles, this sheet is about the construction of angles.

The method is as follows:

- Draw a horizontal line
- Measure the given angle from the end of the drawn line, plotting a point at the angle
- Draw a line from the end of the line to the point
- Label the angle with the letters supplied in the question, remember the vertex is point on the elbow of the angle.
- Draw an arc or sector at the vertex to show the angle is the inside or outside of the angle.
- If the angle is greater than 180° then calculate (360° - the angle) the obtuse/acute angle and use that angle. Draw the sector on the other side of the angle.



Constructing Angles

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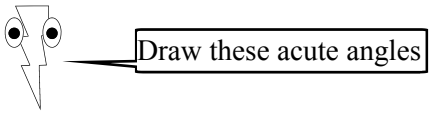
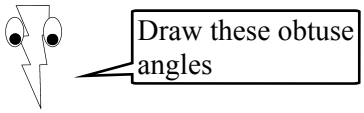


1 Construct a 30° angle
∠ ABC

2 Construct a 60° angle
∠ EVT

3 Construct a 37° angle
∠ NTY

4 Construct an 73° angle
∠ ALR

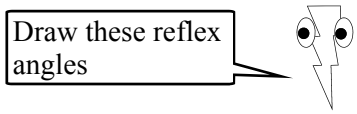


5 Construct a 110° angle
∠ FZT

6 Construct a 165° angle
∠ TRE

7 Construct a 138° angle
∠ JPG

8 Construct an 97° angle
∠ NJD



9 Construct a 240° angle
∠ BYC

10 Construct a 190° angle
∠ XOJ

11 Construct a 337° angle
∠ JVA

12 Construct an 254° angle
∠ PUK

Constructing Angles

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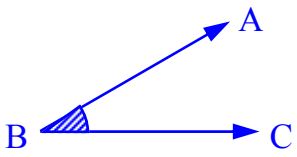
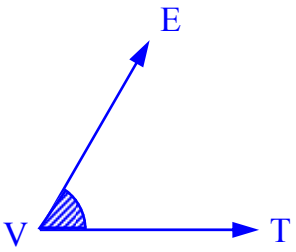
Draw these acute angles

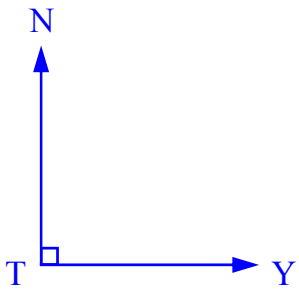
1 Construct a 30° angle
∠ ABC



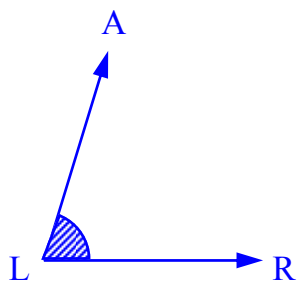
2 Construct a 60° angle
∠ EVT



3 Construct a 90° angle
∠ NTY

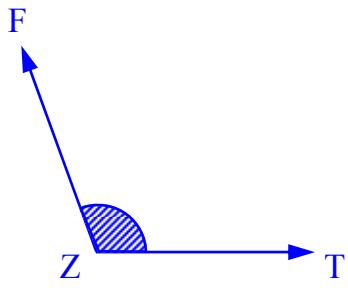


4 Construct an 73° angle
∠ ALR

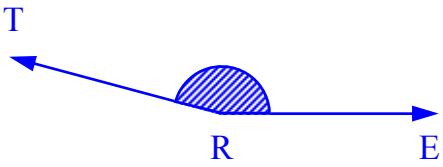


Draw these obtuse angles

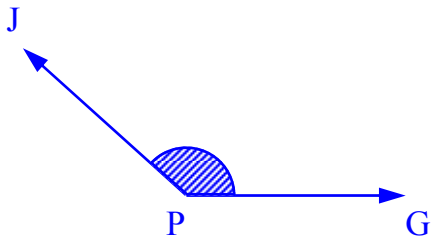
5 Construct a 110° angle
∠ FZT



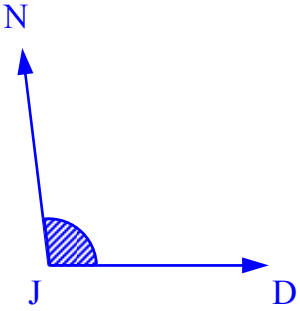
6 Construct a 165° angle
∠ TRE



7 Construct a 138° angle
∠ JPG

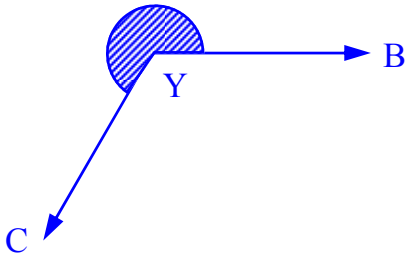


8 Construct an 97° angle
∠ NJD



Draw these reflex angles

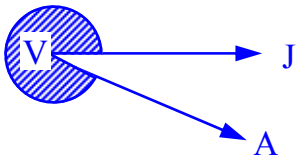
9 Construct a 240° angle
∠ BYC



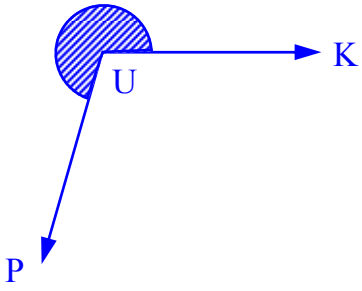
10 Construct a 190° angle
∠ XOJ



11 Construct a 337° angle
∠ JVA



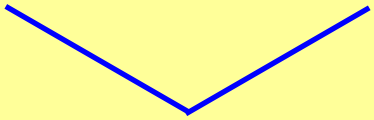
12 Construct an 254° angle
∠ PUK



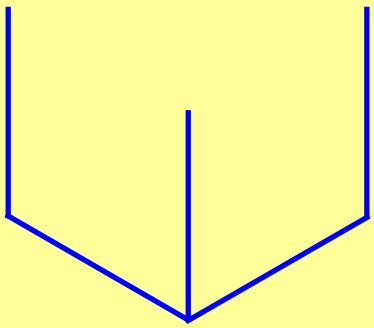
Creating an Isometric Cube

Isometric drawings are done using 30° angles. These are either drawn with a protractor or a 30°/60° set square. Make sure you don't use a 45° set square, as then you will be drawing an oblique drawing. Method:

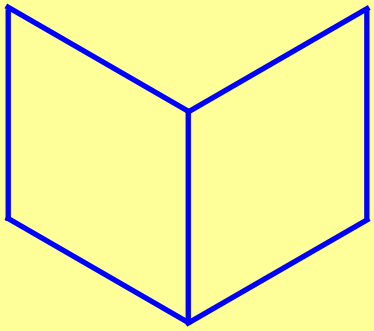
- From the point on the worksheet, or from a point on your page draw a 30° line to the left and another to the right.
- Measure off 10 cm and draw solid lines



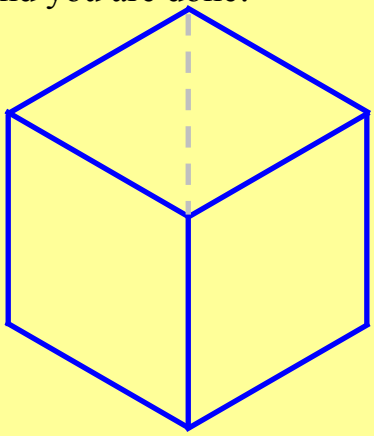
- Draw 3 vertical lines: from the start point and from ends of both lines you have just drawn. Make sure that these lines are vertical by measuring the distance from the side border of the sheet, make sure it is the same distance at the top and the bottom.



- Join the 3 ends together



- Draw a light construction line straight up the centre line, and construct the 30° angled lines from each end. Join up and you are done!



Time Out Activity - Using Angles to Create an Isometric Cube

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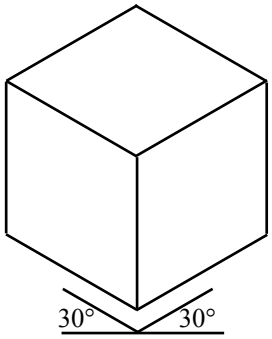
Construct an isometric cube with sides 10 cm using your protractor skills and a ruler. Hint: draw the bottom edges first then the 3 vertical lines.

A smaller version of the finished product



Time Out Activity - Using Angles to Create an Isometric Cube

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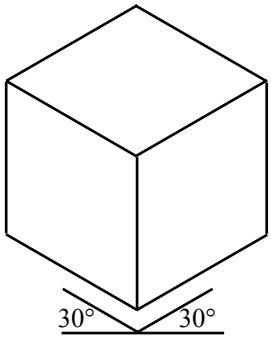
Construct an isometric cube with sides 10 cm using your protractor skills and a ruler. Hint: draw the bottom edges first then the 3 vertical lines.

A smaller version of the finished product



Adjacent Angles

Adjacent means next to, in the case of angles this means that the angle shares a common ray (or arm) and vertex with the angle beside it.

In Column 1 the table requires you to add the two angles together to get a total, the sum of two adjacent angles. This continues down the column only with graphical representation, note that these angles aren't to scale, so don't use the angles as a guide. In Q.3 there are three angles but only two angles have numbers, the third is a right angle which is 90°. It is suggested that you write in the '90°', that way you won't forget it in the addition.

In the 2nd Column Q.6 asks you to name the adjacent angle. This requires you to name the two angles that touch the given angle. As the layout is circular that means each angle has an adjacent angle on each side of it, don't name one angle then the same angle with the letters reversed. Q. 7 is another table which this time gives you the total of two angles, and one of the angles. Subtract the angle from the total and you have the answer.

The same method is used for the rest of the sheet Subtract the known angles from the total to obtain the unknown angle. Except Q.10 & 12, with these use division! Q.10 gives you the hint.

Adjacent Angles

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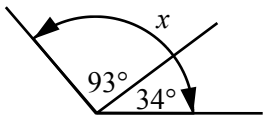
Angles 1 and 2 are adjacent find the sum of the two angles

1

Angle 1	Angle 2	Total
35°	42°	77°
19°	27°	
7°	96°	
115°	153°	
21°	117°	
273°	24°	
311°	46°	

Find the size of the angle formed by adding these adjacent angles.

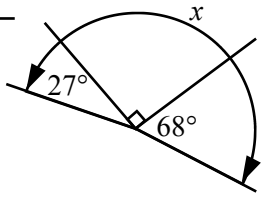
2



x = ____ + ____

x = ____

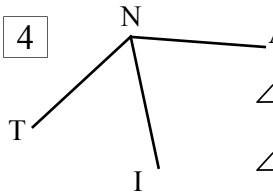
3



x = _____

x = _____

4



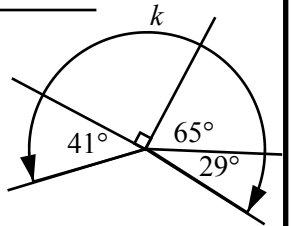
∠ TNI = 68°

∠ ANI = 77°

∠ TNA = _____

_____

5

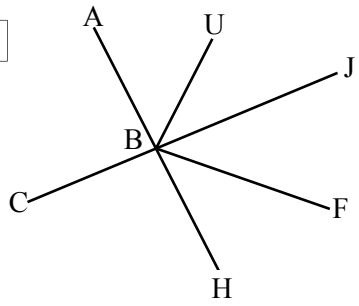


_____

_____

Name two adjacent angles to the following

6



∠ ABC : _____ & _____

∠ HBF : _____ & _____

∠ UBC : _____ & _____

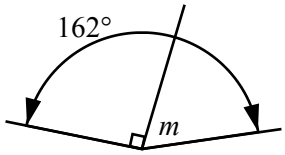
This time subtract. Given the total and one angle, find the other angle.

7

Total	Angle 1	Angle 2
38°	15°	
52°		17°
191°		115°
311°	208°	
187°	11°	
304°	221°	
339°		147°

You are given the total now subtract or divide to find the missing angles.

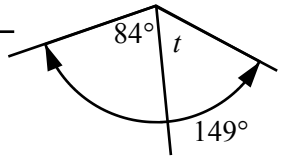
8



m = ____ - ____

m = ____

9



_____

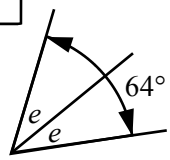
_____

Keep going!

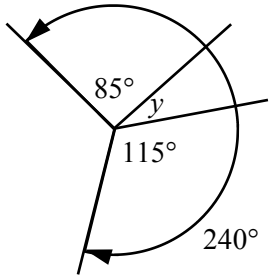
10

$2e$ = ____

e = ____



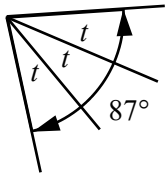
11



_____

_____

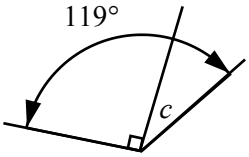
12



_____

_____

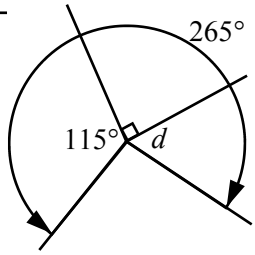
13



_____

_____

14



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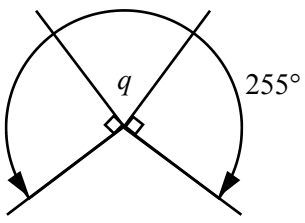
_____

15



_____

_____

Adjacent Angles

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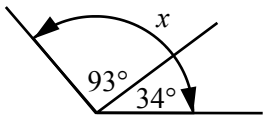
Angles 1 and 2 are adjacent find the sum of the two angles

1

Angle 1	Angle 2	Total
35°	42°	77°
19°	27°	46°
7°	96°	103°
115°	153°	268°
21°	117°	138°
273°	24°	297°
311°	46°	357°

Find the size of the angle formed by adding these adjacent angles.

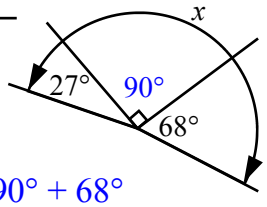
2



x = 93° + 34°

x = 127°

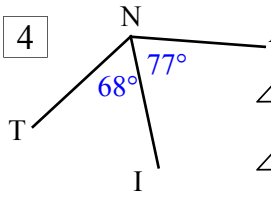
3



x = 27° + 90° + 68°

x = 185°

4



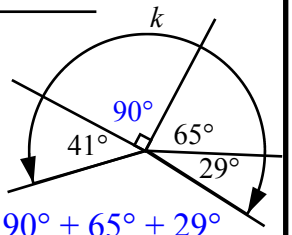
∠ TNI = 68°

∠ ANI = 77°

∠ TNA = 68° + 77°

∠ TNA = 145°

5

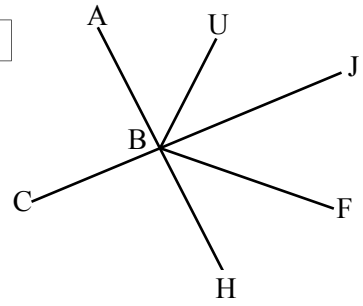


k = 41° + 90° + 65° + 29°

k = 225°

Name two adjacent angles to the following

6



∠ ABC : ∠ ABU & ∠ CBH

∠ HBF : ∠ FBJ & ∠ HBC

∠ UBC : ∠ UBJ & ∠ CBH

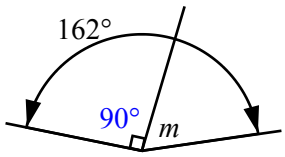
This time subtract. Given the total and one angle, find the other angle.

7

Total	Angle 1	Angle 2
38°	15°	23°
52°	35°	17°
191°	76°	115°
311°	208°	103°
187°	11°	176°
304°	221°	83°
339°	192°	147°

You are given the total now subtract or divide to find the missing angles.

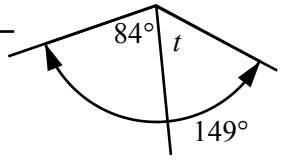
8



m = 162° - 90°

m = 72°

9

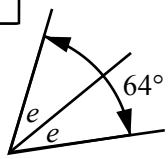


t = 149° - 84°

t = 65°

Keep going!

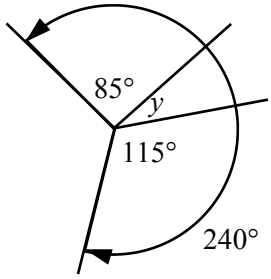
10



2e = 64°

e = 32°

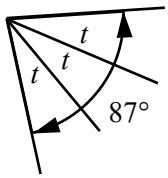
11



y = 240° - 115° - 85°

y = 40°

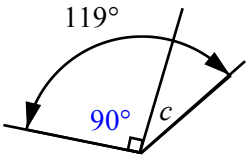
12



3t = 87°

t = 29°

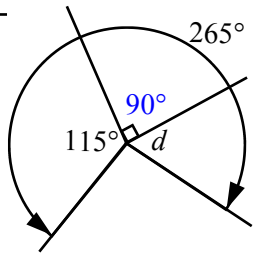
13



c = 119° - 90°

c = 29°

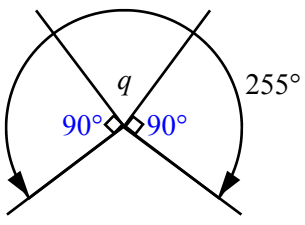
14



d = 265° - 115° - 90°

d = 60°

15



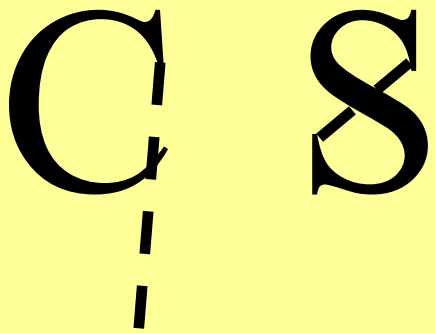
q = 255° - 90° - 90°

q = 75°

Complementary and Supplementary Angles

Complementary angles are angles that add to 90° and supplementary angles are angles that add to 180°. How do you remember which is which? C comes before S in the alphabet, as 90° comes before 180°. Or look at the construction below.

The C can be changed to a 9 (90°) and the S can be changed to an 8 to remember 180°



In Column 1 the first three questions ask you to verify that the two angles are complementary. Add the two angles together, if the sum equals 90° then they are.

For the rest of Column 1 you have to find the angle that when added to the angle given, has a total of 90°. You answer this by subtracting the given angle from 90°. The most common mistake is thinking that a right angle is 100°, or at least using it in calculations, remember 90° not 100°!

The 2nd Column has the same layout as the first but instead of the angles adding to give 90° they should total 180°, to be supplementary. The remainder of the column requires you to subtract the given angle from 180° to get the answer. When a right angle is involved write in 90°, this process should always be done as the angle may be overlooked.

The 3rd column is a mixture of the first 2 columns.

Complementary and Supplementary Angles

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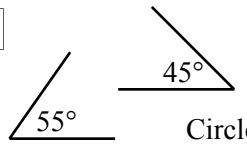
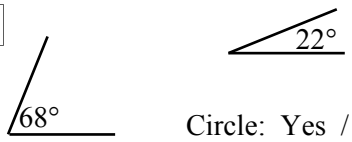
Are the following angles complementary?

1



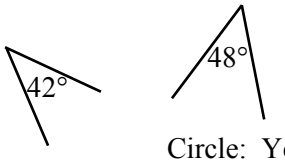
Circle: Yes / No

2



Circle: Yes / No

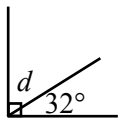
3



Circle: Yes / No

Find the unknown adjacent complementary angle

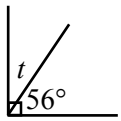
4



$d = 90° -$

$d =$

5



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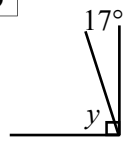

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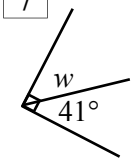

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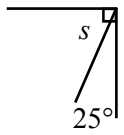

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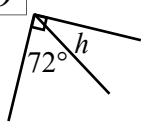
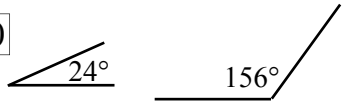


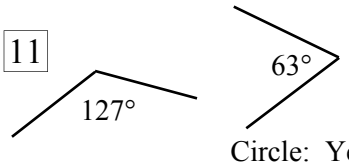
Are the following angles supplementary?

10



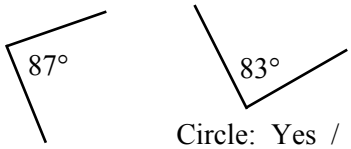
Circle: Yes / No

11



Circle: Yes / No

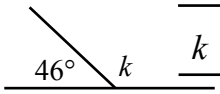
12



Circle: Yes / No

Find the unknown adjacent supplementary angle

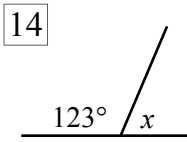
13



$k = 180° -$

$k =$

14



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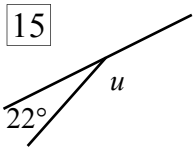

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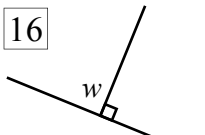

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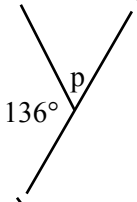

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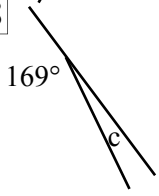

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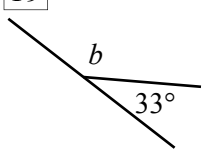
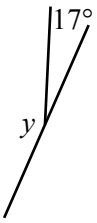


Find the missing angle

20



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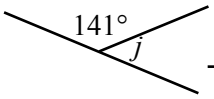

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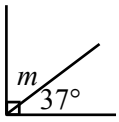

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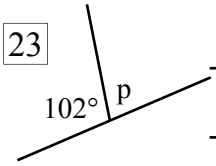

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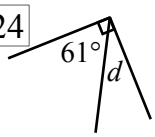

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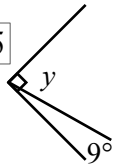

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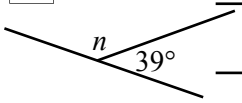

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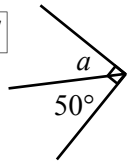

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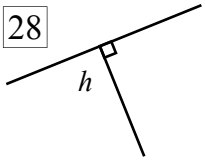

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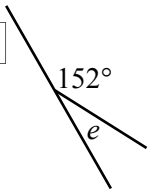

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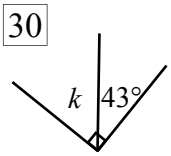

Complementary and Supplementary Angles

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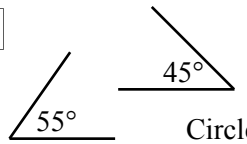
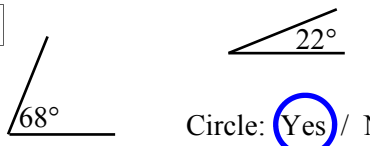
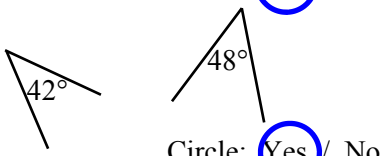
Are the following angles complementary?

1 

Circle: Yes / **No**

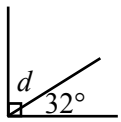
2 

Circle: **Yes** / No

3 

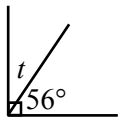
Circle: **Yes** / No

Find the unknown adjacent complementary angle

4 

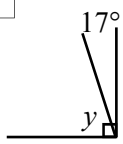
$d = 90° - 32°$

$d = 58°$

5 

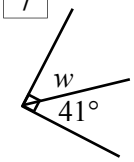
$t = 90° - 56°$

$t = 34°$

6 

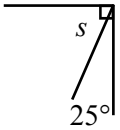
$y = 90° - 17°$

$y = 73°$

7 

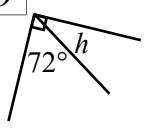
$w = 90° - 41°$

$w = 49°$

8 

$s = 90° - 25°$

$s = 65°$

9 

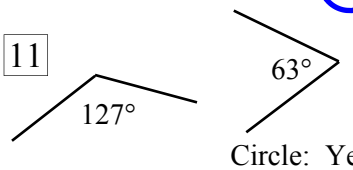
$h = 90° - 72°$

$h = 18°$

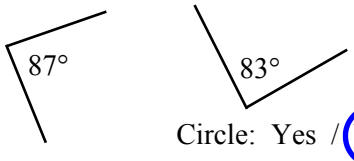
Are the following angles supplementary?

10 

Circle: **Yes** / No

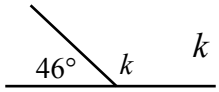
11 

Circle: Yes / **No**

12 

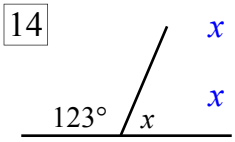
Circle: Yes / **No**

Find the unknown adjacent supplementary angle

13 

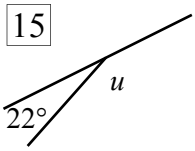
$k = 180° - 46°$

$k = 134°$

14 

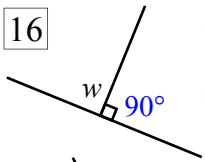
$x = 180° - 123°$

$x = 57°$

15 

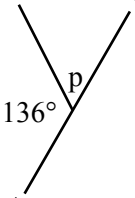
$u = 180° - 22°$

$u = 158°$

16 

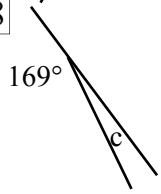
$w = 180° - 90°$

$w = 90°$

17 

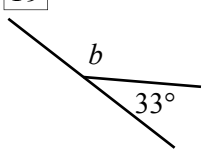
$p = 180° - 136°$

$p = 44°$

18 

$c = 180° - 169°$

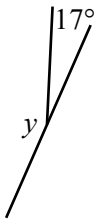
$c = 11°$

19 

$b = 180° - 33°$

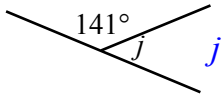
$b = 147°$

Find the missing angle

20 

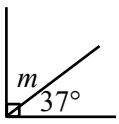
$y = 180° - 17°$

$y = 163°$

21 

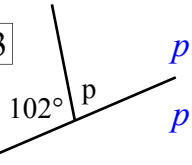
$j = 180° - 141°$

$j = 39°$

22 

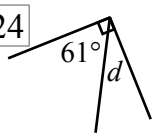
$m = 90° - 37°$

$m = 53°$

23 

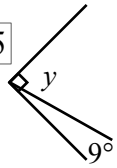
$p = 180° - 102°$

$p = 78°$

24 

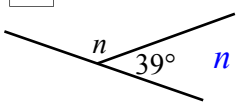
$d = 90° - 61°$

$d = 29°$

25 

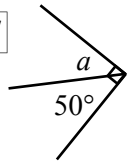
$y = 90° - 9°$

$y = 81°$

26 

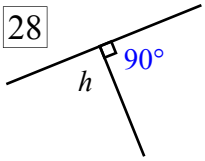
$n = 180° - 139°$

$n = 141°$

27 

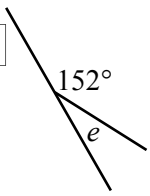
$a = 90° - 50°$

$a = 40°$

28 

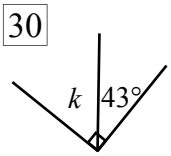
$h = 180° - 90°$

$h = 90°$

29 

$e = 180° - 152°$

$e = 28°$

30 

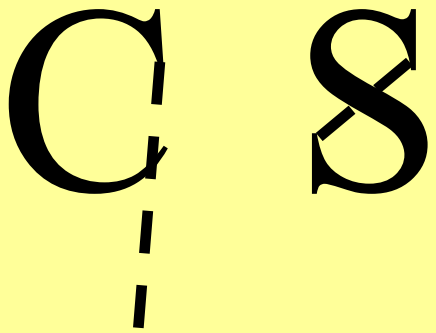
$k = 90° - 43°$

$k = 47°$

Complementary Angles

Complementary angles are angles that add to 90°. Supplementary angles add to 180° (the next sheet). How do you remember which is which? C comes before S in the alphabet, as 90° comes before 180°. Or look at the construction below.

The C can be changed to a 9 (90°) and the S can be changed to an 8 to remember 180°



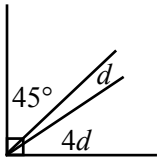
In Column 1 you are asked to find the complement, this means find the angle that when added to the angle given, has a total of 90°. You do this by subtracting the given angle from 90°. This continues down the column, subtract the angle given on the diagram from 90° to find the answer. The most common mistake is thinking that a right angle is 100°, or at least using it in calculations remember 90° not 100°!

The 2nd Column through to Q 16 in the 3rd Column is an extension of this. The same method is used only more than one angle is subtracted from 90°. Question 7 shows the method of working required.

Questions 17 and 18 require you to add the letters together, these equal 90°. Then divide by the number in front of the letter to get the value of the pronumeral. Note that the value of the pronumeral is being found, this isn't necessarily the value of the angle.

Questions 19 and 20 are more difficult questions and require an extra step. The letters are added on the left of the equals sign, and the subtraction of the angle from 90° is put on the right side. See the example below.

Example



$5d = 90° - 45°$

$5d = 45°$

$d = 9°$

Divide by 5

Complementary Angles

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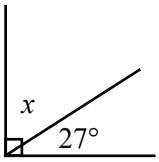
Find the complementary angle to those given in the table below.

1

Angle	Complement
30°	60°
45°	
73°	
21°	
9°	
57°	
84°	

Find the complementary angle for the following

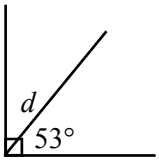
2



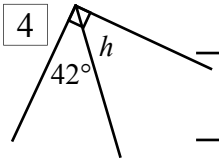
x = 90° - ____

x = ____

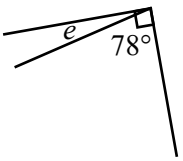
3



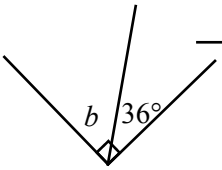
4



5

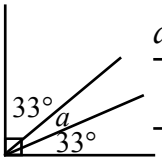


6



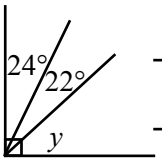
Now there are more than 2 angles. Solve these.

7



a = 90° - 33° - 33°

8



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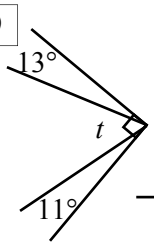

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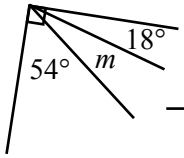

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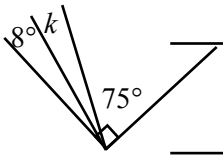

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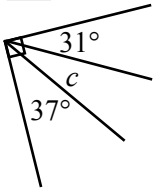

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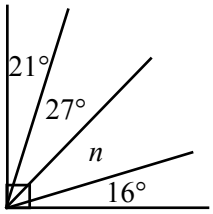
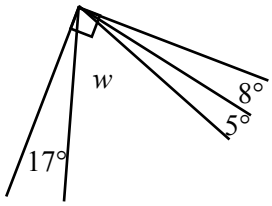


14



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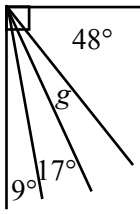
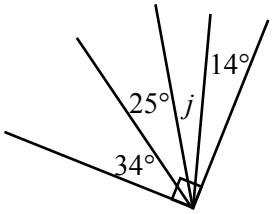
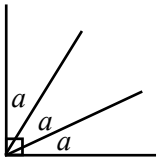


16

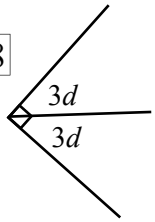


Use division to find the value of the letters.

17



18



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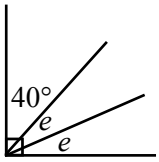

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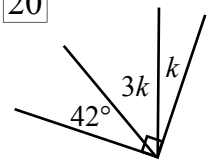

Complementary Angles

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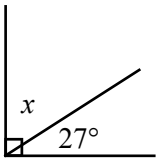
Find the complementary angle to those given in the table below.

1

Angle	Complement
30°	60°
45°	45°
73°	17°
21°	69°
9°	81°
57°	33°
84°	6°

Find the complementary angle for the following

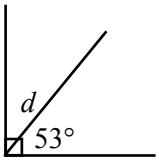
2



$x = 90° - 27°$

$x = 63°$

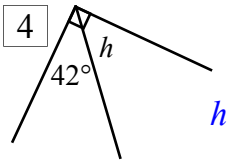
3



$d = 90° - 53°$

$d = 37°$

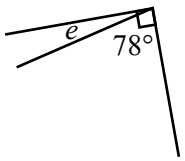
4



$h = 90° - 42°$

$h = 48°$

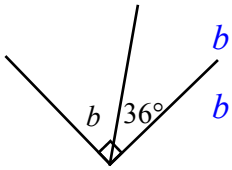
5



$e = 90° - 78°$

$e = 12°$

6

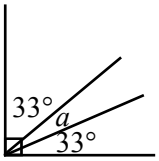


$b = 90° - 36°$

$b = 54°$

Now there are more than 2 angles. Solve these.

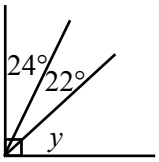
7



$a = 90° - 33° - 33°$

$a = 24°$

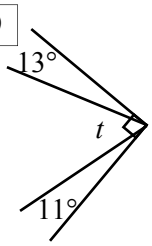
8



$y = 90° - 24° - 22°$

$y = 44°$

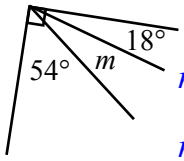
9



$t = 90° - 11° - 13°$

$t = 66°$

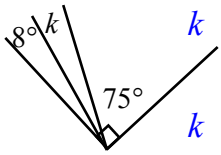
10



$m = 90° - 54° - 18°$

$m = 18°$

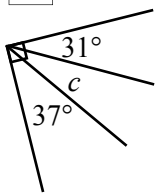
11



$k = 90° - 8° - 75°$

$k = 7°$

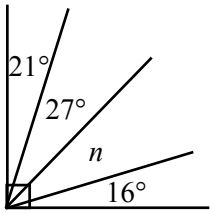
12



$c = 90° - 37° - 31°$

$c = 22°$

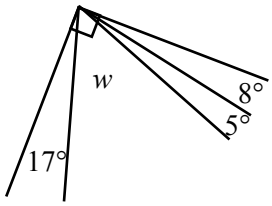
13



$n = 90° - 21° - 27° - 16°$

$n = 26°$

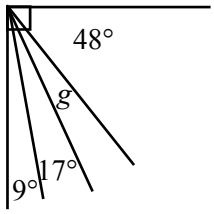
14



$w = 90° - 17° - 8° - 5°$

$w = 60°$

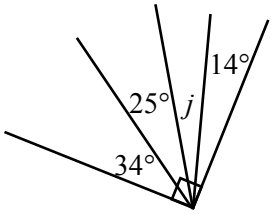
15



$g = 90° - 9° - 17° - 48°$

$g = 16°$

16

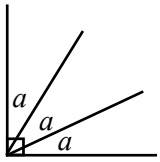


$j = 90° - 34° - 25° - 14°$

$j = 17°$

Use division to find the value of these unknowns.

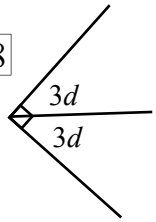
17



$3a = 90°$

$a = 30°$

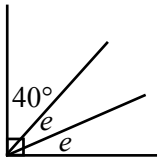
18



$6d = 90°$

$d = 15°$

19

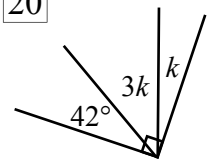


$2e = 90° - 40°$

$2e = 50°$

$e = 25°$

20



$4k = 90° - 42°$

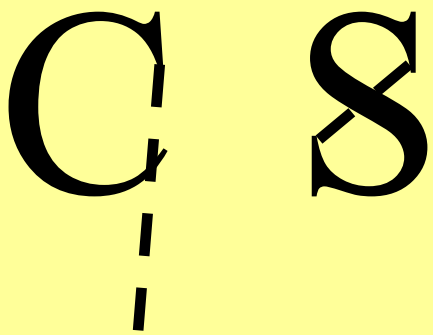
$4k = 48°$

$k = 12°$

Supplementary Angles

Supplementary angles are angles that add to 180°. How do you remember the difference between supplementary and complementary? C comes before S in the alphabet, just as 90° comes before 180°. Or look at the construction below.

The C can be changed to a 9 (90°) and the S can be changed to an 8 to remember 180°



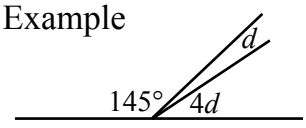
In Column 1 you are asked to find the supplement, this means find the angle that when added to the angle given, has a total of 180°. You do this by subtracting the given angle from 180°. This continues down the column, subtract the angle given on the diagram from 180° to get the answer.

The 2nd Column has a table to complete, this time both the complementary and supplementary angles are required. Subtract the angle from 180° to get the supplement and from 90° for the complement. Q 9 through 16 add more angles into the problem, but the problem is still solved the same way, by subtracting all the given angles from 180°.

Questions 17 and 19 require you to add the letters together, these equal 180°. Then divide by the number in front of the letter to get the value of the pronumeral. Note that the value of the pronumeral is being found which isn't necessarily the value of the angle (see Question 20).

Question 18 is a more difficult question and requires an extra step. The letters are added on the left of the equals sign, and the subtraction of the angle from 90° is put on the right side. See the example below.

Example



$5d = 180° - 145°$

$5d = 35°$

$d = 7°$

Divide by 5

Supplementary Angles

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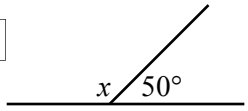
Find the supplementary angle to those given in the table below.

1

Angle	Supplement
110°	70°
45°	
162°	
16°	
123°	
27°	
177°	

Find the supplementary angle for the following

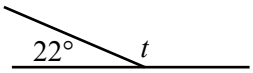
2



x = 180° - ____

x = ____

3



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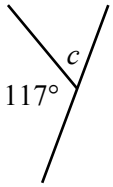

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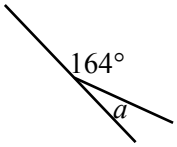

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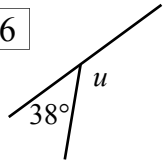

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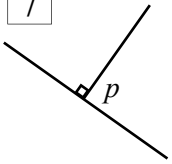


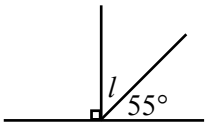
For the angles below find both the supplement and the complement.

8

Angle	Supplement	Complement
25°		
65°		
13°		
79°		
57°		
3°		
41°		

Now there are more than 2 angles. Solve these.

9



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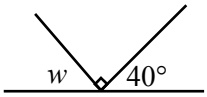

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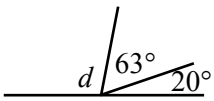

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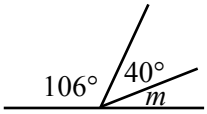
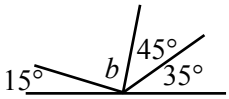


13



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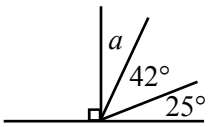
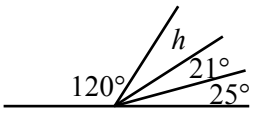
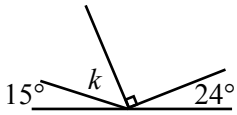


15

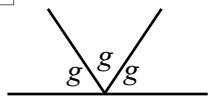


16

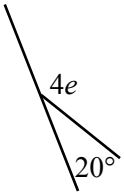


Use division to find the value of these unknowns.

17



18



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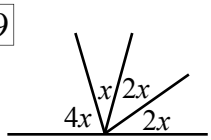

20 With the value of x found above, find the value of:

$2x$ = ____ $4x$ = ____

Supplementary Angles

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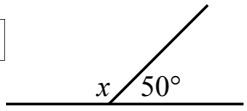
Find the supplementary angle to those given in the table below.

1

Angle	Supplement
110°	70°
45°	135°
162°	18°
16°	164°
123°	57°
27°	153°
177°	3°

Find the supplementary angle for the following

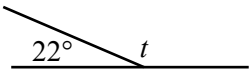
2



x = 180° - 50°

x = 130°

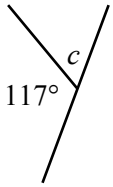
3



t = 180° - 22°

t = 158°

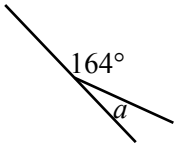
4



c = 180° - 117°

c = 63°

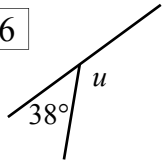
5



a = 180° - 164°

a = 16°

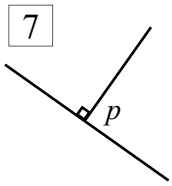
6



u = 180° - 38°

u = 142°

7



p = 180° - 90°

p = 90°

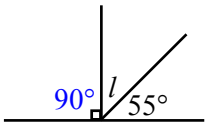
For the angles below find both the supplement and the complement.

8

Angle	Supplement	Complement
25°	155°	65°
65°	115°	25°
13°	167°	77°
79°	101°	11°
57°	123°	33°
3°	177°	87°
41°	139°	49°

Now there are more than 2 angles. Solve these.

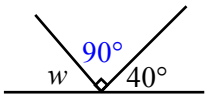
9



l = 180° - 90° - 55°

l = 35°

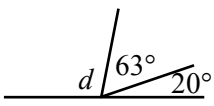
10



w = 180° - 90° - 40°

w = 50°

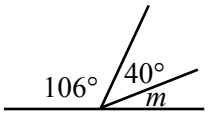
11



d = 180° - 63° - 20°

d = 97°

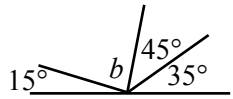
12



m = 180° - 106° - 40°

m = 34°

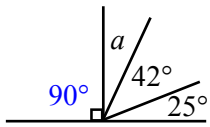
13



b = 180° - 15° - 45° - 35°

b = 85°

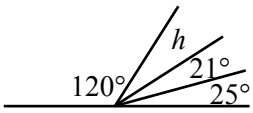
14



a = 180° - 90° - 42° - 25°

a = 23°

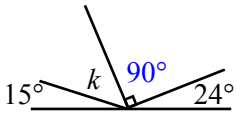
15



h = 180° - 120° - 21° - 25°

h = 14°

16

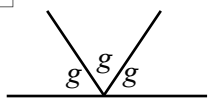


k = 180° - 15° - 90° - 24°

k = 51°

Use division to find the value of these unknowns.

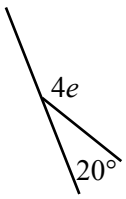
17



$3g$ = 180°

g = 60°

18

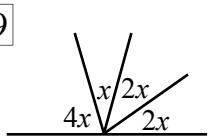


$4e$ = 180° - 20°

$4e$ = 160°

e = 40°

19



$9x$ = 180°

x = 20°

20 With the value of x found above, find the value of:

$2x$ = 40° $4x$ = 80°

Angles at a Point

Just like a circle, the angle sum at a point is 360°. To find the size of an unknown angle subtract all the given angles from 360°.

In Column 1 the unknown angle is given a letter, just write the letter then '=' and subtract all the given angles from 360°. Some angles are right angles, write '90°' in these so that you don't forget to include the right angle in your calculation.

Column 2 requires you to use your algebra skills, add the letters and then divide 360° by the number in front of the letter. Questions 13 through 15 have an additional step, use the same method to find the value of the letter, but then multiply that value by the number in front of the letter for each angle. For example if you find $x = 25°$ and the angle in the question is $3x$, then $3x = 3 \times 25° = 75°$. If the question asks 'to solve for x', then you don't need to do this, it is only when you are asked for the angle, as the angle is $3x$ you must find the size of the angle, as the size of x isn't sufficient.

Column 3 is a harder column and some students may experience difficulty with these. The steps are a combination of the earlier problems.

- add the letters together and write it then an equals, (in question 16. $e + e = 2e$)
- then subtract the given angles (remember the right angles) from 360°
- the next line write the letters sum again only evaluate 360° - the angle
- then use division to solve like earlier problems
- the last two questions are harder, they require multiplication to get the angle (like Q13-15)

Angles at a Point

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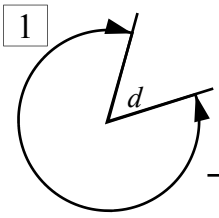
Use subtraction from 360° to find the unknown angle

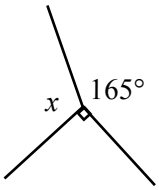
1



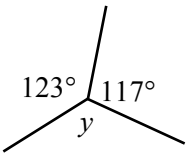
$d = 360° -$

$d =$

2



3



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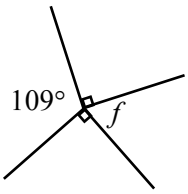

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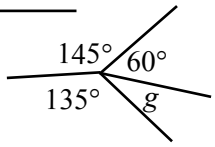

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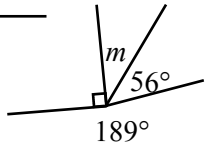

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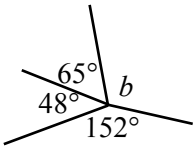
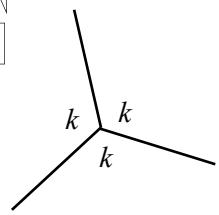


This time use division

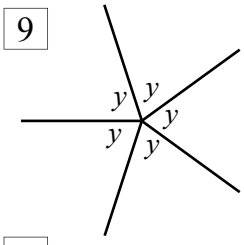
8



$3k =$

$k =$

9



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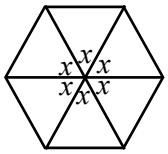

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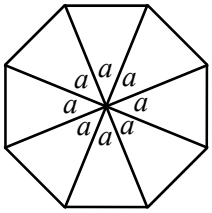
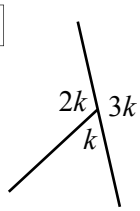


Use the same method but this time find the value of the letter and the angles

12

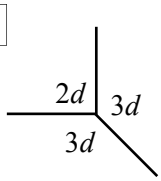


$6k =$

$k =$

$2k =$ $3k =$

13



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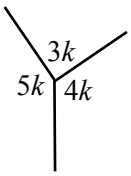


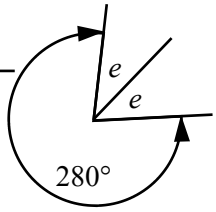
Use subtraction from 360° then division to find the unknown angles

15

$2e = 360° -$

$2e =$

$e =$



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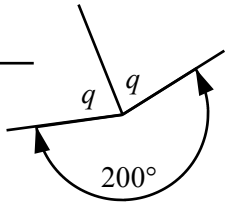
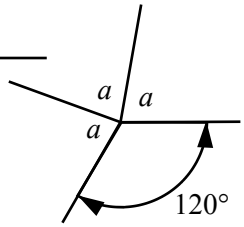
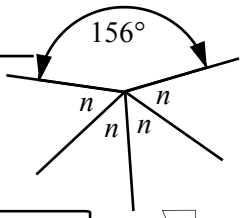


17



19

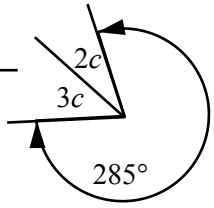


Now the angles are different

20

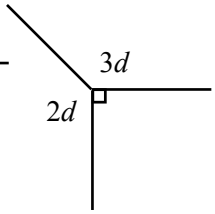
$5c =$

$5c =$



$2c =$ $3c =$

21



Angles at a Point

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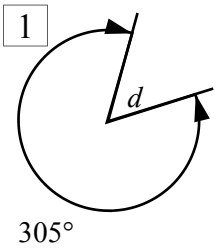
Use subtraction from 360° to find the unknown angle

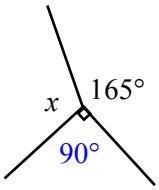
1



$d = 360° - 305°$

$d = 55°$

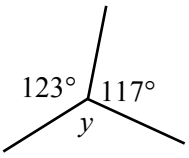
2



$x = 360° - 90° - 165°$

$x = 105°$

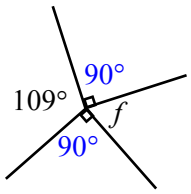
3



$y = 360° - 123° - 117°$

$y = 120°$

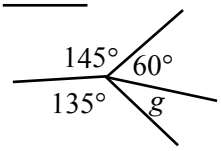
4



$f = 360° - 109° - 90° - 90°$

$f = 71°$

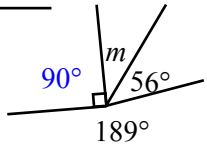
5



$g = 360° - 135° - 145° - 60°$

$g = 20°$

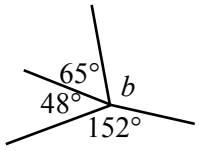
6



$m = 360° - 189° - 90° - 56°$

$m = 25°$

7

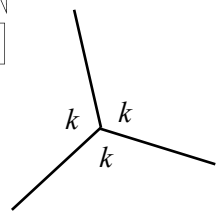


$b = 360° - 152° - 48° - 65°$

$b = 95°$

This time use division

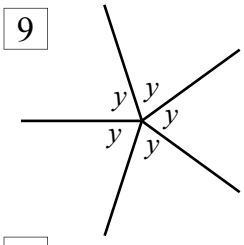
8



$3k = 360°$

$k = 120°$

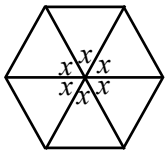
9



$5y = 360°$

$y = 72°$

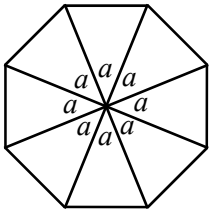
10



$6x = 360°$

$x = 60°$

11

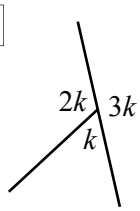


$8a = 360°$

$a = 45°$

Use the same method but this time find the value of the letter and the angles

12

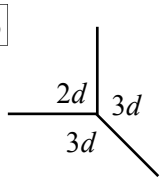


$6k = 360°$

$k = 60°$

$2k = 120°$ $3k = 180°$

13

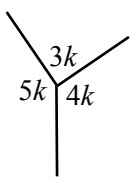


$8d = 360°$

$d = 45°$

$2d = 90°$ $3d = 135°$

14



$12k = 360°$

$k = 30°$

$3k = 90°$ $4k = 120°$

$5k = 150°$

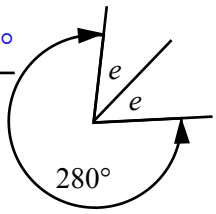
Use subtraction from 360° then division to find the unknown angles

15

$2e = 360° - 280°$

$2e = 80°$

$e = 40°$

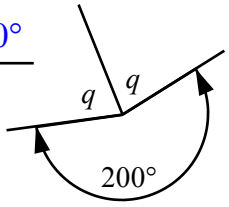


16

$2q = 360° - 200°$

$2q = 160°$

$q = 80°$

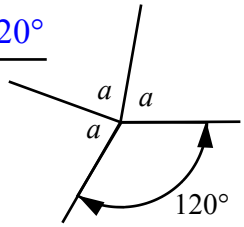


17

$3a = 360° - 120°$

$3a = 240°$

$a = 80°$

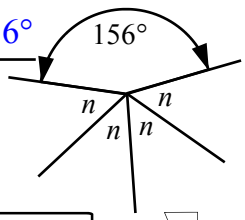


18

$4n = 360° - 156°$

$4n = 204°$

$n = 51°$



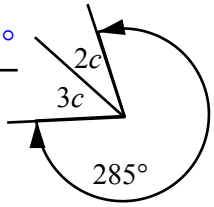
Now the angles are different

19

$5c = 360° - 285°$

$5c = 75°$

$c = 15°$



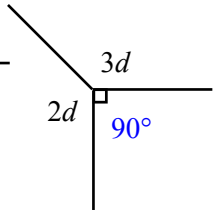
$2c = 30°$ $3c = 45°$

20

$5d = 360° - 90°$

$5d = 270°$

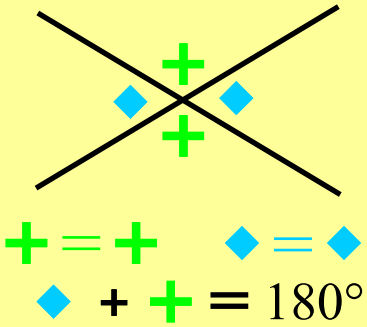
$d = 54°$



$2d = 108°$ $3d = 162°$

Vertically Opposite Angles

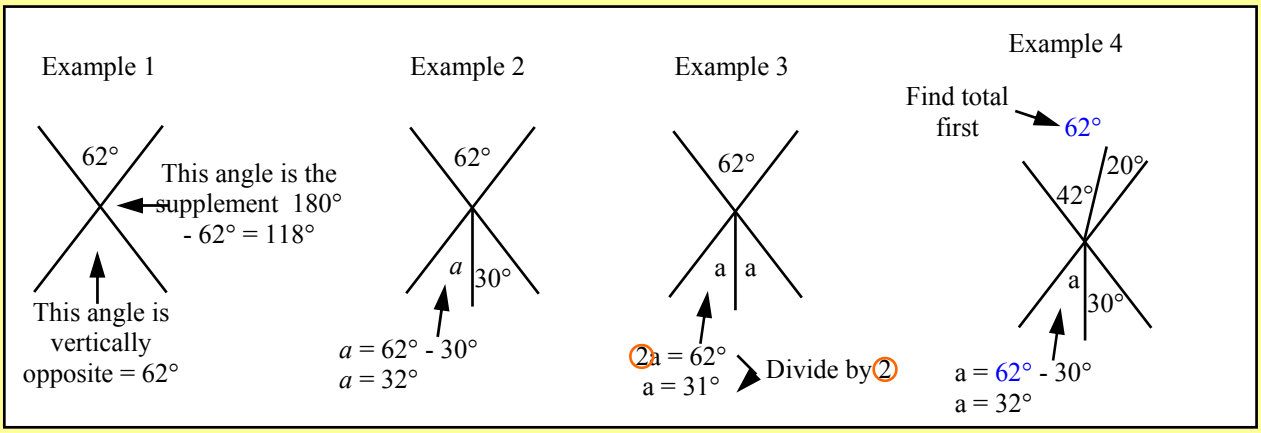
Vertically opposite angles are equal when formed by intersecting straight lines. The diagram below illustrates this. Don't let the word 'vertical' confuse you, as you can see if angles are opposite each other horizontally they are also considered vertically opposite (equal).



In Column 1 identify the vertically opposite angle, by drawing a star in the angle and then also by naming it. Write the name of the angle on the line and colour the star beside it the same colour as the one you placed on the diagram. Q 5 is more difficult as many angles are all on the one diagram so take care with this question.

Column 2 asks you to find the value of the letter, this is just identification, (the answer is in front of you) for Q. 6 through 9. But from Q 10 on…. some mathematics is required, this will always be subtraction or division look at the examples below.

With column 3 Questions 15 and 16 use the same method as the example above them, establishing two angles from identification then the 3rd by supplementary angle methods, look at the example at the top of the column. The last two questions are more difficult and require you to choose vertically opposite or supplementary methods to solve them.



Vertically Opposite Angles

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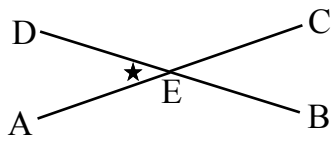
Name and label with a star the vertically opposite angle to the one marked with a star

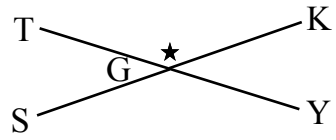
1



Colour this star the same as the one you put in the angle

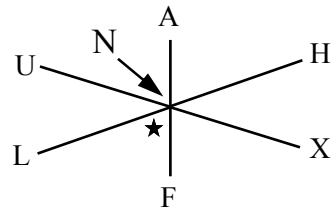
☆ ∠ _____

2



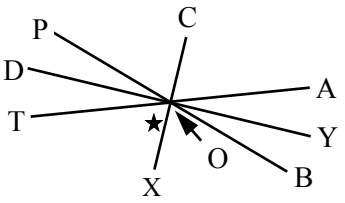
☆ _____

3



☆ _____

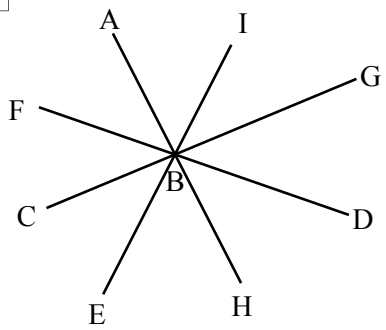
4



☆ _____

Name the vertically opposite angle to the following angles

5



∠ IBG — vertically opposite → _____

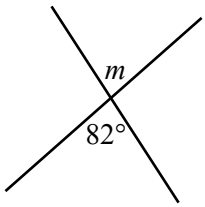
∠ HBD → _____

∠ ABC → _____

∠ CBD → _____

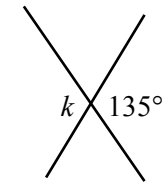
Find the value of the letters below

6



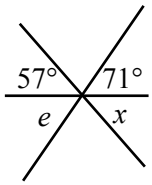
m = _____

7



_____

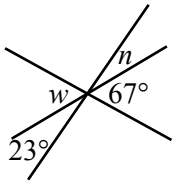
8



_____

_____

9



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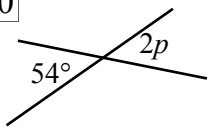

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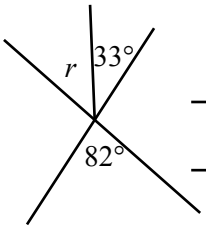

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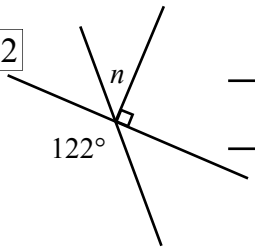
12



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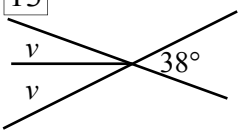
13



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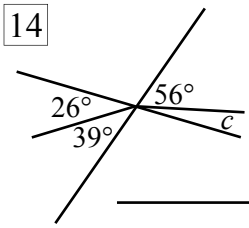
14



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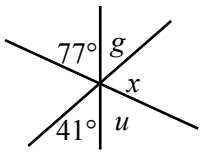
Use vertically opposite and supplementary angle properties to find the value of these letters

Example



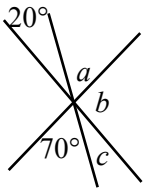
$g = 41°$

$u = 77°$

$x = 180° - 41° - 77°$

$x = 62°$

15



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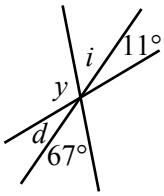
16



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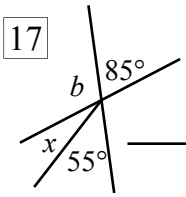
17



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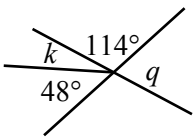
18



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Vertically Opposite Angles

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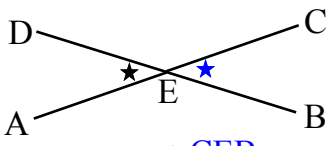
Name and label with a star the vertically opposite angle to the one marked with a star

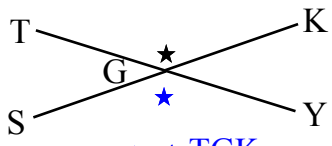
1



Colour this star the same as the one you put in the angle

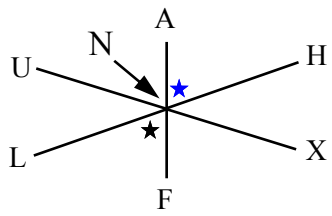
★ ∠ CEB

2



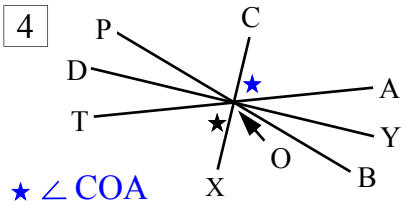
★ ∠ TGK

3



★ ∠ ANH

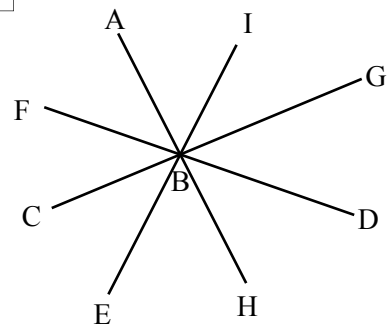
4



★ ∠ COA

Name the vertically opposite angle to the following angles

5



∠ IBG → (vertically opposite) ∠ CBE

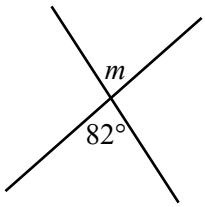
∠ HBD → ∠ ABF

∠ ABC → ∠ GBH

∠ CBD → ∠ FBG

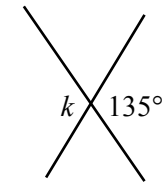
Find the value of the letters below

6



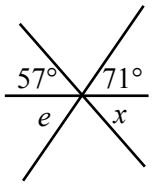
$m = 82°$

7



$k = 135°$

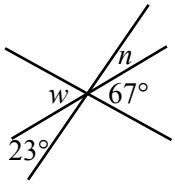
8



$e = 71°$

$x = 57°$

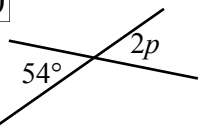
9



$n = 23°$

$w = 67°$

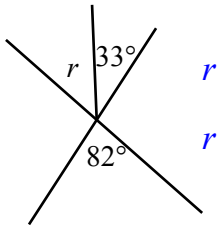
10



$2p = 54°$

$p = 27°$

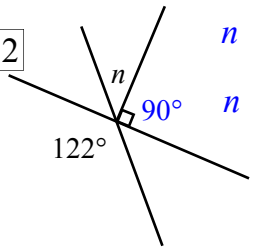
11



$r = 82° - 33°$

$r = 49°$

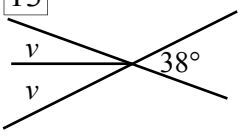
12



$n = 122° - 90°$

$n = 32°$

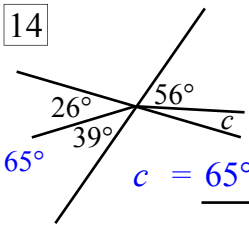
13



$2v = 38°$

$v = 19°$

14

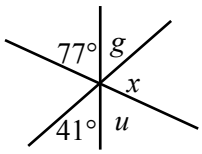


$c = 65° - 56°$

$c = 9°$

Use vertically opposite and supplementary angle properties to find the value of these letters

Example



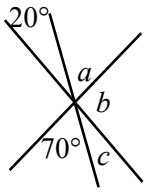
$g = 41°$

$u = 77°$

$x = 180° - 41° - 77°$

$x = 62°$

15



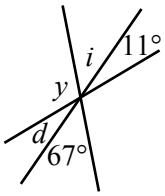
$a = 70°$

$c = 20°$

$b = 180° - 70° - 20°$

$b = 90°$

16



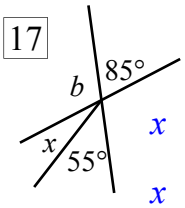
$d = 11°$

$i = 67°$

$y = 180° - 11° - 67°$

$y = 102°$

17



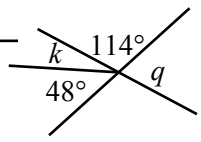
$x = 85° - 55$

$x = 30°$

$b = 180° - 55° - 30°$

$b = 95°$

18



$k = 180° - 114° - 48°$

$k = 18°$

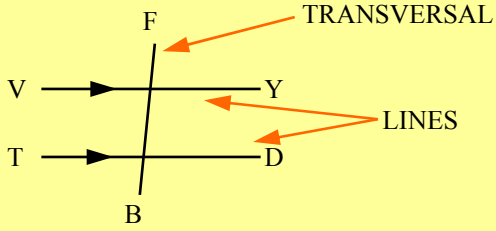
$q = 48° + 18°$

$q = 66°$

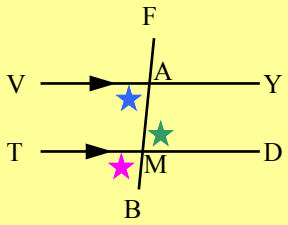
Parallel Lines and the Transversal

When parallel lines are crossed by a line the angles formed are related to each other by properties. This will require you to remember certain words and their meaning.

The first word is 'transversal' this is the name of the line that cuts across parallel lines. In column 1 you are asked to name the transversal and the parallel lines, write the 2 letters that represent the line, this can be done either way as the example below shows. The naming of angles uses the 3 point method, there is 4 possible answers for some questions as the example below shows, realise that if the answer doesn't match your answer you still may be correct, but the middle letter (vertex) must always be the same.



Lines are VY or YV and TD or DT
Transversal is either FB or BF



★ ∠VAM or ∠VAB or ∠MAV or ∠BAV

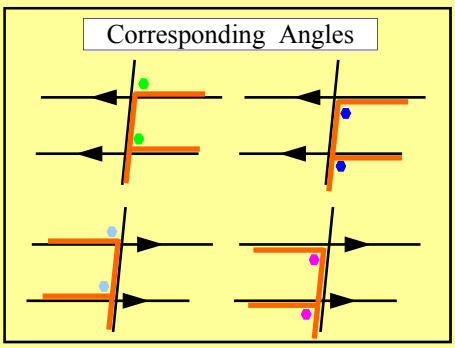
★ ∠AMD or ∠FMD or ∠DMA or ∠DMF

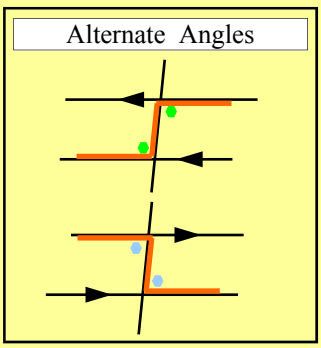
★ ∠TMB or ∠BMT

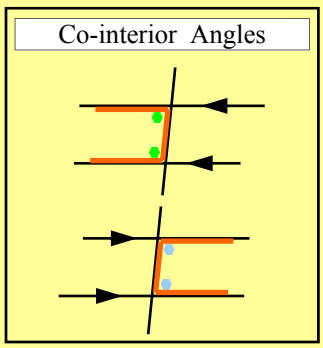
When lines cross, pairs of angles are made. Co-interior angles (called C angles) are the pair of angles formed on one side of the transversal inside the parallel lines, a 'c' can be formed around the angles (one is back to front). There are only two co-interior angle pairs in a standard 2-line 1 transversal problem. Remember the word interior means on the inside (of the parallel lines and the transversal).

Alternate angles (called Z angles) are also on the inside of the parallel lines but unlike co-interior angles they are on opposite sides of the transversal. A 'Z' can be made by the lines that include these angles.

Both alternate and co-interior angles are inside the two parallel lines. Corresponding angles (called F angles) require one point to be on the inside of the parallel lines and the other to be on the outside. An 'F ' or back to front ‘F’ is made by the lines that include these angles. With corresponding angles the angle is in exactly the same position on both points of intersection. Column 3 asks you to write either 'alternate', 'co-interior' or 'corresponding'. Questions 32 through 37 are more challenging but the same method is used.







Parallel Lines and the Transversal

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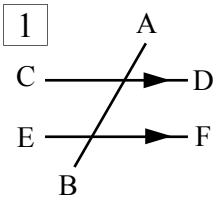
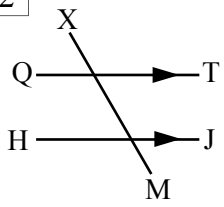
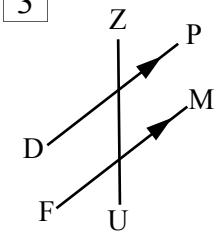
Using the letters name the parallel lines (L) and the transversal (T)

1


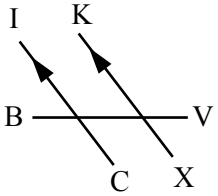
L: _____
L: _____
T: _____

2


L: _____
L: _____
T: _____

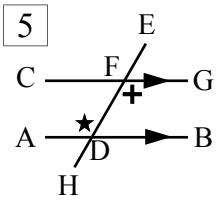
3


L: _____
L: _____
T: _____

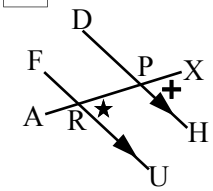
4


L: _____
L: _____
T: _____

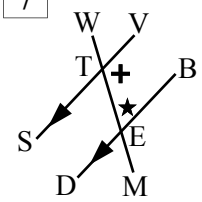
Using the assigned letters name the 2 angles with symbols

5


★ _____
+ _____

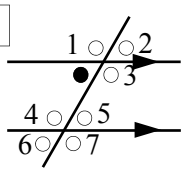
6


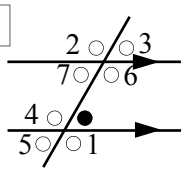
★ _____
+ _____

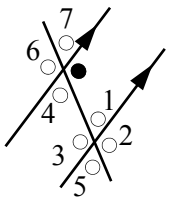
7


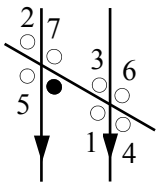
★ _____
+ _____

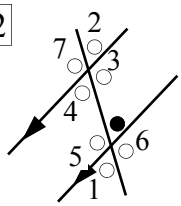
Show the co-interior angle by colouring the circle (the numbers are for marking purposes)

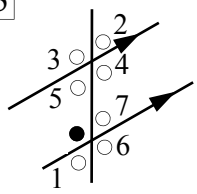
8


9


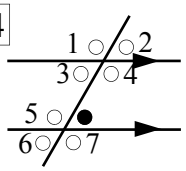
10


11


12


13


Now colour the alternate angle

14


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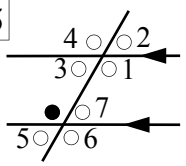

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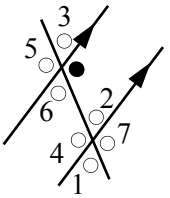

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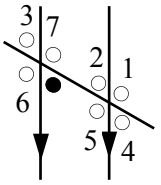

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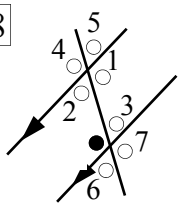

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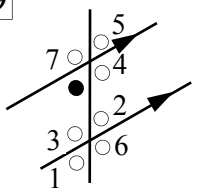

Now colour the corresponding angle

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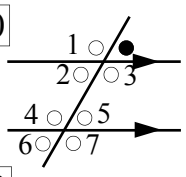

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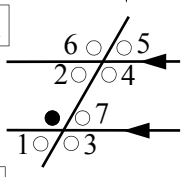

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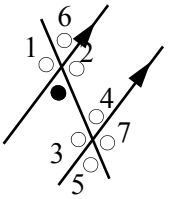

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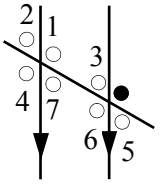

State if these angles are corresponding, alternate or co-interior.

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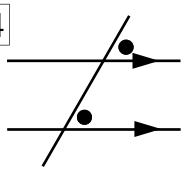

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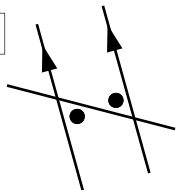

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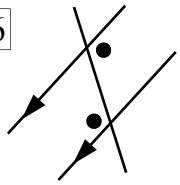

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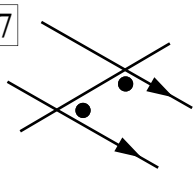

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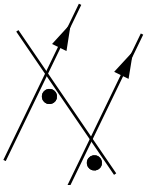

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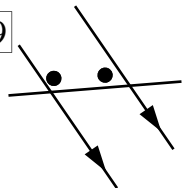

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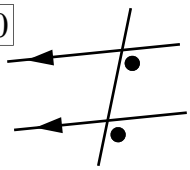

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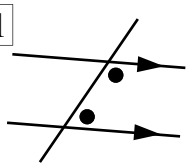

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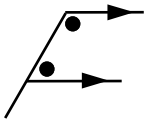

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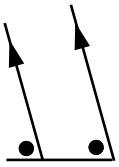

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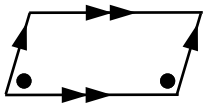

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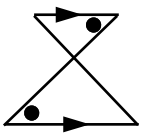

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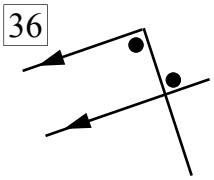

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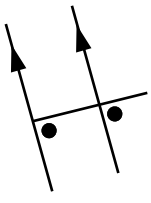

Parallel Lines and the Transversal

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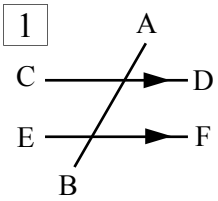
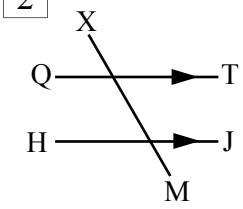
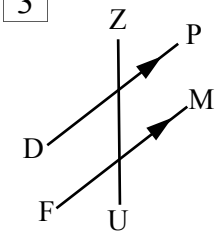
Using the letters name the parallel lines (L) and the transversal (T)

1 

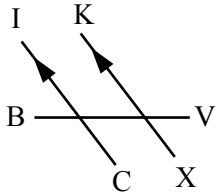
L: CD
L: EF
T: AB

2 

L: QT
L: HJ
T: XM

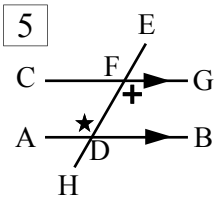
3 

L: DP
L: FM
T: ZU

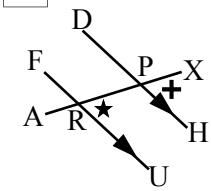
4 

L: IC
L: KX
T: BV

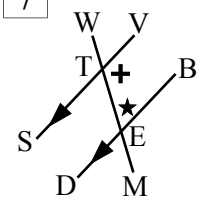
Using the assigned letters name the 2 angles with symbols

5 

★ ∠ ADF
+ ∠ DFG

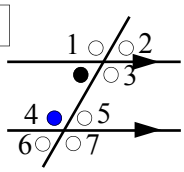
6 

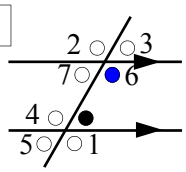
★ ∠ PRU
+ ∠ XPH

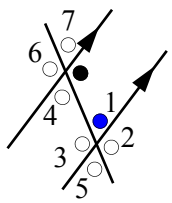
7 

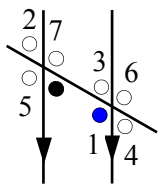
★ ∠ TEB
+ ∠ VTE

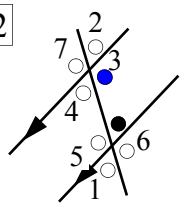
Show the co-interior angle by colouring the circle (the numbers are for marking purposes)

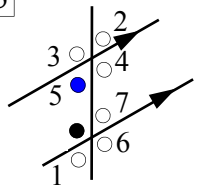
8 

9 

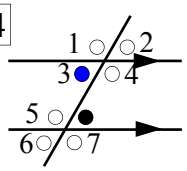
10 

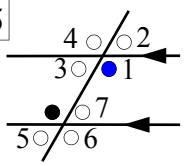
11 

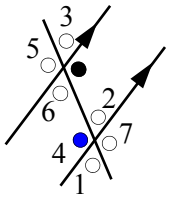
12 

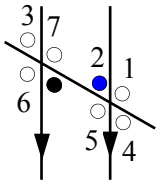
13 

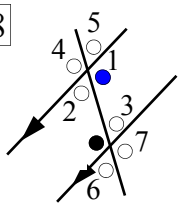
Now colour the alternate angle

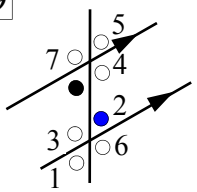
14 

15 

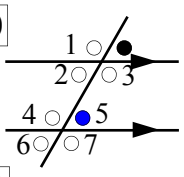
16 

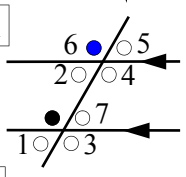
17 

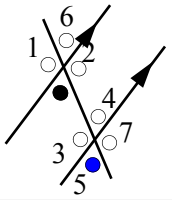
18 

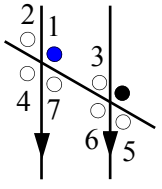
19 

Now colour the corresponding angle

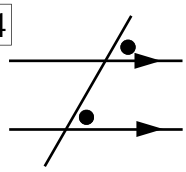
20 

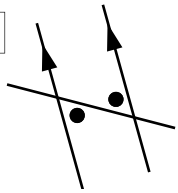
21 

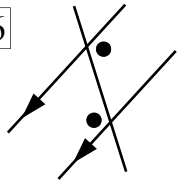
22 

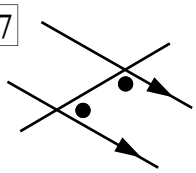
23 

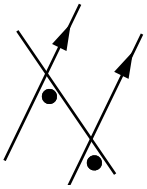
State if these angles are corresponding, alternate or co-interior.

24 
corresponding

25 
alternate

26 
alternate

27 
co-interior

28 
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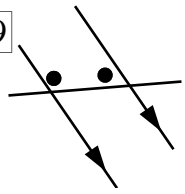
29 
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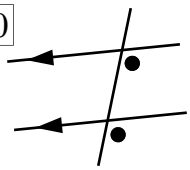
30 
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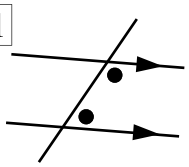
31 
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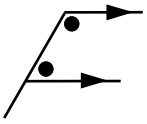
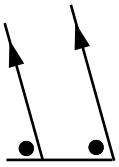
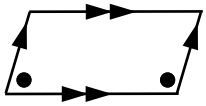
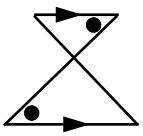
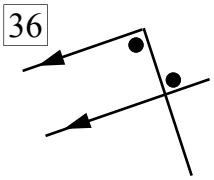
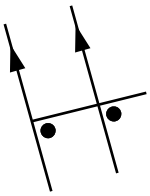
32 
co-interior

33 
corresponding

34 
co-interior

35 
alternate

36 
alternate

37 
corresponding

Parallel Line Angle Properties

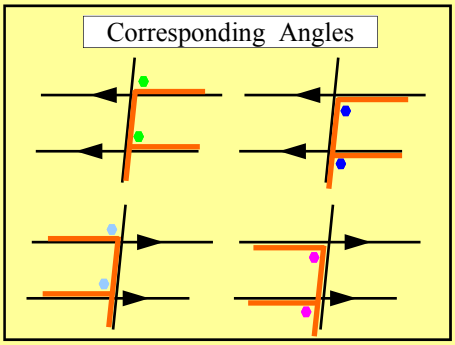
When two parallel lines are crossed by a transversal, pairs of angles are made that we can use mathematically. The previous sheet taught you alternate, co-interior and corresponding angles, now we can use their angle properties, which are:

- corresponding angles are equal if lines are parallel
- alternate angles are equal if lines are parallel
- co-interior angle sum is 180° if lines are parallel (the two angles add to 180°)

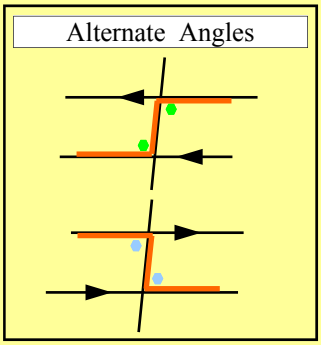
In Column 1 an angle is given and an unknown angle is required. The unknown angle will be either the alternate, corresponding or co-interior angle to the given angle. The first step is identifying which. Once identified if the angle is the alternate angle or the corresponding angle it is the same as the given angle, no working required. If the relationship is co-interior then subtract the given angle from 180° to obtain the answer.

Column 2 gives both angles, identify if the relationship between the 2 angles is either alternate or corresponding. If it is, the angles must be the same for the lines to be parallel, if the angles aren't equal then the lines aren't parallel. If the relationship between the 2 angles is co-interior then the sum of the two angles must equal 180° for the lines to be parallel. If they don't add to 180º then the lines aren't parallel. This includes if the angles are the same, the same co-interior angle means the lines aren't parallel, unless the angles are 90°. See the example at the top of the column.

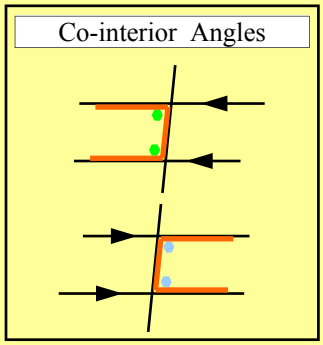
Column 3 is the same as column 1 in procedure, the only difference being that you have to divide the angle by the number in front of the letter to get your answer. The last 3 questions require a subtraction to take place before the division.



Corresponding angles are equal if the lines are parallel



Alternate angles are equal if the lines are parallel



Co-interior angles add to 180° if the lines are parallel

Parallel Lines Angle Properties

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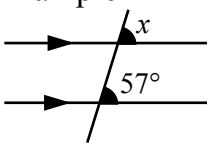
State if alternate, co-interior or corresponding. Then find the value of the letter.

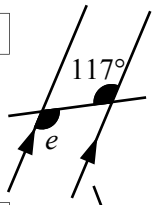
Example



corresponding

$x = 57°$

1



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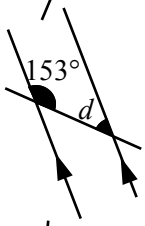

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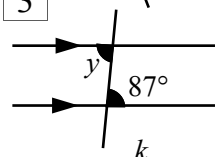

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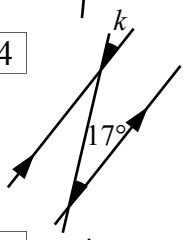

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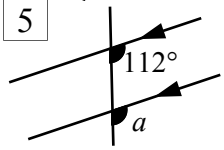

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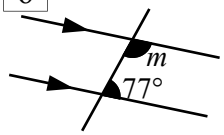

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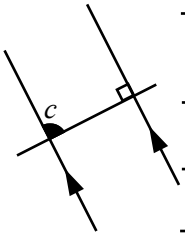

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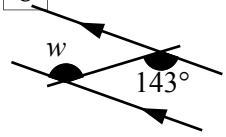

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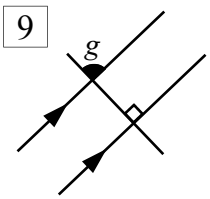
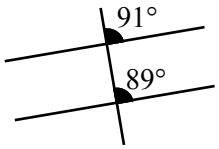


Find if the lines below are parallel, give a brief reason for your decision.

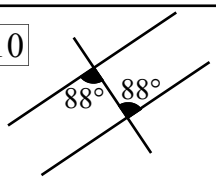
Example



Parallel?
Circle: Yes / No

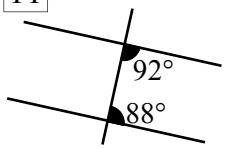
Corresponding angles not equal so lines aren't parallel.

10



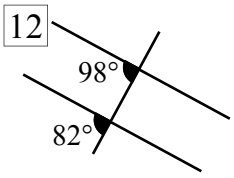
Parallel?
Circle: Yes / No

11



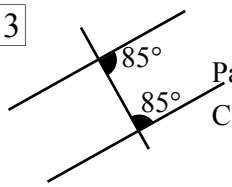
Parallel?
Circle: Yes / No

12



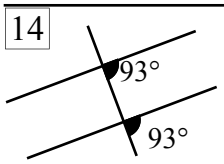
Parallel?
Circle: Yes / No

13



Parallel?
Circle: Yes / No

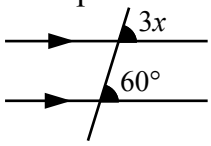
14



Parallel?
Circle: Yes / No

Find the value of x

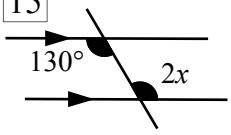
Example



$3x = 60°$

$x = 20°$

15



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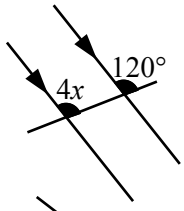

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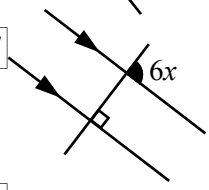

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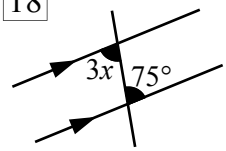

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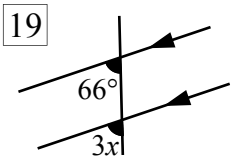
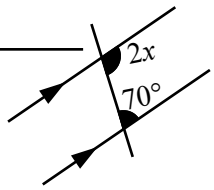


Co-interior angles require an extra step, find the value of x in these

20 $2x = 180° -$

$2x =$



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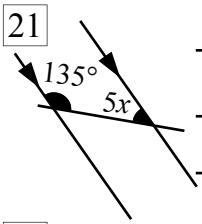

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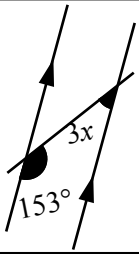

Parallel Lines Angle Properties

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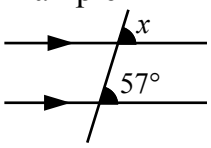
State if alternate, co-interior or corresponding. Then find the value of the letter.

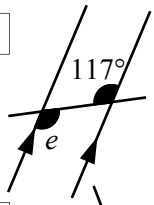
Example



corresponding

$x = 57°$

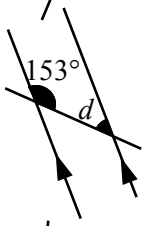
1



alternate

$e = 117°$

2

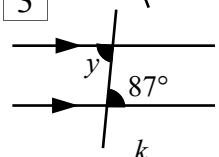


co-interior

$d = 180° - 153°$

$d = 27°$

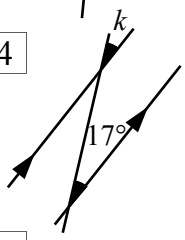
3



alternate

$y = 87°$

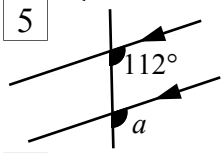
4



corresponding

$k = 17°$

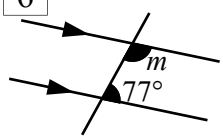
5



corresponding

$a = 112°$

6

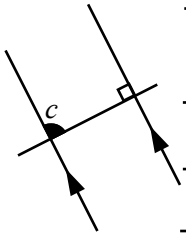


co-interior

$m = 180° - 77°$

$m = 103°$

7

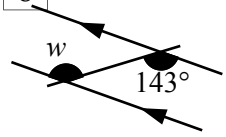


all 3 apply

$c = 180° - 90°$

$c = 90°$

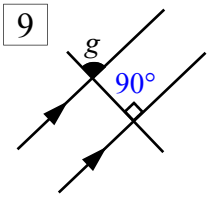
8



alternate

$w = 143°$

9

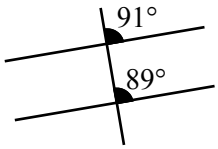


corresponding

$g = 90°$

Find if the lines below are parallel, give a brief reason for your decision.

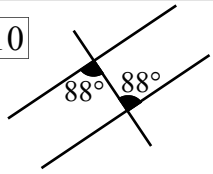
Example



Parallel?
Circle: Yes / No

Corresponding angles not equal so lines aren't parallel.

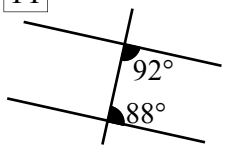
10



Parallel?
Circle: Yes / No

Alternate angles equal so lines are parallel.

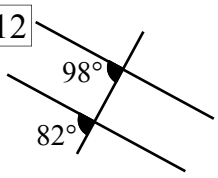
11



Parallel?
Circle: Yes / No

Co-interior angle sum is 180° so lines are parallel

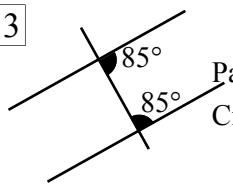
12



Parallel?
Circle: Yes / No

Corresponding angles not equal so lines aren't parallel.

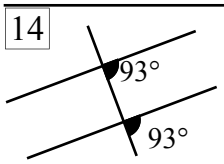
13



Parallel?
Circle: Yes / No

Co-interior angle sum isn't 180° so lines aren't parallel

14

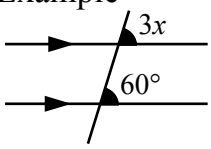


Parallel?
Circle: Yes / No

Corresponding angles equal so lines are parallel.

Find the value of x

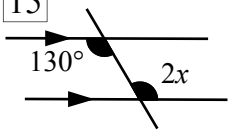
Example



$3x = 60°$

$x = 20°$

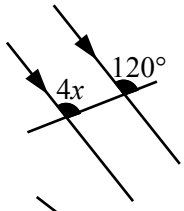
15



$2x = 130°$

$x = 65°$

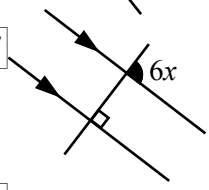
16



$4x = 120°$

$x = 30°$

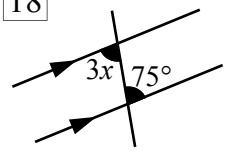
17



$6x = 90°$

$x = 15°$

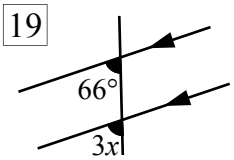
18



$3x = 75°$

$x = 25°$

19



$3x = 66°$

$x = 22°$

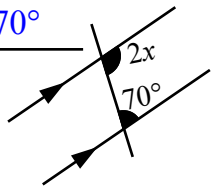
Co-interior angles require an extra step, find the value of x in these

20

$2x = 180° - 70°$

$2x = 110°$

$x = 55°$



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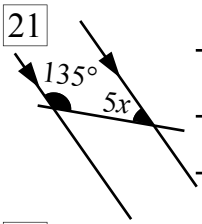


$5x = 180° - 135°$

$5x = 45°$

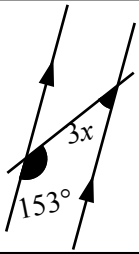
$x = 9°$

22

$3x = 180° - 153°$

$3x = 27°$

$x = 9°$



Further Parallel Lines

If two parallel lines are crossed by a transversal eight angles are formed. Given one angle, all of the other angles can be found by using these properties:

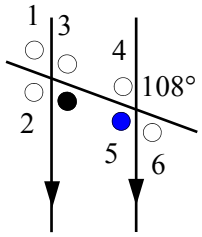
- supplementary, the angles adjacent (next to it) to the angle can be found by subtracting the given angle from 180°
- vertically opposite, the angle opposite the given angle is the same size as the angle
- then parallel line properties are used. Alternate, corresponding and co-interior can be used (it isn't necessary to use them all)

In column 1 a single angle is given, you are required to find the value of the black dotted angle in two solution 'moves'. The first step requires you to use supplementary or vertically opposite methods then for the second step use one of the parallel line properties.

The example below shows the angle given is 108°, vertically opposite was used to find 5 then co-interior to the black dotted angle. Note we could also use two other methods. Supplement to 4 then alternate to black dot or supplement to 6 and corresponding to black dot. Either way is correct. Note that the dots can be coloured, ensure that your colour choice is matched on the solution line.

Column 3 is an extension, this time the seven other angles are required. Use any method you like just make sure you give a reason, like the example below.

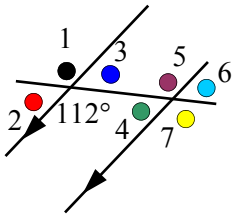
Example



● = 108° as vertically opposite

● = 72° as co-interior to ●

Example



● = 68° as supplement

● = 112° as vertically opposite

● = 68° as supplement

● = 68° as co-interior

● = 112° as alternate

● = 68° as supplement to ●

● = 112° as corresponding

Further Parallel Lines

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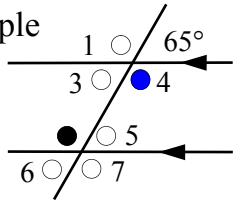
Find the vertically opposite or supplementary angle and use it to solve for the unknown angle, give a reason

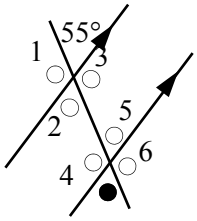
Example



● = 115° supplement to 65°

● = 115° alternate to •

1



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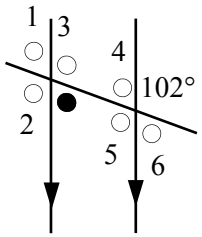

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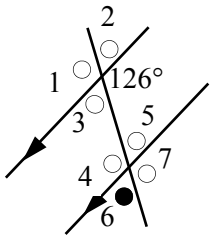

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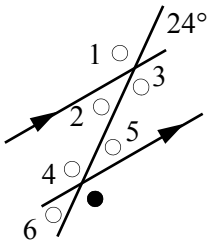

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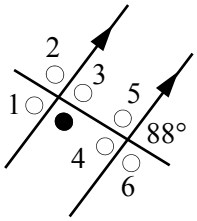

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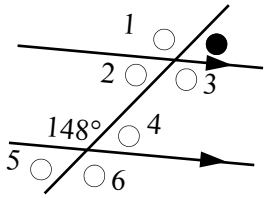

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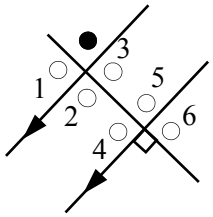

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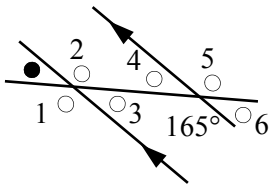

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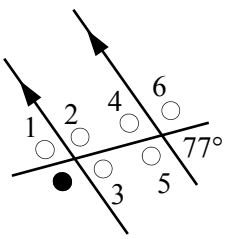
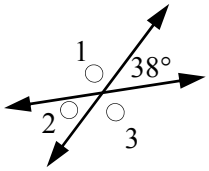


Now using the same skills find all the angles on the diagram, with reasons.

10



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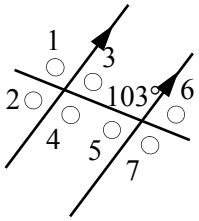

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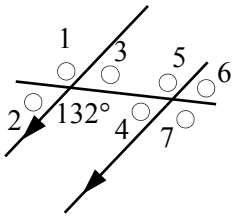

Further Parallel Lines

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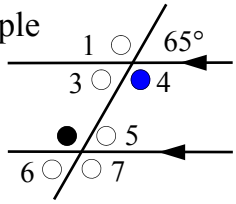
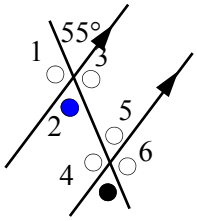
Find the vertically opposite or supplementary angle and use it to solve for the un-known angle, give a reason

Example



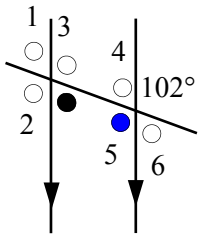
- (4) = 115° as supplement
- (4) = 115° as 4 alternate to •

1



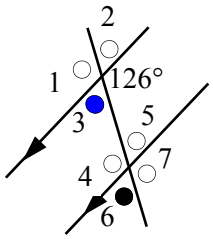
- (2)=55° as vertically opposite
- = 55° as 2 corresponds to •

2



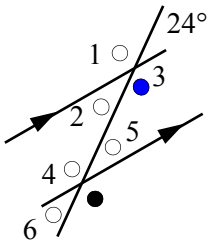
- (5)=102° as vertically opposite
- = 78° as 5 co-interior to •

3



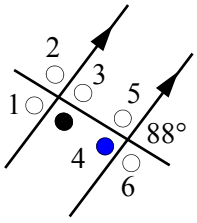
- (3) = 54° as supplement
- = 54° as corresponds to •

4



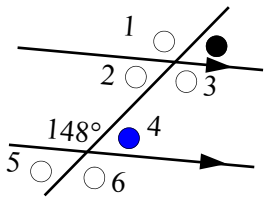
- (3) = 156° as supplement
- = 156° as corresponds to •

5



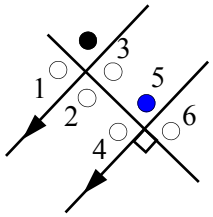
- (4)=92° as vertically opposite
- = 88° as co-interior to •

6



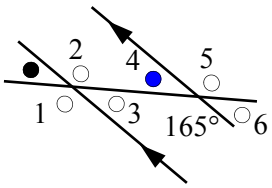
- (4) = 32° as supplement
- = 32° as corresponds to •

7



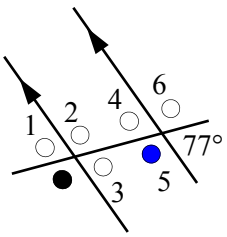
- (5)=90° as vertically opposite
- = 90° as corresponds to •

8



- (4) = 15° as supplement
- = 15° as corresponds to •

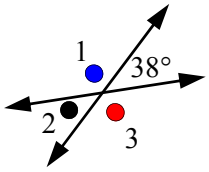
9



- (5) = 103° as supplement
- = 103° as corresponds to •

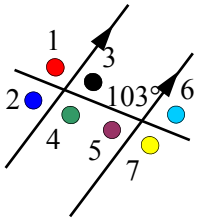
Now using the same skills find all the angles on the diagram, with reasons.

10



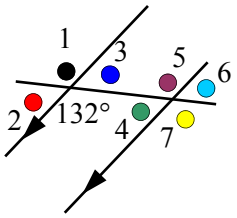
- (1)= 142° as supplement
- (2)=38° as vertically opposite
- (3)= 142° as supplement

11



- (3)= 77° as 5 co-interior
- (5)= 77° as supplement
- (4)= 103° as alternate
- (6)= 77° as supplement
- (7)= 103° as vertically opposite
- (1)= 103° as corresponding
- (2)= 77° as supplement to •

12



- (2)= 48° as supplement
- (1)= 132° as vertically opposite
- (3)= 48° as supplement
- (4)= 48° as co-interior
- (5)= 132° as alternate
- (6)= 48° as supplement to •
- (7)= 132° as corresponding

Measuring Angles in Triangles

This exercise is an introductory exercise to show that the angle sum of a triangle is 180°. Use a protractor to measure each angle in the triangles, note you should expect to experience some error in any measurement exercise, your total may be out by a degree.
Follow these steps:

- measure the angles with a protractor and then using the addition spaces in the bottom left corner add them and compare the total to 180°
- Repeat for all the triangles

Why are there errors? As the angles are not exactly a whole degree you may round them up or down, this will affect your total.

Measuring Angles in Triangles

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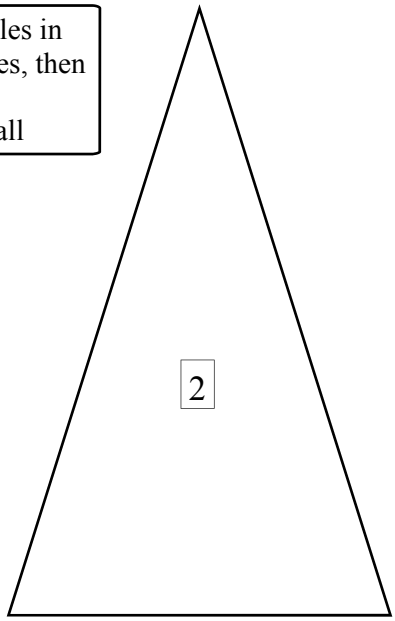
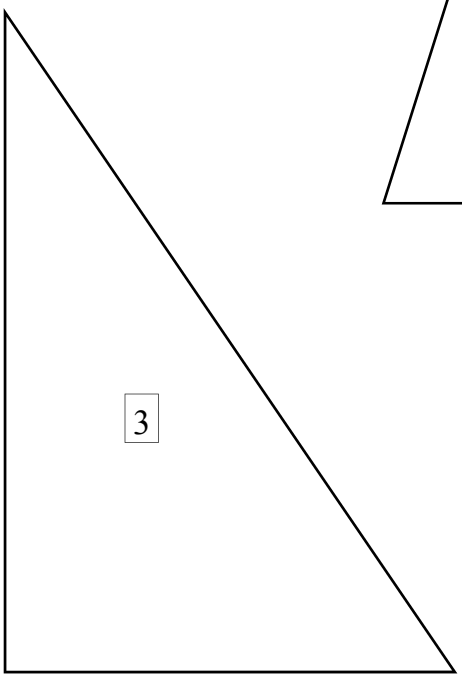
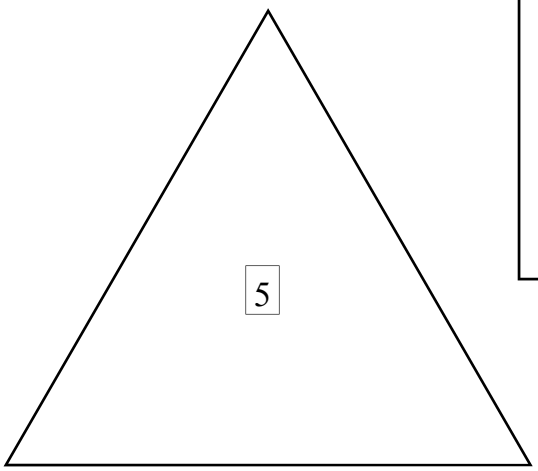
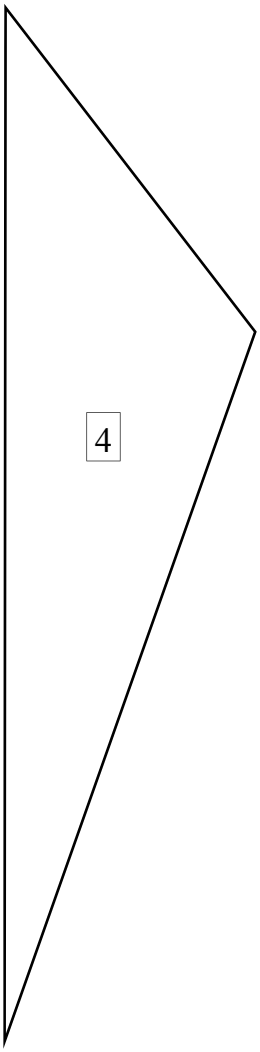
Measure the 3 angles in each of the triangles, then add them to get 180°,allow for small









1 + 2 + 3 +

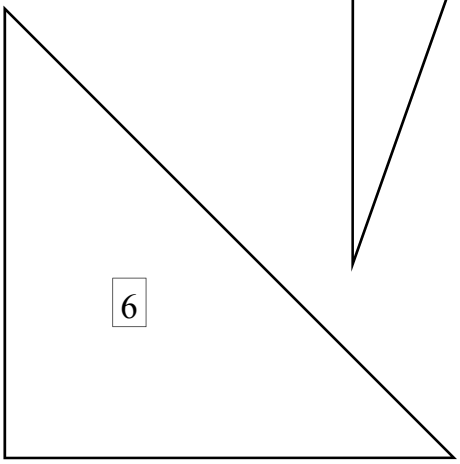
_____ _____ _____

_____ _____ _____

4 + 5 + 6 +

_____ _____ _____

_____ _____ _____



Measuring Angles in Triangles

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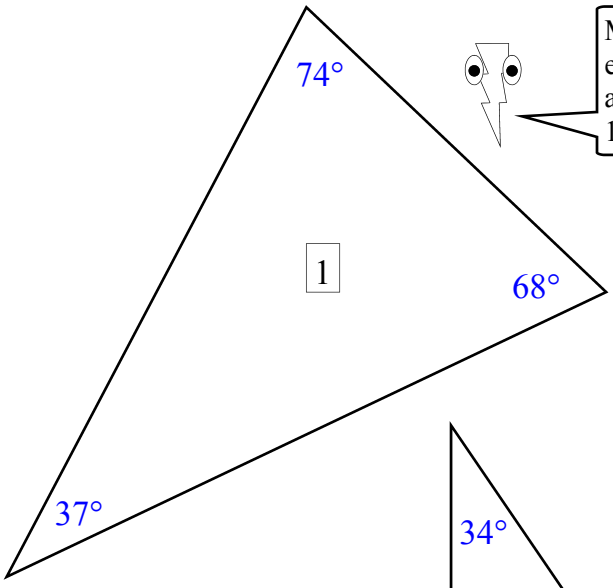
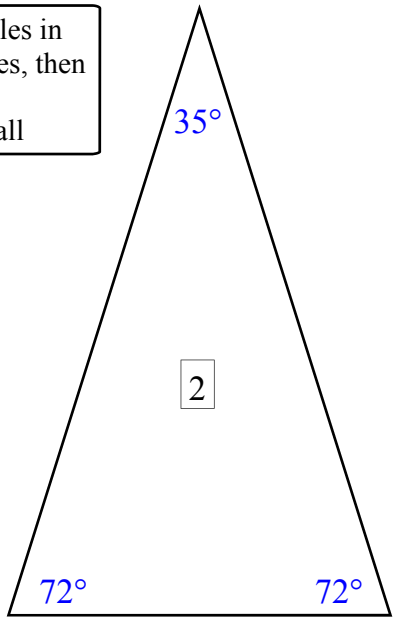
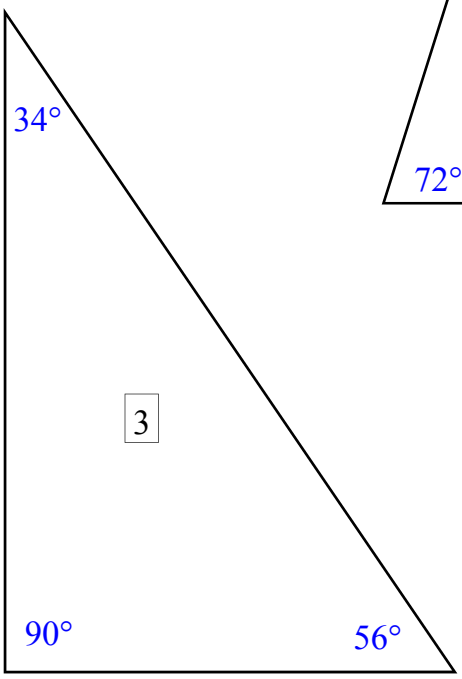
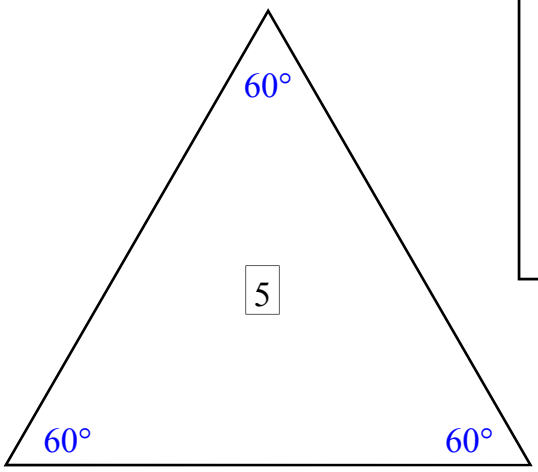
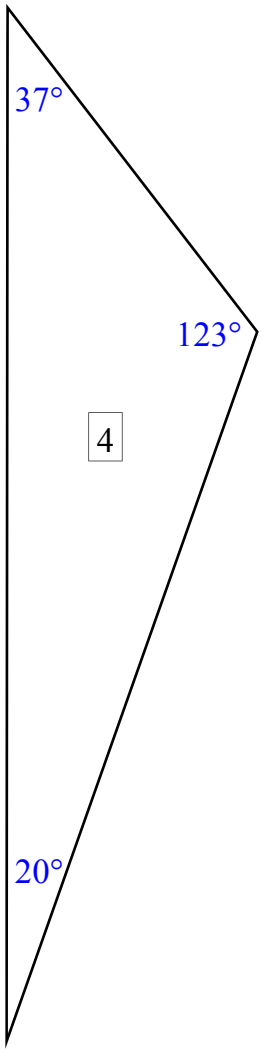


Measure the 3 angles in each of the triangles, then add them to get 180°,allow for small

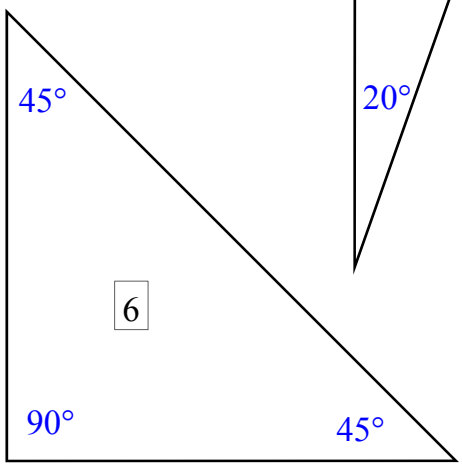








1	37	+	2	1 72	+	3	1 34	+
	68			72			56	
	74			35			90	
	179			179			180	
4	1 37	+	5	60	+	6	1 90	+
	123			60			45	
	20			60			45	
	180			180			180	



Angles in Triangles

The angle sum of a triangle is 180°. This property holds true regardless of the type of triangle. Types of triangles are:

- Equilateral - all sides equal length, all (internal) angles are 60°
- Isosceles - two sides equal length, two angles equal, note that the equal sides are opposite the equal angles
- Right angled triangle - has a right angle in the triangle with the other two angles being unequal
- Right Isosceles triangle - is a right angled triangle with the other two angles being equal (they must be 45°)
- Scalene triangle - all sides and all angles unequal, the words acute and obtuse can be used to further the description
- Acute triangle - all angles are less than 90°
- Obtuse triangle - one angle is greater than 90°, note that you can't have more than one obtuse angle in a triangle

Column 1 requires you to find the missing angle. Two angles are always given, subtract these from 180° to get the answer, (example at top of column). Once you have the 3 angles describe the type of triangle it is from the selection at the top of the sheet. Often the case is to use the word ‘obtuse’ in describing a triangle but not ‘acute’. So that if obtuse isn't used the triangle must be acute. This classification will only apply to scalene and isosceles triangles, right angled triangles and equilateral triangles can't contain an obtuse angle.

Column 3 tests your knowledge of isosceles and equilateral triangles. With equilateral triangles you know the angle is always 60°. So if the angle is $12w$ then that means $12w = 60°$ (÷ by 12) and $w = 5°$. If it was $15t$ then $15t = 60°$ (÷ by 15) and $t = 4°$. Note that in these questions you asked to find the value of the letter, this is different than finding the angle. For example if an angle is $20k$ in the corner of an equilateral triangle, $k = 3°$ but the actual angle is still 60°.

With isosceles triangles the process is the same except that instead of 60° the angle will match another angle on the diagram, question 18 is more challenging.

Angles in Triangles

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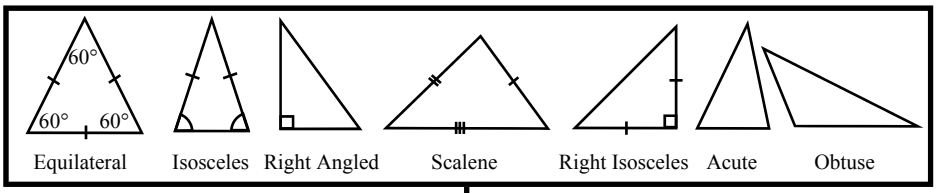
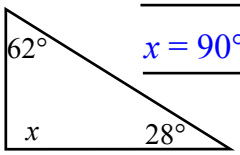


Find the missing angle, then classify (name the type of) the triangle.

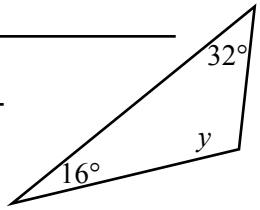
Example $x = 180° - 62° - 28°$

$x = 90°$



Type: Right angled triangle

1



Type:

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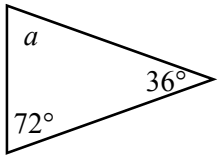
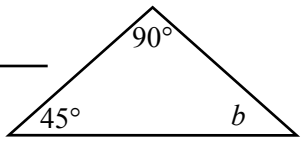


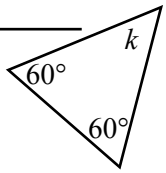
Type:

3



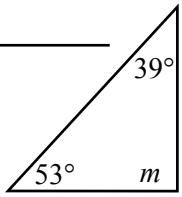
Type:

4



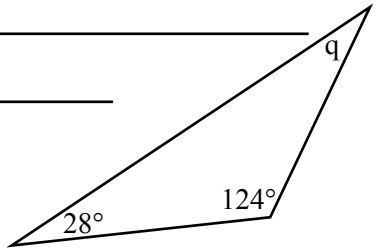
Type:

5



Type:

6



Type:

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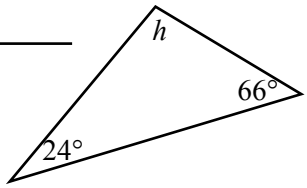


Type:

Use the same method to solve, identify the type of triangle formed. Use '$x =$'

8 A triangle has angles 23° and 46°, find the other angle

Type:

9 A triangle has angles 56° and 68°, find the other angle

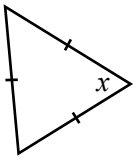
Type:

10 Two identical angles in a triangle total 120°, find the other angle

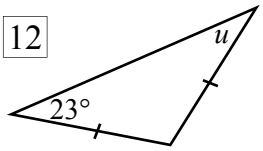
Type:

Use the side lengths to help you choose a method to find the value of the letters

11



12



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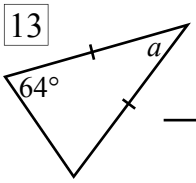

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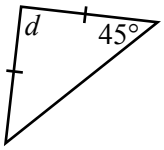

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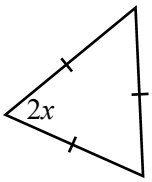
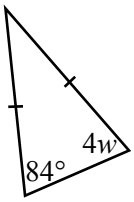


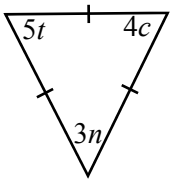
$2x =$

16

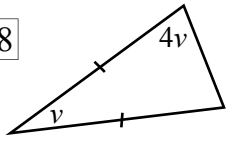


These are harder

17



18



Angles in Triangles

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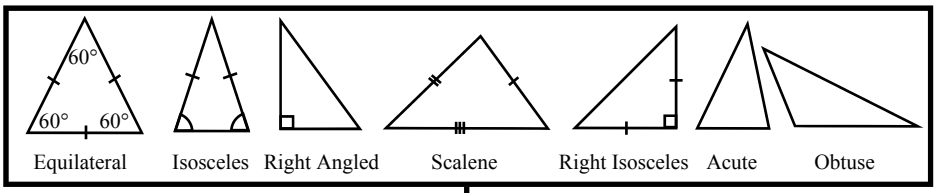
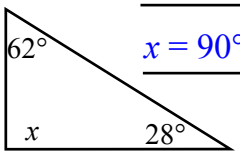


Find the missing angle, then classify (name the type of) the triangle.

Example $x = 180° - 62° - 28°$

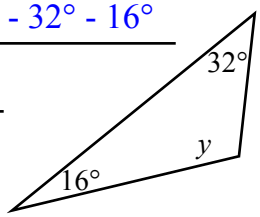
$x = 90°$



Type: Right angled triangle

1 $y = 180° - 32° - 16°$

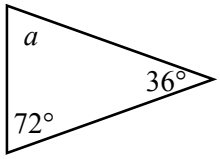
$y = 132°$



Type: Obtuse scalene triangle

2 $a = 180° - 72° - 36°$

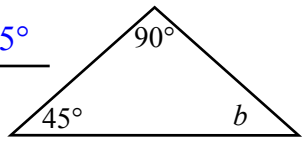
$a = 72°$



Type: Acute Isosceles Triangle

3 $b = 180° - 90° - 45°$

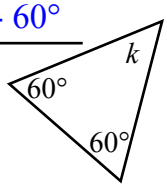
$b = 45°$



Type: Right Isosceles Triangle

4 $k = 180° - 60° - 60°$

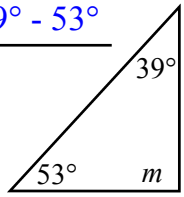
$k = 60°$



Type: Equilateral triangle

5 $m = 180° - 39° - 53°$

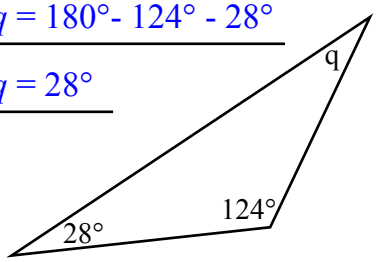
$m = 88°$



Type: Acute scalene triangle

6 $q = 180° - 124° - 28°$

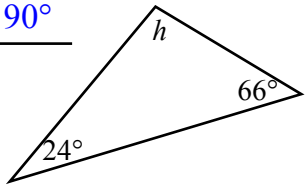
$q = 28°$



Type: Obtuse isosceles triangle

7 $h = 180° - 24° - 66°$

$h = 90°$



Type: Right angled triangle

Use the same method to solve, identify the type of triangle formed. Use '$x =$'

8 A triangle has angles 23° and 46°, find the other angle

$x = 180° - 23° - 46°$

$x = 111°$

Type: Obtuse scalene triangle

9 A triangle has angles 56° and 68°, find the other angle

$x = 180° - 56° - 68°$

$x = 56°$

Type: Acute isosceles triangle

10 Two identical angles in a triangle total 120°, find the other angle

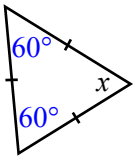
$x = 180° - 120°$

$x = 60°$

Type: Equilateral triangle

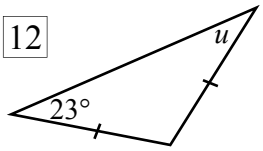
Use the side lengths to help you choose a method to find the value of the letters

11



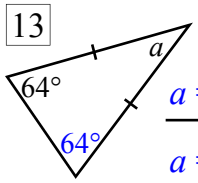
$x = 60°$

12



$u = 23°$

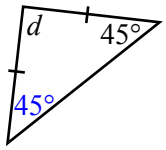
13



$a = 180° - 64° - 64°$

$a = 52°$

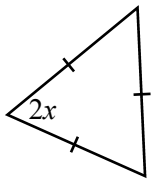
14



$d = 180° - 45° - 45°$

$d = 90°$

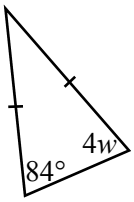
15



$2x = 60°$

$x = 30°$

16

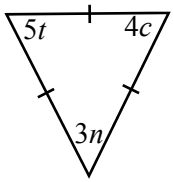


$4w = 84°$

$w = 21°$

These are harder

17



$4c = 60°$

$c = 15°$

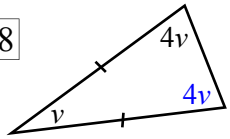
$5t = 60°$

$t = 12°$

$3n = 60°$

$n = 20°$

18



$9v = 180°$

$v = 20°$

Isosceles Triangles

Your knowledge of isosceles triangles and their properties will be tested throughout your mathematics studies, the 3 methods of finding missing angles are dealt with on this sheet. In column 1 your are given one angle and asked to find another. With a scalene triangle this is impossible because you require 2 angles to find the remaining angle, but because these are isosceles triangles you know that the angles opposite the sides with markings are equal.

Column 1 is done all the same way, place the number in the empty corner then you have your 2 angles. Then subtract the 2 angles from 180°. Look at question 1, the unlabelled angle is also 48°, so the working is s = 180° - 48° - 48° or s = 180° - 2 × 48°. The most common mistake when answering these questions is you will forget the unlabelled angle and subtract 48° from 180° instead of 48° doubled.

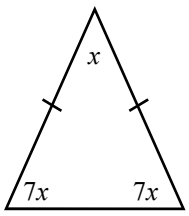
In column 2 the second type of problem is encountered, this time you are given the non-paired angle. This time 2 × (the letter) = 180° - (the given angle). An example is at the top of the column, the entire column is done using the same method. It is up to you if you write the letter in the unlabelled corner, but it is recommended.

Column 3 has written problems, 17 - 19 are the same as the previous columns, you just have to decide which method is used. The last 2 questions are harder, an example is below.

Example (questions 20 and 21)

A triangle has a pair of angles which are seven times the size of the other angle, find the size of all the angles. Let x = the smallest angle.

How this appears as a diagram:



The working required:

$7x + 7x + x = 15x$

$15x = 180°$

$\div 15$

$x = 12°$

$x = 12°$ and $7x = 84°$

Isosceles Triangles

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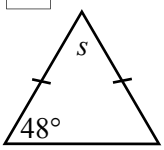
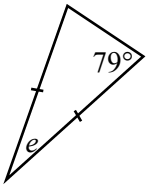
Find the value of the pronumeral

1

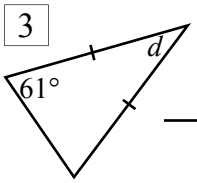


$s = 180° -$

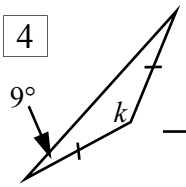
2



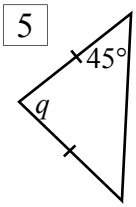
3



4



5



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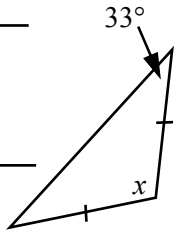

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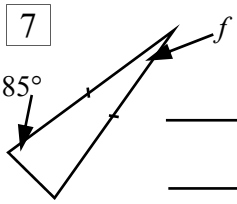

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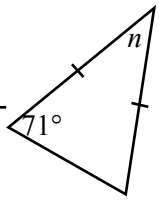
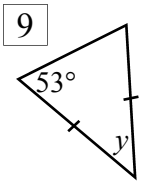


9



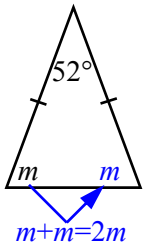
Find the value of the letter, these will take 3 lines to solve.

Example

$2m = 180° - 52°$

$2m = 128°$

$m = 64°$



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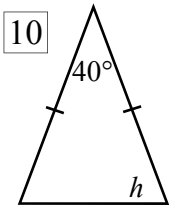

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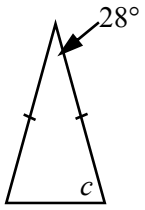

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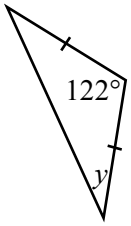

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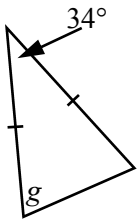

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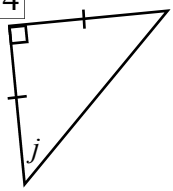

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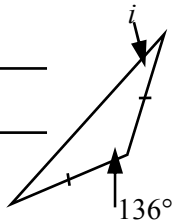

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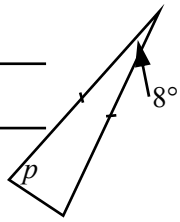

Solve these, you might not always need 3 lines. Use 'x=' in your working.

17 An isosceles triangle has one angle of 102°, find the size of the other angles

18 An isosceles triangle has a pair of angles of 17°, find the size of the other angle

19 An isosceles triangle has a pair of angles of 55°, find the size of the other angle

These are harder!

20 An isosceles triangle has a pair of angles that are twice the size of the other angle find the angles (Hint: use x and $2x$)

$x =$ $2x =$

21 Repeat the above question only this time the angles are 4 times the size of the other.

Isosceles Triangles

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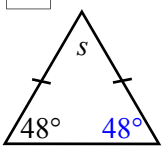
Find the value of the pronumeral

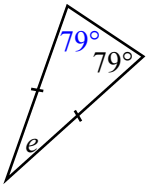
1



$s = 180° - 48° - 48°$

$s = 84°$

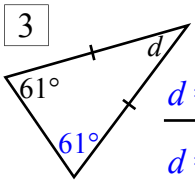
2



$e = 180° - 79° - 79°$

$e = 22°$

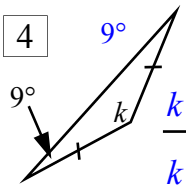
3



$d = 180° - 61° - 61°$

$d = 58°$

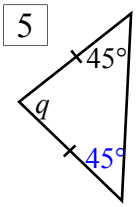
4



$k = 180° - 9° - 9°$

$k = 162°$

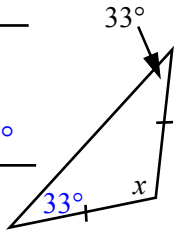
5



$q = 180° - 45° - 45°$

$q = 90°$

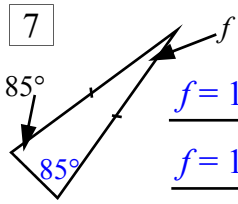
6



$x = 180° - 33° - 33°$

$x = 114°$

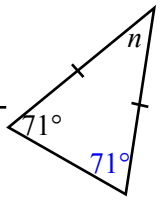
7



$f = 180° - 85° - 85°$

$f = 10°$

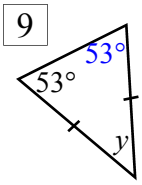
8



$n = 180° - 71° - 71°$

$n = 38°$

9

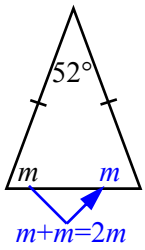


$y = 180° - 53° - 53°$

$y = 74°$

Find the value of the letter, these will take 3 lines to solve.

Example

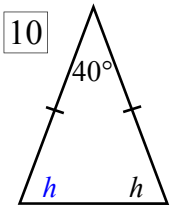


$2m = 180° - 52°$

$2m = 128°$

$m = 64°$

10

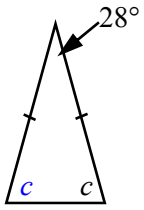


$2h = 180° - 40°$

$2h = 140°$

$h = 70°$

11

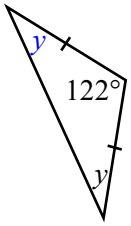


$2c = 180° - 28°$

$2c = 152°$

$c = 76°$

12

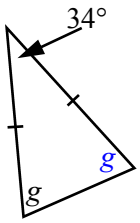


$2y = 180° - 122°$

$2y = 58°$

$y = 29°$

13

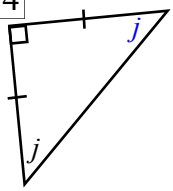


$2g = 180° - 34°$

$2g = 146°$

$g = 73°$

14

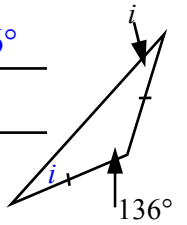


$2j = 180° - 90°$

$2j = 90°$

$j = 45°$

15

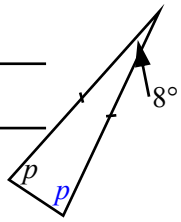


$2i = 180° - 136°$

$2i = 44°$

$i = 22°$

16



$2p = 180° - 8°$

$2p = 172°$

$p = 86°$

Solve these, you might not always need 3 lines. Use 'x=' in your working.

17 An isosceles triangle has one angle of 102°, find the size of the other angles

$2x = 180° - 102°$

$2x = 78°$

$x = 39°$

18 An isosceles triangle has a pair of angles of 17°, find the size of the other angle

$x = 180° - 17° - 17°$

$x = 146°$

19 An isosceles triangle has a pair of angles of 55°, find the size of the other angle

$x = 180° - 55° - 55°$

$x = 70°$

These are harder!

20 An isosceles triangle has a pair of angles that are twice the size of the other angle find the angles (Hint: use x and $2x$)

$5x = 180°$

$x = 36°$

$x = 36°$ $2x = 72°$

21 Repeat the above question only this time the angles are 4 times the size of the other.

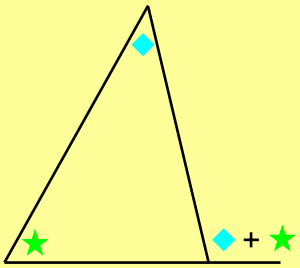
$9x = 180°$

$x = 20°$

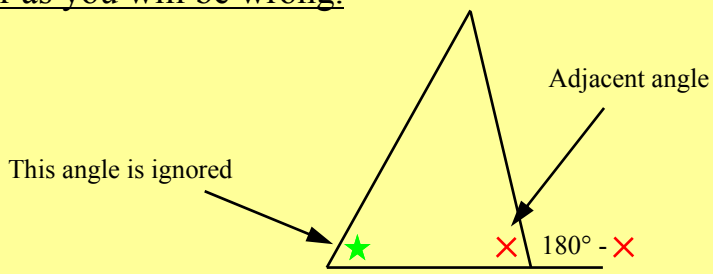
$x = 20°$ $4x = 80°$

Exterior Angles of a Triangle

There is a relationship between the exterior angle of a triangle and the 2 opposite interior angles, the sum of the 2 opposite interior angles = the exterior angle. In other words when you add the 2 opposite interior angles you get the exterior angle. The diagram below shows this graphically. It is important that you realise that the two opposite angles aren't adjacent to (don't touch) the exterior angle.



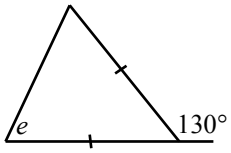
Column 1 Q.1 - 5 require you to add the two interior angles together to get the exterior angle. The adjacent angle is kept out of these questions. An example is at the top of the column. The next 3 questions show that if you have the adjacent internal angle you just take the supplement to get the external angle (subtract from 180°). So if the adjacent angle is one of the 2 angles, then just take the supplement (subtract it from 180°) of the adjacent angle. This is where the most common mistake is made, if the adjacent angle is given don't add it to the other angle to get your answer as you will be wrong.



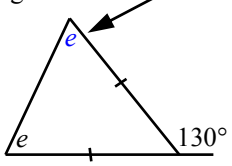
Column 2 works in reverse, you are given the exterior angle and have to calculate the missing interior angle. Do these by subtracting the given interior angle from the exterior angle, see the example at the top of the column.

The third column is challenging as it requires you to use your skills with isosceles and equilateral triangles as well as algebra. There is an example below.

Example : Find the angle



Because the triangle is isosceles the other base angle is also e



Solution:

$2e = 130°$ (opposite angle sum = exterior)

$e = 65°$

Exterior Angles of a Triangle

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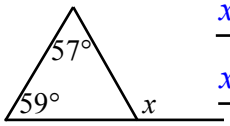
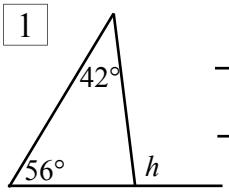
Find the exterior angle for the following triangles.

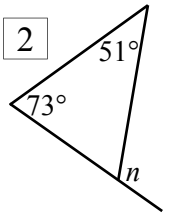
Example

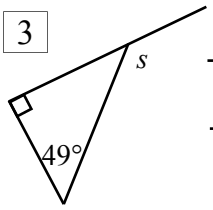


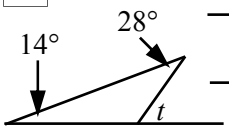
$x = 57° + 59°$

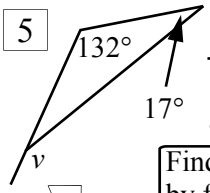
$x = 116°$

1 

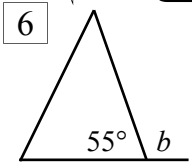
2 

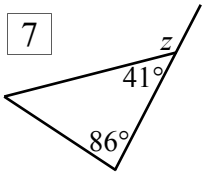
3 

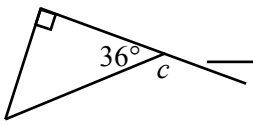
4 

5 

Find the exterior angle by finding the supplement, ignore any extra angles

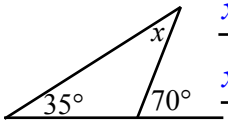
6 

7 

8 

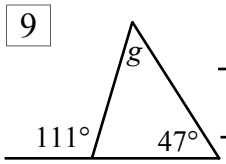
Now the exterior angle is given, use subtraction to find the unknown angle.

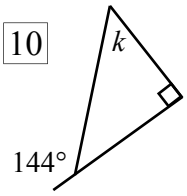
Example

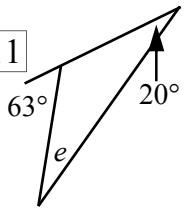


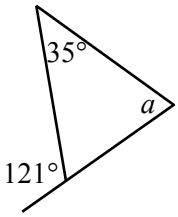
$x = 70° - 35°$

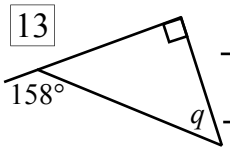
$x = 35°$

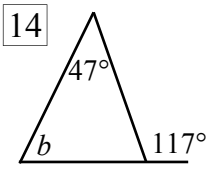
9 

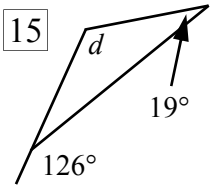
10 

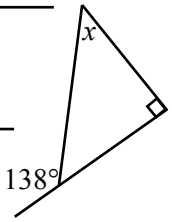
11 

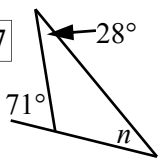
12 

13 

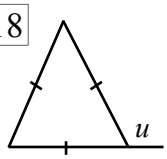
14 

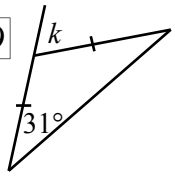
15 

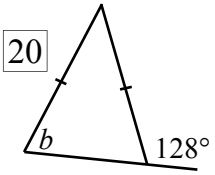
16 

17 

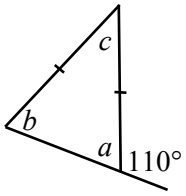
Use your knowledge of triangle properties to find the value of the angles

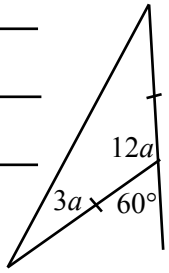
18 

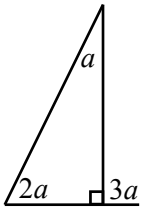
19 

20 

21 Hint: Find a then b then c



22 

23 

Exterior Angles of a Triangle

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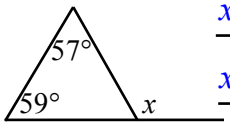
Find the exterior angle for the following triangles.

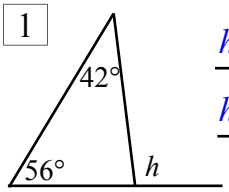
Example



$x = 57° + 59°$

$x = 116°$

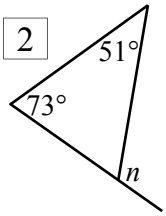
1



$h = 56° + 42°$

$h = 98°$

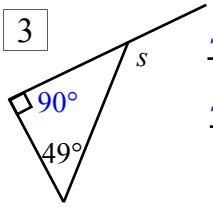
2



$n = 51° + 73°$

$n = 124°$

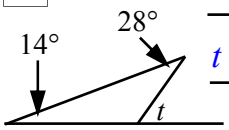
3



$s = 90° + 49°$

$s = 139°$

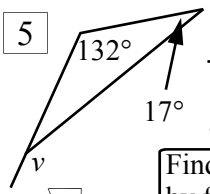
4



$t = 14° + 28°$

$t = 42°$

5

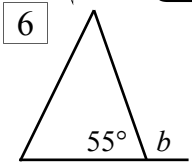


$v = 132° + 17°$

$v = 149°$

Find the exterior angle by finding the supplement, ignore any extra angles

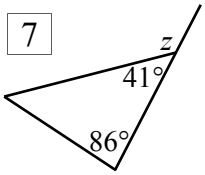
6



$b = 180° - 55°$

$b = 125°$

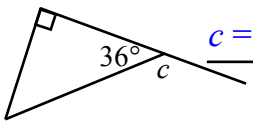
7



$z = 180° - 41°$

$z = 139°$

8

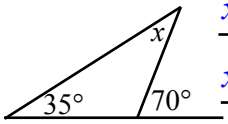


$c = 180° - 36°$

$c = 144°$

Now the exterior angle is given, use subtraction to find the unknown angle.

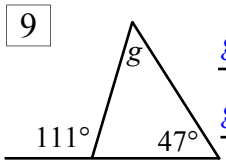
Example



$x = 70° - 35°$

$x = 35°$

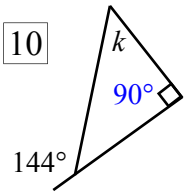
9



$g = 111° - 47°$

$g = 64°$

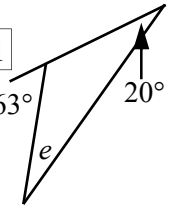
10



$k = 144° - 90°$

$k = 54°$

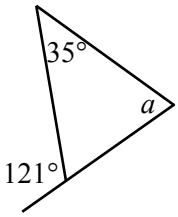
11



$e = 63° - 20°$

$e = 43°$

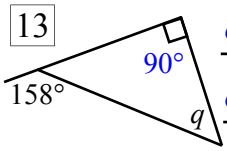
12



$a = 121° - 35°$

$a = 86°$

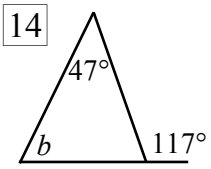
13



$q = 158° - 90°$

$q = 68°$

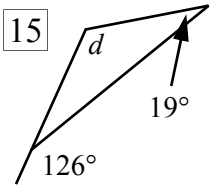
14



$b = 117° - 47°$

$b = 70°$

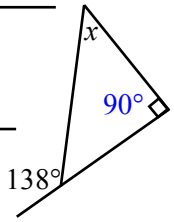
15



$d = 126° - 19°$

$d = 107°$

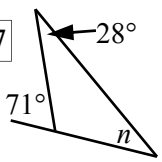
16



$x = 138° - 90°$

$x = 48°$

17

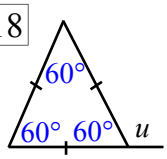


$n = 71° - 28°$

$n = 43°$

Use your knowledge of triangle properties to find the value of the angles

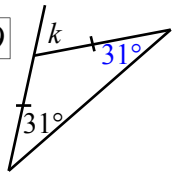
18



$u = 60° + 60°$

$u = 120°$

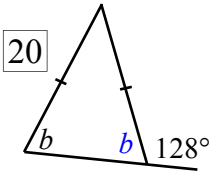
19



$k = 31° + 31°$

$k = 62°$

20



$b = 180° - 128°$

$b = 52°$

21 Hint: Find a then b then c

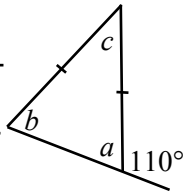
$a = 180° - 110°$

$a = 70°$

$b = 70°$ (isos)

$c = 110° - 70°$

$c = 40°$



22

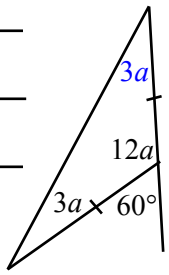
$3a + 3a = 6a$

$6a = 60°$

$a = 10°$

$3a = 30°$

$12a = 120°$



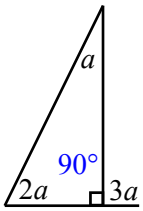
23

$3a = 180° - 90°$

$3a = 90°$

$a = 30°$

$2a = 60°$



Angles in Quadrilaterals

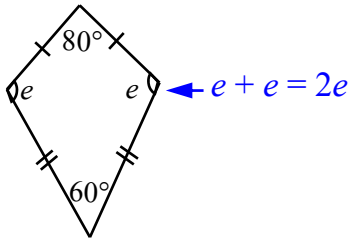
The angle sum of a quadrilateral is 360°, this means that when all the angles inside the shape are added, they total 360°. If you are given 3 angles then to find the missing angle subtract the 3 angles from 360° and what is left is the your answer.

In column 1 through to question 11 the exercises are all done the same way, you are given 3 angles (remember a right angle is 90°), subtract these from 360°. If you write in '90°' in the space next to the right angle sign you have less chance of forgetting it in your calculations. The question at the top of the 1st column shows the setting out for the exercises.

The rest of column 2 involves a small amount of algebra. The exercises are done in the same way as the first column in that the missing angles equal 360° minus the given angles. Because in the questions the 2 unknown angles are the same then you can add the letters together, i.e. $a + a = 2a$, $x + x = 2x$. An example is below. Question 20 is a similar style of question.

Column 3 is the same as the first column only one of the angles given is an exterior angle. To change the exterior angle to an interior angle subtract it from 360°. Then you have 3 interior angles which subtracted from 360° gives you the missing angle. The most common mistake is forgetting to change the exterior angle it to an interior angle in these questions.

Example: Find the value of e



Solution:

$2e = 360° - 80° - 60°$

$2e = 220°$

$e = 110°$

Angles in Quadrilaterals

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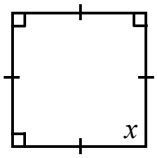
Find the missing angle in these quadrilaterals

1 $x = 360° - 90° - 90° - 90°$

$x =$



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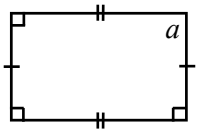

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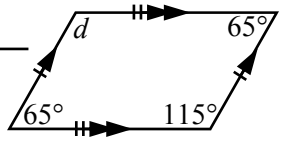

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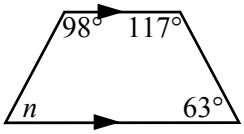

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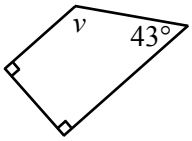

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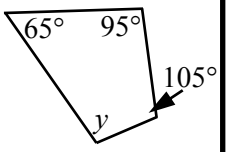

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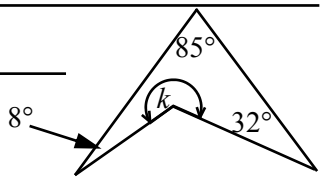

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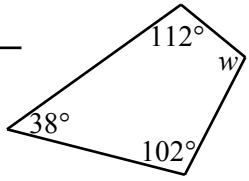

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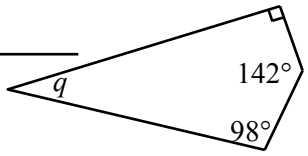

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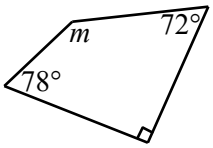
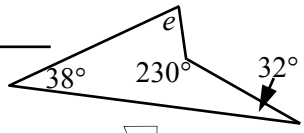


11

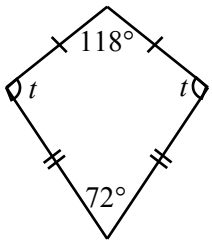


These have 2 unknown angles

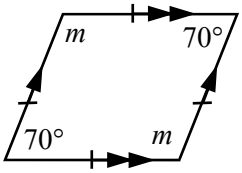
12 $2t = 360° -$

$2t =$

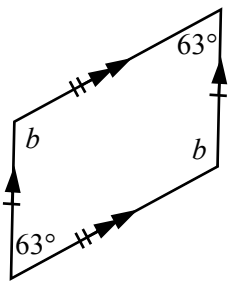
$t =$



13



14



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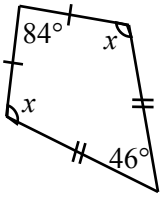
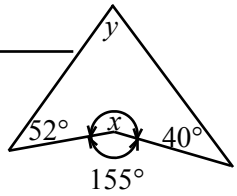


Convert the exterior angles to interior angles to find x then solve for y.

16 $x = 360° -$

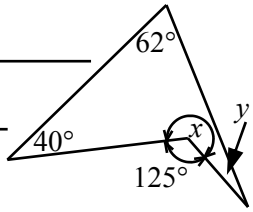
$x =$



$y =$

$y =$

17



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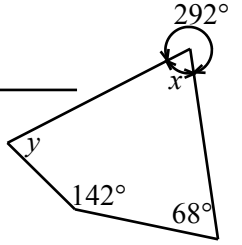

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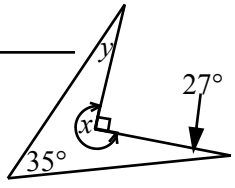
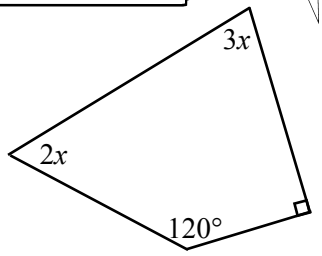


Find the 2 angles

20



$2x =$

$3x =$

Angles in Quadrilaterals

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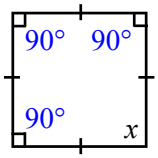
Find the missing angle in these quadrilaterals

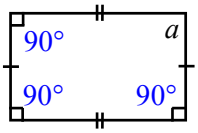
1 $x = 360° - 90° - 90° - 90°$

$x = 90°$



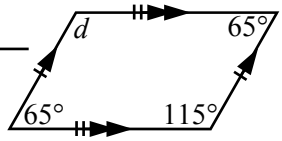
2 $a = 360° - 90° - 90° - 90°$

$a = 90°$



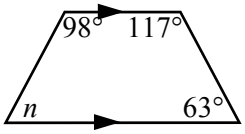
3 $d = 360° - 65° - 115° - 65°$

$d = 115°$



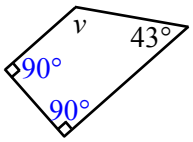
4 $n = 360° - 98° - 117° - 63°$

$n = 82°$



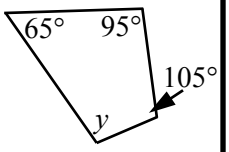
5 $v = 360° - 43° - 90° - 90°$

$v = 137°$



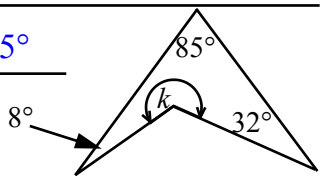
6 $y = 360° - 105° - 65° - 95°$

$y = 95°$



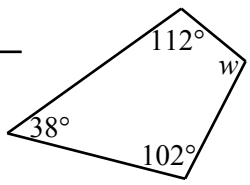
7 $k = 360° - 8° - 32° - 85°$

$k = 235°$



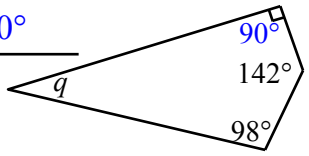
8 $w = 360° - 112° - 38° - 102°$

$w = 108°$



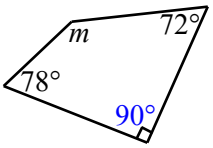
9 $q = 360° - 90° - 142° - 98°$

$q = 30°$



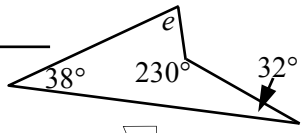
10 $m = 360° - 78° - 90° - 72°$

$m = 120°$



11 $e = 360° - 38° - 32° - 230°$

$e = 60°$

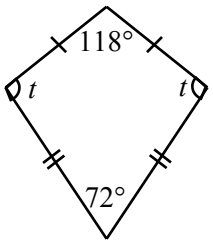


These have 2 unknown angles

12 $2t = 360° - 118° - 72°$

$2t = 170°$

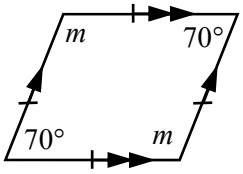
$t = 85°$



13 $2m = 360° - 70° - 70°$

$2m = 220°$

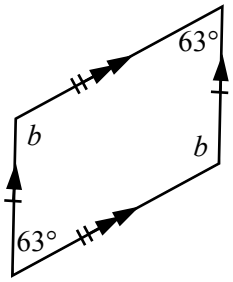
$m = 110°$



14 $2b = 360° - 63° - 63°$

$2b = 234°$

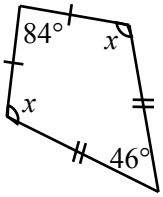
$b = 117°$



15 $2x = 360° - 84° - 46°$

$2x = 230°$

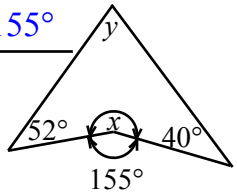
$x = 115°$



Convert the exterior angles to interior angles to find x then solve for y.

16 $x = 360° - 155°$

$x = 205°$

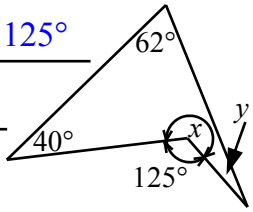


$y = 360° - 205° - 52° - 40°$

$y = 63°$

17 $x = 360° - 125°$

$x = 235°$

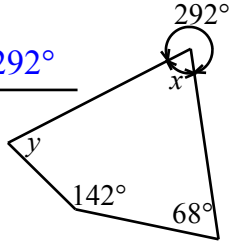


$y = 360° - 235° - 40° - 62°$

$y = 23°$

18 $x = 360° - 292°$

$x = 68°$

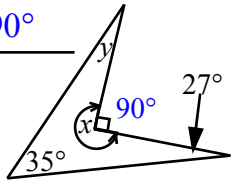


$y = 360° - 68° - 68° - 142°$

$y = 82°$

19 $x = 360° - 90°$

$x = 270°$

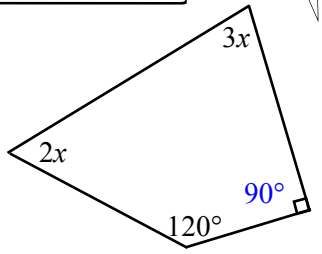


$y = 360° - 270° - 27° - 35°$

$y = 28°$

Find the 2 angles

20



$5x = 360° - 120° - 90°$

$5x = 150°$ $2x = 60°$

$x = 30°$ $3x = 90°$

FREEFALL MATHEMATICS

ALTITUDE

INTEGERS

Directed Numbers

Directed numbers are numbers that have direction assigned, a number will have a positive value (+) in one direction and a negative value in the opposite direction. These are related to the number plane, where up and to the right are positive and down and to the left are negative. So up, North, East, a gaining of something or time after something has occurred are all positive. These can be denoted by a positive (+) sign, but this isn't necessary, just the number alone is sufficient. Down, South, West, a loss of something or time before something is to occur are all negative. These are denoted by a negative (minus) sign (-).

Column 1 uses words that imply either a positive or negative answer. Use the above as a guide if you need to and then write in the answer, note that you don't write the units, for example: $450 profit → 450, $50 loss → -50. So no km, cm, $, min or any other unit is required. A number with a minus sign when necessary is it.

Column 2 asks you to match opposites, the match is between those on the left hand side with those on the right hand side. Draw a line to connect the dots. The second part of the column asks you to make short statements using the number and topic supplied. Show units with these. Use the sign with the number to help you decide on which word to use. (Up, down, before, after etc.)

Column 3 introduces addition and subtraction. The first section asks you to imagine a change in temperature, you are given the starting temperature and then are told the size that the temperature rises or falls. You can use the thermometer diagram beside the questions as a guide. The second section gives you a 'before and after' comparison, you are asked to calculate the amount of the drop or rise in temperature that has occurred. If there is a drop use a negative sign to show that there was a fall in temperature.

Directed Numbers

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Use a directed number to represent the given action

Example

A gain of 300 points 300

1 Up 60 m

2 Loss of $95

3 Increase of 13%

4 Deposit of $20

5 South 400 km

6 Left 15°

7 Dismantling 61 cars

8 Bank fees of $3.80

9 Down 4 storeys

10 Fall of 23°C

11 Profit of $712

12 East 650 m

13 Lost 6 nuts

14 12 s before lift-off

15 Withdrawal of $193

16 12 knot wind assist

17 West 5 paces

18 Penalise 13 strokes

19 Found 14 bolts

20 Constructing 4 buses

21 Account interest $16

22 4 min after ignition

23 Right 85 cm

24 North 33 km

25 Rise of 56%

Match the words used in column 1 with its opposite, with a line

26

Lost •	• Penalise
North •	• Increase
Fall •	• Withdrawal
Decrease •	• Found
Bank fees •	• Profit
Up •	• South
Deposit •	• Down
Left •	• Rise
Assist •	• Interest
Loss •	• Right

Construct sentences that describe the directed numbers, given a subject for the sentence

Example -13: Time

The space shuttle will commence lift off in 13 seconds

27 -4 000: Distance

28 -220: Money

29 40: Percentage

30 -6: Temperature

Find the new temperature when the given change occurs



31 From 6°C drops 8°C

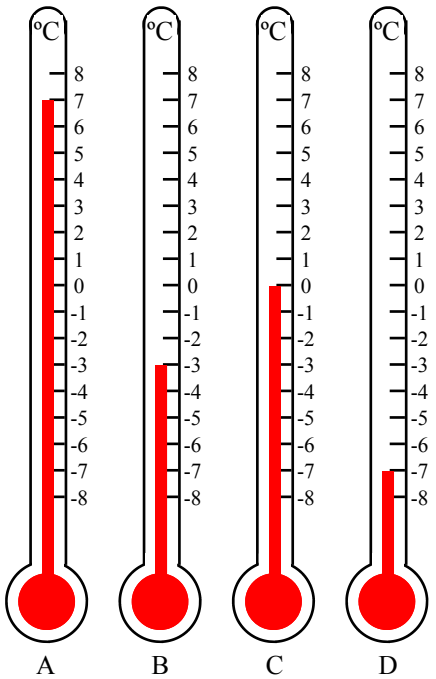
32 From 4°C drops 11°C

33 From -3°C rises 7°C

34 From -5°C rises 3°C

35 From -7°C rises 7°C

Give the temperature difference between the two thermometers, if there is a temperature drop, use '-'.



36 From A to C

37 From B to C

38 From C to D

39 From D to B

40 From A to B

41 From D to A

42 From B to A

Directed Numbers

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Use a directed number to represent the given action

Example

A gain of 300 points — 300

1 Up 60 m — 60

2 Loss of $95 — -95

3 Increase of 13% — 13

4 Deposit of $20 — 20

5 South 400 km — -400

6 Left 15º — -15

7 Dismantling 61 cars — -61

8 Bank fees of $3.80 — -3.80

9 Down 4 storeys — -4

10 Fall of 23ºC — -23

11 Profit of $712 — 712

12 East 650 m — 650

13 Lost 6 nuts — -6

14 12 s before lift-off — -12

15 Withdrawal of $193 — -193

16 12 knot wind assist — 12

17 West 5 paces — -5

18 Penalise 13 strokes — -13

19 Found 14 bolts — 14

20 Constructing 4 buses — 4

21 Account interest $16 — 16

22 4 min after ignition — 4

23 Right 85 cm — 85

24 North 33 km — 33

25 Rise of 56% — 56

Match the words used in column 1 with its opposite, with a line

26

Lost	Penalise
North	Increase
Fall	Withdrawal
Decrease	Found
Bank fees	Profit
Up	South
Deposit	Down
Left	Rise
Assist	Interest
Loss	Right

Construct sentences that describe the directed numbers, given a subject for the sentence

Example -13: Time

The space shuttle will commence lift off in 13 seconds

27 -4 000: Distance

The ship is 4 000 km west of its destination port

28 -220: Money

A withdrawal of $220 was made from a bank account

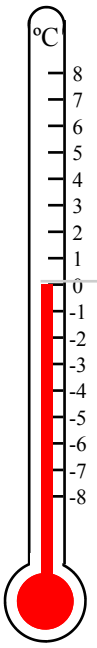
29 40: Percentage

There was a 40% increase in sales from last year

30 -6: Temperature

There was a drop in temperature of 6ºC overnight

Find the new temperature when the given change occurs



31 From 6ºC drops 8ºC — -2º C

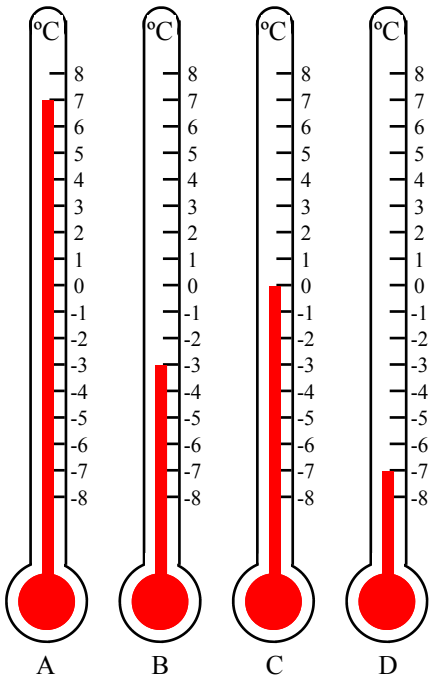
32 From 4ºC drops 11ºC — -7º C

33 From -3ºC rises 7ºC — 4º C

34 From -5ºC rises 3ºC — -2º C

35 From -7ºC rises 7ºC — 0º C

Give the temperature difference between the two thermometers, if there is a temperature drop, use '-'.



36 From A to C — -7º C

37 From B to C — 3º C

38 From C to D — -7º C

39 From D to B — 4º C

40 From A to B — -10º C

41 From D to A — 14º C

42 From B to A — 10º C

Number Line and Magnitude

This sheet deals with the size and position of negative numbers. Positive numbers are all greater than zero and increase with the size of the numbers used. Negative numbers are the opposite, they are all less than 0 and as the number part (ignoring the negative sign) increases the number decreases in size. For example, 200 > 30 but -200 < -30.

Column 1 deals with plotting points on the number line and then reading a number line. You are first asked to plot numbers, this is done by selecting the unfilled circle above the number required and filling it in. You are asked to plot odd and even numbers, an even positive number will also be even if negative. The same applies for odd numbers. The case of zero (0) is that it is neither even or odd. Questions 5 through 8 asks you to write the plotted numbers in ascending order, remember descending is down (highest to lowest), ascending is the opposite. With a number line the lowest values are to the left.

Column 2 asks you to use < and > signs to make the mathematical statement true. Remember that you point the 'arrow' at the smallest number. Then arrange the numbers in descending order in the next section of the column.

Column 3 asks you to circle the largest number and put a square around the smallest number. The last section of the column deals again with inequalities (the use of < and >). Instead of drawing the sign, a sign has been included, if the sign is correct (if it 'points' to the smallest number) then write 'true', if it doesn't, write 'false'.

Number Line and Magnitude

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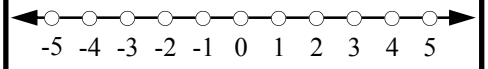
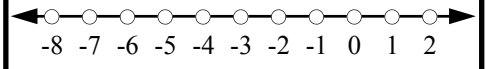
Plot the numbers on the number lines provided (by filling the circles)

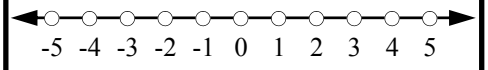
1 -3, -1, 0, 1, 3



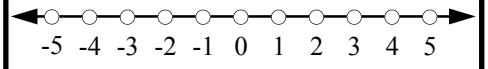
2 All numbers between 0 and -5



3 All even numbers between -4 and 4.

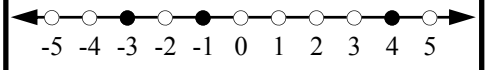


4 All odd numbers between -4 and 4.



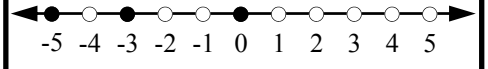
Write the numbers marked on the number lines in ascending order.

5



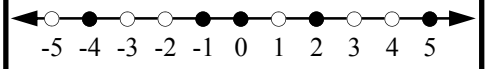
→ _____

6



→ _____

7



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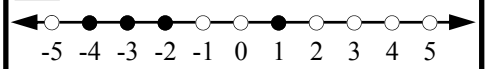
8



→ _____

Fill the box with < or >, to make these true

9	-3	☐	4	10	10	☐	5
11	16	☐	-22	12	-25	☐	-18
13	-3	☐	0	14	12	☐	-4
15	-7	☐	-23	16	15	☐	-40
17	-60	☐	-3	18	-10	☐	0
19	8	☐	6	20	3.5	☐	-7
21	-12	☐	-9	22	8	☐	-45
23	-56	☐	-9	24	0	☐	-3

Arrange these in descending order

25 6, -17, 0, 20, -100

____, ____, ____, ____, ____

26 40, -5, 14, -35, 2

____, ____, ____, ____, ____

27 -7, 0, 127, 13, -6.5

____, ____, ____, ____, ____

Arrange these in ascending order

28 -5.9, -9, 34, -30, 242

____, ____, ____, ____, ____

29 23, 107, -14, 56, -2

____, ____, ____, ____, ____

30 -20, -85, -37, 87, 4

____, ____, ____, ____, ____

Circle the largest number, a square for the smallest.

31	10	-2	15	-20
32	-3	-6	0	3
33	-4	-2	-10	0
34	-3	-6	2	-1
35	0	-1	-5	3
36	-3	-7	-10	-5
37	-34	0	-2	-5
38	-10	-5	30	-20
39	3	0	9	-10
40	-200	-50	5	199
41	3	-6	6	-3
42	-21	22	0	20

Write 'true' or 'false' for the following

43	5	<	10	_____
44	-3	>	-5	_____
45	-7	<	-6	_____
46	-5	>	0	_____
47	8	>	-10	_____
48	6	<	-16	_____
49	-1	>	-6	_____
50	2	<	-1	_____
51	0	<	-3	_____
52	3	>	0	_____
53	0	>	3	_____

Number Line and Magnitude

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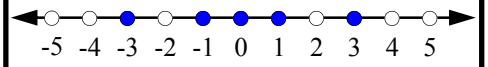
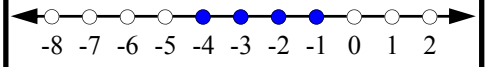
Plot the numbers on the number lines provided (by filling the circles)

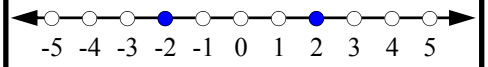
1. -3, -1, 0, 1, 3



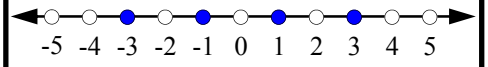
2. All numbers between 0 and -5



3. All even numbers between -4 and 4.

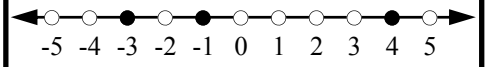


4. All odd numbers between -4 and 4.



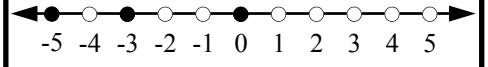
Write the numbers marked on the number lines in ascending order.

5



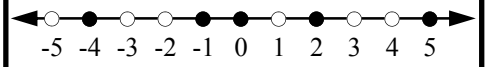
→ -3, -1, 4

6



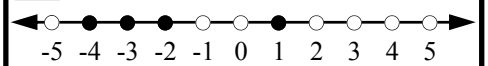
→ -5, -3, 0

7



→ -4, -1, 0, 2, 5

8



→ -4, -3, -2, 1

Fill the box with < or >, to make these true

9	-3	<	4	10	10	>	5
11	16	>	-22	12	-25	<	-18
13	-3	<	0	14	12	>	-4
15	-7	>	-23	16	15	>	-40
17	-60	<	-3	18	-10	<	0
19	8	>	6	20	3.5	>	-7
21	-12	<	-9	22	8	>	-45
23	-56	<	-9	24	0	>	-3

Arrange these in descending order

25 6, -17, 0, 20, -100

20, 6, 0, -17, -100

26 40, -5, 14, -35, 2

40, 14, 2, -5, -35

27 -7, 0, 127, 13, -6.5

127, 13, 0, -6.5, -7

Arrange these in ascending order

28 -5.9, -9, 34, -30, 242

-30, -9, -5.9, 34, 242

29 23, 107, -14, 56, -2

-14, -2, 23, 56, 107

30 -20, -85, -37, 87, 4

-85, -37, -20, 4, 87

Circle the largest number, a square for the smallest.

31	10	-2	15	-20
32	-3	-6	0	3
33	-4	-2	-10	0
34	-3	-6	2	-1
35	0	-1	-5	3
36	-3	-7	-10	-5
37	-34	0	-2	-5
38	-10	-5	30	-20
39	3	0	9	-10
40	-200	-50	5	199
41	3	-6	6	-3
42	-21	22	0	20

Write 'true' or 'false' for the following

43	5	<	10	True
44	-3	>	-5	True
45	-7	<	-6	True
46	-5	>	0	False
47	8	>	-10	True
48	6	<	-16	False
49	-1	>	-6	True
50	2	<	-1	False
51	0	<	-3	False
52	3	>	0	True
53	0	>	3	False

Addition of Integers

Unlike positive numbers (1, 2, 3 …) which when added always give a larger number, the addition of negative numbers (-1, -2, -3….) results in smaller numbers. Negative numbers can challenge students so don't feel frustrated if you don't take to them straight away, it may mean counting to yourself out loud or using your fingers to assist you.

In Column 1 both positive and negative numbers are involved in additions with positive numbers. Questions 1 to 9 and questions 10 to 18 are in 2 groups. These questions give answers that form a number pattern, the pattern can be used to check your answer as you move through the question groups. Note also there is a number line at the top of the page to assist you. Remember that when you subtract you move to the left and addition moves to the right.

Questions 21 to 29 mostly won't fit on the number line so you have to understand the concept of negative numbers for these questions. All the questions start with a negative number and have a positive number added to them. Answer these questions by solving them and then circling the answer.

Column 2 involves the addition of both positive and negative numbers with negative numbers. Don’t get confused by the two signs (+ and -) next to each other. When a plus and minus sign are together the result is a minus sign. So + and - = - (or - and + = -). It is even more easy if you remember unlike signs together = minus. As this rule will be built upon in the next sheets. So really, while we are adding the numbers, the result is a subtraction, I hear you ask 'then why show the + sign?', the answer is that when you substitute negative numbers into positive equations the two signs will occur. This column has the same layout, two groups of questions which have a pattern. Use the pattern or the number line to assist you.

Column 3 involves 3 numbers, that means that you answer the questions in 2 stages. Look at the first 2 numbers, complete the sum, strike them out and write the total above them. There is an example at the top of the column.

Addition of Integers

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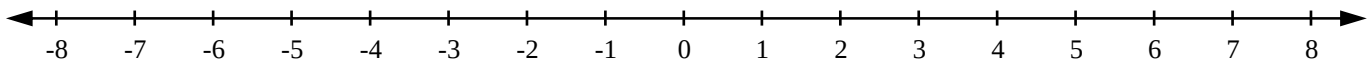

Look at the pattern formed by your answers and the number line to assist you

1 3 + 2 = ____	2 2 + 2 = ____	3 1 + 2 = ____
4 0 + 2 = ____	5 -1 + 2 = ____	6 -2 + 2 = ____
7 -3 + 2 = ____	8 -4 + 2 = ____	9 -5 + 2 = ____
10 -5 + 0 = ____	11 -5 + 1 = ____	12 -5 + 2 = ____
13 -5 + 3 = ____	14 -5 + 4 = ____	15 -5 + 5 = ____
16 -5 + 6 = ____	17 -5 + 7 = ____	18 -5 + 8 = ____

Circle the correct answer

19	-5 + 4 =	-9	9	-1
20	-4 + 3 =	-1	7	-7
21	-10 + 8 =	2	-2	-18
22	-15 + 3 =	-12	12	-18
23	-30 + 25 =	-55	-5	5
24	-14 + 14 =	0	28	-28
25	-5 + 10 =	-15	15	5
26	-7 + 3 =	-4	4	10
27	-20 + 60 =	-40	-80	40
28	-15 + 27 =	-42	-12	12
29	-11 + 4 =	-7	-15	7

When you add a negative number it is like subtracting a positive number

30 5 + 1 = ____	31 5 + 0 = ____	32 5 + -1 = ____
33 5 + -2 = ____	34 5 + -3 = ____	35 5 + -4 = ____
36 5 + -5 = ____	37 5 + -6 = ____	38 5 + -7 = ____
39 4 + -7 = ____	40 3 + -7 = ____	41 2 + -7 = ____
42 1 + -7 = ____	43 0 + -7 = ____	44 -1 + -7 = ____
45 -2 + -7 = ____		46 -3 + -7 = ____

Circle the correct answer

47	-6 + -6 =	0	12	-12
48	3 + -5 =	-8	-2	8
49	-5 + -7 =	2	-12	-2
50	8 + -5 =	3	-3	13
51	10 + -12 =	-2	-22	2
52	-4 + -10 =	6	-6	-14
53	-15 + -8 =	-23	-7	7
54	14 + -12 =	-2	2	26
55	20 + -50 =	30	-70	-30
56	-5 + -25=	20	-20	-30
57	12 + -12 =	0	-24	24

Now find the answer for these

Example

$\cancel{4} + -\cancel{10}^{-6} + 6 =$ 0

58 10 + -5 + 3 = ____

59 8 + 10 + -6 = ____

60 -3 + 9 + -2 = ____

61 -8 + -3 + 6 = ____

62 11 + -5 + -8 = ____

63 8 + -12 + 4 = ____

64 -6 + 5 + -9 = ____

65 12 + 8 + -15 = ____

66 -15 + 3 + 10 = ____

67 -9 + -9 + 20 = ____

68 8 + -15 + -6 = ____

69 -10 + -10 + 50 = ____

70 -30 + 5 + 17 = ____

71 9 + -15 + 8 = ____

72 7 + 12 + -20 = ____

73 13 + -20 + 10 = ____

74 18 + -20 + -2 = ____

75 -25 + 35 + -10 = ____

Addition of Integers

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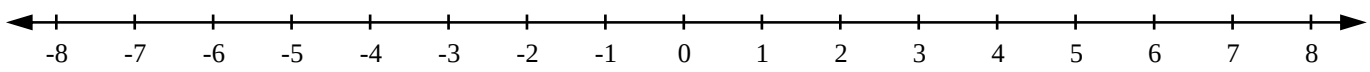

Look at the pattern formed by your answers and the number line to assist you

1 3 + 2 = 5	2 2 + 2 = 4	3 1 + 2 = 3
4 0 + 2 = 2	5 -1 + 2 = 1	6 -2 + 2 = 0
7 -3 + 2 = -1	8 -4 + 2 = -2	9 -5 + 2 = -3
10 -5 + 0 = -5	11 -5 + 1 = -4	12 -5 + 2 = -3
13 -5 + 3 = -2	14 -5 + 4 = -1	15 -5 + 5 = 0
16 -5 + 6 = 1	17 -5 + 7 = 2	18 -5 + 8 = 3

Circle the correct answer

19	-5 + 4 =	-9 9 **-1**
20	-4 + 3 =	**-1** 7 -7
21	-10 + 8 =	2 **-2** -18
22	-15 + 3 =	**-12** 12 -18
23	-30 + 25 =	-55 **-5** 5
24	-14 + 14 =	**0** 28 -28
25	-5 + 10 =	-15 15 **5**
26	-7 + 3 =	**-4** 4 10
27	-20 + 60 =	-40 -80 **40**
28	-15 + 27 =	-42 -12 **12**
29	-11 + 4 =	**-7** -15 7

When you add a negative number it is like subtracting a positive number

30 5 + 1 = 6	31 5 + 0 = 5	32 5 + -1 = 4
33 5 + -2 = 3	34 5 + -3 = 2	35 5 + -4 = 1
36 5 + -5 = 0	37 5 + -6 = -1	38 5 + -7 = -2
39 4 + -7 = -3	40 3 + -7 = -4	41 2 + -7 = -5
42 1 + -7 = -6	43 0 + -7 = -7	44 -1 + -7 = -8
45 -2 + -7 = -9		46 -3 + -7 = -10

Circle the correct answer

47	-6 + -6 =	0 12 **-12**
48	3 + -5 =	-8 **-2** 8
49	-5 + -7 =	2 **-12** -2
50	8 + -5 =	**3** -3 13
51	10 + -12 =	**-2** -22 2
52	-4 + -10 =	6 -6 **-14**
53	-15 + -8 =	**-23** -7 7
54	14 + -12 =	-2 **2** 26
55	20 + -50 =	30 -70 **-30**
56	-5 + -25=	20 -20 **-30**
57	12 + -12 =	**0** -24 24

Now find the answer for these

Example

$4 + -^{-6}10 + 6 =$ 0

58	$10 + -^{5}5 + 3 =$	8
59	$8 + 10 + -^{18}6 =$	12
60	$-3 + 9 + -^{6}2 =$	4
61	$-8 + -^{-11}3 + 6 =$	5
62	$11 + -^{6}5 + -8 =$	-2
63	$8 + -^{-4}12 + 4 =$	0
64	$-6 + 5 + -^{-1}9 =$	-10
65	$12 + 8 + -^{20}15 =$	5
66	$-15 + 3 + -^{-12}10 =$	-2
67	$-9 + -9 + -^{-18}20 =$	2
68	$8 + -15 + -^{-7}6 =$	-13
69	$-10 + -10 + -^{-20}50 =$	30
70	$-30 + 5 + -^{-25}17 =$	-8
71	$9 + -15 + -^{-6}8 =$	2
72	$7 + 12 + -^{19}20 =$	-1
73	$13 + -20 + -^{-7}10 =$	3
74	$18 + -20 + -^{-2}2 =$	-4
75	$-25 + 35 + -^{10}10 =$	0

Subtraction of Integers

Unlike positive numbers which when subtracted always give a smaller number, the subtraction of two negative numbers results in larger number. That is because when two '-' signs are together the result is a '+'.

In column 1 both positive and negative numbers are involved in subtractions with positive numbers. Questions 1 to 9 and questions 10 to 18 are in 2 groups. Note that sometimes numbers have a '+' in front of them and sometimes they don't, but both mean the same thing. E.g. -10 - 2 = -10 - +2 = -12. As you learnt on the previous sheet unlike signs equals a minus, so the minus and the plus next to each other equals a minus. These questions give answers that form a number pattern, this pattern can be used to check your answer as you move through the question groups. Note also there is a number line at the top of the page to assist you.

Questions 21 to 29 mostly won't fit on the number line so you have to understand the concept of negative numbers for these questions. All the questions subtract a positive number. This means they will decrease, circle the correct answer.

Column 2 involves the subtraction of both positive and negative numbers with negative numbers. Don't be confused by the two minus signs (- -) next to each other. When a minus and another minus sign are together the result is a plus '+' sign. So - and - = +. And as you know a + and + = +. It is even more easy if you remember like signs = plus. This column has the same layout, two groups of questions which have a pattern. Use the pattern or the number line to assist you. Then for the lower part of the column circle the correct answer.

Column 3 involves 3 numbers, that means that you answer the questions in 2 stages. Look at the first 2 numbers, complete the sum strike them as you move through. There is an example at the top of the column.

Subtraction of Integers

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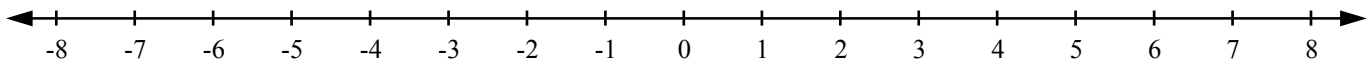

Look at the pattern formed by your answers and the number line to assist you

1 5 - +2 = ____	2 4 - 2 = ____	3 3 - +2 = ____
4 2 - +2 = ____	5 1 - +2 = ____	6 0 - +2 = ____
7 -1 - 2 = ____	8 -2 - 2 = ____	9 -3 - 2 = ____
10 2 - 0 = ____	11 2 - 1 = ____	12 2 - +2 = ____
13 2 - +3 = ____	14 2 - +4 = ____	15 2 - +5 = ____
16 2 - 6 = ____	17 2 - 7 = ____	18 2 - 8 = ____

Circle the correct answer

19	8 - +10 =	-18	-2	2
20	5 - +9 =	4	-13	-4
21	-2 - +6 =	-4	-8	4
22	-10 - 20 =	-30	10	30
23	15 - 30 =	-45	-15	15
24	-9 - +12 =	-21	3	21
25	12 - 12 =	-24	24	0
26	13 - +15 =	2	-2	-28
27	-20 - 7 =	-27	-13	13
28	-14 - 14 =	-28	0	28
29	-11 - 9 =	-2	2	-20

When you subtract a negative number it is like adding a positive number

30 2 - -3 = ____	31 1 - -3 = ____	32 0 - -3 = ____
33 -1- -3 = ____	34 -2 - -3 = ____	35 -3- -3 = ____
36 -4 - -3 = ____	37 -5 - -3 = ____	38 -6 - -3 = ____
39 -4 - -2 = ____	40 -4 - -3 = ____	41 -4 - -4 = ____
42 -4- -5 = ____	43 -4- -6 = ____	44 -4- -7 = ____
45 -4- -8 = ____		46 -4- -9 = ____

Circle the correct answer

47	5 - -2 =	-3	3	7
48	-3 - -7 =	4	-4	-10
49	-10 - -10 =	-20	10	0
50	-14 - - 6 =	-20	-8	8
51	3 - - 13 =	-10	16	-16
52	-24 - -12 =	-12	-36	12
53	5 - - 17 =	-22	-12	22
54	-20 - - 9 =	-29	-11	11
55	15 - - 7 =	-8	22	8
56	8 - - 8 =	16	-8	0
57	0 - - 6 =	-6	0	6

Now find the answer for these

Example

~~5~~ - - 1~~0~~ 15 - 6 = 9

58	20 - +8 - 4 =	____
59	6 - -10 - 18 =	____
60	-40 - 14 - - 6 =	____
61	-12 - +7 - -10 =	____
62	-8 - -13 - -15 =	____
63	8 - +12 - - 4 =	____
64	-20 - - 5 - - 8 =	____
65	20 - 35 - - 3 =	____
66	4 - -4 - -8 =	____
67	4 - -4 - +8 =	____
68	-10 - 40 - - 90 =	____
69	-2 - -17 - +9 =	____
70	10 - 20 - 30 =	____
71	-9 - -10 -4 =	____
72	8 - 19 - - 11 =	____
73	16 - 20 - - 7 =	____
74	-7 - -20 - - 13 =	____
75	-6 - 11 - - 17 =	____

Subtraction of Integers

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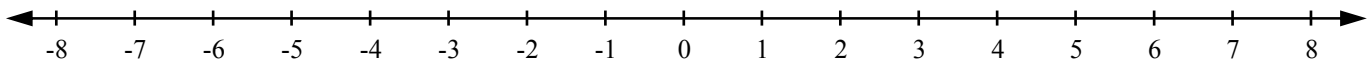


Look at the pattern formed by your answers and the number line to assist you

1 5 - +2 = 3	2 4 - 2 = 2	3 3 - +2 = 1
4 2 - +2 = 0	5 1 - +2 = -1	6 0 - +2 = -2
7 -1 - 2 = -3	8 -2 - 2 = -4	9 -3 - 2 = -5
10 2 - 0 = 2	11 2 - 1 = 1	12 2 - +2 = 0
13 2 - +3 = -1	14 2 - +4 = -2	15 2 - +5 = -3
16 2 - 6 = -4	17 2 - 7 = -5	18 2 - 8 = -6

Circle the correct answer

19	8 - +10 =	-18	(-2)	2
20	5 - +9 =	4	-13	(-4)
21	-2 - +6 =	-4	(-8)	4
22	-10 - 20 =	(-30)	10	30
23	15 - 30 =	-45	(-15)	15
24	-9 - +12 =	(-21)	3	21
25	12 - 12 =	-24	24	(0)
26	13 - +15 =	2	(-2)	-28
27	-20 - 7 =	(-27)	-13	13
28	-14 - 14 =	(-28)	0	28
29	-11 - 9 =	-2	2	(-20)

When you subtract a negative number it is like adding a positive number

30 2 - -3 = 5	31 1 - -3 = 4	32 0 - -3 = 3
33 -1- -3 = 2	34 -2 - -3 = 1	35 -3- -3 = 0
36 -4 - -3 = -1	37 -5 - -3 = -2	38 -6 - -3 = -3
39 -4 - -2 = -2	40 -4 - -3 = -1	41 -4 - -4 = 0
42 -4- -5 = 1	43 -4- -6 = 2	44 -4- -7 = 3
45 -4- -8 = 4		46 -4- -9 = 5

Circle the correct answer

47	5 - -2 =	-3	3	(7)
48	-3 - -7 =	(4)	-4	-10
49	-10 - -10 =	-20	10	(0)
50	-14 - - 6 =	-20	(-8)	8
51	3 - - 13 =	-10	(16)	-16
52	-24 - -12 =	(-12)	-36	12
53	5 - - 17 =	-22	-12	(22)
54	-20 - - 9 =	-29	(-11)	11
55	15 - - 7 =	-8	(22)	8
56	8 - - 8 =	(16)	-8	0
57	0 - - 6 =	-6	0	(6)

Now find the answer for these

Example

	~~5~~ - - 1~~0~~ (15) - 6 =	9
58	~~20~~ - +~~8~~ (12) - 4 =	8
59	~~6~~ - -1~~0~~ (16) - 18 =	-2
60	-4~~0~~ - 1~~4~~ (-54) - - 6 =	-48
61	-1~~2~~ - +~~7~~ (-19) - -10 =	-9
62	-~~8~~ - -1~~3~~ (5) - -15 =	20
63	~~8~~ - +1~~2~~ (-4) - - 4 =	0
64	-2~~0~~ - - ~~5~~ (-15) - - 8 =	-7
65	~~20~~ - 3~~5~~ (-15) - - 3 =	-12
66	~~4~~ - -~~4~~ (8) - -8 =	16
67	~~4~~ - -~~4~~ (8) - +8 =	0
68	-1~~0~~ - 4~~0~~ (-50) - - 90 =	40
69	-~~2~~ - -1~~7~~ (15) - +9 =	24
70	1~~0~~ - 2~~0~~ (-10) - 30 =	-40
71	-~~9~~ - -1~~0~~ (1) -4 =	-3
72	~~8~~ - 1~~9~~ (-11) - - 11 =	0
73	1~~6~~ - 2~~0~~ (-4) - - 7 =	3
74	-~~7~~ - -2~~0~~ (13) - - 13 =	26
75	-~~6~~ - 1~~1~~ (-17) - - 17 =	0

Subtraction/Addition and Using Brackets with Integers

This sheet mixes subtraction and addition together and introduces the use of brackets with integers.

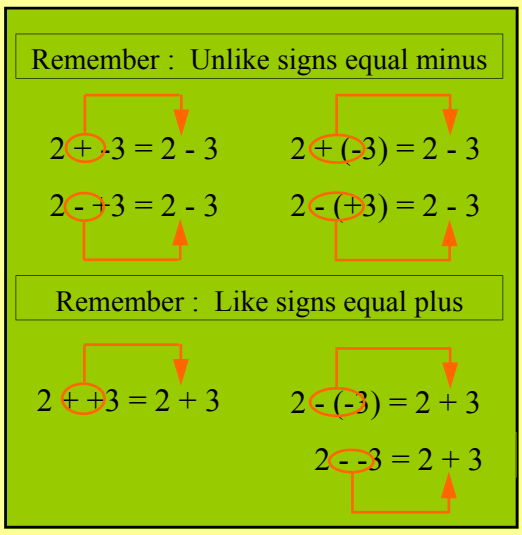
Remember negative numbers are on the left hand side of the number line and as you should know as you move to the left the 'value' of the numbers decrease. So instead of 0 being the smallest number it now lies in the middle of positive and negative numbers. So:

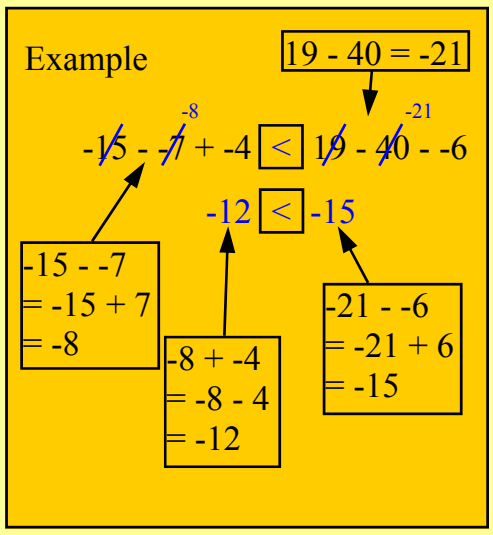
- 0 > (greater than) all negative numbers e.g. 0 > -1, 0 > -8, 0 > -1 008
- The larger the value of the number part of the negative number the smaller it is, e.g. -8 < (less than) -3, -204 < -176 and -82 > -106
- All positive numbers are greater than negative numbers

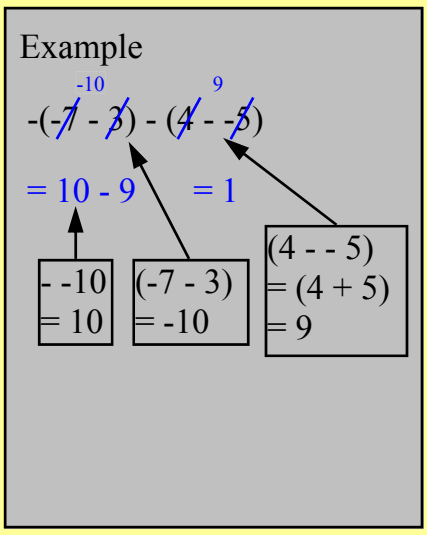
Column 1 asks you to place a <, > or = sign in the boxes. The first two involve 2 numbers on each side. Find the answer for each side and write it in the box below it, then 'point the arrow' at the smallest number. From then on there are 3 numbers on each side. Look at each side separately, add/subtract the first two numbers, strike them out and total them. Repeat for the other side and write in the <, > or = signs. An example is to the left, below.

Column 2 introduces brackets with negative numbers. When a number is in brackets on its own then treat the brackets as if they aren't there. For example -(-10) = - -10 = 10 and -(15) = -15. When there are two or more numbers within brackets evaluate the inside of the brackets first to get a single number, then the above applies again, remove the brackets and solve. For example
-(10 - 13) = -(-3) = 3, -(5 - -11) = -(16) = -16 and -(-6 -7) = -(-13) = 13.

Column 3 is an extension on this with 4 numbers involved so total as you move across. Brackets are also introduced. Again it isn't obvious why we need brackets for single numbers, it will become clear when powers and substitution are introduced. The last 5 questions involve two numbers in brackets, remember with brackets you solve the inside of the brackets first, then the tricky part, how does the sign in front of the brackets affect the number inside? An example is to the right, below.







Subtraction/Addition and Using Brackets with Integers

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Fill the box with <, >, or = to make these true

Example

$-75 - -8 + -4 \;\square\; -180 + 100$ (-67)

$-71 \;\square\; -80$

1 $20 - -4 \;\square\; -5 + 22$

2 $9 - 14 \;\square\; -8 + 14$

3 $13 + -6 - -4 \;\square\; 4 + -7 - -12$

4 $20 - -6 + 8 \;\square\; -8 - -40 + 7$

5 $-17 - -9 + 8 \;\square\; 11 - 17 + 6$

6 $-8 - 7 - -11 \;\square\; -17 - -8 - -3$

7 $14 - 9 - -20 \;\square\; 18 - -12 - 5$

8 $-2 + 5 - 9 \;\square\; -5 - 8 - -7$

9 $-6 - -10 - -4 \;\square\; -3 - +7 + 11$

10 $4 + -7 - +3 \;\square\; -20 - -10 - 4$

11 $13 - 50 - -8 \;\square\; -15 - -8 + 6$

Express the following without the brackets

12 $-(-3)$ = ____

13 $-(-8)$ = ____

14 $-(7)$ = ____

15 $-(-20)$ = ____

16 $-(-30)$ = ____

17 $-(19)$ = ____

18 $(-5 - 10)$ = ____

19 $(-9 - 14)$ = ____

20 $-(-2 - 13)$ = $-(\quad)$ = ____

21 $-(-5 - 6)$ = ____ = ____

22 $-(9 + 3)$ = ____ = ____

23 $-(8 + 3)$ = ____ = ____

24 $-(2 - 7)$ = ____ = ____

25 $-(10 + 7)$ = ____ = ____

26 $-(4 - 9)$ = ____ = ____

27 $-(9 + 4)$ = ____ = ____

28 $(8 - -9)$ = ____ = ____

29 $(10 - -15)$ = ____ = ____

30 $-(8 - -30)$ = ____ = ____

31 $-(6 - -10)$ = ____ = ____

32 $-(-7 - -4)$ = ____ = ____

33 $-(-10 - 9)$ = ____ = ____

These are longer and may include brackets ( ).

Example

$5 - (-7) + 6 - 10$ (12, 18) = 8

34 $-8 - (-3) - 7 + 12$ = ____

35 $-19 + 8 - (3) + 7$ = ____

36 $0 - (-7) - 10 + 3$ = ____

37 $-(-7) + 3 - 20 + 4$ = ____

38 $13 - 20 + -6 - -10$ = ____

39 $-11 - 7 + (-2) + 5$ = ____

40 $-1 - -1 - (+1) - 1$ = ____

41 $37 - (-20) + 9 - 4$ = ____

42 $-15 - 8 - -9 - -7$ = ____

These are harder!

43 $6 + (-3 - 7) - -8$

= ____ = ____

44 $-14 - (-6 - 2) + 6$

= ____ = ____

45 $9 - (-2 - 15) + 17$

= ____ = ____

46 $(-2 - -3) + (3 - 5)$

= ____ = ____

47 $-(5 - 12) - (2 - 5)$

= ____ = ____

Subtraction/Addition and Using Brackets with Integers

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Fill the box with <, >, or = to make these true

Example

$-75 - -8 + -4 \boxed{>} -180 + 100$ (-67)

$-71 \boxed{>} -80$

1 $20 - -4 \boxed{>} -5 + 22$

$24 \boxed{>} 17$

2 $9 - 14 \boxed{<} -8 + 14$

$-5 \boxed{<} 6$

3 $13 + -6 - -4 \boxed{>} 4 + -7 - -12$ (7) (-3)

$11 \boxed{>} 9$

4 $20 - -6 + 8 \boxed{\ } -8 - -40 + 7$ (26) (32)

$34 \boxed{<} 39$

5 $-17 - -9 + 8 \boxed{=} 11 - 17 + 6$ (-8) (-6)

$0 \boxed{=} 0$

6 $-8 - 7 - -11 \boxed{>} -17 - -8 - -3$ (-15) (-9)

$-4 \boxed{>} -6$

7 $14 - 9 - -20 \boxed{=} 18 - -12 - 5$ (5) (30)

$25 \boxed{=} 25$

8 $-2 + 5 - 9 \boxed{=} -5 - 8 - -7$ (3) (-13)

$-6 \boxed{=} -6$

9 $-6 - -10 - -4 \boxed{>} -3 - +7 + 11$ (4) (-10)

$8 \boxed{>} 1$

10 $4 + -7 - +3 \boxed{>} -20 - -10 - 4$ (-3) (-10)

$-6 \boxed{>} -14$

11 $13 - 50 - -8 \boxed{<} -15 - -8 + 6$ (-37) (-7)

$-29 \boxed{<} -1$

Express the following without the brackets

12 $-(-3)$ $= 3$

13 $-(-8)$ $= 8$

14 $-(7)$ $= -7$

15 $-(-20)$ $= 20$

16 $-(-30)$ $= 30$

17 $-(19)$ $= -19$

18 $(-5 - 10)$ $= -15$

19 $(-9 - 14)$ $= -23$

20 $-(-2 - 13)$ $= -(-15)$ $= 15$

21 $-(-5 - 6)$ $= -(-11)$ $= 11$

22 $-(9 + 3)$ $= -(12)$ $= -12$

23 $-(8 + 3)$ $= -(11)$ $= -11$

24 $-(2 - 7)$ $= -(-9)$ $= -9$

25 $-(10 + 7)$ $= -(17)$ $= -17$

26 $-(4 - 9)$ $= -(-5)$ $= 5$

27 $-(9 + 4)$ $= -(13)$ $= -13$

28 $(8 - -9)$ $= (17)$ $= 17$

29 $(10 - -15)$ $= (25)$ $= 25$

30 $-(8 - -30)$ $= -(38)$ $= -38$

31 $-(6 - -10)$ $= -(16)$ $= -16$

32 $-(-7 - -4)$ $= -(-3)$ $= 3$

33 $-(-10 - 9)$ $= -(-19)$ $= 19$



These are longer and may include brackets ( ).

Example

$5 - (-7) + 6 - 10$ (12) (18) $= 8$

34 $-8 - (-3) - 7 + 12$ (-5) (-12) $= 0$

35 $-19 + 8 - (3) + 7$ (-11) (-14) $= -7$

36 $0 - (-7) - 10 + 3$ (7) (-3) $= 0$

37 $-(-7) + 3 - 20 + 4$ (10) (-10) $= -6$

38 $13 - 20 + -6 - -10$ (-7) (-13) $= -3$

39 $-11 - 7 + (-2) + 5$ (-18) (-20) $= -15$

40 $-1 - -1 - (+1) - 1$ (0) (-1) $= -2$

41 $37 - (-20) + 9 - 4$ (57) (66) $= 62$

42 $-15 - 8 - -9 - -7$ (-23) (-14) $= -7$



These are harder!

43 $6 + (-3 - 7) - -8$ (-10)

$= 6 - 10 + 8$ (-4) $= 4$

44 $-14 - (-6 - 2) + 6$ (-8)

$= -14 + 8 + 6$ (-6) $= 0$

45 $9 - (-2 - 15) + 17$ (-17)

$= 9 + 17 + 17$ (26) $= 43$

46 $(-2 - -3) + (3 - 5)$ (1) (-2)

$= 1 - 2$ $= -1$

47 $-(5 - 12) - (2 - 5)$ (-7) (-3)

$= 7 + 3$ $= 10$

Multiplication of Integers

As with addition and subtraction, multiplication has rules concerning negative numbers also. These are:

- a positive × a positive = a positive, this you already know (+ × + = +)
- a negative number × a negative = a positive (- × - = +)

So this means that when like signs are multiplied you get a positive answer, and:

- a negative × a positive = a negative (- × + = -)
- a positive × a negative = a negative (+ × - = -)

So this means that when unlike signs are multiplied you get a negative answer.

In column 1 you are asked to multiply 2 numbers together, one is a negative number, the other a positive, this means the answer will always be negative.

In column 2 down to question 44 you are asked to multiply 2 negative numbers. This means the answer will always be positive. The rest of the 2nd column is a mixture, some are like signs others are unlike. The method is to look at the question and determine the sign, talk it out in your head '….a minus times a minus is a plus' or '…..a minus times a plus is a minus'. Then write the sign and then go back and multiply the numbers as if the signs aren't there.

In column 3 the first 5 questions are to test you know the sign. As above you talk it through mentally, for -5 × -2 × 4 × -3 you would say this: 'a minus times a minus is a plus...a plus times a plus is a plus ... a plus times a minus is a minus. So the sign is negative, circle it. You can use the same method for the next set of questions to get the sign first, then multiply the numbers ignoring the signs. Using the same example above, 5 × 2 is 10, × 4 is 40 and × 3 is 120. So you have -120! There is an example at the top of the column.

Multiplication of Integers

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Try these, use the rules: - × + = - and + × - = -

1 -2 × 3 =

2 4 × -2 =

3 -3 × 4 =

4 5 × -3 =

5 5 × -7 =

6 -5 × (+10) =

7 8 × -4 =

8 -3 × 7 =

9 -2 × (+8) =

10 -5 × 5 =

11 3 × -25 =

12 9 × -5 =

13 -6 × (+4) =

14 5 × -11 =

15 13 × -10 =

16 -12 × 2 =

17 15 × (-4) =

18 6 × -9 =

19 -8 × 3 =

20 -7 × +20 =

21 -3 × 12 =

22 7 × -6 =

23 9 × (-11) =

24 -16 × 2 =

25 -35 × +3 =

26 12 × -7 =

27 -8 × 21 =

28 -4 × 13 =

29 -7 × +8 =

30 34 × (-3) =

Try these, use the rule: - × - = +

31 -2 × -2 =

32 -4 × -5 =

33 -10 × (-3) =

34 -7 × -3 =

35 -6 × -21 =

36 -8 × -4 =

37 (-5) × -9 =

38 -11 × -8 =

39 -4 × (-13) =

40 -22 × -5 =

41 - 1.7 × -2 =

42 $-3\frac{1}{2}$ × -2 =

43 -14 × -5 =

44 -3 × -15 =

These are mixed up

45 -30 × 6 =

46 -2 × -16 =

47 -9 × -4 =

48 11 × -7 =

49 -10 × -23 =

50 -60 × 4 =

51 -15 × -5 =

52 5 × -12 =

53 8 × -4 =

54 -3 × -11 =

55 -8 × -7 =

56 -5 × 50 =

57 -2 × -45 =

58 7 × -21 =

Circle the sign that the answer would have, don't try to solve them

Example

- × + = - × - = +

-5 × 2 × -3 - or (+)

59 -2 × -2 × 3 - or +

60 4 × -2 × 5 - or +

61 -2 × -4 × -4 - or +

62 -5 × -7 × -2 × -1 - or +

63 -(-7) × 4 × -2 - or +

With these, find the sign of the answer first, then multiply for the answer.

Example + × - = -, then - × + = - and then - × - = +. So positive

$\cancel{5}^{1} \times -\cancel{2}^{10} \times \cancel{3}^{30} \times -2$ = 60

64 5 × -4 × -4 =

65 -2 × -6 × -5 =

66 8 × 3 × -2 =

67 -2 × 5 × -2 × -3 =

68 5 × 3 × -1 × 1 =

69 20 × -2 × -2 × 5 =

70 -3 × -4 × -2 × -2 =

71 10 × -2 × -1 × 2 =

72 -4 × -2 × -4 × 2 =

73 2 × -2 × 2 × -2 =

74 -1 × -1 × -1 × -1 =

75 4 × -2 × -5 × 0 =

Multiplication of Integers

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Try these, use the rules: - × + = - and + × - = -

1	-2 × 3 = -6	2	4 × -2 = -8
3	-3 × 4 = -12	4	5 × -3 = -15
5	5 × -7 = -35	6	-5 × (+10) = -50
7	8 × -4 = -32	8	-3 × 7 = -21
9	-2 × (+8) = -16	10	-5 × 5 = -25
11	3 × -25 = -75	12	9 × -5 = -45
13	-6 × (+4) = -24	14	5 × -11 = -55
15	13 × -10 = -130	16	-12 × 2 = -24
17	15 × (-4) = -60	18	6 × -9 = -54
19	-8 × 3 = -24	20	-7 × +20 = -140
21	-3 × 12 = -36	22	7 × -6 = -42
23	9 × (-11) = -99	24	-16 × 2 = -32
25	-35 × +3 = -105	26	12 × -7 = -84
27	-8 × 21 = -168	28	-4 × 13 = -52
29	-7 × +8 = -56	30	34 × (-3) = -102

Try these, use the rule: - × - = +

31	-2 × -2 = 4	32	-4 × -5 = 20
33	-10 × (-3) = 30	34	-7 × -3 = 21
35	-6 × -21 = 126	36	-8 × -4 = 32
37	(-5) × -9 = 45	38	-11 × -8 = 88
39	-4 × (-13) = 52	40	-22 × -5 = 110
41	- 1.7 × -2 = 3.4	42	$-3\frac{1}{2}$ × -2 = 7
43	-14 × -5 = 70	44	-3 × -15 = 45

These are mixed up

45	-30 × 6 = -180	46	-2 × -16 = 32
47	-9 × -4 = 36	48	11 × -7 = -77
49	-10 × -23 = 230	50	-60 × 4 = -240
51	-15 × -5 = 75	52	5 × -12 = -60
53	8 × -4 = -32	54	-3 × -11 = 33
55	-8 × -7 = 56	56	-5 × 50 = -250
57	-2 × -45 = 90	58	7 × -21 = -147

Circle the sign that the answer would have, don't try to solve them

Example

- × + = ⊖ ▶ ⊖ × - = +

-5 × 2 × -3 - or ⊕

59 -2 × -2 × 3 - or ⊕

60 4 × -2 × 5 ⊖ or +

61 -2 × -4 × -4 ⊖ or +

62 -5 × -7 × -2 × -1 - or ⊕

63 -(-7) × 4 × -2 ⊖ or +

With these, find the sign of the answer first, then multiply for the answer.

Example + × - = -, then - × + = - and then - × - = +. So positive

5 × -2 × 3 × -2 = 60 (10, 30)

64 5 × -4 × -4 = 80 (20)

65 -2 × -6 × -5 = -60 (12)

66 8 × 3 × -2 = -48 (24)

67 -2 × 5 × -2 × -3 = -60 (10, 20)

68 5 × 3 × -1 × 1 = -15 (15, 15)

69 20 × -2 × -2 × 5 = 400 (40, 80)

70 -3 × -4 × -2 × -2 = 48 (12, 24)

71 10 × -2 × -1 × 2 = 40 (20, 20)

72 -4 × -2 × -4 × 2 = -64 (8, 32)

73 2 × -2 × 2 × -2 = 16 (4, 8)

74 -1 × -1 × -1 × -1 = 1 (1, 1)

75 4 × -2 × -5 × 0 = 0 (8, 40)

Division of Integers

As with addition, subtraction and multiplication, division has rules concerning negative numbers also. The good news is that it is just the same as the other rules, that is:

- a positive ÷ a positive = a positive, this you already know (+ ÷ + = +)
- a negative number ÷ a negative = a positive (- ÷ - = +)

So this means that when like signs are divided you get a positive answer, and:

- a negative ÷ a positive = a negative (- ÷ + = -)
- a positive ÷ a negative = a negative (+ ÷ - = -)

So this means that when unlike signs are divided you get a negative answer.

In column 1 you are asked to divide 2 numbers, one is a negative number, the other a positive, this means the answer will always be negative. So don't let the signs confuse you, write the negative sign then divide the two numbers, ignoring the signs, to complete the answer.

In Column 2 down to question 40 you are asked to divide two negative numbers. This means the answer will always be positive, so divide the two numbers together ignoring the signs. The rest of the 2nd column is a mixture, some are like signs others are unlike. The method is to look at the question and determine the sign, talk it out in your head '….a minus divided by a minus is a plus' or '…..a minus divided by a plus is a minus'. Then write the sign. Then divide the numbers as if the signs aren't there.

In Column 3 the first 5 questions are to test you know the sign. As above you talk it through mentally, for -40 ÷ -2 ÷ 5 ÷ -2 you would say this: 'a minus divided by a minus is a plus...a plus divided by a plus is a plus ... a plus divided by a minus is a minus. So the sign is negative, so circle it. Use the same method with the rest of the column, find the sign first by talking it through, write it in, then staying with the example above, ignore the signs e.g. 40 ÷ 2 is 20, ÷ 5 is 4 and ÷ 2 is 2. So you have -2! There is an example at the top of the column.

Division of Integers

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Find the quotient for these divisions.

1 $20 \div -4 =$ ____

2 $15 \div -5 =$ ____

3 $30 \div -5 =$ ____

4 $45 \div -9 =$ ____

5 $-3 \overline{)\ 519}$

6 $34 \div -2 =$ ____

7 $\frac{64}{-8} =$ ____

8 $54 \div -6 =$ ____

9 $72 \div -6 =$ ____

10 $-8 \overline{)\ 176}$

11 $28 \div -4 =$ ____

12 $\frac{99}{-11} =$ ____

Now find these. Use: '- ÷ + = -'

13 $-12 \div 3 =$ ____

14 $-60 \div 4 =$ ____

15 $-72 \div 9 =$ ____

16 $-90 \div 6 =$ ____

17 $4 \overline{)\ -556}$

18 $-39 \div 3 =$ ____

19 $-63 \div 7 =$ ____

20 $9 \overline{)\ -423}$

21 $\frac{-75}{5} =$ ____

22 $-68 \div 2 =$ ____

23 $-84 \div 7 =$ ____

24 $\frac{-24}{3} =$ ____

25 $7 \overline{)\ -406}$

26 $-85 \div 5 =$ ____

Now apply the rule : '- ÷ - = +'

27 $-25 \div -5 =$ ____

28 $-35 \div -7 =$ ____

29 $-39 \div -13 =$ ____

30 $\frac{-42}{3} =$ ____

31 $-6 \overline{)\ -942}$

32 $-44 \div -4 =$ ____

33 $-75 \div -15 =$ ____

34 $-3 \overline{)\ -258}$

35 $\frac{-81}{-9} =$ ____

36 $-28 \div -7 =$ ____

37 $-48 \div -6 =$ ____

38 $-80 \div -20 =$ ____

39 $-8 \overline{)\ -504}$

40 $-27 \div -3 =$ ____

Try these they are mixed up.

41 $-56 \div 7 =$ ____

42 $18 \div -3 =$ ____

43 $-40 \div -5 =$ ____

44 $55 \div -11 =$ ____

45 $4 \overline{)\ -344}$

46 $-66 \div -3 =$ ____

47 $\frac{38}{-2} =$ ____

48 $80 \div -5 =$ ____

49 $-20 \div -2 =$ ____

50 $-6 \overline{)\ -258}$

51 $\frac{120}{-4} =$ ____

52 $-100 \div -4 =$ ____

Circle the sign that the answer would have, don't try to solve them

Example

- ÷ + = + ÷ - = +

$-12 \div 6 \div -3$ - or (+)

53 $-30 \div -5 \div -2$ - or +

54 $-24 \div 2 \div -4$ - or +

55 $-18 \div -6 \div -3$ - or +

56 $-60 \div 5 \div -3 \div -2$ - or +

57 $-9 \div -3 \div -3 \div -1$ - or +

Find the sign for the answer first, then divide for the number.

Example

$40 \div -2 \div 5 \div 4 =$ -1

58 $-45 \div -3 \div -5 =$ ____

59 $-60 \div -15 \div -2 =$ ____

60 $-100 \div -5 \div -2 \div -5 =$ ____

61 $-200 \div 5 \div -4 \div 2 =$ ____

62 $70 \div 2 \div -7 \div -1 =$ ____

63 $90 \div -2 \div -5 \div -3 =$ ____

64 $-150 \div -3 \div -5 \div -2 =$ ____

65 $300 \div 5 \div -12 \div 5 =$ ____

66 $-24 \div -2 \div -3 \div -2 =$ ____

67 $500 \div -5 \div 10 \div -5 =$ ____

68 $-80 \div -4 \div -5 \div -2 =$ ____

Division of Integers

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Find the quotient for these divisions.

1 20 ÷ -4 = -5

2 15 ÷ -5 = -3

3 30 ÷ -5 = -6

4 45 ÷ -9 = -5

5 $-3 \overline{)519}$ = -173 (2)

6 34 ÷ -2 = -17

7 $\frac{64}{-8}$ = -8

8 54 ÷ -6 = -9

9 72 ÷ -6 = -12

10 $-8 \overline{)176}$ = -22 (1)

11 28 ÷ -4 = -7

12 $\frac{99}{-11}$ = -9

Now find these. Use: '- ÷ + = -'

13 -12 ÷ 3 = -4

14 -60 ÷ 4 = -15

15 -72 ÷ 9 = -8

16 -90 ÷ 6 = -15

17 $4 \overline{)-556}$ = -139 (1 3)

18 -39 ÷ 3 = -13

19 -63 ÷ 7 = -9

20 $9 \overline{)-423}$ = -47 (6)

21 $\frac{-75}{5}$ = -15

22 -68 ÷ 2 = -34

23 -84 ÷ 7 = -12

24 $\frac{-24}{3}$ = -8

25 $7 \overline{)-406}$ = -58 (5)

26 -85 ÷ 5 = -17

Now apply the rule : '- ÷ - = +'

27 -25 ÷ -5 = 5

28 -35 ÷ -7 = 5

29 -39 ÷ -13 = 3

30 $\frac{-42}{3}$ = 14

31 $-6 \overline{)-942}$ = 157 (3 4)

32 -44 ÷ -4 = 11

33 -75 ÷ -15 = 5

34 $-3 \overline{)-258}$ = 86 (1)

35 $\frac{-81}{-9}$ = 9

36 -28 ÷ -7 = 4

37 -48 ÷ -6 = 8

38 -80 ÷ -20 = 4

39 $-8 \overline{)-504}$ = 63 (2)

40 -27 ÷ -3 = 9

Try these they are mixed up.

41 -56 ÷ 7 = -8

42 18 ÷ -3 = -6

43 -40 ÷ -5 = 8

44 55 ÷ -11 = -5

45 $4 \overline{)-344}$ = -86 (2)

46 -66 ÷ -3 = 22

47 $\frac{38}{-2}$ = -19

48 80 ÷ -5 = -16

49 -20 ÷ -2 = 10

50 $-6 \overline{)-258}$ = 43 (1)

51 $\frac{120}{-4}$ = -30

52 -100 ÷ -4 = 25

Circle the sign that the answer would have, don't try to solve them

Example

- ÷ + = ▶ ÷ - = +

-12 ÷ 6 ÷ -3 - or (+)

53 -30 ÷ -5 ÷ -2 (-) or +

54 -24 ÷ 2 ÷ -4 - or (+)

55 -18 ÷ -6 ÷ -3 (-) or +

56 -60 ÷ 5 ÷ -3 ÷ -2 (-) or +

57 -9 ÷ -3 ÷ -3 ÷ -1 - or (+)

Find the sign for the answer first, then divide for the number.

Example

40 ÷ -2 ÷ 5 ÷ 4 = -1 (20, 4)

58 -45 ÷ -3 ÷ -5 = -3 (15)

59 -60 ÷ -15 ÷ -2 = -2 (4)

60 -100 ÷ -5 ÷ -2 ÷ -5 = 2 (20, 10)

61 -200 ÷ 5 ÷ -4 ÷ 2 = 5 (40, 10)

62 70 ÷ 2 ÷ -7 ÷ -1 = 5 (35, 5)

63 90 ÷ -2 ÷ -5 ÷ -3 = -3 (45, 9)

64 -150 ÷ -3 ÷ -5 ÷ -2 = 5 (50, 10)

65 300 ÷ 5 ÷ -12 ÷ 5 = -1 (60, 5)

66 -24 ÷ -2 ÷ -3 ÷ -2 = 2 (12, 4)

67 500 ÷ -5 ÷ 10 ÷ -5 = 2 (100, 10)

68 -80 ÷ -4 ÷ -5 ÷ -2 = 2 (20, 4)

Mixed Operations and Brackets with Integers

This sheet involves integers in exercises with the four operations. This means that order of operations rules will have to be considered when solving these problems. Questions using brackets are also involved, remember brackets are done first before any other operation.

Column 1 involves integers and +, -, × and ÷. Remember order of operations require that the × and ÷ part of the exercise be done before the + and - part. An example is at the top of the column.

Column 2 has division questions with the numerator and denominator containing operations. Calculate the top row and the bottom row separately first. Then perform the division. The questions involve order of operations from Q.20 so an extra line is supplied for these to perform the order of operation part first. An example is below.

$$\frac{11 \times -6 + 3}{90 \div 10 \div -3} = \frac{-66 + 3}{9 \div -3}$$

$$= \frac{-63}{-3} = 21$$

Column 3 involves using brackets. The method for these is:

- 1st working line: evaluate the brackets and rewrite the first line with the brackets removed
- 2nd working line : perform the order of operations part of the question
- Solve

$$100 + 3 \times (9 - 15)$$

$$= 100 + 3 \times -6$$

$$= 100 - 18$$

$$= 82$$

Mixed Operations and Brackets with Integers

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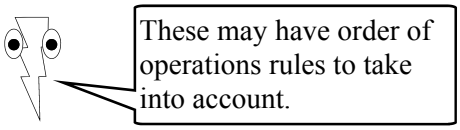


Example

$15 - \cancel{3} \times \overset{-12}{\cancel{-4}}$

$= 15 - -12$

$= 27$

1 $-9 - 17 \times 2$

2 $24 \div -3 \times -4$

3 $-55 + 15 \div 5$

4 $5 \times -10 \div 2$

5 $16 \div -4 \times -2$

6 $7 - -3 \times -4$

7 $8 + -9 \times 2$

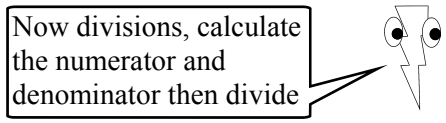
8 $-60 \div -15 \times 4$

9 $100 \div -20 + 5$

10 $7 + 13 \times -5$

11 $19 + 25 \times -3$

12 $11 \times -5 - 10$



Example

$$\frac{-2\cancel{1} - \overset{-18}{\cancel{-3}}}{-\cancel{2} \times -\cancel{3}} = \frac{-18}{6} = -3$$

13 $\dfrac{-50 - -2}{20 - 8} = \_\_\_ = \_\_\_$ (6)

14 $\dfrac{44 \div -2}{-5 - -6} = \_\_\_ = \_\_\_$

15 $\dfrac{8 \times -7}{-9 - 5} = \_\_\_ = \_\_\_$

16 $\dfrac{-18 - -3}{10 - 13} = \_\_\_ = \_\_\_$

17 $\dfrac{-100 + 25}{15 - -10} = \_\_\_ = \_\_\_$

18 $\dfrac{-40 - 32}{-4 \times -2} = \_\_\_ = \_\_\_$

19 $\dfrac{-84 \div -4}{-6 - 5} = \_\_\_ = \_\_\_$

20 $\dfrac{17 - 3 \times -5}{20 \div -2 - 6} = \_\_\_$

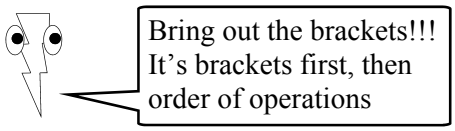
$= \_\_\_ = \_\_\_$

21 $\dfrac{-7 + 6 \times -3}{-50 + 9 \times 5} = \_\_\_$

$= \_\_\_ = \_\_\_$

22 $\dfrac{80 \div -4 + 6}{30 + 8 \times -2} = \_\_\_$

$= \_\_\_ = \_\_\_$



Example

$-(\cancel{1}\overset{6}{\cancel{3}} - 7) \times (-\cancel{2} - \overset{-7}{\cancel{5}})$

$= -6 \times -7$

$= 42$

23 $-3 + (-5\ -7) \times 4$

24 $15 - 7 \times (-3 + 8)$

25 $(1 - 7) \times 4 - 9$

26 $(-11 - 9) - 3 \times -2$

27 $(12 - 30) \div (27 \div -3)$

28 $(-65 + 40) \times (-3\ -8)$

Mixed Operations and Brackets with Integers

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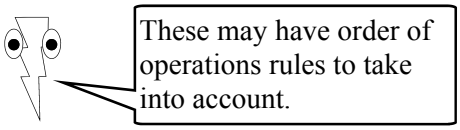
These may have order of operations rules to take into account.



Example

$15 - \cancel{3} \times \cancel{-4}^{\boxed{-12}}$

$= 15 - -12$

$= 27$

1

$-9 - 17 \times 2$

$= -9 - 34$

$= -43$

2

$24 \div -3 \times -4$

$= -8 \times -4$

$= 32$

3

$-55 + 15 \div 5$

$= -55 + 3$

$= -52$

4

$5 \times -10 \div 2$

$= -50 \div 2$

$= -25$

5

$16 \div -4 \times -2$

$= -4 \times -2$

$= 8$

6

$7 - -3 \times -4$

$= 7 - 12$

$= -5$

7

$8 + -9 \times 2$

$= 8 + -18$

$= -10$

8

$-60 \div -15 \times 4$

$= 4 \times 4$

$= 16$

9

$100 \div -20 + 5$

$= -5 + 5$

$= 0$

10

$7 + 12 \times -5$

$= 7 + -60$

$= -53$

11

$19 + 25 \times -3$

$= 19 + -75$

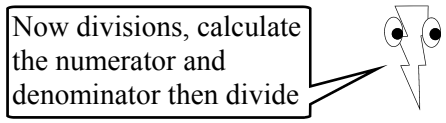
$= -56$

12

$11 \times -5 - 10$

$= -55 - 10$

$= -65$



Now divisions, calculate the numerator and denominator then divide

Example

$$\frac{-2\cancel{1} - -\cancel{3}^{\boxed{-18}}}{-\cancel{2} \times -\cancel{3}_{\boxed{6}}} = \frac{-18}{6} = -3$$

13 $\dfrac{-50 - -2}{20 - 8} = \dfrac{-48}{12} = -4$

14 $\dfrac{44 \div -2}{-5 - -6} = \dfrac{-22}{1} = -22$

15 $\dfrac{8 \times -7}{-9 - 5} = \dfrac{-56}{-14} = 4$

16 $\dfrac{-18 - -3}{10 - 13} = \dfrac{-15}{-3} = 5$

17 $\dfrac{-100 + 25}{15 - -10} = \dfrac{-75}{25} = -3$

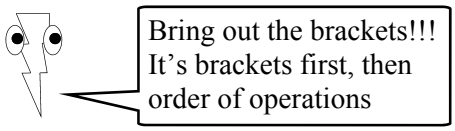
18 $\dfrac{-40 - 32}{-4 \times -2} = \dfrac{-72}{8} = -9$

19 $\dfrac{-88 \div -4}{-6 - 5} = \dfrac{22}{-11} = -2$

20 $\dfrac{17 - 3 \times -5}{20 \div -2 - 6} = \dfrac{17 - -15}{-10 - 6} = \dfrac{32}{-16} = -2$

21 $\dfrac{-7 + 6 \times -3}{-50 + 9 \times 5} = \dfrac{-7 + -18}{-50 + 45} = \dfrac{-25}{-5} = 5$

22 $\dfrac{80 \div -4 + 6}{30 + 8 \times -2} = \dfrac{-20 + 6}{30 + -16} = \dfrac{-14}{14} = -1$



Bring out the brackets!!! It's brackets first, then order of operations

Example

$-(\cancel{13} - \cancel{7}^{\boxed{6}}) \times (-\cancel{2} - \cancel{5}^{\boxed{-7}})$

$= -6 \times -7$

$= 42$

23 $-3 + (-5 - 7) \times 4$

$= -3 + -12 \times 4$

$= -3 - 48$

$= -51$

24 $15 - 7 \times (-3 + 8)$

$= 15 - 7 \times 5$

$= 15 - 35$

$= -20$

25 $(1 - 7) \times 4 - 9$

$= -6 \times 4 - 9$

$= -24 - 9$

$= -33$

26 $(-11 - 9) - 3 \times -2$

$= -20 - -6$

$= -14$

27 $(12 - 30) \div (27 \div -3)$

$= -18 \div -9$

$= 2$

28 $(-65 + 40) \times (-3 - 8)$

$= -25 \times -11$

$= 275$

Powers and Integers

When a negative number is raised to a power the answer is either positive or negative. When the power is even the answer will be positive and when the power is an odd number the answer will be negative. So as a negative number is multiplied by itself, over and over, the answer will alternate (change) between positive and negative.

Column 1 asks you to express the number raised to a power in expanded form. This means no powers and separated by × signs. Write the number in the brackets then a × sign, then the number again and so on, until there are the same amount of numbers as the power. Then solve, find the sign first, then multiply the numbers without signs. Questions 6 to 11 attempt to show you how the answer changes with the power. The last two questions require a single word response.

Column 2 starts with you identifying the rule regarding odd and even powers and negative numbers. If the power is an odd number then the answer will be negative. If the power is even the answer will be positive. Column 2 then returns to expanding and finding the solution. The difference is that the numbers are larger and they require using the multiplication working spaces provided. 'Build up' two numbers from the expansion by replacing 2 or 3 numbers with one larger number, then use the working space. An example of this is above Q 22. When you multiply the two numbers don't include the signs as you should know the sign of the answer from the power in the question being odd or even. These questions continue to the end of the sheet.

Powers and Integers

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Show these in expanded form, then evaluate

Example

$(-2)^3 = -2 \times -2 \times -2 = -8$

1 $(-3)^3$ = _____ = _____

2 $(-5)^3$ = _____ = _____

3 $(-2)^4$ = _____

= _____

4 $(-10)^3$

= _____

= _____

5 $(-2)^6$

= _____

= _____

6 $(-1)^2$ = _____

7 $(-1)^3$ = _____

8 $(-1)^4$ = _____

9 $(-1)^5$ = _____

= _____

10 $(-1)^6$

= _____

= _____

11 $(-1)^7$

= _____

= _____

12 A negative number raised to an even power gives a _____ answer

13 A negative number raised to an odd power gives a _____ answer

Would the answer to these be a positive or a negative number?

Example

Odd power so negative

$(-58)^9$ Positive ○ Negative ●

14 $(-853)^3$ Positive ○ Negative ○

15 $(-17)^8$ Positive ○ Negative ○

16 $(-11)^2$ Positive ○ Negative ○

17 $(-35)^{10}$ Positive ○ Negative ○

18 $(-81)^{95}$ Positive ○ Negative ○

19 $(-60)^{607}$ Positive ○ Negative ○

20 $(-2\,319)^{12}$ Positive ○ Negative ○

21 $(-48\,646)^5$ Positive ○ Negative ○

Expand these and then make two numbers to multiply for the answer

Example $(-3)^6$

$= -3 \times -3 \times -3 \times -3 \times -3 \times -3$ (-27, -27)

$= -27 \times -27$

$= 729$

When you multiply ignore the - signs and multiply 2 positive numbers. The power of 6 tells you the answer will be positive

$$\begin{array}{r} {\scriptstyle 1} \\ 27 \\ \times\ 27 \\ \hline {\scriptstyle 1} \\ 189 \\ 540 \\ \hline 729 \end{array}$$

22 $(-6)^3$

_____ ×

_____ _____

_____ _____

23 $(-8)^3$

_____ ×

_____ _____

_____ _____

24 $(-7)^3$

_____ ×

_____ _____

_____ _____

25 $(-5)^5$

_____ ×

_____ _____

_____ _____

_____ _____

_____ _____

26 $(-4)^6$

_____ ×

_____ _____

_____ _____

_____ _____

_____ _____

27 $(-9)^4$

_____ ×

_____ _____

_____ _____

_____ _____

_____ _____

28 $(-8)^4$

_____ ×

_____ _____

_____ _____

_____ _____

_____ _____

Powers and Integers

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Show these in expanded form, then evaluate

Example

$(-2)^3 = \overset{4}{-2} \times \overset{4}{-2} \times -2 = -8$

1 $(-3)^3 = \overset{9}{-3} \times \overset{9}{-3} \times -3 = -27$

2 $(-5)^3 = \overset{25}{-5} \times \overset{25}{-5} \times -5 = -125$

3 $(-2)^4 = \overset{4}{-2} \times \overset{4}{-2} \times \overset{-8}{-2} \times -2$

$= 16$

4 $(-10)^3$

$= \overset{100}{-10} \times \overset{100}{-10} \times -10$

$= -1\,000$

5 $(-2)^6$

$= \overset{4}{-2} \times \overset{4}{-2} \times \overset{-8}{-2} \times \overset{16}{-2} \times \overset{-32}{-2} \times -2$

$= 64$

6 $(-1)^2 = -1 \times -1 = 1$

7 $(-1)^3 = -1 \times -1 \times -1 = -1$

8 $(-1)^4 = -1 \times -1 \times -1 \times -1 = 1$

9 $(-1)^5 = -1 \times -1 \times -1 \times -1 \times -1$

$= -1$

10 $(-1)^6$

$= -1 \times -1 \times -1 \times -1 \times -1 \times -1$

$= 1$

11 $(-1)^7$

$= -1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1$

$= -1$

12 A negative number raised to an even power gives a positive answer

13 A negative number raised to an odd power gives a negative answer

Would the answer to these be a positive or a negative number?

Example

Odd power so negative

$(-58)^9$ Positive ○ Negative ●

14 $(-853)^3$ Positive ○ Negative ●

15 $(-17)^8$ Positive ● Negative ○

16 $(-11)^2$ Positive ● Negative ○

17 $(-35)^{10}$ Positive ● Negative ○

18 $(-81)^{95}$ Positive ○ Negative ●

19 $(-60)^{607}$ Positive ○ Negative ●

20 $(-2\,319)^{12}$ Positive ● Negative ○

21 $(-48\,646)^5$ Positive ○ Negative ●

Expand these and then make two numbers to multiply for the answer

Example $(-3)^6$

$= \overset{-27}{-3} \times -3 \times \overset{-27}{-3} \times -3 \times -3 \times \overset{-27}{-3}$

$= -27 \times -27$

$= 729$

When you multiply ignore the - signs and multiply 2 positive numbers. The power of 6 tells you the answer will be positive

```
  1
 27 ×
 27
---
189
540
---
729
```

22 $(-6)^3$

$= \overset{36}{-6} \times \overset{36}{-6} \times -6$

$= 36 \times -6$

$= -216$

```
  3
 36 ×
  6
---
216
```

23 $(-8)^3$

$= \overset{64}{-8} \times \overset{64}{-8} \times -8$

$= 64 \times -8$

$= -512$

```
  3
 64 ×
  8
---
512
```

24 $(-7)^3$

$= \overset{49}{-7} \times \overset{49}{-7} \times -7$

$= 49 \times -7$

$= -343$

```
  6
 49 ×
  7
---
343
```

25 (-5)

$= \overset{25}{-5} \times \overset{25}{-5} \times \overset{-125}{-5} \times \overset{25}{-5} \times -5$

$= -125 \times 25$

$= -3\,125$

```
  125 ×
   25
-----
    1
  625
 2500
-----
 3 125
```

26 $(-4)^6$

$= \overset{16}{-4} \times \overset{16}{-4} \times \overset{-64}{-4} \times -4 \times \overset{-64}{-4} \times -4$

$= -64 \times -64$

$= 4\,096$

```
   2
  64 ×
  64
-----
   1
 256
3 840
-----
4 096
```

27 $(-9)^4$

$= \overset{81}{-9} \times \overset{81}{-9} \times -9 \times -9$

$= 81 \times 81$

$= 6\,561$

```
  81 ×
  81
-----
   1
  81
6 480
-----
6 561
```

28 $(-8)^4$

$= \overset{64}{-8} \times \overset{64}{-8} \times -8 \times -8$

$= 64 \times 64$

$= 4\,096$

```
   2
  64 ×
  64
-----
   1
 256
3 840
-----
4 096
```

Absolute Value

An absolute value of a number is the positive value of that number. Absolute value is shown by 'bars' | | , these are straight lines not brackets. These bars mean that you want a positive answer for the number (or operation) inside the bars. For example:

- |5| = 5 → absolute value has no effect on numbers that are already positive
- |-10| = 10 → remove the negative sign and 10 is the absolute value of -10.

Column 1 starts with 10 questions on numbers inside 'bars'. If the number is a positive number then you just rewrite the same number without the bars. If the number is a negative number, write the number without the minus sign. The rest of the column requires you to perform a calculation and then find the absolute value. These questions involve two lines of working. In the first line write the bars and inside them the answer to the operation. Then the third line write the answer. The answer will be the same as the previous line, but no bars, and if there is a minus sign, it is removed.

Column 2 introduces a negative sign outside of the bars. As the sign is outside of the bars it is not affected by them so: -|10| = -10 and -|-10| = -10. So if there is a negative sign before the bars the answer must be negative. The first 10 questions test you understand this, then the next 10 ask you to apply this to some operations. There are two examples, before question 29, showing you how to answer the questions.

Column 3 involves order of operations in the questions. Complete the order of operations step first, then use the next 3 lines to solve the question, use the example at the top of the column as a guide.

Absolute Value

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Find the absolute value of these numbers

Examples

i) $|-12| =$ 12 ii) $|25| =$ 25

1 $|30| =$ ___ 2 $|65| =$ ___

3 $|-73| =$ ___ 4 $|-1.6| =$ ___

5 $|-19| =$ ___ 6 $|141| =$ ___

7 $|-83| =$ ___ 8 $|39| =$ ___

9 $|0| =$ ___ 10 $|-1| =$ ___

Now find the absolute value of these operations

Examples

i) $|43 - -16|$

$= |59|$

$= 59$

ii) $|18 - 30|$

$= |-12|$

$= 12$

11 $|15 - 25|$

12 $|67 - 14|$

13 $|18 - -6|$

14 $|17 - 200|$

15 $|5 \times -8|$

16 $|-4 \times -12|$

17 $|-27 + 9|$

18 $|-5 - -3|$

The - sign outside the brackets is not affected

Examples

i) $-|17| =$ -17 ii) $-|-76| =$ -76

19 $-|8| =$ ___ 20 $-|-61| =$ ___

21 $|-5.9| =$ ___ 22 $-|81| =$ ___

23 $-|-11| =$ ___ 24 $|-35| =$ ___

25 $-|-20| =$ ___ 26 $-|47| =$ ___

27 $-|-5| =$ ___ 28 $|-94| =$ ___

Now try these operations

Examples

i) $|-11| - |-4|$

$= 11 - 4$

$= 7$

ii) $-|13| - |-9|$

$= -13 - 9$

$= -22$

29 $|29| - |-39|$

30 $|12| + |-26|$

31 $-|-2| \times |-7|$

32 $-|-8| \div |-4|$

33 $-|-3| + |-3|$

34 $|-20| - |17|$

35 $|58| - |-12|$

36 $|-41| + |-9|$

37 $-|-11| \times |9|$

38 $-|-60| - |20|$

These may require order of operations rules to be considered. Solve these.

Example

$|12 + 5 \times 3| - |-5 + 7|$ (15)

$= |27| - |2|$

$= 27 - 2$

$= 25$

39 $|15 + 9 \div 3| - |26 - 50|$

40 $-|30 \div 3 \times 2| \times |-9 + 5|$

41 $-|-12 \div 6| \times |5 - 4 \times 7|$

42 $|16 \div 8 - 4| - |-3 + 7 \times 6|$

43 $-|6 - 4 \times 9| \div |12 - 2 \times 3|$

Absolute Value

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Find the absolute value of these numbers

Examples

i) $|-12| = 12$ ii) $|25| = 25$

1 $|30| = 30$ 2 $|65| = 65$

3 $|-73| = 73$ 4 $|-1.6| = 1.6$

5 $|-19| = 19$ 6 $|141| = 141$

7 $|-83| = 83$ 8 $|39| = 39$

9 $|0| = 0$ 10 $|-1| = 1$

Now find the absolute value of these operations

Examples

i) $|43 - -16|$

$= |59|$

$= 59$

ii) $|18 - 30|$

$= |-12|$

$= 12$

11 $|15 - 25|$

$= |-10|$

$= 10$

12 $|67 - 14|$

$= |53|$

$= 53$

13 $|18 - -6|$

$= |24|$

$= 24$

14 $|17 - 200|$

$= |-183|$

$= 183$

15 $|5 \times -8|$

$= |-40|$

$= 40$

16 $|-4 \times -12|$

$= |48|$

$= 48$

17 $|-27 + 9|$

$= |-18|$

$= 18$

18 $|-5 - -3|$

$= |-2|$

$= 2$

The - sign outside the brackets is not affected

Examples

i) $-|17| = -17$ ii) $-|-76| = -76$

19 $-|8| = -8$ 20 $-|-61| = -61$

21 $|-5.9| = 5.9$ 22 $-|81| = -81$

23 $-|-11| = -11$ 24 $|-35| = 35$

25 $-|-20| = -20$ 26 $-|47| = -47$

27 $-|-5| = -5$ 28 $|-94| = 94$

Now try these operations

Examples

i) $|-11| - |-4|$

$= 11 - 4$

$= 7$

ii) $-|13| - |-9|$

$= -13 - 9$

$= -22$

29 $|29| - |-39|$

$= 29 - 39$

$= -10$

30 $|12| + |-26|$

$= 12 + 26$

$= 38$

31 $-|-2| \times |-7|$

$= -2 \times 7$

$= -14$

32 $-|-8| \div |-4|$

$= -8 \div 4$

$= -2$

33 $-|-3| + |-3|$

$= -3 + 3$

$= 0$

34 $|-20| - |17|$

$= 20 - 17$

$= 3$

35 $|58| - |-12|$

$= 58 - 12$

$= 46$

36 $|-41| + |-9|$

$= 41 + 9$

$= 50$

37 $-|-11| \times |9|$

$= -11 \times 9$

$= -99$

38 $-|-60| - |20|$

$= -60 - 20$

$= -80$

These may require order of operations rules to be considered. Solve these.

Example

$|12 + 5 \times 3| - |-5 + 7|$ (15)

$= |27| - |2|$

$= 27 - 2$

$= 25$

39 $|15 + 9 \div 3| - |26 - 50|$ (3)

$= |18| - |-24|$

$= 18 - 24$

$= -6$

40 $-|30 \div 3 \times 2| \times |-9 + 5|$ (10)

$= -|20| \times |-4|$

$= -20 \times 4$

$= -80$

41 $-|-12 \div 6| \times |5 - 4 \times 7|$ (28)

$= -|-2| \times |-23|$

$= -2 \times 23$

$= -46$

42 $|16 \div 8 - 4| - |-3 + 7 \times 6|$ (2) (42)

$= |-2| - |39|$

$= 2 - 39$

$= -37$

43 $-|6 - 4 \times 9| \div |12 - 2 \times 3|$ (36) (6)

$= -|-30| \div |6|$

$= -30 \div 6$

$= -5$

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ALGEBRA 2

Solving One Step Equations
Addition and Subtraction

One Step equations are equations you solve in one line of working. They will involve a letter (pronumeral) with two numbers, you have to solve the equation for the value of the letter.

In column 1 you are given one step equations that you can solve with no working, talk them through if it helps you. Here is an example:

$d + 20 = 30$ …. So something plus 20 equals 30
Try $d = 10$ …. 10 plus 20 equals 30, so $d = 10$

Another way to do these is to think of the equation as scales, as each side is equal you have to maintain balance by finding what number would keep the scales balanced.

Column 2 is the same as column 1 except you are asked to now show working. The goal is to get the letter by itself. One of the examples at the top of the 2nd column is $b + 15 = 85$. To get the letter by itself you have to remove the + 15, how do you remove + 15? You subtract 15. So you minus 15 from the left hand side, and because it's like a set of scales, you must do the same thing on the other side of the equals sign. So you subtract 15 from $b + 15$ and you get b on its own. You subtract 15 from 85 and you get 70. The other example is $a - 9 = 27$. You have to get a by itself, to get rid of the - 9 you have to add 9. So you add 9 to each side and you get $a = 36$.

So the questions are the same as the first column but you are showing you are following a procedure by showing your working out. You may be wondering why you have to do this when often you can see the answer without working. You have to practice the process so that when you encounter more complicated problems in the future, that you can't just see the answer for, you know the correct method to use.

Column 3 is the same as column 2 except the letter is on the right hand side of the equals sign. Solve it in the same way and you get the number equal to the letter. Then rewrite your answer so that the letter is first. Again there are examples at the top of the column.

Solving One Step Equations - Addition/Subtraction

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Look at these equations and give the value for the pronumeral

1 $k + 8 = 10$ $k =$

2 $b + 3 = 15$

3 $d + 4 = 9$

4 $x + 20 = 60$

5 $t - 10 = 5$

6 $m - 8 = 20$

7 $h + 9 = 30$

8 $l - 20 = 70$

9 $w - 15 = 104$

10 $s + 12 = 13$

11 $u - 9 = 9$

12 $q + 7 = 23$

13 $n - 100 = 40$

14 $y + 22 = 30$

15 $a + 16 = 50$

16 $e - 7 = 6$

17 $z - 11 = 5$

18 $c - 10 = 94$

19 $j + 3 = 101$

20 $v + 35 = 100$

21 $t - 4 = 24$

22 $m - 17 = 4$

23 $x + 11 = 100$

Now solve these equations showing working

Examples

$b + 15 = 85$
(-15) (-15)
$b = 70$

$a - 9 = 27$
(+9) (+9)
$a = 36$

24 $c + 15 = 22$

25 $y + 11 = 28$

26 $x - 9 = 34$

27 $t - 7 = 18$

28 $q - 20 = 17$

29 $b + 10 = 11$

30 $n + 26 = 26$

31 $d - 20 = 77$

32 $t - 14 = 33$

33 $a - 11 = 59$

34 $15 + g = 33$

35 $w - 9 = 8$

36 $x - 12 = 38$

37 $18 + c = 27$

38 $e + 7 = 103$

39 $i - 56 = 11$

40 $37 + h = 80$

41 $k - 8 = 109$

42 $d - 90 = 1$

43 $u - 15 = 63$

The letter is on the right and so these need an extra line. Find the value of the letter.

Examples

$16 = 5 + c$
(-5) (-5)
$11 = c$
$c = 11$

Then reverse the equation

$35 = e - 7$
(+7) (+7)
$42 = e$
$e = 42$

44 $20 = f - 11$

45 $19 = m + 4$

46 $83 = r + 16$

47 $12 = n - 36$

48 $64 = 19 + k$

49 $11 = m - 17$

50 $87 = u - 51$

51 $100 = a - 9$

52 $33 = 18 + s$

53 $56 = d - 24$

54 $75 = p + 55$

55 $86 = 37 + x$

56 $11 = j - 11$

57 $62 = 16 + t$

Solving One Step Equations - Addition/Subtraction

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Look at these equations and give the value for the pronumeral



1	$k + 8 = 10$	$k = 2$
2	$b + 3 = 15$	$b = 12$
3	$d + 4 = 9$	$d = 5$
4	$x + 20 = 60$	$x = 40$
5	$t - 10 = 5$	$t = 15$
6	$m - 8 = 20$	$m = 28$
7	$h + 9 = 30$	$h = 21$
8	$l - 20 = 70$	$l = 90$
9	$w - 15 = 104$	$w = 119$
10	$s + 12 = 13$	$s = 1$
11	$u - 9 = 9$	$u = 18$
12	$q + 7 = 23$	$q = 16$
13	$n - 100 = 40$	$n = 140$
14	$y + 22 = 30$	$y = 8$
15	$a + 16 = 50$	$a = 34$
16	$e - 7 = 6$	$e = 13$
17	$z - 11 = 5$	$z = 16$
18	$c - 10 = 94$	$c = 104$
19	$j + 3 = 101$	$j = 88$
20	$v + 35 = 100$	$v = 65$
21	$t - 4 = 24$	$t = 28$
22	$m - 17 = 4$	$m = 21$
23	$x + 11 = 100$	$x = 89$

Now solve these equations showing working



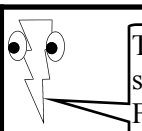
Examples

$b + 15 = 85$
(-15) (-15)
$b = 70$

$a - 9 = 27$
(+9) (+9)
$a = 36$

24	$c + 15 = 22$ - 15 - 15 $c = 7$	25 $y + 11 = 28$ - 11 - 11 $y = 17$
26	$x - 9 = 34$ + 9 + 9 $x = 43$	27 $t - 7 = 18$ + 7 + 7 $t = 25$
28	$q - 20 = 17$ + 20 + 20 $q = 37$	29 $b + 10 = 11$ - 10 - 10 b = 1
30	$n + 26 = 26$ - 26 - 26 $n = 0$	31 $d - 20 = 77$ + 20 + 20 $d = 97$
32	$t - 14 = 33$ + 14 +14 $t = 47$	33 $a - 11 = 59$ + 11 + 11 $a = 70$
34	$15 + g = 33$ - 15 - 15 $g = 18$	35 $w - 9 = 8$ + 9 + 9 $w = 17$
36	$x - 12 = 38$ + 12 + 12 $x = 50$	37 $18 + c = 27$ - 18 - 18 $c = 9$
38	$e + 7 = 103$ - 7 - 7 $e = 96$	39 $i - 56 = 11$ + 56 + 56 $i = 67$
40	$37 + h = 80$ - 37 - 37 $h = 43$	41 $k - 8 = 109$ + 8 + 8 $k = 117$
42	$d - 90 = 1$ + 90 + 90 $d = 91$	43 $u - 15 = 63$ + 15 + 15 $u = 78$

The letter is on the right and so these need an extra line. Find the value of the letter.



Examples

$16 = 5 + c$
(-5) (-5)
$11 = c$
$c = 11$

Then reverse the equation

$35 = e - 7$
(+7) (+7)
$42 = e$
$e = 42$

44	$20 = f - 11$ + 11 + 11 $31 = f$ $f = 31$	45 $19 = m + 4$ - 4 - 4 $15 = m$ $m = 15$
46	$83 = r + 16$ - 16 - 16 $67 = r$ $r = 67$	47 $12 = n - 36$ + 36 + 36 $48 = n$ $n = 48$
48	$64 = 19 + k$ - 19 - 19 $55 = k$ $k = 55$	49 $11 = m - 17$ + 17 + 17 $28 = m$ $m = 28$
50	$87 = u - 51$ + 51 + 51 $138 = u$ $u = 138$	51 $100 = a - 9$ + 9 + 9 $109 = a$ $a = 109$
52	$33 = 18 + s$ - 18 - 18 $15 = s$ $s = 15$	53 $56 = d - 24$ + 24 + 24 $80 = d$ $d = 80$
54	$75 = p + 55$ - 55 - 55 $20 = p$ $p = 20$	55 $86 = 37 + x$ - 37 - 37 $49 = x$ $x = 49$
56	$11 = j - 11$ + 11 + 11 $22 = j$ $j = 22$	57 $62 = 16 + t$ - 16 - 16 $46 = t$ $t = 46$

Further One Step Equations
Adding and Subtracting Negative Numbers

This sheet involves one step equations that involve integers (positive and negative numbers). This sheet may challenge some students. Before you attempt this sheet you should have completed the previous one step equation sheet and the folder "Integers".

Column 1 begins with 10 questions, you are asked to write in the number that satisfies the equation (makes both sides equal). This is a quick refresher on using negative numbers. The next 11 questions are much the same, except the box has been replaced by a pronumeral (letter). If you have trouble with these questions pick a number and replace the letter with it and see if it works. If it doesn't, use another number that you think is a better choice and so on. Make sure you write the letter, an '=' sign and the answer. Not the answer alone.

The second column follows the same method used with the previous one step equation sheet. See the example at the top of the column. The second part of the column has the letter on the right hand side of the equation, solve in the same way then use the additional line to reverse the answer so that the letter is on the left hand side. This is the same method used in the third column on the previous sheet.

Column 3 is different in that the sign in front of the letter is a minus sign. When you solve this equation you will have a negative sign in front of the letter. Remove it by multiplying each side by -1, this will change the signs on both sides. There are a pair of examples at the top of the column.

Just a reminder:

$+ - \;\; = - + \;\; = -$

(a plus sign before or after a minus sign is the same as a minus sign alone)

$- - = +$

(a minus sign together with another minus sign is the same as a plus sign alone)

Further One Step Equations - Add/Sub Negative Numbers

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Find the solution to these and put the number in the box

1 $3 - \square = -1$

2 $8 + \square = 5$

3 $-6 + \square = 11$

4 $-5 + \square = -1$

5 $\square - 8 = -6$

6 $\square + 3 = -10$

7 $\square + -7 = 4$

8 $\square - 8 = -8$

9 $\square + 5 = 7$

10 $\square + -3 = -1$

Now try these without working, test numbers to help you if necessary.

11 $g + 6 = 4$ $g =$

12 $n - 5 = -3$

13 $j - 2 = -10$

14 $v + 4 = -7$

15 $m + 6 = -2$

16 $y + 8 = -8$

17 $d + 10 = -7$

18 $q + 2 = 0$

19 $a - 7 = -2$

20 $e + -5 = 10$

21 $s - 7 = -7$

Now solve these equations showing working

Examples

$6 + z = -5$

(-6) (-6)

$z = -11$

$t - 9 = -4$

(+9) (+9)

$t = 5$

22 $m + 7 = 3$

23 $b - 4 = -6$

24 $n + 11 = -5$

25 $a - 3 = -2$

26 $q + 6 = -1$

27 $9 + r = -5$

28 $t - 4 = -3$

29 $c - -5 = -5$

30 $w - -2 = 0$

31 $n - 7 = -3$

Get the letter on its own then rewrite the equation so that the letter is on the left.

32 $15 = -7 + d$

33 $-3 = y - 3$

Then reverse the equation

34 $-20 = x + 9$

35 $-3 = 10 + u$

36 $5 = -3 + b$

37 $-1 = k + 6$

Solve these then change the sign on both sides of the equation so that it's positive

Examples

$7 - j = -6$

(-7) (-7)

$-j = -13$

$j = 13$

$-h - 4 = 10$

(+4) (+4)

$-h = 14$

$h = -14$

Multiply throughout by -1

38 $8 - m = -11$

39 $-v - 6 = 5$

40 $-t - 6 = 0$

41 $-7 - a = -3$

42 $5 - e = -2$

43 $-x - 11 = 2$

44 $14 - p = 40$

45 $-d - -9 = 12$

46 $-k - 8 = -15$

47 $-t - -6 = 4$

48 $14 - u = 20$

49 $-5 - c = 2$

50 $8 - a = -15$

51 $-80 - f = 35$

Further One Step Equations - Add/Sub Negative Numbers

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Find the solution to these and put the number in the box

1 $3 - \boxed{4} = -1$

2 $8 + \boxed{-3} = 5$

3 $-6 + \boxed{17} = 11$

4 $-5 + \boxed{4} = -1$

5 $\boxed{2} - 8 = -6$

6 $\boxed{-13} + 3 = -10$

7 $\boxed{11} + -7 = 4$

8 $\boxed{0} - 8 = -8$

9 $\boxed{2} + 5 = 7$

10 $\boxed{2} + -3 = -1$

Now try these without working, test numbers to help you if necessary.

11 $g + 6 = 4$ — $g = -2$

12 $n - 5 = -3$ — $n = 2$

13 $j - 2 = -10$ — $j = -8$

14 $v + 4 = -7$ — $v = -11$

15 $m + 6 = -2$ — $m = -8$

16 $y + 8 = -8$ — $y = -16$

17 $d + 10 = -7$ — $d = -17$

18 $q + 2 = 0$ — $q = -2$

19 $a - 7 = -2$ — $a = 5$

20 $e + -5 = 10$ — $e = 15$

21 $s - 7 = -7$ — $s = 0$

Now solve these equations showing working

Examples

$6 + z = -5$
(-6) (-6)
$z = -11$

$t - 9 = -4$
(+9) (+9)
$t = 5$

22 $m + 7 = 3$
- 7 - 7
$m = -4$

23 $b - 4 = -6$
+ 4 + 4
$b = -2$

24 $n + 11 = -5$
- 11 - 11
$n = -16$

25 $a - 3 = -2$
+ 3 + 3
$a = 1$

26 $q + 6 = -1$
- 6 - 6
$q = -7$

27 $9 + r = -5$
- 9 - 9
$r = -14$

28 $t - 4 = -3$
+ 4 + 4
$t = 1$

29 $c - -5 = -5$
- 5 - 5
$c = -10$

30 $w - -2 = 0$
- 2 - 2
$w = -2$

31 $n - 7 = -3$
+ 7 + 7
$n = 4$

Get the letter on its own then rewrite the equation so that the letter is on the left.

32 $15 = -7 + d$
+ 7 +7
$22 = d$
$d = 22$

33 $-3 = y - 3$
+ 3 + 3
$0 = y$
$y = 0$

Then reverse the equation

34 $-20 = x + 9$
- 9 - 9
$-29 = x$
$x = -29$

35 $-3 = 10 + u$
- 10 - 10
$-13 = u$
$u = -13$

36 $5 = -3 + b$
+ 3 + 3
$8 = b$
$b = 8$

37 $-1 = k + 6$
- 6 - 6
$-7 = k$
$k = -7$

Solve these then change the sign on both sides of the equation so that it's positive

Examples

$7 - j = -6$
(-7) (-7)
$-j = -13$
$j = 13$

$-h - 4 = 10$
(+4) (+4)
$-h = 14$
$h = -14$

Multiply throughout by -1

38 $8 - m = -11$
- 8 - 8
$-m = -19$
$m = 19$

39 $-v - 6 = 5$
+ 6 + 6
$-v = 11$
$v = -11$

40 $-t - 6 = 0$
+ 6 + 6
$-t = 6$
$t = -6$

41 $-7 - a = -3$
+ 7 + 7
$-a = 4$
$a = -4$

42 $5 - e = -2$
- 5 - 5
$-e = -7$
$e = 7$

43 $-x - 11 = 2$
+ 11 + 11
$-x = 13$
$x = -13$

44 $14 - p = 40$
- 14 - 14
$-p = 26$
$p = -26$

45 $-d - -9 = 12$
- 9 - 9
$-d = 3$
$d = -3$

46 $-k - 8 = -15$
+ 8 + 8
$-k = -7$
$k = 7$

47 $-t - -6 = 4$
- 6 - 6
$-t = -2$
$t = 2$

48 $14 - u = 20$
- 14 - 14
$-u = 6$
$u = -6$

49 $-5 - c = 2$
+ 5 + 5
$-c = 7$
$c = -7$

50 $8 - a = -15$
- 8 - 8
$-a = -23$
$a = 23$

51 $-80 - f = 35$
+ 80 + 80
$-f = 115$
$f = -115$

Solving One Step Equations - Multiplication

This sheet deals with one step equations that involve multiplication. The sheet starts with a column of questions that you should be able to answer by inspection without showing your working out. From column 2 onwards you are expected to show working, even if you can still solve the problems mentally.

Column 1 involves simple one step equations in that the answers are all cardinal numbers. One way to solve these is to talk them through, for example:

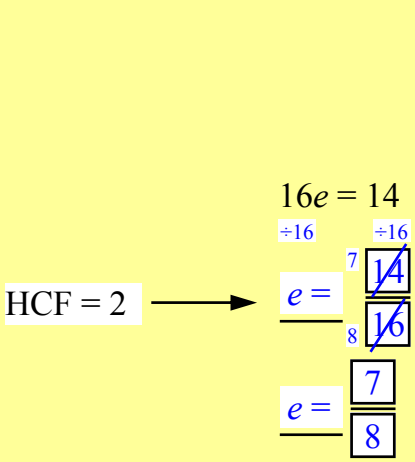
$8m = 24$ …. say 8 times something equals 24
Try $m = 3$ …. say 8 times 3 is 24 so write $m = 3$

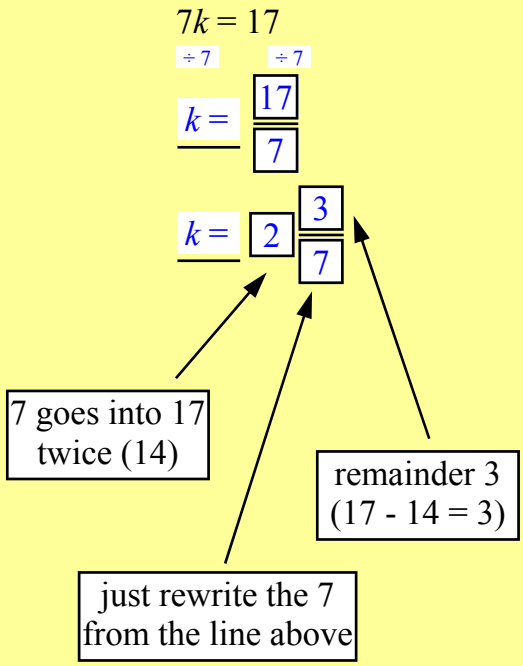
Column 2 asks you to show working and also includes fractions. These fractions won't simplify until Q 34. Follow the examples at the top of the column by dividing the number on the right hand side by the number on the left hand side (in front of the letter). Show your working as the example shows.

From Q 34 it's fractions only, this time they simplify. So use the same method to get the first line, then find the HCF (Highest Common Factor) or the largest number that goes into the top and bottom numbers and divide through.

The third column changes with the last 6 questions. These involve answers that are mixed numerals (number + fraction). Use the same method to get to the first line by dividing through. You will have an improper fraction (the numerator or top number is larger than the denominator). Then convert to mixed numerals….do you remember how? Ask yourself:

- how many times does the bottom number go into the top number, write this in the whole number box at the front
- then write the remainder (amount left over) in the top box.
- the bottom box has the same number as the bottom box in the line above.





Solving One Step Equations - Multiplication

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Look at these equations and give the value for the pronumeral

1 $4d = 8$ $d =$

2 $6h = 12$

3 $5x = 45$

4 $9g = 36$

5 $10n = 120$

6 $7e = 42$

7 $8y = 56$

8 $5t = 55$

9 $11a = 44$

10 $9q = 18$

11 $3z = 39$

12 $2w = 30$

13 $5r = 15$

14 $15p = 45$

15 $4b = 28$

16 $6c = 36$

17 $2k = 48$

18 $9y = 90$

19 $30u = 180$

20 $15x = 60$

21 $2v = 34$

22 $7j = 84$

23 $5t = 0$

Solve these equations only this time show your working out.

Examples

$6p = 54$ (÷6) (÷6) $p = 9$

$5d = 3$ (÷5) (÷5) $d = \frac{3}{5}$

24 $3e = 36$

25 $9u = 4$

26 $8a = 40$

27 $2w = 54$

28 $11q = 9$

29 $17t = 7$

30 $12c = 60$

31 $15x = 13$

32 $20m = 9$

33 $7r = 42$

These will have fraction answers that can be simplified

34 $6j = 3$

35 $8d = 6$

36 $15k = 9$

37 $20e = 12$

38 $40a = 25$

39 $12v = 10$

40 $24f = 6$

41 $20s = 16$

42 $30h = 12$

43 $35x = 28$

These can be changed from improper fractions to mixed numerals

44 $4t = 9$

45 $3c = 11$

46 $8n = 35$

47 $14d = 33$

48 $11z = 50$

49 $4k = 33$

Solving One Step Equations - Multiplication

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Look at these equations and give the value for the pronumeral

1	$4d = 8$	$d = 2$
2	$6h = 12$	$h = 2$
3	$5x = 45$	$x = 9$
4	$9g = 36$	$g = 4$
5	$10n = 120$	$n = 12$
6	$7e = 42$	$e = 6$
7	$8y = 56$	$y = 7$
8	$5t = 55$	$t = 11$
9	$11a = 44$	$a = 4$
10	$9q = 18$	$q = 2$
11	$3z = 39$	$z = 13$
12	$2w = 30$	$w = 15$
13	$5r = 15$	$r = 3$
14	$15p = 45$	$p = 3$
15	$4b = 28$	$b = 7$
16	$6c = 36$	$c = 6$
17	$2k = 48$	$k = 24$
18	$9y = 90$	$y = 10$
19	$30u = 180$	$u = 6$
20	$15x = 60$	$x = 4$
21	$2v = 34$	$v = 17$
22	$7j = 84$	$j = 12$
23	$5t = 0$	$t = 0$

Solve these equations only this time show your working out.

Examples

$6p = 54$ (÷6 ÷6) $p = 9$

$5d = 3$ (÷5 ÷5) $d = \frac{3}{5}$

24 $3e = 36$ (÷3 ÷3) $e = 12$

25 $9u = 4$ (÷9 ÷9) $u = \frac{4}{9}$

26 $8a = 40$ (÷8 ÷8) $a = 5$

27 $2w = 54$ (÷2 ÷2) $w = 27$

28 $11q = 9$ (÷11 ÷11) $q = \frac{9}{11}$

29 $17t = 7$ (÷17 ÷17) $t = \frac{7}{17}$

30 $12c = 60$ (÷12 ÷12) $c = 5$

31 $15x = 13$ (÷15 ÷15) $x = \frac{13}{15}$

32 $20m = 9$ (÷20 ÷20) $m = \frac{9}{20}$

33 $7r = 42$ (÷7 ÷7) $r = 6$

These will have fraction answers that can be simplified

34 $6j = 3$ (÷6 ÷6) $j = \frac{\cancel{3}^{1}}{\cancel{6}_{2}}$ $j = \frac{1}{2}$

35 $8d = 6$ (÷8 ÷8) $d = \frac{\cancel{6}^{3}}{\cancel{8}_{4}}$ $d = \frac{3}{4}$

36 $15k = 9$ (÷15 ÷15) $k = \frac{\cancel{9}^{3}}{\cancel{15}_{5}}$ $k = \frac{3}{5}$

37 $20e = 12$ (÷20) $e = \frac{\cancel{12}^{3}}{\cancel{20}_{5}}$ $e = \frac{3}{5}$

38 $40a = 25$ (÷40 ÷40) $a = \frac{\cancel{25}^{5}}{\cancel{40}_{8}}$ $a = \frac{5}{8}$

39 $12v = 10$ (÷12 ÷12) $v = \frac{\cancel{10}^{5}}{\cancel{12}_{6}}$ $v = \frac{5}{6}$

40 $24f = 6$ (÷24 ÷24) $f = \frac{\cancel{6}^{1}}{\cancel{24}_{4}}$ $f = \frac{1}{4}$

41 $20s = 16$ (÷20 ÷20) $s = \frac{\cancel{16}^{4}}{\cancel{20}_{5}}$ $s = \frac{4}{5}$

42 $30h = 12$ (÷30 ÷30) $h = \frac{\cancel{12}^{2}}{\cancel{30}_{5}}$ $h = \frac{2}{5}$

43 $35x = 28$ (÷35 ÷35) $x = \frac{\cancel{28}^{4}}{\cancel{35}_{5}}$ $x = \frac{4}{5}$

These can be changed from improper fractions to mixed numerals

44 $4t = 9$ (÷4 ÷4) $t = \frac{9}{4}$ $t = 2\frac{1}{4}$

45 $3c = 11$ (÷3 ÷3) $c = \frac{11}{3}$ $c = 3\frac{2}{3}$

46 $8n = 35$ (÷8 ÷8) $n = \frac{35}{8}$ $n = 4\frac{3}{8}$

47 $14d = 33$ (÷14 ÷14) $d = \frac{33}{14}$ $d = 2\frac{5}{14}$

48 $11z = 50$ (÷11 ÷11) $z = \frac{50}{11}$ $z = 4\frac{6}{11}$

49 $4k = 33$ (÷4 ÷4) $k = \frac{33}{4}$ $k = 8\frac{1}{4}$

Further One Step Equations - Multiplication

This sheet involves the use of negative numbers in one step equations. To answer this sheet you should have first completed the 'Integers' folder or already have learnt about negative numbers. The methods used in this sheet are identical to the previous sheet except that the rules for operations with negative numbers are included.

Column 1 begins with revision of operations with negative numbers, you are asked to write in the answer (with the sign) in the box provided. Remember that:

- negative × (or ÷) negative = positive
- negative × (or ÷) positive = negative
- positive × (or ÷) negative = negative

The second part of column 1 is the same as above except the box is replaced by a letter and you have to specify the value of the letter. Talk it through to yourself if it makes it easier. So if you had:

$8b = -40$: say→ eight times something gives me -40. I know 8 times 5 gives me 40, so eight times -5 must give me -40.

so $b = -5$.

Note that you can test numbers to see if they work and modify your number until you get your answer also.

Column 2 uses division by the number (including the sign) that is in front of the letter. Look at the examples at the top of the column. The questions have the letter on the right hand side in the second part of the column. Use the same method then reverse your answer, as shown in the example at the base of this sheet.

Column 3 is the same as the third column on the previous sheet. Simplify the fraction on the first line by dividing through by the HCF. From Q 44 the answers are to be expressed as mixed numerals. Have a look at the examples below.

$-35 = -7x$

÷ -7 ÷ -7

$5 = x$

$x = 5$

$-32u = 56$

÷ -32 ÷ -32

$u = \frac{56}{-32}$ (7, -4)

$u = \frac{-7}{4}$

$-5w = -19$

÷ -5 ÷ -5

$w = \frac{-19}{-5}$

$w = 3\frac{4}{5}$

Further One Step Equations - Multiplication

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Find the solution to these and place the number in the box

1 $3 \times \square = -6$

2 $5 \times \square = -20$

3 $-7 \times \square = -35$

4 $-9 \times \square = 45$

5 $-6 \times \square = -66$

6 $3 \times \square = -15$

7 $2 \times \square = -2$

8 $-5 \times \square = -50$

9 $4 \times \square = -36$

10 $1 \times \square = -1$

Now try these without working, test numbers to help you if necessary.

11 $6r = -24$ $r =$ _____

12 $-9e = 72$ _____

13 $-5q = -25$ _____

14 $2y = -4$ _____

15 $-8v = 32$ _____

16 $-3j = -33$ _____

17 $7k = -28$ _____

18 $-5c = 0$ _____

19 $-4n = -60$ _____

20 $13g = -52$ _____

21 $4p = -4$ _____

Solve these equations, only this time show your working out.

Examples

$6x = -54$

$\div 6$ $\div 6$

$x = -9$

$-3h = -33$

$\div -3$ $\div -3$

$h = 11$

22 $12q = -24$ _____

23 $-7t = -70$ _____

24 $15d = -45$ _____

25 $-4a = 36$ _____

26 $-10h = 50$ _____

27 $-b = 80$ _____

28 $3c = -39$ _____

29 $-8m = -48$ _____

30 $15i = -90$ _____

31 $-2y = 54$ _____

32 $-5e = -75$ _____

33 $4s = -100$ _____

Solve these then reverse the answer so that the letter is on the left.

34 $38 = -2x$ _____

35 $-28 = -4r$ _____

36 $-56 = 7w$ _____

37 $75 = -3f$ _____

The fraction answers will need to be simplified. From Q 44 mixed numeral answers are required.

38 $-20e = -8$

39 $16v = -12$

40 $-45t = 20$

41 $30h = -24$

42 $-6g = -3$

43 $60p = -48$

44 $-3a = -8$

45 $-4w = 11$

46 $5n = -12$

47 $-7b = 24$

48 $-12c = -17$

49 $-3x = 5$

Further One Step Equations - Multiplication

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Find the solution to these and place the number in the box

1 $3 \times \boxed{-2} = -6$

2 $5 \times \boxed{-4} = -20$

3 $-7 \times \boxed{5} = -35$

4 $-9 \times \boxed{-5} = 45$

5 $-6 \times \boxed{11} = -66$

6 $3 \times \boxed{-5} = -15$

7 $2 \times \boxed{-1} = -2$

8 $-5 \times \boxed{10} = -50$

9 $4 \times \boxed{-9} = -36$

10 $1 \times \boxed{-1} = -1$

Now try these without working, test numbers to help you if necessary.



11 $6r = -24$ $r = -4$

12 $-9e = 72$ $e = -8$

13 $-5q = -25$ $q = 5$

14 $2y = -4$ $y = -2$

15 $-8v = 32$ $v = -4$

16 $-3j = -33$ $j = 11$

17 $7k = -28$ $k = -4$

18 $-5c = 0$ $c = 0$

19 $-4n = -60$ $n = 15$

20 $13g = -52$ $g = -4$

21 $4p = -4$ $p = -1$



Solve these equations, only this time show your working out.

Examples

$6x = -54$

$\div 6 \quad \div 6$

$x = -9$

$-3h = -33$

$\div -3 \quad \div -3$

$h = 11$

22 $12q = -24$

$\div 12 \quad \div 12$

$q = -2$

23 $-7t = -70$

$\div -7 \quad \div -7$

$t = 10$

24 $15d = -45$

$\div 15 \quad \div 15$

$d = -3$

25 $-4a = 36$

$\div -4 \quad \div -4$

$a = -9$

26 $-10h = 50$

$\div -10 \quad \div -10$

$h = -5$

27 $-b = 80$

$\div -1 \quad \div -1$

$b = -80$

28 $3c = -39$

$\div 3 \quad \div 3$

$c = -13$

29 $-8m = -48$

$\div -8 \quad \div -8$

$m = 6$

30 $15i = -90$

$\div 15 \quad \div 15$

$i = -6$

31 $-2y = 54$

$\div -2 \quad \div -2$

$y = -27$

32 $-5e = -75$

$\div -5 \quad \div -5$

$e = 15$

33 $4s = -100$

$\div 4 \quad \div 4$

$s = -25$



Solve these then reverse the answer so that the letter is on the left.

34 $38 = -2x$

$\div -2 \quad \div -2$

$-19 = x$

$x = -19$

35 $-28 = -4r$

$\div -4 \quad \div -4$

$7 = r$

$r = 7$

36 $-56 = 7w$

$\div 7 \quad \div 7$

$-8 = w$

$w = -8$

37 $75 = -3f$

$\div -3 \quad \div -3$

$-25 = f$

$f = -25$



The fraction answers will need to be simplified. From Q 44 mixed numeral answers are required.

38 $-20e = -8$

$\div -20 \quad \div -20$

$e = \frac{\cancel{-8}^{2}}{\cancel{-20}_{5}}$

$e = \frac{2}{5}$

39 $16v = -12$

$\div 16 \quad \div 16$

$v = \frac{\cancel{-12}^{-3}}{\cancel{16}_{4}}$

$v = \frac{-3}{4}$

40 $-45t = 20$

$\div -45 \quad \div -45$

$t = \frac{\cancel{20}^{-4}}{\cancel{-45}_{9}}$

$t = \frac{-4}{9}$

41 $30h = -24$

$\div 30 \quad \div 30$

$h = \frac{\cancel{-24}^{-4}}{\cancel{30}_{5}}$

$h = \frac{-4}{5}$

42 $-6g = -3$

$\div -6 \quad \div -6$

$g = \frac{\cancel{-3}^{1}}{\cancel{-6}_{2}}$

$g = \frac{1}{2}$

43 $60p = -48$

$\div 60 \quad \div 60$

$p = \frac{\cancel{-48}^{-4}}{\cancel{60}_{5}}$

$p = \frac{-4}{5}$

44 $-3a = -8$

$\div -3 \quad \div -3$

$a = \frac{-8}{-3}$

$a = 2\frac{2}{3}$

45 $-4w = 11$

$\div -4 \quad \div -4$

$w = \frac{11}{-4}$

$w = -2\frac{3}{4}$

46 $5n = -12$

$\div 5 \quad \div 5$

$n = \frac{-12}{5}$

$n = -2\frac{2}{5}$

47 $-7b = 24$

$\div -7 \quad \div -7$

$b = \frac{24}{-7}$

$b = -3\frac{3}{7}$

48 $-12c = -17$

$\div -12 \quad \div -12$

$c = \frac{-17}{-12}$

$c = 1\frac{5}{12}$

49 $-3x = 5$

$\div -3 \quad \div -3$

$x = \frac{5}{-3}$

$x = -1\frac{2}{3}$

Solving One Step Equations - Division

These one step equations have the letter being divided by a number (or decimal). As with addition, subtraction and multiplication you will use the opposite operation to solve these exercises. The opposite sign to ÷ is ×, so you will use multiplication to solve these questions.

Column 1 starts with questions that you should be able to complete mentally. Talk it through to yourself, for example if you have $^{t}/_{6} = 4$:
$^{t}/_{6} = 4$ ….. say something divided by 6 equals 4
t = 24 ….. say 24 divided by 6 is equal to 4, so $t = 24$

Column 2 then uses multiplication to solve the problems. Look at the number that the letter is being divided by and multiply both sides by it. Examples are at the top of the column.

Column 3 uses the same solving method except decimals are involved. Follow the examples at the top of the column then see if you can answer the first 4 questions mentally. From then on when you multiply, use the multiplication working space to get your answer. Write your answer and remember to write the letter then '=' before your answer.

Solving One Step Equations - Division

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Find the value for the letter that will make these equations true.

1 $\frac{d}{2} = 5$ $d =$ _____

2 $\frac{t}{6} = 3$ _____

3 $\frac{b}{4} = 3$ _____

4 $\frac{c}{5} = 3$ _____

5 $\frac{f}{6} = 4$ _____

6 $\frac{n}{10} = 4$ _____

7 $\frac{p}{7} = 3$ _____

8 $\frac{q}{6} = 8$ _____

9 $\frac{k}{5} = 10$ _____

10 $\frac{j}{2} = 9$ _____

11 $\frac{v}{3} = 8$ _____

12 $\frac{m}{12} = 3$ _____

13 $\frac{y}{8} = 4$ _____

14 $\frac{h}{7} = 6$ _____

15 $\frac{r}{2} = 17$ _____

16 $\frac{u}{20} = 8$ _____



Now solve these equations showing working

Examples

$\frac{d}{7} = 8$ (×7) (×7) $d = 56$

$\frac{v}{11} = 5$ (×11) (×11) $v = 55$

17 $\frac{x}{9} = 6$ _____

18 $\frac{h}{15} = 3$ _____

19 $\frac{q}{12} = 10$ _____

20 $\frac{c}{7} = 9$ _____

21 $\frac{k}{5} = 15$ _____

22 $\frac{w}{20} = 4$ _____

23 $\frac{n}{6} = 21$ _____

24 $\frac{b}{10} = 17$ _____

25 $\frac{m}{53} = 2$ _____

26 $\frac{d}{4} = 25$ _____

27 $\frac{y}{31} = 3$ _____

28 $\frac{a}{5} = 40$ _____

29 $\frac{p}{4} = 12$ _____

30 $\frac{e}{10} = 13$ _____

31 $\frac{f}{9} = 8$ _____

32 $\frac{h}{6} = 30$ _____



This time the questions involve decimals. Try to answer without a calculator, spaces are provided.

Examples

$\frac{k}{4} = 3.2$ (×4) (×4) $k = 12.8$

$\frac{n}{2.1} = 5$ (×2.1) (×2.1) $v = 10.5$

33 $\frac{a}{2} = 3.4$ _____

34 $\frac{e}{4.3} = 3$ _____

35 $\frac{h}{7.1} = 6$ _____

36 $\frac{t}{3} = 5.2$ _____

37 $\frac{n}{2.9} = 7$ _____

2.9 ×
7
_____

38 $\frac{q}{4.7} = 5$ _____

×
_____

39 $\frac{t}{3} = 9.6$ _____

×
_____

40 $\frac{b}{0.8} = 4$ _____

×
_____

41 $\frac{k}{5} = 7.4$ _____

×
_____

Solving One Step Equations - Division

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Find the value for the letter that will make these equations true.

1 $\frac{d}{2} = 5$ — $d = 10$

2 $\frac{t}{6} = 3$ — $t = 18$

3 $\frac{b}{4} = 3$ — $b = 12$

4 $\frac{c}{5} = 3$ — $c = 15$

5 $\frac{f}{6} = 4$ — $f = 24$

6 $\frac{n}{10} = 4$ — $n = 40$

7 $\frac{p}{7} = 3$ — $p = 21$

8 $\frac{q}{6} = 8$ — $q = 48$

9 $\frac{k}{5} = 10$ — $k = 50$

10 $\frac{j}{2} = 9$ — $j = 18$

11 $\frac{v}{3} = 8$ — $v = 24$

12 $\frac{m}{12} = 3$ — $m = 36$

13 $\frac{y}{8} = 4$ — $y = 32$

14 $\frac{h}{7} = 6$ — $h = 42$

15 $\frac{r}{2} = 17$ — $r = 34$

16 $\frac{u}{20} = 8$ — $u = 160$



Now solve these equations showing working

Examples

$\frac{d}{7} = 8$ (×7 ×7) $d = 56$

$\frac{v}{11} = 5$ (×11 ×11) $v = 55$

17 $\frac{x}{9} = 6$ (×9 ×9) $x = 54$

18 $\frac{h}{15} = 3$ (×15 ×15) $h = 45$

19 $\frac{q}{12} = 10$ (×12 ×12) $q = 120$

20 $\frac{c}{7} = 9$ (×7 ×7) $c = 63$

21 $\frac{k}{5} = 15$ (×5 ×5) $k = 75$

22 $\frac{w}{20} = 4$ (×20 ×20) $w = 80$

23 $\frac{n}{6} = 21$ (×6 ×6) $n = 126$

24 $\frac{b}{10} = 17$ (×10 ×10) $b = 170$

25 $\frac{m}{53} = 2$ (×53 ×53) $m = 106$

26 $\frac{d}{4} = 25$ (×4 ×4) $d = 100$

27 $\frac{y}{31} = 3$ (×31 ×31) $y = 93$

28 $\frac{a}{5} = 40$ (×5 ×5) $a = 200$

29 $\frac{p}{4} = 12$ (×4 ×4) $p = 48$

30 $\frac{e}{10} = 13$ (×10 ×10) $e = 130$

31 $\frac{f}{9} = 8$ (×9 ×9) $f = 72$

32 $\frac{h}{6} = 30$ (×6 ×6) $h = 180$



This time the questions involve decimals. Try to answer without a calculator, spaces are provided.

Examples

$\frac{k}{4} = 3.2$ (×4 ×4) $k = 12.8$

$\frac{n}{2.1} = 5$ (×2.1 ×2.1) $v = 10.5$

33 $\frac{a}{2} = 3.4$ (×2 ×2) $a = 6.8$

34 $\frac{e}{4.3} = 3$ (×4.3 ×4.3) $e = 12.9$

35 $\frac{h}{7.1} = 6$ (×7.1 ×7.1) $h = 42.6$

36 $\frac{t}{3} = 5.2$ (×3 ×3) $t = 15.6$

37 $\frac{n}{2.9} = 7$ (×2.9 ×2.9) $n = 20.3$

2.9×7 (6) $= 20.3$

38 $\frac{q}{4.7} = 5$ (×4.7 ×4.7) $q = 23.5$

4.7×5 (3) $= 23.5$

39 $\frac{t}{3} = 9.6$ (×3 ×3) $t = 28.8$

9.6×3 (1) $= 28.8$

40 $\frac{b}{0.8} = 4$ (×0.8 ×0.8) $b = 3.2$

0.8×4 (3) $= 3.2$

41 $\frac{k}{5} = 7.4$ (×5 ×5) $k = 37$

7.4×5 (2) $= 37.0$

Further One Step Division Equations and Solutions to Equations

This sheet involves using negative numbers, you should have covered 'Integers' before you attempt this sheet.

Column 1 carries on from the previous sheet only this time negative numbers are involved. Examples are at the top of the column, the method is the same as the previous sheet.

Column 2 uses substitution, (when the letter is replaced by a number). These questions aren't to find the value for the letter that makes the equation true. A value is already given and you are asked to test to see whether it is a solution. This is done by replacing the letter with the value given in each question, simplifying the left hand side of the equation (L.H.S.) and checking if it equals the right hand side (R.H.S.) of the equation.

If the same number is on both sides of the = sign then the value 'satisfies' the equation (it is a solution). If the numbers aren't the same on either side of the = sign then you write ≠ (not equal to) and say that it isn't a solution. As in the example at the top of column 2.

Column 3 follows on from column 2 except that brackets are introduced. You have to evaluate the brackets before you multiply by the outside number.

Test for $b = 6$

$2(b - 9) = -6$

$2 \times (6 - 9) = -6$ (-3)

$-6 = -6$

LHS = RHS so 6 satisfies the equation.

Further One Step Division Equations and Solutions to Equations

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Find the value of the letter that make these true.

Examples

$\frac{j}{6} = -12$ (× 6, × 6)

$j = -72$

$\frac{n}{-8} = -5$ (× -8, × -8)

$n = 40$

1 $\frac{a}{5} = -6$

2 $\frac{t}{-7} = 9$

3 $\frac{h}{-3} = -15$

4 $\frac{y}{5} = -12$

5 $\frac{t}{7} = -1$

6 $\frac{m}{-6} = -3$

7 $\frac{u}{9} = -5$

8 $\frac{b}{16} = -2$

9 $\frac{d}{-3} = -11$

10 $\frac{g}{-5} = 60$

11 $\frac{f}{20} = -12$

12 $\frac{q}{-7} = -8$

13 $\frac{m}{17} = -3$

14 $\frac{e}{15} = -5$

15 $\frac{x}{19} = -2$

16 $\frac{k}{-2} = -1$



Substitute the value for the letter to see if it is a solution for the equation

Example

$g + 17 = 40$ Try $g = 33$

$33 + 17 = 40$ LHS ≠ RHS so 33

$50 \neq 40$ is not a solution

17 Test for $n = 31$

$n - 12 = 19$

18 Test for $t = 16$

$t + 37 = 63$

19 Test for $m = 7$

$3m = 23$

20 Test for $u = -4$

$8u = 32$

21 Test for $c = -6$

$19 - c = 25$

22 Test for $q = 9$

$-5 - q = 14$

23 Test for $a = -3$

$-6a = 18$

24 Test for $k = -8$

$k^2 = 64$

25 Test for $g = 5$

$7(g + 3) = 56$

26 Test for $d = 12$

$5(9 - d) = -2$

27 Test for $x = -27$

$\frac{x}{-3} = -9$

28 Test for $e = -9$

$2(3 - e) = 24$

29 Test for $v = 3$

$5(v - 8) = -30$

Further One Step Division Equations and Solutions to Equations

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Find the value of the letter that make these true.

Examples

$\frac{j}{6} = -12$ (× 6 / × 6)

$j = -72$

$\frac{n}{-8} = -5$ (× -8 / × -8)

$n = 40$

1 $\frac{a}{5} = -6$ (× 5 / × 5)

$a = -30$

2 $\frac{t}{-7} = 9$ (× -7 / × -7)

$t = -63$

3 $\frac{h}{-3} = -15$ (× -3 / × -3)

$h = 75$

4 $\frac{y}{5} = -12$ (× 5 / × 5)

$y = -60$

5 $\frac{t}{7} = -1$ (× 7 / × 7)

$t = -7$

6 $\frac{m}{-6} = -3$ (× -6 / × -6)

$m = 18$

7 $\frac{u}{9} = -5$ (× 9 / × 9)

$u = -45$

8 $\frac{b}{16} = -2$ (× 16 / × 16)

$b = -32$

9 $\frac{d}{-3} = -11$ (× -3 / × -3)

$d = 33$

10 $\frac{g}{-5} = 60$ (× -5 / × -5)

$g = -300$

11 $\frac{f}{20} = -12$ (× 20 / × 20)

$f = -240$

12 $\frac{q}{-7} = -8$ (× -7 / × -7)

$q = 56$

13 $\frac{m}{17} = -3$ (× 17 / × 17)

$m = -51$

14 $\frac{e}{15} = -5$ (× 15 / × 15)

$e = -75$

15 $\frac{x}{19} = -2$ (× 19 / × 19)

$x = -38$

16 $\frac{k}{-2} = -1$ (× -2 / × -2)

$k = 2$

Substitute the value for the letter to see if it is a solution for the equation

Example

$g + 17 = 40$ Try $g = 33$

$33 + 17 = 40$ LHS ≠ RHS so 33

$50 \neq 40$ is not a solution

17 Test for $n = 31$

$n - 12 = 19$ LHS = RHS so 31

$31 - 12 = 19$ is a solution

$19 = 19$

18 Test for $t = 16$

$t + 37 = 63$ LHS ≠ RHS so 16

$16 + 37 = 63$ is not a solution

$53 \neq 63$

19 Test for $m = 7$

$3m = 23$ LHS ≠ RHS so 7 is

$3 \times 7 = 23$ not a solution

$21 \neq 23$

20 Test for $u = -4$

$8u = 32$ LHS ≠ RHS so

$8 \times -4 = 32$ -4 is not a

$-32 \neq 32$ solution

21 Test for $c = -6$

$19 - c = 25$ LHS = RHS so

$19 - -6 = 25$ -6 is a solution

$25 = 25$

22 Test for $q = 9$

$-5 - q = 14$ LHS ≠ RHS so 9

$-5 - 9 = 14$ is not a solution

$-14 \neq 14$

23 Test for $a = -3$

$-6a = 18$ LHS = RHS so

$-6 \times -3 = 18$ -3 is a solution

$18 = 18$

24 Test for $k = -8$

$k^2 = 64$ LHS = RHS so

$(-8)^2 = 64$ -8 is a solution

$64 = 64$

25 Test for $g = 5$

$7(g + 3) = 56$ LHS = RHS so

$7 \times (5 + 3) = 56$ (8) 5 is a solution

$56 = 56$

26 Test for $d = 12$

$5(9 - d) = -2$ LHS ≠ RHS so

$5 \times (9 - 12) =$ (-3) 12 is not a

$-15 \neq -2$ solution

27 Test for $x = -27$

$\frac{x}{-3} = -9$ LHS ≠ RHS so

-9 is not a

$\frac{-27}{-3} = -9$ solution

$9 \neq 9$

28 Test for $e = -9$

$2(3 - e) = 24$ LHS = RHS so

$2 \times (3 - -9) =$ (12) -9 is a solution

$24 = 24$

29 Test for $v = 3$

$5(v - 8) = -30$ LHS ≠ RHS so 3

$5 \times (3 - 8) = -30$ (-5) is not a solution

$-25 \neq -30$

Powers and Expansion

This sheet has some questions written sideways. To read these questions click on the "View" menu, select the rotate option then "counter clockwise".

When you use powers with pronumerals (letters) it is important that you understand what the power means. Unlike numbers raised to a power, you can't use a calculator to get an answer with letters.

Column 1 gives you expanded expressions (expressions don't have an =, equations have an =) that you are asked to simplify. Expanded form is when the expression is broken up so that there are no powers, with each piece being separated by an × sign. The way you answer these is:

- Multiply all the numbers together and write the number part of the answer
- Count the number of same letters then write the letter and the number straight after it, small and raised, e.g. a^2. If there is only 1 letter then you don't need to write a number.
- Are there other letters in the question? If there are, count the number of those letters and repeat the process.

Column 2 asks you to expand the expressions. Use the opposite method. Write the number then an '=' sign. Then the letters separated by × signs, the number of letters being the power, no power means one letter.

Column 3 is a brief introduction to multiplying with powers. The column starts with asking you to expand out the multiplication, then simplify. It's the same as the previous work. You may notice a rule that applies to multiplying indices. This rule is that you add the powers when you multiply. This is dealt with by the second part of the column, there are examples in the column. Just remember when a letter is by itself, it is as if there is a 1 as its power. So $m = m^1$.

Powers and Expansion

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Simplify these expressions by removing the × signs and rewriting using powers

Example $4 \times j \times v \times 6 \times j \times j = 24vj^3$

1 $y \times y \times y \times y$ =

2 $m \times m \times m \times m \times m$ =

3 $a \times a$ =

4 $h \times h \times h$ =

5 $6 \times t \times t \times t$ =

6 $u \times u \times 4 \times u \times u$ =

7 $2 \times b \times b \times 3 \times b \times b$ =

8 $a \times x \times x$ =

9 $n \times n \times c \times c \times c$ =

10 $8 \times i \times i \times h \times i \times i$ =

11 $d \times a \times d \times d \times a \times a$ =

12 $e \times e \times 3 \times q \times e \times e \times 7 \times q$ =

13 $c \times c \times 5 \times c \times b \times c \times c \times 4$ =

14 $6 \times g \times b \times b \times g \times g \times b \times 9$ =

15 $8 \times c \times b \times a \times c \times b \times c \times 5$ =

16 $s \times t \times s \times t \times v \times 4 \times s \times s$ =

17 $y \times y \times 3 \times a \times 2 \times x \times a \times 4$ =

Now the opposite, expand these removing powers and inserting × signs

Example $6am^2 = 6 \times a \times m \times m$

18 $7b^2$ =

19 $10k^4$ =

20 $2t^3$ =

21 $5qx^3$ =

22 d^2c^4 =

23 $4r^3b^4$ =

24 $57ud^3$ =

25 $8y^4s^3$ =

26 $3r^2d^2$ =

27 ab^2c^3 =

28 $6mr^3t$ =

29 y^2ib^3 =

30 $12x^2y^2$ =

31 $10d^3aw$ =

32 $3m^2qk^3$ =

33 dv^4h^2 =

34 $2s^2rx^3$ =

35 $38q^2vb^2$ =

36 $9k^3g^2e$ =

37 $2cgpq^2$ =

38 x^2atf^3 =

Expand these and then rewrite them as simplified indices

Example $h^2 \times h^2 = h \times h \times h \times h = h^4$ ($h^2 = h \times h$)

39 $a^2 \times a^3$

= =

40 $b^4 \times b^2$

= =

41 $n^3 \times n$

= =

42 $3c^2 \times c^2$

= =

43 $3y^3 \times 4y$

= =

Sum the powers to find the answer to these

Examples $h^7 \times h^9 = h^{16}$ ($7 + 9 = 16$) $k \times k^6 = k^7$ ($k = k^1$, $1 + 6 = 7$)

$5x^5 \times 4x^7 = 20x^{12}$

43 $u^3 \times u^8$ =

44 $t^9 \times t^{11}$ =

45 $c^5 \times c^{17}$ =

46 $3x^{14} \times x^{17}$ =

47 $5d^3 \times 7d$ =

48 $2s^5 \times 4s^5$ =

49 $7q^6 \times 9q^3$ =

50 $5w^4 \times 2w \times 3w^2$ =

51 $4e^4 \times 3e^5 \times e^2$ =

Powers and Expansion

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Simplify these expressions by removing the × signs and rewriting using powers

Example $4 \times j \times v \times 6 \times j \times j = 24vj^3$

1 $y \times y \times y \times y = y^4$

2 $m \times m \times m \times m \times m = m^5$

3 $a \times a = a^2$

4 $h \times h \times h = h^3$

5 $6 \times t \times t \times t = 6t^3$

6 $u \times u \times 4 \times u \times u = 4u^4$

7 $2 \times b \times b \times 3 \times b \times b = 6b^4$

8 $a \times x \times x = ax^2$

9 $n \times n \times c \times c \times c = n^2c^3$

10 $8 \times i \times i \times h \times i \times i = 8i^4h$

11 $d \times a \times d \times d \times a \times a = d^3a^3$

12 $e \times e \times 3 \times q \times e \times e \times 7 \times q = 21e^4q^2$

13 $c \times c \times 5 \times c \times b \times c \times c \times 4 = 20bc^5$

14 $6 \times g \times b \times b \times g \times g \times b \times 9 = 54g^3b^3$

15 $8 \times c \times b \times a \times c \times b \times c \times 5 = 40ab^2c^3$

16 $s \times t \times s \times t \times v \times 4 \times s \times s = 4vs^4t^2$

17 $y \times y \times 3 \times a \times 2 \times x \times a \times 4 = 24a^2xy^2$

Now the opposite, expand these removing powers and inserting × signs

Example $6am^2 = 6 \times a \times m \times m$

18 $7b^2 = 7 \times b \times b$

19 $10k^4 = 10 \times k \times k \times k \times k$

20 $2t^3 = 2 \times t \times t \times t$

21 $5qx^3 = 5 \times q \times x \times x \times x$

22 $d^2c^4 = d \times d \times c \times c \times c \times c$

23 $4r^3b^4 = 4 \times r \times r \times r \times b \times b \times b \times b$

24 $57ud^3 = 57 \times u \times d \times d \times d$

25 $8y^4s^3 = 8 \times y \times y \times y \times y \times s \times s \times s$

26 $3r^2d^2 = 3 \times r \times r \times d \times d$

27 $ab^2c^3 = a \times b \times b \times c \times c \times c$

28 $6mr^3t = 6 \times m \times r \times r \times r \times t$

29 $y^2ib^3 = y \times y \times i \times b \times b \times b$

30 $12x^2y^2 = 12 \times x \times x \times y \times y$

31 $10d^3aw = 10 \times d \times d \times d \times a \times w$

32 $3m^2qk^3 = 3 \times m \times m \times q \times k \times k \times k$

33 $dv^4h^2 = d \times v \times v \times v \times v \times h \times h$

34 $2s^2rx^3 = 2 \times s \times s \times r \times x \times x \times x$

35 $38q^2vb^2 = 38 \times q \times q \times v \times b \times b$

36 $9k^3g^2e = 9 \times k \times k \times k \times g \times g \times e$

37 $2cgpq^2 = 2 \times c \times g \times p \times q \times q$

38 $x^2atf^3 = x \times x \times a \times t \times f \times f \times f$

Expand these and then rewrite them as simplified indices

Example $h^2 \times h^2 = h \times h \times h \times h = h^4$ ($h^2 = h \times h$, $h^2 = h \times h$)

39 $a^2 \times a^3 = a \times a \times a \times a \times a = a^5$

40 $b^4 \times b^2 = b \times b \times b \times b \times b \times b = b^6$

41 $n^3 \times n = n \times n \times n \times n = n^4$

42 $3c^2 \times c^2 = 3 \times c \times c \times c \times c = 3c^4$

43 $3y^3 \times 4y = 3 \times y \times y \times y \times 4 \times y = 12y^4$

Sum the powers to find the answer to these

Examples $h^7 \times h^9 = h^{16}$ ($7 + 9 = 16$), $k \times k^6 = k^7$ ($k = k^1$, $1 + 6 = 7$)

$5x^5 \times 4x^7 = 20x^{12}$

44 $u^3 \times u^8 = u^{11}$

45 $t^9 \times t^{11} = t^{20}$

46 $c^5 \times c^{17} = c^{22}$

47 $3x^{14} \times x^{17} = 3x^{31}$

48 $5d^3 \times 7d = 35d^4$

49 $2s^5 \times 4s^5 = 8s^{10}$

50 $7q^6 \times 9q^3 = 63q^9$

51 $5w^4 \times 2w \times 3w^2 = 30w^7$

52 $4e^4 \times 3e^5 \times e^2 = 12e^{11}$

Substitution

Substitute means to replace with. In mathematics you are often asked to replace a letter with a number and find the answer. There are two important things to remember, the first is when there is no sign between letters then it is as if there is a × between them.
For example wr is $w \times r$, so if $w = 4$ and $r = 2$ then $wr = 4 \times 2 = 8$ not 42.

The second point is powers, so if $w = 4$ then : $w^2 = w \times w = 4 \times 4 = 16$ (or $4^2 = 16$),
not $w^2 = 4 \times 2 = 8$. A frequent mistake.

Column 1 starts with questions that ask you to find the values of two pronumerals. Substitute in the numbers on the second line (after an equals sign), then on the third line write the letter = and your answer. Question 1 has the format shown for you to start you off. From question 4 you may encounter questions that order of operations rules apply. Remember × and ÷ before + and -, be careful.

Column 2 is the same as column 1 except you may find the numbers a little more challenging.

Find the values of a and b
given $y = 12$ and $x = 7$

a)	b)
$a = y + 3x$	$b = 2y - x$
$= 12 + 3 \times 7$ 21	$= 2 \times 12 - 7$ 24
$a = 33$	$b = 17$

Column 3 tests your knowledge of negative numbers, remember:

- a negative number × or ÷ a negative number gives a positive number
- a positive number × or ÷ a negative number gives a negative number
- a negative number × or ÷ a positive number gives a negative number
- + then a - = - (+ - = -)
- - then a + = - (- + = -)
- - then a - = + (- - = +)

Substitution

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Substitute the values for the letters and find the answer

1 Find the value of k and r when $x = 7$ and $m = 6$

a) $k = xm$

$= \quad \times$

$k =$

b) $r = x + m$

$= \quad +$

$r =$

2 Evaluate for h and e when $a = 5$ and $t = 3$

a) $h = at$

b) $e = 2a + t$

3 Solve for b and v given $w = 7$ and $c = 2$

a) $b = w^2$

b) $v = 3cw$

4 Find the values of g and d given $n = 12$ and $q = 5$

a) $g = n + 8q$

b) $d = n - 2q$

5 Solve for c, v and y given $t = 6$ and $e = 3$

a) $c = te^2$

b) $v = t + 5e$

c) $y = c + 2v$

6 Solve for q, v and b given $e = 6$ and $x = 3$

a) $q = 2xe$

b) $v = q - 2e$

c) $b = q + v - x$

7 Find the values of r, c and t given $u = 10$ and $a = 4$

a) $r = 3u - a$

b) $c = a^2 - u$

c) $t = 2r - 3c$

8 Solve for x, y and z given $a = 6$ and $b = 2$

a) $x = ab^2$

b) $y = 2x - 18$

c) $z = (y - x)^2$

9 Find the values of d and p given $c = 24$ and $v = 3$

a) $d = c + 3v$

b) $p = 2d - c$

These questions use negative numbers

10 Evaluate for h and n when $t = -5$ and $c = 3$

a) $h = c + t$

b) $n = 3t$

11 Find the values of s and k given $x = -8$ and $d = -4$

a) $s = x - d$

b) $k = d^2$

12 Find the values of n and b given $r = -10$ and $e = 6$

a) $n = r - 2e$

b) $b = 3r + e$

13 Solve for a and b given $u = -4$ and $y = -7$

a) $a = 2u^2$

b) $b = a + y$

14 Find the values of a and k given $p = -3$ and $q = 9$

a) $a = \frac{2q}{p}$

b) $k = \frac{pq}{p^2}$

Substitution

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Substitute the values for the letters and find the answer

1 Find the value of k and r when $x = 7$ and $m = 6$

a) $k = xm$

$= 7 \times 6$

$k = 42$

b) $r = x + m$

$= 7 + 6$

$r = 13$

2 Evaluate for h and e when $a = 5$ and $t = 3$

a) $h = at$

$= 5 \times 3$

$h = 15$

b) $e = 2a + t$

$= \cancel{2} \times \cancel{5}^{10} + 3$

$e = 13$

3 Solve for b and v given $w = 7$ and $c = 2$

a) $b = w^2$

$= 7 \times 7$

$b = 49$

b) $v = 3cw$

$= \cancel{3} \times \cancel{2}^{6} \times 7$

$v = 42$

4 Find the values of g and d given $n = 12$ and $q = 5$

a) $g = n + 8q$

$= 12 + \cancel{8} \times \cancel{5}^{40}$

$g = 52$

b) $d = n - 2q$

$= 12 - \cancel{2} \times \cancel{5}^{10}$

$d = 2$

5 Solve for c, v and y given $t = 6$ and $e = 3$

a) $c = te^2$

$= \cancel{6} \times \cancel{3}^{18} \times 3$

$c = 54$

b) $v = t + 5e$

$= 6 + \cancel{5} \times \cancel{3}^{15}$

$v = 21$

c) $y = c + 2v$

$= 54 + \cancel{2} \times \cancel{21}^{42}$

$y = 96$

6 Solve for q, v and b given $e = 6$ and $x = 3$

a) $q = 2xe$

$= \cancel{2} \times \cancel{3}^{6} \times 6$

$q = 36$

b) $v = q - 2e$

$= 36 - \cancel{2} \times \cancel{6}^{12}$

$v = 24$

c) $b = q + v - x$

$= 36 + 24 - \cancel{3}^{21}$

$b = 57$

7 Find the values of r, c and t given $u = 10$ and $a = 4$

a) $r = 3u - a$

$= \cancel{3} \times 10^{30} - 4$

$r = 26$

b) $c = a^2 - u$

$= \cancel{4} \times \cancel{4}^{16} - 10$

$c = 6$

c) $t = 2r - 3c$

$= \cancel{2} \times 26^{52} - \cancel{3} \times \cancel{6}^{18}$

$t = 34$

8 Solve for x, y and z given $a = 6$ and $b = 2$

a) $x = ab^2$

$= \cancel{6} \times \cancel{2}^{12} \times 2$

$x = 24$

b) $y = 2x - 18$

$= \cancel{2} \times 24^{48} - 18$

$y = 30$

c) $z = (y - x)^2$

$= (\cancel{30} - 24)^{2\ 6}$

$z = 36$

9 Find the values of d and p given $c = 24$ and $v = 3$

a) $d = c + 3v$

$= 24 + \cancel{3} \times \cancel{3}^{9}$

$d = 33$

b) $p = 2d - c$

$= \cancel{2} \times \cancel{33}^{66} - 24$

$p = 42$

These questions use negative numbers

10 Evaluate for h and n when $t = -5$ and $c = 3$

a) $h = c + t$

$= 3 + -5$

$h = -2$

b) $n = 3t$

$= 3 \times (-5)$

$n = -15$

11 Find the values of s and k given $x = -8$ and $d = -4$

a) $s = x - d$

$= -8 - -4$

$s = -4$

b) $k = d^2$

$= (-4)^2$

$k = 16$

12 Find the values of n and b given $r = -10$ and $e = 6$

a) $n = r - 2e$

$= -10 - \cancel{2} \times \cancel{6}^{12}$

$n = -22$

b) $b = 3r + e$

$= \cancel{3} \times -\cancel{10}^{-30} + 6$

$b = -24$

13 Solve for a and b given $u = -4$ and $y = -7$

a) $a = 2u^2$

$= 2 \times (-\cancel{4})^{2\ 16}$

$a = 32$

b) $b = a + y$

$= 32 + -7$

$b = 25$

14 Find the values of a and k given $p = -3$ and $q = 9$

a) $a = \dfrac{2q}{p}$

$= \dfrac{\cancel{2} \times \cancel{9}^{18}}{-3}$

$a = -6$

b) $k = \dfrac{pq}{p^2}$

$= \dfrac{-\cancel{3} \times \cancel{9}^{-27}}{(-\cancel{3})^{2\ 9}}$

$k = -3$

FREEFALL MATHEMATICS

ALTITUDE

GEOMETRY

Bisecting an Interval

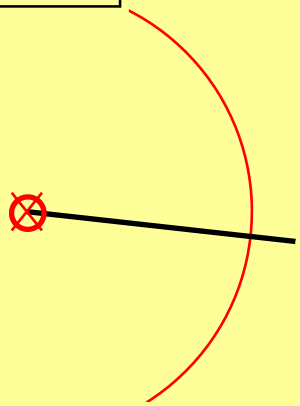
Bisecting something is cutting it into two equal pieces. This sheet deals with bisecting intervals. An interval is a straight line but instead of going on forever it has specific length. The intervals on this sheet have markers at their ends for you to position instruments more effectively.

Column 1 starts with asking you to bisect these five intervals with a ruler. Measure the length of the interval, write your answer in the space, give the answer in mm. Divide this number by 2 and write that answer in the 'half length' space. Mark off half-way with a small stroke.

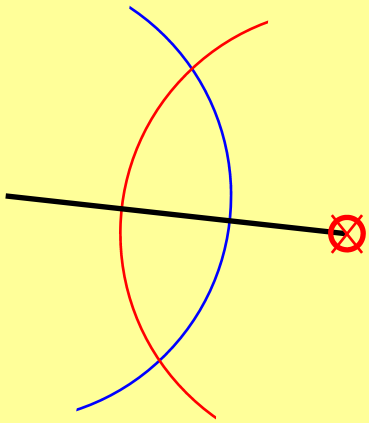
Questions 6 through 10 involve the same 60 mm interval, the questions ask you to break the interval up into a specific number of parts. Divide 60 by the number of parts required in the question and that is the measured gap required between 'marks'.

Columns 2 and 3 are the same, it is asking you to bisect the given intervals using a compass. Below is the process outlined step-by-step.

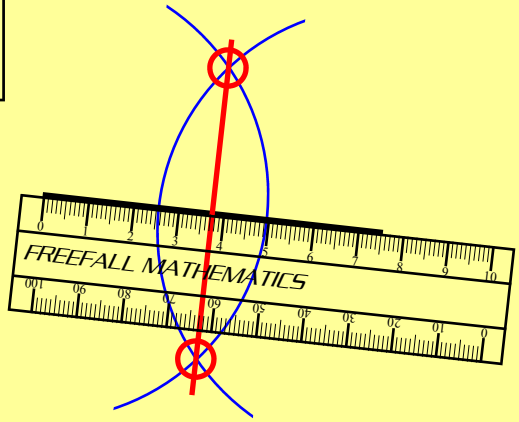
1 Draw an arc from one end of the interval (red circle). Open the compass more than half the interval's length.



2 Repeat for the other end of the interval. Keep the compass set at the same distance.



3 Draw a line through the two intersection points (red circles). Use a ruler to measure the lengths as a check.



Bisecting an Interval

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Using your ruler, measure then bisect these intervals

1 Interval length : _____

Half length : _____

2 Interval length : _____

Half length : _____

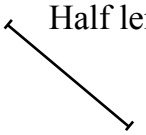
3 Interval length : _____

Half length : _____



4 Interval length : _____

Half length : _____



5 Interval length : _____

Half length : _____



Divide the 60 mm intervals into 3, 4, 5, 6 and 10 parts

6 3 parts, interval = _____ mm

7 4 parts, interval = _____ mm

8 5 parts, interval = _____ mm

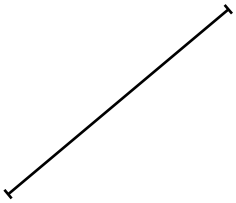
9 6 parts, interval = _____ mm

10 10 parts, interval = _____ mm

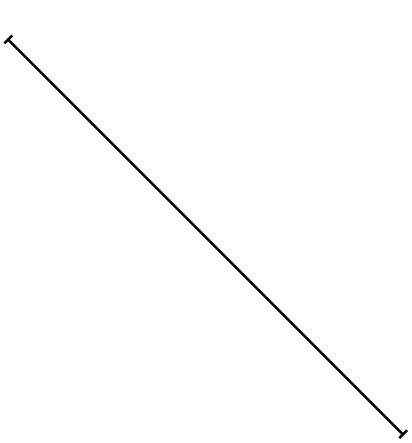
Use a compass to bisect these intervals. Give the measurement from one end to the midpoint (cm)

11 Half length : _____

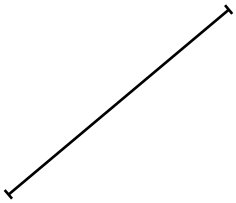
12 Half length : _____



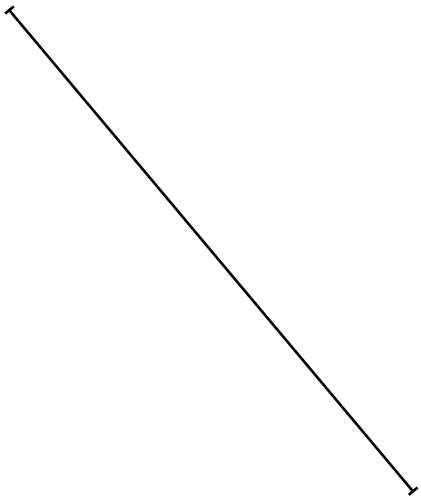
13 Half length : _____



14 Half length : _____



15 Half length : _____



16 Half length : _____

Bisecting an Interval

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Using your ruler, measure then bisect these intervals

1 Interval length : 60 mm

Half length : 30 mm

2 Interval length : 48 mm

Half length : 24 mm

3 Interval length : 36 mm

Half length : 18 mm

4 Interval length : 21 mm

Half length : 10.5 mm

5 Interval length : 42 mm

Half length : 21 mm

Divide the 60 mm intervals into 3, 4, 5, 6 and 10 parts

6 3 parts, interval = 20 mm

7 4 parts, interval = 15 mm

8 5 parts, interval = 12 mm

9 6 parts, interval = 10 mm

10 10 parts, interval = 6 mm

Use a compass to bisect these intervals. Give the measurement from one end to the midpoint (cm)

11 Half length : 2.8 cm

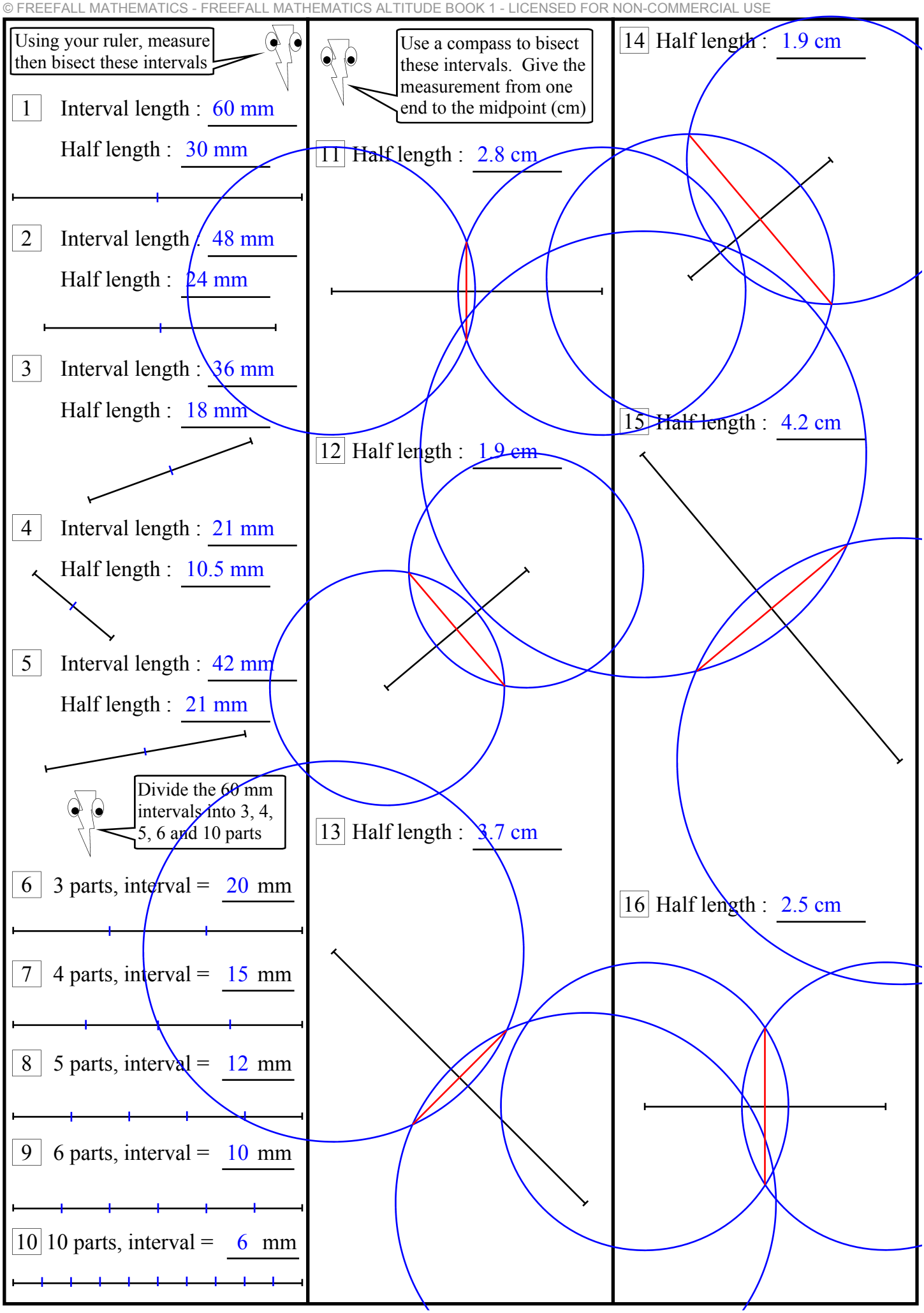
12 Half length : 1.9 cm

13 Half length : 3.7 cm

14 Half length : 1.9 cm

15 Half length : 4.2 cm

16 Half length : 2.5 cm

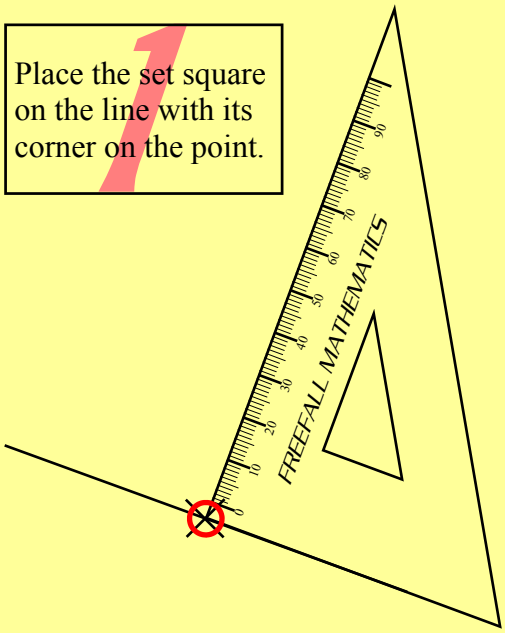


Perpendicular and Parallel Lines

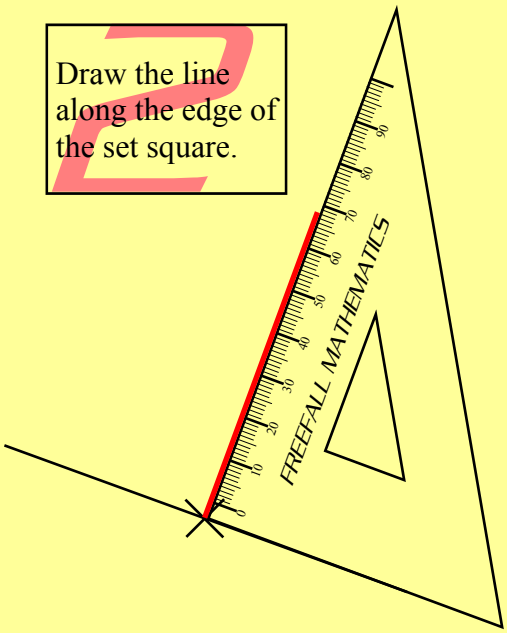
Perpendicular lines are lines that are at right angles to each other, so the angles formed when the lines intersect are 90º. Parallel lines are lines that are the same distance apart throughout their length and so they never intersect (touch each other). This sheet requires the use of a compass and a set square to create parallel and perpendicular lines.

Column 1 - Perpendicular from a point on a line - Set Square

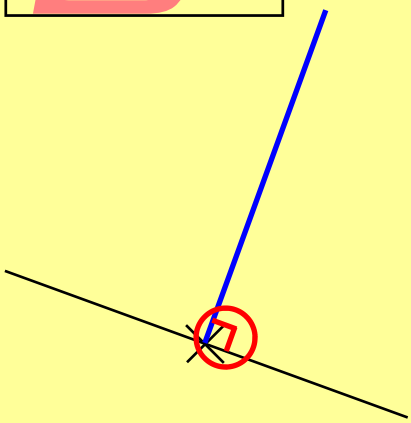
1 Place the set square on the line with its corner on the point.



2 Draw the line along the edge of the set square.

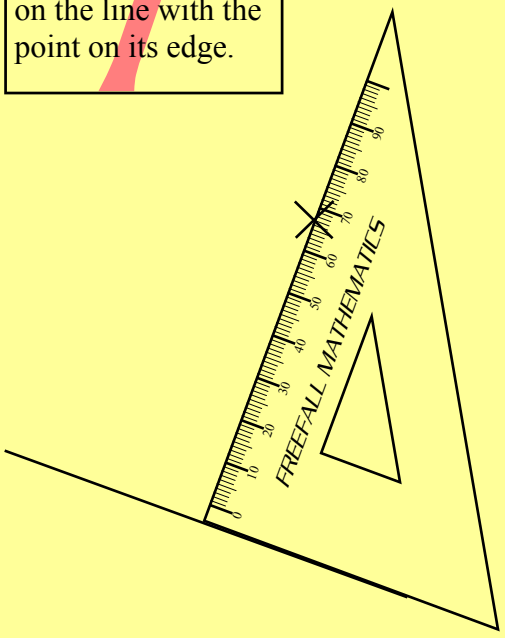


3 Place right angle markings to show it is a perpendicular.

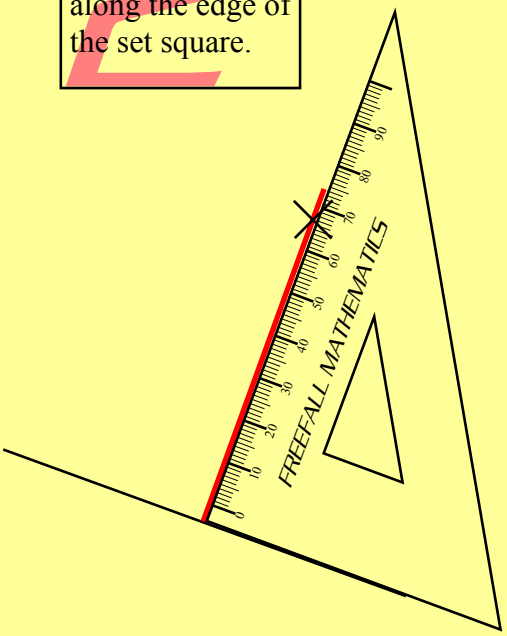


Column 1 - Perpendicular from a point away from a line - Set Square

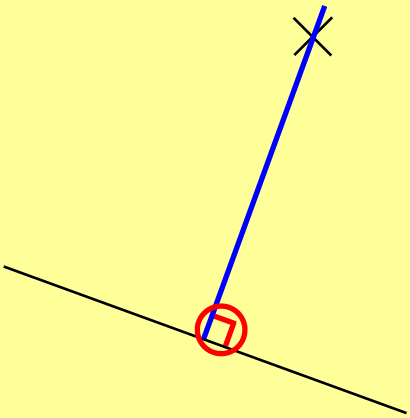
1 Place the set square on the line with the point on its edge.



2 Draw the line along the edge of the set square.

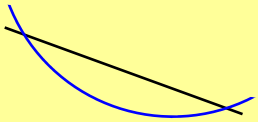


3 Place right angle markings to show it is a perpendicular.

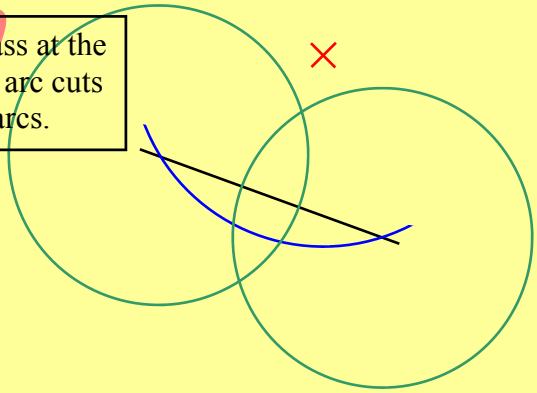


Column 2 - Perpendicular from a point away from on a line - Compass

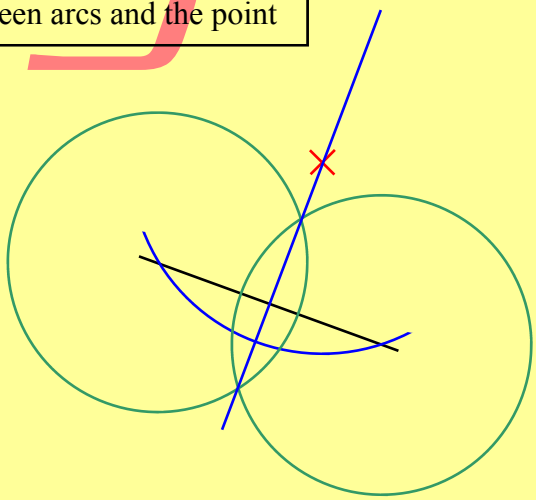
1 Position the compass in the point and draw an arc that cuts the interval in two places



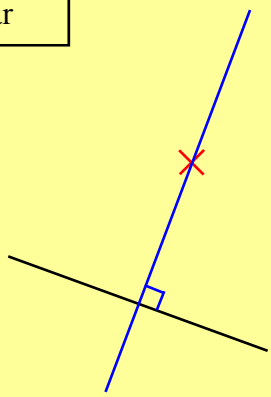
2 Then place the compass at the points where the blue arc cuts the interval, create 2 arcs.



3 Draw a line through the intersection points of the green arcs and the point



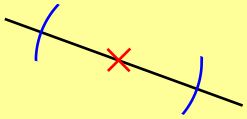
4 Place right angle markings to show it is a perpendicular



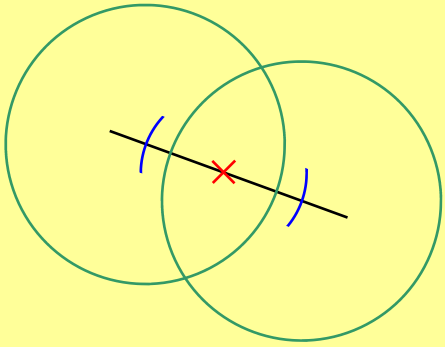
5 Green arcs and blue arcs don't have to be the same size

Column 2 - Perpendicular from a point away from on a line - Compass

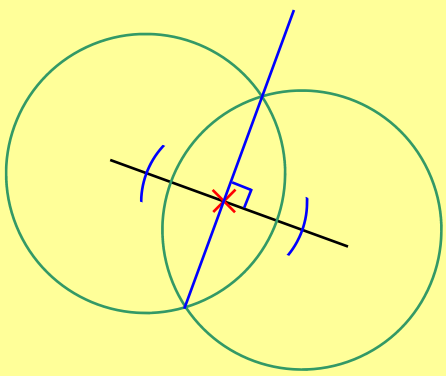
1 Position the compass at the point and draw an arc cutting the interval each side of the point (blue).



2 Position the compass where the blue arcs cut the interval and draw 2 arcs as shown, (green circles).



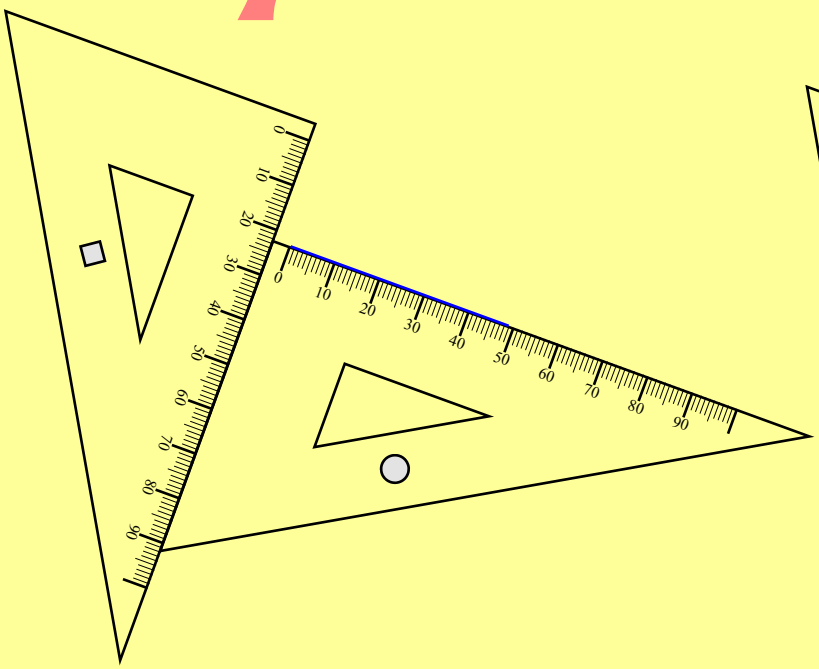
3 Draw a line through the two intersection points and the original point. Add a right angle symbol to complete.



Column 3 - Drawing a Parallel line - Set Square

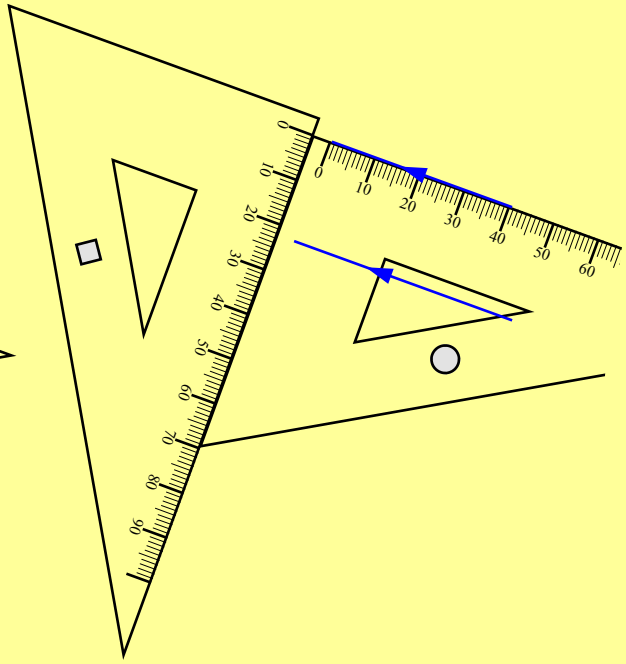
1

Position the set square marked with a circle on the line. Place another set square (square) or ruler on its edge, slide it to the required length (2.2 cm).



2

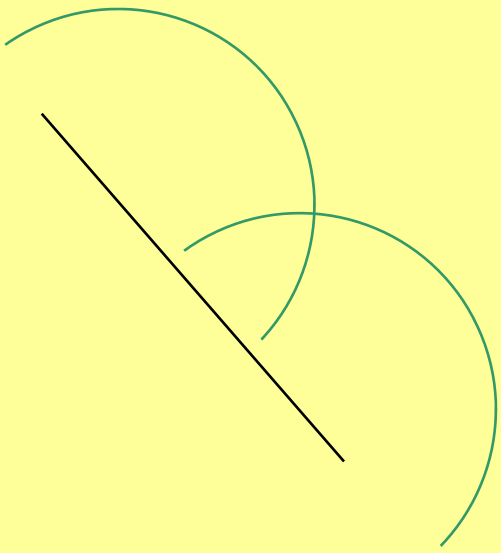
Slide the set square (circle) up to the zero then draw your line. To show the lines are parallel draw an arrow-head on each line.



Column 3 - Drawing a Parallel line - Compass

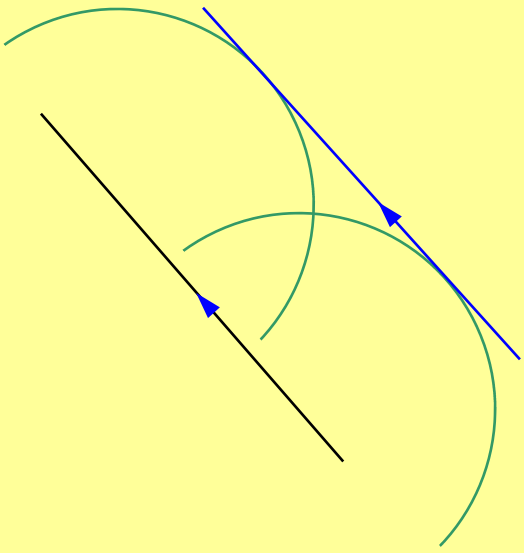
1

Open your compass the required distance and draw two arcs from any position on the interval.



2

Draw a line across the edges of the circles. Place arrow heads on the lines to show they are parallel.



Perpendicular and Parallel Lines

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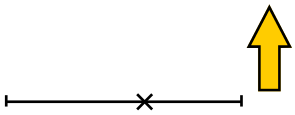
Use a set square to draw a perpendicular to the interval at the X, in the direction of the arrow.

1



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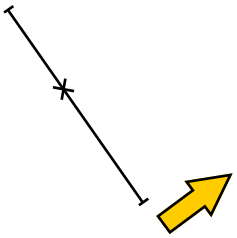

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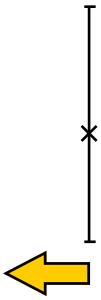
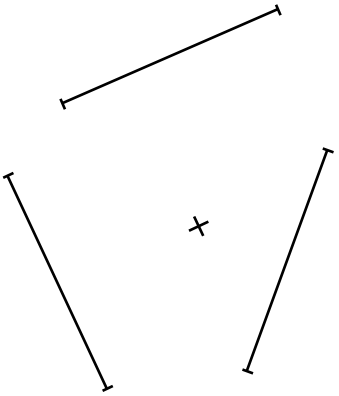


Use a set square to draw a perpendicular from the X to the interval or intervals

4



5



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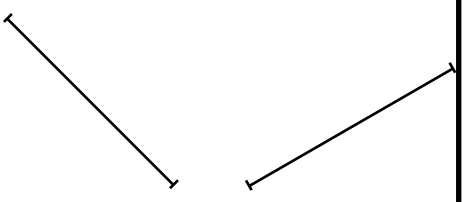
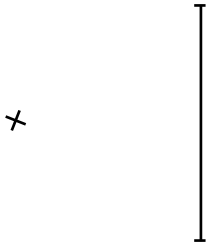


Use a compass and a ruler to draw a perpendicular from the X to the interval

7



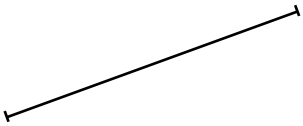
8



9

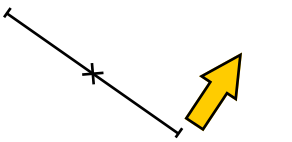


10



Now the X is on the line

11



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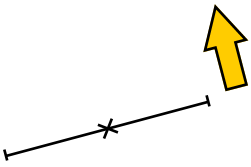
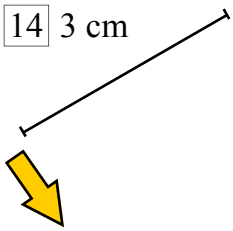


Use a set square and ruler to draw a parallel line to those below, separated by the given distance

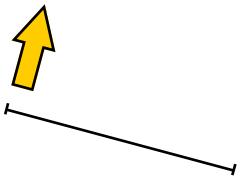
13 2 cm



14 3 cm

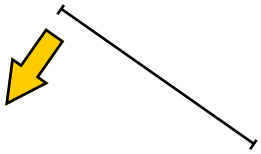


15 1.7 cm

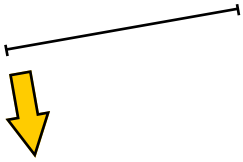


Keep going only this time use a compass and a ruler

16 2.2 cm



17 4.3 cm



Perpendicular and Parallel Lines

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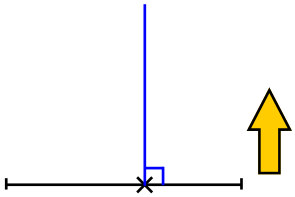
Use a set square to draw a perpendicular to the interval at the X, in the direction of the arrow.

1



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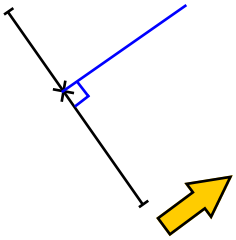

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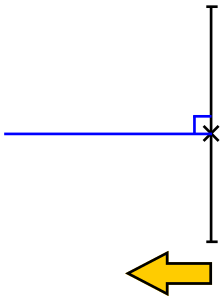
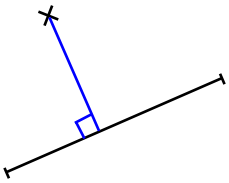


Use a set square to draw a perpendicular from the X to the interval or intervals

4



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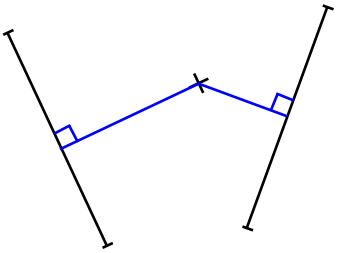

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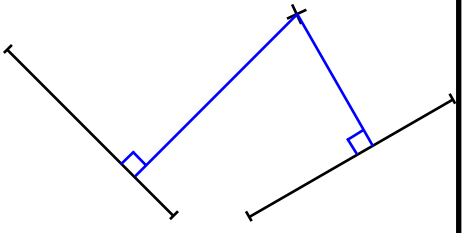
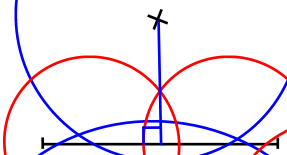


Use a compass and a ruler to draw a perpendicular from the X to the interval

7



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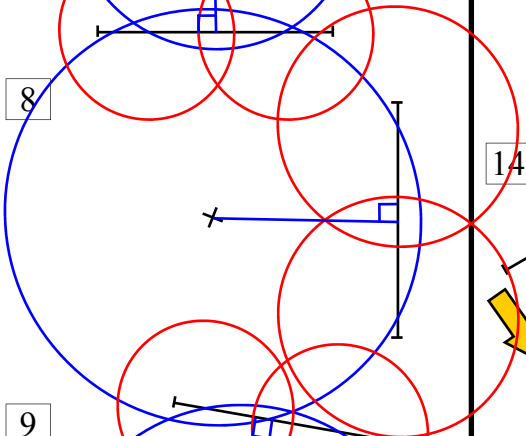

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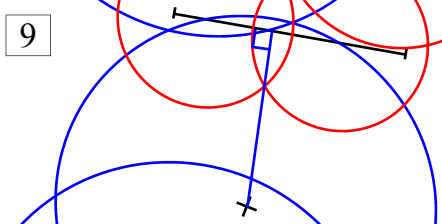

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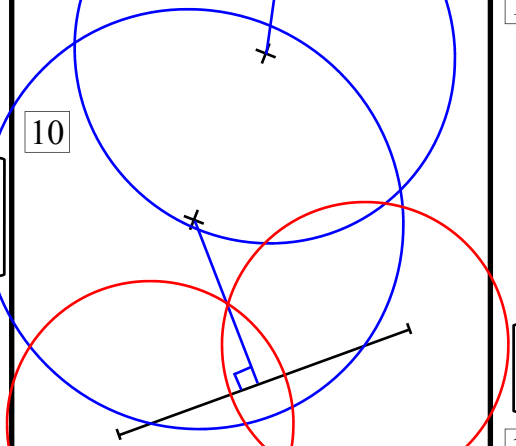
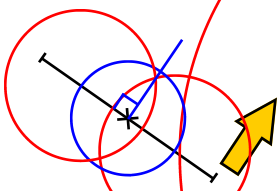


Now the X is on the line

11



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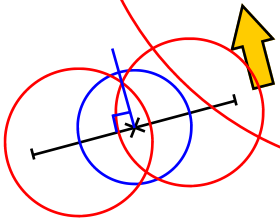
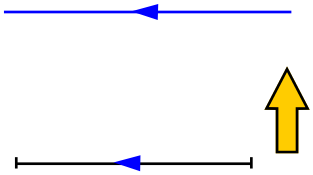
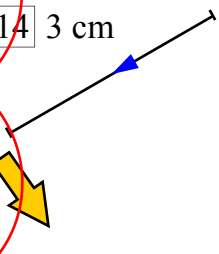


Use a set square and ruler to draw a parallel line to those below, separated by the given distance

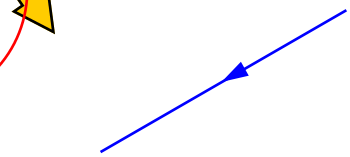
13 2 cm



14 3 cm

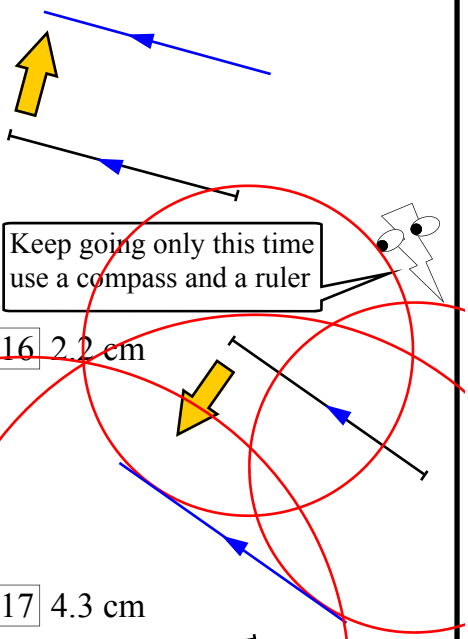


15 1.7 cm

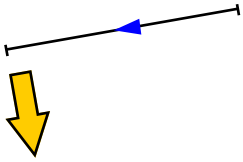


Keep going only this time use a compass and a ruler

16 2.2 cm



17 4.3 cm



Geometry - Spider's Web

This sheet is a line drawing exercise which will make a spider's web. Join the numbers that add up to 8, in branches that face each other, 1 and 7, 2 and 6 and so on. If you prefer to have no numbers on your worksheet, the second worksheet has the numbers removed.

Geometry - Spider's Web

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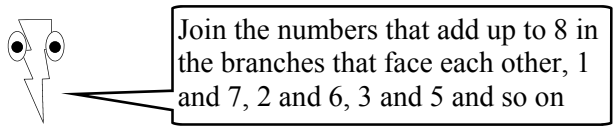
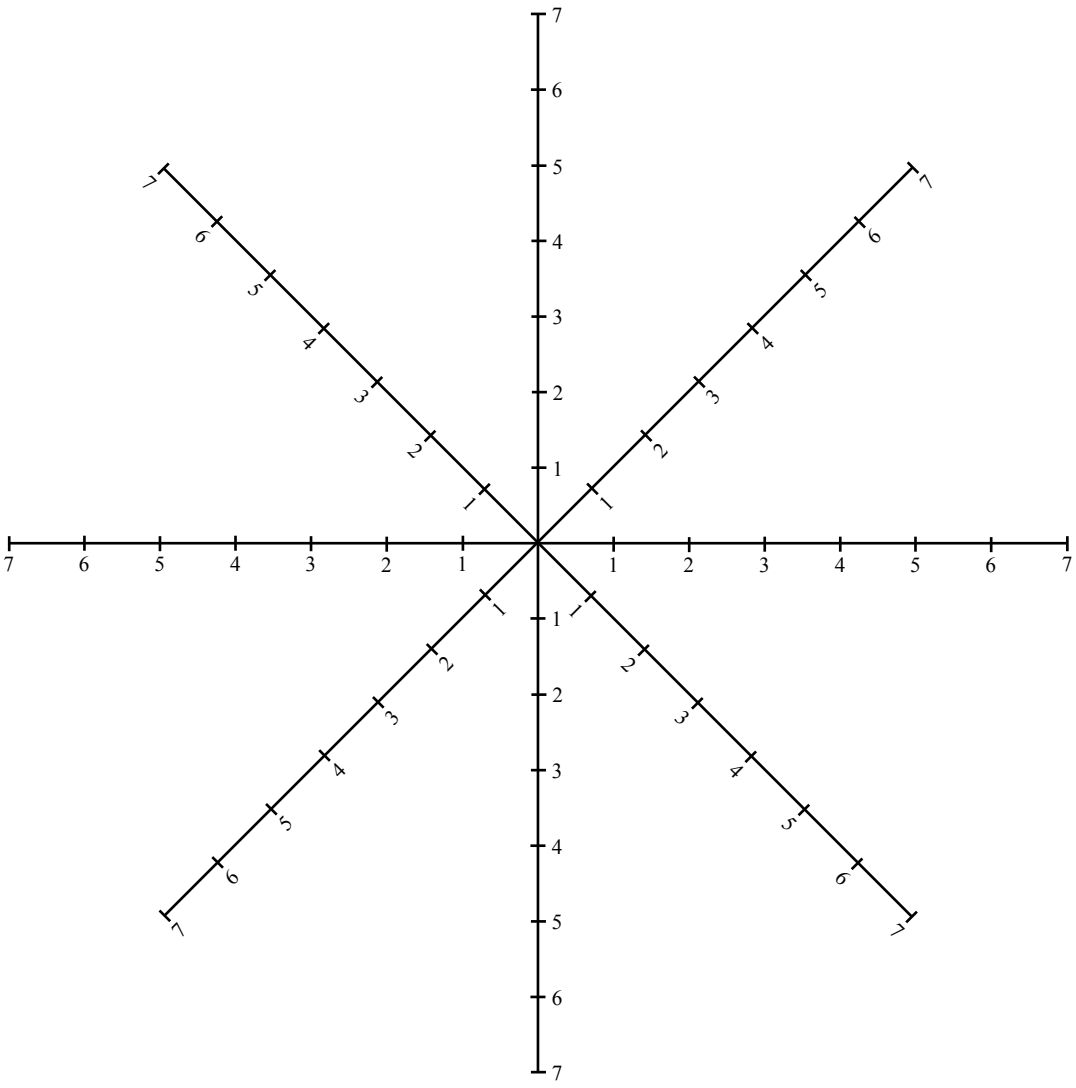


Join the numbers that add up to 8 in the branches that face each other, 1 and 7, 2 and 6, 3 and 5 and so on



Geometry - Spider's Web

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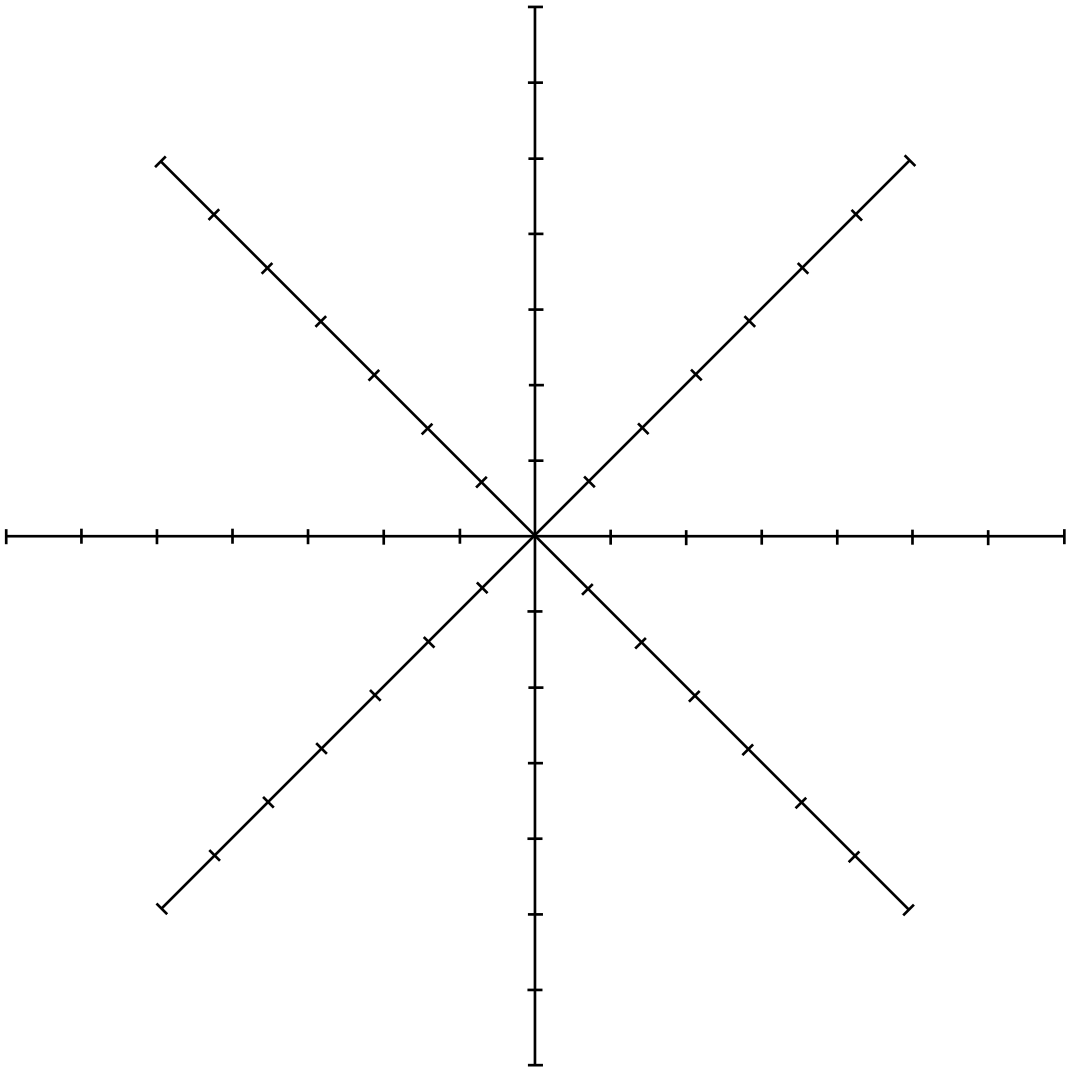
Join the numbers that add up to 8 in the branches that face each other, 1 and 7, 2 and 6, 3 and 5 and so on



Geometry - Spider's Web

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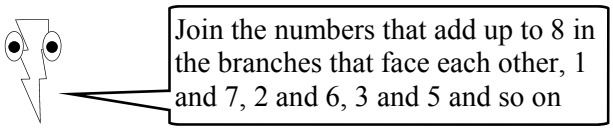
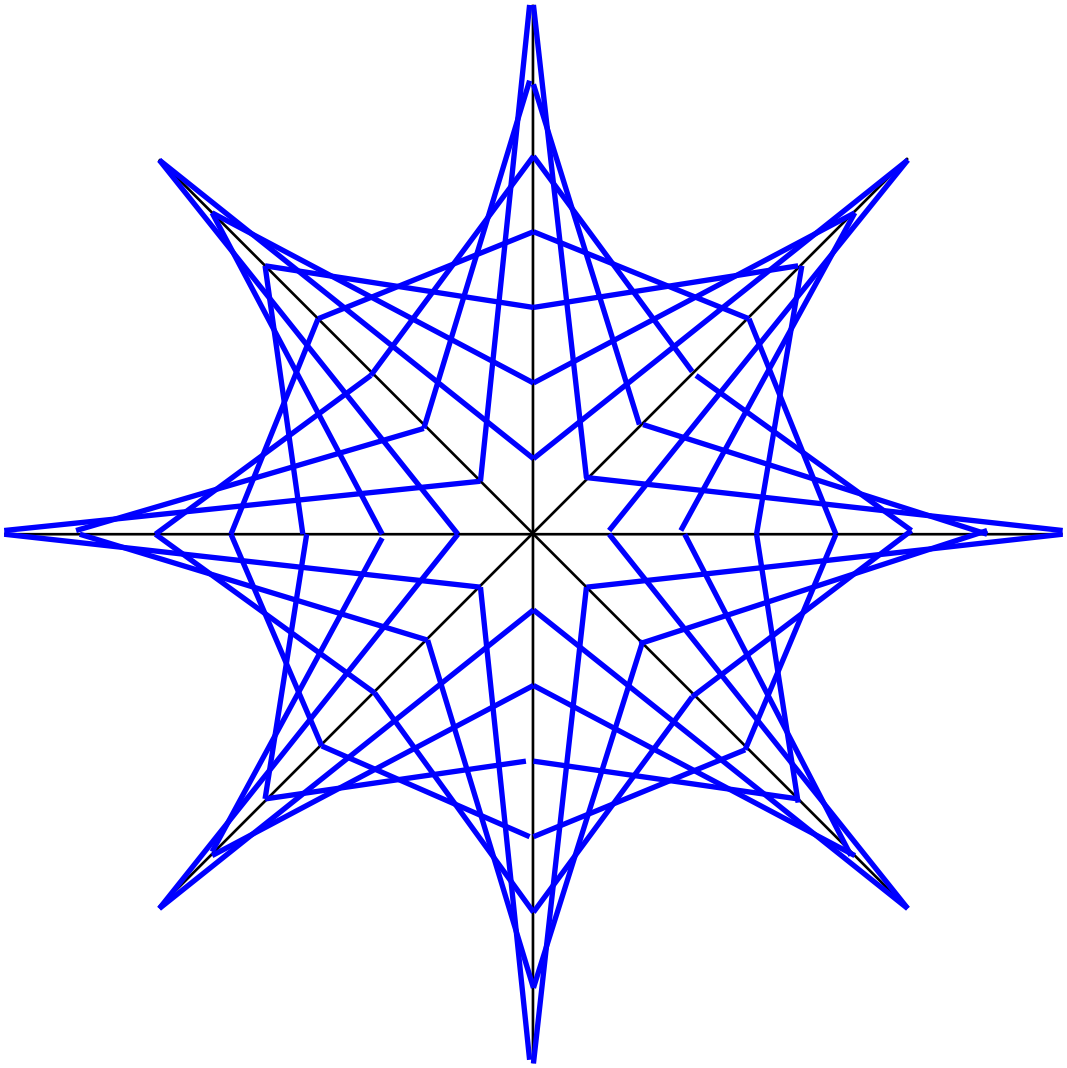




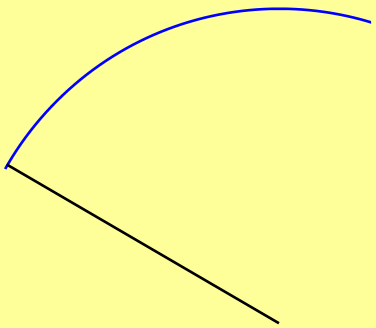
Equilateral and Scalene Triangles

Equilateral triangles are triangles with all sides equal and all interior angles equal (all 60º). This sheet is about constructing equilateral triangles with a compass (and a ruler). This is best described in steps, follow the steps outlined below.

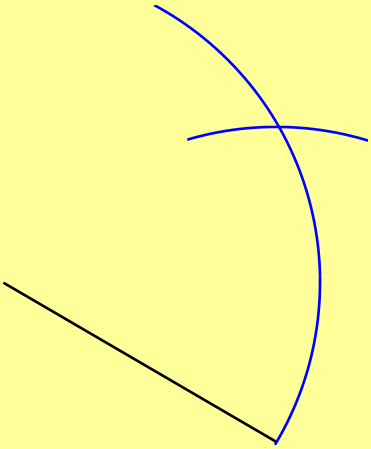
Scalene triangles have no equal sides. Follow the steps on the next page for constructing these triangles. Note that when a scalene triangle has an interior angle that is 90°, it isn't a scalene triangle it is a right (angled) triangle.

Column 1 - Constructing Equilateral Triangles - Compass

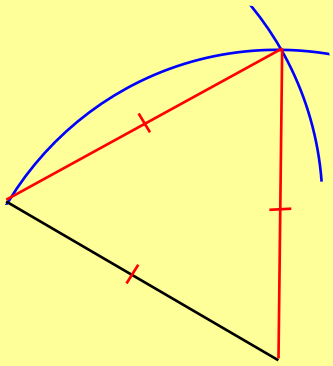
1 Position the compass at the end of the interval and open it to the interval length and draw an arc.



2 Repeat for the other end of the interval to create a point of intersection.

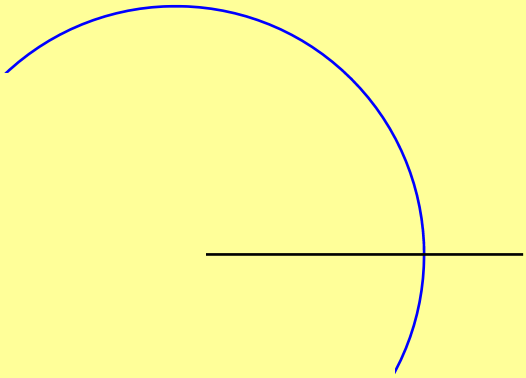


3 Draw a line from the end of the interval to the intersection point and then repeat for the other end of the interval. Mark the sides to show sides are all equal.

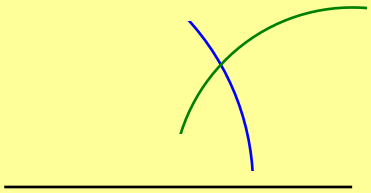


Column 2 - For the second part of the column rule a line first then repeat the above steps.

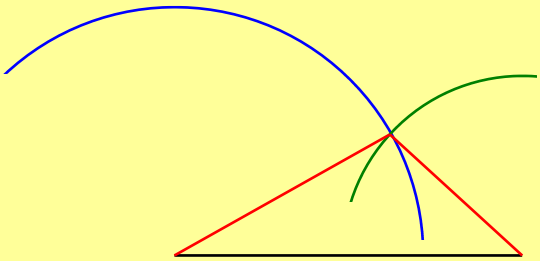
Column 2 - Constructing Scalene Triangles - Compass



1 Position the compass at the end of the interval and open it to the given length and draw an arc.



2 Repeat for the other end of the interval to create a point of intersection.



3 Draw a line from the end of the interval to the intersection point and then repeat for the other end of the interval.

Column 2 - For the second part of the column rule a line first then repeat the above steps. Rule the longest line first, then use your compass to create the other two lines.

Equilateral and Scalene Triangles

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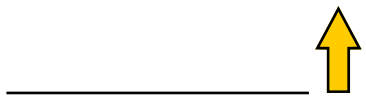
Make an equilateral triangle given a side. Use a compass and a ruler. Construct the triangle in the direction of the arrow.

1



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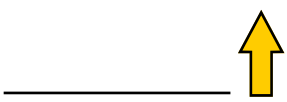

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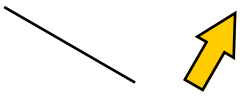
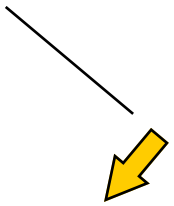


4



5



6



Now construct equilateral triangles with the side lengths given below

7 5.5 cm

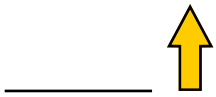
8 6.3 cm

Now make scalene triangles with one side drawn and the other two sides given.

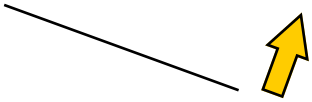
9 4 cm and 3 cm



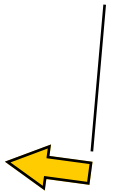
10 2.5 cm and 3.1 cm



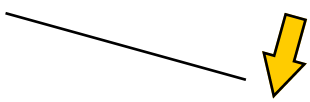
11 2.2 cm and 4.3 cm



12 1.9 cm and 2.7 cm



13 1.3 cm and 4.7 cm



14 5.4 cm and 4.9 cm

Construct scalene triangles with the side lengths given below

15 3 cm, 1.7 cm and 2.4 cm

16 3.8 cm, 2.9 cm and 4.1 cm

Equilateral and Scalene Triangles

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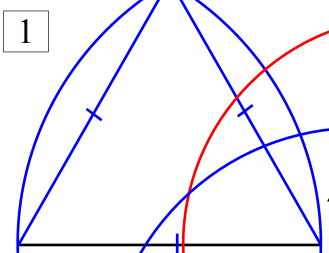
Make an equilateral triangle given a side. Use a compass and a ruler. Construct the triangle in the direction of the arrow.

1



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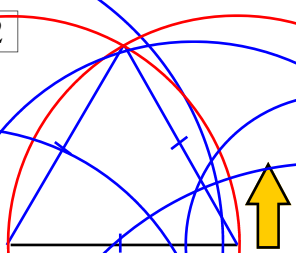

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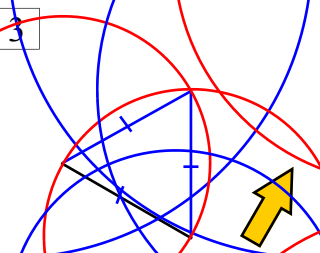

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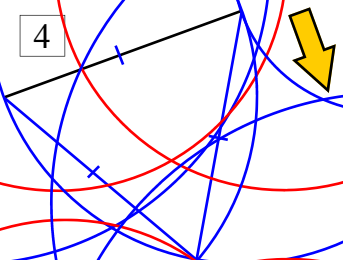

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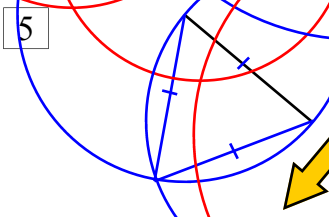

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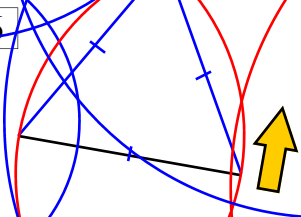
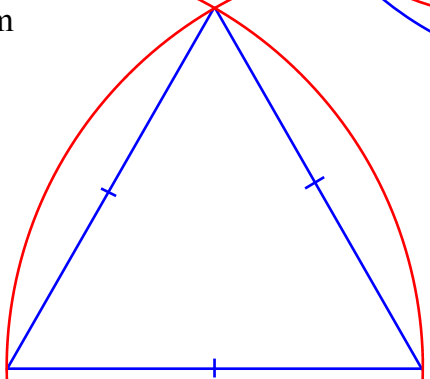
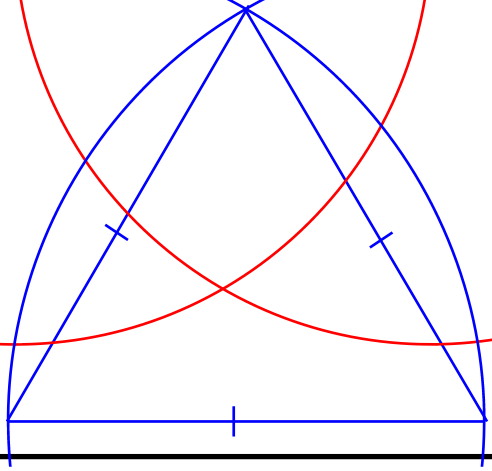


Now construct equilateral triangles with the side lengths given below

7 5.5 cm

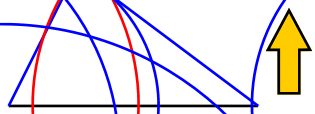


8 6.3 cm

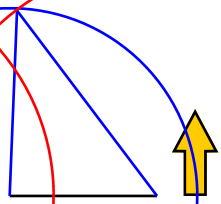


Now make scalene triangles with one side drawn and the other two sides given.

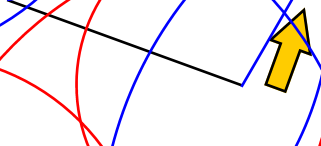
9 2 cm and 3 cm



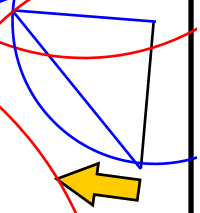
10 2.5 cm and 3.1 cm



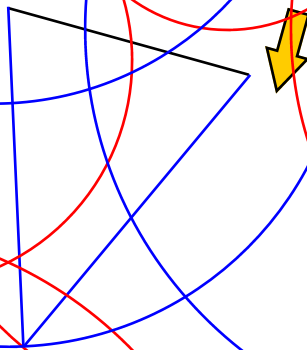
11 2.2 cm and 4.3 cm



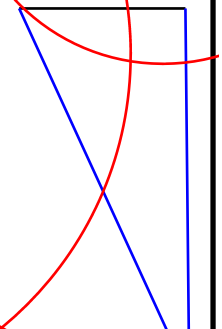
12 1.9 cm and 2.7 cm



13 4.5 cm and 4.7 cm

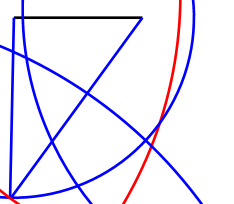


14 5.4 cm and 4.9 cm

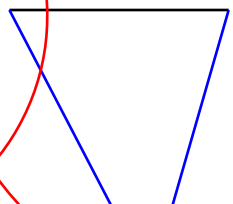


Construct scalene triangles with the side lengths given below

15 3 cm, 1.7 cm and 2.4 cm



16 3.8 cm, 2.9 cm and 4.1 cm



Creating Isosceles and Side-Angle-Side Triangles

Column 1 - Constructing Isosceles Triangles - Compass

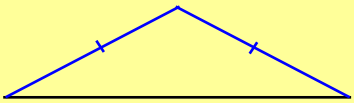
1 Open your compass to the given length then position the compass at the end of the line and draw an arc.

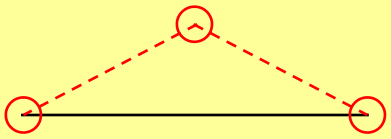
2 Repeat for the other end of the line to create a point of intersection. Note that the second line may not be the same length as the first.



3 Using a ruler, draw lines from the intersection point to the ends of the line.

4 Label the two sides of equal length with dashes.





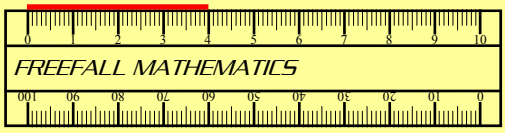
Questions 7 & 8 - Rule a line first then repeat the above steps. Rule the longest line first, then use your compass to construct the other two lines.

Creating Isosceles and Side-Angle-Side Triangles

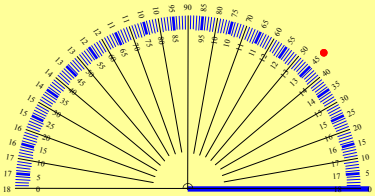
Column 2 - Constructing Triangles - Protractor & Ruler

Example:
4 cm, 3 cm
& 45° angle

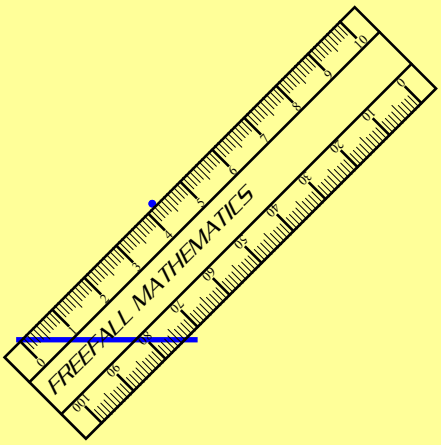
1 Draw the line of longest line first, the 4 cm line in our example.



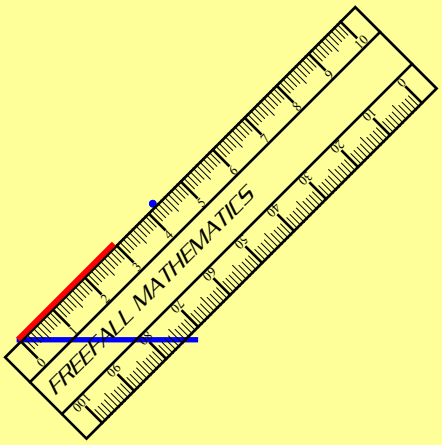
2 Place a protractor at the end of the line and plot a point at the given angle. In our example it is 45º.



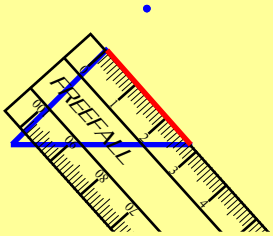
3 Remove the protractor and place a ruler with the zero at the end of the line and the point on the ruler's edge.



4 Rule the other line given in the question. It doesn't have to pass through the point.



5 Complete the final side by using a ruler between the two points



Making Isosceles and Side Angle Side Triangles

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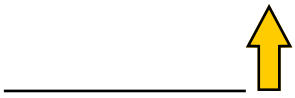
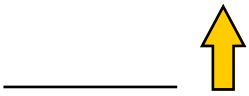
Make an isosceles triangle, one side is already drawn for you. Use the two other measurements to complete it.

1 3 cm & 3 cm

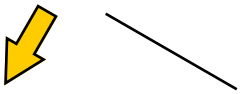
2 2.3 cm & 1.6 cm





3 2.6 cm & 2 cm

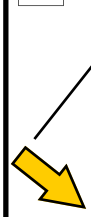
4 4.2 cm & 4.2 cm

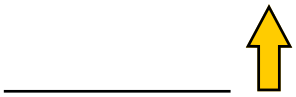




5 3.3 cm & 3.3 cm

6 1.2 cm & 3 cm





Now construct isosceles triangles with the side lengths given below



7 2.6 cm, 4.1 cm and 4.1 cm

8 2.7 cm, 3.8 cm and 3.8 cm



Here are two sides with the angle between them. Use a protractor and a ruler to draw them, then join the ends for the third side



9 4 cm, 3 cm, 60º

10 2.5 cm, 3.2 cm, 40º

11 5.3 cm, 4.7 cm, 25º

12 7.1 cm, 6.3 cm, 38º

Making Isosceles and Side Angle Side Triangles

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Make an isosceles triangle, one side is already drawn for you. Use the two other measurements to complete it.

1 3 cm & 3 cm

2 2.3 cm & 1.6 cm

3 2.6 cm & 2 cm

4 4.2 cm & 4.2 cm

5 3.3 cm & 3.3 cm

6 1.2 cm & 3 cm

Now construct isosceles triangles with the side lengths given below

7 2.6 cm, 4.1 cm and 4.1 cm

8 2.7 cm, 3.8 cm and 3.8 cm

Here are two sides with the angle between them. Use a protractor and a ruler to draw them, then join the ends for the third side

9 4 cm, 3 cm, 60º

10 2.5 cm, 3.2 cm, 40º

11 5.3 cm, 4.7 cm, 25º

12 7.1 cm, 6.3 cm, 38º

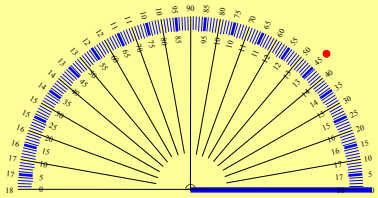
Angle-Angle-Side Triangles and Properties

A triangle can be constructed when you are given two angles and a side. This sheet requires a protractor and a ruler. The steps are below:

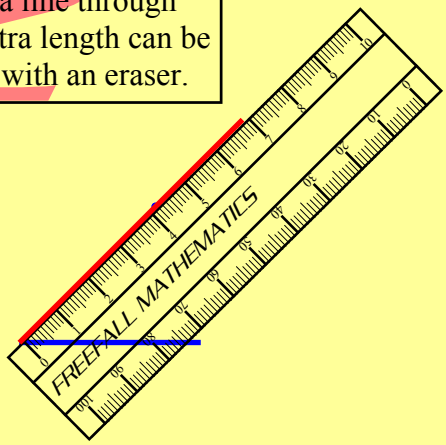
Column 1 - Constructing AAS Triangles - Protractor & Ruler

Example: 45º and 60º Base Angles

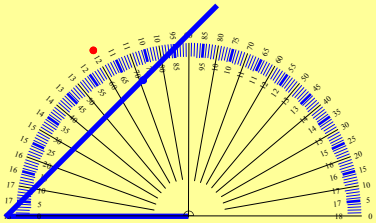
1 Using your protractor place it at one end of the line and measure the first angle. Plot a dot at the first angle.



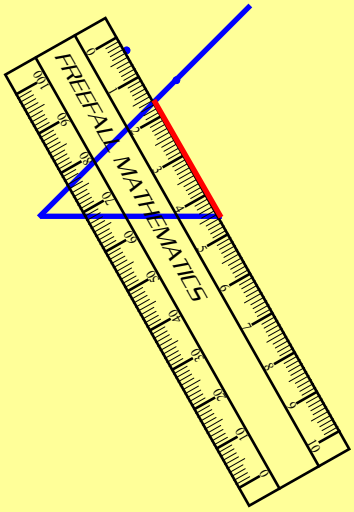
2 Place a ruler at the end of the line and draw a line through the dot, the extra length can be removed later with an eraser.



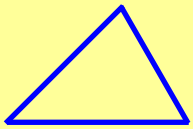
3 Repeat for the other end of the line.



4 Use a ruler to draw the final side. Stop when the line intersects the other side.



5 Erase the extended side length. All done!



Column 2 - Q.8 - Q.11.

Use the same method as above after first ruling the line.

The second part of the sheet aims to outline the relationship between the largest angle of a triangle and the triangle's longest side. Name the longest side and largest angle and in Q.15 explain how the angle and side are always located on a triangle.

Angle-Angle-Side Triangles and Properties

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Use a protractor and a ruler to create triangles with base internal angles as given below

1 40º & 50º

2 23º & 55º

3 70º & 80º

4 52º & 47º

5 15º & 38º

6 117º & 29º

7 60º & 60º

This time draw the given side then use the same method. Be careful where you choose to place the line.

8 2.5 cm, 30º, 40º

9 3.7 cm, 22º, 47º

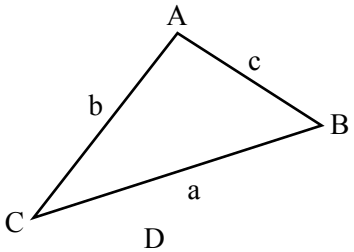
10 4.4 cm, 53º, 35º

11 1.7 cm, 75º, 75º

Name the largest angle and the largest side in these triangles

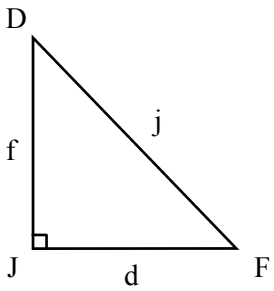
12 Side : ____

Angle : ∠ _____



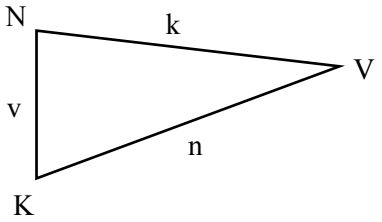
13 Side : ____

Angle : ∠ _____



14 Side : ____

Angle : ∠ _____



15 What is the relationship between the largest angle and the longest side? _____

Angle-Angle-Side Triangles and Properties

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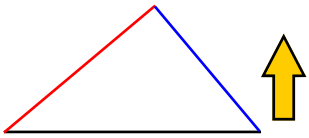
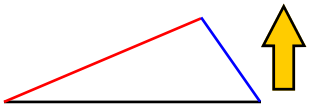
Use a protractor and a ruler to create triangles with base internal angles as given below

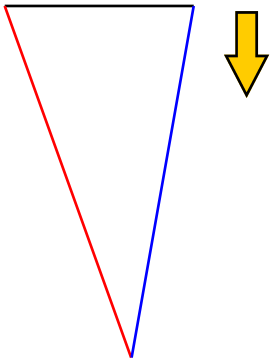
1 40º & 50º



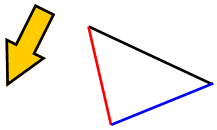
2 23º & 55º



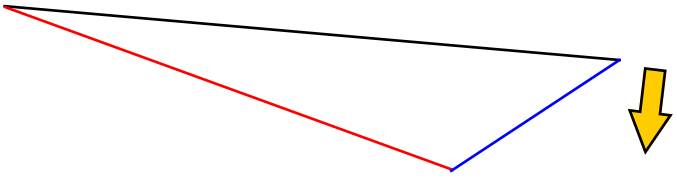
3 70º & 80º



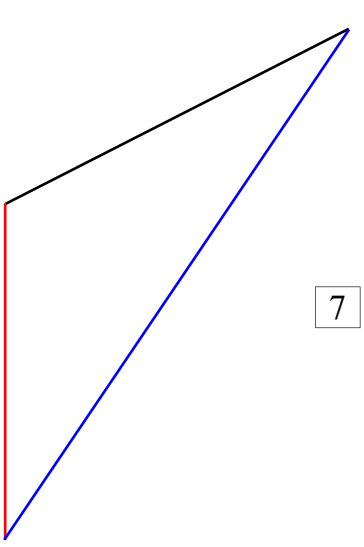
4 52º & 47º



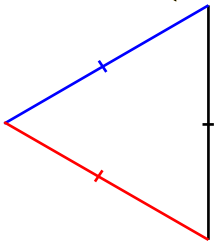
5 15º & 38º



6 117º & 29º

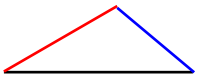


7 60º & 60º

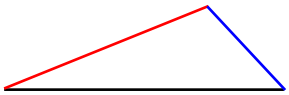


This time draw the given side then use the same method. Be careful where you choose to place the line.

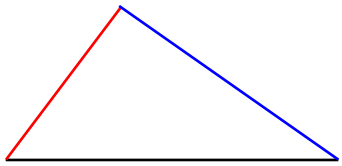
8 2.5 cm, 30º, 40º



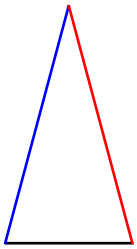
9 3.7 cm, 22º, 47º



10 4.4 cm, 53º, 35º



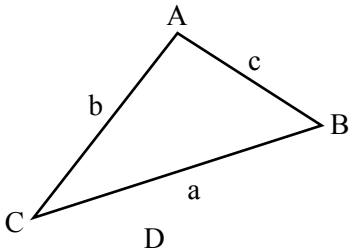
11 1.7 cm, 75º, 75º



Name the largest angle and the largest side in these triangles

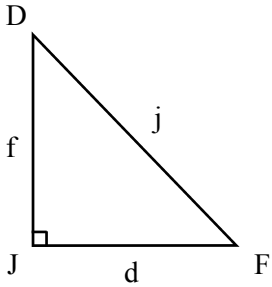
12 Side : a

Angle : ∠ BAC



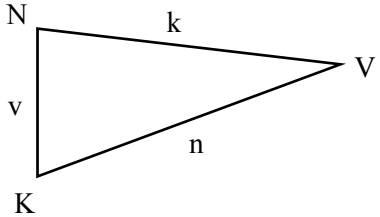
13 Side : j

Angle : ∠ DJF



14 Side : n

Angle : ∠ VNK



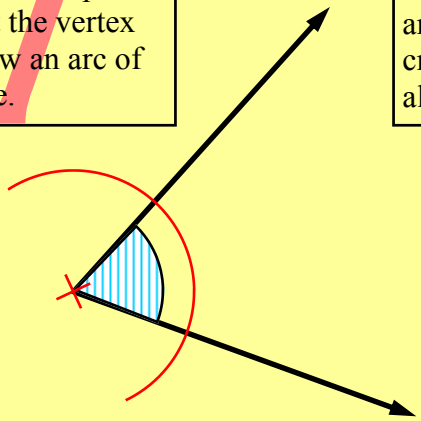
15 What is the relationship between the largest angle and the longest side? They are always opposite each other.

Bisecting Angles and Making 45° Angles

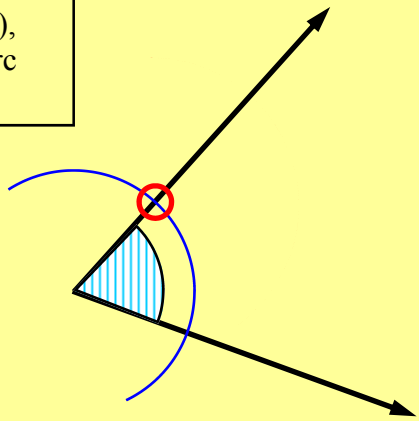
This sheet is about bisecting an angle, this means cutting the angle in half. The second part of the sheet is about creating 45° angles using a compass and a ruler. When you make a 45° angle you are also making a 135° angle, it's the angle's supplement, it will be the adjacent obtuse angle beside the 45° angle.

Column 1 - Bisecting Angles - Protractor & Ruler

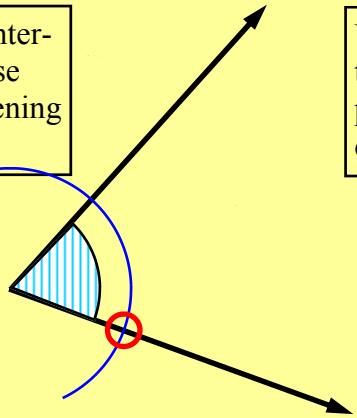
1 Place the compass point at the vertex and draw an arc of any size.



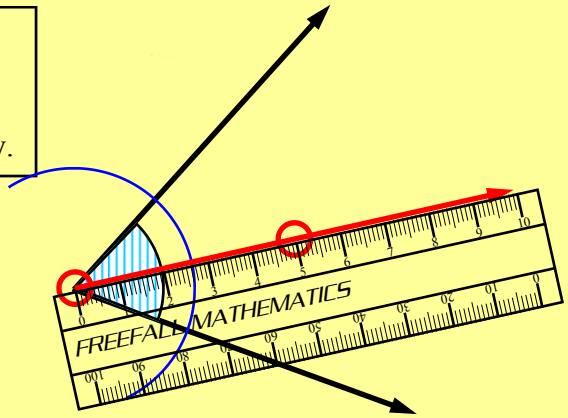
2 Using the intersection of the arc and the ray (red circle), create another arc. This arc also can be of any size.



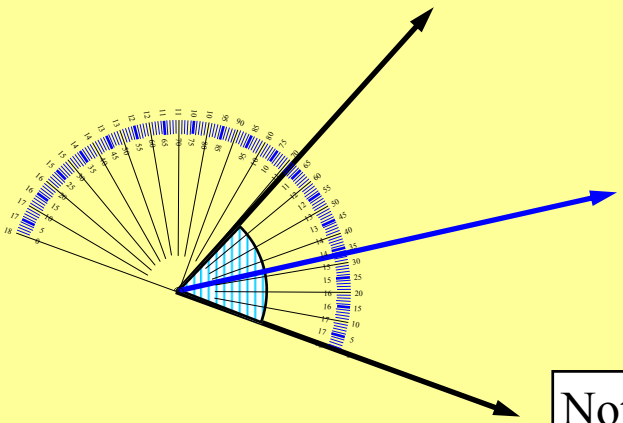
3 Repeat for the other intersection point (red). Use the same compass opening as the previous arc.



4 Using the vertex and the arc intersection point (red circles), draw the bisecting ray.



5 Use a protractor to measure the size of the bisection and original angle to check that your method is correct.



Note Q.4 is a reflex angle so a working space is provided.

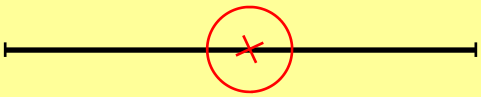
Bisecting Angles and Making 45° Angles

A 45º angle can be constructed by first making a 90º angle and bisecting it. So a right angle is created, then the same process from the previous column is repeated.

Column 2 - Constructing 45º Angles - Compass & Ruler

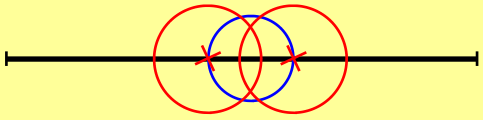
1

Using a compass draw a circle near the centre of the interval.



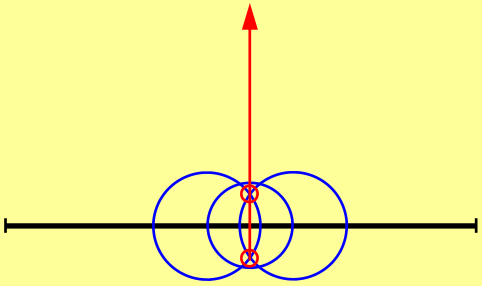
2

Using the intersection points, create two more circles (or arcs).



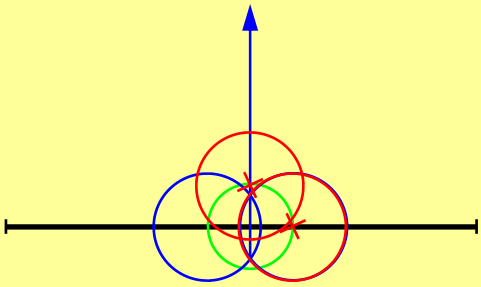
3

Use the intersection points created by the two circles (red), draw a perpendicular line. This is a 90º angle.



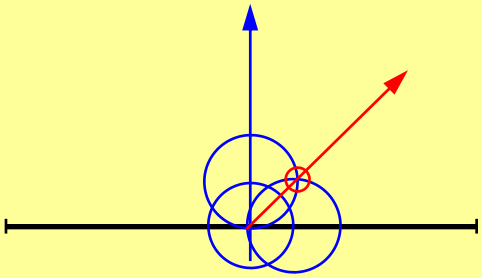
4

The first circle (green) intersects the perpendicular and the interval. Using these point create two arcs or circles (red circles).



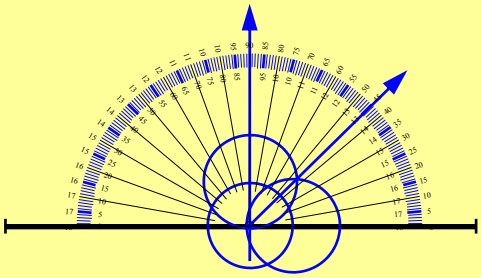
5

Draw a line through the vertex and intersection point (red circle) and its done!



6

Check your work with a protractor.



Bisecting Angles and Making 45° Angles

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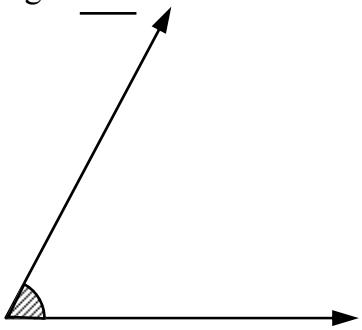
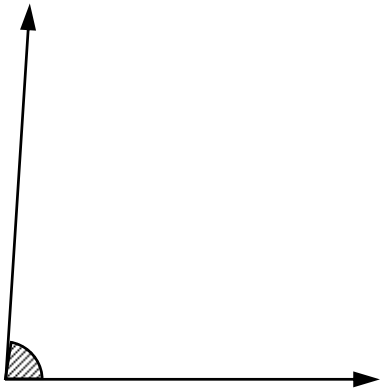
Bisect the angles below using a compass and a ruler. Verify that the bisection is correct using a protractor.

To make a 45° angle create a 90° angle (perpendicular) then bisect it. A 45° angle could also be 135°!

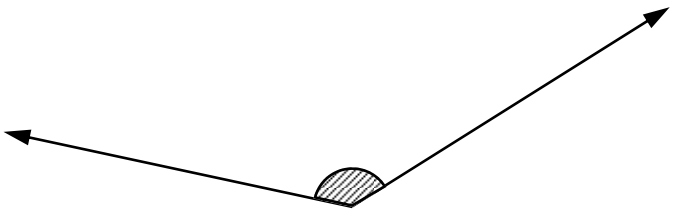
1 Original angle ___° Bisection angle ___°



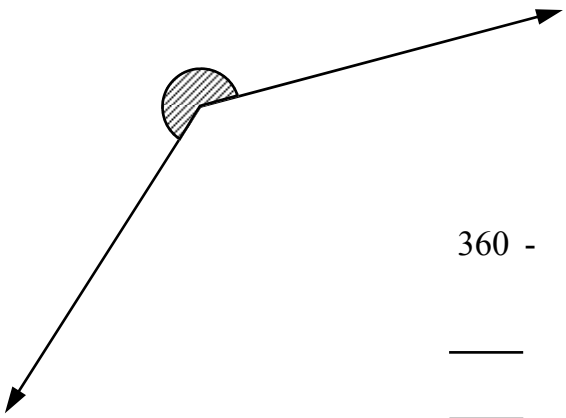
2 Original angle ___ Bisection angle ___



3 Original angle ____ Bisection angle ___



4 Original angle ____ Bisection angle ____



360 -

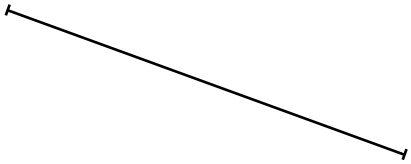
___

___

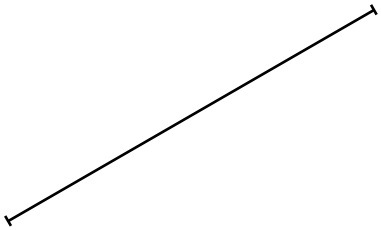
5

6

7



8



Bisecting Angles and Making 45° Angles

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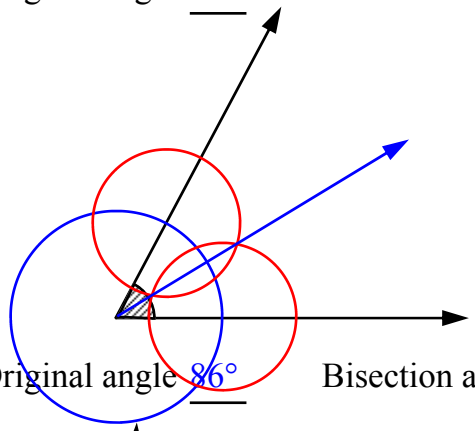
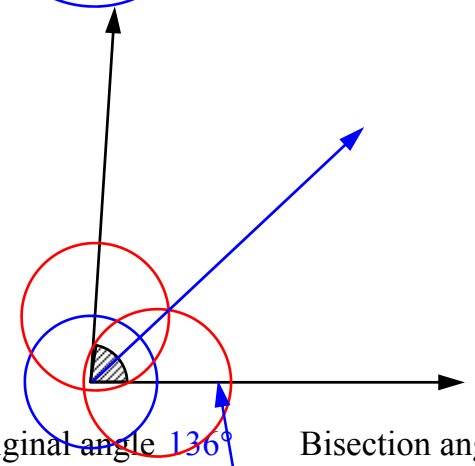
Bisect the angles below using a compass and a ruler. Verify that the bisection is correct using a protractor.

To make a 45° angle create a 90° angle (perpendicular) then bisect it. A 45° angle could also be 135°!

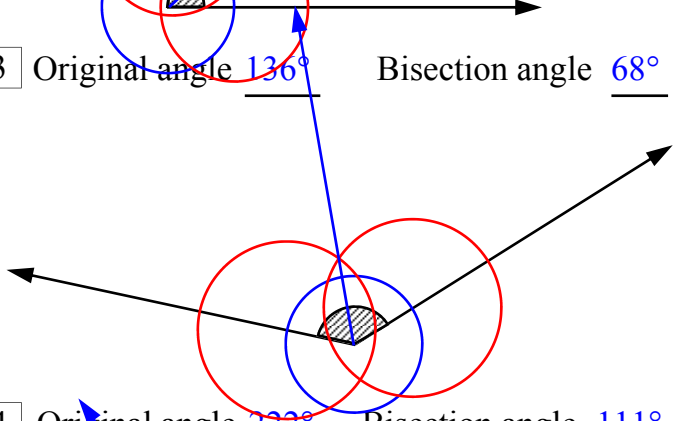
1 Original angle 62 ° Bisection angle 31 °



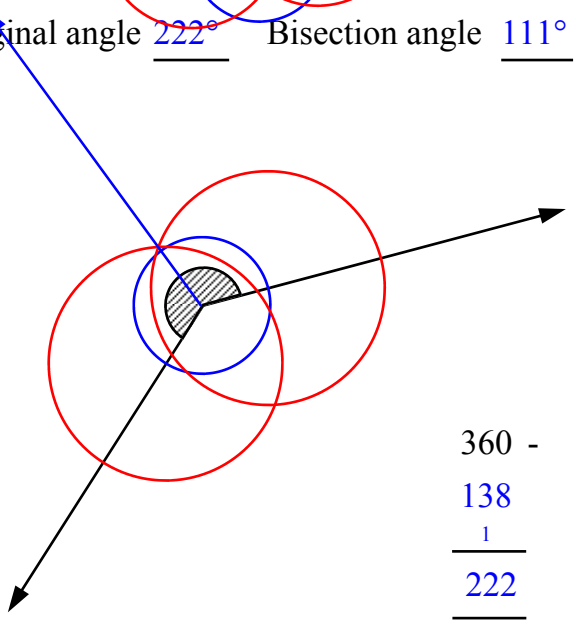
2 Original angle 86° Bisection angle 43°



3 Original angle 136° Bisection angle 68°

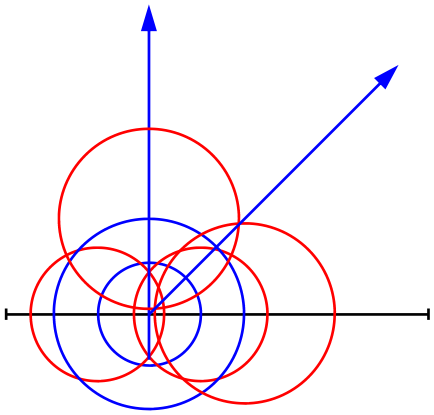


4 Original angle 222° Bisection angle 111°

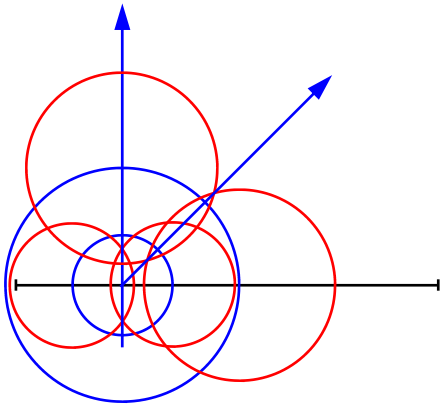


360 - 138 = 222

5



6



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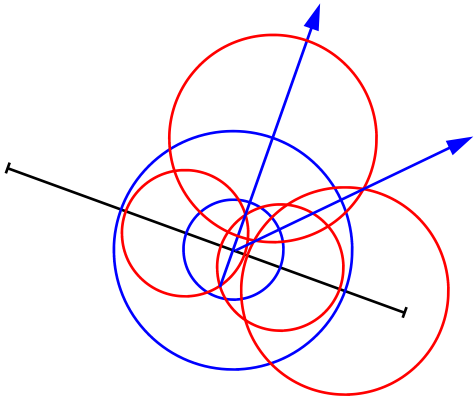

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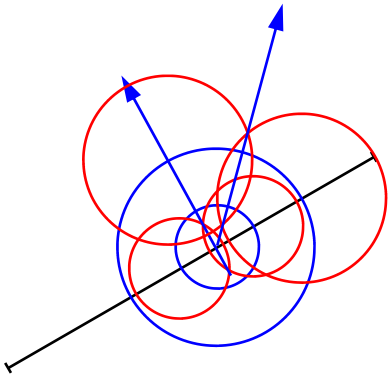


Constructing 60° and 30° Angles

This sheet is about constructing a 60° angle and a 30° angle (a bisected 60° angle), to complete this sheet you require a compass and a ruler. One important thing to remember is that if you make a 60° angle you are also making a 120° angle, it's the supplementary angle. It will be the adjacent obtuse angle beside the 60° angle. Likewise when you make a 30° angle the adjacent obtuse angle will be 150°.

Column 1 - Constructing 60º & 120º Angles - Compass & Ruler

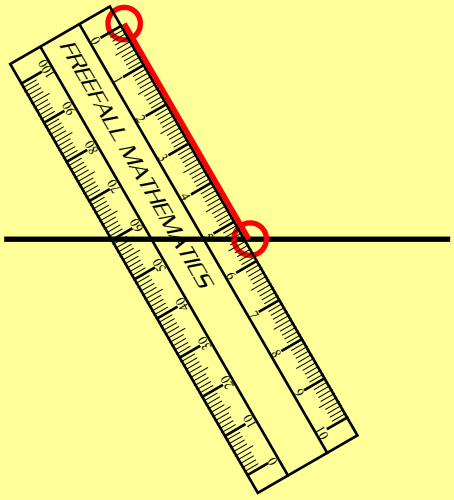
1 Place the compass at the end of the interval, open to any size and draw an arc.



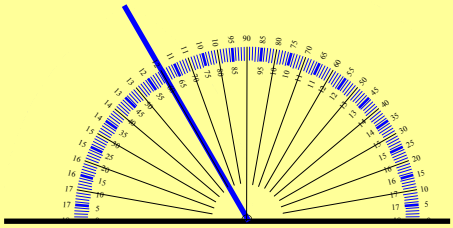
2 Using the intersection of the arc and the interval (red circle), keep the compass at the same opening and draw another arc.



3 Use a ruler to connect the two intersection points (red circles).



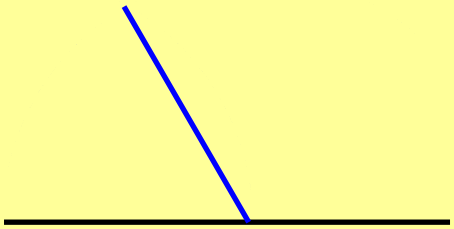
4 Check your work with a protractor.



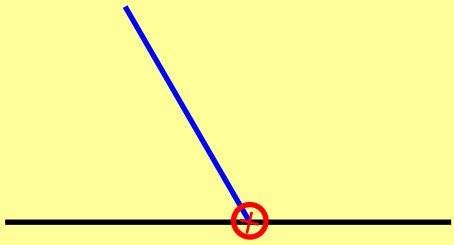
Constructing 60° and 30° Angles

Column 2 - Constructing 30º & 150º Angles - Compass & Ruler

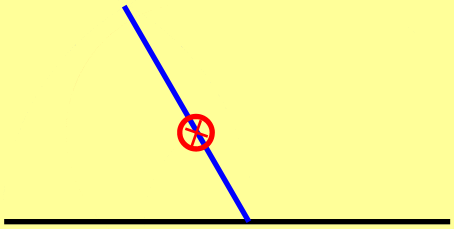
1 Create a 60º angle as outlined on the previous page. A 30º angle is made by bisecting a 60º angle.



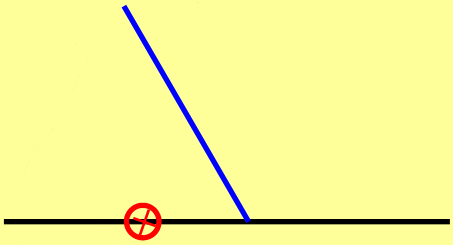
2 Draw an arc from the vertex of the angle (red circle) of any length.



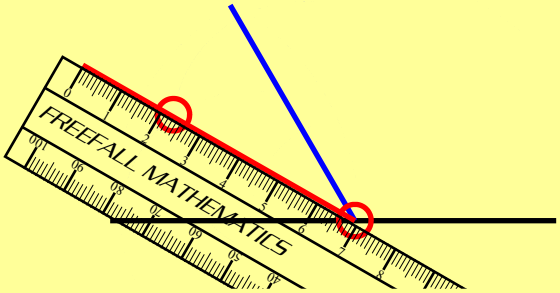
3 Draw an arc from one intersection point (red circle).



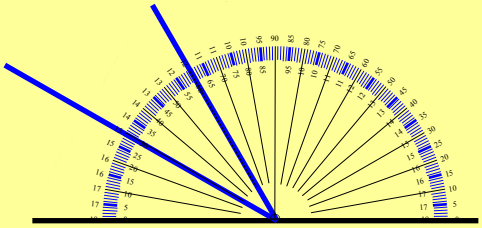
4 Then repeat for the other intersection point (red circle). Keep the compass at the same opening.



5 Use a ruler to connect the intersection point and the vertex (red circles). All done!



6 Using a protractor, check your work.



Constructing 60° and 30° Angles

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To make a 60° angle use the same method as creating an equilateral Δ. A 60° could also be a 120° angle.

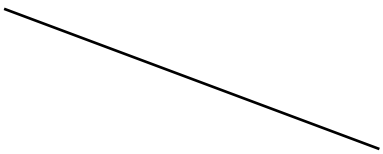
1

2

3



4



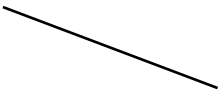


To make a 30° angle create a 60° angle then bisect it. A 30° angle could also be 150°!

5

6

7



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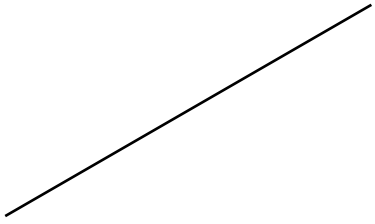

Constructing 60° and 30° Angles

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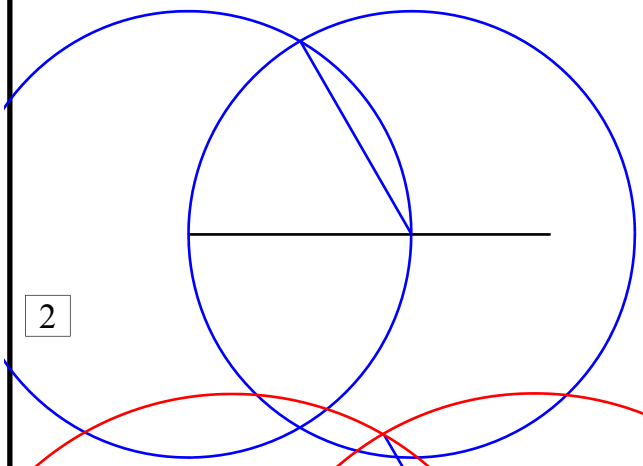
To make a 60° angle use the same method as creating an equilateral Δ. A 60° could also be a 120° angle.

1



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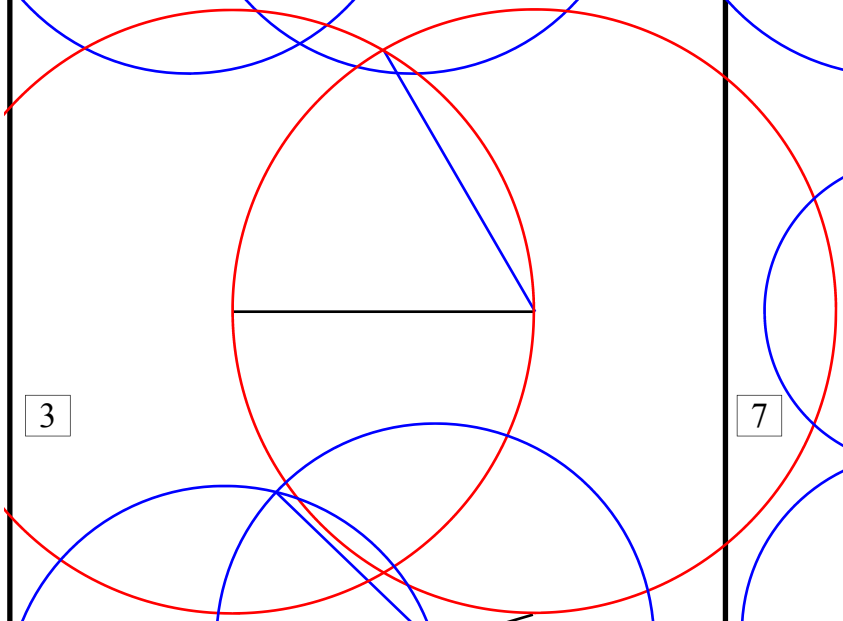

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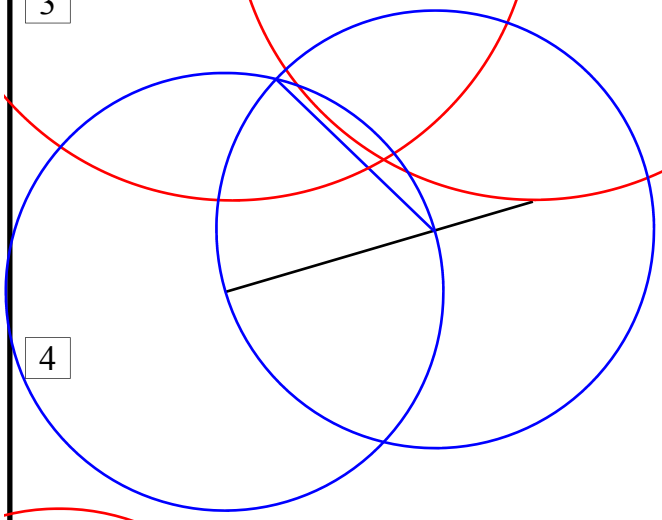

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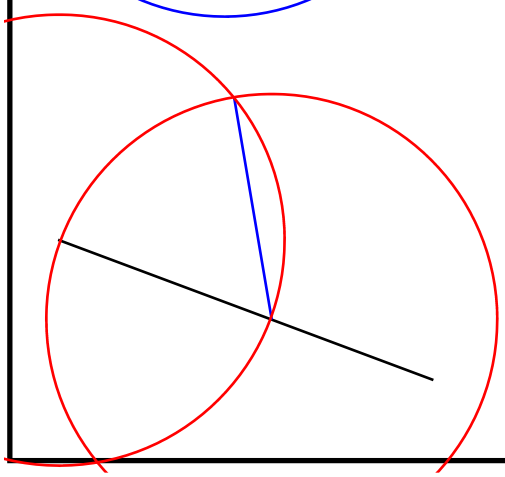
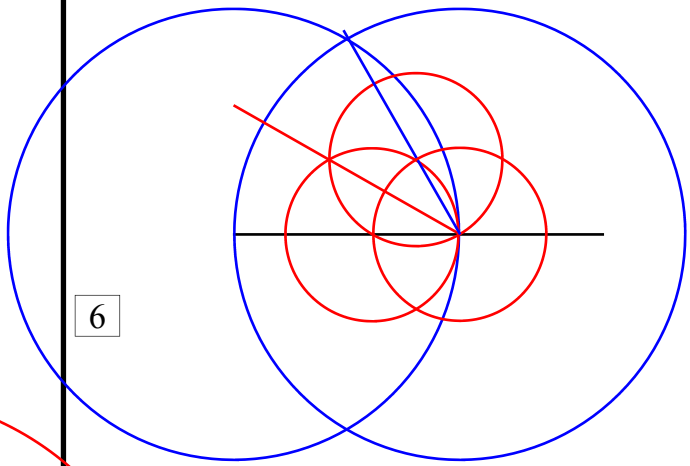


To make a 30° angle create a 60° angle then bisect it. A 30° angle could also be 150°!

5



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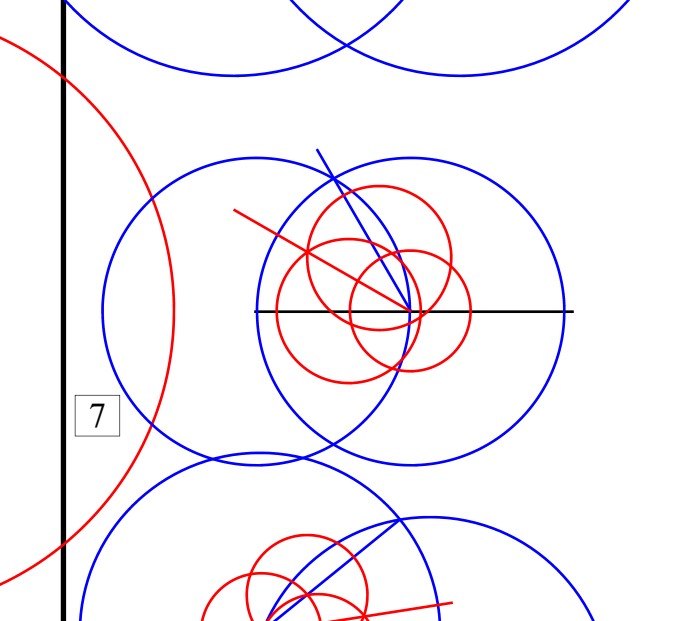

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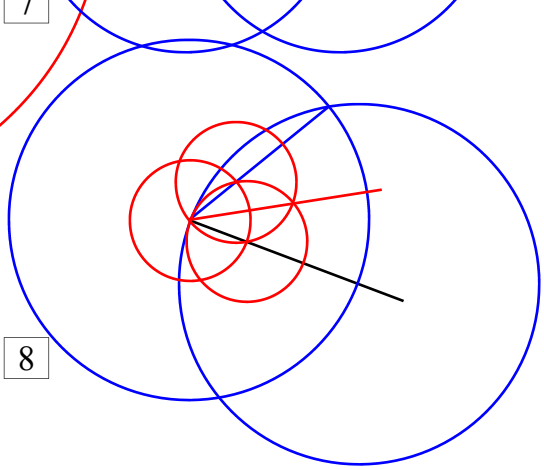

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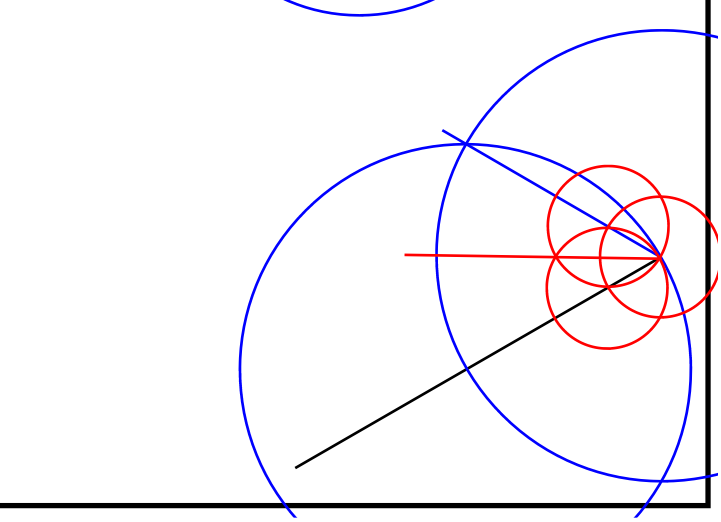
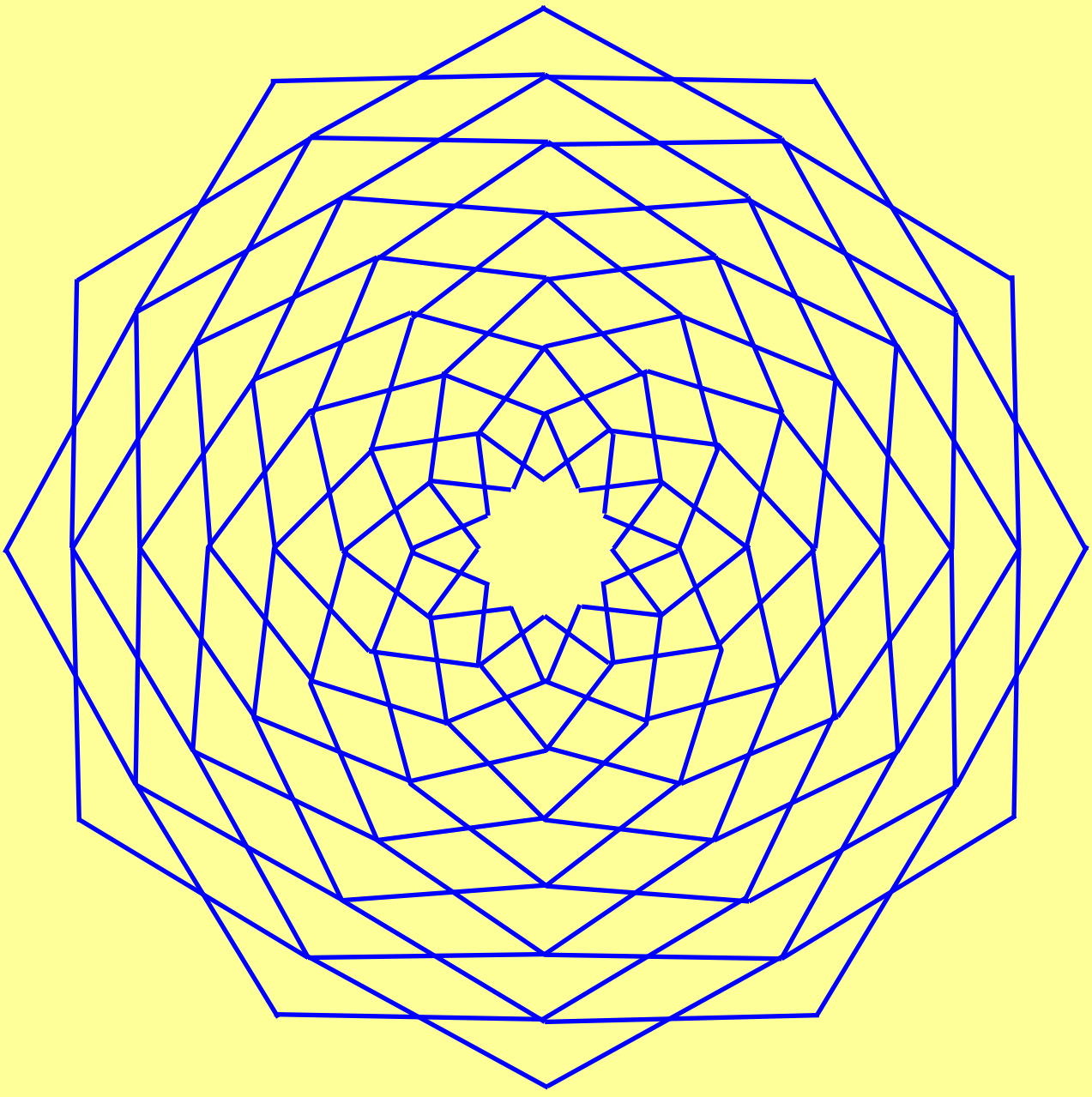


Flower Design

This is a simple design that creates a flower-like effect. Draw lines starting at the smallest circle then moving to the next circle and one position clockwise. Complete all the way to the outside circle then repeat starting from the next position on the inside circle, and so on.

After you have a full revolution you repeat the process only you move anti-clockwise instead. There are two worksheets, the first has one completed clockwise step and a completed anti-clockwise step. The second sheet is completely blank if you don't need the help.

A image of the completed flower is below.



Flower Design

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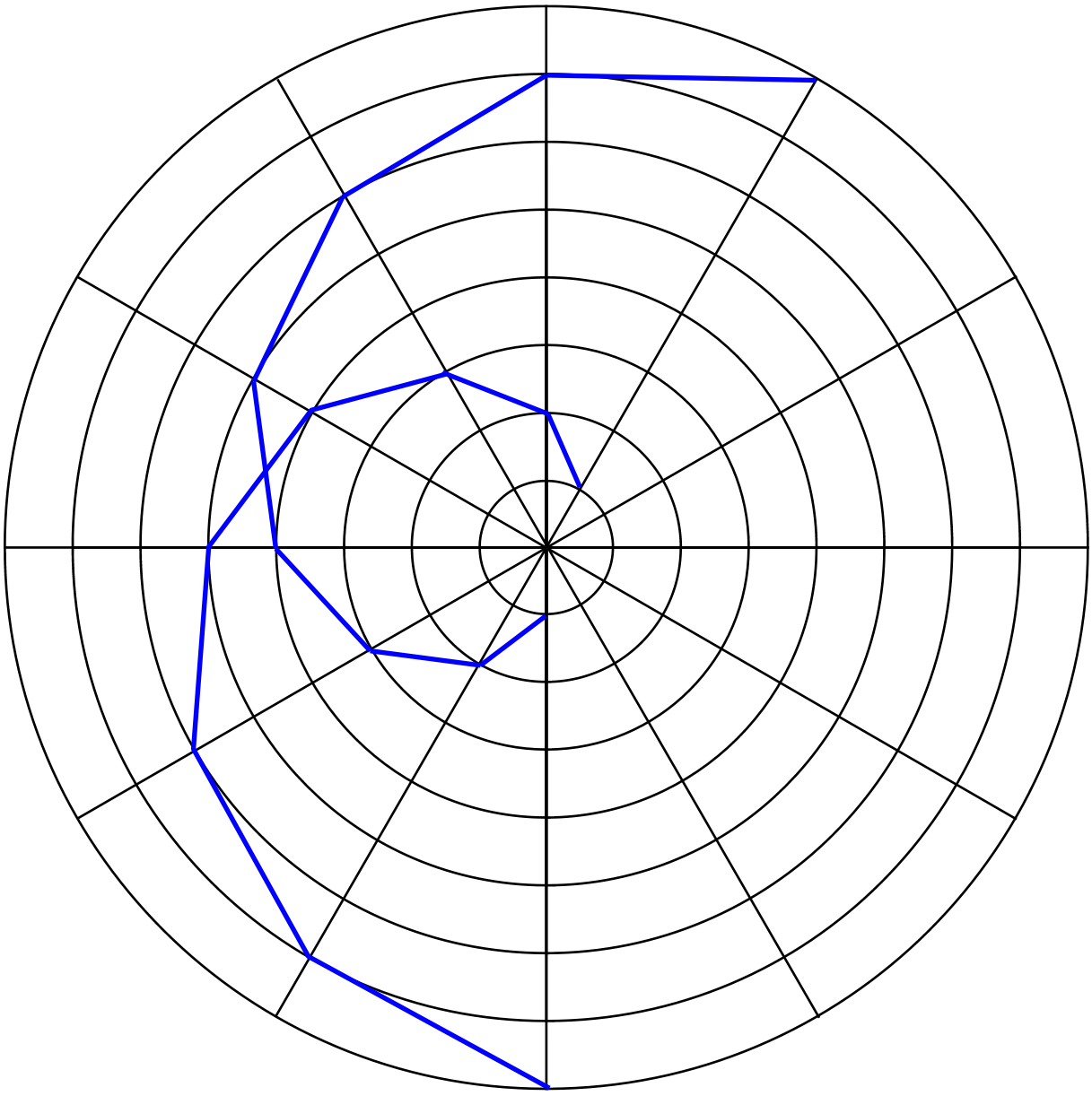
Complete the line pattern, move one circle out and one line clock-wise. Go all the way around, then repeat only move one circle out and one line **anti-clockwise**. Two lines are already drawn for you.



Flower Design

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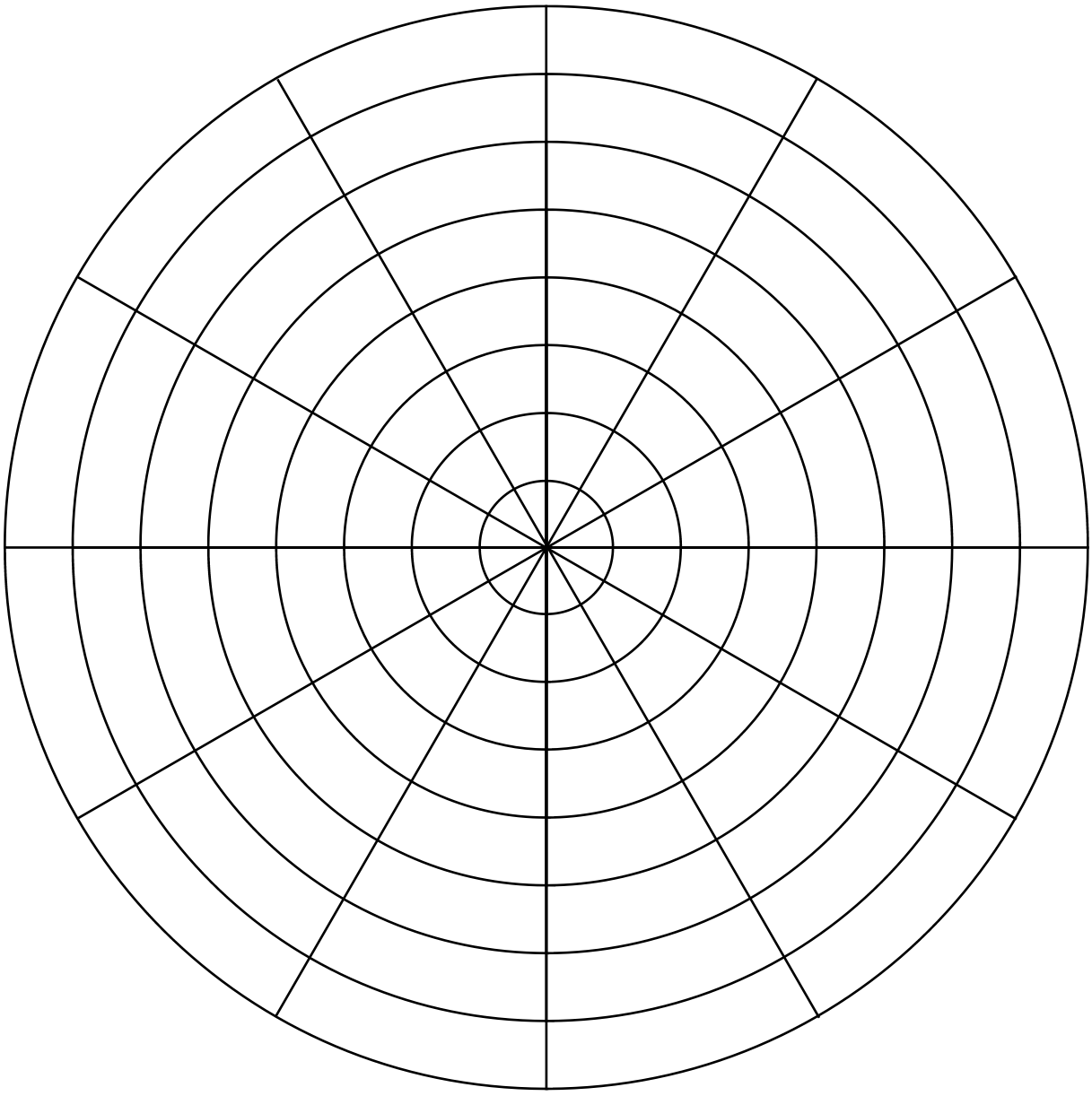
Complete the line pattern, move one circle out and one line clock-wise. Go all the way around, then repeat only move one circle out and one line **anti-clockwise**.



Flower Design

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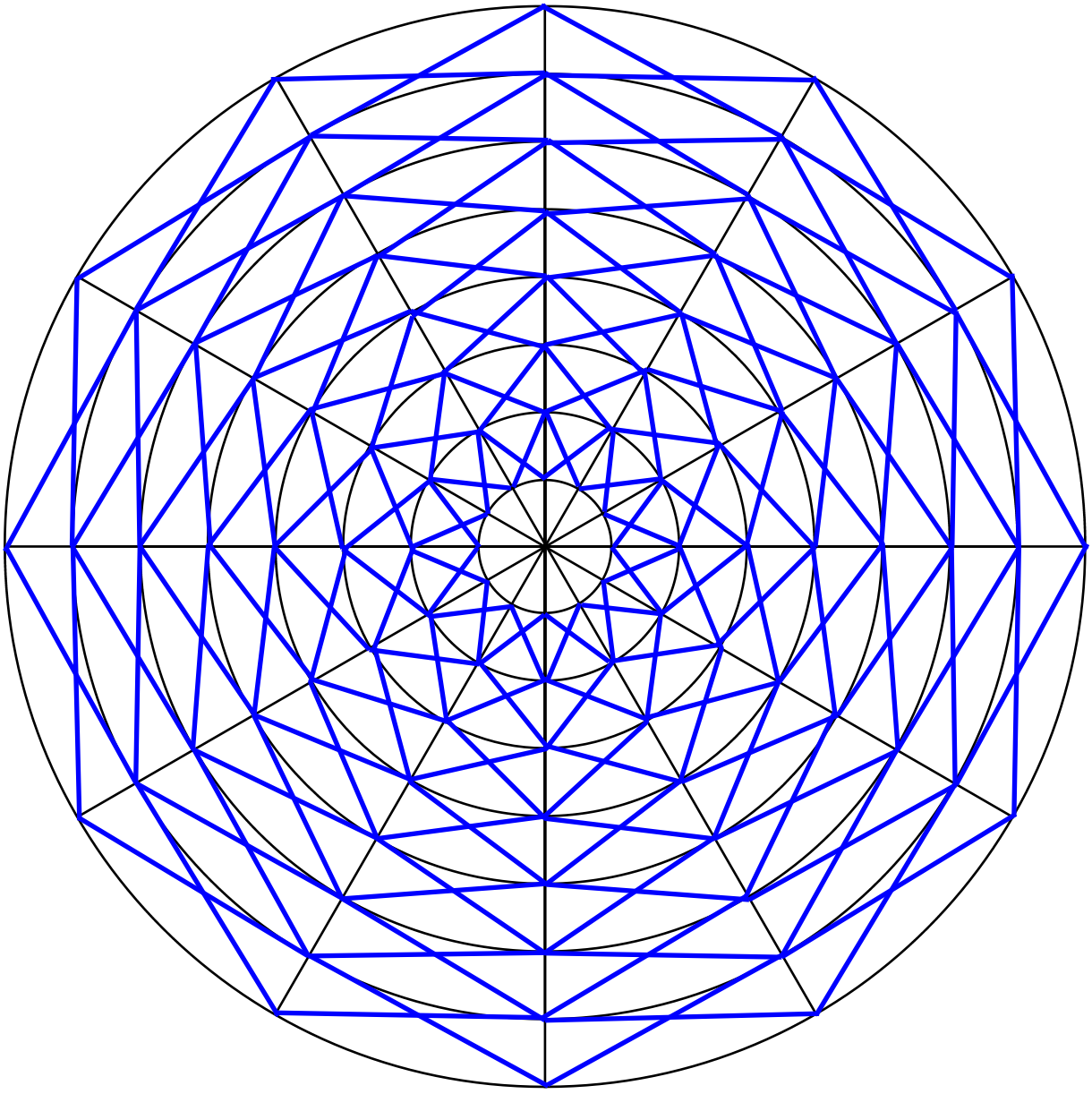
Complete the line pattern, move one circle out and one line clockwise. Go all the way around, then repeat only move one circle out and one line **anti-clockwise**. Two lines are already drawn for you.

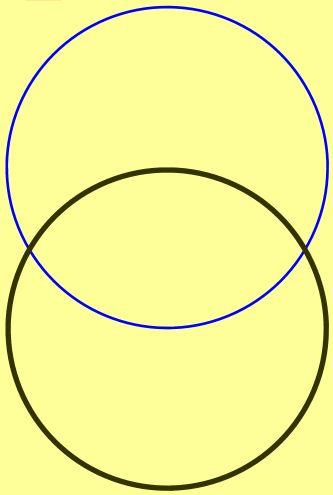


Making a Hexagon

This sheet involves constructing a hexagon using a compass and ruler, if printing this sheet use the dot in the middle of the circle to open your compass to the required radius or use a ruler and open your compass to 7 cm.

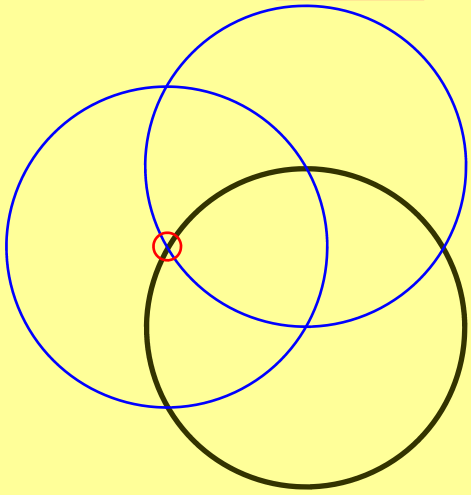
1

Position your compass at any position on the circumference and draw a circle.



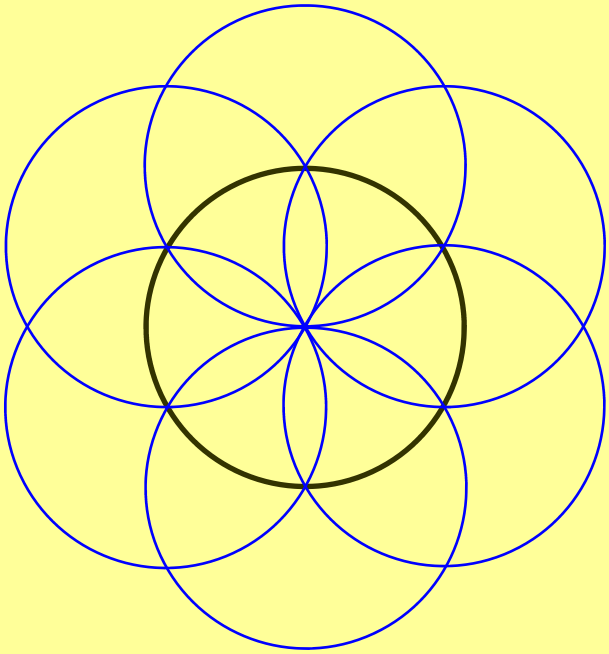
2

Place the compass at one of the points where the blue circle intersects the first circle and create another circle.



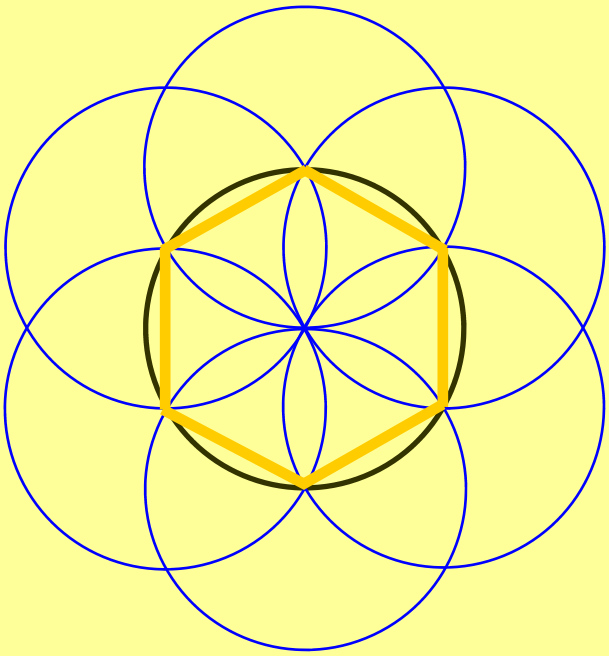
3

Repeat until six circles are completed



4

Join all intersection points to complete a hexagon (yellow).



Making a Hexagon

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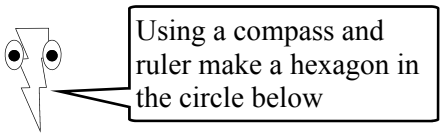

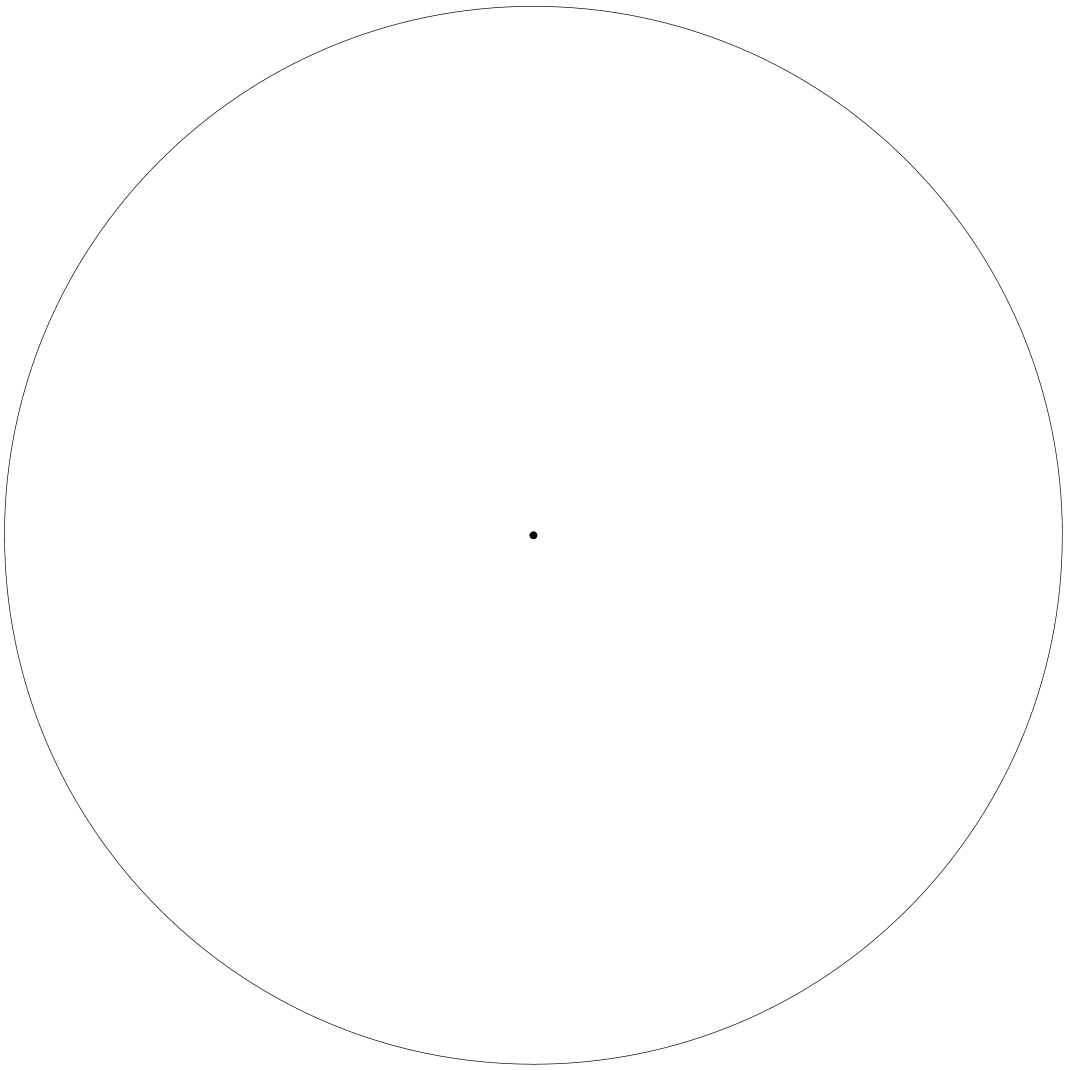

Making a Hexagon

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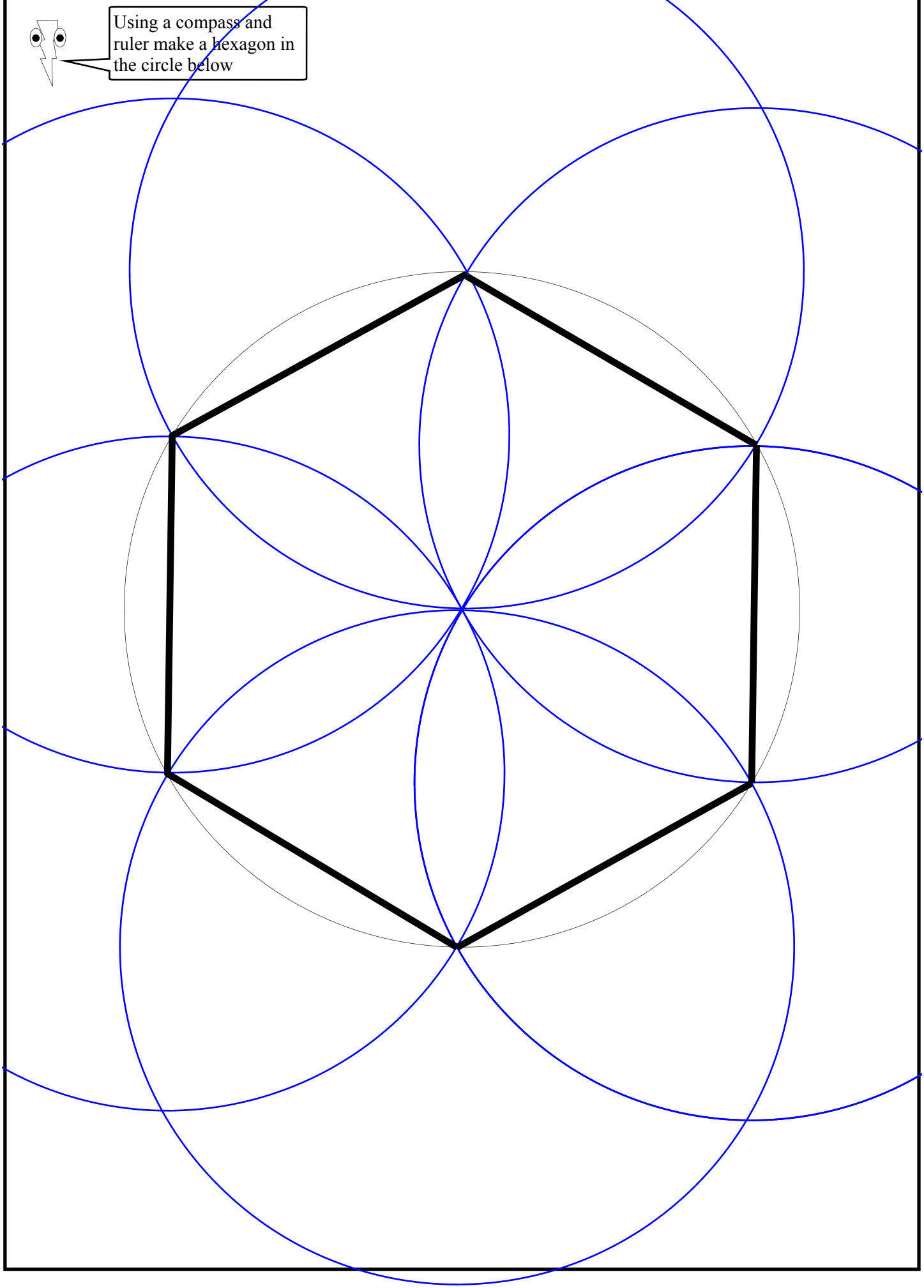


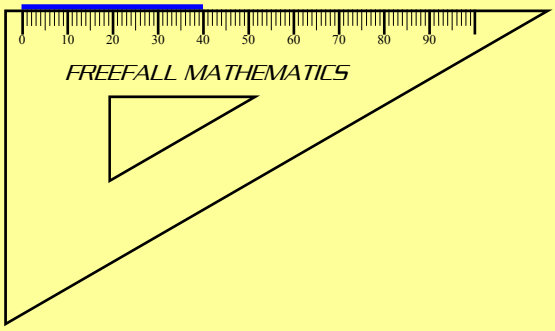
Drawing Quadrilaterals

This sheet involves the construction of four different plane (2-D) shapes. These are squares, rectangles, rhombi (plural of rhombus) and parallelograms.

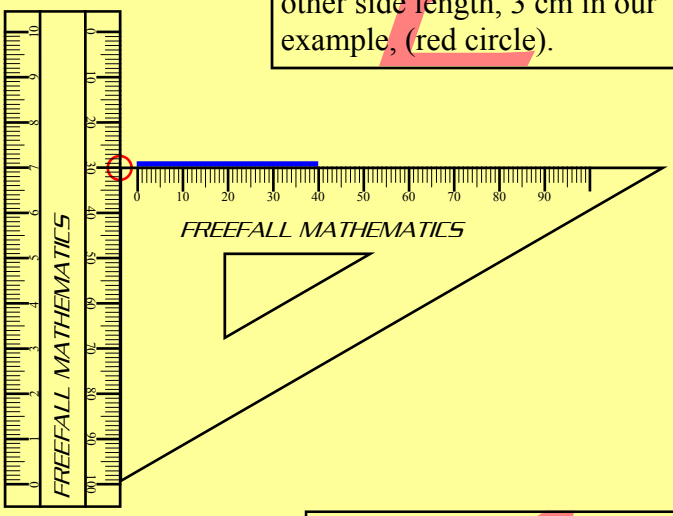
Column 1 - Constructing Squares & Rectangles - Set Square & Ruler

Example:
4 cm × 3 cm
Rectangle

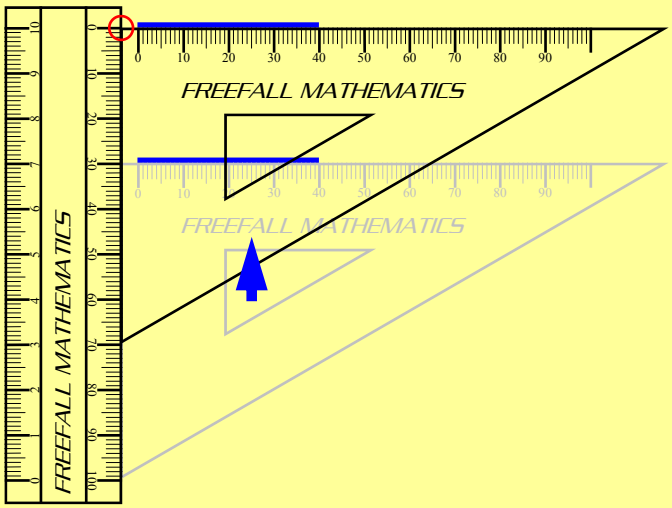
1 Using your set square draw a line the required length. In our example, 4 cm in length.



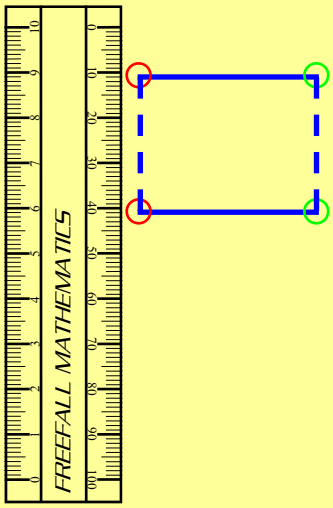
2 Place a ruler on the set square's edge so that the corner is at the other side length, 3 cm in our example, (red circle).

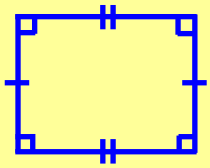


3 Slide the set square along the ruler edge until its corner is at zero, (red circle). Then draw the same length line as previously drawn, 4 cm in our example



4 Use a ruler to draw the remaining two sides. In our example they are 3 cm.





5 Place right angle markings in each corner and mark the opposite equal sides with dashes. All done!

Drawing Quadrilaterals

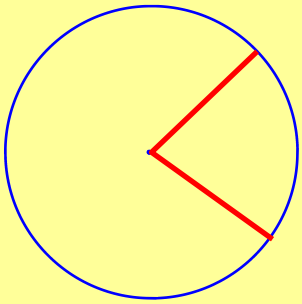
Column 2 - Constructing a Rhombus - Compass & Ruler

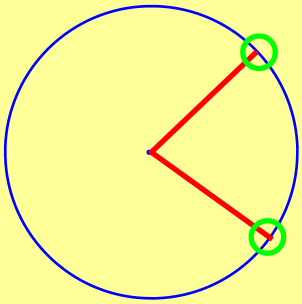
Example:
4 cm Rhombus

1 Draw a point where you wish to start.

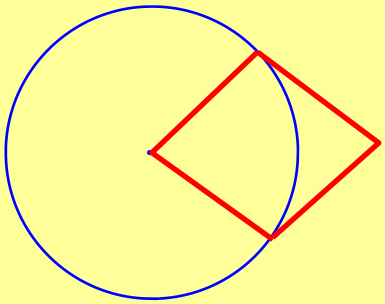
2 Open your compass to the side length (4 cm in our example) and draw a circle. Then draw 2 lines from the centre to the circumference, at any angle you like.

3 Using the compass open at the same length, draw 2 arcs from the intersection points shown below in green.

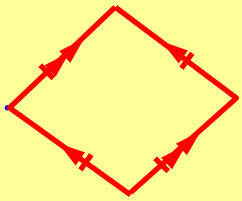




4 Use your ruler to complete the rhombus.



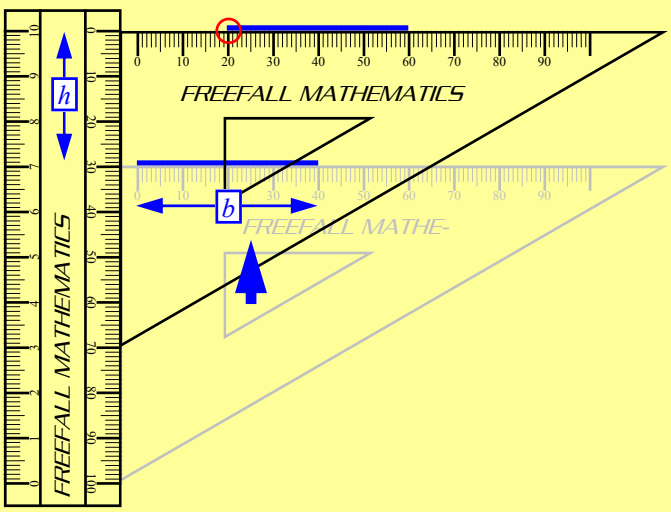
5 Show the parallel sides with arrows. Label all the equal sides with dashes.

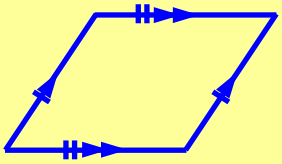


Column 2 - Constructing a Parallelogram - Set Square & Ruler

1 Start by using the same method as with the construction of a square or rectangle. When drawing the second line, instead of starting from zero start from another number, in the example below 2 cm was chosen.

2 Complete the sides and place arrows on parallel sides. You should also put dashes on the sides to show that they are the same length, that way a rhombus isn't confused with a parallelogram.





Drawing Quadrilaterals

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Construct squares using a ruler and a set square. Place side length markings and perpendicular symbols on the shape.

1 $l = 2.5$ cm

2 $l = 4.1$ cm

3 $l = 37$ mm

4 $l = 1.9$ cm

Now construct rectangles with the same length sides marked and perpendicular markings in each corner



5 $l = 4$ cm, $b = 2.5$ cm

6 $l = 2.2$ cm, $b = 3.1$ cm

7 $l = 34$ mm, $b = 6$ cm

8 $l = 1.3$ cm, $b = 5.2$ cm



Use a compass to draw rhombi with the side lengths below, mark the parallel sides.

9 $l = 2.5$ cm

10 $l = 1.9$ cm

11 $l = 2.8$ cm

12 $l = 14$ mm

Construct parallelograms with parallel side markings. The base length and the perpendicular height are given



13 $h = 1.5$ cm, $b = 4$ cm

14 $h = 3.1$ cm, $b = 3.6$ cm

15 $h = 0.8$ cm, $b = 2.6$ cm

16 $h = 1.8$ cm, $b = 3.2$ cm

Drawing Quadrilaterals

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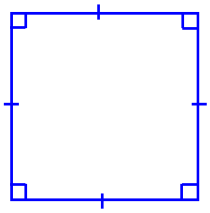
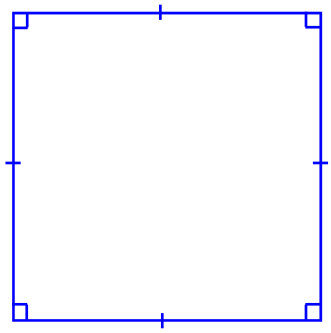
Construct squares using a ruler and a set square. Place side length markings and perpendicular symbols on the shape.

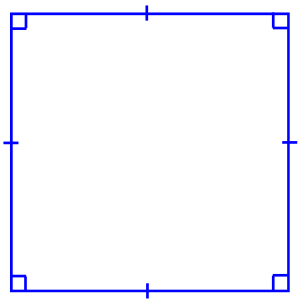
1 $l = 2.5$ cm



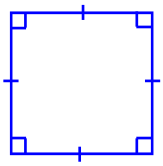
2 $l = 4.1$ cm



3 $l = 37$ mm

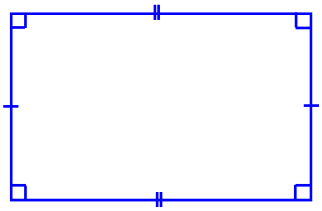


4 $l = 1.9$ cm

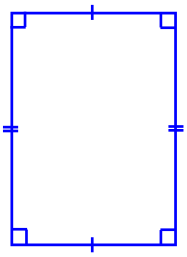


Now construct rectangles with the same length sides marked and perpendicular markings in each corner

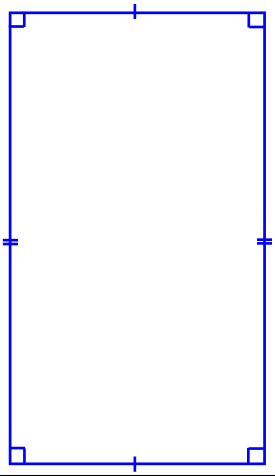
5 $l = 4$ cm, $b = 2.5$ cm



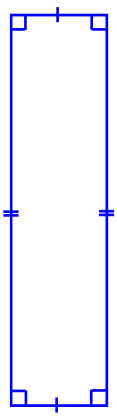
6 $l = 2.2$ cm, $b = 3.1$ cm



7 $l = 34$ mm, $b = 6$ cm

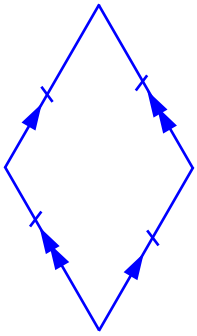


8 $l = 1.3$ cm, $b = 5.2$ cm

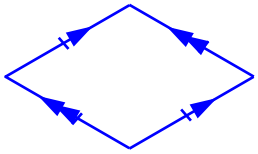


Use a compass to draw rhombi with the side lengths below, mark the parallel sides.

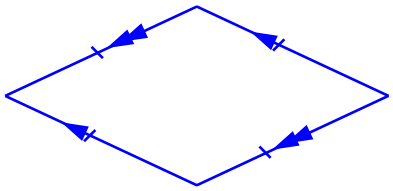
9 $l = 2.5$ cm



10 $l = 1.9$ cm



11 $l = 2.8$ cm

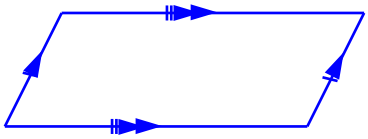


12 $l = 14$ mm

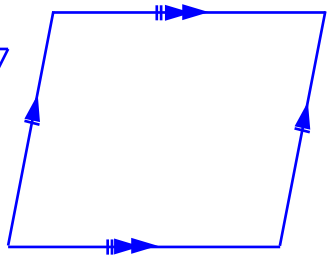


Construct parallelograms with parallel side markings. The base length and the perpendicular height are given

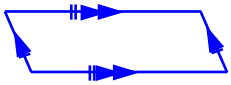
13 $h = 1.5$ cm, $b = 4$ cm



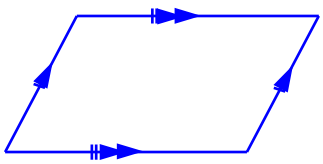
14 $h = 3.1$ cm, $b = 3.6$ cm



15 $h = 0.8$ cm, $b = 2.6$ cm



16 $h = 1.8$ cm, $b = 3.2$ cm



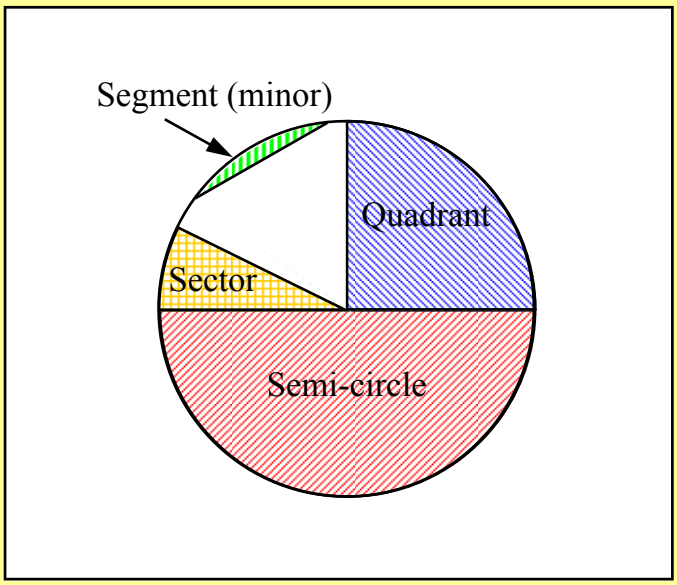
The Circle

This sheet deals with naming parts of a circle, these are areas of circles created by particular lines and also the lines themselves.

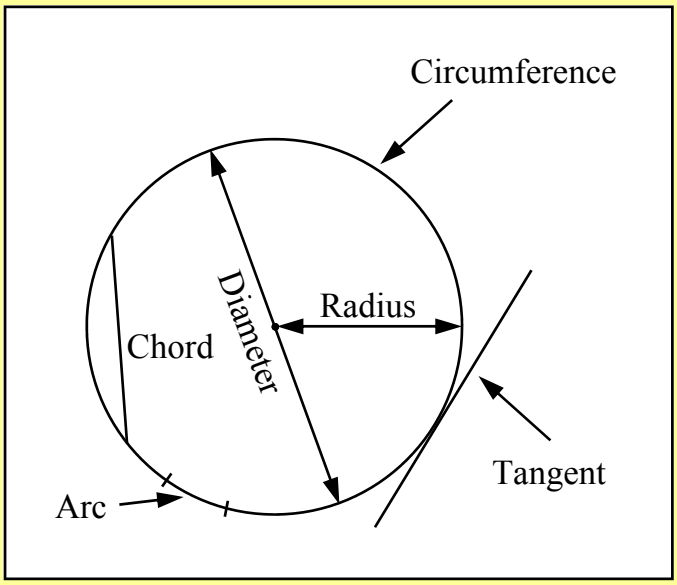
The sheet has 2 columns with diagrams at the top of each. In the spaces provided, name the parts of the circle that are being pointed to.

The questions below the diagrams all refer to the parts that have just been named, so look at the diagrams below if you need help. A word you may not be familiar with is radii - this is the plural of radius (so 1 radius, 2 radii). Also segments may be named minor segments when they are smaller than a semi-circle and major segments when they are larger than a semi-circle. Though for this sheet the word 'segment' alone is adequate, ask your teacher if minor/major is also required.

Column 1 - Circle Areas



Column 2 - Circle Lengths



The Circle

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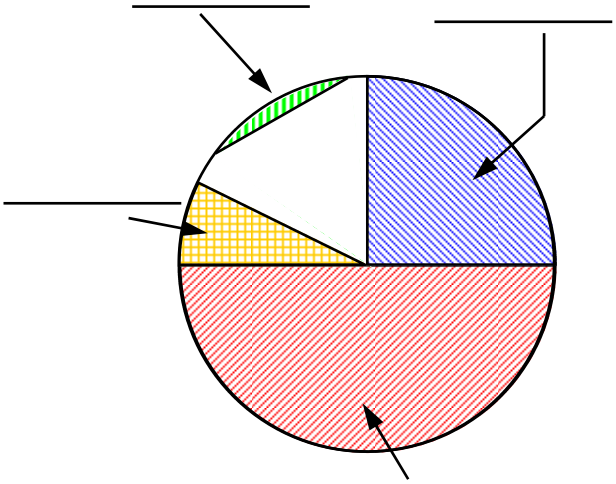
Name the parts of the circle below.



The answers for these will be found on the diagram above

1 Pizzas are sliced into these _____

2 Two of these makes a semi-circle _____

3 This is the only shape that doesn't touch the centre of the circle _____

4 When two or more of these are combined they could make a quadrant _____

5 A chord is used to make this shape _____

6 This shape has the circle's diameter as one of its sides _____

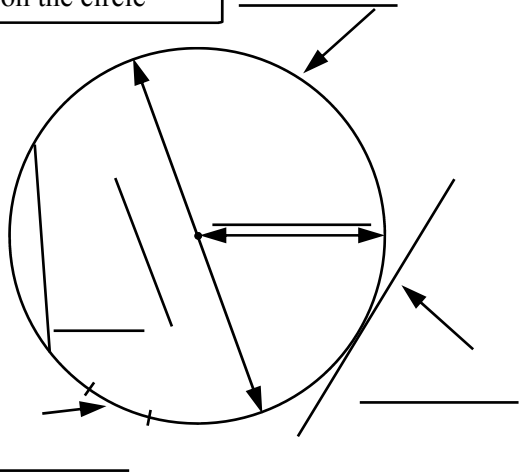
7 This shape has a right angle _____

8 When an arc is combined with two radii the shape that is made is a _____

9 If 6 identical sectors form a quadrant what shape is formed by 12 sectors ? _____

10 If 6 identical sectors form a quadrant what shape is formed by 3 sectors ? _____

Name the lengths on the circle



The answers for these will be one of the lengths shown above

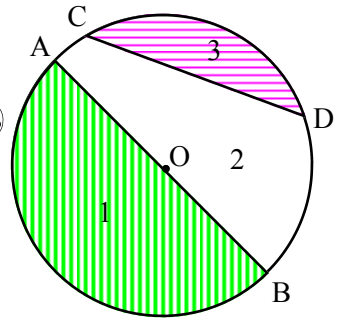
11 This distance is half the diameter _____

12 If a wheel rolled one revolution, the distance traveled would be the _____

13 A right angle is formed where the radius meets this line _____

14 A segment is formed by an arc and one of these _____

Name lengths/areas described by these letters



15 AC: _____

16 OB: _____

17 AB: _____

18 CD: _____

19 OD: _____

20 BC: _____

21 1: _____

22 3: _____

23 1 + 2: _____

The Circle

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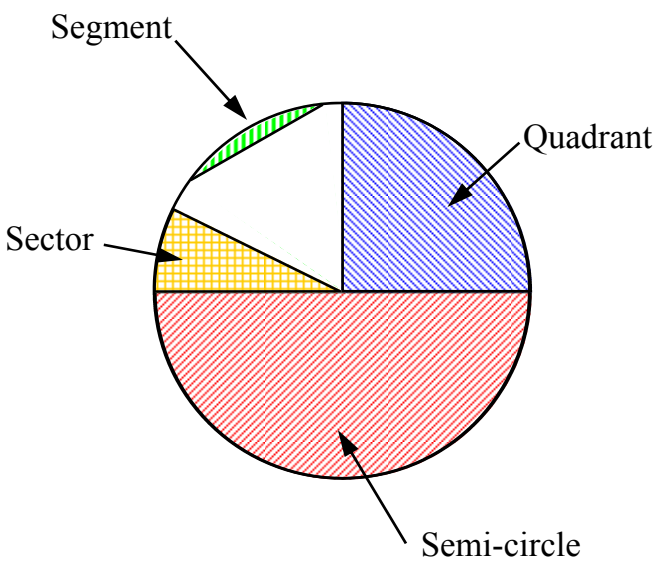
Here are the different areas within a circle



The answers for these will be found on the diagram above

1 Pizzas are sliced into these sectors

2 Two of these makes a semi-circle quadrants

3 This is the only shape that doesn't touch the centre of the circle segment

4 When two or more of these are combined they could make a quadrant sectors

5 A chord is used to make this shape segment

6 This shape has the circle's diameter as one of its sides semi-circle

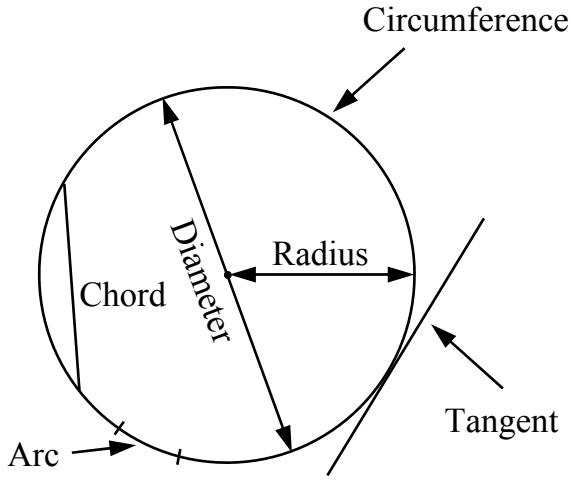
7 This shape has a right angle quadrant

8 When an arc is combined with two radii the shape that is made is a sector or quadrant

9 If 6 identical sectors form a quadrant what shape is formed by 12 sectors ? semi-circle

10 If 6 identical sectors form a quadrant what shape is formed by 3 sectors ? sector

Here are the different distances and features of a circle



The answers for these will be one of the lengths shown above

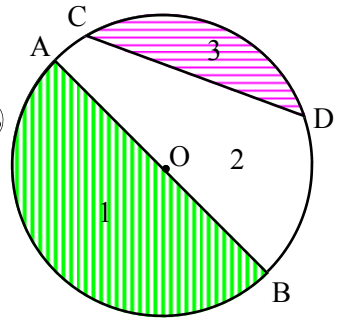
11 This distance is half the diameter radius

12 If a wheel rolled one revolution, the distance traveled would be the circumference

13 A right angle is formed where the radius meets this line tangent

14 A segment is formed by an arc and one of these chord

Name lengths/areas described by these letters



15 AC: arc

16 OB: radius

17 AB: diameter

18 CD: chord

19 OD: radius

20 BC: chord

21 1: semi-circle

22 3: segment

23 1 + 2: major segment

Geometry - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Geometry Find a Word

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Find the words in the puzzle from the wordlist

P	E	R	P	E	N	D	I	C	U	L	A	R	I	I
S	P	A	T	T	E	R	N	Q	B	D	R	E	N	S
U	T	N	E	G	N	A	T	X	E	Z	C	T	T	O
I	B	F	R	M	Q	A	E	G	T	P	D	E	E	S
D	L	V	E	K	X	U	R	C	N	R	T	M	R	C
A	A	B	C	D	F	E	S	D	O	M	O	A	V	E
R	R	K	T	V	E	X	E	H	A	M	P	I	A	L
H	E	X	A	G	O	N	C	Q	Q	U	P	D	L	E
M	T	P	N	S	E	C	T	O	R	J	Q	A	L	S
E	A	N	G	V	E	S	I	E	B	F	M	S	S	C
R	L	T	L	C	Q	K	O	T	L	I	L	G	H	S
T	I	K	E	U	N	F	N	P	T	C	S	Q	P	T
N	U	X	A	U	Q	E	T	A	T	O	R	E	U	X
E	Q	R	P	A	R	A	L	L	E	L	V	I	C	N
C	E	C	N	E	R	E	F	M	U	C	R	I	C	T

WORDLIST

BISECT	CHORD	CENTRE	HEXAGON	SQUARE
DIAMETER	INTERVAL	TANGENT	RADIUS	ISOSCELES
QUADRANT	PARALLEL	SECTOR	COMPASS	RECTANGLE
CIRCLE	ROTATE	ARC	PATTERN	DEGREE
CIRCUMFERENCE	PERPENDICULAR	INTERSECTION	EQUILATERAL	

Geometry Find a Word

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Find the words in the puzzle from the wordlist

P	E	R	P	E	N	D	I	C	U	L	A	R	I	I
S	P	A	T	T	E	R	N	Q	B	D	R	E	N	S
U	T	N	E	G	N	A	T	X	E	Z	C	T	T	O
I	B	F	R	M	Q	A	E	G	T	P	D	E	E	S
D	L	V	E	K	X	U	R	C	N	R	T	M	R	C
A	A	B	C	D	F	E	S	D	O	M	O	A	V	E
R	R	K	T	V	E	X	E	H	A	M	P	I	A	L
H	E	X	A	G	O	N	C	Q	Q	U	P	D	L	E
M	T	P	N	S	E	C	T	O	R	J	Q	A	L	S
E	A	N	G	V	E	S	I	E	B	F	M	S	S	C
R	L	T	L	C	Q	K	O	T	L	I	L	G	H	S
T	I	K	E	U	N	F	N	P	T	C	S	Q	P	T
N	U	X	A	U	Q	E	T	A	T	O	R	E	U	X
E	Q	R	P	A	R	A	L	L	E	L	V	I	C	N
C	E	C	N	E	R	E	F	M	U	C	R	I	C	T

WORDLIST

~~BISECT~~ ~~CHORD~~ ~~CENTRE~~ ~~HEXAGON~~ ~~SQUARE~~

~~DIAMETER~~ ~~INTERVAL~~ ~~TANGENT~~ ~~RADIUS~~ ~~ISOSCELES~~

~~QUADRANT~~ ~~PARALLEL~~ ~~SECTOR~~ ~~COMPASS~~ ~~RECTANGLE~~

~~CIRCLE~~ ~~ROTATE~~ ~~ARC~~ ~~PATTERN~~ ~~DEGREE~~

~~CIRCUMFERENCE~~ ~~PERPENDICULAR~~ ~~INTERSECTION~~ ~~EQUILATERAL~~

FREEFALL MATHEMATICS

ALTITUDE

GRAPHS & TABLES

Picture Graphs

Picture graphs are similar to bar graphs, the data is displayed by pictures or symbols of an item. Graphs usually have an axis to read off the value, Picture graphs don't require this scale as each picture of an item represents a specified number of items. Picture graphs are as accurate as their scale allows, but they are used mainly for quick reference rather than an exact display of data.

Column 1 of this sheet has the graphs drawn for you, you are asked to read the graph and answer the questions. Note the 'key' or 'scale', this tells you the value of each symbol. To increase accuracy, portions of a symbol can be used.

The computer in the first column is either: fully drawn, ¾ drawn, ½ drawn or ¼ drawn. If a fully drawn computer represents 20 computers then part drawn computers equal that fraction of 20.

Graph 2 is a representation of a Café's sales of fresh juices. This time each symbol represents 200 glasses of juice. Note that again the same fractions ¼, ½ and ¾ are used. Questions 6 and 7 are about rounding. While 2 symbols may represent 400 glasses, those same 2 symbols could also represent 390 glasses. Some students will have difficulty with this concept.

Column 2 asks you to construct graphs from data in the tables. The data can be constructed exactly by adding whole and part pieces of a symbol, you don't need to worry about rounding.

Remember that fractions of a picture are always on the end (right hand side) and there can only be one in each line. Don't use several fractions of a symbol, the purpose of a picture graph is that they are easy and quick to read, not confusing!

Picture Graphs

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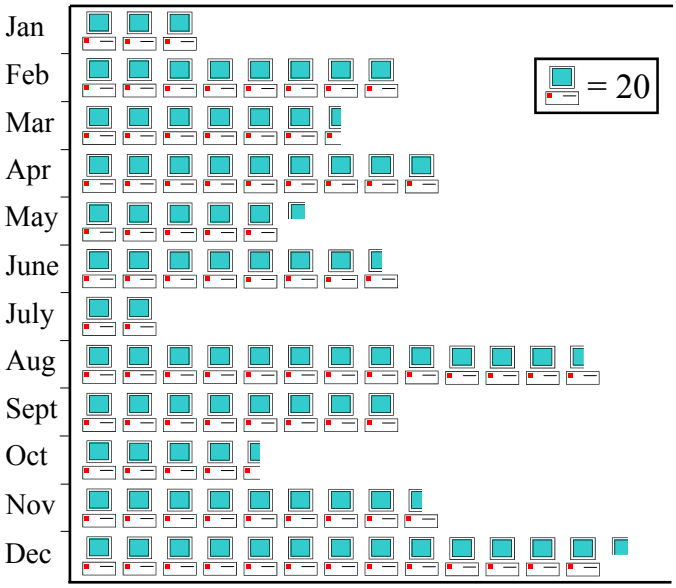
Study the graphs then answer the questions

Graph 1 : Computer Sales in 2002



1 Give the values represented by these symbols:

= ___ = ___ = ___ = ___

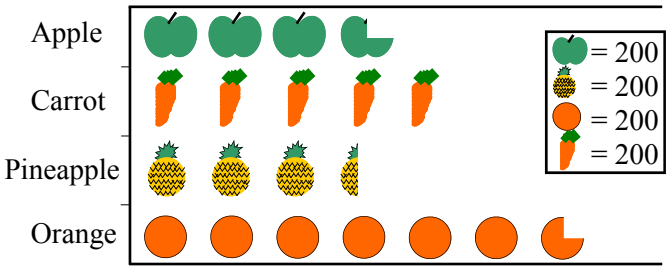
2 Name the months with the same sales figures and give the number of sales _____

3 Give the lowest sales month & sales _____

4 Give the highest sales month & sales figure, give a reason why this month could have the most sales.

_____

Graph 2 : Average Monthly Juice Sales (glass)



5 Name and give sales figures for the:

a) least popular juice _____

b) most popular juice _____

6 Actual sales figures for carrot juice could range from 975 - 1 024 glasses. Briefly explain why.

_____

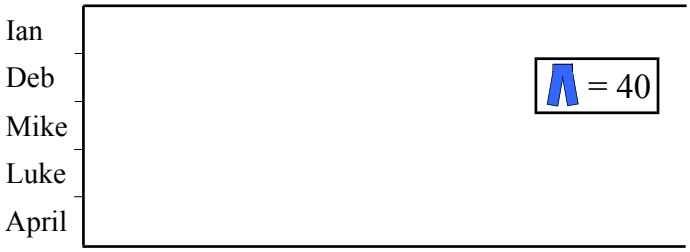
7 Give the largest and smallest possible sales figures for pineapple juice _____

Now place the details in the table on the graph

Table 1 : Jeans Shop Sales Assistant Figures

Name	Ian	Deb	Mike	Luke	April
Sales	240	80	140	170	150

Graph 3 : Jeans Shop Sales Assistant Figures



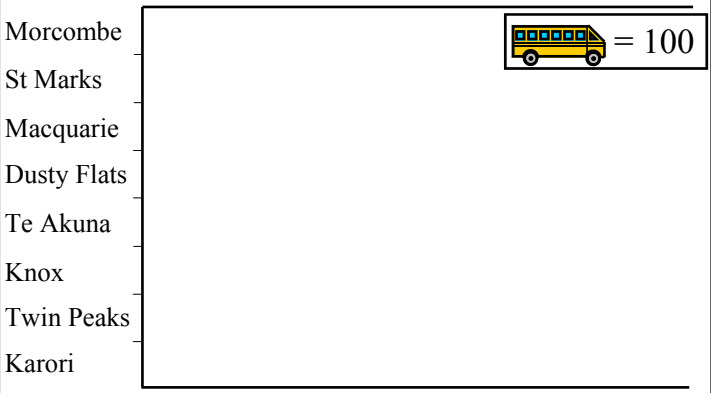
8 Give the values represented by these symbols:

= ___ = ___ = ___ = ___

Table 2 : Students Using Bus Transport

School Name	Student Numbers
Morcombe	250
St Marks	300
Macquarie	175
Dusty Flats	550
Te Akuna	50
Knox	225
Twin Peaks	475
Karori	325

Graph 4 : Students Using Bus Transport



9 Give the values represented by these symbols:

= ___ = ___ = ___ = ___

Picture Graphs

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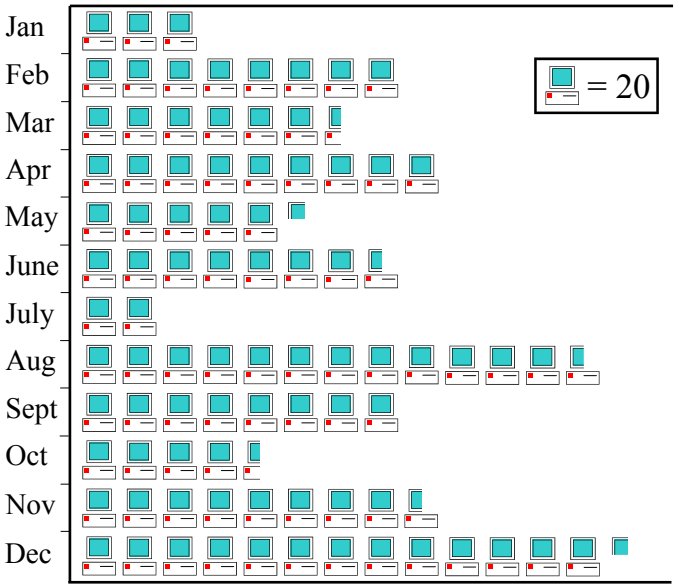
Study the graphs then answer the questions

Graph 1 : Computer Sales in 2002



1 Give the values represented by these symbols:

= 5 = 10 = 15 = 30

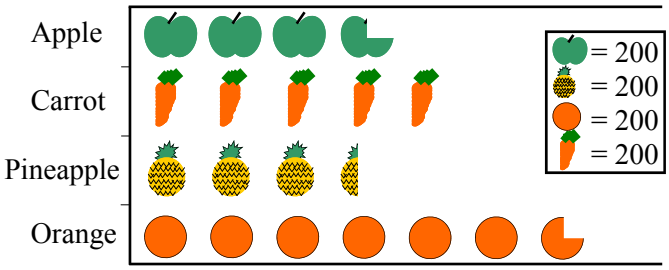
2 Name the months with the same sales figures and give the number of sales February (160) September (160)

3 Give the lowest sales month & sales July (40)

4 Give the highest sales month & sales figure, give a reason why this month could have the most sales.

December (265) - Christmas and Post Christmas sales

Graph 2 : Average Monthly Juice Sales (glass)



5 Name and give sales figures for the:

a) least popular juice Pineapple juice (700)

b) most popular juice Orange juice (1 350)

6 Actual sales figures for carrot juice could range from 975 - 1 024 glasses. Briefly explain why.

974 rounds to 950 and 1 025 rounds to 1 050

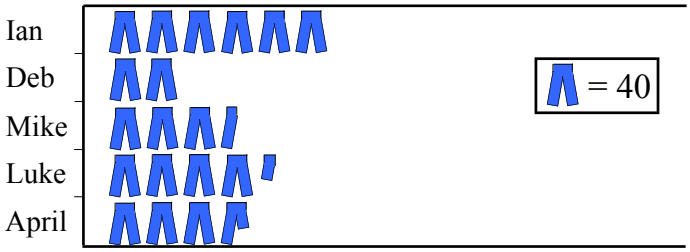
7 Give the largest and smallest possible sales figures for pineapple juice Smallest 675 and largest 724

Now place the details in the table on the graph

Table 1 : Jeans Shop Sales Assistant Figures

Name	Ian	Deb	Mike	Luke	April
Sales	240	80	140	170	150

Graph 3 : Jeans Shop Sales Assistant Figures



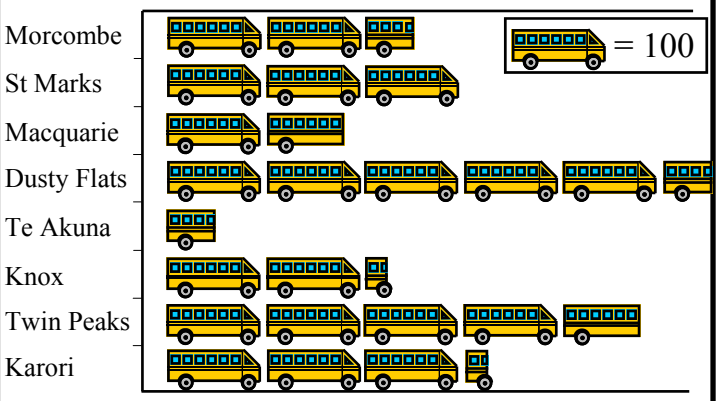
8 Give the values represented by these symbols:

= 10 = 20 = 30 = 70

Table 2 : Students Using Bus Transport

School Name	Student Numbers
Morcombe	250
St Marks	300
Macquarie	175
Dusty Flats	550
Te Akuna	50
Knox	225
Twin Peaks	475
Karori	325

Graph 4 : Students Using Bus Transport



9 Give the values represented by these symbols:

= 25 = 50 = 75 = 125

Column Graphs

Column graphs show data with columns which are read off a vertical scale, with the horizontal axis giving the meaning of the column. This sheet has two columns, the first being reading graphs and the second being constructing graphs (from reading a table).

Column 1 starts with a column graph on the fundraising activity of a charity. It outlines the different methods of fundraising along its horizontal axis and their financial success on the vertical axis. This graph can be read to the nearest $5 000.

The second graph is about the conversion of a chicken battery farm to a free range farm. This time there are two columns for each category. The free range egg figure and the battery laid figure. Note also that the columns are rectangular prisms, these give the graph a different appearance, but be careful when reading the columns. Read the columns off the top edge that is 'closer' to you, not the edge at the back of the column. In other words, use the front face of the prism for measurement.

Column 2 starts with a question about dice. When you roll 2 dice you have 36 different combinations of the two faces of each die. The first table is just like a 6 by 6 addition table, just add the two numbers. Then count how many of each number there are for the next table. Note both tables have been started for you. Then graph the result. Note that the first column is already done for you.

Question 6 involves a survey of traffic, it requires you to read the data straight from the table and place it on the graph.

Note the use of the word frequency, frequency means the number of times something occurs.

Column Graphs

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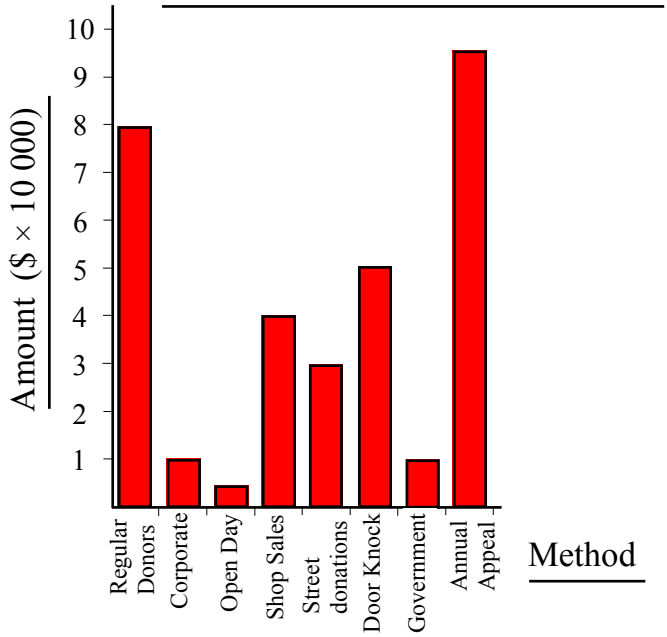
Study the column graphs below then answer the questions

Graph 1 : Fundraising by a Charity



1 Give amount raised by: *a*) Door Knock _____

b) Regular Donors _____ *c*) Shop Sales _____

2 Which method of fundraising:

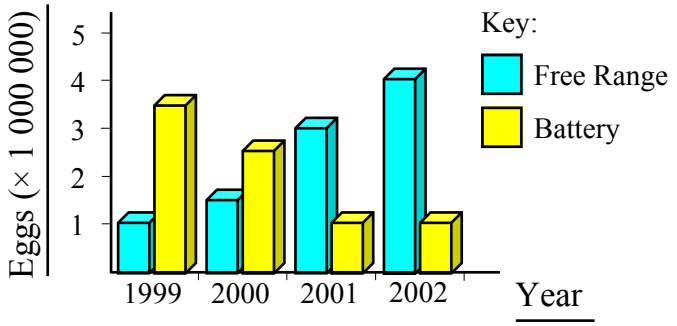
a) is the third most successful _____

b) matches Government donations _____

c) more than triples the street donations _____

d) made $5 000 _____

Graph 2 : Ingall's Hen Farm Egg Production



3 Give the total number of eggs for: *a*) 1999 _____

b) 2000 _____ *c*) 2001 _____

4 Give the year that free range production was:

a) 1 000 000 eggs less then battery laid eggs _____

b) increased and battery laid was unchanged _____

c) three times more than battery laid eggs _____

d) increased by 1 500 000 eggs _____

Complete the tables for the totals possible for 2 dice, then graph them.

5

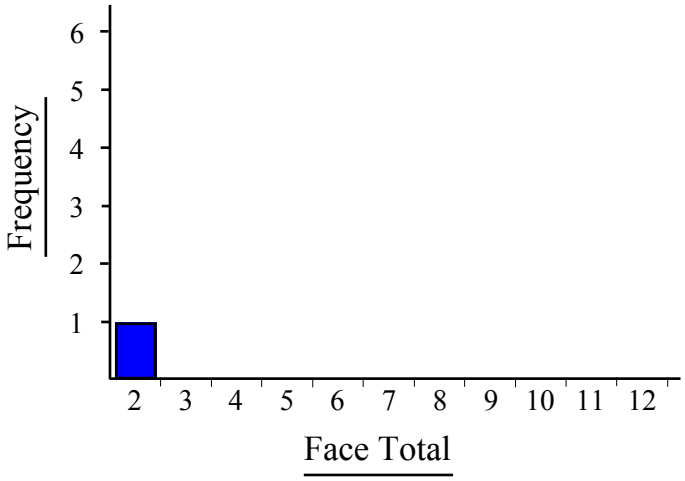
Table 1 : Total of 2 dice

Face No.	1	2	3	4	5	6
1	2	3	4			
2	3	4				
3	4					
4						
5						
6						

Table 2 : Frequency for Total of Two dice

Total	2	3	4	5	6	7	8	9	10	11	12
Freq	1	2	3								

Graph 3 : Face Total : 2 dice

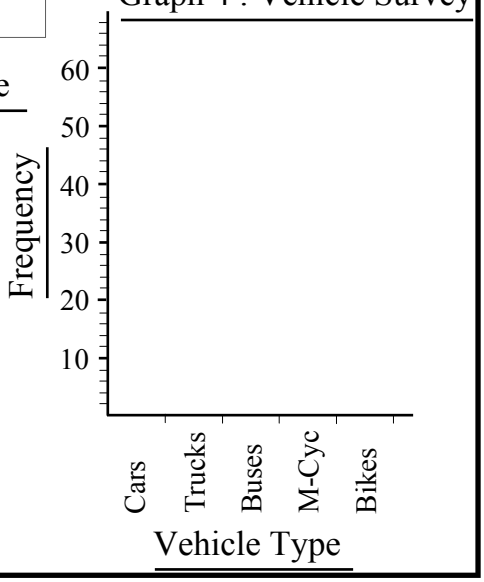


6 Transfer this data to the graph

Table 3 : Vehicle Survey

Type	Freq
Cars	63
Trucks	11
Buses	17
M-cyc	9
Bikes	4

Graph 4 : Vehicle Survey



Column Graphs

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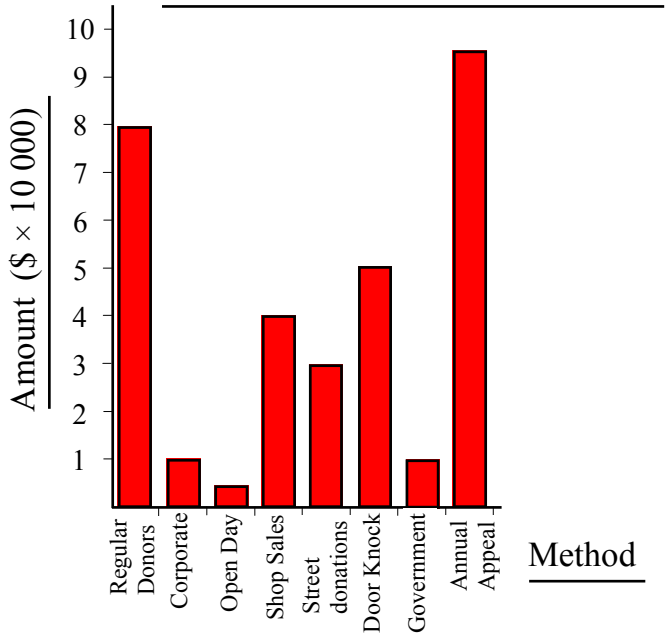
Study the column graphs below then answer the questions

Graph 1 : Fundraising by a Charity



1 Give amount raised by: *a*) Door Knock $50 000

b) Regular Donors $80 000 *c*) Shop Sales $40 000

2 Which method of fundraising:

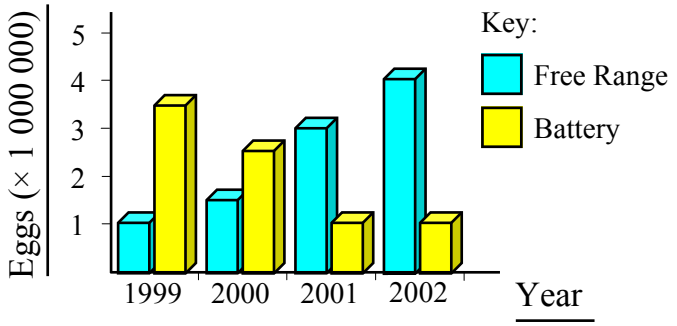
a) is the third most successful Door knock

b) matches Government donations Corporate

c) more than triples the street donations Annual Appeal

d) made $5 000 Open Day

Graph 2 : Ingall's Hen Farm Egg Production



3 Give the total number of eggs for: *a*) 1999 4 500 000

b) 2000 4 000 000 *c*) 2001 4 000 000

4 Give the year that free range production was:

a) 1 000 000 eggs less then battery laid eggs 2 000

b) increased and battery laid was unchanged 2002

c) three times more than battery laid eggs 2001

d) increased by 1 500 000 eggs 2001

Complete the tables for the totals possible for 2 dice, then graph them.

5

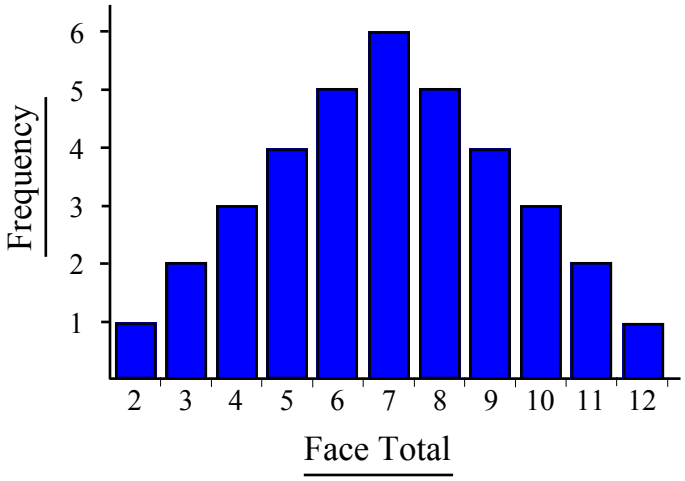
Table 1 : Total of 2 dice

Face No.	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Table 2 : Frequency for Total of Two dice

Total	2	3	4	5	6	7	8	9	10	11	12
Freq	1	2	3	4	5	6	5	4	3	2	1

Graph 3 : Face Total : 2 dice



6 Transfer this data to the graph

Table 3 : Vehicle Survey

Type	Freq
Cars	63
Trucks	11
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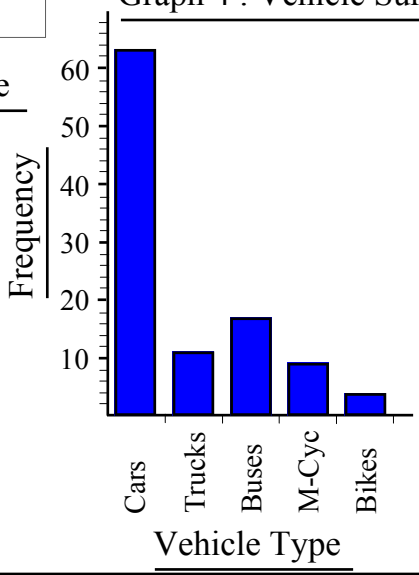

Bar Graphs

Bar graphs display data in the same way as column graphs except the graph is on its side. These graphs all have:

- A title which describes the purpose of the graph
- Axes headings, these describe the scale on each axis
- Scales, these let you to read or construct the bars with reasonably accuracy

Column 1 asks you to extract information from constructed graphs. Both graphs have columns that are either exactly on the number or half way between two numbers on the horizontal scale. The first graph deals with Blue Whale population, Questions 1 and 2 ask you to read figures off the graph, question 3 asks you to write the years that correspond to the change in population.

The second graph refers to the way a student may spend their day. This graph uses ½ hours so write either 0.5 h or ½ h.

The second column asks you to create bar graphs. Write the title, axes headings and complete the scale which has been started for you. Then construct the bars.

Bar Graphs

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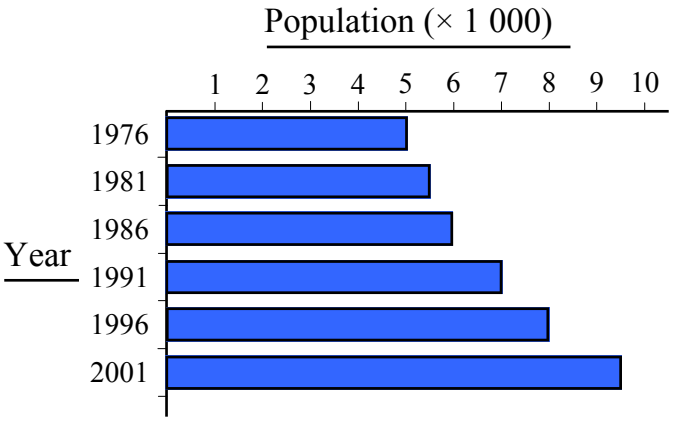
Answer these questions on the use of bar graphs

Graph 1 : Blue Whale Population Estimation



1 Give the estimated number of blue whales in:

a) 1976 _____ *b*) 1986 _____ *c*) 1996 _____

2 Give the increase in whale numbers between

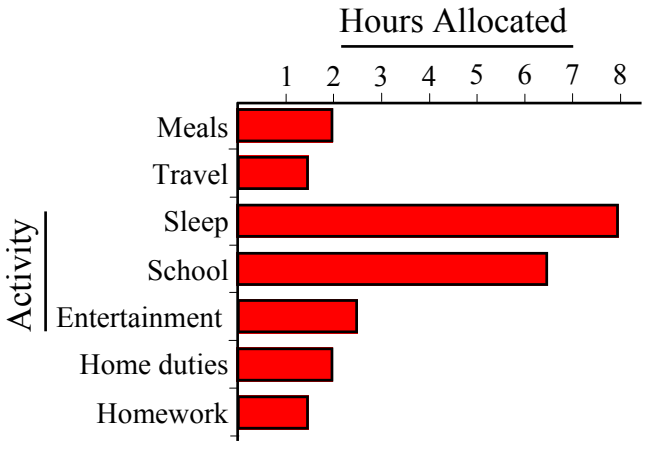
a) 1976 - 1991 _____ *b*) 1981 - 2001 _____

3 Between which years is there a population increase of:

a) 500 _____ *b*) 1 500 _____

c) 3 000 _____ *d*) 4 000 _____

Graph 2 : Analysis of a School Day



4 Give the number of hours spent: *a*) Sleeping ____

b) Eating ____ *c*) At school ____ *d*) Travelling ____

5 Give the difference in time spent on these activities:

a) Entertainment and.. Homework ____ Eating ____

b) Time at School and.. Travel ____ Sleeping ____

c) Home duties and.. School time ____ Travel ____

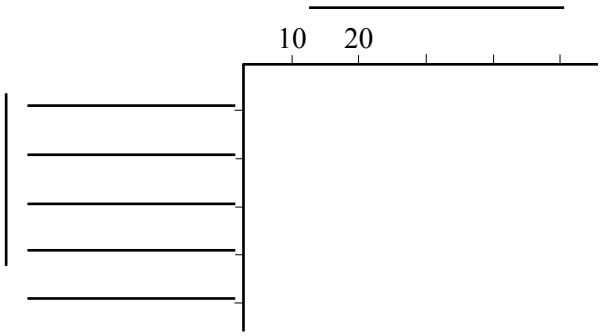
6 How many hours could the above student be at home each weekday _____

Now put this data on the bar graph below

7 Table 1 : Least Painful Home Duty

Home Duty	Student Approval
Mowing Grass	15%
Washing Dishes	10%
Tidy Bedroom	5%
Washing Car	25%
Meal Preparation	45%

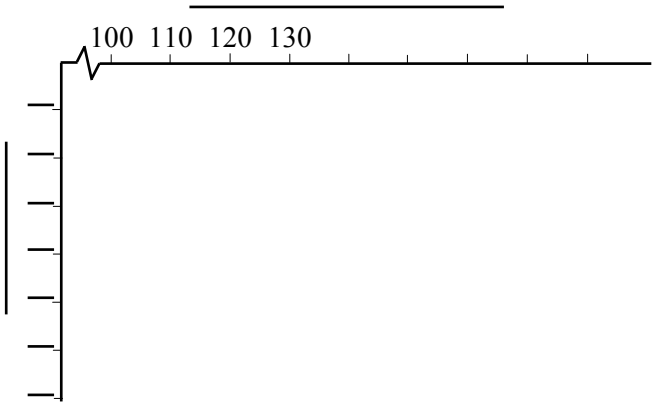
Graph 3 : _____



8 Table 2 : Timing Sunscreen Protection

Sunscreen	Effective Time (min)
A	105
B	130
C	125
D	110
E	180
F	135
G	115

Graph 4 : _____



Bar Graphs

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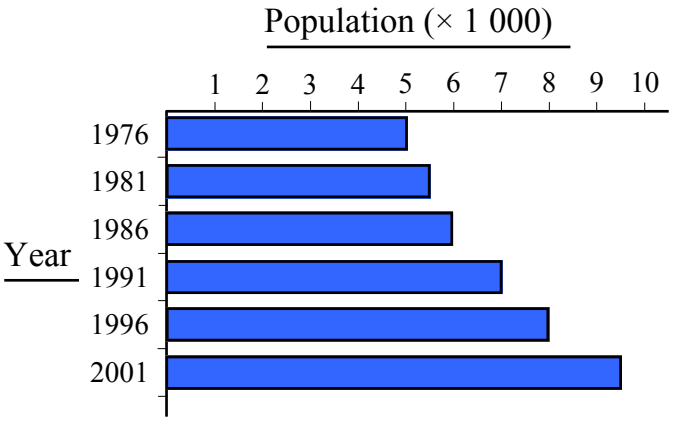
Answer these questions on the use of bar graphs

Graph 1 : Blue Whale Population Estimation



1 Give the estimated number of blue whales in:

a) 1976 5 000 *b*) 1986 6 000 *c*) 1996 8 000

2 Give the increase in whale numbers between

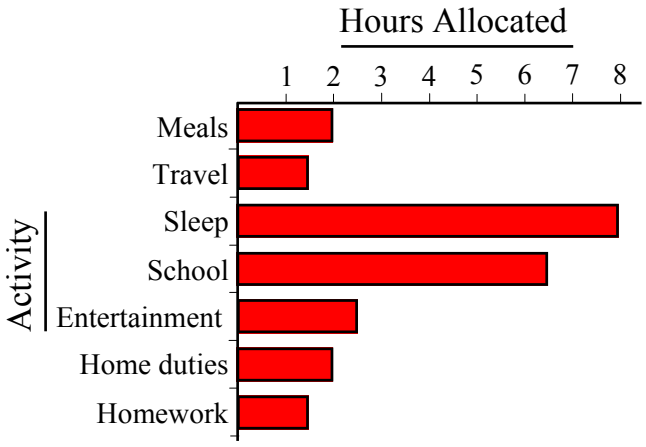
a) 1976 - 1991 2 000 *b*) 1981 - 2001 4 000

3 Between which years is there a population increase of:

a) 500 1976 - 1981, 1981 - 1986 *b*) 1 500 1996 - 2001

c) 3 000 1976 - 1996 *d*) 4 000 1981 - 2001

Graph 2 : Analysis of a School Day



4 Give the number of hours spent: *a*) Sleeping 8

b) Eating 2 *c*) At school 6.5 *d*) Travelling 1.5

5 Give the difference in time spent on these activities:

a) Entertainment and.. Homework 1 h Eating 0.5 h

b) Time at School and.. Travel 5 h Sleeping 1.5 h

c) Home duties and.. School time 4.5 h Travel 0.5 h

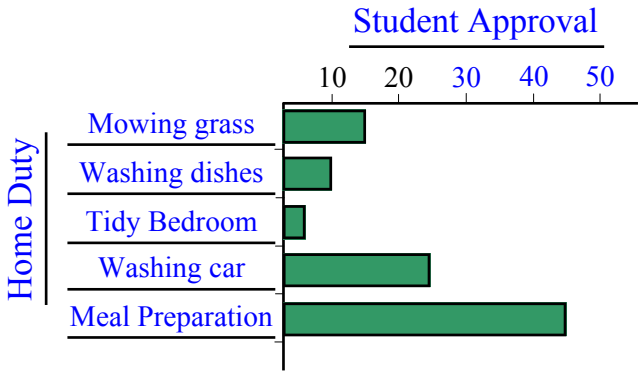
6 How many hours could the above student be at home each weekday 16 h

Now put this data on the bar graph below

7 Table 1 : Least Painful Home Duty

Home Duty	Student Approval
Mowing Grass	15%
Washing Dishes	10%
Tidy Bedroom	5%
Washing Car	25%
Meal Preparation	45%

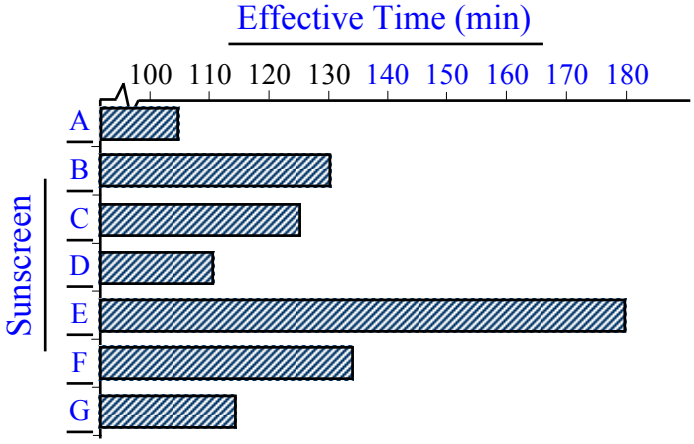
Graph 3 : Least Painful Home Duty



8 Table 2 : Timing Sunscreen Protection

Sunscreen	Effective Time (min)
A	105
B	130
C	125
D	110
E	180
F	135
G	115

Graph 4 : Timing Sunscreen Protection



Line Graphs

Line graphs show data as a continuous line, this allows for estimations to be made between data. For example if population data is assembled every 5 years by using a line graph you can estimate the population in between those years. You should realise that there are limitations to this, as the population won't change in a straight line, it could rapidly rise and fall between actual readings. Another common use of line graphs is to relate two values against each other. This relationship may be a straight line and so values at any point on the line will be correct.

Column 1 starts with a town's population being graphed. Note the scale on the vertical axis is × 10 000, so 1 = 10 000, 1.4 = 14 000 and so on. You would use a ruler to accurately read off the graph. The second graph involves converting currency, between NZ$ and A$ (or NZD and AUD).

Column 2 questions 5, 6 and 7 refer to the column 1 graph, only you are asked to find one value from the graph and then using your number skills, find other conversions without using the graph. Remember when you multiply by 10 you move the decimal point one place to the right, 100 = 2 places and so on. For division move the decimal point to the left.

Graph 3 compares US$ with both Australian and New Zealand currencies. To convert US$ to A$ use a ruler horizontally (flat) then where the ruler intersects the black line, read off the amount on the bottom scale. To convert to from US$ to NZ$ use the same method but use the red line. Note that you could use this graph to convert between A$ and NZ$, you would go up to the red or black line, then across to other line, then down.

Line Graphs

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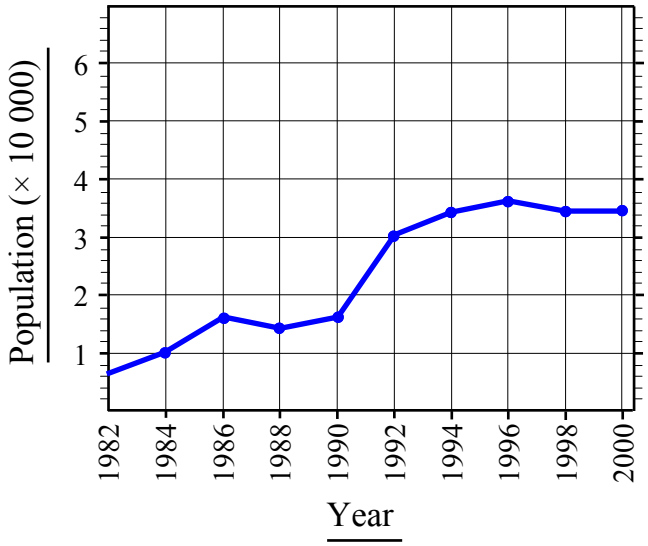
Study the line graphs below then answer the questions

Graph 1 : Population of Mathsville 1982-2000



1 Give the population in: *a*) 1988 _____

b) 1992 _____ *c*) 1996 _____ *d*) 2000 _____

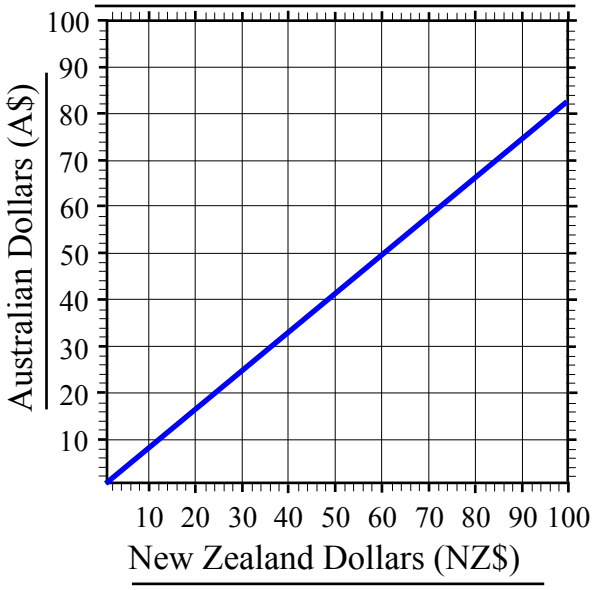
2 Estimate the population in *a*) 1985 _____

b) 1991 _____ *c*) 1995 _____ *d*) 1999 _____

3 Give the year that may have had a population of:

a) 8 000 ____ *b*) 32 000 ____ *c*) 13 000 ____

Graph 2 : NZ$ and A$ Conversion



Complete the table to the nearest $

4 Table 1 : NZ$ and A$ Conversions

NZ$	40	70		30			63			98	
A$			80		56	14		22	42		39

Convert the first figure and use it, not the graph, to find the others.

5 *a*) A$38 = NZ$ ___ *b*) A$3.80 = NZ$ ___

c) A$380 = NZ$ ___ *d*) A$3 800 = NZ$ ___

6 *a*) A$50 = NZ$ ___ *b*) A$0.50 = NZ$ ___

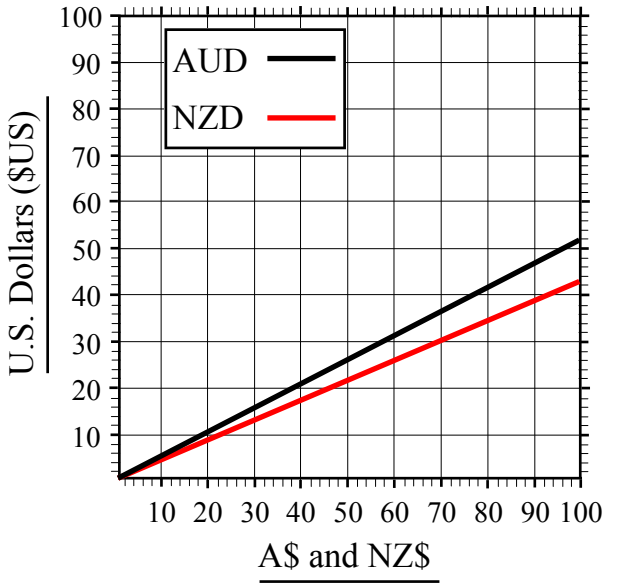
c) A$100 = NZ$ ___ *d*) A$5 000 = NZ$ ___

7 *a*) NZ$100 = A$ ___ *b*) NZ$200 = A$ ___

c) NZ$1 000 = A$ ___ *d*) NZ$1 001 = A$ ___

Use this graph to convert between US$, A$ and NZ$

Graph 3 : Converting US$ to A$ and NZ$



8 Table 2 : US$, A$ & NZ$ Conversions

US$	20	14			40				32		18
A$			80			50		60			
NZ$				60			70			80	

Use the graph for the first conversion then your number skills for the rest.

9 *a*) US$10 = NZ$ ___ *b*) US$100 = NZ$ ___

c) US$1 000 = NZ$ ___ *d*) US$1 = NZ$ ___

10 *a*) US$50 = A$ ___ *b*) US$100 = A$ ___

c) US$1 000 = A$ ___ *d*) US$5 000 = A$ ___

11 *a*) A$40 = US$ ___ *b*) A$400 = US$ ___

c) A$0.40 = US$ ___ *d*) A$400.40 = US$ ___

Line Graphs

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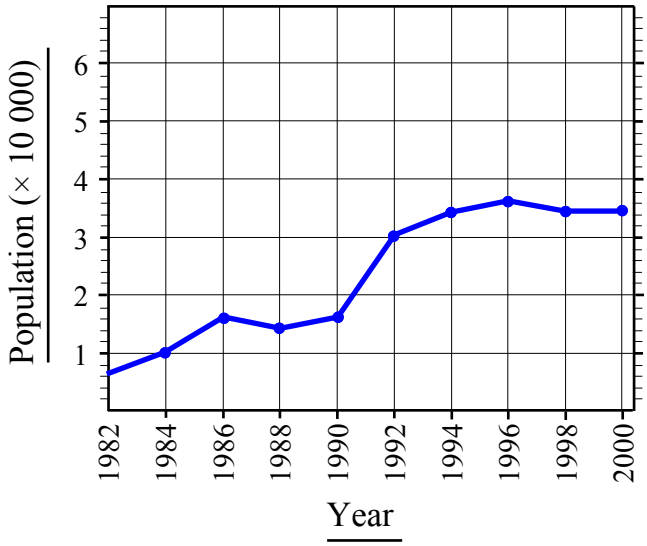
Study the line graphs below then answer the questions

Graph 1 : Population of Mathsville 1982-2000



1 Give the population in: *a*) 1988 14 000

b) 1992 30 000 *c*) 1996 36 000 *d*) 2000 35 000

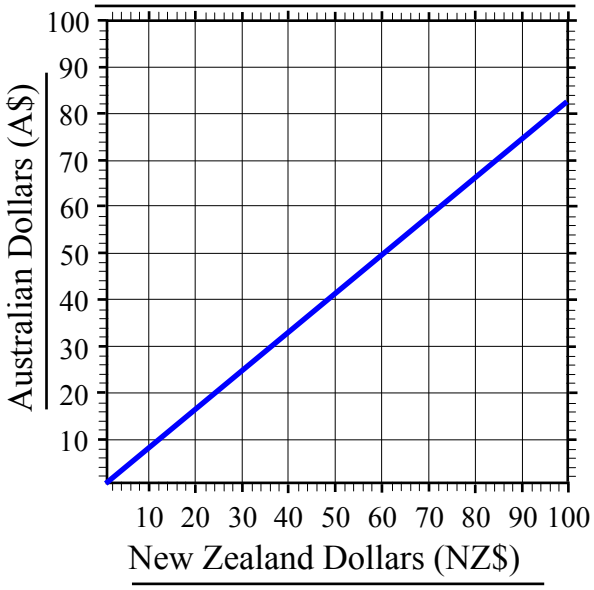
2 Estimate the population in *a*) 1985 13 000

b) 1991 24 000 *c*) 1995 35 000 *d*) 1999 35 000

3 Give the year that may have had a population of:

a) 8 000 1983 *b*) 32 000 1993 *c*) 13 000 1985

Graph 2 : NZ$ and A$ Conversion



Complete the table to the nearest $

4 Table 1 : NZ$ and A$ Conversions

NZ$	40	70	96	30	68	17	63	29	51	98	47
A$	33	58	80	25	56	14	52	22	42	81	39

Convert the first figure and use it, not the graph, to find the others.

5 *a*) A$38 = NZ$ 46 *b*) A$3.80 = NZ$ 4.60

c) A$380 = NZ$ 460 *d*) A$3 800 = NZ$ 4 600

6 *a*) A$50 = NZ$ 60 *b*) A$0.50 = NZ$ 0.60

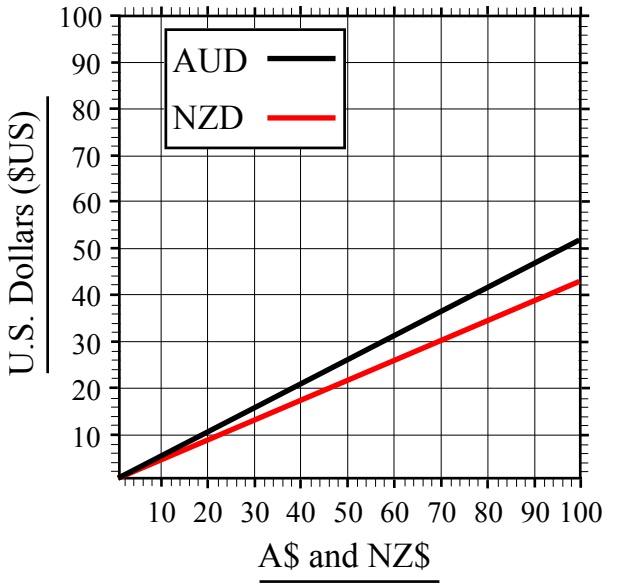
c) A$100 = NZ$ 120 *d*) A$5 000 = NZ$ 6 000

7 *a*) NZ$100 = A$ 82 *b*) NZ$200 = A$ 164

c) NZ$1 000 = A$ 820 *d*) NZ$1 001 = A$ 820.82

Use this graph to convert between US$, A$ and NZ$

Graph 3 : Converting US$ to A$ and NZ$



8 Table 2 : US$, A$ & NZ$ Conversions

US$	20	14	41	26	40	26	30	31	32	35	18
A$	38	26	80	50	77	50	58	60	61	66	34
NZ$	46	31	96	60	93	60	70	72	74	80	41

Use the graph for the first conversion then your number skills for the rest.

9 *a*) US$10 = NZ$ 22 *b*) US$100 = NZ$ 220

c) US$1 000 = NZ$ 2200 *d*) US$1 = NZ$ 2.20

10 *a*) US$50 = A$ 96 *b*) US$100 = A$ 192

c) US$1 000 = A$ 1 920 *d*) US$5 000 = A$ 9 600

11 *a*) A$40 = US$ 21 *b*) A$400 = US$ 210

c) A$0.40 = US$ 0.21 *d*) A$400.40 = US$ 210.21

Composite Graphs

Composite graphs stack different pieces of information together to form the total column (or bar). This method of displaying data allows access to the total as well as the basic parts to form the total.

Column 1 starts with a composite column graph showing a family's phone call history for the past 6 months. The calls are categorised as local calls, national calls, international calls and calls to mobile phones. All columns are rounded to the nearest $10 so that they are easier to read. Note that each part starts where the other stops, so to obtain a value for a specific part read off the amount at its base and subtract that from the amount at its top.

Another way of showing composite graphs is with just a single bar (or column) which is divided up into parts. These are usually to scale so that a ruler can be used to get information. Time lines are commonly in this format. Our graph is a time line about the construction of a hamburger in a fast food outlet. Use a ruler to measure each part and the total of the bar, note that 1 cm = 4 s (seconds). Q 5 shows employees being faster than standard time in certain aspects of burger building, replace the standard time with the new time and then give the reduced total time for the complete build. Q 6 refers to an employee who prefers a better quality of finish and is slower in certain phases of the build.

Column 2 asks you to construct a composite column graph comparing 5 different dentists in the procedure of putting a filling in a tooth. Position columns on top of the first part (the needle injection) which is in place for each column and build them up.

The same method is use for the composite bar graph, note that 1 cm = 10 animals (or 1 mm = 1 animal). You can colour the key the same colour as the blocks and you can write the names on as well.

Composite Graphs

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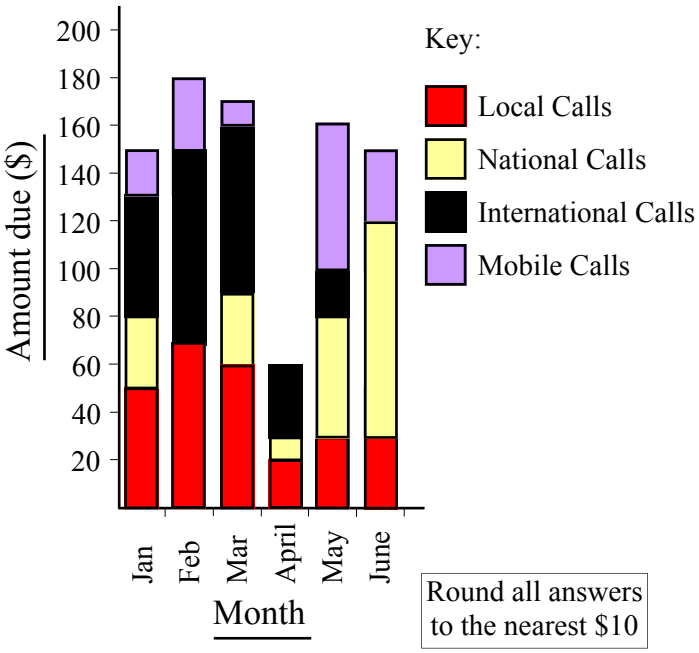
Study the composite graphs then answer the questions

Graph 1 : Forsythe Family Monthly Phone Bill



Round all answers to the nearest $10

1 Give the total amount due in:

a) Jan _____ *b*) Feb _____ *c*) May _____

2 Which month had the lowest number of calls to:

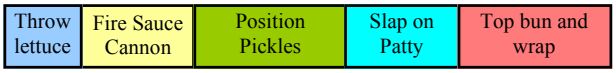
a) Mobile phones _____ *b*) Local Numbers _____

3 Give the amount spent on international calls in:

a) Jan _____ *b*) Feb _____

c) March _____ *d*) April _____

Graph 2 : Standard Times for Building a Burger



1 cm = 4 seconds

4 Give the time taken to:

a) 'Apply' lettuce _____ *b*) Build burger _____

5 If some actions can be done faster, calculate total time

a) Jackie can sauce burgers in 3 s, new total time: _____

b) Oscar can place pickles in 2 s, new total time: _____

c) Jason cannons the sauce in 5 s: new total time: _____

d) Marie slaps on a patty in 4 s, new total time: _____

6 Ryan treats burgers as art, find his total time when he takes 16 s to position the pickles and 12 s to apply sauce, all other times are unchanged _____

Construct a composite column graph for these dentists

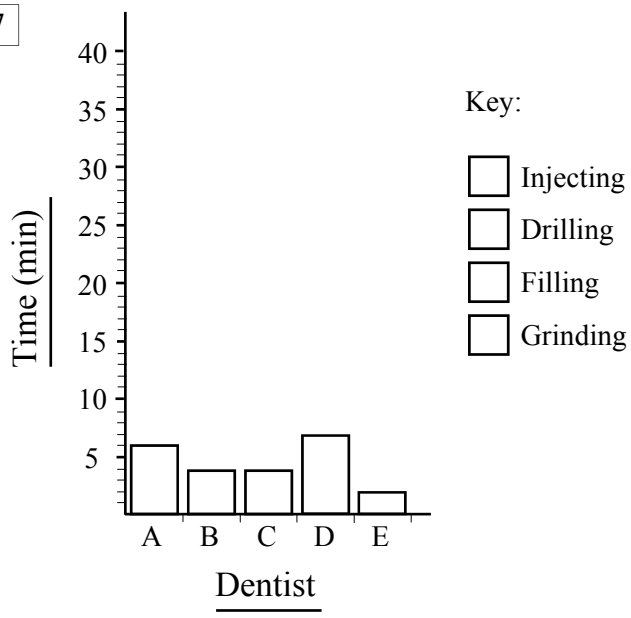
Table 1 : Time Comparison for Filling a Tooth

Action	Dentist A	Dentist B	Dentist C	Dentist D	Dentist E
Injecting	6	4	4	7	2
Drilling	13	8	12	15	14
Filling	5	7	5	4	5
Grinding	9	6	11	10	12

All times above are in min

Graph 3 : Time Comparison for Filling a Tooth

7



Construct a composite bar graph for the details in the table below

Table 2 : Treated Animals for Day

Animal	Dogs	Cats	Birds	Rabbits	Other
Number	16	25	9	7	23

8 Graph 4 : Treated Animals for Day

1 cm = 10 animals

Key:

Dogs Birds Other

Cats Rabbits

Composite Graphs

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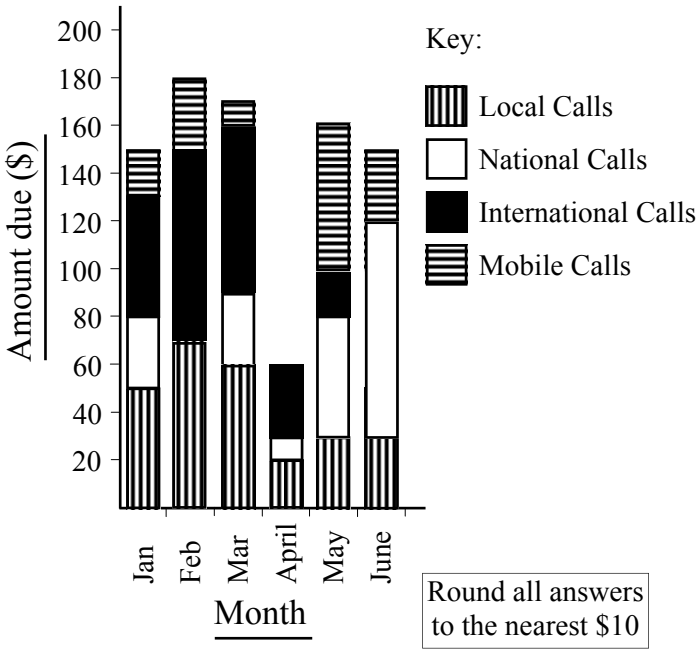
Study the composite graphs then answer the questions

Graph 1 : Forsythe Family Monthly Phone Bill



Round all answers to the nearest $10

1 Give the total amount due in:

a) Jan _____ *b*) Feb _____ *c*) May _____

2 Which month had the lowest number of calls to:

a) Mobile phones _____ *b*) Local Numbers _____

3 Give the amount spent on international calls in:

a) Jan _____ *b*) Feb _____

c) March _____ *d*) April _____

Graph 2 : Standard Times for Building a Burger

Throw lettuce	Fire Sauce Cannon	Position Pickles	Slap on Patty	Top bun and wrap

1 cm = 4 seconds

4 Give the time taken to:

a) 'Apply' lettuce _____ *b*) Build burger _____

5 If some actions can be done faster, calculate total time

a) Jackie can sauce burgers in 3 s, new total time: _____

b) Oscar can place pickles in 2 s, new total time: _____

c) Jason cannons the sauce in 5 s: new total time: _____

d) Marie slaps on a patty in 4 s, new total time: _____

6 Ryan treats burgers as art, find his total time when he takes 16 s to position the pickles and 12 s to apply sauce, all other times are unchanged _____

Construct a composite column graph for these dentists

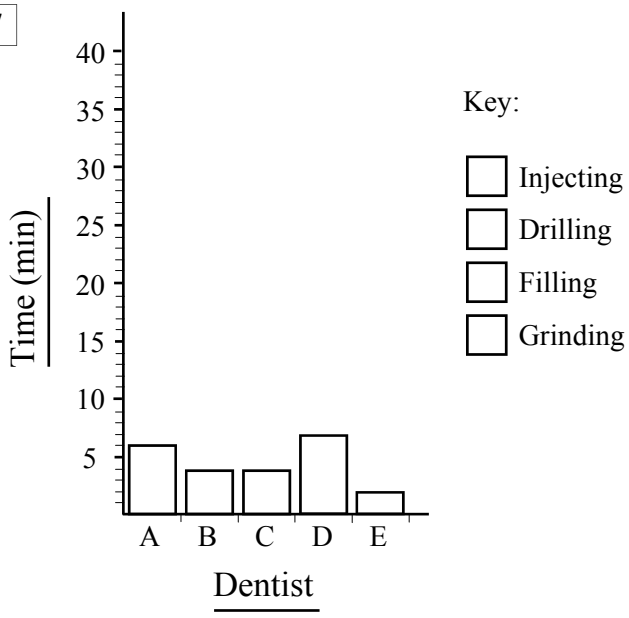
Table 1 : Time Comparison for Filling a Tooth

Action	Dentist A	Dentist B	Dentist C	Dentist D	Dentist E
Injecting	6	4	4	7	2
Drilling	13	8	12	15	14
Filling	5	7	5	4	5
Grinding	9	6	11	10	12

All times above are in min

Graph 3 : Time Comparison for Filling a Tooth

7



Construct a composite bar graph for the details in the table below

Table 2 : Treated Animals for Day

Animal	Dogs	Cats	Birds	Rabbits	Other
Number	16	25	9	7	23

8 Graph 4 : Treated Animals for Day

1 cm = 10 animals

Key:

Dogs Birds Other

Cats Rabbits

Composite Graphs

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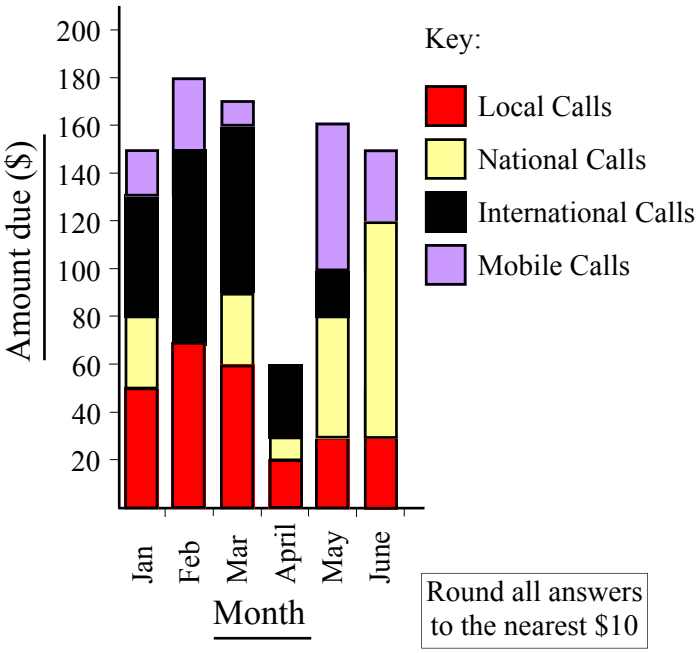
Study the composite graphs then answer the questions

Graph 1 : Forsythe Family Monthly Phone Bill



Round all answers to the nearest $10

1 Give the total amount due in:

a) Jan $150 *b*) Feb $180 *c*) May $160

2 Which month had the lowest number of calls to:

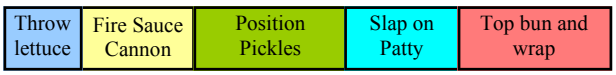
a) Mobile phones April *b*) Local Numbers April

3 Give the amount spent on international calls in:

a) Jan $50 *b*) Feb $80

c) March $70 *d*) April $30

Graph 2 : Standard Times for Building a Burger



1 cm = 4 seconds

4 Give the time taken to:

a) 'Apply' lettuce 4 s *b*) Build burger 32 s

5 If some actions can be done faster, calculate total time

a) Jackie can sauce burgers in 3 s, new total time: 29 s

b) Oscar can place pickles in 2 s, new total time: 26 s

c) Jason cannons the sauce in 5 s: new total time: 31 s

d) Marie slaps on a patty in 4 s, new total time: 30 s

6 Ryan treats burgers as art, find his total time when he takes 16 s to position the pickles and 12 s to apply sauce, all other times are unchanged 46 s

Construct a composite column graph for these dentists

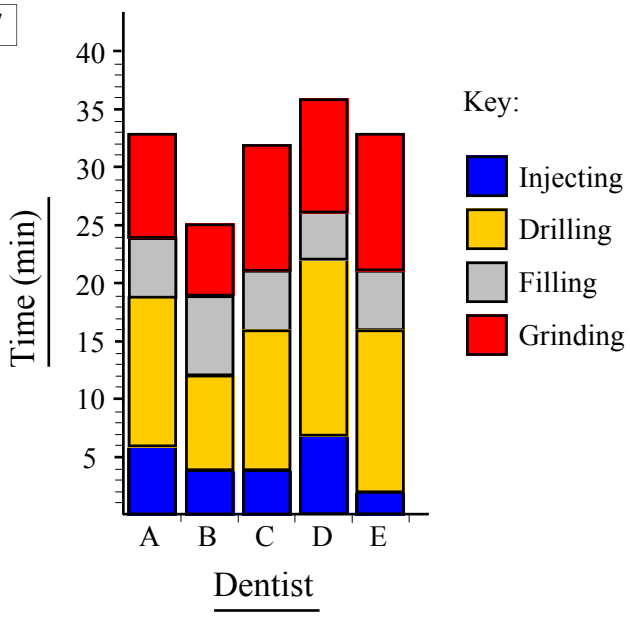
Table 1 : Time Comparison for Filling a Tooth

Action	Dentist A	Dentist B	Dentist C	Dentist D	Dentist E
Injecting	6	4	4	7	2
Drilling	13	8	12	15	14
Filling	5	7	5	4	5
Grinding	9	6	11	10	12

All times above are in min

Graph 3 : Time Comparison for Filling a Tooth

7



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Animal	Dogs	Cats	Birds	Rabbits	Other
Number	16	25	9	7	23

8 Graph 4 : Treated Animals for Day



1 cm = 10 animals

Key:

Dogs Birds Other

Cats Rabbits

Sector Graphs

You may know of them as Pie Graphs, Circle Graphs or Sector Graphs, they are all the same, just different names. Sector graphs display data as portions of a circle, so they are most commonly used with percentages or portions of a whole. At this stage we will deal only with percentages, all of which will have values that are multiples of 5%.

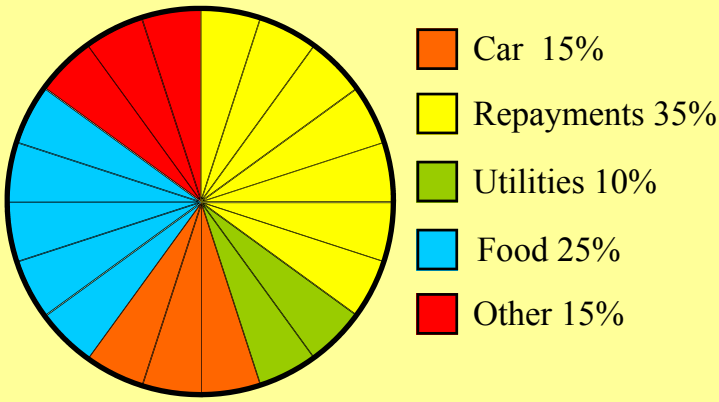
As a circle covers 360º in a revolution, with percentages that means 100% is 360º. So 10% is 36º and so 5% is 18º. Column 1 asks you to give the sector angle for all the multiples of 5%. To do this add 18º to the previous angle and that's your answer. You should be able to check you addition at 50% because it will equal ½ of 360º, which is 180º.

Graph 1 is a sector graph describing travel to school, you are asked to find the percentages of each. You need a protractor to do this, the percentages are multiples of 5 and so your angle will match a percentage given in Q.1. Note that the lines inside the circle may have to be extended to allow you to read them properly, (depending on your protractor type). The second worksheet is supplied for students without protractors, markings are on the graph at every 5º.

Graphs 3 and 4 are the reverse, creating sectors this time. Before you start you may like to change all the percentages to angles, remember use the values in Q.1. Once you have finished each sector remember to label it.

Graphs 5 and 6 are sector graphs that you can colour. They don't need labels as the square in front of the category is coloured the same colour as the sector. It is usually referred to as a 'key'. Look at the example below, it shows a completed sector graph.

Example : Household Expenditure



Colour the sectors, each sector is 5%, all percentages given are in multiples of 5%.

Colour the square, which is the key, the same colour as its respective sector.

Sector Graphs

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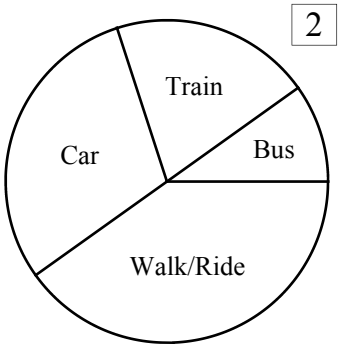
Convert these percentages of a circle to Angles by adding 18º for each 5%

1

5%	= 18°	40%	=	75%	=
10%	= 36°	45%	=	80%	=
15%	= 54°	50%	=	85%	=
20%	=	55%	=	90%	=
25%	=	60%	=	95%	=
30%	=	65%	=	100%	=
35%	=	70%	=		

Now try these

Graph 1 : Type of Transport to School (%)



2 Give the % who travel by:

a) Train _____

b) Car _____

c) Bus _____

d) Foot or Bike _____

3 Give the % of students that:

a) Don't catch a bus _____ *b*) Use a car or bus _____

c) Use public transport _____

Graph 2 : Type of Movie Last Seen by Students

4 Give the % who saw a:

a) Comedy _____

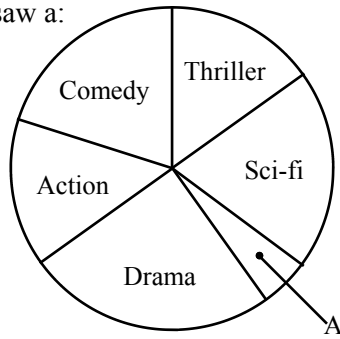
b) Thriller _____

c) Sci-fi _____

d) Drama _____

e) an Action film _____

f) an Animated film _____



5 Give the % of students that:

a) Saw either a Science fiction or Action film _____

b) Didn't see a Thriller _____

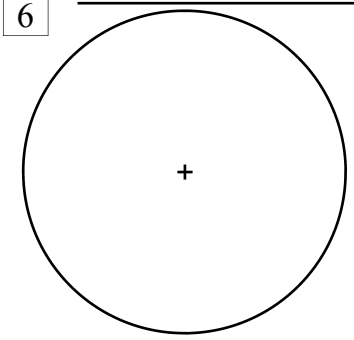
c) Last saw a comedy or drama: _____

d) Didn't see an animated or Sci-fi movie _____

Place this data on the sector graphs below

Graph 3 : Batman's Vehicle Usage (%)

6



Place these on the circle:

Car 60%

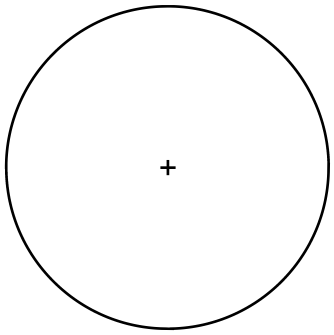
Copter 15%

Bike 20%

Plane 5%

Graph 4 : Karl's Music Preference (%)

7



Place these on the circle:

Alternative 25%

Mainstream 35%

Rap 15%

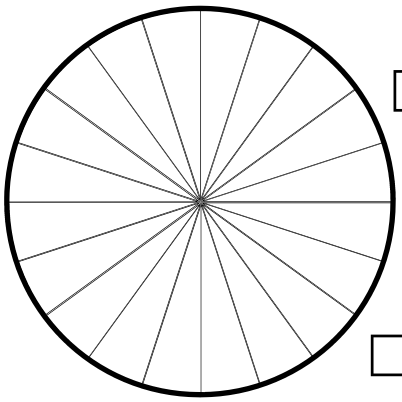
Dance 20%

Classical 5%

Each sector is worth 5%. Colour the sectors the same colour as the key squares

Graph 5 : How Tina spends her Spare Time

8



☐ Watch TV 20%

☐ Read 15%

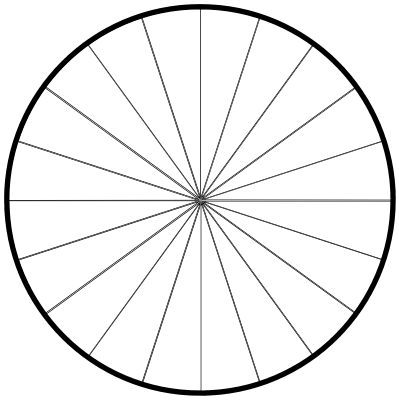
☐ Computer 5%

☐ Sport 20%

☐ With Friends 40%

Graph 6 : What Scares Students the Most

9



☐ Heights 10%

☐ Spiders 15%

☐ Other 20%

☐ Rats 5%

☐ Sharks 15%

☐ Ventriloquist Dummies 35%

Sector Graphs

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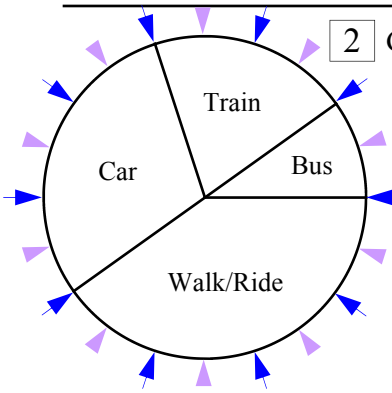
Convert these percentages of a circle to Angles by adding 18° for each 5%

1

5%	= 18°	40%	=	75%	=
10%	= 36°	45%	=	80%	=
15%	= 54°	50%	=	85%	=
20%	=	55%	=	90%	=
25%	=	60%	=	95%	=
30%	=	65%	=	100%	=
35%	=	70%	=		

Now try these

Graph 1 : Type of Transport to School (%)



2 Give the % who travel by:

a) Train _____

b) Car _____

c) Bus _____

d) Foot or Bike _____

3 Give the % of students that:

a) Don't catch a bus _____ *b*) Use a car or bus _____

c) Use public transport _____

Graph 2 : Type of Movie Last Seen by Students

4 Give the % who saw a:

a) Comedy _____

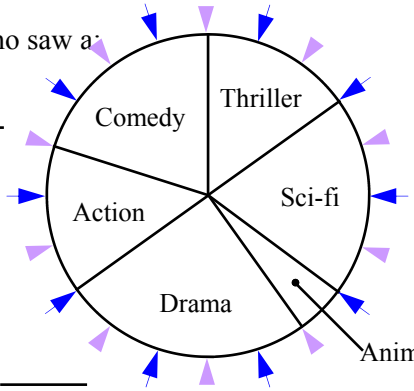
b) Thriller _____

c) Sci-fi _____

d) Drama _____

e) an Action film _____

f) an Animated film _____



5 Give the % of students that:

a) Saw either a Science fiction or Action film _____

b) Didn't see a Thriller _____

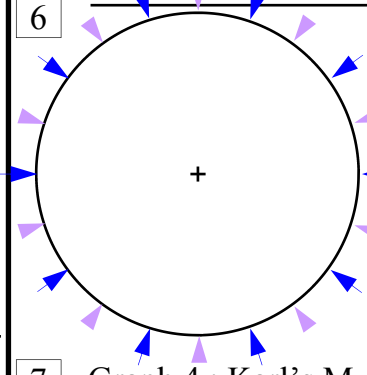
c) Last saw a comedy or drama: _____

d) Didn't see an animated or Sci-fi movie _____

Place this data on the sector graphs below

Graph 3 : Batman's Vehicle Usage (%)

6



Place these on the circle:

Car 60%

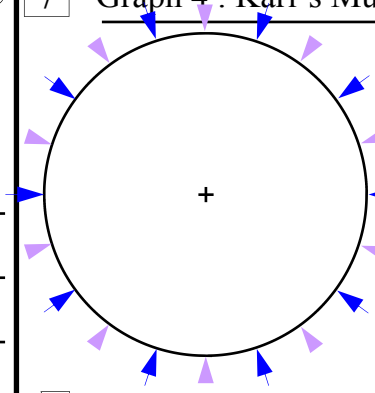
Copter 15%

Bike 20%

Plane 5%

Graph 4 : Karl's Music Preference (%)

7



Place these on the circle:

Alternative 25%

Mainstream 35%

Rap 15%

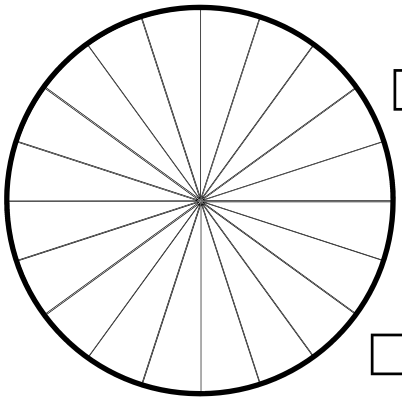
Dance 20%

Classical 5%

Each sector is worth 5%. Colour the sectors the same colour as the key squares

Graph 5 : How Tina spends her Spare Time

8



☐ Watch TV 20%

☐ Read 15%

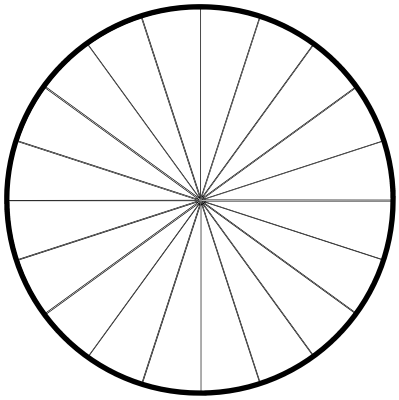
☐ Computer 5%

☐ Sport 20%

☐ With Friends 40%

Graph 6 : What Scares Students the Most

9



☐ Heights 10%

☐ Spiders 15%

☐ Other 20%

☐ Rats 5%

☐ Sharks 15%

☐ Ventriloquist Dummies 35%

Sector Graphs

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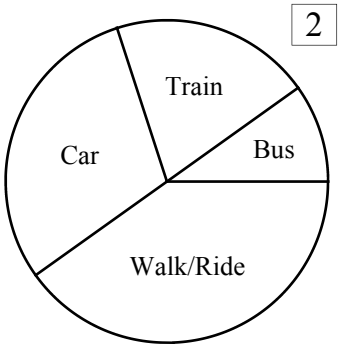
Convert these percentages of a circle to Angles by adding 18° for each 5%

1

5%	= 18°	40%	= 144°	75%	= 270°
10%	= 36°	45%	= 162°	80%	= 288°
15%	= 54°	50%	= 180°	85%	= 306°
20%	= 72°	55%	= 198°	90%	= 324°
25%	= 90°	60%	= 216°	95%	= 342°
30%	= 108°	65%	= 234°	100%	= 360°
35%	= 126°	70%	= 252°		

Graph 1 : Type of Transport to School (%)



2 Give the % who travel by:

a) Train 20%

b) Car 30%

c) Bus 10%

d) Foot or Bike 40%

3 Give the % of students that:

a) Don't catch a bus 90% *b*) Use a car or bus 40%

c) Use public transport 30%

Graph 2 : Type of Movie Last Seen by Students

4 Give the % who saw a:

a) Comedy 20%

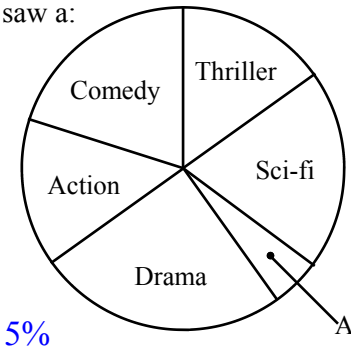
b) Thriller 15%

c) Sci-fi 20%

d) Drama 25%

e) an Action film 15%

f) an Animated film 5%



5 Give the % of students that:

a) Saw either a Science fiction or Action film 35%

b) Didn't see a Thriller 85%

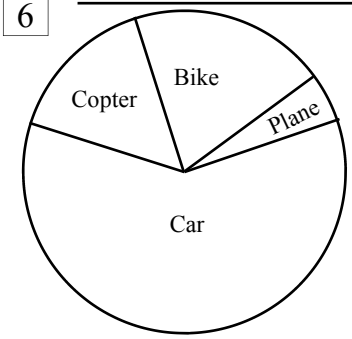
c) Last saw a comedy or drama: 45%

d) Didn't see an animated or Sci-fi movie 75%

Place this data on the sector graphs below

Graph 3 : Batman's Vehicle Usage (%)

6



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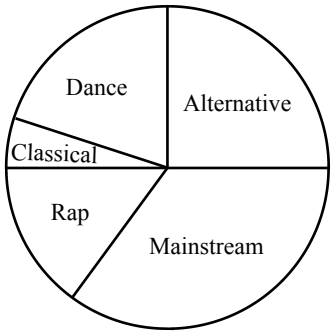

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Dance 20%

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Graph 5 : How Tina spends her Spare Time

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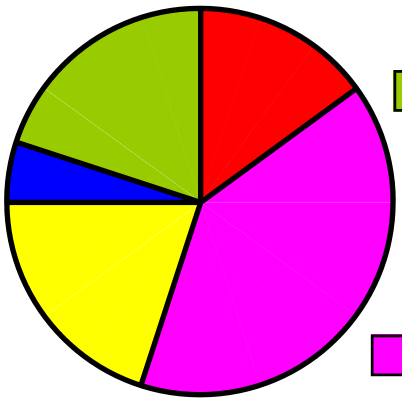

Watch TV 20%

Read 15%

Computer 5%

Sport 20%

With Friends 40%

Graph 6 : What Scares Students the Most

9



Heights 10%

Spiders 15%

Other 20%

Rats 5%

Sharks 15%

Ventriloquist Dummies 35%

FREEFALL MATHEMATICS

ALTITUDE

NUMBER PLANE

Grid Reference - The Hidden Friend

A grid reference system is used to pinpoint locations on a map or diagram. This sheet uses grid referencing to plot diagrams of 2 hidden friends. Grid referencing gives a letter then a number, the letter being along the horizontal axis (the top and bottom) and the number being on the vertical axis, which are the two sides.

Some maps may use a zero, these maps start at 1. To plot the points move across to the letter required then up the column to the number, then colour the square. Then plot the next grid position, after the points are plotted your friends should be revealed.

The co-ordinates are all in the same colour except where you are told to change colour, but if you want to make a multi-coloured one go ahead! You should strike out each grid reference as you plot them incase you need to remember where you are up to.

Grid Reference - The Hidden Friend

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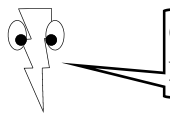
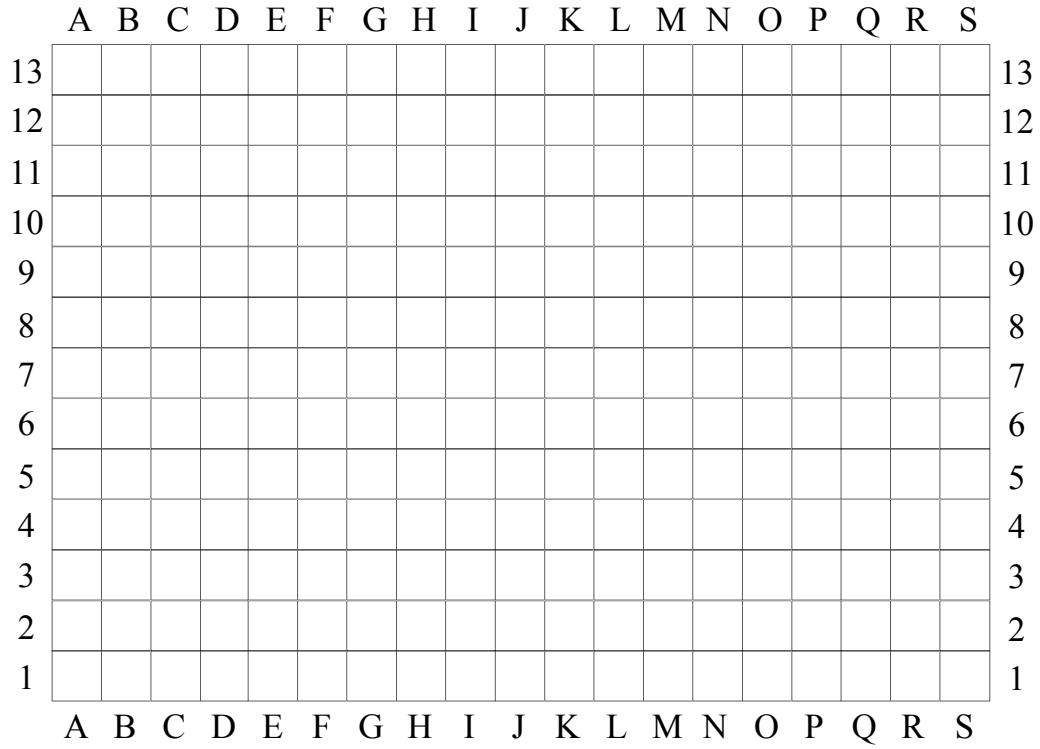


Colour the following squares and you will find a smiling friend



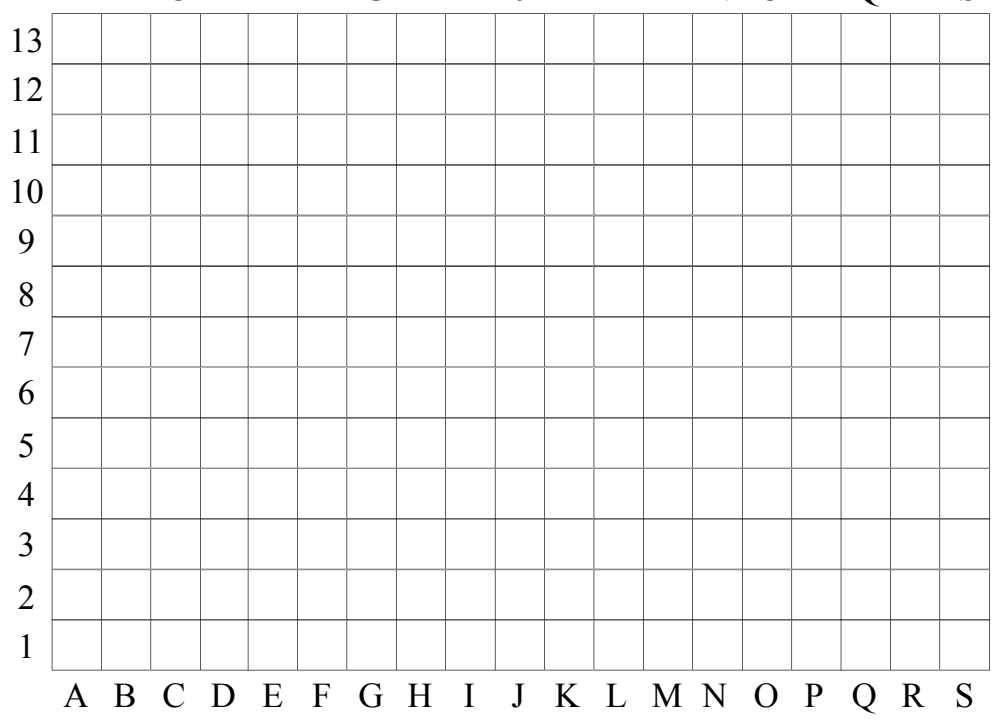
Co-ordinates:

D3 I2 C7
N5 B6 F2
D8 C4 L6
G2 E9 E3
J8 K11 L4
F12 M4 Q7
F4 F9 H12
I13 H2 J3
O8 P9 B5
M6 H9 D12
E13 P6 Q8
J12 K3 S8
G11 O5 K7
I9 R9 E6

Different colour



Colour the following squares in blue (or black and red when indicated) and you will find an enthusiastic friend



Co-ordinates: G11 A4 D1 F7 K12 L3
P6 N8 Q8 A9 I12 S8
C2 O2 E11 S9 H8 L12
M6 M11 N7 R10 O5
P12 R3 Q2 B5 E2
F8 N10 A7 M4 O8
Q1 J13 O11 M5 G4
C7 D8 C8 G5 E5
H12 B10 E3 F10 A8
G6 D12 E4 S4 P1
P8 S6 H3 Q11 S7
C1 B3 O3 C11 Q7
E8 A6 L8 D6 O4
R5

Change Colour:

K2 J1 J2 I2

Change Colour:

L9 J5 I5 J4 H9 K5

Grid Reference - The Hidden Friend

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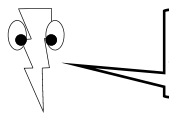
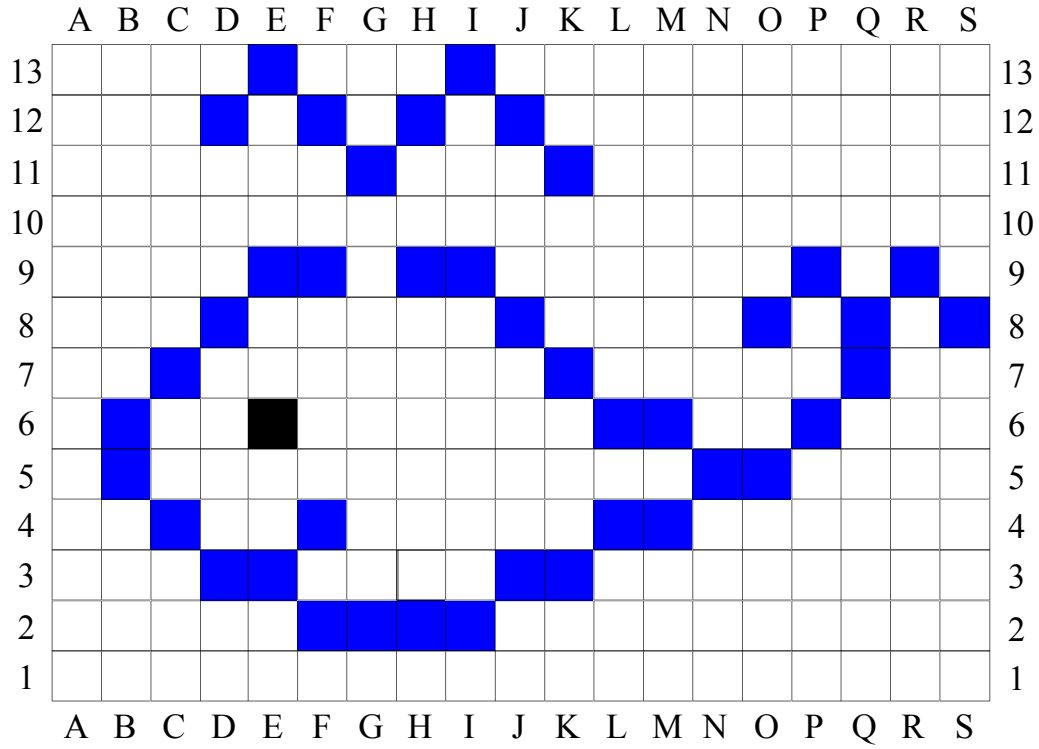


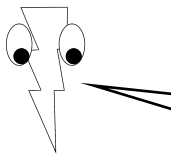
Colour the following squares and you will find a smiling friend



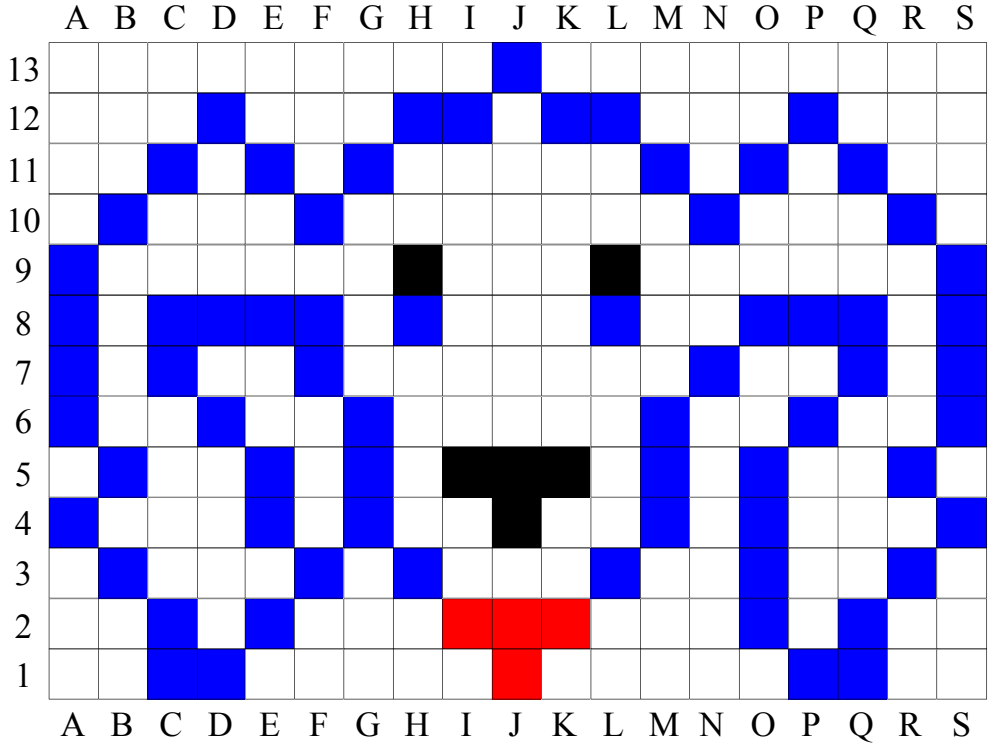
Co-ordinates:

D3 I2 C7
N5 B6 F2
D8 C4 L6
G2 E9 E3
J8 K11 L4
F12 M4 Q7
F4 F9 H12
I13 H2 J3
O8 P9 B5
M6 H9 D12
E13 P6 Q8
J12 K3 S8
G11 O5 K7
I9 R9 E6

Different colour



Colour the following squares in blue (or black and red when indicated) and you will find an enthusiastic friend



Co-ordinates:

G11 A4 D1 F7 K12 L3
P6 N8 Q8 A9 I12 S8
C2 O2 E11 S9 H8 L12
M6 M11 N7 R10 O5
P12 R3 Q2 B5 E2
F8 N10 A7 M4 O8
Q1 J13 O11 M5 G4
C7 D8 C8 G5 E5
H12 B10 E3 F10 A8
G6 D12 E4 S4 P1
P8 S6 H3 Q11 S7
C1 B3 O3 C11 Q7
E8 A6 L8 D6 O4
R5

Change Colour:

K2 J1 J2 I2

Change Colour:

L9 J5 I5 J4 H9 K5

Using a Street Map

A street map uses grid co-ordinates to specify the location of streets and points of interest. The reference system generally uses letters across its horizontal axis (top and bottom) and numbers on its vertical axis (sides). To give a co-ordinate you move across first, then up, so the letter first then the number.

For example D7 would be the green area. Note that some areas could be given more than one co-ordinate, due to its size it could cross into another reference area. If you are attempting this sheet on paper use a ruler or just the eye.

Column 1 asks for the co-ordinates for the black symbols on the map. Look at the map key (sometimes called a 'legend') so that you know what you are looking for. Then using the grid read off the locations.

Column 2 asks you to place buildings and points of interest on the map. So show that artistic talent!

Column 3 asks you to place post boxes, traffic lights and phone boxes in given locations.

You can print either a map with a grid over it or a map with no grid.

Using a Street Map - No Grid

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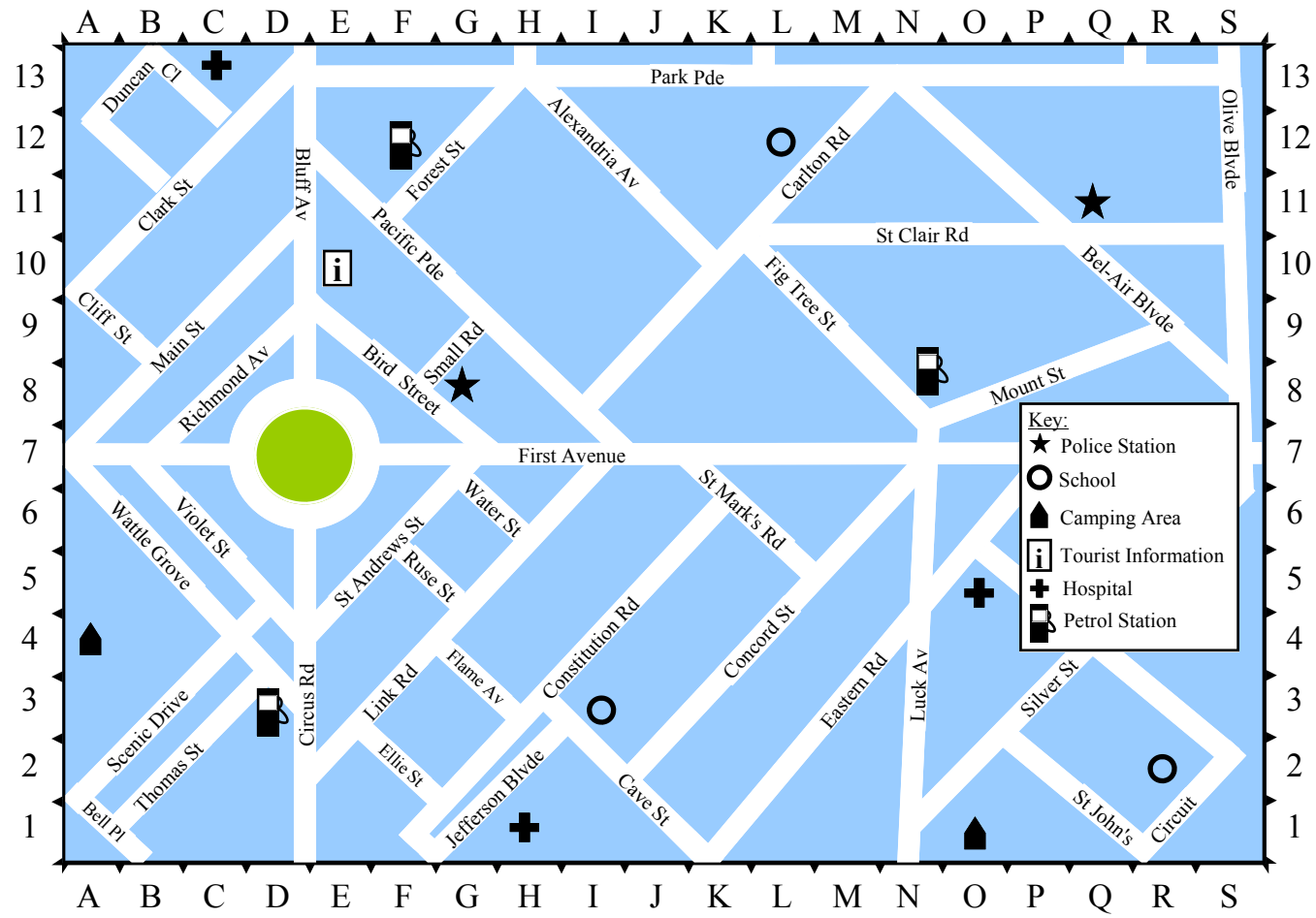

Give the grid references for all of the following

1 Petrol Stations ___ ___ ___

2 Schools ___ ___ ___

3 Camping Grounds ___ ___

4 Tourist Information ___

5 Hospitals ___ ___ ___

6 Intersection of First Av and Bird St ___

7 Intersection of Carlton Rd and Alexandria Av ___

8 Intersection of Scenic Drive & Wattle Grove ___

9 Bell Place ___

Draw these buildings or make symbols for them in the co-ordinates below.

10	Veterinarian	K9
11	St Mary's Cathedral	K5
12	Palace Hotel	N11
13	Imam Ali Mosque	I1
14	Sunshine Circus	C1
15	Council Car Parks	M1 A5 Q9
16	Fuzzy's Café	P12
17	Community Centre	O3
18	Council Parks	H11 D7
19	Aquatic Centre	P10

Put traffic lights, post boxes and phone booths in these locations

20 Traffic Lights Co-ordinates

P3 F8 H3

N7 K10 F2

21 Post Boxes Co-ordinates

I9 R10 S3

K2 B3 M4

22 Phone Booths Co-ordinates

R1 J12 A12

S9 L7 I2

E4 H8 O6

Using a Street Map - Grid

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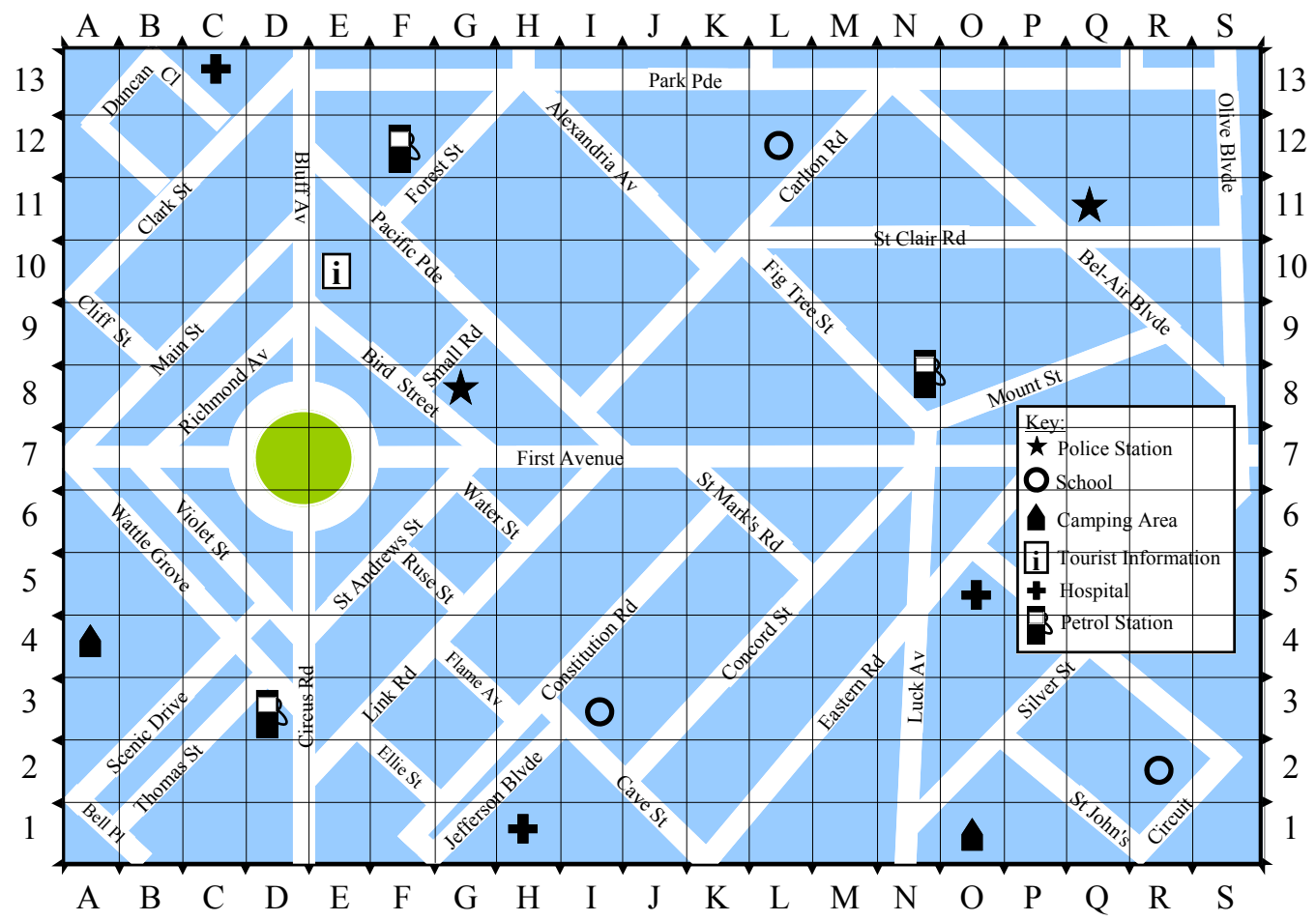

Give the grid references for all of the following

1 Petrol Stations ___ ___ ___

2 Schools ___ ___ ___

3 Camping Grounds ___ ___

4 Tourist Information ___

5 Hospitals ___ ___ ___

6 Intersection of First Av and Bird St ___

7 Intersection of Carlton Rd and Alexandria Av ___

8 Intersection of Scenic Drive & Wattle Grove ___

9 Bell Place ___

Draw these buildings or make symbols for them in the co-ordinates below.

10	Veterinarian	K9
11	St Mary's Cathedral	K5
12	Palace Hotel	N11
13	Imam Ali Mosque	I1
14	Sunshine Circus	C1
15	Council Car Parks	M1 A5 Q9
16	Fuzzy's Café	P12
17	Community Centre	O3
18	Council Parks	H11 D7
19	Aquatic Centre	P10

Put traffic lights, post boxes and phone booths in these locations

20 Traffic Lights Co-ordinates

P3 F8 H3

N7 K10 F2

21 Post Boxes Co-ordinates

I9 R10 S3

K2 B3 M4

22 Phone Booths Co-ordinates

R1 J12 A12

S9 L7 I2

E4 H8 O6

Using a Street Map

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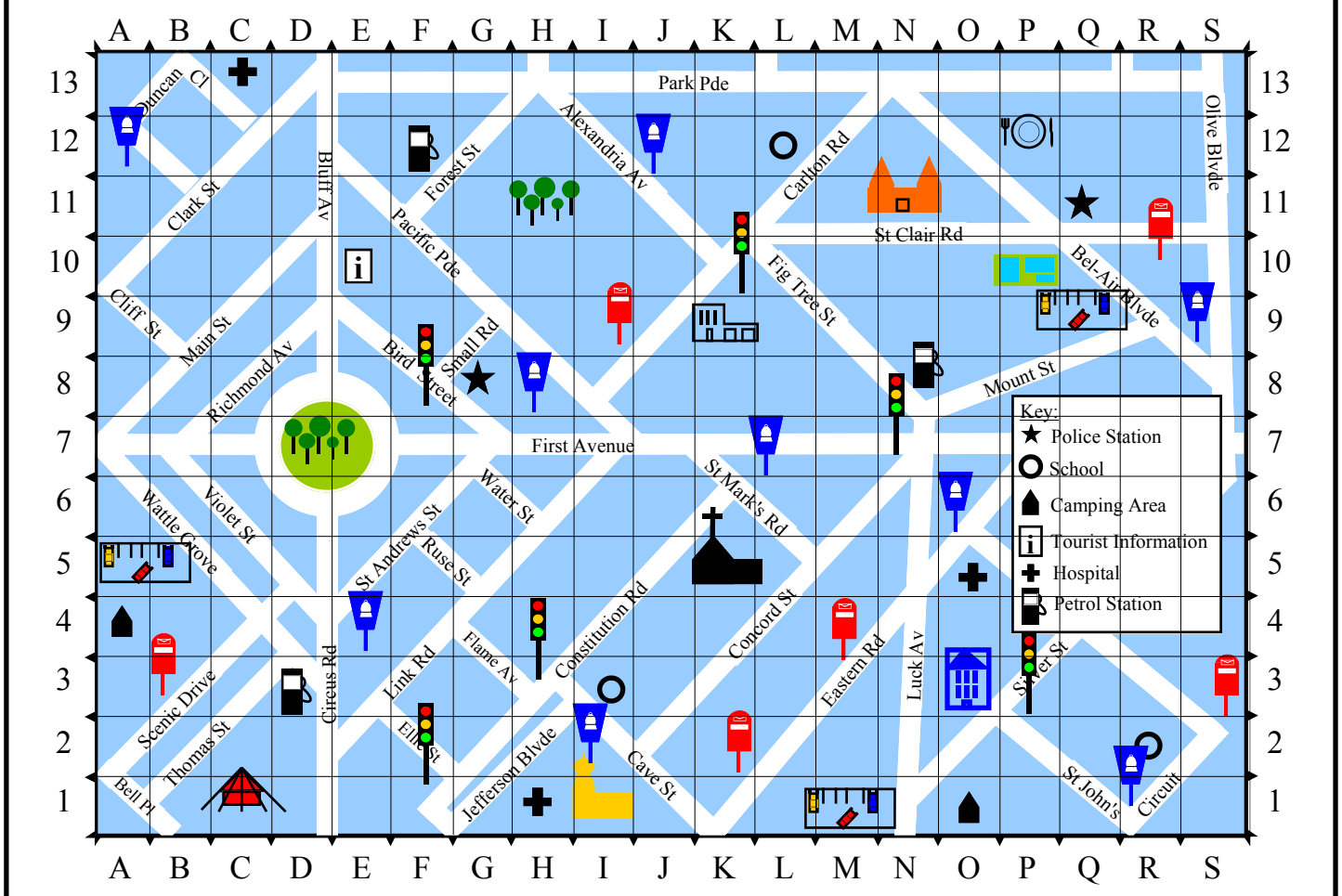


Give the grid references for all of the following

1 Petrol Stations D3 F12 N8

2 Schools I3 L12 R2

3 Camping Grounds A4 O1

4 Tourist Information E10

5 Hospitals C13 H1 O5

6 Intersection of First Av and Bird St G7

7 Intersection of Carlton Rd and Alexandria Av K10

8 Intersection of Scenic Drive & Wattle Grove C4

9 Bell Place A1

Draw these buildings or make symbols for them in the co-ordinates below.

10 Veterinarian K9

11 St Mary's Cathedral K5

12 Palace Hotel N11

13 Imam Ali Mosque I1

14 Sunshine Circus C1

15 Council Car Parks M1 A5 Q9

16 Fuzzy's Café P12

17 Community Centre O3

18 Council Parks H11 D7

19 Aquatic Centre P10

Put traffic lights, post boxes and phone booths in these locations

20 Traffic Lights Co-ordinates

P3 F8 H3

N7 K10 F2

21 Post Boxes Co-ordinates

I9 R10 S3

K2 B3 M4

22 Phone Booths Co-ordinates

R1 J12 A12

S9 L7 I2

E4 H8 O6

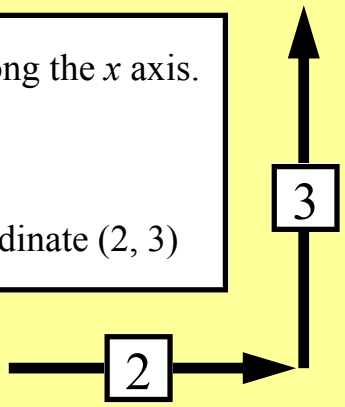
Plotting Points - Positive Quadrant

This sheet involves the plotting of Cartesian co-ordinates. When plotting points on a number plane you are given 2 numbers, referred to as an ordered pair. The first number tells you how much you move across and the second number tells you how much you move up or down. This sheet only moves across to the right and up, negative numbers (not on this sheet) move to the left and down.

Move across first along the *x* axis.

Then move up.

Eg. To plot the co-ordinate (2, 3)



Note that the plane consists of an *x*-axis (horizontal or across) and a *y*-axis which is vertical (up and down). The plural of axis is axes (pronounced ak-seas). The co-ordinate (0, 0) is commonly referred to as the 'origin', the place where measurement starts from. Teachers often ask for the co-ordinates of the origin.

The first column asks you to locate the points and write in their co-ordinates. This is done by opening a bracket writing the *x* co-ordinate, writing a comma, then the *y* co-ordinate and close bracket. You should always place the co-ordinates in brackets.

The second column asks you to plot points. Once the group of points are plotted draw a line between points (in alphabetical order) to form a shape. Then write in the name of the shape plotted in the space provided.

Plotting Points - Positive Quadrant

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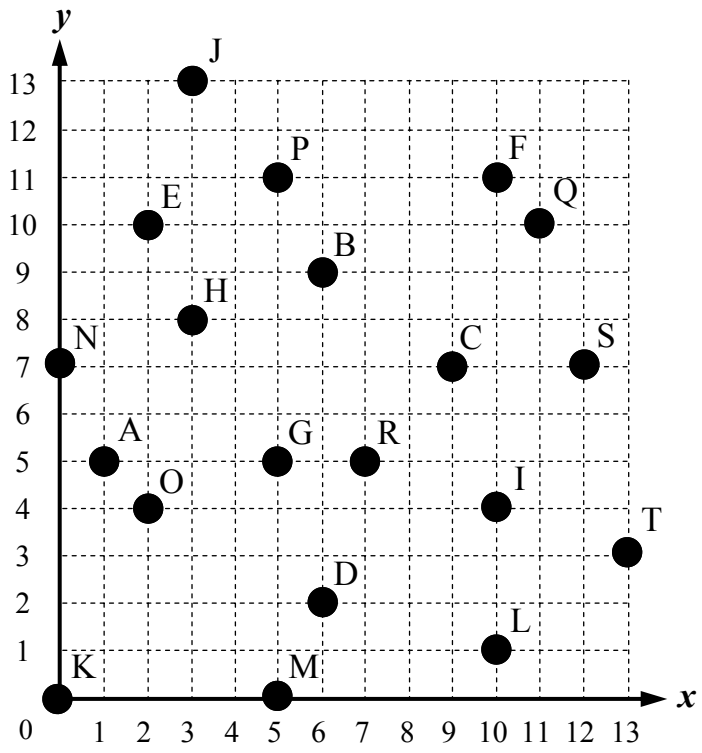
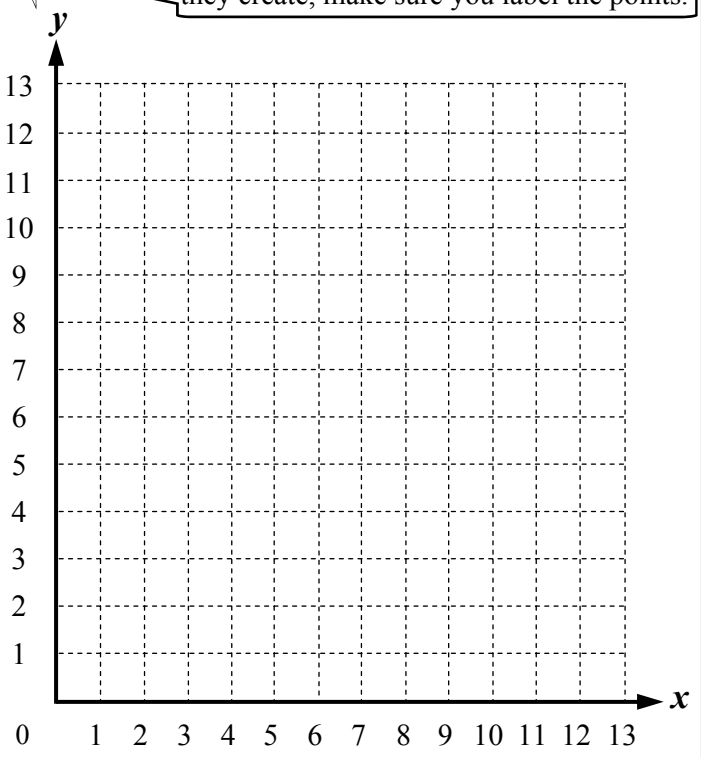
Give the co-ordinates for the following points, place them in brackets separated by a comma



1	Point A _____	11	Point K _____
2	Point B _____	12	Point L _____
3	Point C _____	13	Point M _____
4	Point D _____	14	Point N _____
5	Point E _____	15	Point O _____
6	Point F _____	16	Point P _____
7	Point G _____	17	Point Q _____
8	Point H _____	18	Point R _____
9	Point I _____	19	Point S _____
10	Point J _____	20	Point T _____

Place the groups of points given below on the grid, join the points and name the shape they create, make sure you label the points.



21 Point A (10, 4) Point B (11, 2)
Point C (12, 4) Point D (11, 5)
Shape formed: _____

22 Point E (3, 5) Point F (0, 8)
Point G (5, 13) Point H (8, 10)
Shape formed: _____

23 Point J (5, 4) Point K (9, 6)
Point L (7, 8)
Shape formed: _____
Two word description

24 Point M (2, 3) Point N (8, 3)
Point O (6, 0) Point P (0, 0)
Shape formed: _____

25 Point R (10, 9) Point S (13, 13)
Point T (13, 9)
Shape formed: _____
Two or three word description

Plotting Points - Positive Quadrant

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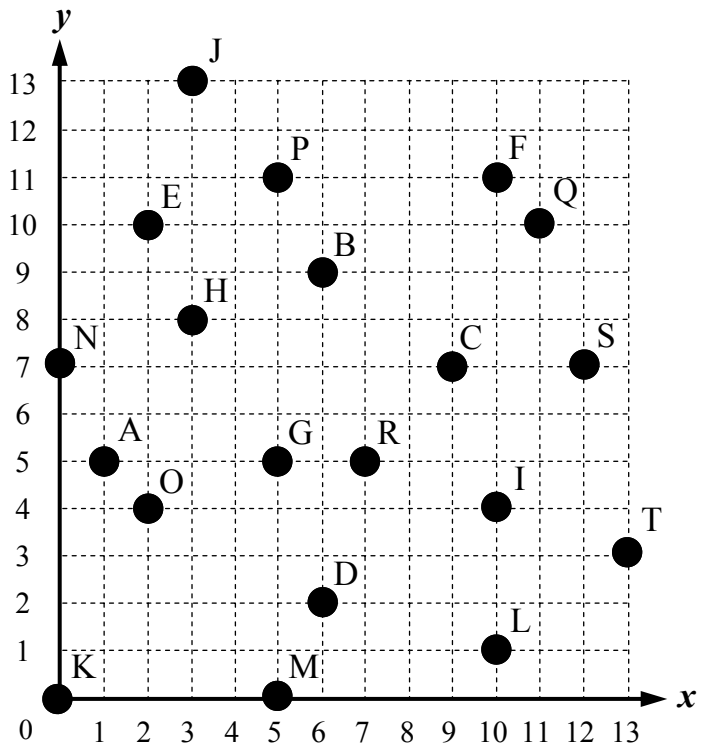
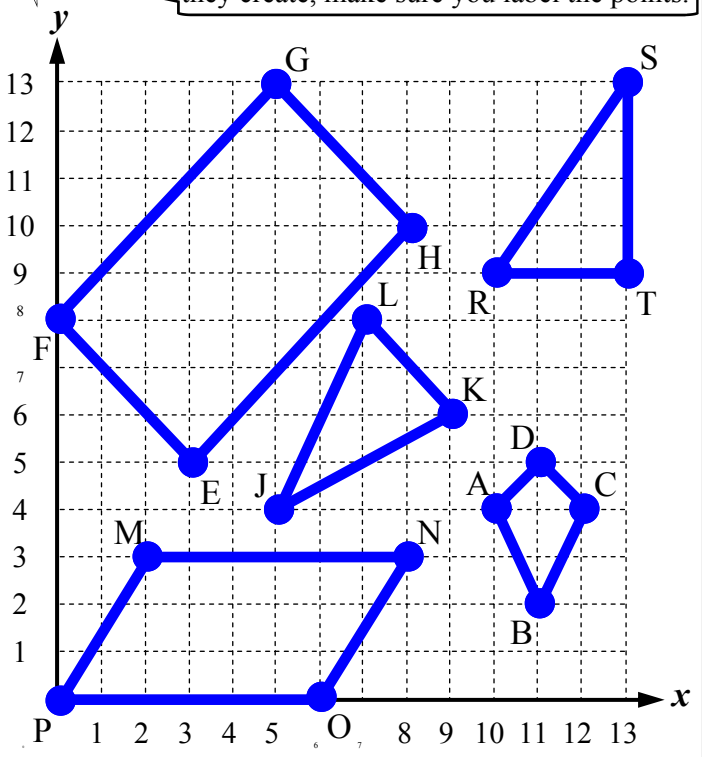
Give the co-ordinates for the following points, place them in brackets separated by a comma



1	Point A	(1, 5)	11	Point K	(0, 0)
2	Point B	(6, 9)	12	Point L	(10, 1)
3	Point C	(9, 7)	13	Point M	(5, 0)
4	Point D	(6, 2)	14	Point N	(0, 7)
5	Point E	(2, 10)	15	Point O	(2, 4)
6	Point F	(10, 11)	16	Point P	(5, 11)
7	Point G	(5, 5)	17	Point Q	(11, 10)
8	Point H	(3, 8)	18	Point R	(7, 5)
9	Point I	(10, 4)	19	Point S	(12, 7)
10	Point J	(3, 13)	20	Point T	(13, 3)

Place the groups of points given below on the grid, join the points and name the shape they create, make sure you label the points.



21 Point A (10, 4) Point B (11, 2)
Point C (12, 4) Point D (11, 5)
Shape formed: Kite

22 Point E (3, 5) Point F (0, 8)
Point G (5, 13) Point H (8, 10)
Shape formed: Rectangle

23 Point J (5, 4) Point K (9, 6)
Point L (7, 8)
Shape formed: Isosceles Triangle
Two word description

24 Point M (2, 3) Point N (8, 3)
Point O (6, 0) Point P (0, 0)
Shape formed: Parallelogram

25 Point R (10, 9) Point S (13, 13)
Point T (13, 9)
Shape formed: Right (Angled) Triangle
Two or three word description

Drawing Animals - Positive Quadrant

This sheet uses your skills in plotting points from the previous sheet. The series of points makes a pair of animals when lines are joined between the points. The way you attempt this exercise is, for the first animal:

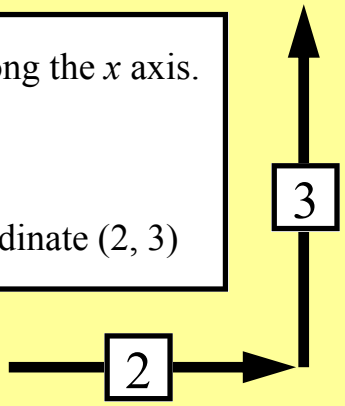
- Plot one point at a time and then draw a line between it and the previous point. Don't try to plot all the points and then draw the lines afterwards as the drawing is too complicated.
- When you reach stop, restart at the new position and start again, DON'T JOIN THE POINTS BETWEEN WHERE YOU STOPPED AND WHERE YOU RESTARTED.

For the next animal follow the same process except use 2 different colours.

Move across first along the *x* axis.

Then move up.

Eg. To plot the co-ordinate (2, 3)



Drawing Animals - Positive Quadrant

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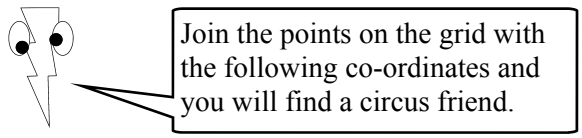
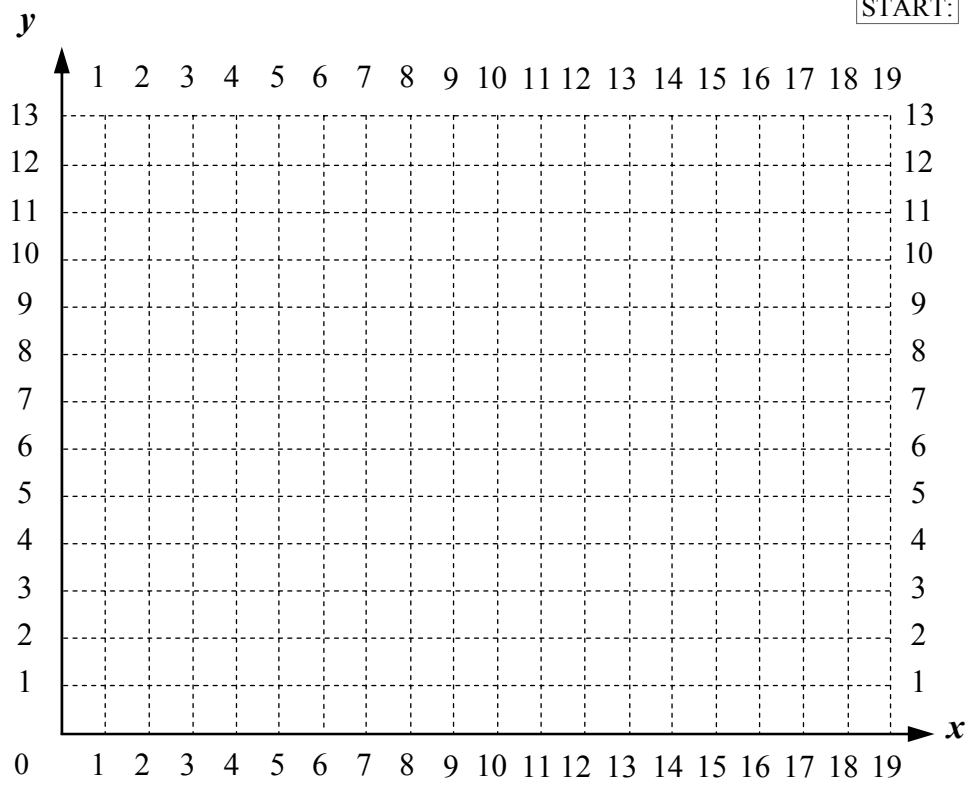
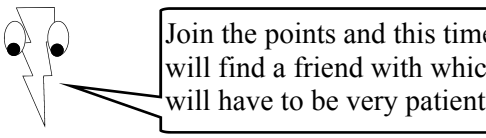


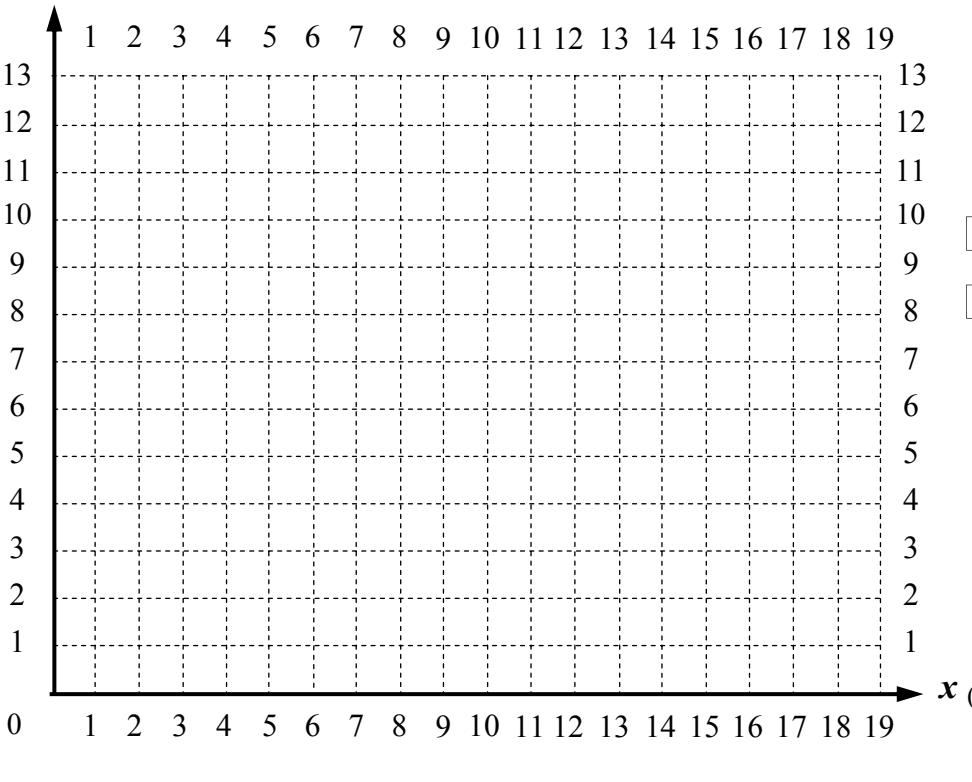


Co-ordinates:

START: (2, 12) , (3, 12) , (4, 11) , (5, 12) ,
(7, 12) , (8, 11) , (9, 12) , (10, 12) ,
(11, 11) , (11, 6) , (10, 3) , (9, 3) ,
(8, 7) , (8, 5) , (7, 7) , (7, 3) ,
(6, 2) , (4, 2) , (4, 3) , (5, 3) ,
(6, 4) , (5, 7) , (4, 5) , (4, 7) ,
(3, 3) , (2, 3) , (1, 6) , (1, 11) ,
(2, 12) STOP!

START:
(11, 10) , (12, 11) , (16, 12) , (18, 10) ,
(19, 7) , (18, 9) , (18, 3) , (15, 3) ,
(16, 4) , (16, 6) , (14, 5) , (11, 5) ,
(11, 4) , (13, 4) , (13, 1) , (12, 2) ,
(9, 2) , (8, 3) , (7, 6) , STOP!





Co-ordinates:

START: (1, 11) , (2, 10) , (2, 9) , (1, 8) ,
(1, 7) , (2, 6) , (2, 5) , (3, 3) ,
(5, 1) , (7, 0) , (13, 0) , (16, 3) ,
(17, 5) , (15, 3) , (12, 2) , (8, 2) ,
(6, 3) , (4, 5) , (3, 6) , (4, 7) ,
(4, 8) , (3, 9) , (3, 10) , (4, 11) ,
STOP!

START:
(11, 6) , (12, 6) , (11, 7) , (10, 6) ,
(11, 5) , (12, 5) , (14, 6) , (14, 7) ,
(12, 9) , (10, 9) , (8, 7) , (8, 5) ,
(10, 3) , (12, 3) , (14, 4) , (15, 6) ,
(15, 7) , (14, 9) , (13, 10) , (10, 10) ,
(8, 9) , (7, 7) , (8, 3) , (11, 2) ,
(12, 2) , (15, 3) , (17, 6) , (17, 7) ,
(15, 11) , (12, 12) , (10, 12) , (7, 11) ,
(5, 9) , (4, 6) , (4, 5) , STOP!

Drawing Animals - Positive Quadrant

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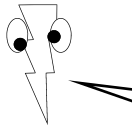
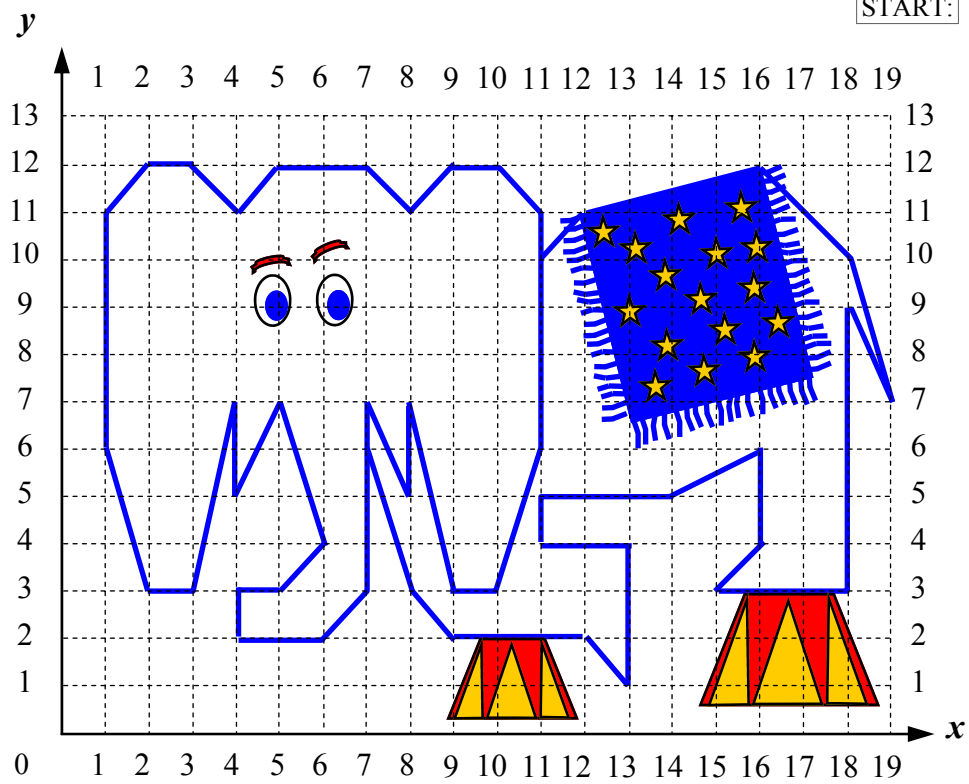


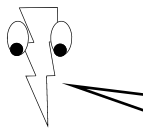
Join the points on the grid with the following co-ordinates and you will find a circus friend.



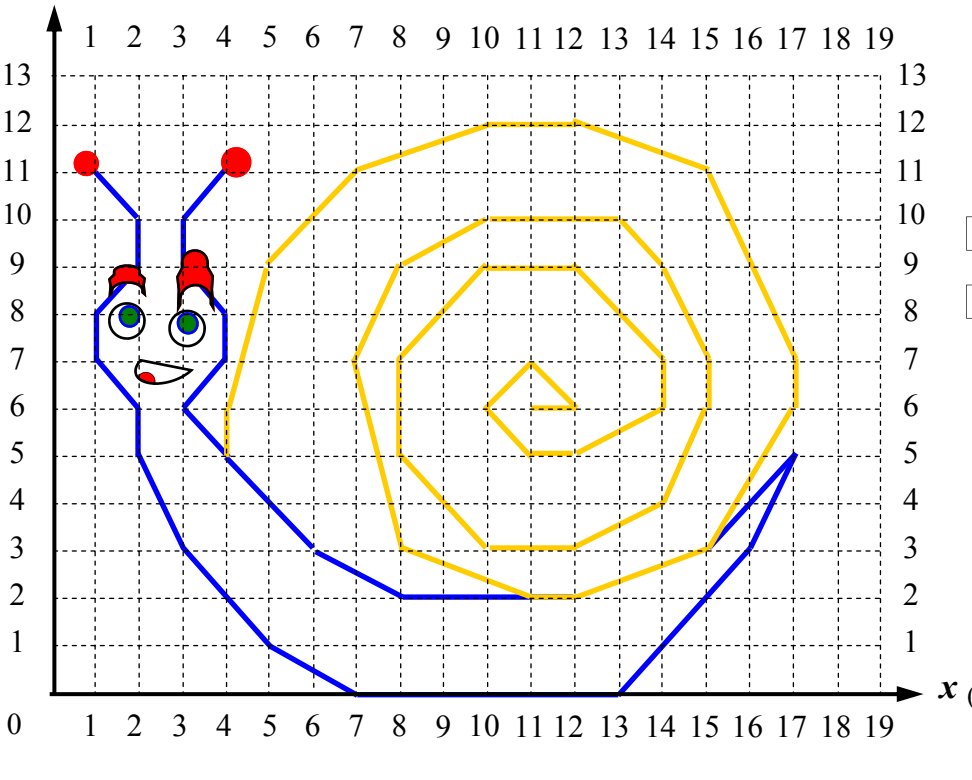
Co-ordinates:

START: (2, 12) , (3, 12) , (4, 11) , (5, 12) ,
(7, 12) , (8, 11) , (9, 12) , (10, 12) ,
(11, 11) , (11, 6) , (10, 3) , (9, 3) ,
(8, 7) , (8, 5) , (7, 7) , (7, 3) ,
(6, 2) , (4, 2) , (4, 3) , (5, 3) ,
(6, 4) , (5, 7) , (4, 5) , (4, 7) ,
(3, 3) , (2, 3) , (1, 6) , (1, 11) ,
(2, 12) STOP!

START:
(11, 10) , (12, 11) , (16, 12) , (18, 10) ,
(19, 7) , (18, 9) , (18, 3) , (15, 3) ,
(16, 4) , (16, 6) , (14, 5) , (11, 5) ,
(11, 4) , (13, 4) , (13, 1) , (12, 2) ,
(9, 2) , (8, 3) , (7, 6) , STOP!



Join the points and this time you will find a friend with which you will have to be very patient.



Co-ordinates:

START: (1, 11) , (2, 10) , (2, 9) , (1, 8) ,
(1, 7) , (2, 6) , (2, 5) , (3, 3) ,
(5, 1) , (7, 0) , (13, 0) , (16, 3) ,
(17, 5) , (15, 3) , (12, 2) , (8, 2) ,
(6, 3) , (4, 5) , (3, 6) , (4, 7) ,
(4, 8) , (3, 9) , (3, 10) , (4, 11) ,
STOP!

START:
(11, 6) , (12, 6) , (11, 7) , (10, 6) ,
(11, 5) , (12, 5) , (14, 6) , (14, 7) ,
(12, 9) , (10, 9) , (8, 7) , (8, 5) ,
(10, 3) , (12, 3) , (14, 4) , (15, 6) ,
(15, 7) , (14, 9) , (13, 10) , (10, 10) ,
(8, 9) , (7, 7) , (8, 3) , (11, 2) ,
(12, 2) , (15, 3) , (17, 6) , (17, 7) ,
(15, 11) , (12, 12) , (10, 12) , (7, 11) ,
(5, 9) , (4, 6) , (4, 5) , STOP!

Plotting Points - 4 Quadrants

This sheet involves the plotting of Cartesian co-ordinates. When plotting points on a number plane you are given two numbers. The first number tells you how much you move across and the second number tells you how much you move up or down. As the numbers are in order, across first, up/down second, these numbers can be referred to as 'ordered pairs', as their order is important. Positive numbers move across to the right if the first number, negative numbers move to the left if the first number. Positive numbers move up if the second number, negative numbers move down if the second number.

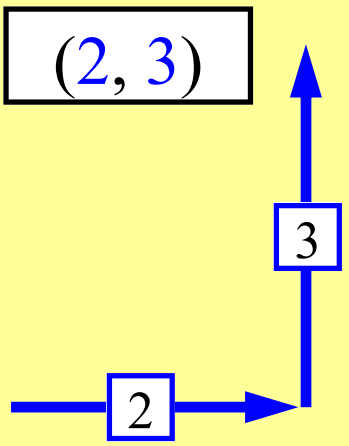
The first column asks you to locate the points and write in their co-ordinates. This is done by opening a bracket writing the *x* co-ordinate, a comma, then the *y* co-ordinate and close bracket. Always place co-ordinates in brackets.

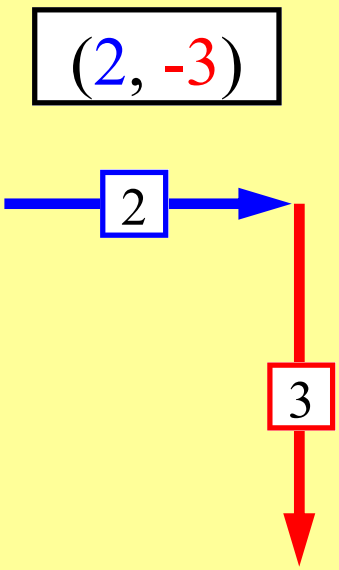
The second column asks you to plot points. Draw filled in circles at the given location and label them with the letter supplied in each question.

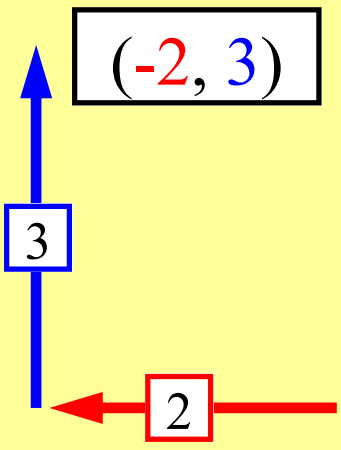
Once you plot the point record down which quadrant it is in. Quadrants start with the 1st Quadrant (top right) and then moving anti-clockwise, are called II (2nd), III (3rd) and IV (4th). If the point is on the axis then record it by filling in the oval around the word 'Axis'.

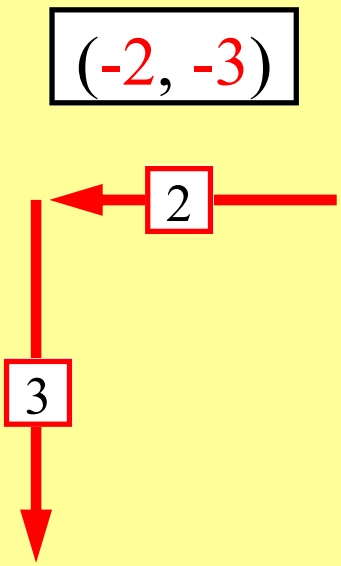
Move across first along the *x* axis.
Negative to the left, Positive to the right.

Then move up or down in the *y* direction.
Negative is down, Positive is up.









Plotting Points - 4 Quadrants

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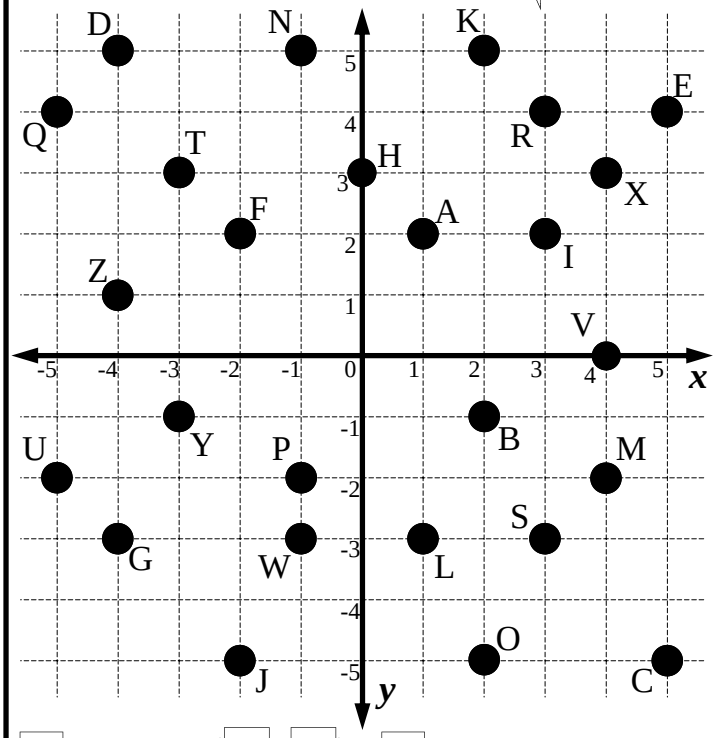
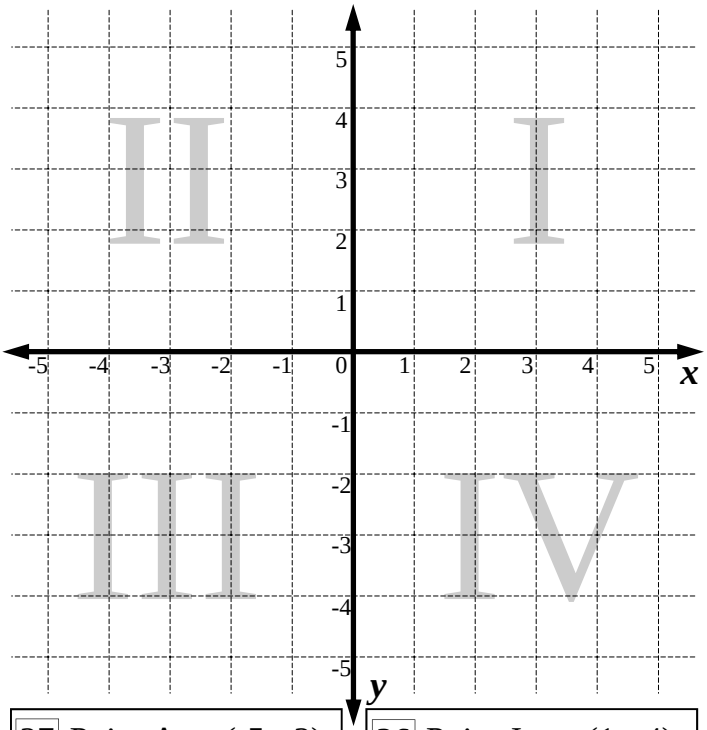
Give the co-ordinates for the points on the number plane below.



1 Point A (,)
2 Point B
3 Point C
4 Point D
5 Point E
6 Point F
7 Point G
8 Point H
9 Point I
10 Point J
11 Point K
12 Point L
13 Point M
14 Point N
15 Point O
16 Point P
17 Point Q
18 Point R
19 Point S
20 Point T
21 Point U
22 Point V
23 Point W
24 Point X
25 Point Y
26 Point Z

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant I II III IV Axis
28 Point B (2, 4) Quadrant I II III IV Axis
29 Point C (4, 0) Quadrant I II III IV Axis
30 Point D (3, -3) Quadrant I II III IV Axis
31 Point E (-3, 5) Quadrant I II III IV Axis
32 Point F (0, 0) Quadrant I II III IV Axis
33 Point G (5, -5) Quadrant I II III IV Axis
34 Point H (-2, 4) Quadrant I II III IV Axis
35 Point I (1, 2) Quadrant I II III IV Axis
36 Point J (1, -4) Quadrant I II III IV Axis
37 Point K (-3, -4) Quadrant I II III IV Axis
38 Point L (-1, 0) Quadrant I II III IV Axis
39 Point M (-4, 2) Quadrant I II III IV Axis
40 Point N (3, -5) Quadrant I II III IV Axis
41 Point O (0, -3) Quadrant I II III IV Axis
42 Point P (-5, -5) Quadrant I II III IV Axis
43 Point R (-2, 1) Quadrant I II III IV Axis
44 Point S (2, -2) Quadrant I II III IV Axis

Plotting Points - 4 Quadrants

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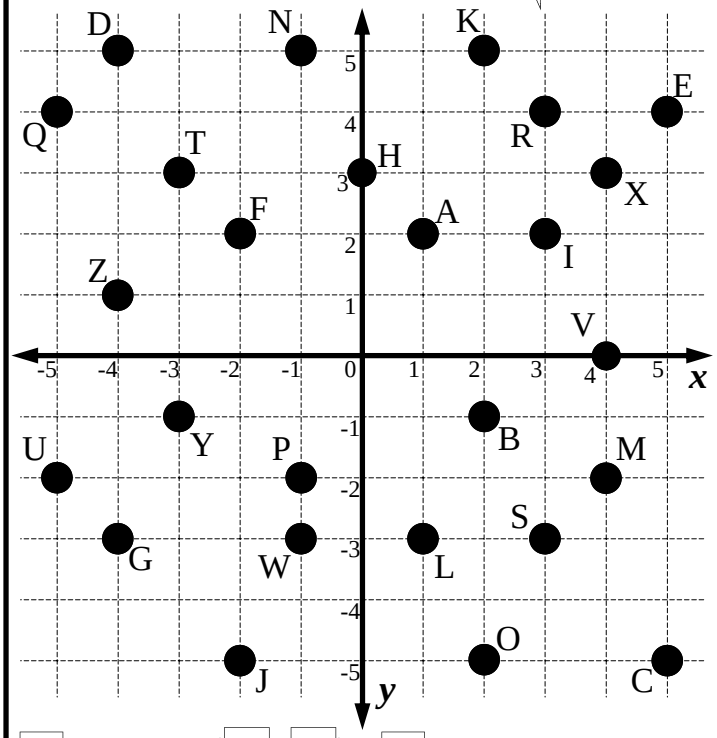
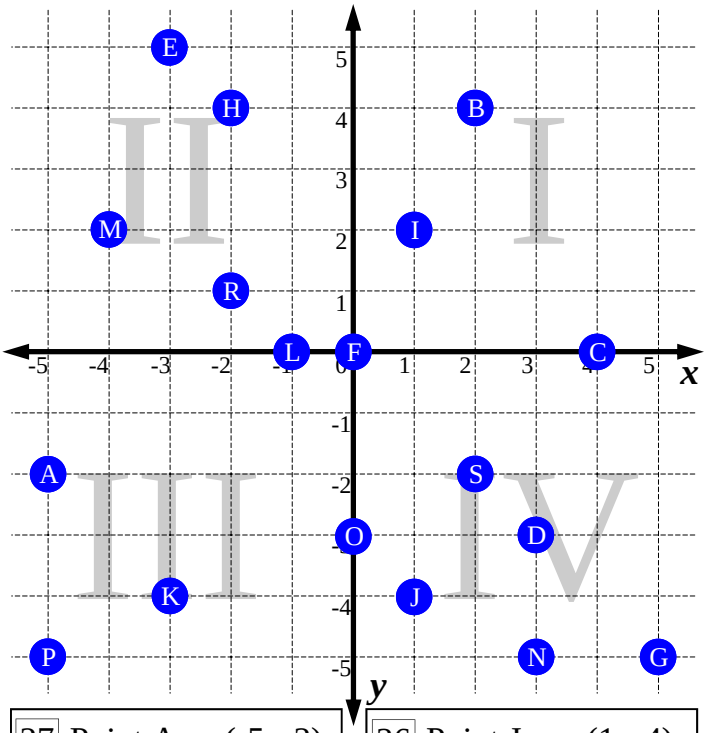
Give the co-ordinates for the points on the number plane below.



1	Point A	(1, 2)	14	Point N	(-1, 5)
2	Point B	(2, -1)	15	Point O	(2, -5)
3	Point C	(5, -5)	16	Point P	(-1, -2)
4	Point D	(-4, 5)	17	Point Q	(-5, 4)
5	Point E	(5, 4)	18	Point R	(3, 4)
6	Point F	(-2, 2)	19	Point S	(3, -3)
7	Point G	(-4, -3)	20	Point T	(-3, 3)
8	Point H	(0, 3)	21	Point U	(-5, -2)
9	Point I	(3, 2)	22	Point V	(4, 0)
10	Point J	(-2, -5)	23	Point W	(-1, -3)
11	Point K	(2, 5)	24	Point X	(4, 3)
12	Point L	(1, -3)	25	Point Y	(-3, -1)
13	Point M	(4, -2)	26	Point Z	(-4, 1)

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant I II **III** IV Axis

28 Point B (2, 4) Quadrant **I** II III IV Axis

29 Point C (4, 0) Quadrant I II III IV **Axis**

30 Point D (3, -3) Quadrant I II **III** IV Axis

31 Point E (-3, 5) Quadrant **II** III IV Axis

32 Point F (0, 0) Quadrant I II III IV **Axis**

33 Point G (5, -5) Quadrant I II **III** IV Axis

34 Point H (-2, 4) Quadrant **II** III IV Axis

35 Point I (1, 2) Quadrant **I** II III IV Axis

36 Point J (1, -4) Quadrant I II III **IV** Axis

37 Point K (-3, -4) Quadrant I II **III** IV Axis

38 Point L (-1, 0) Quadrant I II III IV **Axis**

39 Point M (-4, 2) Quadrant **II** III IV Axis

40 Point N (3, -5) Quadrant I II III **IV** Axis

41 Point O (0, -3) Quadrant I II III IV **Axis**

42 Point P (-5, -5) Quadrant I II **III** IV Axis

43 Point R (-2, 1) Quadrant **II** III IV Axis

44 Point S (2, -2) Quadrant I II III **IV** Axis

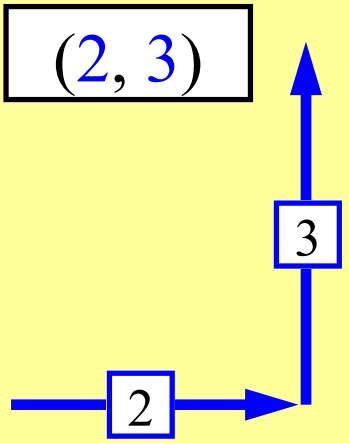
Drawing Animals - All Quadrants

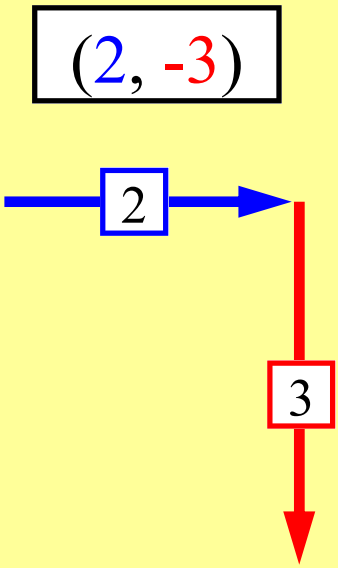
This sheet uses your skills in plotting points from the previous sheet. The series of points makes a pair of animals when lines are joined between the points. The way you attempt this exercise is, for the first animal:

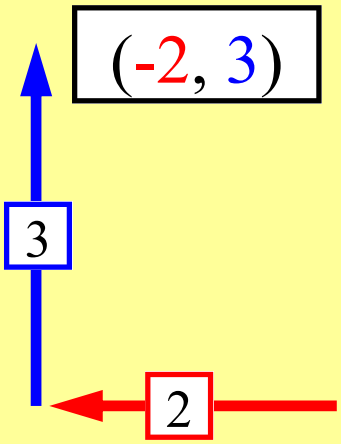
- Plot one point at a time and then draw a line between it and the previous point. Don't try to plot all the points and then draw the lines afterwards as the drawing is too complicated.
- When you reach stop, restart at the new position and start again, DON'T JOIN THE POINTS BETWEEN WHERE YOU STOPPED AND WHERE YOU RESTARTED.

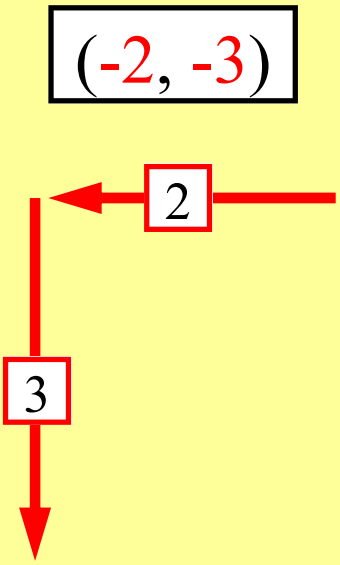
Move across first along the *x* axis.
Negative to the left, Positive to the right.

Then move up or down in the *y* direction.
Negative is down, Positive is up.









Drawing Animals - 4 Quadrants

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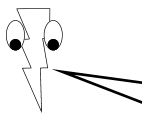
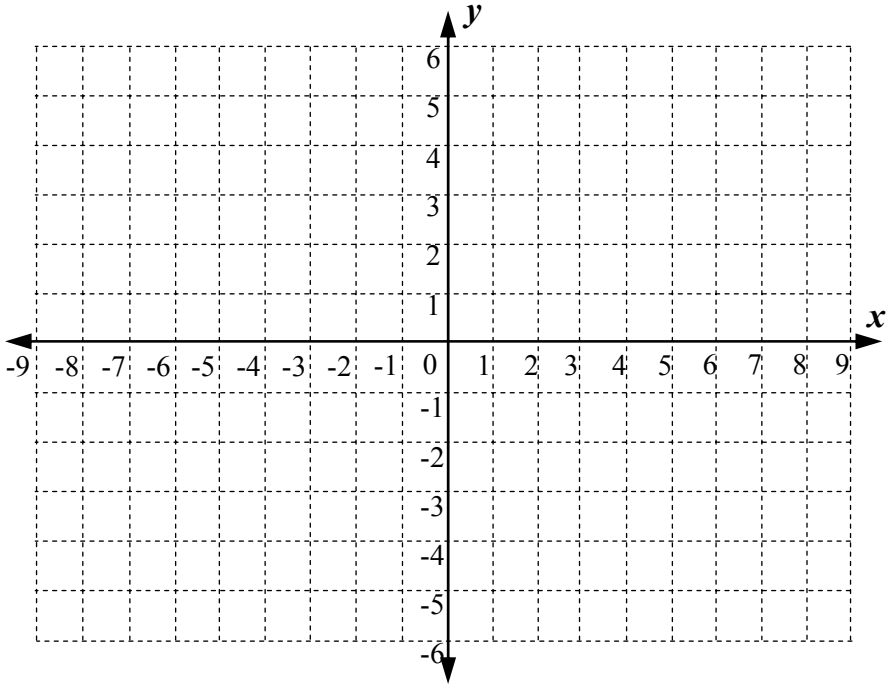


Plot the points and join them with lines to meet one of your more colourful friends

Co-ordinates: START: (-1, 3) , (-1, 4) , (1, 6) , (2, 6) , (6, 5) , (7, 4) , (7, 2) , (6, -1) , (3, -2) , (5,-4) , (5, -5) , (3, -6) , (1, -6) , (-2, -4) , (-4, -6) , (-5, -6) , (-6, -5) , (-6, -4) , (-5, -2) , (-6, -1) , (-8, 2) , (-8, 4) , (-7, 6) , (-6, 6) , (-4, 4) , (-4, 3) , STOP!

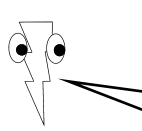


START: (-2, 1) , (1, 4) , (4, 4) , (5, 2) , (3,-1) , (0, -2) , (3, -3) , (3, -5) , (1, -5) , (-1, -4) , (-2, -3) , (-4, -5) , (-5, -5) , (-5, -3) , (-3, -2) , (-5, -1) , (-7, 2) , (-6, 4) , (-5, 4) , (-3,1) , STOP!

START: (-4, 3) , (-3, 4) , (-2, 4) , (-1, 3) , (-2, 2) , (-2, -4) , (-3, 2) , (-4, 3) , STOP!

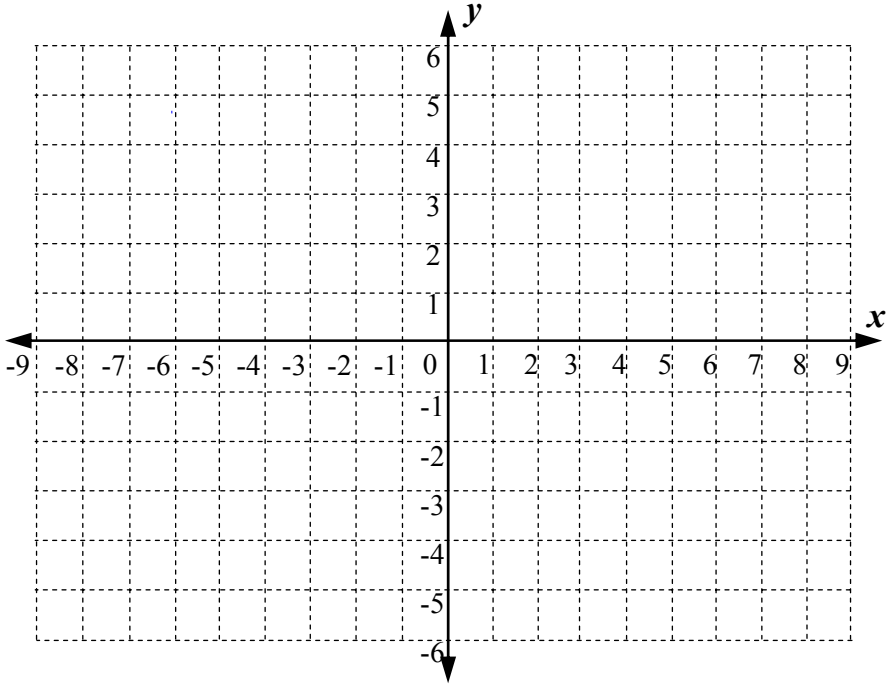
START: (-3, 4) , (-4, 6) , STOP!

START: (-2, 4) , (-1, 6) , STOP!



Plot the points and join them with lines to share your lunch with your vegetarian friend.

Co-ordinates: START: (2, -1) , (3, -3) , (3, -4) , (2, -5) , (0, -6) , (-1, -6) , (-3, -5) , (-4, -4) , (-4, -3) , (-3, -1) , (-6, 2) , (-7, -4) , (-8, 4) , (-7, 6) , (-4, 5) , (-2, -1) , (0, -1) , (3, 4) , (5, 6) , (8, 6) , (9, 4) , (5, 6) , STOP!



START: (7, 5) , (6, 3) , (2, -1) , STOP!

START: (2, 0) , (4, 4) , (6, 5) , (5, 3) , (2, 0) , STOP!

START: (-3, 0) , (-5, 2) , (-6, 4) , (-4, 3) , (-3, 0) , STOP!

START: (-2, -4) , (-6, -4) , STOP!

START: (-2, -4) , (-5, -2) , STOP!

START: (-2, -4) , (-5, -3) , STOP!

START: (1, -4) , (5, -4) , STOP!

START: (1, -4) , (5, -3) , STOP!

START: (1, -4) , (5, -5) , STOP!

Drawing Animals - 4 Quadrants

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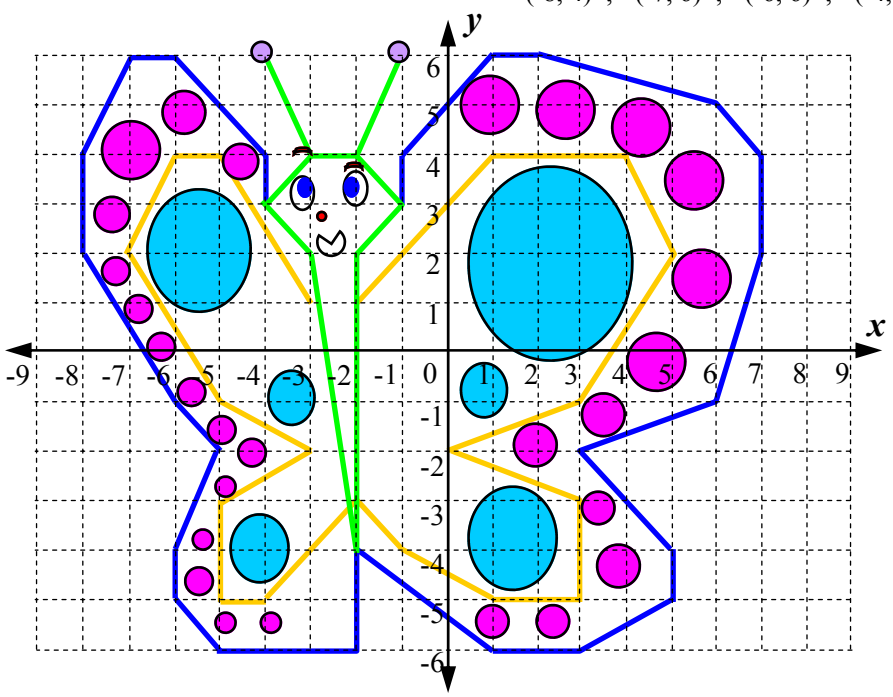
Plot the points and join them with lines to meet one of your more colourful friends

Co-ordinates: START: (-1, 3) , (-1, 4) , (1, 6) , (2, 6) , (6, 5) , (7, 4) , (7, 2) , (6, -1) , (3, -2) , (5,-4) , (5, -5) , (3, -6) , (1, -6) , (-2, -4) , (-4, -6) , (-5, -6) , (-6, -5) , (-6, -4) , (-5, -2) , (-6, -1) , (-8, 2) , (-8, 4) , (-7, 6) , (-6, 6) , (-4, 4) , (-4, 3) , STOP!



START: (-2, 1) , (1, 4) , (4, 4) , (5, 2) , (3,-1) , (0, -2) , (3, -3) , (3, -5) , (1, -5) , (-1, -4) , (-2, -3) , (-4, -5) , (-5, -5) , (-5, -3) , (-3, -2) , (-5, -1) , (-7, 2) , (-6, 4) , (-5, 4) , (-3,1) , STOP!

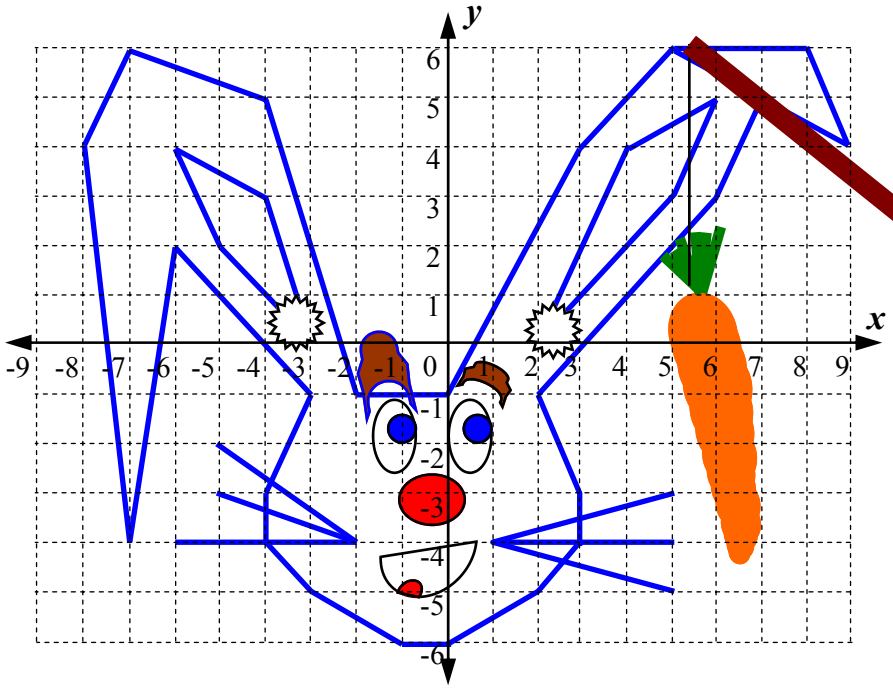
START: (-4, 3) , (-3, 4) , (-2, 4) , (-1, 3) , (-2, 2) , (-2, -4) , (-3, 2) , (-4, 3) , STOP!

START: (-3, 4) , (-4, 6) , STOP!

START: (-2, 4) , (-1, 6) , STOP!

Plot the points and join them with lines to share your lunch with your vegetarian friend.

Co-ordinates: START: (2, -1) , (3, -3) , (3, -4) , (2, -5) , (0, -6) , (-1, -6) , (-3, -5) , (-4, -4) , (-4, -3) , (-3, -1) , (-6, 2) , (-7, -4) , (-8, 4) , (-7, 6) , (-4, 5) , (-2, -1) , (0, -1) , (3, 4) , (5, 6) , (8, 6) , (9, 4) , (5, 6) , STOP!



START: (7, 5) , (6, 3) , (2, -1) , STOP!

START: (2, 0) , (4, 4) , (6, 5) , (5, 3) , (2, 0) , STOP!

START: (-3, 0) , (-5, 2) , (-6, 4) , (-4, 3) , (-3, 0) , STOP!

START: (-2, 4) , (-6, -4) , STOP!

START: (-2, -4) , (5, -2) , STOP!

START: (-2, -4) , (-5, -3) , STOP!

START: (1, -4) , (5, -4) , STOP!

START: (1, -4) , (5, -3) , STOP!

START: (1, -4) , (5, -5) , STOP!

Warship Rules

This game uses your grid reference skills to play a game. This game is called ‘Warship’ and it involves marine military vessels.

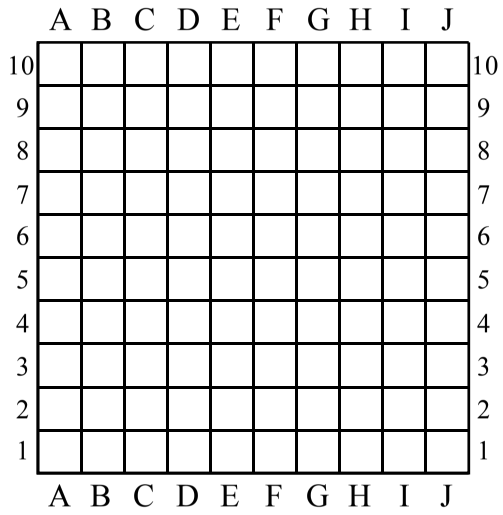
The game:

- The bottom grid is for placing one of each of the 5 craft supplied either horizontally or vertically. For a shorter game don't use the 'zodiac'.
- Your opponent does the same on their separate grid sheet and then you decide who has first shot.
- You pick a co-ordinate and call it out, letter first then number, (like… B3) if it hits you opponent's craft they say 'Hit', if it misses they say 'Miss'. On the top grid you then place a cross in the square if it hits or a dot in the square if its a miss. The dot ensures you don’t call the same co-ordinate again.
- Your opponent then has a shot, if their co-ordinates hit part of a craft then you call 'Hit' and place a cross on the bottom grid, if they miss you don’t have to record their shot.
- Note that your next try will be above/below or to the left/right of the hit if you hit your opponent, until you sink the craft. Your opponent must call out 'Sunk' if the craft is destroyed (a cross over each square that the craft is in). If you didn't hit a craft in your previous try then guess another 'random' location, away from the point called earlier.
- Shots are alternated until one player loses all his or her fleet. You may like to cross out your sunken opponents craft as a reminder of the craft still remaining.
- Note: Sometimes you score a hit and then get 4 more hits but your opponent doesn't call 'sunk'. If they have placed two craft in a line this is possible don't jump to the conclusion that they have made a mistake.
- If you lose a zodiac early you are unlucky and may lose the game because of it, so think carefully about its placement. Also if the zodiac is hit while your opponent is hitting a craft next to your zodiac then you must say that it is the zodiac that was sunk.
- Another way to speed up the game is to give a player another turn if a hit is made including after the craft is sunk.

Don't cheat by changing the location of your craft during play as your opponent will quickly tire of playing you and it spoils the fun for both of you. You would also be surprised at how long one particular craft can be unsuccessfully hunted.

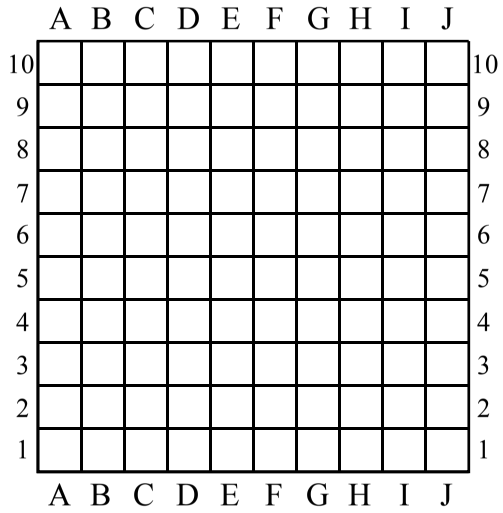
Warship - Printable Sheet

GAME 1 : OPPONENT'S SHIPS

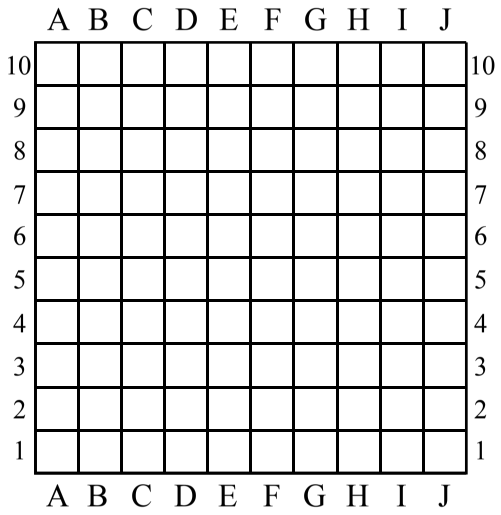


Aircraft Carrier	5 squares
Cruiser	4 squares
Submarine	3 squares
Sub Hunter	2 squares
Zodiac	1 square

GAME 1 : YOUR SHIPS

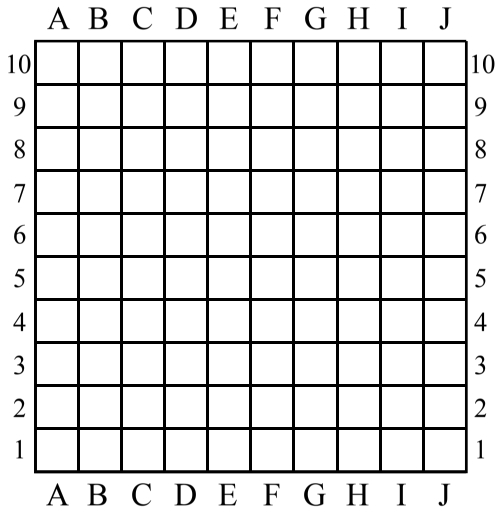


GAME 2 : OPPONENT'S SHIPS

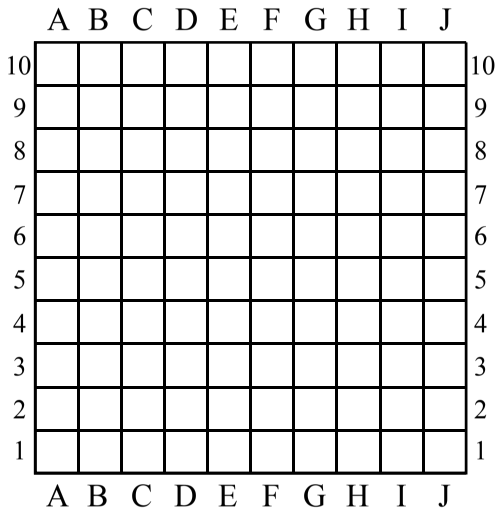


Aircraft Carrier	5 squares
Cruiser	4 squares
Submarine	3 squares
Sub Hunter	2 squares
Zodiac	1 square

GAME 2 : YOUR SHIPS

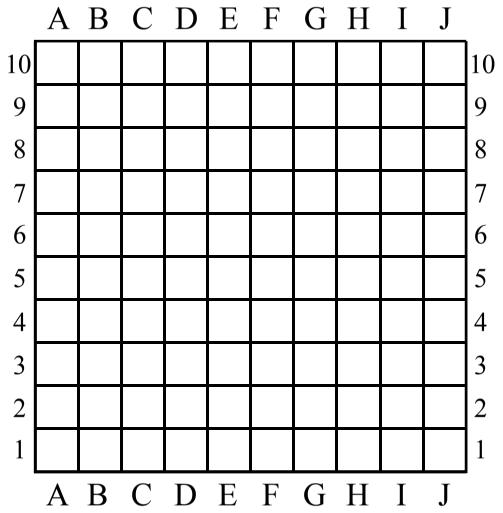


GAME 3 : OPPONENT'S SHIPS



Aircraft Carrier	5 squares
Cruiser	4 squares
Submarine	3 squares
Sub Hunter	2 squares
Zodiac	1 square

GAME 3 : YOUR SHIPS



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Latitude and Longitude

Similar to the number plane, when you give locations on the globe you do so by using 2 co-ordinates. Latitude lines are the horizontal or flat 'lines' (remember lat is flat). Longitude lines are the vertical lines (remember they aren't flat!).

Note that there are no negative numbers, instead direction is used, the equator is when the latitude is 0°. Then moving up (northerly), latitudes increase and are denoted by an N for north. Moving down (southwards) they are denoted by an S. Longitudes are measured from the 0° 'prime meridian' which matches GMT, Greenwich Meridian Time. Then when you move left (West) you denote the movement with a W, moving right (East) you use an E.

When you give these locations unlike a number plane you give the latitude first, you probably remember the compass points by saying 'North South East West' well this applies to the co-ordinates also, give the North or South co-ordinate first then the East or West co-ordinate. You also use a degree sign, separate the co-ordinates with a comma and put them in brackets.

If you look at the point N at the top left of the globe you would give its position as:
(50°N, 120°W).

While the sheet differs from column to column the use of co-ordinates is all that is being dealt with on this sheet. The first column asks you to give the co-ordinates for the given point, open a bracket and write in the latitude, then a comma, then the longitude and close the brackets. Note that 0° latitude or longitude doesn't require a letter, just write it as 0°.

Column 2 is the reverse, given a co-ordinate find the point. Write in the letter.

Column 3 takes this one step further. A word (which will be a city) will be formed by using the co-ordinates to find each letter in order, write the letter as you locate the points. If the city is unrecognisable then you need to go back and check your points. Note that the letters are in order they aren't jumbled.

The most common mistake made apart from giving the co-ordinates reversed is with the longitude. Most people get the North or South right but some mix up their east with their west.

Latitude and Longitude

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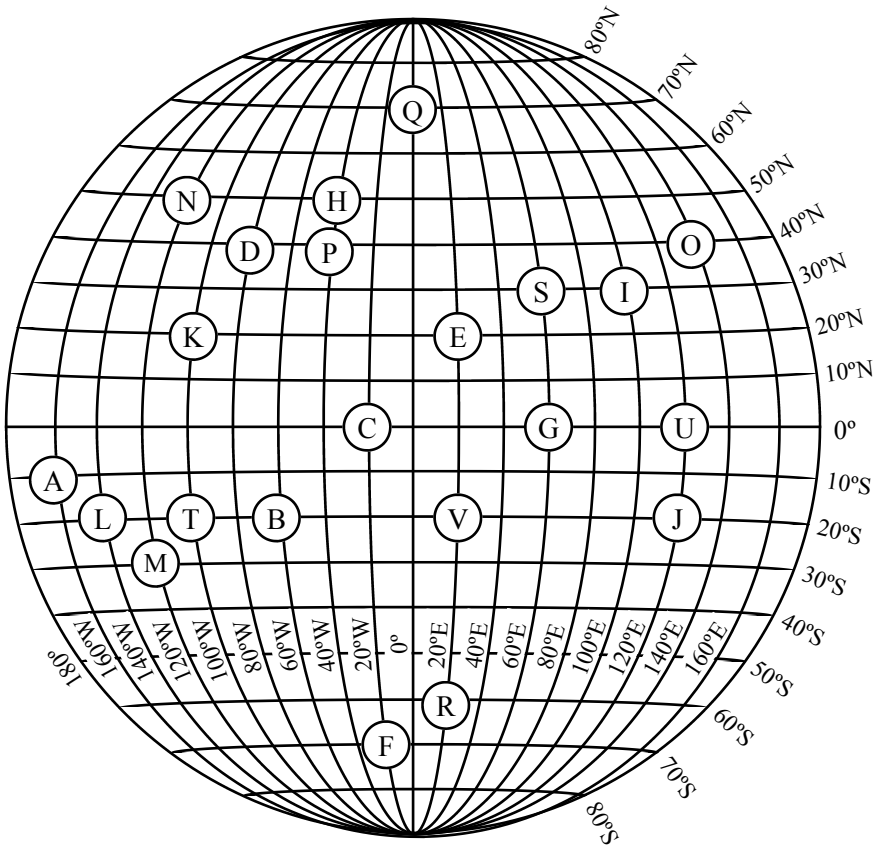

Give the co-ordinates of these points:

1	A	
2	B	
3	C	
4	D	
5	E	
6	F	
7	G	
8	H	
9	I	
10	J	
11	K	

Give the letter at the co-ordinates below:

12	(30°S, 120°W)	
13	(40°N, 40°W)	
14	(30°N, 60°E)	
15	(20°S, 100°W)	
16	(0°, 120°E)	
17	(60°S, 20°E)	
18	(50N°, 120°W)	
19	(20°S, 140°W)	
20	(70°N, 0°)	
21	(40°N, 140°E)	
22	(20°S, 20°E)	

Give the cities that these co-ordinates spell

23 Name City: _____

(40°N, 40°W)
(10°S, 160°W)
(60°S, 20°E)
(30°N, 100°E)
(30°N, 60°E)

24 Name City: _____

(40°N, 80°W)
(20N°, 20°E)
(20°S, 140°W)
(50°N, 40W°)
(30°N, 100°E)

25 Name City: _____

(50°N, 40°W)
(40°N, 140°E)
(0°, 120°E)
(30°N, 60°E)
(20°S, 100°W)
(40°N, 140°E)
(50°N, 120°W)

Latitude and Longitude

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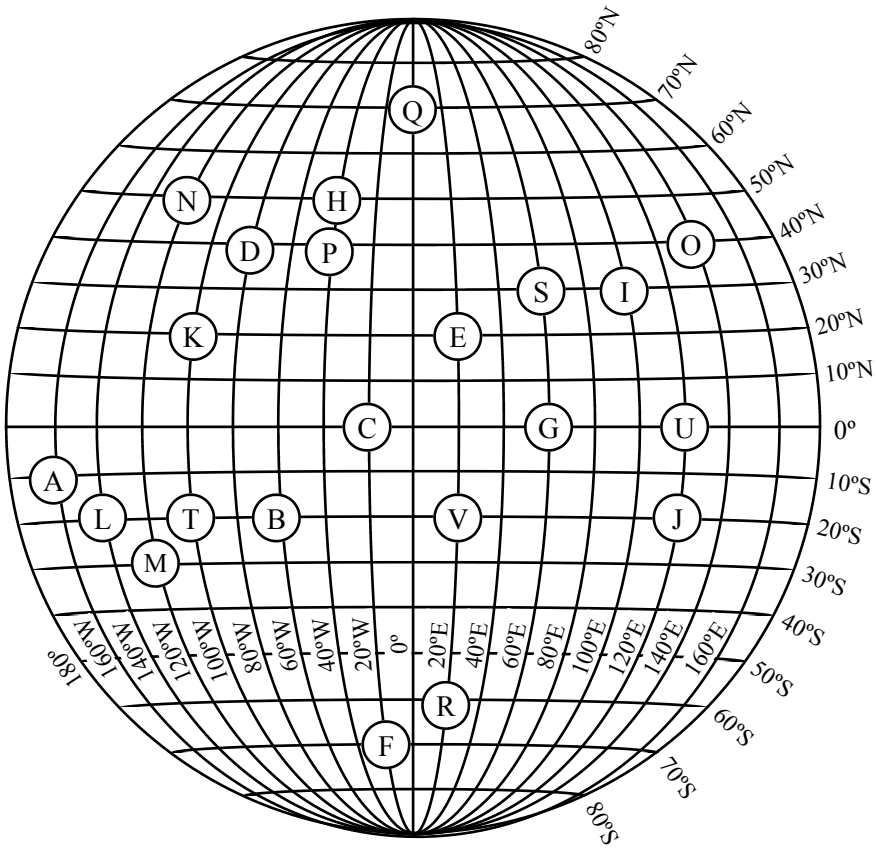

Give the co-ordinates of these points:

1	A	(10°S, 160°W)
2	B	(20°S, 60°W)
3	C	(0°, 20°W)
4	D	(40°N, 80°W)
5	E	(20°N, 20°E)
6	F	(70°S, 20°W)
7	G	(0°, 60°E)
8	H	(50°N, 40°W)
9	I	(30°N, 100°E)
10	J	(20°S, 120°E)
11	K	(20°N, 100°W)

Give the letter at the co-ordinates below:

12	(30°S, 120°W)	M
13	(40°N, 40°W)	P
14	(30°N, 60°E)	S
15	(20°S, 100°W)	T
16	(0°, 120°E)	U
17	(60°S, 20°E)	R
18	(50N°, 120°W)	N
19	(20°S, 140°W)	L
20	(70°N, 0°)	Q
21	(40°N, 140°E)	O
22	(20°S, 20°E)	V

Give the cities that these co-ordinates spell

23 Name City: PARIS

(40°N, 40°W)
(10°S, 160°W)
(60°S, 20°E)
(30°N, 100°E)
(30°N, 60°E)

24 Name City: DELHI

(40°N, 80°W)
(20N°, 20°E)
(20°S, 140°W)
(50°N, 40W°)
(30°N, 100°E)

25 Name City: HOUSTON

(50°N, 40°W)
(40°N, 140°E)
(0°, 120°E)
(30°N, 60°E)
(20°S, 100°W)
(40°N, 140°E)
(50°N, 120°W)

FREEFALL MATHEMATICS

ALTITUDE

PERCENTAGES

Fractions to Percentages

Often a result of a test or survey is expressed as a fraction. Fractions can be difficult to compare and often these are converted to percentages, allowing easier comparison of the result. A fraction is converted to a percentage by multiplying it by 100. This is more easily achieved if the fraction already has a denominator of 100, so finding the equivalent fraction with a denominator of 100 is the first step:

$$\frac{7}{25} \overset{\times 4}{=} \frac{\boxed{28}}{100} = \underline{28\ \%}$$ Why is it 28%?…. Because $\frac{28}{\cancel{100}} \times \cancel{100} = 28\%$

In the example above $^{7}/_{25}$ is changed to $^{28}/_{100}$ by looking at the denominators. Ask yourself what you would multiply 25 by to get 100?… 4! If you multiply the bottom by 4 you have to multiply the top by 4 …. $7 \times 4 = 28$. Then if you multiply $^{28}/_{100}$ by 100, you are left with 28%.

Note that this method only works for factors of 100, denominators of 2, 4, 5, 10, 20, 25, 50. It can also work for multiples of 100, this time by dividing, as long as the numerator also allows the division.

Column 1 starts with writing the shaded fraction of a shape as a fraction with a denominator of 100 (don't simplify), and then as a percentage. Count the painted squares, this is the numerator of the fraction. The same number is also the percentage.

Column 2 is an extension of this. A fraction is given and you are asked to make an equivalent fraction with a denominator of 100. Look at the denominator (bottom number) and find the number you need to multiply it by to get 100. You can write this number above/below the equals sign if you like (see the example, top of column). Then multiply the numerator (top number) by the same number. This will give you the percentage. Note questions 12 and 13 require division.

You may think that 100% is all that you can give. With things like 'effort', 'determination' or 'support', this is the case. With mathematics it is possible you can find 300% of a number, it's 3 times the number.

Column 3 involves converting mixed numbers to percentages. If $1 = {}^{100}/_{100}$, $2 = {}^{200}/_{100}$, $3 = {}^{300}/_{100}$ etc, this means that 1 = 100%, 2 = 200% and 3 = 300%. So with this column first split the whole number from the fraction and write each separately as a fraction over 100, then add them together and change to a percentage. An example is at the top of the column.

Fractions to Percentages

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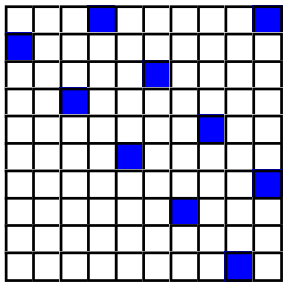
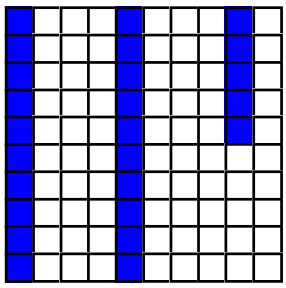
Write the shaded part of the (hundredths) grid as a fraction 'over a hundred', and then as a percentage.

1 

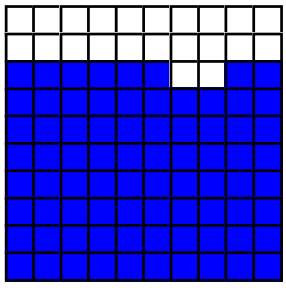
$= \frac{\square}{100}$

$= \_\_\_\_ \%$

2 

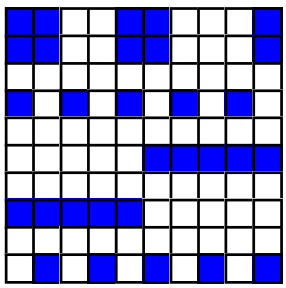
$= \frac{\square}{100}$

$= \_\_\_\_ \%$

3 

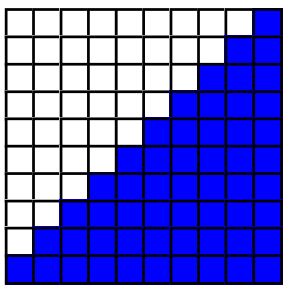
$= \frac{\square}{100}$

$= \_\_\_\_ \%$

4 

$= \frac{\square}{100}$

$= \_\_\_\_ \%$

5 

$= \frac{\square}{100}$

$= \_\_\_\_ \%$

Rewrite the fraction changing the denominator to 100. Then write as a %.

Example

$\frac{3}{20} \overset{\times 5}{=} \frac{\boxed{15}}{100} = 15\ \%$ $\times 5$

6 $\frac{4}{5} = \frac{\square}{100} = \_\_\_\_ \%$

7 $\frac{6}{10} = \frac{\square}{100} = \_\_\_\_ \%$

8 $\frac{7}{20} = \frac{\square}{100} = \_\_\_\_ \%$

9 $\frac{17}{50} = \frac{\square}{100} = \_\_\_\_ \%$

10 $\frac{3}{4} = \frac{\square}{100} = \_\_\_\_ \%$

11 $\frac{7}{4} = \frac{\square}{100} = \_\_\_\_ \%$

12 $\frac{12}{200} = \frac{\square}{100} = \_\_\_\_ \%$

13 $\frac{350}{500} = \frac{\square}{100} = \_\_\_\_ \%$

14 David is $^{3}/_{10}$ of the way home, write as a percentage

$\frac{3}{10} = \frac{\square}{100} = \_\_\_\_ \%$

15 Ellen scored $^{17}/_{20}$ in a test, write this as a percentage.

$\frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_ \%$

16 Ken forgets his lunch 1 day in 5, write as a percentage

$\frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_ \%$

Change these mixed numbers to fractions with 100 as the denominator, then simplify them.

Example

$2\ \frac{1}{4} = \frac{\boxed{200}}{100} + \frac{\boxed{25}}{100} = \frac{\boxed{225}}{100}$

$= 225\ \%$

17 $4\ \frac{7}{10} = \frac{\square}{100} + \frac{\square}{100} = \frac{\square}{100}$

$= \_\_\_\_ \%$

18 $5\ \frac{1}{2} = \frac{\square}{100} + \frac{\square}{100} = \frac{\square}{100}$

$= \_\_\_\_ \%$

19 $7\ \frac{9}{20} = \frac{\square}{100} + \frac{\square}{100} = \frac{\square}{100}$

$= \_\_\_\_ \%$

20 $1\ \frac{34}{200} = \frac{\square}{100} + \frac{\square}{100} = \frac{\square}{100}$

$= \_\_\_\_ \%$

21 $9\ \frac{15}{500} = \frac{\square}{100} + \frac{\square}{100} = \frac{\square}{100}$

$= \_\_\_\_ \%$

22 $6\ \frac{8}{25} = \frac{\square}{100} + \frac{\square}{100} = \frac{\square}{100}$

$= \_\_\_\_ \%$

23 $4\ \frac{2}{50} = \frac{\square}{100} + \frac{\square}{100} = \frac{\square}{100}$

$= \_\_\_\_ \%$

Fractions to Percentages

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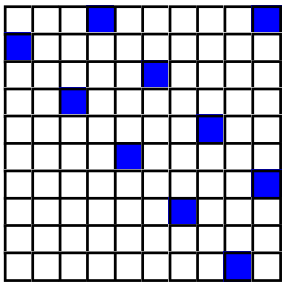
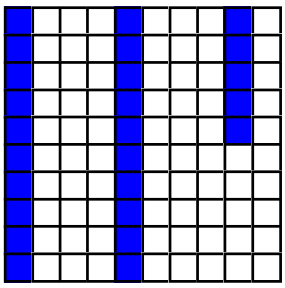
Write the shaded part of the (hundredths) grid as a fraction 'over a hundred', and then as a percentage.

1 

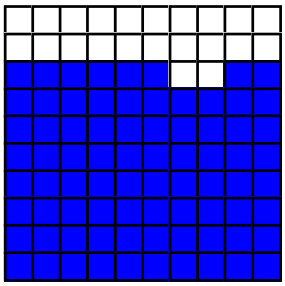
$= \frac{10}{100}$

$= 10\ \%$

2 

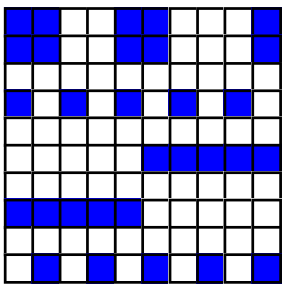
$= \frac{25}{100}$

$= 25\ \%$

3 

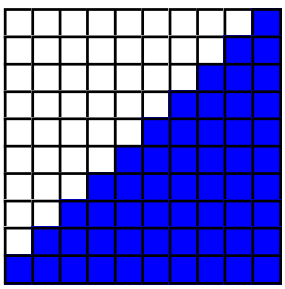
$= \frac{78}{100}$

$= 78\ \%$

4 

$= \frac{30}{100}$

$= 30\ \%$

5 

$= \frac{55}{100}$

$= 55\ \%$

Rewrite the fraction changing the denominator to 100. Then write as a %.

Example

$\frac{3}{20} \overset{\times 5}{=} \frac{15}{100} = 15\ \%$

6 $\frac{4}{5} \overset{\times 20}{=} \frac{80}{100} = 80\ \%$

7 $\frac{6}{10} \overset{\times 10}{=} \frac{60}{100} = 60\ \%$

8 $\frac{7}{20} \overset{\times 5}{=} \frac{35}{100} = 35\ \%$

9 $\frac{17}{50} \overset{\times 2}{=} \frac{34}{100} = 34\ \%$

10 $\frac{3}{4} \overset{\times 25}{=} \frac{75}{100} = 75\ \%$

11 $\frac{7}{4} \overset{\times 25}{=} \frac{175}{100} = 175\ \%$

12 $\frac{\overset{6}{\cancel{12}}}{\underset{100}{\cancel{200}}} = \frac{6}{100} = 6\ \%$

13 $\frac{\overset{70}{\cancel{350}}}{\underset{100}{\cancel{500}}} = \frac{70}{100} = 70\ \%$

14 David is $^3/_{10}$ of the way home, write as a percentage

$\frac{3}{10} \overset{\times 10}{=} \frac{30}{100} = 30\ \%$

15 Ellen scored $^{17}/_{20}$ in a test, write this as a percentage.

$\frac{17}{20} \overset{\times 5}{=} \frac{85}{100} = 85\ \%$

16 Ken forgets his lunch 1 day in 5, write as a percentage

$\frac{1}{5} \overset{\times 20}{=} \frac{20}{100} = 20\ \%$

Change these mixed numbers to fractions with 100 as the denominator, then simplify them.

$2\frac{1}{4} = \frac{200}{100} + \frac{25}{100} = \frac{225}{100}$

$= 225\ \%$

17 $4\frac{7}{10} = \frac{400}{100} + \frac{70}{100} = \frac{470}{100}$

$= 470\ \%$

18 $5\frac{1}{2} = \frac{500}{100} + \frac{50}{100} = \frac{550}{100}$

$= 550\ \%$

19 $7\frac{9}{20} = \frac{700}{100} + \frac{45}{100} = \frac{745}{100}$

$= 745\ \%$

20 $1\frac{34}{200} = \frac{100}{100} + \frac{17}{100} = \frac{117}{100}$

$= 117\ \%$

21 $9\frac{10}{500} = \frac{900}{100} + \frac{2}{100} = \frac{902}{100}$

$= 902\ \%$

22 $6\frac{8}{25} = \frac{600}{100} + \frac{32}{100} = \frac{632}{100}$

$= 632\ \%$

23 $4\frac{2}{5} = \frac{400}{100} + \frac{40}{100} = \frac{440}{100}$

$= 440\ \%$

Percentages to Fractions

To use percentages in calculations the percentage will usually be converted to a decimal or a fraction. This sheet deals with converting percentages to fractions.

Column 1 asks you to write the percentages in simplified form. Start by rewriting the percentage as a fraction over 100. Then fully simplify, this is done by finding the largest number that goes into both numbers, their HCF, then dividing through by that number. An example is at the top of the first column, use the same method throughout the column.

Column 2 is in problem form, start with the same method (except question 16) by writing the percentage as a fraction with 100 as its denominator. Then find the equivalent fraction changing the denominator to the number given in the question. In question 16 you have to find the percentage first, though you may see a quicker way.

Column 3 is much the same as column 1 with the difference being that the percentages are greater than 100% and so the fraction will have a value greater than 1. Start with the same first step, writing the number over 100. Then look for the HCF. Divide through and write the answer as an improper fraction. The answer could be left this way if the question just asked for a fraction, but the question asks for mixed numerals. So convert the improper fraction to a mixed numeral. Look at the mixed numeral when you have finished and make sure it can't be further simplified.

Percentages to Fractions

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Change these percentages to fractions with 100 as the denominator, then simplify them fully.



Example (÷ 5)

$55\ \% = \frac{55}{100} = \frac{11}{20}$

1 $30\ \% = \frac{\square}{100} = \frac{\square}{\square}$

2 $20\ \% = \frac{\square}{100} = \frac{\square}{\square}$

3 $60\ \% = \frac{\square}{100} = \frac{\square}{\square}$

4 $12\ \% = \frac{\square}{100} = \frac{\square}{\square}$

5 $34\ \% = \frac{\square}{100} = \frac{\square}{\square}$

6 $44\ \% = \frac{\square}{100} = \frac{\square}{\square}$

7 $15\ \% = \frac{\square}{100} = \frac{\square}{\square}$

8 $85\ \% = \frac{\square}{100} = \frac{\square}{\square}$

9 $64\ \% = \frac{\square}{100} = \frac{\square}{\square}$

10 Ryan drank 35% of his drink, express as a fraction

$\_\_\_\ \% = \frac{\square}{100} = \frac{\square}{\square}$

11 Write a 16% levy on water rates as a fraction

$\_\_\_\ \% = \frac{\square}{100} = \frac{\square}{\square}$



Use equivalent fractions to solve these

12 Michael achieved 68% in a test. If the test was out of 50, what was his mark?

$68\ \% = \frac{\square}{100} = \frac{\square}{50}$

13 Carla achieved 80% in a test which had a maximum mark of 20, find her mark.

$80\ \% = \frac{\square}{100} = \frac{\square}{\square}$

14 A new apartment building has 300 apartments, 40% have been sold, how many apartments is this?

$\_\_\_\ \% = \frac{\square}{\square} = \frac{\square}{\square}$

15 How many apartments are still available?

16 Jane scored 12 out of 20 in her previous test. If the next test is out of 50 what must she score for the same %?

$\frac{12}{20} = \frac{\square}{\square} = \_\_\_\ \%$

$\_\_\_\ \% = \frac{\square}{\square} = \frac{\square}{\square}$

17 Ian falls out of bed in his sleep 8% of nights. How many times would he fall out of bed in a 25 night period?

$\_\_\_\ \% = \frac{\square}{\square} = \frac{\square}{\square}$



These percentages are larger than 100%. Express them as mixed numerals.

Example (÷ 50) improper

$150\ \% = \frac{150}{100} = \frac{3}{2}$

Rewrite as a mixed numeral $= 1\frac{1}{2}$

18 $120\ \% = \frac{\square}{100} = \frac{\square}{\square} = \square\frac{\square}{\square}$

19 $250\ \% = \frac{\square}{100} = \frac{\square}{\square} = \square\frac{\square}{\square}$

20 $110\ \% = \frac{\square}{100} = \frac{\square}{\square} = \square\frac{\square}{\square}$

21 $180\ \% = \frac{\square}{100} = \frac{\square}{\square} = \square\frac{\square}{\square}$

22 $125\ \% = \frac{\square}{100} = \frac{\square}{\square} = \square\frac{\square}{\square}$

23 $105\ \% = \frac{\square}{100} = \frac{\square}{\square} = \square\frac{\square}{\square}$

Percentages to Fractions

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Change these percentages to fractions with 100 as the denominator, then simplify them fully.

Example (÷ 5)

$55\ \% = \frac{55}{100} = \frac{11}{20}$

1 $30\ \% = \frac{30}{100} = \frac{3}{10}$

2 $20\ \% = \frac{20}{100} = \frac{1}{5}$

3 $60\ \% = \frac{60}{100} = \frac{3}{5}$

4 $12\ \% = \frac{12}{100} = \frac{3}{25}$

5 $34\ \% = \frac{34}{100} = \frac{17}{50}$

6 $44\ \% = \frac{44}{100} = \frac{11}{25}$

7 $15\ \% = \frac{15}{100} = \frac{3}{20}$

8 $85\ \% = \frac{85}{100} = \frac{17}{20}$

9 $64\ \% = \frac{64}{100} = \frac{16}{25}$

10 Ryan drank 35% of his drink, express as a fraction

$35\ \% = \frac{35}{100} = \frac{7}{20}$

11 Write a 16% levy on water rates as a fraction

$16\ \% = \frac{16}{100} = \frac{4}{25}$

Use equivalent fractions to solve these

12 Michael achieved 68% in a test. If the test was out of 50, what was his mark?

$68\ \% = \frac{68}{100} = \frac{34}{50}$

13 Carla achieved 80% in a test which had a maximum mark of 20, find her mark.

$80\ \% = \frac{80}{100} = \frac{16}{20}$

14 A new apartment building has 300 apartments, 40% have been sold, how many apartments is this?

$40\ \% = \frac{40}{100} = \frac{120}{300}$

15 How many apartments are still available?

180 apartments

16 Jane scored 12 out of 20 in her previous test. If the next test is out of 50 what must she score for the same %?

$\frac{12}{20} \overset{\times 5}{=} \frac{60}{100} = 60\ \%$

$60\ \% = \frac{60}{100} = \frac{30}{50}$

17 Ian falls out of bed in his sleep 8% of nights. How many times would he fall out of bed in a 25 night period?

$8\ \% = \frac{8}{100} = \frac{2}{25}$

These percentages are larger than 100%. Express them as mixed numerals.

Example (÷ 50) improper

$150\ \% = \frac{150}{100} = \frac{3}{2}$

Rewrite as a mixed numeral $= 1\frac{1}{2}$

18 $120\ \% = \frac{120}{100} = \frac{6}{5} = 1\frac{1}{5}$

19 $250\ \% = \frac{250}{100} = \frac{5}{2} = 2\frac{1}{2}$

20 $110\ \% = \frac{110}{100} = \frac{11}{10} = 1\frac{1}{10}$

21 $180\ \% = \frac{180}{100} = \frac{9}{5} = 1\frac{4}{5}$

22 $125\ \% = \frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$

23 $105\ \% = \frac{105}{100} = \frac{21}{20} = 1\frac{1}{20}$

Decimals and Percentages

When percentages are used in calculations such as with bank loans and government taxes, they are often converted to decimals. This is done, as with fractions, by dividing by 100. With decimals this step is replaced with moving the decimal point 2 places to the left (dividing by 100). If converting a decimal to a percentage then you multiply by 100, move the decimal point 2 places to the right.

Column 1 has 100 square grid and you are asked to express the shaded area as a decimal and percentage, then repeat for the unshaded part. To express the shaded part as a decimal count the squares and write the number behind 0._ _. For example 53 squares would be 0.53, don't write in a zero when it isn't required, for example 30 squares is 0.3 rather than 0.30. The percentage shaded is the number of shaded squares with the % sign after it. For example 10 squares = 10%. Once you have answered the shaded part then complete the unshaded portion. You can count the white squares, but it would be easier and faster to subtract the number of blue squares from 100. If you want to be really fast then count the smallest number of squares (unshaded or shaded) and answer it first. Your teacher may ask you what the sum (addition) of the two decimals equals and what the sum of the two percentages equals, you should know that this is 1 (decimals) and 100% (percentages).

Column 2 involves changing decimals to percentages, answer by multiplying by 100, so move the decimal point 2 places right. If there isn't a decimal point it's because the number is a whole number, so you assume it's behind the last digit. That means you add 2 zeros to the end. There is an example at the top of the column.

Column 3 is the reverse, converting percentages to decimals. This time move the decimal point 2 places to the left, as you are dividing by 100. Again there is an example at the top. Possibly the most asked question and the one that tricks the most students is to write 0.5% (or ½%) as a decimal, this question is there (Q.45), see how you go.

Decimals and Percentages

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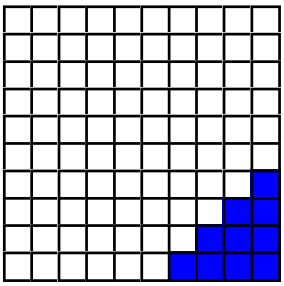
Write the both the shaded and unshaded part of the hundredths grid as a decimal and a percentage.

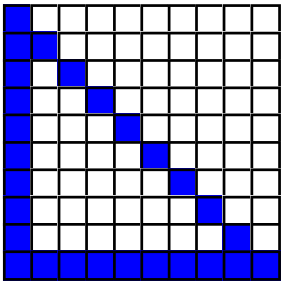
Decimal: write the number of squares behind decimal point ie, 23 sq = 0.23
Percentage: write the number of squares in front of the % sign. (23%)

1



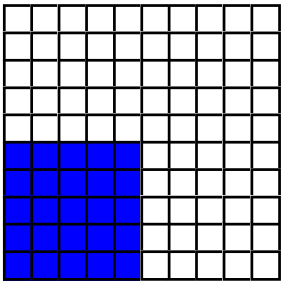
Shaded:
= ____
= ____ %
Unshaded:
= ____
= ____ %

2



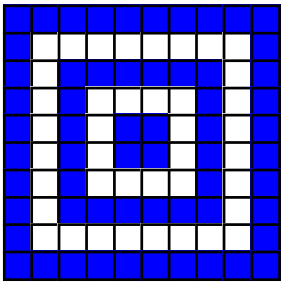
Shaded:
= ____
= ____ %
Unshaded:
= ____
= ____ %

3



Shaded:
= ____
= ____ %
Unshaded:
= ____
= ____ %

4



Shaded:
= ____
= ____ %
Unshaded:
= ____
= ____ %

Convert these decimals to percentages. Multiply by 100.

0.14 = 14% (2 places right)

5	0.04	=	%
6	0.28	=	
7	0.135	=	
8	0.217	=	
9	3.54	=	
10	0.4	=	
11	2	=	
12	1.22	=	
13	8.09	=	
14	3.064	=	
15	2.077	=	
16	4.002	=	
17	1.01	=	
18	0.006	=	
19	7.309	=	
20	10	=	
21	0.407	=	
22	2.6004	=	
23	0.0801	=	
24	0.1001	=	
25	0.3509	=	
26	3.05	=	
27	0.099	=	
28	5.0002	=	
29	1.7071	=	

Now change these percentages to decimals, by dividing by 100

31.8% = 0.318 (2 places left)

30	45%	=
31	11%	=
32	1%	=
33	10%	=
34	120%	=
35	430%	=
36	5.7%	=
37	60.9%	=
38	0.4%	=
39	12.7%	=
40	463%	=
41	800.6%	=
42	100%	=
43	40.7%	=
44	110%	=
45	0.5%	=
46	20.75%	=
47	10.03%	=
48	9.1%	=
49	80.6%	=
50	0.02%	=
51	309.1%	=
52	22.6%	=
53	0%	=
54	1.001%	=

Decimals and Percentages

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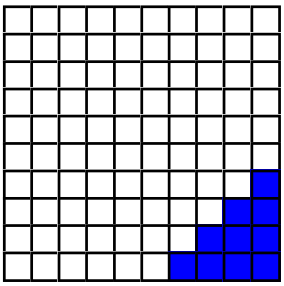
Write the both the shaded and unshaded part of the hundredths grid as a decimal and a percentage.

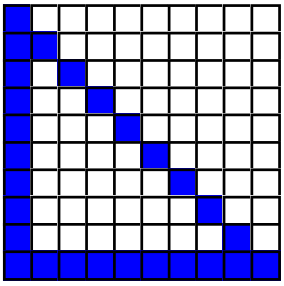
Decimal: write the number of squares behind decimal point ie, 23 sq = 0.23
Percentage: write the number of squares in front of the % sign. (23%)

1



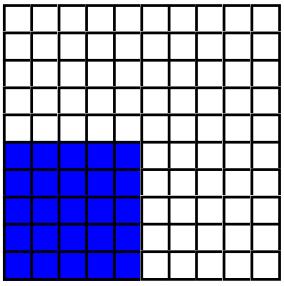
Shaded: = 0.1 = 10 %
Unshaded: = 0.9 = 90 %

2



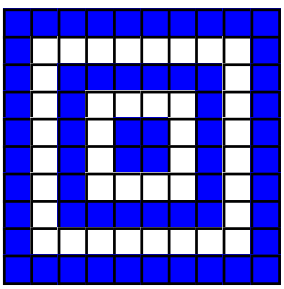
Shaded: = 0.27 = 27 %
Unshaded: = 0.73 = 73 %

3



Shaded: = 0.25 = 25 %
Unshaded: = 0.75 = 75 %

4



Shaded: = 0.6 = 60 %
Unshaded: = 0.4 = 40 %

Convert these decimals to percentages. Multiply by 100.

0.14 = 14% (2 places right)

5	0.04	=	4 %
6	0.28	=	28%
7	0.135	=	13.5%
8	0.217	=	21.7%
9	3.54	=	354%
10	0.4	=	40%
11	2	=	200%
12	1.22	=	122%
13	8.09	=	809%
14	3.064	=	306.4%
15	2.077	=	207.7%
16	4.002	=	400.2%
17	1.01	=	101%
18	0.006	=	0.6%
19	7.309	=	730.9%
20	10	=	1 000%
21	0.407	=	40.7%
22	2.6004	=	260.04%
23	0.0801	=	8.01%
24	0.1001	=	10.01%
25	0.3509	=	35.09%
26	3.05	=	305%
27	0.099	=	9.9%
28	5.0002	=	500.02%
29	1.7071	=	170.71%

Now change these percentages to decimals, by dividing by 100

31.8% = 0.318 (2 places left)

30	45%	=	0.45
31	11%	=	0.11
32	1%	=	0.01
33	10%	=	0.1
34	120%	=	1.2
35	430%	=	4.3
36	5.7%	=	0.057
37	60.9%	=	0.609
38	0.4%	=	0.004
39	12.7%	=	0.127
40	463%	=	4.63
41	800.6%	=	8.006
42	100%	=	1
43	40.7%	=	0.407
44	110%	=	1.1
45	0.5%	=	0.005
46	20.75%	=	0.2075
47	10.03%	=	0.1003
48	9.1%	=	0.091
49	80.6%	=	0.806
50	0.02%	=	0.0002
51	309.1%	=	3.091
52	22.6%	=	0.226
53	0%	=	0
54	1.001%	=	0.01001

Further Fractions to Percentages

Changing fractions to percentages with denominators that are factors of 100 has been covered, but what of the fractions with denominators that aren't factors of 100, this sheet deals with these.

Column 1 deals with the first step in changing a fraction to a percentage, you should look at the fraction and see if you can express it with a denominator of 100. All the fractions in this column can be, and the process is:

- Look at the fraction and change it to have a denominator that is a factor of 100 (2, 4, 5, 10, 20, 25, 50)
- Rewrite the equivalent fraction with the new denominator
- Then write the equivalent fraction with a denominator of 100
- The percentage is the numerator (top number).

An example is at the top of the column. Note that these fractions have been carefully selected, this process only works sometimes, but it is an important method to learn.

Column 2 outlines the method for all other fractions. Using division, change to a decimal, then to a percentage by moving the decimal point 2 places to the right. The method is identical to converting fractions to decimals with the decimal point shifting being the only difference. The answers are to be percentages to 1 d.p. so that means you require a 4 d.p. answer. Why 4? Because the 4th decimal digit is needed for rounding, if the 4th decimal digit is 5 or above then you round up. In the example at the top of the column the 4th d.p. digit is a 0 so there is no rounding.

Column 3 has long division problems, again the percentage is required to be to 1 d.p. so you need a 4 d.p. answer then round up or down as required. An example is shown, in the example the answer is rounded up due to the 4th d.p. digit being a 5.

Further Fractions to Percentages

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Use you knowledge of the factors of 100 to write these fractions as decimals

Example

$\frac{9}{15} = \frac{3}{5} \overset{\times 20}{=} \frac{60}{100} \overset{\times 100}{=} 60\ \%$ (÷ 3)

1 $\frac{6}{8} = \frac{\square}{4} = \frac{\square}{100} = \_\_\_\_\ \%$

2 $\frac{3}{12} = \frac{\square}{4} = \frac{\square}{100} = \_\_\_\_\ \%$

3 $\frac{32}{40} = \frac{\square}{20} = \frac{\square}{100} = \_\_\_\_\ \%$

4 $\frac{12}{16} = \frac{\square}{4} = \frac{\square}{100} = \_\_\_\_\ \%$

5 $\frac{36}{80} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

6 $\frac{24}{120} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

7 $\frac{39}{60} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

8 $\frac{18}{24} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

9 $\frac{33}{110} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

10 $\frac{24}{30} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

11 $\frac{66}{150} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

12 $\frac{30}{75} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

13 $\frac{27}{45} = \frac{\square}{\square} = \frac{\square}{100} = \_\_\_\_\ \%$

Convert these fractions to decimals, then to percentages (to 1 d.p.)

Example

$\frac{5}{8} = 8\overline{)5.0000}$ (add zeros) $= 0.6250$ (change to decimal)

$= 62.5\%$ (multiply by 100 move • 2 places)

14 $\frac{3}{8} = 8\overline{)3}$

$= \_\_\_\_$

15 $\frac{4}{9} = 9\overline{)4}$

$= \_\_\_\_$

16 $\frac{5}{6} = 6\overline{)5}$

$= \_\_\_\_$

17 $\frac{2}{7} = 7\overline{)2}$

$= \_\_\_\_$

18 $\frac{1}{3} = 3\overline{)1}$

$= \_\_\_\_$

19 $\frac{7}{8} = 8\overline{)7}$

$= \_\_\_\_$

20 $\frac{1}{9} = 9\overline{)1}$

$= \_\_\_\_$

21 $\frac{5}{9} = 9\overline{)5}$

$= \_\_\_\_$

22 $\frac{1}{6} = 6\overline{)1}$

$= \_\_\_\_$

These fractions to percentages will require long division (1 d.p.)

Example

$\frac{5}{16} = 31.3\%$

$16\overline{)5.0000} = 0.3125$

48
20
16
40
32
80
80
0

23 $\frac{2}{15} = \_\_\_\_$

$15\overline{)2}$

24 $\frac{4}{17} = \_\_\_\_$

$17\overline{)4}$

25 $\frac{9}{11} = \_\_\_\_$

$11\overline{)9}$

Further Fractions to Percentages

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Use you knowledge of the factors of 100 to write these fractions as decimals

Example

$\frac{9}{15} \overset{\div 3}{=} \frac{3}{5} \overset{\times 20}{=} \frac{60}{100} \overset{\times 100}{=} 60\ \%$

1 $\frac{6}{8} = \frac{3}{4} = \frac{75}{100} = 75\ \%$

2 $\frac{3}{12} = \frac{1}{4} = \frac{25}{100} = 25\ \%$

3 $\frac{32}{40} = \frac{16}{20} = \frac{80}{100} = 80\ \%$

4 $\frac{12}{16} = \frac{3}{4} = \frac{75}{100} = 75\ \%$

5 $\frac{36}{80} = \frac{9}{20} = \frac{45}{100} = 45\ \%$

6 $\frac{24}{120} = \frac{4}{20} = \frac{20}{100} = 20\ \%$

7 $\frac{39}{60} = \frac{13}{20} = \frac{65}{100} = 65\ \%$

8 $\frac{18}{24} = \frac{3}{4} = \frac{75}{100} = 75\ \%$

9 $\frac{33}{110} = \frac{3}{10} = \frac{30}{100} = 30\ \%$

10 $\frac{24}{30} = \frac{4}{5} = \frac{80}{100} = 80\ \%$

11 $\frac{66}{150} = \frac{22}{50} = \frac{44}{100} = 44\ \%$

12 $\frac{30}{75} = \frac{2}{5} = \frac{40}{100} = 40\ \%$

13 $\frac{27}{45} = \frac{3}{5} = \frac{60}{100} = 60\ \%$

Convert these fractions to decimals, then to percentages (to 1 d.p.)

Example

$\frac{5}{8} = 8\overline{)5.0000}$ (24) $= 0.6250$ ← change to decimal ← add zeros

$= 62.5\%$ ← multiply by 100 move • 2 places

14 $\frac{3}{8} = 8\overline{)3.0000}$ (64) $= 0.3750 = 37.5\%$

15 $\frac{4}{9} = 9\overline{)4.0000}$ (444) $= 0.4444 = 44.4\%$

16 $\frac{5}{6} = 6\overline{)5.0000}$ (222) $= 0.8333 = 83.3\%$

17 $\frac{2}{7} = 7\overline{)2.0000}$ (645) $= 0.2857 = 28.6\%$

18 $\frac{1}{3} = 3\overline{)1.0000}$ (111) $= 0.3333 = 33.3\%$

19 $\frac{7}{8} = 8\overline{)7.0000}$ (64) $= 0.8750 = 87.5\%$

20 $\frac{1}{9} = 9\overline{)1.0000}$ (111) $= 0.1111 = 11.1\%$

21 $\frac{5}{9} = 9\overline{)5.0000}$ (555) $= 0.5555 = 55.6\%$

22 $\frac{1}{6} = 6\overline{)1.0000}$ (444) $= 0.1666 = 16.7\%$

These fractions to percentages will require long division (1 d.p.)

Example

$\frac{5}{16} = 31.3\%$

$16\overline{)5.0000} = 0.3125$

48, 20, 16, 40, 32, 80, 80, 0

23 $\frac{2}{15} = 13.3\%$

$15\overline{)2.0000} = 0.1333$

15, 50, 45, 50, 45, 50, 45, 5

24 $\frac{4}{17} = 23.5\ \%$

$17\overline{)4.0000} = 0.2352$

34, 60, 51, 90, 85, 50, 34, 16

25 $\frac{9}{11} = 81.8\%$

$11\overline{)9.0000} = 0.8181$

88, 20, 11, 90, 88, 20, 11, 9

Fractions - Decimals - Percentages

This is a combination of converting between percentages, fractions and decimals. It is assumed that earlier sheets have been completed before this sheet is attempted.

Column 1 asks you to:

- Convert a decimal to a %. To answer these multiply by 100 or move the decimal point 2 places to the right.
- Converting a % to a decimal is the opposite, divide by 100 this time so move the decimal point 2 places to the left.
- Converting fractions to percentages requires multiplying by 100. This will involve 2 steps. First rewrite the fraction (form an equivalent fraction) with a denominator of 100. The % will be the numerator (top number) of the fraction. Rewrite it with a % sign.
- Converting a % to a fraction is the reverse. Write the % over 100 (dividing by 100) then simplify the fraction. Expect improper fraction answers at times.

Column 2 is the same as Column 1 except it is in table form and is jumbled. You are given either a decimal, percentage or fraction and you have to find the other two. An example is at the top of the column.

Fractions - Decimals - Percentages

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Express these decimals as percentages, multiply by 100, so move the decimal point 2 places right.

1 0.34 ____ 2 1.24 ____

3 0.05 ____ 4 0.4 ____

5 7.09 ____ 6 0.091 ____

7 0.002 ____ 8 2 ____

9 0.708 ____ 10 20.06 ____

Now the reverse, change these percentages to decimals. Move • 2 places to the left.

11 86% ____ 12 17% ____

13 9% ____ 14 44.6% ____

15 110% ____ 16 0.5% ____

17 30.02% ____ 18 400.5% ____

19 300% ____ 20 80.3% ____

Express these fractions as percentages, change denominator to 100, numerator = %

21 $\frac{7}{10} = \frac{\square}{100}$ ____ 22 $\frac{1}{4} = \frac{\square}{100}$ ____

23 $\frac{3}{5} = \frac{\square}{100}$ ____ 24 $\frac{9}{20} = \frac{\square}{100}$ ____

25 $\frac{150}{200} = \frac{\square}{100}$ ____ 26 $\frac{18}{25} = \frac{\square}{100}$ ____

27 $\frac{110}{500} = \frac{\square}{100}$ ____ 28 $\frac{11}{5} = \frac{\square}{100}$ ____

Now change these percentages to fractions

29 60% $\frac{\square}{100} = \frac{\square}{\square}$ 30 15% $\frac{\square}{100} = \frac{\square}{\square}$

31 350% $\frac{\square}{100} = \frac{\square}{\square}$ 32 32% $\frac{\square}{100} = \frac{\square}{\square}$

33 ½% $\frac{\square}{100} = \frac{\square}{\square}$ 34 120% $\frac{\square}{100} = \frac{\square}{\square}$

35 104% $\frac{\square}{100} = \frac{\square}{\square}$ 36 42% $\frac{\square}{100} = \frac{\square}{\square}$

Complete the table below filling all the missing spaces, so that the fraction, decimal and percentage are equivalent.

Example

32 % → 0.32 → $\frac{32}{100} = \frac{8}{25}$

	FRACTION	DECIMAL	PERCENTAGE
37	$\frac{\square}{100} = \frac{\square}{\square}$	____	10 %
38	$\frac{\square}{100} = \frac{\square}{\square}$	0.5	%
39	$\frac{\square}{100} = \frac{\square}{\square}$	0.8	%
40	$\frac{\square}{100} = \frac{\square}{\square}$	____	75 %
41	$1\frac{1}{2} = \frac{\square}{100}$	____	%
42	$\frac{\square}{100} = \frac{\square}{\square}$	0.95	%
43	$\frac{9}{50} = \frac{\square}{100}$	____	%
44	$\frac{\square}{100} = \frac{\square}{\square}$	____	120 %
45	$\frac{\square}{100} = \frac{\square}{\square}$	____	44 %
46	$\frac{18}{25} = \frac{\square}{100}$	____	%
47	$\frac{\square}{100} = \frac{\square}{\square}$	2.5	%
48	$\frac{\square}{100} = \frac{\square}{\square}$	0.08	%
49	$\frac{\square}{100} = \frac{\square}{\square}$	____	65 %

Fractions - Decimals - Percentages

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Express these decimals as percentages, multiply by 100, so move the decimal point 2 places right.

1	0.34	34%	2	1.24	124%
3	0.05	5%	4	0.4	40%
5	7.09	709%	6	0.091	9.1%
7	0.002	0.2%	8	2	200%
9	0.708	70.8%	10	20.06	2 006%

Now the reverse, change these percentages to decimals. Move • 2 places to the left.

11	86%	0.86	12	17%	0.17
13	9%	0.09	14	44.6%	0.446
15	110%	1.1	16	0.5%	0.005
17	30.02%	0.3002	18	400.5%	4.005
19	300%	3	20	80.3%	0.803

Express these fractions as percentages, change denominator to 100, numerator = %

21	$\frac{7}{10} = \frac{70}{100}$	70%	22	$\frac{1}{4} = \frac{25}{100}$	25%
23	$\frac{3}{5} = \frac{60}{100}$	60%	24	$\frac{9}{20} = \frac{45}{100}$	45%
25	$\frac{150}{200} = \frac{75}{100}$	75%	26	$\frac{18}{25} = \frac{72}{100}$	72%
27	$\frac{110}{500} = \frac{22}{100}$	22%	28	$\frac{11}{5} = \frac{220}{100}$	220%

Now change these percentages to fractions

29	60%	$\frac{60}{100} = \frac{3}{5}$	30	15%	$\frac{15}{100} = \frac{3}{20}$
31	350%	$\frac{350}{100} = \frac{7}{2}$	32	32%	$\frac{32}{100} = \frac{8}{25}$
33	½%	$\frac{½}{100} = \frac{1}{200}$	34	120%	$\frac{120}{100} = \frac{6}{5}$
35	104%	$\frac{104}{100} = \frac{26}{25}$	36	42%	$\frac{42}{100} = \frac{21}{50}$

Complete the table below filling all the missing spaces, so that the fraction, decimal and percentage are equivalent.

Example

32 % → 0.32 → $\frac{32}{100} = \frac{8}{25}$

	FRACTION	DECIMAL	PERCENTAGE
37	$\frac{10}{100} = \frac{1}{10}$	0.1	10 %
38	$\frac{50}{100} = \frac{1}{2}$	0.5	50 %
39	$\frac{80}{100} = \frac{4}{5}$	0.8	80 %
40	$\frac{75}{100} = \frac{3}{4}$	0.75	75 %
41	$1\frac{1}{2} = \frac{150}{100}$	1.5	150 %
42	$\frac{95}{100} = \frac{19}{20}$	0.95	95 %
43	$\frac{9}{50} = \frac{18}{100}$	0.18	18 %
44	$\frac{120}{100} = 1\frac{1}{5}$	1.2	120 %
45	$\frac{44}{100} = \frac{11}{25}$	0.44	44 %
46	$\frac{18}{25} = \frac{72}{100}$	0.72	72 %
47	$\frac{250}{100} = 2\frac{1}{2}$	2.5	250 %
48	$\frac{8}{100} = \frac{2}{25}$	0.08	8 %
49	$\frac{65}{100} = \frac{13}{20}$	0.65	65 %

Calculating Percentages - Calculator

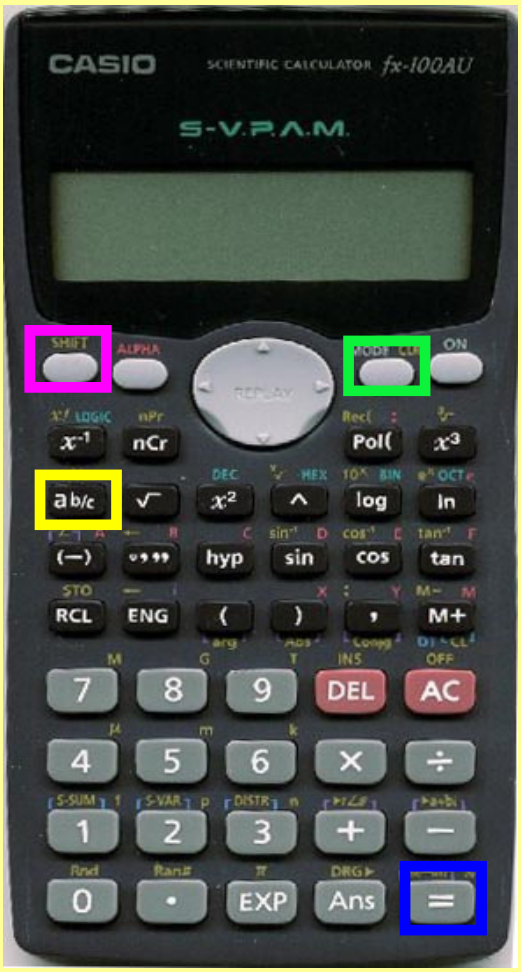
The method of calculating a percentage from a decimal or fraction is to multiply the decimal or fraction by 100. To change a percentage to a decimal or fraction you divide by 100.

A picture of a calculator is shown with below with keys highlighted. The methods used for each column are also listed on the foillowing pages. Where there is a alternative method it is shown, so you can select the method that makes the most sense to you.

Column 1 asks you to convert fractions to percentages to 2 decimal places, then to a specified number of decimal places (in brackets). Fix your calculator to the correct d.p. then multiply the fractions by 100 for the answer. Note that if writing a whole number, such as 45%, when asked to give to 2 d.p. then the answer is 45.00%. Even though the decimal portion adds nothing to the answer, it fulfils the question. If the question said 'show to 2 d.p. where appropriate (or necessary)' then you wouldn't have to show the decimal part.

Column 2 asks you to convert the decimals to percentages. Again multiply the question by 100.

Column 3 deals with changing percentages back to fractions or decimals. In this case divide by 100.



Column 2

Example of calculating a decimal percentage from a decimal

Convert 0.206 to a percentage

Using × 100

0 . 2 0 6 × 1 0 0 =

if you get 20.6 for the question above you are correct

Column 1

Example of calculating a decimal percentage from a fraction

Convert $^{7}/_{8}$ to a percentage

Using % key (not recommended)

7 ÷ 8 Shift %

Using × 100

7 ÷ 8 × 1 0 0 =

Using fraction key

7 $a^b/_c$ 8 × 1 0 0 = $a^b/_c$

if you get 87.5 (no fix) for the question above you are correct

Example of calculating a fraction percentage from a fraction

Convert $^{7}/_{8}$ to a percentage

Using fraction key

7 $a^b/_c$ 8 × 1 0 0 =

Using × 100

7 ÷ 8 × 1 0 0 = $a^b/_c$

if you get 87 ½ for the question above you are correct
note that this style of question isn't on the sheet

Example of calculating a decimal percentage from a mixed numeral

Convert $2\ ^{7}/_{8}$ to a percentage

Using fraction key

2 $a^b/_c$ 7 $a^b/_c$ 8 × 1 0 0 = $a^b/_c$

Using × 100

2 + 7 ÷ 8 = × 1 0 0 =

if you get 287.5 for the question above you are correct

Column 3

Example of converting a decimal percentage to a decimal

Convert 56.32% to a decimal

Using ÷ 100

5 6 . 3 2 ÷ 1 0 0 =

if you get 0.5632 (no fix) for the question above you are correct

Example of converting a fraction percentage to a decimal

Convert 56 ¼% to a decimal

Using the fraction key

5 6 $a^b/_c$ 1 $a^b/_c$ 4 ÷ 1 0 0 = $a^b/_c$

if you get 0.5625 (no fix) for the question above you are correct

Example of converting a decimal percentage to a fraction

Convert 56.25% to a fraction

Using the fraction key Note that the featured calculator can convert decimals to fractions, this may not be a feature on your calculator

5 6 . 2 5 ÷ 1 0 0 = $a^b/_c$

If your calculator has a fraction key but can't convert decimals to fractions then use this method (note you need some mental skills)

5 6 $a^b/_c$ 2 5 $a^b/_c$ 1 0 0 ÷ 1 0 0 =

if you get $^9/_{16}$ for the question above you are correct

Example of converting a fraction percentage to a fraction

Convert 56 ¼% to a fraction

Using the fraction key

5 6 $a^b/_c$ 1 $a^b/_c$ 4 ÷ 1 0 0 =

if you get $^9/_{16}$ for the question above you are correct

Calculating Percentages - Calculator

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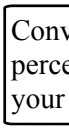
Convert these fractions to percentages (by × 100). Round answer to 2 d.p.

1	$^{2}/_{3}$	=	%
2	$^{5}/_{7}$	=	
3	$^{1}/_{8}$	=	
4	$^{1}/_{12}$	=	
5	$^{4}/_{13}$	=	
6	$1\ ^{56}/_{83}$	=	
7	$^{124}/_{171}$	=	
8	$^{85}/_{302}$	=	
9	$^{206}/_{118}$	=	
10	$4\ ^{357}/_{502}$	=	



Continue converting to percentages but round your answer to the d.p. in the brackets.

11	$^{51}/_{75}$ [1]	=	%
12	$^{131}/_{165}$ [2]	=	
13	$3\ ^{81}/_{97}$ [2]	=	
14	$^{1}/_{3}$ [3]	=	
15	$1\ ^{144}/_{758}$ [1]	=	
16	$^{41}/_{85}$ [3]	=	
17	$^{288}/_{650}$ [3]	=	
18	$^{65}/_{91}$ [1]	=	
19	$2\ ^{32}/_{55}$ [0]	=	
20	$^{193}/_{210}$ [2]	=	
21	$^{5}/_{8}$ [2]	=	
22	$^{249}/_{250}$ [0]	=	



Convert these decimals to percentages (× 100). Give your answer to 1 d.p.

23	0.15	=	%
24	0.439	=	
25	1.266	=	
26	0.78841	=	
27	0.0606	=	
28	3.0295	=	
29	0.0074	=	
30	0.4989	=	
31	1.0907	=	
32	0.9	=	



Round the percentage answer to the d.p. asked

33	0.021141 [3]	=	%
34	1.30055 [2]	=	
35	1.08 [1]	=	
36	0.29992 [2]	=	
37	4.00681 [2]	=	
38	0.0096 [1]	=	
39	11.0016 [1]	=	
40	0.701061 [3]	=	
41	0.349211 [0]	=	
42	0.59097 [2]	=	
43	0.087026 [3]	=	
44	0.389001 [0]	=	
45	1.999802 [1]	=	



Convert these percentages to decimals (÷ 100). Give your answer to 3 d.p.

46	95%	=	
47	102.7%	=	
48	0.13%	=	
49	11.63%	=	
50	225.46%	=	
51	0.3%	=	
52	½%	=	
53	33⅓%	=	
54	1.03%	=	
55	67 ¾%	=	



Write the percentages below as fractions

56	15%	=	$^{\square}/_{\square}$
57	68.4%	=	$^{\square}/_{\square}$
58	12.5%	=	$^{\square}/_{\square}$
59	75.4%	=	$^{\square}/_{\square}$
60	117.5%	=	$\square\ ^{\square}/_{\square}$
61	42.36%	=	$^{\square}/_{\square}$
62	½%	=	$^{\square}/_{\square}$
63	18.8%	=	$^{\square}/_{\square}$

Calculating Percentages - Calculator

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Convert these fractions to percentages (by × 100). Round answer to 2 d.p.

1	$^{2}/_{3}$	= 66.67 %
2	$^{5}/_{7}$	= 71.43%
3	$^{1}/_{8}$	= 12.50%
4	$^{1}/_{12}$	= 8.33%
5	$^{4}/_{13}$	= 30.77%
6	$1\ ^{56}/_{83}$	= 167.47%
7	$^{124}/_{171}$	= 72.51%
8	$^{85}/_{302}$	= 28.15%
9	$^{206}/_{118}$	= 174.58%
10	$4\ ^{357}/_{502}$	= 471.12%

Convert these decimals to percentages (× 100). Give your answer to 1 d.p.

23	0.15	= 15.0 %
24	0.439	= 43.9%
25	1.266	= 126.6%
26	0.78841	= 78.8%
27	0.0606	= 6.1%
28	3.0295	= 303.0%
29	0.0074	= 0.7%
30	0.4989	= 49.9%
31	1.0907	= 109.1%
32	0.9	= 90.0%

Convert these percentages to decimals (÷ 100). Give your answer to 3 d.p.

46	95%	= 0.950
47	102.7%	= 1.027
48	0.13%	= 0.001
49	11.63%	= 0.116
50	225.46%	= 2.255
51	0.3%	= 0.003
52	½%	= 0.005
53	33⅓%	= 0.333
54	1.03%	= 0.010
55	67 ¾%	= 0.678

Continue converting to percentages but round your answer to the d.p. in the brackets.

11	$^{51}/_{75}$ [1]	= 68.0 %
12	$^{131}/_{165}$ [2]	= 79.39%
13	$3\ ^{81}/_{97}$ [2]	= 383.51%
14	$^{1}/_{3}$ [3]	= 33.333%
15	$1\ ^{144}/_{758}$ [1]	= 119.0%
16	$^{41}/_{85}$ [3]	= 48.235%
17	$^{288}/_{650}$ [3]	= 44.308%
18	$^{65}/_{91}$ [1]	= 71.4%
19	$2\ ^{32}/_{55}$ [0]	= 258%
20	$^{193}/_{210}$ [2]	= 91.90%
21	$^{5}/_{8}$ [2]	= 62.50%
22	$^{249}/_{250}$ [0]	= 100%

Round the percentage answer to the d.p. asked

33	0.021141 [3]	= 2.114 %
34	1.30055 [2]	= 130.06%
35	1.08 [1]	= 108.0%
36	0.29992 [2]	= 29.99%
37	4.00681 [2]	= 400.68%
38	0.0096 [1]	= 1.0%
39	11.0016 [1]	= 1 100.2%
40	0.701061 [3]	= 70.106%
41	0.349211 [0]	= 35%
42	0.59097 [2]	= 59.10%
43	0.087026 [3]	= 8.703%
44	0.389001 [0]	= 39%
45	1.999802 [1]	= 200.0%

Write the percentages below as fractions

56	15%	= $^{3}/_{20}$
57	68.4%	= $^{171}/_{250}$
58	12.5%	= $^{1}/_{8}$
59	75.4%	= $^{377}/_{500}$
60	117.5%	= $1\ ^{7}/_{40}$
61	42.36%	= $^{1\,059}/_{2\,500}$
62	½%	= $^{1}/_{200}$
63	18.8%	= $^{47}/_{250}$

FREEFALL MATHEMATICS

ALTITUDE

VOLUME, CAPACITY & MASS

Centicube Volumes

Volume is a measure of the amount of space an object occupies. The units used in measuring volume are cubic units: mm^3, cm^3 and m^3. The solids on this sheet are all constructed from centicubes (each 1 cm^3). This allows the volume to be found by counting the cubes with the volume being the total number of cubes (in cm^3).

The first 2 columns ask you to count the cubes, you have to accept that the cubes that you can't see, due to cubes being in front, are still there. A cube must be on top of another cube to be above the first layer. When you write the volume you must include the units (cm^3).

Column 3 introduces the Volume formula : $V = Ah$. This applies to any shape with a uniform cross-section, (this means the blue area is the same throughout the height of the shape). Count the number of squares that make up the blue area (A) and multiply this by the number of red squares that make up the height (h), this will give you the volume in cm^3.

Don't let the word 'height' confuse you. You may feel that height must go up and down but the word height is used to describe the direction of the uniform cross-section. When you think about it if you turn the page sideways it will be up and down, so the answer will be the same.

Centicube Volumes

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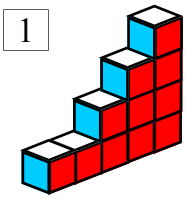
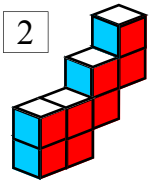
Calculate the volume of these solids built from identical cubes, assume 1 cube = 1cm^3.

1



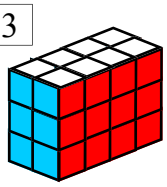
V = ____ cm^3

2



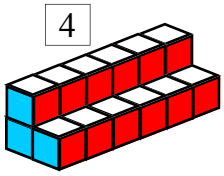
V = ____ cm^3

3



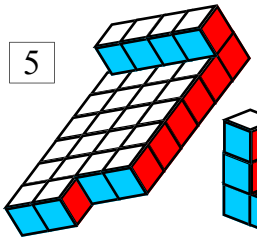
V = _____

4



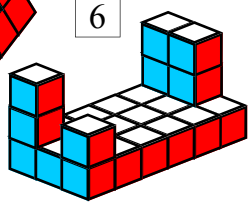
V = _____

5



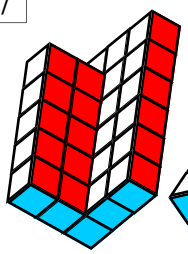
V = _____

6



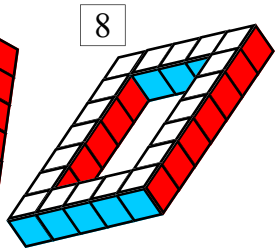
V = _____

7



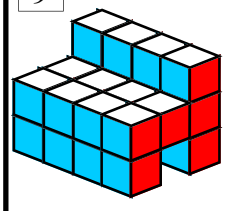
V = _____

8



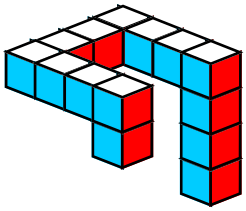
V = _____

9



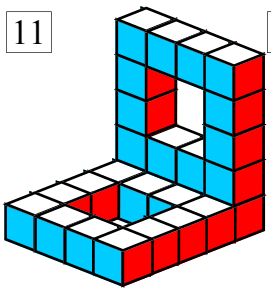
V = _____

10



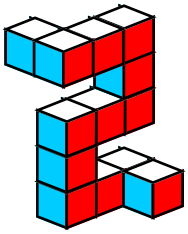
V = _____

11



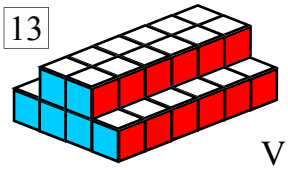
V = _____

12



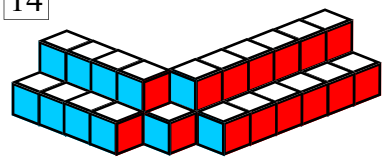
V = _____

13



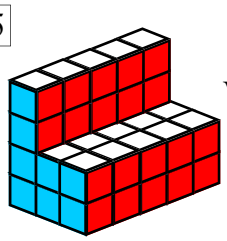
V = _____

14



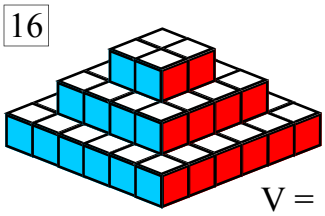
V = _____

15



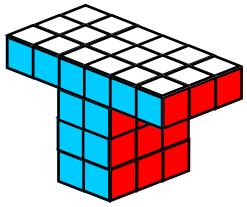
V = _____

16



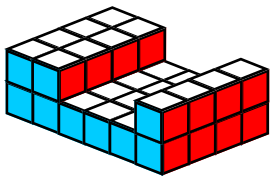
V = _____

17



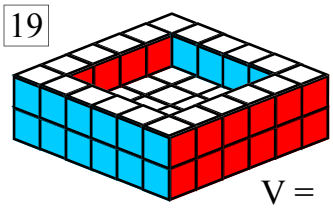
V = _____

18



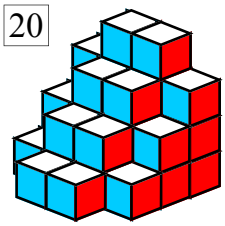
V = _____

19



V = _____

20



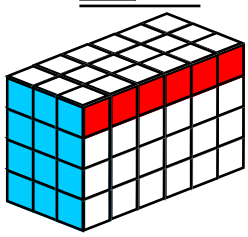
V = _____

Find the area of the front face then multiply it by the height (the distance the shape goes back into the page).

21 Face Area (A) = ____ cm^2

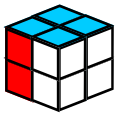
Shape height (h) = ____ cm

A × h = ____ cm^3



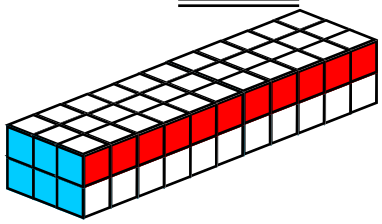
22 A = ____ cm^2 h = ____ cm

V = Ah = ____ cm^3



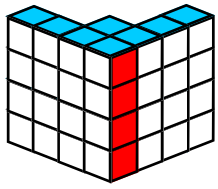
23 A = _____ h = _____

V = Ah = _____



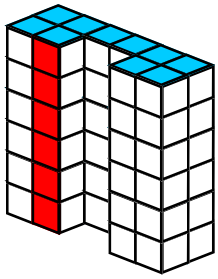
24 A = _____ h = _____

V = Ah = _____



25 A = _____ h = _____

V = Ah = _____



Centicube Volumes

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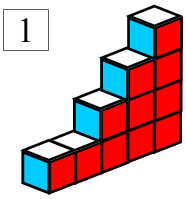
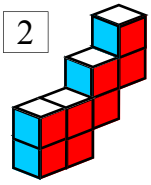
Calculate the volume of these solids built from identical cubes, assume 1 cube = 1cm^3.

1



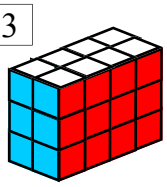
V = 11 cm^3

2



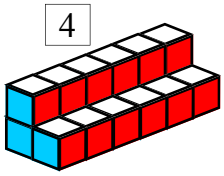
V = 8 cm^3

3



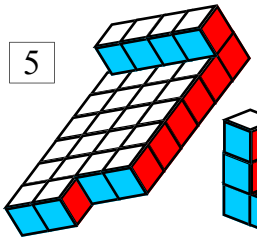
V = 24 cm^3

4



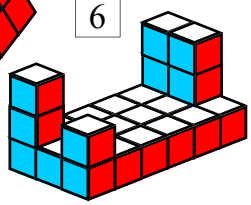
V = 18 cm^3

5



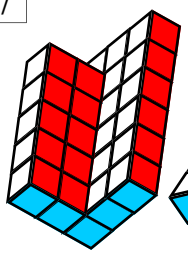
V = 30 cm^3

6



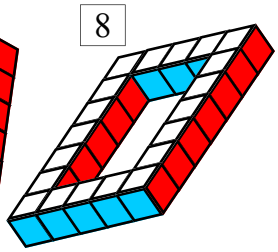
V = 25 cm^3

7



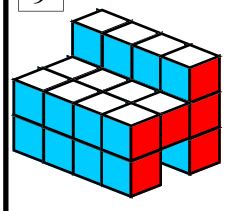
V = 28 cm^3

8



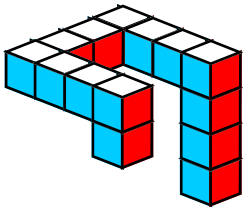
V = 20 cm^3

9



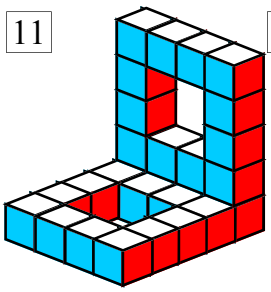
V = 24 cm^3

10



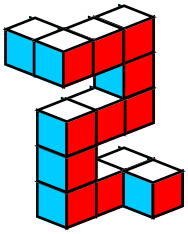
V = 14 cm^3

11



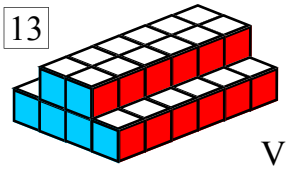
V = 28 cm^3

12



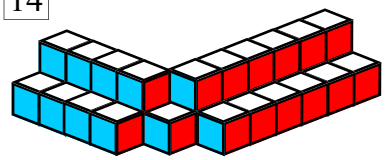
V = 13 cm^3

13



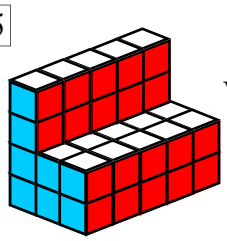
V = 36 cm^3

14



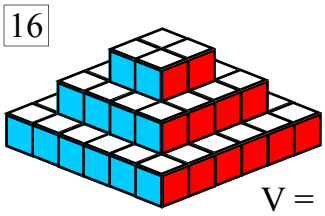
V = 31 cm^3

15



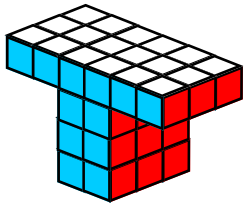
V = 40 cm^3

16



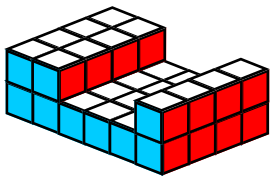
V = 56 cm^3

17



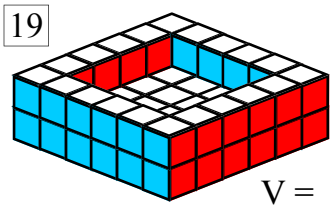
V = 36 cm^3

18



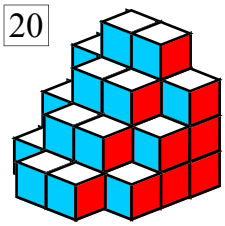
V = 36 cm^3

19



V = 56 cm^3

20



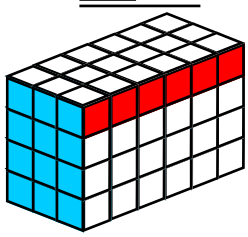
V = 32 cm^3

Find the area of the front face then multiply it by the height (the distance the shape goes back into the page).

21 Face Area (A) = 12 cm^2

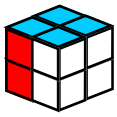
Shape height (h) = 6 cm

A × h = 72 cm^3



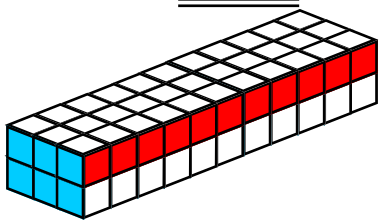
22 A = 4 cm^2 h = 2 cm

V = Ah = 8 cm^3



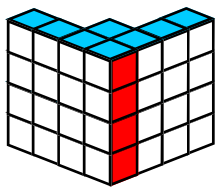
23 A = 6 cm^2 h = 11 cm

V = Ah = 66 cm^3



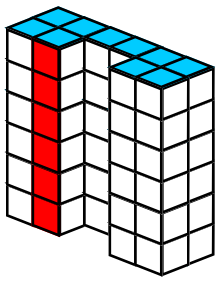
24 A = 8 cm^2 h = 4 cm

V = Ah = 32 cm^3



25 A = 10 cm^2 h = 6 cm

V = Ah = 60 cm^3



Volume, Area and Height

When a solid has a uniform cross section (same shape throughout the height of the solid) then the formula $V = Ah$ can be used to obtain the volume of the solid. The area will be given in square units (mm^2, cm^2, m^2) and the height of the solid will be given in mm, cm and m. Multiply the two together to get your answer. The answers will be in cubic units mm^3, cm^3, m^3.

The entire sheet uses the same method, this is : writing the equation ($V = Ah$), substituting the values (A and h), with an '=' sign before them and a '×' sign between them, then the answer with the cubic units after it.

Volume, Area and Height

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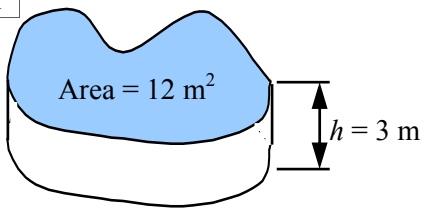
Multiply the area by the height to find the volume

1

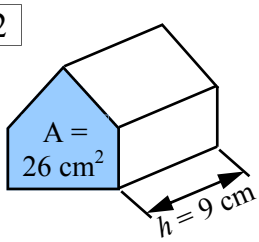


$V = Ah$

$= \quad \times$

$V = \quad m^3$

2



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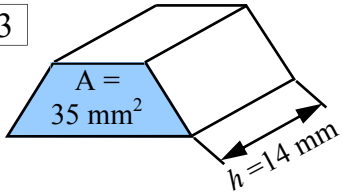

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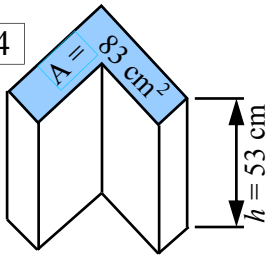

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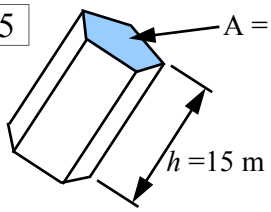

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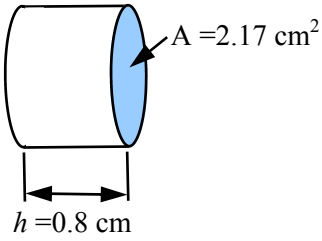

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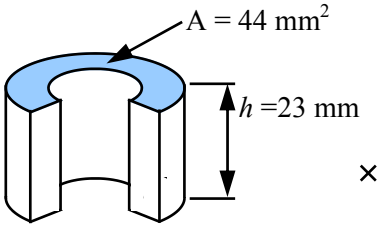

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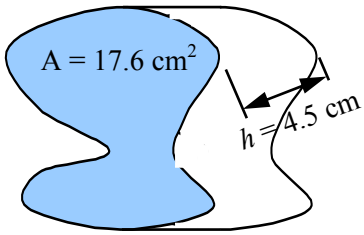

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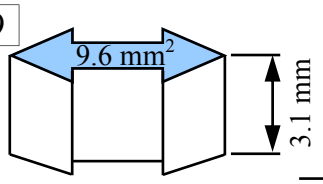

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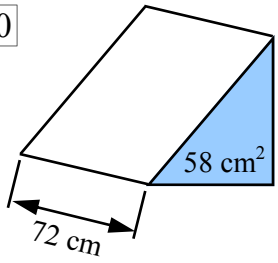

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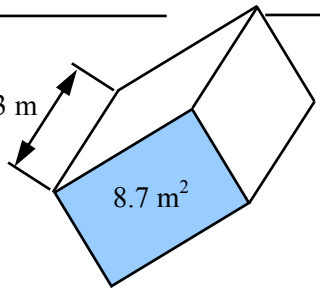
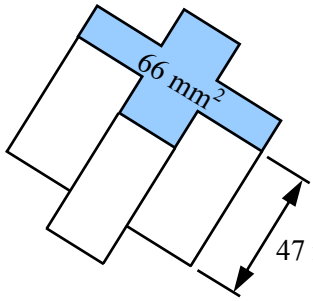


12



Volume, Area and Height

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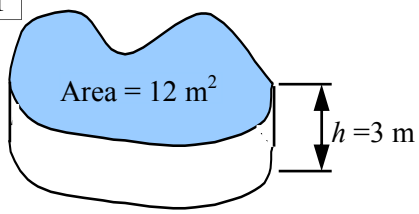
Multiply the area by the height to find the volume

1

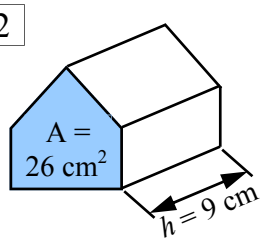


$V = Ah$

$= 12 \times 3$

$V = 36\ m^3$

2



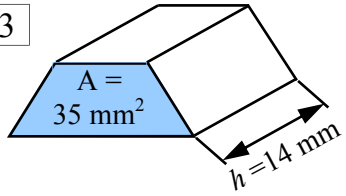
$V = Ah$

$= 26 \times 9$

$V = 234\ cm^3$

$$\begin{array}{r} {\scriptstyle 5}\\ 26 \\ \times\ 9 \\ \hline 234 \end{array}$$

3



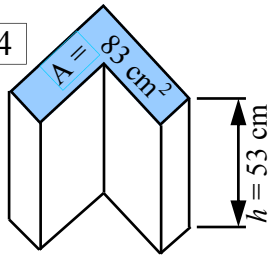
$V = Ah$

$= 35 \times 14$

$V = 490\ mm^3$

$$\begin{array}{r} 35 \\ \times\ 14 \\ \hline 140 \\ 350 \\ \hline 490 \end{array}$$

4



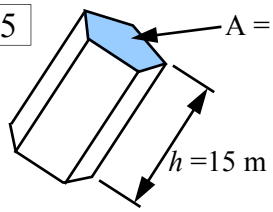
$V = Ah$

$= 83 \times 53$

$V = 4\,399\ cm^3$

$$\begin{array}{r} 83 \\ \times\ 53 \\ \hline 249 \\ 4\,150 \\ \hline 4\,399 \end{array}$$

5



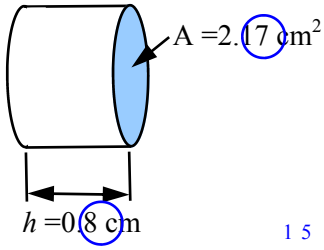
$V = Ah$

$= 17 \times 15$

$V = 255\ m^3$

$$\begin{array}{r} 17 \\ \times\ 15 \\ \hline {\scriptstyle 1}\\ 85 \\ 170 \\ \hline 255 \end{array}$$

6



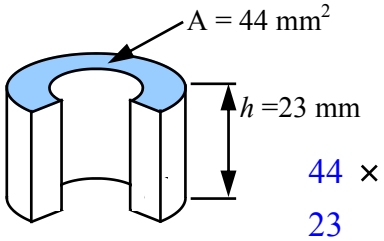
$V = Ah$

$= 2.17 \times 0.8$

$V = 1.736\ cm^3$

$$\begin{array}{r} {\scriptstyle 1\ 5}\\ 217 \\ \times\ 8 \\ \hline 1.736 \end{array}$$

7



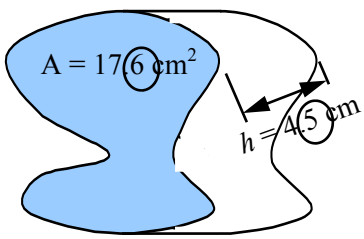
$V = Ah$

$= 44 \times 23$

$V = 1\,012\ mm^3$

$$\begin{array}{r} 44 \\ \times\ 23 \\ \hline {\scriptstyle 1}\\ 132 \\ 880 \\ \hline 1\,012 \end{array}$$

8



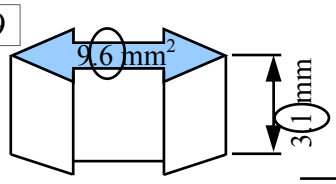
$V = Ah$

$= 17.6 \times 4.5$

$V = 79.2\ cm^3$

$$\begin{array}{r} {\scriptstyle 3\ 2}\\ 176 \\ \times\ 45 \\ \hline {\scriptstyle 1}\\ 880 \\ 7\,040 \\ \hline 79.20 \end{array}$$

9



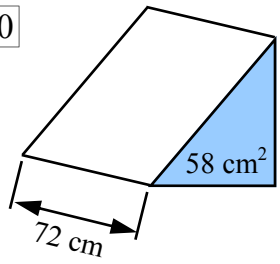
$V = Ah$

$= 9.6 \times 3.1$

$V = 29.76\ mm^3$

$$\begin{array}{r} {\scriptstyle 1}\\ 96 \\ \times\ 31 \\ \hline {\scriptstyle 1}\\ 96 \\ 2\,880 \\ \hline 2\,976 \end{array}$$

10



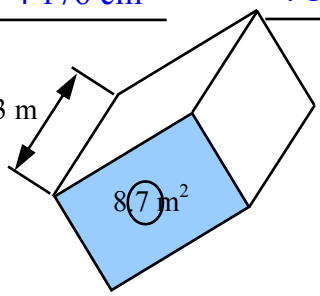
$V = Ah$

$= 58 \times 72$

$V = 4\,176\ cm^3$

$$\begin{array}{r} {\scriptstyle 1}\\ 72 \\ \times\ 58 \\ \hline {\scriptstyle 1}\\ 576 \\ 3\,600 \\ \hline 4\,176 \end{array}$$

11



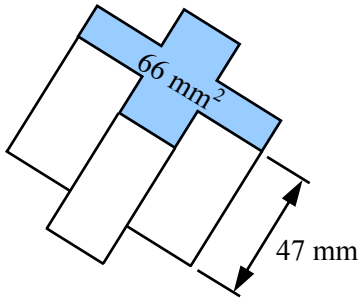
$V = Ah$

$= 8.7 \times 3$

$V = 26.1\ m^3$

$$\begin{array}{r} {\scriptstyle 2}\\ 87 \\ \times\ 3 \\ \hline 26.1 \end{array}$$

12



$V = Ah$

$= 66 \times 47$

$V = 3\,102\ mm^3$

$$\begin{array}{r} {\scriptstyle 2}\\ 66 \\ \times\ 47 \\ \hline {\scriptstyle 1\ 1}\\ 462 \\ 2\,640 \\ \hline 3\,102 \end{array}$$

Volume of Rectangular Prisms

Rectangular prisms are the most common form of packaging, from shipping containers to match boxes they are everywhere. To calculate their volume you multiply the 3 sides together, in any order. Because we have $V = Ah$ as the standard equation for volume with the 'area' part being a rectangle we can replace the A with $l \times b$. So we have $V = l \times b \times h$, or $V = lbh$.

This sheet uses the same method for the entire sheet. Using 3 lines of working:

- Write the equation $V = lbh$
- Substitute in the values (order isn't important), separated by × signs
- Strike out 2 numbers that you can multiply together mentally and write the total above the two numbers
- Use the working space to multiply the non-stroked number with the number above
- Write this answer in the 3rd line with 'V =' at the start and the cubic units at the end (either mm^3, cm^3 or m^3)

An example is at the top of the 1st column.

Volume of Rectangular Prisms

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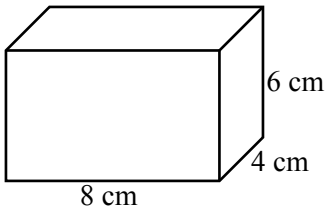
Use V = *lbh* to find the volume of these prisms, remember units!

Example



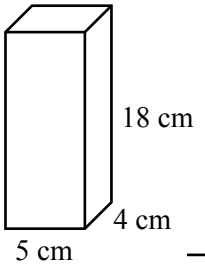
$V = lbh$

$= \not{8} \times \not{6} \times 4$ (48)

$V = 192\ cm^3$

48 × 4 = 192 (3)

1



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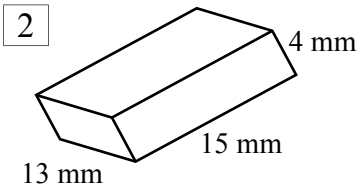

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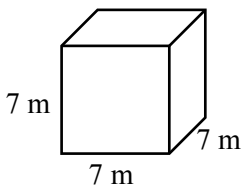

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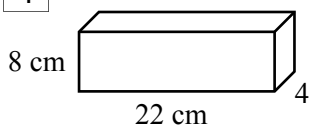

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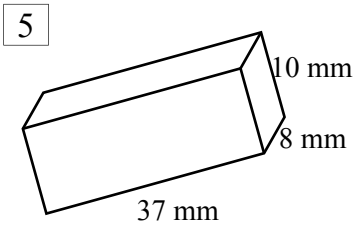

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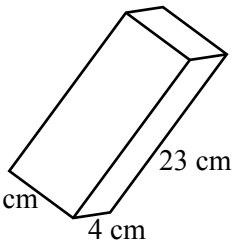

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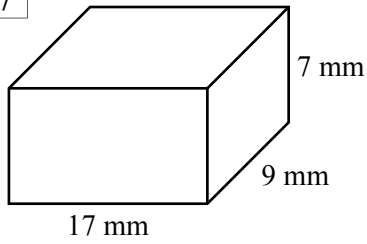
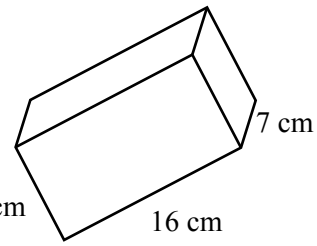


8



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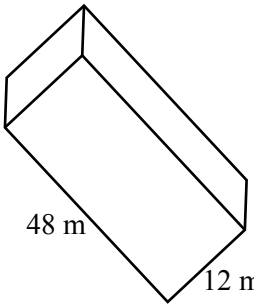

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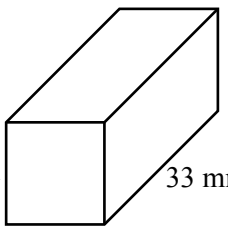

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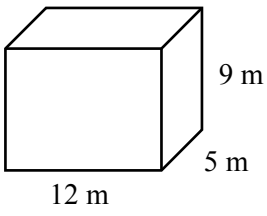

Volume of Rectangular Prisms

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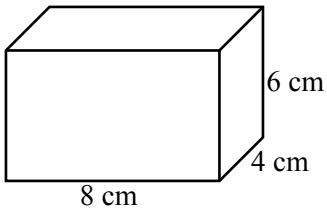
Use V = *lbh* to find the volume of these prisms, remember units!

Example



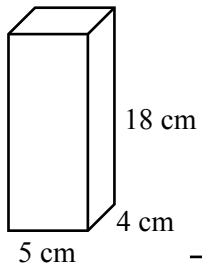
V = *lbh*

$= \cancel{8} \times \cancel{6}^{48} \times 4$

V = 192 cm^3

$\overset{3}{48} \times 4 = 192$

1



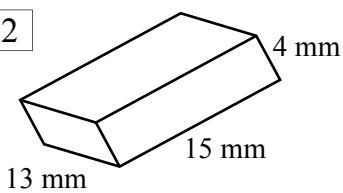
V = *lbh*

$= \cancel{5} \times \cancel{4}^{20} \times 18$

V = 360 cm^3

$\overset{1}{18} \times 20 = 360$

2



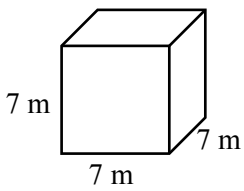
V = *lbh*

$= 13 \times \cancel{4} \times \cancel{15}^{60}$

V = 780 mm^3

$\overset{1}{13} \times 60 = 780$

3



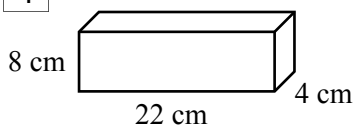
V = *lbh*

$= 7 \times \cancel{7} \times \cancel{7}^{49}$

V = 343 m^3

$\overset{6}{49} \times 7 = 343$

4



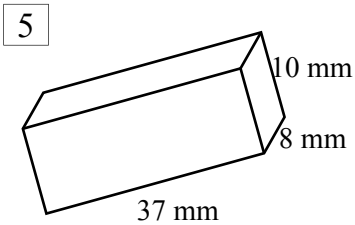
V = *lbh*

$= \cancel{8} \times \cancel{4}^{32} \times 22$

V = 704 cm^3

$32 \times 22 = \overset{1}{64}$, 640, 704

5



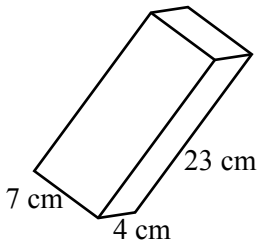
V = *lbh*

$= 37 \times \cancel{8} \times \cancel{10}^{80}$

V = 2960 mm^3

$\overset{5}{37} \times 80 = 2\,960$

6



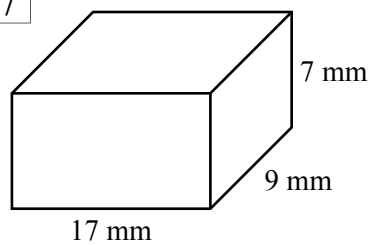
V = *lbh*

$= \cancel{7} \times \cancel{4}^{28} \times 23$

V = 644 cm^3

$28 \times 23 = \overset{1}{84}$, 560, 644

7



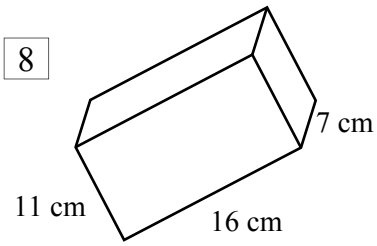
V = *lbh*

$= 17 \times \cancel{9} \times \cancel{7}^{63}$

V = 1 071 mm^3

$63 \times 17 = 441$, 630, $1\,071$

8



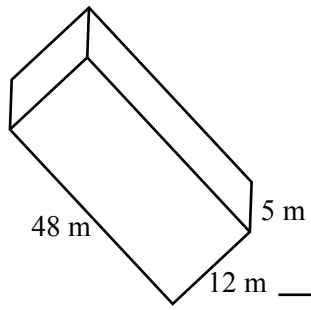
V = *lbh*

$= 16 \times \cancel{7} \times \cancel{11}^{77}$

V = 1 232 cm^3

$77 \times 16 = \overset{1}{462}$, 770, $1\,232$

9



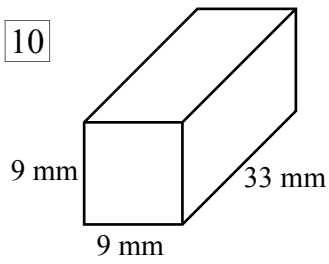
V = *lbh*

$= 48 \times \cancel{5} \times \cancel{12}^{60}$

V = 2 880 m^3

$\overset{4}{48} \times 60 = 2\,880$

10



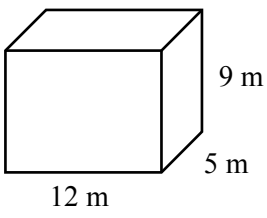
V = *lbh*

$= \cancel{9} \times \cancel{9}^{81} \times 33$

V = 2 673 mm^3

$81 \times 33 = 243$, $2\,430$, $2\,673$

11



V = *lbh*

$= 12 \times \cancel{5} \times \cancel{9}^{45}$

V = 540 m^3

$45 \times 12 = \overset{1}{90}$, 450, 540

Further Volumes of Prisms

The method of the sheet is to first calculate the area of the face (uniform cross section area) then multiply it by the height of the shape. Note that because this sheet deals with triangular prisms you must not confuse the h used in $A = \frac{1}{2}bh$ with the h used in $V = Ah$.

There are six lines of working as the example in column 1 shows. Calculate the area of the triangular face, then using the working space multiply the area with the shape's height to get its volume. Write 3 lines of working for the volume. Don't forget you will have square units (mm^2, cm^2, m^2) for the area and cubic units (mm^3, cm^3, m^3) for the volume.

The 3rd column involves composite shapes. You have to add the 2 areas to get the total area. So find the sum of the two areas (add them) and multiply by the height to get the volume.

There is an example at the top of column 1.

Further Volumes of Prisms

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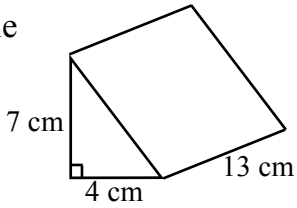
Find the uniform cross section area, use V=A*h* to find the volume

Example



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 7$

$A = 14\ cm^2$

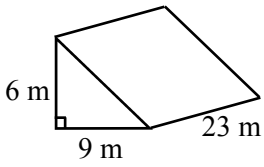
$V = Ah$

$= 14 \times 13$

$V = 182\ cm^3$

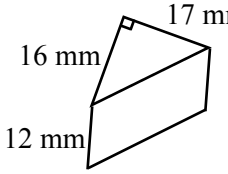
14 ×
13
42
140
182

1



×

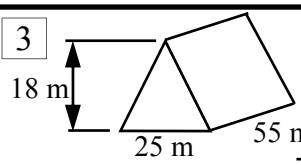
2



×

×

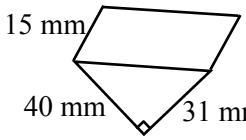
3



×

×

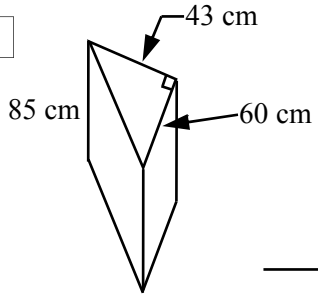
4



×

×

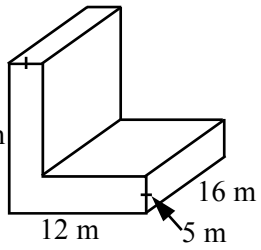
5



×

×

6



+

$A_1 = lb$

$A_2 = lb$

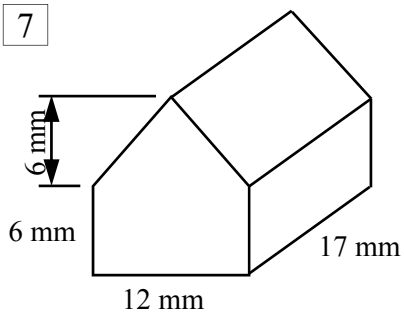
A = +

A = m^2

×

V = A*h*

7



+

×

Further Volumes of Prisms

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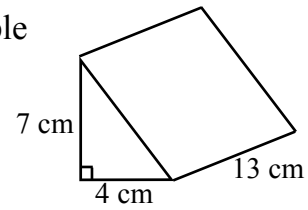
Find the uniform cross section area, use V=A*h* to find the volume

Example



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 7$

$A = 14\ cm^2$

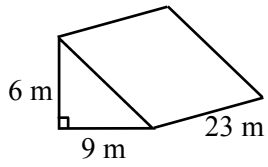
$V = Ah$

$= 14 \times 13$

$V = 182\ cm^3$

	14 ×
	13
	42
	140
	182

1



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 9$

$A = 27\ m^2$

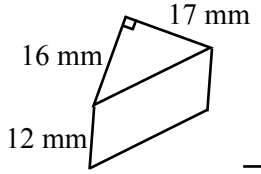
$V = Ah$

$= 27 \times 23$

$V = 621\ m^3$

	1
	27 ×
	23
	1
	81
	540
	621

2



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 16 \times 17$

$A = 136\ mm^2$

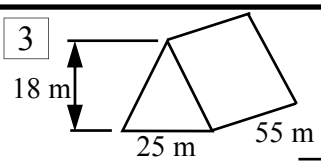
$V = Ah$

$= 136 \times 12$

$V = 1632\ mm^3$

	5
	17 ×
	8
	136
	136 ×
	12
	1
	272
	1 360
	1 632

3



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 25$

$A = 225\ cm^2$

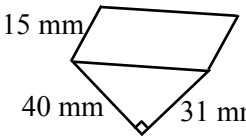
$V = Ah$

$= 225 \times 55$

$V = 12\ 375\ cm^3$

	4
	25 ×
	9
	225
	1 2
	225 ×
	55
	1 125
	11 250
	12 375

4



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 40 \times 31$

$A = 620\ mm^2$

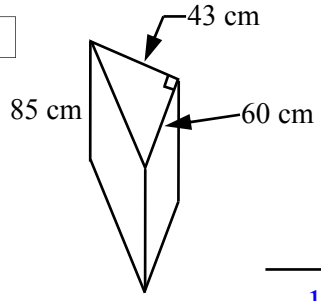
$V = Ah$

$= 620 \times 31$

$V = 19\ 220\ mm^3$

	31 ×
	20
	320
	620 ×
	31
	1
	620
	18 600
	19 220

5



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 60 \times 43$

$A = 1\ 290\ cm^2$

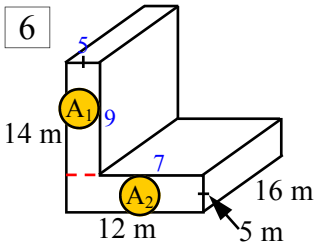
$V = Ah$

$= 1\ 290 \times 85$

$V = 109\ 650\ cm^3$

	43 ×
	30
	1 290
	2 7
	1 290 ×
	85
	6 450
	103 200
	109 650

6



$A_1 = lb$

$= 9 \times 5$

$A_1 = 45\ m^2$

$A_2 = lb$

$= 12 \times 5$

$A_2 = 60\ m^2$

$A = 45 + 60$

$A = 105\ m^2$

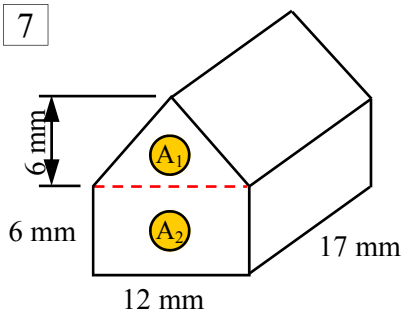
$V = Ah$

$= 105 \times 16$

$V = 1\ 680\ m^3$

	60 +
	45
	105
	105 ×
	16
	630
	1 050
	1 680

7



$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 6$

$A_1 = 36\ mm^2$

$A_2 = lb$

$= 12 \times 6$

$A_2 = 72\ mm^2$

$A = 36 + 72$

$A = 108\ mm^2$

$V = Ah$

$= 108 \times 17$

$V = 1\ 836\ cm^3$

	6
	72 +
	36
	108
	108 ×
	17
	1
	756
	1 080
	1 836

Converting between Volume and Capacity

When you calculate the volume within a solid it's measured in cubic units such as mm^3, cm^3, or m^3. These units are used usually to define empty space such as storage space, shipping containers or packaging, usually when they are to be filled with solid material. Capacity is measured in millilitres (mL), litres (L), kilolitres (kL) and Megalitres (ML). Each unit is 1 000 times larger than the unit before it. While capacity is usually associated with liquids it is also used with air. For example the space inside a car will have its capacity measured in litres.

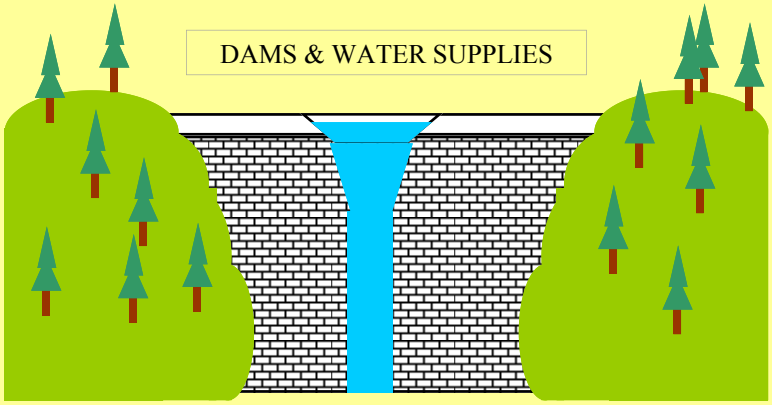
Column 1 starts with comparing mL and cm^3. They are the same, so a measurement in mL, say 5.72 mL is the same in cm^3, 5.72 cm^3. So rewrite the number and change to the other unit. From question 7 you are asked to convert cm^3 to L. As 1 cm^3 is 1 mL, and there are 1 000 mL in a L, then 1 cm^3 must be $^1/_{1000}$ of a L. So divide by 1 000 or move the decimal point 3 places to the left to get your answer. Remember that litres is written with an upper case L.

A common object where both capacity and volume are used is with car motors. The size of the engine could be given in c.c. (cubic centimetres) which is an accepted way of writing cm^3, or litres (L). For example a 1 800 c.c. engine could also be described as a 1.8 L engine.

Column 2 starts with the reverse converting from L to cm^3. This time you multiply by 1 000 or move the decimal point 3 places to the right. The last 6 questions involve the conversion between kL and m^3. Just as with mL and cm^3, they are identical so 1 kilolitre (1 000 L) = 1 cubic metre (m^3). Just rewrite the number and change the unit.

Column 3 asks you first to convert from m^3 to L. Remember that $1m^3$ = 1 kL which is 1 000 L. So multiply by 1 000 or move the decimal point 3 places to the right. The next section of the column reverses the process, converting L to m^3. Reverse the process by dividing by 1 000 or moving the decimal place 3 places to the left.

This sheet doesn't deal with Megalitres (ML) as they are rarely used. But there are times when their use is essential when dealing with large capacities, can you think of where ML would be used?



Converting between Volume and Capacity

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Change the units from mL to cm^3 or the reverse

Example

73 mL = 73 cm^3

1. 350 mL = _____
2. 105 mL = _____
3. 56.3 cm^3 = _____
4. 850 mL = _____
5. 2.43 cm^3 = _____
6. 600 cm^3 = _____

Convert these volumes in cm^3 (c.c.) to litres

850 cm^3 = 0.85 L (3 places left)

7. 1 000 cm^3 = _____
8. 800 cm^3 = _____
9. 450 cm^3 = _____
10. 2 100 cm^3 = _____
11. 1 700 cm^3 = _____
12. 750 cm^3 = _____
13. 1 250 cm^3 = _____
14. 5 000 cm^3 = _____
15. 85 cm^3 = _____
16. 4 302 cm^3 = _____
17. 120.4 cm^3 = _____
18. 333 cm^3 = _____
19. 2 006 cm^3 = _____
20. 500 cm^3 = _____
21. 25.1 cm^3 = _____
22. 1 007 cm^3 = _____
23. 10 cm^3 = _____

Now reverse, change these from L to cm^3

4.3 L = 4 300 cm^3 (3 places right)

24. 2 L = _____
25. 0.5 L = _____
26. 3.1 L = _____
27. 0.2 L = _____
28. 3.5 L = _____
29. 1.75 L = _____
30. 0.04 L = _____
31. 4.03 L = _____
32. 0.05 L = _____
33. 0.01 L = _____
34. 0.2063 L = _____
35. 2.02 L = _____
36. 0.25 L = _____
37. 4.06 L = _____
38. 0.96 L = _____
39. 3.007 L = _____
40. 4.55 L = _____

Change the units from kL to m^3 or the reverse

Example

2.9 m^3 = 2.9 kL

41. 7 m^3 = _____
42. 3.84 kL = _____
43. 43 kL = _____
44. 0.3 m^3 = _____
45. 204 m^3 = _____
46. 7.06 kL = _____

Change these volumes from m^3 to litres.

2.53 m^3 = 2 530 L (3 places right)

47. 7.2 m^3 = _____
48. 0.5 m^3 = _____
49. 3.7 m^3 = _____
50. 0.25 m^3 = _____
51. 16.403 m^3 = _____
52. 0.02 m^3 = _____
53. 5.62 m^3 = _____
54. 1.01 m^3 = _____
55. 7.9035 m^3 = _____
56. 18.006 m^3 = _____
57. 105.12 m^3 = _____

Now change these from litres to cubic metres

2 650 L = 2.65 m^3 (3 places left)

58. 2 000 L = _____
59. 1 700 L = _____
60. 800 L = _____
61. 3 075 L = _____
62. 175 L = _____
63. 422 L = _____
64. 9.06 L = _____
65. 45.6 L = _____
66. 115 L = _____
67. 2 L = _____
68. 500.5 L = _____
69. 3 000.6 L = _____

Converting between Volume and Capacity

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Change the units from mL to cm^3 or the reverse

Example

73 mL = 73 cm^3

1. 350 mL = 350 cm^3
2. 105 mL = 105 cm^3
3. 56.3 cm^3 = 56.3 mL
4. 850 mL = 850 cm^3
5. 2.43 cm^3 = 2.43 mL
6. 600 cm^3 = 600 mL

Convert these volumes in cm^3 (c.c.) to litres

850 cm^3 = 0.85 L (3 places left)

7. 1 000 cm^3 = 1 L
8. 800 cm^3 = 0.8 L
9. 450 cm^3 = 0.45 L
10. 2 100 cm^3 = 2.1 L
11. 1 700 cm^3 = 1.7 L
12. 750 cm^3 = 0.75 L
13. 1 250 cm^3 = 1.25 L
14. 5 000 cm^3 = 5 L
15. 85 cm^3 = 0.085 L
16. 4 302 cm^3 = 4.302 L
17. 120.4 cm^3 = 0.1204 L
18. 333 cm^3 = 0.333 L
19. 2 006 cm^3 = 2.006 L
20. 500 cm^3 = 0.5 L
21. 25.1 cm^3 = 0.0251 L
22. 1 007 cm^3 = 1.007 L
23. 10 cm^3 = 0.01 L

Now reverse, change these from L to cm^3

4.3 L = 4 300 cm^3 (3 places right)

24. 2 L = 2 000 cm^3
25. 0.5 L = 500 cm^3
26. 3.1 L = 3 100 cm^3
27. 0.2 L = 200 cm^3
28. 3.5 L = 3 500 cm^3
29. 1.75 L = 1 750 cm^3
30. 0.04 L = 40 cm^3
31. 4.03 L = 4 030 cm^3
32. 0.05 L = 50 cm^3
33. 0.01 L = 10 cm^3
34. 0.2063 L = 206.3 cm^3
35. 2.02 L = 2 020 cm^3
36. 0.25 L = 250 cm^3
37. 4.06 L = 4 060 cm^3
38. 0.96 L = 960 cm^3
39. 3.007 L = 3 007 cm^3
40. 4.55 L = 4 550 cm^3

Change the units from kL to m^3 or the reverse

Example

2.9 m^3 = 2.9 kL

41. 7 m^3 = 7 kL
42. 3.84 kL = 3.84 m^3
43. 43 kL = 43 m^3
44. 0.3 m^3 = 0.3 kL
45. 204 m^3 = 204 kL
46. 7.06 kL = 7.06 m^3

Change these volumes from m^3 to litres.

2.53 m^3 = 2 530 L (3 places right)

47. 7.2 m^3 = 7 200 L
48. 0.5 m^3 = 500 L
49. 3.7 m^3 = 3 700 L
50. 0.25 m^3 = 250 L
51. 16.403 m^3 = 16 403 L
52. 0.02 m^3 = 20 L
53. 5.62 m^3 = 5 620 L
54. 1.01 m^3 = 1 010 L
55. 7.9035 m^3 = 7 903.5 L
56. 18.006 m^3 = 18 006 L
57. 105.12 m^3 = 105 120 L

Now change these from litres to cubic metres

2 650 L = 2.65 m^3 (3 places left)

58. 2 000 L = 2 m^3
59. 1 700 L = 1.7 m^3
60. 800 L = 0.8 m^3
61. 3 075 L = 3.075 m^3
62. 175 L = 0.175 m^3
63. 422 L = 0.422 m^3
64. 9.06 L = 9 060 m^3
65. 45.6 L = 0.0456 m^3
66. 115 L = 0.115 m^3
67. 2 L = 0.002 m^3
68. 500.5 L = 0.5005 m^3
69. 3 000.6 L = 3.0006 m^3

Units of Capacity

Capacity is another way of expressing volume. Rather than using length measurements, capacity is expressed using measurements normally associated with liquids. Usually mL, L and kL. This sheet deals with the changing of units between these three. As with length, (mm, m and km), capacity units are × 1 000 apart (cm is the exception with length). So you multiply by 1 000, or as hopefully you know, you move the decimal point 3 places to the right (multiplying) and to the left (dividing).

Column 1 deals with changing from a smaller unit to a larger unit. With these you divide by 1 000. So move the decimal point 3 places to the left with these, remember to write the new unit after your answer, it will be the unit inside the brackets. Watch that you don't have unnecessary zeros in your answers when decimal points are involved.

Column 2 is the reverse, this time moving to a smaller unit so you have more of them, so you are multiplying by 1 000. Move the decimal point 3 places to the right to make the number larger. Again remember to write the new units after the number.

Column 3 asks you to find the volume of the prisms, the answer will be in cubic units, either cm^3 or m^3. Convert these to either mL (if cm^3) or kL (if m^3), rewrite the number and change the units. Then rewrite the answer in litres (L), using your skills from the previous 2 columns. Capacity is volume, so you still write 'V =' in your answer.

Note that L is the symbol for litres, it is a capital letter, this may seem unusual but many metric units use capital letters, an example being ‘N’ (Newtons, a measure of force) or ‘W’ (Watt, a measure of electrical power).

Units of Capacity

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Change these measurements to the larger units shown in the brackets

mL → L ÷ 1 000 → 3 places left

L → kL ÷ 1 000 → 3 places left

1 2 000 mL [L] = _____

2 5 000 L [kL] = _____

3 1 500 L [kL] = _____

4 3 400 mL [L] = _____

5 900 L [kL] = _____

6 500 mL [L] = _____

7 1 375 mL [L] = _____

8 250 mL [L] = _____

9 505 L [kL] = _____

10 50 L [kL] = _____

11 10 mL [L] = _____

12 35 mL [L] = _____

13 1 017 L [kL] = _____

14 110 mL [L] = _____

15 4 070 L [kL] = _____

16 1 400 mL [L] = _____

17 60 mL [L] = _____

18 15 L [kL] = _____

19 410 mL [L] = _____

20 5 030 L [kL] = _____

21 303 L [kL] = _____

22 1 L [kL] = _____

Convert the measurements to the smaller units shown in the brackets

kL → L × 1 000 → 3 places right

L → mL × 1 000 → 3 places right

23 1 L [mL] = _____

24 6 L [mL] = _____

25 4.5 kL [L] = _____

26 1.25 L [mL] = _____

27 0.75 L [mL] = _____

28 0.8 kL [L] = _____

29 3.2 L [mL] = _____

30 0.01 kL [L] = _____

31 1.12 kL [L] = _____

32 3.05 kL [L] = _____

33 0.079 L [mL] = _____

34 5.03 L [mL] = _____

35 0.871 kL [L] = _____

36 5.64 kL [L] = _____

37 0.0025 L [mL] = _____

38 2.004 kL [L] = _____

39 0.07 L [mL] = _____

40 0.003 kL [L] = _____

41 0.0005 kL [L] = _____

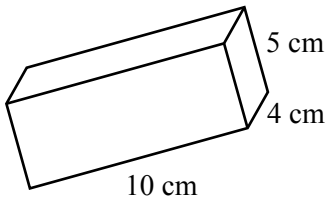
42 1.9 L [mL] = _____

43 0.0087 kL [L] = _____

44 0.053 L [mL] = _____

Calculate the volume in either cm^3 or m^3, write the answer as mL or kL, then rewrite converting to litres

45



$V = l \times b \times h$

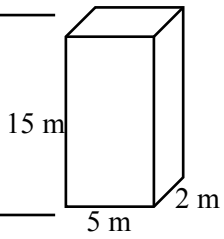
_____

V = _____ cm^3

V = _____ mL

V = _____ L

46



_____

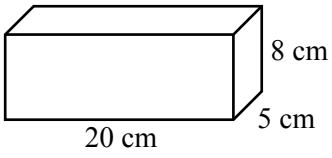
_____

_____

_____ kL

_____ L

47



_____

_____

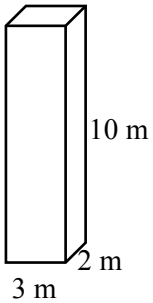
_____

_____

_____

_____

48



_____

_____

_____

_____

_____

Units of Capacity

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Change these measurements to the larger units shown in the brackets

mL → L ÷ 1 000 → 3 places left

L → kL ÷ 1 000 → 3 places left

1	2 000 mL	[L] =	2 L
2	5 000 L	[kL] =	5 kL
3	1 500 L	[kL] =	1.5 kL
4	3 400 mL	[L] =	3.4 L
5	900 L	[kL] =	0.9 kL
6	500 mL	[L] =	0.5 L
7	1 375 mL	[L] =	1.375 L
8	250 mL	[L] =	0.25 L
9	505 L	[kL] =	0.505 L
10	50 L	[kL] =	0.05 kL
11	10 mL	[L] =	0.01 L
12	35 mL	[L] =	0.035 L
13	1 017 L	[kL] =	1.017 kL
14	110 mL	[L] =	0.11 L
15	4 070 L	[kL] =	4.07 L
16	1 400 mL	[L] =	1.4 L
17	60 mL	[L] =	0.06 L
18	15 L	[kL] =	0.015 kL
19	410 mL	[L] =	0.41 L
20	5 030 L	[kL] =	5.03 kL
21	303 L	[kL] =	0.303 kL
22	1 L	[kL] =	0.001 kL

Convert the measurements to the smaller units shown in the brackets

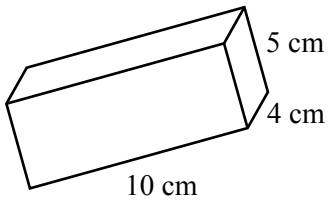
kL → L × 1 000 → 3 places right

L → mL × 1 000 → 3 places right

23	1 L	[mL] =	1 000 mL
24	6 L	[mL] =	6 000 mL
25	4.5 kL	[L] =	4 500 mL
26	1.25 L	[mL] =	1 250 mL
27	0.75 L	[mL] =	750 mL
28	0.8 kL	[L] =	800 L
29	3.2 L	[mL] =	3 200 mL
30	0.01 kL	[L] =	10 L
31	1.12 kL	[L] =	1 120 L
32	3.05 kL	[L] =	3 050 L
33	0.079 L	[mL] =	79 mL
34	5.03 L	[mL] =	5 030 mL
35	0.871 kL	[L] =	871 L
36	5.64 kL	[L] =	5 640 L
37	0.0025 L	[mL] =	2.5 mL
38	2.004 kL	[L] =	2 004
39	0.07 L	[mL] =	70 mL
40	0.003 kL	[L] =	3 L
41	0.0005 kL	[L] =	0.5 L
42	1.9 L	[mL] =	1 900 mL
43	0.0087 kL	[L] =	8.7 L
44	0.053 L	[mL] =	53 mL

Calculate the volume in either cm^3 or m^3, write the answer as mL or kL, then rewrite converting to litres

45



$V = l \times b \times h$

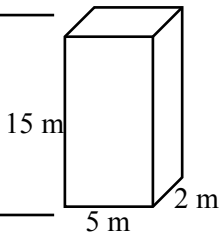
$= 10 \times 5 \times 4$

V = 200 cm^3

V = 200 mL

V = 0.2 L

46



$V = l \times b \times h$

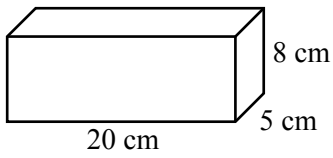
$= 5 \times 15 \times 2$

$V = 150\ m^3$

V = 150 kL

V = 150 000 L

47



$V = l \times b \times h$

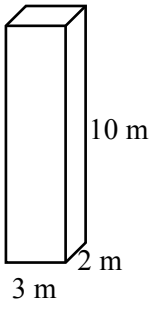
$= 20 \times 8 \times 5$

$V = 800\ cm^3$

V = 800 mL

V = 0.8 L

48



$V = l \times b \times h$

$= 3 \times 10 \times 2$

$V = 60\ m^3$

V = 60 kL

V = 60 000 L

Mass

Mass is a measure of weight, so why is it called mass? The difference is that mass is constant, if you weigh 53 kg at sea level you won't actually weigh 53 kg on top of Mt Everest. This is due to gravity being weaker when you are further from the centre of the planet, so as you move up, you technically weigh less! But your mass is still 53 kg, it stays unchanged from gravitational or outside effects. This is exaggerated when space or moon travel are involved as each planet/moon has a different gravitational pull. So the technical word is mass but unless you are going to Mars or Mt Everest the word weight is usually more commonly used.

Column 1 deals with changing from one unit to a larger unit. In all circumstances the difference is a factor of 1 000, and when you move to a larger unit you divide. But when you divide by 1 000 it is easier to move the decimal point 3 places (count the zeros) to the left. Examples are at the top of the column, don't forget to write the units (mg, g, kg, t). If in your answer you have a zero on the end of the number and it is behind the decimal point, it shouldn't be there, decimal answers should always be in their shortest form.

43.70 kg = 43.7 kg

Don't write the mass with an unnecessary zero, always write decimals in their most simplest form.

Column 2 is the reverse, you are moving to a smaller unit and so you multiply by 1 000. This means you move the decimal point 3 places to the right. Examples are there as well as the 'arrowed' guide, again don't forget the units.

Column 3 is in problem form and asks you to then express (state or give) the answer in two different units. The last two questions ask for the unit weight. The unit weight is the weight of 1 item, so you will have to divide the total mass by the number of items to get the unit weight.

Mass

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Express these masses in the units given in the brackets. All are ÷ 1 000.



50 mg = 0.05 g (3 places left)

950 g = 0.95 kg (3 places left)

4 250 kg = 4.25 t (3 places left)

1 2 000 kg [t] = _____

2 3 600 g [kg] = _____

3 900 g [kg] = _____

4 150 mg [g] = _____

5 305.7 kg [t] = _____

6 100 mg [g] = _____

7 1 750 mg [g] = _____

8 80.2 g [kg] = _____

9 220 kg [t] = _____

10 3 510 kg [t] = _____

11 5 300 mg [g] = _____

12 95 g [kg] = _____

13 750 g [kg] = _____

14 11 mg [g] = _____

15 8 070 kg [t] = _____

16 610 kg [t] = _____

17 8 g [kg] = _____

18 400 mg [g] = _____

19 16 300 kg [t] = _____

20 7 070 mg [g] = _____

21 30 090g [kg] = _____

22 101.3 g [kg] = _____

Now the reverse, multiply by 1 000 to change to the units in the brackets



3.7 t = 3 700 kg (3 places right)

0.52 kg = 520 g (3 places right)

1.06 g = 1 060 mg (3 places right)

23 1.2 kg [g] = _____

24 4.3 g [mg] = _____

25 3.85 t [kg] = _____

26 0.7 kg [g] = _____

27 5.02 t [kg] = _____

28 11 kg [g] = _____

29 0.05 g [mg] = _____

30 1.01 g [mg] = _____

31 10.01 kg [g] = _____

32 0.04 kg [g] = _____

33 13.14 t [kg] = _____

34 0.35 t [kg] = _____

35 9.04 g [mg] = _____

36 46 kg [g] = _____

37 0.01 g [mg] = _____

38 6.702 t [kg] = _____

39 0.65 g [mg] = _____

40 7.15 kg [g] = _____

41 6.06 t [kg] = _____

42 0.05 kg [g] = _____

43 0.2 g [mg] = _____

44 3.71 g [mg] = _____

Calculate the combined mass of these, then convert to kilograms.



45 7 cans of salmon, each 450g

g: _____ ×

kg: _____ _____

46 3 cartons of milk, each one having a mass of 830 g.

g: _____ ×

kg: _____ _____

47 A dozen oranges each having an average mass of 270g

g: _____ ×

kg: _____ _____

Now find the average unit weight for the following



48 Eight tablets with a total mass of 36.48 g

g: _____

mg: _____)_____

49 Four cars with a total mass of 4.3 t.

t: _____

kg: _____)_____

Mass

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Express these masses in the units given in the brackets. All are ÷ 1 000.

50 mg = 0.05 g (3 places left)

950 g = 0.95 kg (3 places left)

4 250 kg = 4.25 t (3 places left)

1	2 000 kg	[t] =	2 t
2	3 600 g	[kg] =	3.6 kg
3	900 g	[kg] =	0.9 kg
4	150 mg	[g] =	0.15 g
5	305.7 kg	[t] =	0.3057 t
6	100 mg	[g] =	0.1 g
7	1 750 mg	[g] =	1.75 g
8	80.2 g	[kg] =	0.0802 kg
9	220 kg	[t] =	0.22 kg
10	3 510 kg	[t] =	3.51 t
11	5 300 mg	[g] =	5.3 g
12	95 g	[kg] =	0.095 kg
13	750 g	[kg] =	0.75 kg
14	11 mg	[g] =	0.011 g
15	8 070 kg	[t] =	8.07 t
16	610 kg	[t] =	0.61 t
17	8 g	[kg] =	0.008 kg
18	400 mg	[g] =	0.4 g
19	16 300 kg	[t] =	16.3 t
20	7 070 mg	[g] =	7.07 g
21	30 090g	[kg] =	30.09 kg
22	101.3 g	[kg] =	0.1013 kg

Now the reverse, multiply by 1 000 to change to the units in the brackets

3.7 t = 3 700 kg (3 places right)

0.52 kg = 520 g (3 places right)

1.06 g = 1 060 mg (3 places right)

23	1.2 kg	[g] =	1 200 g
24	4.3 g	[mg] =	4 300 mg
25	3.85 t	[kg] =	3 850 kg
26	0.7 kg	[g] =	700 g
27	5.02 t	[kg] =	5 020 kg
28	11 kg	[g] =	11 000 g
29	0.05 g	[mg] =	50 mg
30	1.01 g	[mg] =	1 010 mg
31	10.01 kg	[g] =	10 010 g
32	0.04 kg	[g] =	40 g
33	13.14 t	[kg] =	13 140 kg
34	0.35 t	[kg] =	350 kg
35	9.04 g	[mg] =	9 040 mg
36	46 kg	[g] =	46 000 g
37	0.01 g	[mg] =	10 mg
38	6.702 t	[kg] =	6 702 kg
39	0.65 g	[mg] =	650 mg
40	7.15 kg	[g] =	7 150 g
41	6.06 t	[kg] =	6 060 kg
42	0.05 kg	[g] =	50 g
43	0.2 g	[mg] =	200 mg
44	3.71 g	[mg] =	3 710 mg

Calculate the combined mass of these, then convert to kilograms.

45 7 cans of salmon, each 450g

g: 3 150 g

kg: 3.15 kg

$$\begin{array}{r} {\scriptstyle 3}\,450 \\ \times\ 7 \\ \hline 3\,150 \end{array}$$

46 3 cartons of milk, each one having a mass of 830 g.

g: 2 490 g

kg: 2.49 kg

$$\begin{array}{r} 830 \\ \times\ 3 \\ \hline 2\,490 \end{array}$$

47 A dozen oranges each having an average mass of 270g

g: 3 240 g

kg: 3.24 kg

$$\begin{array}{r} {\scriptstyle 1}\,270 \\ \times\ 12 \\ \hline {\scriptstyle 1}\,540 \\ 2\,700 \\ \hline 3\,240 \end{array}$$

Now find the average unit weight for the following

48 Eight tablets with a total mass of 36.44 g

g: 4.56 g

mg: 4 560 mg

$$8\overline{)\,36.48} = 4.56$$

49 Four cars with a total mass of 4.3 t.

t: 1.075 t

kg: 1 075 kg

$$4\overline{)\,4.300} = 1.075$$

Net and Gross Mass

When you buy a product from a shop you want to know how much product you are getting, so that you can compare. For example frozen corn kernels will be in a plastic bag while unfrozen corn kernels will be in a can, the can may weigh more than the bag, but is that because of the can or is there more corn inside? Instead of products having their total weight on them they have their Net weight, this is the weight of the actual product inside the can or bag.

So Net weight is the weight of the contents. The total weight, product + container is called the Gross weight. Gross weight is rarely on the item but you have to calculate it if you are sending an item by mail or paying for its transport. Gross weight is also important in industry which sets a maximum weight that a person can lift, the packaging must be factored into calculations. Remember the environment when you buy an item, try to buy one that minimises the use of packaging.

These three equations are involved with this worksheet:

- Net weight = Gross weight - Container weight
- Gross weight = Net weight + Container weight
- Container weight = Gross weight - Net weight

Column 1 asks you to calculate the gross weight, add the packaging weight to the net weight to get the answer. You should be able to complete the table mentally, watch the units asked for in the answer. Problem style questions then follow, use the working spaces provided then give your answer in the units asked for.

Column 2 asks you to calculate the net weight. This is found by subtracting the packaging weight from the gross weight. This column is in the same format as the previous column, you should be able to do these mentally, then the problems have working spaces provided.

Column 3 asks you to calculate the weight of the packaging, subtract the net weight from the gross weight to answer these. The last question has a table that gives two values in each row, you have to use your skills with these to calculate the third value. Do you add or do you subtract…?



The weight given on a product is the net weight. This is the weight of the tuna inside the can without the can's weight. You only want to know this weight so… 'the net is what you get.'

Net and Gross Mass

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Calculate the gross weight of the items below in the given units

1 Net + Container = Gross

Net	Container	Gross
150 g	30 g	g
225 g	70 g	g
480 g	90 g	g
1 kg	50 g	g
20 kg	750 g	kg
13.2 kg	1050 g	kg

2 A cable car weighing 1.4 t has 8 people board it with an average weight of 77 kg.

i) Calculate the combined weight of the passengers

×

kg: _____

_____

_____

ii) Calculate the current gross mass of the cable car in kg and t

+

kg: _____ _____

t: _____ _____

3 A jar of vitamin pills contains 35 pills at 7 g each. If the jar itself weighs 83 g find the gross weight.

× +

_____ _____

_____ _____

_____

What's in the net is what you get.. calculate the net mass of these

4 Gross - Container = Net

Gross	Container	Net
305 g	60 g	g
495 g	25 g	g
120 g	35 g	g
75 kg	17 kg	kg
1 750 kg	300 kg	t
8.7 t	250 kg	t

5 Tegan has caught a rare finger-swallowing tiger slug. If the bug catcher weighs 469 g and with the slug weighs 718 g, find the mass of the slug.

-

_____ _____

_____ _____

6 A variety pack of cereals has 8 different selections. If the net mass of the pack is 600g find the mass of each selection.

_____)

_____

7 A man on scales weighing 86 kg lifts his daughter up and the new reading is 133 kg. Find the weight of his daughter.

-

_____ _____

_____ _____

8 A rain gauge jar weighs 57 g, after rain it weighs 67 g. The mL of water inside is…

1 ⬭ 10 ⬭ 100 ⬭ 1 000 ⬭

Now calculate the weight of the containers.

9 Gross - Net = Container

Gross	Net	Container
45 g	18 g	g
300 g	225 g	g
705 g	620 g	g
8.3 kg	7.6 kg	g
850 kg	65 kg	t
19.2 t	1.6 t	t

10 Harry's glasses weigh 248g, in their case the weight is 437g. Find the weight of the case.

-

_____ _____

_____ _____

11 Han weighs 75 kg, when he is encased in carbonite he tips the scales at 722 kg. Find the weight of the carbonite.

-

_____ _____

_____ _____

Fill in the blanks… may the gross be with you!

12

Net	Packaging	Gross
200 g	55 g	
	17 g	80 g
20.3 kg		21.9 kg
0.83 t	150 kg	
5.7 t	80 kg	
	97 kg	1.3 t

Net and Gross Mass

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Calculate the gross weight of the items below in the given units

1 Net + Container = Gross

Net	Container	Gross
150 g	30 g	180 g
225 g	70 g	295 g
480 g	90 g	570 g
1 kg	50 g	1 050 g
20 kg	750 g	20.75 kg
13.2 kg	1050 g	14.25 kg

2. A cable car weighing 1.4 t has 8 people inside with an average weight of 77 kg.

i) Calculate the combined weight of the passengers

kg: 616 kg

$^{5}77 \times 8 = 616$

ii) Calculate the current gross mass of the cable car in kg and t

kg: 2 016 kg

t: 2.016 t

$^{1}1\,400 + 616 = 2\,016$

3. A jar of vitamin pills contains 35 pills at 7 g each. If the jar itself weighs 83 g find the gross weight.

$^{3}35 \times 7 = 245$

$^{1}245 + 83 = 328$

Gross weight is 328 g

What's in the net is what you get.. calculate the net mass of these

4 Gross - Container = Net

Gross	Container	Net
305 g	60 g	245 g
495 g	25 g	470 g
120 g	35 g	85 g
75 kg	17 kg	58 kg
1 750 kg	300 kg	1.45 t
8.7 t	250 kg	8.45 t

5 Tegan has caught a rare finger-swallowing tiger slug. If the bug catcher weighs 469 g and with the slug weighs 718 g, find the mass of the slug.

Start running its 249 g!!!!

$^{1\,1}718 - 469 = 249$

6 A variety pack of cereals has 8 different selections. If the net mass of the pack is 600g find the mass of each selection.

Each pack weighs 75 g

$8\overline{)600} = 75$

7 A man on scales weighing 86 kg lifts his daughter up and the new reading is 133 kg. Find the weight of his daughter.

She weighs 47 kg.

$^{1\,1}133 - 86 = 47$

8 A rain gauge jar weighs 57 g, after rain it weighs 67 g. The mL of water inside is…

1 ○ 10 ● 100 ○ 1 000 ○

Now calculate the weight of the containers.

9 Gross - Net = Container

Gross	Net	Container
45 g	18 g	27 g
300 g	225 g	75 g
705 g	620 g	85 g
8.3 kg	7.6 kg	700 g
850 kg	65 kg	0.785 t
19.2 t	1.6 t	17.6 t

10 Harry's glasses weigh 248g, in their case the weight is 437g. Find the weight of the case.

The spectacles case weighs 189 g

$^{1\,1}437 - 248 = 189$

11 Han weighs 75 kg, when he is encased in carbonite he tips the scales at 722 kg. Find the weight of the carbonite.

Its 647 kg! Lucky it hovers.

$^{1\,1}722 - 75 = 647$

Fill in the blanks… may the gross be with you!

12

Net	Packaging	Gross
200 g	55 g	255 g
63 g	17 g	80 g
20.3 kg	1.6 kg	21.9 kg
0.83 t	150 kg	980 kg
5.7 t	80 kg	5.78 t
1.203 t	97 kg	1.3 t

FREEFALL MATHEMATICS

ALTITUDE

BOOK TWO

FREEFALL MATHEMATICS

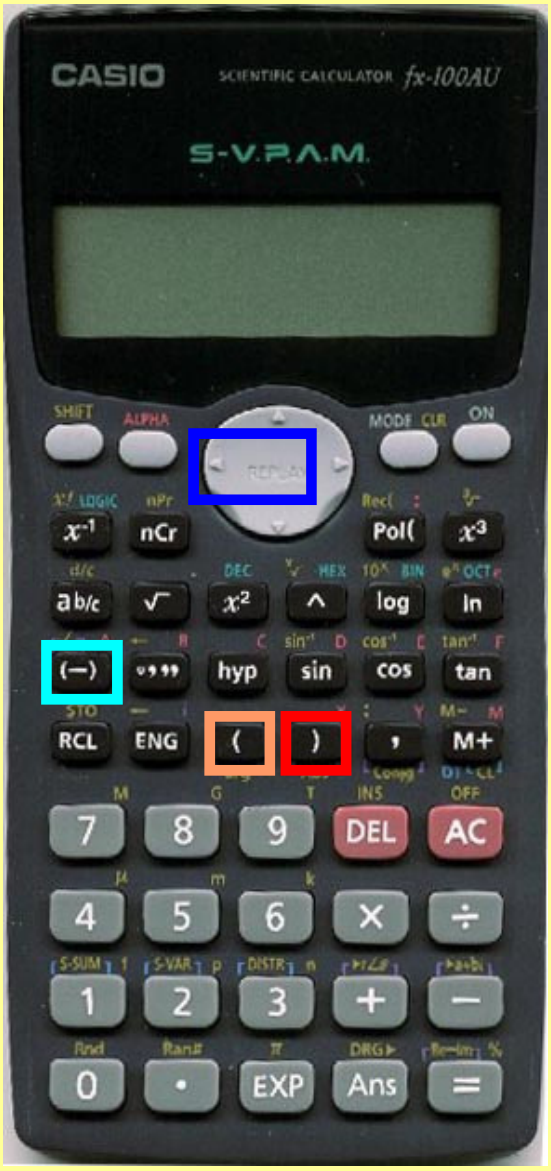
ALTITUDE

CALCULATOR

Replay, Negatives & Brackets - Calculator Applications

This sheet deals with understanding some features of your calculator. Calculator design is always changing, so the button style may be different to the layout below, but an alternative button will exist of\n your calculator.

Column 1 deals with use of the replay key, as referred to on Casio calculators. The "Back Button" is just the same. It is important, especially when using "Guess and Check" methods that you can alter numbers in a long calculation without having to retype the Entire calculation again. So this column gets you to use the replay button to save time.



Note that some replay functions write over the number while some insert the number, so you may have to use the DEL key after you enter the number.

Column 2 starts off with the use of the negative key in calculations. Note that this key could also be a +/- key. Some calculators allow you to use the minus key and you may ask ….. why bother? The big difference is when power keys are used.

Column 2 and three then introduce the brackets key. Brackets keys are important when order of operations properties are important. Column 2 gives you the brackets in the questions, while in Column 3 the brackets aren't there, but you have to use them with your calculator or you will get the wrong answer.

Column 3 ends with some calculations. When you get the answer write the question in the first space. Then turn your calculator upside down, the inverted numbers can be read off like they are letters. Write the word that is spelt out. If the word doesn't make sense, you have probably made a calculator key stroke error.

Replay, Negatives & Brackets - Calculator Applications

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Use your 'Replay' or back arrow to guess & check to find the answer for the box.

1 $56 \times 0.08 \times \square = 62.72$

Try: $\square = 5$ Answer:

Use your REPLAY key

Try: $\square = 9$ Answer:

Use your REPLAY key

Try: $\square = 12$ Answer:

Use your REPLAY key

Try: $\square = 14$ Answer:

2 $850 \times 0.062 \times \square = 421.6$

Try: $\square = 4$ Answer:

Try: $\square = 12$ Answer:

Try: $\square = 8$ Answer:

3 $5\,500 \times 0.04 \times \square = 440$

Try: $\square = 6$ Answer:

Try: $\square = 3$ Answer:

Try: $\square = 2$ Answer:

4 $300 \times 0.075 \times \square = 270$

Try: $\square = 4$ Answer:

Try: $\square = 10$ Answer:

Try: $\square = 12$ Answer:

Find the answer for these, use the replay button to speed things up.

5 $141 \div 3 \times 9 - 15 =$ _____

6 $141 \div 3 \times 7 - 15 =$ _____

7 $141 \div 3 \times 7 - 40 =$ _____

8 $255 \div 3 \times 7 - 40 =$ _____

9 $255 \div 5 \times 7 - 40 =$ _____

Use your NEGATIVE KEY to calculate these.

Calc: [(-)] [1] [8] [×] [7] [=]

10 $-18 \times 7 =$ _____ or [+/−]

11 $35 \times -9 =$ _____

12 $-238 \div -17 =$ _____

13 $-9.2 - 17 =$ _____

14 $17 + -31 =$ _____

15 $-5.5 \div -0.2 =$ _____

16 $-93.6 + -47.1 =$ _____

Now use brackets to solve these.

Calc:

[(] [6] [7] [-] [7] [9] [)] [×] [8] [=]

17 $(67 - 79) \times 8 =$ _____

18 $(-13 - 17) \times 15 =$ _____

19 $(3 - 7) \times (5 - 16) =$ _____

20 $9 + 4 \times (13 + 24) =$ _____

21 $-6 \times (-2 - 13) =$ _____

33 $\dfrac{(104 + 37)}{8} =$ _____

34 $\dfrac{(354 - 172)}{(9 + 5)} =$ _____

35 $\dfrac{(18 + 8 \times 7)}{(16 - 9 \times 2)} =$ _____

36 $\dfrac{(14 \times 9 - 6)}{(-2 \times 18 + 6)} =$ _____

37 $\dfrac{(200 - 13 \times 8)}{(516 \div 43)} =$ _____

You can't see the brackets in these, you have to put them in yourself to get the answer.

Calc:

[(] [8] [5] [+] [6] [7] [)] [÷] [4] [=]

38 $\dfrac{85 + 67}{4} =$ _____

39 $\dfrac{45 - 19}{6 + 7} =$ _____

40 $\dfrac{13 + 9 \times 5}{16 - 6 \times 2} =$ _____

41 $\dfrac{38 - 7 \times 19}{-8 \times 5} =$ _____

Perform these calculations then turn your calculator upside down. Write the number & word formed.

Example

$221 \times 35 =$ 7 735 Sell

42 $(16\,588 + 30\,489) \times 8 =$

_____ _____

43 $1\,933.5 \div (50 \times 50) =$

_____ _____

44 $406 \div 58 \times 113 \times 7 =$

_____ _____

45 $(7\,891 \times 24 + 19) \times 2 =$

_____ _____

46 $(3\,911 - 22\,000) \times -20 =$

_____ _____

47 $5 \times -5 \times 3\,691 \times -5 =$

_____ _____

48 $(10\,000 - 481) \times 4 =$

_____ _____

Replay, Negatives & Brackets - Calculator Applications

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Use your 'Replay' or back arrow to guess & check to find the answer for the box.

1 $56 \times 0.08 \times \square = 62.72$

Try: $\square = 5$ Answer: 22.4

Use your REPLAY key

Try: $\square = 9$ Answer: 40.32

Use your REPLAY key

Try: $\square = 12$ Answer: 53.76

Use your REPLAY key

Try: $\square = 14$ Answer: 62.72

2 $850 \times 0.062 \times \square = 421.6$

Try: $\square = 4$ Answer: 210.8

Try: $\square = 12$ Answer: 632.4

Try: $\square = 8$ Answer: 421.6

3 $5\,500 \times 0.04 \times \square = 440$

Try: $\square = 6$ Answer: 1 320

Try: $\square = 3$ Answer: 660

Try: $\square = 2$ Answer: 440

4 $300 \times 0.075 \times \square = 270$

Try: $\square = 4$ Answer: 90

Try: $\square = 10$ Answer: 225

Try: $\square = 12$ Answer: 270

Find the answer for these, use the replay button to speed things up.

5 $141 \div 3 \times 9 - 15 =$ 408

6 $141 \div 3 \times 7 - 15 =$ 314

7 $141 \div 3 \times 7 - 40 =$ 289

8 $255 \div 3 \times 7 - 40 =$ 555

9 $255 \div 5 \times 7 - 40 =$ 317

Use your NEGATIVE KEY to calculate these.

Calc: [(-)] [1] [8] [×] [7] [=]

10 $-18 \times 7 =$ -126 or [+/−]

11 $35 \times -9 =$ -315

12 $-238 \div -17 =$ 14

13 $-9.2 - 17 =$ -26.2

14 $17 + -31 =$ -14

15 $-5.5 \div -0.2 =$ 27.5

16 $-93.6 + -47.1 =$ -140.7

Now use brackets to solve these.

Calc: [(] [6] [7] [-] [7] [9] [)] [×] [8] [=]

17 $(67 - 79) \times 8 =$ -96

18 $(-13 - 17) \times 15 =$ -450

19 $(3 - 7) \times (5 - 16) =$ 44

20 $9 + 4 \times (13 + 24) =$ 157

21 $-6 \times (-2 - 13) =$ 90

33 $\dfrac{(104 + 37)}{8} =$ 17.625

34 $\dfrac{(354 - 172)}{(9 + 5)} =$ 13

35 $\dfrac{(18 + 8 \times 7)}{(16 - 9 \times 2)} =$ -37

36 $\dfrac{(14 \times 9 - 6)}{(-2 \times 18 + 6)} =$ -4

37 $\dfrac{(200 - 13 \times 8)}{(516 \div 43)} =$ 8

You can't see the brackets in these, you have to put them in yourself to get the answer.

Calc: [(] [8] [5] [+] [6] [7] [)] [÷] [4] [=]

38 $\dfrac{85 + 67}{4} =$ 38

39 $\dfrac{45 - 19}{6 + 7} =$ 2

40 $\dfrac{13 + 9 \times 5}{16 - 6 \times 2} =$ 14.5

41 $\dfrac{38 - 7 \times 19}{-8 \times 5} =$ 2.375

Perform these calculations then turn your calculator upside down. Write the number & word formed.

Example

$221 \times 35 =$ 7 735 Sell

42 $(16\,588 + 30\,489) \times 8 =$

376 616 Giggle

43 $1\,933.5 \div (50 \times 50) =$

0.7734 Hello

44 $406 \div 58 \times 113 \times 7 =$

5 537 Less

45 $(7\,891 \times 24 + 19) \times 2 =$

378 806 Gobble

46 $(3\,911 - 22\,000) \times -20 =$

361 780 Oblige

47 $5 \times -5 \times 3\,691 \times -5 =$

461 375 Sleigh

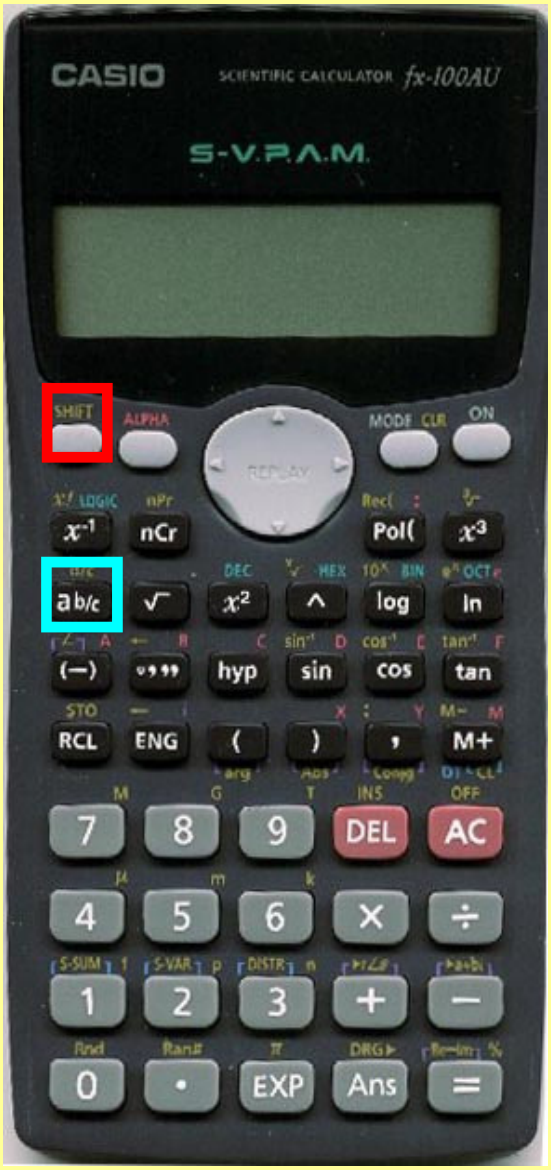
48 $(10\,000 - 481) \times 4 =$

38 076 Globe

Fractions - Calculator Applications

This worksheet deals with the use of your calculator's fraction key. This key could also look like this: $\square\!\!\frac{\square}{\square}$ as well, it depends on the brand and model of your calculator. The first column starts with changing fractions to decimals. All columns have keystroke examples but this may differ from your calculator, depending on brand or the model used.

To change a fraction to a decimal, enter the fraction using the fraction key (example at the top of the column), press '=' and then the fraction key again. Your calculator may have a S→D button. If so press this button instead. To change decimals to a fraction, enter the decimal press the '=' button then the fraction key. (Or the S→D key).



Note that a fraction can be converted to a decimal by just using the division '÷' key as well.

Column 2 deals first with simplifying fractions. Enter the fraction then press the '=' button to simplify. The same process is used to change improper fractions to Mixed Numerals. Just enter the fraction and press '='. (Note that this may vary with some calculators, the S→D key with the Shift key may be required).

Converting mixed numerals to improper fractions is achieved by entering the mixed number, pressing the '=' button, then SHIFT & the fraction key. Some calculators are set to display fractions as improper fractions, this can be changed in the calculators settings.

Column 3 has mixed questions.

Converting fractions to decimals allows easier comparison. Make sure that when you write the decimals that they have the same number of decimal places, this makes it easier to compare them.

Fractions - Calculator Applications

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Change these fractions to decimals using the fraction key on your calculator.

Calc: 3 5 a b/c 4 0 = a b/c

1 $\frac{35}{40}$ = _____ 2 $\frac{1}{4}$ = _____

3 $\frac{18}{25}$ = _____ 4 $\frac{64}{80}$ = _____

5 $\frac{13}{20}$ = _____ 6 $\frac{48}{60}$ = _____

7 $\frac{3}{8}$ = _____ 8 $\frac{9}{40}$ = _____

Convert & round the decimals to 2 d.p.

9 $\frac{2}{3}$ = _____ 10 $\frac{5}{9}$ = _____

11 $\frac{2}{7}$ = _____ 12 $\frac{10}{11}$ = _____

Now the reverse, write these decimals as fractions.

Calc: 0 . 4 4 = a b/c

13 0.44 = ☐/☐ 14 0.65 = ☐/☐

15 0.24 = ☐/☐ 16 0.75 = ☐/☐

17 0.6875 = ☐/☐ 18 0.125 = ☐/☐

19 3.4 = ☐ ☐/☐ 20 4.56 = ☐ ☐/☐

Convert the fractions to decimals to compare, use < or > in the box

21 $\frac{3}{4}$ ☐ $\frac{13}{16}$

Decimal 4 d.p. Decimal

22 $\frac{4}{5}$ ☐ $\frac{42}{48}$

_____ _____

Simplify these fractions.

Calc: 1 6 a b/c 2 0 =

23 $\frac{16}{20}$ = ☐/☐ 24 $\frac{72}{96}$ = ☐/☐

25 $\frac{225}{270}$ = ☐/☐ 26 $\frac{27}{126}$ = ☐/☐

27 $\frac{405}{500}$ = ☐/☐ 28 $\frac{325}{520}$ = ☐/☐

29 $\frac{68}{320}$ = ☐/☐ 30 $\frac{15}{85}$ = ☐/☐

Convert these improper fractions to mixed numerals.

Calc: 1 7 a b/c 4 =

31 $\frac{17}{4}$ = ☐ ☐/☐ 32 $\frac{7}{2}$ = ☐ ☐/☐

33 $\frac{40}{16}$ = ☐ ☐/☐ 34 $\frac{35}{3}$ = ☐ ☐/☐

35 $\frac{26}{7}$ = ☐ ☐/☐ 36 $\frac{35}{19}$ = ☐ ☐/☐

37 $\frac{50}{13}$ = ☐ ☐/☐ 38 $\frac{26}{9}$ = ☐ ☐/☐

Convert these mixed numerals to improper fractions.

Calc: 2 a b/c 3 a b/c 5 = SHIFT a b/c

39 $2\frac{3}{5}$ = ☐/☐ 40 $5\frac{7}{8}$ = ☐/☐

41 $6\frac{4}{7}$ = ☐/☐ 42 $1\frac{8}{13}$ = ☐/☐

43 $5\frac{1}{8}$ = ☐/☐ 44 $10\frac{2}{9}$ = ☐/☐

45 $8\frac{2}{17}$ = ☐/☐ 46 $3\frac{4}{23}$ = ☐/☐

47 $7\frac{3}{13}$ = ☐/☐ 48 $3\frac{24}{99}$ = ☐/☐

Perform these calculations on your calculator.

49 $\frac{1}{2} + \frac{1}{3}$ = ☐/☐

50 $5\frac{7}{8} - 3\frac{2}{5}$ = ☐ ☐/☐

51 $\frac{3}{7} \times \frac{5}{9}$ = ☐/☐

52 $\frac{11}{15} - \frac{2}{3}$ = ☐/☐

53 $\frac{6}{5} \div \frac{8}{15}$ = ☐ ☐/☐

54 $\frac{4}{9} + \frac{2}{3} \times \frac{7}{12}$ = ☐/☐

55 $\frac{1}{5} \div \frac{7}{8} + \frac{3}{10}$ = ☐/☐

56 $4\frac{7}{9} \times 4 - 5\frac{3}{4}$ = ☐ ☐/☐

57 $3\frac{2}{3} \div 1\frac{1}{2} + \frac{7}{10}$ = ☐ ☐/☐

Use decimals to arrange these in descending order

58 $\frac{3}{5}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{10}$

_____ _____ _____ _____

Write the decimals to 3 d.p.

Rewrite in Descending order → ☐/☐ , ☐/☐ , ☐/☐ , ☐/☐ .

59 $\frac{9}{12}$, $\frac{5}{2}$, $\frac{9}{4}$, $\frac{12}{5}$

_____ _____ _____ _____

Rewrite in Descending order → ☐/☐ , ☐/☐ , ☐/☐ , ☐/☐ .

Fractions - Calculator Applications

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Change these fractions to decimals using the fraction key on your calculator.

Calc: 3 5 a b/c 4 0 = a b/c

1 $\frac{35}{40}$ = 0.875 2 $\frac{1}{4}$ = 0.25

3 $\frac{18}{25}$ = 0.875 4 $\frac{64}{80}$ = 0.875

5 $\frac{13}{20}$ = 0.875 6 $\frac{48}{60}$ = 0.875

7 $\frac{3}{8}$ = 0.875 8 $\frac{9}{40}$ = 0.875

Convert & round the decimals to 2 d.p.

9 $\frac{2}{3}$ = 0.67 10 $\frac{5}{9}$ = 0.56

11 $\frac{2}{7}$ = 0.29 12 $\frac{10}{11}$ = 0.91

Now the reverse, write these decimals as fractions.

Calc: 0 . 4 4 = a b/c

13 0.44 = $\frac{11}{25}$ 14 0.65 = $\frac{13}{20}$

15 0.24 = $\frac{6}{25}$ 16 0.75 = $\frac{3}{4}$

17 0.6875 = $\frac{11}{16}$ 18 0.125 = $\frac{1}{8}$

19 3.4 = $3\frac{2}{5}$ 20 4.56 = $4\frac{14}{25}$

Convert the fractions to decimals to compare, use < or > in the box

21 $\frac{3}{4}$ < $\frac{13}{16}$

0.7500 (Decimal 4 d.p.) 0.8125 (Decimal)

22 $\frac{4}{5}$ < $\frac{42}{48}$

0.800 0.875

Simplify these fractions.

Calc: 1 6 a b/c 2 0 =

23 $\frac{16}{20} = \frac{4}{5}$ 24 $\frac{72}{96} = \frac{3}{4}$

25 $\frac{225}{270} = \frac{5}{6}$ 26 $\frac{27}{126} = \frac{3}{14}$

27 $\frac{405}{500} = \frac{81}{100}$ 28 $\frac{325}{520} = \frac{5}{8}$

29 $\frac{68}{320} = \frac{17}{80}$ 30 $\frac{15}{85} = \frac{3}{17}$

Convert these improper fractions to mixed numerals.

Calc: 1 7 a b/c 4 =

31 $\frac{17}{4} = 4\frac{1}{4}$ 32 $\frac{7}{2} = 3\frac{1}{2}$

33 $\frac{40}{16} = 2\frac{1}{2}$ 34 $\frac{35}{3} = 11\frac{2}{3}$

35 $\frac{26}{7} = 3\frac{5}{7}$ 36 $\frac{35}{19} = 1\frac{16}{19}$

37 $\frac{50}{13} = 3\frac{11}{13}$ 38 $\frac{26}{9} = 2\frac{8}{9}$

Convert these mixed numerals to improper fractions.

Calc: 2 a b/c 3 a b/c 5 = SHIFT a b/c

39 $2\frac{3}{5} = \frac{13}{5}$ 40 $5\frac{7}{8} = \frac{47}{8}$

41 $6\frac{4}{7} = \frac{46}{7}$ 42 $1\frac{8}{13} = \frac{21}{13}$

43 $5\frac{1}{8} = \frac{41}{8}$ 44 $10\frac{2}{9} = \frac{92}{9}$

45 $8\frac{2}{17} = \frac{138}{17}$ 46 $3\frac{4}{23} = \frac{73}{23}$

47 $7\frac{3}{13} = \frac{94}{13}$ 48 $3\frac{24}{99} = \frac{107}{33}$

Perform these calculations on your calculator.

49 $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

50 $5\frac{7}{8} - 3\frac{2}{5} = 2\frac{19}{40}$

51 $\frac{3}{7} \times \frac{5}{9} = \frac{5}{21}$

52 $\frac{11}{15} - \frac{2}{3} = \frac{1}{15}$

53 $\frac{6}{5} \div \frac{8}{15} = 2\frac{1}{4}$

54 $\frac{4}{9} + \frac{2}{3} \times \frac{7}{12} = \frac{5}{6}$

55 $\frac{1}{5} \div \frac{7}{8} + \frac{3}{10} = \frac{37}{70}$

56 $4\frac{7}{9} \times 4 - 5\frac{3}{4} = 13\frac{13}{36}$

57 $3\frac{2}{3} \div 1\frac{1}{2} + \frac{7}{10} = 3\frac{13}{90}$

Use decimals to arrange these in descending order

58 $\frac{3}{5}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{10}$

0.600 0.625 0.750 0.700

Write the decimals to 3 d.p.

Rewrite in Descending order → $\frac{3}{4}$, $\frac{7}{10}$, $\frac{5}{8}$, $\frac{3}{5}$.

59 $\frac{9}{12}$, $\frac{5}{2}$, $\frac{9}{4}$, $\frac{12}{5}$

0.750 2.500 2.250 2.400

Rewrite in Descending order → $\frac{5}{2}$, $\frac{12}{5}$, $\frac{9}{4}$, $\frac{9}{12}$.

Powers and Roots - Calculator Applications

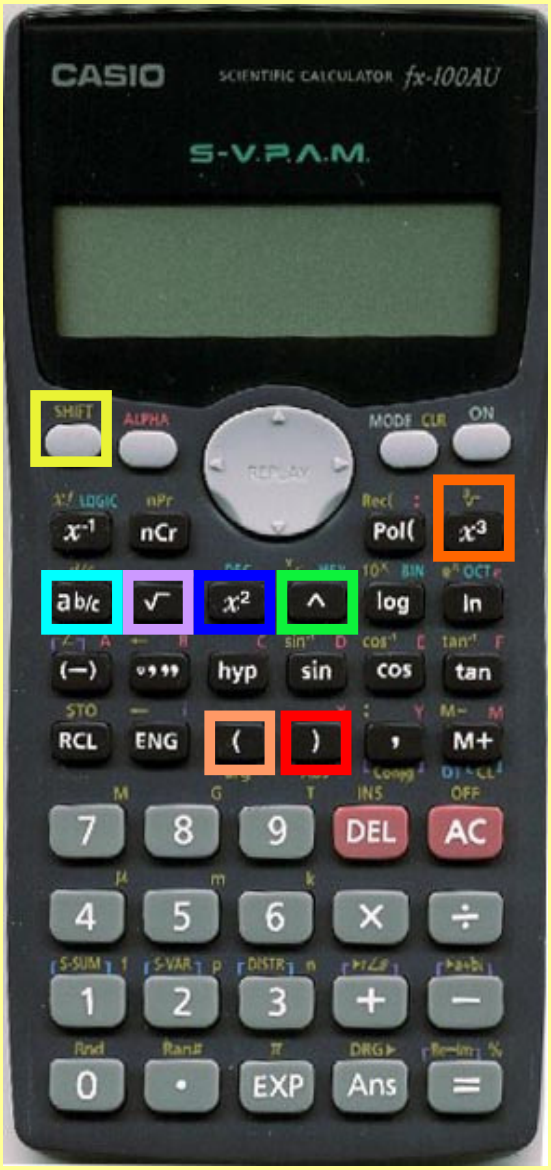
This sheet is about the use of powers and roots on a calculator. There are examples on the next sheet with a calculator image showing you the buttons. This calculator is probably different from yours but the symbols used on the keys should be the same, just look for the symbols.

Column 1 is all square roots, some calculators require you to type the number then the square root, the steps for these calculators are different than those shown, try your calculator on the examples on the next page and see the method you use to get the correct answer. The first part of the column doesn't require brackets, the second part does. Your calculator needs brackets to force order of operations on it, otherwise it will take the square root of the first number only.

Column 2 deals with squares (power of 2) and cubes (power of 3). Some calculators may not have a cube key, if this is your case then use the power key with a power of 3. Some calculators experience difficulty with fractions and powers, with the power being considered on the last number entered only (the denominator), so you may have to put fractions in brackets before entering the power.

Column 3 starts with using the power key, then the use of the cube root key. Your calculator may feature a cube root key or you may have to use shift first.

Some examples are on the next page.



Examples of steps in calculating square roots

$\sqrt{85}$ = √ 8 5 = = 9.22

$\sqrt{5\ ^2/_7}$ = √ 5 a b/c 2 a b/c 7 = a b/c = 2.30

$\sqrt{13.4 + 22 \times 3.6}$ These answers to 2 d.p.

= √ (1 3 . 4 + 2 2 × 3 . 6) = = 9.62

$\sqrt{\dfrac{34}{45 + 19.7}}$

= √ (3 4 ÷ (4 5 + 1 9 . 7)) = = 0.72

Use the same method with the cube root key only press Shift first

Examples of steps in calculating squares and cubes

5.6^2 = 5 . 6 x^2 = = 31.36

$(5\ ^2/_7)^2$ = (5 a b/c 2 a b/c 7) x^2 = a b/c = 27.94

Use the same method with the cube key x^3

These answers to 2 d.p.

Examples of steps using the power key

5.6^8 = 5 . 6 ^ 8 = = 967 173.12

These answers to 2 d.p.

$(5\ ^2/_7)^9$ = (5 a b/c 2 a b/c 7) ^ 9 = a b/c = 3 220 573.07

Powers and Roots - Calculator Applications

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Use the square root key on your calculate to answer these.

Calc: [√] [4] [4] [1] [=]

1 $\sqrt{441}$ = ____ 2 $\sqrt{361}$ = ____

3 $\sqrt{256}$ = ____ 4 $\sqrt{484}$ = ____

5 $\sqrt{289}$ = ____ 6 $\sqrt{729}$ = ____

7 $\sqrt{0.04}$ = ____ 8 $\sqrt{1\ 369}$

9 $\sqrt{0.3136}$ = ____ = ____

Now round your answers to 1 decimal place

10 $\sqrt{32}$ = ____ 11 $\sqrt{77}$ = ____

12 $\sqrt{205}$ = ____ 13 $\sqrt{888}$ = ____

Use the brackets key to answer these, round to 1 d.p when necessary.

Calc: [√] [(] [2] [3] [×] [1] [6] [)] [=]

14 $\sqrt{(23 \times 16)}$ = ____

15 $\sqrt{(27.2 + 48.9)}$ = ____

16 $\sqrt{(15 \div 3.9)}$ = ____

17 $\sqrt{(106 + 45 \times 19)}$ = ____

The brackets aren't in these questions but you still need to use them on your calculator.

18 $\sqrt{19 \times 42 + 102}$ = ____

19 $\sqrt{\frac{133.6}{8.35}}$ = ____

20 $\sqrt{509 - 21 \times 18.6}$ = ____

21 $\sqrt{17.7 \times 30 \times 1.8}$ = ____

22 $\sqrt{49.3 + 112 + 95}$ = ____

23 $\sqrt{\frac{2\ 293.2}{5.2}}$ = ____

Find these squares and cubes, answer to 2 d.p. where required.

24 35^2 = ____

25 9.7^2 = ____

26 20.6^2 = ____

27 $(3\frac{1}{2})^3$ = ____

28 10.5^3 = ____

29 65^2 = ____

30 12^3 = ____

31 $(4\frac{3}{5})^3$ = ____

32 8.09^3 = ____

33 $19.4^2 + 10.9^3$ = ____

34 18.1×11^3 = ____

35 $\frac{52.5^2}{14^3}$ = ____

36 $25^2 - 12^3$ = ____

37 $34 + 6.2^3 \times 13^2$ = ____

38 $\sqrt{77} + 25^2$ = ____

39 $(\sqrt{66})^2$ = ____

40 $(11 - 26.2)^2$ = ____

41 $11 - 26.2^2$ = ____

42 $11^2 - 26.2^2$ = ____

43 $18^3 - 18^2 - 18$ = ____

44 $18^3 - 18^2 \times 18$ = ____

45 $100^3 \div 10^3$ = ____

46 $100^3 \div 10^3 \div 10^3$ = ____

Use the power key [^] for these. Answer to 2 d.p. if required.

47 2^4 = ____

48 5^5 = ____

49 1^7 = ____

50 10^6 = ____

51 5.2^3 = ____

52 2^{12} = ____

53 $(1^2/_7)^6$ = ____

54 $8^9 - 8^8$ = ____

55 $2^{15} \div 2^{13}$ = ____

Use the cube root key [$\sqrt[3]{}$] to answer these. Express your answer to 2 d.p. when required

56 $\sqrt[3]{216}$ = ____

57 $\sqrt[3]{343}$ = ____

58 $\sqrt[3]{1\ 000}$ = ____

Brackets now required

59 $\sqrt[3]{\frac{5\ 375}{43}}$ = ____

60 $\sqrt[3]{452 \times 17.3}$ = ____

61 $\sqrt[3]{88^3 + 88^3}$ = ____

62 $\sqrt[3]{5\ 833 - 18^3}$ = ____

Careful with this one

63 $\sqrt[3]{\frac{5^3 + 2^7}{11^2}}$ = ____

Powers and Roots - Calculator Applications

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Use the square root key on your calculate to answer these.

Calc: [√] [4] [4] [1] [=]

1 $\sqrt{441} = 21$ 2 $\sqrt{361} = 19$

3 $\sqrt{256} = 16$ 4 $\sqrt{484} = 22$

5 $\sqrt{289} = 17$ 6 $\sqrt{729} = 27$

7 $\sqrt{0.04} = 0.2$ 8 $\sqrt{1\ 369} = 37$

9 $\sqrt{0.3136} = 0.56$

Now round your answers to 1 decimal place

10 $\sqrt{32} = 5.7$ 11 $\sqrt{77} = 8.8$

12 $\sqrt{205} = 14.3$ 13 $888 = 29.8$

Use the brackets key to answer these, round to 1 d.p when necessary.

Calc: [√] [(] [2] [3] [×] [1] [6] [)] [=]

14 $\sqrt{(23 \times 16)} = 19.2$

15 $\sqrt{(27.2 + 48.9)} = 8.7$

16 $\sqrt{(15 \div 3.9)} = 2$

17 $\sqrt{(106 + 45 \times 19)} = 31$

The brackets aren't in these questions but you still need to use them on your calculator.

18 $\sqrt{19 \times 42 + 102} = 30$

19 $\sqrt{\frac{133.6}{8.35}} = 4$

20 $\sqrt{509 - 21 \times 18.6} = 10.9$

21 $\sqrt{17.7 \times 30 \times 1.8} = 9.8$

22 $\sqrt{49.3 + 112 + 95} = 16$

23 $\sqrt{\frac{2\ 293.2}{5.2}} = 21$

Find these squares and cubes, answer to 2 d.p. where required.

24 $35^2 = 1\ 225$

25 $9.7^2 = 94.09$

26 $20.6^2 = 424.36$

27 $(3\frac{1}{2})^3 = 42.88$

28 $10.5^3 = 1\ 157.63$

29 $65^2 = 4\ 225$

30 $12^3 = 1\ 728$

31 $(4^3/_5)^3 = 97.34$

32 $8.09^3 = 529.48$

33 $19.4^2 + 10.9^3 = 1\ 671.39$

34 $18.1 \times 11^3 = 24\ 091.1$

35 $\frac{52.5^2}{14^3} = 1$

36 $25^2 - 12^3 = -1\ 103$

37 $34 + 6.2^3 \times 13^2 = 31\ 493.3$

38 $\sqrt{77} + 25^2 = 633.77$

39 $(\sqrt{66})^2 = 66$

40 $(11 - 26.2)^2 = 231.04$

41 $11 - 26.2^2 = -675.44$

42 $11^2 - 26.2^2 = -565.44$

43 $18^3 - 18^2 - 18 = 5\ 490$

44 $18^3 - 18^2 \times 18 = 0$

45 $100^3 \div 10^3 = 1\ 000$

46 $100^3 \div 10^3 \div 10^3 = 1$

Use the power key [^] for these. Answer to 2 d.p. if required.

47 $2^4 = 16$

48 $5^5 = 3\ 125$

49 $1^7 = 1$

50 $10^6 = 1\ 000\ 000$

51 $5.2^3 = 140.61$

52 $2^{12} = 4\ 096$

53 $(1^2/_7)^6 = 4.52$

54 $8^9 - 8^8 = 117\ 440\ 512$

55 $2^{15} \div 2^{13} = 4$

Use the cube root key [∛] to answer these. Express your answer to 2 d.p. when required

56 $\sqrt[3]{216} = 6$

57 $\sqrt[3]{343} = 7$

58 $\sqrt[3]{1\ 000} = 10$

Brackets now required

59 $\sqrt[3]{\frac{5\ 375}{43}} = 5$

60 $\sqrt[3]{452 \times 17.3} = 19.85$

61 $\sqrt[3]{88^3 + 88^3} = 110.87$

62 $\sqrt[3]{5\ 833 - 18^3} = 1$

Careful with this one

63 $\sqrt[3]{\frac{5^3 + 2^7}{11^2}} = 1.45$

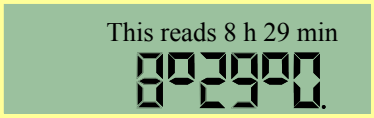
Time - Calculator Applications

Time calculations are performed every day, …..how long until lunch?, ….the bus arrives? When you become a wage earner it is important to be able to check that your hours worked is correct, but because minutes and hours are in groups of 60 min this is not always straight forward. But a calculator makes it easy.

The calculator image on the next page shows the DMS key or the 'bubble button' (the key has an orange border on the next page). This key allows you work with hours and minutes.

IMPORTANT YOU MUST ENTER A 0 (THEN DMS KEY) IF THERE IS ONLY MINUTES (NO HOURS).

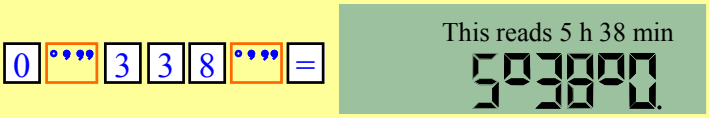
The calculator will always show the hours, minutes and seconds (we won't be using seconds other than in rounding) separated by a degree sign (a small raised o) →°.





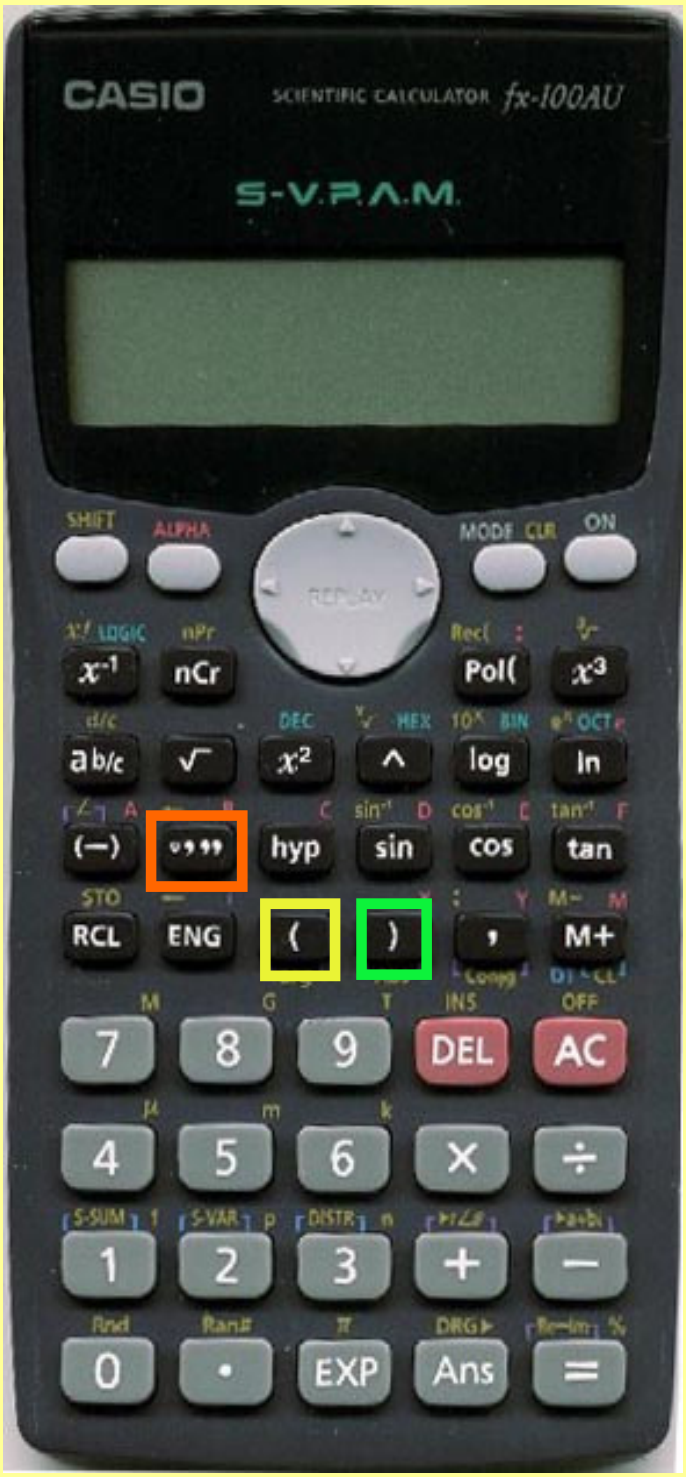


In column 1 you are asked to convert the times given in minutes to hours and minutes. This is done by pressing 0 then the DMS key, then the minutes in the question then the DMS key and then press = and the answer will be displayed. Example, change 338 min to hours and minutes.



Column 2 requires times in decimal format to be changed to h min format. Enter the decimal in your calculator then press the DMS key. Finding the difference between two times is done by subtraction. If times are in a.m. and p.m. then before the subtraction is undertaken you have to convert p.m. times to 24 h format. An example is above Q. 25.

The 3rd column is using × and ÷ these are done no differently just remember that you are multiplying by a number not a time and so only use the DMS on the time. The time sheet (Q.35) requires you to find the time difference for each day between start and finish times, put them in the spaces provided then add them. Monday is already done for you, check you get the same answer and don't forget to include it in your addition. Remember to use 24 h format for p.m. times. The calculation of an average time (Q 37) can be done in two ways refer to the text box below the calculator image for the full key stroke method on the next page.



Example of steps in calculating average time
of 11h 12 min, 8 h 57 min and 10 h 9 min

Using Brackets

(1 1 °’’’ 1 2 °’’’ + 8 °’’’ 5 7 °’’’ + 1 0 °’’’ 9 °’’’) ÷ 3 =

Using = to avoid order of operations

1 1 °’’’ 1 2 °’’’ + 8 °’’’ 5 7 °’’’ + 1 0 °’’’ 9 °’’’ = ÷ 3 =

if you get 10 h 6 min for the question above you are correct

Time - Calculator Applications

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Rewrite the calculator displays in h and min.

1 

2 

3 

4 

Convert the following times in min to h and min, there are two methods you can use.

Method 1

2 1 1 ÷ 6 0 = DMS ° ’ ’’

Method 2

0 DMS ° ’ ’’ 2 1 1 DMS ° ’ ’’ =

5 211 min

6 185 min

7 247 min

8 744 min

9 190 min

10 333 min

11 296 min

12 575 min

13 1 541 min

14 1 080 min

15 1 625 min

Change these times to hours and minutes.

5 • 4 = DMS ° ’ ’’

16 5.4 h

17 0.6 h

18 1.05 h

19 3.8 h

20 4.15 h

21 6.1 h

Perform the division which gives a decimal answer then change to hours and minutes. Round to the nearest minute.

22 22.4 ÷ 7

23 41 ÷ 8

24 165 ÷ 105

Find the time difference between these times that are on the same day. Use 24 hour time format.

Example

1853 h in 24 h format

11.07 a.m. and 6.53 p.m.

1 8 DMS ° ’ ’’ 5 3 DMS ° ’ ’’ -

1 1 DMS ° ’ ’’ 7 DMS ° ’ ’’ = 7 h 46 min

25 10.30 a.m. and 3.56 p.m.

26 3.52 a.m. and 11.27 a.m.

27 4.23 a.m. and 6.17 p.m.

28 1.09 p.m. and 10.57 p.m.

Now use × and ÷ with these times

29 8 × 23 min

30 7 × 1 h 46 min

31 1 h 55 min ÷ 5

32 19 h 57 min ÷ 9

33 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min

34 8 h 45 min ÷ 5 + 5 min × 17 + 1 h 31 min

Complete the time sheet below, then find the total hours worked

35

A. Jolie Time Sheet

Day	Start	Finish	Hours
Mon	9.30 a.m.	4.45 p.m.	7 h 15 min
Tues	10.45 a.m.	2.30 p.m.	h min
Wed	9.30 a.m.	5.00 p.m.	h min
Thur	9.30 a.m.	3.45 p.m.	h min
Fri	1.15 p.m.	6.00 p.m.	h min
		Total hours for the week	h min

Try these time problems

36 Sean watches 3 movies at a 6 hour movie marathon, if two films were 1 h 46 min and 1 h 50 min, find the duration of the other film.

37 Julie's time for return travel to school for 3 days were: 1h 15 min, 1h 22 min and 56 min. Find the average time for her return trip.

Time - Calculator Applications

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Rewrite the calculator displays in h and min.

1  0 h 43 min

2  3 h 7 min

3  6 h 55 min

4  8 h 19 min

Convert the following times in min to h and min, there are two methods you can use.

Method 1

2 1 1 ÷ 6 0 = DMS ° ’ ’’

Method 2

0 DMS ° ’ ’’ 2 1 1 DMS ° ’ ’’ =

5 211 min — 3 h 31 min

6 185 min — 3 h 5 min

7 247 min — 4 h 7 min

8 744 min — 12 h 24 min

9 190 min — 3 h 10 min

10 333 min — 5 h 33 min

11 296 min — 4 h 56 min

12 575 min — 9 h 35 min

13 1 541 min — 25 h 41 min

14 1 080 min — 18 h

15 1 625 min — 27 h 5 min

Change these times to hours and minutes.

5 • 4 = DMS ° ’ ’’

16 5.4 h — 5 h 24 min

17 0.6 h — 36 min

18 1.05 h — 1 h 3 min

19 3.8 h — 3 h 48 min

20 4.15 h — 4 h 9 min

21 6.1 h — 6 h 6 min

Perform the division which gives a decimal answer then change to hours and minutes. Round to the nearest minute.

22 22.4 ÷ 7 — 3 h 12 min

23 41 ÷ 8 — 5 h 8 min (Nearest min)

24 165 ÷ 105 — 1 h 34 min (Nearest min)

Find the time difference between these times that are on the same day. Use 24 hour time format.

Example

1853 h in 24 h format

11.07 a.m. and 6.53 p.m.

1 8 DMS ° ’ ’’ 5 3 DMS ° ’ ’’ -

1 1 DMS ° ’ ’’ 7 DMS ° ’ ’’ = 7 h 46 min

25 10.30 a.m. and 3.56 p.m.

5 h 26 min

26 3.52 a.m. and 11.27 a.m.

7 h 35 min

27 4.23 a.m. and 6.17 p.m.

13 h 54 min

28 1.09 p.m. and 10.57 p.m.

9 h 48 min

Now use × and ÷ with these times

29 8 × 23 min — 3 h 4 min

30 7 × 1 h 46 min — 12 h 22 min

31 1 h 55 min ÷ 5 — 23 min

32 19 h 57 min ÷ 9 — 2 h 13 min

33 54 min × 6 + 3 h 7 min ÷ 11 + 1 h 27 min — 3 h 12 min

34 8 h 45 min ÷ 5 + 5 min × 17 + 1 h 31 min — 4 h 41 min

Complete the time sheet below, then find the total hours worked

35

A. Jolie Time Sheet

Day	Start	Finish	Hours
Mon	9.30 a.m.	4.45 p.m.	7 h 15 min
Tues	10.45 a.m.	2.30 p.m.	3 h 45 min
Wed	9.30 a.m.	5.00 p.m.	7 h 30 min
Thur	9.30 a.m.	3.45 p.m.	6 h 15 min
Fri	1.15 p.m.	6.00 p.m.	4 h 45 min
		Total hours for the week	29 h 30 min

Try these time problems

36 Sean watches 3 movies at a 6 hour movie marathon, if two films were 1 h 46 min and 1 h 50 min, find the duration of the other film.

2 h 24 min

37 Julie's time for return travel to school for 3 days were: 1h 15 min, 1h 22 min and 56 min. Find the average time for her return trip.

1 h 11 min

Calculator - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Calculator - Find A Word

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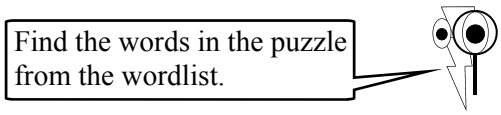

E	C	B	S	M	B	R	A	C	S	S	E	R	P	I
V	A	Y	U	H	U	U	Q	L	I	R	E	V	M	E
I	L	R	B	T	I	L	I	A	E	I	U	P	T	T
T	C	E	T	V	E	F	T	M	E	R	R	O	I	E
Q	U	T	R	A	R	V	T	I	V	O	A	V	H	L
U	L	T	A	D	E	B	U	C	P	V	E	U	B	E
A	A	A	C	D	P	B	R	E	R	L	B	R	Q	D
N	T	B	T	I	L	R	R	D	N	U	Y	R	A	S
O	E	V	I	T	A	G	E	N	T	G	E	T	E	C
I	K	K	O	I	Y	C	A	T	O	O	R	T	D	P
T	E	R	N	O	K	E	O	A	I	R	B	A	I	O
C	C	A	C	N	T	N	I	V	E	M	X	B	V	W
A	R	B	K	E	T	A	U	L	A	V	E	T	I	E
R	A	R	E	T	C	U	B	R	A	B	A	T	D	R
F	B	R	A	C	K	E	T	S	E	T	U	N	I	M

WORDLIST

CALCULATE	TIME	MINUTES	BUTTON	MULTIPLY
EVALUATE	IMPROPER	DIVIDE	SHIFT	PRESS
NEGATIVE	DECIMAL	SQUARE	ROOT	REPLAY
ADDITION	CUBED	FRACTION	HOURS	POWER
SUBTRACTION	ENGRAVE	BATTERY	BRACKETS	DELETE

Calculator - Find A Word

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Find the words in the puzzle from the wordlist.



E	C	B	S	M	B	R	A	C	S	S	E	R	P	I
V	A	Y	U	H	U	U	Q	L	I	R	E	V	M	E
I	L	R	B	T	I	L	I	A	E	I	U	P	T	T
T	C	E	T	V	E	F	T	M	E	R	R	O	I	E
Q	U	T	R	A	R	V	T	I	V	O	A	V	H	L
U	L	T	A	D	E	B	U	C	P	V	E	U	B	E
A	A	A	C	D	P	B	R	E	R	L	B	R	Q	D
N	T	B	T	I	L	R	R	D	N	U	Y	R	A	S
O	E	V	I	T	A	G	E	N	T	G	E	T	E	C
I	K	K	O	I	Y	C	A	T	O	O	R	T	D	P
T	E	R	N	O	K	E	O	A	I	R	B	A	I	O
C	C	A	C	N	T	N	I	V	E	M	X	B	V	W
A	R	B	K	E	T	A	U	L	A	V	E	T	I	E
R	A	R	E	T	C	U	B	R	A	B	A	T	D	R
F	B	R	A	C	K	E	T	S	E	T	U	N	I	M

WORDLIST

~~CALCULATE~~ ~~TIME~~ ~~MINUTES~~ ~~BUTTON~~ ~~MULTIPLY~~

~~EVALUATE~~ ~~IMPROPER~~ ~~DIVIDE~~ ~~SHIFT~~ ~~PRESS~~

~~NEGATIVE~~ ~~DECIMAL~~ ~~SQUARE~~ ~~ROOT~~ ~~REPLAY~~

~~ADDITION~~ ~~CUBED~~ ~~FRACTION~~ ~~HOURS~~ ~~POWER~~

~~SUBTRACTION~~ ~~ENGRAVE~~ ~~BATTERY~~ ~~BRACKETS~~ ~~DELETE~~

FREEFALL MATHEMATICS

ALTITUDE

PERCENTAGES

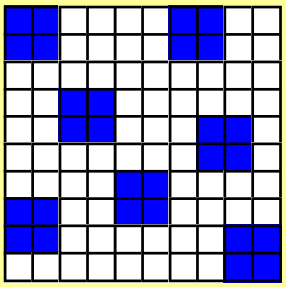
Fractions to Percentages

To convert a fraction to a percentage you multiply by 100 (×100). So that is pretty straight forward. The only difference is when it gets easier! If the fraction has a denominator of 100, then the percentage is simply the numerator. Or in non-mathematical speak, when the bottom number is 100, the percentage is the top number.

Column 1 starts with fractions that have a denominator of 100, you are asked to change them to percentages.

Example

There are 28 squares shaded and as this is a 100 grid, there are 100 - 28 = 72 72 squares unshaded.



Shaded = $\frac{28}{100}$ = 28 %

Unshaded = $\frac{72}{100}$ = 72 %

Denominator is 100 so that means the % is the numerator.

Column 2 involves fractions that have denominators that aren't 100. So with these put the fraction into your calculator and multiply by 100.

Examples $\frac{17}{32}$ = 53.1 % $\frac{15}{7}$ = 214.3 % $1\frac{5}{9}$ = 155.6 %

Using your calculator

[1][7][a b/c][3][2][×][1][0][0][=]

Answer on Screen 53⌟ 1⌟ 8

[a b/c]

Answer on Screen 53.125
Round to 1 decimal place
53.1%

Using your calculator

[1][5][a b/c][7][×][1][0][0][=]

Answer on Screen 214⌟ 2⌟ 7

[a b/c]

Answer on Screen
214.2857143
Round to 1 decimal place
214.3%

Using your calculator

[1][a b/c][5][a b/c][9][×][1][0][0][=]

Answer on Screen 155⌟ 5⌟ 9

[a b/c]

Answer on Screen
155.5555556
Round to 1 decimal place
155.6%

The second part of Column 2 asks you to find a mean percentage. Mean is "the average" so change both fractions to percentages then add them together and divide by 2. There are only two marks in each question so you only divide by two. When you divide by two you must press "=" first then divide by two, or you can use brackets.

Example. Mean (average) of 58.5% and 72.2%.

Using your calculator

Note 2 use of =

[5][8][.][5][+][7][2][.][2][=][÷][2][=]

Or with brackets

[(][5][8][.][5][+][7][2][.][2][)][÷][2][=]

Column 3 starts with changing fractions to percentages. Percentages can be used to compare fractions. Remember Descending (Down), so Ascending means to go up, from lowest to highest.

Fractions to Percentages

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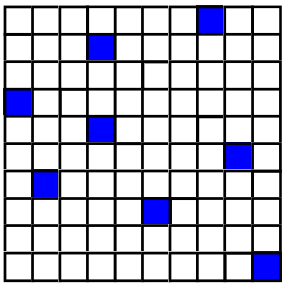
Find the shaded and un-shaded fractions of these 100 grid squares. Then write them as percentages.

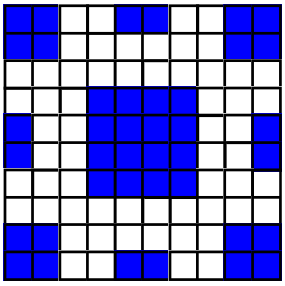
1



Shaded = $\frac{\square}{100}$ Unshaded = $\frac{\square}{100}$

= % = %

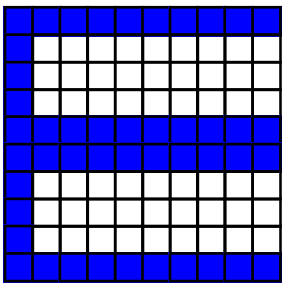
2



Shaded = $\frac{\square}{\square}$ Unshaded = $\frac{\square}{\square}$

= % = %

3



Shaded = $\frac{\square}{\square}$ Unshaded = $\frac{\square}{\square}$

= =

Change these fractions to percentages.

4 $\frac{75}{100}$ = ____

5 $\frac{38}{100}$ = ____

6 $\frac{93}{100}$ = ____

Multiply by 100 to change the fractions to percentages. Round to 1 d.p. if needed.

7 $\frac{16}{25}$ = ____ %

8 $\frac{3}{4}$ = ____ %

9 $\frac{30}{45}$ = ____ %

10 $\frac{5}{8}$ = ____ %

11 $1\frac{2}{5}$ = ____

12 $\frac{15}{11}$ = ____

13 $\frac{54}{78}$ = ____

14 $3\frac{5}{8}$ = ____

15 $\frac{43}{43}$ = ____

16 $\frac{255}{300}$ = ____

Each student sat two tests. Convert each to a % and find the mean percentage.

17 Eric 1: $\frac{15}{25}$ = ____ % 2: $\frac{16}{25}$ = ____ %

Add together and write here

$\bar{x} = \frac{\square}{2}$ = ____ %

18 Jess 1: $\frac{8}{10}$ = ____ 2: $\frac{34}{50}$ = ____

$\bar{x}$ = ____ = ____

19 Kurt 1: $\frac{11}{13}$ = ____ 2: $\frac{17}{38}$ = ____

$\bar{x}$ = ____ = ____

Use percentages to arrange these test results (fractions) in ascending order. (1 d.p.)

20 $\frac{14}{20}$, $\frac{23}{40}$, $\frac{17}{50}$, $\frac{19}{26}$

____ % ____ % ____ % ____ %

Ascending Order ____, ____, ____, ____

21 $\frac{6}{11}$, $\frac{43}{90}$, $\frac{26}{50}$, $\frac{17}{37}$

____ ____ ____ ____

Ascending Order ____, ____, ____, ____

22 $\frac{38}{41}$, $\frac{47}{50}$, $\frac{345}{360}$, $\frac{19}{20}$

____ ____ ____ ____

Ascending Order ____, ____, ____, ____

Now try these. Round all answers to 1 d.p.

23 Rod scored $^{14}/_{22}$ and Ally scored $^{15}/_{23}$. Who attained the highest %, and by how much?

R: ____ A: ____

= % = %

- =

24 April scored $^{55}/_{72}$ and Jacob scored $^{27}/_{34}$. Who attained the highest %, and by how much?

A: ____ J: ____

= =

Fractions to Percentages

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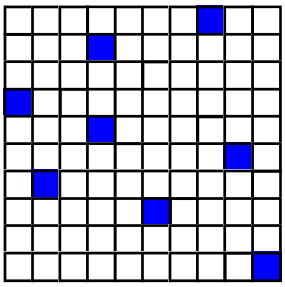
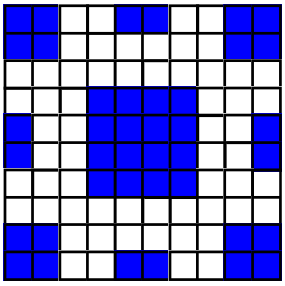
Find the shaded and un-shaded fractions of these 100 grid squares. Then write them as percentages.

1



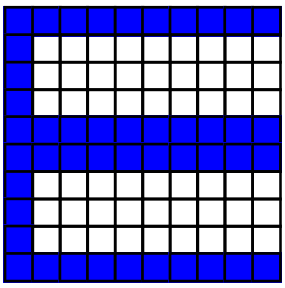
Shaded = $\frac{8}{100}$ = 8 % Unshaded = $\frac{92}{100}$ = 92 %

2



Shaded = $\frac{40}{100}$ = 40 % Unshaded = $\frac{60}{100}$ = 60 %

3



Shaded = $\frac{46}{100}$ = 46 % Unshaded = $\frac{54}{100}$ = 54 %

Change these fractions to percentages.

4 $\frac{75}{100}$ = 75 %

5 $\frac{38}{100}$ = 38 %

6 $\frac{93}{100}$ = 93 %

Multiply by 100 to change the fractions to percentages. Round to 1 d.p. if needed.

7 $\frac{16}{25}$ = 64 %

8 $\frac{3}{4}$ = 75 %

9 $\frac{30}{45}$ = 66.6 %

10 $\frac{5}{8}$ = 62.5 %

11 $1\frac{2}{5}$ = 140 %

12 $\frac{15}{11}$ = 136.4 %

13 $\frac{54}{78}$ = 69.2 %

14 $3\frac{5}{8}$ = 362.5 %

15 $\frac{43}{43}$ = 100 %

16 $\frac{255}{300}$ = 85 %

Each student sat two tests. Convert each to a % and find the mean percentage.

17 Eric 1: $\frac{15}{25}$ = 60 % 2: $\frac{16}{25}$ = 64 %

Add together and write here

$\bar{x} = \frac{124}{2}$ = 62 %

18 Jess 1: $\frac{8}{10}$ = 80 % 2: $\frac{34}{50}$ = 68 %

$\bar{x} = \frac{148}{2}$ = 74 %

19 Kurt 1: $\frac{11}{13}$ = 84.6 % 2: $\frac{17}{38}$ = 44.7 %

$\bar{x} = \frac{129.3}{2}$ = 64.65% (64.7)

Use percentages to arrange these test results (fractions) in ascending order. (1 d.p.)

20 $\frac{14}{20}$, $\frac{23}{40}$, $\frac{17}{50}$, $\frac{19}{26}$

70 % 57.5 % 34 % 73.1 %

Ascending Order → $\frac{17}{50}$, $\frac{23}{40}$, $\frac{14}{20}$, $\frac{19}{26}$

21 $\frac{6}{11}$, $\frac{43}{90}$, $\frac{26}{50}$, $\frac{17}{37}$

54.5 % 47.8 % 52 % 45.9 %

Ascending Order → $\frac{17}{37}$, $\frac{43}{90}$, $\frac{26}{50}$, $\frac{6}{11}$

22 $\frac{38}{41}$, $\frac{47}{50}$, $\frac{345}{360}$, $\frac{19}{20}$

92.7 % 94 % 95.8 % 95 %

Ascending Order → $\frac{38}{41}$, $\frac{47}{50}$, $\frac{19}{20}$, $\frac{345}{360}$

Now try these. Round all answers to 1 d.p.

23 Rod scored $^{14}/_{22}$ and Ally scored $^{15}/_{23}$. Who attained the highest %, and by how much?

R: $\frac{14}{22}$ = 63.6 % A: $\frac{15}{23}$ = 65.2 %

65.2 - 63.6 = 1.6 %

Ally scored highest by 1.6 %.

24 April scored $^{55}/_{72}$ and Jacob scored $^{27}/_{34}$. Who attained the highest %, and by how much?

A: $\frac{55}{72}$ = 76.4 % J: $\frac{27}{34}$ = 79.4 %

79.4 - 76.4 = 3%

Jacob scored highest by 3 %.

Application of Fractions to Percentages

To change a fraction to a percentage you multiply by 100 (× 100). The questions on this worksheet are different because you have to find the fraction first. Once found, multiply it by 100.

Column 1 starts with measuring the length of pencils. If the original length is 95 mm, this number is the denominator (or bottom number) of the fraction. Then measure the length of the pencils and the length (in mm) is the numerator, or top part of the fraction. Now multiply the fraction you have made by 100 to convert to a percentage.

The next part of column 1 is a vertical bar graph. The full height of the graph is the denominator. The height of the red section is the numerator. Convert to a percentage.

Column 2 starts with the four different types of tiles. List how many of each tile there is by counting them and striking them out as you go. Then answer the questions using the data you have collected.

The last part of column 2 is about the percentage of syrup in a milkshake. The amounts of syrup and milk are given in each question, realise that the denominator is found by adding the syrup and the milk together, it isn't just the amount of milk.

Application of Fractions to Percentages

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John loves measuring Mini-Golf pencils. If new they are 95 mm long, find the length of the pencils below as a percentage of the original length (1 d.p.)

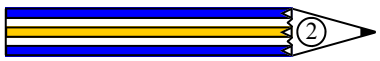
1



Write the length in mm in box and calculate %

$\frac{\square}{95} \times 100 = \underline{\quad}\ \%$

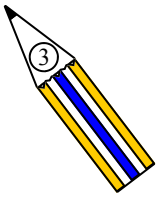
2



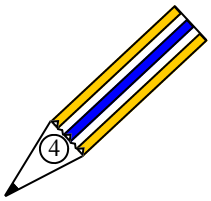
$\frac{\square}{95} \times 100$

$= \underline{\quad}\ \%$

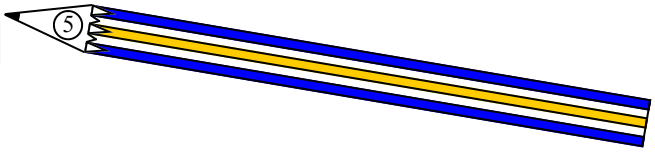
3



4

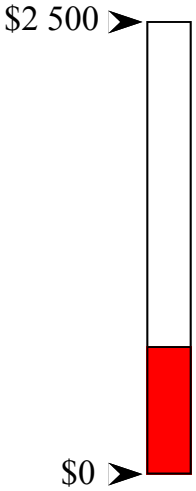


5



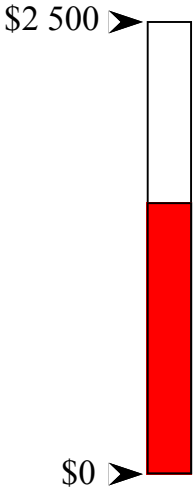
A soccer club is raising $2 500 for better lighting. A scale is displayed showing the funding progress. Find the % achieved at the end of the following months. (1 d.p.)

6



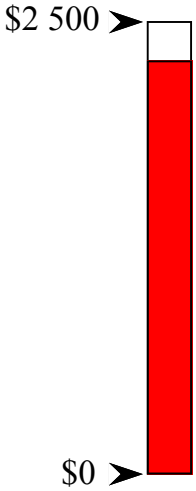
March

7



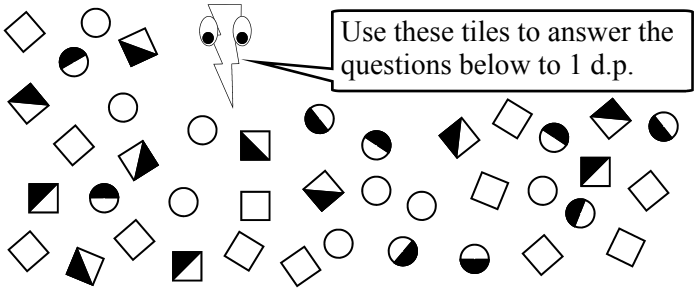
June

8

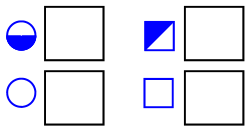


October

Use these tiles to answer the questions below to 1 d.p.



9 What % of the tiles are half-filled?



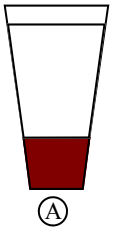
10 What % of the unfilled tiles are round?

11 What % of the half-filled tiles are square?

12 What % of the tiles are round?

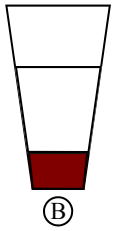
13 What % of the tiles are unfilled and square?

Max is making different milkshakes. Find the percentage of syrup used in each to the nearest whole number. Rank them from strongest to weakest flavour. (1, 2, then 3)



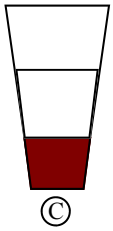
Syrup = 85 mL
Milk = 340 mL.

14 Rank ___



Syrup = 65 mL
Milk = 200 mL.

15 Rank ___



Syrup = 85 mL
Milk = 175 mL.

16 Rank ___

\+

=

Application of Fractions to Percentages

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John loves measuring Mini-Golf pencils. If new they are 95 mm long, find the length of the pencils below as a percentage of the original length (1 d.p.)

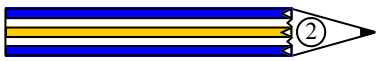
1



Write the length in mm in box and calculate %

$\frac{78}{95} \times 100 = 82.1\ \%$

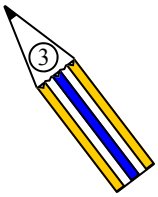
2



$\frac{49}{95} \times 100$

$= 51.6\ \%$

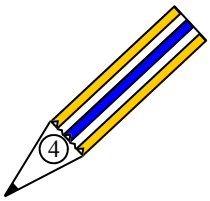
3



$\frac{28}{95} \times 100$

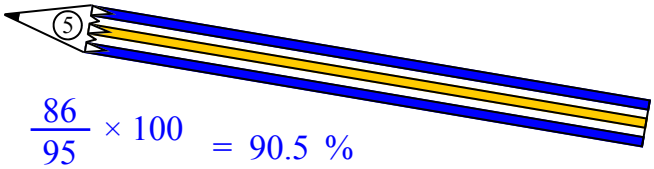
$= 29.5\ \%$

4



$\frac{33}{95} \times 100 = 34.7\ \%$

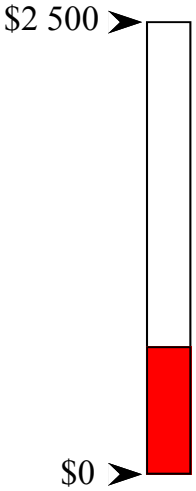
5



$\frac{86}{95} \times 100 = 90.5\ \%$

A soccer club is raising $2 500 for better lighting. A scale is displayed showing the funding progress. Find the % achieved at the end of the following months. (1 d.p.)

6

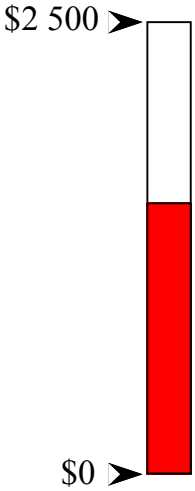


March

$\frac{17}{60} \times 100$

$= 28.3\ \%$

7

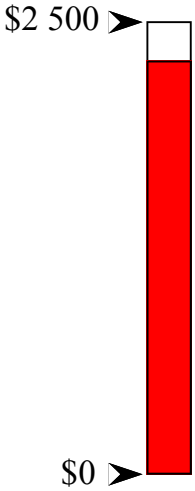


June

$\frac{36}{60} \times 100$

$= 60\%$

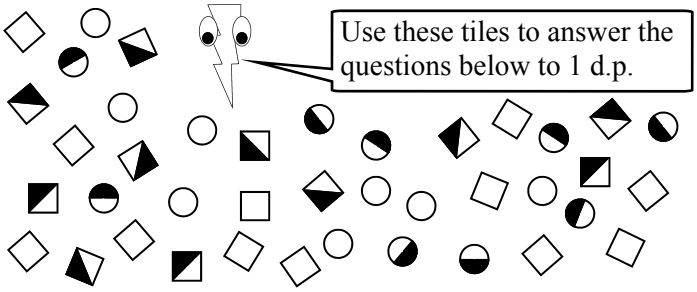
8



October

$\frac{54}{60} \times 100$

$= 90\%$



Use these tiles to answer the questions below to 1 d.p.

9 What % of the tiles are half-filled?

$\frac{20}{40} \times 100 = 50\%$

9 ◒ ◪ 11
8 ○ □ 12
Total = 40

10 What % of the unfilled tiles are round?

$\frac{8}{20} \times 100 = 40\%$

11 What % of the half-filled tiles are square?

$\frac{11}{20} \times 100 = 55\%$

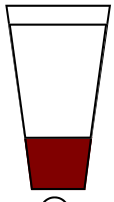
12 What % of the tiles are round?

$\frac{17}{40} \times 100 = 42.5\ \%$

13 What % of the tiles are unfilled and square?

$\frac{12}{40} \times 100 = 30\%$

Max is making different milkshakes. Find the percentage of syrup used in each to the nearest whole number. Rank them from strongest to weakest flavour. (1, 2, then 3)

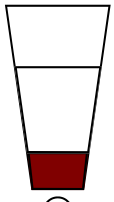


Syrup = 85 mL
Milk = 340 mL.

14 Rank ③

$85 + 340 = 425$ mL

$\frac{85}{425} \times 100 = 20\ \%$

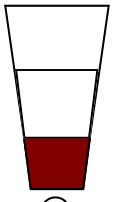


Syrup = 65 mL
Milk = 200 mL.

15 Rank ②

$65 + 200 = 265$ mL

$\frac{65}{265} \times 100 = 24.5\ \%$



Syrup = 85 mL
Milk = 175 mL.

16 Rank ①

$85 + 175 = 260$ mL

$\frac{85}{260} \times 100 = 32.7\ \%$

Percentages, Decimals and Fractions

This sheet is in two halves. The first half deals with changing between decimals and percentages. The second half is changing between decimals, fractions and percentages.

Column 1 starts with changing decimals to percentages. All the questions are answered by multiplying the decimal by 100. So type the number into your calculator then multiply by 100. A quicker way is to move the decimal point 2 places to the right, you won't even need a calculator!

Note that some decimals are shown with a dot above the number, this means that the number repeats itself. These are called repeating or recurring decimals.

Example $0.\dot{4} = 0.4444444444\ldots$
so multiply by 100
$= 44.4444444\ldots$
$= 44.4\%$ (1 d.p.)

Questions 11-20 are the opposite, you are changing percentages to decimals, so this time divide by 100. Questions 21-23 has decimals mixed up with percentages and you have to sort them out. Convert them all to decimals with the same number of decimal places on the line below. Then using those decimals as a guide rewrite them in ascending order. Remember to write the decimals and percentages in order, not the converted decimals as this doesn't answer the question.

Column 2 deals with changing decimals to fractions. If a decimal has one decimal place, then write it over 10. If it has two decimal places, write it over 100. Three places...1 000 and so on.

$$0.8 = \frac{8}{10} = \frac{4}{5} \qquad \div 2$$

Decimal: 1 Decimal Place

$$0.48 = \frac{48}{100} = \frac{12}{25} \qquad \div 4$$

Decimal: 2 Decimal Places

With the table in column 2, to change a percentage to a decimal, divide by 100. To change a fraction to a percentage multiply by 100, likewise with decimals to percentages, multiply by 100.

Percentages, Decimals and Fractions

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To change a decimal to a percentage, multiply the decimal by 100.

Example $0.73 = 0.73 \times 100 = 73\%$

1 0.6 = ____ %

2 0.93 = ____ %

3 2.05 ____

4 0.07 ____

5 1 ____

6 0.005 ____

7 0.75 ____

8 3.5 ____

9 $0.\dot{3}$ ____

10 $0.\dot{6}$ ____

Round answer to 1 d.p.

Round answer to 1 d.p.

To change a percentage to a decimal, divide the percentage by 100.

Example $53.6\% = 53.6 \div 100 = 0.536$

11 48% = ____

12 95% = ____

13 10% ____

14 ½% ____

15 5.3% ____

16 100% ____

17 8½% ____

18 25⅜% ____

19 40.1% ____

20 205% ____

Arrange the decimals and percentages in descending order.

21 48%, 0.409, 4.7, 49.8%, 0.49, 0.4

0.480, 0.409,

Rewrite all as decimals with the same number of decimal places

Rewrite in descending order, remember to write as given in the question

22 13%, 0.31, 30.1%, 1.13, 100%, 0.114

23 5.6, 65%, 6.5%, 0.55, 0.055, 115%

Change these decimals to fractions.

Example 1

$0.40 = \frac{40}{100} = \frac{2}{5}$

Using your calculator

0 . 4 0 = $a^b/_c$

Example 2

$2.25 = 2 + \frac{25}{100} = 2\frac{1}{4}$

Using your calculator

2 . 2 5 = $a^b/_c$

25 0.2 = ☐/☐ = ☐/☐

26 0.15 = ☐/☐ = ☐/☐

27 0.6 = ☐/☐ = ☐/☐

28 0.72 = ☐/☐ = ☐/☐

29 3.36 = ☐ + ☐/☐ = ☐ ☐/☐

30 0.5 = ☐/☐ = ☐/☐

31 0.505 = ☐/☐ = ☐/☐

Complete the table by converting between fractions, decimals and percentages.

Q	Fraction	Decimal	Percentage
32		0.2	
33		0.55	
34			32%
35		1.6	
36			5%
37	$\frac{19}{100}$		
38	$\frac{17}{25}$		
39			240%
40	$\frac{4}{5}$		
41			75%
42	$\frac{1}{3}$		
43		0.005	
44		0.94	
45			66⅔%
46	$1\frac{5}{8}$		

Percentages, Decimals and Fractions

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To change a decimal to a percentage, multiply the decimal by 100.

Example $0.73 = 0.73 \times 100 = 73\%$

1 0.6 = 60%

2 0.93 = 93%

3 2.05 = 205%

4 0.07 = 7%

5 1 = 100%

6 0.005 = 0.5%

7 0.75 = 75%

8 3.5 = 350%

9 $0.\dot{3}$ = 33.3%

10 $0.\dot{6}$ = 66.7%

Round answer to 1 d.p.

Round answer to 1 d.p.

To change a percentage to a decimal, divide the percentage by 100.

Example $53.6\% = 53.6 \div 100 = 0.536$

11 48% = 0.48

12 95% = 0.95

13 10% = 0.1

14 ½% = 0.005

15 5.3% = 0.053

16 100% = 1

17 8½% = 0.085

18 25⅜% = 0.25375

19 40.1% = 0.401

20 205% = 2.05

Arrange the decimals and percentages in descending order.

21 48%, 0.409, 4.7, 49.8%, 0.49, 0.4

0.480, 0.409, 4.700, 0.498, 0.490, 0.400

Rewrite all as decimals with the same number of decimal places

4.7, 49.8%, 0.49, 48%, 0.409, 0.4

Rewrite in descending order, remember to write as given in the question

22 13%, 0.31, 30.1%, 1.13, 100%, 0.114

0.130, 0.310, 0.301, 1.130, 1, 0.114

1.13, 100%, 0.31, 30.1%, 13%, 0.114

23 5.6, 65%, 6.5%, 0.55, 0.055, 115%

5.600, 0.650, 0.065, 0.550, 0.055, 1.150

5.6, 115%, 65%, 0.55, 6.5%, 0.055

Change these decimals to fractions.

Example 1

$0.40 = \frac{40}{100} = \frac{2}{5}$

Using your calculator

0 . 4 0 = $a^b/_c$

Example 2

$2.25 = 2 + \frac{25}{100} = 2\frac{1}{4}$

Using your calculator

2 . 2 5 = $a^b/_c$

25 0.2 = $\frac{2}{10} = \frac{1}{5}$

26 0.15 = $\frac{15}{100} = \frac{3}{20}$

27 0.6 = $\frac{6}{10} = \frac{3}{5}$

28 0.72 = $\frac{72}{100} = \frac{18}{25}$

29 3.36 = $3 + \frac{36}{100} = 3\frac{9}{25}$

30 0.5 = $\frac{5}{10} = \frac{1}{2}$

31 0.505 = $\frac{505}{1\,000} = \frac{101}{200}$

Complete the table by converting between fractions, decimals and percentages.

Q	Fraction	Decimal	Percentage
32	$\frac{1}{5}$	0.2	20%
33	$\frac{11}{20}$	0.55	55%
34	$\frac{8}{25}$	0.32	32%
35	$1\frac{3}{5}$	1.6	160%
36	$\frac{1}{20}$	0.05	5%
37	$\frac{19}{100}$	0.19	19%
38	$\frac{17}{25}$	0.68	68%
39	$1\frac{2}{5}$	2.4	240%
40	$\frac{4}{5}$	0.8	80%
41	$\frac{3}{4}$	0.75	75%
42	$\frac{1}{3}$	$0.\dot{3}$	33⅓%
43	$\frac{1}{200}$	0.005	0.5%
44	$\frac{47}{50}$	0.94	94%
45	$\frac{2}{3}$	$0.\dot{6}$	66⅔%
46	$1\frac{5}{8}$	1.625	162.5%

Percentage of a Quantity

This sheet involves comparing two amounts and finding the percentage one is of the other, as well as finding the percentage of a given quantity.

Column 1 starts with comparing two amounts. This is the same as converting a fraction to a percentage, so all you have to do is multiply by 100.

Example 18 targets from 24 targets

$$\frac{18}{24} \times 100 = 75\%$$

The answer to these questions is a percentage.

Column 2 gives a single quantity and a percentage. To answer these change the percentage to a fraction and multiply by the amount.

Example 62% of 225 L.

$$\frac{62}{100} \times 225 = 139.5 \text{ L}$$

The answer to these questions is a quantity so units are needed

Column 3 are worded questions which are solved by either of the two methods above.

Percentage of a Quantity

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Express the 1st quantity as a percentage of the 2nd. Round to 1 decimal place.

Example. 5 cm of 65 cm

$\frac{5}{65} \times 100$ $= 7.7\ \%$

1 27 points from 35 points.

2 5 pizza slices from 8 slices

3 19 t from 156 t.

4 120 posts from 435 posts.

5 38 frogs out of 196 frogs.

6 \$1 850 from \$7 216.

7 7.6 cm out of 12 cm.

8 650 g of 910 g.

9 14 s out of 15 s.

10 46 ha of 70 ha.

Find the percentage of these quantities. Round to the nearest whole number.

Example. 17% of 300 kg

$\frac{17}{100} \times 300$ $= 51$ kg

11 74% of 820 m.

$\frac{\quad}{\quad} \times 820$

Write % over 100 Remember units

12 45% of 710 mm.

13 85% of 150 kg.

14 15% of \$7 000.

15 125% of 400 mg.

16 71% of 48 m^2.

17 12.5% of \$63.

18 55% of 1 152 kg.

19 98% of 400 m.

20 0.2% of \$120 000.

Answer these, rounding to 1 d.p. when required.

21 38 500 vehicles pass an intersection in 12 hours. If 2% are buses and 87% are cars, find the amount of buses and cars.

Buses:

Cars:

22 The school shot put record is 19 m. If Ian threw a distance of 11.6 m what percentage of the record distance did he throw?

23 Tina has a four coloured spinner. She spins RED 22% of the time, how many times should red occur from 500 spins?

24 \$2 600 is to be split in the following way. Matt 20%, Bess 35% and Karen 45%. Find the amount each will earn.

Matt:

Bess:

Karen:

Percentage of a Quantity

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Express the 1st quantity as a percentage of the 2nd. Round to 1 decimal place.

Example. 5 cm of 65 cm

$\frac{5}{65} \times 100$ $= 7.7\ \%$

1 27 points from 35 points.

$\frac{27}{35} \times 100$ $= 77.1\ \%$

2 5 pizza slices from 8 slices

$\frac{5}{8} \times 100$ $= 62.5\ \%$

3 19 t from 156 t.

$\frac{19}{156} \times 100$ $= 12.2\ \%$

4 120 posts from 435 posts.

$\frac{120}{435} \times 100$ $= 27.6\ \%$

5 38 frogs out of 196 frogs.

$\frac{38}{196} \times 100$ $= 19.4\ \%$

6 \$1 850 from \$7 216.

$\frac{1\,850}{7\,216} \times 100$ $= 25.6\ \%$

7 7.6 cm out of 12 cm.

$\frac{7.6}{12} \times 100$ $= 63.3\ \%$

8 650 g of 910 g.

$\frac{650}{910} \times 100$ $= 71.4\ \%$

9 14 s out of 15 s.

$\frac{14}{15} \times 100$ $= 93.3\ \%$

10 46 ha of 70 ha.

$\frac{46}{70} \times 100$ $= 65.7\ \%$

Find the percentage of these quantities. Round to the nearest whole number.

Example. 17% of 300 kg

$\frac{17}{100} \times 300$ $= 51$ kg

11 74% of 820 m.

$\frac{74}{100} \times 820$ $= 607$ m

Write % over 100 — Remember units

12 45% of 710 mm.

$\frac{45}{100} \times 710$ $= 320$ mm

13 85% of 150 kg.

$\frac{85}{100} \times 150$ $= 128$ kg

14 15% of \$7 000.

$\frac{15}{100} \times 7\,000$ $= \$1\,050$

15 125% of 400 mg.

$\frac{125}{100} \times 400$ $= 500$ mg

16 71% of 48 m^2.

$\frac{71}{100} \times 48$ $= 34\ m^2$

17 12.5% of \$63.

$\frac{12.5}{100} \times 63$ $= \$8$

18 55% of 1 152 kg.

$\frac{55}{100} \times 1\,152$ $= 634$ kg

19 98% of 400 m.

$\frac{98}{100} \times 400$ $= 392$ m

20 0.2% of \$120 000.

$\frac{0.2}{100} \times 120\,000$ $= \$240$

Answer these, rounding to 1 d.p. when required.

21 38 500 vehicles pass an intersection in 12 hours. If 2% are buses and 67% are cars, find the amount of buses and cars.

Buses: $\frac{2}{100} \times 38\,500$

$= 770$ buses

Cars: $\frac{87}{100} \times 38\,500$

$= 33\,495$ cars

22 The school shot put record is 19 m. If Ian threw a distance of 11.6 m what percentage of the record distance did he throw?

$\frac{11.6}{19} \times 100$ $= 61.1\ \%$

23 Tina has a four coloured spinner. She spins RED 22% of the time, how many times should red occur from 500 spins?

$\frac{22}{100} \times 500$

$= 110$ times

24 \$2 600 is to be split in the following way. Matt 20%, Bess 35% and Karen 45%. Find the amount each will earn.

Matt: $\frac{20}{100} \times 2\,600$

$= \$520$

Bess: $\frac{35}{100} \times 2\,600$

$= \$910$

Karen: $\frac{45}{100} \times 2\,600$

$= \$1\,170$

Further Percentage of a Quantity

This sheet involves comparing two amounts and finding the percentage one is of the other, or finding the percentage of a given quantity.

Column 1 starts with changing units from the given unit to the one in brackets. Then from Question 9 two different units are to be compared. It usually the case that the larger unit is changed to the smaller unit, but it can be done in the opposite way with most questions also.

Example 186 m from 3.2 km

3.2 km = 3 200 m

$$\frac{186}{3\,200} \times 100 = 5.8\%$$

The answer to these questions is a percentage.
Round your answer to 1 decimal place.

Question 15 and 16 are the same style of question, the only difference is that you have to calculate the two areas before you can proceed to compare them.

Column 2 gives a single quantity and a percentage. To answer these change the percentage to a fraction and multiply by the amount.

From Question 17 a percentage of a quantity is to be found. To answer these change the percentage to a fraction over 100 then multiply by the quantity. Note that the quantity has to be changed into the units specified.

Example 65% of 0.86 kL. [L]

$$\frac{65}{100} \times 860 = 559 \text{ L}$$

0.86 kL = 860 L

The answer to these questions is a quantity so units are needed.
Round your answer, if needed, to the nearest whole number.

Further Percentage of a Quantity

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Convert these units to the one shown in brackets.

1 7 min = _____ [s]

2 5.2 L = _____ [mL]

3 17 m = _____ [cm]

4 8 weeks = _____ [days]

5 3.25 t = _____ [kg]

6 0.6 kg = _____ [g]

7 230 mm = _____ [m]

8 8 cm^2 = _____ [mm^2]

Express the 1st quantity as a percentage of the 2nd. Round to 1 decimal place.

Example. 24 s out of 3 min.

3 min = 180 s

$\frac{24}{180} \times 100 = 13.3\ \%$

9 420 kg from 3.6 t.

10 27 months from 3 years.

11 150 m from 1.4 km.

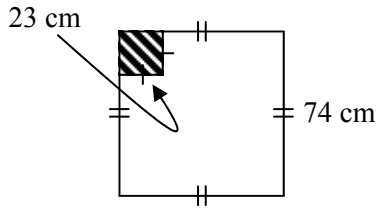
12 8 hrs of 3 days.

13 980 m^2 from 2.6 ha.

14 3 450 mg of 0.8 kg

Find the shaded area as a percentage of the total area Round to 1 decimal place.

15



$A_1 = l^2$ $A_2 = l^2$

16 A room measuring 4.2 m by 3.6 m has a wardrobe measuring 75 cm by 150 cm. Find the percentage of floor space taken by the wardrobe.

Find the percentages of these quantities. Give your answer in the units in the brackets. Round to the nearest whole number.

Example. 43% of 2.4 T [kg]

$\frac{43}{100} \times 2\,400 = 1\,032$ kg

Change T to kg

17 35% of 8.6 m [cm]

×

Change units to cm

18 45% of 9 min. [s]

19 12% of 3.9 L [mL]

20 23% of a year [weeks]

21 85% of 0.85 kg [g]

22 115% of 750 mL [L]

23 30% of 6 doz eggs [eggs]

24 84% of 62 300 m^2 [ha]

Don't round Q 25 and Q 26.

25 A 1 250 mL bottle of drink is increased by 40% extra, for free. Find its new volume in Litres.

26 A plane with 225 seats has a standard ticket price of $450. If 8% of the seats sell at a discount rate of 70% of the standard price, find the earnings for the fully booked flight.

Further Percentage of a Quantity

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Convert these units to the one shown in brackets.

1 7 min = 420 [s]

2 5.2 L = 5 200 [mL]

3 17 m = 1 700 [cm]

4 8 weeks = 56 [days]

5 3.25 t = 3 250 [kg]

6 0.6 kg = 600 [g]

7 230 mm = 0.23 [m]

8 8 cm^2 = 800 [mm^2]

Express the 1st quantity as a percentage of the 2nd. Round to 1 decimal place.

Example. 24 s out of 3 min.

3 min = 180 s

$\frac{24}{180} \times 100$ = 13.3 %

9 420 kg from 3.6 t.

$\frac{420}{3\,600} \times 100$ = 11.7 %

10 27 months from 3 years.

$\frac{27}{36} \times 100$ = 75 %

11 150 m from 1.4 km.

$\frac{150}{1\,400} \times 100$ = 10.7 %

12 8 hrs of 3 days.

$\frac{8}{72} \times 100$ = 11.1 %

13 980 m^2 from 2.6 ha.

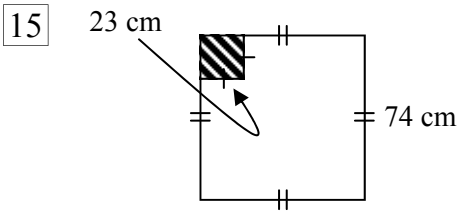
$\frac{980}{26\,000} \times 100$ = 3.8%

14 3 450 mg of 0.8 kg

$\frac{3\,450}{800\,000} \times 100$ = 0.4%

Find the shaded area as a percentage of the total area Round to 1 decimal place.

15



$A_1 = l^2$ $A_2 = l^2$

$= 23^2$ $= 74^2$

$A_1 = 529$ cm^2 $A_2 = 5\,476$ cm^2

$\frac{529}{5\,476} \times 100$ = 9.7%

16 A room measuring 4.2 m by 3.6 m has a wardrobe measuring 75 cm by 150 cm. Find the percentage of floor space taken by the wardrobe.

$A_1 = lb$ $A_2 = lb$

$= 3.6 \times 4.2$ $= 0.75 \times 1.5$

$A_1 = 15.1$ m^2 $A_2 = 1.1$ m^2

$\frac{1.1}{15.1} \times 100$ = 7.3%

Find the percentages of these quantities. Give your answer in the units in the brackets. Round to the nearest whole number.

Example. 43% of 2.4 T [kg]

$\frac{43}{100} \times 2\,400$ = 1 032 kg

Change T to kg

17 35% of 8.6 m [cm]

$\frac{35}{100} \times 860$ = 301 kg

Change units to cm

18 45% of 9 min. [s]

$\frac{45}{100} \times 540$ = 243 s

19 12% of 3.9 L [mL]

$\frac{12}{100} \times 3\,900$ = 468 mL

20 23% of a year [weeks]

$\frac{23}{100} \times 52$ = 12 weeks

21 85% of 0.85 kg [g]

$\frac{88}{100} \times 850$ = 748 g

22 115% of 750 mL [L]

$\frac{115}{100} \times 0.75$ = 1 L

23 30% of 6 doz eggs [eggs]

$\frac{30}{100} \times 72$ = 22 eggs

24 84% of 62 300 m^2 [ha]

$\frac{84}{100} \times 6.23$ = 5 ha

Don't round Q 25 and Q 26.

25 A 1 250 mL bottle of drink is increased by 40% extra, for free. Find its new volume in Litres.

$\frac{40}{100} \times 1.25$ = 0.5 L

1.25 + 0.5 = 1.75L

26 A plane with 225 seats has a standard ticket price of $450. If 8% of the seats sell at a discount rate of 70% of the standard price, find the earnings for the fully booked flight.

$\frac{70}{100} \times 450$ = $315

$\frac{8}{100} \times 225$ = 18 seats

18 × 315 = $5 670

207 × 450 = $93 150

5 670 + 93 150 = $98 820

Increasing and Decreasing by a Percentage

Increasing and decreasing by a percentage is done constantly in business. Prices are often set from percentage 'mark-ups' and discounts are often given by percentages also.

Column 1 asks you to increase by a percentage. These questions are done in 2 steps, the first step is to multiply the amount by the percentage as a fraction. This is the same as finding the percentage of a quantity. So change the percentage to a fraction over 100 ($/_{100}$) and multiply. The additional step is to then add the answer to the original amount. Note that all questions have units, eg. m, mL, $ so make sure your answer includes these.

Example : Increase 46.2 cm by 35%

Multiply amount by percentage as a fraction over 100. So 35% is changed to $\frac{35}{100}$ → $46.2 \times \frac{35}{100}$ → Show units → $= 16.17$ cm

Add the amount in the question to the answer you have just found ← $46.2 + 16.17$ ← Show units ← $= 62.37$ cm

Column 2 asks you to decrease by a percentage. The method starts off the same in that the amount is multiplied by the percentage as a fraction but then instead of adding, you subtract this amount from the original amount.

Example : Decrease 210 kg by 16%

Multiply amount by percentage as a fraction over 100. So 16% is changed to $\frac{16}{100}$ → $210 \times \frac{16}{100}$ → Show units → $= 33.6$ kg

Subtract the answer you have just found from the number in the question. ← $210 - 33.6$ ← Show units ← $= 176.4$ kg

Column 3 is a mixture of the previous 2 columns, if the question is increase then you add the amount of change. If it is decrease you subtract. Colour the relevant circle, increase being addition, decrease being subtraction .

Increasing and Decreasing by a Percentage

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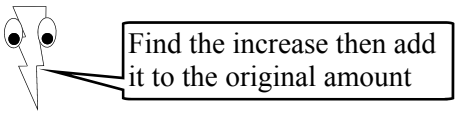


Example

Increase 235 kg by 20%

$235 \times \frac{20}{100}$ $235 + 47$

$= 47$ kg $= 282$ kg

1 Increase 140 L by 22%

2 Increase 23.5 cm by 18%

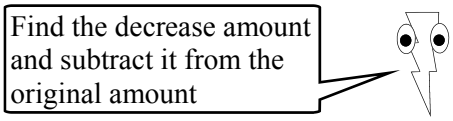
3 Increase \$542 by 134%

4 Increase 1 700 km by 85%

5 Increase 3 620 m by 95%

6 Increase 1 010 m by 64%

7 Increase 6 895 mL by 8%



8 Decrease \$868 by 6%

868 - ☐

9 Decrease 19.2 g by 75%

10 Decrease 45.6 cm by 85%

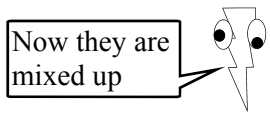
11 Decrease 1 250 mL by 38%

12 Decrease 35.34 sec by 50%

13 Decrease 308.6 kg by 5%

14 Decrease \$76.90 by 30%

15 Decrease 39.5 t by 82%



16 Increase \$3.95 by 20%

Select: ○ Addition ○ Subtraction

17 Decrease 115 mL by 92%

Select: ○ Addition ○ Subtraction

18 Decrease 465 m^3 by 38%

Select: ○ Addition ○ Subtraction

19 Increase 29.4 rpm by 180%

Select: ○ Addition ○ Subtraction

20 Decrease 5.38 m by 15%

Select: ○ Addition ○ Subtraction

21 Increase 373 ha by 60%

Select: ○ Addition ○ Subtraction

22 Increase \$7.50 by 142%

Select: ○ Addition ○ Subtraction

Increasing and Decreasing by a Percentage

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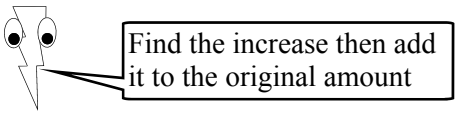


Example

Increase 235 kg by 20%

$235 \times \frac{20}{100}$ = 47 kg

235 + 47 = 282 kg

1 Increase 140 L by 22%

$140 \times \frac{22}{100}$ = 30.8 L

140 + 30.8 = 170.8 L

2 Increase 23.5 cm by 18%

$23.5 \times \frac{18}{100}$ = 4.23 cm

23.5 + 4.23 = 27.73 cm

3 Increase $542 by 134%

$542 \times \frac{134}{100}$ = $726.28

542 + 726.28 = $1 268.28

4 Increase 1 700 km by 85%

$1\,700 \times \frac{85}{100}$ = 1 445 km

1 700 + 1 445 = 3 142 km

5 Increase 3 620 m by 95%

$3\,620 \times \frac{95}{100}$ = 3 439 m

3 620 + 3 439 = 7 059 m

6 Increase 1 010 m by 64%

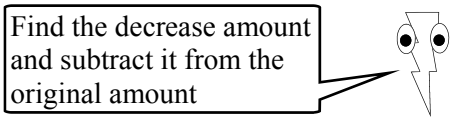
$1\,010 \times \frac{64}{100}$ = 646.4 m

1 010 + 646.4 = 1 656.4 m

7 Increase 6 895 mL by 8%

$6\,895 \times \frac{8}{100}$ = 551.6 mL

6 895 + 551.6 = 7 446.6 mL



8 Decrease $868 by 6%

$868 \times \frac{6}{100}$ = $52.08

868 - 52.08 = $815.92

9 Decrease 19.2 g by 75%

$19.2 \times \frac{75}{100}$ = 14.4 g

19.2 - 14.4 = 4.8 g

10 Decrease 45.6 cm by 85%

$45.6 \times \frac{85}{100}$ = 38.76 cm

45.6 - 38.76 = 6.84 cm

11 Decrease 1 250 mL by 38%

$1\,250 \times \frac{38}{100}$ = 475 mL

1 250 - 475 = 775 mL

12 Decrease 35.34 sec by 50%

$35.34 \times \frac{50}{100}$ = 17.67 sec

35.34 - 17.67 = 17.67 sec

13 Decrease 308.6 kg by 5%

$308.6 \times \frac{5}{100}$ = 15.43 kg

308.6 - 15.43 = 293.17 kg

14 Decrease $76.90 by 30%

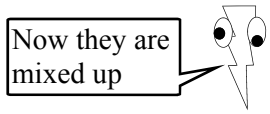
$76.90 \times \frac{30}{100}$ = $23.07

76.90 - 23.07 = $53.83

15 Decrease 39.5 t by 82%

$39.5 \times \frac{82}{100}$ = 32.39 t

39.5 - 32.39 = 7.11 t



16 Increase $3.95 by 20%

Select: ● Addition ○ Subtraction

$3.95 \times \frac{20}{100}$ = $0.79

3.95 + 0.79 = $4.74

17 Decrease 115 mL by 92%

Select: ○ Addition ● Subtraction

$115 \times \frac{92}{100}$ = 105.8 mL

115 - 105.8 = 9.2 mL

18 Decrease 465 m^3 by 38%

Select: ○ Addition ● Subtraction

$465 \times \frac{38}{100}$ = 176.7 m^3

465 - 176.7 = 288.3 m^3

19 Increase 29.4 rpm by 180%

Select: ● Addition ○ Subtraction

$29.4 \times \frac{180}{100}$ =52.92 rpm

29.4 + 52.92 = 82.32 rpm

20 Decrease 5.38 m by 15%

Select: ○ Addition ● Subtraction

$5.38 \times \frac{15}{100}$ = 0.807 m

5.38 - 0.807 = 4.573 m

21 Increase 373 ha by 60%

Select: ● Addition ○ Subtraction

$373 \times \frac{60}{100}$ = 223.8 ha

373 + 223.8 = 596.8 ha

22 Increase $7.50 by 142%

Select: ● Addition ○ Subtraction

$7.50 \times \frac{142}{100}$ = $10.65

7.50 + 10.65 = $18.15

Further Percentage Increase and Decrease

When you increase or decrease by a percentage an alternative method to the previous worksheet is to modify the percentage first before multiplying.

Column 1 is increasing by a percentage. If you increase by a percentage add the percentage to 100% first, then multiply the quantity by this percentage (as a fraction). Question 4 involves increasing by 120%. Be careful, many students forget to add 100% when the percentage is more than 100% in the first place, you still have to add 100% regardless. Remember that these involve quantities, the units must be shown in the answer, which is rounded to 2 decimal places. Money requires 2 decimal places...never just 1 decimal place.

Example : Increase 85 cm by 16%

Multiply changing the percentage to a fraction

Add the increase to 100% →	100% + 16%	85 × $\frac{116}{100}$ ←
Show % sign →	= 116%	= 98.6 cm ← Show units

When you have finished the question, look at the answer … has it increased on the amount in the question? If not recheck your working out.

Column 2 is decreasing, subtract the percentage from 100% first, then multiply the quantity by the new percentage.

Example : Decrease 120 kg by 27%

Multiply changing the percentage to a fraction

Subtract the percentage from 100%. →	100% - 27%	120 × $\frac{73}{100}$ ←
Show % sign →	= 73%	= 87.6 cm ← Show units

Again, check your answer...does it make sense….has it reduced the amount?

Column 3 involves increasing then decreasing a quantity, or the reverse. So apply the process twice. If you increase by a percentage don't use this percentage for the decrease, you always start with '100% +' or '100% -'. These are more difficult in concept and require care.

Example. Increase $540 by 17% then decrease the result by 5%.

So use 100% + 17% = 117% for the increase, then 100% - 5% = 95% for the decrease.
DON'T USE 117% - 5% = 112% decrease. Look below for the working out method.

STEP 1	100% + 17%	540 × $\frac{117}{100}$ STEP 3
	= 117%	= $631.80
STEP 2	100% - 5%	631.80 × $\frac{95}{100}$ STEP 4
	= 95%	= $600.21

Further Percentage Increase and Decrease

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Add the percentage increase to 100%, multiply by the total ÷ 100. Round to 2 d.p.

Example

Increase 135 cm by 42%

$100\% + 42\% = 142\%$

$135 \times \frac{142}{100} = 190.28$ cm

1 Increase \$8.30 by 65%

2 Increase 658 L by 38%

3 Increase 29 kg by 24%

4 Increase 385 m by 120%

5 Increase \$76.20 by 9%

6 Increase 83.65 kg by 230%

7 Increase 3 750 kL by 12%

Subtract the percentage to find the discounted amount. Round to 2 d.p.

Example

Decrease 1.2 m by 38%

$100\% - 38\% = 62\%$

$1.2 \times \frac{62}{100} = 0.74$ m

8 Decrease \$780 by 39%

9 Decrease 43.8 kL by 27%

10 Decrease 9.15 m by 11%

11 Decrease \$66.80 by 8%

12 Decrease 1 610 kg by 57%

13 Decrease 739.6 cm by 83%

14 Decrease \$4 586 by 15%

First increase and then decrease the amount given (or the reverse).

Important : If you increase for example, by 10% and then decrease by 10%, you won't finish with the original amount you started with.

15 Increase \$780 by 20% then decrease the result by 12%

100% +

100% -

16 Increase \$680 by 5% then decrease the result by 95%

17 Decrease \$1 400 by 29% and increase the result by 29%

18 Decrease \$8 360 by 45% and increase the result by 45%

Further Percentage Increase and Decrease

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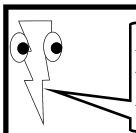


Add the percentage increase to 100%, multiply by the total ÷ 100. Round to 2 d.p.

Example

Increase 135 cm by 42%

100% + 42%	135 × $\frac{142}{100}$
= 142%	= 190.28 cm

1 Increase \$8.30 by 65%

100% + 65%	8.30 × $\frac{165}{100}$
= 165%	= \$13.70

2 Increase 658 L by 38%

100% + 38%	658 × $\frac{138}{100}$
= 138%	= 908.04 L

3 Increase 29 kg by 24%

100% + 24%	29 × $\frac{124}{100}$
= 124%	= 35.96 kg

4 Increase 385 m by 120%

100% + 120%	385 × $\frac{220}{100}$
= 220%	= 847 m

5 Increase \$76.20 by 9%

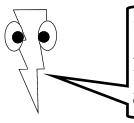
100% + 9%	76.20 × $\frac{109}{100}$
= 109%	= \$83.06

6 Increase 83.65 kg by 230%

100% + 230%	83.65 × $\frac{330}{100}$
= 330%	= 276.05 kg

7 Increase 3 750 kL by 12%

100% + 12%	3 750 × $\frac{112}{100}$
= 112%	= 4 200 kL



Subtract the percentage to find the discounted amount. Round to 2 d.p.

Example

Decrease 1.2 m by 38%

100% - 38%	1.2 × $\frac{62}{100}$
= 62%	= 0.74 m

8 Decrease \$780 by 39%

100% - 39%	780 × $\frac{61}{100}$
= 61%	= \$475.80

9 Decrease 43.8 kL by 27%

100% - 27%	43.8 × $\frac{73}{100}$
= 73%	= 31.97 kL

10 Decrease 9.15 m by 11%

100% - 11%	9.15 × $\frac{89}{100}$
= 89%	= 8.14 m

11 Decrease \$66.80 by 8%

100% - 8%	66.80 × $\frac{92}{100}$
= 92%	= \$61.46

12 Decrease 1 610 kg by 57%

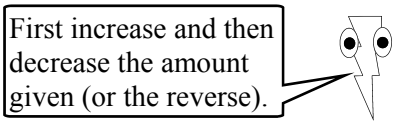
100% - 57%	1 610 × $\frac{43}{100}$
= 43%	= 692.3 kg

13 Decrease 739.6 cm by 83%

100% - 83%	739.6 × $\frac{17}{100}$
= 17%	= 125.73 cm

14 Decrease \$4 586 by 15%

100% - 15%	4 586 × $\frac{85}{100}$
= 85%	= \$3 898.10



First increase and then decrease the amount given (or the reverse).

Important : If you increase for example, by 10% and then decrease by 10%, you won't finish with the original amount you started with.

15 Increase \$780 by 20% then decrease the result by 12%

100% + 20%	780 × $\frac{120}{100}$
= 120%	= \$936
100% - 12%	936 × $\frac{88}{100}$
= 88%	= \$823.68

16 Increase \$680 by 5% then decrease the result by 95%

100% + 5%	680 × $\frac{105}{100}$
= 105%	= \$714
100% - 95%	714 × $\frac{5}{100}$
= 5%	= \$35.70

17 Decrease \$1 400 by 29% and increase the result by 29%

100% - 29%	1 400 × $\frac{71}{100}$
= 71%	= \$994
100% + 29%	994 × $\frac{129}{100}$
= 129%	= \$1 282.26

18 Decrease \$8 360 by 45% and increase the result by 45%

100% - 45%	8 360 × $\frac{55}{100}$
= 55%	= \$4 598
100% + 45%	4 598 × $\frac{145}{100}$
= 145%	= \$6 667.10

Profit, Cost Price and Sale Price

Shops operate by buying goods from suppliers and selling the goods to shoppers, often called consumers. The supplied goods are bought by the shop at "Cost Price" and sold to the consumer at the "Sale Price". The sale price is usually more than the cost price and the difference is the profit, the amount of money the shop makes from the sale.

Profit is usually positive, but it can be negative. Why? Because shops sometimes have to clear old stock fast and they actually sell it for less than the amount that they paid for it. Also, some shops take a loss (negative profit) on certain items to advertise a very cheap price, hoping that consumers will by other items as well, which they then make money on.

The three formulae to remember are:

Profit = Sale Price - Cost Price
Cost Price = Sale Price - Profit
Sale Price = Cost Price + Profit

In column 1 the first table uses the first formula. So find the profit by subtraction. Subtract the Cost Price from the Sale Price. Repeat this process for the second table but you may get negative answers, which is a loss. After you write your answer, colour in the circle "P" for Profit or "L" for Loss.

Questions 17 - 22 ask you to find the Cost price. Again this is achieved by subtraction. Subtract the profit from the Sale price. Make sure you show your working out, remember the dollar sign ($) and write your answer to 2 decimal places if cents are to be shown.

Column 2 asks you to determine the Sale Price to achieve a certain profit. Often shop owners refer to this as the "Mark-Up" - the percentage the cost price is increased by. Some shops will have a uniform mark-up amount on all goods, others will change the mark-up according to the popularity of certain items.

Questions 23 - 26 ask you to mark-up the goods by 40% of the cost price. Using this to calculate the profit and then the Sale Price. The method is as below:

Example Toothpaste: Cost Price $2.10 - Profit is 40% of the cost price.

$\text{Profit} = 2.10 \times \frac{40}{100}$	← Profit is found by multiplying the cost price by 40% ($\frac{40}{100}$)
$\text{Profit} = \$0.84$	← Profit
$SP = 2.10 + 0.84$	← Sale Price (SP for short) = Cost Price + Profit
$SP = \$\ 2.94$	← Complete the addition, dollar sign and 2 decimal places.. DONE!

Question 27 - 32 is the same as the above except the % is 65% rather than 40%. So use $\frac{65}{100}$ instead of $\frac{40}{100}$.

Profit, Cost Price and Sale Price

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Profit = Sale price - Cost price. Complete the table below.

	Sale Price	Cost Price	Profit
1	\$2.58 -	\$1.79 =	
2	\$1.14	\$0.59	
3	\$6.80	\$3.97	
4	\$8.11	\$5.76	
5	\$18.15	\$7.45	
6	\$35	\$26.47	
7	\$17.80	\$8.14	
8	\$25.90	\$21.85	

If Profit is negative it is a 'Loss'. Find the profit (+/-). Is it a loss?

	Sale Price	Cost Price	Profit	P/L
9	\$2.83	\$1.75		P L
10	\$5.24	\$8.89		P L
11	\$0.93	\$1.37		P L
12	\$7.32	\$4.82		P L
13	\$12.42	\$16.15		P L
14	\$3.35	\$2.51		P L
15	\$9.17	\$11.50		P L
16	\$20.10	\$11.95		P L

Cost price = Sale price - Profit. Find the Cost Price (CP) for these below.

17 Sale Price = \$1 360
Profit = \$419

CP = 1 360 -

18 Sale Price = \$1.83
Profit = \$0.37

19 Sale Price = \$2.85
Profit = \$0.64

20 Sale Price = \$9.20
Profit = \$4.47

21 Sale Price = \$417
Profit = \$263.80

22 Sale Price = \$9 999
Profit = \$4 579

Find the Sale Price using a profit of 40% of the Cost Price. First find the profit and then use SP = Cost Price + Profit.

23 Cost Price = \$8.20

Profit = $\times \frac{40}{100}$

Profit =

SP = 8.20 +

SP = \$

24 Cost Price = \$506

25 Cost Price = \$24.20

26 Cost Price = \$765

Now a Profit of 65% of Cost Price is required. Round answer to 2 d.p.

27 Cost Price = \$120

Profit = $\times \frac{65}{100}$

Profit =

SP = 120 +

SP = \$

28 Cost Price = \$14.30

29 Cost Price = \$82.50

30 Cost Price = \$39.70

31 Cost Price = \$118

32 Cost Price = \$1 360

Profit, Cost Price and Sale Price

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Profit = Sale price - Cost price. Complete the table below.

	Sale Price	Cost Price	Profit
1	$2.58 -	$1.79 =	$0.79
2	$1.14	$0.59	$0.55
3	$6.80	$3.97	$2.83
4	$8.11	$5.76	$2.35
5	$18.15	$7.45	$10.70
6	$35	$26.47	$8.53
7	$17.80	$8.14	$9.66
8	$25.90	$21.85	$4.05

If Profit is negative it is a 'Loss'. Find the profit (+/-). Is it a loss?

	Sale Price	Cost Price	Profit	P/L
9	$2.83	$1.75	$1.08	P ● L ○
10	$5.24	$8.89	- $3.65	P ○ L ●
11	$0.93	$1.37	- $0.44	P ○ L ●
12	$7.32	$4.82	$2.50	P ● L ○
13	$12.42	$16.15	- $3.73	P ○ L ●
14	$3.35	$2.51	$0.84	P ● L ○
15	$9.17	$11.50	- $2.33	P ○ L ●
16	$20.10	$11.95	$8.15	P ● L ○

Cost price = Sale price - Profit. Find the Cost Price (CP) for these below.

17 Sale Price = $1 360
Profit = $419

CP = 1 360 - $419

Cost Price = $941

18 Sale Price = $1.83
Profit = $0.37

CP = 1.83 - 0.37

Cost Price = $1.46

19 Sale Price = $2.85
Profit = $0.64

CP = 2.85 - 0.64

Cost Price = $2.21

20 Sale Price = $9.20
Profit = $4.47

CP = 9.20 - 4.47

Cost Price = $4.73

21 Sale Price = $417
Profit = $263.80

CP = 417 - 263.80

Cost Price = $153.20

22 Sale Price = $9 999
Profit = $4 579

CP = 9 999 - 4 579

Cost Price = $5 420

Find the Sale Price using a profit of 40% of the Cost Price. First find the profit and then use SP = Cost Price + Profit.

23 Cost Price = $8.20

Profit = $8.20 \times \frac{40}{100}$

Profit = $3.28

SP = 8.20 + 3.28

SP = $ 11.48

24 Cost Price = $506

Profit = $506 \times \frac{40}{100}$

Profit = $202.40

SP = 506 + 202.40

SP = $708.40

25 Cost Price = $24.20

Profit = $24.20 \times \frac{40}{100}$

Profit = $9.68

SP = 24.20 + 9.68

SP = $33.88

26 Cost Price = $765

Profit = $765 \times \frac{40}{100}$

Profit = $306

SP = 765 + 306

SP = $1 071

Now a Profit of 65% of Cost Price is required. Round answer to 2 d.p.

27 Cost Price = $120

Profit = $120 \times \frac{65}{100}$

Profit = $78

SP = 120 + 78

SP = $ 198

28 Cost Price = $14.30

Profit = $14.30 \times \frac{65}{100}$

Profit = $9.30

SP = 14.30 + 9.30

SP = $23.60

29 Cost Price = $82.50

Profit = $82.50 \times \frac{65}{100}$

Profit = $53.63

SP = 82.50 + 53.63

SP = $136.13

30 Cost Price = $39.70

Profit = $39.70 \times \frac{65}{100}$

Profit = $25.81

SP = 39.70 + 25.81

SP = $65.51

31 Cost Price = $118

Profit = $118 \times \frac{65}{100}$

Profit = $76.70

SP = 118 + 76.70

SP = $194.70

32 Cost Price = $1 360

Profit = $1 360 \times \frac{65}{100}$

Profit = $884

SP = 1 360 + 884

SP = $22.48

Further Profit, Cost Price and Sale Price

It is recommended that this sheet be attempted only after the previous worksheet has been completed.

The three formulae to remember are:

Profit = Sale Price - Cost Price
Cost Price = Sale Price - Profit
Sale Price = Cost Price + Profit

Column 1 starts with a table. You have to either add or subtract the amounts and you have to ensure the order in which you subtract them is correct. The three formulae above are shown at the top of the sheet … take your time and look at each row and ask yourself if it makes sense!

Negative answers for Profit will occur when the Sale Price is less than the Cost Price, show these with a minus sign (-).

Questions 16 to 21 ask you to find the Profit as a percentage of the cost price:

$$\% \text{ Profit} = \frac{\text{Profit}}{\text{Cost Price}} \times 100$$

Which using the table is the same as:

$$\% \text{ Profit} = \frac{\text{2nd Column}}{\text{1st Column}} \times 100$$

Note that question 16 shows you to use the value of B in the top box (numerator) and the value of A in the bottom box (Question 1 in the table). Question 17 the same, only use D and C (Question 4 in the table). Note that this question has a negative profit or Loss. So your answer will include a minus sign (-), round your answer to 1 decimal place and remember the % sign.

Column 2, Questions 22 - 27 ask you to find the Profit as a percentage of the sale price:

$$\% \text{ Profit} = \frac{\text{Profit}}{\text{Sale Price}} \times 100$$

Which using the table is the same as:

$$\% \text{ Profit} = \frac{\text{2nd Column}}{\text{3rd Column}} \times 100$$

Question 22 shows that you are to use E and F, to ensure you don't make a mistake. Refer to the item number shown in the table and use the second and third columns to answer the questions. Your profit always has to be less than the sale price so these percentages can't be more than 100%...but they can be negative!

Question 28 - 30 are worded questions. In all questions first find the profit. Then use either the cost price or sale price formulae above, depending on which is asked for in the question.

Further Profit, Cost Price and Sale Price

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Fill the table with the missing amounts, use:
Cost Price = Sale Price - Profit or,
Sale Price = Cost Price + Profit or,
Profit = Sale Price - Cost Price

Your Profit may be negative!



	Cost Price	Profit	Sale Price
1	A $2.46	B	$4.11
2	$11.20	$4.75	
3		$1.58	$4.75
4	C $19.87	- $2.45 D	
5	$10.25		$13.04
6	$24.65		$19.87
7		$0.58	$1.12
8	$20.20	$16.33	
9		$4.51	$13.87
10	$0.14	$0.54 E	F
11	$5.95		$8.30
12		$3.11	$7.23
13	$31.80	$21.33	
14	$17.46		$11.99
15		$12.94	$41.36

Calculate the Profit as a percentage of the Cost Price, for some of the items above. Round your answer to 1 decimal place.



16 Use Item 1

$\%Profit = \frac{B}{A} \times 100$

%Profit = %

Round to 1 decimal place

17 Use Item 4

$\%Profit = \frac{D}{C} \times$ ☐

%Profit =

Round to 1 decimal place

18 Use Item 12

$\%Profit = \frac{\square}{\square} \times 100$

19 Use Item 7

20 Use Item 9

21 Use Item 14

Calculate the Profit as a percentage of the Sale Price this time. Round your answer to 1 decimal place. Watch for negatives!



22 Use Item 10

$\%Profit = \frac{E}{F} \times 100$

%Profit = %

23 Use Item 3

24 Use Item 11

25 Use Item 5

26 Use Item 15

27 Use Item 6

Now answer these worded questions. Find profit first then the % Profit. Answer to 2 decimal places.



28 Crazy Tom buys TV's at $498 then sells them for $629. Calculate the %Profit on the cost price.

Profit = ☐ - ☐

%Profit =

Profit = ☐

29 A Pet food company sells food at $0.67/can. If costs are $0.34 per can. Find the % Profit of the sale price.

30 Jan buys toasters at $14.76 and kettles at $11.22. If she sells toaster/kettle packs (the two together) for $40, find the % Profit on the cost price for a pack.

Further Profit, Cost Price and Sale Price

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Fill the table with the missing amounts, use:
Cost Price = Sale Price - Profit or,
Sale Price = Cost Price + Profit or,
Profit = Sale Price - Cost Price

Your Profit may be negative!

	Cost Price	Profit	Sale Price
1	A $2.46	$1.65 B	$4.11
2	$11.20	$4.75	$15.95
3	$3.17	$1.58	$4.75
4	C $19.87	- $2.45 D	$17.42
5	$10.25	$2.79	$13.04
6	$24.65	- $4.78	$19.87
7	$0.54	$0.58	$1.12
8	$20.20	$16.33	$36.53
9	$9.36	$4.51	$13.87
10	$0.14	$0.54 E	$0.68 F
11	$5.95	$2.35	$8.30
12	$4.12	$3.11	$7.23
13	$31.80	$21.33	$53.13
14	$17.46	- $5.47	$11.99
15	$28.42	$12.94	$41.36

Calculate the Profit as a percentage of the Cost Price, for some of the items above. Round your answer to 1 decimal place.

16 Use Item 1

$\%Profit = \frac{B\ 1.65}{A\ 2.46} \times 100$

%Profit = 67.1%

Round to 1 decimal place

17 Use Item 4

$\%Profit = \frac{D\ -2.45}{C\ 19.87} \times 100$

%Profit = -12.3%

Round to 1 decimal place

18 Use Item 12

$\%Profit = \frac{3.11}{4.12} \times 100$

%Profit = 75.1%

19 Use Item 7

$\%Profit = \frac{0.58}{0.54} \times 100$

%Profit = 107.4%

20 Use Item 9

$\%Profit = \frac{4.51}{9.36} \times 100$

%Profit = 48.2%

21 Use Item 14

$\%Profit = \frac{-5.47}{17.46} \times 100$

%Profit = -31.3%

Calculate the Profit as a percentage of the Sale Price this time. Round your answer to 1 decimal place. Watch for negatives!

22 Use Item 10

$\%Profit = \frac{E\ 0.54}{F\ 0.68} \times 100$

%Profit = 79.4%

23 Use Item 3

$\%Profit = \frac{1.58}{4.75} \times 100$

%Profit = 33.3%

24 Use Item 11

$\%Profit = \frac{2.35}{8.30} \times 100$

%Profit = 28.3%

25 Use Item 5

$\%Profit = \frac{2.79}{13.04} \times 100$

%Profit = 21.4%

26 Use Item 15

$\%Profit = \frac{12.94}{41.36} \times 100$

%Profit = 31.3%

27 Use Item 6

$\%Profit = \frac{-4.78}{19.87} \times 100$

%Profit = -24.1%

Now answer these worded questions. Find profit first then the % Profit. Answer to 2 decimal places.

28 Crazy Tom buys TV's at $498 then sells them for $629. Calculate the %Profit on the cost price.

Profit = 629 - 498

Profit = $131

$\%Profit = \frac{131}{498} \times 100$

%Profit = 26.31%

29 A Pet food company sells food at $0.67/can. If costs are $0.34 per can. Find the % Profit of the sale price.

Profit = 0.67 - 0.34

Profit = $0.33

$\%Profit = \frac{0.33}{0.67} \times 100$

%Profit = 49.25%

30 Jan buys toasters at $14.76 and kettles at $11.22. If she sells toaster/kettle packs (the two together) for $40, find the % Profit on the cost price for a pack.

CP = 14.76 + 11.22

CP = $25.98

Profit = 40 - 25.98

Profit = $14.02

$\%Profit = \frac{14.02}{25.98} \times 100$

%Profit = 53.97%

The Unitary Method

The Unitary Method is a process of determining a percentage of a quantity. The word 'unitary' meaning that you find the value of a single unit, which is 1%. Once you have found 1% you then use it to find any other percentage that you require.

Column 1 starts with a % and its value, divide both sides by the percentage and that will give you the value of 1%.

Example. If 34% = 612, how much is 1%?

To change 34% to 1%, you have to divide (÷) by 34.

34% = 612

÷ 34 ÷ 34

1% = 18

So divide 612 by 34 and you get your answer. 612 ÷ 34 = 18

The next part of column 1 asks you to find 100% when you are given the 1% value. Multiply both sides by 100. If these steps seem easy then remember all you have to do is combine these two steps and that is the Unitary Method.

Example. If 1% = 5.2, how much is 100%?

To change 1% to 100%, you have to multiply (×) by 100.

1% = 5.2

× 100 × 100

100% = 520

So multiply 5.2 by 100 and you get your answer. 5.2 × 100 = 520

Column 2 gives you a % and you are asked to find 100%. Follow the example by first dividing (by the percentage) to find the value of 1% then multiplying this value by 100 to get 100%. Note that these answers are rounded to 2 decimal places if required.

Note that on the 2nd line the decimal places aren't to 2 d.p, this is the middle step and doesn't have to be to 2 d.p, DON'T CLEAR YOUR SCREEN, use the number on the screen in the multiplication and multiply it by 100 otherwise your answer will be inaccurate.

Example. If 20% = 1 645 kg, how much is 100%?

To change 20% to 1%, you have to divide (÷) by 20.

20% = 1 645 kg

÷ 20 ÷ 20

1% = 82.25 kg

× 100 × 100

100% = 8 225 kg

To change 1% to 100% you have to multiply by 100. So multiply 82.25 by 100 and you get your answer. 82.25 × 100 = 8 225 kg.

The Unitary Method

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Given these percentages find the value of 1%, your answer may be a decimal.

Example. If 45% is 765, then find 1%.

45% = 765

÷ 45 ÷ 45

1% = 17

Example. If 30% is 576, then find 1%.

30% = 576

÷ 30 ÷ 30

1% = 19.2

1 15% = 390 ÷ 15 ÷ 15 1% =

2 7% = 56 ÷ 7 ÷ 7

3 22% = 99

4 72% = 504

5 120% = 288

6 19% = 114

7 4.5% = 90

8 8.2% = 328

9 1.3% = 65

Now given 1% find 100%.

Example. If 1% is 28, find 100%.

1% = 28

× 100 × 100

100% = 2 800

10 1% = 17.1 × 100 × 100 100% =

11 1% = 43.6 × 100 × 100

12 1% = 0.94

13 1% = 131

14 1% = 20.2

15 1% = 0.035

16 1% = 8.536

17 1% = 12.07

Find 100% in 2 steps, first find 1% (by ÷) then find 100% (by ×100), round to 2 d.p.

Example

34% is $15, find 100%.

34% = $15

÷ 34 ÷ 34

1% = $0.44117 ← Don't round, don't clear your calculator screen

× 100 × 100

100% = $44.12 ← Round to 2 d.p.

18 19% = $23.37 ÷ 19 ÷ 19 1% = × 100 × 100 100% =

19 23% = 163.76 m ÷ ÷ 1% = × 100 × 100 100% =

20 86% = 44.72 g

21 55% = 195.91 mm

22 105% = $2 850.75

23 125% = $5 650

24 Airline tickets are being sold at $1 232 which is 88% of the standard price. Find the standard price.

25 An auction is selling goods at 45% of their normal price. If Kim paid $2 790 for a rug, calculate its normal price.

The Unitary Method

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Given these percentages find the value of 1%, your answer may be a decimal.

Example. If 45% is 765, then find 1%.

45% = 765
÷ 45 ÷ 45
1% = 17

Example. If 30% is 576, then find 1%.

30% = 576
÷ 30 ÷ 30
1% = 19.2

1 15% = 390
÷ 15 ÷ 15
1% = 26

2 7% = 56
÷ 7 ÷ 7
1% = 8

3 22% = 99
÷ 22 ÷ 22
1% = 4.5

4 72% = 504
÷ 72 ÷ 72
1% = 7

5 120% = 288
÷ 120 ÷ 120
1% = 2.4

6 19% = 114
÷ 19 ÷ 19
1% = 6

7 4.5% = 90
÷4.5 ÷4.5
1% = 20

8 8.2% = 328
÷8.2 ÷8.2
1% = 40

9 1.3% = 65
÷1.3 ÷1.3
1% = 50

Now given 1% find 100%.

Example. If 1% is 28, find 100%.

1% = 28
× 100 × 100
100% = 2 800

10 1% = 17.1
× 100 × 100
100% = 1 710

11 1% = 43.6
× 100 × 100
100% = 4 360

12 1% = 0.94
× 100 × 100
100% = 94

13 1% = 131
× 100 × 100
100% =13 100

14 1% = 20.2
× 100 × 100
100% = 2 020

15 1% = 0.035
× 100 × 100
100% = 3.5

16 1% = 8.536
× 100 × 100
100% = 853.6

17 1% = 12.07
× 100 × 100
100% = 1 207

Find 100% in 2 steps, first find 1% (by ÷) then find 100% (by ×100), round to 2 d.p.

Example
34% is $15, find 100%.

34% = $15
÷ 34 ÷ 34
1% = $0.44117 ← Don't round, don't clear your calculator screen
× 100 × 100
100% = $44.12 ← Round to 2 d.p.

18
19% = $23.37
÷ 19 ÷ 19
1% = $1.23
× 100 × 100
100% = $123

19
23% = 163.76 m
÷ 23 ÷ 23
1% = 7.12 m
× 100 × 100
100% = 712 m

20
86% = 44.72 g
÷ 86 ÷ 86
1% = 0.52 g
× 100 × 100
100% = 52 g

21
55% = 195.91 mm
÷ 55 ÷ 55
1% = 3.562 mm
× 100 × 100
100% = 356.2 mm

22
105% = $2 850.75
÷105 ÷105
1% = $27.15
× 100 × 100
100% = $2 715

23
125% = $5 650
÷125 ÷125
1% = $45.20
× 100 × 100
100% = $4 520

24
Airline tickets are being sold at $1 232 which is 88% of the standard price. Find the standard price.

88% = $1 232
÷ 88 ÷ 88
1% = $14
× 100 × 100
100% = $1 400

25
An auction is selling goods at 45% of their normal price. If Kim paid $2 790 for a rug, calculate its normal price.

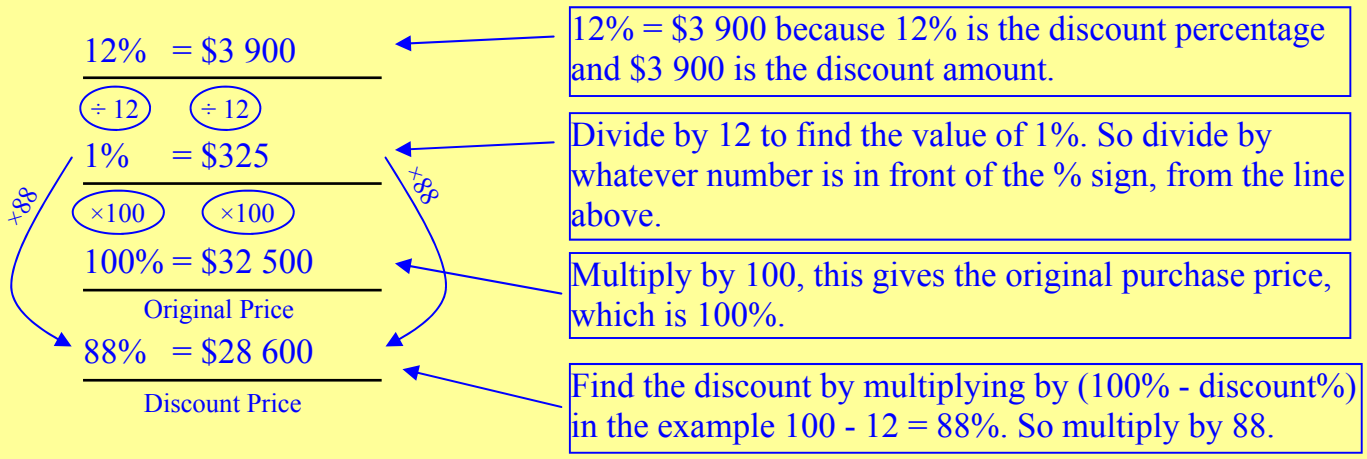
45% = $2 790
÷ 45 ÷ 45
1% = $62
× 100 × 100
100% = $6 200

Further Unitary Method - Discounts and Pricing

This sheet involves discounts, increases, original prices and discounted prices. The most difficult part of these questions is determining what the percentage means...is it the discount or increase?...or is it the amount after the discount or increase? Note that these questions are different to decreasing and incresing by a percentage because you are not given the original amount, you are given either the amount after the increase/decrease has been applied or the increase/decrease amount.

Column 1 involves a discount and its percentage, you aren't given the new price after the discount or the old price. Use this percentage to find the value of 1%, then find the original price (100% ..so multiply by 100) and the discount price, which is 100% minus the discount percentage. Money is always 2 d.p.

Example Car, 12% off, save $3 900



Column 2 gives you the amount after the increase/decrease is applied. This means that if it is an increase you add the increase % to 100%, then solve for 1%. If it is a decrease you subtract the % from 100% and use it to find 1%. The original amount will be 100% and the amount of discount/increase is the percentage in the question.

Your percentage on the first line is
100 + the increase with increase questions
or
100 - the decrease with decreasing questions

In these questions you are given the final amount. You are asked to find the original amount (100%) and then the amount of the increase/decrease (which is the percentage in the question).

Column 3 is a mix of columns 1 and 2, you have to examine the labels and decide what you are being given, the increase/decrease or the amount after the increase/decrease has been applied. This may be challenging so think about these carefully and look at your final answer and ask yourself if the answer looks right.

Further Unitary Method - Discounts and Pricing

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Find the original and the discounted price.

1 Socks, 15% off, save $1.80

15% = $1.80

÷ 15 ÷ 15

1% =

×100 ×100

100% =

Original Price

×85 ×85

85% =

Discount Price

2 Bag, 12% off, save $6.96

12% = $6.96

1% =

100% =

Original Price

88% =

Discount Price

3 Movie tickets, 33⅓% off, save $3.20

Original Price

Discount Price

4 Jacket, 40% off, save $13.

Original Price

Discount Price

Find the original amount and then the increase or decrease, answer to 2 d.p.

5 Recycling increased 18%, to a total of 56 286 t.

118% = 56 286 t

÷ 118 ÷ 118

1% =

×100 ×100

100% =

Original recycling amount

×18 ×18

18% =

Size of Increase

6 Will's second javelin throw decreased 14%, to 38.27 m

Original Distance

Amount of Decrease

7 Fluid level decreased 53%, to 794.3 mL.

Original Fluid Level

Amount of Decrease

Find the original price for these tagged items be careful with these.

8 100% - 20%

80% = $18

1% =

100% =

SALE!!! Save 20%

Pay only $18!!

9

SALE!!! Save 35% That's $13.65 OFF!!

10

SALE!!! Pay only 80%! That's $32 OFF!!

11

Pay only 52%!

You pay $24.57

12

SALE!!! Save 12½%

Pay only $59.50 !

Further Unitary Method - Discounts and Pricing

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Find the original and the discounted price.

1 Socks, 15% off, save $1.80

15% = $1.80
÷ 15 ÷ 15
1% = $0.12
×100 ×100
100% = $12
Original Price
×85 ×85
85% = $10.20
Discount Price

2 Bag, 12% off, save $6.96

12% = $6.96
÷ 12 ÷ 12
1% = $0.58
×100 ×100
100% = $58
Original Price
×88 ×88
88% = $51.04
Discount Price

3 Movie tickets, 33⅓% off, save $3.30

33⅓% = $3.30
÷33⅓ ÷33⅓
1% = $0.099
×100 ×100
100% = $9.90
Original Price
66⅔% = $6.60
Discount Price

4 Jacket, 40% off, save $13.

40% = $13
÷40 ÷40
1% = $0.325
×100 ×100
100% = $32.50
Original Price
60% = $19.50
Discount Price

Find the original amount and then the increase or decrease, answer to 2 d.p.

5 Recycling increased 18%, to a total of 56 286 t.

118% = 56 286 t
÷ 118 ÷ 118
1% = 477 t
×100 ×100
100% = 47 700 t
Original recycling amount
×18 ×18
18% = 8 586 t
Size of Increase

6 Will's second javelin throw decreased 14%, to 38.27 m

86% = 38.27 m
÷ 86 ÷ 86
1% = 0.445 m
×100 ×100
100% = 44.5 m
Original Distance
14% = 6.23 m
Amount of Decrease

7 Fluid level decreased 53%, to 794.3 mL.

47% = 794.3 mL
÷ 47 ÷ 47
1% = 16.9 mL
×100 ×100
100% = 1 690 mL
Original Fluid Level
53% = 895.7 mL
Amount of Decrease

Find the original price for these tagged items be careful with these.

8

100% - 20%
80% = $18
÷ 80 ÷ 80
1% = $0.225
×100 ×100
100% = $22.50

SALE!!! Save 20% Pay only $18!!

9

SALE!!! Save 35% That's $13.65 OFF!!

35% = $13.65
÷ 35 ÷ 35
1% = $0.39
×100 ×100
100% = $39

10

SALE!!! Pay only 80%! That's $32 OFF!!

20% = $32
÷ 20 ÷ 20
1% = $1.60
×100 ×100
100% = $160

11

52% = $24.57
÷ 52 ÷ 52
1% = $0.4725
×100 ×100
100% = $47.25

Pay only 52%! You pay $24.57

12

SALE!!! Save 12½% Pay only $59.50 !

87½% = $59.50
÷87½ ÷87½
1% = $0.68
×100 ×100
100% = $68

Commission on Sales

Commission is when you are paid a percentage of your sales. This means the more you sell, the more you make. So it is a motivational method of payment to pay you more if you make more money for the business. It can result in low pay also for people who aren't good in sales, so commission can be a poor payment option for some people.

Commission is the same as finding a percentage of a quantity, except it is easier because the units are always $, and are always rounded to 2 decimal places.

Column 1 is completed by multiplying the sales amount by the percentage as a decimal. See below.

Example. Sales = $17 315, Rate = 6%

Commission = $17\,315 \times \frac{6}{100}$ ← Multiply the amount by the percentage expressed as a fraction (over 100)

Commission = $1 038.90 ← Write your answer with a ($) and show two decimal places so write 1038.9 as $1 038.90

Column 2 starts with a table. Complete the table the same way as column 1 except no working out is required. If you don't know how to turn the screen click on "View", then "Rotate View" then "Counter Clockwise". Repeat only select "Clockwise" to return to standard view. After you answer the table, rank the earnings of the employees from 1 (Most earned) to 7 (Least earned).

The rest of the sheet, Q17 onwards, is answered the same way. Calculate the commission earned by one or more people and fill in the circle for the person who earned the most. Note that some people are paid a wage (not a commission), their wage requires no calculation, just calculate commission earnings and compare it with the wage earner.

Tom earns $560/wk, Jessica earns 15% comm. on sales of $3 970. ← Tom earns a wage so no calculation is required for Tom. Jessica is paid by commission, so her pay must be calculated.

Jessica = $3\,970 \times \frac{15}{100}$ ← Multiply the amount by the percentage expressed as a fraction (over 100), just as in earlier questions.

Jessica = $595.50

Jessica ● Tom ○ ← Jessica is paid $595.50 and Tom is paid $560. So Jessica earns the most, fill in the circle with her name next to it.

Commission on Sales

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Find the commission on these sales given the rates of commission below. Answer to 2 d.p.

Example

Sales = \$15 540, Rate = 12%

Commission = $15\,540 \times \frac{12}{100}$

Commission = \$1 864.80

1 Sales = \$21 863, Rate = 7%

Commission =

2 Sales = \$11 406, Rate = 4%

3 Sales = \$58 170, Rate = 2%

4 Sales = \$92 011, Rate = 11%

5 Sales = \$12 388, Rate = 7%

6 Sales = \$31 060, Rate = 14%

7 Sales = \$17 202, Rate = 5%

Complete the table by finding the commission payable.

15	14	13	12	11	10	9	8
George	Alicia	Kelly	Mischa	Imran	Anna	Steven	Name
12%	5%	14%	7%	15%	3%	8%	Rate
\$29 810	\$5 104	\$15 773	\$41 051	\$13 022	\$47 997	\$32 694	Sales
							Commission

Rank the above from lowest to highest commission paid (1 = most, 7 = least)

16 (7)

Kelly Alicia George Steven Anna Imran Mischa

Calculate the commission and fill the circle for who earns the most per week.

17 Jan earns \$560/wk, Ian earns 12% comm. on sales of \$4 600

Ian = $4\,600 \times \frac{12}{100}$

Ian =

Jan ◯ Ian ◯

18 Kerrie 7% on \$19 320 sales, Anthia 9% on sales of \$14 450

Kerrie ◯ Anthia ◯

19 Phoebe earns \$930/wk, Kyle earns 16% comm. on \$11 676

Phoebe ◯ Kyle ◯

20 David 3% on \$176 500, Drew 4% on sales of \$143 200

David ◯ Drew ◯

21 Pearce 25% on \$4 760, Bec earns 6% comm. on \$21 300

Pearce ◯ Bec ◯

22 Samantha \$1 050/wk, Corey earns 2% on sales of \$93 000

Samantha ◯ Corey ◯

23 Michelle \$910/wk, Geoff 8% comm. on sales of \$9 470 and Jessica 15% on sales of \$5 320

Michelle ◯ Geoff ◯ Jessica ◯

Commission on Sales

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Find the commission on these sales given the rates of commission below. Answer to 2 d.p.

Example

Sales = \$15 540, Rate = 12%

Commission = $15\,540 \times \frac{12}{100}$

Commission = \$1 864.80

1 Sales = \$21 863, Rate = 7%

Commission = $21\,863 \times \frac{7}{100}$

Commission = \$1 530.41

2 Sales = \$11 406, Rate = 4%

Commission = $11\,406 \times \frac{4}{100}$

Commission = \$456.24

3 Sales = \$58 170, Rate = 2%

Commission = $58\,170 \times \frac{2}{100}$

Commission = \$1 163.40

4 Sales = \$92 011, Rate = 11%

Commission = $92\,011 \times \frac{11}{100}$

Commission = \$10 121.21

5 Sales = \$12 388, Rate = 7%

Commission = $12\,388 \times \frac{7}{100}$

Commission = \$867.16

6 Sales = \$31 060, Rate = 14%

Commission = $31\,060 \times \frac{14}{100}$

Commission = \$4 348.40

7 Sales = \$17 202, Rate = 5%

Commission = $17\,202 \times \frac{5}{100}$

Commission = \$860.10

Complete the table by finding the commission payable.

	15	14	13	12	11	10	9	8
Name	George	Alicia	Kelly	Mischa	Imran	Anna	Steven	
Rate	12%	5%	14%	7%	15%	3%	8%	
Sales	\$29 810	\$5 104	\$15 773	\$41 051	\$13 022	\$47 997	\$32 694	
Commission	\$3 577.20	\$255.20	\$2 208.22	\$2 873.57	\$1 953.30	\$1 439.91	\$2 615.52	

Rank the above from lowest to highest commission paid (1 = most, 7 = least)

16 Kelly (4) Alicia (7) George (1) Steven (3) Anna (6) Imran (5) Mischa (2)

Calculate the commission and fill the circle for who earns the most per week.

17 Jan earns \$560/wk, Ian earns 12% comm. on sales of \$4 600

Ian = $4\,600 \times \frac{12}{100}$

Ian = \$552

Jan ● Ian ○

18 Kerrie 7% on \$19 320 sales, Anthia 9% on sales of \$14 450

Kerrie = $19320 \times \frac{7}{100}$

Kerrie = \$1 352.40

Anthia = $14\,450 \times \frac{9}{100}$

Anthia = \$1 300.50

Kerrie ● Anthia ○

19 Phoebe earns \$930/wk, Kyle earns 16% comm. on \$11 676

Kyle = $11\,676 \times \frac{16}{100}$

Kyle = \$1 868.16

Phoebe ○ Kyle ●

20 David 3% on \$176 500, Drew 4% on sales of \$143 200

David = $176\,500 \times \frac{3}{100}$

David = \$5 295

Drew = $143\,200 \times \frac{4}{100}$

Drew = \$5 728

David ○ Drew ●

21 Pearce 25% on \$4 760, Bec earns 6% comm. on \$21 300

Pearce = $4\,760 \times \frac{25}{100}$

Pearce = \$1 190

Bec = $21\,300 \times \frac{6}{100}$

Bec = \$1 278

Pearce ○ Bec ●

22 Samantha \$1 050/wk, Corey earns 2% on sales of \$93 000

Corey = $93\,000 \times \frac{2}{100}$

Corey = \$1 860

Samantha ○ Corey ●

23 Michelle \$910/wk, Geoff 8% comm. on sales of \$9 470 and Jessica 15% on sales of \$5 320

Geoff = $9\,470 \times \frac{8}{100}$

Geoff = \$757.60

Jessica = $5\,320 \times \frac{15}{100}$

Jessica = \$798

Michelle ● Geoff ○ Jessica ○

Further Commission and Base Salary

An alternative to being paid "Straight Commission" which is commission only, is to receive a base salary each week but receive commission as well, as an incentive to sell more and increase your pay.

Column 1 gives you the base salary and then a sales figure and commission rate. To calculate the pay earned add the base salary to the commission earned. See example below.

Example. Base salary of $415/wk plus 11% comm. on sales of $6 770

Pay = $415 + 6\,770 \times \frac{11}{100}$ ← Base salary + sales × percentage as a fraction (over 100)

Pay = $1 159.70 ← Base salary + sales × percentage as a fraction (over 100)

Note that Q6 and Q7 have two different commission rates, so calculate them separately then add them to the base salary to get the total pay.

Column 2 (from Q8) and Column 3 use the Real Estate agent's sliding scale shown above Q8. This scale starts with a fee of $3000. Then the sale amount of the property determines the rate or rates of commission that apply.

Sale Price $362 000

$362 500

$150 000 $\times \frac{3}{100}$ ← The first $150 000 of the sale price has a commission of 3% payable. So multiply $150 000 by 3% expressed as a fraction.

= $4 500

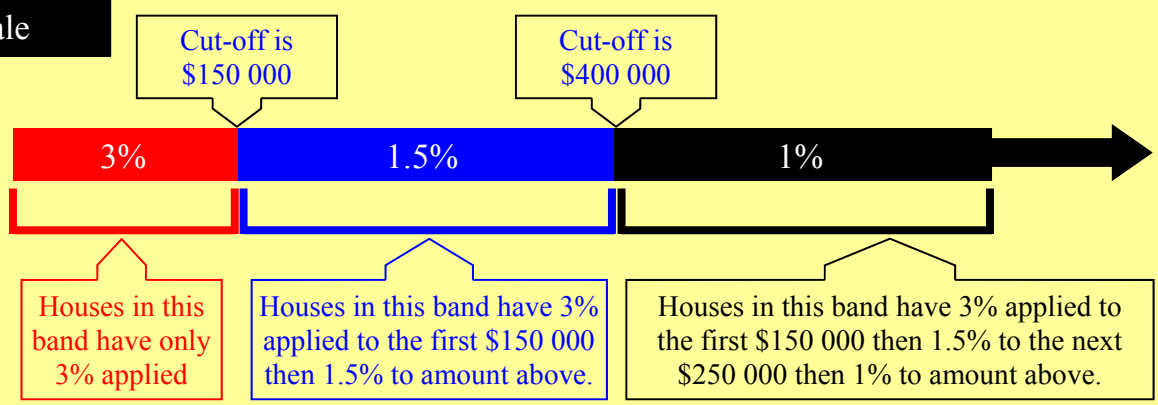
$216 000 $\times \frac{1.5}{100}$ ← $362 000 - $150 000 = $216 000 the second rate of commission applies for the next $250 000, so all of the $216 000 has a rate of 1.5% payable

= $3 240

Comm = 3 000 + 4 500 + 3 240

= $10 740 ← Remember that the commission must include the $3000 basic fee then the other amounts added to it.

Graphical Representation of sliding scale



Further Commission and Base Salary

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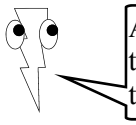

Add the commission to the base salary to find the total weekly pay.

1 Base salary of $210/wk plus 4% comm. on sales of $14 656

Pay = $210 + 14\,656 \times \frac{4}{100}$

Pay =

2 Base salary of $135/wk plus 7% comm. on sales of $9 898

Pay =

Pay =

3 Base salary of $340/wk plus 2% comm. on sales of $6 470

4 Base salary of $175/wk plus 5% comm. on sales of $19 176

5 Base salary of $338/wk plus 1.5% comm. on sales of $42 376

6 Base salary of $643/wk plus 0.5% comm. on sales of $35 600 and 0.75% on sales of $11 200

0.5% comm. 0.75% comm.

= =

= $ = $

Pay = 643 + +

Pay =

7 Base salary of $371/wk plus 0.3% comm. on sales of $18 299 and 0.45% on sales of $38 011.

0.3% comm. 0.45% comm.



Use the real estate agent fee schedule below to find the commission payable

Basic commission of $3 000, then add:
3% on first $150 000 of sale price
1.5% on next $250 000 of sale price
1% on remaining balance

8 Sale Price $98 950

Comm = $3\,000 + 98\,950 \times \frac{3}{100}$

Comm =

9 Sale Price $175 000

$175 000

$150 000 \times \frac{3}{100}$

=

$25 000 \times \frac{1.5}{100}$

=

Comm = 3 000 + ☐

\+ ☐ = ☐

10 Sale Price $345 000

$345 000

$\times \frac{3}{100}$

=

$\times \frac{1.5}{100}$

=

Comm = 3 000 + ☐

\+ ☐ = ☐

11 Sale Price $535 000

$535 000

$\times \frac{3}{100}$

=

$\times \frac{1.5}{100}$

=

$\times \frac{1}{100}$

=

12 Sale Price $1 410 000

$1 410 000

13 Sale Price $282 500

$282 500

Further Commission and Base Salary

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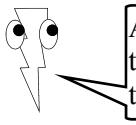

Add the commission to the base salary to find the total weekly pay.

1 Base salary of $210/wk plus 4% comm. on sales of $14 656

Pay = $210 + 14\,656 \times \frac{4}{100}$

Pay = $796.24

2 Base salary of $135/wk plus 7% comm. on sales of $9 898

Pay = $135 + 9\,898 \times \frac{7}{100}$

Pay = $827.86

3 Base salary of $340/wk plus 2% comm. on sales of $6 470

Pay = $340 + 6\,470 \times \frac{2}{100}$

Pay = $469.40

4 Base salary of $175/wk plus 5% comm. on sales of $19 176

Pay = $175 + 19\,176 \times \frac{5}{100}$

Pay = $1 133.80

5 Base salary of $338/wk plus 1.5% comm. on sales of $42 376

Pay = $338 + 42\,376 \times \frac{1.5}{100}$

Pay = $973.64

6 Base salary of $643/wk plus 0.5% comm. on sales of $35 600 and 0.75% on sales of $11 200

0.5% comm. $= 35\,600 \times \frac{0.5}{100}$ = $ 178

0.75% comm. $= 11\,200 \times \frac{.75}{100}$ = $ 84

Pay = 643 + 178 + 84

Pay = $905

7 Base salary of $371/wk plus 0.3% comm. on sales of $18 299 and 0.45% on sales of $38 011.

0.3% comm. $= 18\,299 \times \frac{0.3}{100}$ = $ 54.90

0.45% comm. $= 38\,011 \times \frac{.45}{100}$ =$ 171.05

Pay = 371 + 54.90 + 171.05

Pay = $596.95



Use the real estate agent fee schedule below to find the commission payable

Basic commission of $3 000, then add:
3% on first $150 000 of sale price
1.5% on next $250 000 of sale price
1% on remaining balance

8 Sale Price $98 950

Comm = $3\,000 + 98\,950 \times \frac{3}{100}$

Comm = $5 968.50

9 Sale Price $175 000

$175 000

$150 000 × 3/100 = $4 500

$25 000 × 1.5/100 = $375

Comm = 3 000 + 4 500 + 375 = $7 875

10 Sale Price $345 000

$345 000

$150 000 × 3/100 = $4 500

$195 000 × 1.5/100 = $2 925

Comm = 3 000 + 4 500 + 2 925 = $10 425

11 Sale Price $535 000

$535 000

$150 000 × 3/100 = $4 500

$250 000 × 1.5/100 = $3 750

$135 000 × 1/100 = $1 350

Comm = 3 000 + 4 500 + 3 750 + 1 350

Comm = $12 600

12 Sale Price $1 410 000

$1 410 000

$150 000 × 3/100 = $4 500

$250 000 × 1.5/100 = $3 750

$1 010 000 × 1/100 = $10 100

Comm = 3 000 + 4 500 + 3 750 + 10 100

Comm = $21 350

13 Sale Price $282 500

$282 500

$150 000 × 3/100 = $4 500

$132 500 × 1.5/100 = $1 987.50

Comm = 3 000 + 4 500 + 1 987.50 =$9 487.50

Simple Interest

It's a big formula, so what is so simple about it? It is called simple interest because at the end of each year no interest is paid, it is only paid when the total time period of the investment is reached. So interest isn't paid at the end of the each year, just when the investment is completed.

There are 4 variables in the formula, these are:

$$I = \frac{PRT}{100}$$

I = Interest earned on the investment (or payable on a loan)
P = Principle, the amount invested (or borrowed if a loan)
R = interest rate applied to the investment (or loan)
T = the term, the time period the investment (or loan) is over

To find the interest payable, multiply the given Principal, rate and term then divide by 100. There is an example below. As you are dealing with money remember to round your answer to 2 decimal places (2 d.p.)

Example. $710 at 5.85% for 6 years ← P = $710, r = 5.85% and T = 6

$$I = \frac{PrT}{100} = \frac{710 \times 5.85 \times 6}{100}$$

$$I = \$249.21$$

Using your calculator

7	1	0	×	5	.	8	5	×	6	÷	1	0	0	=

In column 3 you are asked to calculate the account balance, to do this, use the same method as Columns 1 and 2 except you add the Interest to the Principle as a last step. So don't clear your calculator, add the principle to the answer on your calculator screen. This gives you the Account Balance or "A". For the example above.

$$A = 710 + 249.21$$

$$A = \$959.21$$

A = P + I
or
Acc Bal = Principle + Interest

Warning - you may prefer to use the fraction key rather than divide. If you do, you may get a fraction answer, after you press the equals (=) button press the fraction key again and it will change the answer back to a decimal.

Simple Interest

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Calculate the interest earned on the investments below, round your answer to 2 d.p.

Example

P T R

$800 for 3 years at 6% p.a.

$$I = \frac{PRT}{100} = \frac{800 \times 6 \times 3}{100}$$

Calculator

8 0 0 × 6 × 3 ÷ 1 0 0 =

$I = \$144$

1 $235 for 5 years at 8% p.a.

$$I = \frac{PRT}{100} = \_\_\_\_\_$$

2 $788 for 9 weeks at 0.2% per week.

3 $771 for 7 months at 1.4% per month.

4 $5 370 for 200 days at 0.015% per day.

5 $65 at 9.25% p.a. for a year

6 $428 for 3 years at 4.3% p.a.

7 $43 200 at 3% p.a. for 7 years.

8 $400 for 9 weeks at 0.17% per week.

9 $8 509 for 3 quarters at 3.9% per quarter.

10 0.02% per day is paid on a $86 700 investment for 75 days.

11 $580 is invested at 0.9% per month for 10 months.

Add the interest (I) to the principle (P) to calculate the account balance (A).

Principle	+ Interest	= Balance
$1 200	$855.20	$2 055.20
$375	$212.72	$587.72
$7 700	$2 795.10	$10 495.10
$95 676	$17 392	$113 068

12 $5 700 for 4 years at 8% p.a.

$$I = \frac{PRT}{100} = \_\_\_\_\_$$

$A = 5700 +$ _____

$A =$ _____

13 $750 for 3 years at 4.3% p.a.

14 $175 at 9.3% p.a. for 2 years

15 $900 for 4 months at 0.64% per month.

Simple Interest

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Calculate the interest earned on the investments below, round your answer to 2 d.p.

Example

P T R

$800 for 3 years at 6% p.a.

$$I = \frac{PRT}{100} = \frac{800 \times 6 \times 3}{100}$$

Calculator

8 0 0 × 6 × 3 ÷ 1 0 0 =

$I = \$144$

1 $235 for 5 years at 8% p.a.

$$I = \frac{PRT}{100} = \frac{235 \times 8 \times 5}{100}$$

$I = \$94$

2 $788 for 9 weeks at 0.2% per week.

$$I = \frac{PRT}{100} = \frac{788 \times 0.2 \times 9}{100}$$

$I = \$14.18$

3 $771 for 7 months at 1.4% per month.

$$I = \frac{PRT}{100} = \frac{771 \times 1.4 \times 7}{100}$$

$I = \$75.56$

4 $5 370 for 200 days at 0.015% per day.

$$I = \frac{PRT}{100}$$

$$= \frac{5\,370 \times 0.015 \times 200}{100}$$

$I = \$161.10$

5 $65 at 9.25% p.a. for a year

$$I = \frac{PRT}{100} = \frac{65 \times 9.25 \times 1}{100}$$

$I = \$6.01$

6 $428 for 3 years at 4.3% p.a.

$$I = \frac{PRT}{100} = \frac{428 \times 4.3 \times 3}{100}$$

$I = \$55.21$

7 $43 200 at 3% p.a. for 7 years.

$$I = \frac{PRT}{100} = \frac{43\,200 \times 3 \times 7}{100}$$

$I = \$9\,072$

8 $400 for 9 weeks at 0.17% per week.

$$I = \frac{PRT}{100} = \frac{400 \times 0.17 \times 9}{100}$$

$I = \$6.12$

9 $8 509 for 3 quarters at 3.9% per quarter.

$$I = \frac{PRT}{100} = \frac{8\,509 \times 3.9 \times 3}{100}$$

$I = \$995.55$

10 0.02% per day is paid on a $86 700 investment for 75 days.

$$I = \frac{PRT}{100}$$

$$= \frac{86\,700 \times 0.02 \times 75}{100}$$

$I = \$1\,300.50$

11 $580 is invested at 0.9% per month for 10 months.

$$I = \frac{PRT}{100} = \frac{580 \times 0.9 \times 10}{100}$$

$I = \$52.20$

Add the interest (I) to the principle (P) to calculate the account balance (A).

Principle	+ Interest	= Balance
$1 200	$855.20	$2 055.20
$375	$212.72	$587.72
$7 700	$2 795.10	$10 495.10
$95 676	$17 392	$113 068

12 $5 700 for 4 years at 8% p.a.

$$I = \frac{PRT}{100} = \frac{5\,700 \times 8 \times 4}{100}$$

$I = \$1\,824$

$A = 5700 + 1\,824$

$A = \$7\,524$

13 $750 for 3 years at 4.3% p.a.

$$I = \frac{PRT}{100} = \frac{750 \times 4.3 \times 3}{100}$$

$I = \$96.75$

$A = 750 + 96.75$

$A = \$846.75$

14 $175 at 9.3% p.a. for 2 years

$$I = \frac{PRT}{100} = \frac{175 \times 9.3 \times 2}{100}$$

$I = \$32.55$

$A = 175 + 32.55$

$A = \$207.55$

15 $900 for 4 months at 0.64% per month.

$$I = \frac{PRT}{100} = \frac{900 \times 0.64 \times 4}{100}$$

$I = \$23.04$

$A = 900 + 23.04$

$A = \$923.04$

Further Simple Interest

This sheet uses periods of investment that don't match the interest rate period. The previous simple interest sheet should have been completed before this sheet is attempted.

Column 1 uses weeks or months but the interest rate is always p.a., so write the number of weeks as a fraction of a year (/52) or write months as a fraction of a year (/12). look at the examples below.

Example 1. $6 050 at 9% for 34 weeks

$$I = \frac{PRT}{100} = \frac{6\,050 \times 9 \times \frac{34}{52}}{100}$$

$I = \$356.02$

Example 2. $3 100 at 7.7% for 8 months

$$I = \frac{PRT}{100} = \frac{3\,100 \times 7.7 \times \frac{8}{12}}{100}$$

$I = \$159.13$

Note that time periods can exceed 1 year. For example you could have an investment for 75 weeks. This would be (75/52). Likewise with months, 20 month investment would use (20/12).

Column 2 starts by using days so write the days over 365 (/365). Then a combination of days, months and years are used. Write the years as the whole number part of the mixed numeral, then make the fraction up using either days, weeks or months (/365 /52 or /12). Examples are below.

Example 1. $800 at 4% for 170 days

$$I = \frac{PRT}{100} = \frac{800 \times 4 \times \frac{170}{365}}{100}$$

$I = \$14.90$

Example 2. $1 950 at 5.6% for 4 yrs 6 wks

$$I = \frac{PRT}{100} = \frac{1\,950 \times 5.6 \times 4\frac{6}{52}}{100}$$

$I = \$449.40$

Column 3 asks you to find the account balance, complete the interest calculation just the same way but then add the interest and principle together.

$A = P + I$
or
Acc Bal = Principle + Interest

Warning - if you use the fraction key for the term (ie: 4 months = 4/12) don't use the calculator fraction key again to divide by 100. If you do, the calculator will change the $^{4}/_{12}$ to $4\ ^{12}/_{100}$. So use ÷ 100 instead of the fraction key to avoid this.

Further Simple Interest

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For investments less than 1 year use fractions. Put weeks over 52 and months over 12. So 3 weeks = $^{3}/_{52}$.

1 $750 at 5% p.a. for 7 months

$I = \frac{PRT}{100} = \frac{\quad \times \quad \times \frac{7}{12}}{}$

Calculator

7 5 0 × 5 × 7 $a^b/_c$ 1 2 = 1 0 0 =

$I =$ _____

2 $1 400 for 9 weeks at 4% p.a.

$I =$ _____ $= \frac{\quad \times \quad \times \frac{9}{52}}{}$

$I =$ _____

3 $850 at 5.7% p.a. for 4 months

_____ _____

_____

4 $3 210 at 6% p.a. for 11 months

_____ _____

_____

5 $975 for 25 weeks at 7.3% p.a.

_____ _____

_____

6 $6 320 for 18 months at 9% p.a.

_____ _____

_____

Change days to fractions (/365) and years & months to mixed numbers.

7 $2 830 at 7.6% p.a. for 210 days

_____ $\frac{\quad \times \quad \times \frac{210}{365}}{}$

_____

8 $9 500 at 8.7% p.a. for 85 days

_____ _____

_____

9 $5 810 at 11% p.a. for 303 days

_____ _____

_____

10 $600 at 3% p.a. for 2 years and 4 months

_____ $\times 2\frac{4}{12}$

_____

11 $850 at 8% p.a for 6 years and 9 weeks

_____ _____

_____

12 $3 095 at 12% p.a for 3 years and 7 months

_____ _____

_____

Find the interest then add it to the principle to find the new balance

Principle	+ Interest	= Balance
$13 954	$2 883	
$5 793	$1 609	
$36 271	$7 030	
$19 430	$890.68	
$948	$56.94	
$29 300	$42 339	

13 $532 at 4% p.a. for 5 years and 2 months.

_____ _____

_____

$A =$ 532 +

$A =$ _____

14 $8 225 for 172 days at 7.6% p.a.

_____ _____

_____

_____

_____

15 $3 777 at 6% p.a. for 2 years and 94 days.

_____ _____

_____

_____

_____

Further Simple Interest

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For investments less than 1 year use fractions. Put weeks over 52 and months over 12. So 3 weeks = $^3/_{52}$.

1 $750 at 5% p.a. for 7 months

$$I = \frac{PRT}{100} = \frac{750 \times 5 \times \frac{7}{12}}{100}$$

Calculator

7 5 0 × 5 × 7 $a^b/_c$ 1 2 = 1 0 0 =

$I = \$21.88$

2 $1 400 for 9 weeks at 4% p.a.

$$I = \frac{PRT}{100} = \frac{1\,400 \times 4 \times \frac{9}{52}}{100}$$

$I = \$9.69$

3 $850 at 5.7% p.a. for 4 months

$$I = \frac{PRT}{100} = \frac{850 \times 5.7 \times \frac{4}{12}}{100}$$

$I = \$16.15$

4 $3 210 at 6% p.a. for 11 months

$$I = \frac{PRT}{100} = \frac{3\,210 \times 6 \times \frac{11}{12}}{100}$$

$I = \$176.55$

5 $975 for 25 weeks at 7.3% p.a.

$$I = \frac{PRT}{100} = \frac{975 \times 7.3 \times \frac{25}{52}}{100}$$

$I = \$34.22$

6 $6 320 for 18 months at 9% p.a.

$$I = \frac{PRT}{100} = \frac{6\,320 \times 9 \times \frac{18}{12}}{100}$$

$I = \$853.20$

Change days to fractions (/365) and years & months to mixed numbers.

7 $2 830 at 7.6% p.a. for 210 days

$$I = \frac{PRT}{100} = \frac{2\,830 \times 7.6 \times \frac{210}{365}}{100}$$

$I = \$123.74$

8 $9 500 at 8.7% p.a. for 85 days

$$I = \frac{PRT}{100} = \frac{9\,500 \times 8.7 \times \frac{85}{365}}{100}$$

$I = \$192.47$

9 $5 810 at 11% p.a. for 303 days

$$I = \frac{PRT}{100} = \frac{5\,810 \times 11 \times \frac{303}{365}}{100}$$

$I = \$530.54$

10 $600 at 3% p.a. for 2 years and 4 months

$$I = \frac{PRT}{100} = \frac{600 \times 3 \times 2\frac{4}{12}}{100}$$

$I = \$42$

11 $850 at 8% p.a for 6 years and 9 weeks

$$I = \frac{PRT}{100} = \frac{850 \times 8 \times 6\frac{9}{52}}{100}$$

$I = \$419.77$

12 $3 095 at 12% p.a for 3 years and 7 months

$$I = \frac{PRT}{100} = \frac{3\,095 \times 12 \times 3\frac{7}{12}}{100}$$

$I = \$1\,330.85$

Find the interest then add it to the principle to find the new balance

Principle	+ Interest	= Balance
$13 954	$2 883	$14 237
$5 793	$1 609	$7 402
$36 271	$7 030	$43 301
$19 430	$890.68	$20 320.68
$948	$56.94	$1 004.94
$29 300	$42 339	$71 639

13 $532 at 4% p.a. for 5 years and 2 months.

$$I = \frac{PRT}{100} = \frac{532 \times 4 \times 5\frac{2}{12}}{100}$$

$I = \$109.95$

$A = 532 + 109.95$

$A = \$641.95$

14 $8 225 for 172 days at 7.6% p.a.

$$I = \frac{PRT}{100} = \frac{8\,225 \times 7.6 \times \frac{172}{365}}{100}$$

$I = \$294.57$

$A = 8\,225 + 294.57$

$A = \$8\,519.57$

15 $3 777 at 6% p.a. for 2 years and 94 days.

$$I = \frac{PRT}{100} = \frac{3\,777 \times 6 \times 2\frac{94}{365}}{100}$$

$I = \$511.60$

$A = 3\,777 + 511.60$

$A = \$4\,288.60$

GST Calculations - Unitary Method - Australia GST 10%

GST is short for Goods and Services Tax, it is a tax that is applied to day to day items at a rate of 10%*. While there is another way of calculating GST on an item (÷ 11), this method won't work when the rate changes to another amount or you travel to a country such as New Zealand which has a different rate, 12.5%*.

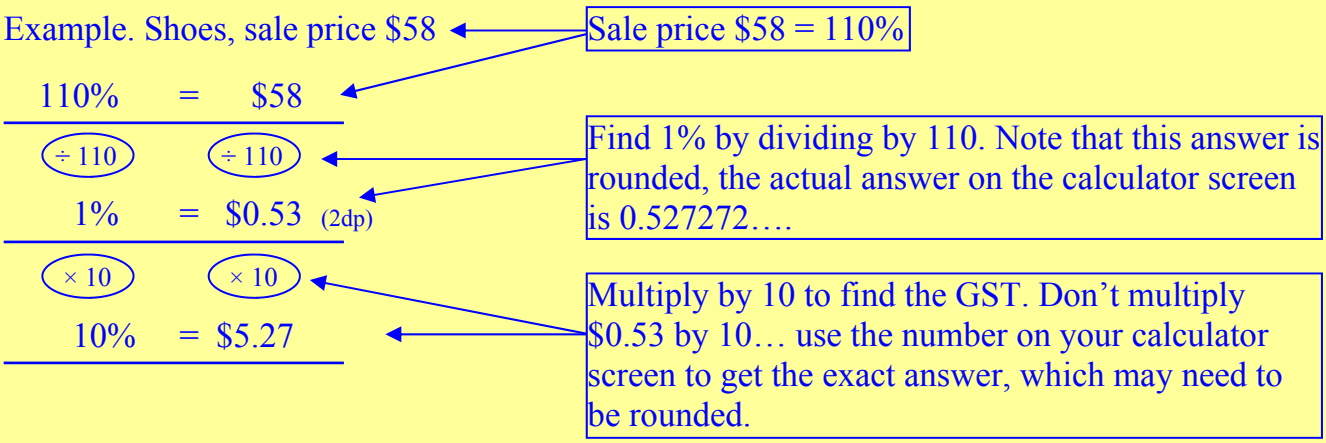
The following describes GST:

Ex-GST price + GST = Sale Price

Ex-GST = Price excluding GST GST = TAX Sale Price = Price paid for item

Prices calculated may need to be rounded, the customer gets a rounded GST figure to the nearest cent.

Column 1 uses the unitary method to find the GST in an item. Because the sale price is the item vale (100%) + GST (10%) the sale price is effectively 110% of the item's value. So using the unitary method:



Column 2 asks you to calculate the sale price given the ex-GST price. Because there is no tax the percentage is 100%. So you are given 100% and have to find 110%. Follow the ovals in Questions 8 and 9 to get you started.

Column 3 is just the reverse of Column 2. This time you are given the sale price and have to find the ex-GST price. So you are given 110% and have to find 100%. Follow questions 16 and 17 tips to get the method.

* GST is 10% in Australia and 12.5% in New Zealand at the time of software release.

GST Calculations - Unitary Method - Australia GST 10%

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Calculate the GST included in these prices. Round 2nd and 3rd lines to 2 d.p.

Example: Jacket, price $84.15

110% = $84.15

÷ 110 ÷ 110

1% = $0.75 (2dp)

Use number on Calculator screen not the rounded answer shown here.

× 10 × 10

10% = $7.65

1 $12.54

÷ 110 ÷ 110

× 10 × 10

2 $41.25

÷ 110 ÷ 110

× 10 × 10

3 $31.79

4 $8 558

5 $352

6 $1.32

7 Find the GST included in a bracelet which sells for $95

Find the sale price, given the ex-GST price. So given 100% find 110%.

8 $704

100% =

÷ 100 ÷ 100

1% =

× 110 × 110

110% = $

9 $46.20

÷ 100 ÷ 100

× 110 × 110

10 $61.60

11 $2.86

12 $17.82

13 $96.25

14 A lounge suite has a ex-GST price of $3 949, find the sale price

15 A T-shirt has a ex-GST price of $15.29, find the sale price.

Find the ex-GST price, given the sale price. So given 110% find 100%.

16 $4.13

110% =

÷ 110 ÷ 110

1% =

× 100 × 100

100% = $

17 $1.89

÷ 110 ÷ 110

× 100 × 100

18 $0.96

19 $244

20 $7.73

21 $1.08

22 A TV sells for $843. Find the ex-GST price.

23 A poster sells for $12.50. Find the ex-GST price.

GST Calculations - Unitary Method - Australia GST 10%

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Calculate the GST included in these prices. Round 2nd and 3rd lines to 2 d.p.

Example: Jacket, price $84.15

110% = $84.15

÷ 110 ÷ 110

1% = $0.75 (2dp)

Use number on Calculator screen not the rounded answer shown here.

× 10 × 10

10% = $7.65

1 $12.54

110% = $12.54

÷ 110 ÷ 110

1% = $0.11

× 10 × 10

10% = $1.14

2 $41.25

110% = $41.25

÷ 110 ÷ 110

1% = $0.38 (2dp)

× 10 × 10

10% = $3.75

3 $31.79

110% = $31.79

÷ 110 ÷ 110

1% = $0.29 (2dp)

× 10 × 10

10% = $2.89

4 $8 558

110% = $8 558

÷ 110 ÷ 110

1% = $77.80

× 10 × 10

10% = $778

5 $352

110% = $352

÷ 110 ÷ 110

1% = $3.20

× 10 × 10

10% = $32

6 $1.32

110% = $1.32

÷ 110 ÷ 110

1% = $0.01 (2dp)

× 10 × 10

10% = $0.12

7 Find the GST included in a bracelet which sells for $95

110% = $95

÷ 110 ÷ 110

1% = $0.86 (2dp)

× 10 × 10

10% = $8.64

Find the sale price, given the ex-GST price. So given 100% find 110%.

8 $704

100% = $704

÷ 100 ÷ 100

1% = $7.04

× 110 × 110

110% = $774.40

9 $46.20

100% = $46.20

÷ 100 ÷ 100

1% = $0.46 (2dp)

× 110 × 110

110% = $50.82

10 $61.60

100% = $61.60

÷ 100 ÷ 100

1% = $0.62 (2dp)

× 110 × 110

110% = $67.76

11 $2.86

100% = $2.86

÷ 100 ÷ 100

1% = $0.03 (2dp)

× 110 × 110

110% = $3.15

12 $17.82

100% = $17.82

÷ 100 ÷ 100

1% = $0.18 (2dp)

× 110 × 110

110% = $19.60

13 $96.25

100% = $96.25

÷ 100 ÷ 100

1% = $0.96

× 110 × 110

110%=$105.88

14 A lounge suite has a ex-GST price of $3 949, find the sale price

100% = $3 949

÷ 100 ÷ 100

1% = $39.49

× 110 × 110

110% = $4 343.90

15 A T-shirt has a ex-GST price of $15.29, find the sale price.

100% = $15.29

÷ 100 ÷ 100

1% = $0.15 (2dp)

× 110 × 110

110% = $16.82

Find the ex-GST price, given the sale price. So given 110% find 100%.

16 $4.13

110% = $4.13

÷ 110 ÷ 110

1% = $0.04 (2dp)

× 100 × 100

100% = $3.75

17 $1.89

110% = $1.89

÷ 110 ÷ 110

1% = $0.02 (2dp)

× 100 × 100

100% = $1.72

18 $0.96

110% = $0.96

÷ 110 ÷ 110

1% = $0.01 (2dp)

× 100 × 100

100% = $0.87

19 $244

110% = $244

÷ 110 ÷ 110

1% = $2.22 (2dp)

× 100 × 100

100%=$221.82

20 $7.73

110% = $7.73

÷ 110 ÷ 110

1% = $0.07 (2dp)

× 100 × 100

100% = $7.03

21 $1.08

110% = $1.08

÷ 110 ÷ 110

1% = $0.01 (2dp)

× 100 × 100

100% = $0.98

22 A TV sells for $843. Find the ex-GST price.

110% = $843

÷ 110 ÷ 110

1% = $7.66 (2dp)

× 100 × 100

100%=$766.36

23 A poster sells for $12.50. Find the ex-GST price.

110% = $12.50

÷ 110 ÷ 110

1% = $0.11 (2dp)

× 100 × 100

100% = $11.36

GST Calculations - Unitary Method - New Zealand GST 15%

GST is short for Goods and Services Tax, it is a tax that is applied to day to day items at a rate of 15%*. There is a quick method to calculate GST and that is to multiply by $^{3}/_{23}$, but the rate can change and this rule won't work if the rate changes.

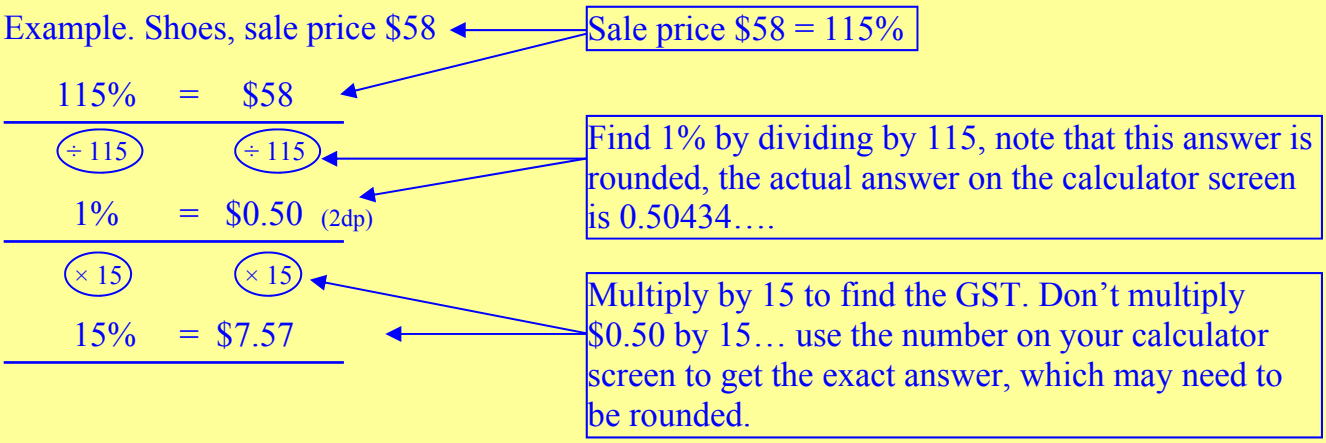
The following describes GST:

Ex-GST price + GST = Sale Price

Ex-GST = Price excluding GST GST = TAX Sale Price = Price paid for item

Prices calculated may need to be rounded, the customer gets a rounded GST figure to the nearest cent.

Column 1 uses the unitary method to find the GST in an item. Because the sale price is the item vale (100%) + GST (15%) the sale price is effectively 115% of the item's value. So using the unitary method:



Column 2 starts by asking you to calculate the sale price given the ex-GST price. Because there is no tax the percentage is 100%. So you are given 100% and have to find 115%. Follow the ovals in Questions 10 and 11 to get you started.

The second group of questions in Column 2 is just the reverse of the previous questions. This time you are given the sale price and have to find the ex-GST price. So you are given 115% and have to find 100%. Follow questions 14 and 15 tips to get the method.

* GST is 12.5% in New Zealand at the time of software release. But it has been suggested that from October 2010, it will rise to 15%.

GST Calculations - Unitary Method - New Zealand GST 15%

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Calculate the GST included in these prices. Round 2nd and 3rd lines to 2 d.p.

Example: Scuba Gear
price $725

115% = $725
÷ 115 ÷ 115
1% = $6.30 (2dp)
× 15 × 15
15% = $94.57

Use the number on your Calculator screen not the rounded answer shown here.

1 Watch, $63.25

115% =
÷ 115 ÷ 115
1% =
× 15 × 15
15% =

2 Table, $851

÷ 115 ÷ 115
× 15 × 15

3 Jeans, $138

4 Dinner, $68.20

5 Ticket, $35

6 Taxi fare, $53

7 Drink, $2.75

8 Concert, $160

9 Sport fee, $75

Find the sale price, given the ex-GST price. So given 100% find 115%.

10 Phone, $85

100% = $85
÷ 100 ÷ 100
1% =
× 115 × 115
115% =

11 Fare, $6.49

÷ 100 ÷ 100
× 115 × 115

12 Headphones, $30

13 Ice cream, $4.27

Find the ex-GST price, given the sale price. So given 115% find 100%.

14 Sunscreen, $8.05

115% = $8.05
÷ 115 ÷ 115
1% =
× 100 × 100
100% =

15 Camera, $103.45

÷ 115 ÷ 115
× 100 × 100

16 TV, $1 306.80

17 Rucksack, $72

18 Book, $8.55

GST Calculations - Unitary Method - New Zealand GST 15%

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Calculate the GST included in these prices. Round 2nd and 3rd lines to 2 d.p.

Example: Scuba Gear price $725

115% = $725
÷ 115 ÷ 115
1% = $6.30 (2dp)
× 15 × 15
15% = $94.57

Use the number on your Calculator screen not the rounded answer shown here.

1 Watch, $63.25

115% = 63.25
÷ 115 ÷ 115
1% = $0.55
× 15 × 15
15% = $8.25

2 Table, $851

115% = $851
÷ 115 ÷ 115
1% = $7.40
× 15 × 15
15% = $111

3 Jeans, $138

115% = $138
÷ 115 ÷ 115
1% = $1.20
× 15 × 15
15% = $18

4 Dinner, $68.20

115% = $68.20
÷ 115 ÷ 115
1% = $0.59 (2dp)
× 15 × 15
15% = $8.90

5 Ticket, $35

115% = $35
÷ 115 ÷ 115
1% = $0.30 (2dp)
× 15 × 15
15% = $4.57

6 Taxi fare, $53

115% = $53
÷ 115 ÷ 115
1% = $0.46 (2dp)
× 15 × 15
15% = $6.91

7 Drink, $2.75

115% = $2.75
÷ 115 ÷ 115
1% = $0.02 (2dp)
× 15 × 15
15% = $0.36

8 Concert, $160

115% = $160
÷ 115 ÷ 115
1% = $1.39 (2dp)
× 15 × 15
15% = $20.87

9 Sport fee, $75

115% = $75
÷ 115 ÷ 115
1% = $0.65 (2dp)
× 15 × 15
15% = $9.78

Find the sale price, given the ex-GST price. So given 100% find 115%.

10 Phone, $85

100% = $85
÷ 100 ÷ 100
1% = $0.85
× 115 × 115
115% = $97.75

11 Fare, $6.49

100% = $6.49
÷ 100 ÷ 100
1% = $0.06 (2dp)
× 115 × 115
115% = $7.46

12 Headphones, $30

100% = $30
÷ 100 ÷ 100
1% = $0.30
× 115 × 115
115% = $34.50

13 Ice cream, $4.27

100% = $4.27
÷ 100 ÷ 100
1% = 0.04 (2dp)
× 115 × 115
115% = $4.91

Find the ex-GST price, given the sale price. So given 115% find 100%.

14 Sunscreen, $8.05

115% = $8.05
÷ 115 ÷ 115
1% = $0.07
× 100 × 100
100% = $7.00

15 Camera, $103.45

115% = $103.45
÷ 115 ÷ 115
1% = $0.90 (2dp)
× 100 × 100
100% = $89.96

16 TV, $1 306.80

115% = $1 306.80
÷ 115 ÷ 115
1% = $11.36 (2dp)
× 100 × 100
100% = $1 136.35

17 Rucksack, $72

115% = $72
÷ 115 ÷ 115
1% = $0.63 (2dp)
× 100 × 100
100% = $62.61

18 Book, $8.55

115% = $8.55
÷ 115 ÷ 115
1% = $0.07 (2dp)
× 100 × 100
100% = $7.43

Depreciation

When you buy an item it goes down in value each year. This is called depreciation. Items depreciate at different rates, depending on how popular or rare they are or their expected life. Depreciation is given as a percentage and it is calculated just the same as finding a percentage of a quantity.

Column 1 involves calculating the depreciation of an item for its first three years of life. An example is shown below.

Car \$39 400 value depreciates by 14% p.a. ← Each year the value of this car will drop by 14%. Note that p.a. stands for per annum which means "per year".

Depreciation = $39\,400 \times \frac{14}{100}$

Depreciation = \$5 516 ← This is the amount of depreciation. So each year the car will go down by this amount. (\$5 516)

Yr 1: 39 400 - 5 516 = \$33 884 ← At the end of the first year the car will have this value.

Yr 2: 33 884 - 5 516 = \$28 368 ← At the end of the second year the car will have this value.

Yr 3: 28 368 - 5 516 = \$22 852 ← At the end of the third year the car will have this value.

Depreciation is only a mathematical model. For example with the car above, would the value be the same after three years if the car wasn't looked after? ..... If the car was considered rare? So its application ignores other factors that could affect the value.

Column 2 is the same as Column 1 except that the values are plotted. So complete all of the calculations first, then:

On Year 0 (The vertical axis) : Go up to the value of the new item, as shown with the Jet Ski in Q5. Plot a point, this has been done for the Jet Ski.

Year 1: Go up to the year 1 value that you calculated and plot a point.

Repeat for Years 2 and 3.

Then draw a line through the points the full size of the graph. At Year 5 draw a line up until it hits your sketched line then draw the line across and read off the value on the scale. This will give the value at the end of the fifth year. Because it is done graphically answers will vary.

You might also like to consider after what amount of time the item will be worth nothing. When the value is zero it is referred to as being "Written Off".

Depreciation

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Find the value of the items below after 1, 2 and 3 years

1 Car $15 900 value depreciates by 12% p.a.

Depreciation = $15\,900 \times \frac{12}{100}$

Depreciation = $1 908

Yr 1: 15 900 - 1 908 = $13 992

Yr 2: 13 992 - 1 908 =

Yr 3: - =

2 Boat $6 700 value depreciates by 22% p.a.

Depreciation =

Yr 1:

Yr 2:

Yr 3:

3 Video Camera $1 850 depreciates by 16% p.a.

Yr 1:

Yr 2:

Yr 3:

4 Forklift $8 300 depreciates by 30% p.a.

Yr 1:

Yr 2:

Yr 3:

Find the value after depreciation after 1, 2 and 3 years. Plot the values on the graph then draw a line. Estimate the value after 5 years using the graph.

5 Jet ski $7 950 value depreciates by 16% p.a.

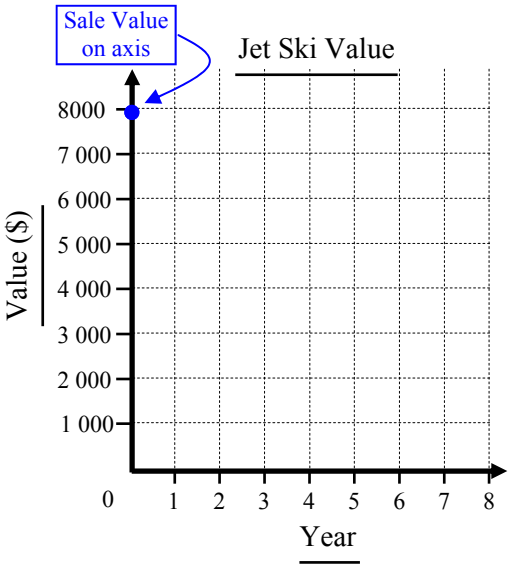
Depreciation =

Yr 1:

Yr 2:

Yr 3:

Yr 5 (Graph):



6 Motorcycle $9 900 value depreciates by 18% p.a.

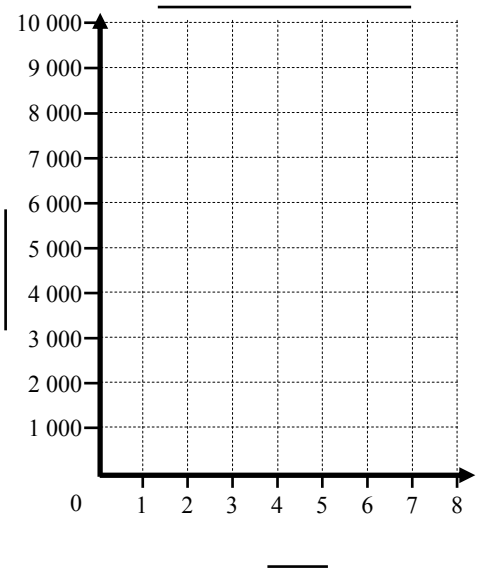
Depreciation =

Yr 1:

Yr 2:

Yr 3:

Yr 5 (Graph):



7 Phone $420 value depreciates by 20% p.a.

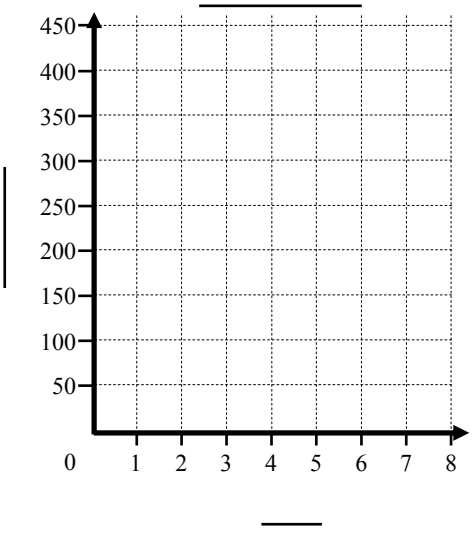
Depreciation =

Yr 1:

Yr 2:

Yr 3:

Yr 5 (Graph):



Depreciation

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Find the value of the items below after 1, 2 and 3 years

1 Car $15 900 value depreciates by 12% p.a.

Depreciation = $15\,900 \times \frac{12}{100}$

Depreciation = $1 908

Yr 1: 15 900 - 1 908 = $13 992

Yr 2: 13 992 - 1 908 = $12 084

Yr 3: 12 084 - 1908 = $10 176

2 Boat $6 700 value depreciates by 22% p.a.

Depreciation = $6\,700 \times \frac{22}{100}$

Depreciation = $1 474

Yr 1: 6 700 - 1 474 = $5 226

Yr 2: 5 226 - 1 474 = $3 752

Yr 3: 3 752 - 1 474 = $2 278

3 Video Camera $1 850 depreciates by 16% p.a.

Depreciation = $1\,850 \times \frac{16}{100}$

Depreciation = $296

Yr 1: 1 850 - 296 = $1 554

Yr 2: 1 554 - 296 = $1 258

Yr 3: 1 258 - 296 = $962

4 Forklift $8 300 depreciates by 30% p.a.

Depreciation = $8\,300 \times \frac{30}{100}$

Depreciation = $2 490

Yr 1: 8 300 - 2 490 = $5 810

Yr 2: 5 810 - 2 490 = $3 320

Yr 3: 3 320 - 2 490 = $830

Find the value after depreciation after 1, 2 and 3 years. Plot the values on the graph then draw a line. Estimate the value after 5 years using the graph.

5 Jet ski $7 950 value depreciates by 16% p.a.

Depreciation = $7\,950 \times \frac{16}{100}$

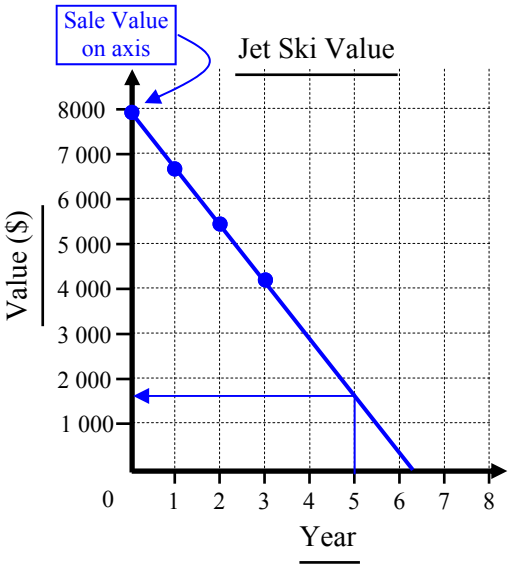
Depreciation = $1 272

Yr 1: 7 950 - 1 272 = $6 678

Yr 2: 6 678 - 1 272 = $5 406

Yr 3: 5 406 - 1 272 = $4 134

Yr 5 (Graph): ≈ $1 600



6 Motorcycle $9 900 value depreciates by 18% p.a.

Depreciation = $9\,900 \times \frac{18}{100}$

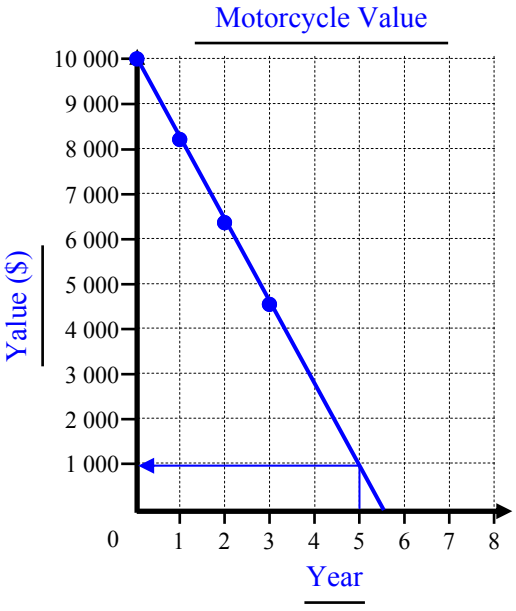
Depreciation = $1 782

Yr 1: 9 900 - 1 782 = $8 118

Yr 2: 8 118 - 1 782 = $6 336

Yr 3: 6 336 - 1 782 = $4 554

Yr 5 (Graph): ≈ $1 000



7 Phone $420 value depreciates by 20% p.a.

Depreciation = $420 \times \frac{20}{100}$

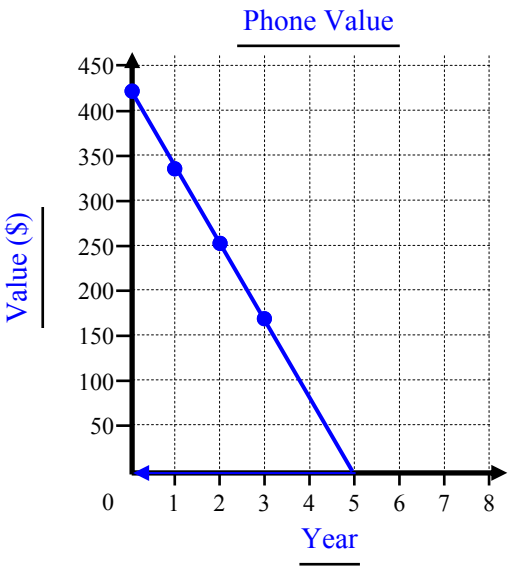
Depreciation = $84

Yr 1: 420 - 84 = $336

Yr 2: 336 - 84 = $252

Yr 3: 252 - 84 = $168

Yr 5 (Graph): ≈ $0



Percentages - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Percentages - Find a Word

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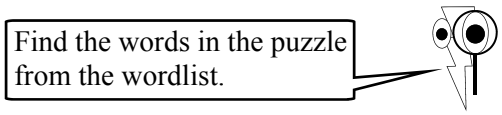

M	E	C	N	A	L	A	B	H	U	N	D	R	E	D
U	N	N	O	I	T	C	A	R	F	F	T	S	Q	T
L	O	C	O	M	M	I	S	S	I	O	N	A	U	N
T	I	E	X	B	M	D	A	P	X	S	O	L	A	E
I	T	E	G	Q	D	E	C	U	D	E	R	A	D	M
P	A	L	S	A	X	D	S	V	Q	T	A	R	V	T
L	I	A	Q	A	T	L	U	S	M	A	A	Y	E	S
Y	C	M	U	M	E	N	M	R	I	R	S	S	D	E
T	E	I	N	T	E	R	E	S	T	O	A	P	I	V
I	R	C	I	I	S	T	C	C	C	E	N	B	V	N
T	P	E	T	F	Q	S	O	N	R	X	K	N	I	I
N	E	D	A	O	F	S	O	C	I	E	X	A	D	S
A	D	T	R	R	T	Z	E	L	P	T	P	O	A	G
U	X	Z	Y	P	Z	D	Y	L	O	T	S	L	Y	O
Q	D	Y	P	R	I	N	C	I	P	L	E	Z	Q	L

WORDLIST

PERCENTAGE	INTEREST	SALARY	SALE	LOSS
DECREASE	FRACTION	RATE	DECIMAL	PRINCIPLE
INVESTMENT	INCREASE	COST	HUNDRED	REDUCE
COMMISSION	UNITARY	MULTIPLY	QUANTITY	DIVIDE
DEPRECIATION	PROFIT	LOAN	BALANCE	TERM

Percentages - Find a Word

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Find the words in the puzzle from the wordlist.



M	E	C	N	A	L	A	B	H	U	N	D	R	E	D
U	N	N	O	I	T	C	A	R	F	F	T	S	Q	T
L	O	C	O	M	M	I	S	S	I	O	N	A	U	N
T	I	E	X	B	M	D	A	P	X	S	O	L	A	E
I	T	E	G	Q	D	E	C	U	D	E	R	A	D	M
P	A	L	S	A	X	D	S	V	Q	T	A	R	V	T
L	I	A	Q	A	T	L	U	S	M	A	A	Y	E	S
Y	C	M	U	M	E	N	M	R	I	R	S	S	D	E
T	E	I	N	T	E	R	E	S	T	O	A	P	I	V
I	R	C	I	I	S	T	C	C	C	E	N	B	V	N
T	P	E	T	F	Q	S	O	N	R	X	K	N	I	I
N	E	D	A	O	F	S	O	C	I	E	X	A	D	S
A	D	T	R	R	T	Z	E	L	P	T	P	O	A	G
U	X	Z	Y	P	Z	D	Y	L	O	T	S	L	Y	O
Q	D	Y	P	R	I	N	C	I	P	L	E	Z	Q	L

WORDLIST

PERCENTAGE
DECREASE
INVESTMENT
COMMISSION
DEPRECIATION
INTEREST
FRACTION
INCREASE
UNITARY
PROFIT
SALARY
RATE
COST
MULTIPLY
LOAN
SALE
DECIMAL
HUNDRED
QUANTITY
BALANCE
LOSS
PRINCIPLE
REDUCE
DIVIDE
TERM

FREEFALL MATHEMATICS

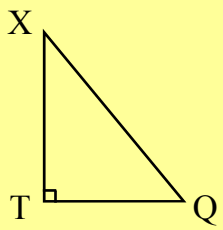
ALTITUDE

PYTHAGORAS THEOREM

The Hypotenuse

The hypotenuse is the longest side of a right-angled triangle. So look for the longest side or, use the right angle symbol to help you, as the hypotenuse is always opposite (it doesn't touch) the right angle.

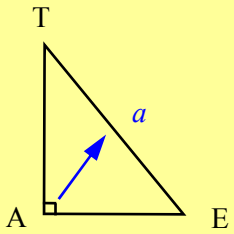
Column 1 starts with asking you to name the triangle and then the hypotenuse. The name for the triangle can be answered starting from any point and moving either clockwise or anti-clockwise, so answers will vary. The hypotenuse only has two alternatives using the letters at each end, forwards or backwards.



$\frac{\Delta QTX}{\text{Triangle}}$

$\frac{XQ \text{ or } QX}{\text{Hypotenuse}}$

Column 2 keeps on with naming but this time instead of using the capital letters on the triangle to name the hypotenuse we will use one single lower case letter. Using the letter that is on the right angle, name the hypotenuse the same letter, except in lower case. All sides can be labeled in this way, side '*a*' is always opposite angle 'A' and so on.



$\frac{\Delta ATE}{\text{Triangle}}$

$\frac{a}{\text{Hypotenuse}}$

Column 3 asks you to construct triangles with the names provided. So draw the triangles first, make sure you include the right angle symbol. Once it is drawn then put the letters on the diagram. Label the hypotenuse first as it has been specified so make sure your drawing matches! Then put the other letter on the last angle, which is of course the right angle.

The Hypotenuse

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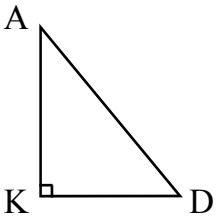
Name the triangle and the side that is opposite the right angle (hypotenuse)

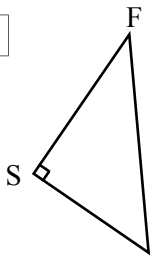
Example



ΔAKD
Triangle

AD or DA
Hypotenuse

1

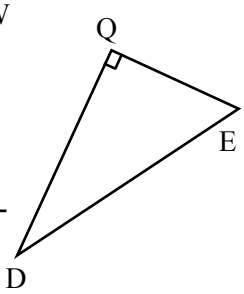


_____ Triangle

or

_____ Hypotenuse

2

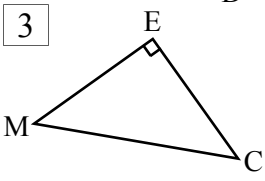


_____ Triangle

or

_____ Hypotenuse

3

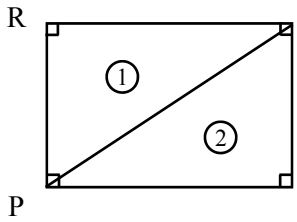


_____ Triangle

or

_____ Hypotenuse

4



① _____ Triangle

or

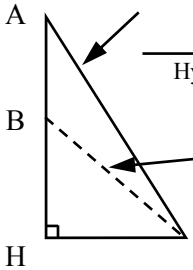
_____ Hypotenuse

② _____ Triangle

or

_____ Hypotenuse

5



_____ Triangle

or

_____ Hypotenuse

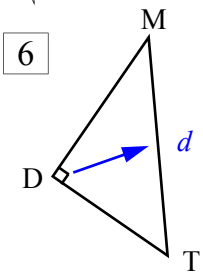
_____ Triangle

or

_____ Hypotenuse

Name the hypotenuse with a single lower case letter

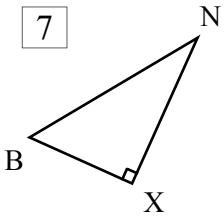
6



_____ Triangle

_____ Hypotenuse

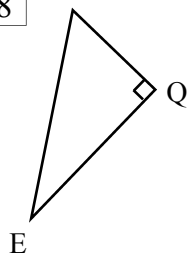
7



_____ Triangle

_____ Hypotenuse

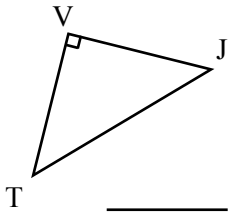
8



_____ Triangle

_____ Hypotenuse

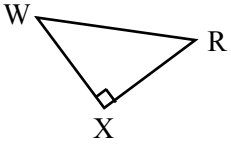
9



_____ Triangle

_____ Hypotenuse

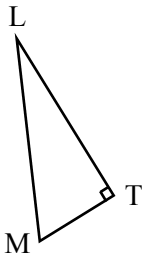
10



_____ Triangle

_____ Hypotenuse

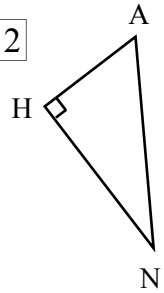
11



_____ Triangle

_____ Hypotenuse

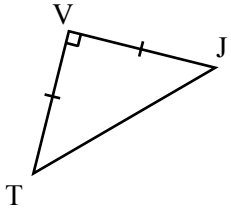
12



_____ Triangle

_____ Hypotenuse

13



_____ Triangle

_____ Hypotenuse

Construct these right angled triangles, label with letters and use a ⌞ symbol.

14 ΔPDG with hypotenuse PD

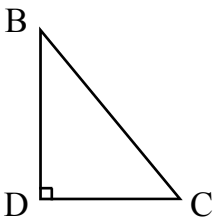
15 ΔGYT with hypotenuse *y*

16 ΔMRD with hypotenuse *d*

17 ΔABF with hypotenuse FB

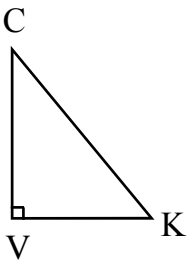
18 ΔABC with hypotenuse BC

draw your Δ touching the one already there



19 ΔDVC with hypotenuse CV

draw your Δ touching the one already there



The Hypotenuse

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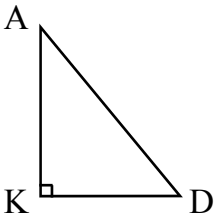
Name the triangle and the side that is opposite the right angle (hypotenuse)

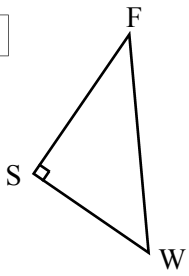
Example



ΔAKD — Triangle

AD or DA — Hypotenuse

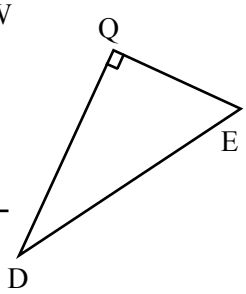
1



ΔFSW — Triangle

FW or WF — Hypotenuse

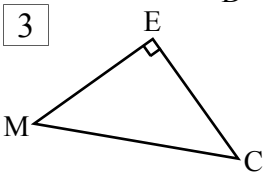
2



ΔQED — Triangle

ED or DE — Hypotenuse

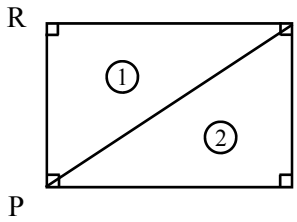
3



ΔMCE — Triangle

MC or CM — Hypotenuse

4



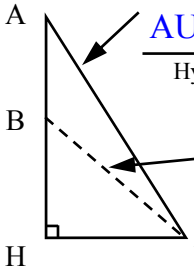
① ΔRYP — Triangle

PY or YP — Hypotenuse

② ΔHYP — Triangle

YP or PY — Hypotenuse

5



ΔAUH — Triangle

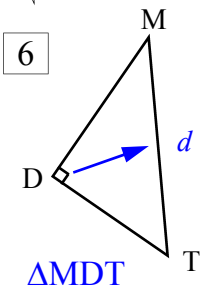
AU or UA — Hypotenuse

ΔBUH — Triangle

UB or BU — Hypotenuse

Name the hypotenuse with a single lower case letter

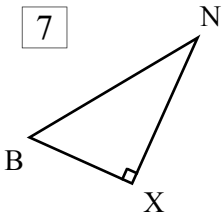
6



ΔMDT — Triangle

d — Hypotenuse

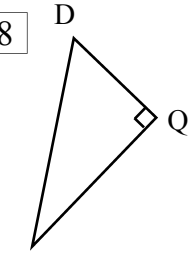
7



ΔXBN — Triangle

x — Hypotenuse

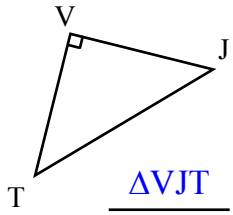
8



ΔDQE — Triangle

q — Hypotenuse

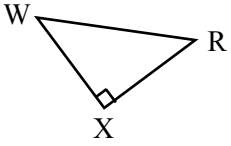
9



ΔVJT — Triangle

v — Hypotenuse

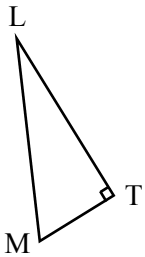
10



ΔWRX — Triangle

x — Hypotenuse

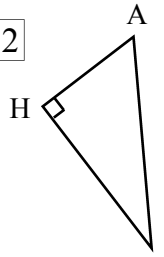
11



ΔMLT — Triangle

t — Hypotenuse

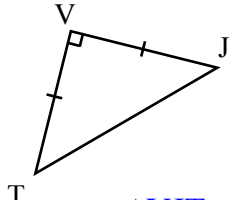
12



ΔAHN — Triangle

h — Hypotenuse

13

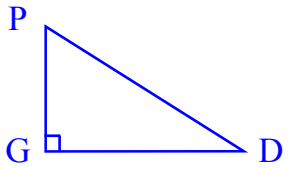


ΔVJT — Triangle

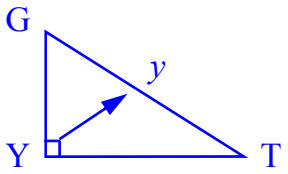
v — Hypotenuse

Construct these right angled triangles, label with letters and use a ⌞ symbol.

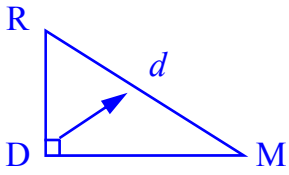
14 ΔPDG with hypotenuse PD



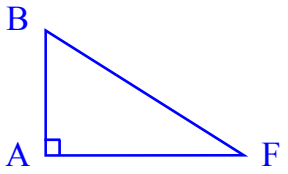
15 ΔGYT with hypotenuse *y*



16 ΔMRD with hypotenuse *d*

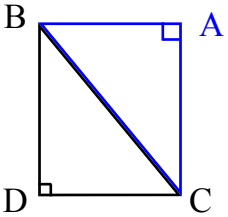


17 ΔABF with hypotenuse FB



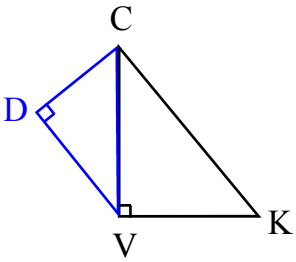
18 ΔABC with hypotenuse BC

draw your Δ touching the one already there



19 ΔDVC with hypotenuse CV

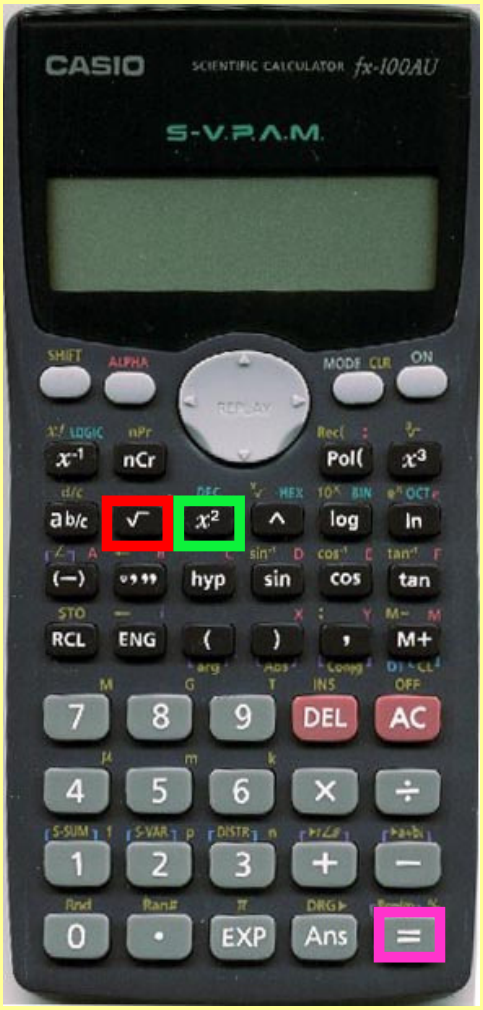
draw your Δ touching the one already there



Calculator Use and Rounding

To complete the Pythagoras Theorem calculations to be introduced in the worksheets that follow you must be able to:

- Square a number (Multiply it by itself)
- Square root a number (find the number that when multiplied by itself, equals the number)
- Round a decimal (reduce a decimal in size using rounding methods)



Squaring a Number Calculator Steps:	Square rooting a Number Calculator Steps:
Find 72^2 7 2 x^2 =	Find $\sqrt{361}$ √ 3 6 1 =
Answer is 5 184	Answer is 19

Column 1 introduces squaring numbers. Using the x^2 key on your calculator, enter the number to be squared, press the x^2 key, press '=' and you are done. The first four questions you should know and allow you to check the answer obtained with your calculator.

Square roots are then shown from Q.11. To answer these press the √ key, then the number, then the '=' key, done. You will probably know the answers to the first three questions, use that knowledge to ensure you are using your calculator correctly.

Column 2 introduces rounding decimals. To round to one decimal place you look at the 2nd number after the decimal point.

Rounding to 1 decimal place (1 d.p.)

Example 1. Round 9.567 to 1 decimal place. 9.(5)[6]7

Answer could be 9.5 or 9.6
Because the boxed number is a 6, which is 5 or more, the answer is 9.6

Example 2. Round 6.246 to 1 decimal place. 6.(2)[4]6

Answer could be 6.2 or 6.3
Because the boxed number is a 4, which is less than 5, the answer is 6.2

Rounding to 2 decimal places (2 d.p.)

Example 3. Round 35.708 to 2 decimal places. 35.7(0)[8]

Answer could be 35.70 or 35.71
Because the boxed number is an 8, which is 5 or more, the answer is 35.71

Calculator Use and Rounding

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Use your calculator's x^2 key to find these.

Example 18^2

[1] [8] [x^2] [=] 324

1. 5^2
2. 8^2
3. 7^2
4. 9^2
5. 17^2
6. 26^2
7. 19^2
8. 45^2
9. 34^2
10. 53^2

Now find these square roots use your calculator's $\sqrt{\ }$ key.

Example $\sqrt{1\,936}$

[$\sqrt{\ }$] [1] [9] [3] [6] [=] 44

11. $\sqrt{100}$
12. $\sqrt{25}$
13. $\sqrt{49}$
14. $\sqrt{529}$
15. $\sqrt{1\,369}$
16. $\sqrt{441}$
17. $\sqrt{6\,084}$
18. $\sqrt{7\,225}$
19. $\sqrt{9\,801}$
20. $\sqrt{4\,624}$

Round these decimals to one decimal place (1 d.p.)

Example $2.7\,9\,31$ 2.8

Look at the digit two numbers behind the decimal point (box around it). If it is 5 or more round up the first number behind the decimal point (circled number changes to an '8' and answer becomes 2.8 If < 5 the decimal would be shortened to 2.7

21. 5.87
22. 9.315
23. 27.249
24. 11.191
25. 16.3333
26. 35.6666
27. 117.062
28. 34.9601
29. 12.209
30. 75.0499

Now use your calculator & round the result to 1 d.p.

31. $\sqrt{19}$
32. $\sqrt{84}$
33. 2.5^2
34. $\sqrt{131}$
35. 9.6^2
36. 8.18^2
37. $\sqrt{98.32}$
38. $\sqrt{37}$
39. $\sqrt{63.5}$
40. 24.77^2
41. $\sqrt{398.4}$

Round these decimals to two decimal places (2 d.p.)

Example $5.3\,2\,6\,04$ 5.33

Look at the digit three numbers behind the decimal point (box around it). If it is 5 or more round up the second number behind the decimal point (circled number changes to an '3' and answer becomes 5.33 If < 5 the decimal would be changed to 5.32

42. 0.9527
43. 8.39823
44. 56.2371
45. 80.8081
46. 28.9962
47. 14.2229
48. 19.0993
49. 43.8172
50. 39.9964
51. 20.2217

Use your calculator & round the result to 2 d.p.

52. 4.39^2
53. $\sqrt{308}$
54. 8.76^2
55. $\sqrt{5}$
56. $\sqrt{41}$
57. $\sqrt{19.6}$
58. 32.17^2
59. $\sqrt{4\,000}$
60. 45.75^2
61. $\sqrt{138.2}$
62. $\sqrt{38}$
63. $\sqrt{23}$

Calculator Use and Rounding

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Use your calculator's x^2 key to find these.

Example 18^2

[1] [8] [x^2] [=] 324

1	5^2	25
2	8^2	64
3	7^2	49
4	9^2	81
5	17^2	289
6	26^2	676
7	19^2	361
8	45^2	2 025
9	34^2	1 156
10	53^2	2 809

Now find these square roots use your calculator's [$\sqrt{\ }$] key.

Example $\sqrt{1\ 936}$

[$\sqrt{\ }$] [1] [9] [3] [6] [=] 44

11	$\sqrt{100}$	10
12	$\sqrt{25}$	5
13	$\sqrt{49}$	7
14	$\sqrt{529}$	23
15	$\sqrt{1\ 369}$	37
16	$\sqrt{441}$	21
17	$\sqrt{6\ 084}$	78
18	$\sqrt{7\ 225}$	85
19	$\sqrt{9\ 801}$	99
20	$\sqrt{4\ 624}$	68

Round these decimals to one decimal place (1 d.p.)

Example 2.7931 2.8

Look at the digit two numbers behind the decimal point (box around it). If it is 5 or more round up the first number behind the decimal point (circled number changes to an '8' and answer becomes 2.8 If < 5 the decimal would be shortened to 2.7

21	5.87	5.9
22	9.315	9.3
23	27.249	27.2
24	11.191	11.2
25	16.3333	16.3
26	35.6666	35.7
27	117.062	117.1
28	34.9601	35 / 35.0
29	12.209	12.2
30	75.0499	75 / 75.0

Now use your calculator & round the result to 1 d.p.

31	$\sqrt{19}$	4.4
32	$\sqrt{84}$	9.2
33	2.5^2	6.3
34	$\sqrt{131}$	11.4
35	9.6^2	92.2
36	8.18^2	66.9
37	$\sqrt{98.32}$	9.9
38	$\sqrt{37}$	6.1
39	$\sqrt{63.5}$	8 / 8.0
40	24.77^2	613.6
41	$\sqrt{398.4}$	20 / 20.0

Round these decimals to two decimal places (2 d.p.)

Example 5.32604 5.33

Look at the digit three numbers behind the decimal point (box around it). If it is 5 or more round up the second number behind the decimal point (circled number changes to an '3' and answer becomes 5.33 If < 5 the decimal would be changed to 5.32

42	0.9527	0.95
43	8.39823	8.4/8.40
44	56.2371	56.24
45	80.8081	80.81
46	28.9962	29/29.00
47	14.2229	14.22
48	19.0993	19.10
49	43.8172	43.82
50	39.9964	40/40.00
51	20.2217	20.22

Use your calculator & round the result to 2 d.p.

52	4.39^2	19.27
53	$\sqrt{308}$	17.55
54	8.76^2	76.74
55	$\sqrt{5}$	2.24
56	$\sqrt{41}$	6.40
57	$\sqrt{19.6}$	4.43
58	32.17^2	1 034.91
59	$\sqrt{4\ 000}$	63.25
60	45.75^2	2 093.06
61	$\sqrt{138.2}$	11.76
62	$\sqrt{38}$	6.16
63	$\sqrt{23}$	4.80

Squares and Square Roots

This sheet practices the skills required to perform Pythagoras Theorem. You need to know how to find the square root of a number and how to square a number.

Column 1 starts with squaring numbers, using a calculator. To answer the questions follow the method at the top of the column. So enter the number into your calculator, press the x^2 key, then the equals key and you are done.

Column 2 is the reverse. finding the square root of a number. The square root of a number is finding the 'number that when multiplied by itself, will equal the first number'. With a calculator press the square root key then the number then the equals button.

The table may look complicated but it isn't. The value in the column 'a^2' is found by finding the square of the number in the first column. The value 'b^2' is found by finding the square of the number in the second column. Now add the two values you calculated, don't clear your calculator screen, just press '+' and retype in the first value you calculated, press '=' and you are done. Now leave that number on your calculator screen and press the square root key then the '=' key, all done.

The answer above, and in column 3, is to be rounded to 2 decimal places, so look at the number 3 places behind the decimal point, if it is 5 or larger round your answer up, otherwise …..don't. Or just fix your calculator to 2 decimal places.

Column 3 asks you to square two numbers and add them. This is entered into the calculator as the example at the top of the column shows. Next you are asked to find the square root of the number you just found. To do this don't clear your calculator screen, press the square root key then the '=' sign. The answer is to be rounded to 2 decimal places, so look at the number 3 places behind the decimal point, if it is 5 or larger round your answer up, otherwise …..don't. Or fix your calculator to 2 decimal places.

Squares and Square Roots

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Square these numbers, use your calculator's x^2 key

Example 12.6^2

1 2 • 6 x^2 = 158.76

1. 13^2
2. 16^2
3. 29.2^2
4. 85.4^2
5. 39.6^2
6. 4.7^2
7. 55.3^2
8. 71.8^2
9. 62.5^2
10. 43.1^2

Find the square root, use your calculator's $\sqrt{\ }$ key

Example $\sqrt{11.56}$

$\sqrt{\ }$ 1 1 • 5 6 = 3.4

11. $\sqrt{169}$
12. $\sqrt{1\ 225}$
13. $\sqrt{31.36}$
14. $\sqrt{2\ 025}$
15. $\sqrt{123.21}$
16. $\sqrt{1\ 274.49}$
17. $\sqrt{7\ 482.25}$
18. $\sqrt{558.8496}$
19. $\sqrt{2\ 323.24}$
20. $\sqrt{948.64}$

Add the squares of these numbers then square root the total (answer to 2 d.p.)

Example $15^2 + 20^2$

1 5 x^2 + 2 0 x^2 = 625

Don't clear your calculator screen $\sqrt{\ }$ = 25

32. $14^2 + 18^2$
    Now square root the answer above
    2 decimal places
33. $12^2 + 20^2$
    Now square root the answer above
34. $11^2 + 17^2$
    Now square root the answer above
35. $6.3^2 + 9.2^2$
    Now square root the answer above
36. $13.4^2 + 8.3^2$
    Now square root the answer above
37. $45^2 + 26^2$
    Now square root the answer above
38. $46.5^2 + 35^2$
    Now square root the answer above
39. $1.4^2 + 2.7^2$
    Now square root the answer above
40. $122^2 + 106^2$
    Now square root the answer above
41. $0.82^2 + 0.6^2$
    Now square root the answer above
42. $490^2 + 350^2$
    Now square root the answer above

Complete the table by working across each row.
Column 3 = (Column 1)^2. Column 4 = (Column 2)^2.
Column 5 = Column 3 + Column 4 and the last step,
Column 6 = $\sqrt{\text{Column 5}}$ (answer to 2 d.p. if needed)

	Column 1 a	Column 2 b	Column 3 a^2	Column 4 b^2	Column 5 $a^2 + b^2$	Column 6 $\sqrt{a^2 + b^2}$
21	10	24				
22	27	36				
23	25	15				
24	9	16				
25	78	35				
26	29	55				
27	52	44				
28	78	108				
29	210	504				
30	186	322				
31	555	740				

Squares and Square Roots

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Square these numbers, use your calculator's x^2 key

Example 12.6^2

1 2 • 6 x^2 = 158.76

1	13^2	169
2	16^2	256
3	29.2^2	852.64
4	85.4^2	7 293.16
5	39.6^2	1 568.16
6	4.7^2	22.09
7	55.3^2	3 058.09
8	71.8^2	5 155.24
9	62.5^2	3 906.25
10	43.1^2	1 857.61

Find the square root, use your calculator's $\sqrt{\ }$ key

Example $\sqrt{11.56}$

$\sqrt{\ }$ 1 1 • 5 6 = 3.4

11	$\sqrt{169}$	13
12	$\sqrt{1\ 225}$	35
13	$\sqrt{31.36}$	5.6
14	$\sqrt{2\ 025}$	45
15	$\sqrt{123.21}$	11.1
16	$\sqrt{1\ 274.49}$	35.7
17	$\sqrt{7\ 482.25}$	86.5
18	$\sqrt{558.8496}$	23.96
19	$\sqrt{2\ 323.24}$	48.2
20	$\sqrt{948.64}$	30.8

Add the squares of these numbers then square root the total (answer to 2 d.p.)

Example $15^2 + 20^2$

1 5 x^2 + 2 0 x^2 = 625

Don't clear your calculator screen $\sqrt{\ }$ = 25

32	$14^2 + 18^2$	= 520
	Now square root the answer above	= 22.80
		2 decimal places
33	$12^2 + 20^2$	= 544
	Now square root the answer above	= 23.32
34	$11^2 + 17^2$	= 410
	Now square root the answer above	= 20.25
35	$6.3^2 + 9.2^2$	= 124.33
	Now square root the answer above	= 11.15
36	$13.4^2 + 8.3^2$	= 248.45
	Now square root the answer above	= 15.76
37	$45^2 + 26^2$	= 2 701
	Now square root the answer above	= 51.97
38	$46.5^2 + 35^2$	= 3 387.25
	Now square root the answer above	= 58.20
39	$1.4^2 + 2.7^2$	= 9.25
	Now square root the answer above	= 3.04
40	$122^2 + 106^2$	= 26 120
	Now square root the answer above	= 161.62
41	$0.82^2 + 0.6^2$	= 1.03
	Now square root the answer above	= 1.02
42	$490^2 + 350^2$	= 362 600
	Now square root the answer above	= 602.16

Complete the table by working across each row.
Column 3 = (Column 1)^2. Column 4 = (Column 2)^2.
Column 5 = Column 3 + Column 4 and the last step,
Column 6 = $\sqrt{\text{Column 5}}$ (answer to 2 d.p. if needed)

	Column 1 a	Column 2 b	Column 3 a^2	Column 4 b^2	Column 5 $a^2 + b^2$	Column 6 $\sqrt{a^2 + b^2}$
21	10	24	100	576	676	26
22	27	36	729	1 296	2 025	45
23	25	15	625	225	850	29.15
24	9	16	81	256	337	18.36
25	78	35	6 084	1 225	7 309	85.49
26	29	55	841	3 025	3 866	62.18
27	52	44	2 704	1 936	4 640	68.11
28	78	108	6 084	11 664	17 748	133.22
29	210	504	44 100	254 016	298 116	546
30	186	322	34 596	103 684	138 280	371.86
31	555	740	308 025	547 600	855 625	925

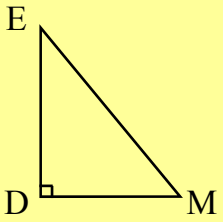
Pythagorean Result + Right-Angled Triangle Proof

Pythagoras found that with right-angled triangles, if you square the hypotenuse it is equal to the square of the two short sides added together.

Column 1 askes you to find the Pythagorean Result, which as there are no side lengths given, this uses the sides themselves.

$$EM^2 = ED^2 + DM^2$$

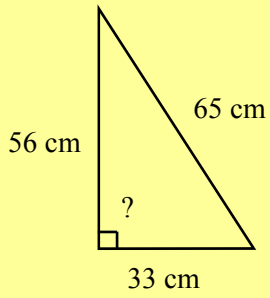
$$\text{Hypotenuse}^2 = \text{Short Side}^2 + \text{Short side}^2$$



The letters could also be back to front, and it doesn't matter which of the short sides is shown first

Questions 5 and 6 ask you to use lower case letters so write the sides in lower case, using the opposite angle to name each side.

Column 2 (and 3) deal with proving if a triangle is right-angled or not. As Pythagoras Theorem applies only to right-angled triangles, it can be used to determine if a triangle is right-angled. So as with the previous column, hypotenuse^2 = short side^2 + short side^2. In the example below, the hypotenuse is 65 and the two short sides are 33 and 56. So $65^2 = 33^2 + 56^2$.



$65^2 = 33^2 + 56^2$

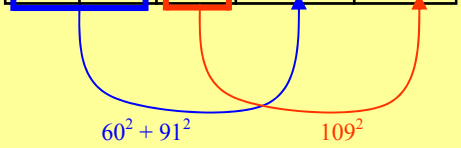
$4\,225 = 4\,225$

LHS = RHS Δ is right-angled

Calculate 65^2, write '=' then calculate $33^2 + 56^2$. Are they equal? If they are you have a right angled triangle. Make sure you say "LHS = RHS", this is the statement of proof. That the left hand side (of the equals sign) is equal to the right hand side (of the equals sign). If they aren't equal strike out the equals sign, to make 'not equal to' which is $\neq$. Of course you also have to answer the question which is "Yes the triangle is right angled" or the opposite.

Column 3 continues with asking if the three sides given would make a right-angled triangle. Find $a^2 + b^2$ then find c^2. If they are equal, you have a right angled triangle.

a	b	c	$a^2 + b^2$	c^2
60	91	109	11 881	11 881



Yes No

Pythagorean Result + Right-Angled Triangle Proof

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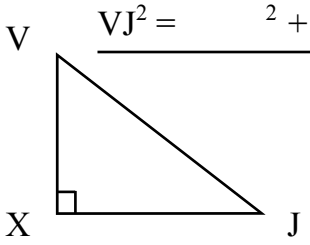
Write the Pythagorean result for the triangles below

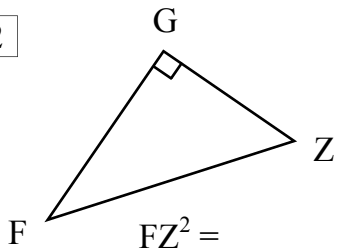
$(\text{hypotenuse})^2 = (\text{short side})^2 + (\text{short side})^2$

1



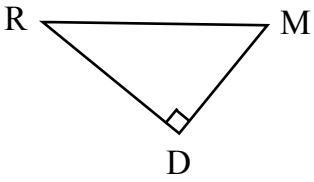
$VJ^2 = \quad ^2 + \quad ^2$

2

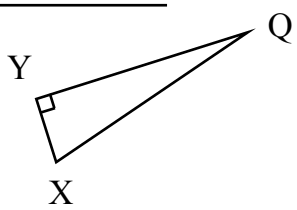


$FZ^2 =$

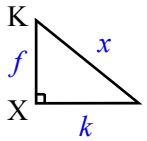
3



4

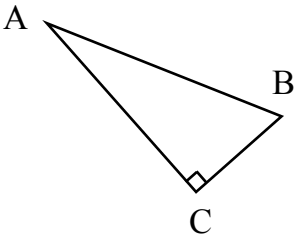


Now use lower case letters

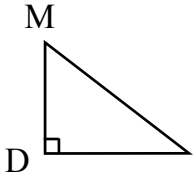


$x^2 = k^2 + f^2$

5

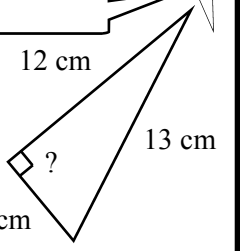


6



Apply Pythagoras Theorem to check if these triangles are right-angled.

Example

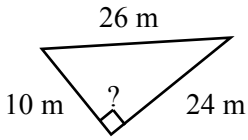


$13^2 = 7^2 + 12^2$

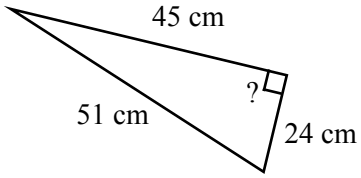
$169 \neq 193$

LHS $\neq$ RHS Δ is not right-angled

7



8



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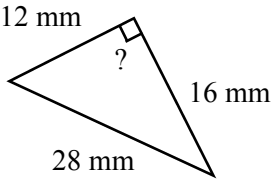
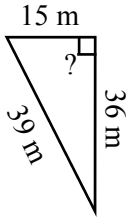
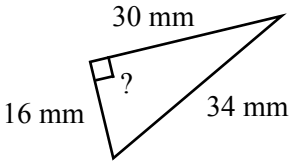


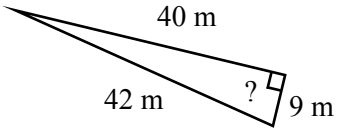
10



11



12



Are these the sides of a right angled triangle? Find $a^2 + b^2$ and c^2 to support your answer.

13

a	b	c	$a^2 + b^2$	c^2
21	72	75		

○ Yes ○ No

14

a	b	c	$a^2 + b^2$	c^2
3	15	18		

○ Yes ○ No

15

a	b	c	$a^2 + b^2$	c^2
15	24	39		

○ Yes ○ No

Pythagorean Result + Right-Angled Triangle Proof

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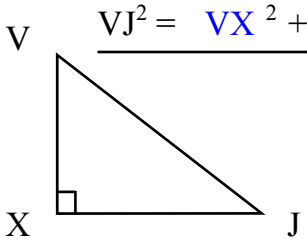
Write the Pythagorean result for the triangles below

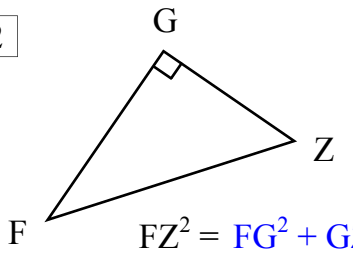
$(\text{hypotenuse})^2 = (\text{short side})^2 + (\text{short side})^2$

1



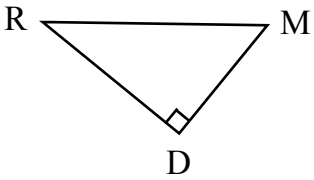
$VJ^2 = VX^2 + XJ^2$

2



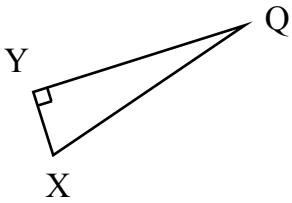
$FZ^2 = FG^2 + GZ^2$

3



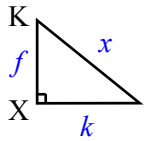
$RM^2 = RD^2 + MD^2$

4



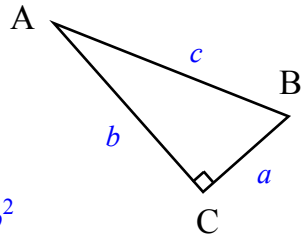
$QX^2 = YQ^2 + YX^2$

Now use lower case letters



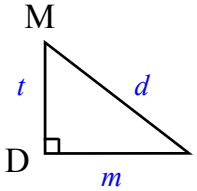
$x^2 = k^2 + f^2$

5



$c^2 = a^2 + b^2$

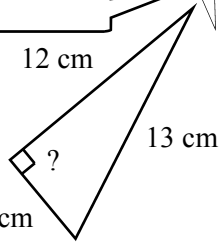
6



$d^2 = t^2 + m^2$

Apply Pythagoras Theorem to check if these triangles are right-angled.

Example

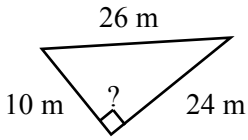


$13^2 = 7^2 + 12^2$

$169 \neq 193$

LHS ≠ RHS Δ is not right-angled

7

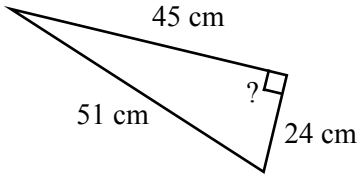


$26^2 = 10^2 + 24^2$

$676 = 676$

LHS = RHS Δ is right-angled

8

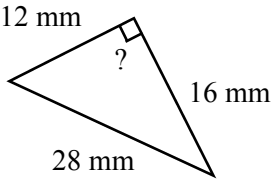


$51^2 = 45^2 + 24^2$

$2\,601 = 2\,601$

LHS = RHS Δ is right-angled

9

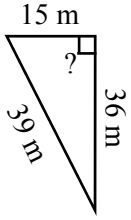


$28^2 = 12^2 + 16^2$

$784 \neq 400$

LHS ≠ RHS Δ is not right-angled

10

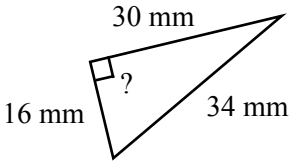


$39^2 = 15^2 + 36^2$

$1\,521 = 1\,521$

LHS = RHS Δ is right-angled

11

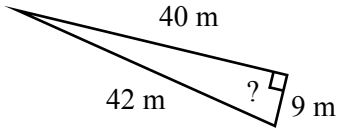


$34^2 = 16^2 + 30^2$

$1\,156 = 1\,156$

LHS = RHS Δ is right-angled

12



$42^2 = 9^2 + 40^2$

$1\,764 \neq 1\,681$

LHS ≠ RHS Δ is not right-angled

Are these the sides of a right angled triangle? Find $a^2 + b^2$ and c^2 to support your answer.

13

a	b	c	$a^2 + b^2$	c^2
21	72	75	5 625	5 625

● Yes ○ No

14

a	b	c	$a^2 + b^2$	c^2
3	15	18	234	324

○ Yes ● No

15

a	b	c	$a^2 + b^2$	c^2
15	24	39	801	1 521

○ Yes ● No

Graphical Application of Pythagoras Theorem

Pythagoras' Theorem $c^2 = a^2 + b^2$ applies to all right-angled triangles. This sheet contains 7 right-angled triangles. The question is …… does Pythagoras' Theorem apply to them?

To apply P.T. to these triangles measure the length of each side of the triangle. There are boxes on each side of these triangles, but this isn't for writing the length inside, it is for writing the 'square of' the length inside. So measure the side then either using your grey matter or your calculator square the length, then write it in the box. Once all three sides are done, check that when you add the square of the two short sides you get the square of the long side.

Or in other words: $c^2 = a^2 + b^2$.

The first question has been started for you, the base of the triangle is 3 cm long, $3^2 = 9$ or if you prefer $3 \times 3 = 9$ so a '9' has been written in the box.

The side lengths are in whole centimetres, so if you measure a length as 16.9 cm, consider it as being 17 cm.

MAKE SURE THAT WHEN YOU PRINT THIS SHEET YOU HAVE 'NONE' IN THE PAGE SCALING WINDOW, OR THE SHEET WON'T PRINT OUT CORRECTLY.

Graphical Application of Pythagoras Theorem

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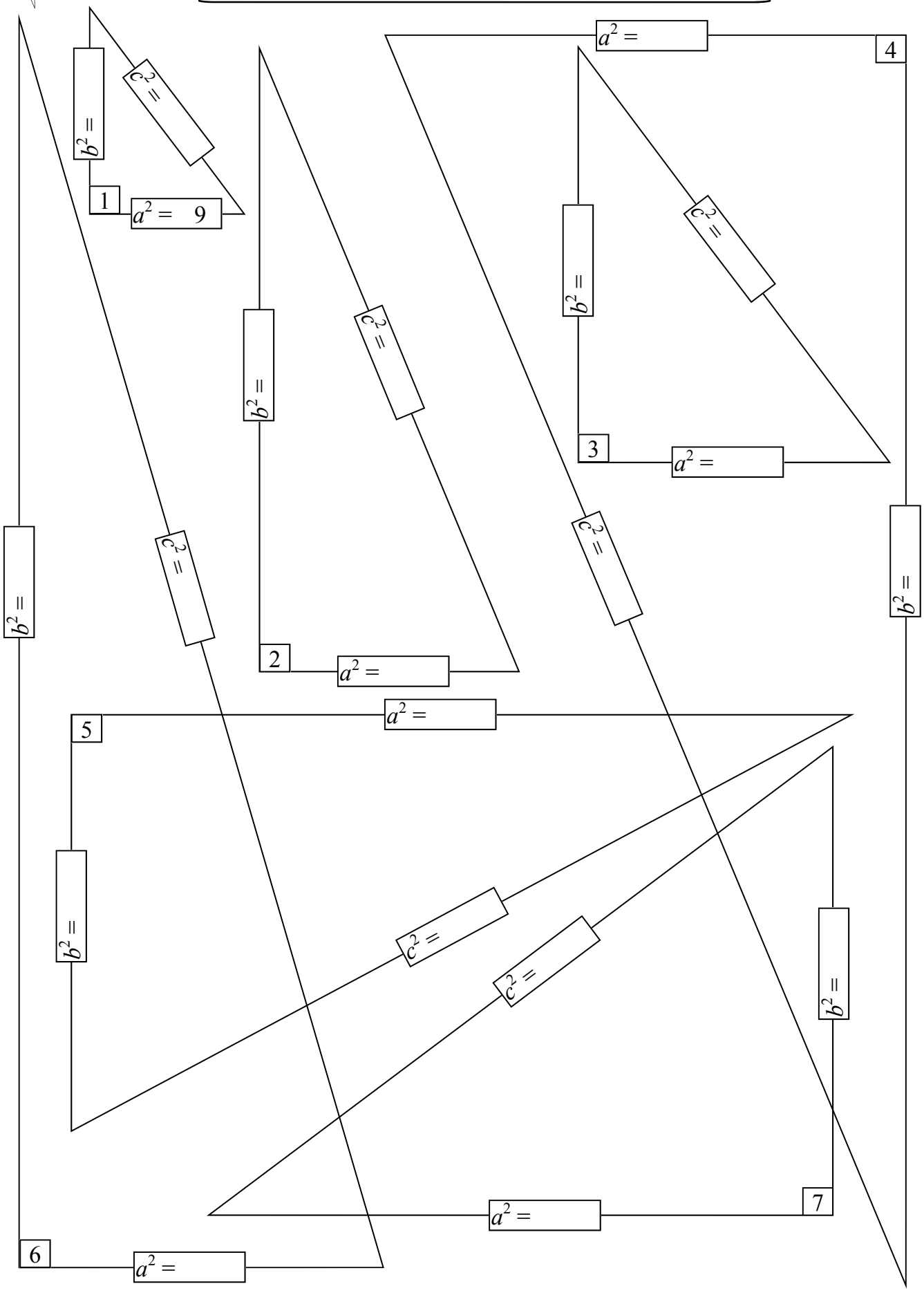
Measure each side of the right-angled triangles below to the nearest centimetre, then using a calculator, square each of the side lengths. Write your 'squared' answer in the boxes. Check that $c^2 = a^2 + b^2$.



Graphical Application of Pythagoras Theorem

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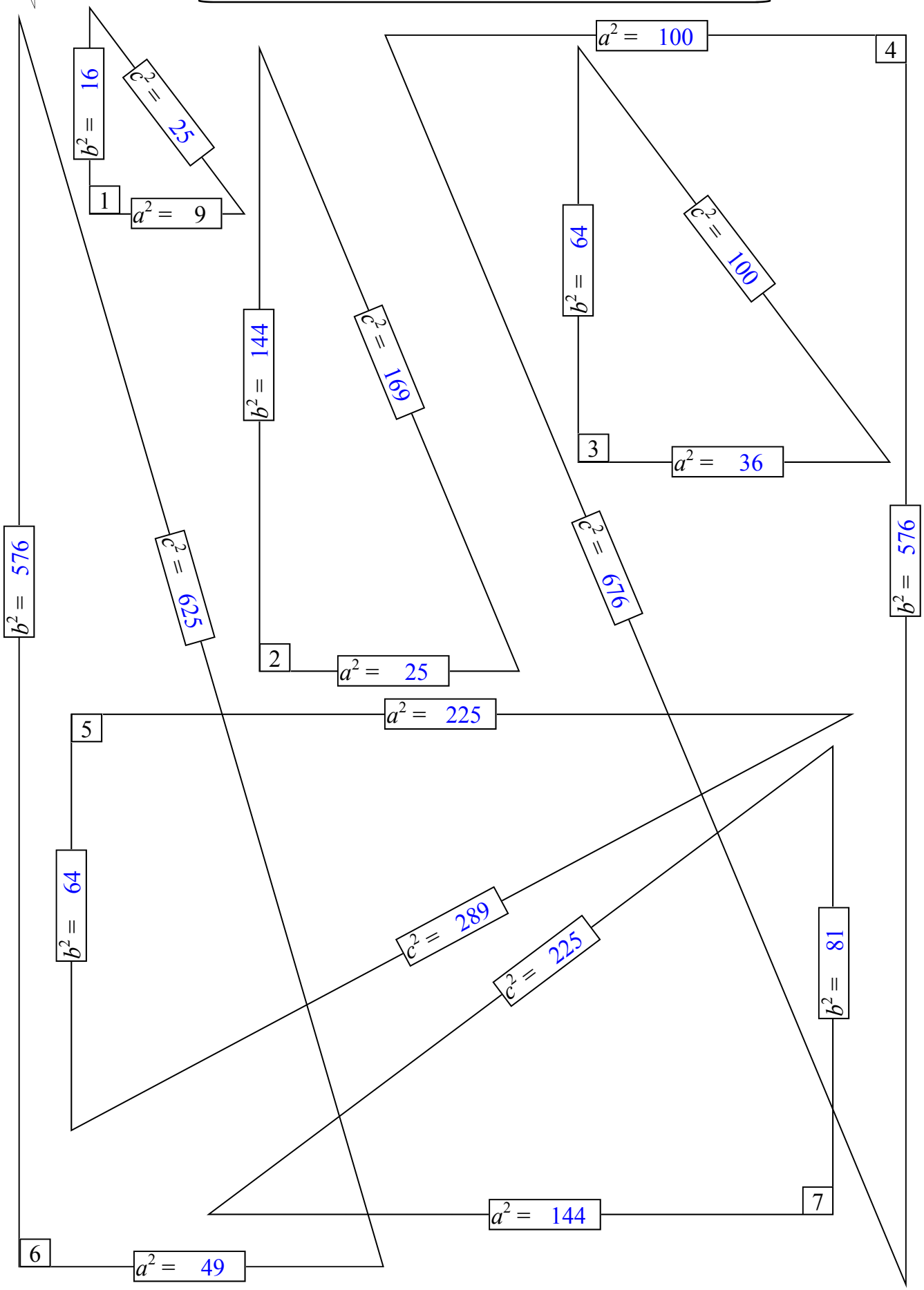
Measure each side of the right-angled triangles below to the nearest centimetre, then using a calculator, square each of the side lengths. Write your 'squared' answer in the boxes. Check that $c^2 = a^2 + b^2$.



Counting Squares + Triples

Pythagoras Theorem relates the squares of the side lengths of a right angled triangle. By drawing squares on each short side you can find the size of the square on the hypotenuse, its size is found by adding the two smaller squares together.

In column 1 you are given triangles with squares drawn on each side. The two small squares called a^2 and b^2 have grids inside, to find c^2 count the number of grid squares in a^2 and b^2 and add them together. You can count all the grid squares or you can count the grid squares along the side of the square …and square it!

Column 2 is the same as column 1 but this time after finding c^2, take the square root of it to find just c, which is the length of the hypotenuse. The numbers of grid squares is larger in these questions, so as you count the squares in a^2 and b^2 write them in the spaces. Add the two numbers together and that is c^2, like before. The next step is to square root this number to find c, you may need a calculator with the larger squares. Now that you know c, you can construct the grid squares along the hypotenuse, then grid the entire square.

Column 3 deals with Pythagorean triples. These are the three sides of right triangles that have whole number lengths on all three sides, no decimal answers. The numbers in black at the top of the table are know as Pythagorean ‘triples’ or ‘triads’, by multiplying these by 2, 3, 4 and so on …. you can create other triples. Remember that you can add down the column instead of multiplying to find each successive triple, that way you can use your mental skills rather than a calculator. Just add the top black triple to the previous blue triple to solve for the next row.

Counting Squares + Triples

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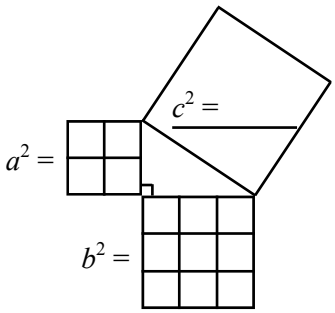
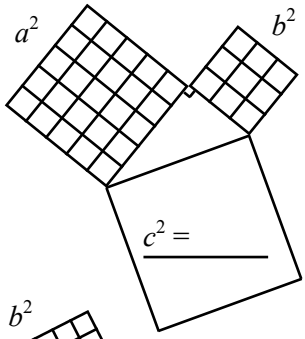
Count the squares and add them, the total is c^2. So…. $c^2 = a^2 + b^2$.

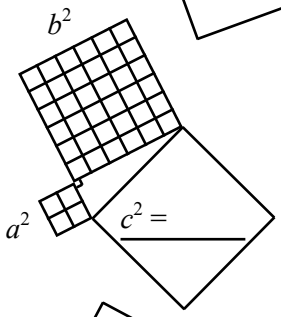
1



2



3



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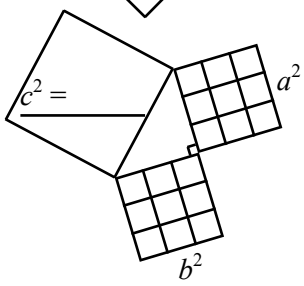

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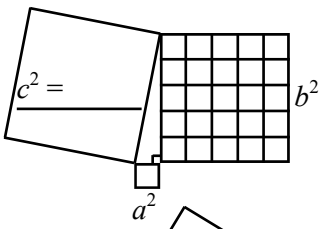

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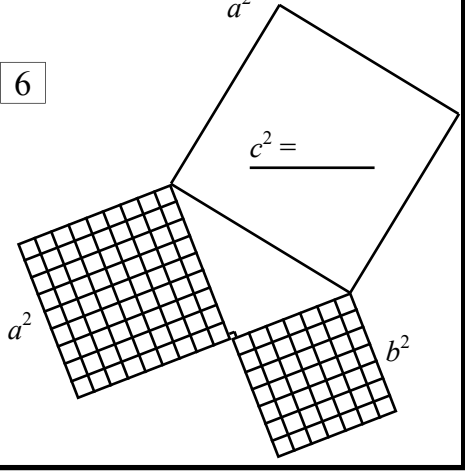
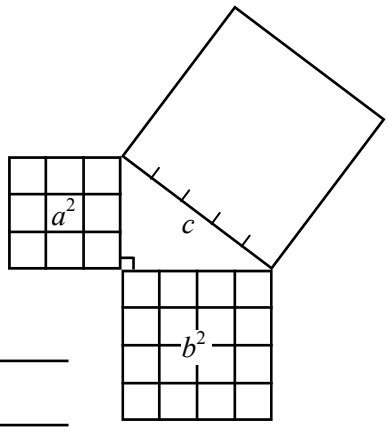


Draw in the square grid and find c^2, take the square root to get c, the side length

7

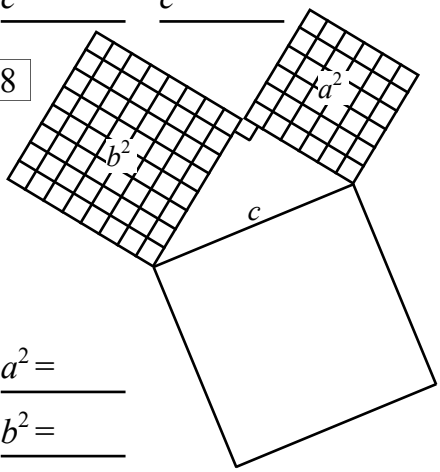


$a^2 =$ _____

$b^2 =$ _____

$c^2 =$ _____ $c =$ _____

8

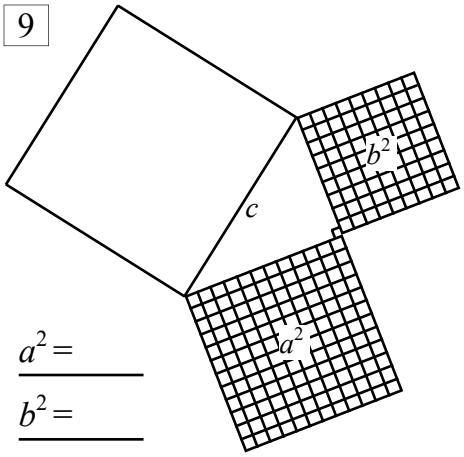


$a^2 =$ _____

$b^2 =$ _____

$c^2 =$ _____ $c =$ _____

9

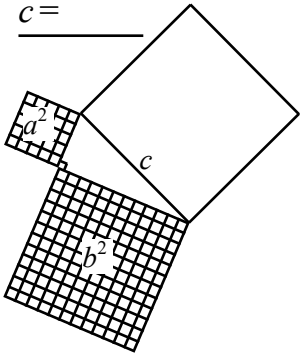


$a^2 =$ _____

$b^2 =$ _____

$c^2 =$ _____ $c =$ _____

10



$a^2 =$ _____

$b^2 =$ _____

$c^2 =$ _____ $c =$ _____

Complete the tables to find some common Pythagorean Triples

Multiply the top row of numbers by 2 then 3, 4 and so on. These form right triangles with whole number sides.

11

	a	b	c
	3	4	5
× 2	6		
× 3			
× 4			
× 5			
× 6			
× 7			
× 8			
× 9			
× 10			

Before you grab that calculator remember that you can just add 12 and 13 to the row above

12

	a	b	c
	5	12	13
× 2	10		
× 3			
× 4			
× 5			
× 6			
× 7			
× 8			

13

	a	b	c
	8	15	17
× 2	16		
× 3			
× 4			
× 5			
× 6			

Counting Squares + Triples

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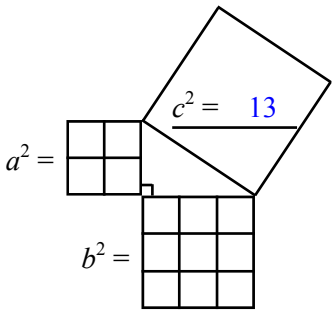
Count the squares and add them, the total is c^2. So…. $c^2 = a^2 + b^2$.

1



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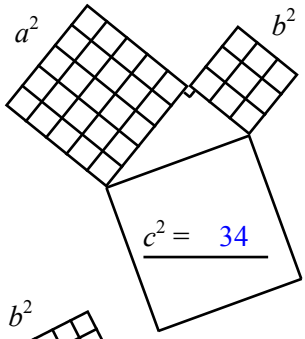

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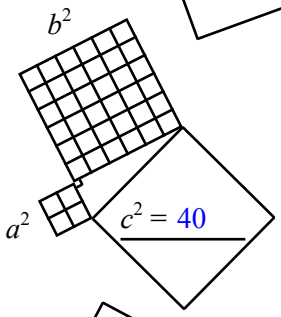

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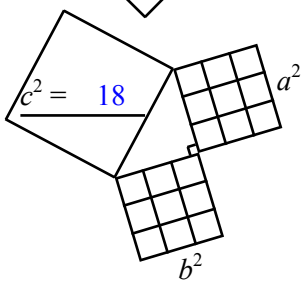

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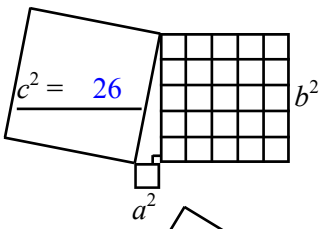

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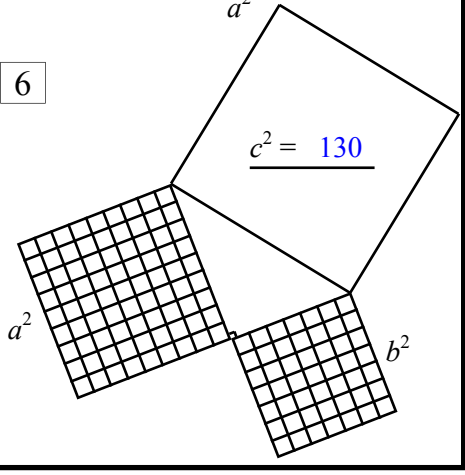
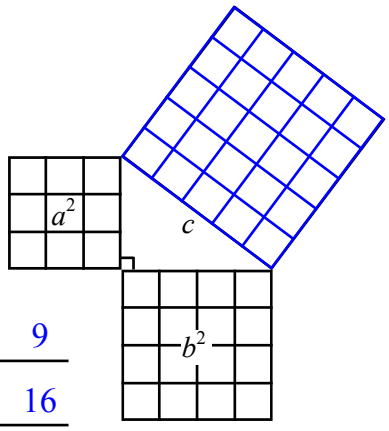


Draw in the square grid and find c^2, take the square root to get c, the side length

7

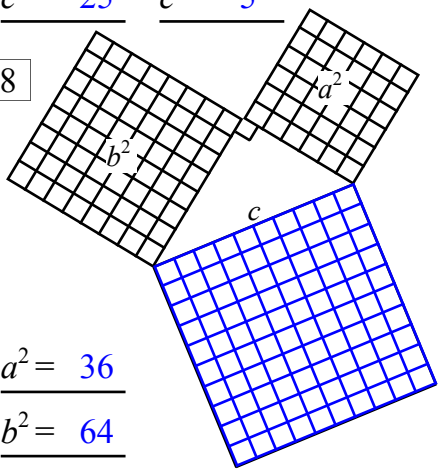


$a^2 =$ 9

$b^2 =$ 16

$c^2 =$ 25 $c =$ 5

8

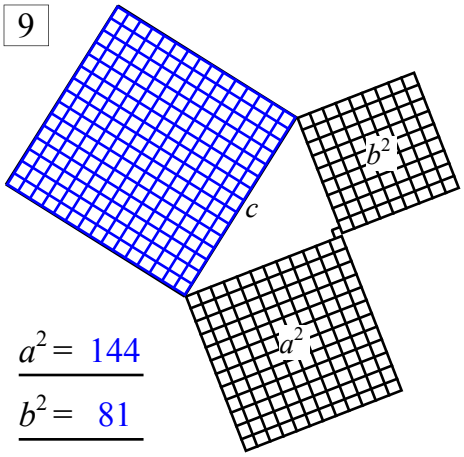


$a^2 =$ 36

$b^2 =$ 64

$c^2 =$ 100 $c =$ 10

9

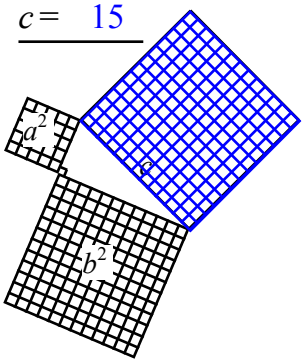


$a^2 =$ 144

$b^2 =$ 81

$c^2 =$ 225 $c =$ 15

10



$a^2 =$ 25

$b^2 =$ 144

$c^2 =$ 169 $c =$ 13

Complete the tables to find some common Pythagorean Triples

Multiply the top row of numbers by 2 then 3, 4 and so on. These form right triangles with whole number sides.

11

	a	b	c
	3	4	5
× 2	6	8	10
× 3	9	12	15
× 4	12	16	20
× 5	15	20	25
× 6	18	24	30
× 7	21	28	35
× 8	24	32	40
× 9	27	36	45
× 10	30	40	50

Before you grab that calculator remember that you can just add 12 and 13 to the row above

12

	a	b	c
	5	12	13
× 2	10	24	26
× 3	15	36	39
× 4	20	48	52
× 5	25	60	65
× 6	30	72	78
× 7	35	84	91
× 8	40	96	104

13

	a	b	c
	8	15	17
× 2	16	30	34
× 3	24	45	51
× 4	32	60	68
× 5	40	75	85
× 6	48	90	102

Further Creating Pythagorean Triples

When a right-angled triangle has three sides that are all whole numbers, the lengths can be called Pythagorean triples.

To create these triples, three algebraic formulae can be used. These are:

When m and n are consecutive counting numbers, where $m = n + 1$:

Hypotenuse length is found by $m^2 + n^2$.
Short side length is found by $m^2 - n^2$.
Other short side length is found by $2mn$.

So using $m = 2$ and $n = 1$, would generate the first triple. Then use $m = 3$ and $n = 2$ to generate the next. Continue on to make as many triples as you want always using a value of n that is one less then m.

The first triple is shown as an example at the top of the first column. Continue the pattern to create seven more triples. The third column asks you to check that the triples are correct. Use the Pythagorean proof:

$$c^2 = a^2 + b^2$$

To show that the triples you created in questions 1, 2, 3 and 4 are correct complete the Pythagorean proof for each question. An example is shown at the top of the column, showing the proof being used on the example question from the first column.

Further Creating Pythagorean Triples

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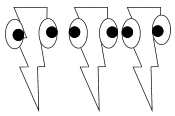


Use m and n to find a, b, and c. I feel like I am being watched!

Example. Use $m = 2$ and $n = 1$

$a = m^2 - n^2$ $\quad$ $b = 2mn$

$a = 2^2 - 1^2$ $\quad$ $b = 2 \times 2 \times 1$

$a = 3$ $\quad$ $b = 4$

$c = m^2 + n^2$ $\quad$ a, b, c are:

$c = 2^2 + 1^2$ $\quad$ 3 ,4 and 5.

$c = 5$

1 Use $m = 3$ and $n = 2$

$a = m^2 - n^2$ $\quad$ $b = 2mn$

$a = 3^2 - 2^2$ $\quad$ $b = 2 \times 3 \times 2$

$a =$ $\quad$ $b =$

$c = m^2 + n^2$ $\quad$ a, b, c are:

$c = 3^2 + 2^2$

$c =$

2 Use $m = 4$ and $n = 3$

$a = m^2 - n^2$ $\quad$ $b = 2mn$

$a =$ $\quad$ $b =$

$a =$ $\quad$ $b =$

$c = m^2 + n^2$ $\quad$ a, b, c are:

$c =$

$c =$

3 Use $m = 5$ and $n = 4$

$a =$ $\quad$ $b =$

$a =$ $\quad$ $b =$

$a =$ $\quad$ $b =$

$c =$ $\quad$ a, b, c are:

$c =$

$c =$

4 Use $m = 6$ and $n = 5$

5 Use $m = 7$ and $n = 6$

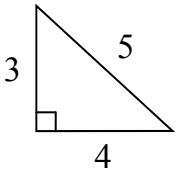
6 Use $m = 8$ and $n = 7$

7 Use $m = 9$ and $n = 8$

Draw 4 right triangles, put the side lengths on them that you found in Columns 1 and 2. Then apply P.T. proof to check.



Example. Use example answer.



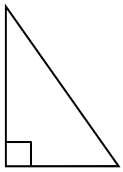
$5^2 = 3^2 + 4^2$

$25 = 25$

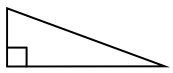
LHS = RHS

$\therefore \Delta$ is right-angled

8 Now Use question 1



9 Now Use question 2



10 Now Use question 3

11 Now Use question 4

Further Creating Pythagorean Triples

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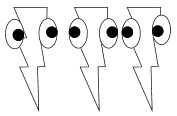


Use m and n to find a, b, and c. I feel like I am being watched!

Example. Use $m = 2$ and $n = 1$

$a = m^2 - n^2$

$a = 2^2 - 1^2$

$a = 3$

$b = 2mn$

$b = 2 \times 2 \times 1$

$b = 4$

$c = m^2 + n^2$

$c = 2^2 + 1^2$

$c = 5$

a, b, c are:

3 ,4 and 5.

1 Use $m = 3$ and $n = 2$

$a = m^2 - n^2$

$a = 3^2 - 2^2$

$a = 5$

$b = 2mn$

$b = 2 \times 3 \times 2$

$b = 12$

$c = m^2 + n^2$

$c = 3^2 + 2^2$

$c = 13$

a, b, c are:

5, 12 and 13.

2 Use $m = 4$ and $n = 3$

$a = m^2 - n^2$

$a = 4^2 - 3^2$

$a = 7$

$b = 2mn$

$b = 2 \times 4 \times 3$

$b = 24$

$c = m^2 + n^2$

$c = 4^2 + 3^2$

$c = 25$

a, b, c are:

7, 24 and 25.

3 Use $m = 5$ and $n = 4$

$a = m^2 - n^2$

$a = 5^2 - 4^2$

$a = 9$

$b = 2mn$

$b = 2 \times 5 \times 4$

$b = 40$

$c = m^2 + n^2$

$c = 5^2 + 4^2$

$c = 41$

a, b, c are:

9, 40 and 41.

4 Use $m = 6$ and $n = 5$

$a = m^2 - n^2$

$a = 6^2 - 5^2$

$a = 11$

$b = 2mn$

$b = 2 \times 6 \times 5$

$b = 60$

$c = m^2 + n^2$

$c = 6^2 + 5^2$

$c = 61$

a, b, c are:

11, 60 and 61.

5 Use $m = 7$ and $n = 6$

$a = m^2 - n^2$

$a = 7^2 - 6^2$

$a = 13$

$b = 2mn$

$b = 2 \times 7 \times 6$

$b = 84$

$c = m^2 + n^2$

$c = 7^2 + 6^2$

$c = 85$

a, b, c are:

13, 84 and 85.

6 Use $m = 8$ and $n = 7$

$a = m^2 - n^2$

$a = 8^2 - 7^2$

$a = 15$

$b = 2mn$

$b = 2 \times 8 \times 7$

$b = 112$

$c = m^2 + n^2$

$c = 8^2 + 7^2$

$c = 113$

a, b, c are:

15, 112 & 113

7 Use $m = 9$ and $n = 8$

$a = m^2 - n^2$

$a = 9^2 - 8^2$

$a = 17$

$b = 2mn$

$b = 2 \times 9 \times 8$

$b = 144$

$c = m^2 + n^2$

$c = 9^2 + 8^2$

$c = 145$

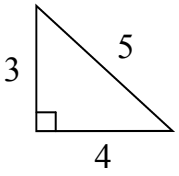
a, b, c are:

17, 144 & 145

Draw 4 right triangles, put the side lengths on them that you found in Columns 1 and 2. Then apply P.T. proof to check.



Example. Use example answer.



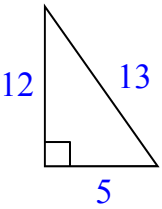
$5^2 = 3^2 + 4^2$

$25 = 25$

LHS = RHS

$\therefore \Delta$ is right-angled

8 Now Use question 1



$13^2 = 5^2 + 12^2$

$169 = 169$

LHS = RHS

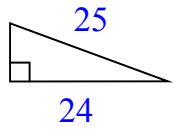
$\therefore \Delta$ is right-angled

9 Now Use question 2

$25^2 = 7^2 + 24^2$

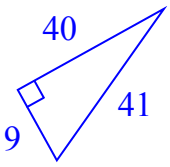
$625 = 625$

LHS = RHS



$\therefore \Delta$ is right-angled

10 Now Use question 3



$41^2 = 9^2 + 40^2$

$1\,681 = 1\,681$

LHS = RHS

$\therefore \Delta$ is right-angled

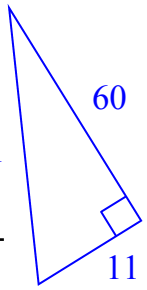
11 Now Use question 4

$61^2 = 11^2 + 60^2$

$3\,721 = 3\,721$

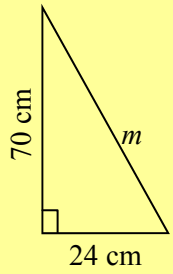
LHS = RHS

$\therefore \Delta$ is right-angled



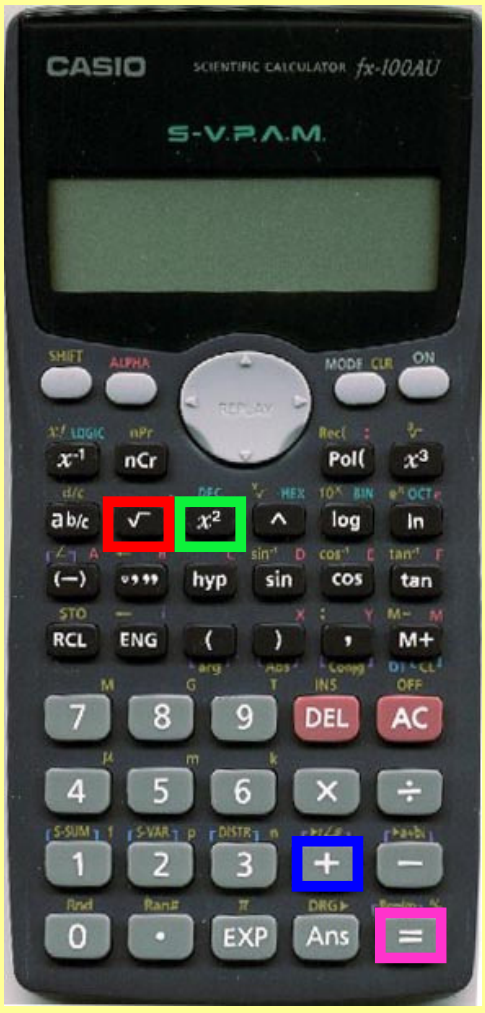
Finding the Length of the Hypotenuse - Whole Number Answers

Pythagoras' Theorem $c^2 = a^2 + b^2$ can be used to find the length of the hypotenuse given the two short sides of the right-angled triangle. The process is shown in the example below.



$m^2 = 24^2 + 70^2$	This is P.T. with c replaced with the letter m, and a and b replaced with the short side lengths.
$m^2 = 5\,476$	Rewrite m^2 then calculate the sum of the squares.
$m = \sqrt{5\,476}$	Rewrite the line above but change m^2 to m and write the number in a square root sign.
$m = 74$ cm	Solve the square root, on this sheet all the answers are whole numbers, include the units.

The entire sheet is answered in the same way as the method above. The diagrams in column 3 include triangles within other shapes but the process is just the same. Except with Q.14 which requires a little extra work before you can apply Pythagoras Theorem.



$m^2 = 24^2 + 70^2$	Calculator Steps:
$m^2 = 5\,476$	2 4 x^2 + 7 0 x^2 =
$m = \sqrt{5\,476}$	$\sqrt{\ }$ =
$m = 74$ cm	

Finding the Length of the Hypotenuse - Whole Number Answers

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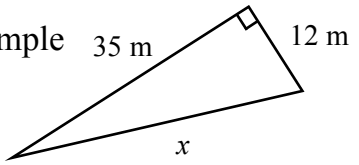
Find the length of the hypotenuse, the answer will be a whole number.

Example



$x^2 = 12^2 + 35^2$

First two lines are x^2

$x^2 = 1\,369$

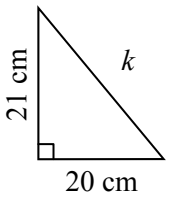
$x = \sqrt{1\,369}$

Last two lines are x

Write the units

$x = 37$ m

1



$k^2 = \square^2 + \square^2$

$k^2 =$

$k = \sqrt{\ }$

$k =$

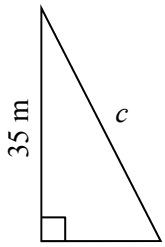
2

$c^2 =$

$c^2 =$

$c = \sqrt{\ }$

$c =$



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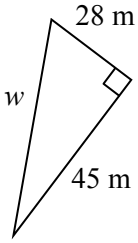

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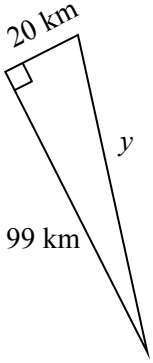

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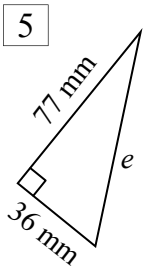

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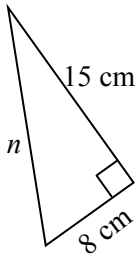

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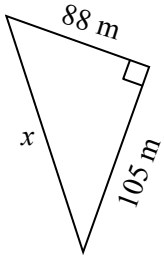

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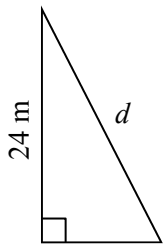

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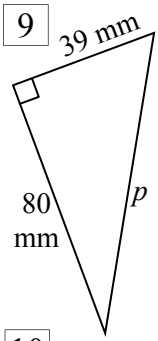

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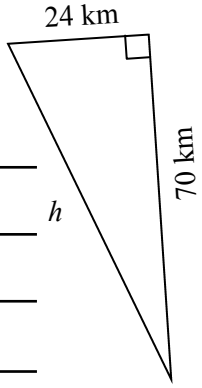
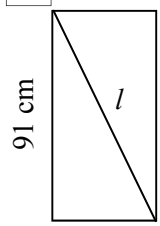


Find the length of the unknown side(s), answers are all whole numbers.

11



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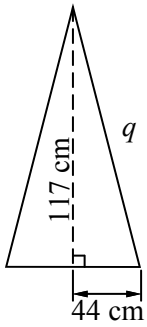

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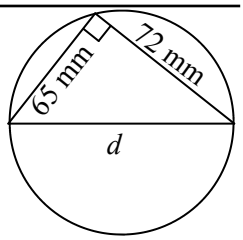

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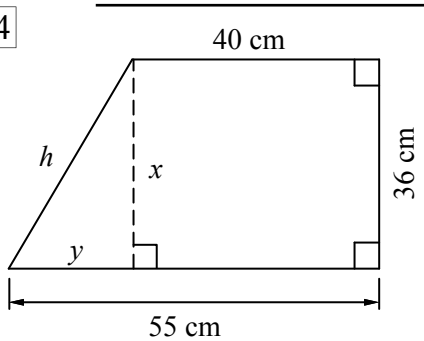

$x =$

$y = \square - \square =$

Now use x and y to find h

$h^2 =$

Finding the Length of the Hypotenuse - Whole Number Answers

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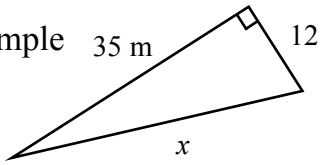
Find the length of the hypotenuse, the answer will be a whole number.

Example



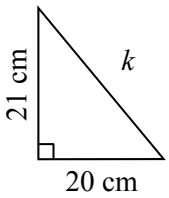
$x^2 = 12^2 + 35^2$ (First two lines are x^2)

$x^2 = 1\,369$

$x = \sqrt{1\,369}$ (Last two lines are x)

$x = 37$ m (Write the units)

1



$k^2 = 20^2 + 21^2$

$k^2 = 841$

$k = \sqrt{841}$

$k = 29$ cm

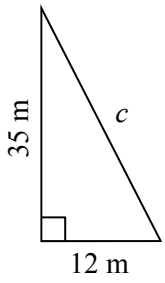
2

$c^2 = 12^2 + 35^2$

$c^2 = 1\,369$

$c = \sqrt{1\,369}$

$c = 37$ m



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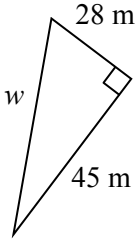


$w^2 = 28^2 + 45^2$

$w^2 = 2\,809$

$w = \sqrt{2\,809}$

$w = 53$ m

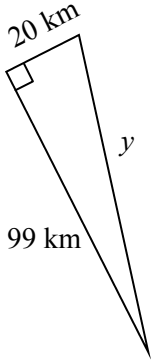
4

$y^2 = 99^2 + 20^2$

$y^2 = \sqrt{10\,201}$

$y = 10\,201$

$y = 101$ km



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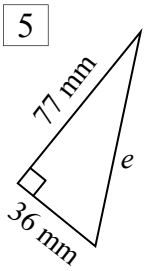


$e^2 = 36^2 + 77^2$

$e^2 = 7\,225$

$e = \sqrt{7\,225}$

$e = 85$ mm

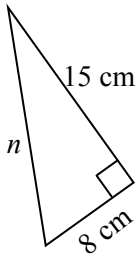
6

$n^2 = 8^2 + 15^2$

$n^2 = 289$

$n = \sqrt{289}$

$n = 17$ cm



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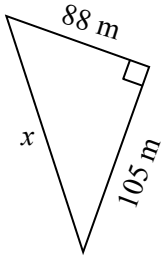


$x^2 = 88^2 + 105^2$

$x^2 = 18\,769$

$x = \sqrt{18\,769}$

$x = 137$ m

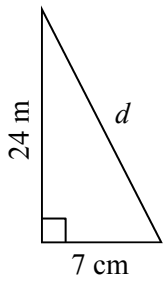
8

$d^2 = 7^2 + 24^2$

$d^2 = 625$

$d = \sqrt{625}$

$d = 25$ m



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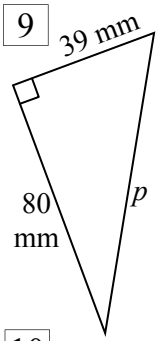


$p^2 = 39^2 + 80^2$

$p^2 = 7\,921$

$p = \sqrt{7\,921}$

$p = 89$ mm

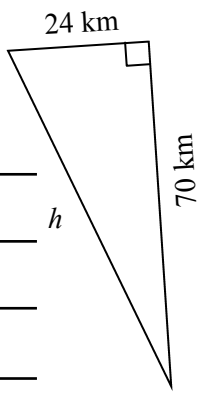
10

$h^2 = 24^2 + 70^2$

$h^2 = 5\,476$

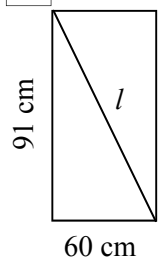
$h = \sqrt{5\,476}$

$h = 74$ km



Find the length of the unknown side(s), answers are all whole numbers.

11



$l^2 = 60^2 + 91^2$

$l^2 = 11\,881$

$l = \sqrt{11\,881}$

$l = 109$ cm

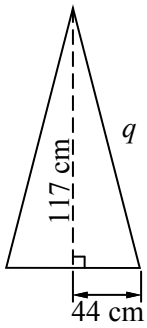
12

$q^2 = 44^2 + 117^2$

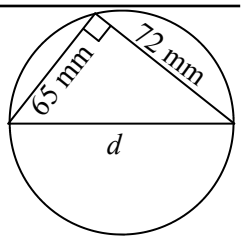
$q^2 = 15\,625$

$q = \sqrt{15\,625}$

$q = 125$ cm



13



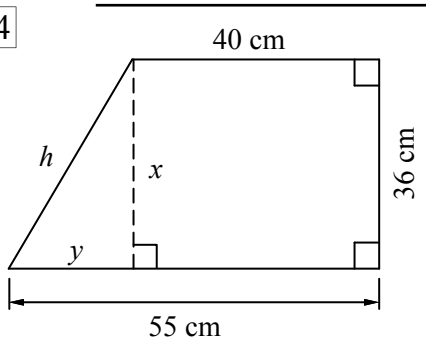
$d^2 = 65^2 + 72^2$

$d^2 = 9\,409$

$d = \sqrt{9\,409}$

$d = 97$ mm

14



$x = 36$ cm

$y = 55 - 40 = 15$ cm

Now use x and y to find h

$h^2 = 36^2 + 15^2$

$h^2 = 1\,521$

$h = \sqrt{1\,521}$

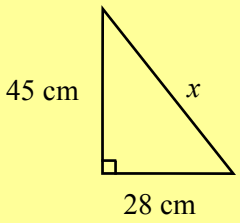
$h = 39$ cm

Finding the Hypotenuse - Rounding

This sheet uses Pythagoras theorem to find the length of the hypotenuse, the longest side of a right-angled triangle.

Pythagoras Theorem: $c^2 = a^2 + b^2$

Column 1 starts with an example that shows the method required. The hypotenuse is called x so the equation is x^2 = short side^2 + short side^2. Or put simply, square the two sides you are given and add them together. After you do this you square root the number to get the answer.



$x^2 = 28^2 + 45^2$ ← Write the letter on the hypotenuse squared. Then write the sum of the square of the two sides.

$x^2 = 2\,809$ ← Write '$x^2 =$' then calculate the right hand side and write the answer.

$x = \sqrt{2\,809}$ ← Change '$x^2 =$' to '$x =$' then write the answer in line two within a square root symbol or 'radical'.

$x = 53$ cm ← Complete the square root, check if rounding is required then write the units to complete.

Note that there are 4 lines of working, the first 2 lines are x^2, the next 2 are just x, try to remember this as some students forget. Look at the example above if you need help.

Column 2 is the same as column 1 except answers are to be rounded to 1 decimal place and the distance to be found isn't x, the letter used in each question varies.

Column 3 again the same method as column 1 but round to 2 decimal places.

Finding the Hypotenuse - Rounding

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Round these decimals to 1 decimal place. (1 d.p.)

1 8.87

2 15.273

3 24.319

4 11.6812

Now round to 2 d.p.

5 29.3609

6 73.8681

7 31.9961

8 5.00799

Complete the square roots then round

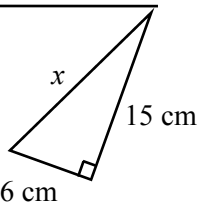
9 $\sqrt{18}$ ☐ (1 d.p.)

10 $\sqrt{70}$ ☐ (1 d.p.)

11 $\sqrt{650}$ ☐ (2 d.p.)

Find 'x' to 1 dp.

Example



First two lines are x^2

$x^2 = 6^2 + 15^2$

$x^2 = 261$

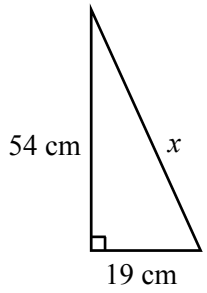
Last two lines are x

$x = \sqrt{261}$

$x = 16.2$ cm

Round to 1 d.p. and write the units

12



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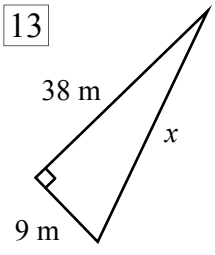
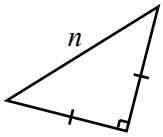


Keep going, round your answer to 1 d.p.

14



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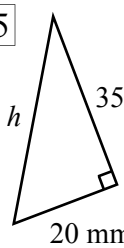

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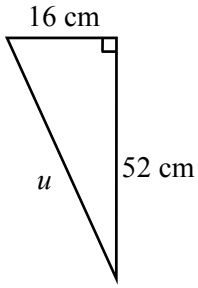

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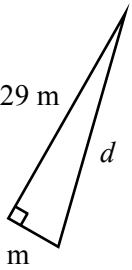
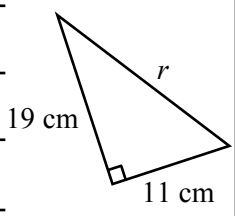
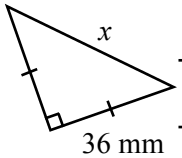


18



19



Given the two short sides find the hypotenuse (call it x), answer to 2 d.p.

20 17.6 mm and 22.9 mm

21 53 m and 19.8 m

22 173 cm and 104.5 cm

23 2.09 m and 4.63 m

24 574 mm and 422 mm

Finding the Hypotenuse - Rounding

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Round these decimals to 1 decimal place. (1 d.p.)

1. 8.87 — 8.9
2. 15.273 — 15.3
3. 24.319 — 24.3
4. 11.6812 — 11.7

Now round to 2 d.p.

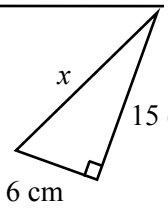
5. 29.3609 — 29.36
6. 73.8681 — 73.87
7. 31.9961 — 32/32.00
8. 5.00799 — 5.01

Complete the square roots then round

9. $\sqrt{18}$ — 4.2 (1 d.p.)
10. $\sqrt{70}$ — 8.4 (1 d.p.)
11. $\sqrt{650}$ — 25.50 (2 d.p.)

Find 'x' to 1 dp.

Example



First two lines are x^2

$x^2 = 6^2 + 15^2$

$x^2 = 261$

Last two lines are x

$x = \sqrt{261}$

$x = 16.2$ cm

Round to 1 d.p. and write the units

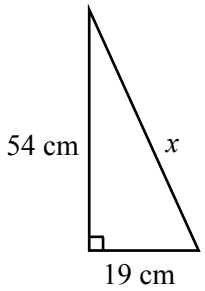
12

$x^2 = 19^2 + 54^2$

$x^2 = 3\,277$

$x = \sqrt{3\,277}$

$x = 57.2$ cm



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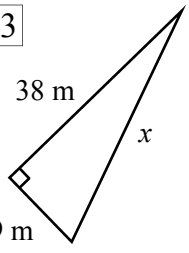


$x^2 = 9^2 + 38^2$

$x^2 = 1\,525$

$x = \sqrt{1\,525}$

$x = 39.1$ m

Keep going, round your answer to 1 d.p.

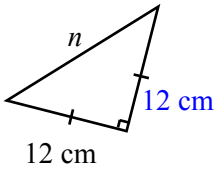
14

$n^2 = 12^2 + 12^2$

$n^2 = 288$

$n = \sqrt{288}$

$n = 17.0$ cm



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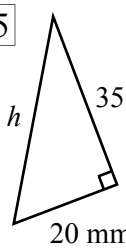


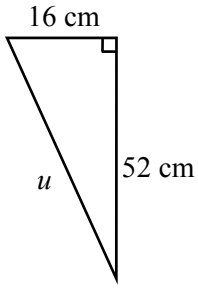
$h^2 = 20^2 + 35^2$

$h^2 = 1\,625$

$h = \sqrt{1\,625}$

$h = 40.3$ mm

16



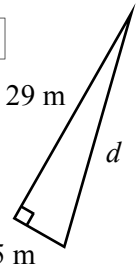
$u^2 = 16^2 + 52^2$

$u^2 = 2\,960$

$u = \sqrt{2\,960}$

$u = 54.4$ cm

17



$d^2 = 5^2 + 29^2$

$d^2 = 866$

$d = \sqrt{866}$

$d = 29.4$ m

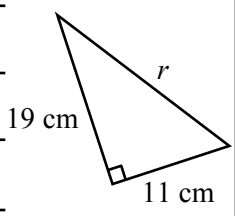
18

$r^2 = 19^2 + 11^2$

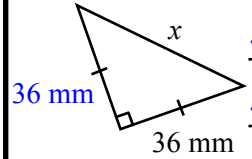
$r^2 = 482$

$r = \sqrt{482}$

$r = 22.0$ cm



19



$x^2 = 36^2 + 36^2$

$x^2 = 2\,592$

$x = \sqrt{2\,592}$

$x = 50.9$ mm

Given the two short sides find the hypotenuse (call it x), answer to 2 d.p.

20 17.6 mm and 22.9 mm

$x^2 = 17.6^2 + 22.9^2$

$x^2 = 834.17$

$x = \sqrt{834.17}$

$x = 28.88$ mm

21 53 m and 19.8 m

$x^2 = 53^2 + 19.8^2$

$x^2 = 3\,201.04$

$x = \sqrt{3\,201.04}$

$x = 56.58$ m

22 173 cm and 104.5 cm

$x^2 = 173^2 + 104.5^2$

$x^2 = 40\,849.25$

$x = \sqrt{40\,849.25}$

$x = 202.11$ cm

23 2.09 m and 4.63 m

$x^2 = 2.09^2 + 4.63^2$

$x^2 = 25.81$

$x = \sqrt{25.81}$

$x = 5.08$ m

24 574 mm and 422 mm

$x^2 = 574^2 + 422^2$

$x^2 = 507\,560$

$x = \sqrt{507\,560}$

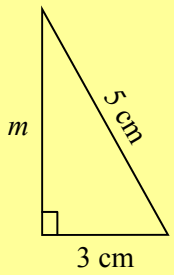
$x = 712.43$ mm

Finding a Short Side - Whole Number Answers

Pythagoras' Theorem $c^2 = a^2 + b^2$ can be used to find the length of the hypotenuse given the two short sides of the right-angled triangle. The process can be reversed when given the length of the hypotenuse and one of the short sides of the triangle.

$$b^2 = c^2 - a^2$$

In other words, use subtraction instead of addition. Just remember to use the hypotenuse length as 'c' as if you don't you will get a calculator error.

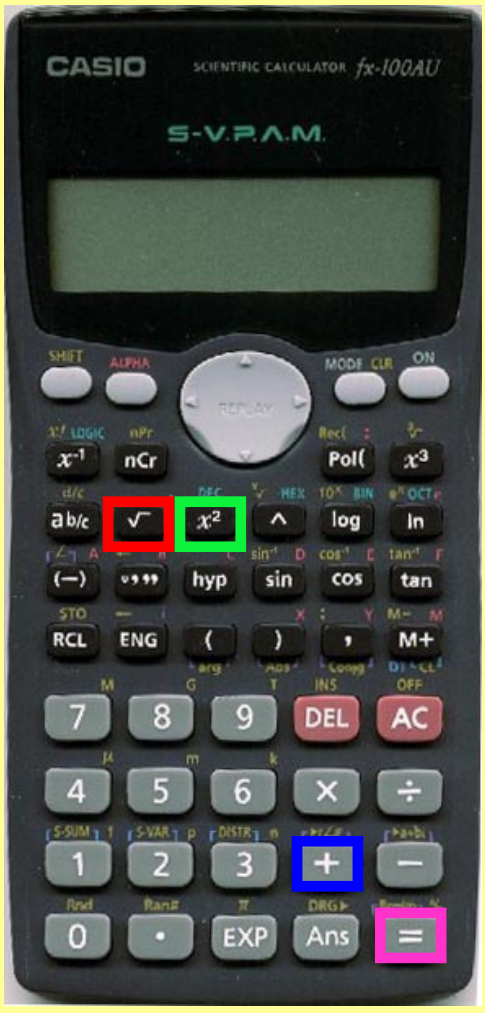


$m^2 = 5^2 - 3^2$ ← Remember to use '-' not '+' and it is the hypotenuse squared minus the short side squared.

$m^2 = 16$ ← Rewrite m^2 then subtract the two squared numbers.

$m = \sqrt{16}$ ← Rewrite the line above but change m^2 to m and write the number in a square root sign.

$m = 4$ cm ← Solve the square root, on this sheet all the answers are whole numbers, include the units.



The entire sheet is answered in the same way as the method above. The diagrams in column 3 include triangles within other shapes but the process is just the same. Except with Q.15 which requires a little extra work before you can apply Pythagoras Theorem.

$m^2 = 74^2 - 24^2$

$m^2 = 4\,900$

$m = \sqrt{4\,900}$

$m = 70$ cm

Calculator Steps:
7 4 x^2 - 2 4 x^2 =
$\sqrt{\ }$ =

Finding a Short Side - Whole Number Answers

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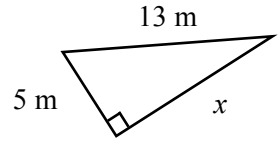
Find the length of the short side, the answer will be a whole number.

Example



$x^2 = 13^2 - 5^2$

First two lines are x^2

$x^2 = 144$

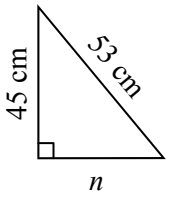
$x = \sqrt{144}$

Last two lines are x

Write the units

$x = 12$ m

1



$n^2 = \square^2 - \square^2$

$n^2 =$

$n = \sqrt{\ }$

$n =$

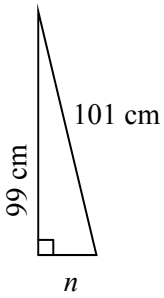
2

$n^2 = \square^2 - \square^2$

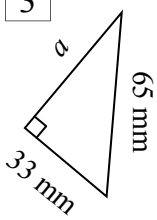
$n^2 =$

$n = \sqrt{\ }$

$n =$



3



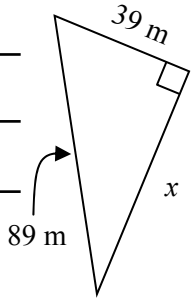
$a^2 =$

$a^2 =$

$a =$

$a =$

4



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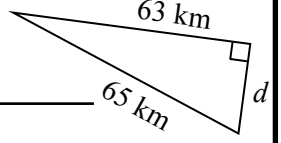

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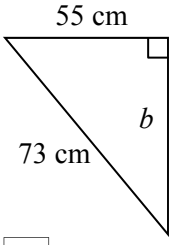
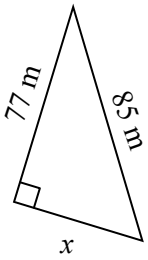


7



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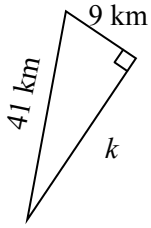

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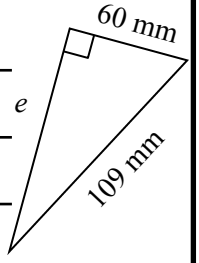

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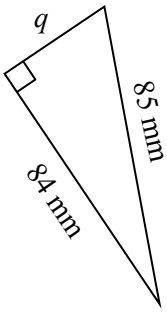
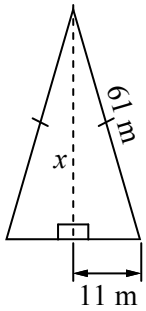


Find 'x' the unknown length, the answer will be a whole number.

11



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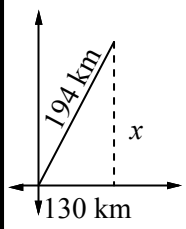
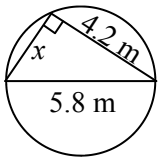
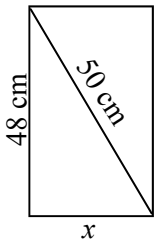


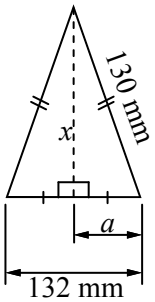
13



14



15 $a = \square \div \square =$



$x^2 =$

Finding a Short Side - Whole Number Answers

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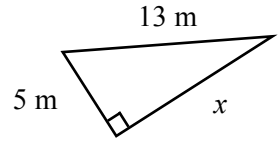
Find the length of the short side, the answer will be a whole number.

Example



$x^2 = 13^2 - 5^2$

First two lines are x^2

$x^2 = 144$

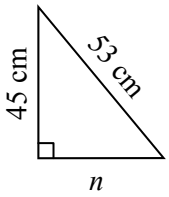
$x = \sqrt{144}$

Last two lines are x

Write the units

$x = 12$ m

1



$n^2 = \boxed{53}^2 - \boxed{45}^2$

$n^2 = 784$

$n = \sqrt{784}$

$n = 28$ cm

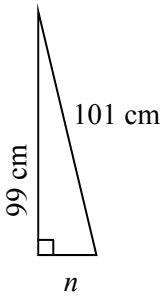
2

$n^2 = \boxed{101}^2 - \boxed{99}^2$

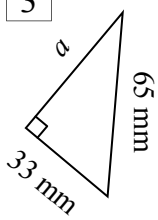
$n^2 = 400$

$n = \sqrt{400}$

$n = 20$ cm



3



$a^2 = 65^2 - 33^2$

$a^2 = 3\,136$

$a = \sqrt{3\,136}$

$a = 56$ mm

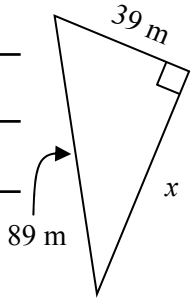
4

$x^2 = 89^2 - 39^2$

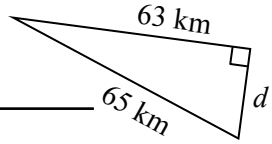
$x^2 = 6\,400$

$x = \sqrt{6\,400}$

$x = 80$ m



5



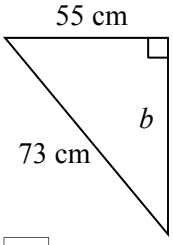
$d^2 = 65^2 - 63^2$

$d^2 = 256$

$d = \sqrt{256}$

$d = 16$ km

6



$b^2 = 73^2 - 55^2$

$b^2 = 2\,304$

$b = \sqrt{2\,304}$

$b = 48$ cm

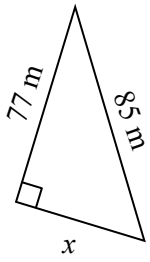
7

$x^2 = 85^2 - 77^2$

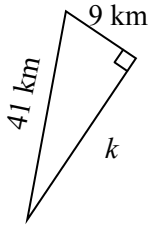
$x^2 = 1\,296$

$x = \sqrt{1\,296}$

$x = 36$ m



8



$k^2 = 41^2 - 9^2$

$k^2 = 1\,600$

$k = \sqrt{1\,600}$

$k = 40$ km

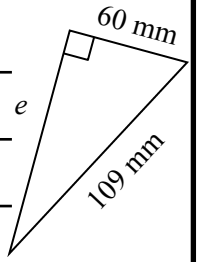
9

$e^2 = 109^2 - 60^2$

$e^2 = 8\,281$

$e = \sqrt{8\,281}$

$e = 91$ mm



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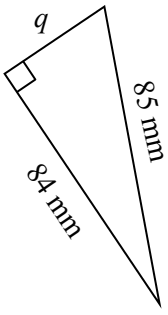


$q^2 = 85^2 - 84^2$

$q^2 = 169$

$q = \sqrt{169}$

$q = 13$ mm

Find 'x' the unknown length, the answer will be a whole number.

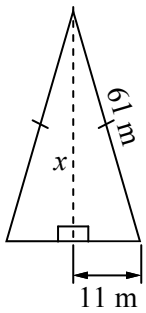
11

$x^2 = 61^2 - 11^2$

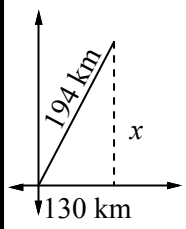
$x^2 = 3\,600$

$x = \sqrt{3\,600}$

$x = 60$ m



12



$x^2 = 194^2 - 130^2$

$x^2 = 20\,736$

$x = \sqrt{20\,736}$

$x = 144$ km

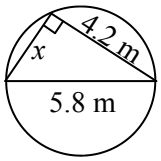
13

$x^2 = 5.8^2 - 4.2^2$

$x^2 = 16$

$x = \sqrt{16}$

$x = 4$ m



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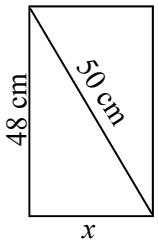


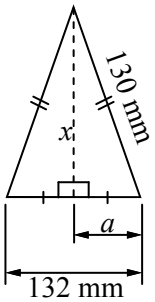
$x^2 = 50^2 - 48^2$

$x^2 = 196$

$x = \sqrt{196}$

$x = 14$ cm

15 $a = \boxed{132} \div \boxed{2} = 66$ mm



$x^2 = 130^2 - 66^2$

$x^2 = 12\,544$

$x = \sqrt{12\,544}$

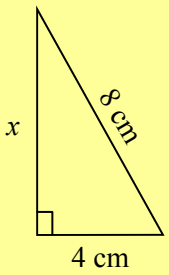
$x = 112$ mm

Finding a Short Side - Rounding

Pythagoras' Theorem $c^2 = a^2 + b^2$ can be used to find the length of the hypotenuse given the two short sides of the right-angled triangle. The process can be reversed when given the length of the hypotenuse and one of the short sides of the triangle.

$$b^2 = c^2 - a^2$$

In other words, use subtraction instead of addition. Just remember to use the hypotenuse length as 'c' as if you don't you will get a calculator error.

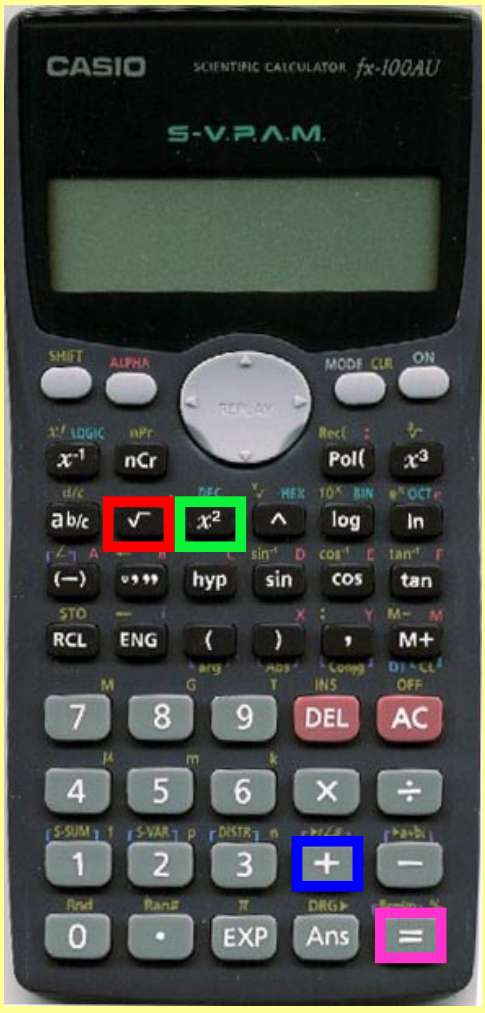


$x^2 = 8^2 - 4^2$ ← Remember to use '-' not '+' and it is the hypotenuse squared minus the short side squared.

$x^2 = 48$ ← Rewrite x^2 then subtract the two squared numbers.

$x = \sqrt{48}$ ← Rewrite the line above but change x^2 to x and write the number in a square root sign.

$x = 6.9$ cm ← Solve the square root, on this sheet the answers will require rounding, include the units.



The entire sheet is answered in the same way as the method above. Diagrams in column 3 have to be drawn. Instead of solving for 'x' you will be solving for a side defined by its vertices, so use the two letters like 'BC' for example.

$m^2 = 86^2 - 24^2$

$m^2 = 6\,820$

$m = \sqrt{6\,820}$

$m = 82.6$ cm

Calculator Steps:

8 6 x^2 - 2 4 x^2 =

$\sqrt{\ }$ =

Then round your answer

Finding the Short Side - Rounding

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Complete the square roots then round

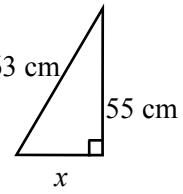
1 $\sqrt{208}$ _____ (1 d.p.)

2 $\sqrt{11.3}$ _____ (1 d.p.)

3 $\sqrt{228}$ _____ (2 d.p.)

Find 'x' to 1 dp.

Example



First two lines are x^2

$x^2 = 63^2 - 55^2$

$x^2 = 944$

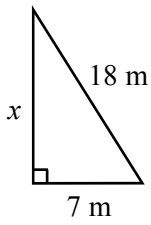
Last two lines are x

$x = \sqrt{944}$

$x = 30.7$ cm

Round to 1 d.p. and write the units

4



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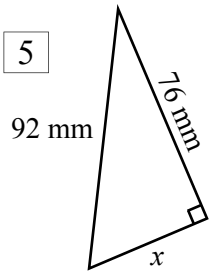

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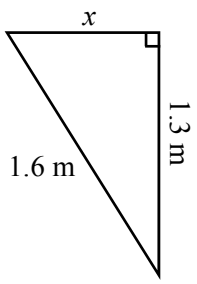

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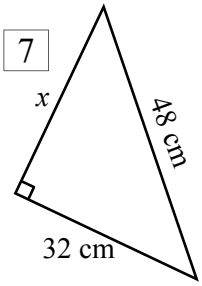
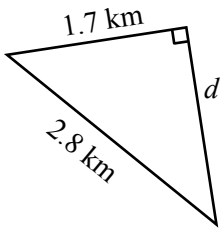


Now round your answer to 2 d.p.

8



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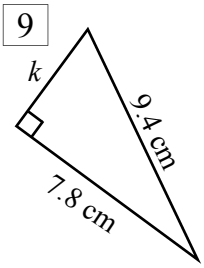

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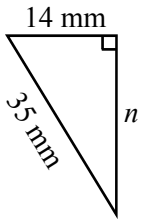

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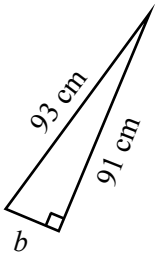

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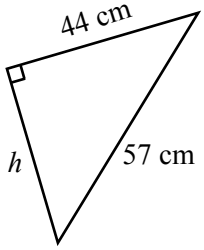

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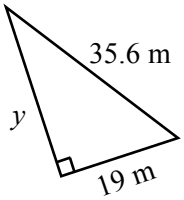

Construct your own triangles, I've taken the lot. Answer to 1 d.p.

14 Hypotenuse AC = 15 cm, AB = 11 cm, Find BC.

Draw Diagram

15 Hypotenuse VX = 52.8 m, XQ = 27 m, Find VQ.

Draw Diagram

16 Hypotenuse AM = 34 mm, TA = 32 mm, Find MT.

Draw Diagram

Finding the Short Side - Rounding

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Complete the square roots then round

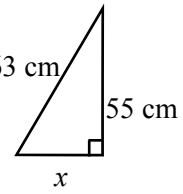
1 $\sqrt{208}$ = 14.4 (1 d.p.)

2 $\sqrt{11.3}$ = 3.4 (1 d.p.)

3 $\sqrt{228}$ = 15.1 (2 d.p.)

Find 'x' to 1 dp.

Example



First two lines are x^2

$x^2 = 63^2 - 55^2$

$x^2 = 944$

Last two lines are x

$x = \sqrt{944}$

$x = 30.7$ cm

Round to 1 d.p. and write the units

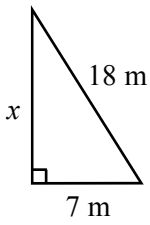
4

$x^2 = 18^2 - 7^2$

$x^2 = 275$

$x = \sqrt{275}$

$x = 16.6$ m



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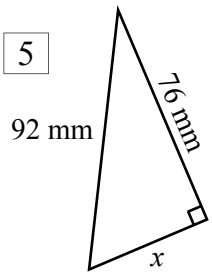


$x^2 = 92^2 - 76^2$

$x^2 = 2\,688$

$x = \sqrt{2\,688}$

$x = 51.8$ mm

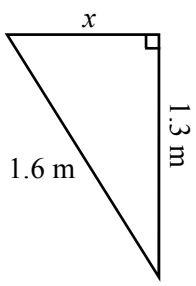
6

$x^2 = 1.6^2 - 1.3^2$

$x^2 = 0.87$

$x = \sqrt{0.87}$

$x = 0.9$ m



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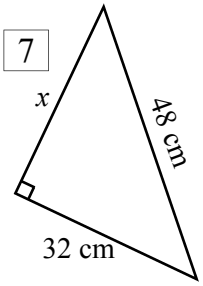


$x^2 = 48^2 - 32^2$

$x^2 = 1\,280$

$x = \sqrt{1\,280}$

$x = 35.8$ cm

Now round your answer to 2 d.p.

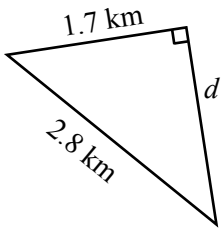
8

$d^2 = 2.8^2 - 1.7^2$

$d^2 = 4.95$

$d = \sqrt{4.95}$

$d = 2.22$ km



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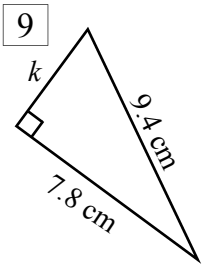


$k^2 = 9.4^2 - 7.8^2$

$k^2 = 27.52$

$k = \sqrt{27.52}$

$k = 5.25$ cm

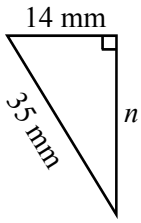
10

$n^2 = 35^2 - 14^2$

$n^2 = 1\,029$

$n = \sqrt{1\,029}$

$n = 32.08$ mm



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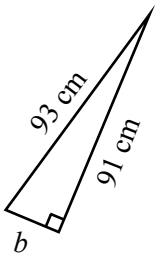


$b^2 = 93^2 - 91^2$

$b^2 = 368$

$b = \sqrt{368}$

$b = 19.18$ cm

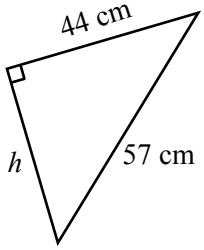
12

$h^2 = 57^2 - 44^2$

$h^2 = 1\,313$

$h = \sqrt{1\,313}$

$h = 36.24$ cm



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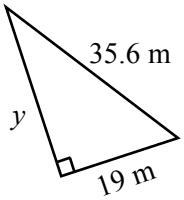


$y^2 = 35.6^2 - 19^2$

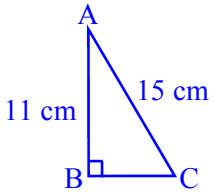
$y^2 = 906.36$

$y = \sqrt{906.36}$

$y = 30.11$ m

Construct your own triangles, I've taken the lot. Answer to 1 d.p.

14 Hypotenuse AC = 15 cm, AB = 11 cm, Find BC.



Draw Diagram

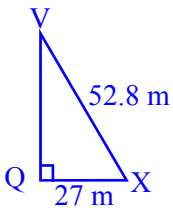
$BC^2 = 15^2 - 11^2$

$BC^2 = 104$

$BC = \sqrt{104}$

$BC = 10.2$ cm

15 Hypotenuse VX = 52.8 m, XQ = 27 m, Find VQ.



Draw Diagram

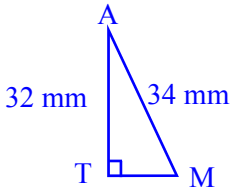
$VQ^2 = 52.8^2 - 27^2$

$VQ^2 = 2\,058.84$

$VQ = \sqrt{2\,058.84}$

$VQ = 45.4$ m

16 Hypotenuse AM = 34 mm, TA = 32 mm, Find MT.



Draw Diagram

$MT^2 = 34^2 - 32^2$

$MT^2 = 132$

$MT = \sqrt{132}$

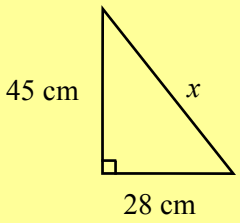
$MT = 11.5$ mm

Pythagoras - Mixed Rounding and Surds

This sheet should only be attempted after finding the hypotenuse and finding the short side worksheets have been completed.

Column 1 is a mixture of finding the short side and finding the hypotenuse, so some are subtraction and some are addition. Note that the pronumeral (or letter) changes, it isn't always 'x'.

Example - Finding the Hypotenuse



$x^2 = 28^2 + 45^2$

$x^2 = 2\,809$

$x = \sqrt{2\,809}$

$x = 53$ cm

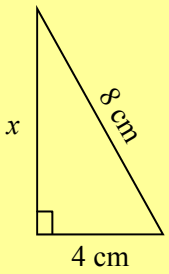
Write the letter on the hypotenuse squared. Then write the sum of the square of the two sides.

Write '$x^2 =$' then calculate the right hand side and write the answer.

Change '$x^2 =$' to '$x =$' then write the answer in line two within a square root symbol or 'radical'.

Complete the square root, check if rounding is required then write the units to complete.

Example - Finding the Short side



$x^2 = 8^2 - 4^2$

$x^2 = 48$

$x = \sqrt{48}$

$x = 6.9$ cm

Remember to use '-' not '+' and it is the hypotenuse squared minus the short side squared.

Rewrite x^2 then subtract the two squared numbers.

Rewrite the line above but change x^2 to x and write the number in a square root sign.

Solve the square root, on this sheet the answers will require rounding, include the units.

Column 2 asks to give the answers in surd form. These are easier as you don't have to calculate the square root, the 4th line is left out. Remember units, and if none are supplied then write 'units'.

Lower in Column 2 a truss is shown, a truss is a structure used to support load, such as in building roof structures and bridges. The lengths of all the external members are given, you have to calculate the lengths of the internal members. You have to start from the right hand side (R.H.S) and move left through the truss. This continues through Column 3 to complete all six of the internal members.

Pythagoras - Mixed Rounding and Surds

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Find the unknown length on these triangles, they may be subtraction or addition. Round your answer to 1 d.p.

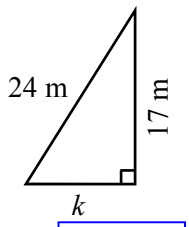
1

$k^2 =$ -

$k^2 =$

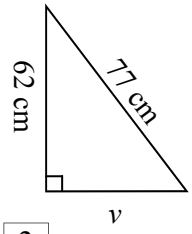
$k =$ √

$k =$ m



Remember the units

2



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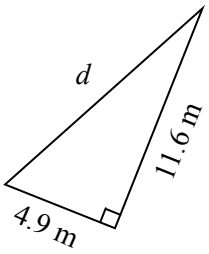

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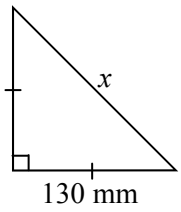

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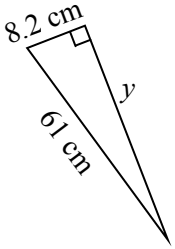

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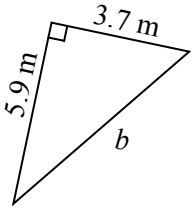


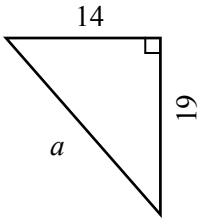
Leave your answer as a surd. That means don't square root. Write 'units' if none shown.

Example

$a^2 = 14^2 + 19^2$

$a^2 = 557$

$a = \sqrt{557}$ units



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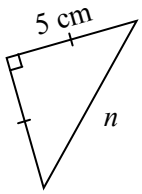

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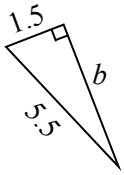

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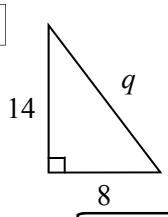
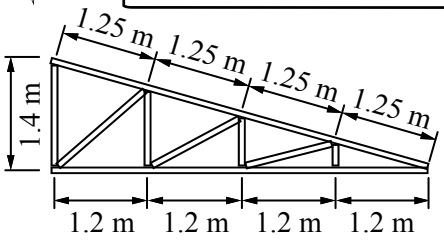
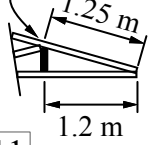


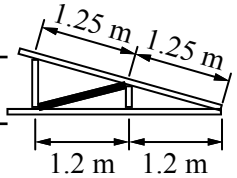
Find the lengths of the internal members of the truss below, round to 2 d.p.



10 Black vertical



11 Use your answer from Q. 10



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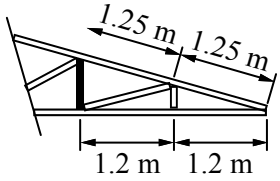

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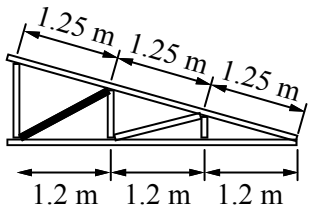

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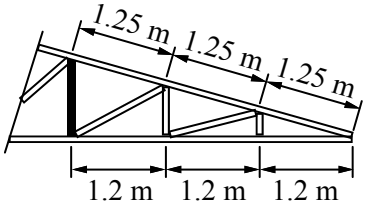

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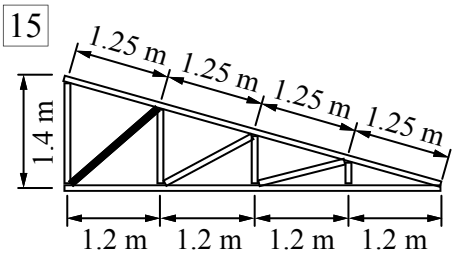

Pythagoras - Mixed Rounding and Surds

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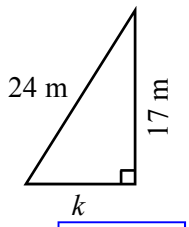
Find the unknown length on these triangles, they may be subtraction or addition. Round your answer to 1 d.p.

1



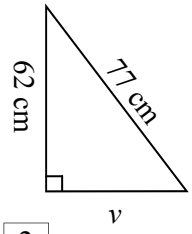
$k^2 = 24^2 - 17^2$

$k^2 = 287$

$k = \sqrt{287}$

$k = 16.9$ m ← Remember the units

2



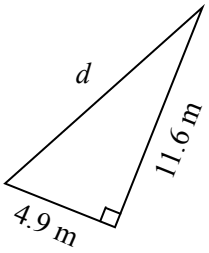
$v^2 = 77^2 - 62^2$

$v^2 = 2\,085$

$v = \sqrt{2\,085}$

$v = 45.7$ cm

3



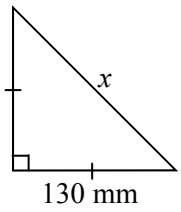
$d^2 = 4.9^2 + 11.6^2$

$d^2 = 158.57$

$d = \sqrt{158.57}$

$d = 45.7$ m

4



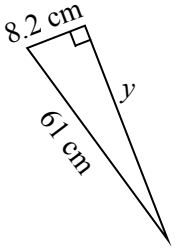
$x^2 = 130^2 + 130^2$

$x^2 = 33\,800$

$x = \sqrt{33\,800}$

$x = 183.8$ mm

5



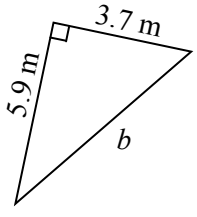
$y^2 = 61^2 - 8.2^2$

$y^2 = 3\,653.76$

$y = \sqrt{3\,653.76}$

$y = 60.4$ cm

6



$b^2 = 5.9^2 + 3.7^2$

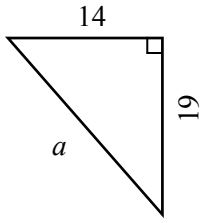
$b^2 = 48.5$

$b = \sqrt{48.5}$

$b = 7.0$ cm

Leave your answer as a surd. That means don't square root. Write 'units' if none shown.

Example

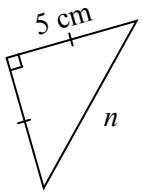


$a^2 = 14^2 + 19^2$

$a^2 = 557$

$a = \sqrt{557}$ units

7

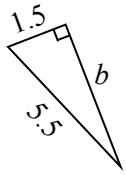


$n^2 = 5^2 + 5^2$

$n^2 = 50$

$n = \sqrt{50}$ cm

8

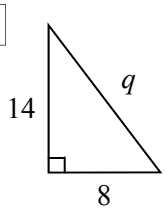


$b^2 = 5.5^2 - 1.5^2$

$b^2 = 28$

$b = \sqrt{28}$ units

9

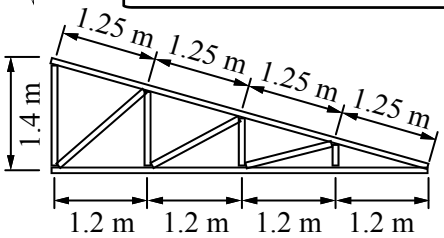


$q^2 = 14^2 + 8^2$

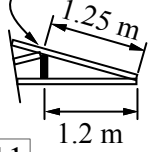
$q^2 = 260$

$q = \sqrt{260}$ units

Find the lengths of the internal members of the truss below, round to 2 d.p.



10 Black vertical



$x^2 = 1.25^2 - 1.2^2$

$x^2 = 0.1225$

$x = \sqrt{0.1225}$

$x = 0.35$ m

11

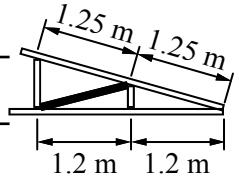
$x^2 = 1.2^2 + 0.35^2$

$x^2 = 1.5625$

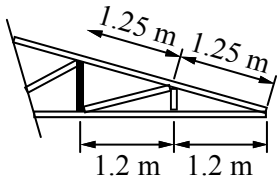
$x = \sqrt{1.5625}$

$x = 1.25$ m

Use your answer from Q. 10



12



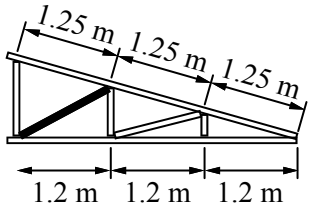
$x^2 = 2.5^2 - 2.4^2$

$x^2 = 0.49$

$x = \sqrt{0.49}$

$x = 0.7$ m

13



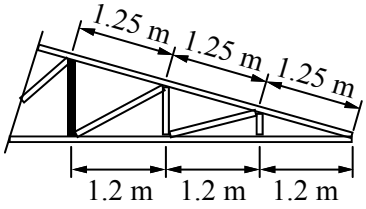
$x^2 = 0.7^2 + 1.2^2$

$x^2 = 1.93$

$x = \sqrt{1.93}$

$x = 1.39$ m

14



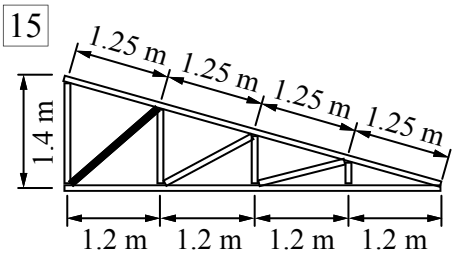
$x^2 = 3.75^2 - 3.6^2$

$x^2 = 1.1025$

$x = \sqrt{1.1025}$

$x = 1.05$ m

15



$x^2 = 1.05^2 + 1.2^2$

$x^2 = 2.5425$

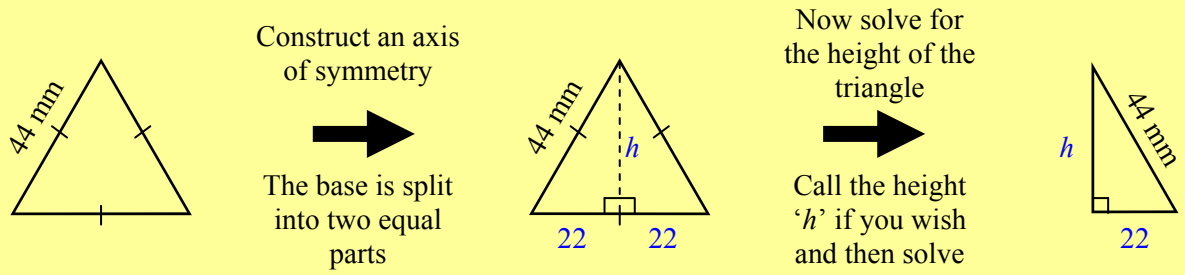
$x = \sqrt{2.5425}$

$x = 1.59$ m

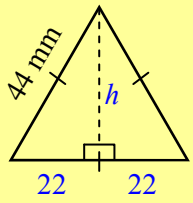
Pythagoras Theorem - Equilateral and Isosceles Triangles

Equilateral triangles and Non-Right angled Isosceles triangles don't have a right angle. So Pythagoras Theorem can't be applied to them unless a right angle is introduced. Because there are matching sides an axis of symmetry can be drawn through the triangle to create two right-angles triangles. Then Pythagoras can be applied.

Example



Solution



$h^2 = 44^2 - 22^2$

$h^2 = 1\ 452$

$h = \sqrt{1\ 452}$

$h = 38.11$ mm (2 d.p.)

Column 1 uses the above method, but Column 2 gives you the height and a side length and asks you to calculate the base length. Again use the method above but realise that you will be calculating 'half the base length' with some questions and may need to double the answer. The last two questions are more challenging.

Column 3 is an extension of the previous work and ties in algebra. This uses index laws and is more difficult. A full worked solution is at the top of the column, the good news is the algebra procedure doesn't change, so you can follow the process with just a different number in the questions below the example.

Pythagoras Theorem - Equilateral and Isosceles Triangles

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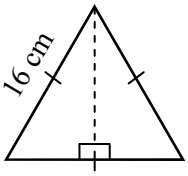
Find the height of these triangles. Make sure you ½ the base length. (1 d.p.)

1



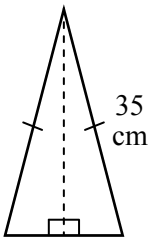
$h^2 = 16^2 - 8^2$

$h^2 =$

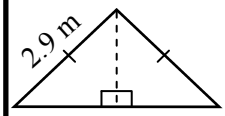
$h =$

$h =$

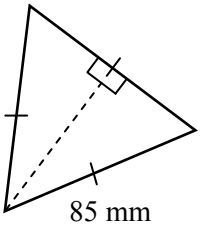
2



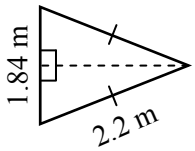
3



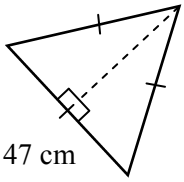
4



5

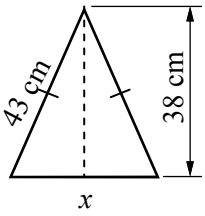


6

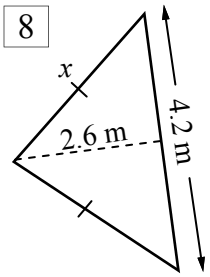


Now find the missing side length. Round to 1 d.p.

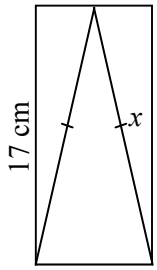
7



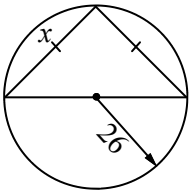
8



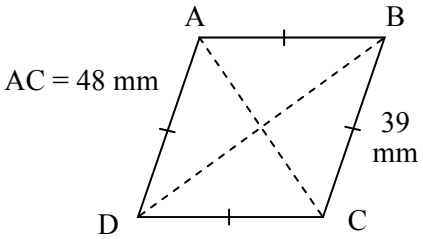
9



10



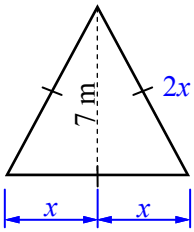
11 Find the length DB



Now find the side length of these equilateral triangles. It's algebra time! Round to 2 d.p.

Example

Let the side length = $2x$
So half side length = x



$(2x)^2 = 7^2 + x^2$

$4x^2 = 49 + x^2$

$-x^2$ $-x^2$

$3x^2 = 49$

$\div 3$ $\div 3$

$x^2 = 16.33....$calc

$\sqrt{\ }$ $\sqrt{\ }$

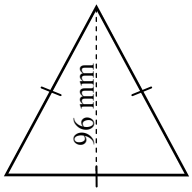
Side length

$x = 4.04....$calc

$= 8.08$ m

$2x = 8.08$ m

12



13 Find the side length of an equilateral triangle which has a height of 25 cm.

Diagram

Pythagoras Theorem - Equilateral and Isosceles Triangles

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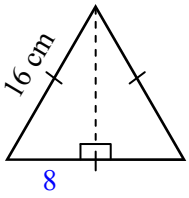
Find the height of these triangles. Make sure you ½ the base length. (1 d.p.)

1



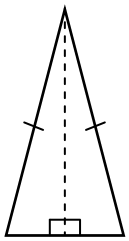
$h^2 = 16^2 - 8^2$

$h^2 = 192$

$h = \sqrt{192}$

$h = 13.9$ cm

2



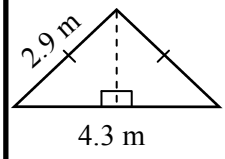
$h^2 = 35^2 - 9^2$

$h^2 = 1\ 144$

$h = \sqrt{1\ 144}$

$h = 33.8$ cm

3



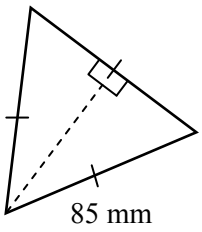
$h^2 = 2.9^2 - 2.15^2$

$h^2 = 3.7875$

$h = \sqrt{3.7875}$

$h = 1.9$ m

4



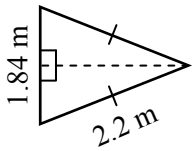
$h^2 = 85^2 - 42.5^2$

$h^2 = 5418.75$

$h = \sqrt{5418.75}$

$h = 73.6$ mm

5



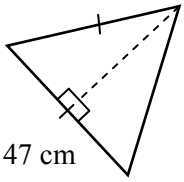
$h^2 = 2.2^2 - 0.92^2$

$h^2 = 3.9936$

$h = \sqrt{3.9936}$

$h = 2$ m

6



$h^2 = 47^2 - 23.5^2$

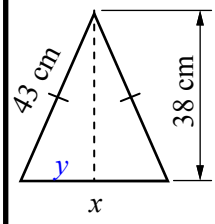
$h^2 = 1656.75$

$h = \sqrt{1656.75}$

$h = 40.7$ cm

Now find the missing side length. Round to 1 d.p.

7



$y^2 = 43^2 - 38^2$

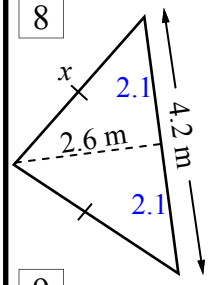
$y^2 = 405$

$y = \sqrt{405}$

$y = 20.1$ cm

$x = 2 \times 20.1 = 40.2$ cm

8



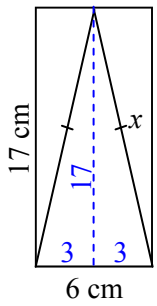
$x^2 = 2.6^2 + 2.1^2$

$x^2 = 11.17$

$x = \sqrt{11.17}$

$x = 3.3$ m

9



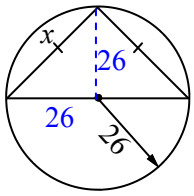
$x^2 = 3^2 + 17^2$

$x^2 = 298$

$x = \sqrt{298}$

$x = 17.3$ cm

10



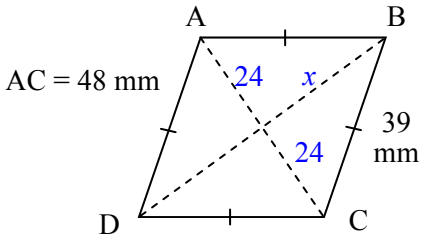
$x^2 = 26^2 + 26^2$

$x^2 = 1\ 352$

$x = \sqrt{1\ 352}$

$x = 36.8$ units

11 Find the length DB



$x^2 = 39^2 - 24^2$

$x^2 = 945$

$x = \sqrt{945}$

$x = 30.7$ mm

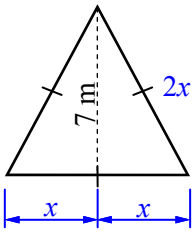
$DB = 2x$

$= 61.4$ mm

Now find the side length of these equilateral triangles. It's algebra time! Round to 2 d.p.

Example

Let the side length = $2x$
So half side length = x



$(2x)^2 = 7^2 + x^2$

$4x^2 = 49 + x^2$

$-x^2$ $-x^2$

$3x^2 = 49$

$\div 3$ $\div 3$

$x^2 = 16.33....$calc

$\sqrt{}$ $\sqrt{}$

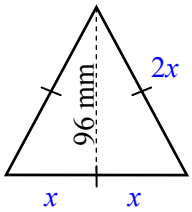
$x = 4.04....$calc

Side length

$= 8.08$ m

$2x = 8.08$ m

12



$(2x)^2 = 96^2 + x^2$

$4x^2 = 9\ 216 + x^2$

$-x^2$ $-x^2$

$3x^2 = 9\ 216$

$\div 3$ $\div 3$

$x^2 = 3\ 072$

$\sqrt{}$ $\sqrt{}$

$x = 55.42....$calc

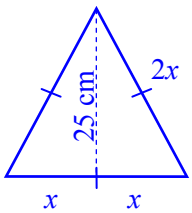
Side length

$=110.85$ mm

$2x = 110.85$ mm

13 Find the side length of an equilateral triangle which has a height of 25 cm.

Diagram



$(2x)^2 = 25^2 + x^2$

$4x^2 = 625 + x^2$

$-x^2$ $-x^2$

$3x^2 = 625$

$\div 3$ $\div 3$

$x^2 = 208.3....$calc

$\sqrt{}$ $\sqrt{}$

$x = 14.43....$calc

Side length

$= 28.87$ cm

$2x = 28.87$ cm

Pythagoras Theorem - Problems

These are problems involving Pythagoras Theorem. These questions should be only attempted after previous worksheets have been completed.

The questions are just like 'triangle' questions, the only difference is that you may have to:

- Interpret how a diagram converts to a triangle
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Pythagoras Theorem - Problems

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Solve these problems using Pythagoras Theorem, answer to 2 decimal places.

1 A 2.3 m ladder is resting against a wall as shown. If the foot of the ladder is 0.8 m from the base of the wall, how far up the wall will the ladder reach? Start by filling in the boxes.

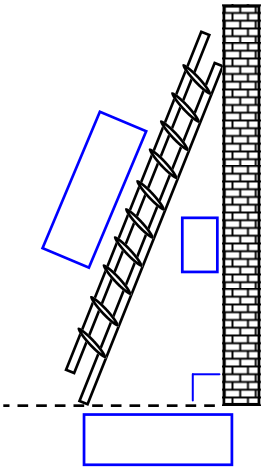
$h^2 =$

$h^2 =$

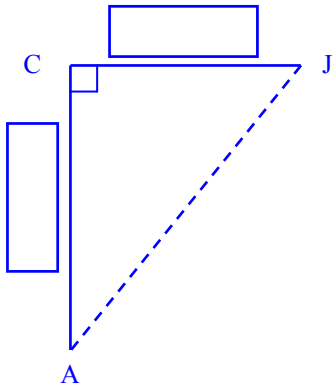
$h = \sqrt{}$

$h =$

The ladder will reach



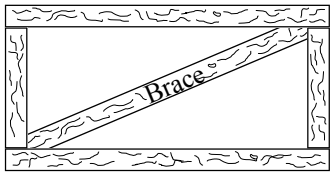
2 Jake and Amy left the same campsite (C). Jake (J) walked 4.2 km East while Amy (A) walked 5.6 km South. How far apart are they?



Answer using a sentence

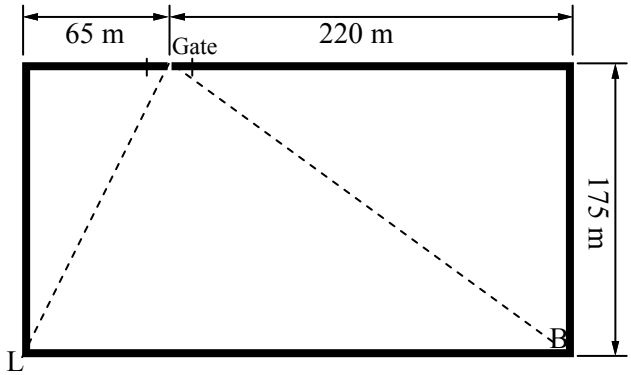
3 A kite is on a 35 m string and is a horizontal distance of 22 m from the flier. How high is the kite above the ground? Construct a diagram showing all relevant details.

4 A driveway gate (3.4 m × 1.2 m) is shown, calculate the length of the wooden brace.



5 Calculate the cost to build the gate given that the wood costs $7.30 per metre.

6 Leaping Larry always takes a shortcut through Farmer Fred's paddock. Bezerko the bull chases Larry because Larry wears red socks. If Larry is at point L and Bezerko is at B, how far does each have to run to get to the gate?



Larry:

Berzerko:

7 Find Larry's distance as a percentage of Berzerko's, round to the nearest whole number.

Pythagoras Theorem - Problems

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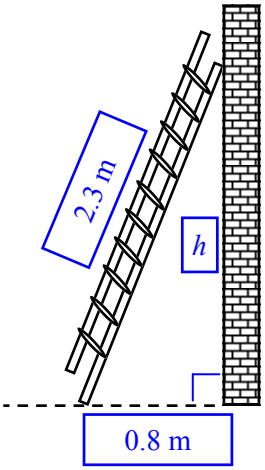
$h^2 = 2.3^2 - 0.8^2$

$h^2 = 4.65$

$h = \sqrt{4.65}$

$h = 2.16$ m

The ladder will reach 2.16 m up the wall.



2 Jake and Amy left the same campsite (C). Jake (J) walked 4.2 km East while Amy (A) walked 5.6 km South. How far apart are they?

$JA^2 = 5.6^2 + 4.2^2$

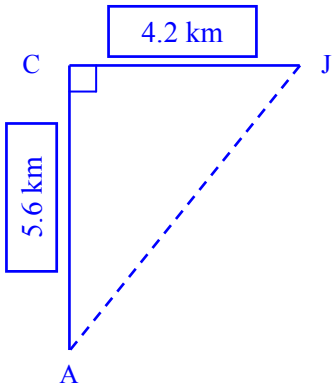
$JA^2 = 49$

$JA = \sqrt{49}$

$JA = 7$ km

They are 7 km apart.

Answer using a sentence



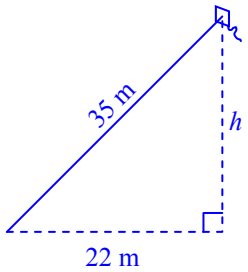
3 A kite is on a 35 m string and is a horizontal distance of 22 m from the flier. How high is the kite above the ground? Construct a diagram showing all relevant details.

$h^2 = 35^2 - 22^2$

$h^2 = 741$

$h = \sqrt{741}$

$h = 27.22$ m



The kite is 27.22 m above the ground.

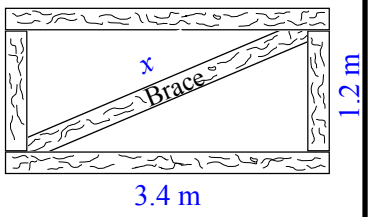
4 A driveway gate (3.4 m × 1.2 m) is shown, calculate the length of the wooden brace.

$x^2 = 3.4^2 + 1.2^2$

$x^2 = 13$

$x = \sqrt{13}$

$x = 3.61$ m



The brace is 3.61 m long.

5 Calculate the cost to build the gate given that the wood costs $7.30 per metre.

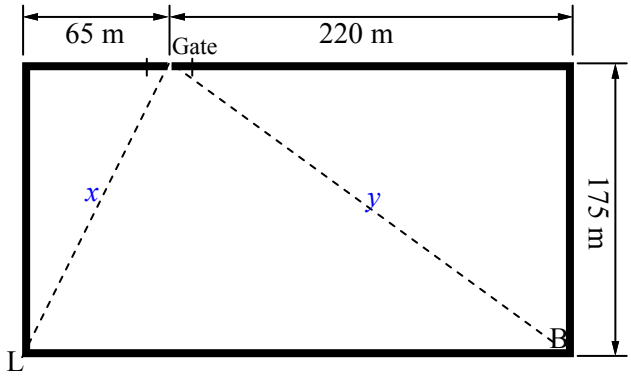
$P = 2 \times 3.4 + 2 \times 1.2$

$+ 3.61 = 12.81$ m

Cost @ 12.81=$93.51

Cost @ 13 m=$94.90

6 Leaping Larry always takes a shortcut through Farmer Fred's paddock. Bezerko the bull chases Larry because Larry wears red socks. If Larry is at point L and Bezerko is at B, how far does each have to run to get to the gate?



Larry:

$x^2 = 175^2 + 65^2$

$x^2 = 34\,850$

$x = \sqrt{34\,850}$

$x = 186.68$ m

Larry has to run a distance of 186.68 m.

Berzerko:

$y^2 = 175^2 + 220^2$

$y^2 = 79\,025$

$y = \sqrt{79\,025}$

$y = 281.11$ m

Berzerko charges a distance of 281.11 m.

7 Find Larry's distance as a percentage of Berzerko's, round to the nearest whole number.

$\frac{186.68}{281.11} \times 100 = 66.4\ldots$ calc $= 66\%$

Pythagoras Theorem - Further Problems

These are challenging problems involving Pythagoras Theorem. These questions should be only attempted after previous worksheets have been completed.

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Pythagoras Theorem - Further Problems

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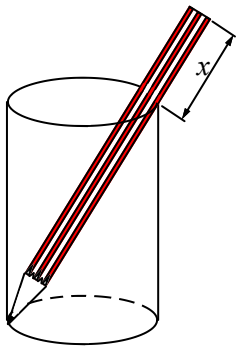
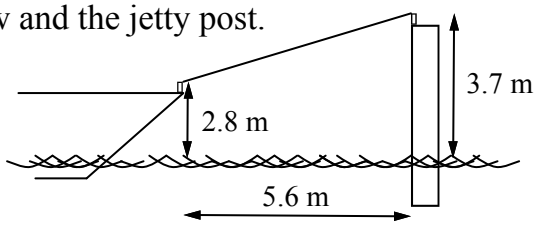
Solve these problems using Pythagoras Theorem, answer to 2 decimal places.

1 The glass below is 65 mm wide and 9.2 cm tall. If the pencil is 14 cm long, what length of the pencil is above the rim of the glass. Ignore the thickness of the pencil in your calculation.



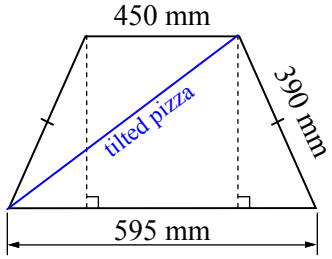
2 Calculate the length of rope between the boat's bow and the jetty post.



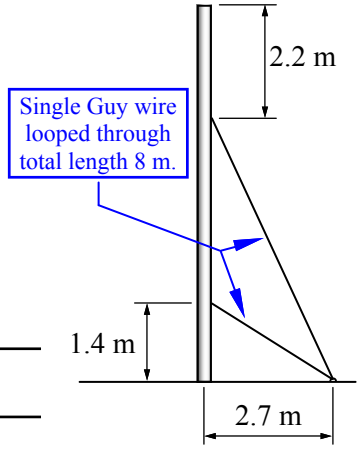
3 Alec walked South 1.42 km, West 850 m then North 2.01 km. How far from his starting location is he? Answer to the nearest metre.

Diagram

4 Lou's pizza prides itself on the largest pizza in its area. A competitor has released a 60 cm pizza, which is larger than the width of his pizza oven door, shown below. Lou thinks that if he tilts the pizza through the door, as shown, he can increase the size of his pizza. What size pizza can he make to the nearest cm, if he does so?



5 A flagpole shown below has a single guy wire attached to it 2.2 m from its tip. The 8 m guy wire is anchored 2.7 m from the base of the flagpole, loops through an eyelet and reconnects to the flagpole 1.4 m above the ground. Find the height of the flagpole.



Pythagoras Theorem - Further Problems

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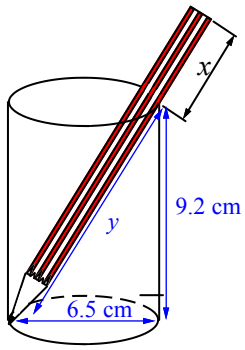


$y^2 = 6.5^2 + 9.2^2$

$y^2 = 126.89$

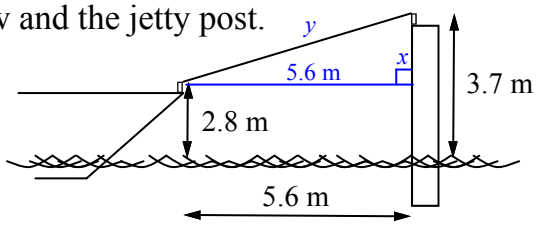
$y = \sqrt{126.89}$

$y = 11.26$ cm

$x = 14 - 11.26 = 2.74$ cm

2.74 cm of pencil is protruding

2 Calculate the length of rope between the boat's bow and the jetty post.



$x = 3.7 - 2.8 = 0.9$ m

$y^2 = 0.9^2 + 5.6^2$

$y^2 = 32.17$

$y = \sqrt{32.17}$

$y = 5.67$ m

The length of rope is 5.67 m.

3 Alec walked South 1.42 km, West 850 m then North 2.01 km. How far from his starting location is he? Answer to the nearest metre.

$x = 2\,010 - 1\,420$

$x = 590$ m

$y^2 = 590^2 + 850^2$

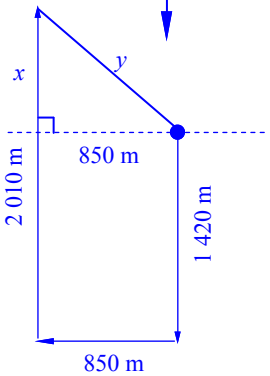
$y^2 = 1\,070\,600$

$y = \sqrt{1\,070\,600}$

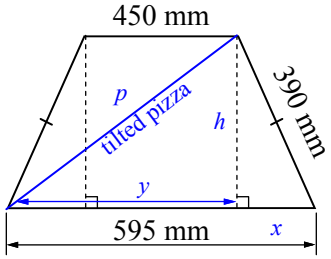
$y = 1\,035$ m

Alec is 1 035 m away from his original position.

Diagram



4 Lou's pizza prides itself on the largest pizza in its area. A competitor has released a 60 cm pizza, which is larger than the width of his pizza oven door, shown below. Lou thinks that if he tilts the pizza through the door, as shown, he can increase the size of his pizza. What size pizza can he make to the nearest cm, if he does so?



$h^2 = 390^2 - 72.5^2$

$h^2 = 146\,843.75$

$h = \sqrt{146\,843.75}$

$h = 383.20$ mm

$p^2 = 383.2^2 + 522.5^2$

$p^2 = 419\,848.49$

$p = \sqrt{419\,848.49}$

$p = 647.96$ mm

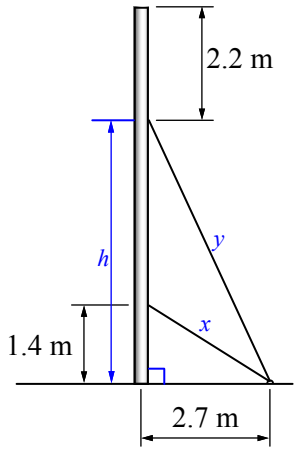
$x = (595 - 450) \div 2$

$x = 72.5$ mm

$y = 595 - 72.5 = 522.5$ mm

Largest pizza size is 64 cm

5 A flagpole shown below has a single guy wire attached to it 2.2 m from its tip. The 8 m guy wire is anchored 2.7 m from the base of the flagpole, loops through an eyelet and reconnects to the flagpole 1.4 m above the ground. Find the height of the flagpole.



$x^2 = 1.4^2 + 2.7^2$

$x^2 = 9.25$

$x = \sqrt{9.25}$

$x = 3.04$ m

$y = 8 - 3.04$

$y = 4.96$ m

$h^2 = 4.96^2 - 2.7^2$

$h^2 = 17.3116$

$h = \sqrt{17.3116}$

$h = 4.16$ m

Flagpole height = 4.16 + 2.2 = 6.36 m

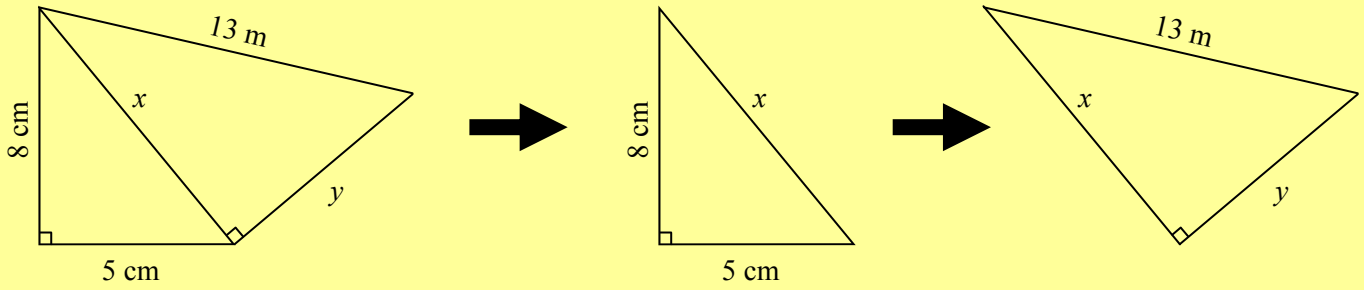
Pythagoras Theorem - Two Triangle Application

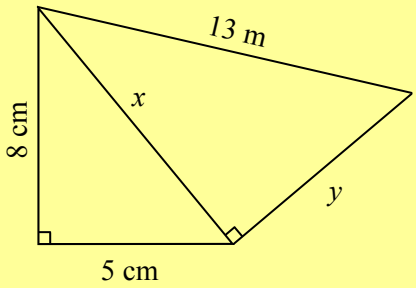
These questions require a double application of Pythagoras Theorem to solve. So apply P.T. to get your first answer, leave it as a surd for accuracy, then apply P.T. the second time.

Example - Find the length 'y'

First Application
Find x
(Leave as a surd)

Second Application
Use x and the other
given side to find y





First Application

$x^2 = 5^2 + 8^2$

$x^2 = 89$

$x = \sqrt{89}$ m

Leave this answer as a surd

Second Application

$y^2 = 13^2 - (\sqrt{89})^2$

$y^2 = 80$

$y = \sqrt{80}$

$y = 8.9$ m (1 d.p.)

Pythagoras Theorem - Two Triangle Application

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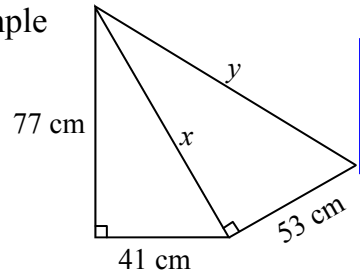
Use Pythagoras Theorem to find y to 1 d.p. by first finding x as a surd.

Example



The squaring of a square root simply removes the square root.
So $(\sqrt{7\,610})^2 = 7\,610$

$x^2 = 41^2 + 77^2$

$x^2 = 7\,610$

$x = \sqrt{7\,610}$ cm

Leave this answer as a surd

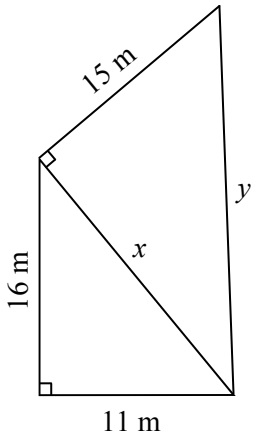
$y^2 = 53^2 + (\sqrt{7\,610})^2$

$y^2 = 10\,419$

$y = \sqrt{10\,419}$

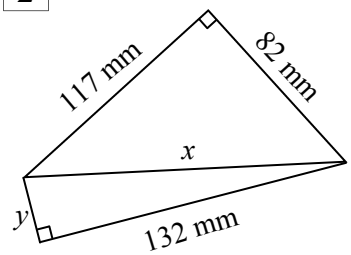
$y = 102.1$ cm

1



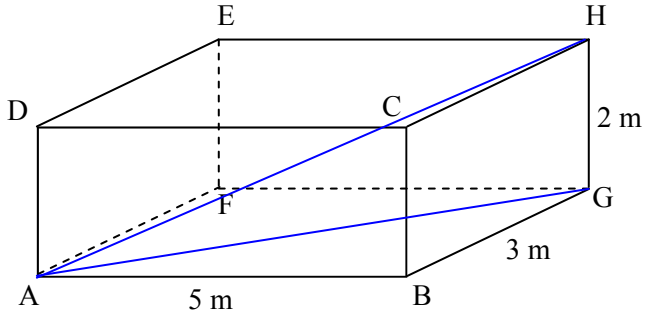
Leave this answer as a surd

2

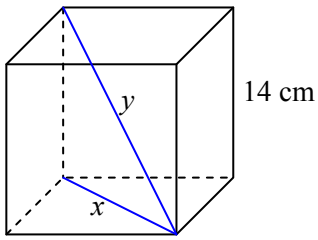


3 Calculate the perimeter of the shape above.

4 Find AG as a surd then calculate AH (1 d.p.)

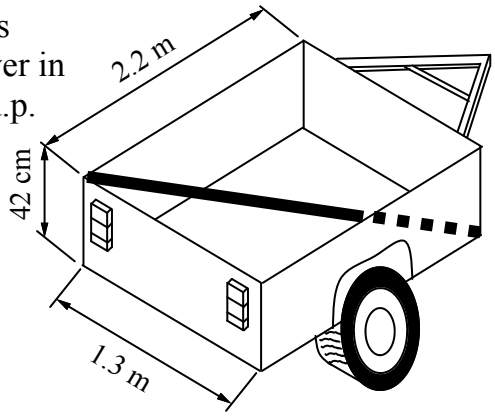


5 Find x as a surd then calculate y. (1 d.p.)



Cube, side length of 14 cm.

6 Find the maximum length iron bar that can fit in Tucky's trailer. Answer in metres to 2 d.p.



Pythagoras Theorem - Two Triangle Application

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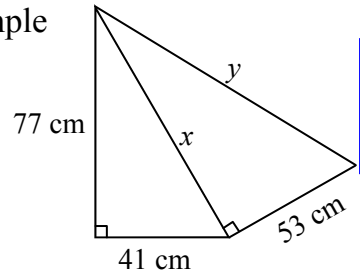
Use Pythagoras Theorem to find y to 1 d.p. by first finding x as a surd.



Example



The squaring of a square root simply removes the square root.
So $(\sqrt{7\,610})^2 = 7\,610$

$x^2 = 41^2 + 77^2$

$x^2 = 7\,610$

$x = \sqrt{7\,610}$ cm

Leave this answer as a surd

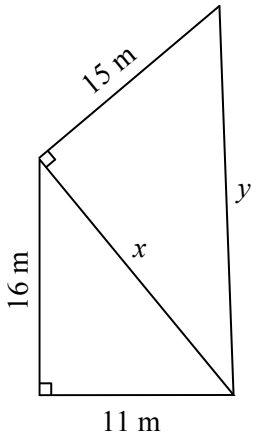
$y^2 = 53^2 + (\sqrt{7\,610})^2$

$y^2 = 10\,419$

$y = \sqrt{10\,419}$

$y = 102.1$ cm

1



$x^2 = 11^2 + 16^2$

$x^2 = 377$

$x = \sqrt{377}$ m

Leave this answer as a surd

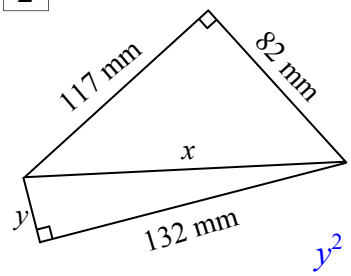
$y^2 = 15^2 + (\sqrt{377})^2$

$y^2 = 602$

$y = \sqrt{602}$

$y = 24.5$ m

2



$x^2 = 82^2 + 117^2$

$x^2 = 20\,413$

$x = \sqrt{20\,413}$ mm

$y^2 = (\sqrt{20\,413})^2 - 132^2$

$y^2 = 2\,989$

$y = \sqrt{2\,989}$

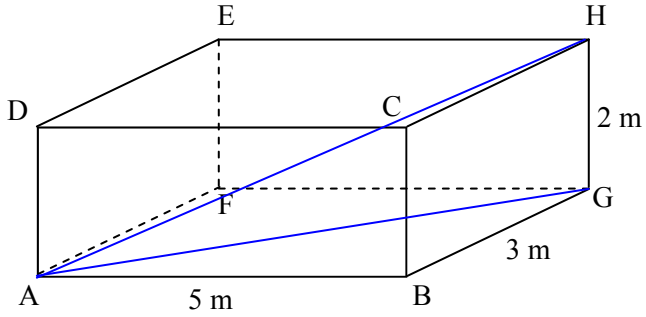
$y = 54.7$ mm

3 Calculate the perimeter of the shape above.

$P = 117 + 82 + 132 + 54.7$

$P = 385.7$ mm

4 Find AG as a surd then calculate AH (1 d.p.)



$AG^2 = 5^2 + 3^2$

$AG^2 = 34$

$AG = \sqrt{34}$ m

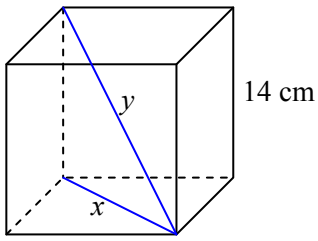
$AH^2 = 2^2 + (\sqrt{34})^2$

$AH^2 = 38$

$AH = \sqrt{38}$

$AH = 6.2$ m

5 Find x as a surd then calculate y. (1 d.p.)



Cube, side length of 14 cm.

$x^2 = 14^2 + 14^2$

$x^2 = 392$

$x = \sqrt{392}$ cm

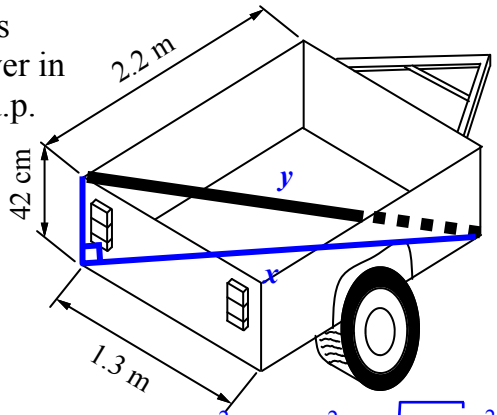
$y^2 = 14^2 + (\sqrt{392})^2$

$y^2 = 588$

$y = \sqrt{588}$

$y = 24.2$ cm

6 Find the maximum length iron bar that can fit in Tucky's trailer. Answer in metres to 2 d.p.



$x^2 = 1.3^2 + 2.2^2$

$x^2 = 6.53$

$x = \sqrt{6.53}$ m

$y^2 = 0.42^2 + (\sqrt{6.53})^2$

$y^2 = 6.7064$

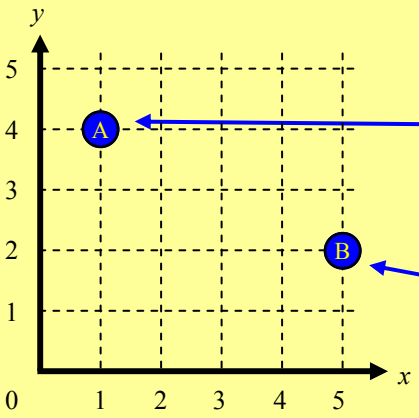
$y = \sqrt{6.7064}$

$y = 2.59$ m

Pythagoras Co-Ordinate Geometry Application - Positive Quadrant

This sheet uses Pythagoras Theorem to find the distance between two points. When two points are plotted on a cartesian plane a triangle can be created after a third point is plotted. The steps are shown below.

Example A (1, 4) and B (5, 2). Label right angle C.

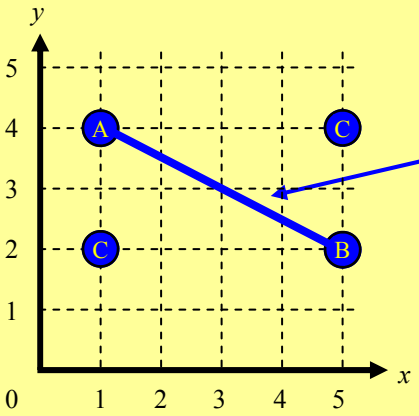


STEP 1

Plot point 'A', from the origin (0,0) move across 1 and up 4. Plot the point with an 'A' inside.

STEP 2

Plot point 'B', from the origin (0,0) move across 5 and up 2. Plot the point with an 'B' inside.

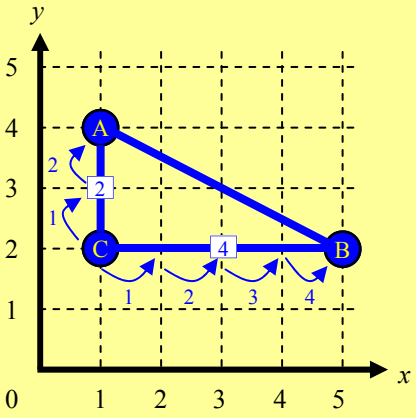


STEP 3

Draw a line between the two points

STEP 4

Now plot point 'C' to form a right-angle triangle. It can be in either of the two positions shown. Not both.



STEP 5

Construct the two short sides of the Triangle.

STEP 6

Count off the lengths to find the length of both of the short sides. In the example one side is 2 units, the other is 4 units.

STEP 7

Now apply Pythagoras Theorem to the triangle to find the hypotenuse. Answer to 2 d.p. and the length is in 'units'.

AC = 2

CB = 4

$AB^2 = 2^2 + 4^2$

$AB^2 = 20$

$AB = \sqrt{20}$

AB = 4.47 units (2 d.p.)

Pythagoras Co-Ordinate Geometry Application - Positive Quadrant

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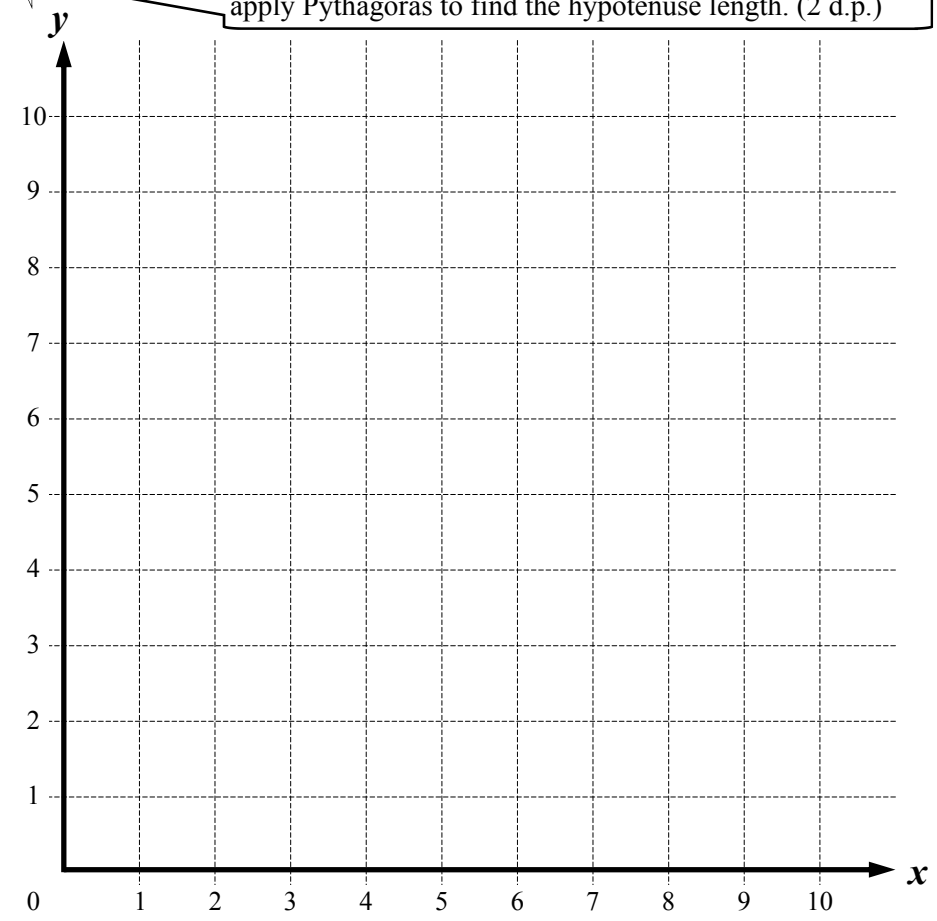
Plot the two points on the graph and draw a line between them, this is the hypotenuse. Draw the two short sides then apply Pythagoras to find the hypotenuse length. (2 d.p.)



1 A (4, 3) and B (6, 7).
Label right angle C.

AC = ☐ BC = ☐

$AB^2 = \square^2 + \square^2$

$AB^2 =$

$AB = \sqrt{}$

$AB =$

2 K (10, 0) and L (9, 10).
Label right angle J.

JK = ☐ JL = ☐

3 S (1, 8) and R (6, 10).
Label right angle T.

TR = ☐ TS = ☐

4 V (6, 1) and X (8, 0).
Label right angle Y.

YV = ☐ YX = ☐

5 F (0, 0) and T (5, 9).
Label right angle N.

NF = ☐ NT = ☐

6 U (2, 2) and K (10, 5).
Label right angle V.

VU = ☐ VK = ☐

7 Z (1, 7) and A (4, 1).
Label right angle K.

KZ = ☐ KA = ☐

8 P (0, 5) and V (3, 7).
Label right angle E.

EP = ☐ EV = ☐

Pythagoras Co-Ordinate Geometry Application - Positive Quadrant

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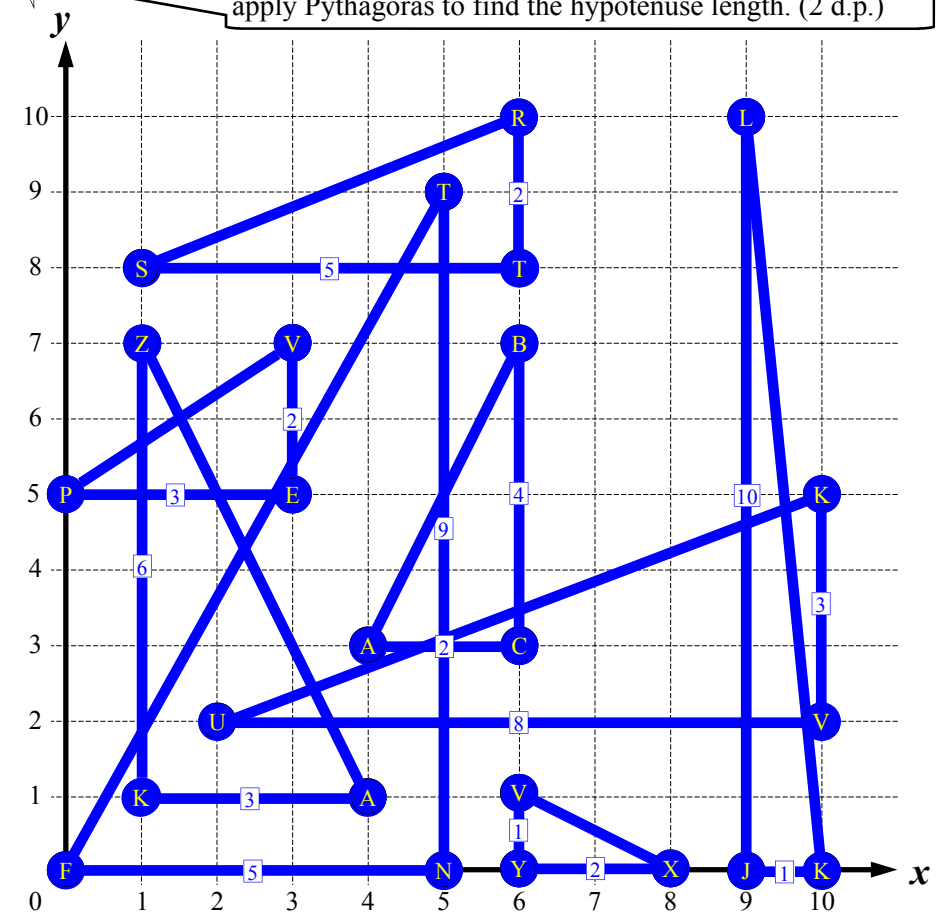
Plot the two points on the graph and draw a line between them, this is the hypotenuse. Draw the two short sides then apply Pythagoras to find the hypotenuse length. (2 d.p.)



1 A (4, 3) and B (6, 7). Label right angle C.

AC = 2 BC = 4

$AB^2 = 2^2 + 4^2$

$AB^2 = 20$

$AB = \sqrt{20}$

AB = 4.47 units

2 K (10, 0) and L (9, 10). Label right angle J.

JK = 1 JL = 10

$KL^2 = 1^2 + 10^2$

$KL^2 = 101$

$KL = \sqrt{101}$

KL = 10.05 units

3 S (1, 8) and R (6, 10). Label right angle T.

TR = 2 TS = 5

$SR^2 = 2^2 + 5^2$

$SR^2 = 29$

$SR = \sqrt{29}$

SR = 5.39 units

4 V (6, 1) and X (8, 0). Label right angle Y.

YV = 1 YX = 2

$VX^2 = 1^2 + 2^2$

$VX^2 = 5$

$VX = \sqrt{5}$

VX = 2.24 units

5 F (0, 0) and T (5, 9). Label right angle N.

NF = 5 NT = 9

$FT^2 = 5^2 + 9^2$

$FT^2 = 106$

$FT = \sqrt{106}$

FT = 10.30 units

6 U (2, 2) and K (10, 5). Label right angle V.

VU = 8 VK = 3

$UK^2 = 8^2 + 3^2$

$UK^2 = 73$

$UK = \sqrt{73}$

UK = 8.54 units

7 Z (1, 7) and A (4, 1). Label right angle K.

KZ = 6 KA = 3

$AZ^2 = 6^2 + 3^2$

$AZ^2 = 45$

$AZ = \sqrt{45}$

AZ = 6.71 units

8 P (0, 5) and V (3, 7). Label right angle E.

EP = 3 EV = 2

$PV^2 = 3^2 + 2^2$

$PV^2 = 13$

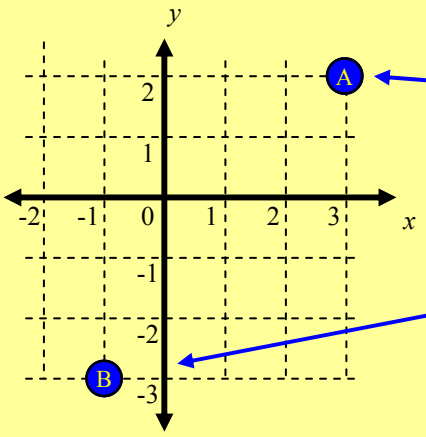
$PV = \sqrt{13}$

PV = 3.61 units

Pythagoras Co-Ordinate Geometry Application - All Quadrants

This sheet uses Pythagoras Theorem to find the distance between two points. When two points are plotted on a cartesian plane a triangle can be created after a third point is plotted. The steps are shown below.

Example A (3, 2) and B (-1, -3). Label right angle C.

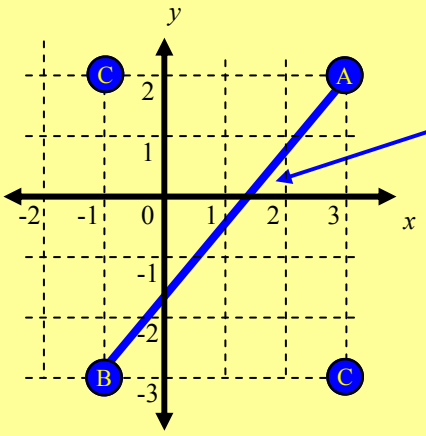


STEP 1

Plot point 'A', from the origin (0,0) move right, across 3, then up 2. Plot the point with an 'A' inside.

STEP 2

Plot point 'B', from the origin (0,0) move left across 1, to -1, and then down 3, to -3. Plot the point with an 'B' inside.

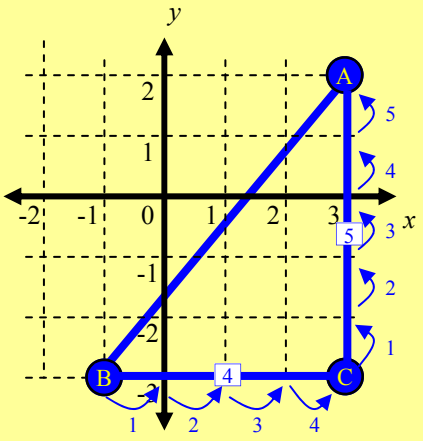


STEP 3

Draw a line between the two points

STEP 4

Now plot point 'C' to form a right-angle triangle. It can be in either of the two positions shown. Not both.



STEP 5

Construct the two short sides of the Triangle.

STEP 6

Count off the lengths to find the length of both of the short sides. In the example one side is 5 units, the other is 4 units.

STEP 7

Now apply Pythagoras Theorem to the triangle to find the hypotenuse. Answer to 2 d.p. and the length is in 'units'.

AC = 5

CB = 4

$AB^2 = 5^2 + 4^2$

$AB^2 = 41$

$AB = \sqrt{41}$

AB = 6.40 units (2 d.p.)

Pythagoras Co-Ordinate Geometry Application - All Quadrants

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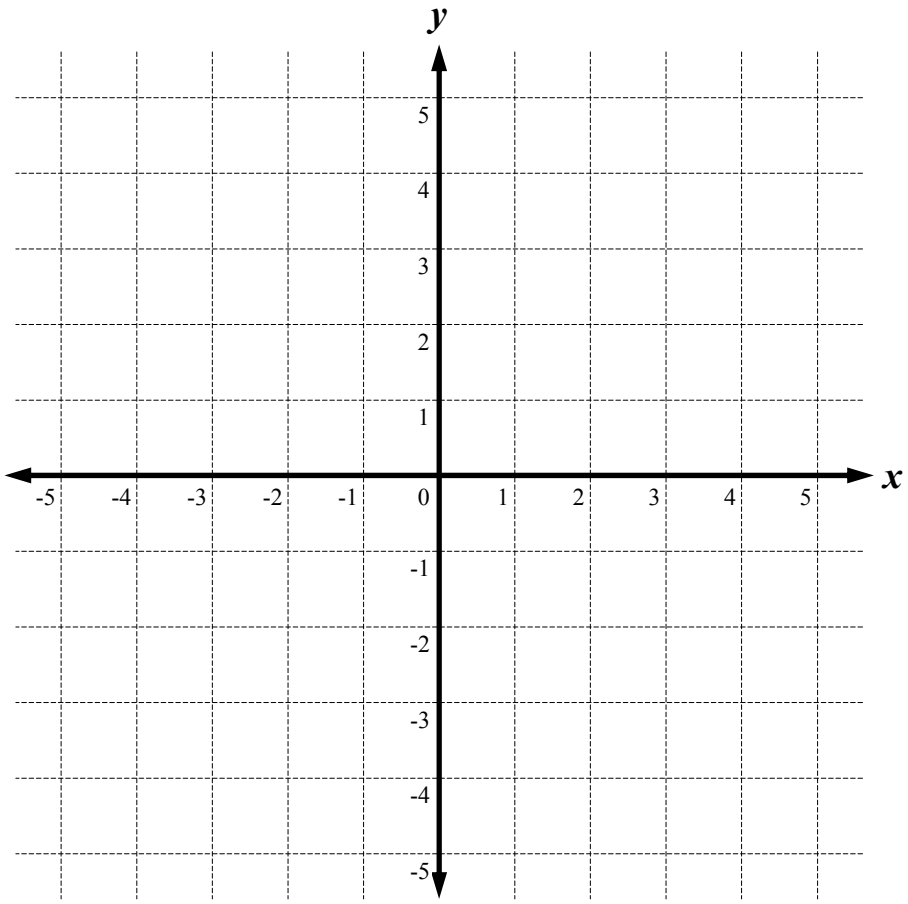
Plot the two points on the graph and draw a line between them, this is the hypotenuse. Draw the two short sides then apply Pythagoras to find the hypotenuse length. (2 d.p.)



1 A (-5, 4) and B (-3, -2). Label right angle C.

AC = ☐ BC = ☐

$AB^2 =$ ☐$^2 +$ ☐^{2}

$AB^2 =$

$AB = \sqrt{}$

AB =

2 K (-3, 3) and L (4, 5). Label right angle J.

JK = ☐ JL = ☐

3 S (-2, -4) and R (2, 2). Label right angle T.

TR = ☐ TS = ☐

4 V (-5, -4) and X (-3, -3). Label right angle Y.

YV = ☐ YX = ☐

5 F (1, 5) and T (5, -3). Label right angle N.

NF = ☐ NT = ☐

6 U (0, 5) and K (-3, -5). Label right angle V.

VU = ☐ VK = ☐

7 K (4, -2) and T (2, -5). Label right angle D.

KD = ☐ TD = ☐

8 P (-2, 2) and V (3, -1). Label right angle E.

EP = ☐ EV = ☐

Pythagoras Co-Ordinate Geometry Application - All Quadrants

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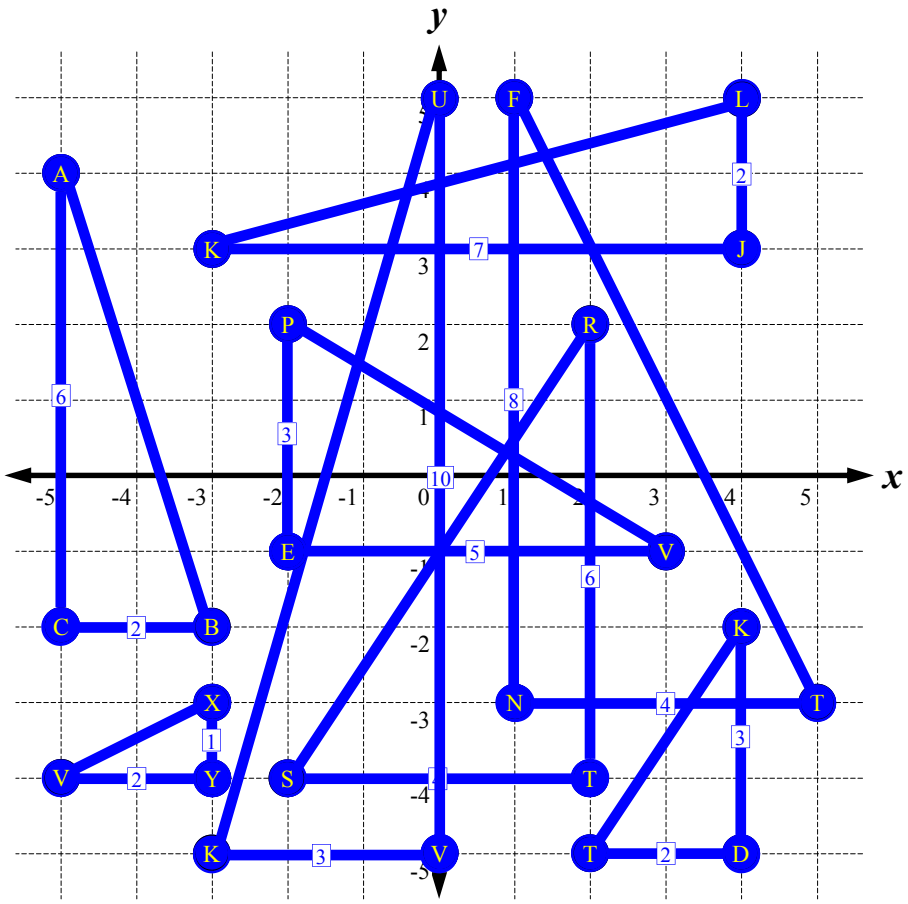
Plot the two points on the graph and draw a line between them, this is the hypotenuse. Draw the two short sides then apply Pythagoras to find the hypotenuse length. (2 d.p.)



1 A (-5, 4) and B (-3, -2). Label right angle C.

AC = 6 BC = 2

$AB^2 = 6^2 + 2^2$

$AB^2 = 40$

$AB = \sqrt{40}$

AB = 6.32 units

2 K (-3, 3) and L (4, 5). Label right angle J.

JK = 7 JL = 2

$KL^2 = 7^2 + 2^2$

$KL^2 = 53$

$KL = \sqrt{53}$

KL = 7.28 units

3 S (-2, -4) and R (2, 2). Label right angle T.

TR = 6 TS = 4

$SR^2 = 6^2 + 4^2$

$SR^2 = 52$

$SR = \sqrt{52}$

SR = 7.21 units

4 V (-5, -4) and X (-3, -3). Label right angle Y.

YV = 2 YX = 1

$VX^2 = 2^2 + 1^2$

$VX^2 = 5$

$VX = \sqrt{5}$

VX = 2.24 units

5 F (1, 5) and T (5, -3). Label right angle N.

NF = 8 NT = 4

$FT^2 = 8^2 + 4^2$

$FT^2 = 80$

$FT = \sqrt{80}$

FT = 8.94 units

6 U (0, 5) and K (-3, -5). Label right angle V.

VU = 10 VK = 3

$UK^2 = 10^2 + 3^2$

$UK^2 = 109$

$UK = \sqrt{109}$

UK = 10.44 units

7 K (4, -2) and T (2, -5). Label right angle D.

KD = 3 TD = 2

$KT^2 = 3^2 + 2^2$

$KT^2 = 13$

$KT = \sqrt{13}$

KT = 3.61 units

8 P (-2, 2) and V (3, -1). Label right angle E.

EP = 3 EV = 5

$PV^2 = 3^2 + 5^2$

$PV^2 = 34$

$PV = \sqrt{34}$

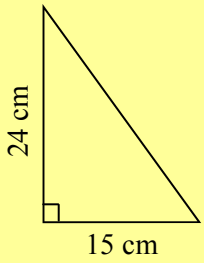
PV = 5.83 units

Pythagoras - Area and Perimeter Applications

Perimeter is measurement of distance, the distance around a shape. So to find the perimeter of a triangle you add all the side lengths together. An example is at the top of column 1 of the worksheet.

The area of a triangle is found by $A = \frac{1}{2}bh$ which is 'a half times the base times the height'. Note that ½ = 0.5, it may be easier to type '0.5' into your calculator, also note that the 'base' isn't necessarily on the bottom of the triangle, due to rotation. Below is an example of finding the area of a triangle.

Example - Area of a Triangle



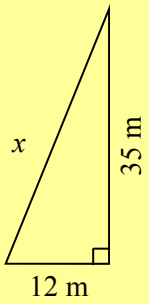
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 15 \times 24$

$A = 180 \text{ cm}^2$

Column 2 is all calculating the perimeter of a triangle. Note that you are given two sides only, but to calculate perimeter you need to know all three sides. So first calculate the unknown side using P.T., once you have calculated the side, add all three sides together. That gives you the perimeter. Remember that you may have subtraction or addition using Pythagoras, it depends if you are finding the length of the hypotenuse (addition or +) or finding a short side (subtraction or -). The first question reminds you of the process but there is also an example below.

Example - Using P.T. then finding the perimeter.



$x^2 = 35^2 + 12^2$

$x^2 = 1\,369$

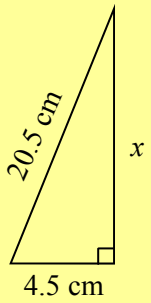
$x = \sqrt{1\,369}$

$x = 37 \text{ m}$

$P = 12 + 35 + 37 = 84 \text{ m}$

Column 3 deals with finding the area of a triangle. Again, like above you will need to use P.T. first to calculate the unknown side. Then use the area formula: $A = \frac{1}{2}bh$.

Example - Using P.T. then finding the area.



$x^2 = 20.5^2 - 4.5^2$

$x^2 = 400$

$x = \sqrt{400}$

$x = 20 \text{ cm}$

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4.5 \times 20$

$A = 45 \text{ cm}^2$

Pythagoras - Area and Perimeter Applications

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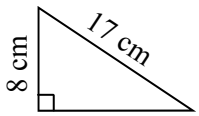
Find the perimeter for these triangles.

Example

Add all the sides

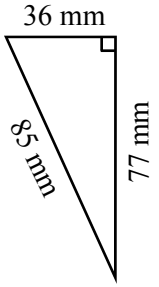


$P = 8 + 15 + 17$

$P = 40$ cm

Remember the units

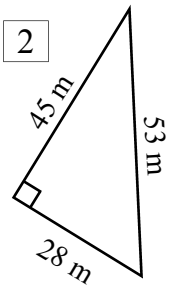
1



$P =$

$P =$ mm

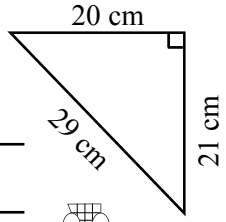
2



$P =$

$P =$

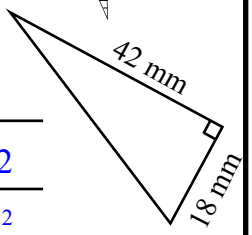
3



Now find the area.

Remember ½ = 0.5

4

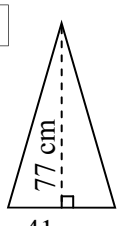


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 42$

$A =$ mm^2

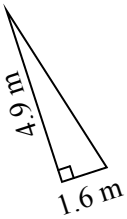
5



$A =$

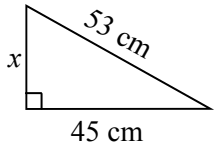
$A =$ cm^2

6



Calculate the missing side using Pythagoras Theorem then find the perimeter.

7



$x^2 =$

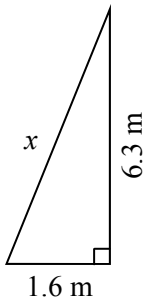
$x^2 =$

$x = \sqrt{}$

$x =$

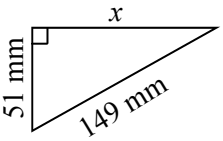
$P =$ + + = cm

8



$P =$

9



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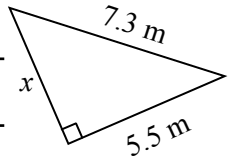
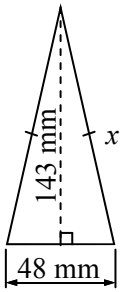
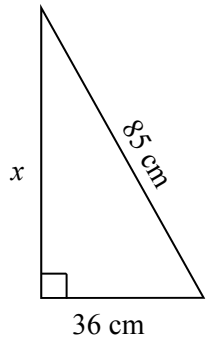


11



Now find the area. You need the base and the perpendicular height.

12



$x^2 =$

$x^2 =$

$x =$

$x =$

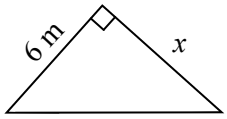
$A = \frac{1}{2}bh$

$= \times \times$

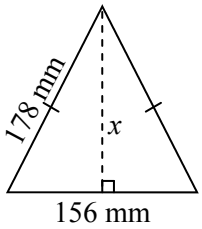
$A =$ cm^2

Area is measured in square units

13



14



Pythagoras - Area and Perimeter Applications

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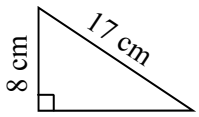
Find the perimeter for these triangles.

Example



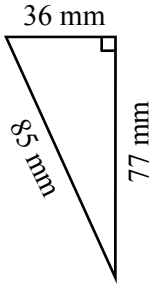
Add all the sides

$P = 8 + 15 + 17$

$P = 40$ cm

Remember the units

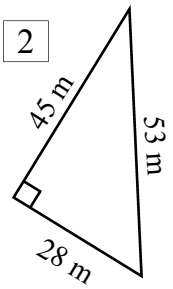
1



$P = 36 + 77 + 85$

$P = 198$ mm

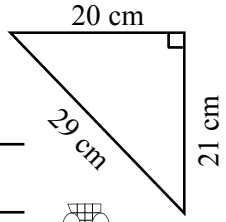
2



$P = 45 + 28 + 53$

$P = 126$ m

3



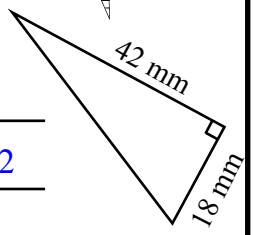
$P = 20 + 21 + 29$

$P = 70$ cm

Now find the area.



4 Remember ½ = 0.5

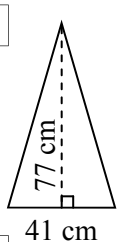


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 42$

$A = 378$ mm^2

5

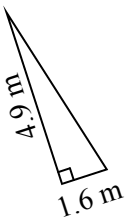


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 41 \times 77$

$A = 1\,578.5$ cm^2

6



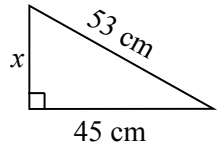
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 1.6 \times 4.9$

$A = 3.92$ m^2

Calculate the missing side using Pythagoras Theorem then find the perimeter.

7



$x^2 = 53^2 - 45^2$

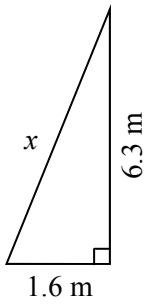
$x^2 = 784$

$x = \sqrt{784}$

$x = 28$ cm

$P = 28 + 45 + 53 = 126$ cm

8



$x^2 = 1.6^2 + 6.3^2$

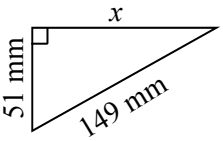
$x^2 = 42.25$

$x = \sqrt{42.25}$

$x = 6.5$ m

$P = 1.6 + 6.3 + 6.5 = 14.4$ m

9



$x^2 = 149^2 - 51^2$

$x^2 = 19\,600$

$x = \sqrt{19\,600}$

$x = 140$ mm

$P = 51 + 140 + 149 = 340$ mm

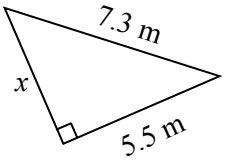
10

$x^2 = 7.3^2 - 5.5^2$

$x^2 = 23.04$

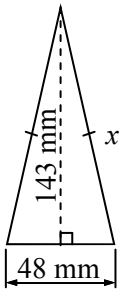
$x = \sqrt{23.04}$

$x = 4.8$ m



$P = 5.5 + 7.3 + 4.8 = 17.6$ m

11



$x^2 = 24^2 + 143^2$

$x^2 = 21\,025$

$x = \sqrt{21\,025}$

$x = 145$ mm

$P = 48 + 2 \times 145 = 338$ mm

Now find the area. You need the base and the perpendicular height.

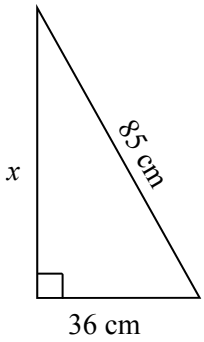
12

$x^2 = 85^2 - 36^2$

$x^2 = 5\,929$

$x = \sqrt{5\,929}$

$x = 77$ cm



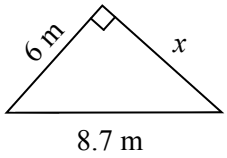
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 36 \times 77$

$A = 1\,386$ cm^2

Area is measured in square units

13



$x^2 = 8.7^2 - 6^2$

$x^2 = 39.69$

$x = \sqrt{39.69}$

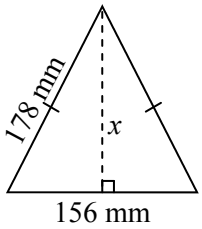
$x = 6.3$ m

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 6.3$

$A = 18.9$ cm^2

14



$x^2 = 178^2 - 78^2$

$x^2 = 25\,600$

$x = \sqrt{25\,600}$

$x = 160$ mm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 156 \times 160$

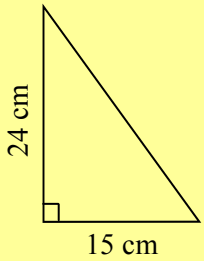
$A = 12\,480$ mm^2

Pythagoras - Further Area and Perimeter Applications

Perimeter is measurement of distance, the distance around a shape. So to find the perimeter of a triangle you add all the side lengths together. This sheet features problems which will require subtraction or some other calculation before applying Pythagoras Theorem. The questions can be challenging and so the previous 'Perimeter and Area' sheet should b attempted first.

The area of a triangle is found by $A = \frac{1}{2}bh$ which is 'a half times the base times the height'. Note that ½ = 0.5, it may be easier to type '0.5' into your calculator, also note that the 'base' isn't necessarily on the bottom of the triangle, due to rotation. Below is an example of finding the area of a triangle.

Example - Area of a Triangle



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 15 \times 24$

$A = 180 \text{ cm}^2$

Column 1 is calculating the perimeter of a shape. These are composite shapes which require you to calculate either two or three unknown lengths before you can find the perimeter.

Column 2 and Column 3 are area questions, calculate all required sides first, then calculate the two areas individually and add them together. Question 6 is a subtraction question, with two identical triangles being removed from a rectangle to form the composite shape.

Pythagoras - Further Area and Perimeter Applications

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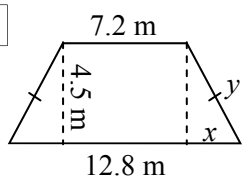
Find the missing side length(s) then find the perimeter in metres.

1



Find x first then use P.T. to find y

$x = \dfrac{\text{-}}{2}$ $\quad y^2 =$

$y^2 =$

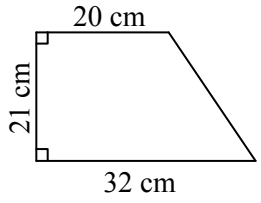
$x =$ m $\quad y = \sqrt{}$

$y =$

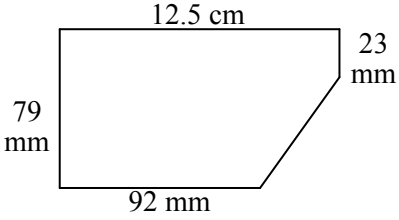
P = + + 2 ×

P = m

2

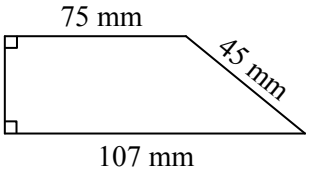


3



Now find the area. Answer to 1 d.p. when required.

4



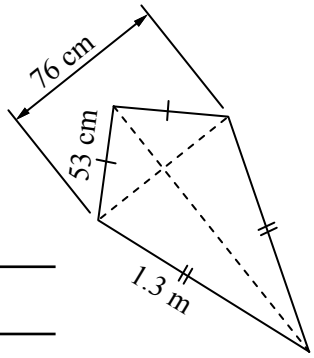
$A_1 = lb$ $\quad A_2 = \frac{1}{2}bh$

$=$ $\quad =$

$A_1 =$ mm^2 $\quad A_2 =$ mm^2

$A_T =$ + = mm^2

5



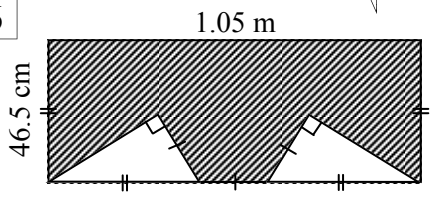
$A_1 =$ $\quad A_2 =$

$A_1 =$ $\quad A_2 =$

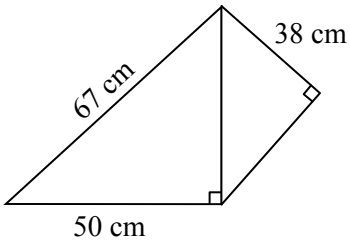
$A_T =$

Find the area, shaded area in Q6, to 1 d.p.

6



7



Pythagoras - Further Area and Perimeter Applications

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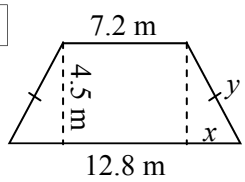
Find the missing side length(s) then find the perimeter in metres.

1



Find x first then use P.T. to find y

$x = \dfrac{12.8 - 7.2}{2}$

$x = 2.8$ m

$y^2 = 2.8^2 + 4.5^2$

$y^2 = 28.09$

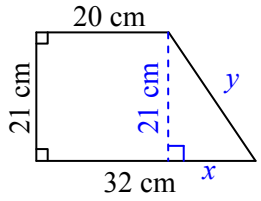
$y = \sqrt{28.09}$

$y = 5.3$ m

$P = 12.8 + 7.2 + 2 \times 5.3$

$P = 30.6$ m

2



$x = 32 - 20$

$x = 12$ cm

$y^2 = 12^2 + 21^2$

$y^2 = 585$

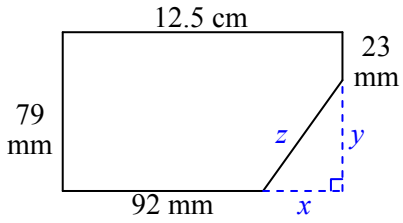
$y = \sqrt{585}$

$y = 24.2$ m

$P = 20 + 24.2 + 32 + 21$

$P = 97.2$ cm $= 0.972$ m

3



$x = 125 - 92$

$x = 33$ mm

$z^2 = 33^2 + 56^2$

$z^2 = 4\,225$

$z = \sqrt{4\,225}$

$z = 65$ mm

$y = 79 - 23$

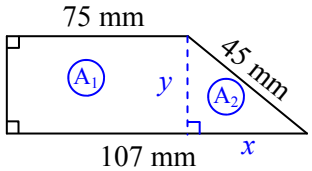
$y = 56$ mm

$P = 79 + 125 + 23 + 65 + 92$

$P = 384$ mm $= 0.384$ m

Now find the area. Answer to 1 d.p. when required.

4



$x = 107 - 75$

$x = 32$ mm

$y^2 = 45^2 - 32^2$

$y^2 = 1\,001$

$y = \sqrt{1\,001}$

$y = 31.6$ mm

$A_1 = lb$

$= 75 \times 31.6$

$A_1 = 2\,370$ mm^2

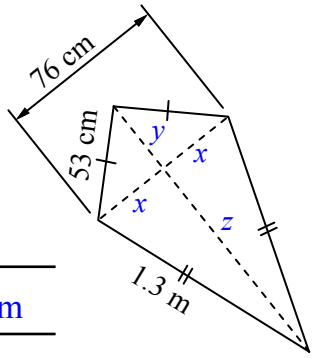
$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 32 \times 31.6$

$A_2 = 505.6$ mm^2

$A_T = 2\,370 + 505.6 = 2875.6$ mm^2

5



$2x = 76$

$x = 38$ cm

$y^2 = 53^2 - 38^2$

$y^2 = 1\,365$

$y = \sqrt{1\,365}$

$y = 36.9$ cm

$z^2 = 130^2 - 38^2$

$z^2 = 15\,456$

$z = \sqrt{15\,456}$

$z = 124.3$ cm

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 76 \times 36.9$

$A_1 = 1\,402.2$ cm^2

$A_2 = \frac{1}{2}bh$

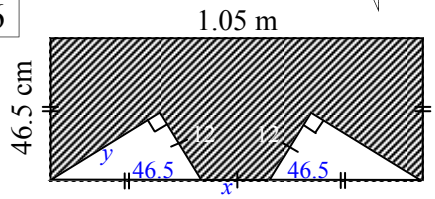
$= \frac{1}{2} \times 76 \times 124.3$

$A_2 = 4\,723.4$ cm^2

$A_T = 1402.2 + 4\,723.4 = 6\,125.6$ cm^2

Find the area, shaded area in Q6, to 1 d.p.

6



$x = 105 - 2 \times 46.5$

$x = 12$ cm

$y^2 = 46.5^2 - 12^2$

$y^2 = 2\,018.25$

$y = \sqrt{2\,018.5}$

$y = 44.9$ cm

$A_1 = lb$

$= 105 \times 46.5$

$A_1 = 4\,882.5$ cm^2

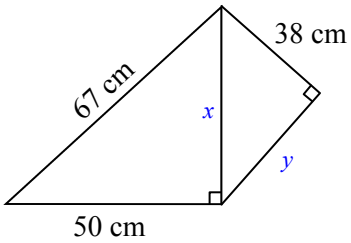
$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 44.9 \times 12$

$A_2 = 269.4$ cm^2

$A_S = 4\,882.5 - 2 \times 269.4 = 4343.7$ cm^2

7



$x^2 = 67^2 - 50^2$

$x^2 = 1\,989$

$x = \sqrt{1\,989}$

$x = 44.6$ cm

$y^2 = 44.6^2 - 38^2$

$y^2 = 545.16$

$y = \sqrt{545.16}$

$y = 23.3$ cm

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 50 \times 44.6$

$A_1 = 1\,115$ cm^2

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 38 \times 23.3$

$A_2 = 442.7$ cm^2

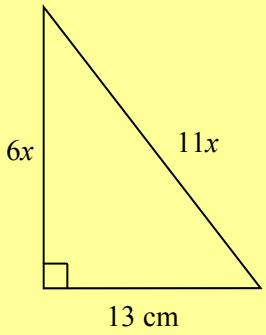
$A_T = 1\,115 + 442.7 = 1\,557.7$ cm^2

Pythagoras Theorem - Algebra Extension

This is an extension and should only be attempted once the core of the Pythagoras Theorem chapter has been completed.

This worksheet involves finding two unknown sides on a right-angled triangle. The unknown sides have ratio applied to them, such that one is a multiple of the other. In other words if one is 'x' then the other may be '$5x$'. So one side is 5 times longer than the other.

Example



$(6x)^2 + 13^2 = (11x)^2$ — Write the Pythagorean Result for the Triangle, it can be: $a^2 + b^2 = c^2$ Or the reverse: $c^2 = a^2 + b^2$

$36x^2 + 169 = 121x^2$ — Simplify the equation.

$-36x^2$ $-36x^2$

$169 = 85x^2$ — Get the x^2 term on one side. In this case by $-36x^2$ from each side.

$85x^2 = 169$ — Swap the equation so that the x^2 is on the left hand side.

÷ 85 ÷ 85

$x^2 = 19.882\ldots$(calc) — Divide by 85 to get x^2 on its own.

$\sqrt{}$ $\sqrt{}$

$x = 1.41$ cm (2 d.p.) — Square root to find x.

$6x = 8.46$ cm (2 d.p.)

$11x = 15.51$ cm (2 d.p.)

Once you have x, you can multiply by 6 to get a value for $6x$ and multiply by 11 to get $11x$.

FINISHED!!!! ….. Not quite. This is a complex process and you may have made a mistake along the way, so to ensure you have found the correct answer, do a quick check to see that P.T. applies.

$15.51^2 = 13^2 + 8.46^2$

$240.5601 = 240.5716$

LHS ≈ RHS

Note that answers aren't exact due to the rounding that has taken place, but they are close enough to verify that the solution is correct.

The left hand side of the equation is approximately equal to the right hand side of the equation.

Pythagoras Theorem - Algebra Extension

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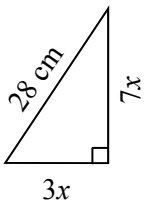
Find the side lengths of these triangles. (2 d.p.)

Example



$(3x)^2 + (7x)^2 = 28^2$

$9x^2 + 49x^2 = 784$

$58x^2 = 784$

$\div 58$ $\div 58$

$x^2 = 13.517\ldots$(calc)

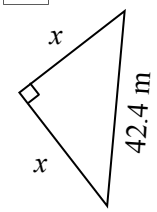
$\sqrt{\ }$ $\sqrt{\ }$

$x = 3.68$ cm (2 d.p.)

$3x = 11.04$ cm (2 d.p.)

$7x = 25.76$ cm (2 d.p.)

1



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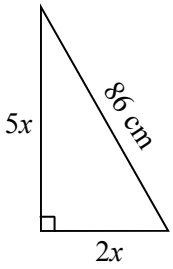

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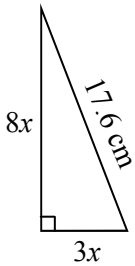


Now answer these problems. Answer to 2 d.p.

4 Jason walked North x km. At the same time Amy walked East twice as far, leaving from the same point. If they are 3.2 km apart, sketch a diagram and find the distance each has walked.

5 Find the width and height of a 106 cm TV, given it has a ratio of 16:9. TV size is the diagonal length across the screen.

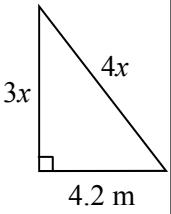
H: W:

6 Repeat for a 120 cm screen.

H: W:

Find the side lengths of these triangles. (2 d.p.)

7



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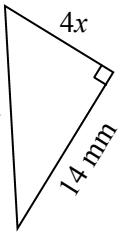

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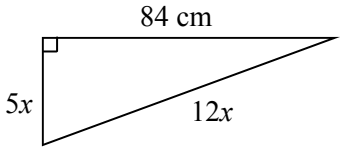

Pythagoras Theorem - Algebra Extension

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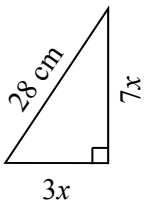
Find the side lengths of these triangles. (2 d.p.)

Example



$(3x)^2 + (7x)^2 = 28^2$

$9x^2 + 49x^2 = 784$

$58x^2 = 784$

$\div 58$ $\div 58$

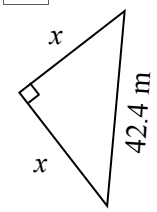
$x^2 = 13.517\ldots$(calc)

$\sqrt{\ }$ $\sqrt{\ }$

$x = 3.68$ cm (2 d.p.)

$3x = 11.04$ cm $7x = 25.76$ cm

1



$x^2 + x^2 = 42.4^2$

$2x^2 = 1\ 797.76$

$x^2 = 898.88$

$x = 29.98$ cm (2 d.p.)

2

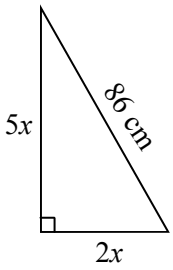
$(2x)^2 + (5x)^2 = 86^2$

$4x^2 + 25x^2 = 7\ 396$

$29x^2 = 7\ 396$

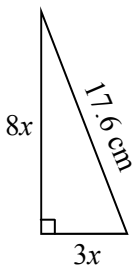
$x^2 = 255.03$ .. (calc)

$x = 15.97$ cm (2 d.p.)



$2x = 31.94$ cm $5x = 25.76$ cm

3



$(3x)^2 + (8x)^2 = 17.6^2$

$9x^2 + 64x^2 = 309.76$

$73x^2 = 309.76$

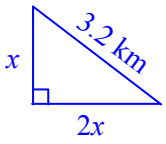
$x^2 = 4.243\ldots$ (calc)

$x = 2.06$ cm (2 d.p.)

$3x = 6.18$ cm $8x = 16.48$ cm

Now answer these problems. Answer to 2 d.p.

4 Jason walked North x km. At the same time Amy walked East twice as far, leaving from the same point. If they are 3.2 km apart, sketch a diagram and find the distance each has walked.



$(x)^2 + (2x)^2 = 3.2^2$

$x^2 + 4x^2 = 10.24$

$5x^2 = 10.24$

$x^2 = 2.048$

$x = 1.43$ km (2 d.p.)

Jason's walk is 1.43 km.

Amy's walk is 2.86 km.

5 Find the width and height of a 106 cm TV, given it has a ratio of 16:9. TV size is the diagonal length across the screen.

$(9x)^2 + (16x)^2 = 106^2$

$81x^2 + 256x^2 = 11\ 236$

$337x^2 = 11\ 236$

$x^2 = 33.341\ldots$ (calc)

$x = 5.77$ cm (2 d.p.)

H: 51.93 cm W: 92.32 cm

6 Repeat for a 120 cm screen.

$(9x)^2 + (16x)^2 = 120^2$

$81x^2 + 256x^2 = 14\ 400$

$337x^2 = 14\ 400$

$x^2 = 42.729\ldots$ (calc)

$x = 6.54$ cm (2 d.p.)

H: 58.86 cm W: 104.64 cm

Find the side lengths of these triangles. (2 d.p.)

7

$(3x)^2 + 4.2^2 = (4x)^2$

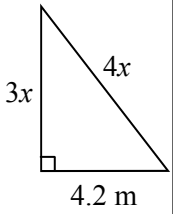
$9x^2 + 17.62 = 16x^2$

$17.62 = 7x^2$

$7x^2 = 17.62$

$x^2 = 2.517\ldots$(calc)

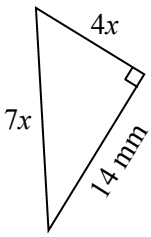
$x = 1.59$ m (2 d.p.)



$3x = 4.77$ m

$4x = 6.36$ m

8



$(4x)^2 + 14^2 = (7x)^2$

$16x^2 + 196 = 49x^2$

$196 = 33x^2$

$33x^2 = 196$

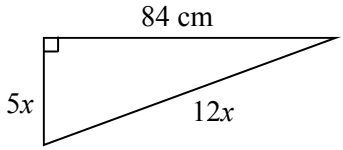
$x^2 = 5.939\ldots$(calc)

$x = 2.44$ mm (2 d.p.)

$4x = 9.76$ mm

$7x = 17.08$ mm

9



$(5x)^2 + 84^2 = (12x)^2$

$25x^2 + 7\ 056 = 144x^2$

$7\ 056 = 119x^2$

$119x^2 = 7\ 056$

$x^2 = 59.294\ldots$(calc)

$x = 7.70$ cm (2 d.p.)

$5x = 38.5$ cm

$12x = 92.40$ cm

Pythagoras Theorem - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don't colour it in too dark (using a texta for example) so that you can still read it.

Pythagoras Theorem - Find a Word

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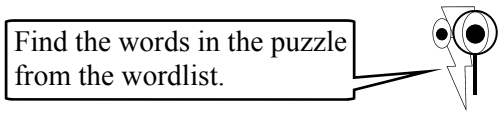

A	D	D	I	T	I	O	N	L	A	C	I	D	A	R
D	D	R	O	U	N	D	I	N	G	Y	P	D	D	I
I	P	P	X	A	D	D	E	C	I	M	A	L	D	G
L	Y	E	T	A	N	I	D	R	O	O	C	E	E	H
A	T	R	R	U	Q	S	E	L	G	N	A	I	R	T
R	H	I	M	I	B	E	M	L	A	N	N	C	U	A
E	A	M	E	X	M	L	U	C	D	B	N	K	S	N
T	G	E	R	U	L	E	N	G	T	H	E	E	A	G
A	O	T	O	E	Q	C	T	A	U	T	R	L	E	L
L	R	E	E	B	W	S	I	E	T	A	R	O	M	E
I	A	R	H	T	X	O	S	Y	U	S	T	O	O	R
U	S	Y	T	U	K	S	P	Q	X	B	T	O	H	P
Q	U	X	K	E	T	I	S	O	P	P	O	I	R	S
E	Y	E	S	U	N	E	T	O	P	Y	H	R	N	T
S	I	D	E	T	C	A	R	T	B	U	S	R	O	U

WORDLIST

LENGTH	RIGHTANGLE	DECIMAL	ISOSCELES	RADICAL
SIDE	COORDINATE	UNITS	POWER	ADDITION
TRIANGLE	EQUILATERAL	PERIMETER	ROUNDING	LABEL
SQUARE	PYTHAGORAS	OPPOSITE	ROOT	THEOREM
HYPOTENUSE	SUBTRACT	SHORT	MEASURE	AREA

Pythagoras Theorem - Find a Word

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Find the words in the puzzle from the wordlist.

A	D	D	I	T	I	O	N	L	A	C	I	D	A	R
D	D	R	O	U	N	D	I	N	G	Y	P	D	D	I
I	P	P	X	A	D	D	E	C	I	M	A	L	D	G
L	Y	E	T	A	N	I	D	R	O	O	C	E	E	H
A	T	R	R	U	Q	S	E	L	G	N	A	I	R	T
R	H	I	M	I	B	E	M	L	A	N	N	C	U	A
E	A	M	E	X	M	L	U	C	D	B	N	K	S	N
T	G	E	R	U	L	E	N	G	T	H	E	E	A	G
A	O	T	O	E	Q	C	T	A	U	T	R	L	E	L
L	R	E	E	B	W	S	I	E	T	A	R	O	M	E
I	A	R	H	T	X	O	S	Y	U	S	T	O	O	R
U	S	Y	T	U	K	S	P	Q	X	B	T	O	H	P
Q	U	X	K	E	T	I	S	O	P	P	O	I	R	S
E	Y	E	S	U	N	E	T	O	P	Y	H	R	N	T
S	I	D	E	T	C	A	R	T	B	U	S	R	O	U

WORDLIST

LENGTH
SIDE
TRIANGLE
SQUARE
HYPOTENUSE
RIGHTANGLE
COORDINATE
EQUILATERAL
PYTHAGORAS
SUBTRACT
DECIMAL
UNITS
PERIMETER
OPPOSITE
SHORT
ISOSCELES
POWER
ROUNDING
ROOT
MEASURE
RADICAL
ADDITION
LABEL
THEOREM
AREA

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ALTITUDE

Worksheets in this chapter are also in Chapters 4, 7 and 16.

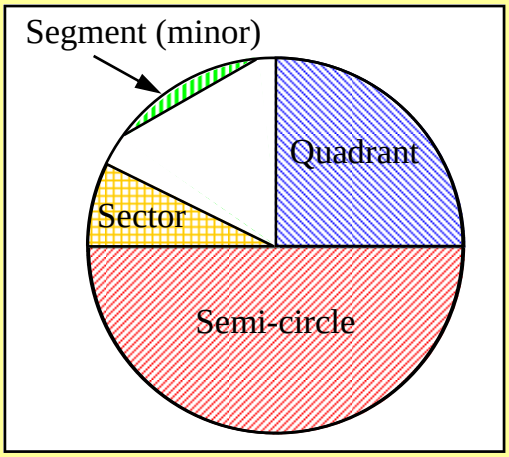
THE CIRCLE

The Circle (Sheet 1)

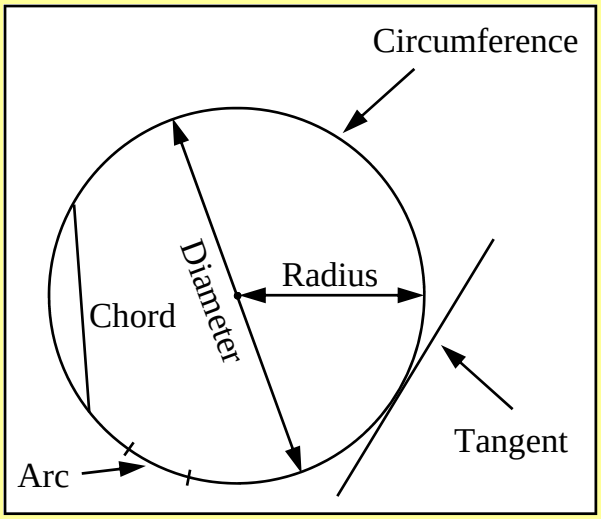
This sheet deals with naming parts of a circle, these are areas of circles created by particular lines and also the lines themselves.

Column 1 starts with matching diagrams showing the features of the circle with the words on the right of the column. Draw a line between the diagram and the correct word. The areas and lengths are shown below.

Circle Areas



Circle Lengths



Column 2 asks you to draw lines between points on the circle, this will create areas. Name the lines (using words like radius, diameter ….) as well as the areas.

Questions 29 and 30 require you to draw the lines and label points yourself, you will need a protractor to answer question 30.

The Circle (Sheet 1)

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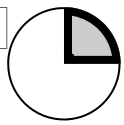
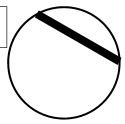
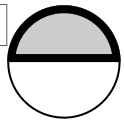
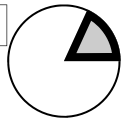
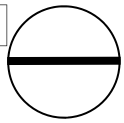
The diagram on the left shows a shaded area or a feature in black. Match the diagram with the list on the right, using a ruler to draw a line between the two black dots.

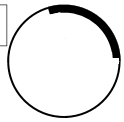
1 

2 

3 

4 

5 

6 

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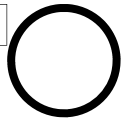
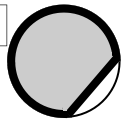
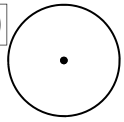
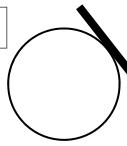
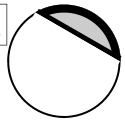
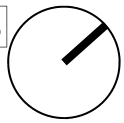
8 

9 

10 

11 

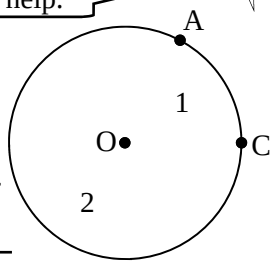
12 

13 

- Arc
- Minor Segment
- Radius
- Major Sector
- Quadrant
- tangent
- Centre
- Sector
- Diameter
- Chord
- Circumference
- Semi-Circle
- Major Segment

Given points on each diagram, draw the intervals and then name the features. Use the word list in the left column for help.

14 Connect O to A, then O to C and shade the two areas.

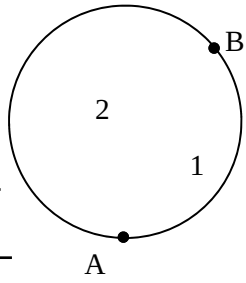


15 Area 1 _____

16 OA _____

17 AC _____ 18 Area 2 _____

19 Connect A to B and shade the two areas.

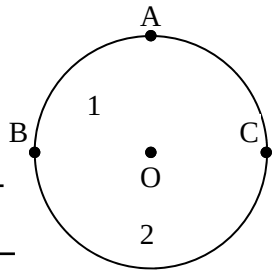


20 Area 1 _____

21 AB _____
Straight line

22 AB _____ 23 Area 2 _____
Curve

24 Draw a straight line BOC, then perpendicular OA

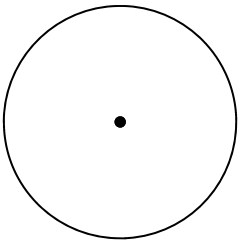


25 Area 1 _____

26 BC _____

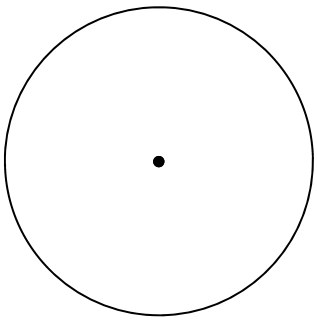
27 OA _____ 28 Area 2 _____

Use the circles below to make the following constructions.



29 Plot a point 'C' on the circumference of the circle, draw a tangent touching the circle at C. Draw a line from C to the centre of the circle O.

30 Name the point 'O' at the centre of the circle. Draw two radii, OA and OB, with an acute angle between the two. Plot a point C away from points A and B. Draw chords AC & BC. Protractor?



∠AOB = _____ ∠ACB = _____

The Circle (Sheet 1)

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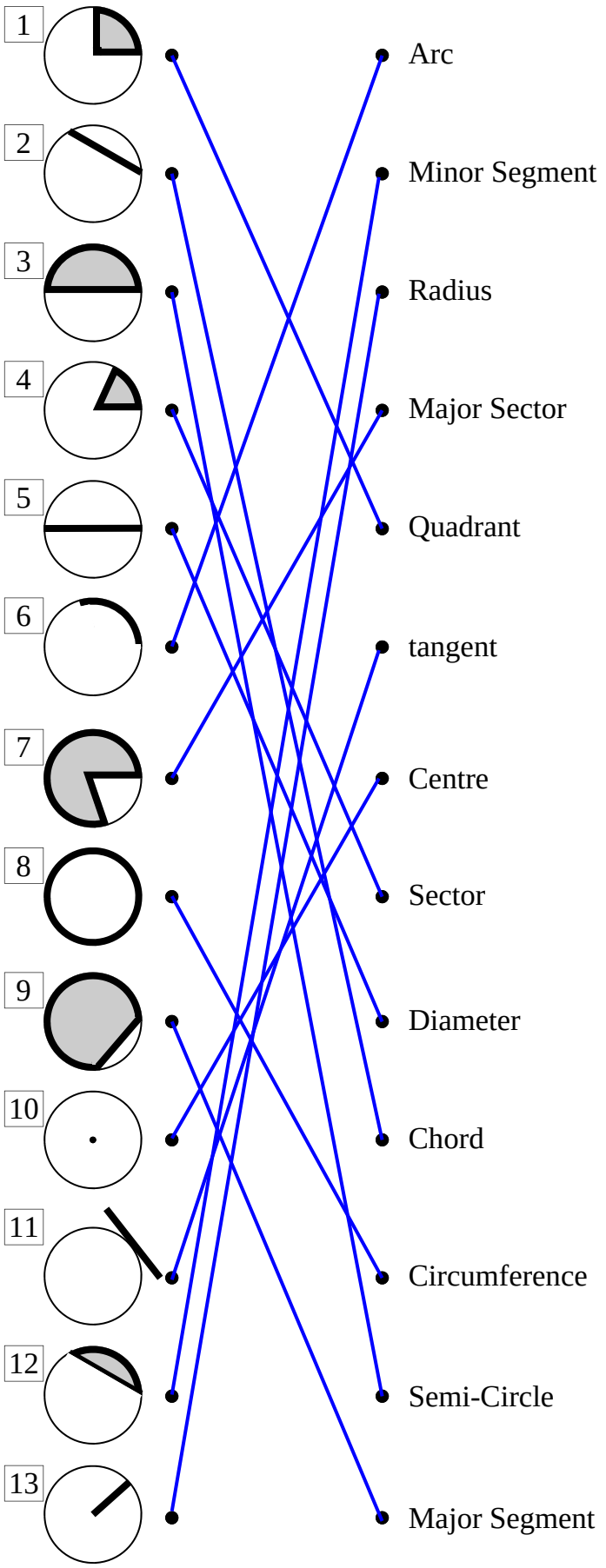
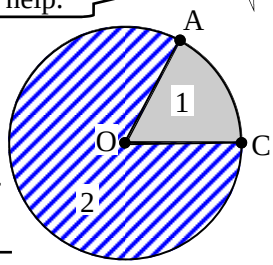
The diagram on the left shows a shaded area or a feature in black. Match the diagram with the list on the right, using a ruler to draw a line between the two black dots.



Given points on each diagram, draw the intervals and then name the features. Use the word list in the left column for help.

14 Connect O to A, then O to C and shade the two areas.



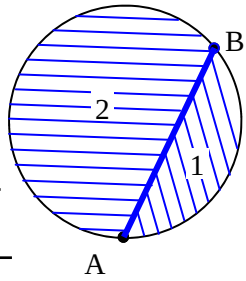
15 Area 1 Minor Sector

16 OA Radius

17 AC Arc

18 Area 2 Major Sector

19 Connect A to B and shade the two areas.



20 Area 1 Minor Segment

21 AB Chord

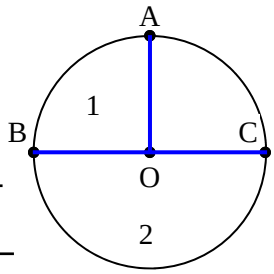
Straight line

22 AB Arc

Curve

23 Area 2 Major Segment

24 Draw a straight line BOC, then perpendicular OA



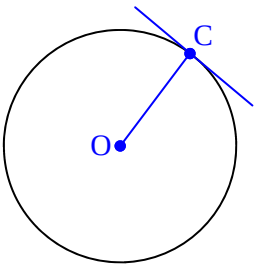
25 Area 1 Quadrant

26 BC Diameter

27 OA Radius

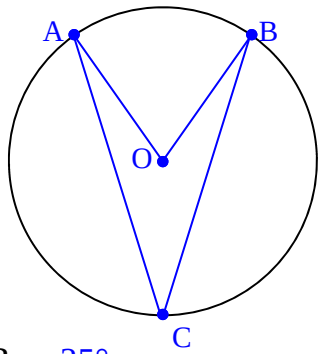
28 Area 2 Semi-circle

Use the circles below to make the following constructions.



29 Plot a point 'C' on the circumference of the circle, draw a tangent touching the circle at C. Draw a line from C to the centre of the circle O.

30 Name the point 'O' at the centre of the circle. Draw two radii, OA and OB, with an acute angle between the two. Plot a point C away from points A and B. Draw chords AC & BC. Protractor?



∠AOB = 70° ∠ACB = 35°

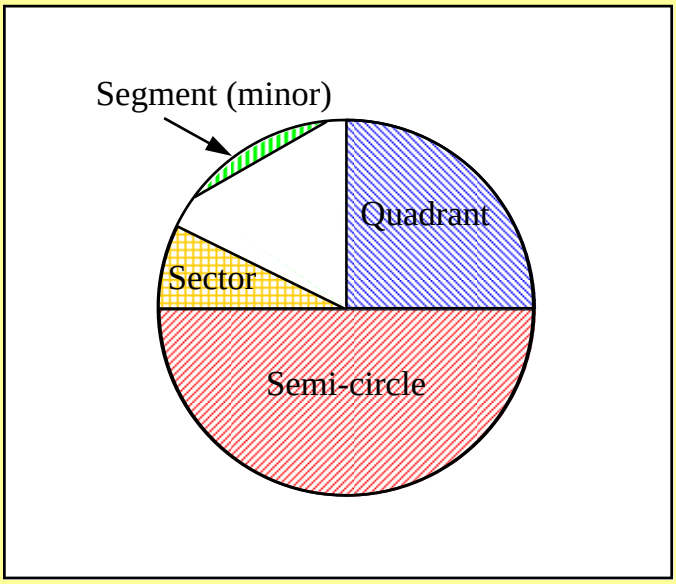
The Circle (Sheet 2)

This sheet deals with naming parts of a circle, these are areas of circles created by particular lines and also the lines themselves.

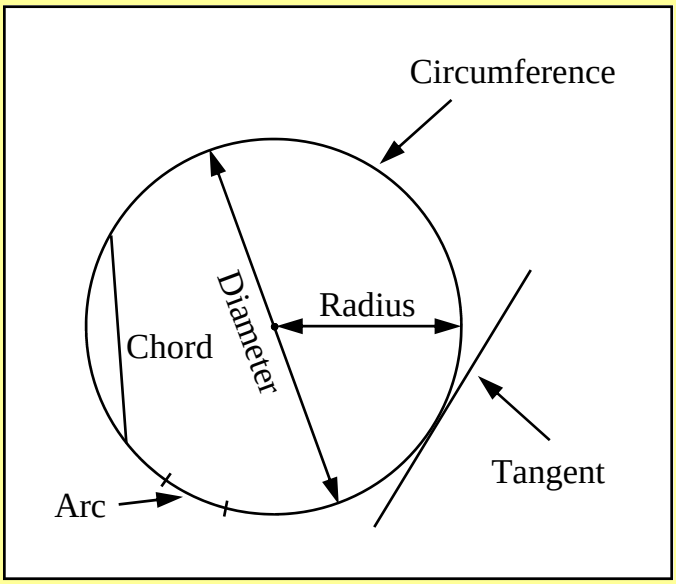
The sheet has 2 columns with diagrams at the top of each. In the spaces provided, name the parts of the circle that are being pointed to.

The questions below the diagrams all refer to the parts that have just been named, so look at the diagrams below if you need help. A word you may not be familiar with is radii - this is the plural of radius (so 1 radius, 2 radii). Also segments may be named minor segments when they are smaller than a semi-circle and major segments when they are larger than a semi-circle. Though for this sheet the word 'segment' alone is adequate, ask your teacher if minor/major is also required.

Column 1 - Circle Areas



Column 2 - Circle Lengths



The Circle (Sheet 2)

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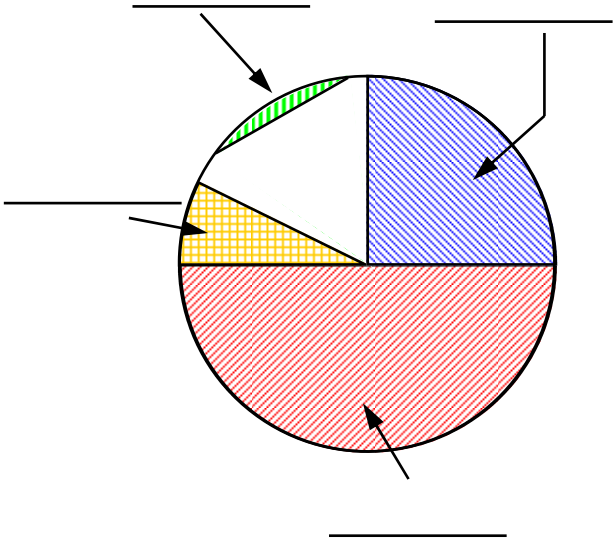
Name the parts of the circle below.



The answers for these will be found on the diagram above

1 Pizzas are sliced into these _____

2 Two of these makes a semi-circle _____

3 This is the only shape that doesn't touch the centre of the circle _____

4 When two or more of these are combined they could make a quadrant _____

5 A chord is used to make this shape _____

6 This shape has the circle's diameter as its straight side _____

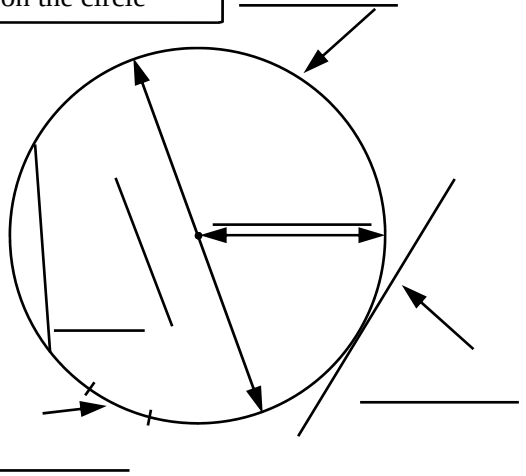
7 This shape has a right angle _____

8 When an arc is combined with two radii the shape that is made is a _____

9 If 6 identical sectors form a quadrant what shape is formed by 12 sectors ? _____

10 If 6 identical sectors form a quadrant what shape is formed by 3 sectors ? _____

Name the lengths on the circle



The answers for these will be one of the lengths shown above

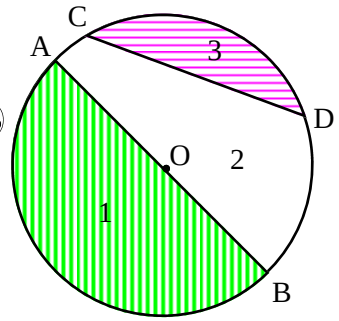
11 This distance is half the diameter _____

12 If a wheel rolled one revolution, the distance traveled would be the _____

13 A right angle is formed where the radius meets this line _____

14 A segment is formed by an arc and one of these _____

Name lengths/areas described by these letters



15 AC: _____

16 OB: _____

17 AB: _____

18 CD: _____

19 OD: _____

20 BC: _____

21 1: _____

22 3: _____

23 1 + 2: _____

The Circle (Sheet 2)

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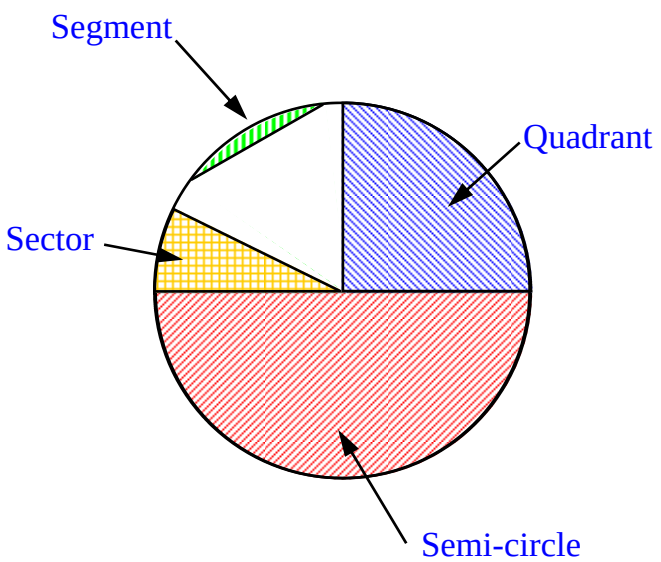
Here are the different areas within a circle



The answers for these will be found on the diagram above

1 Pizzas are sliced into these sectors

2 Two of these makes a semi-circle quadrants

3 This is the only shape that doesn't touch the centre of the circle segment

4 When two or more of these are combined they could make a quadrant sectors

5 A chord is used to make this shape segment

6 This shape has the circle's diameter as its straight side semi-cicle

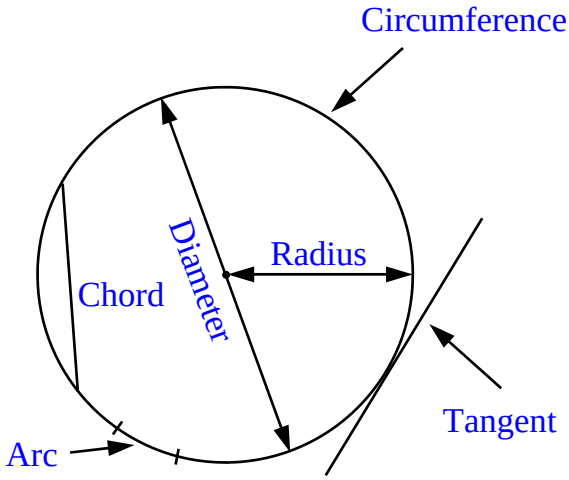
7 This shape has a right angle quadrant

8 When an arc is combined with two radii the shape that is made is a sector or quadrant

9 If 6 identical sectors form a quadrant what shape is formed by 12 sectors ? semi-circle

10 If 6 identical sectors form a quadrant what shape is formed by 3 sectors ? sector

Here are the different distances and features of a circle



The answers for these will be one of the lengths shown above

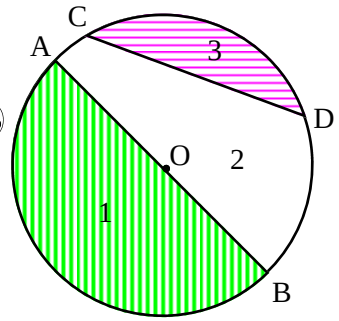
11 This distance is half the diameter radius

12 If a wheel rolled one revolution, the distance traveled would be the circumference

13 A right angle is formed where the radius meets this line tangent

14 A segment is formed by an arc and one of these chord

Name lengths/areas described by these letters



15 AC: arc

16 OB: radius

17 AB: diameter

18 CD: chord

19 OD: radius

20 BC: chord

21 1: semi-circle

22 3: segment

23 1 + 2: major segment

Exact & Estimation - Circumference

The circumference of a circle is its perimeter, the distance around its outside. The way to measure circumference when you don't know how to calculate it is to place a mark on the circle roll it one revolution and see how far the mark has travelled. But you can use this formula:

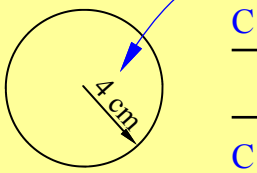
$$C = 2\pi r \text{ or } C = \pi d$$

Where: C = the circumference (length is same units as the radius/diameter)
π = Pi, it's a value on your calculator 3.14159….. Which is a bit more than 3!
r = the radius, the length from the centre to any point on the circumference
d = the diameter, the width of the circle.

Note that if life is complicated enough, just use $C = 2\pi r$ only, then remember to always use the radius. You don't have to remember both formulae. If given a diameter, halve it!

Column 1 starts with circles that have a radius shown, you will require a ruler for question 4. To find the circumference in terms of π, also called the exact length, use the value of r given in the circle. To find the estimate, use the exact circumference and substitute 3 for π.

Example



$C = 2\pi r$

$= 2\times\pi\times4$ ← Replace r with 4, (the radius of the circle)

$C = 8\pi$ cm ← $2 \times 4 = 8$, leave π as is. Write the units.

So the circumference is 8π cm. This is the exact length, to estimate the length without a calculator use $\pi = 3$. This is less than the actual value for π, but it gives a fairly accurate answer without too much calculation.

Estimate:

$C = 8\pi$ cm ← Rewrite the circumference from previous step

$\approx 8 \times 3$ ← Replace π with 3 and = with ≈

$C \approx 24$ cm ← Solve, use either ≈ or ≒ to indicate 'approximately'.

Column 2 uses diameters. You can either halve the diameter for the radius and solve as before, or use $C = \pi d$. So just multiply the diameter by 3 for an estimate of the circumference.

At the bottom of the page are four circles. Measure the diameter accurate to the nearest 0.5 cm, then calculate the exact circumference (in terms of π) and then an estimate, again using $\pi = 3$. Use the working out spaces in the third column to answer these.

Exact & Estimation - Circumference

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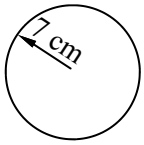
Find the circumference in terms of π. Then using π = 3, estimate the circumference.

1 $C = 2\pi r$

$= 2 \times \pi \times \square$

$C = \square\ \pi$ cm

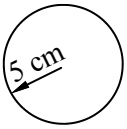


Estimate: $C = \square\ \pi$ cm

$\approx \square \times 3$

$C \approx \square$ cm

2



$C = 2\pi r$

$=$

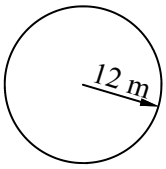
$C =$

Estimate: $C =$

$\approx$

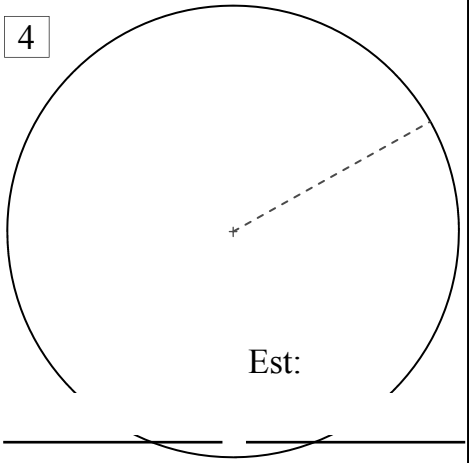
$C \approx$

3



Estimate:

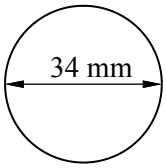
4



Est:

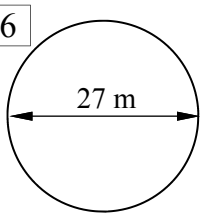
Divide the diameter by 2 or use $C = \pi d$ to find the circumference & estimate.

5



Estimate:

6



Estimate:

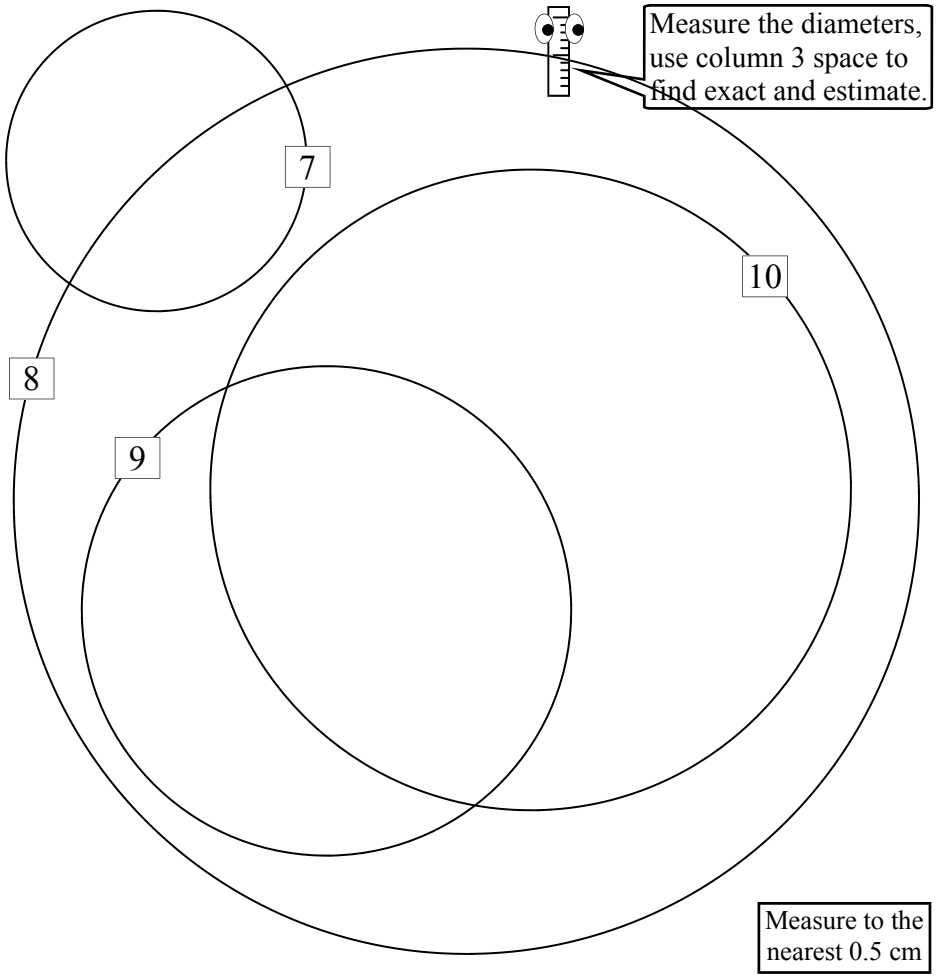
7

8

9

10

Measure the diameters, use column 3 space to find exact and estimate.



Measure to the nearest 0.5 cm

Exact & Estimation - Circumference

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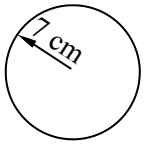
Find the circumference in terms of π. Then using π = 3, estimate the circumference.

1 $C = 2\pi r$

$= 2 \times \pi \times 7$

$C = 14\pi$ cm

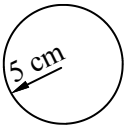


Estimate: $C = 14\pi$ cm

$\approx 14 \times 3$

$C \approx 42$ cm

2



$C = 2\pi r$

$= 2 \times \pi \times 5$

$C = 10\pi$ cm

Estimate: $C = 10\pi$ cm

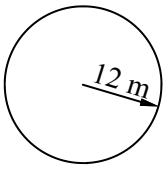
$\approx 10 \times 3$

$C \approx 30$ cm

3 $C = 2\pi r$

$= 2 \times \pi \times 12$

$C = 24\pi$ m

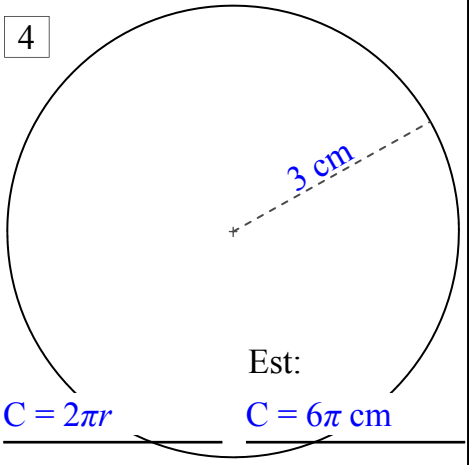


Estimate: $C = 24\pi$ cm

$\approx 24 \times 3$

$C \approx 72$ m

4



$C = 2\pi r$

$= 2 \times \pi \times 3$

$C = 6\pi$ cm

Est:

$C = 6\pi$ cm

$\approx 6 \times 3$

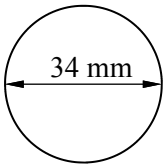
$C \approx 18$ cm

Divide the diameter by 2 or use $C = \pi d$ to find the circumference & estimate.

5 $C = \pi d$

$= \pi \times 34$

$C = 34\pi$ mm

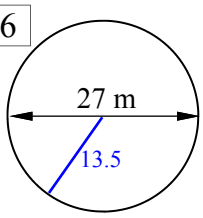


Estimate: $C = 34\pi$ mm

$\approx 34 \times 3$

$C \approx 102$ mm

6



$C = \pi d$

$= \pi \times 27$

$C = 27\pi$ m

Estimate: $C = 27\pi$ m

$\approx 27 \times 3$

$C \approx 81$ m

7

$C = \pi d$

$= \pi \times 4$

$C = 4\pi$ cm

$C = 4\pi$ cm

$\approx 4 \times 3$

$C \approx 12$ cm

8

$C = \pi d$

$= \pi \times 12$

$C = 12\pi$ cm

$C = 12\pi$ cm

$\approx 12 \times 3$

$C \approx 36$ cm

9

$C = \pi d$

$= \pi \times 6.5$

$C = 6.5\pi$ cm

$C = 6.5\pi$ cm

$\approx 6.5 \times 3$

$C \approx 19.5$ cm

10

$C = \pi d$

$= \pi \times 8.5$

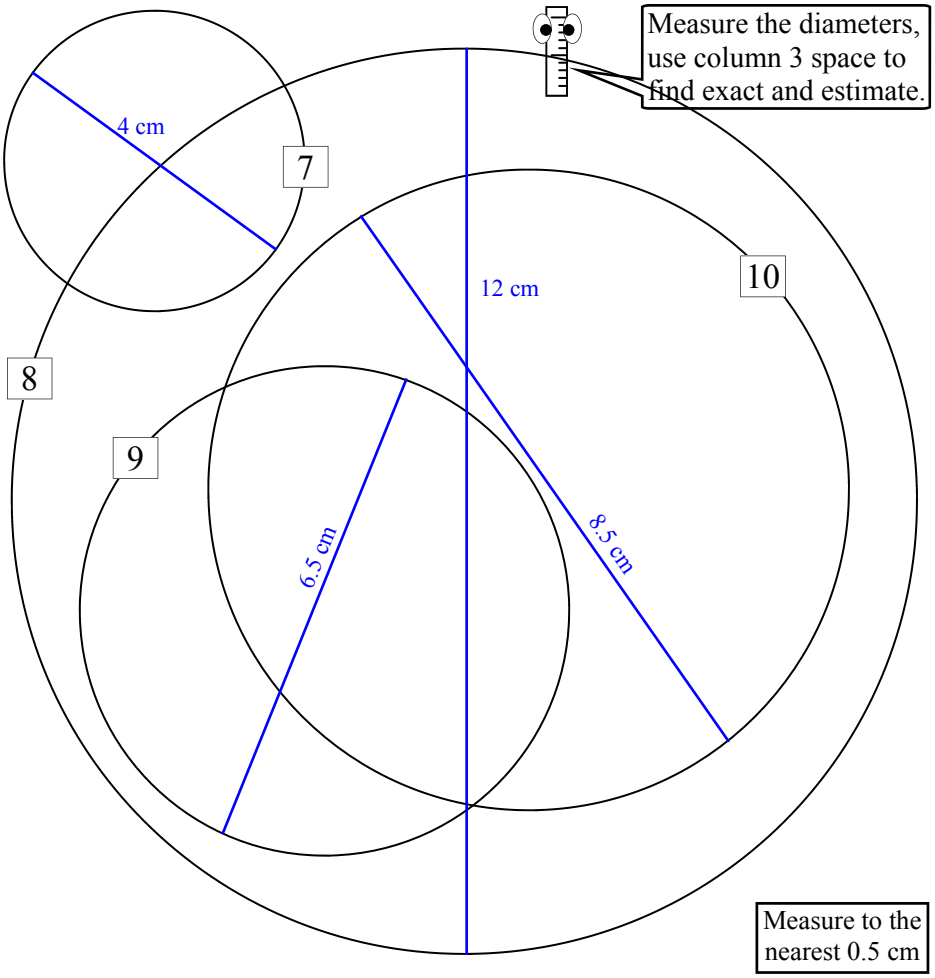
$C = 8.5\pi$ cm

$C = 8.5\pi$ cm

$\approx 8.5 \times 3$

$C \approx 25.5$ cm

Measure the diameters, use column 3 space to find exact and estimate.



Measure to the nearest 0.5 cm

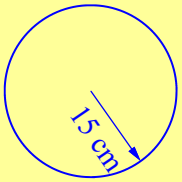
Circumference (Sheet 1)

The circumference of a circle is its perimeter, the distance around the outside of the circle. This distance is found by using this formula:

$$C = 2\pi r$$

Where: C = Circumference of circle
r = radius of the circle

Example



$C = 2\pi r$ ← Write the formula.

$= 2 \times \pi \times 15$ ← Show × signs and replace r with the radius (15 on diagram).

$C = 94.2$ cm (1 d.p.) ← Calculate, round and show units.

Calculator 2 × π × 1 5 × = Answer on Screen 94.24777961

Round to 1 decimal place 94.2 cm (1 d.p.)

Column 1 uses the above process for all questions.

Column 2 gives you a diameter rather than a radius. So you can divide by the diameter by 2 to find the radius then use $C = 2\pi r$, but an easier way is to use:

$$C = \pi d$$

Where: C = Circumference of circle
d = diameter of the circle (note that $d = 2r$)

Column 3 are problem style questions using the circumference of a circle.

Circumference (Sheet 1)

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Use $C=2\pi r$ to find the circumference of these circles to 1 decimal place.

Example

$C = 2\pi r$

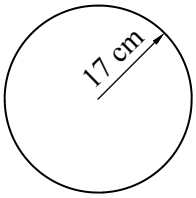
Write formula

$= 2 \times \pi \times 17$

Substitute value for r

$C = 106.8$ cm

Answer 1 d.p., show units



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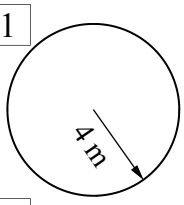

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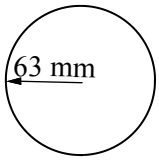

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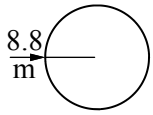

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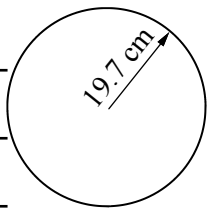

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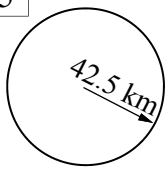

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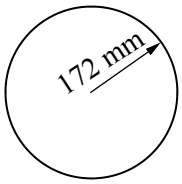

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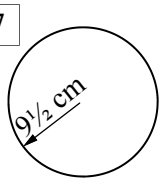
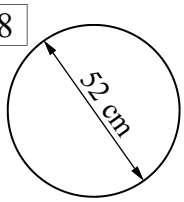


These use diameter. Either divide by 2 for the radius *OR* use $C=\pi d$. Answer to 2 d.p.

8



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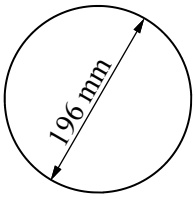

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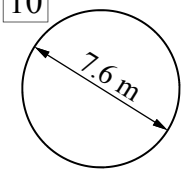

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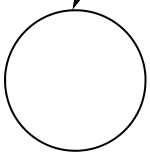
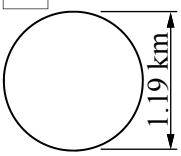


$*\phi$ = *'phi'* = d

12



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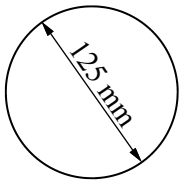

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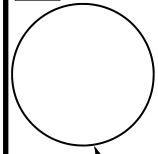
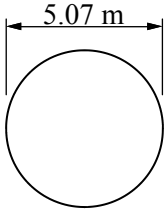


15

$\phi = 7.89$ m



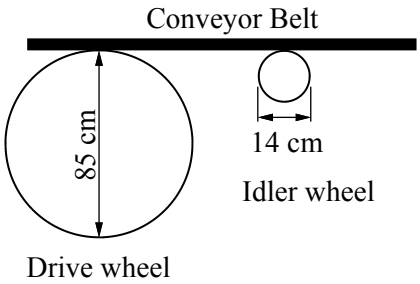
Now apply your skills to these problems. Answer to 2 d.p. where appropriate.

16 A car tyre has a diameter of 63 cm. Find its circumference. Give your answer in metres.

17 How far does the tyre travel when it rotates 150 times?

18 How many complete rotations of the tyre will occur if 1 km is travelled?

19 Calculate the amount of conveyor belt fed by 1 rotation of the drive wheel.



20 1 rotation of the idler wheel

21 Complete:

$$\frac{C_{(\text{Drive})}}{C_{(\text{Idler})}} = \frac{\square}{\square} = \_\_\_\_$$

Circumference (Sheet 1)

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Use $C=2\pi r$ to find the circumference of these circles to 1 decimal place.

Example

$C = 2\pi r$

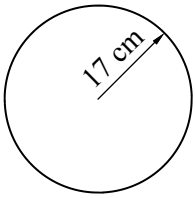
Write formula

$= 2 \times \pi \times 17$

Substitute value for r

$C = 106.8$ cm

Answer 1 d.p., show units



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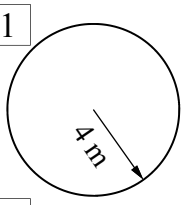


$C = 2\pi r$

$= 2 \times \pi \times 4$

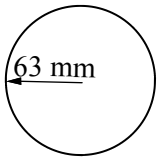
$C = 25.1$ m

2

$C = 2\pi r$

$= 2 \times \pi \times 63$

$C = 395.8$ mm



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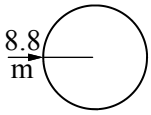


$C = 2\pi r$

$= 2 \times \pi \times 8.8$

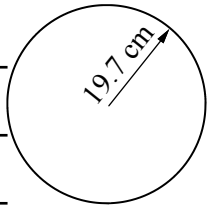
$C = 55.3$ m

4

$C = 2\pi r$

$= 2 \times \pi \times 19.7$

$C = 123.8$ cm



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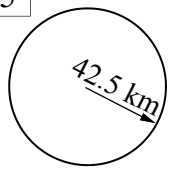


$C = 2\pi r$

$= 2 \times \pi \times 42.5$

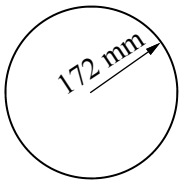
$C = 267$ km

6

$C = 2\pi r$

$= 2 \times \pi \times 172$

$C = 1\,080.7$ mm



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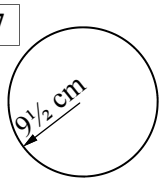


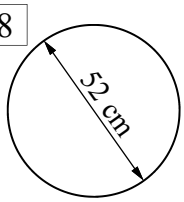
$C = 2\pi r$

$= 2 \times \pi \times 9.5$

$C = 59.7$ cm

These use diameter. Either divide by 2 for the radius *OR* use $C=\pi d$. Answer to 2 d.p.

8



$C = \pi d$

$= \pi \times 52$

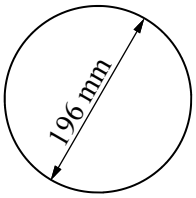
$C = 163.36$ cm

9

$C = \pi d$

$= \pi \times 196$

$C = 615.75$ mm



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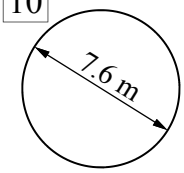


$C = \pi d$

$= \pi \times 7.6$

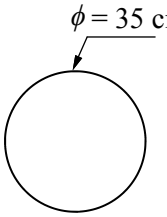
$C = 23.88$ m

11

$C = \pi d$

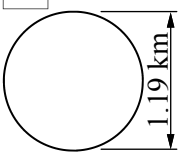
$= \pi \times 35$

$C = 109.96$ cm



* ϕ = 'phi' = d

12



$C = \pi d$

$= \pi \times 1.19$

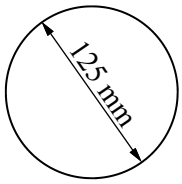
$C = 3.74$ km

13

$C = \pi d$

$= \pi \times 125$

$C = 392.70$ mm



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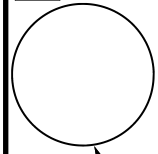


$C = \pi d$

$= \pi \times 7.89$

$C = 24.79$ m

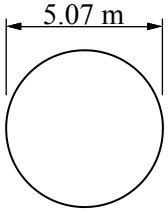
15

$\phi = 7.89$ m

$C = \pi d$

$= \pi \times 5.07$

$C = 15.93$ m



Now apply your skills to these problems. Answer to 2 d.p. where appropriate.

16 A car tyre has a diameter of 63 cm. Find its circumference. Give your answer in metres.

$C = \pi d$

$= \pi \times 0.63$

$C = 1.98$ m

17 How far does the tyre travel when it rotates 150 times?

Distance $= 150 \times 1.98$

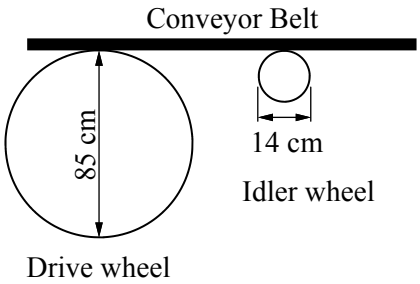
Distance $= 297$ m (296.88 m If 16 unrounded)

18 How many complete rotations of the tyre will occur if 1 km is travelled?

Rotations $= 1\,000 \div 1.98$

Rotations $= 505$

19 Calculate the amount of conveyor belt fed by 1 rotation of the drive wheel.



$C = \pi d$

$= \pi \times 85$

$C = 267.04$ cm

20 1 rotation of the idler wheel

$C = \pi d$

$= \pi \times 14$

$C = 43.98$ cm

21 Complete:

$$\frac{C_{(\text{Drive})}}{C_{(\text{Idler})}} = \frac{267.04}{43.98} = 6.07$$

Circumference (Sheet 2)

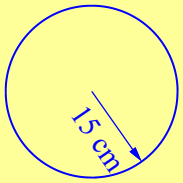
The circumference of a circle is its perimeter, the distance around the outside of the circle. This distance is found by using this formula:

$$C = 2\pi r$$

Where: C = Circumference of circle
r = radius of the circle

The usual process is to use your π key, but there are two common approximations for π. These are $\pi = 3.14$ (as a decimal) and $\pi = {}^{22}/_{7}$ (as a fraction).

Example - $\pi = 3.14$

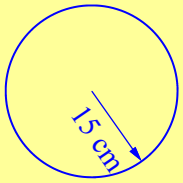


$C = 2\pi r$ ← Write the formula.

$= 2 \times 3.14 \times 15$ ← Show × signs and replace π with 3.14 and r with the radius .

$C = 94.2$ cm (1 d.p.) ← Calculate, round and show units.

Example - $\pi = {}^{22}/_{7}$



$C = 2\pi r$ ← Write the formula.

$= 2 \times {}^{22}/_{7} \times 15$ ← Show × signs and replace π with $^{22}/_{7}$ and r with the radius .

$C = 94.3$ cm (1 d.p.) ← Calculate, round and show units.

Column 1 uses the above process for all questions.

Column 2 gives you a diameter rather than a radius. Use:

$$C = \pi d$$

Where: C = Circumference of circle
d = diameter of the circle (note that $d = 2r$)

There are two tables at the base of the column, just write the answer with units, no working out required.

Column 3 finds the diameter (and later radius) given the circumference. If you divide the circumference by π then you find the diameter. If you halve this answer you get the radius.

$$C = \pi d$$

$\div \pi \quad \div \pi$

$$\frac{C}{\pi} = d$$

$$d = \frac{C}{\pi}$$

Circumference (Sheet 2)

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Use $C=2\pi r$ with $\pi = 3.14$, to find the circumference. Round your answer to 1 decimal place.

Example

$C = 2\pi r$

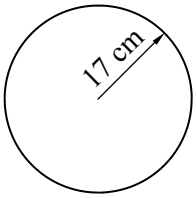
Write formula

$= 2 \times 3.14 \times 17$

Substitute 3.14 and r

$C = 106.8$ cm

Answer 1 d.p., show units



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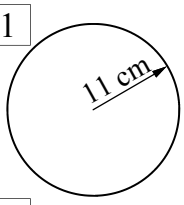

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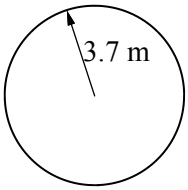

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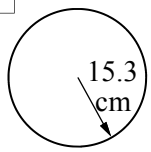
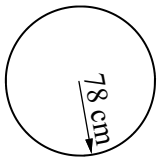


Now use $\pi = {}^{22}/_{7}$, to find the circumference. Round to the nearest whole number.

4



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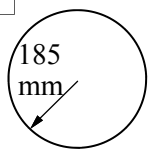

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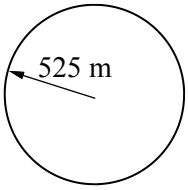
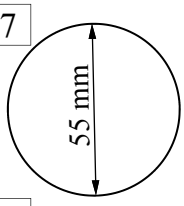


Now use $C = \pi d$ & $\pi = 3.14$, find the circumference and round to 1 decimal place.

7



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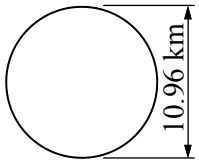

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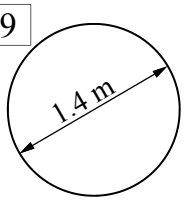

Now use $C = \pi d$ & $\pi = {}^{22}/_{7}$, to complete the table to 1 dp No working out required.

Q.	d	Circumference
10	23 cm	
11	83.2 m	
12	19.6 mm	
13	16.07 km	
14	208 m	
15	485 km	

Now use $C = \pi d$ & the π key on your calculator, to complete the table to 1 d.p.

Q.	d	Circumference
16	5.6 m	
17	0.9 km	
18	93 mm	
19	408 m	
20	317 mm	
21	13.68 km	
22	80.1 cm	

This time you are given the circumference. Divide C by π (π key) to find d to 1 d.p.

23 Find d if C = 27.3 cm.

$d = \frac{C}{\pi} = \frac{\square}{\pi}$ $d =$ _____

24 Find d if C = 1.6 km.

$d = \frac{C}{\pi} = \frac{\square}{\pi}$ $d =$ _____

25 Find d if C = 192.3 mm.

A cylindrical spool spins 400 times and gathers 125 m of thread. Find, to 2 d.p. :

26 The amount gathered in 1 revolution. Answer in cm.

27 Find the spool's diameter.

$d \div 2 = r$

This time you want the radius. Divide C by π to get d but then halve your answer to find r. Answer to one decimal place.

Q.	C	Radius (r)
28	17 m	
29	85 mm	
30	116 cm	
31	70.4 m	
32	41 km	
33	250 mm	
34	60 cm	
35	782 mm	
36	98 m	
37	410 cm	

Circumference (Sheet 2)

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Use $C=2\pi r$ with $\pi = 3.14$, to find the circumference. Round your answer to 1 decimal place.

Example

$C = 2\pi r$

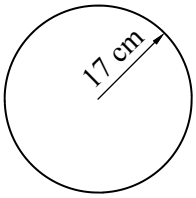
Write formula

$= 2 \times 3.14 \times 17$

Substitute 3.14 and r

$C = 106.8$ cm

Answer 1 d.p., show units



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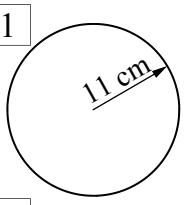
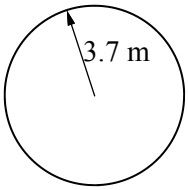


$C = 2\pi r$

$= 2 \times 3.14 \times 11$

$C = 69.1$ cm

2

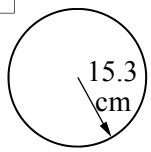


$C = 2\pi r$

$= 2 \times 3.14 \times 3.7$

$C = 23.2$ m

3



$C = 2\pi r$

$= 2 \times 3.14 \times 15.3$

$C = 96.1$ cm

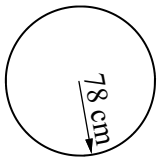
Now use $\pi = {}^{22}/_{7}$, to find the circumference. Round to the nearest whole number.

4

$C = 2\pi r$

$= 2 \times {}^{22}/_{7} \times 78$

$C = 490$ cm



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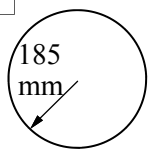


$C = 2\pi r$

$= 2 \times {}^{22}/_{7} \times 185$

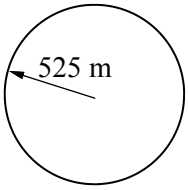
$C = 1\,163$ mm

6

$C = 2\pi r$

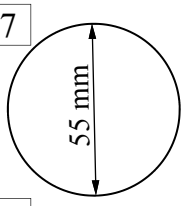
$= 2 \times {}^{22}/_{7} \times 525$

$C = 3\,300$ m



Now use $C = \pi d$ & $\pi = 3.14$, find the circumference and round to 1 decimal place.

7



$C = \pi d$

$= 3.14 \times 55$

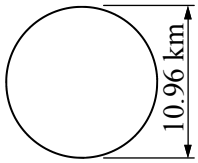
$C = 172.7$ mm

8

$C = \pi d$

$= 3.14 \times 10.96$

$C = 34.4$ km



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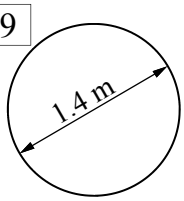


$C = \pi d$

$= 3.14 \times 1.4$

$C = 4.4$ m

Now use $C = \pi d$ & $\pi = {}^{22}/_{7}$, to complete the table to 1 dp No working out required.

Q.	d	Circumference
10	23 cm	72.3 cm
11	83.2 m	261.5 m
12	19.6 mm	61.6 mm
13	16.07 km	50.5 km
14	208 m	653.7 m
15	485 km	1 524.3 km

Now use $C = \pi d$ & the π key on your calculator, to complete the table to 1 d.p.

Q.	d	Circumference
16	5.6 m	17.6 m
17	0.9 km	2.8 km
18	93 mm	292.2 mm
19	408 m	1 281.8 m
20	317 mm	995.9 mm
21	13.68 km	43.0 km
22	80.1 cm	251.6 cm

This time you are given the circumference. Divide C by π (π key) to find d to 1 d.p.

23 Find d if C = 27.3 cm.

$d = \frac{C}{\pi} = \frac{27.3}{\pi}$ $d =$ 8.7 cm

24 Find d if C = 1.6 km.

$d = \frac{C}{\pi} = \frac{1.6}{\pi}$ $d =$ 0.5 km

25 Find d if C = 192.3 mm.

$d = \frac{C}{\pi} = \frac{192.3}{\pi}$ $d =$ 61.2 mm

A cylindrical spool spins 400 times and gathers 125 m of thread. Find, to 2 d.p. :

26 The amount gathered in 1 revolution. Answer in cm.

$125 \div 400 = 0.3125$ m $= 31.25$ cm

27 Find the spool's diameter.

$d = \frac{C}{\pi} = \frac{31.25}{\pi}$ $d =$ 9.95 cm

$d \div 2 = r$

This time you want the radius. Divide C by π to get d but then halve your answer to find r. Answer to one decimal place.

Q.	C	Radius (r)
28	17 m	2.7 cm
29	85 mm	13.5 mm
30	116 cm	18.5 cm
31	70.4 m	11.2 m
32	41 km	6.5 km
33	250 mm	39.8 mm
34	60 cm	9.5 cm
35	782 mm	124.5 mm
36	98 m	15.6 m
37	410 cm	65.3 cm

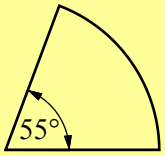
Fractions of a Circle and Arc Length

This sheet finds the length of an arc of a sector. The arc is the curved length which is a fraction of a whole circle's circumference. So in other words, this sheet deals with the circumference of a circle but multiplied by a fraction.

Column 1 starts with sectors (including quadrants and semi-circles) and finding the fraction of the circle that they represent. This requires you to identify the internal angle and write it 'over' 360°. That answers the question but the next step is to simplify the fraction. This can be done in your head by dividing by the LCM, or using a calculator.

Example

$\frac{55}{360} = \frac{11}{72}$ (÷ 5 above, ÷ 5 below)



Calculator 5 5 $a^b/_c$ 3 6 0 =

Answer on Screen 11⌟72 But you write $\frac{11}{72}$

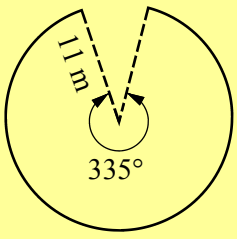
Column 2 relates the fraction to the arc length. The circumference of a circle is $C = 2\pi r$, but as these are sectors then the arc length is a fraction of the circumference (if the circle was whole).

$$l = \frac{\theta}{360} \times 2\pi r$$

Where:

l = the arc length (curved length)
θ = the internal angle of the sector
r = radius of sector

Example



$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{335}{360} \times 2 \times \pi \times 11$

$l = 64.3$ m (1 d.p.)

Column 3 asks you to find the perimeter of the sectors. So the method is just the same as above, but then add on the straight lengths as well. The straight lines are the radii, there are two of them. So work out the arc length, add two radii and you are done.

$$l = \frac{\theta}{360} \times 2\pi r + 2r$$

Fractions of a Circle and Arc Length

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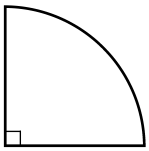
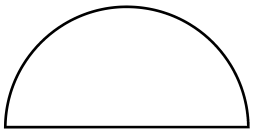
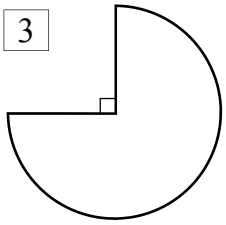
Calculate the fraction of a circle shown by these sectors, simplify if possible.

1 

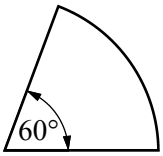
$\frac{\square}{360} = \frac{\square}{4}$

2 

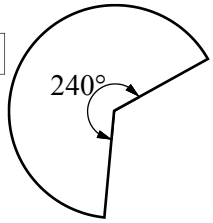
$\frac{\square}{360} = \frac{\square}{\square}$

3 

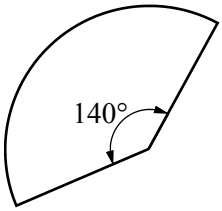
$\frac{\square}{\square} = \frac{\square}{\square}$

4 

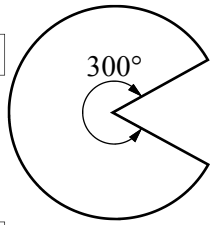
$\frac{\square}{\square} = \frac{\square}{\square}$

5 

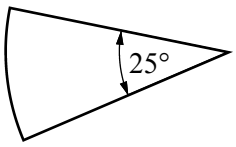
$\frac{\square}{\square} = \frac{\square}{\square}$

6 

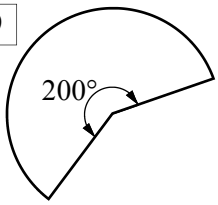
$\frac{\square}{\square} = \frac{\square}{\square}$

7 

$\frac{\square}{\square} = \frac{\square}{\square}$

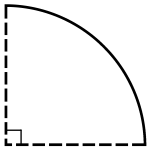
8 

$\frac{\square}{\square} = \frac{\square}{\square}$

9 

$\frac{\square}{\square} = \frac{\square}{\square}$

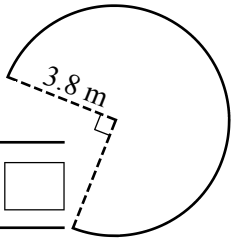
Find the fraction of a circle and multiply by $2\pi r$ to find the arc length 'l' to 1 d.p.

10 

$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{\square}{360} \times 2 \times \pi \times \square$

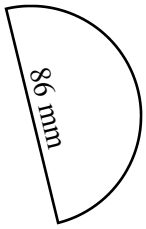
$l =$ cm

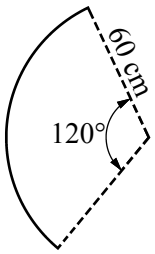
11 

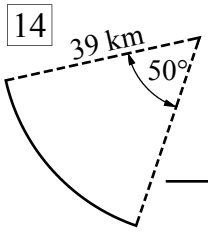
$l = \frac{\theta}{360} \times 2\pi r$

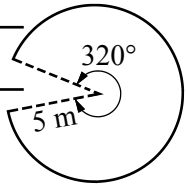
$= \frac{\square}{\square} \times 2 \times \pi \times \square$

$l =$

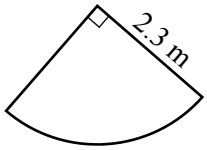
12 

13 

14 

15 

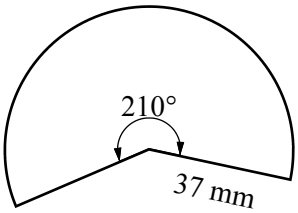
Now find the perimeter, to 1 d.p. Include the radii.

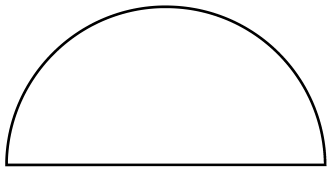
16 

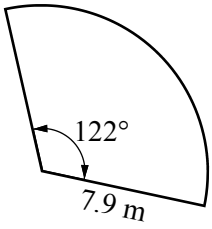
$l = \frac{\theta}{360} \times 2\pi r + 2r$

$= \frac{\square}{360} \times 2 \times \pi \times \square + 2 \times \square$

$l =$

17 

18 

19 

Fractions of a Circle and Arc Length

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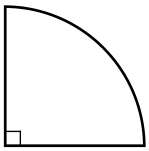
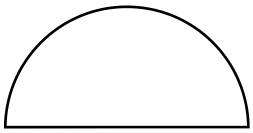
Calculate the fraction of a circle shown by these sectors, simplify if possible.

1



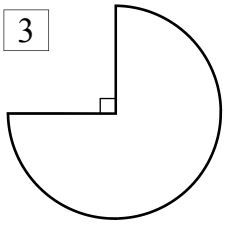
$\frac{90}{360} = \frac{1}{4}$

2



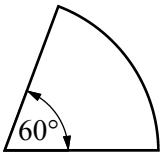
$\frac{180}{360} = \frac{1}{2}$

3



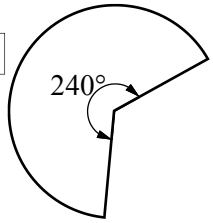
$\frac{270}{360} = \frac{3}{4}$

4



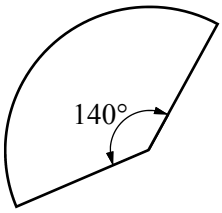
$\frac{60}{360} = \frac{1}{6}$

5



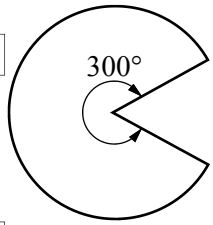
$\frac{240}{360} = \frac{1}{3}$

6



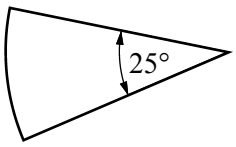
$\frac{140}{360} = \frac{7}{18}$

7



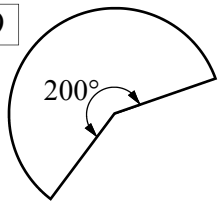
$\frac{300}{360} = \frac{5}{6}$

8



$\frac{25}{360} = \frac{5}{72}$

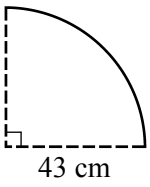
9



$\frac{200}{360} = \frac{5}{9}$

Find the fraction of a circle and multiply by $2\pi r$ to find the arc length 'l' to 1 d.p.

10

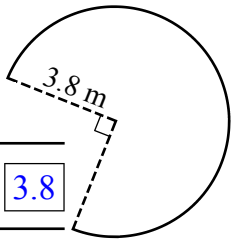


$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{90}{360} \times 2 \times \pi \times 43$

$l = 67.5$ cm

11

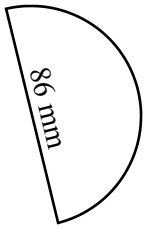


$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{270}{360} \times 2 \times \pi \times 3.8$

$l = 17.9$ m

12

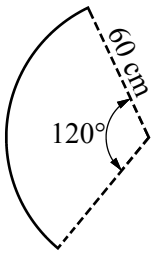


$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{180}{360} \times 2 \times \pi \times 86$

$l = 270.2$ mm

13

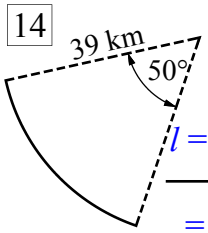


$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{120}{360} \times 2 \times \pi \times 60$

$l = 125.7$ cm

14



$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{50}{360} \times 2 \times \pi \times 39$

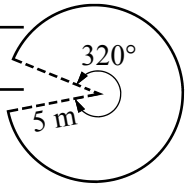
$l = 34$ km

15

$l = \frac{\theta}{360} \times 2\pi r$

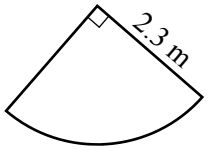
$= \frac{320}{360} \times 2 \times \pi \times 5$

$l = 27.9$ m



Now find the perimeter, to 1 d.p. Include the radii.

16

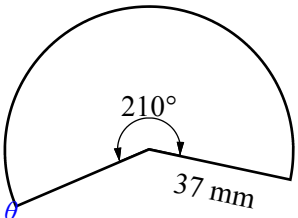


$l = \frac{\theta}{360} \times 2\pi r + 2r$

$= \frac{90}{360} \times 2 \times \pi \times 2.3 + 2 \times 2.3$

$l = 8.2$ m

17

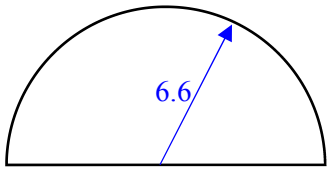


$l = \frac{\theta}{360} \times 2\pi r + 2r$

$= \frac{210}{360} \times 2 \times \pi \times 37 + 2 \times 37$

$l = 209.6$ mm

18

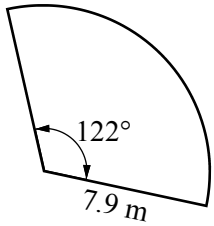


$l = \frac{\theta}{360} \times 2\pi r + d$

$= \frac{180}{360} \times 2 \times \pi \times 6.6 + 13.2$

$l = 33.9$ cm

19



$l = \frac{\theta}{360} \times 2\pi r + 2r$

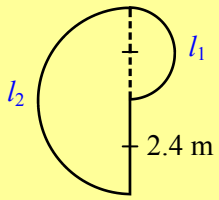
$= \frac{122}{360} \times 2 \times \pi \times 7.9 + 2 \times 7.9$

$l = 32.6$ m

Perimeter of Composite Shapes

This sheet shouldn't be attempted before the previous sheet on Arc Length is attempted. The circumference of a circle is found by $C = 2\pi r$ or $C = \pi d$. But with composite shapes parts of a circle are involved with other shapes/lengths, so you need to use fractions to find the arc length, then add that length to the other sides of the shape.

Example - Finding the perimeter to 1 d.p.



Label l_1 on the diagram and calculate the arc length.

$l_1 = {}^1/_2 \times \pi d$

$= {}^1/_2 \times \pi \times 2.4$

$l_1 = 3.8$ m

$l_2 = {}^1/_2 \times \pi d$

$= {}^1/_2 \times \pi \times 4.8$

$l_2 = 7.5$ m

Label l_2 on the diagram and calculate the arc length.

Add the three lengths. That is the two arc lengths and the 2.4 m straight length.

$P = 3.8 + 7.5 + 2.4$

$P = 13.7$ m

This sheet is answered by the above method, except sometimes quadrants are involved. So instead of using ${}^1/_2$ you will use ${}^1/_4$ to find the arc length.

Column 3 has the last two questions requiring an extra step, these require Pythagoras Theorem before solving.

Perimeter of Composite Shapes

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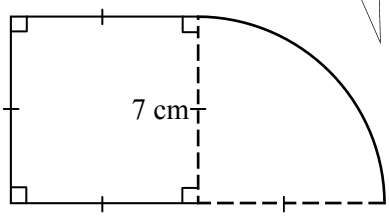
Find the perimeter to 1 d.p.

1



$l = {}^1/_4 \times 2\pi r$

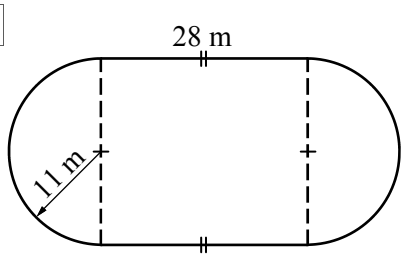
$= {}^1/_4 \times 2 \times \pi \times$ ☐

$l =$ ☐ cm (1 d.p.)

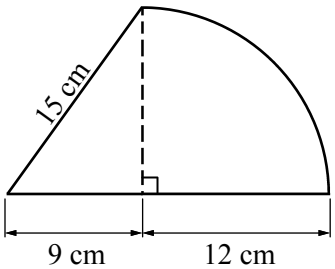
P = ☐ + 4 × ☐

P = ☐ cm

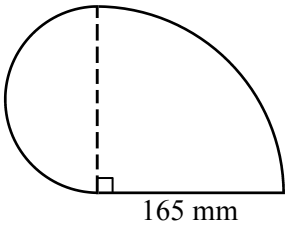
2



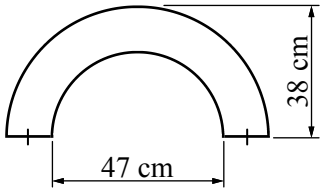
3



4



5



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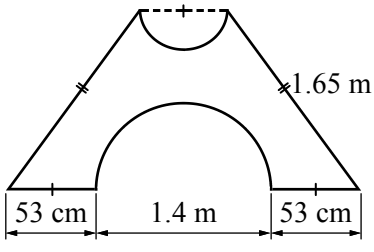
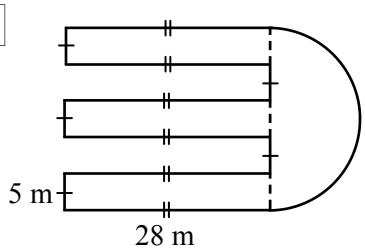
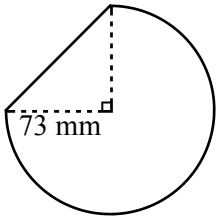


7

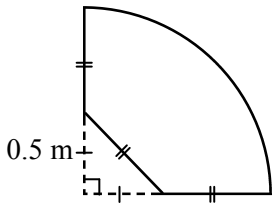


Pythagoras required, round to 2 d.p.

8



9



Perimeter of Composite Shapes

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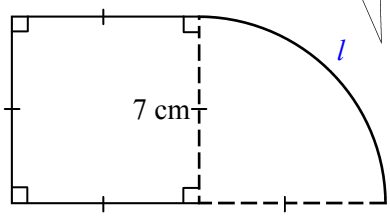
Find the perimeter to 1 d.p.

1



$l = {}^1/_4 \times 2\pi r$

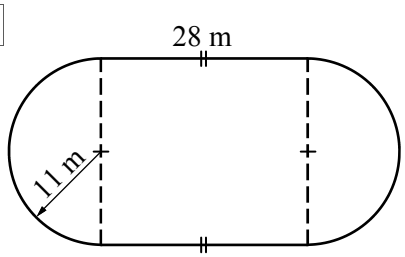
$= {}^1/_4 \times 2 \times \pi \times$ 7

$l =$ 11 cm (1 d.p.)

P = 11 + 4 × 7

P = 39 cm

2



$C = 2\pi r$

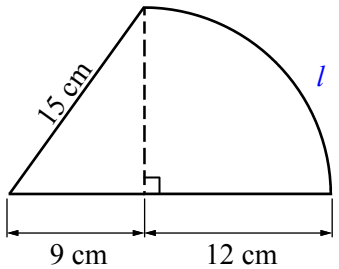
$= 2 \times \pi \times 11$

C = 69.1 m (1 d.p.)

P = 69.1 + 2 × 28

P = 125.1 m

3



$l = {}^1/_4 \times 2\pi r$

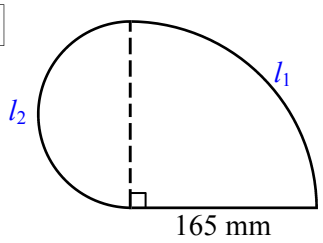
$= {}^1/_4 \times 2 \times \pi \times 12$

l = 18.8 cm (1 d.p.)

P = 18.8 + 15 + 9 + 12

P = 54.8 cm

4



$l_1 = {}^1/_4 \times 2\pi r$ $l_2 = {}^1/_2 \times 2\pi r$

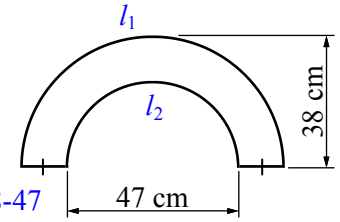
$= {}^1/_4 \times 2 \times \pi \times 165$ $= {}^1/_2 \times 2 \times \pi \times 82.5$

l_1 = 259.2 mm l_2 = 259.2 mm

P = 2 × 259.2 + 165

P = 683.4 mm

5



2×38-47 = 29

$l_1 = {}^1/_2 \times 2\pi r$ $l_2 = {}^1/_2 \times \pi d$

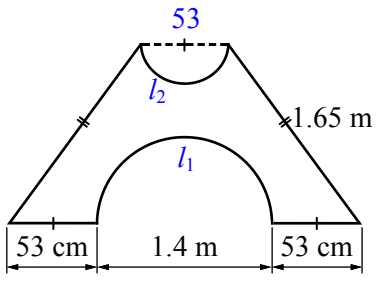
$= {}^1/_2 \times 2 \times \pi \times 38$ $= {}^1/_2 \times \pi \times 47$

l_1 = 119.4 cm l_2 = 73.8 cm

P = 119.4 + 73.8 + 29

P = 222.2 cm

6



$l_1 = {}^1/_2 \times \pi d$ $l_2 = {}^1/_2 \times \pi d$

$= {}^1/_2 \times \pi \times 1.4$ $= {}^1/_2 \times \pi \times 0.53$

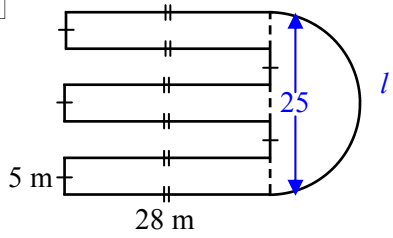
l_1 = 2.2 m (1 d.p) l_2 = 0.8 m (1 d.p)

P = 2×0.53 + 2.2 + 2×1.65 + 0.8

P = 7.4 m (1 d.p) or

P = 739.2 cm

7



$l = {}^1/_2 \times \pi d$

$= {}^1/_2 \times \pi \times 25$

l = 39.27 m

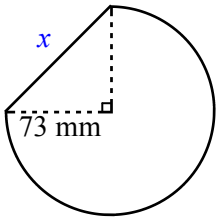
P = 39.27 + 3 × (5 + 2 × 28) + 10

P = 39.27 + 183 + 10

P = 232.27 m

Pythagoras required, round to 2 d.p.

8



$x^2 = 73^2 + 73^2$ $l = {}^3/_4 \times 2\pi r$

x^2 = 10 658 $= {}^3/_4 \times 2 \times \pi \times 73$

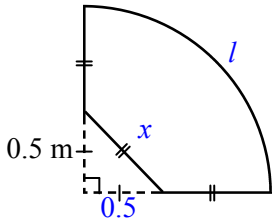
$x = \sqrt{10\,658}$ l = 344 mm

x = 103.24 mm

P = 103.24 + 344

P = 447.24 mm

9



$x^2 = 0.5^2 + 0.5^2$ r=0.5+0.71=1.21

$x^2 = 0.5$ $l = {}^1/_4 \times 2\pi r$

$x = \sqrt{0.5}$ $= {}^1/_4 \times 2 \times \pi \times 1.21$

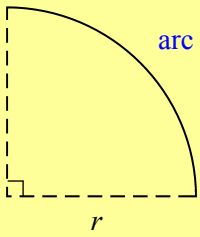
x = 0.71 m l = 1.9 m

P = 1.9 + 3 × 0.71

P = 4.03 m

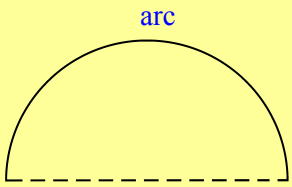
Further Perimeter Circles (Exact)

When finding the perimeter of a shape all the sides are added together. With the shapes in this worksheet an arc length is included, which has to be calculated in terms of π. This arc length will always be a fraction of a circle. For example:



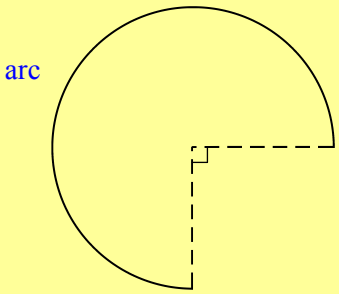
The arc length is:

$\frac{1}{4} \times 2\pi r$



The arc length is:

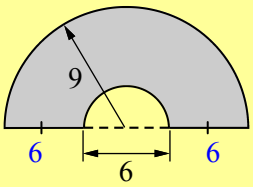
$\frac{1}{2} \times 2\pi r$ *or* $\frac{1}{2} \times \pi d$



The arc length is:

$\frac{3}{4} \times 2\pi r$

Example - Find the perimeter of the shape below



Because 18 - 6 = 12
shared equally = 6

$P = \frac{1}{2} \times 2\pi r + \frac{1}{2} \times \pi d + 2 \times 6$

Outside arc | Inside arc | Straight lengths

$= \frac{1}{2} \times 2 \times \pi \times 9 + \frac{1}{2} \times \pi \times 6 + 12$

Sub $r = 9$ | Sub $d = 6$ | $2 \times 6 = 12$

$= 9\pi + 3\pi + 12$

Solve each part

$P = 12\pi + 12$ units

Gather like terms and include 'units'

Column 1 starts with finding the perimeter of sectors, quadrants and semi-circles. These are all solved by finding the fraction of a circle shown and multiplying this by $2\pi r$. Then add the arc length with the other straight side lengths to get the perimeter.

Column 2 involve composite shapes and so include more sides in the calculation, watch for repetition to speed up your calculation remembering that two halves will make a whole circle. Note that Pythagoras Theorem is required at times.

Column 3 uses algebra-style questions, where letters are introduced rather than numbers. These may be more challenging.

Further Perimeter Circles (Exact)

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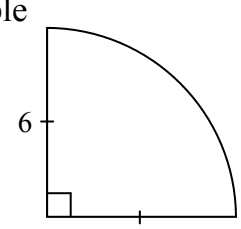
Find the perimeter in terms of π for these shapes.

Example

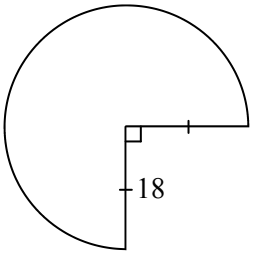


$P = \frac{1}{4}\times 2\pi r + 2r$

$= \frac{1}{4}\times 2\times\pi\times 6 + 2\times 6$

$P = 3\pi + 12$ units

1



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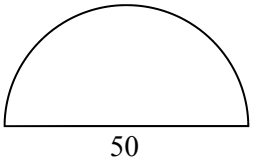

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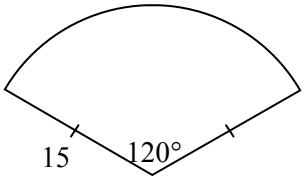

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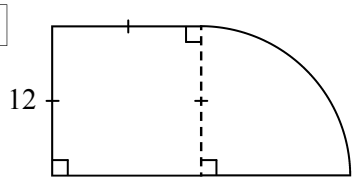

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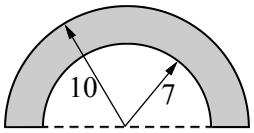

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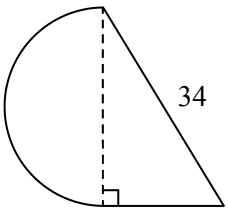

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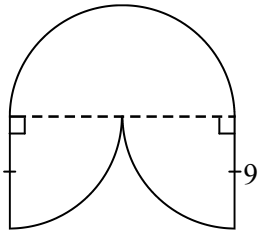
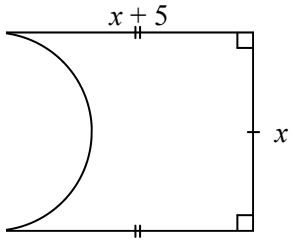


Keep finding the perimeter but this time using algebra.

8



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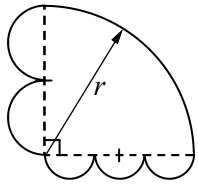

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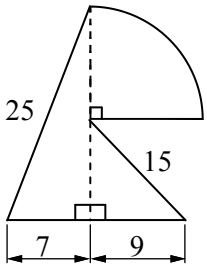

Further Perimeter Circles (Exact)

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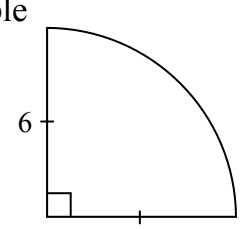
Find the perimeter in terms of π for these shapes.

Example

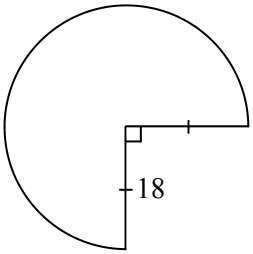


$P = \frac{1}{4}\times 2\pi r + 2r$

$= \frac{1}{4}\times 2\times\pi\times 6 + 2\times 6$

$P = 3\pi + 12$ units

1

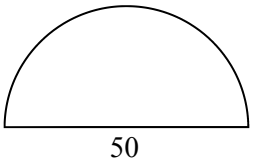


$P = \frac{3}{4}\times 2\pi r + 2r$

$= \frac{3}{4}\times 2\times\pi\times 18 + 2\times 18$

$P = 27\pi + 36$ units

2

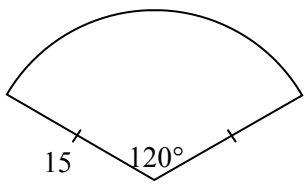


$P = \frac{1}{2}\times\pi d + d$

$= \frac{1}{2}\times\pi\times 50 + 50$

$P = 25\pi + 50$ units

3

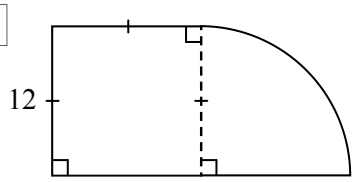


$P = \frac{1}{3}\times 2\times\pi\times r + 2\times r$

$= \frac{1}{3}\times 2\times\pi\times 15 + 2\times 15$

$P = 10\pi + 30$ units

4

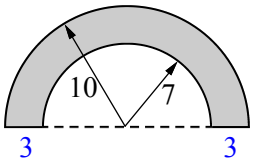


$P = \frac{1}{4}\times 2\pi r + 4r$

$= \frac{1}{4}\times 2\times\pi\times 12 + 4\times 12$

$P = 6\pi + 48$ units

5



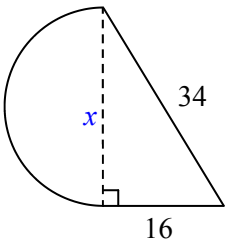
$P = \frac{1}{2}\times 2\pi R + \frac{1}{2}\times 2\pi r + 2\times 3$

$= \frac{1}{2}\times 2\times\pi\times 10 + \frac{1}{2}\times 2\times\pi\times 7 + 6$

$= 10\pi + 7\pi + 6$ units

$P = 17\pi + 6$ units

6



$x^2 = 34^2 - 16^2$

$x^2 = 900$

$x = \sqrt{900}$

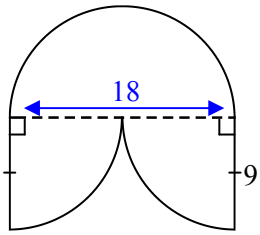
$x = 30$ units

$P = \frac{1}{2}\times\pi d + 16 + 34$

$= \frac{1}{2}\times\pi\times 30 + 50$

$P = 15\pi + 50$ units

7



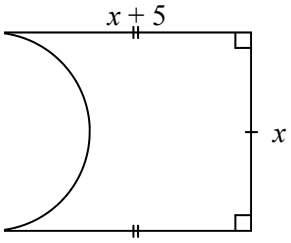
$P = \pi d + 2\times 9$

$= \pi\times 18 + 18$

$P = 18\pi + 18$ units

Keep finding the perimeter but this time using algebra.

8

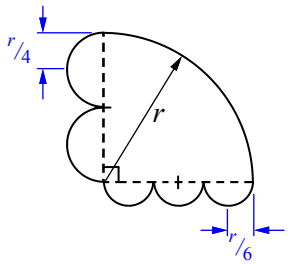


$P = \frac{1}{2}\times\pi d + 2(x + 5) + x$

$= \frac{1}{2}\times\pi\times x + 2x + 10 + x$

$P = \frac{1}{2}\pi x + 3x + 10$ units

9

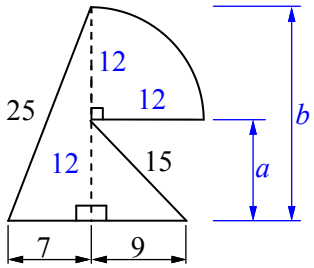


$P = \frac{1}{4}\times 2\pi r + 2\pi(\frac{r}{4}) + 1\frac{1}{2}\times 2\pi(\frac{r}{6})$

$= \frac{1}{2}\pi r + \frac{1}{2}\pi r + \frac{1}{2}\pi r$

$= \frac{3}{2}\pi r$ units or $1\frac{1}{2}\pi r$ units

10



$a^2 = 15^2 - 9^2$ | $b^2 = 25^2 - 7^2$

$a^2 = 144^2$ | $b^2 = 576^2$

$a = \sqrt{144}$ | $b = \sqrt{576}$

$a = 12$ units | $b = 24$ units

$P = \frac{1}{4}\times 2\pi r + 25 + 16 + 15 + r$

$= \frac{1}{4}\times 2\times\pi\times 12 + 56 + 12$

$P = 6\pi + 68$ units

Exact & Estimate - Area

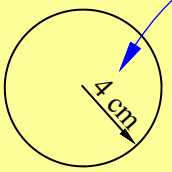
The area of a circle is the measurement of space inside the circle. Formula:

$$A = \pi r^2$$

Where: A = the area (square units, same as the radius/diameter except ^{2}, ie. cm^2, m^2, mm^2)
π = Pi, it's a value on your calculator 3.14159….. Which is a bit more than 3!
r = the radius, the length from the centre to any point on the circumference

Column 1 starts with circles that have a radius shown, to find the area in terms of π (also called the exact area) use the value of r given in the circle. To make an estimate, use the exact area and substitute 3 for π.

Example



$A = \pi r^2$

$= \pi \times 4^2$ ← Replace r with 4, (the radius of the circle)

$A = 16\pi$ cm^2 ← $4^2 = 16$, remember ^{2} is the number times itself not the number times 2. Leave π as is. Write the units.

So the area is 16π cm^2. This is the exact area, to estimate the area without a calculator use $\pi = 3$. This is less than the actual value for π, but it gives a fairly accurate answer without too much calculation.

Estimate:

$A = 16\pi$ cm^2 ← Rewrite the area from previous step

$\approx 16 \times 3$ ← Replace π with 3 and = with ≈

$A \approx 48$ cm^2 ← Solve, use either ≈ or ≒ to indicate 'approximately'.

Column 2 uses diameters. Halve the diameter for the radius and solve as before, there is a formula for calculating the area using d. But it won't be introduced on this sheet.

At the bottom of the page are four circles. Measure the diameter accurate to the nearest cm, then calculate the exact area (in terms of π) and then an estimate, again using $\pi = 3$. Use the working out spaces in the third column to answer these.

Exact and Estimate - Area

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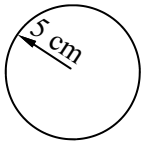
Find the area in terms of π. Then using π = 3, estimate the area.

1 $A = \pi r^2$

$= \pi \times \square^2$

$A = \square\ \pi\ cm^2$

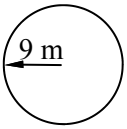


Estimate: $A = \square\ \pi\ cm^2$

$\approx \square \times 3$

$A \approx \square\ cm^2$

2



$A = \pi r^2$

$=$

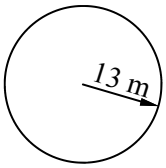
$A =$

Estimate: $A =$

$\approx$

$A \approx$

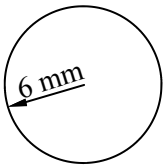
3



Estimate:

$\square \times 3$

4



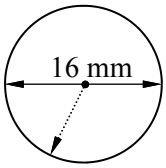
Estimate:

5 Circle radius of 7 km

Exact: Estimate:

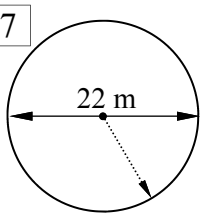
Divide the diameter by 2 to find the radius, then find the exact and estimate.

6



Estimate:

7



Estimate:

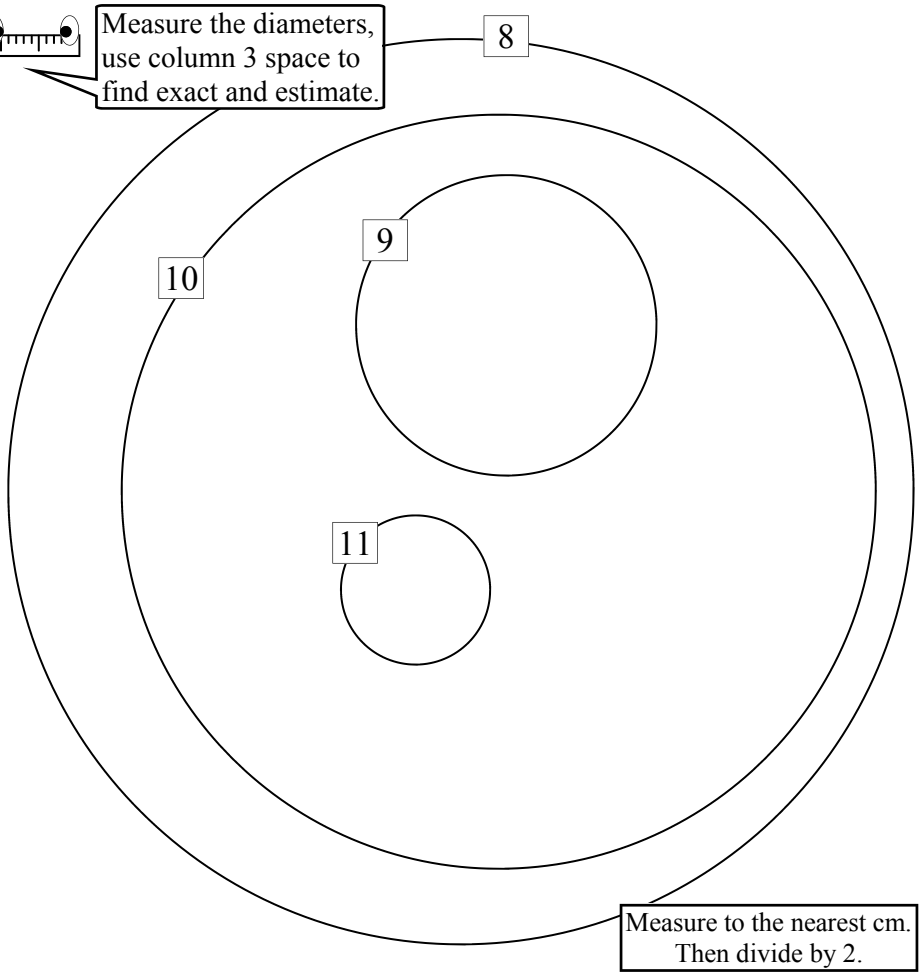
8

9

10

11

Measure the diameters, use column 3 space to find exact and estimate.



Measure to the nearest cm. Then divide by 2.

Exact and Estimate - Area

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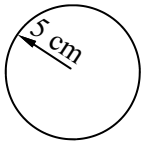
Find the area in terms of π. Then using π = 3, estimate the area.

1

$A = \pi r^2$

$= \pi \times 5^2$

$A = 25\pi$ cm^2

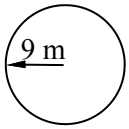


Estimate: $A = 25\pi$ cm^2

$\approx 25 \times 3$

$A \approx 75$ cm^2

2



$A = \pi r^2$

$= \pi \times 9^2$

$A = 81\pi$ m^2

Estimate: $A = 81\pi$ m^2

$\approx 81 \times 3$

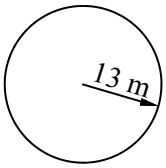
$A \approx 243$ m^2

3

$A = \pi r^2$

$= \pi \times 13^2$

$A = 169\pi$ m^2



$169 \times 3 = 507$

$A = 169\pi$ m^2

$\approx 169 \times 3$

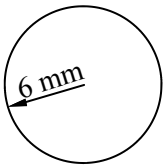
$A \approx 243$ m^2

4

$A = \pi r^2$

$= \pi \times 6^2$

$A = 36\pi$ mm^2



Estimate: $A = 36\pi$ m^2

$\approx 36 \times 3$

$A \approx 108$ mm^2

5 Circle radius of 7 km

Exact:

$A = \pi r^2$

$= \pi \times 7^2$

$A = 49\pi$ km^2

Estimate:

$A = 49\pi$ m^2

$\approx 49 \times 3$

$A \approx 147$ km^2

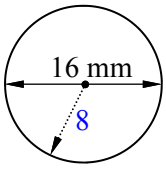
Divide the diameter by 2 to find the radius, then find the exact and estimate.

6

$A = \pi r^2$

$= \pi \times 8^2$

$A = 64\pi$ mm^2

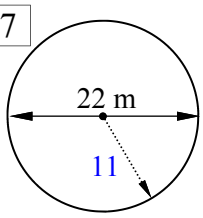


Estimate: $A = 64\pi$ mm^2

$\approx 64 \times 3$

$A \approx 192$ mm^2

7



$A = \pi r^2$

$= \pi \times 11^2$

$A = 121\pi$ m^2

Estimate: $A = 121\pi$ m^2

$\approx 121 \times 3$

$A \approx 363$ m^2

8

$A = \pi r^2$

$= \pi \times 6^2$

$A = 36\pi$ cm^2

$A = 36\pi$ cm^2

$\approx 36 \times 3$

$A \approx 108$ cm^2

9

$A = \pi r^2$

$= \pi \times 2^2$

$A = 4\pi$ cm^2

$A = 4\pi$ cm^2

$\approx 4 \times 3$

$A \approx 12$ cm^2

10

$A = \pi r^2$

$= \pi \times 5^2$

$A = 25\pi$ cm^2

$A = 25\pi$ cm^2

$\approx 25 \times 3$

$A \approx 75$ cm^2

11

$A = \pi r^2$

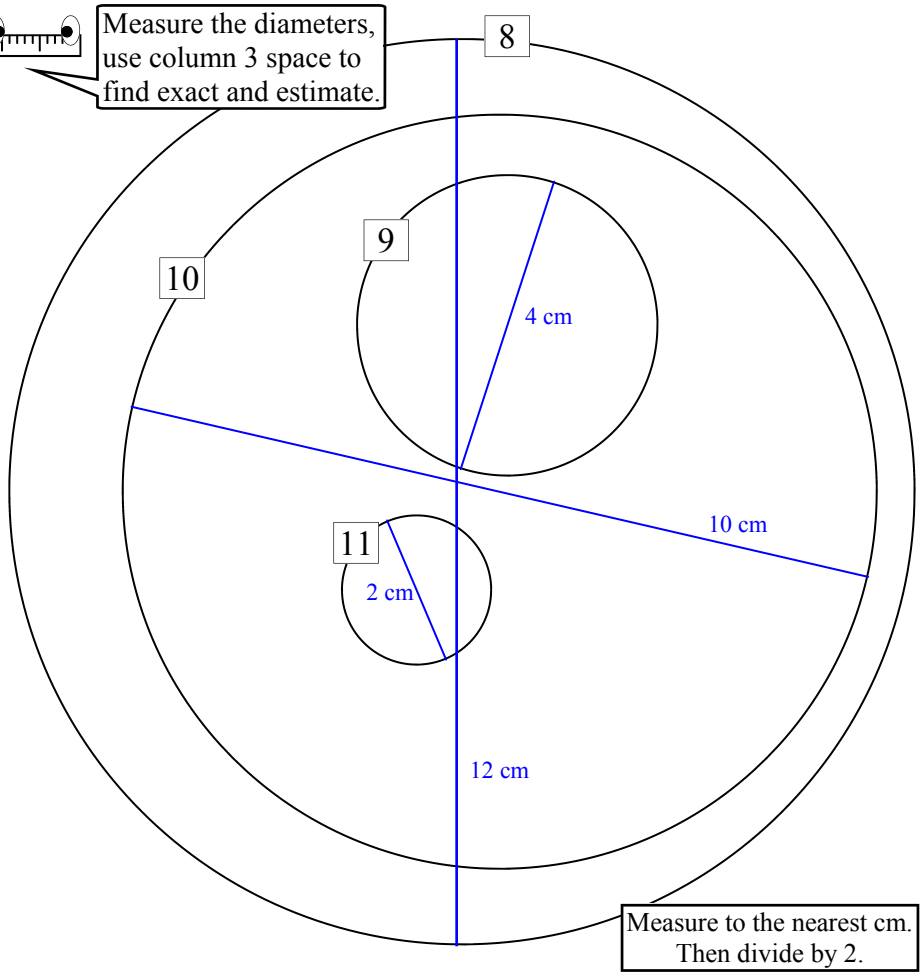
$= \pi \times 1^2$

$A = \pi$ cm^2

$A = \pi$ cm^2

$A \approx 3$ cm^2

Measure the diameters, use column 3 space to find exact and estimate.



Measure to the nearest cm. Then divide by 2.

Area of a Circle (Sheet 1)

The formula for calculating the area of a circle is:

$$A = \pi r^2$$

Where:

A = Area of the circle, in the same units as the radius, but squared. (cm^2, m^2, mm^2)
r = The radius of the circle (distance from centre to circumference).

Column 1 features circles with the radius shown. Use the area formula above to calculate the answer. The answer should be rounded to 2 decimal places and the units should be shown as square units. There is an example at the top of the column.

Column 2 is the same as Column 1 except for one additional step. Instead of the radius you are given the diameter. So convert the diameter to a radius by dividing it by 2, write the radius in the space provided, either in or beside the circle. Then solve the questions the same way, but round to one decimal place this time.

Column 3 starts with a mixture of questions that are either radius or diameter, so the questions are solved the same way as either Column 1 or Column 2.

At the bottom of the column a circle is fit perfectly into a square, so the circle's diameter is the same as the side length of the square. Calculate the square's total area (as if the circle wasn't there), then calculate the circle's area. Then calculate the %Area that the circle accommodates within the square, by multiplying the fraction by 100. Did you expect the circle to take up more space, or less?

Area of a Circle (Sheet 1)

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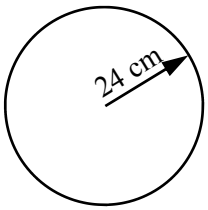
Calculate the area of these circles, round to 2 decimal places.

Example

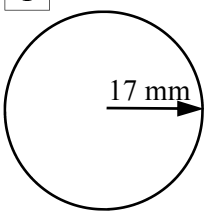


$A = \pi r^2$

$= \pi \times 24^2$

$A = 1\,809.56 \text{ cm}^2$

1



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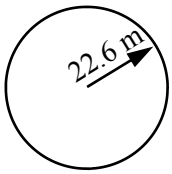

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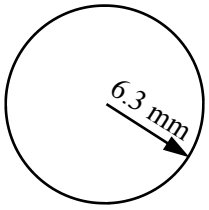

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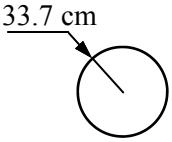

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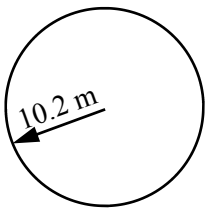

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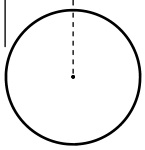
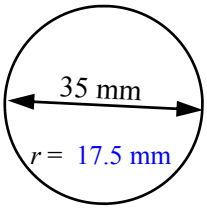


Calculate the radius then the area of these circles, round to 1 d.p.

Example



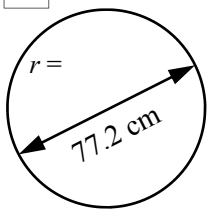
$r = 17.5$ mm

$A = \pi r^2$

$= \pi \times 17.5^2$

$A = 962.1 \text{ mm}^2$

7



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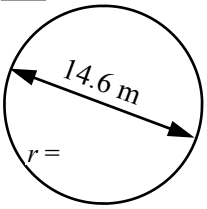

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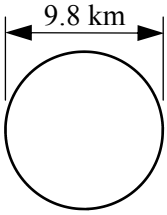
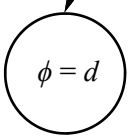


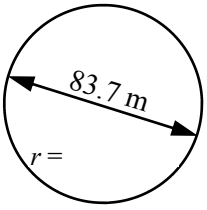
$r =$

10



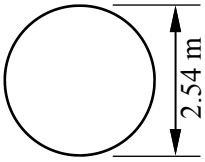
$r =$

11



$r =$

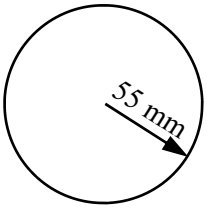
12



$r =$

Find the area to 1 d.p., watch for diameters.

13



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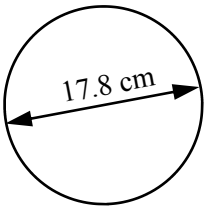

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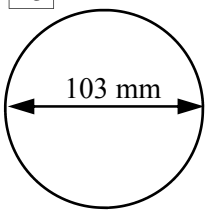

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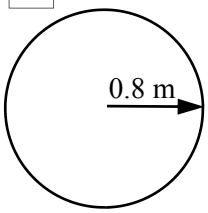
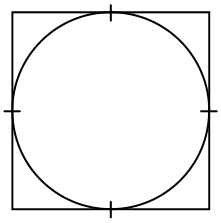


Find the area of the …..

17 Square

$A = l^2$



18 Circle

19 Find $\frac{A_{CIRCLE}}{A_{SQUARE}}$ as a percentage

Round to 1 d.p.

Area of a Circle (Sheet 1)

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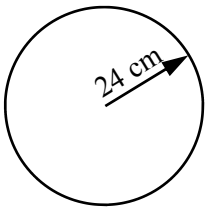
Calculate the area of these circles, round to 2 decimal places.

Example

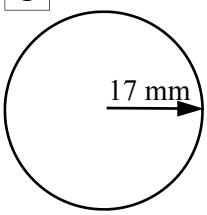


$A = \pi r^2$

$= \pi \times 24^2$

$A = 1\,809.56 \text{ cm}^2$

1

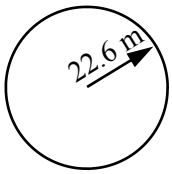


$A = \pi r^2$

$= \pi \times 17^2$

$A = 907.93 \text{ mm}^2$

2



$A = \pi r^2$

$= \pi \times 22.6^2$

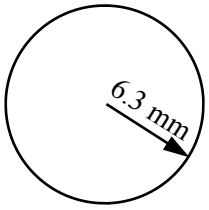
$A = 1\,604.60 \text{ m}^2$

3

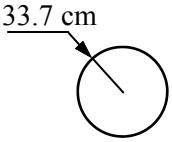
$A = \pi r^2$

$= \pi \times 6.3^2$

$A = 124.70 \text{ mm}^2$



4



$A = \pi r^2$

$= \pi \times 33.7^2$

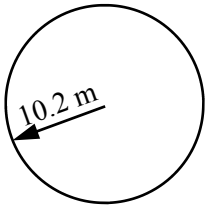
$A = 3\,567.88 \text{ cm}^2$

5

$A = \pi r^2$

$= \pi \times 10.2^2$

$A = 326.85 \text{ m}^2$

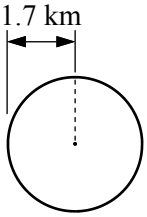


6

$A = \pi r^2$

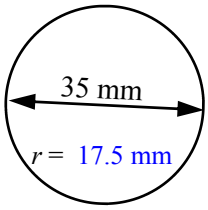
$= \pi \times 1.7^2$

$A = 9.08 \text{ km}^2$



Calculate the radius then the area of these circles, round to 1 d.p.

Example

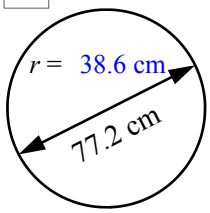


$A = \pi r^2$

$= \pi \times 17.5^2$

$A = 962.1 \text{ mm}^2$

7

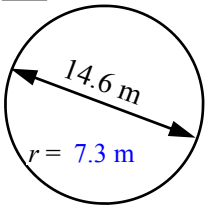


$A = \pi r^2$

$= \pi \times 38.6^2$

$A = 4\,680.85 \text{ m}^2$

8



$A = \pi r^2$

$= \pi \times 7.3^2$

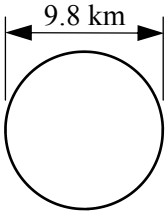
$A = 167.42 \text{ m}^2$

9

$A = \pi r^2$

$= \pi \times 4.9^2$

$A = 75.43 \text{ km}^2$



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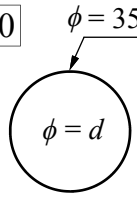


$A = \pi r^2$

$= \pi \times 17.5^2$

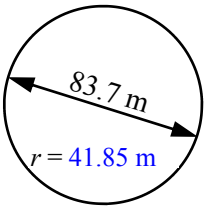
$A = 962.11 \text{ cm}^2$

11

$A = \pi r^2$

$= \pi \times 41.85^2$

$A = 5\,502.26 \text{ m}^2$

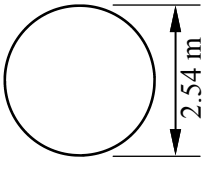


12

$A = \pi r^2$

$= \pi \times 1.27^2$

$A = 5.07 \text{ m}^2$



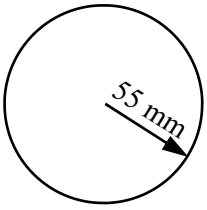
Find the area to 1 d.p., watch for diameters.

13

$A = \pi r^2$

$= \pi \times 55^2$

$A = 9\,503.3 \text{ m}^2$

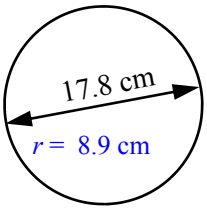


14

$A = \pi r^2$

$= \pi \times 8.9^2$

$A = 248.8 \text{ cm}^2$



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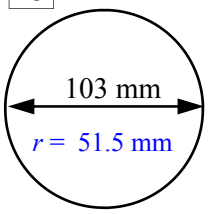
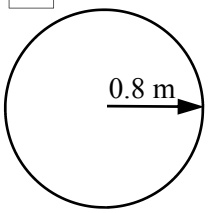


$A = \pi r^2$

$= \pi \times 51.5^2$

$A = 8\,332.3 \text{ mm}^2$

16



$A = \pi r^2$

$= \pi \times 0.8^2$

$A = 2 \text{ m}^2$

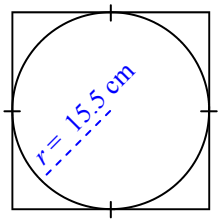
Find the area of the …..

17 Square

$A = l^2$

$= 31^2$

$A = 961 \text{ cm}^2$



18 Circle $A = \pi r^2$

$= \pi \times 15.5^2$

$A = 754.8 \text{ cm}^2$

19 Find $\frac{A_{CIRCLE}}{A_{SQUARE}}$ as a percentage

$\text{Percentage} = \frac{754.8}{961} \times 100$

$\text{Percentage} = 78.5\%$

Round to 1 d.p.

Area of a Circle (Sheet 2)

The formula for calculating the area of a circle is:

$$A = \pi r^2$$

Where:

A = Area of the circle, in the same units as the radius, but squared. (cm^2, m^2, mm^2)
r = The radius of the circle (distance from centre to circumference).

This sheet uses three values for π: Decimal 3.14; Fraction $^{22}/_{7}$ and Calculator 3.14159….

Column 1 features circles with the radius shown. Use the area formula above to calculate the answer. But for the first part of the column use $\pi = 3.14$ to calculate your answer. Then round to 1 decimal place. The second part of the column use $\pi = {}^{22}/_{7}$ to find your answer rounded to the nearest whole number. Note that your answer could be an improper fraction or a mixed number depending on the calculator you use. Press the fraction key after the fractional answer is displayed to convert it to a decimal. Decimals are easier to round for most students.

Column 2 requires you to halve the diameter first to get a radius, write the radius on the circle next to the dotted line, or draw your own line. Then solve the same way as column 1 except round to 2 decimal places, note that you use $\pi = 3.14$.

Two tables are then shown. Divide the diameter by 2 to get the radius column, then calculate the area. Note that the first table uses $\pi = {}^{22}/_{7}$, all lengths are in cm. The second table requires you to use the π key on your calculator, answer in m^2 rounded to 2 decimal places.

Column 3 asks you to find the radius given the area. First it asks you to manipulate the area formula to make r the subject. The steps are shown, see how you go. The rest of the column uses the formula you created to answer the rest of the column. If you couldn't rearrange the formula don't worry, Fizzle tells you the formula above question 24.

Note that you have to use brackets as the calculator step shows in question 24, or work out the inside first, press the 'equals button' then the 'square root button' and 'equals' again.

Area of a Circle (Sheet 2)

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Use $A=\pi r^2$ with $\pi = 3.14$, to find the area. Round your answer to 1 decimal place.

Example

$A = \pi r^2$

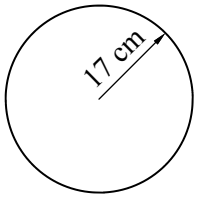
Write formula

$= 3.14 \times 17^2$

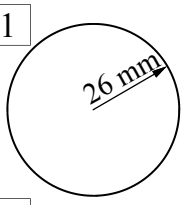
Substitute 3.14 and r

$A = 907.5\ cm^2$

Answer 1 d.p., show units^2



1



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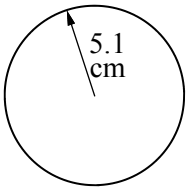

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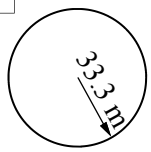
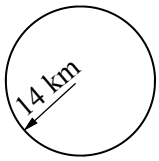


Now use $\pi = {}^{22}/_{7}$, to find the area. Round to the nearest whole number.

4

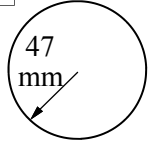
$A = \pi r^2$

$= {}^{22}/_{7} \times$



Press $a^{b}/_{c}$ key to change fractions to decimals

5



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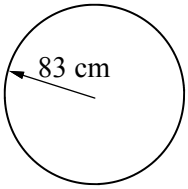
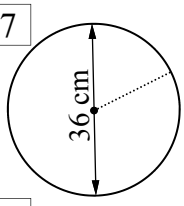


These are diameters, halve them then use $A = \pi r^2$ & $\pi = 3.14$, to find area. 2 d.p.

7



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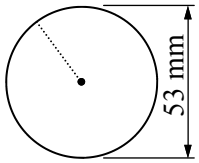

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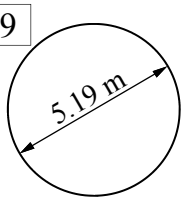

Find r then use $\pi = {}^{22}/_{7}$, to complete the table to 1 d.p. All lengths are in cm.

Q.	d	r	Area (cm^2)
10	15		
11	87		
12	11.9		
13	41		
14	109		
15	60.7		

Now use the π key, to complete the table to 2 d.p. All units are in metres.

Q.	d	r	Area (m^2)
16	3.9		
17	27		
18	8.3		
19	65		
20	11.3		
21	56.8		
22	70.2		

Rearrange the area formula to make r the subject. The steps are shown in circles.

23 $A = \pi r^2$

$\div \pi$ $\div \pi$ Divide both sides by π

$\sqrt{\ }$ $\sqrt{\ }$ Square root both sides

$\Rightarrow$

Rewrite with r on the L.H.S

Using $r = \sqrt{\frac{A}{\pi}}$ find the radius given the area. To 1 d.p.

24 Circle area = 132.7 m^2

$\sqrt{\ }$ (1 3 2 • 7 ÷ π) =

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{\square}{\pi}}$

$=$ m

25 Circle area = 2 273.3 cm^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{\square}{\pi}}$

26 Circle area = 422.7 mm^2

27 Circle area = 2 498.3 m^2

radius diameter

28 Circle area = 23 506.2 km^2

radius diameter

Area of a Circle (Sheet 2)

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Use $A=\pi r^2$ with $\pi = 3.14$, to find the area. Round your answer to 1 decimal place.

Example

$A = \pi r^2$

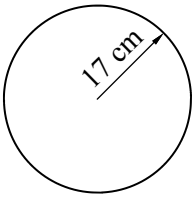
Write formula

$= 3.14 \times 17^2$

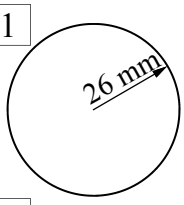
Substitute 3.14 and r

$A = 907.5 \text{ cm}^2$

Answer 1 d.p., show units^2



1



$A = \pi r^2$

$= 3.14 \times 26^2$

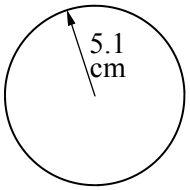
$A = 2\,122.6 \text{ mm}^2$

2

$A = \pi r^2$

$= 3.14 \times 5.1^2$

$A = 81.7 \text{ cm}^2$



3



$A = \pi r^2$

$= 3.14 \times 33.3^2$

$A = 3\,481.9 \text{ m}^2$

Now use $\pi = {}^{22}/_{7}$, to find the area. Round to the nearest whole number.

4

$A = \pi r^2$

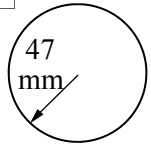
$= {}^{22}/_{7} \times 14^2$

$A = 616 \text{ km}^2$



Press $a^b/_c$ key to change fractions to decimals

5



$A = \pi r^2$

$= {}^{22}/_{7} \times 47^2$

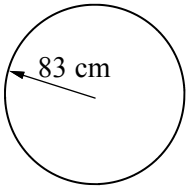
$A = 6\,943 \text{ mm}^2$

6

$A = \pi r^2$

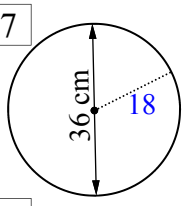
$= {}^{22}/_{7} \times 83^2$

$A = 21\,651 \text{ cm}^2$



These are diameters, halve them then use $A = \pi r^2$ & $\pi = 3.14$, to find area. 2 d.p.

7



$A = \pi r^2$

$= 3.14 \times 18^2$

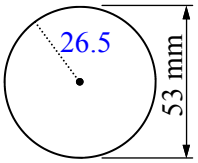
$A = 1\,017.36 \text{ cm}^2$

8

$A = \pi r^2$

$= 3.14 \times 26.5^2$

$A = 2\,205.07 \text{ mm}^2$



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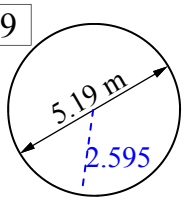


$A = \pi r^2$

$= 3.14 \times 2.595^2$

$A = 21.14 \text{ m}^2$

Find r then use $\pi = {}^{22}/_{7}$, to complete the table to 1 d.p. All lengths are in cm.

Q.	d	r	Area (cm^2)
10	15	7.5	176.8 cm^2
11	87	43.5	5 947.1 cm^2
12	11.9	5.95	111.3 cm^2
13	41	20.5	1 320.8 cm^2
14	109	54.5	9 335.1 cm^2
15	60.7	30.35	2 895 cm^2

Now use the π key, to complete the table to 2 d.p. All units are in metres.

Q.	d	r	Area (m^2)
16	3.9	1.95	11.95 m^2
17	27	13.5	572.56 m^2
18	8.3	4.15	54.11 m^2
19	65	32.5	3 318.31 m^2
20	11.3	5.65	100.29 m^2
21	56.8	28.4	2 533.88 m^2
22	70.2	35.1	3 870.47 m^2

Rearrange the area formula to make r the subject. The steps are shown in circles.

23 $A = \pi r^2$

$\div \pi$ $\div \pi$ Divide both sides by π

$\frac{A}{\pi} = r^2$

$\sqrt{\ }$ $\sqrt{\ }$ Square root both sides

$\sqrt{\frac{A}{\pi}} = r \Rightarrow r = \sqrt{\frac{A}{\pi}}$

Rewrite with r on the L.H.S

Using $r = \sqrt{\frac{A}{\pi}}$ find the radius given the area. To 1 d.p.

24 Circle area = 132.7 m^2

$\sqrt{\ }$ (1 3 2 • 7 ÷ π) =

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{132.7}{\pi}}$

$= 6.5 \text{ m}$ (1d.p.)

25 Circle area = 2 273.3 cm^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{2\,273.3}{\pi}}$

$= 26.9 \text{ cm}$ (1d.p.)

26 Circle area = 422.7 mm^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{422.7}{\pi}}$

$= 11.6 \text{ mm}$ 1d.p.

27 Circle area = 2 498.3 m^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{2\,498.3}{\pi}}$

$= 28.2 \text{ m}$ (1d.p.) radius

$= 56.4 \text{ m}$ (1d.p.) diameter

28 Circle area = 23 506.2 km^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{23\,506.2}{\pi}}$

$= 86.5 \text{ km}$ (1d.p.) radius

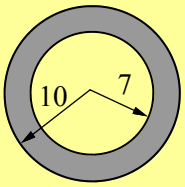
$= 173 \text{ km}$ (1d.p.) diameter

The Annulus

An annulus is a ring-like figure, in other words a circle within another circle. Most students recognise the similarity with the word 'anus', as well as the similar appearance. For this reason it is usually an easy word to remember!

Column 1 deals with calculating the area of an annulus. The process is to calculate the outside circle's area, then calculate the inside circle's area, then subtract them. An example is below.

Example - Area of an Annulus



Lengths are in cm

$A_1 = \pi R^2$

$= \pi \times 10^2$

$A_1 = 314.2 \text{ cm}^2$ (1 dp)

Capital *R*, means 'big'.

So A_1 is the area of the outside circle.

$A_2 = \pi r^2$

$= \pi \times 7^2$

$A_2 = 153.9 \text{ cm}^2$ (1 dp)

Lower case *r*, means 'small'.

So A_2 is the area of the inside circle.

$A = A_1 - A_2 = 314.2 - 153.9$

$A = 160.3 \text{ cm}^2$

Then subtract for the annulus area. The larger area - smaller area.

Column 2 introduces the annulus formula. This is a faster way of solving exercises but it is important that you understand the process above as well. The formula is:

$$A = \pi (R^2 - r^2)$$

Where:

R = outside circle's radius
r = inside circle's radius

This formula is created by factorising $\pi R^2 - \pi r^2$. If you take out π you get the above formula.

Column 2 starts with assistance in showing the working out. After the first 3 questions fractions need to be used because the area for 'fractions of' annuli are required. Use the internal angle within the annulus, then multiply by the angle over 360°.

So if the angle was 90°, you would use $A = {}^{90}/_{360} \times \pi(R^2 - r^2)$ but you may also recognize that this is the same as $A = \frac{1}{4} \times \pi(R^2 - r^2)$.

The Annulus

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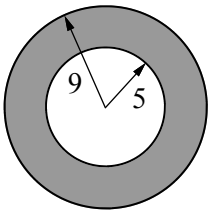
Find the outside area and the inside area, then subtract to find the shaded area. (1 d.p.)

All measurements are in cm

1 $A_1 = \pi R^2$ $= \pi \times 9^2$ $A_1 =$ ☐ cm^2 (1 dp)

$A_2 = \pi r^2$ $= \pi \times 5^2$ $A_2 =$ ☐ cm^2 (1 dp)

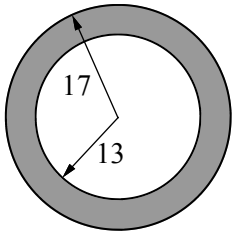


$A = A_1 - A_2 =$ ☐ - ☐

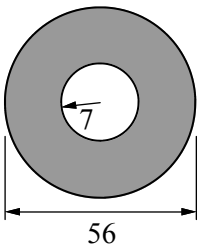
$A =$ ☐ cm^2 (1 dp)

2 $A_1 = \pi R^2$ $= \pi \times$ ☐^{2} $A_1 =$ ☐ cm^2 (1 dp)

$A_2 = \pi r^2$ $= \pi \times$ ☐^{2} $A_2 =$ ☐ cm^2 (1 dp)



3 Watch for diameters!



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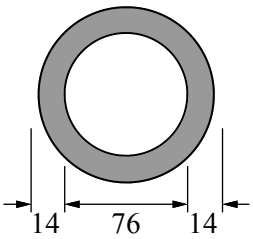
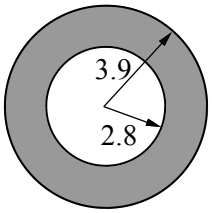


Now use the Annulus formula $A = \pi(R^2 - r^2)$. All lengths are in metres. Answer to 1 d.p.

5

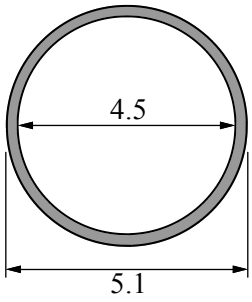


$A = \pi(R^2 - r^2)$

$= \pi \times$ (☐^{2} - ☐^{2})

$A =$ ☐ m^2 (1 dp)

6



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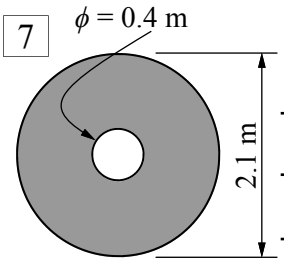
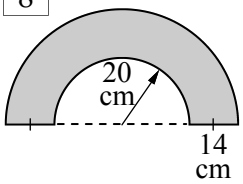


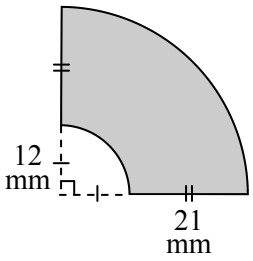
Use the Annulus formula, but put a fraction before the π, to find the shaded area. (1 d.p.)

8



$A =$ ☐/☐ $\times \pi(R^2 - r^2)$

9



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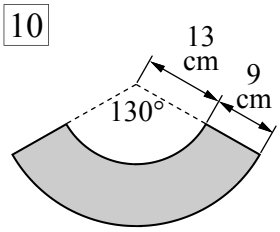

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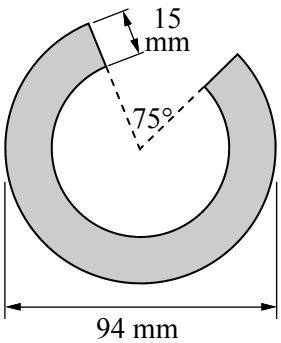

The Annulus

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Find the outside area and the inside area, then subtract to find the shaded area. (1 d.p.)

All measurements are in cm

1

$A_1 = \pi R^2$

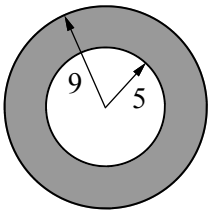
$= \pi \times 9^2$

$A_1 =$ 254.5 cm^2 (1 dp)

$A_2 = \pi r^2$

$= \pi \times 5^2$

$A_2 =$ 78.5 cm^2 (1 dp)



$A = A_1 - A_2 =$ 254.5 - 78.5

$A =$ 176 cm^2 (1 dp)

2

$A_1 = \pi R^2$

$= \pi \times$ 17 ^{2}

$A_1 =$ 907.9 cm^2 (1 dp)

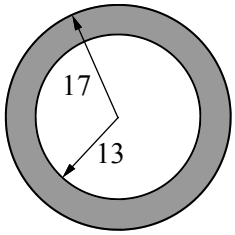
$A_2 = \pi r^2$

$= \pi \times$ 13 ^{2}

$A_2 =$ 530.9 cm^2 (1 dp)

$A = A_1 - A_2 = 907.9 - 530.9$

$A = 377\ cm^2$



3 Watch for diameters!

$A_1 = \pi R^2$

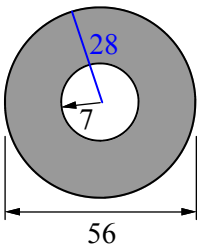
$= \pi \times 28^2$

$A_1 = 2\,463\ cm^2$ (1 dp)

$A_2 = \pi r^2$

$= \pi \times 7^2$

$A_2 = 153.9\ cm^2$ (1 dp)



$A = A_1 - A_2 = 2\,463 - 153.9$

$A = 2\,309.1\ cm^2$

4

$A_1 = \pi R^2$

$= \pi \times 52^2$

$A_1 = 8\,494.9\ cm^2$ (1 dp)

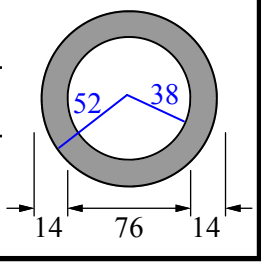
$A_2 = \pi r^2$

$= \pi \times 38^2$

$A_2 = 4\,536.5\ cm^2$ (1 dp)

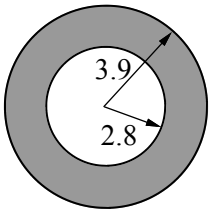
$A = A_1 - A_2 = 8\,494.9 - 4\,536.5$

$A = 3\,958.4\ cm^2$



Now use the Annulus formula $A = \pi(R^2 - r^2)$. All lengths are in metres. Answer to 1 d.p.

5



$A = \pi(R^2 - r^2)$

$= \pi \times$ (3.9 ^{2} - 2.8 ^{2})

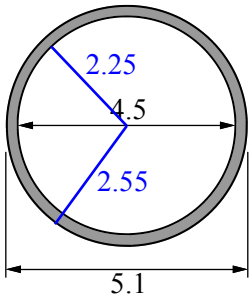
$A =$ 23.2 m^2 (1 dp)

6

$A = \pi(R^2 - r^2)$

$= \pi \times (2.55^2 - 2.25^2)$

$A = 4.5\ m^2$



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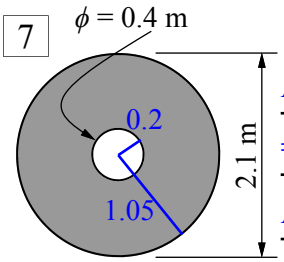


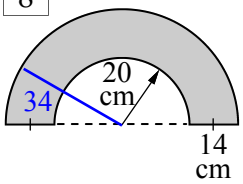
$A = \pi(R^2 - r^2)$

$= \pi \times (1.05^2 - 0.2^2)$

$A = 3.3\ m^2$

Use the Annulus formula, but put a fraction before the π, to find the shaded area. (1 d.p.)

8



$A = \frac{1}{2} \times \pi(R^2 - r^2)$

$= ½ \times \pi \times (34^2 - 20^2)$

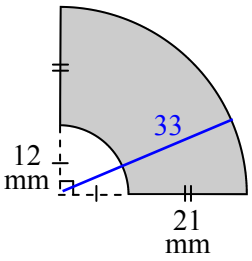
$A = 1\,187.5\ cm^2$

9

$A = ¼ \times \pi(R^2 - r^2)$

$= ¼ \times \pi \times (33^2 - 12^2)$

$A = 742.2\ mm^2$



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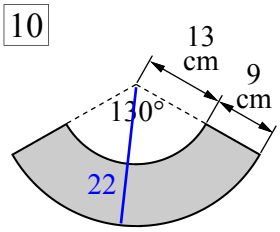


$A = {}^{130}/_{360} \times \pi(R^2 - r^2)$

$= {}^{130}/_{360} \times \pi \times (22^2 - 13^2)$

$A = 357.4\ cm^2$

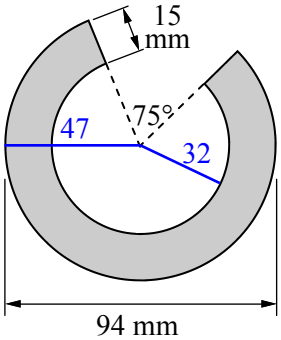
11

Internal angle = 360 - 75 = 285°

$A = {}^{285}/_{360} \times \pi(R^2 - r^2)$

$= {}^{285}/_{360} \times \pi \times (47^2 - 32^2)$

$A = 2\,947.2\ mm^2$



Area of Sectors, Quadrants And Semi-Circles

If you know how to multiply by a fraction and how to find the area of a circle, then this worksheet simply blends the two.

The formula for calculating the area of a circle is:

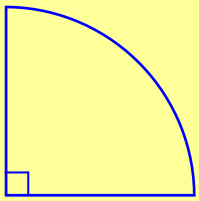
$$A = \pi r^2$$

Where:

A = Area of the circle, in the same units as the radius, but squared. (cm^2, m^2, mm^2)
r = The radius of the circle (distance from centre to circumference).

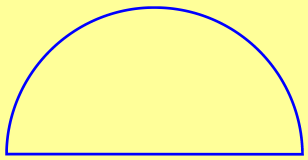
But with semi-circles (half a circle), quadrants (quarter of a circle) and sectors the above formula is multiplied by a fraction.

Quadrants



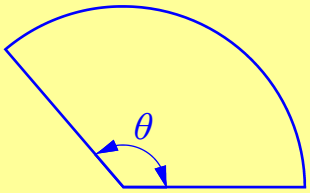
$$A = \frac{1}{4}\pi r^2$$

Semi-Circles



$$A = \frac{1}{2}\pi r^2$$

Sectors



$$A = \frac{\theta}{360} \times \pi r^2$$

Column 1 starts with semi-circles, so use the middle formula above, in other words multiply πr^2 by ½. Question 5 sees a change to quadrants, use the first formula shown above, multiply πr^2 by ¼.

Column 2 again features semi-circles, but this time instead of being given the radius you are given a diameter, so find the radius (by dividing the diameter by 2) then solve as before.

Sectors are then introduced at the bottom of the column. The internal angle (commonly called 'pheta') is written over 360, then simplified with your calculator. The calculator steps for simplifying are shown on the sheet if you have forgotten the process.

Then sector area is required to be found. Use the third formula above, so make a fraction from the internal angle and 360 as the denominator. Multiply this by the area formula and you are done!

Area of Sectors, Quadrants And Semi-Circles

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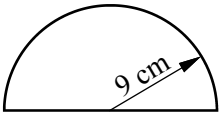
Calculate the area inside these semi-circles and quadrants. (1 d.p.)

1



$A = \frac{1}{2}\pi r^2$

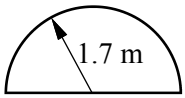
$= \frac{1}{2} \times \pi \times \square^2$

$A = \square$ cm^2

Units are square units

2

$A = \frac{1}{2}\pi r^2$



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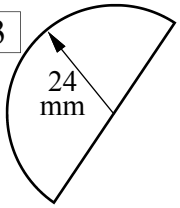

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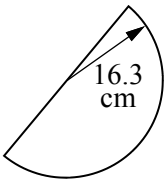

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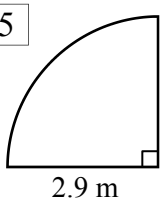
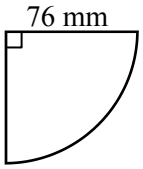
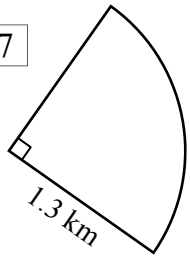


$A = \frac{1}{4}\pi r^2$

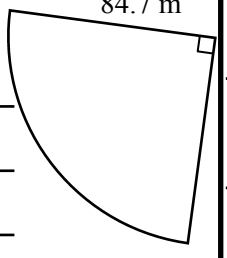
6



7

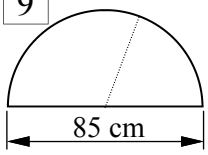


8



Find the area, but find the radius first. Answer to 1 dp.

9



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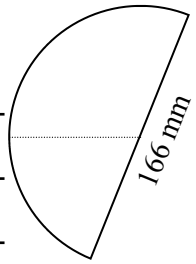

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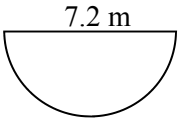
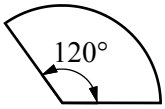


Write the fraction of a circle with a denominator of 360, then simplify.

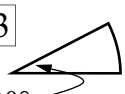
Calc: 1 2 0 $a^b/_c$ 3 6 0 =

12



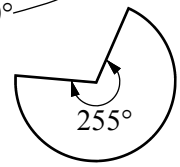
$\frac{\square}{360} = \frac{\square}{\square}$

13



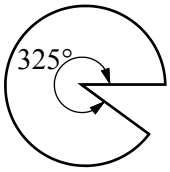
$\frac{\square}{\square} = \frac{\square}{\square}$

14



$\frac{\square}{\square} = \frac{\square}{\square}$

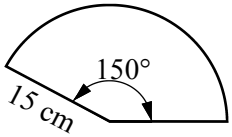
15



$\frac{\square}{\square} = \frac{\square}{\square}$

Find the areas of these sectors. Round to 2 d.p.

16

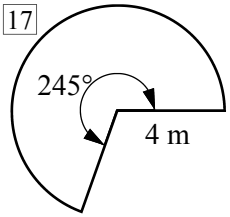


$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{\square}{\square} \times \pi \times \square^2$

$A =$

17



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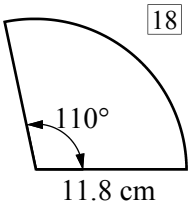

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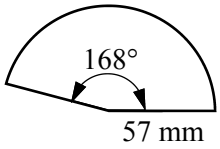

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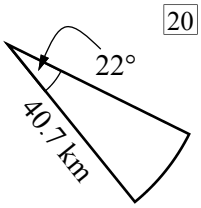

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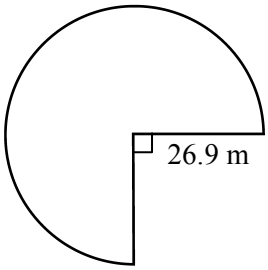

Area of Sectors, Quadrants And Semi-Circles

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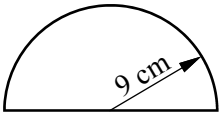
Calculate the area inside these semi-circles and quadrants. (1 d.p.)

1



$A = \frac{1}{2}\pi r^2$

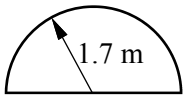
$= \frac{1}{2} \times \pi \times 9^2$

$A = 127.2$ cm^2

Units are square units

2

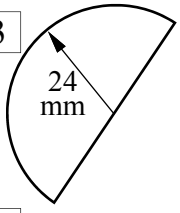
$A = \frac{1}{2}\pi r^2$



$= \frac{1}{2} \times \pi \times 1.7^2$

$A = 4.5$ m^2

3



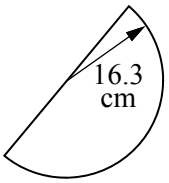
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 24^2$

$A = 904.8$ mm^2

4

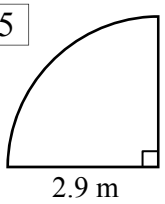
$A = \frac{1}{2}\pi r^2$



$= \frac{1}{2} \times \pi \times 16.3^2$

$A = 417.3$ cm^2

5



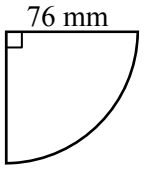
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 2.9^2$

$A = 6.6$ m^2

6

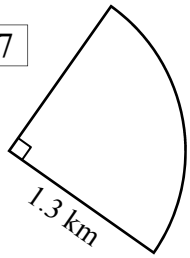
$A = \frac{1}{4}\pi r^2$



$= \frac{1}{4} \times \pi \times 76^2$

$A = 4\,536.5$ mm^2

7

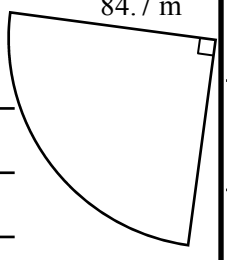


$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 1.3^2$

$A = 1.3$ km^2

8



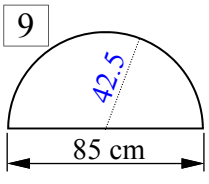
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 84.7^2$

$A = 5\,634.5$ m^2

Find the area, but find the radius first. Answer to 1 dp.

9



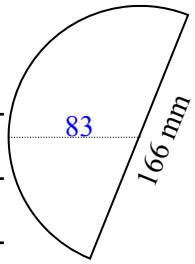
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 42.5^2$

$A = 2\,837.3$ cm^2

10

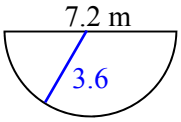
$A = \frac{1}{2}\pi r^2$



$= \frac{1}{2} \times \pi \times 83^2$

$A = 10\,821.2$ mm^2

11



$A = \frac{1}{2}\pi r^2$

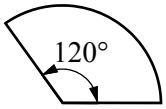
$= \frac{1}{2} \times \pi \times 3.6^2$

$A = 20.4$ m^2

Write the fraction of a circle with a denominator of 360, then simplify.

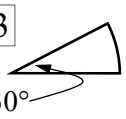
Calc: 1 2 0 a$^b/_c$ 3 6 0 =

12



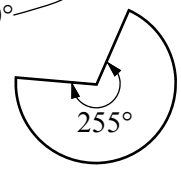
$\frac{120}{360} = \frac{1}{3}$

13



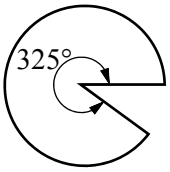
$\frac{30}{360} = \frac{1}{12}$

14



$\frac{255}{360} = \frac{17}{24}$

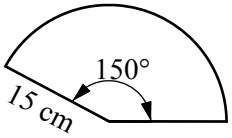
15



$\frac{325}{360} = \frac{65}{72}$

Find the areas of these sectors. Round to 2 d.p.

16



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{150}{360} \times \pi \times 15^2$

$A = 294.52$ cm^2

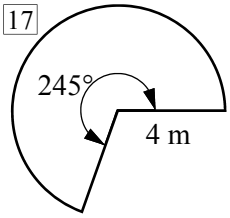
17

$A = {}^{\theta}/_{360} \times \pi r^2$

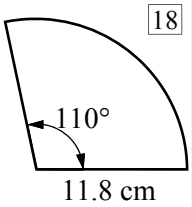
$= {}^{245}/_{360} \times \pi \times 4^2$

$A = 34.21$ m^2

17



18



18

$A = {}^{\theta}/_{360} \times \pi r^2$

$= {}^{110}/_{360} \times \pi \times 11.8^2$

$A = 3133.66$ cm^2

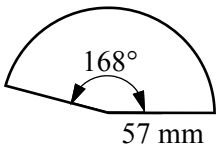
19

$A = {}^{\theta}/_{360} \times \pi r^2$

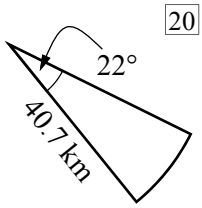
$= {}^{168}/_{360} \times \pi \times 57^2$

$A = 4\,763.28$ mm^2

19



20



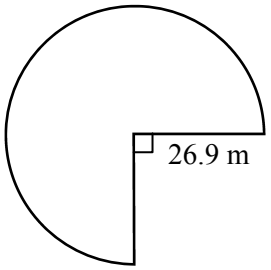
20

$A = {}^{\theta}/_{360} \times \pi r^2$

$= {}^{22}/_{360} \times \pi \times 40.7^2$

$A = 318.02$ km^2

21



$A = {}^{\theta}/_{360} \times \pi r^2$

$= {}^{270}/_{360} \times \pi \times 26.9^2$

$A = 1\,704.97$ m^2

Area of Composite Shapes

A composite shape is a shape that is composed of two or more plane shapes. These two shapes can add together to create an area, or be subtracted. Before this sheet is attempted you should have completed earlier sheets on area of a circle.

Column 1 are all exercises using addition. So calculate the individual areas and then add them together. The process is shown in question 1.

Column 2 is essentially the same process, find the areas that make the shape but then subtract the areas. Question 8 will require Pythagoras Theorem to first be applied.

Area of Composite Shapes

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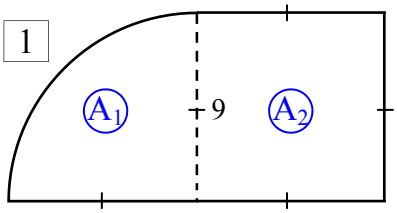
Find the areas, sum for the total area. Round to 1 d.p. All lengths are cm.

1



$A_1 = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A_1 = \square$ cm^2 (1 dp)

$A_2 = l^2$

$= \square^2$

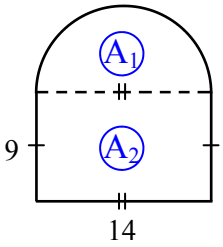
$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square$

$A = \square$ cm^2

2

$A_1 = \frac{1}{2}\pi r^2$

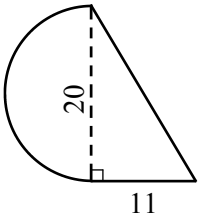


$A_2 = lb$

$A = A_1 + A_2 = \square + \square$

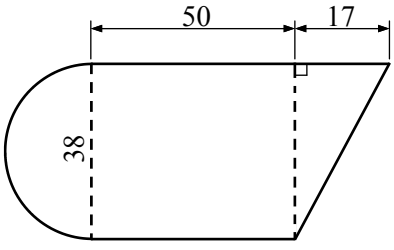
$A = \square$

3



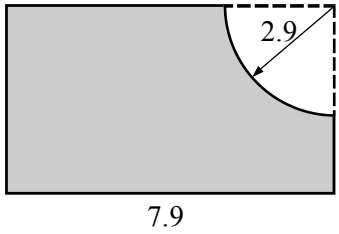
$A_2 = \frac{1}{2}bh$

4

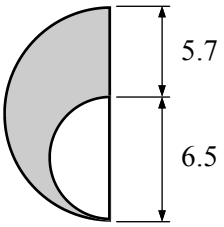


This time subtract the areas. Round to 1 d.p. All measurements in metres.

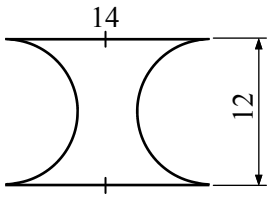
5



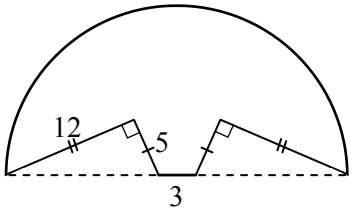
6



7



8



Area of Composite Shapes

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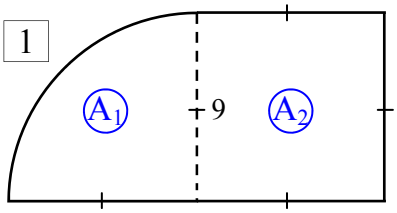
Find the areas, sum for the total area. Round to 1 d.p. All lengths are cm.

1



$A_1 = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 9^2$

$A_1 = 63.6 \text{ cm}^2$ (1 dp)

$A_2 = l^2$

$= 9^2$

$A_2 = 81 \text{ cm}^2$

$A = A_1 + A_2 = 63.6 + 81$

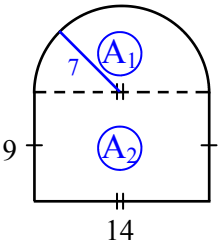
$A = 144.6 \text{ cm}^2$

2

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 7^2$

$A_1 = 77 \text{ cm}^2$ (1 d.p.)



$A_2 = lb$

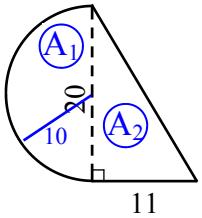
$= 14 \times 9$

$A_2 = 126 \text{ cm}^2$

$A = A_1 + A_2 = 77 + 126$

$A = 203 \text{ cm}^2$

3



$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 10^2$

$A_1 = 157.1 \text{ cm}^2$ (1 d.p.)

$A_2 = \frac{1}{2}bh$

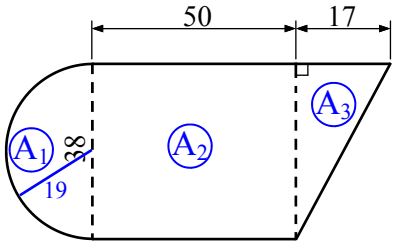
$= \frac{1}{2} \times 11 \times 20$

$A_2 = 110 \text{ cm}^2$

$A = A_1 + A_2 = 157.1 + 110$

$A = 267.1 \text{ cm}^2$

4



$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 19^2$

$A_1 = 567.1 \text{ cm}^2$ (1 d.p.)

$A_2 = lb$

$= 50 \times 38$

$A_2 = 1\,900 \text{ cm}^2$

$A_3 = \frac{1}{2}bh$

$= \frac{1}{2} \times 38 \times 17$

$A_3 = 323 \text{ cm}^2$

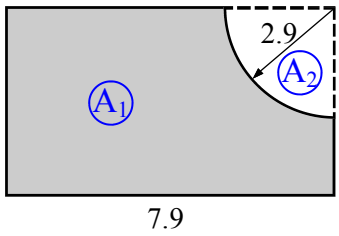
$A = A_1 + A_2 + A_3$

$= 567.1 + 1\,900 + 323$

$A = 2\,790.1 \text{ cm}^2$

This time subtract the areas. Round to 1 d.p. All measurements in metres.

5



$A_1 = lb$

$= 7.9 \times 4.7$

$A_1 = 37.1 \text{ m}^2$ (1 d.p.)

$A_2 = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 2.9^2$

$A_2 = 6.6 \text{ m}^2$ (1 d.p.)

$A = A_1 - A_2 = 37.1 - 6.6$

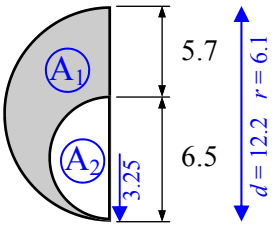
$A = 30.5 \text{ m}^2$

6

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 6.1^2$

$A_1 = 58.4 \text{ m}^2$ (1 d.p.)



$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 3.25^2$

$A_2 = 16.6 \text{ m}^2$ (1 d.p.)

$A = A_1 - A_2 = 58.4 - 16.6$

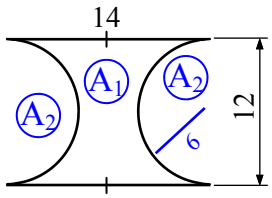
$A = 41.8 \text{ m}^2$

7

$A_1 = lb$

$= 14 \times 12$

$A_1 = 168 \text{ m}^2$



$A_2 = \pi r^2$

$= \pi \times 6^2$

$A_2 = 113.1 \text{ m}^2$ (1 d.p.)

$A = A_1 - A_2 = 168 - 113.1$

$A = 54.9 \text{ m}^2$

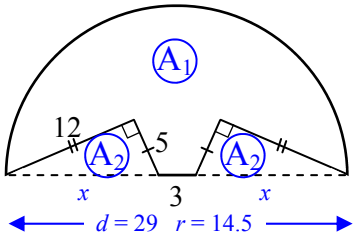
8

$x^2 = 5^2 + 12^2$

$x^2 = 169$

$x = \sqrt{169}$

$x = 13 \text{ m}$



$A_2 = 2 \times \frac{1}{2}bh$

$= 2 \times \frac{1}{2} \times 5 \times 12$

$A_2 = 60 \text{ m}^2$

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 14.5^2$

$A_1 = 330.3 \text{ m}^2$ (1 d.p.)

$A = A_1 - A_2 = 330.3 - 60$

$A = 270.3 \text{ m}^2$

Area and Perimeter Problems

These are problem based questions based on circles or composite solids with circular elements. So before attempting this sheet ensure you have covered composite areas.

Column 1 are area questions, so use $A = \pi r^2$ to find the area, then apply other skills to solve the questions.

Column 2 are circumference questions, so use $C = \pi d$ *or* $C = 2\pi r$ to find the circumference or use fractions when arcs are involved.

As these questions require earlier exercises to be completed first, no examples have been included.

Area and Perimeter Problems

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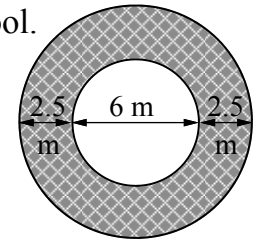
Use $A = \pi r^2$ to find the area and then apply your skills to answer these. Answer to 1 decimal place when needed.

A circular pool shown below in plan view has a diameter of 6 m. The pool fence encloses a 2.5 m wide paved area around the pool. Find:

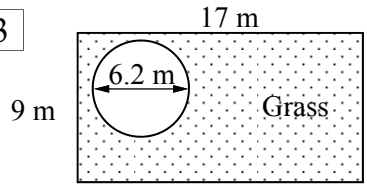
1 The area occupied by the pool.



2 The area occupied by the tiles.

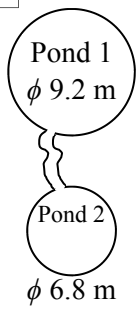
Tom has built a 6.2 m diameter pergola in his backyard. The rest of his backyard is covered by grass. Calculate the percentage of his backyard that has to be mowed.

3



Laura is constructing a wastewater system using two round settling ponds. Pond 1 is to have reeds covering 80% of its area, Pond 2 is to have 34% of its area covered with reeds. Calculate the amount of reeds (in m^2) required.

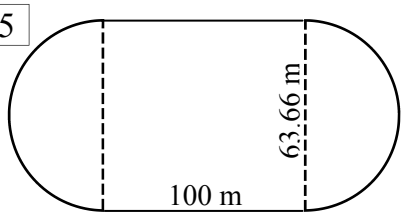
4



Use $C = 2\pi r$ or $C = \pi d$ to find the circle's circumference then apply your skills to answer these. Answer to 1 decimal place.

Calculate the distance around the track field shown below.

5

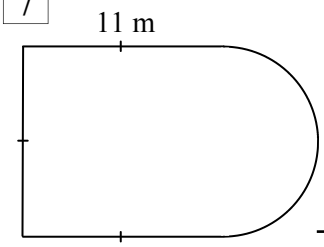


If Eve can run at an average speed of 3 m/s calculate the time, in minutes and seconds, that it would take her to complete one lap of the track above.

6

A tiger exhibit at the zoo is shown below. The curved length is a glass viewing wall costing $1 450 per metre. While the remaining three walls are concrete and cost $845 per metre. Find the cost of constructing the walls.

7



Jen's hamster 'Spartacus' runs in his 30 cm diameter exercise wheel. Jen has fitted a counter to the wheel and finds that Spartacus completed 800 revolutions one day, 560 the next and 725 on the third day. Find, in m, the:

8 Total distance Spartacus covers in three days.

9 Average distance travelled per day.

Area and Perimeter Problems

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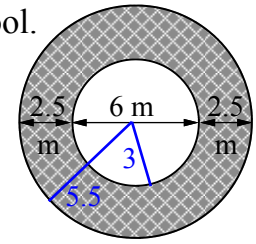
Use $A = \pi r^2$ to find the area and then apply your skills to answer these. Answer to 1 decimal place when needed.

A circular pool shown below in plan view has a diameter of 6 m. The pool fence encloses a 2.5 m wide paved area around the pool. Find:

1 The area occupied by the pool.



$A = \pi r^2$

$= \pi \times 3^2$

$A = 28.3 \text{ m}^2$

2 The area occupied by the tiles.

$A = \pi r^2$

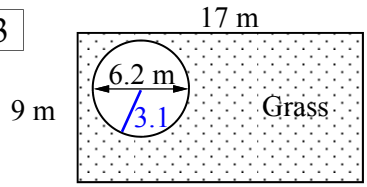
$= \pi \times 5.5^2$

$A = 95 \text{ m}^2$

$A = 95 - 28.3 = 66.7 \text{ m}^2$

Tom has built a 6.2 m diameter pergola in his backyard. The rest of his backyard is covered by grass. Calculate the percentage of his backyard that has to be mowed.

3



$A = \pi r^2$

$= \pi \times 3.1^2$

$A = 30.2 \text{ m}^2$

$A = lb$

$= 17 \times 9$

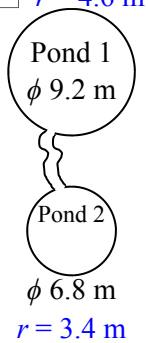
$A = 153 \text{ m}^2$

$\%\text{ Mowed} = \frac{30.2}{153} \times 100$

$\%\text{ Mowed} = 19.7\%$

Laura is constructing a wastewater system using two round settling ponds. Pond 1 is to have reeds covering 80% of its area, Pond 2 is to have 34% of its area covered with reeds. Calculate the amount of reeds (in m^2) required.

4



$r = 4.6$ m

$r = 3.4$ m

$A_1 = \pi r^2$

$= \pi \times 4.6^2$

$A_1 = 66.5 \text{ m}^2$

$A_2 = \pi r^2$

$= \pi \times 3.4^2$

$A_2 = 36.3 \text{ m}^2$

$\text{Pond 1} = 66.5 \times 0.80$

$\text{Pond 1} = 53.2 \text{ m}^2$

$\text{Pond 2} = 36.3 \times 0.34$

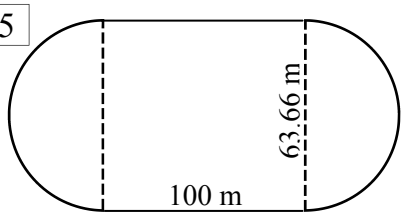
$\text{Pond 2} = 12.3 \text{ m}^2$

$\text{Total Reeds} = 53.2 + 12.3 = 65.5 \text{ m}^2$

Use $C = 2\pi r$ or $C = \pi d$ to find the circle's circumference then apply your skills to answer these. Answer to 1 decimal place.

Calculate the distance around the track field shown below.

5



$C = \pi d$

$= \pi \times 63.66$

$C = 200 \text{ m}$

$\text{Distance} = 200 + 2 \times 100$

$\text{Distance} = 400 \text{ m}$

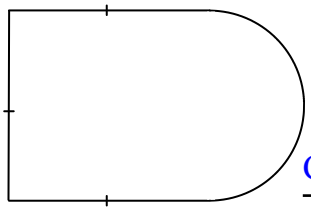
If Eve can run at an average speed of 3 m/s calculate the time, in minutes and seconds, that it would take her to complete one lap of the track above.

6 $\text{Time} = \frac{400}{3}$

$\text{Time} = 133.\dot{3} \text{ secs} = 2 \text{ m } 13 \text{ s}$

A tiger exhibit at the zoo is shown below. The curved length is a glass viewing wall costing \$1 450 per metre. While the remaining three walls are concrete and cost \$845 per metre. Find the cost of constructing the walls.

7



$l = \frac{1}{2}\pi d$

$= \frac{1}{2} \times \pi \times 11$

$l = 17.3 \text{ m}$

$\text{Cost}_{\text{GLASS}} = 17.3 \times 1\,450$

$\text{Cost}_{\text{GLASS}} = \$25\,085$

$\text{Cost}_{\text{CONCRETE}} = 11 \times 3 \times 845$

$\text{Cost}_{\text{CONCRETE}} = \$27\,885$

Jen's hamster 'Spartacus' runs in his 30 cm diameter exercise wheel. Jen has fitted a counter to the wheel and finds that Spartacus completed 800 revolutions one day, 560 the next and 725 on the third day. Find, in m, the:

8 Total distance Spartacus covers in three days.

$C = \pi d$

$= \pi \times 0.3$

$C = 0.9 \text{ m}$

$D = 2\,085 \times 0.9$

$D = 1\,876.5 \text{ m}$

Or 1 965.1 m (No rounding)

9 Average distance travelled per day.

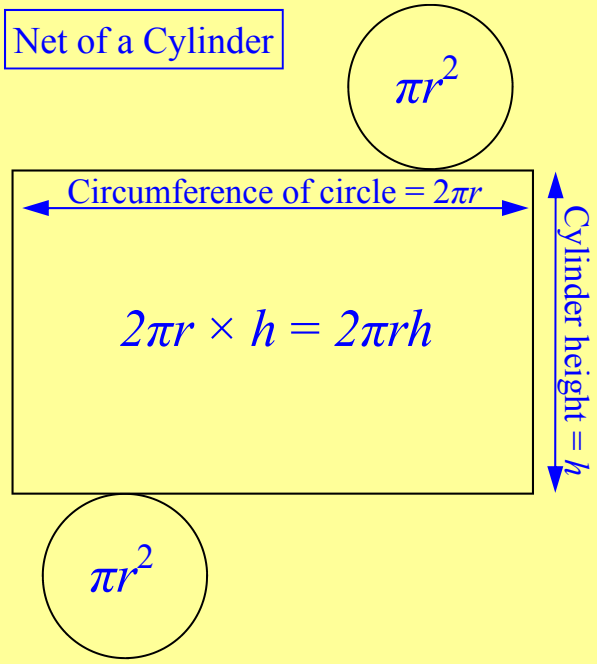
$\text{Average Dist} = 1\,876.5 \div 3 = 625.5 \text{ m}$

Or No Rounding $\text{Average Dist} = 1\,965.1 \div 3 = 655 \text{ m}$

Estimation/Terms of π - Surface Area of Cylinders

When you calculate the SA of a cylinder, or the area of a cylinder wall (the curved part of a cylinder) you will generally use a calculator. A calculator uses an approximation for π because π is irrational, its decimal component goes on forever. So if an exact value is required, the answer must be expressed 'in terms of π'.

Net of a Cylinder



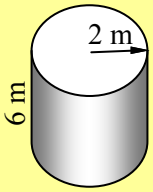
As the net shows, each circle has an area of πr^2. So the two circles added gives an area of $2\pi r^2$.

The cylinder wall, the rectangle in the net, wraps around the circle so one of its sides is the same length as the circumference of the circular ends, which is $2\pi r$. Its height is the height of the cylinder, which is h. As this area is a rectangle, its area is one side times the other, or $2\pi r \times h = 2\pi rh$.

SA of a Cylinder = $2\pi r^2 + 2\pi rh$

SA of a Cylinder Wall = $2\pi rh$

Example - Exact Area of a Cylinder Wall & Estimation of area (using π =3).



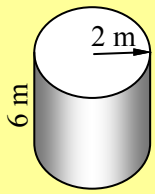
$A = 2\pi rh$	Write the formula
$= 2\times\pi\times2\times6$	Substitute r and h
$A = 24\pi$ m^2	Find area

Estimate:

$A = 24\pi$ m^2	
$\approx 24 \times 3$	Replace π with 3
$A \approx 72$ m^2	

Example - Exact Surface Area of a Cylinder & Estimation of area (using π =3).

$SA = 2\pi r^2 + 2\pi rh$	Write the formula
$= 2\times\pi\times2^2 + 2\times\pi\times2\times6$	Substitute r and h
$= 8\pi + 24\pi$	Simplify parts
$SA = 32\pi$ m^2	Find SA



Estimate: using $\pi = 3$

$SA = 32\pi$ m^2
$\approx 32 \times 3$
$SA \approx 96$ m^2

Note that as π = 3.14 to 2 d.p., using π = 3 will always give an answer that is below the actual surface area/area. But it is a simple estimate and a quick way to check your calculations.

Estimation/Terms of π - Surface Area of Cylinders

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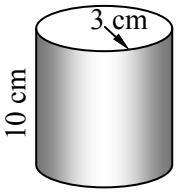
Find the area of the cylinder walls in terms of π, then estimate the area using $\pi = 3$

Example



$A = 2\pi rh$

$= 2\times\pi\times3\times10$

$A = 60\pi$ cm^2

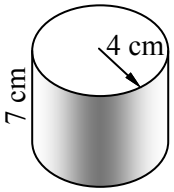
units^2 must match

Estimate: $A = 60\pi$ cm^2

$\approx 60 \times 3$

$A \approx 180$ cm^2

1



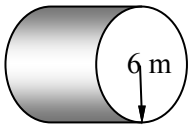
$A = 2\pi rh$

$=$

$A =$

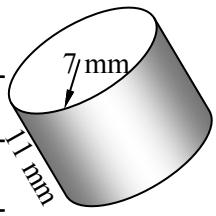
Estimate:

2



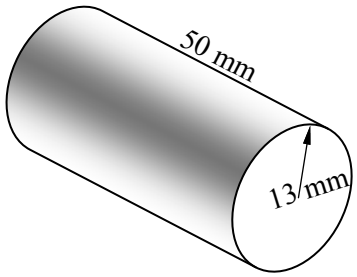
Estimate:

3



Estimate:

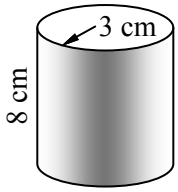
4



Estimate:

Now repeat the same process except this time for the SA of a cylinder.

5



$SA = 2\pi r^2 + 2\pi rh$

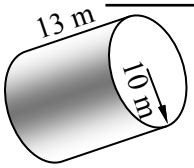
$= 2\times\pi\times\square^2 + 2\times\pi\times\square\times\square$

$= \square\ \pi + \square\ \pi$

SA = $\square\ \pi$ cm^2

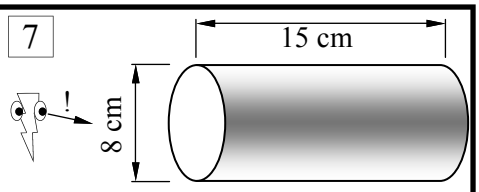
Estimate: using $\pi = 3$

6



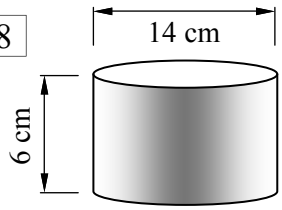
Estimate: using $\pi = 3$

7



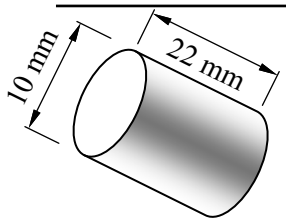
Estimate: using $\pi = 3$

8



Estimate: using $\pi = 3$

9



Estimate: using $\pi = 3$

Estimation/Terms of π - Surface Area of Cylinders

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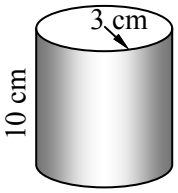
Find the area of the cylinder walls in terms of π, then estimate the area using $\pi = 3$

Example



$A = 2\pi rh$

$= 2\times\pi\times3\times10$

$A = 60\pi$ cm^2

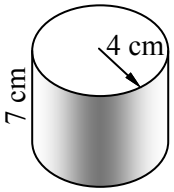
units^2 must match

Estimate: $A = 60\pi$ cm^2

$\approx 60 \times 3$

$A \approx 180$ cm^2

1



$A = 2\pi rh$

$= 2\times\pi\times4\times7$

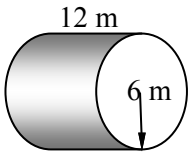
$A = 56\pi$ cm^2

Estimate: $A = 56\pi$ cm^2

$\approx 56 \times 3$

$A \approx 168$ cm^2

2



$A = 2\pi rh$

$= 2\times\pi\times6\times12$

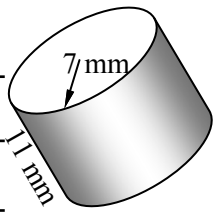
$A = 144\pi$ m^2

Estimate: $A = 144\pi$ m^2

$\approx 144 \times 3$

$A \approx 432$ m^2

3



$A = 2\pi rh$

$= 2\times\pi\times7\times11$

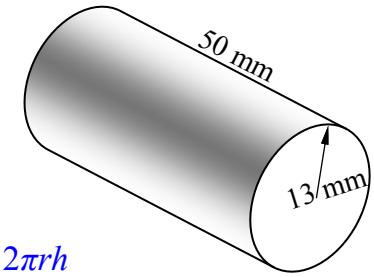
$A = 154\pi$ mm^2

Estimate: $A = 154\pi$ mm^2

$\approx 154 \times 3$

$A \approx 462$ mm^2

4



$A = 2\pi rh$

$= 2\times\pi\times13\times50$

$A = 1\,300\pi$ mm^2

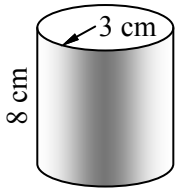
Estimate: $A = 1\,300\pi$ mm^2

$\approx 1\,300 \times 3$

$A \approx 3\,900$ mm^2

Now repeat the same process except this time for the SA of a cylinder.

5



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times\boxed{3}^2 + 2\times\pi\times\boxed{3}\times\boxed{8}$

$= \boxed{18}\ \pi + \boxed{48}\ \pi$

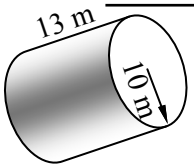
$SA = \boxed{66}\ \pi$ cm^2

Estimate: $SA = 66\pi$ cm^2

$\approx 66 \times 3$

$SA \approx 198$ cm^2

6



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times10^2 + 2\times\pi\times10\times13$

$= 200\pi + 260\pi$

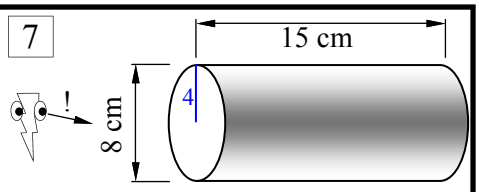
$SA = 460\pi$ m^2

Estimate: $SA = 460\pi$ m^2

$\approx 460 \times 3$

$SA \approx 1\,380$ m^2

7



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times4^2 + 2\times\pi\times4\times15$

$= 32\pi + 120\pi$

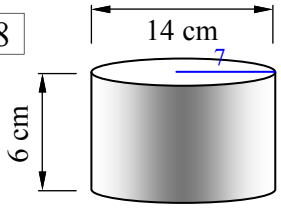
$SA = 152\pi$ cm^2

Estimate: $SA = 152\pi$ cm^2 using $\pi = 3$

$\approx 152 \times 3$

$SA \approx 456$ cm^2

8



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times7^2 + 2\times\pi\times7\times6$

$= 98\pi + 84\pi$

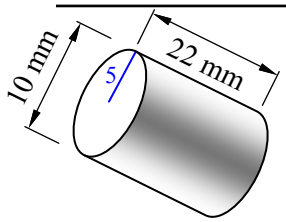
$SA = 182\pi$ cm^2

Estimate: $SA = 182\pi$ cm^2 using $\pi = 3$

$\approx 182 \times 3$

$SA \approx 546$ cm^2

9



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times5^2 + 2\times\pi\times5\times22$

$= 50\pi + 220\pi$

$SA = 270\pi$ mm^2

Estimate: $SA = 270\pi$ mm^2 using $\pi = 3$

$\approx 270 \times 3$

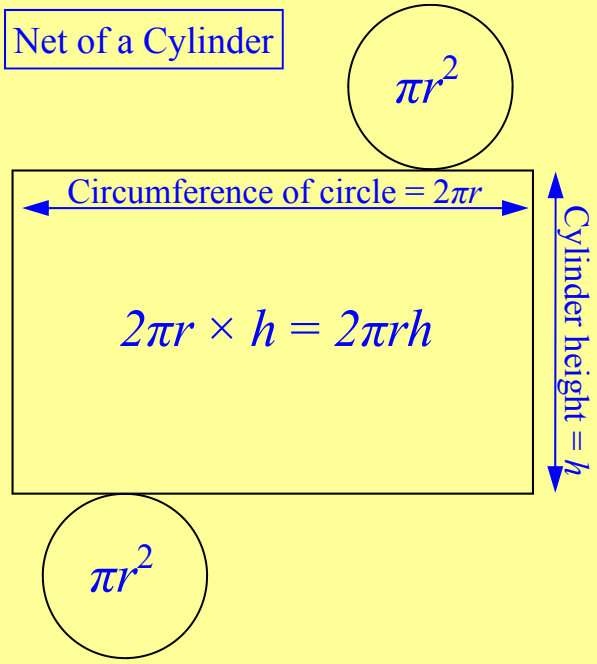
$SA \approx 810$ mm^2

Surface Area of Cylinders

The surface area of a cylinder is the sum of the areas of the solid. As the net of a cylinder below shows, each circle has an area of πr^2. So the two circles added together gives an area of $2\pi r^2$. The cylinder wall, (also called the curved surface) is the rectangle in the net, it wraps around the circles so one of its sides is the same length as the circumference of the circular ends, which is $2\pi r$. Its height is the height of the cylinder, which is h.

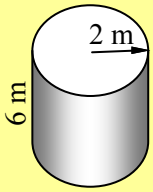
As this area is a rectangular area, its area is one side times the other, or $2\pi r \times h = 2\pi rh$.

Net of a Cylinder



SA of a Cylinder = $2\pi r^2 + 2\pi rh$

Example - SA of a cylinder - Using $\pi = {}^{22}/_{7}$



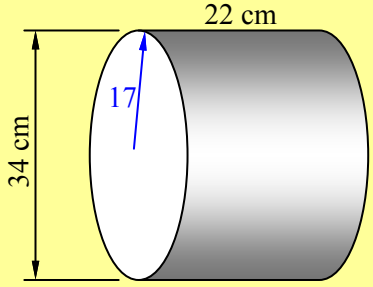
$SA = 2\pi r^2 + 2\pi rh$	Write the formula
$= 2\times{}^{22}/_{7}\times 2^2 + 2\times{}^{22}/_{7}\times 2\times 6$	Substitute r and h, Change π to ${}^{22}/_{7}$.
$SA = 100.57\ m^2$ (2 d.p.)	Find surface area

Calculator 2 × 2 2 $a^b/_c$ 7 × 2 x^2 + 2 × 2 2 $a^b/_c$ 7 × 2 × 6 =

Answer on Screen 100⌟4⌟7

Calculator $a^b/_c$ Answer on Screen 100.5714286 Round to 2 decimal places $100.57\ m^2$ (2 d.p.)

Example - SA of a cylinder - Given the diameter instead of radius - Using π key



Divide the diameter by two. Then on the diagram draw the radius.

Use that number in your calculation. It may be a decimal.

$SA = 2\pi r^2 + 2\pi rh$	Write the formula
$= 2\times\pi\times 17^2 + 2\times\pi\times 17\times 22$	Substitute r and h
$SA = 4\ 165.75\ cm^2$ (2 d.p.)	Find surface area

Calculator 2 × π × 1 7 x^2 + 2 × π × 1 7 × 2 2 =

Answer on Screen 4165.751859 Round to 2 decimal places $4\ 165.75\ cm^2$ (2 d.p.)

Surface Area of Cylinders

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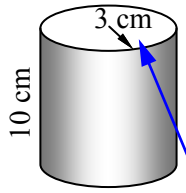
Find the surface area of these cylinders.
Use π = 3.14, not the π key. Round to 2 dp.

Example

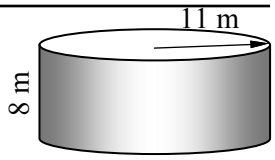


$SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times3^2 + 2\times3.14\times3\times10$

$SA = 244.92\ cm^2$ (2 d.p.)

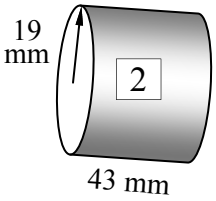
$units^2$ must match

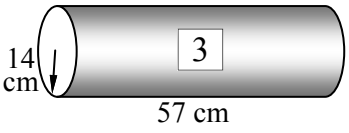


1 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times\square^2 + 2\times3.14\times\square\times\square$

$SA =$ m^2





2 $SA = 2\pi r^2 + 2\pi rh$

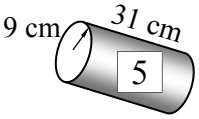
$=$

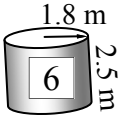
$SA =$

3

Now use π = $^{22}/_{7}$. Use the fraction key after '=' to change to a decimal, round to 1 d.p.





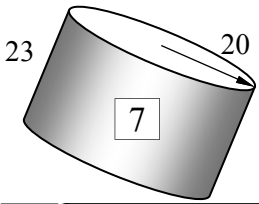


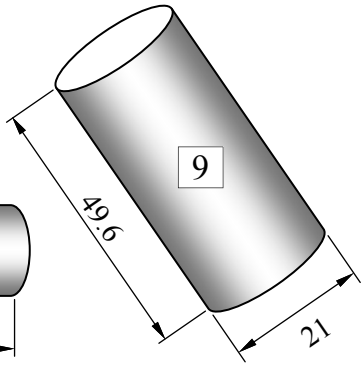
4

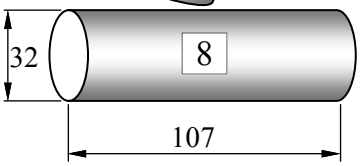
5

6

Now use your calculator's π key. These questions can include diameters. All measurements are in cm. Round to 1 d.p.







7 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times\square^2 + 2\times\pi\times\square\times\square$

8

9

10 Bubblin' Bobs cylindrical water tanks are 4.2 m high and 5.1 m in diameter. Metal sheeting costs \$9.10/m^2, find the cost to build 4 tanks.

11 Slow & Slim's steam rollers have a cylinder for a front wheel that is 1.9 m diameter and 3.1 m wide (height). The roller needs painting, if a tin of paint covers 11 m^2 how many tins are required to paint the roller.

Surface Area of Cylinders

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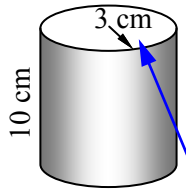
Find the surface area of these cylinders.
Use $\pi = 3.14$, not the π key. Round to 2 dp.

Example

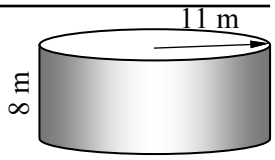


$SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times3^2 + 2\times3.14\times3\times10$

$SA = 244.92 \text{ cm}^2$ (2 d.p.)

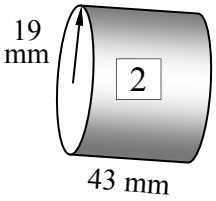
units^2 must match

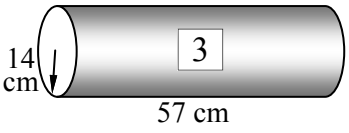


1 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times\boxed{11}^2 + 2\times3.14\times\boxed{11}\times\boxed{8}$

$SA = 1\,312.52 \text{ m}^2$ (2 d.p.)





2 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times19^2 + 2\times3.14\times19\times43$

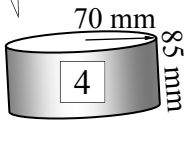
$SA = 7\,397.84 \text{ mm}^2$ (2 d.p.)

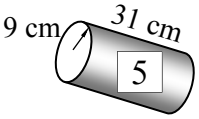
3 $SA = 2\pi r^2 + 2\pi rh$

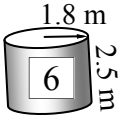
$= 2\times3.14\times14^2 + 2\times3.14\times14\times57$

$SA = 7\,121.52 \text{ mm}^2$ (2 d.p.)

Now use $\pi = {}^{22}/_{7}$. Use the fraction key after '=' to change to a decimal, round to 1 d.p.







4 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times{}^{22}/_{7}\times70^2 + 2\times{}^{22}/_{7}\times70\times85$

$SA = 68\,200 \text{ mm}^2$

5 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times{}^{22}/_{7}\times9^2 + 2\times{}^{22}/_{7}\times9\times31$

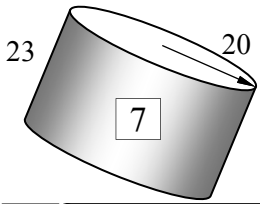
$SA = 2\,262.9 \text{ cm}^2$ (1 d.p.)

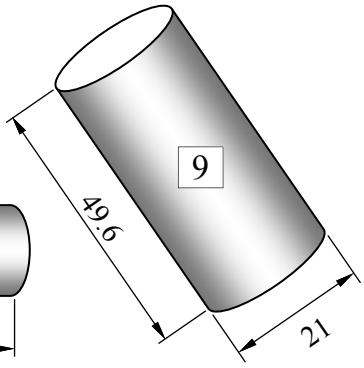
6 $SA = 2\pi r^2 + 2\pi rh$

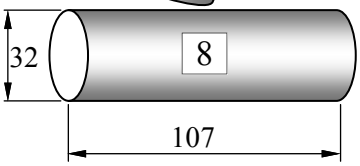
$= 2\times{}^{22}/_{7}\times1.8^2 + 2\times{}^{22}/_{7}\times1.8\times2.5$

$SA = 48.7 \text{ m}^2$ (1 d.p.)

Now use your calculator's π key. These questions can include diameters. All measurements are in cm. Round to 1 d.p.







7 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times\boxed{20}^2 + 2\times\pi\times\boxed{20}\times\boxed{23}$

$SA = 5\,403.5 \text{ cm}^2$ (1 d.p.)

8 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times16^2 + 2\times\pi\times16\times107$

$SA = 12\,365.3 \text{ cm}^2$ (1 d.p.)

9 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times10.5^2 + 2\times\pi\times10.5\times49.6$

$SA = 3\,965 \text{ cm}^2$ (1 d.p.)

10 Bubblin' Bobs cylindrical water tanks are 4.2 m high and 5.1 m in diameter. Metal sheeting costs \$9.10/m^2, find the cost to build 4 tanks.

$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times2.55^2 + 2\times\pi\times2.55\times4.2$

$SA = 108.15 \text{ m}^2$ (2 d.p.)

$Cost = 108.15 \times 4 \times 9.10 = \$3\,936.66$ \$3 936.64 No rounding

11 Slow & Slim's steam rollers have a cylinder for a front wheel that is 1.9 m diameter and 3.1 m wide (height). The roller needs painting, if a tin of paint covers 11 m^2 how many tins are required to paint the roller.

$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times0.95^2 + 2\times\pi\times0.95\times3.1$

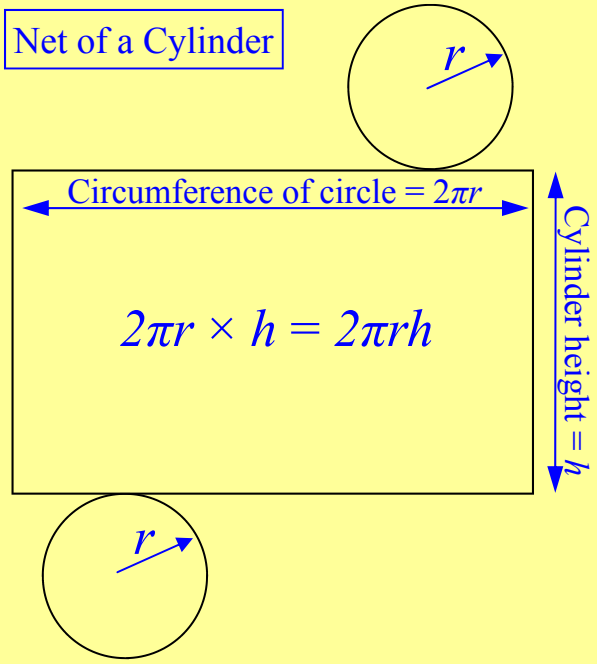
$SA = 24.17 \text{ m}^2$ (2 d.p.)

$Cost = 24.17 \div 11 = 2.19 \ldots = 3$ tins required

Area of Curved Surface - Cylinders

The cylinder wall of a cylinder, (also called the curved surface) is the rectangle shown in the net below, it wraps around the circular ends so one of its sides is the same length as the circumference of the circular ends, which is $2\pi r$. Its height is the height of the cylinder, which is h.

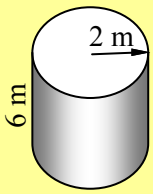
Net of a Cylinder



As this area is a rectangular area, its area is one side times the other,
or $2\pi r \times h = 2\pi rh$.

Area = $2\pi rh$

Example - Area of a cylinder wall - Using $\pi = {}^{22}/_{7}$



SA = $2\pi rh$	Write the formula
= $2\times{}^{22}/_{7}\times2\times6$	Substitute r and h, Change π to ${}^{22}/_{7}$.
SA = 75.4 m^2 (1 d.p.)	Find area of the curved surface

Calculator 2 × 2 2 $a^b/_c$ 7 × 2 × 6 =

Answer on Screen 75⌟ 3⌟ 7

Calculator $a^b/_c$ Answer on Screen 75.42857143 Round to 1 decimal place 75.4 m^2

Column 1 uses $\pi = 3.14$ first. So use the formula $A = 2\pi rh$ and calculate the area. The lower part of the sheet uses $\pi = {}^{22}/_{7}$, solve as above in the example.

Column 2 uses the π key on your calculator, also you may be given the radius or the diameter. If you are given the diameter then halve it (÷ 2) to change the length to the radius, then solve as before. There is a table at the bottom of the column, this doesn't require working out, just answers. The last two questions are different and require division and brackets, see how you go.

Column 3 features problem style questions, they require an extra step or two to complete.

Area of Curved Surface - Cylinders

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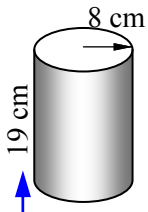
Find the area of the curved surface of these cylinders. Use $\pi = 3.14$. Round to 2 d.p.

Example



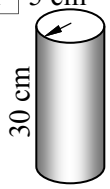
$A = 2\pi rh$

$= 2 \times 3.14 \times 8 \times 19$

$A = 954.56$ cm^2

units^2 must match

1



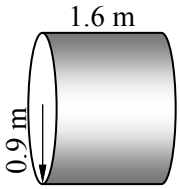
$A = 2\pi rh$

$= 2 \times 3.14 \times \square \times \square$

$A =$ cm^2

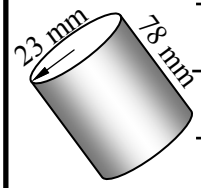
2

$A = 2\pi rh$



$A =$ m^2

3



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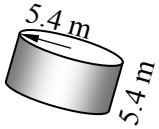
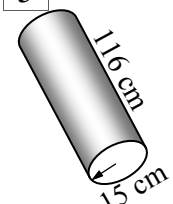


Now use $\pi = {}^{22}/_{7}$. Round to 1 decimal place.

5



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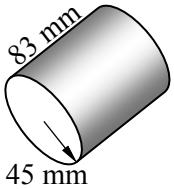
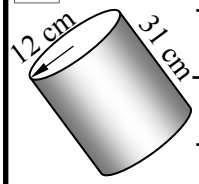
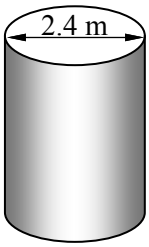


Use the π key for these and round to 1 d.p. Divide the diameter by 2 for the radius.

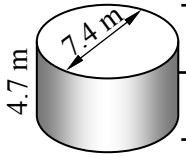
7



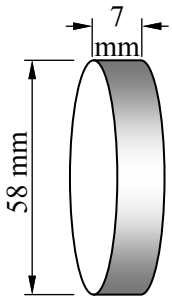
8



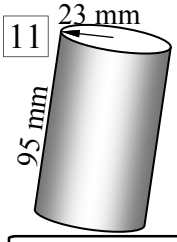
9



10



11

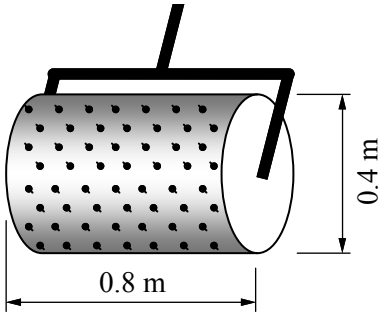


Use the π key and fill all the gaps, round to 1 d.p. All lengths are in cm.

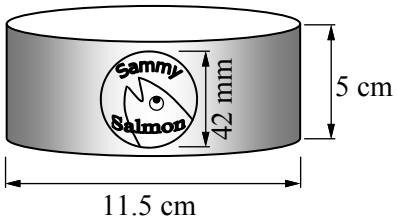
Q	r	d	h	$A = 2\pi rh$
12	8		6	
13		18	25	
14		14	19	
15	17		20	
16		12		565.5 cm^2
17			13	735.1 cm^2

Use the π key for these and round to 2 d.p.

18 A turf roller is 0.8 m wide with a diameter of 0.4 m. Find the area of lawn covered, in m^2, if the roller rotates 20 times.



19 A tin of salmon has a label covering its curved surface. If the label has a circular fish logo, what percentage of the label's area is used by the fish logo?



Area of Curved Surface - Cylinders

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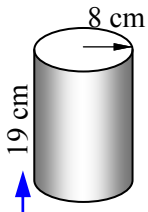
Find the area of the curved surface of these cylinders. Use $\pi = 3.14$. Round to 2 d.p.

Example



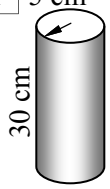
$A = 2\pi rh$

$= 2\times3.14\times8\times19$

$A = 954.56$ cm^2

units^2 must match

1

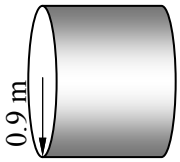


$A = 2\pi rh$

$= 2\times3.14\times$ [5] $\times$ [30]

$A = 942$ cm^2

2

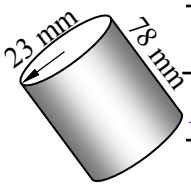


$A = 2\pi rh$

$= 2\times3.14\times0.9\times1.6$

$A = 9.04$ m^2

3

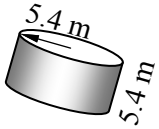


$A = 2\pi rh$

$= 2\times3.14\times23\times78$

$A = 11\,266.32$ mm^2

4



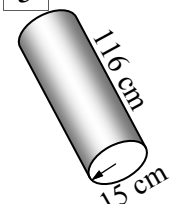
$A = 2\pi rh$

$= 2\times3.14\times5.4^2$

$A = 183.12$ m^2

Now use $\pi = {}^{22}/_{7}$. Round to 1 decimal place.

5

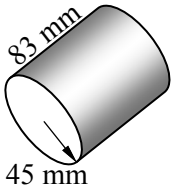


$A = 2\pi rh$

$= 2\times{}^{22}/_{7}\times15\times116$

$A = 10\,937.1$ cm^2

6



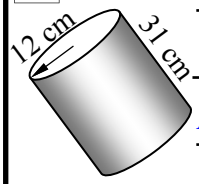
$A = 2\pi rh$

$= 2\times{}^{22}/_{7}\times45\times83$

$A = 23\,477.1$ mm^2

Use the π key for these and round to 1 d.p. Divide the diameter by 2 for the radius.

7

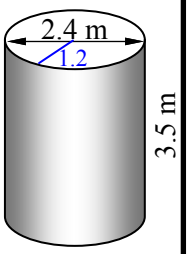


$A = 2\pi rh$

$= 2\times\pi\times12\times31$

$A = 2\,337.3$ cm^2

8

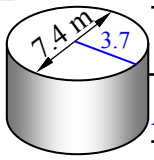


$A = 2\pi rh$

$= 2\times\pi\times1.2\times3.5$

$A = 26.4$ m^2

9

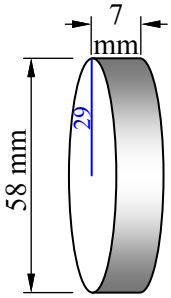


$A = 2\pi rh$

$= 2\times\pi\times3.7\times4.7$

$A = 109.3$ m^2

10

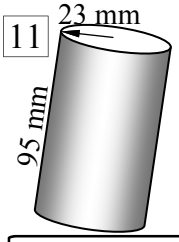


$A = 2\pi rh$

$= 2\times\pi\times29\times7$

$A = 1\,275.5$ mm^2

11



$A = 2\pi rh$

$= 2\times\pi\times23\times95$

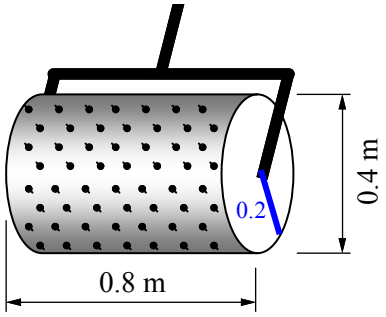
$A = 13\,728.8$ mm^2

Use the π key and fill all the gaps, round to 1 d.p. All lengths are in cm.

Q	r	d	h	$A = 2\pi rh$
12	8	16	6	301.6 cm^2
13	9	18	25	1 413.7 cm^2
14	7	14	19	835.7 cm^2
15	17	34	20	2 136.3 cm^2
16	6	12	15	565.5 cm^2
17	9	18	13	735.1 cm^2

Use the π key for these and round to 2 d.p.

18 A turf roller is 0.8 m wide with a diameter of 0.4 m. Find the area of lawn covered, in m^2, if the roller rotates 20 times.



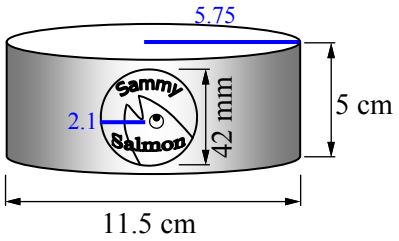
$A = 2\pi rh$

$= 2\times\pi\times0.2\times0.8$

$A = 1.01$ m^2

Area $= 20 \times 1.01 = 20.2$ m^2

19 A tin of salmon has a label covering its curved surface. If the label has a circular fish logo, what percentage of the label's area is used by the fish logo?



$A = 2\pi rh$

$= 2\times\pi\times5.75\times5$

$A = 189.64$ cm^2

$A = \pi r^2$

$= \pi\times2.1^2$

$A = 13.85$ cm^2

Percentage $= \frac{13.85}{189.64} \times 100$

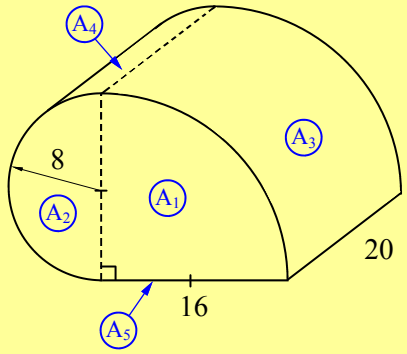
Percentage $= 7.30$ %

Surface Area of Composite Solids - Circles

Finding the SA of a composite solids can be difficult as it requires visualizing the solid, identifying all the face/curved surface areas and then a complicated process of adding them all. The questions on this worksheet all prisms except for question 2. The method is to:

- Find the area of the front face and double it (as the rear face is the same - prisms only)
- Find the face/curved surface areas around the sides of the solid
- Add the areas together

Example - Find the SA of the solid below, measurements are in cm.



STEP 1

Break the front face up into 2 areas, label all faces/curved surfaces around the outside of the solid.

$A_1 = \frac{1}{4} \times \pi \times 16^2$

$A_1 = 201.06 \text{ cm}^2$

$A_2 = \frac{1}{2} \times \pi \times 8^2$

$A_2 = 100.53 \text{ cm}^2$

$A_3 = \frac{1}{4} \times 2 \times \pi \times 16 \times 20$

$A_3 = 502.65 \text{ cm}^2$

$A_4 = \frac{1}{2} \times 2 \times \pi \times 8 \times 20$

$A_4 = 502.65 \text{ cm}^2$

$A_5 = 16 \times 20 = 320 \text{ cm}^2$

STEP 2

Calculate the individual areas.

$SA = 2 \times (201.06 + 100.53) + 502.65 + 502.65 + 320$

$SA = 1\,568.48 \text{ cm}^2$

STEP 3

Find the SA by adding each of the areas.

Surface Area of Composite Solids - Circles

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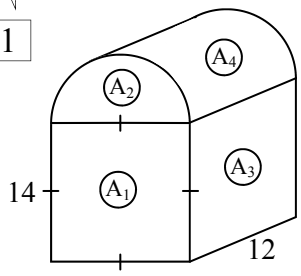
Find the SA for the composite solids. All measurements are in cm. Round to 2 d.p.

1



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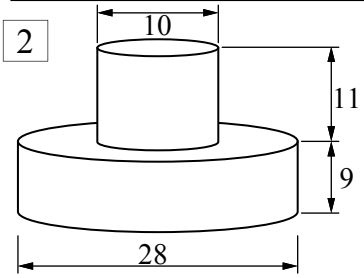

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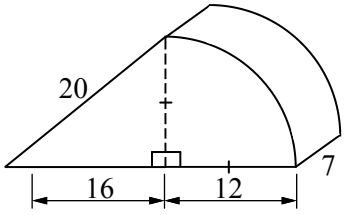

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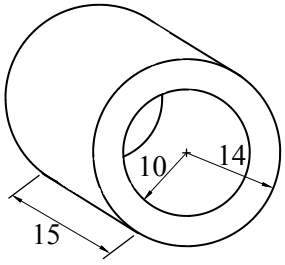

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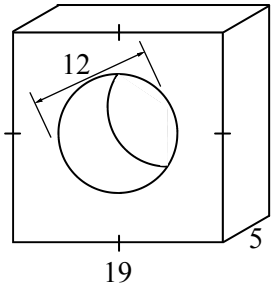

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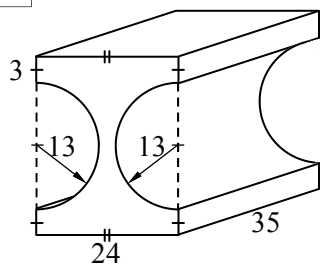
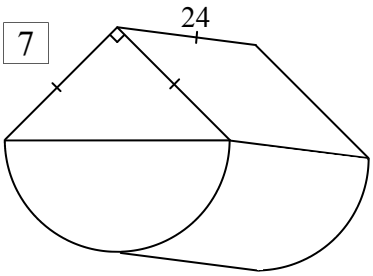


Pythagoras needed!

7



Surface Area of Composite Solids - Circles

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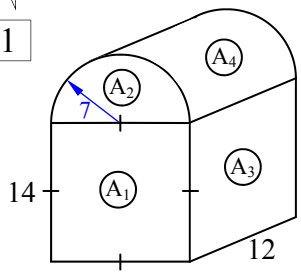
Find the SA for the composite solids. All measurements are in cm. Round to 2 d.p.

1



$A_1 = 14^2 = 196$ cm^2

$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 7^2 = 76.97$ cm^2

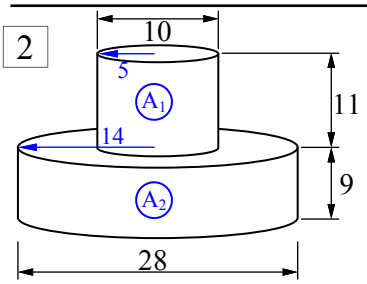
$A_3 = 12^2 = 144$ cm^2

$A_4 = \frac{1}{2} \times 2\pi rh = \frac{1}{2} \times 2 \times \pi \times 7 \times 12 = 263.89$ cm^2

SA = 2 × (196 + 76.97) + 3 × 144 + 263.89

SA = 1 241.83 cm^2

2



$A_1 = 2\pi rh$

$= 2 \times \pi \times 5 \times 11$

$= 345.58$ cm^2

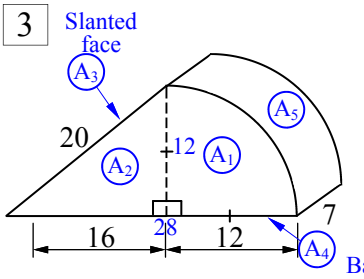
$A_2 = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 14^2 + 2 \times \pi \times 14 \times 9 = 2\,023.19$ cm^2

SA = 345.58 + 2 023.19

SA = 2 368.77 cm^2 (or 2 368.76 cm^2 in 1 step)

3



$A_1 = \frac{1}{4} \times \pi \times 12^2$

$= 113.10$ cm^2

$A_2 = \frac{1}{2} \times 16 \times 12 = 96$ cm^2

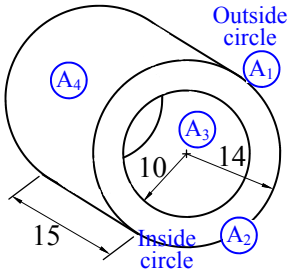
$A_3 = 20 \times 7 = 140$ cm^2

$A_4 = 28 \times 7 = 196$ cm^2 A_5: $\frac{1}{4} \times 2 \times \pi \times 12 \times 7 = 131.94$ cm^2

SA = 2 × (113.10 + 96) + 140 + 196 + 131.94

SA = 886.14 cm^2

4



$A_1 = \pi \times 14^2 = 615.75$ cm^2

$A_2 = \pi \times 10^2 = 314.16$ cm^2

$A_3 = 2 \times \pi \times 10 \times 15$

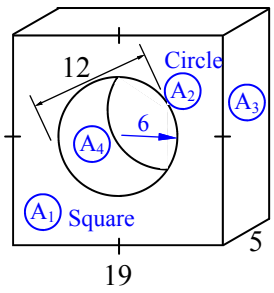
$= 942.48$ cm^2

$A_4 = 2 \times \pi \times 14 \times 15 = 1\,319.47$ cm^2

SA = 2 × (615.75 - 314.16) + 942.48 + 1 319.47

SA = 2 865.13 cm^2 (or 2 865.14 cm^2 in 1 step)

5



$A_1 = 19^2 = 361$ cm^2

$A_2 = \pi r^2 = \pi \times 6^2$

$= 113.10$ cm^2

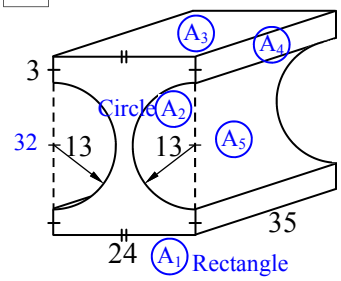
$A_3 = 19 \times 5 = 95$ cm^2

$A_4 = 2\pi rh = 2 \times \pi \times 6 \times 5 = 188.50$ cm^2

SA = 2 × (361 - 113.10) + 4 × 95 + 188.50

SA = 1 064.30 cm^2

6



$A_1 = 24 \times 32 = 768$ cm^2

$A_2 = \pi r^2 = \pi \times 13^2$

$= 530.93$ cm^2

$A_3 = 24 \times 35 = 840$ cm^2

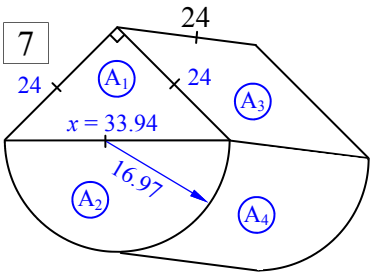
$A_4 = 3 \times 35 = 105$ cm^2

$A_5 = 2\pi rh = 2 \times \pi \times 13 \times 35 = 2\,858.85$ cm^2

SA = 2×(768 - 530.93)+2×840+4×105+2 858.85

SA = 5 432.99 cm^2

7



Pythagoras needed!

$x^2 = 24^2 + 24^2$

$x^2 = 1\,152$

$x = \sqrt{1\,152}$

$x = 33.94$ cm (2 d.p.)

$A_1 = \frac{1}{2} \times 24 \times 24 = 288$ cm^2

$A_2 = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 16.97^2 = 452.36$ cm^2

$A_3 = 24^2 = 576$ cm^2

$A_4 = \frac{1}{2} \times 2 \times \pi \times 16.97 \times 24 = 1\,279.51$ cm^2

SA = 2 × (288 + 452.36) + 2 × 576 + 1 279.51

SA = 3 912.23cm^2

Estimation/Terms of π - Volume of Cylinders

While mostly a calculator will be used to calculate the volume of a cylinder, it may be that you are asked to express the volume 'in terms of π' or the 'exact volume'. Both of these requests mean that π is left in the answer.

Being able to estimate volume is a valuable skill, because you can get a rough idea of the answer without a calculator, or before using a calculator. It also is a good 'self-check' mechanism in case you make a mistake using the calculator.

Column 1 has three cylinders in the column. The procedure is the same as the example at the top of the column. Use the Volume formula:

$$V = \pi r^2 h$$

Where: V = the volume of the prism
r = the radius of the circular face
h = the height of the cylinder

This will give an answer in terms of π, in other words, the exact volume.

The second step is to make an estimate, write the volume out again but then substitute the value of 3 for π. Multiply the two numbers and you have your estimate.

Because π is 3.14159..... It is more than 3, so when you use 3 as an estimate the answer will always be slightly less than the answer achieved by using a calculator.

Columns 2 and 3 use the diagrams at the base of the page. Measure the diameter, halve it for the radius, then measure the height. Record these lengths in the spaces provided on each Cylinder. Once you have done that solve the same way as Column 1.

Estimation/Terms of π - Volume of Cylinders

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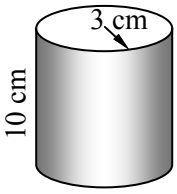
Find the volume of the cylinder in terms of π, then estimate the volume by substituting $\pi = 3$.

Example

$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$V = 90\pi \text{ cm}^3$



units^3 must match

Estimate: $V = 90\pi \text{ cm}^3$

$\approx 90 \times 3$

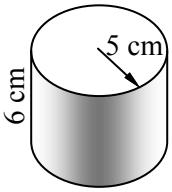
$V \approx 270 \text{ cm}^3$

1

$V = \pi r^2 h$

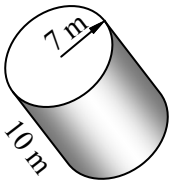
$=$

$V =$



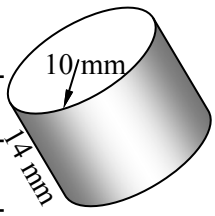
Estimate:
using
$\pi = 3$

2



Estimate:
using
$\pi = 3$

3



Estimate:
using
$\pi = 3$

Measure the cylinders below, find d and halve for r, then find h. All to the nearest cm. Then express the volume in terms of π and then estimate the volume using $\pi = 3$. Use the spaces that match numbers.

4



Estimate:
using
$\pi = 3$

5

Estimate:
using
$\pi = 3$

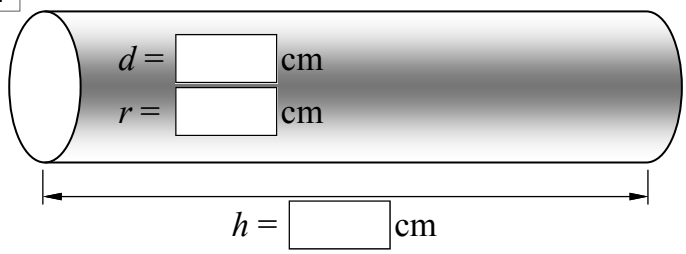
6

Estimate:
using
$\pi = 3$

7

Estimate:
using
$\pi = 3$

4



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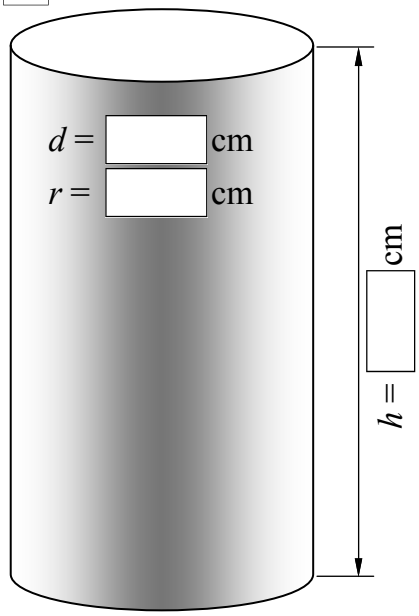

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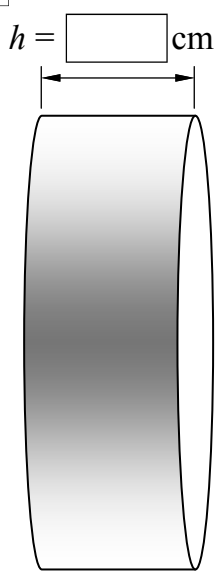

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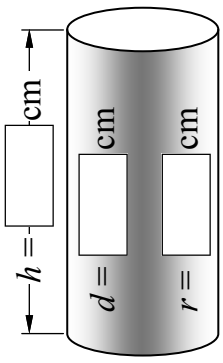

Estimation/Terms of π - Volume of Cylinders

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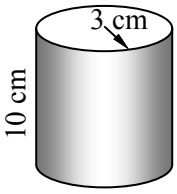
Find the volume of the cylinder in terms of π, then estimate the volume by substituting $\pi = 3$.

Example



$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$V = 90\pi$ cm^3

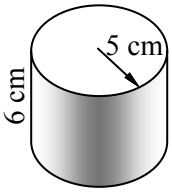
units^3 must match

Estimate: $V = 90\pi$ cm^3

$\approx 90 \times 3$

$V \approx 270$ cm^3

1



$V = \pi r^2 h$

$= \pi \times 5^2 \times 6$

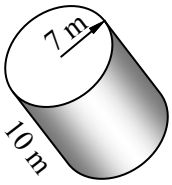
$V = 150\pi$ cm^3

Estimate: $V = 150\pi$ cm^3

using $\pi = 3$ $\approx 150 \times 3$

$V \approx 450$ cm^3

2



$V = \pi r^2 h$

$= \pi \times 7^2 \times 10$

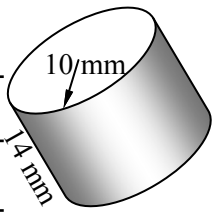
$V = 490\pi$ m^3

Estimate: $V = 490\pi$ m^3

using $\pi = 3$ $\approx 490 \times 3$

$V \approx 1\,470$ m^3

3



$V = \pi r^2 h$

$= \pi \times 10^2 \times 14$

$V = 1\,400\pi$ mm^3

Estimate: $V = 1\,400\pi$ m^3

using $\pi = 3$ $\approx 1\,400 \times 3$

$V \approx 4\,200$ mm^3

Measure the cylinders below, find d and halve for r, then find h. All to the nearest cm. Then express the volume in terms of π and then estimate the volume using $\pi = 3$. Use the spaces that match numbers.

4

$V = \pi r^2 h$

$= \pi \times 1^2 \times 8$

$V = 8\pi$ cm^3

Estimate: $V = 8\pi$ cm^3

using $\pi = 3$ $\approx 8 \times 3$

$V \approx 24$ cm^3

5

$V = \pi r^2 h$

$= \pi \times 2^2 \times 7$

$V = 28\pi$ cm^3

Estimate: $V = 28\pi$ cm^3

using $\pi = 3$ $\approx 28 \times 3$

$V \approx 84$ cm^3

6

$V = \pi r^2 h$

$= \pi \times 3^2 \times 2$

$V = 18\pi$ cm^3

Estimate: $V = 18\pi$ cm^3

using $\pi = 3$ $\approx 18 \times 3$

$V \approx 54$ cm^3

7

$V = \pi r^2 h$

$= \pi \times 1^2 \times 4$

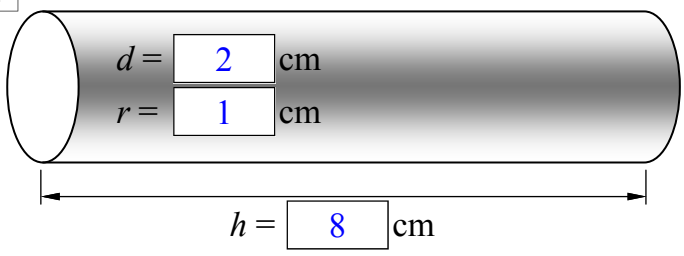
$V = 4\pi$ cm^3

Estimate: $V = 4\pi$ cm^3

using $\pi = 3$ $\approx 4 \times 3$

$V \approx 12$ cm^3

4



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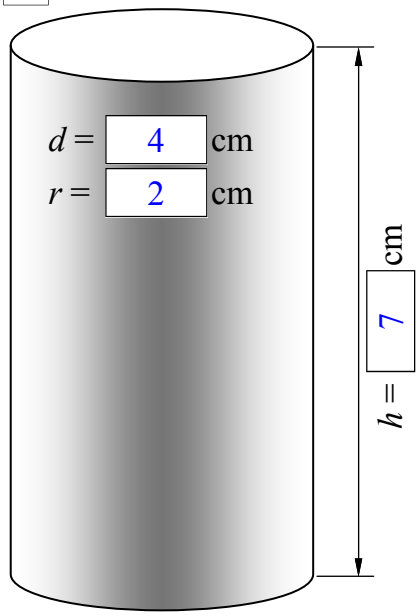

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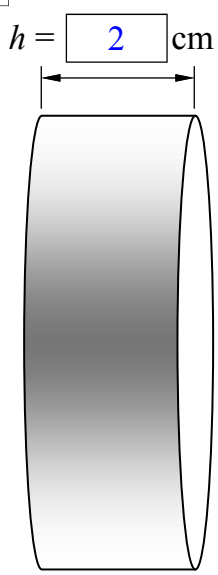

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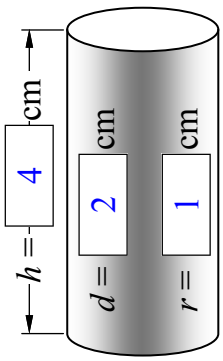


Volume of Cylinders

The standard formula for a prism is V = Ah.

Where: V = the volume of the prism
A = the uniform cross sectional area
h = the height of the prism

But with cylinders the cross sectional area is a circle, which has an area of πr^2. So if you replace A with πr^2 you have:

The formula for a cylinder is V = $\pi r^2 h$.

Where: V = the volume of the prism
r = the radius of the circular face
h = the height of the cylinder

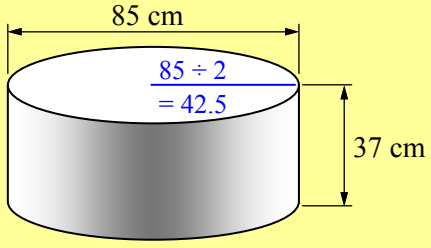
Column 1 starts with an example showing the formula line, substitution and then answer. Make sure you show cubic units. Eg. mm^3, cm^3 and m^3.

Column 2 has an extra step, you are given the diameter of the circle. So before you start, divide the diameter by 2 to get the radius and then use the formula as you did in column 1.

Column 3 applies what you have learned to some worded problems. There is also a table at the bottom of the column, no working required, just answers.

Below is an example of finding the volume when given the cylinder height and diameter.

Example - Find the volume to 1 decimal place.



r = 42.5 cm

$V = \pi r^2 h$ ← Write the formula

$= \pi \times 42.5^2 \times 37$ ← Substitute r and h

$V = 209\ 956.6\ cm^3$ ← Solve showing cubic units

Volume of Cylinders

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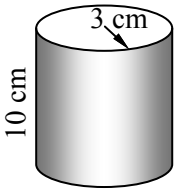
Find the volume of these cylinders using the π key. Round to 1 decimal place.

Example

$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$V = 282.7 \text{ cm}^3$

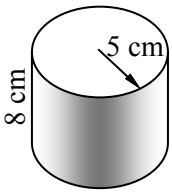


units^3 must match

1

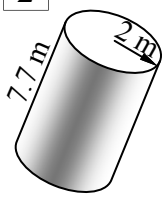
$V = \pi r^2 h$

$V =$ cm^3

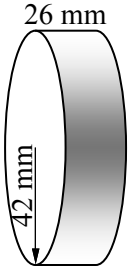


2

$V = \pi r^2 h$



3



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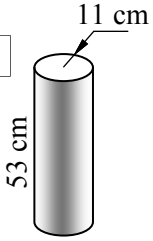

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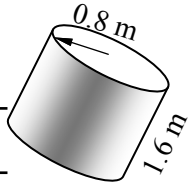

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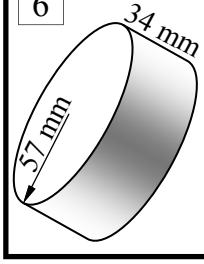


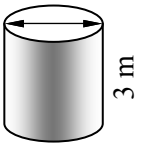
Careful! Diameters in use. Divide by 2 first for the radius. Answer to 2 d.p.

6

$V = \pi r^2 h$

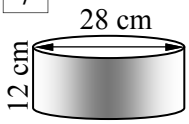
$= \pi \times \square^2 \times 3$

$V =$ m^2



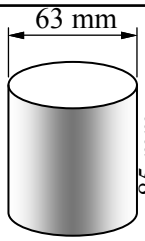
$r =$

7



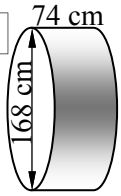
$r =$

8



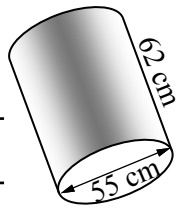
$r =$

9



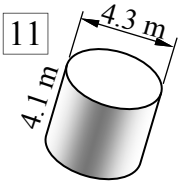
$r =$

10



$r =$

11



$r =$

12

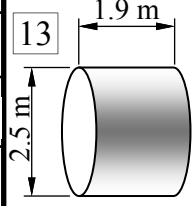
$d = 87$ mm

$h = 15$ mm



$r =$

13

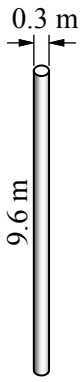


$r =$

Solve these problems involving cylinders.

Calculate the volume of concrete in the cylindrical pylon below to 2 d.p. Calculate its weight (W) given the concrete weighs 2.4 T/m^3.

14

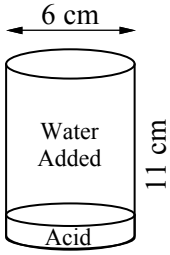


$V =$

$W =$

The glass below contains 45 cm^3 of HCL acid. If water is added to fill the glass find, to 1 decimal place, the:

15 Volume of the empty glass (cm^3).



16 Percentage of acid in the full glass.

Complete the table, which is in cm. Find the volume to the nearest cm^3.

Q.	17	18	19	20	21	22
d	17			106	82	
r		45	65			33
h	25	75	20	55	27	85
Volume (cm^3)						

Volume of Cylinders

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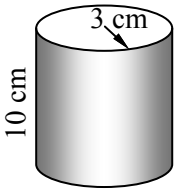
Find the volume of these cylinders using the π key. Round to 1 decimal place.

Example



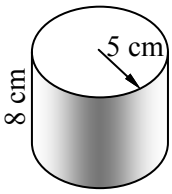
$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$V = 282.7 \text{ cm}^3$

units^3 must match

1

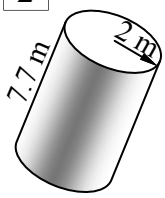


$V = \pi r^2 h$

$= \pi \times 5^2 \times 8$

$V = 628.3 \text{ cm}^3$

2

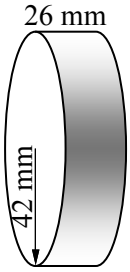


$V = \pi r^2 h$

$= \pi \times 2^2 \times 7.7$

$V = 96.8 \text{ m}^3$

3

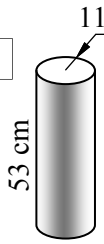


$V = \pi r^2 h$

$= \pi \times 42^2 \times 26$

$V = 144\,086 \text{ mm}^3$

4

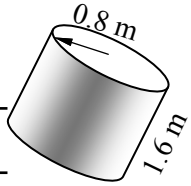


$V = \pi r^2 h$

$= \pi \times 11^2 \times 53$

$V = 20\,147 \text{ cm}^3$

5

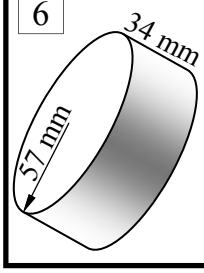


$V = \pi r^2 h$

$= \pi \times 0.8^2 \times 1.6$

$V = 3.2 \text{ m}^3$

6



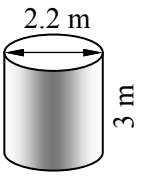
$V = \pi r^2 h$

$= \pi \times 57^2 \times 34$

$V = 347\,039.2 \text{ mm}^3$

Careful! Diameters in use. Divide by 2 first for the radius. Answer to 2 d.p.

6



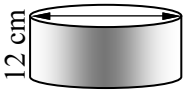
$r = 1.1$ m

$V = \pi r^2 h$

$= \pi \times 1.1^2 \times 3$

$V = 11.4 \text{ m}^2$

7



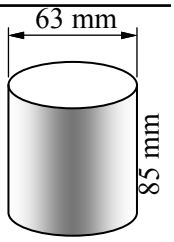
$r = 14$ cm

$V = \pi r^2 h$

$= \pi \times 14^2 \times 12$

$V = 7\,389 \text{ cm}^3$

8



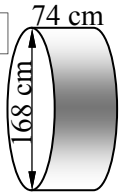
$r = 31.5$ mm

$V = \pi r^2 h$

$= \pi \times 31.5^2 \times 85$

$V = 264\,965.9 \text{ mm}^3$

9



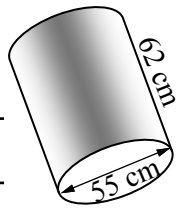
$r = 84$ cm

$V = \pi r^2 h$

$= \pi \times 84^2 \times 74$

$V = 1\,640\,363.8 \text{ cm}^3$

10



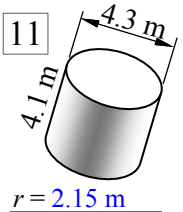
$r = 27.5$ cm

$V = \pi r^2 h$

$= \pi \times 27.5^2 \times 62$

$V = 147\,301.4 \text{ cm}^3$

11



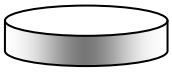
$r = 2.15$ m

$V = \pi r^2 h$

$= \pi \times 2.15^2 \times 4.1$

$V = 59.5 \text{ m}^3$

12



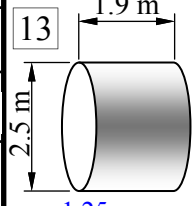
$r = 43.5$ mm

$V = \pi r^2 h$

$= \pi \times 43.5^2 \times 15$

$V = 89\,170.2 \text{ mm}^3$

13



$r = 1.25$ m

$V = \pi r^2 h$

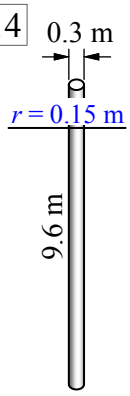
$= \pi \times 1.25^2 \times 1.9$

$V = 9.3 \text{ m}^3$

Solve these problems involving cylinders.

Calculate the volume of concrete in the cylindrical pylon below to 2 d.p. Calculate its weight (W) given the concrete weighs 2.4 T/m^3.

14



$V = \pi r^2 h$

$= \pi \times 0.15^2 \times 9.6$

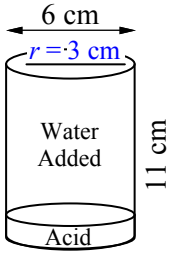
$V = 0.68 \text{ m}^3$

$W = 0.68 \times 2.4$

$W = 1.63 \text{ T}$

The glass below contains 45 cm^3 of HCL acid. If water is added to fill the glass find, to 1 decimal place, the:

15 Volume of the empty glass (cm^3).



$V = \pi r^2 h$

$= \pi \times 3^2 \times 11$

$V = 311 \text{ cm}^3$

16 Percentage of acid in the full glass.

$\%_{ACID} = \frac{45}{311} \times 100$

$\%_{ACID} = 14.5\%$

Complete the table, which is in cm. Find the volume to the nearest cm^3.

Q.	17	18	19	20	21	22
d	17	90	130	106	82	66
r	8.5	45	65	53	41	33
h	25	75	20	55	27	85
Volume (cm^3)	1 135 cm^3	477 129 cm^3	265 465 cm^3	485 360 cm^3	142 587 cm^3	290 802 cm^3

Volume of Cylinders - Different Units, finding *r* and *h*.

Imagine you require a container to hold a specific amount but it must also be a specific height. Finding the radius required needs the formula to be rearranged, that is what this sheet deals with.

Column 1 starts with asking you to rearrange the question given the steps. Then the new rearranged formula is used to find the height of cylinders given the volume and radius (or diameter).

Column 2 is a break away from rearranging/use of formulae. It is finding the volume but the answer is required in cm3, while the dimensions are in various units. So change the units to cm and solve. The last question in the column requires you to rearrange $C=2\pi r$ first, then use the volume formula.

Column 3 gives you the height and the volume and asks you to rearrange the volume formula to find the radius. Then apply to solve for *r*.

Volume of Cylinders - Different Units, finding *r* and *h*.

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Rearrange the volume formula to make *h* the subject. Follow the steps.

1 $V = \pi r^2 h$

$\div \pi r^2$ $\div \pi r^2$ Divide both sides by πr^2.

Rewrite with *h* on the L.H.S

Using $h = \frac{V}{\pi r^2}$ find the height of the cylinder given the volume and radius. To 1 d.p.

2 $V = 550$ cm^3, $r = 5$ cm.

5 5 0 ÷ (π × 5 x^2) =

$h = \frac{V}{\pi r^2} = \frac{\square}{\pi \times \square^2}$

$h =$

3 $V = 11\,680$ cm^3, $r = 13$ cm.

$h = \frac{V}{\pi r^2} = \frac{\square}{\pi \times \square^2}$

$h =$

4 $V = 2\,386$ mm^3, $r = 7$ mm.

5 $V = 5\,917$ mm^3, $d = 12.4$ mm

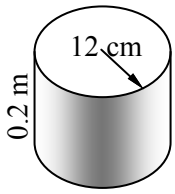
6 $V = 10\,781$ cm^3, $d = 34.4$ cm.

Find the volume of these cylinders in cm^3. Round to 2 decimal places.

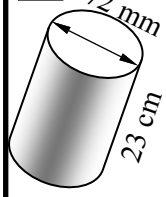
7

$V = \pi r^2 h$

$V =$ cm^3



8



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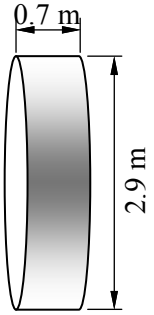
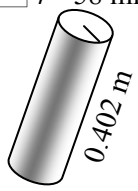


10 $r = 58$ mm



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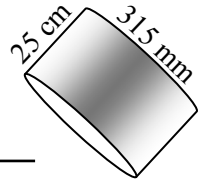

Find the volume (2 d.p.) of a cylinder which has a 15 cm height and a circumference of 0.75 m around its circular face.

12

Rearrange the volume formula to make *r* the subject. Follow the steps.

13 $V = \pi r^2 h$

$\div \pi h$ $\div \pi h$ Divide both sides by πh

$\sqrt{\ }$ $\sqrt{\ }$ Square root both sides

$\Rightarrow$

Rewrite with *r* on the L.H.S

Using $r = \sqrt{\frac{V}{\pi h}}$ find the radius given V and *h*. To 1 d.p.

14 $V = 282$ cm^3, $h = 10$ cm.

2 8 2 ÷ (π × 1 0) =

Then press $\sqrt{\ }$ =

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{\square}{\pi \times \square}}$

$r =$

15 $V = 905$ cm^3, $h = 8$ cm.

$r = \sqrt{\frac{V}{\pi h}}$

16 $V = 28.1$ m^3, $h = 3.5$ m.

17 $V = 29\,050$ mm^3, $h = 32$ mm.

18 $V = 7\,581$ cm^3, $h = 15.2$ cm.

Volume of Cylinders - Different Units, finding r and h.

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Rearrange the volume formula to make h the subject. Follow the steps.

1 $V = \pi r^2 h$

$\div \pi r^2$ $\div \pi r^2$ Divide both sides by πr^2.

$\frac{V}{\pi r^2} = h$

$h = \frac{V}{\pi r^2}$ Rewrite with h on the L.H.S

Using $h = \frac{V}{\pi r^2}$ find the height of the cylinder given the volume and radius. To 1 d.p.

2 $V = 550$ cm^3, $r = 5$ cm.

5 5 0 ÷ (π × 5 x^2) =

$h = \frac{V}{\pi r^2} = \frac{550}{\pi \times 5^2}$

$h = 7$ cm

3 $V = 11\,680$ cm^3, $r = 13$ cm.

$h = \frac{V}{\pi r^2} = \frac{11\,680}{\pi \times 13^2}$

$h = 22$ cm

4 $V = 2\,386$ mm^3, $r = 7$ mm.

$h = \frac{V}{\pi r^2} = \frac{2\,386}{\pi \times 7^2}$

$h = 15.5$ mm

5 $V = 5\,917$ mm^3, $d = 12.4$ mm

$h = \frac{V}{\pi r^2} = \frac{5\,917}{\pi \times 6.2^2}$

$h = 49$ mm

6 $V = 10\,781$ cm^3, $d = 34.4$ cm.

$h = \frac{V}{\pi r^2} = \frac{10\,781}{\pi \times 17.2^2}$

$h = 11.6$ cm

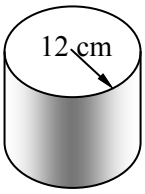
Find the volume of these cylinders in cm^3. Round to 2 decimal places.

7

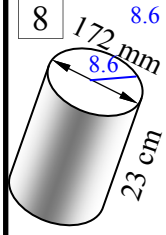
$V = \pi r^2 h$

$= \pi \times 12^2 \times 20$

$V = 9\,047.79$ cm^3



8

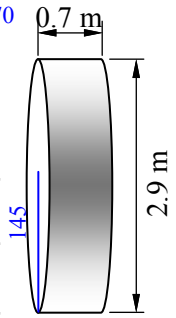


$V = \pi r^2 h$

$= \pi \times 8.6^2 \times 23$

$V = 5\,344.10$ cm^3

9



$V = \pi r^2 h$

$= \pi \times 145^2 \times 70$

$V = 4\,623\,638.99$ cm^3

10

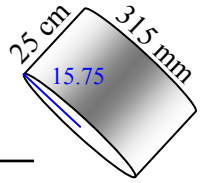


$V = \pi r^2 h$

$= \pi \times 5.8^2 \times 40.2$

$V = 4\,248.46$ mm^3

11



$V = \pi r^2 h$

$= \pi \times 15.75^2 \times 25$

$V = 19\,482.78$ cm^3

Find the volume (2 d.p.) of a cylinder which has a 15 cm height and a circumference of 0.75 m around its circular face.

12 $C = 2\pi r$ $r = {}^{C}/_{2\pi}$

$r = {}^{75}/_{2\pi}$

$r = 11.94$ cm

$V = \pi r^2 h$

$= \pi \times 11.94^2 \times 15$

$V = 6\,718.15$ cm^3

Rearrange the volume formula to make r the subject. Follow the steps.

13 $V = \pi r^2 h$

$\div \pi h$ $\div \pi h$ Divide both sides by πh

$\frac{V}{\pi h} = r^2$

$\sqrt{\ }$ $\sqrt{\ }$ Square root both sides

$\sqrt{\frac{V}{\pi h}} \Rightarrow r = \sqrt{\frac{V}{\pi h}}$

Rewrite with r on the L.H.S

Using $r = \sqrt{\frac{V}{\pi h}}$ find the radius given V and h. To 1 d.p.

14 $V = 282$ cm^3, $h = 10$ cm.

2 8 2 ÷ (π × 1 0) =

Then press $\sqrt{\ }$ =

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{282}{\pi \times 10}}$

$r = 3$ cm

15 $V = 905$ cm^3, $h = 8$ cm.

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{905}{\pi \times 8}}$

$r = 6$ cm

16 $V = 28.1$ m^3, $h = 3.5$ m.

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{28.1}{\pi \times 3.5}}$

$r = 1.6$ m

17 $V = 29\,050$ mm^3, $h = 32$ mm.

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{29\,050}{\pi \times 32}}$

$r = 5.4$ mm

18 $V = 7\,581$ cm^3, $h = 15.2$ cm.

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{7\,581}{\pi \times 15.2}}$

$r = 12.6$ cm

Volume of Composite Solids - Circles

Composite solids are solids that are made by combining two or more solids. To calculate the total volume, separate them into the two volumes (except question 4 - which has 3 solids) and calculate them individually. Then add them together, or subtract them.

Column 1 of the worksheet is answered by adding the volumes together. So calculate the individual volumes and sum them.

Question 1 starts you off with the formulae written and the solid's parts being labeled with V_1 and V_2. Make sure you label each solid. Then the volumes as individual volumes are calculated. Then the last stage is adding the two together. So if you can work out the volume of a cylinder then you just do this twice. Add them and you are done.

Remember to look for when you are given a radius or a diameter, it also may be that you need to calculate some lengths by adding sides together.

Column 2 is also answered by calculating individual volumes but instead of addition use subtraction. It is like a hole has been 'punched' through the solids and because these holes have been removed from the solid, subtraction is used.

Question 5 starts with a rectangular block which has two holes drilled through it. So calculate the volume of the block as if the holes weren't there. Then calculate the volume of one cylindrical hole. The volume will be found by subtracting two 'hole' volumes from the rectangular prism.

You will notice that all of these questions are prisms except the first two questions. Another method of calculating volume is to find the **AREA** of the front face of the prism, then use $\mathbf{V = A}\boldsymbol{h}$ to find the volume. Both methods will result in the same answer.

Volume of Composite Solids - Circles

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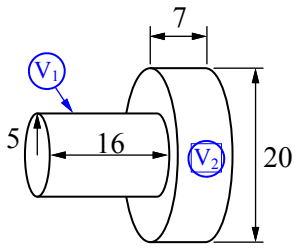
Find the volume to 1 d.p. All lengths are in cm.

1



$V_1 = \pi r^2 h$

$= \pi \times \square^2 \times \square$

$V_1 = \square$ cm^3

$V_2 = \pi r^2 h$

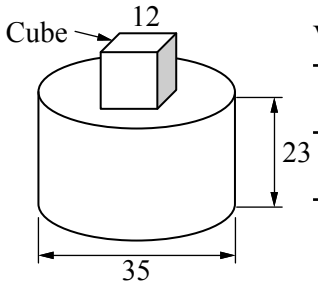
$= \pi \times \square^2 \times \square$

$V_2 = \square$ cm^3

$V = \square + \square$

$V = \square$ cm^3

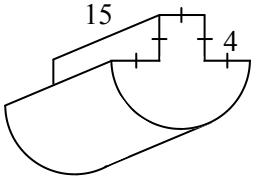
2



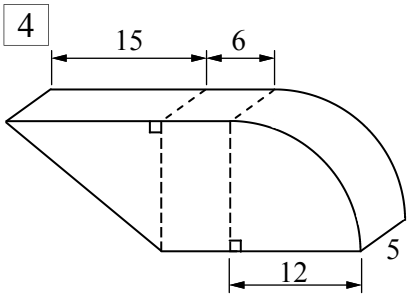
$V_1 = \pi r^2 h$

$V_2 = l^3$

3

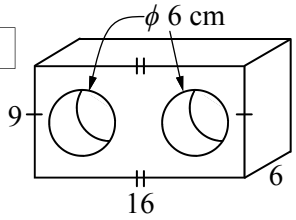


4



Find the volume to 1 d.p. All lengths are in cm. These questions use subtraction.

5

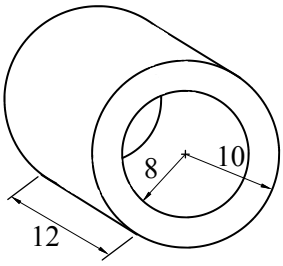


$V_1 = lbh$

$V_2 = \pi r^2 h$

$V = \square - 2 \times \square$

6



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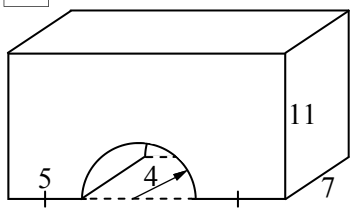

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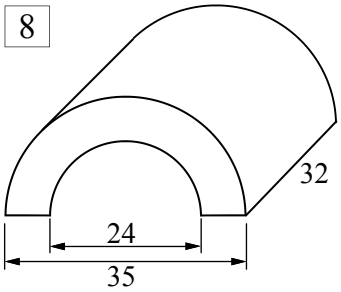

Volume of Composite Solids - Circles

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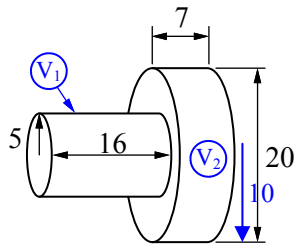
Find the volume to 1 d.p. All lengths are in cm.

1



$V_1 = \pi r^2 h$

$= \pi \times 5^2 \times 16$

$V_1 = 1\,256.6 \text{ cm}^3$

$V_2 = \pi r^2 h$

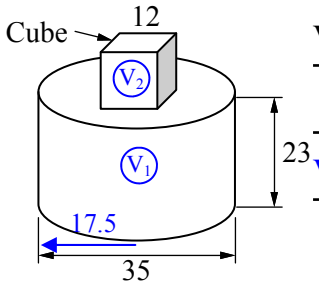
$= \pi \times 10^2 \times 7$

$V_2 = 2\,199.1 \text{ cm}^3$

$V = 1\,256.6 + 2\,199.1$

$V = 3\,455.7 \text{ cm}^3$

2



$V_1 = \pi r^2 h$

$= \pi \times 17.5^2 \times 23$

$V_1 = 22\,128.6 \text{ cm}^3$

$V_2 = L^3$

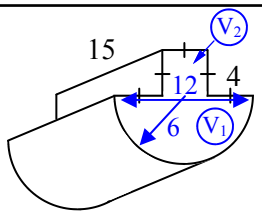
$= 12^3$

$V_2 = 1\,728 \text{ cm}^3$

$V = 22\,128.6 + 1\,728$

$V = 23\,856.6 \text{ cm}^3$

3



$V_1 = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2} \times \pi \times 6^2 \times 15$

$V_1 = 848.2 \text{ cm}^3$

$V_2 = lbh$

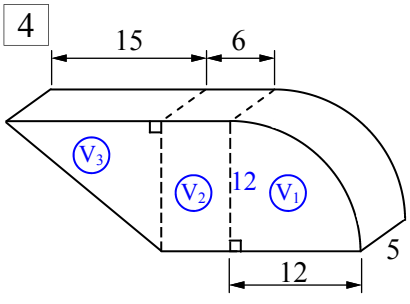
$= 4 \times 4 \times 15$

$V_2 = 240 \text{ cm}^3$

$V = 848.2 + 240$

$V = 1\,088.2 \text{ cm}^3$

4



$V_1 = \frac{1}{4}\pi r^2 h$

$= \frac{1}{4} \times \pi \times 12^2 \times 5$

$V_1 = 565.5 \text{ cm}^3$

$V_2 = lbh$

$= 12 \times 6 \times 5$

$V_2 = 360 \text{ cm}^3$

$V_3 = \frac{1}{2}bh \times H$

$= \frac{1}{2} \times 15 \times 12 \times 5$

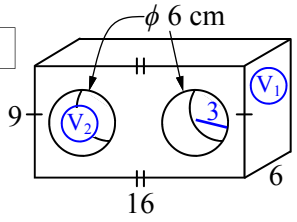
$V_3 = 450 \text{ cm}^3$

$V = 565.5 + 360 + 450$

$V = 1\,375.5 \text{ cm}^3$

Find the volume to 1 d.p. All lengths are in cm. These questions use subtraction.

5



$V_1 = lbh$

$= 16 \times 6 \times 9$

$V_1 = 864 \text{ cm}^3$

$V_2 = \pi r^2 h$

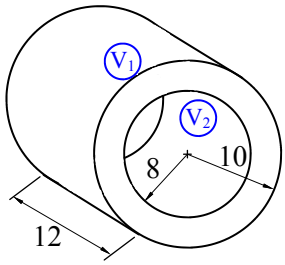
$= \pi \times 3^2 \times 6$

$V_2 = 169.6 \text{ cm}^3$

$V = 864 - 2 \times 169.6$

$V = 524.8 \text{ cm}^3$

6



$V_1 = \pi r^2 h$

$= \pi \times 10^2 \times 12$

$V_1 = 3\,769.9 \text{ cm}^3$

$V_2 = \pi r^2 h$

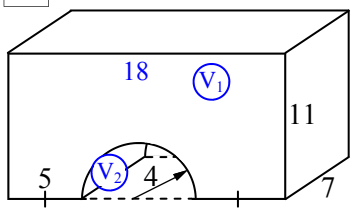
$= \pi \times 8^2 \times 12$

$V_2 = 2\,412.7 \text{ cm}^3$

$V = 3\,769.9 - 2\,412.7$

$V = 1\,357.2 \text{ cm}^3$

7



$V_1 = lbh$

$= 18 \times 11 \times 7$

$V_1 = 1\,386 \text{ cm}^3$

$V_2 = \frac{1}{2}\pi r^2 h$

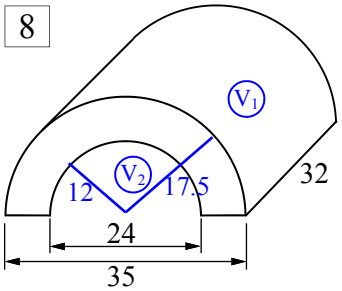
$= \frac{1}{2} \times \pi \times 4^2 \times 7$

$V_2 = 175.9 \text{ cm}^3$

$V = 1\,386 - 175.9$

$V = 1\,210.1 \text{ cm}^3$

8



$V_1 = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2} \times \pi \times 17.5^2 \times 32$

$V_1 = 15\,393.8 \text{ cm}^3$

$V_2 = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2} \times \pi \times 12^2 \times 32$

$V_2 = 7\,238.2 \text{ cm}^3$

$V = 15\,393.8 - 7\,238.2$

$V = 8\,155.6 \text{ cm}^3$

Circle - Find a Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Circle - Find a Word

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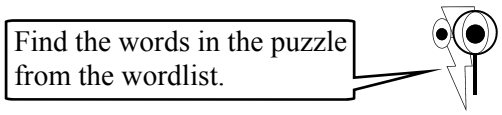

M	I	N	O	R	N	N	P	I	I	K	L	I	M	A
P	E	T	I	S	O	P	M	O	C	D	A	U	Q	R
Q	U	R	A	E	L	C	R	I	C	I	M	E	S	E
W	V	O	P	G	R	I	I	M	U	Q	C	A	T	V
X	R	T	E	M	O	U	A	D	A	N	G	L	E	O
Q	E	A	L	E	T	S	T	S	E	J	E	R	T	L
U	T	T	C	N	C	V	N	R	I	O	O	K	J	U
A	E	I	R	T	E	C	E	N	T	R	E	R	C	T
D	M	O	I	B	S	F	G	R	T	L	M	L	U	I
R	I	N	C	D	M	N	N	S	U	I	D	A	R	O
A	R	Q	U	U	A	R	A	N	T	L	B	D	V	N
N	E	U	C	D	W	R	T	V	Y	X	C	R	E	W
T	P	R	R	V	O	L	U	M	E	W	R	O	D	I
T	I	A	A	N	N	U	L	U	S	Y	A	H	I	D
C	N	N	B	R	E	T	E	M	A	I	D	C	I	I

WORDLIST

AREA	RADII	ANGLE	TANGENT	CENTRE
SECTOR	MINOR	CURVED	VOLUME	COMPOSITE
PI	ANNULUS	CIRCLE	MAJOR	RADIUS
REVOLUTION	QUADRANT	PERIMETER	SEGMENT	SEMICIRCLE
DIAMETER	CHORD	ARC	ROTATION	CIRCUMFERENCE

Circle - Find a Word

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Find the words in the puzzle from the wordlist.



M	I	N	O	R	N	N	P	I	I	K	L	I	M	A
P	E	T	I	S	O	P	M	O	C	D	A	U	Q	R
Q	U	R	A	E	L	C	R	I	C	I	M	E	S	E
W	V	O	P	G	R	I	I	M	U	Q	C	A	T	V
X	R	T	E	M	O	U	A	D	A	N	G	L	E	O
Q	E	A	L	E	T	S	T	S	E	J	E	R	T	L
U	T	T	C	N	C	V	N	R	I	O	O	K	J	U
A	E	I	R	T	E	C	E	N	T	R	E	R	C	T
D	M	O	I	B	S	F	G	R	T	L	M	L	U	I
R	I	N	C	D	M	N	N	S	U	I	D	A	R	O
A	R	Q	U	U	A	R	A	N	T	L	B	D	V	N
N	E	U	C	D	W	R	T	V	Y	X	C	R	E	W
T	P	R	R	V	O	L	U	M	E	W	R	O	D	I
T	I	A	A	N	N	U	L	U	S	Y	A	H	I	D
C	N	N	B	R	E	T	E	M	A	I	D	C	I	I

WORDLIST

~~AREA~~ ~~RADII~~ ~~ANGLE~~ ~~TANGENT~~ ~~CENTRE~~

~~SECTOR~~ ~~MINOR~~ ~~CURVED~~ ~~VOLUME~~ ~~COMPOSITE~~

~~PI~~ ~~ANNULUS~~ ~~CIRCLE~~ MAJOR ~~RADIUS~~

~~REVOLUTION~~ ~~QUADRANT~~ ~~PERIMETER~~ ~~SEGMENT~~ ~~SEMICIRCLE~~

~~DIAMETER~~ ~~CHORD~~ ~~ARC~~ ROTATION ~~CIRCUMFERENCE~~

FREEFALL MATHEMATICS

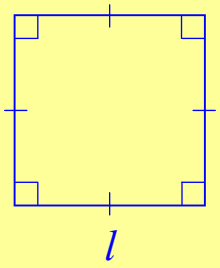
ALTITUDE

AREA & PERIMETER

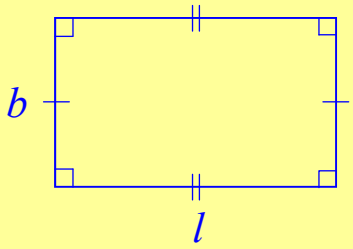
Area of Squares and Rectangles

The area of a square and rectangle is found in the same way. Multiply one side by the other. As squares have sides all the same length, this means you just 'square' the side length.

Square $A = l^2$

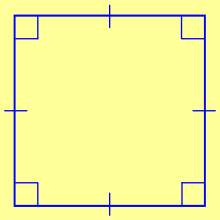


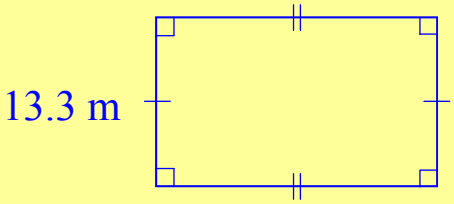
Rectangle $A = lb$



While the answer isn't affected, the length is usually considered the longest length in a rectangle.

Examples - Find the area to 1 decimal place.





$A = l^2$	Write the formula	$A = lb$
$= 17.1^2$	Substitute l (or l & b)	$= 25.2 \times 13.3$
$A = 292.4$ cm^2 (1 d.p.)	Solve showing square units. Round to 1 decimal place.	$A = 335.2$ m^2 (1 d.p.)

In the 2nd column watch for units requiring conversion. Always change the units before multiplying, don't multiply the numbers as they are, then change the answer. Area conversion is more difficult than length conversion.

The last two questions in the third column have the area given, you have to find the side lengths. When finding the side length of a square, square root the area. With rectangles, given one side and the area, divide the area by the given side.

Area of Squares and Rectangles

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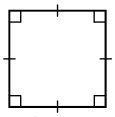
Use $A=l^2$ to find the area of these squares, to 1 d.p. Show a ^{2} as area is in square units.

1



$A = l^2$

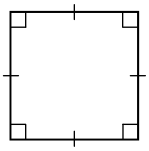
$= \square^2$

Calc: 9 x^2 =

$A = \square$ cm^2

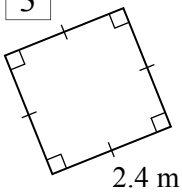
2

A =



A = ☐ cm^2

3



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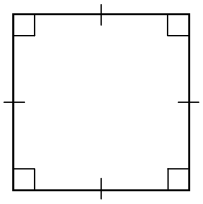

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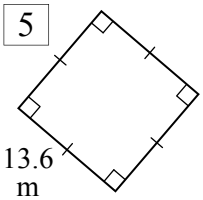

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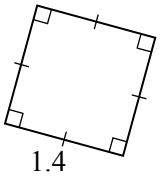

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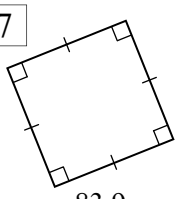
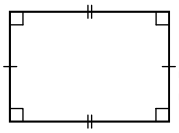


8



Now rectangles. Use $A=lb$ to find the area, round to 1 d.p. when required.

9

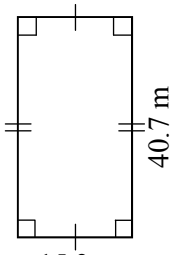


$A = lb$

$= \square \times \square$

A =

10



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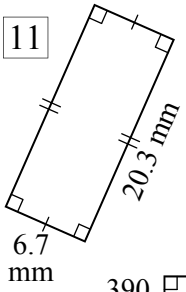

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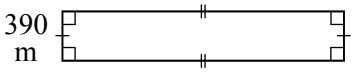
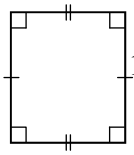
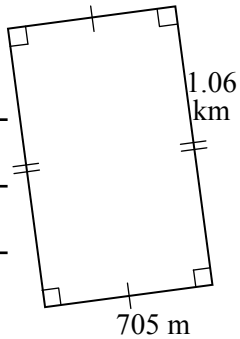


Answer in km^2

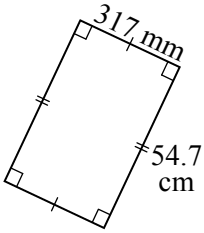
13 Answer in cm^2.



14 In m^2.



15 In mm^2.



Apply your area skills to these questions.

16 How much would it cost to tile a square room with sides of 3.5 m, if tiles cost \$73.20/m^2 ?

Cost =

Cost =

17 A rectangular field has sides 980 m and 660m. Find its area in ha given 1 ha = 10 000 m^2.

18 Office paper often weighs 80 grams per m^2. Find the weight of an A4 sheet, which has sides 29.7 cm and 21 cm. Answer to the nearest gram.

19 A square has an area of 275.56 cm^2. What is its side length?

20 A rectangle has an area of 630.8 mm^2. Find its breadth in cm if it has a length of 16.6 mm.

Area of Squares and Rectangles

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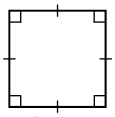
Use $A=l^2$ to find the area of these squares, to 1 d.p. Show a^2 as area is in square units.

1



$A = l^2$

$= 9^2$

Calc: 9 x^2 =

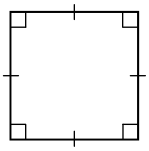
$A = 81$ cm^2

2

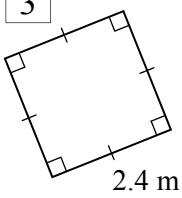
$A = l^2$

$= 12^2$

$A = 144$ cm^2



3

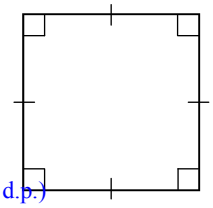


$A = l^2$

$= 2.4^2$

$A = 5.76$ m^2

4

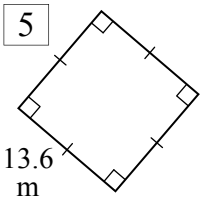


$A = l^2$

$= 11.7^2$

$A = 136.9$ mm^2 (1 d.p.)

5

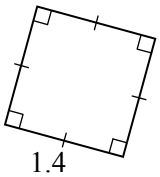


$A = l^2$

$= 13.6^2$

$A = 185$ m^2 (1 d.p.)

6

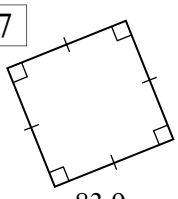


$A = l^2$

$= 1.4^2$

$A = 2$ km^2 (1 d.p.)

7



$A = l^2$

$= 83.9^2$

$A = 7\,039.2$ cm^2 (1 d.p.)

8



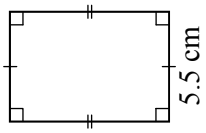
$A = l^2$

$= 7.5^2$

$A = 56.3$ km^2 (1 d.p.)

Now rectangles. Use $A=lb$ to find the area, round to 1 d.p. when required.

9

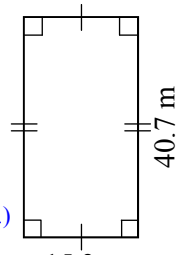


$A = lb$

$= 8 \times 5.5$

$A = 44$ cm^2

10

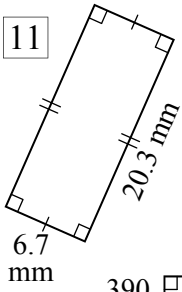


$A = lb$

$= 40.7 \times 15.3$

$A = 622.7$ m^2 (1 d.p.)

11

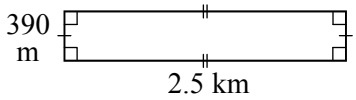


$A = lb$

$= 20.3 \times 6.7$

$A = 136$ mm^2 (1 d.p.)

12



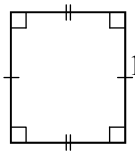
$A = lb$

$= 2.5 \times 0.39$

$A = 1$ km^2 (1 d.p.)

Answer in km^2

13 Answer in cm^2.

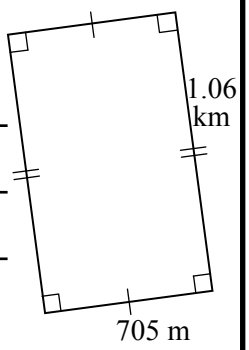


$A = lb$

$= 11.3 \times 8.2$

$A = 92.7$ cm^2 (1 d.p.)

14 In m^2.

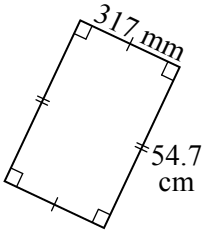


$A = lb$

$= 1\,060 \times 705$

$A = 747\,300$ m^2

15 In mm^2.



$A = lb$

$= 547 \times 317$

$A = 173\,399$ mm^2

Apply your area skills to these questions.

16 How much would it cost to tile a square room with sides of 3.5 m, if tiles cost \$73.20/m^2 ?

$A = l^2$

$= 3.5^2$

$A = 12.25$ m^2

Cost $= 12.25 \times 73.20$

Cost = \$896.70

17 A rectangular field has sides 980 m and 660m. Find its area in ha given 1 ha = 10 000 m^2.

$A = lb$

$= 980 \times 660$

$A = 646\,800$ m^2

$A = \frac{646\,800}{10\,000}$

$A = 64.68$ ha

18 Office paper often weighs 80 grams per m^2. Find the weight of an A4 sheet, which has sides 29.7 cm and 21 cm. Answer to the nearest gram.

$A = lb$

$= 0.297 \times 0.21$

$A = 0.06237$ m^2

Weight $= 0.06237 \times 80 = 5$ g Nearest g

19 A square has an area of 275.56 cm^2. What is its side length?

$l = \sqrt{A}$

$= \sqrt{275.56}$

$l = 16.6$ cm

20 A rectangle has an area of 630.8 mm^2. Find its length in cm if it has a breadth of 16.6 mm.

$l = A \div B$

$= \frac{630.8}{16.6}$

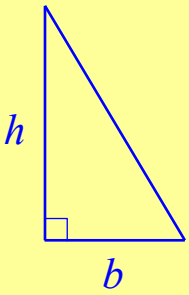
$l = 38$ mm

$l = 3.8$ cm

Area of Triangles

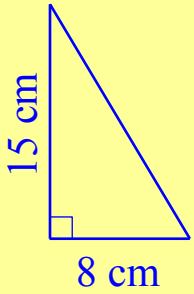
The area of a triangle is found by "half times the base times the perpendicular height." A triangle is half of a rectangle, so it makes sense that the formula is the same as that of a rectangle except it is halved.

Triangle $A = \frac{1}{2}bh$



Where:
b = base length
h = perpendicular height

Note that the height is the perpendicular height, the base and the height must always be at right angles to each other. Below is an example of finding the area of a triangle.



$A = \frac{1}{2}bh$ ← Write the formula

$= \frac{1}{2} \times 8 \times 15$ ← Substitute b and h

$A = 60 \text{ cm}^2$ ← Solve showing square units. Round answer if required.

Column 1 asks you to find the area for a triangle. Remember to watch for units.

Column 2 continues finding the area of a triangle but this time you have to select which numbers you multiply, remember that the height must be perpendicular to the base length.

The end of column 2 and start of column 3 has two worded questions for you to attempt.

Column 3 then has a field that has been broken up into separate areas. Find the areas of the triangles, square and rectangle. Add them together for a total area for the field.

Area of Triangles

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Find the area of these triangles. Units are square units, e.g. cm^2.

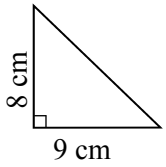
1

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

Calc: 0 . 5 × 9 × 8 =

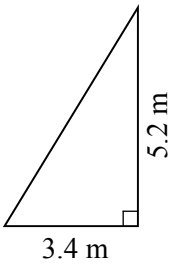
$A = \square\ cm^2$



2

$A =$

$A = \square\ m^2$



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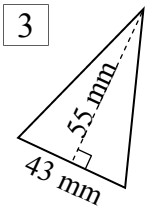

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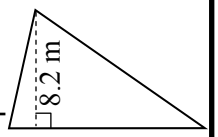

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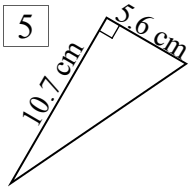

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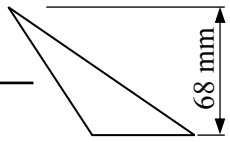
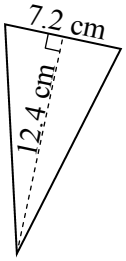


7



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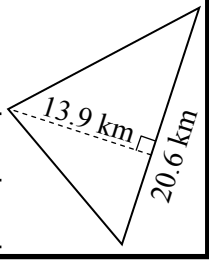
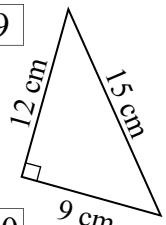


Find the area of these triangles. Cross out the lengths you don't need to use.

9



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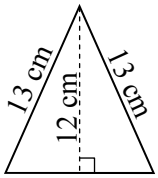

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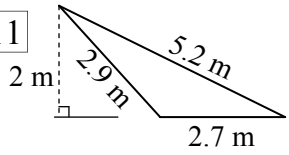

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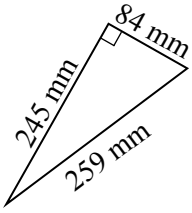

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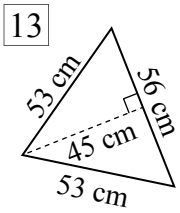


Now apply your skills to these.

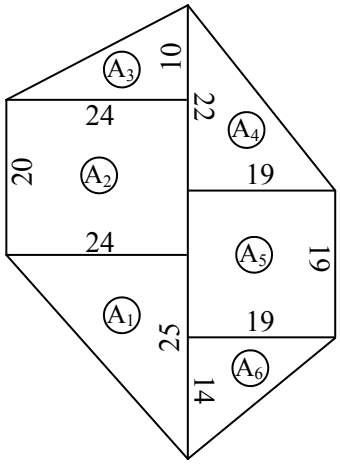
14 Find the area of a triangular paddock with base 205 m and 'height' 307 m, convert the area to hectares (2 d.p.) [1 ha = 10 000 m^2]

15 A yacht jib is a right angled triangle. It is 3.3 m high and has a 2.1 m base. Find the sail area.

16 A race sponsor of the yacht agrees to pay \$8 000 per m^2 of the jib. Find the cost to sponsor the full jib for the yacht above.

Find the area of this field, it is broken up into triangles, squares and rectangles. All lengths in m.

17



$A_1 = \frac{1}{2}bh$ $A_2 = lb$

$A_1 = \square\ m^2$ $A_2 = \square\ m^2$

$A_3 =$ $A_4 =$

$A_3 =$ $A_4 =$

$A_5 = l^2$ $A_6 =$

$A_5 =$ $A_6 =$

Area =

Area of Triangles

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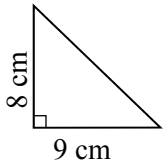
Find the area of these triangles. Units are square units, e.g. cm^2.

1



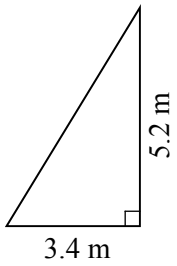
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 9 \times 8$

Calc: 0 . 5 × 9 × 8 =

$A = 36$ cm^2

2

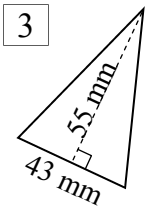


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3.4 \times 5.2$

$A = 8.84$ m^2

3

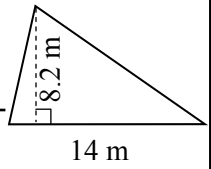


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 43 \times 55$

$A = 1\,182.5$ mm^2

4

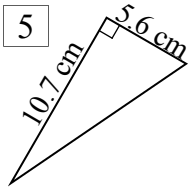


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 14 \times 8.2$

$A = 57.4$ m^2

5

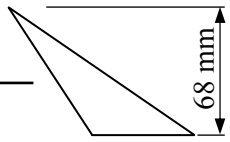


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5.6 \times 10.7$

$A = 29.96$ cm^2

6

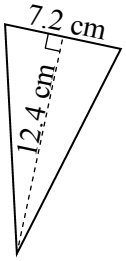


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 46 \times 68$

$A = 1\,564$ mm^2

7

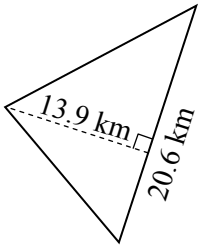


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 7.2 \times 12.4$

$A = 44.64$ cm^2

8



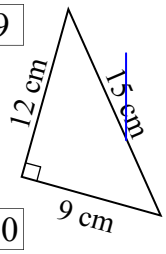
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 20.6 \times 13.9$

$A = 143.17$ km^2

Find the area of these triangles. Cross out the lengths you don't need to use.

9

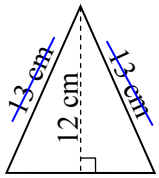


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 9 \times 12$

$A = 54$ cm^2

10

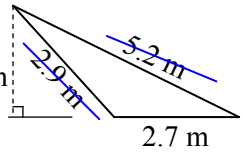


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 10 \times 12$

$A = 60$ cm^2

11

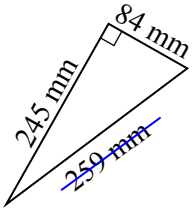


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 2.7 \times 2$

$A = 2.7$ m^2

12

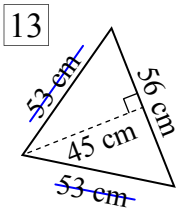


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 84 \times 245$

$A = 10\,290$ mm^2

13



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 56 \times 45$

$A = 1\,260$ cm^2

Now apply your skills to these.

14 Find the area of a triangular paddock with base 205 m and 'height' 307 m, convert the area to hectares (2 d.p.) [1 ha = 10 000 m^2]

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 205 \times 307$

$A = 31\,467.5$ m^2

Hectares: $A = \frac{31\,467.5}{10\,000}$

$A = 3.15$ ha (2 d.p.)

15 A yacht jib is a right angled triangle. It is 3.3 m high and has a 2.1 m base. Find the sail area.

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 2.1 \times 3.3$

$A = 3.465$ m^2

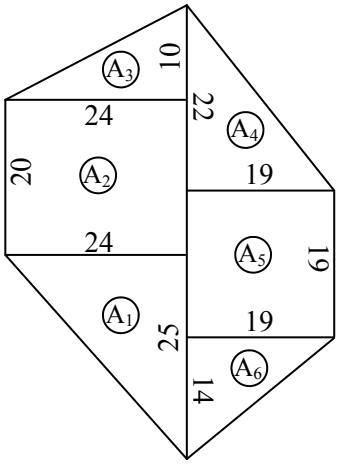
16 A race sponsor of the yacht agrees to pay \$8 000 per m^2 of the jib. Find the cost to sponsor the full jib for the yacht above.

Cost = $3.465 \times 8\,000$

Cost = \$27 720

Find the area of this field, it is broken up into triangles, squares and rectangles. All lengths in m.

17



$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 25 \times 24$

$A_1 = 300$ m^2

$A_2 = lb$

$= 24 \times 20$

$A_2 = 480$ m^2

$A_3 = \frac{1}{2}bh$

$= \frac{1}{2} \times 24 \times 10$

$A_3 = 120$ m^2

$A_4 = \frac{1}{2}bh$

$= \frac{1}{2} \times 22 \times 19$

$A_4 = 209$ m^2

$A_5 = l^2$

$= 19^2$

$A_5 = 361$ m^2

$A_6 = \frac{1}{2}bh$

$= \frac{1}{2} \times 14 \times 19$

$A_6 = 133$ m^2

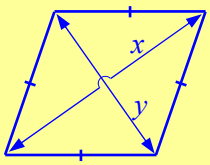
Area = 300+480+120+209+361

+ 133 = 1 603 m^2

Area of Parallelograms and Rhombuses

A parallelogram looks like a rectangle that has been pushed. A rhombus, also called a diamond, looks like a square that has been pushed over. So a parallelogram has two pairs of parallel sides of the same length, like a rectangle. A rhombus has four sides all the same length, with two pairs of parallel sides.

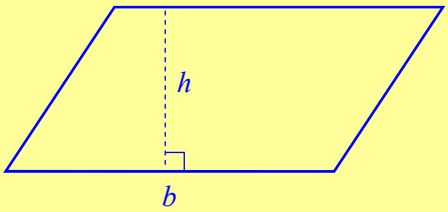
Rhombus $A = \frac{1}{2}xy$



Where:

x and y are the lengths of the diagonals.

Parallelogram $A = bh$

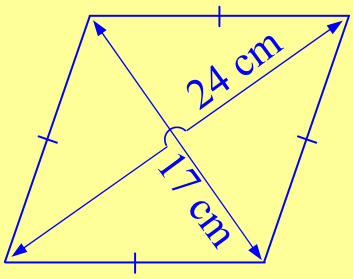


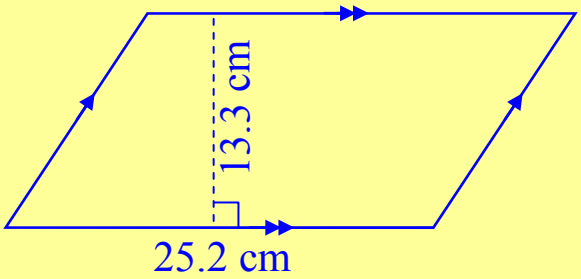
Where:

b = base length

h = perpendicular height

Below are two examples.





$A = \frac{1}{2}xy$	Write the formula	$A = bh$
$= \frac{1}{2} \times 17 \times 24$	Substitute x & y (or b & h)	$= 25.2 \times 13.3$
$A = 204$ cm^2	Solve showing square units. Round to 1 decimal place.	$A = 335.2$ m^2 (1 d.p.)

What happens if you aren't given the diagonals of a rhombus, but instead base and its perpendicular height? Solve just like it is a parallelogram. By definition all rhombuses (or rhombi) are parallelograms.

Area of Parallelograms and Rhombuses

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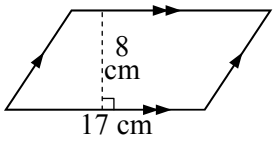
Find the area of these parallelograms. Round to 1 d.p. if needed.

Example

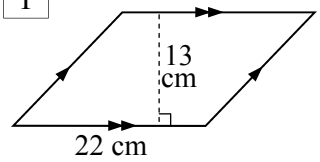


$A = bh$

$= 17 \times 8$

$A = 136 \text{ cm}^2$

1

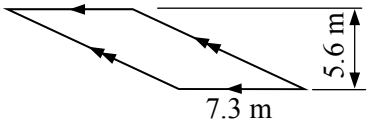


$A = bh$

$= \square \times \square$

$A = \square \text{ cm}^2$

2



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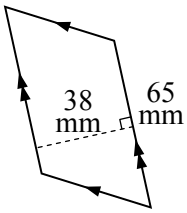

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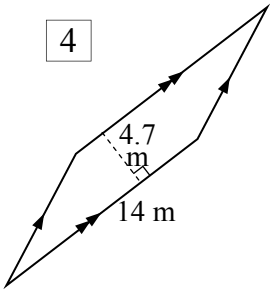
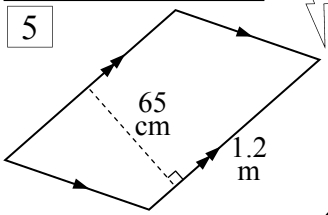


Find the area in cm^2.

5



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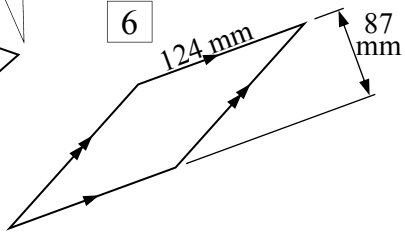

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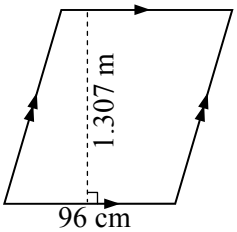

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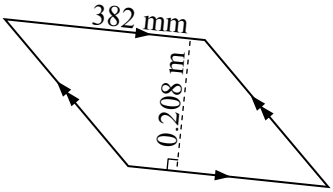
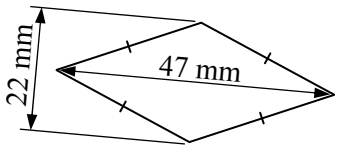


A rhombus is like a square that has been pushed over. Find the area, 1 d.p. when needed.

Example



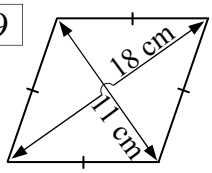
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 2 2 × 4 7 =

$= \frac{1}{2} \times 47 \times 22$

$A = 517 \text{ mm}^2$

9

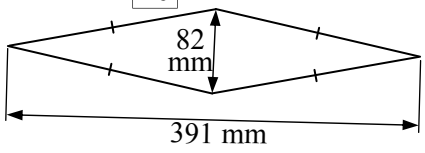


$A = \frac{1}{2}xy$

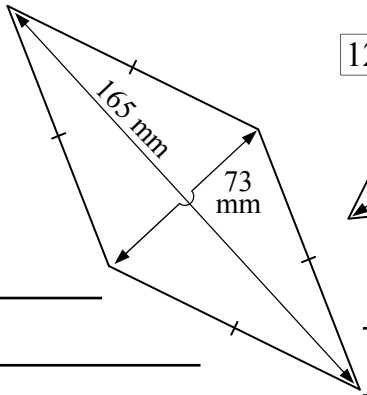
$= \frac{1}{2} \times \square \times \square$

$A = \square \text{ cm}^2$

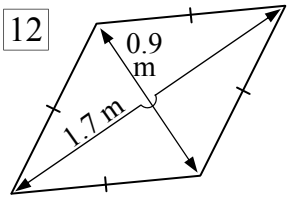
10



11

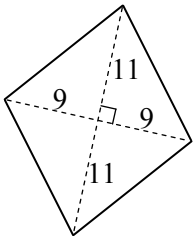


12



All units are in cm.

13



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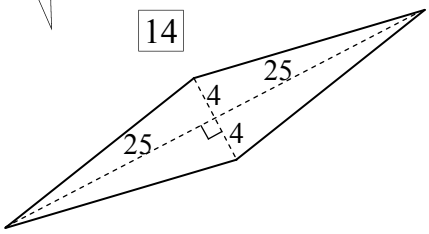
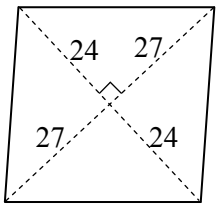
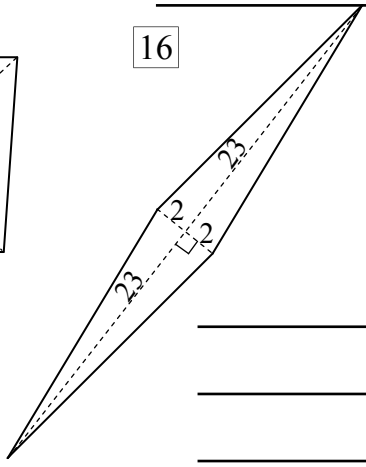


15



16



Area of Parallelograms and Rhombuses

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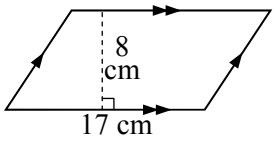
Find the area of these parallelograms. Round to 1 d.p. if needed.

Example

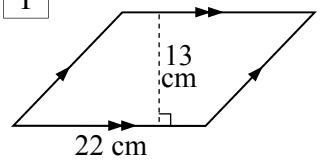


$A = bh$

$= 17 \times 8$

$A = 136 \text{ cm}^2$

1

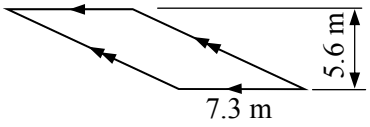


$A = bh$

$= 22 \times 13$

$A = 286 \text{ cm}^2$

2

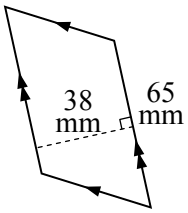


$A = bh$

$= 7.3 \times 5.6$

$A = 40.9 \text{ m}^2$ (1 d.p.)

3

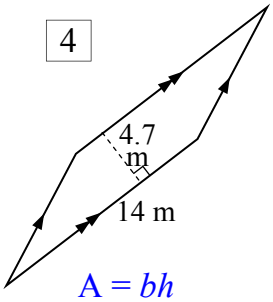


$A = bh$

$= 65 \times 38$

$A = 2\,470 \text{ mm}^2$

4



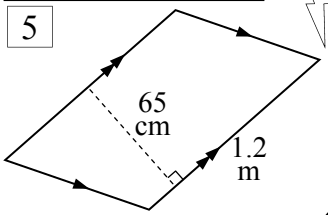
$A = bh$

$= 14 \times 4.7$

$A = 65.8 \text{ m}^2$

Find the area in cm^2.

5

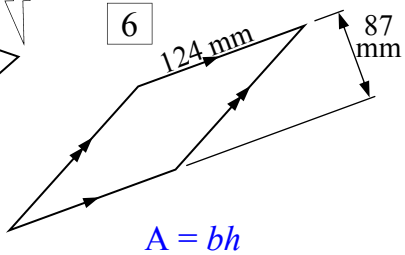


$A = bh$

$= 120 \times 65$

$A = 7\,800 \text{ cm}^2$

6

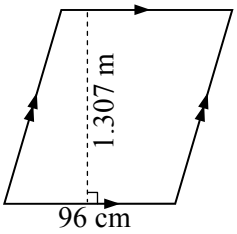


$A = bh$

$= 12.4 \times 8.7$

$A = 107.9 \text{ cm}^2$

7

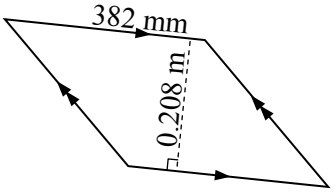


$A = bh$

$= 96 \times 130.7$

$A = 12\,547.2 \text{ cm}^2$

8



(1 d.p.)

$A = bh$

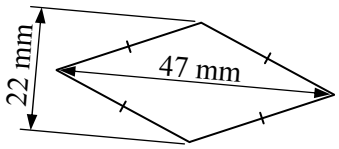
$= 38.2 \times 20.8$

$A = 794.6 \text{ cm}^2$

(1 d.p.)

A rhombus is like a square that has been pushed over. Find the area, 1 d.p. when needed.

Example



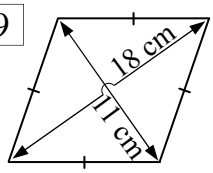
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 2 2 × 4 7 =

$= \frac{1}{2} \times 47 \times 22$

$A = 517 \text{ mm}^2$

9

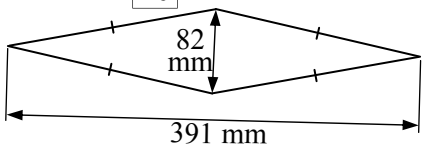


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 11 \times 18$

$A = 99 \text{ cm}^2$

10

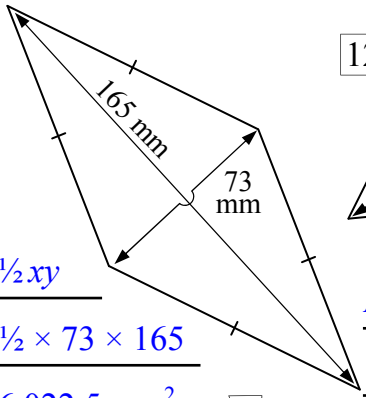


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 82 \times 391$

$A = 16\,031 \text{ mm}^2$

11

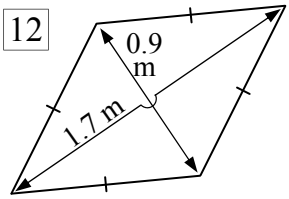


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 73 \times 165$

$A = 6\,022.5 \text{ mm}^2$

12



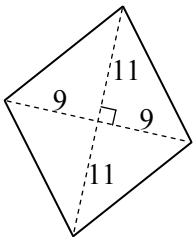
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 1.7 \times 0.9$

$A = 0.8 \text{ m}^2$ (1 d.p.)

All units are in cm.

13

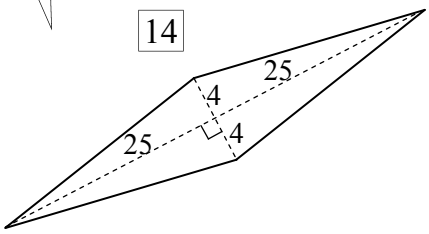


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 18 \times 22$

$A = 198 \text{ cm}^2$

14

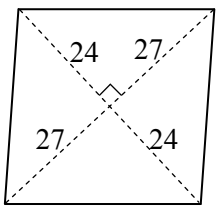


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 50 \times 8$

$A = 200 \text{ cm}^2$

15

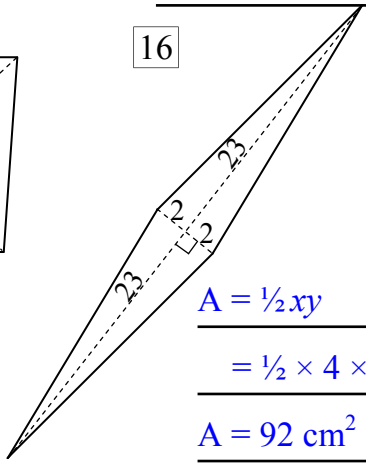


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 54 \times 48$

$A = 1\,296 \text{ cm}^2$

16



$A = \frac{1}{2}xy$

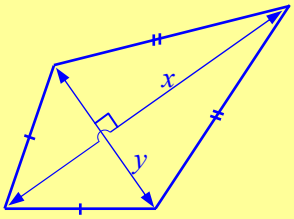
$= \frac{1}{2} \times 4 \times 46$

$A = 92 \text{ cm}^2$

Area of Kites and Trapeziums

A kite is the easiest shape to remember because its name is the same as the kite that you see flying. To find its area the same formula as the rhombus formula is used, that is, its internal diagonal lengths are multiplied together and by a half. Trapeziums are similar to rectangles in a way. The height is the perpendicular height like in a rectangle, but the base is the average of the top and bottom lengths. This average is then multiplied by the height. The formulae are below:

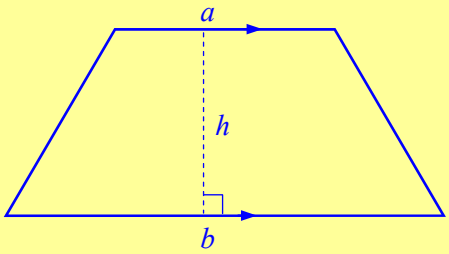
Kite $A = \frac{1}{2}xy$



Where:

x and y are the lengths of the diagonals.

Trapezium $A = \frac{1}{2}(a + b)h$

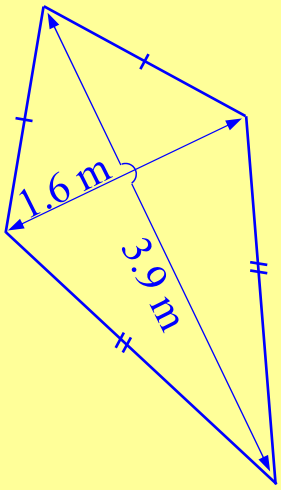


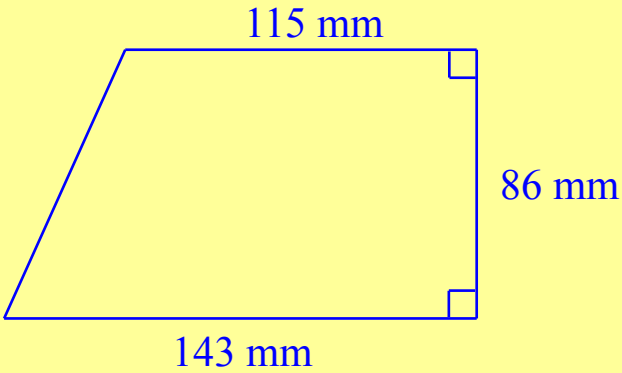
Where:

a, b = parallel side lengths

h = perpendicular height

Here are two examples. With trapeziums you need to use the brackets keys on your calculator.





$A = \frac{1}{2}xy$	Write the formula	$A = \frac{1}{2}(a + b)h$
$= \frac{1}{2} \times 3.9 \times 1.6$	Substitute x & y (or a, b & h)	$= \frac{1}{2} \times (115 + 143) \times 86$
$A = 3.1$ m^2 (1 d.p.)	Solve showing square units. Round to 1 decimal place.	$A = 11\,094$ mm^2

Area of Kites and Trapeziums

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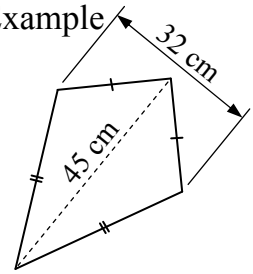
Find the area of these kites. Round to 1 d.p. if needed.

Example



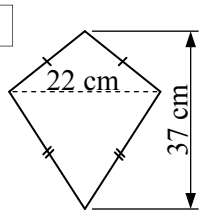
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 3 2 × 4 5 =

$= \frac{1}{2} \times 32 \times 45$

$A = 720 \text{ cm}^2$

1

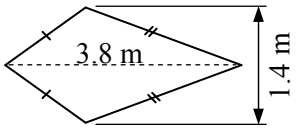


$A = \frac{1}{2}xy$

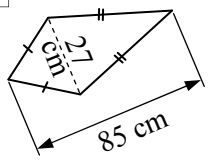
$= \frac{1}{2} \times \square \times \square$

$A = \square \text{ cm}^2$

2



3



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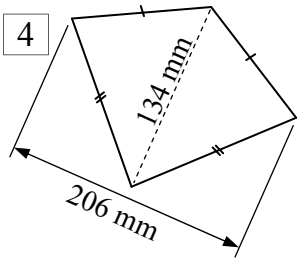
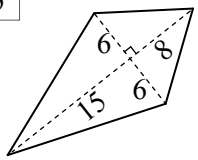
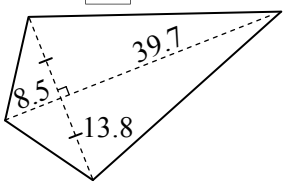


From Q.5 all units are in cm.

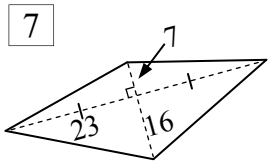
5



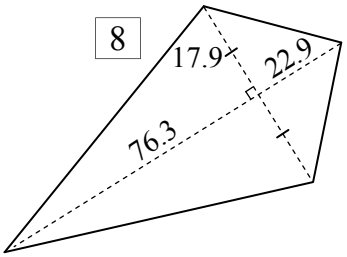
6



7

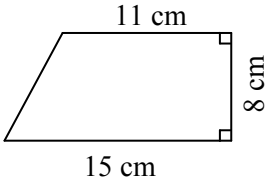


8



Find the area of the trapezia below. Don't forget square units. Eg. mm^2, cm^2, m^2. Round to 1 d.p.

Example



$A = \frac{1}{2}(a + b)\,h$

Calc: 0 . 5 × (1 1 + 1 5) × 8 =

$= \frac{1}{2} \times (11 + 15) \times 8$

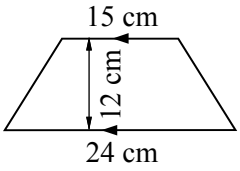
$A = 104 \text{ cm}^2$

9

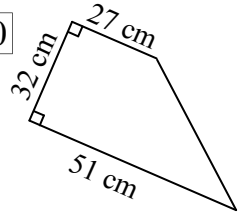
$A = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (\square + \square) \times \square$

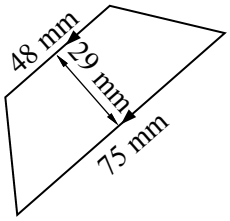
$A = \square \text{ cm}^2$



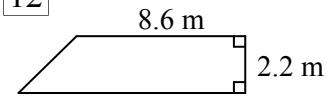
10



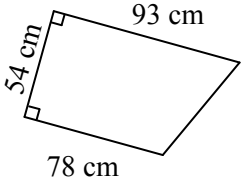
11



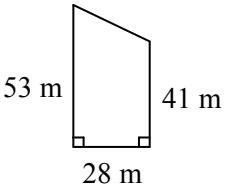
12



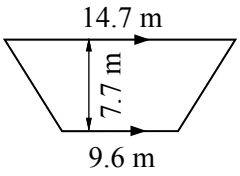
13



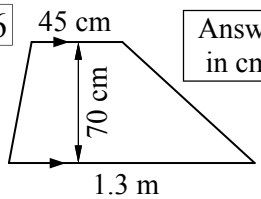
14



15



16



Answer in cm^2

Area of Kites and Trapeziums

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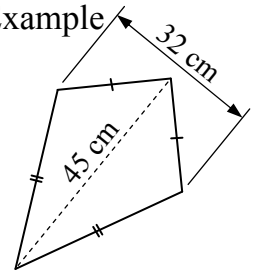
Find the area of these kites. Round to 1 d.p. if needed.

Example



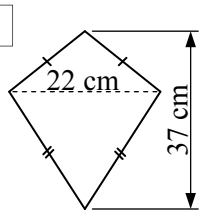
$A = \frac{1}{2}xy$

Calc: 0 . 5 × 3 2 × 4 5 =

$= \frac{1}{2} \times 32 \times 45$

$A = 720 \text{ cm}^2$

1

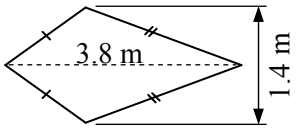


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 22 \times 37$

$A = 407 \text{ cm}^2$

2

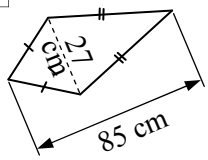


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 1.4 \times 3.8$

$A = 2.66 \text{ m}^2$

3

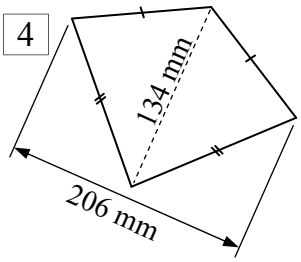


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 27 \times 85$

$A = 1\,147.5 \text{ cm}^2$

4



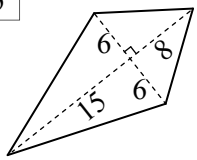
$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 206 \times 134$

$A = 13\,802 \text{ mm}^2$

From Q.5 all units are in cm.

5

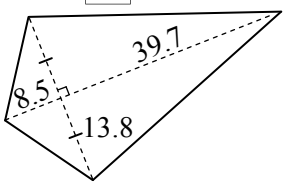


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 12 \times 23$

$A = 138 \text{ cm}^2$

6

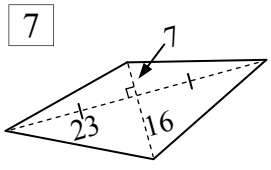


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 27.6 \times 48.2$

$A = 665.2 \text{ cm}^2$ (1 d.p.)

7

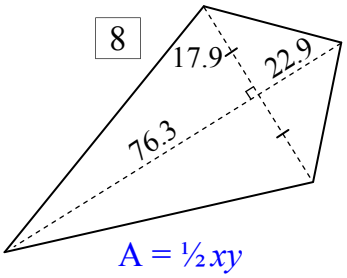


$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 23 \times 46$

$A = 529 \text{ cm}^2$

8



$A = \frac{1}{2}xy$

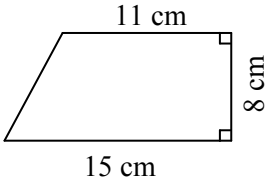
$= \frac{1}{2} \times 35.8 \times 99.2$

$A = 1\,775.7 \text{ cm}^2$

(1 d.p.)

Find the area of the trapezia below. Don't forget square units. Eg. mm^2, cm^2, m^2. Round to 1 d.p.

Example



$A = \frac{1}{2}(a + b)h$

Calc: 0 . 5 × (1 1 + 1 5) × 8 =

$= \frac{1}{2} \times (11 + 15) \times 8$

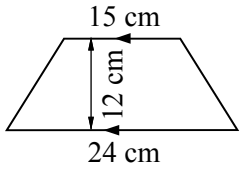
$A = 104 \text{ cm}^2$

9

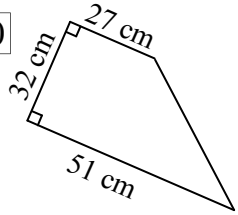
$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (15 + 24) \times 12$

$A = 234 \text{ cm}^2$



10



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (27 + 51) \times 32$

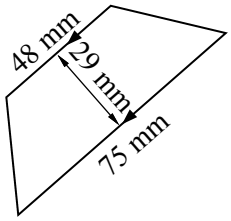
$A = 1\,248 \text{ cm}^2$

11

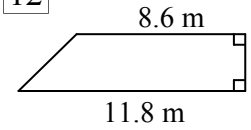
$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (48 + 75) \times 29$

$A = 1\,783.5 \text{ mm}^2$



12



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (8.6 + 11.8) \times 2.2$

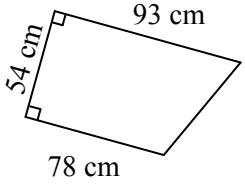
$A = 22.4 \text{ m}^2$ (1 d.p.)

13

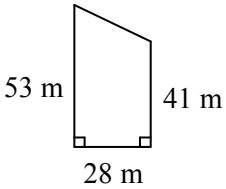
$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (78 + 93) \times 54$

$A = 4\,617 \text{ cm}^2$



14



$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (41 + 53) \times 28$

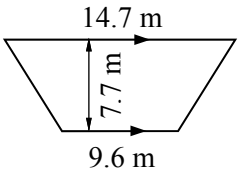
$A = 1\,3\,16 \text{ m}^2$

15

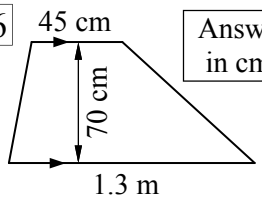
$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (9.6 + 14.7) \times 7.7$

$A = 93.6 \text{ m}^2$ (1 d.p.)



16



Answer in cm^2

$A = \frac{1}{2}(a + b)h$

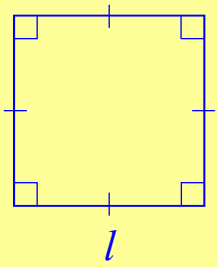
$= \frac{1}{2} \times (45 + 130) \times 70$

$A = 6\,125 \text{ cm}^2$

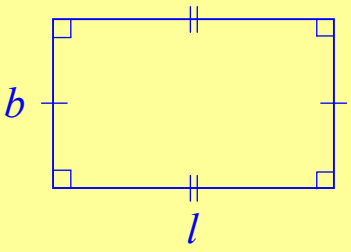
Area of Composite Shapes - Non Circular

Composite shapes are shapes that are made by combining one or more shapes. The area is found by either adding or subtracting the areas. The shapes involved are shown below.

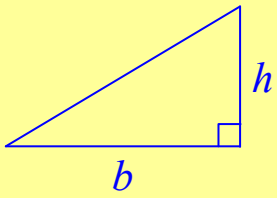
Square: $A = l^2$



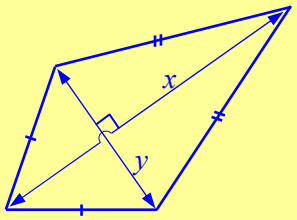
Rectangle: $A = lb$



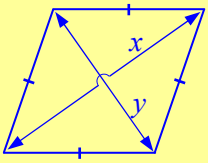
Triangle $A = \frac{1}{2}bh$



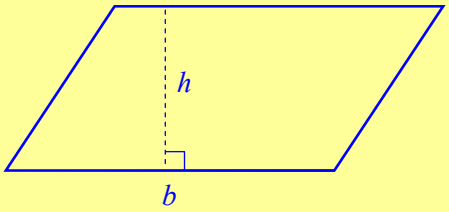
Kite: $A = \frac{1}{2}xy$



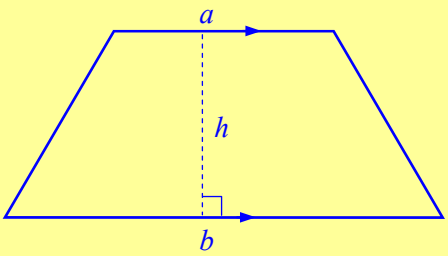
Rhombus $A = \frac{1}{2}xy$



Parallelogram $A = bh$

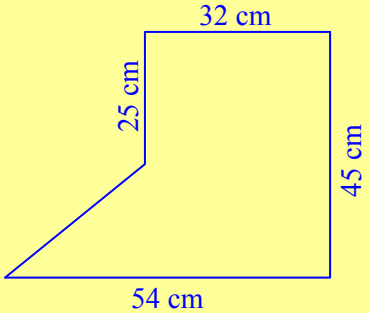


Trapezium $A = \frac{1}{2}(a + b)h$

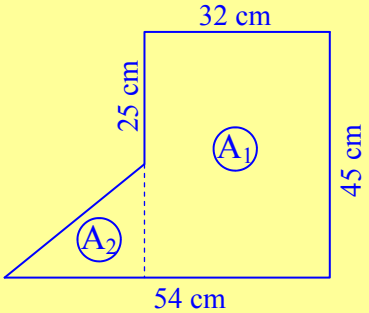


The process for the sheet is to label the areas in the composite shape A_1, A_2 and so on. This may require the shape to have lines drawn inside it to break it up into the plane shapes. Below is an example.

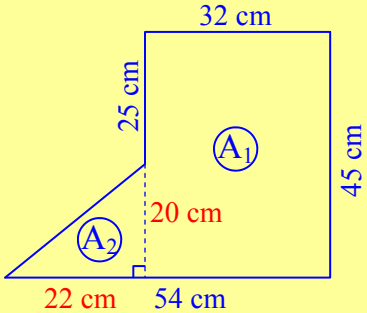
Find the area of the shape.



Break the shape up and label the areas



Find missing lengths (in red)



Solve by finding individual areas and then adding them.

$A_1 = lb$

$= 32 \times 45$

$A_1 = 1\,440 \text{ cm}^2$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 22 \times 20$

$A_2 = 220 \text{ cm}^2$

$A = A_1 + A_2 = 1\,440 + 220$

$A = 1\,660 \text{ cm}^2$

Area of Composite Shapes - Non Circular

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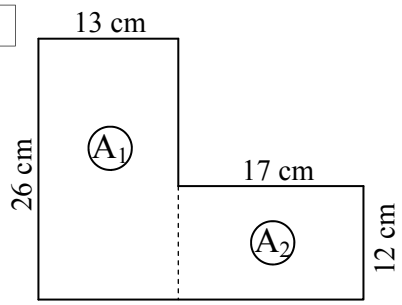
Composite areas are found by dividing the shape into two or more plane shapes, finding their areas then adding or subtracting them. Try these, round to 2 d.p.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

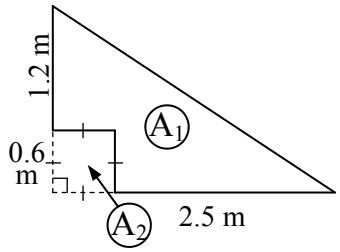
$= \square \times \square$

$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square$

$A = \square$ cm^2

2



$A_1 =$

$=$

$A_1 = \square$ m^2

$A_2 =$

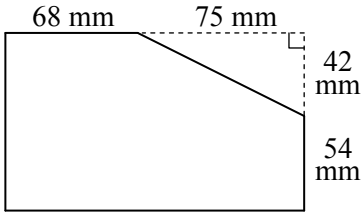
$=$

$A_2 = \square$ m^2

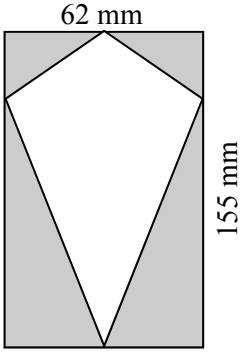
$A = A_1 - A_2 = \square - \square$

$A = \square$ m^2

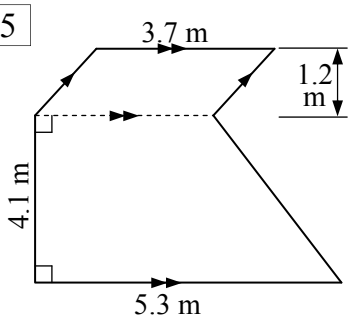
3



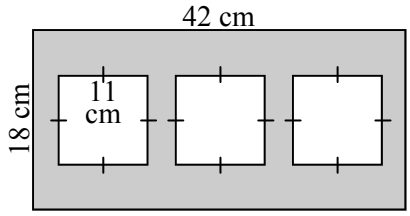
4 Find the shaded area.



5

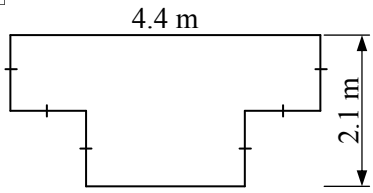


6 Find the shaded area.

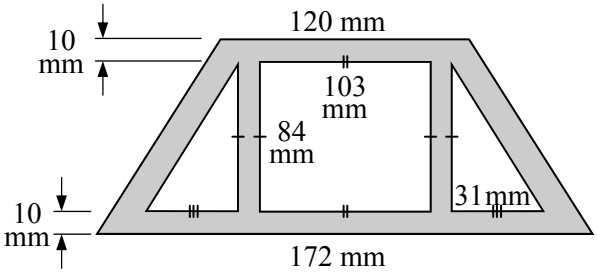


$A = A_1 - 3A_2 =$

7



8 Find the shaded area.



Area of Composite Shapes - Non Circular

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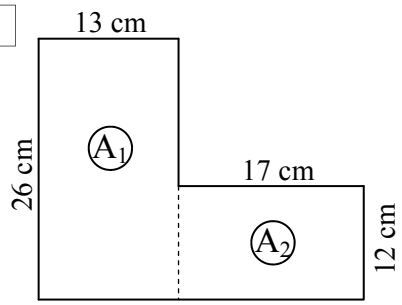
Composite areas are found by dividing the shape into two or more plane shapes, finding their areas then adding or subtracting them. Try these, round to 2 d.p.

1



$A_1 = lb$

$= 26 \times 13$

$A_1 = 338$ cm^2

$A_2 = lb$

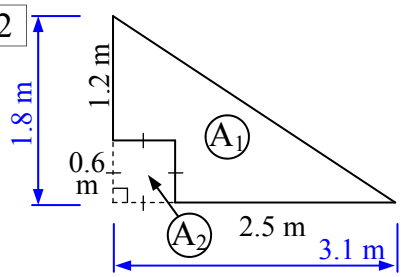
$= 17 \times 12$

$A_2 = 204$ cm^2

$A = A_1 + A_2 = 338 + 204$

$A = 542$ cm^2

2



$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 3.1 \times 1.8$

$A_1 = 2.79$ m^2

$A_2 = l^2$

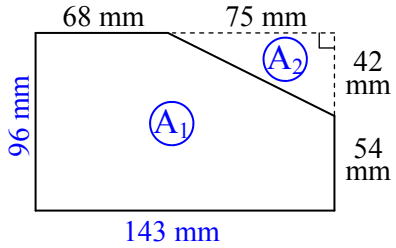
$= 0.6^2$

$A_2 = 0.36$ m^2

$A = A_1 - A_2 = 2.79 - 0.36$

$A = 2.43$ m^2

3



$A_1 = lb$

$= 143 \times 96$

$A_1 = 13\,728$ mm^2

$A_2 = \frac{1}{2}bh$

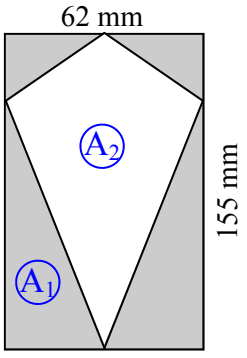
$= \frac{1}{2} \times 42 \times 75$

$A_2 = 1\,575$ mm^2

$A = A_1 - A_2 = 13\,728 - 1\,575$

$A = 12\,153$ mm^2

4 Find the shaded area.



$A_1 = lb$

$= 155 \times 62$

$A_1 = 9\,610$ mm^2

$A_2 = \frac{1}{2}xy$

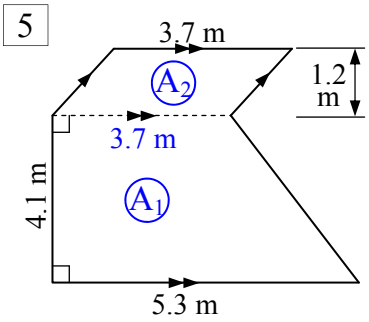
$= \frac{1}{2} \times 62 \times 155$

$A_2 = 4\,805$ mm^2

$A = A_1 - A_2 = 9\,610 - 4\,805$

$A = 4\,805$ mm^2

5



$A_1 = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (5.3 + 3.7) \times 4.1$

$A_1 = 18.45$ m^2

$A_2 = bh$

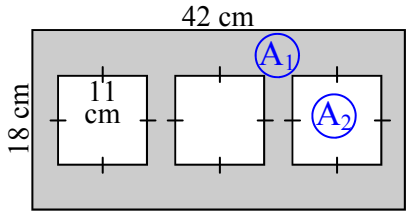
$= 3.7 \times 1.2$

$A_2 = 4.44$ m^2

$A = A_1 + A_2 = 18.45 + 4.44$

$A = 22.89$ m^2

6 Find the shaded area.



$A_1 = lb$

$= 42 \times 18$

$A_1 = 756$ cm^2

$A_2 = l^2$

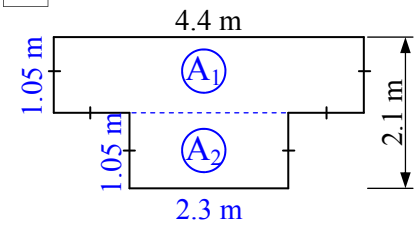
$= 11^2$

$A_2 = 121$ cm^2

$A = A_1 - 3A_2 = 756 - 3 \times 121$

$A = 393$ cm^2

7



$A_1 = lb$

$= 4.4 \times 1.05$

$A_1 = 4.62$ m^2

$A_2 = lb$

$= 2.3 \times 1.05$

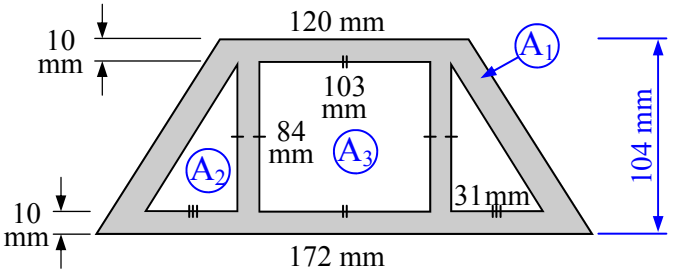
$A_2 = 2.42$ m^2

(2 d.p.)

$A = A_1 + A_2 = 4.62 + 2.42$

$A = 7.04$ m^2

8 Find the shaded area.



$A_1 = \frac{1}{2}(a + b)h$

$= \frac{1}{2} \times (172 + 120) \times 104$

$A_1 = 15\,184$ mm^2

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 31 \times 84$

$A_2 = 1\,302$ mm^2

$A_3 = lb$

$= 103 \times 84$

$A_3 = 8\,652$ mm^2

$A = A_1 - 2 \times A_2 - A_3 = 15\,184 - 2 \times 1\,302 - 8\,652$

$A = 3\,928$ mm^2

Exact & Estimate - Area

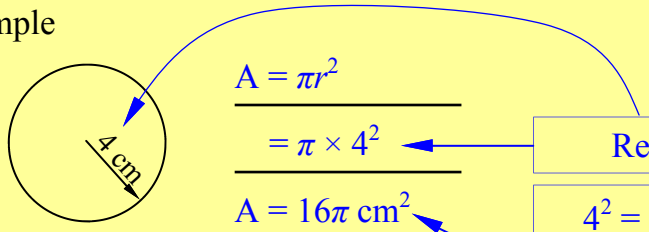
The area of a circle is the measurement of space inside the circle. Formula:

$$A = \pi r^2$$

Where: A = the area (square units, same as the radius/diameter except ^{2}, ie. cm^2, m^2, mm^2)
π = Pi, it's a value on your calculator 3.14159….. Which is a bit more than 3!
r = the radius, the length from the centre to any point on the circumference

Column 1 starts with circles that have a radius shown, to find the area in terms of π (also called the exact area) use the value of r given in the circle. To make an estimate, use the exact area and substitute 3 for π.

Example



$A = \pi r^2$

$= \pi \times 4^2$ ← Replace r with 4, (the radius of the circle)

$A = 16\pi$ cm^2 ← $4^2 = 16$, remember ^{2} is the number times itself not the number times 2. Leave π as is. Write the units.

So the area is 16π cm^2. This is the exact area, to estimate the area without a calculator use $\pi = 3$. This is less than the actual value for π, but it gives a fairly accurate answer without too much calculation.

Estimate:

$A = 16\pi$ cm^2 ← Rewrite the area from previous step

$\approx 16 \times 3$ ← Replace π with 3 and = with ≈

$A \approx 48$ cm^2 ← Solve, use either ≈ or ≒ to indicate 'approximately'.

Column 2 uses diameters. Halve the diameter for the radius and solve as before, there is a formula for calculating the area using d. But it won't be introduced on this sheet.

At the bottom of the page are four circles. Measure the diameter accurate to the nearest cm, then calculate the exact area (in terms of π) and then an estimate, again using $\pi = 3$. Use the working out spaces in the third column to answer these.

Exact and Estimate - Area

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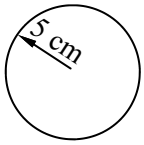
Find the area in terms of π. Then using π = 3, estimate the area.

1 $A = \pi r^2$

$= \pi \times \square^2$

$A = \square\ \pi\ cm^2$

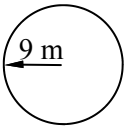


Estimate: $A = \square\ \pi\ cm^2$

$= \square \times 3$

$A \approx \square\ cm^2$

2



$A = \pi r^2$

$=$

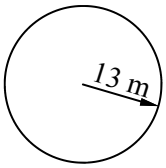
$A =$

Estimate: $A =$

$=$

$A \approx$

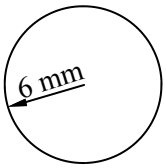
3



Estimate:

$\square \times 3$

4



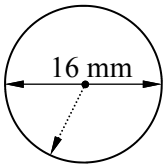
Estimate:

5 Circle radius of 7 km

Exact: Estimate:

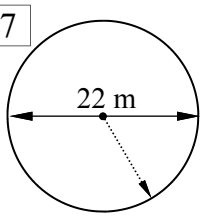
Divide the diameter by 2 to find the radius, then find the exact and estimate.

6



Estimate:

7



Estimate:

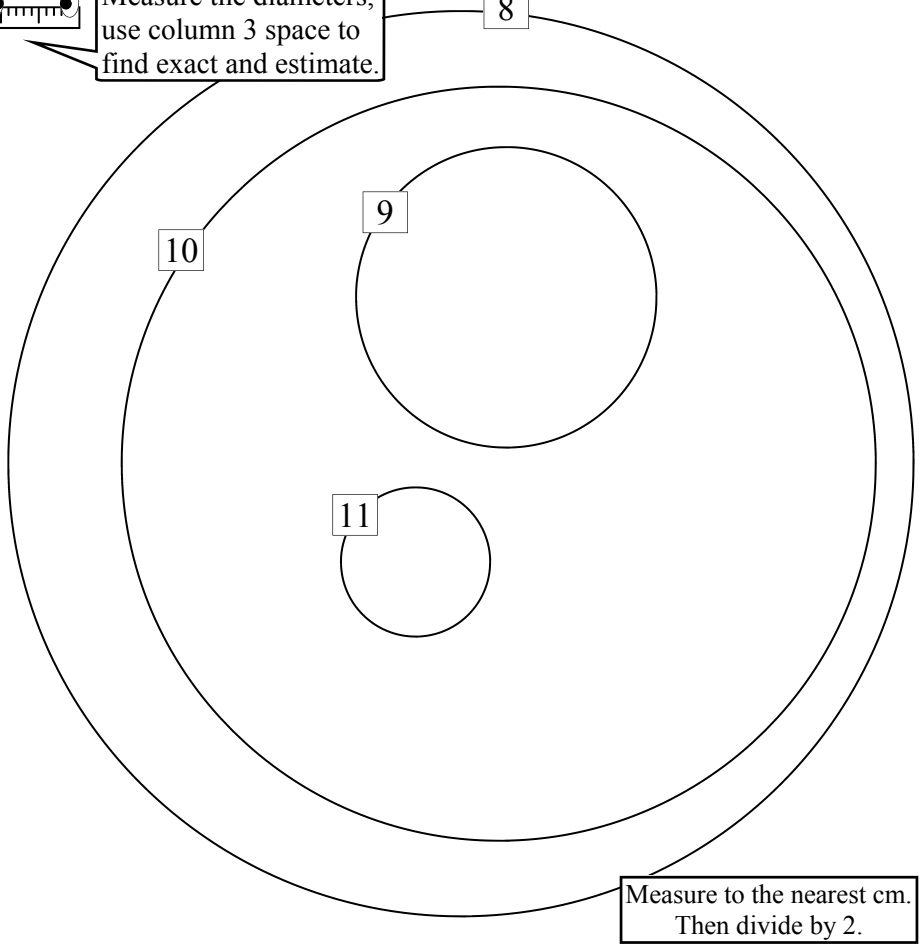
8

9

10

11

Measure the diameters, use column 3 space to find exact and estimate.



Measure to the nearest cm. Then divide by 2.

Exact and Estimate - Area

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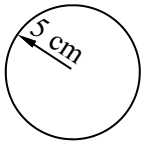
Find the area in terms of π. Then using π = 3, estimate the area.

1

$A = \pi r^2$

$= \pi \times 5^2$

$A = 25\pi$ cm^2

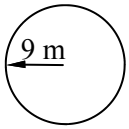


Estimate: $A = 25\pi$ cm^2

$\approx 25 \times 3$

$A \approx 75$ cm^2

2



$A = \pi r^2$

$= \pi \times 9^2$

$A = 81\pi$ m^2

Estimate: $A = 81\pi$ m^2

$\approx 81 \times 3$

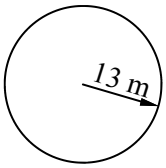
$A \approx 243$ m^2

3

$A = \pi r^2$

$= \pi \times 13^2$

$A = 169\pi$ m^2



$169 \times 3 = 507$

$A = 169\pi$ m^2

$\approx 169 \times 3$

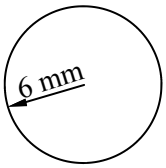
$A \approx 243$ m^2

4

$A = \pi r^2$

$= \pi \times 6^2$

$A = 36\pi$ mm^2



Estimate: $A = 36\pi$ m^2

$\approx 36 \times 3$

$A \approx 108$ mm^2

5 Circle radius of 7 km

Exact:

$A = \pi r^2$

$= \pi \times 7^2$

$A = 49\pi$ km^2

Estimate:

$A = 49\pi$ m^2

$\approx 49 \times 3$

$A \approx 147$ km^2

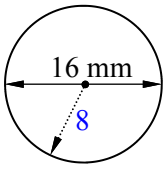
Divide the diameter by 2 to find the radius, then find the exact and estimate.

6

$A = \pi r^2$

$= \pi \times 8^2$

$A = 64\pi$ mm^2

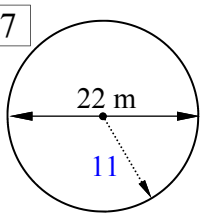


Estimate: $A = 64\pi$ mm^2

$\approx 64 \times 3$

$A \approx 192$ mm^2

7



$A = \pi r^2$

$= \pi \times 11^2$

$A = 121\pi$ m^2

Estimate: $A = 121\pi$ m^2

$\approx 121 \times 3$

$A \approx 363$ m^2

8

$A = \pi r^2$

$= \pi \times 6^2$

$A = 36\pi$ cm^2

$A = 36\pi$ cm^2

$\approx 36 \times 3$

$A \approx 108$ cm^2

9

$A = \pi r^2$

$= \pi \times 2^2$

$A = 4\pi$ cm^2

$A = 4\pi$ cm^2

$\approx 4 \times 3$

$A \approx 12$ cm^2

10

$A = \pi r^2$

$= \pi \times 5^2$

$A = 25\pi$ cm^2

$A = 25\pi$ cm^2

$\approx 25 \times 3$

$A \approx 75$ cm^2

11

$A = \pi r^2$

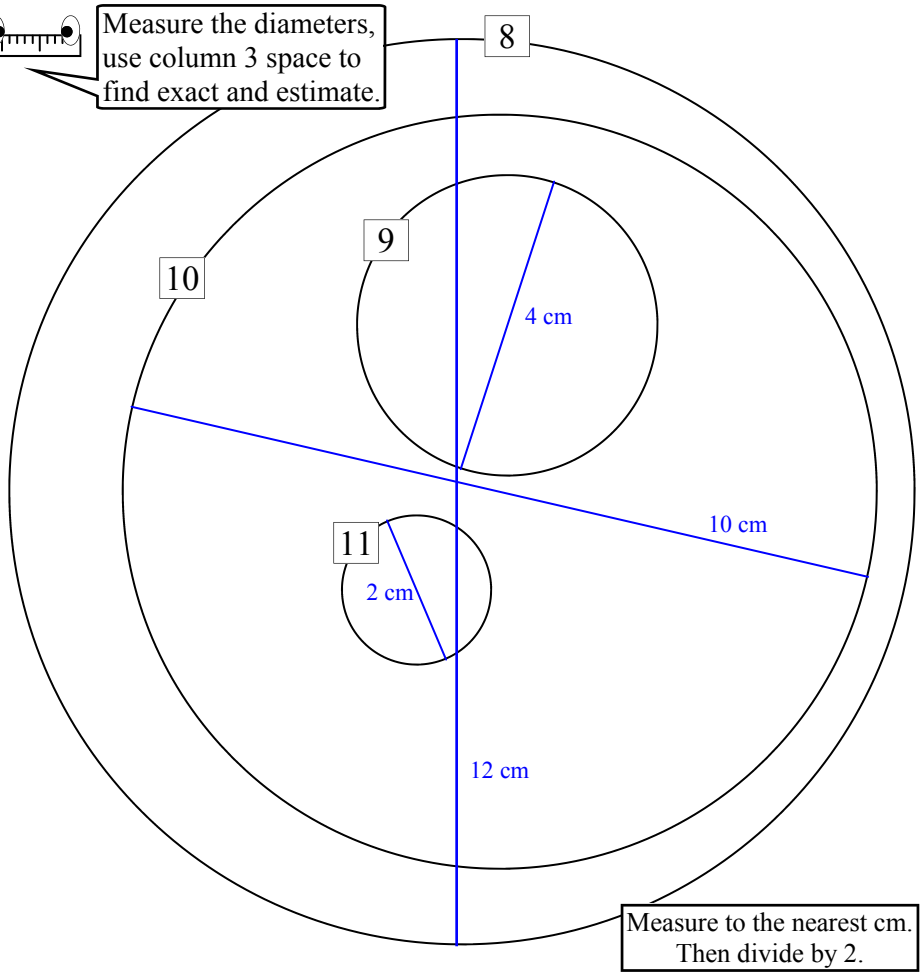
$= \pi \times 1^2$

$A = \pi$ cm^2

$A = \pi$ cm^2

$A \approx 3$ cm^2

Measure the diameters, use column 3 space to find exact and estimate.



Measure to the nearest cm. Then divide by 2.

Area of a Circle (Sheet 1)

The formula for calculating the area of a circle is:

$$A = \pi r^2$$

Where:

A = Area of the circle, in the same units as the radius, but squared. (cm^2, m^2, mm^2)
r = The radius of the circle (distance from centre to circumference).

Column 1 features circles with the radius shown. Use the area formula above to calculate the answer. The answer should be rounded to 2 decimal places and the units should be shown as square units. There is an example at the top of the column.

Column 2 is the same as Column 1 except for one additional step. Instead of the radius you are given the diameter. So convert the diameter to a radius by dividing it by 2, write the radius in the space provided, either in or beside the circle. Then solve the questions the same way, but round to one decimal place this time.

Column 3 starts with a mixture of questions that are either radius or diameter, so the questions are solved the same way as either Column 1 or Column 2.

At the bottom of the column a circle is fit perfectly into a square, so the circle's diameter is the same as the side length of the square. Calculate the square's total area (as if the circle wasn't there), then calculate the circle's area. Then calculate the %Area that the circle accommodates within the square, by multiplying the fraction by 100. Did you expect the circle to take up more space, or less?

Area of a Circle (Sheet 1)

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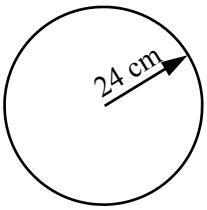
Calculate the area of these circles, round to 2 decimal places.

Example

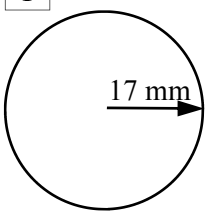


$A = \pi r^2$

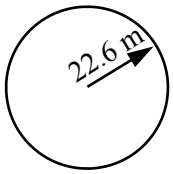
$= \pi \times 24^2$

$A = 1\,809.56 \text{ cm}^2$

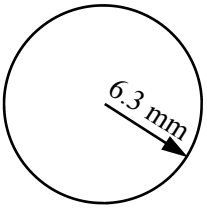
1



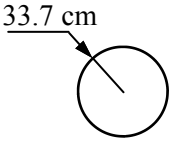
2



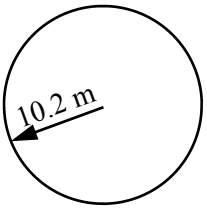
3



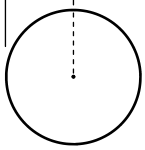
4



5

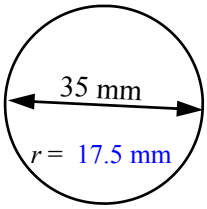


6



Calculate the radius then the area of these circles, round to 1 d.p.

Example



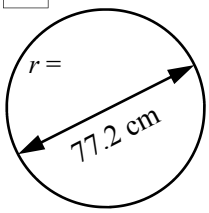
$r = 17.5$ mm

$A = \pi r^2$

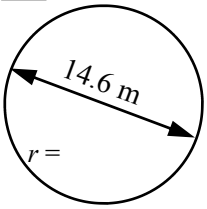
$= \pi \times 17.5^2$

$A = 962.1 \text{ mm}^2$

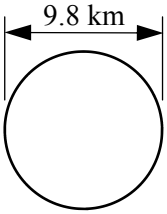
7



8

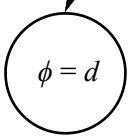


9



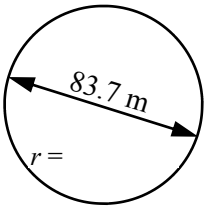
$r =$

10



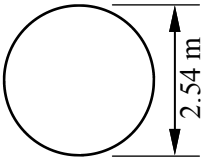
$r =$

11



$r =$

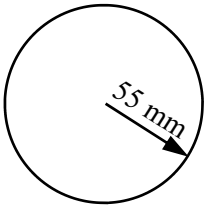
12



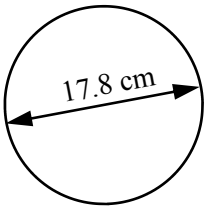
$r =$

Find the area to 1 d.p., watch for diameters.

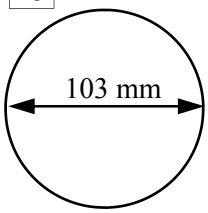
13



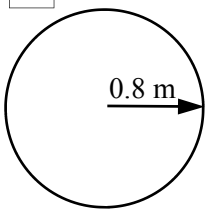
14



15



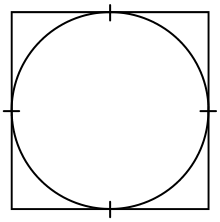
16



Find the area of the …..

17 Square

$A = l^2$



18 Circle

19 Find $\frac{A_{CIRCLE}}{A_{SQUARE}}$ as a percentage

Round to 1 d.p.

Area of a Circle (Sheet 1)

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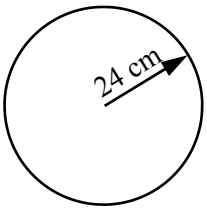
Calculate the area of these circles, round to 2 decimal places.

Example

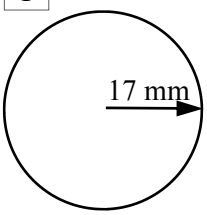


$A = \pi r^2$

$= \pi \times 24^2$

$A = 1\,809.56 \text{ cm}^2$

1

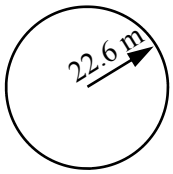


$A = \pi r^2$

$= \pi \times 17^2$

$A = 907.93 \text{ mm}^2$

2



$A = \pi r^2$

$= \pi \times 22.6^2$

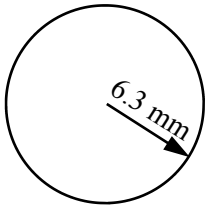
$A = 1\,604.60 \text{ m}^2$

3

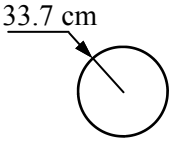
$A = \pi r^2$

$= \pi \times 6.3^2$

$A = 124.70 \text{ mm}^2$



4



$A = \pi r^2$

$= \pi \times 33.7^2$

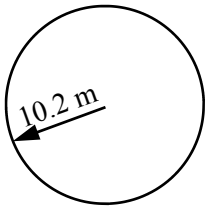
$A = 3\,567.88 \text{ cm}^2$

5

$A = \pi r^2$

$= \pi \times 10.2^2$

$A = 326.85 \text{ m}^2$

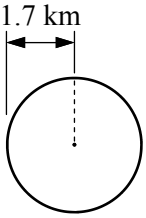


6

$A = \pi r^2$

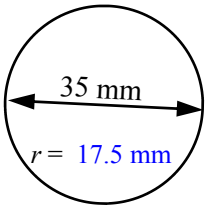
$= \pi \times 1.7^2$

$A = 9.08 \text{ km}^2$



Calculate the radius then the area of these circles, round to 1 d.p.

Example



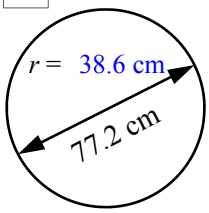
$r = 17.5 \text{ mm}$

$A = \pi r^2$

$= \pi \times 17.5^2$

$A = 962.1 \text{ mm}^2$

7



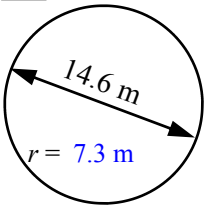
$r = 38.6 \text{ cm}$

$A = \pi r^2$

$= \pi \times 38.6^2$

$A = 4\,680.85 \text{ m}^2$

8



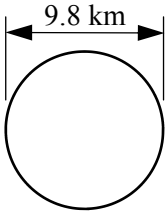
$r = 7.3 \text{ m}$

$A = \pi r^2$

$= \pi \times 7.3^2$

$A = 167.42 \text{ m}^2$

9



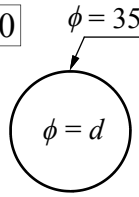
$r = 4.9 \text{ km}$

$A = \pi r^2$

$= \pi \times 4.9^2$

$A = 75.43 \text{ km}^2$

10



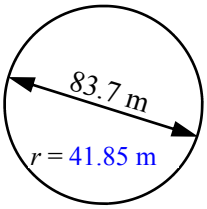
$r = 17.5 \text{ cm}$

$A = \pi r^2$

$= \pi \times 17.5^2$

$A = 962.11 \text{ cm}^2$

11



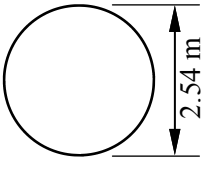
$r = 41.85 \text{ m}$

$A = \pi r^2$

$= \pi \times 41.85^2$

$A = 5\,502.26 \text{ m}^2$

12



$r = 1.27 \text{ m}$

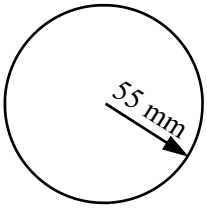
$A = \pi r^2$

$= \pi \times 1.27^2$

$A = 5.07 \text{ m}^2$

Find the area to 1 d.p., watch for diameters.

13

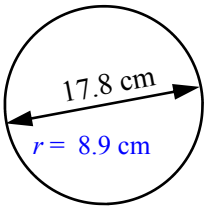


$A = \pi r^2$

$= \pi \times 55^2$

$A = 9\,503.3 \text{ m}^2$

14

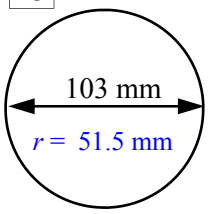


$A = \pi r^2$

$= \pi \times 8.9^2$

$A = 248.8 \text{ cm}^2$

15

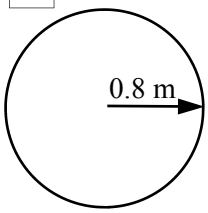


$A = \pi r^2$

$= \pi \times 51.5^2$

$A = 8\,332.3 \text{ mm}^2$

16



$A = \pi r^2$

$= \pi \times 0.8^2$

$A = 2 \text{ m}^2$

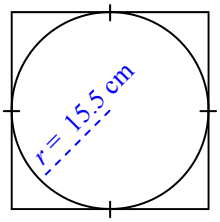
Find the area of the …..

17 Square

$A = l^2$

$= 31^2$

$A = 961 \text{ cm}^2$



18 Circle $A = \pi r^2$

$= \pi \times 15.5^2$

$A = 754.8 \text{ cm}^2$

19 Find $\frac{A_{CIRCLE}}{A_{SQUARE}}$ as a percentage

$\text{Percentage} = \frac{754.8}{961} \times 100$

$\text{Percentage} = 78.5\%$

Round to 1 d.p.

Area of a Circle (Sheet 2)

The formula for calculating the area of a circle is:

$$A = \pi r^2$$

Where:

A = Area of the circle, in the same units as the radius, but squared. (cm^2, m^2, mm^2)
r = The radius of the circle (distance from centre to circumference).

This sheet uses three values for π: Decimal 3.14; Fraction $^{22}/_{7}$ and Calculator 3.14159….

Column 1 features circles with the radius shown. Use the area formula above to calculate the answer. But for the first part of the column use $\pi = 3.14$ to calculate your answer. Then round to 1 decimal place. The second part of the column use $\pi = {}^{22}/_{7}$ to find your answer rounded to the nearest whole number. Note that your answer could be an improper fraction or a mixed number depending on the calculator you use. Press the fraction key after the fractional answer is displayed to convert it to a decimal. Decimals are easier to round for most students.

Column 2 requires you to halve the diameter first to get a radius, write the radius on the circle next to the dotted line, or draw your own line. Then solve the same way as column 1 except round to 2 decimal places, note that you use $\pi = 3.14$.

Two tables are then shown. Divide the diameter by 2 to get the radius column, then calculate the area. Note that the first table uses $\pi = {}^{22}/_{7}$, all lengths are in cm. The second table requires you to use the π key on your calculator, answer in m^2 rounded to 2 decimal places.

Column 3 asks you to find the radius given the area. First it asks you to manipulate the area formula to make r the subject. The steps are shown, see how you go. The rest of the column uses the formula you created to answer the rest of the column. If you couldn't rearrange the formula don't worry, Fizzle tells you the formula above question 24.

Note that you have to use brackets as the calculator step shows in question 24, or work out the inside first, press the 'equals button' then the 'square root button' and 'equals' again.

Area of a Circle (Sheet 2)

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Use $A=\pi r^2$ with $\pi = 3.14$, to find the area. Round your answer to 1 decimal place.

Example

$A = \pi r^2$

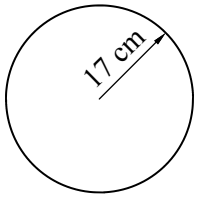
Write formula

$= 3.14 \times 17^2$

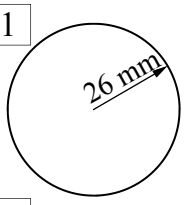
Substitute 3.14 and r

$A = 907.5\ cm^2$

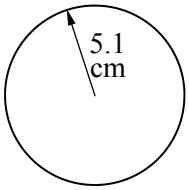
Answer 1 d.p., show units^2



1



2



3

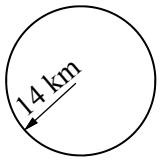


Now use $\pi = {}^{22}/_{7}$, to find the area. Round to the nearest whole number.

4

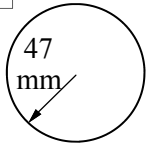
$A = \pi r^2$

$= {}^{22}/_{7} \times$

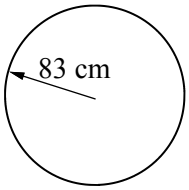


Press $a^{b}/_{c}$ key to change fractions to decimals

5

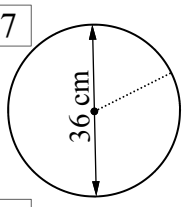


6

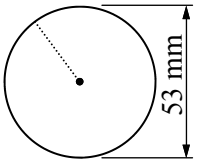


These are diameters, halve them then use $A = \pi r^2$ & $\pi = 3.14$, to find area. 2 d.p.

7



8



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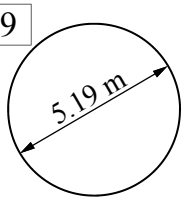

Find r then use $\pi = {}^{22}/_{7}$, to complete the table to 1 d.p. All lengths are in cm.

Q.	d	r	Area (cm^2)
10	15		
11	87		
12	11.9		
13	41		
14	109		
15	60.7		

Now use the π key, to complete the table to 2 d.p. All units are in metres.

Q.	d	r	Area (m^2)
16	3.9		
17	27		
18	8.3		
19	65		
20	11.3		
21	56.8		
22	70.2		

Rearrange the area formula to make r the subject. The steps are shown in circles.

23 $A = \pi r^2$

$\div \pi$ $\div \pi$ Divide both sides by π

$\sqrt{\ }$ $\sqrt{\ }$ Square root both sides

$\Rightarrow$

Rewrite with r on the L.H.S

Using $r = \sqrt{\frac{A}{\pi}}$ find the radius given the area. To 1 d.p.

24 Circle area = 132.7 m^2

$\sqrt{\ }$ (1 3 2 • 7 ÷ π) =

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{\square}{\pi}}$

$=$ m

25 Circle area = 2 273.3 cm^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{\square}{\pi}}$

26 Circle area = 422.7 mm^2

27 Circle area = 2 498.3 m^2

radius diameter

28 Circle area = 23 506.2 km^2

radius diameter

Area of a Circle (Sheet 2)

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Use $A=\pi r^2$ with $\pi = 3.14$, to find the area. Round your answer to 1 decimal place.

Example

$A = \pi r^2$

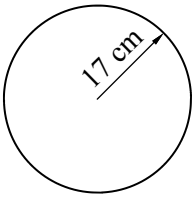
Write formula

$= 3.14 \times 17^2$

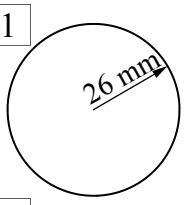
Substitute 3.14 and r

$A = 907.5 \text{ cm}^2$

Answer 1 d.p., show units^2



1



$A = \pi r^2$

$= 3.14 \times 26^2$

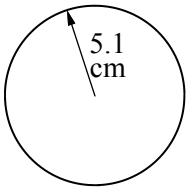
$A = 2\,122.6 \text{ mm}^2$

2

$A = \pi r^2$

$= 3.14 \times 5.1^2$

$A = 81.7 \text{ cm}^2$



3



$A = \pi r^2$

$= 3.14 \times 33.3^2$

$A = 3\,481.9 \text{ m}^2$

Now use $\pi = {}^{22}/_{7}$, to find the area. Round to the nearest whole number.

4

$A = \pi r^2$

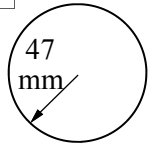
$= {}^{22}/_{7} \times 14^2$

$A = 616 \text{ km}^2$



Press $a^b/_c$ key to change fractions to decimals

5



$A = \pi r^2$

$= {}^{22}/_{7} \times 47^2$

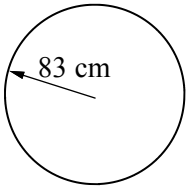
$A = 6\,943 \text{ mm}^2$

6

$A = \pi r^2$

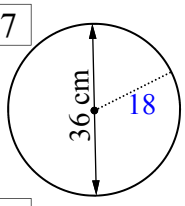
$= {}^{22}/_{7} \times 83^2$

$A = 21\,651 \text{ cm}^2$



These are diameters, halve them then use $A = \pi r^2$ & $\pi = 3.14$, to find area. 2 d.p.

7



$A = \pi r^2$

$= 3.14 \times 18^2$

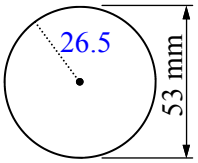
$A = 1\,017.36 \text{ cm}^2$

8

$A = \pi r^2$

$= 3.14 \times 26.5^2$

$A = 2\,205.07 \text{ mm}^2$



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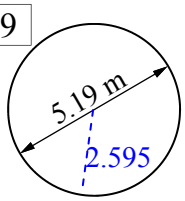


$A = \pi r^2$

$= 3.14 \times 2.595^2$

$A = 21.14 \text{ m}^2$

Find r then use $\pi = {}^{22}/_{7}$, to complete the table to 1 d.p. All lengths are in cm.

Q.	d	r	Area (cm^2)
10	15	7.5	176.8 cm^2
11	87	43.5	5 947.1 cm^2
12	11.9	5.95	111.3 cm^2
13	41	20.5	1 320.8 cm^2
14	109	54.5	9 335.1 cm^2
15	60.7	30.35	2 895 cm^2

Now use the π key, to complete the table to 2 d.p. All units are in metres.

Q.	d	r	Area (m^2)
16	3.9	1.95	11.95 m^2
17	27	13.5	572.56 m^2
18	8.3	4.15	54.11 m^2
19	65	32.5	3 318.31 m^2
20	11.3	5.65	100.29 m^2
21	56.8	28.4	2 533.88 m^2
22	70.2	35.1	3 870.47 m^2

Rearrange the area formula to make r the subject. The steps are shown in circles.

23 $A = \pi r^2$

$\div \pi$ $\div \pi$ Divide both sides by π

$\frac{A}{\pi} = r^2$

$\sqrt{\ }$ $\sqrt{\ }$ Square root both sides

$\sqrt{\frac{A}{\pi}} = r \Rightarrow r = \sqrt{\frac{A}{\pi}}$

Rewrite with r on the L.H.S

Using $r = \sqrt{\frac{A}{\pi}}$ find the radius given the area. To 1 d.p.

24 Circle area = 132.7 m^2

$\sqrt{\ }$ (1 3 2 • 7 ÷ π) =

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{132.7}{\pi}}$

$= 6.5 \text{ m}$ (1d.p.)

25 Circle area = 2 273.3 cm^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{2\,273.3}{\pi}}$

$= 26.9 \text{ cm}$ (1d.p.)

26 Circle area = 422.7 mm^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{422.7}{\pi}}$

$= 11.6 \text{ mm}$ 1d.p.

27 Circle area = 2 498.3 m^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{2\,498.3}{\pi}}$

$= 28.2 \text{ m}$ (1d.p.) radius

$= 56.4 \text{ m}$ (1d.p.) diameter

28 Circle area = 23 506.2 km^2

$r = \sqrt{\frac{A}{\pi}}$ $= \sqrt{\frac{23\,506.2}{\pi}}$

$= 86.5 \text{ km}$ (1d.p.) radius

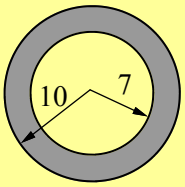
$= 173 \text{ km}$ (1d.p.) diameter

The Annulus

An annulus is a ring-like figure, in other words a circle within another circle. Most students recognise the similarity with the word 'anus', as well as the similar appearance. For this reason it is usually an easy mathematical word to remember!

Column 1 deals with calculating the area of an annulus. The process is to calculate the outside circle's area, then calculate the inside circle's area, then subtract them. An example is below.

Example - Area of an Annulus



Lengths are in cm

$A_1 = \pi R^2$

$= \pi \times 10^2$

$A_1 = 314.2 \text{ cm}^2$ (1 dp)

Capital R, means 'big' radius.

So A_1 is the area of the outside circle.

$A_2 = \pi r^2$

$= \pi \times 7^2$

$A_2 = 153.9 \text{ cm}^2$ (1 dp)

Lower case r, means 'small' radius.

So A_2 is the area of the inside circle.

$A = A_1 - A_2 = 314.2 - 153.9$

$A = 160.3 \text{ cm}^2$

Then subtract for the annulus area. The larger area - smaller area.

Column 2 introduces the annulus formula. This is a faster way of solving exercises but it is important that you understand the process above as well. The formula is:

$$A = \pi (R^2 - r^2)$$

Where:

R = outside circle's radius
r = inside circle's radius

This formula is created by factorising $\pi R^2 - \pi r^2$. If you take out π you get the above formula.

Column 2 starts with assistance in showing the working out. After the first 3 questions fractions need to be used because the area for 'fractions of' annuli are required. Use the internal angle within the annulus, then multiply by the angle over 360°.

So if the angle was 90°, you would use $A = {}^{90}/_{360} \times \pi(R^2 - r^2)$ but you may also recognize that this is the same as $A = \frac{1}{4} \times \pi(R^2 - r^2)$.

The Annulus

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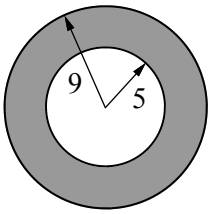
Find the outside area and the inside area, then subtract to find the shaded area. (1 d.p.)

All measurements are in cm

1 $A_1 = \pi R^2$ $A_2 = \pi r^2$

$= \pi \times 9^2$ $= \pi \times 5^2$

$A_1 =$ ☐ cm^2 (1 dp) $A_2 =$ ☐ cm^2 (1 dp)



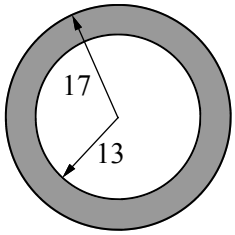
$A = A_1 - A_2 =$ ☐ - ☐

$A =$ ☐ cm^2 (1 dp)

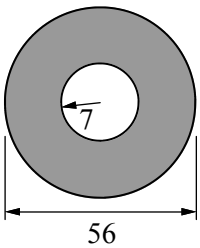
2 $A_1 = \pi R^2$ $A_2 = \pi r^2$

$= \pi \times$ ☐^{2} $= \pi \times$ ☐^{2}

$A_1 =$ ☐ cm^2 (1 dp) $A_2 =$ ☐ cm^2 (1 dp)



3 Watch for diameters!



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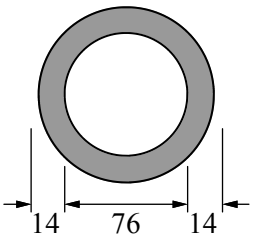
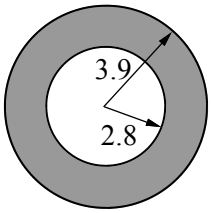


Now use the Annulus formula $A = \pi(R^2 - r^2)$. All lengths are in metres. Answer to 1 d.p.

5

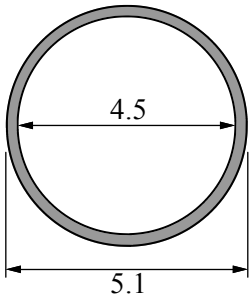


$A = \pi(R^2 - r^2)$

$= \pi \times$ (☐^{2} - ☐^{2})

$A =$ ☐ m^2 (1 dp)

6



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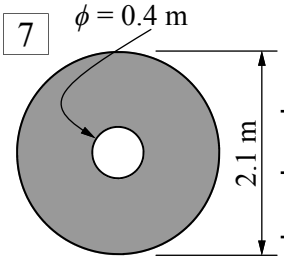
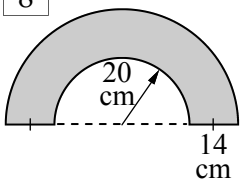


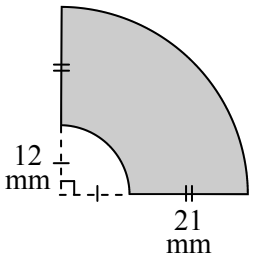
Use the Annulus formula, but put a fraction before the π, to find the shaded area. (1 d.p.)

8



$A =$ ☐/☐ $\times \pi(R^2 - r^2)$

9



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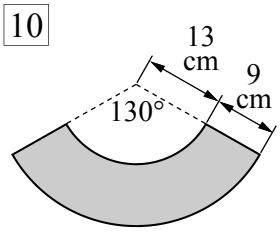

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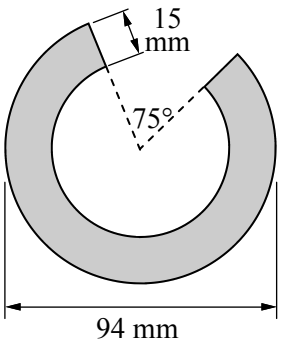

The Annulus

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Find the outside area and the inside area, then subtract to find the shaded area. (1 d.p.)

All measurements are in cm

1

$A_1 = \pi R^2$

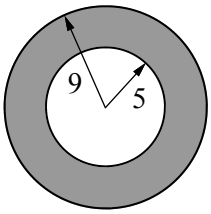
$= \pi \times 9^2$

$A_1 =$ 254.5 cm^2 (1 dp)

$A_2 = \pi r^2$

$= \pi \times 5^2$

$A_2 =$ 78.5 cm^2 (1 dp)



$A = A_1 - A_2 =$ 254.5 - 78.5

$A =$ 176 cm^2 (1 dp)

2

$A_1 = \pi R^2$

$= \pi \times$ 17 ^{2}

$A_1 =$ 907.9 cm^2 (1 dp)

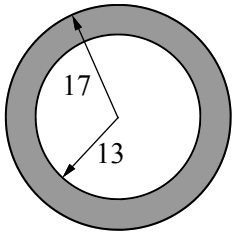
$A_2 = \pi r^2$

$= \pi \times$ 13 ^{2}

$A_2 =$ 530.9 cm^2 (1 dp)

$A = A_1 - A_2 = 907.9 - 530.9$

$A = 377\ cm^2$



3 Watch for diameters!

$A_1 = \pi R^2$

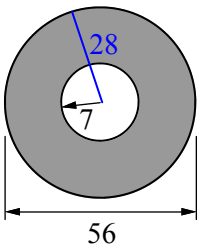
$= \pi \times 28^2$

$A_1 = 2\,463\ cm^2$ (1 dp)

$A_2 = \pi r^2$

$= \pi \times 7^2$

$A_2 = 153.9\ cm^2$ (1 dp)



$A = A_1 - A_2 = 2\,463 - 153.9$

$A = 2\,309.1\ cm^2$

4

$A_1 = \pi R^2$

$= \pi \times 52^2$

$A_1 = 8\,494.9\ cm^2$ (1 dp)

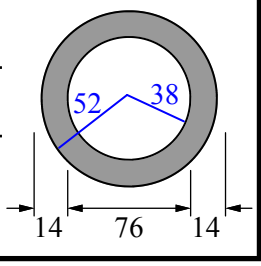
$A_2 = \pi r^2$

$= \pi \times 38^2$

$A_2 = 4\,536.5\ cm^2$ (1 dp)

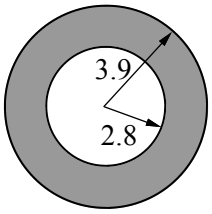
$A = A_1 - A_2 = 8\,494.9 - 4\,536.5$

$A = 3\,958.4\ cm^2$



Now use the Annulus formula $A = \pi(R^2 - r^2)$. All lengths are in metres. Answer to 1 d.p.

5



$A = \pi(R^2 - r^2)$

$= \pi \times$ (3.9 ^{2} - 2.8 ^{2})

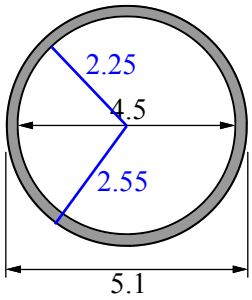
$A =$ 23.2 m^2 (1 dp)

6

$A = \pi(R^2 - r^2)$

$= \pi \times (2.55^2 - 2.25^2)$

$A = 4.5\ m^2$



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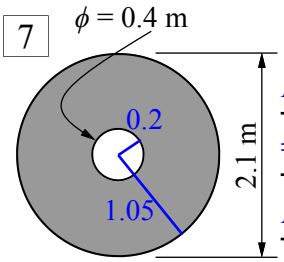


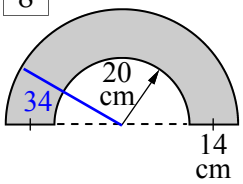
$A = \pi(R^2 - r^2)$

$= \pi \times (1.05^2 - 0.2^2)$

$A = 3.3\ m^2$

Use the Annulus formula, but put a fraction before the π, to find the shaded area. (1 d.p.)

8



$A =$ 1/2 $\times \pi(R^2 - r^2)$

$= ½ \times \pi \times (34^2 - 20^2)$

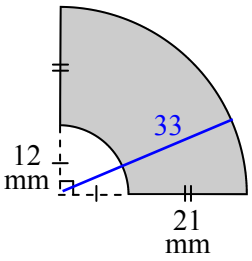
$A = 1\,187.5\ cm^2$

9

$A = ¼ \times \pi(R^2 - r^2)$

$= ¼ \times \pi \times (33^2 - 12^2)$

$A = 742.2\ mm^2$



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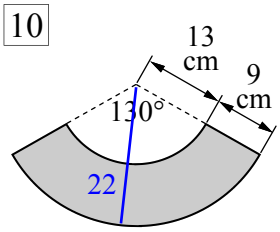


$A = {}^{130}/_{360} \times \pi(R^2 - r^2)$

$= {}^{130}/_{360} \times \pi \times (22^2 - 13^2)$

$A = 357.4\ cm^2$

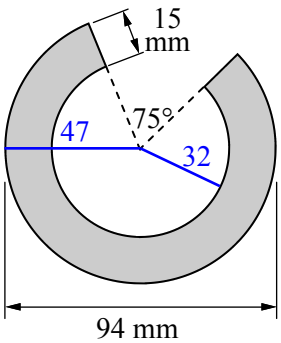
11

Internal angle = 360 - 75 = 285°

$A = {}^{285}/_{360} \times \pi(R^2 - r^2)$

$= {}^{285}/_{360} \times \pi \times (47^2 - 32^2)$

$A = 2\,947.2\ mm^2$



Area of Sectors, Quadrants And Semi-Circles

If you know how to multiply by a fraction and how to find the area of a circle, then this worksheet simply blends the two.

The formula for calculating the area of a circle is:

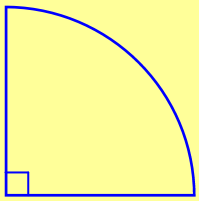
$$A = \pi r^2$$

Where:

A = Area of the circle, in the same units as the radius, but squared. (cm^2, m^2, mm^2)
r = The radius of the circle (distance from centre to circumference).

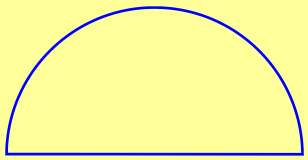
But with semi-circles (half a circle), quadrants (quarter of a circle) and sectors the above formula is multiplied by a fraction.

Quadrants



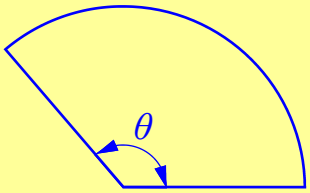
$$A = \frac{1}{4}\pi r^2$$

Semi-Circles



$$A = \frac{1}{2}\pi r^2$$

Sectors



$$A = \frac{\theta}{360} \times \pi r^2$$

Column 1 starts with semi-circles, so use the middle formula above, in other words multiply πr^2 by ½. Question 5 sees a change to quadrants, use the first formula shown above, multiply πr^2 by ¼.

Column 2 again features semi-circles, but this time instead of being given the radius you are given a diameter, so find the radius (by dividing the diameter by 2) then solve as before.

Sectors are then introduced at the bottom of the column. The internal angle (commonly called 'pheta') is written over 360, then simplified with your calculator. The calculator steps for simplifying are shown on the sheet if you have forgotten the process.

Then sector area is required to be found. Use the third formula above, so make a fraction from the internal angle and use 360 as the denominator. Multiply this by the area formula and you are done!

Area of Sectors, Quadrants And Semi-Circles

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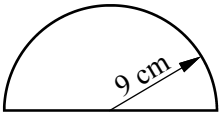
Calculate the area inside these semi-circles and quadrants. (1 d.p.)

1



$A = \frac{1}{2}\pi r^2$

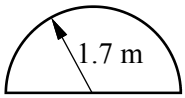
$= \frac{1}{2} \times \pi \times \square^2$

$A = \square\ cm^2$

Units are square units

2

$A = \frac{1}{2}\pi r^2$



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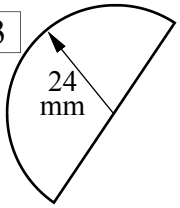

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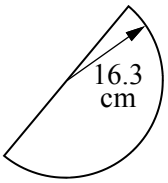

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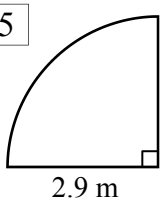
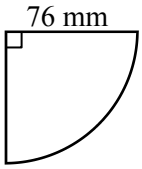


$A = \frac{1}{4}\pi r^2$

6



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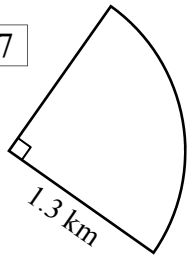

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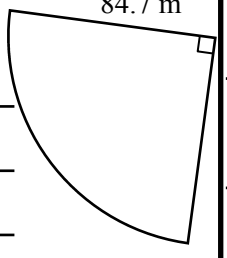
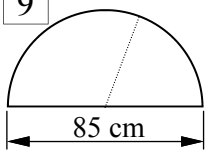


Find the area, but find the radius first. Answer to 1 dp.

9



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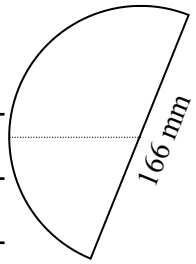

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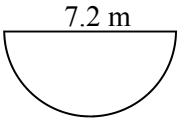
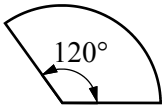


Write the fraction of a circle with a denominator of 360, then simplify.

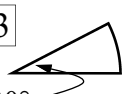
Calc: 1 2 0 $a^b/_c$ 3 6 0 =

12



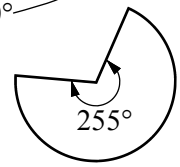
$\frac{\square}{360} = \frac{\square}{\square}$

13



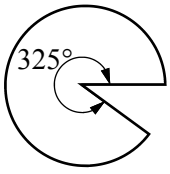
$\frac{\square}{\square} = \frac{\square}{\square}$

14



$\frac{\square}{\square} = \frac{\square}{\square}$

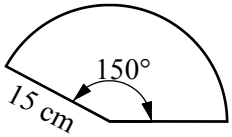
15



$\frac{\square}{\square} = \frac{\square}{\square}$

Find the areas of these sectors. Round to 2 d.p.

16

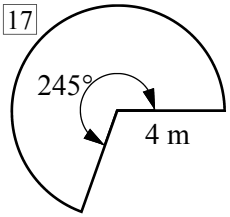


$A = \frac{\theta}{360} \times \pi r^2$

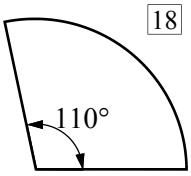
$= \frac{\square}{\square} \times \pi \times \square^2$

$A =$

17



18



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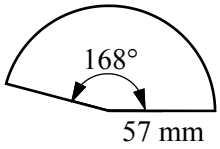

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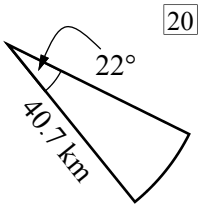

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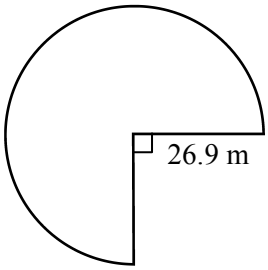

Area of Sectors, Quadrants And Semi-Circles

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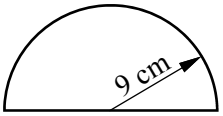
Calculate the area inside these semi-circles and quadrants. (1 d.p.)

1



$A = \frac{1}{2}\pi r^2$

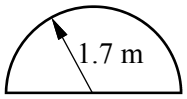
$= \frac{1}{2} \times \pi \times 9^2$

$A = 127.2$ cm^2

Units are square units

2

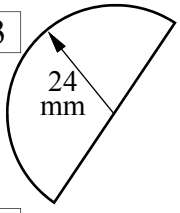
$A = \frac{1}{2}\pi r^2$



$= \frac{1}{2} \times \pi \times 1.7^2$

$A = 4.5$ m^2

3



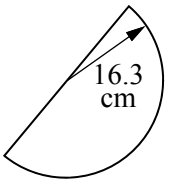
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 24^2$

$A = 904.8$ mm^2

4

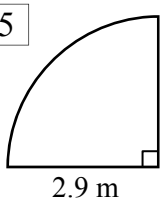
$A = \frac{1}{2}\pi r^2$



$= \frac{1}{2} \times \pi \times 16.3^2$

$A = 417.3$ cm^2

5



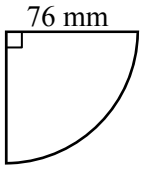
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 2.9^2$

$A = 6.6$ m^2

6

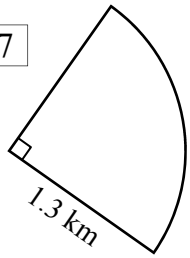
$A = \frac{1}{4}\pi r^2$



$= \frac{1}{4} \times \pi \times 76^2$

$A = 4\,536.5$ mm^2

7

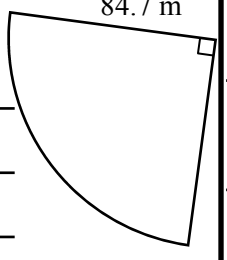


$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 1.3^2$

$A = 1.3$ km^2

8



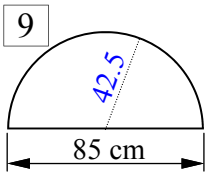
$A = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 84.7^2$

$A = 5\,634.5$ m^2

Find the area, but find the radius first. Answer to 1 dp.

9



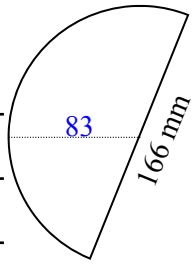
$A = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 42.5^2$

$A = 2\,837.3$ cm^2

10

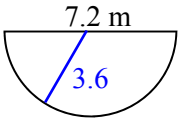
$A = \frac{1}{2}\pi r^2$



$= \frac{1}{2} \times \pi \times 83^2$

$A = 10\,821.2$ mm^2

11



$A = \frac{1}{2}\pi r^2$

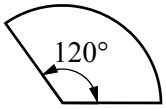
$= \frac{1}{2} \times \pi \times 3.6^2$

$A = 20.4$ m^2

Write the fraction of a circle with a denominator of 360, then simplify.

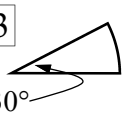
Calc: 1 2 0 a$^b/_c$ 3 6 0 =

12



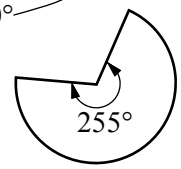
$\frac{120}{360} = \frac{1}{3}$

13



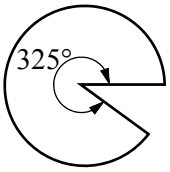
$\frac{30}{360} = \frac{1}{12}$

14



$\frac{255}{360} = \frac{17}{24}$

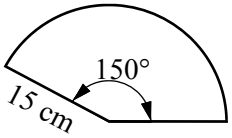
15



$\frac{325}{360} = \frac{65}{72}$

Find the areas of these sectors. Round to 2 d.p.

16



$A = \frac{\theta}{360} \times \pi r^2$

$= \frac{150}{360} \times \pi \times 15^2$

$A = 294.52$ cm^2

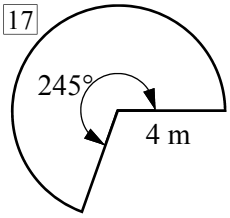
17

$A = {}^{\theta}/_{360} \times \pi r^2$

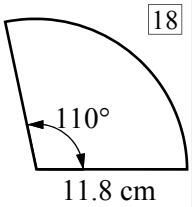
$= {}^{245}/_{360} \times \pi \times 4^2$

$A = 34.21$ m^2

17



18



18

$A = {}^{\theta}/_{360} \times \pi r^2$

$= {}^{110}/_{360} \times \pi \times 11.8^2$

$A = 3133.66$ cm^2

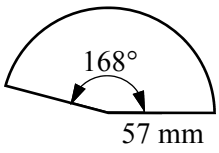
19

$A = {}^{\theta}/_{360} \times \pi r^2$

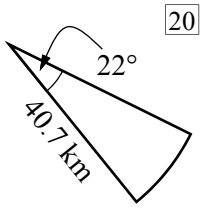
$= {}^{168}/_{360} \times \pi \times 57^2$

$A = 4\,763.28$ mm^2

19



20



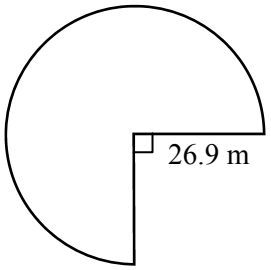
20

$A = {}^{\theta}/_{360} \times \pi r^2$

$= {}^{22}/_{360} \times \pi \times 40.7^2$

$A = 318.02$ km^2

21



$A = {}^{\theta}/_{360} \times \pi r^2$

$= {}^{270}/_{360} \times \pi \times 26.9^2$

$A = 1\,704.97$ m^2

Area of Composite Shapes

A composite shape is a shape that is composed of two or more plane shapes. These two shapes can add together to create an area, or be subtracted. Before this sheet is attempted you should have completed earlier sheets on area of a circle.

Column 1 are all exercises using addition. So calculate the individual areas and then add them together. The process is shown in question 1.

Column 2 is essentially the same process, find the areas that make the shape but then subtract the areas. Question 8 will require Pythagoras Theorem to first be applied.

Area of Composite Shapes

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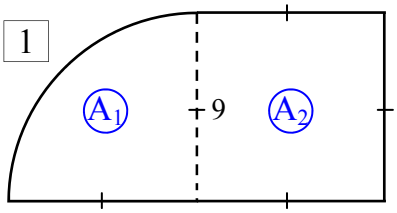
Find the areas, sum for the total area. Round to 1 d.p. All lengths are cm.

1



$A_1 = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times \square^2$

$A_1 = \square$ cm^2 (1 dp)

$A_2 = l^2$

$= \square^2$

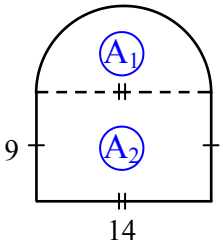
$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square$

$A = \square$ cm^2

2

$A_1 = \frac{1}{2}\pi r^2$

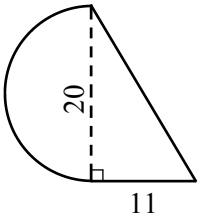


$A_2 = lb$

$A = A_1 + A_2 = \square + \square$

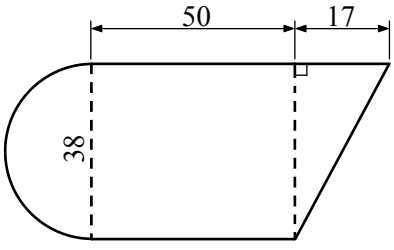
$A = \square$

3



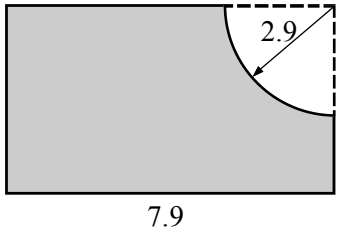
$A_2 = \frac{1}{2}bh$

4

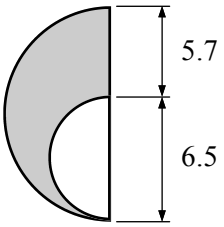


This time subtract the areas. Round to 1 d.p. All measurements in metres.

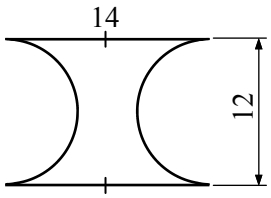
5



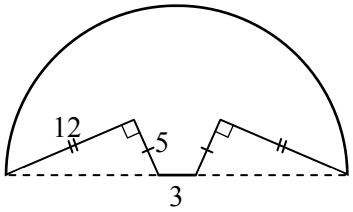
6



7



8



Area of Composite Shapes

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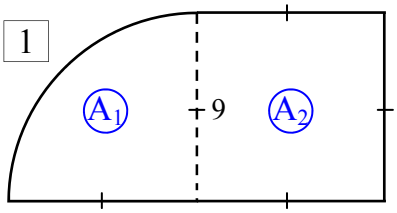
Find the areas, sum for the total area. Round to 1 d.p. All lengths are cm.

1



$A_1 = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 9^2$

$A_1 = 63.6 \text{ cm}^2$ (1 dp)

$A_2 = l^2$

$= 9^2$

$A_2 = 81 \text{ cm}^2$

$A = A_1 + A_2 = 63.6 + 81$

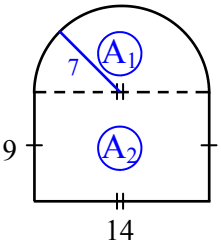
$A = 144.6 \text{ cm}^2$

2

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 7^2$

$A_1 = 77 \text{ cm}^2$ (1 d.p.)



$A_2 = lb$

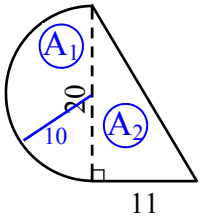
$= 14 \times 9$

$A_2 = 126 \text{ cm}^2$

$A = A_1 + A_2 = 77 + 126$

$A = 203 \text{ cm}^2$

3



$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 10^2$

$A_1 = 157.1 \text{ cm}^2$ (1 d.p.)

$A_2 = \frac{1}{2}bh$

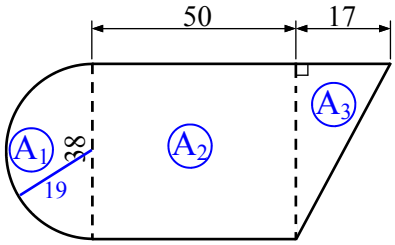
$= \frac{1}{2} \times 11 \times 20$

$A_2 = 110 \text{ cm}^2$

$A = A_1 + A_2 = 157.1 + 110$

$A = 267.1 \text{ cm}^2$

4



$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 19^2$

$A_1 = 567.1 \text{ cm}^2$ (1 d.p.)

$A_2 = lb$

$= 50 \times 38$

$A_2 = 1\,900 \text{ cm}^2$

$A_3 = \frac{1}{2}bh$

$= \frac{1}{2} \times 38 \times 17$

$A_3 = 323 \text{ cm}^2$

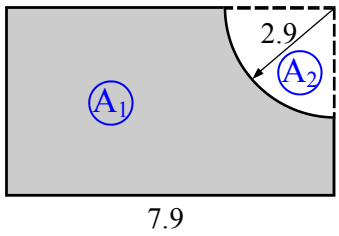
$A = A_1 + A_2 + A_3$

$= 567.1 + 1\,900 + 323$

$A = 2\,790.1 \text{ cm}^2$

This time subtract the areas. Round to 1 d.p. All measurements in metres.

5



$A_1 = lb$

$= 7.9 \times 4.7$

$A_1 = 37.1 \text{ m}^2$ (1 d.p.)

$A_2 = \frac{1}{4}\pi r^2$

$= \frac{1}{4} \times \pi \times 2.9^2$

$A_2 = 6.6 \text{ m}^2$ (1 d.p.)

$A = A_1 - A_2 = 37.1 - 6.6$

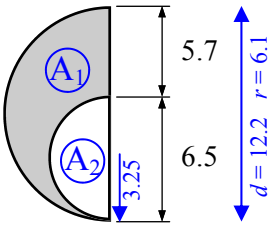
$A = 30.5 \text{ m}^2$

6

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 6.1^2$

$A_1 = 58.4 \text{ m}^2$ (1 d.p.)



$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 3.25^2$

$A_2 = 16.6 \text{ m}^2$ (1 d.p.)

$A = A_1 - A_2 = 58.4 - 16.6$

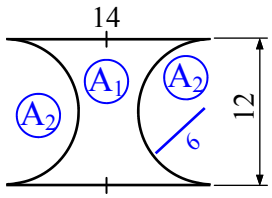
$A = 41.8 \text{ m}^2$

7

$A_1 = lb$

$= 14 \times 12$

$A_1 = 168 \text{ m}^2$



$A_2 = \pi r^2$

$= \pi \times 6^2$

$A_2 = 113.1 \text{ m}^2$ (1 d.p.)

$A = A_1 - A_2 = 168 - 113.1$

$A = 54.9 \text{ m}^2$

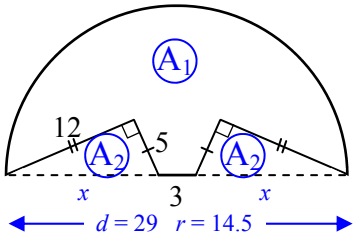
8

$x^2 = 5^2 + 12^2$

$x^2 = 169$

$x = \sqrt{169}$

$x = 13 \text{ m}$



$A_2 = 2 \times \frac{1}{2}bh$

$= 2 \times \frac{1}{2} \times 5 \times 12$

$A_2 = 60 \text{ m}^2$

$A_1 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 14.5^2$

$A_1 = 330.3 \text{ m}^2$ (1 d.p.)

$A = A_1 - A_2 = 330.3 - 60$

$A = 270.3 \text{ m}^2$

Changing Units of Area

This is one of the easiest parts of area calculations to make a mistake. Students often forget this work. The reason is that because there are 10 mm in 1 cm, students automatically use this to convert mm^2 to cm^2, but this isn't the case. There is infact 100 mm^2 in 1 cm^2. The same with converting cm^2 to m^2, there is 10 000 cm^2 in a m^2, not 100 cm^2.

Column 1 first converts areas from cm^2 to mm^2. Note the diagram at the top of the column. A 1 cm square is redrawn with grid lines at each millimetre. The square has 10 mm sides and as you know the area of a square is its side length squared. As $10^2 = 100$ there are 100 1 mm^2 squares in a square cm. So to convert the measurements multiply each number by 100. This means move the decimal point 2 places to the right. So 5 cm^2 would be 500 mm^2, 5.5 cm^2 would be 550 mm^2 and 5.05 cm^2 would be 505 mm^2. Questions 11- 20 are the reverse, move the decimal point 2 places to the left. So 8 mm^2 is 0.08 cm^2, 130 mm^2 is 1.3 cm^2 and 130.02 mm^2 would be 1.3002 cm^2.

Column 2 then deals with converting between m^2 and cm^2. Look at the diagram and note that the same method is used to calculate that there are (100×100) 10 000 cm^2 in 1 m^2. So to change cm^2 to m^2 multiply the number by 10 000 or move the decimal point 4 places to the right. So 8 m^2 would be 80 000 cm^2 and 8.02 m^2 would be 80 200 cm^2. Then the method is reversed for the second part of the column. When you divide by 10 000 move the decimal place 4 places to the left. So 9 cm^2 would be 0.0009 m^2 and 136 cm^2 would be 0.0136 m^2.

Column 3 are worder questions that you solve using the units as given in the question. Then after you answer the question express the answer in the other unit that is requested.

Changing Units of Area

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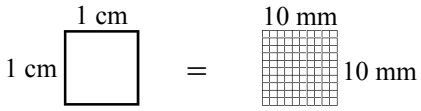
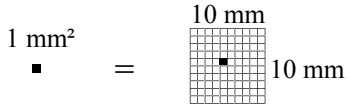
Convert these areas from cm² to mm²



1 cm² = 100 mm²

1 3 cm² _____
2 8 cm² _____
3 7.5 cm² _____
4 18 cm² _____
5 29 cm² _____
6 45.6 cm² _____
7 65.32 cm² _____
8 82.09 cm² _____
9 100 cm² _____
10 100.1 cm² _____

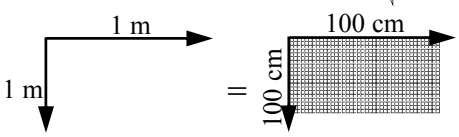
Convert these areas from mm² to cm²



1 mm² = 0.01 cm²

11 5 mm² _____
12 10 mm² _____
13 25 mm² _____
14 125 mm² _____
15 170.1 mm² _____
16 205.4 mm² _____
17 30.03 mm² _____
18 450 mm² _____
19 500 mm² _____
20 500.6 mm² _____

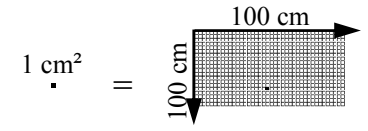
Convert these areas from m² to cm²



1 m² = 10 000 cm²

21 2 m² _____
22 3 m² _____
23 0.3 m² _____
24 0.03 m² _____
25 3.3 m² _____
26 0.725 m² _____
27 0.203 m² _____
28 0.8963 m² _____
29 0.902 m² _____
30 0.0009 m² _____

Convert these areas from cm² to m²



1 cm² = 0.0001 m²

31 7 cm² _____
32 11 cm² _____
33 54 cm² _____
34 154 cm² _____
35 296 cm² _____
36 875 cm² _____
37 1 500 cm² _____
38 8 700 cm² _____
39 12 000 cm² _____
40 15 796 cm² _____

Answer these questions using the methods from the first two columns. Answer to 2 d.p.

41 A rectangular piece of fabric is size 1.5 m × 2.7 m. Find its area in m² and then convert to cm².

A = *lb*

_____ Fabric area:
_____ m²: _____
_____ cm²: _____

42 A kite has axis lengths of 58 cm and 85 cm. Find its area in cm^2, then convert this to m^2.

_____ Kite area:
_____ cm²: _____
_____ m²: _____

43 A triangular sail has a base length of 2.2 m and a height 4.1 m. Find the area of the sail in m^2 & cm^2.

_____ Sail area:
_____ m²: _____
_____ cm²: _____

A square steel plate with sides 8.2 cm has a rectangular slot cut into it with sides 52 mm and 14 mm. Find:

44 The size of the plate

A = l^2

_____ Plate area:
_____ cm²: _____
_____ mm²: _____

45 The size of the slot.

_____ Slot area:
_____ mm²: _____
_____ cm²: _____

46 The total area of the plate

mm²: _____
cm²: _____

Changing Units of Area

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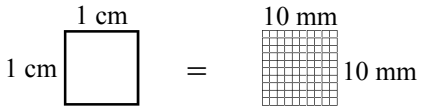
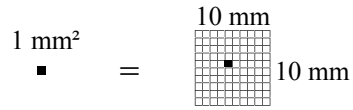
Convert these areas from cm² to mm²



1 cm² = 100 mm²

1	3 cm²	300 mm²
2	8 cm²	800 mm²
3	7.5 cm²	750 mm²
4	18 cm²	1 800 mm²
5	29 cm²	2 900 mm²
6	45.6 cm²	4 560 mm²
7	65.32 cm²	6 532 mm²
8	82.09 cm²	8 209 mm²
9	100 cm²	10 000 mm²
10	100.1 cm²	10 010 mm²

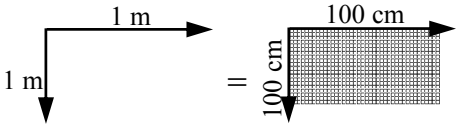
Convert these areas from mm² to cm²



1 mm² = 0.01 cm²

11	5 mm²	0.05 cm²
12	10 mm²	0.1 cm²
13	25 mm²	0.25 cm²
14	125 mm²	1.25 cm²
15	170.1 mm²	1.701 cm²
16	205.4 mm²	2.054 cm²
17	30.03 mm²	0.3003 cm²
18	450 mm²	4.5 cm²
19	500 mm²	5 cm²
20	500.6 mm²	5.006 cm²

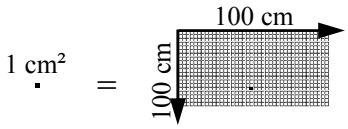
Convert these areas from m² to cm²



1 m² = 10 000 cm²

21	2 m²	20 000 cm²
22	3 m²	30 000 cm²
23	0.3 m²	3 000 cm²
24	0.03 m²	300 cm²
25	3.3 m²	33 000 cm²
26	0.725 m²	7 250 cm²
27	0.203 m²	2 030 cm²
28	0.8963 m²	8 963 cm²
29	0.902 m²	9 020 cm²
30	0.0009 m²	9 cm²

Convert these areas from cm² to m²



1 cm² = 0.0001 m²

31	7 cm²	0.0007 m²
32	11 cm²	0.0011 m²
33	54 cm²	0.0054 m²
34	154 cm²	0.0154 m²
35	296 cm²	0.0296 m²
36	875 cm²	0.0875 m²
37	1 500 cm²	0.15 m²
38	8 700 cm²	0.87 m²
39	12 000 cm²	1.2 m²
40	15 796 cm²	1.5796 m²

Answer these questions using the methods from the first two columns. Answer to 2 d.p.

41 A rectangular piece of fabric is size 1.5 m × 2.7 m. Find its area in m² and then convert to cm².

$A = lb$

$= 1.5 \times 2.7$

$A = 4.05$ m²

Fabric area:

m²: 4.05 m²

cm²: 40 500 cm²

42 A kite has axis lengths of 58 cm and 85 cm. Find its area in cm², then convert this to m².

$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 58 \times 85$

$A = 2\,465$ cm²

Kite area:

cm²: 2 465 cm²

m²: 0.2465 m²

43 A triangular sail has a base length of 2.2 m and a height 4.1 m. Find the area of the sail in m² & cm².

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 2.2 \times 4.1$

$A = 4.51$ m²

Sail area:

m²: 4.51 m²

cm²: 45 100 cm²

A square steel plate with sides 8.2 cm has a rectangular slot cut into it with sides 52 mm and 14 mm. Find:

44 The size of the plate

$A = l^2$

$= 8.2^2$

$A = 67.24$ cm²

Plate area:

cm²: 67.24 cm²

mm²: 6 724 mm²

45 The size of the slot.

$A = lb$

$= 52 \times 14$

$A = 728$ mm²

Slot area:

mm²: 728 mm²

cm²: 7.28 cm²

46 The total area of the plate

mm²: A = 6 724-728 = 5 996 mm²

cm²: A = 67.24-7.28 = 6 cm² (2 d.p.)

Using a Ruler - Line Measurement

When a line is measured using a ruler the answer can be expressed in mm or cm. If the ruler has both units on it you can use the side needed for the answer. However you should also be able to convert between the two units. You should know that:

1 cm = 10 mm, so 2 cm = 20 mm, and decimals 3.8 cm = 38 mm etc.

Column 1 has horizontal lines and vertical lines. It is the thick blue lines you are measuring, don't confuse them with the thin black lines for writing your answer on. The first column asks for answers in cm (to 1 d. p.), that means 3.7 cm, 11.2 cm and so on. The screw and nut at the bottom of the page can be measured either across the object or use the lines with the double-ended arrow between them. The second column is much the same except the lines are angled, note that measurement for this column is in mm.

Use the same method with column 3 only write the size of each side and then add them for the total.

Using a Ruler - Line Measurement

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Use a ruler to measure these, express the answer in cm (to 1 d. p.)

1 _____

2 _____

3 _____

4 _____

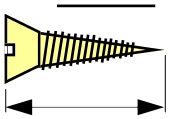
5 _____

6 _____

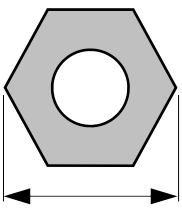
7 _____

13 12 11 10 9 8

14 _____



15 _____



Measure these angled lines to the nearest mm, express your answer in mm

16 _____

17 _____

18 _____

19 _____

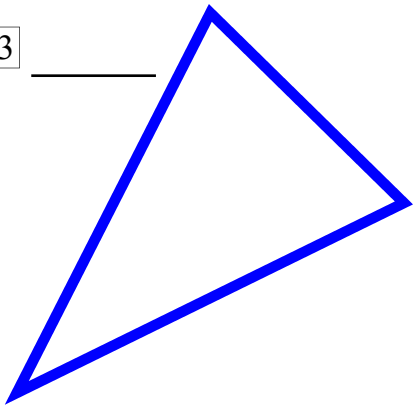
20 _____

21 _____

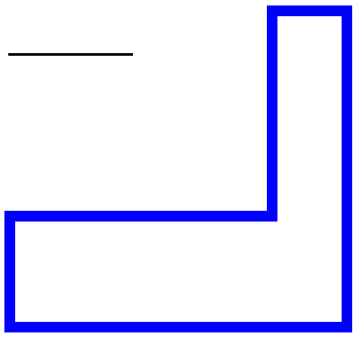
22 _____

Measure the perimeter of these shapes to the nearest mm, write the side length on each side, then total answer in mm.

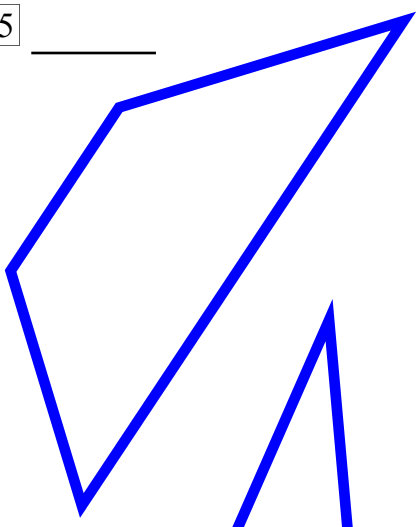
23 _____



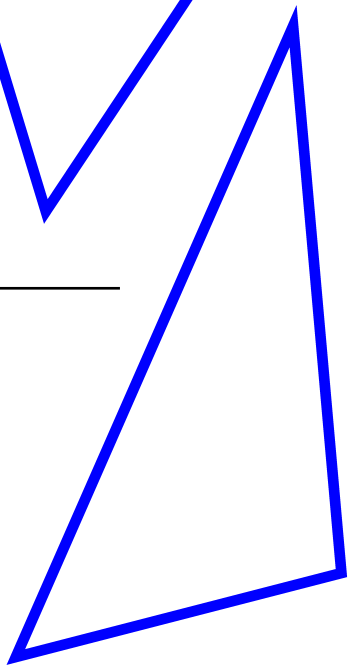
24 _____



25 _____



26 _____



Using a Ruler - Line Measurement

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Use a ruler to measure these, express the answer in cm (to 1 d. p.)

1 1.1 cm

2 0.4 cm

3 1.6 cm

4 0.8 cm

5 2.7 cm

6 2.5 cm

7 3.9 cm

13 14.8 cm

12 13.4 cm

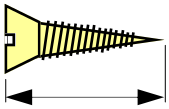
11 16.2 cm

10 7.7 cm

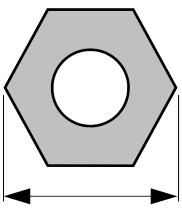
9 9.0 cm

8 5.8 cm

14 2.1 cm



15 2.3 cm



Measure these angled lines to the nearest mm, express your answer in mm

16 32 mm

17 19 mm

18 54 mm

19 46 mm

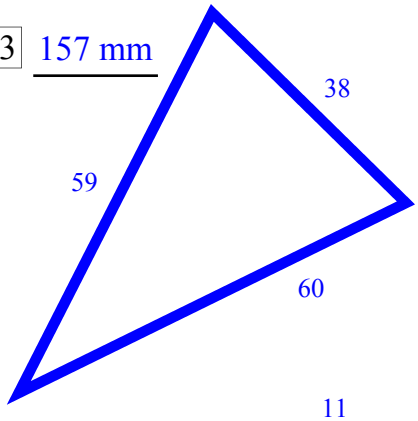
20 98 mm

21 66 mm

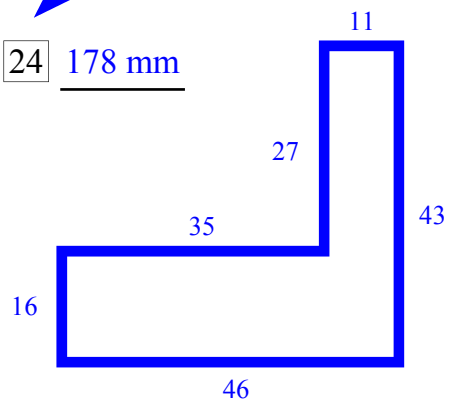
22 53 mm

Measure the perimeter of these shapes to the nearest mm, write the side length on each side, then total answer in mm.

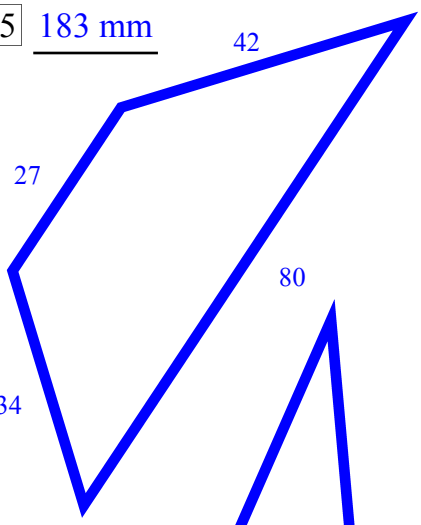
23 157 mm



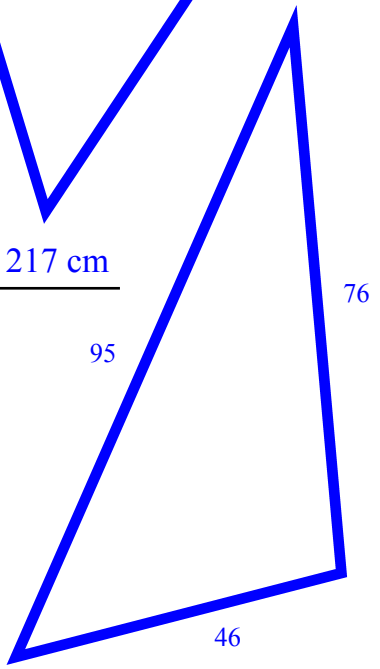
24 178 mm



25 183 mm



26 217 cm



Units of Measurement

When units of measurement are changed, multiply or divide by 10, 100, 1 000 or more. To change to a smaller unit multiply, to convert to a larger unit divide. Some students have difficulty with this, they think that if the unit is larger you must multiply. This isn't the case. Imagine you have 1 m or 100 cm, both are the same distance, there are more of the smaller unit, 1→100, so:

- changing to a smaller unit means - multiply
- changing to a larger unit means - divide

The number used to multiply or divide is the number of small units in the larger unit. For example 2 m to cm, there are 100 cm in a metre, changing to a smaller unit so multiply…so multiply by 100. As only 10, 100 or 1 000 are used to get the answer … move the decimal point. Multiply by 10 move the decimal place 1 position to the right, 100 move 2 positions and 1 000 move the decimal place 3 positions to the right.

Division is the same number of moves, only this time it's to the left. Divide by 10 and move the decimal place 1 position to the left, 100 move it 2 positions and 1 000 moves 3 positions to the left. A reminder is at the top of each column.

Column 1 starts with changing mm to cm. You should be able to talk it through to your-self….mm to cm is changing to a larger unit...that means division….there are 10 mm in a cm that means I divide by 10….divide by 10 means I move the decimal point 1 place to the left. That means 18 mm = 1.8 cm 113 mm = 11.3 cm and 0.9 mm = 0.09 cm. The next part of the column is the reverse, multiply by 10 means move the decimal place 1 position to the right. So 54 cm = 540 mm, 1.3 cm = 13 mm and 123 cm = 1 230 mm.

Column 2 deals with converting between cm and m. This time the change is by dividing or multiplying by 100. That means a 2 decimal place movement. For cm to m (divide by 100) 12 cm = 0.12 m, 106 cm = 1.06 m and 5.78 cm = 0.0578 m. Then the reverse, m to cm means multiplying by 100 or a 2 decimal place shift to the right: 3 m = 300 cm, 0.7 m = 70 cm and 10.05 m = 1 005 cm.

Column 3 is m to km, this involves multiplying or dividing by 1 000, or a 3 decimal place movement. That would mean 300 m = 0.3 km, 2 305 m = 2.305 km and 4.7 m = 0.0047 km. Then the reverse, changing km to m. That's 3 decimal places to the right: 1.2 km = 1 200m, 0.85 km = 850 m and 0.02 km = 20 m.

Units of Measurement

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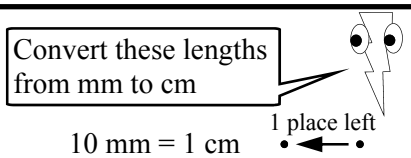
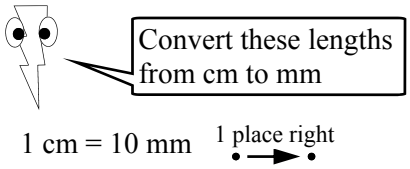


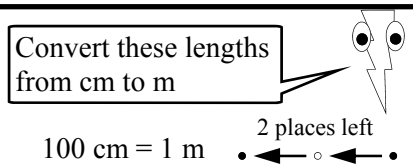
10 mm = 1 cm

1. 20 mm
2. 50 mm
3. 130 mm
4. 55 mm
5. 92 mm
6. 200 mm
7. 18.6 mm
8. 2 020 mm
9. 435 mm
10. 0.95 mm
11. 5.47 mm
12. 906.2 mm



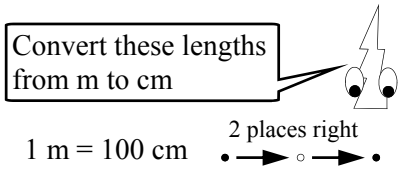
1 cm = 10 mm

13. 5 cm
14. 18 cm
15. 6.3 cm
16. 74.8 cm
17. 356.7 cm
18. 8.12 cm
19. 0.4 cm
20. 0.566 cm
21. 155 cm
22. 829.3 cm
23. 1.002 cm
24. 19.91 cm



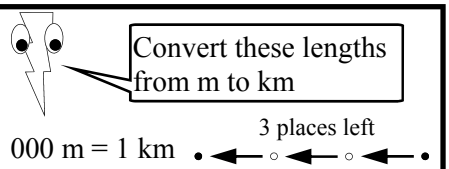
100 cm = 1 m

25. 200 cm
26. 600 cm
27. 150 cm
28. 370 cm
29. 12 cm
30. 86.5 cm
31. 7 300 cm
32. 56.05 cm
33. 3 cm
34. 6.7 cm
35. 8 002 cm
36. 27.08 cm



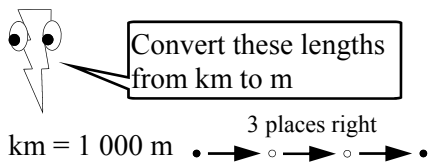
1 m = 100 cm

37. 3 m
38. 7 m
39. 2.2 m
40. 5.9 m
41. 0.3 m
42. 0.75 m
43. 0.363 m
44. 5.02 m
45. 10.7 m
46. 0.006 m
47. 5.071 m
48. 30.262 m



1 000 m = 1 km

49. 1 500 m
50. 1 220 m
51. 4 930 m
52. 110 m
53. 550.6 m
54. 1 003 m
55. 85 m
56. 4.7 m
57. 309.7 m
58. 12 032 m
59. 8 771.3 m
60. 763.2 m



1 km = 1 000 m

61. 3 km
62. 8 km
63. 3.2 km
64. 14.7 km
65. 10.6 km
66. 0.9 km
67. 0.673 km
68. 5.12 km
69. 1.007 km
70. 0.4503 km
71. 4.019 km
72. 0.06 km

Units of Measurement

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Convert these lengths from mm to cm

10 mm = 1 cm

1 place left

1	20 mm	2 cm
2	50 mm	5 cm
3	130 mm	13 cm
4	55 mm	5.5 cm
5	92 mm	9.2 cm
6	200 mm	20 cm
7	18.6 mm	1.86 cm
8	2 020 mm	202 cm
9	435 mm	43.5 cm
10	0.95 mm	0.095 cm
11	5.47 mm	0.547 cm
12	906.2 mm	90.62 cm

Convert these lengths from cm to m

100 cm = 1 m

2 places left

25	200 cm	2 m
26	600 cm	6 m
27	150 cm	1.5 m
28	370 cm	3.7 m
29	12 cm	0.12 m
30	86.5 cm	0.865 m
31	7 300 cm	73 m
32	56.05 cm	0.5605 m
33	3 cm	0.03 m
34	6.7 cm	0.067 m
35	8 002 cm	80.02 m
36	27.08 cm	0.2708 m

Convert these lengths from m to km

1 000 m = 1 km

3 places left

49	1 500 m	1.5 km
50	1 220 m	1.22 km
51	4 930 m	4.93 km
52	110 m	0.11 km
53	550.6 m	0.5506 km
54	1 003 m	1.003 km
55	85 m	0.085 km
56	4.7 m	0.0047 km
57	309.7 m	0.3097 km
58	12 032 m	12.032 km
59	8 771.3 m	8.7713 km
60	763.2 m	0.7632 km

Convert these lengths from cm to mm

1 cm = 10 mm

1 place right

13	5 cm	50 mm
14	18 cm	180 mm
15	6.3 cm	63 mm
16	74.8 cm	748 mm
17	356.7 cm	3567 mm
18	8.12 cm	81.2 mm
19	0.4 cm	4 mm
20	0.566 cm	5.66 mm
21	155 cm	1 550 mm
22	829.3 cm	8 293 mm
23	1.002 cm	10.02 mm
24	19.91 cm	199.1 mm

Convert these lengths from m to cm

1 m = 100 cm

2 places right

37	3 m	300 cm
38	7 m	700 cm
39	2.2 m	220 cm
40	5.9 m	590 cm
41	0.3 m	30 cm
42	0.75 m	75 cm
43	0.363 m	36.3 cm
44	5.02 m	502 cm
45	10.7 m	1 070 cm
46	0.006 m	0.6 cm
47	5.071 m	507.1 cm
48	30.262 m	3 026.2 cm

Convert these lengths from km to m

1 km = 1 000 m

3 places right

61	3 km	3 000 m
62	8 km	8 000 m
63	3.2 km	3 200 m
64	14.7 km	14 700 m
65	10.6 km	10 600 m
66	0.9 km	900 m
67	0.673 km	673 m
68	5.12 km	5 120 m
69	1.007 km	1 007 m
70	0.4503 km	450.3 m
71	4.019 km	4 019 m
72	0.06 km	60 m

Further Units of Measurement

The processes are the same as on the previous sheet 'Units of Measurement' except that this time the conversions aren't in groups of units and multiplication/division. Its all jumbled so you need to know whether you divide or multiply and how many decimal places you shift the decimal point. There is also the inclusion of mm to m and the reverse. This is a 3 decimal place shift (× or ÷ by 1 000).

Further Units of Measurement

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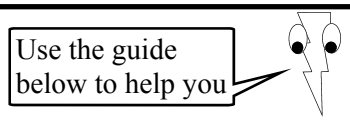
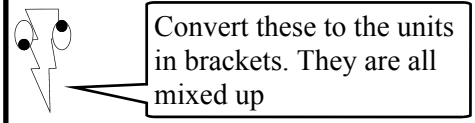


mm → cm	1 place left
mm → m	3 places left
cm → mm	1 place right
cm → m	2 places left
m → mm	3 places right
m → cm	2 places right
m → km	3 places left
km → m	3 places right



1. 3.2 m [cm] _____
2. 4.3 cm [mm] _____
3. 0.98 m [cm] _____
4. 200 mm [m] _____
5. 860 m [km] _____
6. 118 mm [cm] _____
7. 1 033 m [km] _____
8. 27.2 cm [mm] _____
9. 185 mm [m] _____
10. 206.5 mm [m] _____
11. 0.35 cm [mm] _____
12. 686 cm [m] _____
13. 0.496 m [mm] _____
14. 4.03 km [m] _____
15. 65.7 cm [m] _____
16. 825 mm [cm] _____
17. 11.3 m [cm] _____
18. 5.062 km [m] _____
19. 0.838 km [m] _____
20. 3.07 cm [mm] _____
21. 121 mm [m] _____
22. 7 777 m [km] _____
23. 918 cm [m] _____
24. 4.2 cm [m] _____
25. 0.5 cm [m] _____
26. 0.89 m [cm] _____
27. 765 mm [cm] _____
28. 19.6 cm [mm] _____
29. 16 010 m [km] _____
30. 78 mm [m] _____
31. 2.1 m [mm] _____
32. 1.93 m [cm] _____
33. 82.6 cm [mm] _____
34. 16 m [cm] _____
35. 1.8 km [m] _____
36. 3.8 mm [cm] _____
37. 9.021 km [m] _____
38. 56.5 cm [m] _____
39. 28.8 m [km] _____
40. 0.83 cm [mm] _____
41. 0.6 m [mm] _____
42. 0.03 m [mm] _____
43. 41 m [cm] _____
44. 850 mm [m] _____
45. 182.5 cm [m] _____
46. 1 405 mm [m] _____
47. 0.95 m [mm] _____
48. 1 727.4 m [km] _____
49. 0.052 cm [mm] _____
50. 39 cm [m] _____
51. 0.2004 km [m] _____
52. 4.52 m [cm] _____
53. 80.05 cm [mm] _____
54. 293 m [km] _____
55. 0.302 m [mm] _____
56. 13 mm [m] _____
57. 43.9 mm [cm] _____
58. 0.67 m [cm] _____
59. 850 mm [m] _____
60. 2 211 mm [m] _____
61. 549.3 m [km] _____
62. 400.3 cm [m] _____
63. 0.74 km [m] _____
64. 4.96 cm [mm] _____
65. 113.2 cm [mm] _____
66. 333.7 cm [m] _____
67. 17 mm [cm] _____
68. 0.005 m [cm] _____
69. 17 mm [cm] _____
70. 0.02 km [m] _____
71. 680 mm [m] _____
72. 82.4 cm [m] _____
73. 0.65 mm [cm] _____
74. 32.6 mm [m] _____
75. 0.006 km [m] _____

Further Units of Measurement

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Use the guide below to help you

mm → cm	1 place left
mm → m	3 places left
cm → mm	1 place right
cm → m	2 places left
m → mm	3 places right
m → cm	2 places right
m → km	3 places left
km → m	3 places right

Convert these to the units in brackets. They are all mixed up

1	3.2 m	[cm]	320 cm
2	4.3 cm	[mm]	43 mm
3	0.98 m	[cm]	98 cm
4	200 mm	[m]	0.2 m
5	860 m	[km]	0.86 km
6	118 mm	[cm]	11.8 cm
7	1 033 m	[km]	1.033 km
8	27.2 cm	[mm]	272 mm
9	185 mm	[m]	0.185 m
10	206.5 mm	[m]	0.2065 m
11	0.35 cm	[mm]	3.5 mm
12	686 cm	[m]	6.86 m
13	0.496 m	[mm]	496 mm
14	4.03 km	[m]	4 030 m
15	65.7 cm	[m]	0.657 m
16	825 mm	[cm]	82.5 cm
17	11.3 m	[cm]	1 130 cm
18	5.062 km	[m]	5 062 m
19	0.838 km	[m]	838 m
20	3.07 cm	[mm]	30.7 mm
21	121 mm	[m]	0.121 m
22	7 777 m	[km]	7.777 km
23	918 cm	[m]	9.18 m
24	4.2 cm	[m]	0.042 m
25	0.5 cm	[m]	0.005 m
26	0.89 m	[cm]	89 cm
27	765 mm	[cm]	76.5 cm
28	19.6 cm	[mm]	196 mm
29	16 010 m	[km]	16.01 km
30	78 mm	[m]	0.078 m
31	2.1 m	[mm]	2 100 mm
32	1.93 m	[cm]	193 cm
33	82.6 cm	[mm]	826 mm
34	16 m	[cm]	1 600 cm
35	1.8 km	[m]	1 800 m
36	3.8 mm	[cm]	0.38 cm
37	9.021 km	[m]	9 021 m
38	56.5 cm	[m]	0.565 m
39	28.8 m	[km]	0.0288 km
40	0.83 cm	[mm]	8.3 mm
41	0.6 m	[mm]	600 mm
42	0.03 m	[mm]	30 mm
43	41 m	[cm]	4 100 cm
44	850 mm	[m]	0.85 m
45	182.5 cm	[m]	1.825 m
46	1 405 mm	[m]	1.405 m
47	0.95 m	[mm]	950 mm
48	1 727.4 m	[km]	1.7274 km
49	0.052 cm	[mm]	0.52 mm
50	39 cm	[m]	0.39 m
51	0.2004 km	[m]	200.4 m
52	4.52 m	[cm]	452 cm
53	80.05 cm	[mm]	800.5 mm
54	293 m	[km]	0.293 km
55	0.302 m	[mm]	302 mm
56	13 mm	[m]	0.013 m
57	43.9 mm	[cm]	4.39 cm
58	0.67 m	[cm]	67 cm
59	850 mm	[m]	0.85 m
60	2 211 mm	[m]	2.211 m
61	549.3 m	[km]	0.5493 km
62	400.3 cm	[m]	4.003 m
63	0.74 km	[m]	740 m
64	4.96 cm	[mm]	49.6 mm
65	113.2 cm	[mm]	1 132 mm
66	333.7 cm	[m]	3.337 m
67	17 mm	[cm]	1.7 cm
68	0.005 m	[cm]	0.5 cm
69	17 mm	[cm]	1.7 cm
70	0.02 km	[m]	20 m
71	680 mm	[m]	0.68 m
72	82.4 cm	[m]	0.824 m
73	0.65 mm	[cm]	0.065 cm
74	32.6 mm	[m]	0.0326 m
75	0.006 km	[m]	6 m

Perimeter of Shapes

The perimeter of a shape is the distance around the outside of the shape. The units of measurement are mm, cm, m and km. Shapes often have side lengths that are the same, rather than writing this distance out again a mark is put on the identical sides to show they are the same length. If there are a number of different pairs of sides that are the same length, 2 marks together will be used and so on, such as with a rectangle.

To answer columns 1 and 2 use the same method, this is:

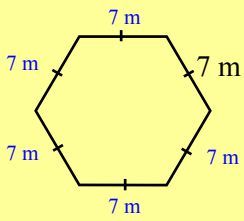
- Look at the side markings and write side lengths on every side it is up to you if you write the units as well
- The first line of working is the sum of the sides of the shape, write 'P =' then the sides separated by + signs. Note that to avoid missing sides don't jump around in the addition. Start on a side then move around the shape in a clockwise or anti-clockwise direction.
- If the sum is challenging to complete mentally, total the sum as you move through, look at the example below.
- The second line is the answer line, again write 'P =' then the answer, then the units, either mm, cm or m.

Column 3 introduces formulae used for squares and rectangles. These are:

- Squares: $P = 4l$ (which is 4 times the side length)
- Rectangles: $P = 2l + 2b$ (which is 2 times the length + 2 times the breadth)

The questions are in tables. With squares, multiply the side length by 4 and write the answer with units. The rectangles are completed in stages, l is given and you double it to get the value of $2l$, b is given and you double it to get the value of $2b$. Then add the values for $2l$ and $2b$ together to get the answer, write in the units (all of which are cm.)

Fill in all the side lengths to avoid missing them



P = 7 + 7 + 7 + 7 + 7 + 7

P = 42 m

Perimeter of Shapes

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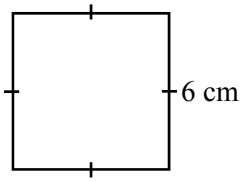
Put measurements on all sides then add them to find the perimeter.

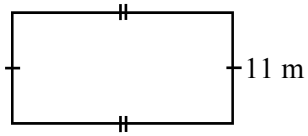
1



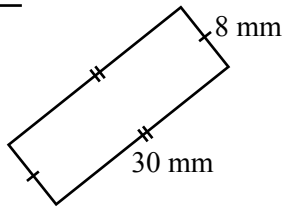
P = 6 + + +

P = cm

2



3



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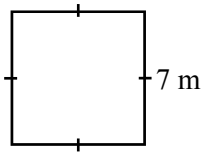
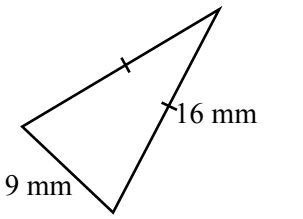
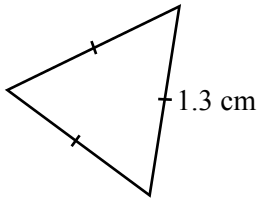


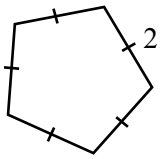
5



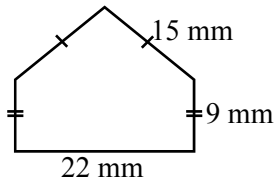
6



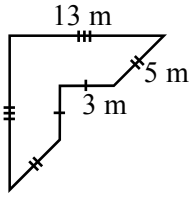
7



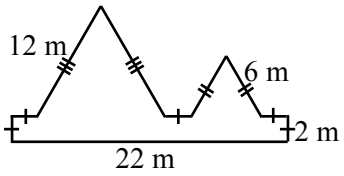
8



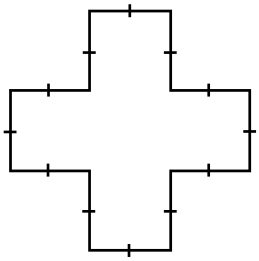
9



10



11

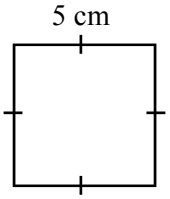


P = ×

Complete the tables, all measurements are in cm.

Perimeter Formula for Squares :

$P = 4l$ where l = side length



$P = 4l$

$= 4 \times 5$

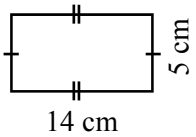
$P = 20$ cm

12

Length (l)	Perimeter ($4l$)	Length (l)	Perimeter ($4l$)
5	20 cm	15	
8		11	
10		14	
9		30	
3		50	
4		80	
12		55	
20		42	

Perimeter Formula for Rectangles :

$P = 2l + 2b$ where : l = length and b = breadth



$P = 2l + 2b$

$= 2 \times 14 + 2 \times 5$

$P = 38$ cm

13

Length (l)	$2l$	Breadth (b)	$2b$	Perimeter ($2l + 2b$)
14	28	5	10	38 cm
8		11		
12		5		
9		6		
15		25		
20		16		
7		19		
13		17		

Perimeter of Shapes

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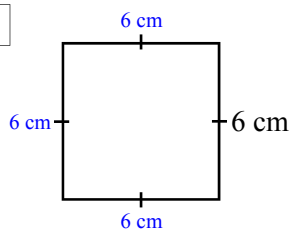
Put measurements on all sides then add them to find the perimeter.

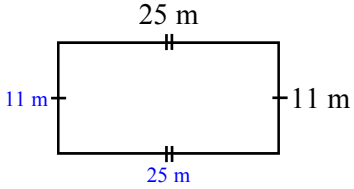
1



P = 6 + 6 + 6 + 6

P = 24 cm

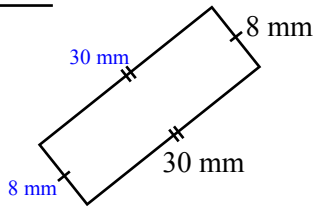
2



P = 25 + 25 + 11 + 11

P = 72 m

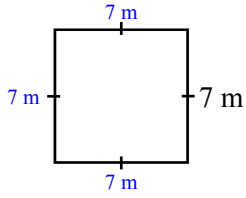
3



P = 30 + 30 + 8 + 8

P = 76 mm

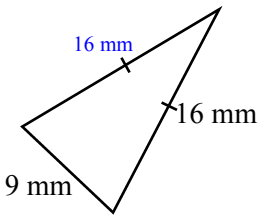
4



P = 7 + 7 + 7 + 7

P = 28 m

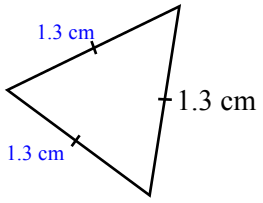
5



P = 16 + 16 + 9

P = 41 mm

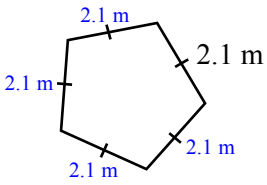
6



P = 1.3 + 1.3 + 1.3

P = 3.9 cm

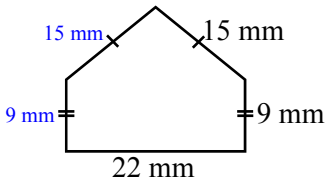
7



P = 2.1 + 2.1 + 2.1 + 2.1 + 2.1

P = 10.5 m

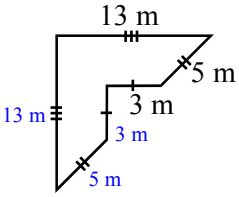
8



P = 22 + 9 + 15 + 15 + 9

P = 70 mm

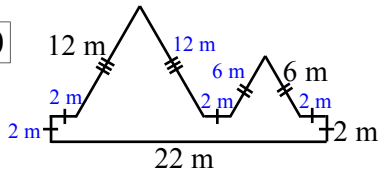
9



P = 5 + 13 + 13 + 5 + 3 + 3

P = 42 m

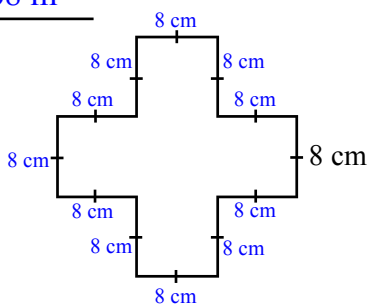
10



P = 22 + 2 + 2 + 12 + 12 + 2 + 6 + 6 + 2 + 2

P = 68 m

11



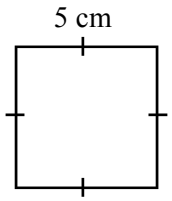
P = 8 × 12

P = 96 cm

Complete the tables, all measurements are in cm.

Perimeter Formula for Squares :

P = 4*l* where *l* = side length



P = 4*l*

= 4 × 5

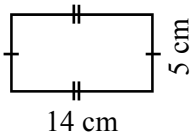
P = 20 cm

12

Length (*l*)	Perimeter (4*l*)	Length (*l*)	Perimeter (4*l*)
5	20 cm	15	60 cm
8	32 cm	11	44 cm
10	40 cm	14	56 cm
9	36 cm	30	120 cm
3	12 cm	50	200 cm
4	16 cm	80	320 cm
12	48 cm	55	220 cm
20	80 cm	42	168 cm

Perimeter Formula for Rectangles :

P = 2*l* + 2*b* where : *l* = length and *b* = breadth



P = 2*l* + 2*b*

= 2×14 + 2×5

P = 38 cm

13

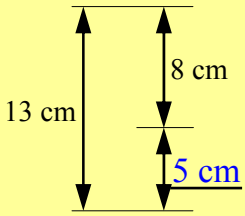
Length (*l*)	2*l*	Breadth (*b*)	2*b*	Perimeter (2*l* + 2*b*)
14	28	5	10	38 cm
8	16	11	22	38 cm
12	24	5	10	34 cm
9	18	6	12	30 cm
15	30	25	50	80 cm
20	40	16	32	72 cm
7	14	19	38	52 cm
13	26	17	34	60 cm

Perimeter - Finding a Missing Side

Sometimes before the perimeter of a shape can be calculated the missing sides have to be found. The missing sides will either have side markings that indicate that they are the same size as another side or they will be found by adding or subtracting sides that you know the value of. Note that this applies due to all the angles being right angles, the corners are not marked but they are 90º.

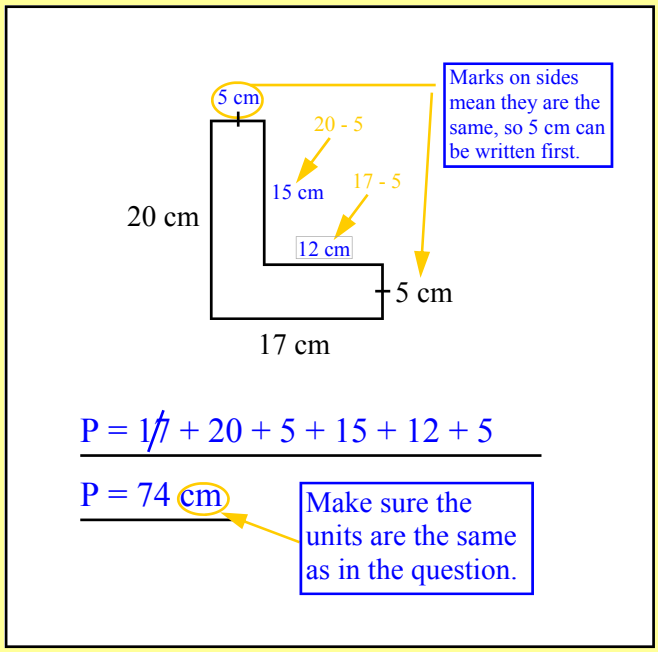
In column 1 questions 1 - 5, two different ways are used to measure the same distance. This means that they must be equal. Use addition or subtraction to find the value. In the example below a distance is measured as 13 cm. The same distance is then broken up into to parts, one of those parts is 8 cm. Ask yourself, 'What number adds to 8 to give 13?', the answer is 5, so the missing length is 5 cm. From question 6, through the column, shapes are given and you use the same method to find the missing sides. The method is:

- If the shape has side markings write in all the sides that you know.
- Use the given lengths to then calculate the other side lengths



Columns 2 and 3 add one step to column 1, the perimeter is calculated. Use 2 lines of working, the first line being the addition of all the sides and the second line being the answer with the units (mm, cm, m etc). Writing the side lengths on all sides of the shape will help avoid mistakes. Some students like to miss this step because they know the value of the side and don't need to write it, the problem is that when they add the sides to get the perimeter they often forget to include them. A method to avoid this problem is to label every side with its measurement then look at the addition and check off the numbers against the numbers on the sides of the shape. If there are different numbers or there is too many or too few you have to find the problem. Note that if you add around the sides of the shape in the same way, clock-wise or an anti-clockwise direction, it makes it less confusing.

Be careful with question 19 it is different and may trick you.



Perimeter - Finding a Missing Side

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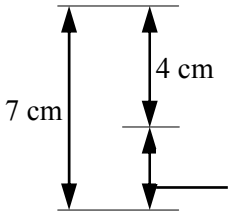
Calculate the missing lengths, all angles are right angles

1



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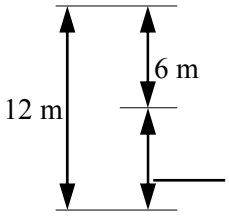

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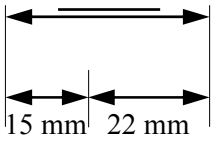

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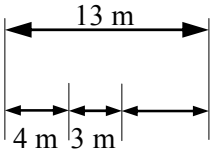

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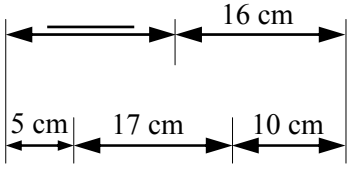

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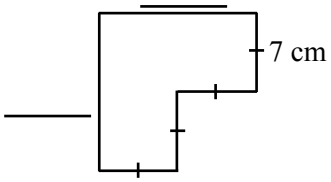

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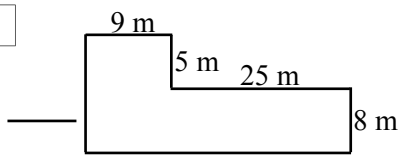

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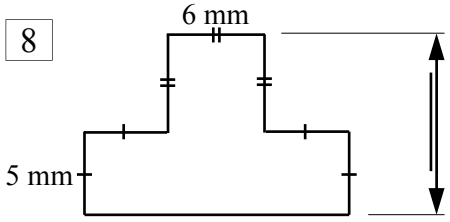

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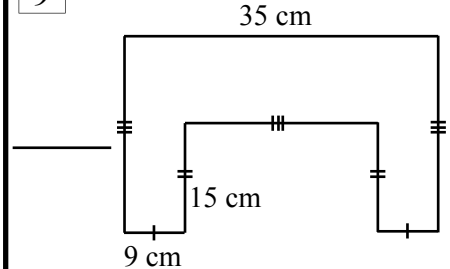
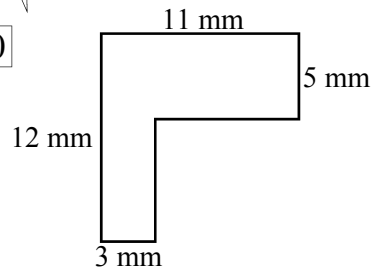


Find the missing sides then find the perimeter

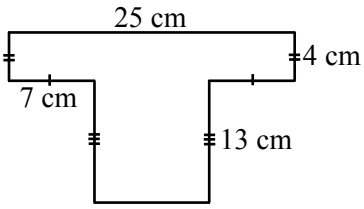
10



P = 3 + 12 + 11 + 5 + + +

P = mm

11



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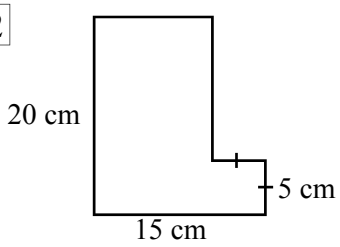

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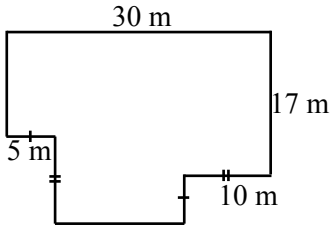

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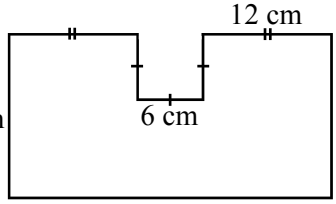

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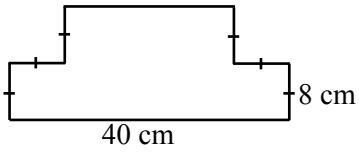

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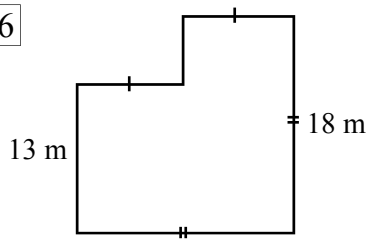

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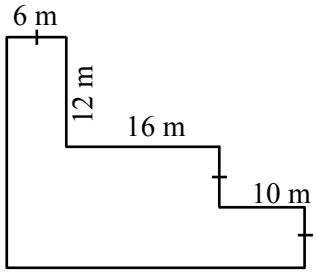

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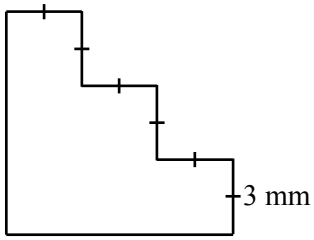

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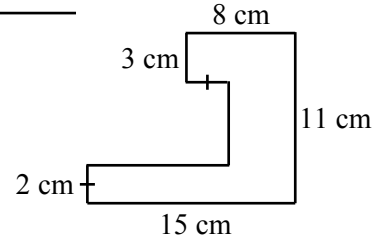

Perimeter - Finding a Missing Side

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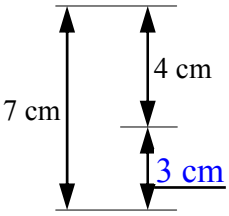
Calculate the missing lengths, all angles are right angles

1



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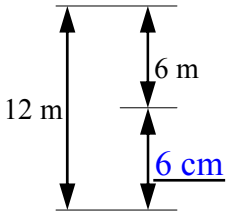

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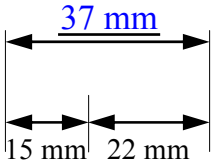

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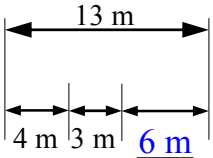

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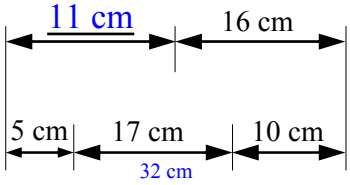

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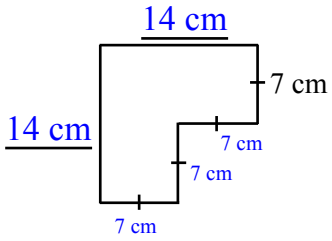

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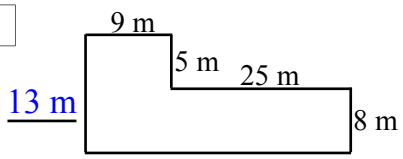
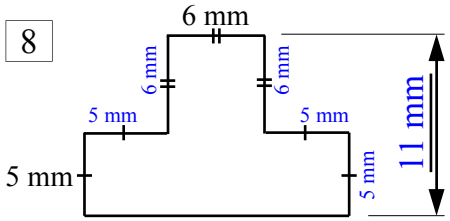


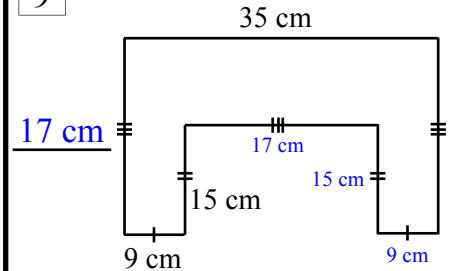
34 m

8



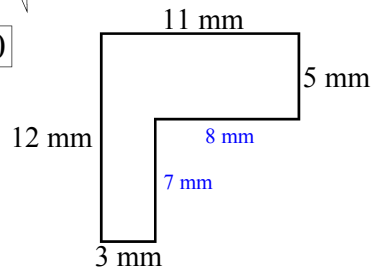
16 mm

9



Find the missing sides then find the perimeter

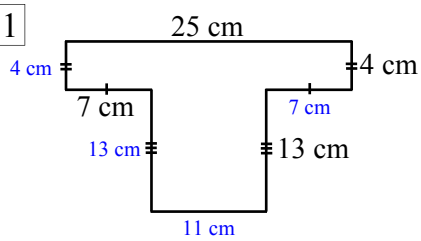
10



P = 3 + 12 + 11 + 5 + 8 + 7

P = 46 mm

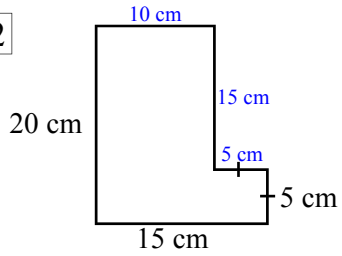
11



P = 11 + 13 + 7 + 4 + 25 + 4 + 7 + 13

P = 84 cm

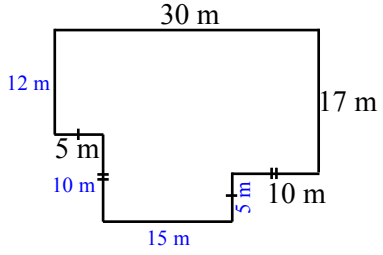
12



P = 15 + 20 + 10 + 15 + 5 + 5

P = 70 cm

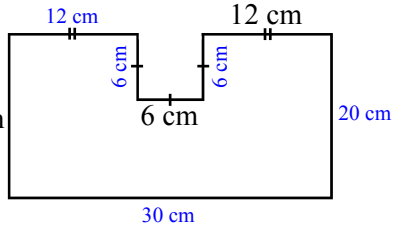
13



P = 15 + 10 + 5 + 12 + 30 + 17 + 10 + 5

P = 104 m

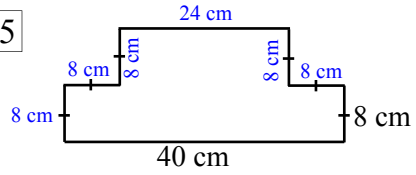
14



P = 30 + 20 + 12 + 6 + 6 + 6 + 12 + 20

P = 112 cm

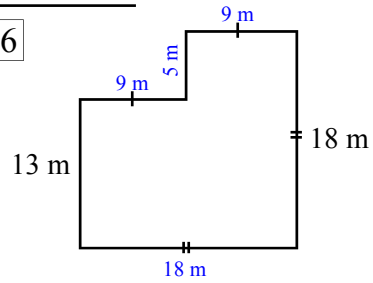
15



P = 40 + 8 + 8 + 8 + 24 + 8 + 8 + 8

P = 112 cm

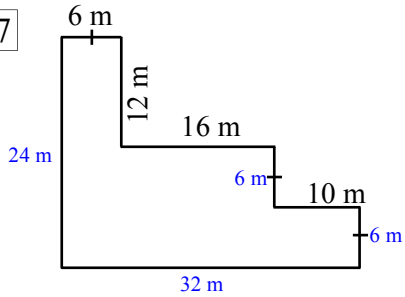
16



P = 18 + 13 + 9 + 5 + 9 + 18

P = 72 m

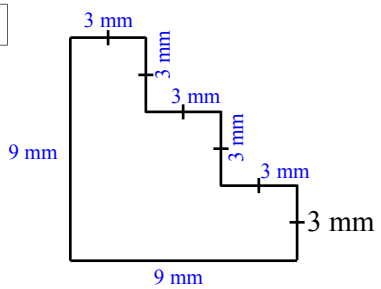
17



P = 32 + 24 + 6 + 12 + 16 + 6 + 10 + 6

P = 112 m

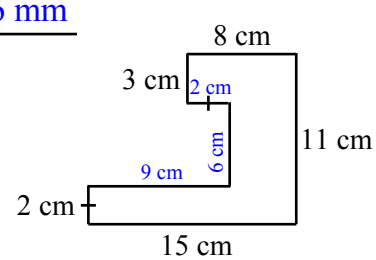
18



P = 9 + 9 + 3 + 3 + 3 + 3 + 3 + 3

P = 36 mm

19



P = 15 + 2 + 9 + 6 + 2 + 3 + 8 + 11

P = 56 cm

Exact & Estimation - Circumference

The circumference of a circle is its perimeter, the distance around its outside. The way to measure circumference when you don't know how to calculate it is to place a mark on the circle roll it one revolution and see how far the mark has travelled. But you can use this formula:

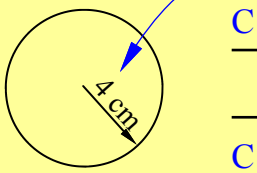
$$C = 2\pi r \text{ or } C = \pi d$$

Where: C = the circumference (length is same units as the radius/diameter)
π = Pi, it's a value on your calculator 3.14159….. Which is a bit more than 3!
r = the radius, the length from the centre to any point on the circumference
d = the diameter, the width of the circle.

Note that if life is complicated enough, just use $C = 2\pi r$ only, then remember to always use the radius. You don't have to remember both formulae. If given a diameter, halve it!

Column 1 starts with circles that have a radius shown, you will require a ruler for question 4. To find the circumference in terms of π, also called the exact length, use the value of r given in the circle. To find the estimate, use the exact circumference and substitute 3 for π.

Example



$C = 2\pi r$

$= 2\times\pi\times4$ ← Replace r with 4, (the radius of the circle)

$C = 8\pi$ cm ← $2 \times 4 = 8$, leave π as is. Write the units.

So the circumference is 8π cm. This is the exact length, to estimate the length without a calculator use $\pi = 3$. This is less than the actual value for π, but it gives a fairly accurate answer without too much calculation.

Estimate:

$C = 8\pi$ cm ← Rewrite the circumference from previous step

$\approx 8 \times 3$ ← Replace π with 3 and = with ≈

$C \approx 24$ cm ← Solve, use either ≈ or ≒ to indicate 'approximately'.

Column 2 uses diameters. You can either halve the diameter for the radius and solve as before, or use $C = \pi d$. So just multiply the diameter by 3 for an estimate of the circumference.

At the bottom of the page are four circles. Measure the diameter accurate to the nearest 0.5 cm, then calculate the exact circumference (in terms of π) and then an estimate, again using $\pi = 3$. Use the working out spaces in the third column to answer these.

Exact & Estimation - Circumference

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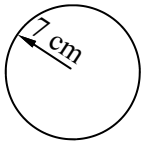
Find the circumference in terms of π. Then using π = 3, estimate the circumference.

1 $C = 2\pi r$

$= 2 \times \pi \times \square$

$C = \square\ \pi$ cm

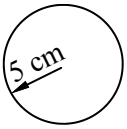


Estimate: $C = \square\ \pi$ cm

$\approx \square \times 3$

$C \approx \square$ cm

2



$C = 2\pi r$

$=$

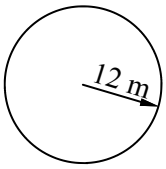
$C =$

Estimate: $C =$

$\approx$

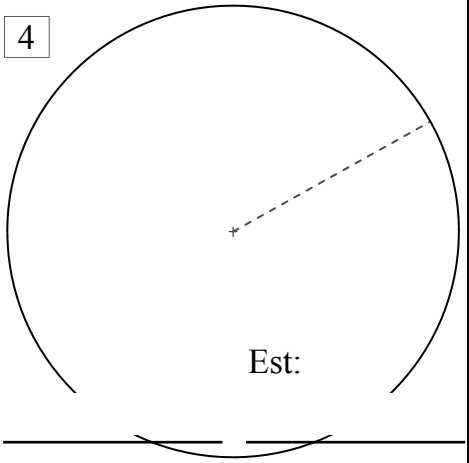
$C \approx$

3



Estimate:

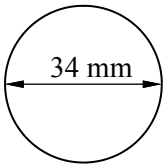
4



Est:

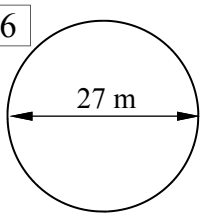
Divide the diameter by 2 or use $C = \pi d$ to find the circumference & estimate.

5



Estimate:

6



Estimate:

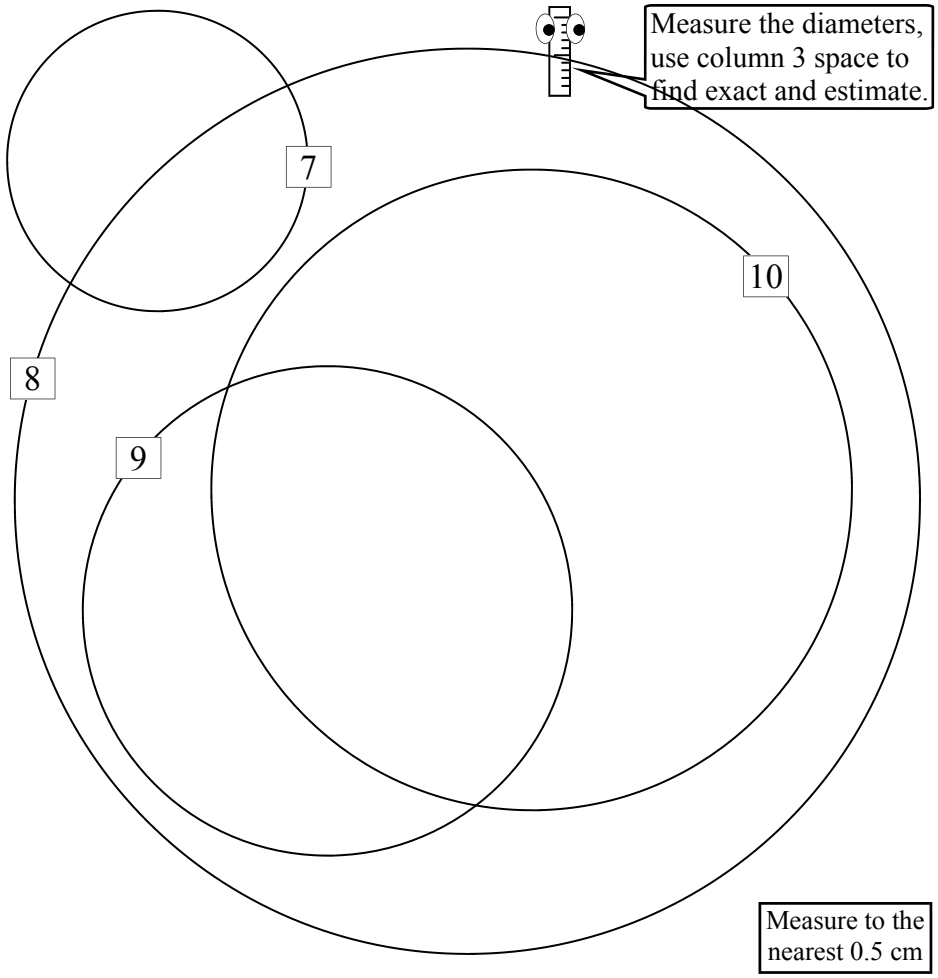
7

8

9

10

Measure the diameters, use column 3 space to find exact and estimate.



Measure to the nearest 0.5 cm

Exact & Estimation - Circumference

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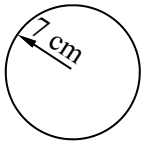
Find the circumference in terms of π. Then using π = 3, estimate the circumference.

1 $C = 2\pi r$

$= 2 \times \pi \times 7$

$C = 14\pi$ cm

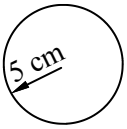


Estimate: $C = 14\pi$ cm

$\approx 14 \times 3$

$C \approx 42$ cm

2



$C = 2\pi r$

$= 2 \times \pi \times 5$

$C = 10\pi$ cm

Estimate: $C = 10\pi$ cm

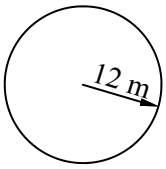
$\approx 10 \times 3$

$C \approx 30$ cm

3 $C = 2\pi r$

$= 2 \times \pi \times 12$

$C = 24\pi$ m

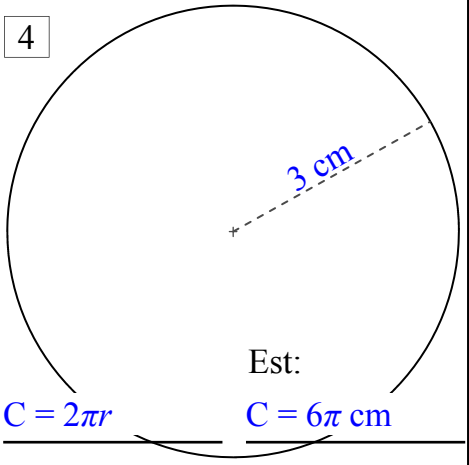


Estimate: $C = 24\pi$ cm

$\approx 24 \times 3$

$C \approx 72$ m

4



Est:

$C = 2\pi r$ | $C = 6\pi$ cm

$= 2 \times \pi \times 3$ | $\approx 6 \times 3$

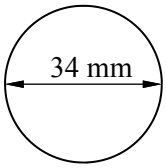
$C = 6\pi$ cm | $C \approx 18$ cm

Divide the diameter by 2 or use $C = \pi d$ to find the circumference & estimate.

5 $C = \pi d$

$= \pi \times 34$

$C = 34\pi$ mm

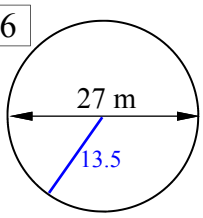


Estimate: $C = 34\pi$ mm

$\approx 34 \times 3$

$C \approx 102$ mm

6



$C = \pi d$

$= \pi \times 27$

$C = 27\pi$ m

Estimate: $C = 27\pi$ m

$\approx 27 \times 3$

$C \approx 81$ m

7

$C = \pi d$ | $C = 4\pi$ cm

$= \pi \times 4$ | $\approx 4 \times 3$

$C = 4\pi$ cm | $C \approx 12$ cm

8

$C = \pi d$ | $C = 12\pi$ cm

$= \pi \times 12$ | $\approx 12 \times 3$

$C = 12\pi$ cm | $C \approx 36$ cm

9

$C = \pi d$ | $C = 6.5\pi$ cm

$= \pi \times 6.5$ | $\approx 6.5 \times 3$

$C = 6.5\pi$ cm | $C \approx 19.5$ cm

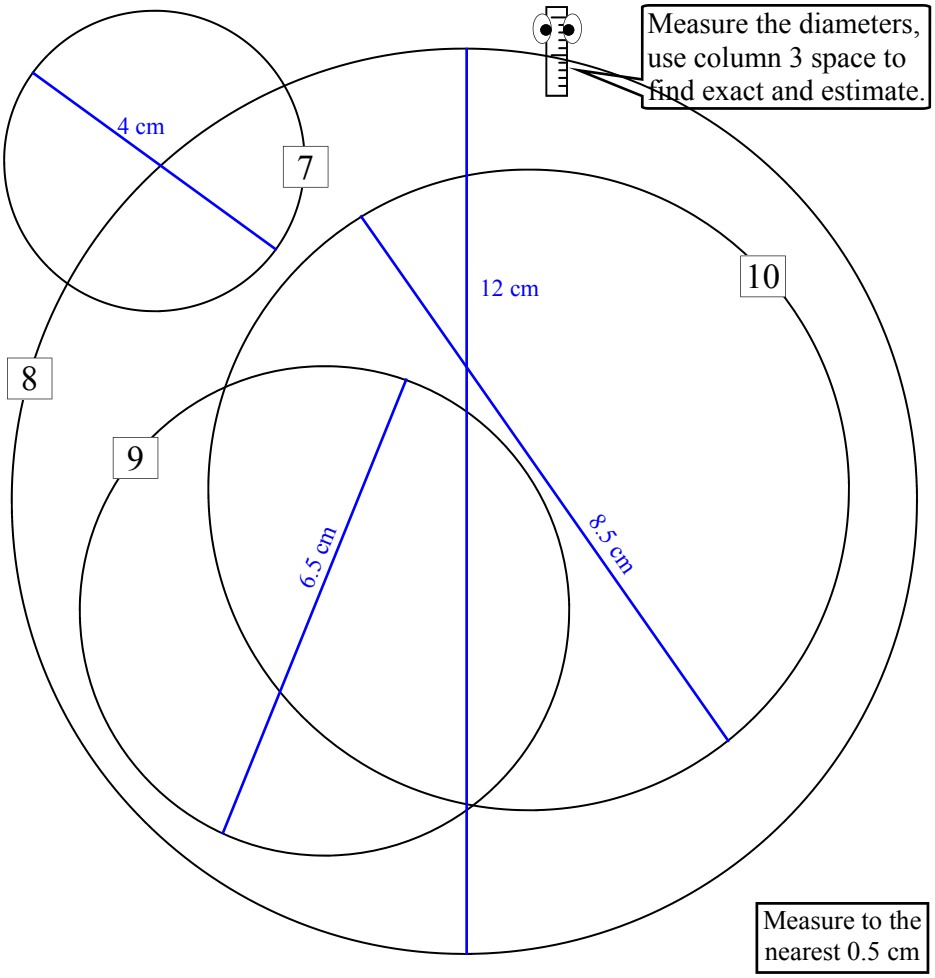
10

$C = \pi d$ | $C = 8.5\pi$ cm

$= \pi \times 8.5$ | $\approx 8.5 \times 3$

$C = 8.5\pi$ cm | $C \approx 25.5$ cm

Measure the diameters, use column 3 space to find exact and estimate.



Measure to the nearest 0.5 cm

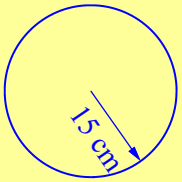
Circumference (Sheet 1)

The circumference of a circle is its perimeter, the distance around the outside of the circle. This distance is found by using this formula:

$$C = 2\pi r$$

Where: C = Circumference of circle
r = radius of the circle

Example



$C = 2\pi r$ ← Write the formula.

$= 2 \times \pi \times 15$ ← Show × signs and replace r with the radius (15 on diagram).

$C = 94.2$ cm (1 d.p.) ← Calculate, round and show units.

Calculator 2 × π × 1 5 = Answer on Screen 94.24777961

Round to 1 decimal place 94.2 cm (1 d.p.)

Column 1 uses the above process for all questions.

Column 2 gives you a diameter rather than a radius. So you can divide by the diameter by 2 to find the radius then use $C = 2\pi r$, but an easier way is to use:

$$C = \pi d$$

Where: C = Circumference of circle
d = diameter of the circle (note that $d = 2r$)

Column 3 are problem style questions using the circumference of a circle.

Circumference (Sheet 1)

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Use $C=2\pi r$ to find the circumference of these circles to 1 decimal place.

Example

$C = 2\pi r$

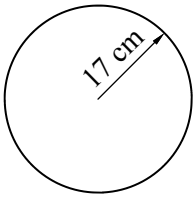
Write formula

$= 2 \times \pi \times 17$

Substitute value for r

$C = 106.8$ cm

Answer 1 d.p., show units



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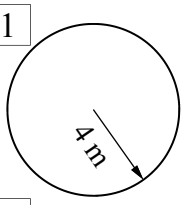

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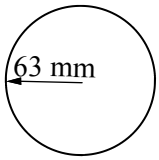

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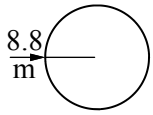

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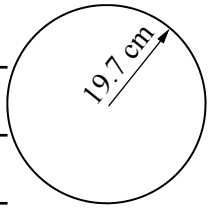

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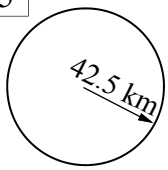

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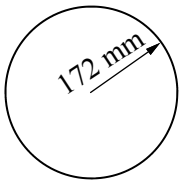

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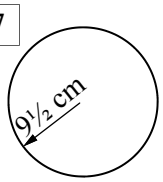
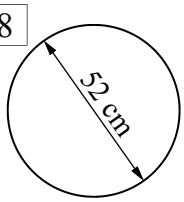


These use diameter. Either divide by 2 for the radius *OR* use $C=\pi d$. Answer to 2 d.p.

8



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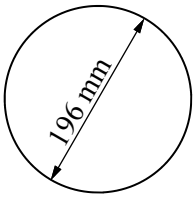

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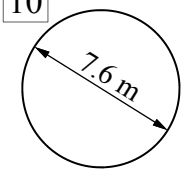

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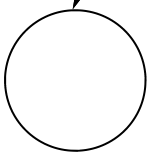
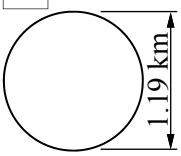


*ϕ = 'phi' = d

12



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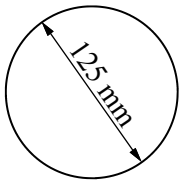
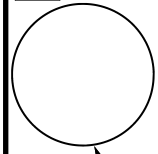
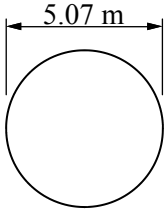


14



15

$\phi = 7.89$ m



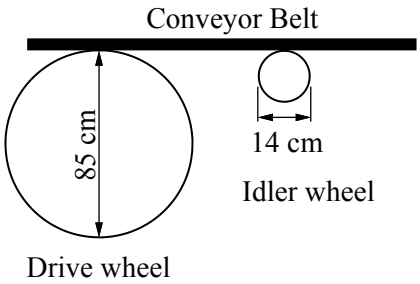
Now apply your skills to these problems. Answer to 2 d.p. where appropriate.

16 A car tyre has a diameter of 63 cm. Find its circumference. Give your answer in metres.

17 How far does the tyre travel when it rotates 150 times?

18 How many complete rotations of the tyre will occur if 1 km is travelled?

19 Calculate the amount of conveyor belt fed by 1 rotation of the drive wheel.



20 1 rotation of the idler wheel

21 Complete:

$$\frac{C_{(\text{Drive})}}{C_{(\text{Idler})}} = \frac{\square}{\square} = \_\_\_\_$$

Circumference (Sheet 1)

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Use $C=2\pi r$ to find the circumference of these circles to 1 decimal place.

Example

$C = 2\pi r$

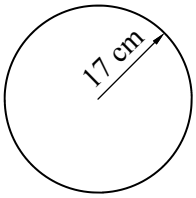
Write formula

$= 2 \times \pi \times 17$

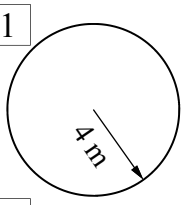
Substitute value for r

$C = 106.8$ cm

Answer 1 d.p., show units



1



$C = 2\pi r$

$= 2 \times \pi \times 4$

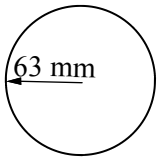
$C = 25.1$ m

2

$C = 2\pi r$

$= 2 \times \pi \times 63$

$C = 395.8$ mm



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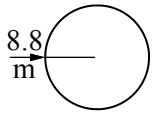


$C = 2\pi r$

$= 2 \times \pi \times 8.8$

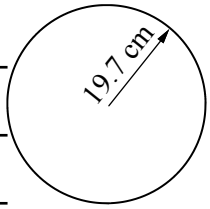
$C = 55.3$ m

4

$C = 2\pi r$

$= 2 \times \pi \times 19.7$

$C = 123.8$ cm



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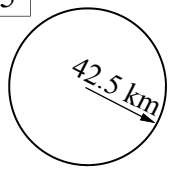


$C = 2\pi r$

$= 2 \times \pi \times 42.5$

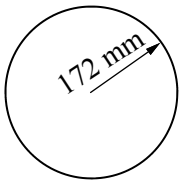
$C = 267$ km

6

$C = 2\pi r$

$= 2 \times \pi \times 172$

$C = 1\,080.7$ mm



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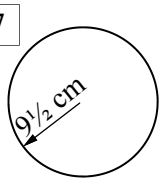


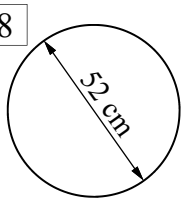
$C = 2\pi r$

$= 2 \times \pi \times 9.5$

$C = 59.7$ cm

These use diameter. Either divide by 2 for the radius *OR* use $C=\pi d$. Answer to 2 d.p.

8



$C = \pi d$

$= \pi \times 52$

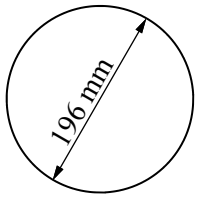
$C = 163.36$ cm

9

$C = \pi d$

$= \pi \times 196$

$C = 615.75$ mm



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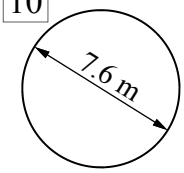


$C = \pi d$

$= \pi \times 7.6$

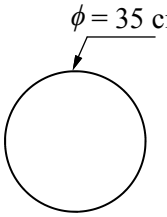
$C = 23.88$ m

11

$C = \pi d$

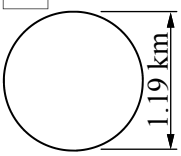
$= \pi \times 35$

$C = 109.96$ cm



* ϕ = 'phi' = d

12



$C = \pi d$

$= \pi \times 1.19$

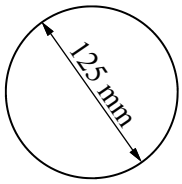
$C = 3.74$ km

13

$C = \pi d$

$= \pi \times 125$

$C = 392.70$ mm



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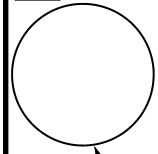


$C = \pi d$

$= \pi \times 7.89$

$C = 24.79$ m

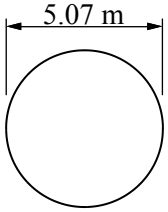
15

$\phi = 7.89$ m

$C = \pi d$

$= \pi \times 5.07$

$C = 15.93$ m



Now apply your skills to these problems. Answer to 2 d.p. where appropriate.

16 A car tyre has a diameter of 63 cm. Find its circumference. Give your answer in metres.

$C = \pi d$

$= \pi \times 0.63$

$C = 1.98$ m

17 How far does the tyre travel when it rotates 150 times?

Distance $= 150 \times 1.98$

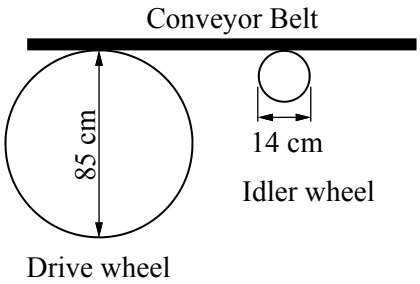
Distance $= 297$ m (296.88 m If 16 unrounded)

18 How many complete rotations of the tyre will occur if 1 km is travelled?

Rotations $= 1\,000 \div 1.98$

Rotations $= 505$

19 Calculate the amount of conveyor belt fed by 1 rotation of the drive wheel.



$C = \pi d$

$= \pi \times 85$

$C = 267.04$ cm

20 1 rotation of the idler wheel

$C = \pi d$

$= \pi \times 14$

$C = 43.98$ cm

21 Complete:

$$\frac{C_{(\text{Drive})}}{C_{(\text{Idler})}} = \frac{267.04}{43.98} = 6.07$$

Circumference (Sheet 2)

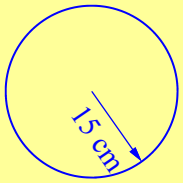
The circumference of a circle is its perimeter, the distance around the outside of the circle. This distance is found by using this formula:

$$C = 2\pi r$$

Where: C = Circumference of circle
r = radius of the circle

The usual process is to use your π key, but there are two common approximations for π. These are $\pi = 3.14$ (as a decimal) and $\pi = {}^{22}/_{7}$ (as a fraction).

Example - $\pi = 3.14$

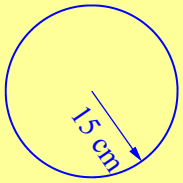


$C = 2\pi r$ ← Write the formula.

$= 2 \times 3.14 \times 15$ ← Show × signs and replace π with 3.14 and r with the radius .

$C = 94.2$ cm (1 d.p.) ← Calculate, round and show units.

Example - $\pi = {}^{22}/_{7}$



$C = 2\pi r$ ← Write the formula.

$= 2 \times {}^{22}/_{7} \times 15$ ← Show × signs and replace π with $^{22}/_{7}$ and r with the radius .

$C = 94.3$ cm (1 d.p.) ← Calculate, round and show units.

Column 1 uses the above process for all questions.

Column 2 gives you a diameter rather than a radius. Use:

$$C = \pi d$$

Where: C = Circumference of circle
d = diameter of the circle (note that $d = 2r$)

There are two tables at the base of the column, just write the answer with units, no working out required.

Column 3 finds the diameter (and later radius) given the circumference. If you divide the circumference by π then you find the diameter. If you halve this answer you get the radius.

$$C = \pi d$$
$$\div \pi \quad \div \pi$$
$$\frac{C}{\pi} = d$$
$$d = \frac{C}{\pi}$$

Circumference (Sheet 2)

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Use $C=2\pi r$ with $\pi = 3.14$, to find the circumference. Round your answer to 1 decimal place.

Example

$C = 2\pi r$

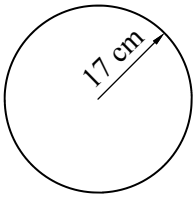
Write formula

$= 2 \times 3.14 \times 17$

Substitute 3.14 and r

$C = 106.8$ cm

Answer 1 d.p., show units



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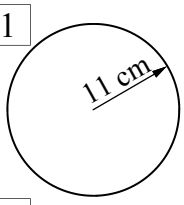

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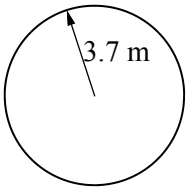

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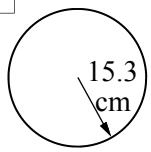
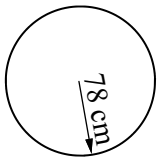


Now use $\pi = {}^{22}/_{7}$, to find the circumference. Round to the nearest whole number.

4



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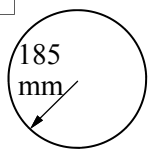

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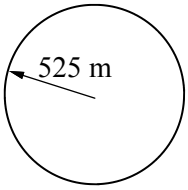
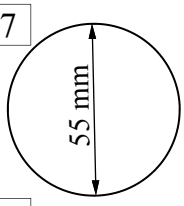


Now use $C = \pi d$ & $\pi = 3.14$, find the circumference and round to 1 decimal place.

7



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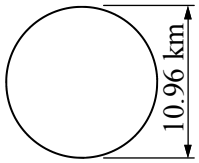

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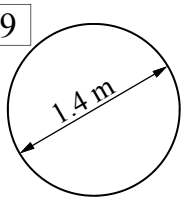

Now use $C = \pi d$ & $\pi = {}^{22}/_{7}$, to complete the table to 1 dp No working out required.

Q.	d	Circumference
10	23 cm	
11	83.2 m	
12	19.6 mm	
13	16.07 km	
14	208 m	
15	485 km	

Now use $C = \pi d$ & the π key on your calculator, to complete the table to 1 d.p.

Q.	d	Circumference
16	5.6 m	
17	0.9 km	
18	93 mm	
19	408 m	
20	317 mm	
21	13.68 km	
22	80.1 cm	

This time you are given the circumference. Divide C by π (π key) to find d to 1 d.p.

23 Find d if C = 27.3 cm.

$d = \frac{C}{\pi} = \frac{\square}{\pi}$ $d =$ _____

24 Find d if C = 1.6 km.

$d = \frac{C}{\pi} = \frac{\square}{\pi}$ $d =$ _____

25 Find d if C = 192.3 mm.

A cylindrical spool spins 400 times and gathers 125 m of thread. Find, to 2 d.p. :

26 The amount gathered in 1 revolution. Answer in cm.

27 Find the spool's diameter.

$d \div 2 = r$

This time you want the radius. Divide C by π to get d but then halve your answer to find r. Answer to one decimal place.

Q.	C	Radius (r)
28	17 m	
29	85 mm	
30	116 cm	
31	70.4 m	
32	41 km	
33	250 mm	
34	60 cm	
35	782 mm	
36	98 m	
37	410 cm	

Circumference (Sheet 2)

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Use $C=2\pi r$ with $\pi = 3.14$, to find the circumference. Round your answer to 1 decimal place.

Example

$C = 2\pi r$

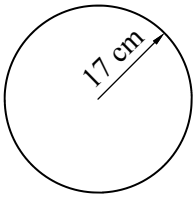
Write formula

$= 2 \times 3.14 \times 17$

Substitute 3.14 and r

$C = 106.8$ cm

Answer 1 d.p., show units



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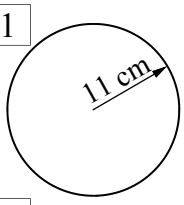
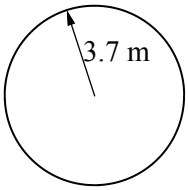


$C = 2\pi r$

$= 2 \times 3.14 \times 11$

$C = 69.1$ cm

2

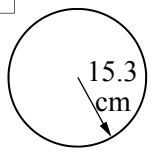


$C = 2\pi r$

$= 2 \times 3.14 \times 3.7$

$C = 23.2$ m

3



$C = 2\pi r$

$= 2 \times 3.14 \times 15.3$

$C = 96.1$ cm

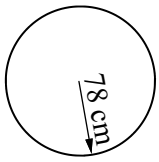
Now use $\pi = {}^{22}/_{7}$, to find the circumference. Round to the nearest whole number.

4

$C = 2\pi r$

$= 2 \times {}^{22}/_{7} \times 78$

$C = 490$ cm



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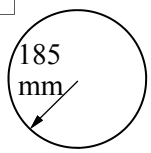


$C = 2\pi r$

$= 2 \times {}^{22}/_{7} \times 185$

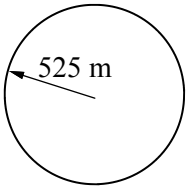
$C = 1\,163$ mm

6

$C = 2\pi r$

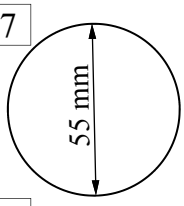
$= 2 \times {}^{22}/_{7} \times 525$

$C = 3\,300$ m



Now use $C = \pi d$ & $\pi = 3.14$, find the circumference and round to 1 decimal place.

7



$C = \pi d$

$= 3.14 \times 55$

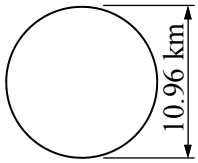
$C = 172.7$ mm

8

$C = \pi d$

$= 3.14 \times 10.96$

$C = 34.4$ km



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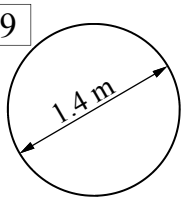


$C = \pi d$

$= 3.14 \times 1.4$

$C = 4.4$ m

Now use $C = \pi d$ & $\pi = {}^{22}/_{7}$, to complete the table to 1 dp No working out required.

Q.	d	Circumference
10	23 cm	72.3 cm
11	83.2 m	261.5 m
12	19.6 mm	61.6 mm
13	16.07 km	50.5 km
14	208 m	653.7 m
15	485 km	1 524.3 km

Now use $C = \pi d$ & the π key on your calculator, to complete the table to 1 d.p.

Q.	d	Circumference
16	5.6 m	17.6 m
17	0.9 km	2.8 km
18	93 mm	292.2 mm
19	408 m	1 281.8 m
20	317 mm	995.9 mm
21	13.68 km	43.0 km
22	80.1 cm	251.6 cm

This time you are given the circumference. Divide C by π (π key) to find d to 1 d.p.

23 Find d if C = 27.3 cm.

$d = \frac{C}{\pi} = \frac{27.3}{\pi}$ $d =$ 8.7 cm

24 Find d if C = 1.6 km.

$d = \frac{C}{\pi} = \frac{1.6}{\pi}$ $d =$ 0.5 km

25 Find d if C = 192.3 mm.

$d = \frac{C}{\pi} = \frac{192.3}{\pi}$ $d =$ 61.2 mm

A cylindrical spool spins 400 times and gathers 125 m of thread. Find, to 2 d.p. :

26 The amount gathered in 1 revolution. Answer in cm.

$125 \div 400 = 0.3125$ m $= 31.25$ cm

27 Find the spool's diameter.

$d = \frac{C}{\pi} = \frac{31.25}{\pi}$ $d =$ 9.95 cm

$d \div 2 = r$

This time you want the radius. Divide C by π to get d but then halve your answer to find r. Answer to one decimal place.

Q.	C	Radius (r)
28	17 m	2.7 cm
29	85 mm	13.5 mm
30	116 cm	18.5 cm
31	70.4 m	11.2 m
32	41 km	6.5 km
33	250 mm	39.8 mm
34	60 cm	9.5 cm
35	782 mm	124.5 mm
36	98 m	15.6 m
37	410 cm	65.3 cm

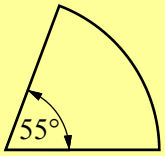
Fractions of a Circle and Arc Length

This sheet finds the length of an arc of a sector. The arc is the curved length which is a fraction of a whole circle's circumference. So in other words, this sheet deals with the circumference of a circle but multiplied by a fraction.

Column 1 starts with sectors (including quadrants and semi-circles) and finding the fraction of the circle that they represent. This requires you to identify the internal angle and write it 'over' 360°. That answers the question but the next step is to simplify the fraction. This can be done in your head by dividing by the LCM, or using a calculator.

Example

$\frac{55}{360} = \frac{11}{72}$ (÷ 5 above and below)



Calculator 5 5 $a^b/_c$ 3 6 0 =

Answer on Screen 11⌟72 But you write $\frac{11}{72}$

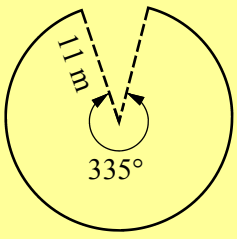
Column 2 relates the fraction to the arc length. The circumference of a circle is $C = 2\pi r$, but as these are sectors then the arc length is a fraction of the circumference (if the circle was whole).

$$l = \frac{\theta}{360} \times 2\pi r$$

Where:

l = the arc length (curved length)
θ = the internal angle of the sector
r = radius of sector

Example



$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{335}{360} \times 2 \times \pi \times 11$

$l = 64.3$ m (1 d.p.)

Column 3 asks you to find the perimeter of the sectors. So the method is just the same as above, but then add on the straight lengths as well. The straight lines are the radii, there are two of them. So work out the arc length, add two radii and you are done.

$$l = \frac{\theta}{360} \times 2\pi r + 2r$$

Fractions of a Circle and Arc Length

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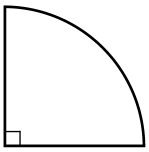
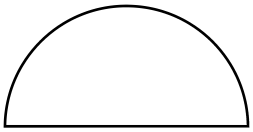
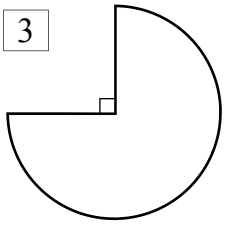
Calculate the fraction of a circle shown by these sectors, simplify if possible.

1 

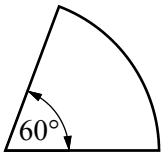
$\frac{\square}{360} = \frac{\square}{4}$

2 

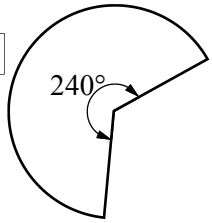
$\frac{\square}{360} = \frac{\square}{\square}$

3 

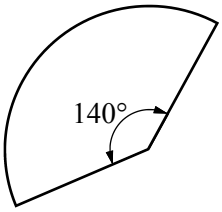
$\frac{\square}{\square} = \frac{\square}{\square}$

4 

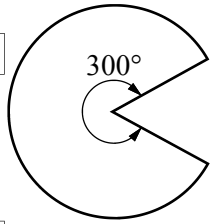
$\frac{\square}{\square} = \frac{\square}{\square}$

5 

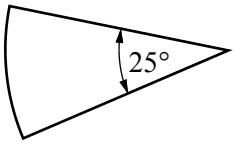
$\frac{\square}{\square} = \frac{\square}{\square}$

6 

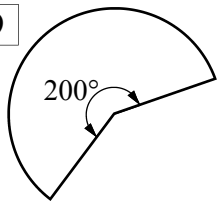
$\frac{\square}{\square} = \frac{\square}{\square}$

7 

$\frac{\square}{\square} = \frac{\square}{\square}$

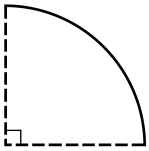
8 

$\frac{\square}{\square} = \frac{\square}{\square}$

9 

$\frac{\square}{\square} = \frac{\square}{\square}$

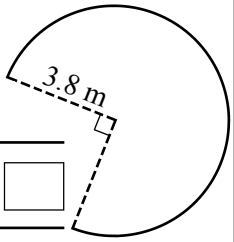
Find the fraction of a circle and multiply by $2\pi r$ to find the arc length 'l' to 1 d.p.

10 

$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{\square}{360} \times 2 \times \pi \times \square$

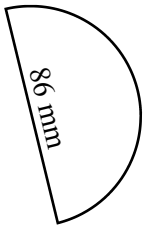
$l =$ cm

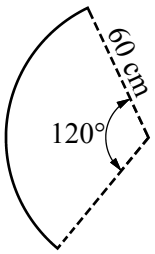
11 

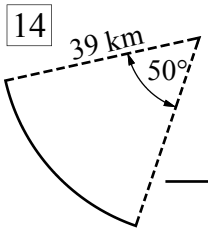
$l = \frac{\theta}{360} \times 2\pi r$

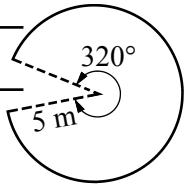
$= \frac{\square}{\square} \times 2 \times \pi \times \square$

$l =$

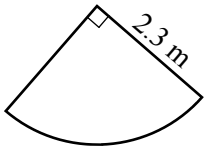
12 

13 

14 

15 

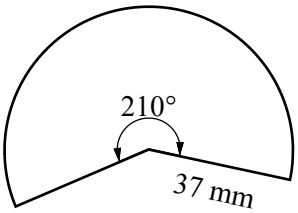
Now find the perimeter, to 1 d.p. Include the radii.

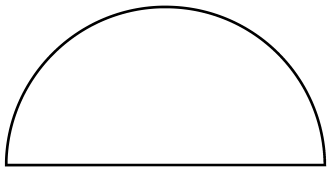
16 

$l = \frac{\theta}{360} \times 2\pi r + 2r$

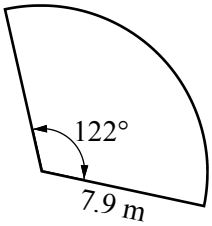
$= \frac{\square}{360} \times 2 \times \pi \times \square + 2 \times \square$

$l =$

17 

18 

13.2 cm

19 

Fractions of a Circle and Arc Length

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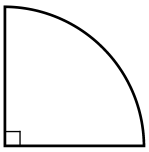
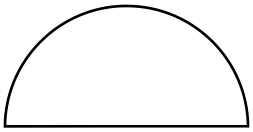
Calculate the fraction of a circle shown by these sectors, simplify if possible.

1



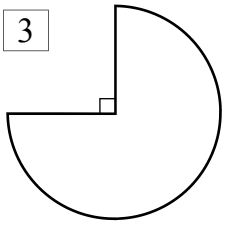
$\frac{90}{360} = \frac{1}{4}$

2



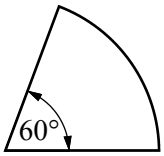
$\frac{180}{360} = \frac{1}{2}$

3



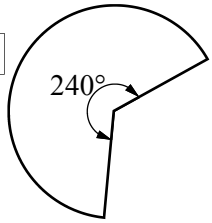
$\frac{270}{360} = \frac{3}{4}$

4



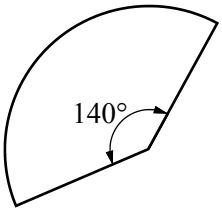
$\frac{60}{360} = \frac{1}{6}$

5



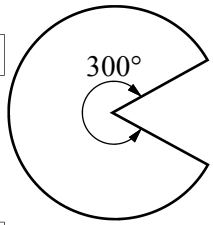
$\frac{240}{360} = \frac{1}{3}$

6



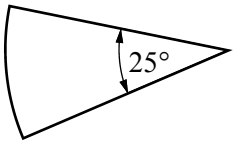
$\frac{140}{360} = \frac{7}{18}$

7



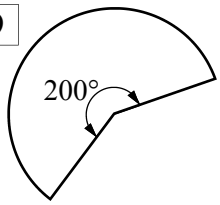
$\frac{300}{360} = \frac{5}{6}$

8



$\frac{25}{360} = \frac{5}{72}$

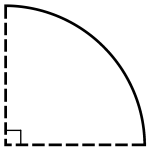
9



$\frac{200}{360} = \frac{5}{9}$

Find the fraction of a circle and multiply by $2\pi r$ to find the arc length 'l' to 1 d.p.

10

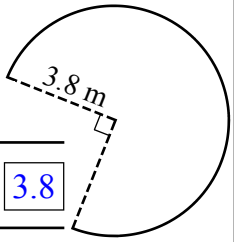


$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{90}{360} \times 2 \times \pi \times 43$

$l = 67.5$ cm

11

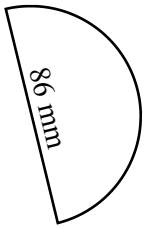


$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{270}{360} \times 2 \times \pi \times 3.8$

$l = 17.9$ m

12

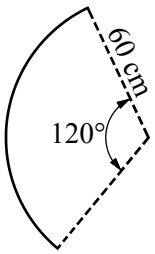


$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{180}{360} \times 2 \times \pi \times 86$

$l = 270.2$ mm

13

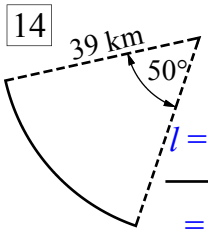


$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{120}{360} \times 2 \times \pi \times 60$

$l = 125.7$ cm

14



$l = \frac{\theta}{360} \times 2\pi r$

$= \frac{50}{360} \times 2 \times \pi \times 39$

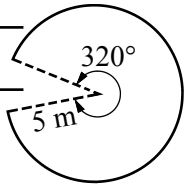
$l = 34$ km

15

$l = \frac{\theta}{360} \times 2\pi r$

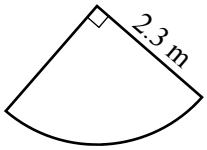
$= \frac{320}{360} \times 2 \times \pi \times 5$

$l = 27.9$ m



Now find the perimeter, to 1 d.p. Include the radii.

16

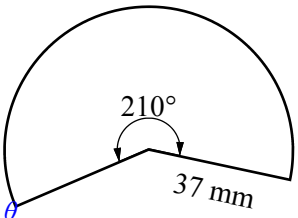


$l = \frac{\theta}{360} \times 2\pi r + 2r$

$= \frac{90}{360} \times 2 \times \pi \times 2.3 + 2 \times 2.3$

$l = 8.2$ m

17

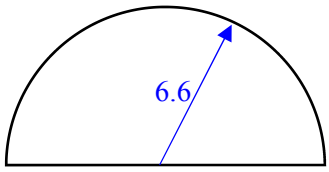


$l = \frac{\theta}{360} \times 2\pi r + 2r$

$= \frac{210}{360} \times 2 \times \pi \times 37 + 2 \times 37$

$l = 209.6$ mm

18

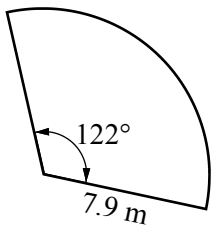


$l = \frac{\theta}{360} \times 2\pi r + d$

$= \frac{180}{360} \times 2 \times \pi \times 6.6 + 13.2$

$l = 33.9$ cm

19



$l = \frac{\theta}{360} \times 2\pi r + 2r$

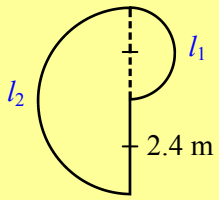
$= \frac{122}{360} \times 2 \times \pi \times 7.9 + 2 \times 7.9$

$l = 32.6$ m

Perimeter of Composite Shapes

This sheet shouldn't be attempted before the previous sheet on Arc Length is attempted. The circumference of a circle is found by $C = 2\pi r$ or $C = \pi d$. But with composite shapes parts of a circle are involved with other shapes/lengths, so you need to use fractions to find the arc length, then add that length to the other sides of the shape.

Example - Finding the perimeter to 1 d.p.



Label l_1 on the diagram and calculate the arc length.

$l_1 = {}^1/_2 \times \pi d$

$= {}^1/_2 \times \pi \times 2.4$

$l_1 = 3.8$ m

$l_2 = {}^1/_2 \times \pi d$

$= {}^1/_2 \times \pi \times 4.8$

$l_2 = 7.5$ m

Label l_2 on the diagram and calculate the arc length.

Add the three lengths. That is the two arc lengths and the 2.4 m straight length.

$P = 3.8 + 7.5 + 2.4$

$P = 13.7$ m

This sheet is answered by the above method, except sometimes quadrants are involved. So instead of using ${}^1/_2$ you will use ${}^1/_4$ to find the arc length.

Column 3 has the last two questions requiring an extra step, these require Pythagoras Theorem before solving.

Perimeter of Composite Shapes

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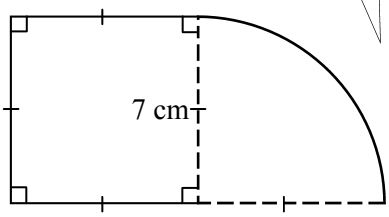
Find the perimeter to 1 d.p.

1



$l = {}^1/_4 \times 2\pi r$

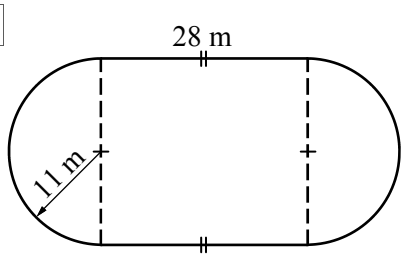
$= {}^1/_4 \times 2 \times \pi \times$ ☐

$l =$ ☐ cm (1 d.p.)

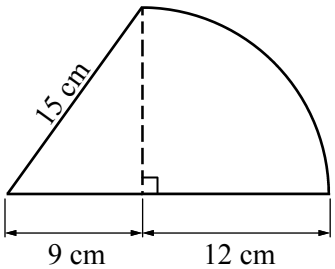
P = ☐ + 4 × ☐

P = ☐ cm

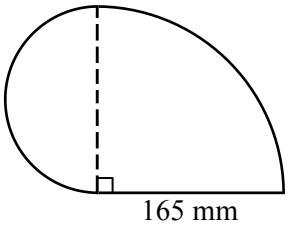
2



3



4



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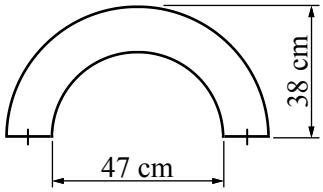

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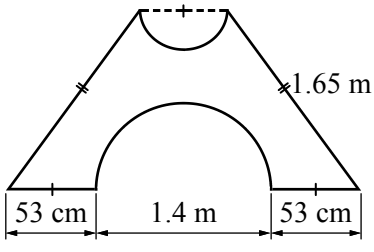

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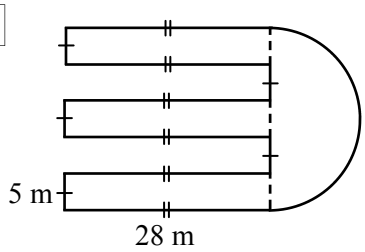
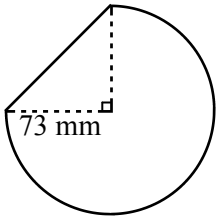


Pythagoras required, round to 2 d.p.

8



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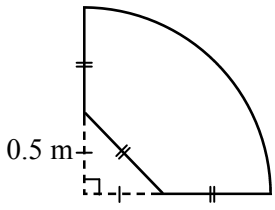

Perimeter of Composite Shapes

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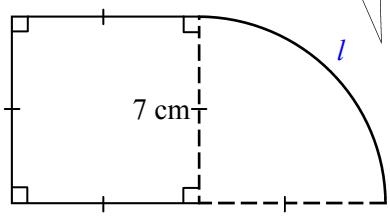
Find the perimeter to 1 d.p.

1



$l = {}^1/_4 \times 2\pi r$

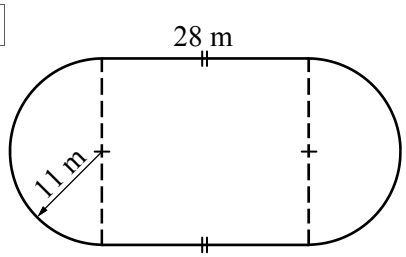
$= {}^1/_4 \times 2 \times \pi \times$ 7

$l =$ 11 cm (1 d.p.)

P = 11 + 4 × 7

P = 39 cm

2



$C = 2\pi r$

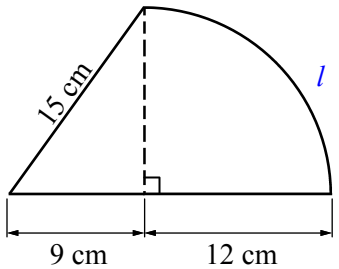
$= 2 \times \pi \times 11$

C = 69.1 m (1 d.p.)

P = 69.1 + 2 × 28

P = 125.1 m

3



$l = {}^1/_4 \times 2\pi r$

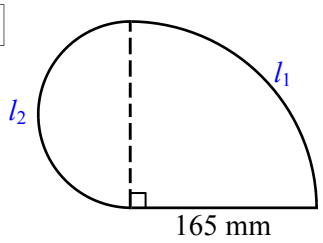
$= {}^1/_4 \times 2 \times \pi \times 12$

l = 18.8 cm (1 d.p.)

P = 18.8 + 15 + 9 + 12

P = 54.8 cm

4



$l_1 = {}^1/_4 \times 2\pi r$ $l_2 = {}^1/_2 \times 2\pi r$

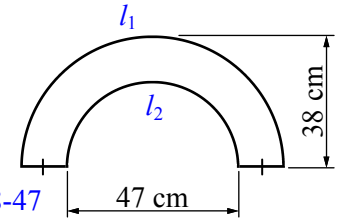
$= {}^1/_4 \times 2 \times \pi \times 165$ $= {}^1/_2 \times 2 \times \pi \times 82.5$

l_1 = 259.2 mm l_2 = 259.2 mm

P = 2 × 259.2 + 165

P = 683.4 mm

5



2×38-47 = 29

$l_1 = {}^1/_2 \times 2\pi r$ $l_2 = {}^1/_2 \times \pi d$

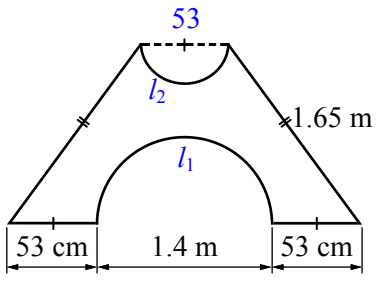
$= {}^1/_2 \times 2 \times \pi \times 38$ $= {}^1/_2 \times \pi \times 47$

l_1 = 119.4 cm l_2 = 73.8 cm

P = 119.4 + 73.8 + 29

P = 222.2 cm

6



$l_1 = {}^1/_2 \times \pi d$ $l_2 = {}^1/_2 \times \pi d$

$= {}^1/_2 \times \pi \times 1.4$ $= {}^1/_2 \times \pi \times 0.53$

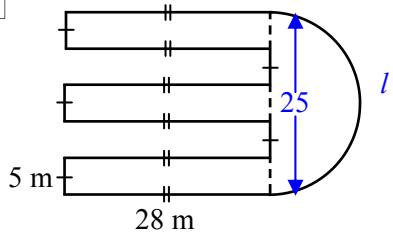
l_1 = 2.2 m (1 d.p) l_2 = 0.8 m (1 d.p)

P = 2×0.53 + 2.2 + 2×1.65 + 0.8

P = 7.4 m (1 d.p) or

P = 739.2 cm

7



$l = {}^1/_2 \times \pi d$

$= {}^1/_2 \times \pi \times 25$

l = 39.27 m

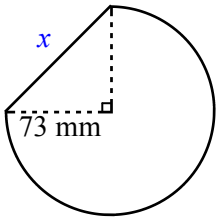
P = 39.27 + 3 × (5 + 2 × 28) + 10

P = 39.27 + 183 + 10

P = 232.27 m

Pythagoras required, round to 2 d.p.

8



$x^2 = 73^2 + 73^2$ $l = {}^3/_4 \times 2\pi r$

x^2 = 10 658 $= {}^3/_4 \times 2 \times \pi \times 73$

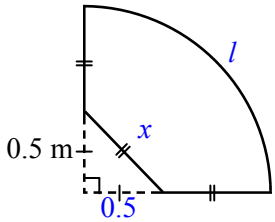
$x = \sqrt{10\,658}$ l = 344 mm

x = 103.24 mm

P = 103.24 + 344

P = 447.24 mm

9



$x^2 = 0.5^2 + 0.5^2$ r=0.5+0.71=1.21

$x^2 = 0.5$ $l = {}^1/_4 \times 2\pi r$

$x = \sqrt{0.5}$ $= {}^1/_4 \times 2 \times \pi \times 1.21$

x = 0.71 m l = 1.9 m

P = 1.9 + 3 × 0.71

P = 4.03 m

Perimeter Problems

These problems are an extension of the exercises covered earlier with an inclusion of cost. A common use of perimeter is in the calculation of cost of fences and walls. To calculate this cost calculate the perimeter first, then multiply the answer by the cost of the fence/wall by its unit cost, (the cost per metre).

In all the problems two lines of working are expected. The first line being the sum of the side lengths:
P = _ + _ + _ + etc

the second line being the total length and the units
P = _____ m (or cm).

Perimeter Problems

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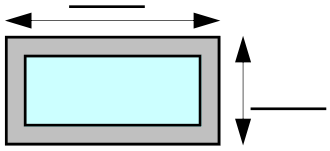
Answer the perimeter problems below.

1 A rectangular swimming pool has sides with lengths 12 m and 4 m. Find the distance around the pool.

2 A path 1 m wide surrounds the same pool (Q1) calculate the length and breadth of the pool and the path combined, put the dimensions on the diagram below and find the perimeter



3 If the pool above has a safety fence around the outside of the path find the cost of fencing the pool when the fence costs $27 per metre. ×

4 In a science experiment Travis noted that on average an ant stopped every 5 cm. If the viewing tray was 30 cm by 20 cm, find its perimeter.

How many times would the ant stop:

i) in 1 lap

ii) in 5 laps

iii) over a 9 m distance

5 Three hobby farmers are each given small garden areas. The plots all occupy the same land area (16 m²) but have different side lengths, find the perimeter of each.

Plot 1 : 16 m by 1 m

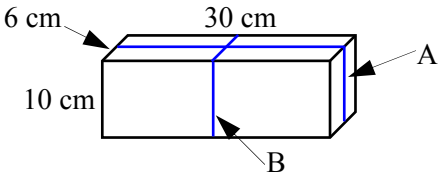
Plot 2 : 8 m by 2 m

Plot 3 : 4 m by 4 m

6 If you were offered a plot and had to pay to fence the area, would it be cheaper to have a square area or a rectangular area?

Circle : square or rectangle

7 A parcel has 2 strings around it as shown below. Calculate the length of the string A and string B, then add them to find the total length.



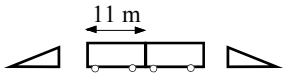
A:

B:

8 Scott says that when you double the side length of a square you get 4 times the perimeter, is he right?

For Not a

9 Dare-devil Dan is about to jump 2 buses, lined up end to end (see below). Dan is superstitious and walks 1 lap around the buses before the jump, how far does he walk?



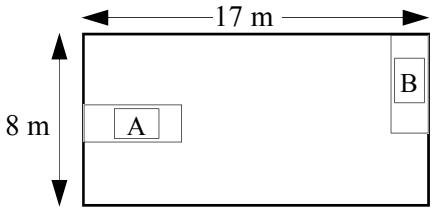
The buses are 11 m long and 3 m wide

10 An inventor created a synthetic material that will stretch to 18 times its length before breaking. If the original length is 7 m, find its maximum length. Will it stretch around a rectangular building with sides 25 m and 35 m? (Find P then circle answer)

Max length ×

Yes / No

11 Leon's dog 'Oblong' always runs around the perimeter of the backyard when he sees Leon arrive home. Find the distance he runs around the empty yard. Then calculate the distances he runs when Leon's car is in position A or in position B, (not both together). Leon's car is 2 m wide and 5 m long.



Position A is in the centre of the yard

No car:

Car A :

Car B :

Perimeter Problems

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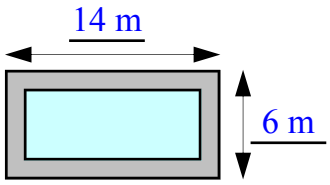
Answer the perimeter problems below.

1 A rectangular swimming pool has sides with lengths 12 m and 4 m. Find the distance around the pool.

$P = 12 + 12 + 4 + 4$

$P = 32$ m

2 A path 1 m wide surrounds the same pool (Q1) calculate the length and breadth of the pool and the path combined, put the dimensions on the diagram below and find the perimeter



$P = 14 + 6 + 14 + 6$

$P = 40$ m

3 If the pool above has a safety fence around the outside of the path find the cost of fencing the pool when the fence costs $27 per metre.

$\text{Cost} = 40 \times 27$

$\text{Cost} = \$1\,080$

$$\begin{array}{r} {\scriptstyle 2} \\ 27 \\ \times\ 40 \\ \hline 1\,080 \end{array}$$

4 In a science experiment Travis noted that on average an ant stopped every 5 cm. If the viewing tray was 30 cm by 20 cm, find its perimeter.

$P = 30 + 30 + 20 + 20$

$P = 100$ cm

How many times would the ant stop:

i) in 1 lap — 20 times

ii) in 5 laps — 100 times

iii) over a 9 m distance — 180 times

5 Three hobby farmers are each given small garden areas. The plots all occupy the same land area (16 m²) but have different side lengths, find the perimeter of each.

Plot 1 : 16 m by 1 m

$P = 16 + 16 + 1 + 1$

$P = 34$ m

Plot 2 : 8 m by 2 m

$P = 8 + 8 + 2 + 2$

$P = 20$ m

Plot 3 : 4 m by 4 m

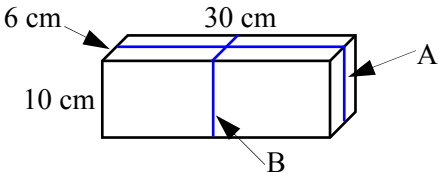
$P = 4 + 4 + 4 + 4$

$P = 16$ m

6 If you were offered a plot and had to pay to fence the area, would it be cheaper to have a square area or a rectangular area?

Circle : (square) or rectangle

7 A parcel has 2 strings around it as shown below. Calculate the length of the string A and string B, then add them to find the total length.



A: $P = 30 + 30 + 10 + 10$

$P = 80$ cm

B: $P = 10 + 10 + 6 + 6$

$P = 32$ cm

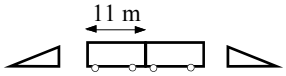
$P = 80 + 32$

$P = 112$ cm

8 Scott says that when you double the side length of a square you get 4 times the perimeter, is he right?

For ○ Not a ●

9 Dare-devil Dan is about to jump 2 buses, lined up end to end (see below). Dan is superstitious and walks 1 lap around the buses before the jump, how far does he walk?



The buses are 11 m long and 3 m wide

$P = 11 + 11 + 3 + 11 + 11 + 3$

$P = 50$ m

10 An inventor created a synthetic material that will stretch to 18 times its length before breaking. If the original length is 7 m, find its maximum length. Will it stretch around a rectangular building with sides 25 m and 35 m? (Find P then circle answer)

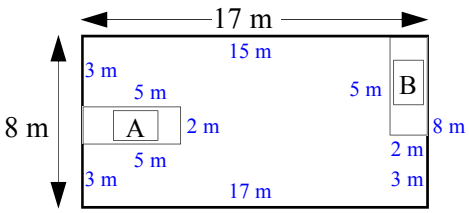
Max length 126 m

$P = 35 + 35 + 25 + 25$

$P = 120$ m (Yes) / No

$$\begin{array}{r} {\scriptstyle 5} \\ 18 \\ \times\ 7 \\ \hline 126 \end{array}$$

11 Leon's dog 'Oblong' always runs around the perimeter of the backyard when he sees Leon arrive home. Find the distance he runs around the empty yard. Then calculate the distances he runs when Leon's car is in position A or in position B, (not both together). Leon's car is 2 m wide and 5 m long.



Position A is in the centre of the yard

No car: $P = 17 + 17 + 8 + 8$

$P = 50$ m

Car A :

$P = 17 + 3 + 5 + 2 + 5 + 3 + 17 + 8$

$P = 60$ m

Car B :

$P = 17 + 8 + 15 + 5 + 2 + 3$

$P = 50$ m

Area and Perimeter Problems

These are problem based questions based on circles or composite solids with circular elements. So before attempting this sheet ensure you have covered composite areas.

Column 1 are area questions, so use $A = \pi r^2$ to find the area, then apply other skills to solve the questions.

Column 2 are circumference questions, so use $C = \pi d$ *or* $C = 2\pi r$ to find the circumference or use fractions when arcs are involved.

As these questions require earlier exercises to be completed first, no examples have been included.

Area and Perimeter Problems

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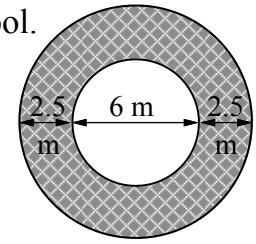
Use $A = \pi r^2$ to find the area and then apply your skills to answer these. Answer to 1 decimal place when needed.

A circular pool shown below in plan view has a diameter of 6 m. The pool fence encloses a 2.5 m wide paved area around the pool. Find:

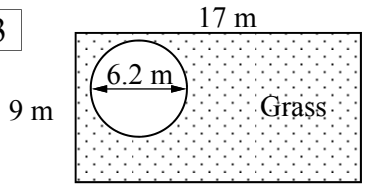
1 The area occupied by the pool.



2 The area occupied by the tiles.

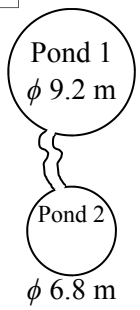
Tom has built a 6.2 m diameter pergola in his backyard. The rest of his backyard is covered by grass. Calculate the percentage of his backyard that has to be mowed.

3



Laura is constructing a wastewater system using two round settling ponds. Pond 1 is to have reeds covering 80% of its area, Pond 2 is to have 34% of its area covered with reeds. Calculate the amount of reeds (in m^2) required.

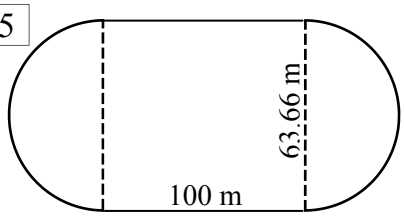
4



Use $C = 2\pi r$ or $C = \pi d$ to find the circle's circumference then apply your skills to answer these. Answer to 1 decimal place.

Calculate the distance around the track field shown below.

5

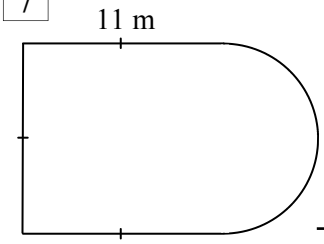


If Eve can run at an average speed of 3 m/s calculate the time, in minutes and seconds, that it would take her to complete one lap of the track above.

6

A tiger exhibit at the zoo is shown below. The curved length is a glass viewing wall costing $1 450 per metre. While the remaining three walls are concrete and cost $845 per metre. Find the cost of constructing the walls.

7



Jen's hamster 'Spartacus' runs in his 30 cm diameter exercise wheel. Jen has fitted a counter to the wheel and finds that Spartacus completed 800 revolutions one day, 560 the next and 725 on the third day. Find, in m, the:

8 Total distance Spartacus covers in three days.

9 Average distance travelled per day.

Area and Perimeter Problems

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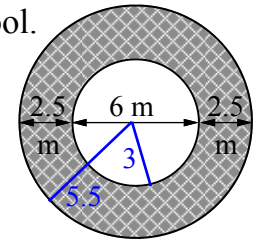
Use $A = \pi r^2$ to find the area and then apply your skills to answer these. Answer to 1 decimal place when needed.

A circular pool shown below in plan view has a diameter of 6 m. The pool fence encloses a 2.5 m wide paved area around the pool. Find:

1 The area occupied by the pool.



$A = \pi r^2$

$= \pi \times 3^2$

$A = 28.3 \text{ m}^2$

2 The area occupied by the tiles.

$A = \pi r^2$

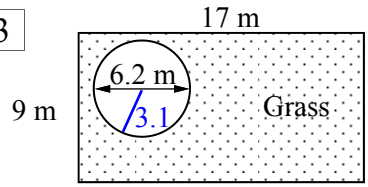
$= \pi \times 5.5^2$

$A = 95 \text{ m}^2$

$A = 95 - 28.3 = 66.7 \text{ m}^2$

Tom has built a 6.2 m diameter pergola in his backyard. The rest of his backyard is covered by grass. Calculate the percentage of his backyard that has to be mowed.

3



$A = \pi r^2$

$= \pi \times 3.1^2$

$A = 30.2 \text{ m}^2$

$A = lb$

$= 17 \times 9$

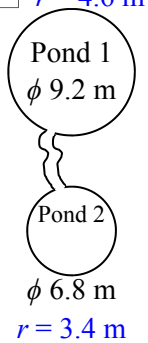
$A = 153 \text{ m}^2$

$\%\text{ Mowed} = \frac{30.2}{153} \times 100$

$\%\text{ Mowed} = 19.7\%$

Laura is constructing a wastewater system using two round settling ponds. Pond 1 is to have reeds covering 80% of its area, Pond 2 is to have 34% of its area covered with reeds. Calculate the amount of reeds (in m^2) required.

4



$r = 4.6$ m

$r = 3.4$ m

$A_1 = \pi r^2$

$= \pi \times 4.6^2$

$A_1 = 66.5 \text{ m}^2$

$A_2 = \pi r^2$

$= \pi \times 3.4^2$

$A_2 = 36.3 \text{ m}^2$

$\text{Pond 1} = 66.5 \times 0.80$

$\text{Pond 1} = 53.2 \text{ m}^2$

$\text{Pond 2} = 36.3 \times 0.34$

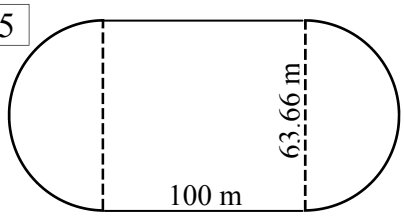
$\text{Pond 2} = 12.3 \text{ m}^2$

$\text{Total Reeds} = 53.2 + 12.3 = 65.5 \text{ m}^2$

Use $C = 2\pi r$ or $C = \pi d$ to find the circle's circumference then apply your skills to answer these. Answer to 1 decimal place.

Calculate the distance around the track field shown below.

5



$C = \pi d$

$= \pi \times 63.66$

$C = 200 \text{ m}$

$\text{Distance} = 200 + 2 \times 100$

$\text{Distance} = 400 \text{ m}$

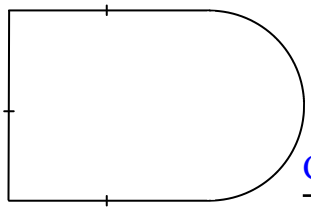
If Eve can run at an average speed of 3 m/s calculate the time, in minutes and seconds, that it would take her to complete one lap of the track above.

6 $\text{Time} = \frac{400}{3}$

$\text{Time} = 133.\dot{3} \text{ secs} = 2 \text{ m } 13 \text{ s}$

A tiger exhibit at the zoo is shown below. The curved length is a glass viewing wall costing \$1 450 per metre. While the remaining three walls are concrete and cost \$845 per metre. Find the cost of constructing the walls.

7



$l = \frac{1}{2}\pi d$

$= \frac{1}{2} \times \pi \times 11$

$l = 17.3 \text{ m}$

$\text{Cost}_{\text{GLASS}} = 17.3 \times 1\,450$

$\text{Cost}_{\text{GLASS}} = \$25\,085$

$\text{Cost}_{\text{CONCRETE}} = 11 \times 3 \times 845$

$\text{Cost}_{\text{CONCRETE}} = \$27\,885$

Jen's hamster 'Spartacus' runs in his 30 cm diameter exercise wheel. Jen has fitted a counter to the wheel and finds that Spartacus completed 800 revolutions one day, 560 the next and 725 on the third day. Find, in m, the:

8 Total distance Spartacus covers in three days.

$C = \pi d$

$= \pi \times 0.3$

$C = 0.9 \text{ m}$

$D = 2\,085 \times 0.9$

$D = 1\,876.5 \text{ m}$

Or 1 965.1 m (No rounding)

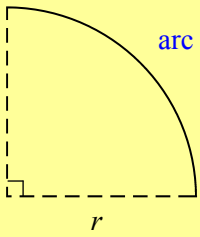
9 Average distance travelled per day.

$\text{Average Dist} = 1\,876.5 \div 3 = 625.5 \text{ m}$

Or No Rounding $\text{Average Dist} = 1\,965.1 \div 3 = 655 \text{ m}$

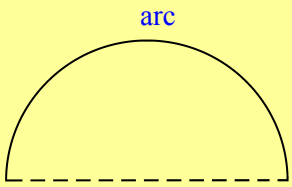
Further Perimeter Circles (Exact)

When finding the perimeter of a shape all the sides are added together. With the shapes in this worksheet an arc length is included, which has to be calculated in terms of π. This arc length will always be a fraction of a circle. For example:



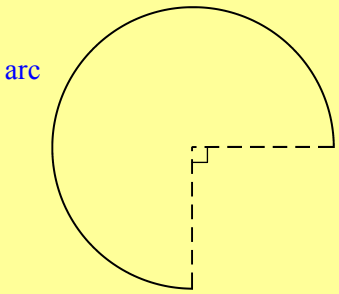
The arc length is:

$\frac{1}{4} \times 2\pi r$



The arc length is:

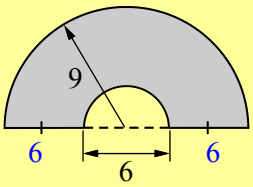
$\frac{1}{2} \times 2\pi r$ *or* $\frac{1}{2} \times \pi d$



The arc length is:

$\frac{3}{4} \times 2\pi r$

Example - Find the perimeter of the shape below



Because 18 - 6 = 12
shared equally = 6

$P = \frac{1}{2} \times 2\pi r + \frac{1}{2} \times \pi d + 2 \times 6$

Outside arc | Inside arc | Straight lengths

$= \frac{1}{2} \times 2 \times \pi \times 9 + \frac{1}{2} \times \pi \times 6 + 12$

Sub $r = 9$ | Sub $d = 6$ | $2 \times 6 = 12$

$= 9\pi + 3\pi + 12$

Solve each part

$P = 12\pi + 12$ units

Gather like terms and include 'units'

Column 1 starts with finding the perimeter of sectors, quadrants and semi-circles. These are all solved by finding the fraction of a circle shown and multiplying this by $2\pi r$. Then add the arc length with the other straight side lengths to get the perimeter.

Column 2 involve composite shapes and so include more sides in the calculation, watch for repetition to speed up your calculation remembering that two halves will make a whole circle. Note that Pythagoras Theorem is required at times.

Column 3 uses algebra-style questions, where letters are introduced rather than numbers. These may be more challenging.

Further Perimeter Circles (Exact)

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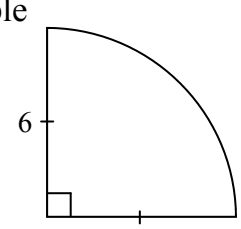
Find the perimeter in terms of π for these shapes.

Example

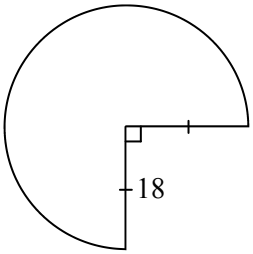


$P = \frac{1}{4}\times 2\pi r + 2r$

$= \frac{1}{4}\times 2\times\pi\times 6 + 2\times 6$

$P = 3\pi + 12$ units

1



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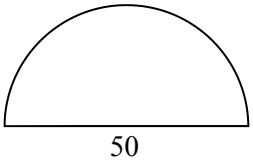
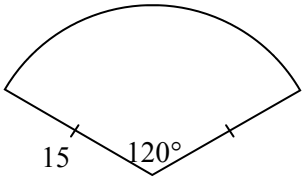


3



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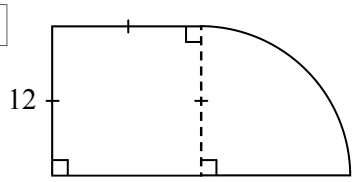

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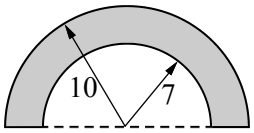

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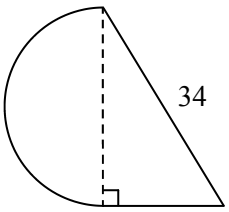
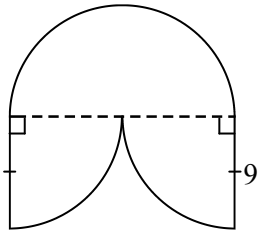
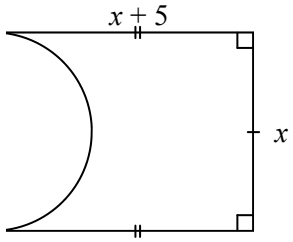


7



Keep finding the perimeter but this time using algebra.

8



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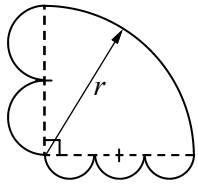

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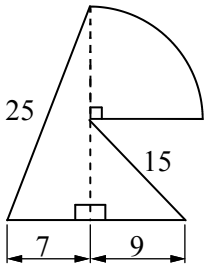

Further Perimeter Circles (Exact)

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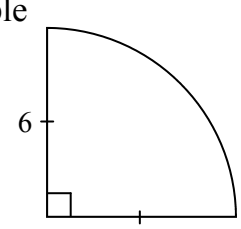
Find the perimeter in terms of π for these shapes.

Example

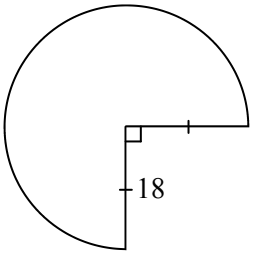


$P = \frac{1}{4}\times 2\pi r + 2r$

$= \frac{1}{4}\times 2\times\pi\times 6 + 2\times 6$

$P = 3\pi + 12$ units

1

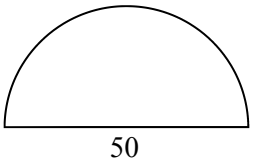


$P = \frac{3}{4}\times 2\pi r + 2r$

$= \frac{3}{4}\times 2\times\pi\times 18 + 2\times 18$

$P = 27\pi + 36$ units

2

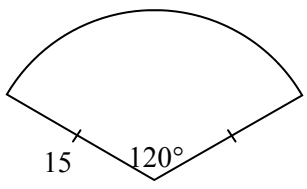


$P = \frac{1}{2}\times\pi d + d$

$= \frac{1}{2}\times\pi\times 50 + 50$

$P = 25\pi + 50$ units

3

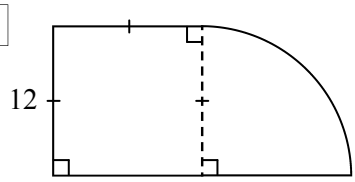


$P = \frac{1}{3}\times 2\times\pi\times r + 2\times r$

$= \frac{1}{3}\times 2\times\pi\times 15 + 2\times 15$

$P = 10\pi + 30$ units

4

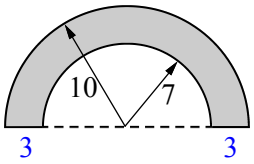


$P = \frac{1}{4}\times 2\pi r + 4r$

$= \frac{1}{4}\times 2\times\pi\times 12 + 4\times 12$

$P = 6\pi + 48$ units

5



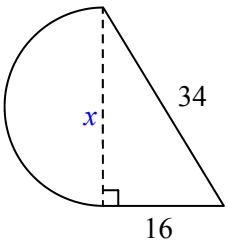
$P = \frac{1}{2}\times 2\pi R + \frac{1}{2}\times 2\pi r + 2\times 3$

$= \frac{1}{2}\times 2\times\pi\times 10 + \frac{1}{2}\times 2\times\pi\times 7 + 6$

$= 10\pi + 7\pi + 6$ units

$P = 17\pi + 6$ units

6



$x^2 = 34^2 - 16^2$

$x^2 = 900$

$x = \sqrt{900}$

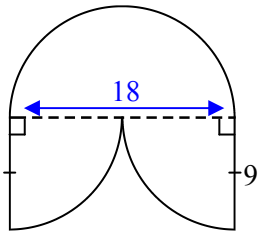
$x = 30$ units

$P = \frac{1}{2}\times\pi d + 16 + 34$

$= \frac{1}{2}\times\pi\times 30 + 50$

$P = 15\pi + 50$ units

7



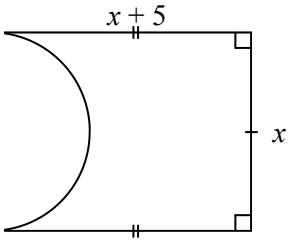
$P = \pi d + 2\times 9$

$= \pi\times 18 + 18$

$P = 18\pi + 18$ units

Keep finding the perimeter but this time using algebra.

8

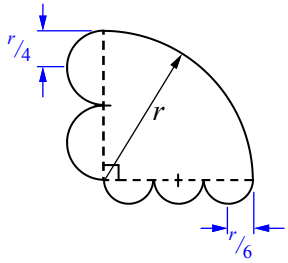


$P = \frac{1}{2}\times\pi d + 2(x + 5) + x$

$= \frac{1}{2}\times\pi\times x + 2x + 10 + x$

$P = \frac{1}{2}\pi x + 3x + 10$ units

9

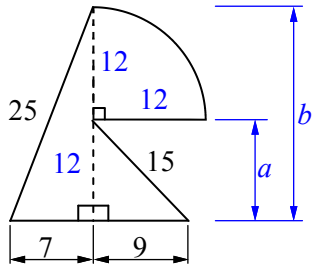


$P = \frac{1}{4}\times 2\pi r + 2\pi(\frac{r}{4}) + 1\frac{1}{2}\times 2\pi(\frac{r}{6})$

$= \frac{1}{2}\pi r + \frac{1}{2}\pi r + \frac{1}{2}\pi r$

$= \frac{3}{2}\pi r$ units or $1\frac{1}{2}\pi r$ units

10



$a^2 = 15^2 - 9^2$ | $b^2 = 25^2 - 7^2$

$a^2 = 144^2$ | $b^2 = 576^2$

$a = \sqrt{144}$ | $b = \sqrt{576}$

$a = 12$ units | $b = 24$ units

$P = \frac{1}{4}\times 2\pi r + 25 + 16 + 15 + r$

$= \frac{1}{4}\times 2\times\pi\times 12 + 56 + 12$

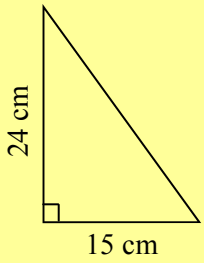
$P = 6\pi + 68$ units

Pythagoras - Area and Perimeter Applications

Perimeter is measurement of distance, the distance around a shape. So to find the perimeter of a triangle you add all the side lengths together. An example is at the top of column 1 of the worksheet.

The area of a triangle is found by $A = \frac{1}{2}bh$ which is 'a half times the base times the height'. Note that ½ = 0.5, it may be easier to type '0.5' into your calculator, also note that the 'base' isn't necessarily on the bottom of the triangle, due to rotation. Below is an example of finding the area of a triangle.

Example - Area of a Triangle



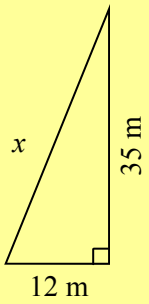
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 15 \times 24$

$A = 180 \text{ cm}^2$

Column 2 is all calculating the perimeter of a triangle. Note that you are given two sides only, but to calculate perimeter you need to know all three sides. So first calculate the unknown side using P.T., once you have calculated the side, add all three sides together. That gives you the perimeter. Remember that you may have subtraction or addition using Pythagoras, it depends if you are finding the length of the hypotenuse (addition or +) or finding a short side (subtraction or -). The first question reminds you of the process but there is also an example below.

Example - Using P.T. then finding the perimeter.



$x^2 = 35^2 + 12^2$

$x^2 = 1\,369$

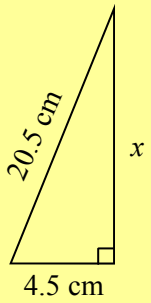
$x = \sqrt{1\,369}$

$x = 37 \text{ m}$

$P = 12 + 35 + 37 = 84 \text{ m}$

Column 3 deals with finding the area of a triangle. Again, like above you will need to use P.T. first to calculate the unknown side. Then use the area formula: $A = \frac{1}{2}bh$.

Example - Using P.T. then finding the area.



$x^2 = 20.5^2 - 4.5^2$

$x^2 = 400$

$x = \sqrt{400}$

$x = 20 \text{ cm}$

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4.5 \times 20$

$A = 45 \text{ cm}^2$

Pythagoras - Area and Perimeter Applications

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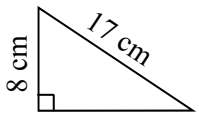
Find the perimeter for these triangles.

Example

Add all the sides

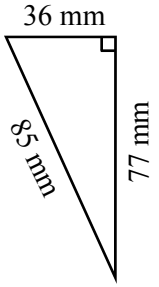


$P = 8 + 15 + 17$

$P = 40$ cm

Remember the units

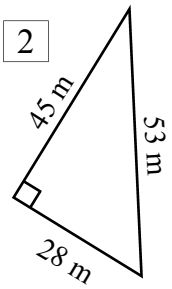
1



$P =$

$P =$ mm

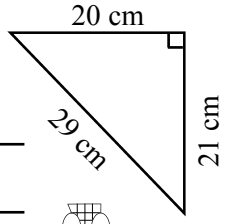
2



$P =$

$P =$

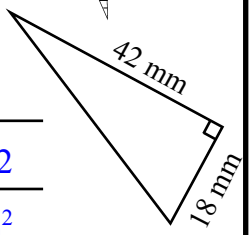
3



Now find the area.

Remember ½ = 0.5

4

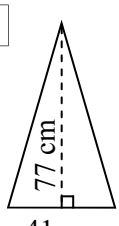


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 42$

$A =$ mm^2

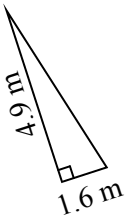
5



$A =$

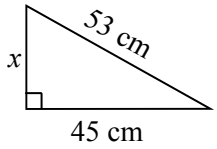
$A =$ cm^2

6



Calculate the missing side using Pythagoras Theorem then find the perimeter.

7



$x^2 =$

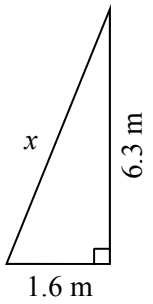
$x^2 =$

$x = \sqrt{}$

$x =$

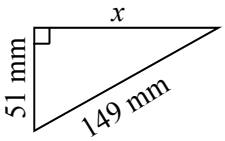
$P =$ + + = cm

8



$P =$

9



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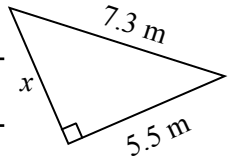
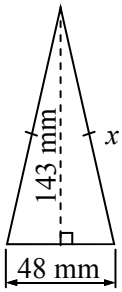
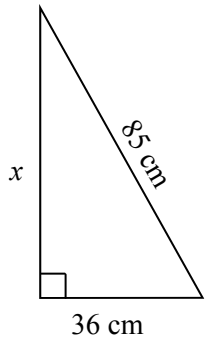


11



Now find the area. You need the base and the perpendicular height.

12



$x^2 =$

$x^2 =$

$x =$

$x =$

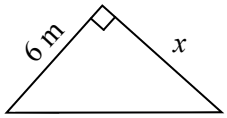
$A = \frac{1}{2}bh$

$= \quad \times \quad \times$

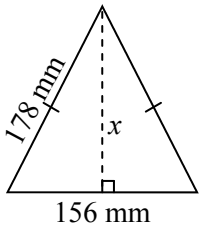
$A =$ cm^2

Area is measured in square units

13



14



Pythagoras - Area and Perimeter Applications

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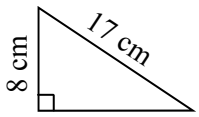
Find the perimeter for these triangles.

Example



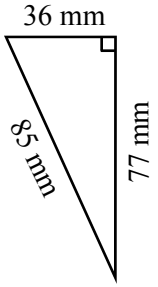
Add all the sides

$P = 8 + 15 + 17$

$P = 40$ cm

Remember the units

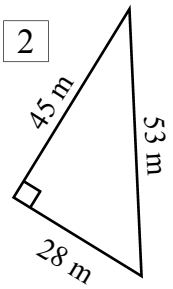
1



$P = 36 + 77 + 85$

$P = 198$ mm

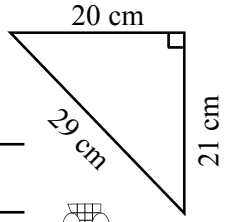
2



$P = 45 + 28 + 53$

$P = 126$ m

3



$P = 20 + 21 + 29$

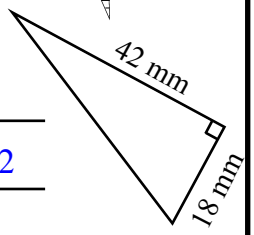
$P = 70$ cm

Now find the area.



Remember ½ = 0.5

4

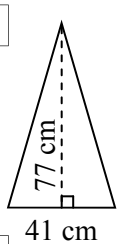


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 18 \times 42$

$A = 378$ mm^2

5

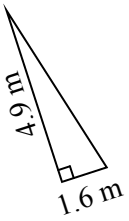


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 41 \times 77$

$A = 1\,578.5$ cm^2

6



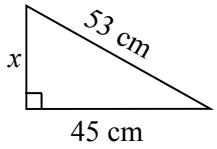
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 1.6 \times 4.9$

$A = 3.92$ m^2

Calculate the missing side using Pythagoras Theorem then find the perimeter.

7



$x^2 = 53^2 - 45^2$

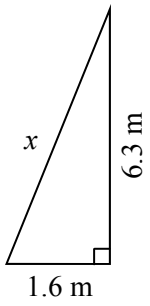
$x^2 = 784$

$x = \sqrt{784}$

$x = 28$ cm

$P = 28 + 45 + 53 = 126$ cm

8



$x^2 = 1.6^2 + 6.3^2$

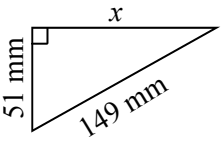
$x^2 = 42.25$

$x = \sqrt{42.25}$

$x = 6.5$ m

$P = 1.6 + 6.3 + 6.5 = 14.4$ m

9



$x^2 = 149^2 - 51^2$

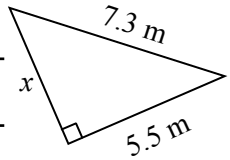
$x^2 = 19\,600$

$x = \sqrt{19\,600}$

$x = 140$ mm

$P = 51 + 140 + 149 = 340$ mm

10



$x^2 = 7.3^2 - 5.5^2$

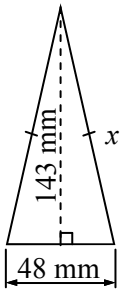
$x^2 = 23.04$

$x = \sqrt{23.04}$

$x = 4.8$ m

$P = 5.5 + 7.3 + 4.8 = 17.6$ m

11



$x^2 = 24^2 + 143^2$

$x^2 = 21\,025$

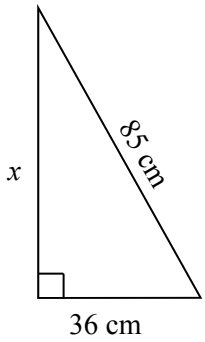
$x = \sqrt{21\,025}$

$x = 145$ mm

$P = 48 + 2 \times 145 = 338$ mm

Now find the area. You need the base and the perpendicular height.

12



$x^2 = 85^2 - 36^2$

$x^2 = 5\,929$

$x = \sqrt{5\,929}$

$x = 77$ cm

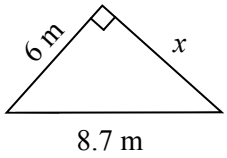
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 36 \times 77$

$A = 1\,386$ cm^2

Area is measured in square units

13



$x^2 = 8.7^2 - 6^2$

$x^2 = 39.69$

$x = \sqrt{39.69}$

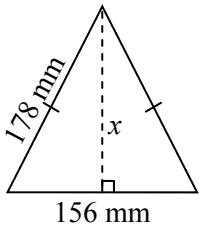
$x = 6.3$ m

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 6 \times 6.3$

$A = 18.9$ cm^2

14



$x^2 = 178^2 - 78^2$

$x^2 = 25\,600$

$x = \sqrt{25\,600}$

$x = 160$ mm

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 156 \times 160$

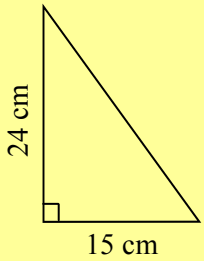
$A = 12\,480$ mm^2

Pythagoras - Further Area and Perimeter Applications

Perimeter is measurement of distance, the distance around a shape. So to find the perimeter of a triangle you add all the side lengths together. This sheet features problems which will require subtraction or some other calculation before applying Pythagoras Theorem. The questions can be challenging and so the previous 'Perimeter and Area' sheet should be attempted first.

The area of a triangle is found by $A = \frac{1}{2}bh$ which is 'a half times the base times the height'. Note that ½ = 0.5, it may be easier to type '0.5' into your calculator, also note that the 'base' isn't necessarily on the bottom of the triangle, due to rotation. Below is an example of finding the area of a triangle.

Example - Area of a Triangle



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 15 \times 24$

$A = 180 \text{ cm}^2$

Column 1 is calculating the perimeter of a shape. These are composite shapes which require you to calculate either two or three unknown lengths before you can find the perimeter.

Column 2 and Column 3 are area questions, calculate all required sides first, then calculate the two areas individually and add them together. Question 6 is a subtraction question, with two identical triangles being removed from a rectangle to form the composite shape.

Pythagoras - Further Area and Perimeter Applications

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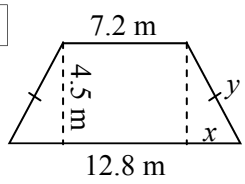
Find the missing side length(s) then find the perimeter in metres.

1



Find x first then use P.T. to find y

$x = \dfrac{\text{-}}{2}$ $\quad y^2 =$

$y^2 =$

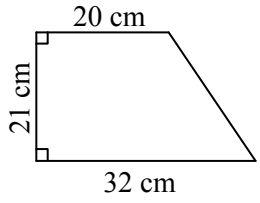
$x =$ m $\quad y = \sqrt{}$

$y =$

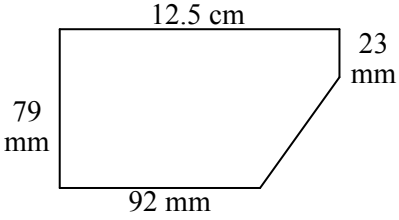
P = + + 2 ×

P = m

2

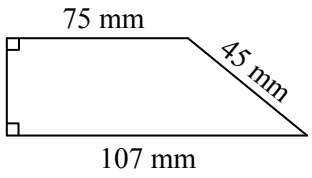


3



Now find the area. Answer to 1 d.p. when required.

4



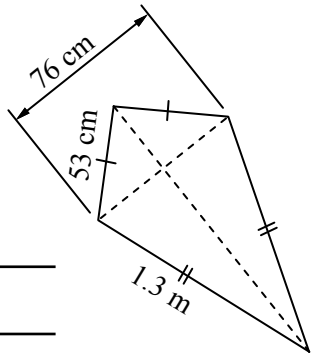
$A_1 = lb$ $\quad A_2 = \frac{1}{2}bh$

= $\quad$ =

$A_1 =$ mm^2 $\quad A_2 =$ mm^2

$A_T =$ + = mm^2

5



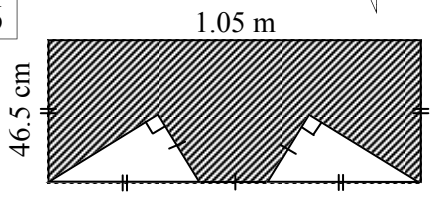
$A_1 =$ $\quad A_2 =$

$A_1 =$ $\quad A_2 =$

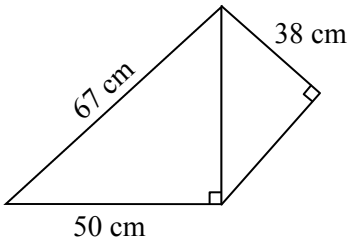
$A_T =$

Find the area, shaded area in Q6, to 1 d.p.

6



7



Pythagoras - Further Area and Perimeter Applications

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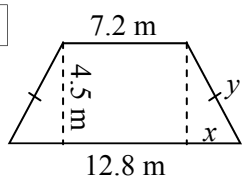
Find the missing side length(s) then find the perimeter in metres.

1



Find x first then use P.T. to find y

$x = \dfrac{12.8 - 7.2}{2}$

$x = 2.8$ m

$y^2 = 2.8^2 + 4.5^2$

$y^2 = 28.09$

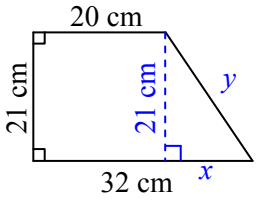
$y = \sqrt{28.09}$

$y = 5.3$ m

$P = 12.8 + 7.2 + 2 \times 5.3$

$P = 30.6$ m

2



$x = 32 - 20$

$x = 12$ cm

$y^2 = 12^2 + 21^2$

$y^2 = 585$

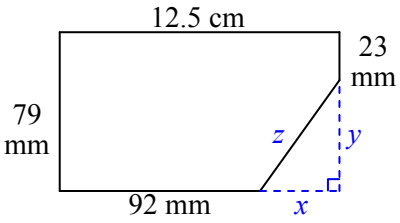
$y = \sqrt{585}$

$y = 24.2$ m

$P = 20 + 24.2 + 32 + 21$

$P = 97.2$ cm $= 0.972$ m

3



$x = 125 - 92$

$x = 33$ mm

$y = 79 - 23$

$y = 56$ mm

$z^2 = 33^2 + 56^2$

$z^2 = 4\,225$

$z = \sqrt{4\,225}$

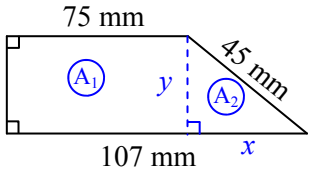
$z = 65$ mm

$P = 79 + 125 + 23 + 65 + 92$

$P = 384$ mm $= 0.384$ m

Now find the area. Answer to 1 d.p. when required.

4



$x = 107 - 75$

$x = 32$ mm

$y^2 = 45^2 - 32^2$

$y^2 = 1\,001$

$y = \sqrt{1\,001}$

$y = 31.6$ mm

$A_1 = lb$

$= 75 \times 31.6$

$A_1 = 2\,370$ mm^2

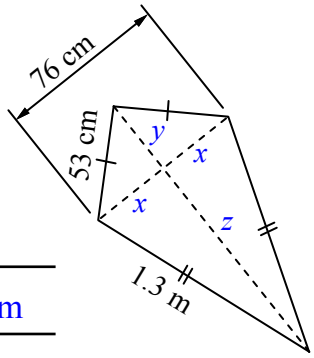
$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 32 \times 31.6$

$A_2 = 505.6$ mm^2

$A_T = 2\,370 + 505.6 = 2875.6$ mm^2

5



$2x = 76$

$x = 38$ cm

$y^2 = 53^2 - 38^2$

$y^2 = 1\,365$

$y = \sqrt{1\,365}$

$y = 36.9$ cm

$z^2 = 130^2 - 38^2$

$z^2 = 15\,456$

$z = \sqrt{15\,456}$

$z = 124.3$ cm

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 76 \times 36.9$

$A_1 = 1\,402.2$ cm^2

$A_2 = \frac{1}{2}bh$

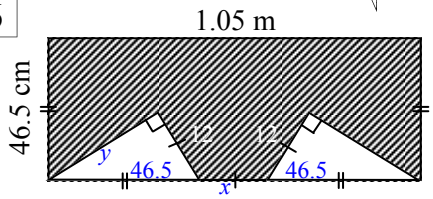
$= \frac{1}{2} \times 76 \times 124.3$

$A_2 = 4\,723.4$ cm^2

$A_T = 1402.2 + 4\,723.4 = 6\,125.6$ cm^2

Find the area, shaded area in Q6, to 1 d.p.

6



$x = 105 - 2 \times 46.5$

$x = 12$ cm

$y^2 = 46.5^2 - 12^2$

$y^2 = 2\,018.25$

$y = \sqrt{2\,018.5}$

$y = 44.9$ cm

$A_1 = lb$

$= 105 \times 46.5$

$A_1 = 4\,882.5$ cm^2

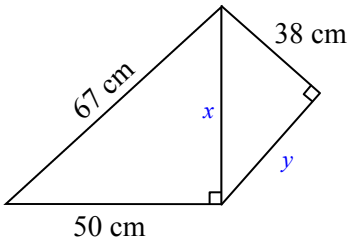
$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 44.9 \times 12$

$A_2 = 269.4$ cm^2

$A_S = 4\,882.5 - 2 \times 269.4 = 4343.7$ cm^2

7



$x^2 = 67^2 - 50^2$

$x^2 = 1\,989$

$x = \sqrt{1\,989}$

$x = 44.6$ cm

$y^2 = 44.6^2 - 38^2$

$y^2 = 545.16$

$y = \sqrt{545.16}$

$y = 23.3$ cm

$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 50 \times 44.6$

$A_1 = 1\,115$ cm^2

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 38 \times 23.3$

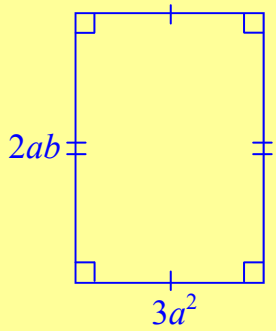
$A_2 = 442.7$ cm^2

$A_T = 1\,115 + 442.7 = 1\,557.7$ cm^2

Further Area and Perimeter - Algebraic Application

This sheet applies Algebraic skills to Area and Perimeter exercises. The skills to complete this worksheet are taught in the Algebra chapter (Chapter 5). This sheet is a sheet built up from the last columns of three algebra sheets, if you require a refresher on the skills required then you should refer to the Further Adding, Further Multiplying and Further Expanding worksheets. There is an example in column 1 for finding the perimeter, below are some examples for the second and third column.

Column 2 Example - Finding the Area



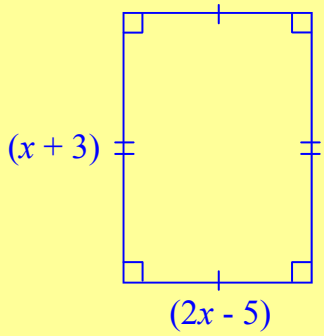
$A = lb$ ← Write the Area formula

$= 2ab \times 3a^2$ ← Substitute the side lengths

$A = 6a^3b$ units^2 ← Simplify, remember the units.

Column 3 Example - Finding the Area

This column requires you to use the 'foil' method to find the area.



$A = lb = (x + 3)(2x - 5)$ ← Write the Area formula and substitute

$= 2x^2 - 5x + 6x - 15$ ← Expand the expression using FOIL

$A = 2x^2 + x - 15$ units^2 ← Collect like terms, remember the units.

Further Area and Perimeter - Algebraic Application

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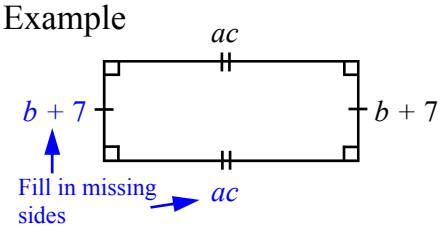
Use your algebra skills to give an expression for the perimeter of the shapes below. Collect like terms.

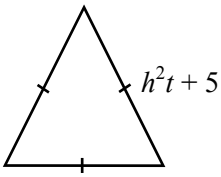
Example



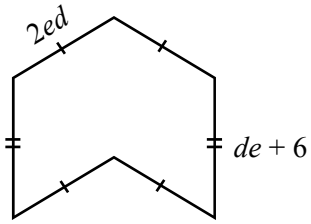
$P = ac + ac + b + 7 + b + 7$

$P = 2ac + 2b + 14$ units

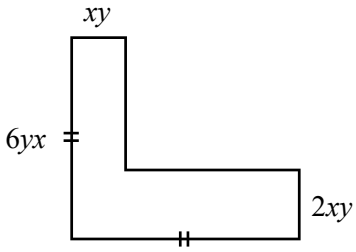
1



2

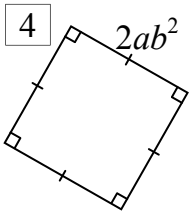


3



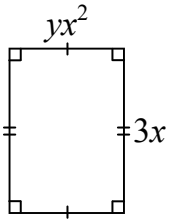
Give an algebraic expression for the areas of these shapes.

4

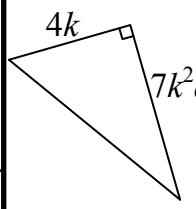


$A = l^2$

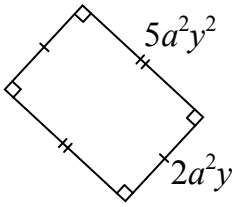
5



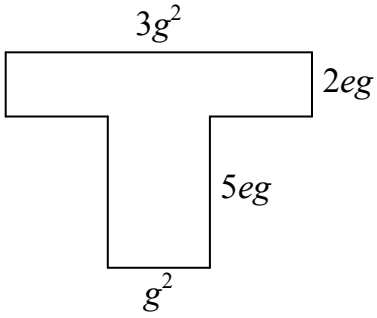
6



7



8

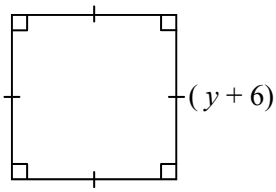


A = ☐ + ☐

A =

Find an expression for the area of these shapes, fully expand and simplify your answer.

9

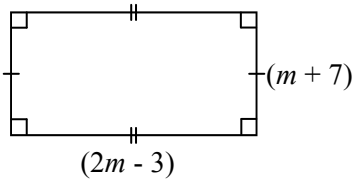


$A = l^2 = (\quad)(\quad)$

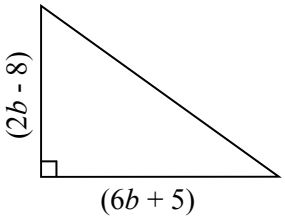
$=$

$A =$

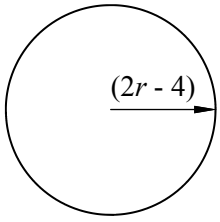
10



11



12



Further Area and Perimeter - Algebraic Application

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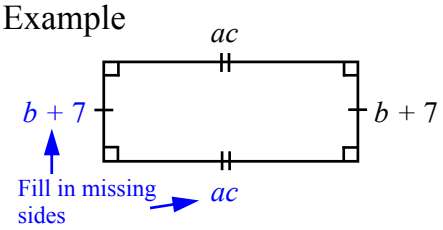
Use your algebra skills to give an expression for the perimeter of the shapes below. Collect like terms.

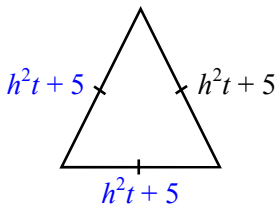
Example



$P = ac + ac + b + 7 + b + 7$

$P = 2ac + 2b + 14$ units

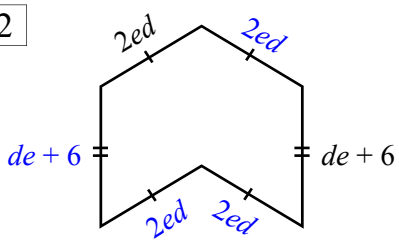
1



$P = h^2t + 5 + h^2t + 5 + h^2t + 5$

$P = 3h^2t + 15$ units

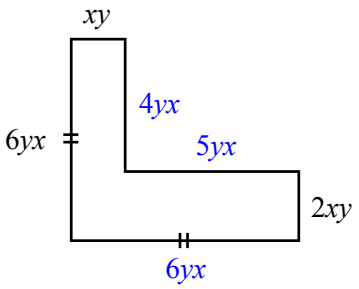
2



$P = 4 \times 2ed + de + 6 + de + 6$

$P = 10ed + 12$ units

3



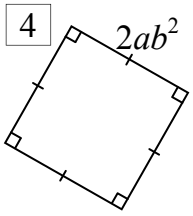
$P = xy + 4xy + 5xy + 2xy + 6xy$

$+ 6xy$

$P = 12xy$ units

Give an algebraic expression for the areas of these shapes.

4

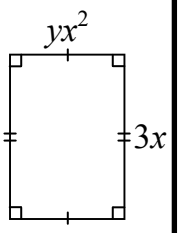


$A = l^2$

$= 2ab^2 \times 2ab^2$

$A = 4a^2b^4$ units^2

5

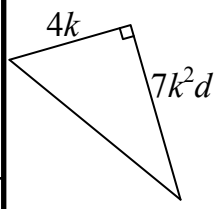


$A = lb$

$= 3x \times yx^2$

$A = 3yx^3$ units^2

6

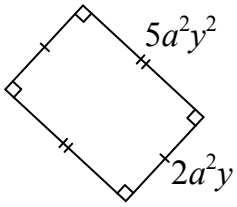


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4k \times 7k^2d$

$A = 14k^3d$ units^2

7

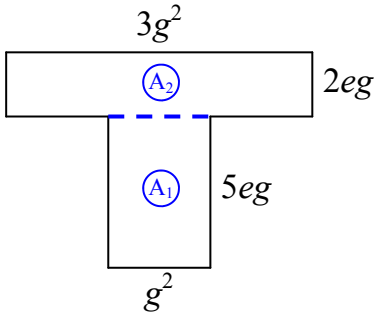


$A = lb$

$= 5a^2y^2 \times 2a^2y$

$A = 10a^4y^3$ units^2

8



$A_1 = lb$

$= 5eg \times g^2$

$A_1 = 5eg^2$ units^2

$A_2 = lb$

$= 2eg \times 3g^2$

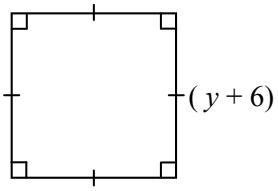
$A_2 = 6eg^2$ units^2

$A = 5eg^2 + 6eg^2$

$A = 11eg^2$ units^2

Find an expression for the area of these shapes, fully expand and simplify your answer.

9

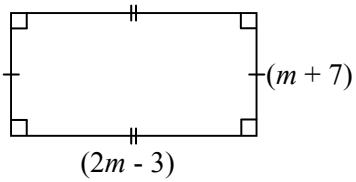


$A = l^2 = (y + 6)(y + 6)$

$= y^2 + 6y + 6y + 36$

$A = y^2 + 12y + 36$ units^2

10

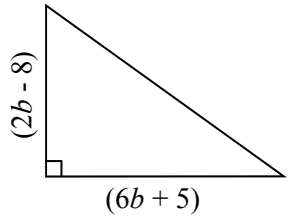


$A = lb = (m + 7)(2m - 3)$

$= 2m^2 - 3m + 14m - 21$

$A = 2m^2 + 11m - 21$ units^2

11



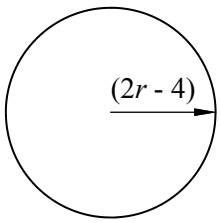
$A = \frac{1}{2}bh = \frac{1}{2} \times (2b - 8)(6b + 5)$

$= (b - 4)(6b + 5)$

$= 6b^2 + 5b - 24b - 20$

$A = 6b^2 - 19b - 20$ units^2

12



$A = \pi r^2 = \pi \times (2r - 4)(2r - 4)$

$= (2\pi r - 4\pi)(2r - 4)$

$= 4\pi r^2 - 8\pi r - 8\pi r + 16\pi$

$A = 4\pi r^2 - 16\pi r + 16\pi$ units^2

Perimeter/Area - Find a Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don't colour it in too dark (using a texta for example) so that you can still read it.

Perimeter/Area - Find a Word

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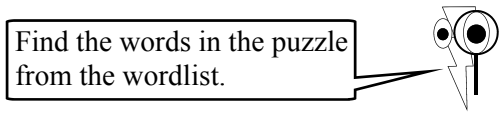

A	R	E	A	T	N	E	M	E	R	U	S	A	E	M
D	E	R	A	U	Q	S	B	R	E	A	D	T	H	T
I	M	E	L	N	M	U	I	Z	E	P	A	R	T	C
A	A	L	L	C	O	M	P	O	S	I	T	E	I	Q
M	R	G	E	E	T	I	K	I	T	I	K	R	E	U
E	G	N	N	H	K	R	T	I	T	K	C	H	L	A
T	O	A	G	E	S	A	Q	A	R	U	Q	O	G	D
E	L	I	T	C	Q	L	U	O	M	N	U	M	N	R
R	E	R	H	T	U	U	T	F	A	I	E	B	A	A
H	L	T	I	A	C	C	E	K	I	T	T	U	T	N
K	L	T	I	R	P	R	E	P	R	S	I	S	C	T
E	A	C	T	E	E	I	A	R	R	O	T	C	E	S
S	R	C	O	N	T	C	E	S	U	I	D	A	R	C
C	A	E	C	R	O	R	E	T	E	M	I	R	E	P
E	P	E	R	P	E	N	D	I	C	U	L	A	R	A

WORDLIST

CIRCUMFERENCE	SQUARED	LENGTH	TRAPEZIUM	PERIMETER
PARALLELOGRAM	CIRCULAR	RECTANGLE	UNITS	SECTOR
PERPENDICULAR	DIAMETER	AREA	HECTARE	RHOMBUS
ESTIMATION	COMPOSITE	ARC	HEIGHT	BREADTH
MEASUREMENT	QUADRANT	KITE	RADIUS	TRIANGLE

Perimeter/Area - Find a Word

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Find the words in the puzzle from the wordlist.



A	R	E	A	T	N	E	M	E	R	U	S	A	E	M
D	E	R	A	U	Q	S	B	R	E	A	D	T	H	T
I	M	E	L	N	M	U	I	Z	E	P	A	R	T	C
A	A	L	L	C	O	M	P	O	S	I	T	E	I	Q
M	R	G	E	E	T	I	K	I	T	I	K	R	E	U
E	G	N	N	H	K	R	T	I	T	K	C	H	L	A
T	O	A	G	E	S	A	Q	A	R	U	Q	O	G	D
E	L	I	T	C	Q	L	U	O	M	N	U	M	N	R
R	E	R	H	T	U	U	T	F	A	I	E	B	A	A
H	L	T	I	A	C	C	E	K	I	T	T	U	T	N
K	L	T	I	R	P	R	E	P	R	S	I	S	C	T
E	A	C	T	E	E	I	A	R	R	O	T	C	E	S
S	R	C	O	N	T	C	E	S	U	I	D	A	R	C
C	A	E	C	R	O	R	E	T	E	M	I	R	E	P
E	P	E	R	P	E	N	D	I	C	U	L	A	R	A

WORDLIST

~~CIRCUMFERENCE~~ ~~SQUARED~~ ~~LENGTH~~ ~~TRAPEZIUM~~ ~~PERIMETER~~

~~PARALLELOGRAM~~ ~~CIRCULAR~~ ~~RECTANGLE~~ ~~UNITS~~ ~~SECTOR~~

~~PERPENDICULAR~~ ~~DIAMETER~~ ~~AREA~~ ~~HECTARE~~ ~~RHOMBUS~~

~~ESTIMATION~~ ~~COMPOSITE~~ ~~ARC~~ ~~HEIGHT~~ ~~BREADTH~~

~~MEASUREMENT~~ ~~QUADRANT~~ ~~KITE~~ ~~RADIUS~~ ~~TRIANGLE~~

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ALTITUDE

ALGEBRA

Completing Number Patterns

To complete number patterns either a "rule" is required which tells you how the number pattern is created, or, enough numbers (usually 4) are needed to determine the rule yourself. Once the rule has been found the last number is used to find the next one and so on.

In column 1 the rule is given, use the last number given and put it in the rule to get the first missing number. Then using the new number, put it in the rule and get the next missing value, and so on.

In column 2 you are asked to create the next pattern by building it with matchsticks. Then find the missing numbers to extend the pattern. Then write the rule in words as you were given in the first column.

Column 3 has number patterns, note how the numbers are changing and carry on the process for the unknown numbers in the boxes. The last few questions are more difficult and require two operations, for example multiplying by two an subtracting one.

Completing Number Patterns

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Complete the patterns by finding the missing numbers

1 Start with 2, "add" 3

→ 2, 5, 8, ☐, ☐

2 Start with 21, "subtract" 3

→ 21, 18, 15, ☐, ☐

3 Start with 5, "add" 5

→ 5, 10, ☐, 20, ☐

4 Start with 5, "multiply by" 2

→ 5, 10, ☐, 40, ☐, ☐

5 Start with 80, "divide by" 2

→ 80, 40, ☐, ☐, ☐

6 Start with 25, "subtract" 5

→ 25, 20, ☐, ☐, ☐

7 Start with 2, "multiply by" 2 then add 1

→ 2, 5, 11, ☐, ☐, ☐

8 Start with 5, "multiply by" 3 then subtract 5

→ 5, 10, ☐, 70, ☐, ☐

9 Start with 6, "multiply by" 2 then subtract 4

→ 6, 8, ☐, ☐, ☐, ☐

10 Start with 3, "multiply by" 2 then add 1

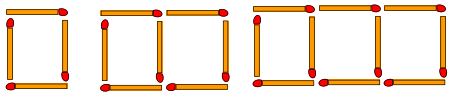
→ 3, 7, ☐, ☐, ☐, ☐

11 Start with 100, "divide by" 2 then add 2

→ 100, 52, ☐, ☐, ☐

Draw the next match-stick pattern then complete the numbers and pattern in words below it.

12

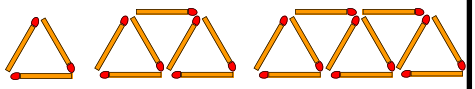


4th → pattern

4, 7, 10, ☐, ☐, ☐, ☐

Start with ☐ "add" ☐

13

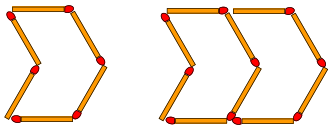


4th → pattern

3, 7, ☐, ☐, ☐, ☐

Start with ☐ ___ ☐

14

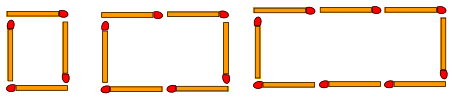


3rd → pattern

6, 10, ☐, ☐, ☐, ☐

___ ☐ ___ ☐

15



4th → pattern

☐, ☐, ☐, ☐, ☐

___ ☐ ___ ☐

Complete the patterns you will at times get answers which include decimals.

16 4, 9, 14, 19, ☐, ☐, ☐

17 20, 35, 50, 65, ☐, ☐

18 1, 2, 4, 8, ☐, ☐, ☐

19 320, 160, 80, 40, ☐, ☐

20 $\frac{1}{5}$, 1, 5, 25, ☐, ☐

21 88, 99, 110, 121, ☐, ☐

22 5, 14, 23, 32, ☐, ☐

23 87, 91, 95, 99, ☐, ☐

(decimal answers)

24 1 000, 100, 10, 1, ☐, ☐

25 1, 7, 13, 19, ☐, ☐, ☐

26 15, 10, 5, 0, ☐, ☐, ☐

27 197, 157, 117, 77, ☐, ☐

28 54, 63, 72, 81, ☐, ☐

These are harder, you will have to multiply by a number then add/subtract.

29 1, 4, 10, 22, ☐, ☐

30 2, 4, 10, 28, ☐, ☐

31 1, 4, 13, 40, ☐, ☐

32 2½, 10, 25, 55, ☐, ☐

33 1, 2, 5, 14, ☐, ☐

34 1, 6, 16, 36, ☐, ☐

Completing Number Patterns

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Complete the patterns by finding the missing numbers

1 Start with 2, "add" 3

→ 2, 5, 8, [11], [14]

2 Start with 21, "subtract" 3

→ 21, 18, 15, [12], [9]

3 Start with 5, "add" 5

→ 5, 10, [15], 20, [25]

4 Start with 5, "multiply by" 2

→ 5, 10, [20], 40, [80], [160]

5 Start with 80, "divide by" 2

→ 80, 40, [20], [10], [5]

6 Start with 25, "subtract" 5

→ 25, 20, [15], [10], [5]

7 Start with 2, "multiply by" 2 then add 1

→ 2, 5, 11, [23], [47], [95]

8 Start with 5, "multiply by" 3 then subtract 5

→ 5, 10, [25], 70, [205], [610]

9 Start with 6, "multiply by" 2 then subtract 4

→ 6, 8, [12], [20], [36], [68]

10 Start with 3, "multiply by" 2 then add 1

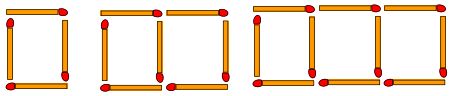
→ 3, 7, [15], [31], [63], [127]

11 Start with 100, "divide by" 2 then add 2

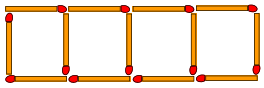
→ 100, 52, [28], [16], [10]

Draw the next match-stick pattern then complete the numbers and pattern in words below it.

12



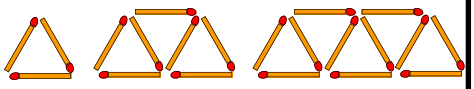
4th pattern →



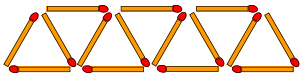
4, 7, 10, [13], [16], [19], [22]

Start with [4] "add" [3]

13



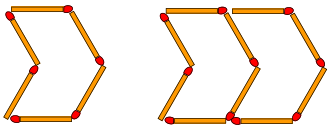
4th pattern →



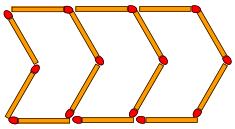
3, 7, [11], [15], [19], [23]

Start with [3] "add" [4]

14



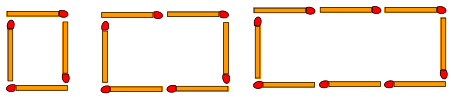
3rd pattern →



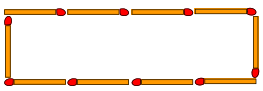
6, 10, [14], [18], [22], [26]

Start with [6] "add" [4]

15



4th pattern →



[4], [6], [8], [10], [12]

Start with [4] "add" [2]

Complete the patterns you will at times get answers which include decimals.

16 4, 9, 14, 19, [24], [29], [34]

17 20, 35, 50, 65, [80], [95]

18 1, 2, 4, 8, [16], [32], [64]

19 320, 160, 80, 40, [20], [10]

20 $\frac{1}{5}$, 1, 5, 25, [125], [625]

21 88, 99, 110, 121, [132], [143]

22 5, 14, 23, 32, [41], [50]

23 87, 91, 95, 99, [103], [107]

(decimal answers)

24 1 000, 100, 10, 1, [0.1], [0.01]

25 1, 7, 13, 19, [25], [31], [37]

26 15, 10, 5, 0, [-5], [-10], [-15]

27 197, 157, 117, 77, [37], [-3]

28 54, 63, 72, 81, [90], [99]

These are harder, you will have to multiply by a number then add/subtract.

29 1, 4, 10, 22, [46], [94]

30 2, 4, 10, 28, [82], [244]

31 1, 4, 13, 40, [121], [364]

32 2½, 10, 25, 55, [115], [235]

33 1, 2, 5, 14, [41], [122]

34 1, 6, 16, 36, [76], [156]

Writing Algebraic Expressions

An algebraic expression involves pronumerals (which are letters), like a, b, etc. Why use letters in mathematics? A letter can be used to represent different numbers, so it is a quick way to use the same 'formula' over and over.

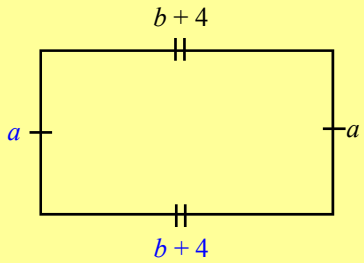
To write expressions in words there are several words that should be understood:
addition - increase, raise, sum of, plus, and add.
subtraction - decrease, reduce, subtract, minus and take away.
division - divide, over, find the quotient (answer after division).
multiplication - times, product of, multiply, lots of
squared - square of, multiplied by itself

In Column 1 write the algebraic expression that the sentence describes, this is sometimes not straight-forward. E.g. Subtract g^2 from the product of 9 and $m \rightarrow 9 \times m - g^2$
While g^2 is written first in the sentence, it is the last mathematical operation to be completed so it is written last in the expression.

Column 2 is the reverse of column 1, construct the sentences given the expression. Some will require order of operations to be considered so be careful.

Column 3 requires an expression for the perimeter of the shapes, write each side separated by a '+' sign, then add the numbers together to get a total but leave the letters as they are. Different letters are left with plus signs between them, you are about to learn how to add pronumerals, but for this exercise this is not required.

Example.



$P = b + 4 + a + b + 4 + a$
$= b + b + a + a + 8$ (leave your answer like this or simplify as below)
$= 2b + 2a + 8$

Writing Algebraic Expressions

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Read the sentences and write algebraic expressions to describe them.

Example

Increase 15 by t squared

$15 + t^2$

1 The sum of g and 20

2 Decrease w by 29

3 Multiply 5 by d

4 From 15 subtract u

5 The sum of a, b and c

6 Raise 16 by r squared

7 Reduce b by 870

8 Subtract 38 from the sum of w and a

9 Reduce the quotient of 20 and q by 8

10 From the product of m and 9 subtract 6

This time write a sentence that matches the algebraic expressions below

Use words like: squared, times, increase, decrease, raise, reduce, sum of, subtract, product, plus, divide, multiply, minus and add.

11 $2 \times m$

12 $k + 5 - d$

* order of operations

13 $a + 2 \times b$*

14 $854 \div 3 \times f$

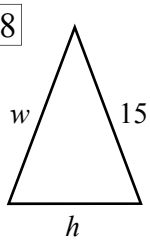
15 $9 + h \times c$

16 $a^2 + 25 - q$

17 $x^2 - 3 \times a$

Construct an algebraic expression for the perimeter of the shapes below.

18



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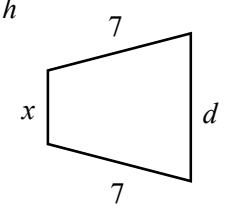

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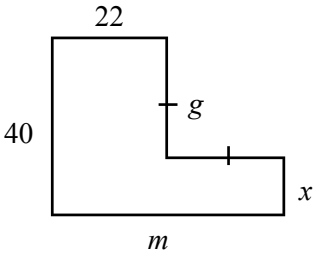
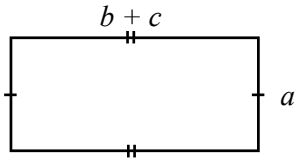


These two are harder

21



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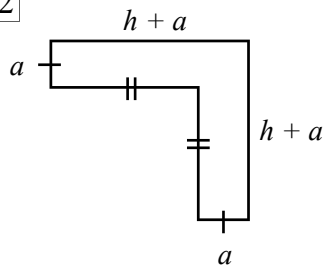

Writing Algebraic Expressions

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Read the sentences and write algebraic expressions to describe them.

Example

Increase 15 by t squared

$15 + t^2$

1 The sum of g and 20

$g + 20$

2 Decrease w by 29

$w - 29$

3 Multiply 5 by d

$5 \times d$

4 From 15 subtract u

$15 - u$

5 The sum of a, b and c

$a + b + c$

6 Raise 16 by r squared

$16 + r^2$

7 Reduce b by 870

$b - 870$

8 Subtract 38 from the sum of w and a

$w + a - 38$

9 Reduce the quotient of 20 and q by 8

$20 \div q - 8$

10 From the product of m and 9 subtract 6

$m \times 9 - 6$

This time write a sentence that matches the algebraic expressions below

Use words like: squared, times, increase, decrease, raise, reduce, sum of, subtract, product, plus, divide, multiply, minus and add.

11 $2 \times m$

The product of 2 and m

12 $k + 5 - d$

From the sum of k and 5

subtract d

* order of operations

13 $a + 2 \times b$*

To the product of 2 and b add a

14 $854 \div 3 \times f$

Divide 854 by 3 then multiply

the quotient by f

15 $9 + h \times c$

To the product of h and c add 9

16 $a^2 + 25 - q$

From the sum of a squared and

25, subtract q

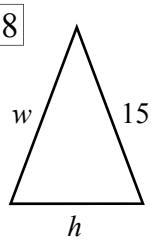
17 $x^2 - 3 \times a$

From x squared subtract the

product of 3 and a

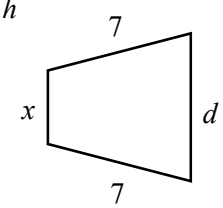
Construct an algebraic expression for the perimeter of the shapes below.

18



$P = w + h + 15$

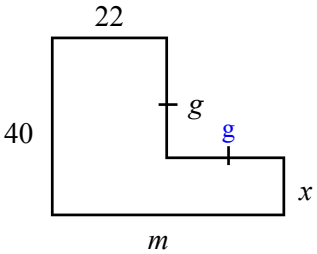
19



$P = x + d + 7 + 7$

$P = x + d + 14$

20

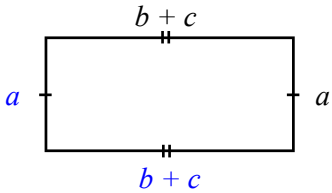


$P = 40 + 22 + g + g + x + m$

$P = 62 + 2g + x + m$

These two are harder

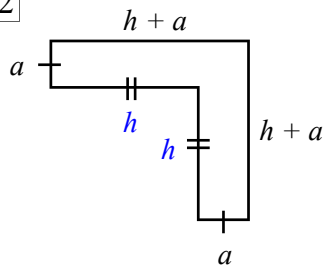
21



$P = a + b + c + a + b + c$

$P = 2a + 2b + 2c$

22



$P = a + h + a + h + a + a + h + h$

$P = 4a + 4h$

Algebraic Expressions - Adding and Subtracting

Think of algebra as being items. Standing at a street corner you may see trucks, cars and buses. If you see 3 trucks ($3t$) and 2 buses ($2b$) you can't add these together, so: $3t + 2b$ remains as $3t + 2b$. If you see 3 cars then 2 buses then 4 more cars, you have seen a total of 7 cars and 2 buses, or $3c + 2b + 4c = 7c + 2b$.

So 'like' terms (the same letter) can be added and subtracted but not different letters.

The same with numbers and terms. Numbers can be collected together and the letters can be collected together but letters can't be added/subtracted with numbers.
For example,

$3 + 4c + 4 + 5c - 2 \quad = 9c + 5 \quad (4c + 5c = 9c \quad \text{and} \quad 3 + 4 - 2 = 5)$

A common mistake is to write a '1' in front of a single letter, this isn't fully simplifying your answer.
For example,

$5b - 7 + 2b - 6b \quad = b - 7$ (don't write this as $1b - 7$).

Just like numbers, when the same term is subtracted from itself it will give zero.
Example,

$5f + 2 - 3f - 2f = 2 \quad (\text{as } 5f - 3f - 2f = 0)$

Algebraic Expressions - Adding and Subtracting

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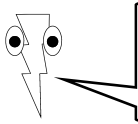


Simplify these, remember that you don't write a 1 in front of a letter, $1m = m$

1 $t + t$

2 $k + 2k$

3 $6m - 4m$

4 $12i + 3i$

5 $2j + 5j$

6 $16e - 9e$

7 $9x + 5x + 3x$

8 $12b + 9b + 4b$

9 $6n + 2n + n$

10 $t + 6t - 2t$

11 $3e + 5e - 4e$

12 $12r - 7r + 2r$

13 $11k - 7k - 4k$

14 $24h - 9h - 14h$

15 $2c + 5c + 7c$

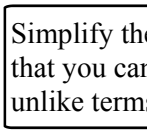
16 $u + 5u - 2u$

17 $6d - 3d - 2d$

18 $14q - 11q - 3q$

19 $8v - 2v - 3v$

20 $12s + 3s - 14s$



Simplify these, remember that you can't add/subtract unlike terms.

21 $a + g + a$

22 $2e + c + e$

23 $4h + 3h + 2d$

24 $10m - 3n - m$

25 $2c - 7q + 3c$

26 $9k + 3k + n$

27 $3e + 2f - f$

28 $m + p - m$

29 $4d - 2m + d$

30 $6m + 3b - 5m$

31 $2d + 3d - h$

32 $t + 5e - t$

33 $5e - 2e + q$

34 $3a - 2d - 2a$

35 $4c + 2b - b$

36 $6w + n - 6w$

37 $6n - 3n + 2d$

38 $2f + 3f + f - 2d$

39 $a + b + a + b$

40 $d + 3d + 3e$



Simplify these, remember that you can't add/subtract numbers and terms.

41 $2a + 3 + 2a$

42 $5e + 4 - 4e$

43 $10d - 3d - 6$

44 $3 + 4q + 2 - q$

45 $4y + 12 - 3y$

46 $4k + 2 + 5 - k$

47 $a + 2 + 2a - 1$

48 $2d - 5 + 4d$

49 $5 + 3e - 2 + e$

50 $t + 4 - t - 2$

51 $3v - 6 - 2v$

52 $10 + 2b - 5$

53 $a + 8 - a + 8$

54 $6x - 10 + 4x$

55 $9m + 6 - 8m$

56 $5 + b - 4 + b$

57 $2f - 10 - f$

58 $10 + k + 15$

59 $h + 3 + h + 1$

60 $y + 2y - 3 + y$

Algebraic Expressions - Adding and Subtracting

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Simplify these, remember that you don't write a 1 in front of a letter, $1m = m$

1	$t + t$	$2t$
2	$k + 2k$	$3k$
3	$6m - 4m$	$2m$
4	$12i + 3i$	$15i$
5	$2j + 5j$	$7j$
6	$16e - 9e$	$7e$
7	$9x + 5x + 3x$	$17x$
8	$12b + 9b + 4b$	$25b$
9	$6n + 2n + n$	$9n$
10	$t + 6t - 2t$	$5t$
11	$3e + 5e - 4e$	$4e$
12	$12r - 7r + 2r$	$7r$
13	$11k - 7k - 4k$	0
14	$24h - 9h - 14h$	h
15	$2c + 5c + 7c$	$14c$
16	$u + 5u - 2u$	$4u$
17	$6d - 3d - 2d$	d
18	$14q - 11q - 3q$	0
19	$8v - 2v - 3v$	$3v$
20	$12s + 3s - 14s$	s

Simplify these, remember that you can't add/subtract unlike terms.

21	$a + g + a$	$2a + g$
22	$2e + c + e$	$3e + c$
23	$4h + 3h + 2d$	$7h + 2d$
24	$10m - 3n - m$	$9m - 3n$
25	$2c - 7q + 3c$	$5c - 7q$
26	$9k + 3k + n$	$12k + n$
27	$3e + 2f - f$	$3e + f$
28	$m + p - m$	p
29	$4d - 2m + d$	$5d - 2m$
30	$6m + 3b - 5m$	$m + 3b$
31	$2d + 3d - h$	$5d - h$
32	$t + 5e - t$	$5e$
33	$5e - 2e + q$	$3e + q$
34	$3a - 2d - 2a$	$a - 2d$
35	$4c + 2b - b$	$4c + b$
36	$6w + n - 6w$	n
37	$6n - 3n + 2d$	$3n + 2d$
38	$2f + 3f + f - 2d$	$6f - 2d$
39	$a + b + a + b$	$2a + 2b$
40	$d + 3d + 3e$	$4d + 3e$

Simplify these, remember that you can't add/subtract numbers and terms.

41	$2a + 3 + 2a$	$4a + 3$
42	$5e + 4 - 4e$	$e + 4$
43	$10d - 3d - 6$	$7d - 6$
44	$3 + 4q + 2 - q$	$3q + 5$
45	$4y + 12 - 3y$	$y + 12$
46	$4k + 2 + 5 - k$	$3k + 7$
47	$a + 2 + 2a - 1$	$3a + 1$
48	$2d - 5 + 4d$	$6d - 5$
49	$5 + 3e - 2 + e$	$4e + 3$
50	$t + 4 - t - 2$	2
51	$3v - 6 - 2v$	$v - 6$
52	$10 + 2b - 5$	$2b + 5$
53	$a + 8 - a + 8$	16
54	$6x - 10 + 4x$	$10x - 10$
55	$9m + 6 - 8m$	$m + 6$
56	$5 + b - 4 + b$	$2b + 1$
57	$2f - 10 - f$	$f - 10$
58	$10 + k + 15$	$k + 25$
59	$h + 3 + h + 1$	$2h + 4$
60	$y + 2y - 3 + y$	$4y - 3$

Algebraic Expressions - Further Adding and Subtracting

To add and subtract terms they must be like. For example:
$5a + 2a = 7a$
$7bc - 6bc = bc$
$3ab + 2ba = 5ba$ (or $5ab$)

However unlike terms can't be added together:
$4c + 3a = 4c + 3a$
$7qm - 3q = 7qm - 3q$
$3ab - a - 3b = 3ab - a - 3b$

The order of the letters doesn't matter: $4bc = 4cb$, $5xqm = 5qxm = 5mqx$ so terms can be added when the letters are the same but in a different order.

The same applies to indexed terms, terms that have a power. Such as $3b^2$, $5d^2f$ and $2x^2y^2$. Terms with different powers can't be added or subtracted. But you can add/subtract these:
$a^2 + 3a^2 = 4a^2$
$5x^2y - 2yx^2 = 3x^2y$

But not these :
$a^2 + 2a = a^2 + 2a$,
$3e^2 - 3ed = 3e^2 - 3ed$
$5x^2y - 2xy^2 = 5x^2y - 2xy^2$

Remember that the order doesn't matter $3k^2mw^2 = 3mk^2w^2 = 3w^2mk^2$, so long as the powers are on the same letters the terms are like.

The final column requires the missing sides to be found and then the sides added together to get the perimeter. Follow the example at the top of the column.

Algebraic Expressions - Further Adding and Subtracting

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Collect like terms and simplify.

Example

$bx + b + 5 + 2xb + 3b + 7$

$= 3xb + 4b + 12$

1 $3a + 5t + 3 - a + 6 + 4t$

2 $12d + 2a + 16 - 5d - 9 + 3a$

3 $5 + a + b + 6 + 2ab + 3b$

4 $30ue + 15e - 5e - u$

5 $17 + 20as - 3 - 15sa - 3as$

6 $5 + 5m + 21 - 4m + 3ma$

Watch out for negatives

7 $4ux + 3xa - 7xu - 3ax$

8 $at - 2ta - 4at + 5 + a$

9 $-4kd - 13 - 5dk + 8 - kd$

10 $5tu + 4 - 12ut - t - 9$

11 $at - 5a - 11 - 5ta + 11a$

12 $-3mx - 18 + 4xm + 6 - mx$

Collect like terms, this time using powers. (^{2})

Example

$50 + a^2 + 3a + 17 - 2a + 3a^2$

$= 4a^2 - a + 67$

13 $17 + 2m^2 - m + m^2 + 14m$

14 $2h^2 - 8 - 3h^2 + 4 - 4h^2$

15 $9k^2 + 14k - 12 - 8k^2 - 7 - 6k$

16 $9r^2 + 5tr + 7 - 3r^2 - 16 - 2rt$

17 $13e - 5e^2 - 8e - 2e^2 + 3 - 4e$

18 $9fg^2 + 3f^2g + 3g^2f$

19 $10yt^2 + 5t^2y + 3y^2t$

20 $5a^2 + 3a^2t - 3a^2 - 2ta^2 - at^2$

21 $y^2 + 2xy^2 + 3y^2 - y^2x^2 - y^2x$

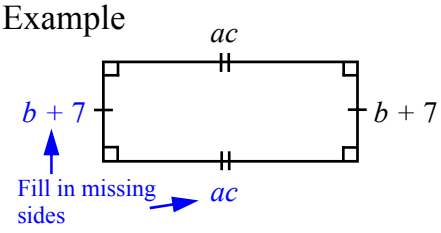
22 $-ab^2 + ba^2 - 5 + 2a^2 - 4a^2b$

23 $x^3y^2z - zy^3x^2 - x^2zy^3 + y^3zx^3$

24 $20t^2s^3 + s^2t^3 - 15s^3t^2 - 9t^3s^2$

Use your algebra skills to give an expression for the perimeter of the shapes below. Collect like terms.

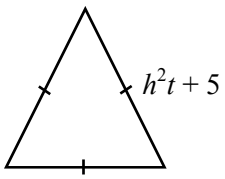
Example



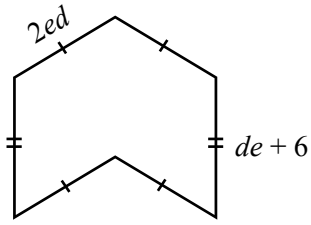
$P = ac + ac + b + 7 + b + 7$

$P = 2ac + 2b + 14$ units

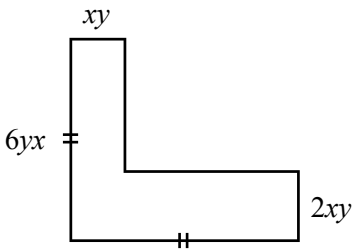
25



26



27



Algebraic Expressions - Further Adding and Subtracting

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Collect like terms and simplify.

Example

$bx + b + 5 + 2xb + 3b + 7$

$= 3xb + 4b + 12$

1 $3a + 5t + 3 - a + 6 + 4t$

$= 2a + 9t + 9$

2 $12d + 2a + 16 - 5d - 9 + 3a$

$= 7d + 5a + 7$

3 $5 + a + b + 6 + 2ab + 3b$

$= 11 + a + 4b + 2ab$

4 $30ue + 15e - 5e - u$

$= 30ue + 10e - u$

5 $17 + 20as - 3 - 15sa - 3as$

$= 14 + 2as$

6 $5 + 5m + 21 - 4m + 3ma$

$= 26 + m + 3ma$

Watch out for negatives

7 $4ux + 3xa - 7xu - 3ax$

$= -3ux$

8 $at - 2ta - 4at + 5 + a$

$= -5at + 5 + a$

9 $-4kd - 13 - 5dk + 8 - kd$

$= -10kd - 5$

10 $5tu + 4 - 12ut - t - 9$

$= -8tu - 5 - t$

11 $at - 5a - 11 - 5ta + 11a$

$= -4at + 6a - 11$

12 $-3mx - 18 + 4xm + 6 - mx$

$= -12$

Collect like terms, this time using powers. (^{2})

Example

$50 + a^2 + 3a + 17 - 2a + 3a^2$

$= 4a^2 - a + 67$

13 $17 + 2m^2 - m + m^2 + 14m$

$= 17 + 3m^2 + 13m$

14 $2h^2 - 8 - 3h^2 + 4 - 4h^2$

$= -5h^2 - 4$

15 $9k^2 + 14k - 12 - 8k^2 - 7 - 6k$

$= k^2 + 8k - 19$

16 $9r^2 + 5tr + 7 - 3r^2 - 16 - 2rt$

$= 6r^2 + 3tr - 9$

17 $13e - 5e^2 - 8e - 2e^2 + 3 - 4e$

$= -7e^2 + e + 3$

18 $9fg^2 + 3f^2g + 3g^2f$

$= 12fg^2 + 3f^2g$

19 $10yt^2 + 5t^2y + 3y^2t$

$= 15yt^2 + 3y^2t$

20 $5a^2 + 3a^2t - 3a^2 - 2ta^2 - at^2$

$= 2a^2 + a^2t - 3a^2 - at^2$

21 $y^2 + 2xy^2 + 3y^2 - y^2x^2 - y^2x$

$= 4y^2 + xy^2 - y^2x^2$

22 $-ab^2 + ba^2 - 5 + 2a^2 - 4a^2b$

$= -ab^2 - 3ba^2 - 5 - 2a^2$

23 $x^3y^2z - zy^3x^2 - x^2zy^3 + y^3zx^3$

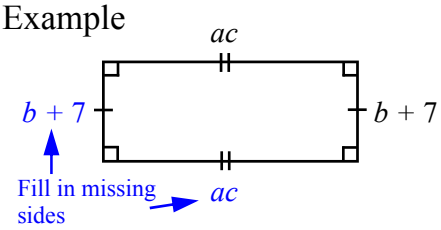
$= x^3y^2z - 2x^2zy^3 + y^3zx^3$

24 $20t^2s^3 + s^2t^3 - 15s^3t^2 - 9t^3s^2$

$= 5t^2s^3 - 8t^3s^2$

Use your algebra skills to give an expression for the perimeter of the shapes below. Collect like terms.

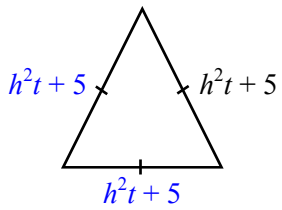
Example



$P = ac + ac + b + 7 + b + 7$

$P = 2ac + 2b + 14$ units

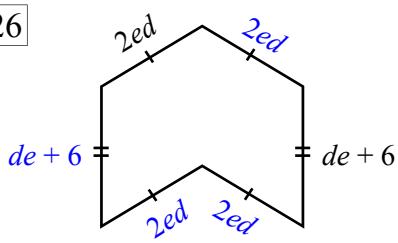
25



$P = h^2t + 5 + h^2t + 5 + h^2t + 5$

$P = 3h^2t + 15$ units

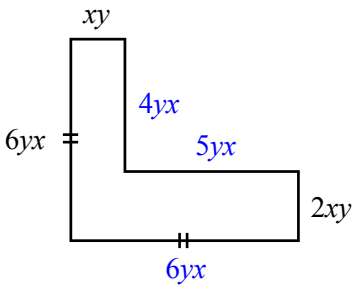
26



$P = 4 \times 2ed + de + 6 + de + 6$

$P = 10ed + 12$ units

27



$P = xy + 4xy + 5xy + 2xy + 6xy$

$+ 6xy$

$P = 12xy$ units

Multiplying Algebraic Terms

We show multiplication between numbers and pronumerals as the number first then the letter. For example, $2 \times a = 2a$, $5 \times d = 5d$ and $4 \times a \times d = 4ad$ (or $4da$).

When several numbers and letters are mixed up, calculate the product of the numbers first, then the letters. For example, $5e \times 2g = 10eg$. Note that when the same letter is featured twice a squared term will result. E.g. $3q \times 5q = 15q^2$. A letter times itself is that letter squared. ($a \times a = a^2$, $c \times c = c^2$ etc.)

Column 1 starts with multiplications as above, Q. 9 features a times table, multiply just as you would with a normal number times table, only the answers will include algebraic terms. From Q.15 negative numbers are involved. When you multiply negative numbers …..

Negative × Positive = Negative (different signs = negative)
Positive × Negative = Negative (different signs = negative)
Negative × Negative = Positive (same signs = Positive)

Remember that three negatives swap the sign back to negative again.

Negative × Negative × Negative = Positive × Negative = Negative

Column 2 builds on the previous column. This time an addition or subtraction are included and you have to consider order of operations principles. Remember to do the × before the + or -. There is an example at the top of the column.

Column 3 is the same as the first two columns, except that it involves shapes. To find the area of squares and rectangles you multiply the sides. So these are multiplication questions like Column 1. Remember that area is measured in square units, as there are no units given you write 'units^2' in the final answer. The last two questions are composite areas, so these questions are much like Column 2.

Multiplying Algebraic Terms

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Multiply these terms. Write numbers first. Simplify.

1 $4 \times x$ ____ 2 $a \times n$ ____

3 $x \times x$ ____ 4 $3 \times h$ ____

5 $g \times 5$ ____ 6 $a \times ax$ ____

7 $m \times 2x$ ____ 8 $3c \times c$ ____

9

×	3	m	ab	$2c$
3				
m				
ab				
$2c$				

10 $3x \times 2xy$ ____

11 $4bc \times 4b$ ____

12 $3m \times 2 \times my$ ____

13 $2abc \times 5bc$ ____

14 $2am \times 4e \times em$ ____

Negative terms change the sign. Multiply these.

15 $-2 \times f$ ____ 16 $g \times (-g)$ ____

17 $-e \times (-e)$ ____ 18 $-ak \times k$ ____

19 $-gh \times agh$ ____

20 $-3dv \times (-5v)$ ____

21 $-2b \times 3a \times (-3ab)$ ____

22 $-f \times mj \times 5mjf$ ____

23 $-ab \times (-b) \times (-ca)$ ____

24 $-3a \times (-2d) \times (-4a)$ ____

25 $ekg \times (-5k) \times 7e$ ____

26 $(-y) \times (-y) \times (-y)$ ____

Addition & subtraction now involved. Simplify these.

Example $3am \times 2m - a \times 2m^2$

$= 6am^2 - 2am^2$

$= 4am^2$

27 $7e \times 2 - 4 \times 3e$

$=$ ____ $-$ ____

$=$ ____

28 $p \times 4a + 5a \times 2p$

$=$ ____ $+$ ____

$=$ ____

29 $3ab \times 3a - 8a^2 \times (-b)$

$=$ ____

$=$ ____

30 $10k - 5k \times 2 + 7$

31 $-2g \times ag + 3a \times 5g^2$

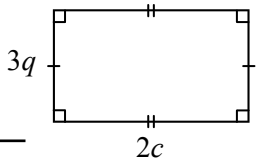
32 $6ac \times 2 - 3a \times (-4c)$

33 $6em - 7e \times 2m + 8e$

34 $3 \times b - b \times (-3) + 3 \times (-b)$

Find the area of these shapes.

Example

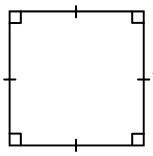


$A = lb$

$= 3q \times 2c$

$A = 6qc$ units^2

35

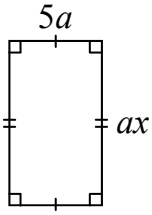


$A = l^2$

$=$ ____ $\times$ ____

$A =$ ____ units^2

36

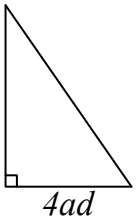


$A =$

$=$ ____ $\times$ ____

$A =$ ____ units^2

37

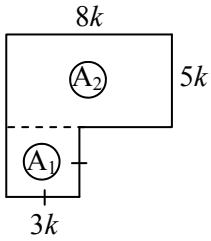


$A = \frac{1}{2}bh$

$=$ ____ $\times$ ____ $\times$ ____

$A =$ ____

38



$A_1 =$

$=$

$A_1 =$

$A_2 =$

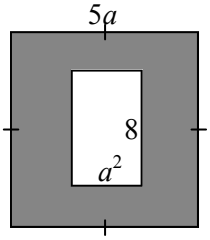
$=$

$A_2 =$

$A =$ ____ $+$ ____

$A =$

39 *Shaded Area



Multiplying Algebraic Terms

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Multiply these terms. Write numbers first. Simplify.

1 $4 \times x$ $4x$

2 $a \times n$ an

3 $x \times x$ x^2

4 $3 \times h$ $3h$

5 $g \times 5$ $5g$

6 $a \times ax$ a^2x

7 $m \times 2x$ $2xm$

8 $3c \times c$ $3c^2$

9

×	3	m	ab	$2c$
3	9	$3m$	$3ab$	$6c$
m	$3m$	m^2	mab	$2cm$
ab	$3ab$	mab	a^2b^2	$2cab$
$2c$	$6c$	$2cm$	$2cab$	$4c^2$

10 $3x \times 2xy$ $6x^2y$

11 $4bc \times 4b$ $16b^2c$

12 $3m \times 2 \times my$ $6m^2y$

13 $2abc \times 5bc$ $10ab^2c^2$

14 $2am \times 4e \times em$ $8ae^2m^2$

Negative terms change the sign. Multiply these.

15 $-2 \times f$ $-2f$

16 $g \times (-g)$ $-g^2$

17 $-e \times (-e)$ e^2

18 $-ak \times k$ $-ak^2$

19 $-gh \times agh$ $-ag^2h^2$

20 $-3dv \times (-5v)$ $15dv^2$

21 $-2b \times 3a \times (-3ab)$ $18a^2b^2$

22 $-f \times mj \times 5mjf$ $-5m^2j^2f^2$

23 $-ab \times (-b) \times (-ca)$ $-a^2b^2c$

24 $-3a \times (-2d) \times (-4a)$ $-24a^2d$

25 $ekg \times (-5k) \times 7e$ $-35e^2k^2g$

26 $(-y) \times (-y) \times (-y)$ $-y^3$

Addition & subtraction now involved. Simplify these.

Example $3am \times 2m - a \times 2m^2$

$= 6am^2 - 2am^2$

$= 4am^2$

27 $7e \times 2 - 4 \times 3e$

$=$ $14e$ $-$ $12e$

$= 2e$

28 $p \times 4a + 5a \times 2p$

$=$ $4ap$ $+$ $10ap$

$= 14ap$

29 $3ab \times 3a - 8a^2 \times (-b)$

$= 9a^2b - (-8a^2b)$

$= 17a^2b$

30 $10k - 5k \times 2 + 7$

$= 10k - 10k + 7$

$= 7$

31 $-2g \times ag + 3a \times 5g^2$

$= -2ag^2 + 15ag^2$

$= 13ag^2$

32 $6ac \times 2 - 3a \times (-4c)$

$= 12ac - (-12ac)$

$= 24ac$

33 $6em - 7e \times 2m + 8e$

$= 6em - 14em + 8e$

$= -8em + 8e$

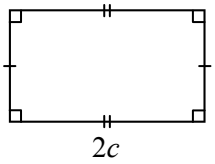
34 $3 \times b - b \times (-3) + 3 \times (-b)$

$= 3b - (-3b) + (-3b)$

$= 3b$

Find the area of these shapes.

Example

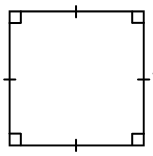


$A = lb$

$= 3q \times 2c$

$A = 6qc$ units^2

35

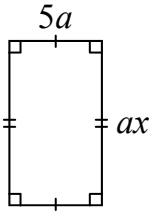


$A = l^2$

$=$ $7x$ $\times$ $7x$

$A =$ $49x^2$ units^2

36

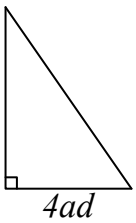


$A = lb$

$=$ $5a$ $\times$ ax

$A =$ $5a^2x$ units^2

37

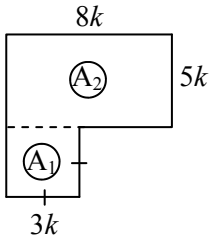


$A = \frac{1}{2}bh$

$=$ $\frac{1}{2}$ $\times$ $4ad$ $\times$ de

$A =$ $2ad^2e$ units^2

38



$A_1 = l^2$

$= (3k)^2$

$A_1 = 9k^2$ units^2

$A_2 = lb$

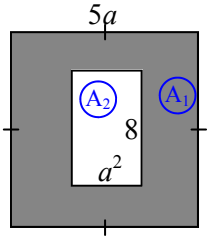
$= 8k \times 5k$

$A_2 = 40k^2$ units^2

$A =$ $9k^2$ $+$ $40k^2$

$A = 40k^2$ units^2

39 *Shaded Area



$A_1 = l^2$

$= (5a)^2$

$A_1 = 25a^2$ units^2

$A_2 = lb$

$= 8 \times a^2$

$A_2 = 8a^2$ units^2

$A = 25a^2 - 8a^2$

$A_2 = 17a^2$ units^2

Multiplying Algebraic Terms - Powers

When you multiply algebraic terms you write the terms next to each other.
Example $t \times a \times g = tag$

But when letters are the same, powers are involved.
Example $t \times g \times t = tgt$ but this is equal to t^2g or gt^2 Both answers are correct.

There are 2 t's above and that results in a power of 2. So by counting the number of terms the power is found.
Example $t \times t \times t \times t \times t \times t = t^6$ There are six t's and the power is 6. EASY!

The above can be rewritten as $t \times t \times t \times t \times t \times t = t^2 \times t^2 \times t^2 = t^6$
Notice that when you add the 2, 2 and 2 it equals 6. So when you multiply terms with powers you add the powers. You just have to remember that when there is no power it is as if there is a '1' there.

Examples

$t^2 \times t^5 = t^7$ (as 2 + 5 = 7)
$t^6 \times t = t^7$ (as 6 + 1 = 7)
$e^3n^2 \times en^5t = e^4n^7t$ (e: 3 + 1 = 4 n: 2 + 5 = 7 t: no change)

When numbers are involved, multiply the numbers …. Don't add them.

Examples
$2a^2b^6 \times a^3b = 2a^5b^7$ (2: no change a: 2 + 3 = 5 b: 6 + 1 = 7)
$5a^7q^3 \times 2aq^2w = 10a^8q^5w$ (5 × 2 =10 a: 7 + 1 = 8 q: 3 + 2 = 5 w: no change)

Column 1 starts with multiplying single terms together without powers. So count each term as 1 to find the power to use. Q. 11-16 are expansions, this is the opposite to the questions above, in other words …. Put in × signs between the terms. Q.17 asks you to join the dots together for matching pairs. The pairs are in opposite columns.

Column 2 has multiplications which involve powers, remember to add the powers on matching terms, and no power means 'the power of 1'. There is also a multiplication table to fill in as well as negative numbers at the base of the column. Remember the properties of multiplying negatives. These are:

- × - = +	- × + = -	+ × - = -
Same sign = +	Different sign = -	Different sign = -

Column 3 applies what you have learned about multiplying terms and applies it to area Calculation. So these are exactly the same as Column 2 except that you are working out an area so write the formula, substitute, then answer. The answer should have units^2 after it.

Multiplying Algebraic Terms - Powers

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Simplify these.

Example $k \times k \times k \times a$ k^3a

1 $x \times x$ ___ 2 $x \times x \times x$ ___

3 $a \times a \times a \times a \times a$ ___

4 $m \times m \times e \times e$ ___

5 $w \times x \times x \times x \times x$ ___

6 $t \times t \times g \times g \times t \times g$ ___

7 $2 \times a \times a \times b \times c$ ___

8 $5 \times q \times q \times q \times t$ ___

9 $m \times m \times 3 \times m \times d$ ___

10 $4 \times y \times y \times (-2) \times t$ ___

Rewrite these in expanded form.

Example $6ma^2$ $6 \times m \times a \times a$

11 k^3 ___

12 c^5 ___

13 $3ad^3$ ___

14 $7e^2k^2$ ___

15 $-4ay^3$ ___

16 $-10b^2c$ ___

Join the matching pairs

17 Join a dot on the left to a dot on the right

x^2y	$2 \times x \times x \times 2$
$2xy^2$	$-2 \times x \times -2 \times -2$
x^2y^2	$x \times y \times x$
$4x^2$	$y \times x \times y \times -2$
$-2xy^2$	$2 \times x \times y \times 2$
$4xy$	$y \times x \times x \times y$
$8xy^2$	$x \times 2 \times y \times y$
$-8x$	$y \times x \times 8 \times y$

Multiply these terms. Write numbers first and then add the powers on common terms.

Example $4k^2m \times 2mk$ $8k^3m^2$

18 $r^2 \times r^3$ ___ 19 $c^5 \times c$ ___

20 $a^2 \times a^7$ ___ 21 $n \times n^3$ ___

22 $e^2 \times eh^2$ ___ 23 $a^3 \times ca$ ___

24 $ef \times fe^2$ ___ 25 $xy \times y$ ___

26 $u^2t \times t^2uv^3$ ___

27 $su \times t^3o \times u^2st$ ___

28 $2a^3b \times ab$ ___

29 $t^4e^6 \times 4t^2e^3$ ___

30 $2mq \times 2q^3m$ ___

31 $7n^2s^3 \times 3sn^2$ ___

32 $ab^2c \times 3a^4bc^2$ ___

33

×	$6a^4$	a^3b	$3ab^2$
a			
b^2a			
$2a^2$			
$5b^3a^2$			

Now negatives are involved.

34 $2c^2b \times (-bc)$ ___

35 $-gt \times (-g) \times (-t^5)$ ___

36 $-q \times (-qv) \times v^2$ ___

37 $9e^2 \times (-4ce)$ ___

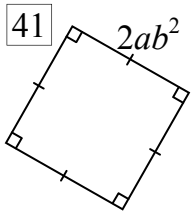
38 $-kt^4y \times (-2k)$ ___

39 $-b \times (-b) \times (-q)$ ___

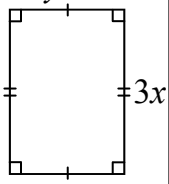
40 $-ab^3c^2d \times c^2d$ ___

Give an algebraic expression for the areas of these shapes.

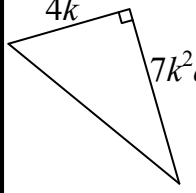
41 $2ab^2$ $A = L^2$



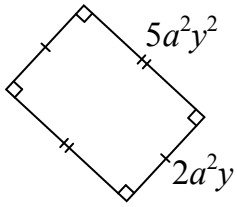
42 yx^2 $3x$



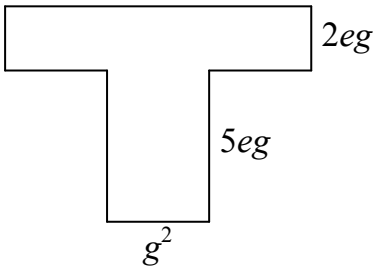
43 $4k$ $7k^2d$



44 $5a^2y^2$ $2a^2y$



44 $3g^2$ $2eg$ $5eg$ g^2



A = ___ + ___

A = ___

Multiplying Algebraic Terms - Powers

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Simplify these.

Example $k \times k \times k \times a$ k^3a

1 $x \times x$ x^2 2 $x \times x \times x$ x^3

3 $a \times a \times a \times a \times a$ a^5

4 $m \times m \times e \times e$ m^2e^2

5 $w \times x \times x \times x \times x$ wx^4

6 $t \times t \times g \times g \times t \times g$ t^3g^3

7 $2 \times a \times a \times b \times c$ $2a^2bc$

8 $5 \times q \times q \times q \times t$ $5q^3t$

9 $m \times m \times 3 \times m \times d$ $3m^3d$

10 $4 \times y \times y \times (-2) \times t$ $-8y^2t$

Rewrite these in expanded form.

Example $6ma^2$ $6 \times m \times a \times a$

11 k^3 $k \times k \times k$

12 c^5 $c \times c \times c \times c \times c$

13 $3ad^3$ $3 \times a \times d \times d \times d$

14 $7e^2k^2$ $7 \times e \times e \times k \times k$

15 $-4ay^3$ $-4 \times a \times y \times y \times y$

16 $-10b^2c$ $-10 \times b \times b \times c$

Join the matching pairs

17 Join a dot on the left to a dot on the right

x^2y	$2 \times x \times x \times 2$
$2xy^2$	$-2 \times x \times -2 \times -2$
x^2y^2	$x \times y \times x$
$4x^2$	$y \times x \times y \times -2$
$-2xy^2$	$2 \times x \times y \times 2$
$4xy$	$y \times x \times x \times y$
$8xy^2$	$x \times 2 \times y \times y$
$-8x$	$y \times x \times 8 \times y$

Multiply these terms. Write numbers first and then add the powers on common terms.

Example $4k^2m \times 2mk$ $8k^3m^2$

18 $r^2 \times r^3$ r^5 19 $c^5 \times c$ c^6

20 $a^2 \times a^7$ a^9 21 $n \times n^3$ n^4

22 $e^2 \times eh^2$ e^3h^2 23 $a^3 \times ca$ a^4c

24 $ef \times fe^2$ e^3f^2 25 $xy \times y$ xy^2

26 $u^2t \times t^2uv^3$ $u^3t^3v^3$

27 $su \times t^3o \times u^2st$ $s^2u^3t^4o$

28 $2a^3b \times ab$ $2a^4b^2$

29 $t^4e^6 \times 4t^2e^3$ $4e^9t^6$

30 $2mq \times 2q^3m$ $4m^2q^4$

31 $7n^2s^3 \times 3sn^2$ $21n^4s^4$

32 $ab^2c \times 3a^4bc^2$ $3a^5b^3c^3$

33

×	$6a^4$	a^3b	$3ab^2$
a	$6a^5$	a^4b	$3a^2b^2$
b^2a	$6a^5b^2$	a^4b^3	$3a^2b^4$
$2a^2$	$12a^6$	$2a^5b$	$6a^3b^2$
$5b^3a^2$	$30a^6b^3$	$5a^5b^4$	$15a^3b^5$

Now negatives are involved.

34 $2c^2b \times (-bc)$ $-2c^3b^2$

35 $-gt \times (-g) \times (-t^5)$ $-g^2t^6$

36 $-q \times (-qv) \times v^2$ q^2v^3

37 $9e^2 \times (-4ce)$ $-36e^3c$

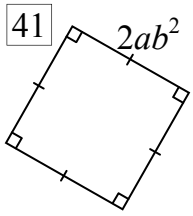
38 $-kt^4y \times (-2k)$ $2k^2t^4y$

39 $-b \times (-b) \times (-q)$ $-b^2q$

40 $-ab^3c^2d \times c^2d$ $-ab^3c^4d^2$

Give an algebraic expression for the areas of these shapes.

41

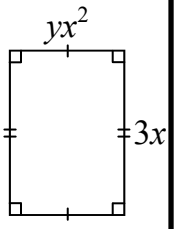


$A = L^2$

$= 2ab^2 \times 2ab^2$

$A = 4a^2b^4$ units^2

42

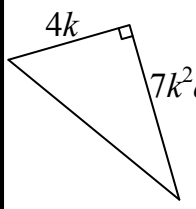


$A = LB$

$= 3x \times yx^2$

$A = 3yx^3$ units^2

43

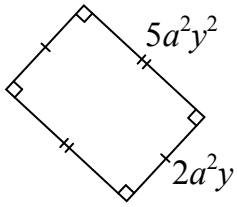


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 4k \times 7k^2d$

$A = 14k^3d$ units^2

44

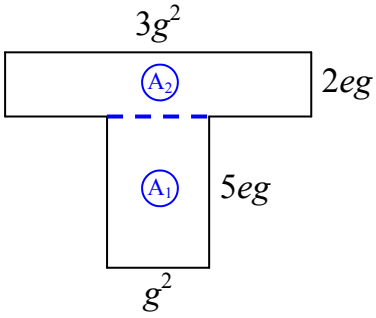


$A = LB$

$= 5a^2y^2 \times 2a^2y$

$A = 10a^4y^3$ units^2

44



$A_1 = LB$ $A_2 = LB$

$= 5eg \times g^2$ $= 2eg \times 3g^2$

$A_1 = 5eg^3$ units^2 $A_1 = 6eg^3$ units^2

$A = 5eg^3 + 6eg^3$

$A = 11eg^3$ units^2

Dividing Algebraic Terms

Division can be shown with a '÷' or a vinculum. A vinculum is a horizontal line that separates the numerator (top part of a fraction) from the denominator (bottom part). So you have to know both ways that a division can be written.

Example

If terms are separated then the ÷ only applies to the term after it.

Brackets gather the terms and so both terms become the denominator

$a \div b = \frac{a}{b}$ $\quad 2 \div ab = \frac{2}{ab}$ $\quad 2 \div a \times b = \frac{2b}{a}$ $\quad 2 \div (a \times b) = \frac{2}{ab}$

The opposite process is covered further down Column 1, it also requires you to complete an expansion, so all numbers and pronumerals are separated by × signs.

Example

If a pronumeral has a power on it, then remove the power and expand.

$\frac{a}{b} = a \div b$ $\quad \frac{2a}{b} = 2 \times a \div b$ $\quad \frac{5qv^2}{2m} = 5 \times q \times v \times v \div (2 \times m)$

Brackets are needed to group the $2 \times m$.

Simplifying algebraic expressions means reducing the expression to its most simplest form. The column starts with the pronumerals being ignored (as they can't be simplified) and concentrating solely on the numbers. An example is at the top of the column. You have to find the HCF (highest common factor) and divide both top and bottom numbers by the HCF. You don't write '1' except in the top line if there are no pronumerals in that line.

The second part of Column 2 allows the pronumerals to be simplified but not the numbers. So the numbers stay the same and the letters are cancelled top and bottom. There are two Examples above Q. 26. Note that squared terms have the power removed but the letter stays.

Column 3 brings the top operations of Column 2 together. So you have to simplify both the numbers and the pronumerals in the questions. Examples are at the top of the column, remember to simplify the numbers first, then cancel the pronumerals.

Dividing Algebraic Terms

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Write these expressions without the ÷ sign.

1 $e \div 4 = \frac{\square}{\square}$

2 $d \div j = \frac{\square}{\square}$

3 $5 \div k = \frac{\square}{\square}$

4 $q \div k^2 = \frac{\square}{\square}$

5 $b^3 \div f = \frac{\square}{\square}$

6 $c \div 2i = \frac{\square}{\square}$

7 $wm \div t \times 5 = \frac{\square}{\square}$

8 $wm \div 5t = \frac{\square}{\square}$

9 $wm \div (t \times 5) = \frac{\square}{\square}$

10 $5 \times t \div 7 \times a = \frac{\square}{\square}$

Now write using ÷ signs, then expand your answer.

Example

$\frac{5y}{3x} = 5y \div 3x = 5 \times y \div (3 \times x)$ *expanded*

11 $\frac{2w}{y} = \_\_\_\_ = \_\_\_\_$ *expanded*

12 $\frac{d}{nb} = \_\_\_\_ = \_\_\_\_$ *expanded*

13 $\frac{3}{5a^2} = \_\_\_\_ = \_\_\_\_$ *expanded*

14 $\frac{2vc}{9m^2} = \_\_\_\_$

$= \_\_\_\_$ *expanded*

15 $\frac{7c^2b}{8yw^2} = \_\_\_\_$

$= \_\_\_\_$ *expanded*

The numbers in these questions can be simplified.

Example $\frac{\overset{1}{\cancel{5}}f}{\underset{2}{\cancel{10}}e} = \frac{f}{2e}$ ($\div 5$, $\div 5$)

16 $\frac{6k}{8m} = \frac{\square}{\square}$

17 $\frac{9c}{6} = \frac{\square}{\square}$

18 $\frac{10}{12x} = \frac{\square}{\square}$

19 $\frac{4t}{8} = \frac{\square}{\square}$

20 $\frac{9w}{15ay} = \frac{\square}{\square}$

21 $\frac{6tu^2}{3} = \frac{\square}{\square}$

22 $\frac{5r^2}{25d} = \frac{\square}{\square}$

23 $\frac{12e^2}{18h} = \frac{\square}{\square}$

24 $\frac{8xy^2}{2} = \frac{\square}{\square}$

25 $\frac{12kd}{15z} = \frac{\square}{\square}$

Numbers can't simplify, this time it's the pronumerals.

Examples

$\frac{3\cancel{f}}{7e\cancel{f}} = \frac{3}{7e}$ $\qquad \frac{5\cancel{a}b^{\cancel{2}\,1}}{9\cancel{a}\cancel{b}} = \frac{5b}{9}$

26 $\frac{4m}{5m} = \frac{\square}{\square}$

27 $\frac{3c}{7ec} = \frac{\square}{\square}$

28 $\frac{2d^2}{d} = \frac{\square}{\square}$

29 $\frac{9fg}{7g^2} = \frac{\square}{\square}$

30 $\frac{t}{3t} = \frac{\square}{\square}$

31 $\frac{6q^2y}{11q} = \frac{\square}{\square}$

32 $\frac{abc}{ab^2c} = \frac{\square}{\square}$

33 $\frac{x^2}{5x} = \frac{\square}{\square}$

34 $\frac{8a^2n}{n^2a} = \frac{\square}{\square}$

35 $\frac{x^2y^2}{xw} = \frac{\square}{\square}$

Watch for both numbers and pronumerals in these. Simplify.

Examples

$\frac{\overset{1}{\cancel{3}}\cancel{f}g}{\underset{3}{\cancel{9}}e\cancel{f}} = \frac{g}{3e}$ $\qquad \frac{\overset{5}{\cancel{20}}\cancel{v}r^{\cancel{2}\,1}}{\underset{1}{\cancel{4}}\cancel{v}\cancel{r}} = 5r$

36 $\frac{4t}{6t} = \frac{\square}{\square}$

37 $\frac{10k}{2kn} = \frac{\square}{\square}$

38 $\frac{3ab}{12b} = \frac{\square}{\square}$

39 $\frac{15h}{5h^2} = \frac{\square}{\square}$

40 $\frac{4a^2v}{6v^2} = \frac{\square}{\square}$

41 $\frac{8abc}{2ab} = \frac{\square}{\square}$

42 $\frac{3a}{6a^2} = \frac{\square}{\square}$

43 $\frac{24xy}{36x} = \frac{\square}{\square}$

44 $20by^2 \div 4y = \frac{\square}{\square}$

45 $\frac{15c}{9vc^2} = \frac{\square}{\square}$

46 $\frac{10bx}{16ab} = \frac{\square}{\square}$

47 $\frac{6q^2}{2dq} = \frac{\square}{\square}$

48 $\frac{18a^2}{12ab} = \frac{\square}{\square}$

49 $5xt \div 25x^2 = \frac{\square}{\square}$

50 $\frac{6e^2c}{2ec} = \frac{\square}{\square}$

51 $\frac{8cv^2}{18v} = \frac{\square}{\square}$

52 $6xy^2z \div 14x^2yz = \frac{\square}{\square}$

53 $8a^2b \div 16ab^2c = \frac{\square}{\square}$

Dividing Algebraic Terms

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Write these expressions without the ÷ sign.

1 $e \div 4 = \frac{e}{4}$

2 $d \div j = \frac{d}{j}$

3 $5 \div k = \frac{5}{k}$

4 $q \div k^2 = \frac{q}{k^2}$

5 $b^3 \div f = \frac{b^3}{f}$

6 $c \div 2i = \frac{c}{2i}$

7 $wm \div t \times 5 = \frac{5wm}{t}$

8 $wm \div 5t = \frac{wm}{5t}$

9 $wm \div (t \times 5) = \frac{wm}{5t}$

10 $5 \times t \div 7 \times a = \frac{5ta}{7}$

Now write using ÷ signs, then expand your answer.

Example

$\frac{5y}{3x} = 5y \div 3x = 5 \times y \div (3 \times x)$ *expanded*

11 $\frac{2w}{y} = 2w \div y = 2 \times w \div y$ *expanded*

12 $\frac{d}{nb} = d \div nb = d \div (n \times b)$ *expanded*

13 $\frac{3}{5a^2} = 3 \div 5a^2 = 3 \div (5 \times a \times a)$ *expanded*

14 $\frac{2vc}{9m^2} = 2vc \div 9m^2 = 2 \times v \times c \div (9 \times m \times m)$ *expanded*

15 $\frac{7c^2b}{8yw^2} = 7c^2b \div 8yw^2 = 7 \times c \times c \times b \div (8 \times y \times w \times w)$ *expanded*

Numbers in these questions can be simplified. Letters stay as is.

Example $\frac{\overset{1}{\cancel{5}f}}{\underset{2}{\cancel{10}e}} \overset{\div 5}{\underset{\div 5}{=}} \frac{f}{2e}$

16 $\frac{6k}{8m} = \frac{3k}{4m}$

17 $\frac{9c}{6} = \frac{3c}{2}$

18 $\frac{10}{12x} = \frac{5}{6x}$

19 $\frac{4t}{8} = \frac{t}{2}$

20 $\frac{9w}{15ay} = \frac{3w}{5ay}$

21 $\frac{6tu^2}{3} = 2tu^2$

22 $\frac{5r^2}{25d} = \frac{r^2}{5d}$

23 $\frac{12e^2}{18h} = \frac{2e^2}{3h}$

24 $\frac{8xy^2}{2} = 4xy^2$

25 $\frac{12kd}{15z} = \frac{4kd}{5z}$

Numbers can't simplify, this time it's the pronumerals.

Examples

$\frac{3\cancel{f}}{7e\cancel{f}} = \frac{3}{7e}$ $\qquad \frac{5\cancel{ab}^{1}}{9\cancel{ab}} = \frac{5b}{9}$

26 $\frac{4m}{5m} = \frac{4}{5}$

27 $\frac{3c}{7ec} = \frac{3}{7e}$

28 $\frac{2d^2}{d} = 2d$

29 $\frac{9fg}{7g^2} = \frac{9f}{7g}$

30 $\frac{t}{3t} = \frac{1}{3}$

31 $\frac{6q^2y}{11q} = \frac{6qy}{11}$

32 $\frac{abc}{ab^2c} = \frac{1}{b}$

33 $\frac{x^2}{5x} = \frac{x}{5}$

34 $\frac{8a^2n}{n^2a} = \frac{8a}{n}$

35 $\frac{x^2y^2}{xw} = \frac{xy^2}{w}$

Watch for both numbers and pronumerals in these. Simplify.

Examples

$\frac{\overset{1}{\cancel{3}fg}}{\underset{3}{\cancel{9}ef}} = \frac{g}{3e}$ $\qquad \frac{\overset{5}{\cancel{20}}vr^{\overset{1}{\cancel{2}}}}{\underset{1}{\cancel{4}}\cancel{xv}} = 5r$

36 $\frac{4t}{6t} = \frac{2}{3}$

37 $\frac{10k}{2kn} = \frac{5}{n}$

38 $\frac{3ab}{12b} = \frac{a}{4}$

39 $\frac{15h}{5h^2} = \frac{3}{h}$

40 $\frac{4a^2v}{6v^2} = \frac{2a^2}{3v}$

41 $\frac{8abc}{2ab} = 4c$

42 $\frac{3a}{6a^2} = \frac{1}{2a}$

43 $\frac{24xy}{36x} = \frac{2y}{3}$

44 $20by^2 \div 4y = 5by$

45 $\frac{15c}{9vc^2} = \frac{5}{3vc}$

46 $\frac{10bx}{16ab} = \frac{5x}{8a}$

47 $\frac{6q^2}{2dq} = \frac{3q}{d}$

48 $\frac{18a^2}{12ab} = \frac{3a}{2b}$

49 $5xt \div 25x^2 = \frac{t}{5x}$

50 $\frac{6e^2c}{2ec} = 3e$

51 $\frac{8cv^2}{18v} = \frac{4cv}{9}$

52 $6xy^2z \div 14x^2yz = \frac{3y}{7x}$

53 $8a^2b \div 16ab^2c = \frac{a}{2bc}$

Further Dividing Algebraic Terms - With Powers

This sheet, particularly the middle and last columns are difficult in concept for some students. When you multiply like terms with powers, you add the powers. Division is the opposite to multiplication, so as you would expect, when you divide like terms with powers then you subtract the powers.

$$a^x \div a^n = a^{x-n} \qquad or \qquad \frac{a^x}{a^n} = a^{x-n}$$

How do you apply the above? Here are some examples.

$$\frac{a^7}{a^2} = a^5 \qquad k^9 \div k^4 = k^5 \qquad \frac{a^2}{a^5} = \frac{1}{a^3}$$

When there is more than one pronumeral, then they are cancelled against the same pronumeral. Examples.

$$\frac{cn^7}{n^2} = cn^5 \qquad a^3b^9 \div a^2b^7 = ab^2 \qquad \frac{x^3ya^2}{x^5a^9} = \frac{y}{x^2a^7}$$

Also numbers can be included. While you subtract powers, when the coefficients (numbers) are divided, they are divided … not subtracted. Examples.

$$\frac{4n^7}{2n^2} = 2n^5 \qquad 6a^3b^9 \div 12a^2b^7 = \frac{ab^2}{2} \qquad \frac{16x^3y}{-4xy^9} = \frac{-4x^2}{y^8}$$

Column 1 starts with expanding expressions and cancelling like terms. This is another way of approaching the question. So expand both the numerator and denominator (top and bottom) and then strike out matching term, terms that are in both the top and bottom rows, one for one. What is left is the answer, if you have nothing left then write '1'.

From Q. 5 apply the index law outlined above, by subtracting the powers. There are no coefficients (numbers) in these questions just pronumerals (letters).

Column 2 introduces coefficients and then negative coefficients. So remember to divide the numbers and subtract the powers.

Column 3 reverses the process and asks you to place the answer in the box that makes the statement true. These can be very challenging so concentrate when you attempt these. The final part of the column is the same as column 2 only with more terms.

Further Dividing Algebraic Terms - With Powers

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Show the expansion then strike out the letters to simplify.

Example

$\frac{e^5}{e^2} = \frac{\cancel{e} \times \cancel{e} \times e \times e \times e}{\cancel{e} \times \cancel{e}} = e^3$

1 $\frac{k^4}{k} = \frac{\quad}{\quad} =$ ____

2 $\frac{a^5}{a^4} = \frac{\quad}{\quad} =$ ____

3 $\frac{x^2}{x^4} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

4 $\frac{c^2a^3}{ca} = \frac{\quad}{\quad} =$ ____

When you divide (÷), subtract the powers. Simplify these.

Examples

$n^7 \div n^4 = n^{7-4}$
$= n^3$

$\frac{an^5}{a^6n^2} = \frac{n^3}{a^5}$

5 $\frac{k^4}{k^2} =$ ____ 6 $\frac{a^5}{a^4} =$ ____

7 $b^5 \div b^2 =$ ____

8 $\frac{v^2}{v^7} = \frac{\quad}{\quad}$ 9 $\frac{d^3}{d^{10}} = \frac{\quad}{\quad}$

10 $h^2 \div h^8 = \frac{\quad}{\quad}$ 11 $i^5 \div i^9 = \frac{\quad}{\quad}$

12 $\frac{r^6}{r^5} =$ ____ 13 $\frac{m^3}{m^9} = \frac{\quad}{\quad}$

14 $\frac{e^5t^2}{e^2t} =$ ____ 15 $\frac{a^2b^4}{ab^7} = \frac{\quad}{\quad}$

16 $\frac{xy^3}{x^2y^4} = \frac{\quad}{\quad}$ 17 $\frac{c^3k^2}{k^7c} = \frac{\quad}{\quad}$

18 $\frac{hu^2c}{hc^2} = \frac{\quad}{\quad}$ 19 $\frac{z^2x^2}{z^3kx^5} = \frac{\quad}{\quad}$

20 $\frac{d^3y^3}{ay^2} = \frac{\quad}{\quad}$ 21 $\frac{xy^2z}{xyz} =$ ____

22 $q^4tp^2 \div q^7p = \frac{\quad}{\quad}$

These include numbers. Simplify these.

Examples

$8n^7 \div 2n^4$
$= 4n^3$

$\frac{6an^5}{14a^6n^2} = \frac{3n^3}{7a^5}$

23 $\frac{4y^3}{y^2} =$ ____ 24 $\frac{6}{2a^2} = \frac{\quad}{\quad}$

25 $2w^2 \div 4w^5 = \frac{\quad}{\quad}$

26 $\frac{10n^4}{5n} =$ ____ 27 $\frac{3xm}{9m^2} = \frac{\quad}{\quad}$

28 $8t^2 \div 12st^2 = \frac{\quad}{\quad}$

29 $16j^5p^2 \div 4j^4 =$ ____

30 $\frac{5c^6}{20nc^2} = \frac{\quad}{\quad}$ 31 $\frac{18ey}{12a^2} = \frac{\quad}{\quad}$

32 $\frac{5r^3x}{5r^3} =$ ____ 33 $\frac{35k}{20k^2} = \frac{\quad}{\quad}$

Watch for negatives. Simplify these.

34 $\frac{-6r^3d^2}{3r^2d^3} = \frac{\quad}{\quad}$

35 $9a^3y^5 \div 3y^2a =$ ____

36 $\frac{-2x^5y}{12x^6y} = \frac{\quad}{\quad}$

37 $2u^2v^4x \div (-14v^6ux^2) = \frac{\quad}{\quad}$

38 $-12q^2d^4 \div (-dq^2) =$ ____

39 $-a^5x^3j \div 2a^6ux^3 = \frac{\quad}{\quad}$

40 $15b^5e^3t \div (-45bt^3) = \frac{\quad}{\quad}$

41 $\frac{-50p^2m^7}{-35e^2m^4p} = \frac{\quad}{\quad}$

42 $\frac{2a^3bc^4}{-8a^6b^2c^3} = \frac{\quad}{\quad}$

Find the term that goes into the box.

43 $\frac{2c^3}{\quad} = 2c$ 44 $\frac{w^4}{\quad} = \frac{w^3}{m}$

45 $\frac{\quad}{3a} = 4$ 46 $\frac{4e^5}{\quad} = \frac{e^3}{2}$

47 $\frac{\quad}{\quad} \div 2k = 3k$

48 $6a^2b \div \frac{\quad}{\quad} = 3ab$

49 $\frac{\quad}{\quad} \div y = \frac{3x}{y}$

50 $\frac{\quad}{2j^3} = \frac{7}{j}$

51 $\frac{5ux}{\quad} = \frac{1}{2w}$

52 $\frac{\quad}{\quad} \div 4v^2y = \frac{-5y^3}{2v}$

53 $\frac{\quad}{-2hg^3} = \frac{h}{g}$

54 $16x^3k \div \frac{\quad}{\quad} = \frac{-8}{3x}$

55 $\frac{\quad}{\quad} \div 2y^2 = 9x$

56 $\frac{-8ka^3}{\quad} = \frac{2k}{a^3}$

57 $\frac{\quad}{\quad} \div 4ey^4 = \frac{-1}{y^2}$

These are larger, but the same process applies. Simplify.

58 $\frac{12c^8y^3d^2}{28kdy^5c^5} = \frac{\quad}{\quad}$

59 $\frac{54t^5b^3a}{36b^2t^3w} = \frac{\quad}{\quad}$

60 $\frac{-q^2e^3y^7}{-3b^2q^6ye^5} = \frac{\quad}{\quad}$

61 $\frac{-18fdh^3}{14hd^4} = \frac{\quad}{\quad}$

Further Dividing Algebraic Terms - With Powers

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Show the expansion then strike out the letters to simplify.

Example

$\frac{e^5}{e^2} = \frac{\cancel{e} \times \cancel{e} \times e \times e \times e}{\cancel{e} \times \cancel{e}} = e^3$

1 $\frac{k^4}{k} = \frac{k \times k \times k \times k}{k} = k^3$

2 $\frac{a^5}{a^4} = \frac{a \times a \times a \times a \times a}{a \times a \times a \times a} = a$

3 $\frac{x^2}{x^4} = \frac{x \times x}{x \times x \times x \times x} = \frac{1}{x^2}$

4 $\frac{c^2a^3}{ca} = \frac{c \times c \times a \times a \times a}{c \times a} = ca^2$

When you divide (÷), subtract the powers. Simplify these.

Examples

$n^7 \div n^4 = n^{7-4} = n^3$

$\frac{an^5}{a^6n^2} = \frac{n^3}{a^5}$

5 $\frac{k^4}{k^2} = k^2$

6 $\frac{a^5}{a^4} = a$

7 $b^5 \div b^2 = b^3$

8 $\frac{v^2}{v^7} = \frac{1}{v^5}$

9 $\frac{d^3}{d^{10}} = \frac{1}{d^7}$

10 $h^2 \div h^8 = \frac{1}{h^6}$

11 $i^5 \div i^9 = \frac{1}{i^4}$

12 $\frac{r^6}{r^5} = r$

13 $\frac{m^3}{m^9} = \frac{1}{m^6}$

14 $\frac{e^5t^2}{e^2t} = e^3t$

15 $\frac{a^2b^4}{ab^7} = \frac{a}{b^3}$

16 $\frac{xy^3}{x^2y^4} = \frac{1}{xy}$

17 $\frac{c^3k^2}{k^7c} = \frac{c^2}{k^5}$

18 $\frac{hu^2c}{hc^2} = \frac{u^2}{c}$

19 $\frac{z^2x^2}{z^3kx^5} = \frac{1}{zkx^3}$

20 $\frac{d^3y^3}{ay^2} = \frac{d^3y}{a}$

21 $\frac{xy^2z}{xyz} = y$

22 $q^4tp^2 \div q^7p = \frac{tp}{q^3}$

These include numbers. Simplify these.

Examples

$8n^7 \div 2n^4 = 4n^3$

$\frac{6an^5}{14a^6n^2} = \frac{3n^3}{7a^5}$

23 $\frac{4y^3}{y^2} = 4y$

24 $\frac{6}{2a^2} = \frac{3}{a^2}$

25 $2w^2 \div 4w^5 = \frac{1}{2w^3}$

26 $\frac{10n^4}{5n} = 2n^3$

27 $\frac{3xm}{9m^2} = \frac{x}{3m}$

28 $8t^2 \div 12st^2 = \frac{2}{3s}$

29 $16j^5p^2 \div 4j^4 = 4jp^2$

30 $\frac{5c^6}{20nc^2} = \frac{c^4}{4n}$

31 $\frac{18ey}{12a^2} = \frac{3ey}{2a^2}$

32 $\frac{5r^3x}{5r^3} = x$

33 $\frac{35k}{20k^2} = \frac{7}{4k}$

Watch for negatives. Simplify these.

34 $\frac{-6r^3d^2}{3r^2d^3} = \frac{-2r}{d}$

35 $9a^3y^5 \div 3y^2a = 3a^2y^3$

36 $\frac{-2x^5y}{12x^6y} = \frac{-1}{6x}$

37 $2u^2v^4x \div (-14v^6ux^2) = \frac{-u}{7v^2x}$

38 $-12q^2d^4 \div (-dq^2) = 12d^3$

39 $-a^5x^3j \div 2a^6ux^3 = \frac{-j}{2au}$

40 $15b^5e^3t \div (-45bt^3) = \frac{-b^4e^3}{3t^2}$

41 $\frac{-50p^2m^7}{-35e^2m^4p} = \frac{10pm^3}{7e^2}$

42 $\frac{2a^3bc^4}{-8a^6b^2c^3} = \frac{-c}{4a^3b}$

Find the term that goes into the box.

43 $\frac{2c^3}{c^2} = 2c$

44 $\frac{w^4}{wm} = \frac{w^3}{m}$

45 $\frac{12a}{3a} = 4$

46 $\frac{4e^5}{8e^2} = \frac{e^3}{2}$

47 $6k^2 \div 2k = 3k$

48 $6a^2b \div 2a = 3ab$

49 $3x \div y = \frac{3x}{y}$

50 $\frac{14j^2}{2j^3} = \frac{7}{j}$

51 $\frac{5ux}{10uxw} = \frac{1}{2w}$

52 $-10vy^4 \div 4v^2y = \frac{-5y^3}{2v}$

53 $\frac{-2h^2g^2}{-2hg^3} = \frac{h}{g}$

54 $16x^3k \div -6x^4k = \frac{-8}{3x}$

55 $18xy^2 \div 2y^2 = 9x$

56 $\frac{-8ka^3}{-4a^6} = \frac{2k}{a^3}$

57 $-4ey^2 \div 4ey^4 = \frac{-1}{y^2}$

These are larger, but the same process applies. Simplify.

58 $\frac{12c^8y^3d^2}{28kdy^5c^5} = \frac{3c^3d}{7ky^2}$

59 $\frac{54t^5b^3a}{36b^2t^3w} = \frac{3t^2ba}{2w}$

60 $\frac{-q^2e^3y^7}{-3b^2q^6ye^5} = \frac{y^6}{3q^4e^2}$

61 $\frac{-18fdh^3}{14hd^4} = \frac{-9fh^2}{7d^3}$

Multiplying and Dividing Algebraic Terms - With Powers

When you multiply indices you add the powers. When you divide indices you subtract the powers. Here are some examples.

Multiplication:

$r^2 \times r^3 = r^5$

Add the powers
2 + 3 = 5

$e^2 \times e = e^3$

If there is no power on a letter, it is '1'.
2 + 1 = 3

$e^2r^3 \times er^4 = e^3r^7$

Two or more letters are answered in the same way.

$2e^2r^3 \times 3r^4e = 6e^3r^7$

Coefficients (numbers) are still multiplied, not added.
2 × 3 = 6

Division:

$r^5 \div r^2 = r^3$

Subtract the powers
5 - 2 = 3

$e^2 \div e = e$

If there is no power on a letter, it is '1'.
2 - 1 = 1

$\frac{e^4r^3}{e^2r} = e^2r^2$

Questions can use the 'over' bar called a vinculum

$\frac{e^2r^3}{e^9r^7} = \frac{1}{e^7r^4}$

Subtract the highest from the lowest, so letters can be removed from the top line, leaving a numerator of 1.

$10e^8 \div 5e^2 = 2e^6$

Coefficients (numbers) are still divided, not subtracted.
10 ÷ 5 = 2

$\frac{2e^2r^3}{8e^9r^7} = \frac{1}{4e^7r^4}$

Denominators can be larger than numerators

$\frac{8e^2r^3}{2e^9r^7} = \frac{4}{e^7r^4}$

Numerators can be larger than denominators

Column 1 starts with multiplications and divisions that have no powers greater than 2, so this has already been experienced with previous sheets. These are two stage questions. First multiply the top row and bottom row. Then divide.

Column 2 then introduces multiplication involving powers. Remember to add the powers but multiply coefficients (if there are any). Q. 23 reminds you of showing indices in expanded form, separate the term and place '×' signs between each pronumeral/coefficient. Q. 28 introduces division, remember to subtract powers but still divide coefficients.

Column 3 from Q. 41 blends multiplication with division. Complete the multiplication first by multiplying across the top row, then the bottom. Write the answer in the first boxes, then divide for the final answer.

Multiplying and Dividing Algebraic Terms - With Powers

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Multiply, then divide these.

Example $\dfrac{4n \times 3}{2n} = \dfrac{\overset{6}{\cancel{12}}\cancel{n}}{\underset{1}{\cancel{2}}\cancel{n}} = 6$

1 $\dfrac{5k \times 4}{2} = \dfrac{\quad}{\quad} = \quad$

2 $\dfrac{4 \times a}{2a \times 2} = \dfrac{\quad}{\quad} = \quad$

3 $\dfrac{n \times m}{2 \times m} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

4 $\dfrac{e \times 9}{e \times e} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

5 $\dfrac{x \times 9 \times b}{x \times 3} = \dfrac{\quad}{\quad} = \quad$

6 $\dfrac{2e \times 6k}{e \times 8} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

7 $\dfrac{5t \times 4t}{g \times 2t} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

8 $\dfrac{4d \times 8n}{2n \times 2d} = \dfrac{\quad}{\quad} = \quad$

9 $\dfrac{t \times 3t \times h}{h \times 2h \times t} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

10 $\dfrac{5n \times 4b \times 3b}{2b \times 6n} = \dfrac{\quad}{\quad}$

$= \quad$

11 $\dfrac{2e \times y \times 3h}{4h \times 2e} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

When you multiply you add powers. Multiply these.

Examples $am^3 \times a^2m^2 = a^3m^5$

$m^3 \times m^2 = m^5$ $\quad 2m^3 \times 3m^2 = 6m^5$

12 $d^3 \times d^5 = \quad$

13 $w^3 \times w^7 \times w^2 = \quad$

14 $f^3 \times f^5 \times f^3 = \quad$

15 $c \times a^3 \times c^2 \times a^5 = \quad$

16 $2e^2 \times 4e^4 = \quad$

17 $3r^3 \times 5r^4 = \quad$

18 $ab^3 \times a^2b^4 = \quad$

19 $x^2yz^3 \times x^5y^2z = \quad$

20 $2uv \times 5vu^2 = \quad$

21 $3y^3b^2 \times 6b^4y = \quad$

22 $5ac^3 \times 2a^4b^4 = \quad$

Expand these terms.

Example x^3: $x \times x \times x$

23 a^5: $\quad$

24 k^4: $\quad$

25 $4n^4$: $\quad$

26 $\dfrac{qe^3}{a^2} = \dfrac{\quad}{\quad}$

27 $\dfrac{2nt^2}{3n^4} = \dfrac{\quad}{\quad}$

When you divide you subtract powers. Complete these.

Examples

$m^3 \div m^2 = m$ $\quad \dfrac{8m^7}{2m^5} = 4m^2$

28 $a^8 \div a^2 = \quad$

29 $e^6 \div e^5 = \quad$

30 $k^6 \div k^3 \div k^2 = \quad$

31 $c^3x^6 \div c^2x^3 = \quad$

32 $n^2m^3 \div m^2n^2 = \quad$

33 $\dfrac{e^7v^2}{e^6v^7} = \dfrac{\quad}{\quad}$ $\quad$ 34 $\dfrac{y^2z^3}{z^5} = \dfrac{\quad}{\quad}$

35 $\dfrac{6m^5}{3m^2} = \quad$ $\quad$ 36 $\dfrac{12a^3}{4a^5} = \dfrac{\quad}{\quad}$

37 $\dfrac{2x}{8x^3} = \dfrac{\quad}{\quad}$ $\quad$ 38 $\dfrac{14n^7}{6n^3} = \dfrac{\quad}{\quad}$

39 $\dfrac{9e^2t^5}{3et^3} = \quad$ $\quad$ 40 $\dfrac{4xp^3}{2x^4p} = \dfrac{\quad}{\quad}$

Multiply first, then divide for the answer.

41 $\dfrac{v^5 \times v^7}{v^3 \times v^6} = \dfrac{\quad}{\quad} = \quad$

42 $\dfrac{w^2 \times w^4}{w \times w^3} = \dfrac{\quad}{\quad} = \quad$

43 $\dfrac{r^5 \times r}{r^3 \times r^4} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

44 $\dfrac{3q^2 \times 2q^5}{2q \times q^4} = \dfrac{\quad}{\quad} = \quad$

45 $\dfrac{5ay \times 2y^3}{4a^3y \times ay^4} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

46 $\dfrac{2ge^3 \times g^4e^2}{5g^6 \times 4e^4} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

47 $\dfrac{3b^2t \times 4v}{2t^3v \times 3vb^3} = \dfrac{\quad}{\quad} = \dfrac{\quad}{\quad}$

48 $\dfrac{2abc \times 8b^3c}{4ac^2 \times b^4} = \dfrac{\quad}{\quad} = \quad$

Multiplying and Dividing Algebraic Terms - With Powers

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Multiply, then divide these.

Example $\frac{4n \times 3}{2n} = \frac{12n}{2n} = 6$

1 $\frac{5k \times 4}{2} = \frac{20k}{2} = 10k$

2 $\frac{4 \times a}{2a \times 2} = \frac{4a}{4a} = 1$

3 $\frac{n \times m}{2 \times m} = \frac{nm}{2m} = \frac{n}{2}$

4 $\frac{e \times 9}{e \times e} = \frac{9e}{e^2} = \frac{9}{e}$

5 $\frac{x \times 9 \times b}{x \times 3} = \frac{9xb}{3x} = 3b$

6 $\frac{2e \times 6k}{e \times 8} = \frac{12ek}{8e} = \frac{3k}{2}$

7 $\frac{5t \times 4t}{g \times 2t} = \frac{20t^2}{2gt} = \frac{10t}{g}$

8 $\frac{4d \times 8n}{2n \times 2d} = \frac{32dn}{4dn} = 8$

9 $\frac{t \times 3t \times h}{h \times 2h \times t} = \frac{3t^2h}{2th^2} = \frac{3t}{2h}$

10 $\frac{5n \times 4b \times 3b}{2b \times 6n} = \frac{60nb^2}{12nb} = 5b$

11 $\frac{2e \times y \times 3h}{4h \times 2e} = \frac{6eyh}{8eh} = \frac{3y}{4}$

When you multiply you add powers. Multiply these.

Examples $am^3 \times a^2m^2 = a^3m^5$

$m^3 \times m^2 = m^5$ $2m^3 \times 3m^2 = 6m^5$

12 $d^3 \times d^5 = d^8$

13 $w^3 \times w^7 \times w^2 = w^{12}$

14 $f^3 \times f^5 \times f^3 = f^{11}$

15 $c \times a^3 \times c^2 \times a^5 = c^3a^8$

16 $2e^2 \times 4e^4 = 8e^6$

17 $3r^3 \times 5r^4 = 15r^7$

18 $ab^3 \times a^2b^4 = a^3b^7$

19 $x^2yz^3 \times x^5y^2z = x^7y^3z^4$

20 $2uv \times 5vu^2 = 10u^3v^2$

21 $3y^3b^2 \times 6b^4y = 18y^4b^6$

22 $5ac^3 \times 2a^4b^4 = 10a^5b^4c^3$

Expand these terms.

Example x^3: $x \times x \times x$

23 a^5: $a \times a \times a \times a \times a$

24 k^4: $k \times k \times k \times k$

25 $4n^4$: $4 \times n \times n \times n \times n$

26 $\frac{qe^3}{a^2} = \frac{q \times e \times e \times e}{a \times a}$

27 $\frac{2nt^2}{3n^4} = \frac{2 \times n \times t \times t}{3 \times n \times n \times n \times n}$

When you divide you subtract powers. Complete these.

Examples $m^3 \div m^2 = m$ $\frac{8m^7}{2m^5} = 4m^2$

28 $a^8 \div a^2 = a^6$

29 $e^6 \div e^5 = e$

30 $k^6 \div k^3 \div k^2 = k$

31 $c^3x^6 \div c^2x^3 = cx^3$

32 $n^2m^3 \div m^2n^2 = m$

33 $\frac{e^7v^2}{e^6v^7} = \frac{e}{v^5}$

34 $\frac{y^2z^3}{z^5} = \frac{y^2}{z^2}$

35 $\frac{6m^5}{3m^2} = 2m^3$

36 $\frac{12a^3}{4a^5} = \frac{3}{a^2}$

37 $\frac{2x}{8x^3} = \frac{1}{4x^2}$

38 $\frac{14n^7}{6n^3} = \frac{7n^4}{3}$

39 $\frac{9e^2t^5}{3et^3} = 3et^2$

40 $\frac{4xp^3}{2x^4p} = \frac{2p^2}{x^3}$

Multiply first, then divide for the answer.

41 $\frac{v^5 \times v^7}{v^3 \times v^6} = \frac{v^{12}}{v^9} = v^3$

42 $\frac{w^2 \times w^4}{w \times w^3} = \frac{w^6}{w^4} = w^2$

43 $\frac{r^5 \times r}{r^3 \times r^4} = \frac{r^6}{r^7} = \frac{1}{r}$

44 $\frac{3q^2 \times 2q^5}{2q \times q^4} = \frac{6q^7}{2q^5} = 3q^2$

45 $\frac{5ay \times 2y^3}{4a^3y \times ay^4} = \frac{10ay^4}{4a^4y^5} = \frac{5}{2a^3y}$

46 $\frac{2ge^3 \times g^4e^2}{5g^6 \times 4e^4} = \frac{2g^5e^5}{20g^6e^4} = \frac{e}{10g}$

47 $\frac{3b^2t \times 4v}{2t^3v \times 3vb^3} = \frac{12b^2tv}{6b^3t^3v^2} = \frac{2}{bt^2v}$

48 $\frac{2abc \times 8b^3c}{4ac^2 \times b^4} = \frac{16ab^4c^2}{4ab^4c^2} = 4$

Adding and Subtracting Algebraic Fractions

Just like with number fractions, when algebraic fractions are added/subtracted, the denominators (bottom number/term) must be the same.

Column 1 starts with fractions that have the same denominator. So the denominator stays the same in the answer and the operation is applied only to the numerator (top number/term).

Example $\frac{c}{11} + \frac{8c}{11} = \frac{9c}{11}$

The denominator is 11 in both fractions, so the answer is found by adding straight across the top.
DON'T ADD THE DOMINATORS IT STAYS AS 11.

From Q. 7 the answers can be simplified. So the same working as above but then the answer has to be simplified by dividing through by the HCF (Highest Common Factor).

Example $\frac{2k}{9} + \frac{4k}{9} = \frac{6k}{9} = \frac{2k}{3}$

Divide both the top and bottom numbers by 3

Column 2 starts with reminding you about equivalent fractions. These are fractions of equal value but one has been multiplied or divided by a number.

Example $\frac{2k}{5} = \frac{\square}{10}$ (×2 top, ×2 bottom)

Look at the denominators and ask yourself: "What do you multiply 5 by to get 10?" .... The answer is 2.
So multiply the top term by 2, you get $4k$.

$\frac{2k}{5} = \frac{4k}{10}$ (×2 top, ×2 bottom)

The rest of column 2 applies your equivalent fraction skills. The fractions to be added have different denominators, So you have to change the denominators to match each other, again by equivalent fraction methods. Then add/subtract the fractions as they then have the same denominator. The good news is that the LCM (lowest common multiple) has been found for you, it is already written for you in the second row. So look at the denominators and find what number you need to multiply by.

Example $\frac{3a}{4} - \frac{2a}{5}$

Look at the denominators and ask yourself:
"What is the smallest number that both 4 and 5 go in to?"
The answer is 20.

$= \frac{\square}{20} - \frac{\square}{20}$ (×5 left, ×4 right)

Left Fraction: You have to multiply 4 by 5 to get 20.
Right Fraction: You have to multiply 5 by 4 to get 20.

$= \frac{15a}{20} - \frac{8a}{20} = \frac{7a}{20}$

Once you have the fractions with matching denominators perform the operation on the numerators.

Column 3 is the same except you have to find the LCM yourself, so they are a little harder.

Adding and Subtracting Algebraic Fractions

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Work on the top row only. These all have the same denominator.

1 $\frac{3x}{10} + \frac{4x}{10} = \frac{\square}{10}$

2 $\frac{5a}{8} - \frac{2a}{8} = \frac{\square}{\square}$

3 $\frac{4c}{7} + \frac{2c}{7} - \frac{c}{7} = \frac{\square}{\square}$

4 $\frac{3x}{16} + \frac{5x}{16} + \frac{7x}{16} = \frac{\square}{\square}$

5 $\frac{9e}{11} - \frac{4e}{11} - \frac{e}{11} = \frac{\square}{\square}$

6 $\frac{5}{3k} - \frac{2}{3k} + \frac{4}{3k} = \frac{\square}{\square}$

Same denominator still, but the answer can be simplified.

Example

$\frac{5x}{10} + \frac{3x}{10} = \frac{8x}{10} = \frac{4x}{5}$ (÷ 2 above, ÷ 2 below)

7 $\frac{a}{8} + \frac{a}{8} = \frac{\square}{\square} = \frac{\square}{\square}$

8 $\frac{3n}{4} + \frac{n}{4} = \frac{\square}{\square} = \frac{\square}{\square}$

9 $\frac{8}{9x} - \frac{2}{9x} = \frac{\square}{\square} = \frac{\square}{\square}$

10 $\frac{3c}{2} + \frac{5c}{2} = \frac{\square}{\square} = \frac{\square}{\square}$

11 $\frac{11h}{18} - \frac{h}{18} = \frac{\square}{\square} = \frac{\square}{\square}$

12 $\frac{11}{3x} - \frac{2}{3x} = \frac{\square}{\square} = \frac{\square}{\square}$

13 $\frac{5y}{12} + \frac{3y}{12} = \frac{\square}{\square} = \frac{\square}{\square}$

14 $\frac{13a}{18} - \frac{4a}{18} = \frac{\square}{\square} = \frac{\square}{\square}$

Complete these equivalent algebraic fractions.

15 $\frac{3c}{5} = \frac{\square}{10}$

16 $\frac{k}{3} = \frac{\square}{9}$

17 $\frac{v}{2} = \frac{\square}{16}$

18 $\frac{3e}{4} = \frac{\square}{12}$

19 $\frac{7a}{8} = \frac{\square}{24}$

20 $\frac{5n}{6} = \frac{\square}{42}$

Use equivalent fractions to solve these questions which have different denominators.

21 $\frac{a}{4} + \frac{a}{2} = \frac{a}{4} + \frac{\square}{4} = \frac{\square}{\square}$

22 $\frac{r}{3} + \frac{r}{4} = \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{\square}$

23 $\frac{5t}{6} - \frac{2t}{3} = \frac{\square}{6} - \frac{\square}{6} = \frac{\square}{\square}$

24 $\frac{m}{4} + \frac{5m}{6} = \frac{\square}{12} + \frac{\square}{12} = \frac{\square}{\square}$

25 $\frac{3b}{5} - \frac{b}{2} = \frac{\square}{10} - \frac{\square}{10} = \frac{\square}{\square}$

26 $\frac{3a}{4} - \frac{2a}{5} + \frac{3a}{10} = \frac{\square}{20} - \frac{\square}{20} + \frac{\square}{20} = \frac{\square}{\square}$

27 $\frac{7q}{12} - \frac{q}{3} = \frac{\square}{12} - \frac{\square}{12} = \frac{\square}{\square} = \frac{\square}{\square}$ Simplify

28 $\frac{g}{5} + \frac{2g}{3} - \frac{g}{6} = \frac{\square}{30} + \frac{\square}{30} - \frac{\square}{30} = \frac{\square}{\square} = \frac{\square}{\square}$

Find the (LCM) lowest common multiple and then simplify.

29 $\frac{5t}{6} - \frac{t}{2}$ Simplify

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

30 $\frac{4d}{5} + \frac{7d}{10}$ Simplify

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

31 $\frac{n}{2} + \frac{5n}{12}$

$= \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square}$

32 $\frac{5c}{8} - \frac{c}{3}$

$= \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

33 $\frac{2q}{3} + \frac{q}{2} - \frac{3q}{4}$

$= \frac{\square}{\square} + \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square}$

34 $\frac{7e}{9} - \frac{e}{2} + \frac{5e}{6}$

$= \frac{\square}{\square} - \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

35 $\frac{4v}{5} + \frac{v}{4} - \frac{3v}{10}$

$= \frac{\square}{\square} + \frac{\square}{\square} - \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

36 $\frac{3p}{4} - \frac{2p}{3} + \frac{p}{6}$

$= \frac{\square}{\square} - \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

37 $\frac{y}{3} - \frac{5y}{6} + \frac{4y}{7}$

$= \frac{\square}{\square} - \frac{\square}{\square} + \frac{\square}{\square} = \frac{\square}{\square} = \frac{\square}{\square}$

Adding and Subtracting Algebraic Fractions

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Work on the top row only. These all have the same denominator.

1 $\frac{3x}{10} + \frac{4x}{10} = \frac{7x}{10}$

2 $\frac{5a}{8} - \frac{2a}{8} = \frac{3a}{8}$

3 $\frac{4c}{7} + \frac{2c}{7} - \frac{c}{7} = \frac{5c}{7}$

4 $\frac{3x}{16} + \frac{5x}{16} + \frac{7x}{16} = \frac{15x}{16}$

5 $\frac{9e}{11} - \frac{4e}{11} - \frac{e}{11} = \frac{4e}{11}$

6 $\frac{5}{3k} - \frac{2}{3k} + \frac{4}{3k} = \frac{7}{3k}$

Same denominator still, but the answer can be simplified.

Example

$\frac{5x}{10} + \frac{3x}{10} = \frac{8x}{10} = \frac{4x}{5}$ (÷ 2 above, ÷ 2 below)

7 $\frac{a}{8} + \frac{a}{8} = \frac{2a}{8} = \frac{a}{4}$

8 $\frac{3n}{4} + \frac{n}{4} = \frac{4n}{4} = n$

9 $\frac{8}{9x} - \frac{2}{9x} = \frac{6}{9x} = \frac{2}{3x}$

10 $\frac{3c}{2} + \frac{5c}{2} = \frac{8c}{2} = 4c$

11 $\frac{11h}{18} - \frac{h}{18} = \frac{10h}{18} = \frac{5h}{9}$

12 $\frac{11}{3x} - \frac{2}{3x} = \frac{9}{3x} = \frac{3}{x}$

13 $\frac{5y}{12} + \frac{3y}{12} = \frac{8y}{12} = \frac{2y}{3}$

14 $\frac{13a}{18} - \frac{4a}{18} = \frac{9a}{18} = \frac{a}{2}$

Complete these equivalent algebraic fractions.

15 $\frac{3c}{5} = \frac{6c}{10}$

16 $\frac{k}{3} = \frac{3k}{9}$

17 $\frac{v}{2} = \frac{8v}{16}$

18 $\frac{3e}{4} = \frac{9e}{12}$

19 $\frac{7a}{8} = \frac{21a}{24}$

20 $\frac{5n}{6} = \frac{35n}{42}$

Use equivalent fractions to solve these questions which have different denominators.

21 $\frac{a}{4} + \frac{a}{2} = \frac{a}{4} + \frac{2a}{4} = \frac{3a}{4}$

22 $\frac{r}{3} + \frac{r}{4} = \frac{4r}{12} + \frac{3r}{12} = \frac{7r}{12}$

23 $\frac{5t}{6} - \frac{2t}{3} = \frac{5t}{6} - \frac{4t}{6} = \frac{t}{6}$

24 $\frac{m}{4} + \frac{5m}{6} = \frac{3m}{12} + \frac{10m}{12} = \frac{13m}{12}$

25 $\frac{3b}{5} - \frac{b}{2} = \frac{6b}{10} - \frac{5b}{10} = \frac{b}{10}$

26 $\frac{3a}{4} - \frac{2a}{5} + \frac{3a}{10} = \frac{15a}{20} - \frac{8a}{20} + \frac{6a}{20} = \frac{13a}{20}$

27 $\frac{7q}{12} - \frac{q}{3} = \frac{7q}{12} - \frac{4q}{12} = \frac{3q}{12} = \frac{q}{4}$ Simplify

28 $\frac{g}{5} + \frac{2g}{3} - \frac{g}{6} = \frac{6g}{30} + \frac{20g}{30} - \frac{5g}{30} = \frac{21g}{30} = \frac{7g}{10}$

Find the (LCM) lowest common multiple and then simplify.

29 $\frac{5t}{6} - \frac{t}{2}$ Simplify

$= \frac{5t}{6} - \frac{3t}{6} = \frac{2t}{6} = \frac{t}{3}$

30 $\frac{4d}{5} + \frac{7d}{10}$ Simplify

$= \frac{8d}{10} + \frac{7d}{10} = \frac{15d}{10} = \frac{3d}{2}$

31 $\frac{n}{2} + \frac{5n}{12}$

$= \frac{6n}{12} + \frac{5n}{12} = \frac{11n}{12}$

32 $\frac{5c}{8} - \frac{c}{3}$

$= \frac{15c}{24} - \frac{8c}{24} = \frac{7c}{24}$

33 $\frac{2q}{3} + \frac{q}{2} - \frac{3q}{4}$

$= \frac{8q}{12} + \frac{6q}{12} - \frac{9q}{12} = \frac{5q}{12}$

34 $\frac{7e}{9} - \frac{e}{2} + \frac{5e}{6}$

$= \frac{14e}{18} - \frac{9e}{18} + \frac{15e}{18} = \frac{20e}{18} = \frac{10e}{9}$

35 $\frac{4v}{5} + \frac{v}{4} - \frac{3v}{10}$

$= + \frac{5v}{20} - \frac{6v}{20} = \frac{15v}{20} = \frac{3v}{4}$

36 $\frac{3p}{4} - \frac{2p}{3} + \frac{p}{6}$

$= \frac{9p}{12} - \frac{8p}{12} + \frac{2p}{12} = \frac{3p}{12} = \frac{p}{4}$

37 $\frac{y}{3} - \frac{5y}{6} + \frac{4y}{7}$

$= \frac{14y}{42} - \frac{35y}{42} + \frac{24y}{42} = \frac{3y}{42} = \frac{y}{14}$

Multiplying & Dividing Algebraic Fractions

When you multiply algebraic fractions the denominators don't have to be the same. Just multiply straight across the top and straight across the bottom.

Examples

$$\frac{p}{2} \times \frac{m}{3} = \frac{pm}{6} \qquad p \times m = pm \qquad 2 \times 3 = 6$$

$$\frac{3t}{5n} \times \frac{2x}{7} = \frac{6tx}{35n} \qquad 3t \times 2x = 6tx \qquad 5n \times 7 = 35n$$

From Q. 7 in Column 1 the answer will be able to be simplified. So look at the numbers … can they be simplified? Then look at the pronumerals that are common on the top and bottom, if they can be cancelled.

Examples

$$\frac{qp}{2} \times \frac{p}{q} = \frac{qp^2}{2q} = \frac{p^2}{2}$$

$$\frac{3t}{5x} \times \frac{2x}{6} = \frac{6tx}{30x} = \frac{t}{5}$$

Column 2 introduces the reciprocal. These are when the number/fraction/term are turned upside-down (inverted). So write the denominator as the numerator, and the opposite. The only tricky thing is whole numbers and '^{1}/' fractions. A whole number turns into '^{1}/' the number. While '^{1}/' turns into a whole number.

Q. 21 introduces division. Algebraic fractions are solved similarly to normal fractions, division isn't used to solve them. Instead the reciprocal is found for the term behind the ÷ sign and the ÷ sign is replace by a × sign. So these are essentially the same as multiplication, except with an 'inverted twist.' There is an example above Q. 21.

The final column increases the number of terms to three. Remember that you find the reciprocal only for the term behind the ÷ sign, the other terms are unchanged. There is an example at the top of the column.

Multiplying & Dividing Algebraic Fractions

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Multiply these algebraic fractions. Watch for powers.

Example $\frac{a}{3} \times \frac{2n}{5} = \frac{2an}{15}$

1 $\frac{x}{2} \times \frac{y}{3} =$

2 $\frac{e}{5} \times \frac{k}{y} =$

3 $\frac{n}{2} \times \frac{n}{3} =$

4 $\frac{3p}{5} \times \frac{4p}{7} =$

5 $\frac{e}{2} \times \frac{t}{3} \times \frac{5e}{4} =$

6 $\frac{2}{k} \times \frac{m}{3} \times \frac{4m}{5} =$

These can be simplified. Look for letters to cancel and numbers that divide through.

Example $\frac{2a}{5} \times \frac{3n}{4} \overset{\div 2}{=} \frac{\cancel{6}an}{\cancel{20}} = \frac{3an}{10}$

7 $\frac{m}{2} \times \frac{x}{m} = = $ Simplify

8 $\frac{a}{3} \times \frac{6}{a} = = $

9 $\frac{2c}{3} \times \frac{x}{4} = = $

10 $\frac{ux}{2} \times \frac{x}{u} = = $

11 $\frac{5n}{px} \times \frac{2}{nx} = = $

12 $\frac{3}{k} \times \frac{2}{k} \times \frac{ak}{2} = = $

13 $\frac{3f}{2} \times 4 \times \frac{h}{6f} = = $

14 $\frac{2a}{b} \times \frac{b}{2c} \times \frac{2c}{a} = = $

Find the reciprocal for these. Invert the fractions. *(invert = turn upside-down)*

15 $\frac{x}{2} \rightarrow$

16 $\frac{3}{y} \rightarrow$

17 $\frac{1}{3} \rightarrow$

18 $5 \rightarrow$

19 $\frac{a}{b} \rightarrow$

20 $\frac{c}{2e} \rightarrow$

Divide these by finding the reciprocal and multiplying.

Example $\frac{c}{2} \div \frac{3}{v} = \frac{c}{2} \times \frac{v}{3} = \frac{cv}{6}$ ($\div \rightarrow \times$, reciprocal)

21 $\frac{a}{c} \div \frac{d}{b} = \times = $

22 $\frac{2}{u} \div \frac{u}{2} = \times = $

23 $\frac{d}{3} \div \frac{4}{d} = \times = $

24 $\frac{y}{3} \div \frac{1}{2} = \times = $

25 $\frac{2n}{3} \div \frac{5}{x} = \times = $

26 $\frac{3c}{m} \div 5 = \times = $

27 $\frac{4k}{3} \div \frac{8}{3} = \times = = $ Simplify

28 $\frac{6y}{5} \div \frac{3}{4g} = \times = = $ Simplify

These have both ÷ and × in them. Only invert the term behind the ÷ sign and change the sign to ×.

Example

$\frac{a}{3} \times \frac{n}{5} \div \frac{a}{2}$

$= \frac{a}{3} \times \frac{n}{5} \times \frac{2}{a} = \frac{2an}{15a} = \frac{2n}{15}$ Simplify

($\div \rightarrow \times$, reciprocal)

29 $\frac{k}{3} \div \frac{2}{3} \times \frac{4}{k}$

$= \times \times = = $

30 $\frac{4}{e} \times \frac{2e}{3} \div \frac{1}{e}$

$= \times \times = = $

31 $\frac{ch}{2} \div \frac{1}{2} \times \frac{a}{h}$

$= \times \times = = $

32 $\frac{2v}{5} \times \frac{3}{2} \div \frac{v^2}{3}$

$= \times \times = = $

33 $\frac{1}{d} \times \frac{3d}{4} \div \frac{1}{d}$

$= \times \times = = $

34 $\frac{q}{p} \times \frac{aq}{2p} \div \frac{4q}{5p}$

$= \times \times = = $

35 $\frac{3}{10} \div \frac{v}{t} \times \frac{4t}{k}$

$= \times \times = = $

Multiplying & Dividing Algebraic Fractions

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Multiply these algebraic fractions. Watch for powers.

Example $\frac{a}{3} \times \frac{2n}{5} = \frac{2an}{15}$

1 $\frac{x}{2} \times \frac{y}{3} = \frac{xy}{6}$

2 $\frac{e}{5} \times \frac{k}{y} = \frac{ek}{5y}$

3 $\frac{n}{2} \times \frac{n}{3} = \frac{n^2}{6}$

4 $\frac{3p}{5} \times \frac{4p}{7} = \frac{12p^2}{35}$

5 $\frac{e}{2} \times \frac{t}{3} \times \frac{5e}{4} = \frac{5e^2t}{24}$

6 $\frac{2}{k} \times \frac{m}{3} \times \frac{4m}{5} = \frac{8m^2}{15k}$

These can be simplified. Look for letters to cancel and numbers that divide through.

Example $\frac{2a}{5} \times \frac{3n}{4} = \frac{\overset{\div 2}{\cancel{6}an}}{\underset{\div 2}{\cancel{20}}} = \frac{3an}{10}$

7 $\frac{m}{2} \times \frac{x}{m} = \frac{mx}{2m} = \frac{x}{2}$ Simplify

8 $\frac{a}{3} \times \frac{6}{a} = \frac{6a}{3a} = 2$

9 $\frac{2c}{3} \times \frac{x}{4} = \frac{2cx}{12} = \frac{cx}{6}$

10 $\frac{ux}{2} \times \frac{x}{u} = \frac{ux^2}{2u} = \frac{x^2}{2}$

11 $\frac{5n}{px} \times \frac{2}{nx} = \frac{10n}{pnx^2} = \frac{10}{px^2}$

12 $\frac{3}{k} \times \frac{2}{k} \times \frac{ak}{2} = \frac{6ak}{2k^2} = \frac{3a}{k}$

13 $\frac{3f}{2} \times 4 \times \frac{h}{6f} = \frac{12fh}{12f} = h$

14 $\frac{2a}{b} \times \frac{b}{2c} \times \frac{2c}{a} = \frac{4abc}{2abc} = 2$

Find the reciprocal for these. Invert the fractions. *(invert = turn upside-down)*

15 $\frac{x}{2} \rightarrow \frac{2}{x}$

16 $\frac{3}{y} \rightarrow \frac{y}{3}$

17 $\frac{1}{3} \rightarrow 3$

18 $5 \rightarrow \frac{1}{5}$

19 $\frac{a}{b} \rightarrow \frac{b}{a}$

20 $\frac{c}{2e} \rightarrow \frac{2e}{c}$

Divide these by finding the reciprocal and multiplying.

Example $\frac{c}{2} \div \frac{3}{v} = \frac{c}{2} \times \frac{v}{3} = \frac{cv}{6}$ ($\div \rightarrow \times$, reciprocal)

21 $\frac{a}{c} \div \frac{d}{b} = \frac{a}{c} \times \frac{b}{d} = \frac{ab}{cd}$

22 $\frac{2}{u} \div \frac{u}{2} = \frac{2}{u} \times \frac{2}{u} = \frac{4}{u^2}$

23 $\frac{d}{3} \div \frac{4}{d} = \frac{d}{3} \times \frac{d}{4} = \frac{d^2}{12}$

24 $\frac{y}{3} \div \frac{1}{2} = \frac{y}{3} \times 2 = \frac{2y}{3}$

25 $\frac{2n}{3} \div \frac{5}{x} = \frac{2n}{3} \times \frac{x}{5} = \frac{2nx}{15}$

26 $\frac{3c}{m} \div 5 = \frac{3c}{m} \times \frac{1}{5} = \frac{3c}{5m}$

27 $\frac{4k}{3} \div \frac{8}{3} = \frac{4k}{3} \times \frac{3}{8} = \frac{12k}{24} = \frac{k}{2}$ Simplify

28 $\frac{6y}{5} \div \frac{3}{4g} = \frac{6y}{5} \times \frac{4g}{3} = \frac{24yg}{15} = \frac{8yg}{5}$ Simplify

These have both ÷ and × in them. Only invert the term behind the ÷ sign and change the sign to ×.

Example

$\frac{a}{3} \times \frac{n}{5} \div \frac{a}{2} = \frac{a}{3} \times \frac{n}{5} \times \frac{2}{a} = \frac{2an}{15a} = \frac{2n}{15}$ Simplify

($\div \rightarrow \times$, reciprocal)

29 $\frac{k}{3} \div \frac{2}{3} \times \frac{4}{k} = \frac{k}{3} \times \frac{3}{2} \times \frac{4}{k} = \frac{12k}{6k} = 2$

30 $\frac{4}{e} \times \frac{2e}{3} \div \frac{1}{e} = \frac{4}{e} \times \frac{2e}{3} \times e = \frac{8e^2}{3e} = \frac{8e}{3}$

31 $\frac{ch}{2} \div \frac{1}{2} \times \frac{a}{h} = \frac{ch}{2} \times 2 \times \frac{h}{a} = \frac{2ch^2}{2a} = \frac{ch^2}{a}$

32 $\frac{2v}{5} \times \frac{3}{2} \div \frac{v^2}{3} = \frac{2v}{5} \times \frac{3}{2} \times \frac{3}{v^2} = \frac{18v}{10v^2} = \frac{9}{5v}$

33 $\frac{1}{d} \times \frac{3d}{4} \div \frac{1}{d} = \frac{1}{d} \times \frac{3d}{4} \times d = \frac{3d^2}{4d} = \frac{3d}{4}$

34 $\frac{q}{p} \times \frac{aq}{2p} \div \frac{4q}{5p} = \frac{q}{p} \times \frac{aq}{2p} \times \frac{5p}{4q} = \frac{5paq^2}{8p^2q} = \frac{5aq}{8p}$

35 $\frac{3}{10} \div \frac{v}{t} \times \frac{4t}{k} = \frac{3}{10} \times \frac{t}{v} \times \frac{4t}{k} = \frac{12t^2}{10vk} = \frac{6t^2}{5vk}$

Expanding and Collecting Like Terms

Expanding and 'Removing the brackets' are the same process. This involves multiplying the outside term with the inside terms. Here are two examples.

Outside term multiplied by first term inside — Outside term multiplied by second term inside

$3(x + 2) = 3(x + 2) + 3(x + 2)$

$= 3x + 6$

Outside term multiplied by first term inside — Outside term multiplied by second term inside

$4(n + 5) = 4(n + 5) + 4(n + 5)$

$= 4n + 20$

If the sign in the brackets is a '-' then this changes the answer. Examples.

$5(y - 3) = 5y - 15$ $\qquad$ $2(6 - k) = 12 - 2k$ $\qquad$ $a(a - b) = a^2 - ab$

Column 1 starts with completing expansions using a manual method. By repeating additions of the bracketed expression the answer can be achieved without multiplying, this process is for increasing understanding it shouldn't be used to solve questions. Q. 6 outlines the use of multiplication and asks you to show the working out.

Column 2 is multiplication without working out, so they are quick. But watch out for 'powers' on some terms. The column then introduces negative outside terms, which have the effect of swapping the sign. There are examples at the start of the questions.

Column 3 extends the expansion process by including 'collecting like terms'. Performing a double expansion then gathering terms that can be added/subtracted. There is an example at the top of the column. Remember to watch for negatives, especially in front of the second bracketed expression, which changes the sign and subsequently your answer.

Expanding and Collecting Like Terms

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Expand these questions in steps, collecting like terms to get your answer.



Example

$3(x + 3) = (x+3) + (x+3) + (x+3)$

$= 3x + 9$

1 $2(y + 5) = (\quad) + (\quad)$

$= \quad$

2 $2(e + 7) = (\quad) + (\quad)$

$= \quad$

3 $3(m + k)$

$= (\quad) + (\quad) + (\quad)$

$= \quad$

4 $3(r - 4)$

$= (\quad) + (\quad) + (\quad)$

$= \quad$

5 $3(3x - 8)$

$= (\quad) + (\quad) + (\quad)$

$= \quad$



Expand these questions using multiplication. Hula!

Example

$3(x - 12) = 3 \times x \; - \; 3 \times 12$

$= 3x - 36$

6 $5(j + 3) = 5 \times \square + 5 \times \square$

$= \quad$

7 $7(n - 9) = 7 \times \square - 7 \times \square$

$= \quad$

8 $k(5 - t) = \quad$

$= \quad$

9 $a(a + b) = \quad$

$= \quad$

10 $2x(x + 5) = \quad$

$= \quad$

Expand these, no working out is required.



Examples $4(x - 3) = 4x - 12$

$5(2x + 4) = 10x + 20$

$2a(x + 5a) = 2ax + 10a^2$

11 $4(e + 7) = \quad$

12 $5(n + 2) = \quad$

13 $3(d - 5) = \quad$

14 $a(n + 10) = \quad$

15 $k(k - m) = \quad$

16 $3(14 - e) = \quad$

17 $8(3c + 2d) = \quad$

18 $4n(n - 6) = \quad$

19 $xy(x + y) = \quad$

20 $5t(3t - n) = \quad$

21 $9x(5x - 2xy) = \quad$

Negatives change the sign, expand the following.



Examples $-(x - 4) = -x + 4$

$-7(y + 2) = -7y - 14$

22 $-5(t + 2) = \quad$

23 $-2(m - 7) = \quad$

24 $-(k - j) = \quad$

25 $-3x(2 + x) = \quad$

26 $-a(2a + ab) = \quad$

27 $-c^2(m + c) = \quad$

28 $-6(7y - 4h) = \quad$

29 $-2uv(2u - v^3) = \quad$

30 $-5(2d - dr^2) = \quad$

31 $-4(a - b - c^2) = \quad$

32 $-ax(x^2a - 2x) = \quad$

Expand then collect like terms. Watch for negatives.



Example

$2(y + 7) + 4(y - 3)$

$= 2y + 14 + 4y - 12 = 6y + 2$

33 $9(h + 8) + 7h$

$= \quad + 7h = \quad$

34 $5(3c - 4) - 13$

$= \quad = \quad$

35 $3(x + 2) + 4(x + 5)$

$= \quad = \quad$

36 $4(n + 7) + 2(n - 3)$

$= \quad = \quad$

37 $5(k - 2) + 7(k + 9)$

$= \quad = \quad$

38 $v(v + t) + v(v - t)$

$= \quad = \quad$

39 $6(m + 3) + 5(m - 7)$

$= \quad = \quad$

40 $2(e + 6) - 4(e + 4)$

$= \quad = \quad$

41 $a(b + 2c) + a(4b + c)$

$= \quad$

$= \quad$

42 $2(b + 4) - 3(b - 6)$

$= \quad = \quad$

43 $8(5 - c) - 2(c - 6)$

$= \quad = \quad$

44 $2y(3x - 4) - y(3x + 5)$

$= \quad$

$= \quad$

Expanding and Collecting Like Terms

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Expand these questions in steps, collecting like terms to get your answer.

Example

$3(x + 3) = (x+3) + (x+3) + (x+3)$

$= 3x + 9$

1 $2(y + 5) = (\ y + 5\ ) + (\ y + 5\ )$

$= 2y + 10$

2 $2(e + 7) = (\ e + 7\ ) + (\ e + 7\ )$

$= 2e + 14$

3 $3(m + k)$

$= (\ m + k\ ) + (\ m + k\ ) + (\ m + k\ )$

$= 3m + 3k$

4 $3(r - 4)$

$= (\ r - 4\ ) + (\ r - 4\ ) + (\ r - 4\ )$

$= 3r - 12$

5 $3(3x - 8)$

$= (3x - 8) + (3x - 8) + (3x - 8)$

$= 9x - 24$

Expand these questions using multiplication. Hula!

Example

$3(x - 12) = 3 \times x - 3 \times 12$

$= 3x - 36$

6 $5(j + 3) = 5 \times \boxed{j} + 5 \times \boxed{3}$

$= 5j + 15$

7 $7(n - 9) = 7 \times \boxed{n} - 7 \times \boxed{9}$

$= 7n - 63$

8 $k(5 - t) = k \times 5 - k \times t$

$= 5k - kt$

9 $a(a + b) = a \times a + a \times b$

$= a^2 + ab$

10 $2x(x + 5) = 2x \times x + 2x \times 5$

$= 2x^2 + 10x$

Expand these, no working out is required.

Examples $4(x - 3) = 4x - 12$

$5(2x + 4) = 10x + 20$

$2a(x + 5a) = 2ax + 10a^2$

11 $4(e + 7) = 4e + 28$

12 $5(n + 2) = 5n + 10$

13 $3(d - 5) = 3d - 15$

14 $a(n + 10) = an + 10a$

15 $k(k - m) = k^2 - km$

16 $3(14 - e) = 42 - 3e$

17 $8(3c + 2d) = 24c + 16d$

18 $4n(n - 6) = 4n^2 - 24n$

19 $xy(x + y) = x^2y + xy^2$

20 $5t(3t - n) = 15t^2 - 5tn$

21 $9x(5x - 2xy) = 45x^2 - 18x^2y$

Negatives change the sign, expand the following.

Examples $-(x - 4) = -x + 4$

$-7(y + 2) = -7y - 14$

22 $-5(t + 2) = -5t - 10$

23 $-2(m - 7) = -2m + 14$

24 $-(k - j) = -k + j$

25 $-3x(2 + x) = -6x - 3x^2$

26 $-a(2a + ab) = -2a^2 - a^2b$

27 $-c^2(m + c) = -c^2m - c^3$

28 $-6(7y - 4h) = -42y + 24h$

29 $-2uv(2u - v^3) = -4u^2v + 2uv^4$

30 $-5(2d - dr^2) = -10d + 5dr^2$

31 $-4(a - b - c^2) = -4a + 4b + 4c^2$

32 $-ax(x^2a - 2x) = -a^2x^3 + 2x^2a$

Expand then collect like terms. Watch for negatives.

Example

$2(y + 7) + 4(y - 3)$

$= 2y + 14 + 4y - 12 = 6y + 2$

33 $9(h + 8) + 7h$

$= 9h + 72 + 7h = 16h + 72$

34 $5(3c - 4) - 13$

$= 15c - 20 - 13 = 15c - 33$

35 $3(x + 2) + 4(x + 5)$

$= 3x + 6 + 4x + 20 = 7x + 26$

36 $4(n + 7) + 2(n - 3)$

$= 4n + 28 + 2n - 6 = 6n + 22$

37 $5(k - 2) + 7(k + 9)$

$= 5k - 10 + 7k + 63 = 12k + 53$

38 $v(v + t) + v(v - t)$

$= v^2 + vt + v^2 - vt = 2v^2$

39 $6(m + 3) + 5(m - 7)$

$= 6m + 18 + 5m - 35 = 11m - 17$

40 $2(e + 6) - 4(e + 4)$

$= 2e + 12 - 4e - 16 = -2e - 4$

41 $a(b + 2c) + a(4b + c)$

$= ab + 2ac + 4ab + ac$

$= 5ab + 3ac$

42 $2(b + 4) - 3(b - 6)$

$= 2b + 8 - 3b + 18 = -b + 26$

43 $8(5 - c) - 2(c - 6)$

$= 40 - 8c - 2c + 12 = -10c + 52$

44 $2y(3x - 4) - y(3x + 5)$

$= 6xy - 8y - 3xy - 5y$

$= 3xy - 13y$

Further Expanding

Expanding binomial expressions is remembered best by the word FOIL. This stands for:

F - The front terms are multiplied first

FRONT

$(y + 2)(y + 3) = y^2$

O - The two outside terms are then multiplied

OUTSIDE

$(y + 2)(y + 3) = 3y$

I - The two inside terms are then multiplied

INSIDE

$(y + 2)(y + 3) = 2y$

L - The last two terms in each pair of brackets are multiplied

LAST

$(y + 2)(y + 3) = 6$

So $(y + 2)(y + 3) = y^2 + 3y + 2y + 6$ (then gathering like terms) $= y^2 + 5y + 6$

So when you are working out a question, the process looks like this:

$(a - 2)(a + 3)$

$= a^2 + 3a - 2a - 6$

$= a^2 + a - 6$

Watch out for negative terms and the effect on the sign in the expansion. Column 3 relates expansions to area of a shape. The last two questions involve multiplying by an additional fraction/constant, you can apply this after the expansion or to one of the bracketed expressions. See how you go.

Further Expanding

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Expand these questions using FOIL, then collecting like terms to get your answer.

Example $(x + 2)(x + 5)$

$= x^2 + 5x + 2x + 10$

$= x^2 + 7x + 10$

1 $(y + 3)(y + 5)$

2 $(m + 6)(m + 4)$

3 $(e + 2)(e - 4)$

4 $(w - 5)(w + 7)$

5 $(d - 2x)(d + x)$

6 $(3k + 2)(k - 2)$

7 $(4x - 3)(4x - 8)$

8 $(3c - k)(5c + 2k)$

Rewrite squared expressions, expand and collect like terms.

Example $(x + 2)^2$

$= (x + 2)(x + 2)$

Rewrite the expression without the ^{2}.

$= x^2 + 2x + 2x + 4$

Expand using FOIL.

$= x^2 + 4x + 4$

Collect like terms.

9 $(b + 4)^2$

10 $(h - 6)^2$

11 $(u + 3x)^2$

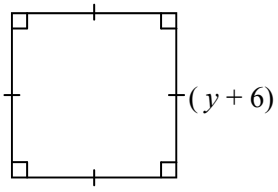
12 $(3a + 9)^2$

13 $(11 - 5q)^2$

14 $(6f - 2g)^2$

Find an expression for the area of these shapes, fully expand and simplify your answer.

15

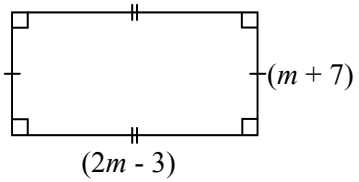


A = l^2 = ( )( )

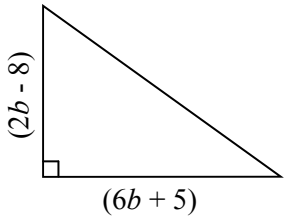
=

A =

16



17



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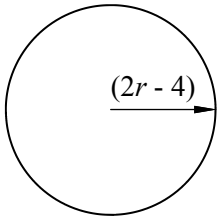

Further Expanding

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Expand these questions using FOIL, then collecting like terms to get your answer.

Example $(x + 2)(x + 5)$

$= x^2 + 5x + 2x + 10$

$= x^2 + 7x + 10$

1 $(y + 3)(y + 5)$

$= y^2 + 5y + 3y + 15$

$= y^2 + 8y + 15$

2 $(m + 6)(m + 4)$

$= m^2 + 4m + 6m + 24$

$= m^2 + 10m + 24$

3 $(e + 2)(e - 4)$

$= e^2 - 4e + 2e - 8$

$= e^2 - 2e - 8$

4 $(w - 5)(w + 7)$

$= w^2 + 7w - 5w - 35$

$= w^2 + 2w - 35$

5 $(d - 2x)(d + x)$

$= d^2 + dx - 2xd - 2x^2$

$= d^2 - dx - 2x^2$

6 $(3k + 2)(k - 2)$

$= 3k^2 - 6k + 2k - 4$

$= 3k^2 - 4k - 4$

7 $(4x - 3)(4x - 8)$

$= 16x^2 - 32x - 12x + 24$

$= 16x^2 - 44x + 24$

8 $(3c - k)(5c + 2k)$

$= 15c^2 + 6ck - 5ck - 2k^2$

$= 15c^2 + ck - 2k^2$

Rewrite squared expressions, expand and collect like terms.

Example $(x + 2)^2$

$= (x + 2)(x + 2)$

Rewrite the expression without the ^{2}.

$= x^2 + 2x + 2x + 4$

Expand using FOIL.

$= x^2 + 4x + 4$

Collect like terms.

9 $(b + 4)^2$

$= (b + 4)(b + 4)$

$= b^2 + 4b + 4b + 16$

$= b^2 + 8b + 16$

10 $(h - 6)^2$

$= (h - 6)(h - 6)$

$= h^2 - 6h - 6h + 36$

$= h^2 - 12h + 36$

11 $(u + 3x)^2$

$= (u + 3x)(u + 3x)$

$= u^2 + 3ux + 3ux + 9x^2$

$= u^2 + 6ux + 9x^2$

12 $(3a + 9)^2$

$= (3a + 9)(3a + 9)$

$= 9a^2 + 27a + 27a + 81$

$= 9a^2 + 54a + 81$

13 $(11 - 5q)^2$

$= (11 - 5q)(11 - 5q)$

$= 121 - 55q - 55q + 25q^2$

$= 121 - 110q + 25q^2$

14 $(6f - 2g)^2$

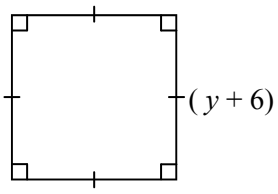
$= (6f - 2g)(6f - 2g)$

$= 36f^2 - 12fg - 12fg + 4g^2$

$= 36f^2 - 24fg + 4g^2$

Find an expression for the area of these shapes, fully expand and simplify your answer.

15

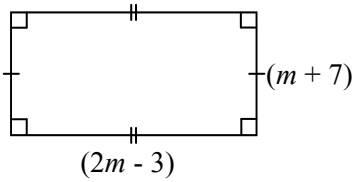


$A = l^2 = (y + 6)(y + 6)$

$= y^2 + 6y + 6y + 36$

$A = y^2 + 12y + 36$ units^2

16

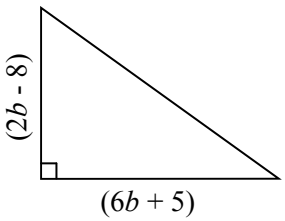


$A = lb = (m + 7)(2m - 3)$

$= 2m^2 - 3m + 14m - 21$

$A = 2m^2 + 11m - 21$ units^2

17



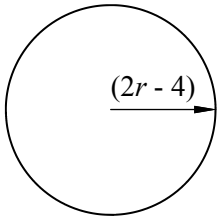
$A = \frac{1}{2}bh = \frac{1}{2} \times (2b - 8)(6b + 5)$

$= (b - 4)(6b + 5)$

$= 6b^2 + 5b - 24b - 20$

$A = 6b^2 - 19b - 20$ units^2

18



$A = \pi r^2 = \pi \times (2r - 4)(2r - 4)$

$= (2\pi r - 4\pi)(2r - 4)$

$= 4\pi r^2 - 8\pi r - 8\pi r + 16\pi$

$A = 4\pi r^2 - 16\pi r + 16\pi$ units^2

Factorising Algebraic Expressions

Factorising is the opposite to expanding. So you are showing an expression using brackets. To factorise an expression you have to take the HCF outside the brackets. The HCF is the highest common factor.

Examples - Finding the HCF of two terms

$2, 4x$ HCF = 2 $8x, 6x$ HCF = $2x$ $2x^2, 4x$ HCF = $2x$

Examples - Factorising an Expression

$2x + 12 = 2(x + 6)$ $kx - 5x = x(k - 5)$ $4x - 6x^2 = 2x(2 - 3x)$

Column 1 starts with a brief section on finding the HCF. Look at the numbers first, then look if there are any common letters. Then factorising starts, but the HCF has been found for you and is shown outside the brackets, so you have to just find what is inside the brackets. From Q. 15 you are only given the sign inside the brackets, you have to find all three boxes. The outside term is only a number or single pronumeral. From Q. 21 the outside term is a term with two pronumerals (letters) or a coefficient and a pronumeral (number and a letter).

Column 2 is still factorising but without assistance, further down the column are questions with a negative first term. When you factorise the first term inside the brackets should always be positive, so look at the example before Q. 40 and remember how the sign change affects the sign in front of the second term in the brackets.

Column 3 relates factorising to perimeter. There is an example at the top of the column.

Factorising Algebraic Expressions

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Find the HCF (the largest number or term that divides into both) for these.

1 $4x$, 2 ____ 2 $6y$, 9 ____

3 $5q$, 15 ____ 4 ce, e ____

5 v^2, $4v$ ____ 6 $2xy$, $6x$ ____

Find the value for the boxes to make these statements true.

7 $5j + 10 = 5(\square + \square)$

8 $3x + 3y = 3(\square + \square)$

9 $xk + xc = x(\square + \square)$

10 $4t - 6 = 2(\square - \square)$

11 $ab + 3a = a(\square + \square)$

12 $6bc - 2b = 2b(\square - \square)$

13 $c^2v + 5c = c(\square + \square)$

14 $6xk - 12k^2 = 6k(\square - \square)$

Now factorise these expressions.

15 $2k + 10 = \square(\square + \square)$

16 $yk + yr = \square(\square + \square)$

17 $4t - 6 = \square(\square - \square)$

18 $ab + b^2 = \square(\square + \square)$

19 $9q - 6n = \square(\square - \square)$

20 $10 - 4k = \square(\square - \square)$

Factorise these, terms outside the brackets have 2 pronumerals or a coefficient & a pronumeral.

21 $2y + 4y^2 = \square(\square + \square)$

22 $6xb + 8b = \square(\square + \square)$

23 $abc - a^2b = \square(\square - \square)$

24 $5u^2 + 5u = \square(\square + \square)$

25 $b^2c - c^2b = \square(\square - \square)$

Factorise these like you have never factorised before!

26 $2y + 8 =$ ____

27 $am + pm =$ ____

28 $6k - 18 =$ ____

29 $eu - u^2 =$ ____

30 $5xy + 2y =$ ____

31 $n^2 - 4n =$ ____

32 $\pi R^2 - \pi r^2 =$ ____

33 $a^2 + a^3 =$ ____

34 $12a - 2a^2b =$ ____

35 $ac^2b - cba^2 =$ ____

36 $4xd + 14d^2 =$ ____

37 $2\pi r^2 + 2\pi rh =$ ____

38 $15x - 3h^2x =$ ____

39 $n^2 - 4n + an^3 =$ ____

The front term **in** the brackets must be positive. Factorise.

Example

$-2x + 6 = -2(x - 3)$ ***not*** $2(-x + 3)$

40 $-6 + 12p =$ ____

41 $-3t - 9c =$ ____

42 $-a - ab =$ ____

43 $-16q + 8p =$ ____

44 $-t^2 - tc^2 =$ ____

45 $-20xe + 8x =$ ____

46 $-15ux - 10u^2 =$ ____

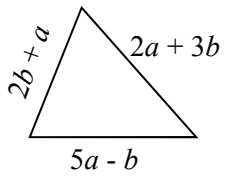
47 $-xa - 3x^2 =$ ____

48 $-10t - 8tc^2 =$ ____

49 $-4a^2b + 8a^3bc =$ ____

Find the perimeter of these shapes. Then factorise the answer.

Example

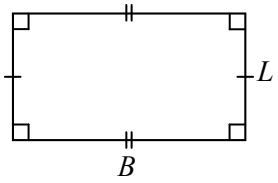


$P = 2b + a + 2a + 3b + 5a - b$

$P = 8a + 4b$ units

$P = 4(2a + b)$ units

50

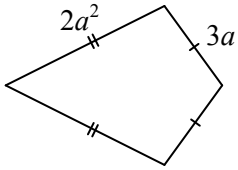


P = ____

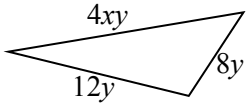
P = ____ units

P = ____ units

51



52



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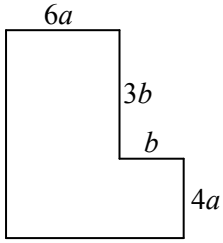

Extra line for the first addition

Factorising Algebraic Expressions

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Find the HCF (the largest number or term that divides into both) for these.

1 $4x$, 2 — 2
2 $6y$, 9 — 3
3 $5q$, 15 — 5
4 ce, e — e
5 v^2, $4v$ — v
6 $2xy$, $6x$ — $2x$

Find the value for the boxes to make these statements true.

7 $5j + 10 = 5($ [j] + [2] $)$
8 $3x + 3y = 3($ [x] + [y] $)$
9 $xk + xc = x($ [k] + [c] $)$
10 $4t - 6 = 2($ [$2t$] - [3] $)$
11 $ab + 3a = a($ [b] + [3] $)$
12 $6bc - 2b = 2b($ [$3c$] - [1] $)$
13 $c^2v + 5c = c($ [cv] + [5] $)$
14 $6xk - 12k^2 = 6k($ [x] - [$2k$] $)$

Now factorise these expressions.

15 $2k + 10 =$ [2] $($ [k] + [5] $)$
16 $yk + yr =$ [y] $($ [k] + [r] $)$
17 $4t - 6 =$ [2] $($ [$2t$] - [3] $)$
18 $ab + b^2 =$ [b] $($ [a] + [b] $)$
19 $9q - 6n =$ [3] $($ [$3q$] - [$2n$] $)$
20 $10 - 4k =$ [2] $($ [5] - [$2k$] $)$

Factorise these, terms outside the brackets have 2 pronumerals or a coefficient & a pronumeral.

21 $2y + 4y^2 =$ [$2y$] $($ [1] + [$2y$] $)$
22 $6xb + 8b =$ [$2b$] $($ [$3x$] + [4] $)$
23 $abc - a^2b =$ [ab] $($ [c] - [a] $)$
24 $5u^2 + 5u =$ [$5u$] $($ [u] + [1] $)$
25 $b^2c - c^2b =$ [bc] $($ [b] - [c] $)$

Factorise these like you have never factorised before!

26 $2y + 8 =$ $2(y + 4)$
27 $am + pm =$ $m(a + p)$
28 $6k - 18 =$ $6(k - 3)$
29 $eu - u^2 =$ $u(e - u)$
30 $5xy + 2y =$ $y(5x + 2)$
31 $n^2 - 4n =$ $n(n - 4)$
32 $\pi R^2 - \pi r^2 =$ $\pi(R^2 - r^2)$
33 $a^2 + a^3 =$ $a^2(1 + a)$
34 $12a - 2a^2b =$ $2a(6 - ab)$
35 $ac^2b - cba^2 =$ $abc(c - a)$
36 $4xd + 14d^2 =$ $2d(2x + 7d)$
37 $2\pi r^2 + 2\pi rh =$ $2\pi r(r + h)$
38 $15x - 3h^2x =$ $3x(5 - h^2)$
39 $n^2 - 4n + an^3 =$ $n(n - 4 + an^2)$

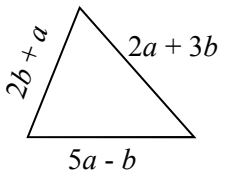
The front term **in** the brackets must be positive. Factorise.

Example
$-2x + 6 = -2(x - 3)$ ***not*** $2(-x + 3)$

40 $-6 + 12p =$ $-6(1 - 2p)$
41 $-3t - 9c =$ $-3(t + 3c)$
42 $-a - ab =$ $-a(1 + b)$
43 $-16q + 8p =$ $-8(2q - p)$
44 $-t^2 - tc^2 =$ $-t(t + c^2)$
45 $-20xe + 8x =$ $-4x(5e - 2)$
46 $-15ux - 10u^2 =$ $-5u(3x + 2u)$
47 $-xa - 3x^2 =$ $-x(a + 3x)$
48 $-10t - 8tc^2 =$ $-2t(5 + 4c^2)$
49 $-4a^2b + 8a^3bc =$ $-4a^2b(1 - 2ac)$

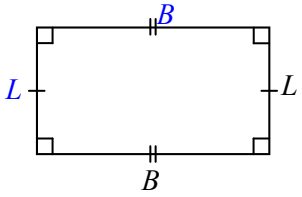
Find the perimeter of these shapes. Then factorise the answer.

Example



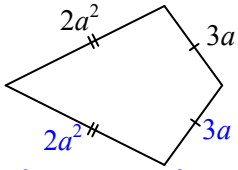
$P = 2b + a + 2a + 3b + 5a - b$
$P = 8a + 4b$ units
$P = 4(2a + b)$ units

50



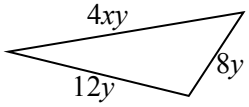
$P = L + B + L + B$
$P = 2L + 2B$ units
$P = 2(L + B)$ units

51



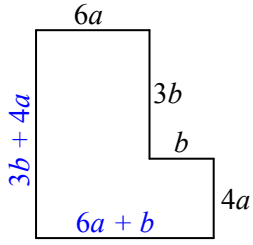
$P = 2a^2 + 3a + 2a^2 + 3a$
$P = 4a^2 + 6a$ units
$P = 2a(2a + 3)$ units

52



$P = 12y + 4xy + 8y$
$P = 20y + 4xy$ units
$P = 4y(5 + x)$ units

53



Extra line for the first addition

$P = 6a + b + 3b + 4a + 6a$
$+ 3b + b + 4a$
$P = 20a + 8b$ units
$P = 4(5a + 2b)$ units

Algebra - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Algebra - Find a Word

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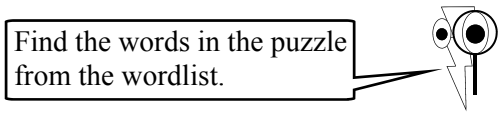

P	F	A	C	T	O	R	I	S	E	T	E	Q	O	T
Y	L	P	I	T	L	U	M	X	P	C	X	X	R	C
P	R	Y	N	X	P	R	R	E	W	O	P	T	D	A
L	E	N	O	I	P	X	E	P	X	E	A	N	E	R
U	C	W	I	D	E	R	T	P	S	F	N	E	R	T
L	I	O	T	E	O	U	U	I	C	F	D	L	Y	B
R	P	N	C	N	U	Q	M	O	S	I	I	A	I	U
O	R	K	A	O	U	P	M	T	Q	C	N	V	N	S
T	O	N	R	M	L	M	E	Q	D	I	V	I	D	E
A	C	U	F	I	O	K	X	U	Q	E	E	U	I	V
R	A	Y	F	N	C	E	L	U	R	N	R	Q	C	Y
E	L	Y	L	A	X	E	P	R	X	T	T	E	E	L
M	P	X	R	T	L	A	R	E	M	U	N	O	R	P
U	X	B	U	O	Q	C	I	A	R	B	E	G	L	A
N	E	S	N	R	E	T	T	A	P	Y	L	I	K	E

WORDLIST

EQUIVALENT	TERM	UNKNOWN	COMMON	SUBTRACT
COEFFICIENT	FACTORISE	ALGEBRAIC	MULTIPLY	EXPAND
PRONUMERAL	LIKE	BRACKETS	PATTERNS	INVERT
RULE	ORDER	SIMPLIFY	FRACTION	INDICE
DENOMINATOR	RECIPROCAL	POWER	DIVIDE	NUMERATOR

Algebra - Find a Word

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Find the words in the puzzle from the wordlist.



P	F	A	C	T	O	R	I	S	E	T	E	Q	O	T
Y	L	P	I	T	L	U	M	X	P	C	X	X	R	C
P	R	Y	N	X	P	R	R	E	W	O	P	T	D	A
L	E	N	O	I	P	X	E	P	X	E	A	N	E	R
U	C	W	I	D	E	R	T	P	S	F	N	E	R	T
L	I	O	T	E	O	U	U	I	C	F	D	L	Y	B
R	P	N	C	N	U	Q	M	O	S	I	I	A	I	U
O	R	K	A	O	U	P	M	T	Q	C	N	V	N	S
T	O	N	R	M	L	M	E	Q	D	I	V	I	D	E
A	C	U	F	I	O	K	X	U	Q	E	E	U	I	V
R	A	Y	F	N	C	E	L	U	R	N	R	Q	C	Y
E	L	Y	L	A	X	E	P	R	X	T	T	E	E	L
M	P	X	R	T	L	A	R	E	M	U	N	O	R	P
U	X	B	U	O	Q	C	I	A	R	B	E	G	L	A
N	E	S	N	R	E	T	T	A	P	Y	L	I	K	E

WORDLIST

~~EQUIVALENT~~ ~~TERM~~ ~~UNKNOWN~~ ~~COMMON~~ ~~SUBTRACT~~

~~COEFFICIENT~~ ~~FACTORISE~~ ~~ALGEBRAIC~~ ~~MULTIPLY~~ ~~EXPAND~~

~~PRONUMERAL~~ ~~LIKE~~ ~~BRACKETS~~ ~~PATTERNS~~ ~~INVERT~~

~~RULE~~ ~~ORDER~~ ~~SIMPLIFY~~ ~~FRACTION~~ ~~INDICE~~

~~DENOMINATOR~~ ~~RECIPROCAL~~ ~~POWER~~ ~~DIVIDE~~ ~~NUMERATOR~~

FREEFALL MATHEMATICS

ALTITUDE

VOLUME & CAPACITY

Centicube Volumes

Volume is a measure of the amount of space an object occupies. The units used in measuring volume are cubic units: mm^3, cm^3 and m^3. The solids on this sheet are all constructed from centicubes (each 1 cm^3). This allows the volume to be found by counting the cubes with the volume being the total number of cubes (in cm^3).

The first 2 columns ask you to count the cubes, you have to accept that the cubes that you can't see, due to cubes being in front, are still there. A cube must be on top of another cube to be above the first layer. When you write the volume you must include the units (cm^3).

Column 3 introduces the Volume formula : $V = Ah$. This applies to any shape with a uniform cross-section, (this means the blue area is the same throughout the height of the shape). Count the number of squares that make up the blue area (A) and multiply this by the number of red squares that make up the height (h), this will give you the volume in cm^3.

Don't let the word 'height' confuse you. You may feel that height must go up and down but the word height is used to describe the direction of the uniform cross-section. When you think about it if you turn the page sideways it will be up and down, so the answer will be the same.

Centicube Volumes

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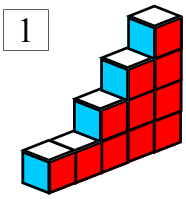
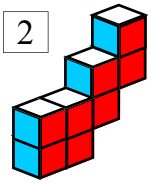
Calculate the volume of these solids built from identical cubes, assume 1 cube = 1cm^3.

1



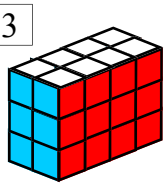
V = ____ cm^3

2



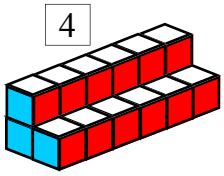
V = ____ cm^3

3



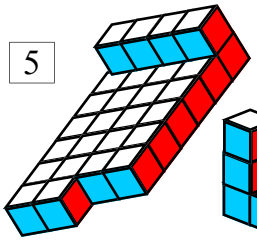
V = _____

4



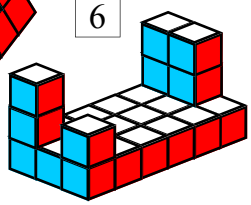
V = _____

5



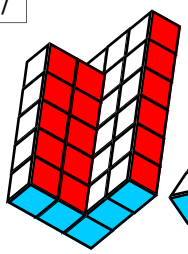
V = _____

6



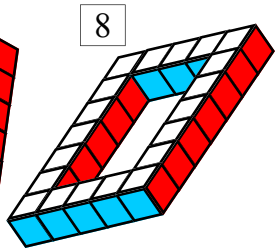
V = _____

7



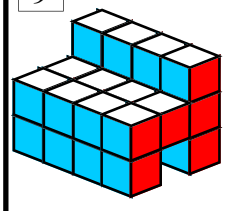
V = _____

8



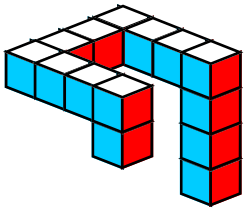
V = _____

9



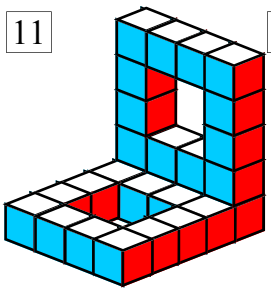
V = _____

10



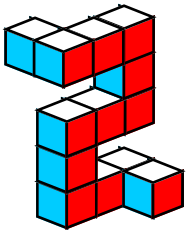
V = _____

11



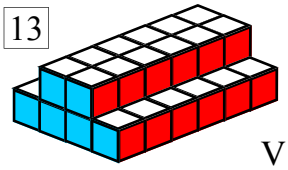
V = _____

12



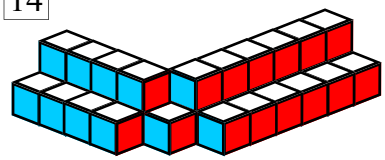
V = _____

13



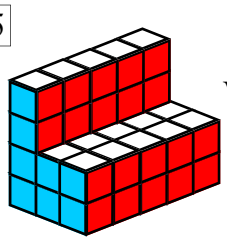
V = _____

14



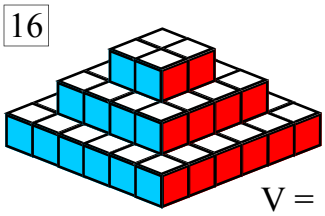
V = _____

15



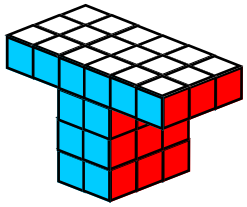
V = _____

16



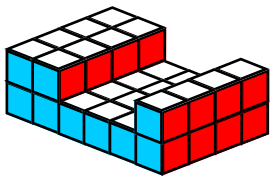
V = _____

17



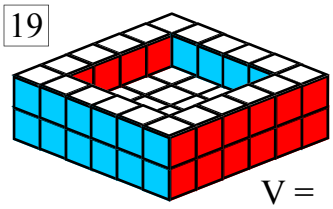
V = _____

18



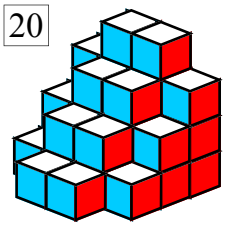
V = _____

19



V = _____

20



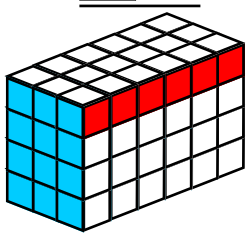
V = _____

Find the area of the front face then multiply it by the height (the distance the shape goes back into the page).

21 Face Area (A) = ____ cm^2

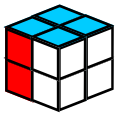
Shape height (h) = ____ cm

A × h = ____ cm^3



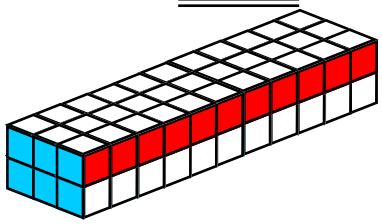
22 A = ____ cm^2 h = ____ cm

V = Ah = ____ cm^3



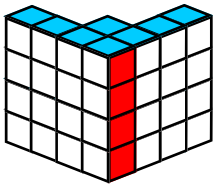
23 A = _____ h = _____

V = Ah = _____



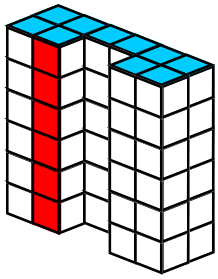
24 A = _____ h = _____

V = Ah = _____



25 A = _____ h = _____

V = Ah = _____



Centicube Volumes

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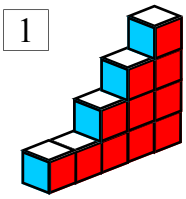
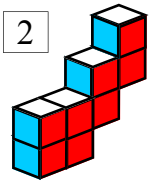
Calculate the volume of these solids built from identical cubes, assume 1 cube = 1cm^3.

1



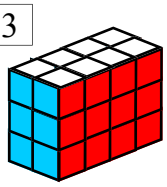
V = 11 cm^3

2



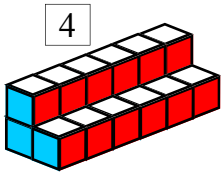
V = 8 cm^3

3



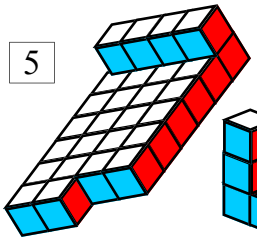
V = 24 cm^3

4



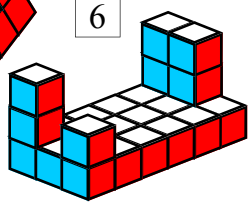
V = 18 cm^3

5



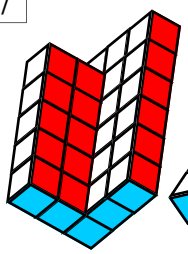
V = 30 cm^3

6



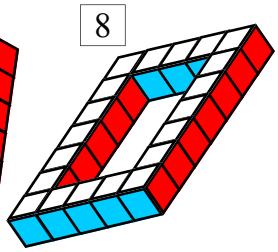
V = 25 cm^3

7



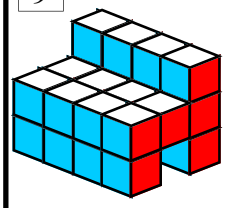
V = 28 cm^3

8



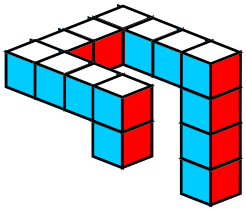
V = 20 cm^3

9



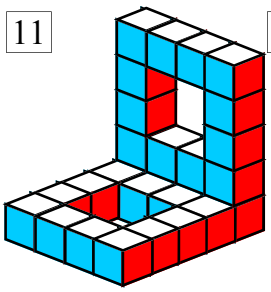
V = 24 cm^3

10



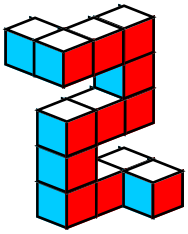
V = 14 cm^3

11



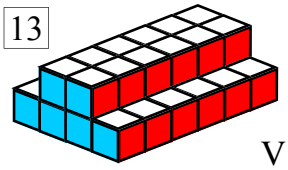
V = 28 cm^3

12



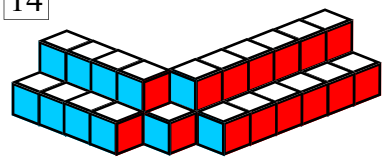
V = 13 cm^3

13



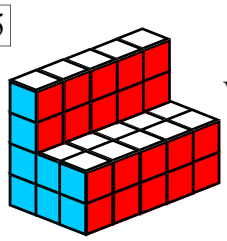
V = 36 cm^3

14



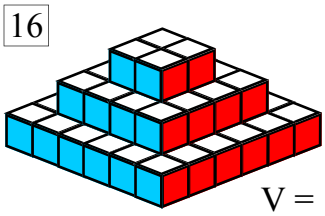
V = 31 cm^3

15



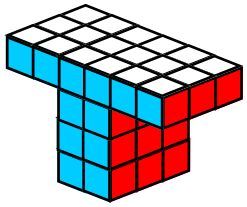
V = 40 cm^3

16



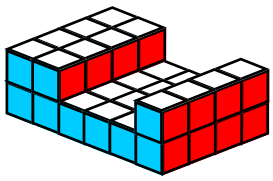
V = 56 cm^3

17



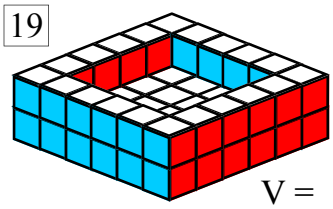
V = 36 cm^3

18



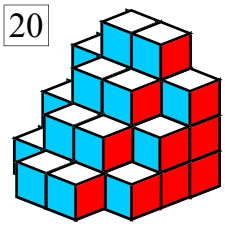
V = 36 cm^3

19



V = 56 cm^3

20



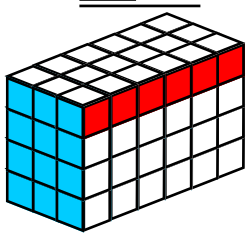
V = 32 cm^3

Find the area of the front face then multiply it by the height (the distance the shape goes back into the page).

21 Face Area (A) = 12 cm^2

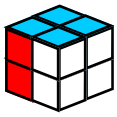
Shape height (h) = 6 cm

A × h = 72 cm^3



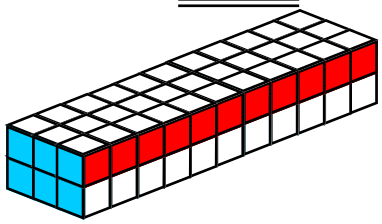
22 A = 4 cm^2 h = 2 cm

V = Ah = 8 cm^3



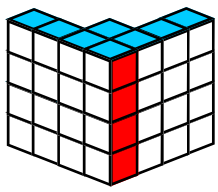
23 A = 6 cm^2 h = 11 cm

V = Ah = 66 cm^3



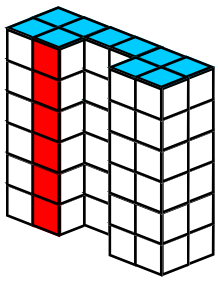
24 A = 8 cm^2 h = 4 cm

V = Ah = 32 cm^3



25 A = 10 cm^2 h = 6 cm

V = Ah = 60 cm^3

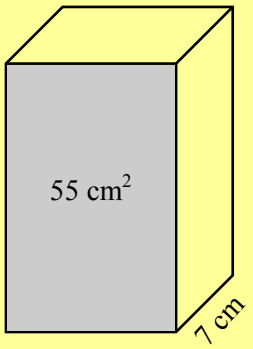


Volume, Area and Height - Volume of a Prism

When a solid has a uniform cross section (same shape throughout the height of the solid) then it is a prism. The formula V = *Ah* can be used to obtain the volume of the solid. The area will be given in square units (mm^2, cm^2, m^2) and the height of the solid will be given in mm, cm and m. Multiply the two together to get your answer. The answers will be in cubic units mm^3, cm^3, m^3.

The first columns uses the same method as shown in the example below, this is: writing the formula (V = *Ah*), substituting the values (for *A* and *h*), with an '=' sign before them and a '×' sign between them, then the answer with the appropriate cubic units after it.

Example



V = A*h*

= 55 × 7

V = 385 cm^3 ← Volume is measured in cubic units

Column 2 is the same as column 1 except that the height unit doesn't match the area unit. So you need to change the height to same units as the area. Then solve as before.

Column 3 applies your skills to a duck pond, the ducks are getting their bills all muddy when they dive down, it's time to give them a little more depth.

Volume, Area and Height - Volume of a Prism

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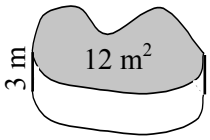
Multiply the area by the height to find the volume

1

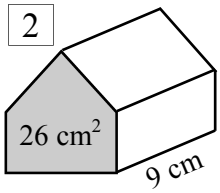


$V = Ah$

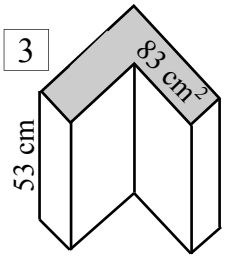
$= \square \times \square$

$V = \square$ m³

2



3



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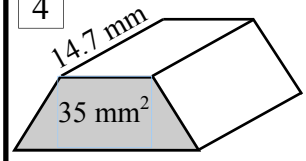

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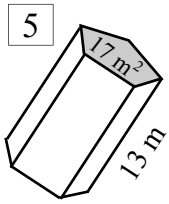

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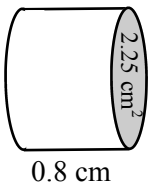

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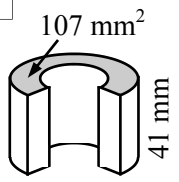

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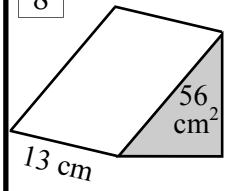
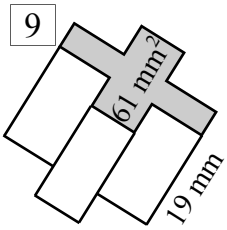
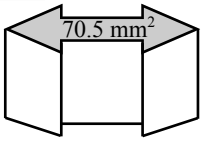


9

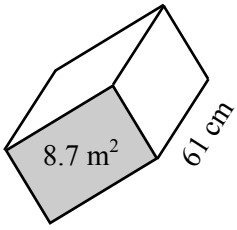


Change the height unit to match the area units. Then multiply for the volume. Round to 2 d.p.

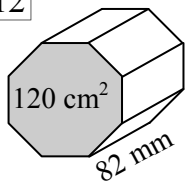
10



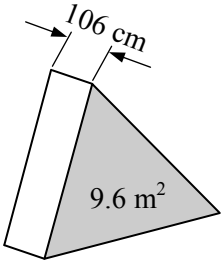
11



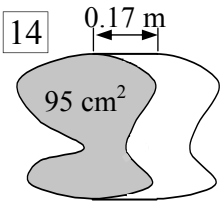
12



13



14



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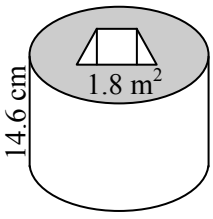
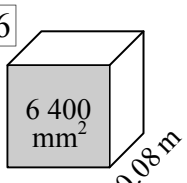
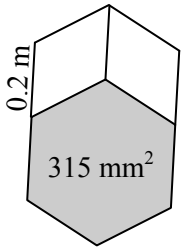


16

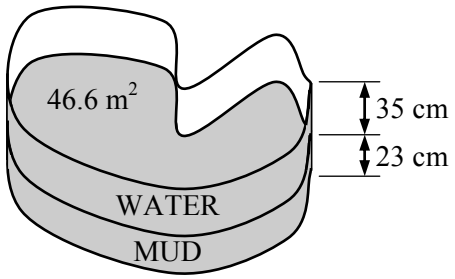


17



Apply your volume skills to these questions. The bill for this job is too much to wear!

A council duck pond has to have the mud build-up removed. It has to be drained, excavated and refilled.



18 Find the mud volume in m³.

19 Find the water volume in m³.

20 Mud excavation/removal costs $712 per m³ (or part m³). Find the cost for mud removal.

21 Find the volume of water in the mud cleared pond in m³.

22 Find the percentage increase for the volume of water. (1 d.p)

$$\% \text{ Increase} = \frac{H_2O\ Vol_{NEW}}{H_2O\ Vol_{OLD}} \times 100 - 100$$

$$= \frac{\square}{\square} \times 100 - 100$$

% Increase = _____

Volume, Area and Height - Volume of a Prism

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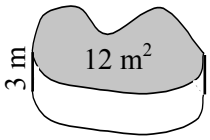
Multiply the area by the height to find the volume

1

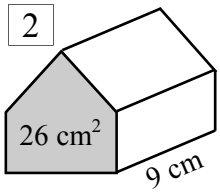


$V = Ah$

$= 12 \times 3$

$V = 36$ m³

2

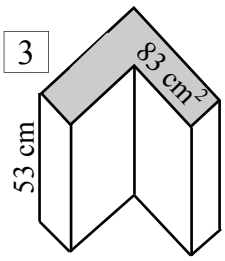


$V = Ah$

$= 26 \times 9$

$V = 234$ cm³

3

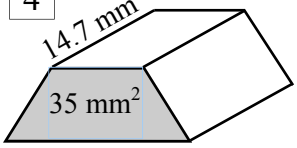


$V = Ah$

$= 83 \times 53$

$V = 4\,399$ cm³

4

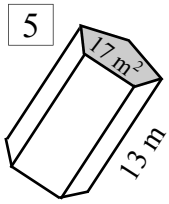


$V = Ah$

$= 35 \times 14.7$

$V = 514.5$ mm³

5

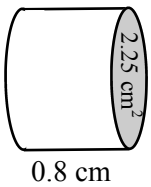


$V = Ah$

$= 17 \times 13$

$V = 221$ m³

6

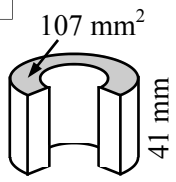


$V = Ah$

$= 2.25 \times 0.8$

$V = 1.8$ cm³

7

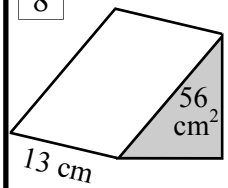


$V = Ah$

$= 107 \times 41$

$V = 4\,387$ mm³

8

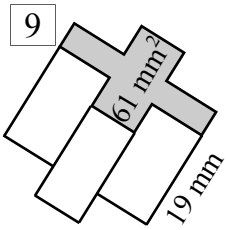


$V = Ah$

$= 56 \times 13$

$V = 728$ cm³

9



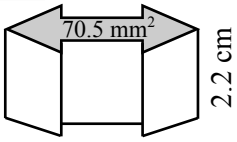
$V = Ah$

$= 61 \times 19$

$V = 1\,159$ mm³

Change the height unit to match the area units. Then multiply for the volume. Round to 2 d.p.

10

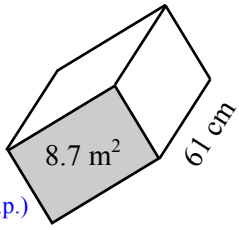


$V = Ah$

$= 70.5 \times 22$

$V = 1\,551$ mm³

11

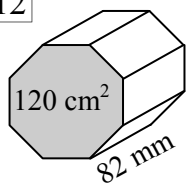


$V = Ah$

$= 8.7 \times 0.61$

$V = 5.31$ m³ (2 d.p.)

12

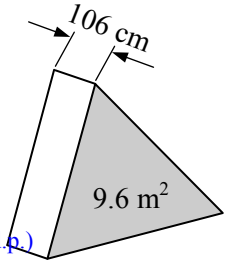


$V = Ah$

$= 120 \times 8.2$

$V = 984$ cm³

13

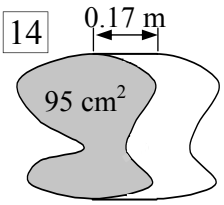


$V = Ah$

$= 9.6 \times 1.06$

$V = 10.18$ m³ (2 d.p.)

14

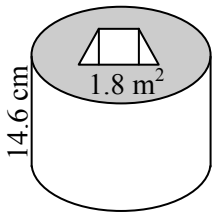


$V = Ah$

$= 95 \times 17$

$V = 1\,615$ cm³

15

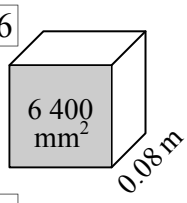


$V = Ah$

$= 1.8 \times 0.146$

$V = 0.26$ m³

16

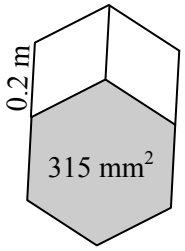


$V = Ah$

$= 6\,400 \times 80$

$V = 512\,000$ mm³

17



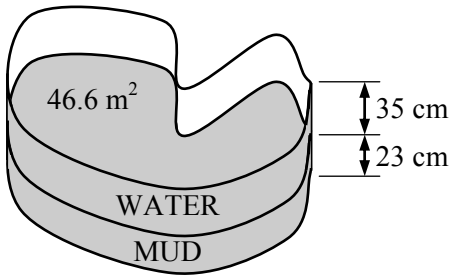
$V = Ah$

$= 315 \times 200$

$V = 63\,000$ mm³

Apply your volume skills to these questions. The bill for this job is too much to wear!

A council duck pond has to have the mud build-up removed. It has to be drained, excavated and refilled.



18 Find the mud volume in m³.

$V = Ah$

$= 46.6 \times 0.23$

$V = 10.718$ m³

19 Find the water volume in m³.

$V = Ah$

$= 46.6 \times 0.35$

$V = 16.31$ m³

20 Mud excavation/removal costs \$712 per m³ (or part m³). Find the cost for mud removal.

Cost $= 11 \times 712$

Cost $=$ \$7 832

21 Find the volume of water in the mud cleared pond in m³.

$V = Ah$

$= 46.6 \times 0.58$

$V = 27.028$ m³

22 Find the percentage increase for the volume of water. (1 d.p)

$$\%\ \text{Increase} = \frac{H_2O\ Vol_{NEW}}{H_2O\ Vol_{OLD}} \times 100 - 100$$

$$= \frac{27.028}{16.31} \times 100 - 100$$

% Increase $=$ 65.7% (1 d.p.)

Volume of Cubes and Rectangular Prisms (Sheet 1)

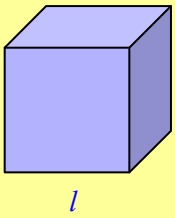
This sheet deals with cubes and rectangular prisms. Sheet 2 is harder and includes changing units and finding lengths given volume.

Rectangular prisms are the most common form of packaging, from shipping containers to match boxes they are everywhere. To calculate their volume you multiply the 3 sides together, in any order. Because we have V = Ah as the standard formula for volume for a prism with the Area being a rectangle, we can replace the A with $l \times b$. So we have V = $l \times b \times h$, or V = lbh.

As a cube has all lengths the same it is like the formula V = $l \times b \times h$ is changed to V = $l \times l \times l$, which can be simplified to V = l^3. Sometimes this is written as V = s^3, where s is the side length. It is the same thing, just a different way of writing it.

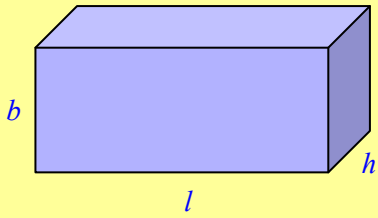
Cubes

$V = l^3$



Rectangular Prisms

$V = lbh$

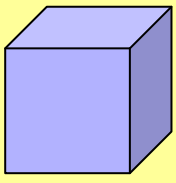


Example

$V = l^3$

$= 18^3$

$V = 5\,832\ cm^3$

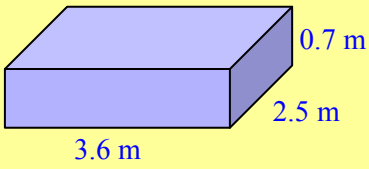


Example

$V = lbh$

$= 3.6 \times 2.5 \times 0.7$

$V = 6.3\ m^3$



Volume is measured in cubic units, that means that your answer should have a ^{3} above the unit when written in the answer. So mm^3, cm^3 and m^3.

Volume of Cubes and Rectangular Prisms (Sheet 1)

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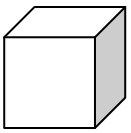
Use $V=l^3$ to find the volume of these cubes. Remember to show a ^{3} on the units.

1



$V = l^3$

$= \square^3$

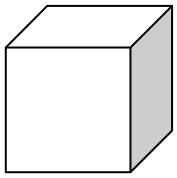
$V = \square$ cm^3

2

$V = l^3$

$=$

$V =$



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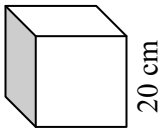

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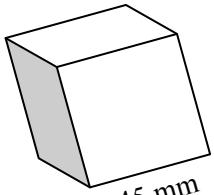
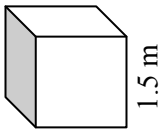


Round these answers to 2 d.p.

5



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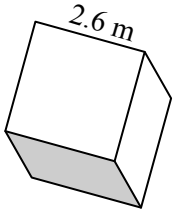

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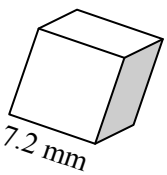
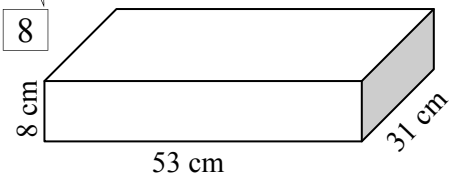


Use $V=lbh$ to find the volume of these rectangular prisms.

8

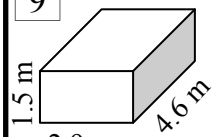


$V = lbh$

$= \square \times \square \times \square$

$V = \square$ cm^3

9



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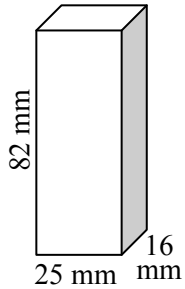

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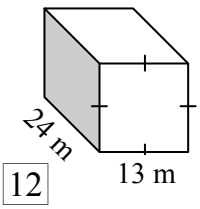

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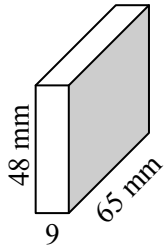

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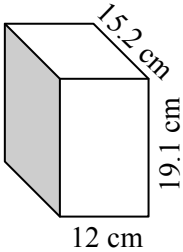

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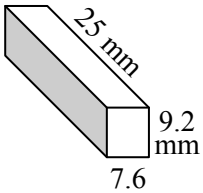

Now apply your skills to these. Round to 1 d.p. if required.

15 Ron is hiring a storage unit. It has sides 8 m, 7.3 m and 5.7 m. Find its volume to the nearest m^3.

16 If storage costs \$0.60/m^3 per mth, how much should Ron expect to pay for the storage unit per month?

17 Lilly's favorite glass is shaped like a rectangular prism. It is 12 cm tall with both sides 7 cm. Find its volume.

18 Lilly's drops 5 ice cubes in the empty glass. Each cube has a side length of 1.8 cm. Find volume of ice.

19 Find the amount of drink that can be added to the glass, in cm^3.

20 Calculate the volume of ice as a percentage of the glass volume.

$$\% \text{ Ice} = \frac{\square}{\square} \times 100$$

$\%$ Ice $=$

Volume of Cubes and Rectangular Prisms (Sheet 1)

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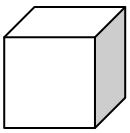
Use $V=l^3$ to find the volume of these cubes. Remember to show a ^{3} on the units.

1

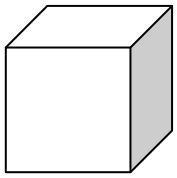


$V = l^3$

$= 5^3$

$V = 125$ cm^3

2

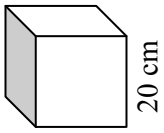


$V = l^3$

$= 9^3$

$V = 729$ m^3

3

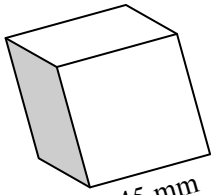


$V = l^3$

$= 20^3$

$V = 8\,000$ cm^3

4



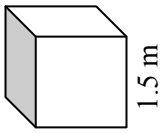
$V = l^3$

$= 45^3$

$V = 91\,125$ mm^3

Round these answers to 2 d.p.

5

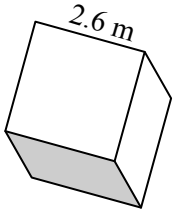


$V = l^3$

$= 1.5^3$

$V = 3.38$ m^3 (2 d.p.)

6

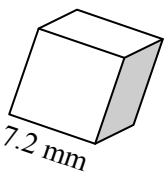


$V = l^3$

$= 2.6^3$

$V = 17.58$ m^3 (2 d.p.)

7



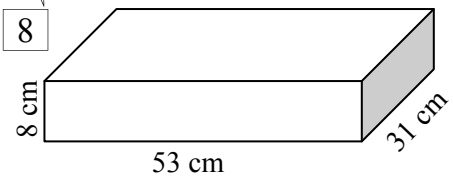
$V = l^3$

$= 7.2^3$

$V = 373.25$ mm^3 (2 d.p.)

Use $V=lbh$ to find the volume of these rectangular prisms.

8

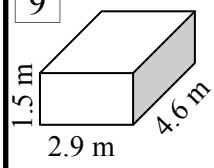


$V = lbh$

$= 8 \times 53 \times 31$

$V = 13\,144$ cm^3

9

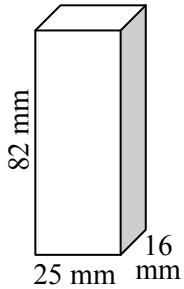


$V = lbh$

$= 1.2 \times 2.9 \times 4.6$

$V = 20.01$ m^3

10

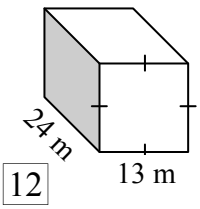


$V = lbh$

$= 82 \times 25 \times 16$

$V = 32\,800$ mm^3

11

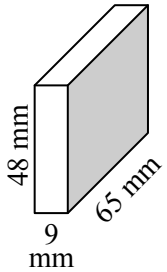


$V = lbh$

$= 13 \times 24 \times 13$

$V = 4\,056$ m^3

12

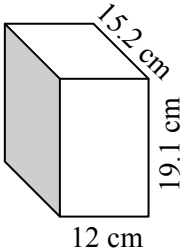


$V = lbh$

$= 9 \times 48 \times 65$

$V = 28\,080$ mm^3

13

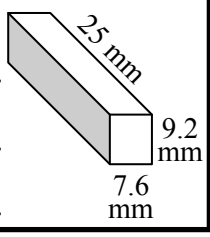


$V = lbh$

$=12 \times 15.2 \times 19.1$

$V = 3\,483.84$ cm^3

14



$V = lbh$

$=25 \times 7.6 \times 9.2$

$V = 1\,748$ mm^3

Now apply your skills to these. Round to 1 d.p. if required.

15 Ron is hiring a storage unit. It has sides 8 m, 7.3 m and 5.7 m. Find its volume to the nearest m^3.

$V = lbh$

$= 8 \times 7.3 \times 5.7$

$V = 333$ m^3 (nearest m^3)

16 If storage costs \$0.60/m^3 per mth, how much should Ron expect to pay for the storage unit per month?

Cost $= 333 \times 0.60$

Cost = \$199.80 per month

17 Lilly's favorite glass is shaped like a rectangular prism. It is 12 cm tall with both sides 7 cm. Find its volume.

$V = lbh$

$= 12 \times 7 \times 7$

$V = 588$ cm^3

18 Lilly's drops 5 ice cubes in the empty glass. Each cube has a side length of 1.8 cm. Find volume of ice.

$V = l^3$

$= 1.8^3$

$V = 5.832$ cm^3

Total $= 5 \times 5.832 = 29.16$ cm^3

19 Find the amount of drink that can be added to the glass, in cm^3.

$588 - 29.16 = 558.84$ cm^3

20 Calculate the volume of ice as a percentage of the glass volume.

$$\% \text{ Ice} = \frac{29.16}{588} \times 100$$

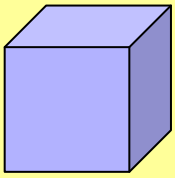
% Ice = 4.96% (2 d.p.)

Volume of Cubes and Rectangular Prisms (Sheet 2)

This sheet deals with cubes and rectangular prisms. Sheet 2 is more challenging than Sheet 1 as it includes changing units and finding lengths given volume.

Cubes

$V = l^3$



Rectangular Prisms

$V = lbh$

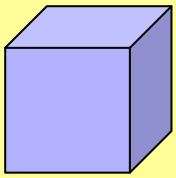


Example

$V = l^3$

$= 18^3$

$V = 5\,832\ cm^3$

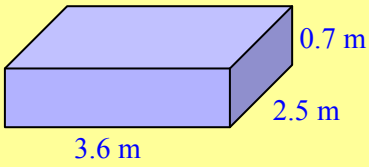


Example

$V = lbh$

$= 3.6 \times 2.5 \times 0.7$

$V = 6.3\ m^3$



Volume is measured in cubic units, that means that your answer should have a ^{3} above the unit when written in the answer. So mm^3, cm^3 and m^3.

Column 1 involves finding the volume for cubes. So use the method shown above (left), note that Q.6 and Q.7 require units to be changed first before the calculation.

Column 2 also is about cubes but you are given the volume. This time you are required to find the side length of the cube. To do this, cube root the volume, this will give you the side length. Column 2 then introduces rectangular prisms, solve these using the method above (right). Watch for units not matching, make sure you change the units to a common unit before solving.

Column 3 ends with the volume of a rectangular prism being given. You have to find the height given two other side lengths. The process is solved by multiplying the two sides you are given to find the face area. As $V = Ah$ for a prism, $h = {}^{V}/_{A}$. So divide the volume by the area to find the height.

Volume of Cubes and Rectangular Prisms (Sheet 2)

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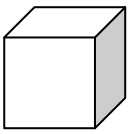
Use $V=l^3$ to find the volume of these cubes. Round to 1 d.p. if needed.

1



10 cm

$V = l^3$

$= \square^3$

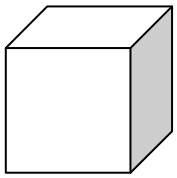
$V = \square$ cm^3

2

$V = l^3$

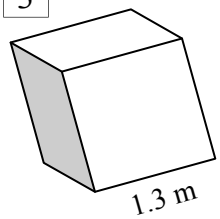
$=$

$V =$



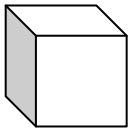
82 mm

3



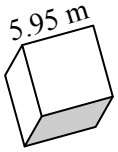
1.3 m

4



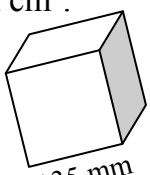
47 cm

5



5.95 m

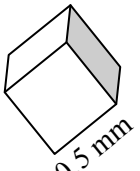
6 Give Volume in cm^3.



135 mm

$=$ cm

7 Give Volume in cm^3.

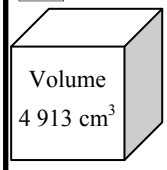


20.5 mm

$=$ cm

Cube root the volume to find the cube's side length. 1 d.p.

7



Volume 4 913 cm^3

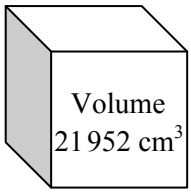
$l = \sqrt[3]{\text{V}}$

$= \sqrt[3]{}$

Calc: $\sqrt[3]{}$ 4 9 1 3 =

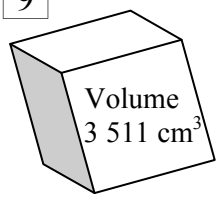
$l =$

8



Volume 21 952 cm^3

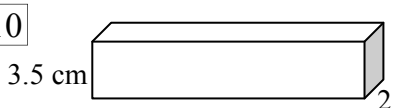
9



Volume 3 511 cm^3

Use $V=lbh$ to find the volume for these rectangular prisms. So multiply the three sides!

10



3.5 cm

15 cm

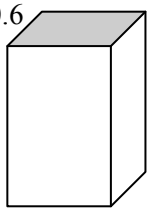
2 cm

$V = lbh$

$= \square \times \square \times \square$

$V = \square$ cm^3

11 Lengths in m.



0.6

2.5

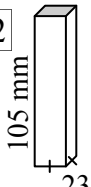
1.6

$V = lbh$

$=$

$V =$

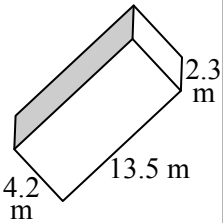
12



105 mm

23 mm

13



2.3 m

13.5 m

4.2 m

14 Find the volume of a room with lengths 4.3 m, 6.2 m and 2.7 m.

15 Find the volume of a refrigerator with sides 1.58 m, 95 cm and 65 cm. Answer in m^3 to 2 decimal places.

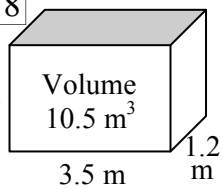
An air-filled platform with lengths of 16.8 m, 11.2 m and 85 cm is to be pumped up at a play centre. Find the:

16 Volume of the Platform (m^3)

17 Time taken to inflate the platform to the nearest minute, given a pump operates at 2.2 m^3/min.

Find the height of these if the volume and base lengths are given. $V = Ah \rightarrow h = V \div A$

18



h

Volume 10.5 m^3

3.5 m

1.2 m

Find Area of Base

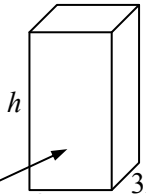
$A = \square \times \square$

$A =$ m^2

$h = V \div A$

$= \square \div \square$

$h =$



h

85 cm

32 cm

19 $V = 312\,800$ cm^3

$h =$

$A =$ $=$

$A =$ $h =$

Volume of Cubes and Rectangular Prisms (Sheet 2)

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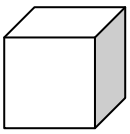
Use $V=l^3$ to find the volume of these cubes. Round to 1 d.p. if needed.

1



$V = l^3$

$= 10^3$

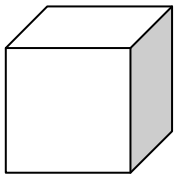
$V = 1\,000$ cm^3

2

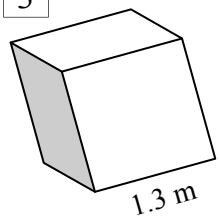
$V = l^3$

$= 82^3$

$V = 551\,368$ mm^3



3



$V = l^3$

$= 1.3^3$

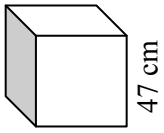
$V = 2.2$ m^3 (1 d.p)

4

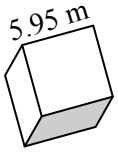
$V = l^3$

$= 47^3$

$V = 103\,823$ cm^3



5



$V = l^3$

$= 5.95^3$

$V = 210.6$ m^3

(1 d.p.)

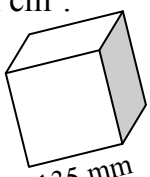
6 Give Volume in cm^3.

$V = l^3$

$= 13.5^3$

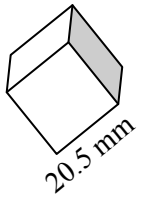
$V = 2\,460.4$ cm^3

(1 d.p.)



$= 13.5$ cm

7 Give Volume in cm^3.



$V = l^3$

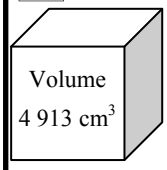
$= 2.05^3$

$V = 8.6$ cm^3

$= 2.05$ cm

Cube root the volume to find the cube's side length. 1 d.p.

7



$l = \sqrt[3]{V}$

$= \sqrt[3]{4\,913}$

Calc: $\sqrt[3]{}$ 4 9 1 3 =

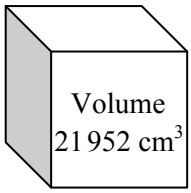
$l = 17$ cm

8

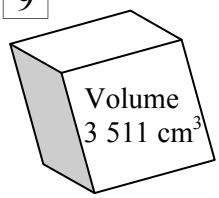
$l = \sqrt[3]{V}$

$= \sqrt[3]{21\,952}$

$l = 28$ cm



9



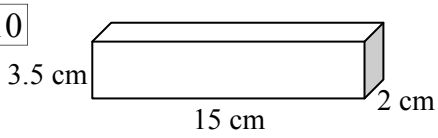
$l = \sqrt[3]{V}$

$= \sqrt[3]{3\,511}$

$l = 15.2$ cm (1 d.p.)

Use $V=lbh$ to find the volume for these rectangular prisms. So multiply the three sides!

10



$V = lbh$

$= 15 \times 3.5 \times 2$

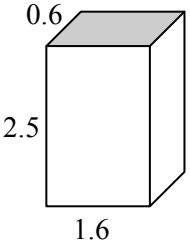
$V = 105$ cm^3

11 Lengths in m.

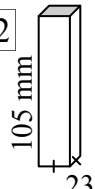
$V = lbh$

$= 1.6 \times 0.6 \times 2.5$

$V = 2.4$ m^3



12



$V = lbh$

$= 105 \times 23 \times 23$

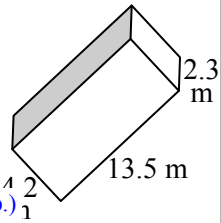
$V = 55\,545$ mm^3

13

$V = lbh$

$= 4.2 \times 13.5 \times 2.3$

$V = 130.4$ m^3 (1 d.p.)



14 Find the volume of a room with lengths 4.3 m, 6.2 m and 2.7 m.

$V = lbh$

$= 4.3 \times 6.2 \times 2.7$

$V = 71.98$ m^3 (2 d.p.)

15 Find the volume of a refrigerator with sides 1.58 m, 95 cm and 65 cm. Answer in m^3 to 2 decimal places.

$V = lbh$

$= 1.58 \times 0.95 \times 0.65$

$V = 0.98$ m^3 (2 d.p.)

An air-filled platform with lengths of 16.8 m, 11.2 m and 85 cm is to be pumped up at a play centre. Find the:

16 Volume of the Platform (m^3)

$V = lbh$

$= 16.8 \times 11.2 \times 0.85$

$V = 159.936$ m^3

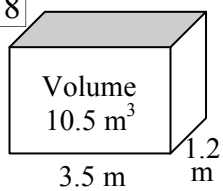
17 Time taken to inflate the platform to the nearest minute, given a pump operates at 2.2 m^3/min.

Time $= 159.936 \div 2.2$

Time $= 73$ mins (Nearest min) 1 h 13m

Find the height of these if the volume and base lengths are given. $V = Ah \rightarrow h = V \div A$

18



Find Area of Base

$A = 3.5 \times 1.2$

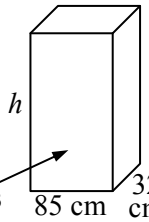
$A = 4.2$ m^2

$h = V \div A$

$= 10.5 \div 4.2$

$h = 2.5$ m

19 $V = 312\,800$ cm^3



$h = V \div A$

$A = 85 \times 32$

$= 312\,800 \div 2\,720$

$A = 2\,720$ cm^3

$h = 115$ cm

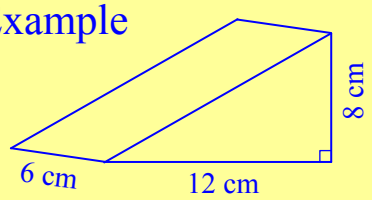
Volume of Triangular Prisms

There are two ways of calculating the volume of a triangular prism. You can calculate the Triangular face area and multiply by the height of the solid, or you can treat the prism as being half of a rectangular prism of the same size.

Area then Volume Method

$A = \frac{1}{2}bh$ then $V = Ah$

Example



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 12 \times 8$

$A = 48 \text{ cm}^2$

This is area so it is measured in square units

$V = Ah$

$= 48 \times 6$

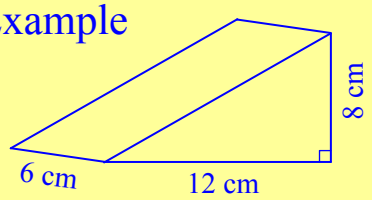
$V = 288 \text{ cm}^3$

This is volume so it is measured in cubic units

Volume Method

$V = \frac{1}{2}lbh$

Example



$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 8 \times 12 \times 6$

$V = 288 \text{ m}^3$

This is volume so it is measured in cubic units

The second method finds the answer in one step so it is quicker. But the first method is essential to learn as it is a method that will be used regularly with prisms.

Column 1 starts with a reminder about the method of finding the area of a triangle. The remainder of the column is using the above left method. It may be that you prefer the second method, but apply the area then volume process regardless, it is a skill you will need.

Column 2 is the method shown above right. So it is quicker as it is solved using one formula/ substitution.

Column 3 can be solved using either method and are worded questions. Watch that all units are the same when finding the volume.

Volume of Triangular Prisms

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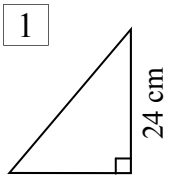
Use A=½*bh* to find the area of these triangles.

1



A = ½*bh*

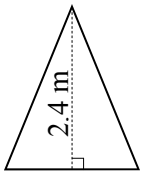
= ½ × ☐ × ☐

0 . 5 × 1 9 × 2 4 =

A = ☐ cm²

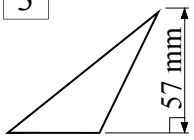
2

A =



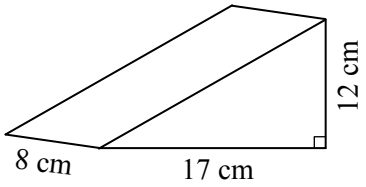
A =

3



Find the volume using 2 steps, find Area first, using A=½*bh*, then the Volume using V = A*h*.

4



A = ½*bh*

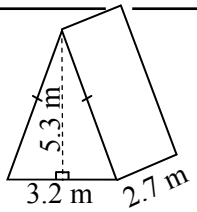
V = A*h*

= ☐ × 8

cm²

cm³

5



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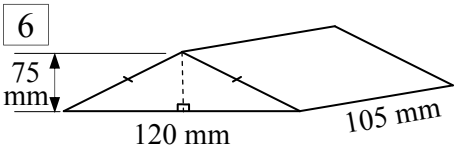


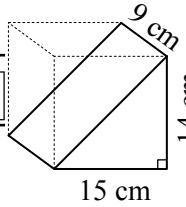
A triangular prism is half of a rectangular Prism. So use V=½*lbh* to find the volume.

7

V = ½*lbh*

= ½× ☐ × ☐ × ☐

V = ☐ cm³



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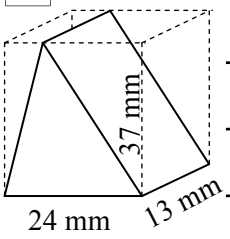

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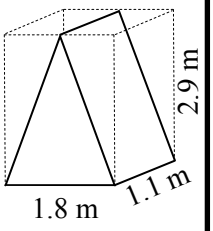

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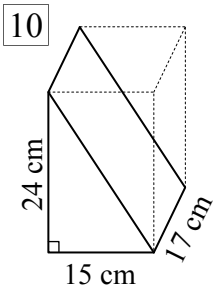

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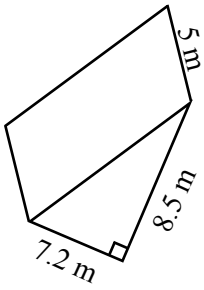

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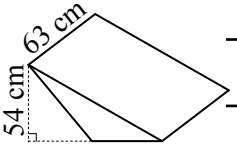

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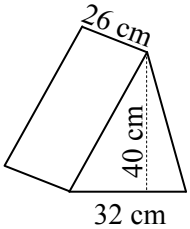
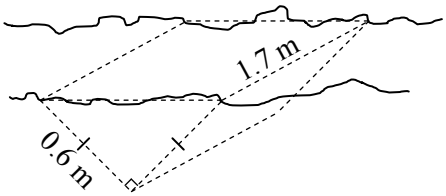
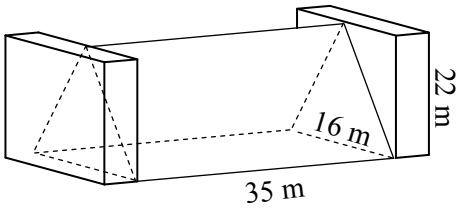


Now apply your skills to these questions.

14 Jane is burying a drainage pipe in her yard. Find the volume of soil to be removed (in m³).

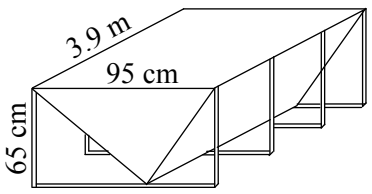


15 A mining company has a coal pile between two retaining walls. Find the volume in m³.



16 If the coal has a density of 845 kg/m³. Find the mass of the coal in T. (1 T = 1 000 kg).

17 A steel cow-feeding trough, is shown below, find its volume in m³ to 2 d.p.



Volume of Triangular Prisms

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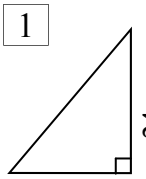
Use $A=\frac{1}{2}bh$ to find the area of these triangles.

1



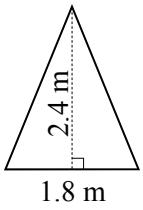
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 19 \times 24$

0 . 5 × 1 9 × 2 4 =

$A = 228 \text{ cm}^2$

2

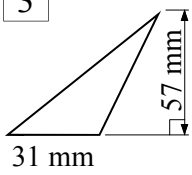


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 1.8 \times 2.4$

$A = 2.16 \text{ m}^2$

3



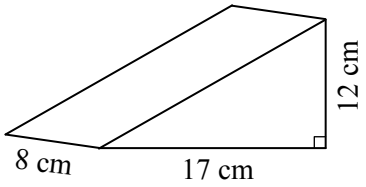
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 31 \times 57$

$A = 883.5 \text{ m}^2$

Find the volume using 2 steps, find Area first, using $A=\frac{1}{2}bh$, then the Volume using $V = AH$.

4



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 17 \times 12$

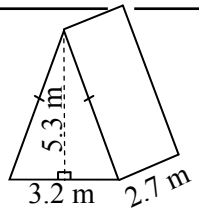
$A = 102 \text{ cm}^2$

$V = Ah$

$= 102 \times 8$

$A = 816 \text{ cm}^3$

5



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 3.2 \times 5.3$

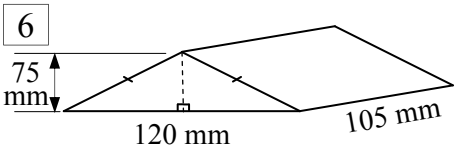
$A = 8.48 \text{ m}^2$

$V = Ah$

$= 8.48 \times 2.7$

$V = 22.896 \text{ m}^3$

6



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 120 \times 75$

$A = 4\,500 \text{ mm}^2$

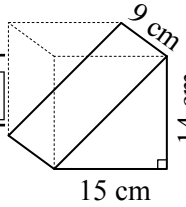
$V = Ah$

$= 4\,500 \times 105$

$V = 472\,500 \text{ mm}^3$

A triangular prism is half of a rectangular Prism. So use $V=\frac{1}{2}lbh$ to find the volume.

7

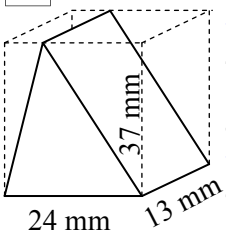


$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 15 \times 14 \times 9$

$V = 945 \text{ cm}^3$

8

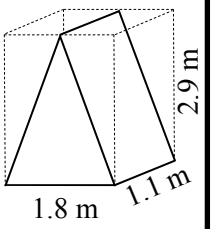


$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 24 \times 13 \times 37$

$V = 5\,772 \text{ mm}^3$

9

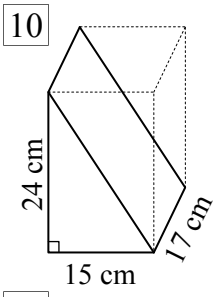


$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 1.8 \times 1.1 \times 2.9$

$V = 2.871 \text{ m}^3$

10

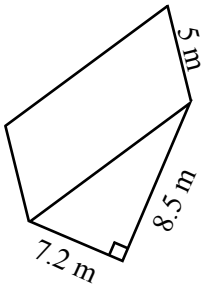


$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 15 \times 24 \times 17$

$V = 3\,060 \text{ cm}^3$

11

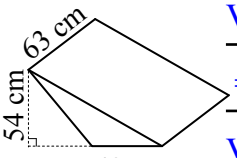


$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 7.2 \times 8.5 \times 5$

$V = 153 \text{ m}^3$

12

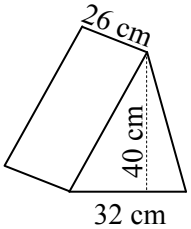


$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 42 \times 54 \times 63$

$V = 71\,442 \text{ cm}^3$

13



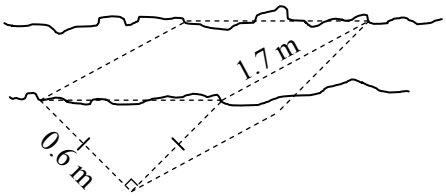
$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 32 \times 40 \times 26$

$V = 16\,640 \text{ cm}^3$

Now apply your skills to these questions.

14 Jane is burying a drainage pipe in her yard. Find the volume of soil to be removed (in m^3).

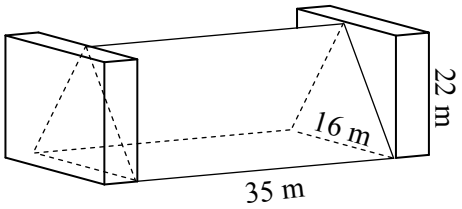


$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 0.6 \times 0.6 \times 1.7$

$V = 0.306 \text{ m}^3$

15 A mining company has a coal pile between two retaining walls. Find the volume in m^3.



$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 35 \times 16 \times 22$

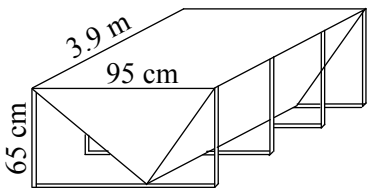
$V = 6\,160 \text{ m}^3$

16 If the coal has a density of 845 kg/m^3. Find the mass of the coal in T. (1 T = 1 000 kg).

Mass = $6\,160 \times 0.845$

Mass = 5 205.5 T

17 A steel cow-feeding trough, is shown below, find its volume in m^3 to 2 d.p.



$V = \frac{1}{2}lbh$

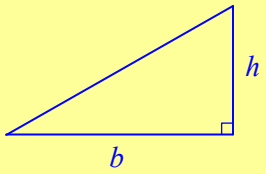
$= \frac{1}{2} \times 0.65 \times 0.95 \times 3.9$

$V = 1.20 \text{ m}^3$ (2 d.p.)

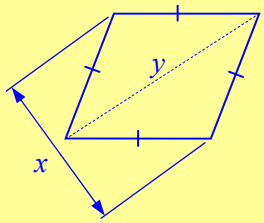
Volume of Various Prisms

Finding the volume of a prism is found by finding the area of the uniform cross section and multiplying it by the height of the solid. This sheet deals with non-circular prisms. The face that requires the area calculation is always the front face with this sheet, the formula used will be one of the following:

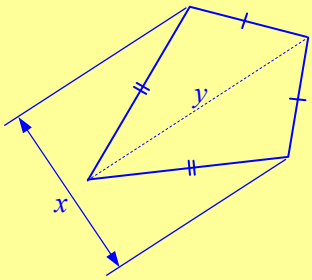
Triangle $A = \frac{1}{2}bh$



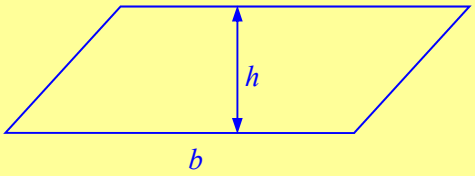
Rhombus $A = \frac{1}{2}xy$



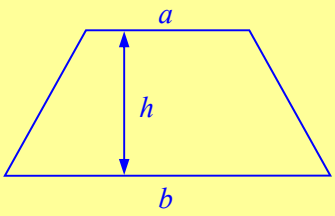
Kite $A = \frac{1}{2}xy$

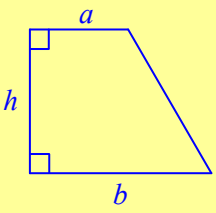


Parallelogram $A = bh$

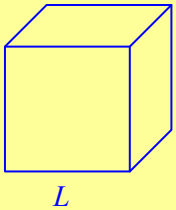


Trapezium $A = \frac{1}{2}(a + b)h$

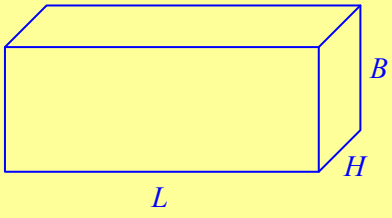




Cubes $V = l^3$

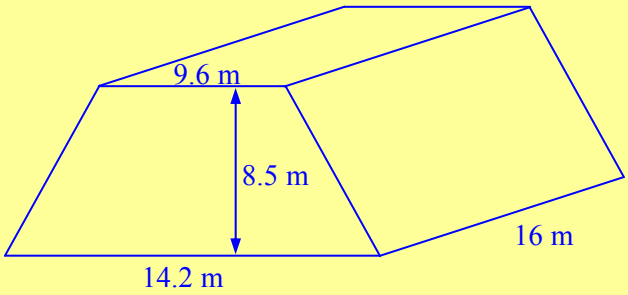


Rectangular Prisms $V = lbh$



Cubes, rectangular prisms and triangular prisms can be solved using their specific formulae. However the other solids require you to first calculate the area of the front face then use this answer to find the volume. An example for a trapezoidal prism (prism with a trapezium as its uniform cross section) is shown below.

Example - Find the volume.



$A = \frac{1}{2}(a + b) \times h$

$= \frac{1}{2} \times (9.6 + 14.2) \times 8.5$

$A = 101.15 \text{ m}^2$

This is the **area** of the front face so it is measured in square units.

$V = Ah$

$= 101.15 \times 16$

$V = 1\,618.4 \text{ m}^3$

This is volume so it is measured in cubic units

Volume of Various Prisms

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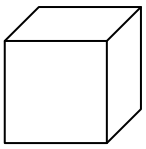
These shapes will often require you to find the face area first, then use V = A*h* for the volume. Round to 2 d.p. when needed.

1 Cube



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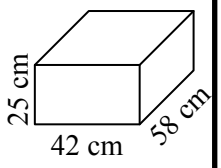
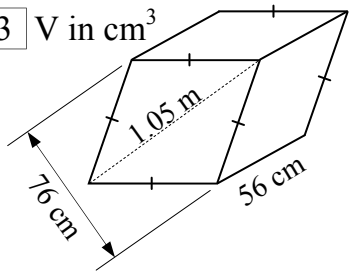
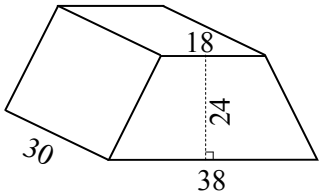


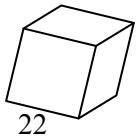
3 V in cm^3



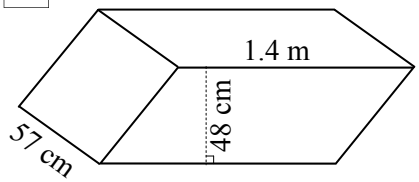
4 Units are in mm.



5 Cube



6 V in cm^3.



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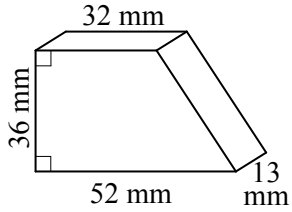
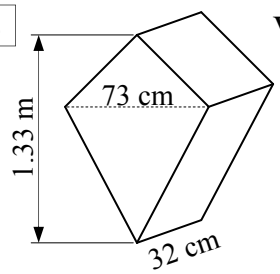


8 V in cm^3.



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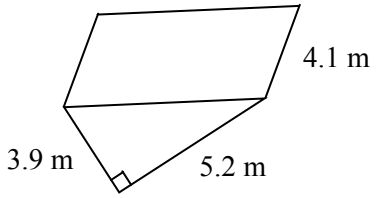

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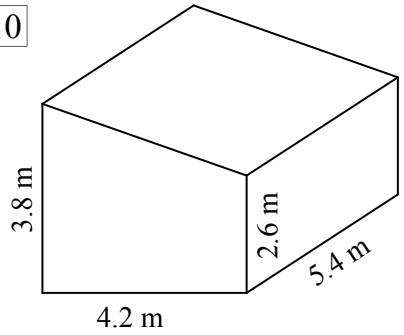
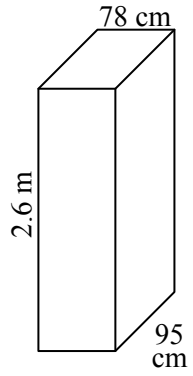
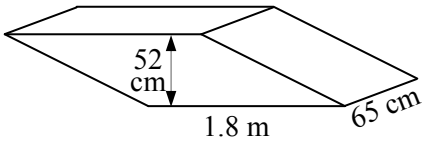


11 V in m^3.



12 V in m^3.



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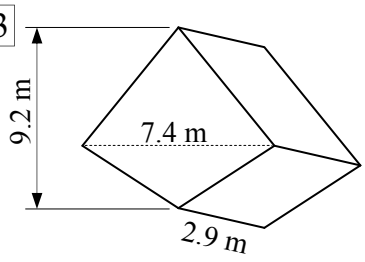

Volume of Various Prisms

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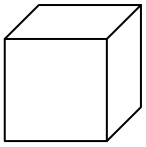
These shapes will often require you to find the face area first, then use V = Ah for the volume. Round to 2 d.p. when needed.

1 Cube

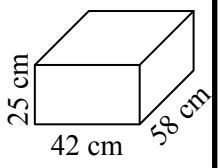
$V=l^3$



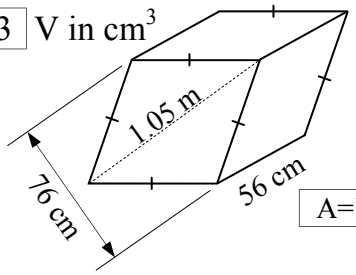
16 cm

2

$V=lbh$



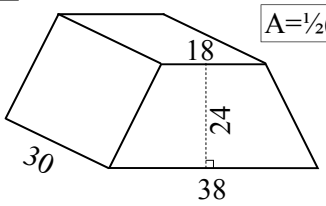
3 V in cm^3



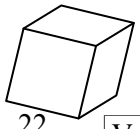
$A=\frac{1}{2}xy$

4 Units are in mm.

$A=\frac{1}{2}(a+b)h$



5 Cube

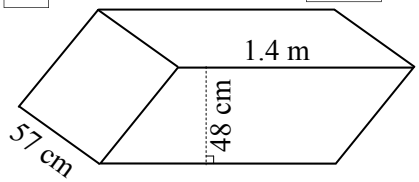


22 mm

$V=l^3$

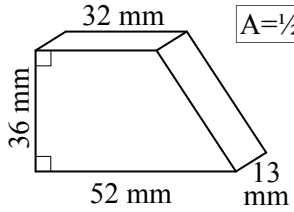
6 V in cm^3.

$A=bh$



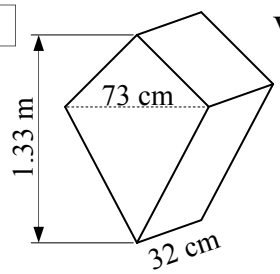
7

$A=\frac{1}{2}(a+b)h$



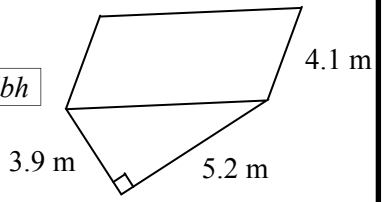
8 V in cm^3.

$A=\frac{1}{2}xy$



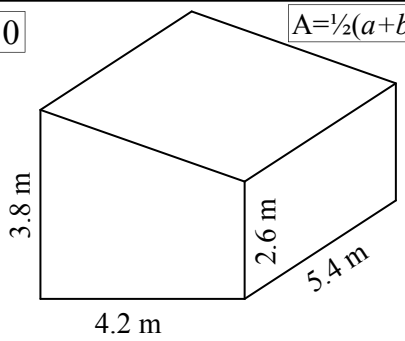
9

$V=\frac{1}{2}lbh$



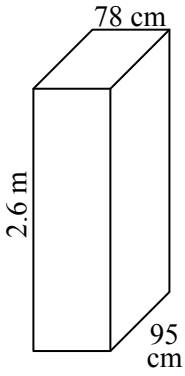
10

$A=\frac{1}{2}(a+b)h$

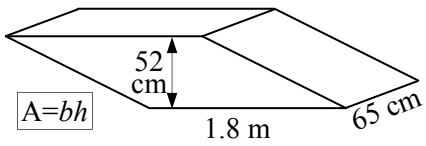


11 V in m^3.

$V=lbh$

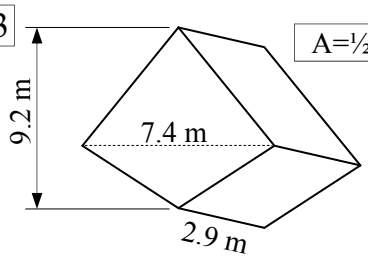


12 V in m^3.



13

$A=\frac{1}{2}xy$



Volume of Various Prisms

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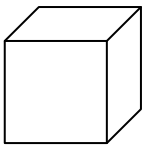
These shapes will often require you to find the face area first, then use V = AH for the volume. Round to 2 d.p. when needed.

1 Cube



$V = l^3$

$= 16^3$

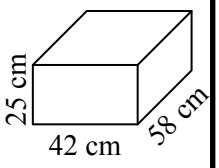
$V = 4\,096\ cm^3$

2

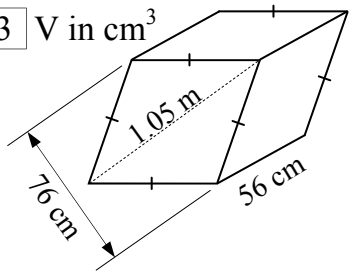
$V = lbh$

$= 25 \times 42 \times 58$

$V = 60\,900\ cm^3$



3 V in cm^3.



$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 76 \times 105$

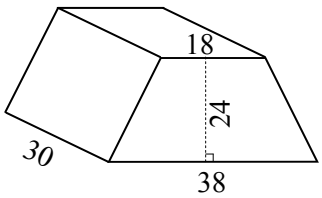
$A = 3\,990\ cm^2$

$V = Ah$

$= 3\,990 \times 56$

$V = 223\,440\ cm^3$

4 Units are in mm.



$A = \frac{1}{2}(a + b) \times h$

$= \frac{1}{2} \times (38 + 18) \times 24$

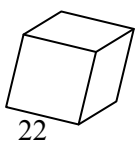
$A = 672\ mm^2$

$V = Ah$

$= 672 \times 30$

$V = 20\,160\ mm^3$

5 Cube

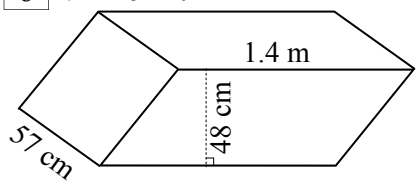


$V = l^3$

$= 22^3$

$V = 10\,648\ mm^3$

6 V in cm^3.



$A = bh$

$= 140 \times 48$

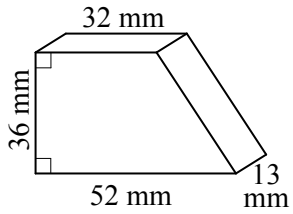
$A = 6\,720\ cm^2$

$V = Ah$

$= 6\,720 \times 57$

$V = 383\,040\ cm^3$

7



$A = \frac{1}{2}(a + b) \times h$

$= \frac{1}{2} \times (52 + 32) \times 36$

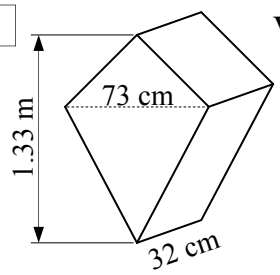
$A = 1\,512\ mm^2$

$V = Ah$

$= 1\,512 \times 13$

$V = 19\,656\ mm^3$

8 V in cm^3.



$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 73 \times 133$

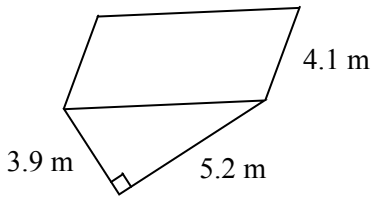
$A = 4\,854.5\ cm^2$

$V = Ah$

$= 4\,854.5 \times 32$

$V = 155\,344\ cm^3$

9

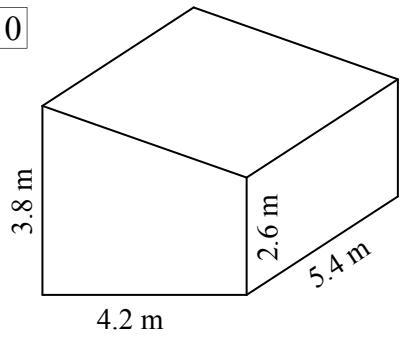


$V = \frac{1}{2}lbh$

$= \frac{1}{2} \times 5.2 \times 3.9 \times 4.1$

$V = 41.57\ m^3$ (2 d.p.)

10



$A = \frac{1}{2}(a + b) \times h$

$= \frac{1}{2} \times (3.8 + 2.6) \times 4.2$

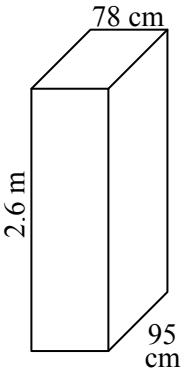
$A = 13.44\ m^2$

$V = Ah$

$= 13.44 \times 5.4$

$V = 72.58\ m^3$ (2 d.p.)

11 V in m^3.

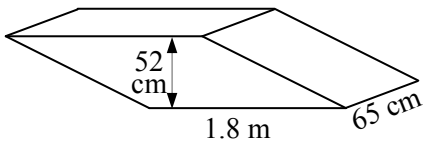


$V = lbh$

$= 2.6 \times 0.95 \times 0.78$

$V = 1.93\ m^3$ (2 d.p.)

12 V in m^3.



$A = bh$

$= 1.8 \times 0.52$

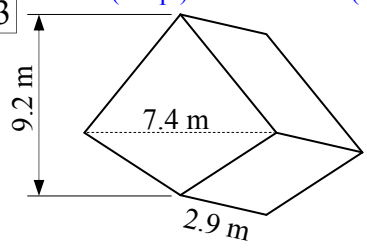
$A = 0.94\ m^2$ (2 d.p.)

$V = Ah$

$= 0.94 \times 0.65$

$V = 0.61\ m^3$ (2 d.p.)

13



$A = \frac{1}{2}xy$

$= \frac{1}{2} \times 7.4 \times 9.2$

$A = 34.04\ m^2$

$V = Ah$

$= 34.04 \times 2.9$

$V = 98.72\ m^3$ (2 d.p.)

Estimation/Terms of π - Volume of Cylinders

While mostly a calculator will be used to calculate the volume of a cylinder, it may be that you are asked to express the volume 'in terms of π' or the 'exact volume'. Both of these requests mean that π is left in the answer.

Being able to estimate volume is a valuable skill, because you can get a rough idea of the answer without a calculator, or before using a calculator. It also is a good 'self-check' mechanism in case you make a mistake using the calculator.

Column 1 has three cylinders in the column. The procedure is the same as the example at the top of the column. Use the Volume formula:

$$V = \pi r^2 h$$

Where: V = the volume of the prism
r = the radius of the circular face
h = the height of the cylinder

This will give an answer in terms of π, in other words, the exact volume.

The second step is to make an estimate, write the volume out again but then substitute the value of 3 for π. Multiply the two numbers and you have your estimate.

Because π is 3.14159..... It is more than 3, so when you use 3 as an estimate the answer will always be slightly less than the answer achieved by using a calculator.

Columns 2 and 3 use the diagrams at the base of the page. Measure the diameter, halve it for the radius, then measure the height. Record these lengths in the spaces provided on each cylinder. Once you have done that solve the same way as Column 1.

Estimation/Terms of π - Volume of Cylinders

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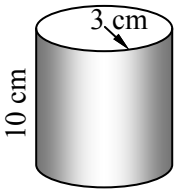
Find the volume of the cylinder in terms of π, then estimate the volume by substituting $\pi = 3$.

Example

$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$V = 90\pi \text{ cm}^3$



units^3 must match

Estimate: $V = 90\pi \text{ cm}^3$

$\approx 90 \times 3$

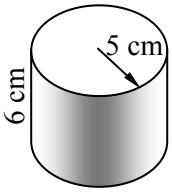
$V \approx 270 \text{ cm}^3$

1

$V = \pi r^2 h$

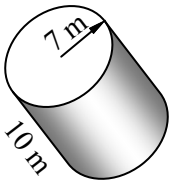
$=$

$V =$



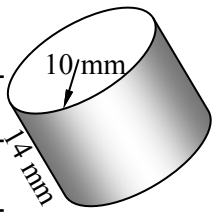
Estimate:
using
$\pi = 3$

2



Estimate:
using
$\pi = 3$

3



Estimate:
using
$\pi = 3$

Measure the cylinders below, find d and halve for r, then find h. All to the nearest cm. Then express the volume in terms of π and then estimate the volume using $\pi = 3$. Use the spaces that match numbers.

4



Estimate:
using
$\pi = 3$

5

Estimate:
using
$\pi = 3$

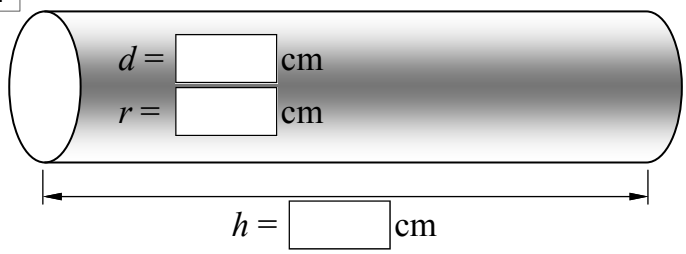
6

Estimate:
using
$\pi = 3$

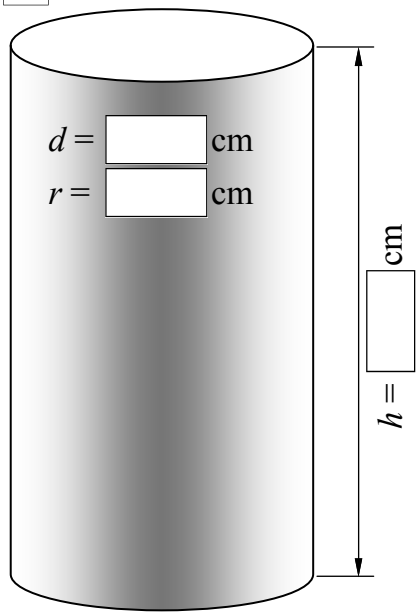
7

Estimate:
using
$\pi = 3$

4



5



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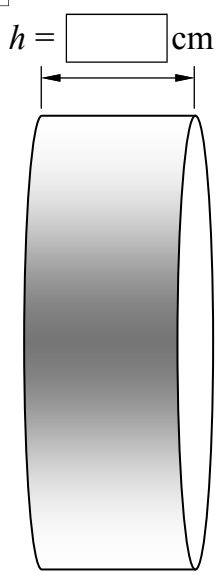

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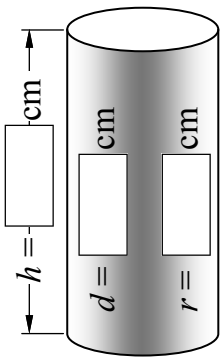

Estimation/Terms of π - Volume of Cylinders

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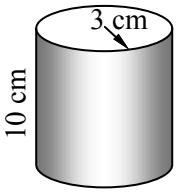
Find the volume of the cylinder in terms of π, then estimate the volume by substituting $\pi = 3$.

Example



$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$V = 90\pi$ cm^3

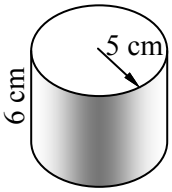
units^3 must match

Estimate: $V = 90\pi$ cm^3

$\approx 90 \times 3$

$V \approx 270$ cm^3

1



$V = \pi r^2 h$

$= \pi \times 5^2 \times 6$

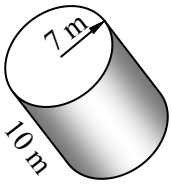
$V = 150\pi$ cm^3

Estimate: $V = 150\pi$ cm^3

using $\pi = 3$ $\approx 150 \times 3$

$V \approx 450$ cm^3

2



$V = \pi r^2 h$

$= \pi \times 7^2 \times 10$

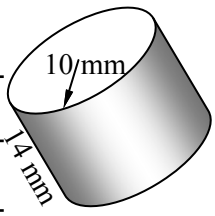
$V = 490\pi$ m^3

Estimate: $V = 490\pi$ m^3

using $\pi = 3$ $\approx 490 \times 3$

$V \approx 1\,470$ m^3

3



$V = \pi r^2 h$

$= \pi \times 10^2 \times 14$

$V = 1\,400\pi$ mm^3

Estimate: $V = 1\,400\pi$ m^3

using $\pi = 3$ $\approx 1\,400 \times 3$

$V \approx 4\,200$ mm^3

Measure the cylinders below, find d and halve for r, then find h. All to the nearest cm. Then express the volume in terms of π and then estimate the volume using $\pi = 3$. Use the spaces that match numbers.

4

$V = \pi r^2 h$

$= \pi \times 1^2 \times 8$

$V = 8\pi$ cm^3

Estimate: $V = 8\pi$ cm^3

using $\pi = 3$ $\approx 8 \times 3$

$V \approx 24$ cm^3

5

$V = \pi r^2 h$

$= \pi \times 2^2 \times 7$

$V = 28\pi$ cm^3

Estimate: $V = 28\pi$ cm^3

using $\pi = 3$ $\approx 28 \times 3$

$V \approx 84$ cm^3

6

$V = \pi r^2 h$

$= \pi \times 3^2 \times 2$

$V = 18\pi$ cm^3

Estimate: $V = 18\pi$ cm^3

using $\pi = 3$ $\approx 18 \times 3$

$V \approx 54$ cm^3

7

$V = \pi r^2 h$

$= \pi \times 1^2 \times 4$

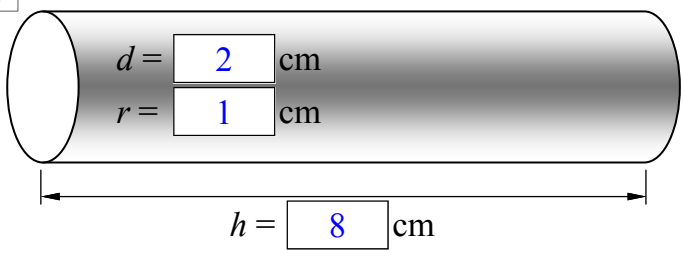
$V = 4\pi$ cm^3

Estimate: $V = 4\pi$ cm^3

using $\pi = 3$ $\approx 4 \times 3$

$V \approx 12$ cm^3

4



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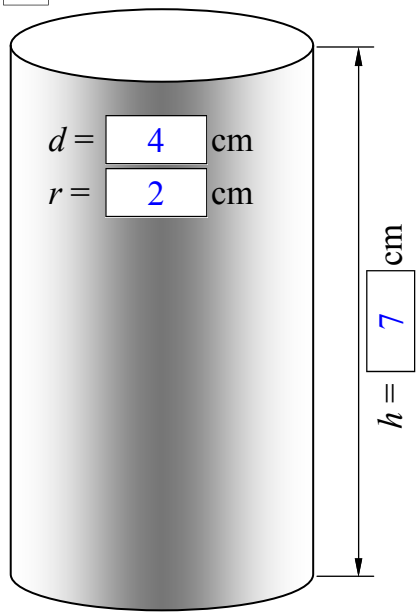

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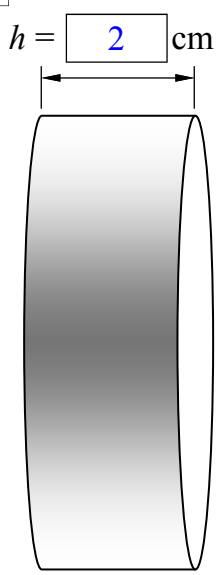

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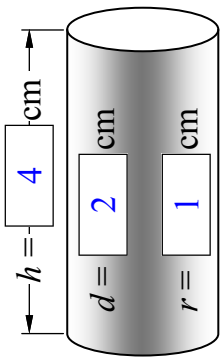


Volume of Cylinders

The standard formula for a prism is V = Ah.

Where: V = the volume of the prism
A = the uniform cross sectional area
h = the height of the prism

But with cylinders the cross sectional area is a circle, which has an area of πr^2. So if you replace A with πr^2 you have:

The formula for a cylinder is V = $\pi r^2 h$.

Where: V = the volume of the prism
r = the radius of the circular face
h = the height of the cylinder

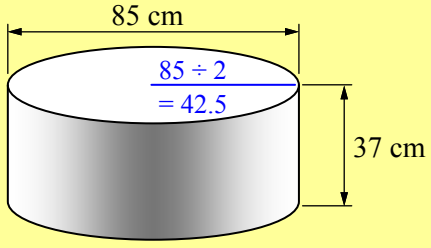
Column 1 starts with an example showing the formula line, substitution and then answer. Make sure you show cubic units. Eg. mm^3, cm^3 and m^3.

Column 2 has an extra step, you are given the diameter of the circle. So before you start, divide the diameter by 2 to get the radius and then use the formula as you did in column 1.

Column 3 applies what you have learned to some worded problems. There is also a table at the bottom of the column, no working required, just answers.

Below is an example of finding the volume when given the cylinder height and diameter.

Example - Find the volume to 1 decimal place.



r = 42.5 cm

$V = \pi r^2 h$ ← Write the formula

$= \pi \times 42.5^2 \times 37$ ← Substitute r and h

$V = 209\ 956.6\ cm^3$ ← Solve showing cubic units

Volume of Cylinders

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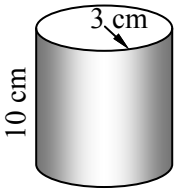
Find the volume of these cylinders using the π key. Round to 1 decimal place.

Example

$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$V = 282.7 \text{ cm}^3$

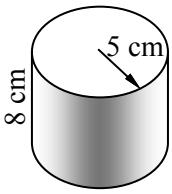


units^3 must match

1

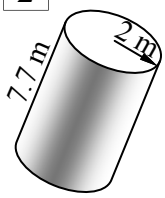
$V = \pi r^2 h$

$V =$ cm^3



2

$V = \pi r^2 h$



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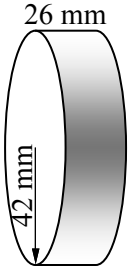

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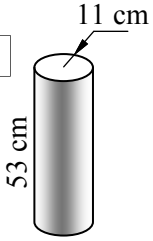

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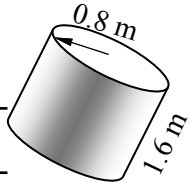

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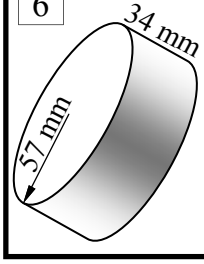


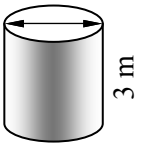
Careful! Diameters in use. Divide by 2 first for the radius. Answer to 2 d.p.

6

$V = \pi r^2 h$

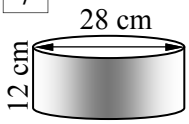
$= \pi \times \square^2 \times 3$

$V =$ m^2



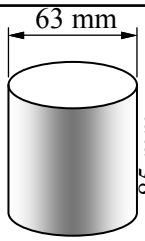
$r =$

7



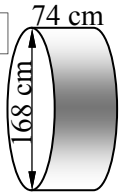
$r =$

8



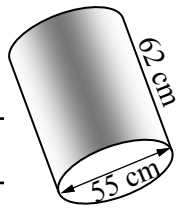
$r =$

9



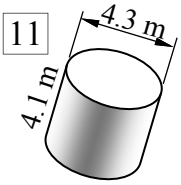
$r =$

10



$r =$

11



$r =$

12

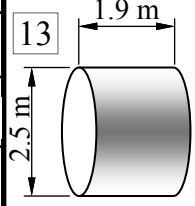
$d = 87$ mm

$h = 15$ mm



$r =$

13

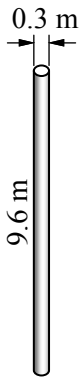


$r =$

Solve these problems involving cylinders.

Calculate the volume of concrete in the cylindrical pylon below to 2 d.p. Calculate its weight (W) given the concrete weighs 2.4 T/m^3.

14

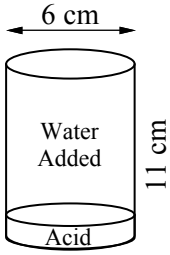


$V =$

$W =$

The glass below contains 45 cm^3 of HCL acid. If water is added to fill the glass find, to 1 decimal place, the:

15 Volume of the empty glass (cm^3).



16 Percentage of acid in the full glass.

Complete the table, which is in cm. Find the volume to the nearest cm^3.

Q.	17	18	19	20	21	22
d	17			106	82	
r		45	65			33
h	25	75	20	55	27	85
Volume (cm^3)						

Volume of Cylinders

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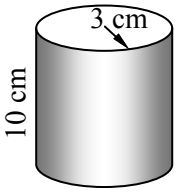
Find the volume of these cylinders using the π key. Round to 1 decimal place.

Example



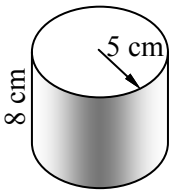
$V = \pi r^2 h$

$= \pi \times 3^2 \times 10$

$V = 282.7 \text{ cm}^3$

units^3 must match

1

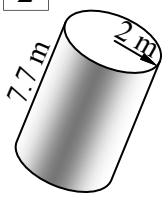


$V = \pi r^2 h$

$= \pi \times 5^2 \times 8$

$V = 628.3 \text{ cm}^3$

2

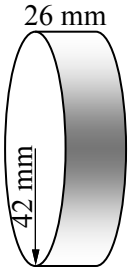


$V = \pi r^2 h$

$= \pi \times 2^2 \times 7.7$

$V = 96.8 \text{ m}^3$

3

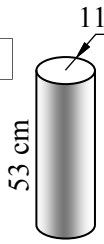


$V = \pi r^2 h$

$= \pi \times 42^2 \times 26$

$V = 144\,086 \text{ mm}^3$

4

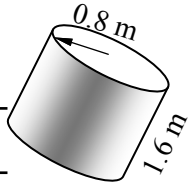


$V = \pi r^2 h$

$= \pi \times 11^2 \times 53$

$V = 20\,147 \text{ cm}^3$

5

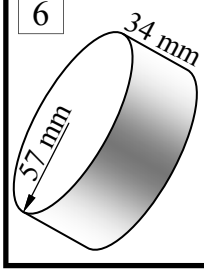


$V = \pi r^2 h$

$= \pi \times 0.8^2 \times 1.6$

$V = 3.2 \text{ m}^3$

6



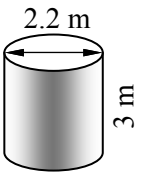
$V = \pi r^2 h$

$= \pi \times 57^2 \times 34$

$V = 347\,039.2 \text{ mm}^3$

Careful! Diameters in use. Divide by 2 first for the radius. Answer to 2 d.p.

6



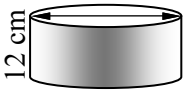
$r = 1.1$ m

$V = \pi r^2 h$

$= \pi \times 1.1^2 \times 3$

$V = 11.4 \text{ m}^2$

7



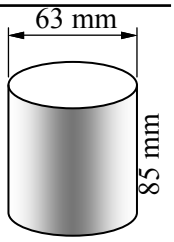
$r = 14$ cm

$V = \pi r^2 h$

$= \pi \times 14^2 \times 12$

$V = 7\,389 \text{ cm}^3$

8



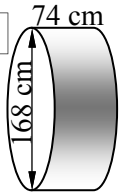
$r = 31.5$ mm

$V = \pi r^2 h$

$= \pi \times 31.5^2 \times 85$

$V = 264\,965.9 \text{ mm}^3$

9



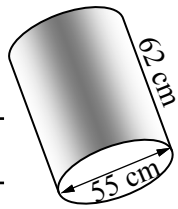
$r = 84$ cm

$V = \pi r^2 h$

$= \pi \times 84^2 \times 74$

$V = 1\,640\,363.8 \text{ cm}^3$

10



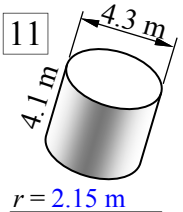
$r = 27.5$ cm

$V = \pi r^2 h$

$= \pi \times 27.5^2 \times 62$

$V = 147\,301.4 \text{ cm}^3$

11



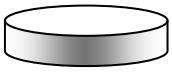
$r = 2.15$ m

$V = \pi r^2 h$

$= \pi \times 2.15^2 \times 4.1$

$V = 59.5 \text{ m}^3$

12



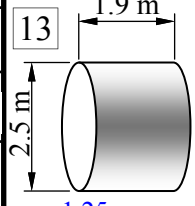
$r = 43.5$ mm

$V = \pi r^2 h$

$= \pi \times 43.5^2 \times 15$

$V = 89\,170.2 \text{ mm}^3$

13



$r = 1.25$ m

$V = \pi r^2 h$

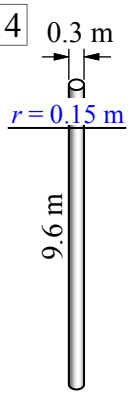
$= \pi \times 1.25^2 \times 1.9$

$V = 9.3 \text{ m}^3$

Solve these problems involving cylinders.

Calculate the volume of concrete in the cylindrical pylon below to 2 d.p. Calculate its weight (W) given the concrete weighs 2.4 T/m^3.

14



$V = \pi r^2 h$

$= \pi \times 0.15^2 \times 9.6$

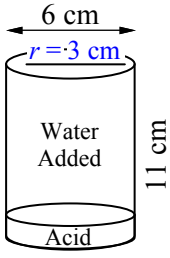
$V = 0.68 \text{ m}^3$

$W = 0.68 \times 2.4$

$W = 1.63 \text{ T}$

The glass below contains 45 cm^3 of HCL acid. If water is added to fill the glass find, to 1 decimal place, the:

15 Volume of the empty glass (cm^3).



$V = \pi r^2 h$

$= \pi \times 3^2 \times 11$

$V = 311 \text{ cm}^3$

16 Percentage of acid in the full glass.

$\%_{ACID} = \frac{45}{311} \times 100$

$\%_{ACID} = 14.5\%$

Complete the table, which is in cm. Find the volume to the nearest cm^3.

Q.	17	18	19	20	21	22
d	17	90	130	106	82	66
r	8.5	45	65	53	41	33
h	25	75	20	55	27	85
Volume (cm^3)	1 135 cm^3	477 129 cm^3	265 465 cm^3	485 360 cm^3	142 587 cm^3	290 802 cm^3

Volume of Cylinders - Different Units, finding *r* and *h*.

Imagine you require a container to hold a specific amount but it must also be a specific height. Finding the radius required needs the formula to be rearranged, that is what this sheet deals with.

Column 1 starts with asking you to rearrange the question given the steps. Then the new rearranged formula is used to find the height of cylinders given the volume and radius (or diameter).

Column 2 is a break away from rearranging/use of formulae. It is finding the volume but the answer is required in cm^3, while the dimensions are in various units. So change the units to cm and solve. The last question in the column requires you to rearrange $C=2\pi r$ first, then use the volume formula.

Column 3 gives you the height and the volume and asks you to rearrange the volume formula to find the radius. Then apply to solve for *r*.

Volume of Cylinders - Different Units, finding *r* and *h*.

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Rearrange the volume formula to make *h* the subject. Follow the steps.

1 $V = \pi r^2 h$

$\div \pi r^2$ $\div \pi r^2$ Divide both sides by πr^2.

Rewrite with *h* on the L.H.S

Using $h = \frac{V}{\pi r^2}$ find the height of the cylinder given the volume and radius. To 1 d.p.

2 $V = 550$ cm^3, $r = 5$ cm.

5 5 0 ÷ (π × 5 x^2) =

$h = \frac{V}{\pi r^2} = \frac{\square}{\pi \times \square^2}$

$h =$

3 $V = 11\,680$ cm^3, $r = 13$ cm.

$h = \frac{V}{\pi r^2} = \frac{\square}{\pi \times \square^2}$

$h =$

4 $V = 2\,386$ mm^3, $r = 7$ mm.

5 $V = 5\,917$ mm^3, $d = 12.4$ mm

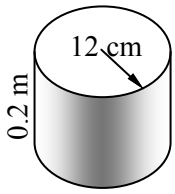
6 $V = 10\,781$ cm^3, $d = 34.4$ cm.

Find the volume of these cylinders in cm^3. Round to 2 decimal places.

7

$V = \pi r^2 h$

$V =$ cm^3



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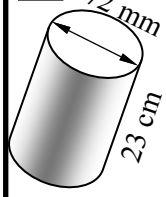

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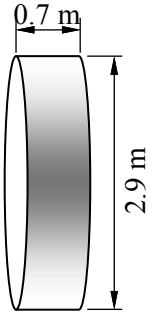
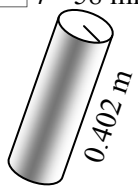
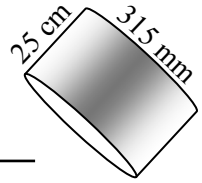


10 $r = 58$ mm



11



Find the volume (2 d.p.) of a cylinder which has a 15 cm height and a circumference of 0.75 m around its circular face.

12

Rearrange the volume formula to make *r* the subject. Follow the steps.

13 $V = \pi r^2 h$

$\div \pi h$ $\div \pi h$ Divide both sides by πh

$\sqrt{\ }$ $\sqrt{\ }$ Square root both sides

$\Rightarrow$

Rewrite with *r* on the L.H.S

Using $r = \sqrt{\frac{V}{\pi h}}$ find the radius given V and *h*. To 1 d.p.

14 $V = 282$ cm^3, $h = 10$ cm.

2 8 2 ÷ (π × 1 0) =

Then press $\sqrt{\ }$ =

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{\square}{\pi \times \square}}$

$r =$

15 $V = 905$ cm^3, $h = 8$ cm.

$r = \sqrt{\frac{V}{\pi h}}$

16 $V = 28.1$ m^3, $h = 3.5$ m.

17 $V = 29\,050$ mm^3, $h = 32$ mm.

18 $V = 7\,581$ cm^3, $h = 15.2$ cm.

Volume of Cylinders - Different Units, finding r and h.

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Rearrange the volume formula to make h the subject. Follow the steps.

1 $V = \pi r^2 h$

$\div \pi r^2$ $\div \pi r^2$ Divide both sides by πr^2.

$\frac{V}{\pi r^2} = h$

$h = \frac{V}{\pi r^2}$ Rewrite with h on the L.H.S

Using $h = \frac{V}{\pi r^2}$ find the height of the cylinder given the volume and radius. To 1 d.p.

2 $V = 550$ cm^3, $r = 5$ cm.

5 5 0 ÷ (π × 5 x^2) =

$h = \frac{V}{\pi r^2} = \frac{550}{\pi \times 5^2}$

$h = 7$ cm

3 $V = 11\,680$ cm^3, $r = 13$ cm.

$h = \frac{V}{\pi r^2} = \frac{11\,680}{\pi \times 13^2}$

$h = 22$ cm

4 $V = 2\,386$ mm^3, $r = 7$ mm.

$h = \frac{V}{\pi r^2} = \frac{2\,386}{\pi \times 7^2}$

$h = 15.5$ mm

5 $V = 5\,917$ mm^3, $d = 12.4$ mm

$h = \frac{V}{\pi r^2} = \frac{5\,917}{\pi \times 6.2^2}$

$h = 49$ mm

6 $V = 10\,781$ cm^3, $d = 34.4$ cm.

$h = \frac{V}{\pi r^2} = \frac{10\,781}{\pi \times 17.2^2}$

$h = 11.6$ cm

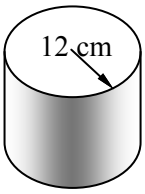
Find the volume of these cylinders in cm^3. Round to 2 decimal places.

7

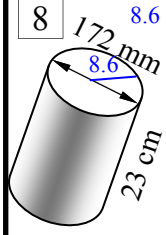
$V = \pi r^2 h$

$= \pi \times 12^2 \times 20$

$V = 9\,047.79$ cm^3



8

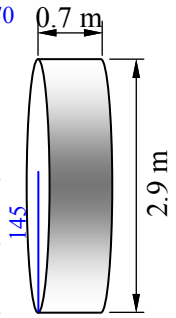


$V = \pi r^2 h$

$= \pi \times 8.6^2 \times 23$

$V = 5\,344.10$ cm^3

9



$V = \pi r^2 h$

$= \pi \times 145^2 \times 70$

$V = 4\,623\,638.99$ cm^3

10

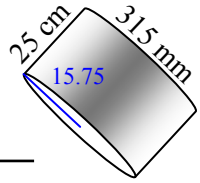


$V = \pi r^2 h$

$= \pi \times 5.8^2 \times 40.2$

$V = 4\,248.46$ mm^3

11



$V = \pi r^2 h$

$= \pi \times 15.75^2 \times 25$

$V = 19\,482.78$ cm^3

Find the volume (2 d.p.) of a cylinder which has a 15 cm height and a circumference of 0.75 m around its circular face.

12 $C = 2\pi r$ $r = {}^{C}/_{2\pi}$

$r = {}^{75}/_{2\pi}$

$r = 11.94$ cm

$V = \pi r^2 h$

$= \pi \times 11.94^2 \times 15$

$V = 6\,718.15$ cm^3

Rearrange the volume formula to make r the subject. Follow the steps.

13 $V = \pi r^2 h$

$\div \pi h$ $\div \pi h$ Divide both sides by πh

$\frac{V}{\pi h} = r^2$

$\sqrt{\ }$ $\sqrt{\ }$ Square root both sides

$\sqrt{\frac{V}{\pi h}} \Rightarrow r = \sqrt{\frac{V}{\pi h}}$

Rewrite with r on the L.H.S

Using $r = \sqrt{\frac{V}{\pi h}}$ find the radius given V and h. To 1 d.p.

14 $V = 282$ cm^3, $h = 10$ cm.

2 8 2 ÷ (π × 1 0) =

Then press $\sqrt{\ }$ =

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{282}{\pi \times 10}}$

$r = 3$ cm

15 $V = 905$ cm^3, $h = 8$ cm.

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{905}{\pi \times 8}}$

$r = 6$ cm

16 $V = 28.1$ m^3, $h = 3.5$ m.

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{28.1}{\pi \times 3.5}}$

$r = 1.6$ m

17 $V = 29\,050$ mm^3, $h = 32$ mm.

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{29\,050}{\pi \times 32}}$

$r = 5.4$ mm

18 $V = 7\,581$ cm^3, $h = 15.2$ cm.

$r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{7\,581}{\pi \times 15.2}}$

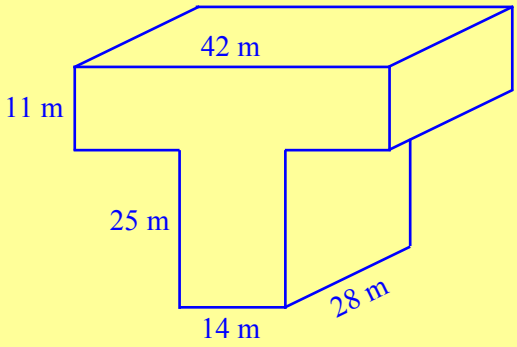
$r = 12.6$ cm

Finding Area and Volume - Composite Shapes

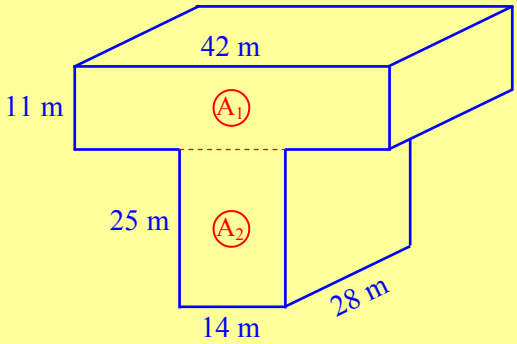
The volume of a prism is found using the formula $V = Ah$. So if you know how to find the area, you only have to multiply by the height to get the volume.

This sheet has two columns, the left column are questions finding the area. The right column are questions finding the volume. But the columns are related because the questions match in layout, for example, Question 1 is a composite shape made from two rectangles, Question 5 is a composite solid made from two rectangular prisms. The method of solving is exactly the same in both questions, the only difference is the extra step in finding the volume. Below is an example.

Example - Find the volume of the shape below.



STEP 1 Here is the solid, find its volume.



STEP 2 Break the front face up into two rectangles. Label the rectangles A_1 and A_2.

$A_1 = lb$

$= 11 \times 42$

$A_1 = 462 \text{ m}^2$

$A_2 = lb$

$= 14 \times 25$

$A_2 = 350 \text{ m}^2$

STEP 3 Calculate the two separate areas. As this is area, the units are square units.

$A = A_1 + A_2 = 462 + 350 = 812 \text{ m}^2$

STEP 4 Add the two areas to get the total area of the front face.

$V = Ah = 812 \times 28 = 22\,736 \text{ m}^3$

STEP 5 Multiply the area by the height to get the volume. Volume is in cubic units.

Finding Area and Volume - Composite Shapes

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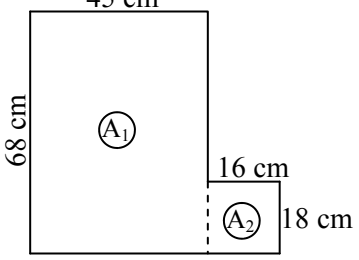
Calculate the area of these shapes by breaking them up into parts first. All angles are right angles, remember to use square units, e.g. cm^2.

1



$A_1 = lb$

$= \square \times \square$

$A_1 = \square$ cm^2

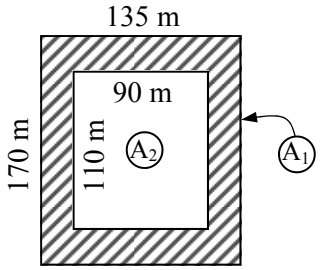
$A_2 = lb$

$= \square \times \square$

$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square = \square$ cm^2

2 Find the shaded area



$A_1 =$

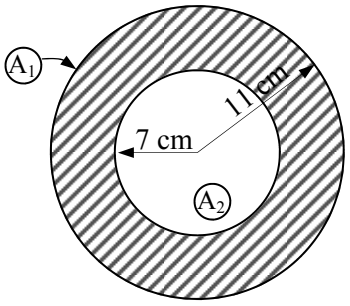
$A_1 =$

$A_2 =$

$A_2 =$

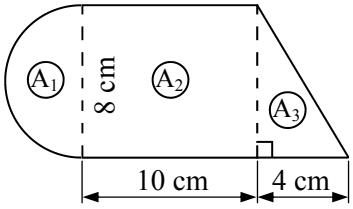
$A = A_1 - A_2 = \square - \square = \square$ m^2

3 Shaded area, to 2 d.p.



$A = A_1 - A_2 = \square - \square = \square$ cm^2

4 Round to 2 d.p.



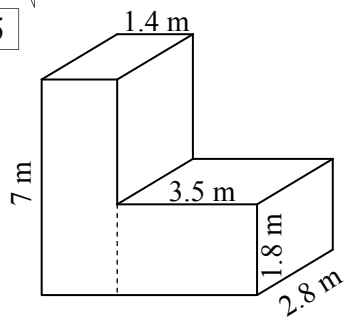
$A = A_1 + A_2 + A_3$

$= \square + \square + \square$

$A =$

Now apply your skills to find the volume. Use the same method as in the left column, except add one more line $V = Ah$ to find the volume.

5



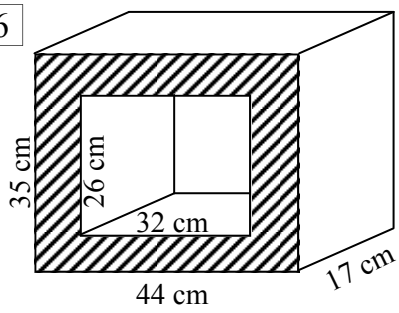
$A_1 =$

$A_2 =$

$A = A_1 + A_2 = \square + \square = \square$ m^2

$V = Ah = \square \times 2.8 = \square$ m^3

6



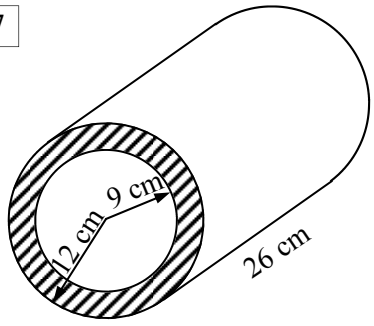
$A_1 =$

$A_2 =$

$A = A_1 - A_2 =$

$V = Ah =$

7



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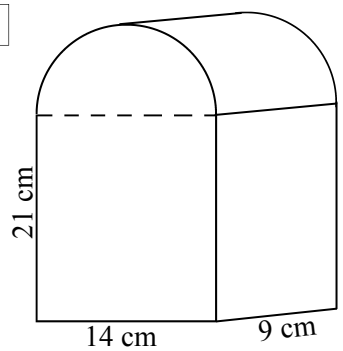

Finding Area and Volume - Composite Shapes

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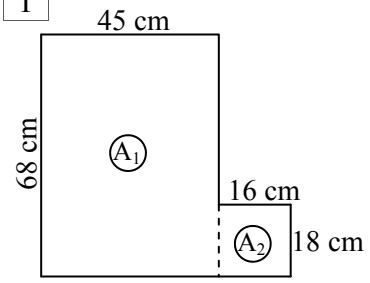
Calculate the area of these shapes by breaking them up into parts first. All angles are right angles, remember to use square units, e.g. cm^2.

1



$A_1 = lb$

$= 68 \times 45$

$A_1 = 3\,060 \text{ cm}^2$

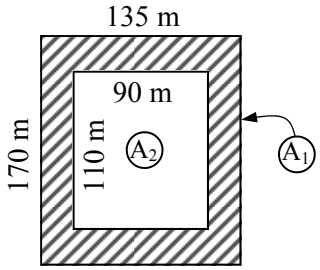
$A_2 = lb$

$= 16 \times 18$

$A_2 = 288 \text{ cm}^2$

$A = A_1 + A_2 = 3\,060 + 288 = 3\,348 \text{ cm}^2$

2 Find the shaded area



$A_1 = lb$

$= 170 \times 135$

$A_1 = 22\,950 \text{ m}^2$

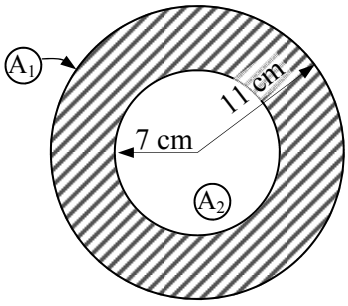
$A_2 = lb$

$= 110 \times 90$

$A_2 = 9\,900 \text{ m}^2$

$A = A_1 - A_2 = 22\,950 - 9\,900 = 13\,050 \text{ m}^2$

3 Shaded area, to 2 d.p.



$A_1 = \pi r^2$

$= \pi \times 11^2$

$A_1 = 380.13 \text{ cm}^2$ (2 d.p.)

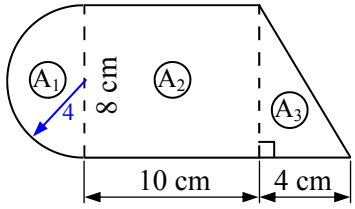
$A_2 = \pi r^2$

$= \pi \times 7^2$

$A_2 = 153.94 \text{ cm}^2$ (2 d.p.)

$A = A_1 - A_2 = 380.13 - 153.94 = 226.19 \text{ cm}^2$

4 Round to 2 d.p.



$A_1 = \pi r^2 \div 2$

$= \pi \times 4^2 \div 2$

$A_1 = 25.13 \text{ cm}^2$ (2 d.p.)

$A_2 = lb$

$= 10 \times 8$

$A_2 = 80 \text{ cm}^2$

$A_3 = \frac{1}{2}bh$

$= \frac{1}{2} \times 4 \times 8$

$A_3 = 16 \text{ cm}^2$

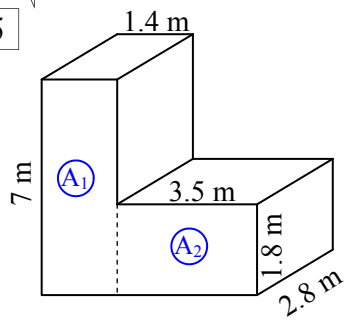
$A = A_1 + A_2 + A_3$

$= 25.13 + 80 + 16$

$A = 121.13 \text{ cm}^2$

Now apply your skills to find the volume. Use the same method as in the left column, except add one more line V=AH to find the volume.

5



$A_1 = lb$

$= 7 \times 1.4$

$A_1 = 9.8 \text{ m}^2$

$A_2 = lb$

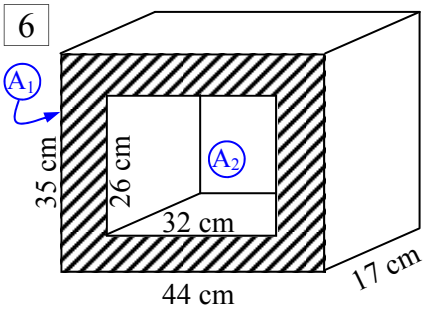
$= 3.5 \times 1.8$

$A_2 = 6.3 \text{ m}^2$

$A = A_1 + A_2 = 9.8 + 6.3 = 16.1 \text{ m}^2$

$V = Ah = 16.1 \times 2.8 = 45.08 \text{ m}^3$

6



$A_1 = lb$

$= 35 \times 44$

$A_1 = 1\,540 \text{ cm}^2$

$A_2 = lb$

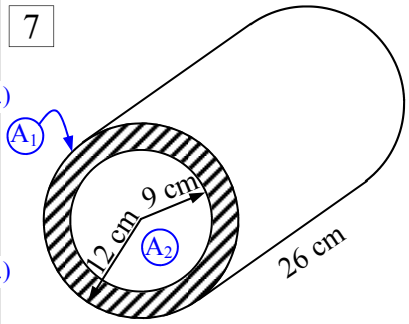
$= 32 \times 26$

$A_2 = 832 \text{ cm}^2$

$A = A_1 - A_2 = 1\,540 - 832 = 708 \text{ cm}^2$

$V = Ah = 708 \times 17 = 12\,036 \text{ cm}^3$

7



$A_1 = \pi r^2$

$= \pi \times 12^2$

$A_1 = 452.39 \text{ cm}^2$ (2 d.p.)

$A_2 = \pi r^2$

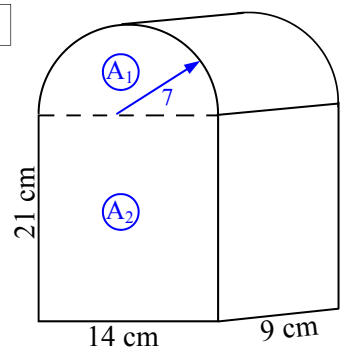
$= \pi \times 9^2$

$A_2 = 254.47 \text{ cm}^2$ (2 d.p.)

$A = A_1 - A_2 = 452.39 - 254.47 = 197.92 \text{ cm}^2$

$V = Ah = 197.92 \times 26 = 5\,145.92 \text{ cm}^3$

8



$A_1 = \pi r^2$

$= \pi \times 7^2$

$A_1 = 153.94 \text{ cm}^2$ (2 d.p.)

$A_2 = lb$

$= 21 \times 14$

$A_2 = 294 \text{ cm}^2$

$A = A_1 + A_2 = 153.94 + 294 = 447.94 \text{ cm}^2$

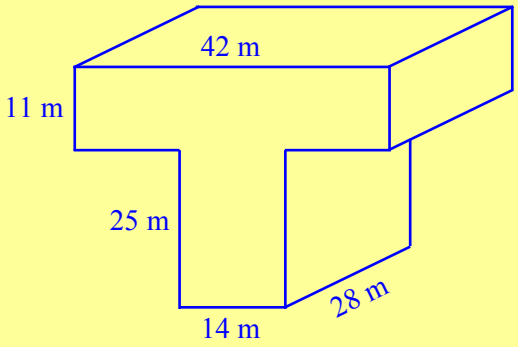
$V = Ah = 447.94 \times 9 = 4\,031.46 \text{ cm}^3$

Volume of Composite Solids - Prisms - Non Circular

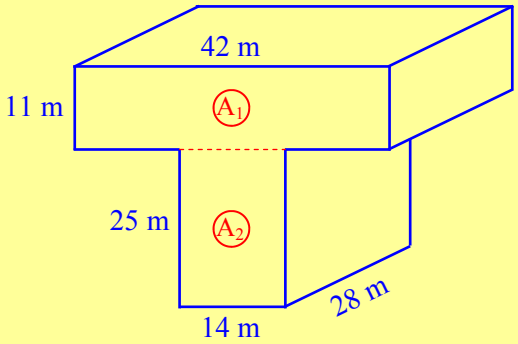
Composite solids are solids built from common solids being added together, or subtracted from each other, to form a new solid. The solids on the first worksheet are all prisms, that means that the front face is the same all through the solid. Because of the uniform cross sectional area these questions can be solved by finding the front face area then multiplying by the height of the solid.

The volume can also be found by finding volumes separately and then adding them together. The two different methods of calculating volume are shown below.

Volume by Finding Area First



STEP 1 Here is the solid, find its volume.



STEP 2 Break the front face up into two rectangles. Label the rectangles A_1 and A_2.

$A_1 = lb$

$= 11 \times 42$

$A_1 = 462 \text{ m}^2$

$A_2 = lb$

$= 14 \times 25$

$A_2 = 350 \text{ m}^2$

STEP 3 Calculate the two separate areas. As this is area, the units are square units.

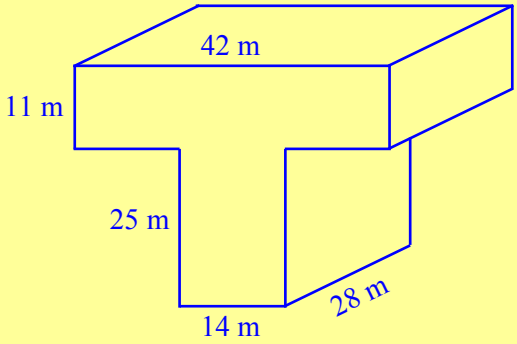
$A = A_1 + A_2 = 462 + 350 = 812 \text{ m}^2$

STEP 4 Add the two areas to get the total area of the front face.

$V = Ah = 812 \times 28 = 22\,736 \text{ m}^3$

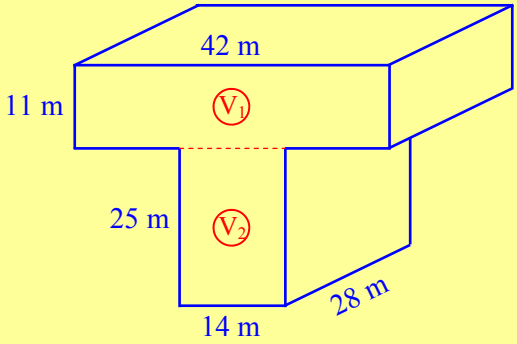
STEP 5 Multiply the area by the height to get the volume. Volume is in cubic units.

Volume by adding Individual Volumes



STEP 1

Here is the solid, find its volume.



STEP 2

Break the front face up into two rectangular prisms. Label them V_1 and V_2.

$V_1 = lbh$

$= 11 \times 42 \times 28$

$V_1 = 12\,936\ m^2$

$V_2 = lbh$

$= 14 \times 25 \times 28$

$V_2 = 9\,800\ m^2$

STEP 3

Calculate the two separate volumes. As this is volume, the units are cubic units.

$V = V_1 + V_2 = 12\,936 + 9\,800 = 22\,736\ m^2$

STEP 4

Add the two volumes to get the total volume.

Volume of Composite Solids - Prisms - Non Circular

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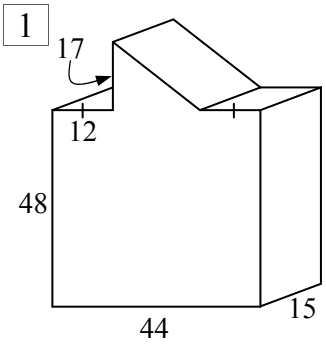
Find the volume of these solids. Round to 2 decimal places if required. All units are in centimetres.

1



$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times \square \times \square$

$A_1 = \square$ cm^2

$A_2 = lb$

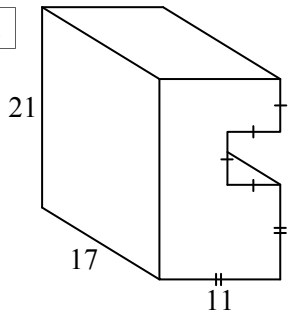
$= \square \times \square$

$A_2 = \square$ cm^2

$A = A_1 + A_2 = \square + \square = \square$ cm^2

$V = Ah = \square \times \square = \square$ cm^3

2



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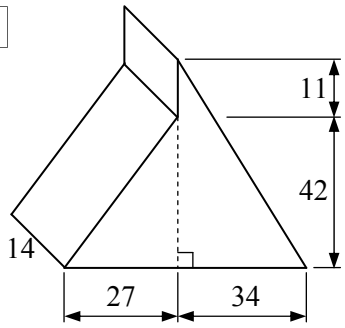

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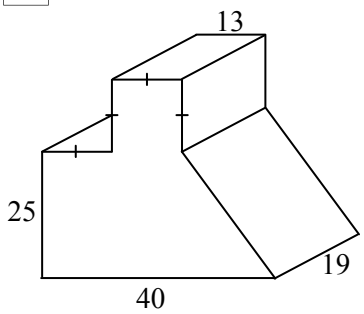

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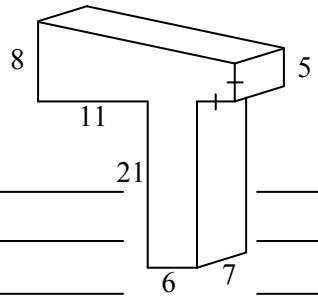

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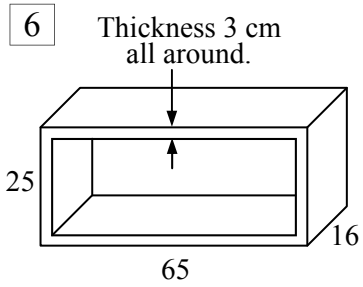

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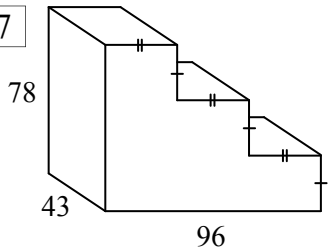

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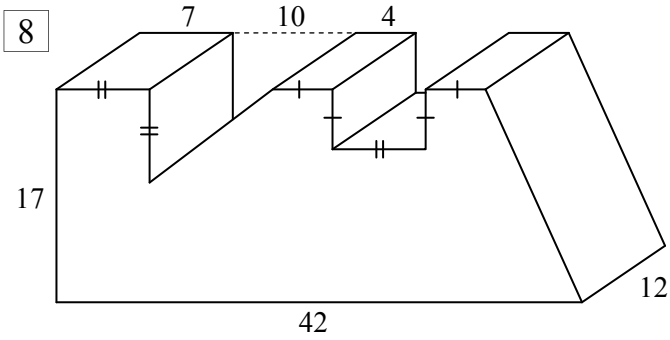

Volume of Composite Solids - Prisms - Non Circular

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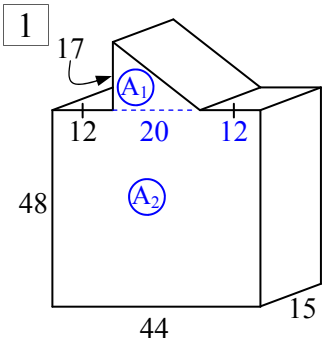
Find the volume of these solids. Round to 2 decimal places if required. All units are in centimetres.

1



$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 20 \times 17$

$A_1 = 170 \text{ cm}^2$

$A_2 = lb$

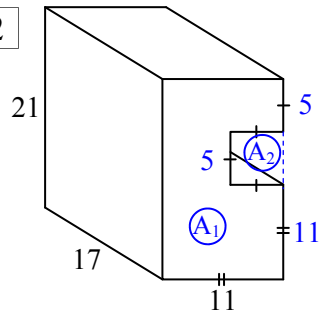
$= 48 \times 44$

$A_2 = 2\,112 \text{ cm}^2$

$A = A_1 + A_2 = 170 + 2\,112 = 2\,282 \text{ cm}^2$

$V = Ah = 2\,282 \times 15 = 34\,230 \text{ cm}^3$

2



$A_1 = lb$

$= 11 \times 21$

$A_1 = 231 \text{ cm}^2$

$A_2 = l^2$

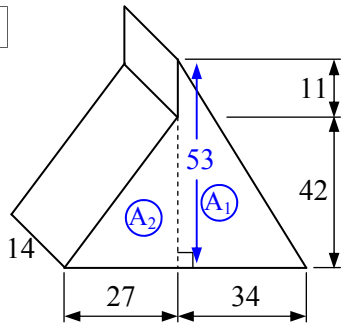
$= 5^2$

$A_2 = 25 \text{ cm}^2$

$A = A_1 - A_2 = 231 - 25 = 206 \text{ cm}^2$

$V = Ah = 206 \times 17 = 3\,502 \text{ cm}^3$

3



$A_1 = \frac{1}{2}bh$

$= \frac{1}{2} \times 34 \times 53$

$A_1 = 901 \text{ cm}^2$

$A_2 = \frac{1}{2}bh$

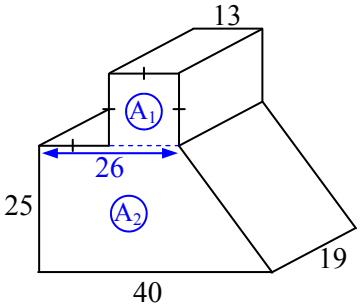
$= \frac{1}{2} \times 27 \times 42$

$A_2 = 567 \text{ cm}^2$

$A = A_1 + A_2 = 901 + 567 = 1\,468 \text{ cm}^2$

$V = Ah = 1\,468 \times 14 = 20\,552 \text{ cm}^3$

4



$A_1 = l^2$

$= 13^2$

$A_1 = 169 \text{ cm}^2$

$A_2 = \frac{1}{2}(a + b)\,h$

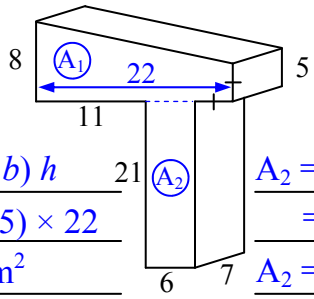
$= \frac{1}{2} \times (40 + 26) \times 25$

$A_2 = 825 \text{ cm}^2$

$A = A_1 + A_2 = 169 + 825 = 994 \text{ cm}^2$

$V = Ah = 994 \times 19 = 18\,886 \text{ cm}^3$

5



$A_1 = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (8 + 5) \times 22$

$A_1 = 143 \text{ cm}^2$

$A_2 = lb$

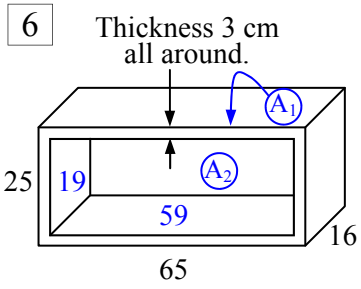
$= 6 \times 21$

$A_2 = 126 \text{ cm}^2$

$A = A_1 + A_2 = 143 + 126 = 269 \text{ cm}^2$

$V = Ah = 269 \times 7 = 1\,883 \text{ cm}^3$

6



$A_1 = lb$

$= 65 \times 25$

$A_1 = 1\,625 \text{ cm}^2$

$A_2 = lb$

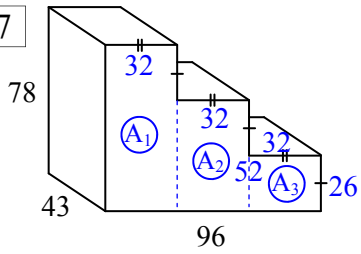
$= 59 \times 19$

$A_2 = 1\,121 \text{ cm}^2$

$A = A_1 - A_2 = 1\,625 - 1\,121 = 504 \text{ cm}^2$

$V = Ah = 504 \times 16 = 8\,064 \text{ cm}^3$

7



$A_1 = lb$

$= 32 \times 26$

$A_1 = 832 \text{ cm}^2$

$A_2 = lb$

$= 32 \times 52$

$A_2 = 1\,664 \text{ cm}^2$

$A_3 = lb$

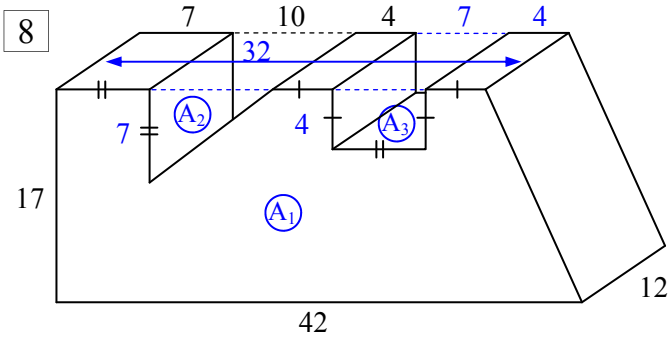
$= 32 \times 78$

$A_3 = 2\,496 \text{ cm}^2$

$A = A_1 + A_2 + A_3 = 832 + 1\,664 + 2\,496 = 4\,992 \text{ cm}^2$

$V = Ah = 4\,992 \times 43 = 214\,656 \text{ cm}^3$

8



$A_1 = \frac{1}{2}(a + b)\,h$

$= \frac{1}{2} \times (32 + 42) \times 17$

$A_1 = 629 \text{ cm}^2$

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 7 \times 10$

$A_2 = 35 \text{ cm}^2$

$A_3 = lb$

$= 4 \times 7$

$A_3 = 28 \text{ cm}^2$

$A = A_1 - A_2 - A_3 = 629 - 35 - 28 = 566 \text{ cm}^2$

$V = Ah = 566 \times 12 = 6\,792 \text{ cm}^3$

Volume of Composite Solids - Circles

Composite solids are solids that are made by combining two or more solids. To calculate the total volume, separate them into the two volumes (except question 4 - which has 3 solids) and calculate them individually. Then add them together, or subtract them.

Column 1 of the worksheet is answered by adding the volumes together. So calculate the individual volumes and sum them.

Question 1 starts you off with the formulae written and the solid's parts being labeled with V_1 and V_2. Make sure you label each solid. Then the volumes as individual volumes are calculated. Then the last stage is adding the two together. So if you can work out the volume of a cylinder then you just do this twice. Add them and you are done.

Remember to look for when you are given a radius or a diameter, it also may be that you need to calculate some lengths by adding sides together.

Column 2 is also answered by calculating individual volumes but instead of addition use subtraction. It is like a hole has been 'punched' through the solids and because these holes have been removed from the solid, subtraction is used.

Question 5 starts with a rectangular block which has two holes drilled through it. So calculate the volume of the block as if the holes weren't there. Then calculate the volume of one cylindrical hole. The volume will be found by subtracting two 'hole' volumes from the rectangular prism.

You will notice that all of these questions are prisms except the first two questions. Another method of calculating volume is to find the **AREA** of the front face of the prism, then use $\mathbf{V = A}\boldsymbol{h}$ to find the volume. Both methods will result in the same answer.

Volume of Composite Solids - Circles

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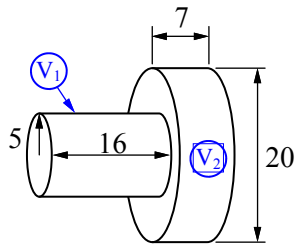
Find the volume to 1 d.p. All lengths are in cm.

1



$V_1 = \pi r^2 h$

$= \pi \times \square^2 \times \square$

$V_1 = \square$ cm^3

$V_2 = \pi r^2 h$

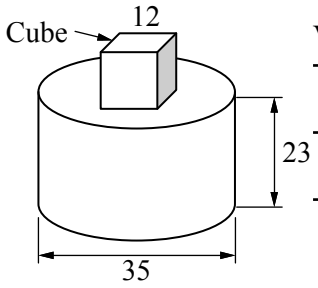
$= \pi \times \square^2 \times \square$

$V_2 = \square$ cm^3

$V = \square + \square$

$V = \square$ cm^3

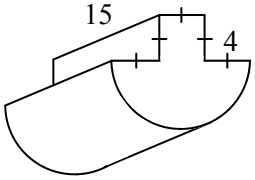
2



$V_1 = \pi r^2 h$

$V_2 = l^3$

3



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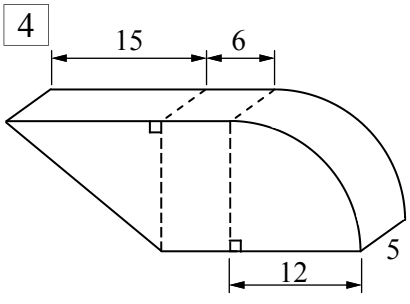
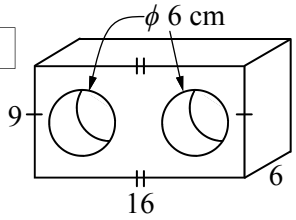


Find the volume to 1 d.p. All lengths are in cm. These questions use subtraction.

5

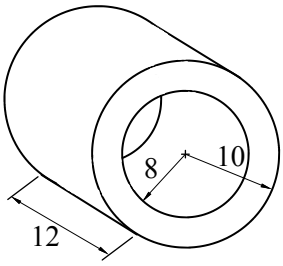


$V_1 = lbh$

$V_2 = \pi r^2 h$

$V = \square - 2 \times \square$

6



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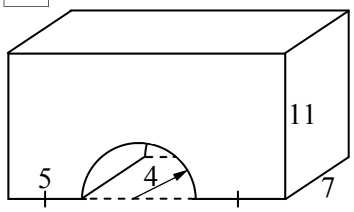

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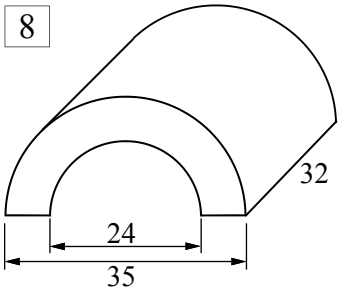

Volume of Composite Solids - Circles

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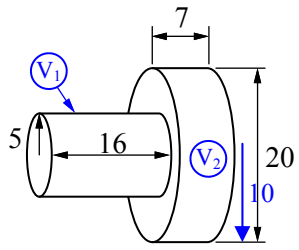
Find the volume to 1 d.p. All lengths are in cm.

1



$V_1 = \pi r^2 h$

$= \pi \times 5^2 \times 16$

$V_1 = 1\,256.6 \text{ cm}^3$

$V_2 = \pi r^2 h$

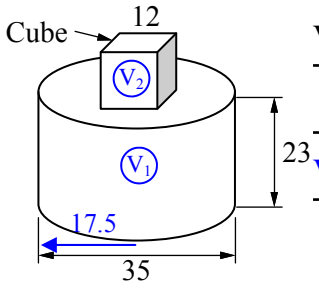
$= \pi \times 10^2 \times 7$

$V_2 = 2\,199.1 \text{ cm}^3$

$V = 1\,256.6 + 2\,199.1$

$V = 3\,455.7 \text{ cm}^3$

2



$V_1 = \pi r^2 h$

$= \pi \times 17.5^2 \times 23$

$V_1 = 22\,128.6 \text{ cm}^3$

$V_2 = l^3$

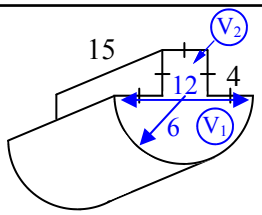
$= 12^3$

$V_2 = 1\,728 \text{ cm}^3$

$V = 22\,128.6 + 1\,728$

$V = 23\,856.6 \text{ cm}^3$

3



$V_1 = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2} \times \pi \times 6^2 \times 15$

$V_1 = 848.2 \text{ cm}^3$

$V_2 = lbh$

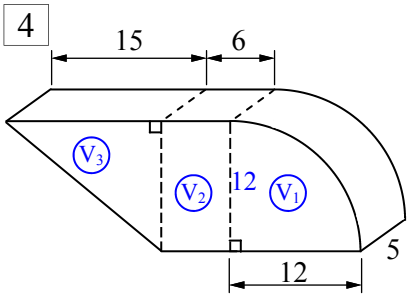
$= 4 \times 4 \times 15$

$V_2 = 240 \text{ cm}^3$

$V = 848.2 + 240$

$V = 1\,088.2 \text{ cm}^3$

4



$V_1 = \frac{1}{4}\pi r^2 h$

$= \frac{1}{4} \times \pi \times 12^2 \times 5$

$V_1 = 565.5 \text{ cm}^3$

$V_2 = lbh$

$= 12 \times 6 \times 5$

$V_2 = 360 \text{ cm}^3$

$V_3 = \frac{1}{2}lbh$

$= \frac{1}{2} \times 15 \times 12 \times 5$

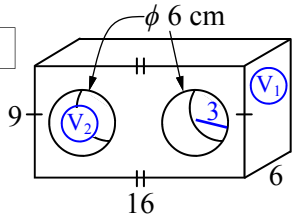
$V_3 = 450 \text{ cm}^3$

$V = 565.5 + 360 + 450$

$V = 1\,375.5 \text{ cm}^3$

Find the volume to 1 d.p. All lengths are in cm. These questions use subtraction.

5



$V_1 = lbh$

$= 16 \times 6 \times 9$

$V_1 = 864 \text{ cm}^3$

$V_2 = \pi r^2 h$

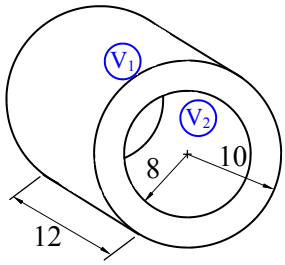
$= \pi \times 3^2 \times 6$

$V_2 = 169.6 \text{ cm}^3$

$V = 864 - 2 \times 169.6$

$V = 524.8 \text{ cm}^3$

6



$V_1 = \pi r^2 h$

$= \pi \times 10^2 \times 12$

$V_1 = 3\,769.9 \text{ cm}^3$

$V_2 = \pi r^2 h$

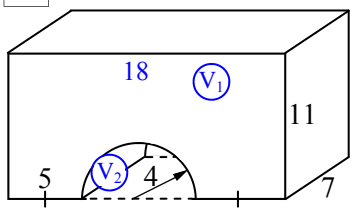
$= \pi \times 8^2 \times 12$

$V_2 = 2\,412.7 \text{ cm}^3$

$V = 3\,769.9 - 2\,412.7$

$V = 1\,357.2 \text{ cm}^3$

7



$V_1 = lbh$

$= 18 \times 11 \times 7$

$V_1 = 1\,386 \text{ cm}^3$

$V_2 = \frac{1}{2}\pi r^2 h$

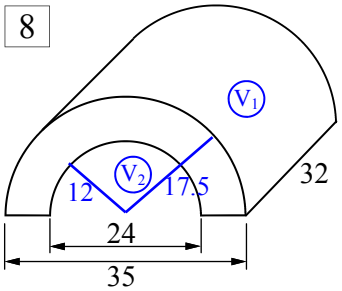
$= \frac{1}{2} \times \pi \times 4^2 \times 7$

$V_2 = 175.9 \text{ cm}^3$

$V = 1\,386 - 175.9$

$V = 1\,210.1 \text{ cm}^3$

8



$V_1 = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2} \times \pi \times 17.5^2 \times 32$

$V_1 = 15\,393.8 \text{ cm}^3$

$V_2 = \frac{1}{2}\pi r^2 h$

$= \frac{1}{2} \times \pi \times 12^2 \times 32$

$V_2 = 7\,238.2 \text{ cm}^3$

$V = 15\,393.8 - 7\,238.2$

$V = 8\,155.6 \text{ cm}^3$

Converting between Volume and Capacity

When you calculate the volume within a solid it's measured in cubic units such as mm^3, cm^3, or m^3. These units are used usually to define empty space such as storage space, shipping containers or packaging, usually when they are to be filled with solid material. Capacity is measured in millilitres (mL), litres (L), kilolitres (kL) and Megalitres (ML). Each unit is 1 000 times larger than the unit before it. While capacity is usually associated with liquids it is also used with air. For example the space inside a car will have its capacity measured in litres.

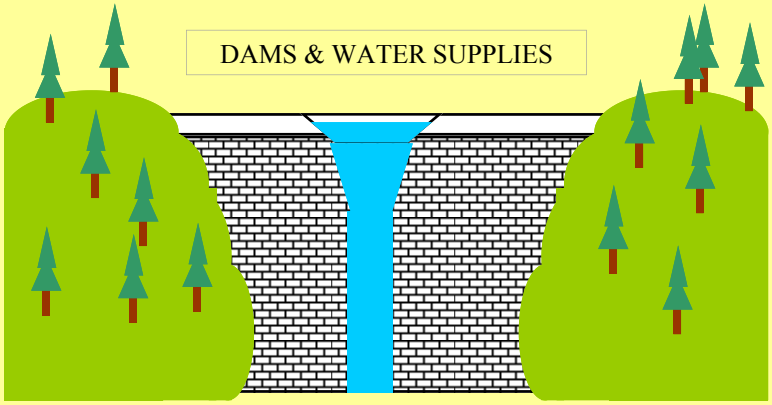
Column 1 starts with comparing mL and cm^3. They are the same, so a measurement in mL, say 5.72 mL is the same in cm^3, 5.72 cm^3. So rewrite the number and change to the other unit. From question 7 you are asked to convert cm^3 to L. As 1 cm^3 is 1 mL, and there are 1 000 mL in a L, then 1 cm^3 must be $^1/_{1000}$ of a L. So divide by 1 000 or move the decimal point 3 places to the left to get your answer. Remember that litres is written with an upper case L.

A common object where both capacity and volume are used is with car motors. The size of the engine could be given in c.c. (cubic centimetres) which is an accepted way of writing cm^3, or litres (L). For example a 1 800 c.c. engine could also be described as a 1.8 L engine.

Column 2 starts with the reverse, converting from L to cm^3. This time you multiply by 1 000 or move the decimal point 3 places to the right. The last 6 questions involve the conversion between kL and m^3. Just as with mL and cm^3, they are identical so 1 kilolitre (1 000 L) = 1 cubic metre (m^3). Just rewrite the number and change the unit.

Column 3 asks you first to convert from m^3 to L. Remember that $1m^3$ = 1 kL which is 1 000 L. So multiply by 1 000 or move the decimal point 3 places to the right. The next section of the column reverses the process, converting L to m^3. Reverse the process by dividing by 1 000 or moving the decimal place 3 places to the left.

This sheet doesn't deal with Megalitres (ML) as they are rarely used. But there are times when their use is essential when dealing with large capacities, can you think of where ML would be used?



Converting between Volume and Capacity

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Change the units from mL to cm^3 or the reverse

Example

73 mL = 73 cm^3

1. 350 mL = _____
2. 105 mL = _____
3. 56.3 cm^3 = _____
4. 850 mL = _____
5. 2.43 cm^3 = _____
6. 600 cm^3 = _____

Convert these volumes in cm^3 (c.c.) to litres

850 cm^3 = 0.85 L (3 places left)

7. 1 000 cm^3 = _____
8. 800 cm^3 = _____
9. 450 cm^3 = _____
10. 2 100 cm^3 = _____
11. 1 700 cm^3 = _____
12. 750 cm^3 = _____
13. 1 250 cm^3 = _____
14. 5 000 cm^3 = _____
15. 85 cm^3 = _____
16. 4 302 cm^3 = _____
17. 120.4 cm^3 = _____
18. 333 cm^3 = _____
19. 2 006 cm^3 = _____
20. 500 cm^3 = _____
21. 25.1 cm^3 = _____
22. 1 007 cm^3 = _____
23. 10 cm^3 = _____

Now reverse, change these from L to cm^3

4.3 L = 4 300 cm^3 (3 places right)

24. 2 L = _____
25. 0.5 L = _____
26. 3.1 L = _____
27. 0.2 L = _____
28. 3.5 L = _____
29. 1.75 L = _____
30. 0.04 L = _____
31. 4.03 L = _____
32. 0.05 L = _____
33. 0.01 L = _____
34. 0.2063 L = _____
35. 2.02 L = _____
36. 0.25 L = _____
37. 4.06 L = _____
38. 0.96 L = _____
39. 3.007 L = _____
40. 4.55 L = _____

Change the units from kL to m^3 or the reverse

Example

2.9 m^3 = 2.9 kL

41. 7 m^3 = _____
42. 3.84 kL = _____
43. 43 kL = _____
44. 0.3 m^3 = _____
45. 204 m^3 = _____
46. 7.06 kL = _____

Change these volumes from m^3 to litres.

2.53 m^3 = 2 530 L (3 places right)

47. 7.2 m^3 = _____
48. 0.5 m^3 = _____
49. 3.7 m^3 = _____
50. 0.25 m^3 = _____
51. 16.403 m^3 = _____
52. 0.02 m^3 = _____
53. 5.62 m^3 = _____
54. 1.01 m^3 = _____
55. 7.9035 m^3 = _____
56. 18.006 m^3 = _____
57. 105.12 m^3 = _____

Now change these from litres to cubic metres

2 650 L = 2.65 m^3 (3 places left)

58. 2 000 L = _____
59. 1 700 L = _____
60. 800 L = _____
61. 3 075 L = _____
62. 175 L = _____
63. 422 L = _____
64. 9.06 L = _____
65. 45.6 L = _____
66. 115 L = _____
67. 2 L = _____
68. 500.5 L = _____
69. 3 000.6 L = _____

Converting between Volume and Capacity

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Change the units from mL to cm^3 or the reverse

Example

73 mL = 73 cm^3

1. 350 mL = 350 cm^3
2. 105 mL = 105 cm^3
3. 56.3 cm^3 = 56.3 mL
4. 850 mL = 850 cm^3
5. 2.43 cm^3 = 2.43 mL
6. 600 cm^3 = 600 mL

Convert these volumes in cm^3 (c.c.) to litres

850 cm^3 = 0.85 L (3 places left)

7. 1 000 cm^3 = 1 L
8. 800 cm^3 = 0.8 L
9. 450 cm^3 = 0.45 L
10. 2 100 cm^3 = 2.1 L
11. 1 700 cm^3 = 1.7 L
12. 750 cm^3 = 0.75 L
13. 1 250 cm^3 = 1.25 L
14. 5 000 cm^3 = 5 L
15. 85 cm^3 = 0.085 L
16. 4 302 cm^3 = 4.302 L
17. 120.4 cm^3 = 0.1204 L
18. 333 cm^3 = 0.333 L
19. 2 006 cm^3 = 2.006 L
20. 500 cm^3 = 0.5 L
21. 25.1 cm^3 = 0.0251 L
22. 1 007 cm^3 = 1.007 L
23. 10 cm^3 = 0.01 L

Now reverse, change these from L to cm^3

4.3 L = 4 300 cm^3 (3 places right)

24. 2 L = 2 000 cm^3
25. 0.5 L = 500 cm^3
26. 3.1 L = 3 100 cm^3
27. 0.2 L = 200 cm^3
28. 3.5 L = 3 500 cm^3
29. 1.75 L = 1 750 cm^3
30. 0.04 L = 40 cm^3
31. 4.03 L = 4 030 cm^3
32. 0.05 L = 50 cm^3
33. 0.01 L = 10 cm^3
34. 0.2063 L = 206.3 cm^3
35. 2.02 L = 2 020 cm^3
36. 0.25 L = 250 cm^3
37. 4.06 L = 4 060 cm^3
38. 0.96 L = 960 cm^3
39. 3.007 L = 3 007 cm^3
40. 4.55 L = 4 550 cm^3

Change the units from kL to m^3 or the reverse

Example

2.9 m^3 = 2.9 kL

41. 7 m^3 = 7 kL
42. 3.84 kL = 3.84 m^3
43. 43 kL = 43 m^3
44. 0.3 m^3 = 0.3 kL
45. 204 m^3 = 204 kL
46. 7.06 kL = 7.06 m^3

Change these volumes from m^3 to litres.

2.53 m^3 = 2 530 L (3 places right)

47. 7.2 m^3 = 7 200 L
48. 0.5 m^3 = 500 L
49. 3.7 m^3 = 3 700 L
50. 0.25 m^3 = 250 L
51. 16.403 m^3 = 16 403 L
52. 0.02 m^3 = 20 L
53. 5.62 m^3 = 5 620 L
54. 1.01 m^3 = 1 010 L
55. 7.9035 m^3 = 7 903.5 L
56. 18.006 m^3 = 18 006 L
57. 105.12 m^3 = 105 120 L

Now change these from litres to cubic metres

2 650 L = 2.65 m^3 (3 places left)

58. 2 000 L = 2 m^3
59. 1 700 L = 1.7 m^3
60. 800 L = 0.8 m^3
61. 3 075 L = 3.075 m^3
62. 175 L = 0.175 m^3
63. 422 L = 0.422 m^3
64. 9.06 L = 9 060 m^3
65. 45.6 L = 0.0456 m^3
66. 115 L = 0.115 m^3
67. 2 L = 0.002 m^3
68. 500.5 L = 0.5005 m^3
69. 3 000.6 L = 3.0006 m^3

Units of Capacity

Capacity is another way of expressing volume. Rather than using length measurements, capacity is expressed using measurements normally associated with liquids. Usually mL, L and kL. This sheet deals with the changing of units between these three. As with length, (mm, m and km), capacity units are × 1 000 apart (cm is the exception with length). So you multiply by 1 000, or as hopefully you know, you move the decimal point 3 places to the right (multiplying) and to the left (dividing).

Column 1 deals with changing from a smaller unit to a larger unit. With these you divide by 1 000. So move the decimal point 3 places to the left with these, remember to write the new unit after your answer, it will be the unit inside the brackets. Watch that you don't have unnecessary zeros in your answers when decimal points are involved.

Column 2 is the reverse, this time moving to a smaller unit so you have more of them, so you are multiplying by 1 000. Move the decimal point 3 places to the right to make the number larger. Again remember to write the new units after the number.

Column 3 asks you to find the volume of the prisms, the answer will be in cubic units, either cm^3 or m^3. Convert these to either mL (if cm^3) or kL (if m^3), rewrite the number and change the units. Then rewrite the answer in litres (L), using your skills from the previous 2 columns. Capacity is volume, so you still write 'V =' in your answer.

Note that L is the symbol for litres, it is a capital letter, this may seem unusual but many metric units use capital letters, an example being ‘N’ (Newtons, a measure of force) or ‘W’ (Watt, a measure of electrical power).

Units of Capacity

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Change these measurements to the larger units shown in the brackets

mL → L ÷ 1 000 → 3 places left

L → kL ÷ 1 000 → 3 places left

1 2 000 mL [L] = _____

2 5 000 L [kL] = _____

3 1 500 L [kL] = _____

4 3 400 mL [L] = _____

5 900 L [kL] = _____

6 500 mL [L] = _____

7 1 375 mL [L] = _____

8 250 mL [L] = _____

9 505 L [kL] = _____

10 50 L [kL] = _____

11 10 mL [L] = _____

12 35 mL [L] = _____

13 1 017 L [kL] = _____

14 110 mL [L] = _____

15 4 070 L [kL] = _____

16 1 400 mL [L] = _____

17 60 mL [L] = _____

18 15 L [kL] = _____

19 410 mL [L] = _____

20 5 030 L [kL] = _____

21 303 L [kL] = _____

22 1 L [kL] = _____

Convert the measurements to the smaller units shown in the brackets

kL → L × 1 000 → 3 places right

L → mL × 1 000 → 3 places right

23 1 L [mL] = _____

24 6 L [mL] = _____

25 4.5 kL [L] = _____

26 1.25 L [mL] = _____

27 0.75 L [mL] = _____

28 0.8 kL [L] = _____

29 3.2 L [mL] = _____

30 0.01 kL [L] = _____

31 1.12 kL [L] = _____

32 3.05 kL [L] = _____

33 0.079 L [mL] = _____

34 5.03 L [mL] = _____

35 0.871 kL [L] = _____

36 5.64 kL [L] = _____

37 0.0025 L [mL] = _____

38 2.004 kL [L] = _____

39 0.07 L [mL] = _____

40 0.003 kL [L] = _____

41 0.0005 kL [L] = _____

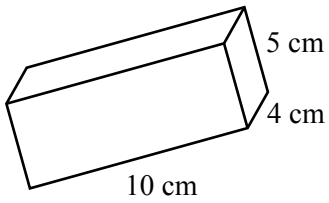
42 1.9 L [mL] = _____

43 0.0087 kL [L] = _____

44 0.053 L [mL] = _____

Calculate the volume in either cm^3 or m^3, write the answer as mL or kL, then rewrite converting to litres

45



V = *lbh*

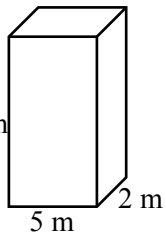
=

V = cm³

V = mL

V = L

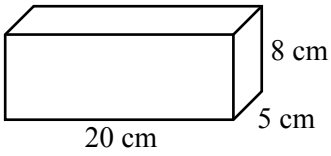
46



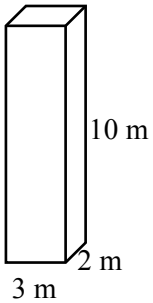
kL

L

47



48



Units of Capacity

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Change these measurements to the larger units shown in the brackets

mL → L : ÷ 1 000 → 3 places left

L → kL : ÷ 1 000 → 3 places left

1	2 000 mL	[L] =	2 L
2	5 000 L	[kL] =	5 kL
3	1 500 L	[kL] =	1.5 kL
4	3 400 mL	[L] =	3.4 L
5	900 L	[kL] =	0.9 kL
6	500 mL	[L] =	0.5 L
7	1 375 mL	[L] =	1.375 L
8	250 mL	[L] =	0.25 L
9	505 L	[kL] =	0.505 L
10	50 L	[kL] =	0.05 kL
11	10 mL	[L] =	0.01 L
12	35 mL	[L] =	0.035 L
13	1 017 L	[kL] =	1.017 kL
14	110 mL	[L] =	0.11 L
15	4 070 L	[kL] =	4.07 L
16	1 400 mL	[L] =	1.4 L
17	60 mL	[L] =	0.06 L
18	15 L	[kL] =	0.015 kL
19	410 mL	[L] =	0.41 L
20	5 030 L	[kL] =	5.03 kL
21	303 L	[kL] =	0.303 kL
22	1 L	[kL] =	0.001 kL

Convert the measurements to the smaller units shown in the brackets

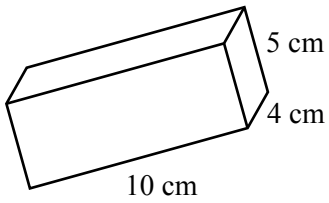
kL → L : × 1 000 → 3 places right

L → mL : × 1 000 → 3 places right

23	1 L	[mL] =	1 000 mL
24	6 L	[mL] =	6 000 mL
25	4.5 kL	[L] =	4 500 mL
26	1.25 L	[mL] =	1 250 mL
27	0.75 L	[mL] =	750 mL
28	0.8 kL	[L] =	800 L
29	3.2 L	[mL] =	3 200 mL
30	0.01 kL	[L] =	10 L
31	1.12 kL	[L] =	1 120 L
32	3.05 kL	[L] =	3 050 L
33	0.079 L	[mL] =	79 mL
34	5.03 L	[mL] =	5 030 mL
35	0.871 kL	[L] =	871 L
36	5.64 kL	[L] =	5 640 L
37	0.0025 L	[mL] =	2.5 mL
38	2.004 kL	[L] =	2 004
39	0.07 L	[mL] =	70 mL
40	0.003 kL	[L] =	3 L
41	0.0005 kL	[L] =	0.5 L
42	1.9 L	[mL] =	1 900 mL
43	0.0087 kL	[L] =	8.7 L
44	0.053 L	[mL] =	53 mL

Calculate the volume in either cm^3 or m^3, write the answer as mL or kL, then rewrite converting to litres

45



$V = lbh$

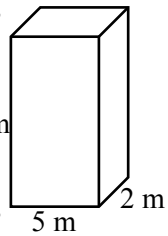
$= 10 \times 5 \times 4$

$V = 200 \text{ cm}^3$

$V = 200 \text{ mL}$

$V = 0.2 \text{ L}$

46



$V = lbh$

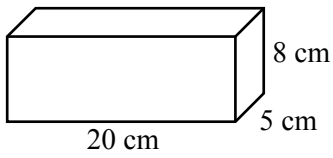
$= 5 \times 15 \times 2$

$V = 150 \text{ m}^3$

$V = 150 \text{ kL}$

$V = 150\,000 \text{ L}$

47



$V = lbh$

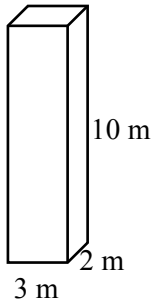
$= 20 \times 8 \times 5$

$V = 800 \text{ cm}^3$

$V = 800 \text{ mL}$

$V = 0.8 \text{ L}$

48



$V = lbh$

$= 3 \times 10 \times 2$

$V = 60 \text{ m}^3$

$V = 60 \text{ kL}$

$V = 60\,000 \text{ L}$

Volume and Capacity

This sheet involves both volume units (mm^3, cm^3 and m^3) and capacity units (mL, L and kL).

Column 1 starts with solids built from cube blocks. Each cube is 10 cm^3 in volume. As 1 cm^3 is equal to 1 mL, that also means that each block has a capacity of 10mL. So count the blocks and multiply by 10 for the volume and capacity. The second part of the column gives you the area of the face of the prism and its height. Use $V = Ah$ to find the volume. After you find the volume convert the cm^3 volume to mL and the m^3 volumes to kL.

Column 2 involves finding the volumes for cubes and rectangular prisms.

Volume of a Rectangular Prism:	$V = lbh$
Volume of a Cube:	$V = l^3$

To change the units remember that:

$1\ cm^3 = 1\ mL$

$1\ m^3 = 1\ kL$

Column 3 deals with triangular prisms and cylinders. The formulae to use for these is:

Volume of a Triangular Prism:	$V = \frac{1}{2}lbh$
Volume of a Cylinder:	$V = \pi r^2 h$

Remember to watch for when you are given a diameter rather than a radius. Halve the diameter (÷2) to get the radius, then use the radius in the formula.

Volume and Capacity

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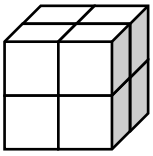
Find the volume of these solids then rewrite in mL. Each cube has a volume of 10 cm^3.

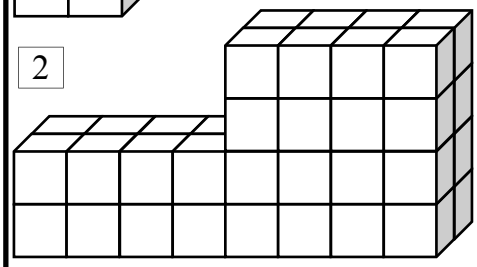
1



V = ☐ cm^3

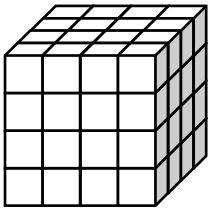
Capacity = ☐ mL

2



V = Capacity =

3

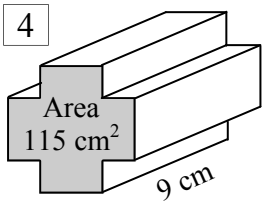


V =

Capacity =

Use V = AH to find the volume of these solids. Then convert m^3 to kL or cm^3 to mL.

4



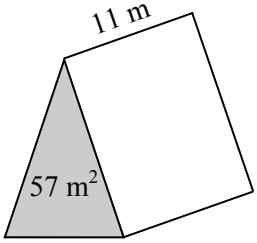
V = A*h*

= ☐ × ☐

V =

Capacity =

5



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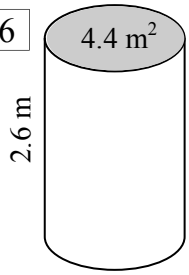
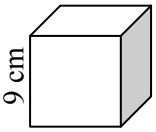


If these solids were immersed in water, what volume of water would be displaced? To 1 d.p.

7 Cube



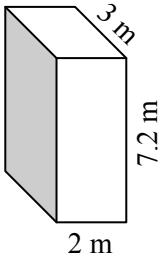
V =

=

V =

Capacity =

8



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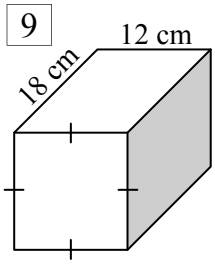
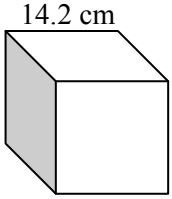


10 Cube



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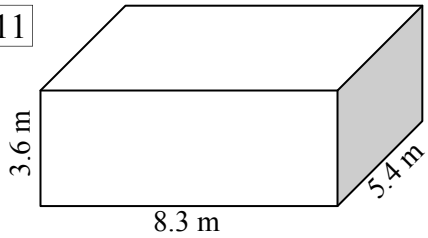

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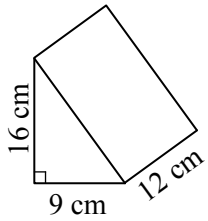

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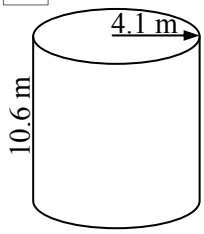

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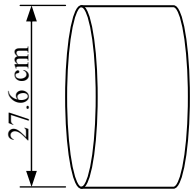

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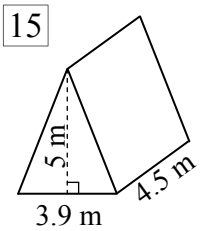

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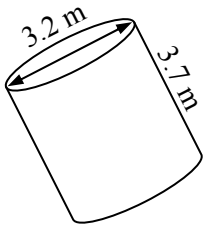
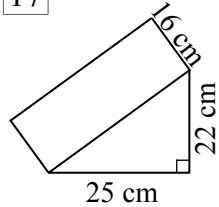


17



Volume and Capacity

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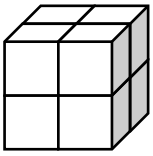
Find the volume of these solids then rewrite in mL. Each cube has a volume of 10 cm^3.

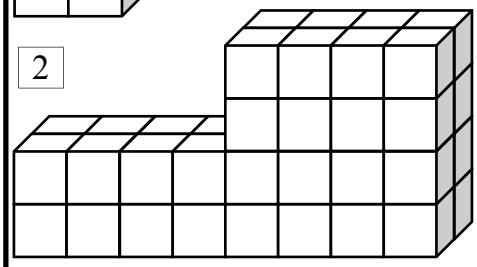
1



V = 80 cm^3

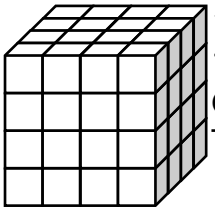
Capacity = 80 mL

2



V = 480 cm^3 Capacity = 480 mL

3

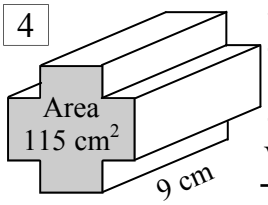


V = 640 cm^3

Capacity = 640 mL

Use V = AH to find the volume of these solids. Then convert m^3 to kL or cm^3 to mL.

4



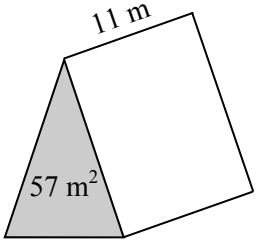
V = A*h*

= 115 × 9

V = 1 035 cm^3

Capacity = 1 035 mL

5



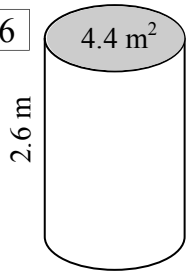
V = A*h*

= 57 × 11

V = 627 m^3

Capacity = 627 kL

6



V = A*h*

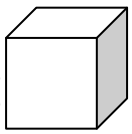
= 4.4 × 2.6

V = 11.44 m^3

Capacity = 11.44 kL

If these solids were immersed in water, what volume of water would be displaced? To 1 d.p.

7 Cube



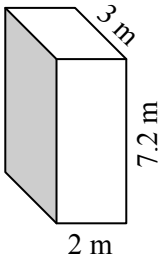
V = l^3

= 9^3

V = 729 cm^3

Capacity = 729 mL

8



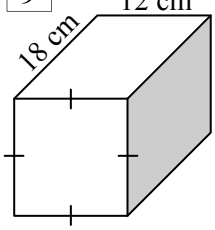
V = *lbh*

= 2 × 7.2 × 3

V = 43.2 m^3

Capacity = 43.2 kL

9



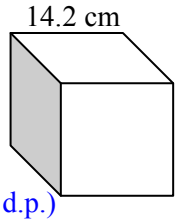
V = *lbh*

= 12×12×18

V = 2 592 cm^3

Capacity = 2 592 mL

10 Cube



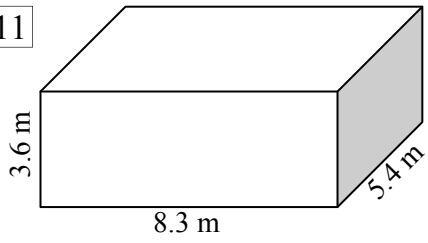
V = l^3

= 14.2^3

V = 2 863.3 cm^3 (1 d.p.)

Capacity = 2 863.3 mL

11



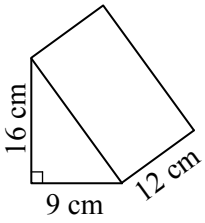
V = *lbh*

= 3.6 × 8.3 × 5.4

V = 161.4 m^3 (1 d.p.)

Capacity = 161.4 kL

12



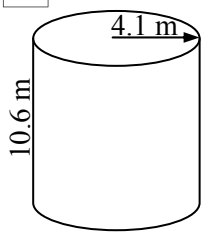
V = ½*lbh*

= ½×9×16×12

V = 864 cm^3

Capacity = 864 mL

13



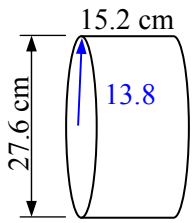
V = $\pi r^2 h$

= $\pi \times 4.1^2 \times 10.6$

V = 559.8 m^3 (1 d.p.)

Capacity = 559.8 kL

14



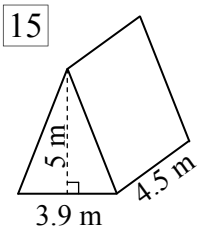
V = $\pi r^2 h$

= $\pi \times 13.8^2 \times 15.2$

V = 9 093.9 cm^3 (1 d.p.)

Capacity = 9 093.9 mL

15



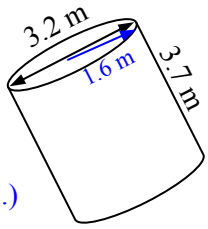
V = ½*lbh*

= ½×3.9×5×4.5

V = 43.9 m^3 (1 d.p.)

Capacity = 43.9 kL

16



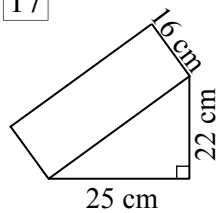
V = $\pi r^2 h$

= $\pi \times 1.6^2 \times 3.7$

V = 29.8 m^3 (1 d.p.)

Capacity = 29.8 kL

17



V = ½*lbh*

= ½×25×22×16

V = 4 400 cm^3

Capacity = 4 400 mL

Further Volume and Capacity

Volume and capacity are essentially the same thing. The only difference is that volume uses cubic length units (mm^3, cm^3, m^3) while capacity uses liquid units (ml, L, kL). The good news is that:

$1\ cm^3 = 1\ mL$
and
$1\ m^3 = 1\ kL$

Note that there is no unit conversion for litres. So if an answer requires litres then the units have to be converted from mL or kL.

Column 1 shows objects that are commonly measured in litres. But the dimensions of the objects are in cm. So the process to answer these questions is to first find the volume in cm^3. Then rewrite this only using mL. Finally divide this answer by 1 000 to convert to Litres.

Columns 2 and 3 involve objects which are measured in metres. So you will get a volume in m^3. This can be rewritten in kL, then multiplied by 1 000 to find the answer in Litres.

Further Volume and Capacity

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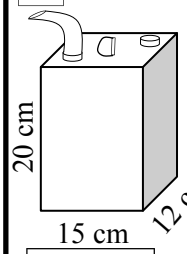
Find the volume of these objects, then convert to mL and Litres. Round to 2 d.p. when required.

1



$V = lbh$

$= \square \times \square \times \square$

$V = \square$ cm^3

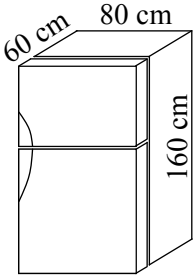
$\square$ cm^3 = $\square$ mL

Convert to mL: 1 cm^3 = 1 mL

Capacity = $\square$ L

Convert to L: 1 000 mL = 1 L

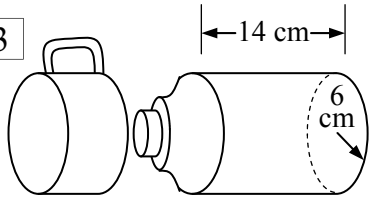
2



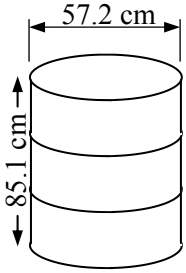
$\square$ cm^3 = $\square$ mL

Capacity = $\square$ L

3

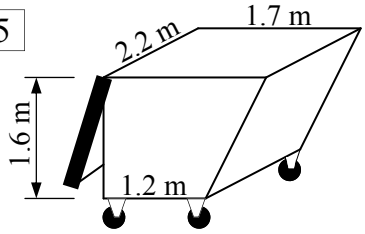


4



Use V=A*h* to find the volume for these, then give the capacity in kL, then in L. Round to 2 d.p.

5



A =

V = A*h*

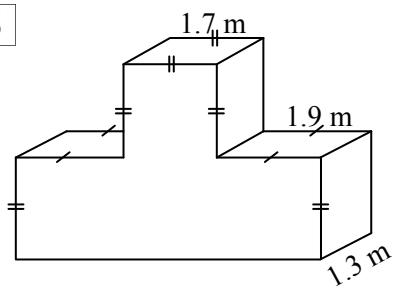
$\square$ m^3 = $\square$ kL

Convert to kL: 1 m^3 = 1 kL

Capacity = $\square$ L

Convert to L: 1 kL = 1 000 L

6



A_1 = A_2 =

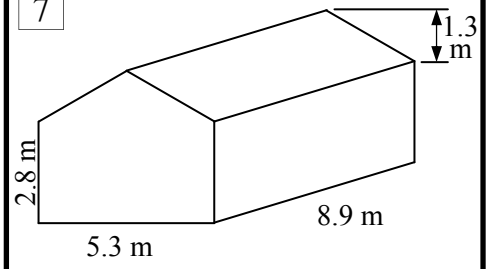
$A = A_1 + A_2 =$

V = A*h*

$\square$ m^3 = $\square$ kL

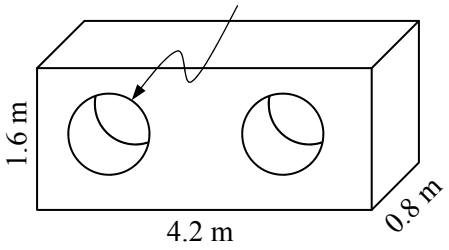
Capacity = $\square$ L

7



8

Both holes 1 m diameter, completely through solid.



Further Volume and Capacity

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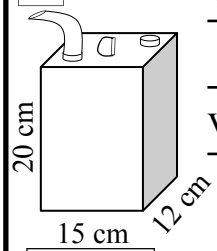
Find the volume of these objects, then convert to mL and Litres. Round to 2 d.p. when required.

1



$V = lbh$

$= 20 \times 15 \times 12$

$V = 3\,600$ cm^3

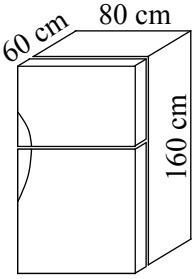
3 600 cm^3 = 3 600 mL

Convert to mL: 1 cm^3 = 1 mL

Capacity = 3.6 L

Convert to L: 1 000 mL = 1 L

2



$V = lbh$

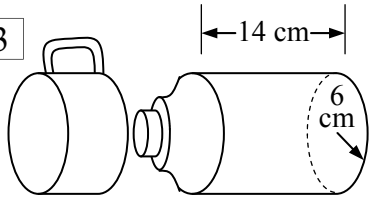
$= 60 \times 80 \times 160$

$V = 768\,000$ cm^3

768 000 cm^3 = 768 000 mL

Capacity = 768 L

3



$V = \pi r^2 h$

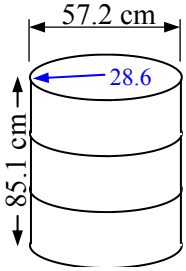
$= \pi \times 6^2 \times 14$

$V = 1\,583.36$ cm^3 (2 d.p.)

1 583.36 cm^3 = 1 583.36 mL

Capacity = 1.58 L (2 d.p.)

4



$V = \pi r^2 h$

$= \pi \times 28.6^2 \times 85.1$

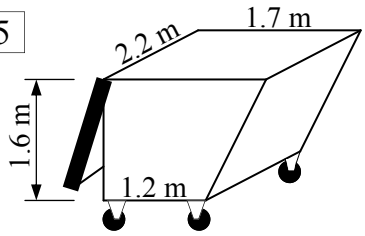
$V = 218\,681.23$ cm^3 (2 d.p.)

218 681.23 cm^3 = 218 681.23 mL

Capacity = 218.68 L (2 d.p.)

Use V=A*h* to find the volume for these, then give the capacity in kL, then in L. Round to 2 d.p.

5



$A = \frac{1}{2}(a + b) \times h$

$= \frac{1}{2} \times (1.2 + 1.7) \times 1.6$

$A = 2.32$ m^3

$V = Ah$

$= 2.32 \times 2.2$

$V = 5.10$ m^3 (2 d.p.)

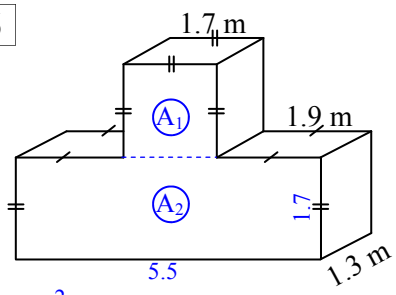
5.1 m^3 = 5.1 kL

Convert to kL: 1 m^3 = 1 kL

Capacity = 5 100 L

Convert to L: 1 kL = 1 000 L

6



$A_1 = l^2$

$= 1.7^2$

$A_1 = 2.89$ m^2

$A_2 = lb$

$= 1.7 \times 5.5$

$A_2 = 9.35$ m^2

$A = A_1 + A_2 = 2.89 + 9.35 = 12.24$ m^2

$V = Ah$

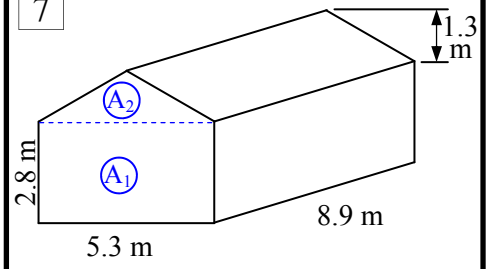
$= 12.24 \times 1.3$

$V = 15.91$ m^3 (2 d.p.)

15.91 m^3 = 15.91 kL

Capacity = 15 910 L

7



$A_1 = lb$

$= 2.8 \times 5.3$

$A_1 = 14.84$ m^2

$A_2 = \frac{1}{2}bh$

$= \frac{1}{2} \times 1.3 \times 5.3$

$A_2 = 3.45$ m^2 (2 d.p.)

$A = A_1 + A_2 = 14.84 + 3.45 = 18.29$ m^2

$V = Ah$

$= 18.29 \times 8.9$

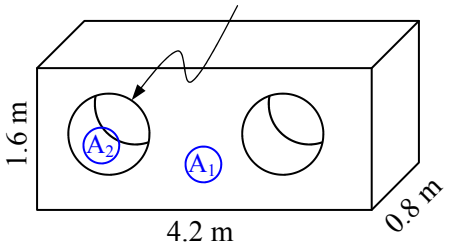
$V = 162.78$ m^3 (2 d.p.)

162.78 m^3 = 162.78 kL

Capacity = 162 780 L

8

Both holes 1 m diameter, completely through solid.



$A_1 = lb$

$= 1.6 \times 4.2$

$A_1 = 6.72$ m^2

$A_2 = \pi r^2$

$= \pi \times 0.5^2$

$A_2 = 0.79$ m^2 (2 d.p.)

$A = 6.72 - 2 \times 0.79 = 5.14$ m^2

$V = Ah$

$= 5.14 \times 0.8$

$V = 4.11$ m^3 (2 d.p.)

4.11 m^3 = 4.11 kL

Capacity = 4 110 L

Volume and Capacity Problems

These are worded questions using skills learned on previous sheets. Previous sheets should be attempted before this sheet.

Formulae required for this sheet are:

Volume of a Rectangular Prism: $V = lbh$

Volume of a Cylinder: $V = \pi r^2 h$

Volume of a Prism: $V = Ah$

Area of a Trapezium: $A = \frac{1}{2}(a + b)\, h$

Volume and Capacity Problems

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Use your volume and capacity skills to answer these, to 1 d.p.

A cylindrical rainwater tank with radius 3.4 m and height 5.2 m is the water source at a property. The tank is full.

1 Draw the tank

2 Calculate the capacity of the tank, answer in litres.

3 If the family uses an average of 630 L per day, how many days supply is there without rain?

Dave is selling drinks at a school fete. The rectangular prism-shaped container has sides of 18 cm, 26 cm and 30 cm.

4 Find its volume in litres.

5 If the mix requires 15% cordial and 85% water, calculate the amount of each required.

Joanne has found an old paint can in the garage. The label is removed so she can't read its capacity. The can is 90% full and has a 22 cm diameter and 25 cm height.

6 Find the capacity of the tank when full.

7 Find the amount of paint in the can.

8 If one litre covers 14 m^2, find the coverage, to the nearest m^2, that Joanne can expect.

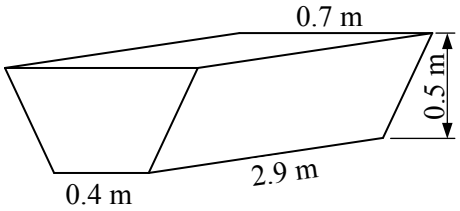
Ian has to fill a rectangular prism shaped fish tank with a cylindrical bucket. The tank has sides 60 cm, 38 cm & 25 cm, the bucket has a diameter of 23 cm and height of 34 cm. Find:

9 The tank capacity in L.

10 The bucket capacity in L.

11 How many bucketfuls will be required to fill the tank?

A water trough made from concrete is shown below. It weighs 125 kg when empty. If water weighs 1 T/m^3, find :



12 The tough's volume, (3 d.p.)

13 The weight of the trough, in kg, if it is full of water.

Amy's garden tap is dripping. If it leaks at 55 mL per minute, find:

14 The water lost after 3 hours.

15 The time taken to lose 30 L of water. (nearest minute)

16 The time taken, to the nearest min, to fill a cylindrical bucket of diameter 35 cm and height 40 cm.

Volume and Capacity Problems

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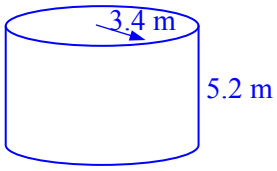
Use your volume and capacity skills to answer these, to 1 d.p.

A cylindrical rainwater tank with radius 3.4 m and height 5.2 m is the water source at a property. The tank is full.

1 Draw the tank



2 Calculate the capacity of the tank, answer in litres.

$V = \pi r^2 h$

$= \pi \times 3.4^2 \times 5.2$

$V = 188.8 \text{ m}^3$ (1 d.p.)

Capacity is 188 800 L (188 847.4 L)

3 If the family uses an average of 630 L per day, how many days supply is there without rain?

188 800 ÷ 630 = 299.7 days 1d.p.

(or 188 847.4 ÷ 630 = 299.8 days) (1 d.p.)

Dave is selling drinks at a school fete. The rectangular prism-shaped container has sides of 18 cm, 26 cm and 30 cm.

4 Find its volume in litres.

$V = lbh$

$= 18 \times 26 \times 30$

$V = 14\,040 \text{ cm}^3$

Capacity = 14 litres (1 d.p.)

5 If the mix requires 15% cordial and 85% water, calculate the amount of each required.

Cordial Amount: 0.15 × 14

= 2.1 litres

Water Amount: 14 - 2.1

= 11.9 litres

Joanne has found an old paint can in the garage. The label is removed so she can't read its capacity. The can is 90% full and has a 22 cm diameter and 25 cm height.

6 Find the capacity of the tank when full.

$V = \pi r^2 h$

$= \pi \times 11^2 \times 25$

$V = 9\,503.3 \text{ cm}^3$ (1 d.p.)

Capacity is 9.5 L

7 Find the amount of paint in the can.

Paint: 0.9 × 9.5

= 8.6 L (1 d.p.)

8 If one litre covers 14 m^2, find the coverage, to the nearest m^2, that Joanne can expect.

Coverage: 8.6 × 14

= 120 m^2 (nearest m^2)

Ian has to fill a rectangular prism shaped fish tank with a cylindrical bucket. The tank has sides 60 cm, 38 cm & 25 cm, the bucket has a diameter of 23 cm and height of 34 cm. Find:

9 The tank capacity in L.

$V = lbh$

$= 60 \times 38 \times 25$

$V = 57\,000 \text{ cm}^3$

Capacity = 57 litres.

10 The bucket capacity in L.

$V = \pi r^2 h$

$= \pi \times 11.5^2 \times 34$

$V = 14\,126.2 \text{ cm}^3$ (1 d.p.)

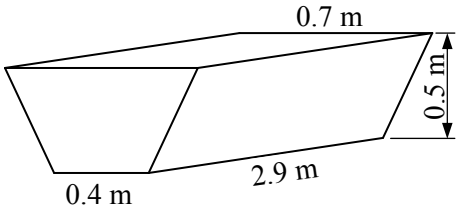
Capacity = 14.1 litres. (1 d.p.)

11 How many bucketfuls will be required to fill the tank?

Buckets Req'd = 57 ÷ 14.1

= 4.04…. So 5 buckets required

A water trough made from concrete is shown below. It weighs 125 kg when empty. If water weighs 1 T/m^3, find :



12 The tough's volume, (3 d.p.)

$A = \frac{1}{2}(a + b) \times h$

$= \frac{1}{2}(0.4 + 0.7) \times 0.5$

$A = 0.275 \text{ m}^2$

$V = Ah$

$= 0.275 \times 2.9$

$V = 0.798 \text{ m}^3$ (3 d.p.)

13 The weight of the trough, in kg, if it is full of water.

Weight Water = 0.798 × 1 000

= 798 kg

Total = 798 + 125 = 923 kg

Amy's garden tap is dripping. If it leaks at 55 mL per minute, find:

14 The water lost after 3 hours.

Water Lost = 55 × (3 × 60)

Water Lost = 9 900 mL (9.9 L)

15 The time taken to lose 30 L of water. (nearest minute)

Time = 30 000 ÷ 55

Time = 545 mins (9 h 5 min)

16 The time taken, to the nearest min, to fill a cylindrical bucket of diameter 35 cm and height 40 cm.

$V = \pi r^2 h$

$= \pi \times 17.5^2 \times 40$

$V = 38\,484.5 \text{ cm}^3$ (1 d.p.)

Time = 38 484.5 ÷ 55

Time = 700 min (nearest min)

(11 h 40 min)

Volume - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Volume - Find a Word

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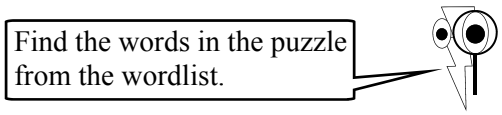

D	R	N	I	L	Y	C	Y	L	I	N	D	E	R	L
I	P	A	C	R	A	L	U	G	N	A	I	R	T	Y
C	A	S	D	R	S	X	E	R	H	O	M	B	I	C
C	R	X	E	I	Q	E	X	C	V	X	E	M	Y	O
Y	A	A	M	C	U	E	H	O	Z	M	N	I	R	M
L	L	C	U	A	A	S	E	N	O	S	S	L	E	P
I	L	B	L	P	R	F	I	C	I	I	I	L	T	O
N	E	M	O	A	E	H	G	A	A	R	O	I	E	S
R	L	R	V	C	X	T	H	V	L	P	N	L	M	I
I	X	O	C	I	C	G	T	E	A	L	S	I	A	T
C	C	F	O	T	O	N	Z	A	Z	A	K	T	I	E
A	U	I	N	Y	N	E	S	T	I	N	U	R	D	X
L	B	N	V	K	I	L	O	L	I	T	R	E	X	E
C	I	U	E	R	A	L	U	G	N	A	T	C	E	R
E	C	E	X	L	A	D	I	O	Z	E	P	A	R	T

WORDLIST

RECTANGULAR	SQUARE	LENGTH	CYLINDER	UNITS
CYLINDRICAL	PRISM	CONVEX	VOLUME	CUBE
DIMENSIONS	UNIFORM	MILLILITRE	AREA	KILOLITRE
TRAPEZOIDAL	HEIGHT	DIAMETER	CONCAVE	RADIUS
TRIANGULAR	SPACE	CAPACITY	COMPOSITE	CUBIC
	RHOMBIC	PARALLEL	FACES	

Volume - Find a Word

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Find the words in the puzzle from the wordlist.



D	R	N	I	L	Y	C	Y	L	I	N	D	E	R	L
I	P	A	C	R	A	L	U	G	N	A	I	R	T	Y
C	A	S	D	R	S	X	E	R	H	O	M	B	I	C
C	R	X	E	I	Q	E	X	C	V	X	E	M	Y	O
Y	A	A	M	C	U	E	H	O	Z	M	N	I	R	M
L	L	C	U	A	A	S	E	N	O	S	S	L	E	P
I	L	B	L	P	R	F	I	C	I	I	I	L	T	O
N	E	M	O	A	E	H	G	A	A	R	O	I	E	S
R	L	R	V	C	X	T	H	V	L	P	N	L	M	I
I	X	O	C	I	C	G	T	E	A	L	S	I	A	T
C	C	F	O	T	O	N	Z	A	Z	A	K	T	I	E
A	U	I	N	Y	N	E	S	T	I	N	U	R	D	X
L	B	N	V	K	I	L	O	L	I	T	R	E	X	E
C	I	U	E	R	A	L	U	G	N	A	T	C	E	R
E	C	E	X	L	A	D	I	O	Z	E	P	A	R	T

WORDLIST

~~RECTANGULAR~~
~~CYLINDRICAL~~
~~DIMENSIONS~~
~~TRAPEZOIDAL~~
~~TRIANGULAR~~
~~SQUARE~~
~~PRISM~~
~~UNIFORM~~
~~HEIGHT~~
~~SPACE~~
~~RHOMBIC~~
~~LENGTH~~
~~CONVEX~~
~~MILLILITRE~~
~~DIAMETER~~
~~CAPACITY~~
~~PARALLEL~~
~~CYLINDER~~
~~VOLUME~~
~~AREA~~
~~CONCAVE~~
~~COMPOSITE~~
~~FACES~~
~~UNITS~~
~~CUBE~~
~~KILOLITRE~~
~~RADIUS~~
~~CUBIC~~

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ALTITUDE

REVIEW:
GRAPHS

Picture Graphs

Picture graphs are similar to bar graphs, the data is displayed by pictures or symbols of an item. Graphs usually have an axis to read off the value, Picture graphs don't require this scale as each picture of an item represents a specified number of items. Picture graphs are as accurate as their scale allows, but they are used mainly for quick reference rather than an exact display of data.

Column 1 of this sheet has the graphs drawn for you, you are asked to read the graph and answer the questions. Note the 'key' or 'scale', this tells you the value of each symbol. To increase accuracy, portions of a symbol can be used.

The computer in the first column is either: fully drawn, ¾ drawn, ½ drawn or ¼ drawn. If a fully drawn computer represents 20 computers then part drawn computers equal that fraction of 20.

Graph 2 is a representation of a Café's sales of fresh juices. This time each symbol represents 200 glasses of juice. Note that again the same fractions ¼, ½ and ¾ are used. Questions 6 and 7 are about rounding. While 2 symbols may represent 400 glasses, those same 2 symbols could also represent 390 glasses. Some students will have difficulty with this concept.

Column 2 asks you to construct graphs from data in the tables. The data can be constructed exactly by adding whole and part pieces of a symbol, you don't need to worry about rounding.

Remember that fractions of a picture are always on the end (right hand side) and there can only be one in each line. Don't use several fractions of a symbol, the purpose of a picture graph is that they are easy and quick to read, not confusing!

Picture Graphs

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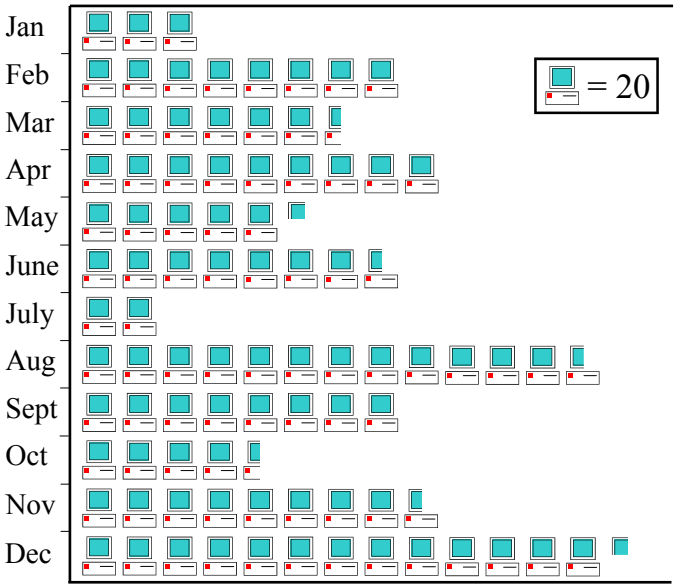
Study the graphs then answer the questions

Graph 1 : Computer Sales in 2002



1 Give the values represented by these symbols:

= ___ = ___ = ___ = ___

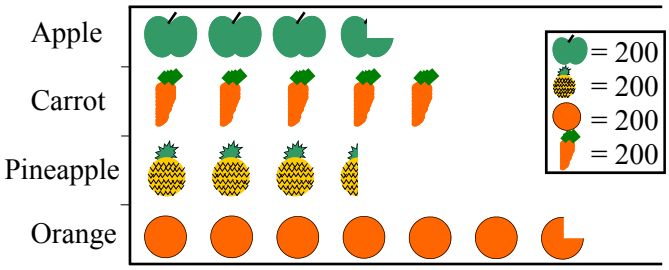
2 Name the months with the same sales figures and give the number of sales _____

3 Give the lowest sales month & sales _____

4 Give the highest sales month & sales figure, give a reason why this month could have the most sales.

_____

Graph 2 : Average Monthly Juice Sales (glass)



5 Name and give sales figures for the:

a) least popular juice _____

b) most popular juice _____

6 Actual sales figures for carrot juice could range from 975 - 1 024 glasses. Briefly explain why.

_____

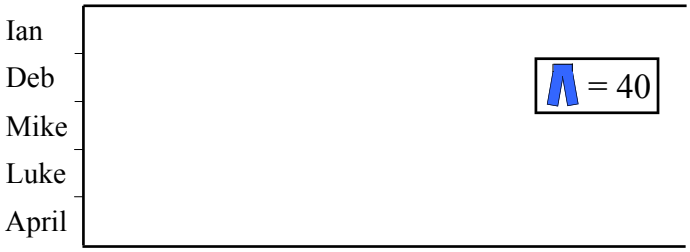
7 Give the largest and smallest possible sales figures for pineapple juice _____

Now place the details in the table on the graph

Table 1 : Jeans Shop Sales Assistant Figures

Name	Ian	Deb	Mike	Luke	April
Sales	240	80	140	170	150

Graph 3 : Jeans Shop Sales Assistant Figures



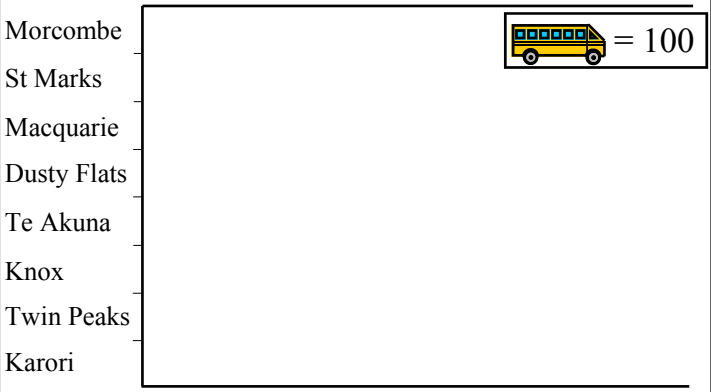
8 Give the values represented by these symbols:

= ___ = ___ = ___ = ___

Table 2 : Students Using Bus Transport

School Name	Student Numbers
Morcombe	250
St Marks	300
Macquarie	175
Dusty Flats	550
Te Akuna	50
Knox	225
Twin Peaks	475
Karori	325

Graph 4 : Students Using Bus Transport



9 Give the values represented by these symbols:

= ___ = ___ = ___ = ___

Picture Graphs

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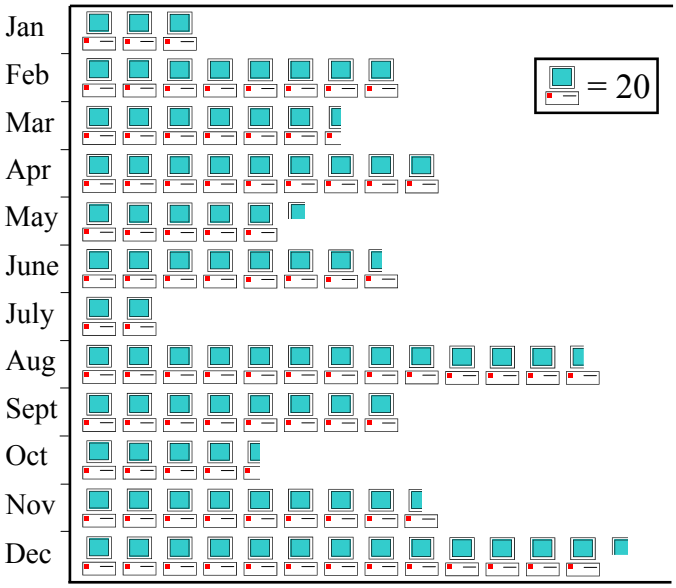
Study the graphs then answer the questions

Graph 1 : Computer Sales in 2002



1 Give the values represented by these symbols:

= 5 = 10 = 15 = 30

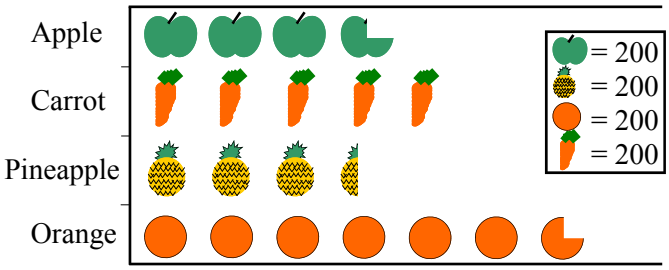
2 Name the months with the same sales figures and give the number of sales February (160) September (160)

3 Give the lowest sales month & sales July (40)

4 Give the highest sales month & sales figure, give a reason why this month could have the most sales.

December (265) - Christmas and Post Christmas sales

Graph 2 : Average Monthly Juice Sales (glass)



5 Name and give sales figures for the:

a) least popular juice Pineapple juice (700)

b) most popular juice Orange juice (1 350)

6 Actual sales figures for carrot juice could range from 975 - 1 024 glasses. Briefly explain why.

974 rounds to 950 and 1 025 rounds to 1 050

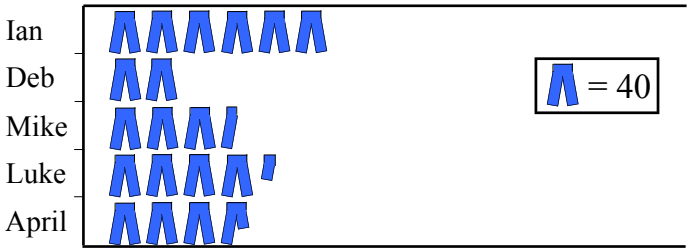
7 Give the largest and smallest possible sales figures for pineapple juice Smallest 675 and largest 724

Now place the details in the table on the graph

Table 1 : Jeans Shop Sales Assistant Figures

Name	Ian	Deb	Mike	Luke	April
Sales	240	80	140	170	150

Graph 3 : Jeans Shop Sales Assistant Figures



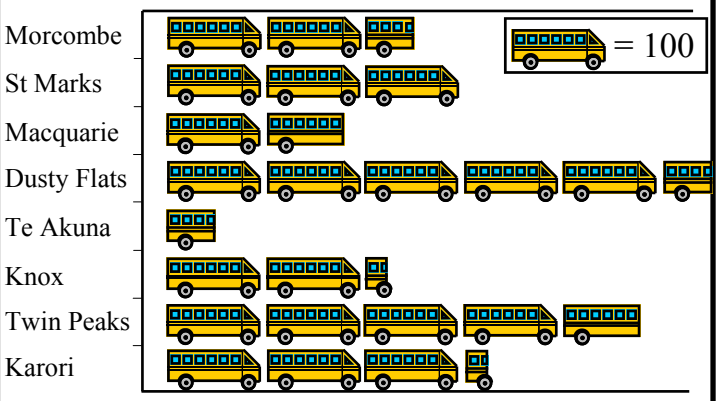
8 Give the values represented by these symbols:

= 10 = 20 = 30 = 70

Table 2 : Students Using Bus Transport

School Name	Student Numbers
Morcombe	250
St Marks	300
Macquarie	175
Dusty Flats	550
Te Akuna	50
Knox	225
Twin Peaks	475
Karori	325

Graph 4 : Students Using Bus Transport



9 Give the values represented by these symbols:

= 25 = 50 = 75 = 125

Column Graphs

Column graphs show data with columns which are read off a vertical scale, with the horizontal axis giving the meaning of the column. This sheet has two columns, the first being reading graphs and the second being constructing graphs (from reading a table).

Column 1 starts with a column graph on the fundraising activity of a charity. It outlines the different methods of fundraising along its horizontal axis and their financial success on the vertical axis. This graph can be read to the nearest $5 000.

The second graph is about the conversion of a chicken battery farm to a free range farm. This time there are two columns for each category. The free range egg figure and the battery laid figure. Note also that the columns are rectangular prisms, these give the graph a different appearance, but be careful when reading the columns. Read the columns off the top edge that is 'closer' to you, not the edge at the back of the column. In other words, use the front face of the prism for measurement.

Column 2 starts with a question about dice. When you roll 2 dice you have 36 different combinations of the two faces of each die. The first table is just like a 6 by 6 addition table, just add the two numbers. Then count how many of each number there are for the next table. Note both tables have been started for you. Then graph the result. Note that the first column is already done for you.

Question 6 involves a survey of traffic, it requires you to read the data straight from the table and place it on the graph.

Note the use of the word frequency, frequency means the number of times something occurs.

Column Graphs

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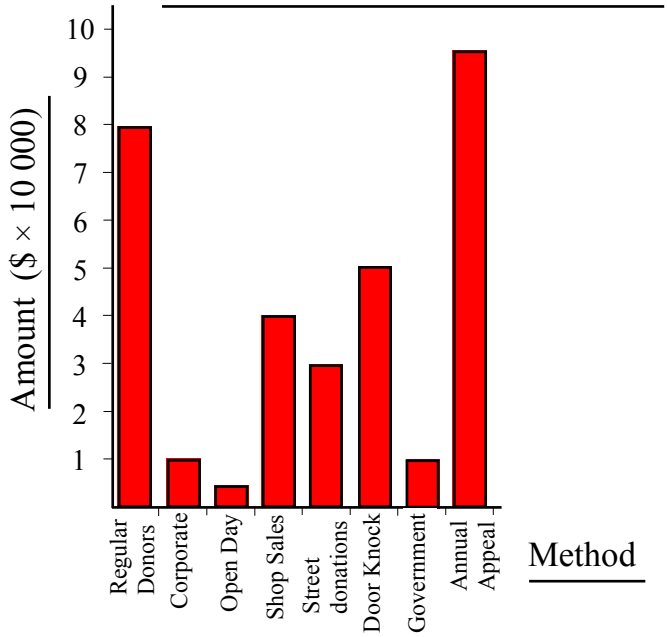
Study the column graphs below then answer the questions

Graph 1 : Fundraising by a Charity



1 Give amount raised by: *a*) Door Knock _____

b) Regular Donors _____ *c*) Shop Sales _____

2 Which method of fundraising:

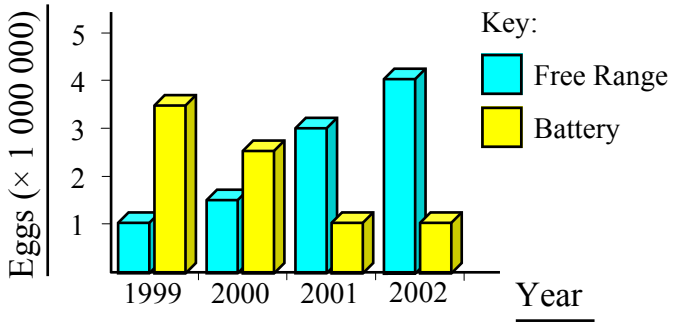
a) is the third most successful _____

b) matches Government donations _____

c) more than triples the street donations _____

d) made $5 000 _____

Graph 2 : Ingall's Hen Farm Egg Production



3 Give the total number of eggs for: *a*) 1999 _____

b) 2000 _____ *c*) 2001 _____

4 Give the year that free range production was:

a) 1 000 000 eggs less then battery laid eggs _____

b) increased and battery laid was unchanged _____

c) three times more than battery laid eggs _____

d) increased by 1 500 000 eggs _____

Complete the tables for the totals possible for 2 dice, then graph them.

5

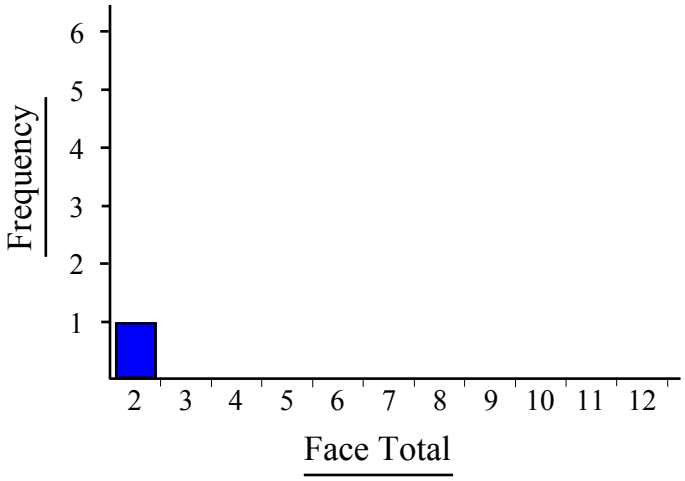
Table 1 : Total of 2 dice

Face No.	1	2	3	4	5	6
1	2	3	4			
2	3	4				
3	4					
4						
5						
6						

Table 2 : Frequency for Total of Two dice

Total	2	3	4	5	6	7	8	9	10	11	12
Freq	1	2	3								

Graph 3 : Face Total : 2 dice

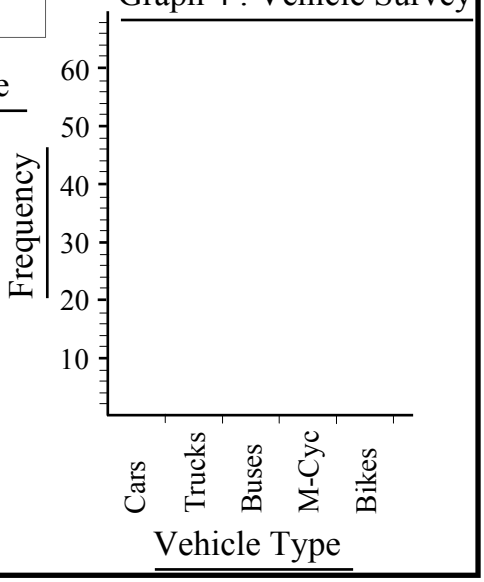


6 Transfer this data to the graph

Table 3 : Vehicle Survey

Type	Freq
Cars	63
Trucks	11
Buses	17
M-cyc	9
Bikes	4

Graph 4 : Vehicle Survey



Column Graphs

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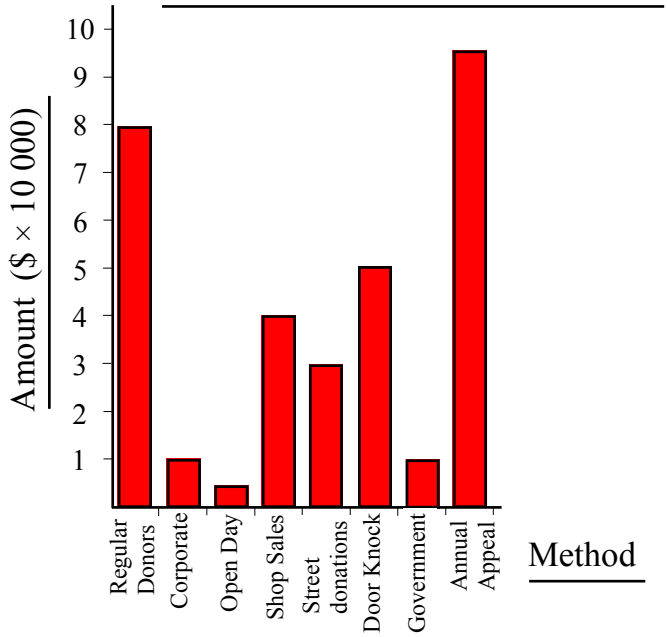
Study the column graphs below then answer the questions

Graph 1 : Fundraising by a Charity



1 Give amount raised by: *a*) Door Knock $50 000

b) Regular Donors $80 000 *c*) Shop Sales $40 000

2. Which method of fundraising:

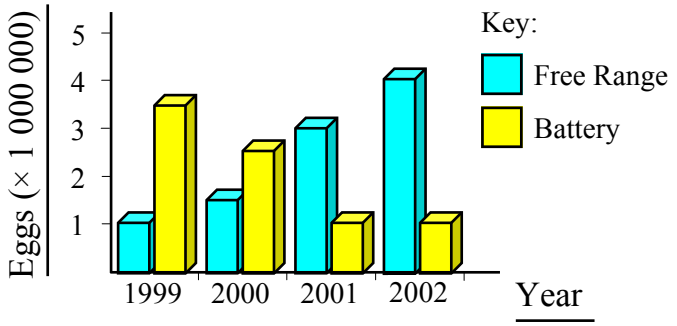
a) is the third most successful Door knock

b) matches Government donations Corporate

c) more than triples the street donations Annual Appeal

d) made $5 000 Open Day

Graph 2 : Ingall's Hen Farm Egg Production



4 Give the total number of eggs for: *a*) 1999 4 500 000

b) 2000 4 000 000 *c*) 2001 4 000 000

5 Give the year that free range production was:

a) 1 000 000 eggs less then battery laid eggs 2 000

b) increased and battery laid was unchanged 2002

c) three times more than battery laid eggs 2001

d) increased by 1 500 000 eggs 2001

Complete the tables for the totals possible for 2 dice, then graph them.

6

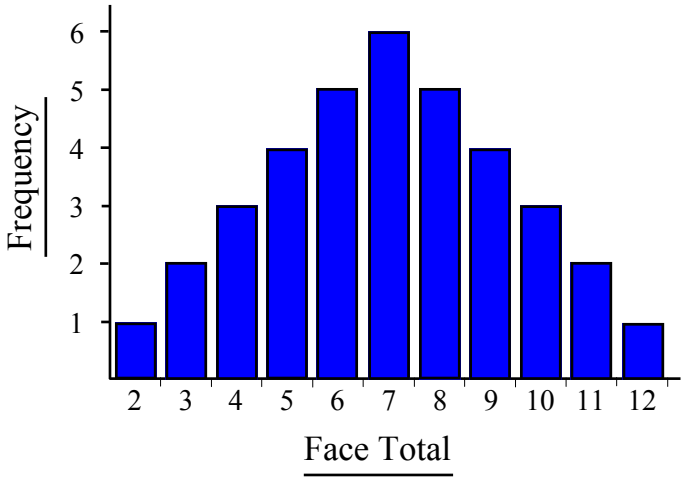
Table 1 : Total of 2 dice

Face No.	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Table 2 : Frequency for Total of Two dice

Total	2	3	4	5	6	7	8	9	10	11	12
Freq	1	2	3	4	5	6	5	4	3	2	1

Graph 3 : Face Total : 2 dice

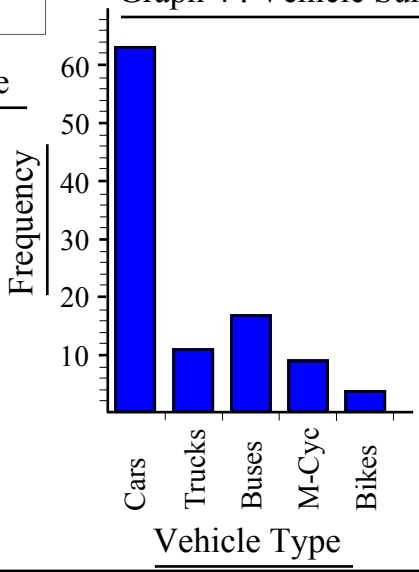


7 Transfer this data to the graph

Table 3 : Vehicle Survey

Type	Freq
Cars	63
Trucks	11
Buses	17
M-cyc	9
Bikes	4

Graph 4 : Vehicle Survey



Bar Graphs

Bar graphs display data in the same way as column graphs except the graph is on its side. These graphs all have:

- A title which describes the purpose of the graph
- Axes headings, these describe the scale on each axis
- Scales, these let you to read or construct the bars with reasonably accuracy

Column 1 asks you to extract information from constructed graphs. Both graphs have columns that are either exactly on the number or half way between two numbers on the horizontal scale. The first graph deals with Blue Whale population, Questions 1 and 2 ask you to read figures off the graph, question 3 asks you to write the years that correspond to the change in population.

The second graph refers to the way a student may spend their day. This graph uses ½ hours so write either 0.5 h or ½ h.

The second column asks you to create bar graphs. Write the title, axes headings and complete the scale which has been started for you. Then construct the bars.

Bar Graphs

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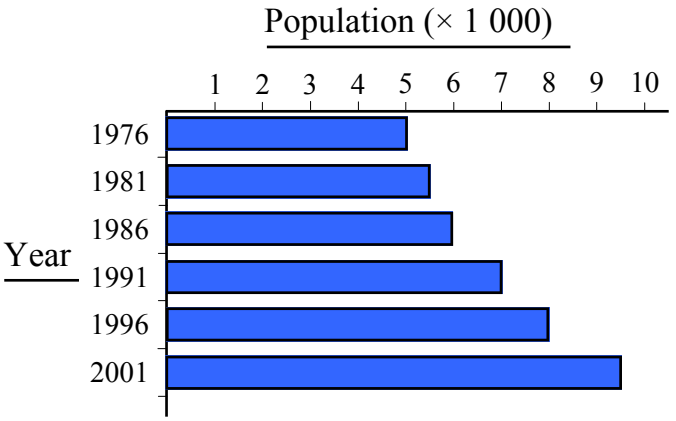
Answer these questions on the use of bar graphs

Graph 1 : Blue Whale Population Estimation



1 Give the estimated number of blue whales in:

a) 1976 _____ *b*) 1986 _____ *c*) 1996 _____

2 Give the increase in whale numbers between

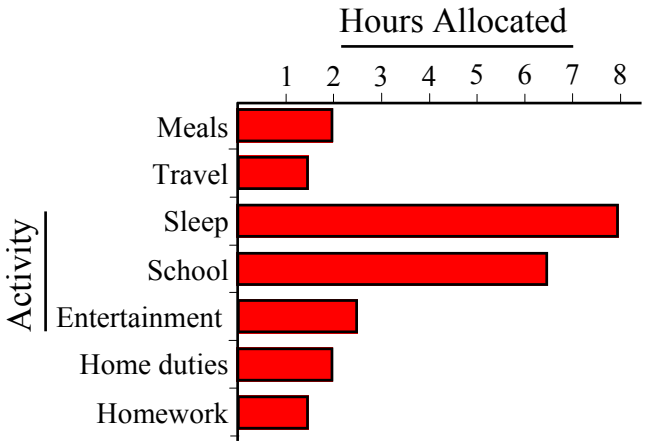
a) 1976 - 1991 _____ *b*) 1981 - 2001 _____

3 Between which years is there a population increase of:

a) 500 _____ *b*) 1 500 _____

c) 3 000 _____ *d*) 4 000 _____

Graph 2 : Analysis of a School Day



4 Give the number of hours spent: *a*) Sleeping ____

b) Eating ____ *c*) At school ____ *d*) Travelling ____

5 Give the difference in time spent on these activities:

a) Entertainment and.. Homework ____ Eating ____

b) Time at School and.. Travel ____ Sleeping ____

c) Home duties and.. School time ____ Travel ____

6 How many hours could the above student be at home each weekday _____

Now put this data on the bar graph below

Table 1 : Least Painful Home Duty

Home Duty	Student Approval
Mowing Grass	15%
Washing Dishes	10%
Tidy Bedroom	5%
Washing Car	25%
Meal Preparation	45%

Graph 3 :

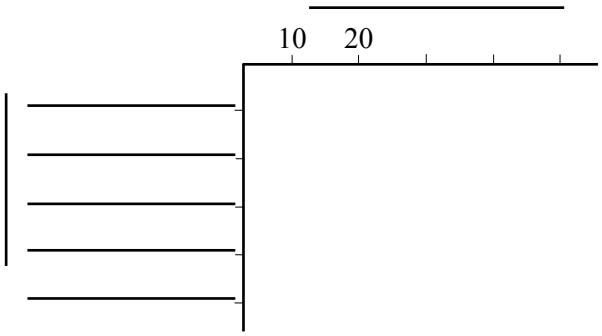
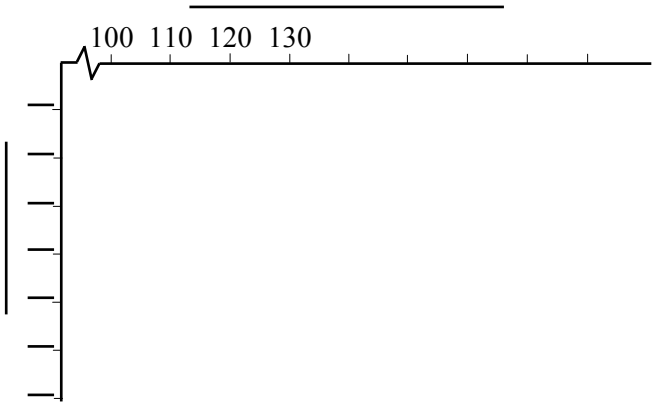


Table 2 : Timing Sunscreen Protection

Sunscreen	Effective Time (min)
A	105
B	130
C	125
D	110
E	180
F	135
G	115

Graph 4 :



Bar Graphs

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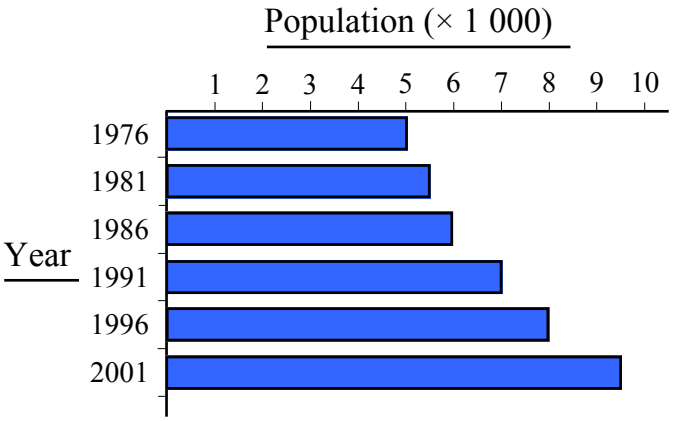
Answer these questions on the use of bar graphs

Graph 1 : Blue Whale Population Estimation



1 Give the estimated number of blue whales in:

a) 1976 5 000 *b*) 1986 6 000 *c*) 1996 8 000

2 Give the increase in whale numbers between

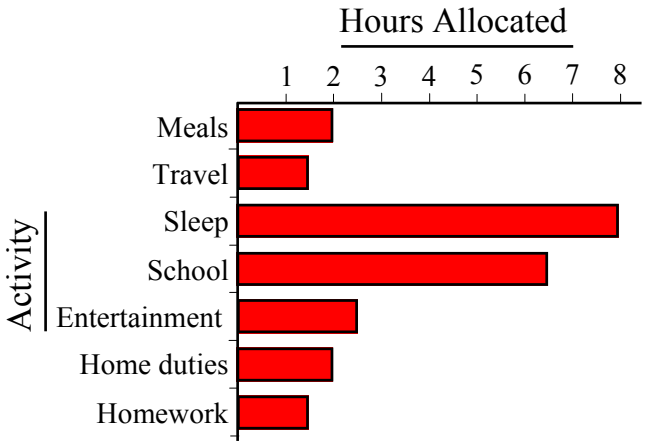
a) 1976 - 1991 2 000 *b*) 1981 - 2001 4 000

3 Between which years is there a population increase of:

a) 500 1976 - 1981, 1981 - 1986 *b*) 1 500 1996 - 2001

c) 3 000 1976 - 1996 *d*) 4 000 1981 - 2001

Graph 2 : Analysis of a School Day



4 Give the number of hours spent: *a*) Sleeping 8

b) Eating 2 *c*) At school 6.5 *d*) Travelling 1.5

5 Give the difference in time spent on these activities:

a) Entertainment and.. Homework 1 h Eating 0.5 h

b) Time at School and.. Travel 5 h Sleeping 1.5 h

c) Home duties and.. School time 4.5 h Travel 0.5 h

6 How many hours could the above student be at home each weekday 16 h

Now put this data on the bar graph below

Table 1 : Least Painful Home Duty

Home Duty	Student Approval
Mowing Grass	15%
Washing Dishes	10%
Tidy Bedroom	5%
Washing Car	25%
Meal Preparation	45%

Graph 3 : Least Painful Home Duty

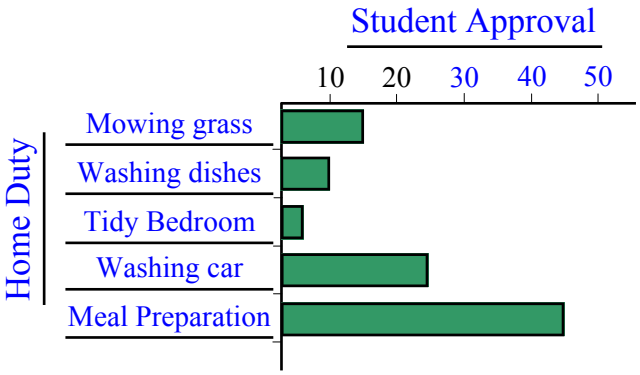
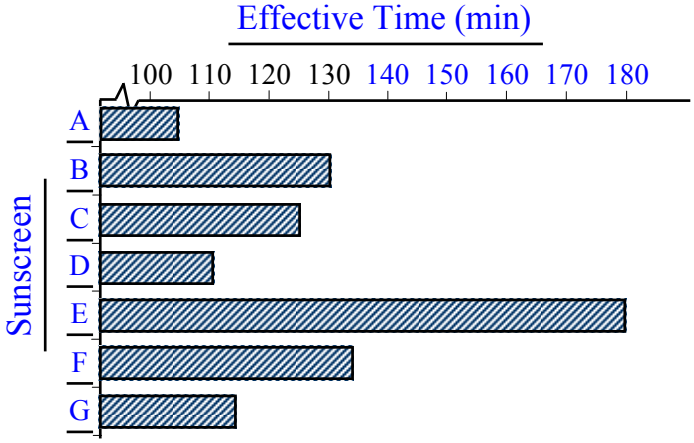


Table 2 : Timing Sunscreen Protection

Sunscreen	Effective Time (min)
A	105
B	130
C	125
D	110
E	180
F	135
G	115

Graph 4 : Timing Sunscreen Protection



Line Graphs

Line graphs show data as a continuous line, this allows for estimations to be made between data. For example if population data is assembled every 5 years by using a line graph you can estimate the population in between those years. You should realise that there are limitations to this, as the population won't change in a straight line, it could rapidly rise and fall between actual readings. Another common use of line graphs is to relate two values against each other. This relationship may be a straight line and so values at any point on the line will be correct.

Column 1 starts with a town's population being graphed. Note the scale on the vertical axis is × 10 000, so 1 = 10 000, 1.4 = 14 000 and so on. You would use a ruler to accurately read off the graph. The second graph involves converting currency, between NZ$ and A$ (or NZD and AUD).

Column 2 questions 5, 6 and 7 refer to the column 1 graph, only you are asked to find one value from the graph and then using your number skills, find other conversions without using the graph. Remember when you multiply by 10 you move the decimal point one place to the right, 100 = 2 places and so on. For division move the decimal point to the left.

Graph 3 compares US$ with both Australian and New Zealand currencies. To convert US$ to A$ use a ruler horizontally (flat) then where the ruler intersects the black line, read off the amount on the bottom scale. To convert to from US$ to NZ$ use the same method but use the red line. Note that you could use this graph to convert between A$ and NZ$, you would go up to the red or black line, then across to other line, then down.

Line Graphs

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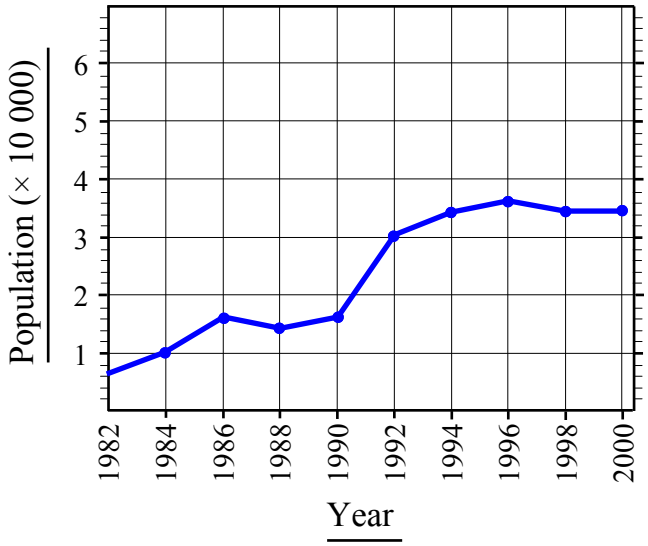
Study the line graphs below then answer the questions

Graph 1 : Population of Mathsville 1982-2000



1 Give the population in: *a*) 1988 _____

b) 1992 _____ *c*) 1996 _____ *d*) 2000 _____

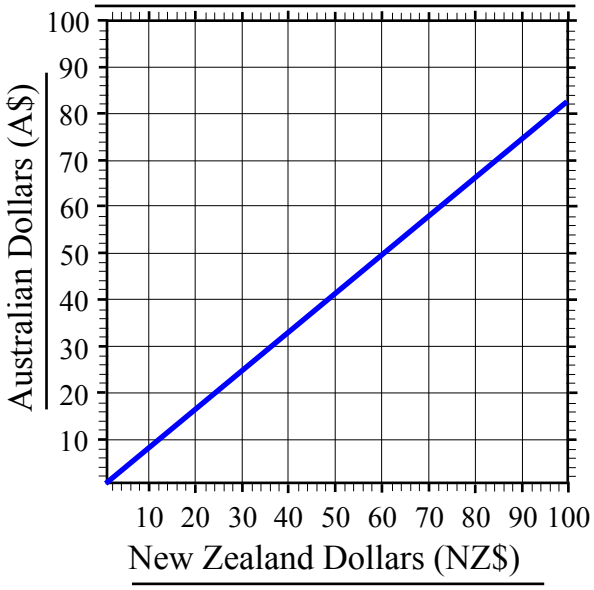
2 Estimate the population in *a*) 1985 _____

b) 1991 _____ *c*) 1995 _____ *d*) 1999 _____

3 Give the year that may have had a population of:

a) 8 000 ____ *b*) 32 000 ____ *c*) 13 000 ____

Graph 2 : NZ$ and A$ Conversion



Complete the table to the nearest $

4 Table 1 : NZ$ and A$ Conversions

NZ$	40	70		30			63			98	
A$			80		56	14		22	42		39

Convert the first figure and use it, not the graph, to find the others.

5 *a*) A$38 = NZ$ ___ *b*) A$3.80 = NZ$ ___

c) A$380 = NZ$ ___ *d*) A$3 800 = NZ$ ___

6 *a*) A$50 = NZ$ ___ *b*) A$0.50 = NZ$ ___

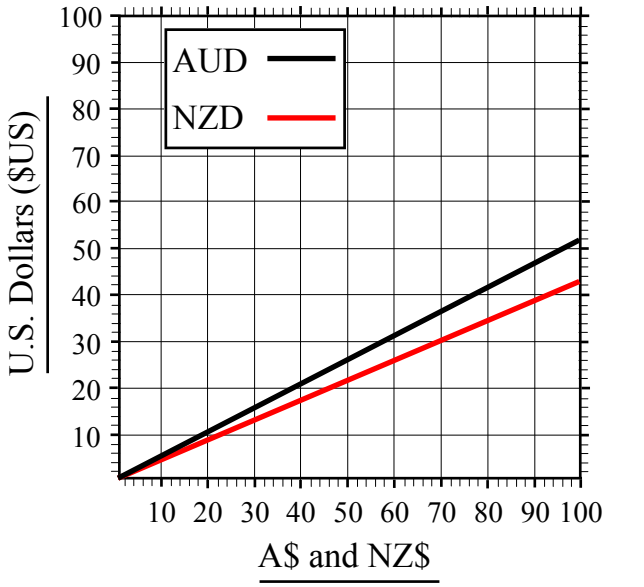
c) A$100 = NZ$ ___ *d*) A$5 000 = NZ$ ___

7 *a*) NZ$100 = A$ ___ *b*) NZ$200 = A$ ___

c) NZ$1 000 = A$ ___ *d*) NZ$1 001 = A$ ___

Use this graph to convert between US$, A$ and NZ$

Graph 3 : Converting US$ to A$ and NZ$



8 Table 2 : US$, A$ & NZ$ Conversions

US$	20	14			40				32		18
A$			80			50		60			
NZ$				60			70			80	

Use the graph for the first conversion then your number skills for the rest.

9 *a*) US$10 = NZ$ ___ *b*) US$100 = NZ$ ___

c) US$1 000 = NZ$ ___ *d*) US$1 = NZ$ ___

10 *a*) US$50 = A$ ___ *b*) US$100 = A$ ___

c) US$1 000 = A$ ___ *d*) US$5 000 = A$ ___

11 *a*) A$40 = US$ ___ *b*) A$400 = US$ ___

c) A$0.40 = US$ ___ *d*) A$400.40 = US$ ___

Line Graphs

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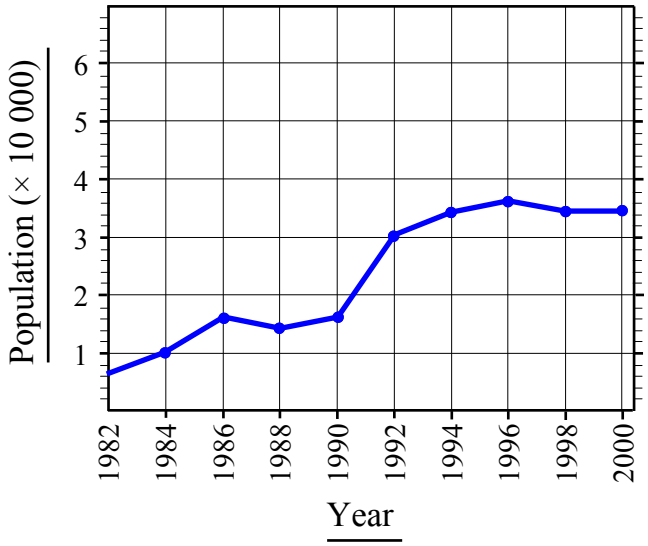
Study the line graphs below then answer the questions

Graph 1 : Population of Mathsville 1982-2000



1 Give the population in: *a*) 1988 14 000

b) 1992 30 000 *c*) 1996 36 000 *d*) 2000 35 000

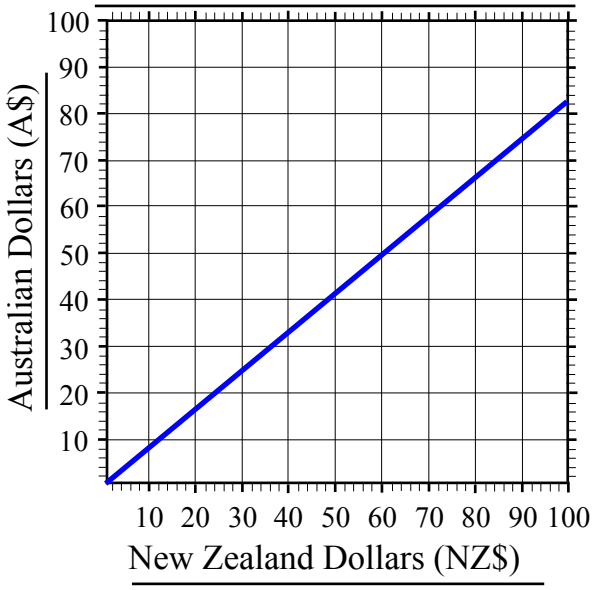
2 Estimate the population in *a*) 1985 13 000

b) 1991 24 000 *c*) 1995 35 000 *d*) 1999 35 000

3 Give the year that may have had a population of:

a) 8 000 1983 *b*) 32 000 1993 *c*) 13 000 1985

Graph 2 : NZ$ and A$ Conversion



Complete the table to the nearest $

4 Table 1 : NZ$ and A$ Conversions

NZ$	40	70	96	30	68	17	63	29	51	98	47
A$	33	58	80	25	56	14	52	22	42	81	39

Convert the first figure and use it, not the graph, to find the others.

5 *a*) A$38 = NZ$ 46 *b*) A$3.80 = NZ$ 4.60

c) A$380 = NZ$ 460 *d*) A$3 800 = NZ$ 4 600

6 *a*) A$50 = NZ$ 60 *b*) A$0.50 = NZ$ 0.60

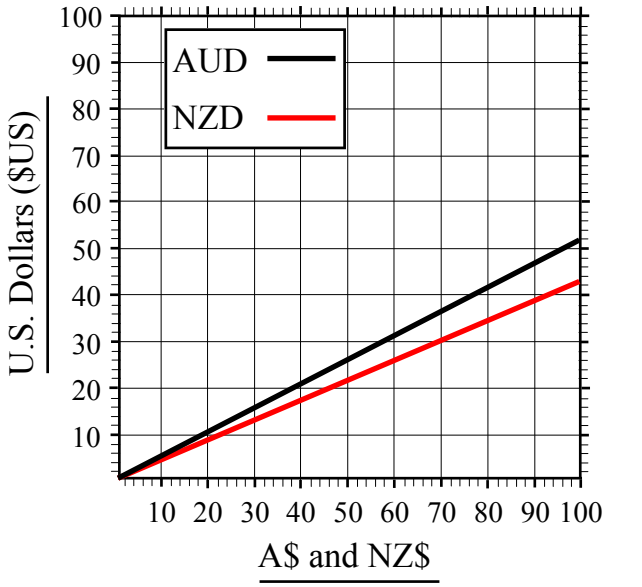
c) A$100 = NZ$ 120 *d*) A$5 000 = NZ$ 6 000

7 *a*) NZ$100 = A$ 82 *b*) NZ$200 = A$ 164

c) NZ$1 000 = A$ 820 *d*) NZ$1 001 = A$ 820.82

Use this graph to convert between US$, A$ and NZ$

Graph 3 : Converting US$ to A$ and NZ$



8 Table 2 : US$, A$ & NZ$ Conversions

US$	20	14	41	26	40	26	30	31	32	35	18
A$	38	26	80	50	77	50	58	60	61	66	34
NZ$	46	31	96	60	93	60	70	72	74	80	41

Use the graph for the first conversion then your number skills for the rest.

9 *a*) US$10 = NZ$ 22 *b*) US$100 = NZ$ 220

c) US$1 000 = NZ$ 2200 *d*) US$1 = NZ$ 2.20

10 *a*) US$50 = A$ 96 *b*) US$100 = A$ 192

c) US$1 000 = A$ 1 920 *d*) US$5 000 = A$ 9 600

11 *a*) A$40 = US$ 21 *b*) A$400 = US$ 210

c) A$0.40 = US$ 0.21 *d*) A$400.40 = US$ 210.21

Composite Graphs

Composite graphs stack different pieces of information together to form the total column (or bar). This method of displaying data allows access to the total as well as the basic parts to form the total.

Column 1 starts with a composite column graph showing a family's phone call history for the past 6 months. The calls are categorised as local calls, national calls, international calls and calls to mobile phones. All columns are rounded to the nearest $10 so that they are easier to read. Note that each part starts where the other stops, so to obtain a value for a specific part read off the amount at its base and subtract that from the amount at its top.

Another way of showing composite graphs is with just a single bar (or column) which is divided up into parts. These are usually to scale so that a ruler can be used to get information. Time lines are commonly in this format. Our graph is a time line about the construction of a hamburger in a fast food outlet. Use a ruler to measure each part and the total of the bar, note that 1 cm = 4 s (seconds). Q 5 shows employees being faster than standard time in certain aspects of burger building, replace the standard time with the new time and then give the reduced total time for the complete build. Q 6 refers to an employee who prefers a better quality of finish and is slower in certain phases of the build.

Column 2 asks you to construct a composite column graph comparing 5 different dentists in the procedure of putting a filling in a tooth. Position columns on top of the first part (the needle injection) which is in place for each column and build them up.

The same method is use for the composite bar graph, note that 1 cm = 10 animals (or 1 mm = 1 animal). You can colour the key the same colour as the blocks and you can write the names on as well.

Composite Graphs

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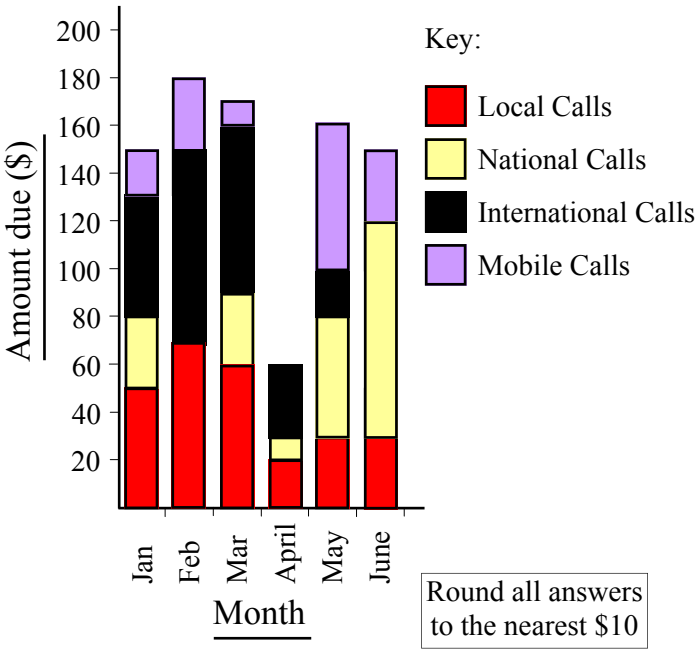
Study the composite graphs then answer the questions

Graph 1 : Forsythe Family Monthly Phone Bill



Round all answers to the nearest $10

1 Give the total amount due in:

a) Jan _____ *b*) Feb _____ *c*) May _____

2 Which month had the lowest number of calls to:

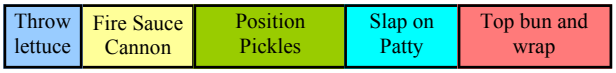
a) Mobile phones _____ *b*) Local Numbers _____

3 Give the amount spent on international calls in:

a) Jan _____ *b*) Feb _____

c) March _____ *d*) April _____

Graph 2 : Standard Times for Building a Burger



1 cm = 4 seconds

4 Give the time taken to:

a) 'Apply' lettuce _____ *b*) Build burger _____

5 If some actions can be done faster, calculate total time

a) Jackie can sauce burgers in 3 s, new total time: _____

b) Oscar can place pickles in 2 s, new total time: _____

c) Jason cannons the sauce in 5 s: new total time: _____

d) Marie slaps on a patty in 4 s, new total time: _____

6 Ryan treats burgers as art, find his total time when he takes 16 s to position the pickles and 12 s to apply sauce, all other times are unchanged _____

Construct a composite column graph for these dentists

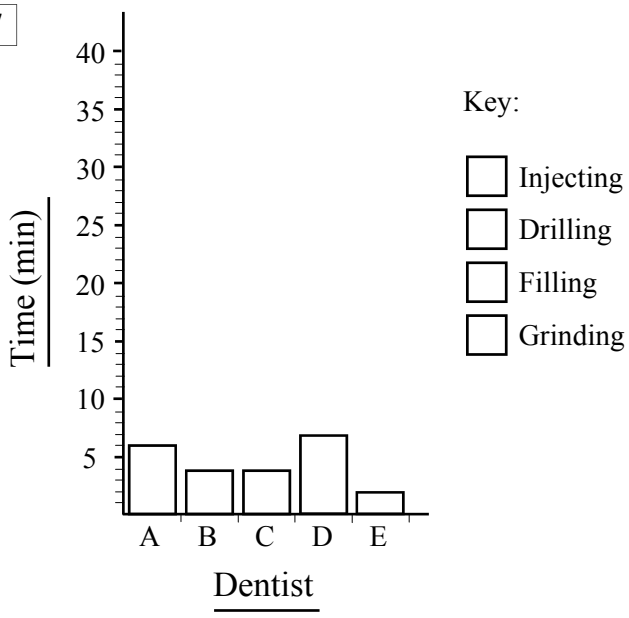
Table 1 : Time Comparison for Filling a Tooth

Action	Dentist A	Dentist B	Dentist C	Dentist D	Dentist E
Injecting	6	4	4	7	2
Drilling	13	8	12	15	14
Filling	5	7	5	4	5
Grinding	9	6	11	10	12

All times above are in min

Graph 3 : Time Comparison for Filling a Tooth

7



Construct a composite bar graph for the details in the table below

Table 2 : Treated Animals for Day

Animal	Dogs	Cats	Birds	Rabbits	Other
Number	16	25	9	7	23

8 Graph 4 : Treated Animals for Day

1 cm = 10 animals

Key:

Dogs Birds Other

Cats Rabbits

Composite Graphs

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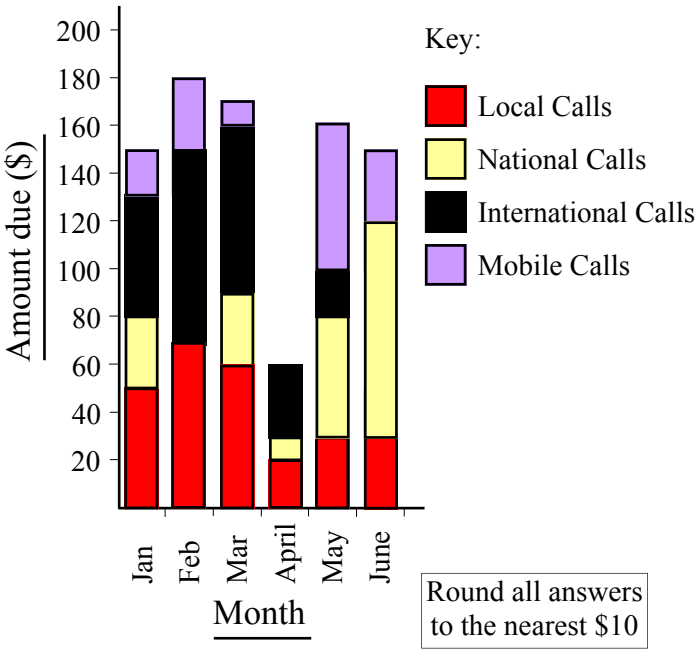
Study the composite graphs then answer the questions

Graph 1 : Forsythe Family Monthly Phone Bill



Round all answers to the nearest $10

1 Give the total amount due in:

a) Jan $150 *b*) Feb $180 *c*) May $160

2 Which month had the lowest number of calls to:

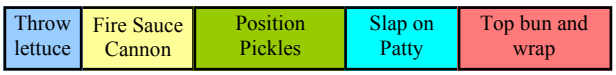
a) Mobile phones April *b*) Local Numbers April

3 Give the amount spent on international calls in:

a) Jan $50 *b*) Feb $80

c) March $70 *d*) April $30

Graph 2 : Standard Times for Building a Burger



1 cm = 4 seconds

4 Give the time taken to:

a) 'Apply' lettuce 4 s *b*) Build burger 32 s

5 If some actions can be done faster, calculate total time

a) Jackie can sauce burgers in 3 s, new total time: 29 s

b) Oscar can place pickles in 2 s, new total time: 26 s

c) Jason cannons the sauce in 5 s: new total time: 31 s

d) Marie slaps on a patty in 4 s, new total time: 30 s

6 Ryan treats burgers as art, find his total time when he takes 16 s to position the pickles and 12 s to apply sauce, all other times are unchanged 46 s

Construct a composite column graph for these dentists

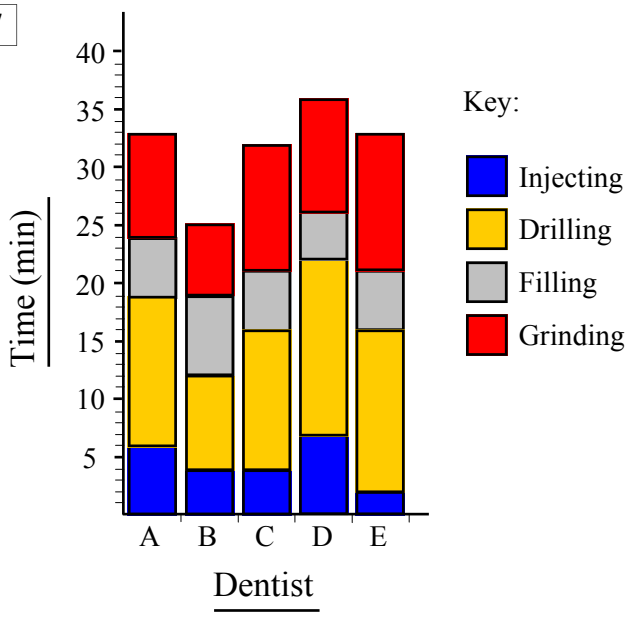
Table 1 : Time Comparison for Filling a Tooth

Action	Dentist A	Dentist B	Dentist C	Dentist D	Dentist E
Injecting	6	4	4	7	2
Drilling	13	8	12	15	14
Filling	5	7	5	4	5
Grinding	9	6	11	10	12

All times above are in min

Graph 3 : Time Comparison for Filling a Tooth

7



Construct a composite bar graph for the details in the table below

Table 2 : Treated Animals for Day

Animal	Dogs	Cats	Birds	Rabbits	Other
Number	16	25	9	7	23

8 Graph 4 : Treated Animals for Day



1 cm = 10 animals

Key:

Dogs Birds Other

Cats Rabbits

Frequency Tables

The frequency of something is how often it occurs. So a Frequency Table records how many times numbers in a data set occur. There are three columns to a frequency table, when the data is not already entered. These are the 'Score', 'Tally' and 'Frequency'. Score can be replaced with a title when the data is categorical (which means not numbers), for example 'Car Type' or 'Holiday Type', it can also just be called 'x'. The frequency column is regularly written just as 'f'. Note the use of Σf at the bottom of the frequency column. Σ means 'the sum of' so Σf means add all the column up.

Example

Data: 6, 7, 5, 4, 6, 7, 10, 5, 6, 4, 9, 6, 5, 7, 6.

Score	Tally	Frequency
4		
5		
6		
7		
8		
9		
10		
		$\Sigma f =$

STEP 1

Circle the lowest score and the highest score in the data set.

STEP 2

In the score column of the table write in the lowest score at the top, increasing down the column until the highest score.

!

Note that there is no score of '8' in the data set. The number '8' is still included in the score column anyway.

Data: 6, 7, 5, 4, 6, 7, 10, 5, 6, 4, 9, 6, 5, 7, 6.

Score	Tally	Frequency
4		
5		
6	|	
7		
8		
9		
10		
		$\Sigma f =$

STEP 3

Data is now entered number by number Cross out the 6 and put a mark in the Tally column, then the 7 and so on.

!

Don't count all the 4's then all the 5's. This method is more likely to cause mistakes in large data sets. It also doesn't effectively use the Tally process.

Score	Tally	Frequency
4	||	2
5	|||	3
6	||||	5
7	|||	3
8		0
9	|	1
10	|	1
		$\Sigma f =$ 15

The 5th score is a cross stroke

STEP 4

Now change the Tally into numbers in the Frequency column.

STEP 5

Total the Frequency column to get Σf it has to equal the number of scores in the data set. If it doesn't ….start searching!

Frequency Tables

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Place the data below in a frequency table.

Before you start
Circle the smallest number
Circle the largest number

1 Data: 0, 3, 2, 6, 5, 3, 4, 5, 1, 6, 3, 4, 2, 5, 3, 2, 0, 5, 1, 2, 3, 5, 4, 3, 2, 4, 5, 2, 1, 5, 2, 3, 2, 5, 6, 1, 4, 3, 2, 2.

Score	Tally	Frequency
0	|	
		$\Sigma f =$

Count the numbers in the data, does it equal Σf?

2 Ages of Students on an Excursion.
Data: 15, 17, 12, 17, 16, 17, 12, 18, 13, 16, 17, 15, 11, 13, 13, 14, 12, 15, 17, 17, 15, 14, 13, 15, 16, 17, 12, 16, 14, 13, 15, 17, 17, 17, 16, 12, 14.

Score	Tally	Frequency
		$\Sigma f =$

3 What was the age that occurred most? _____

4 How many students were older that 14? _____

5 How many teenage students were there? _____

6 What percentage of the students were 16 or older?

_____ = _____
Show working out Answer to 1 d.p.

Student eye colour was recorded with the data shown, Br = Brown, Bl = Blue, G = Green and H = Hazel. Tabulate the results.

7 Data: Br, Br, Br, G, Bl, Br, Br, Br, Bl, H, H, Br, Br, Br, Br, Bl, H, Bl, Br, Br, H, Bl, G, Br, Br, Br, H, H, Bl.

8 What was most common eye colour? _____

9 How many students had hazel eyes? _____

10 What percentage of the students had blue eyes?

_____ = _____
Answer to 1 d.p.

11 What percentage of the students had either brown or hazel eyes?

_____ = _____
Answer to 1 d.p.

12 Place the spelling test results in the table.
Data: 9, 6, 8, 3, 6, 8, 10, 8, 5, 6, 9, 7, 9, 8, 7, 3, 8, 10, 6, 9, 8, 7, 9, 5, 6, 9, 8, 8, 8, 7, 5, 9, 6, 3, 7, 9, 6, 10, 8, 7, 10, 6, 5, 5, 3, 7, 9, 9, 10, 5.

Frequency Tables

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Place the data below in a frequency table.

Before you start
Circle the smallest number
Circle the largest number

1 Data: 0, 3, 2, 6, 5, 3, 4, 5, 1, 6, 3, 4, 2, 5, 3, 2, 0, 5, 1, 2, 3, 5, 4, 3, 2, 4, 5, 2, 1, 5, 2, 3, 2, 5, 6, 1, 4, 3, 2, 2.

40 scores

Score	Tally	Frequency
0	||	2
1	||||	4
2	𝍸 |||	10
3	𝍸 |||	8
4	𝍸	5
5	𝍸 |||	8
6	|||	3
		$\Sigma f = 40$

check

Count the numbers in the data, does it equal Σf?

2 Ages of Students on an Excursion.

Data: 15, 17, 12, 17, 16, 17, 12, 18, 13, 16, 17, 15, 11, 13, 13, 14, 12, 15, 17, 17, 15, 14, 13, 15, 16, 17, 12, 16, 14, 13, 15, 17, 17, 17, 16, 12, 14.

Ages of Students on an Excursion

Score	Tally	Frequency
11	|	1
12	𝍸	5
13	𝍸	5
14	||||	4
15	𝍸 |	6
16	𝍸	5
17	𝍸 𝍸	10
18	|	1
		$\Sigma f = 37$

3 What was the age that occurred most? 17

4 How many students were older that 14? 22

5 How many teenage students were there? 31

6 What percentage of the students were 16 or older?

$\frac{16}{37} \times 100$ = 43.2%

Show working out | Answer to 1 d.p.

Student eye colour was recorded with the data shown, Br = Brown, Bl = Blue, G = Green and H = Hazel. Tabulate the results.

7 Data: Br, Br, Br, G, Bl, Br, Br, Br, Bl, H, H, Br, Br, Br, Br, Bl, H, Bl, Br, Br, H, Bl, G, Br, Br, Br, H, H, Bl.

Student Eye Colour

Colour	Tally	Frequency
Brown	𝍸 𝍸 𝍸	15
Blue	𝍸 |	6
Green	||	2
Hazel	𝍸 |	6
		$\Sigma f = 29$

8 What was most common eye colour? Brown

9 How many students had hazel eyes? 6

10 What percentage of the students had blue eyes?

$\frac{6}{29} \times 100$ = 20.7%

Answer to 1 d.p.

11 What percentage of the students had either brown or hazel eyes?

$\frac{21}{29} \times 100$ = 72.4%

Answer to 1 d.p.

12 Place the spelling test results in the table.

Data: 9, 6, 8, 3, 6, 8, 10, 8, 5, 6, 9, 7, 9, 8, 7, 3, 8, 10, 6, 9, 8, 7, 9, 5, 6, 9, 8, 8, 8, 7, 5, 9, 6, 3, 7, 9, 6, 10, 8, 7, 10, 6, 5, 5, 3, 7, 9, 9, 10, 5.

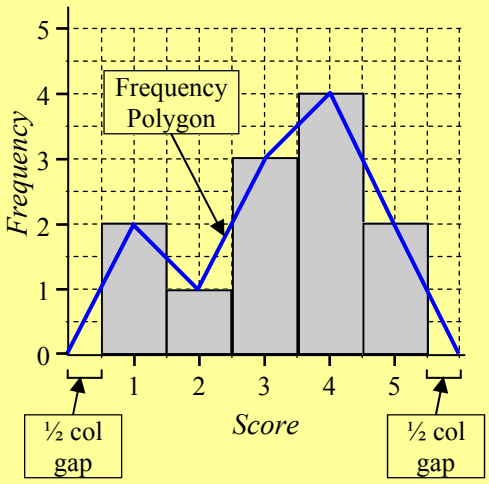
Spelling Test results

Score	Tally	Frequency
3	||||	4
4		0
5	𝍸 |	6
6	𝍸 |||	8
7	𝍸 ||	7
8	𝍸 𝍸	10
9	𝍸 𝍸	10
10	𝍸	5
		$\Sigma f = 50$

Frequency Histograms and Frequency Polygons

Frequency Histograms are very similar to column graphs, the difference being that:

- A half-column gap is left between the first column and the vertical axis.
- The columns touch each other, there is no gap between them.
- A half-column gap is left after the last column.



Frequency Polygons are similar to line graphs, they:

- Always start and finish at the horizontal axis.
- They inter-connect the centres of the top of columns.

Because a half-column gap is required, it makes sense to use columns that are '2 grids' wide so that the grids help you to draw the graph. You shouldn't make a column '1 grid wide' and the gap '½ a grid wide' because your columns won't use the grid lines, it will be harder to create and use.

Column 1 starts with transferring the data from a Frequency table to the graph. This should be straight forward. Just remember the key points. Columns touching, polygon inter-connecting the centre of the top of each column and of course the half-column gap.

Column 2 adds a step. A data set is given and a Frequency table has to be constructed first. More has to be done with the graph, a title, labeling the axes, a scale on the axes is also required.

Frequency Histograms and Frequency Polygons

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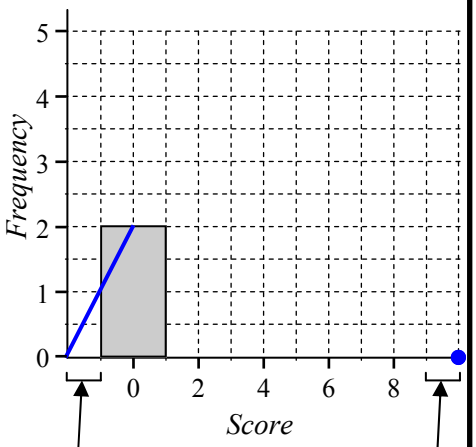
Use a ruler to construct a Frequency Histogram and Polygon for the given data.

1

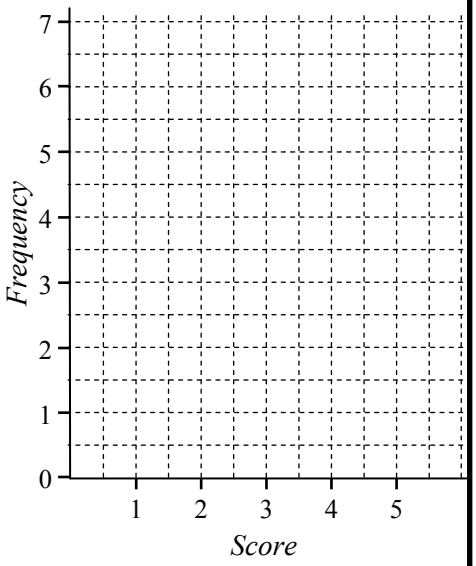
Score	*Frequency*
0	2
2	3
4	5
6	1
8	4



Columns touch each other - no gaps between
There is a ½ column gap at the start and at the finish of the columns.
Polygon ends at the dot.

2

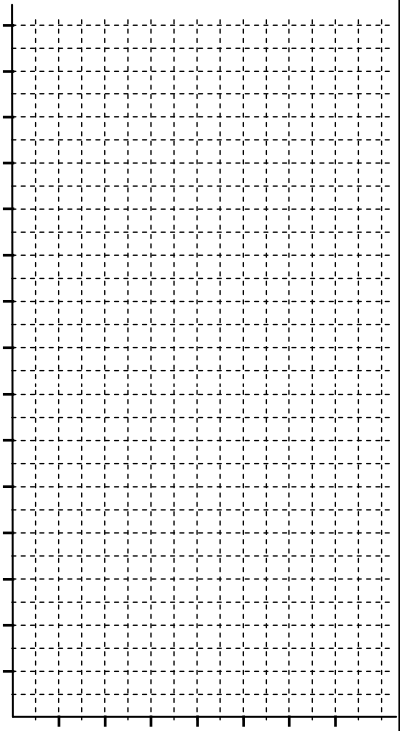
Score	*Frequency*
1	4
2	2
3	7
4	0
5	6



Given a data set, place the data into a Frequency Table then construct a Frequency Histogram and Frequency Polygon.

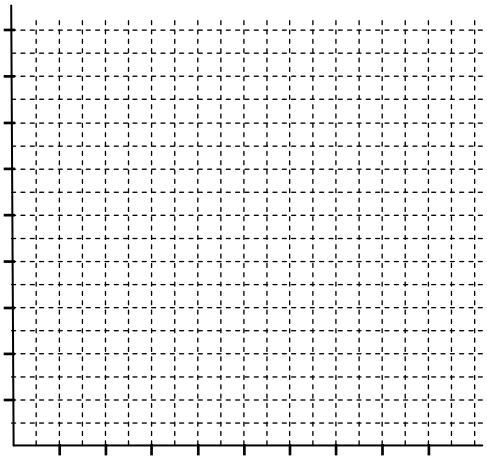
3 Family Size

Data: 4, 3, 4, 2, 3, 7, 4, 5, 7, 6, 5, 3, 4, 8, 5, 4, 5, 3, 4, 3, 5, 4, 4, 2, 8, 6, 4, 4, 3, 5, 4, 4, 5, 6, 3, 4, 4, 3, 4.



4 100 m Sprint times (*s*)

Data:
17, 19, 22, 16, 18, 19, 15, 21, 17, 22, 18, 16, 22, 17, 18, 20, 17, 19, 22, 16, 23, 18, 21, 19, 15, 17, 17, 19, 22, 21, 15, 17, 16, 16, 20, 23, 19, 19, 17, 18.



Frequency Histograms and Frequency Polygons

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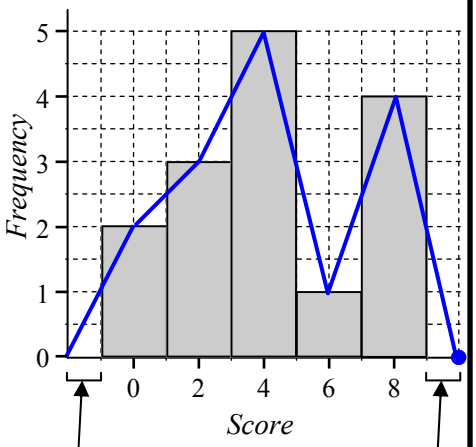
Use a ruler to construct a Frequency Histogram and Polygon for the given data.

1

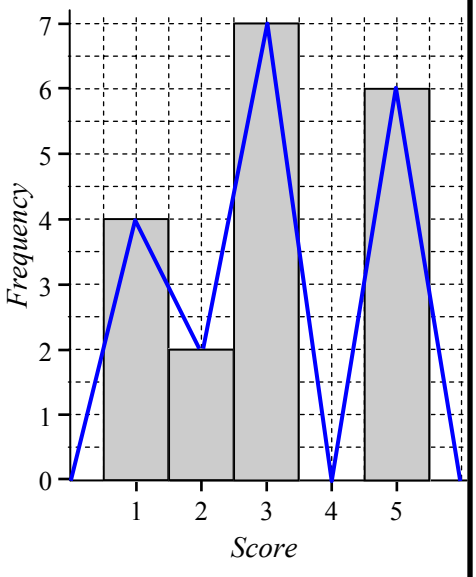
Score	*Frequency*
0	2
2	3
4	5
6	1
8	4



Columns touch each other - no gaps between
There is a ½ column gap at the start and at the finish of the columns.
Polygon ends at the dot.

2

Score	*Frequency*
1	4
2	2
3	7
4	0
5	6



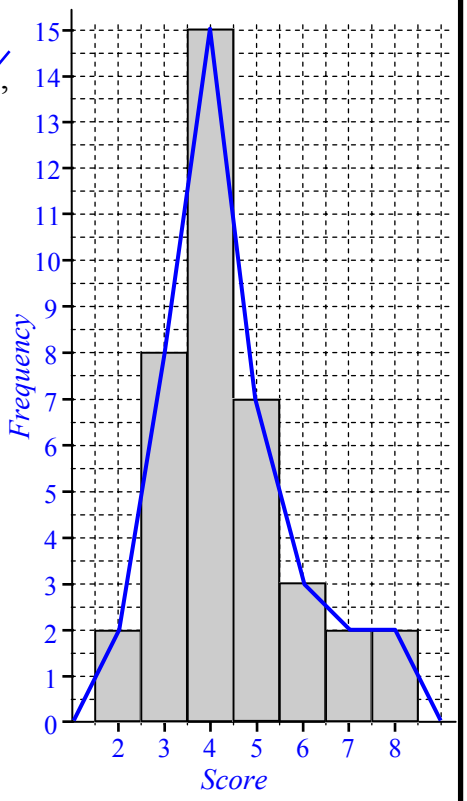
Given a data set, place the data into a Frequency Table then construct a Frequency Histogram and Frequency Polygon.

3 Family Size

Data: 4, 3, 4, 2, 3, 7, 4, 5, 7, 6, 5, 3, 4, 8, 5, 4, 5, 3, 4, 3, 5, 4, 4, 2, 8, 6, 4, 4, 3, 5, 4, 4, 5, 6, 3, 4, 4, 3, 4.

Family Size

Size	*Tally*	*Frequency*
2	||	2
3	𝍸 |||	8
4	𝍸 𝍸 𝍸	15
5	𝍸 ||	7
6	|||	3
7	||	2
8	||	2
		$\Sigma f = 39$



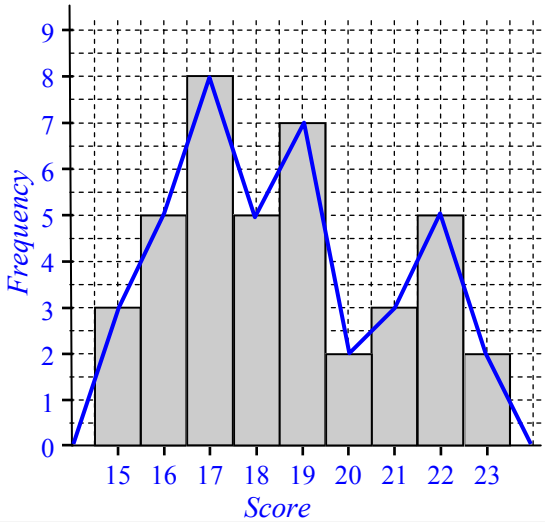
4 100 m Sprint times (s)

Data:
17, 19, 22, 16, 18, 19, 15, 21, 17, 22, 18, 16, 22, 17, 18, 20, 17, 19, 22, 16, 23, 18, 21, 19, 15, 17, 17, 19, 22, 21, 15, 17, 16, 16, 20, 23, 19, 19, 17, 18.

100 m Sprint Times (s)

Time (s)	*Tally*	*Frequency*
15	|||	3
16	𝍸	5
17	𝍸 |||	8
18	𝍸	5
19	𝍸 ||	7
20	||	2
21	|||	3
22	𝍸	5
23	||	2
		$\Sigma f = 40$

100 m Sprint Times (s)



Frequency Tables with Classes

A frequency table is used when there is high repetition of a certain number of scores. Such as with a die, where there are only 6 outcomes. But when there are a large range of scores then the data is best grouped together in classes.

The process is essentially the same as a frequency table, using a tally column and then changing the tally to a number in the frequency column. The only difference is that the 'Score' column is replaced with the word 'Classes' which cover a range of scores rather than just a single score.

Example. 'BarknFlea' Dog Show - Points listing

Data: ~~15~~, ~~26~~, 11, 31, 27, 44, 28, 31, 46, 23, 5,

Cross out the first score and place a stroke in the tally column, 15 is in the 10-19 class.

Repeat for 26, which is in the 20-29 class.

Class	Tally	Frequency
0 - 9		
10 - 19	|	
20 - 29	|	
30 - 39		
40 - 49		
		Σf =

Data: ~~15~~, ~~26~~, ~~11~~, ~~31~~, ~~27~~, ~~44~~, ~~28~~, ~~31~~, ~~46~~, ~~23~~, ~~5~~,

Place all scores into the tally column then change the strokes to numbers in the frequency column.

Add the frequency column up to find Σf and check that the number of scores is the same as Σf.

You are done!

Class	Tally	Frequency
0 - 9	|	1
10 - 19	||	2
20 - 29	||||	4
30 - 39	||	2
40 - 49	||	2
		$\Sigma f = 11$

11 scores correct

Count the number of scores in the data, does it equal Σf?

Column 2 is the same as Column 1 except you have to draw up the table and specify the class intervals yourself. Make sure that the class intervals are all the same size.

Frequency Tables with Classes

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Place the data below in the frequency table, using the suggested class intervals.

1 Data: 25, 16, 32, 41, 28, 7, 11, 36, 27, 20, 34, 43, 19, 31, 7, 48, 9, 20, 5, 35, 22, 10, 33, 3, 21, 24, 17, 28, 2, 36, 32, 12, 26, 43, 7, 21, 7, 5.

Class	Tally	Frequency
0 - 9		
10 - 19		
20 - 29		
30 - 39		
40 - 49		
		$\Sigma f =$

Count the number of scores in the data, does it equal Σf?

2 'Appleshot' Archery Club Championship

Points: 80, 72, 83, 67, 94, 92, 85, 73, 79, 90, 65, 77, 97, 93, 89, 65, 85, 71, 62, 83, 79, 64, 92, 70, 62, 84, 75, 81, 98, 75, 73, 63, 84, 91, 99, 98, 76, 78, 85, 96, 70, 74, 76, 62, 68, 83, 95, 100, 72, 81.

Points	Tally	Frequency
61 - 65		
66 - 70		
71 - 75		
76 - 80		
81 - 85		
86 - 90		
91 - 95		
96 - 100		

3 How many archers were there? _____

4 How many archers scored 75 or less? _____

5 What was the most popular class? _____

6 What percentage of the archers scored more than 90?

_____ = _____

Show working out

7 What percentage scored between 65 and 81?

_____ = _____

Now use your own classes, you have to distribute the data, so use at least 5 classes.

8 Data: 6, 15, 20, 11, 19, 22, 18, 2, 11, 10, 18, 24, 16, 23, 2, 18, 9, 14, 28, 9, 14, 13, 25, 14, 17, 29, 4, 15, 1, 23, 19, 12, 17, 25, 10, 14, 19, 24, 16.

Class	Tally	Frequency

Don't forget Σf

9 Student Height (cm)

Data: 155, 172, 163, 158, 168, 174, 183, 165, 180, 161, 174, 168, 170, 181, 163, 162, 171, 155, 183, 175, 169, 149, 170, 162, 181, 172, 176, 162, 158, 166, 157, 184, 175, 166, 163, 159, 172, 170.

Class	Tally	Frequency

Frequency Tables with Classes

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Place the data below in the frequency table, using the suggested class intervals.

1 Data: 25, 16, 32, 41, 28, 7, 11, 36, 27, 20, 34, 43, 19, 31, 7, 48, 9, 20, 5, 35, 22, 10, 33, 3, 21, 24, 17, 28, 2, 36, 32, 12, 26, 43, 7, 21, 7, 5.

Class	Tally	Frequency
0 - 9	𝍸 ||||	9
10 - 19	𝍸 |	6
20 - 29	𝍸 𝍸 |	11
30 - 39	𝍸 |||	8
40 - 49	||||	4
		Σf = 38

Count the number of scores in the data, does it equal Σf?

2 'Appleshot' Archery Club Championship

Points: 80, 72, 83, 67, 94, 92, 85, 73, 79, 90, 65, 77, 97, 93, 89, 65, 85, 71, 62, 83, 79, 64, 92, 70, 62, 84, 75, 81, 98, 75, 73, 63, 84, 91, 99, 98, 76, 78, 85, 96, 70, 74, 76, 62, 68, 83, 95, 100, 72, 81.

Points	Tally	Frequency
61 - 65	𝍸 ||	7
66 - 70	||||	4
71 - 75	𝍸 |||	8
76 - 80	𝍸 ||	7
81 - 85	𝍸 𝍸	10
86 - 90	||	2
91 - 95	𝍸 |	6
96 - 100	𝍸 |	6
		Σf = 50

3 How many archers were there? 50

4 How many archers scored 75 or less? 19

5 What was the most popular class? 81 - 85

6 What percentage of the archers scored more than 90? $\frac{12}{50} \times 100$ = 24%

7 What percentage scored between 65 and 81? $\frac{19}{50} \times 100$ = 38%

Now use your own classes, you have to distribute the data, so use at least 5 classes.

8 Data: 6, 15, 20, 11, 19, 22, 18, 2, 11, 10, 18, 24, 16, 23, 2, 18, 9, 14, 28, 9, 14, 13, 25, 14, 17, 29, 4, 15, 1, 23, 19, 12, 17, 25, 10, 14, 19, 24, 16.

Class	Tally	Frequency
1 - 5	||||	4
6 - 10	𝍸	5
11 - 15	𝍸 𝍸	10
16 - 20	𝍸 𝍸 |	11
21 - 25	𝍸 ||	7
26 - 30	||	2
		Σf = 39

Don't forget Σf

9 Student Height (cm)

Data: 155, 172, 163, 158, 168, 174, 183, 165, 180, 161, 174, 168, 170, 181, 163, 162, 171, 155, 183, 175, 169, 149, 170, 162, 181, 172, 176, 162, 158, 166, 157, 184, 175, 166, 163, 159, 172, 170.

Class	Tally	Frequency
145 - 149	|	1
150 - 154		0
155 - 159	𝍸 |	6
160 - 164	𝍸 ||	7
165 - 169	𝍸 |	6
170 - 174	𝍸 ||||	9
175 - 179	|||	3
180 - 184	𝍸 |	6
		Σf = 38

Frequency Histograms and Polygons with Classes

The display of data in a Histogram/Polygon with classes uses exactly the same method as without classes, with one difference. Instead of displaying the score across the horizontal axis, you display the class centre.

The class centre is found by averaging the two scores shown in the class.

Example

$\frac{20+1}{2} = 10.5$

$\frac{21+40}{2} = 30.5$

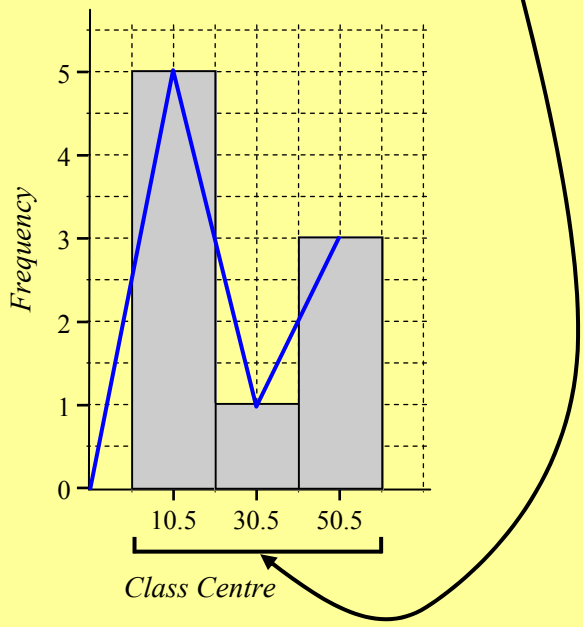
$\frac{41+60}{2} = 50.5$

c.c. is short for class centre

Class	*c.c.*	Frequency
1 - 20	10.5	5
21 - 40	30.5	1
41 - 60	50.5	3

Remember that when you divide the sum of the two numbers, press '=' before you '÷2' or you will get the wrong answer, due to order of operations principles.

Plot the data using a histogram and polygon, the only difference being that the class centres are used.



Frequency Histograms and Polygons with Classes

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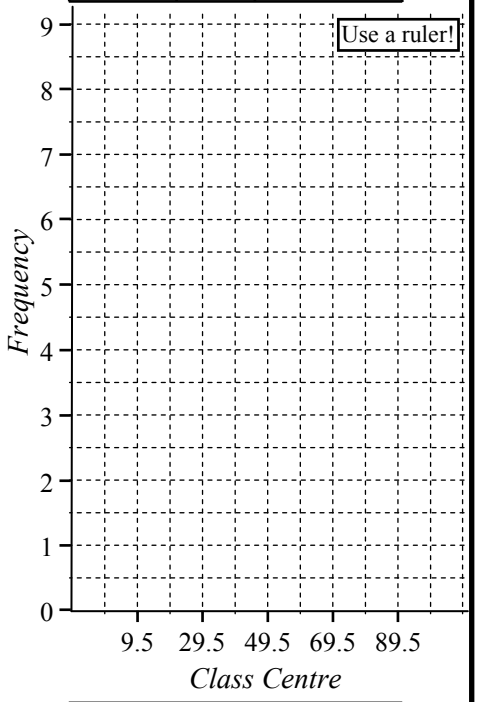
Display the data below using a frequency histogram and polygon

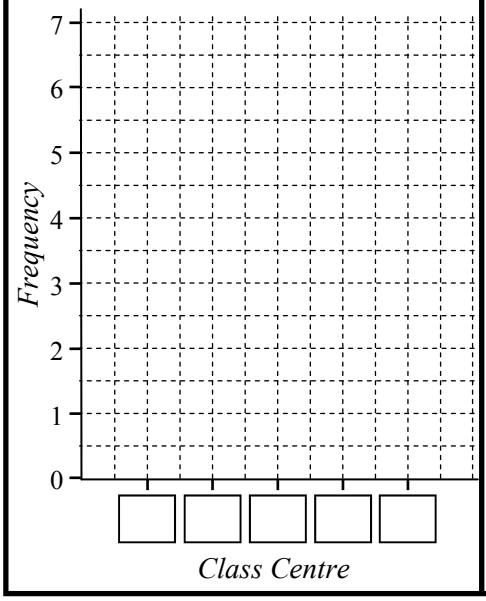
1

Class	*c.c.*	Frequency
0 - 19	9.5	6
20 - 39	29.5	9
40 - 59	49.5	3
60 - 79	69.5	5
80 - 99	89.5	2



2

Class	*c.c.*	Frequency
1 - 5		4
6 - 10		7
11 - 15		0
16 - 20		2
21 - 25		5

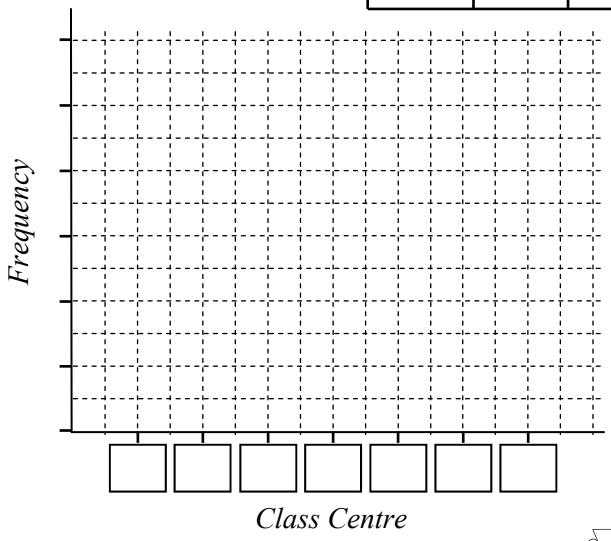


Place the data in the Frequency Table using classes 0 - 9, 10 -19 and so on. Then show the data as a Frequency Histogram and Polygon.

3 Data: 63, 9, 33, 43, 9, 63, 10, 67, 54, 3, 68, 16, 56, 13, 21, 15, 4, 50, 58, 8, 4, 40, 48, 17, 65, 59, 10, 44, 50, 23, 26, 25, 37, 31.

Class	*c.c.*	Tally	*f*
0 - 9			
10 - 19			
			$\Sigma f =$

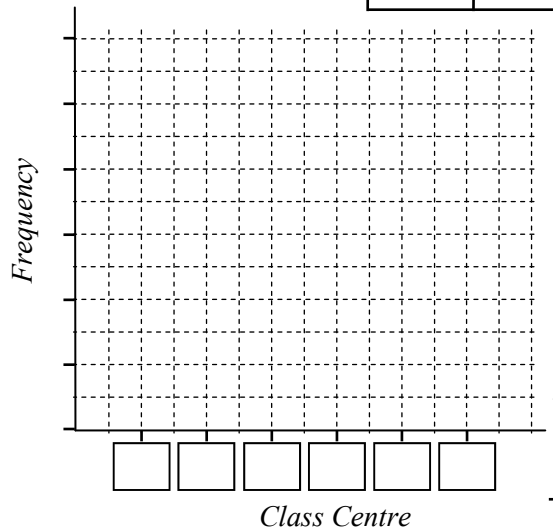
Count the number of scores in the data, does it equal Σf?



Repeat the process except this time use classes 1 - 5, 6 - 10, 11 - 15 and so on.

4 Data: 15, 30, 10, 25, 1, 30, 6, 10, 2, 21, 3, 12, 14, 27, 28, 6, 11, 16, 9, 15, 21, 17, 15, 20, 25.

Class	*c.c.*	Tally	*f*
			$\Sigma f =$



5 How many scores are larger than 15? _____

6 How many scores between 5 and 21? _____

7 What percentage of the scores were 20 or less?

_____

Frequency Histograms and Polygons with Classes

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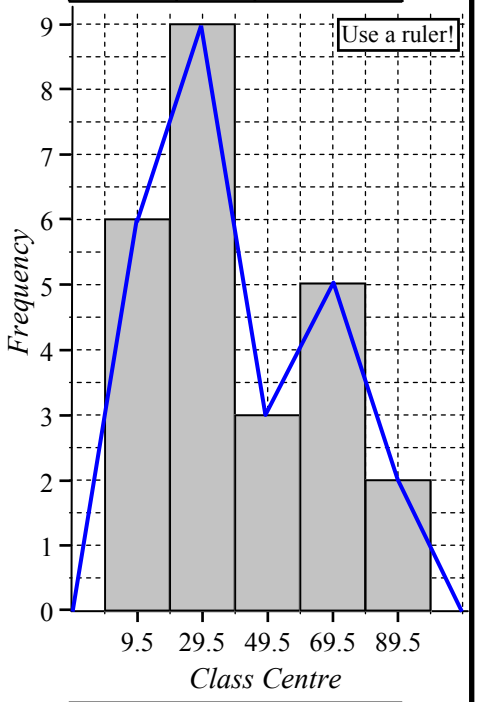
Display the data below using a frequency histogram and polygon

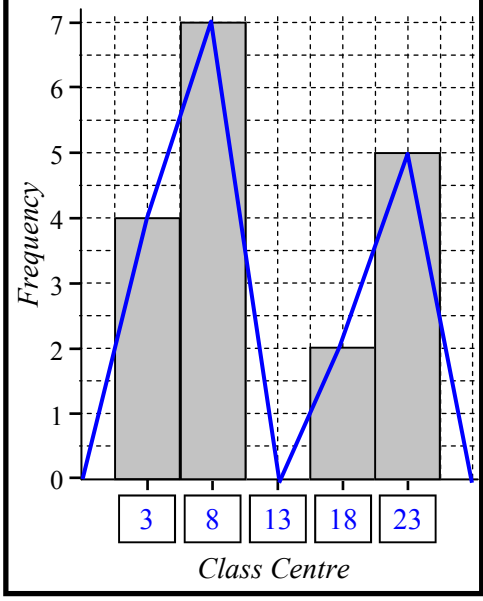
1

Class	*c.c.*	Frequency
0 - 19	9.5	6
20 - 39	29.5	9
40 - 59	49.5	3
60 - 79	69.5	5
80 - 99	89.5	2



2

Class	*c.c.*	Frequency
1 - 5	3	4
6 - 10	8	7
11 - 15	13	0
16 - 20	18	2
21 - 25	23	5

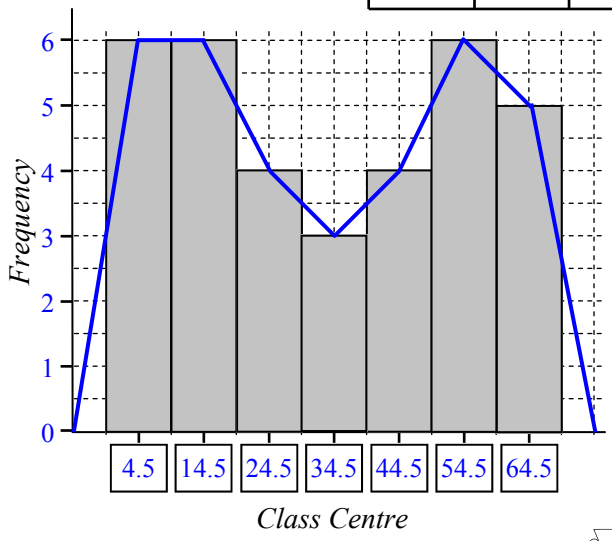


Place the data in the Frequency Table using classes 0 - 9, 10 -19 and so on. Then show the data as a Frequency Histogram and Polygon.

3 Data: 63, 9, 33, 43, 9, 63, 10, 67, 54, 3, 68, 16, 56, 13, 21, 15, 4, 50, 58, 8, 4, 40, 48, 17, 65, 59, 10, 44, 50, 23, 26, 25, 37, 31.

Class	*c.c.*	Tally	*f*
0 - 9	4.5	𝍸 |	6
10 - 19	14.5	𝍸 |	6
20 - 29	24.5	𝍸	4
30 - 39	34.5	|||	3
40 - 49	44.5	𝍸	4
50 - 59	54.5	𝍸 |	6
60 - 69	64.5	𝍸	5
			$\Sigma f = 34$

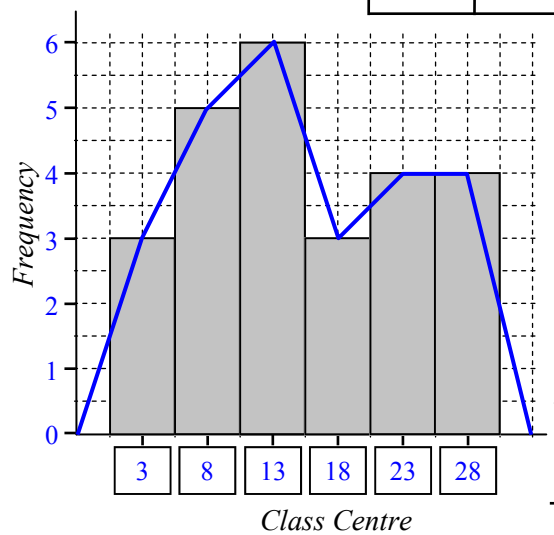
Count the number of scores in the data, does it equal Σf?



Repeat the process except this time use classes 1 - 5, 6 - 10, 11 - 15 and so on.

4 Data: 15, 30, 10, 25, 1, 30, 6, 10, 2, 21, 3, 12, 14, 27, 28, 6, 11, 16, 9, 15, 21, 17, 15, 20, 25.

Class	*c.c.*	Tally	*f*
1 - 5	3	|||	3
6 - 10	8	𝍸	5
11 - 15	13	𝍸 |	6
16 - 20	18	|||	3
21 - 25	23	||||	4
26 - 30	28	||||	4
			$\Sigma f = 25$



5 How many scores are larger than 15? 11

6 How many scores between 5 and 21? 14

7 What percentage of the scores were 20 or less?

$\frac{17}{25} \times 100 = 68\%$

Dot Plots - Frequency Tables

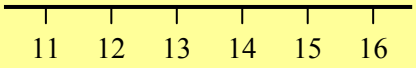
Dot Plots represent data with dots! Amazing! The beauty of a dot plot is it is simple. There is one axis which shows the score, or category and then dots are stacked above them. Dot plots can also be plotted sideways just like a column graph. Two critical features are:

- The dots have to have the same height, for example a column of four dots shouldn't be as high as a column of five dots, so be careful as you plot them.
- If you have a vertical scale then use it effectively. Because dot plots are supposed to be fast to read, a vertical scale can be included so that you don't have to count all the dots in a column.

Example

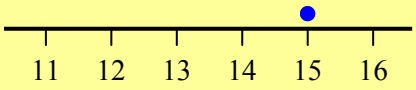
Data: 15, 12, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.

STEP 1 Circle the lowest score and the highest score in the data set.



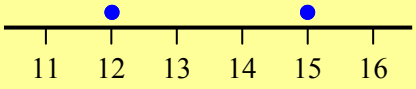
STEP 2 Number the axis from lowest to highest, note there is no 14 score, but it is still included on the axis.

Data: ~~15~~, 12, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.



STEP 3 Cross out the first score and place the dot above the number on the axis.

Data: ~~15~~, ~~12~~, 13, 11, 16, 15, 16, 12, 12, 15,
11, 13, 12, 11, 15, 16, 12, 13, 15, 11.



STEP 4 And so on until all the scores are represented with dots

Dot Plots

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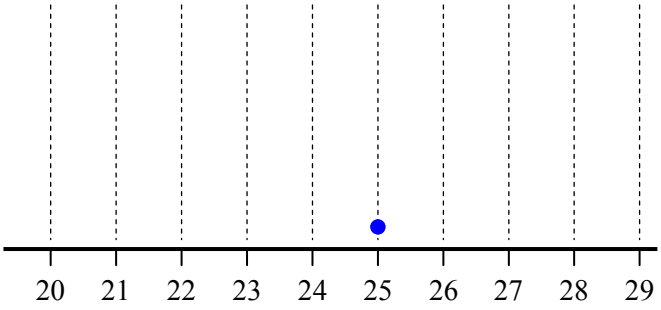
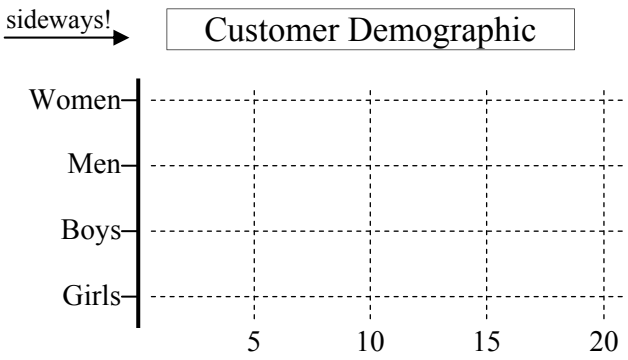
Display the data in a dot plot answering the questions from the plot.

1 Data: 25, 28, 23, 24, 23, 24, 27, 26, 25, 25, 22, 22, 20, 29, 26, 25, 27, 26, 28, 21, 25, 25, 27, 21, 25, 27, 26, 27, 22, 26, 23, 20, 22, 28.



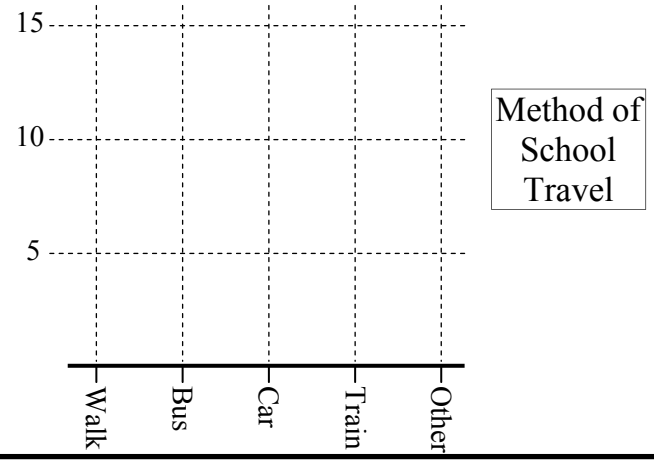
2 Women, Men, Boys and Girls entering a store have been recorded. Display on a dot plot.

Data:
MBGWB GGBWW BBWWW
MMGWB MWGBW GBWBB MMWGW
BGGBW WGMWB MWWGB WWGBM.



3 A student survey recorded five different methods of travel to school, they are: Walking, Bus, Car and Train, Other. Display on a dot plot.

Data:
TBWWO BCCTC BCTTT
WWCBB TCBBT TBCWO
BCWOW TWBTT OCTBC
CCBCO TTBCO TTBBB



4 How many students were surveyed? _____

5 What percentage of students walked to school?

_____ = _____
Show working out Answer to 1 d.p.

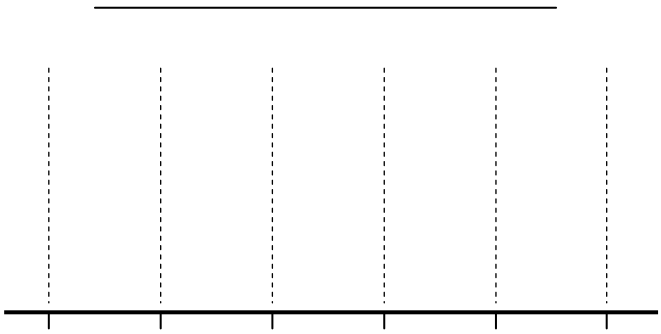
6 What percentage of students used public transport?

_____ = _____
Answer to 1 d.p.



A die was rolled with the results shown below. Display the data as a dot plot, then in a frequency table.

7 Data: 3, 1, 5, 6, 3, 4, 2, 2, 1, 5, 3, 6, 2, 3, 2, 5, 5, 6, 4, 5, 2, 3, 6, 1, 1, 6, 5, 4, 1, 6, 2, 2, 4, 5.



8

Score	Frequency
	$\Sigma f =$

9 How many times was a number less than 3 rolled? _____

10 How many times was an even number rolled? _____

11 How many times was a prime number rolled? _____

12 What percentage of die rolls resulted in a multiple of 3?

_____ = _____
Answer to 1 d.p.

13 What percentage of die rolls resulted in a factor of 12?

_____ = _____
Answer to 1 d.p.

Dot Plots

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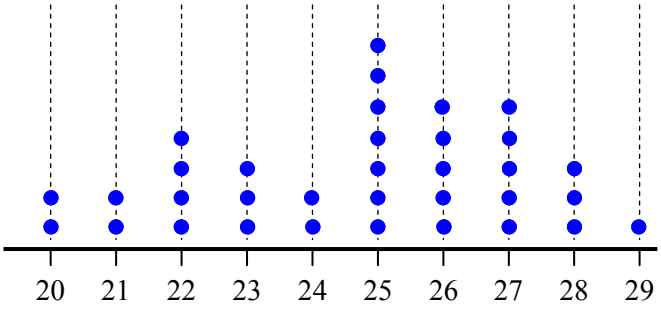
Display the data in a dot plot answering the questions from the plot.

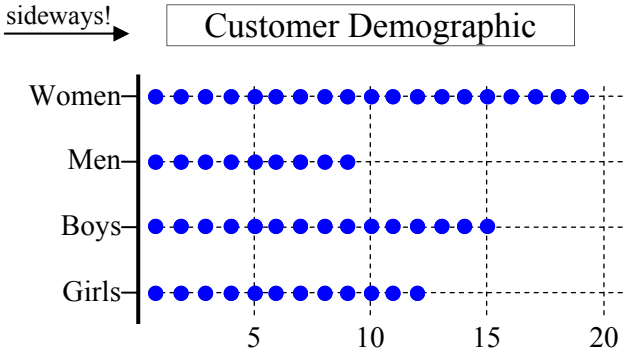
1 Data: 25, 28, 23, 24, 23, 24, 27, 26, 25, 25, 22, 22, 20, 29, 26, 25, 27, 26, 28, 21, 25, 25, 27, 21, 25, 27, 26, 27, 22, 26, 23, 20, 22, 28.



2 Women, Men, Boys and Girls entering a store have been recorded. Display on a dot plot.

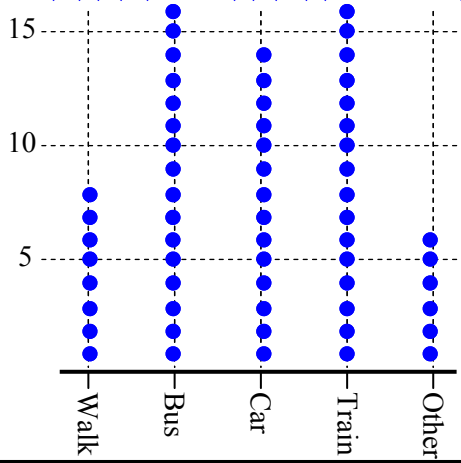
Data:
MBGWB GGBWW BBWWW
MMGWB MWGBW GBWBB MMWGW
BGGBW WGMWB MWWGB WWGBM.

sideways!



3 A student survey recorded five different methods of travel to school, they are: Walking, Bus, Car and Train, Other. Display on a dot plot.

Data:
TBWWO WWCBB BCWOW CCBCO
BCCTC TCBBT TWBTT TTBCO
BCTTT TBCWO OCTBC TTBBB



Method of School Travel

4 How many students were surveyed? 60

5 What percentage of students walked to school?

$\frac{8}{60} \times 100 = 13.3\%$

Answer to 1 d.p.

6 What percentage of students used public transport?

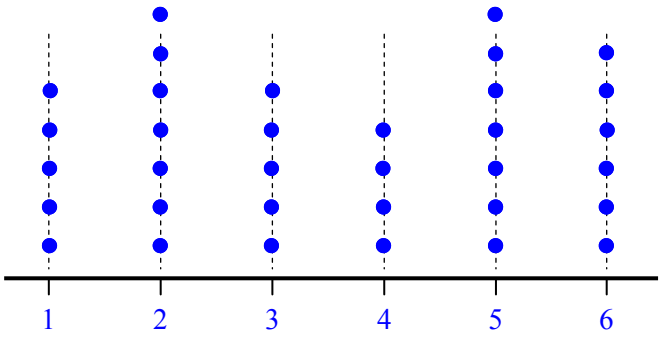
$\frac{32}{60} \times 100 = 53.3\%$

Answer to 1 d.p.

A die was rolled with the results shown below. Display the data as a dot plot, then in a frequency table.

7 Data: 3, 1, 5, 6, 3, 4, 2, 2, 1, 5, 3, 6, 2, 3, 2, 5, 5, 6, 4, 5, 2, 3, 6, 1, 1, 6, 5, 4, 1, 6, 2, 2, 4, 5.

Dot Plot of Die Rolling Results



8

Score	Frequency
1	5
2	7
3	5
4	4
5	7
6	6
	$\Sigma f = 34$

9 How many times was a number less than 3 rolled? 12 times

10 How many times was an even number rolled? 17 times

11 How many times was a prime number rolled? 19 times

12 What percentage of die rolls resulted in a multiple of 3?

$\frac{11}{34} \times 100 = 32.4\%$

Answer to 1 d.p.

13 What percentage of die rolls resulted in a factor of 12?

$\frac{27}{34} \times 100 = 79.4\%$

Answer to 1 d.p.

Ordered Stem and Leaf Plots

Stem and leaf plots separate data into its units component and its tens (and above) component. They then display the 'units' on the right in the leaf section and the 'tens' on left in a stem column. The effect is to break the data up into classes and also display a simple column graph (on its side) of numbers showing the way the data is spread.

Numbers have their units removed so:

- 23 becomes 'a 2' and 'a 3'
- 15 becomes 'a 1' and 'a 5'
- 9 (is seen as 09) becomes 'a 0' and 'a 9'

Stem	Leaf
0	9
1	5
2	3

In a plot this looks like the plot shown above. But what about larger numbers?

- 106 becomes 'a 10' and 'a 6'
- 117 becomes 'a 11' and 'a 7'
- 125 becomes 'a 12' and 'a 5'
- 120 becomes 'a 12' and 'a 0'
- 110 becomes 'a 11' and 'a 0'

Stem	Leaf
10	6
11	70
12	50

Because the units have two digits in front, both digits are shown in the Stem column.

When all data has been entered into the plot, the plot then has to be rewritten in order. The stem stays the same, but the leaf is put into ascending order. That is it!

Stem	Leaf
10	6
11	70
12	50

Leaf in order →

Stem	Leaf
10	6
11	07
12	05

Negative numbers occur in data and they can also be displayed in stem and leaf plots.

Example. Place the data into a Stem and Leaf Plot

Data: 5, 8, 12, -3, -7, 17, -8, **25**, 10, **-23**, -5, 0

The lowest value is -23 so the stem starts at -2
The highest value is 25 so the stem ends at 2.

Note that with negative numbers, the stem has the minus sign, the leaf just has positive numbers.

Stem	Leaf
-2	3
-1	
-0	3785
0	580
1	270
2	5

Leaf in order →

Stem	Leaf
-2	3
-1	
-0	3578
0	058
1	027
2	5

Ordered Stem and Leaf Plots

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Display the data sets in Raw then Ordered Stem and Leaf plots. Don't use commas!



1 Data: 23, 17, 35, 21, 30, 8, 11, 24, 6, 9, 17, 21, 32, 3, 11, 13.

RAW → ORDERED

Tens	Units
0	
1	
2	
3	

Tens	Units
0	
1	
2	
3	

2 Data: 74, 97, 70, 106, 92, 100, 86, 91, 97, 85, 110, 83, 101, 95, 79, 90, 103, 85.

RAW → ORDERED

Stem	Leaf
7	
8	
9	
10	
11	

Stem	Leaf

3 Data: 5, 19, -3, 13, 27, -12, 8, 17, -6, 28, 0, -10, 18, -1, -26, 14, 11, -15, 20, -8.

Negative data uses the negative tens in the stem but are written in the units (or leaf) as positive numbers.

Stem	Leaf
-2	
-1	
-0	
0	
1	
2	

Stem	Leaf

4 Data: 220, 195, 206, 212, 184, 177, 209, 195, 191, 204, 173, 168, 188, 228, 219, 176, 192, 213, 201, 180, 171.

Stem	Leaf

→

Stem	Leaf

5 How many scores were over 200? _____

6 How many scores were even? _____

7 What percentage of scores were less than 180?

_____ = _____

Answer to 2 d.p.



Round the data to the nearest whole number then display the rounded data in a Stem & Leaf plot.

8 Data: 16.5, 4.07, 10.3, 5.91, 21.6, -3.5, -7, -17.1, 37.7, -4, 11.2, -19.55, 18.48, -9, 38.64, 2.2, 13.36, -9.94, -7, 18.57, 20.8, 13.25, -4.81, 0.43, -2.88, 35.95, -9.46, -19.3, 14.19, 20.72, 13.06.

Stem	Leaf

→

Stem	Leaf

Ordered Stem and Leaf Plots

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Display the data sets in Raw then Ordered Stem and Leaf plots. Don't use commas!

1 Data: 23, 17, 35, 21, 30, 8, 11, 24, 6, 9, 17, 21, 32, 3, 11, 13.

RAW → ORDERED

Tens	Units
0	8693
1	71713
2	3141
3	502

Tens	Units
0	3689
1	11377
2	1134
3	025

2 Data: 74, 97, 70, 106, 92, 100, 86, 91, 97, 85, 110, 83, 101, 95, 79, 90, 103, 85.

RAW → ORDERED

Stem	Leaf
7	409
8	6535
9	721750
10	6013
11	0

Stem	Leaf
7	049
8	3556
9	012577
10	0136
11	0

3 Data: 5, 19, -3, 13, 27, -12, 8, 17, -6, 28, 0, -10, 18, -1, -26, 14, 11, -15, 20, -8.

Negative data uses the negative tens in the stem but are written in the units (or leaf) as positive numbers.

Stem	Leaf
-2	6
-1	205
-0	3618
0	580
1	937841
2	780

Stem	Leaf
-2	6
-1	025
-0	1368
0	058
1	134789
2	078

4 Data: 220, 195, 206, 212, 184, 177, 209, 195, 191, 204, 173, 168, 188, 228, 219, 176, 192, 213, 201, 180, 171.

Stem	Leaf
16	8
17	7361
18	480
19	5512
20	6941
21	293
22	08

Stem	Leaf
16	8
17	1367
18	048
19	1255
20	1469
21	239
22	08

5 How many scores were over 200? 9 scores

6 How many scores were even? 11 scores

7 What percentage of scores were less than 180?

$\frac{5}{21} \times 100 = 23.81\ \%$

Answer to 2 d.p.

Round the data to the nearest whole number then display the rounded data in a Stem & Leaf plot.

8 Data: 16.5, 4.07, 10.3, 5.91, 21.6, -3.5, -7, -17.1, 37.7, -4, 11.2, -19.55, 18.48, -9, 38.64, 2.2, 13.36, -9.94, -7, 18.57, 20.8, 13.25, -4.81, 0.43, -2.88, 35.95, -9.46, -19.3, 14.19, 20.72, 13.06.

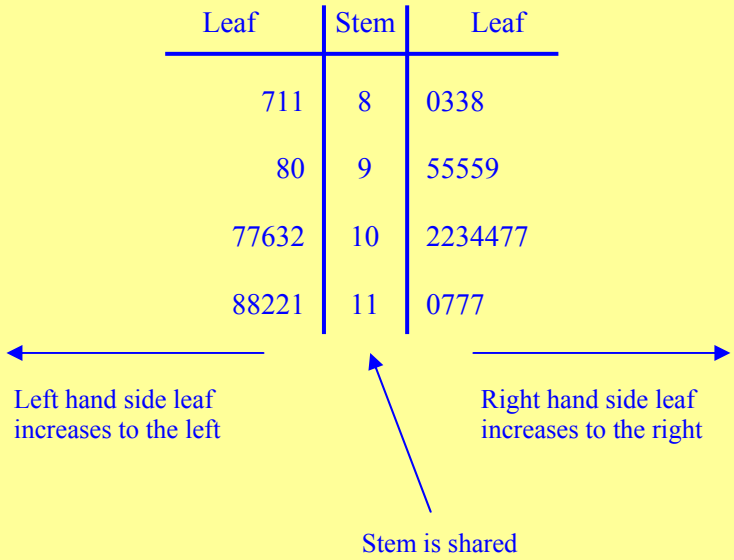
Stem	Leaf
-2	0
-1	709
-0	47497539
0	4620
1	701839343
2	211
3	896

Stem	Leaf
-2	0
-1	079
-0	34457799
0	0246
1	013334789
2	112
3	689

Back-to-Back Stem and Leaf Plots

Back-to-back stem and leaf plots are used to display 2 different groups of data together in the one plot. The groups are displayed separately but share a common stem. One group is displayed on the left hand side, the other on the R.H.S.

The L.H.S has its leaf in ascending order (going up) with larger numbers to the left, while the R.H.S. leaf has its numbers ascending, as with single leaf stem and leaf plots, up to the right.



This sheet shouldn't be attempted before completing the previous stem and leaf plot exercise. The process is the same other than the process being performed twice.

Two mistakes are regularly made with Stem and Leaf plots. These are:

- Commas are used in the plots, like data sets. Commas shouldn't be used in a stem and leaf plot.
- A mistake is made finding the smallest 'tens' to put in the stem, so the number is written at the bottom of the stem column. The stem has to be in order, so if you miss the smallest 'tens' then the plot should be redone.

Back-to-Back Stem and Leaf Plots

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Rewrite this stem and leaf plot in order.

Same data shown in order

65440	0	04456
955	1	559
7200	2	0027

1 RAW

Leaf	Stem	Leaf
83026	0	00285
7339	1	77116579
780411	2	203884
5443986	3	9265813

ORDERED

Leaf	Stem	Leaf
	0	
	1	
	2	
	3	

Enter the data into the back-to-back stem and leaf plot, firstly raw, then ordered. Girls on the left, boys on the right. Swing those partners!

2 School Barn Dance Ticket Sales (Hourly)

Girls: 15, 35, 11, 28, 29, 36, 12, 15, 31, 36, 28, 11, 19, 26, 24, 24, 42, 18, 22, 38, 16, 25.

Boys: 26, 11, 19, 13, 38, 12, 19, 15, 36, 20, 37, 10, 19, 33, 14, 17, 38, 33, 25, 16, 15, 48.

RAW

Girls	Stem	Boys
	1	
	2	
	3	
	4	

ORDERED

Girls	Stem	Boys
	1	
	2	
	3	
	4	

A single plane air service delivers freight and passengers. Weekly hours are shown for 6 months, display the data in a B-2-B S&L plot.

3 Weekly Flight Time (hours) KA-223

Passengers: 90, 86, 42, 84, 73, 36, 58, 80, 78, 32, 59, 85, 87, 43, 84, 55, 90, 75, 43, 89, 46, 76, 76, 42, 31, 71.

Freight: 36, 40, 84, 42, 53, 92, 68, 46, 48, 94, 67, 41, 39, 83, 42, 71, 36, 51, 83, 37, 80, 50, 50, 84, 95, 55.

How many weeks had ....

4 Over 60 hours of freight transport? _____

5 More than 80 passenger hours? _____

6 What percentage of weeks had less than 50 hours freight time?

_____ = _____

Answer to 1 d.p.

Back-to-Back Stem and Leaf Plots

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Rewrite this stem and leaf plot in order.

Same data shown in order

65440	0	04456
955	1	559
7200	2	0027

1

RAW Leaf	Stem	Leaf
83026	0	00285
7339	1	77116579
780411	2	203884
5443986	3	9265813

ORDERED Leaf	Stem	Leaf
86320	0	00258
9733	1	11567779
874110	2	023488
9865443	3	1235689

Enter the data into the back-to-back stem and leaf plot, firstly raw, then ordered. Girls on the left, boys on the right. Swing those partners!

2 School Barn Dance Ticket Sales (Hourly)

Girls: 15, 35, 11, 28, 29, 36, 12, 15, 31, 36, 28, 11, 19, 26, 24, 24, 42, 18, 22, 38, 16, 25.

Boys: 26, 11, 19, 13, 38, 12, 19, 15, 36, 20, 37, 10, 19, 33, 14, 17, 38, 33, 25, 16, 15, 48.

RAW Girls	Stem	Boys
68915215	1	193295094765
52446898	2	605
86165	3	867383
2	4	8

ORDERED Girls	Stem	Boys
98655211	1	012345567999
98865442	2	056
86651	3	336788
2	4	8

A single plane air service delivers freight and passengers. Weekly hours are shown for 6 months, display the data in a B-2-B S&L plot.

3 Weekly Flight Time (hours) KA-223

Passengers: 90, 86, 42, 84, 73, 36, 58, 80, 78, 32, 59, 85, 87, 43, 84, 55, 90, 75, 43, 89, 46, 76, 76, 42, 31, 71.

Freight: 36, 40, 84, 42, 53, 92, 68, 46, 48, 94, 67, 41, 39, 83, 42, 71, 36, 51, 83, 37, 80, 50, 50, 84, 95, 55.

Passengers	Stem	Freight
126	3	6967
26332	4	026812
598	5	31005
	6	87
166583	7	1
9475046	8	43304
00	9	245

Passengers	Stem	Freight
621	3	6679
63322	4	012268
985	5	00135
	6	78
866531	7	1
9765440	8	03344
00	9	245

How many weeks had ....

4 Over 60 hours of freight transport? 11 weeks

5 More than 80 passenger hours? 8 weeks

6 What percentage of weeks had less than 50 hours freight time?

$\frac{10}{26} \times 100 = 38.5\%$

Answer to 1 d.p.

Sector Graphs

Sector graphs, Pie charts, Circle graphs are all different names for the same graph. Sector graphs are generally used to display something that has been broken up into parts. Such as the time spent doing something or the way sales were made.

A sector is a fraction of a circle and the angle of the sector is found by multiplying fractions with 360°.

Example. Graph shows how Bill's time playing sport was spent. Create a sector graph.

Sport	Weeks	Sector Angle (Nearest °)
Cricket	11	$\frac{11}{50} \times 360 = 79°$
Squash	5	$\frac{5}{50} \times 360 = 36°$
Football	9	$\frac{9}{50} \times 360 = 65°$
Swimming	20	$\frac{20}{50} \times 360 = 144°$
Tennis	1	$\frac{1}{50} \times 360 = 7°$
Other	4	$\frac{4}{50} \times 360 = 29°$
	Σ = 50	ΣAngles = 360°

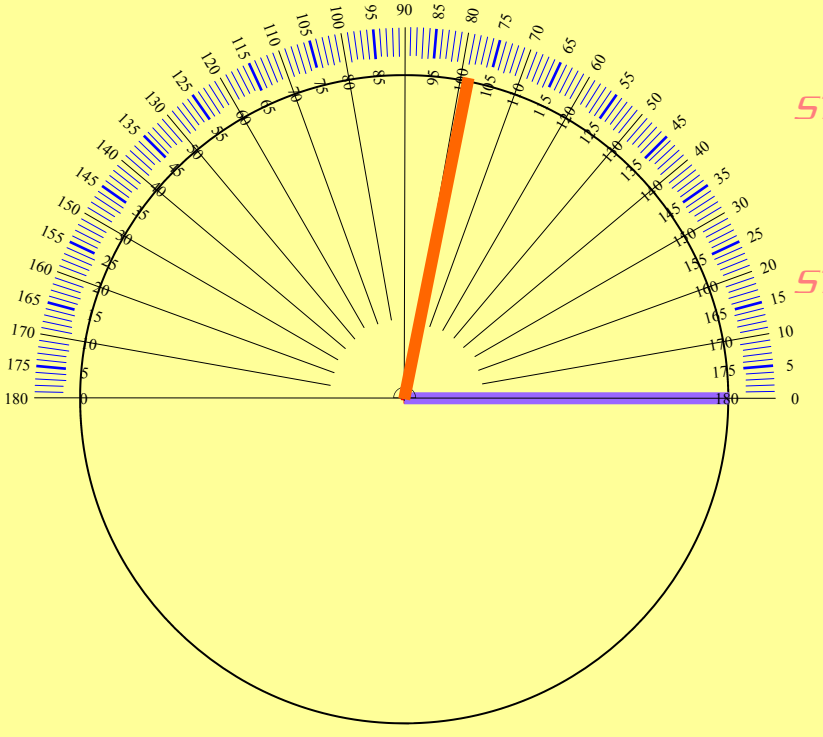
STEP 1 Add the column and write the total at the bottom of the column where the Σ is shown.

STEP 2 Make a fraction and multiply the fraction by 360°. In this case it is 11 over 50, then × 360.

STEP 3 Repeat for all the angles so that you have a sector angle for each different category (sport).

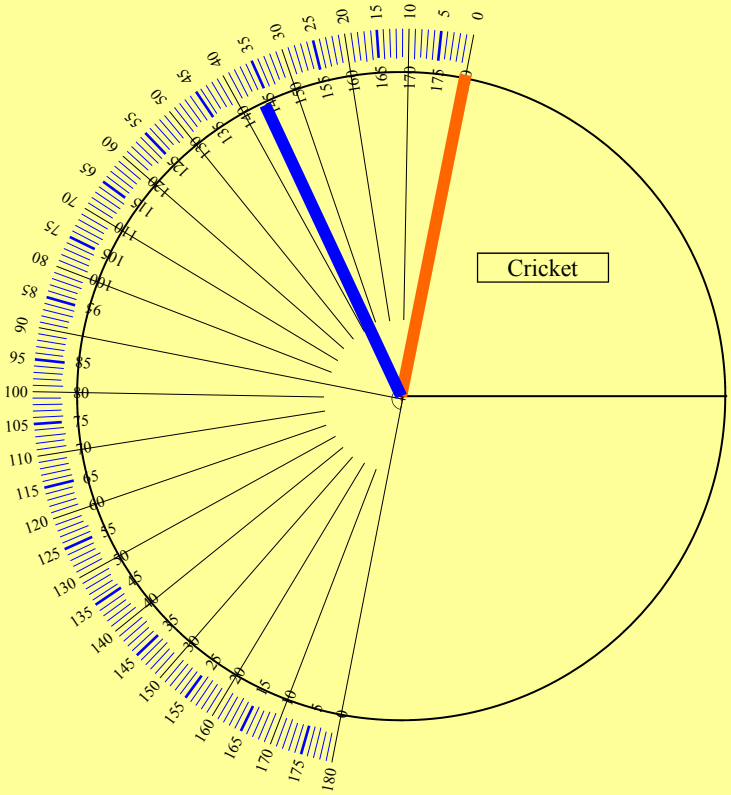
STEP 4 Sum (add) all the angles up and the total should be 360°. It may be out by a degree or two due to rounding.

Now the table has to be converted to a sector graph.



STEP 1 Draw a horizontal line from the centre of the circle to the circumference. Purple line.

STEP 2 Use a protractor to create the first sector. Shown in orange. (79° from the table).



STEP 3

Label the sector with the category name, which is cricket.

STEP 4

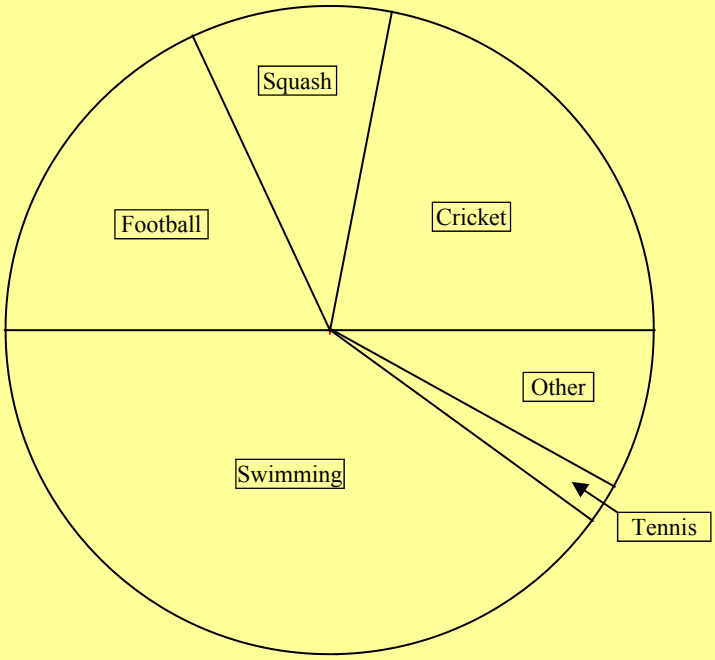
Rotate the protractor, the 0° line being on the last line drawn. The next sector is 36°. Blue line.

STEP 5

Label the sector and rotate the protractor and keep going!

STEP 6

The finished product has all the sectors labeled. Colour it in to improve the finished product.



Sector Graphs

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Calculate the sector angle that each subject would require if Robert's timetable were shown on a sector graph.

1 Robert's Timetable

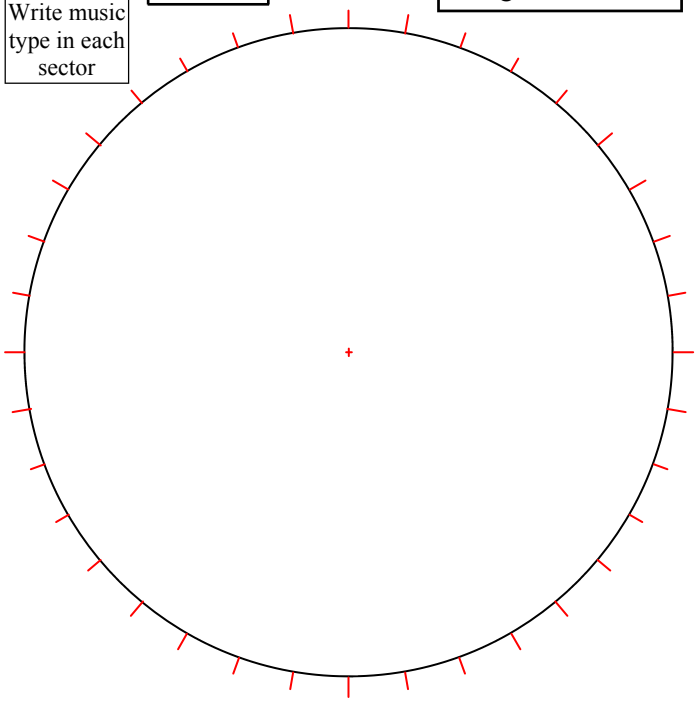
Subject	Periods	Sector Angle (°)
English	6	$\frac{6}{\square} \times 360 = \square$
Maths	5	
Science	5	
History	4	
Metal	3	
I.T.	3	
Sport/P.E.	4	
	Σ	ΣAngles =

Find the sector angle and create a sector graph, each division is 10°. No Protractor needed.

2 Favorite Music

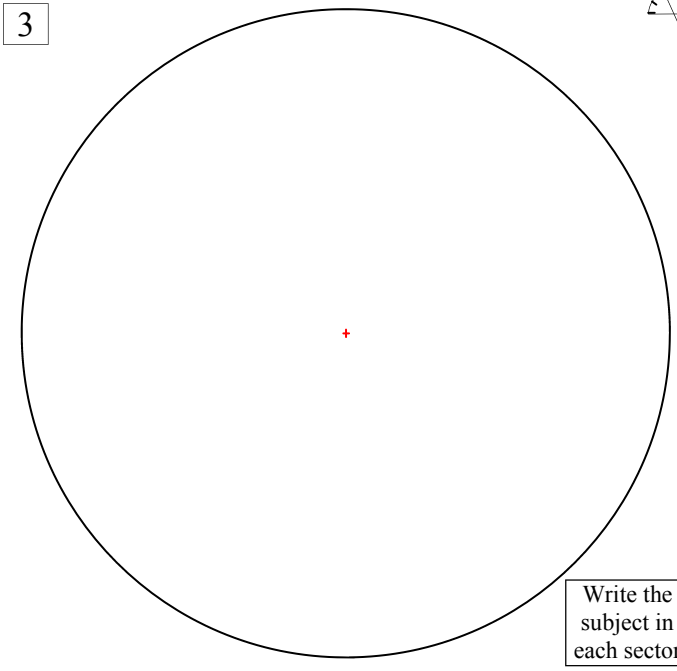
Type	Students	Sector Angle
Rock	5	
Dance	3	
Rap	2	
Metal	4	
Hip Hop	1	
Other	3	
	Σ	ΣAngles =

Write music type in each sector



Plot a sector graph for Robert's timetable from Q.1. You need a protractor for Q.3. and Q.4.

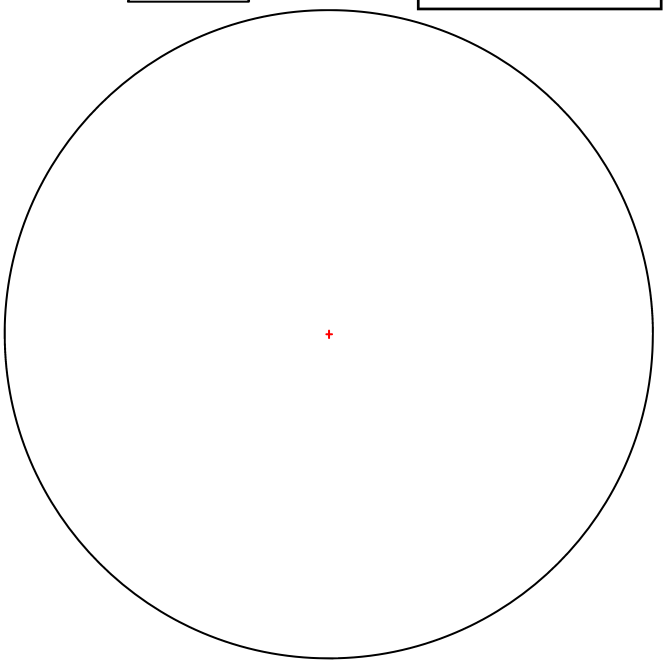
3



Write the subject in each sector

4 Amy's Weekend Time Usage

Type	Hours	Sector Angle (Nearest °)
T.V.	6	
Computer	5	
Read	2	
Music	1	
Friends	7	
Sport	3	
Family	4	
	Σ	ΣAngles =



Sector Graphs

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Calculate the sector angle that each subject would require if Robert's timetable were shown on a sector graph.

1 Robert's Timetable

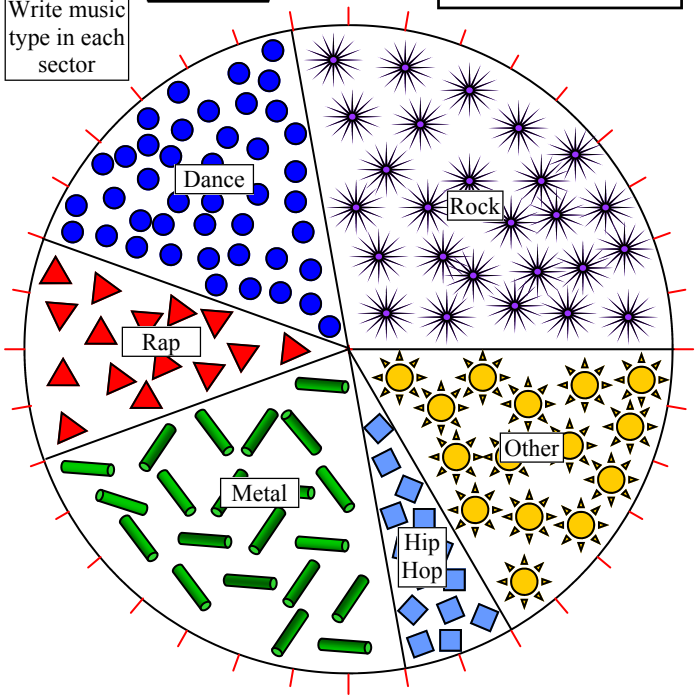
Subject	Periods	Sector Angle (°)
English	6	$\frac{6}{30} \times 360 = 72°$
Maths	5	$\frac{5}{30} \times 360 = 60°$
Science	5	$\frac{5}{30} \times 360 = 60°$
History	4	$\frac{4}{30} \times 360 = 48°$
Metal	3	$\frac{3}{30} \times 360 = 36°$
I.T.	3	$\frac{3}{30} \times 360 = 36°$
Sport/P.E.	4	$\frac{4}{30} \times 360 = 48°$
	$\Sigma P = 30$	ΣAngles = 360° ✓

Find the sector angle and create a sector graph, each division is 10°. No Protractor needed.

2 Favorite Music

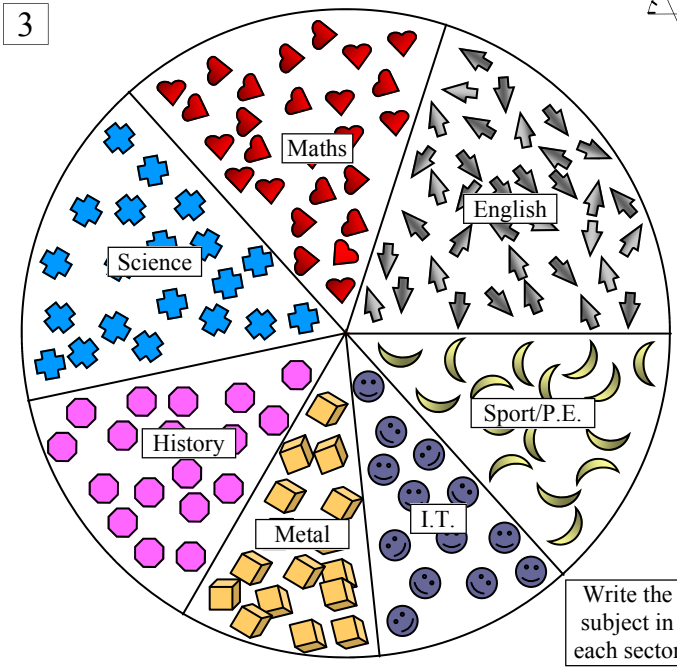
Type	Students	Sector Angle
Rock	5	$\frac{5}{18} \times 360 = 100°$
Dance	3	$\frac{3}{18} \times 360 = 60°$
Rap	2	$\frac{2}{18} \times 360 = 40°$
Metal	4	$\frac{4}{18} \times 360 = 80°$
Hip Hop	1	$\frac{1}{18} \times 360 = 20°$
Other	3	$\frac{3}{18} \times 360 = 60°$
	$\Sigma S = 18$	ΣAngles = 360° ✓

Write music type in each sector



Plot a sector graph for Robert's timetable from Q.1. You need a protractor for Q.3. and Q.4.

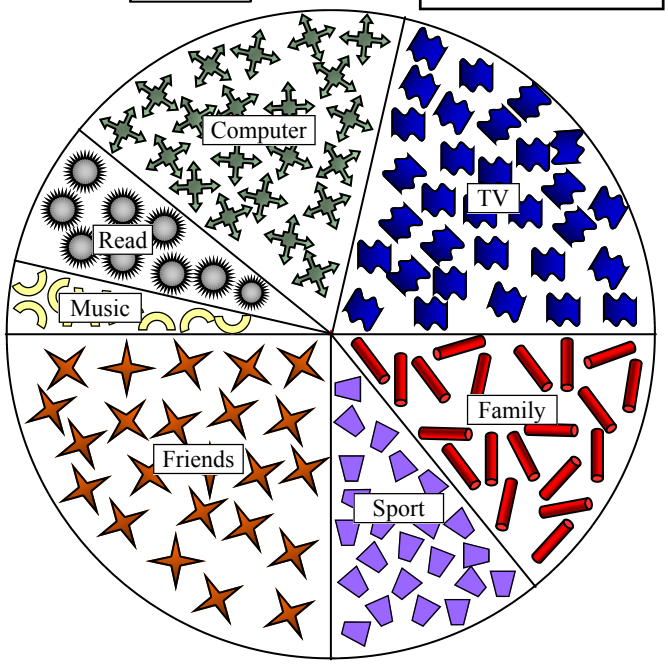
3



Write the subject in each sector

4 Amy's Weekend Time Usage

Type	Hours	Sector Angle (Nearest °)
T.V.	6	$\frac{6}{28} \times 360 = 77°$
Computer	5	$\frac{5}{28} \times 360 = 64°$
Read	2	$\frac{2}{28} \times 360 = 26°$
Music	1	$\frac{1}{28} \times 360 = 13°$
Friends	7	$\frac{7}{28} \times 360 = 90°$
Sport	3	$\frac{3}{28} \times 360 = 39°$
Family	4	$\frac{4}{28} \times 360 = 51°$
	$\Sigma H = 28$	ΣAngles = 360° ✓



Sector Graphs - Sheet 2

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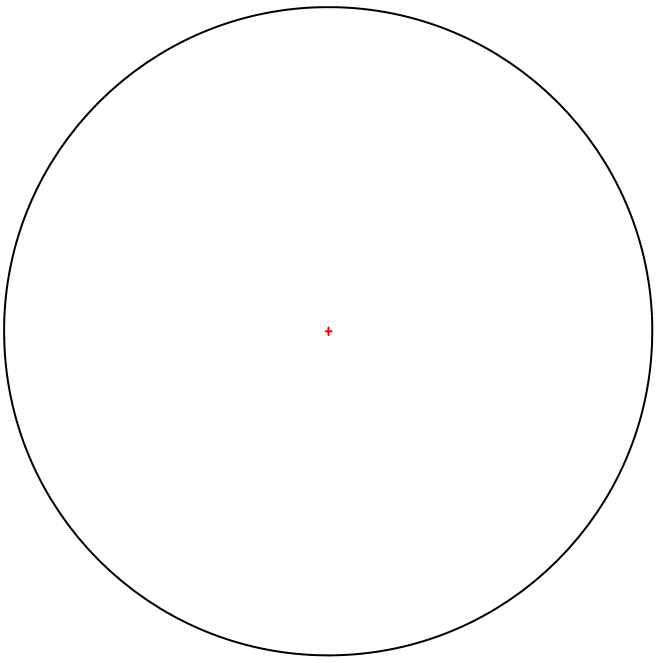
75 people were trialed for their preferred breakfast cereal, place the results in a sector graph, colour it in and use a key.

1

ABFDA	FBBCD	DBCCE	ABBDE
EFCCB	ADFEB	CCBBC	DFECC
DFBBC	BAFED	FBCBA	CBDAE
DDBBC	BCCDA	AEFCC	

Cereal	Tally	f	Sector Angle (nearest degree)
Type 'A'			
Type 'B'			
Type 'C'			
Type 'D'			
Type 'E'			
Type 'F'			
		$\Sigma f =$	ΣAngles =



KEY:

☐ TYPE A ☐ TYPE B ☐ TYPE C

☐ TYPE D ☐ TYPE E ☐ TYPE F

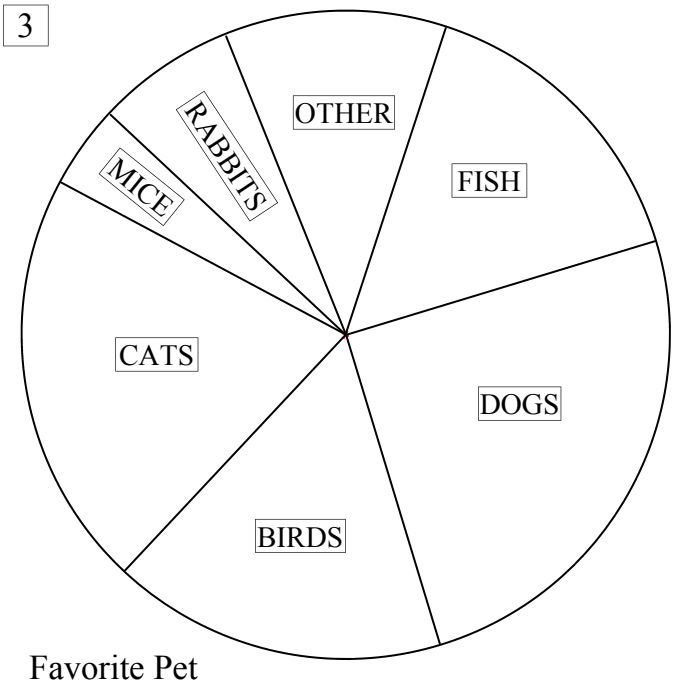
2 Calculate the percentage of people preferring:

Type B _____ _____

Type E _____ _____

1 800 people were polled about their family pet. Use a protractor to extract the data from the sector graph and display it in a frequency table.

3



Favorite Pet

Pet	Angle	Frequency
Rabbits		
Mice		
Cats		
Birds		
Dogs		
Fish		
Other		
	$\Sigma =$	$\Sigma f =$

4 List the animals from most to least favorite.

_____

_____

5 What percentage of people prefer either cats or dogs?

_____

6 If 3 000 were polled and their views were the same as the graph, how many would prefer dogs?

_____

7 How many of the 3 000 would prefer mice or rabbits?

_____

Sector Graphs - Sheet 2

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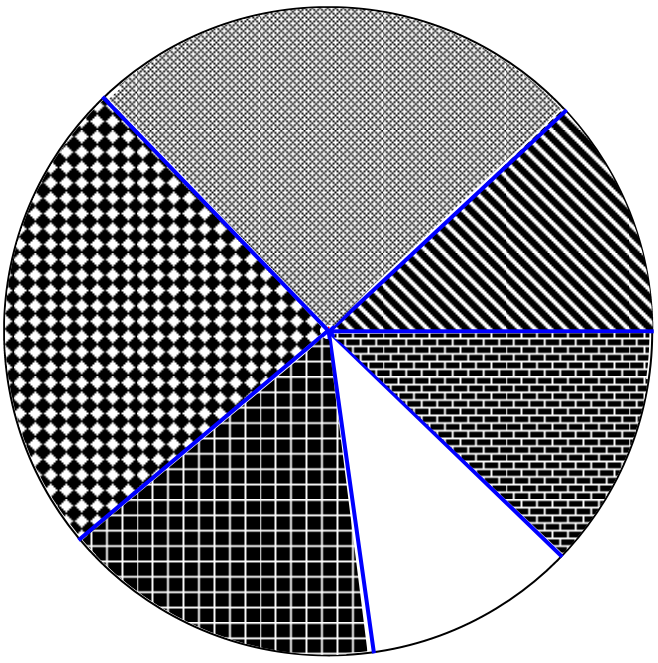
75 people were trialed for their preferred breakfast cereal, place the results in a sector graph, colour it in and use a key.

1

ABFDA FBBCD DBCCE ABBDE
EFCCB ADFEB CCBBC DFECC
DFBBC BAFED FBCBA CBDAE
DDBBC BCCDA AEFCC

Cereal	Tally	f	Sector Angle (nearest degree)
Type 'A'	𝍸 ||||	9	$\frac{9}{75} \times 360 = 43°$
Type 'B'	𝍸 𝍸 𝍸 ||||	19	$\frac{19}{75} \times 360 = 91°$
Type 'C'	𝍸 𝍸 𝍸 |||	18	$\frac{18}{75} \times 360 = 86°$
Type 'D'	𝍸 𝍸 ||	12	$\frac{12}{75} \times 360 = 58°$
Type 'E'	𝍸 |||	8	$\frac{8}{75} \times 360 = 38°$
Type 'F'	𝍸 ||||	9	$\frac{9}{75} \times 360 = 43°$
		$\Sigma f = 75$	ΣAngles = 359°

Preferred Breakfast Cereal



KEY:

TYPE A TYPE B TYPE C

TYPE D TYPE E TYPE F

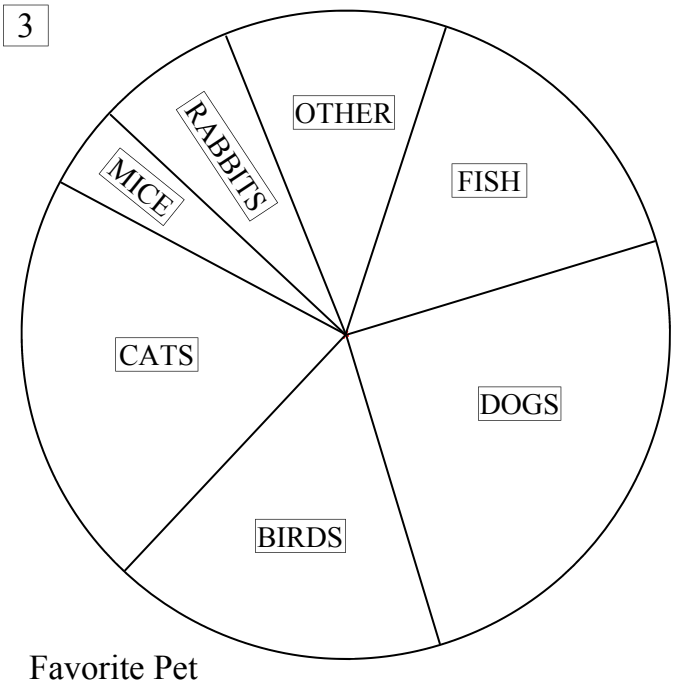
2 Calculate the percentage of people preferring:

Type B: $\frac{19}{75} \times 100$ = 25.3% (1 d.p.)

Type E: $\frac{8}{75} \times 100$ = 10.7% (1 d.p.)

1 800 people were polled about their family pet. Use a protractor to extract the data from the sector graph and display it in a frequency table.

3



Favorite Pet

Pet	Angle	Frequency
Rabbits	25°	$\frac{25}{360} \times 1\,800 = 125$
Mice	15°	$\frac{15}{360} \times 1\,800 = 75$
Cats	75°	$\frac{75}{360} \times 1\,800 = 375$
Birds	60°	$\frac{60}{360} \times 1\,800 = 300$
Dogs	90°	$\frac{90}{360} \times 1\,800 = 450$
Fish	55°	$\frac{55}{360} \times 1\,800 = 275$
Other	40°	$\frac{40}{360} \times 1\,800 = 200$
	$\Sigma = 360°$	$\Sigma f = 1\,800$

4 List the animals from most to least favorite.

Dogs, cats, birds, fish, other, rabbits, mice.

5 What percentage of people prefer either cats or dogs? $\frac{165}{360} \times 100 = 45.8\%$ (1 d.p.)

6 If 3 000 were polled and their views were the same as the graph, how many would prefer dogs?

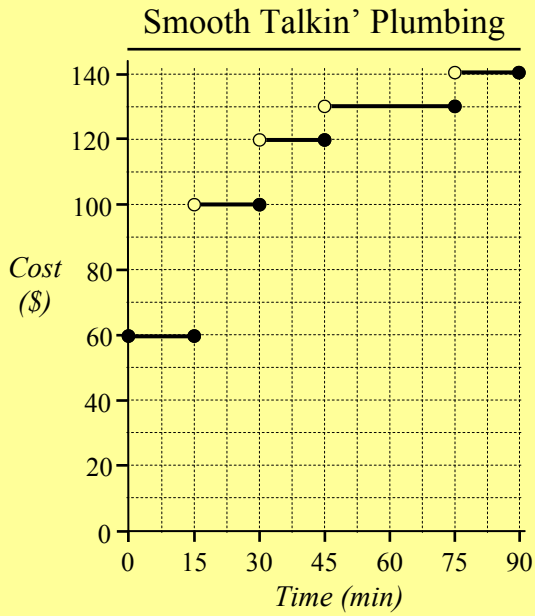
$\frac{90}{360} \times 3\,000 = 750$ people

7 How many of the 3 000 would prefer mice or rabbits? $\frac{40}{360} \times 3\,000 =$ about 333 people

Step Graphs

Not everything is smooth in practice and Step Graphs represent a way in which data can be applied in a 'jumpy' or 'broken up' way. By having different bands, separate graphs that apply costs are brought together in one graph.

In the "Smooth Talkin' Plumbing" fee scale shown to the right, you can see that for 0 min to 15 min the fee is $60.



But the circle shows this line ends at 15 min, so from 16 min through to 30 min then the charge is $100.

Again after the 30th minute is finished the fee lifts to $120. And so on.

So the fee structure is in stages, this is for simplicity and also to cover 'getting to the job' that is, travel time has to be covered by the more expensive starting rate.

Note the use of the circles. A filled-in circle means 'stop here' while a open circle means don't stop here or 'pass through'. Because the $60 first charge has a filled in circle on its right end, it means 15 min is included. If it was open, then the 15 min would travel to the next rate which is $100. Of course the circle in the $100 would have to be filled in to say 15 min is included.

Column 1 deals with obtaining information from the step graph provided. You have to use the graph two ways, to find the weight a package could be…. given price, or the price paid given a weight.

Column 2 starts off with a step graph on hiring a forklift, again working both ways, obtaining time given cost, or cost given time. The final question requires you to create the step graph from the information provided.

Step Graphs

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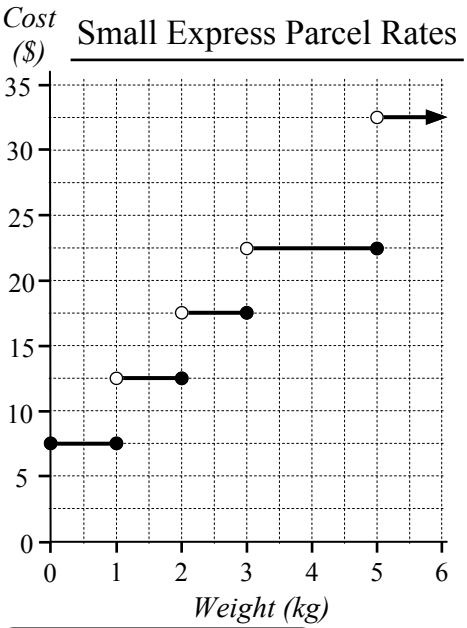
Use the step graph to answer the questions below.



What is the cost to send parcels weighing .....

1 1 kg _____

2 3 kg _____

3 4.5 kg _____

4 750 g and 1.35 kg

_____ = _____

5 4 kg and 1.75 kg

_____ = _____

6 5.9 kg and 3.2 kg

_____ = _____

What is the heaviest parcel that can be sent if you have .....

7 $22.50 _____

8 $7.50 _____

9 $20 _____

10 $11.60 _____

11 $23.90 _____

Day and hourly rates for the hire of a Forklift are shown, answer the questions using the graph.



Find the cost for :

12 7 hrs hire _____ 13 2 hrs hire _____ 14 5 hrs hire _____

Find the maximum time you can hire the forklift if you have :

15 $150 _____ 16 $275 _____ 17 $430 _____

If a job requires a forklift for 13 hours. Find the difference in cost between hiring the forklift for 13 hours in one day or .....

18 6 hr one day, 7 hr the next _____

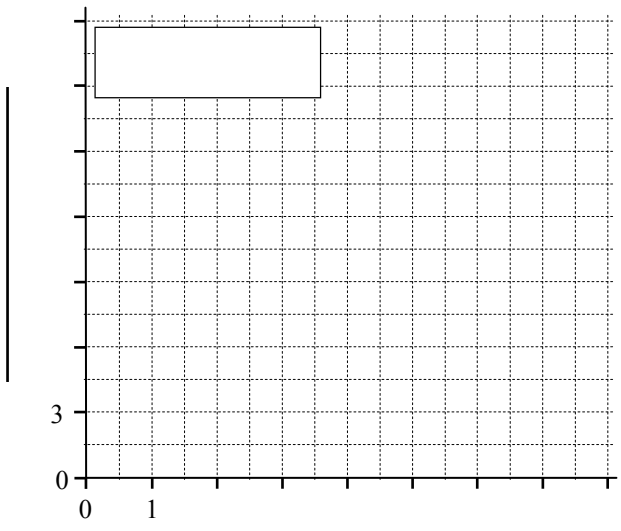
19 1 hr one day, 12 hr the next _____

20 5 hr one day, 4 hr two days _____

21 5 hr two days, 3 hr the next _____

StackEmUp Car Parking charges $6 for the first hour then $3 per additional hour. After 5 hours the day rate of $20 (only) is charged. Construct a step graph to represent the fee structure.

22



_____

Step Graphs

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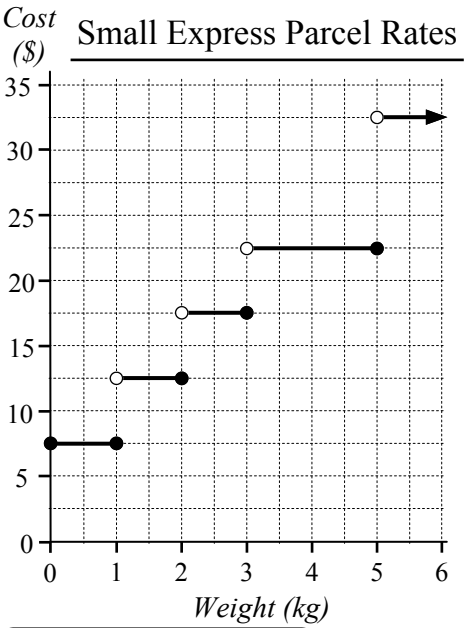
Use the step graph to answer the questions below.



What is the cost to send parcels weighing .....

1 1 kg $7.50

2 3 kg $17.50

3 4.5 kg $22.50

4 750 g and 1.35 kg

$7.50 + $12.50 = $20

5 4 kg and 1.75 kg

$22.50 + $12.50 = $35

6 5.9 kg and 3.2 kg

$32.50 + $22.50 = $55

What is the heaviest parcel that can be sent if you have .....

7 $22.50 5 kg

8 $7.50 1 kg

9 $20 3 kg

10 $11.60 1 kg

11 $23.90 5 kg

Day and hourly rates for the hire of a Forklift are shown, answer the questions using the graph.



Find the cost for :

12 7 hrs hire $200 13 2 hrs hire $150 14 5 hrs hire $200

Find the maximum time you can hire the forklift if you have :

15 $150 4 h 59 m 16 $275 11 h 59 m 17 $430 25 h 59 m

If a job requires a forklift for 13 hours. Find the difference in cost between hiring the forklift for 13 hours in one day or .....

18 6 hr one day, 7 hr the next 200 + 200 - 325 = $75

19 1 hr one day, 12 hr the next 100 + 325 - 325 = $100

20 5 hr one day, 4 hr two days

200 + 2 × 150 - 325

= $175

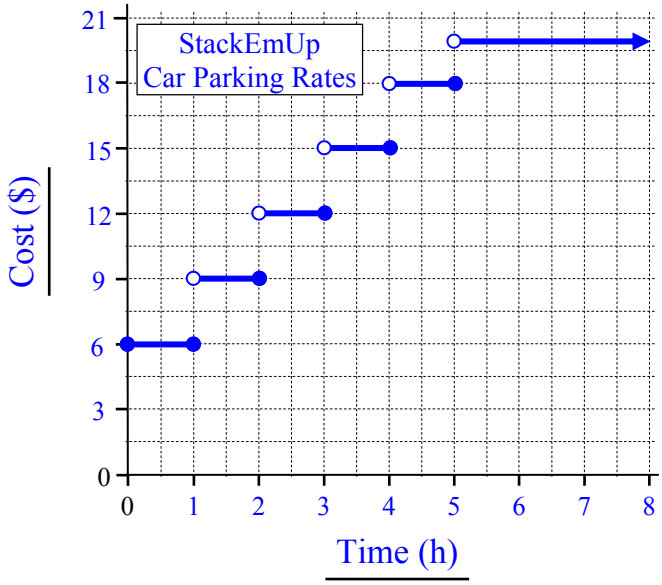
21 5 hr two days, 3 hr the next

2 × 200 + 150 - 325

= $225

StackEmUp Car Parking charges $6 for the first hour then $3 per additional hour. After 5 hours the day rate of $20 (only) is charged. Construct a step graph to represent the fee structure.

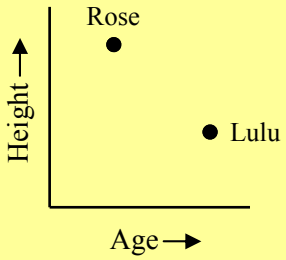
22



Scatter Diagrams

Scatter diagrams plot two different qualities against each other. These can be related, such as the amount of wood used compared with the heat generated from a fire. They can also be unrelated, such as the shoe size of a student compared with their eyesight quality.

This means that the one plotted point gives two answers, as below.



Rose and Lulu are shown on the scatter diagram, which compares age with height.

Lulu is further to the right than Rose, that means that she is older than Rose. Because as you move right, the age increases.

But Rose is taller than Lulu. Because height is measured up the vertical scale, the higher up the point, the taller the person is. Note that there is no scale, Rose could be two years younger, or three weeks younger, without a scale, you can't put a number on, or estimate the difference.

Column 1 has four graphs like the one shown above, only this time four people are compared within each graph. Read the questions and use the graph that is relevant and look at how the dots are spaced.

Column 2 asks you to plot points on a scatter diagram, a diagram about Back-burning operations. The forest names start with A and end with G, so plot the point and place a capital letter beside them, representing the forest name. Then answer the questions.

Scatter Diagrams

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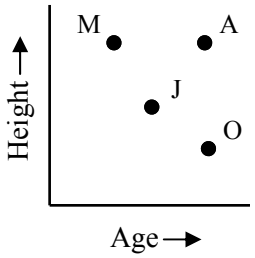
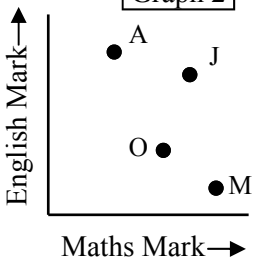
Four students are compared in the scatter diagrams below, use the graphs to answer the questions.

A = Amy, O = Oscar, M = Michael and J = Jessica

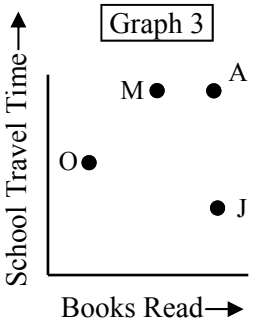
Graph 1



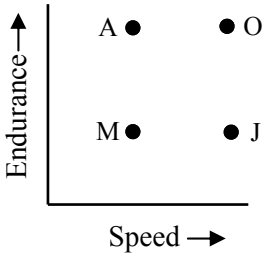
Graph 2



Graph 3



Graph 4



1 Who is the same age as Amy? _____

2 Who is the best at Maths? _____

3 Who is taller than Jessica? _____

4 Who travels the same amount of time to get to school as Amy? _____

5 Who probably reads more at home? _____

6 Who do you think is the best athlete? _____

7 List the students according to their ability:

In Maths: _____
Lowest to Highest Ability

In English: _____
Lowest to Highest Ability

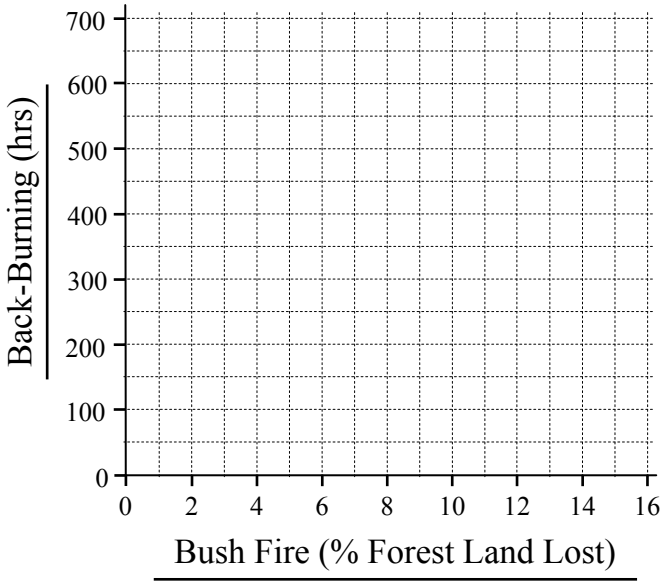
8 Name two things that Oscar is better at than Amy. _____

9 Name two things that Amy is better at than Michael. _____

Plot the data in the table on the scatter diagram grid. Then answer the questions.

Back-Burning, Forest Loss to Bushfire Data

Forest	B/Burn Hours	Forest % lost
Allenby	150	8
Bellamy	150	12
Centfall	600	2
Domino	450	5
Everturn	300	6
Fleckle	550	2
Gritten	95	16



10 As back-burning hours increase explain the result. _____

11 Which forest, Bellamy or Allenby, would you suggest is the larger? Explain why.

12 More allocation of hours is required when the Bushfire index exceeds 10. Add a column called 'B.I.' to the table above, calculate the B.I. given:

$$\text{B.I.} = \frac{\%\ \text{Forest Land Lost} \times 175}{\text{Back Burn Hours}}$$

Round to the nearest whole number

13 Which forests qualify for more allocated hours? Do you have any suggestions?

Scatter Diagrams

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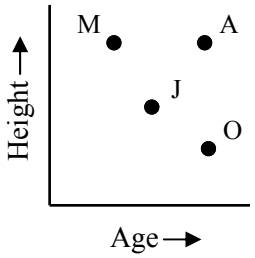
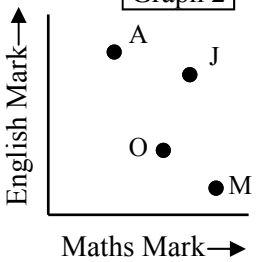
Four students are compared in the scatter diagrams below, use the graphs to answer the questions.

A = Amy, O = Oscar, M = Michael and J = Jessica

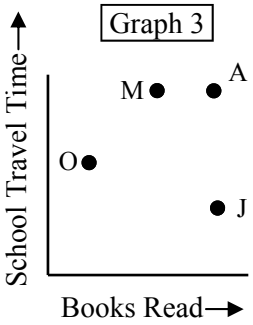
Graph 1



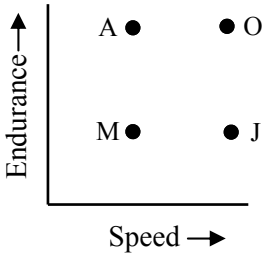
Graph 2



Graph 3



Graph 4



1 Who is the same age as Amy? Oscar

2 Who is the best at Maths? Michael

3 Who is taller than Jessica? Michael, Amy.

4 Who travels the same amount of time to get to school as Amy? Michael

5 Who probably reads more at home? Jessica

6 Who do you think is the best athlete? Oscar

7 List the students according to their ability:

In Maths: Amy, Oscar, Jessica then Michael.
Lowest to Highest Ability

In English: Michael, Oscar, Jessica then Amy.
Lowest to Highest Ability

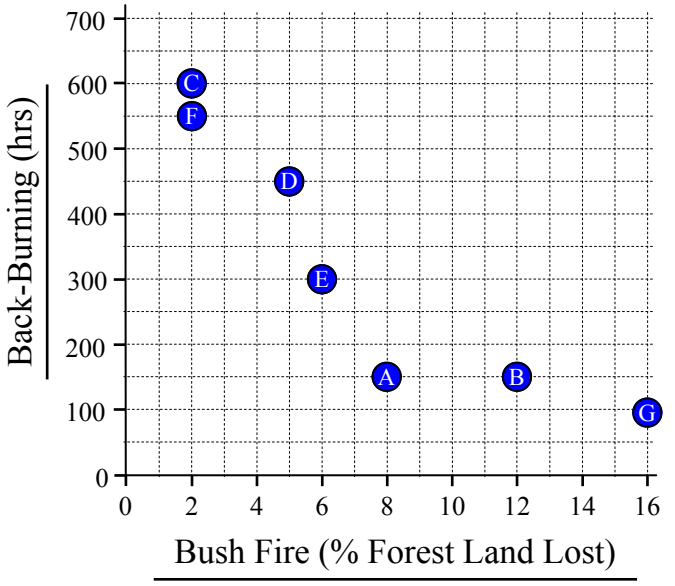
8 Name two things that Oscar is better at than Amy. Faster and better at maths.

9 Name two things that Amy is better at than Michael. More endurance, English and Reading

Plot the data in the table on the scatter diagram grid. Then answer the questions.

Back-Burning, Forest Loss to Bushfire Data

Forest	B/Burn Hours	Forest % lost	B.I.
Allenby	150	8	9
Bellamy	150	12	14
Centfall	600	2	1
Domino	450	5	2
Everturn	300	6	4
Fleckle	550	2	1
Gritten	95	16	29



10 As back-burning hours increase explain the result. There is less forest land lost to bush fires.

As back burning increases, forest loss decreases.

11 Which forest, Bellamy or Allenby, would you suggest is the larger? Explain why.

Bellamy is most likely larger as more forest was lost than Allenby despite the same hours spent.

12 More allocation of hours is required when the Bushfire index exceeds 10. Add a column called 'B.I.' to the table above, calculate the B.I. given:

$$\text{B.I.} = \frac{\%\ \text{Forest Land Lost} \times 175}{\text{Back Burn Hours}}$$

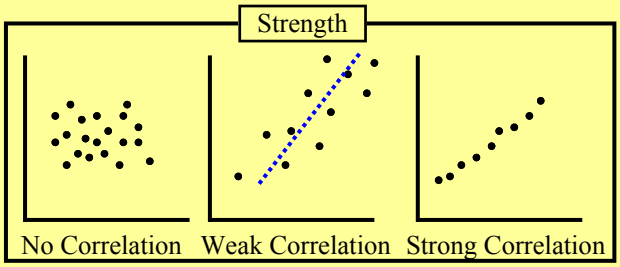
Round to the nearest whole number

13 Which forests qualify for more allocated hours? Do you have any suggestions?

Gritten and Bellamy, but Allenby also included.

Scatter Diagrams - Correlation

Correlation means ‘is there a connection’ between the data shown in a scatter diagram. This sheet introduces three categories used to describe scatter diagrams. You just have to pick which apply to the diagram in each question.



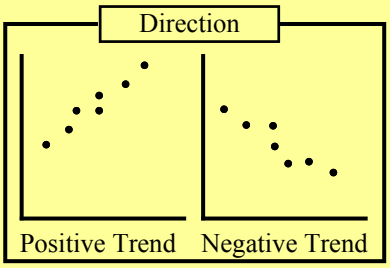
Firstly strength. There is a choice of three different words available to describe strength.

No Correlation means that the dots are scattered ‘randomly’, such as graphing two completely unrelated things.

IF THERE IS NO CORRELATION THEN NO FURTHER DESCRIPTION IS REQUIRED.

Weak correlation means that the two things have a relationship, where a line or curve can be drawn through them, but it isn’t exact. This doesn’t mean that the blue line has to pass through the points, it is saying, this is where the points are generally heading.

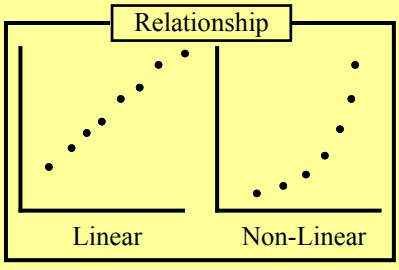
Strong correlation means that the two are very closely related and the dots clearly show this.



Secondly Direction. This requires you to indicate whether the diagram’s dots are going up to the right, which is called a positive trend.

If the dots are going down to the right then this is called a negative trend.

Lastly Relationship. If the dots are forming a line then this is called a linear relationship. If the dots are forming a curve then this is a non-linear relationship.



The first column asks you to fill in the appropriate circle describing the correlation. Remember, no correlation means you only have to fill the ‘No Correlation’ circle, then you are done.

The second column describes comparing two events and you have to create your own scatter diagram. These will may differ from person to another, once you have completed the diagram, describe it in words, of course using the three characteristics you have learned.

Scatter Diagrams - Correlation

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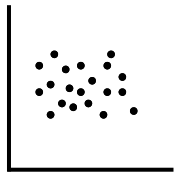
Strength



No Correlation



Weak Correlation



Strong Correlation

Direction

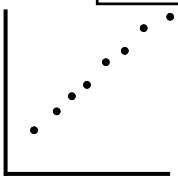


Positive Trend

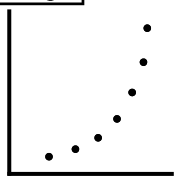


Negative Trend

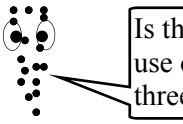
Relationship



Linear

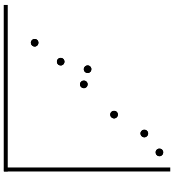


Non-Linear



Is there correlation? If so, use one of each of the three descriptors above.

Example



Strength: ○ No Correlation ○ Weak Correlation ● Strong Correlation

Direction: ○ Positive Trend ● Negative Trend

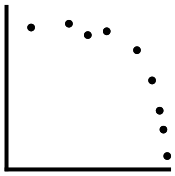
Relationship: ● Linear ○ Non-Linear

1

Strength: ○ No Correlation ○ Weak Correlation ○ Strong Correlation

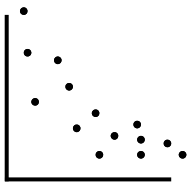
Direction: ○ Positive Trend ○ Negative Trend

Relationship: ○ Linear ○ Non-Linear



2

Strength: ○ No Correlation ○ Weak Correlation ○ Strong Correlation



Direction: ○ Positive Trend ○ Negative Trend

Relationship: ○ Linear ○ Non-Linear

3

Strength: ○ No Correlation ○ Weak Correlation ○ Strong Correlation

Direction: ○ Positive Trend ○ Negative Trend

Relationship: ○ Linear ○ Non-Linear

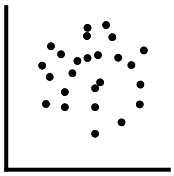


4

Strength: ○ No Correlation ○ Weak Correlation ○ Strong Correlation

Direction: ○ Positive Trend ○ Negative Trend

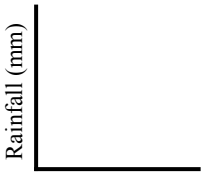
Relationship: ○ Linear ○ Non-Linear



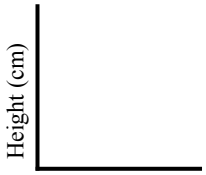


Given the situation create a possible scatter diagram and then describe the diagram as before, only with words.

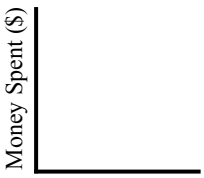
5 Rainfall vs Number of Umbrellas used.



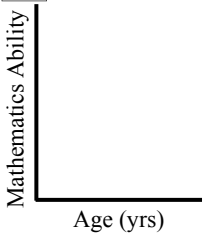
6 Student Height vs Eyesight Quality.



7 Given $20, Money Spent vs Money Saved.



8 Mathematics Ability vs Your Age



Scatter Diagrams - Correlation

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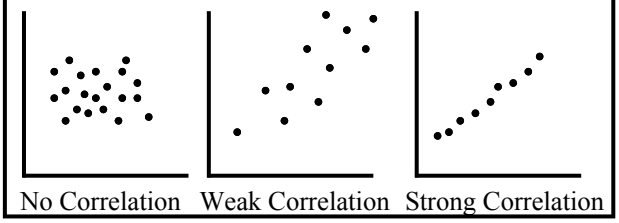
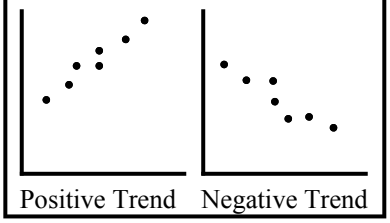
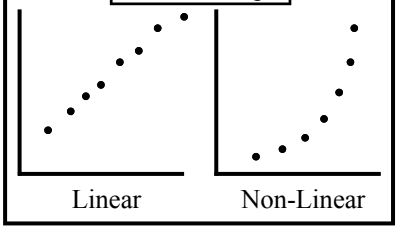
Strength



Direction



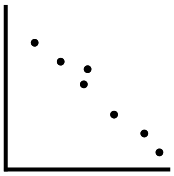
Relationship





Is there correlation? If so, use one of each of the three descriptors above.

Example

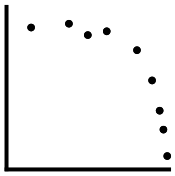


Strength: ○ No Correlation ○ Weak Correlation ● Strong Correlation

Direction: ○ Positive Trend ● Negative Trend

Relationship: ● Linear ○ Non-Linear

1

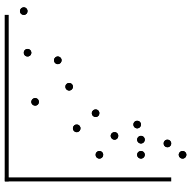


Strength: ○ No Correlation ○ Weak Correlation ● Strong Correlation

Direction: ○ Positive Trend ● Negative Trend

Relationship: ○ Linear ● Non-Linear

2



Strength: ○ No Correlation ● Weak Correlation ○ Strong Correlation

Direction: ○ Positive Trend ● Negative Trend

Relationship: ○ Linear ● Non-Linear

3



Strength: ○ No Correlation ○ Weak Correlation ● Strong Correlation

Direction: ● Positive Trend ○ Negative Trend

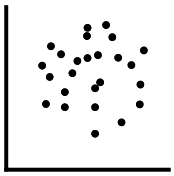
Relationship: ● Linear ○ Non-Linear

4

Strength: ● No Correlation ○ Weak Correlation ○ Strong Correlation

Direction: ○ Positive Trend ○ Negative Trend

Relationship: ○ Linear ○ Non-Linear

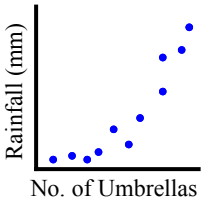




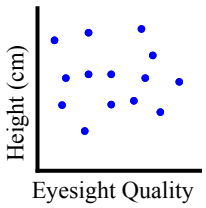
Given the situation create a possible scatter diagram and then describe the diagram as before, only with words.

5 Rainfall vs Number of Umbrellas used.

There is a weak or strong correlation, a positive trend, with a non-linear relationship.



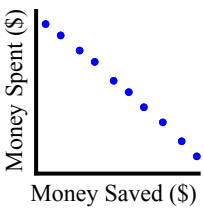
6 Student Height vs Eyesight Quality.



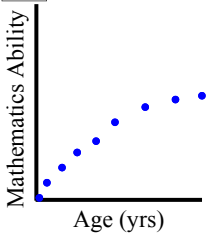
There is no correlation between student height and eyesight.

7 Given $20, Money Spent vs Money Saved.

Strong correlation with a negative trend, with a linear relationship.



8 Your Mathematics Ability vs Your Age



Hopefully strong correlation, a non-linear relationship and a positive trend.

Cumulative Frequency

Cumulative frequency 'builds' up with the frequency column, these graphs are used to help you to find the median from a frequency table using a graphical solution, this will become more clear when you learn to analyse data.

The column is simple to complete, but because it is next to the Frequency (or *f*) column it is easy to make a mistake.

Example - Adding a Cumulative Frequency Column (or *c.f.* for short)

Score	*f*	*c.f.*
0	4	4
1	2	6
2	7	13
3	4	17
$\Sigma f =$	17	check

BEFORE YOU START TOTAL THE *f* COLUMN

Write the '4' from the *f* column straight across into the *c.f.* column. This means so far there have been 4 scores.

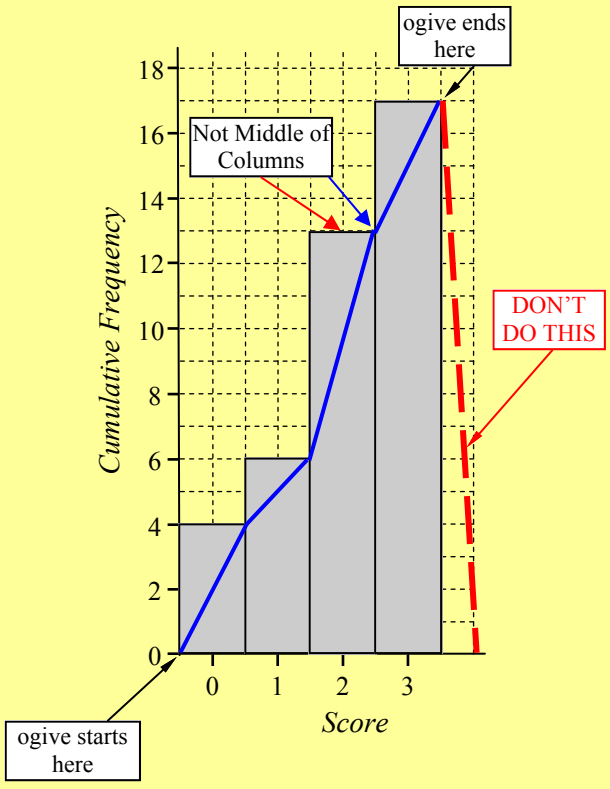
A score of '1' occurs twice. So add '2' to the '4' from the *c.f.* column. This means so far there have been 6 scores.

A score of '2' occurs seven times. So add '7' to the '6' from the *c.f.* column. This means so far there have been 13 scores.

A score of '3' occurs four times. So add '4' to the '13' from the *c.f.* column. This means there has been 17 scores.

As a final check, make sure that Σf is equal to the last number in the *c.f.* column. If they don't match, look for an error in your calculation.

When you display a cumulative frequency graph, you use a 'Cumulative Frequency Histogram'. You also include a 'Cumulative Frequency Polygon', also known as an 'Ogive'.



There are a couple of differences between a Frequency Histogram/Polygon and a Cumulative Frequency Histogram and Ogive:

- A half column gap at the start isn't required, unlike the Frequency Histogram.
- The columns always increase or stay the same height, they never reduce in height.
- The ogive interconnects the top right hand corner of the columns, not their centres.
- The ogive stops at the top right hand corner of the last column. It doesn't drop down.

Cumulative Frequency Histogram and Ogive

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Complete the cumulative frequency column in these frequency tables.

1

Score	f	$c.f.$
0	6	6
2	5	
4	3	
6	7	
8	4	
10	2	
$\Sigma f =$		

check

2

Score	f	$c.f.$
0	2	
1	5	
2	6	
3	0	
4	8	
5	5	
6	4	
7	3	
$\Sigma f =$		

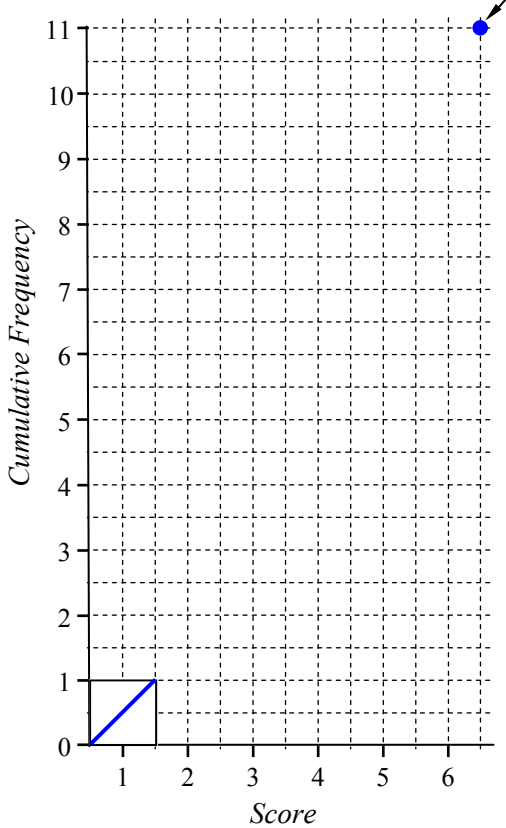
check

Use your ruler to add a $c.f.$ column then construct a $c.f.$ histogram and ogive.

3

Score	f	
1	1	
2	2	
3	0	
4	3	
5	4	
6	1	
$\Sigma f =$		

ogive ends here

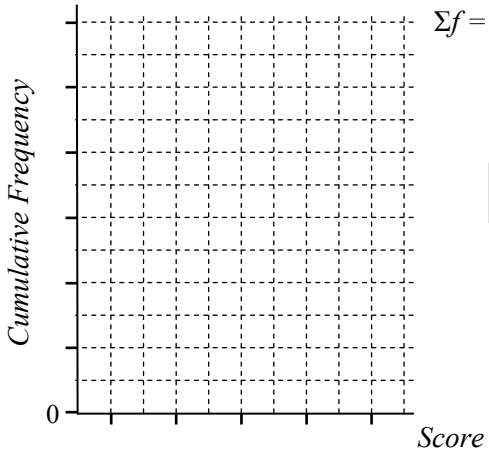


4

Score	f	
0	1	
5	1	
10	2	
15	0	
20	2	
$\Sigma f =$		

5

Score	f	
2	3	
4	1	
6	2	
8	0	
10	2	
12	1	
$\Sigma f =$		

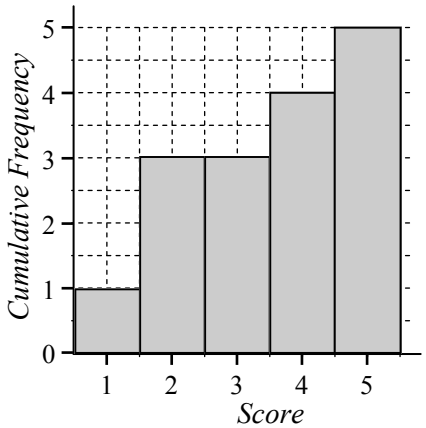


Grid for Q. 4.

Grid for Q. 5.

Given a cumulative frequency histogram, complete the frequency table. Careful!

6



Score	f
1	
2	
3	
4	
5	

Cumulative Frequency Histogram and Ogive

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Complete the cumulative frequency column in these frequency tables.

1

Score	f	$c.f.$
0	6	6
2	5	11
4	3	14
6	7	21
8	4	25
10	2	27
$\Sigma f =$	27	check

2

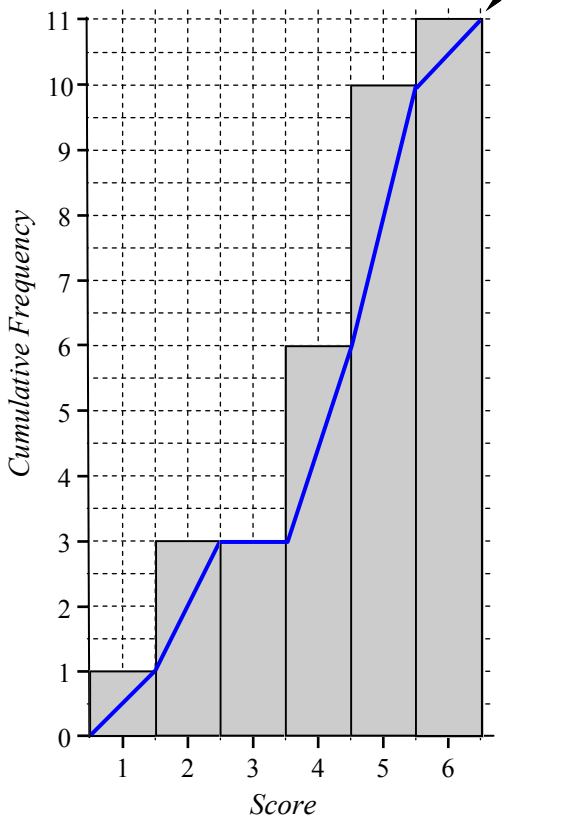
Score	f	$c.f.$
0	2	2
1	5	7
2	6	13
3	0	13
4	8	21
5	5	26
6	4	30
7	3	33
$\Sigma f =$	33	check

Use your ruler to add a *c.f.* column then construct a *c.f.* histogram and ogive.

3

Score	f	$c.f.$
1	1	1
2	2	3
3	0	3
4	3	6
5	4	10
6	1	11
$\Sigma f =$	11	check

ogive ends here

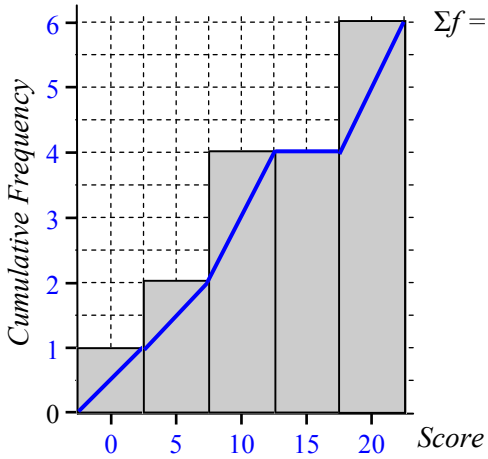


4

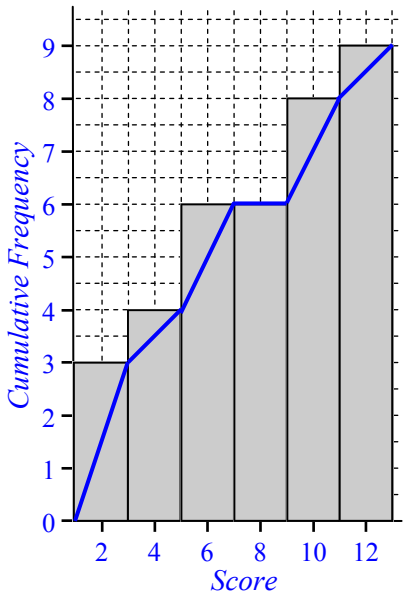
Score	f	$c.f.$
0	1	1
5	1	2
10	2	4
15	0	4
20	2	6
$\Sigma f =$	6	check

5

Score	f	$c.f.$
2	3	3
4	1	4
6	2	6
8	0	6
10	2	8
12	1	9
$\Sigma f =$	9	check



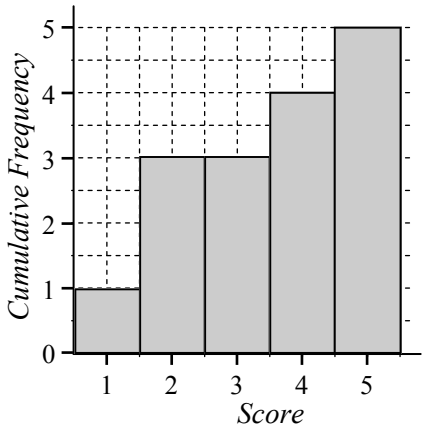
Grid for Q. 4.



Grid for Q. 5.

Given a cumulative frequency histogram, complete the frequency table. Careful!

6



Score	f
1	1
2	2
3	0
4	1
5	1

Cumulative Frequency Histogram and Ogive - With Classes

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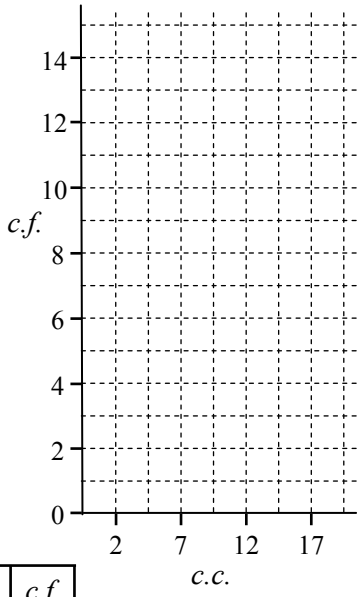
Complete the table, then display the data using a Cumulative Frequency Histogram and Ogive.

1

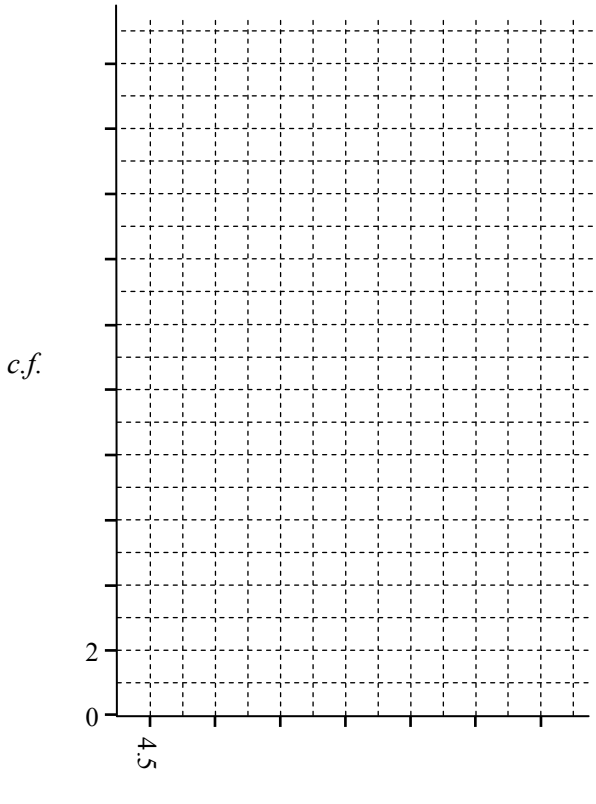
Class	*c.c.*	*f*	*c.f.*
0 - 4	2	2	
5 - 9	7	4	
10 - 14	12	3	
15 - 19	17	5	
	$\Sigma f =$		

use a ruler



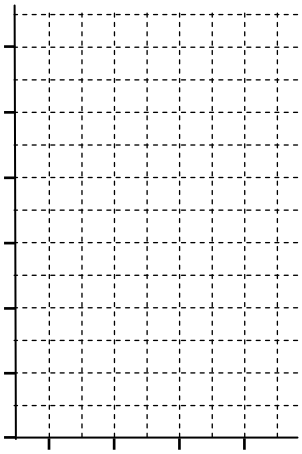
2

Class	*c.c.*	*f*	*c.f.*
0 - 9		4	
10 - 19		3	
20 - 29		2	
30 - 39		0	
40 - 49		1	
50 - 59		2	
60 - 69		5	



3

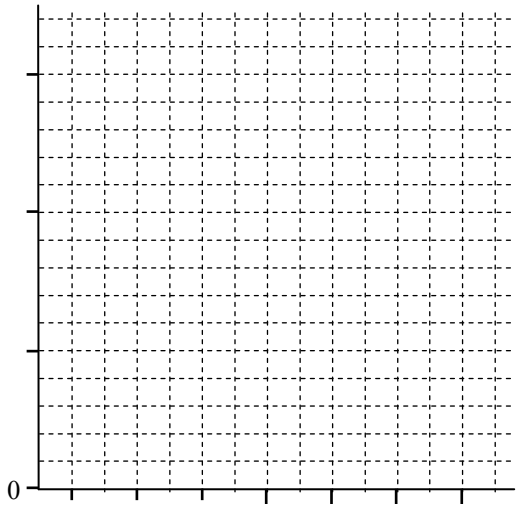
Class	*c.c.*	*f*	*c.f.*
1 - 5		3	
6 - 10		2	
11 - 15		0	
16 - 20		1	



Class test results are shown below, place them in a Frequency table with classes 31 - 40, 41 - 50, 51 - 60 and so on. Then complete a Cumulative Frequency Histogram and Ogive.

4 Data: 92, 65, 89, 64, 74, 89, 52, 95, 65, 53, 37, 57, 90, 57, 79, 64, 74, 67, 68, 66, 65, 39, 45, 83, 42, 56, 75, 48, 70, 76.

Class	*c.c.*	Tally	*f*	*c.f.*



Cumulative Frequency Histogram and Ogive - With Classes

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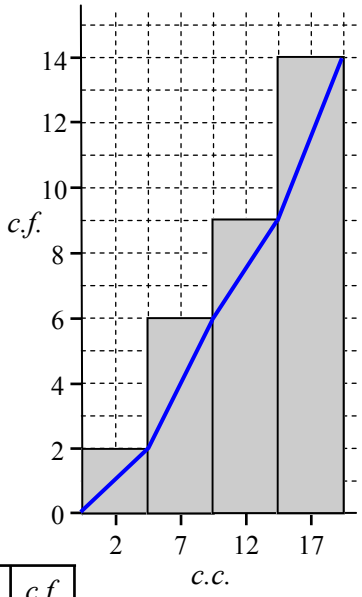
Complete the table, then display the data using a Cumulative Frequency Histogram and Ogive.

1

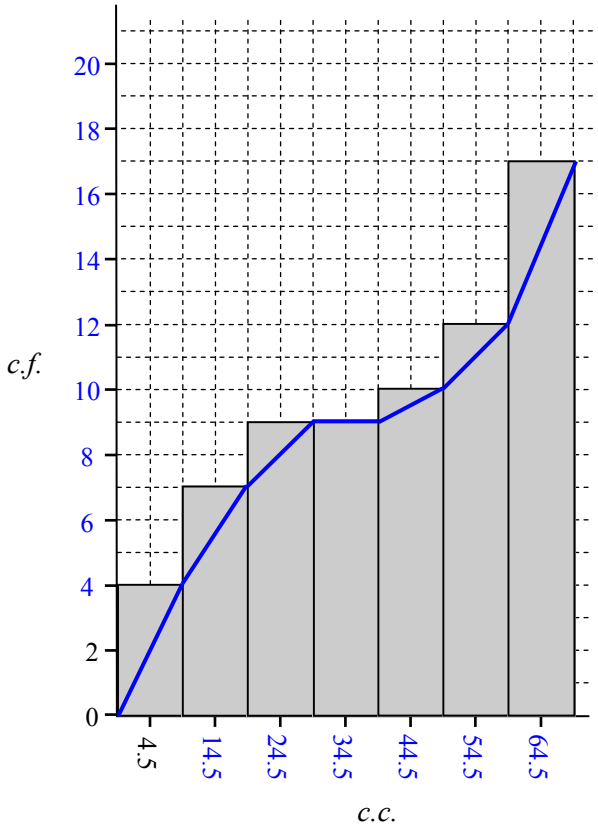
Class	*c.c.*	*f*	*c.f.*
0 - 4	2	2	2
5 - 9	7	4	6
10 - 14	12	3	9
15 - 19	17	5	14
	$\Sigma f =$	14	

use a ruler



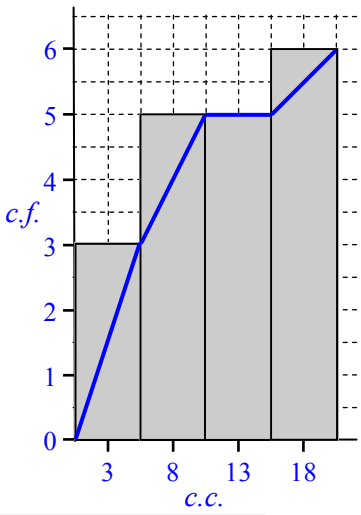
2

Class	*c.c.*	*f*	*c.f.*
0 - 9	4.5	4	4
10 - 19	14.5	3	7
20 - 29	24.5	2	9
30 - 39	34.5	0	9
40 - 49	44.5	1	10
50 - 59	54.5	2	12
60 - 69	64.5	5	17
	$\Sigma f =$	17	



3

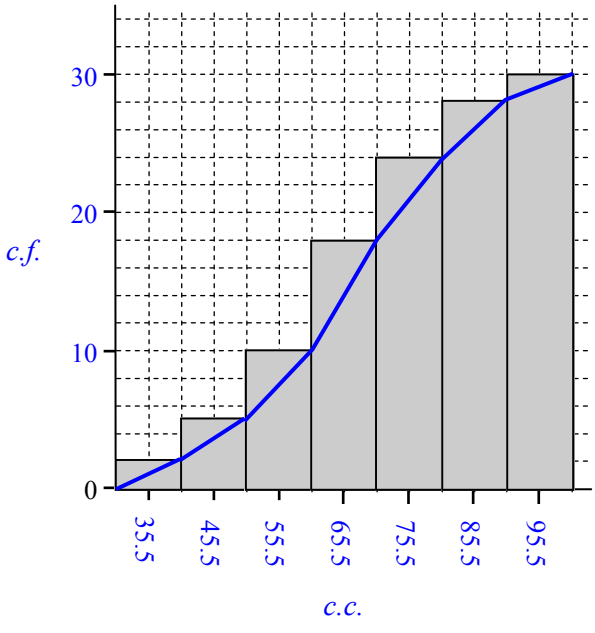
Class	*c.c.*	*f*	*c.f.*
1 - 5	3	3	3
6 - 10	8	2	5
11 - 15	13	0	5
16 - 20	18	1	6
	$\Sigma f =$	6	



Class test results are shown below, place them in a Frequency table with classes 31 - 40, 41 - 50, 51 - 60 and so on. Then complete a Cumulative Frequency Histogram and Ogive.

4 Data: 92, 65, 89, 64, 74, 89, 52, 95, 65, 53, 37, 57, 90, 57, 79, 64, 74, 67, 68, 66, 65, 39, 45, 83, 42, 56, 75, 48, 70, 76.

Class	*c.c.*	Tally	*f*	*c.f.*
31 - 40	35.5	||	2	2
41 - 50	45.5	|||	3	5
51 - 60	55.5	𝍸	5	10
61 - 70	65.5	𝍸 |||	8	18
71 - 80	75.5	𝍸 |	6	24
81 - 90	85.5	||||	4	28
91 - 100	95.5	||	2	30
		$\Sigma f =$	30	



Organising Data - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don't colour it in too dark (using a texta for example) so that you can still read it.

Organising Data - Find a Word

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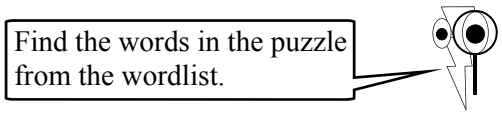

P	N	E	X	A	V	G	N	I	D	N	E	C	S	A
Q	O	O	L	S	E	S	S	A	L	C	X	A	T	A
U	G	L	C	U	M	U	L	A	T	I	V	E	V	X
P	Y	L	O	W	E	S	T	S	E	U	Q	V	E	V
L	L	C	X	W	A	R	E	U	M	U	Q	I	O	K
O	O	O	N	A	S	H	T	B	A	E	R	G	R	M
L	P	R	X	E	G	X	A	N	T	P	T	O	D	A
P	L	G	X	I	U	Y	U	M	E	A	T	S	E	R
T	V	A	H	M	A	Q	S	E	N	C	L	E	R	G
O	Q	N	B	L	T	U	E	C	A	Q	U	L	S	O
L	U	I	P	M	E	B	G	R	A	P	H	C	Y	T
P	E	S	E	L	B	A	T	X	F	T	O	K	Q	S
T	I	E	G	B	M	O	T	B	A	R	T	U	T	I
D	X	N	V	M	R	O	T	C	E	S	E	E	V	H
X	A	X	E	P	D	A	T	A	L	T	P	T	R	A

WORDLIST

SECTOR	TALLY	DATA	CLASSES	ORGANISE
DISPLAY	OGIVE	SET	POLYGON	LEAF
STEM	GRAPH	ORDER	TABLE	RAW
HISTOGRAM	DOT	FREQUENCY	STEP	SCATTER
AXES	CUMULATIVE	ANGLE	PLOT	SCORE
ASCENDING	PROTRACTOR	LOWEST	CENTRE	HIGHEST

Organising Data - Find a Word

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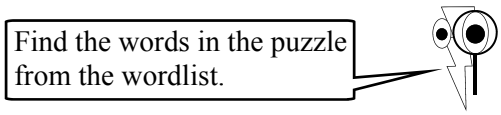

P	N	E	X	A	V	G	N	I	D	N	E	C	S	A
Q	O	O	L	S	E	S	S	A	L	C	X	A	T	A
U	G	L	C	U	M	U	L	A	T	I	V	E	V	X
P	Y	L	O	W	E	S	T	S	E	U	Q	V	E	V
L	L	C	X	W	A	R	E	U	M	U	Q	I	O	K
O	O	O	N	A	S	H	T	B	A	E	R	G	R	M
L	P	R	X	E	G	X	A	N	T	P	T	O	D	A
P	L	G	X	I	U	Y	U	M	E	A	T	S	E	R
T	V	A	H	M	A	Q	S	E	N	C	L	E	R	G
O	Q	N	B	L	T	U	E	C	A	Q	U	L	S	O
L	U	I	P	M	E	B	G	R	A	P	H	C	Y	T
P	E	S	E	L	B	A	T	X	F	T	O	K	Q	S
T	I	E	G	B	M	O	T	B	A	R	T	U	T	I
D	X	N	V	M	R	O	T	C	E	S	E	E	V	H
X	A	X	E	P	D	A	T	A	L	T	P	T	R	A

WORDLIST

~~SECTOR~~ ~~TALLY~~ ~~DATA~~ ~~CLASSES~~ ~~ORGANISE~~

~~DISPLAY~~ ~~OGIVE~~ ~~SET~~ ~~POLYGON~~ ~~LEAF~~

~~STEM~~ ~~GRAPH~~ ~~ORDER~~ ~~TABLE~~ ~~RAW~~

~~HISTOGRAM~~ ~~DOT~~ ~~FREQUENCY~~ ~~STEP~~ ~~SCATTER~~

~~AXES~~ ~~CUMULATIVE~~ ~~ANGLE~~ ~~PLOT~~ ~~SCORE~~

~~ASCENDING~~ ~~PROTRACTOR~~ ~~LOWEST~~ ~~CENTRE~~ ~~HIGHEST~~

FREEFALL MATHEMATICS

ALTITUDE

EQUATIONS & INEQUATIONS

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ALTITUDE

REVIEW:

1-STEP EQUATIONS

Solving One Step Equations
Addition and Subtraction

One Step equations are equations you solve in one line of working. They will involve a letter (pronumeral) with two numbers, you have to solve the equation for the value of the letter.

In column 1 you are given one step equations that you can solve with no working, talk them through if it helps you. Here is an example:

$d + 20 = 30$ …. So something plus 20 equals 30
Try $d = 10$ …. 10 plus 20 equals 30, so $d = 10$

Another way to do these is to think of the equation as scales, as each side is equal you have to maintain balance by finding what number would keep the scales balanced.

Column 2 is the same as column 1 except you are asked to now show working. The goal is to get the letter by itself. One of the examples at the top of the 2nd column is $b + 15 = 85$. To get the letter by itself you have to remove the + 15, how do you remove + 15? You subtract 15. So you minus 15 from the left hand side, and because it's like a set of scales, you must do the same thing on the other side of the equals sign. So you subtract 15 from $b + 15$ and you get b on its own. You subtract 15 from 85 and you get 70. The other example is $a - 9 = 27$. You have to get a by itself, to get rid of the - 9 you have to add 9. So you add 9 to each side and you get $a = 36$.

So the questions are the same as the first column but you are showing you are following a procedure by showing your working out. You may be wondering why you have to do this when often you can see the answer without working. You have to practice the process so that when you encounter more complicated problems in the future, that you can't just see the answer for, you know the correct method to use.

Column 3 is the same as column 2 except the letter is on the right hand side of the equals sign. Solve it in the same way and you get the number equal to the letter. Then rewrite your answer so that the letter is first. Again there are examples at the top of the column.

Solving One Step Equations - Addition/Subtraction

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Look at these equations and give the value for the pronumeral



1 $k + 8 = 10$ $k =$ _____

2 $b + 3 = 15$ _____

3 $d + 4 = 9$ _____

4 $x + 20 = 60$ _____

5 $t - 10 = 5$ _____

6 $m - 8 = 20$ _____

7 $h + 9 = 30$ _____

8 $l - 20 = 70$ _____

9 $w - 15 = 104$ _____

10 $s + 12 = 13$ _____

11 $u - 9 = 9$ _____

12 $q + 7 = 23$ _____

13 $n - 100 = 40$ _____

14 $y + 22 = 30$ _____

15 $a + 16 = 50$ _____

16 $e - 7 = 6$ _____

17 $z - 11 = 5$ _____

18 $c - 10 = 94$ _____

19 $j + 3 = 101$ _____

20 $v + 35 = 100$ _____

21 $t - 4 = 24$ _____

22 $m - 17 = 4$ _____

23 $x + 11 = 100$ _____

Now solve these equations showing working



Examples

$b + 15 = 85$
(-15) (-15)
$b = 70$

$a - 9 = 27$
(+9) (+9)
$a = 36$

24 $c + 15 = 22$ _____

25 $y + 11 = 28$ _____

26 $x - 9 = 34$ _____

27 $t - 7 = 18$ _____

28 $q - 20 = 17$ _____

29 $b + 10 = 11$ _____

30 $n + 26 = 26$ _____

31 $d - 20 = 77$ _____

32 $t - 14 = 33$ _____

33 $a - 11 = 59$ _____

34 $15 + g = 33$ _____

35 $w - 9 = 8$ _____

36 $x - 12 = 38$ _____

37 $18 + c = 27$ _____

38 $e + 7 = 103$ _____

39 $i - 56 = 11$ _____

40 $37 + h = 80$ _____

41 $k - 8 = 109$ _____

42 $d - 90 = 1$ _____

43 $u - 15 = 63$ _____

The letter is on the right and so these need an extra line. Find the value of the letter.



Examples

$16 = 5 + c$
(-5) (-5)
$11 = c$
$c = 11$

Then reverse the equation

$35 = e - 7$
(+7) (+7)
$42 = e$
$e = 42$

44 $20 = f - 11$ _____

45 $19 = m + 4$ _____

46 $83 = r + 16$ _____

47 $12 = n - 36$ _____

48 $64 = 19 + k$ _____

49 $11 = m - 17$ _____

50 $87 = u - 51$ _____

51 $100 = a - 9$ _____

52 $33 = 18 + s$ _____

53 $56 = d - 24$ _____

54 $75 = p + 55$ _____

55 $86 = 37 + x$ _____

56 $11 = j - 11$ _____

57 $62 = 16 + t$ _____

Solving One Step Equations - Addition/Subtraction

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Look at these equations and give the value for the pronumeral



1	$k + 8 = 10$	$k = 2$
2	$b + 3 = 15$	$b = 12$
3	$d + 4 = 9$	$d = 5$
4	$x + 20 = 60$	$x = 40$
5	$t - 10 = 5$	$t = 15$
6	$m - 8 = 20$	$m = 28$
7	$h + 9 = 30$	$h = 21$
8	$l - 20 = 70$	$l = 90$
9	$w - 15 = 104$	$w = 119$
10	$s + 12 = 13$	$s = 1$
11	$u - 9 = 9$	$u = 18$
12	$q + 7 = 23$	$q = 16$
13	$n - 100 = 40$	$n = 140$
14	$y + 22 = 30$	$y = 8$
15	$a + 16 = 50$	$a = 34$
16	$e - 7 = 6$	$e = 13$
17	$z - 11 = 5$	$z = 16$
18	$c - 10 = 94$	$c = 104$
19	$j + 3 = 101$	$j = 88$
20	$v + 35 = 100$	$v = 65$
21	$t - 4 = 24$	$t = 28$
22	$m - 17 = 4$	$m = 21$
23	$x + 11 = 100$	$x = 89$

Now solve these equations showing working



Examples

$b + 15 = 85$
(-15) (-15)
$b = 70$

$a - 9 = 27$
(+9) (+9)
$a = 36$

24	$c + 15 = 22$ - 15 - 15 $c = 7$	25 $y + 11 = 28$ - 11 - 11 $y = 17$
26	$x - 9 = 34$ + 9 + 9 $x = 43$	27 $t - 7 = 18$ + 7 + 7 $t = 25$
28	$q - 20 = 17$ + 20 + 20 $q = 37$	29 $b + 10 = 11$ - 10 - 10 b = 1
30	$n + 26 = 26$ - 26 - 26 $n = 0$	31 $d - 20 = 77$ + 20 + 20 $d = 97$
32	$t - 14 = 33$ + 14 +14 $t = 47$	33 $a - 11 = 59$ + 11 + 11 $a = 70$
34	$15 + g = 33$ - 15 - 15 $g = 18$	35 $w - 9 = 8$ + 9 + 9 $w = 17$
36	$x - 12 = 38$ + 12 + 12 $x = 50$	37 $18 + c = 27$ - 18 - 18 $c = 9$
38	$e + 7 = 103$ - 7 - 7 $e = 96$	39 $i - 56 = 11$ + 56 + 56 $i = 67$
40	$37 + h = 80$ - 37 - 37 $h = 43$	41 $k - 8 = 109$ + 8 + 8 $k = 117$
42	$d - 90 = 1$ + 90 + 90 $d = 91$	43 $u - 15 = 63$ + 15 + 15 $u = 78$



The letter is on the right and so these need an extra line. Find the value of the letter.

Examples

$16 = 5 + c$
(-5) (-5)
$11 = c$
$c = 11$

Then reverse the equation

$35 = e - 7$
(+7) (+7)
$42 = e$
$e = 42$

44	$20 = f - 11$ + 11 + 11 $31 = f$ $f = 31$	45 $19 = m + 4$ - 4 - 4 $15 = m$ $m = 15$
46	$83 = r + 16$ - 16 - 16 $67 = r$ $r = 67$	47 $12 = n - 36$ + 36 + 36 $48 = n$ $n = 48$
48	$64 = 19 + k$ - 19 - 19 $55 = k$ $k = 55$	49 $11 = m - 17$ + 17 + 17 $28 = m$ $m = 28$
50	$87 = u - 51$ + 51 + 51 $138 = u$ $u = 138$	51 $100 = a - 9$ + 9 + 9 $109 = a$ $a = 109$
52	$33 = 18 + s$ - 18 - 18 $15 = s$ $s = 15$	53 $56 = d - 24$ + 24 + 24 $80 = d$ $d = 80$
54	$75 = p + 55$ - 55 - 55 $20 = p$ $p = 20$	55 $86 = 37 + x$ - 37 - 37 $49 = x$ $x = 49$
56	$11 = j - 11$ + 11 + 11 $22 = j$ $j = 22$	57 $62 = 16 + t$ - 16 - 16 $46 = t$ $t = 46$

Further One Step Equations
Adding and Subtracting Negative Numbers

This sheet involves one step equations that involve integers (positive and negative numbers). This sheet may challenge some students. Before you attempt this sheet you should have completed the previous one step equation sheet and the folder "Integers".

Column 1 begins with 10 questions, you are asked to write in the number that satisfies the equation (makes both sides equal). This is a quick refresher on using negative numbers. The next 11 questions are much the same, except the box has been replaced by a pronumeral (letter). If you have trouble with these questions pick a number and replace the letter with it and see if it works. If it doesn't, use another number that you think is a better choice and so on. Make sure you write the letter, an '=' sign and the answer. Not the answer alone.

The second column follows the same method used with the previous one step equation sheet. See the example at the top of the column. The second part of the column has the letter on the right hand side of the equation, solve in the same way then use the additional line to reverse the answer so that the letter is on the left hand side. This is the same method used in the third column on the previous sheet.

Column 3 is different in that the sign in front of the letter is a minus sign. When you solve this equation you will have a negative sign in front of the letter. Remove it by multiplying each side by -1, this will change the signs on both sides. There are a pair of examples at the top of the column.

Just a reminder:

$+ - \;\; = - + \;\; = -$

(a plus sign before or after a minus sign is the same as a minus sign alone)

$- - = +$

(a minus sign together with another minus sign is the same as a plus sign alone)

Further One Step Equations - Add/Sub Negative Numbers

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Find the solution to these and put the number in the box

1 $3 - \square = -1$

2 $8 + \square = 5$

3 $-6 + \square = 11$

4 $-5 + \square = -1$

5 $\square - 8 = -6$

6 $\square + 3 = -10$

7 $\square + -7 = 4$

8 $\square - 8 = -8$

9 $\square + 5 = 7$

10 $\square + -3 = -1$

Now try these without working, test numbers to help you if necessary.

11 $g + 6 = 4$ $g =$

12 $n - 5 = -3$

13 $j - 2 = -10$

14 $v + 4 = -7$

15 $m + 6 = -2$

16 $y + 8 = -8$

17 $d + 10 = -7$

18 $q + 2 = 0$

19 $a - 7 = -2$

20 $e + -5 = 10$

21 $s - 7 = -7$

Now solve these equations showing working

Examples

$6 + z = -5$ (-6) (-6) $z = -11$

$t - 9 = -4$ (+9) (+9) $t = 5$

22 $m + 7 = 3$

23 $b - 4 = -6$

24 $n + 11 = -5$

25 $a - 3 = -2$

26 $q + 6 = -1$

27 $9 + r = -5$

28 $t - 4 = -3$

29 $c - -5 = -5$

30 $w - -2 = 0$

31 $n - 7 = -3$

Get the letter on its own then rewrite the equation so that the letter is on the left.

32 $15 = -7 + d$

33 $-3 = y - 3$

Then reverse the equation

34 $-20 = x + 9$

35 $-3 = 10 + u$

36 $5 = -3 + b$

37 $-1 = k + 6$

Solve these then change the sign on both sides of the equation so that it's positive

Examples

$7 - j = -6$ (-7) (-7) $-j = -13$ $j = 13$

$-h - 4 = 10$ (+4) (+4) $-h = 14$ $h = -14$

Multiply throughout by -1

38 $8 - m = -11$

39 $-v - 6 = 5$

40 $-t - 6 = 0$

41 $-7 - a = -3$

42 $5 - e = -2$

43 $-x - 11 = 2$

44 $14 - p = 40$

45 $-d - -9 = 12$

46 $-k - 8 = -15$

47 $-t - -6 = 4$

48 $14 - u = 20$

49 $-5 - c = 2$

50 $8 - a = -15$

51 $-80 - f = 35$

Further One Step Equations - Add/Sub Negative Numbers

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Find the solution to these and put the number in the box

1. 3 - [4] = - 1
2. 8 + [-3] = 5
3. -6 + [17] = 11
4. -5 + [4] = -1
5. [2] - 8 = -6
6. [-13] + 3 = -10
7. [11] + -7 = 4
8. [0] - 8 = -8
9. [2] + 5 = 7
10. [2] + -3 = -1

Now try these without working, test numbers to help you if necessary.

11. $g + 6 = 4$ — $g = -2$
12. $n - 5 = -3$ — $n = 2$
13. $j - 2 = -10$ — $j = -8$
14. $v + 4 = -7$ — $v = -11$
15. $m + 6 = -2$ — $m = -8$
16. $y + 8 = -8$ — $y = -16$
17. $d + 10 = -7$ — $d = -17$
18. $q + 2 = 0$ — $q = -2$
19. $a - 7 = -2$ — $a = 5$
20. $e + -5 = 10$ — $e = 15$
21. $s - 7 = -7$ — $s = 0$

Now solve these equations showing working

Examples

$6 + z = -5$
(-6) (-6)
$z = -11$

$t - 9 = -4$
(+9) (+9)
$t = 5$

22. $m + 7 = 3$ (-7 -7) → $m = -4$
23. $b - 4 = -6$ (+4 +4) → $b = -2$
24. $n + 11 = -5$ (-11 -11) → $n = -16$
25. $a - 3 = -2$ (+3 +3) → $a = 1$
26. $q + 6 = -1$ (-6 -6) → $q = -7$
27. $9 + r = -5$ (-9 -9) → $r = -14$
28. $t - 4 = -3$ (+4 +4) → $t = 1$
29. $c - -5 = -5$ (-5 -5) → $c = -10$
30. $w - -2 = 0$ (-2 -2) → $w = -2$
31. $n - 7 = -3$ (+7 +7) → $n = 4$

Get the letter on its own then rewrite the equation so that the letter is on the left.

32. $15 = -7 + d$ (+7 +7) → $22 = d$ → $d = 22$
33. $-3 = y - 3$ (+3 +3) → $0 = y$ → $y = 0$

Then reverse the equation

34. $-20 = x + 9$ (-9 -9) → $-29 = x$ → $x = -29$
35. $-3 = 10 + u$ (-10 -10) → $-13 = u$ → $u = -13$
36. $5 = -3 + b$ (+3 +3) → $8 = b$ → $b = 8$
37. $-1 = k + 6$ (-6 -6) → $-7 = k$ → $k = -7$

Solve these then change the sign on both sides of the equation so that it's positive

Examples

$7 - j = -6$
(-7) (-7)
$-j = -13$
$j = 13$

$-h - 4 = 10$
(+4) (+4)
$-h = 14$
$h = -14$

Multiply throughout by -1

38. $8 - m = -11$ (-8 -8) → $-m = -19$ → $m = 19$
39. $-v - 6 = 5$ (+6 +6) → $-v = 11$ → $v = -11$
40. $-t - 6 = 0$ (+6 +6) → $-t = 6$ → $t = -6$
41. $-7 - a = -3$ (+7 +7) → $-a = 4$ → $a = -4$
42. $5 - e = -2$ (-5 -5) → $-e = -7$ → $e = 7$
43. $-x - 11 = 2$ (+11 +11) → $-x = 13$ → $x = -13$
44. $14 - p = 40$ (-14 -14) → $-p = 26$ → $p = -26$
45. $-d - -9 = 12$ (-9 -9) → $-d = 3$ → $d = -3$
46. $-k - 8 = -15$ (+8 +8) → $-k = -7$ → $k = 7$
47. $-t - -6 = 4$ (-6 -6) → $-t = -2$ → $t = 2$
48. $14 - u = 20$ (-14 -14) → $-u = 6$ → $u = -6$
49. $-5 - c = 2$ (+5 +5) → $-c = 7$ → $c = -7$
50. $8 - a = -15$ (-8 -8) → $-a = -23$ → $a = 23$
51. $-80 - f = 35$ (+80 +80) → $-f = 115$ → $f = -115$

Solving One Step Equations - Multiplication

This sheet deals with one step equations that involve multiplication. The sheet starts with a column of questions that you should be able to answer by inspection without showing your working out. From column 2 onwards you are expected to show working, even if you can still solve the problems mentally.

Column 1 involves simple one step equations in that the answers are all cardinal numbers. One way to solve these is to talk them through, for example:

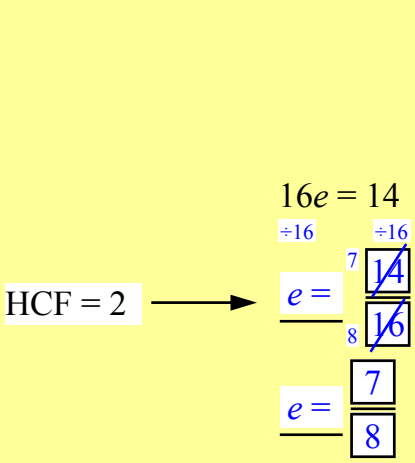
$8m = 24$ …. say 8 times something equals 24
Try $m = 3$ …. say 8 times 3 is 24 so write $m = 3$

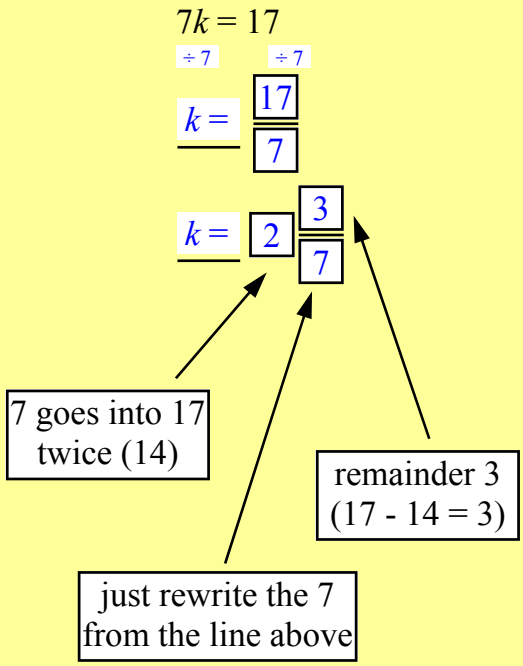
Column 2 asks you to show working and also includes fractions. These fractions won't simplify until Q 34. Follow the examples at the top of the column by dividing the number on the right hand side by the number on the left hand side (in front of the letter). Show your working as the example shows.

From Q 34 it's fractions only, this time they simplify. So use the same method to get the first line, then find the HCF (Highest Common Factor) or the largest number that goes into the top and bottom numbers and divide through.

The third column changes with the last 6 questions. These involve answers that are mixed numerals (number + fraction). Use the same method to get to the first line by dividing through. You will have an improper fraction (the numerator or top number is larger than the denominator). Then convert to mixed numerals….do you remember how? Ask yourself:

- how many times does the bottom number go into the top number, write this in the whole number box at the front
- then write the remainder (amount left over) in the top box.
- the bottom box has the same number as the bottom box in the line above.





Solving One Step Equations - Multiplication

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Look at these equations and give the value for the pronumeral

1 $4d = 8$ $d =$

2 $6h = 12$

3 $5x = 45$

4 $9g = 36$

5 $10n = 120$

6 $7e = 42$

7 $8y = 56$

8 $5t = 55$

9 $11a = 44$

10 $9q = 18$

11 $3z = 39$

12 $2w = 30$

13 $5r = 15$

14 $15p = 45$

15 $4b = 28$

16 $6c = 36$

17 $2k = 48$

18 $9y = 90$

19 $30u = 180$

20 $15x = 60$

21 $2v = 34$

22 $7j = 84$

23 $5t = 0$

Solve these equations only this time show your working out.

Examples

$6p = 54$ (÷6) (÷6) $p = 9$

$5d = 3$ (÷5) (÷5) $d = \frac{3}{5}$

24 $3e = 36$

25 $9u = 4$

26 $8a = 40$

27 $2w = 54$

28 $11q = 9$

29 $17t = 7$

30 $12c = 60$

31 $15x = 13$

32 $20m = 9$

33 $7r = 42$

These will have fraction answers that can be simplified

34 $6j = 3$

35 $8d = 6$

36 $15k = 9$

37 $20e = 12$

38 $40a = 25$

39 $12v = 10$

40 $24f = 6$

41 $20s = 16$

42 $30h = 12$

43 $35x = 28$

These can be changed from improper fractions to mixed numerals

44 $4t = 9$

45 $3c = 11$

46 $8n = 35$

47 $14d = 33$

48 $11z = 50$

49 $4k = 33$

Solving One Step Equations - Multiplication

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Look at these equations and give the value for the pronumeral

1	$4d = 8$	$d = 2$
2	$6h = 12$	$h = 2$
3	$5x = 45$	$x = 9$
4	$9g = 36$	$g = 4$
5	$10n = 120$	$n = 12$
6	$7e = 42$	$e = 6$
7	$8y = 56$	$y = 7$
8	$5t = 55$	$t = 11$
9	$11a = 44$	$a = 4$
10	$9q = 18$	$q = 2$
11	$3z = 39$	$z = 13$
12	$2w = 30$	$w = 15$
13	$5r = 15$	$r = 3$
14	$15p = 45$	$p = 3$
15	$4b = 28$	$b = 7$
16	$6c = 36$	$c = 6$
17	$2k = 48$	$k = 24$
18	$9y = 90$	$y = 10$
19	$30u = 180$	$u = 6$
20	$15x = 60$	$x = 4$
21	$2v = 34$	$v = 17$
22	$7j = 84$	$j = 12$
23	$5t = 0$	$t = 0$

Solve these equations only this time show your working out.

Examples

$6p = 54$ (÷6 ÷6) $p = 9$

$5d = 3$ (÷5 ÷5) $d = \frac{3}{5}$

24 $3e = 36$ (÷3 ÷3) $e = 12$

25 $9u = 4$ (÷9 ÷9) $u = \frac{4}{9}$

26 $8a = 40$ (÷8 ÷8) $a = 5$

27 $2w = 54$ (÷2 ÷2) $w = 27$

28 $11q = 9$ (÷11 ÷11) $q = \frac{9}{11}$

29 $17t = 7$ (÷17 ÷17) $t = \frac{7}{17}$

30 $12c = 60$ (÷12 ÷12) $c = 5$

31 $15x = 13$ (÷15 ÷15) $x = \frac{13}{15}$

32 $20m = 9$ (÷20 ÷20) $m = \frac{9}{20}$

33 $7r = 42$ (÷7 ÷7) $r = 6$

These will have fraction answers that can be simplified

34 $6j = 3$ (÷6 ÷6) $j = \frac{\cancel{3}^{1}}{\cancel{6}_{2}}$ $j = \frac{1}{2}$

35 $8d = 6$ (÷8 ÷8) $d = \frac{\cancel{6}^{3}}{\cancel{8}_{4}}$ $d = \frac{3}{4}$

36 $15k = 9$ (÷15 ÷15) $k = \frac{\cancel{9}^{3}}{\cancel{15}_{5}}$ $k = \frac{3}{5}$

37 $20e = 12$ (÷20) $e = \frac{\cancel{12}^{3}}{\cancel{20}_{5}}$ $e = \frac{3}{5}$

38 $40a = 25$ (÷40 ÷40) $a = \frac{\cancel{25}^{5}}{\cancel{40}_{8}}$ $a = \frac{5}{8}$

39 $12v = 10$ (÷12 ÷12) $v = \frac{\cancel{10}^{5}}{\cancel{12}_{6}}$ $v = \frac{5}{6}$

40 $24f = 6$ (÷24 ÷24) $f = \frac{\cancel{6}^{1}}{\cancel{24}_{4}}$ $f = \frac{1}{4}$

41 $20s = 16$ (÷20 ÷20) $s = \frac{\cancel{16}^{4}}{\cancel{20}_{5}}$ $s = \frac{4}{5}$

42 $30h = 12$ (÷30 ÷30) $h = \frac{\cancel{12}^{2}}{\cancel{30}_{5}}$ $h = \frac{2}{5}$

43 $35x = 28$ (÷35 ÷35) $x = \frac{\cancel{28}^{4}}{\cancel{35}_{5}}$ $x = \frac{4}{5}$

These can be changed from improper fractions to mixed numerals

44 $4t = 9$ (÷4 ÷4) $t = \frac{9}{4}$ $t = 2\frac{1}{4}$

45 $3c = 11$ (÷3 ÷3) $c = \frac{11}{3}$ $c = 3\frac{2}{3}$

46 $8n = 35$ (÷8 ÷8) $n = \frac{35}{8}$ $n = 4\frac{3}{8}$

47 $14d = 33$ (÷14 ÷14) $d = \frac{33}{14}$ $d = 2\frac{5}{14}$

48 $11z = 50$ (÷11 ÷11) $z = \frac{50}{11}$ $z = 4\frac{6}{11}$

49 $4k = 33$ (÷4 ÷4) $k = \frac{33}{4}$ $k = 8\frac{1}{4}$

Further One Step Equations - Multiplication

This sheet involves the use of negative numbers in one step equations. To answer this sheet you should have first completed the 'Integers' folder or already have learnt about negative numbers. The methods used in this sheet are identical to the previous sheet except that the rules for operations with negative numbers are included.

Column 1 begins with revision of operations with negative numbers, you are asked to write in the answer (with the sign) in the box provided. Remember that:

- negative × (or ÷) negative = positive
- negative × (or ÷) positive = negative
- positive × (or ÷) negative = negative

The second part of column 1 is the same as above except the box is replaced by a letter and you have to specify the value of the letter. Talk it through to yourself if it makes it easier. So if you had:

$8b = -40$: say→ eight times something gives me -40. I know 8 times 5 gives me 40, so eight times -5 must give me -40.

so $b = -5$.

Note that you can test numbers to see if they work and modify your number until you get your answer also.

Column 2 uses division by the number (including the sign) that is in front of the letter. Look at the examples at the top of the column. The questions have the letter on the right hand side in the second part of the column. Use the same method then reverse your answer, as shown in the example at the base of this sheet.

Column 3 is the same as the third column on the previous sheet. Simplify the fraction on the first line by dividing through by the HCF. From Q 44 the answers are to be expressed as mixed numerals. Have a look at the examples below.

$-35 = -7x$

÷ -7 ÷ -7

$5 = x$

$x = 5$

$-32u = 56$

÷ -32 ÷ -32

$u = \frac{56}{-32}$ (7, -4)

$u = -\frac{7}{4}$

$-5w = -19$

÷ -5 ÷ -5

$w = \frac{-19}{-5}$

$w = 3\frac{4}{5}$

Further One Step Equations - Multiplication

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Find the solution to these and place the number in the box

1 $3 \times \square = -6$

2 $5 \times \square = -20$

3 $-7 \times \square = -35$

4 $-9 \times \square = 45$

5 $-6 \times \square = -66$

6 $3 \times \square = -15$

7 $2 \times \square = -2$

8 $-5 \times \square = -50$

9 $4 \times \square = -36$

10 $1 \times \square = -1$

Now try these without working, test numbers to help you if necessary.

11 $6r = -24$ $r =$ _____

12 $-9e = 72$ _____

13 $-5q = -25$ _____

14 $2y = -4$ _____

15 $-8v = 32$ _____

16 $-3j = -33$ _____

17 $7k = -28$ _____

18 $-5c = 0$ _____

19 $-4n = -60$ _____

20 $13g = -52$ _____

21 $4p = -4$ _____

Solve these equations, only this time show your working out.

Examples

$6x = -54$ (÷6) (÷6) $x = -9$

$-3h = -33$ (÷-3) (÷-3) $h = 11$

22 $12q = -24$ _____

23 $-7t = -70$ _____

24 $15d = -45$ _____

25 $-4a = 36$ _____

26 $-10h = 50$ _____

27 $-b = 80$ _____

28 $3c = -39$ _____

29 $-8m = -48$ _____

30 $15i = -90$ _____

31 $-2y = 54$ _____

32 $-5e = -75$ _____

33 $4s = -100$ _____

Solve these then reverse the answer so that the letter is on the left.

34 $38 = -2x$ _____

35 $-28 = -4r$ _____

36 $-56 = 7w$ _____

37 $75 = -3f$ _____

The fraction answers will need to be simplified. From Q 44 mixed numeral answers are required.

38 $-20e = -8$

39 $16v = -12$

40 $-45t = 20$

41 $30h = -24$

42 $-6g = -3$

43 $60p = -48$

44 $-3a = -8$

45 $-4w = 11$

46 $5n = -12$

47 $-7b = 24$

48 $-12c = -17$

49 $-3x = 5$

Further One Step Equations - Multiplication

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Find the solution to these and place the number in the box

1 $3 \times \boxed{-2} = -6$

2 $5 \times \boxed{-4} = -20$

3 $-7 \times \boxed{5} = -35$

4 $-9 \times \boxed{-5} = 45$

5 $-6 \times \boxed{11} = -66$

6 $3 \times \boxed{-5} = -15$

7 $2 \times \boxed{-1} = -2$

8 $-5 \times \boxed{10} = -50$

9 $4 \times \boxed{-9} = -36$

10 $1 \times \boxed{-1} = -1$

Now try these without working, test numbers to help you if necessary.



11 $6r = -24$ $r = -4$

12 $-9e = 72$ $e = -8$

13 $-5q = -25$ $q = 5$

14 $2y = -4$ $y = -2$

15 $-8v = 32$ $v = -4$

16 $-3j = -33$ $j = 11$

17 $7k = -28$ $k = -4$

18 $-5c = 0$ $c = 0$

19 $-4n = -60$ $n = 15$

20 $13g = -52$ $g = -4$

21 $4p = -4$ $p = -1$



Solve these equations, only this time show your working out.

Examples

$6x = -54$

$\div 6 \quad \div 6$

$x = -9$

$-3h = -33$

$\div -3 \quad \div -3$

$h = 11$

22 $12q = -24$

$\div 12 \quad \div 12$

$q = -2$

23 $-7t = -70$

$\div -7 \quad \div -7$

$t = 10$

24 $15d = -45$

$\div 15 \quad \div 15$

$d = -3$

25 $-4a = 36$

$\div -4 \quad \div -4$

$a = -9$

26 $-10h = 50$

$\div -10 \quad \div -10$

$h = -5$

27 $-b = 80$

$\div -1 \quad \div -1$

$b = -80$

28 $3c = -39$

$\div 3 \quad \div 3$

$c = -13$

29 $-8m = -48$

$\div -8 \quad \div -8$

$m = 6$

30 $15i = -90$

$\div 15 \quad \div 15$

$i = -6$

31 $-2y = 54$

$\div -2 \quad \div -2$

$y = -27$

32 $-5e = -75$

$\div -5 \quad \div -5$

$e = 15$

33 $4s = -100$

$\div 4 \quad \div 4$

$s = -25$



Solve these then reverse the answer so that the letter is on the left.

34 $38 = -2x$

$\div -2 \quad \div -2$

$-19 = x$

$x = -19$

35 $-28 = -4r$

$\div -4 \quad \div -4$

$7 = r$

$r = 7$

36 $-56 = 7w$

$\div 7 \quad \div 7$

$-8 = w$

$w = -8$

37 $75 = -3f$

$\div -3 \quad \div -3$

$-25 = f$

$f = -25$



The fraction answers will need to be simplified. From Q 44 mixed numeral answers are required.

38 $-20e = -8$

$\div -20 \quad \div -20$

$e = \frac{\cancel{-8}^{2}}{\cancel{-20}_{5}}$

$e = \frac{2}{5}$

39 $16v = -12$

$\div 16 \quad \div 16$

$v = \frac{\cancel{-12}^{-3}}{\cancel{16}_{4}}$

$v = \frac{-3}{4}$

40 $-45t = 20$

$\div -45 \quad \div -45$

$t = \frac{\cancel{20}^{-4}}{\cancel{-45}_{9}}$

$t = \frac{-4}{9}$

41 $30h = -24$

$\div 30 \quad \div 30$

$h = \frac{\cancel{-24}^{-4}}{\cancel{30}_{5}}$

$h = \frac{-4}{5}$

42 $-6g = -3$

$\div -6 \quad \div -6$

$g = \frac{\cancel{-3}^{1}}{\cancel{-6}_{2}}$

$g = \frac{1}{2}$

43 $60p = -48$

$\div 60 \quad \div 60$

$p = \frac{\cancel{-48}^{-4}}{\cancel{60}_{5}}$

$p = \frac{-4}{5}$

44 $-3a = -8$

$\div -3 \quad \div -3$

$a = \frac{-8}{-3}$

$a = 2\frac{2}{3}$

45 $-4w = 11$

$\div -4 \quad \div -4$

$w = \frac{11}{-4}$

$w = -2\frac{3}{4}$

46 $5n = -12$

$\div 5 \quad \div 5$

$n = \frac{-12}{5}$

$n = -2\frac{2}{5}$

47 $-7b = 24$

$\div -7 \quad \div -7$

$b = \frac{24}{-7}$

$b = -3\frac{3}{7}$

48 $-12c = -17$

$\div -12 \quad \div -12$

$c = \frac{-17}{-12}$

$c = 1\frac{5}{12}$

49 $-3x = 5$

$\div -3 \quad \div -3$

$x = \frac{5}{-3}$

$x = -1\frac{2}{3}$

Solving One Step Equations - Division

These one step equations have the letter being divided by a number (or decimal). As with addition, subtraction and multiplication you will use the opposite operation to solve these exercises. The opposite sign to ÷ is ×, so you will use multiplication to solve these questions.

Column 1 starts with questions that you should be able to complete mentally. Talk it through to yourself, for example if you have $^{t}/_{6} = 4$:
$^{t}/_{6} = 4$ ….. say something divided by 6 equals 4
t = 24 ….. say 24 divided by 6 is equal to 4, so *t* = 24

Column 2 then uses multiplication to solve the problems. Look at the number that the letter is being divided by and multiply both sides by it. Examples are at the top of the column.

Column 3 uses the same solving method except decimals are involved. Follow the examples at the top of the column then see if you can answer the first 4 questions mentally. From then on when you multiply, use the multiplication working space to get your answer. Write your answer and remember to write the letter then '=' before your answer.

Solving One Step Equations - Division

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Find the value for the letter that will make these equations true.

1 $\frac{d}{2} = 5$ $d =$ _____

2 $\frac{t}{6} = 3$ _____

3 $\frac{b}{4} = 3$ _____

4 $\frac{c}{5} = 3$ _____

5 $\frac{f}{6} = 4$ _____

6 $\frac{n}{10} = 4$ _____

7 $\frac{p}{7} = 3$ _____

8 $\frac{q}{6} = 8$ _____

9 $\frac{k}{5} = 10$ _____

10 $\frac{j}{2} = 9$ _____

11 $\frac{v}{3} = 8$ _____

12 $\frac{m}{12} = 3$ _____

13 $\frac{y}{8} = 4$ _____

14 $\frac{h}{7} = 6$ _____

15 $\frac{r}{2} = 17$ _____

16 $\frac{u}{20} = 8$ _____

Now solve these equations showing working

Examples

$\frac{d}{7} = 8$ (×7) (×7)

$d = 56$

$\frac{v}{11} = 5$ (×11) (×11)

$v = 55$

17 $\frac{x}{9} = 6$ _____

18 $\frac{h}{15} = 3$ _____

19 $\frac{q}{12} = 10$ _____

20 $\frac{c}{7} = 9$ _____

21 $\frac{k}{5} = 15$ _____

22 $\frac{w}{20} = 4$ _____

23 $\frac{n}{6} = 21$ _____

24 $\frac{b}{10} = 17$ _____

25 $\frac{m}{53} = 2$ _____

26 $\frac{d}{4} = 25$ _____

27 $\frac{y}{31} = 3$ _____

28 $\frac{a}{5} = 40$ _____

29 $\frac{p}{4} = 12$ _____

30 $\frac{e}{10} = 13$ _____

31 $\frac{f}{9} = 8$ _____

32 $\frac{h}{6} = 30$ _____

This time the questions involve decimals. Try to answer without a calculator, spaces are provided.

Examples

$\frac{k}{4} = 3.2$ (×4) (×4)

$k = 12.8$

$\frac{n}{2.1} = 5$ (×2.1) (×2.1)

$v = 10.5$

33 $\frac{a}{2} = 3.4$ _____

34 $\frac{e}{4.3} = 3$ _____

35 $\frac{h}{7.1} = 6$ _____

36 $\frac{t}{3} = 5.2$ _____

37 $\frac{n}{2.9} = 7$ _____

2.9 ×
7
_____

38 $\frac{q}{4.7} = 5$ _____

×
_____

39 $\frac{t}{3} = 9.6$ _____

×
_____

40 $\frac{b}{0.8} = 4$ _____

×
_____

41 $\frac{k}{5} = 7.4$ _____

×
_____

Solving One Step Equations - Division

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Find the value for the letter that will make these equations true.

1 $\frac{d}{2} = 5$ $d = 10$

2 $\frac{t}{6} = 3$ $t = 18$

3 $\frac{b}{4} = 3$ $b = 12$

4 $\frac{c}{5} = 3$ $c = 15$

5 $\frac{f}{6} = 4$ $f = 24$

6 $\frac{n}{10} = 4$ $n = 40$

7 $\frac{p}{7} = 3$ $p = 21$

8 $\frac{q}{6} = 8$ $q = 48$

9 $\frac{k}{5} = 10$ $k = 50$

10 $\frac{j}{2} = 9$ $j = 18$

11 $\frac{v}{3} = 8$ $v = 24$

12 $\frac{m}{12} = 3$ $m = 36$

13 $\frac{y}{8} = 4$ $y = 32$

14 $\frac{h}{7} = 6$ $h = 42$

15 $\frac{r}{2} = 17$ $r = 34$

16 $\frac{u}{20} = 8$ $u = 160$



Now solve these equations showing working

Examples

$\frac{d}{7} = 8$ (× 7, × 7) $d = 56$

$\frac{v}{11} = 5$ (× 11, × 11) $v = 55$

17 $\frac{x}{9} = 6$ (× 9, × 9) $x = 54$

18 $\frac{h}{15} = 3$ (× 15, × 15) $h = 45$

19 $\frac{q}{12} = 10$ (× 12, × 12) $q = 120$

20 $\frac{c}{7} = 9$ (× 7, × 7) $c = 63$

21 $\frac{k}{5} = 15$ (× 5, × 5) $k = 75$

22 $\frac{w}{20} = 4$ (× 20, × 20) $w = 80$

23 $\frac{n}{6} = 21$ (× 6, × 6) $n = 126$

24 $\frac{b}{10} = 17$ (× 10, × 10) $b = 170$

25 $\frac{m}{53} = 2$ (× 53, × 53) $m = 106$

26 $\frac{d}{4} = 25$ (× 4, × 4) $d = 100$

27 $\frac{y}{31} = 3$ (× 31, × 31) $y = 93$

28 $\frac{a}{5} = 40$ (× 5, × 5) $a = 200$

29 $\frac{p}{4} = 12$ (× 4, × 4) $p = 48$

30 $\frac{e}{10} = 13$ (× 10, × 10) $e = 130$

31 $\frac{f}{9} = 8$ (× 9, × 9) $f = 72$

32 $\frac{h}{6} = 30$ (× 6, × 6) $h = 180$



This time the questions involve decimals. Try to answer without a calculator, spaces are provided.

Examples

$\frac{k}{4} = 3.2$ (× 4, × 4) $k = 12.8$

$\frac{n}{2.1} = 5$ (× 2.1, × 2.1) $v = 10.5$

33 $\frac{a}{2} = 3.4$ (× 2, × 2) $a = 6.8$

34 $\frac{e}{4.3} = 3$ (× 4.3, × 4.3) $e = 12.9$

35 $\frac{h}{7.1} = 6$ (× 7.1, × 7.1) $h = 42.6$

36 $\frac{t}{3} = 5.2$ (× 3, × 3) $t = 15.6$

37 $\frac{n}{2.9} = 7$ (× 2.9, × 2.9) $n = 20.3$

$$\begin{array}{r} {\scriptstyle 6} \\ 2.9 \times \\ 7 \\ \hline 20.3 \end{array}$$

38 $\frac{q}{4.7} = 5$ (× 4.7, × 4.7) $q = 23.5$

$$\begin{array}{r} {\scriptstyle 3} \\ 4.7 \times \\ 5 \\ \hline 23.5 \end{array}$$

39 $\frac{t}{3} = 9.6$ (× 3, × 3) $t = 28.8$

$$\begin{array}{r} {\scriptstyle 1} \\ 9.6 \times \\ 3 \\ \hline 28.8 \end{array}$$

40 $\frac{b}{0.8} = 4$ (× 0.8, × 0.8) $b = 3.2$

$$\begin{array}{r} {\scriptstyle 3} \\ 0.8 \times \\ 4 \\ \hline 3.2 \end{array}$$

41 $\frac{k}{5} = 7.4$ (× 5, × 5) $k = 37$

$$\begin{array}{r} {\scriptstyle 2} \\ 7.4 \times \\ 5 \\ \hline 37.0 \end{array}$$

Further One Step Division Equations and Solutions to Equations

This sheet involves using negative numbers, you should have covered 'Integers' before you attempt this sheet.

Column 1 carries on from the previous sheet only this time negative numbers are involved. Examples are at the top of the column, the method is the same as the previous sheet.

Column 2 uses substitution, (when the letter is replaced by a number). These questions aren't to find the value for the letter that makes the equation true. A value is already given and you are asked to test to see whether it is a solution. This is done by replacing the letter with the value given in each question, simplifying the left hand side of the equation (L.H.S.) and checking if it equals the right hand side (R.H.S.) of the equation.

If the same number is on both sides of the = sign then the value 'satisfies' the equation (it is a solution). If the numbers aren't the same on either side of the = sign then you write ≠ (not equal to) and say that it isn't a solution. As in the example at the top of column 2.

Column 3 follows on from column 2 except that brackets are introduced. You have to evaluate the brackets before you multiply by the outside number.

Test for $b = 6$

$2(b - 9) = -6$

$2 \times (6 - 9) = -6$ (-3)

$-6 = -6$

LHS = RHS so 6 satisfies the equation.

Further One Step Division Equations and Solutions to Equations

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Find the value of the letter that make these true.

Examples

$\frac{j}{6} = -12$ (× 6, × 6)

$j = -72$

$\frac{n}{-8} = -5$ (× -8, × -8)

$n = 40$

1 $\frac{a}{5} = -6$

2 $\frac{t}{-7} = 9$

3 $\frac{h}{-3} = -15$

4 $\frac{y}{5} = -12$

5 $\frac{t}{7} = -1$

6 $\frac{m}{-6} = -3$

7 $\frac{u}{9} = -5$

8 $\frac{b}{16} = -2$

9 $\frac{d}{-3} = -11$

10 $\frac{g}{-5} = 60$

11 $\frac{f}{20} = -12$

12 $\frac{q}{-7} = -8$

13 $\frac{m}{17} = -3$

14 $\frac{e}{15} = -5$

15 $\frac{x}{19} = -2$

16 $\frac{k}{-2} = -1$



Substitute the value for the letter to see if it is a solution for the equation

Example

$g + 17 = 40$ Try $g = 33$

$33 + 17 = 40$ LHS ≠ RHS so 33

$50 \neq 40$ is not a solution

17 Test for $n = 31$

$n - 12 = 19$

18 Test for $t = 16$

$t + 37 = 63$

19 Test for $m = 7$

$3m = 23$

20 Test for $u = -4$

$8u = 32$

21 Test for $c = -6$

$19 - c = 25$

22 Test for $q = 9$

$-5 - q = 14$

23 Test for $a = -3$

$-6a = 18$

24 Test for $k = -8$

$k^2 = 64$

25 Test for $g = 5$

$7(g + 3) = 56$

26 Test for $d = 12$

$5(9 - d) = -2$

27 Test for $x = -27$

$\frac{x}{-3} = -9$

28 Test for $e = -9$

$2(3 - e) = 24$

29 Test for $v = 3$

$5(v - 8) = -30$

Further One Step Division Equations and Solutions to Equations

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Find the value of the letter that make these true.

Examples

$\frac{j}{6} = -12$ (× 6 / × 6)

$j = -72$

$\frac{n}{-8} = -5$ (× -8 / × -8)

$n = 40$

1 $\frac{a}{5} = -6$ (× 5 / × 5)

$a = -30$

2 $\frac{t}{-7} = 9$ (× -7 / × -7)

$t = -63$

3 $\frac{h}{-3} = -15$ (× -3 / × -3)

$h = 75$

4 $\frac{y}{5} = -12$ (× 5 / × 5)

$y = -60$

5 $\frac{t}{7} = -1$ (× 7 / × 7)

$t = -7$

6 $\frac{m}{-6} = -3$ (× -6 / × -6)

$m = 18$

7 $\frac{u}{9} = -5$ (× 9 / × 9)

$u = -45$

8 $\frac{b}{16} = -2$ (× 16 / × 16)

$b = -32$

9 $\frac{d}{-3} = -11$ (× -3 / × -3)

$d = 33$

10 $\frac{g}{-5} = 60$ (× -5 / × -5)

$g = -300$

11 $\frac{f}{20} = -12$ (× 20 / × 20)

$f = -240$

12 $\frac{q}{-7} = -8$ (× -7 / × -7)

$q = 56$

13 $\frac{m}{17} = -3$ (× 17 / × 17)

$m = -51$

14 $\frac{e}{15} = -5$ (× 15 / × 15)

$e = -75$

15 $\frac{x}{19} = -2$ (× 19 / × 19)

$x = -38$

16 $\frac{k}{-2} = -1$ (× -2 / × -2)

$k = 2$

Substitute the value for the letter to see if it is a solution for the equation

Example

$g + 17 = 40$ Try $g = 33$

$33 + 17 = 40$ LHS ≠ RHS so 33

$50 \neq 40$ is not a solution

17 Test for $n = 31$

$n - 12 = 19$ LHS = RHS so 31

$31 - 12 = 19$ is a solution

$19 = 19$

18 Test for $t = 16$

$t + 37 = 63$ LHS ≠ RHS so 16

$16 + 37 = 63$ is not a solution

$53 \neq 63$

19 Test for $m = 7$

$3m = 23$ LHS ≠ RHS so 7 is

$3 \times 7 = 23$ not a solution

$21 \neq 23$

20 Test for $u = -4$

$8u = 32$ LHS ≠ RHS so

$8 \times -4 = 32$ -4 is not a

$-32 \neq 32$ solution

21 Test for $c = -6$

$19 - c = 25$ LHS = RHS so

$19 - -6 = 25$ -6 is a solution

$25 = 25$

22 Test for $q = 9$

$-5 - q = 14$ LHS ≠ RHS so 9

$-5 - 9 = 14$ is not a solution

$-14 \neq 14$

23 Test for $a = -3$

$-6a = 18$ LHS = RHS so

$-6 \times -3 = 18$ -3 is a solution

$18 = 18$

24 Test for $k = -8$

$k^2 = 64$ LHS = RHS so

$(-8)^2 = 64$ -8 is a solution

$64 = 64$

25 Test for $g = 5$

$7(g + 3) = 56$ LHS = RHS so

$7 \times (5 + 3) = 56$ (8) 5 is a solution

$56 = 56$

26 Test for $d = 12$

$5(9 - d) = -2$ LHS ≠ RHS so

$5 \times (9 - 12) =$ (-3) 12 is not a

$-15 \neq -2$ solution

27 Test for $x = -27$

$\frac{x}{-3} = -9$ LHS ≠ RHS so

-9 is not a

$\frac{-27}{-3} = -9$ solution

$9 \neq 9$

28 Test for $e = -9$

$2(3 - e) = 24$ LHS = RHS so

$2 \times (3 - -9) =$ (12) -9 is a solution

$24 = 24$

29 Test for $v = 3$

$5(v - 8) = -30$ LHS ≠ RHS so 3

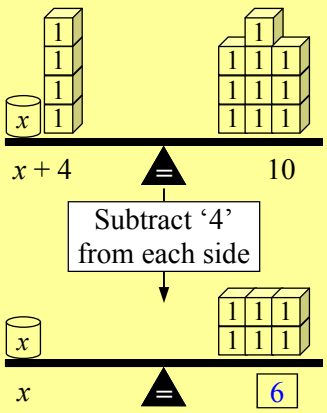
$5 \times (3 - 8) = -30$ (-5) is not a solution

$-25 \neq -30$

Equation Process Diagrams

This sheet is an introduction sheet to solving equations. As equations have an 'equals sign' in them, this means that both sides of the equation are equal, or balanced. So imagine a balance, or scales, or seesaw, with possibly different objects on each side but still weighing the same amount. If you take off the same amount from both sides, the balance is maintained. If you halve each side, or multiply each side by the same amount then the balance is maintained.

Example



The goal is to get x by itself. If you remove the 4 cubes on the left, that are with the x, then the x is left by itself, so subtract 4, but subtract from both sides to maintain the balance.

You have your answer for x. After the 4 is removed you are left with six cubes. As the weight of x and the cubes are the same, x must be 6.

The same process above with the scales is then applied to questions without the scales being there. But the approach to solve them is the same. You were told to 'subtract 4' from each side of the equation, the questions in column 2 also tell you to subtract, or divide, or add in the same way. Just follow the process.

Examples - Column 2 questions

$x - 6 = 13$

$+6 \quad +6$

$x = 19$

This question has '-6' with the x. How do you remove the -6? Add 6 and it becomes 0. So '+6' to each side and x is by itself. $13 + 6 = 19$, so $x = 19$.

$5x = 45$

$\div 5 \quad \div 5$

$x = 9$

This time there is a 5 in front of the x. This is the same as '$5 \times x$' or if you like '$x \times 5$'. The opposite to multiplying by 5 is dividing by 5. So divide the other side by 5 as well and you have your answer. $45 \div 5 = 9$, so $x = 9$.

Column 3 starts with questions that require two-steps to solve. This means that they are solved by adding or subtracting a number first, then dividing by a number to solve. So the questions are like the two questions above … only joined together.

The last questions in column 3 gives you the answer to the equation, you just have to write in the steps that have been taken to get that answer.

Equation Process Diagrams

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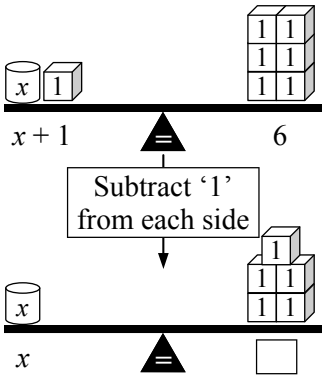
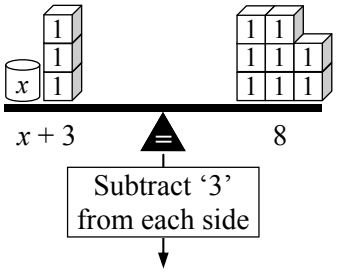
To find the mass 'x', follow the steps using the balance.

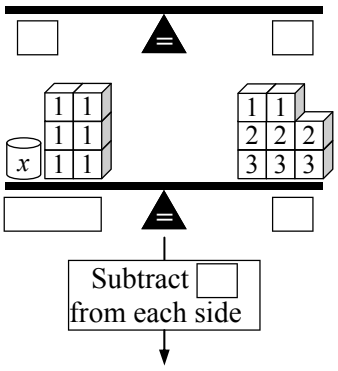
1



2

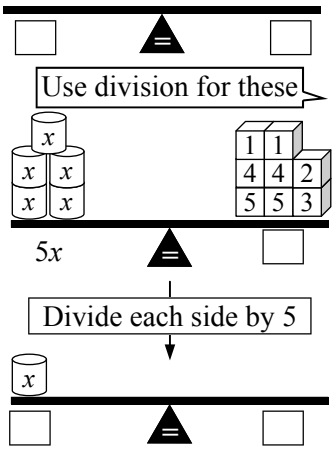


3

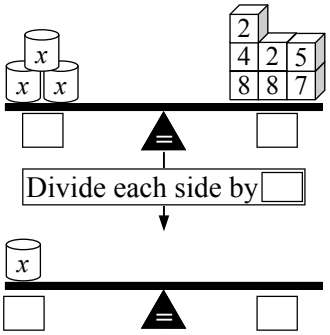


Use division for these

4

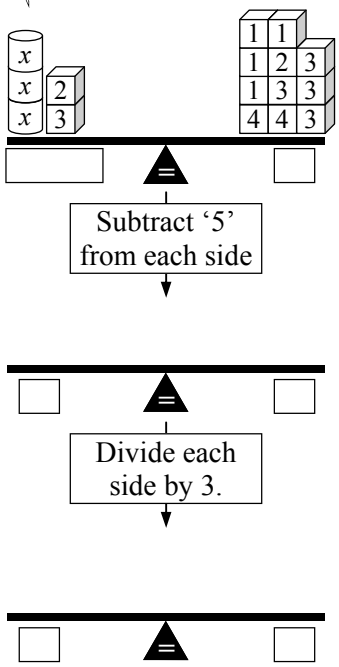


5



Combine the subtraction and division process.

6



The balance is gone, but the process is the same. Find x.

7 $x + 3 = 15$ (−3 / −3)

8 $x + 6 = -5$ (−6 / −6)

9 $x - 4 = 1$ (+4 / +4)

10 $x - 8 = 0$ (+8 / +8)

11 $x + 9 = -6$ (−9 / −9)

12 $x - 4 = -2$ (+4 / +4)

13 $6x = 18$ (÷6 / ÷6)

14 $4x = -12$ (÷4 / ÷4)

15 $7x = 35$ (÷7 / ÷7)

16 $5x = -25$ (÷5 / ÷5)

Now combine the two.

17 $2x + 3 = 17$ (−3 / −3, ÷2 / ÷2)

18 $5x - 4 = 26$ (+4 / +4, ÷5 / ÷5)

19 $6x - 2 = 4$ (+2 / +2, ÷6 / ÷6)

20 $7x + 11 = -10$ (−11 / −11, ÷7 / ÷7)

21 $10 + 2x = 4$ (−10 / −10, ÷2 / ÷2)

22 $18 - 3x = 33$ (−18 / −18, ÷-3 / ÷-3)

23 $12 + 3x = 0$ (−12 / −12, ÷3 / ÷3)

24 $16 - 2x = -2$ (−16 / −16, ÷-2 / ÷-2)

The process needs to be filled in this time. What has been done to get to the line below?

25 $x + 7 = 10$ → $x = 3$

26 $k - 5 = 9$ → $k = 14$

27 $2y + 4 = 10$ → $2y = 6$ → $y = 3$

28 $4t - 2 = 26$ → $4t = 28$ → $t = 7$

Equation Process Diagrams

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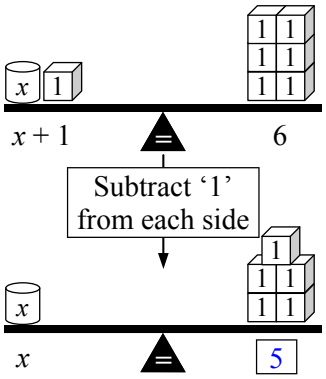
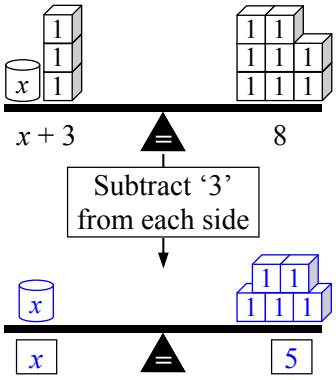
To find the mass 'x', follow the steps using the balance.

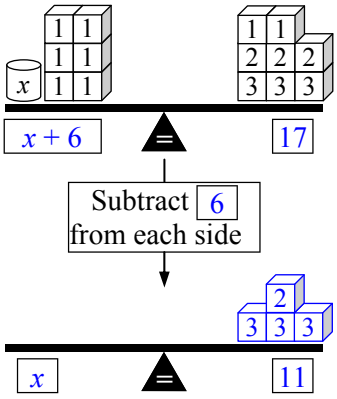
1



2

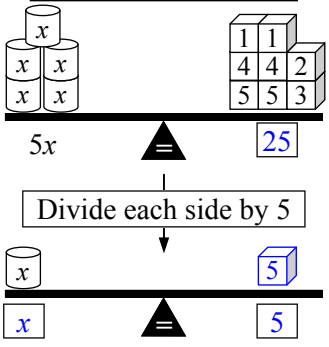


3

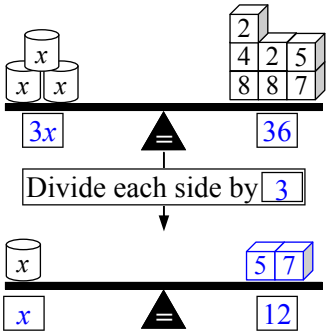


Use division for these

4

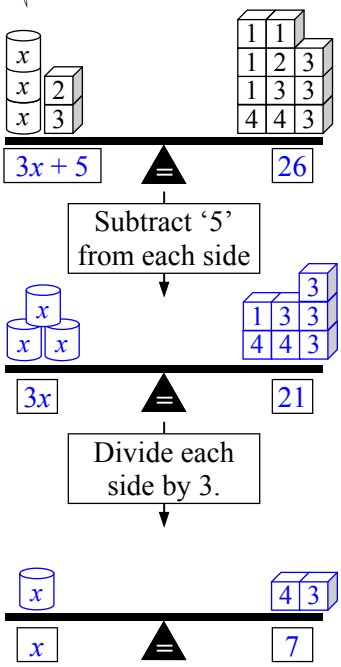


5



Combine the subtraction and division process.

6



The balance is gone, but the process is the same. Find x.

7

$x + 3 = 15$

$x = 12$

8

$x + 6 = -5$

$x = -11$

9

$x - 4 = 1$

$x = 5$

10

$x - 8 = 0$

$x = 8$

11

$x + 9 = -6$

$x = -15$

12

$x - 4 = -2$

$x = 2$

13

$6x = 18$

$x = 3$

14

$4x = -12$

$x = -3$

15

$7x = 35$

$x = 5$

16

$5x = -25$

$x = -5$

Now combine the two.

17

$2x + 3 = 17$

$2x = 14$

$x = 7$

18

$5x - 4 = 26$

$5x = 30$

$x = 6$

19

$6x - 2 = 4$

$6x = 6$

$x = 1$

20

$7x + 11 = -10$

$7x = -21$

$x = -3$

21

$10 + 2x = 4$

$2x = -6$

$x = -3$

22

$18 - 3x = 33$

$-3x = 15$

$x = -5$

23

$12 + 3x = 0$

$3x = -12$

$x = -4$

24

$16 - 2x = -2$

$-2x = -18$

$x = -9$

The process needs to be filled in this time. What has been done to get to the line below?

25

$x + 7 = 10$

$x = 3$

26

$k - 5 = 9$

$k = 14$

27

$2y + 4 = 10$

$2y = 6$

$y = 3$

28

$4t - 2 = 26$

$4t = 28$

$t = 7$

One Step Equations - Review

You may have covered 1-Step equations already and you just need a quick refresher. Well if that is the case, do we have a sheet for you! Examples are at the top of each column.

1-Step Equations - Review

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Remember to remove a +, you minus. To remove a -, you add. Solve these + and - questions.

Examples

$t - 4 = -3$ → (+4) (+4) → $t = 1$ (Show your working)

$c + 5 = 9$ → (-5) (-5) → $c = 4$ (Show your working)

1 $k + 7 = 2$

2 $d + 9 = 25$

3 $y - 3 = 11$

4 $f + 3 = 0$

5 $5 + n = -8$

6 $h - 9 = -91$

7 $w + 2 = -2$

8 $t + 8 = -7$

9 $u - 3 = 0.7$

10 $g - 1.6 = -2$

11 $1 + c = 5\frac{1}{2}$

12 $b - 4 = -\frac{2}{3}$

13 $t + 20 = -9$

14 $k - 2 = 0$

15 $e - 7 = -3.1$

16 $q - 0.2 = 0.8$

Remember to remove a ×, you divide. To remove a ÷, you multiply. Solve these.

Examples

$4y = -12$ → (÷4) (÷4) → $y = -3$ (Show your working)

$\frac{a}{5} = 6$ → (×5) (×5) → $a = 30$ (Show your working)

17 $\frac{r}{3} = 2$

18 $8y = 192$

19 $\frac{t}{6} = -7$

20 $\frac{a}{9} = 1$

21 $2c = 5$

22 $y \div 11 = 5$

23 $4k = -30$

24 $-6d = 18$

25 $\frac{v}{8} = -0.4$

26 $-3x = -24$

27 $-7u = -17.5$

28 $\frac{p}{-5} = -1$

29 $-\frac{h}{3} = \frac{2}{3}$

30 $\frac{-a}{4} = 0.85$

31 $0.3m = 12$

32 $-5.5y = -88$

These are a mix of the first two columns. Solve these.

33 $y - 15 = -12$

34 $3b = -165$

35 $\frac{m}{7} = 5.25$

36 $d + 7 = -9$

37 $e - 9 = -9$

38 $\frac{t}{15} = -5.3$

39 $-c = 20$

40 $f + 11 = -4$

The letter is on the R.H.S., solve & reverse the equation.

41 $11 = h - 3$ → (+3) (+3) → $\square = h$, $h = \square$

42 $20 = -8q$ → $\square = q$, $q = \square$

43 $0.5 = i + 7$

44 $-16 = \frac{w}{-2}$

Turn these upside-down before you do anything. Solve.

45 $\frac{3}{q} = \frac{1}{2}$ → $\frac{\square}{\square} = \square$

46 $\frac{4}{c} = \frac{2}{11}$ → $\frac{\square}{\square} = \frac{\square}{\square}$

1-Step Equations - Review

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Remember to remove a +, you minus. To remove a -, you add. Solve these + and - questions.

Examples

$t - 4 = -3$ (+4) (+4) → $t = 1$ (Show your working)

$c + 5 = 9$ (-5) (-5) → $c = 4$

1 $k + 7 = 2$ (-7) (-7) → $k = -5$

2 $d + 9 = 25$ (-9) (-9) → $d = 16$

3 $y - 3 = 11$ (+3) (+3) → $y = 14$

4 $f + 3 = 0$ (-3) (-3) → $f = -3$

5 $5 + n = -8$ (-5) (-5) → $n = -13$

6 $h - 9 = -91$ (+9) (+9) → $h = -82$

7 $w + 2 = -2$ (-2) (-2) → $w = -4$

8 $t + 8 = -7$ (-8) (-8) → $t = -15$

9 $u - 3 = 0.7$ (+3) (+3) → $u = 3.7$

10 $g - 1.6 = -2$ (+1.6) (+1.6) → $g = -0.4$

11 $1 + c = 5\frac{1}{2}$ (-1) (-1) → $c = 4\frac{1}{2}$

12 $b - 4 = -\frac{2}{3}$ (+4) → $b = 3\frac{1}{3}$

13 $t + 20 = -9$ (-20) (-20) → $t = -29$

14 $k - 2 = 0$ (+2) (+2) → $k = 2$

15 $e - 7 = -3.1$ (+7) (+7) → $e = 3.9$

16 $q - 0.2 = 0.8$ (+0.2) (+0.2) → $q = 1$

Remember to remove a ×, you divide. To remove a ÷, you multiply. Solve these.

Examples

$4y = -12$ (÷4) (÷4) → $y = -3$ (Show your working)

$\frac{a}{5} = 6$ (×5) (×5) → $a = 30$

17 $\frac{r}{3} = 2$ (×3) (×3) → $r = 6$

18 $8y = 192$ (÷8) (÷8) → $y = 24$

19 $\frac{t}{6} = -7$ (×6) (×6) → $t = -42$

20 $\frac{a}{9} = 1$ (×9) (×9) → $a = 9$

21 $2c = 5$ (÷2) (÷2) → $c = 2.5$

22 $y \div 11 = 5$ (×11) (×11) → $y = 55$

23 $4k = -30$ (÷4) (÷4) → $k = -7.5$

24 $-6d = 18$ (÷-6) (÷-6) → $d = -3$

25 $\frac{v}{8} = -0.4$ (×8) (×8) → $v = -3.2$

26 $-3x = -24$ (÷-3) (÷-3) → $x = 8$

27 $-7u = -17.5$ (÷-7) (÷-7) → $u = 2.5$

28 $\frac{p}{-5} = -1$ (×-5) (×-5) → $p = 5$

29 $-\frac{h}{3} = \frac{2}{3}$ (×-3) (×-3) → $h = -2$

30 $\frac{-a}{4} = 0.85$ (×-4) (×-4) → $a = -3.4$

31 $0.3m = 12$ (÷0.3) (÷0.3) → $m = 40$

32 $-5.5y = -88$ (÷-5.5) (÷-5.5) → $y = 16$

These are a mix of the first two columns. Solve these.

33 $y - 15 = -12$ (+15) (+15) → $y = 3$

34 $3b = -165$ (÷3) (÷3) → $b = -55$

35 $\frac{m}{7} = 5.25$ (×7) (×7) → $m = 36.75$

36 $d + 7 = -9$ (-7) (-7) → $d = -16$

37 $e - 9 = -9$ (+9) (+9) → $e = 0$

38 $\frac{t}{15} = -5.3$ (×15) (×15) → $7 = -79.5$

39 $-c = 20$ (÷-1) (÷-1) → $c = -20$

40 $f + 11 = -4$ (-11) (-11) → $f = -15$

The letter is on the R.H.S., solve & reverse the equation.

41 $11 = h - 3$ (+3) (+3) → $\boxed{14} = h$ → $h = \boxed{14}$

42 $20 = -8q$ (÷-8) (÷-8) → $\boxed{-2.5} = q$ → $q = \boxed{-2.5}$

43 $0.5 = i + 7$ (-7) (-7) → $-6.5 = i$ → $i = -6.5$

44 $-16 = \frac{w}{-2}$ (×-2) (×-2) → $32 = w$ → $w = 32$

Turn these upside-down before you do anything. Solve.

45 $\frac{3}{q} = \frac{1}{2}$ → $\frac{\boxed{q}}{\boxed{3}} = \boxed{2}$ (×3) (×3) → $q = 6$

46 $\frac{4}{c} = \frac{2}{11}$ → $\frac{\boxed{c}}{\boxed{4}} = \frac{\boxed{11}}{\boxed{2}}$ (×4) (×4) → $c = 22$

Two Step Equations (Sheet 1)

You solve two-step equations in two steps …. pick yourself up off the floor. The equations in this worksheet have an algebraic term (number with a letter) and a constant, (a number by itself). The process is to first remove the constant, then the coefficient (number in front of or below the letter). There are two different types of equations in this sheet, examples are below.

Example, Solve $4n - 2 = 10$.

$4n - 2 = 10$ — Remove the constant first, so remove the -2. So what is the opposite to -2? It's +2!

$(+2) \quad (+2)$

$4n = 12$

$(\div 4) \quad (\div 4)$ — Remove the coefficient, so get rid of 4! Use ÷4.

$n = 3$

Example, Solve $4n + 2 = 10$.

$4n + 2 = 10$ — Remove the constant first, so remove the +2. So what is the opposite to +2? It's -2!

$(-2) \quad (-2)$

$4n = 8$

$(\div 4) \quad (\div 4)$ — Remove the coefficient, so get rid of 4! Use ÷4.

$n = 2$

Example, Solve $-4n - 2 = 10$.

$-4n - 2 = 10$ — Remove the constant first, so remove the -2. So what is the opposite to -2? It's +2!

$(+2) \quad (+2)$

$-4n = 12$

$(\div -4) \quad (\div -4)$ — Remove the coefficient, so get rid of -4! Use ÷-4.

$n = -3$

Example, Solve $-4n + 2 = -10$.

$-4n + 2 = -10$ — Remove the constant first, so remove the +2. So what is the opposite to +2? It's -2!

$(-2) \quad (-2)$

$-4n = -12$

$(\div -4) \quad (\div -4)$ — Remove the coefficient, so get rid of -4! Use ÷-4.

$n = 3$

The second type of equation has a fractional coefficient. So there is a number under the letter.

Example, Solve $\frac{t}{2} - 4 = 6$

$\frac{t}{2} - 4 = 6$ — Remove the constant first, so remove the -4. So what is the opposite to -4? It's +4!

$(+4) \quad (+4)$

$\frac{t}{2} = 10$

$(\times 2) \quad (\times 2)$ — Remove the denominator, so get rid of the ÷2! Use ×2.

$t = 20$

Example, Solve $\frac{t}{2} + 4 = 6$

$\frac{t}{2} + 4 = 6$ — Remove the constant first, so remove the +4. So what is the opposite to +4? It's -4!

$(-4) \quad (-4)$

$\frac{t}{2} = 2$

$(\times 2) \quad (\times 2)$ — Remove the denominator, so get rid of the ÷2! Use ×2.

$t = 4$

Example, Solve $-\frac{t}{2} - 4 = 6$

$-\frac{t}{2} - 4 = 6$ — Remove the constant first, so remove the -4. So what is the opposite to -4? It's +4!

$(+4) \quad (+4)$

$\frac{t}{-2} = 10$ — Write the negative sign on the denominator to remind you what you have to multiply by.

$(\times -2) \quad (\times -2)$

$t = -20$ — Remove the denominator, so get rid of the ÷-2! Use ×-2.

Example, Solve $-\frac{t}{2} + 4 = -6$

$-\frac{t}{2} + 4 = -6$ — Remove the constant first, so remove the +4. So what is the opposite to +4? It's -4!

$(-4) \quad (-4)$

$\frac{t}{-2} = -10$

$(\times -2) \quad (\times -2)$ — Remove the denominator, so get rid of the ÷-2! Use ×-2.

$t = 20$

The columns start off with examples and also questions with the operation required already supplied. Column 3 is a mixture of the first two columns.

Two Step Equations (Sheet 1)

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Follow the operation in the circle to solve these 2-Step Equations.

Example

$3x + 5 = 23$

(-5) (-5) ← Remove the number that is with the algebraic term first, get rid of 5!

$3x = 18$

(÷3) (÷3) ← Remove the coefficient, so get rid of 3! Use ÷.

$x = 6$

1 $2n + 3 = 17$

(-3) (-3)

(÷2) (÷2)

2 $7y - 4 = -32$

(+4) (+4)

(÷7) (÷7)

3 $-5k - 2 = -12$

(+2) (+2)

(÷-5) (÷-5)

4 $9w - 7 = 11$

(+7) (+7)

(÷9) (÷9)

5 $6t + 9 = -15$

(-9) (-9)

(÷6) (÷6)

6 $4 - 2x = -46$

(-4) (-4)

(÷-2) (÷-2)

Now attempt these without the hints.

9 $3g + 15 = -12$

8 $-4j + 8 = 20$

9 $10t - 9 = -29$

10 $-6c + 12 = -12$

This time the coefficient is a fraction. Solve these using '×'.

Example

$\frac{k}{8} - 2 = 9$

(+2) (+2) ← Remove the number that is with the algebraic term first, get rid of the -2.

$\frac{k}{8} = 11$

(×8) (×8) ← Remove the fraction, so multiply by 8.

$k = 88$

11 $\frac{n}{4} + 5 = 12$

(-5) (-5)

$\frac{\square}{\square} = \square$

(×4) (×4)

12 $\frac{d}{5} - 3 = -4$

(+3) (+3)

$\frac{\square}{\square} = \square$

(×5) (×5)

13 $-\frac{t}{6} + 4 = -8$

(-4) (-4)

$\frac{\square}{\square} = \square$

(×-6) (×-6)

14 $\frac{-c}{3} - 10 = -6$

(+10) (+10)

$\frac{\square}{\square} = \square$

(×-3) (×-3)

15 $\frac{e}{2} - 9 = 5$

16 $\frac{h}{7} + 12 = 3$

17 $-\frac{u}{5} + 13 = 0$

18 $\frac{m}{-2} + 4 = -11$

19 $\frac{w}{15} - 8 = -5$

20 $\frac{-a}{6} + 7 = -2$

These are a mix of the first two columns. Solve.

21 $7k - 9 = -23$

22 $\frac{t}{8} - 3 = 0$

23 $-2y + 35 = -9$

24 $\frac{n}{3} - 14 = 3$

25 $-\frac{a}{2} + 7 = -2$

26 $\frac{k}{5} - 12 = -10$

27 $9n + 13 = -14$

28 $-y + 9 = -15$

29 $\frac{c}{-2} - 8 = -8$

30 $23 - 6d = -7$

31 $6 - 5h = -9$

32 $-11 - \frac{w}{3} = 7$

Two Step Equations (Sheet 1)

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Follow the operation in the circle to solve these 2-Step Equations.

Example

$3x + 5 = 23$

(-5) (-5) ← Remove the number that is with the algebraic term first, get rid of 5!

$3x = 18$

(÷3) (÷3) ← Remove the coefficient, so get rid of 3! Use ÷.

$x = 6$

1

$2n + 3 = 17$

(-3) (-3)

$2n = 14$

(÷2) (÷2)

$n = 7$

2

$7y - 4 = -32$

(+4) (+4)

$7y = -28$

(÷7) (÷7)

$y = -4$

3

$-5k - 2 = -12$

(+2) (+2)

$-5k = -10$

(÷-5) (÷-5)

$k = 2$

4

$9w - 7 = 11$

(+7) (+7)

$9w = 18$

(÷9) (÷9)

$w = 2$

5

$6t + 9 = -15$

(-9) (-9)

$6t = -24$

(÷6) (÷6)

$t = -4$

6

$4 - 2x = -46$

(-4) (-4)

$-2x = -50$

(÷-2) (÷-2)

$x = 25$

Now attempt these without the hints.

9

$3g + 15 = -12$

(-15) (-15)

$3g = -27$

(÷3) (÷3)

$g = -9$

8

$-4j + 8 = 20$

(-8) (-8)

$-4j = 12$

(÷-4) (÷-4)

$j = -3$

9

$10t - 9 = -29$

(+9) (+9)

$10t = -20$

(÷10) (÷10)

$t = -2$

10

$-6c + 12 = -12$

(-12) (-12)

$-6c = -24$

(÷-6) (÷-6)

$c = 4$

This time the coefficient is a fraction. Solve these using '×'.

Example

$\frac{k}{8} - 2 = 9$

(+2) (+2) ← Remove the number that is with the algebraic term first, get rid of the -2.

$\frac{k}{8} = 11$

(×8) (×8) ← Remove the fraction, so multiply by 8.

$k = 88$

11

$\frac{n}{4} + 5 = 12$

(-5) (-5)

$\frac{n}{4} = 7$

(×4) (×4)

$n = 28$

12

$\frac{d}{5} - 3 = -4$

(+3) (+3)

$\frac{d}{5} = -1$

(×5) (×5)

$d = -5$

13

$-\frac{t}{6} + 4 = -8$

(-4) (-4)

$\frac{t}{-6} = -12$

(×-6) (×-6)

$t = 72$

14

$\frac{-c}{3} - 10 = -6$

(+10) (+10)

$\frac{-c}{3} = 4$

(×-3) (×-3)

$c = -12$

15

$\frac{e}{2} - 9 = 5$

(+9) (+9)

$\frac{e}{2} = 14$

(×2) (×2)

$e = 28$

16

$\frac{h}{7} + 12 = 3$

(-12) (-12)

$\frac{h}{7} = -9$

(×7) (×7)

$h = -63$

17

$-\frac{u}{5} + 13 = 0$

(-13) (-13)

$\frac{u}{-5} = -13$

(×-5) (×-5)

$u = 65$

18

$\frac{m}{-2} + 4 = -11$

(-4) (-4)

$\frac{m}{-2} = -15$

(×-2) (×-2)

$m = 30$

19

$\frac{w}{15} - 8 = -5$

(+8) (+8)

$\frac{w}{15} = 3$

(×15) (×15)

$w = 45$

20

$\frac{-a}{6} + 7 = -2$

(-7) (-7)

$\frac{-a}{6} = -9$

(×-6) (×-6)

$a = 54$

These are a mix of the first two columns. Solve.

21

$7k - 9 = -23$

(+9) (+9)

$7k = -14$

(÷7) (÷7)

$k = -2$

22

$\frac{t}{8} - 3 = 0$

(+3) (+3)

$\frac{t}{8} = 3$

(×8) (×8)

$t = 24$

23

$-2y + 35 = -9$

(-35) (-35)

$-2y = -44$

(÷-2) (÷-2)

$y = 22$

24

$\frac{n}{3} - 14 = 3$

(+14) (+14)

$\frac{n}{3} = 17$

(×3) (×3)

$n = 51$

25

$-\frac{a}{2} + 7 = -2$

(-7) (-7)

$\frac{a}{-2} = -9$

(×-2) (×-2)

$a = 18$

26

$\frac{k}{5} - 12 = -10$

(+12) (+12)

$\frac{k}{5} = 2$

(×5) (×5)

$k = 10$

27

$9n + 13 = -14$

(-13) (-13)

$9n = -27$

(÷9) (÷9)

$n = -3$

28

$-y + 9 = -15$

(-9) (-9)

$-y = -24$

(÷-1) (÷-1)

$y = 24$

29

$\frac{c}{-2} - 8 = -8$

(+8) (+8)

$\frac{c}{-2} = 0$

(×-2) (×-2)

$c = 0$

30

$23 - 6d = -7$

(-23) (-23)

$-6d = -30$

(÷-6) (÷-6)

$d = 5$

31

$6 - 5h = -9$

(-6) (-6)

$-5h = -15$

(÷-5) (÷-5)

$h = 3$

32

$-11 - \frac{w}{3} = 7$

(+11) (+11)

$\frac{w}{-3} = 18$

(×-3) (×-3)

$w = -54$

Two Step Equations (Sheet 2)

This sheet involves division in two-step equations. The whole L.H.S. (Left-hand-side) of the equation has a denominator, so the process is to remove the denominator first. The previous sheet had you removing the constant first then the denominator. So the process is reversed, so try to remember the difference. Below are examples of the two different types of equations you will be solving on the worksheet.

Column 1 Typical Questions

Example, Solve $\frac{n-8}{7} = 2$

$\frac{n-8}{7} = 2$

$\times 7 \quad \times 7$ ← Remove the denominator first, so remove the 7. So what is the opposite to ÷7? It's ×7!

$n - 8 = 16$

$+8 \quad +8$ ← It is now a 1-Step Equation, remove the -8 by +8.

$n = 24$

Example, Solve $\frac{n-8}{-7} = 2$

$\frac{n-8}{-7} = 2$

$\times -7 \quad \times -7$ ← Remove the denominator first, so remove the -7. So what is the opposite to ÷-7? It's ×-7!

$n - 8 = -14$

$+8 \quad +8$ ← It is now a 1-Step Equation, remove the -8 by +8.

$n = -6$

Column 2 Typical Questions

Example, Solve $\frac{2m}{3} = 4$

$\frac{2m}{3} = 4$

$\times 3 \quad \times 3$ ← Remove the denominator first, so remove the 3. So what is the opposite to ÷3? It's ×3!

$2m = 12$

$\div 2 \quad \div 2$ ← It is now a 1-Step Equation, remove the 2 by ÷2.

$m = 6$

Example, Solve $\frac{-6m}{5} = -12$

$\frac{-6m}{5} = -12$

$\times 5 \quad \times 5$ ← Remove the denominator first, so remove the 5. So what is the opposite to ÷5? It's ×5!

$-6m = -60$

$\div -6 \quad \div -6$ ← It is now a 1-Step Equation, remove the -6 by ÷-6.

$m = 10$

Column 3 is a mix of the first two columns.

Two Step Equations (Sheet 2)

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Follow the operation in the circle to solve these 2-Step Equations.

Example

$\frac{x+4}{3} = 9$

$\times 3$ $\times 3$ ← Remove the denominator first. It becomes a 1-Step!

$x + 4 = 27$

-4 -4 ← Remove the constant.

$x = 23$

1 $\frac{a+2}{5} = 3$

$\times 5$ $\times 5$

-2 -2

2 $\frac{n-6}{2} = 8$

$\times 2$ $\times 2$

$+6$ $+6$

3 $\frac{k-5}{7} = -9$

$\times 7$ $\times 7$

$+5$ $+5$

4 $\frac{3+t}{-4} = -2$

$\times -4$ $\times -4$

-3 -3

5 $\frac{b+2}{3} = 5$

6 $\frac{w-3}{2} = 8$

7 $\frac{c-5}{7} = -4$

8 $\frac{5+y}{-2} = -6$

9 $\frac{1+d}{-5} = -3$

10 $\frac{t-2}{-2} = 0$

Solve these 2-Step Equations by removing the denominator, then the numerator.

Example

$\frac{2x}{3} = 6$

$\times 3$ $\times 3$ ← Remove the denominator first.

$2x = 18$

$\div 2$ $\div 2$ ← Then remove the coefficient to solve.

$x = 9$

11 $\frac{3g}{5} = 6$

$\times 5$ $\times 5$

$\div 3$ $\div 3$

12 $\frac{-f}{9} = 1$

$\times 9$ $\times 9$

$\div -1$ $\div -1$

13 $\frac{2e}{7} = 6$

14 $\frac{6k}{5} = -12$

15 $\frac{4c}{-3} = -8$

16 $\frac{-7w}{2} = 17.5$

You can answer these faster by using the reciprocal. Invert that fraction!

17 $\frac{2b}{5} = 4$

$\times \frac{5}{2}$ $\times \frac{5}{2}$

$b =$ ☐

18 $\frac{3u}{2} = -6$

$\times \frac{2}{3}$ $\times \frac{2}{3}$

$u =$ ☐

19 $\frac{3x}{8} = 12$

20 $\frac{5h}{6} = 10$

21 $\frac{-3a}{4} = -9$

22 $\frac{8v}{-3} = 16$

Now for a mix, solve these.

23 $\frac{x+7}{3} = 9$

24 $\frac{2n}{7} = 6$

You may not need to use the 3rd line.

25 $\frac{5h}{3} = 15$

26 $\frac{u-3}{2} = -11$

27 $\frac{k+6}{-4} = 3$

28 $\frac{-2y}{5} = 8$

29 $\frac{-a}{3} = -4$

30 $\frac{3e}{-5} = -6$

31 $\frac{2+y}{6} = 2.5$

32 $\frac{a-5}{10} = 3.1$

33 $\frac{-u}{5} = 1$

34 $\frac{q-1}{-8} = 5.5$

Two Step Equations (Sheet 2)

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Follow the operation in the circle to solve these 2-Step Equations.

Example

$\frac{x+4}{3} = 9$ (×3) (×3)

$x + 4 = 27$ (-4) (-4)

$x = 23$

Remove the denominator first. It becomes a 1-Step!

Remove the constant.

1 $\frac{a+2}{5} = 3$ (×5) (×5) $a + 2 = 15$ (-2) (-2) $a = 13$

2 $\frac{n-6}{2} = 8$ (×2) (×2) $n - 6 = 16$ (+6) (+6) $n = 22$

3 $\frac{k-5}{7} = -9$ (×7) (×7) $k - 5 = -63$ (+5) (+5) $k = -58$

4 $\frac{3+t}{-4} = -2$ (×-4) (×-4) $3 + t = 8$ (-3) (-3) $t = 5$

5 $\frac{b+2}{3} = 5$ (×3) (×3) $b + 2 = 15$ (-2) (-2) $b = 13$

6 $\frac{w-3}{2} = 8$ (×2) (×2) $w - 3 = 16$ (+3) (+3) $w = 19$

7 $\frac{c-5}{7} = -4$ (×7) (×7) $c - 5 = -28$ (+5) (+5) $c = -23$

8 $\frac{5+y}{-2} = -6$ (×-2) (×-2) $5 + y = 12$ (-5) (-5) $y = 7$

9 $\frac{1+d}{-5} = -3$ (×-5) (×-5) $1 + d = 15$ (-1) (-1) $d = 14$

10 $\frac{t-2}{-2} = 0$ (×-2) (×-2) $t - 2 = 0$ (+2) (+2) $t = 2$

Solve these 2-Step Equations by removing the denominator, then the numerator.

Example

$\frac{2x}{3} = 6$ (×3) (×3)

$2x = 18$ (÷2) (÷2)

$x = 9$

Remove the denominator first.

Then remove the coefficient to solve.

11 $\frac{3g}{5} = 6$ (×5) (×5) $3g = 30$ (÷3) (÷3) $g = 10$

12 $\frac{-f}{9} = 1$ (×9) (×9) $-f = 9$ (÷-1) (÷-1) $f = -9$

13 $\frac{2e}{7} = 6$ (×7) (×7) $2e = 42$ (÷2) (÷2) $e = 21$

14 $\frac{6k}{5} = -12$ (×5) (×5) $6k = -60$ (÷6) (÷6) $k = -10$

15 $\frac{4c}{-3} = -8$ (×-3) (×-3) $4c = 24$ (÷4) (÷4) $c = 6$

16 $\frac{-7w}{2} = 17.5$ (×2) (×2) $-7w = 35$ (÷-7) (÷-7) $w = -5$

You can answer these faster by using the reciprocal. Invert that fraction!

17 $\frac{2b}{5} = 4$ (×$\frac{5}{2}$) (×$\frac{5}{2}$) $b =$ 10

18 $\frac{3u}{2} = -6$ (×$\frac{2}{3}$) (×$\frac{2}{3}$) $u =$ -4

19 $\frac{3x}{8} = 12$ (×$\frac{8}{3}$) (×$\frac{8}{3}$) $x = 32$

20 $\frac{5h}{6} = 10$ (×$\frac{6}{5}$) (×$\frac{6}{5}$) $h = 12$

21 $\frac{-3a}{4} = -9$ (×$\frac{-4}{3}$) (×$\frac{-4}{3}$) $a = 12$

22 $\frac{8v}{-3} = 16$ (×$\frac{-3}{8}$) (×$\frac{-3}{8}$) $v = -6$

Now for a mix, solve these.

23 $\frac{x+7}{3} = 9$ (×3) (×3) $x + 7 = 27$ (-7) (-7) $x = 20$

24 $\frac{2n}{7} = 6$ (×7) (×7) $2n = 42$ (÷2) (÷2) $n = 21$

You may not need to use the 3rd line.

25 $\frac{5h}{3} = 15$ (×3) (×3) $5h = 45$ (÷5) (÷5) $h = 9$

26 $\frac{u-3}{2} = -11$ (×2) (×2) $u - 3 = -22$ (+3) (+3) $u = -19$

27 $\frac{k+6}{-4} = 3$ (×-4) (×-4) $k + 6 = -12$ (-6) (-6) $k = -18$

28 $\frac{-2y}{5} = 8$ (×5) (×5) $-2y = 40$ (÷-2) (÷-2) $y = -20$

29 $\frac{-a}{3} = -4$ (×3) (×3) $-a = -12$ (÷-1) (÷-1) $a = 12$

30 $\frac{3e}{-5} = -6$ (×-5) (×-5) $3e = 30$ (÷3) (÷3) $e = 10$

31 $\frac{2+y}{6} = 2.5$ (×6) (×6) $2 + y = 15$ (-2) (-2) $y = 13$

32 $\frac{a-5}{10} = 3.1$ (×10) (×10) $a - 5 = 31$ (+5) (+5) $a = 36$

33 $\frac{-u}{5} = 1$ (×5) (×5) $-u = 5$ (÷-1) (÷-1) $u = -5$

34 $\frac{q-1}{-8} = 5.5$ (×-8) (×-8) $q - 1 = -44$ (+1) (+1) $q = -43$

Further Two-Step Equations

The previous sheets on Two-Step Equations involved integer answers. This sheet has answers that are either decimals or fractions.

Column 1 has questions that will give decimal answers. The process for solving these is the same as the previous sheets, the only difference is that the answer is a decimal.

Example, Solve $4n - 2 = 8$.

$4n - 2 = 8$

$\oplus 2 \quad \oplus 2$ ← Remove the constant first, so remove the -2. So what is the opposite to -2? It's +2!

$4n = 10$

$\div 4 \quad \div 4$ ← Remove the coefficient, so get rid of 4! Use ÷4.

$n = 2.5$

Calculator Steps:

1 0 ÷ 4 =

Calculator Screen:

2.5

OR

Calculator Steps:

1 0 $a^b/_c$ 4 =

Calculator Screen: 2 ⌟ 1 ⌟ 2

$a^b/_c$

Calculator Screen: 2.5

Column 2 requires the answers to be fractions. So give your answer as a proper or improper fraction. These questions include an extra step. Rather than achieving your answer and moving to the next question, these questions require you to check your answer, by substituting the answer back into the equation. This makes the question more time consuming, but it is important to be able to check your answer, especially when tests are involved.

Example. Solve $3k - 5 = 6$, perform a substitution to verify your answer.

$3k - 5 = 6$

$\oplus 5 \quad \oplus 5$

$3k = 11$

$\div 3 \quad \div 3$

$k = {}^{11}/_3$

SOLVE
THEN
SUBSTITUTE

Substitute into eq^n.

$3k - 5 = 6$ ← Write the equation from the question again.

$3 \times {}^{11}/_3 - 5 = 6$ ← Substitute your answer for the pronumeral, in this case it is $k = {}^{11}/_3$

$6 = 6$ ✓ ← If the L.H.S. = R.H.S. then your answer is correct

Column 3 also involves fractions, this time you can answer as proper, improper or mixed numerals.

Further Two-Step Equations

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Solve these equations. Leave your answers as decimals.

1 $6j + 3 = 0$

(-3) (-3)

(÷6) (÷6)

2 $-10 + 5y = -2$

3 $\frac{k}{7} - 4 = 2.2$

4 $\frac{5a}{7} = 6.2$

5 $\frac{c + 12}{3} = 9.2$

6 $-\frac{t}{3} + 7 = 0.4$

7 $1.2 - 12d = 5.7$

8 $8 - 8x = -2$

9 $\frac{-5a}{3} = -0.18$

10 $\frac{h - 5.7}{0.2} = -3$

11 $1 - 4y = 0$

12 $\frac{1 + b}{7} = 8.6$

Solve, answering with fractions. Then substitute your answer to verify that it is correct.

13 $3m + 2 = 4$

(-2) (-2)

(÷3) (÷3)

Substitute into eqn.

$3m + 2 = 4$

$3 \times \square + 2 = 4$

$\square = 4$ ✓

14 $7d - 3 = 12$

Substitute into eqn.

$7d - 3 = 12$

$7 \times \frac{\square}{\square} - 3 = 12$

15 $9 - 5c = -3$

Substitute into eqn.

16 $10d - 8 = 8$

Substitute into eqn.

17 $25 - 100p = 5$

Substitute into eqn.

18 $8f - 5 = 7$

Substitute into eqn.

Solve these, leaving the answers as fractions or mixed numerals.

19 $\frac{t + 3}{2} = \frac{4}{5}$

(×2) (×2)

(-3) (-3)

20 $\frac{2k}{3} = 5$

21 $\frac{7e}{4} = -2$

22 $\frac{u - 7}{4} = \frac{2}{3}$

23 $\frac{j + 4}{5} = \frac{1}{2}$

24 $\frac{3t}{8} = 5$

25 $\frac{-2n}{5} = -9$

26 $\frac{8e}{-3} = -1$

27 $\frac{2 + y}{7} = \frac{4}{5}$

28 $\frac{a - 5}{9} = \frac{7}{6}$

29 $\frac{-6h}{5} = -3$

30 $\frac{m - 7}{2} = \frac{1}{5}$

Further Two-Step Equations

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Solve these equations. Leave your answers as decimals.

1

$6j + 3 = 0$

$-3 \quad -3$

$6j = -3$

$\div 6 \quad \div 6$

$j = -0.5$

2

$-10 + 5y = -2$

$+10 \quad +10$

$5y = 8$

$\div 5 \quad \div 5$

$y = 1.6$

3

$\frac{k}{7} - 4 = 2.2$

$+4 \quad +4$

$\frac{k}{7} = 6.2$

$\times 7 \quad \times 7$

$k = 43.4$

4

$\frac{5a}{7} = 6.2$

$\times 7 \quad \times 7$

$5a = 43.4$

$\div 5 \quad \div 5$

$a = 8.68$

5

$\frac{c + 12}{3} = 9.2$

$\times 3 \quad \times 3$

$c + 12 = 27.6$

$-12 \quad -12$

$c = 15.6$

6

$-\frac{t}{3} + 7 = 0.4$

$-7 \quad -7$

$-\frac{t}{3} = -6.6$

$\times -3 \quad \times -3$

$t = 19.8$

7

$1.2 - 12d = 5.7$

$-1.2 \quad -1.2$

$-12d = 4.5$

$\div -12 \quad \div -12$

$d = -0.375$

8

$8 - 8x = -2$

$-8 \quad -8$

$-8x = -10$

$\div -8 \quad \div -8$

$x = 1.25$

9

$\frac{-5a}{3} = -0.18$

$\times 3 \quad \times 3$

$-5a = -0.54$

$\div -5 \quad \div -5$

$a = 0.108$

10

$\frac{h - 5.7}{0.2} = -3$

$\times 0.2 \quad \times 0.2$

$h - 5.7 = -0.6$

$+5.7 \quad +5.7$

$h = 5.1$

11

$1 - 4y = 0$

$-1 \quad -1$

$-4y = -1$

$\div -4 \quad \div -4$

$y = 0.25$

12

$\frac{1 + b}{7} = 8.6$

$\times 7 \quad \times 7$

$1 + b = 60.2$

$-1 \quad -1$

$b = 59.2$

Solve, answering with fractions. Then substitute your answer to verify that it is correct.

13

$3m + 2 = 4$

$-2 \quad -2$

$3m = 2$

$\div 3 \quad \div 3$

$m = \frac{2}{3}$

Substitute into eqn.

$3m + 2 = 4$

$3 \times \frac{2}{3} + 2 = 4$

$4 = 4$ ✓

14

$7d - 3 = 12$

$+3 \quad +3$

$7d = 15$

$\div 7 \quad \div 7$

$d = \frac{15}{7}$

Substitute into eqn.

$7d - 3 = 12$

$7 \times \frac{15}{7} - 3 = 12$

$12 = 12$ ✓

15

$9 - 5c = -3$

$-9 \quad -9$

$-5c = -12$

$\div -5 \quad \div -5$

$c = \frac{12}{5}$

Substitute into eqn.

$9 - 5c = -3$

$9 - 5 \times \frac{12}{5} = -3$

$-3 = -3$ ✓

16

$10d - 8 = 8$

$+8 \quad +8$

$10d = 16$

$\div 10 \quad \div 10$

$d = \frac{8}{5}$

Substitute into eqn.

$10d - 8 = 8$

$10 \times \frac{8}{5} - 8 = 8$

$8 = 8$ ✓

17

$25 - 100p = 5$

$-25 \quad -25$

$-100p = -20$

$\div -100 \quad \div -100$

$d = \frac{1}{5}$

Substitute into eqn.

$25 - 100p = 5$

$25 - 100 \times \frac{1}{5} = 5$

$5 = 5$ ✓

18

$8f - 5 = 7$

$+5 \quad +5$

$8f = 12$

$\div 8 \quad \div 8$

$f = \frac{3}{2}$

Substitute into eqn.

$8f - 5 = 7$

$8 \times \frac{3}{2} - 5 = 7$

$7 = 7$ ✓

Solve these, leaving the answers as fractions or mixed numerals.

19

$\frac{t + 3}{2} = \frac{4}{5}$

$\times 2 \quad \times 2$

$t + 3 = \frac{8}{5}$

$-3 \quad -3$

$t = \frac{-7}{5}$ $(-1\frac{2}{5})$

20

$\frac{2k}{3} = 5$

$\times 3 \quad \times 3$

$2k = 15$

$\div 2 \quad \div 2$

$k = \frac{15}{2}$ $(7\frac{1}{2})$

21

$\frac{7e}{4} = -2$

$\times 4 \quad \times 4$

$7e = -8$

$\div 7 \quad \div 7$

$e = \frac{-8}{7}$ $(-1\frac{1}{7})$

22

$\frac{u - 7}{4} = \frac{2}{3}$

$\times 4 \quad \times 4$

$u - 7 = \frac{8}{3}$

$+7 \quad +7$

$u = \frac{29}{3}$ $(9\frac{2}{3})$

23

$\frac{j + 4}{5} = \frac{1}{2}$

$\times 5 \quad \times 5$

$j + 4 = \frac{5}{2}$

$-4 \quad -4$

$j = \frac{-3}{2}$ $(-1\frac{1}{2})$

24

$\frac{3t}{8} = 5$

$\times 8 \quad \times 8$

$3t = 40$

$\div 3 \quad \div 3$

$t = \frac{40}{3}$ $(13\frac{1}{3})$

25

$\frac{-2n}{5} = -9$

$\times 5 \quad \times 5$

$-2n = -45$

$\div -2 \quad \div -2$

$n = \frac{45}{2}$ $(22\frac{1}{2})$

26

$\frac{8e}{-3} = -1$

$\times -3 \quad \times -3$

$8e = 3$

$\div 8 \quad \div 8$

$e = \frac{3}{8}$

27

$\frac{2 + y}{7} = \frac{4}{5}$

$\times 7 \quad \times 7$

$2 + y = \frac{28}{5}$

$-2 \quad -2$

$y = \frac{18}{5}$ $(3\frac{3}{5})$

28

$\frac{a - 5}{9} = \frac{7}{6}$

$\times 9 \quad \times 9$

$a - 5 = \frac{63}{6}$

$+5 \quad +5$

$a = \frac{31}{2}$ $(15\frac{1}{2})$

29

$\frac{-6h}{5} = -3$

$\times 5 \quad \times 5$

$-6h = -15$

$\div -6 \quad \div -6$

$h = \frac{5}{2}$ $(2\frac{1}{2})$

30

$\frac{m - 7}{2} = \frac{1}{5}$

$\times 2 \quad \times 2$

$m - 7 = \frac{2}{5}$

$+7 \quad +7$

$m = \frac{37}{5}$ $(7\frac{2}{5})$

Unknowns on Both Sides

These equations has algebraic terms on both sides of the '=' sign, they often also have a constant (number) on each side. If you are after an answer like $x = 4$, then you can see that the x is on the left hand side (L.H.S.) and that the 4 is on the R.H.S.

So to answer these questions you have to get the numbers to the R.H.S. and the terms to the L.H.S. Once you do that you have a 1-step equation to solve. There is an example at the top of the column for you to follow, as well as the first 4 questions having the process shown.

The second column asks you to substitute the answer that you found back into the equation, if the L.H.S. and the R.H.S. of the equation is equal, your solution is correct.

Unknowns on Both Sides

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To solve these you have to get the number on the right hand side, and the term (letter) on the left.

All answers are integers in this column.

Example $4x + 2 = 20 - 2x$

-2 -2 ← Remove the number on the LHS.

$4x = 18 - 2x$

$+2x$ $+2x$ ← Remove the term on the RHS.

$6x = 18$

$\div 6$ $\div 6$ ← Divide through to solve the equation.

$x = 3$

1 $3c = c + 32$

$-c$ $-c$

$\div 2$ $\div 2$

2 $9a = 2a - 14$

$-2a$ $-2a$

$\div 7$ $\div 7$

3 $8k - 21 = 5k$

$+21$ $+21$

$-5k$ $-5k$

$\div 3$ $\div 3$

4 $5y + 5 = 3y - 5$

-5 -5

$-3y$ $-3y$

$\div 2$ $\div 2$

5 $6n - 17 = 2n + 3$

6 $6p + 2 = 16 - p$

7 $k - 4 = 5k + 36$

8 $20 - d = -4 - 3d$

9 $-13 - g = 37 + 9g$

10 $4b + 23 = 23 - 4b$

Fraction answers this time, solve then substitute your answer to test that you are correct.

11 $8n = 2n + 5$

12 $8n = 2n + 5$

$8 \times \square = 2 \times \square + 5$

$\frac{\square}{\square} = \frac{\square}{\square}$

L.H.S. = R.H.S.

$\therefore \square$ is a solution

13 $3v - 2 = 5 - v$

14 $3v - 2 = 5 - v$

$3 \times \frac{\square}{\square} - 2 = 5 - \frac{\square}{\square}$

$\frac{\square}{\square} = \frac{\square}{\square}$

L.H.S. = R.H.S.

$\therefore \square$ is a solution

15 $15 - 2x = x + 7$

16

17 $10 - 3d = 4 + 2d$

18

19 $-v - 7 = -9 - 4v$

20

Unknowns on Both Sides

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To solve these you have to get the number on the right hand side, and the term (letter) on the left.

All answers are integers in this column.

Example

$4x + 2 = 20 - 2x$

-2 -2 ← Remove the number on the LHS.

$4x = 18 - 2x$

$+2x$ $+2x$ ← Remove the term on the RHS.

$6x = 18$

$\div 6$ $\div 6$ ← Divide through to solve the equation.

$x = 3$

1 $3c = c + 32$

$-c$ $-c$

$2c = 32$

$\div 2$ $\div 2$

$c = 16$

2 $9a = 2a - 14$

$-2a$ $-2a$

$7a = -14$

$\div 7$ $\div 7$

$a = -2$

3 $8k - 21 = 5k$

$+21$ $+21$

$8k = 5k + 21$

$-5k$ $-5k$

$3k = 21$

$\div 3$ $\div 3$

$k = 7$

4 $5y + 5 = 3y - 5$

-5 -5

$5y = 3y - 10$

$-3y$ $-3y$

$2y = -10$

$\div 2$ $\div 2$

$y = -5$

5 $6n - 17 = 2n + 3$

$+17$ $+17$

$6n = 2n + 20$

$-2n$ $-2n$

$4n = 20$

$\div 4$ $\div 4$

$n = 5$

6 $6p + 2 = 16 - p$

-2 -2

$6p = 14 - p$

$+p$ $+p$

$7p = 14$

$\div 7$ $\div 7$

$p = 2$

7 $k - 4 = 5k + 36$

$+4$ $+4$

$k = 5k + 40$

$-5k$ $-5k$

$-4k = 40$

$\div -4$ $\div -4$

$k = -10$

8 $20 - d = -4 - 3d$

-20 -20

$-d = -24 - 3d$

$+3d$ $+3d$

$2d = -24$

$\div 2$ $\div 2$

$d = -12$

9 $-13 - g = 37 + 9g$

$+13$ $+13$

$-g = 50 + 9g$

$-9g$ $-9g$

$-10g = 50$

$\div -10$ $\div -10$

$g = -5$

10 $4b + 23 = 23 - 4b$

-23 -23

$4b = -4b$

$+4b$ $+4b$

$8b = 0$

$\div 8$ $\div 8$

$b = 0$

Fraction answers this time, solve then substitute your answer to test that you are correct.

11 $8n = 2n + 5$

$-2n$ $-2n$

$6n = 5$

$\div 6$ $\div 6$

$n = {}^{5}/_{6}$

12 $8n = 2n + 5$

$8 \times \frac{5}{6} = 2 \times \frac{5}{6} + 5$

$6\frac{2}{3} = 6\frac{2}{3}$

L.H.S. = R.H.S.

$\therefore \frac{5}{6}$ is a solution

13 $3v - 2 = 5 - v$

$+2$ $+2$

$3v = 7 - v$

$+v$ $+v$

$4v = 7$

$\div 4$ $\div 4$

$v = {}^{7}/_{4}$

14 $3v - 2 = 5 - v$

$3 \times \frac{7}{4} - 2 = 5 - \frac{7}{4}$

$3\frac{1}{4} = 3\frac{1}{4}$

L.H.S. = R.H.S.

$\therefore \frac{7}{4}$ is a solution

15 $15 - 2x = x + 7$

-15 -15

$-2x = x - 8$

$-x$ $-x$

$-3x = -8$

$\div -3$ $\div -3$

$x = {}^{8}/_{3}$

16 $15 - 2x = x + 7$

$15 - 2 \times {}^{8}/_{3} = {}^{8}/_{3} + 7$

$9^{2}/_{3} = 9^{2}/_{3}$

L.H.S. = R.H.S.

$\therefore {}^{8}/_{3}$ is a solution

17 $10 - 3d = 4 + 2d$

-10 -10

$-3d = -6 + 2d$

$-2d$ $-2d$

$-5d = -6$

$\div -5$ $\div -5$

$d = {}^{6}/_{5}$

18 $10 - 3d = 4 + 2d$

$10 - 3 \times {}^{6}/_{5} = 4 + 2 \times {}^{6}/_{5}$

$6^{2}/_{5} = 6^{2}/_{5}$

L.H.S. = R.H.S.

$\therefore {}^{6}/_{5}$ is a solution

19 $-v - 7 = -9 - 4v$

$+7$ $+7$

$-v = -2 - 4v$

$+4v$ $+4v$

$3v = -2$

$\div 3$ $\div 3$

$v = {}^{-2}/_{3}$

20 $-v - 7 = -9 - 4v$

$-({}^{-2}/_{3}) - 7 = -9 - 4 \times ({}^{-2}/_{3})$

$-6^{1}/_{3} = -6^{1}/_{3}$

L.H.S. = R.H.S.

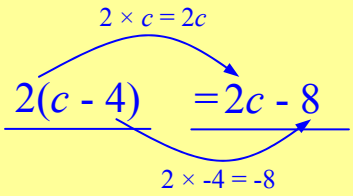
$\therefore {}^{-2}/_{3}$ is a solution

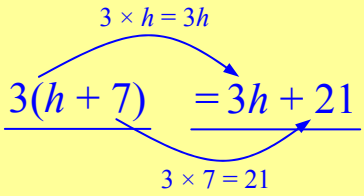
Equations with Expansions

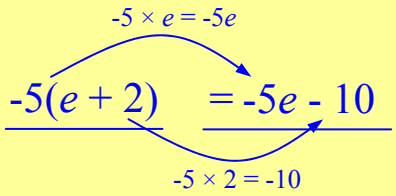
This worksheet combines 2-step equation skills with expansion. So the process isn't new, it is just two methods you have already learned …. combined.

Column 1 starts with expanding expressions, this should be revision for you. Remember that you multiply the outside term with the inside terms. There are examples at the top of the column but here are a few more.

Example - Expanding algebraic expressions.







So a negative number in front of the brackets will change the sign when multiplied.

Then Column 1 and 2 deals with solving equations that include expansions. There is an example before Q.9 and the next 3 questions give you the required operations to help you along. Column 2 is just the same except there are no hints and negatives are introduced.

A term can be outside the brackets as well. Column 3 has an example which shows that the term or constant outside of the brackets isn't included in the expansion. Once you have expanded you can then collect like terms and ….. you have a 2-step equation again. So it is just an extra step.

Equations with Expansions

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Expand these expressions.

Examples

$4(x + 2)$ $= 4x + 8$

$3(k - 4)$ $= 3k - 12$

$-3(n - 2)$ $= -3n + 6$

1 $2(y + 5) = 2\square + \square$

2 $4(p - 6) = 4\square - \square$

3 $3(t - 2) =$

4 $5(b + 7) =$

5 $9(c - 4) =$

6 $-2(f + 8) =$

7 $-(d - 2) =$

8 $-5(g - 4) =$

Expand these bracketed expressions, then solve just like a 2-Step equation. All answers are integers.

Example

$2(y + 7) = 10$

$2y + 14 = 10$ ← Expand the expression

(-14) (-14) ← Remove constant LHS

$2y = -4$

(÷2) (÷2) ← Divide for solution.

$y = -2$

9

$3(m + 8) = 51$

Expand expression

(-24) (-24) Subtract LHS constant

(÷3) (÷3) Divide by coefficient

10

$4(h - 3) = 24$

(+12) (+12)

(÷4) (÷4)

11

$2(v + 5) = 6$

(-10) (-10)

(÷2) (÷2)

Keep going … remember to expand, remove the constant and then divide through.

10 $7(p + 9) = -7$

11 $5(e - 9) = -35$

12 $4(t - 4) = -8$

13 $6(k + 3) = -18$

Now negatives - Remember change the sign!

14 $-3(u + 2) = 12$

15 $-8(h - 4) = 24$

16 $-5(y - 6) = -15$

17 $-2(a - 7) = 56$

18 $-(x + 9) = -7$

19 $-10(b - 3) = 0$

If a term or constant is outside the brackets, it isn't included in the expansion. Solve these.

Example

$2(k + 5) + 5k = 31$

$2k + 10 + 5k = 31$ ← $5k$ stays as is.

$7k + 10 = 31$ ← Collect the like terms

(-10) (-10)

$7k = 21$

(÷7) (÷7) 2-Step Equation! Solve as before.

$k = 3$

20 $3(y - 8) + 9y = 36$

21 $7(c + 4) - 19 = -26$

22 $5t + 3(t - 3) = 47$

23 $10g - 4(g - 2) + 11 = 1$

Equations with Expansions

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Expand these expressions.

Examples

$4(x + 2) = 4x + 8$

$3(k - 4) = 3k - 12$

$-3(n - 2) = -3n + 6$

1. $2(y + 5) = 2y + 10$
2. $4(p - 6) = 4p - 24$
3. $3(t - 2) = 3t - 6$
4. $5(b + 7) = 5b + 35$
5. $9(c - 4) = 9c - 36$
6. $-2(f + 8) = -2f - 16$
7. $-(d - 2) = -d + 2$
8. $-5(g - 4) = -5g + 20$

Expand these bracketed expressions, then solve just like a 2-Step equation. All answers are integers.

Example

$2(y + 7) = 10$

$2y + 14 = 10$ ← Expand the expression

$-14 \quad -14$ ← Remove constant LHS

$2y = -4$

$\div 2 \quad \div 2$ ← Divide for solution.

$y = -2$

9

$3(m + 8) = 51$

$3m + 24 = 51$ Expand expression

$-24 \quad -24$ Subtract LHS constant

$3m = 27$

$\div 3 \quad \div 3$ Divide by coefficient

$m = 9$

10

$4(h - 3) = 24$

$4h - 12 = 24$

$+12 \quad +12$

$4h = 36$

$\div 4 \quad \div 4$

$h = 9$

11

$2(v + 5) = 6$

$2v + 10 = 6$

$-10 \quad -10$

$2v = -4$

$\div 2 \quad \div 2$

$v = -2$

Keep going … remember to expand, remove the constant and then divide through.

10

$7(p + 9) = -7$

$7p + 63 = -7$

$-63 \quad -63$

$7p = -70$

$\div 7 \quad \div 7$

$p = -10$

11

$5(e - 9) = -35$

$5e - 45 = -35$

$+45 \quad +45$

$5e = 10$

$\div 5 \quad \div 5$

$e = 2$

12

$4(t - 4) = -8$

$4t - 16 = -8$

$+16 \quad +16$

$4t = 8$

$\div 4 \quad \div 4$

$t = 2$

13

$6(k + 3) = -18$

$6k + 18 = -18$

$-18 \quad -18$

$6k = -36$

$\div 6 \quad \div 6$

$k = -6$

Now negatives - Remember change the sign!

14

$-3(u + 2) = 12$

$-3u - 6 = 12$

$+6 \quad +6$

$-3u = 18$

$\div -3 \quad \div -3$

$u = -6$

15

$-8(h - 4) = 24$

$-8h + 32 = 24$

$-32 \quad -32$

$-8h = -8$

$\div -8 \quad \div -8$

$h = 1$

16

$-5(y - 6) = -15$

$-5y + 30 = -15$

$-30 \quad -30$

$-5y = -45$

$\div -5 \quad \div -5$

$y = 9$

17

$-2(a - 7) = 56$

$-2a + 14 = 56$

$-14 \quad -14$

$-2a = 42$

$\div -2 \quad \div -2$

$a = -21$

18

$-(x + 9) = -7$

$-x - 9 = -7$

$+9 \quad +9$

$-x = 2$

$\div -1 \quad \div -1$

$x = -2$

19

$-10(b - 3) = 0$

$-10b + 30 = 0$

$-30 \quad -30$

$-10b = -30$

$\div -10 \quad \div -10$

$b = 3$

If a term or constant is outside the brackets, it isn't included in the expansion. Solve these.

Example

$2(k + 5) + 5k = 31$

$2k + 10 + 5k = 31$ ← $5k$ stays as is.

$7k + 10 = 31$ ← Collect the like terms

$-10 \quad -10$

$7k = 21$

$\div 7 \quad \div 7$

$k = 3$

2-Step Equation! Solve as before.

20

$3(y - 8) + 9y = 36$

$3y - 24 + 9y = 36$

$12y - 24 = 36$

$+24 \quad +24$

$12y = 60$

$\div 12 \quad \div 12$

$y = 5$

21

$7(c + 4) - 19 = -26$

$7c + 28 - 19 = -26$

$7c + 9 = -26$

$-9 \quad -9$

$7c = -35$

$\div 7 \quad \div 7$

$c = -5$

22

$5t + 3(t - 3) = 47$

$5t + 3t - 9 = 47$

$8t - 9 = 47$

$+9 \quad +9$

$8t = 56$

$\div 8 \quad \div 8$

$t = 7$

23

$10g - 4(g - 2) + 11 = 1$

$10g - 4g + 8 + 11 = 1$

$6g + 19 = 1$

$-19 \quad -19$

$6g = -18$

$\div 6 \quad \div 6$

$g = -3$

Further Expansion - Solving Equations

This is an extension sheet involving double expansions, the previous Expansion sheet should be completed before attempting this sheet. An example for column 1 questions is below.

Example - Column 1 - Double Expansion - Same Side of Equation

$5(k + 2) - 3(4k - 4) = 8$ ← The Question to be solved

$5k + 10 - 12k + 12 = 8$ ← Expand the expression. Note the effect of the -3 in front of the second brackets, it changes the sign.

$-7k + 22 = 8$ ← Collect like terms

(-22) (-22)

$-7k = -14$

(÷-7) (÷-7)

$k = 2$

Solve the 2-Step Equation

Example - Column 2 - Double Expansion - Opposite Sides of Equation

$4(h - 2) = -2(h - 6)$ ← The Question to be solved

$4h - 8 = -2h + 12$ ← Expand the expression. Note the effect of the -2 on the RHS, it changes the sign.

(+8) (+8)

$4h = -2h + 20$

(+2h) (+2h)

$6h = 20$

(÷6) (÷6)

$h = {}^{10}/_{3}$

Solve the 'unknown on both sides' Equation.

Column three has two questions that apply the skills above to area applications. Q.10 uses the same skills as Column 1, Q. 11 uses the same skills as Column 2.

Further Expansion - Solving Equations

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Expand, gather like terms and then solve the equations below. Answers may involve fractions.

1 $2(y + 2) + 3(y - 4) = 7$

Expand the expression

Collect like terms

+8 +8

÷5 ÷5

2 $6(a - 5) + 3(a + 1) = -6$

3 $5(4 + d) - 2(d + 3) = 20$

4 $3(4x - 5) - 3(1 - 5x) = 36$

5 $12c - 6(1 - 4c) + 16 = 31$

Now the expansions are on both sides of the equation. Solve.

6 $2(x - 6) = 5(x + 2)$

Expand the expression

+12 +12

-5x -5x

÷-3 ÷-3

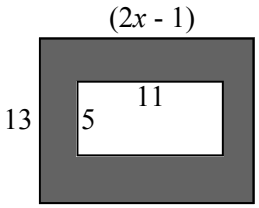
7 $7(k - 5) = 3(8 - k)$

8 $-3(b + 7) = 4(6 - 2b)$

9 $-7(r - 9) = 2(12 - 5r)$

The shaded area is 140 cm^2, find the perimeter of the outside rectangle. Lengths are in cm.

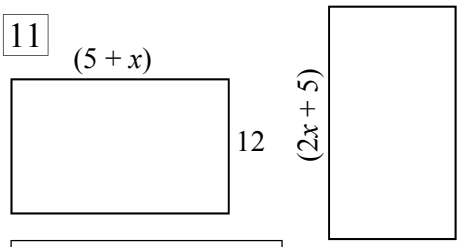
10



Not to scale

These two rectangles have the same area, find the area.

11



Drawings not to scale

Further Expansion - Solving Equations

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Expand, gather like terms and then solve the equations below. Answers may involve fractions.

1 $2(y + 2) + 3(y - 4) = 7$

$2y + 4 + 3y - 12 = 7$

Expand the expression

$5y - 8 = 7$

Collect like terms

(+8) (+8)

$5y = 15$

(÷5) (÷5)

$y = 3$

2 $6(a - 5) + 3(a + 1) = -6$

$6a - 30 + 3a + 3 = -6$

$9a - 27 = -6$

(+27) (+27)

$9a = 21$

(÷9) (÷9)

$a = {}^{7}/_{3}$

3 $5(4 + d) - 2(d + 3) = 20$

$20 + 5d - 2d - 6 = 20$

$3d + 14 = 20$

(-14) (-14)

$3d = 6$

(÷3) (÷3)

$d = 2$

4 $3(4x - 5) - 3(1 - 5x) = 36$

$12x - 15 - 3 + 15x = 36$

$27x - 18 = 36$

(+18) (+18)

$27x = 54$

(÷27) (÷27)

$x = 2$

5 $12c - 6(1 - 4c) + 16 = 31$

$12c - 6 + 24c + 16 = 31$

$36c + 10 = 31$

(-10) (-10)

$36c = 21$

(÷36) (÷36)

$c = {}^{7}/_{12}$

Now the expansions are on both sides of the equation. Solve.

6 $2(x - 6) = 5(x + 2)$

$2x - 12 = 5x + 10$

Expand the expression

(+12) (+12)

$2x = 5x + 22$

(-5x) (-5x)

$-3x = 22$

(÷-3) (÷-3)

$x = {}^{-22}/_{3}$

7 $7(k - 5) = 3(8 - k)$

$7k - 35 = 24 - 3k$

(+35) (+35)

$7k = 59 - 3k$

(+3k) (+3k)

$10k = 59$

(÷10) (÷10)

$k = {}^{59}/_{10}$

8 $-3(b + 7) = 4(6 - 2b)$

$-3b - 21 = 24 - 8b$

(+21) (+21)

$-3b = 45 - 8b$

(+8b) (+8b)

$5b = 45$

(÷5) (÷5)

$b = 9$

9 $-7(r - 9) = 2(12 - 5r)$

$-7r + 63 = 24 - 10r$

(-63) (-63)

$-7r = -39 - 10r$

(+10r) (+10r)

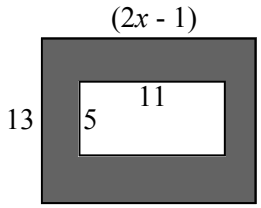
$3r = -39$

(÷3) (÷3)

$r = -13$

The shaded area is 140 cm^2, find the perimeter of the outside rectangle. Lengths are in cm.

10



Not to scale

$13(2x - 1) - 11 \times 5 = 140$

$26x - 13 - 55 = 140$

$26x - 68 = 140$

(+68) (+68)

$26x = 208$

(÷26) (÷26)

$x = 8$

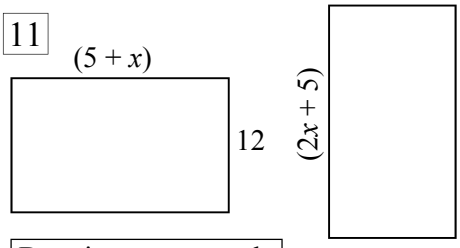
$P = 2(2x - 1) + 2 \times 13$

$= 2(2 \times 8 - 1) + 26$

$P = 56$ cm

These two rectangles have the same area, find the area.

11



Drawings not to scale

$12(5 + x) = 9(2x + 5)$

$60 + 12x = 18x + 45$

(-60) (-60)

$12x = 18x - 15$

(-18x) (-18x)

$-6x = -15$

(÷-6) (÷-6)

$x = 2.5$ units or ${}^{5}/_{2}$ units

If LHS used	If RHS used
$A = 12(5 + x)$	$A = 9(2x + 5)$
$= 12(5 + 2.5)$	$= 9(2 \times 2.5 + 5)$
$A = 90$ units^2	$A = 90$ units^2

Further Expansion - Substitution Page

The next page has a substitution page for checking solutions from the previous worksheet. It has two identical sides, so it is designed to be cut up and shared between two students.

It has three spare working out spaces if required for other exercises or if mistakes are made.

Further Expansion - Substitution Page

1 $2(y + 2) + 3(y - 4) = 7$

$2 \times (\square + 2) + 3 \times (\square - 4) = 7$

$\square = 7$ ✓ ✗

2 $6(a - 5) + 3(a + 1) = -6$

$6 \times (\square - 5) + 3 \times (\square + 1) = -6$

$\square = \square$ ✓ ✗

3 $5(4 + d) - 2(d + 3) = 20$

$5 \times (4 + \square) - 2 \times (\square + 3) = 20$

$\square = \square$ ✓ ✗

4 $3(4x - 5) - 3(1 - 5x) = 36$

$3 \times (4 \times \square - 5) - 3 \times (1 - 5 \times \square) = 36$

$\square = \square$ ✓ ✗

5 $12c - 6(1 - 4c) + 16 = 31$

$12 \times \square - 6 \times (1 - 4 \times \square) + 16 = 31$

$\square = \square$ ✓ ✗

6 $2(x - 6) = 5(x + 2)$

$2 \times (\square - 6) = 5 \times (\square + 2)$

$\square = \square$ ✓ ✗

7 $7(k - 5) = 3(8 - k)$

$7 \times (\square - 5) = 3 \times (8 - \square)$

$\square = \square$ ✓ ✗

8 $-3(b + 7) = 4(6 - 2b)$

$-3 \times (\square + 7) = 4 \times (6 - 2 \times \square)$

$\square = \square$ ✓ ✗

9 $-7(r - 9) = 2(12 - 5r)$

$-7 \times (\square - 9) = 2 \times (12 - 5 \times \square)$

$\square = \square$ ✓ ✗

□

$\square = \square$ ✓ ✗

□

$\square = \square$ ✓ ✗

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Further Expansion - Substitution Page

1 $2(y + 2) + 3(y - 4) = 7$

$2 \times (\square + 2) + 3 \times (\square - 4) = 7$

$\square = 7$ ✓ ✗

2 $6(a - 5) + 3(a + 1) = -6$

$6 \times (\square - 5) + 3 \times (\square + 1) = -6$

$\square = \square$ ✓ ✗

3 $5(4 + d) - 2(d + 3) = 20$

$5 \times (4 + \square) - 2 \times (\square + 3) = 20$

$\square = \square$ ✓ ✗

4 $3(4x - 5) - 3(1 - 5x) = 36$

$3 \times (4 \times \square - 5) - 3 \times (1 - 5 \times \square) = 36$

$\square = \square$ ✓ ✗

5 $12c - 6(1 - 4c) + 16 = 31$

$12 \times \square - 6 \times (1 - 4 \times \square) + 16 = 31$

$\square = \square$ ✓ ✗

6 $2(x - 6) = 5(x + 2)$

$2 \times (\square - 6) = 5 \times (\square + 2)$

$\square = \square$ ✓ ✗

7 $7(k - 5) = 3(8 - k)$

$7 \times (\square - 5) = 3 \times (8 - \square)$

$\square = \square$ ✓ ✗

8 $-3(b + 7) = 4(6 - 2b)$

$-3 \times (\square + 7) = 4 \times (6 - 2 \times \square)$

$\square = \square$ ✓ ✗

9 $-7(r - 9) = 2(12 - 5r)$

$-7 \times (\square - 9) = 2 \times (12 - 5 \times \square)$

$\square = \square$ ✓ ✗

□

$\square = \square$ ✓ ✗

□

$\square = \square$ ✓ ✗

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Substitution

Substitution is the replacement of a pronumeral (letter) with a number. Substitution is used often with formulae where the letters in the formula represent a measurement of distance, time, speed, etc.

Column 1 starts with algebraic expressions. You are given the values of each letter, replace the letter with the number and evaluate. Remember two important features:

When pronumerals are beside each other, it is like there is a '×' in between them.
So: $ab = a \times b$

If powers are involved, the power is only on the pronumeral it follows.
So $ab^2 = a \times b^2$ not $a^2 \times b^2$ or $(ab)^2$

There are two examples at the top of Column 1, note that there are three steps. Write the expression (already in the question), make the substitution, solve.

Columns 2 and 3 involve using formulae, some of which you may have experienced already. There are 3 lines required for each question, write the formula on the first line, make the substitution on the second line and solve (showing the units) on the third line. Some of these questions use units you may not have experienced before, like Joules and Newtons. The units are given to you, just write them after your answer, which is to be rounded to 1 decimal place.

Substitution

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If $a = 2$, $b = 3$ and $c = -4$, find the value of these expressions.

Examples

$c + a$

$= -4 + 2$

$= -2$

$ba - c$

$= 3 \times 2 - (-4)$

$= 9$

1 $a + 6$

$= \square + \square$

$=$

2 ac

$= \square \times \square$

$=$

3 abc

$=$

$=$

4 $a^2 - b^2$

$=$

$=$

5 $\frac{bc}{a}$

$=$

$=$

6 $\frac{6c}{ab}$

$=$

$=$

7 $c(10 - b)$

$=$

$=$

8 $9c^2 - 7a$

$=$

$=$

Find the unknown's value, then use that value in other questions. Use $x = 7$, $k = -2$, $y = 8$.

9 $w = 2xy - k$

$=$

$w =$

10 $c = kx - 2w$

$=$

$c =$

11 $g = wy + 5c^2$

12 $b = gk \div y$

Equations are used regularly in mathematics and science, use these formulae. Round to 1 d.p.

13 Rectangle Area: $A = lb$

Find A *in* m^2, *given*:

$l = 10.9$ m and $b = 8.6$ m.

$A = lb$

$=$

$A =$

14 Square's Perimeter: $P = 4l$

Find P *in* cm, *given*: $l = 13.7$ cm

15 Force: $F = ma$

Find F *in* N (Newtons), *given*:

$m = 85$ kg and $a = 9.8$ m/s^2.

16 Volume Rect. Prism: $V = lbh$

Find V *in* m^3, *given*:

$l = 3.4$ m, $b = 2.7$ m & $h = 4.2$ m

17 Kinetic Energy: $KE = \frac{1}{2}mv^2$

Find KE *in* J (Joules), *given*:

$m = 6.2$ kg and $v = 12$ m/s.

18 Converting Temperature:

$$C = \frac{5}{9}(F - 32)$$

Find C *in* °C, *given* F = 100°F.

19 Trapezium Area:

$$A = \frac{1}{2}(a + b)h$$

Find A *in* cm^2, *given*:

$a = 35$ cm, $b = 16$ cm & $h = 9$ cm

20 Speed: $S = \frac{D}{T}$

Find S *in* km/h, *given*:

D = 252 km and T = 4.5 h

21 Annulus Area: $A = \pi(R^2 - r^2)$

Find A *in* mm^2, *given*:

$R = 45$ mm and $r = 16$ mm

22 Rectangle Perimeter:

$$P = 2l + 2b$$

Find P *in* m, *given*:

$l = 27.6$ m and $b = 18.3$ m

Substitution

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If $a = 2$, $b = 3$ and $c = -4$, find the value of these expressions.

Examples

$c + a$

$= -4 + 2$

$= -2$

$ba - c$

$= 3 \times 2 - (-4)$

$= 10$

1 $a + 6$

$= 2 + 6$

$= 8$

2 ac

$= 2 \times -4$

$= -8$

3 abc

$= 2 \times 3 \times (-4)$

$= -24$

4 $a^2 - b^2$

$= 2^2 - 3^2$

$= -5$

5 $\frac{bc}{a}$

$= \frac{3 \times (-4)}{2}$

$= -6$

6 $\frac{6c}{ab}$

$= \frac{6 \times (-4)}{2 \times 3}$

$= -4$

7 $c(10 - b)$

$= -4(10 - 3)$

$= -28$

8 $9c^2 - 7a$

$= 9(-4)^2 - 7 \times 2$

$= 130$

Find the unknown's value, then use that value in other questions. Use $x = 7$, $k = -2$, $y = 8$.

9 $w = 2xy - k$

$= 2 \times 7 \times 8 - (-2)$

$w = 114$

10 $c = kx - 2w$

$= -2 \times 7 - 2 \times 114$

$c = -242$

11 $g = wy + 5c^2$

$= 114 \times 8 + 5 \times (-242)^2$

$g = 293\,732$

12 $b = gk \div y$

$= 293\,732 \times (-2) \div 8$

$b = -73\,433$

Equations are used regularly in mathematics and science, use these formulae. Round to 1 d.p.

13 Rectangle Area: $A = lb$

Find A *in* m^2, *given*:

$l = 10.9$ m and $b = 8.6$ m.

$A = lb$

$= 10.9 \times 8.6$

$A = 93.7\ m^2$ (1 d.p.)

14 Square's Perimeter: $P = 4l$

Find P *in* cm, *given*: $l = 13.7$ cm

$P = 4l$

$= 4 \times 13.7$

$P = 54.8$ cm

15 Force: $F = ma$

Find F *in* N (Newtons), *given*:

$m = 85$ kg and $a = 9.8$ m/s^2.

$F = ma$

$= 85 \times 9.8$

$F = 833$ N

16 Volume Rect. Prism: $V = lbh$

Find V *in* m^3, *given*:

$l = 3.4$ m, $b = 2.7$ m & $h = 4.2$ m

$V = lbh$

$= 3.4 \times 2.7 \times 4.2$

$F = 38\,556\ m^3$

17 Kinetic Energy: $KE = \frac{1}{2}mv^2$

Find KE *in* J (Joules), *given*:

$m = 6.2$ kg and $v = 12$ m/s.

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2} \times 6.2 \times 12^2$

$KE = 446.4$ J

18 Converting Temperature:

$C = \frac{5}{9}(F - 32)$

Find C *in* °C, *given* F = 100°F.

$C = \frac{5}{9}(F - 32)$

$= \frac{5}{9}(100 - 32)$

$C = 37.8$ °C (1 d.p.)

19 Trapezium Area:

$A = \frac{1}{2}(a + b)h$

Find A *in* cm^2, *given*:

$a = 35$ cm, $b = 16$ cm & $h = 9$ cm

$A = \frac{1}{2}(a + b)h$

$= \frac{1}{2}(35 + 16) \times 9$

$A = 229.5\ cm^2$

20 Speed: $S = \frac{D}{T}$

Find S *in* km/h, *given*:

D = 252 km and T = 4.5 h

$S = \frac{D}{T}$

$= \frac{252}{4.5}$

$S = 56$ km/h

21 Annulus Area: $A = \pi(R^2 - r^2)$

Find A *in* mm^2, *given*:

$R = 45$ mm and $r = 16$ mm

$A = \pi(R^2 - r^2)$

$= \pi(45^2 - 16^2)$

$A = 5\,557.5\ mm^2$ (1 d.p.)

22 Rectangle Perimeter:

$P = 2l + 2b$

Find P *in* m, *given*:

$l = 27.6$ m and $b = 18.3$ m

$P = 2l + 2b$

$= 2 \times 27.6 + 2 \times 18.3$

$A = 91.8$ m

Further Substitution

This sheet is difficult in concept for some students, before it is attempted the previous sheet on 'Substitution' should be completed. This sheet deals with using formulae, but unlike the previous sheet the unknown value is on the RHS not the LHS of the '=' sign. This means that rather than a substitution and solve exercise, it is a substitution, perform an operation (or two) then solve.

The entire sheet is the same in layout, the only difference between questions is that there may be one or two operations to carry out. Below is an example, there is an example at the top of the first column also.

Example - Finding the height of a rectangular prism, given its length and breadth.

Rectangular Prism Volume: $V = lbh$ ← The formula to use is shown here.

Find h in cm, given: ← The unknown that you need to find, and units.

$V = 11\ 730\ cm^3$, $l = 34$ cm and $b = 15$ cm. ← Values to substitute into the formula.

$V = lbh$ ← First step is to write the formula.

$11\ 730 = 34 \times 15 \times h$ ← Substitute all the given values.

$11\ 730 = 510 \times h$ ← Perform calculations to simplify.

$\div 510$ $\div 510$

$23 = h$ ← Get the unknown by itself. In this case we remove the 510 by $\div 510$.

$h = 23$ cm ← Rewrite with the 'letter' on the LHS. Write the units after the value.

Column 1 has support to assist you in solving the questions, as does the first question in Column 2.

Further Substitution

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Using formulae may require some manipulation. Solve.

Example

Rectangle Area: $A = lb$

Find b in m, given:

$A = 34.32$ m^2 and $l = 5.2$ m.

$A = lb$ ← Write the formula

$34.32 = 5.2 \times b$ ← Substitute

$\div 5.2$ $\div 5.2$ ← Pronumeral must be by itself. So Remove the × 5.2

$6.6 = b$

$b = 6.6$ m ← Reverse solution

1 Rectangle Area: $A = lb$

Find l in cm, given:

$A = 810$ cm^2 and $b = 22.5$ cm.

$A = lb$

$\square = l \times \square$

$\div$ $\div$

$\square = l$

$l = \square$ cm

2 Square Perimeter: $P = 4l$

Find l in cm, given: $P = 70$ cm

$P = 4l$

$\square = 4 \times l$

$\div$ $\div$

$\square = l$

$l = \square$ cm

3 Area of a Triangle: $A = \frac{1}{2}bh$

Find h in mm, given:

$A = 2\,890$ mm^2 and $b = 68$ mm.

$A = \frac{1}{2}bh$

$\square = \frac{1}{2} \times \square \times h$

$\square = \square \times h$

$\div$ $\div$

$\square = h$

$h = \square$ mm

4 Line : $y = 4x + 2$

Find x, given: $y = -2$.

$-$ $-$

$\div$ $\div$

5 Line : $y = -3x + b$

Find x, given: $y = 13$ & $b = -8$.

6 Speed: $S = \frac{D}{T}$

Find D in km, given:

S = 68 km/h and T = 8.5 h.

7 Force on a Object: $F = ma$

Find m in kg, given:

$F = 548.8$ N and $a = 9.8$ m/s^2.

8 Line : $y = mx + b$

Find x, if $y = 5$, $m = 2$ & $b = -9$

9 Trapezium Area:

$A = \frac{1}{2}(a + b)h$

Find h in cm, given: $a = 41$ cm, $b = 63$ cm and $A = 2\,444$ cm^2.

10 Pythagoras theorem:

$c^2 = a^2 + b^2$

Find b in cm, given:

$a = 28$ cm and $c = 53$ cm.

Further Substitution

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Using formulae may require some manipulation. Solve.

Example — Rectangle Area: $A = lb$

Find b in m, given:

$A = 34.32$ m^2 and $l = 5.2$ m.

$A = lb$ ← Write the formula

$34.32 = 5.2 \times b$ ← Substitute

$\div 5.2 \quad \div 5.2$

$6.6 = b$ ← Pronumeral must be by itself. So Remove the × 5.2

$b = 6.6$ m ← Reverse solution

1 Rectangle Area: $A = lb$

Find l in cm, given:

$A = 810$ cm^2 and $b = 22.5$ cm.

$A = lb$

$810 = l \times 22.5$

$\div 22.5 \quad \div 22.5$

$36 = l$

$l = 36$ cm

2 Square Perimeter: $P = 4l$

Find l in cm, given: $P = 70$ cm

$P = 4l$

$70 = 4 \times l$

$\div 4 \quad \div 4$

$17.5 = l$

$l = 17.5$ cm

3 Area of a Triangle: $A = \frac{1}{2}bh$

Find h in mm, given:

$A = 2\,890$ mm^2 and $b = 68$ mm.

$A = \frac{1}{2}bh$

$2\,890 = \frac{1}{2} \times 68 \times h$

$2\,890 = 34 \times h$

$\div 34 \quad \div 34$

$85 = h$

$h = 85$ mm

4 Line : $y = 4x + 2$

Find x, given: $y = -2$.

$y = 4x + 2$

$-2 = 4x + 2$

$-2 \quad -2$

$-4 = 4x$

$\div 4 \quad \div 4$

$-1 = x$

$x = -1$

5 Line : $y = -3x + b$

Find x, given: $y = 13$ & $b = -8$.

$y = -3x + b$

$13 = -3x - 8$

$+8 \quad +8$

$21 = -3x$

$\div -3 \quad \div -3$

$-7 = x$

$x = -7$

6 Speed: $S = \frac{D}{T}$

Find D in km, given:

$S = 68$ km/h and $T = 8.5$ h.

$S = {}^{D}/_{T}$

$68 = \frac{D}{8.5}$

$\times 8.5 \quad \times 8.5$

$578 = D$

$D = 578$ km

7 Force on a Object: $F = ma$

Find m in kg, given:

$F = 548.8$ N and $a = 9.8$ m/s^2.

$F = ma$

$548.8 = m \times 9.8$

$\div 9.8 \quad \div 9.8$

$56 = m$

$m = 56$ kg

8 Line : $y = mx + b$

Find x, if $y = 5$, $m = 2$ & $b = -9$

$y = mx + b$

$5 = 2x - 9$

$+9 \quad +9$

$14 = 2x$

$\div 2 \quad \div 2$

$7 = x$

$x = 7$

9 Trapezium Area:

$A = \frac{1}{2}(a + b)h$

Find h in cm, given: $a = 41$ cm, $b = 63$ cm and $A = 2\,444$ cm^2.

$A = \frac{1}{2}(a + b)h$

$2\,444 = \frac{1}{2} \times (41 + 63) \times h$

$2\,444 = 52 \times h$

$\div 52 \quad \div 52$

$47 = h$

$h = 47$ cm

10 Pythagoras theorem:

$c^2 = a^2 + b^2$

Find b in cm, given:

$a = 28$ cm and $c = 53$ cm.

$c^2 = a^2 + b^2$

$53^2 = 28^2 + b^2$

$2\,809 = 784 + b^2$

$-784 \quad -784$

$2\,025 = b^2$

$b^2 = 2025$

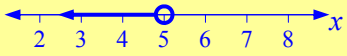
$\sqrt{\ } \quad \sqrt{\ }$

$b = 45$ cm

Inequations

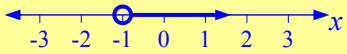
Inequations are just like equations except rather than using an '=' they use <, >, ⩽ and ⩾. The solution to the inequation can be also shown on a number line. Below are some examples of showing your solution on a number line.

Examples



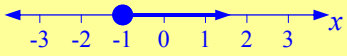
$x < 5$

This number line shows x as all numbers less than 5. Note that the circle on the 5 is hollow, so 5 isn't included. The < sign in $x < 5$ looks like an arrow pointing left, and that is the way the arrow on the number line is graphed, so they match ... look at the inequality symbol like it is an arrow.



$x > -1$

This number line shows x as all numbers greater than -1. Note the arrow and the > (greater than) sign match.



$x \geqslant -1$

This number line shows x as all numbers greater than or equal to -1. Note the arrow and the > still match, but the circle on the -1 is now filled in. This means it is included.

Column 1 starts with graphs that are already drawn, so just write down the inequation, remember the arrow direction and if the circle is filled or hollow. There is a reminder at the top of the column about the arrow and circle use. From Q. 6 you are asked to graph the inequation, try to position the number where the arrow starts in the centre of the graph, look at the arrow direction and fill in the circle if needed.

Column 2 deals with solving 1-step and 2-step inequations. Use the same method as with solving equations, just replace the = sign with an inequality sign. Then graph your result.

Inequations

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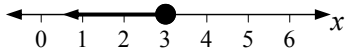
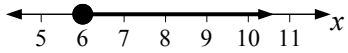
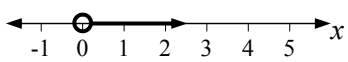
Give the inequation that is shown in the following graphs.

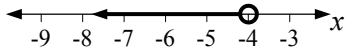
< If arrow points left and ○
⩽ If arrow points left and ●
> If arrow points right and ○
⩾ If arrow points right and ●

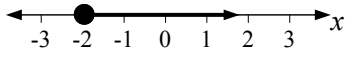
1 

$x \leqslant \square$

2 

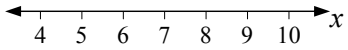
3 

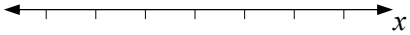
4 

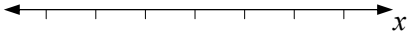
5 

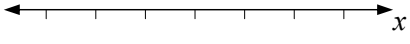
Now the opposite, given the inequation ….. graph it.

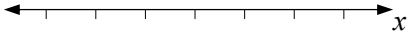
Write the number near the centre of the graph

6 $x > 7$


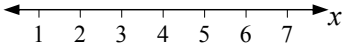
7 $x < -2$


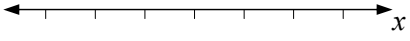
8 $x \leqslant 9$


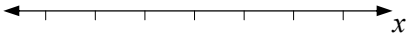
9 $x > -3$


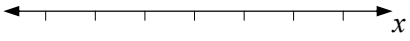
10 $x \geqslant -8$


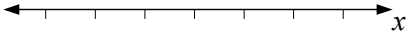
Solve the inequations and then graph the result.

11 $x + 5 \leqslant 9$
(-5) (-5)
$x \leqslant \square$


12 $x - 7 > -3$
(+7) (+7)
$x > \square$
For 12: 

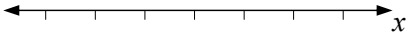
For 13: 

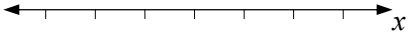
13 $5x < 30$
(÷5) (÷5)
For 14: 

For 15: 

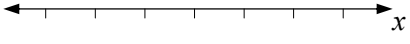
14 $5 + x \leqslant -3$

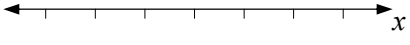
15 $\frac{x}{4} > 2$

16 $3x \geqslant 54$
For 16: 

For 17: 

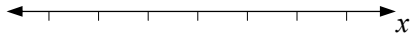
17 $x - 8 > -13$

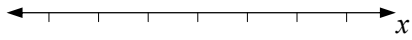
18 $\frac{x}{2} < -1$
For 18: 

For 19: 

19 $x - 14 \geqslant 0$

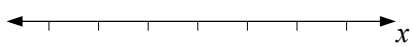
These are two step inequations, solve and graph the result.

20 $2x - 3 \leqslant 15$
(+3) (+3)
(÷2) (÷2)
For 20: 

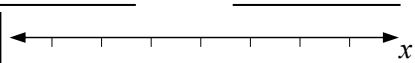
For 21: 

21 $3x + 20 > 11$
(-20) (-20)
(÷3) (÷3)

22 $\frac{x}{5} + 4 > -2$
$\frac{\square}{\square} > \square$
For 22: 

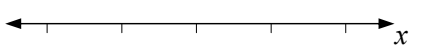
For 23: 

23 $\frac{x}{8} - 1 \geqslant 0$
$\frac{\square}{\square} \geqslant \square$

24 $\frac{x+1}{3} < 2$
For 24: 

For 25: 

25 $\frac{3x}{5} \geqslant 6$

26 $3(x + 9) \leqslant -6$
Expand and then solve


Inequations

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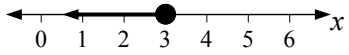
Give the inequation that is shown in the following graphs.

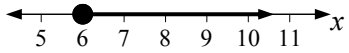
< If arrow points left and ○
⩽ If arrow points left and ●
> If arrow points right and ○
⩾ If arrow points right and ●

1



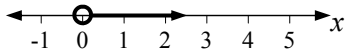
$x \leqslant 3$

2



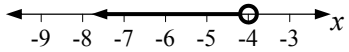
$x \geqslant 6$

3



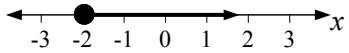
$x > 0$

4



$x < -4$

5

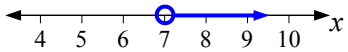


$x \geqslant -2$

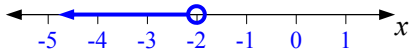
Now the opposite, given the inequation ….. graph it.

Write the number near the centre of the graph

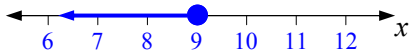
6 $x > 7$



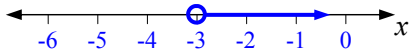
7 $x < -2$



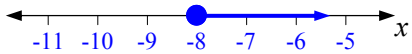
8 $x \leqslant 9$



9 $x > -3$



10 $x \geqslant -8$

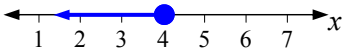


Solve the inequations and then graph the result.

11 $x + 5 \leqslant 9$

(-5) (-5)

$x \leqslant 4$

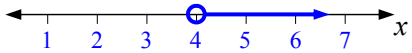


12 $x - 7 > -3$

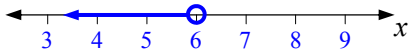
(+7) (+7)

$x > 4$

For 12:



For 13:



13 $5x < 30$

(÷5) (÷5)

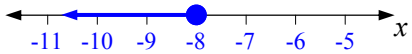
$x < 6$

14 $5 + x \leqslant -3$

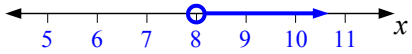
(-5) (-5)

$x \leqslant -8$

For 14:



For 15:



15 $\frac{x}{4} > 2$

(×4) (×4)

$x > 8$

16 $3x \geqslant 54$

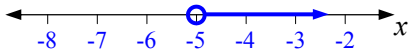
(÷3) (÷3)

$x \geqslant 18$

For 16:



For 17:



17 $x - 8 > -13$

(+8) (+8)

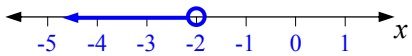
$x > -5$

18 $\frac{x}{2} < -1$

(×2) (×2)

$x < -2$

For 18:



For 19:



19 $x - 14 \geqslant 0$

(+14) (+14)

$x \geqslant 14$

These are two step inequations, solve and graph the result.

20 $2x - 3 \leqslant 15$

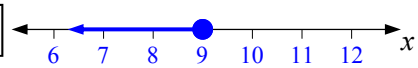
(+3) (+3)

$2x \leqslant 18$

(÷2) (÷2)

$x \leqslant 9$

For 20:



21 $3x + 20 > 11$

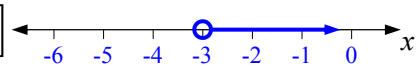
(-20) (-20)

$3x > -9$

(÷3) (÷3)

$x > -3$

For 21:



22 $\frac{x}{5} + 4 > -2$

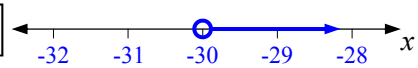
(-4) (-4)

$\frac{x}{5} > -6$

(×5) (×5)

$x > -30$

For 22:



23 $\frac{x}{8} - 1 \geqslant 0$

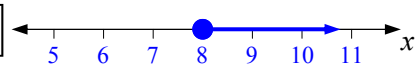
(+1) (+1)

$\frac{x}{8} \geqslant 1$

(×8) (×8)

$x \geqslant 8$

For 23:



24 $\frac{x+1}{3} < 2$

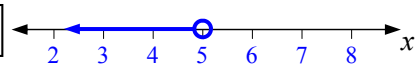
(×3) (×3)

$x + 1 < 6$

(-1) (-1)

$x < 5$

For 24:



25 $\frac{3x}{5} \geqslant 6$

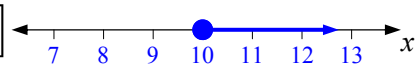
(×5) (×5)

$3x \geqslant 30$

(÷3) (÷3)

$x \geqslant 10$

For 25:



26 $3(x + 9) \leqslant -6$

$3x + 27 \leqslant -6$

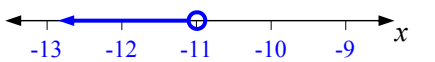
(-27) (-27)

$3x \leqslant -33$

(÷3) (÷3)

$x \leqslant -11$

Expand and then solve



Further Inequations

This sheet deals inequations with a negative coefficient. What does that mean? It means that the number in front of the x now has a negative sign. So you divide or multiply by -1 and it changes the sign. Except with inequations when you multiply or divide by a negative number, the direction of the inequality symbol is reversed. It points the opposite direction.

Example - Solve and then graph the inequation

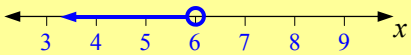
$20 - 2x > 8$ ← Solve. First remove the constant, 20.

$-20 \quad -20$

$-2x > -12$ ← Then divide by -2 to get x by itself.

$\div -2 \quad \div -2$

$x < 6$ ← Because you have divided by a negative number (-2) the inequality is reversed.

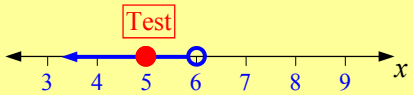


While the above isn't a huge earth shattering piece of information, it is easy to forget. You are busy following the process of solving the equation and forget to reverse the sign. In a test especially, it makes sense to check your answer and this is performed by using a test point.

In the graph above it includes all values to the left of 6, so pick a number to the left of the 6 and substitute it into the inequation, if the substitution is correct, then the direction of your 'arrow' is also correct. This test process features in Column 3 of the worksheet.

Example - Testing a Point

Note that 5 is to the left of 6, (on the arrow) so substitute 5 and check that the inequation is satisfied.



$20 - 2x > 8$ ← Rewrite the original inequation.

$20 - 2 \times 5 > 8$ ← Replace x with the test point, in this case, 5.

$10 > 8$ ← Perform the calculation

$\therefore$ Correct ← 10 is greater than 8, so you are correct. If you obtain a false statement then your arrow direction or your inequation solution are incorrect.

Further Inequations

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Changing the sign of the variable reverses the direction of the inequality symbols. Rewrite, then plot on the number line.

1 $-x \leqslant -5 \longrightarrow x \geqslant \square$

x

2 $-x > 8 \longrightarrow$ _____

x

3 $-x \geqslant -7 \longrightarrow$ _____

x

4 $-x < -10 \longrightarrow$ _____

x

Once the sign is changed, reverse the inequality symbol. Solve & graph the solution.

5 $2 - x \leqslant -6$

(-2) (-2)

(÷-1) (÷-1)

For 5: x

6 $-x + 7 > 14$

For 6: x

7 $\frac{-x}{9} < -2$

For 7: x

8 $-2x \geqslant 8$

For 8: x

9 $-3x + 7 \geqslant -2$

(-7) (-7)

(÷-3) (÷-3)

For 9: x

10 $-1 - \frac{x}{5} < -4$

For 10: x

11 $\frac{1 - x}{2} \leqslant 5$

For 11: x

12 $11 - 2x > -11$

For 12: x

13 $-4(x - 4) > 12$

For 13: x

14 $-(24 - x) \leqslant 3x$

For 14: x

Test a point to check your answer. Solve the inequation, graph it, then test your answer.

Step 1: Solve the inequation
Step 2: Graph the inequation
Step 3: Pick test point ON arrow to check
Step 4: If incorrect, check arrow direction or your inequation solution.

15 $-4x < 8$

Test using $x = 1$

$-4x < 8$

$-4 \times \square < 8$

< 8

$\therefore$ Correct

For 15: x

16 $\frac{2 - 5x}{6} \leqslant -8$

Test using $x =$ ☐

$\frac{2 - 5x}{6} \leqslant -8$

$\frac{2 - 5 \times \square}{-6} \leqslant -8$

For 16: x

17 $3(x - 4) \geqslant 5x$

Test using $x =$ ☐

For 17: x

Further Inequations

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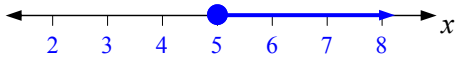
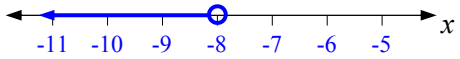
Changing the sign of the variable reverses the direction of the inequality symbols. Rewrite, then plot on the number line.

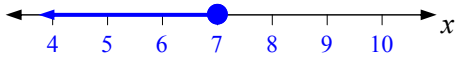
1 $-x \leqslant -5 \longrightarrow x \geqslant 5$



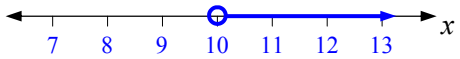
2 $-x > 8 \longrightarrow x < -8$



3 $-x \geqslant -7 \longrightarrow x \leqslant 7$



4 $-x < -10 \longrightarrow x > 10$



Once the sign is changed, reverse the inequality symbol. Solve & graph the solution.

5

$2 - x \leqslant -6$

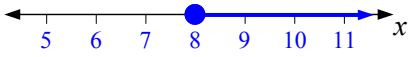
$(-2) \quad (-2)$

$-x \leqslant -8$

$(\div -1) \quad (\div -1)$

$x \geqslant 8$

For 5:



6

$-x + 7 > 14$

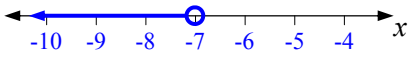
$(-7) \quad (-7)$

$-x > 7$

$(\div -1) \quad (\div -1)$

$x < -7$

For 6:



7

$\frac{-x}{9} < -2$

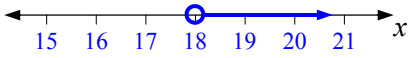
$(\times 9) \quad (\times 9)$

$-x < -18$

$(\div -1) \quad (\div -1)$

$x > 18$

For 7:



8

$-2x \geqslant 8$

$(\div -2) \quad (\div -2)$

$x \leqslant -4$

For 8:



9

$-3x + 7 \geqslant -2$

$(-7) \quad (-7)$

$-3x \geqslant -9$

$(\div -3) \quad (\div -3)$

$x \leqslant 3$

For 9:



10

$-1 - \frac{x}{5} < -4$

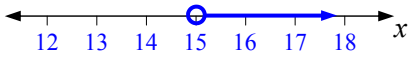
$(+1) \quad (+1)$

$\frac{x}{-5} < -3$

$(\times -5) \quad (\times -5)$

$x > 15$

For 10:



11

$\frac{1 - x}{2} \leqslant 5$

$(\times 2) \quad (\times 2)$

$1 - x \leqslant 10$

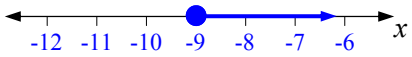
$(-1) \quad (-1)$

$-x \leqslant 9$

$(\div -1) \quad (\div -1)$

$x \geqslant -9$

For 11:



12

$11 - 2x > -11$

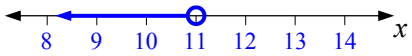
$(-11) \quad (-11)$

$-2x > -22$

$(\div -2) \quad (\div -2)$

$x < 11$

For 12:



13

$-4(x - 4) > 12$

$-4x + 16 > 12$

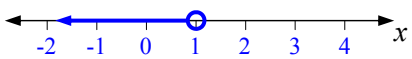
(-16)

$-4x > -4$

$(\div -4) \quad (\div -4)$

$x < 1$

For 13:



14

$-(24 - x) \leqslant 3x$

$-24 + x \leqslant 3x$

$(+24) \quad (+24)$

$x \leqslant 3x + 24$

$(-3x) \quad (-3x)$

$-2x \leqslant 24$

$(\div -2) \quad (\div -2)$

$x \geqslant -12$

For 14:



Test a point to check your answer. Solve the inequation, graph it, then test your answer.

Step 1: Solve the inequation
Step 2: Graph the inequation
Step 3: Pick test point ON arrow to check
Step 4: If incorrect, check arrow direction or your inequation solution.

15

$-4x < 8$

$(\div -4) \quad (\div -4)$

$x > -2$

Test using $x = 1$

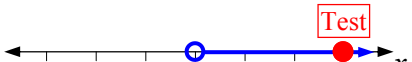
$-4x < 8$

$-4 \times 1 < 8$

$-4 < 8$

$\therefore$ Correct

For 15:



16

$\frac{2 - 5x}{6} \leqslant -8$

$(\times 6) \quad (\times 6)$

$2 - 5x \leqslant -48$

$(-2) \quad (-2)$

$-5x \leqslant -50$

$(\div -5) \quad (\div -5)$

$x \geqslant 10$

Test using $x =$ 12

$\frac{2 - 5x}{6} \leqslant -8$

$\frac{2 - 5 \times 12}{-6} \leqslant -8$

$-9\frac{2}{3} \leqslant -8$

$\therefore$ Correct

For 16:



17

$3(x - 4) \geqslant 5x$

$3x - 12 \geqslant 5x$

$(+12) \quad (+12)$

$3x \geqslant 5x + 12$

$(-5x) \quad (-5x)$

$-2x \geqslant 12$

$(\div -2) \quad (\div -2)$

$x \leqslant -6$

Test using $x =$ -8

$3(x - 4) \geqslant 5x$

$3(-8 - 4) \geqslant 5(-8)$

$-36 \geqslant -40$

$\therefore$ Correct

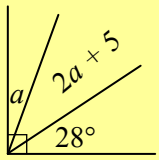
For 17:



Application of Equations

This sheet uses equations to solve questions involving angles, perimeter, area and number operations. Below is an example of the use of equations.

Example Finding the unknown angles.



$a + 2a + 5 + 28 = 90$ ← All the angles add to give a right angle, which is 90°. So add all angles, then '= 90'.

$3a + 33 = 90$ ← Gather like terms.

$-33 \quad -33$

$3a = 57$ ← Solve. First remove constant.

$\div 3 \quad \div 3$

$a = 19°$ ← Then remove the coefficient.

Note that this doesn't answer the question. One angle is 'a' so it is 19°, but there is another angle '$2a + 5$'. You need to substitute $a = 19$, to calculate this angle.

$2a + 5 = 2 \times 19 + 5$

$2a + 5 = 43°$

Questions vary but the method of solution is the same process. Note that you can perform a check to make sure your solution is correct. For the above: $a = 19°$, $2a + 5 = 43°$, so add the angles and they should total 90°.

$19 + 43 + 28 = 90$

Application of Equations

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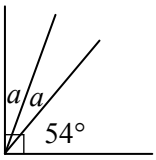
Use your equation skills to solve for the missing angles.

Example



$2a + 54 = 90$

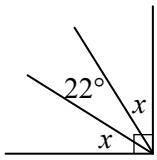
$-54 \quad -54$

$2a = 36$

$\div 2 \quad \div 2$

$a = 18°$

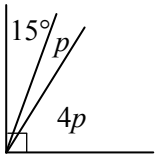
1



$-22 \quad -22$

$\div 2 \quad \div 2$

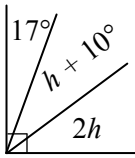
2



$4p = 4 \times \square = \square$

3

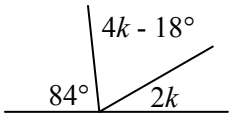
$\square + \square + \square = \square$



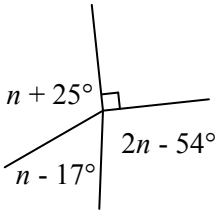
$2h = 2 \times \square = \square$

$h + 10 = \square + 10 = \square$

4



5



How old are these people? Use equations to solve.

6 Jane is 'x' years old. Dan is twice as old as Jane, Pete is 4 years older than Dan. Find the age of each person if together their ages total 89 years.

$x + 2x + 2x + 4 = 89$

Jane is $\square$ years old

Dan $= 2 \times \square$

Dan is $\square$ years old

Pete $= \square + 4$

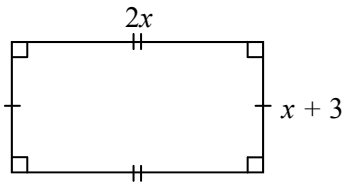
Pete is $\square$ years old

7 Joe is 4 years older than Tom. Dave is twice as old as Joe. Their ages add to 120 years. Find the age of each person.

The equation above should feature brackets

Given the perimeter or area, find the unknown side lengths.

8 Perimeter = 72 cm



$\square + \square + \square + \square = 72$

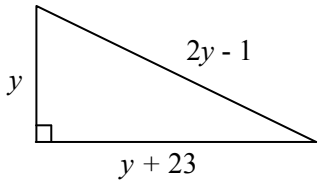
$2x = 2 \times \square$

$2x = \square$ cm

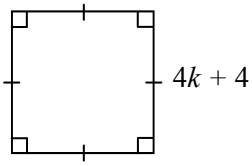
$x + 3 = \square + 3$

$x + 3 = \square$ cm

9 Perimeter = 154 mm



10 Area = 256 cm^2



Application of Equations

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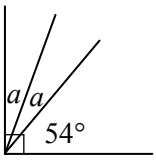
Use your equation skills to solve for the missing angles.

Example



$2a + 54 = 90$

$-54 \quad -54$

$2a = 36$

$\div 2 \quad \div 2$

$a = 18°$

1

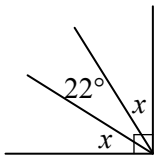
$2x + 22 = 90$

$-22 \quad -22$

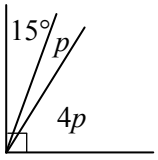
$2x = 68$

$\div 2 \quad \div 2$

$x = 34°$



2



$5p + 15 = 90$

$-15 \quad -15$

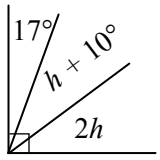
$5p = 75$

$\div 5 \quad \div 5$

$p = 15°$

$4p = 4 \times \boxed{15} = \boxed{60°}$

3 $\boxed{17} + \boxed{h + 10} + \boxed{2h} = \boxed{90}$



$3h + 27 = 90$

$-27 \quad -27$

$3h = 63$

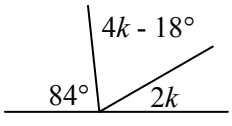
$\div 3 \quad \div 3$

$h = 21°$

$2h = 2 \times \boxed{21} = \boxed{42°}$

$h + 10 = \boxed{21} + 10 = \boxed{31°}$

4 $84 + 4k - 18 + 2k = 180$



$6k + 66 = 180$

$-66 \quad -66$

$6k = 114$

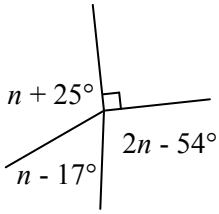
$\div 6 \quad \div 6$

$k = 19°$

$4k - 18 = 4 \times 19 - 18 = 58°$

$2k = 2 \times 19 = 38°$

5 $n + 25 + n - 17 + 2n - 54 + 90 = 360$



$4n + 44 = 360$

$-44 \quad -44$

$4n = 316$

$\div 4 \quad \div 4$

$n = 79°$

$n + 25 = 79 + 25 = 104°$

$n - 17 = 79 - 17 = 62°$

$2n - 54 = 2 \times 79 - 54 = 104°$

How old are these people? Use equations to solve.

6 Jane is 'x' years old. Dan is twice as old as Jane, Pete is 4 years older than Dan. Find the age of each person if together their ages total 89 years.

$x + 2x + 2x + 4 = 89$

$5x + 4 = 89$

$-4 \quad -4$

$5x = 85$

$\div 5 \quad \div 5$

$x = 17$

Jane is $\boxed{17}$ years old

Dan $= 2 \times \boxed{17}$

Dan is $\boxed{34}$ years old

Pete $= \boxed{34} + 4$

Pete is $\boxed{38}$ years old

7 Joe is 4 years older than Tom. Dave is twice as old as Joe. Their ages add to 120 years. Find the age of each person.

$t + t + 4 + 2(t + 4) = 120$

The equation above should feature brackets

$t + t + 4 + 2t + 8 = 120$

$4t + 12 = 120$

$-12 \quad -12$

$4t = 108$

$\div 4 \quad \div 4$

$t = 27$

Tom is 27 years old

Joe $= 27 + 4$

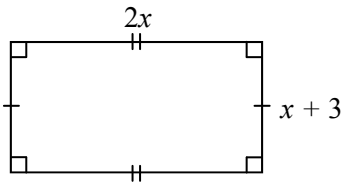
Joe is 31 years old

Dave $= 2 \times 31$

Dave is 62 years old

Given the perimeter or area, find the unknown side lengths.

8 Perimeter = 72 cm



$\boxed{2x} + \boxed{x + 3} + \boxed{2x} + \boxed{x + 3} = 72$

$6x + 6 = 72$

$-6 \quad -6$

$6x = 66$

$\div 6 \quad \div 6$

$x = 11$

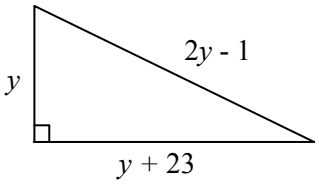
$2x = 2 \times \boxed{11}$

$2x = \boxed{22}$ cm

$x + 3 = \boxed{11} + 3$

$x + 3 = \boxed{14}$ cm

9 Perimeter = 154 mm



$y + y + 23 + 2y - 1 = 154$

$4y + 22 = 154$

$-22 \quad -22$

$4y = 132$

$\div 4 \quad \div 4$

$y = 33$ cm

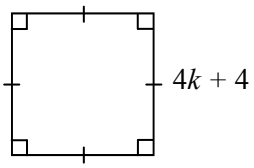
$y + 23 = 33 + 23$

$y + 22 = 56$ cm

$2y - 1 = 2 \times 33 - 1$

$2y - 1 = 65$ cm

10 Area = 256 cm^2



$4k + 4 = \sqrt{256}$

$4k + 4 = 16$

$-4 \quad -4$

$4k = 12$

$\div 4 \quad \div 4$

$k = 3$

$4k + 4 = 4 \times 3 + 4$

$4k + 4 = 16$ cm

Equations & Inequations - Find a Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Equations & Inequations - Find a Word

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Find the words in the puzzle from the wordlist.



E	T	U	T	I	T	S	B	U	S	N	A	P	X	E
X	K	D	E	X	P	O	Y	L	P	I	T	L	U	M
P	D	I	T	C	E	L	L	O	C	E	P	S	E	T
A	V	I	L	E	N	V	E	X	P	X	S	V	N	E
N	E	S	U	Q	W	E	F	C	N	E	E	A	P	X
D	Y	M	U	E	O	Q	U	O	L	E	T	Q	S	E
I	T	R	Y	R	N	Q	I	M	R	S	E	E	U	G
N	I	E	F	E	K	S	B	M	N	M	Q	B	B	N
G	L	T	I	T	N	T	R	O	T	B	U	U	T	A
B	A	C	L	A	U	U	C	N	U	U	A	L	R	R
E	U	T	P	E	B	R	A	C	K	E	T	S	A	R
U	Q	X	M	R	T	C	E	D	I	V	I	D	C	A
L	E	P	I	G	E	V	I	T	I	S	O	P	T	E
A	N	A	S	E	V	I	T	A	G	E	N	E	X	R
V	I	N	P	T	N	E	I	C	I	F	F	E	O	C

WORDLIST

REARRANGE	POSITIVE	DIVIDE	EQUATION	TERMS
SUBSTITUTE	SUBTRACT	COLLECT	BRACKETS	COMMON
INEQUALITY	UNKNOWN	GREATER	ADD	NEGATIVE
COEFFICIENT	SOLVE	CONSTANT	SIMPLIFY	LESS
EXPANSION	FORMULA	LIKE	MULTIPLY	VALUE
		EXPANDING		

Equations & Inequations - Find a Word

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Find the words in the puzzle from the wordlist.



E	T	U	T	I	T	S	B	U	S	N	A	P	X	E
X	K	D	E	X	P	O	Y	L	P	I	T	L	U	M
P	D	I	T	C	E	L	L	O	C	E	P	S	E	T
A	V	I	L	E	N	V	E	X	P	X	S	V	N	E
N	E	S	U	Q	W	E	F	C	N	E	E	A	P	X
D	Y	M	U	E	O	Q	U	O	L	E	T	Q	S	E
I	T	R	Y	R	N	Q	I	M	R	S	E	E	U	G
N	I	E	F	E	K	S	B	M	N	M	Q	B	B	N
G	L	T	I	T	N	T	R	O	T	B	U	U	T	A
B	A	C	L	A	U	U	C	N	U	U	A	L	R	R
E	U	T	P	E	B	R	A	C	K	E	T	S	A	R
U	Q	X	M	R	T	C	E	D	I	V	I	D	C	A
L	E	P	I	G	E	V	I	T	I	S	O	P	T	E
A	N	A	S	E	V	I	T	A	G	E	N	E	X	R
V	I	N	P	T	N	E	I	C	I	F	F	E	O	C

WORDLIST

~~REARRANGE~~ ~~POSITIVE~~ ~~DIVIDE~~ ~~EQUATION~~ ~~TERMS~~

~~SUBSTITUTE~~ ~~SUBTRACT~~ ~~COLLECT~~ ~~BRACKETS~~ ~~COMMON~~

~~INEQUALITY~~ ~~UNKNOWN~~ ~~GREATER~~ ~~ADD~~ ~~NEGATIVE~~

~~COEFFICIENT~~ ~~SOLVE~~ ~~CONSTANT~~ ~~SIMPLIFY~~ ~~LESS~~

~~EXPANSION~~ ~~FORMULA~~ ~~LIKE~~ ~~MULTIPLY~~ ~~VALUE~~

~~EXPANDING~~

FREEFALL MATHEMATICS

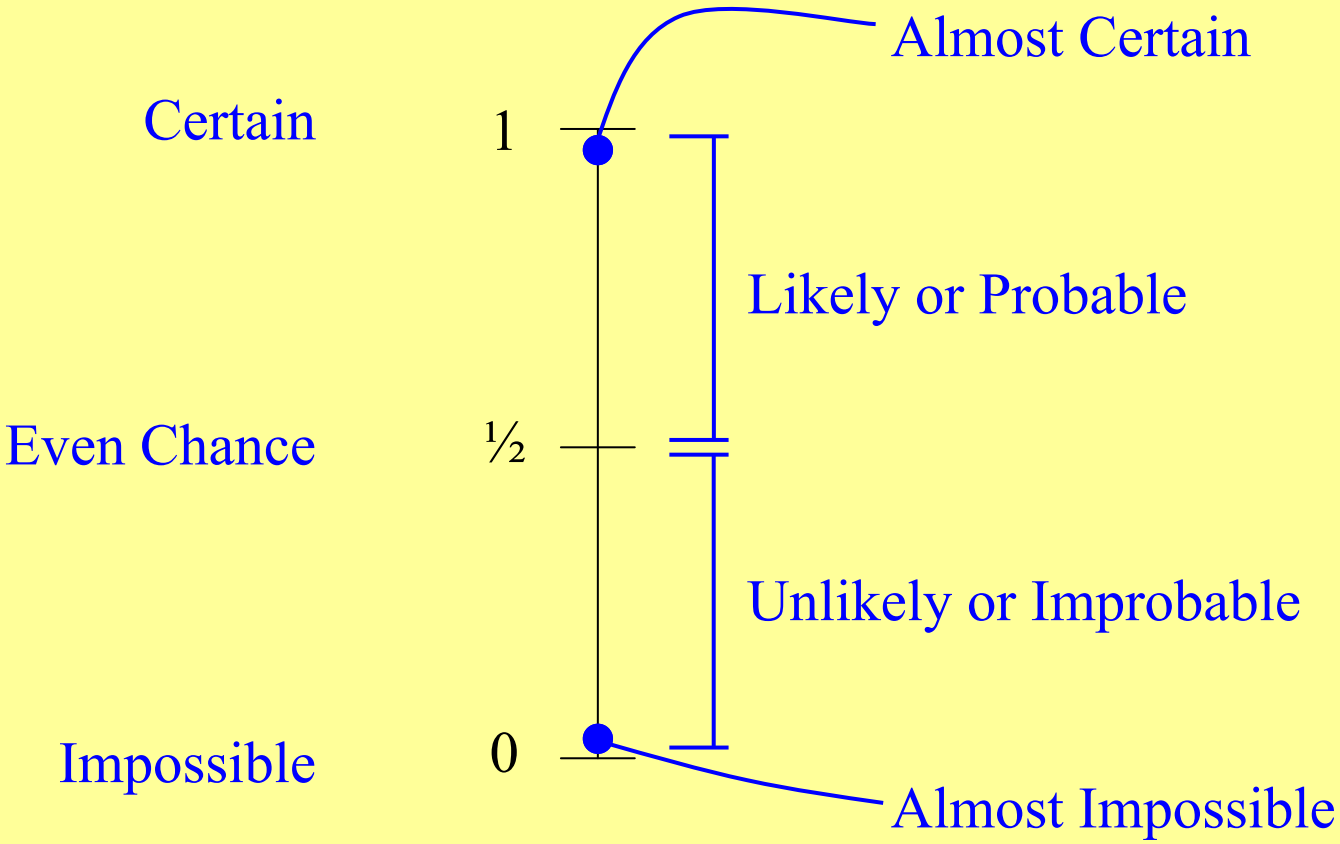
ALTITUDE

PROBABILITY

The Language of Chance

The chance of something happening, or the probability of something happening….means the same thing. In Mathematics the measurement of chance can be through the use of numbers, but also the use of words.

Probability varies from zero to one (0 → 1), or 0% to 100% if percentages are used. But what if you can't describe something exactly, if there are no numbers to base your answer on? Then you need to use words instead. Look at the probability scale below, which shows where these words occur.



So an event that is certain has a probability of 1, an event that is impossible has a probability of 0. An even chance has a probability of ½.

When something is almost certain it is extremely close to1, and when an event is almost impossible is almost 0. To the right of the scale are the two ranges of likely (above an even chance but not certain) and unlikely (below an even chance but not impossible).

The Language of Chance

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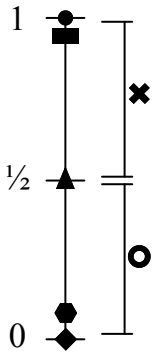
Place the appropriate word that matches the shape in the probability scale.

Use this word list for the symbols: Impossible, likely, unlikely, even chance, certain, highly unlikely and almost certain.



1 ● = _____

2 ▲ = _____

3 ◆ = _____

4 ⬢ = _____

5 ○ = _____

6 ▬ = _____

7 ✖ = _____

Place the question numbers on the probability scale below, where you think they go.

8 The rolling of a number less than 3 with a fair die.

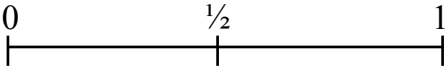
9 Selecting a red card from a deck of cards

10 The chance of buying one ticket and winning the lottery.

11 Selecting a number card from a deck of cards

12 Rolling a die and getting a number less than 7

Place numbers below the scale



Explain the meaning of an event having ...

13 A Probability of 1.

_____

_____

14 A Probability of ½.

_____

_____

15 A Probability of 0.

_____

_____

Using a bag of marbles that contains either blue or green marbles, or a mix of both, describe an event that is ….

16 Almost impossible.

_____

_____

_____

17 An even chance.

_____

_____

_____

18 Certain. _____

_____

_____

19 Impossible.

_____

_____

_____

Draw a line between the word and its match.

20

Certain	Even Chance
Likely	Impossible
Should Happen	50/50 Chance
Almost Impossible	Has to happen
Cannot happen	Highly Unlikely

Use the words above to answer these sentences.

21 The chance you will celebrate your 53rd birthday tomorrow. _____

22 The chance you will use your eraser today. _____

23 The chance you will loan money to the person seated closest to you. _____

24 The chance you will answer this question with your non-writing hand. _____

25 The chance the next baby born at the local hospital will be a boy. _____

26 The chance the next birth at your local hospital will be triplets. _____

27 The chance of reading this question while patting a live mammoth. _____

28 The chance of tossing a coin and getting a head or a tail.

_____

The Language of Chance

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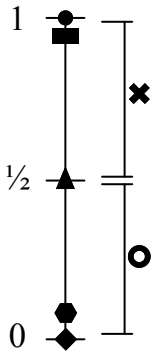
Place the appropriate word that matches the shape in the probability scale.

Use this word list for the symbols: Impossible, likely, unlikely, even chance, certain, highly unlikely and almost certain.



1 ● = Certain

2 ▲ = Even Chance

3 ◆ = Impossible

4 ⬢ = Highly Unlikely

5 ○ = Unlikely

6 ▬ = Almost Certain

7 ✖ = Likely

Place the question numbers on the probability scale below, where you think they go.

8 The rolling of a number less than 3 with a fair die.

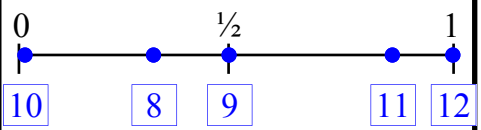
9 Selecting a red card from a deck of cards

10 The chance of buying one ticket and winning the lottery.

11 Selecting a number card from a deck of cards

12 Rolling a die and getting a number less than 7

Place numbers below the scale



10 8 9 11 12

Explain the meaning of an event having ...

13 A Probability of 1.

The event has to happen, it is certain.

14 A Probability of ½.

The event has an equal chance of happening or not happening.

15 A Probability of 0.

The event cannot happen, it is impossible.

Using a bag of marbles that contains either blue or green marbles, or a mix of both, describe an event that is ….

16 Almost impossible.

Selecting a blue marble from a bag that contains 1 000 green marbles and 1 blue marble.

17 An even chance.

Selecting a green marble from a bag that contains 5 green and 5 blue marbles.

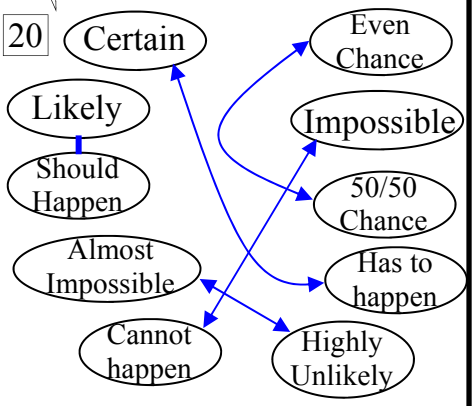
18 Certain. Selecting a green marble from a bag that only contains green marbles.

19 Impossible.

Selecting a blue marble from a bag that contains only green marbles.

Draw a line between the word and its match.

20



Use the words above to answer these sentences.

21 The chance you will celebrate your 53rd birthday tomorrow. Impossible. (student)

22 The chance you will use your eraser today. Likely.

23 The chance you will loan money to the person seated closest to you. Answer will vary

24 The chance you will answer this question with your non-writing hand. Highly unlikely? or certain

25 The chance the next baby born at the local hospital will be a boy. Even chance.

26 The chance the next birth at your local hospital will be triplets. Highly unlikely.

27 The chance of reading this question while patting a live mammoth. Impossible.

28 The chance of tossing a coin and getting a head or a tail. Certain.

Sample Space and Theoretical Probability

Sample space is a list of all the possible outcomes for an event. The sample space is normally listed using braces and commas, eg. Sample space of a coin = {H, T} or {Head, Tail}.
So look at all the outcomes and list them. If one outcome could happen twice then list it twice.

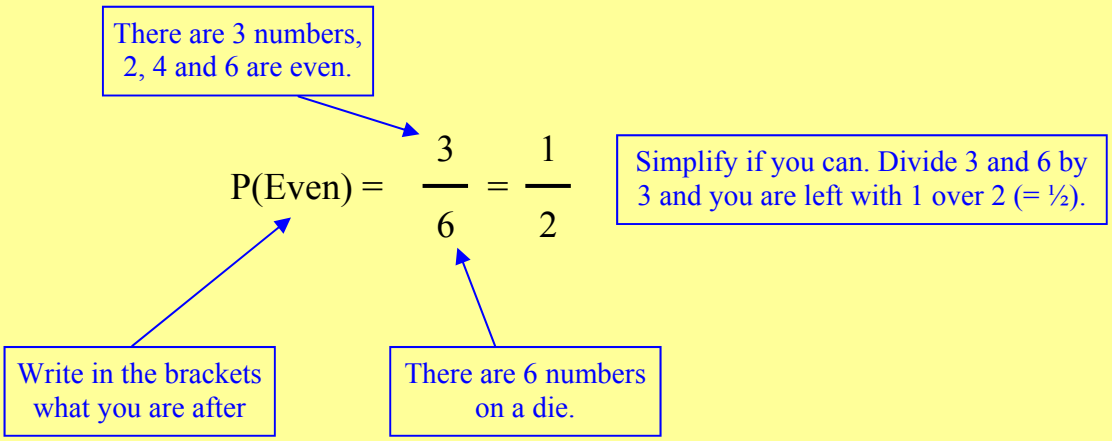
Theoretical probability is the theoretical chance of an event occurring it is found by:

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

If the above is a bit confusing then think of it as:

$$P(E) = \frac{\text{How many of what you are after}}{\text{Size of sample space}}$$

Example - Find the probability of rolling an even number with a die.



Column 1 starts with finding the sample space. Write 'Sample Space = {' then list all the possibilities, separated by commas. At the bottom of column 1 is a few questions on simplifying. To answer these put the fraction into your calculator and press '=' or divide the top and bottom (numerator and denominator) by the largest number that goes into both, (HCF).

Column 2 and 3 are all probability questions. Remember to write what you are looking for in the brackets, then write the fraction and simplify if possible. Question 28 is a little tricky, see how you go.

Sample Space and Theoretical Probability

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List the sample space for the following.

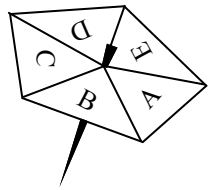
1 A coin.

Sample Space is { _____ }

2 A die.

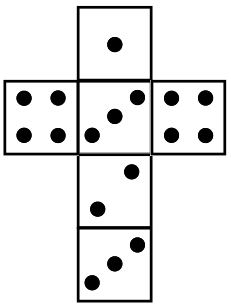
Sample Space is { _____ }

3 The spinner shown.



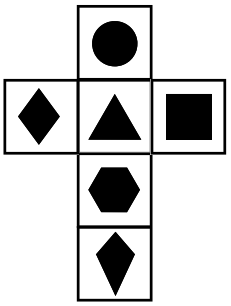
_____

4 The dice with nets shown.



_____

5



_____

_____

_____

Simplify these fractions.

6 $\frac{5}{20} = \frac{\square}{\square}$

7 $\frac{18}{24} = \frac{\square}{\square}$

8 $\frac{45}{75} = \frac{\square}{\square}$

9 $\frac{32}{48} = \frac{\square}{\square}$

Find the probability for these events.

A hat contains 3 blue marbles, 7 red and 6 green marbles. Find the probability (simplify) of selecting a:

10 Green marble P(G) = _____

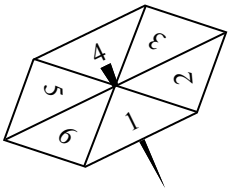
11 Blue marble _____

12 Black marble _____

13 Red or blue marble

_____

The spinner below is numbered 1 through 6. If the spinner is spun once find the probability of spinning:



14 An even number

_____

15 A number less than 5

_____

16 Spinning a factor of 8

_____

17 Not spinning a 2

_____

A standard deck of cards has 4 suits with 13 cards in each suit. There are no jokers included. For a standard deck of cards, find the probability (simplify your answer) of selecting a:

18 Black card _____

19 King _____

20 Red Ace or a 2 or 3 from any suite

_____

A lottery has 60 tickets, numbered 1 to 60. If a single ticket is selected, calculate the probability that the selected ticket will be a number that:

21 Is less than 20 _____

22 Is a single digit number

_____

23 Is a multiple of 7

_____

24 Has at least one 5 as a digit

_____

A carnival dart throwing stall gives a prize if more than 20 points are scored from 3 throws. If David's first 2 throws total 12 points. Calculate the probability of him winning with his last throw. The dart board is as below.

Carnival Dart Board

5	2	10	4	8	9
10	3	7	3	9	8
3	10	2	5	5	10
8	5	8	7	9	3
2	5	9	10	2	9
9	8	2	5	10	7

25 _____

From a deck of cards (no jokers) calculate the probability of selecting:

26 A card that isn't red or a picture card.

_____

27 A black king or a 'spades' card.

_____

A bag contains 6 red discs, 3 yellow discs and an unknown number of black discs. The probability of selecting a red disc is 2/5 how many black discs are in the bag?

28 _____

Sample Space and Theoretical Probability

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List the sample space for the following.

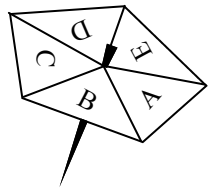
1 A coin.

Sample Space is { H, T. }

2 A die.

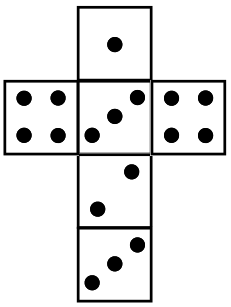
Sample Space is { 1, 2, 3, 4, 5, 6. }

3 The spinner shown.



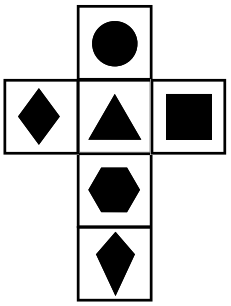
Sample space is {A, B, C, D, E}

4 The dice with nets shown.



Sample space is {1, 2, 3, 3, 4, 4}

5



Sample space is {Circle, Kite, Square, Triangle, Hexagon, Rhombus or diamond}

Simplify these fractions.

6 $\frac{5}{20} = \frac{1}{4}$

7 $\frac{18}{24} = \frac{3}{4}$

8 $\frac{45}{75} = \frac{3}{5}$

9 $\frac{32}{48} = \frac{2}{3}$

Find the probability for these events.

A hat contains 3 blue marbles, 7 red and 6 green marbles. Find the probability (simplify) of selecting a:

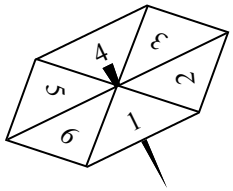
10 Green marble $P(G) = \frac{6}{16} = \frac{3}{8}$

11 Blue marble $P(B) = \frac{3}{16}$

12 Black marble $P(\text{Black}) = 0$

13 Red or blue marble $P(\text{Red, Blue}) = \frac{10}{16} = \frac{5}{8}$

The spinner below is numbered 1 through 6. If the spinner is spun once find the probability of spinning:



14 An even number $P(\text{Even}) = \frac{3}{6} = \frac{1}{2}$

15 A number less than 5 $P(<5) = \frac{4}{6} = \frac{2}{3}$

16 Spinning a factor of 8 $P(\text{Factor of 8}) = \frac{3}{6} = \frac{1}{2}$

17 Not spinning a 2 $P(\widetilde{2}) = \frac{5}{6}$

A standard deck of cards has 4 suits with 13 cards in each suit. There are no jokers included. For a standard deck of cards, find the probability (simplify your answer) of selecting a:

18 Black card $P(\text{Black}) = \frac{1}{2}$

19 King $P(\text{King}) = \frac{4}{52} = \frac{1}{13}$

20 Red Ace or a 2 or 3 from any suite $P(\text{R.Ace, 2, 3}) = \frac{10}{52} = \frac{5}{26}$

A lottery has 60 tickets, numbered 1 to 60. If a single ticket is selected, calculate the probability that the selected ticket will be a number that:

21 Is less than 20 $P(<20) = \frac{19}{60}$

22 Is a single digit number $P(<10) = \frac{9}{60} = \frac{3}{20}$

23 Is a multiple of 7 $P(\text{Multiple of 7}) = \frac{8}{60} = \frac{2}{15}$

24 Has at least one 5 as a digit $P(\text{5 as a digit}) = \frac{15}{60} = \frac{1}{4}$

A carnival dart throwing stall gives a prize if more than 20 points are scored from 3 throws. If David's first 2 throws total 12 points. Calculate the probability of him winning with his last throw. The dart board is as below.

Carnival Dart Board

5	2	(10)	4	8	(9)
(10)	3	7	3	(9)	8
3	(10)	2	5	5	(10)
8	5	8	7	(9)	3
2	5	(9)	(10)	2	(9)
(9)	8	2	5	(10)	7

25 $P(\text{9 or more}) = \frac{12}{36} = \frac{1}{3}$

From a deck of cards (no jokers) calculate the probability of selecting:

26 A card that isn't red or a picture card. $P(\text{Not Red or Pic}) = \frac{20}{52} = \frac{5}{13}$

27 A black king or a 'spades' card. $P(\text{B.King or Spades}) = \frac{14}{52} = \frac{7}{26}$

A bag contains 6 red discs, 3 yellow discs and an unknown number of black discs. The probability of selecting a red disc is 2/5 how many black discs are in the bag?

28 $\frac{2}{5} = \frac{6}{x}$ 15 - 6 - 3 = 6 black

Theoretical Probability - Sheet 2

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Complete the table for the tossing of two coins. Then answer the questions.

Coin 1

Coin 2	Head	Tail
Head	(Head, Head)	
Tail		

1 How many outcomes are possible? _____

2 Find P(T,T) _____ 3 Find P(H,H) _____

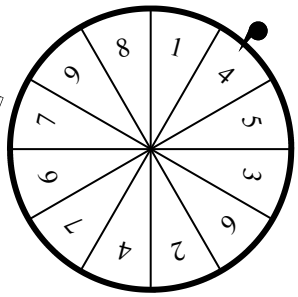
4 Find P(H,T) _____ 5 Find P(T,H) _____

If two coins were tossed 100 times, how many times would you expect to get...

6 Two Heads? _____ 7 Two Tails? _____

8 A head and a tail in either order? _____

The carnival is in town and Don's wheel is in sideshow alley. It costs $2 to spin, if you spin an even number you get $3, (your $2 + $1), spin an odd number and you lose the $2.



9 List the sample space for the wheel.

_____

10 Find P(Win) and P(Lose). _____

_____

11 Is this a fair bet? Explain how it is or isn't.

_____

_____

_____

12 If the wheel was spun 400 times in a night how much money is Don likely to make?

_____

_____

_____

The grid below is for the sum of the face numbers of two rolled dice. So it is an addition table. Fill er' up!

Dice 1

Dice 2

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

13 How many outcomes are possible? _____

14 Find P(10) _____ 15 Find P(3) _____

16 Find P(7) _____ 17 Find P(2) _____

18 Find P(Even Number) _____

19 Find P(Number < 10) _____

20 Find P(Number > 5) _____

21 Find P(Multiple of 3) _____

Now use the grid to find the following.

22 If the two dice were rolled 108 times how many times would you expect to roll a total of 7?

_____

23 If the two dice were rolled 300 times how many times would you expect to roll a total of 4?

_____

If the two dice were rolled 1 000 times how many times would you expect to roll a total that was….

24 A multiple of 4? _____

25 10 or more? _____

_____

Theoretical Probability - Sheet 2

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Complete the table for the tossing of two coins. Then answer the questions.

Coin 1

Coin 2	Head	Tail
Head	(Head, Head)	(Head, Tail)
Tail	(Tail, Head)	(Tail, Tail)

1 How many outcomes are possible? 4

2 Find P(T,T) $\frac{1}{4}$ 3 Find P(H,H) $\frac{1}{4}$

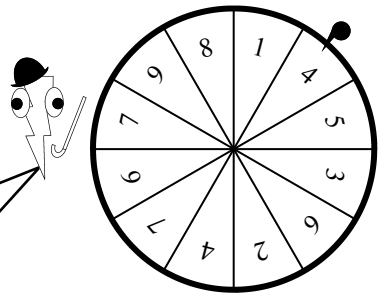
4 Find P(H,T) $\frac{1}{4}$ 5 Find P(T,H) $\frac{1}{4}$

If two coins were tossed 100 times, how many times would you expect to get...

6 Two Heads? 25 7 Two Tails? 25

8 A head and a tail in either order? 50

The carnival is in town and Don's wheel is in sideshow alley. It costs $2 to spin, if you spin an even number you get $3, (your $2 + $1), spin an odd number and you lose the $2.



9 List the sample space for the wheel.

Sample space = {1, 2, 3, 4, 4, 5, 6, 6, 7, 7, 8, 9}

10 Find P(Win) and P(Lose). P(Win) = $\frac{6}{12} = \frac{1}{2}$

P(Lose) = $\frac{6}{12} = \frac{1}{2}$

11 Is this a fair bet? Explain how it is or isn't.

It isn't a fair bet because while there is an equal chance of winning and losing, when you win you only gain $1 while losing costs you $2.

12 If the wheel was spun 400 times in a night how much money is Don likely to make?

200 wins will earn Don $400. 200 × 2 = $400

200 losses will cost Don $200 200 × 1 = $200

So he should make about $200. 400 - 200 = $200

The grid below is for the sum of the face numbers of two rolled dice. So it is an addition table. Fill er' up!

Dice 1

Dice 2

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

13 How many outcomes are possible? 36

14 Find P(10) $\frac{3}{36} = \frac{1}{12}$ 15 Find P(3) $\frac{2}{36} = \frac{1}{18}$

16 Find P(7) $\frac{6}{36} = \frac{1}{6}$ 17 Find P(2) $\frac{1}{36}$

18 Find P(Even Number) $\frac{18}{36} = \frac{1}{2}$

19 Find P(Number < 10) $\frac{30}{36} = \frac{5}{6}$

20 Find P(Number > 5) $\frac{26}{36} = \frac{13}{18}$

21 Find P(Multiple of 3) $\frac{12}{36} = \frac{1}{3}$

Now use the grid to find the following.

22 If the two dice were rolled 108 times how many times would you expect to roll a total of 7?

$108 \times \frac{1}{6}$ = 18 times

23 If the two dice were rolled 300 times how many times would you expect to roll a total of 4?

$300 \times \frac{3}{36}$ = 25 times.

If the two dice were rolled 1 000 times how many times would you expect to roll a total that was….

24 A multiple of 4? $1\,000 \times \frac{9}{36}$ = 250 times.

25 10 or more? $1\,000 \times \frac{6}{36}$ = 166⅔

So about 166 times.

Experimental Probability

Experimental probability bases theory on results that are achieved. So if you roll a dice 10 times and you don't roll 'a 6' then the probability of rolling a 6 is P(6) = 0. Of course you know that the chance of rolling 'a 6' isn't zero, but you are acting on the experimental results obtained.

It is important to remember that experimental probability is very dependant upon the size of the sample space. The more you roll the dice, toss the coin or spin the spinner, the more the experimental results will tend towards the theoretical results. But a reduced amount of trials could result in experimental probabilities that suggest unrealistic or inaccurate probabilities.

As experimental probability requires a number of trials, the results are normally displayed in a table. This is a Frequency table but it has an additional column called 'Relative Frequency' or '*r.f.*' for short.

$$r.f. = \frac{\text{frequency for event}}{\text{sum of frequency column}} = \frac{f}{\Sigma f}$$

Colour	f	$r.f.$
Red	35	$\frac{35}{75} = \frac{7}{15}$
Blue	24	$\frac{24}{75} = \frac{8}{25}$
Green	16	$\frac{16}{75}$
	$\Sigma f = 75$	

STEP 2

Make a fraction by writing 'f' over Σf. If you can simplify the fraction, do so.

STEP 1

Add the column and write the total at the bottom of the column where the Σ is shown.

Once you have the relative frequency for each event you can then apply this to the questions. For example the sample space was 75 for the table above. If the sample space was 150 how many people would choose red? 150 is double 75 so you would expect to double the frequency for Red, so 2 × 35 = 70. But using the relative frequency this would be:

Relative frequency → $\frac{7}{15} \times 150 = 70$

New sample space

So if the sample space was 150, Red would occur 70 times….. but would it? Remember that experimental frequency is indicating a probability. It may not happen, but based on the results, this is the most likely outcome. But the result isn't certain.

Experimental Probability

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Find the relative frequency (as a fraction) for the traffic results shown in the table.

1 Table: Traffic Type

Vehicle Type	*f*	*r.f.*
Cars	150	
Trucks	28	
Buses	6	
Motorcycles	3	
Utilities	18	
Vans	5	
	$\Sigma f =$	

What is the probability that the next vehicle to pass by would ..

2 Be a truck or a van? _____

3 Be a bus or a car? _____

4 Not be a Utility? _____

If 11 655 vehicles used the street in a week, how many of the vehicles would you expect..

5 To be cars? _____

_____

6 To be buses? _____

_____

7 To be trucks or vans?

_____

If 450 cars used the street in a day, estimate how many…..

8 Buses used the street.

_____

9 Trucks used the street.

_____

A die is rolled 60 times with the results shown. Place the data into a frequency table, with a relative frequency column (*r.f.*) expressed as a fraction. Then answer the questions.

Data:

53142	23561	33456
61224	43125	54522
36412	53361	32453
41222	52431	42332

Face No.	Tally	*f*	*r.f.*
		$\Sigma f =$	

Basing your answers from the table, estimate the probability of ….

10 Rolling a 2 or a 6? _____

11 Rolling an even number? _____

12 Not rolling a 5? _____

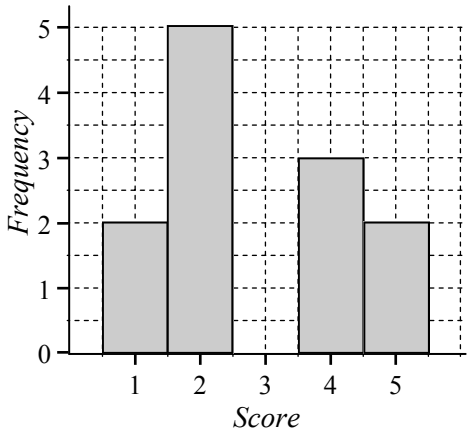
13 Rolling a composite number? _____

14 Would you consider this a fair die? Outline your reasoning.

_____

_____

A spinner numbered 1to 5 is spun with the results shown in the frequency histogram below. Examine the graph then answer the questions.



15 Which spinner number occurred the most? _____

16 How many times was the spinner spun? _____

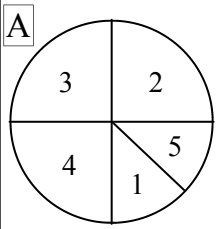
17 Find P(4) _____

18 Find P(1) _____

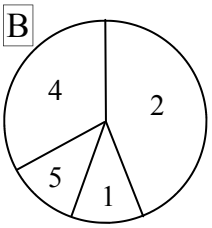
19 Find P(3) _____

20 Which spinner based on the results is the one most likely used?

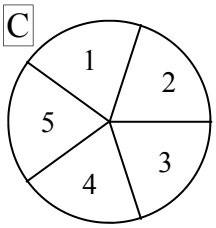
A



B



C



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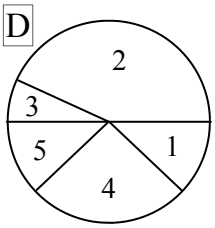

Experimental Probability

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Find the relative frequency (as a fraction) for the traffic results shown in the table.

1 Table: Traffic Type

Vehicle Type	f	$r.f.$
Cars	150	$\frac{150}{210} = \frac{5}{7}$
Trucks	28	$\frac{28}{210} = \frac{2}{15}$
Buses	6	$\frac{6}{210} = \frac{1}{35}$
Motorcycles	3	$\frac{3}{210} = \frac{1}{70}$
Utilities	18	$\frac{18}{210} = \frac{3}{35}$
Vans	5	$\frac{5}{210} = \frac{1}{42}$
	$\Sigma f = 210$	

What is the probability that the next vehicle to pass by would ..

2 Be a truck or a van? $\frac{33}{210} = \frac{11}{70}$

3 Be a bus or a car? $\frac{156}{210} = \frac{26}{35}$

4 Not be a Utility? $\frac{192}{210} = \frac{32}{35}$

If 11 655 vehicles used the street in a week, how many of the vehicles would you expect..

5 To be cars? $\frac{5}{7} \times 11\,655$

$= 8\,325$ cars

6 To be buses? $\frac{1}{35} \times 11\,655$

$= 333$ buses

7 To be trucks or vans?

$\frac{33}{210} \times 11\,655 \approx 1\,831$ vehicles

If 450 cars used the street in a day, estimate how many…..

8 Buses used the street.

$\frac{450}{150} \times 6 = 18$ buses

9 Trucks used the street.

$\frac{450}{150} \times 28 = 84$ trucks

A die is rolled 60 times with the results shown. Place the data into a frequency table, with a relative frequency column ($r.f.$) expressed as a fraction. Then answer the questions.

Data:

53142 23561 33456
61224 43125 54522
36412 53361 32453
41222 52431 42332

Face No.	Tally	f	$r.f.$
1	𝍸 |	8	$\frac{8}{60} = \frac{2}{15}$
2	𝍸 𝍸	15	$\frac{15}{60} = \frac{1}{4}$
3	𝍸 𝍸 |||	13	$\frac{13}{60}$
4	𝍸 ||||	10	$\frac{10}{60} = \frac{1}{6}$
5	𝍸 ||||	9	$\frac{9}{60} = \frac{3}{20}$
6	𝍸	5	$\frac{5}{60} = \frac{1}{12}$
		$\Sigma f = 60$	

Basing your answers from the table, estimate the probability of ….

10 Rolling a 2 or a 6?

$P(2,6) = \frac{20}{60} = \frac{1}{3}$

11 Rolling an even number?

$P(\text{Even}) = \frac{30}{60} = \frac{1}{2}$

12 Not rolling a 5?

$P(\widetilde{5}) = \frac{51}{60}$

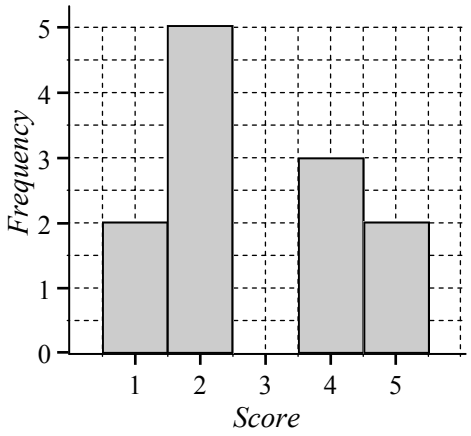
13 Rolling a composite number?

$P(4,6) = \frac{15}{60} = \frac{1}{4}$

14 Would you consider this a fair die? Outline your reasoning.

From the results only no it isn't fair, but a larger sample space is required to effectively indicate whether the die is fair or not.

A spinner numbered 1to 5 is spun with the results shown in the frequency histogram below. Examine the graph then answer the questions.



15 Which spinner number occurred the most? 2

16 How many times was the spinner spun? 12 times

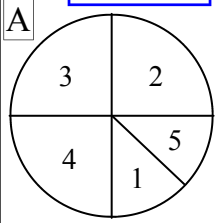
17 Find P(4) $= \frac{3}{12} = \frac{1}{4}$

18 Find P(1) $= \frac{2}{12} = \frac{1}{6}$

19 Find P(3) $= \frac{0}{12} = 0$

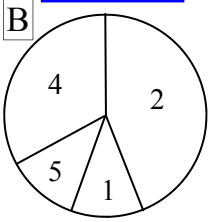
20 2 is too small, 3 is too large

A



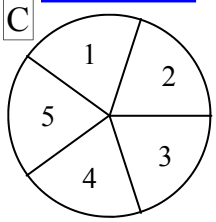
3 still must be featured

B

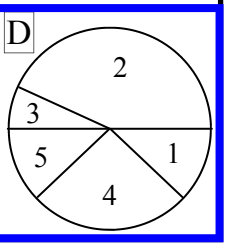


Doesn't allow for results

C



D



Probability Experiment Sheet

This sheet is designed to compare experimental results with theoretical results. There are two sheets as the first sheet has the "Sum of two Dice" grid left blank, the second grid has the numbers filled in, to avoid mistakes if this is an issue. As this is an experiment sheet there are no answers.

Column 1 has a table for recording the tossing of two coins. This requires either 2 different coins so that one is called 'Coin 1' and the other 'Coin 2', or use a single coin and toss it twice. The table has a tally and frequency column to complete, find $\sum f$ at the base of the column to verify the correct number of tosses has occurred. The data can then be transferred to the column graph below the table which has the theoretical results already graphed.

Compare the results and describe how your results related to the theoretical, indicate how the sample space size (the number of times the coins are tossed) affects the results.

Column 2 requires you to roll two dice 72 times. The dice can be rolled at the same time. Once rolled, add the two upper face numbers together to get a total and record the sum in the table using the tally column, converting to a frequency on completion. Again find the two $\sum f$'s and add them to verify that you have rolled the dice 72 times.

Now complete the theoretical probability grid (if blank) by adding the two numbers. You should notice a pattern along the diagonals. Plot the experimental result, then plot the theoretical result by counting all the same numbers in the grid, but because you rolled the dice 72 times then you have to double this number. Use different coloured columns for experimental and theoretical results.

Compare the results and describe how your results related to the theoretical, indicate how the sample space size (the number of times the dice are rolled) affects the results.

Probability Experiment Sheet

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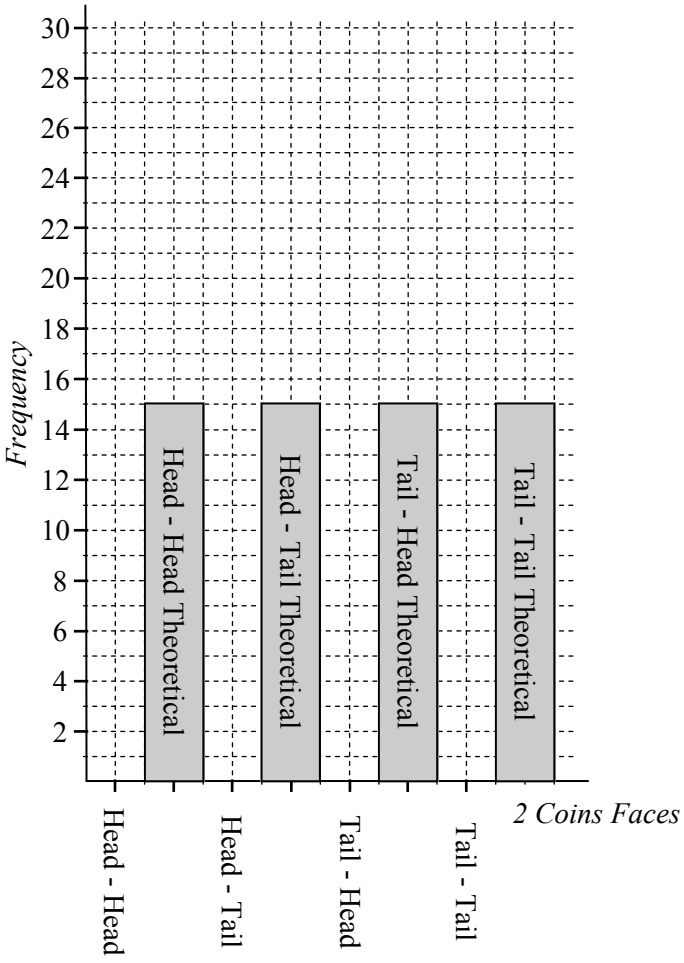
Toss two coins 60 times and record the results below. Toss one coin first then the second, or use two different value coins and call one Coin 1 and the other Coin 2 and toss them together.

Coin 1	Coin 2	Tally	*f*
Head	Head		
Head	Tail		
Tail	Head		
Tail	Tail		
			$\Sigma f =$

Column Graph: Two Tossed Coins Theoretical and Experimental



Explain how the experimental results compared with the theoretical results. Was the sample space large enough to get the expected result?

Now dice. Roll two dice 72 times adding the two face numbers together to get a total. Complete the table and grid, refer to grid to graph theoretical results. Compare the results with the theoretical.

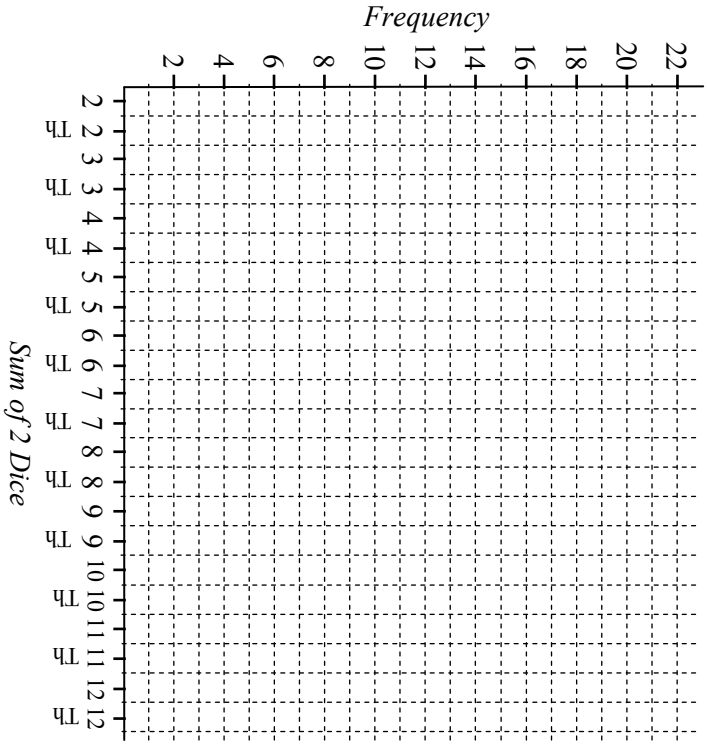
Total	Tally	*f*
2		
3		
4		
5		
6		
	$\Sigma f =$	

Total	Tally	*f*
7		
8		
9		
10		
11		
12		
	$\Sigma f =$	

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

Theoretical Probability Grid

This grid has 36 outcomes, so with 72 rolls, double the frequency of each number in the grid and use that as your expected theoretical frequency for the columns below.



Explain how the experimental results compared with the theoretical results.

Probability Experiment Sheet

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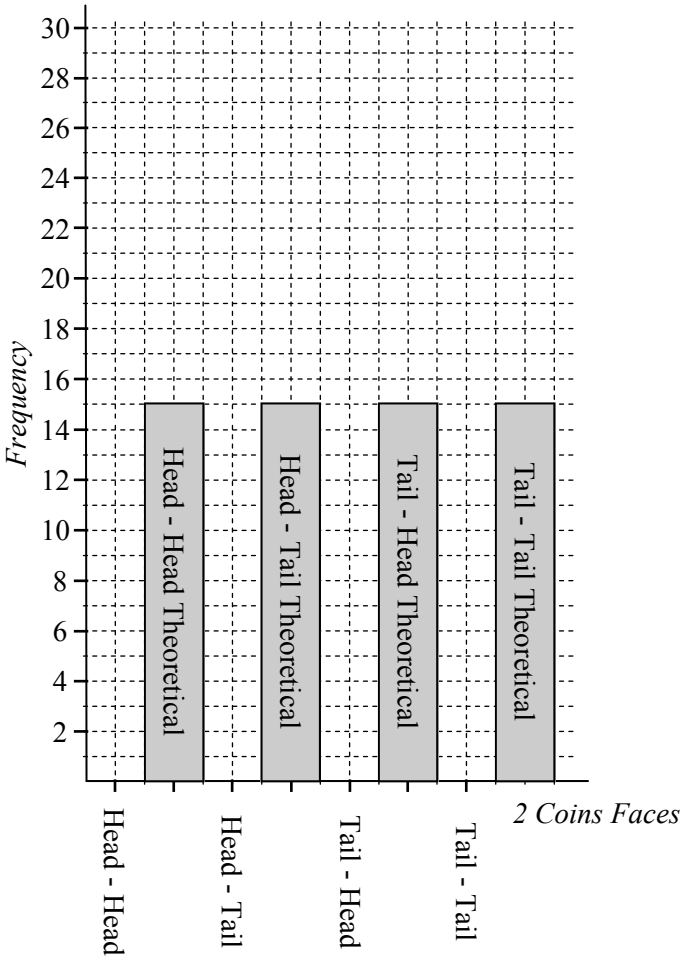
Toss two coins 60 times and record the results below. Toss one coin first then the second, or use two different value coins and call one Coin 1 and the other Coin 2 and toss them together.

Coin 1	Coin 2	Tally	*f*
Head	Head		
Head	Tail		
Tail	Head		
Tail	Tail		
			$\Sigma f =$

Column Graph: Two Tossed Coins Theoretical and Experimental



Explain how the experimental results compared with the theoretical results. Was the sample space large enough to get the expected result?

Now dice. Roll two dice 72 times adding the two face numbers together to get a total. Complete the table then refer to grid to graph the theoretical results. Compare the results with the theoretical.

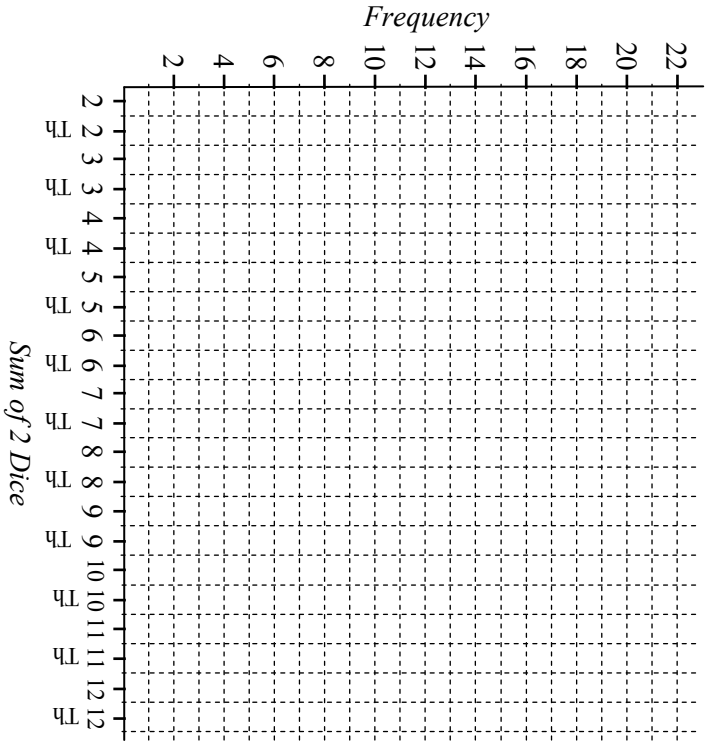
Total	Tally	*f*
2		
3		
4		
5		
6		
	$\Sigma f =$	

Total	Tally	*f*
7		
8		
9		
10		
11		
12		
	$\Sigma f =$	

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Theoretical Probability Grid

This grid has 36 outcomes, so with 72 rolls, double the frequency of each number in the grid and use that as your expected theoretical frequency for the columns below.



Explain how the experimental results compared with the theoretical results.

Probability with Decimals and Percentages - Non Replacement

Probability can be shown as a fraction but also as a decimal and a percentage. To convert to a decimal calculate the probability as a fraction, and either use the calculator division key, or the fraction key.

Example - Rolling a die and getting a 6, P(6) = $^1/_6$

Using the Fraction key

1 $a^b/_c$ 6 =

Answer on Screen 1⌟ 6

$a^b/_c$

Answer on Screen 0.1666..
Round to 2 decimal places
0.17

Using the Division key

1 ÷ 6 =

Answer on Screen 0.1666..
Round to 2 decimal places
0.17

Faster Method

It also helps you to remember that $a^b/_c$ = ÷

If you are asked to change to a percentage, then multiply the fraction (or decimal) by 100.

Example - Rolling a die and getting a 6, P(6) = $^1/_6$

Using the Fraction key

1 $a^b/_c$ 6 × 1 0 0 =

Answer on Screen 16⌟ 2⌟ 3

$a^b/_c$

Answer on Screen 16.6666...
Round to 1 decimal place
16.7%

Using the Division key

1 ÷ 6 × 1 0 0 =

Answer on Screen 16.6666...
Round to 1 decimal place
16.7%

Faster Method

It also helps you to remember that $a^b/_c$ = ÷

This sheet also includes non-replacement. This is when a selection is made and the sample space is reduced, then another selection is made. If you have 5 cards with 1, 2, 3, 4, and 5 being on separate cards.

Then to select a 3: P(3) = 1/5 = 0.2 (decimal)

If the 3 isn't replaced the sample space is reduced to 4 cards.

Then to select a 5: P(5) = 1/4 = 0.25 (decimal)

So if a card, or cards are removed the sample space reduces and the chance of selecting another card increases because the denominator in the fraction is getting smaller each time.

Probability with Decimals and Percentages - Non Replacement

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The word IRRESISTIBLE is written with each letter on a card and placed face down on a table. Find the probability, as a percentage of selecting ...

Example

An 'I' $P(I) = \frac{3}{12} \times 100 = 25\%$

Round to 1 decimal place when required

1 An 'E' _____

2 A 'T' _____

3 A vowel

_____

4 An 'I' or 'S' or 'E'

_____

5 If an 'E' was selected and not replaced, find P(T).

_____

If an 'E', 'B' and 'I' were selected and not replaced, find:

6 P(I) _____

7 P(E) _____

8 P(R) _____

9 P(B) _____

If the cards with vowels were removed, find the probability as a decimal (to 2 d.p.) of:

10 P(T) _____

11 P(S) _____

12 P(R, S)

_____

13 P(B, T, L)

_____

Two darts are thrown at the board below, if you get the same number twice, the second dart is removed and re-thrown. If you get 10 points or more, you win a prize.

5	3	2	4
6	1	2	8
5	1	3	7
2	1	2	2

Answer as a percentage Round to 1 d.p. when required

14 If a '4' was scored with the first dart, find P(Win).

_____

15 If a '3' was scored with the first dart, find P(Win).

_____

16 If a '2' was scored with the first dart, find P(Win).

_____

17 If an '8' was scored with the first dart, find P(Lose).

_____

A marble bag contains 4 blue, 7 red, 6 white and 3 green marbles. Find as a decimal to 2 d.p. ……

18 P(Blue) _____

19 P(B, R) _____

20 P(Not White) _____

21 P(Yellow) _____

If a marble was selected from the bag and replaced 32 times, how often would you expect to select a marble that was :

22 Green _____

_____

23 Blue or red _____

_____

If one person is chosen, show the probabilities as a column graph below.

24

Name	Sex	Age
Elizabeth	Female	50
Johan	Male	51
Lars	Male	17
Kristi	Female	8
James	Male	13

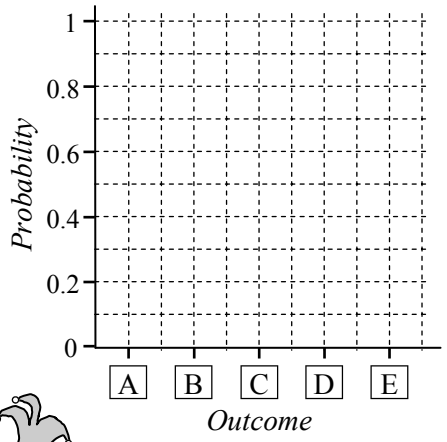
A The person is male

B The person is older than 8 years.

C The person is a teenager.

D The persons name has 2 vowels.

E The person is male or female.



A card is selected from a standard deck (no jokers), express the probability as a percentage (to 1 d.p.).

25 P(King) _____

_____

26 P(J, Q, K , A) _____

_____

27 P(Red Even Number Card)

_____

28 P(10,5,2) if a 10, a 5 and a 7 have been selected/not replaced.

_____

_____

Probability with Decimals and Percentages - Non Replacement

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The word IRRESISTIBLE is written with each letter on a card and placed face down on a table. Find the probability, as a percentage of selecting ...

Example

An 'I' $P(I) = \frac{3}{12} \times 100 = 25\%$

Round to 1 decimal place when required

1 An 'E' $P(E) = \frac{2}{12} \times 100 = 16.7\%$

2 A 'T' $P(T) = \frac{1}{12} \times 100 = 8.3\%$

3 A vowel

$P(\text{Vowel}) = \frac{5}{12} \times 100 = 41.7\%$

4 An 'I' or 'S' or 'E'

$P(I, S, E) = \frac{7}{12} \times 100 = 58.3\%$

5 If an 'E' was selected and not replaced, find P(T).

$P(T) = \frac{1}{11} \times 100 = 9.1\%$

If an 'E', 'B' and 'I' were selected and not replaced, find:

6 P(I) $P(I) = \frac{2}{9} \times 100 = 22.2\%$

7 P(E) $P(E) = \frac{1}{9} \times 100 = 11.1\%$

8 P(R) $P(R) = \frac{2}{9} \times 100 = 22.2\%$

9 P(B) $P(B) = \frac{0}{9} \times 100 = 0\%$

If the cards with vowels were removed, find the probability as a decimal (to 2 d.p.) of:

10 P(T) $P(T) = \frac{1}{7} = 0.14$

11 P(S) $P(S) = \frac{2}{7} = 0.29$

12 P(R, S)

$P(R, S) = \frac{4}{7} = 0.57$

13 P(B, T, L)

$P(B, T, L) = \frac{3}{7} = 0.43$

Two darts are thrown at the board below, if you get the same number twice, the second dart is removed and re-thrown. If you get 10 points or more, you win a prize.

5	3	2	4
6	1	2	8
5	1	3	7
2	1	2	2

Answer as a percentage Round to 1 d.p. when required

14 If a '4' was scored with the first dart, find P(Win).

$P(\text{Win}) = \frac{3}{15} \times 100 = 20\%$

15 If a '3' was scored with the first dart, find P(Win).

$P(\text{Win}) = \frac{2}{14} \times 100 = 20\%$

16 If a '2' was scored with the first dart, find P(Win).

$P(\text{Win}) = \frac{1}{11} \times 100 = 9.1\%$

17 If an '8' was scored with the first dart, find P(Lose).

$P(\text{Lose}) = \frac{3}{15} \times 100 = 20\%$

A marble bag contains 4 blue, 7 red, 6 white and 3 green marbles. Find as a decimal to 2 d.p. ……

18 P(Blue) $P(\text{Blue}) = \frac{4}{20} = 0.20$

19 P(B, R) $P(B, R) = \frac{11}{20} = 0.55$

20 P(Not White) $= \frac{14}{20} = 0.70$

21 P(Yellow) $P(Y) = \frac{0}{20} = 0$

If a marble was selected from the bag and replaced 32 times, how often would you expect to select a marble that was :

22 Green $P(G) = \frac{3}{20} = 0.15$

$0.15 \times 32 = 4.8 \approx 5$ times

23 Blue or red $P(B, R) = 0.55$

$0.55 \times 32 = 17.6 \approx 18$ times

24 If one person is chosen, show the probabilities as a column graph below.

Name	Sex	Age
Elizabeth	Female	50
Johan	Male	51
Lars	Male	17
Kristi	Female	8
James	Male	13

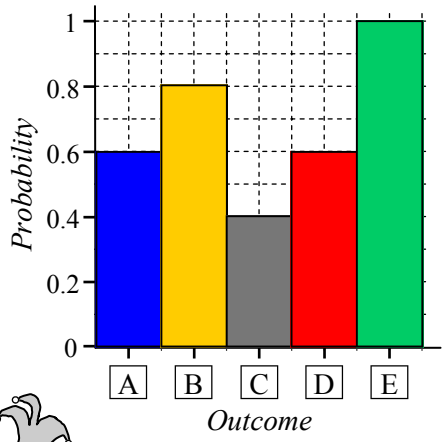
A The person is male

B The person is older than 8 years.

C The person is a teenager.

D The persons name has 2 vowels.

E The person is male or female.



A card is selected from a standard deck (no jokers), express the probability as a percentage (to 1 d.p.).

25 P(King) $P(\text{King}) = \frac{4}{52} \times 100$

$P(\text{King}) = 7.7\%$

26 P(J, Q, K , A)

$P(J, Q, K, A) = \frac{16}{52} \times 100 = 30.8\%$

27 P(Red Even Number Card)

$P(\text{Red/Even}) = \frac{10}{52} \times 100 = 19.2\%$

28 P(10,5,2) if a 10, a 5 and a 7 have been selected/not replaced.

$P(10, 5, 2) = \frac{10}{49} \times 100 = 20.4\%$

Complementary Events

The complement to an event is essentially the opposite to the event. For example the complement to “Rolling a 6 with a die”, would be “Not rolling a 6 with a die”. Note that the probability of rolling a 6 is $^1/_6$ while the probability not of rolling a 6 is $^5/_6$. When you add these probabilities you get a total of 1.

$$P(E) + P(\widetilde{E}) = 1 \qquad (100\%)$$

Percentage

Probability of an event

Complement to the event

Note that the complement is shown with a wavy line. Some examples are below:

$P(\widetilde{6})$ - Probability of not rolling a 6

$P(\widetilde{Car})$ - Probability of the vehicle not being a car

$P(\widetilde{wet\ day})$ - Probability of the day not being wet

$P(\widetilde{King})$ - Probability of not selecting a King

Column 1 starts with sentences that require you to write the complement, this is fairly straight forward, just consider what the opposite is and write that as a sentence. In most cases it just requires one word being changed.

From Q.6. onwards, the questions are probability questions, the only difference being that at times you are finding the complementary event. Just watch for the wavy line and if you see it then just imagine finding the opposite, or ***NOT*** finding what is in the brackets.

Remember to simplify your fraction answers and note that some questions require non-fraction answers. Questions 14 - 19 require percentage answers, while 31 - 35 are decimal answers (or change them to fractions or percentages as the answering method isn’t specified in the question).

Complementary Events

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Write the complement to these events.

1 It will rain all day today.

2 This team has no chance of winning the final.

3 It is unlikely this car will break down.

4 The train is certain to arrive on time.

5 Its likely my next pizza will have mushrooms.

Use complementary events to answer some of these questions.

In the word HYPOTENUSE, find the fraction for:

6 P(Y) = $\frac{\square}{\square}$

7 P($\widetilde{E}$) = $\frac{\square}{\square}$

8 P(vowel) = $\frac{\square}{\square}$

9 P($\widetilde{O,T,E}$) = $\frac{\square}{\square}$

A fund-raising raffle has sold 100 tickets which are numbered 1 to 100, one ticket is chosen, find the probability the ticket:

10 Will be numbered 70 or more. $\frac{\square}{\square}$

11 Won't be 57. $\frac{\square}{\square}$

12 Will have a 5 as its 2nd digit. $\frac{\square}{\square}$

13 Won't end in zero. $\frac{\square}{\square}$

A coffee shop critic finds coffee could be too hot, too cold or perfect. Probability is P(Hot) = 0.3 and P(cold) = 0.06. Find as a percentage:

14 P($\widetilde{Hot}$) = _____

15 P(Perfect) = _____

16 P($\widetilde{Cold}$) = _____

17 P($\widetilde{Perfect}$) = _____

18 P($\widetilde{Cold}$) + P(Cold) = _____

19 P(Perfect) - P($\widetilde{Cold}$) + P(Hot) = _____

The Kings, Queens and Jacks are removed from a deck of cards and placed face down on a table, one card can then be turned over. Find the probability, as a fraction, of :

20 Selecting a Queen. $\frac{\square}{\square}$

21 Selecting a Red King. $\frac{\square}{\square}$

22 Not selecting a Jack or a Red card. $\frac{\square}{\square}$

A fair die is rolled. Find the probability, as a fraction, for the following:

23 P(1, 5) = $\frac{\square}{\square}$

24 P(Odd) = $\frac{\square}{\square}$

25 P(Prime) = $\frac{\square}{\square}$

26 P($\widetilde{<5}$) = $\frac{\square}{\square}$

27 P($\widetilde{0}$) = $\frac{\square}{\square}$ ____

28 P($\widetilde{3}$) - P(3) = $\frac{\square}{\square}$

Peter, Anne, Jerry and Deb are in a foot race. The odds of winning the event are: P(Peter) = 0.1, P(Anne) = 0.27. If Jerry is twice as likely to win when compared with Deb. Find the chance of:

29 Deb winning. _____

30 One of the boys winning. _____

31 Anne not winning. _____

32 Jerry or Anne not winning. _____

33 A hat contains 20 marbles that are blue, red and black. If 6 are blue and P($\widetilde{Black}$) = 3/5, how many red marbles are there?

_____ _____

_____ _____

Complementary Events

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Write the complement to these events.

1 It will rain all day today.

It won't rain at all today.

2 This team has no chance of winning the final.

This team will win the final.

3 It is unlikely this car will break down.

It is likely this car will break down.

4 The train is certain to arrive on time.

The train will not arrive on time.

5 Its likely my next pizza will have mushrooms.

It is unlikely my next pizza will have mushrooms.

Use complementary events to answer some of these questions.

In the word HYPOTENUSE, find the fraction for:

6 P(Y) = $\frac{1}{10}$

7 P($\widetilde{E}$) = $\frac{4}{5}$

8 P(vowel) = $\frac{2}{5}$

9 P($\widetilde{O,T,E}$) = $\frac{6}{10}$

A fund-raising raffle has sold 100 tickets which are numbered 1 to 100, one ticket is chosen, find the probability the ticket:

10 Will be numbered 70 or more. P(>69)= $\frac{31}{100}$

11 Won't be 57. P($\widetilde{57}$)= $\frac{99}{100}$

12 Will have a 5 as its 2nd digit. P(_5) = $\frac{9}{100}$

13 Won't end in zero. P($\widetilde{\text{end 0}}$) = $\frac{90}{100}$

A coffee shop critic finds coffee could be too hot, too cold or perfect. Probability is P(Hot) = 0.3 and P(cold) = 0.06. Find as a percentage:

14 P($\widetilde{\text{Hot}}$) = 70%

15 P(Perfect) = 64%

16 P($\widetilde{\text{Cold}}$) = 94%

17 P($\widetilde{\text{Perfect}}$) = 36%

18 P($\widetilde{\text{Cold}}$) + P(Cold) = 100%

19 P(Perfect) - P($\widetilde{\text{Cold}}$) + P(Hot) = 0%

The Kings, Queens and Jacks are removed from a deck of cards and placed face down on a table, one card can then be turned over. Find the probability, as a fraction, of :

20 Selecting a Queen. P(Q) = $\frac{1}{3}$

21 Selecting a Red King. P(RK)= $\frac{1}{6}$

22 Not selecting a Jack or a Red card. P($\widetilde{\text{J or red}}$) = $\frac{1}{3}$

A fair die is rolled. Find the probability, as a fraction, for the following:

23 P(1, 5) = $\frac{1}{3}$

24 P(Odd) = $\frac{1}{2}$

25 P(Prime) = $\frac{1}{2}$

26 P($\widetilde{<5}$) = $\frac{1}{3}$

27 P($\widetilde{0}$) = $\frac{6}{6}$ = 1

28 P($\widetilde{3}$) - P(3) = $\frac{2}{3}$

Peter, Anne, Jerry and Deb are in a foot race. The odds of winning the event are: P(Peter) = 0.1, P(Anne) = 0.27. If Jerry is twice as likely to win when compared with Deb. Find the chance of:

29 Deb winning. 1 - 0.1 - 0.27 = 0.63

$\frac{0.63}{3}$ = 0.21 P(Deb) = 0.21

30 One of the boys winning. 0.1 + 2×0.21 = 0.52

P(Peter, Jerry) = 0.52

31 Anne not winning. P($\widetilde{A}$) = 1 - 0.27 = 0.73

32 Jerry or Anne not winning. P($\widetilde{J, A}$) = 0.31

33 A hat contains 20 marbles that are blue, red and black. If 6 are blue and P($\widetilde{\text{Black}}$) = 3/5, how many red marbles are there?

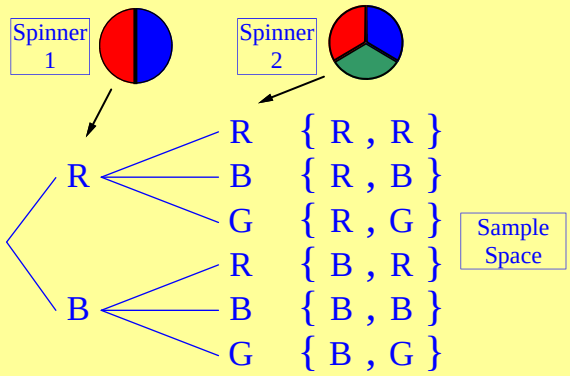
$\frac{3}{5} = \frac{12}{20}$ 20 - 12 = 8 So 8 black marbles.

20 - 6 - 8 = 6 There are 6 red marbles

Multi-Staged Events - Tree diagrams

A tree diagram can be used to show a multi-staged event, which is an event that has more than one event within it. For example, rolling a die and tossing a coin has two events. A tree diagram displays the process graphically allowing you to construct a sample space for all the outcomes for the event.

Example - Spinning two spinners.



The tree diagram shows the first spinner with two options, either red or blue. Note that Spinner 2 has been shown twice, it has been shown the first time if a red is spun on Spinner 1, it has been shown a second time if a blue was spun on Spinner 1.

The sample space is then shown, by following the tree's branches in a path.

Column 1 has the trees constructed for you, you just have to put in all the outcomes and then list the sample space. The first question is for the rolling of a die and tossing of a coin. The second question is for spinning two spinners, after you list the sample space it might be easier to add the two numbers and write the sum to the right of each pair, it will help you to answer the questions below the tree diagram.

Column 2 and 3 ask you to draw up your own trees and list the sample space. Remember to show the outcome at the end of each branch and when you list the sample space make sure you use braces {} and a comma , to display them. Question 19 has a three stage event on the tossing of three coins, so give your self room when drawing the tree.

Multi-Staged Events - Tree diagrams

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A die is rolled and a coin is tossed, complete the tree diagram and show the sample space.

1

Die | Coin | Sample Space

Wow! Look two spinners!

Spinner 1: 1, 2, 4, 3

Spinner 2: 1, 2, 3

2

Sample Space

Add the two numbers on the spinners to find …..

3 P(Sum 4) = ☐/☐

4 P(Odd Sum) = ☐/☐

5 P(Sum < 6) = ☐/☐

6 P(Sum Factor of 8) = ☐/☐

Draw a tree diagram for tossing two coins, then show the sample space.

7 *Use H & T

Tree Diagram | Sample Space

8 How many outcomes are there? _____

9 Find P(H, H) = ☐/☐

10 P(H, T) = ☐/☐

11 P(H & T) any order = ☐/☐

12 P(Tails) = ☐/☐

Draw a tree diagram for the two spinners. Then answer the questions.

Spinner 1: Red, Blue, Grey

Spinner 2: Red, Blue, Grey, Red

13 *Use R, B & G

Tree Diagram | Sample Space

These questions are on the spinners. (Q.13)

14 How many outcomes are there? _____

15 Find P(B, R) = ☐/☐

16 Find P(B) = ☐/☐

17 Find P(2 same colours) = ☐/☐

18 Find P(G) = ☐/☐

Draw a tree diagram for the tossing of three coins. Draw it sideways.

19

Tree Diagram

Sample Space

20 How many outcomes are there? _____

21 Find P(H, T, H) = ☐/☐

22 Find P(At least 2 tails) = ☐/☐

23 Find P(Heads) = ☐/☐

24 P(H & 2T) any order = ☐/☐

25 Find P(3H or 3T) = ☐/☐

Multi-Staged Events - Tree diagrams

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A die is rolled and a coin is tossed, complete the tree diagram and show the sample space.

1

Die	Coin	Sample Space
1	H	{1, H}
	T	{1, T}
2	H	{2, H}
	T	{2, T}
3	H	{3, H}
	T	{3, T}
4	H	{4, H}
	T	{4, T}
5	H	{5, H}
	T	{5, T}
6	H	{6, H}
	T	{6, T}

Wow! Look two spinners!

Spinner 1: 1, 2, 4, 3

Spinner 2: 1, 2, 3

2

		Sample Space	
1	1	{1, 1}	2
	2	{1, 2}	3
	3	{1, 3}	4
2	1	{2, 1}	3
	2	{2, 2}	4
	3	{2, 3}	5
3	1	{3, 1}	4
	2	{3, 2}	5
	3	{3, 3}	6
4	1	{4, 1}	5
	2	{4, 2}	6
	3	{4, 3}	7

Add the two numbers on the spinners to find …..

3 P(Sum 4) = $\frac{1}{4}$

4 P(Odd Sum) = $\frac{1}{2}$

5 P(Sum < 6) = $\frac{3}{4}$

6 P(Sum Factor of 8) = $\frac{1}{3}$

Draw a tree diagram for tossing two coins, then show the sample space.

7 *Use H & T

Tree Diagram: H → H, T; T → H, T

Sample Space: {H, H}, {H, T}, {T, H}, {T, T}

8 How many outcomes are there? 4 outcomes.

9 Find P(H, H) = $\frac{1}{4}$

10 P(H, T) = $\frac{1}{4}$

11 P(H & T) any order = $\frac{1}{2}$

12 P(Tails) = $\frac{1}{4}$

Draw a tree diagram for the two spinners. Then answer the questions.

Spinner 1: Red, Blue, Grey

Spinner 2: Red, Blue, Grey, Red

13 *Use R, B & G

Tree Diagram: R → R, B, R, G; B → R, B, R, G; G → R, B, R, G

Sample Space: {R, R}, {R, B}, {R, R}, {R, G}, {B, R}, {B, B}, {B, R}, {B, G}, {G, R}, {G, B}, {G, R}, {G, G}

These questions are on the spinners. (Q.13)

14 How many outcomes are there? 12 outcomes

15 Find P(B, R) = $\frac{1}{6}$

16 Find P(B) = $\frac{1}{2}$

17 Find P(2 same colours) = $\frac{1}{3}$

18 Find P(Ğ) = $\frac{1}{2}$

Draw a tree diagram for the tossing of three coins. Draw it sideways.

19

Tree Diagram: T → T, H; H → T, H; T → T, H; H → T, H; T → T, H; H → T, H; T → T, H; H → T, H

Sample Space: {T, T, T}, {T, T, H}, {T, H, T}, {T, H, H}, {H, T, T}, {H, T, H}, {H, H, T}, {H, H, H}

20 How many outcomes are there? 8 Outcomes

21 Find P(H, T, H) = $\frac{1}{8}$

22 Find P(At least 2 tails) = $\frac{1}{2}$

23 Find P(Heads) = $\frac{1}{8}$

24 P(H & 2T) any order = $\frac{3}{8}$

25 Find P(3H or 3T) = $\frac{1}{4}$

Probability - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Probability - Find a Word

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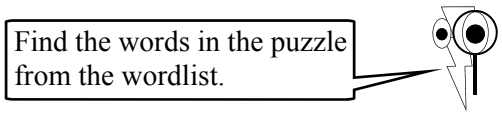

E	E	F	R	E	Q	U	E	N	C	Y	V	E	V	R
X	X	A	T	N	E	M	E	L	P	M	O	C	E	I
Y	T	I	L	I	B	A	B	O	R	P	H	C	V	A
L	H	L	Y	A	T	N	E	V	E	O	N	V	V	F
E	E	T	L	Y	T	S	K	X	O	A	V	E	E	V
K	O	P	V	M	P	N	Q	S	H	V	V	L	V	X
I	R	X	E	A	T	I	E	C	E	L	Y	B	I	E
L	E	N	C	V	C	V	X	M	Y	L	Y	I	T	Q
V	T	E	O	E	E	S	O	O	I	O	D	S	A	S
T	I	X	X	I	R	C	S	K	X	R	E	S	L	A
C	C	A	C	D	T	T	E	O	A	X	E	O	E	M
E	A	M	I	U	A	C	X	C	T	Y	V	P	R	P
L	L	L	O	R	I	K	A	K	Y	T	E	M	X	L
E	C	I	C	X	N	A	B	R	T	X	Y	I	X	E
S	N	I	O	C	M	T	B	Y	F	E	D	I	C	E

WORDLIST

ROLL	SAMPLE	SELECT	LIKELY	DICE
DIE	CARD	EVENT	FREQUENCY	EVEN
EXPERIMENTAL	FRACTION	RELATIVE	CHOOSE	COMPLEMENT
IMPOSSIBLE	OUTCOME	SPACE	TOSS	CERTAIN
PROBABILITY	FAIR	COINS	CHANCE	THEORETICAL

Probability - Find a Word

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Find the words in the puzzle from the wordlist.



E	E	F	R	E	Q	U	E	N	C	Y	V	E	V	R
X	X	A	T	N	E	M	E	L	P	M	O	C	E	I
Y	T	I	L	I	B	A	B	O	R	P	H	C	V	A
L	H	L	Y	A	T	N	E	V	E	O	N	V	V	F
E	E	T	L	Y	T	S	K	X	O	A	V	E	E	V
K	O	P	V	M	P	N	Q	S	H	V	V	L	V	X
I	R	X	E	A	T	I	E	C	E	L	Y	B	I	E
L	E	N	C	V	C	V	X	M	Y	L	Y	I	T	Q
V	T	E	O	E	E	S	O	O	I	O	D	S	A	S
T	I	X	X	I	R	C	S	K	X	R	E	S	L	A
C	C	A	C	D	T	T	E	O	A	X	E	O	E	M
E	A	M	I	U	A	C	X	C	T	Y	V	P	R	P
L	L	L	O	R	I	K	A	K	Y	T	E	M	X	L
E	C	I	C	X	N	A	B	R	T	X	Y	I	X	E
S	N	I	O	C	M	T	B	Y	F	E	D	I	C	E

WORDLIST

~~ROLL~~ ~~SAMPLE~~ ~~SELECT~~ ~~LIKELY~~ ~~DICE~~

~~DIE~~ ~~CARD~~ ~~EVENT~~ ~~FREQUENCY~~ ~~EVEN~~

~~EXPERIMENTAL~~ ~~FRACTION~~ ~~RELATIVE~~ ~~CHOOSE~~ ~~COMPLEMENT~~

~~IMPOSSIBLE~~ ~~OUTCOME~~ ~~SPACE~~ ~~TOSS~~ ~~CERTAIN~~

~~PROBABILITY~~ ~~FAIR~~ ~~COINS~~ ~~CHANCE~~ ~~THEORETICAL~~

FREEFALL MATHEMATICS

ALTITUDE

GEOMETRIC CONSTRUCTIONS

FREEFALL MATHEMATICS

ALTITUDE

REVIEW:
GEOMETRY

Bisecting an Interval

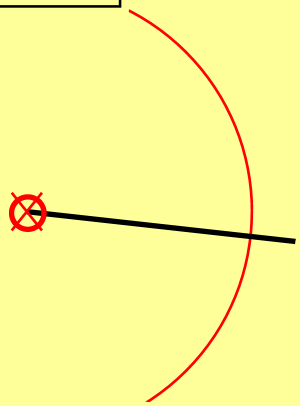
Bisecting something is cutting it into two equal pieces. This sheet deals with bisecting intervals. An interval is a straight line but instead of going on forever it has specific length. The intervals on this sheet have markers at their ends for you to position instruments more effectively.

Column 1 starts with asking you to bisect these five intervals with a ruler. Measure the length of the interval, write your answer in the space, give the answer in mm. Divide this number by 2 and write that answer in the ‘half length’ space. Mark off half-way with a small stroke.

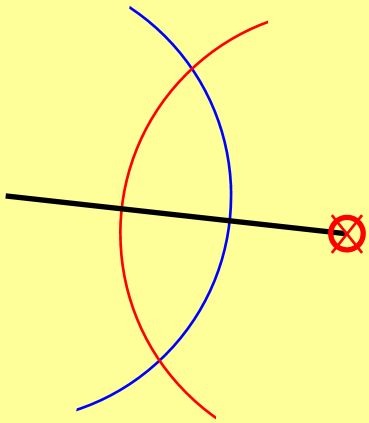
Questions 6 through 10 involve the same 60 mm interval, the questions ask you to break the interval up into a specific number of parts. Divide 60 by the number of parts required in the question and that is the measured gap required between 'marks'.

Columns 2 and 3 are the same, it is asking you to bisect the given intervals using a compass. Below is the process outlined step-by-step.

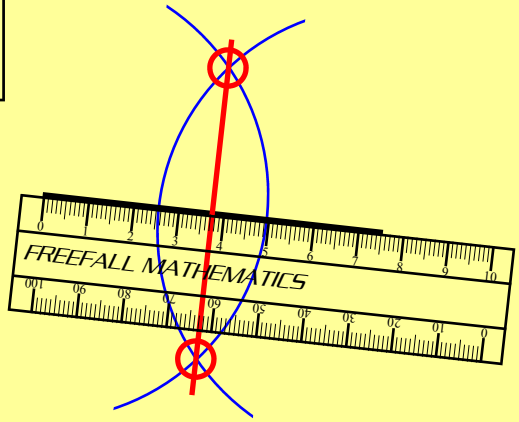
1 Draw an arc from one end of the interval (red circle). Open the compass more than half the interval’s length.



2 Repeat for the other end of the interval. Keep the compass set at the same distance.



3 Draw a line through the two intersection points (red circles). Use a ruler to measure the lengths as a check.



Bisecting an Interval

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Using your ruler, measure then bisect these intervals

1 Interval length : _____

Half length : _____

2 Interval length : _____

Half length : _____

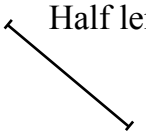
3 Interval length : _____

Half length : _____



4 Interval length : _____

Half length : _____



5 Interval length : _____

Half length : _____



Divide the 60 mm intervals into 3, 4, 5, 6 and 10 parts

6 3 parts, interval = _____ mm

7 4 parts, interval = _____ mm

8 5 parts, interval = _____ mm

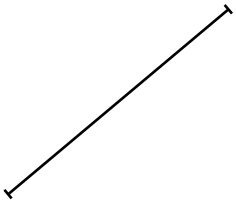
9 6 parts, interval = _____ mm

10 10 parts, interval = _____ mm

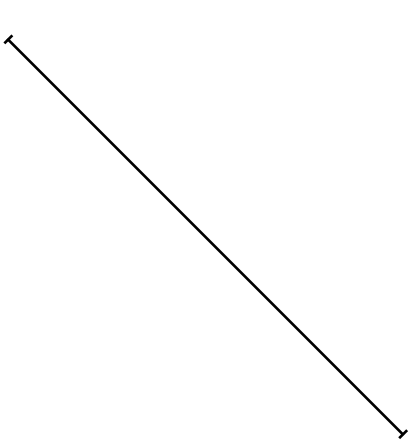
Use a compass to bisect these intervals. Give the measurement from one end to the midpoint (cm)

11 Half length : _____

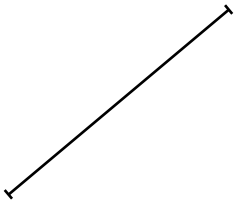
12 Half length : _____



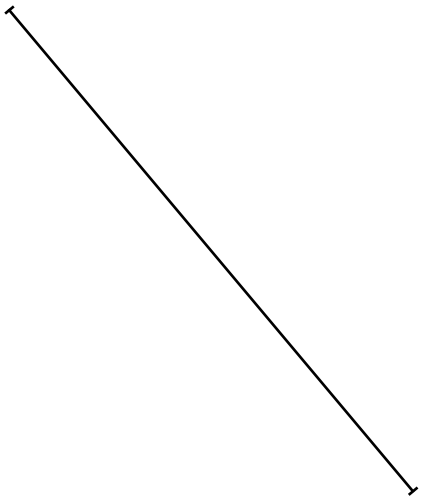
13 Half length : _____



14 Half length : _____



15 Half length : _____



16 Half length : _____

Bisecting an Interval

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Using your ruler, measure then bisect these intervals

1 Interval length : 60 mm

Half length : 30 mm

2 Interval length : 48 mm

Half length : 24 mm

3 Interval length : 36 mm

Half length : 18 mm

4 Interval length : 21 mm

Half length : 10.5 mm

5 Interval length : 42 mm

Half length : 21 mm

Divide the 60 mm intervals into 3, 4, 5, 6 and 10 parts

6 3 parts, interval = 20 mm

7 4 parts, interval = 15 mm

8 5 parts, interval = 12 mm

9 6 parts, interval = 10 mm

10 10 parts, interval = 6 mm

Use a compass to bisect these intervals. Give the measurement from one end to the midpoint (cm)

11 Half length : 2.8 cm

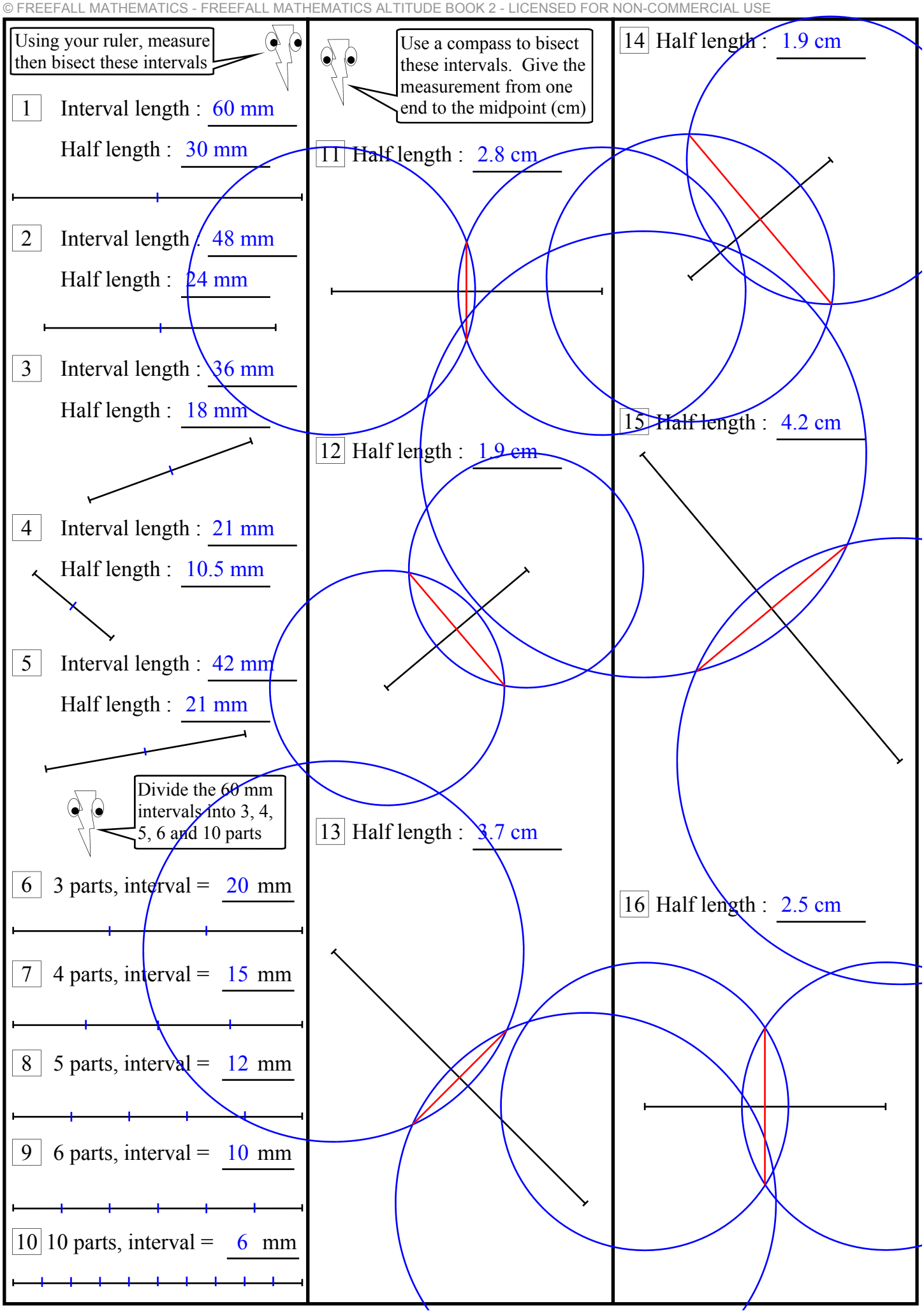
12 Half length : 1.9 cm

13 Half length : 3.7 cm

14 Half length : 1.9 cm

15 Half length : 4.2 cm

16 Half length : 2.5 cm

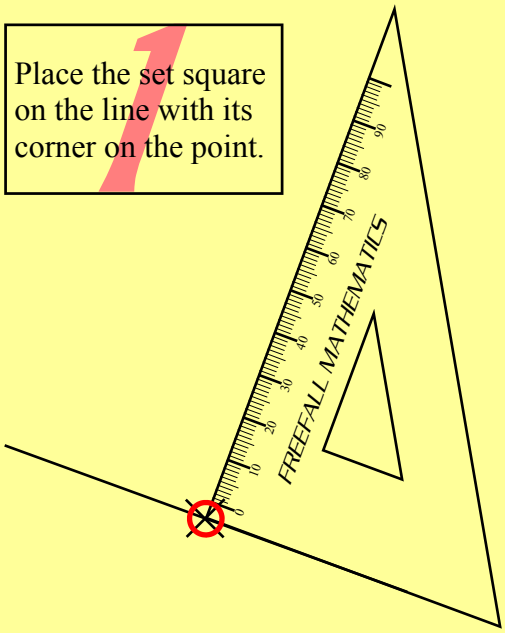


Perpendicular and Parallel Lines

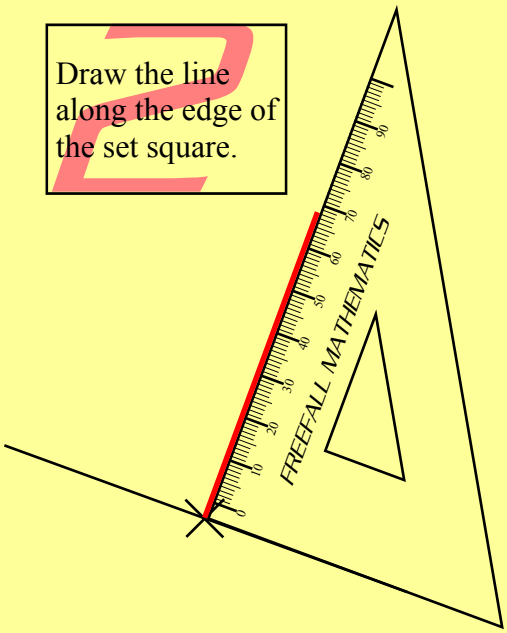
Perpendicular lines are lines that are at right angles to each other, so the angles formed when the lines intersect are 90º. Parallel lines are lines that are the same distance apart throughout their length and so they never intersect (touch each other). This sheet requires the use of a compass and a set square to create parallel and perpendicular lines.

Column 1 - Perpendicular from a point on a line - Set Square

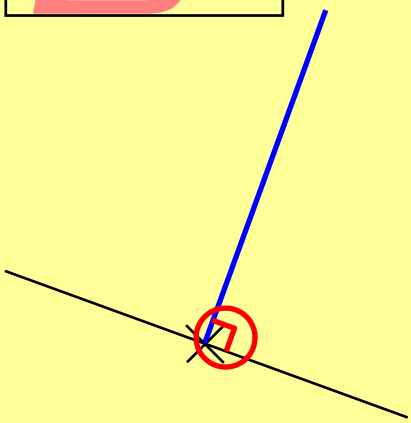
1 Place the set square on the line with its corner on the point.



2 Draw the line along the edge of the set square.

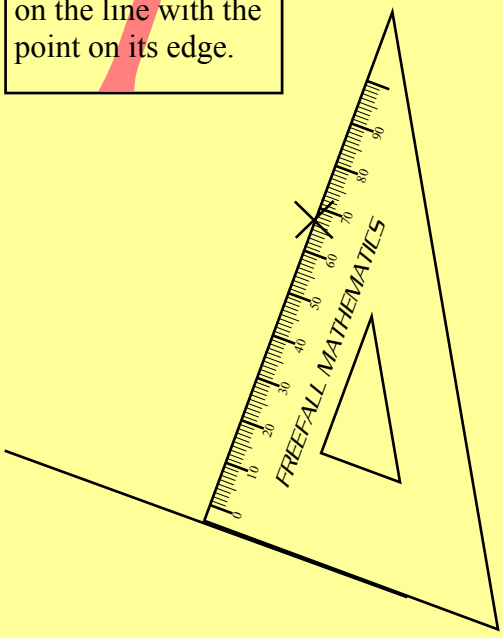


3 Place right angle markings to show it is a perpendicular.

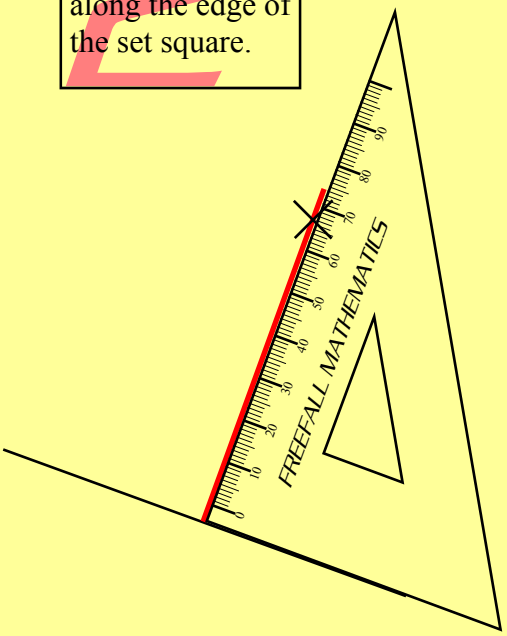


Column 1 - Perpendicular from a point away from a line - Set Square

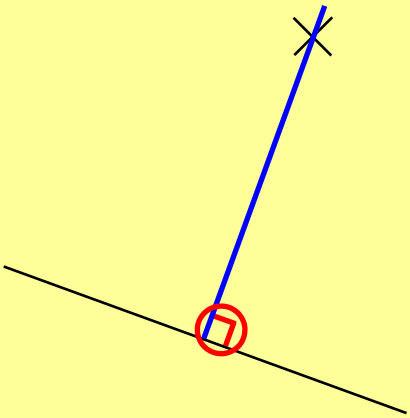
1 Place the set square on the line with the point on its edge.



2 Draw the line along the edge of the set square.

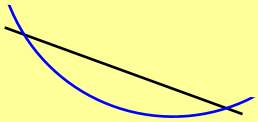


3 Place right angle markings to show it is a perpendicular.

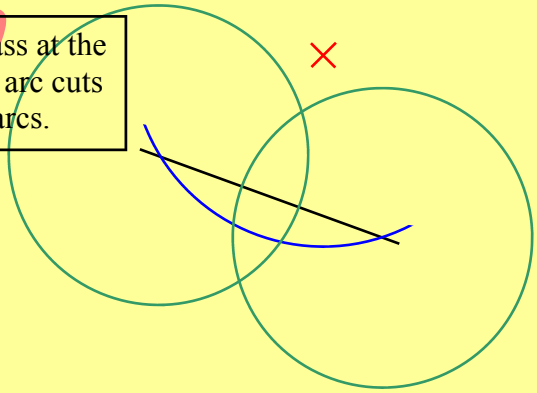


Column 2 - Perpendicular from a point away from on a line - Compass

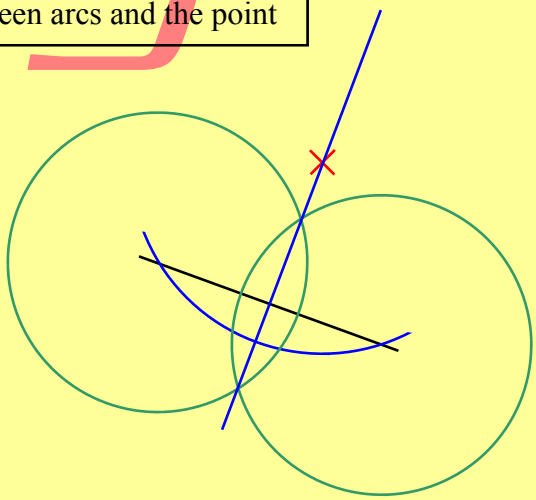
1 Position the compass in the point and draw an arc that cuts the interval in two places



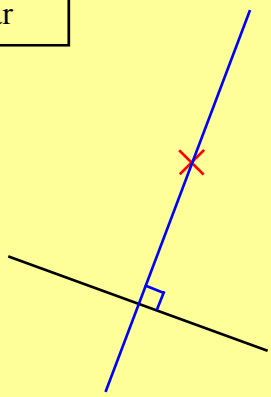
2 Then place the compass at the points where the blue arc cuts the interval, create 2 arcs.



3 Draw a line through the intersection points of the green arcs and the point



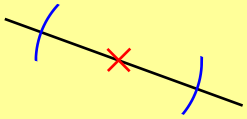
4 Place right angle markings to show it is a perpendicular



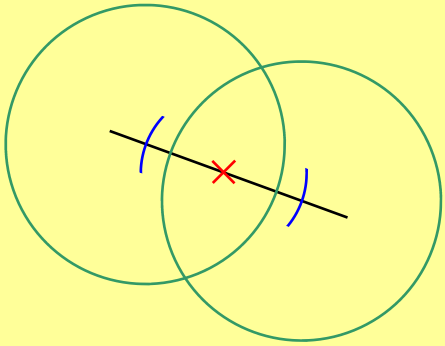
5 Green arcs and blue arcs don't have to be the same size

Column 2 - Perpendicular from a point away from on a line - Compass

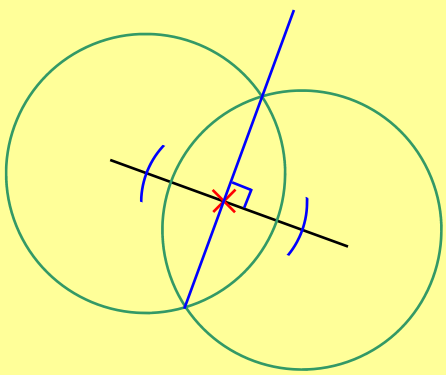
1 Position the compass at the point and draw an arc cutting the interval each side of the point (blue).



2 Position the compass where the blue arcs cut the interval and draw 2 arcs as shown, (green circles).



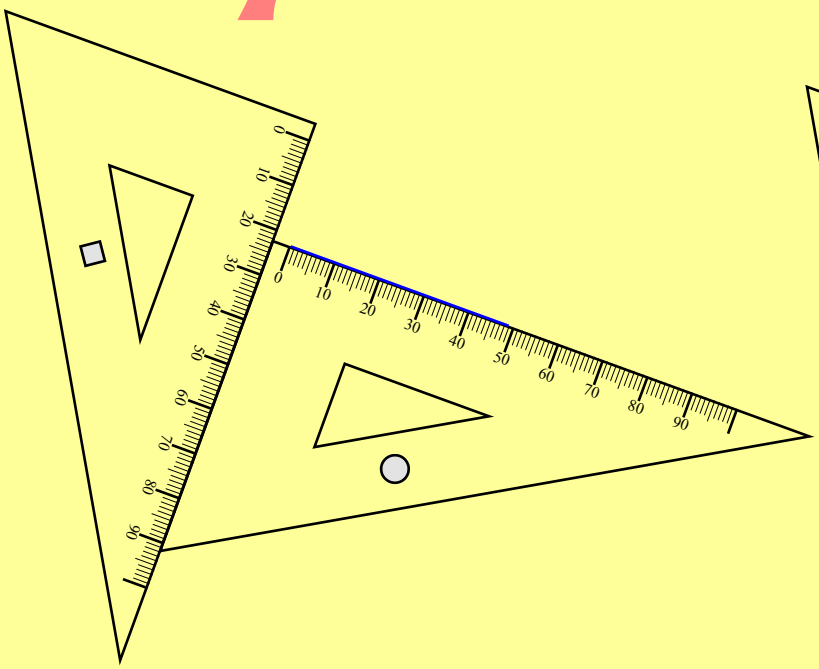
3 Draw a line through the two intersection points and the original point. Add a right angle symbol to complete.



Column 3 - Drawing a Parallel line - Set Square

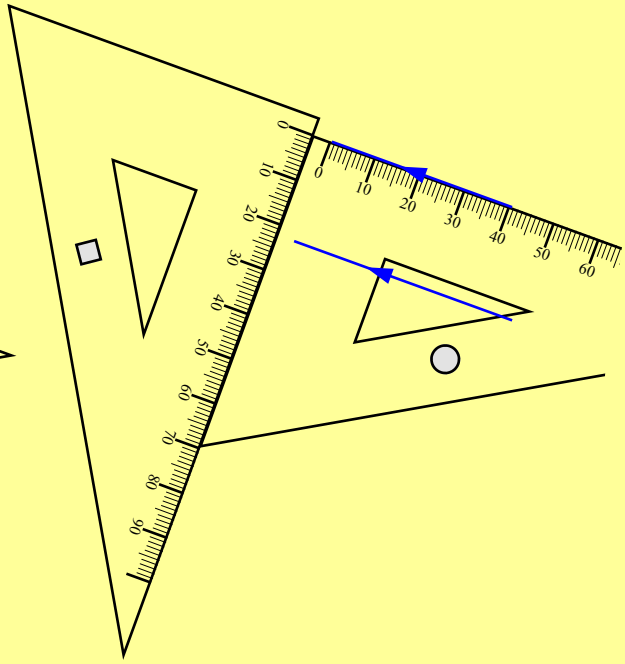
1

Position the set square marked with a circle on the line. Place another set square (square) or ruler on its edge, slide it to the required length (2.2 cm).



2

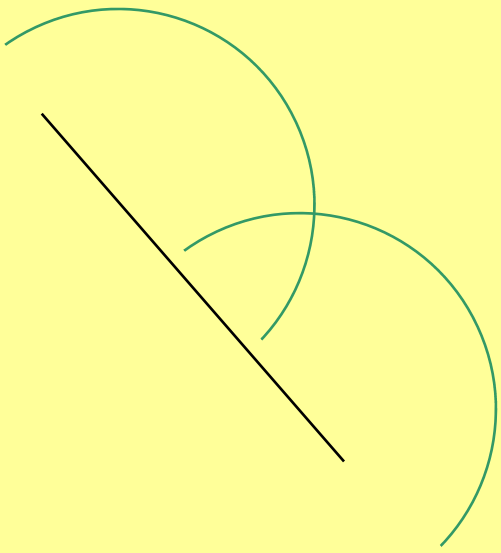
Slide the set square (circle) up to the zero then draw your line. To show the lines are parallel draw an arrow-head on each line.



Column 3 - Drawing a Parallel line - Compass

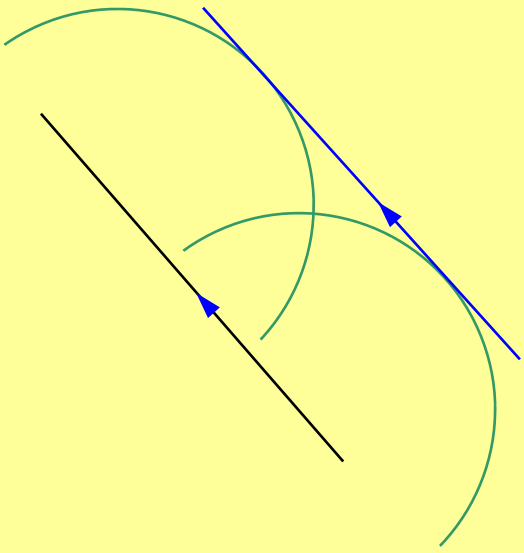
1

Open your compass the required distance and draw two arcs from any position on the interval.



2

Draw a line across the edges of the circles. Place arrow heads on the lines to show they are parallel.



Perpendicular and Parallel Lines

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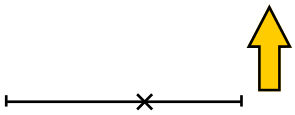
Use a set square to draw a perpendicular to the interval at the X, in the direction of the arrow.

1



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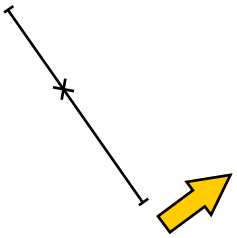

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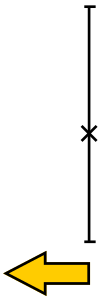
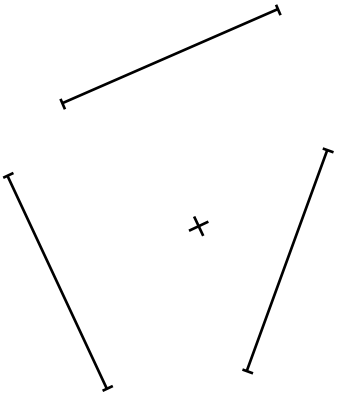


Use a set square to draw a perpendicular from the X to the interval or intervals

4



5



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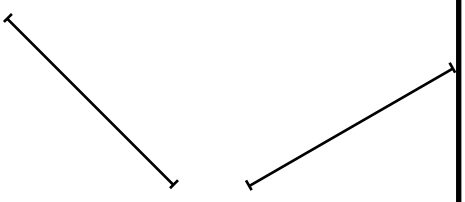
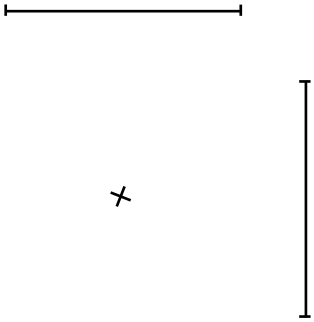


Use a compass and a ruler to draw a perpendicular from the X to the interval

7



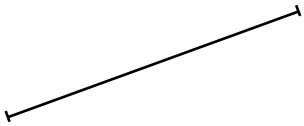
8



9

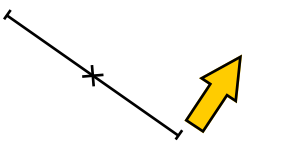


10



Now the X is on the line

11



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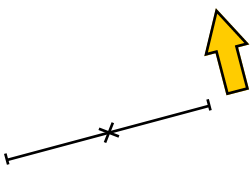
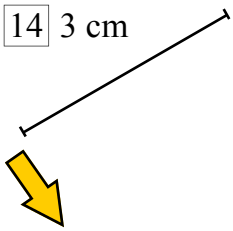


Use a set square and ruler to draw a parallel line to those below, separated by the given distance

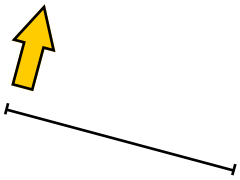
13 2 cm



14 3 cm

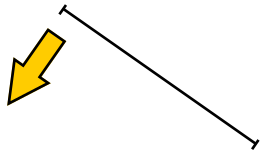


15 1.7 cm

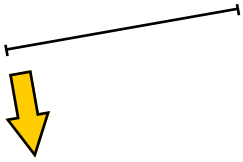


Keep going only this time use a compass and a ruler

16 2.2 cm



17 4.3 cm



Perpendicular and Parallel Lines

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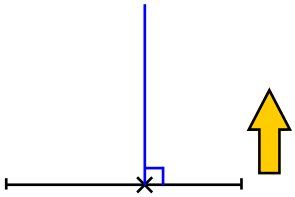
Use a set square to draw a perpendicular to the interval at the X, in the direction of the arrow.

1



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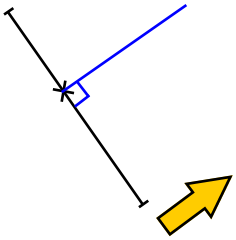

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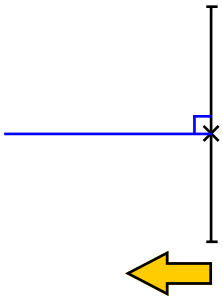
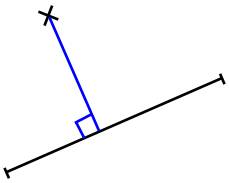


Use a set square to draw a perpendicular from the X to the interval or intervals

4



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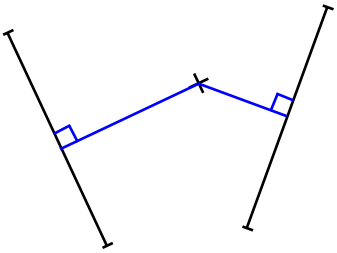

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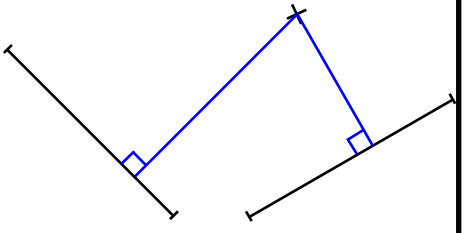
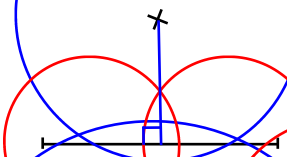


Use a compass and a ruler to draw a perpendicular from the X to the interval

7



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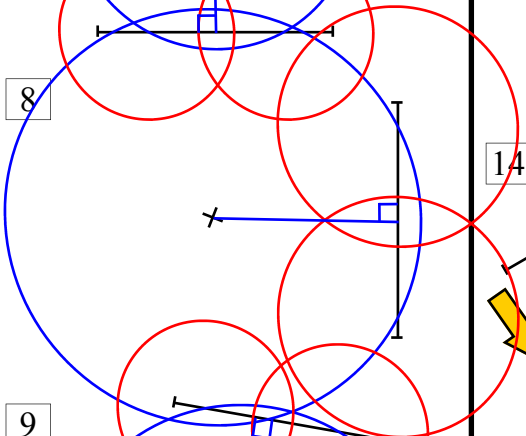

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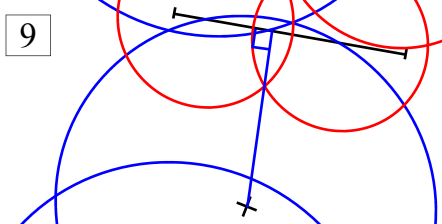

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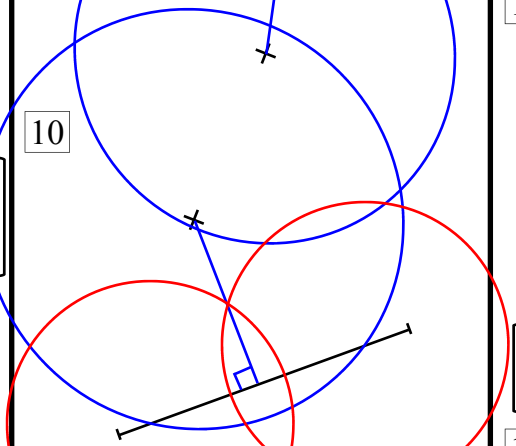
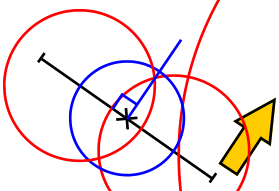
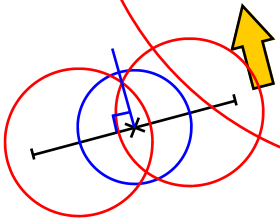


Now the X is on the line

11

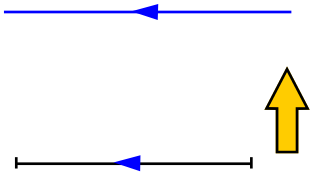


12

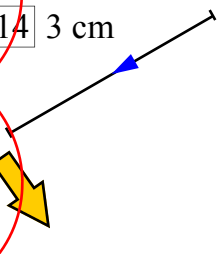


Use a set square and ruler to draw a parallel line to those below, separated by the given distance

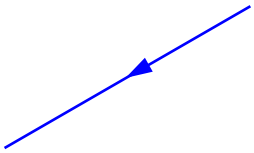
13 2 cm



14 3 cm

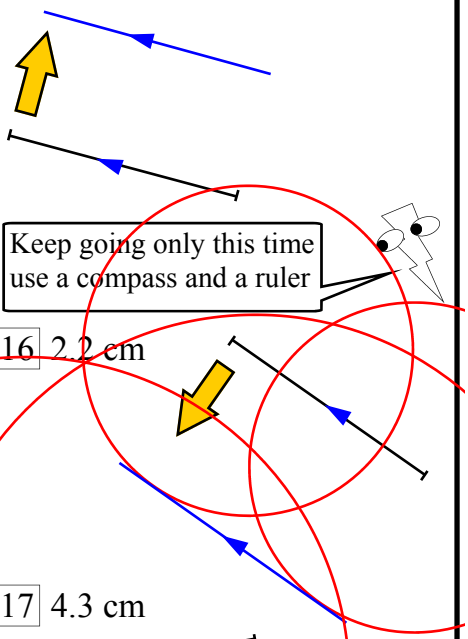


15 1.7 cm

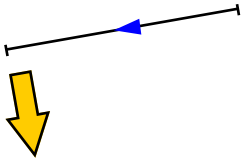


Keep going only this time use a compass and a ruler

16 2.2 cm



17 4.3 cm



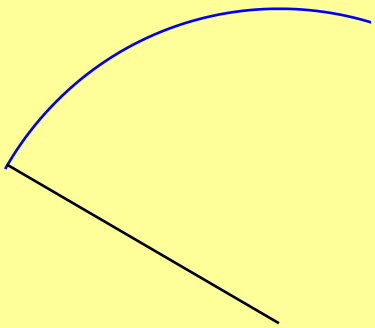
Equilateral and Scalene Triangles

Equilateral triangles are triangles with all sides equal and all interior angles equal (all 60º). This sheet is about constructing equilateral triangles with a compass (and a ruler). This is best described in steps, follow the steps outlined below.

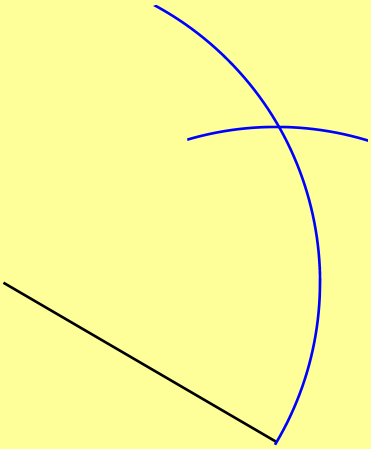
Scalene triangles have no equal sides. Follow the steps on the next page for constructing these triangles. Note that when a scalene triangle has an interior angle that is 90°, it isn't a scalene triangle it is a right (angled) triangle.

Column 1 - Constructing Equilateral Triangles - Compass

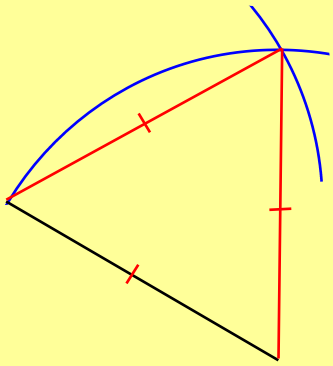
1 Position the compass at the end of the interval and open it to the interval length and draw an arc.



2 Repeat for the other end of the interval to create a point of intersection.

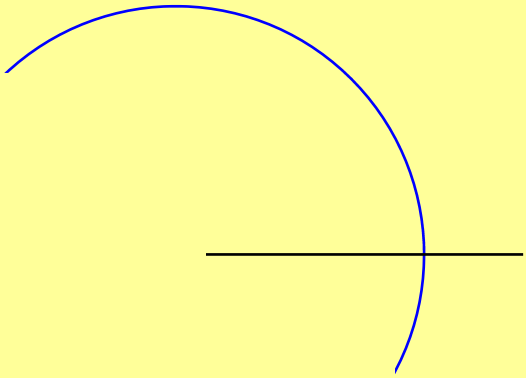


3 Draw a line from the end of the interval to the intersection point and then repeat for the other end of the interval. Mark the sides to show sides are all equal.

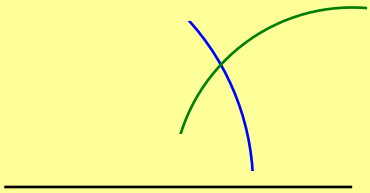


Column 2 - For the second part of the column rule a line first then repeat the above steps.

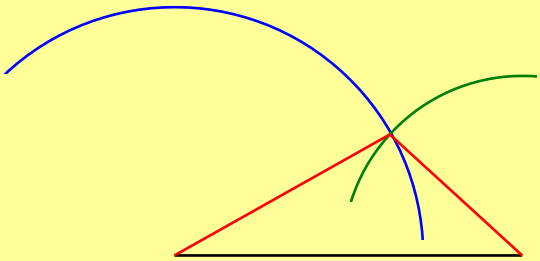
Column 2 - Constructing Scalene Triangles - Compass



1 Position the compass at the end of the interval and open it to the given length and draw an arc.



2 Repeat for the other end of the interval to create a point of intersection.



3 Draw a line from the end of the interval to the intersection point and then repeat for the other end of the interval.

Column 2 - For the second part of the column rule a line first then repeat the above steps. Rule the longest line first, then use your compass to create the other two lines.

Equilateral and Scalene Triangles

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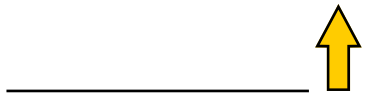
Make an equilateral triangle given a side. Use a compass and a ruler. Construct the triangle in the direction of the arrow.

1



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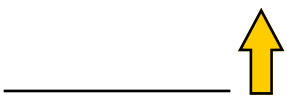

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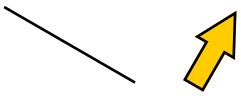

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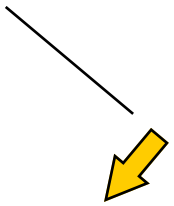


6



Now construct equilateral triangles with the side lengths given below

7 5.5 cm

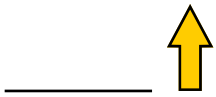
8 6.3 cm

Now make scalene triangles with one side drawn and the other two sides given.

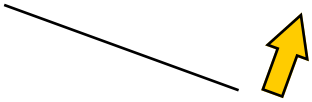
9 4 cm and 3 cm



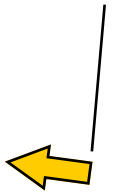
10 2.5 cm and 3.1 cm



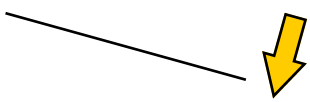
11 2.2 cm and 4.3 cm



12 1.9 cm and 2.7 cm



13 1.3 cm and 4.7 cm



14 5.4 cm and 4.9 cm

Construct scalene triangles with the side lengths given below

15 3 cm, 1.7 cm and 2.4 cm

16 3.8 cm, 2.9 cm and 4.1 cm

Equilateral and Scalene Triangles

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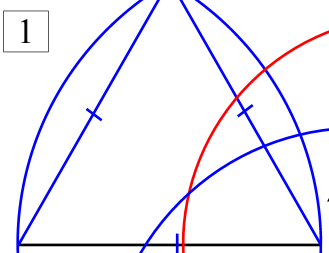
Make an equilateral triangle given a side. Use a compass and a ruler. Construct the triangle in the direction of the arrow.

1



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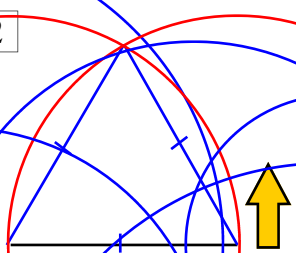

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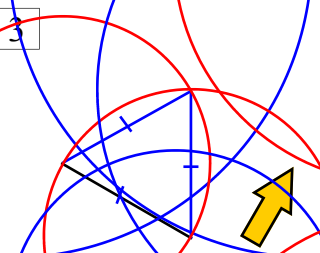

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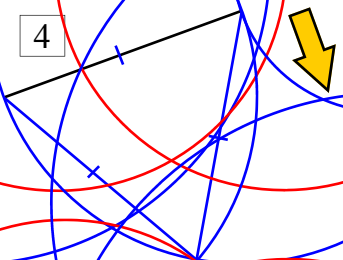

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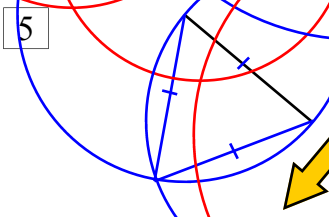

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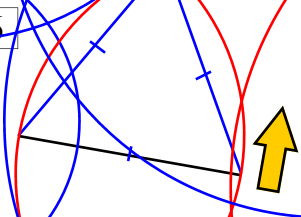
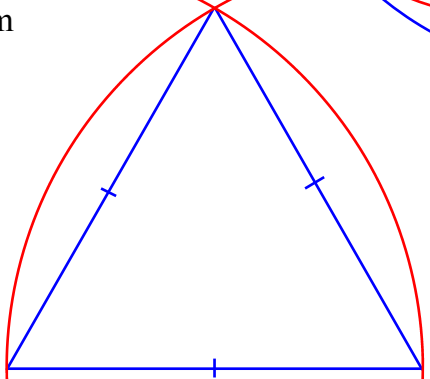
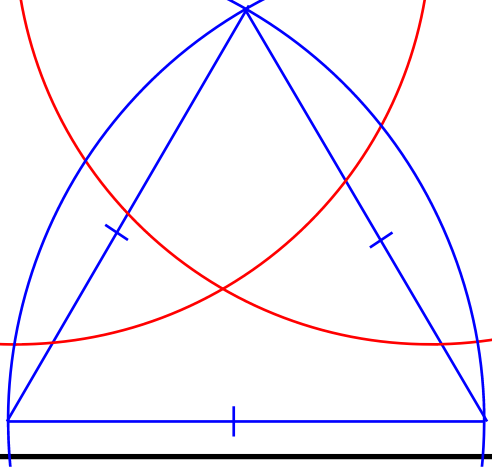


Now construct equilateral triangles with the side lengths given below

7 5.5 cm

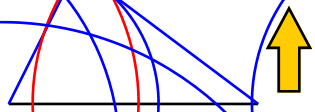


8 6.3 cm

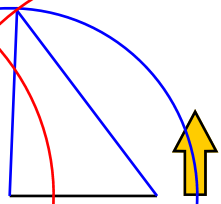


Now make scalene triangles with one side drawn and the other two sides given.

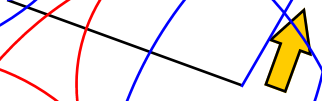
9 2 cm and 3 cm



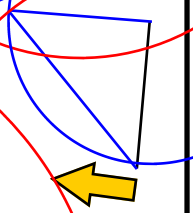
10 2.5 cm and 3.1 cm



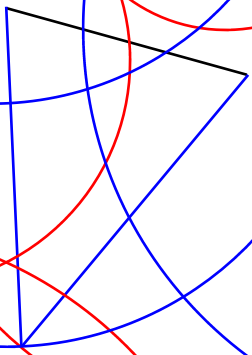
11 2.2 cm and 4.3 cm



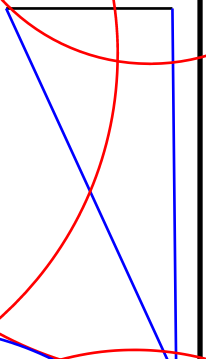
12 1.9 cm and 2.7 cm



13 4.5 cm and 4.7 cm

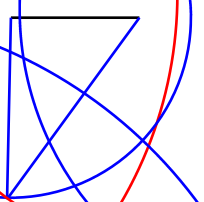


14 5.4 cm and 4.9 cm

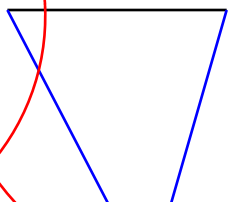


Construct scalene triangles with the side lengths given below

15 3 cm, 1.7 cm and 2.4 cm



16 3.8 cm, 2.9 cm and 4.1 cm



Creating Isosceles and Side-Angle-Side Triangles

Column 1 - Constructing Isosceles Triangles - Compass

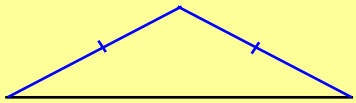
1 Open your compass to the given length then position the compass at the end of the line and draw an arc.

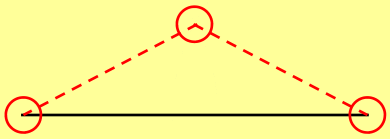
2 Repeat for the other end of the line to create a point of intersection. Note that the second line may not be the same length as the first.



3 Using a ruler, draw lines from the intersection point to the ends of the line.

4 Label the two sides of equal length with dashes.





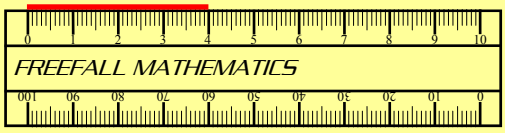
Questions 7 & 8 - Rule a line first then repeat the above steps. Rule the longest line first, then use your compass to construct the other two lines.

Creating Isosceles and Side-Angle-Side Triangles

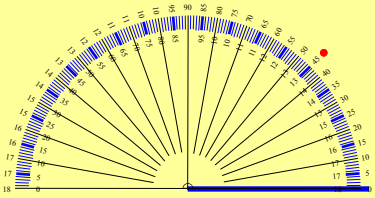
Column 2 - Constructing Triangles - Protractor & Ruler

Example:
4 cm, 3 cm
& 45° angle

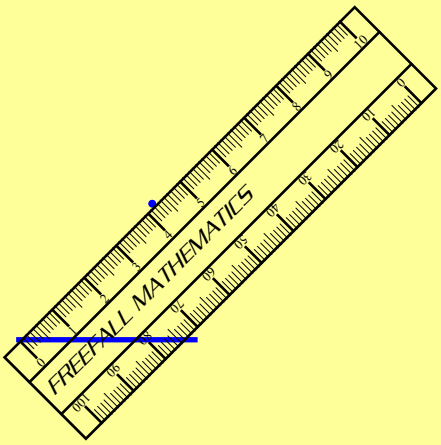
1 Draw the line of longest line first, the 4 cm line in our example.



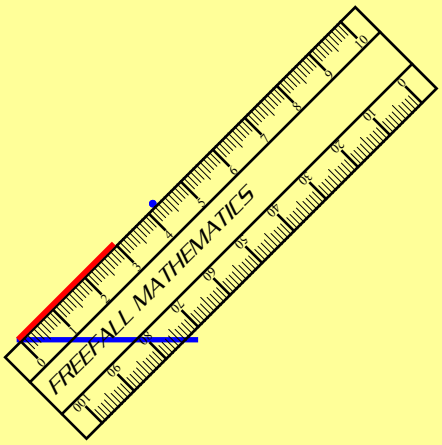
2 Place a protractor at the end of the line and plot a point at the given angle. In our example it is 45º.



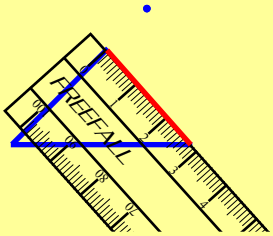
3 Remove the protractor and place a ruler with the zero at the end of the line and the point on the ruler's edge.



4 Rule the other line given in the question. It doesn't have to pass through the point.



5 Complete the final side by using a ruler between the two points



Making Isosceles and Side Angle Side Triangles

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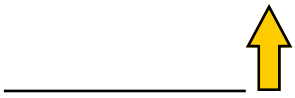
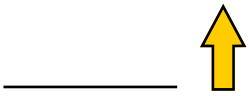
Make an isosceles triangle, one side is already drawn for you. Use the two other measurements to complete it.

1 3 cm & 3 cm

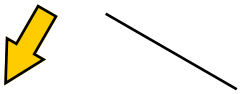
2 2.3 cm & 1.6 cm





3 2.6 cm & 2 cm

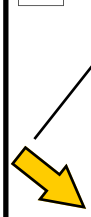
4 4.2 cm & 4.2 cm

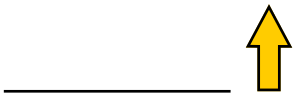




5 3.3 cm & 3.3 cm

6 1.2 cm & 3 cm





Now construct isosceles triangles with the side lengths given below



7 2.6 cm, 4.1 cm and 4.1 cm

8 2.7 cm, 3.8 cm and 3.8 cm





Here are two sides with the angle between them. Use a protractor and a ruler to draw them, then join the ends for the third side

9 4 cm, 3 cm, 60º

10 2.5 cm, 3.2 cm, 40º

11 5.3 cm, 4.7 cm, 25º

12 7.1 cm, 6.3 cm, 38º

Making Isosceles and Side Angle Side Triangles

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Make an isosceles triangle, one side is already drawn for you. Use the two other measurements to complete it.

1 3 cm & 3 cm

2 2.3 cm & 1.6 cm

3 2.6 cm & 2 cm

4 4.2 cm & 4.2 cm

5 3.3 cm & 3.3 cm

6 1.2 cm & 3 cm

Now construct isosceles triangles with the side lengths given below

7 2.6 cm, 4.1 cm and 4.1 cm

8 2.7 cm, 3.8 cm and 3.8 cm

Here are two sides with the angle between them. Use a protractor and a ruler to draw them, then join the ends for the third side

9 4 cm, 3 cm, 60º

10 2.5 cm, 3.2 cm, 40º

11 5.3 cm, 4.7 cm, 25º

12 7.1 cm, 6.3 cm, 38º

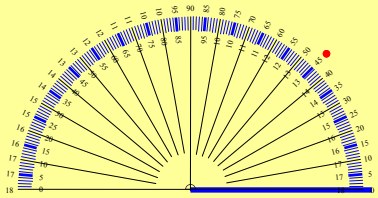
Angle-Angle-Side Triangles and Properties

A triangle can be constructed when you are given two angles and a side. This sheet requires a protractor and a ruler. The steps are below:

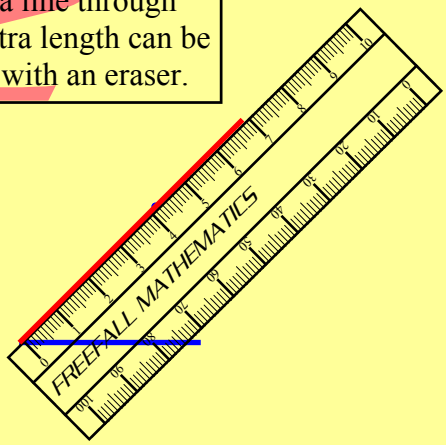
Column 1 - Constructing AAS Triangles - Protractor & Ruler

Example: 45° and 60° Base Angles

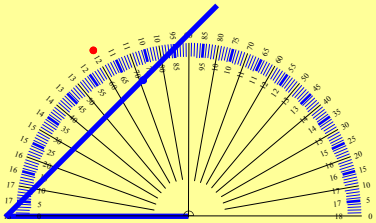
1 Using your protractor place it at one end of the line and measure the first angle. Plot a dot at the first angle.



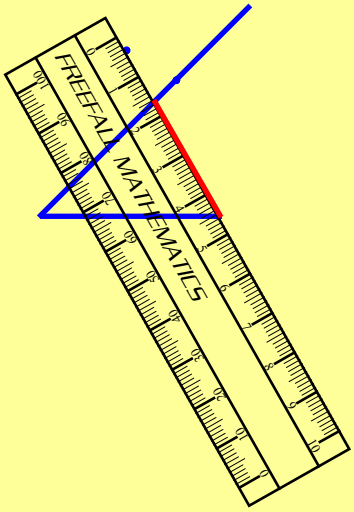
2 Place a ruler at the end of the line and draw a line through the dot, the extra length can be removed later with an eraser.



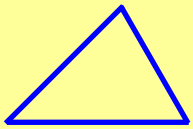
3 Repeat for the other end of the line.



4 Use a ruler to draw the final side. Stop when the line intersects the other side.



5 Erase the extended side length. All done!



Column 2 - Q.8 - Q.11.

Use the same method as above after first ruling the line.

The second part of the sheet aims to outline the relationship between the largest angle of a triangle and the triangle's longest side. Name the longest side and largest angle and in Q.15 explain how the angle and side are always located on a triangle.

Angle-Angle-Side Triangles and Properties

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Use a protractor and a ruler to create triangles with base internal angles as given below

1 40º & 50º

2 23º & 55º

3 70º & 80º

4 52º & 47º

5 15º & 38º

6 117º & 29º

7 60º & 60º

This time draw the given side then use the same method. Be careful where you choose to place the line.

8 2.5 cm, 30º, 40º

9 3.7 cm, 22º, 47º

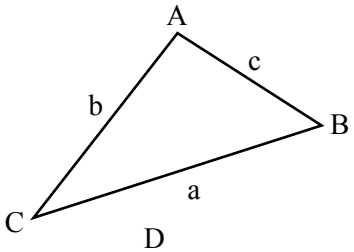
10 4.4 cm, 53º, 35º

11 1.7 cm, 75º, 75º

Name the largest angle and the largest side in these triangles

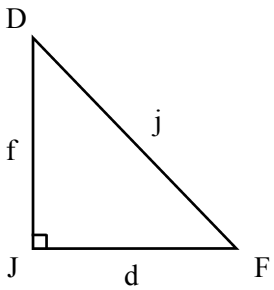
12 Side : ____

Angle : ∠ _____



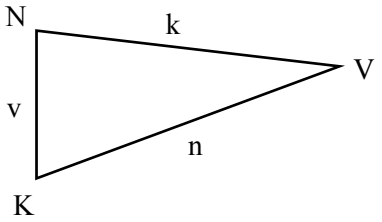
13 Side : ____

Angle : ∠ _____



14 Side : ____

Angle : ∠ _____



15 What is the relationship between the largest angle and the longest side? _____

Angle-Angle-Side Triangles and Properties

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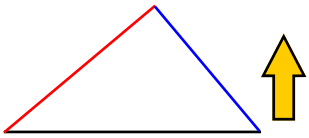
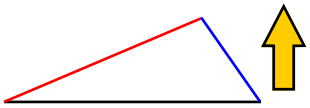
Use a protractor and a ruler to create triangles with base internal angles as given below

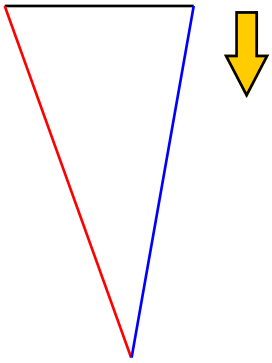
1 40º & 50º



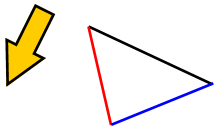
2 23º & 55º



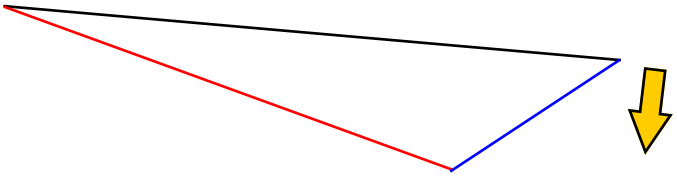
3 70º & 80º



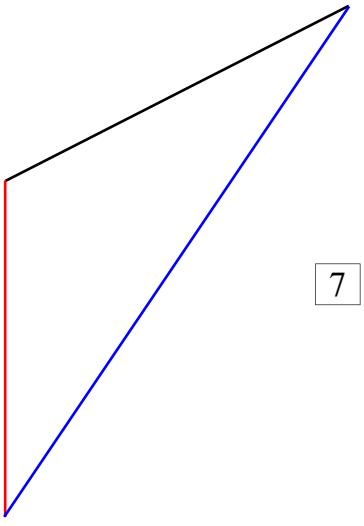
4 52º & 47º



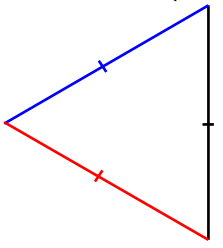
5 15º & 38º



6 117º & 29º

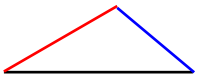


7 60º & 60º

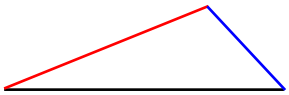


This time draw the given side then use the same method. Be careful where you choose to place the line.

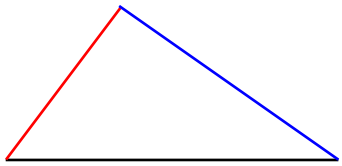
8 2.5 cm, 30º, 40º



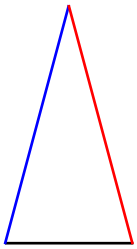
9 3.7 cm, 22º, 47º



10 4.4 cm, 53º, 35º



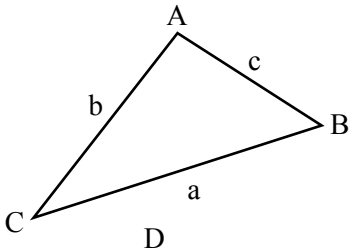
11 1.7 cm, 75º, 75º



Name the largest angle and the largest side in these triangles

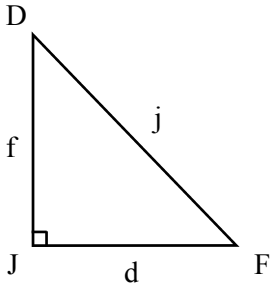
12 Side : a

Angle : ∠ BAC



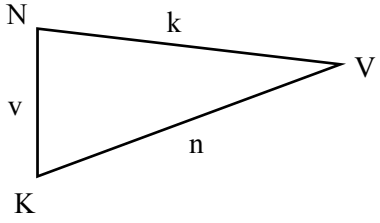
13 Side : j

Angle : ∠ DJF



14 Side : n

Angle : ∠ VNK



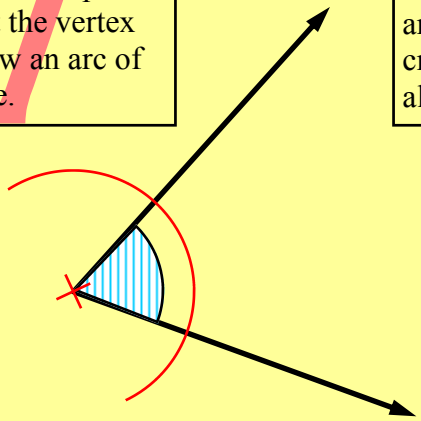
15 What is the relationship between the largest angle and the longest side? They are always opposite each other.

Bisecting Angles and Making 45° Angles

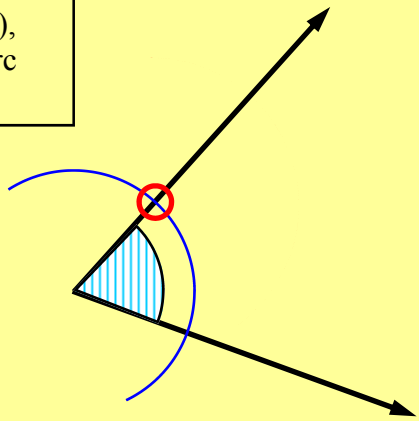
This sheet is about bisecting an angle, this means cutting the angle in half. The second part of the sheet is about creating 45° angles using a compass and a ruler. When you make a 45° angle you are also making a 135° angle, it's the angle's supplement, it will be the adjacent obtuse angle beside the 45° angle.

Column 1 - Bisecting Angles - Protractor & Ruler

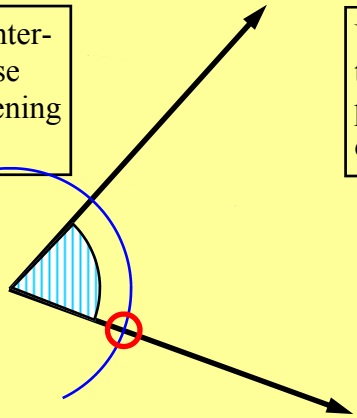
1 Place the compass point at the vertex and draw an arc of any size.



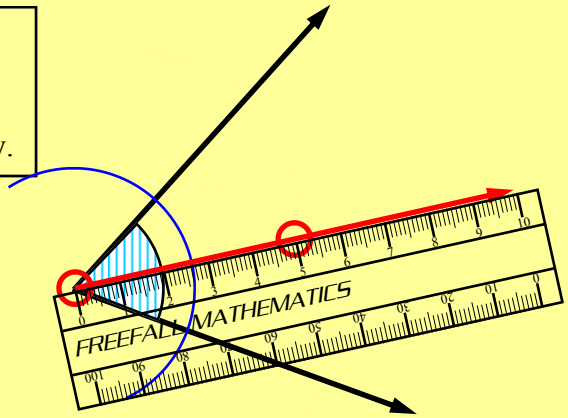
2 Using the intersection of the arc and the ray (red circle), create another arc. This arc also can be of any size.



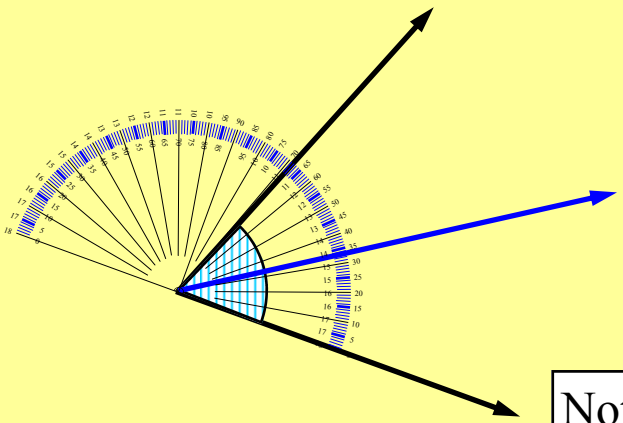
3 Repeat for the other intersection point (red). Use the same compass opening as the previous arc.



4 Using the vertex and the arc intersection point (red circles), draw the bisecting ray.



5 Use a protractor to measure the size of the bisection and original angle to check that your method is correct.



Note Q.4 is a reflex angle so a working space is provided.

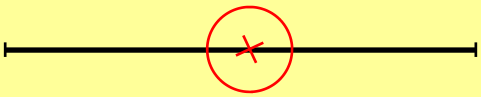
Bisecting Angles and Making 45° Angles

A 45º angle can be constructed by first making a 90º angle and bisecting it. So a right angle is created, then the same process from the previous column is repeated.

Column 2 - Constructing 45º Angles - Compass & Ruler

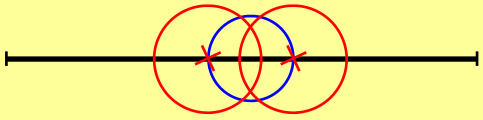
1

Using a compass draw a circle near the centre of the interval.



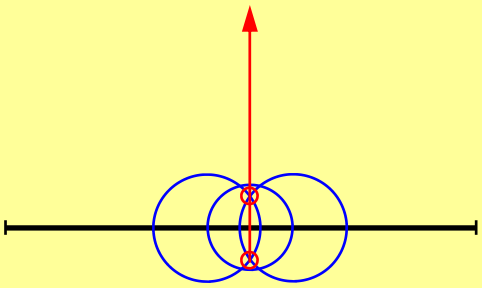
2

Using the intersection points, create two more circles (or arcs).



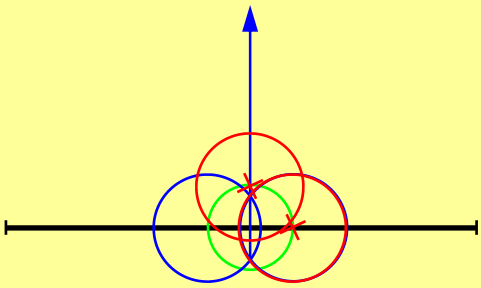
3

Use the intersection points created by the two circles (red), draw a perpendicular line. This is a 90º angle.



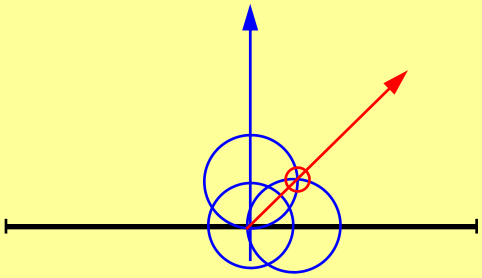
4

The first circle (green) intersects the perpendicular and the interval. Using these point create two arcs or circles (red circles).



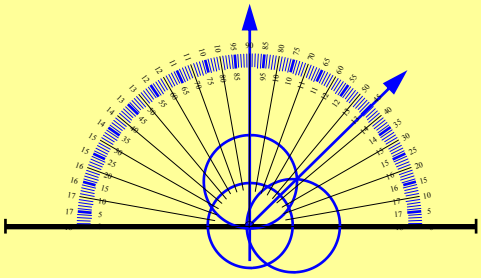
5

Draw a line through the vertex and intersection point (red circle) and its done!



6

Check your work with a protractor.



Bisecting Angles and Making 45° Angles

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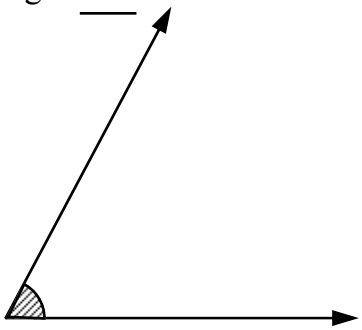
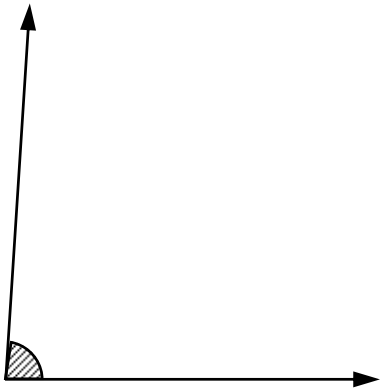
Bisect the angles below using a compass and a ruler. Verify that the bisection is correct using a protractor.

To make a 45° angle create a 90° angle (perpendicular) then bisect it. A 45° angle could also be 135°!

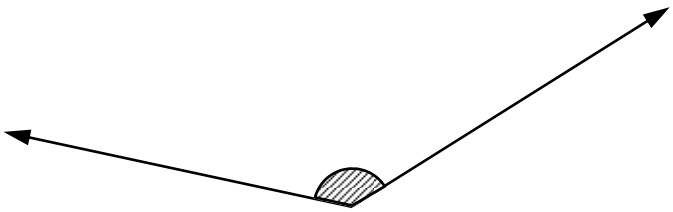
1 Original angle ___° Bisection angle ___°



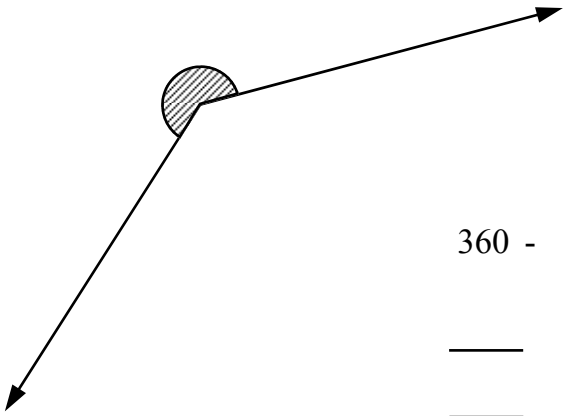
2 Original angle ___ Bisection angle ___



3 Original angle ____ Bisection angle ___



4 Original angle ____ Bisection angle ____



360 -

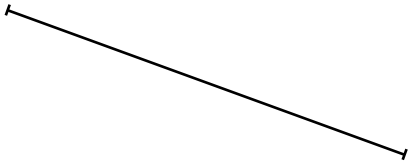
___

___

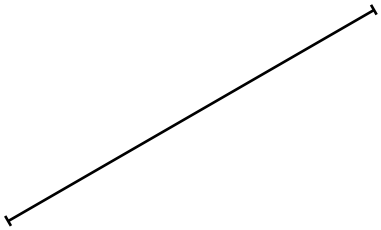
5

6

7



8



Bisecting Angles and Making 45° Angles

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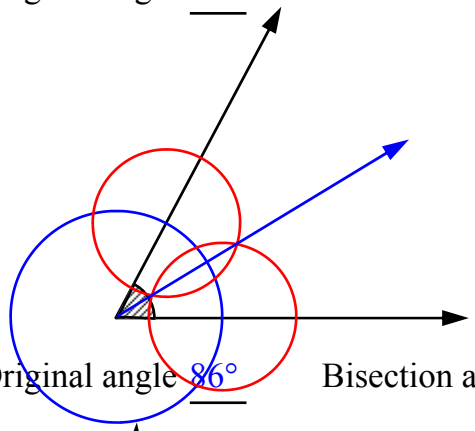
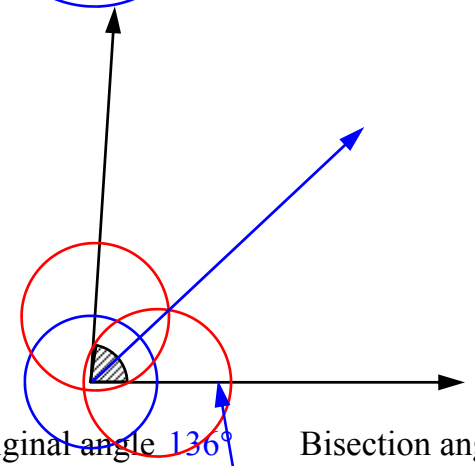
Bisect the angles below using a compass and a ruler. Verify that the bisection is correct using a protractor.

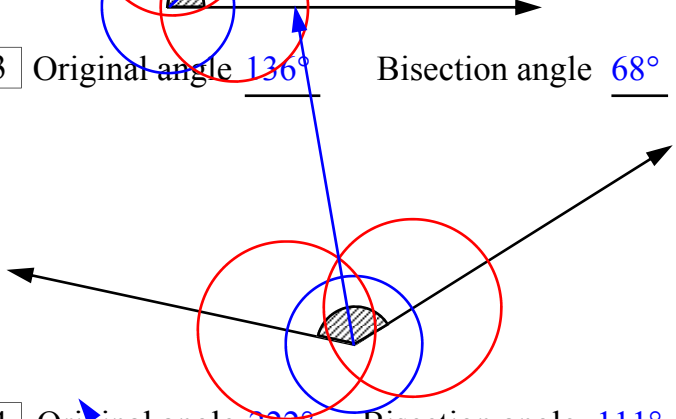
1 Original angle 62 ° Bisection angle 31 °



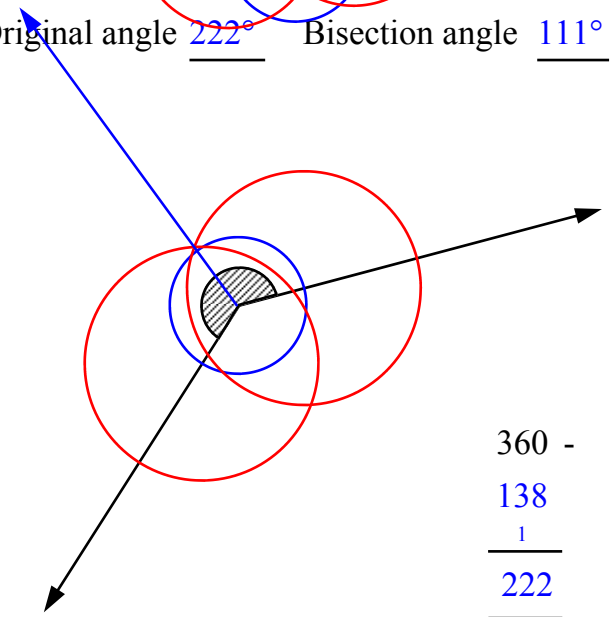
2 Original angle 86° Bisection angle 43°



3 Original angle 136° Bisection angle 68°



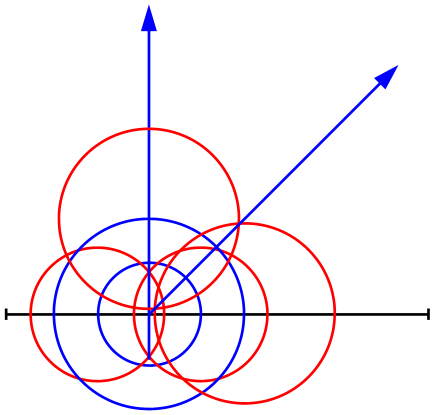
4 Original angle 222° Bisection angle 111°



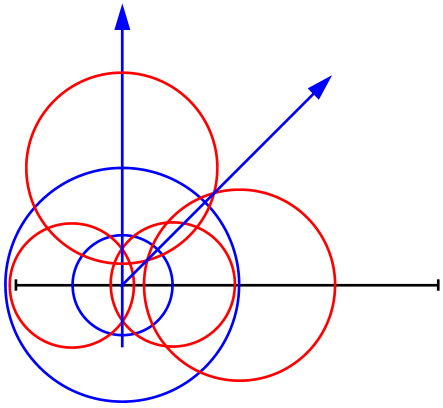
360 - 138 = 222

To make a 45° angle create a 90° angle (perpendicular) then bisect it. A 45° angle could also be 135°!

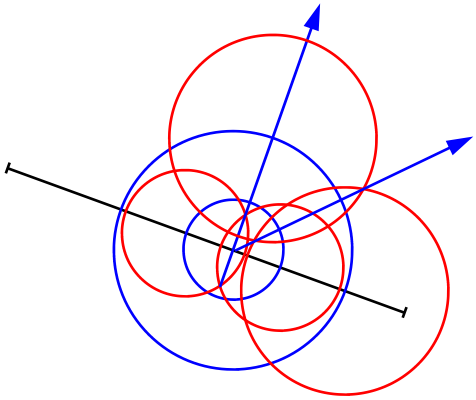
5



6



7



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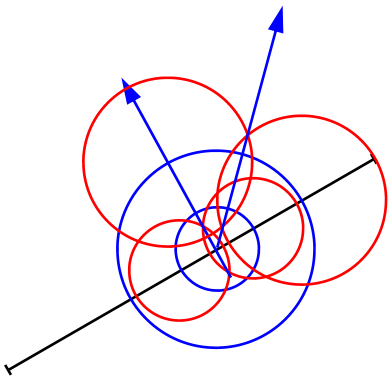


Constructing 60° and 30° Angles

This sheet is about constructing a 60° angle and a 30° angle (a bisected 60° angle), to complete this sheet you require a compass and a ruler. One important thing to remember is that if you make a 60° angle you are also making a 120° angle, it's the supplementary angle. It will be the adjacent obtuse angle beside the 60° angle. Likewise when you make a 30° angle the adjacent obtuse angle will be 150°.

Column 1 - Constructing 60º & 120º Angles - Compass & Ruler

1

Place the compass at the end of the interval, open to any size and draw an arc.



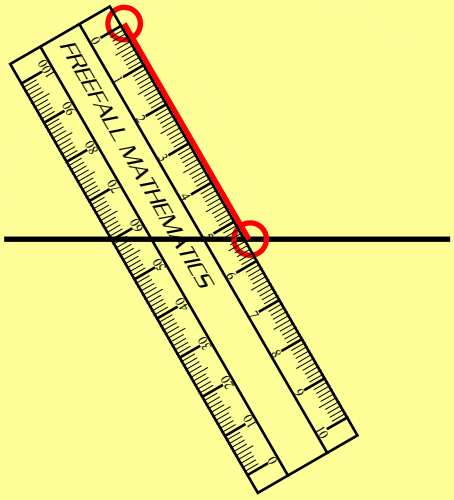
2

Using the intersection of the arc and the interval (red circle), keep the compass at the same opening and draw another arc.



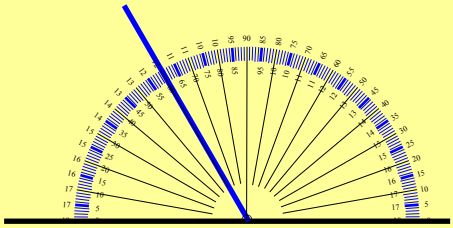
3

Use a ruler to connect the two intersection points (red circles).



4

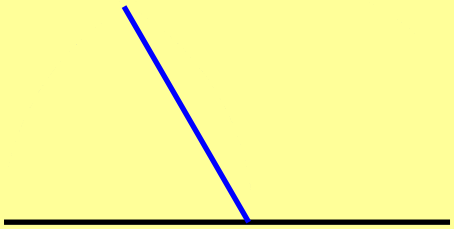
Check your work with a protractor.



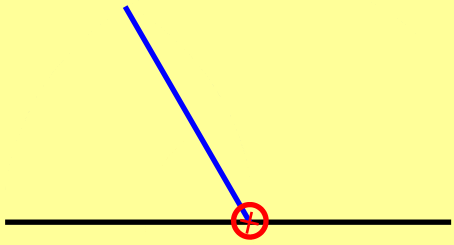
Constructing 60° and 30° Angles

Column 2 - Constructing 30º & 150º Angles - Compass & Ruler

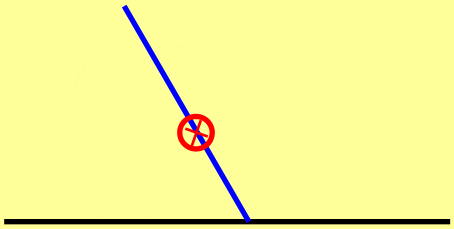
1 Create a 60º angle as outlined on the previous page. A 30º angle is made by bisecting a 60º angle.



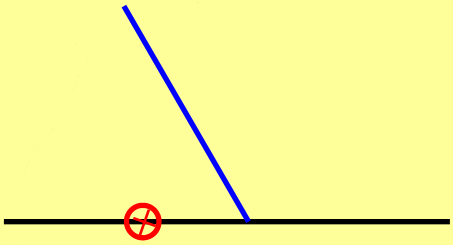
2 Draw an arc from the vertex of the angle (red circle) of any length.



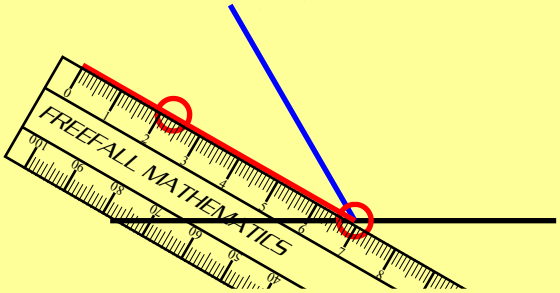
3 Draw an arc from one intersection point (red circle).



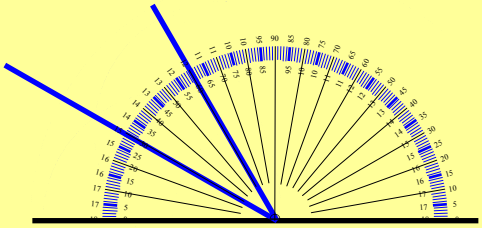
4 Then repeat for the other intersection point (red circle). Keep the compass at the same opening.



5 Use a ruler to connect the intersection point and the vertex (red circles). All done!



6 Using a protractor, check your work.



Constructing 60° and 30° Angles

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To make a 60° angle use the same method as creating an equilateral Δ. A 60° could also be a 120° angle.

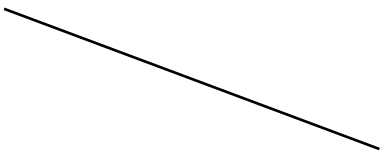
1

2

3



4



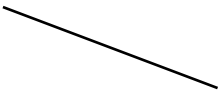


To make a 30° angle create a 60° angle then bisect it. A 30° angle could also be 150°!

5

6

7



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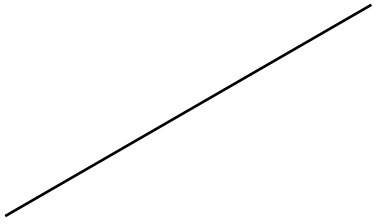

Constructing 60° and 30° Angles

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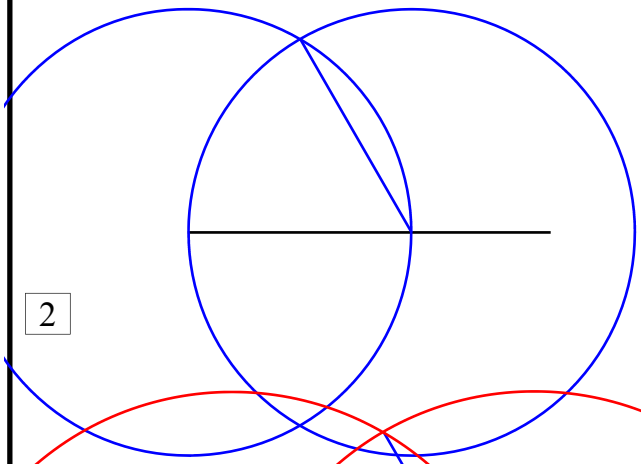
To make a 60° angle use the same method as creating an equilateral Δ. A 60° could also be a 120° angle.

1



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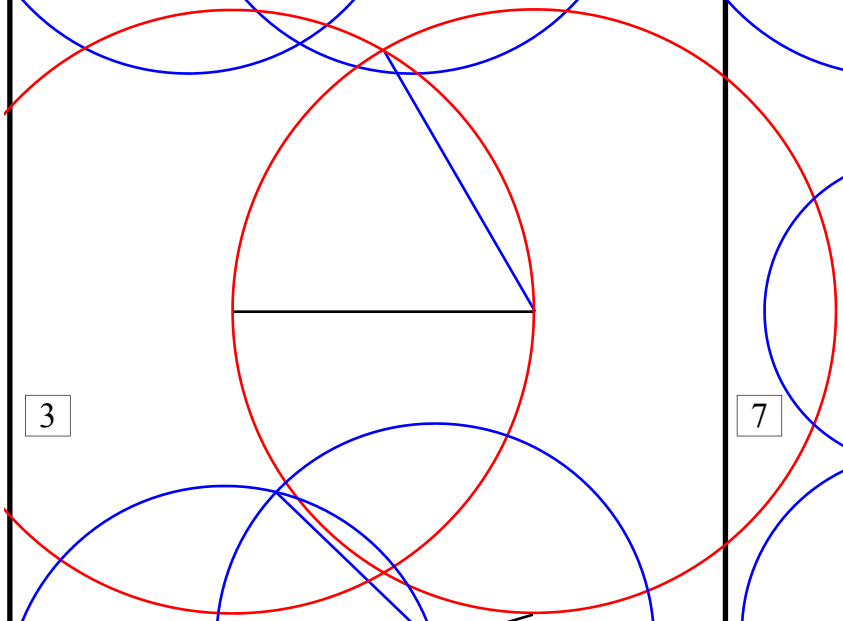

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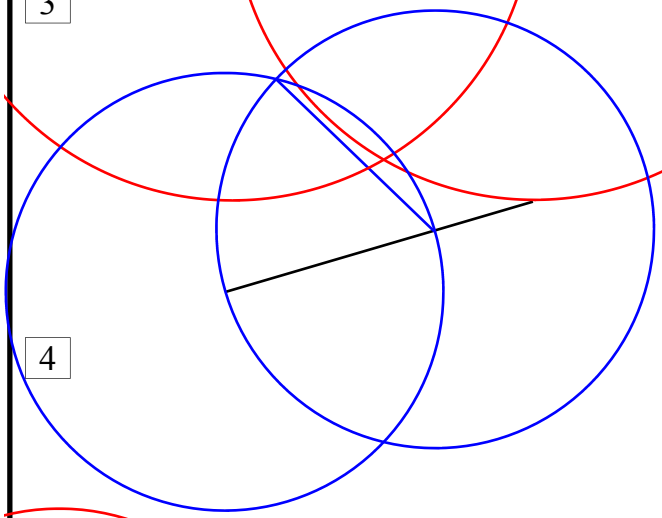

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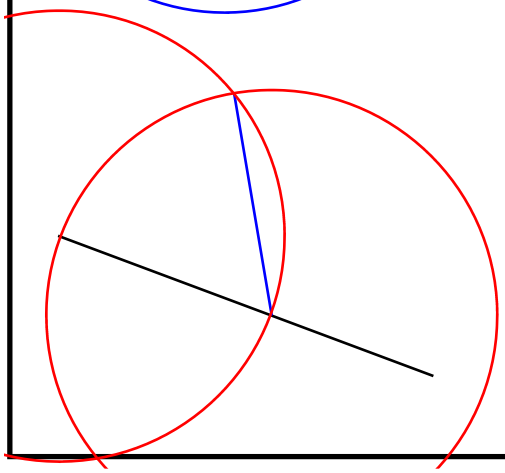
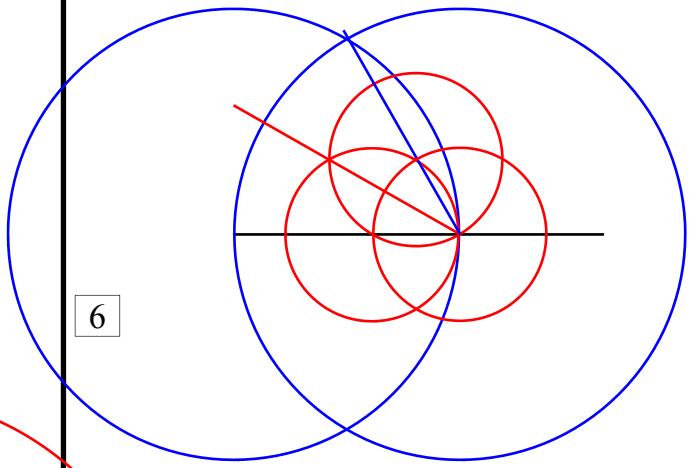


To make a 30° angle create a 60° angle then bisect it. A 30° angle could also be 150°!

5



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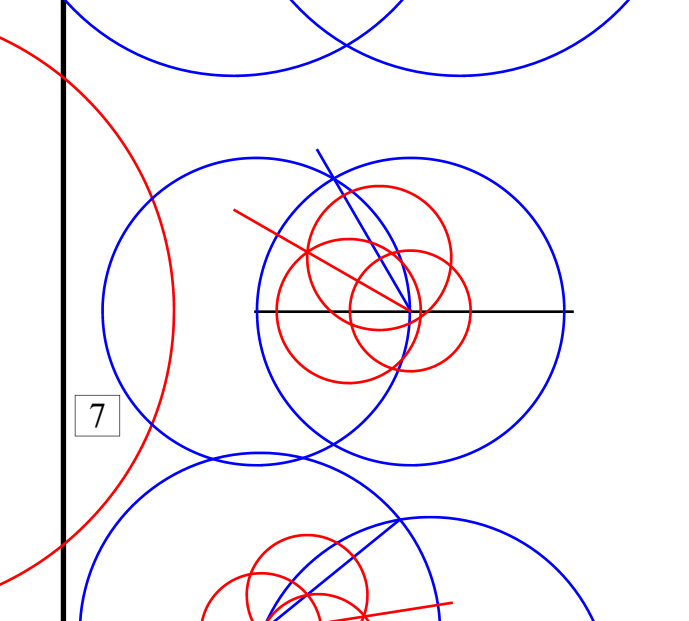

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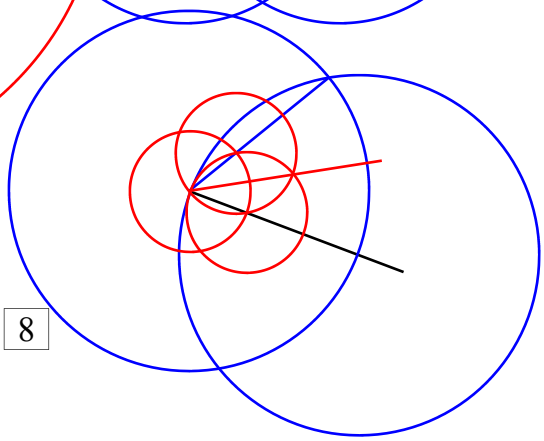
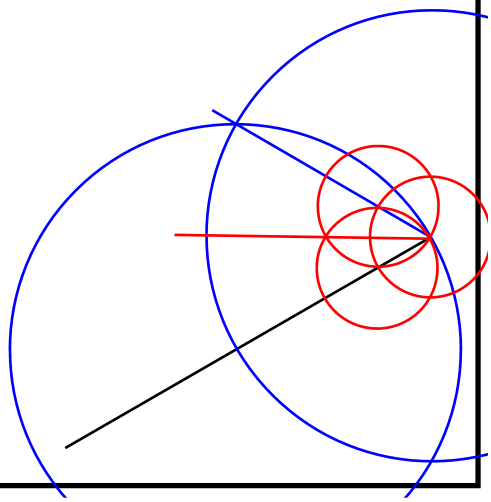
8





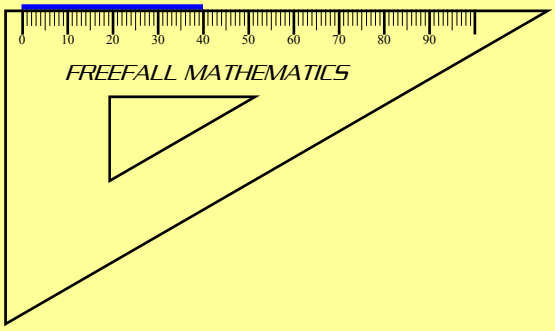
Drawing Quadrilaterals

This sheet involves the construction of four different plane (2-D) shapes. These are squares, rectangles, rhombi (plural of rhombus) and parallelograms.

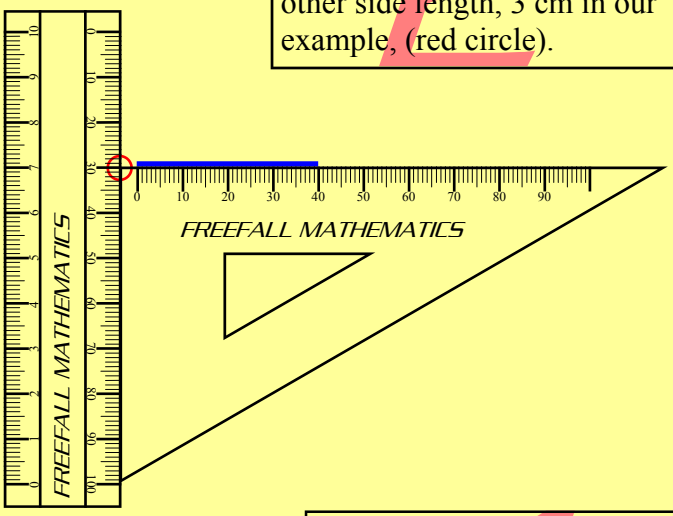
Column 1 - Constructing Squares & Rectangles - Set Square & Ruler

Example:
4 cm × 3 cm
Rectangle

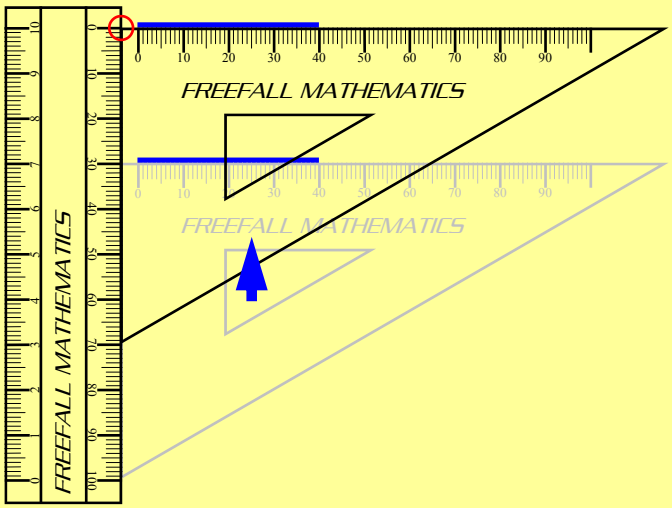
1 Using your set square draw a line the required length. In our example, 4 cm in length.



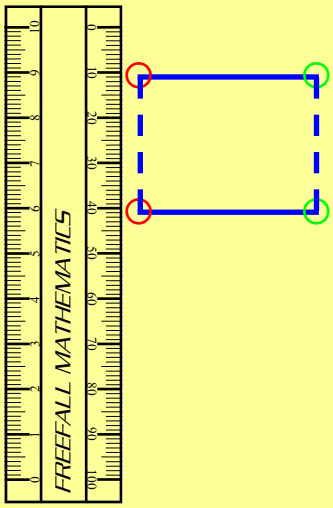
2 Place a ruler on the set square's edge so that the corner is at the other side length, 3 cm in our example, (red circle).

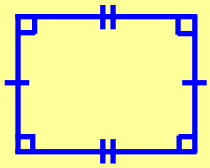


3 Slide the set square along the ruler edge until its corner is at zero, (red circle). Then draw the same length line as previously drawn, 4 cm in our example



4 Use a ruler to draw the remaining two sides. In our example they are 3 cm.





5 Place right angle markings in each corner and mark the opposite equal sides with dashes. All done!

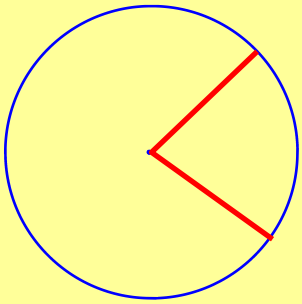
Drawing Quadrilaterals

Column 2 - Constructing a Rhombus - Compass & Ruler

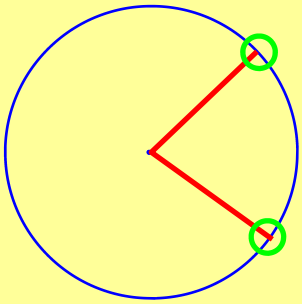
Example:
4 cm Rhombus

1 Draw a point where you wish to start.

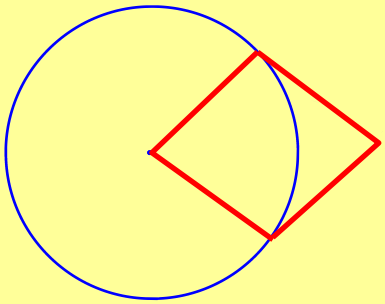
2 Open your compass to the side length (4 cm in our example) and draw a circle. Then draw 2 lines from the centre to the circumference, at any angle you like.



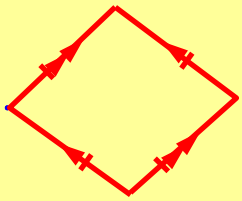
3 Using the compass open at the same length, draw 2 arcs from the intersection points shown below in green.



4 Use your ruler to complete the rhombus.

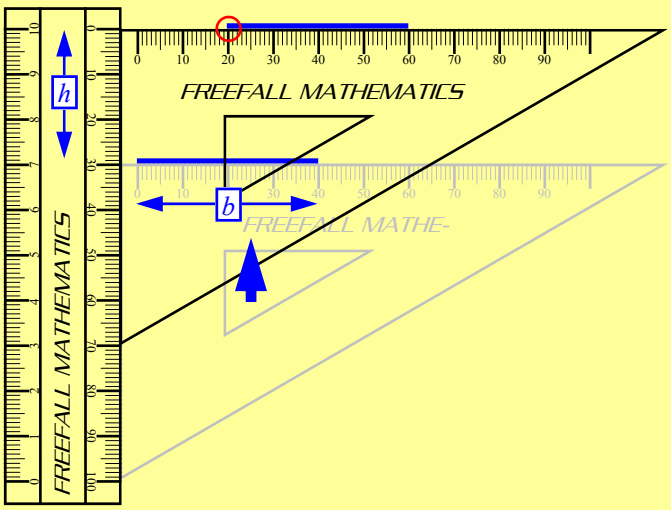


5 Show the parallel sides with arrows. Label all the equal sides with dashes.

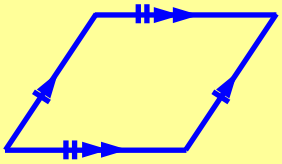


Column 2 - Constructing a Parallelogram - Set Square & Ruler

1 Start by using the same method as with the construction of a square or rectangle. When drawing the second line, instead of starting from zero start from another number, in the example below 2 cm was chosen.



2 Complete the sides and place arrows on parallel sides. You should also put dashes on the sides to show that they are the same length, that way a rhombus isn't confused with a parallelogram.



Drawing Quadrilaterals

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Construct squares using a ruler and a set square. Place side length markings and perpendicular symbols on the shape.

1 $l = 2.5$ cm

2 $l = 4.1$ cm

3 $l = 37$ mm

4 $l = 1.9$ cm

Now construct rectangles with the same length sides marked and perpendicular markings in each corner



5 $l = 4$ cm, $b = 2.5$ cm

6 $l = 2.2$ cm, $b = 3.1$ cm

7 $l = 34$ mm, $b = 6$ cm

8 $l = 1.3$ cm, $b = 5.2$ cm



Use a compass to draw rhombi with the side lengths below, mark the parallel sides.

9 $l = 2.5$ cm

10 $l = 1.9$ cm

11 $l = 2.8$ cm

12 $l = 14$ mm

Construct parallelograms with parallel side markings. The base length and the perpendicular height are given



13 $h = 1.5$ cm, $b = 4$ cm

14 $h = 3.1$ cm, $b = 3.6$ cm

15 $h = 0.8$ cm, $b = 2.6$ cm

16 $h = 1.8$ cm, $b = 3.2$ cm

Drawing Quadrilaterals

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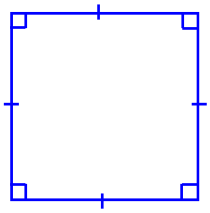
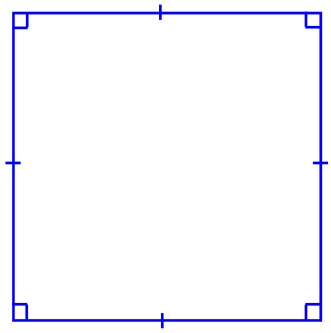
Construct squares using a ruler and a set square. Place side length markings and perpendicular symbols on the shape.

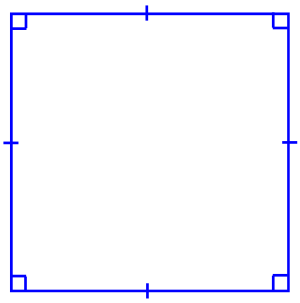
1 $l = 2.5$ cm



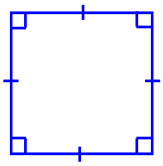
2 $l = 4.1$ cm



3 $l = 37$ mm

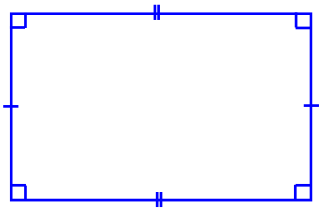


4 $l = 1.9$ cm

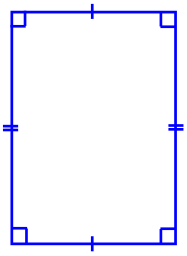


Now construct rectangles with the same length sides marked and perpendicular markings in each corner

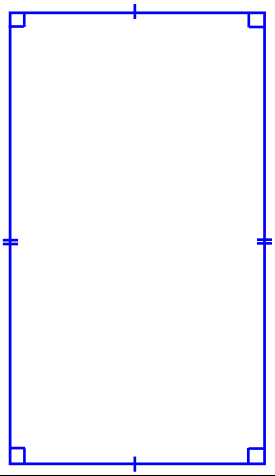
5 $l = 4$ cm, $b = 2.5$ cm



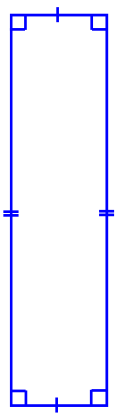
6 $l = 2.2$ cm, $b = 3.1$ cm



7 $l = 34$ mm, $b = 6$ cm

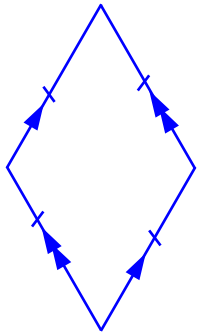


8 $l = 1.3$ cm, $b = 5.2$ cm

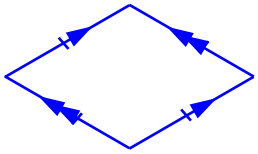


Use a compass to draw rhombi with the side lengths below, mark the parallel sides.

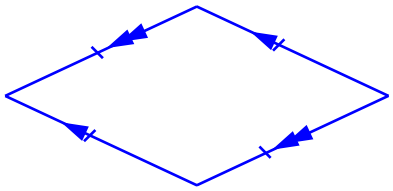
9 $l = 2.5$ cm



10 $l = 1.9$ cm



11 $l = 2.8$ cm

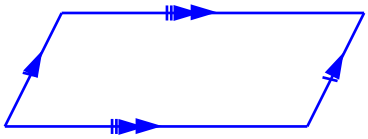


12 $l = 14$ mm

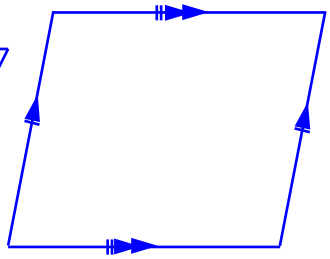


Construct parallelograms with parallel side markings. The base length and the perpendicular height are given

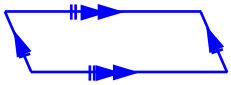
13 $h = 1.5$ cm, $b = 4$ cm



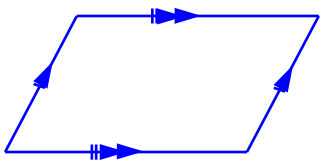
14 $h = 3.1$ cm, $b = 3.6$ cm



15 $h = 0.8$ cm, $b = 2.6$ cm



16 $h = 1.8$ cm, $b = 3.2$ cm



FREEFALL MATHEMATICS

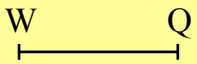
ALTITUDE

REVIEW:
ANGLES

Naming Angles

Naming angles allows you to explain which angle or part of a shape you're dealing with.

The first column introduces intervals, lines and rays. With intervals and lines you can name them in either direction. The interval below could be WQ or QW. You are asked to give both the names in these questions.



With rays, name them in the direction of the arrow only. The ray below is called HG only, not GH.



In column 2 you are asked to name the rays and also the vertex, this is the meeting point of the two rays, the 'elbow' if you like. The vertex is named by the letter at the point alone. To show it is a vertex we put a hat on it, like an upside-down 'v'.

To name an angle we generally use 3 letters, moving from ray-to vertex-to ray. The vertex is always in the middle. To show that it is an angle you put an angle sign in front of it or write the letters placing a hat over the vertex.

$\angle ABC$ or, $\hat{A}BC$

When you name a shape (in column 3) you start at any point then go around the shape the one way. If the letters are in alphabetical order you might like to start at the earliest letter. No symbol is put before the name of a shape, though sometimes this is done with triangles. (e.g. ΔABC)

Naming Angles

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Give the 2 possible names for these intervals.

1 A B or _____

2 K Z or _____

3 D L or _____

4 U W or _____

Give the 2 possible names for these lines.

5 T Q or _____

6 C S or _____

7 H I or _____

8 E K i) V ii) R

i) _____ or _____ ii) _____ or _____

Name these rays

9 X F _____

10 N Y _____

11 P M _____

12 J H _____

Name the 2 rays and the vertex for these angles

13 K Y V ^

ray _____ ray _____ vertex _____

14 T P A ^

ray _____

ray _____

vertex _____

15 A E L S Z ★ +

★ ray _____ ray _____ vertex _____

+ ray _____ ray _____ vertex _____

Name these angles, with the vertex as the middle letter. Use either ^ or ∠

16 X A B ∠ _____

17 F Z L T ★ _____ + _____

18 M K A O Y ♦ _____ ★ _____ + _____

Name these shapes using their letters. Then name the angle with the symbol.

19 A D C B + _____ + _____

20 R H T K V _____ + _____

21 A F E D B C _____ + _____ ★ _____

22 P J E N K T _____ + _____ ★ _____ ♦ _____

Place the given symbol in the following angles

23 C D F G B A I H E

Note E is the point of contact

▲ ∠ BCD _____ ♥ ∠ AED _____

★ ∠ FED _____ + ∠ AEI _____

♦ ∠ IHG _____ × ∠ ABC _____

Naming Angles

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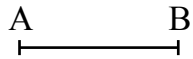
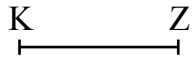
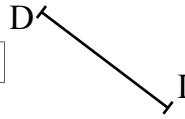
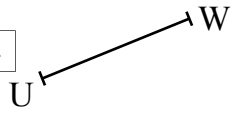
Give the 2 possible names for these intervals.

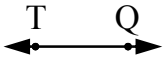
1  AB or BA

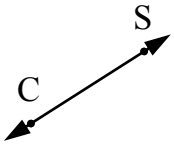
2  KZ or ZK

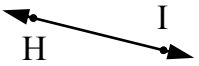
3  DL or LD

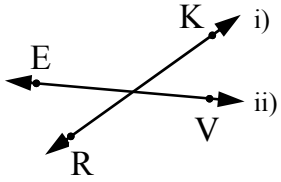
4  UW or WU

Give the 2 possible names for these lines.

5  TQ or QT

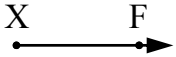
6  CS or SC

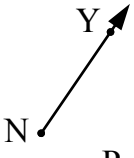
7  HI or IH

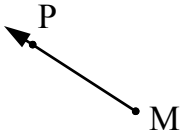
8 

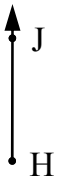
i) RK or KR ii) EV or VE

Name these rays

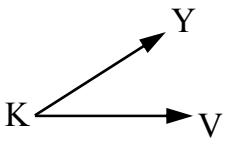
9  XF

10  NY

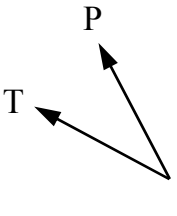
11  MP

12  HJ

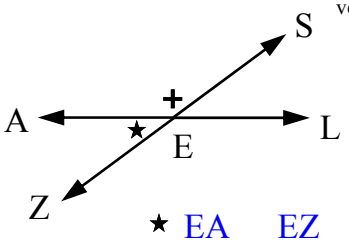
Name the 2 rays and the vertex for these angles

13 

KV ray, KY ray, ^K vertex

14 

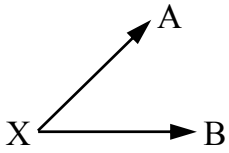
AT ray, AP ray, ^A vertex

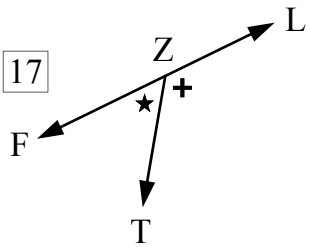
15 

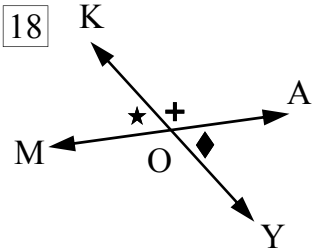
★ EA ray, EZ ray, ^E vertex

+ EA ray, ES ray, ^E vertex

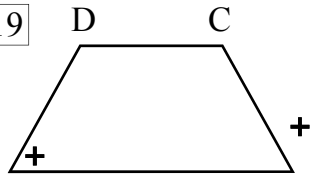
Name these angles, with the vertex as the middle letter. Use either ^ or ∠

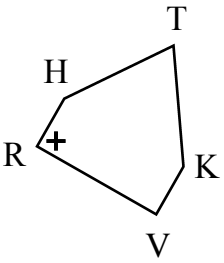
16  ∠ AXB

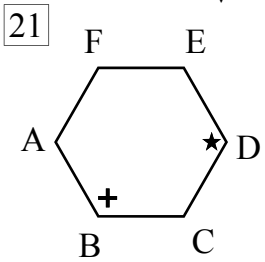
17  ★ ∠ FZT, + ∠ TZL

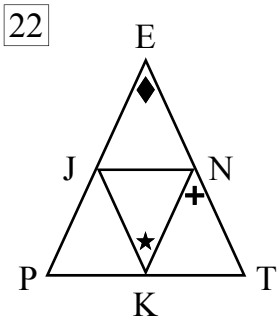
18  ♦ ∠ AOY, ★ ∠ KOM, + ∠ AOK

Name these shapes using their letters. Then name the angle with the symbol.

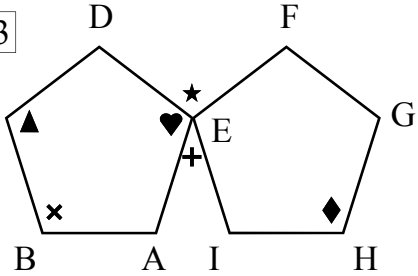
19  ABCD, + ∠ DAB

20  TKVRH, + ∠ HRV

21  ABCDEF, + ∠ ABC, ★ ∠ CDE

22  ENTKPJ, + ∠ TNK, ★ ∠ JKN, ♦ ∠ JEN

Place the given symbol in the following angles

23 

Note E is the point of contact

▲ ∠ BCD ♥ ∠ AED

★ ∠ FED + ∠ AEI

♦ ∠ IHG × ∠ ABC

Classifying Angles

Classification means to group into categories. There are six classifications in all:

- acute angles are between 0° and 90°
- a right angle is exactly 90°
- obtuse angles are between 90° and 180°
- a straight angle is exactly 180°
- reflex angles are between 180° and 360°
- and a revolution is 360°

How do you remember all these? You should be able to remember the straight angle, right angle and revolution, it is the others you could mix up. Remember that as the angle increases you move through the alphabet. A (acute) is before O(Obtuse) which is before R(reflex). There is a guide at the top of the page to refer to. With the angles, the measured side is the side marked with the arc (part circle) or the right angle symbol.

When asked to name angles remember that you can use either the hat or the angle sign method, as below.

$\angle ABC$ or, $\hat{ABC}$

In column 3 the word internal is used. What does internal mean? It means the inside (interior), so the internal angles are the angles on the inside of a shape.

Classifying Angles

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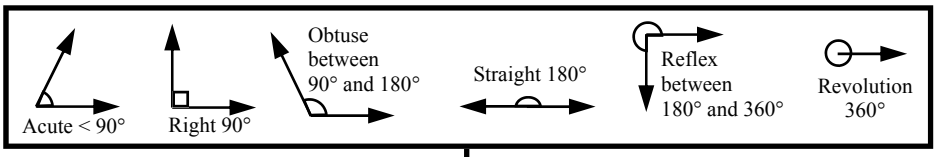


Classify the following angles

1 _____

2 _____

3 _____

4 _____

5 _____

6 _____

7 _____

8 _____

9 _____

10 _____

11 _____

12 _____

13 _____

14 _____

15 _____

16 _____

17 _____

18 _____

19 _____

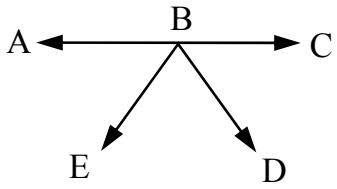
Complete the table below

20

Angle	Classification
115°	
333°	
73°	
165°	
180°	
200°	
360°	

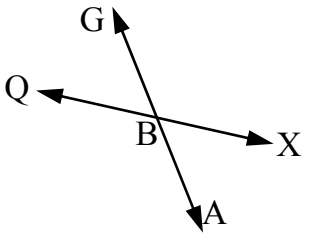
Using the ray AB in all angles name an acute, obtuse and straight angle

21



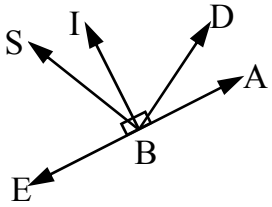
acute _____ obtuse _____ straight _____

22



acute _____ obtuse _____ straight _____

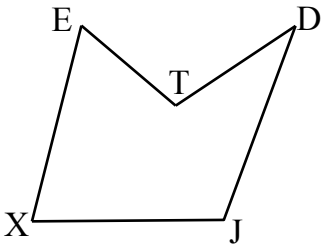
23



acute _____ obtuse _____ straight _____

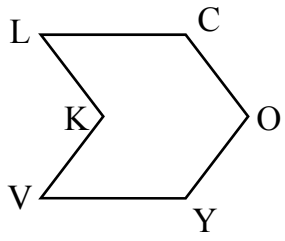
For the two shapes below name internal angles that are acute, obtuse or reflex.

24



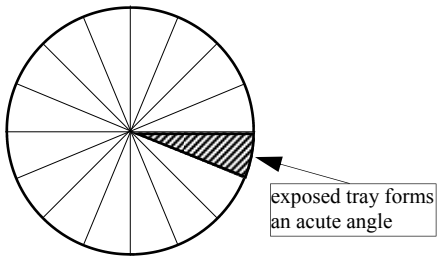
acute _____ obtuse _____ reflex _____

25



acute _____ obtuse _____ reflex _____

26 A family size pizza has the first piece eaten forming an acute angle. Each piece is the same size and when removed increases the size of the angle. Find the number of:



i) acute angles that can be made _____

ii) pieces eaten to make a right angle _____

iii) pieces eaten to make the largest obtuse angle _____

iv) reflex angles that can be made _____

Classifying Angles

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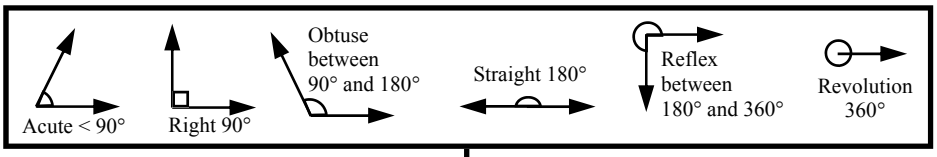


Classify the following angles

1 acute

2 obtuse

3 revolution

4 right

5 reflex

6 straight

7 acute

8 reflex

9 revolution

10 obtuse

11 right

12 obtuse

13 acute

14 acute

15 reflex

16 obtuse

17 straight

18 obtuse

19 reflex

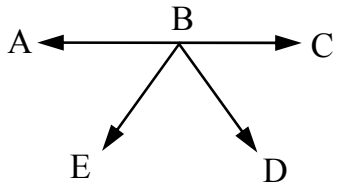
Complete the table below

20

Angle	Classification
115°	obtuse
333°	reflex
73°	acute
165°	obtuse
180°	straight
200°	reflex
360°	revolution

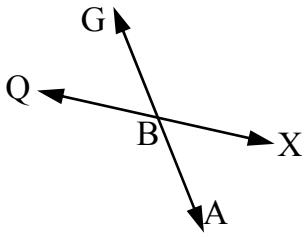
Using the ray AB in all angles name an acute, obtuse and straight angle

21



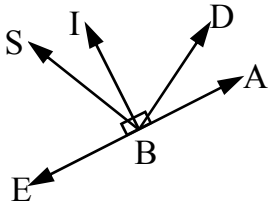
∠ ABE acute ∠ ABD obtuse ∠ ABC straight

22



∠ ABX acute ∠ ABQ obtuse ∠ ABG straight

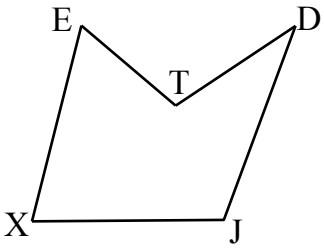
23



∠ ABD acute ∠ ABS obtuse ∠ ABE straight

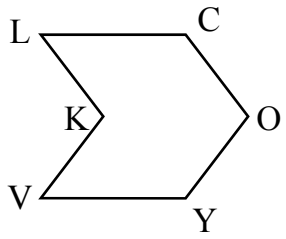
For the two shapes below name internal angles that are acute, obtuse or reflex.

24



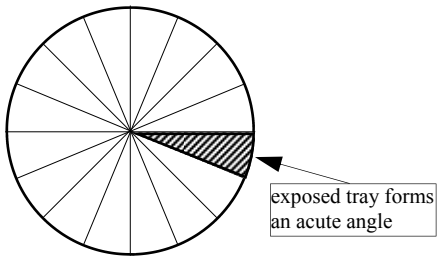
∠ XET acute ∠ DJX obtuse ∠ ETD reflex

25



∠ KLC acute ∠ LCO obtuse ∠ LKV reflex

26 A family size pizza has the first piece eaten forming an acute angle. Each piece is the same size and when removed increases the size of the angle. Find the number of:



i) acute angles that can be made 3

ii) pieces eaten to make a right angle 4

iii) pieces eaten to make the largest obtuse angle 7

iv) reflex angles that can be made 7

Estimating and Measuring Angles

An angle is used to measure rotation, a protractor is required for this worksheet.

To answer this sheet look at each angle and estimate its size, estimate meaning a skilled guess, so it is unlikely to guess the exact answer, but within say 15° would be very good. Don't use a protractor until you have estimated all of the angles, use the guide at the top right of the sheet if you need the help.

When using a standard protractor ensure you use the correct scale (inside or outside, make sure the scale used starts at 0°) and if the angle is a reflex (greater than 180°), measure the outside of the angle then subtract the angle from 360°. If an angle is measured and it is above 90°, then it must be larger than a right angle, if it isn't you have made a mistake. Use your estimation as a guide as well, and then see how close you were.

Estimating and Measuring Angles

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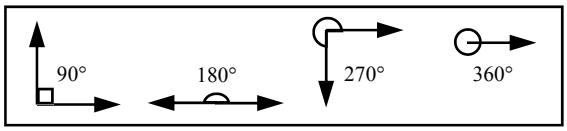
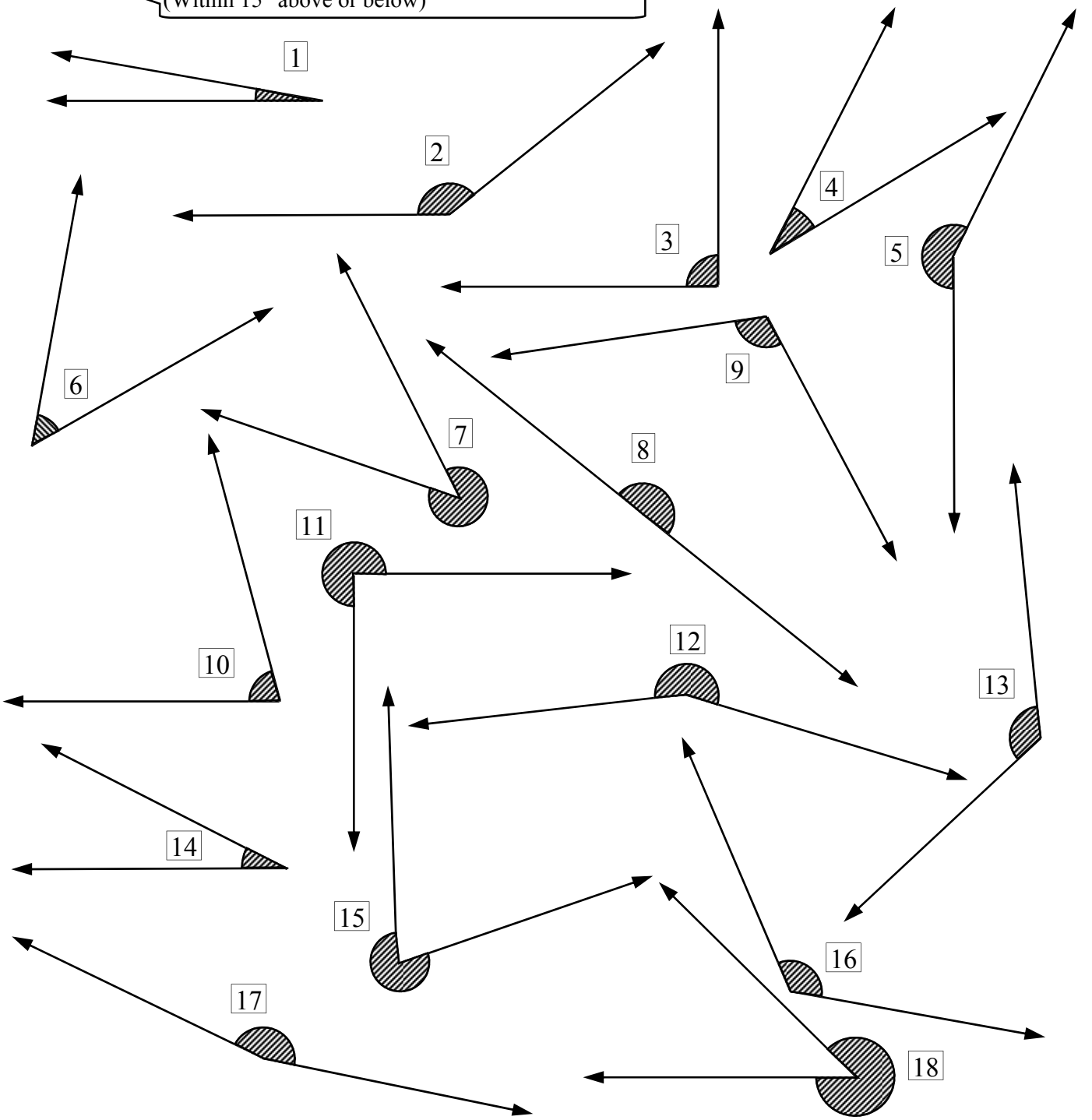
Estimate the size of all the shaded angles and place your answers in the table. Then using a protractor measure all the angles. See how well you estimated. (Within 15° above or below)





No	Estimate	Measured	No	Estimate	Measured	No	Estimate	Measured
1			7			13		
2			8			14		
3			9			15		
4			10			16		
5			11			17		
6			12			18		

Estimating and Measuring Angles

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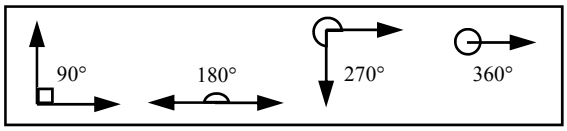
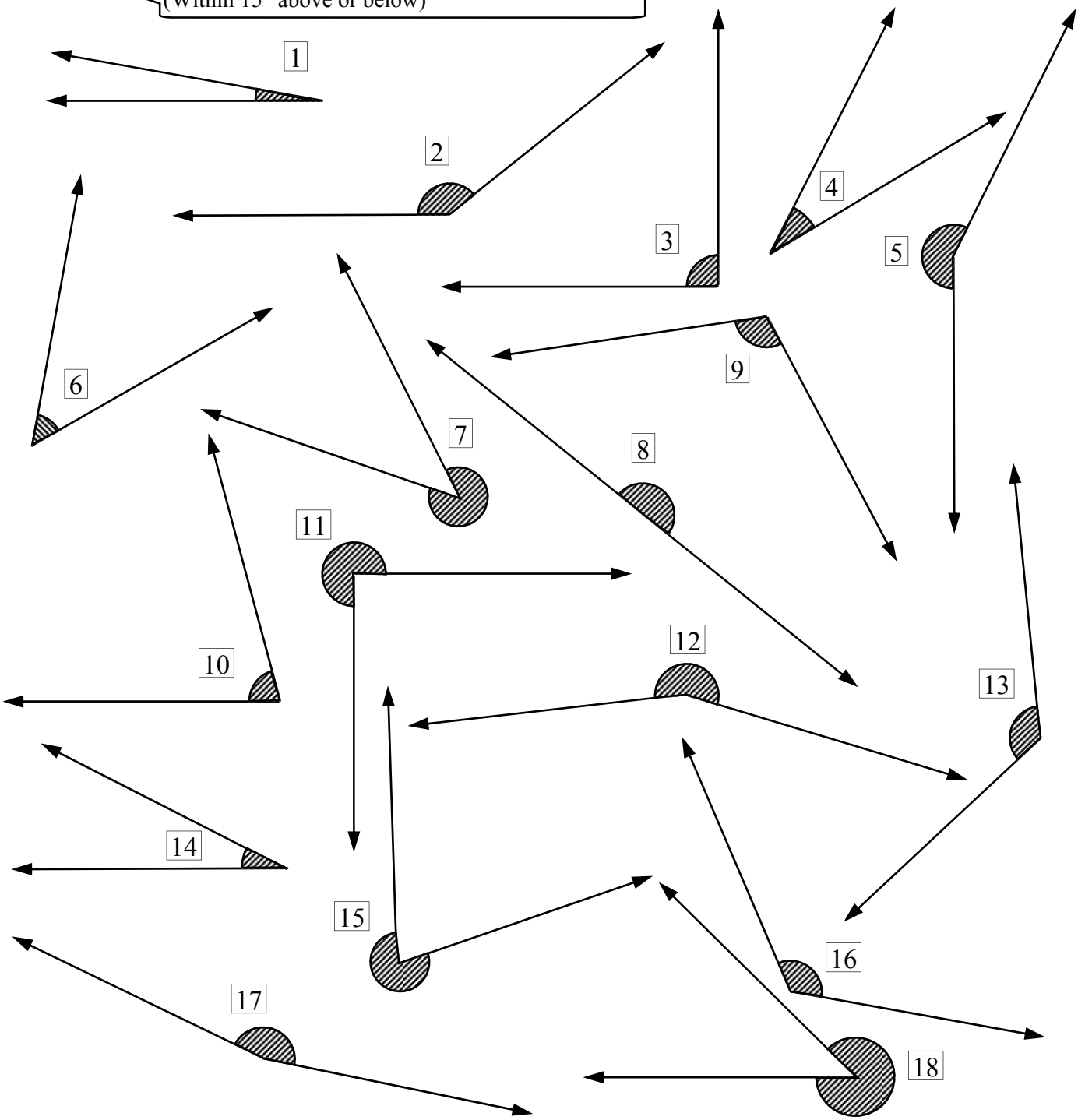
Estimate the size of all the shaded angles and place your answers in the table. Then using a protractor measure all the angles. See how well you estimated. (Within 15° above or below)





No	Estimate	Measured	No	Estimate	Measured	No	Estimate	Measured
1	5 ~ 25°	10°	7	300 ~ 330°	316°	13	115 ~ 140°	127°
2	125 ~ 155°	141°	8	180°	180°	14	20 ~ 35°	27°
3	85 ~ 95°	90°	9	95 ~ 125°	110°	15	270 ~ 300°	287°
4	20 ~ 40°	32°	10	65 ~ 85°	75°	16	110 ~ 135°	123°
5	190 ~ 220°	206°	11	265 ~ 275°	270°	17	150 ~ 170°	166°
6	40 ~ 60°	50°	12	190 ~ 220°	203°	18	300 ~ 330°	316°

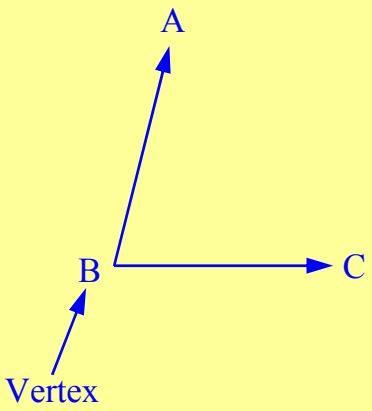
Constructing Angles

The previous sheet involved measuring angles, this sheet is about the construction of angles.

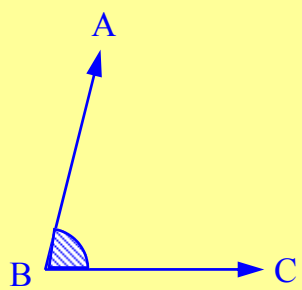
The method is as follows:

- Draw a horizontal line
- Measure the given angle from the end of the drawn line, plotting a point at the angle
- Draw a line from the end of the line to the point
- Label the angle with the letters supplied in the question, remember the vertex is point on the elbow of the angle.
- Draw an arc or sector at the vertex to show the angle is the inside or outside of the angle.
- If the angle is greater than 180° then calculate (360° - the angle) the obtuse/acute angle and use that angle. Draw the sector on the other side of the angle.

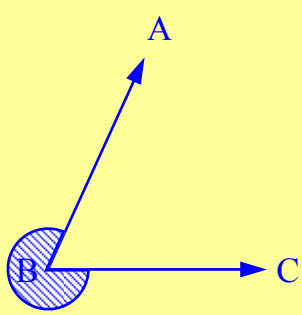
A completed angle



A completed angle with sector



A completed reflex angle with sector



Constructing Angles

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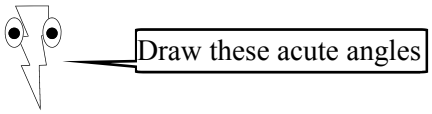
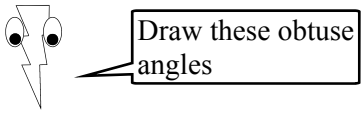


1 Construct a 30° angle
∠ ABC

2 Construct a 60° angle
∠ EVT

3 Construct a 37° angle
∠ NTY

4 Construct an 73° angle
∠ ALR

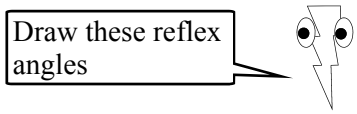


5 Construct a 110° angle
∠ FZT

6 Construct a 165° angle
∠ TRE

7 Construct a 138° angle
∠ JPG

8 Construct an 97° angle
∠ NJD



9 Construct a 240° angle
∠ BYC

10 Construct a 190° angle
∠ XOJ

11 Construct a 337° angle
∠ JVA

12 Construct an 254° angle
∠ PUK

Constructing Angles

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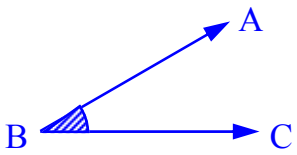
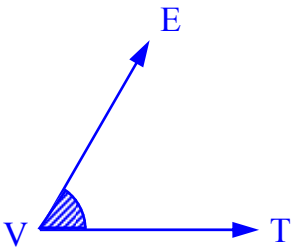
Draw these acute angles

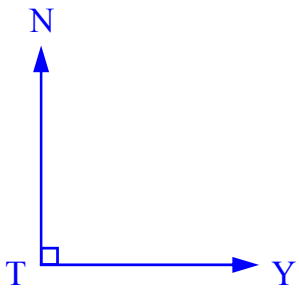
1 Construct a 30° angle
∠ ABC



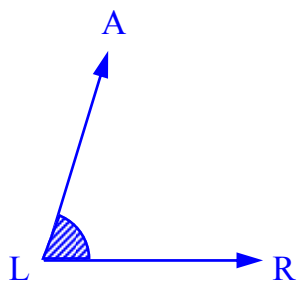
2 Construct a 60° angle
∠ EVT



3 Construct a 90° angle
∠ NTY

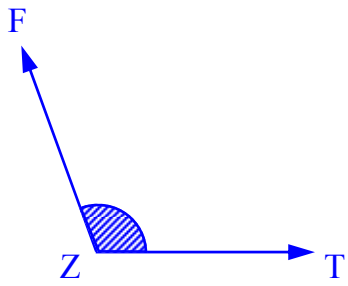


4 Construct an 73° angle
∠ ALR

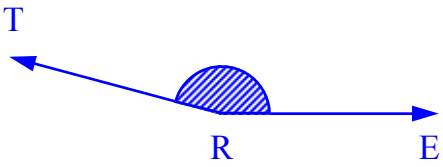


Draw these obtuse angles

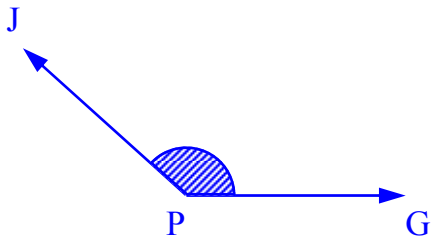
5 Construct a 110° angle
∠ FZT



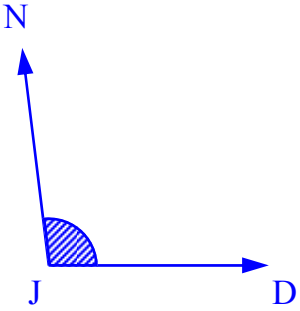
6 Construct a 165° angle
∠ TRE



7 Construct a 138° angle
∠ JPG

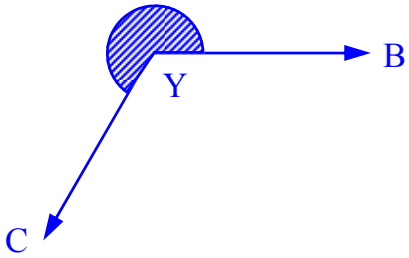


8 Construct an 97° angle
∠ NJD



Draw these reflex angles

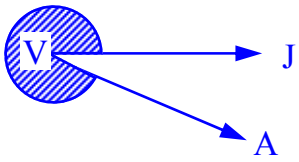
9 Construct a 240° angle
∠ BYC



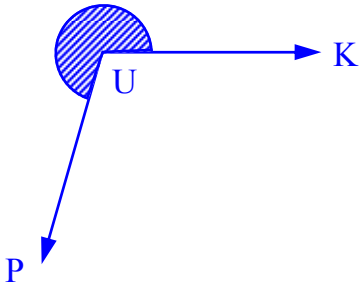
10 Construct a 190° angle
∠ XOJ



11 Construct a 337° angle
∠ JVA



12 Construct an 254° angle
∠ PUK



Adjacent Angles

Adjacent means next to, in the case of angles this means that the angle shares a common ray (or arm) and vertex with the angle beside it.

In column 1 the table requires you to add the two angles together to get a total, the sum of two adjacent angles. This continues down the column only with graphical representation, note that these angles aren't to scale, so don't use the angles as a guide. In question 3 there are 3 angles but only 2 angles have numbers, the third is a right angle which is 90°. It is suggested that you write in the '90°', that way you won't forget it in the addition.

In the 2nd column question 6 asks you to name the adjacent angle. This requires you to name the 2 angles that touch the given angle. As the layout is circular that means each angle has an adjacent angle on each side of it, don't name one angle then the same angle with the letters reversed. Question 7 is another table which this time gives you the total of two angles, and one of the angles. Subtract the angle from the total and you have the answer.

The same method is used for the rest of the sheet Subtract the known angles from the total to obtain the unknown angle. Except questions 10 and 12, with these use division! Question 10 gives you the hint.

Adjacent Angles

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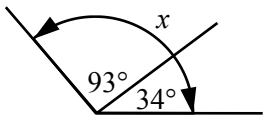
Angles 1 and 2 are adjacent find the sum of the two angles

1

Angle 1	Angle 2	Total
35°	42°	77°
19°	27°	
7°	96°	
115°	153°	
21°	117°	
273°	24°	
311°	46°	

Find the size of the angle formed by adding these adjacent angles.

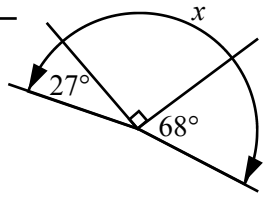
2



x = ____ + ____

x = ____

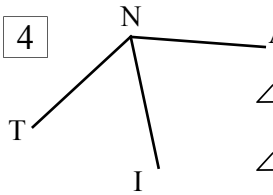
3



x = _____

x = _____

4



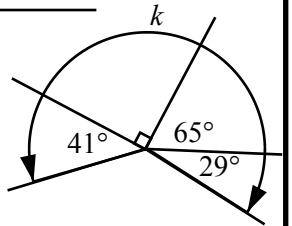
∠ TNI = 68°

∠ ANI = 77°

∠ TNA = _____

_____

5

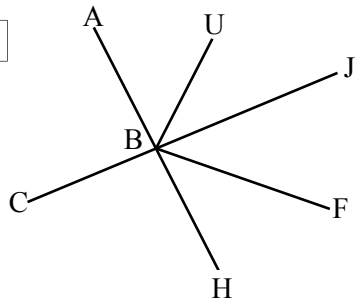


_____

_____

Name two adjacent angles to the following

6



∠ ABC : _____ & _____

∠ HBF : _____ & _____

∠ UBC : _____ & _____

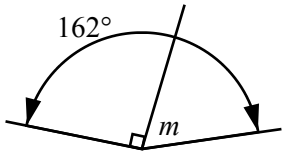
This time subtract. Given the total and one angle, find the other angle.

7

Total	Angle 1	Angle 2
38°	15°	
52°		17°
191°		115°
311°	208°	
187°	11°	
304°	221°	
339°		147°

You are given the total now subtract or divide to find the missing angles.

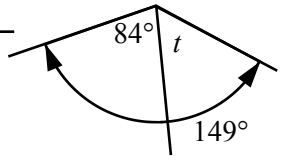
8



m = ____ - ____

m = ____

9

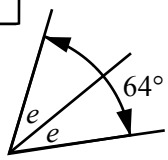


_____

_____

Keep going!

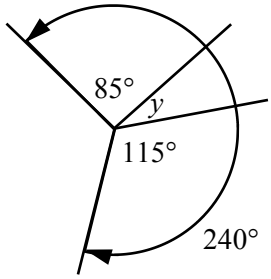
10



$2e$ = ____

e = ____

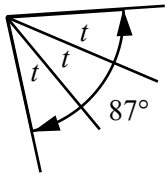
11



_____

_____

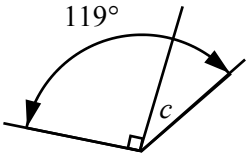
12



_____

_____

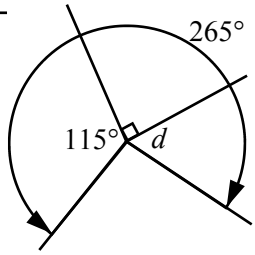
13



_____

_____

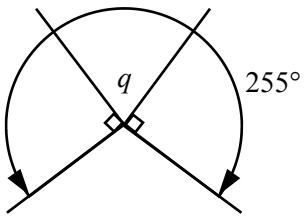
14



_____

_____

15



_____

_____

Adjacent Angles

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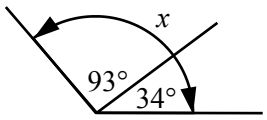
Angles 1 and 2 are adjacent find the sum of the two angles

1

Angle 1	Angle 2	Total
35°	42°	77°
19°	27°	46°
7°	96°	103°
115°	153°	268°
21°	117°	138°
273°	24°	297°
311°	46°	357°

Find the size of the angle formed by adding these adjacent angles.

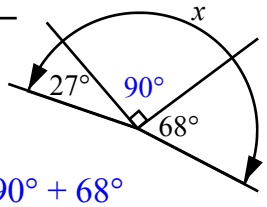
2



x = 93° + 34°

x = 127°

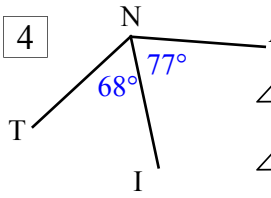
3



x = 27° + 90° + 68°

x = 185°

4



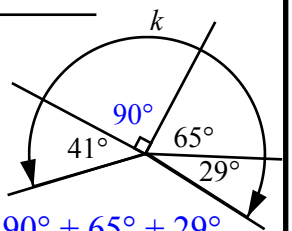
∠ TNI = 68°

∠ ANI = 77°

∠ TNA = 68° + 77°

∠ TNA = 145°

5

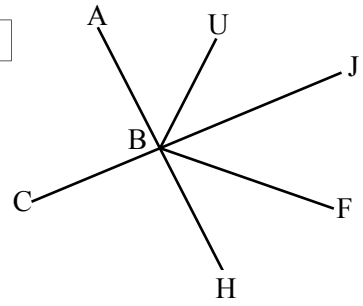


k = 41° + 90° + 65° + 29°

k = 225°

Name two adjacent angles to the following

6



∠ ABC : ∠ ABU & ∠ CBH

∠ HBF : ∠ FBJ & ∠ HBC

∠ UBC : ∠ UBJ & ∠ CBH

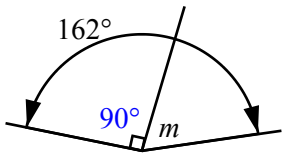
This time subtract. Given the total and one angle, find the other angle.

7

Total	Angle 1	Angle 2
38°	15°	23°
52°	35°	17°
191°	76°	115°
311°	208°	103°
187°	11°	176°
304°	221°	83°
339°	192°	147°

You are given the total now subtract or divide to find the missing angles.

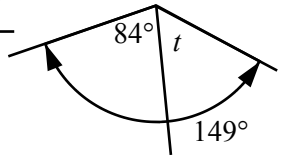
8



m = 162° - 90°

m = 72°

9

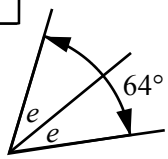


t = 149° - 84°

t = 65°

Keep going!

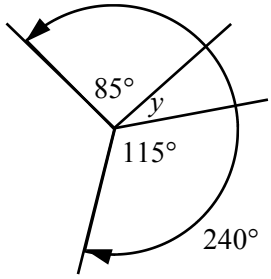
10



2e = 64°

e = 32°

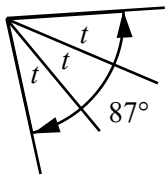
11



y = 240° - 115° - 85°

y = 40°

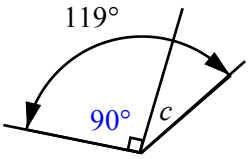
12



3t = 87°

t = 29°

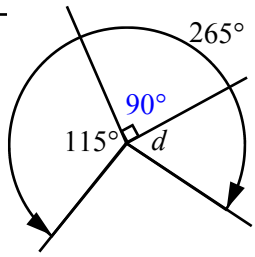
13



c = 119° - 90°

c = 29°

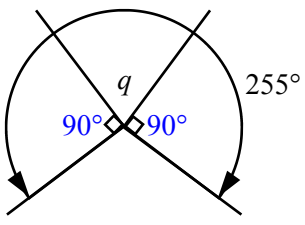
14



d = 265° - 115° - 90°

d = 60°

15



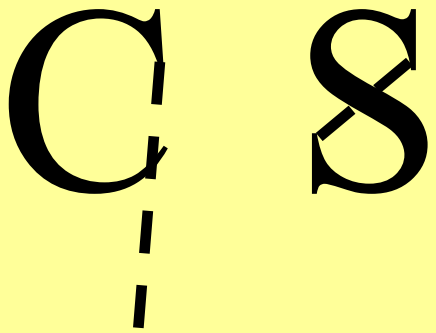
q = 255° - 90° - 90°

q = 75°

Complementary Angles

Complementary angles are angles that add to 90°. Supplementary angles add to 180° (the next sheet). How do you remember which is which? C comes before S in the alphabet, as 90° comes before 180°. Or look at the construction below.

The C can be changed to a 9 (90°) and the S can be changed to an 8 to remember 180°



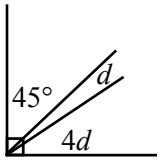
In Column 1 you are asked to find the complement, this means find the angle that when added to the angle given, has a total of 90°. You do this by subtracting the given angle from 90°. This continues down the column, subtract the angle given on the diagram from 90° to find the answer. The most common mistake is thinking that a right angle is 100°, or at least using it in calculations remember 90 not 100!

The 2nd column through to question 16 in the 3rd column is an extension of this. The same method is used only more than one angle is subtracted from 90°. Question 7 shows the method of working required.

Questions 17 and 18 require you to add the letters together, these equal 90°. Then divide by the number in front of the letter to get the value of the pronumeral. Note that the value of the pronumeral is being found, this isn't necessarily the value of the angle.

Questions 19 and 20 are more difficult questions and require an extra step. The letters are added on the left of the equals sign, and the subtraction of the angle from 90° is put on the right side. See the example below.

Example



$5d = 90° - 45°$

$5d = 45°$

$d = 9°$

Divide by 5

Complementary Angles

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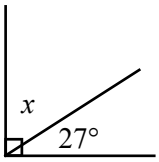
Find the complementary angle to those given in the table below.

1

Angle	Complement
30°	60°
45°	
73°	
21°	
9°	
57°	
84°	

Find the complementary angle for the following

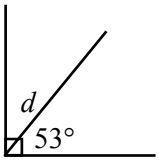
2



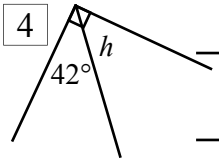
x = 90° - ____

x = ____

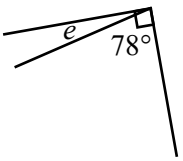
3



4



5



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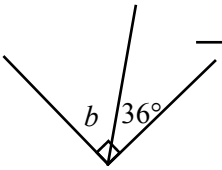
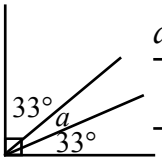


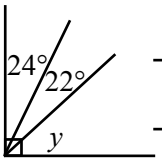
Now there are more than 2 angles. Solve these.

7



a = 90° - 33° - 33°

8



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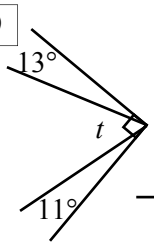

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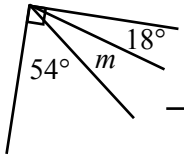

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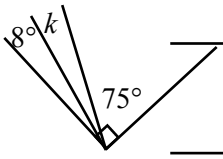

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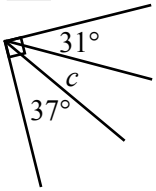

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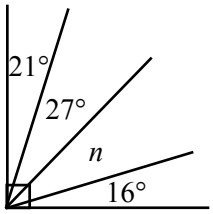

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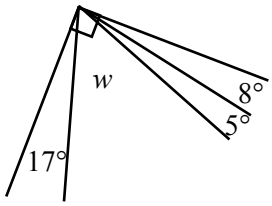

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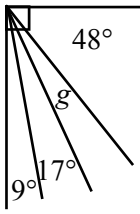

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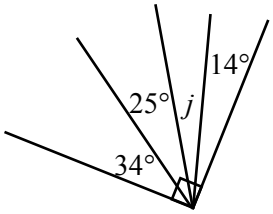
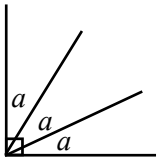


Use division to find the value of the letters.

17



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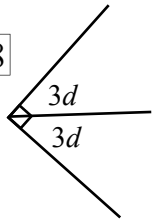

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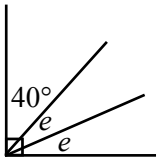

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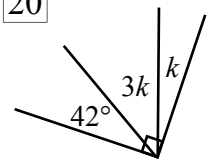

Complementary Angles

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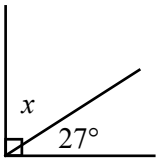
Find the complementary angle to those given in the table below.

1

Angle	Complement
30°	60°
45°	45°
73°	17°
21°	69°
9°	81°
57°	33°
84°	6°

Find the complementary angle for the following

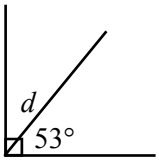
2



$x = 90° - 27°$

$x = 63°$

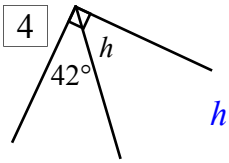
3



$d = 90° - 53°$

$d = 37°$

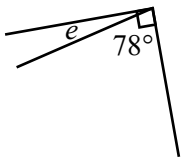
4



$h = 90° - 42°$

$h = 48°$

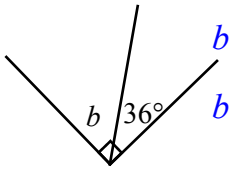
5



$e = 90° - 78°$

$e = 12°$

6

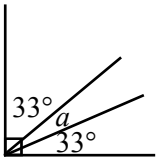


$b = 90° - 36°$

$b = 54°$

Now there are more than 2 angles. Solve these.

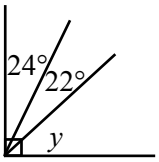
7



$a = 90° - 33° - 33°$

$a = 24°$

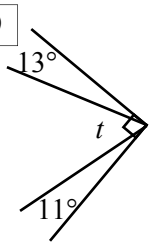
8



$y = 90° - 24° - 22°$

$y = 44°$

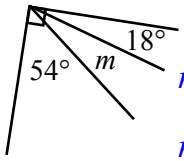
9



$t = 90° - 11° - 13°$

$t = 66°$

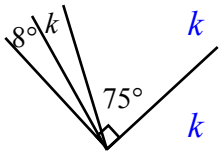
10



$m = 90° - 54° - 18°$

$m = 18°$

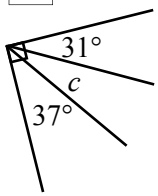
11



$k = 90° - 8° - 75°$

$k = 7°$

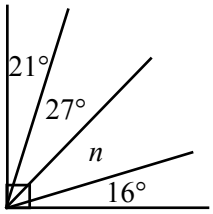
12



$c = 90° - 37° - 31°$

$c = 22°$

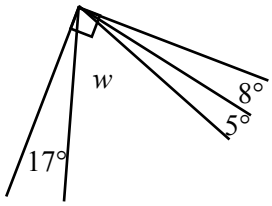
13



$n = 90° - 21° - 27° - 16°$

$n = 26°$

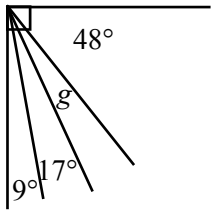
14



$w = 90° - 17° - 8° - 5°$

$w = 60°$

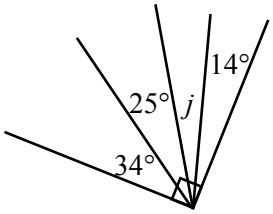
15



$g = 90° - 9° - 17° - 48°$

$g = 16°$

16

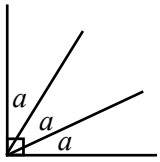


$j = 90° - 34° - 25° - 14°$

$j = 17°$

Use division to find the value of these unknowns.

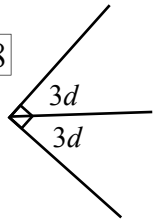
17



$3a = 90°$

$a = 30°$

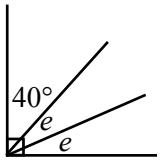
18



$6d = 90°$

$d = 15°$

19

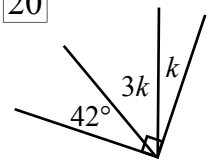


$2e = 90° - 40°$

$2e = 50°$

$e = 25°$

20



$4k = 90° - 42°$

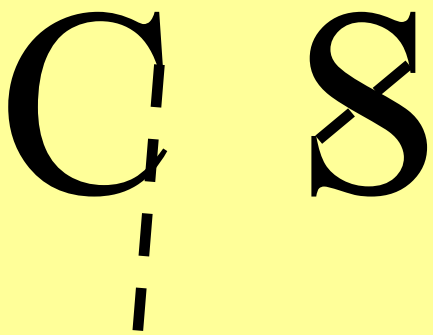
$4k = 48°$

$k = 12°$

Supplementary Angles

Supplementary angles are angles that add to 180°. How do you remember the difference between supplementary and complementary? C comes before S in the alphabet, just as 90° comes before 180°. Or look at the construction below.

The C can be changed to a 9 (90°) and the S can be changed to an 8 to remember 180°



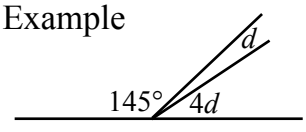
In column 1 you are asked to find the supplement, this means find the angle that when added to the angle given, has a total of 180°. You do this by subtracting the given angle from 180°. This continues down the column, subtract the angle given on the diagram from 180° to get the answer.

The 2nd column has a table to complete, this time both the complementary and supplementary angles are required. Subtract the angle from 180° to get the supplement and from 90° for the complement. Questions 9 through 16 add more angles into the problem, but the problem is still solved the same way, by subtracting all the given angles from 180°.

Questions 17 and 19 requires you to add the letters together, these equal 180°. Then divide by the number in front of the letter to get the value of the pronumeral. Note that the value of the pronumeral is being found which isn't necessarily the value of the angle (see question 20).

Question 18 is a more difficult question and requires an extra step. The letters are added on the left of the equals sign, and the subtraction of the angle from 90° is put on the right side. See the example below.

Example



$5d = 180° - 145°$

$5d = 35°$

$d = 7°$

Divide by 5

Supplementary Angles

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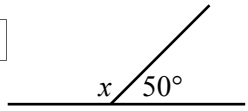
Find the supplementary angle to those given in the table below.

1

Angle	Supplement
110°	70°
45°	
162°	
16°	
123°	
27°	
177°	

Find the supplementary angle for the following

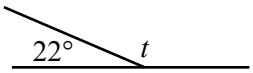
2



x = 180° - ____

x = ____

3



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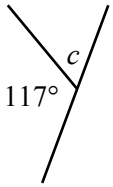

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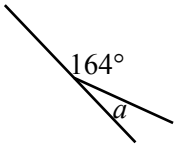

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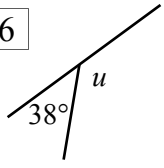

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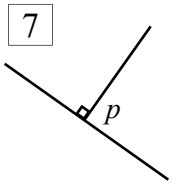


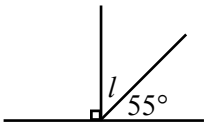
For the angles below find both the supplement and the complement.

8

Angle	Supplement	Complement
25°		
65°		
13°		
79°		
57°		
3°		
41°		

Now there are more than 2 angles. Solve these.

9



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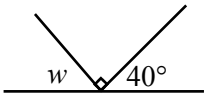

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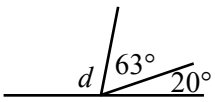

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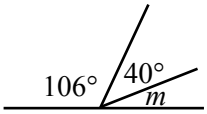

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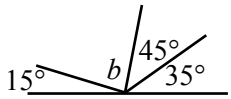

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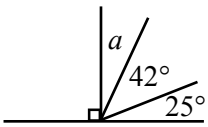

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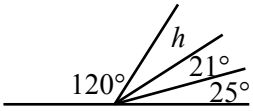

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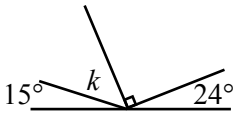
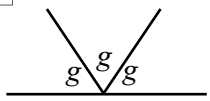


Use division to find the value of these unknowns.

17



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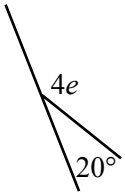

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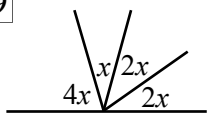

20 With the value of x found above, find the value of:

$2x$ = ____ $4x$ = ____

Supplementary Angles

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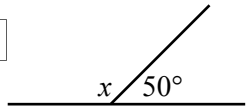
Find the supplementary angle to those given in the table below.

1

Angle	Supplement
110°	70°
45°	135°
162°	18°
16°	164°
123°	57°
27°	153°
177°	3°

Find the supplementary angle for the following

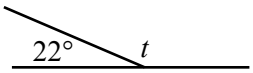
2



x = 180° - 50°

x = 130°

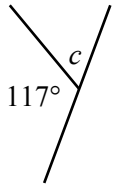
3



t = 180° - 22°

t = 158°

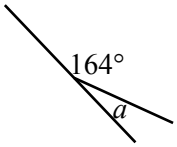
4



c = 180° - 117°

c = 63°

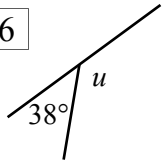
5



a = 180° - 164°

a = 16°

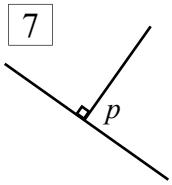
6



u = 180° - 38°

u = 142°

7



p = 180° - 90°

p = 90°

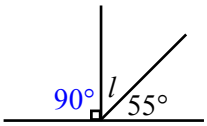
For the angles below find both the supplement and the complement.

8

Angle	Supplement	Complement
25°	155°	65°
65°	115°	25°
13°	167°	77°
79°	101°	11°
57°	123°	33°
3°	177°	87°
41°	139°	49°

Now there are more than 2 angles. Solve these.

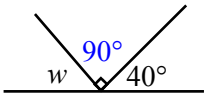
9



l = 180° - 90° - 55°

l = 35°

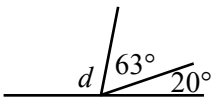
10



w = 180° - 90° - 40°

w = 50°

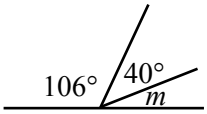
11



d = 180° - 63° - 20°

d = 97°

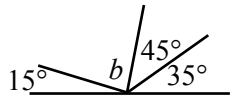
12



m = 180° - 106° - 40°

m = 34°

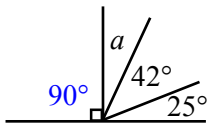
13



b = 180° - 15° - 45° - 35°

b = 85°

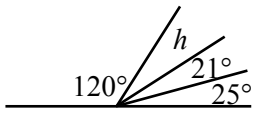
14



a = 180° - 90° - 42° - 25°

a = 23°

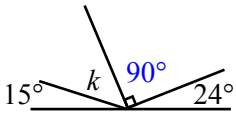
15



h = 180° - 120° - 21° - 25°

h = 14°

16

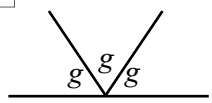


k = 180° - 15° - 90° - 24°

k = 51°

Use division to find the value of these unknowns.

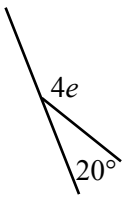
17



$3g$ = 180°

g = 60°

18

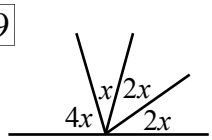


$4e$ = 180° - 20°

$4e$ = 160°

e = 40°

19



$9x$ = 180°

x = 20°

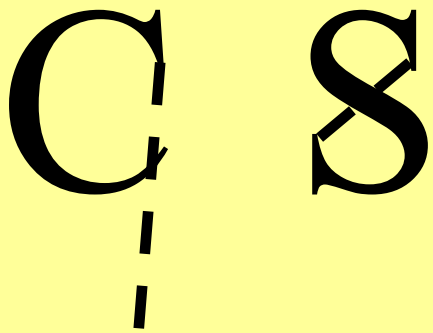
20 With the value of x found above, find the value of:

$2x$ = 40° $4x$ = 80°

Complementary and Supplementary Angles

Complementary angles are angles that add to 90°, and supplementary angles are angles that add to 180°. How do you remember which is which? C comes before S in the alphabet, as 90° comes before 180°. Or look at the construction below.

The C can be changed to a 9 (90°) and the S can be changed to an 8 to remember 180°



In column 1 the first 3 questions ask you to verify that the two angles are complementary. Add the two angles together, if the sum equals 90° then yes they are.

For the rest of column 1 you have to find the angle that when added to the angle given, has a total of 90°. You answer this by subtracting the given angle from 90°. The most common mistake is thinking that a right angle is 100°, or at least using it in calculations, remember 90 not 100!

The 2nd column has the same layout as the first but instead of the angles adding to give 90° they should total 180°, to be supplementary. The remainder of the column requires you to subtract the given angle from 180° to get the answer. When a right angle is involved write in 90°, this process should always be done as the angle may be overlooked.

The 3rd column is a mixture of the first 2 columns.

Complementary and Supplementary Angles

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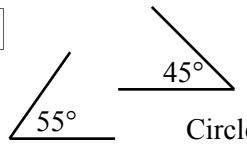
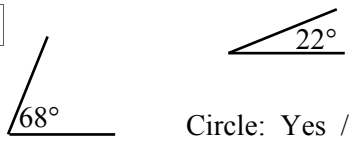
Are the following angles complementary?

1



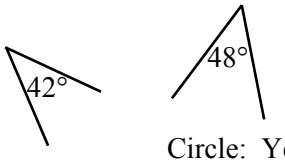
Circle: Yes / No

2



Circle: Yes / No

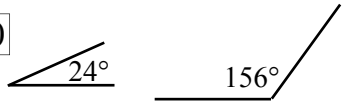
3



Circle: Yes / No

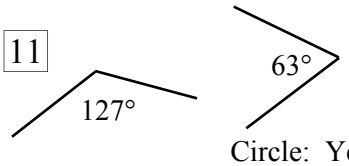
Are the following angles supplementary?

10



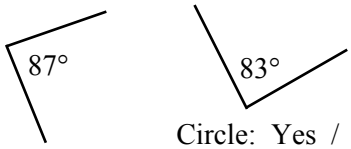
Circle: Yes / No

11



Circle: Yes / No

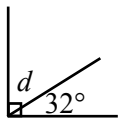
12



Circle: Yes / No

Find the unknown adjacent complementary angle

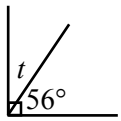
4



$d = 90° -$

$d =$

5



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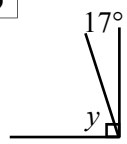

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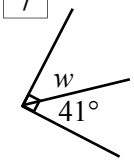

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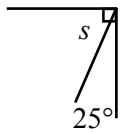

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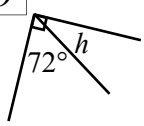
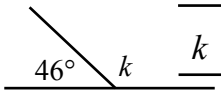


Find the unknown adjacent supplementary angle

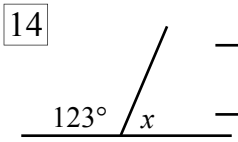
13



$k = 180° -$

$k =$

14



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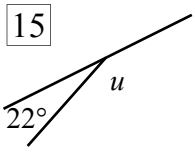

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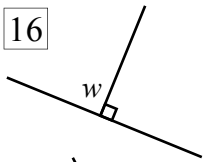

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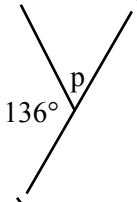

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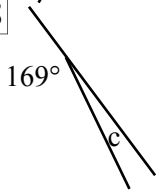

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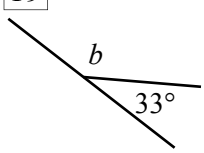
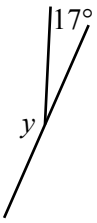


Find the missing angle

20



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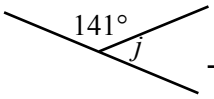

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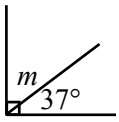

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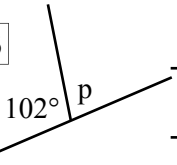

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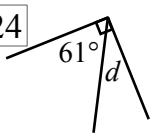

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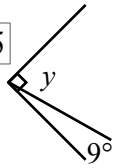

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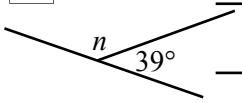
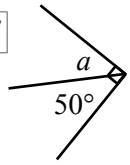


27



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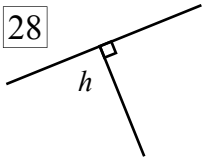

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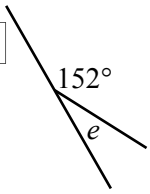

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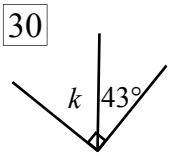

Complementary and Supplementary Angles

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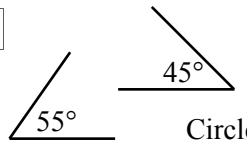
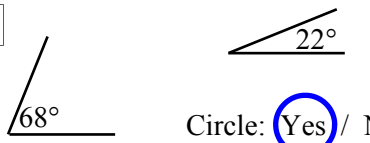
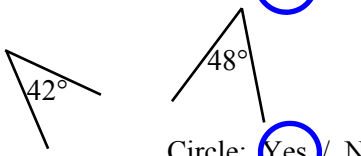
Are the following angles complementary?

1 

Circle: Yes / **No**

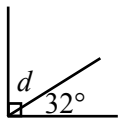
2 

Circle: **Yes** / No

3 

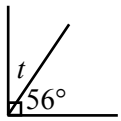
Circle: **Yes** / No

Find the unknown adjacent complementary angle

4 

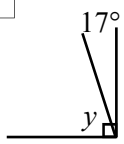
$d = 90° - 32°$

$d = 58°$

5 

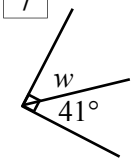
$t = 90° - 56°$

$t = 34°$

6 

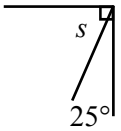
$y = 90° - 17°$

$y = 73°$

7 

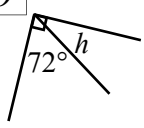
$w = 90° - 41°$

$w = 49°$

8 

$s = 90° - 25°$

$s = 65°$

9 

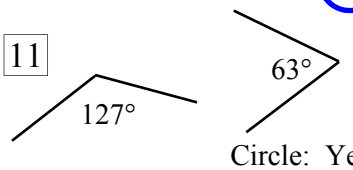
$h = 90° - 72°$

$h = 18°$

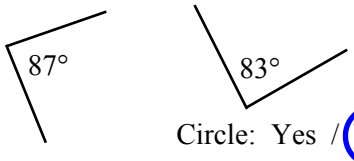
Are the following angles supplementary?

10 

Circle: **Yes** / No

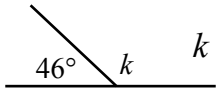
11 

Circle: Yes / **No**

12 

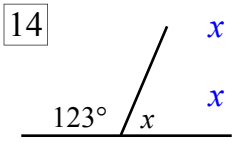
Circle: Yes / **No**

Find the unknown adjacent supplementary angle

13 

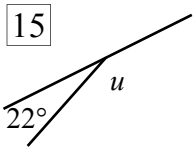
$k = 180° - 46°$

$k = 134°$

14 

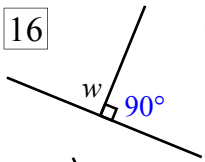
$x = 180° - 123°$

$x = 57°$

15 

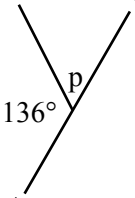
$u = 180° - 22°$

$u = 158°$

16 

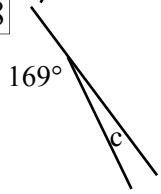
$w = 180° - 90°$

$w = 90°$

17 

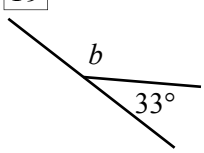
$p = 180° - 136°$

$p = 44°$

18 

$c = 180° - 169°$

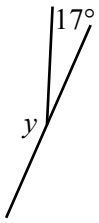
$c = 11°$

19 

$b = 180° - 33°$

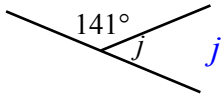
$b = 147°$

Find the missing angle

20 

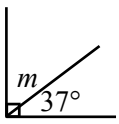
$y = 180° - 17°$

$y = 163°$

21 

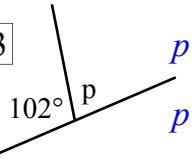
$j = 180° - 141°$

$j = 39°$

22 

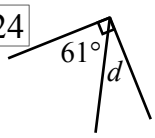
$m = 90° - 37°$

$m = 53°$

23 

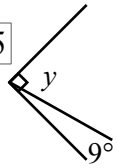
$p = 180° - 102°$

$p = 78°$

24 

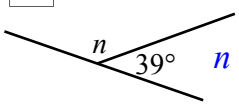
$d = 90° - 61°$

$d = 29°$

25 

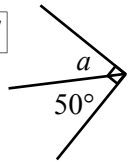
$y = 90° - 9°$

$y = 81°$

26 

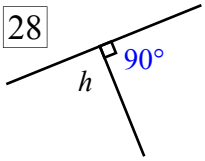
$n = 180° - 139°$

$n = 141°$

27 

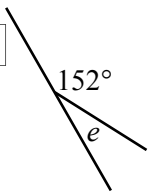
$a = 90° - 50°$

$a = 40°$

28 

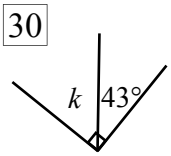
$h = 180° - 90°$

$h = 90°$

29 

$e = 180° - 152°$

$e = 28°$

30 

$k = 90° - 43°$

$k = 47°$

Angles at a Point

Just like a circle, the angle sum at a point is 360°. To find the size of an unknown angle subtract all the given angles from 360°.

In column 1 the unknown angle is given a letter, just write the letter then '=' and subtract all the given angles from 360°. Some angles are right angles, write '90°' in these so that you don't forget to include the angle in your calculation.

Column 2 requires you to use your algebra skills, add the letters and then divide 360° by the number in front of the letter. Questions 13 through 15 have an additional step, use the same method to find the value of the letter, but then multiply that value by the number in front of the letter for each angle. For example if you find $x = 25°$ and the angle in the question is $3x$, then $3x = 3 \times 25° = 75°$. If the question asks 'to solve for x', then you don't need to do this, it is only when you are asked for the angle, as the angle is $3x$ you must find the size of the angle, as the size of x isn't sufficient.

Column 3 is a harder column and some students may experience difficulty with these. The steps are a combination of the earlier problems.

- add the letters together and write it then an equals, (in question 16. $e + e = 2e$)
- then subtract the given angles (remember the right angles) from 360°
- the next line write the letters sum again only evaluate 360° - the angle
- then use division to solve like earlier problems
- the last 2 questions are harder, they require multiplication to get the angle (like questions 13 though 15)

Angles at a Point

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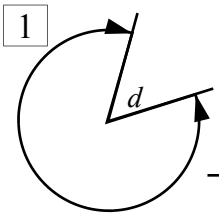
Use subtraction from 360° to find the unknown angle

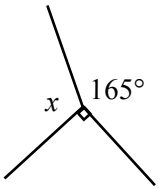
1



d = 360° -

d =

2



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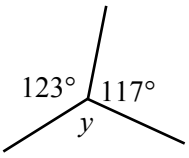

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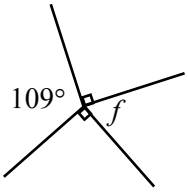

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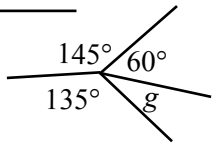

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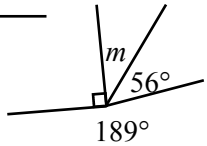
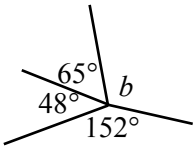
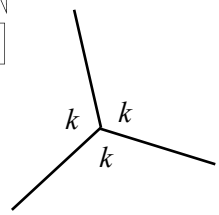


7



This time use division

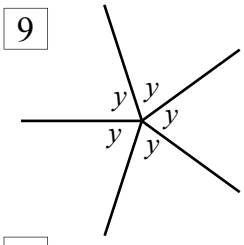
8



$3k$ =

k =

9



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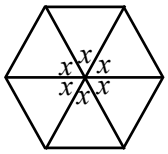

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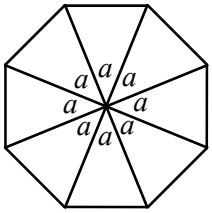
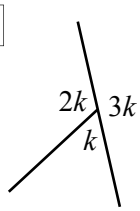


Use the same method but this time find the value of the letter and the angles

12



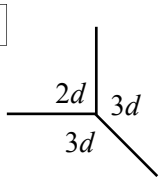
$6k$ =

k =

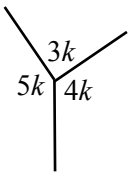
$2k$ =

$3k$ =

13



14



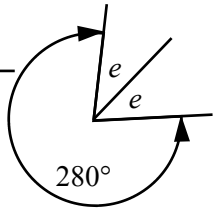
Use subtraction from 360° then division to find the unknown angles

15

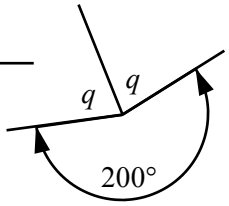
$2e$ = 360° -

$2e$ =

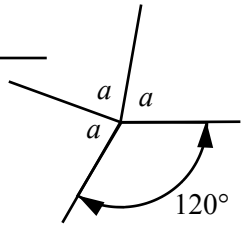
e =



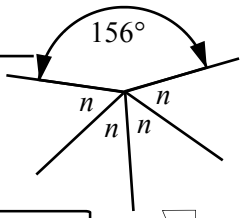
16



17



19

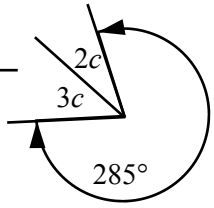


Now the angles are different

20

$5c$ =

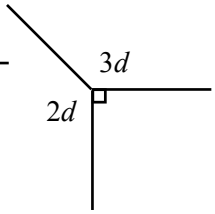
$5c$ =



$2c$ =

$3c$ =

21



Angles at a Point

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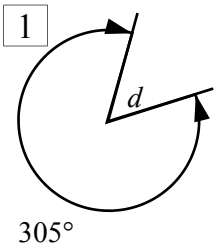
Use subtraction from 360° to find the unknown angle

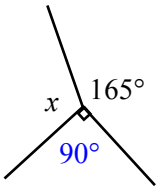
1



$d = 360° - 305°$

$d = 55°$

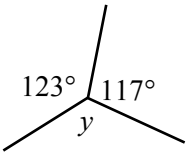
2



$x = 360° - 90° - 165°$

$x = 105°$

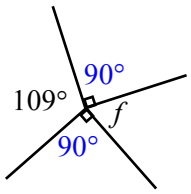
3



$y = 360° - 123° - 117°$

$y = 120°$

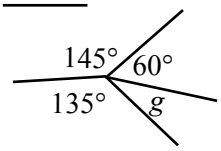
4



$f = 360° - 109° - 90° - 90°$

$f = 71°$

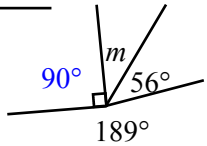
5



$g = 360° - 135° - 145° - 60°$

$g = 20°$

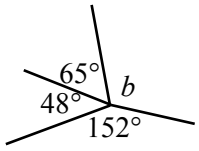
6



$m = 360° - 189° - 90° - 56°$

$m = 25°$

7

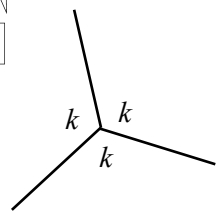


$b = 360° - 152° - 48° - 65°$

$b = 95°$

This time use division

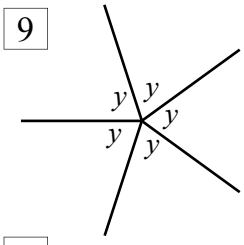
8



$3k = 360°$

$k = 120°$

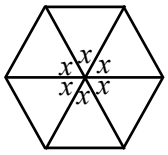
9



$5y = 360°$

$y = 72°$

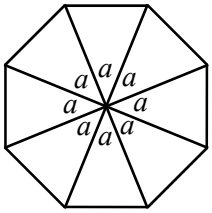
10



$6x = 360°$

$x = 60°$

11

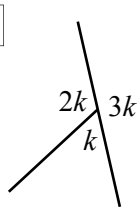


$8a = 360°$

$a = 45°$

Use the same method but this time find the value of the letter and the angles

12

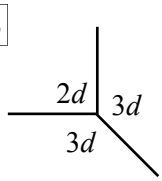


$6k = 360°$

$k = 60°$

$2k = 120°$ $3k = 180°$

13

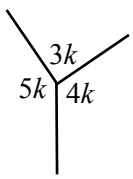


$8d = 360°$

$d = 45°$

$2d = 90°$ $3d = 135°$

14



$12k = 360°$

$k = 30°$

$3k = 90°$ $4k = 120°$

$5k = 150°$

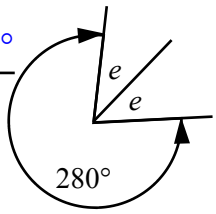
Use subtraction from 360° then division to find the unknown angles

15

$2e = 360° - 280°$

$2e = 80°$

$e = 40°$

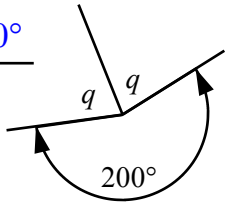


16

$2q = 360° - 200°$

$2q = 160°$

$q = 80°$

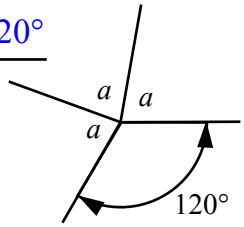


17

$3a = 360° - 120°$

$3a = 240°$

$a = 80°$

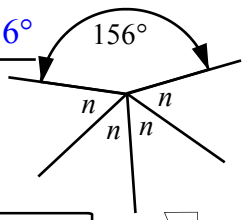


18

$4n = 360° - 156°$

$4n = 204°$

$n = 51°$



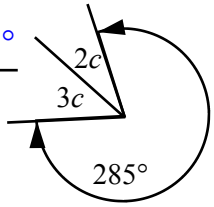
Now the angles are different

19

$5c = 360° - 285°$

$5c = 75°$

$c = 15°$



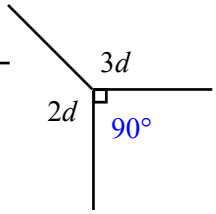
$2c = 30°$ $3c = 45°$

20

$5d = 360° - 90°$

$5d = 270°$

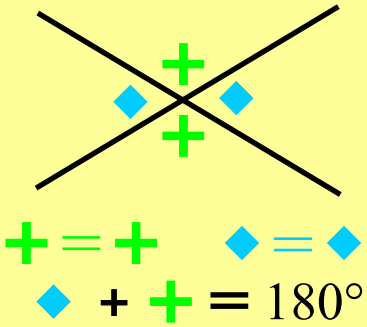
$d = 54°$



$2d = 108°$ $3d = 162°$

Vertically Opposite Angles

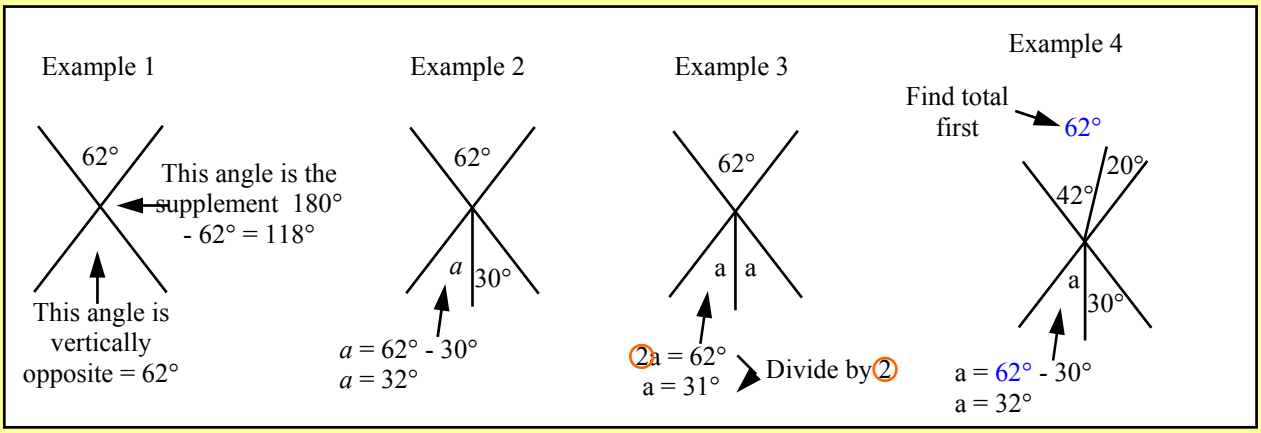
Vertically opposite angles are equal when formed by intersecting straight lines. The diagram below illustrates this. Don't let the word 'vertical' confuse you, as you can see if angles are opposite each other horizontally they are also considered vertically opposite (equal).



In column 1 identify the vertically opposite angle, by drawing a star in the angle and then also by naming it. Write the name of the angle on the line and colour the star beside it the same colour as the one you placed on the diagram. Question 5 is more difficult as many angles are all on the one diagram so take care with this question.

Column 2 asks you to find the value of the letter, this is just identification, (the answer is in front of you) for questions 6 through 9. But from Question 10 on…. some mathematics is required, this will always be subtraction or division look at the examples below.

With column 3 questions 15 and 16 use the same method as the example above them, establishing 2 angles from identification then the 3rd by supplementary angle methods, look at the example at the top of the column. The last 2 questions are more difficult and require you to choose vertically opposite or supplementary methods to solve them.



Vertically Opposite Angles

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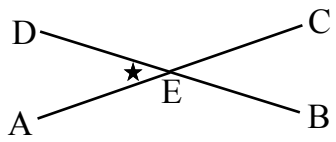
Name and label with a star the vertically opposite angle to the one marked with a star

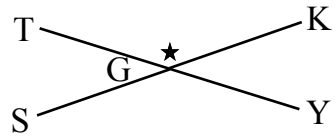
1



Colour this star the same as the one you put in the angle

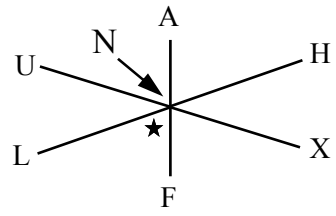
☆ ∠ _____

2



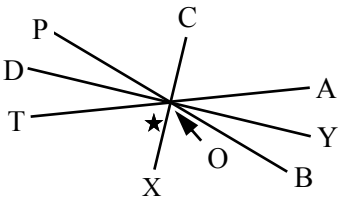
☆ _____

3



☆ _____

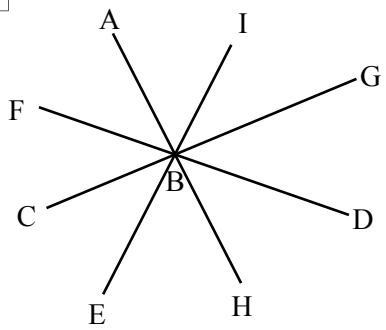
4



☆ _____

Name the vertically opposite angle to the following angles

5



∠ IBG — vertically opposite → _____

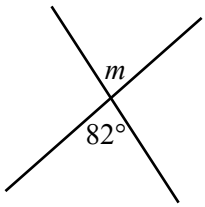
∠ HBD → _____

∠ ABC → _____

∠ CBD → _____

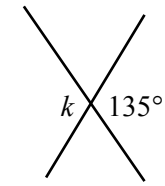
Find the value of the letters below

6



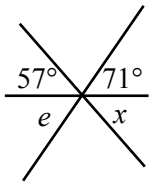
m = _____

7



_____

8



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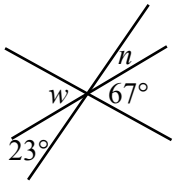
9



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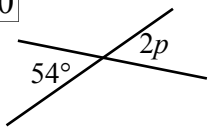
10



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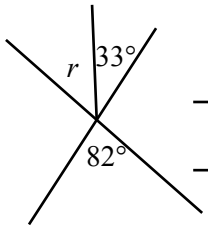
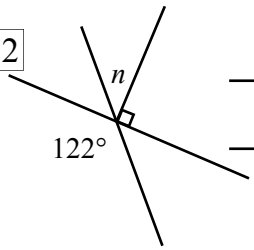
11



_____

_____

12



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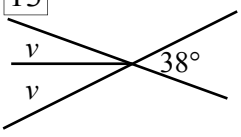
13



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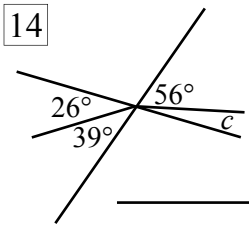
14



_____

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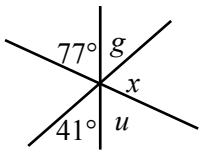
Use vertically opposite and supplementary angle properties to find the value of these letters

Example



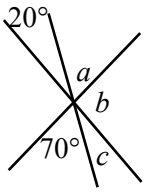
$g = 41°$

$u = 77°$

$x = 180° - 41° - 77°$

$x = 62°$

15



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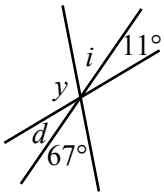
16



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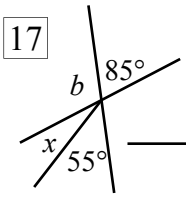
17



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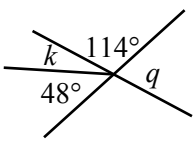
18



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Vertically Opposite Angles

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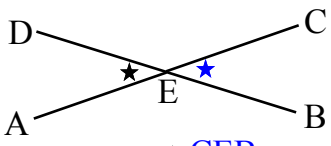
Name and label with a star the vertically opposite angle to the one marked with a star

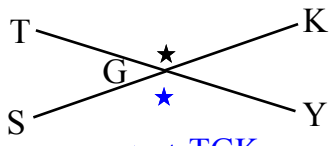
1



Colour this star the same as the one you put in the angle

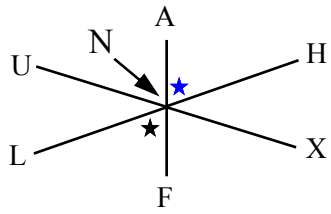
★ ∠ CEB

2



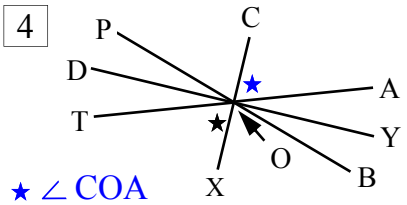
★ ∠ TGK

3



★ ∠ ANH

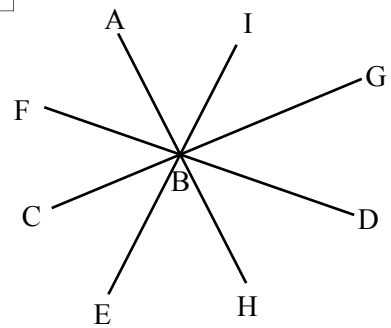
4



★ ∠ COA

Name the vertically opposite angle to the following angles

5



∠ IBG → (vertically opposite) ∠ CBE

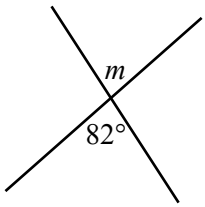
∠ HBD → ∠ ABF

∠ ABC → ∠ GBH

∠ CBD → ∠ FBG

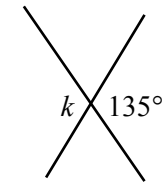
Find the value of the letters below

6



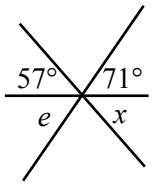
$m = 82°$

7



$k = 135°$

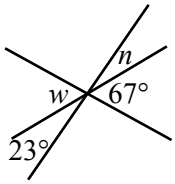
8



$e = 71°$

$x = 57°$

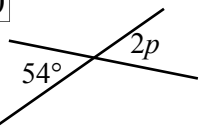
9



$n = 23°$

$w = 67°$

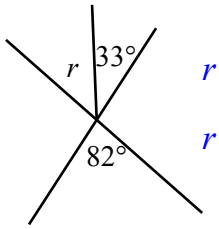
10



$2p = 54°$

$p = 27°$

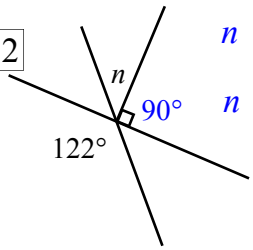
11



$r = 82° - 33°$

$r = 49°$

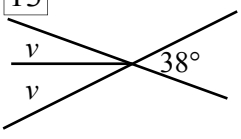
12



$n = 122° - 90°$

$n = 32°$

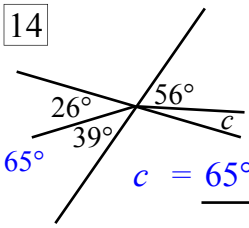
13



$2v = 38°$

$v = 19°$

14

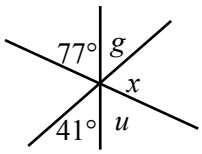


$c = 65° - 56°$

$c = 9°$

Use vertically opposite and supplementary angle properties to find the value of these letters

Example



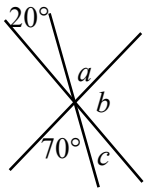
$g = 41°$

$u = 77°$

$x = 180° - 41° - 77°$

$x = 62°$

15



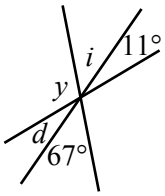
$a = 70°$

$c = 20°$

$b = 180° - 70° - 20°$

$b = 90°$

16



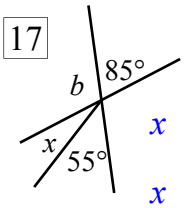
$d = 11°$

$i = 67°$

$y = 180° - 11° - 67°$

$y = 102°$

17



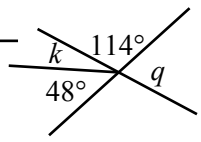
$x = 85° - 55$

$x = 30°$

$b = 180° - 55° - 30°$

$b = 95°$

18



$k = 180° - 114° - 48°$

$k = 18°$

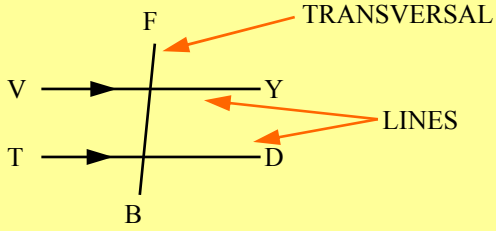
$q = 48° + 18°$

$q = 66°$

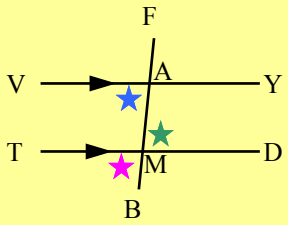
Parallel Lines and the Transversal

When parallel lines are crossed by a line the angles formed are related to each other by properties. This will require you to remember certain words and their meaning.

The first word is 'transversal' this is the name of the line that cuts across parallel lines. In column 1 you are asked to name the transversal and the parallel lines, write the 2 letters that represent the line, this can be done either way as the example below shows. The naming of angles uses the 3 point method, there is 4 possible answers for some questions as the example below shows, realise that if the answer doesn't match your answer you still may be correct, but the middle letter (vertex) must always be the same.



Lines are VY or YV and TD or DT
Transversal is either FB or BF



★ ∠VAM or ∠VAB or ∠MAV or ∠BAV

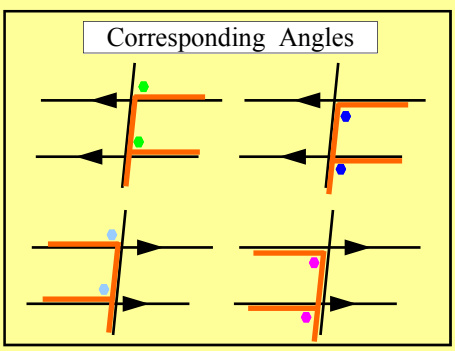
★ ∠AMD or ∠FMD or ∠DMA or ∠DMF

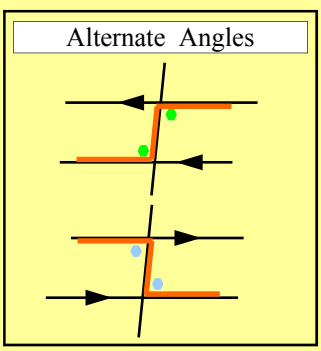
★ ∠TMB or ∠BMT

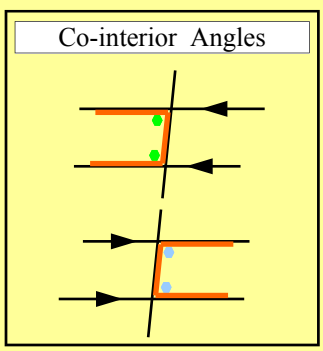
When lines cross, pairs of angles are made. Co-interior angles (called C angles) are the pair of angles formed on one side of the transversal inside the parallel lines, a 'c' can be formed around the angles (one is back to front). There are only two co-interior angle pairs in a standard 2-line 1 transversal problem. Remember the word interior means on the inside (of the parallel lines and the transversal).

Alternate angles (called Z angles) are also on the inside of the parallel lines but unlike co-interior angles they are on opposite sides of the transversal. A 'Z' can be made by the lines that include these angles.

Both alternate and co-interior angles are inside the two parallel lines. Corresponding angles (called F angles) require one point to be on the inside of the parallel lines and the other to be on the outside. An 'F ' or back to front ‘F’ is made by the lines that include these angles. With corresponding angles the angle is in exactly the same position on both points of intersection. Column 3 asks you to write either 'alternate', 'co-interior' or 'corresponding'. Questions 32 through 37 are more challenging but the same method is used.







Parallel Lines and the Transversal

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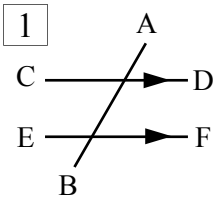
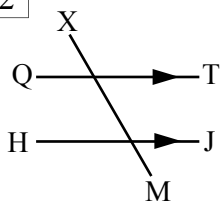
Using the letters name the parallel lines (L) and the transversal (T)

1



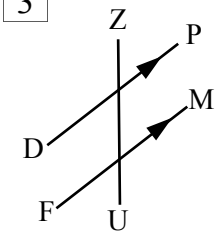
L: _____
L: _____
T: _____

2



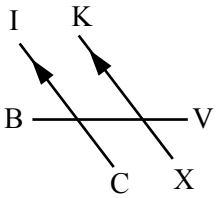
L: _____
L: _____
T: _____

3



L: _____
L: _____
T: _____

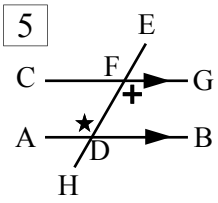
4



L: _____
L: _____
T: _____

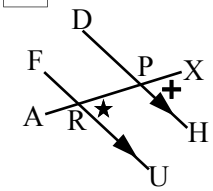
Using the assigned letters name the 2 angles with symbols

5



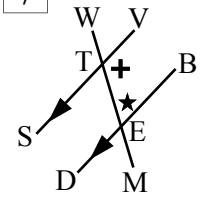
★ _____
+ _____

6



★ _____
+ _____

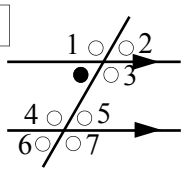
7



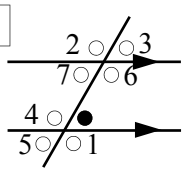
★ _____
+ _____

Show the co-interior angle by colouring the circle (the numbers are for marking purposes)

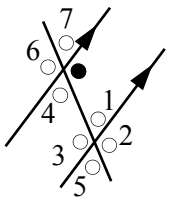
8



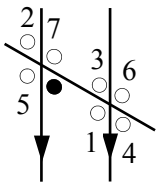
9



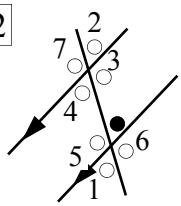
10



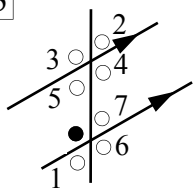
11



12

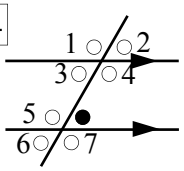


13

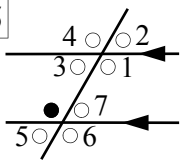


Now colour the alternate angle

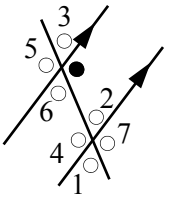
14



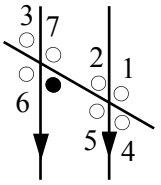
15



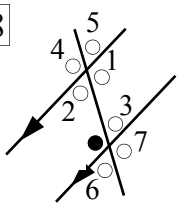
16



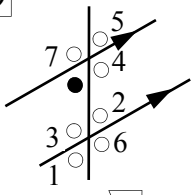
17



18

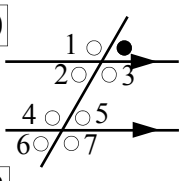


19

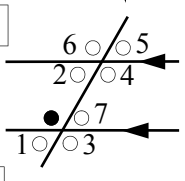


Now colour the corresponding angle

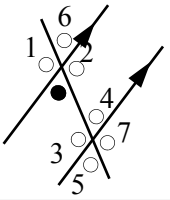
20



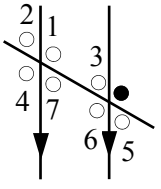
21



22

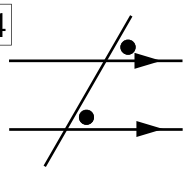


23



State if these angles are corresponding, alternate or co-interior.

24



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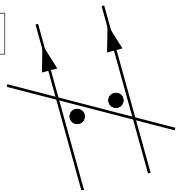
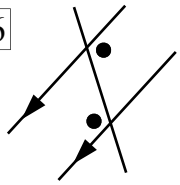
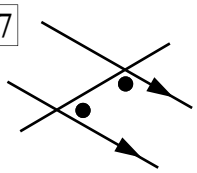


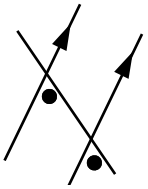
26



27



28



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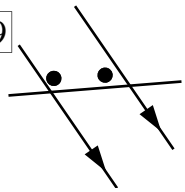

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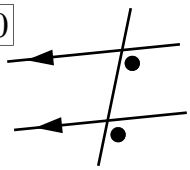

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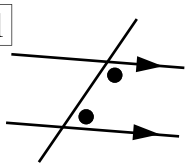

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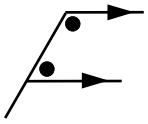

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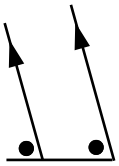

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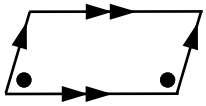

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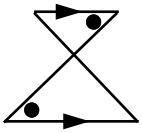

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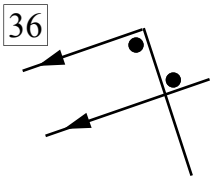

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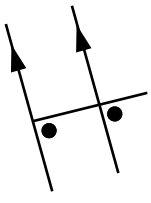

Parallel Lines and the Transversal

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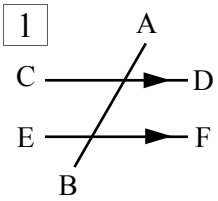
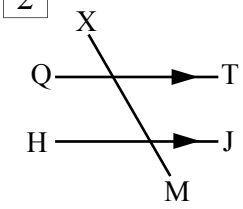
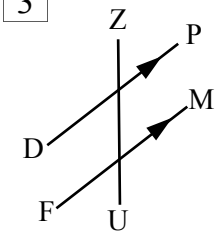
Using the letters name the parallel lines (L) and the transversal (T)

1 

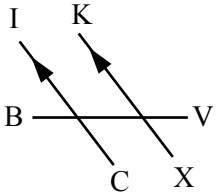
L: CD
L: EF
T: AB

2 

L: QT
L: HJ
T: XM

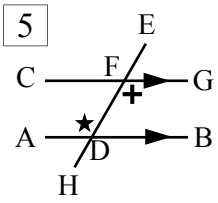
3 

L: DP
L: FM
T: ZU

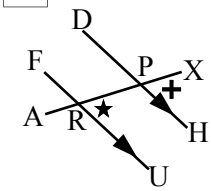
4 

L: IC
L: KX
T: BV

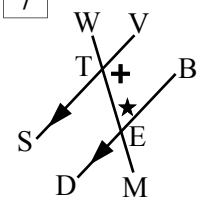
Using the assigned letters name the 2 angles with symbols

5 

★ ∠ ADF
+ ∠ DFG

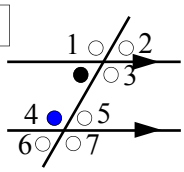
6 

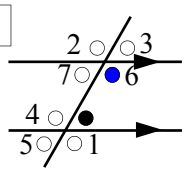
★ ∠ PRU
+ ∠ XPH

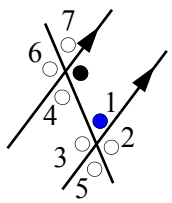
7 

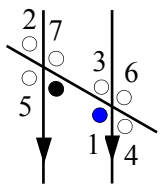
★ ∠ TEB
+ ∠ VTE

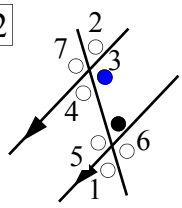
Show the co-interior angle by colouring the circle (the numbers are for marking purposes)

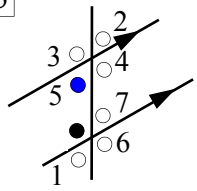
8 

9 

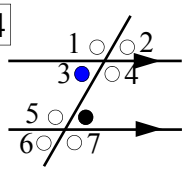
10 

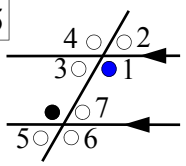
11 

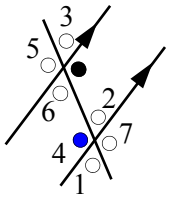
12 

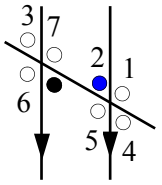
13 

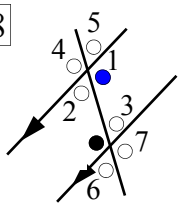
Now colour the alternate angle

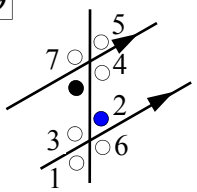
14 

15 

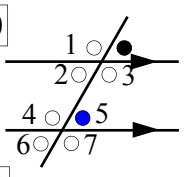
16 

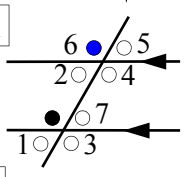
17 

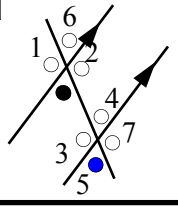
18 

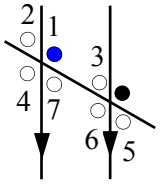
19 

Now colour the corresponding angle

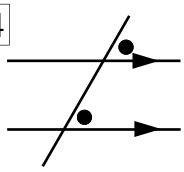
20 

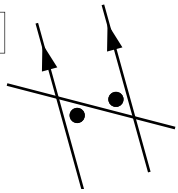
21 

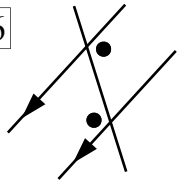
22 

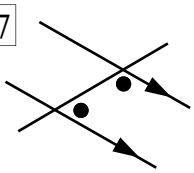
23 

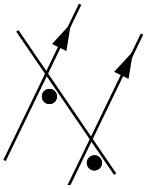
State if these angles are corresponding, alternate or co-interior.

24 
corresponding

25 
alternate

26 
alternate

27 
co-interior

28 
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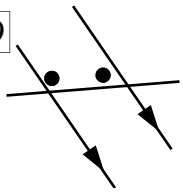
29 
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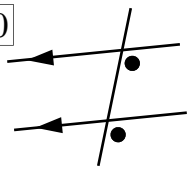
30 
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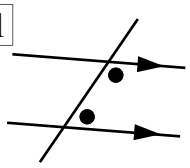
31 
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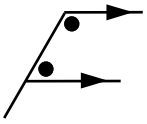
32 
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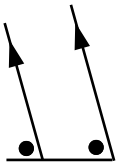
33 
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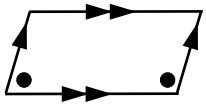
34 
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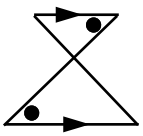
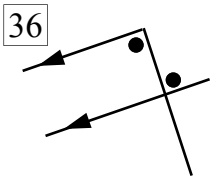
35 
alternate

36 
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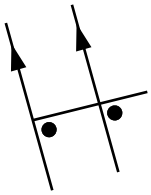
37 
corresponding

Parallel Line Angle Properties

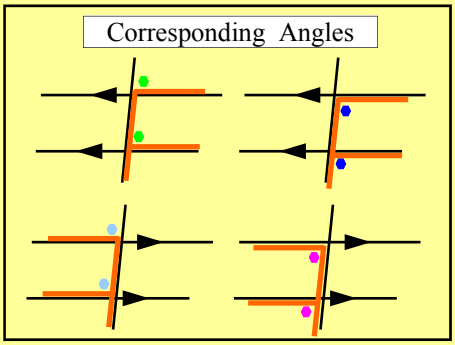
When two parallel lines are crossed by a transversal, pairs of angles are made that we can use mathematically. The previous sheet taught you alternate, co-interior and corresponding angles, now we can use their angle properties, which are:

- corresponding angles are equal if lines are parallel
- alternate angles are equal if lines are parallel
- co-interior angle sum is 180° if lines are parallel (the two angles add to 180°)

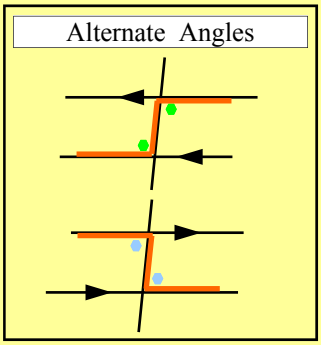
In Column 1 an angle is given and an unknown angle is required. The unknown angle will be either the alternate, corresponding or co-interior angle to the given angle. The first step is identifying which. Once identified if the angle is the alternate angle or the corresponding angle it is the same as the given angle, no working required. If the relationship is co-interior then subtract the given angle from 180° to obtain the answer.

Column 2 gives both angles, identify if the relationship between the 2 angles is either alternate or corresponding. If it is, the angles must be the same for the lines to be parallel, if the angles aren't equal then the lines aren't parallel. If the relationship between the 2 angles is co-interior then the sum of the two angles must equal 180° for the lines to be parallel. If they don't add to 180º then the lines aren't parallel. This includes if the angles are the same, the same co-interior angle means the lines aren't parallel, unless the angles are 90°. See the example at the top of the column.

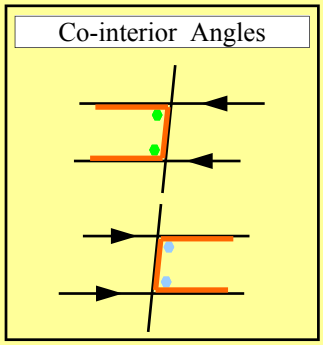
Column 3 is the same as column 1 in procedure, the only difference being that you have to divide the angle by the number in front of the letter to get your answer. The last 3 questions require a subtraction to take place before the division.



Corresponding angles are equal if the lines are parallel



Alternate angles are equal if the lines are parallel



Co-interior angles add to 180° if the lines are parallel

Parallel Lines Angle Properties

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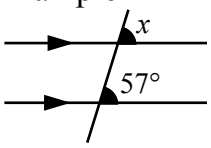
State if alternate, co-interior or corresponding. Then find the value of the letter.

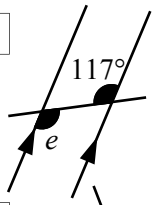
Example



corresponding

$x = 57°$

1



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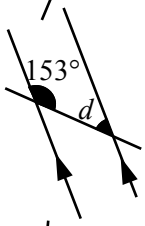
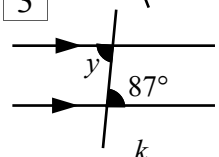


3



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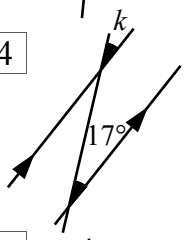

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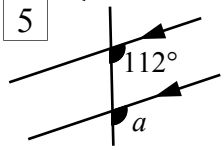

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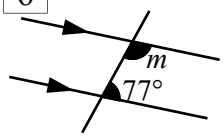

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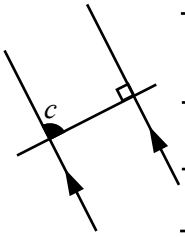

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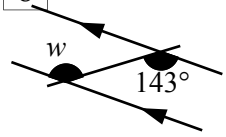

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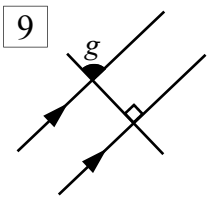
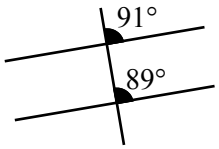


Find if the lines below are parallel, give a brief reason for your decision.

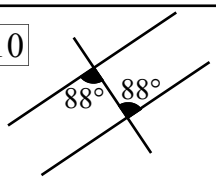
Example



Parallel?
Circle: Yes / No

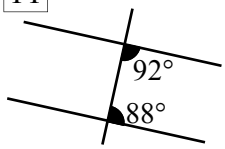
Corresponding angles not equal so lines aren't parallel.

10



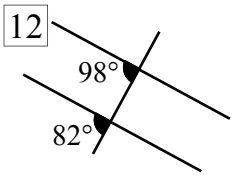
Parallel?
Circle: Yes / No

11



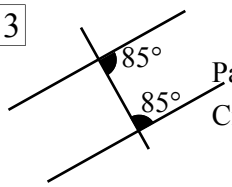
Parallel?
Circle: Yes / No

12



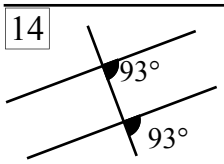
Parallel?
Circle: Yes / No

13



Parallel?
Circle: Yes / No

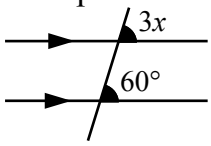
14



Parallel?
Circle: Yes / No

Find the value of x

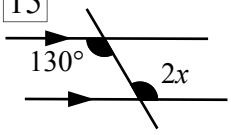
Example



$3x = 60°$

$x = 20°$

15



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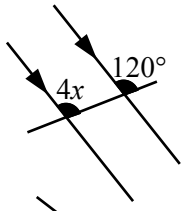

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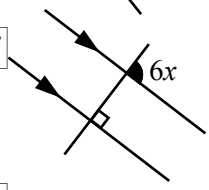

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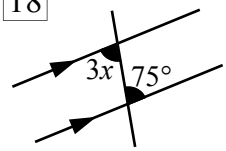

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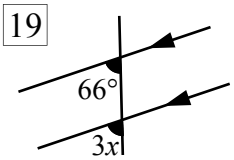
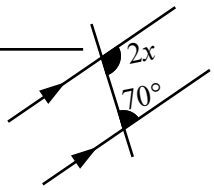


Co-interior angles require an extra step, find the value of x in these

20 $2x = 180° -$

$2x =$



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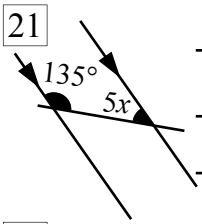

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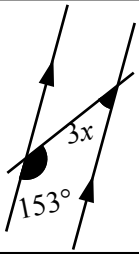

Parallel Lines Angle Properties

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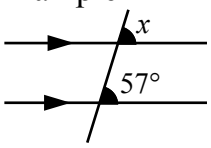
State if alternate, co-interior or corresponding. Then find the value of the letter.

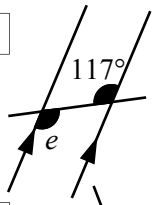
Example



corresponding

$x = 57°$

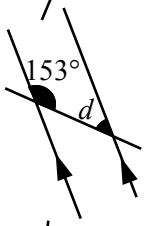
1



alternate

$e = 117°$

2

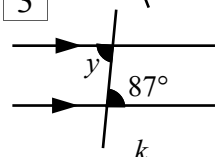


co-interior

$d = 180° - 153°$

$d = 27°$

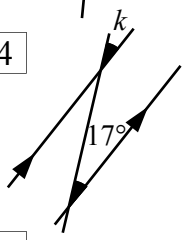
3



alternate

$y = 87°$

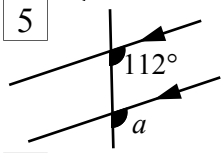
4



corresponding

$k = 17°$

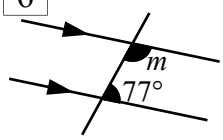
5



corresponding

$a = 112°$

6

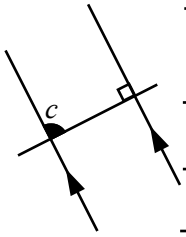


co-interior

$m = 180° - 77°$

$m = 103°$

7

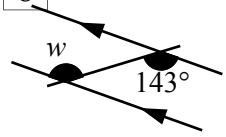


all 3 apply

$c = 180° - 90°$

$c = 90°$

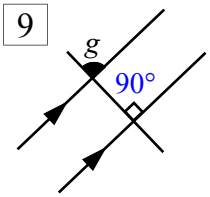
8



alternate

$w = 143°$

9

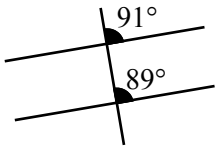


corresponding

$g = 90°$

Find if the lines below are parallel, give a brief reason for your decision.

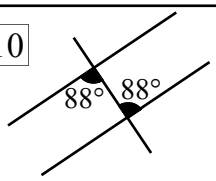
Example



Parallel?
Circle: Yes / No

Corresponding angles not equal so lines aren't parallel.

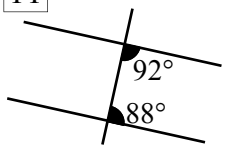
10



Parallel?
Circle: Yes / No

Alternate angles equal so lines are parallel.

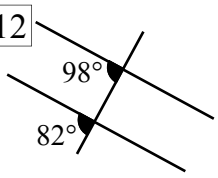
11



Parallel?
Circle: Yes / No

Co-interior angle sum is 180° so lines are parallel

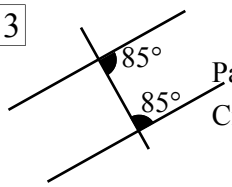
12



Parallel?
Circle: Yes / No

Corresponding angles not equal so lines aren't parallel.

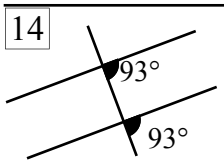
13



Parallel?
Circle: Yes / No

Co-interior angle sum isn't 180° so lines aren't parallel

14

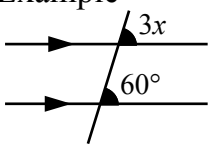


Parallel?
Circle: Yes / No

Corresponding angles equal so lines are parallel.

Find the value of x

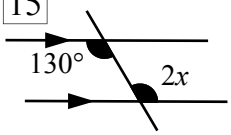
Example



$3x = 60°$

$x = 20°$

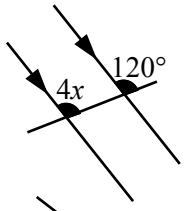
15



$2x = 130°$

$x = 65°$

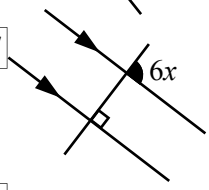
16



$4x = 120°$

$x = 30°$

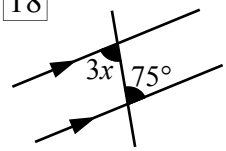
17



$6x = 90°$

$x = 15°$

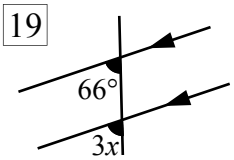
18



$3x = 75°$

$x = 25°$

19



$3x = 66°$

$x = 22°$

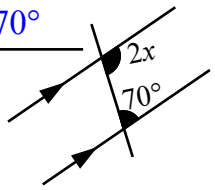
Co-interior angles require an extra step, find the value of x in these

20

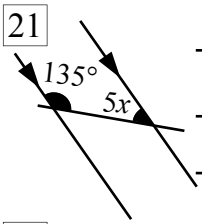
$2x = 180° - 70°$

$2x = 110°$

$x = 55°$



21



$5x = 180° - 135°$

$5x = 45°$

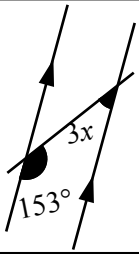
$x = 9°$

22

$3x = 180° - 153°$

$3x = 27°$

$x = 9°$



Further Parallel Lines

If two parallel lines are crossed by a transversal eight angles are formed. Given one angle, all of the other angles can be found by using these properties:

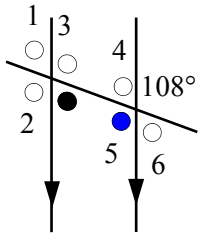
- supplementary, the angles adjacent (next to it) to the angle can be found by subtracting the given angle from 180°
- vertically opposite, the angle opposite the given angle is the same size as the angle
- then parallel line properties are used. Alternate, corresponding and co-interior can be used (it isn't necessary to use them all)

In column 1 a single angle is given, you are required to find the value of the black dotted angle in two solution 'moves'. The first step requires you to use supplementary or vertically opposite methods then for the second step use one of the parallel line properties.

The example below shows the angle given is 108°, vertically opposite was used to find 5 then co-interior to the black dotted angle. Note we could also use two other methods. Supplement to 4 then alternate to black dot or supplement to 6 and corresponding to black dot. Either way is correct. Note that the dots can be coloured, ensure that your colour choice is matched on the solution line.

Column 3 is an extension, this time the seven other angles are required. Use any method you like just make sure you give a reason, like the example below.

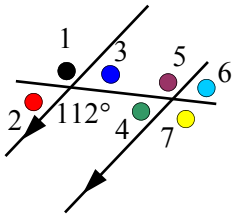
Example



● = 108° as vertically opposite

● = 72° as co-interior to ●

Example



● = 68° as supplement

● = 112° as vertically opposite

● = 68° as supplement

● = 68° as co-interior

● = 112° as alternate

● = 68° as supplement to ●

● = 112° as corresponding

Further Parallel Lines

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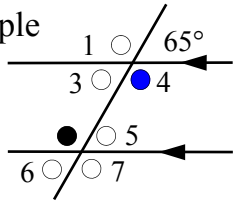
Find the vertically opposite or supplementary angle and use it to solve for the unknown angle, give a reason

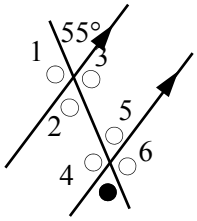
Example



● = 115° supplement to 65°

● = 115° alternate to •

1



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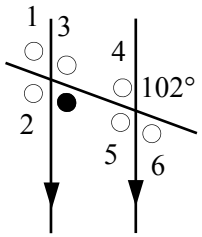

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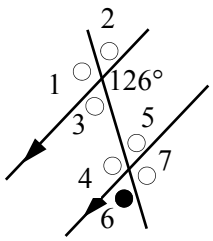

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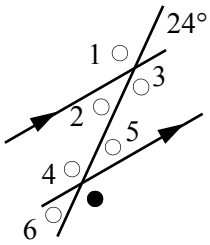

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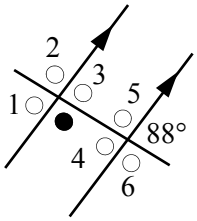

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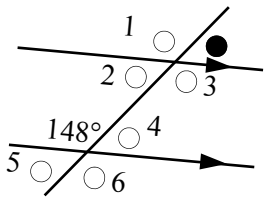

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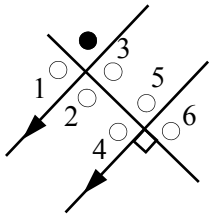

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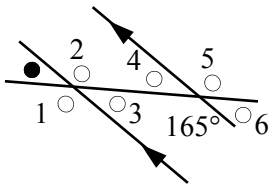

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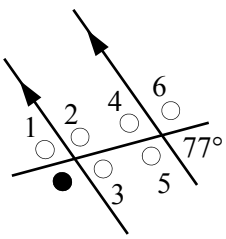
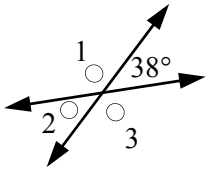


Now using the same skills find all the angles on the diagram, with reasons.

10



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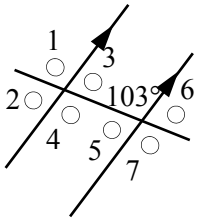

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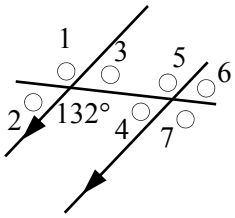

Further Parallel Lines

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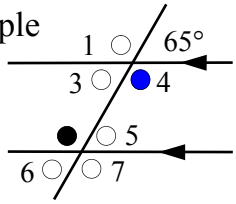
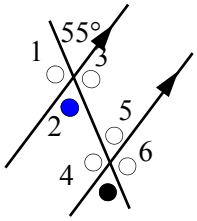
Find the vertically opposite or supplementary angle and use it to solve for the un-known angle, give a reason

Example



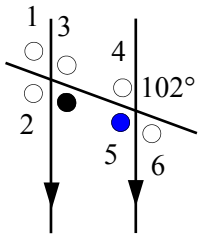
- (4) = 115° as supplement
- (4) = 115° as 4 alternate to •

1



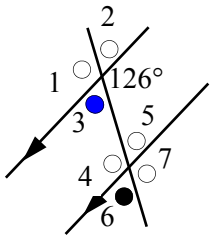
- (2)=55° as vertically opposite
- = 55° as 2 corresponds to •

2



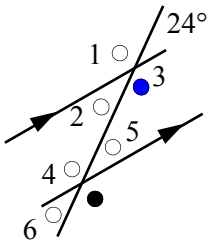
- (5)=102° as vertically opposite
- = 78° as 5 co-interior to •

3



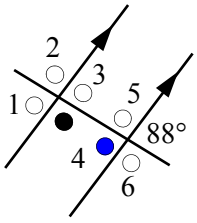
- (3) = 54° as supplement
- = 54° as corresponds to •

4



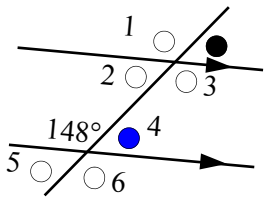
- (3) = 156° as supplement
- = 156° as corresponds to •

5



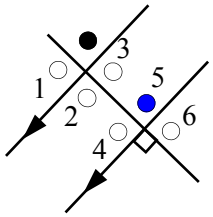
- (4)=92° as vertically opposite
- = 88° as co-interior to •

6



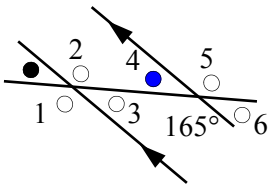
- (4) = 32° as supplement
- = 32° as corresponds to •

7



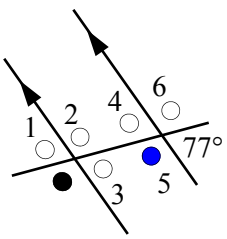
- (5)=90° as vertically opposite
- = 90° as corresponds to •

8



- (4) = 15° as supplement
- = 15° as corresponds to •

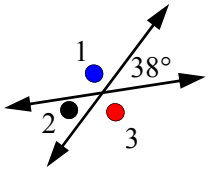
9



- (5) = 103° as supplement
- = 103° as corresponds to •

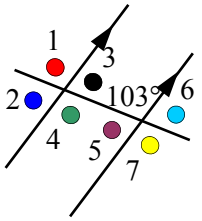
Now using the same skills find all the angles on the diagram, with reasons.

10



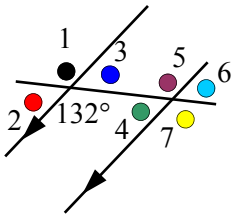
- (1)= 142° as supplement
- (2)=38° as vertically opposite
- (3)= 142° as supplement

11



- (3)= 77° as 5 co-interior
- (5)= 77° as supplement
- (4)= 103° as alternate
- (6)= 77° as supplement
- (7)= 103° as vertically opposite
- (1)= 103° as corresponding
- (2)= 77° as supplement to •

12



- (2)= 48° as supplement
- (1)= 132° as vertically opposite
- (3)= 48° as supplement
- (4)= 48° as co-interior
- (5)= 132° as alternate
- (6)= 48° as supplement to •
- (7)= 132° as corresponding

Measuring Angles in Triangles

This exercise is an introductory exercise to show that the angle sum of a triangle is 180°. Use a protractor to measure each angle in the triangles, note you should expect to experience some error in any measurement exercise, your total may be out by a degree.
Follow these steps:

- measure the angles with a protractor and then using the addition spaces in the bottom left corner add them and compare the total to 180°
- Repeat for all the triangles

Why are there errors? As the angles are not exactly a whole degree you may round them up or down, this will affect your total.

Measuring Angles in Triangles

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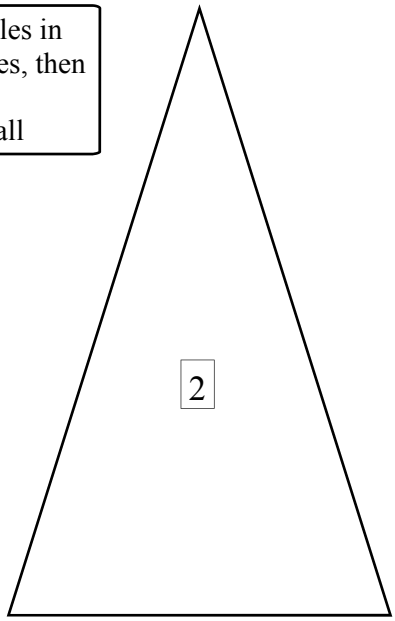
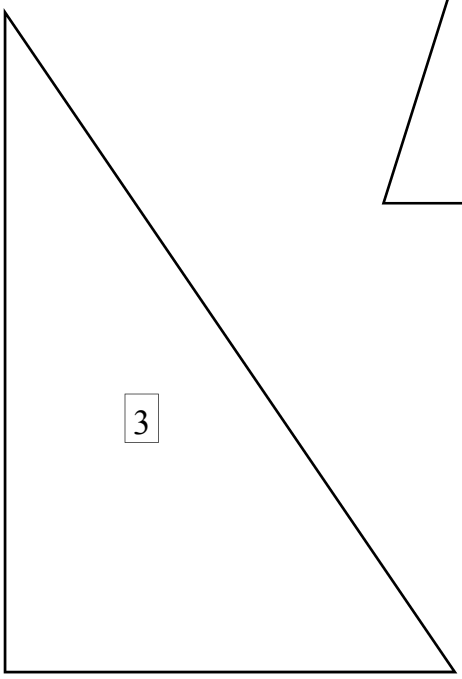
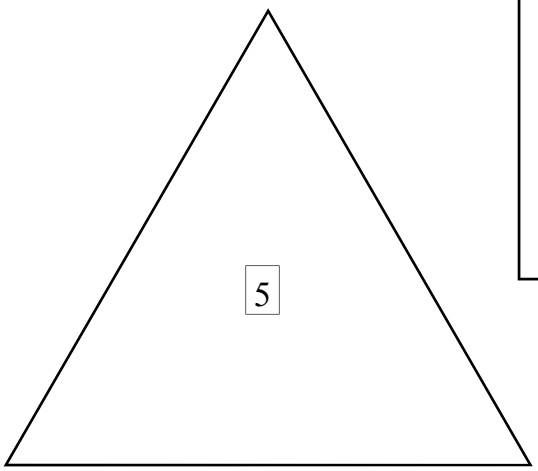
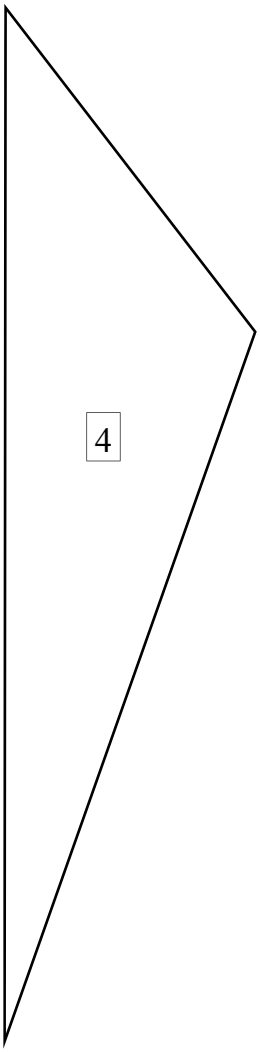
Measure the 3 angles in each of the triangles, then add them to get 180°,allow for small









1 + 2 + 3 +

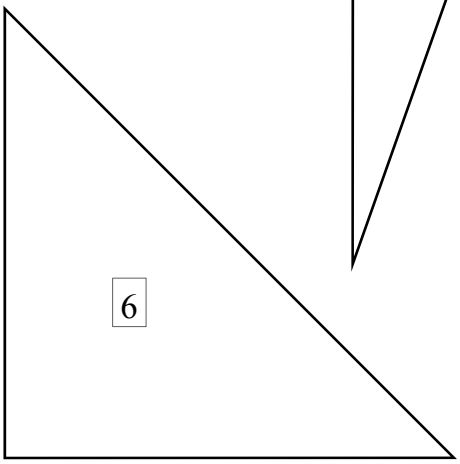
_____ _____ _____

_____ _____ _____

4 + 5 + 6 +

_____ _____ _____

_____ _____ _____



Measuring Angles in Triangles

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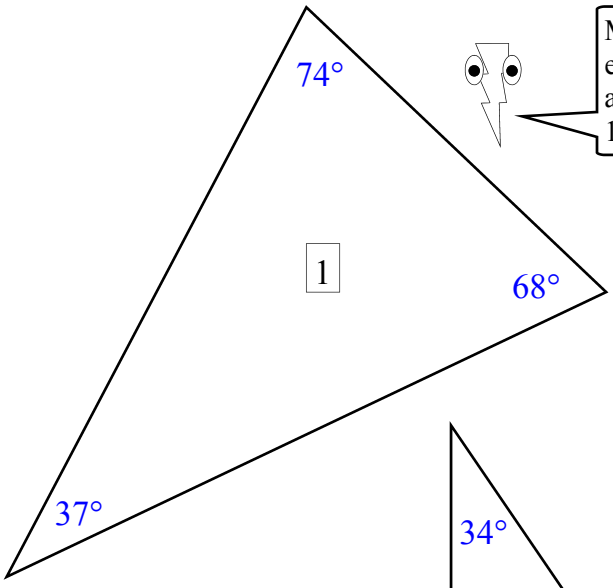
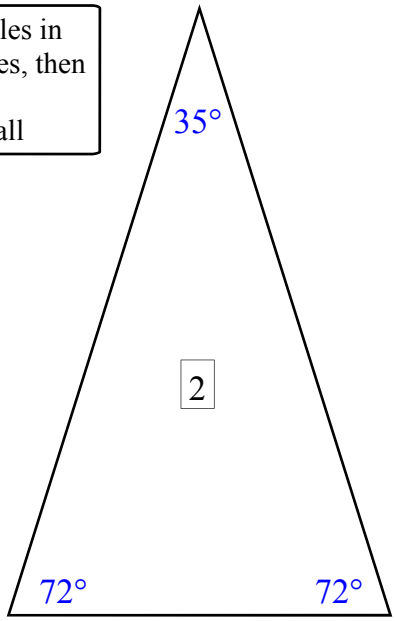
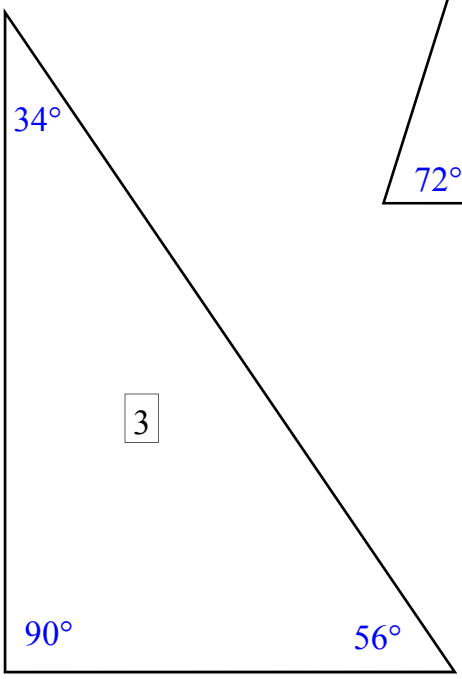
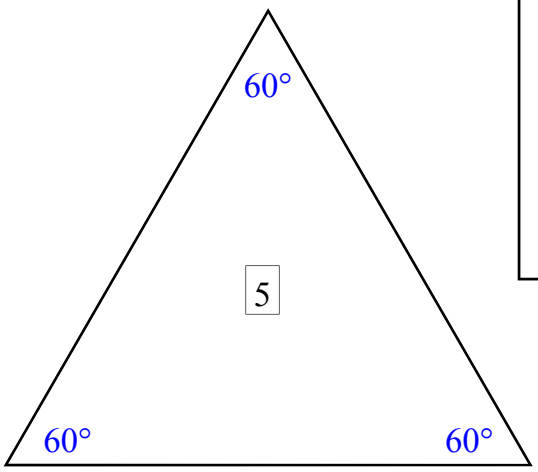
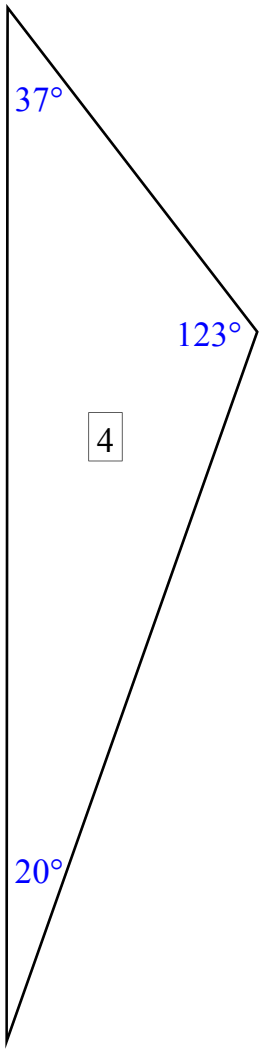


Measure the 3 angles in each of the triangles, then add them to get 180°,allow for small

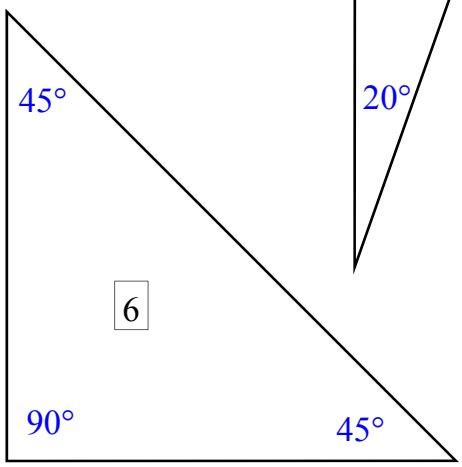








1	37	+	2	1 72	+	3	1 34	+
	68			72			56	
	74			35			90	
	179			179			180	
4	1 37	+	5	60	+	6	1 90	+
	123			60			45	
	20			60			45	
	180			180			180	



Angles in Triangles

The angle sum of a triangle is 180°. This property holds true regardless of the type of triangle. Types of triangles are:

- Equilateral - all sides equal length, all (internal) angles are 60°
- Isosceles - two sides equal length, two angles equal, note that the equal sides are opposite the equal angles
- Right angled triangle - has a right angle in the triangle with the other two angles being unequal
- Right Isosceles triangle - is a right angled triangle with the other two angles being equal (they must be 45°)
- Scalene triangle - all sides and all angles unequal, the words acute and obtuse can be used to further the description
- Acute triangle - all angles are less than 90°
- Obtuse triangle - one angle is greater than 90°, note that you can't have more than one obtuse angle in a triangle

Column 1 requires you to find the missing angle. Two angles are always given, subtract these from 180° to get the answer, (example at top of column). Once you have the 3 angles describe the type of triangle it is from the selection at the top of the sheet. Often the case is to use the word ‘obtuse’ in describing a triangle but not ‘acute’. So that if obtuse isn't used the triangle must be acute. This classification will only apply to scalene and isosceles triangles, right angled triangles and equilateral triangles can't contain an obtuse angle.

Column 3 tests your knowledge of isosceles and equilateral triangles. With equilateral triangles you know the angle is always 60°. So if the angle is $12w$ then that means $12w = 60°$ (÷ by 12) and $w = 5°$. If it was $15t$ then $15t = 60°$ (÷ by 15) and $t = 4°$. Note that in these questions you asked to find the value of the letter, this is different than finding the angle. For example if an angle is $20k$ in the corner of an equilateral triangle, $k = 3°$ but the actual angle is still 60°.

With isosceles triangles the process is the same except that instead of 60° the angle will match another angle on the diagram, question 18 is more challenging.

Angles in Triangles

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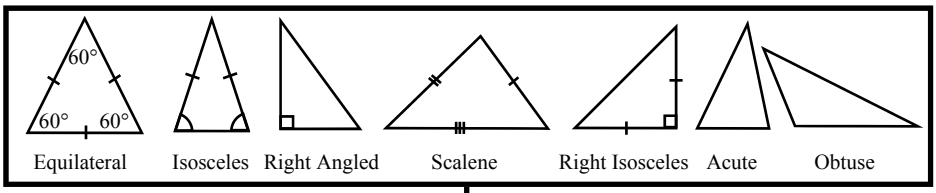
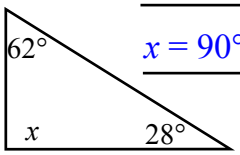


Find the missing angle, then classify (name the type of) the triangle.

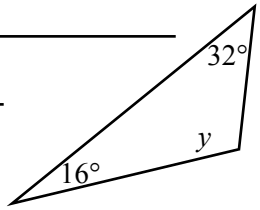
Example $x = 180° - 62° - 28°$

$x = 90°$



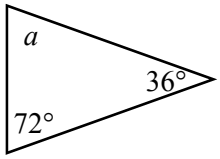
Type: Right angled triangle

1



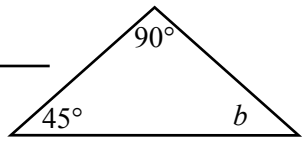
Type:

2



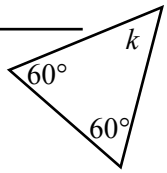
Type:

3



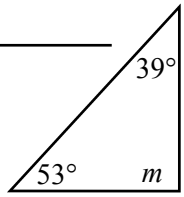
Type:

4



Type:

5



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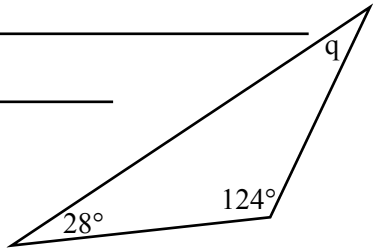
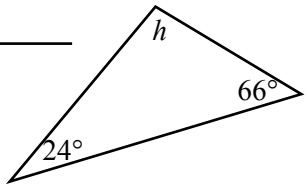
6



Type:

7



Type:

Use the same method to solve, identify the type of triangle formed. Use '$x =$'

8 A triangle has angles 23° and 46°, find the other angle

Type:

9 A triangle has angles 56° and 68°, find the other angle

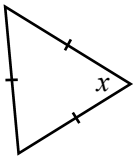
Type:

10 Two identical angles in a triangle total 120°, find the other angle

Type:

Use the side lengths to help you choose a method to find the value of the letters

11



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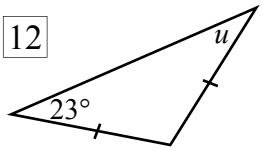

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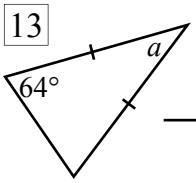

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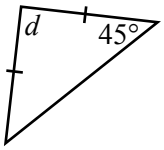

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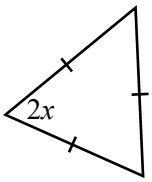
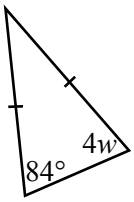


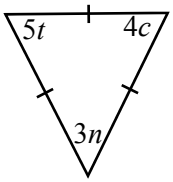
$2x =$

16



These are harder

17



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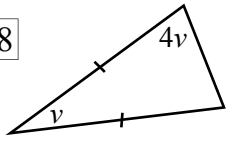

Angles in Triangles

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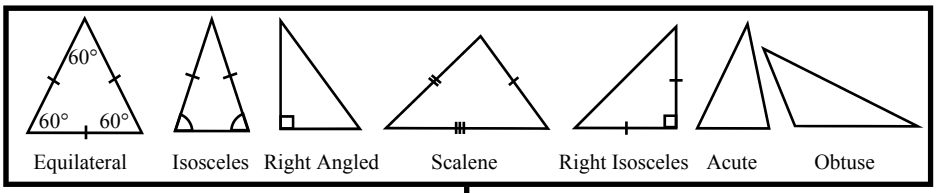
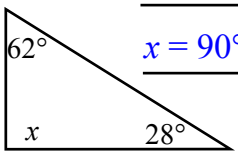


Find the missing angle, then classify (name the type of) the triangle.

Example $x = 180° - 62° - 28°$

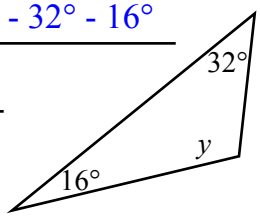
$x = 90°$



Type: Right angled triangle

1 $y = 180° - 32° - 16°$

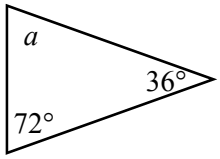
$y = 132°$



Type: Obtuse scalene triangle

2 $a = 180° - 72° - 36°$

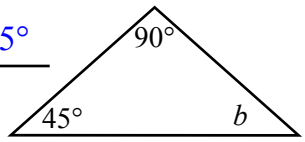
$a = 72°$



Type: Acute Isosceles Triangle

3 $b = 180° - 90° - 45°$

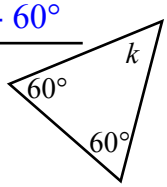
$b = 45°$



Type: Right Isosceles Triangle

4 $k = 180° - 60° - 60°$

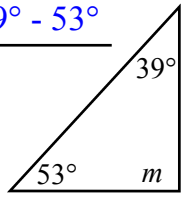
$k = 60°$



Type: Equilateral triangle

5 $m = 180° - 39° - 53°$

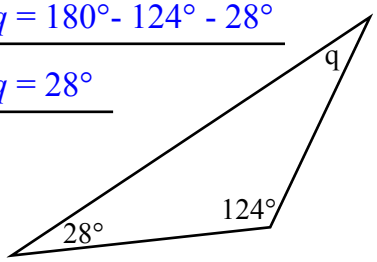
$m = 88°$



Type: Acute scalene triangle

6 $q = 180° - 124° - 28°$

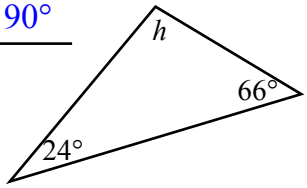
$q = 28°$



Type: Obtuse isosceles triangle

7 $h = 180° - 24° - 66°$

$h = 90°$



Type: Right angled triangle

Use the same method to solve, identify the type of triangle formed. Use '$x =$'

8 A triangle has angles 23° and 46°, find the other angle

$x = 180° - 23° - 46°$

$x = 111°$

Type: Obtuse scalene triangle

9 A triangle has angles 56° and 68°, find the other angle

$x = 180° - 56° - 68°$

$x = 56°$

Type: Acute isosceles triangle

10 Two identical angles in a triangle total 120°, find the other angle

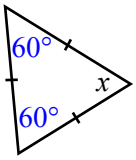
$x = 180° - 120°$

$x = 60°$

Type: Equilateral triangle

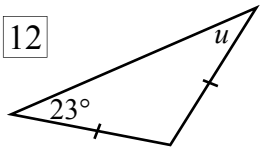
Use the side lengths to help you choose a method to find the value of the letters

11



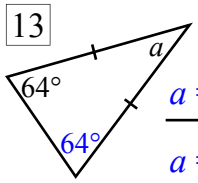
$x = 60°$

12



$u = 23°$

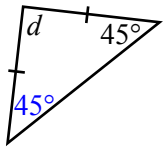
13



$a = 180° - 64° - 64°$

$a = 52°$

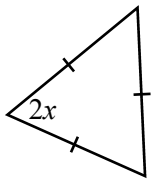
14



$d = 180° - 45° - 45°$

$d = 90°$

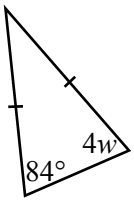
15



$2x = 60°$

$x = 30°$

16

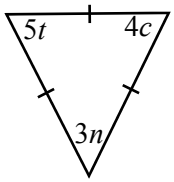


$4w = 84°$

$w = 21°$

These are harder

17



$4c = 60°$

$c = 15°$

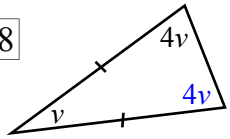
$5t = 60°$

$t = 12°$

$3n = 60°$

$n = 20°$

18



$9v = 180°$

$v = 20°$

Isosceles Triangles

Your knowledge of isosceles triangles and their properties will be tested throughout your mathematics studies, the 3 methods of finding missing angles are dealt with on this sheet. In column 1 your are given one angle and asked to find another. With a scalene triangle this is impossible because you require 2 angles to find the remaining angle, but because these are isosceles triangles you know that the angles opposite the sides with markings are equal.

Column 1 is done all the same way, place the number in the empty corner then you have your 2 angles. Then subtract the 2 angles from 180°. Look at question 1, the unlabelled angle is also 48°, so the working is s = 180° - 48° - 48° or s = 180° - 2 × 48°. The most common mistake when answering these questions is you will forget the unlabelled angle and subtract 48° from 180° instead of 48° doubled.

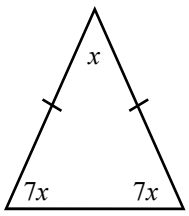
In column 2 the second type of problem is encountered, this time you are given the non-paired angle. This time 2 × (the letter) = 180° - (the given angle). An example is at the top of the column, the entire column is done using the same method. It is up to you if you write the letter in the unlabelled corner, but it is recommended.

Column 3 has written problems, 17 - 19 are the same as the previous columns, you just have to decide which method is used. The last 2 questions are harder, an example is below.

Example (questions 20 and 21)

A triangle has a pair of angles which are seven times the size of the other angle, find the size of all the angles. Let x = the smallest angle.

How this appears as a diagram:



The working required:

$7x + 7x + x = 15x$

$15x = 180°$

$\div 15$

$x = 12°$

$x = 12°$ and $7x = 84°$

Isosceles Triangles

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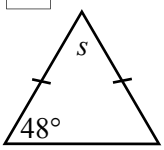
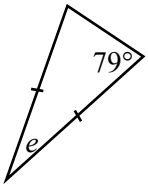
Find the value of the pronumeral

1



$s = 180° -$

2



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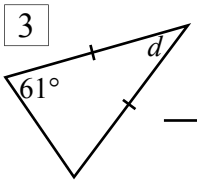

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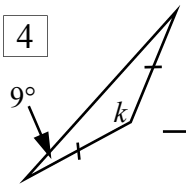

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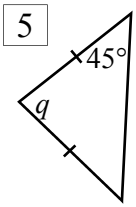

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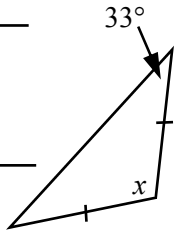

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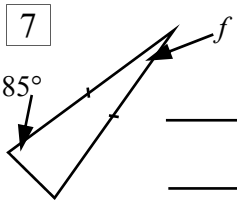

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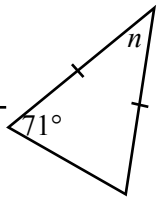

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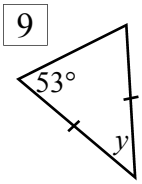


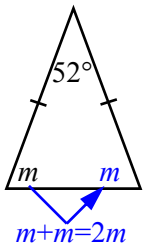
Find the value of the letter, these will take 3 lines to solve.

Example

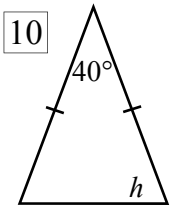
$2m = 180° - 52°$

$2m = 128°$

$m = 64°$



10



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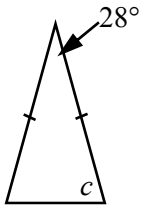

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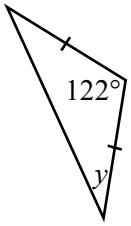

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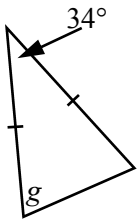

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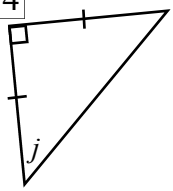

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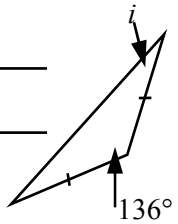
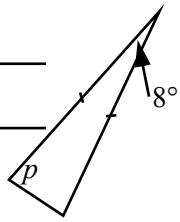


16



Solve these, you might not always need 3 lines. Use 'x=' in your working.

17 An isosceles triangle has one angle of 102°, find the size of the other angles

18 An isosceles triangle has a pair of angles of 17°, find the size of the other angle

19 An isosceles triangle has a pair of angles of 55°, find the size of the other angle

These are harder!

20 An isosceles triangle has a pair of angles that are twice the size of the other angle find the angles (Hint: use x and $2x$)

$x =$ $2x =$

21 Repeat the above question only this time the angles are 4 times the size of the other.

Isosceles Triangles

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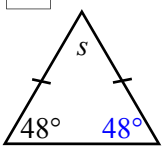
Find the value of the pronumeral

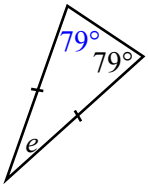
1



$s = 180° - 48° - 48°$

$s = 84°$

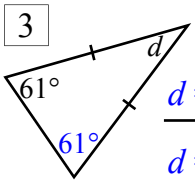
2



$e = 180° - 79° - 79°$

$e = 22°$

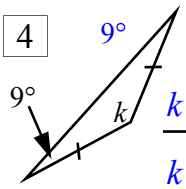
3



$d = 180° - 61° - 61°$

$d = 58°$

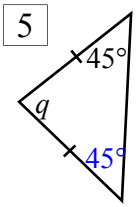
4



$k = 180° - 9° - 9°$

$k = 162°$

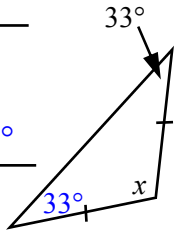
5



$q = 180° - 45° - 45°$

$q = 90°$

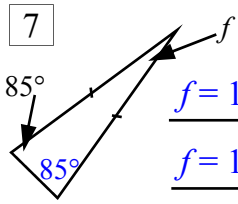
6



$x = 180° - 33° - 33°$

$x = 114°$

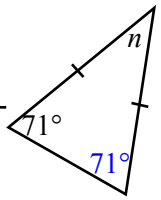
7



$f = 180° - 85° - 85°$

$f = 10°$

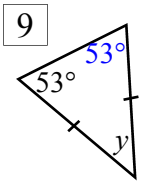
8



$n = 180° - 71° - 71°$

$n = 38°$

9



$y = 180° - 53° - 53°$

$y = 74°$

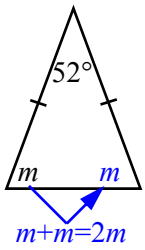
Find the value of the letter, these will take 3 lines to solve.

Example

$2m = 180° - 52°$

$2m = 128°$

$m = 64°$



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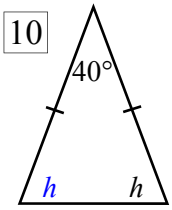
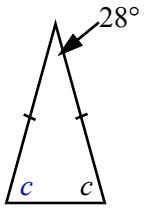


$2h = 180° - 40°$

$2h = 140°$

$h = 70°$

11

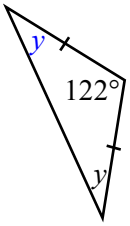


$2c = 180° - 28°$

$2c = 152°$

$c = 76°$

12

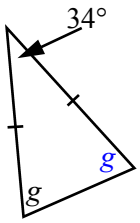


$2y = 180° - 122°$

$2y = 58°$

$y = 29°$

13

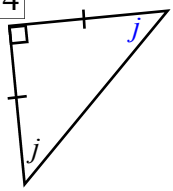


$2g = 180° - 34°$

$2g = 146°$

$g = 73°$

14

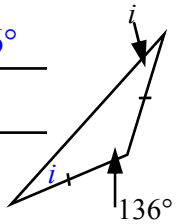


$2j = 180° - 90°$

$2j = 90°$

$j = 45°$

15

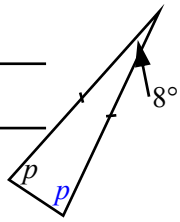


$2i = 180° - 136°$

$2i = 44°$

$i = 22°$

16



$2p = 180° - 8°$

$2p = 172°$

$p = 86°$

Solve these, you might not always need 3 lines. Use 'x=' in your working.

17 An isosceles triangle has one angle of 102°, find the size of the other angles

$2x = 180° - 102°$

$2x = 78°$

$x = 39°$

18 An isosceles triangle has a pair of angles of 17°, find the size of the other angle

$x = 180° - 17° - 17°$

$x = 146°$

19 An isosceles triangle has a pair of angles of 55°, find the size of the other angle

$x = 180° - 55° - 55°$

$x = 70°$

These are harder!

20 An isosceles triangle has a pair of angles that are twice the size of the other angle find the angles (Hint: use x and $2x$)

$5x = 180°$

$x = 36°$

$x = 36°$ $2x = 72°$

21 Repeat the above question only this time the angles are 4 times the size of the other.

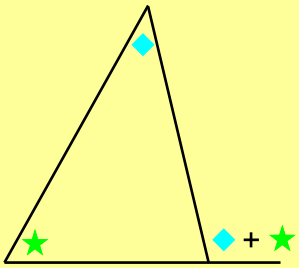
$9x = 180°$

$x = 20°$

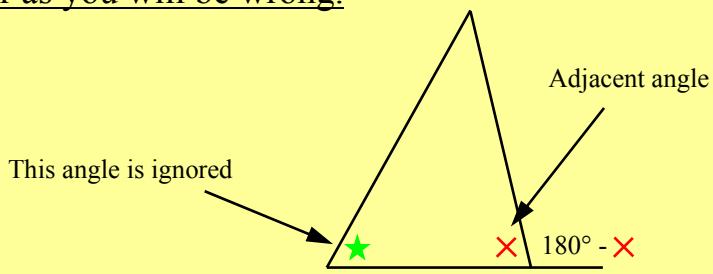
$x = 20°$ $4x = 80°$

Exterior Angles of a Triangle

There is a relationship between the exterior angle of a triangle and the 2 opposite interior angles, the sum of the 2 opposite interior angles = the exterior angle. In other words when you add the 2 opposite interior angles you get the exterior angle. The diagram below shows this graphically. It is important that you realise that the two opposite angles aren't adjacent to (don't touch) the exterior angle.



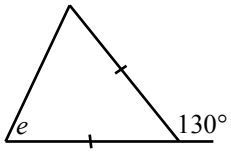
Column 1 Q.1 - 5 require you to add the two interior angles together to get the exterior angle. The adjacent angle is kept out of these questions. An example is at the top of the column. The next 3 questions show that if you have the adjacent internal angle you just take the supplement to get the external angle (subtract from 180°). So if the adjacent angle is one of the 2 angles, then just take the supplement (subtract it from 180°) of the adjacent angle. This is where the most common mistake is made, if the adjacent angle is given don't add it to the other angle to get your answer as you will be wrong.



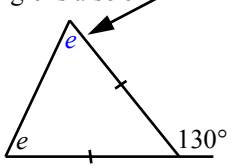
Column 2 works in reverse, you are given the exterior angle and have to calculate the missing interior angle. Do these by subtracting the given interior angle from the exterior angle, see the example at the top of the column.

The third column is challenging as it requires you to use your skills with isosceles and equilateral triangles as well as algebra. There is an example below.

Example : Find the angle



Because the triangle is isosceles the other base angle is also e



Solution:

$2e = 130°$ (opposite angle sum = exterior)

$e = 65°$

Exterior Angles of a Triangle

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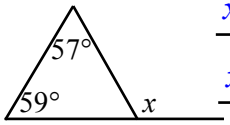
Find the exterior angle for the following triangles.

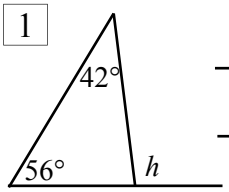
Example



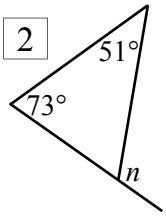
$x = 57 + 59$

$x = 116°$

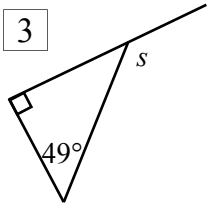
1



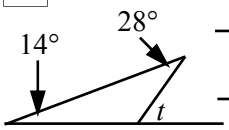
2



3



4



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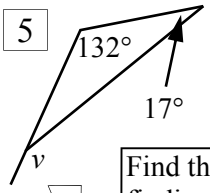
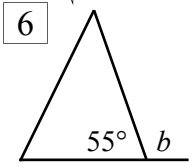
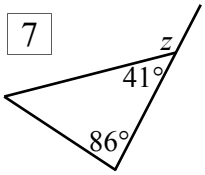


Find the exterior angle by finding the supplement, ignore any extra angles.

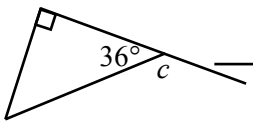
6



7

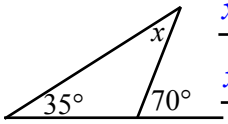


8



Now the exterior angle is given, use subtraction to find the unknown angle.

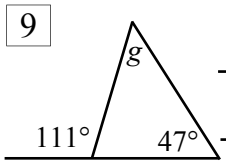
Example



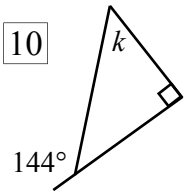
$x = 70 - 35$

$x = 35°$

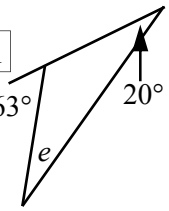
9



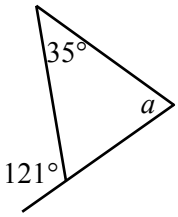
10



11



12



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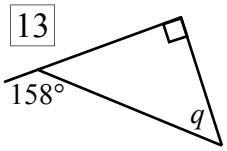

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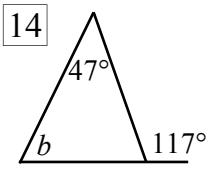

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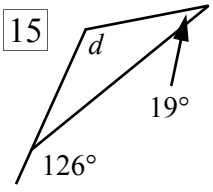
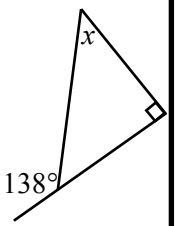
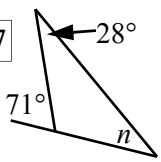


16

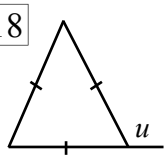


17



Use your knowledge of triangle properties to find the value of the angles

18



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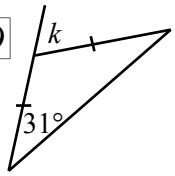

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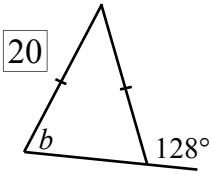
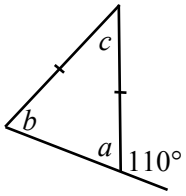


21 Hint: Find a then b then c



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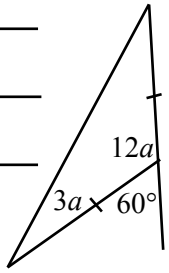
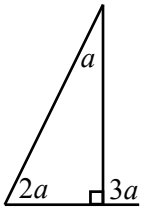


23



Exterior Angles of a Triangle

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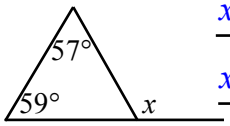
Find the exterior angle for the following triangles.

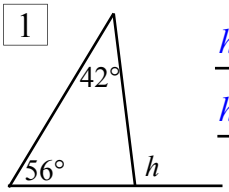
Example



$x = 57 + 59$

$x = 116°$

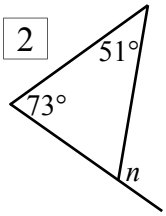
1



$h = 56 + 42$

$h = 98°$

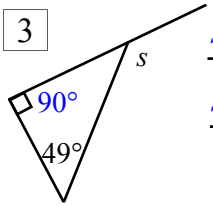
2



$n = 51 + 73$

$n = 124°$

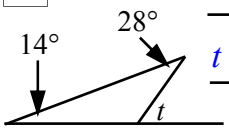
3



$s = 90 + 49$

$s = 139°$

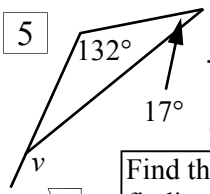
4



$t = 14 + 28$

$t = 42°$

5

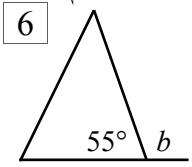


$v = 132 + 17$

$v = 149°$

Find the exterior angle by finding the supplement, ignore any extra angles.

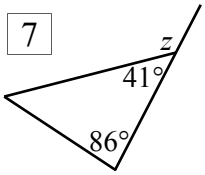
6



$b = 180 - 55$

$b = 125°$

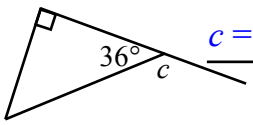
7



$z = 180 - 41$

$z = 139°$

8

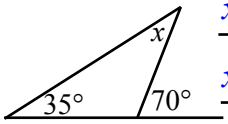


$c = 180 - 36$

$c = 144°$

Now the exterior angle is given, use subtraction to find the unknown angle.

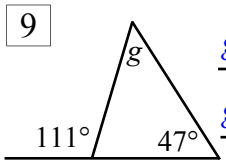
Example



$x = 70 - 35$

$x = 35°$

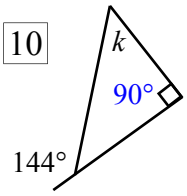
9



$g = 111 - 47$

$g = 64°$

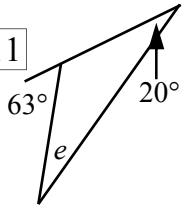
10



$k = 144 - 90$

$k = 54°$

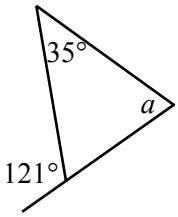
11



$e = 63 - 20$

$e = 43°$

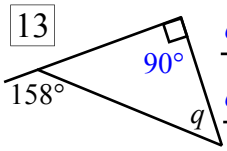
12



$a = 121 - 35$

$a = 86°$

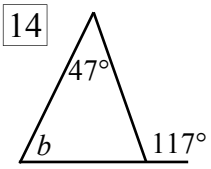
13



$q = 158 - 90$

$q = 68°$

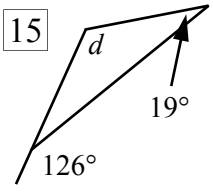
14



$b = 117 - 47$

$b = 70°$

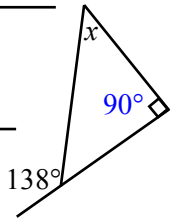
15



$d = 126 - 19$

$d = 107°$

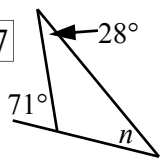
16



$x = 138 - 90$

$x = 48°$

17

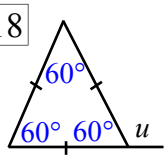


$n = 71 - 28$

$n = 43°$

Use your knowledge of triangle properties to find the value of the angles

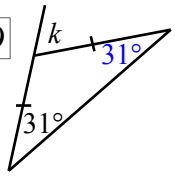
18



$u = 60 + 60$

$u = 120°$

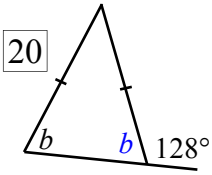
19



$k = 31 + 31$

$k = 62°$

20



$b = 180 - 128$

$b = 52°$

21 Hint: Find a then b then c

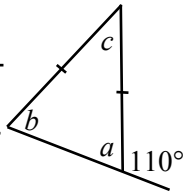
$a = 180 - 110$

$a = 70°$

$b = 70°$ (isos)

$c = 110 - 70$

$c = 40°$



22

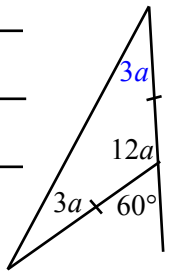
$3a + 3a = 6a$

$6a = 60$

$a = 10°$

$3a = 30°$

$12a = 120°$



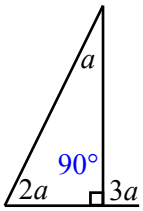
23

$3a = 180 - 90$

$3a = 90$

$a = 30°$

$2a = 60°$



Angles in Quadrilaterals

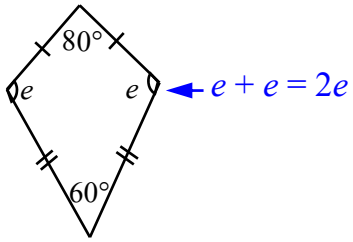
The angle sum of a quadrilateral is 360°, this means that when all the angles inside the shape are added, they total 360°. If you are given 3 angles then to find the missing angle subtract the 3 angles from 360° and what is left is the your answer.

In column 1 through to question 11 the exercises are all done the same way, you are given 3 angles (remember a right angle is 90°), subtract these from 360°. If you write in '90°' in the space next to the right angle sign you have less chance of forgetting it in your calculations. The question at the top of the 1st column shows the setting out for the exercises.

The rest of column 2 involves a small amount of algebra. The exercises are done in the same way as the first column in that the missing angles equal 360° minus the given angles. Because in the questions the 2 unknown angles are the same then you can add the letters together, i.e. $a + a = 2a$, $x + x = 2x$. An example is below. Question 20 is a similar style of question.

Column 3 is the same as the first column only one of the angles given is an exterior angle. To change the exterior angle to an interior angle subtract it from 360°. Then you have 3 interior angles which subtracted from 360° gives you the missing angle. The most common mistake is forgetting to change the exterior angle it to an interior angle in these questions.

Example: Find the value of e



Solution:

$2e = 360° - 80° - 60°$

$2e = 220°$

$e = 110°$

Angles in Quadrilaterals

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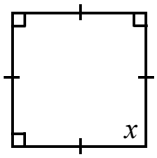
Find the missing angle in these quadrilaterals

1 $x = 360° - 90° - 90° - 90°$

$x =$



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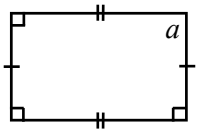

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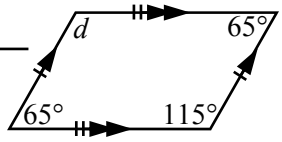

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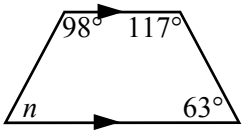

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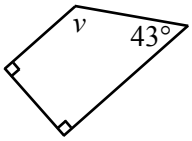

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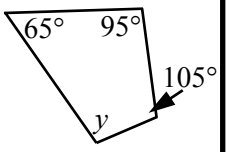

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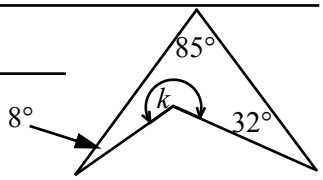

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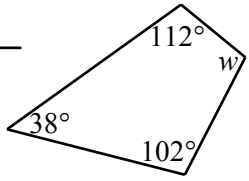

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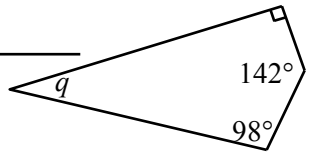

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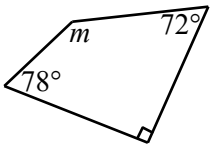

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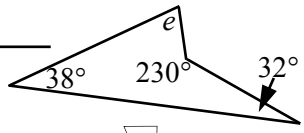
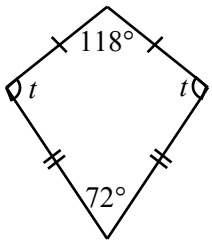


These have 2 unknown angles

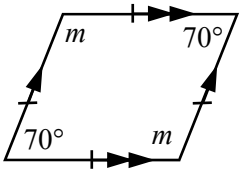
12 $2t = 360° -$

$2t =$

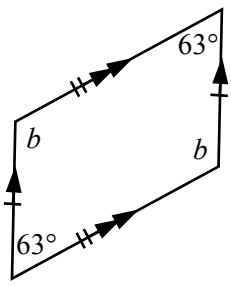
$t =$



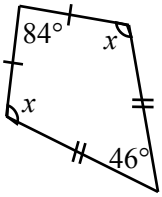
13



14



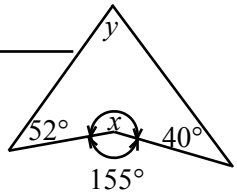
15



Convert the exterior angles to interior angles to find x then solve for y.

16 $x = 360° -$

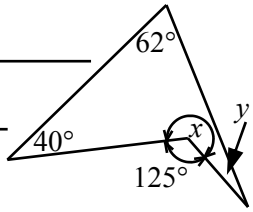
$x =$



$y =$

$y =$

17



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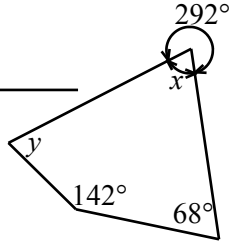
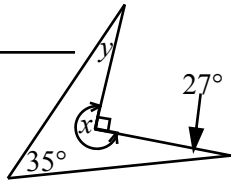
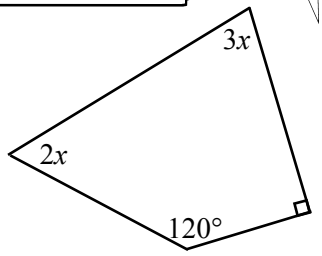


19



Find the 2 angles

20



$2x =$

$3x =$

Angles in Quadrilaterals

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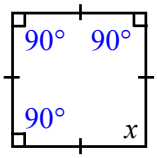
Find the missing angle in these quadrilaterals

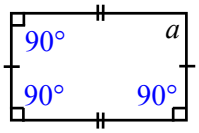
1 $x = 360° - 90° - 90° - 90°$

$x = 90°$



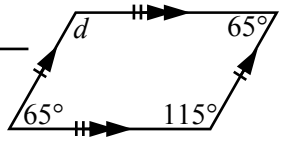
2 $a = 360° - 90° - 90° - 90°$

$a = 90°$



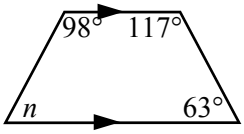
3 $d = 360° - 65° - 115° - 65°$

$d = 115°$



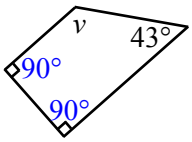
4 $n = 360° - 98° - 117° - 63°$

$n = 82°$



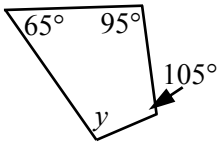
5 $v = 360° - 43° - 90° - 90°$

$v = 137°$



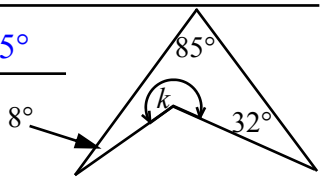
6 $y = 360° - 105° - 65° - 95°$

$y = 95°$



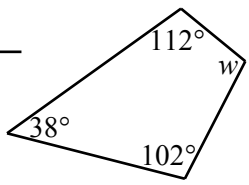
7 $k = 360° - 8° - 32° - 65°$

$k = 255°$



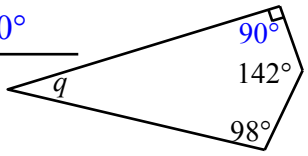
8 $w = 360° - 112° - 38° - 102°$

$w = 108°$



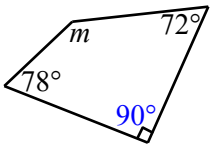
9 $q = 360° - 90° - 142° - 98°$

$q = 30°$



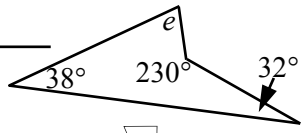
10 $m = 360° - 78° - 90° - 72°$

$m = 120°$



11 $e = 360° - 38° - 32° - 230°$

$e = 60°$

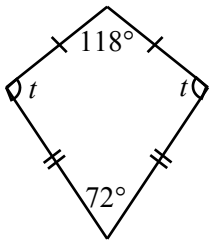


These have 2 unknown angles

12 $2t = 360° - 118° - 72°$

$2t = 170°$

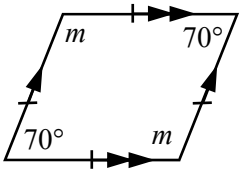
$t = 85°$



13 $2m = 360° - 70° - 70°$

$2m = 220°$

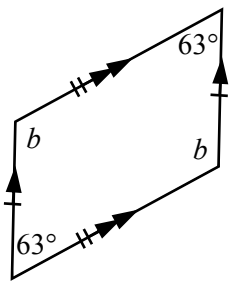
$m = 110°$



14 $2b = 360° - 63° - 63°$

$2b = 234°$

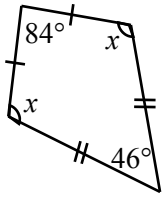
$b = 117°$



15 $2x = 360° - 84° - 46°$

$2x = 230°$

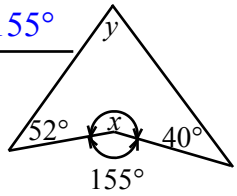
$x = 115°$



Convert the exterior angles to interior angles to find x then solve for y.

16 $x = 360° - 155°$

$x = 205°$

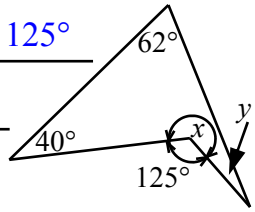


$y = 360° - 205° - 52° - 40°$

$y = 63°$

17 $x = 360° - 125°$

$x = 235°$

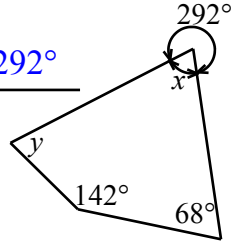


$y = 360° - 235° - 40° - 62°$

$y = 23°$

18 $x = 360° - 292°$

$x = 68°$

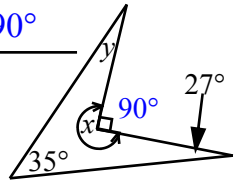


$y = 360° - 68° - 68° - 142°$

$y = 82°$

19 $x = 360° - 90°$

$x = 270°$

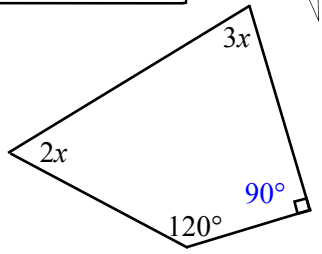


$y = 360° - 270° - 27° - 35°$

$y = 28°$

Find the 2 angles

20



$5x = 360° - 120° - 90°$

$5x = 150°$ $2x = 60°$

$x = 30°$ $3x = 90°$

Classifying, Naming and Measuring Angles

Classification means to group into categories. There are six classifications in all:

- acute angles are between 0° and 90°
- a right angle is exactly 90°
- obtuse angles are between 90° and 180°
- a straight angle is exactly 180°
- reflex angles are between 180° and 360°
- and a revolution is 360°

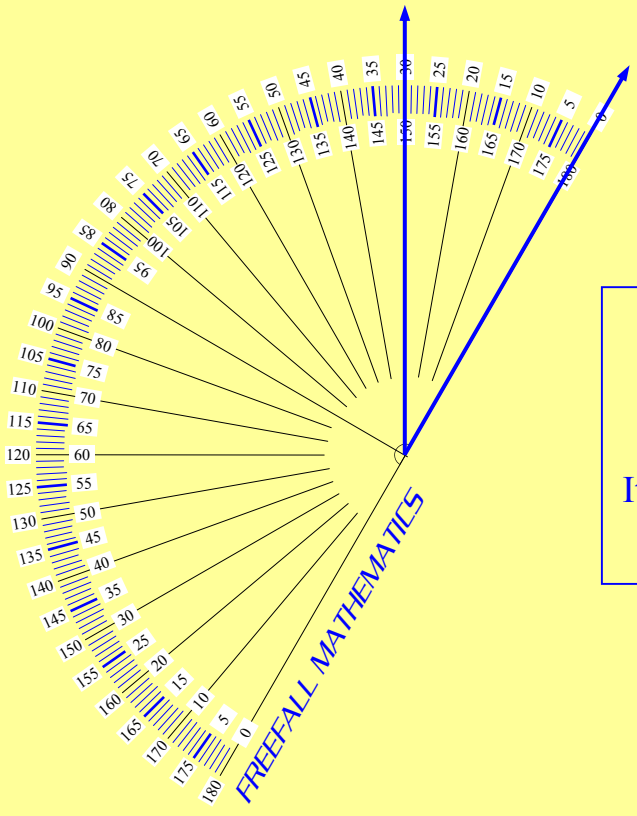
How do you remember all these? You should be able to remember the straight angle, right angle and revolution, it is the others you could mix up. Remember that as the angle increases you move through the alphabet. A (acute) is before O(Obtuse) which is before R(reflex).

To name an angle we generally use 3 letters, moving from ray-to vertex-to ray. The vertex is always in the middle. To show that it is an angle you put an angle sign in front of it or write the letters placing a hat over the vertex.

$\angle ABC$ or, $\hat{ABC}$

A protractor is required for the second column of this worksheet.

When using a standard protractor ensure you use the correct scale (inside or outside, make sure the scale used starts at 0°) and if the angle is a reflex (greater than 180°), measure the outside of the angle then subtract the angle from 360°. If an angle is measured and it is above 90°, then it must be larger than a right angle, so when you record the size of the angle, look at the angle and check in relation to 90° and 180°.



This angle would be 30° not 150°. Look at where the 0° is, that is the scale you use.

It is also less than a right angle, so it can't be more than 90°.

Classifying, Naming and Measuring Angles

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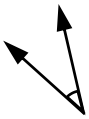
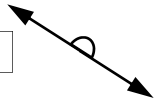
Classify these angles as: Acute, Right, Obtuse, Straight, Reflex and Revolution.

1 

2 

3 

4 

5 

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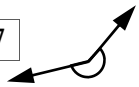
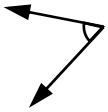
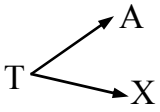
7 

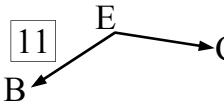
8 

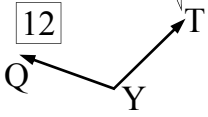
9 

Name the angles shown below. Use three letters to name the angle, with the vertex in the middle.

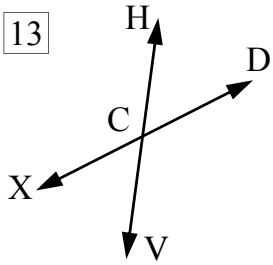


10 

11 

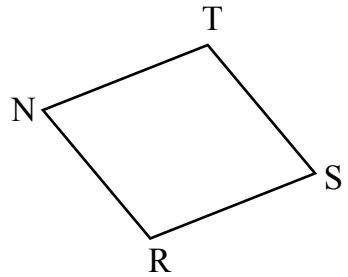
12 



13 

acute | acute | obtuse

obtuse | straight | straight

14 

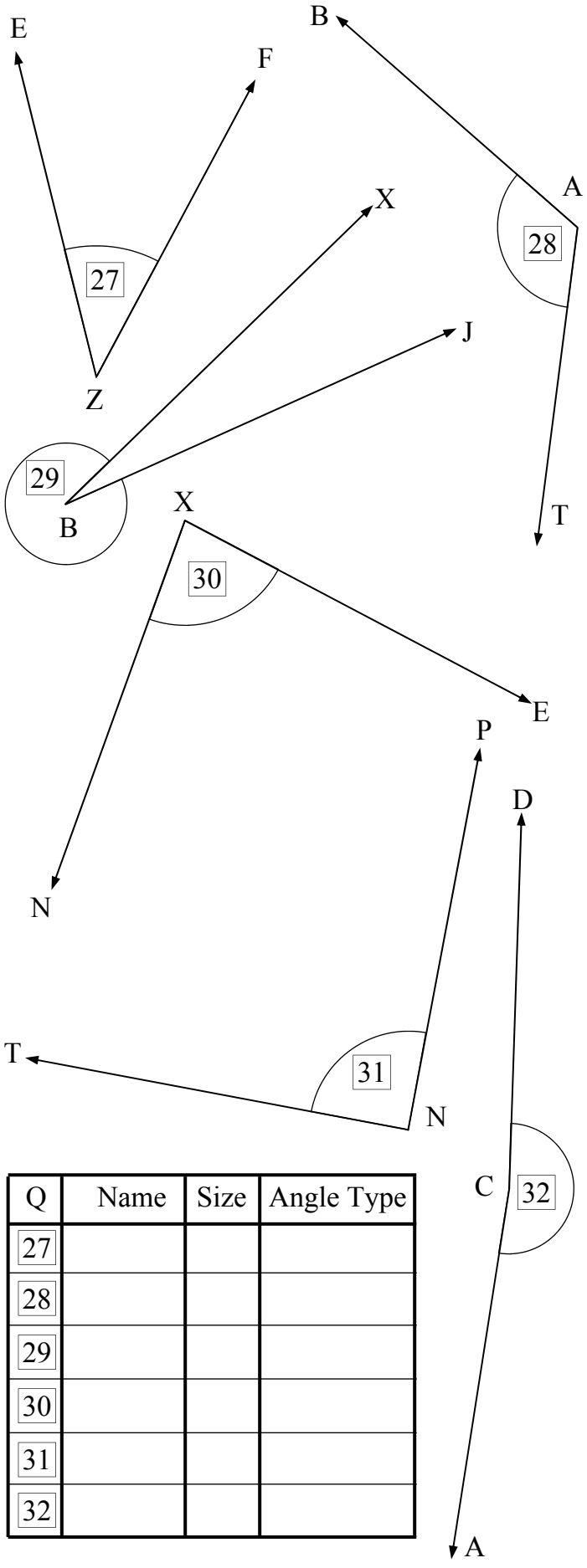
acute | acute

obtuse | obtuse

Complete the table by classifying these angles, acute, reflex etc.

Q	Size	Angle Type	Q	Size	Angle Type
15	135°		21	100°	
16	220°		22	300°	
17	77°		23	90°	
18	110°		24	175°	
19	360°		25	270°	
20	23°		26	180°	

Measure these angles with a protractor, record the name of the angle, its size and classify it. Remember to subtract reflex angles from 360°.



Q	Name	Size	Angle Type
27			
28			
29			
30			
31			
32			

Classifying, Naming and Measuring Angles

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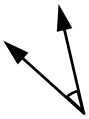
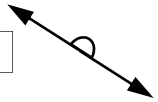
Classify these angles as: Acute, Right, Obtuse, Straight, Reflex and Revolution.

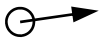
1  Reflex

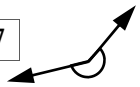
2  Obtuse

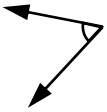
3  Right

4  Acute

5  Straight

6  Revolution

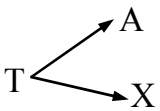
7  Reflex

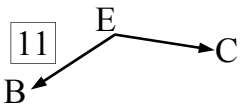
8  Acute

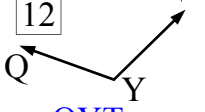
9  Obtuse

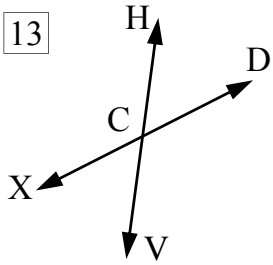
Name the angles shown below. Use three letters to name the angle, with the vertex in the middle.



10  ∠ATX

11  ∠BEC

12  ∠QYT

13 

∠ HDC — acute

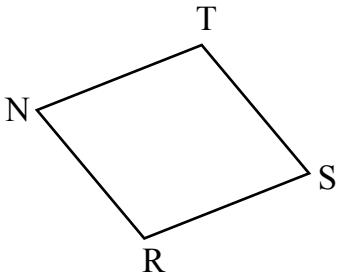
∠ XCV — acute

∠ XCH — obtuse

∠ DCV — obtuse

∠ XCD — straight

∠ HCV — straight

14 

∠ TNR — acute

∠ TSR — acute

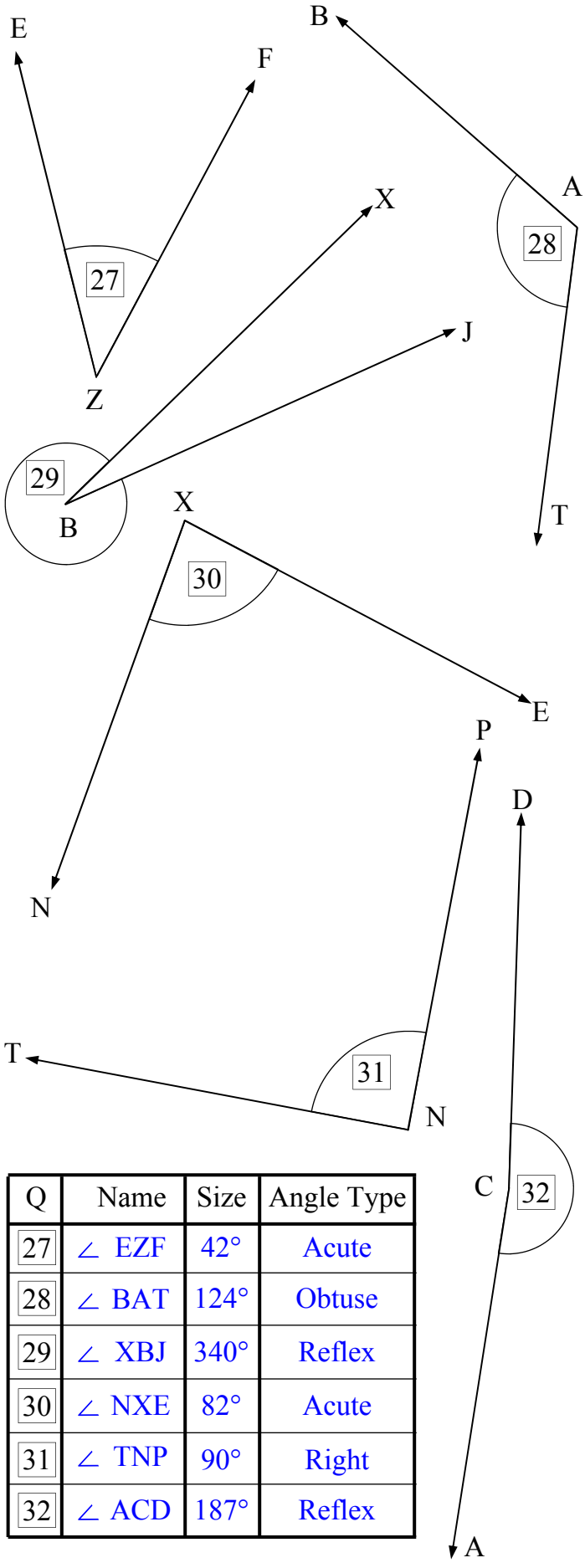
∠ NRS — obtuse

∠ NTS — obtuse

Complete the table by classifying these angles, acute, reflex etc.

Q	Size	Angle Type	Q	Size	Angle Type
15	135°	Obtuse	21	100°	Obtuse
16	220°	Reflex	22	300°	Reflex
17	77°	Acute	23	90°	Right
18	110°	Obtuse	24	175°	Obtuse
19	360°	Revolution	25	270°	Reflex
20	23°	Acute	26	180°	Straight

Measure these angles with a protractor, record the name of the angle, its size and classify it. Remember to subtract reflex angles from 360°.



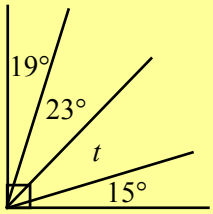
Q	Name	Size	Angle Type
27	∠ EZF	42°	Acute
28	∠ BAT	124°	Obtuse
29	∠ XBJ	340°	Reflex
30	∠ NXE	82°	Acute
31	∠ TNP	90°	Right
32	∠ ACD	187°	Reflex

Adjacent, Complementary and Supplementary Angles

Adjacent means next to, in the case of angles this means that the angle shares a common ray (or arm) and vertex with the angle beside it. Complementary angles add to give 90°, while supplementary angles add to 180°.

Here are a few examples of Column 1 style questions. Find the unknown angles.

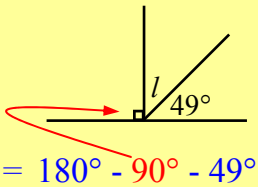
Complementary Angles add to 90°



t = 90° - 19° - 23° - 15°

t = 33°

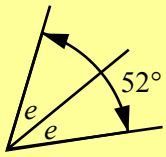
Supplementary Angles add to 180°



l = 180° - 90° - 49°

l = 41°

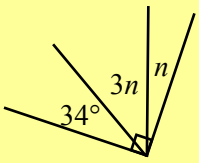
Adjacent Angles add to the total of the angles



$2e$ = 52°

e = 26°

The second column involves the use of 2-step equations. Below is an example.



$n + 3n + 34 = 90$ ← Add up all the angles, the total will equal 90° because the angles sum to a right angle.

$4n + 34 = 90$ ← Collect like terms: ($n + 3n = 4n$)

-34 -34 ← Remove the number from the LHS, so subtract 34 from both sides.

$4n = 56°$

÷4 ÷4 ← Remove the coefficient from the LHS, so Divide both sides by 4

$n = 14°$

$3n = 42°$ ← The value of n has been found, which is one of the angles. The other angle is $3n$, so multiply n by 3 to find $3n$. $3 \times 14 = 42°$.

Adjacent, Complementary and Supplementary Angles

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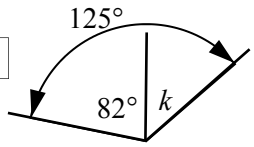
Find the size of the unknown angles

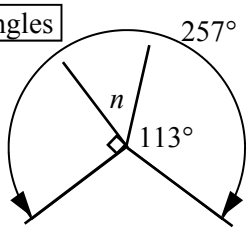
1



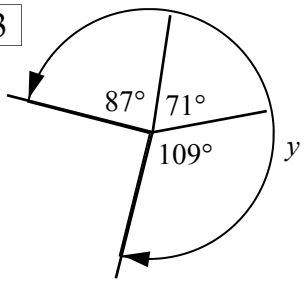
$k = 125 - \square$

$k =$

2



3



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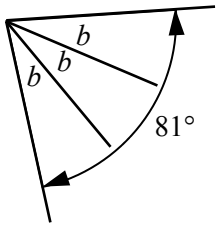

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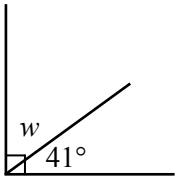
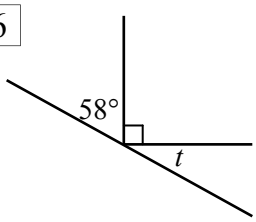
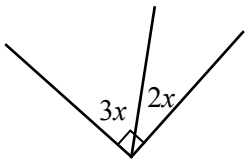


6

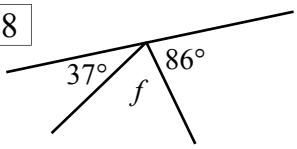


7



$3x = \square$ $2x = \square$

8



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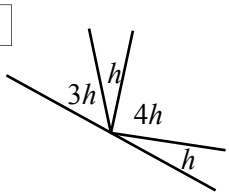

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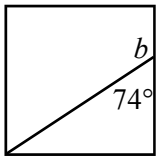
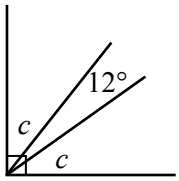


Find the size of the unknown angles. Collect like terms then use your 2-step equation skills.

11

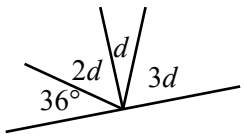


$c + c + 12 = \square$

(-12) (-12)

(÷2) (÷2)

12



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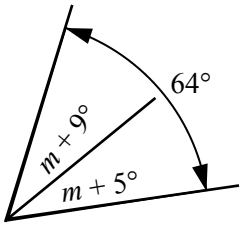

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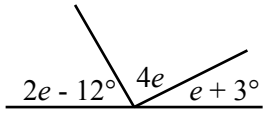
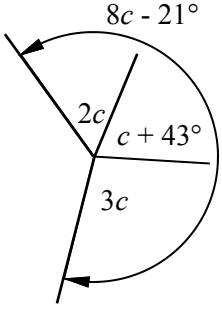


OK …. x is on both sides of the equation.

15



Adjacent, Complementary and Supplementary Angles

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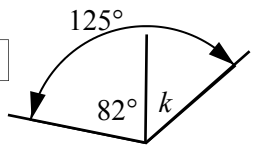
Find the size of the unknown angles

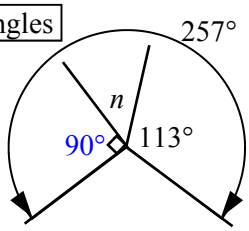
1



$k = 125 -$ 82

$k = 43°$

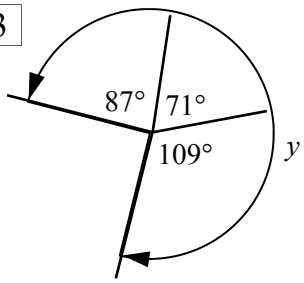
2



$n = 257 - 90 - 113$

$n = 54°$

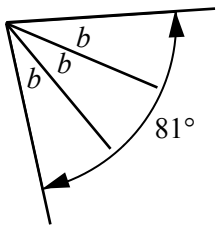
3



$y = 87 + 71 + 109$

$y = 267°$

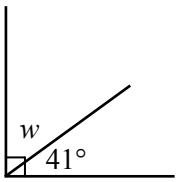
4



$3b = 81$

$b = 27°$

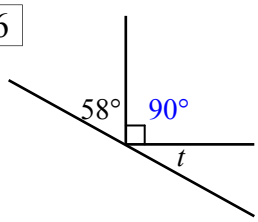
5



$w = 90 - 41$

$w = 49°$

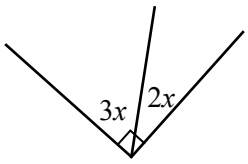
6



$t = 180 - 90 - 58$

$w = 32°$

7

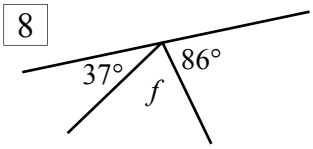


$5x = 90$

$x = 18°$

$3x =$ 54° $2x =$ 36°

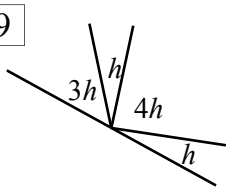
8



$f = 180 - 37 - 86$

$f = 57°$

9

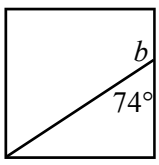


$9h = 180$

$h = 20°$

$3h = 60°$ $4h = 80°$

10

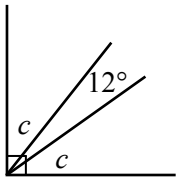


$b = 180 - 74$

$b = 106°$

Find the size of the unknown angles. Collect like terms then use your 2-step equation skills.

11



$c + c + 12 =$ 90

$2c + 12 = 90$

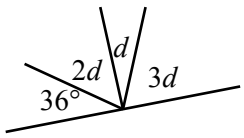
(-12) (-12)

$2c = 78$

(÷2) (÷2)

$c = 39°$

12



$2d + d + 3d + 36 = 180$

$6d + 36 = 180$

(-36) (-36)

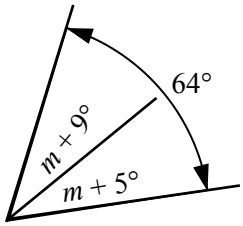
$6d = 144$

(÷6) (÷6)

$d = 24°$

$2d = 48°$ $3d = 72°$

13



$m + 9 + m + 5 = 64$

$2m + 14 = 64$

(-14) (-14)

$2m = 50$

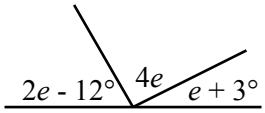
(÷2) (÷2)

$m = 25°$

$m + 9 = 34°$

$m + 5 = 30°$

14



$2e - 12 + 4e + e + 3 = 180$

$7e - 9 = 180$

(+9) (+9)

$7e = 189$

(÷7) (÷7)

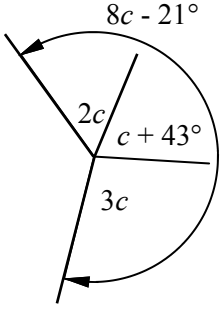
$e = 27°$

$2e - 12 = 42°$ $4e = 108°$

$e + 3 = 30°$

OK …. x is on both sides of the equation.

15



$2c + c + 43 + 3c = 8c - 21$

$6c + 43 = 8c - 21$

(-43) (-43)

$6c = 8c - 64$

(-8c) (-8c)

$-2c = -64$

(÷-2) (÷-2)

$c = 32°$

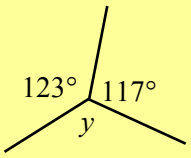
$2c = 64°$ $c + 43 = 75°$

$3c = 96°$ $8c - 21 = 235°$

Angles at a Point & Vertically Opposite Angles

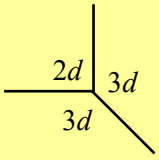
Angles at a point sum to 180°. These questions are answered the same way as complementary angles and supplementary angles, the difference being instead of a 90° or 180° total, it is 360°. Vertically opposite angles are equal. So when two straight lines intersect, the angles formed that are opposite each other are equal. Because these angles are formed by straight lines, the adjacent angle can be found by finding its supplement. (Subtracting it from 180°.)

Some examples of Angle at a point questions:



$y = 360° - 123° - 117°$

$y = 120°$

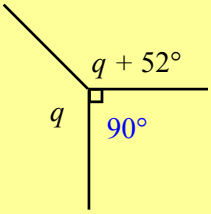


$2d = 90°$

$8d = 360°$

$d = 45°$

$3d = 135°$



$q + q + 52 + 90 = 360$

$2q + 142 = 360$

$-142 \quad -142$

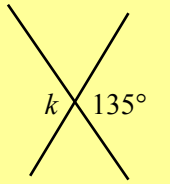
$2q = 218$

$\div 2 \quad \div 2$

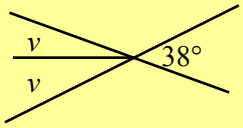
$q = 109°$

$q + 52 = 161°$

Some examples of Vertically opposite angles questions:

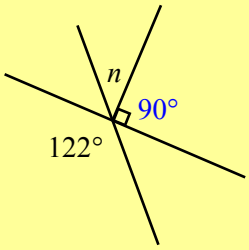


$k = 135°$



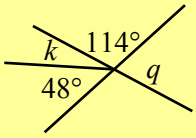
$2v = 38°$

$v = 19°$



$n = 122° - 90°$

$n = 32°$



$k = 180° - 114° - 48°$

$k = 18°$

$q = 48° + 18°$

$q = 66°$

Angles at a Point & Vertically Opposite Angles

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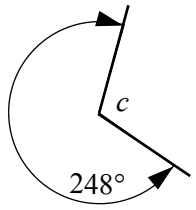
Angles at a point total 360°, find all unknown angles

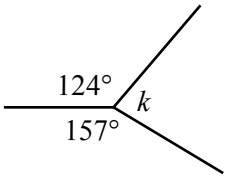
1



$c = 360° -$ ☐

$c =$ ☐ °

2



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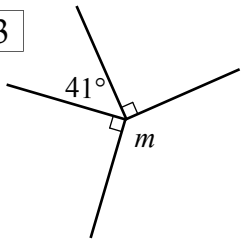

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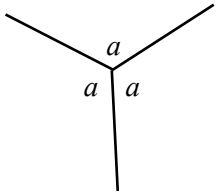

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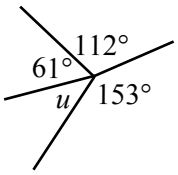

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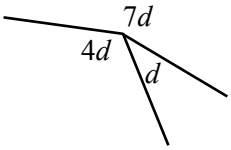
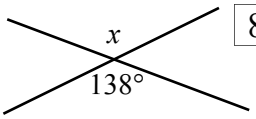


$4d =$ $7d =$

Vertically opposite angles are equal. Find the value of all the unknown angles in these.

7



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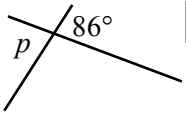

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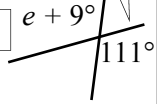

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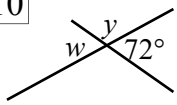
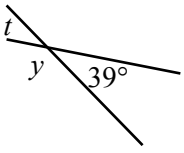


$w =$

$y = 180 -$ ☐

$y =$

11



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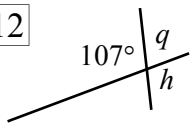

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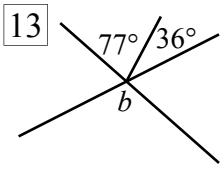

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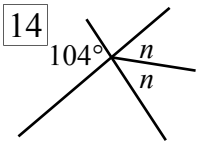

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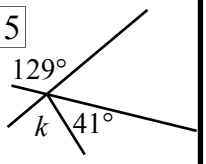
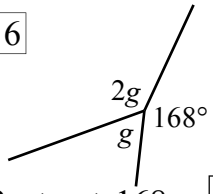


Two-step equations needed to solve these. Find the value of all unknown angles. I'm taking the elevator.

16



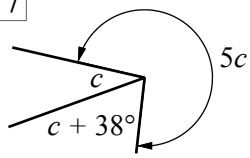
$2g + g + 168 =$ ☐

-168 -168

÷3 ÷3

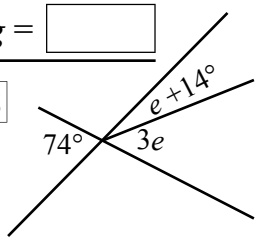
$2g =$ ☐

17



$5c =$ $c + 38° =$

18



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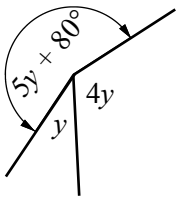
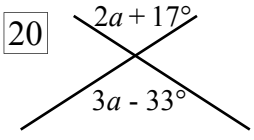


Unknowns on both sides now. Find all unknown angles.

20



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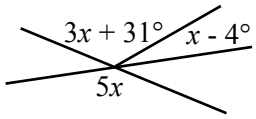

Angles at a Point & Vertically Opposite Angles

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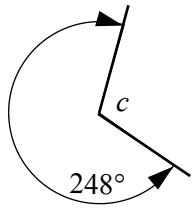
Angles at a point total 360°, find all unknown angles

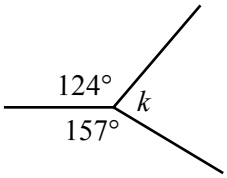
1



$c = 360° - 248$

$c = 112°$

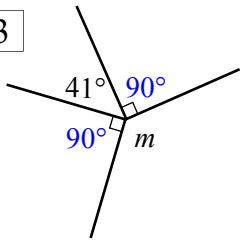
2



$k = 360 - 124 - 157$

$k = 79°$

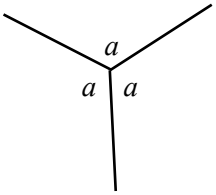
3



$m = 360 - 41 - 90 - 90$

$m = 139°$

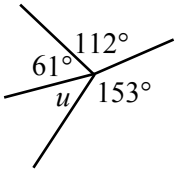
4



$3a = 360$

$a = 120°$

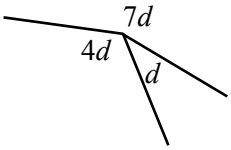
5



$u = 360 - 61 - 112 - 153$

$u = 34°$

6



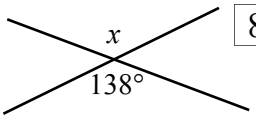
$12d = 360$

$d = 30°$

$4d = 120°$ $7d = 210°$

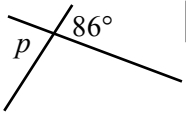
Vertically opposite angles are equal. Find the value of all the unknown angles in these.

7



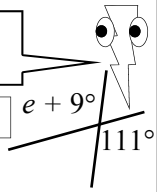
$x = 138°$

8



$p = 86°$

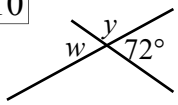
9



$e + 9 = 111$

$e = 102°$

10

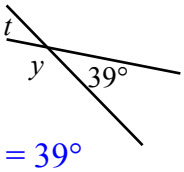


$w = 72°$

$y = 180 - 72$

$y = 108°$

11

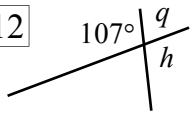


$t = 39°$

$y = 180 - 39$

$y = 141°$

12

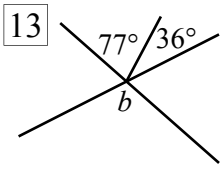


$h = 107°$

$q = 180 - 107$

$q = 73°$

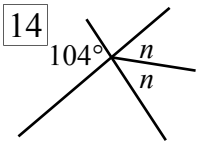
13



$b = 77 + 36$

$b = 113°$

14

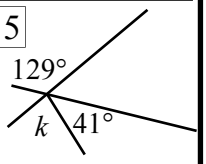


$2n = 104$

$\div 2$ $\div 2$

$n = 52°$

15



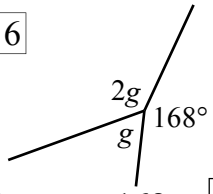
$k + 41 = 129$

-41 -41

$k = 88°$

Two-step equations needed to solve these. Find the value of all unknown angles. I'm taking the elevator.

16



$2g + g + 168 = 360$

$3g + 168 = 360$

-168 -168

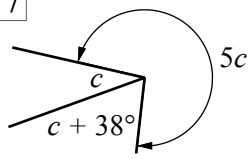
$3g = 192$

$\div 3$ $\div 3$

$g = 64°$

$2g = 128°$

17



$5c + c + c + 38 = 360$

$7c + 38 = 360$

-38 -38

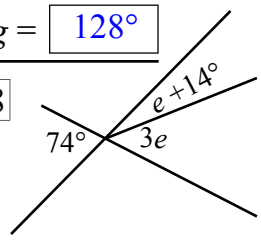
$7c = 322$

$\div 7$ $\div 7$

$c = 46°$

$5c = 230°$ $c + 38° = 84°$

18



$3e + e + 14 = 74$

$4e + 14 = 74$

-14 -14

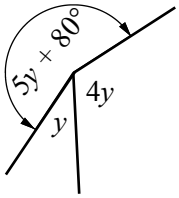
$4e = 60$

$\div 4$ $\div 4$

$e = 15$

$e + 14 = 29°$ $3e = 45°$

19



$5y + 80 + y + 4y = 360$

$10y + 80 = 360$

-80 -80

$10y = 280$

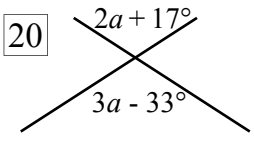
$\div 10$ $\div 10$

$y = 28°$

$4y = 112°$ $5y + 80 = 220°$

Unknowns on both sides now. Find all unknown angles.

20



$2a + 17 = 3a - 33$

-17 -17

$2a = 3a - 50$

$-3a$ $-3a$

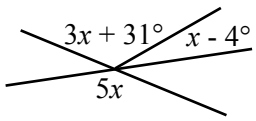
$-a = -50$

$a = 50°$

$2a + 17 = 117°$

$3a - 33 = 117°$

21



$3x + 31 + x - 4 = 5x$

$4x + 27 = 5x$

-27 -27

$4x = 5x - 27$

$-5x$ $-5x$

$-x = -27$

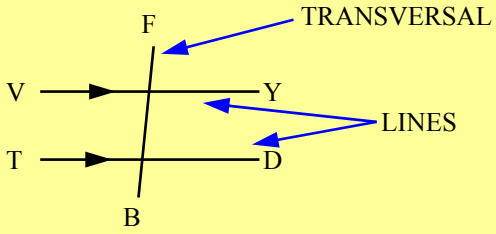
$x = 27°$

$5x = 135°$ $x - 4 = 23°$

$3x + 31 = 112°$

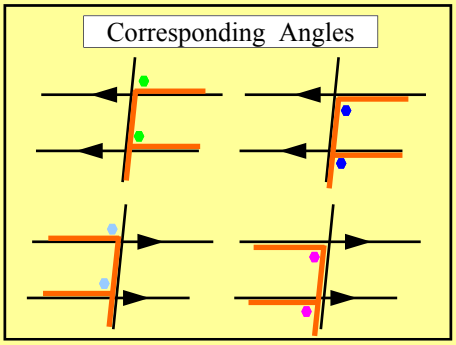
Parallel Line Properties

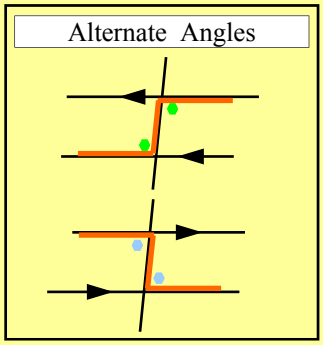
When two lines are parallel they are an equal distance apart for their full length and never touch. A transversal is a line that cuts across parallel lines, by crossing the lines 8 angles are created.

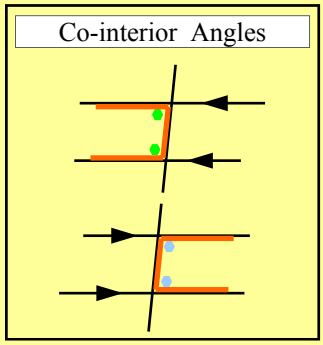


Lines are VY or YV and TD or DT
Transversal is either FB or BF

These angles can be paired off with each other in the following way:







Corresponding angles (called F angles) require one point to be on the inside of the parallel lines and the other to be on the outside. An 'F ' or back to front ‘F’ is made by the lines that include these angles.

Alternate angles (called Z angles) are also on the inside of the parallel lines but unlike co-interior angles they are on opposite sides of the transversal. A 'Z' can be made by the lines that include these angles.

Co-interior angles (called C angles) are the pair of angles formed on one side of the transversal inside the parallel lines, a 'c' can be formed around the angles (one is back to front). There are only two co-interior angle pairs in a standard 2-line 1 transversal exercise. Remember the word interior means on the inside (of the parallel lines and the transversal).

Column 1 of the worksheet asks you to name the transversal and parallel lines. Use the letters at the end of the lines, it doesn’t matter which way you name them. The next questions ask you to say whether the marked angles are corresponding, co-interior or alternate angles. So look at the positions of the square and circle and then choose. The last questions ask you to mark the corresponding (alternate or co-interior) angle by colouring in a circle.

Column 2 asks you to find the angles and state if they are corresponding etc. Corresponding and Alternate angles are equal when lines are parallel. Co-interior angles are supplementary, that is, given one angle subtract it from 180° to find the other.

Parallel Line Properties

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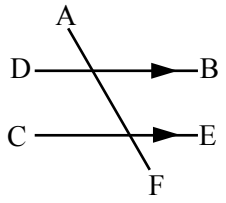
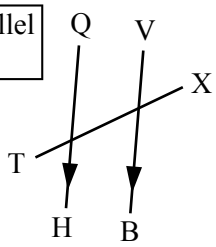
Name the transversal and the parallel lines using the letters supplied.

1



L: ____ L: ____ T: ____

2

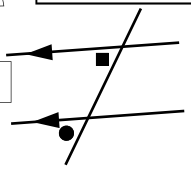


L: ____ L: ____

T: ____

Name the angles as either Corresponding, Alternate or Co-interior.

3



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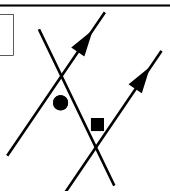

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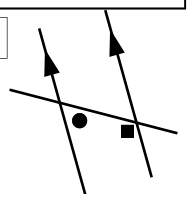

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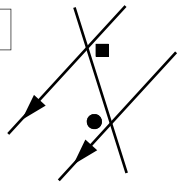

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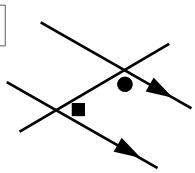

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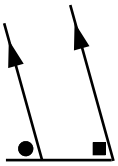

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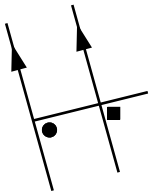

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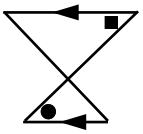

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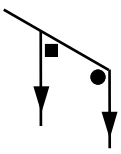

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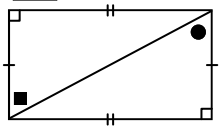

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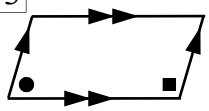

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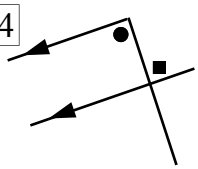
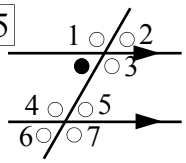


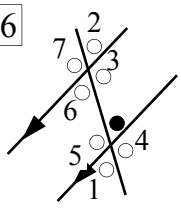
Show the appropriate angle by colouring the circle.

15



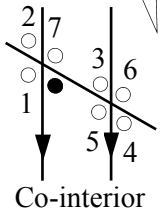
Corresponding

16



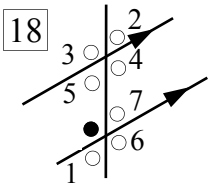
Alternate

17



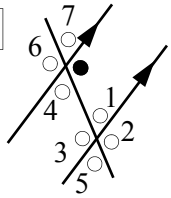
Co-interior

18



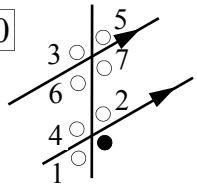
Alternate

19



Co-interior

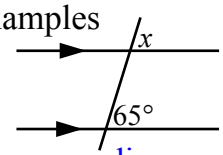
20



Corresponding

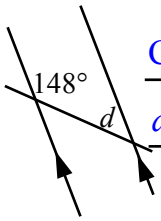
State if the angles are Corresponding, Alternate or Co-interior. Then give the value of the pronumeral.

Examples



Corresponding

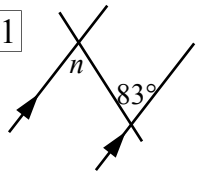
$x = 65°$



Co-interior

$d = 180 - 148 = 27°$

21



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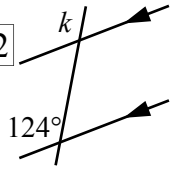

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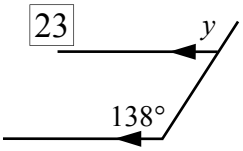

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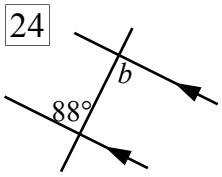

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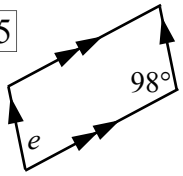

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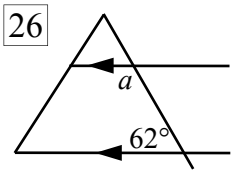

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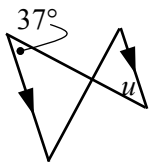

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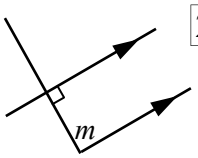

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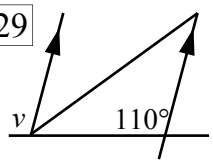
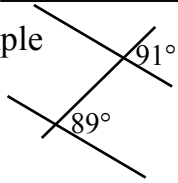


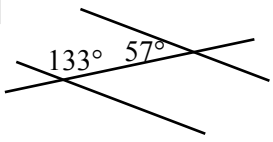
Are these lines parallel? Explain why they are or aren't, using angle properties in your description.

Example

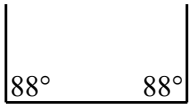


Lines aren't parallel because corresponding angles aren't equal.

30



31



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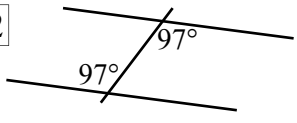

Parallel Line Properties

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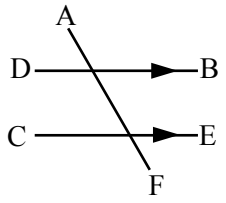
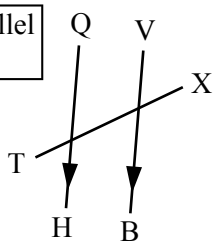
Name the transversal and the parallel lines using the letters supplied.

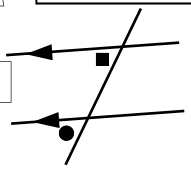
1 

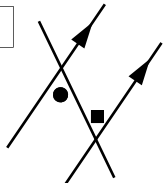
L: DB L: CE T: AF

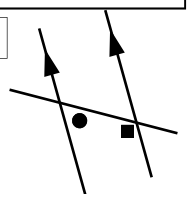
2 

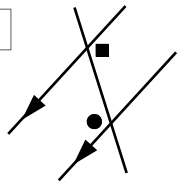
L: QH L: VB T: TX

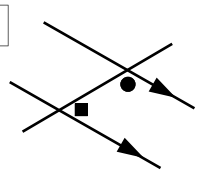
Name the angles as either Corresponding, Alternate or Co-interior.

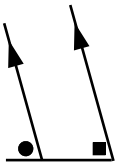
3 
Corresponding

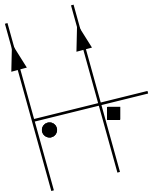
4 
Alternate

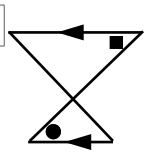
5 
Co-interior

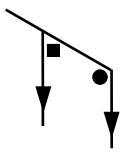
6 
Alternate

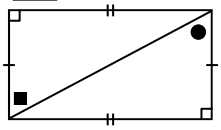
7 
Co-interior

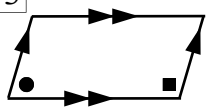
8 
Corresponding

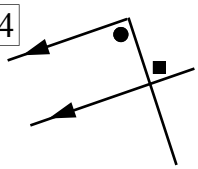
9 
Corresponding

10 
Alternate

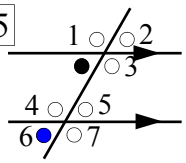
11 
Co-interior

12 
Alternate

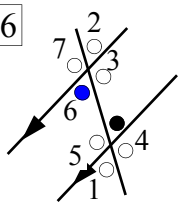
13 
Co-interior

14 
Alternate

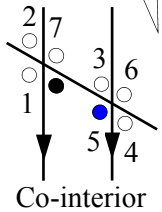
Show the appropriate angle by colouring the circle.

15 

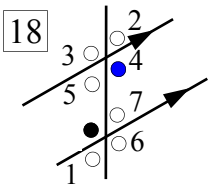
Corresponding

16 

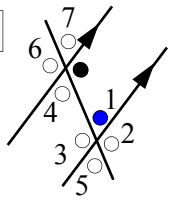
Alternate

17 

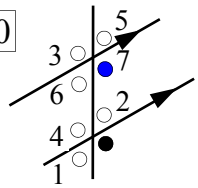
Co-interior

18 

Alternate

19 

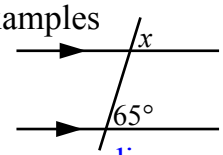
Co-interior

20 

Corresponding

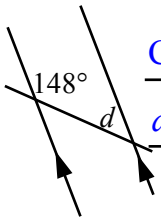
State if the angles are Corresponding, Alternate or Co-interior. Then give the value of the pronumeral.

Examples



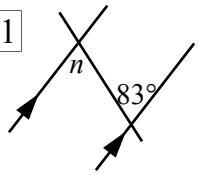
Corresponding

$x = 65°$



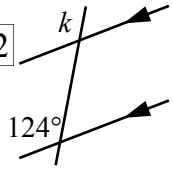
Co-interior

$d = 180 - 148 = 27°$

21 

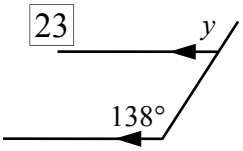
Alternate

$n = 83°$

22 

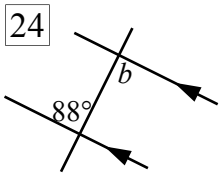
Corresponding

$k = 124°$

23 

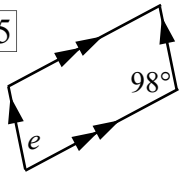
Corresponding

$y = 138°$

24 

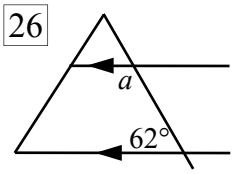
Alternate

$b = 88°$

25 

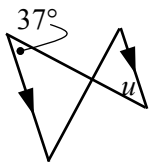
Co-interior

$e = 180 - 98 = 82°$

26 

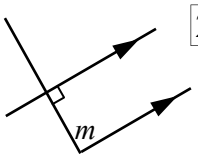
Co-interior

$a = 180 - 62 = 118°$

27 

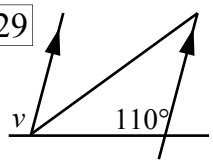
Alternate

$u = 37°$

28 

Co-interior

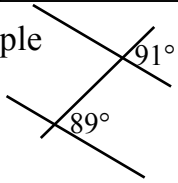
$m = 180 - 90 = 90°$

29 

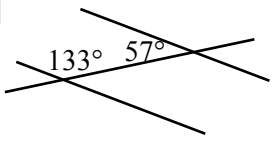
Corresponding

$v = 110°$

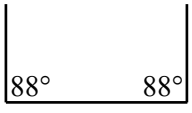
Are these lines parallel? Explain why they are or aren't, using angle properties in your description.

Example 

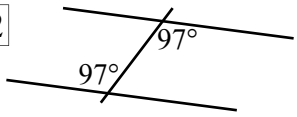
Lines aren't parallel because corresponding angles aren't equal.

30 

Lines aren't parallel because co-interior angle sum isn't 180°.

31 

Lines aren't parallel because co-interior angle sum isn't 180°.

32 

Lines are parallel as co-interior angles are equal.

Parallel Lines

The previous sheet dealt parallel line properties. When a transversal intersects a pair of parallel lines then co-interior, corresponding and alternate angles are formed. This sheet uses those properties to calculate other angles. The sheet also uses vertically opposite angles, angles at a point, supplementary angles and angle sum of a triangle.

The requirement is that you solve a question using the above properties, give your answer, then indicate which one of the properties you used to get the answer. In other words you are giving a reason why the angle is the size you have calculated.

There is an example at the top of the column, note that you can get the same answer by using different methods. For example a student may use alternate angles when another uses corresponding angles. Both can be correct, as they can be using a different angle to get to their answer.

Parallel Lines

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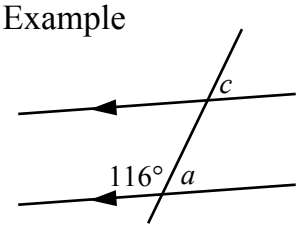
Find the unknown angles. Give a reason for how you found the angle using, for example: Co-interior angles, Corresponding angles, Alternate angles, Vertically opposite, Supplementary, etc.

Example



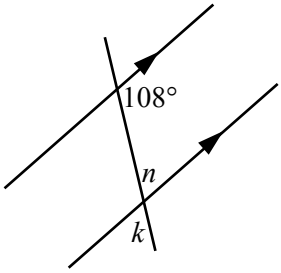
$a = 180 - 116 = 64°$

Supplementary angles.

$c = 64°$

Corresponding angles.

1



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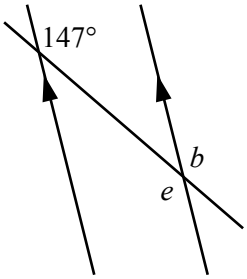

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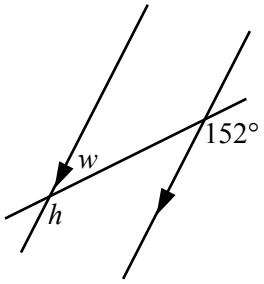

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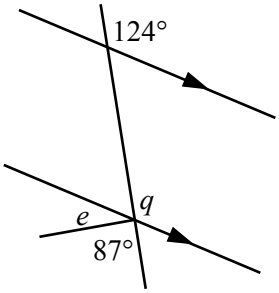

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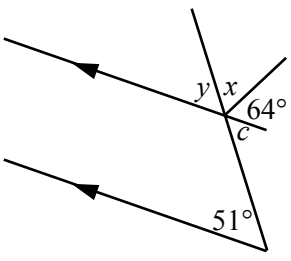

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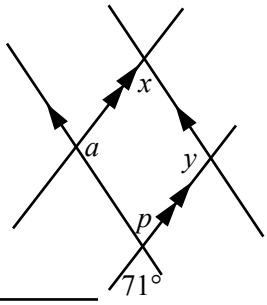

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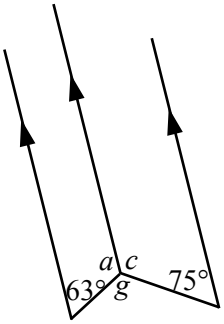

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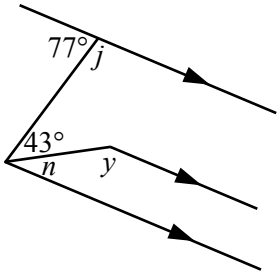

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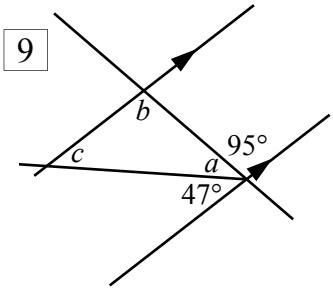

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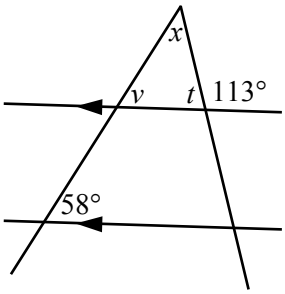

Parallel Lines

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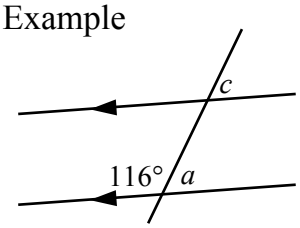
Find the unknown angles. Give a reason for how you found the angle using, for example: Co-interior angles, Corresponding angles, Alternate angles, Vertically opposite, Supplementary, etc.

Example



$a = 180 - 116 = 64°$

Supplementary angles.

$c = 64°$

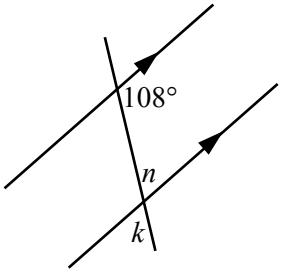
Corresponding angles.

1 $n = 180 - 108 = 72°$

Co-interior angles.

$k = 72°$

Vertically opposite n



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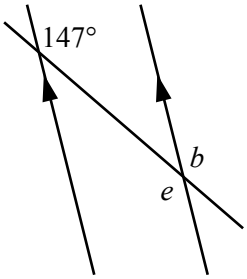


$e = 147°$

Alternate angles.

$b = 147°$

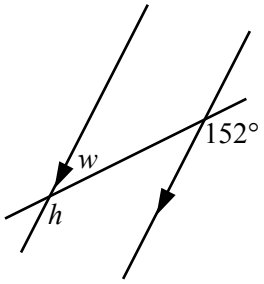
Vertically opposite e

3 $h = 152°$

Corresponding angles.

$w = 180 - 152 = 28°$

Supplementary angles.



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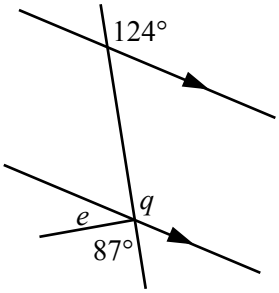


$q = 124°$

Corresponding angles.

$e = 124 - 87 = 37°$

Adjacent angles are vertically opposite q

5 $c = 51°$

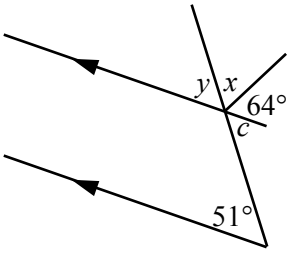
Alternate angles.

$y = 51°$

Vertically opposite c

$x = 180 - 64 - 51 = 65°$

Supplementary angles.



6 $p = 71°$

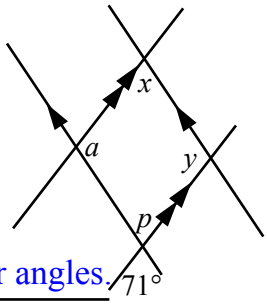
Vertically opposite.

$a = 180 - 71 = 109°$

Co-interior angles.

$y = 180 - 71 = 109°$ Co-interior angles.

$y = 180 - 109 = 71°$ Co-interior angles.



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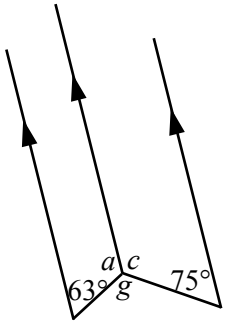


$c = 180 - 75 = 105°$

Co-interior angles.

$a = 180 - 63 = 117°$

$g = 360 - 117 - 105 = 138°$

Angle sum at a point is 360°.

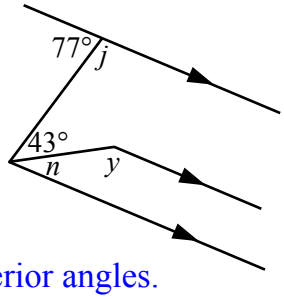
8 $j = 180 - 77 = 103°$

Supplementary angles.

$n = 77 - 43 = 34°$

Alternate angles.

$y = 180 - 34 = 146°$ Co-interior angles.



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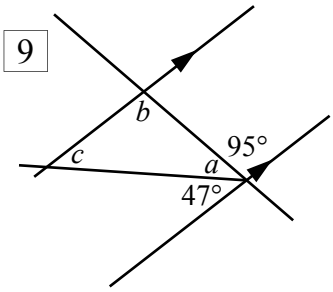


$a = 180 - 47 - 95 = 38°$

Supplementary angles.

$b = 95°$

Alternate angles.

$c = 47°$

Alternate angles.

10 $v = 58°$

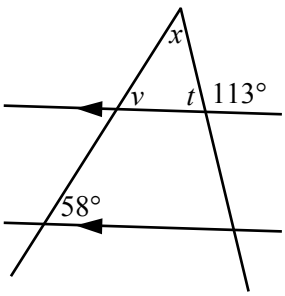
Corresponding angles.

$t = 180 - 113 = 67°$

Supplementary angles.

$x = 180 - 58 - 67 = 55°$

Angle sum of a triangle is 180°



Angles in Triangles

The angle sum of a triangle is 180°. This property holds true regardless of the type of triangle.

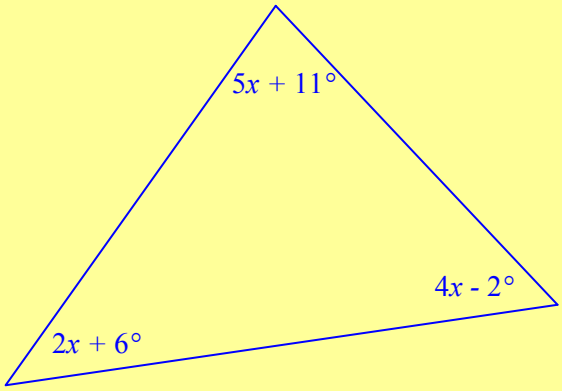
Column 1 requires you to find the missing angle. Two angles are always given, unless the triangle has a particular property such as equilateral triangles. Subtract the given angles from 180° to get the answer, (example at top of column). Note that you may have to use your vertically opposite and supplementary geometry skills.

Column 2 deals with isosceles triangles, base angles are the same with isosceles triangles. Place the same number in the empty corner then you have your two angles. Then subtract the angles from 180°.

In column 2 from question 11 the second type of problem is encountered. This time you are given the non-paired angle. This time 2 × (the letter) + the given angle = 180°. So a 2-Step equation is used to solve these. Question 11 guides you through the process.

The last two questions in Column 3 require you to collect like terms, then solve the resulting 2 -Step equation. An example is below.

Example - Find the angles in the triangle below.



$2x + 6 + 5x + 11 + 4x - 2 = 180$ ← Sum of internal angles = 180°

$11x + 15 = 180$ ← Collect like terms

$-15 \quad -15$

$11x = 165$ ← Remove the constant

$\div 11 \quad \div 11$

$x = 15°$ ← Remove the coefficient

$2x + 6 = 36°$ $\quad 5x + 11 = 86°$ ← Now find the individual angles

$4x - 2 = 58°$

Use your calculator to add the three angles to make sure that the angle sum is 180°.

Angles in Triangles

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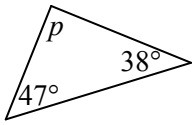
Angles in triangles sum to 180°. Find the value of the unknown in these triangles.

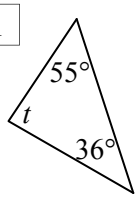
Example



$p = 180 - 47 - 38$

$p = 95°$

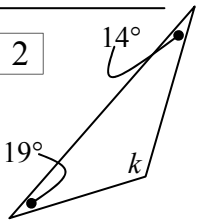
1



1 → $t = 180 - \square - \square$

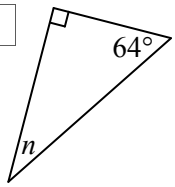
$t =$

2



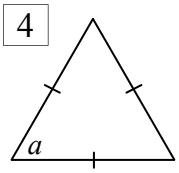
2 →

3



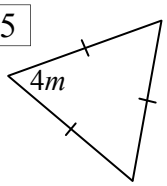
3 →

4



4 →

5



5 →

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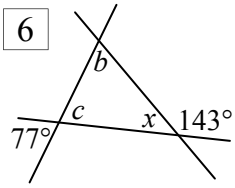
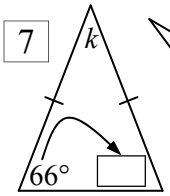


6 →

For these isosceles triangles, write in the matching angle, then find the unknown value.

7



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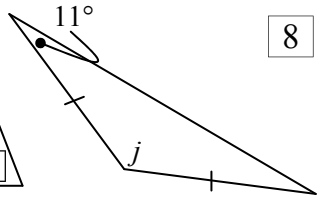
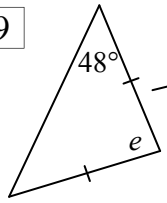


7 → $k = 180 - \square - \square$

$k =$

8 →

9



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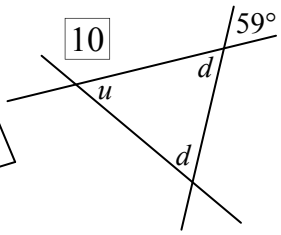
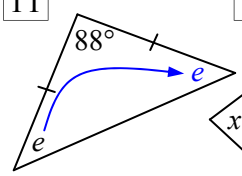


9 →

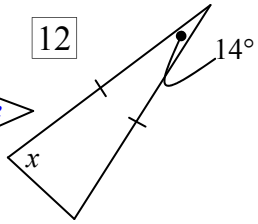
10 →

Use 2-step equations to find the value of the unknown.

11



12



11 → $2e + 88 = 180$

$-88 \quad -88$

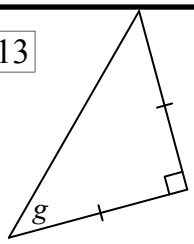
$2e = \square$

$\div 2 \quad \div 2$

$e =$

12 →

13



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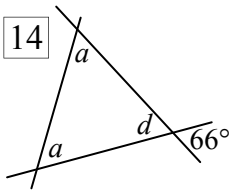
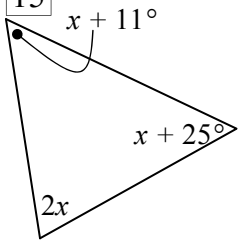


13 →

14 →

These are more complicated. Find all angles in the triangles.

15



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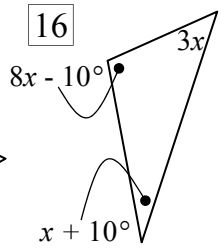

$= 180$

$2x =$ $x+11° =$ $x+25° =$

16 →

$3x =$ $x+10° =$ $8x-10° =$

Angles in Triangles

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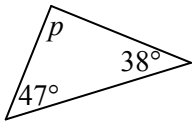
Angles in triangles sum to 180°. Find the value of the unknown in these triangles.

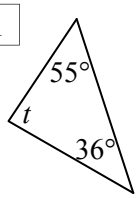
Example



$p = 180 - 47 - 38$

$p = 95°$

1



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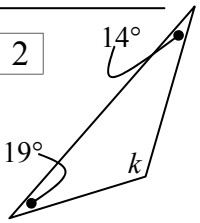


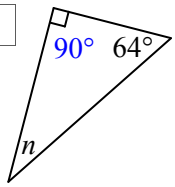
1 → $t = 180 - \boxed{55} - \boxed{36}$

$t = 89°$

2 → $k = 180 - 14 - 19$

$k = 147°$

3



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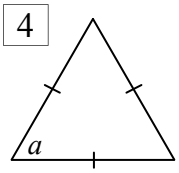


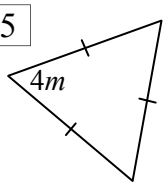
3 → $n = 180 - 90 - 64$

$n = 26°$

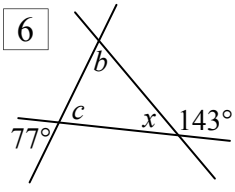
4 → $a = 60°$ or $3a = 180$

$a = 60°$

5



6



5 → $4m = 60°$

$m = 15°$

6 → $c = 77°$

$x = 180 - 143$

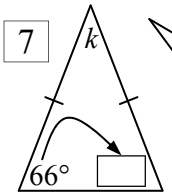
$x = 37°$

$b = 180 - 77 - 37$

$b = 66°$

For these isosceles triangles, write in the matching angle, then find the unknown value.

7



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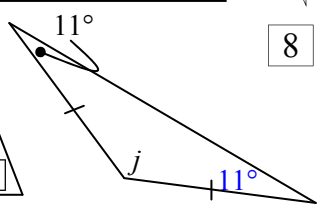


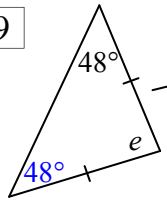
7 → $k = 180 - \boxed{66} - \boxed{66}$

$k = 48°$

8 → $j = 180 - 11 - 11$

$j = 158°$

9



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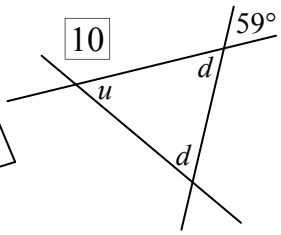


9 → $e = 180 - 48 - 48$

$e = 84°$

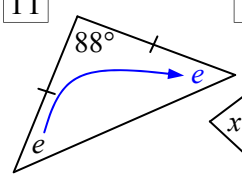
10 → $d = 59°$

$u = 180 - 59 - 59$

$u = 62°$

Use 2-step equations to find the value of the unknown.

11



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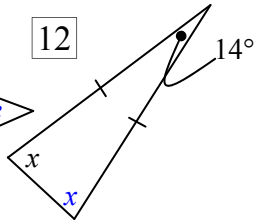


11 → $2e + 88 = 180$

$-88 \quad -88$

$2e = \boxed{92}$

$\div 2 \quad \div 2$

$e = 46°$

12 → $2x + 14 = 180$

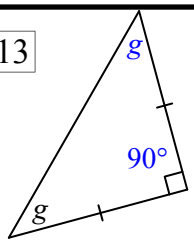
$-14 \quad -14$

$2x = 166$

$\div 2 \quad \div 2$

$x = 83°$

13



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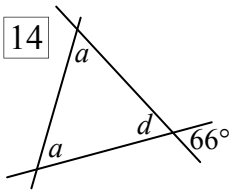


13 → $2g + 90 = 180$

$-90 \quad -90$

$2g = 90$

$\div 2 \quad \div 2$

$g = 45°$

14 → $d = 66°$

$2a + 66 = 180$

$-66 \quad -66$

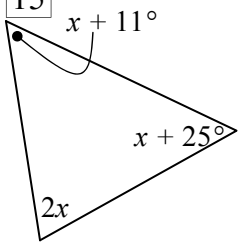
$2a = 114$

$\div 2 \quad \div 2$

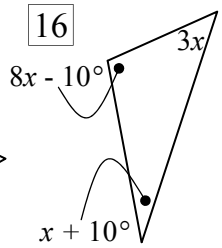
$a = 57°$

These are more complicated. Find all angles in the triangles.

15



16



$x + 11 + x + 25 + 2x = 180$

$4x + 36 = 180$

$-36 \quad -36$

$4x = 144$

$\div 4 \quad \div 4$

$x = 36°$

$2x = 72°$ $x+11° = 45°$ $x+25° = 61°$

16 →

$x + 10 + 3x + 8x - 10 = 180$

$12x = 180$

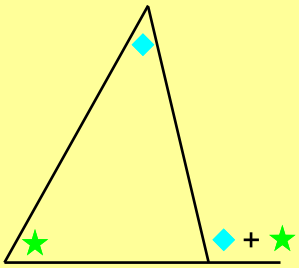
$\div 12 \quad \div 12$

$x = 15°$

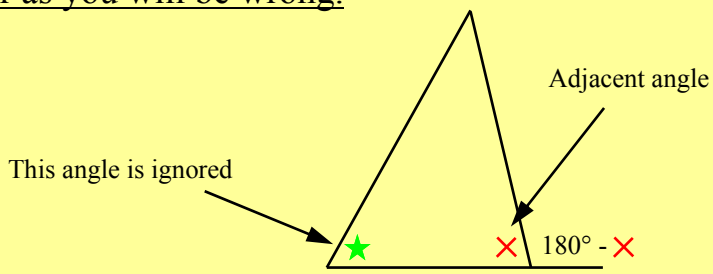
$3x = 45°$ $x+10° = 25°$ $8x-10° = 110°$

Exterior Angles of a Triangle

There is a relationship between the exterior angle of a triangle and the 2 opposite interior angles, the sum of the 2 opposite interior angles = the exterior angle. In other words when you add the 2 opposite interior angles you get the exterior angle. The diagram below shows this graphically. It is important that you realise that the two opposite angles aren't adjacent to (don't touch) the exterior angle.



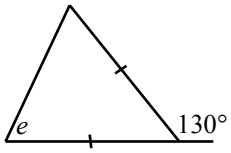
Column 1 Q.1 - 5 require you to add the two interior angles together to get the exterior angle. The adjacent angle is kept out of these questions. An example is at the top of the column. The next 3 questions show that if you have the adjacent internal angle you just take the supplement to get the external angle (subtract from 180°). So if the adjacent angle is one of the 2 angles, then just take the supplement (subtract it from 180°) of the adjacent angle. This is where the most common mistake is made, if the adjacent angle is given don't add it to the other angle to get your answer as you will be wrong.



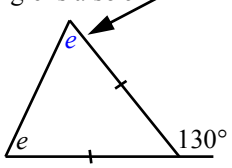
Column 2 works in reverse, you are given the exterior angle and have to calculate the missing interior angle. Do these by subtracting the given interior angle from the exterior angle, see the example at the top of the column.

The third column is challenging as it requires you to use your skills with isosceles and equilateral triangles as well as algebra. There is an example below.

Example : Find the angle



Because the triangle is isosceles the other base angle is also e



Solution:

$2e = 130°$ (opposite angle sum = exterior)

$e = 65°$

Exterior Angles of a Triangle

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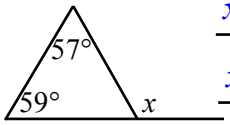
Find the exterior angle for the following triangles.

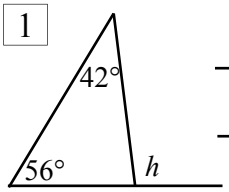
Example



$x = 57 + 59$

$x = 116°$

1



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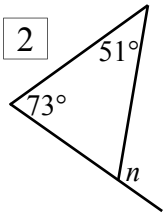

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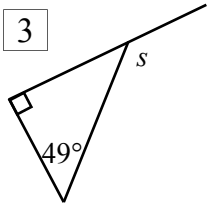

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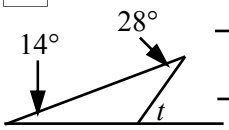

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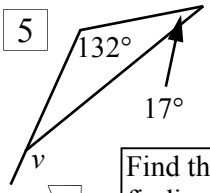
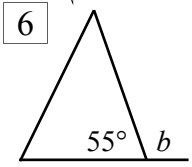


Find the exterior angle by finding the supplement, ignore any extra angles.

6



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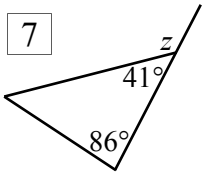

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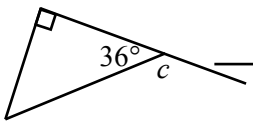
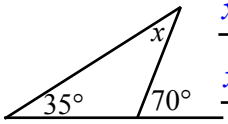


Now the exterior angle is given, use subtraction to find the unknown angle.

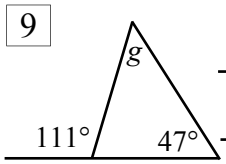
Example



$x = 70 - 35$

$x = 35°$

9



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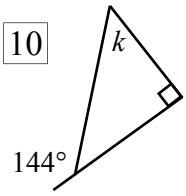

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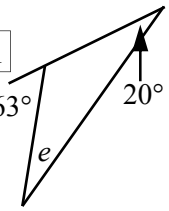

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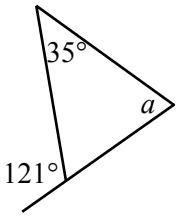

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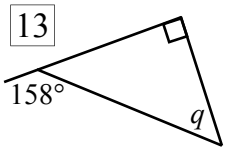

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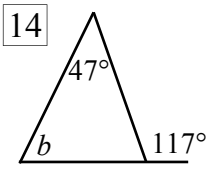

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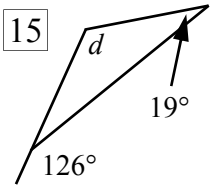

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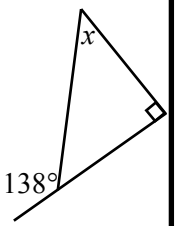

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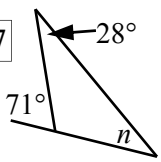
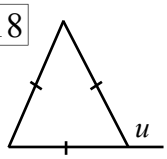


Use your knowledge of triangle properties to find the value of the angles

18



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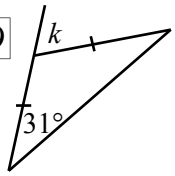

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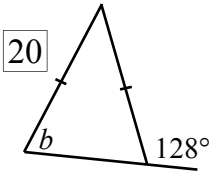
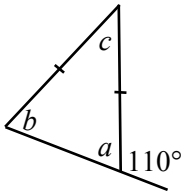


21 Hint: Find a then b then c



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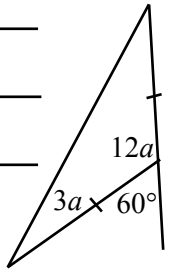

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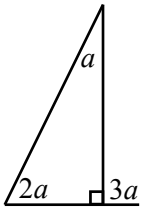

Exterior Angles of a Triangle

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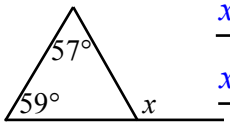
Find the exterior angle for the following triangles.

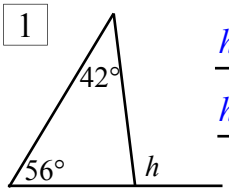
Example



$x = 57 + 59$

$x = 116°$

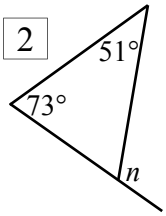
1



$h = 56 + 42$

$h = 98°$

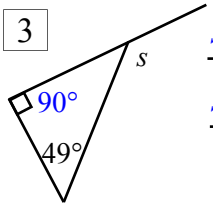
2



$n = 51 + 73$

$n = 124°$

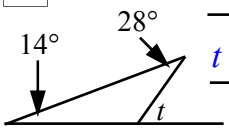
3



$s = 90 + 49$

$s = 139°$

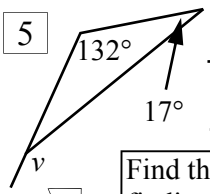
4



$t = 14 + 28$

$t = 42°$

5

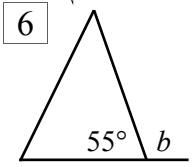


$v = 132 + 17$

$v = 149°$

Find the exterior angle by finding the supplement, ignore any extra angles.

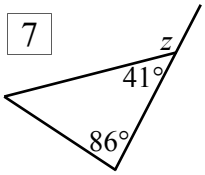
6



$b = 180 - 55$

$b = 125°$

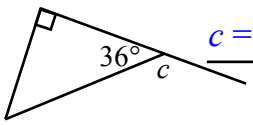
7



$z = 180 - 41$

$z = 139°$

8

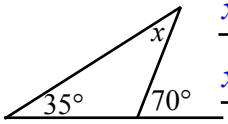


$c = 180 - 36$

$c = 144°$

Now the exterior angle is given, use subtraction to find the unknown angle.

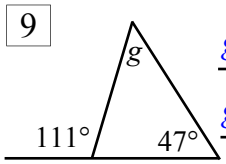
Example



$x = 70 - 35$

$x = 35°$

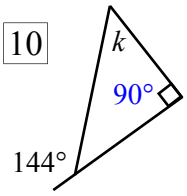
9



$g = 111 - 47$

$g = 64°$

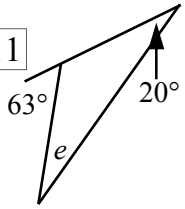
10



$k = 144 - 90$

$k = 54°$

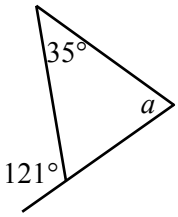
11



$e = 63 - 20$

$e = 43°$

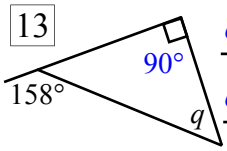
12



$a = 121 - 35$

$a = 86°$

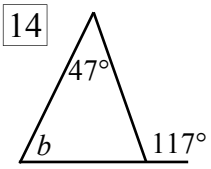
13



$q = 158 - 90$

$q = 68°$

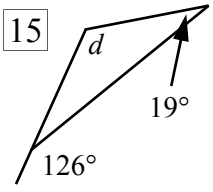
14



$b = 117 - 47$

$b = 70°$

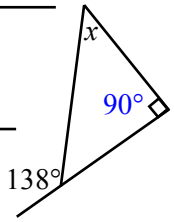
15



$d = 126 - 19$

$d = 107°$

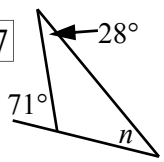
16



$x = 138 - 90$

$x = 48°$

17

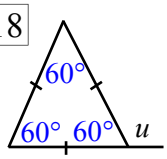


$n = 71 - 28$

$n = 43°$

Use your knowledge of triangle properties to find the value of the angles

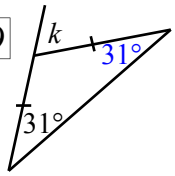
18



$u = 60 + 60$

$u = 120°$

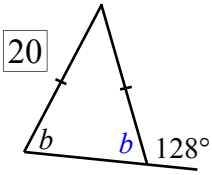
19



$k = 31 + 31$

$k = 62°$

20



$b = 180 - 128$

$b = 52°$

21 Hint: Find a then b then c

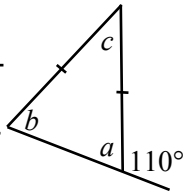
$a = 180 - 110$

$a = 70°$

$b = 70°$ (isos)

$c = 110 - 70$

$c = 40°$



22

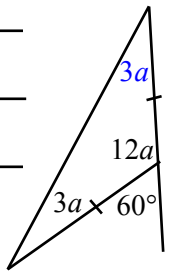
$3a + 3a = 6a$

$6a = 60$

$a = 10°$

$3a = 30°$

$12a = 120°$



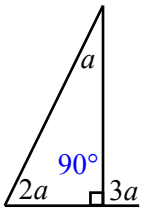
23

$3a = 180 - 90$

$3a = 90$

$a = 30°$

$2a = 60°$



Angles in Quadrilaterals

The angle sum of a quadrilateral is 360°, this means that when all the angles inside the shape are added, they total 360°. If you are given 3 angles then to find the missing angle subtract the 3 angles from 360° and what is left is the your answer.

In column 1 through to question 11 the exercises are all done the same way, you are given 3 angles (remember a right angle is 90°), subtract these from 360°. If you write in '90°' in the space next to the right angle sign you have less chance of forgetting it in your calculations. The question at the top of the 1st column shows the setting out for the exercises.

The rest of column 2 involves a small amount of algebra. The exercises are done in the same way as the first column in that the angles total 360°. But a 2-step equation is to be solved.

Column 3 is the same as the first column only one of the angles given is an exterior angle. To change the exterior angle to an interior angle subtract it from 360°. Then you have 3 interior angles which subtracted from 360° gives you the missing angle. The most common mistake is forgetting to change the exterior angle it to an interior angle in these questions.

Angles in Quadrilaterals

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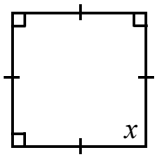
Find the missing angle in these quadrilaterals

1 $x = 360 - 90 - 90 - 90$

$x =$ ____ °



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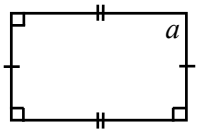

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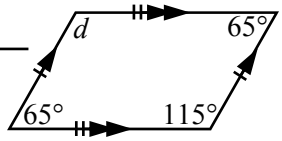

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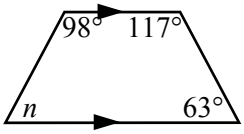

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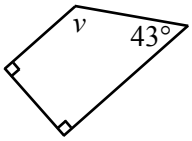

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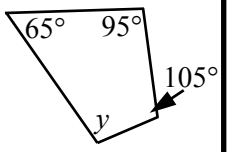

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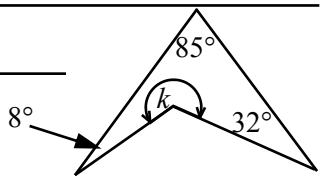

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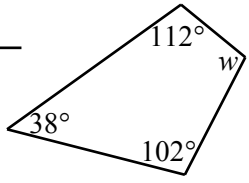

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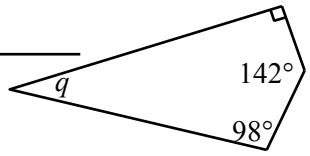

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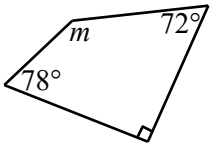

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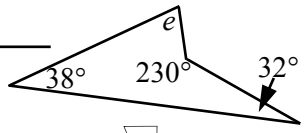


These have 2 unknown angles

12 $t + \square + t + \square = 360$

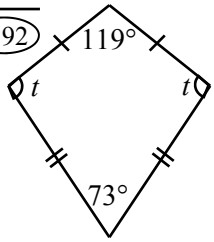
$2t + \square = 360$

-192 -192

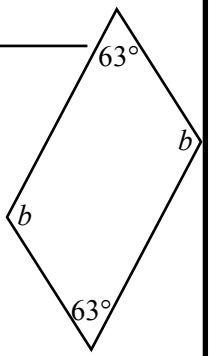
$2t = \square$

÷2 ÷2

$t = \square$ °



13



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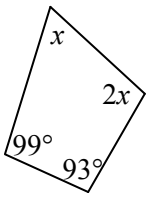
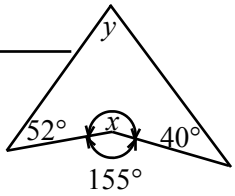


$2x =$ ____

Convert the exterior angles to interior angles to find x then solve for y.

15 $x = 360 -$ ____

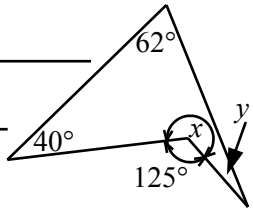
$x =$ ____



$y =$ ____

$y =$ ____

16



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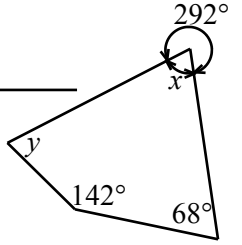

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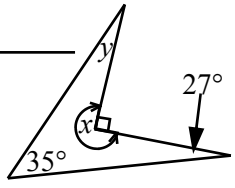
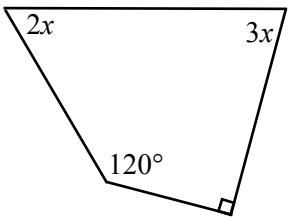


Find the 2 angles

19



$2x =$ ____

$3x =$ ____

Angles in Quadrilaterals

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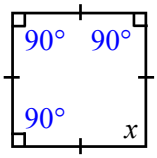
Find the missing angle in these quadrilaterals

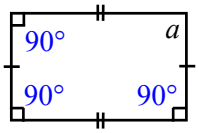
1 $x = 360 - 90 - 90 - 90$

$x = 90°$



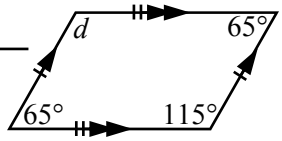
2 $a = 360 - 90 - 90 - 90$

$a = 90°$



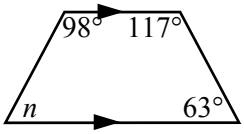
3 $d = 360 - 65 - 115 - 65$

$d = 115°$



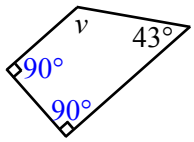
4 $n = 360 - 98 - 117 - 63$

$n = 82°$



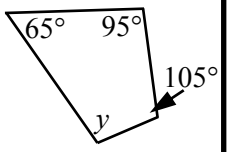
5 $v = 360 - 43 - 90 - 90$

$v = 137°$



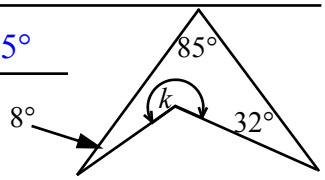
6 $y = 360 - 105 - 65 - 95$

$y = 95°$



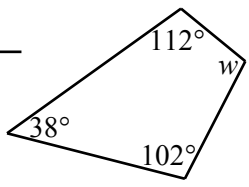
7 $k = 360 - 8 - 32 - 65$

$k = 255°$



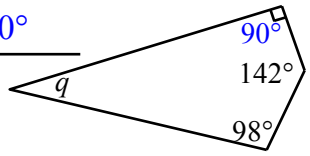
8 $w = 360 - 112 - 38 - 102$

$w = 108°$



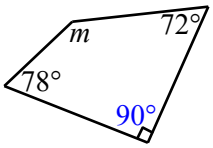
9 $q = 360 - 90 - 142 - 98$

$q = 30°$



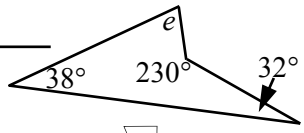
10 $m = 360 - 78 - 90 - 72$

$m = 120°$



11 $e = 360 - 38 - 32 - 230$

$e = 60°$



These have 2 unknown angles

12 $t + \boxed{73} + t + \boxed{119} = 360$

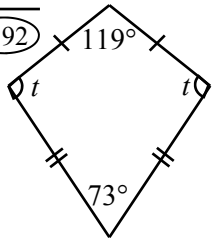
$2t + \boxed{192} = 360$

(-192) (-192)

$2t = \boxed{168}$

(÷2) (÷2)

$t = \boxed{84°}$



13 $b + 63 + b + 63 = 360$

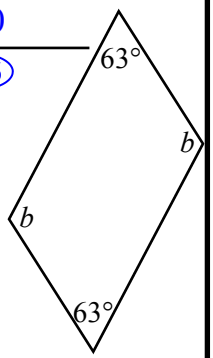
$2b + 126 = 360$

(-126) (-126)

$2b = 234$

(÷2) (÷2)

$b = 117°$



14 $x + 99 + 93 + 2x = 360$

$3x + 192 = 360$

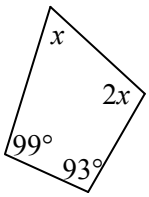
(-192) (-192)

$3x = 168$

(÷3) (÷3)

$x = 56°$

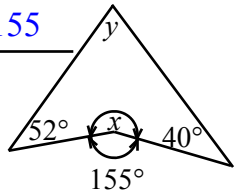
$2x = 112°$



Convert the exterior angles to interior angles to find x then solve for y.

15 $x = 360 - 155$

$x = 205°$

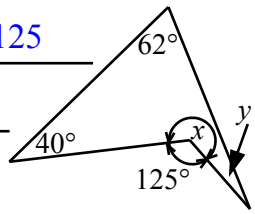


$y = 360 - 205 - 52 - 40$

$y = 63°$

16 $x = 360 - 125$

$x = 235°$

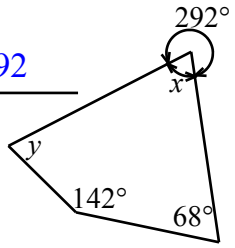


$y = 360 - 235 - 40 - 62$

$y = 23°$

17 $x = 360 - 292$

$x = 68°$

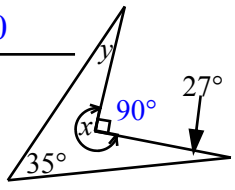


$y = 360 - 68 - 68 - 142$

$y = 82°$

18 $x = 360 - 90$

$x = 270°$

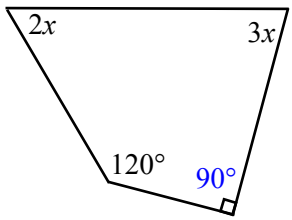


$y = 360 - 270 - 27 - 35$

$y = 28°$

Find the 2 angles

19



$2x + 3x + 90 + 120 = 360$

$5x + 210 = 360$

(-210) (-210)

$5x = 150°$

(÷5) (÷5)

$x = 30°$

$2x = 60°$

$3x = 90°$

Geometric Constructions - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Geometric Constructions - Find A Word

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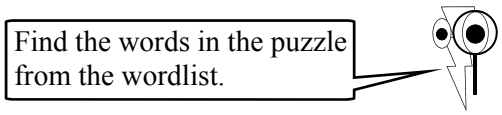

D	E	G	R	E	E	X	L	E	L	L	A	R	A	P
S	X	E	T	R	I	A	N	G	L	E	X	Y	R	I
U	S	T	R	A	I	G	H	T	H	G	I	R	E	S
P	N	C	A	L	T	E	R	N	A	T	E	A	F	O
P	O	E	N	X	R	E	X	N	Y	U	Q	T	L	S
L	I	S	S	E	L	F	G	E	A	A	U	N	E	C
E	T	I	V	Y	L	L	A	C	I	T	R	E	V	E
M	U	B	E	R	E	N	U	D	A	U	Q	M	E	L
E	L	U	R	S	G	T	E	X	E	L	F	E	R	E
N	O	A	S	Q	E	E	S	U	T	B	O	L	T	S
T	V	D	A	D	J	A	C	E	N	T	B	P	E	G
A	E	X	L	E	T	I	S	O	P	P	O	M	X	R
R	R	T	C	E	S	R	E	T	N	I	U	O	Q	E
Y	G	E	G	R	E	E	R	G	N	S	N	C	E	E
Q	U	A	D	R	I	L	A	T	E	R	A	L	R	G

WORDLIST

SUPPLEMENTARY	VERTICALLY	ALTERNATE	OPPOSITE	TRIANGLE
QUADRILATERAL	ACUTE	INTERSECT	DEGREE	BISECT
COMPLEMENTARY	PARALLEL	STRAIGHT	OBTUSE	RAY
REVOLUTION	REFLEX	SUM	ANGLES	POINT
TRANSVERSAL	ADJACENT	ISOSCELES	VERTEX	RIGHT

Geometric Constructions - Find A Word

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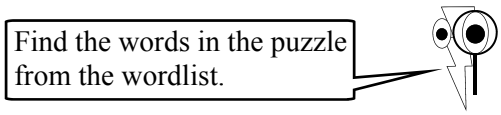

D	E	G	R	E	E	X	L	E	L	L	A	R	A	P
S	X	E	T	R	I	A	N	G	L	E	X	Y	R	I
U	S	T	R	A	I	G	H	T	H	G	I	R	E	S
P	N	C	A	L	T	E	R	N	A	T	E	A	F	O
P	O	E	N	X	R	E	X	N	Y	U	Q	T	L	S
L	I	S	S	E	L	F	G	E	A	A	U	N	E	C
E	T	I	V	Y	L	L	A	C	I	T	R	E	V	E
M	U	B	E	R	E	N	U	D	A	U	Q	M	E	L
E	L	U	R	S	G	T	E	X	E	L	F	E	R	E
N	O	A	S	Q	E	E	S	U	T	B	O	L	T	S
T	V	D	A	D	J	A	C	E	N	T	B	P	E	G
A	E	X	L	E	T	I	S	O	P	P	O	M	X	R
R	R	T	C	E	S	R	E	T	N	I	U	O	Q	E
Y	G	E	G	R	E	E	R	G	N	S	N	C	E	E
Q	U	A	D	R	I	L	A	T	E	R	A	L	R	G

WORDLIST

~~SUPPLEMENTARY~~ ~~VERTICALLY~~ ~~ALTERNATE~~ ~~OPPOSITE~~ ~~TRIANGLE~~

~~QUADRILATERAL~~ ~~ACUTE~~ ~~INTERSECT~~ ~~DEGREE~~ ~~BISECT~~

~~COMPLEMENTARY~~ ~~PARALLEL~~ ~~STRAIGHT~~ ~~OBTUSE~~ ~~RAY~~

~~REVOLUTION~~ ~~REFLEX~~ ~~SUM~~ ~~ANGLES~~ ~~POINT~~

~~TRANSVERSAL~~ ~~ADJACENT~~ ~~ISOSCELES~~ ~~VERTEX~~ ~~RIGHT~~

FREEFALL MATHEMATICS

ALTITUDE

RATES & RATIOS

Ratios - Introduction

Ratios are a comparison. For example the number of steps to the height of the stairs. This sheet will deal with comparing lengths, numbers of objects etc. So you will measure or count and write the number in the box. So the numbers in the ratio will match the amount counted or measured, once you learn about simplifying ratios you will learn that the ratio doesn't necessarily match the amount. So this sheet is just an introduction.

Column 1 starts with counting squares that make a shape. There are shaded, unshaded and the total number of both that build the shape. Count the squares and write the answer in the box in the same order as the 'worded' ratio, ratios can have a larger first number and smaller second number, or the reverse. Ratios aren't limited to two numbers, they can compare an unlimited amount of items. Measure the lines in column 1, write the length in the box then compare using ratios.

The rest of the sheet is answered in the same way. The last questions in column 3 ask you to write ratios in fraction form. The first number in the ratio is the numerator, the second number the denominator.

There is a similarity between fractions and ratios. Both are simplified in the same way, and equivalence also is found using the same method.

Ratios - Introduction

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Give ratios for the following.

1

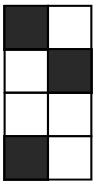


Shaded : Total = ☐ : ☐

Shaded : Unshaded = ☐ : ☐

Total : Unshaded = ☐ : ☐

2

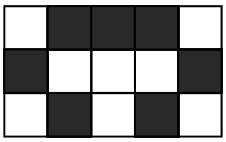


Shaded : Total = ☐ : ☐

Shaded : Unshaded = ☐ : ☐

Total : Unshaded = ☐ : ☐

3

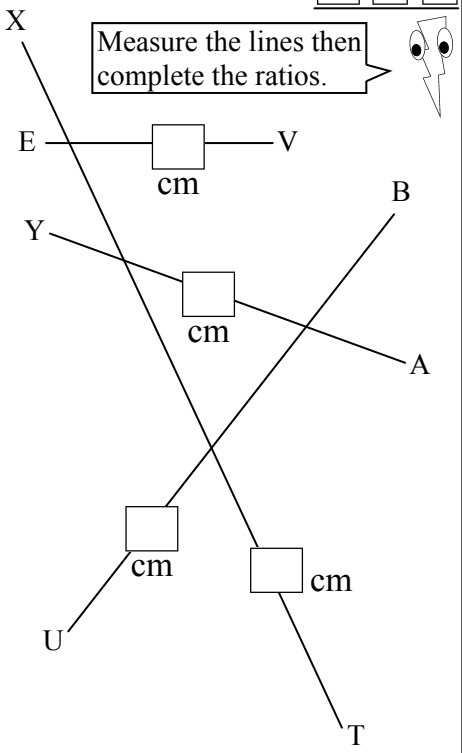


Shaded : Unshaded = ☐ : ☐

Total : Unshaded = ☐ : ☐

Total : Shaded : Unshaded = ☐ : ☐ : ☐

Measure the lines then complete the ratios.



4 TX : UB = ☐ : ☐ No Units required

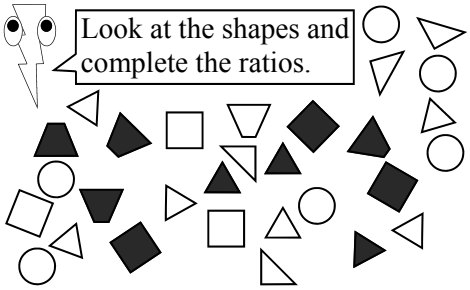
5 AY : EV = ☐ : ☐

6 BU : VE = ☐ : ☐

7 YA : BU = ☐ : ☐

8 XT : EV = ☐ : ☐

Look at the shapes and complete the ratios.



9 Squares : Triangles = ☐ : ☐

10 Circles : Trapeziums = ☐ : ☐

11 Right Triangles : Equilateral Triangles

= ☐ : ☐

12 Black Squares : White Triangles

= ☐ : ☐

13 Black Shapes : White Shapes

= ☐ : ☐

14 Shapes : Shapes With a Right Angle

= ☐ : ☐

15 Black Triangles : White Squares

= ☐ : ☐

A class is comprised of 13 boys and 16 girls. Find these ratios.

16 Girls : Boys = ☐ : ☐

17 Girls : Students = ☐ : ☐

18 Students : Boys = ☐ : ☐

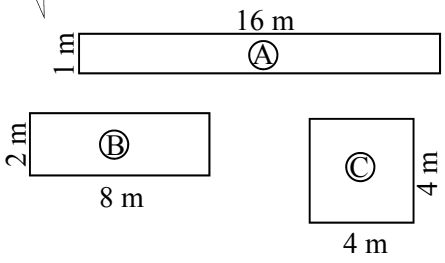
19 Teacher : Students = ☐ : ☐

A school has a teacher : student ratio average of 1 : 24. Answer these questions.

20 Are there more students or more teachers?

21 If another school has a ratio of 1 : 22, are there more teachers at that school?

The rectangles and squares have the same area. Find the ratios.



22 Area of A : Perimeter of A

= ☐ : ☐

23 Area of B : Perimeter of B

= ☐ : ☐

24 Area of C : Perimeter of C

= ☐ : ☐

25 Perimeter of C : Perimeter of A

= ☐ : ☐

26 Perimeter of B : Perimeter of A

= ☐ : ☐

Ratios can also be expressed as fractions. Express as fractions.

Kwa's pencil case has 5 blue pens, 2 red pens, 4 black pens and a green pen. Complete these ratios:

27 Blue : Black = ☐/☐

28 Blue : Red = ☐/☐

29 Green : Red = ☐/☐

30 Blue : All = ☐/☐

31 Green & Red : All = ☐/☐

32 Blue : Green & Blue = ☐/☐

33 Not Blue : Not Green = ☐/☐

Ratios - Introduction

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Give ratios for the following.

1

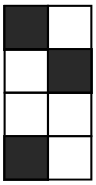


Shaded : Total = 2 : 3

Shaded : Unshaded = 2 : 1

Total : Unshaded = 3 : 1

2

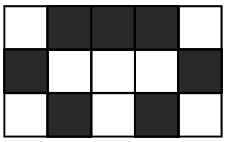


Shaded : Total = 3 : 8

Shaded : Unshaded = 3 : 5

Total : Unshaded = 8 : 5

3

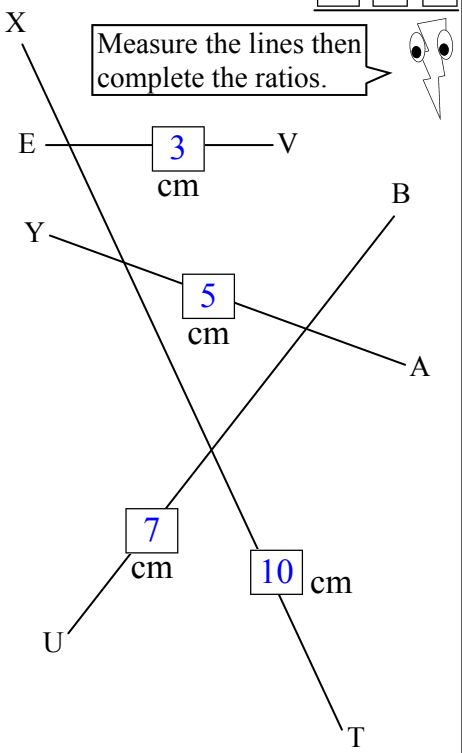


Shaded : Unshaded = 7 : 8

Total : Unshaded = 15 : 8

Total : Shaded : Unshaded = 15 : 7 : 8

Measure the lines then complete the ratios.



4 TX : UB = 10 : 7 (No Units required)

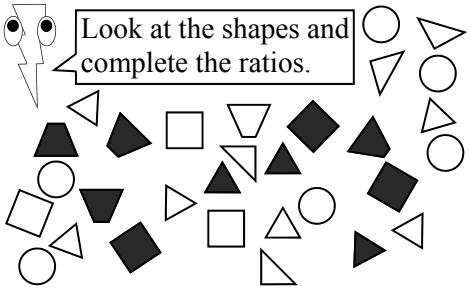
5 AY : EV = 5 : 3

6 BU : VE = 7 : 3

7 YA : BU = 5 : 7

8 XT : EV = 10 : 3

Look at the shapes and complete the ratios.



9 Squares : Triangles = 6 : 13

10 Circles : Trapeziums = 6 : 5

11 Right Triangles : Equilateral Triangles = 4 : 9

12 Black Squares : White Triangles = 3 : 10

13 Black Shapes : White Shapes = 10 : 20

14 Shapes : Shapes With a Right Angle = 30 : 10

15 Black Triangles : White Squares = 3 : 3

A class is comprised of 13 boys and 16 girls. Find these ratios.

16 Girls : Boys = 16 : 13

17 Girls : Students = 16 : 29

18 Students : Boys = 29 : 13

19 Teacher : Students = 1 : 29

A school has a teacher : student ratio average of 1 : 24. Answer these questions.

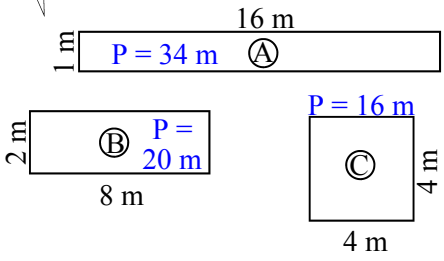
20 Are there more students or more teachers?

There are more students.

21 If another school has a ratio of 1 : 22, are there more teachers at that school?

Can't tell, ratios don't indicate.

The rectangles and squares have the same area. Find the ratios.



22 Area of A : Perimeter of A = 16 : 34

23 Area of B : Perimeter of B = 16 : 20

24 Area of C : Perimeter of C = 16 : 16

25 Perimeter of C : Perimeter of A = 16 : 20

26 Perimeter of B : Perimeter of A = 20 : 32

Ratios can also be expressed as fractions. Express as fractions.

Kwa's pencil case has 5 blue pens, 2 red pens, 4 black pens and a green pen. Complete these ratios:

27 Blue : Black = $\frac{5}{4}$

28 Blue : Red = $\frac{5}{2}$

29 Green : Red = $\frac{1}{2}$

30 Blue : All = $\frac{5}{12}$

31 Green & Red : All = $\frac{3}{12}$

32 Blue : Green & Blue = $\frac{5}{6}$

33 Not Blue : Not Green = $\frac{7}{11}$

Simplifying Ratios - No Decimals/Fractions

There are two sheets in this topic, the second sheet being more complex.
It is suggested that only sheet should be chosen as the first column is similar in both.

Ratios compare two (or more) values. Simplifying ratios is important because ratios could be applied to other amounts and applying a complicated ratio makes things harder. So how do you simplify a ratio? Ratios can be simplified using the same process as with fractions. Pick the HCF (Highest Common Factor) and divide through.

Example - Simplifying Ratios

$$9 : 15 \quad \underbrace{\boxed{3}}_{\text{HCF}} = \underbrace{\boxed{3} : \boxed{5}}_{\substack{9 \div \text{HCF} = 9 \div 3 \\ 15 \div \text{HCF} = 15 \div 3}}$$

Column 1 starts with questions that ask you to find the HCF. Look at the two numbers and ask yourself what is the largest number that divides into both numbers. Q.14 onwards asks you to find the HCF and then divide through, the process is shown above. This is how you simplify ratios. Note that ratios can have more than two numbers, ratios with three numbers are shown in the last column.

Column 2 introduces use of the ratios with shapes and line measurement. Count the shaded squares, that is the first number in the ratio. Then count the squares that make the shape, both shaded and unshaded. This is the second number in the ratio. Then simplify.

With the line measurement, measure the lines with a ruler and record the length (in whole cm) in the box on the line. Then use the line lengths to create ratios in the spaces provided, these can all be simplified.

Column 3 introduces calculator use. There are three outcomes when using a calculator's fraction key to solve ratios. You could get a proper fraction answer, a mixed numeral or a whole number. If you get a proper fraction, this is the answer, using ':' rather than a ⌟ .

If you get a whole number write the answer ': 1'. For example the answer 6 would be written down as '6 : 1'.

If you get a mixed numeral then use 'SHIFT' or the '2nd F' key with the fraction key to convert it to an improper fraction then write that as your answer.

Note that the calculator can only be used to solve ratios with two numbers. If there are more than two numbers then you need to use the HCF method and solve them mentally.

Simplifying Ratios - No Decimals/Fractions

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Find the HCF for these numbers, which is the largest number that will divide into both numbers.

Examples

4, 6. = 2 9, 12. = 3

1 3, 6. = 2 2, 8. =

3 9, 12. = 4 5, 20. =

5 12, 14. = 6 9, 15.=

7 6, 12. = 8 15, 30.=

Now use the HCF to simplify these ratios. Use division.

9 15 : 10 5 HCF = (15÷5 : 10÷5)

10 8 : 12 4 HCF = (8÷4 : 12÷4)

11 14 : 35 7 HCF =

12 24 : 18 6 HCF =

13 10 : 100 10 HCF =

Find the HCF then use it to simplify these ratios.

14 4 : 8 HCF =

15 9 : 6 HCF =

16 12 : 20 HCF =

17 30 : 15 HCF =

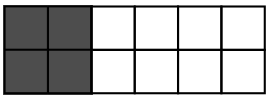
18 12 : 24 HCF =

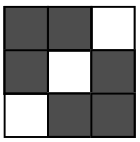
19 100 : 50 HCF =

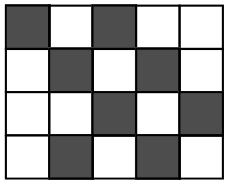
20 18 : 30 HCF =

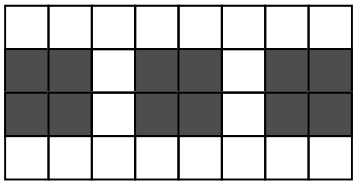
21 24 : 32 HCF =

Find the non-simplified ratio for these grids, then simplify. Find the ratio for shaded : total.

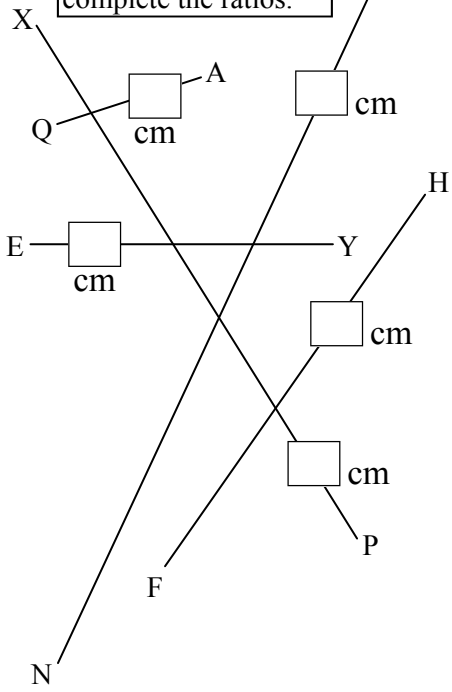
22  : 12 = : Simplify

23  : = :

24  : = :

25  : = :

Measure the lines then complete the ratios.



26 QA : PX = : = :

27 NB : HF = : = :

28 YE : XP = : = :

29 AQ : BN = : = :

30 FH : HF = : = :

Simplify these, you can use your calculator's fraction key. You have to solve triples mentally.

1st number is smaller than the 2nd number

Examples 8 : 12 = 2 : 3

Screen: '2⌟3'

Calc: 8 $a^b/_c$ 1 2 =

A whole number answer is written ':1'

12 : 3 = 4 : 1

Screen: '4'

Calc: 1 2 $a^b/_c$ 3 =

Change mixed numeral to improper fraction

Screen: '1⌟1⌟3'

12 : 9 = 4 : 3

Calc: 1 2 $a^b/_c$ 9 = SHIFT OR 2nd F $a^b/_c$

31 16 : 48 = :

32 24 : 36 = :

33 30 : 6 = :

34 60 : 45 = :

35 18 : 20 = :

36 85 : 50 = :

37 36 : 27 = :

38 49 : 21 = :

39 100 : 95 = :

40 50 : 2 = :

41 500 : 50 = :

42 45 : 18 = :

43 4 : 12 : 8 = : :

44 9 : 6 : 3 = : :

45 25 : 5 : 20 = : :

46 16 : 8 : 24 = : :

47 10 : 50 : 100 = : :

48 20 : 40 : 50 = : :

Simplifying Ratios - No Decimals/Fractions

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Find the HCF for these numbers, which is the largest number that will divide into both numbers.

Examples

4, 6. = 2 9, 12. = 3

1 3, 6. = 3 2 2, 8. = 2

3 9, 12. = 3 4 5, 20. = 5

5 12, 14. = 2 6 9, 15.= 3

7 6, 12. = 6 8 15, 30.= 15

Now use the HCF to simplify these ratios. Use division.

9 15 : 10 5 HCF = 15÷5 : 10÷5 = 3 : 2

10 8 : 12 4 HCF = 8÷4 : 12÷4 = 2 : 3

11 14 : 35 7 HCF = 2 : 5

12 24 : 18 6 HCF = 4 : 3

13 10 : 100 10 HCF = 1 : 10

Find the HCF then use it to simplify these ratios.

14 4 : 8 4 HCF = 1 : 2

15 9 : 6 3 HCF = 3 : 2

16 12 : 20 4 HCF = 3 : 5

17 30 : 15 15 HCF = 2 : 1

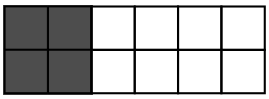
18 12 : 24 12 HCF = 1 : 2

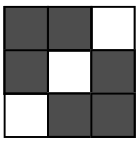
19 100 : 50 50 HCF = 2 : 1

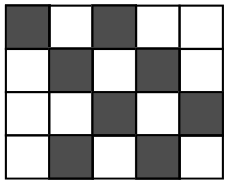
20 18 : 30 6 HCF = 3 : 5

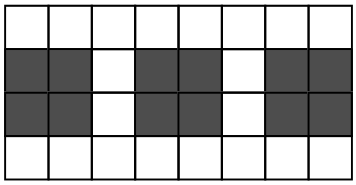
21 24 : 32 8 HCF = 3 : 4

Find the non-simplified ratio for these grids, then simplify. Find the ratio for shaded : total.

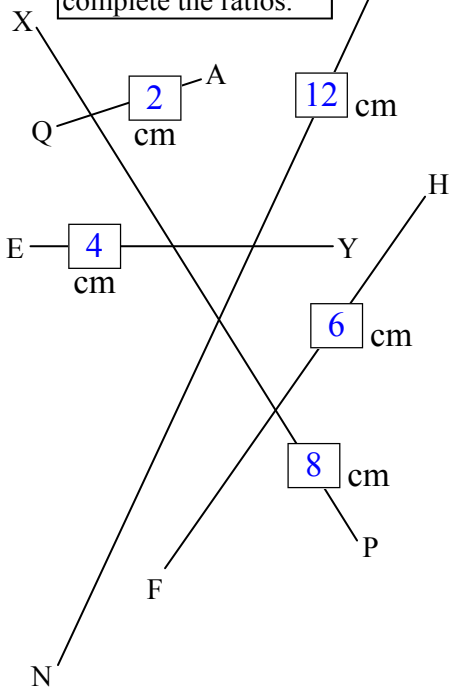
22  = 4 : 12 = 1 : 3 Simplify

23  = 6 : 9 = 2 : 3

24  = 8 : 20 = 2 : 5

25  = 12 : 32 = 3 : 8

Measure the lines then complete the ratios.



26 QA : PX = 2 : 8 = 1 : 4

27 NB : HF = 12 : 6 = 2 : 1

28 YE : XP = 4 : 8 = 1 : 2

29 AQ : BN = 2 : 12 = 1 : 6

30 FH : HF = 6 : 6 = 1 : 1

Simplify these, you can use your calculator's fraction key. You have to solve triples mentally.

1st number is smaller than the 2nd number

Examples 8 : 12 = 2 : 3

Screen: '2⌟3' Calc: 8 $a^b/_c$ 1 2 =

A whole number answer is written ':1'

12 : 3 = 4 : 1

Screen: '4' Calc: 1 2 $a^b/_c$ 3 =

Change mixed numeral to improper fraction

Screen: '1⌟1⌟3' 12 : 9 = 4 : 3

Calc: 1 2 $a^b/_c$ 9 = SHIFT OR 2nd F $a^b/_c$

31 16 : 48 = 1 : 3

32 24 : 36 = 2 : 3

33 30 : 6 = 5 : 1

34 60 : 45 = 4 : 3

35 18 : 20 = 9 : 10

36 85 : 50 = 17 : 10

37 36 : 27 = 4 : 3

38 49 : 21 = 7 : 3

39 100 : 95 = 20 : 19

40 50 : 2 = 25 : 1

41 500 : 50 = 10 : 1

42 45 : 18 = 5 : 2

43 4 : 12 : 8 = 1 : 3 : 2

44 9 : 6 : 3 = 3 : 2 : 1

45 25 : 5 : 20 = 5 : 1 : 4

46 16 : 8 : 24 = 2 : 1 : 3

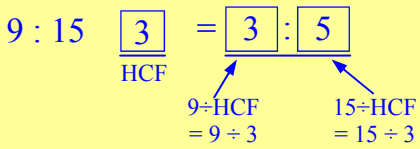
47 10 : 50 : 100 = 1 : 5 : 10

48 20 : 40 : 50 = 2 : 4 : 5

Simplifying Ratios

Ratios compare two (or more) values. Simplifying ratios is important because ratios could be applied to other amounts and applying a complicated ratio makes things harder. So how do you simplify a ratio? Ratios can be simplified using the same process as with fractions. Pick the HCF (Highest Common Factor) and divide through.

Example - Simplifying Ratios



Column 1 starts with questions that ask you to find the HCF. Look at the two numbers and ask yourself what is the largest number that divides into both numbers. Q.9 onwards asks you to find the HCF and then divide through, the process is shown above. This is how you simplify ratios. Note that ratios can have more than two numbers, ratios with three numbers are shown lower in the column.

Column 2 introduces use of the calculator to solve ratios. Note that the calculator can only be used to solve ratios with two numbers. If there are more than two numbers then you need to use the HCF method and solve them mentally.

Decimals and fractions are also introduced, again these can be solved with a calculator if only two numbers (fractions/decimals) are involved. Calculator steps are shown at the top of the column.

Column 3 ends with comparing two quantities. If the units used are like then you can solve straight away using the HCF or calculator method. If the units are different though, you need to convert the measurements to the same unit. It doesn't matter which way you convert, either to the smaller or larger of the two units. Once you have converted …. simplify.

Simplifying Ratios

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Find the HCF for these numbers, which is the largest number that will divide into both numbers.

Examples

4, 6. = 2 9, 12. = 3

1 8, 12. = 2 9, 6. =

3 25, 15. = 4 8, 10. =

5 12, 18. = 6 30, 24.=

7 50, 35. = 8 12, 60.=

Now use the HCF to simplify these ratios.

9 6 : 10 HCF = 6÷HCF : 10÷HCF

10 12 : 9 HCF = 12÷HCF : 9÷HCF

11 15 : 20 HCF =

12 18 : 12 HCF =

13 16 : 24 HCF =

Keep going, but you don't have to show the HCF this time.

14 16 : 24 = :

15 15 : 10 = :

16 120 : 20 = :

17 30 : 45 = :

18 15 : 60 = :

19 75 : 100 = :

20 3 : 12 : 9 = : :

21 15 : 5 : 20 = : :

22 16 : 24 : 12 = : :

23 9 : 30 : 18 = : :

24 60 : 36 : 48 = : :

Use your calculator's fraction key to simplify these. There are three different results. Careful!

Examples 8 : 12 = 2 : 3

Screen: '2⌟3' Calc: 8 $a^b/_c$ 1 2 =

A whole number answer is written ':1'

12 : 3 = 4 : 1

Screen: '4' Calc: 1 2 $a^b/_c$ 3 =

Change mixed numeral to improper fraction

Screen: '1⌟1⌟3' 12 : 9 = 4 : 3

Calc: 1 2 $a^b/_c$ 9 = SHIFT OR 2nd F $a^b/_c$

25 108 : 189 = :

26 68 : 14 = :

27 390 : 65 = :

28 297 : 216 = :

29 258 : 215 = :

30 14 : 350 = :

31 12 : 378 = :

32 270 : 81 = :

33 56 : 136 = :

34 288 : 48 = :

Decimals: Use fraction key & again after '='.

35 1 : 0.12 = :

36 0.55 : 6.6 = :

37 2.2 : 4.2 = :

38 0.7 : 0.05 = :

39 0.4 : 2.04 = :

40 1.9 : 0.1 = :

41 6.4 : 80 = :

42 12 : 2.4 = :

43 0.005 : 0.2 = :

44 0.1 : 0.01 = :

Simplify these …. use the ÷ key to simplify fractions.

Examples $\frac{1}{5} : \frac{3}{4}$ = 4 : 15

Screen: '4⌟15' Calc: 1 $a^b/_c$ 5 ÷ 3 $a^b/_c$ 4 =

Convert Answer to Improper if required

$2\frac{1}{2} : \frac{3}{4}$ = 10 : 3

Calc: 2 $a^b/_c$ 1 $a^b/_c$ 2 ÷ 3 $a^b/_c$ 4 =

Screen: '3⌟1⌟3' → SHIFT OR 2nd F $a^b/_c$ → Screen: '10⌟3'

45 $\frac{2}{3} : \frac{5}{6}$ = :

46 $1\frac{1}{2} : \frac{3}{4}$ = :

47 $\frac{1}{4} : \frac{3}{10}$ = :

48 $1\frac{3}{4} : \frac{5}{8}$ = :

49 $2\frac{1}{2} : 3\frac{3}{4}$ = :

50 $\frac{1}{2} : \frac{1}{16}$ = :

Simplify. When different units are used, convert them to match.

51 650 m : 150 m = :

52 250 g : 400 g = :

53 1 m : 25 cm = cm : cm = :

54 42 mm : 6 cm = mm : mm = :

55 3 min : 45 s = s : s = :

56 800 kg : 5 T = : = :

57 750 mL : 4 L = : = :

58 2 cm^2 : 40 mm^2 = : = :

Simplifying Ratios

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Find the HCF for these numbers, which is the largest number that will divide into both numbers.

Examples

4, 6. = 2 9, 12. = 3

1 8, 12. = 4 2 9, 6. = 3

3 25, 15. = 5 4 8, 10. = 2

5 12, 18. = 6 6 30, 24. = 6

7 50, 35. = 5 8 12, 60. = 12

Now use the HCF to simplify these ratios.

9 6 : 10 — 2 (HCF) = 6÷HCF : 10÷HCF = 3 : 5

10 12 : 9 — 3 (HCF) = 12÷HCF : 9÷HCF = 4 : 3

11 15 : 20 — 5 (HCF) = 3 : 4

12 18 : 12 — 6 (HCF) = 3 : 2

13 16 : 24 — 8 (HCF) = 2 : 3

Keep going, but you don't have to show the HCF this time.

14 16 : 24 = 2 : 3

15 15 : 10 = 3 : 2

16 120 : 20 = 6 : 1

17 30 : 45 = 2 : 3

18 15 : 60 = 1 : 4

19 75 : 100 = 3 : 4

20 3 : 12 : 9 = 1 : 4 : 3

21 15 : 5 : 20 = 3 : 1 : 4

22 16 : 24 : 12 = 4 : 6 : 3

23 9 : 30 : 18 = 3 : 10 : 6

24 60 : 36 : 48 = 5 : 3 : 4

Use your calculator's fraction key to simplify these. There are three different results. Careful!

Examples 8 : 12 = 2 : 3

Screen: '2⌟3' Calc: 8 $a^b/_c$ 1 2 =

A whole number answer is written ':1'

12 : 3 = 4 : 1

Screen: '4' Calc: 1 2 $a^b/_c$ 3 =

Change mixed numeral to improper fraction

Screen: '1⌟1⌟3' 12 : 9 = 4 : 3

Calc: 1 2 $a^b/_c$ 9 = SHIFT OR 2nd F $a^b/_c$

25 108 : 189 = 4 : 7

26 68 : 14 = 34 : 7

27 390 : 65 = 6 : 1

28 297 : 216 = 11 : 8

29 258 : 215 = 6 : 5

30 14 : 350 = 1 : 25

31 12 : 378 = 2 : 63

32 270 : 81 = 10 : 3

33 56 : 136 = 7 : 17

34 288 : 48 = 6 : 1

Decimals: Use fraction key & again after '='.

35 1 : 0.12 = 25 : 3

36 0.55 : 6.6 = 1 : 12

37 2.2 : 4.2 = 11 : 21

38 0.7 : 0.05 = 14 : 1

39 0.4 : 2.04 = 10 : 51

40 1.9 : 0.1 = 19 : 1

41 6.4 : 80 = 2 : 25

42 12 : 2.4 = 5 : 1

43 0.005 : 0.2 = 1 : 40

44 0.1 : 0.01 = 10 : 1

Simplify these …. use the ÷ key to simplify fractions.

Examples $\frac{1}{5} : \frac{3}{4}$ = 4 : 15

Screen: '4⌟15' Calc: 1 $a^b/_c$ 5 ÷ 3 $a^b/_c$ 4 =

Convert Answer to Improper if required

$2\frac{1}{2} : \frac{3}{4}$ = 10 : 3

Calc: 2 $a^b/_c$ 1 $a^b/_c$ 2 ÷ 3 $a^b/_c$ 4 =

Screen: '3⌟1⌟3' → SHIFT OR 2nd F $a^b/_c$ → Screen: '10⌟3'

45 $\frac{2}{3} : \frac{5}{6}$ = 4 : 5

46 $1\frac{1}{2} : \frac{3}{4}$ = 2 : 1

47 $\frac{1}{4} : \frac{3}{10}$ = 5 : 6

48 $1\frac{3}{4} : \frac{5}{8}$ = 14 : 5

49 $2\frac{1}{2} : 3\frac{3}{4}$ = 2 : 3

50 $\frac{1}{2} : \frac{1}{16}$ = 8 : 1

Simplify. When different units are used, convert them to match.

51 650 m : 150 m = 13 : 3

52 250 g : 400 g = 5 : 8

53 1 m : 25 cm = 100 cm : 25 cm = 4 : 1

54 42 mm : 6 cm = 42 mm : 60 mm = 7 : 10

55 3 min : 45 s = 180 s : 45 s = 4 : 1

56 800 kg : 5 T = 800 kg : 5 000 kg = 4 : 25

57 750 mL : 4 L = 750 mL : 4 000 mL = 3 : 16

58 2 cm^2 : 40 mm^2 = 200 mm^2 : 40 mm^2 = 5 : 1

Further Simplifying Ratios

Ratios can be solved mentally and with a calculator. This worksheet concentrates on mentally solving ratios. You can still use a calculator to assist you, but the calculator can't be used to complete the whole process.

Column 1 starts with decimals. The method is to remove the decimal point completely, so you are working with whole numbers. So multiply by 10, 100 or 1 000 … whatever it takes to remove the decimal point from all the decimals. Then solve by finding the HCF (highest common factor). The ratios are triples so a calculator can't be used to solve them. There is an example at the top of the column.

Further in column 1 are some questions on changing ratios so that the first number is 1. This means 'If I have one of these how many of these do I have?' To solve these, divide the numbers in the ratio by the first number. That makes the first number '1' and the second number either a number or fraction or mixed numeral. There is an example above Q.6.

Column 2 involves fractions. To simplify these you need to make equivalent fractions by matching the denominators for all the fractions. The top row of numbers (the numerators) will become the ratio. Check that it can't be simplified.

Column 3 features ratios with different units, they are triples again so a calculator fraction key can't be used. Use the first line to rewrite the ratio so that the units match, so change the units to the most suitable unit. Your choice won't affect the answer but may not fit in the space provided, so keep this in mind. Once converted to the same unit, divide through by the HCF and simplify the ratios. Use a calculator to assist you with some of the divisions if required.

Further Simplifying Ratios

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Decimals in ratios are removed by moving the decimal point until integers remain. Simplify.

Example

$0.2 : 3 : 0.8$ (×10 ×10 ×10)

$= 2 : 30 : 8$ (Rewrite - no decimals) $= 1 : 15 : 4$ (Simplify)

1 $0.6 : 0.09 : 3$

$= \_ : \_ : \_ = \_ : \_ : \_$ (Remove decimals (×100); Simplify)

2 $2.2 : 4 : 1.05$

$= \_ : \_ : \_ = \_ : \_ : \_$

3 $0.04 : 0.008 : 0.1$

$= \_ : \_ : \_ = \_ : \_ : \_$

4 $0.8 : 2 : 0.16$

$= \_ : \_ : \_ = \_ : \_ : \_$

5 $0.7 : 2.1 : 0.35$

$= \_ : \_ : \_ = \_ : \_ : \_$

A ratio may have to be expressed where one number is 1. Use division to change to 1 : x form.

Example

$\frac{1}{2} : \frac{2}{3} = 1 : 1\frac{1}{3}$ (½ ÷ ½; ⅔ ÷ ½)

Divide both fractions (or numbers) by the first fraction (or number).

6 $2 : 7 = 1 : \frac{\square}{\square}$

7 $\frac{1}{4} : 8 = \_ : \_$

8 $\frac{1}{3} : \frac{5}{6} = \_ : \frac{\square}{\square}$

9 Two geared wheels interconnect. One gear spins at 56 rpm the other spins at 136 rpm. If the first gear rotates once, how much has the other gear rotated?

$\_ : \_ = \_ : \frac{\square}{\square}$

Simplify these ratios. Make the denominators match, the numerators become the ratio.

Example

$\frac{1}{2} : \frac{2}{3} = \frac{3}{6} : \frac{4}{6} = 3 : 4$

10 $\frac{3}{4} : \frac{1}{3} = \frac{\square}{12} : \frac{\square}{12} = \_ : \_$

11 $\frac{3}{5} : \frac{7}{10} = \frac{\square}{10} : \frac{\square}{10} = \_ : \_$

12 $\frac{1}{4} : \frac{2}{5} = \frac{\square}{\square} : \frac{\square}{\square} = \_ : \_$

13 $\frac{3}{8} : \frac{5}{6} = \frac{\square}{\square} : \frac{\square}{\square} = \_ : \_$

14 $\frac{3}{4} : \frac{1}{2} : \frac{2}{3} = \frac{\square}{\square} : \frac{\square}{\square} : \frac{\square}{\square}$

$= \_ : \_ : \_$

15 $\frac{4}{5} : \frac{1}{3} : \frac{1}{4} = \frac{\square}{\square} : \frac{\square}{\square} : \frac{\square}{\square}$

$= \_ : \_ : \_$

16 $3\frac{1}{2} : \frac{2}{5} : \frac{9}{10} = \frac{\square}{\square} : \frac{\square}{\square} : \frac{\square}{\square}$

Change mixed numerals to improper fractions first

$= \frac{\square}{\square} : \frac{\square}{\square} : \frac{\square}{\square}$

$= \_ : \_ : \_$

17 $2\frac{3}{4} : \frac{1}{3} : 1\frac{5}{6} = \frac{\square}{\square} : \frac{\square}{\square} : \frac{\square}{\square}$

$= \frac{\square}{\square} : \frac{\square}{\square} : \frac{\square}{\square}$

$= \_ : \_ : \_$

18 $2\frac{1}{2} : 1\frac{2}{5} : 3\frac{1}{4} = \frac{\square}{\square} : \frac{\square}{\square} : \frac{\square}{\square}$

$= \frac{\square}{\square} : \frac{\square}{\square} : \frac{\square}{\square}$

$= \_ : \_ : \_$

Convert units so they match and then simplify the ratios.

19 15 cm : 0.6 m : 30 mm

$=$ cm : cm : cm

$= \_ : \_ : \_$ (No units in final answer)

20 2 m : 45 s : 1 m 25 sec

$= \_ : \_ : \_$

$= \_ : \_ : \_$

21 1.2 kg : 800 g : 0.4 kg

$= \_ : \_ : \_$

$= \_ : \_ : \_$

22 600 L : 4.2 kL : 0.9 kL

$= \_ : \_ : \_$

$= \_ : \_ : \_$

23 \$3.60 : 45c : \$2.07

$= \_ : \_ : \_$

$= \_ : \_ : \_$

24 3 weeks : 6 days : June

$= \_ : \_ : \_$

$= \_ : \_ : \_$

25 6 000 mg : 12 g : 0.06 kg

$= \_ : \_ : \_$

$= \_ : \_ : \_$

26 10 cm^2 : 200 mm^2 : 12 cm^2

$= \_ : \_ : \_$

$= \_ : \_ : \_$

27 600 m^2 : 2 ha : 0.8 ha

$= \_ : \_ : \_$

$= \_ : \_ : \_$

Further Simplifying Ratios

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Decimals in ratios are removed by moving the decimal point until integers remain. Simplify.

Example

$0.2 : 3 : 0.8$ (×10 ×10 ×10)

$= 2 : 30 : 8 = 1 : 15 : 4$

Rewrite - no decimals; Simplify

1 $0.6 : 0.09 : 3$

$= 60 : 9 : 300 = 20 : 3 : 100$

Remove decimals (×100); Simplify

2 $2.2 : 4 : 1.05$

$= 220 : 400 : 105 = 44 : 80 : 21$

3 $0.04 : 0.008 : 0.1$

$= 40 : 8 : 100 = 10 : 2 : 25$

4 $0.8 : 2 : 0.16$

$= 80 : 200 : 16 = 10 : 25 : 2$

5 $0.7 : 2.1 : 0.35$

$= 70 : 210 : 35 = 2 : 6 : 1$

A ratio may have to be expressed where one number is 1. Use division to change to 1 : x form.

Example

$\frac{1}{2} : \frac{2}{3} = 1 : 1\frac{1}{3}$ (½ ÷ ½; ⅔ ÷ ½)

Divide both fractions (or numbers) by the first fraction (or number).

6 $2 : 7 = 1 : 3\frac{1}{2}$

7 $\frac{1}{4} : 8 = 1 : 32$

8 $\frac{1}{3} : \frac{5}{6} = 1 : 2\frac{1}{2}$

9 Two geared wheels interconnect. One gear spins at 56 rpm the other spins at 136 rpm. If the first gear rotates once, how much has the other gear rotated?

$56 : 136 = 1 : 2\frac{3}{7}$

It has rotated $2\frac{3}{7}$ revolutions.

Simplify these ratios. Make the denominators match, the numerators become the ratio.

Example

$\frac{1}{2} : \frac{2}{3} = \frac{3}{6} : \frac{4}{6} = 3 : 4$

10 $\frac{3}{4} : \frac{1}{3} = \frac{9}{12} : \frac{4}{12} = 9 : 4$

11 $\frac{3}{5} : \frac{7}{10} = \frac{6}{10} : \frac{7}{10} = 6 : 7$

12 $\frac{1}{4} : \frac{2}{5} = \frac{5}{20} : \frac{8}{20} = 5 : 8$

13 $\frac{3}{8} : \frac{5}{6} = \frac{9}{24} : \frac{20}{24} = 9 : 20$

14 $\frac{3}{4} : \frac{1}{2} : \frac{2}{3} = \frac{9}{12} : \frac{6}{12} : \frac{8}{12} = 9 : 6 : 8$

15 $\frac{4}{5} : \frac{1}{3} : \frac{1}{4} = \frac{48}{60} : \frac{20}{60} : \frac{15}{60} = 48 : 20 : 15$

16 $3\frac{1}{2} : \frac{2}{5} : \frac{9}{10} = \frac{7}{2} : \frac{2}{5} : \frac{9}{10} = \frac{35}{10} : \frac{4}{10} : \frac{9}{10} = 35 : 4 : 9$

Change mixed numerals to improper fractions first

17 $2\frac{3}{4} : \frac{1}{3} : 1\frac{5}{6} = \frac{11}{4} : \frac{1}{3} : \frac{11}{6} = \frac{66}{24} : \frac{8}{24} : \frac{44}{24} = 33 : 4 : 22$

18 $2\frac{1}{2} : 1\frac{2}{5} : 3\frac{1}{4} = \frac{5}{2} : \frac{7}{5} : \frac{13}{4} = \frac{50}{20} : \frac{28}{20} : \frac{65}{20} = 50 : 28 : 65$

Convert units so they match and then simplify the ratios.

19 15 cm : 0.6 m : 30 mm

$= 15\text{ cm} : 60\text{ cm} : 3\text{ cm}$

$= 5 : 20 : 1$ ← No units in final answer

20 2 m : 45 s : 1 m 25 sec

$= 120\text{ s} : 45\text{ s} : 85\text{ s}$

$= 24 : 9 : 17$

21 1.2 kg : 800 g : 0.4 kg

$= 1\,200\text{ g} : 800\text{ g} : 400\text{ g}$

$= 3 : 2 : 1$

22 600 L : 4.2 kL : 0.9 kL

$= 600\text{ L} : 4\,200\text{ L} : 900\text{ L}$

$= 2 : 14 : 3$

23 \$3.60 : 45c : \$2.07

$= 360\text{ c} : 45\text{ c} : 207\text{ c}$

$= 40 : 5 : 23$

24 3 weeks : 6 days : June

$= 21\text{ days} : 6\text{ days} : 30\text{ days}$

$= 7 : 2 : 10$

25 6 000 mg : 12 g : 0.06 kg

$= 6\text{ g} : 12\text{ g} : 60\text{ g}$

$= 1 : 2 : 10$

26 $10\text{ cm}^2 : 200\text{ mm}^2 : 12\text{ cm}^2$

$= 10\text{ cm}^2 : 2\text{ cm}^2 : 12\text{ cm}^2$

$= 5 : 1 : 6$

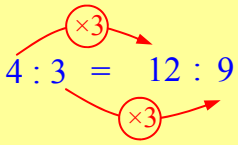
27 $600\text{ m}^2 : 2\text{ ha} : 0.8\text{ ha}$

$= 600\text{ m}^2 : 20\,000\text{ m}^2 : 8\,000\text{m}^2$

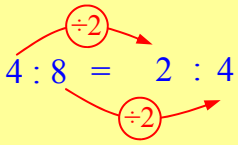
$= 3 : 100 : 40$

Equivalent Ratios (Sheet 1)

When a ratio is multiplied by a number, or divided by a number, an equivalent ratio is created. So to create an equivalent ratio, multiply a ratio by another number. Or you can divide by a number. Below are some examples.

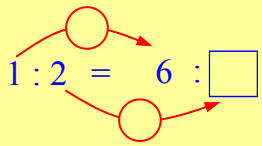


Multiplying: By multiplying the ratio 4 : 3 by 3, you get 12 : 9.

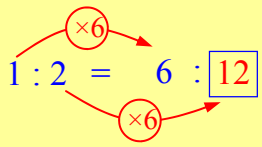


Dividing: By dividing the ratio 4 : 8 by 2, you get 2 : 4. Note that this may or may not be the same as simplifying.

Column 1 starts with 6 questions that ask you to multiply the ratio by a number. But from Q.7 you have to calculate the number you need to multiply by. For example:

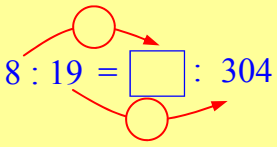


The first ratio has a '1' in the front position. The second ratio has a '6' in the front position. What did I multiply '1' by to get '6'? You multiplied by 6, so write a '×6' in both of the circles.

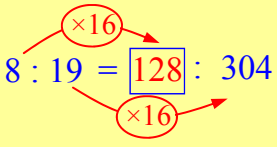


You now have a '×6' in both circles. So find the number in the second position by multiplying '2' by '6'. You get 12. Done.

You could be given the second number in the second ratio. If that is the case then you match the second numbers.



So in this case you match '19' and '304'. You probably don't know what 19 is multiplied by to get 304. So to find the answer divide '304' by '19', this gives '16'. Then multiply '8' by '16'.



You now have a '×16' in both circles. So find the number in the first position by multiplying '8' by '16'. You get 128. Done.

Column 3 introduces division. So when the second ratio is smaller than the first, use division to find the missing values. Again you can use your calculator to assist you, realise that you can also look at the questions backwards and they become multiplication. So if you don't like division …. grab a mirror.

Sheet 2 uses the same methods as above, it features worded questions to apply your skills to.

Equivalent Ratios (Sheet 1)

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Multiply these ratios by the given number to create equivalent ratios.

1 $3 : 4 = \square : \square$ (×5)

2 $2 : 7 = \square : \square$ (×3)

3 $5 : 1 = \square : \square$ (×6)

4 $15 : 4 = \square : \square$ (×2)

5 $3 : 8 : 2 = \square : \square : \square$ (×3)

6 $1 : 7 : 4 = \square : \square : \square$ (×6)

Find the value in the top circle, write it in the lower circle as well, then calculate the box.

7 $1 : 2 = 2 : \square$ (×)

8 $4 : 3 = 8 : \square$

9 $3 : 7 = 9 : \square$

10 $10 : 3 = 50 : \square$

Now the lower circle first.

11 $14 : 9 = \square : 27$ (×)

12 $8 : 5 = \square : 25$

13 $1 : 6 = \square : 42$

14 $10 : 9 = \square : 81$

Find the box values that make equivalent ratios.

15 $4 : 7 = 12 : \square$

16 $3 : 5 = \square : 30$

17 $8 : 1 = \square : 7$

18 $6 : 5 = 36 : \square$

19 $9 : 100 = \square : 1\,000$

20 $20 : 3 = 60 : \square$

21 $13 : 4 = 52 : \square$

22 $11 : 12 = \square : 60$

23 $15 : 4 = 165 : \square$

Instead of boxes, its algebra. There are also some triples.

24 $4 : 9 = x : 36$ $x =$ _____

25 $13 : 7 = y : 21$ $y =$ _____

26 $1 : 19 = 5 : j$ _____

27 $20 : 13 = d : 65$ _____

28 $5 : 2 : 10 = 20 : a : b$

$a =$ _____ $b =$ _____

29 $3 : 7 : 4 = k : 21 : h$

$k =$ _____ $h =$ _____

30 $5 : 9 : 1 = e : t : 8$

_____ _____

31 $2 : 7 : 7 = 14 : x : y$

_____ _____

32 $20 : 3 : 14 = b : a : 126$

_____ _____

33 $100 : 1 : 10 = w : 10 : x$

_____ _____

Now use division, because the RHS ratios are smaller than the LHS ratios. Circles then box.

34 $12 : 6 = 6 : \square$ (÷)

35 $24 : 32 = 6 : \square$

36 $64 : 16 = \square : 2$

37 $27 : 9 = \square : 3$

38 $36 : 18 = 4 : \square$

39 $100 : 125 = 20 : \square$

40 $60 : 48 = \square : 12$

41 $81 : 54 = \square : 2$

42 $144 : 64 = 18 : \square$

Now division & multiplication required. Find the unknowns for these ratios to be equivalent

43 $5 : 3 : x = 15 : b : 12$

$x =$ _____ $b =$ _____

44 $4 : 1 : a = 24 : y : 42$

$a =$ _____ $y =$ _____

45 $36 : c : 54 = q : 12 : 6$

_____ _____

46 $2 : d : e = 4 : e : 20$

_____ _____

47 $a : b : 12 = 100 : a : 60$

_____ _____

Equivalent Ratios (Sheet 1)

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Multiply these ratios by the given number to create equivalent ratios.

1 $3 : 4 = \boxed{15} : \boxed{20}$ (×5)

2 $2 : 7 = \boxed{6} : \boxed{21}$ (×3)

3 $5 : 1 = \boxed{30} : \boxed{6}$ (×6)

4 $15 : 4 = \boxed{30} : \boxed{8}$ (×2)

5 $3 : 8 : 2 = \boxed{9} : \boxed{24} : \boxed{6}$ (×3)

6 $1 : 7 : 4 = \boxed{6} : \boxed{42} : \boxed{24}$ (×6)

Find the value in the top circle, write it in the lower circle as well, then calculate the box.

7 $1 : 2 = 2 : \boxed{4}$ (×2)

8 $4 : 3 = 8 : \boxed{6}$ (×2)

9 $3 : 7 = 9 : \boxed{21}$ (×3)

10 $10 : 3 = 50 : \boxed{15}$ (×5)

Now the lower circle first.

11 $14 : 9 = \boxed{42} : 27$ (×3)

12 $8 : 5 = \boxed{40} : 25$ (×5)

13 $1 : 6 = \boxed{7} : 42$ (×7)

14 $10 : 9 = \boxed{90} : 81$ (×9)

Find the box values that make equivalent ratios.

15 $4 : 7 = 12 : \boxed{21}$

16 $3 : 5 = \boxed{18} : 30$

17 $8 : 1 = \boxed{56} : 7$

18 $6 : 5 = 36 : \boxed{30}$

19 $9 : 100 = \boxed{90} : 1\,000$

20 $20 : 3 = 60 : \boxed{9}$

21 $13 : 4 = 52 : \boxed{16}$

22 $11 : 12 = \boxed{55} : 60$

23 $15 : 4 = 165 : \boxed{44}$

Instead of boxes, its algebra. There are also some triples.

24 $4 : 9 = x : 36$ $x = 16$

25 $13 : 7 = y : 21$ $y = 39$

26 $1 : 19 = 5 : j$ $j = 95$

27 $20 : 13 = d : 65$ $d = 100$

28 $5 : 2 : 10 = 20 : a : b$

$a = 8$ $b = 40$

29 $3 : 7 : 4 = k : 21 : h$

$k = 9$ $h = 12$

30 $5 : 9 : 1 = e : t : 8$

$e = 40$ $t = 72$

31 $2 : 7 : 7 = 14 : x : y$

$x = 49$ $y = 49$

32 $20 : 3 : 14 = b : a : 126$

$b = 180$ $a = 27$

33 $100 : 1 : 10 = w : 10 : x$

$w = 1\,000$ $x = 100$

Now use division, because the RHS ratios are smaller than the LHS ratios. Circles then box.

34 $12 : 6 = 6 : \boxed{3}$ (÷2)

35 $24 : 32 = 6 : \boxed{8}$ (÷4)

36 $64 : 16 = \square : 2$ (÷8)

37 $27 : 9 = \boxed{9} : 3$ (÷3)

38 $36 : 18 = 4 : \boxed{2}$

39 $100 : 125 = 20 : \boxed{25}$

40 $60 : 48 = \boxed{15} : 12$

41 $81 : 54 = \boxed{3} : 2$

42 $144 : 64 = 18 : \boxed{8}$

Now division & multiplication required. Find the unknowns for these ratios to be equivalent

43 $5 : 3 : x = 15 : b : 12$

$x = 4$ $b = 9$

44 $4 : 1 : a = 24 : y : 42$

$a = 7$ $y = 6$

45 $36 : c : 54 = q : 12 : 6$

$c = 108$ $q = 4$

46 $2 : d : e = 4 : e : 20$

$e = 10$ $d = 5$

47 $a : b : 12 = 100 : a : 60$

$a = 20$ $b = 4$

Equivalent Ratios (Sheet 2)

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Find the missing values.

1 $3 : \square = 12 : 24$

2 $\square : 15 = 2 : 5$

3 $8 : 28 = 42 : \square$

4 $4 : \square = 18 : 54$

5 $\square : 20 = 9 : 6$

6 $6 : 8 = \square : 84$

7 $\square : 3 = 172 : 43$

8 $5 : \square = 36 : 144$

9 $12 : 8 : 4 = 39 : a : b$

$a =$ _____ $b =$ _____

10 $3 : 12 : n = q : 78 : 26$

$n =$ _____ $q =$ _____

11 $16 : x : 4 = e : 225 : 45$

$e =$ _____ $x =$ _____

12 $b : 45 : b = d : 6 : 100$

$b =$ _____ $d =$ _____

Use equivalent ratios to solve these.

13 Sam's birthday balloons are coloured silver and gold in the ratio 3 : 5. If there are 27 silver balloons, how many gold balloons are there?

□ : □ = □ : □

Write your answer in words

14 How many balloons were there altogether?

15 Two-stroke fuel for a lawn mower requires an oil : petrol ratio of 1: 40. How much petrol is added to 50 mL of oil?

□ : □ = □ : □

Write your answer showing units

16 How much fuel is there? Give your answer in litres.

17 Sandra is an experienced electrician. She can install 7 light fittings while her apprentice fits 2 in the same time. If Sandra installs 125 lights, how many has the apprentice installed?

□ : □ = □ : □

18 A street directory has a scale of 1 : 20 000. If a street is 9.2 cm long in the directory, give its actual length in metres.

19 If a street was 720 m in length, how long would the street be in the directory. Give your answer in cm.

Rangers use 'Capture & Release' techniques and ratios to calculate animal numbers. Use tag results to estimate animal populations.

20 Wildlife rangers catch 48 deer, tag them and release them. Two months later 29 deer are caught of which 3 have tags. Estimate the park's deer population, 'n'.

1st Capture:		2nd Capture:	
48	n	3	29
Tag	Pop	Tag	Pop

21 Lake Scaley rangers catch 100 trout, tag them, then release them back into the lake. Three weeks later they catch 55 trout of which 4 are tagged. Estimate the number of trout in the lake.

1st Capture:		2nd Capture:	
Tag	Pop	Tag	Pop

22 Mount Bugs National Park rangers are monitoring rabbits. They catch 205, tag and release them. Later they catch 320 rabbits of which 8 are tagged. Estimate the rabbit population.

1st Capture:		2nd Capture:	
Tag	Pop	Tag	Pop

Equivalent Ratios (Sheet 2)

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Find the missing values.

1 3 : 6 = 12 : 24

2 6 : 15 = 2 : 5

3 8 : 28 = 42 : 147

4 4 : 12 = 18 : 54

5 30 : 20 = 9 : 6

6 6 : 8 = 63 : 84

7 12 : 3 = 172 : 43

8 5 : 20 = 36 : 144

9 $12 : 8 : 4 = 39 : a : b$

$a = 26$ $b = 13$

10 $3 : 12 : n = q : 78 : 26$

$n = 4$ $q = 19.5$

11 $16 : x : 4 = e : 225 : 45$

$e = 180$ $x = 20$

12 $b : 45 : b = d : 6 : 100$

$b = 750$ $d = 100$

Use equivalent ratios to solve these.

13 Sam's birthday balloons are coloured silver and gold in the ratio 3 : 5. If there are 27 silver balloons, how many gold balloons are there?

3 : 5 = 27 : g

There are 45 gold balloons.

Write your answer in words

14 How many balloons were there altogether?

27 + 45 = 72

There are 72 balloons.

15 Two-stroke fuel for a lawn mower requires an oil : petrol ratio of 1: 40. How much petrol is added to 50 mL of oil?

1 : 40 = 50 : p

2 000 ml of petrol is added.

Write your answer showing units

16 How much fuel is there? Give your answer in litres.

2 000 + 50 = 2 050 mL

There is 2.05 L of fuel.

17 Sandra is an experienced electrician. She can install 7 light fittings while her apprentice fits 2 in the same time. If Sandra installs 125 lights, how many has the apprentice installed?

7 : 2 = 125 : a

Apprentice has fitted 35.7 (1 dp)

lights, so 35 lights installed.

18 A street directory has a scale of 1 : 20 000. If a street is 9.2 cm long in the directory, give its actual length in metres.

$1 : 20\,000 = 9.2 : l$

Length = 184 000 cm

Length of street is 1 840 m.

19 If a street was 720 m in length, how long would the street be in the directory. Give your answer in cm.

$1 : 20\,000 = l : 720$

Length = 0.036 m

Length in book is 3.6 cm.

Rangers use 'Capture & Release' techniques and ratios to calculate animal numbers. Use tag results to estimate animal populations.

20 Wildlife rangers catch 48 deer, tag them and release them. Two months later 29 deer are caught of which 3 have tags. Estimate the park's deer population, 'n'.

1st Capture:		2nd Capture:	
48	n	3	29
Tag	Pop	Tag	Pop

$16 \times 29 = 464$ deer.

There are approximately 464 deer in the park.

21 Lake Scaley rangers catch 100 trout, tag them, then release them back into the lake. Three weeks later they catch 55 trout of which 4 are tagged. Estimate the number of trout in the lake.

1st Capture:		2nd Capture:	
100	n	4	55
Tag	Pop	Tag	Pop

$25 \times 55 = 1\,375$ trout.

There are approximately 1 375 trout in the lake.

22 Mount Bugs National Park rangers are monitoring rabbits. They catch 205, tag and release them. Later they catch 320 rabbits of which 8 are tagged. Estimate the rabbit population.

1st Capture:		2nd Capture:	
205	n	8	320
Tag	Pop	Tag	Pop

$25.625 \times 320 = 8\,200$ rabbits.

There are approximately 8 200 rabbits in the park.

Further Equivalent Ratios

Ratios compare one quantity against another. This sheet deals with problems that feature three or four different measurements that all affect each other. Normally with equivalent ratios the entire ratio is multiplied by the same amount, or divided by the same amount. This sheet picks out two quantities and depending on their affect on each other, may see opposite operations applied.

There is an example at the top of the first column which shows how these questions are solved. Below is an explanation on using your calculator to convert time.

If you get an answer like '$^{7}/_{9}$' (in hours) how do you convert that to minutes?

Using your calculator	Calculator Display:	So you answer is
7 $a^b/_c$ 9 = ° ' ''	0° 46° 40	46 min 40 s (the hours are 0)

If you get an answer like '$3^{4}/_{5}$' (in hours) how do you convert that to minutes (or hours/min)?

Using your calculator	Calculator Display:	So you answer is
3 $a^b/_c$ 4 $a^b/_c$ 5 = ° ' ''	3° 48° 0	3 h 48 min

If you want to convert this to minutes:

Using your calculator	Calculator Display:	So you answer is
× 6 0 =	228	228 min

Further Equivalent Ratios

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Alter these ratios step by step to answer the question. Make sense? Follow the first example. Increased work means increased time. This is the critical issue.

Example It takes 3 plumbers 2 hours to install 5 toilets. How long would it take 2 plumbers to install 9 toilets, operating at the same work rate?

$3 : 5 : 2$ — Write out ratio, show time last

$\times {}^{9}/_{5}$ $\times {}^{9}/_{5}$

$3 : 9 : {}^{18}/_{5}$ — Change 5 → 9 toilets. Time will increase with more installations.

$\times {}^{2}/_{3}$ $\times {}^{3}/_{2}$

$2 : 9 : {}^{27}/_{5}$ — Change 3 → 2 plumbers. Time will increase with less plumbers.

${}^{27}/_{5}$ h = 324 min or 5 h 24 min.

---

1 It takes 6 trucks of 15 tonne capacity 3 hours to remove rocks from a quarry blast. How long would it take 4 trucks with a 18 tonne capacity to remove the same amount?

Trucks	Capacity	Time
$\times {}^{4}/_{6}$		$\times {}^{6}/_{4}$
	$\times {}^{18}/_{15}$	$\times {}^{15}/_{18}$

Less trucks = more time

More capacity = less time

2 It takes 3 bakers 6 hours to make 1 200 loaves of bread. How long would it take 5 bakers to bake 1 500 loaves working at the same rate?

Bakers	Loaves	Time

3 Two graders can smooth 700 m of road in 5 hours. How long would it take for 5 graders to smooth out 2.6 km of road, working at the same rate?

Now 4 variables. Did you see where my sandwich went?

4 It takes 2 700 ants 4 hours to carry 2 jam sandwiches 37 metres. How long would it take 8 100 ants to transport 6 jam sandwiches 16 m?

Ants	S'wiches	Distance	Time

---

5 It takes 3 hours for 2 conveyor belts with a velocity of 4 m/s to transport the coal from 3 trains. If three new belts replaced the two, and operate at 10 m/s, how long would it take to unload 10 trains?

---

6 If three tugs travel 6 M to dock 4 freighters in 7 hours, how long would it take 6 tugs to travel 11 M and dock 9 freighters? (M = nautical mile).

Further Equivalent Ratios

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Alter these ratios step by step to answer the question. Make sense? Follow the first example. Increased work means increased time. This is the critical issue.

Example It takes 3 plumbers 2 hours to install 5 toilets. How long would it take 2 plumbers to install 9 toilets, operating at the same work rate?

3 : 5 : 2 — Write out ratio, show time last

(×9/5) (×9/5)

3 : 9 : 18/5 — Change 5 → 9 toilets. Time will increase with more installations.

(×2/3) (×3/2)

2 : 9 : 27/5 — Change 3 → 2 plumbers. Time will increase with less plumbers.

27/5 h = 324 min or 5 h 24 min.

---

1 It takes 6 trucks of 15 tonne capacity 3 hours to remove rocks from a quarry blast. How long would it take 4 trucks with a 18 tonne capacity to remove the same amount?

Trucks	Capacity	Time	
6	15	3	Less trucks = more time; More capacity = less time
(×4/6)		(×6/4)	
4	15	9/2	15/4 h = 225 min = 3 h 45 min
		(×15/18)	
4	18	15/4	

2 It takes 3 bakers 6 hours to make 1 200 loaves of bread. How long would it take 5 bakers to bake 1 500 loaves working at the same rate?

Bakers	Loaves	Time	
3	1 200	6	More bakers = less time; More loaves = more time
(×5/3)		(×3/5)	
5	1 200	18/5	9/2 h = 270 min = 4 h 30 min
1 500/1 200 = 5/4 (×5/4)		(×5/4)	
5	1 500	9/2	

3 Two graders can smooth 700 m of road in 5 hours. How long would it take for 5 graders to smooth out 2.6 km of road, working at the same rate?

Graders	Distance	Time	
2	700	5	More graders = less time; More distance = more time
(×5/2)		(×2/5)	
5	700	2	52/7 h = 445 5/7 min
2 600/700 = 26/7 (×26/7)		(×26/7)	
5	2 600	52/7	7 h 26 min (nearest minute)

(7 h 25 min 42.9 s)

Now 4 variables. Did you see where my sandwich went?

4 It takes 2 700 ants 4 hours to carry 2 jam sandwiches 37 metres. How long would it take 8 100 ants to transport 6 jam sandwiches 16 m?

Ants	S'wiches	Distance	Time	
2 700	2	37	4	More ants less time
(×3)			(÷3)	
8 100	2	37	4/3	More sand more time
	(×3)		(×3)	
8 100	6	37	4	Less dist less time
		(×16/37)	(×16/37)	
8 100	6	16	64/37	

64/37 h = 103 29/37 min or 1 h 44 min. (nearest min)

---

5 It takes 3 hours for 2 conveyor belts with a velocity of 4 m/s to transport the coal from 3 trains. If three new belts replaced the two, and operate at 10 m/s, how long would it take to unload 10 trains?

Belts	Velocity	Trains	Time	
2	4	3	3	More belts less time
(×5/3)			(×3/5)	
3	4	3	9/5	More speed less time
	(×5/2)		(×2/5)	
3	10	3	18/25	More trains more time
		(×10/3)	(×10/3)	
3	10	10	12/5	

12/5 h = 144 min or 2 h 24 min.

---

6 If three tugs travel 6 M to dock 4 freighters in 7 hours, how long would it take 6 tugs to travel 11 M and dock 9 freighters? (M = nautical mile).

Tugs	Distance	Freighter	Time	
3	6	4	7	More tugs less time
(×2)			(÷2)	
6	6	4	7/2	More dist more time
	(×11/6)		(×11/6)	
6	11	4	77/12	More ships more time
		(×9/4)	(×9/4)	
6	11	9	231/16	

231/16 h = 866 1/4 min or 14 h 26 min. (nearest min)

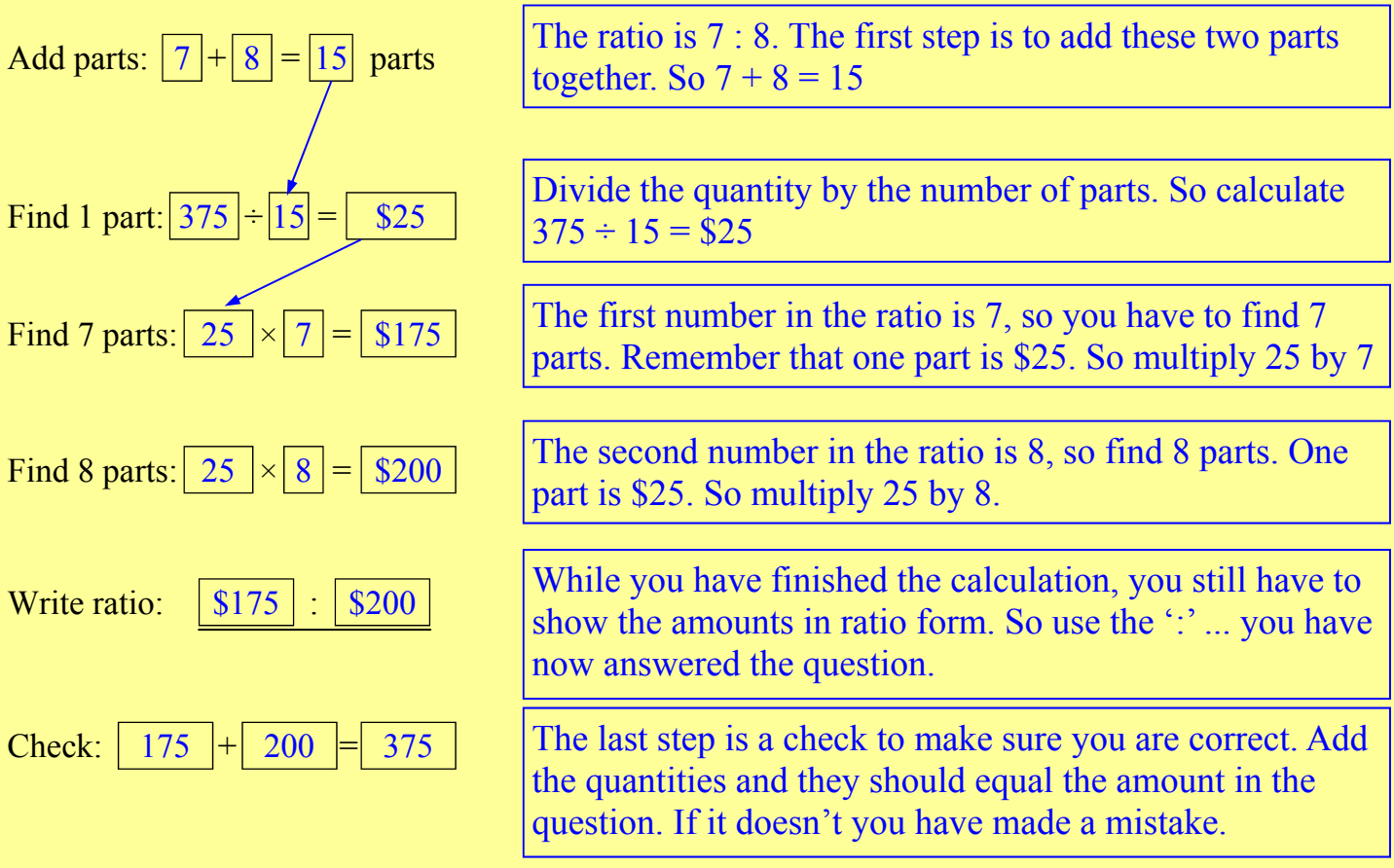
Ratio of a Quantity - Method 1

You can break something in two and share it, but what about sharing things in unequal amounts? Ratios are used to break up a quantity into more complicated portions. There are two common ways of applying a ratio to a quantity, this is Method 1, which uses a unitary method approach. Finding the value of one part then using that to calculate the ratio.

Column 1 starts with some straight forward questions. Questions 1 to 4 ask you to divide a quantity up into equal pieces. All you have to do is divide the quantity by the given number of pieces. The next three questions give you what one 'part' is and then asks you to find 5 parts, or 6 parts etc., so multiply the amount by the number of parts.

At the bottom of column 1 (Q.8) is the first question on dividing quantities by a ratio, these questions are the same for the rest of the sheet. Below is an example.

Divide \$375 in the ratio 7 : 8.



There are two sheets for this topic, the first sheet involves questions shown in the same style as that above. The second sheet are worded questions that use the same method to solve but may include an additional step.

Ratio of a Quantity - Method 1 (Sheet 1)

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Divide these quantities into the number of equal parts specified.

1 Divide 558 m into 6 equal parts. ☐ ÷ ☐ = ☐ m

2 Divide $172.40 into 4 equal parts.

3 Divide 1 125 g into 12 equal parts.

4 Divide 6.32 L into 8 equal parts. Give your answer in mL.

These will use multiplication. Given 1 part, find the quantity.

5 If 1 part is 18 kg, find the mass for 15 parts.

6 If 1 part is 42 s, find the time (min/s) made from 6 parts.

7 If 1 part is 26 cm, find the length (m) made from 38 parts.

Now ratios, follow the steps to find 1 part then the divided quantity.

8 Divide $168 in the ratio 5 : 3.

Add parts: 5 + 3 = ☐ parts

Find 1 part: 168 ÷ ☐ = $☐

Find 5 parts: ☐ × 5 = $☐

Find 3 parts: ☐ × 3 = $☐

Write ratio: $☐ : $☐

Check: ☐ + ☐ = 168

9 Divide 798 kg in the ratio 3 : 4.

Add parts: ☐ + ☐ = ☐ parts

Find 1 part: ☐ ÷ ☐ = ☐

Find 3 parts: ☐ × ☐ = ☐

Find 4 parts: ☐ × ☐ = ☐

Write ratio: ☐ kg : ☐ kg

Check: ☐ + ☐ = 798

10 Divide 1 590 mL in the ratio 11 : 4.

Add parts: ☐ + ☐ = ☐ parts

1 part: ☐ ÷ ☐ = ☐

11 parts: ☐ × ☐ = ☐

4 parts: ☐ × ☐ = ☐

Ratio: ☐ : ☐

Check: ☐ + ☐ = 1 590

11 Divide 3.4 T in the ratio 2 : 3. Answer in kg.

Add parts: ☐ + ☐ = ☐ parts

1 part: ☐ ÷ ☐ = ☐

2 parts: ☐ × ☐ = ☐

3 parts: ☐ × ☐ = ☐

Ratio: ☐ : ☐

Check: ☐ + ☐ = ☐

12 Divide $49.05 into the ratio 7 : 2.

Add parts:

1 part:

7 parts:

2 parts:

Ratio:

Check:

Now three valued ratios.

13 Divide 7.28 kg into the ratio 3 : 7 : 4. Answer in grams.

Add parts:

1 part:

3 parts:

7 parts:

4 parts:

Ratio:

Check

14 Divide 97.5 cm into the ratio 6 : 3 : 4. Answer in mm.

15 Divide 1.06 km into the ratio 2 : 5 : 1. Answer in m.

Ratio of a Quantity - Method 1 (Sheet 1)

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Divide these quantities into the number of equal parts specified.

1 Divide 558 m into 6 equal parts. 558 ÷ 6 = 93 m

2 Divide $172.40 into 4 equal parts. 172.40 ÷ 4 = $43.10

3 Divide 1 125 g into 12 equal parts. 1 125 ÷ 12 = 93.75 g

4 Divide 6.32 L into 8 equal parts. Give your answer in mL.

6 320 ÷ 8 = 790 mL

These will use multiplication. Given 1 part, find the quantity.

5 If 1 part is 18 kg, find the mass for 15 parts.

18 × 15 = 270 kg

6 If 1 part is 42 s, find the time (min/s) made from 6 parts.

42 × 6 = 252 s = 4 min 12 s

7 If 1 part is 26 cm, find the length (m) made from 38 parts.

26 × 38 = 988 cm = 9.88 m

Now ratios, follow the steps to find 1 part then the divided quantity.

8 Divide $168 in the ratio 5 : 3.

Add parts: 5 + 3 = 8 parts

Find 1 part: 168 ÷ 8 = $21

Find 5 parts: 21 × 5 = $105

Find 3 parts: 21 × 3 = $63

Write ratio: $105 : $63

Check: 105 + 63 = 168

9 Divide 798 kg in the ratio 3 : 4.

Add parts: 3 + 4 = 7 parts

Find 1 part: 798 ÷ 7 = 114 kg

Find 3 parts: 114 × 3 = 342 kg

Find 4 parts: 114 × 4 = 456 kg

Write ratio: 342 kg : 456 kg

Check: 342 + 456 = 798

10 Divide 1 590 mL in the ratio 11 : 4.

Add parts: 11 + 4 = 15 parts

1 part: 1 590 ÷ 15 = 106 mL

11 parts: 106 × 11 = 1 166 mL

4 parts: 106 × 4 = 424 mL

Ratio: 1 166 mL : 424 mL

Check: 1 166 + 424 = 1 590

11 Divide 3.4 T in the ratio 2 : 3. Answer in kg.

Add parts: 2 + 3 = 5 parts

1 part: 3 400 ÷ 5 = 680 kg

2 parts: 680 × 2 = 1 360 kg

3 parts: 680 × 3 = 2 040 kg

Ratio: 1 360 kg : 2 040 kg

Check: 1 360 + 2 040 = 3 400

12 Divide $49.05 into the ratio 7 : 2.

Add parts: 7 + 2 = 9 parts

1 part: 49.05 ÷ 9 = $5.45

7 parts: 5.45 × 7 = $38.15

2 parts: 5.45 × 2 = $10.90

Ratio: $38.15 : $10.90

Check: 38.15 + 10.90 = 49.05

Now three valued ratios.

13 Divide 7.28 kg into the ratio 3 : 7 : 4. Answer in grams.

Add parts: 3 + 7 + 4 = 14 parts

1 part: 7 280 ÷ 14 = 520 g

3 parts: 520 × 3 = 1 560 g

7 parts: 520 × 7 = 3 640 g

4 parts: 520 × 4 = 2 080 g

Ratio: 1 560 g:3 640 g:2080 g

Check: 1 560 + 3 640 + 2080 = 7 280 g

14 Divide 97.5 cm into the ratio 6 : 3 : 4. Answer in mm.

Add parts: 6 + 3 + 4 = 13 parts

1 part: 975 ÷ 13 = 75 mm

6 parts: 75 × 6 = 450 mm

3 parts: 75 × 3 = 225 mm

4 parts: 75 × 4 = 300 mm

450 mm : 225 mm : 300 mm

Check: 450 + 225 + 300 = 975

15 Divide 1.06 km into the ratio 2 : 5 : 1. Answer in m.

Add parts: 2 + 5 + 1 = 8 parts

1 part: 1 060 ÷ 8 = 132.5 m

2 parts: 132.5 × 2 = 265 m

5 parts: 132.5 × 5 = 662.5 m

265 m : 662.5 m : 132.5 m

Check: 265 + 662.5 + 132.5 = 1 060 m

Ratio of a Quantity - Method 1 (Sheet 2)

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Use ratios to answer these.

1 The "Treerific Dew-O" are landscape gardeners. On a job, Paul worked 7 h, while Grace worked 11 h. Paid $954 total, how much should each receive?

Add:

1 part:

Paul:

Grace:

Check:

2 Two pylons support a beam. If the 7 000 kg load is shared in the ratio 3 : 5 by Pylon 1 and Pylon 2, find the load in each.

3 Dave saves money from his pay by splitting his income. He divides his pay as spendable and savings in the ratio 7 : 2. Find the amount he spends and saves from his net pay of $1 098.

Now ratios with 3 values

4 A stadium is painted gold, blue and red in the ratio 5 : 4 : 2. If it is estimated that the stadium has a paintable area of 445.5 m^2. If 1 litre of paint covers 15 m^2, find the amount required of each paint colour. (in whole litres)

5 Groucho the cat sleeps 75% of the day. The rest of his day is spent eating, hissing and growling in the ratio 1 : 9 : 2. Find the amount of time spent on each of his 4 activities.

6 A cruise ship operator notes that its passenger balance is children : adults : retirees in the ratio 2 : 9 : 20. The ship has a total capacity of 2 945 people, which has a crew : passengers ratio of 1 : 4. Find the number of crew and of each passenger category on board of a full ship.

7 A town's population ratio of males : females is 49 : 51. If the town has a current population of 17 600, estimate the increase in males and females in 10 years time if the town's population is expected to increase by 30%.

Ratio of a Quantity - Method 1 (Sheet 2)

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Use ratios to answer these.

1 The "Treerific Dew-O" are landscape gardeners. On a job, Paul worked 7 h, while Grace worked 11 h. Paid $954 total, how much should each receive?

Add: 7 + 11 = 18 parts

1 part: 954 ÷ 18 = $53

Paul: 7 × 53 = $371

Grace: 11 × 53 = $583

Check: 371 + 583 = 954

2 Two pylons support a beam. If the 7 000 kg load is shared in the ratio 3 : 5 by Pylon 1 and Pylon 2, find the load in each.

Add: 3 + 5 = 8 parts

1 part: 7 000 ÷ 8 = 875 kg

Pylon 1: 3 × 875 = 2 625 kg

Pylon 2: 5 × 875 = 4 375 kg

Check: 2 625 + 4 375 = 7 000

3 Dave saves money from his pay by splitting his income. He divides his pay as spendable and savings in the ratio 7 : 2. Find the amount he spends and saves from his net pay of $1 098.

Add: 7 + 2 = 9 parts

1 part: 1 098 ÷ 9 = $122

Spendable: 7 × 122 = $854

Savings: 2 × 122 = $244

Check: 854 + 244 = 1 098

Now ratios with 3 values

4 A stadium is painted gold, blue and red in the ratio 5 : 4 : 2. If it is estimated that the stadium has a paintable area of 445.5 m^2. If 1 litre of paint covers 15 m^2, find the amount required of each paint colour. (in whole litres)

Paint = 445.5 ÷ 15 = 29.7 L

Add: 5 + 4 + 2 = 11 parts

1 part: 29.7 ÷ 11 = 2.7 L

Gold: 5 × 2.7 = 13.5 L

= 14 L (whole L)

Blue: 4 × 2.7 = 10.8 L

= 11 L (whole L)

Red: 2 × 2.7 = 5.4 L

= 6 L (whole L)

5 Groucho the cat sleeps 75% of the day. The rest of his day is spent eating, hissing and growling in the ratio 1 : 9 : 2. Find the amount of time spent on each of his 4 activities.

Sleep time = 0.75 × 24 = 18 h

Awake time = 24 - 18 = 6 h

Add: 1 + 9 + 2 = 12 parts

1 part: 6 ÷ 12 = 30 min

Eating: 30 min

Hissing: 9 × 30 = 4 h 30 min

Growling: 2 × 30 = 1 h

Check: 18 + 30 min + 4 h 30 min

+ 1 h = 24 h

6 A cruise ship operator notes that its passenger balance is children : adults : retirees in the ratio 2 : 9 : 20. The ship has a total capacity of 2 945 people, which has a crew : passengers ratio of 1 : 4. Find the number of crew and of each passenger category on board of a full ship.

Add: 1 + 4 = 5 parts

1 part: 2 945 ÷ 5 = 589

Crew: 589 staff

Passengers: 4 × 589 = 2 356

Add: 2 + 9 + 20 = 31 parts

1 part: 2 356 ÷ 31 = 76 pass'gers

Children: 2 × 76 = 152 pas'gers

Adults: 9 × 76 = 684 pas'gers

Retirees: 20 × 76 = 1 520 ppl.

Check: 589+152+684+1 520 = 2 945

7 A town's population ratio of males : females is 49 : 51. If the town has a current population of 17 600, estimate the increase in males and females in 10 years time if the town's population is expected to increase by 30%.

0.3 × 17 600 = 5 280 increase

Add: 49 + 51 = 100 parts

1 part: 5 280 ÷ 100 = 52.8

Males: 49 × 52.8 = 2 587.2

≈ 2 587 males

Females: 51 × 52.8 = 2 692.8

≈ 2 693 females

Ratio of a Quantity - Method 2

This method uses fractions rather than finding the value of a single 'part' in a ratio. The process is also quicker that the unitary method. Below is an example, there is also an example in the first column, the entire sheet uses the one method to solve the questions.

Example: Divide 550 m in the ratio 2 : 9.

Add parts: 2 + 9 = 11 parts

The ratio is 2 : 9. The first step is to add these two parts together. So 2 + 9 = 11

2 parts: $\frac{2}{11} \times 550 = 100$ m

First the 2 in the ratio. Using the 11 calculated above, form a fraction using 11 as the denominator. So $^{2}/_{11} \times 550$.

9 parts: $\frac{9}{11} \times 550 = 450$ m

Next the 9 in the ratio. Solve as with the previous step. So $^{9}/_{11} \times 550$. Remember the units.

Ratio: 100 m : 450 m

Rewrite your answers above in ratio form, this fully answers the question.

Check: 100 + 450 = 550

A check can be performed at the end to verify your answers are correct.

Ratio of a Quantity - Method 2

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Divide these quantities by the ratio given. Here is an example.

Divide 549 mL in the ratio 2 : 7.

Add parts: 2 + 7 = 9 parts

2 parts: $\frac{2}{9} \times 549 = 122$ mL

The 9 parts becomes the denomintor

7 parts: $\frac{7}{9} \times 549 = 427$ mL

Ratio: 122 mL : 427 mL

Check: 122 + 427 = 549

1 Divide \$750 in the ratio 5:1

Add parts: ☐ + ☐ = ☐ parts

5 parts: $\frac{\square}{\square} \times \square = \square$

1 part: $\frac{\square}{\square} \times \square = \square$

Ratio: ☐ : ☐

Check: ☐ + ☐ = ☐

2 Divide 6.1 m in the ratio 3 : 2

Add parts: ☐ + ☐ = ☐ parts

3 parts: $\frac{\square}{\square} \times \square = \square$

2 parts: $\frac{\square}{\square} \times \square = \square$

Ratio: ☐ : ☐

Check: ☐ + ☐ = ☐

3 Divide 984 g in the ratio 7:5

Add parts: ☐ + ☐ = ☐ parts

7 parts: $\frac{\square}{\square} \times \square = \square$

5 parts: $\frac{\square}{\square} \times \square = \square$

Ratio: ☐ : ☐

Check: ☐ + ☐ = ☐

4 Divide 4.23 km into the ratio 7 : 2. Answer in m.

5 Divide 9.1 T into the ratio 3 : 10. Answer in kg.

6 Divide 67 cm into the ratio 22: 13. Answer in mm to 1 d.p.

7 Divide 19 km into the ratio 1 : 5 : 3. Round to 2 d.p.

8 Pam's Petworld finds that clients with dogs outnumber those with cats in the ratio 5 : 3. If she sees 27 animals, how many would you expect of each?

9 Prize money of \$12 000 is shared between Ben, Sarah and Jess in the ratio 2 : 5 : 8. Find each person's share.

10 Shade this rectangle in the ratio 2 : 3, shaded : unshaded.

Ratio of a Quantity - Method 2

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Divide these quantities by the ratio given. Here is an example.

Divide 549 mL in the ratio 2 : 7.

Add parts: 2 + 7 = 9 parts

2 parts: $\frac{2}{9} \times 549 = 122$ mL

The 9 parts becomes the denomintor

7 parts: $\frac{7}{9} \times 549 = 427$ mL

Ratio: 122 mL : 427 mL

Check: 122 + 427 = 549

1 Divide \$750 in the ratio 5:1

Add parts: 5 + 1 = 6 parts

5 parts: $\frac{5}{6} \times 750 = \$625$

1 part: $\frac{1}{6} \times 750 = \$125$

Ratio: \$625 : \$125

Check: 625 + 125 = 750

2 Divide 6.1 m in the ratio 3 : 2

Add parts: 3 + 2 = 5 parts

3 parts: $\frac{3}{5} \times 6.1 = 3.66$ m

2 parts: $\frac{2}{5} \times 6.1 = 2.44$ m

Ratio: 3.66 m : 2.44 m

Check: 3.66 + 2.44 = 6.1

3 Divide 984 g in the ratio 7:5

Add parts: 7 + 5 = 12 parts

7 parts: $\frac{7}{12} \times 984 = 574$ g

5 parts: $\frac{5}{12} \times 984 = 410$ g

Ratio: 574 g : 410 g

Check: 574 + 410 = 984

4 Divide 4.23 km into the ratio 7 : 2. Answer in m.

Add parts: 7 + 2 = 9

7 parts: $^7/_9 \times 4\,230 = 3\,290$ m

2 parts: $^2/_9 \times 4\,230 = 940$ m

3 290 m : 940 m

Check: 3 290 + 940 = 4 230

5 Divide 9.1 T into the ratio 3 : 10. Answer in kg.

Add parts: 3 + 10 = 13

3 parts: $^3/_{13} \times 9\,100 = 2\,100$ kg

10 parts: $^{10}/_{13} \times 9\,100 = 7\,000$ kg

2 100 kg : 7 000 kg

Check: 2 100 + 7 000 = 9 100

6 Divide 67 cm into the ratio 22: 13. Answer in mm to 1 d.p.

Add parts: 22 + 13 = 35

22 parts: $^{22}/_{35} \times 670 = 421.1$ mm

13 parts: $^{13}/_{35} \times 670 = 248.9$ mm

421.1 mm : 248.9 mm

Check: 421.1 + 248.9 = 670

7 Divide 19 km into the ratio 1 : 5 : 3. Round to 2 d.p.

Add parts: 1 + 5 + 3 = 9

1 part: $^1/_9 \times 19 = 2.11$ km

5 parts: $^5/_9 \times 19 = 10.56$ km

3 parts: $^3/_9 \times 19 = 6.33$ km

2.11 km : 10.56 km : 6.33 km

Check: 2.11 + 10.56 + 6.33 = 19

8 Pam's Petworld finds that clients with dogs outnumber those with cats in the ratio 5 : 3. If she sees 27 animals, how many would you expect of each?

Add parts: 5 + 3 = 8

Dogs: $^5/_8 \times 27 = 17$ dogs (0 d.p.)

Cats: $^3/_8 \times 27 = 10$ cats (0 d.p.)

She should see 17 dogs & 10 cats

Check: 17 + 10 = 27

9 Prize money of \$12 000 is shared between Ben, Sarah and Jess in the ratio 2 : 5 : 8. Find each person's share.

Add parts: 2 + 5 + 8 = 15

Ben: $^2/_{15} \times 12\,000 = \$1\,600$

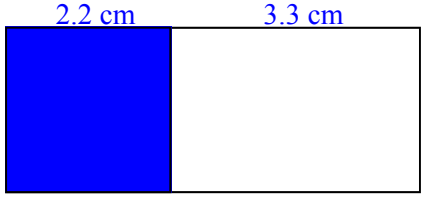
Sarah: $^5/_{15} \times 12\,000 = \$4\,000$

Jess: $^8/_{15} \times 12\,000 = \$6\,400$

Ben \$1 600, Sarah \$4 000 and Jess \$6 400.

Ck:1 600+4 000+6 400 = 12 000

10 Shade this rectangle in the ratio 2 : 3, shaded : unshaded.



Length is 5.5 cm

Add parts: 2 + 3 = 5

Shaded: $^2/_5 \times 5.5 = 2.2$ cm

Unshaded: $^3/_5 \times 5.5 = 3.3$ cm

Check: 2.2 + 3.3 = 5.5

Scale Drawing

Scale drawings allow us to use images to measure distances. By multiplying by the scale any length on a scale diagram can be used to determine the actual length.

Column 1 features diagrams and images of animals/insects in nature and man-made objects, as well as the scale ratio applied to the image. How do you find the actual length? Measure the length in mm, then multiply by the second number in the ratio, then divide by the first number.

If the first number is larger than the second then the actual length is smaller than the image. That means the image was an enlargement.

If the first number is smaller than the second then the actual length is larger than the image. That means the image was an reduction.

Write the measured length in the table at the base of the column, using mm. Apply the scale and write the actual length in the RHS of the table. Again use mm, but if the column asks you to convert this to cm or m, then divide by 10 or 1 000, and rewrite in the units requested.

Column 2 involves travel between two wharves. One person drives on land, the other uses a ferry. Use the scale to measure the distance between the two points requested in km. Then using the speed of the ferry complete a timetable for its operation.

Further down Column 2 is a map with roads connecting towns. Measure the straight line distances between the points, then apply the scale. You should notice a quick way to apply the scale rather than multiplying and dividing by the large numbers. The lengths aren't exact in mm, sometimes they are slightly out, so answers will vary by small amounts. Write the distances in the boxes.

Scale Drawing

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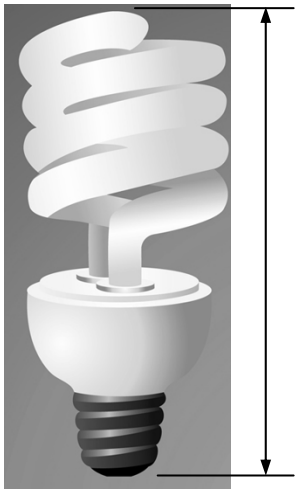
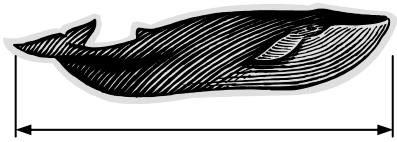
Measure the lengths in mm, using the arrowed length. Write the length in the 'image' box below. Apply the scale to find the actual length. Multiply by the second number in the ratio, divide by the first.

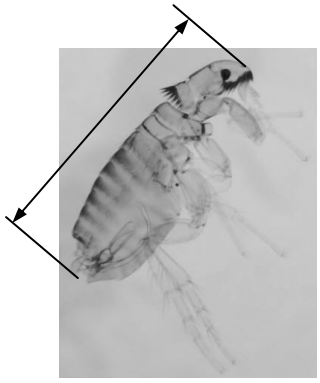
1 Scale = 1 : 2



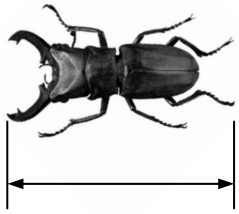
2 Scale = 1 : 660



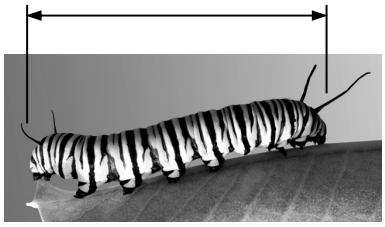
3 Scale = 9 : 1



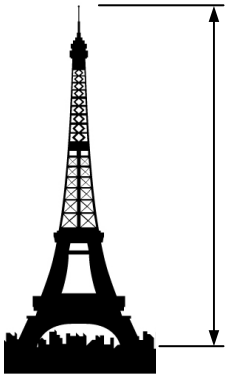
4 Scale = 3 : 2



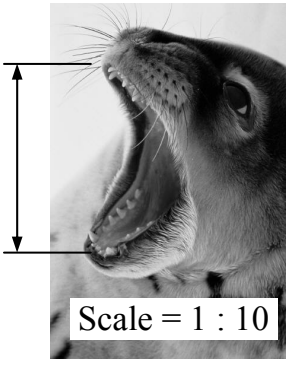
5 Scale = 5 : 3



6 Scale = 1 : 7 200

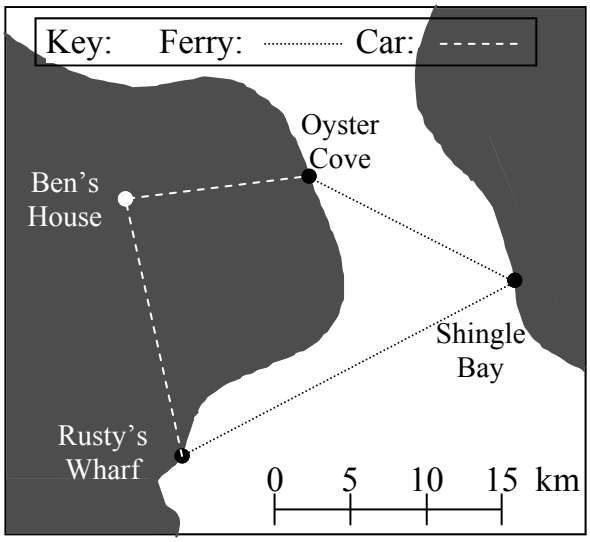


7



Q	Image Length (mm)	Actual Length (mm)
1		
2		mm = m
3		
4		
5		mm = cm
6		mm = m
7		mm = cm

Ben drove from Rusty's Wharf to Oyster Cove, while Tim caught the ferry. Use the scale to answer these questions.



Find the distance in km between:

8 Rusty's Wharf and Shingle Bay _____

9 Shingle Bay and Oyster Cove _____

10 Rusty's Wharf and Ben's House _____

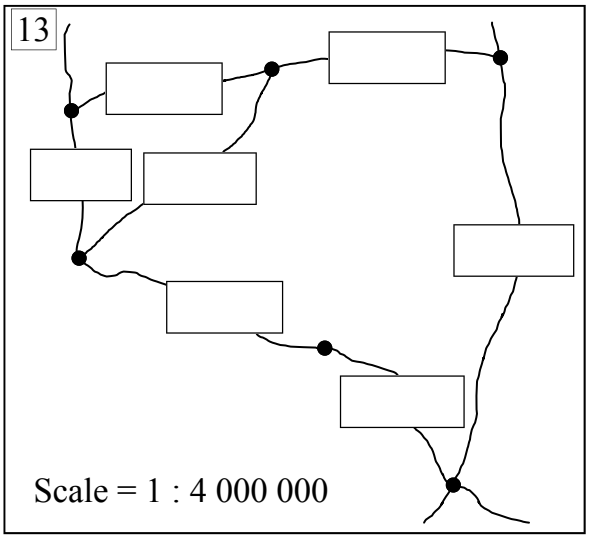
11 Ben's house and Oyster Cove _____

12 The ferry averages 20 km/h (5 km in 15 min), and it stops at all wharves for 10 min. Complete the time table below.

Location	Arrives	Departs
Rusty's Wharf	12.25 pm.	
Shingle Bay		
Oyster Cove		

Measure straight lengths in mm (ignore curves). Use the scale, then write the distances (km) in the boxes.

13



mm → m → km = ÷ 1 000 ÷ 1 000

Scale Drawing

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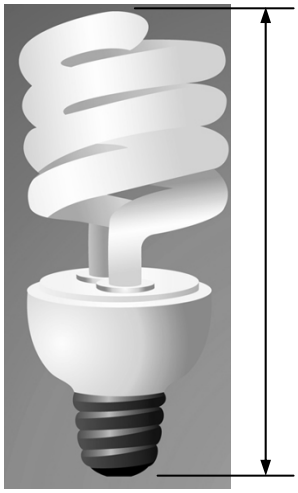
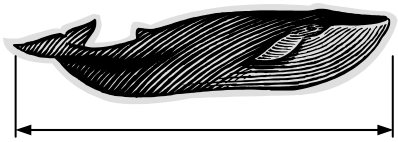
Measure the lengths in mm, using the arrowed length. Write the length in the 'image' box below. Apply the scale to find the actual length. Multiply by the second number in the ratio, divide by the first.

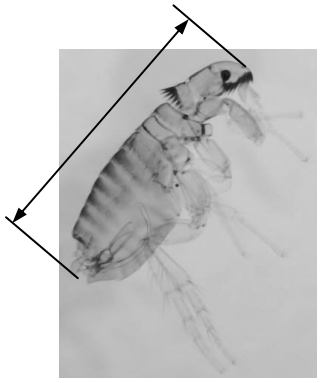
1 Scale = 1 : 2



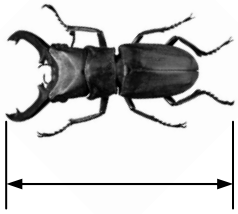
2 Scale = 1 : 660



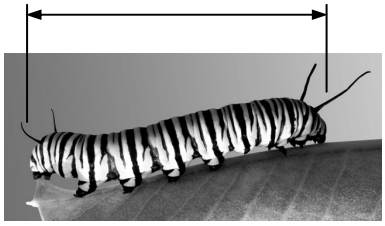
3 Scale = 9 : 1



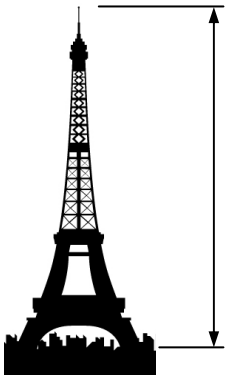
4 Scale = 3 : 2



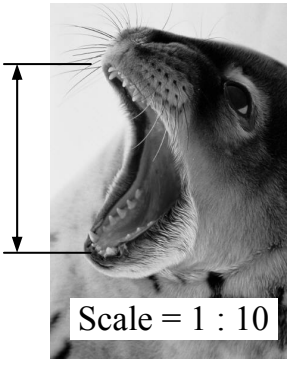
5 Scale = 5 : 3



6 Scale = 1 : 7 200

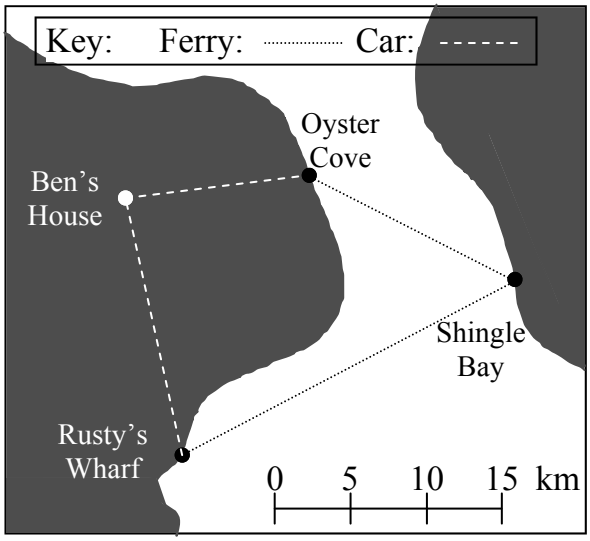


7



Q	Image Length (mm)	Actual Length (mm)
1	62 mm	124 mm
2	50 mm	33 000 mm = 33 m
3	36 mm	4 mm
4	30 mm	20 mm
5	40 mm	24 mm = 2.4 cm
6	45 mm	324 000 mm = 324 m
7	25 mm	250 mm mm = 25 cm

Ben drove from Rusty's Wharf to Oyster Cove, while Tim caught the ferry. Use the scale to answer these questions.



Find the distance in km between:

8 Rusty's Wharf and Shingle Bay 25 km

9 Shingle Bay and Oyster Cove 15 km

10 Rusty's Wharf and Ben's House 17.5 km

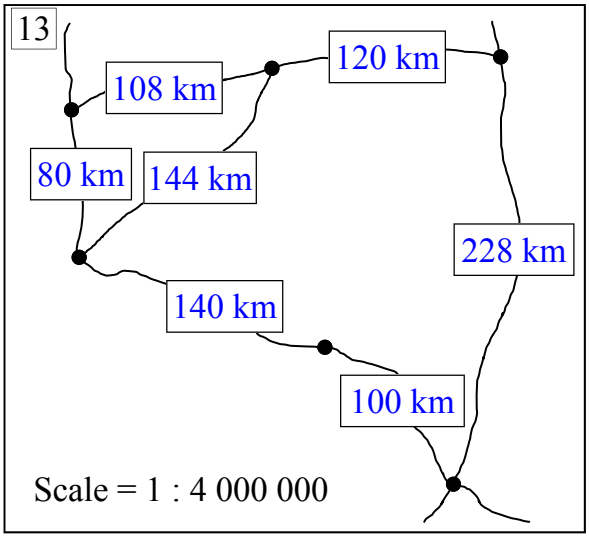
11 Ben's house and Oyster Cove 12.5 km

12 The ferry averages 20 km/h (5 km in 15 min), and it stops at all wharves for 10 min. Complete the time table below.

Location	Arrives	Departs
Rusty's Wharf	12.25 pm.	12.35 pm.
Shingle Bay	1.50 pm.	2.00 pm.
Oyster Cove	2.45 pm.	2.55 pm.

Measure straight lengths in mm (ignore curves). Use the scale, then write the distances (km) in the boxes.

13



mm → m → km = ÷ 1 000 ÷ 1 000

Scale Diagrams - Construction Plan Views

When a building is designed it is shown in two dimensional views. It can be shown from the sides (elevation and end elevation) or from above, which is called the plan view. This sheet deals with scale drawing diagrams in plan view.

So the roof has been ripped off and you are like a giant bent over the room looking down. The kitchen shown on the worksheet contains a pantry cupboard, a sink (wet area), a cooking area and other cupboards. While you can't see how tall they are, you can measure them and find their length and width.

The scale of this diagram is 1 : 50. So every measurement you make has to be multiplied by 50. Once you do that, change the length from cm to m. All the lengths on the sheet are either exact cm lengths, or 0.5 cm lengths.

So measure the lengths, make the conversion using the scale and answer the questions. You should notice an easier way to convert than using your calculator all the time. Nothing that a giant couldn't manage, as long as they have a ruler of course.

Scale Diagrams - Construction Plan Views

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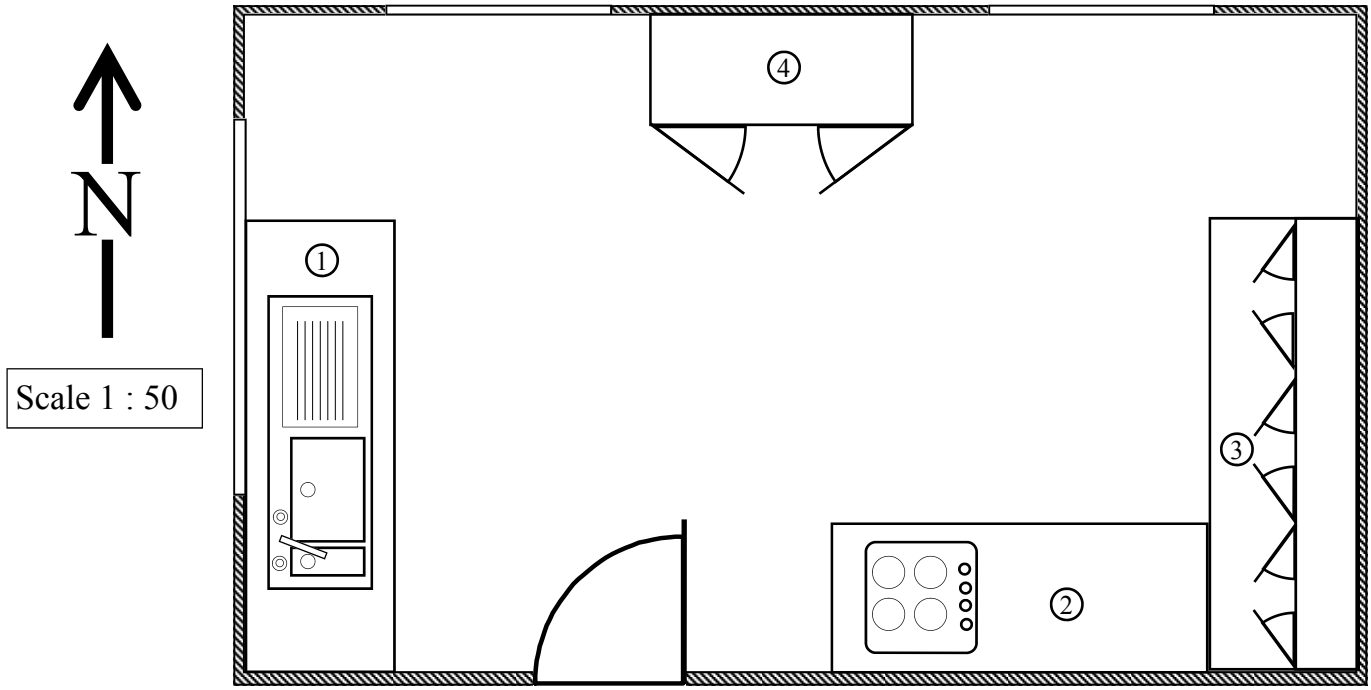

Get out your ruler and measure lengths in cm, to the nearest 0.5 cm. Then use the scale to find the actual length and answer the questions. Round to 2 d.p.

1 The length of the eastern wall (in m). _____

2 The length of the northern wall (in m). _____

3 The floor area (empty) of the room (m^2).

Length = ☐ m Breadth = ☐ m

$A = lb$

$= ☐ \times ☐$

$A = ☐ m^2$

4 The length, breadth and floor area of the pantry ④

Length = ☐ m _____

Breadth = ☐ m _____

_____

5 The length, breadth and floor area of the sink bench ①

Length = ☐ m _____

Breadth = ☐ m _____

_____

6 The length, breadth and floor area of the cook-top bench ②

_____

Length = ☐ m _____

Breadth = ☐ m _____

7 The length, breadth and floor area of the cupboards ③

Length = ☐ m _____

Breadth = ☐ m _____

_____

8 Find the total covered floor area. = ① + ② + ③ + ④

_____

9 Find the total uncovered floor area.

_____

10 Estimate the cost of tiling the uncovered floor area when tiles cost \$35/m^2. Available in whole m^2 only.

_____

11 If the pantry ④ is 2 m tall, calculate the wall area it occupies.

Height = ☐ m _____

Width = ☐ m _____

_____

12 There are three windows in the plan, if they are all 1.2 m high, find the total window area in the room.

Sink Window	Northern Wall Windows	Total

Scale Diagrams - Construction Plan Views

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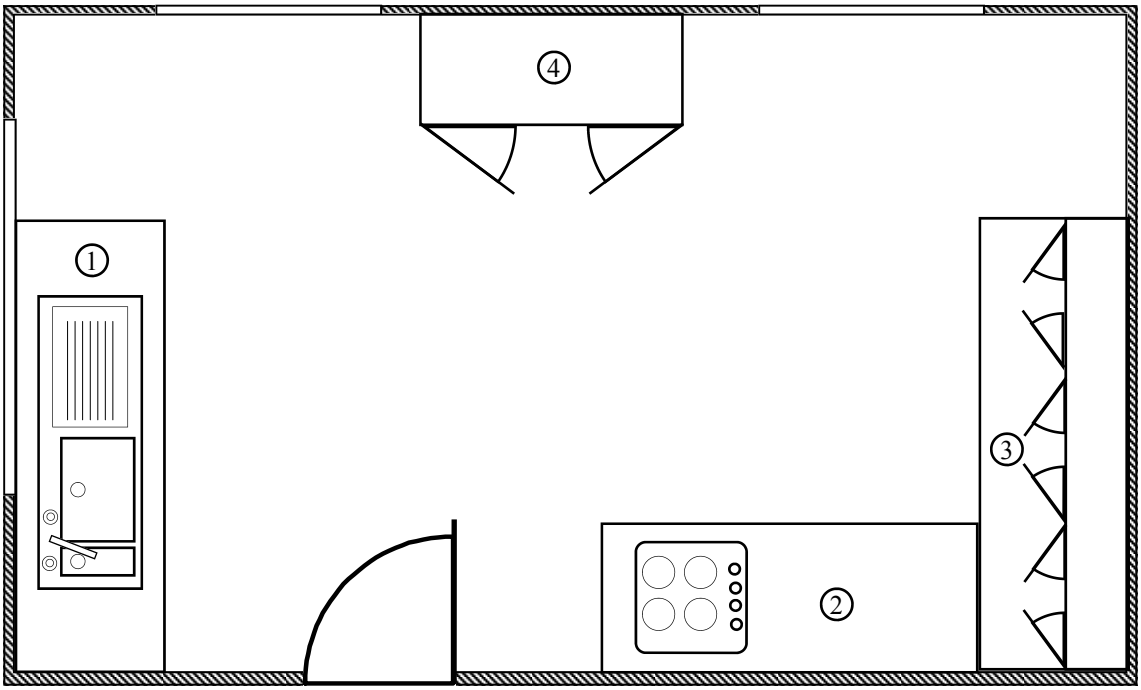

Get out your ruler and measure lengths in cm, to the nearest 0.5 cm. Then use the scale to find the actual length and answer the questions. Round to 2 d.p.

1 The length of the eastern wall (in m). 4.5 m

2 The length of the northern wall (in m). 7.5 m

3 The floor area (empty) of the room (m^2).

Length = 7.5 m Breadth = 4.5 m

$A = lb$

$= 7.5 \times 4.5$

$A = 33.75\ m^2$

4 The length, breadth and floor area of the pantry ④

Length = 1.75 m

Breadth = 0.75 m

$A = lb$

$= 1.75 \times 0.75$

$A = 1.31\ m^2$ (2 d.p.)

5 The length, breadth and floor area of the sink bench ①

Length = 3 m

Breadth = 1 m

$A = lb$

$= 3 \times 1$

$A = 3\ m^2$

6 The length, breadth and floor area of the cook-top bench ②

Length = 2.5 m

Breadth = 1 m

$A = lb$

$= 2.5 \times 1$

$A = 2.5\ m^2$

7 The length, breadth and floor area of the cupboards ③

Length = 3 m

Breadth = 1 m

$A = lb$

$= 3 \times 1$

$A = 3\ m^2$

8 Find the total covered floor area. = ① + ② + ③ + ④

$A = 1.31 + 3 + 2.5 + 3 = 9.81\ m^2$

9 Find the total uncovered floor area.

$A = 33.75 - 9.81 = 23.94\ m^2$

10 Estimate the cost of tiling the uncovered floor area when tiles cost \$35/m^2. Available in whole m^2 only.

$Cost = 24 \times 35 = \$840$

11 If the pantry ④ is 2 m tall, calculate the wall area it occupies.

Height = 2 m

Width = 1.75 m

$A = lb$

$= 2 \times 1.75$

$A = 3.5\ m^2$

12 There are three windows in the plan, if they are all 1.2 m high, find the total window area in the room.

Sink Window	Northern Wall Windows	Total
$A = lb$	$A = lb$	Area = 2 × 1.8
$= 2.5 \times 1.2$	$= 1.5 \times 1.2$	+ 3 = 6.6 m^2
$A = 3\ m^2$	$A = 1.8\ m^2$	

Finding the Scale

Scale drawings are used on maps, construction diagrams and scientific diagrams amongst other applications. The scale can be written on these diagrams as a ratio, such as 1 : 4 000 or a measured scale where a length on the image (diagram or map) represents the actual length. Converting this scale to a ratio is the aim of these worksheet.

Column 1 starts with a review on simplifying ratios, use your calculator's fraction key to simplify these ratios.

Column 2 then deals with measured lengths on a diagram with the actual length. These are again answered with the fraction key of the calculator. Write the image : actual ratio, then simplify. Later in the column the units of these two measurements are different. So convert them to match and then repeat the calculator's fraction key process to simplify.

Column 3 is the same as column 2 except that a ruler is used first. Measure the length of the scale line, then compare that with the actual length. Change the units and solve by the same method as above.

Finding the Scale

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Simplify the ratios below by using your calculator's fraction key

Example 25:65 = 5:13

2 5 $a^b/_c$ 6 5 =

1 20 : 110 = ☐ : ☐

2 72 : 189 = ☐ : ☐

3 86 : 168 =

4 200 : 1 400 =

5 75 : 350 =

6 45 : 270 =

7 24 : 56 =

8 12 : 63 =

9 10 : 1 000 =

10 15 : 650 =

These give mixed or whole number answers. Convert them to improper fractions.

3 6 $a^b/_c$ 2 1 = SHIFT $a^b/_c$

11 36 : 21 = :

12 200 : 75 =

13 350 : 75 =

14 600 : 125 =

15 550 : 120 =

16 900 : 320 =

17 1 500 : 6 =

18 55 : 15 =

19 340 : 85 =

20 10 000 : 20 =

Find the scale given both the scaled or 'image' size and the actual size.

21 Image size 25 mm

Actual size 200 mm

Scale = ☐ : ☐ (image, actual) = ☐ : ☐ (simplify)

22 Image size 3 cm

Actual size 72 cm

Scale = ☐ : ☐ (image, actual) = ☐ : ☐ (simplify)

23 Image size 35 cm

Actual size 12 cm

Scale = ☐ : ☐ = ☐ : ☐

Change the larger unit to match the smaller unit and calculate the scale.

24 Image size 15 cm

Actual size 1.2 m = cm

Scale = ☐ : ☐ (image, actual) = ☐ : ☐ (simplify)

25 Image size 32 mm

Actual size 2.06 m = mm

Scale =

26 Image size 5 cm

Actual size:

175 km =

Scale =

Just write simplified ratio

27 Image size 18 cm

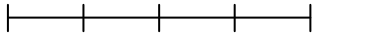
Actual size:

54 km =

Scale =

Rewrite the scale below as a ratio in its simplest form

28

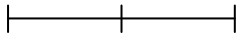


☐ cm = ☐ m

☐ cm = ☐ cm

☐ : ☐

29

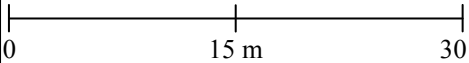


☐ cm = ☐ km

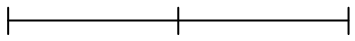
☐ cm = ☐ cm

☐ : ☐

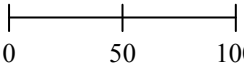
30



31



32



Finding the Scale

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Simplify the ratios below by using your calculator's fraction key

Example 25:65 = 5:13

2 5 $a^b/_c$ 6 5 =

1 20 : 110 = 2 : 11

2 72 : 189 = 8 : 21

3 86 : 168 = 43 : 84

4 200 : 1 400 = 1 : 7

5 75 : 350 = 3 : 14

6 45 : 270 = 1 : 6

7 24 : 56 = 3 : 7

8 12 : 63 = 4 : 21

9 10 : 1 000 = 1 : 100

10 15 : 650 = 3 : 130

These give mixed or whole number answers. Convert them to improper fractions.

3 6 $a^b/_c$ 2 1 = SHIFT $a^b/_c$

11 36 : 21 = 12 : 7

12 200 : 75 = 8 : 3

13 350 : 75 = 14 : 3

14 600 : 125 = 24 : 5

15 550 : 120 = 55 : 12

16 900 : 320 = 45 : 16

17 1 500 : 6 = 250 : 1

18 55 : 15 = 11 : 3

19 340 : 85 = 4 : 1

20 10 000 : 20 = 500 : 1

Find the scale given both the scaled or 'image' size and the actual size.

21 Image size 25 mm

Actual size 200 mm

Scale = 25 (image) : 200 (actual) = 1 : 8 (simplify)

22 Image size 3 cm

Actual size 72 cm

Scale = 3 (image) : 72 (actual) = 1 : 24 (simplify)

23 Image size 35 cm

Actual size 12 cm

Scale = 35 : 12 = 35 : 12

Change the larger unit to match the smaller unit and calculate the scale.

24 Image size 15 cm

Actual size 1.2 m = 120 cm

Scale = 15 (image) : 120 (actual) = 1 : 8 (simplify)

25 Image size 32 mm

Actual size 2.06 m = 2 060 mm

Scale = 32 : 2 060 = 8 : 515

26 Image size 5 cm

Actual size:

175 km = 17 500 000

Scale = 1 : 3 500 000

Just write simplified ratio

27 Image size 18 cm

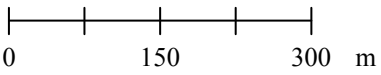
Actual size:

54 km = 5 400 000

Scale = 1 : 300 000

Rewrite the scale below as a ratio in its simplest form

28

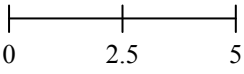


3 cm = 300 m

3 cm = 30 000 cm

1 : 10 000

29

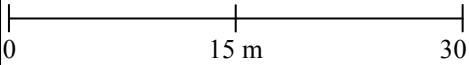


4 cm = 5 km

4 cm = 500 000 cm

1 : 125 000

30

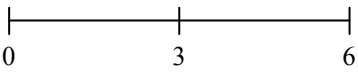


6 cm = 30 m

6 cm = 3 000 cm

1 : 500

31

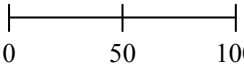


4.5 cm = 6 mm

45 mm = 6 mm

15 : 2

32



2 cm = 100 km

2 cm = 10 000 000 cm

1 : 5 000 000

Rates

Rates compare quantities with each other. This rate can be simplified, it can also be applied to larger quantities.

For example, if you can sharpen four pencils in 2 minutes, then you can sharpen 2 pencils in one minute. And if you sharpen pencils for 8 minutes you can apply the rate to calculate how many pencils you have sharpened. A forward slash separates rate quantities so pencils/min or m/s or kg/min etc. are how rates are shown.

Column 1 starts with simplifying ratios. Divide the first quantity by the second, then show your rate with a '/'. Q.7 - 12 you are supplied the rate, but instead are asked to change the rates units. So this requires you to multiply or divide by the appropriate conversion amount.

Example. Convert 15 T/min to T/s

15 T/min [T/s]

$15 \div 60 = 0.25$ T/s ← There are 60 seconds in a minute. And with less time there will be less weight, so divide by 60.

Column 2 is like the above question but there are two or more conversions to be made. The above example will be attempted again, but this time the answer is to be in kg/s.

Example. Convert 15 T/min to kg/s

15 T/min [kg/s]

$15 \times 1\,000 = 15\,000$ kg/min ← If you have to multiply, complete the multiplication first. So convert T to kg.

$15\,000 \div 60 = 250$ kg/s ← Now complete the second conversion, from min to s. So divide by 60.

The rest of Column 2 and Column 3 are questions that require you to apply a ratio to a given problem. These questions will mostly require either division or multiplication. Look at the question and ask yourself "After the conversion will I have more of less of them?" then use that answer to help you to decide on division or multiplication.

Rates

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Use division to express these rates in their simplest form.

Example

12 truckloads in 3 hours

12 ÷ 3 = 4 truckloads/h

1 85 cm in 5 seconds

□ ÷ □ = □ cm/s

2 285 shells in 19 m^2

3 871 people in 13 entrances

4 $132 for 8 hours work

5 112 nails in 7 m

6 381 km in 5 hours

Express in the given units.

7 8.4 T/h [kg/h]

□ × □ = □ kg/h

8 2.5 km/s [km/min]

9 936 people/year [people/week]

10 9 m/day [m/h]

11 0.8 kg/m^2 [kg/ha]

1 ha = 10 000 m^2 rpm = rev/min

12 504 rpm [rev/s]

Convert these rates to the bracketed units. Multiple conversions required!

13 12 cm/s [m/min]

= □ × □ = □ cm/min

= □ ÷ □ = □ m/min

14 3.4 kg/min [T/h]

= □ × □ = □ kg/h

= □ ÷ □ = □ T/h

15 72 km/h [m/s]

m/h

m/min

m/s

16 15 m/s [km/h]

Apply these rates in these questions.

17 Jerry can fit a car tyre in 5 min. How many can he fit in 20 min?

18 In 1 hour?

19 How long would he take to fit 40 tyres?

20 Sara is paid $942/week. How much would she be paid in a year?

21 After how many complete weeks has she been paid $6 000?

22 A block of cheese costs $13/kg. How much would 2.2 kg cost?

23 What amount of cheese, in kg, could be purchased for $20. (to 2 d.p.)

24 A car emits 109g/km of CO_2. How much CO_2 is emitted in kg when the car has travelled 85 km.

25 If the average distance covered by a motorist is 20 000 km p.a. Find the amount of CO_2 emitted in T, in a year.

25 100 g of toothpaste contains 0.1 g of fluoride. How much fluoride is there in a 225 g tube?

26 SlateFence make toothpaste. They produce a new tube every 0.08 s. How many tubes are produced in a minute?

27 A medicine guidelines are 10 mL, plus 2 mL for every 10 kg of body weight. What amount of this medicine should be taken by a 90 kg person?

Rates

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Use division to express these rates in their simplest form.

Example

12 truckloads in 3 hours

12 ÷ 3 = 4 truckloads/h

1 85 cm in 5 seconds

85 ÷ 5 = 17 cm/s

2 285 shells in 19 m^2

285 ÷ 19 = 15 shells/m^2

3 871 people in 13 entrances

871 ÷ 13 = 67 people/entrance

4 $132 for 8 hours work

132 ÷ 8 = $16.50/h

5 112 nails in 7 m

112 ÷ 7 = 16 nails/m

6 381 km in 5 hours

381 ÷ 5 = 76.2 km/h

Express in the given units.

7 8.4 T/h [kg/h]

8.4 × 1 000 = 8 400 kg/h

8 2.5 km/s [km/min]

2.5 × 60 = 150 km/min

9 936 people/year [people/week]

936 ÷ 52 = 18 people/week

10 9 m/day [m/h]

9 ÷ 24 = 0.375 m/day

11 0.8 kg/m^2 [kg/ha]

0.8 × 10 000 = 8 000 kg/ha

1 ha = 10 000 m^2 rpm = rev/min

12 504 rpm [rev/s]

504 ÷ 60 = 8.4 rev/s

Convert these rates to the bracketed units. Multiple conversions required!

13 12 cm/s [m/min]

= 12 × 60 = 720 cm/min

= 720 ÷ 100 = 7.2 m/min

14 3.4 kg/min [T/h]

= 3.4 × 60 = 204 kg/h

= 204 ÷ 1 000 = 0.204 T/h

15 72 km/h [m/s]

= 72 × 1 000 = 72 000 m/h

= 72 000 ÷ 60 = 1 200 m/min

= 1 200 ÷ 60 = 20 m/s

16 15 m/s [km/h]

= 15 × 60 = 900 m/min

= 900 × 60 = 54 000 m/h

= 54 000 ÷ 1 000 = 54 km/h

Apply these rates in these questions.

17 Jerry can fit a car tyre in 5 min. How many can he fit in 20 min?

$^{20}/_{5}$ = 4 tyres

18 In 1 hour? $^{60}/_{5}$ = 12 tyres

19 How long would he take to fit 40 tyres? 40 × 5 = 200 min (3 h 20 min)

20 Sara is paid $942/week. How much would she be paid in a year?

942 × 52 = $48 984

21 After how many complete weeks has she been paid $6 000?

$^{6\,000}/_{942}$ = 6.36 … = 7 weeks

22 A block of cheese costs $13/kg. How much would 2.2 kg cost?

Cost = 2.2 × 13

Cost = $28.60

23 What amount of cheese, in kg, could be purchased for $20. (to 2 d.p.)

Cheese = $^{20}/_{13}$

Cheese = 1.53 kg (2 d.p.)

24 A car emits 109g/km of CO_2. How much CO_2 is emitted in kg when the car has travelled 85 km.

CO_2 = 109 × 85

CO_2 = 9 265 g = 9.265 kg

25 If the average distance covered by a motorist is 20 000 km p.a. Find the amount of CO_2 emitted in T, in a year.

CO_2 = 20 000 × 109

= 2 180 000 g/year

CO_2 = 2.18 T/year

25 100 g of toothpaste contains 0.1 g of fluoride. How much fluoride is there in a 225 g tube?

Fluoride = $^{225}/_{100}$ × 0.1

Fluoride = 0.225 g

26 SlateFence make toothpaste. They produce a new tube every 0.08 s. How many tubes are produced in a minute?

Tubes = $^{60}/_{0.08}$

Tubes = 750 tubes/s

27 A medicine guidelines are 10 mL, plus 2 mL for every 10 kg of body weight. What amount of this medicine should be taken by a 90 kg person?

Medicine = 10 + $^{90}/_{10}$ × 2

Medicine = 28 mL

Conversion of Speed

Speed can be measured in a variety of units, like cm/h, km/s etc. The most commonly used metric speeds are km/h and m/s. It is important that you are able to convert speed measurements between these two units, this sheet shows you how.

Column 1 starts with using a methodical step-by-step approach to change between units. This method changes one unit at a time and takes three steps, there is an example at the top of the column but there is also one below.

Example - Change 16 m/s to km/h

16 m/s

m/s → m/min

m/min → m/h

m/h → km/h

16 × 60 = 960 m/min

960 × 60 = 57 600 m/h

57 600 ÷ 1 000 = 57.6 km/h

First change m/s to m/min, because there are 60 seconds in a minute multiply the 16 by 60.

Now change m/min to m/h, because there are 60 minutes in an hour, multiply the 960 by 60.

Now change m/h to km/h, because there are 1 000 m in a km, divide the 57 600 by 1 000.

The second column of the sheet answers the questions in a faster process. Rather than changing one unit at a time, it does the lot in one go. There are 1 000 m in a km and there are 60 × 60 (3 600) seconds in an hour, these are used altogether to make the conversion. There is an example above Q.8, while Q.11 and Q.12 help you get started.

The last part of Column 2 involves speed conversion to applying the brakes in a car. When a hazard or emergency arises in a car, the driver has to move their foot from the accelerator pedal to the brake pedal. This can be done in less than a second by racing drivers or drivers with fast reactions, but it is considered that 2.5 seconds will cover 90% of drivers. By converting the speed from km/h you get a speed in m/s. That will tell you how far the car is travelling each second. So multiply that by 2.5 and you get an idea of how far a car travels before it starts to break, let alone the breaking distance.

Notice the difference in distance between a car travelling at 60 km/h and 80 km/h and you cab see why authorities try to reduce speed around schools and built-up areas.

Conversion of Speed

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Use a step by step approach to change these speeds from km/h to m/s. Round to 1 d.p. when needed.

Example: Convert 50 km/h to m/s.

km/h → m/h
m/h → m/min
m/min → m/s

$50 \times 1\,000 = 50\,000$ m/h

$50\,000 \div 60 = 833.\dot{3}$ m/min

$833.\dot{3} \div 60 = 13.9$ m/s (1 d.p.)

1 36 km/h

2 140 km/h

3 4.6 km/h

Now the reverse, change these speeds in m/s to km/h

4 27 m/s

m/s → m/min
m/min → m/h
m/h → km/h

□ × 60 = □ m/min

□ × 60 = □ m/h

□ ÷ 1 000 = □ km/h

5 11 m/s

6 6.3 m/s

7 225 m/s

Now convert the 'Speedy' way. By converting all in one line. Change these to m/s. (1 d.p.)

Example: Convert 70 km/h to m/s.

$$70 \text{ km/h} = \frac{(70 \times 1\,000)}{(60 \times 60)} = 19.4 \text{ m/s (1 d.p.)}$$

8 85 km/h $= \dfrac{(\square \times 1\,000)}{(60 \times 60)} = \square$ m/s (1 d.p.)

9 65 km/h = _____ = _____

10 860 km/h = _____ = _____

Now convert from m/s to km/h. (1 d.p.)

11 14 m/s $= \dfrac{(\square \div 1\,000)}{(1 \div 60 \div 60)} = \square$ km/h

12 343 m/s Speed of Sound $= \dfrac{(\square \div 1\,000)}{(1 \div 60 \div 60)} =$ _____

13 464 m/s = _____ = _____
Earth Rotation Equator

14 1.2 m/s = _____ = _____
Walking speed

How far have these vehicles travelled during the 'reaction time' of the driver. Find the speed in m/s then multiply by 2.5 for the distance travelled.

Brake reaction time, is the time that elapses between when a hazard is seen and the brakes of a vehicle are applied by the driver. It is assumed to be an average of 2.5 seconds.

15 Find the distance travelled at 60 km/h.

60 km/h = _____ = _____ (1 d.p.)
Change km/h to m/s

Distance = □ × 2.5 = □ m (1 d.p.)

16 Find the distance travelled at 80 km/h.

80 km/h = _____ = _____ (1 d.p.)
Change km/h to m/s

Distance = □ × 2.5 = □ m (1 d.p.)

Conversion of Speed

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Use a step by step approach to change these speeds from km/h to m/s. Round to 1 d.p. when needed.

Example: Convert 50 km/h to m/s.

km/h → m/h
m/h → m/min
m/min → m/s

50 × 1 000 = 50 000 m/h

50 000 ÷ 60 = 833.3̇ m/min

833.3̇ ÷ 60 = 13.9 m/s (1 d.p.)

1 36 km/h

km/h → m/h
m/h → m/min
m/min → m/s

36 × 1 000 = 36 000 m/h

36 000 ÷ 60 = 600 m/min

600 ÷ 60 = 10 m/s

2 140 km/h

km/h → m/h
m/h → m/min
m/min → m/s

140 × 1 000 = 140 000 m/h

140 000 ÷ 60 = 2 333.3̇ m/min

2 333.3̇ ÷ 60 = 38.9 m/s (1 d.p.)

3 4.6 km/h

4.6 × 1 000 = 4 600 m/h

4 600 ÷ 60 = 76.6̇ m/min

76.6̇ ÷ 60 = 1.3 m/s (1 d.p.)

Now the reverse, change these speeds in m/s to km/h

4 27 m/s

m/s → m/min
m/min → m/h
m/h → km/h

27 × 60 = 1 620 m/min

1 620 × 60 = 97 200 m/h

97 200 ÷ 1 000 = 97.2 km/h

5 11 m/s

m/s → m/min
m/min → m/h
m/h → km/h

11 × 60 = 660 m/min

660 × 60 = 39 600 m/h

39 600 ÷ 1 000 = 39.6 km/h

6 6.3 m/s

6.3 × 60 = 378 m/min

378 × 60 = 22 680 m/h

22 680 ÷ 1 000 = 22.7 km/h (1 d.p.)

7 225 m/s

225 × 60 = 13 500 m/min

13 500 × 60 = 810 000 m/h

810 000 ÷ 1 000 = 810 km/h

Now convert the 'Speedy' way. By converting all in one line. Change these to m/s. (1 d.p.)

Example: Convert 70 km/h to m/s.

$$70 \text{ km/h} = \frac{(70 \times 1\,000)}{(60 \times 60)} = 19.4 \text{ m/s (1 d.p.)}$$

8 85 km/h $= \frac{(85 \times 1\,000)}{(60 \times 60)} =$ 23.6 m/s (1 d.p.)

9 65 km/h $= \frac{(65 \times 1\,000)}{(60 \times 60)} = 18.1$ m/s (1 d.p.)

10 860 km/h $= \frac{(860 \times 1\,000)}{(60 \times 60)} = 238.9$ m/s (1 d.p.)

Now convert from m/s to km/h. (1 d.p.)

11 14 m/s $= \frac{(14 \div 1\,000)}{(1 \div 60 \div 60)} =$ 50.4 km/h

12 343 m/s (Speed of Sound) $= \frac{(343 \div 1\,000)}{(1 \div 60 \div 60)} = 1\,234.8$ km/h

13 464 m/s (Earth Rotation Equator) $= \frac{(464 \div 1\,000)}{(1 \div 60 \div 60)} = 1\,670.4$ km/h

14 1.2 m/s (Walking speed) $= \frac{(1.2 \div 1\,000)}{(1 \div 60 \div 60)} = 4.3$ km/h (1 d.p.)

How far have these vehicles travelled during the 'reaction time' of the driver. Find the speed in m/s then multiply by 2.5 for the distance travelled.

Brake reaction time, is the time that elapses between when a hazard is seen and the brakes of a vehicle are applied by the driver. It is assumed to be an average of 2.5 seconds.

15 Find the distance travelled at 60 km/h.

60 km/h (Change km/h to m/s) $= \frac{(60 \times 1\,000)}{(60 \times 60)} = 16.7$ m/s (1 d.p.) (1 d.p.)

Distance = 16.7 × 2.5 = 41.8 m (1 d.p.)

16 Find the distance travelled at 80 km/h.

80 km/h (Change km/h to m/s) $= \frac{(80 \times 1\,000)}{(60 \times 60)} = 22.2$ m/s (1 d.p.) (1 d.p.)

Distance = 22.2 × 2.5 = 55.5 m (1 d.p.)

Rates and Food

When you buy fruit, vegetables and meat you use rates. How much you pay for an item is a rate involving cost and weight, for example $4.98/kg. This sheet deals with rates and food and the three ways that rates can be used with food.

Column 1 deals with the most common use of rates:

If it costs this much per kg, how much will I pay if I buy this much?

This column uses multiplication to solve these. So if Ham costs $19.95/kg, how much will 1.8 kg of ham cost? An example is below:

Ham costs $19.95/kg, find the price paid for 1.8 kg.

Cost = $1.8 \times 19.95 = \$35.91$

Column 2 calculates the amount of an item that was purchased:

If it costs this much per kg, and I spent this much …. how much did I buy?

This column uses division to solve these. So if Ham costs $19.95/kg, how much ham have you bought if you spent $23? An example is below:

Ham costs $19.95/kg, find the amount purchased, to 2 d.p., if $23 was spent.

$23 \div 19.95 = 1.15$ (2 d.p.)

1.15 kg of ham was purchased

Column 3 calculates the cost of an item that was purchased:

If I paid this much, and I received this much…. how much did I pay per kg?

This column uses division to solve these. So if you bought 350 g of Ham and it cost $8.55, how much did the ham cost per kg? An example is below:

350 g of Ham costs $8.55, find the price per kg.

$8.55 \div 0.35 = 24.43$ (2 d.p.)

If given a weight in grams, you have to convert it to kg.

Ham costs $24.43/kg.

Rates and Food

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Use multiplication to calculate the cost of the food below.

Example - One banana costs $1.30, how much would 5 cost?

Cost = $5 \times 1.30 = \$6.50$ ← Money is always 2 d.p.

1 One can of salmon is $6.40, how much would 7 cans cost?

Cost = ☐ × ☐ = $ ☐

2 A loaf of bread costs $3.20, how much is paid for 3 loaves?

Cost =

3 Carrots cost $3.65/kg, find the price paid for 4 kg.

4 Apples cost $5.95/kg, how much would 1.6 kg cost?

Rounding required now! Money rounds to 2 d.p.

5 Potatoes cost $6.99/kg. Find the price of 0.8 kg of potatoes.

6 Beans cost $7.19/kg, how much would 0.235 kg cost?

7 Prawns cost $29.97/kg, how much you pay for 400 g?

8 Onions cost $7.67/kg. Find the cost for 328 g of onions.

Now given the cost, find the weight or number of items you can buy. Use division this time.

9 BBQ sauce $4.45/ bottle. If $13.35 was spent, how many bottles were purchased?

☐ ÷ ☐ = ☐

☐ Bottles were purchased.

10 Plum jam $4.15/jar. If $20.75 was spent, how many jars were bought?

11 Calamari rings cost 75c each. If $8.25 was spent, how many were purchased?

12 Tomatoes cost $4.98/kg. If $7.84 was spent, what weight was purchased?

13 Ham costs $18.50/kg. If $10.36 was spent, what amount was purchased? Answer in g.

14 Mince costs $12.95/kg. If a pack costs $15.09, what amount of mince is in the pack? Answer in kg to 2 decimal places.

Use division to find the cost per item, or the price per kg ($/kg) for these products.

15 3 hamburgers cost $15.75. How much are they each?

☐ ÷ ☐ = ☐

Each hamburger costs ☐

16 6 bottles of chlorine cost $52.62. How much is one bottle?

17 2.6 kg of grapes cost $9.23. Find the amount paid per kg.

18 1.4 kg of sausages costs $12.59. Find the cost per kg.

19 775 g of peas cost $6.15. Find the amount paid per kg.

20 905 g of beans cost $6.43. Find the cost per kg.

21 220 g pack of walnuts cost $3.39. Find the cost paid per kg.

Rates and Food

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Use multiplication to calculate the cost of the food below.

Example - One banana costs $1.30, how much would 5 cost?

Cost = $5 \times 1.30 = \$6.50$ ← Money is always 2 d.p.

1 One can of salmon is $6.40, how much would 7 cans cost?

Cost = 7 × 6.40 = $44.80

2 A loaf of bread costs $3.20, how much is paid for 3 loaves?

Cost = $3 \times 3.20 = \$9.60$

3 Carrots cost $3.65/kg, find the price paid for 4 kg.

$\text{Cost} = 4 \times 3.65 = \$14.60$

4 Apples cost $5.95/kg, how much would 1.6 kg cost?

$\text{Cost} = 1.6 \times 5.95 = \$9.52$

Rounding required now! Money rounds to 2 d.p.

5 Potatoes cost $6.99/kg. Find the price of 0.8 kg of potatoes.

$\text{Cost} = 0.8 \times 6.99 = \$5.59$

6 Beans cost $7.19/kg, how much would 0.235 kg cost?

$\text{Cost} = 0.235 \times 7.19 = \$1.69$

7 Prawns cost $29.97/kg, how much you pay for 400 g?

$\text{Cost} = 0.4 \times 29.97 = \$11.99$

8 Onions cost $7.67/kg. Find the cost for 328 g of onions.

$\text{Cost} = 0.328 \times 7.67 = \$2.52$

Now given the cost, find the weight or number of items you can buy. Use division this time.

9 BBQ sauce $4.45/ bottle. If $13.35 was spent, how many bottles were purchased?

13.35 ÷ 4.45 = 3

3 Bottles were purchased.

10 Plum jam $4.15/jar. If $20.75 was spent, how many jars were bought?

$20.75 \div 4.15 = 5$

5 jars of jam were purchased.

11 Calamari rings cost 75c each. If $8.25 was spent, how many were purchased?

$8.25 \div 0.75 = 11$

11 calamari rings were bought.

12 Tomatoes cost $4.98/kg. If $7.84 was spent, what weight was purchased?

$7.84 \div 4.98 = 11$

11 calamari rings were bought.

13 Ham costs $18.50/kg. If $10.36 was spent, what amount was purchased? Answer in g.

$10.36 \div 18.50 = 0.56$

560 g of ham was bought.

14 Mince costs $12.95/kg. If a pack costs $15.09, what amount of mince is in the pack? Answer in kg to 2 decimal places.

$15.09 \div 12.95 = 1.17$ (2 d.p.)

1.17 kg of mince is in the pack.

Use division to find the cost per item, or the price per kg ($/kg) for these products.

15 3 hamburgers cost $15.75. How much are they each?

15.75 ÷ 3 = 5.25

Each hamburger costs $5.25

16 6 bottles of chlorine cost $52.62. How much is one bottle?

$52.62 \div 6 = 8.77$

One bottle of chlorine costs $8.77

17 2.6 kg of grapes cost $9.23. Find the amount paid per kg.

$9.23 \div 2.6 = 3.55$

Grapes cost $3.55/kg.

18 1.4 kg of sausages costs $12.59. Find the cost per kg.

$12.59 \div 1.4 = 8.99$ (2 d.p.)

Sausages cost $8.99/kg.

19 775 g of peas cost $6.15. Find the amount paid per kg.

$6.15 \div 0.775 = 7.94$ (2 d.p.)

Peas cost $7.94/kg.

20 905 g of beans cost $6.43. Find the cost per kg.

$6.43 \div 0.905 = 7.10$ (2 d.p.)

Beans cost $7.10/kg.

21 220 g pack of walnuts cost $3.39. Find the cost paid per kg.

$3.39 \div 0.22 = 15.41$ (2 d.p.)

Pack of walnuts cost $15.41/kg.

Best Buy and Unit Pricing

When you go into a shop to buy eggs it is easy to compare the price if all the eggs are in packs of a dozen. But goods are in different sized packs and sometimes the cheaper product may appear more expensive simply because of its weight. So it is important that you able to judge the best value, of course realise that the quality of the product has to be considered in making a choice, but this sheet deals with the mathematics side only.

There is a push for retailers to display 'unit pricing' which converts everything to a price 'per kg', or 'per 100 g' so that you can compare easily. This sheet shows you how to compare goods by creating your own unit pricing model.

Column 1 starts with questions on best buys, that is, making the best value purchase. To solve these questions convert the money to cents and divide by the weight (or volume). That will give you an answer of 'c/g' or 'c/mL'. You can then compare, the lowest price being the best buy. There is an example in the first column. Why change the $ to cents? Because it stops you being confused with decimals.

Column 2 continues on the same way except the units are different between the goods. One may be in grams, the other a larger pack, may be in kg. Change kg → g, so that they can be compared, then the process is the same.

Column 3 is different, it introduces unit pricing. In this case you are asked to convert the prices from c/g to c/ 100g. Why the need? Again it removes decimals and so it is less confusing to shoppers. All you have to do is multiply your price of c/g or c/mL by 100 to convert to the unit pricing model. Answers are in cents, without any decimal places.

Best Buy and Unit Pricing

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Ignoring quality, the best product is the cheapest. Use Unit Pricing to choose the best buy.

Example: Pick the best buy

500 g frozen peas costs $7.95

or 770 g frozen peas costs $9.60

500g: 795 ÷ 500 = 1.59 c/g

Divide cost in cents by the weight or volume. Round to 2 d.p. for comparison

770g: 960 ÷ 770 = 1.25 c/g

The best buy is the 770 g pack.

1 450 g Corn kernels for $2.35

or

375 g Corn kernels for $1.82

450g: ☐ ÷ ☐ = ☐ c/g

375g: ☐ ÷ ☐ = ☐ c/g

The best buy is the ☐ g can

2



$4.85



$6.30

1.25 L:

2 L:

3 150 g Peanuts for $2.59

or

375 g Peanuts for $6.77

4 12 Oysters for $15.50

or

9 Oysters for $11

These product weights and volumes must be changed to like units, then find the best buy.

5 650 mL Olive Oil for $5.52

or

1.5 L Olive Oil for $16.10

6 300 g Mushrooms for $4.14

or

1.3 kg Mushrooms for $18.62

7 350 g Carrots for $2.10

or

2.6 kg Carrots for $13.84

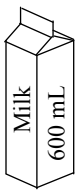
or

1.04 kg Carrots for $5.12

8



$6.50



$1.75



$8.30

Some stores supply unit pricing, but it is usually in amounts like 100 g or 100 mL. Convert these prices to c/100 g or c/100 mL.

Example: Change to c/100g

445 g Marshmallows for $5.30

Cost: 530 ÷ 445 = 1.191…..

Don't clear your calculator screen. Multiply by 100 and round to nearest cent.

Cost: 119 c/100 g (nearest cent)

9 Bleach 750 mL for $8.68

10 Lettuce Mix 150 g for $3.95

Convert both products to c/100 g then find the best buy.

11 250 g Cheese Puffs for $3.40

or

90 g Cheese Puffs for $1.20

12 400 g Sliced Ham for $11.20

or

560 g Sliced Ham for $14.85

Best Buy and Unit Pricing

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Ignoring quality, the best product is the cheapest. Use Unit Pricing to choose the best buy.

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500g: 795 ÷ 500 = 1.59 c/g

Divide cost in cents by the weight or volume. Round to 2 d.p. for comparison

770g: 960 ÷ 770 = 1.25 c/g

The best buy is the 770 g pack.

1 450 g Corn kernels for $2.35

or

375 g Corn kernels for $1.82

450g: 235 ÷ 450 = 0.52 c/g

375g: 182 ÷ 375 = 0.49 c/g

The best buy is the 375 g can

2



$4.85



$6.30

1.25 L: 485 ÷ 1.25 = 388 c/L

2 L: 630 ÷ 2 = 315 c/L

The best buy is the 2 L bottle.

3 150 g Peanuts for $2.59

or

375 g Peanuts for $6.77

150 g: 259 ÷ 150 = 1.73 c/g

375 g: 677 ÷ 375 = 1.81 c/g

The best buy is the 150 g pack.

4 12 Oysters for $15.50

or

9 Oysters for $11

12 : 1 550 ÷ 12 = 129 c/oyster

9 : 1 100 ÷ 9 = 122 c/oyster

The best buy is the 9 oysters.

These product weights and volumes must be changed to like units, then find the best buy.

5 650 mL Olive Oil for $5.52

or

1.5 L Olive Oil for $16.10

650 mL: 552 ÷ 650 = 0.85 c/g

1.5 L: 1 610 ÷ 1 500 = 1.07 c/g

Best buy is the 650 mL bottle.

6 300 g Mushrooms for $4.14

or

1.3 kg Mushrooms for $18.62

300 g: 414 ÷ 300 = 1.38 c/g

1.3 kg: 1 862 ÷ 1 300 = 1.43 c/g

Best buy is the 300 g pack.

7 350 g Carrots for $2.10

or

2.6 kg Carrots for $13.84

or

1.04 kg Carrots for $5.12

350 g: 210 ÷ 350 = 0.60 c/g

2.6 kg: 1 384 ÷ 2 600 = 0.53 c/g

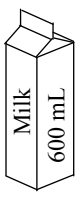
1.04 kg: 512 ÷ 1 040 = 0.49 c/g

Best buy is the 1.04 kg pack.

8



$6.50



$1.75



$8.30

2 L: 650 ÷ 2 000 = 0.33 c/mL

600 mL: 175 ÷ 600 = 0.29 c/mL

3 L: 830 ÷ 3 000 = 0.28 c/mL

Best buy is the 3 L bottle.

Some stores supply unit pricing, but it is usually in amounts like 100 g or 100 mL. Convert these prices to c/100 g or c/100 mL.

Example: Change to c/100g

445 g Marshmallows for $5.30

Cost: 530 ÷ 445 = 1.191…..

Don't clear your calculator screen. Multiply by 100 and round to nearest cent.

Cost: 119 c/100 g (nearest cent)

9 Bleach 750 mL for $8.68

Bleach: 868 ÷ 750 = 1.157…..

Bleach: 116 c/100 mL

10 Lettuce Mix 150 g for $3.95

Lettuce: 395 ÷ 150 = 2.633…..

Lettuce: 263 c/100 g

Convert both products to c/100 g then find the best buy.

11 250 g Cheese Puffs for $3.40

or

90 g Cheese Puffs for $1.20

250 g: 340 ÷ 250 = 1.36

250 g: 136 c/100 g

90 g: 120 ÷ 90 = 1.333….

90 g: 133 c/100 g

Best buy is the 90 g pack.

12 400 g Sliced Ham for $11.20

or

560 g Sliced Ham for $14.85

400 g: 1 120 ÷ 400 = 2.8

400 g: 280 c/100 g

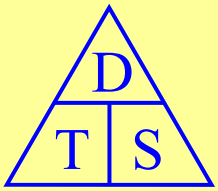
560 g: 1 485 ÷ 560 = 2.65…..

560 g: 265 c/100 g

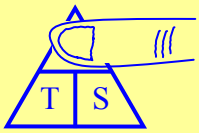
Best buy is the 400 g pack.

Speed - Distance - Time

There are three formulae to remember with these but you can also remember a stupid sentence and a triangle. The sentence is : "Dad sits on the Toilet Seat". So in the triangle shown left, the D, which stands for distance is on top (like Dad). The T and S which are Time and Speed are below the D, like the toilet seat.

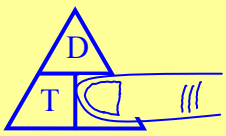


So after you draw the triangle … how do you use it? You place your finger over what you are after and build the formula. Here goes …



Cover the 'D' to find the distance formula. Because T and S are next to each other, write the formula with the letters next to each other.

$D = TS$ or Distance = Time × Speed



Cover the 'S' to find the speed formula. Because D is on top of T, write the formula with D over T.

$S = {}^{D}/_{T}$ or Speed = Distance ÷ Time



Cover the 'T' to find the time formula. Because D is on top of S, write the formula with D over S.

$T = {}^{D}/_{S}$ or Time = Distance ÷ Speed

There are two sheets for this exercise. The first sheet has the skills separated into three columns, Fizzle gives you the formulae to use at the top of each column. The second worksheet are worded problems, are mixed up and are more complicated.

These sheets require you to use time periods that aren't in exact hours. To use time mathematically you have two options.

Use the fraction key: 57 min = ${}^{57}/_{60}$, 2 h 35 min = $2\ {}^{35}/_{60}$.
Or
Use the DMS or 'Bubble Button', (whichever your calculator features)

57 min = 0 DMS ° ' '' 5 7 DMS ° ' ''

2 h 35 min = 2 DMS ° ' '' 3 5 DMS ° ' ''

Example: Terri walks for 35 min at 4.5 km/h. Find the distance walked in metres.

$D = TS$
$= {}^{35}/_{60} \times 4.5$ (you can write time as a fraction)
$D = 2.625$ km = 2 625 m.

Speed - Distance - Time

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Distance = Time × Speed. Find the distance. Time units must match.



1 Speed of 85 km/h for 5 hours

D = TS

= ☐ × ☐

D = ☐ km

2 Speed of 54 km/h for 7 hours

D = TS

= ☐ × ☐

D = ☐ km

3 17 m/s for 35 s

4 8 m/s for 2 min 12 s

Change time to seconds

5 45 km/h for 53 min

4 5 × 0 DMS ° ’ ’’ 5 3 DMS ° ’ ’’ = DMS ° ’ ’’

6 37 cm/h for 3 h 19 min

Round to 1 d.p.

7 658 km/h for 18 h 44 min

Round to 1 d.p.

Speed = Distance ÷ Time. Find the average speed for these. Round to 1 d.p. if needed.



8 522 km in 9 hours

$S = \frac{D}{T}$

$= \frac{\square}{\square}$

S = ☐ km/h

9 117.6 m in 28 s

$S = \frac{D}{T}$

$= \frac{\square}{\square}$

S = ☐ m/s

10 345 km in 4 h 18 min

11 19 cm in 5 min (in cm/min)

12 Convert above to cm/h

13 Convert above to m/h

14 735 km in 1 day 2 h 49 min in km/h.

Time = Distance ÷ Speed. Find the time taken for these journeys. To the nearest minute.



15 273 km at 78 km/h

2 7 3 ÷ 7 8 = DMS ° ’ ’’

$T = \frac{D}{S}$

$= \frac{\square}{\square}$

T = ☐ h ☐ min

16 780 m at 12 m/s (in seconds)

17 Convert above to min & s.

18 4.48 m at 7.8 cm/min

19 130 km at 95 km/h

20 1.6 km at 32 m/s (in seconds)

21 420 m at 85 km/h (nearest s)

Speed - Distance - Time

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Distance = Time × Speed. Find the distance. Time units must match.

1 Speed of 85 km/h for 5 hours

D = TS

= 5 × 85

D = 425 km

2 Speed of 54 km/h for 7 hours

D = TS

= 7 × 54

D = 378 km

3 17 m/s for 35 s

D = TS

= 35 × 17

D = 595 m

ms^{-1} = m/s

4 8 m/s for 2 min 12 s

D = TS

= 132 × 8

D = 1 056 m

2 min 12 s = 120 + 12 = 132 s

5 45 km/h for 53 min

4 5 × 0 DMS ° ’ ’’ 5 3 DMS ° ’ ’’ = DMS ° ’ ’’

D = TS

$= {}^{53}/_{60} \times 45$

D = 39.75 km

6 37 cm/h for 3 h 19 min

D = TS

$= 3\ {}^{19}/_{60} \times 37$

D = 122.7 cm (1 d.p.)

Round to 1 d.p.

7 658 km/h for 18 h 44 min

D = TS

$= 18\ {}^{44}/_{60} \times 658$

D = 12 326.5 km (1 d.p.)

Round to 1 d.p.

Speed = Distance ÷ Time. Find the average speed for these. Round to 1 d.p. if needed.

8 522 km in 9 hours

$S = \frac{D}{T}$

$= \frac{522}{9}$

S = 58 km/h

9 117.6 m in 28 s

$S = \frac{D}{T}$

$= \frac{117.6}{28}$

S = 4.2 m/s

10 345 km in 4 h 18 min

$S = {}^{D}/_{T}$

$= 345 \div (4\ {}^{18}/_{60})$

S = 80.2 km/h

11 19 cm in 5 min (in cm/min)

$S = {}^{D}/_{T}$

$= 19 \div 5$

S = 3.8 cm/min

12 Convert above to cm/h

S = 3.8 × 60

S = 228 cm/h

13 Convert above to m/h

S = 228 ÷ 100

S = 2.28 m/h

14 735 km in 1 day 2 h 49 min in km/h.

1 d 2 h 49 min = 26 h 49 min

$S = {}^{D}/_{T}$

$= 735 \div (26\ {}^{49}/_{60})$

S = 27.4 km/h

Time = Distance ÷ Speed. Find the time taken for these journeys. To the nearest minute.

15 273 km at 78 km/h

2 7 3 ÷ 7 8 = DMS ° ’ ’’

$T = \frac{D}{S}$

$= \frac{273}{78}$

T = 3 h 30 min

16 780 m at 12 m/s (in seconds)

$T = {}^{D}/_{S}$

$= 780 \div 12$

T = 65 s

17 Convert above to min & s.

T = 1 min 5 s

18 4.48 m at 7.8 cm/min

$T = {}^{D}/_{S}$

$= 448 \div 7.8$

T = 57 min (nearest min)

19 130 km at 95 km/h

$T = {}^{D}/_{S}$

$= 130 \div 95$

T = 1 h 22 min (nearest min)

20 1.6 km at 32 m/s (in seconds)

$T = {}^{D}/_{S}$

$= 1\ 600 \div 32$

T = 50 s

21 420 m at 85 km/h (nearest s)

$T = {}^{D}/_{S}$

$= 420 \div 85$

T = 18 s (nearest second)

Speed - Distance - Time (Worded Questions)

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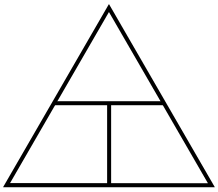
Use the DTS triangle to answer these. Round to 1 d.p. if needed, round time to nearest minute.

1 Complete



S = $\frac{\square}{\square}$

D = $\square$

T = $\frac{\square}{\square}$

2 A car is travelling at an average speed 86 km/h. How far has it travelled after 2 h 25 min?

D = TS

= $\square \times \square$

D = $\square$ km (1 d.p.)

The car has travelled $\square$

3 How long will it take a bicyclist to travel 25 km when she maintains an average speed of 14 km/h?

4 Pete walks at 5 km/h for 20 min then jogs at 9 km/h for 5 min. How far has he travelled?

5 What is his average speed for the 25 min?

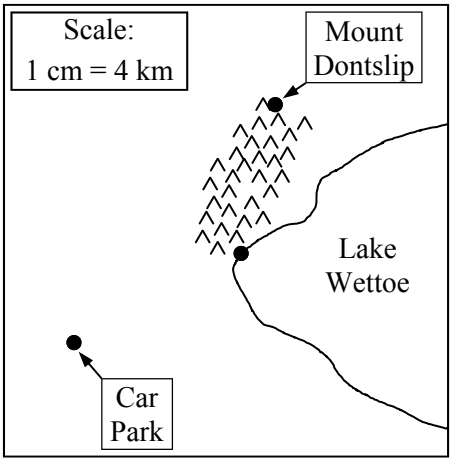
6 It took Anne 8 min to walk around a square paddock. If she is walking at an average speed of 4.2 km/h, find the width of the paddock in metres.

7 Jamie drives for 4 h 10 min at an average speed of 86 km/h. Liam finishes the same trip in 4 h 35 min, find Liam's average speed.

8 A car delivers fuel economy of 6.2 L/100 km on the highway. If the car has used 32.8 L while driving for 5 h 45 min on the highway, find its average speed.

9 Phil and Dave are the drivers in two cars, both driving to the snow from Dave's house. Phil leaves first. Dave leaves 8 min later and catches up to Phil in 48 min. If Dave averages 65 km/h, calculate Phil's average speed.

10 Steve is going for a bush walk. He plans to park his car and walk at 5 km/h to the lake, stop for an hour then walk to Mount Dontslip at 3 km/h. If he leaves the car park at 10.30 a.m., at what time does he reach Mount Dontslip?



Speed - Distance - Time (Worded Questions)

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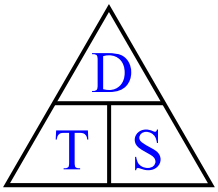
Use the DTS triangle to answer these. Round to 1 d.p. if needed, round time to nearest minute.

1 Complete



$S = \frac{D}{T}$

$D = TS$

$T = \frac{D}{S}$

2 A car is travelling at an average speed 86 km/h. How far has it travelled after 2 h 25 min?

$D = TS$

$= 2\,{}^{25}/_{60} \times 86$

$D = 207.8$ km (1 d.p.)

The car has travelled 207.8 km.

3 How long will it take a bicyclist to travel 25 km when she maintains an average speed of 14 km/h?

$T = {}^{D}/_{S}$

$= 25 \div 14$

$T = 1$ h 47 min (nearest min)

The car has travelled 207.8 km.

4 Pete walks at 5 km/h for 20 min then jogs at 9 km/h for 5 min. How far has he travelled?

$D = TS$

$= {}^{20}/_{60} \times 5 + {}^{5}/_{60} \times 9$

$D = 2.4$ km (1 d.p.)

Pete has travelled 2.4 km.

5 What is his average speed for the 25 min?

$S = {}^{D}/_{T}$

$= 2.4 \div {}^{25}/_{60}$

$S = 5.8$ km/h (1 d.p.)

Pete's average speed is 5.8 km/h

6 It took Anne 8 min to walk around a square paddock. If she is walking at an average speed of 4.2 km/h, find the width of the paddock in metres.

$D = TS$

$= {}^{8}/_{60} \times 4.2$

$D = 0.28$ km $= 280$ m

$P = 4l$

$280 = 4l$ (÷4) (÷4)

$l = 70$ m

The paddock is 70 m wide.

7 Jamie drives for 4 h 10 min at an average speed of 86 km/h. Liam finishes the same trip in 4 h 35 min, find Liam's average speed.

$D = TS$

$= 4\,{}^{10}/_{60} \times 86$

$D = 358.3$ km (1 d.p.)

$S = {}^{D}/_{T}$

$= 358.3 \div 4\,{}^{35}/_{60}$

$S = 78.2$ km/h (1 d.p.)

Liam's ave speed is 78.2 km/h

8 A car delivers fuel economy of 6.2 L/100 km on the highway. If the car has used 32.8 L while driving for 5 h 45 min on the highway, find its average speed.

$D = 32.8 \div 6.2 \times 100$

$= 529$ km (1 d.p.)

$S = {}^{D}/_{T}$

$= 529 \div 5\,{}^{45}/_{60}$

$S = 92$ km/h

Car's average speed is 92 km/h

9 Phil and Dave are the drivers in two cars, both driving to the snow from Dave's house. Phil leaves first. Dave leaves 8 min later and catches up to Phil in 48 min. If Dave averages 65 km/h, calculate Phil's average speed.

$D = TS$

$= {}^{48}/_{60} \times 65$

$D = 52$ km

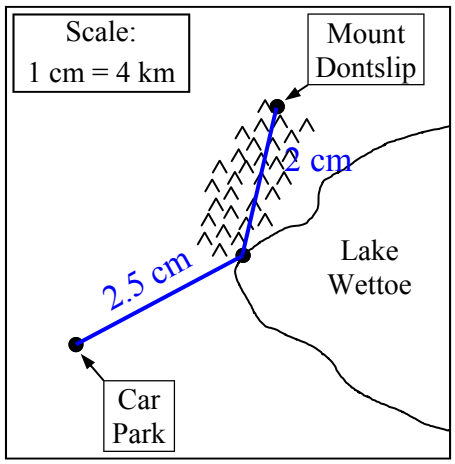
$S = {}^{D}/_{T}$

$= 52 \div {}^{56}/_{60}$

$S = 55.7$ km/h (1 d.p.)

Phil's ave speed is 55.7 km/h

10 Steve is going for a bush walk. He plans to park his car and walk at 5 km/h to the lake, stop for an hour then walk to Mount Dontslip at 3 km/h. If he leaves the car park at 10.30 a.m., at what time does he reach Mount Dontslip?



Dist $= 2.5 \times 4 = 10$ km

$T = {}^{D}/_{S} = 10 \div 5$

$T = 2$ h

Dist $= 2 \times 4 = 8$ km

$T = {}^{D}/_{S} = 8 \div 3$

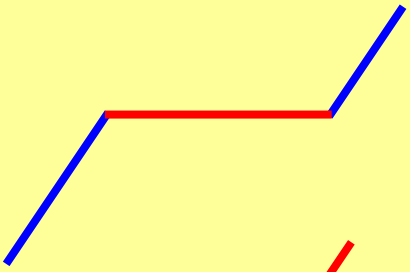
$T = 2$ h 40 min

10.30 + 2 + 1 + 2 h 40 = 4.10 pm.

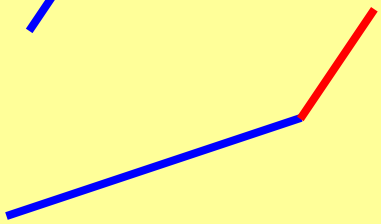
He should arrive at 4.10 pm.

Travel Graphs

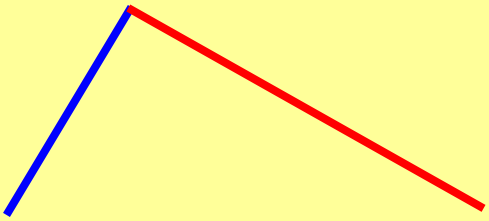
Travel graphs represent travel just like a line graph. Key features are:



A Flat line means that you are not moving, or aren't changing your distance from your starting point.



A Steep line means that you have increased speed. It doesn't mean you are going up a hill.



A line with a downward slope doesn't mean that you are going down a hill. It means you are travelling back toward your starting point. In this example you are returning at a slower rate as the line is less steep.

Column 1 starts with a travel graph that is fully constructed. You have to extract information from the graph. At the base of the column are question involving speed. The formula for speed is:

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

This sheet uses time intervals of 15 min. So if you have a time like 2 h 15 min, it becomes 2.25 when using your calculator. 1 h 45 would become 1.75.

The other way of using time is to use the DMS key or ° ' '' (bubble) button. For example if you travelled 100 km in 1 h 45 min.

Using your calculator:

1 0 0 ÷ 1 DMS ° ' '' 4 5 DMS ° ' '' = DMS ° ' ''

Calculator screen:

57.14285714 Rounding to 1 decimal place: 57.1 km/h

Travel Graphs

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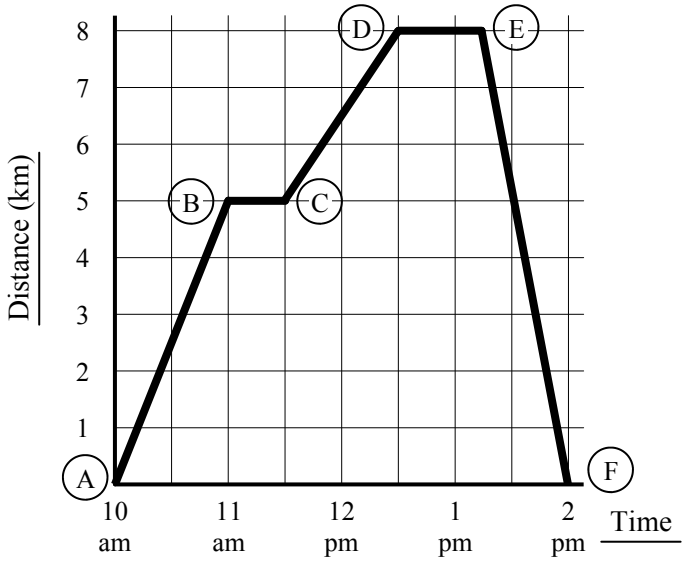
The travel graph below outlines Red's travel from her home at A to the basket shop and then to Granny's house for lunch, then home again.



1 What is the distance from Red's home to the basket shop?

2 What was the total distance Red covered for the entire journey?

3 At what time did Red arrive at Granny's house?

4 Why is the line BC flat?

5 What was the total time Red was stopped?

6 How far apart are the shop and Granny's house?

7 Red bought a basket of goodies. How did this affect her travel?

8 Red left Granny's house in a hurry, how long did it take her to get home?

Calculate Red's average speed for these parts of her journey. Use Speed = Distance ÷ Time. (1 d.p.)

9 AB: Speed $= \frac{D}{T} = \frac{\square}{\square}$

Speed = ____ km/h

10 AD: Speed $= \frac{D}{T} = \frac{\square}{\square}$

Speed = ____ km/h

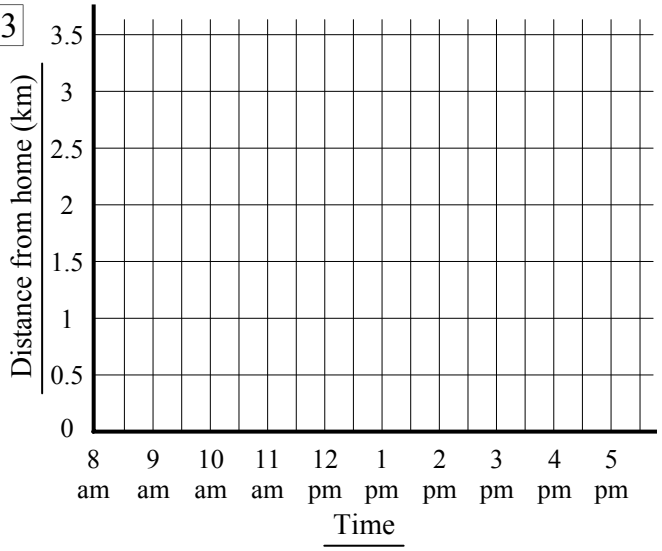
11 AF: Speed $= \frac{\square}{\square} = \frac{\square}{\square} =$ ____

12 EF: Speed $= \frac{\square}{\square} = \frac{\square}{\square} =$ ____

Display Tom's travel on a travel graph.

Details: Leaves home at 8 am., walks 45 min to school, which is 2.5 km away. Leaves school at 3.30 pm. Walks to the soccer oval which is 1 km back towards home, in 15 min. Stays at soccer for 1 hr 15 min. Then he is driven home, which takes 10 min.

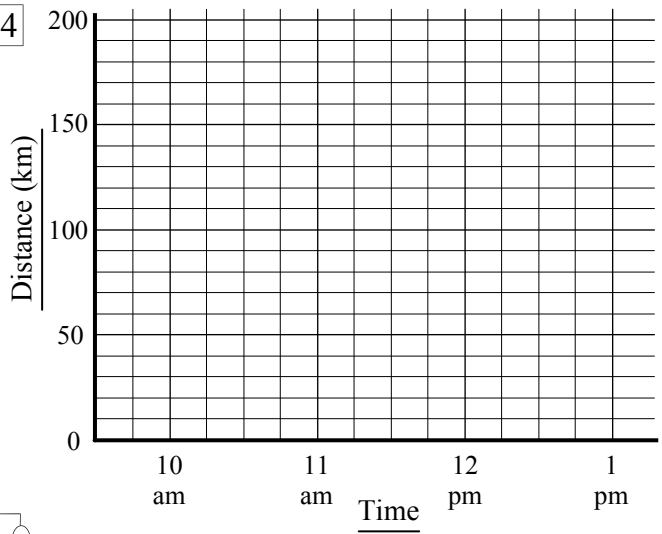
13



A car is tested by a reporter over a 200 km distance, in both city and country driving. It is a one way trip. Display the results in a travel graph.

Details: Leaves at 9.30 am. Travels 50 km in 45 min. Stops for 15 min. Travels next 140 km in 1 h 30 min. Stops for 30 min. Travels 10 km in 30 min. Stops.

14



Calculate the average speed (1 d.p.) for the ……

15 Entire trip

Speed $= \frac{\square}{\square} = \frac{\square}{\square}$

Speed = ____

16 Driving legs of trip

Speed $= \frac{\square}{\square} = \frac{\square}{\square}$

Speed = ____

Travel Graphs

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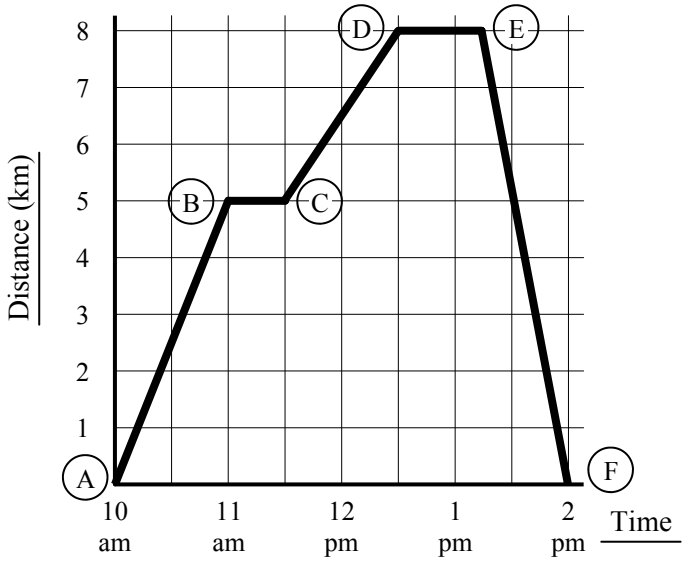
The travel graph below outlines Red's travel from her home at A to the basket shop and then to Granny's house for lunch, then home again.



1 What is the distance from Red's home to the basket shop?

5 km

2 What was the total distance Red covered for the entire journey?

8 + 8 = 16 km

3 At what time did Red arrive at Granny's house?

D: 12.30 pm.

4 Why is the line BC flat? She is shopping, so she is stopped.

5 What was the total time Red was stopped?

30 + 45 = 1 h 15 min

6 How far apart are the shop and Granny's house?

8 - 5 = 3 km

7 Red bought a basket of goodies. How did this affect her travel? She walked 5 km in 1 h to the shop. Then she walked 3 km in 1 h. So she is slower.

8 Red left Granny's house in a hurry, how long did it take her to get home? 45 min.

Calculate Red's average speed for these parts of her journey. Use Speed = Distance ÷ Time. (1 d.p.)

9 AB: Speed $= \frac{D}{T} = \frac{5}{1}$

Speed = 5 km/h

10 AD: Speed $= \frac{D}{T} = \frac{8}{2.5}$

Speed = 3.2 km/h

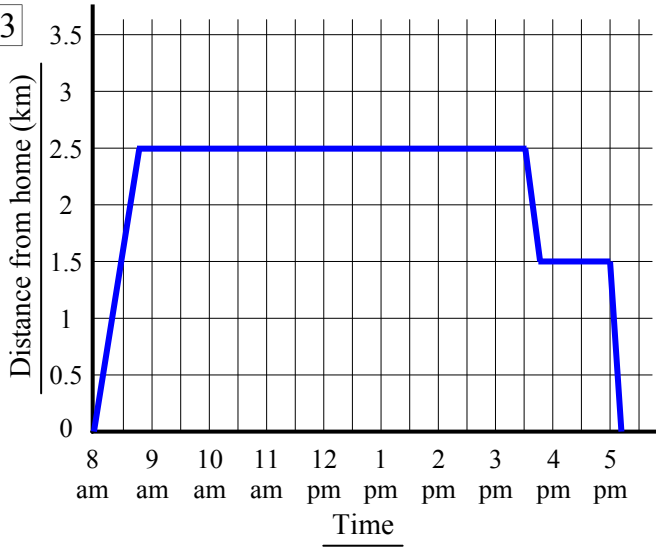
11 AF: Speed $= \frac{D}{T} = \frac{16}{4} =$ 4 km/h

12 EF: Speed $= \frac{D}{T} = \frac{8}{0.75} =$ 10.7 km/h (1 d.p.)

Display Tom's travel on a travel graph.

Details: Leaves home at 8 am., walks 45 min to school, which is 2.5 km away. Leaves school at 3.30 pm. Walks to the soccer oval which is 1 km back towards home, in 15 min. Stays at soccer for 1 hr 15 min. Then he is driven home, which takes 10 min.

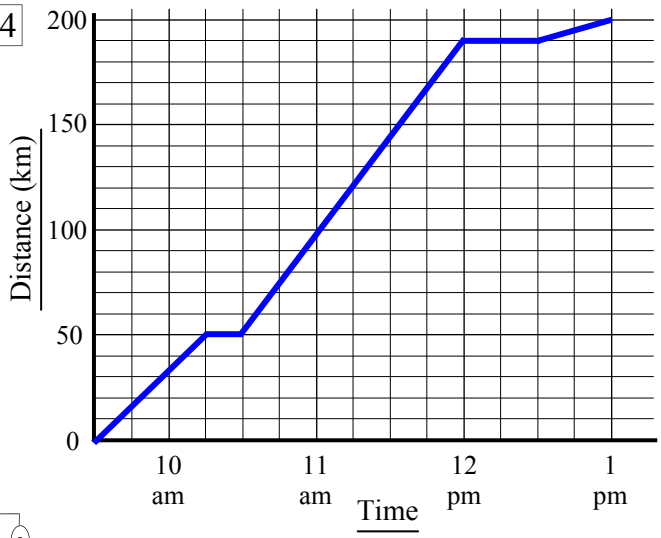
13



A car is tested by a reporter over a 200 km distance, in both city and country driving. It is a one way trip. Display the results in a travel graph.

Details: Leaves at 9.30 am. Travels 50 km in 45 min. Stops for 15 min. Travels next 140 km in 1 h 30 min. Stops for 30 min. Travels 10 km in 30 min. Stops.

14



Calculate the average speed (1 d.p.) for the ……

15 Entire trip

Speed $= \frac{D}{T} = \frac{200}{3.5}$

Speed = 57.1 km/h (1 d.p.)

16 Driving legs of trip

Speed $= \frac{D}{T} = \frac{200}{2.75}$

Speed = 72.7 km/h (1 d.p.)

Rates and Ratio - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don't colour it in too dark (using a texta for example) so that you can still read it.

Rates and Ratio - Find A Word

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Find the words in the puzzle from the wordlist.



A	C	C	E	L	E	R	A	T	E	Q	U	Q	U	E
E	U	C	W	T	N	E	L	A	V	I	U	Q	E	Q
Q	N	O	N	I	Q	E	R	T	E	M	O	L	I	K
U	A	N	O	L	U	L	C	O	I	T	A	R	L	Q
I	U	V	I	K	A	N	O	L	O	C	K	E	U	S
V	Q	E	T	U	N	I	M	K	S	K	V	A	T	E
X	D	R	A	N	T	K	P	I	K	A	I	A	S	Q
Y	I	S	C	I	I	G	A	L	R	K	T	L	E	N
F	S	I	I	T	T	N	R	T	L	I	K	S	B	E
I	T	O	L	A	Y	I	E	I	O	D	E	E	P	S
L	A	N	P	R	Q	W	K	N	D	I	S	C	N	E
P	N	L	P	Y	U	A	A	Q	U	A	N	O	N	E
M	C	I	A	M	A	R	G	O	L	I	K	N	Y	M
I	E	K	I	L	Y	D	I	U	Q	E	O	D	U	I
S	E	T	A	R	L	I	K	I	L	T	E	Q	B	T

WORDLIST

CONVERSION	TRAVEL	RATES	QUANTITY	DRAWING
EQUIVALENT	UNITARY	COLON	SPEED	BEST
STATIONARY	KILOGRAM	COMPARE	KILOMETRE	BUY
ACCELERATE	SCALE	SECOND	RATIO	TIME
APPLICATION	DISTANCE	TONNE	MINUTE	SIMPLIFY

Rates and Ratio - Find A Word

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Find the words in the puzzle from the wordlist.



A	C	C	E	L	E	R	A	T	E	Q	U	Q	U	E
E	U	C	W	T	N	E	L	A	V	I	U	Q	E	Q
Q	N	O	N	I	Q	E	R	T	E	M	O	L	I	K
U	A	N	O	L	U	L	C	O	I	T	A	R	L	Q
I	U	V	I	K	A	N	O	L	O	C	K	E	U	S
V	Q	E	T	U	N	I	M	K	S	K	V	A	T	E
X	D	R	A	N	T	K	P	I	K	A	I	A	S	Q
Y	I	S	C	I	I	G	A	L	R	K	T	L	E	N
F	S	I	I	T	T	N	R	T	L	I	K	S	B	E
I	T	O	L	A	Y	I	E	I	O	D	E	E	P	S
L	A	N	P	R	Q	W	K	N	D	I	S	C	N	E
P	N	L	P	Y	U	A	A	Q	U	A	N	O	N	E
M	C	I	A	M	A	R	G	O	L	I	K	N	Y	M
I	E	K	I	L	Y	D	I	U	Q	E	O	D	U	I
S	E	T	A	R	L	I	K	I	L	T	E	Q	B	T

WORDLIST

~~CONVERSION~~ ~~TRAVEL~~ ~~RATES~~ ~~QUANTITY~~ ~~DRAWING~~

~~EQUIVALENT~~ ~~UNITARY~~ ~~COLON~~ ~~SPEED~~ ~~BEST~~

~~STATIONARY~~ ~~KILOGRAM~~ ~~COMPARE~~ ~~KILOMETRE~~ ~~BUY~~

~~ACCELERATE~~ ~~SCALE~~ ~~SECOND~~ ~~RATIO~~ ~~TIME~~

~~APPLICATION~~ ~~DISTANCE~~ ~~TONNE~~ ~~MINUTE~~ ~~SIMPLIFY~~

FREEFALL MATHEMATICS

ALTITUDE

CONGRUENCE & SIMILARITY

Enlargements - Drawing Exercises

There are two worksheets for this exercise. The first is a simplified sketch of a ladybird beetle (or ladybug). This is an easier exercise, the second drawing is of King Tutankhamun's Death Mask. This sketch is more complicated.

Enlargements - Drawing Exercise (Sheet 1)

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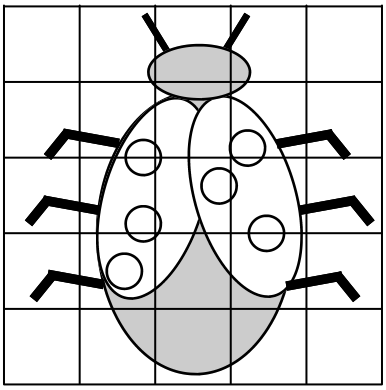

Reproduce the Ladybird beetle in the space below. Once you are done colour the wings red and the body and spots black.



Enlargements - Drawing Exercise (Sheet 2)

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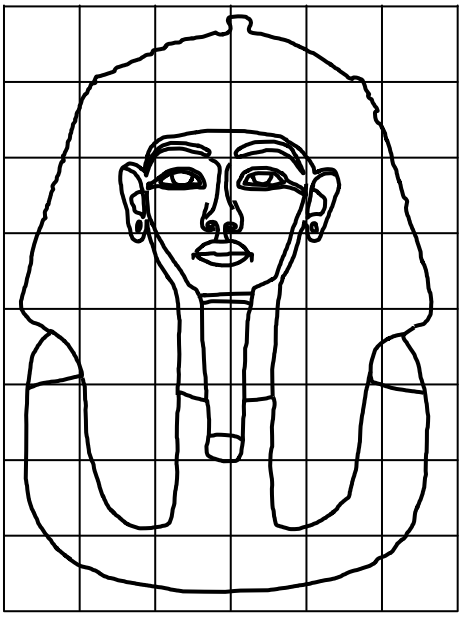


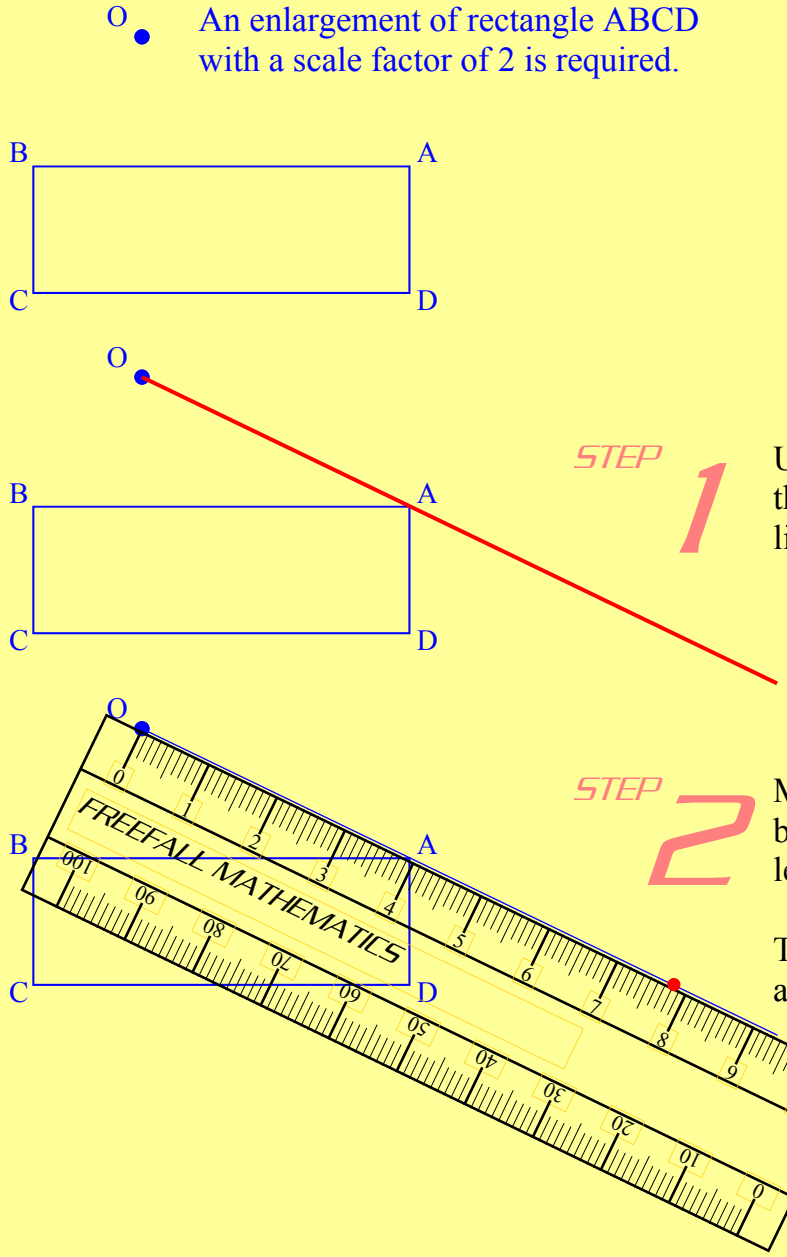
Reproduce the Death Mask of King Tutankhamun in the space below. Once you are done perform an image search on the web to colour it in correctly.



Enlargements (Sheet 1)

One way to make a similar figure is to create an enlargement of the shape. A point called a centre of enlargement (CoE) is used by drawing lines from it to all the vertices (corners) of the shape. The length from the CoE is measured then multiplied by the scale factor to move that point along the line. The scale factors on this sheet are all more than 1, that means that the enlarged shape will be bigger than the original shape.

The CoE can be placed anywhere. It can be inside the shape (Q.2) and outside the shape. It can also be on one of the lines that make the shape. Below are the steps to follow to make an enlargement.



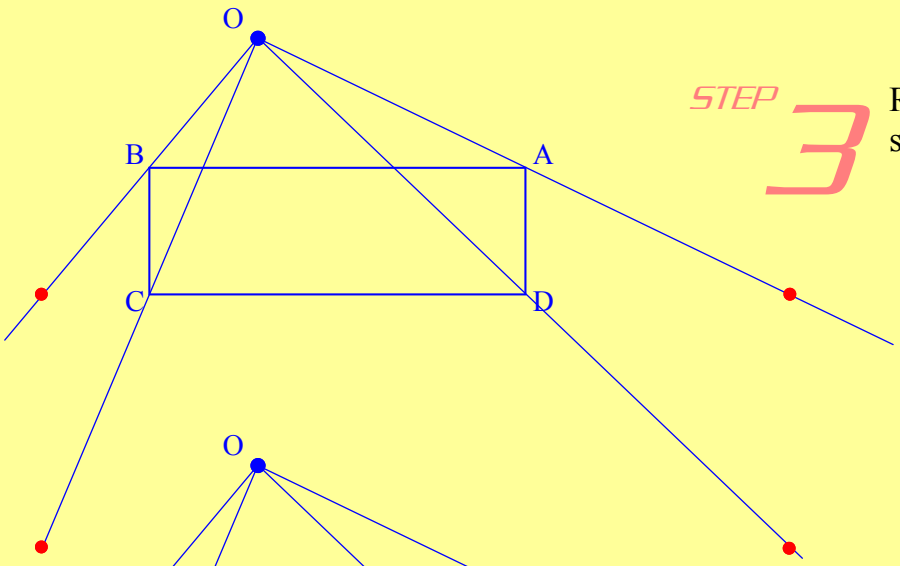
STEP 1

Use a ruler to draw a line from the CoE through one point. In this case it is the red line from O through A.

STEP 2

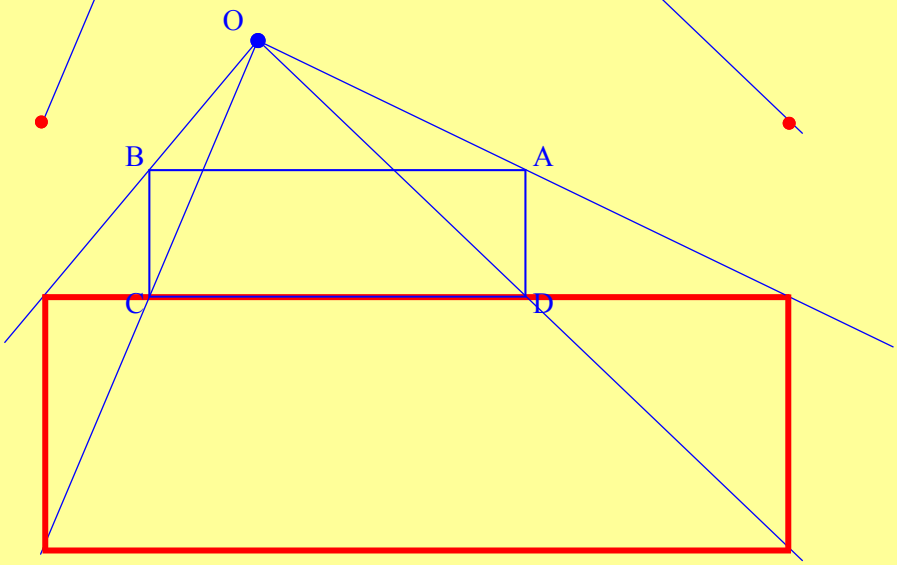
Measure the length OA and then double it, because the scale factor is 2. Mark the length on the constructed line.

The length was 3.9 cm, so a mark is placed at 7.8 cm.



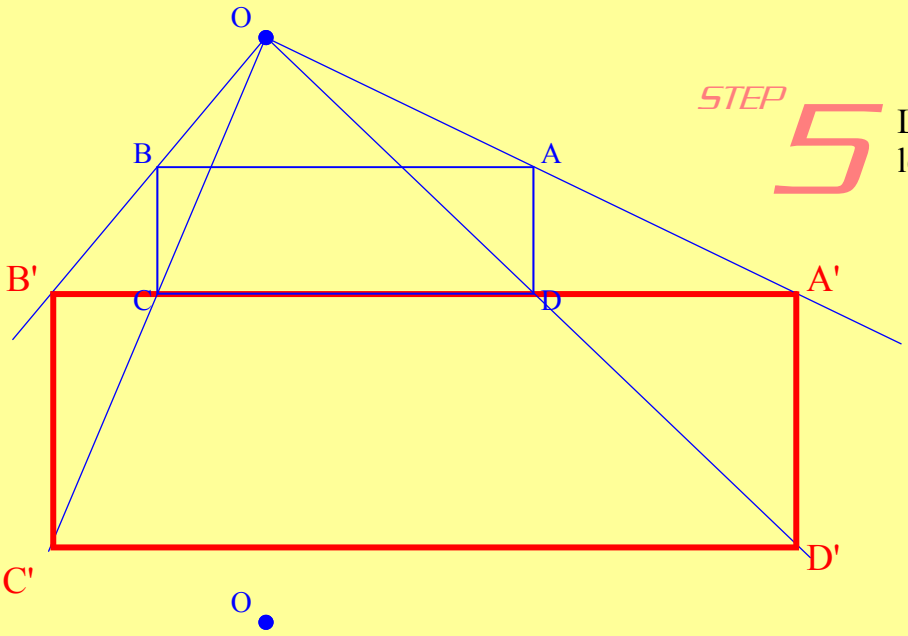
STEP 3

Repeat for the three other corners of the shape.



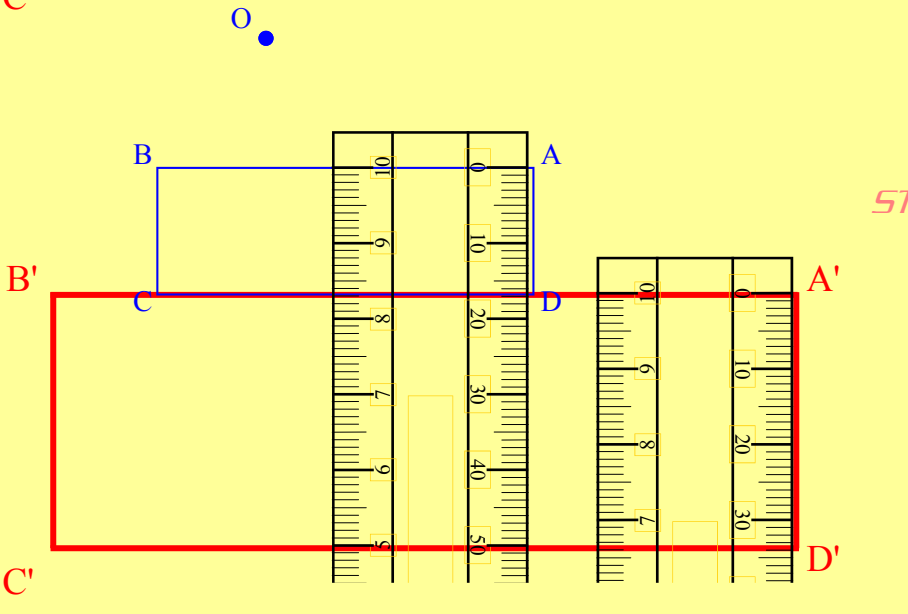
STEP 4

Draw the lines to make the shape.



STEP 5

Label the sides of the shape, use the same letters and add a dash. You are all done!



STEP 6

Measure matching sides, one should be double the other.

AD = 1.7 cm

A'D' = 3.4 cm Looks good.

Enlargements (Sheet 1)

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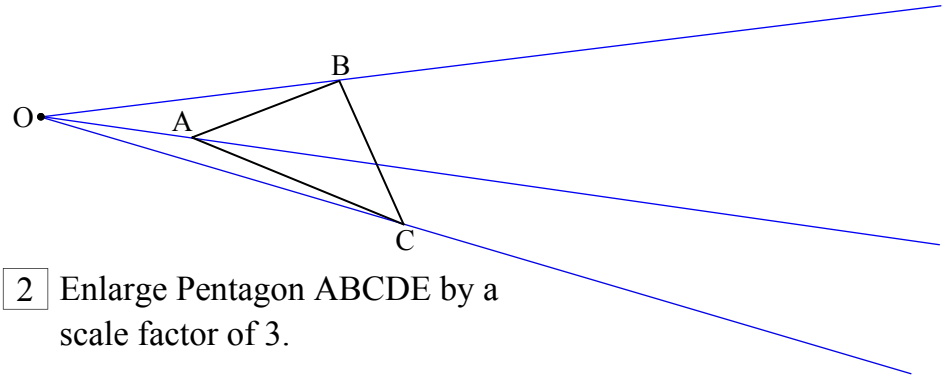
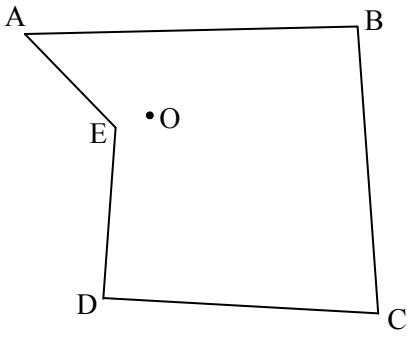
Enlarge the shapes shown using 'O' as the centre of enlargement. Label the enlarged shape as A'B'C', and so on.



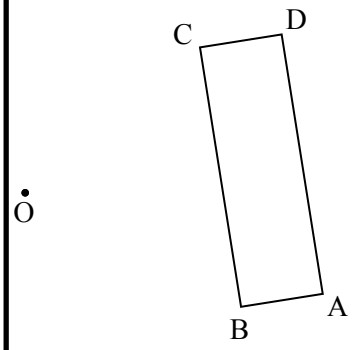
1 Enlarge Triangle ABC by a scale factor of 2.



2 Enlarge Pentagon ABCDE by a scale factor of 3.



3 Enlarge Rectangle ABCD by a scale factor of 2.5.



Enlargements (Sheet 1)

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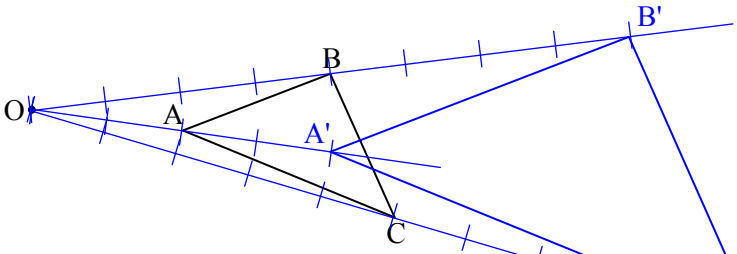
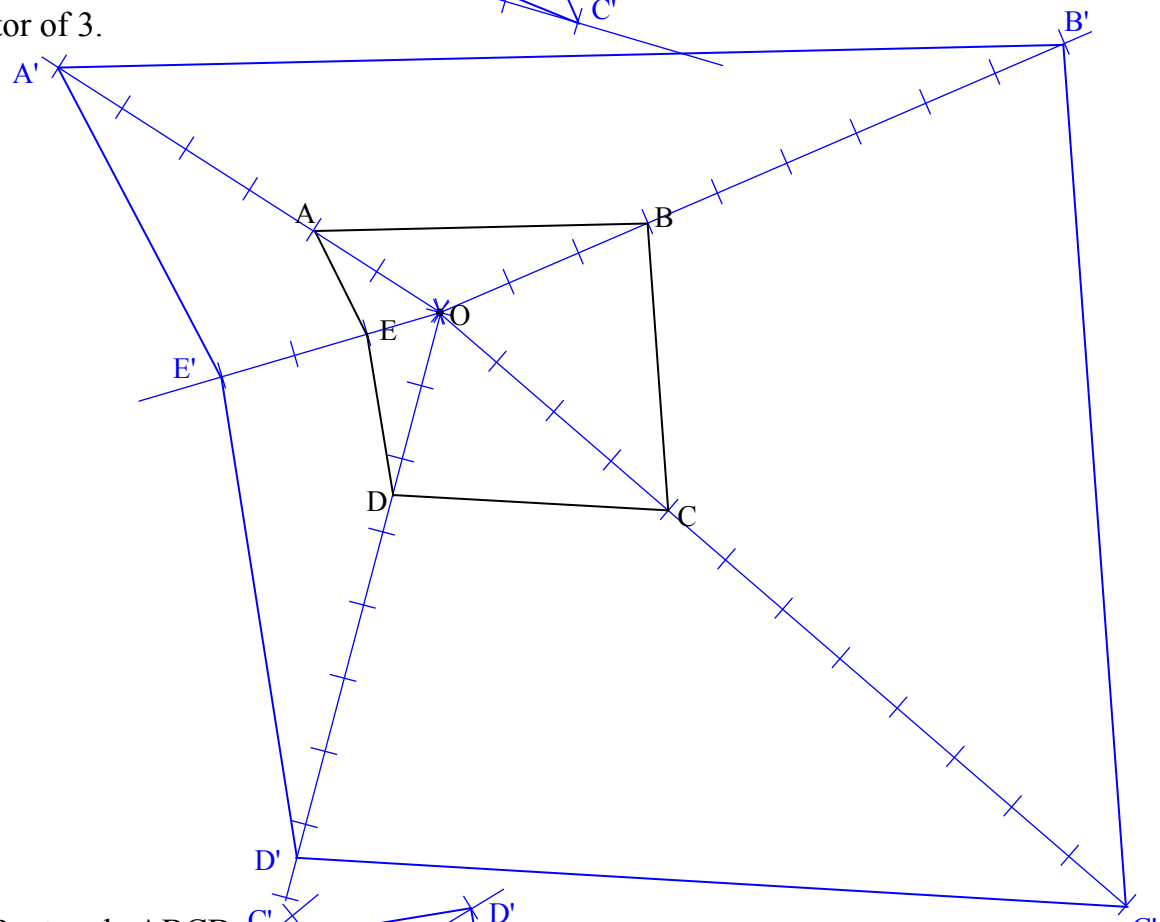
1 Enlarge Triangle ABC by a scale factor of 2.



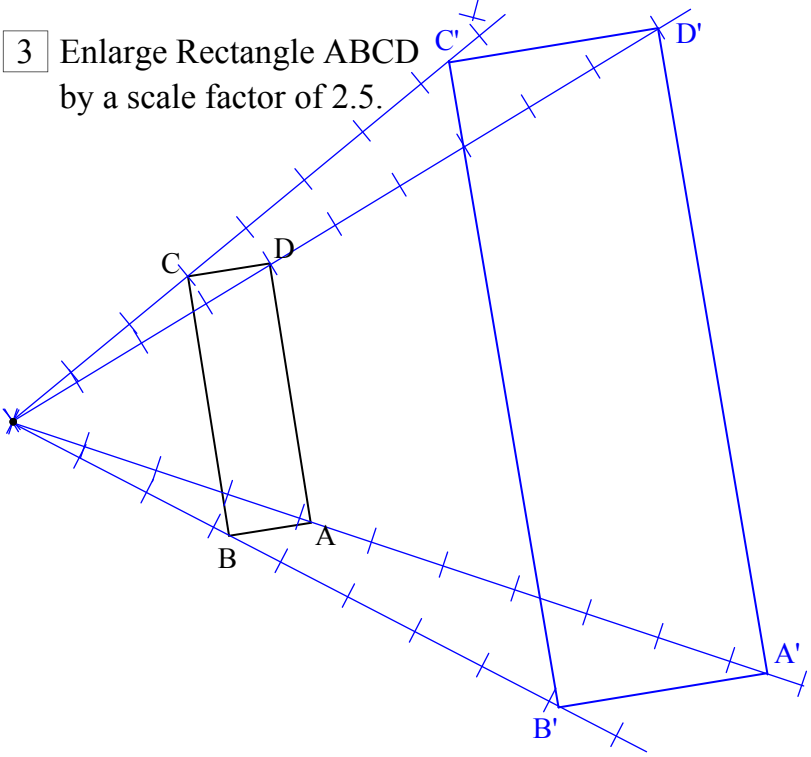
Enlarge the shapes shown using 'O' as the centre of enlargement. Label the enlarged shape as A'B'C', and so on.



2 Enlarge Pentagon ABCDE by a scale factor of 3.



3 Enlarge Rectangle ABCD by a scale factor of 2.5.



Enlargements (Sheet 1) - With Ratios

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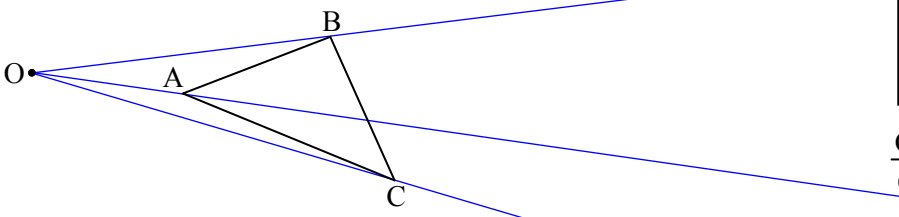
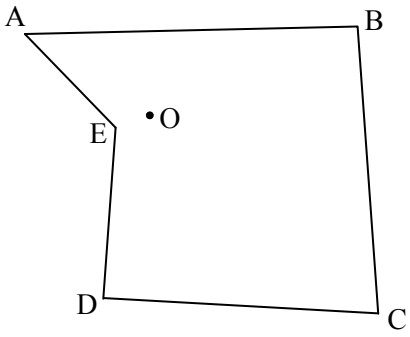
1 Enlarge Triangle ABC by a scale factor of 2.



Enlarge the shapes shown using 'O' as the centre of enlargement. Label the enlarged shape as A'B'C', and so on. Then complete the ratios to verify scale factor properties.

$\frac{OB'}{OB} = \frac{\square}{\square} = \square$ $\frac{A'B'}{AB} = \frac{\square}{\square} = \square$

2 Enlarge Pentagon ABCDE by a scale factor of 3.

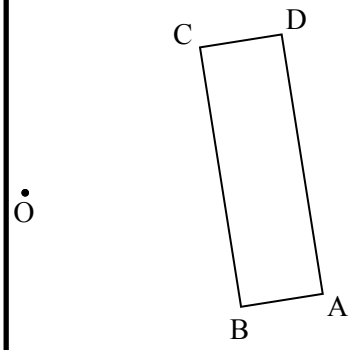


$\frac{OA'}{OA} = \frac{\square}{\square} = \square$

$\frac{B'C'}{BC} = \frac{\square}{\square} = \square$

$\frac{C'D'}{CD} = \frac{\square}{\square} = \square$

3 Enlarge Rectangle ABCD by a scale factor of 2.5.



$\frac{OD'}{OD} = \frac{\square}{\square} = \square$

$\frac{A'D'}{AD} = \frac{\square}{\square} = \square$

$\frac{B'D'}{BD} = \frac{\square}{\square} = \square$

Enlargements (Sheet 1) - With Ratios

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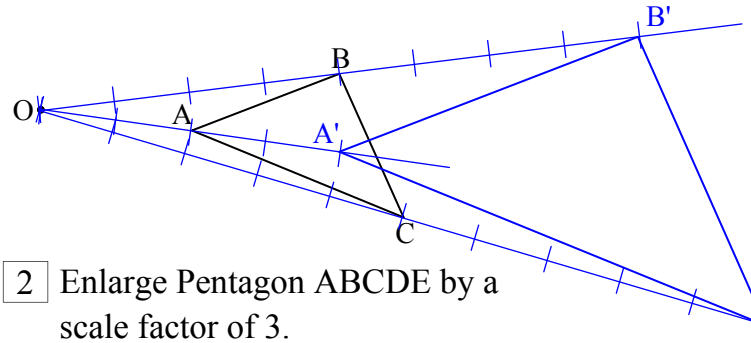
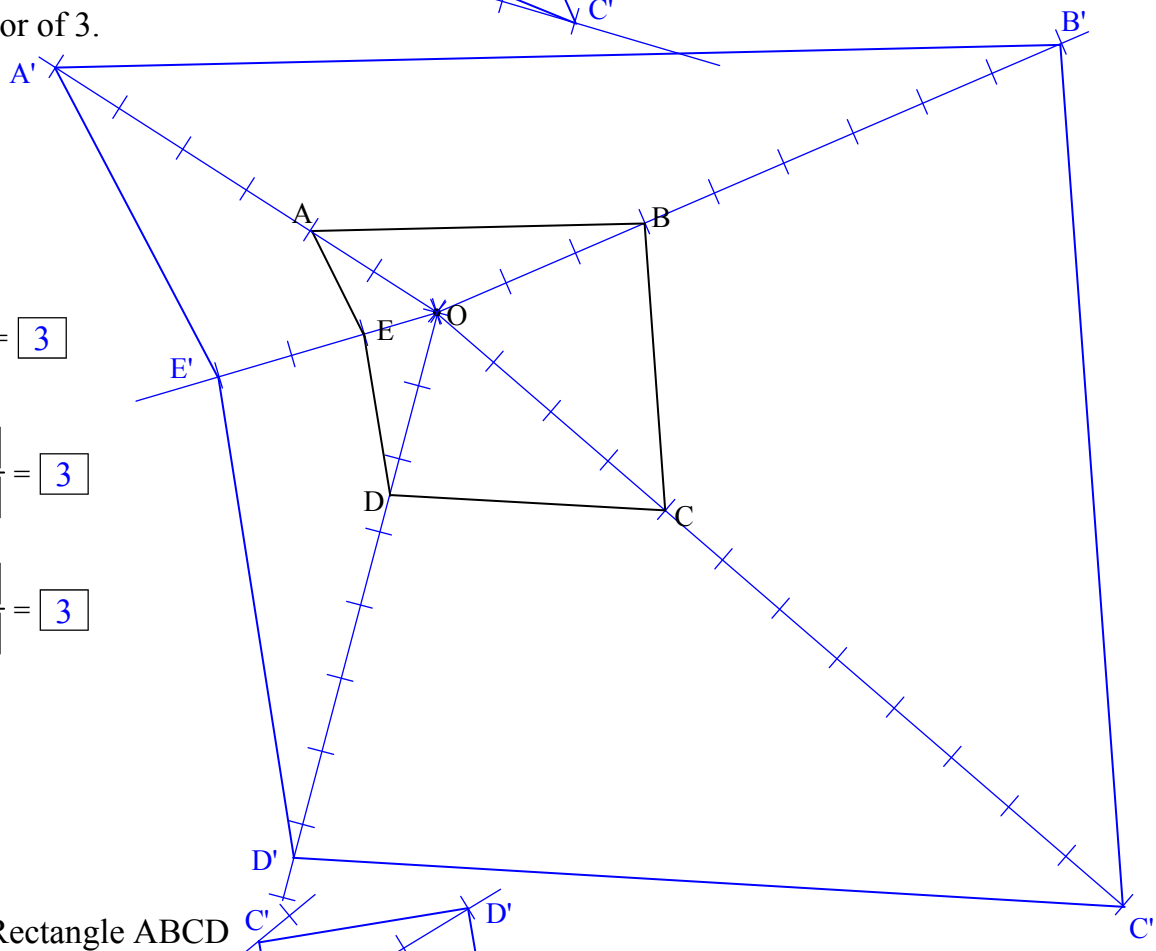
1 Enlarge Triangle ABC by a scale factor of 2.



Enlarge the shapes shown using 'O' as the centre of enlargement. Label the enlarged shape as A'B'C', and so on. Then complete the ratios to verify scale factor properties.

$\frac{OB'}{OB} = \frac{80}{40} = 2$ $\quad \frac{A'B'}{AB} = \frac{42}{21} = 2$

2 Enlarge Pentagon ABCDE by a scale factor of 3.

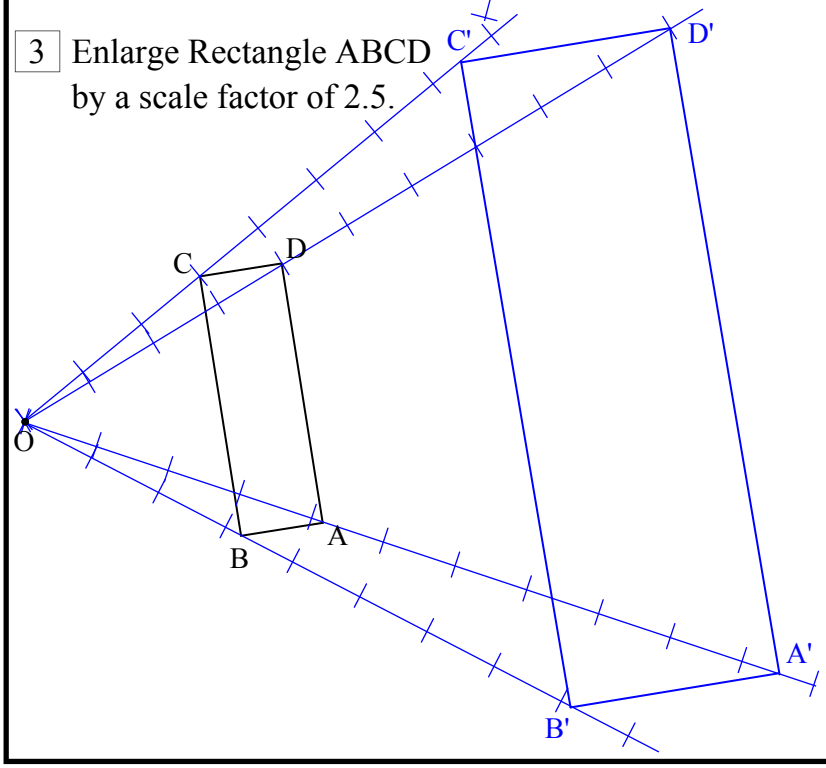


$\frac{OA'}{OA} = \frac{6}{2} = 3$

$\frac{B'C'}{BC} = \frac{114}{38} = 3$

$\frac{C'D'}{CD} = \frac{110}{36.5} = 3$

3 Enlarge Rectangle ABCD by a scale factor of 2.5.



$\frac{OD'}{OD} = \frac{100}{40} = 2.5$

$\frac{A'D'}{AD} = \frac{86}{34} = 2.5$

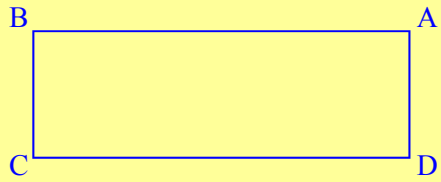
$\frac{B'D'}{BD} = \frac{90}{36} = 2.5$

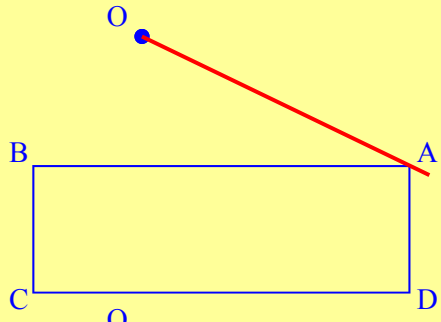
Enlargements (Sheet 2)

This sheet uses the same process as the previous sheet, the difference is that the scale factor is less than 1, so instead of the enlargement resulting in a bigger shape, it results in a smaller shape. Below is an example, the method is much the same as the previous sheet.

O

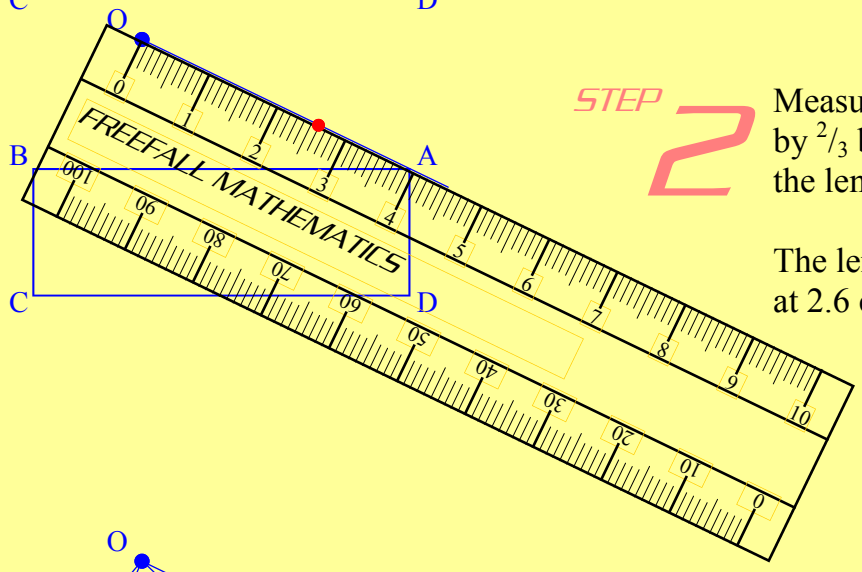
An enlargement of rectangle ABCD with a scale factor of $^2/_3$ is required.





STEP 1

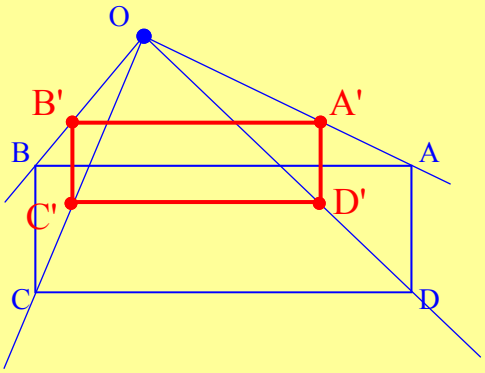
Use a ruler to draw a line from the CoE through one vertex. In this case it is the red line from O through A.



STEP 2

Measure the length OA and then multiply it by $^2/_3$ because the scale factor is $^2/_3$. Mark the length on the constructed line.

The length was 3.9 cm, so a mark is placed at 2.6 cm. Expect to round decimal answers



STEP 3

Repeat for all other vertices and join to make the similar shape.

Label the sides and compare lengths of matching sides to check.

Enlargements (Sheet 2)

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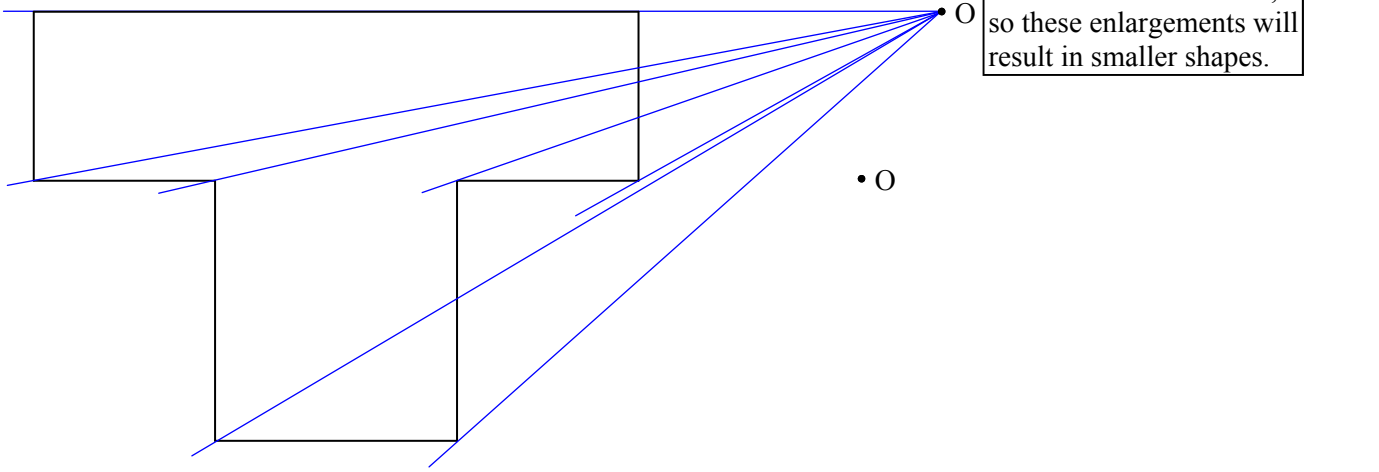
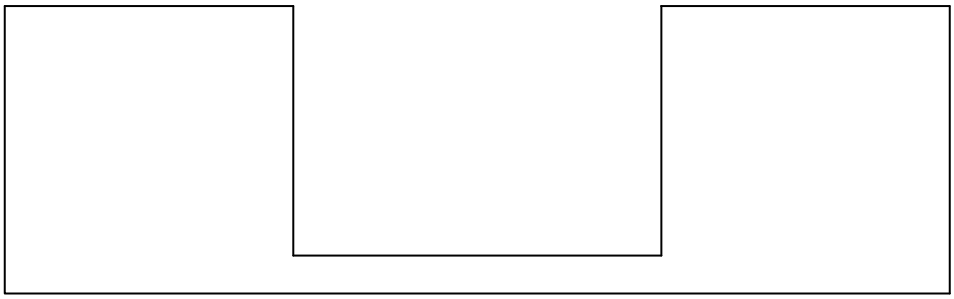
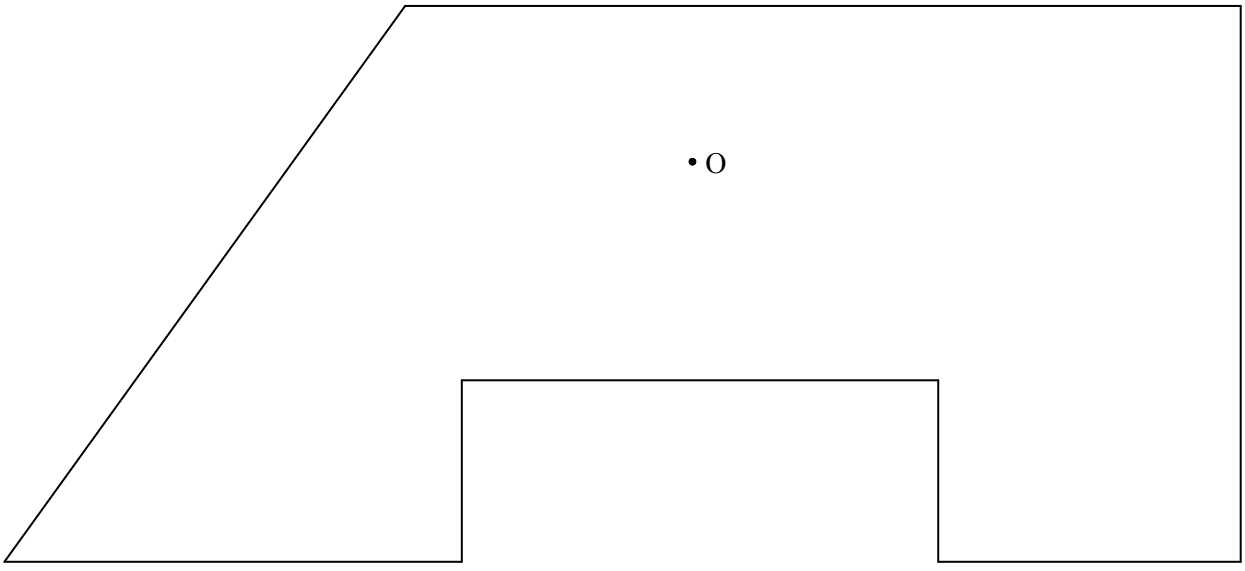
1 Enlarge the shape below by a scale factor of ½.



2 Enlarge this shape by a scale factor of ⅓.



3 Enlarge this shape by a scale factor of ¾.



Enlargements (Sheet 2)

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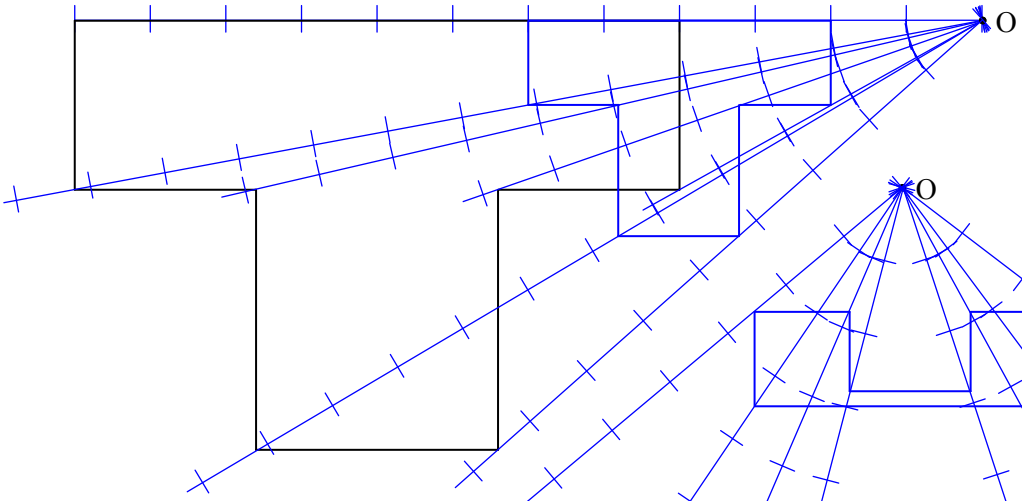
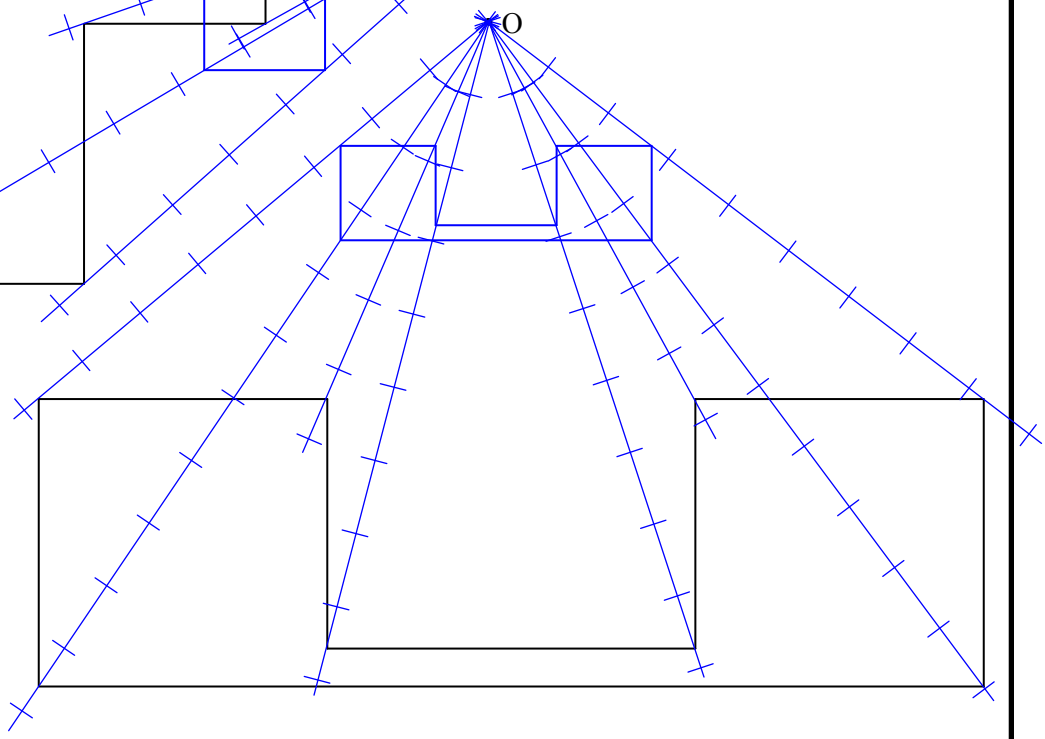
1 Enlarge the shape below by a scale factor of ½.

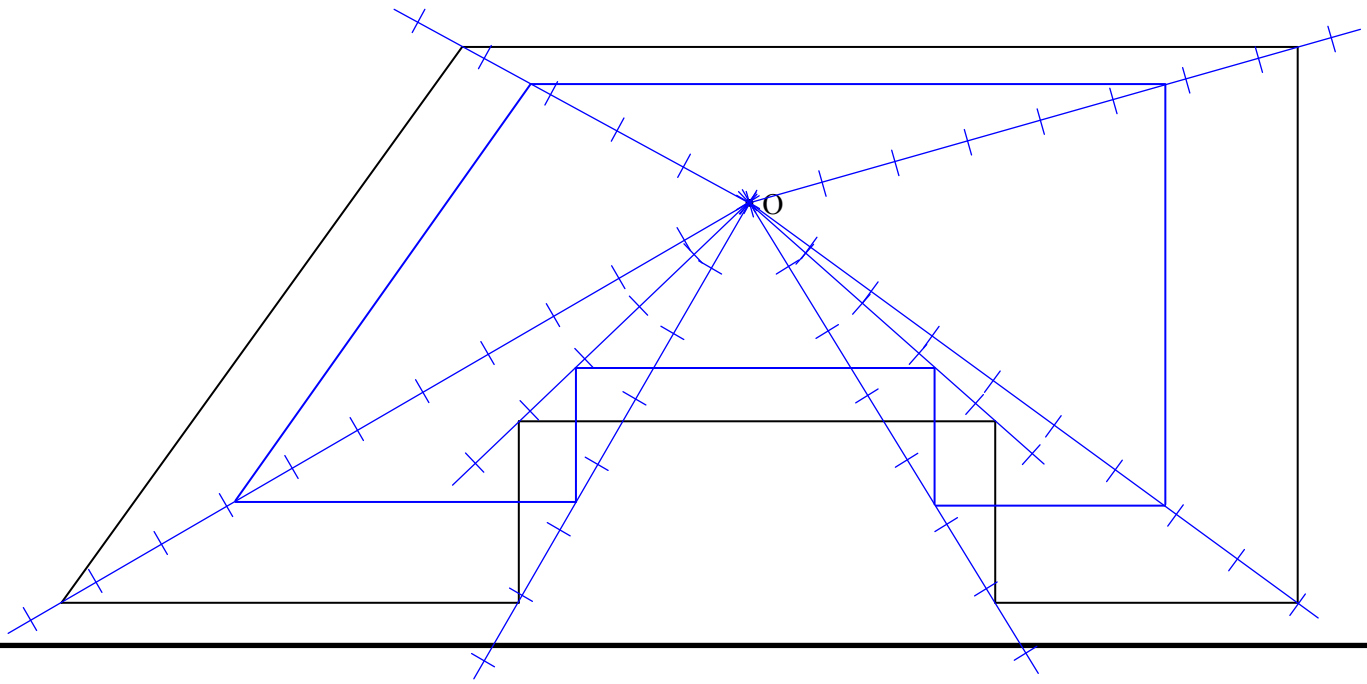
Enlarge the shapes shown using 'O' as the centre of enlargement. The scale factors are less than one, so these enlargements will result in smaller shapes.





2 Enlarge this shape by a scale factor of ⅓.

3 Enlarge this shape by a scale factor of ¾.



Similar Figures & Similarity Statements

Similar figures and shapes are the same shape only a scale factor has been applied to enlarge one of the shapes. Angles inside the shape are identical, but the side lengths are different by a scale factor. Similarity is stated by using three vertical strokes, ||| .

Column 1 starts with pairs of similar shapes scattered around the column. Find the similar pairs, that is, identical shapes just one is larger than the other. When you have found them, write the answer in the boxes at the base of the column. Strike each shape out with a lead pencil that you can rub out later, because you can colour the matching pairs the same colour. There is one shape that doesn't have a match, write its letter/number in the box at the bottom of the column.

Column 2 requires you to write similarity statements. When you name a shape you start at one vertex (corner) of the shape and go around it either in a clockwise or anti-clockwise direction. With similarity (and later congruence) statements you must start at matching vertices and move around the shape always matching vertices. Because angles match in similar figures, you can check that the angles are the same when identifying matching vertices. So check that the internal angles are the same to make sure you are listing the letters the correct way.

Similar Figures & Similarity Statements

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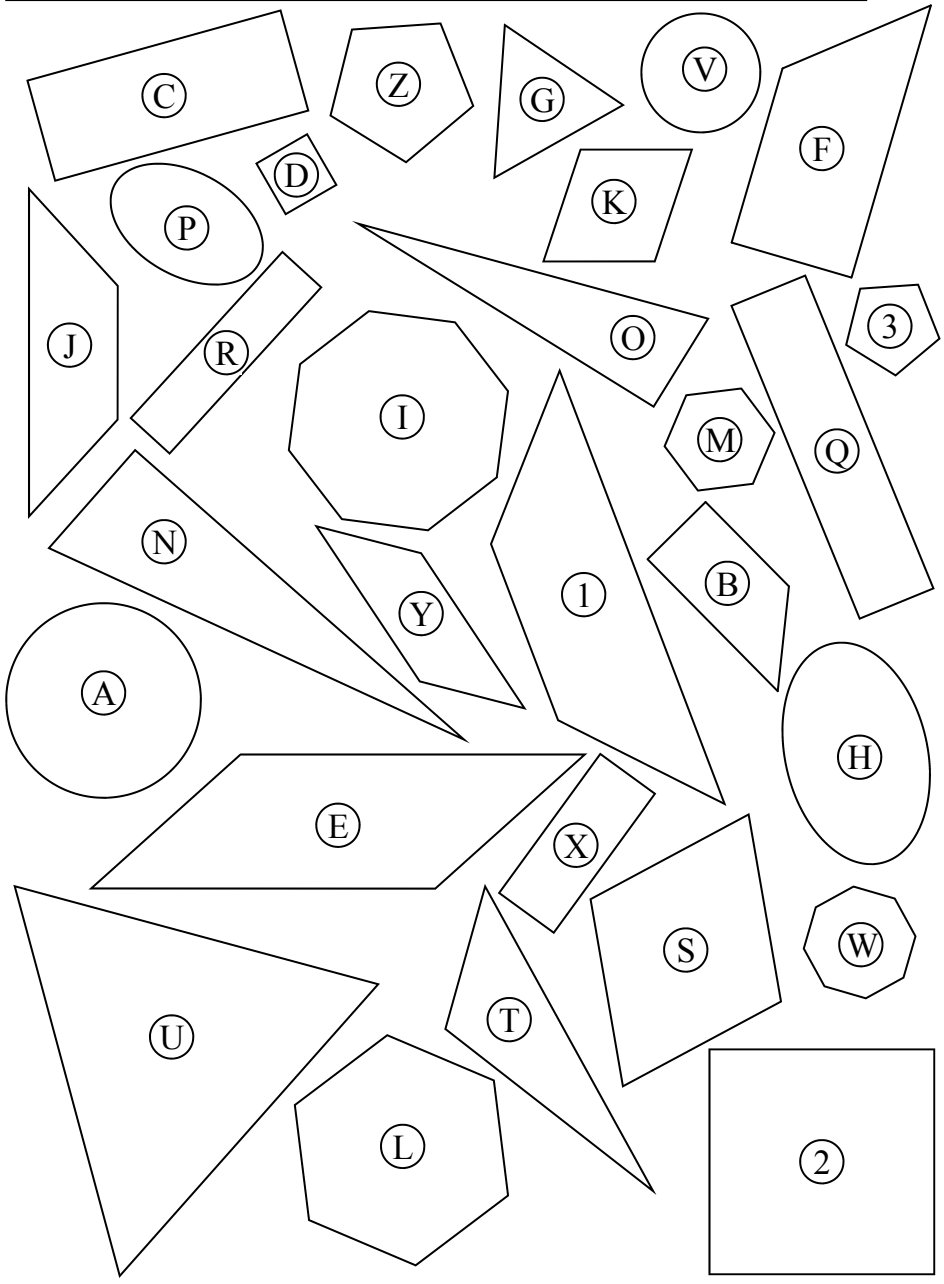
Similar figures have matching internal angles but have a scale factor applied to their side lengths. In other words, one is an enlargement of the other. Find all the similar pairs below, there is one non-paired shape.



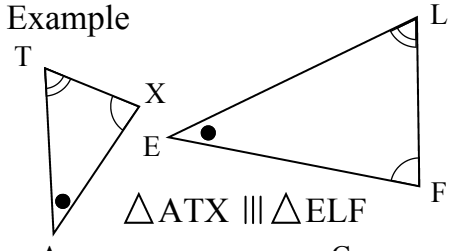
Write your matches below, once you are finished colour the similar shapes the same colour.

☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
	☐ & ☐	☐ & ☐	

There is one unmatched shape, it is: ☐

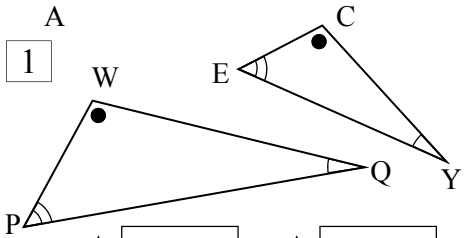
Write similarity statements for these similar shapes. Start at the same vertex and maintain order, or you could end up in here.

Example



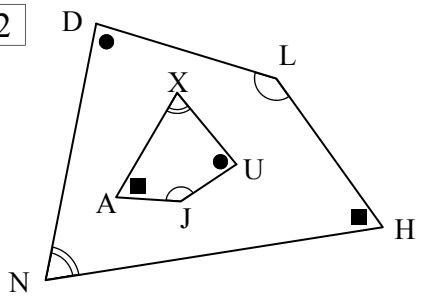
△ATX ||| △ELF

1



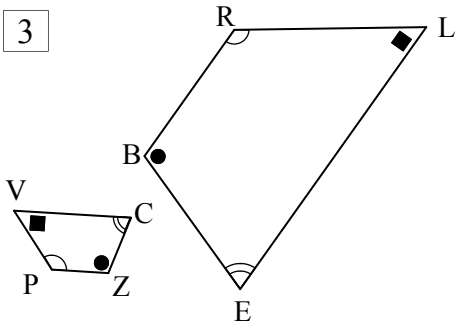
△ ☐ ||| △ ☐

2



☐ ||| ☐

3



Now list all matching angles and all matching sides from the two shapes shown above (Q.3).

4 Angles:

∠V = ∠L ∠C = ∠☐

∠☐ = ∠☐ ∠☐ = ∠☐

5 Sides:

VC & ☐ ☐ & ☐

☐ & ☐ ☐ & ☐

Similar Figures & Similarity Statements

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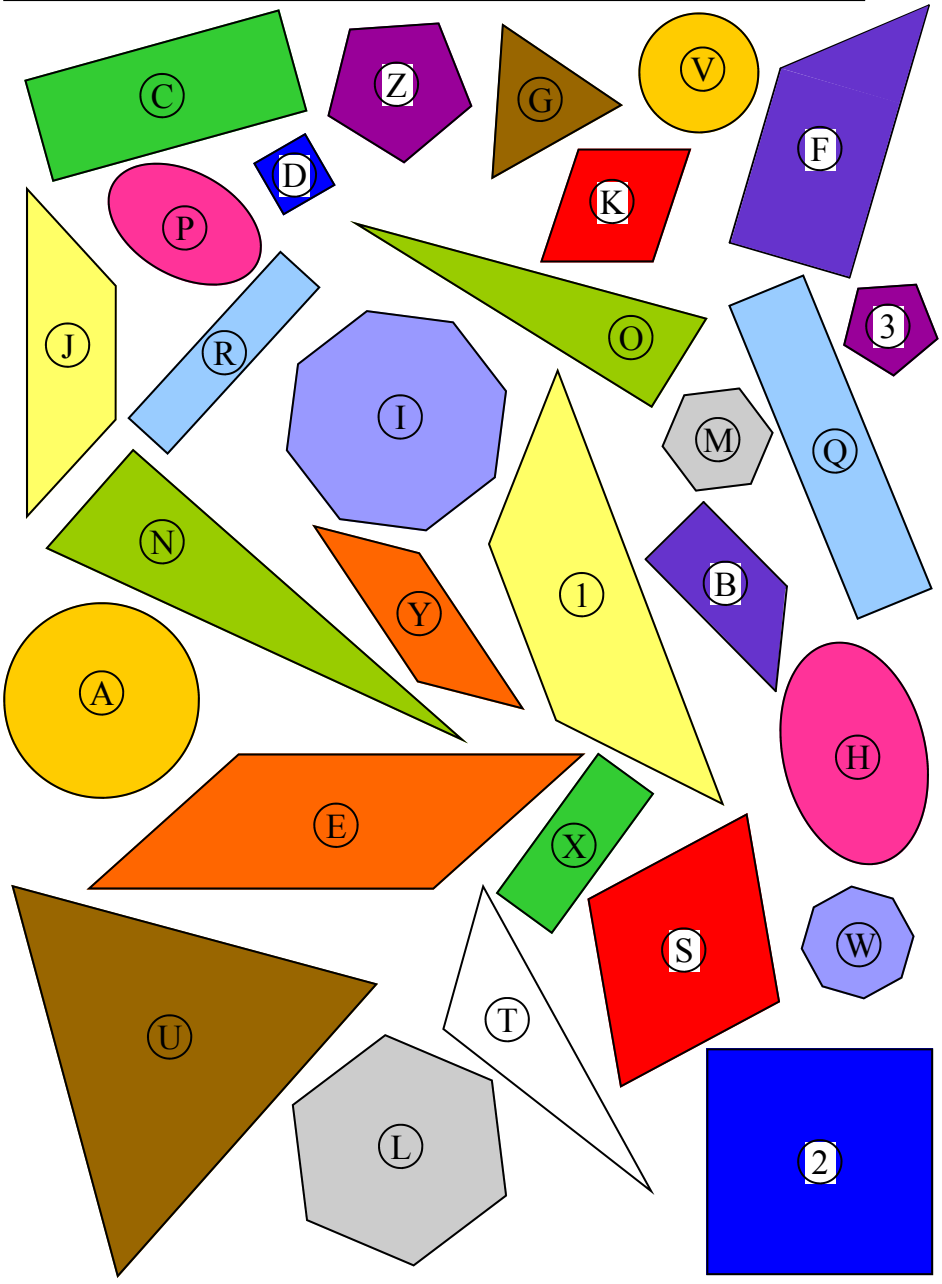
Similar figures have matching internal angles but have a scale factor applied to their side lengths. In other words, one is an enlargement of the other. Find all the similar pairs below, there is one non-paired shape.



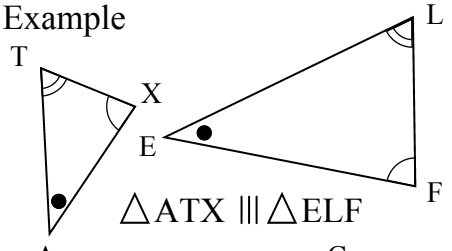
Write your matches below, once you are finished colour the similar shapes the same colour.

E & Y	K & S	J & 1	L & M
C & X	D & 2	A & V	O & N
Q & R	P & H	F & B	W & I
	U & G	Z & 3	

There is one unmatched shape, it is: T

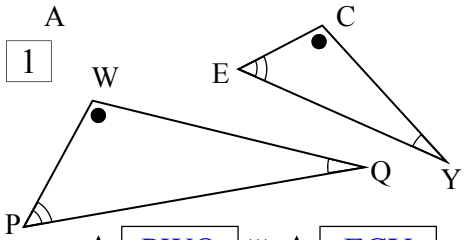
Write similarity statements for these similar shapes. Start at the same vertex and maintain order, or you could end up in here.

Example



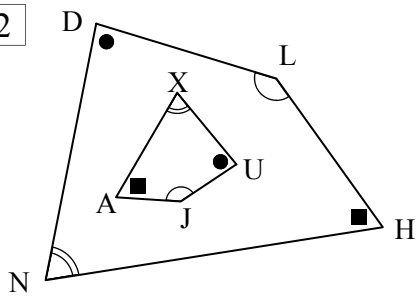
△ATX ||| △ELF

1



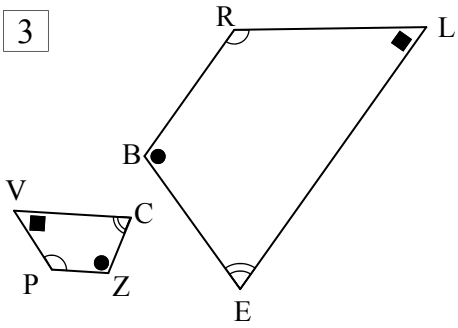
△ PWQ ||| △ ECY

2



NDLH ||| XUJA

3



VCZP ||| LEBR

Now list all matching angles and all matching sides from the two shapes shown above (Q.3).

4 Angles:

∠V = ∠L ∠C = ∠E

∠Z = ∠B ∠P = ∠R

5 Sides:

VC & LE CZ & EB

ZP & BR PV & RL

Similarity - Finding Sides Using Scale Factor

Similar figures are created by applying a scale factor to get the same shape only its larger or smaller. The internal angles are the same in both shapes. By comparing matching sides of the shapes a scale factor can be calculated, then once you have the scale factor you can apply this to all lengths on the shapes.

Column 1 starts with equivalent ratios. Sides are found using equivalent ratios, so this is a refresher on calculating them. Use multiplication or division to find the value of the pronumeral. Questions 7 - 9 involve finding the scale factor between two similar figures. Divide the second shape's side by the first. There are two measurements given on some shapes, make sure you use the one's that match. You can double check by using the other measurement as well. Note that scale factors are larger than 1 when the shape is enlarged, when it is reduced in size the scale factor will be a fraction.

Column 2 doesn't ask you to find the scale factor, but you need to do it anyway. So divide matching sides to get the scale factor, then multiply using the scale factor to get all missing sides.

Column 3 carries on with the same process, but the triangles are created by parallel lines. So parallel line theory questions are also in the column. Triangles are similar when all internal angles are the same, all of these questions feature this quality. Question 19 and 21 are different as the triangle is within the other triangle. Question 21 is also more difficult because you may have to calculate a second scale factor to solve for b.

Similarity - Finding Sides Using Scale Factor

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Lets start with a quick review of equivalent ratios. Find the value of the pronumerals in the ratios.

1 $5 : 1 = 30 : x$ (×6, ×6)

$x =$ ☐

2 $4 : 3 = 16 : y$

$y =$ ☐

3 $3 : 2 = k : 22$

$k =$ ☐

4 $a : 24 = 10 : 3$

$a =$ ☐

5 $20 : 7 : 13 = x : 35 : y$

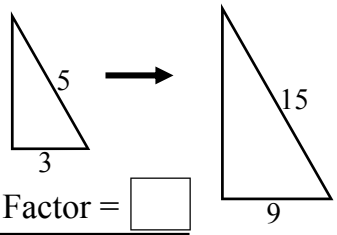
$x =$ ☐ $y =$ ☐

6 $b : 4 : 36 = 4 : 1 : f$

$b =$ ☐ $f =$ ☐

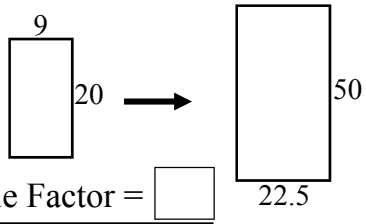
Find the scale factor applied to the first shape to get the second.

7



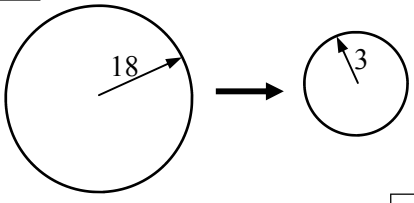
Scale Factor = ☐

8



Scale Factor = ☐

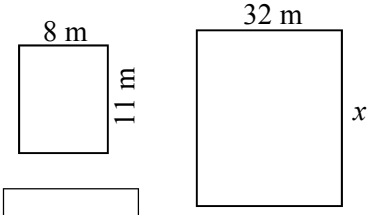
9



Scale Factor = ☐

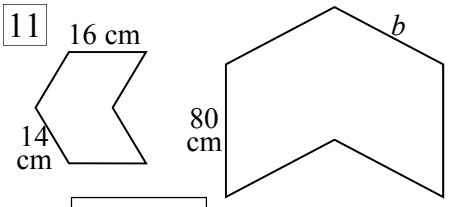
Use your ratio skills to find the unknown sides on these similar figures. Drawings not to scale.

10



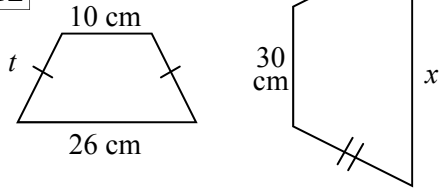
$x =$ ☐

11



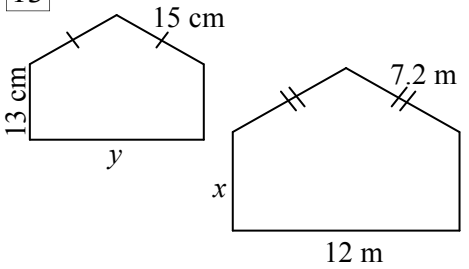
$b =$ ☐

12



$x =$ ☐ $t =$ ☐

13

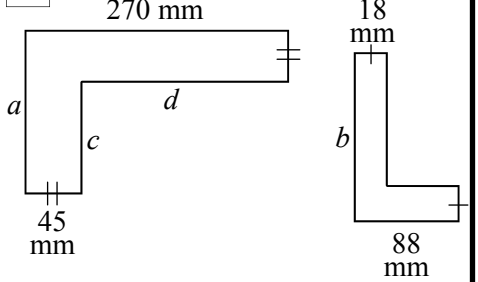


$x =$ ☐ $y =$ ☐

Find the perimeter of the larger shape

14

15

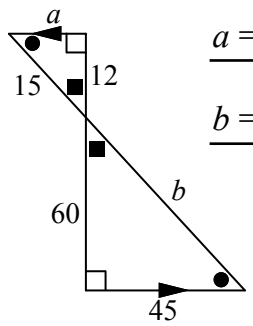


$a =$ ☐ $b =$ ☐

$c =$ ☐ $d =$ ☐

These triangles are similar. Find the value of a and b and give some angle theory reasoning.

16



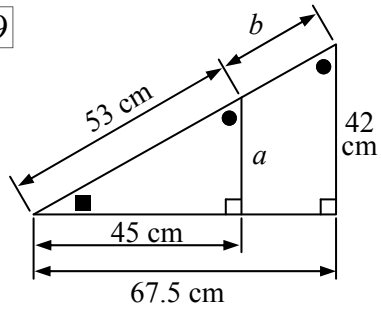
$a =$

$b =$

17 What angle property exists between the angles marked with a ● ?

18 What angle property exists between the angles marked with a ■ ?

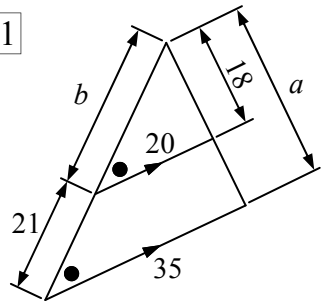
19



$a =$ $b =$

20 Why are the angles marked with a ● equal?

21



$a =$ $b =$

22 Why are the angles marked with a ● equal?

Similarity - Finding Sides Using Scale Factor

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Lets start with a quick review of equivalent ratios. Find the value of the pronumerals in the ratios.

1 $5 : 1 = 30 : x$ (×6, ×6)

$x =$ 6

2 $4 : 3 = 16 : y$

$y =$ 12

3 $3 : 2 = k : 22$

$k =$ 33

4 $a : 24 = 10 : 3$

$a =$ 80

5 $20 : 7 : 13 = x : 35 : y$

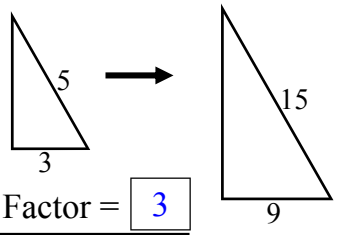
$x =$ 100 $y =$ 65

6 $b : 4 : 36 = 4 : 1 : f$

$b =$ 16 $f =$ 9

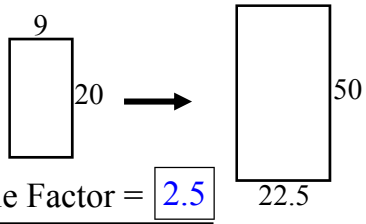
Find the scale factor applied to the first shape to get the second.

7



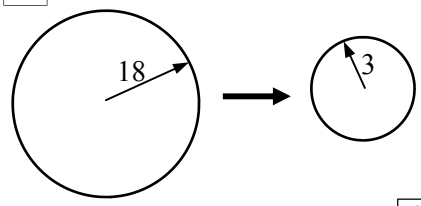
Scale Factor = 3

8



Scale Factor = 2.5

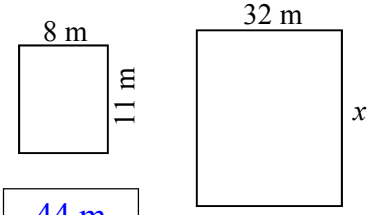
9



Scale Factor = $\frac{1}{6}$

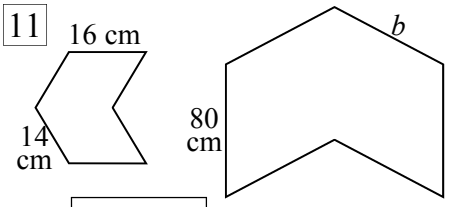
Use your ratio skills to find the unknown sides on these similar figures. Drawings not to scale.

10



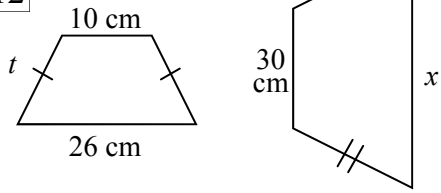
$x =$ 44 m

11



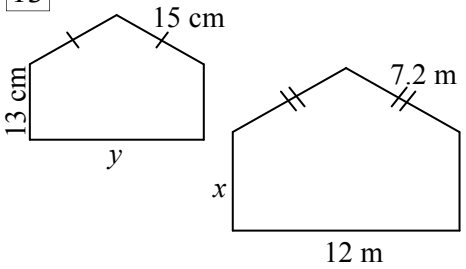
$b =$ 70 cm

12



$x =$ 78 cm $t =$ 17 cm

13



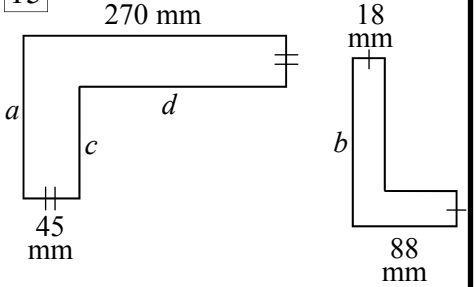
$x =$ 6.24 m $y =$ 25 cm

Find the perimeter of the larger shape

14 $P = 2 \times 6.24 + 2 \times 7.2 + 12$

$P = 38.88$ m

15

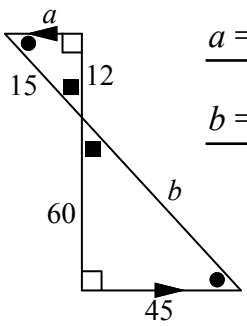


$a =$ 220 mm $b =$ 108 mm

$c =$ 175 mm $d =$ 225 mm

These triangles are similar. Find the value of a and b and give some angle theory reasoning.

16



$a =$ 9 units

$b =$ 75 units

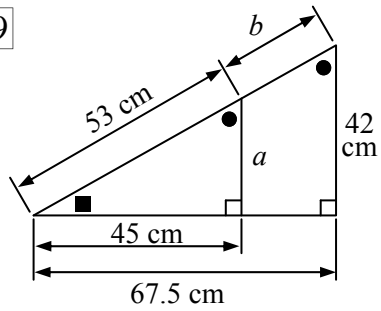
17 What angle property exists between the angles marked with a ● ?

They are alternate angles.

18 What angle property exists between the angles marked with a ■ ?

They are vertically opposite.

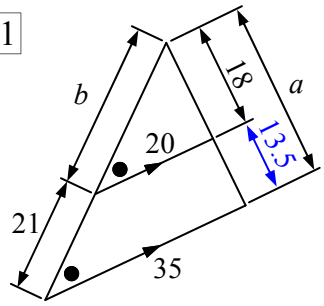
19



$a =$ 28 cm $b =$ 26.5 cm

20 Why are the angles marked with a ● equal? Because they are corresponding angles. or angle sum of Δ = 180°

21



$a =$ 31.5 cm $b =$ 28 cm

22 Why are the angles marked with a ● equal? Because they are corresponding angles.

Tessellations

Tessellations are patterns made by the repeated use of a shape, without leaving gaps.

Question 1 in the first column features tessellated triangles. Colour the adjacent triangles (that share a common side) to build the shapes shown. Then colour the square in front of the shape's name the same colour. Like a key or legend in a map.

Column 1 then has three tessellated diagrams, complete the patterns. If you want to use different colours then print the second worksheet, which is unpainted.

Column 2 consists of irregular tessellations. This is when two or more different shapes are used. Colour them to form patterns.

The final tessellation requires you to complete it by using the dots as guides. Once you have drawn it colour in the stars that have been formed.

Tessellations

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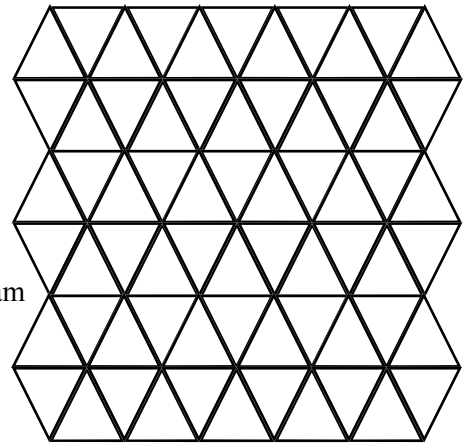
Create the shapes below on the grid by colouring the triangles. Then match that colour in one of the key squares before the shape name.

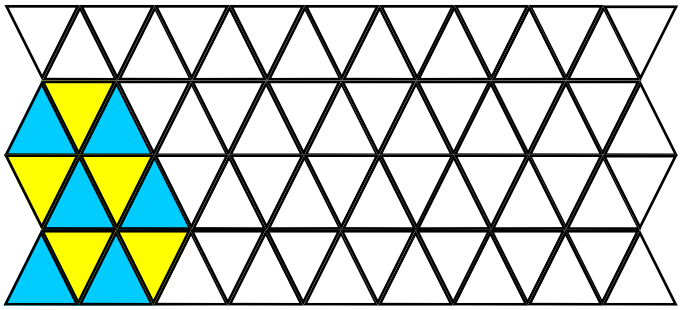
1



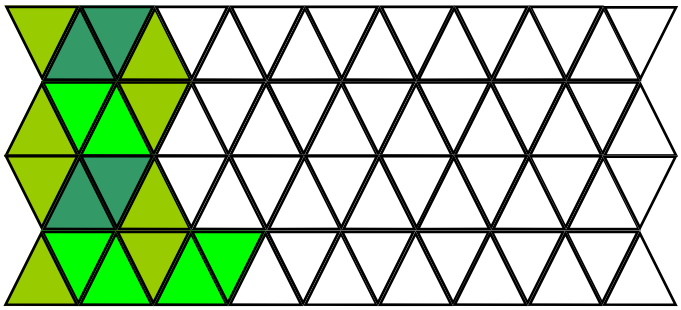
- ☐ Hexagon
- ☐ Rhombus
- ☐ Parallelogram
- ☐ Trapezium

Complete these regular tessellations

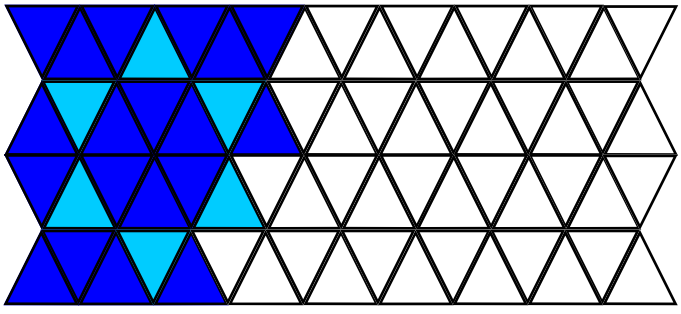
2 Triangle



3 Rhombus

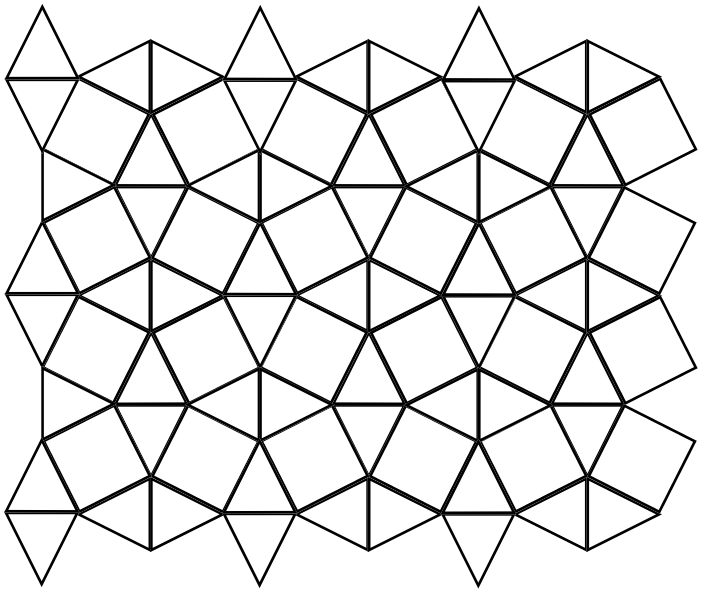


4 Hexagon

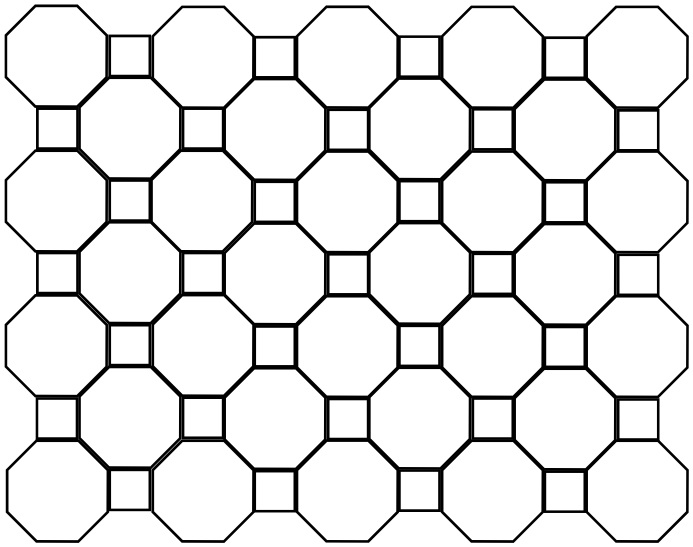


Colour-in these irregular tessellations.

5 Triangles and Squares

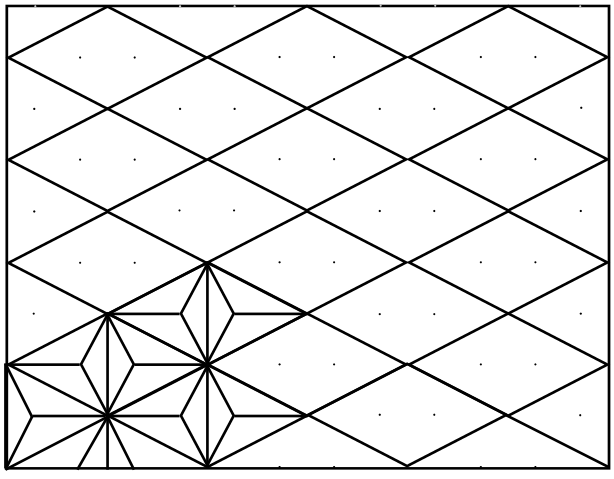


5 Octagons and Squares



Complete this irregular tessellation, then colour-in the stars

7



Tessellations

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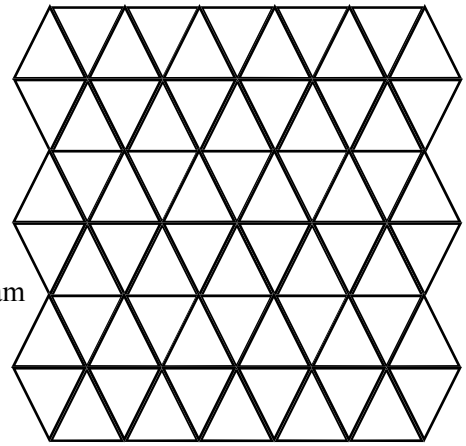
Create the shapes below on the grid by colouring the triangles. Then match that colour in one of the key squares before the shape name.

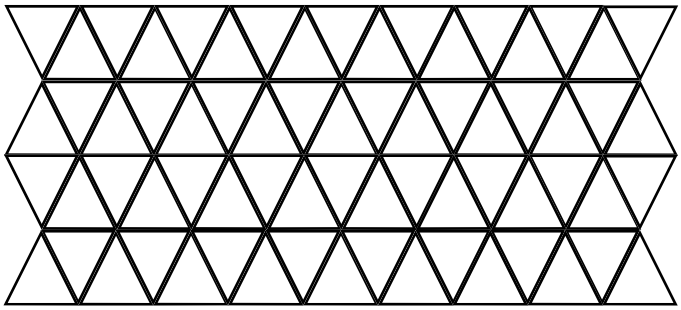
1



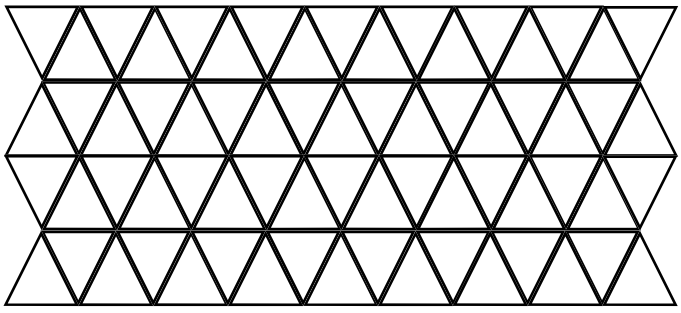
- ☐ Hexagon
- ☐ Rhombus
- ☐ Parallelogram
- ☐ Trapezium

Complete these regular tessellations

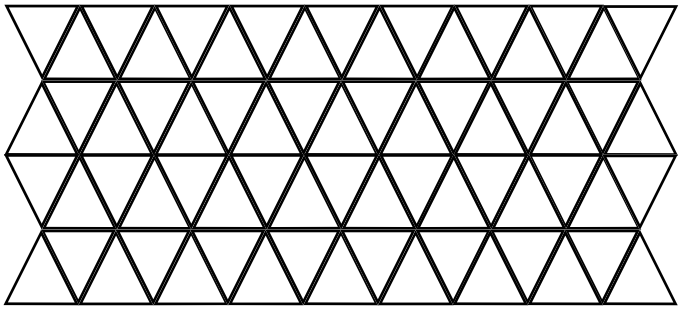
2 Triangle



3 Rhombus

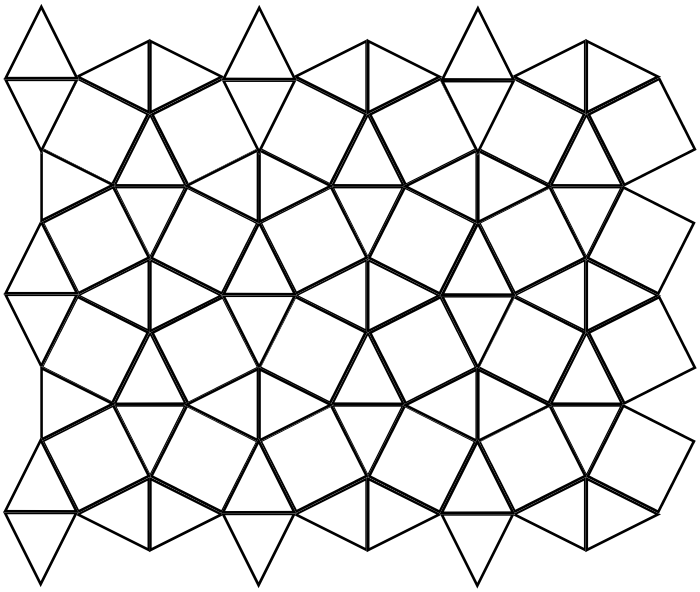


4 Hexagon

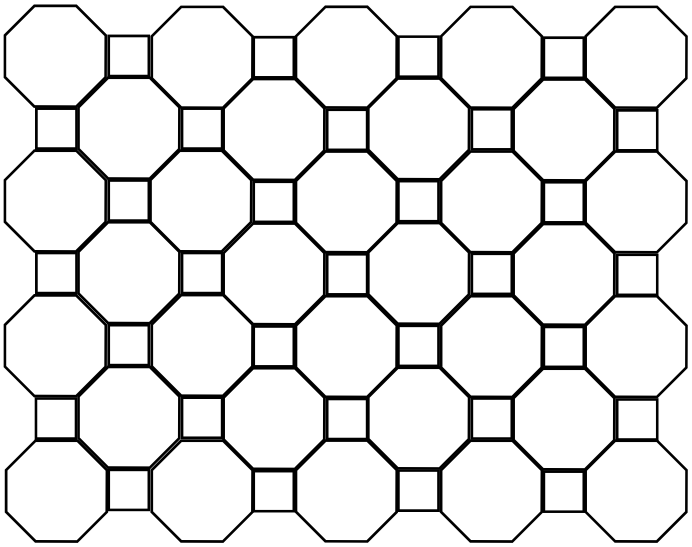


Colour-in these irregular tessellations.

5 Triangles and Squares

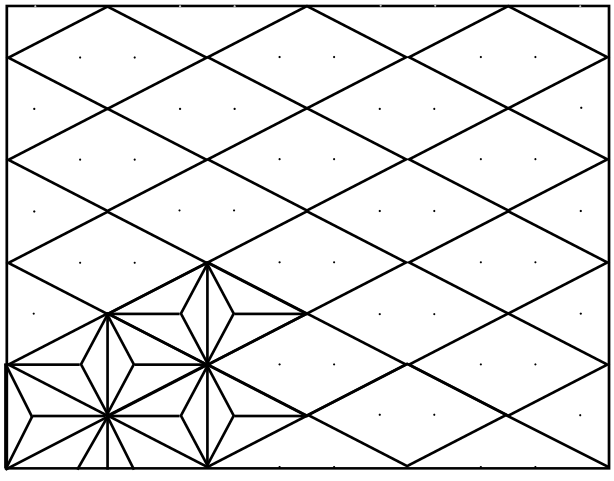


5 Octagons and Squares



Complete this irregular tessellation, then colour-in the stars

7



Tessellations

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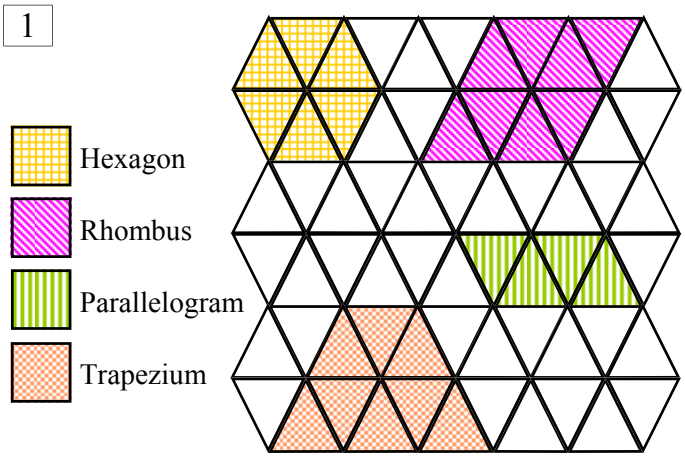
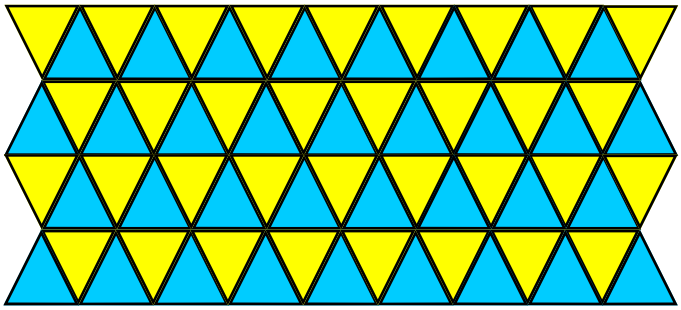
Create the shapes below on the grid by colouring the triangles. Then match that colour in one of the key squares before the shape name.

1

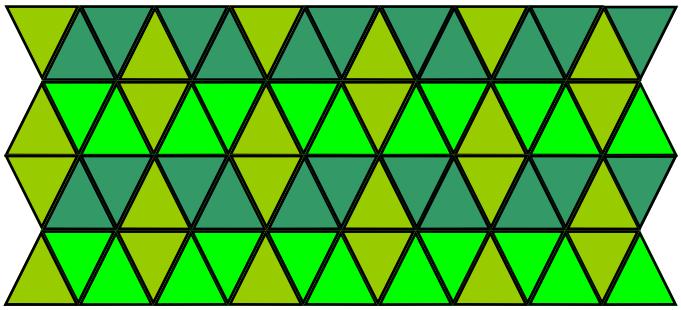


Complete these regular tessellations

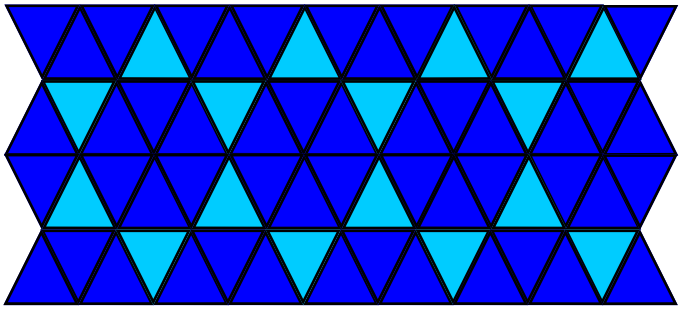
2 Triangle



3 Rhombus

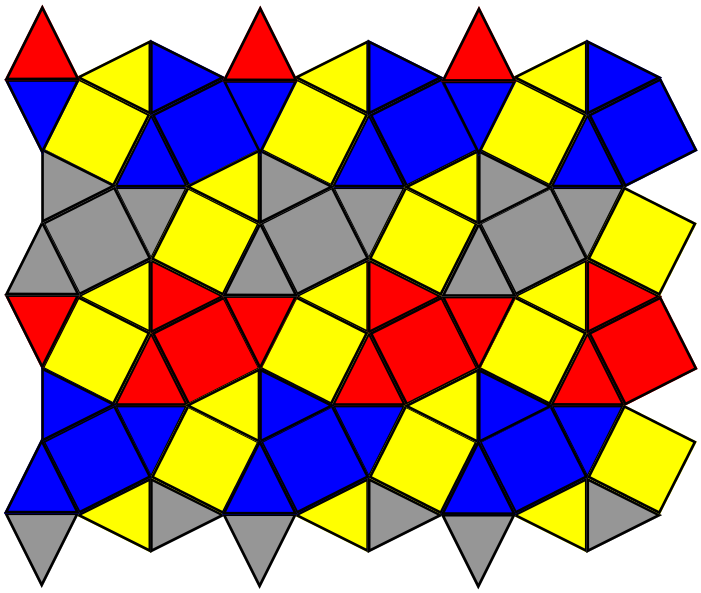


4 Hexagon

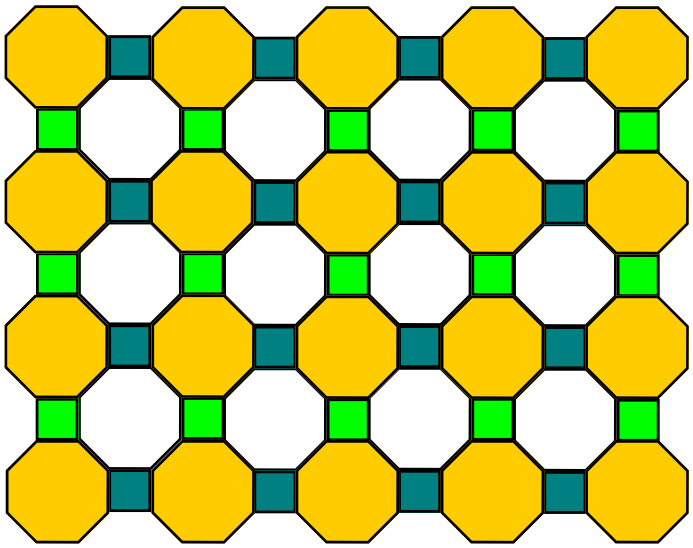


Colour-in these irregular tessellations.

5 Triangles and Squares

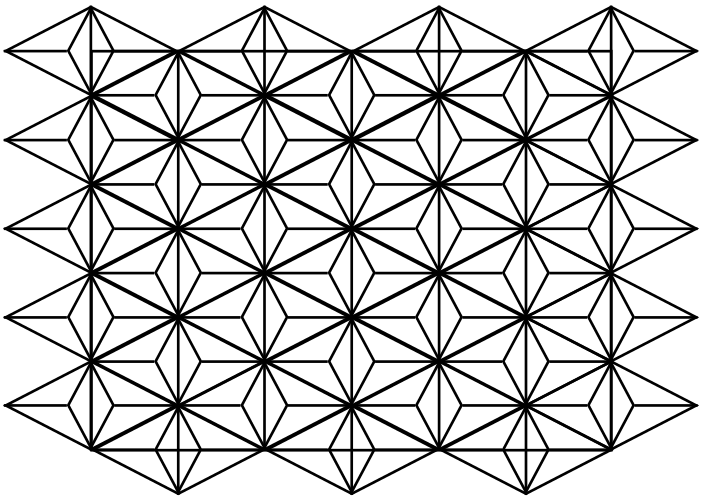


5 Octagons and Squares



Complete this irregular tessellation, then colour-in the stars

7



Axis of Symmetry

An axis of symmetry is a line which divides a shape into two congruent (identical) shapes. To complete this sheet on paper cut the shapes out and fold the shape on top of itself, the crease made by the fold is the axis of symmetry, count the creases and write the number in the box inside the shape.

If working on screen, imagine a line cutting the shape and check whether the shape either side of the imaginary line is an identical image.

Axis of Symmetry

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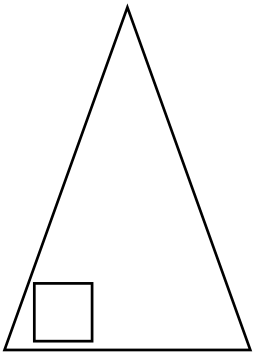
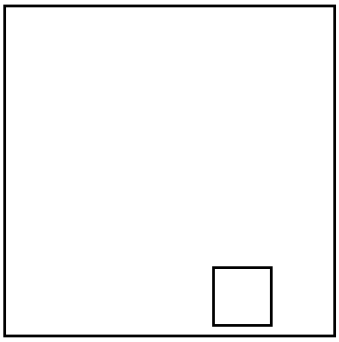
Cut the shapes out and fold them. If the shape doubles back on itself so that the fold mark creates two shapes that are the same, then that is an axis of symmetry. Repeat until all are found. Then write the number found in the square of the shape and stick the shape and its name in your book.

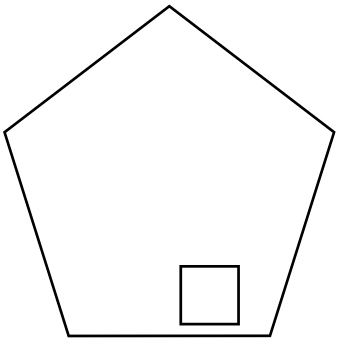
1 Isosceles triangle



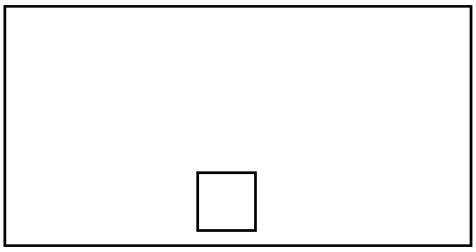
2 Square



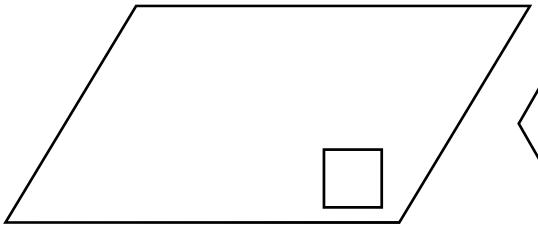
3 Pentagon



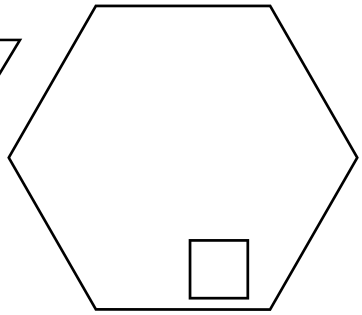
4 Rectangle



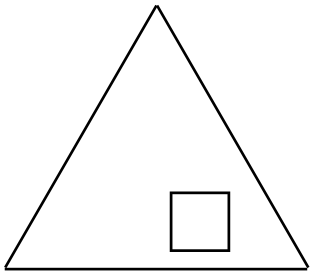
5 Parallelogram



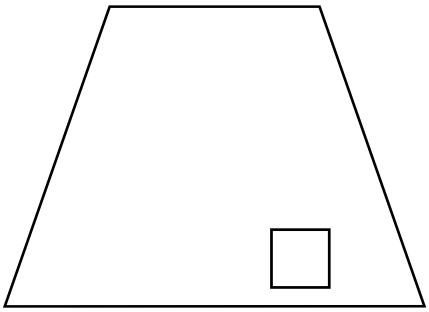
6 Hexagon



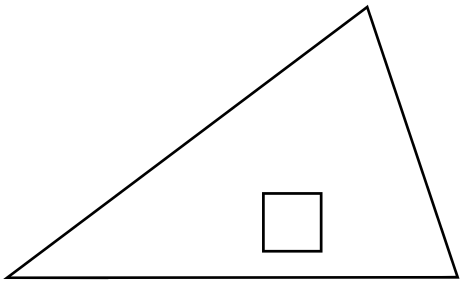
7 Equilateral triangle



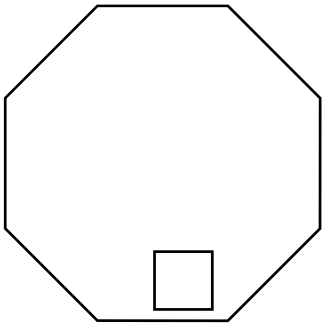
8 Trapezium



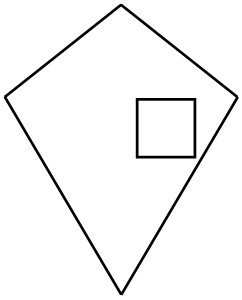
9 Scalene triangle



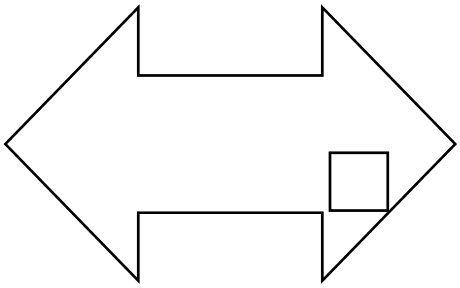
10 Octagon



11 Kite



12 Irregular Decagon



Axis of Symmetry

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Cut the shapes out and fold them. If the shape doubles back on itself so that the fold mark creates two shapes that are the same, then that is an axis of symmetry. Repeat until all are found. Then write the number found in the square of the shape and stick the shape and its name in your book.

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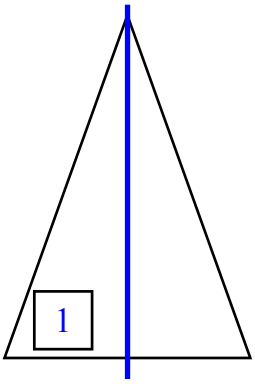
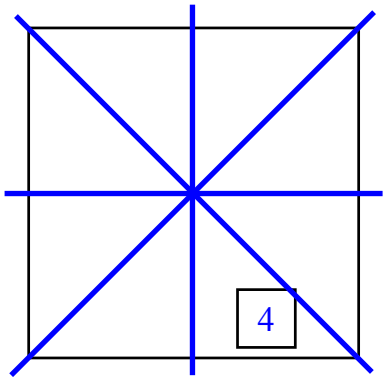
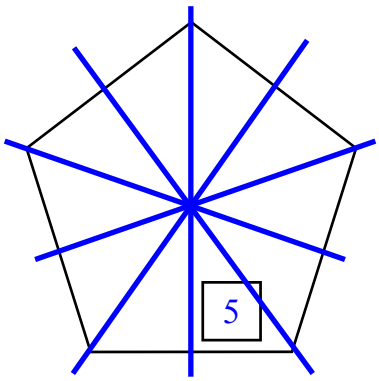


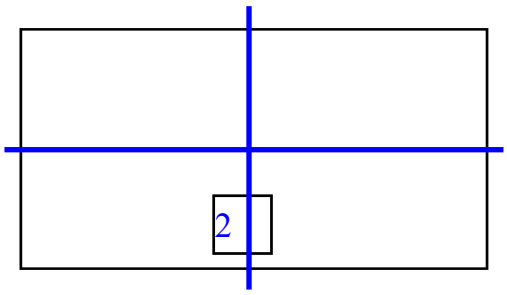
2 Square



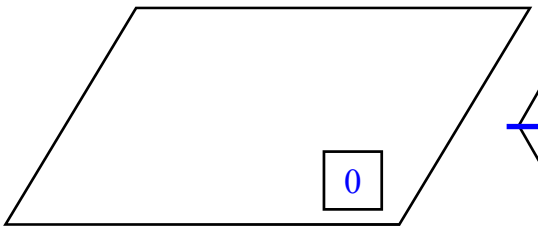
3 Pentagon



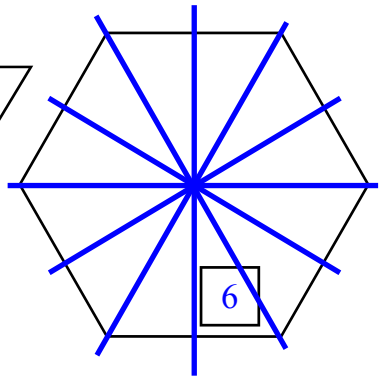
4 Rectangle



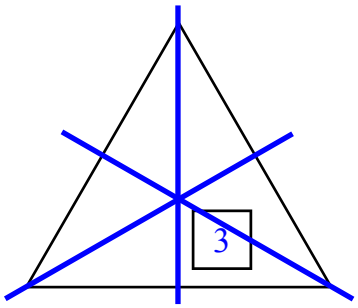
5 Parallelogram



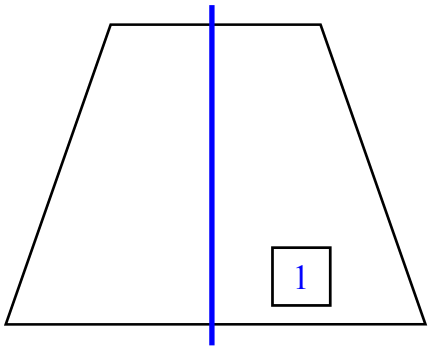
6 Hexagon



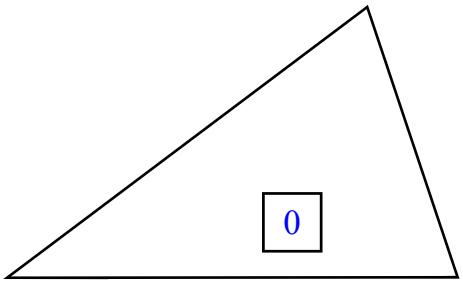
7 Equilateral triangle



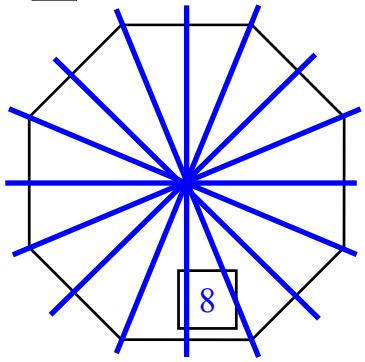
8 Trapezium



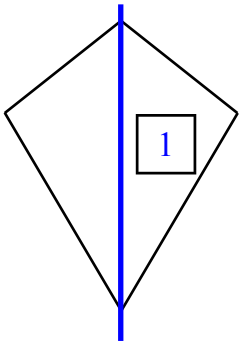
9 Scalene triangle



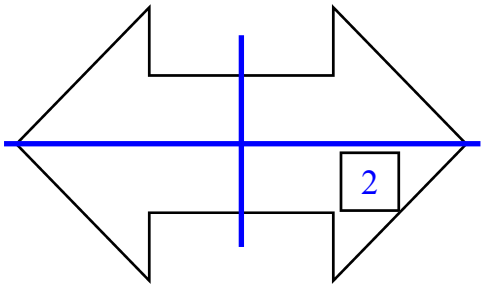
10 Octagon



11 Kite



12 Irregular Decagon



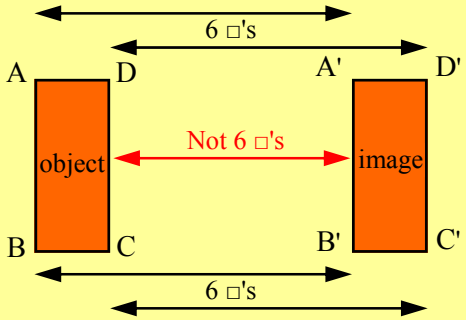
Transformations

When something is transformed it is changed in some way. In mathematics there are 3 different ways to transform a shape.

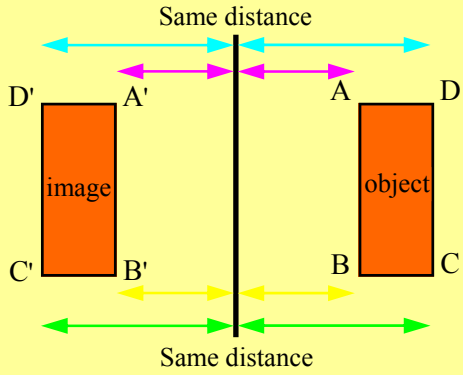
- Translation - this simply means to move, on the worksheet the shapes are on a grid and you are asked to move the object a certain number of squares on the grid.
- Refection - this is like a mirror image, the shape is reflected across a plane (line). The new shape will be the same distance away from the plane as the original, only upside-down or back-to-front.
- Rotation means to spin around a point, the point could be on the shape or away from the shape, this type of transformation requires the most thought.

With positioning the letters on the shape, the letters stay in the same position with translation, reversed in reflection and will be rotated in rotation, look at the examples below.

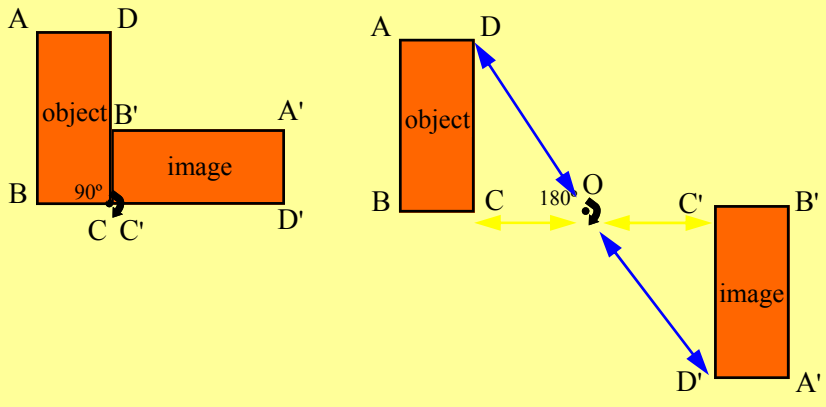
Translation means to move, the letters stay in the same position on the image. If you move the shape 6 □'s to the left then the distance between the same points is 6 □'s. The gap between the object and its image isn't 6 □'s, the most frequent mistake made.



Reflection means to copy back-to-front across a plane, the letters stay the same distance from the plane which means they appear to be reversed.



Rotation means to spin around a point, the point can be on the object or away from the object. The direction of rotation is given as either clockwise or anticlockwise. The distance of a point on the object from the rotation point remains the same once rotated.



Transformations

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Create congruent shapes by completing the following transformations.

1 Reflect shape (1) in the line AB.

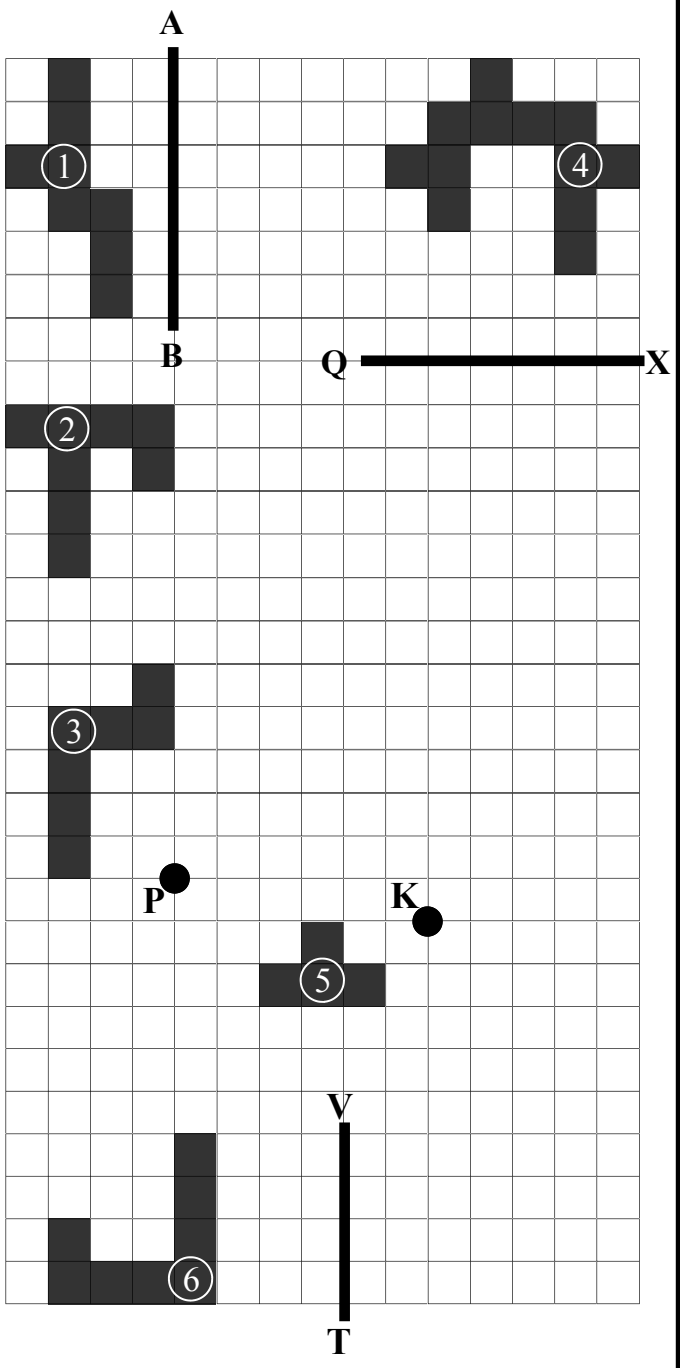
2 Translate shape (2) 5 squares right.

3 Rotate shape (3) 90° clockwise at P.

4 Reflect shape (4) in the line QX.

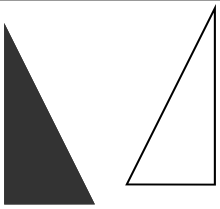
5 Rotate shape (5) 180° clockwise at K.

6 Translate shape (6) 2 squares right, then reflect the translated shape in the line VT.



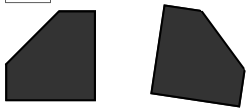
These solid shapes have had two transformations applied to them to create a congruent shape, write 'translation', 'reflection' or 'rotation' in the spaces

7



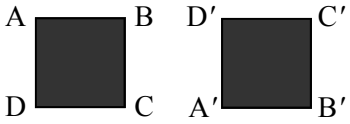
_____ & _____

8



_____ & _____

9



_____ & _____

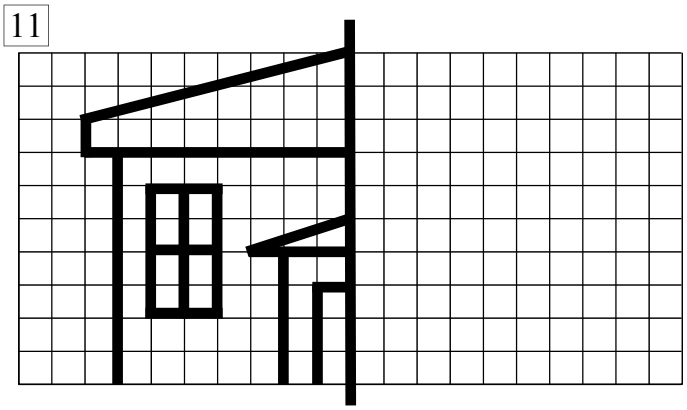
10



_____ & _____

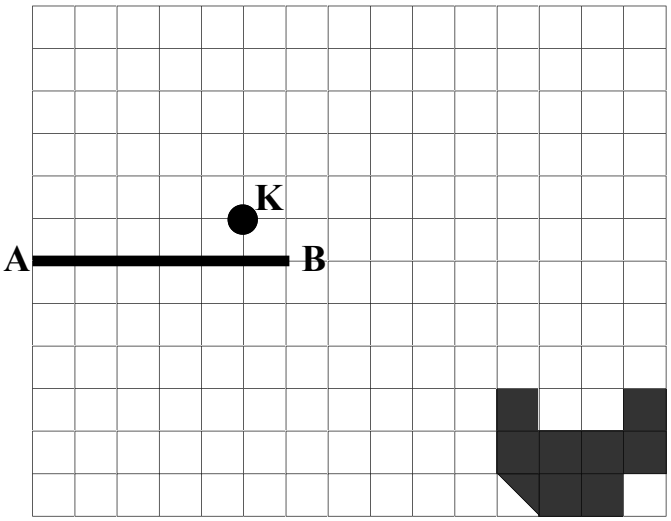
Draw the rest of the house as a reflection across the axis of symmetry.

11



Translate the shape below seven squares up and five squares left, rotate 90° anti-clockwise at K then reflect in the line AB.

12



Transformations

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Create congruent shapes by completing the following transformations.

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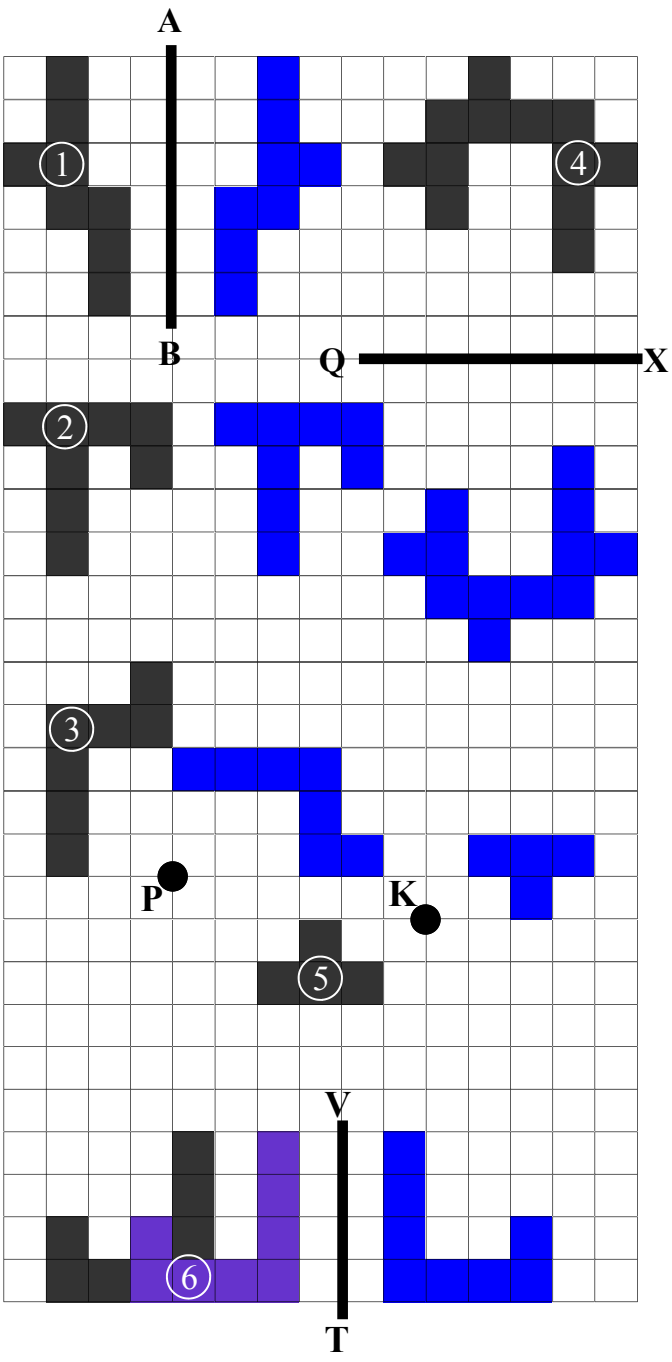
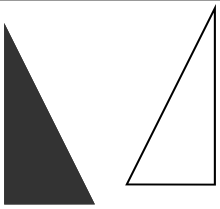
6 Translate shape (6) 2 squares right, then reflect the translated shape in the line VT.



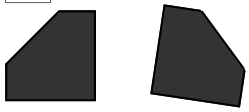
These solid shapes have had two transformations applied to them to create a congruent shape, write 'translation', 'reflection' or 'rotation' in the spaces

7



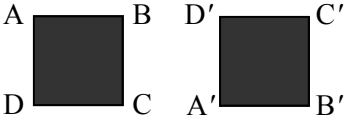
Reflection & Translation

8



Reflection & Rotation

9



Rotation & Reflection

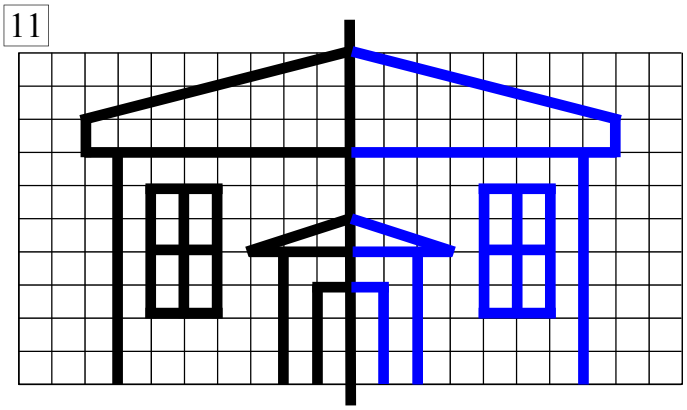
10



Translation & Rotation

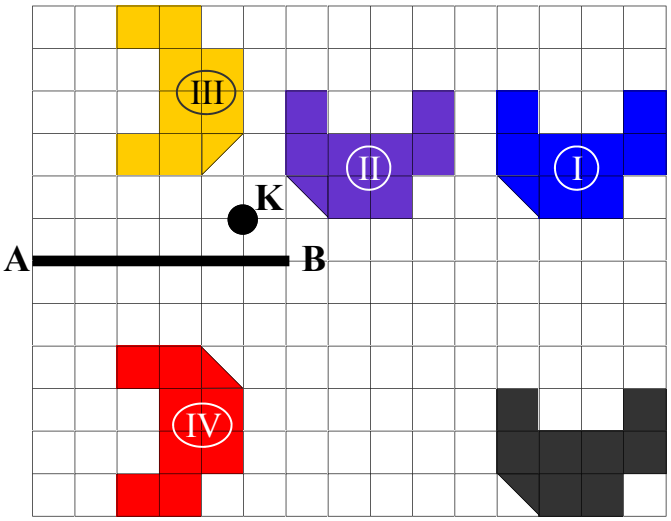
Draw the rest of the house as a reflection across the axis of symmetry.

11



Translate the shape below seven squares up and five squares left, rotate 90° anti-clockwise at K then reflect in the line AB.

12



Cut and Paste - Congruent Shapes

This worksheet involves cutting out the shapes in the right hand column and sticking them on the character on the left hand side. Before you stick them on, look at the number before you cover it up. Write the letter in the matching cut out shape into the table, next to the number.

Cut and Paste - Congruent Shapes

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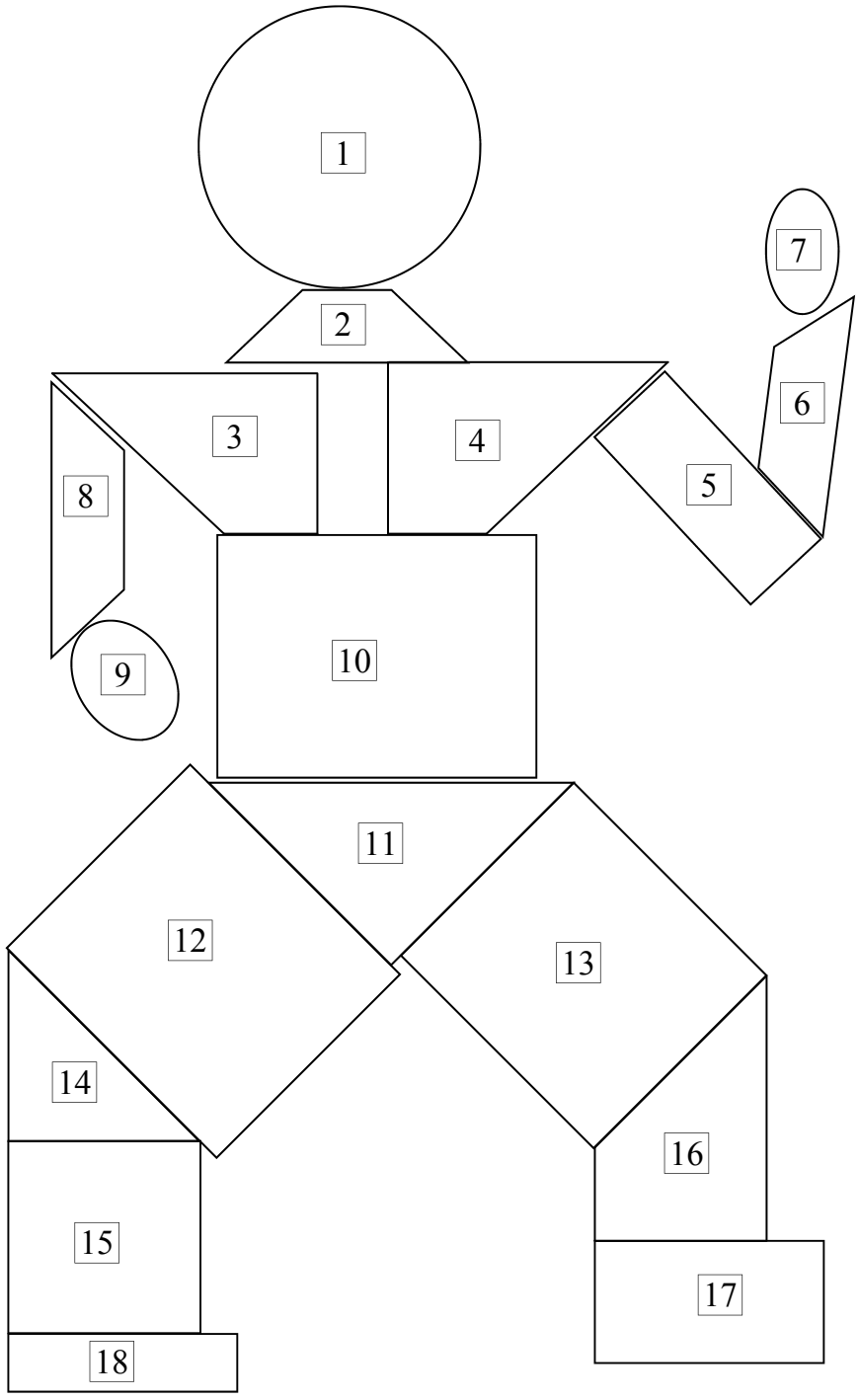
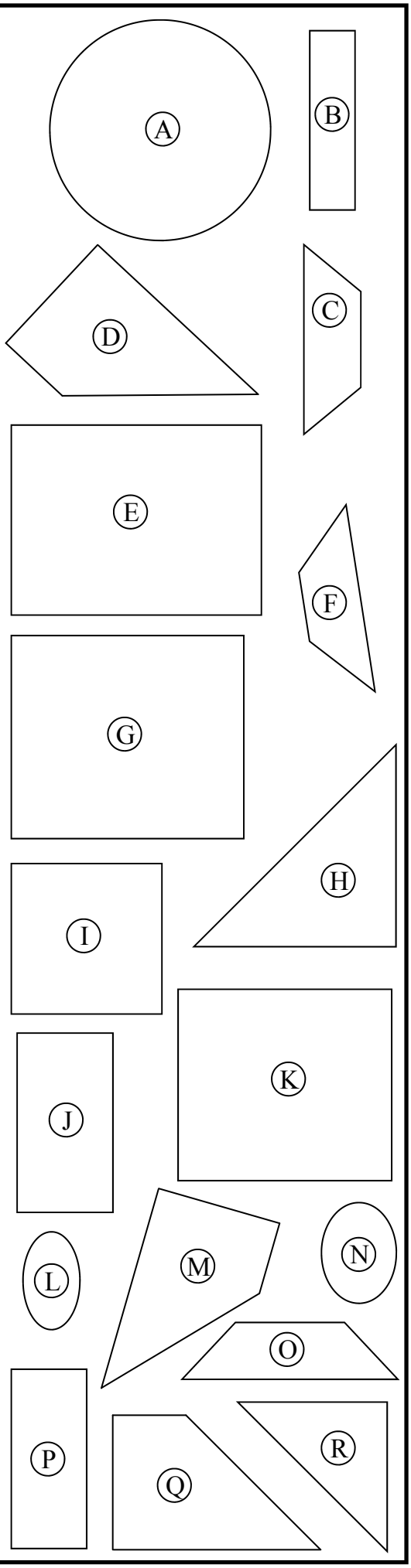
Cut the shapes out from the left hand side column and stick them onto the drawing below. Write down the letter that matches its congruent mate in the table below.

Number	Letter	Number	Letter	Number	Letter
1		7		13	
2		8		14	
3		9		15	
4		10		16	
5		11		17	
6		12		18	





Cut and Paste - Congruent Shapes

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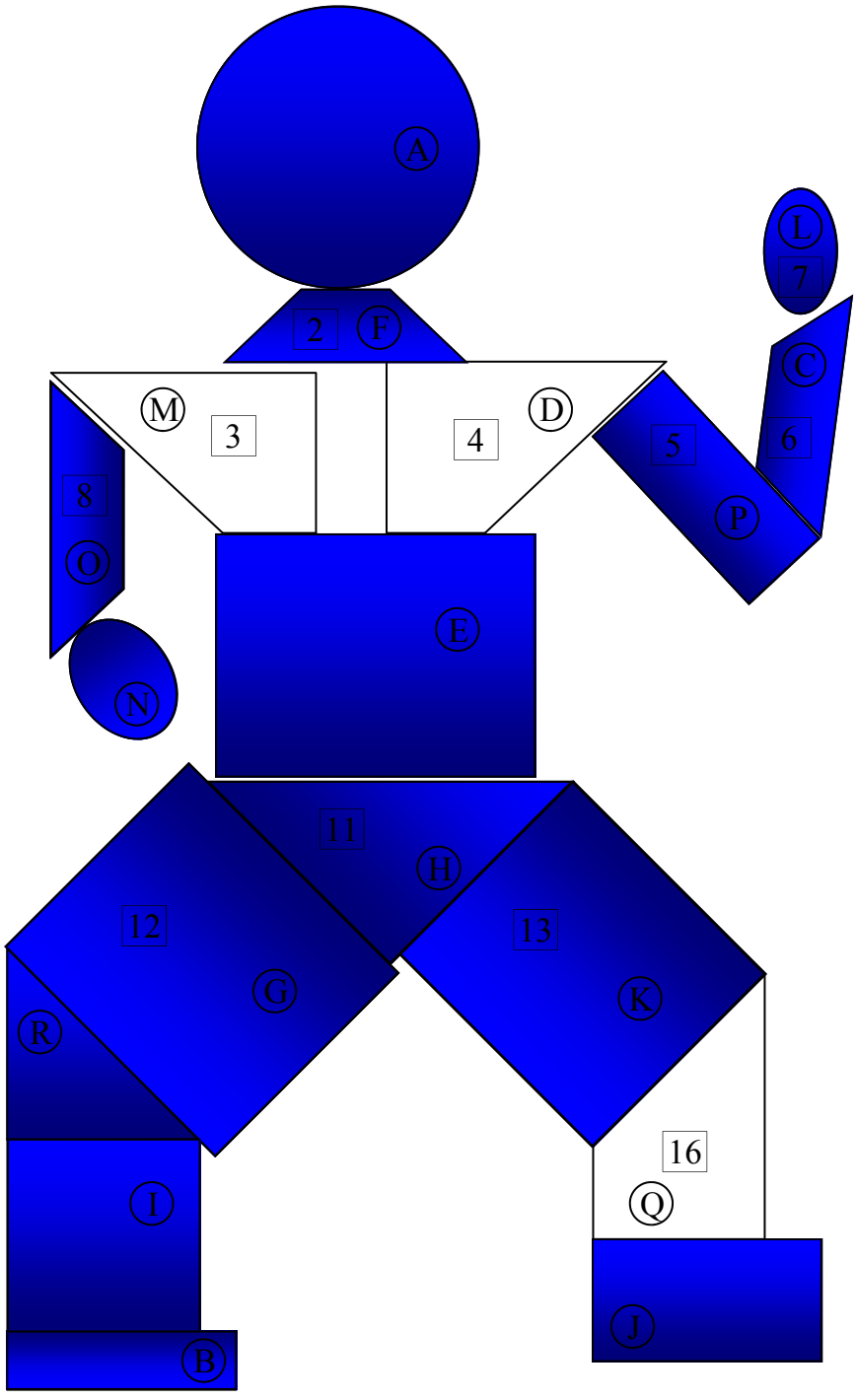
Cut the shapes out from the left hand side column and stick them onto the drawing below. Write down the letter that matches its congruent mate in the table below.

Number	Letter	Number	Letter	Number	Letter
1	A	7	L	13	K
2	F	8	O	14	R
3	M	9	N	15	I
4	D	10	E	16	Q
5	P	11	H	17	J
6	C	12	G	18	B



Matching Congruent Shapes

This sheet compares shapes and asks you if they are congruent. When shapes are congruent they are identical, same internal angles and same side lengths. Note that a congruent shape can be formed by reflection.

Column 1 starts with three questions and asks you to use a ruler and write down all the side lengths. Once you do this if all the sides match, and are in the same position, then the shapes are congruent. Fill in the yes circle if they are.

Column 2 has a number of shapes labeled, A to Z and 1 - 9. So there are 35 shapes. There are 17 pairs of matching shapes and one ‘non-matching shape’. Some shapes are easy as there is only one possible match, but others aren’t so straight forward. With slight variations and rotation as well it is difficult to tell. You have three options:

- Reach for your ruler.
- Use a piece of tracing paper to trace the more difficult shapes, then move the trace over the possible matching shapes.
- Or you teacher may give you a transparency which has all the shapes on it. Use the transparency to check for congruence.

Once you have found your pairs write them in the boxes below the shapes. After you have found all the matches there will be one shape left, write that at the base of the page. Your last step is to colour the shapes that match the same colour.

Matching Congruent Shapes

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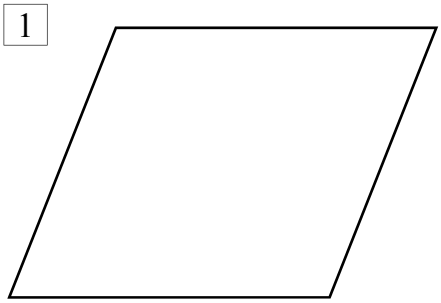
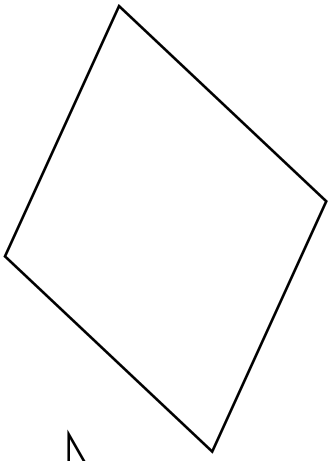
Using a ruler, measure and then write the side lengths on the shapes. Are they congruent?

1

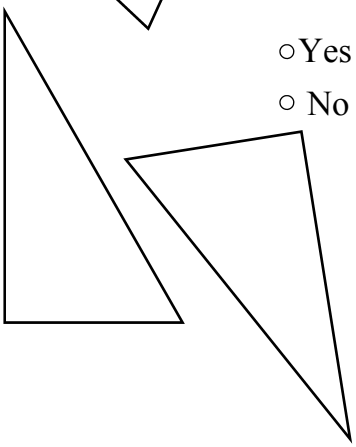


○ Yes
○ No

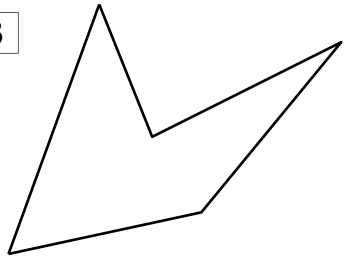


2

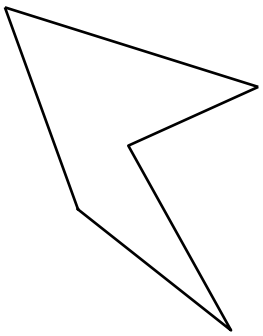
○ Yes
○ No



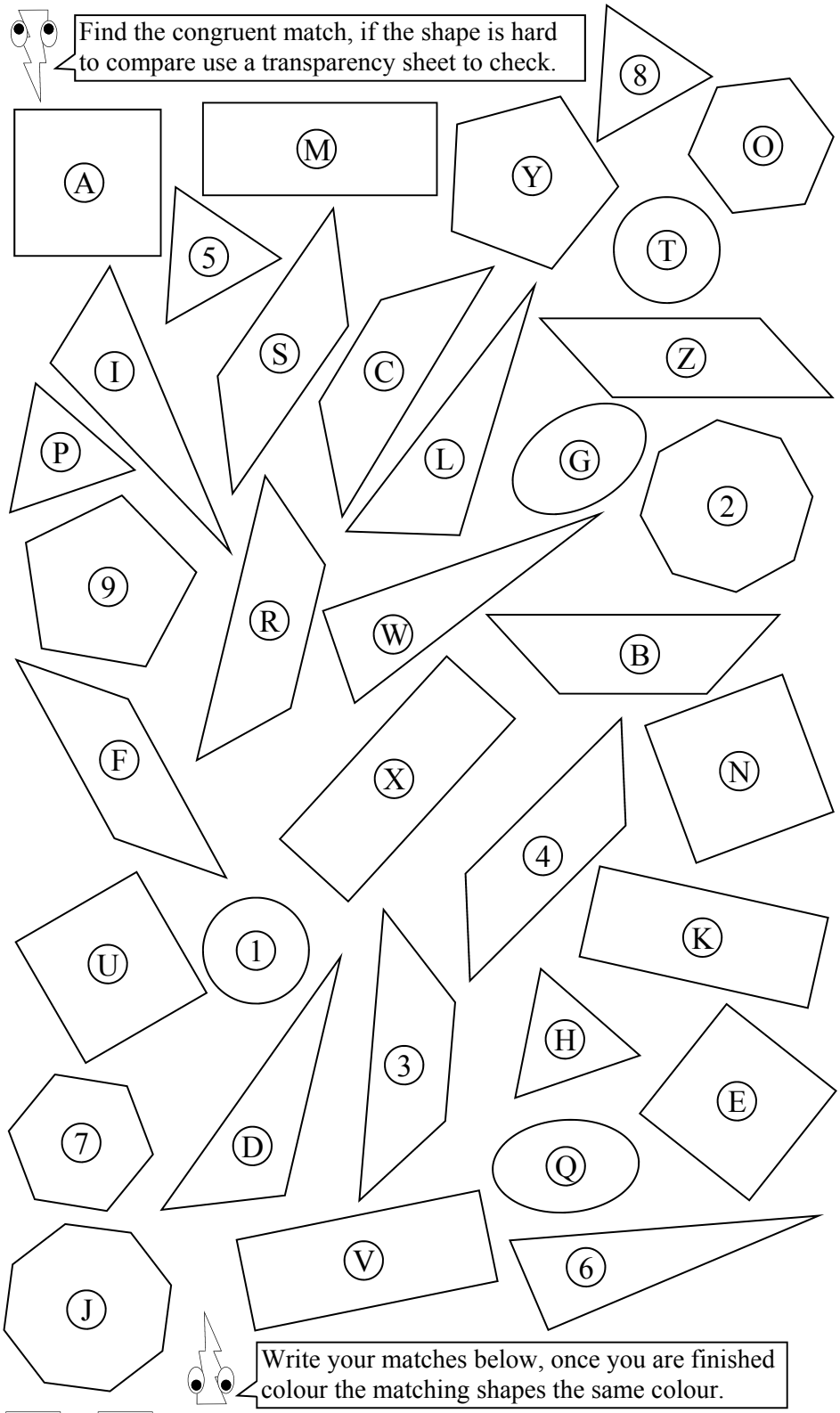
3



○ Yes
○ No



Find the congruent match, if the shape is hard to compare use a transparency sheet to check.



Write your matches below, once you are finished colour the matching shapes the same colour.

☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐	☐ & ☐	☐ & ☐	☐ & ☐
☐ & ☐			

There is one unmatched shape, it is: ☐

Matching Congruent Shapes

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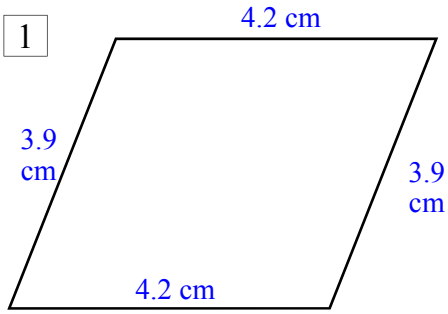
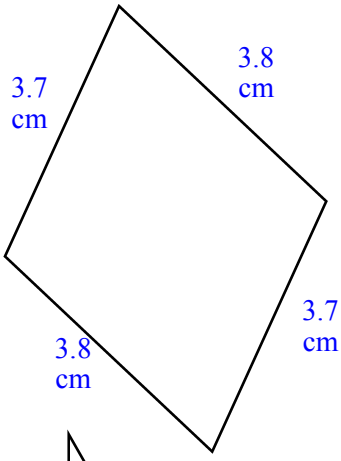
Using a ruler, measure and then write the side lengths on the shapes. Are they congruent?

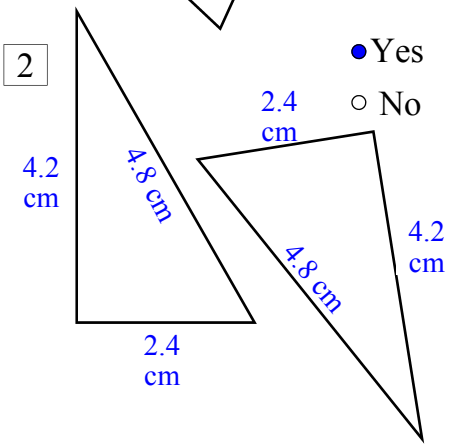
1





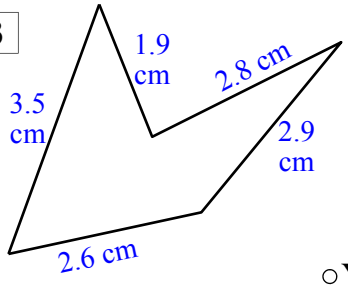
○ Yes
● No

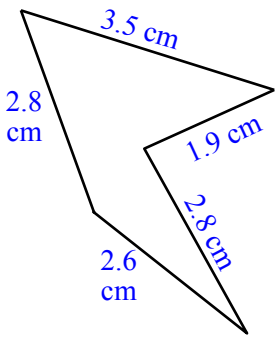
2



● Yes
○ No

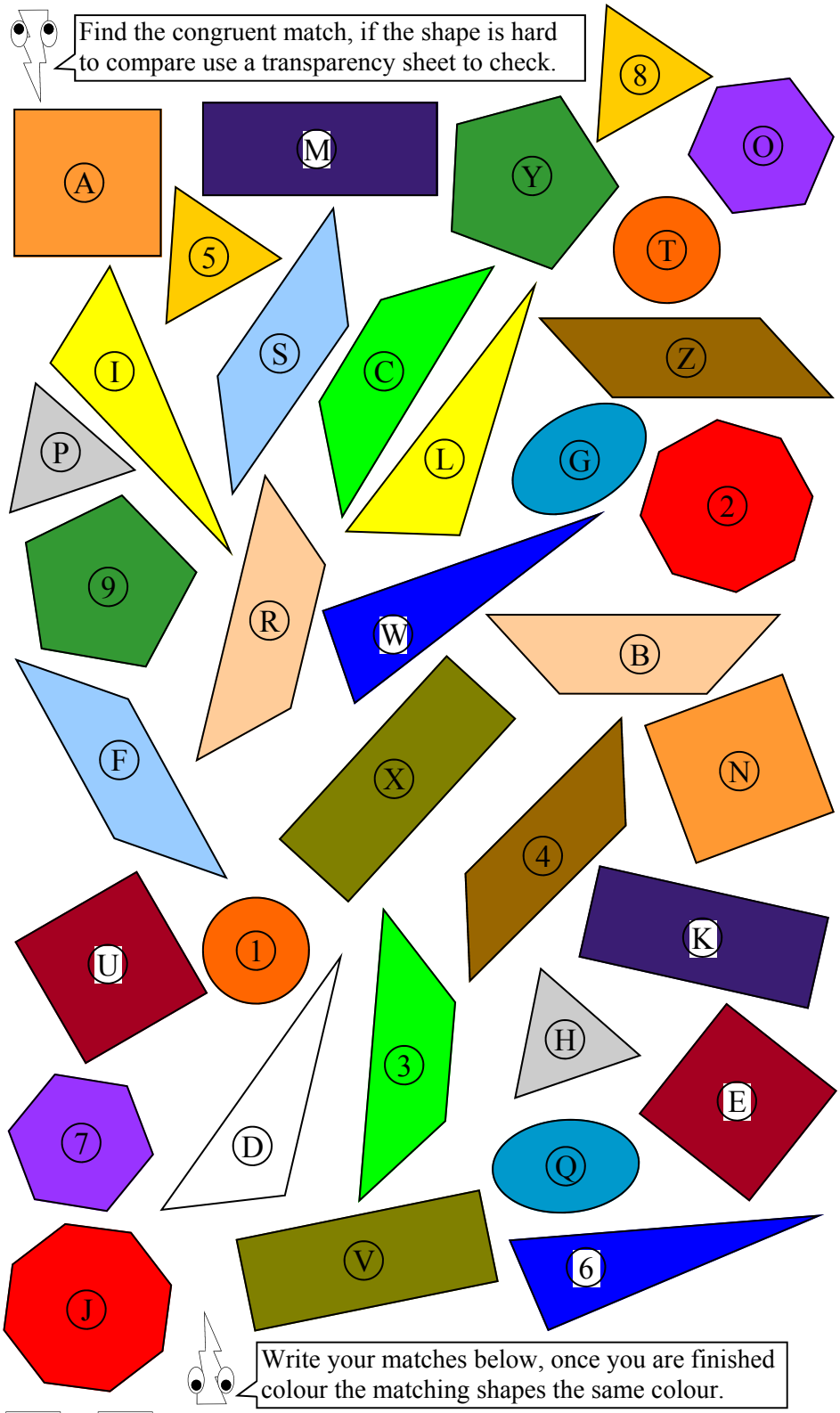
3





○ Yes
● No

Find the congruent match, if the shape is hard to compare use a transparency sheet to check.



Write your matches below, once you are finished colour the matching shapes the same colour.

T & 1	H & P	E & U	M & K
G & Q	A & N	C & 3	W & 6
J & 2	R & B	X & V	5 & 8
W & Y	F & S	I & L	Z & 4
7 & O			

There is one unmatched shape, it is: D

Matching Congruent Shapes - Transparency Printout Sheet

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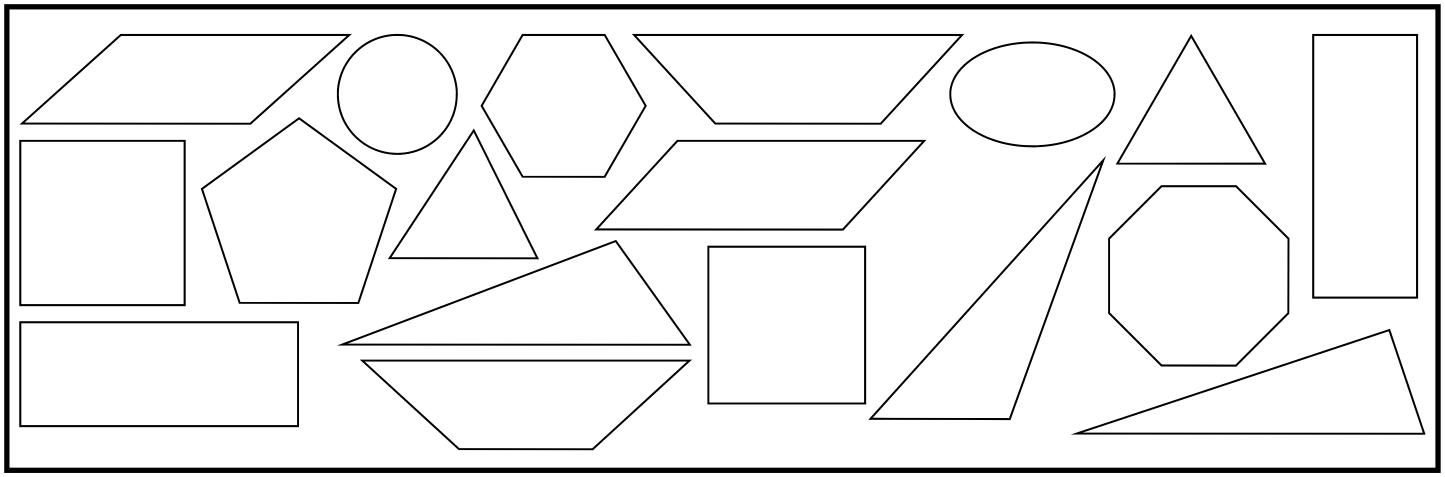

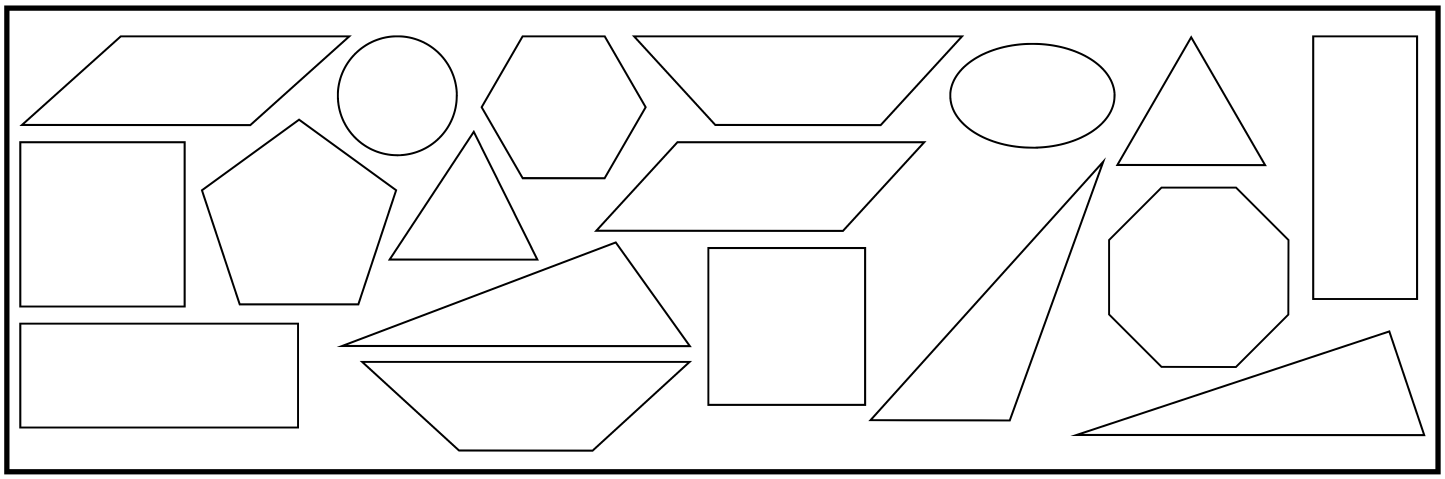

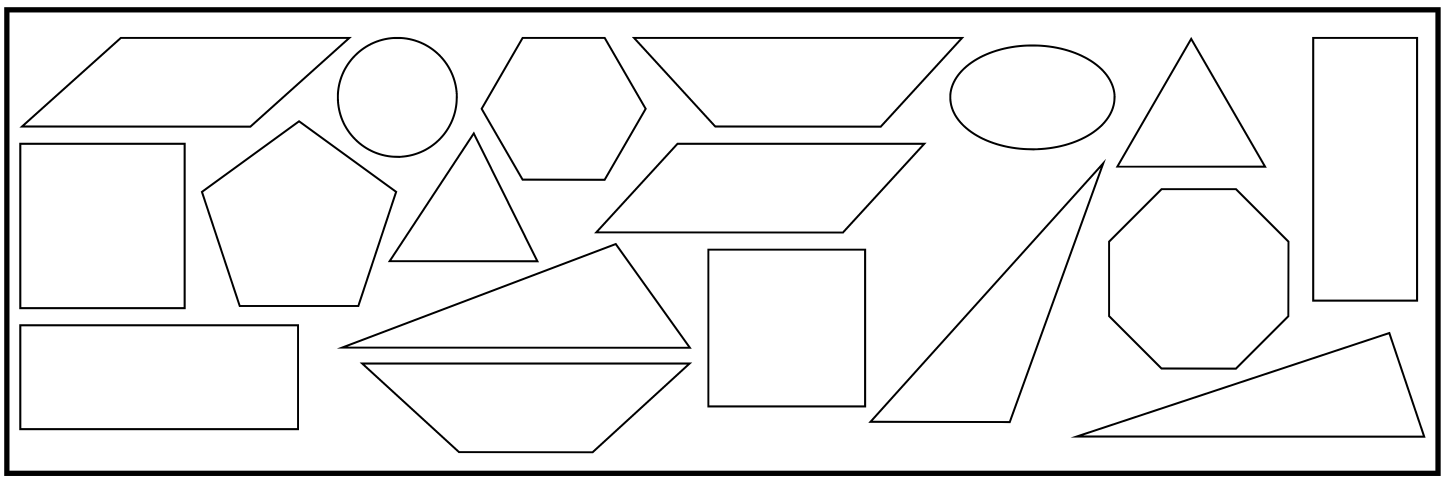

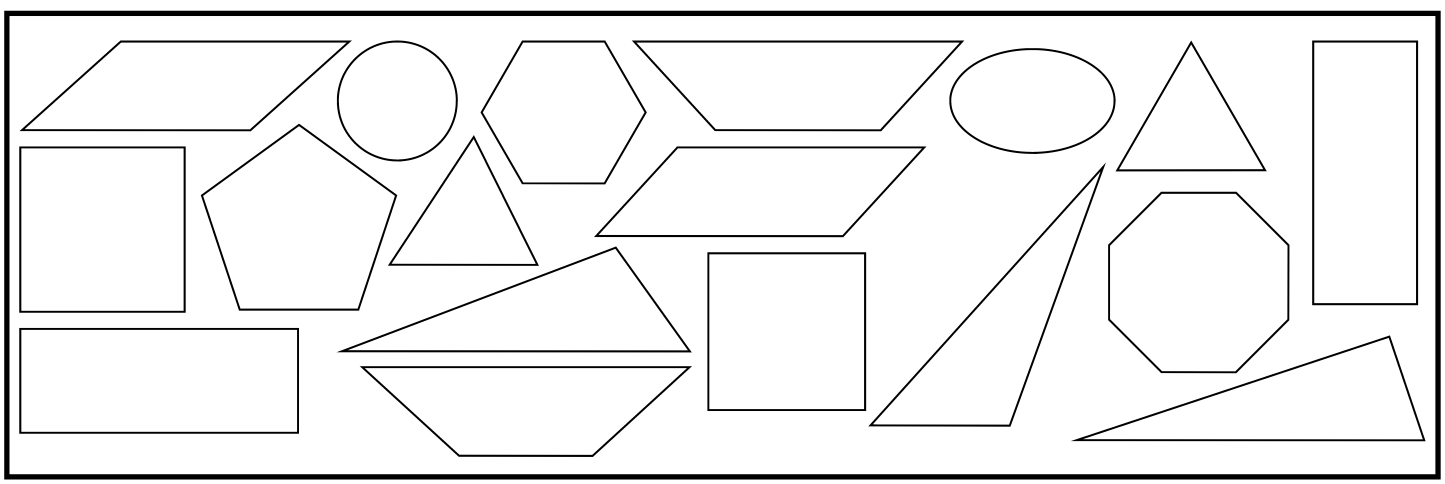

Congruence Statements

If two shapes are identical then they are congruent. This worksheet doesn't require you to prove that the shapes are congruent … they all are. So you may ask why do you need to say they are congruent when they are? Because you are showing that you can identify the properties of congruent shapes. If a shape is congruent then this angle must equal that angle, or this side must equal that side.

Column 1 starts with a series of matching shapes. You just need to name them. This seems straight forward but there is one problem. When you make a congruence statement that one shape is congruent to another you have to name the vertices in the same order and start at the same matching vertex (or corner). Note that the symbol for congruence is $\equiv$, it can also be $\cong$. The column has shapes to assist you in naming the shapes the correct way.

Column 2 asks you to show which angles are equal and which sides are equal and make a congruence statement.

Column 3 continues with similar questions then asks you to find values for pronumerals. This is again matching one piece of data on one triangle with its position on the other, you may also need to perform a calculation.

Congruence Statements

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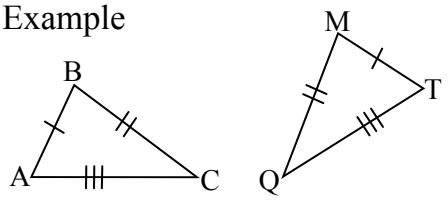
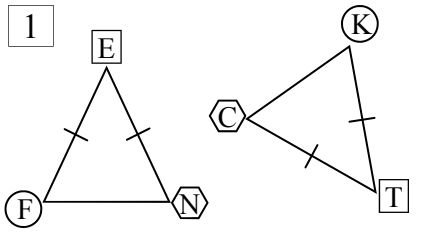
A congruence statement has the matching vertices named in the same order. Write a congruence statement for these.

Example



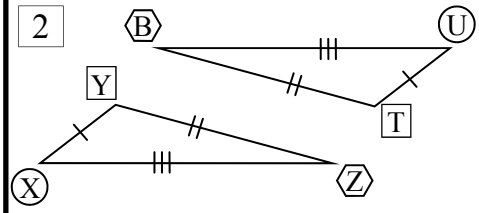
$\triangle ABC \equiv \triangle TMQ$

1



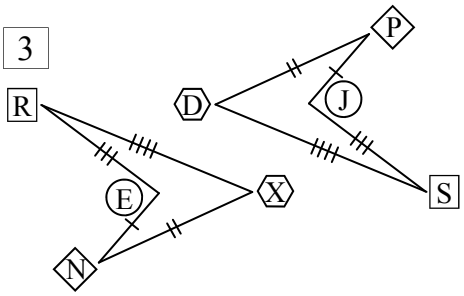
$\triangle\bigcirc\square\hexagon \equiv \triangle\bigcirc\square\hexagon$

2



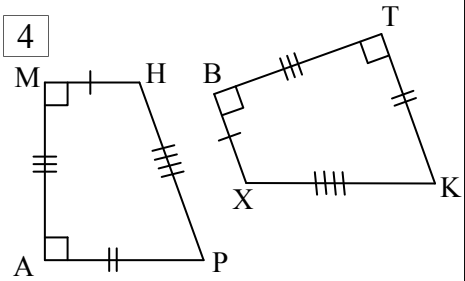
$\triangle\bigcirc\square\hexagon \equiv \triangle\bigcirc\square\hexagon$

3

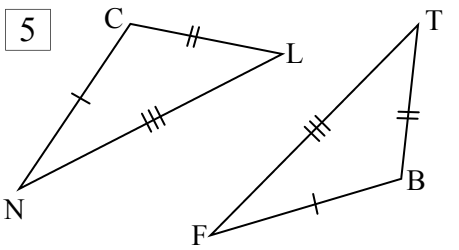


$\bigcirc\square\hexagon\diamond \equiv \bigcirc\square\hexagon\diamond$

4

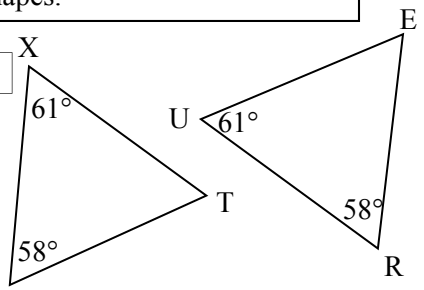


5



State which angles and sides are equal & complete congruence statements for these congruent shapes.

6



$\angle P = \angle R$ $\quad \angle X = \angle\square$

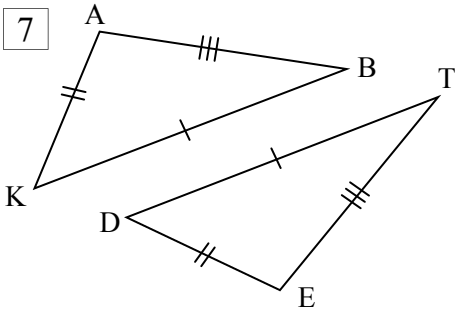
$\angle T = \angle\square$ $\quad XP = \square$

$PT = \square$ $\quad XT = \square$

$\triangle\square \equiv \triangle\square$

Remember same starting point & order.

7



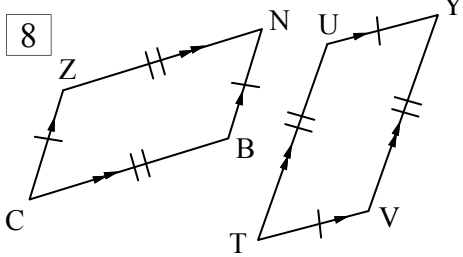
$\angle K = \angle\square$ $\quad \angle\square = \angle\square$

$\angle\square = \angle\square$ $\quad AK = \square$

$\square = \square$ $\quad \square = \square$

$\triangle\square \equiv \triangle\square$

8



$\angle\square = \angle\square = \angle\square = \angle\square$

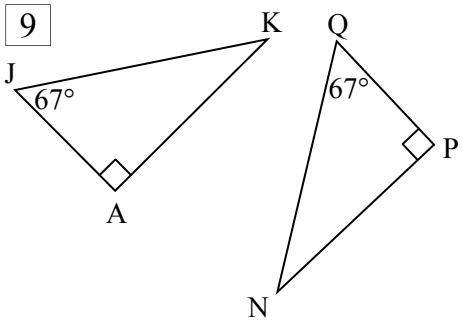
$\angle\square = \angle\square = \angle\square = \angle\square$

$\square = \square = \square = \square$

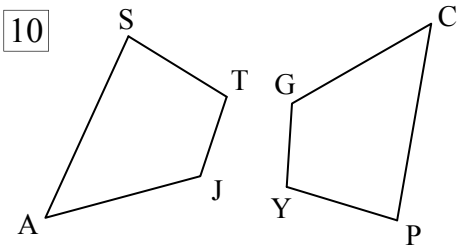
$\square = \square = \square = \square$

$\square \equiv \square$

9

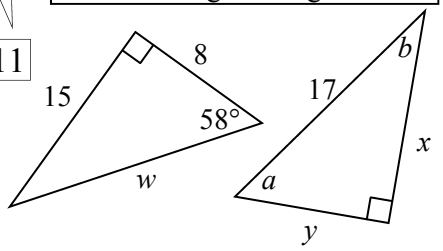


10



Find the value of all unknowns, for these congruent figures.

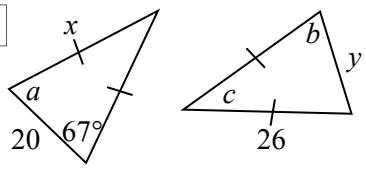
11



$a =$ $\quad b =$ $\quad w =$

$x =$ $\quad y =$

12



$a =$ $\quad b =$ $\quad c =$

$x =$ $\quad y =$

Congruence Statements

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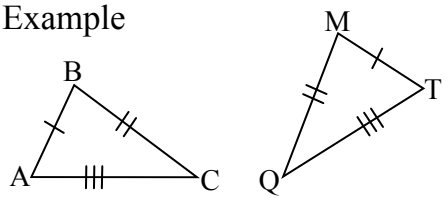
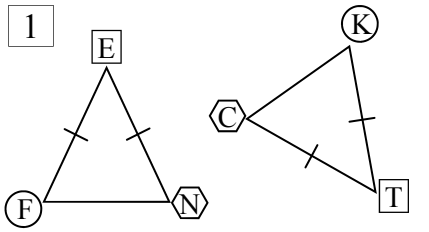
A congruence statement has the matching vertices named in the same order. Write a congruence statement for these.

Example



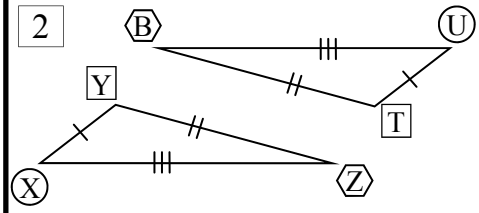
$\triangle ABC \equiv \triangle TMQ$

1



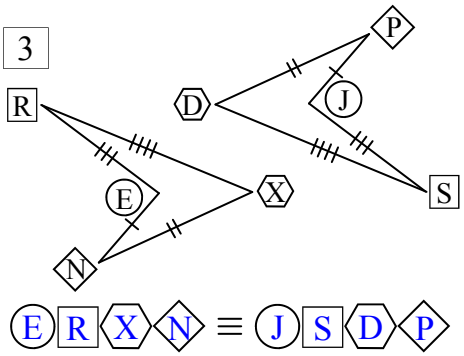
$\triangle FEN \equiv \triangle KTC$

2



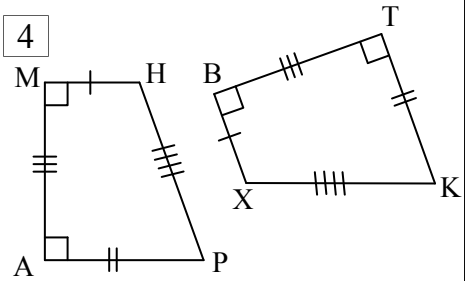
$\triangle XYZ \equiv \triangle UTB$

3



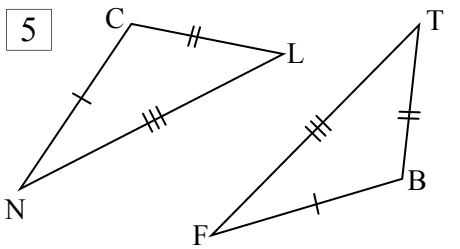
$ERXN \equiv JSDP$

4



$AMHP \equiv TBXK$

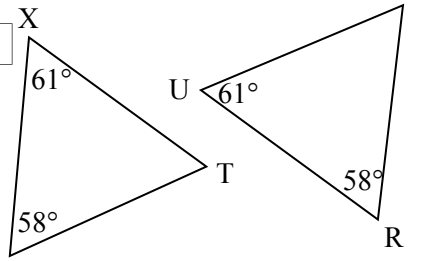
5



$\triangle CLN \equiv \triangle BTF$

State which angles and sides are equal & complete congruence statements for these congruent shapes.

6



$\angle P = \angle R$ $\quad \angle X = \angle U$

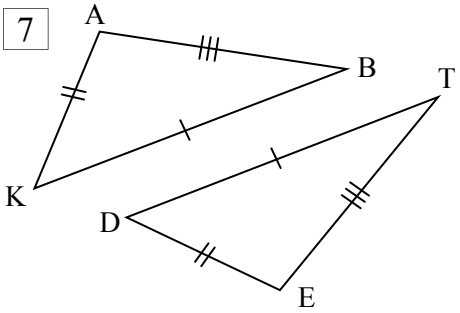
$\angle T = \angle E$ $\quad XP = UR$

$PT = RE$ $\quad XT = UE$

$\triangle PXT \equiv \triangle RUE$

Remember same starting point & order.

7



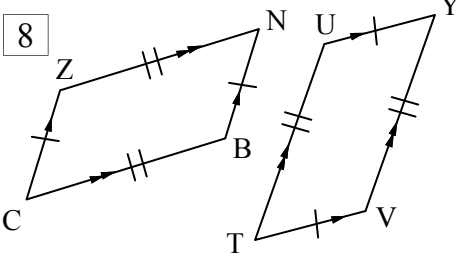
$\angle K = \angle D$ $\quad \angle A = \angle E$

$\angle B = \angle T$ $\quad AK = ED$

$AB = ET$ $\quad TD = BK$

$\triangle ABK \equiv \triangle ETD$

8



$\angle C = \angle N = \angle T = \angle Y$

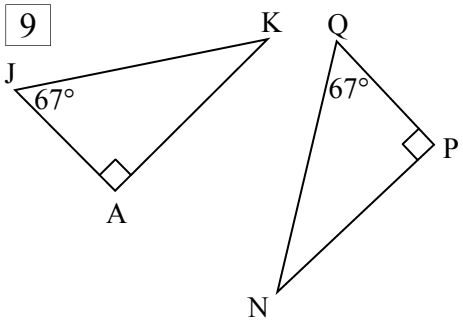
$\angle Z = \angle B = \angle U = \angle V$

$ZC = NB = YU = VT$

$ZN = CB = YV = UT$

$CZNB \equiv TUYV$

9



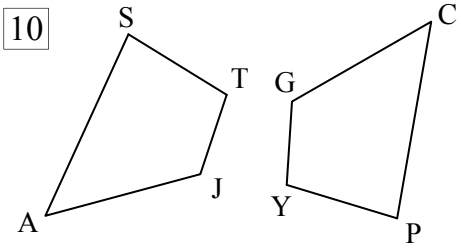
$\angle A = \angle P$ $\quad \angle N = \angle K$

$\angle J = \angle Q$ $\quad JA = QP$

$AK = QP$ $\quad QN = JK$

$\triangle AJK \equiv \triangle PQN$

10



$\angle A = \angle C$ $\quad \angle J = \angle G$

$\angle T = \angle Y$ $\quad \angle S = \angle P$

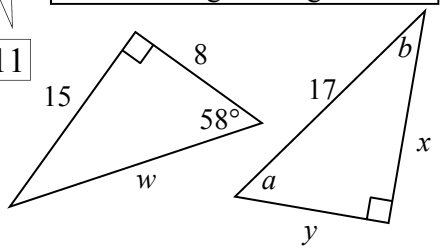
$AS = CP$ $\quad ST = PY$

$TJ = YG$ $\quad JA = GC$

$ASTJ \equiv CPYG$

Find the value of all unknowns, for these congruent figures.

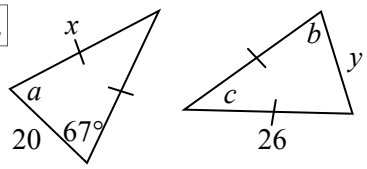
11



$a = 58°$ $\quad b = 32°$ $\quad w = 17$

$x = 15$ $\quad y = 8$

12



$a = 67°$ $\quad b = 67°$ $\quad c = 46°$

$x = 26$ $\quad y = 20$

Congruence & Similarity - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Congruence & Similarity - Find A Word

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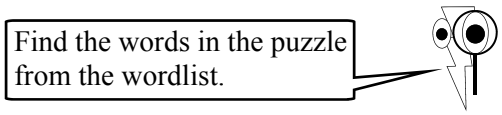

A	X	S	S	Y	L	P	I	T	L	U	M	H	G	N
X	X	N	O	I	T	A	L	S	N	A	R	T	H	O
S	I	M	I	L	A	R	I	T	Y	T	E	G	N	I
R	S	S	Y	R	T	E	M	M	Y	S	D	N	N	T
O	X	A	A	E	Q	U	Q	E	S	E	I	E	Q	C
T	E	Q	I	U	N	U	Q	E	Q	O	V	L	U	E
A	C	E	D	V	U	L	L	U	L	Q	I	Q	S	L
T	N	R	E	E	F	L	A	A	U	A	D	T	T	F
I	E	A	N	R	A	L	V	R	I	V	C	V	A	E
O	U	P	T	T	C	Z	A	N	G	L	E	S	T	R
N	R	M	I	I	T	D	I	V	I	E	D	D	E	S
A	G	O	C	C	O	V	H	C	T	A	M	L	M	I
U	N	C	A	E	R	X	S	X	X	I	U	E	E	X
Q	O	C	L	S	M	E	A	S	U	R	E	E	N	A
E	C	N	O	I	T	A	T	N	E	I	R	O	T	T

WORDLIST

TESSELLATION	FACTOR	IDENTICAL	LENGTH	MULTIPLY
ENLARGEMENT	SYMMETRY	DIVIDE	REFLECTION	VERTICES
TRANSLATION	RULER	COMPARE	SCALE	ROTATION
ORIENTATION	ANGLES	SIMILARITY	RATIO	MATCH
CONGRUENCE	STATEMENT	EQUAL	MEASURE	AXIS

Congruence & Similarity - Find A Word

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Find the words in the puzzle from the wordlist.



A	X	S	S	Y	L	P	I	T	L	U	M	H	G	N
X	X	N	O	I	T	A	L	S	N	A	R	T	H	O
S	I	M	I	L	A	R	I	T	Y	T	E	G	N	I
R	S	S	Y	R	T	E	M	M	Y	S	D	N	N	T
O	X	A	A	E	Q	U	Q	E	S	E	I	E	Q	C
T	E	Q	I	U	N	U	Q	E	Q	O	V	L	U	E
A	C	E	D	V	U	L	L	U	L	Q	I	Q	S	L
T	N	R	E	E	F	L	A	A	U	A	D	T	T	F
I	E	A	N	R	A	L	V	R	I	V	C	V	A	E
O	U	P	T	T	C	Z	A	N	G	L	E	S	T	R
N	R	M	I	I	T	D	I	V	I	E	D	D	E	S
A	G	O	C	C	O	V	H	C	T	A	M	L	M	I
U	N	C	A	E	R	X	S	X	X	I	U	E	E	X
Q	O	C	L	S	M	E	A	S	U	R	E	E	N	A
E	C	N	O	I	T	A	T	N	E	I	R	O	T	T

WORDLIST

~~TESSELLATION~~ ~~FACTOR~~ ~~IDENTICAL~~ ~~LENGTH~~ ~~MULTIPLY~~

~~ENLARGEMENT~~ ~~SYMMETRY~~ ~~DIVIDE~~ ~~REFLECTION~~ ~~VERTICES~~

~~TRANSLATION~~ ~~RULER~~ ~~COMPARE~~ ~~SCALE~~ ~~ROTATION~~

~~ORIENTATION~~ ~~ANGLES~~ ~~SIMILARITY~~ ~~RATIO~~ ~~MATCH~~

~~CONGRUENCE~~ ~~STATEMENT~~ ~~EQUAL~~ ~~MEASURE~~ ~~AXIS~~

FREEFALL MATHEMATICS

ALTITUDE

LINEAR RELATIONSHIPS

FREEFALL MATHEMATICS

ALTITUDE

REVIEW:

COORD GEOMETRY

Grid Reference - The Hidden Friend

A grid reference system is used to pinpoint locations on a map or diagram. This sheet uses grid referencing to plot diagrams of two hidden friends. Grid referencing gives a letter then a number, the letter being along the horizontal axis (the top and bottom) and the number being on the vertical axis, which are the two sides.

Some maps may use a zero, these maps start at 1. To plot the points move across to the letter required then up the column to the number, then colour the square. Then plot the next grid position, after the points are plotted your friends should be revealed.

The co-ordinates are all in the same colour except where you are told to change colour, but if you want to make a multi-coloured one go ahead! You should strike out each grid reference as you plot them incase you need to remember where you are up to.

Grid Reference - The Hidden Friend

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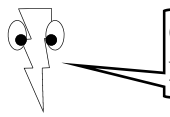
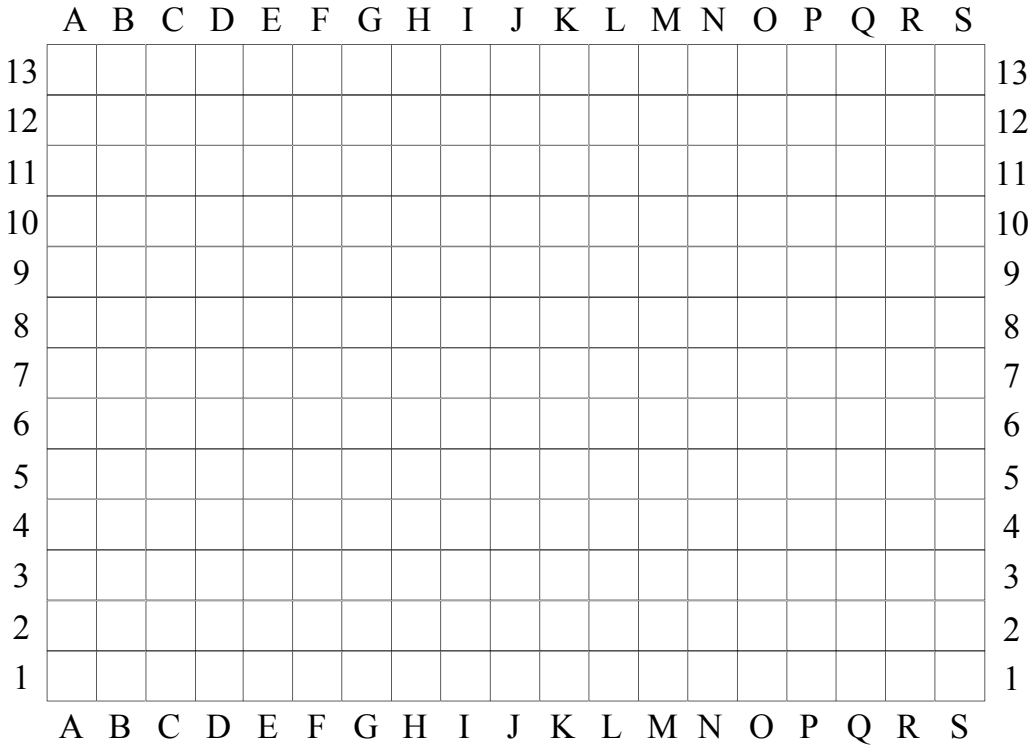


Colour the following squares and you will find a smiling friend



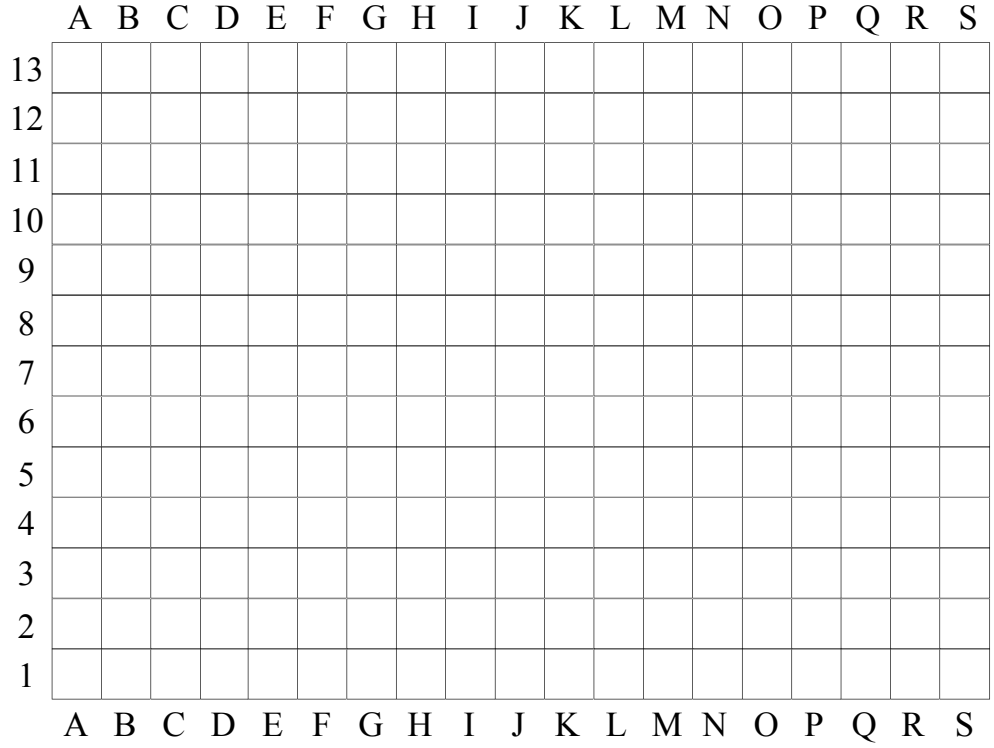
Co-ordinates:

D3 I2 C7
N5 B6 F2
D8 C4 L6
G2 E9 E3
J8 K11 L4
F12 M4 Q7
F4 F9 H12
I13 H2 J3
O8 P9 B5
M6 H9 D12
E13 P6 Q8
J12 K3 S8
G11 O5 K7
I9 R9 E6

Different colour



Colour the following squares in blue (or black and red when indicated) and you will find an enthusiastic friend



Co-ordinates:

G11 A4 D1 F7 K12 L3
P6 N8 Q8 A9 I12 S8
C2 O2 E11 S9 H8 L12
M6 M11 N7 R10 O5
P12 R3 Q2 B5 E2
F8 N10 A7 M4 O8
Q1 J13 O11 M5 G4
C7 D8 C8 G5 E5
H12 B10 E3 F10 A8
G6 D12 E4 S4 P1
P8 S6 H3 Q11 S7
C1 B3 O3 C11 Q7
E8 A6 L8 D6 O4
R5

Change Colour:

K2 J1 J2 I2

Change Colour:

L9 J5 I5 J4 H9 K5

Grid Reference - The Hidden Friend

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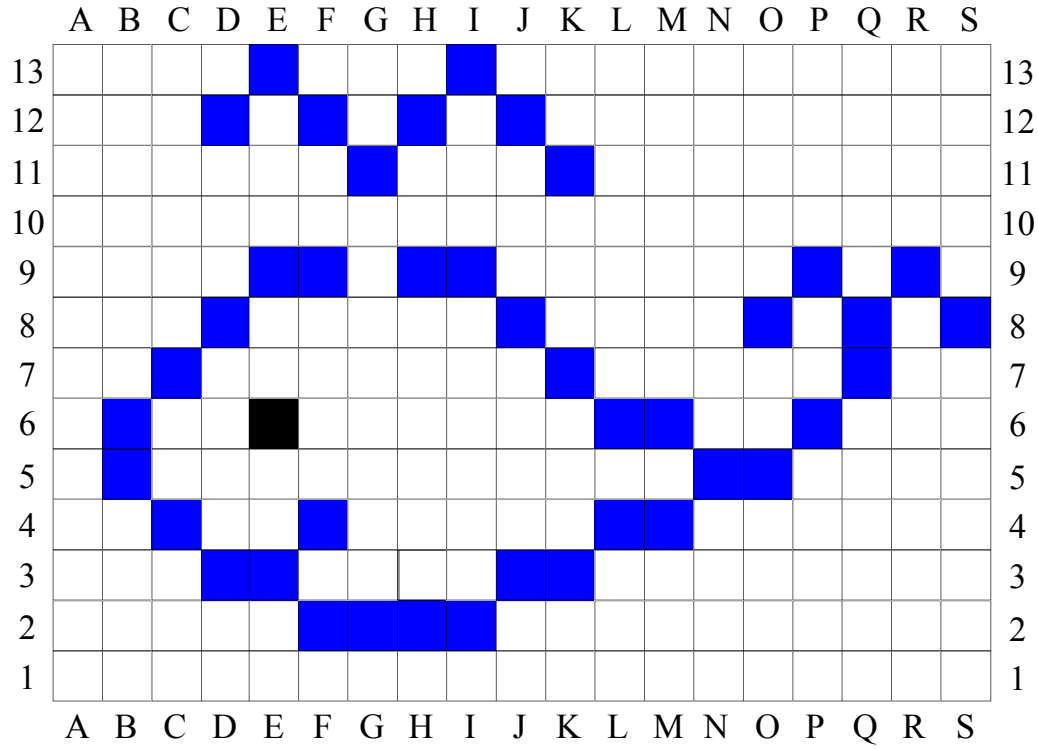
Colour the following squares and you will find a smiling friend

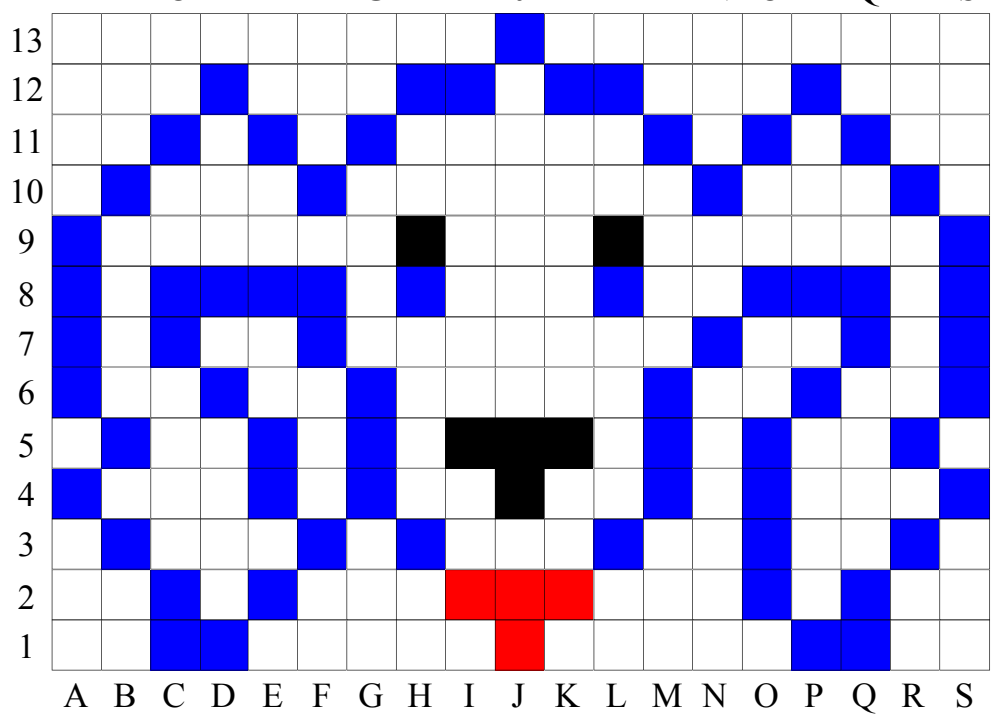


Co-ordinates:

D3 I2 C7
N5 B6 F2
D8 C4 L6
G2 E9 E3
J8 K11 L4
F12 M4 Q7
F4 F9 H12
I13 H2 J3
O8 P9 B5
M6 H9 D12
E13 P6 Q8
J12 K3 S8
G11 O5 K7
I9 R9 E6

Different colour

Colour the following squares in blue (or black and red when indicated) and you will find an enthusiastic friend



Co-ordinates:

G11 A4 D1 F7 K12 L3
P6 N8 Q8 A9 I12 S8
C2 O2 E11 S9 H8 L12
M6 M11 N7 R10 O5
P12 R3 Q2 B5 E2
F8 N10 A7 M4 O8
Q1 J13 O11 M5 G4
C7 D8 C8 G5 E5
H12 B10 E3 F10 A8
G6 D12 E4 S4 P1
P8 S6 H3 Q11 S7
C1 B3 O3 C11 Q7
E8 A6 L8 D6 O4
R5

Change Colour:

K2 J1 J2 I2

Change Colour:

L9 J5 I5 J4 H9 K5

Using a Street Map

A street map uses grid co-ordinates to specify the location of streets and points of interest. The reference system generally uses letters across its horizontal axis (top and bottom) and numbers on its vertical axis (sides). To give a co-ordinate you move across first, then up, so the letter first then the number.

For example D7 would be the green area. Note that some areas could be given more than one co-ordinate, due to its size it could cross into another reference area. If you are attempting this sheet on paper use a ruler or just the eye.

Column 1 asks for the co-ordinates for the black symbols on the map. Look at the map key (sometimes called a 'legend') so that you know what you are looking for. Then using the grid read off the locations.

Column 2 asks you to place buildings and points of interest on the map. So show that artistic talent!

Column 3 asks you to place post boxes, traffic lights and phone boxes in given locations.

You can print either a map with a grid over it, or a map with no grid.

Using a Street Map - No Grid

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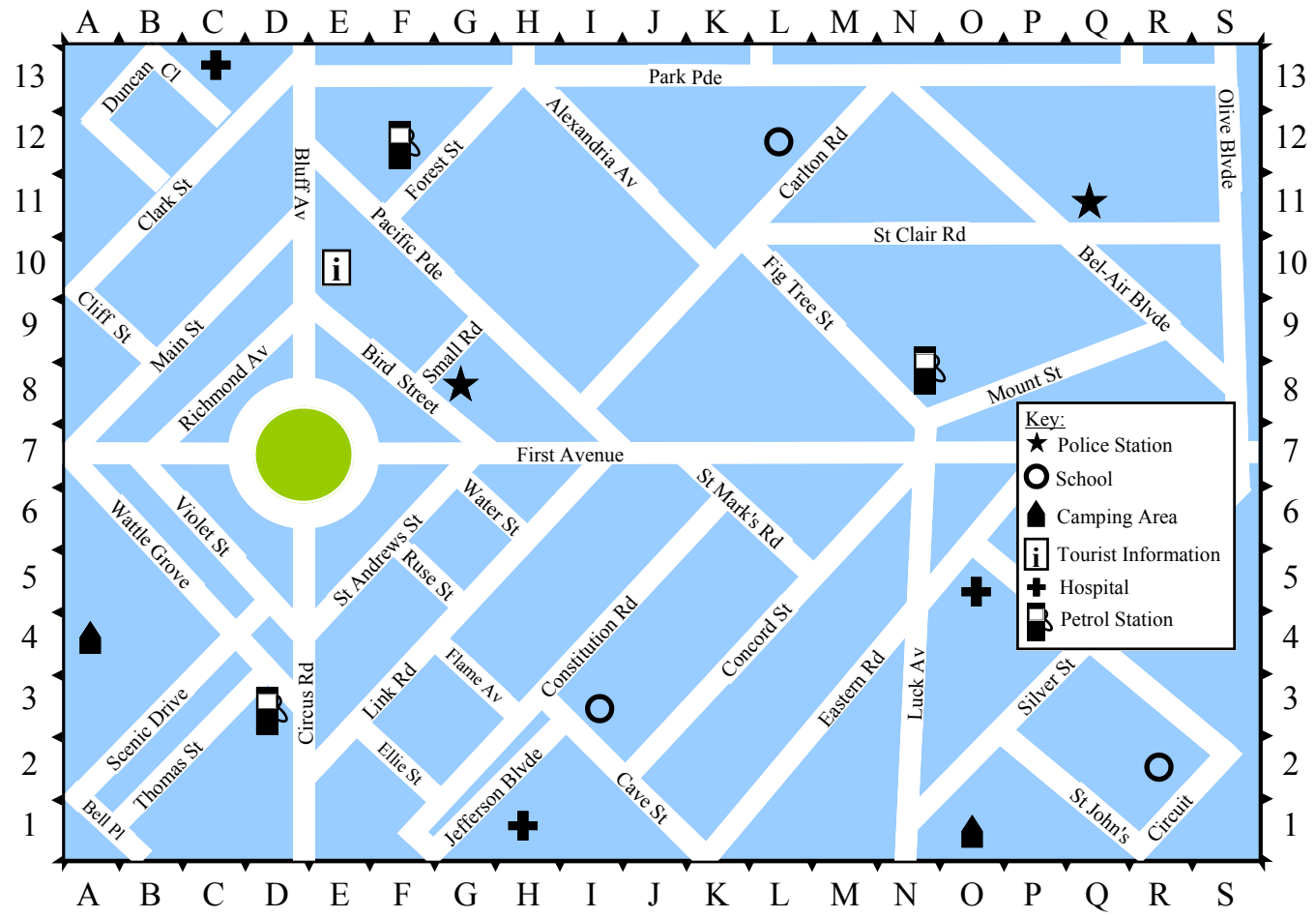

Give the grid references for all of the following

1 Petrol Stations ___ ___ ___

2 Schools ___ ___ ___

3 Camping Grounds ___ ___

4 Tourist Information ___

5 Hospitals ___ ___ ___

6 Intersection of First Av and Bird St ___

7 Intersection of Carlton Rd and Alexandria Av ___

8 Intersection of Scenic Drive & Wattle Grove ___

9 Bell Place ___

Draw these buildings or make symbols for them in the co-ordinates below.

10	Veterinarian	K9
11	St Mary's Cathedral	K5
12	Palace Hotel	N11
13	Imam Ali Mosque	I1
14	Sunshine Circus	C1
15	Council Car Parks	M1 A5 Q9
16	Fuzzy's Café	P12
17	Community Centre	O3
18	Council Parks	H11 D7
19	Aquatic Centre	P10

Put traffic lights, post boxes and phone booths in these locations

20 Traffic Lights Co-ordinates

P3 F8 H3

N7 K10 F2

21 Post Boxes Co-ordinates

I9 R10 S3

K2 B3 M4

22 Phone Booths Co-ordinates

R1 J12 A12

S9 L7 I2

E4 H8 O6

Using a Street Map - Grid

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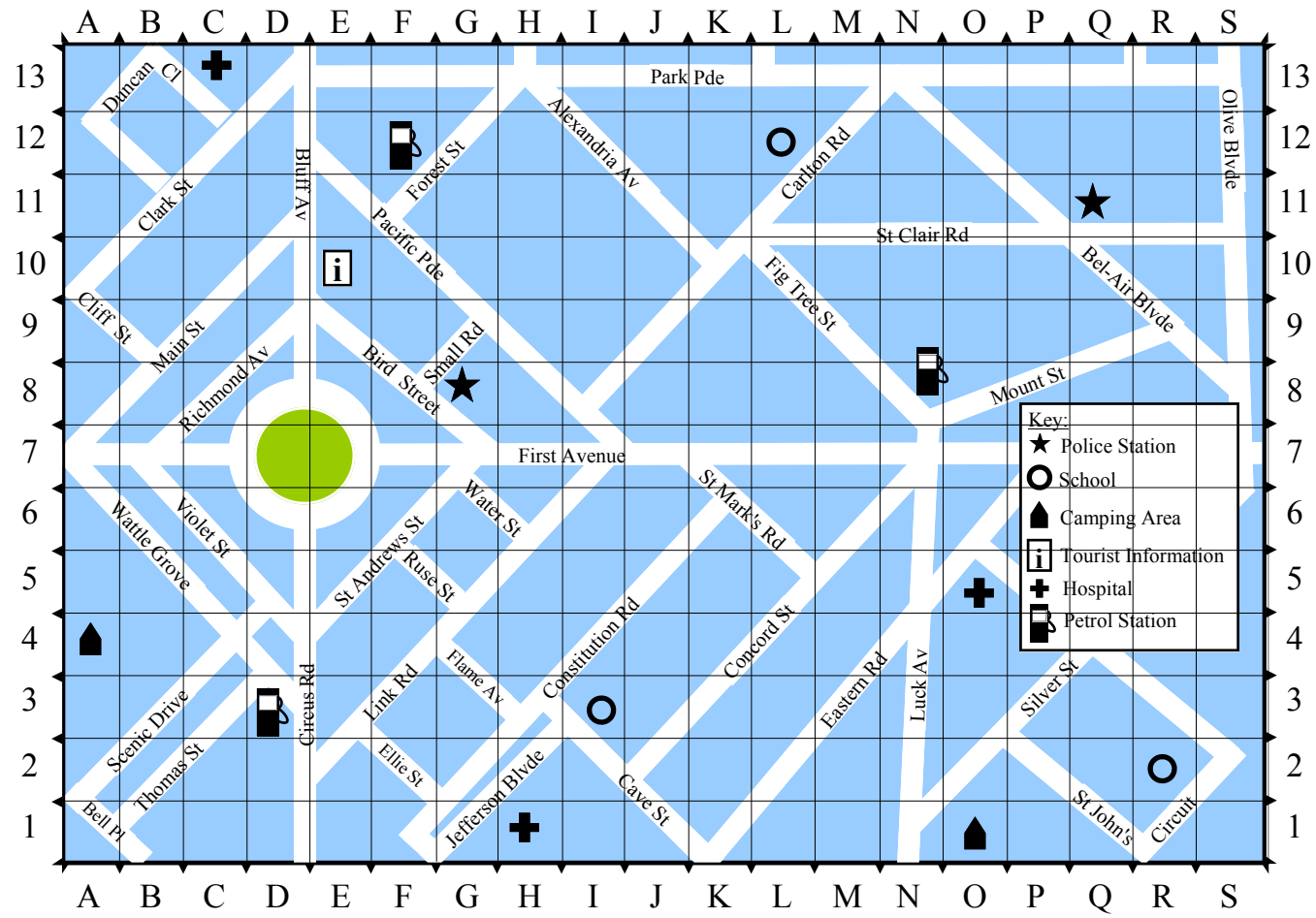

Give the grid references for all of the following

1 Petrol Stations ___ ___ ___

2 Schools ___ ___ ___

3 Camping Grounds ___ ___

4 Tourist Information ___

5 Hospitals ___ ___ ___

6 Intersection of First Av and Bird St ___

7 Intersection of Carlton Rd and Alexandria Av ___

8 Intersection of Scenic Drive & Wattle Grove ___

9 Bell Place ___

Draw these buildings or make symbols for them in the co-ordinates below.

10 Veterinarian K9

11 St Mary's Cathedral K5

12 Palace Hotel N11

13 Imam Ali Mosque I1

14 Sunshine Circus C1

15 Council Car Parks M1 A5 Q9

16 Fuzzy's Café P12

17 Community Centre O3

18 Council Parks H11 D7

19 Aquatic Centre P10

Put traffic lights, post boxes and phone booths in these locations

20 Traffic Lights Co-ordinates

P3 F8 H3

N7 K10 F2

21 Post Boxes Co-ordinates

I9 R10 S3

K2 B3 M4

22 Phone Booths Co-ordinates

R1 J12 A12

S9 L7 I2

E4 H8 O6

Using a Street Map

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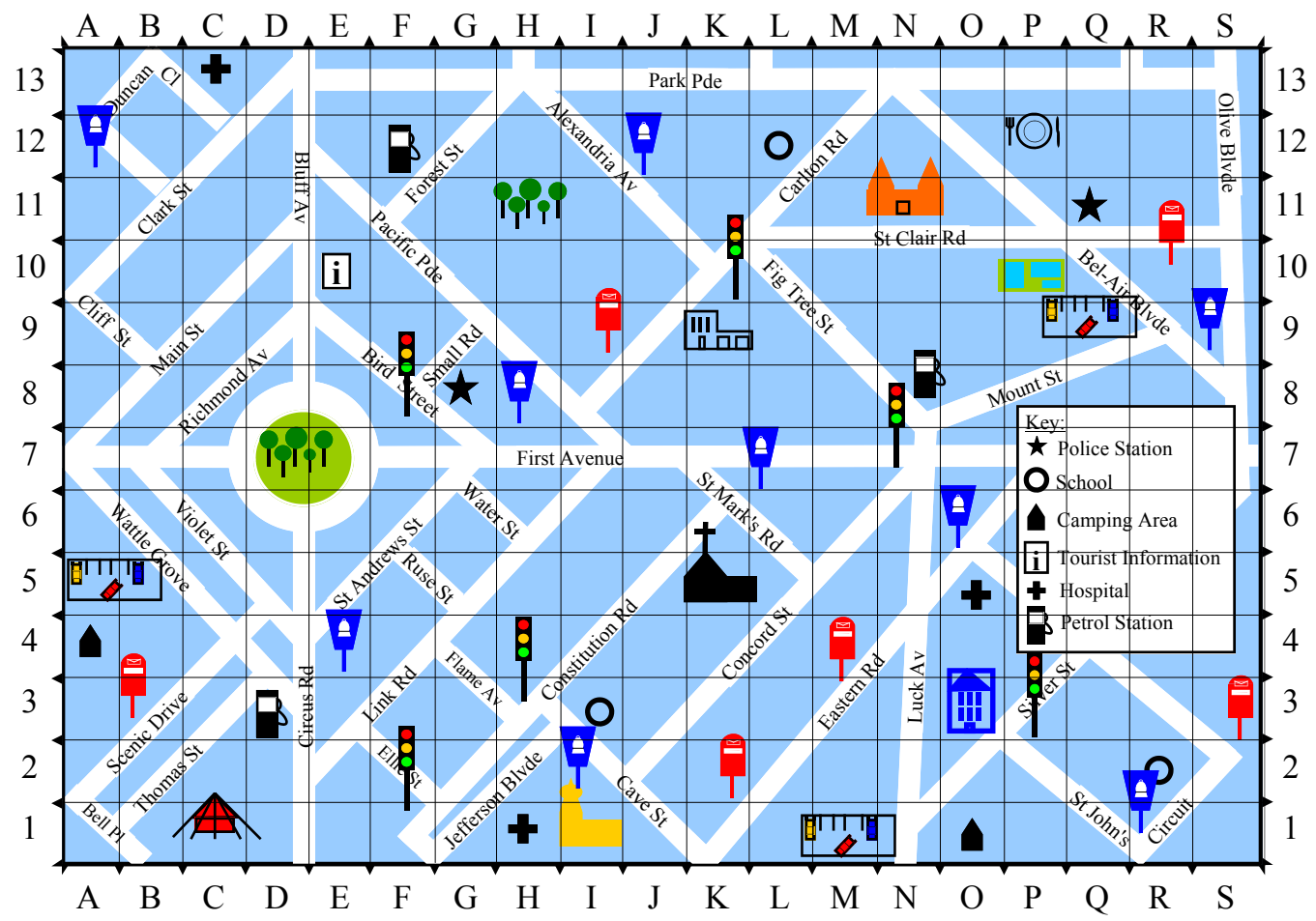

Give the grid references for all of the following

1 Petrol Stations D3 F12 N8

2 Schools I3 L12 R2

3 Camping Grounds A4 O1

4 Tourist Information E10

5 Hospitals C13 H1 O5

6 Intersection of First Av and Bird St G7

7 Intersection of Carlton Rd and Alexandria Av K10

8 Intersection of Scenic Drive & Wattle Grove C4

9 Bell Place A1

Draw these buildings or make symbols for them in the co-ordinates below.

10 Veterinarian K9

11 St Mary's Cathedral K5

12 Palace Hotel N11

13 Imam Ali Mosque I1

14 Sunshine Circus C1

15 Council Car Parks M1 A5 Q9

16 Fuzzy's Café P12

17 Community Centre O3

18 Council Parks H11 D7

19 Aquatic Centre P10

Put traffic lights, post boxes and phone booths in these locations

20 Traffic Lights Co-ordinates

P3 F8 H3

N7 K10 F2

21 Post Boxes Co-ordinates

I9 R10 S3

K2 B3 M4

22 Phone Booths Co-ordinates

R1 J12 A12

S9 L7 I2

E4 H8 O6

Latitude and Longitude

Similar to the number plane, when you give locations on the globe you do so by using a pair of co-ordinates. Latitude lines are the horizontal or flat 'lines' (remember lat is flat). Longitude lines are the vertical lines (remember they aren't flat!).

Note that there are no negative numbers, instead direction is used, the equator is when the latitude is 0°. Then moving up (northerly), latitudes increase and are denoted by an N for north. Moving down (southwards) they are denoted by an S. Longitudes are measured from the 0° 'prime meridian' which matches GMT, Greenwich Meridian Time. Then when you move left (West) you denote the movement with a W, moving right (East) you use an E.

When you give these locations unlike a number plane you give the latitude first, you probably remember the compass points by saying 'North South East West' well this applies to the co-ordinates also, give the North or South co-ordinate first then the East or West co-ordinate. You also use a degree sign, separate the co-ordinates with a comma and put them in brackets.

If you look at the point N at the top left of the globe on the worksheet you would give its position as: (50°N, 120°W).

While the sheet differs from column to column the use of co-ordinates is all that is being dealt with on this sheet. The first column asks you to give the co-ordinates for the given point, open a bracket and write in the latitude, then a comma, then the longitude and close the brackets. Note that 0° latitude or longitude doesn't require a letter, just write it as 0°.

Column 2 is the reverse, given a co-ordinate find the point. Write in the letter.

Column 3 takes this one step further. A word (which will be a city) will be formed by using the co-ordinates to find each letter in order, write the letter as you locate the points. If the city is unrecognisable then you need to go back and check your points. Note that the letters are in order they aren't jumbled.

The most common mistake made apart from giving the co-ordinates reversed is with the longitude. Most people get the North or South right but some mix up their East with their West.

Latitude and Longitude

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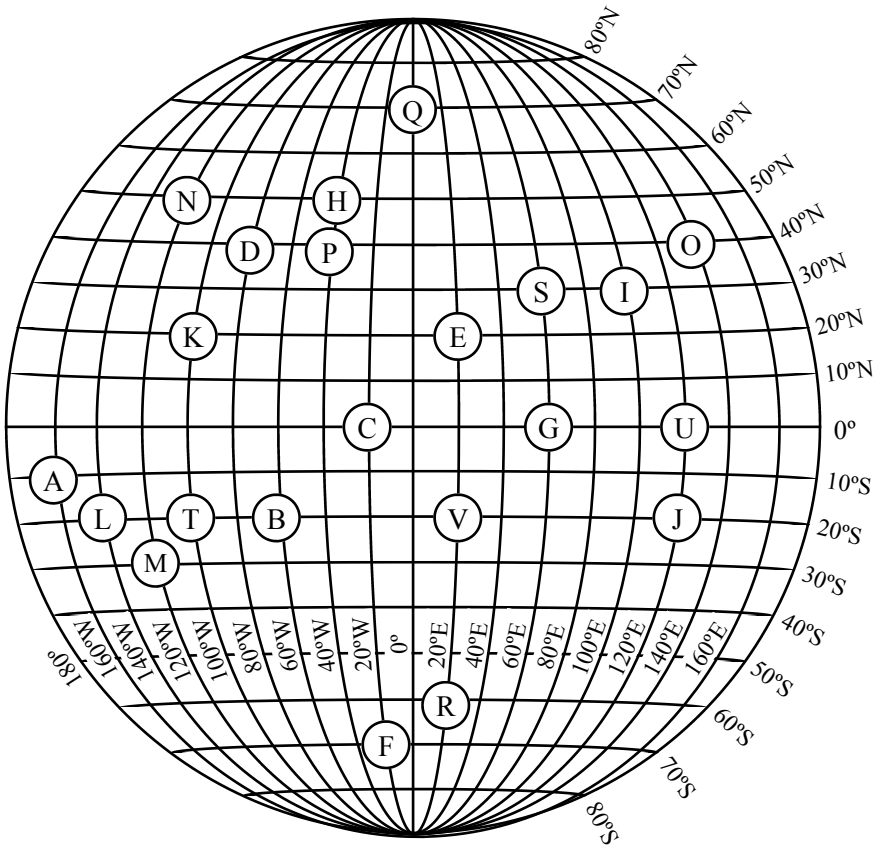

Give the co-ordinates of these points:

1	A	
2	B	
3	C	
4	D	
5	E	
6	F	
7	G	
8	H	
9	I	
10	J	
11	K	

Give the letter at the co-ordinates below:

12	(30°S, 120°W)	
13	(40°N, 40°W)	
14	(30°N, 60°E)	
15	(20°S, 100°W)	
16	(0°, 120°E)	
17	(60°S, 20°E)	
18	(50N°, 120°W)	
19	(20°S, 140°W)	
20	(70°N, 0°)	
21	(40°N, 140°E)	
22	(20°S, 20°E)	

Give the cities that these co-ordinates spell

23 Name City: _____

(40°N, 40°W)
(10°S, 160°W)
(60°S, 20°E)
(30°N, 100°E)
(30°N, 60°E)

24 Name City: _____

(40°N, 80°W)
(20N°, 20°E)
(20°S, 140°W)
(50°N, 40W°)
(30°N, 100°E)

25 Name City: _____

(50°N, 40°W)
(40°N, 140°E)
(0°, 120°E)
(30°N, 60°E)
(20°S, 100°W)
(40°N, 140°E)
(50°N, 120°W)

Latitude and Longitude

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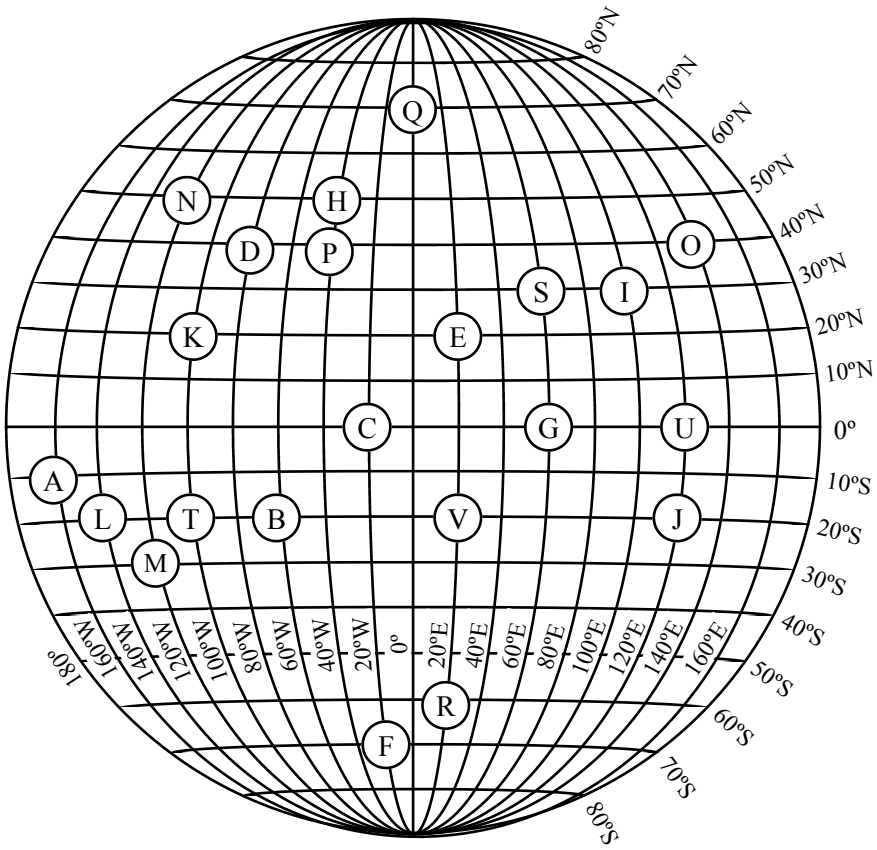


Give the co-ordinates of these points:

1	A	(10°S, 160°W)
2	B	(20°S, 60°W)
3	C	(0°, 20°W)
4	D	(40°N, 80°W)
5	E	(20°N, 20°E)
6	F	(70°S, 20°W)
7	G	(0°, 60°E)
8	H	(50°N, 40°W)
9	I	(30°N, 100°E)
10	J	(20°S, 120°E)
11	K	(20°N, 100°W)

Give the letter at the co-ordinates below:

12	(30°S, 120°W)	M
13	(40°N, 40°W)	P
14	(30°N, 60°E)	S
15	(20°S, 100°W)	T
16	(0°, 120°E)	U
17	(60°S, 20°E)	R
18	(50N°, 120°W)	N
19	(20°S, 140°W)	L
20	(70°N, 0°)	Q
21	(40°N, 140°E)	O
22	(20°S, 20°E)	V

Give the cities that these co-ordinates spell

23 Name City: PARIS

(40°N, 40°W)
(10°S, 160°W)
(60°S, 20°E)
(30°N, 100°E)
(30°N, 60°E)

24 Name City: DELHI

(40°N, 80°W)
(20N°, 20°E)
(20°S, 140°W)
(50°N, 40W°)
(30°N, 100°E)

25 Name City: HOUSTON

(50°N, 40°W)
(40°N, 140°E)
(0°, 120°E)
(30°N, 60°E)
(20°S, 100°W)
(40°N, 140°E)
(50°N, 120°W)

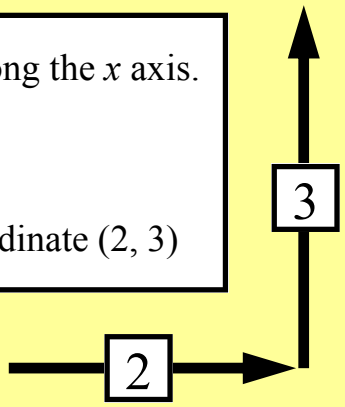
Plotting Points - Positive Quadrant

This sheet involves the plotting of Cartesian co-ordinates. When plotting points on a number plane you are given 2 numbers, referred to as an ordered pair. The first number tells you how much you move across and the second number tells you how much you move up or down. This sheet only moves across to the right and up, negative numbers (not on this sheet) move to the left and down.

Move across first along the *x* axis.

Then move up.

Eg. To plot the co-ordinate (2, 3)



Note that the plane consists of an *x*-axis (horizontal or across) and a *y*-axis which is vertical (up and down). The plural of axis is axes (pronounced ak-seas). The co-ordinate (0, 0) is commonly referred to as the 'origin', the place where measurement starts from. Teachers often ask for the co-ordinates of the origin.

The first column asks you to locate the points and write in their co-ordinates. This is done by opening a bracket writing the *x* co-ordinate, writing a comma, then the *y* co-ordinate and close bracket. You should always place the co-ordinates in brackets.

The second column asks you to plot points. Once the group of points are plotted draw a line between points (in alphabetical order) to form a shape. Then write in the name of the shape plotted in the space provided.

Plotting Points - Positive Quadrant

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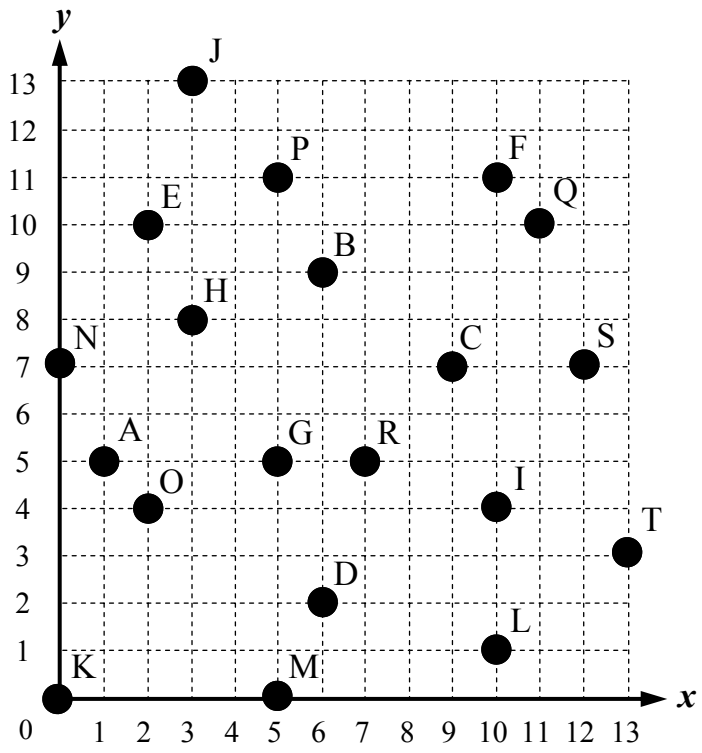
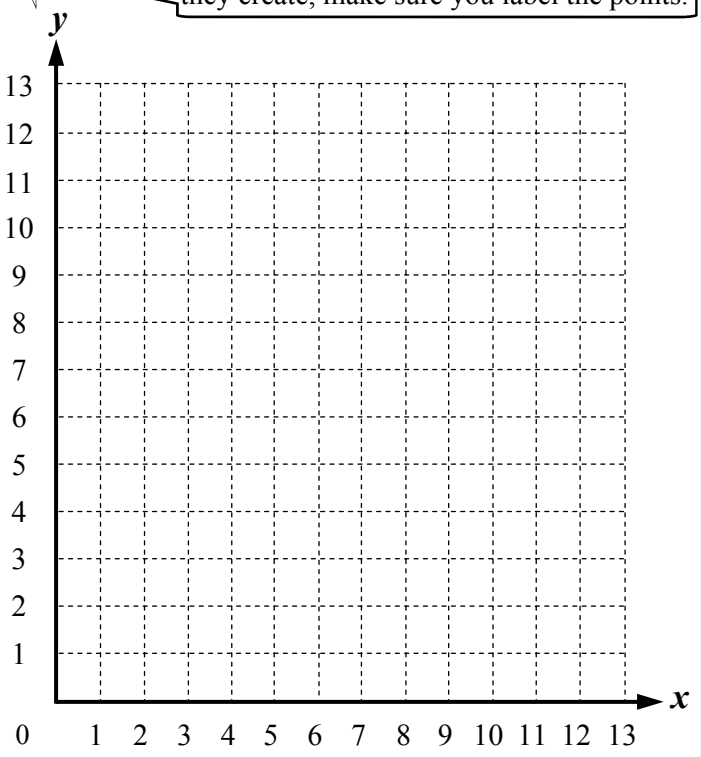
Give the co-ordinates for the following points, place them in brackets separated by a comma



1	Point A _____	11	Point K _____
2	Point B _____	12	Point L _____
3	Point C _____	13	Point M _____
4	Point D _____	14	Point N _____
5	Point E _____	15	Point O _____
6	Point F _____	16	Point P _____
7	Point G _____	17	Point Q _____
8	Point H _____	18	Point R _____
9	Point I _____	19	Point S _____
10	Point J _____	20	Point T _____

Place the groups of points given below on the grid, join the points and name the shape they create, make sure you label the points.



21 Point A (10, 4) Point B (11, 2)
Point C (12, 4) Point D (11, 5)
Shape formed: _____

22 Point E (3, 5) Point F (0, 8)
Point G (5, 13) Point H (8, 10)
Shape formed: _____

23 Point J (5, 4) Point K (9, 6)
Point L (7, 8)
Shape formed: _____
Two word description

24 Point M (2, 3) Point N (8, 3)
Point O (6, 0) Point P (0, 0)
Shape formed: _____

25 Point R (10, 9) Point S (13, 13)
Point T (13, 9)
Shape formed: _____
Two or three word description

Plotting Points - Positive Quadrant

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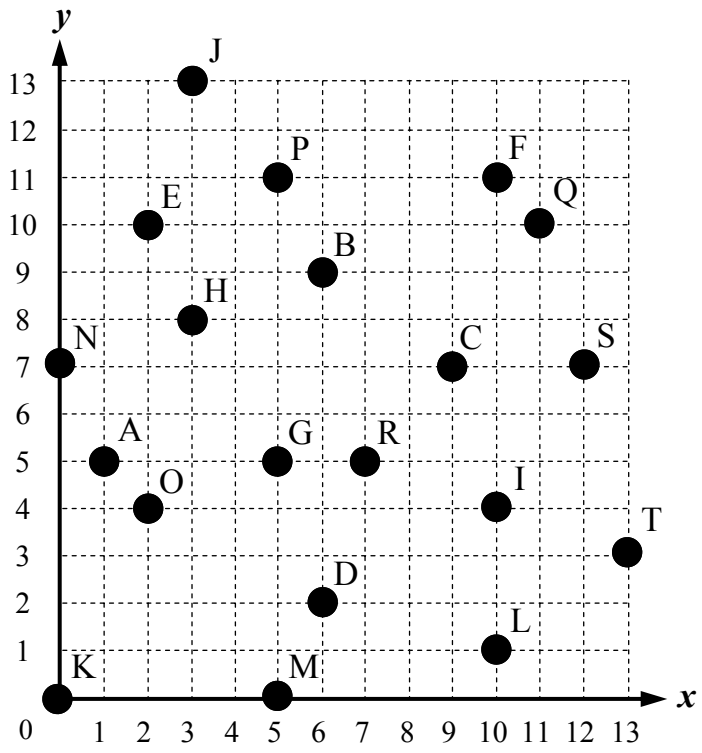
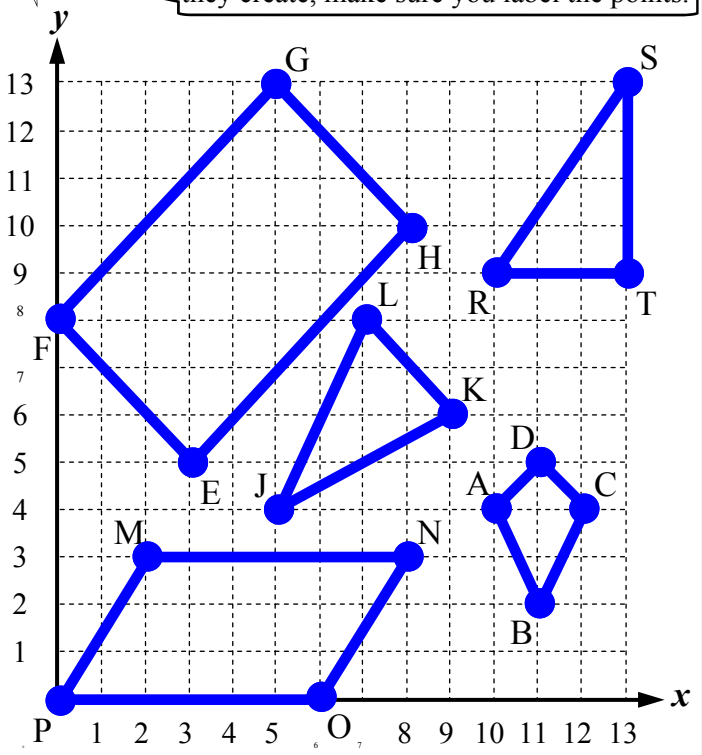
Give the co-ordinates for the following points, place them in brackets separated by a comma



1	Point A	(1, 5)	11	Point K	(0, 0)
2	Point B	(6, 9)	12	Point L	(10, 1)
3	Point C	(9, 7)	13	Point M	(5, 0)
4	Point D	(6, 2)	14	Point N	(0, 7)
5	Point E	(2, 10)	15	Point O	(2, 4)
6	Point F	(10, 11)	16	Point P	(5, 11)
7	Point G	(5, 5)	17	Point Q	(11, 10)
8	Point H	(3, 8)	18	Point R	(7, 5)
9	Point I	(10, 4)	19	Point S	(12, 7)
10	Point J	(3, 13)	20	Point T	(13, 3)

Place the groups of points given below on the grid, join the points and name the shape they create, make sure you label the points.



21 Point A (10, 4) Point B (11, 2)
Point C (12, 4) Point D (11, 5)
Shape formed: Kite

22 Point E (3, 5) Point F (0, 8)
Point G (5, 13) Point H (8, 10)
Shape formed: Rectangle

23 Point J (5, 4) Point K (9, 6)
Point L (7, 8)
Shape formed: Isosceles Triangle
Two word description

24 Point M (2, 3) Point N (8, 3)
Point O (6, 0) Point P (0, 0)
Shape formed: Parallelogram

25 Point R (10, 9) Point S (13, 13)
Point T (13, 9)
Shape formed: Right (Angled) Triangle
Two or three word description

Drawing Animals - Positive Quadrant

This sheet uses your skills in plotting points from the previous sheet. The series of points makes a pair of animals when lines are joined between the points. The way you attempt this exercise is, for the first animal:

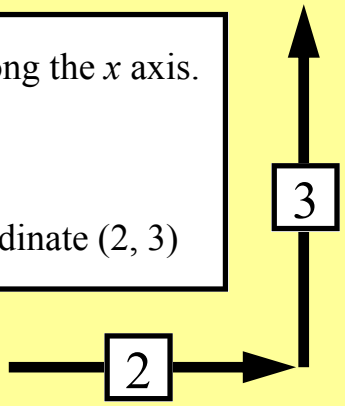
- Plot one point at a time and then draw a line between it and the previous point. Don't try to plot all the points and then draw the lines afterwards as the drawing is too complicated.
- When you reach stop, restart at the new position and start again, DON'T JOIN THE POINTS BETWEEN WHERE YOU STOPPED AND WHERE YOU RESTARTED.

For the next animal follow the same process except use 2 different colours.

Move across first along the *x* axis.

Then move up.

Eg. To plot the co-ordinate (2, 3)



Drawing Animals - Positive Quadrant

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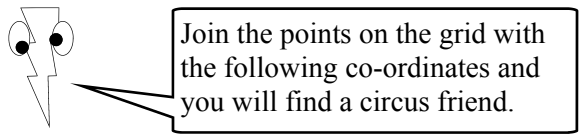
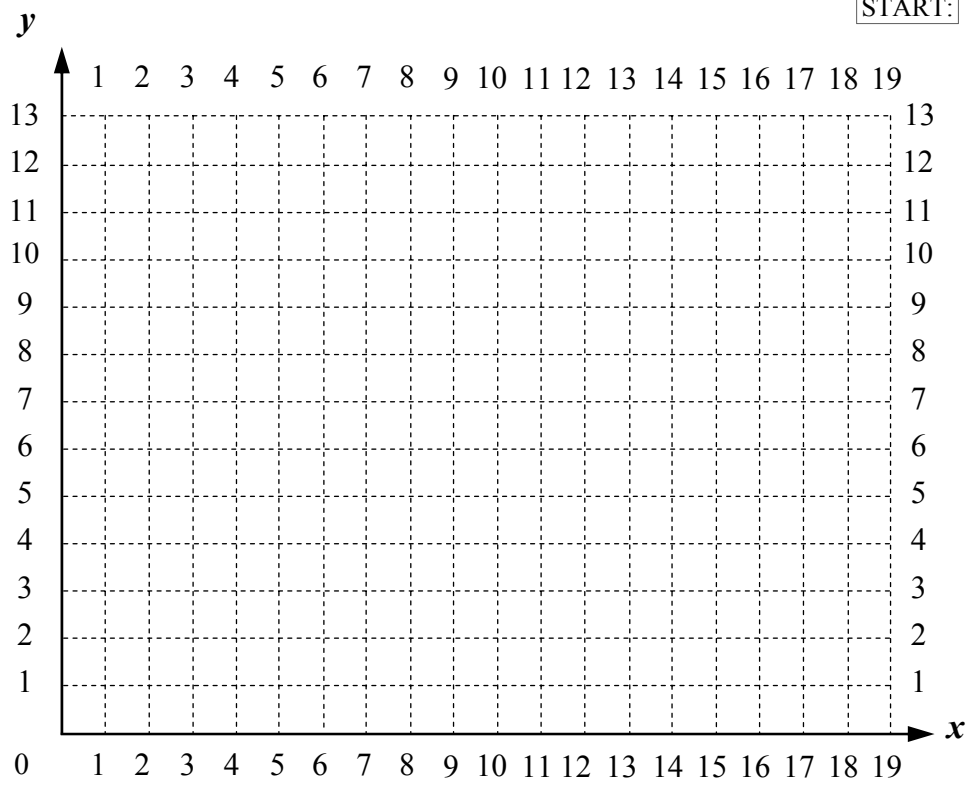
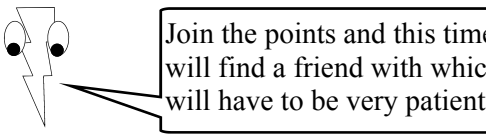


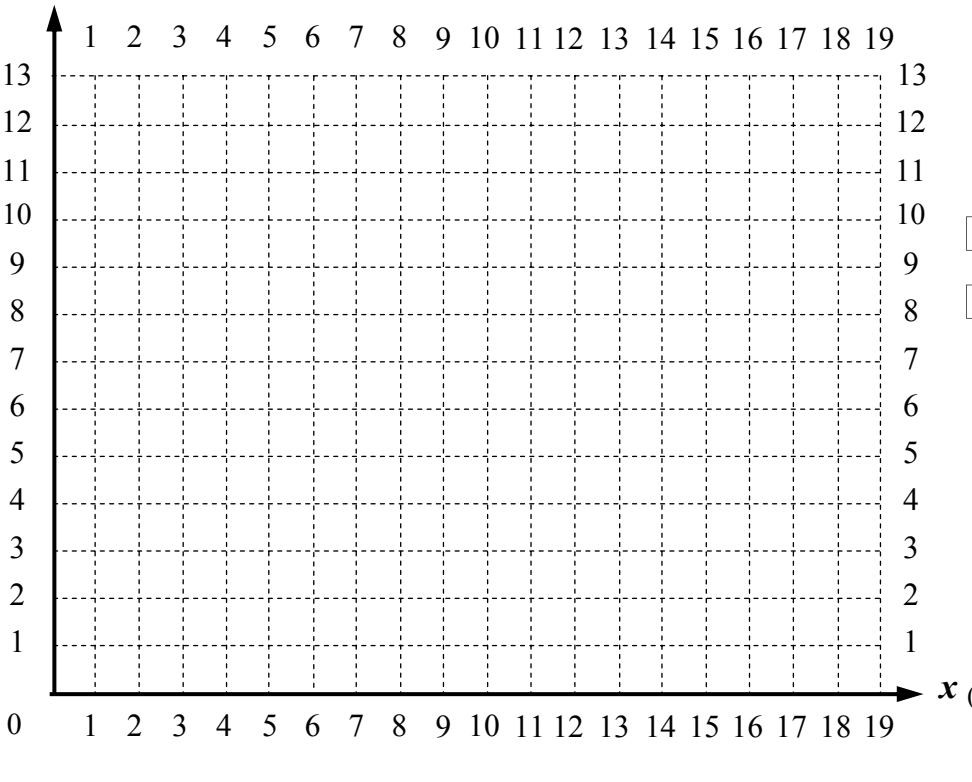


Co-ordinates:

START: (2, 12) , (3, 12) , (4, 11) , (5, 12) ,
(7, 12) , (8, 11) , (9, 12) , (10, 12) ,
(11, 11) , (11, 6) , (10, 3) , (9, 3) ,
(8, 7) , (8, 5) , (7, 7) , (7, 3) ,
(6, 2) , (4, 2) , (4, 3) , (5, 3) ,
(6, 4) , (5, 7) , (4, 5) , (4, 7) ,
(3, 3) , (2, 3) , (1, 6) , (1, 11) ,
(2, 12) STOP!

START:
(11, 10) , (12, 11) , (16, 12) , (18, 10) ,
(19, 7) , (18, 9) , (18, 3) , (15, 3) ,
(16, 4) , (16, 6) , (14, 5) , (11, 5) ,
(11, 4) , (13, 4) , (13, 1) , (12, 2) ,
(9, 2) , (8, 3) , (7, 6) , STOP!





Co-ordinates:

START: (1, 11) , (2, 10) , (2, 9) , (1, 8) ,
(1, 7) , (2, 6) , (2, 5) , (3, 3) ,
(5, 1) , (7, 0) , (13, 0) , (16, 3) ,
(17, 5) , (15, 3) , (12, 2) , (8, 2) ,
(6, 3) , (4, 5) , (3, 6) , (4, 7) ,
(4, 8) , (3, 9) , (3, 10) , (4, 11) ,
STOP!

START:
(11, 6) , (12, 6) , (11, 7) , (10, 6) ,
(11, 5) , (12, 5) , (14, 6) , (14, 7) ,
(12, 9) , (10, 9) , (8, 7) , (8, 5) ,
(10, 3) , (12, 3) , (14, 4) , (15, 6) ,
(15, 7) , (14, 9) , (13, 10) , (10, 10) ,
(8, 9) , (7, 7) , (8, 3) , (11, 2) ,
(12, 2) , (15, 3) , (17, 6) , (17, 7) ,
(15, 11) , (12, 12) , (10, 12) , (7, 11) ,
(5, 9) , (4, 6) , (4, 5) , STOP!

Drawing Animals - Positive Quadrant

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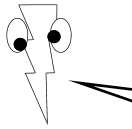
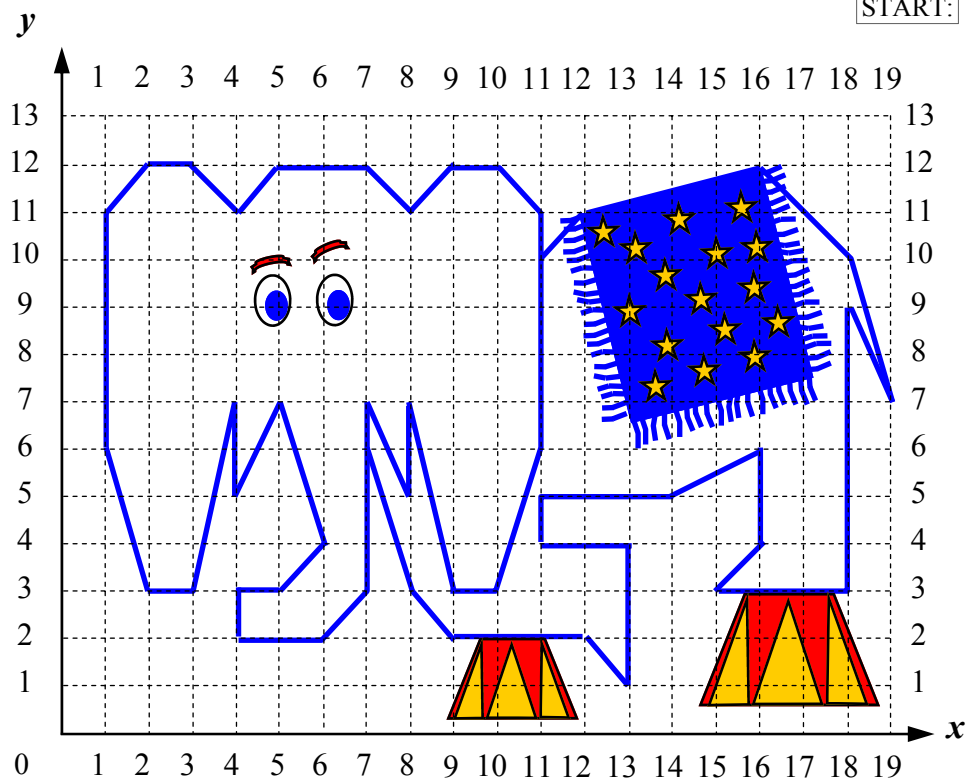


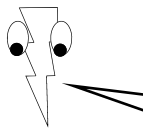
Join the points on the grid with the following co-ordinates and you will find a circus friend.



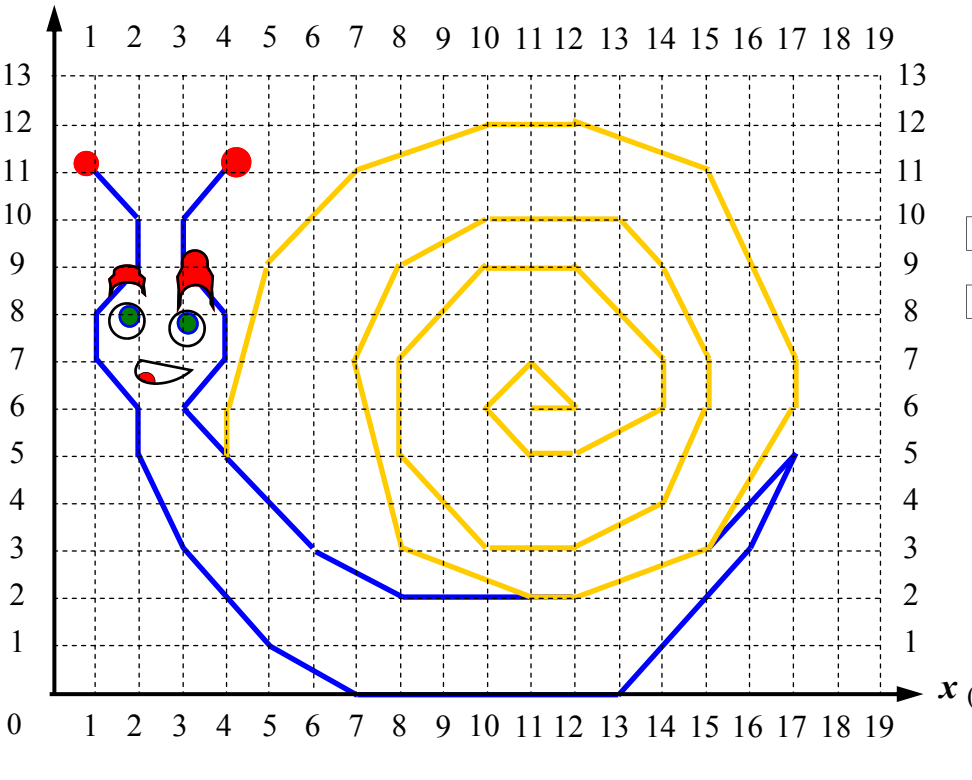
Co-ordinates:

START: (2, 12) , (3, 12) , (4, 11) , (5, 12) ,
(7, 12) , (8, 11) , (9, 12) , (10, 12) ,
(11, 11) , (11, 6) , (10, 3) , (9, 3) ,
(8, 7) , (8, 5) , (7, 7) , (7, 3) ,
(6, 2) , (4, 2) , (4, 3) , (5, 3) ,
(6, 4) , (5, 7) , (4, 5) , (4, 7) ,
(3, 3) , (2, 3) , (1, 6) , (1, 11) ,
(2, 12) STOP!

START:
(11, 10) , (12, 11) , (16, 12) , (18, 10) ,
(19, 7) , (18, 9) , (18, 3) , (15, 3) ,
(16, 4) , (16, 6) , (14, 5) , (11, 5) ,
(11, 4) , (13, 4) , (13, 1) , (12, 2) ,
(9, 2) , (8, 3) , (7, 6) , STOP!



Join the points and this time you will find a friend with which you will have to be very patient.



Co-ordinates:

START: (1, 11) , (2, 10) , (2, 9) , (1, 8) ,
(1, 7) , (2, 6) , (2, 5) , (3, 3) ,
(5, 1) , (7, 0) , (13, 0) , (16, 3) ,
(17, 5) , (15, 3) , (12, 2) , (8, 2) ,
(6, 3) , (4, 5) , (3, 6) , (4, 7) ,
(4, 8) , (3, 9) , (3, 10) , (4, 11) ,
STOP!

START:
(11, 6) , (12, 6) , (11, 7) , (10, 6) ,
(11, 5) , (12, 5) , (14, 6) , (14, 7) ,
(12, 9) , (10, 9) , (8, 7) , (8, 5) ,
(10, 3) , (12, 3) , (14, 4) , (15, 6) ,
(15, 7) , (14, 9) , (13, 10) , (10, 10) ,
(8, 9) , (7, 7) , (8, 3) , (11, 2) ,
(12, 2) , (15, 3) , (17, 6) , (17, 7) ,
(15, 11) , (12, 12) , (10, 12) , (7, 11) ,
(5, 9) , (4, 6) , (4, 5) , STOP!

Plotting Points - 4 Quadrants

This sheet involves the plotting of Cartesian co-ordinates. When plotting points on a number plane you are given two numbers. The first number tells you how much you move across and the second number tells you how much you move up or down. As the numbers are in order, across first, up/down second, these numbers can be referred to as 'ordered pairs', as their order is important. Positive numbers move across to the right if the first number, negative numbers move to the left if the first number. Positive numbers move up if the second number, negative numbers move down if the second number.

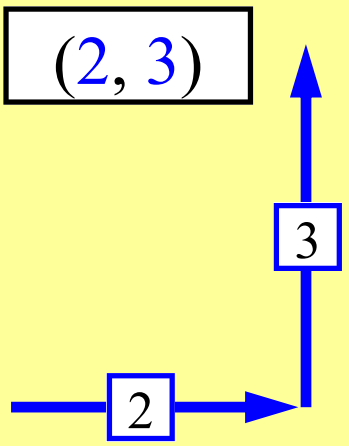
The first column asks you to locate the points and write in their co-ordinates. This is done by opening a bracket writing the *x* co-ordinate, a comma, then the *y* co-ordinate and close bracket. Always place co-ordinates in brackets.

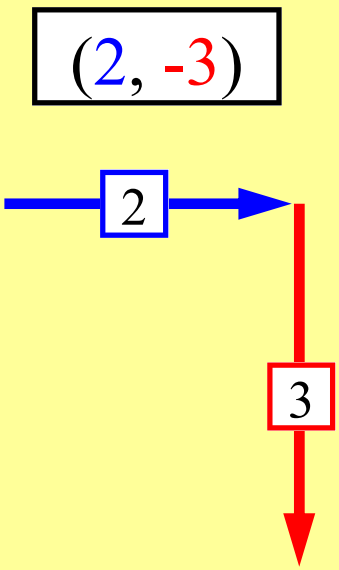
The second column asks you to plot points. Draw filled in circles at the given location and label them with the letter supplied in each question.

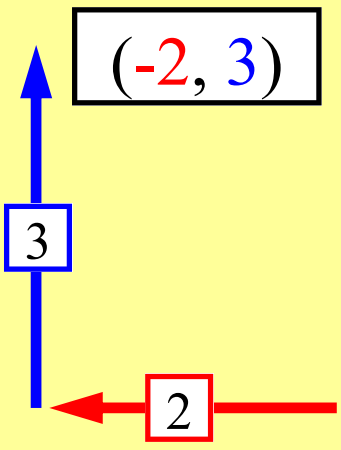
Once you plot the point record down which quadrant it is in. Quadrants start with the 1st Quadrant (top right) and then moving anti-clockwise, are called II (2nd), III (3rd) and IV (4th). If the point is on the axis then record it by filling in the oval around the word 'Axis'.

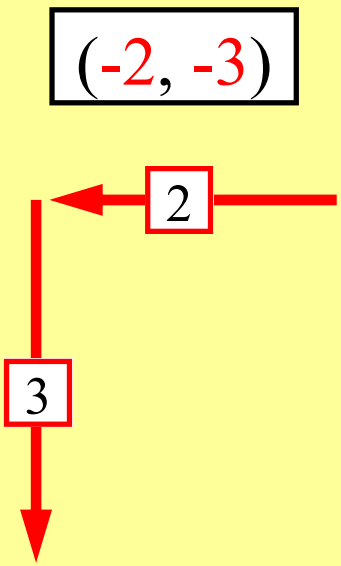
Move across first along the *x* axis.
Negative to the left, Positive to the right.

Then move up or down in the *y* direction.
Negative is down, Positive is up.









Plotting Points - 4 Quadrants

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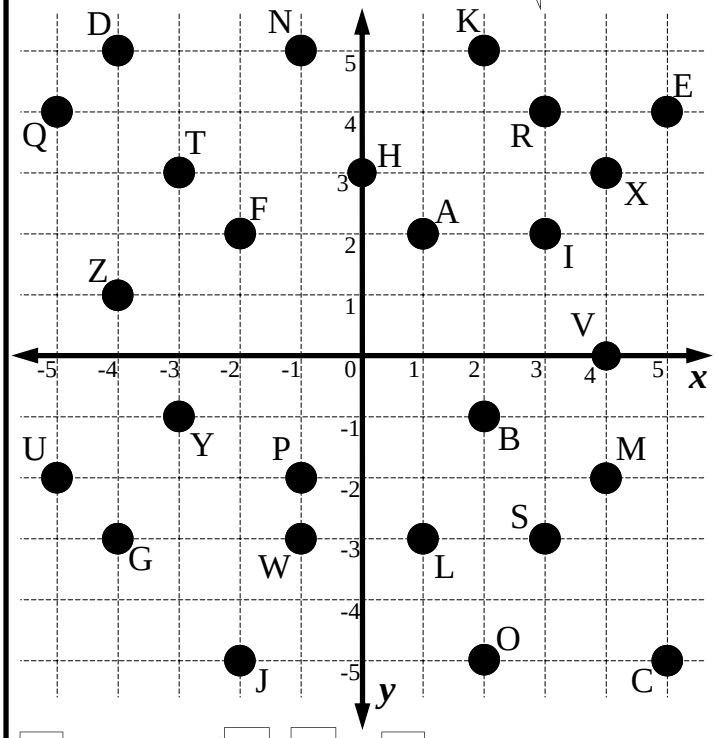
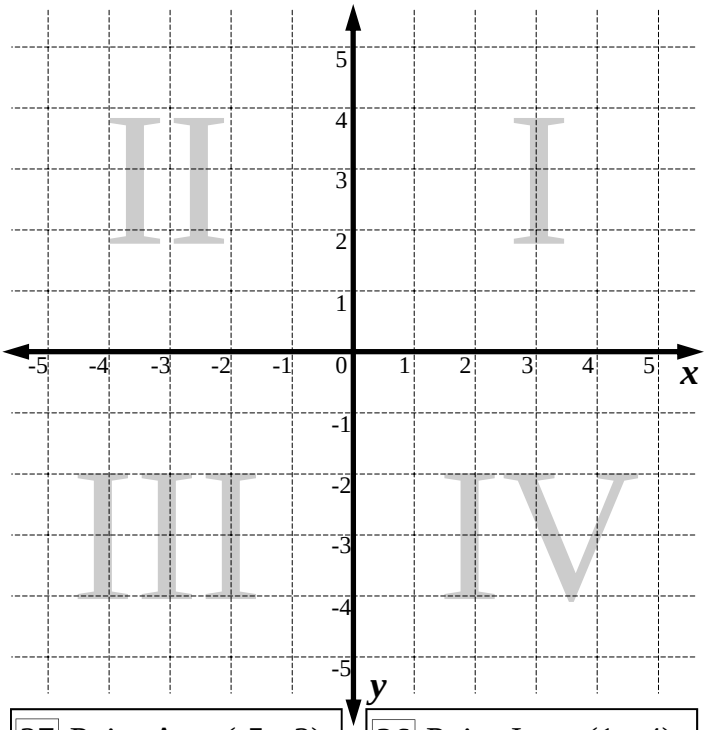
Give the co-ordinates for the points on the number plane below.



1 Point A (,)
2 Point B
3 Point C
4 Point D
5 Point E
6 Point F
7 Point G
8 Point H
9 Point I
10 Point J
11 Point K
12 Point L
13 Point M
14 Point N
15 Point O
16 Point P
17 Point Q
18 Point R
19 Point S
20 Point T
21 Point U
22 Point V
23 Point W
24 Point X
25 Point Y
26 Point Z

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant I II III IV Axis
28 Point B (2, 4) Quadrant I II III IV Axis
29 Point C (4, 0) Quadrant I II III IV Axis
30 Point D (3, -3) Quadrant I II III IV Axis
31 Point E (-3, 5) Quadrant I II III IV Axis
32 Point F (0, 0) Quadrant I II III IV Axis
33 Point G (5, -5) Quadrant I II III IV Axis
34 Point H (-2, 4) Quadrant I II III IV Axis
35 Point I (1, 2) Quadrant I II III IV Axis
36 Point J (1, -4) Quadrant I II III IV Axis
37 Point K (-3, -4) Quadrant I II III IV Axis
38 Point L (-1, 0) Quadrant I II III IV Axis
39 Point M (-4, 2) Quadrant I II III IV Axis
40 Point N (3, -5) Quadrant I II III IV Axis
41 Point O (0, -3) Quadrant I II III IV Axis
42 Point P (-5, -5) Quadrant I II III IV Axis
43 Point R (-2, 1) Quadrant I II III IV Axis
44 Point S (2, -2) Quadrant I II III IV Axis

Plotting Points - 4 Quadrants

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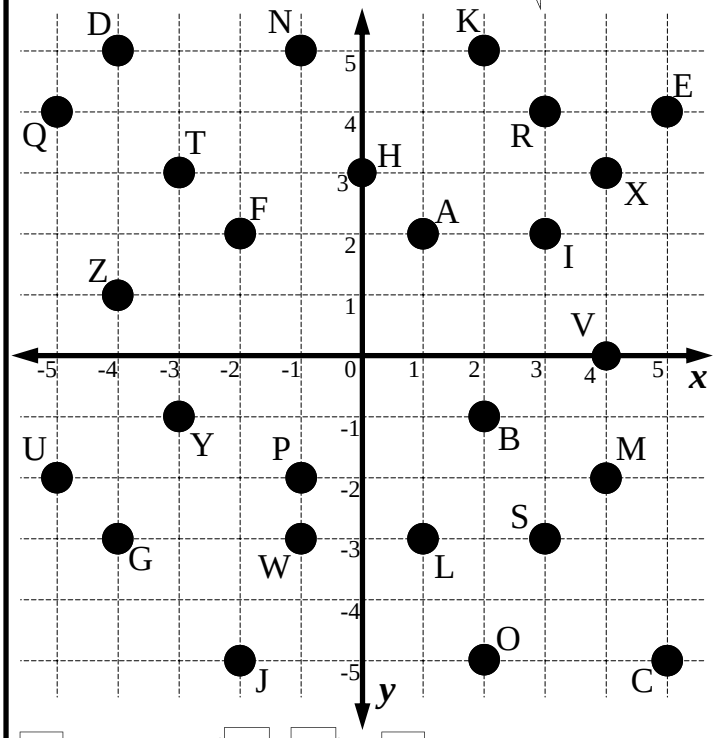
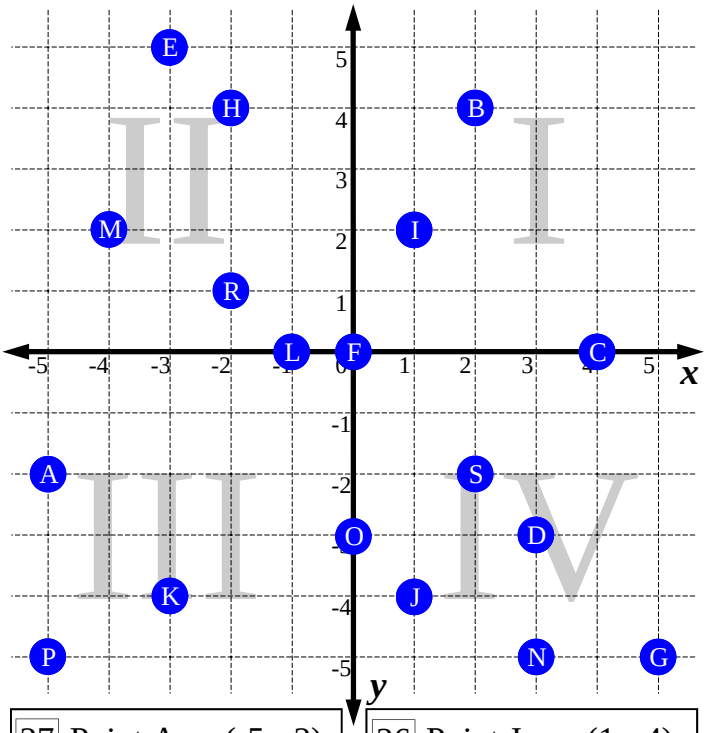
Give the co-ordinates for the points on the number plane below.



1	Point A	(1, 2)	14	Point N	(-1, 5)
2	Point B	(2, -1)	15	Point O	(2, -5)
3	Point C	(5, -5)	16	Point P	(-1, -2)
4	Point D	(-4, 5)	17	Point Q	(-5, 4)
5	Point E	(5, 4)	18	Point R	(3, 4)
6	Point F	(-2, 2)	19	Point S	(3, -3)
7	Point G	(-4, -3)	20	Point T	(-3, 3)
8	Point H	(0, 3)	21	Point U	(-5, -2)
9	Point I	(3, 2)	22	Point V	(4, 0)
10	Point J	(-2, -5)	23	Point W	(-1, -3)
11	Point K	(2, 5)	24	Point X	(4, 3)
12	Point L	(1, -3)	25	Point Y	(-3, -1)
13	Point M	(4, -2)	26	Point Z	(-4, 1)

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant I II **III** IV Axis

28 Point B (2, 4) Quadrant **I** II III IV Axis

29 Point C (4, 0) Quadrant I II III IV **Axis**

30 Point D (3, -3) Quadrant I II **III** IV Axis

31 Point E (-3, 5) Quadrant **II** III IV Axis

32 Point F (0, 0) Quadrant I II III IV **Axis**

33 Point G (5, -5) Quadrant I II **III** IV Axis

34 Point H (-2, 4) Quadrant **II** III IV Axis

35 Point I (1, 2) Quadrant **I** II III IV Axis

36 Point J (1, -4) Quadrant I II III **IV** Axis

37 Point K (-3, -4) Quadrant I II **III** IV Axis

38 Point L (-1, 0) Quadrant I II III IV **Axis**

39 Point M (-4, 2) Quadrant **II** III IV Axis

40 Point N (3, -5) Quadrant I II III **IV** Axis

41 Point O (0, -3) Quadrant I II III IV **Axis**

42 Point P (-5, -5) Quadrant I II **III** IV Axis

43 Point R (-2, 1) Quadrant **II** III IV Axis

44 Point S (2, -2) Quadrant I II III **IV** Axis

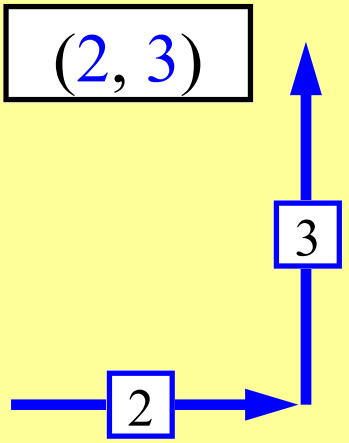
Drawing Animals - All Quadrants

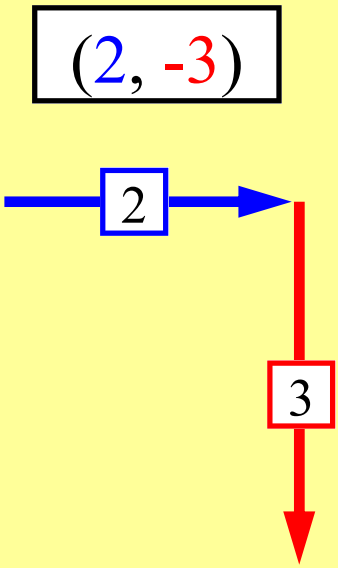
This sheet uses your skills in plotting points from the previous sheet. The series of points makes a pair of animals when lines are joined between the points. The way you attempt this exercise is, for the first animal:

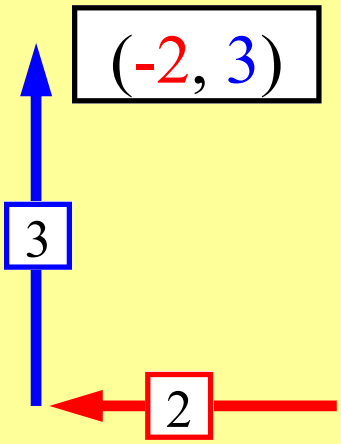
- Plot one point at a time and then draw a line between it and the previous point. Don't try to plot all the points and then draw the lines afterwards as the drawing is too complicated.
- When you reach stop, restart at the new position and start again, DON'T JOIN THE POINTS BETWEEN WHERE YOU STOPPED AND WHERE YOU RESTARTED.

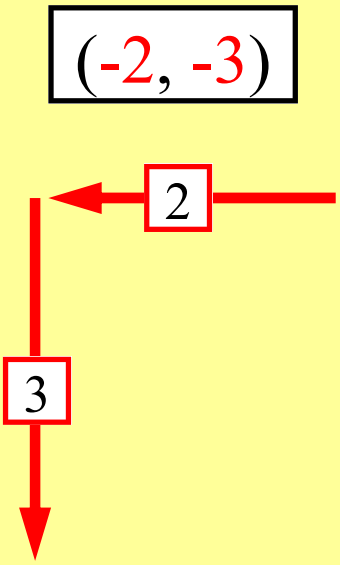
Move across first along the *x* axis.
Negative to the left, Positive to the right.

Then move up or down in the *y* direction.
Negative is down, Positive is up.









Drawing Animals - 4 Quadrants

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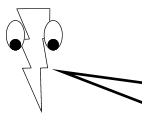
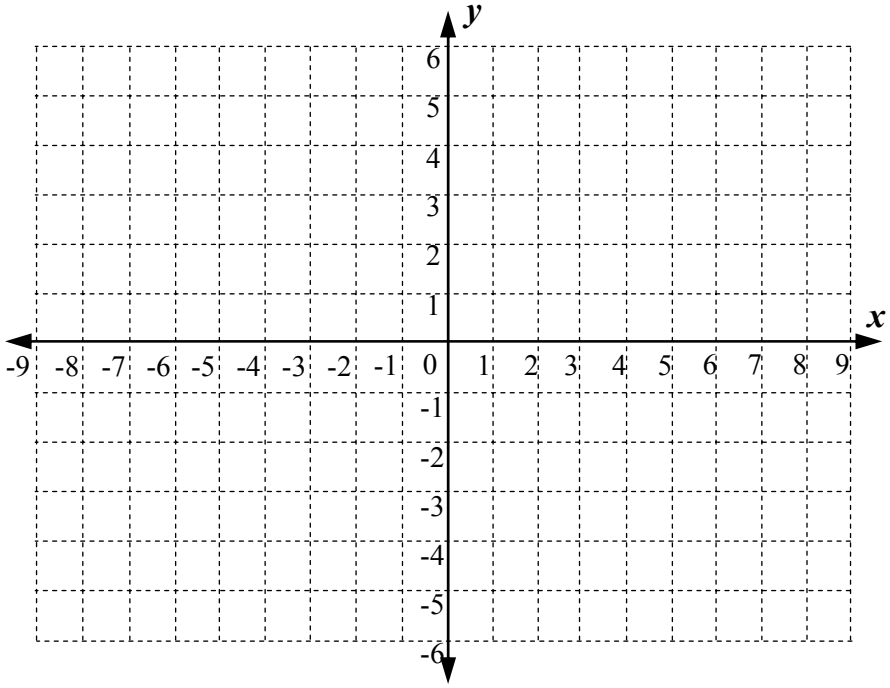


Plot the points and join them with lines to meet one of your more colourful friends

Co-ordinates: START: (-1, 3) , (-1, 4) , (1, 6) , (2, 6) , (6, 5) , (7, 4) , (7, 2) , (6, -1) , (3, -2) , (5,-4) , (5, -5) , (3, -6) , (1, -6) , (-2, -4) , (-4, -6) , (-5, -6) , (-6, -5) , (-6, -4) , (-5, -2) , (-6, -1) , (-8, 2) , (-8, 4) , (-7, 6) , (-6, 6) , (-4, 4) , (-4, 3) , STOP!

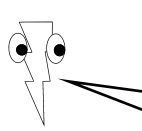


START: (-2, 1) , (1, 4) , (4, 4) , (5, 2) , (3,-1) , (0, -2) , (3, -3) , (3, -5) , (1, -5) , (-1, -4) , (-2, -3) , (-4, -5) , (-5, -5) , (-5, -3) , (-3, -2) , (-5, -1) , (-7, 2) , (-6, 4) , (-5, 4) , (-3,1) , STOP!

START: (-4, 3) , (-3, 4) , (-2, 4) , (-1, 3) , (-2, 2) , (-2, -4) , (-3, 2) , (-4, 3) , STOP!

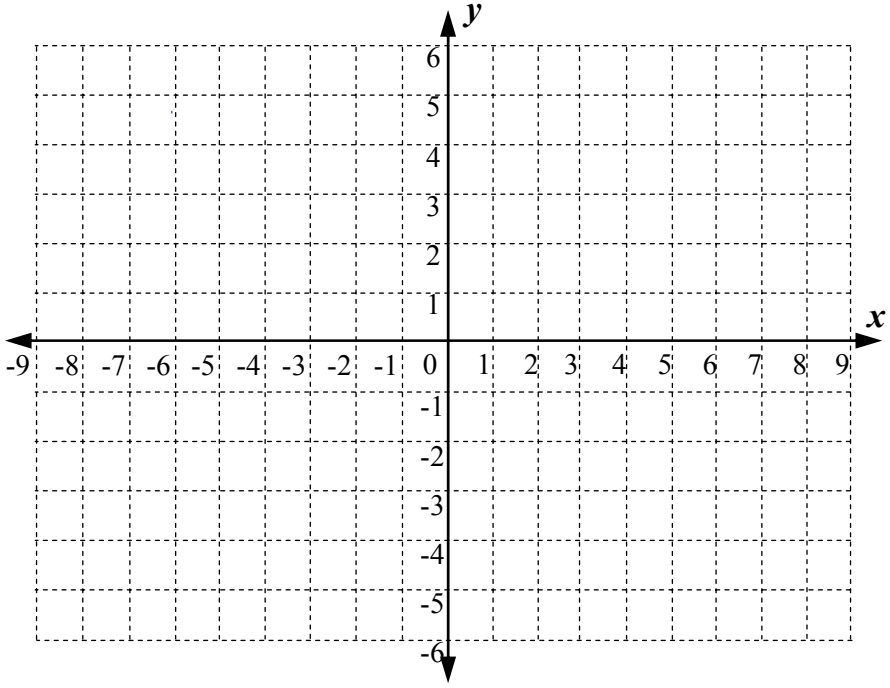
START: (-3, 4) , (-4, 6) , STOP!

START: (-2, 4) , (-1, 6) , STOP!



Plot the points and join them with lines to share your lunch with your vegetarian friend.

Co-ordinates: START: (2, -1) , (3, -3) , (3, -4) , (2, -5) , (0, -6) , (-1, -6) , (-3, -5) , (-4, -4) , (-4, -3) , (-3, -1) , (-6, 2) , (-7, -4) , (-8, 4) , (-7, 6) , (-4, 5) , (-2, -1) , (0, -1) , (3, 4) , (5, 6) , (8, 6) , (9, 4) , (5, 6) , STOP!



START: (7, 5) , (6, 3) , (2, -1) , STOP!

START: (2, 0) , (4, 4) , (6, 5) , (5, 3) , (2, 0) , STOP!

START: (-3, 0) , (-5, 2) , (-6, 4) , (-4, 3) , (-3, 0) , STOP!

START: (-2, -4) , (-6, -4) , STOP!

START: (-2, -4) , (-5, -2) , STOP!

START: (-2, -4) , (-5, -3) , STOP!

START: (1, -4) , (5, -4) , STOP!

START: (1, -4) , (5, -3) , STOP!

START: (1, -4) , (5, -5) , STOP!

Drawing Animals - 4 Quadrants

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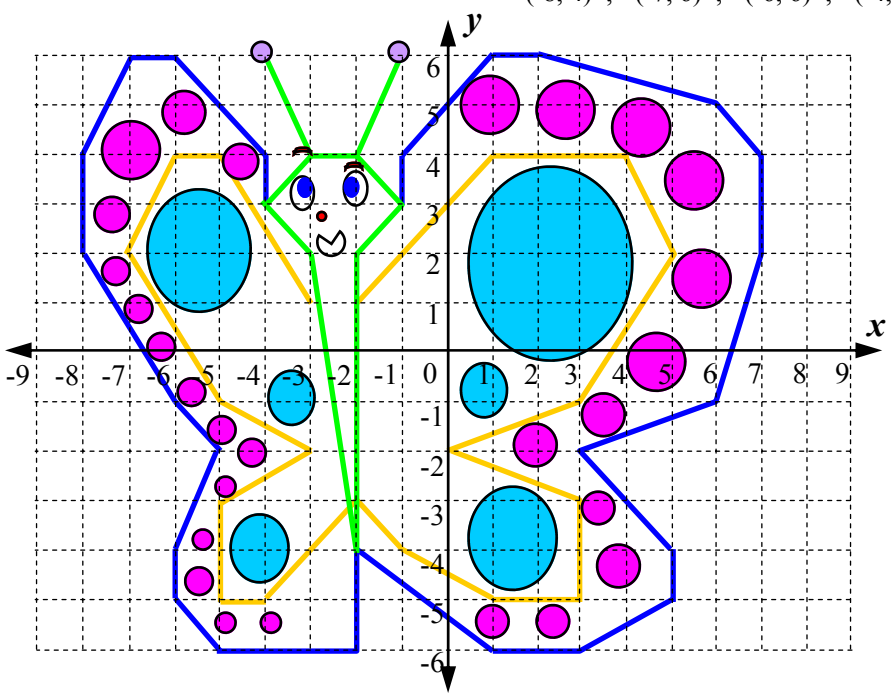
Plot the points and join them with lines to meet one of your more colourful friends

Co-ordinates: START: (-1, 3) , (-1, 4) , (1, 6) , (2, 6) , (6, 5) , (7, 4) , (7, 2) , (6, -1) , (3, -2) , (5,-4) , (5, -5) , (3, -6) , (1, -6) , (-2, -4) , (-4, -6) , (-5, -6) , (-6, -5) , (-6, -4) , (-5, -2) , (-6, -1) , (-8, 2) , (-8, 4) , (-7, 6) , (-6, 6) , (-4, 4) , (-4, 3) , STOP!



START: (-2, 1) , (1, 4) , (4, 4) , (5, 2) , (3,-1) , (0, -2) , (3, -3) , (3, -5) , (1, -5) , (-1, -4) , (-2, -3) , (-4, -5) , (-5, -5) , (-5, -3) , (-3, -2) , (-5, -1) , (-7, 2) , (-6, 4) , (-5, 4) , (-3,1) , STOP!

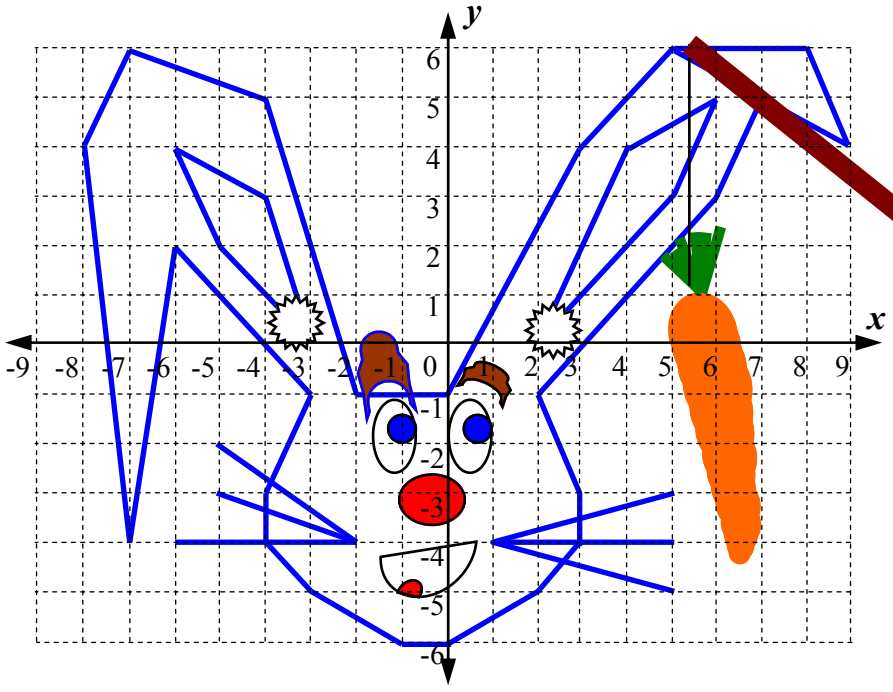
START: (-4, 3) , (-3, 4) , (-2, 4) , (-1, 3) , (-2, 2) , (-2, -4) , (-3, 2) , (-4, 3) , STOP!

START: (-3, 4) , (-4, 6) , STOP!

START: (-2, 4) , (-1, 6) , STOP!

Plot the points and join them with lines to share your lunch with your vegetarian friend.

Co-ordinates: START: (2, -1) , (3, -3) , (3, -4) , (2, -5) , (0, -6) , (-1, -6) , (-3, -5) , (-4, -4) , (-4, -3) , (-3, -1) , (-6, 2) , (-7, -4) , (-8, 4) , (-7, 6) , (-4, 5) , (-2, -1) , (0, -1) , (3, 4) , (5, 6) , (8, 6) , (9, 4) , (5, 6) , STOP!



START: (7, 5) , (6, 3) , (2, -1) , STOP!

START: (2, 0) , (4, 4) , (6, 5) , (5, 3) , (2, 0) , STOP!

START: (-3, 0) , (-5, 2) , (-6, 4) , (-4, 3) , (-3, 0) , STOP!

START: (-2, 4) , (-6, -4) , STOP!

START: (-2, -4) , (5, -2) , STOP!

START: (-2, -4) , (-5, -3) , STOP!

START: (1, -4) , (5, -4) , STOP!

START: (1, -4) , (5, -3) , STOP!

START: (1, -4) , (5, -5) , STOP!

Plotting Points - 4 Quadrants

This sheet involves the plotting of Cartesian co-ordinates. When plotting points on a number plane you are given two numbers. The first number tells you how much you move across and the second number tells you how much you move up or down. As the numbers are in order, across first, up/down second, these numbers can be referred to as 'ordered pairs', as their order is important. Positive numbers move across to the right if the first number, negative numbers move to the left if the first number. Positive numbers move up if the second number, negative numbers move down if the second number.

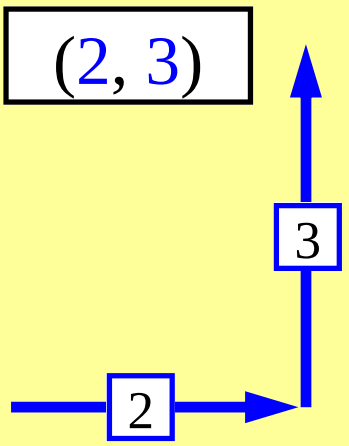
The first column asks you to locate the points and write in their co-ordinates. This is done by opening a bracket writing the *x* co-ordinate, a comma, then the *y* co-ordinate and close bracket. Always place co-ordinates in brackets.

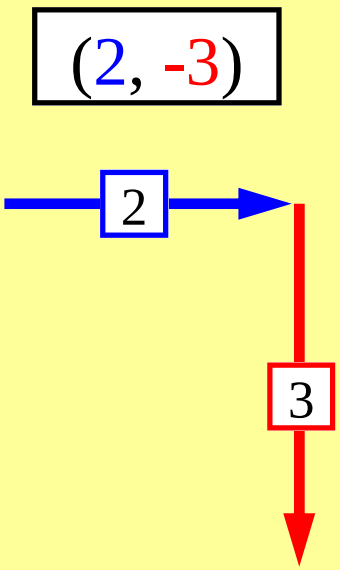
The second column asks you to plot points. Draw filled in circles at the given location and label them with the letter supplied in each question.

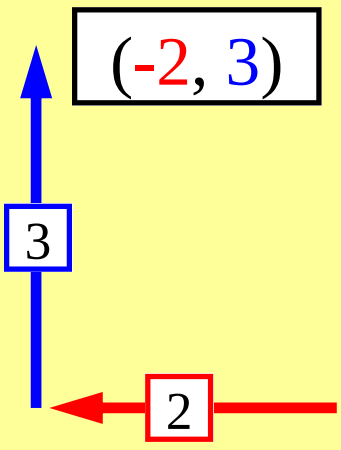
Once you plot the point record down which quadrant it is in. Quadrants start with the 1st Quadrant (top right) and then moving anti-clockwise, are called II (2nd), III (3rd) and IV (4th). If the point is on the axis then record it by filling in the oval around the word 'Axis'.

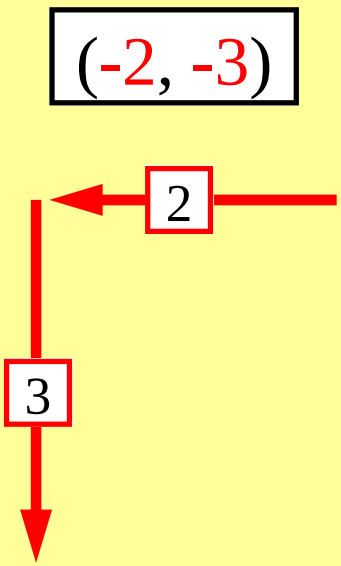
Move across first along the *x* axis.
Negative to the left, Positive to the right.

Then move up or down in the *y* direction.
Negative is down, Positive is up.









Plotting Points - 4 Quadrants

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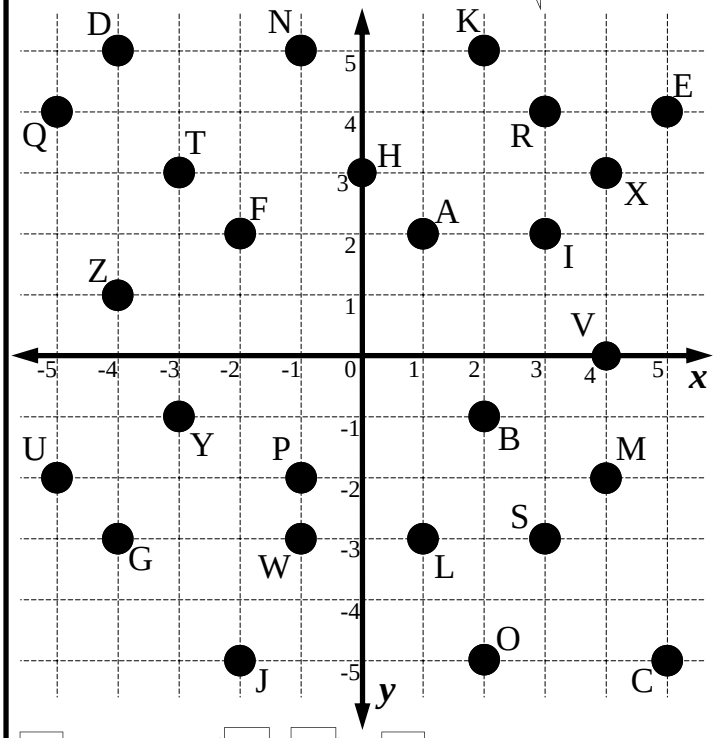
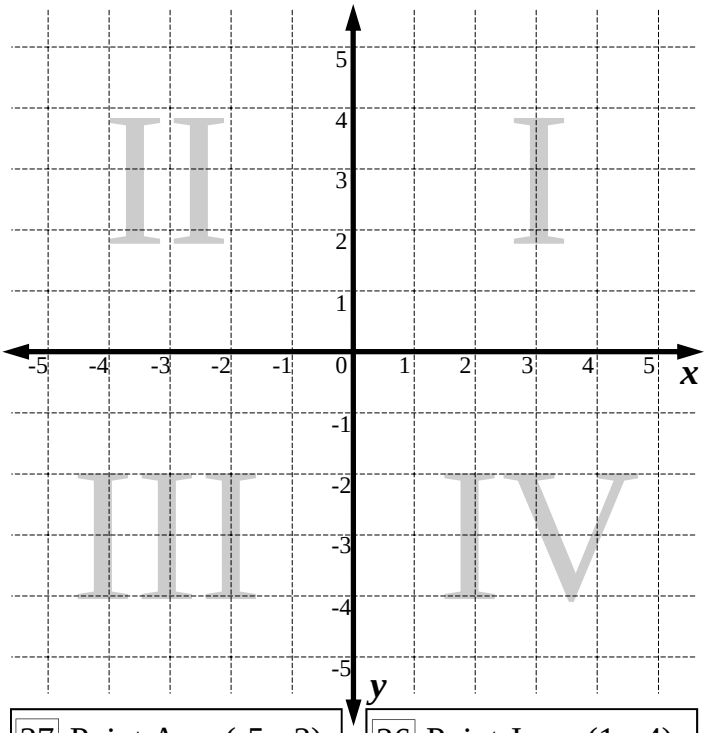
Give the co-ordinates for the points on the number plane below.



1 Point A (,)
2 Point B _____
3 Point C _____
4 Point D _____
5 Point E _____
6 Point F _____
7 Point G _____
8 Point H _____
9 Point I _____
10 Point J _____
11 Point K _____
12 Point L _____
13 Point M _____
14 Point N _____
15 Point O _____
16 Point P _____
17 Point Q _____
18 Point R _____
19 Point S _____
20 Point T _____
21 Point U _____
22 Point V _____
23 Point W _____
24 Point X _____
25 Point Y _____
26 Point Z _____

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant (I) (II) (III) (IV) (Axis)
28 Point B (2, 4) Quadrant (I) (II) (III) (IV) (Axis)
29 Point C (4, 0) Quadrant (I) (II) (III) (IV) (Axis)
30 Point D (3, -3) Quadrant (I) (II) (III) (IV) (Axis)
31 Point E (-3, 5) Quadrant (I) (II) (III) (IV) (Axis)
32 Point F (0, 0) Quadrant (I) (II) (III) (IV) (Axis)
33 Point G (5, -5) Quadrant (I) (II) (III) (IV) (Axis)
34 Point H (-2, 4) Quadrant (I) (II) (III) (IV) (Axis)
35 Point I (1, 2) Quadrant (I) (II) (III) (IV) (Axis)
36 Point J (1, -4) Quadrant (I) (II) (III) (IV) (Axis)
37 Point K (-3, -4) Quadrant (I) (II) (III) (IV) (Axis)
38 Point L (-1, 0) Quadrant (I) (II) (III) (IV) (Axis)
39 Point M (-4, 2) Quadrant (I) (II) (III) (IV) (Axis)
40 Point N (3, -5) Quadrant (I) (II) (III) (IV) (Axis)
41 Point O (0, -3) Quadrant (I) (II) (III) (IV) (Axis)
42 Point P (-5, -5) Quadrant (I) (II) (III) (IV) (Axis)
43 Point R (-2, 1) Quadrant (I) (II) (III) (IV) (Axis)
44 Point S (2, -2) Quadrant (I) (II) (III) (IV) (Axis)

Plotting Points - 4 Quadrants

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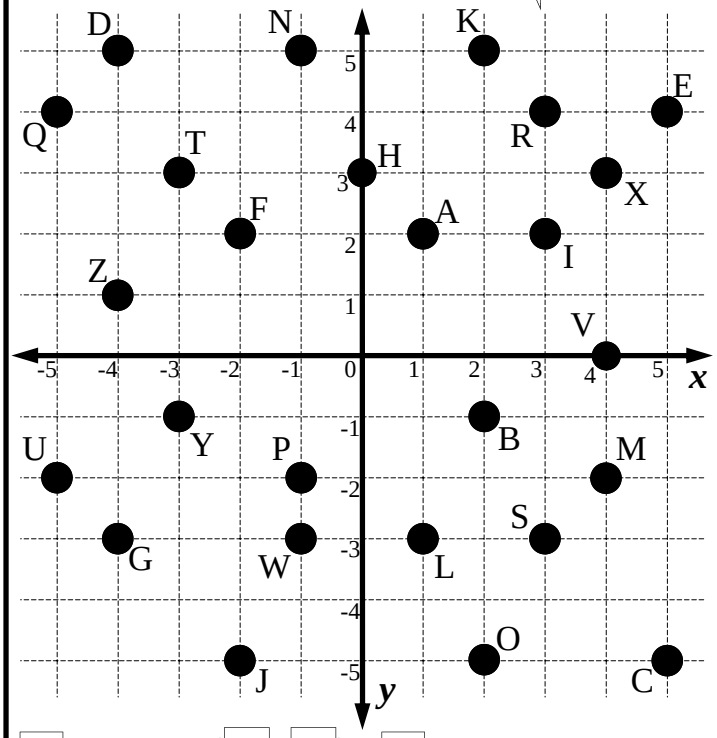
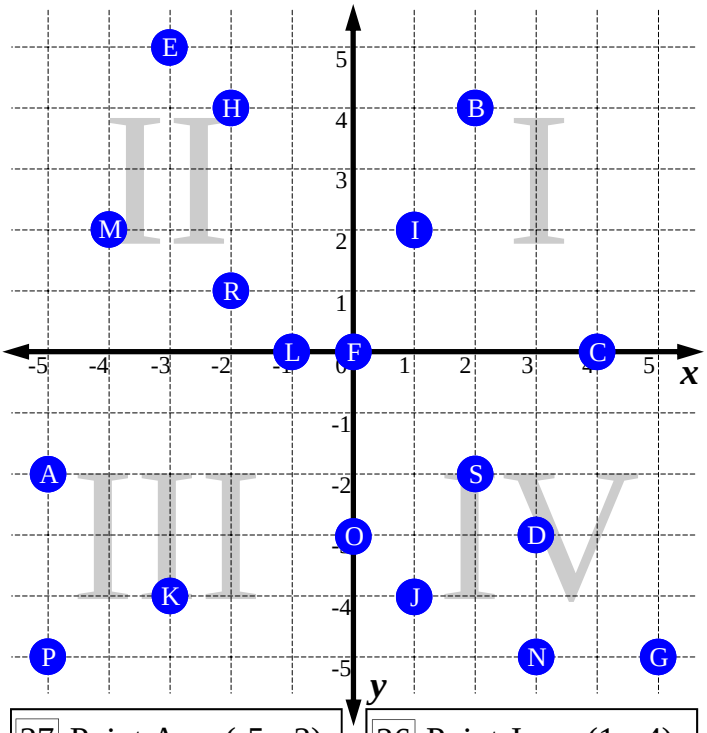
Give the co-ordinates for the points on the number plane below.



1	Point A	(1, 2)	14	Point N	(-1, 5)
2	Point B	(2, -1)	15	Point O	(2, -5)
3	Point C	(5, -5)	16	Point P	(-1, -2)
4	Point D	(-4, 5)	17	Point Q	(-5, 4)
5	Point E	(5, 4)	18	Point R	(3, 4)
6	Point F	(-2, 2)	19	Point S	(3, -3)
7	Point G	(-4, -3)	20	Point T	(-3, 3)
8	Point H	(0, 3)	21	Point U	(-5, -2)
9	Point I	(3, 2)	22	Point V	(4, 0)
10	Point J	(-2, -5)	23	Point W	(-1, -3)
11	Point K	(2, 5)	24	Point X	(4, 3)
12	Point L	(1, -3)	25	Point Y	(-3, -1)
13	Point M	(4, -2)	26	Point Z	(-4, 1)

Plot the following points on the number plane below, label each point and give the quadrant that the point lies in.



27 Point A (-5, -2) Quadrant I II **III** IV Axis

28 Point B (2, 4) Quadrant **I** II III IV Axis

29 Point C (4, 0) Quadrant I II III IV **Axis**

30 Point D (3, -3) Quadrant I II **III** IV Axis

31 Point E (-3, 5) Quadrant **II** III IV Axis

32 Point F (0, 0) Quadrant I II III IV **Axis**

33 Point G (5, -5) Quadrant I II **III** IV Axis

34 Point H (-2, 4) Quadrant **II** III IV Axis

35 Point I (1, 2) Quadrant **I** II III IV Axis

36 Point J (1, -4) Quadrant I II III **IV** Axis

37 Point K (-3, -4) Quadrant I II **III** IV Axis

38 Point L (-1, 0) Quadrant I II III IV **Axis**

39 Point M (-4, 2) Quadrant **II** III IV Axis

40 Point N (3, -5) Quadrant I II III **IV** Axis

41 Point O (0, -3) Quadrant I II III IV **Axis**

42 Point P (-5, -5) Quadrant I II **III** IV Axis

43 Point R (-2, 1) Quadrant **II** III IV Axis

44 Point S (2, -2) Quadrant I II III **IV** Axis

Substitution & Table of Values

Substitution means replacing a pronumeral (letter) with a number. You are given the number to substitute, either in the question or in a table. Here are some examples.

Example - Substitution

If $a = 4$ and $b = 5$, find the value for:

$a + b = 4 + 5$
$= 9$

$3ab = 3 \times 4 \times 5$
$= 60$

$a^2b = 4^2 \times 5$
$= 80$

Example - Completing a table of values

$y = 3x + 2$

x	0	1	2	3	4
y	2	5	8	11	14

$y = 3 \times 0 + 2 = 2$
$y = 3 \times 1 + 2 = 5$
$y = 3 \times 2 + 2 = 8$
$y = 3 \times 3 + 2 = 11$
$y = 3 \times 4 + 2 = 14$

Completing a table of values is just like the substitution above except the value changes in steps.

So you replace the x with 0 for the first value. Then replace x with 1 for the second value.

Notice that there is a pattern, the y-values are increasing by three. You should use this as a check, if the numbers aren't increasing or decreasing in a regular pattern, then you have made a mistake.

The last column features questions that are different in appearance. Often these have x and y on the one side of the '=' sign. Here is an example.

$x + y = 15$

x	-2	-1	0	1	2
y	17	16	15	14	13

Ask yourself: What do I add to -2 to get to 15?

If I add 2 to -2 I get zero, then 15 more will be 15. So I add 17 to -2 to get 15. $y = 17$

Ask yourself: What do I add to -1 to get to 15?

If I add 1 to -1 I get zero, then 15 more will be 15. So I add 16 to -1 to get 15. $y = 16$

$-2 + y = 15$
$-1 + y = 15$
$0 + y = 15$
$1 + y = 15$
$2 + y = 15$

Substitution & Table of Values

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If $a = 2$, $b = 3$ and $c = 5$, substitute the values into the expressions and solve.

Examples $c - a$ $= 5 - 2$ $= 3$

$2a + c$ $= 2 \times 2 + 5$ $= 9$

1 $4a$ $= \square \times \square$ $=$ _____

2 bc $= \square \times \square$ $=$ _____

3 $3c + 2$ $= \square \times \square + \square$ $=$ _____

4 $b^2 - a$ $= \square^2 - \square$ $=$ _____

5 $ac + bc$ = _____ = ___

6 $c - 3b$ = _____ = ___

7 $\frac{5b + c}{a} = \frac{\square}{\square}$ = ___

8 $a^2 - b^2$ = _____ = ___

9 $a(a - c)$ = _____ = ___

10 $10 - ac$ = _____ = ___

Use $x = 0, 1, 2, \ldots$ And so on to complete these.

Using $y = 2x$, find y when:

10 $x = 0$: $2 \times \square$ = ___

11 $x = 1$: $\square \times \square$ = ___

12 $x = 2$: $\square \times \square$ = ___

13 $x = 3$: $\square \times \square$ = ___

Using $y = 3x + 2$, find y when:

14 $x = 0$: $\square \times \square + \square$ = ___

15 $x = 1$: $\square \times \square + \square$ = ___

16 $x = 2$: $\square \times \square + \square$ = ___

17 $x = 3$: $\square \times \square + \square$ = ___

18 $x = 4$: $\square \times \square + \square$ = ___

Complete these table of values.

19 $y = x + 5$

x	0	1	2	3	4
y					

20 $y = 2x + 3$

x	0	1	2	3	4
y					

21 $y = 5x - 1$

x	0	1	2	3	4
y					

22 $y = -x + 6$

x	0	1	2	3	4
y					

23 $y = 12 - 2x$

x	0	1	2	3	4
y					

Note that the x values have changed

24 $y = x + 10$

x	-2	-1	0	1	2
y					

25 $y = -x - 7$

x	-2	-1	0	1	2
y					

26 $y = -4x - 7$

x	-2	-1	0	1	2
y					

27 $y = 3x - 9$

x	-4	-2	0	2	4
y					

28 $y = -5x + 11$

x	-4	-2	0	2	4
y					

These are a bit different

29 $y - x = 5$

x	0	1	2	3	4
y					

30 $x + y = 11$

x	0	1	2	3	4
y					

31 $y - 7 = x$

x	0	1	2	3	4
y					

32 $x - y = 0$

x	0	1	2	3	4
y					

Note that the x values have changed

33 $x = 8 - y$

x	-2	-1	0	1	2
y					

34 $y = -x$

x	-2	-1	0	1	2
y					

Substitution & Table of Values

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If $a = 2$, $b = 3$ and $c = 5$, substitute the values into the expressions and solve.

Examples $c - a$ $= 5 - 2$ $= 3$ $\qquad$ $2a + c$ $= 2 \times 2 + 5$ $= 9$

1 $4a$ $= 4 \times 2$ $= 8$

2 bc $= 3 \times 5$ $= 15$

3 $3c + 2$ $= 3 \times 5 + 2$ $= 17$

4 $b^2 - a$ $= 3^2 - 2$ $= 7$

5 $ac + bc = 2 \times 5 + 3 \times 5 = 25$

6 $c - 3b = 5 - 3 \times 3 = -4$

7 $\frac{5b + c}{a} = \frac{5 \times 3 + 5}{2} = 10$

8 $a^2 - b^2 = 2^2 - 3^2 = -5$

9 $a(a - c) = 2 \times (2 - 5) = -6$

10 $10 - ac = 10 - 2 \times 5 = 0$

Use $x = 0, 1, 2, \ldots$ And so on to complete these.

Using $y = 2x$, find y when:

10 $x = 0$: $2 \times 0 = 0$

11 $x = 1$: $2 \times 1 = 2$

12 $x = 2$: $2 \times 2 = 4$

13 $x = 3$: $2 \times 3 = 6$

Using $y = 3x + 2$, find y when:

14 $x = 0$: $3 \times 0 + 2 = 2$

15 $x = 1$: $3 \times 1 + 2 = 5$

16 $x = 2$: $3 \times 2 + 2 = 8$

17 $x = 3$: $3 \times 3 + 2 = 11$

18 $x = 4$: $3 \times 4 + 2 = 14$

Complete these table of values.

19 $y = x + 5$

x	0	1	2	3	4
y	5	6	7	8	9

20 $y = 2x + 3$

x	0	1	2	3	4
y	3	5	7	9	11

21 $y = 5x - 1$

x	0	1	2	3	4
y	-1	4	9	14	19

22 $y = -x + 6$

x	0	1	2	3	4
y	6	5	4	3	2

23 $y = 12 - 2x$

x	0	1	2	3	4
y	12	10	8	6	4

Note that the x values have changed

24 $y = x + 10$

x	-2	-1	0	1	2
y	8	9	10	11	12

25 $y = -x - 7$

x	-2	-1	0	1	2
y	-5	-6	-7	-8	-9

26 $y = -4x - 7$

x	-2	-1	0	1	2
y	1	-3	-7	-11	-15

27 $y = 3x - 9$

x	-4	-2	0	2	4
y	-21	-15	-9	-3	3

28 $y = -5x + 11$

x	-4	-2	0	2	4
y	31	21	11	1	-9

These are a bit different

29 $y - x = 5$

x	0	1	2	3	4
y	5	6	7	8	9

30 $x + y = 11$

x	0	1	2	3	4
y	11	10	9	8	7

31 $y - 7 = x$

x	0	1	2	3	4
y	7	8	9	10	11

32 $x - y = 0$

x	0	1	2	3	4
y	0	1	2	3	4

Note that the x values have changed

33 $x = 8 - y$

x	-2	-1	0	1	2
y	10	9	8	7	6

34 $y = -x$

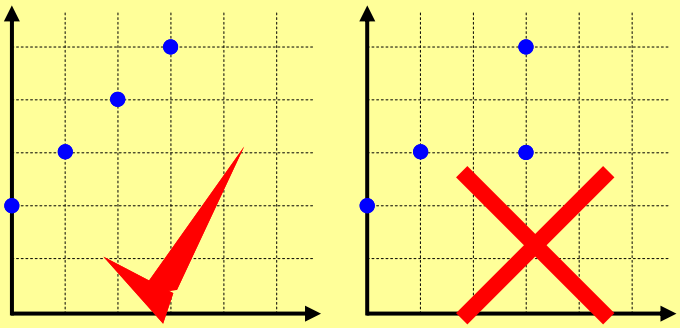
x	-2	-1	0	1	2
y	2	1	0	-1	-2

Plotting Points & Drawing Straight Lines

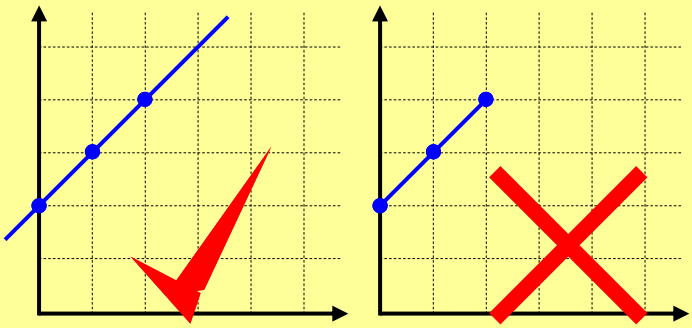
If you can plot points on a grid, drawing straight lines is just drawing a line through them. This sheet removes calculation from the process and just relies on you plotting points correctly and then drawing a line correctly.

There are three key features of drawing a straight line. These are:

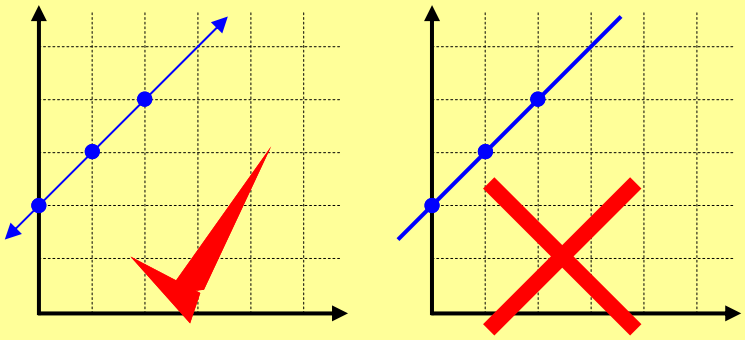
STEP 1 Make sure that your points are in a straight line.



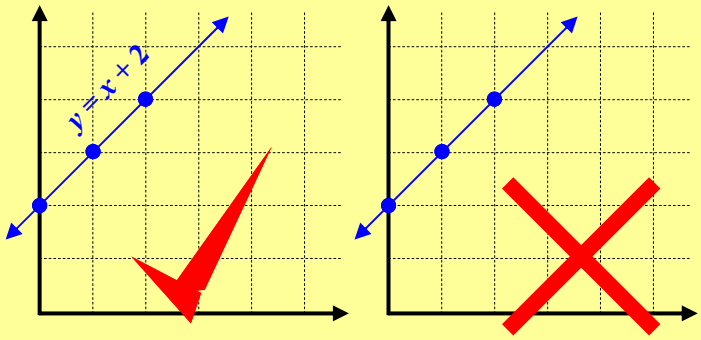
STEP 2 Draw a line the full size of the grid, not just between the first and last dot.



STEP 3 Put arrow heads on each end of the line.



STEP 4 Write the formula for the line on top of the line. Write it in pencil as another line may plot over it later and you will have to move the name.



Column 1 gives you the points to plot as ordered pairs (x co-ord, y co-ord). Plot the points then draw a line through them, keeping the above in mind. The second graph asks you to list all the points where an intersection of lines occurs.

Column 2 is the same except that rather than the points being given as ordered pairs they are in tables. The x value for the point is in the top row, the y value is in the bottom row.

Plotting Points & Drawing Straight Lines

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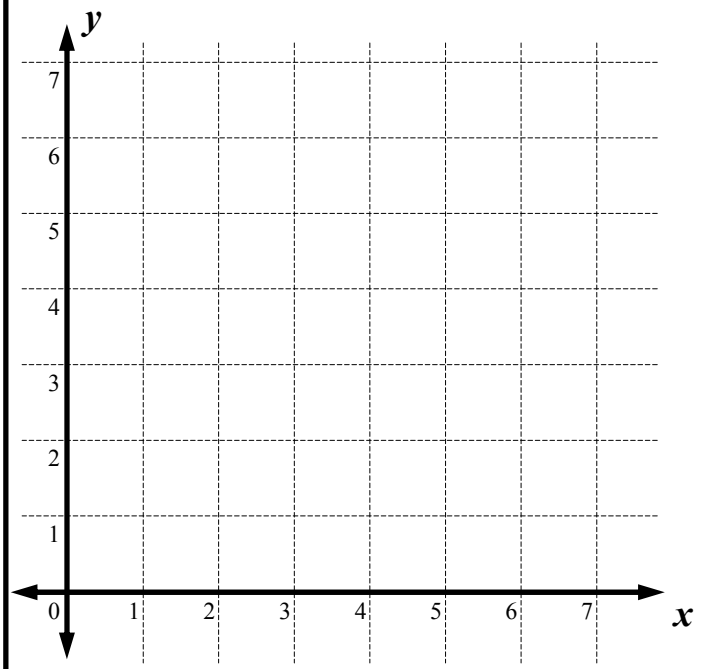
Plot the points then draw a line through them. Use a ruler, show arrow heads and write the equation of the line on the line. The line should be the full size of the graph, not stop at the last point.

1 $y = 2x + 1$ Plot Points: (0, 1), (1, 3), (2, 5).

2 $y = -x + 6$ Plot Points: (0, 6), (2, 4), (4, 2).

3 $y = x - 3$ Plot Points: (3, 0), (4, 1), (5, 2).

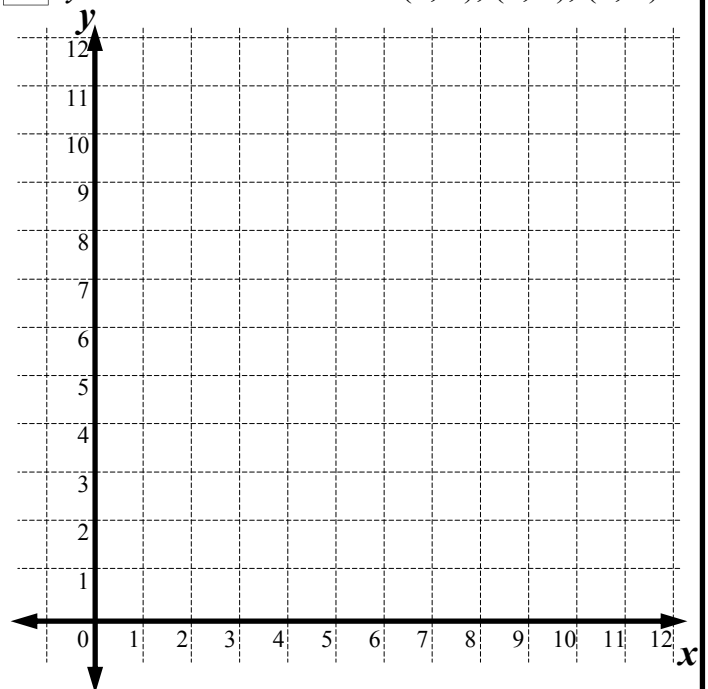


Use the lower grid for these

4 $y = 12 - x$ Plot Points: (0, 12), (2, 10), (4, 8)

5 $y = 3x$ Plot Points: (0, 0), (2, 6), (4, 12)

6 $y = x + 2$ Plot Points: (0, 2), (2, 4), (4, 6)



7 List 3 points where lines intersect.

(□, □), (□, □), (□, □).

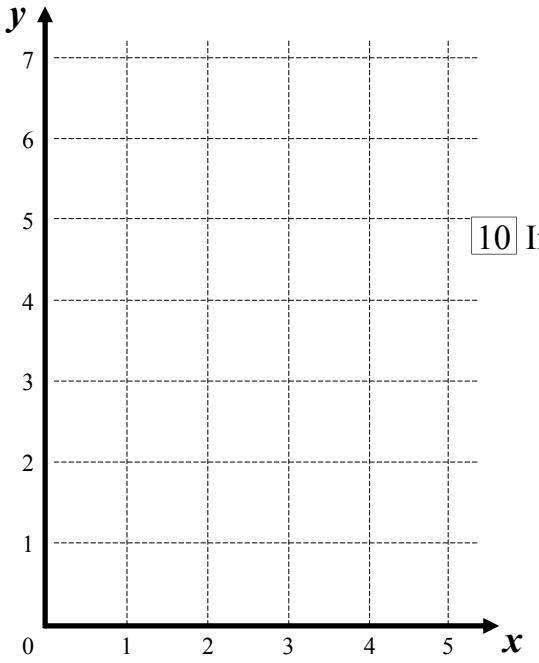
Now use the table of values to plot the lines. Go across 'x' and up 'y'.

8 $y = x + 2$

x	0	1	2	3	4
y	2	3	4	5	6

9 $y = -x + 6$

x	0	1	2	3	4
y	6	5	4	3	2



10 Intersection (□, □) Point

11 $x + y = 5$

x	0	1	2	3	4
y	5	4	3	2	1

12 $y = 2x - 4$

x	0	1	2	3	4
y	-4	-2	0	2	4

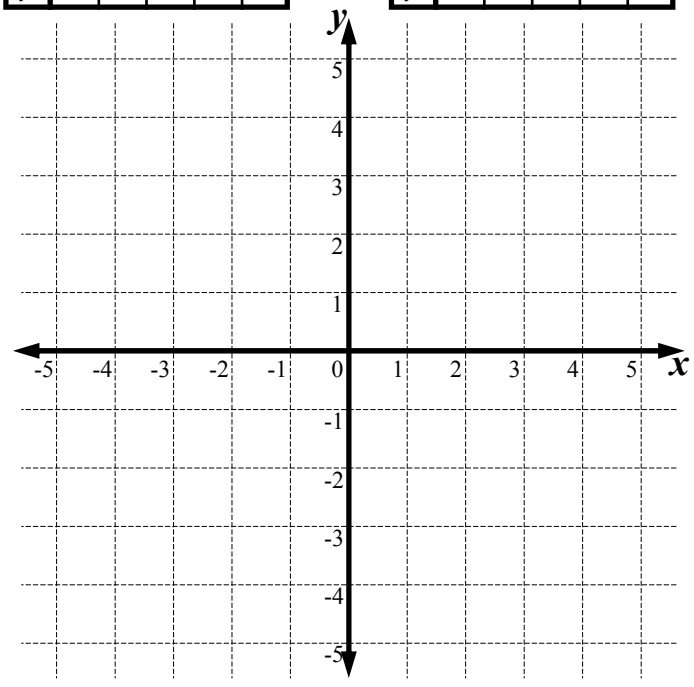
Use the lower grid for these

13 $y = ½x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4

14 $y = -x - 4$

x	-4	-3	-2	-1	0
y	0	-1	-2	-3	-4



15 List the 5 points where lines intersect.

(□, □), (□, □), (□, □), (□, □), (□, □).

Plotting Points & Drawing Straight Lines

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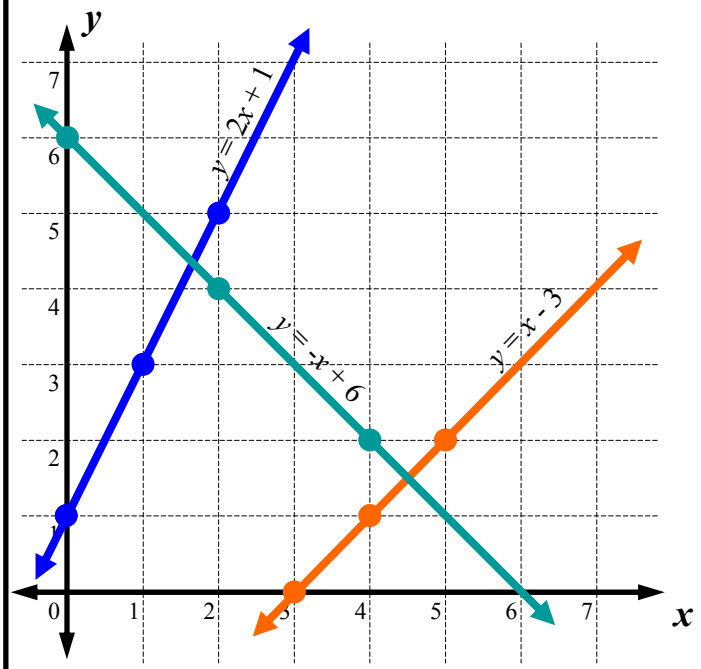
Plot the points then draw a line through them. Use a ruler, show arrow heads and write the equation of the line on the line. The line should be the full size of the graph, not stop at the last point.

1 $y = 2x + 1$ Plot Points: (0, 1), (1, 3), (2, 5).

2 $y = -x + 6$ Plot Points: (0, 6), (2, 4), (4, 2).

3 $y = x - 3$ Plot Points: (3, 0), (4, 1), (5, 2).

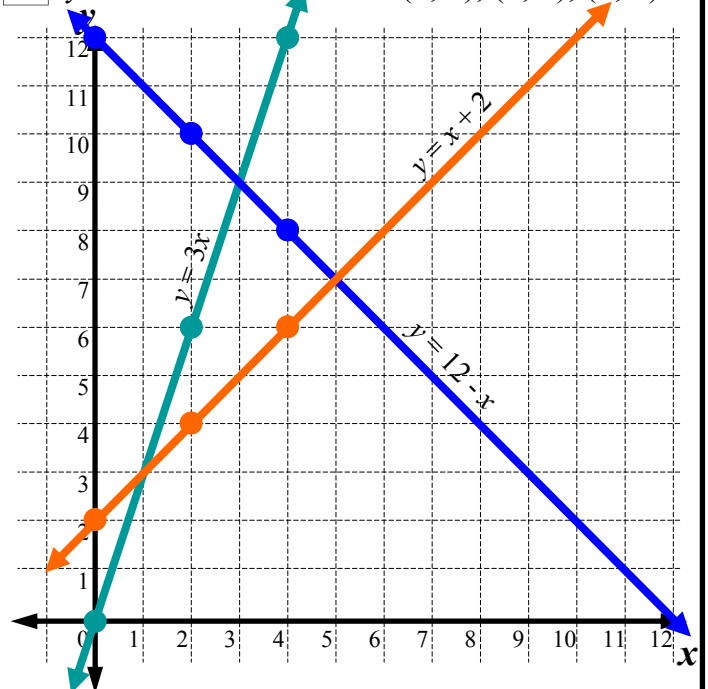


Use the lower grid for these

4 $y = 12 - x$ Plot Points: (0, 12), (2, 10), (4, 8)

5 $y = 3x$ Plot Points: (0, 0), (2, 6), (4, 12)

6 $y = x + 2$ Plot Points: (0, 2), (2, 4), (4, 6)



7 List 3 points where lines intersect.

(5, 7), (1, 3), (3, 9).

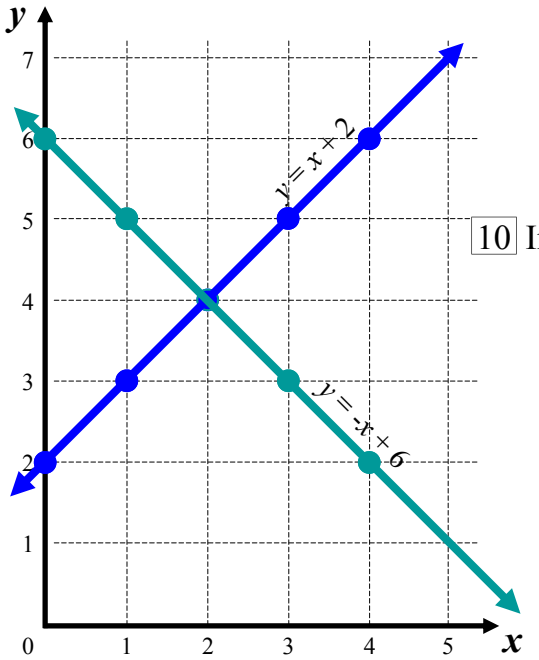
Now use the table of values to plot the lines. Go across 'x' and up 'y'.

8 $y = x + 2$

x	0	1	2	3	4
y	2	3	4	5	6

9 $y = -x + 6$

x	0	1	2	3	4
y	6	5	4	3	2



10 Intersection (2, 4) Point

11 $x + y = 5$

x	0	1	2	3	4
y	5	4	3	2	1

12 $y = 2x - 4$

x	0	1	2	3	4
y	-4	-2	0	2	4

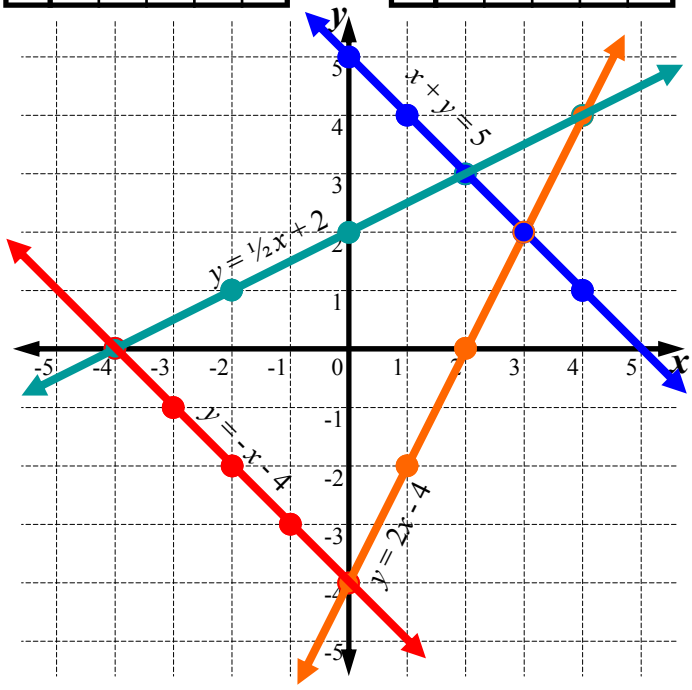
Use the lower grid for these

13 $y = \frac{1}{2}x + 2$

x	-4	-2	0	2	4
y	0	1	2	3	4

14 $y = -x - 4$

x	-4	-3	-2	-1	0
y	0	-1	-2	-3	-4



15 List the 5 points where lines intersect.

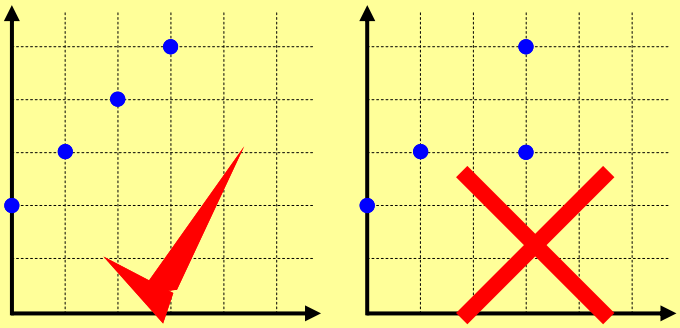
(4, 4), (2, 3), (3, 2), (0, -4), (-4, 0).

Straight Lines - Positive Quadrant & Cost Functions

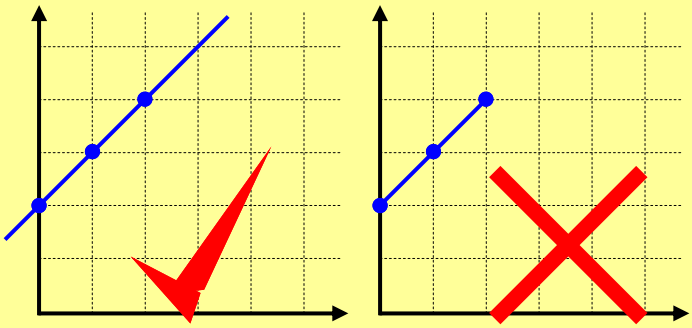
This sheet requires you to complete a table of values and then plot a straight line. Straight lines can represent a mathematical equation, they are also used in business to represent sales, costs and profit.

Column 1 asks you to plot the linear equations supplied. Complete the table of values by substituting *x* with the value from the table. Solve and write the answer as the *y* value. Then plot the points and complete the straight line. Remember the process shown below.

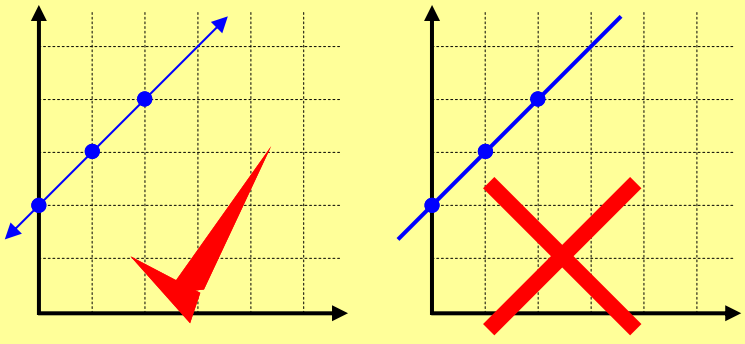
STEP 1 Make sure that your points are in a straight line.



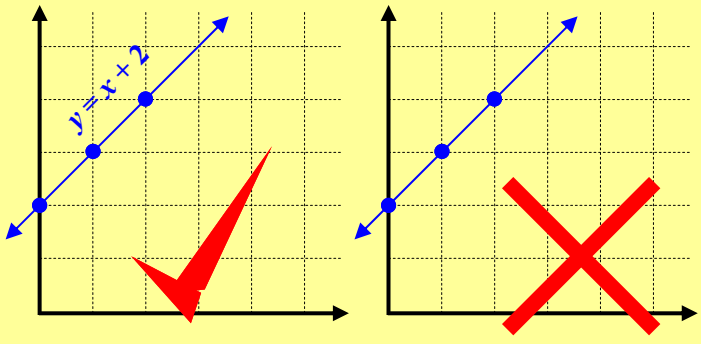
STEP 2 Draw a line the full size of the grid, not just between the first and last dot.



STEP 3 Put arrow heads on each end of the line.



STEP 4 Write the formula for the line on top of the line. Write it in pencil as another line may plot over it later and you will have to move the name.



Column 2 is a cost function, plotting the results for a cricket bat manufacturer. This grid will have two lines on it, one for the cost of manufacture, the other for the income from the sales. Don't confuse income with profit. Profit is : Profit = Sales - Costs. The term 'break-even' is the amount of production required to start making a profit, or to recoup costs. If the sales don't result in the break-even point being met, the venture will result in a loss.

Complete the tables and the graph, plot the lines in different colours, you can use a broken line for the income and then make it solid after break-even. Match the line colour in the key. A cost function line doesn't necessarily require a label or arrow heads, but ask your teacher.

Straight Lines - Positive Quadrant & Cost Functions

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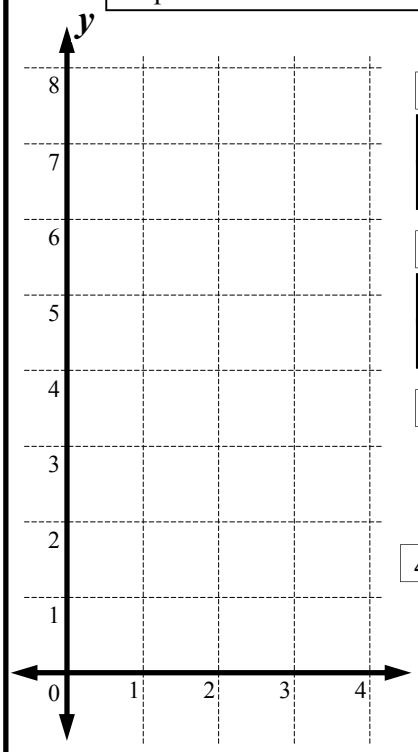
Complete the table of values then plot the points. Draw the line and remember to show arrow heads and the name of the line. List the coordinates for the points of intersection when required.



1 $y = 2x$

x	0	1	2	3	4
y					

2 $y = x + 3$

x	0	1	2	3	4
y					

3 $y = 3 - x$

x	0	1	2	3
y				

4 Points of Intersection:

$(\square, \square), (\square, \square),$

$(\square, \square).$

Use the lower grid for these

5 $y = x - 3$

x	3	4	5	6	7
y					

6 $y = 3x - 5$

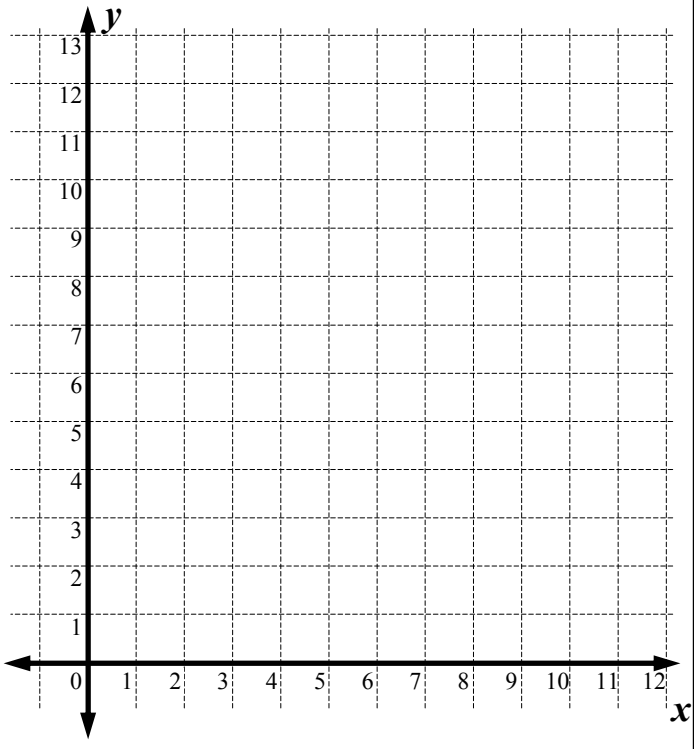
x	2	3	4	5	6
y					

7 $y = \frac{1}{2}x + 4$

x	0	2	4	6	8
y					

8 $x + y = 10$

x	0	2	4	6	8
y					



"Mite-e-Willow" manufacture cricket bats. The manufacture cost function and sales function are shown. Complete the table of values and plot the lines. The intersection is the break even point.

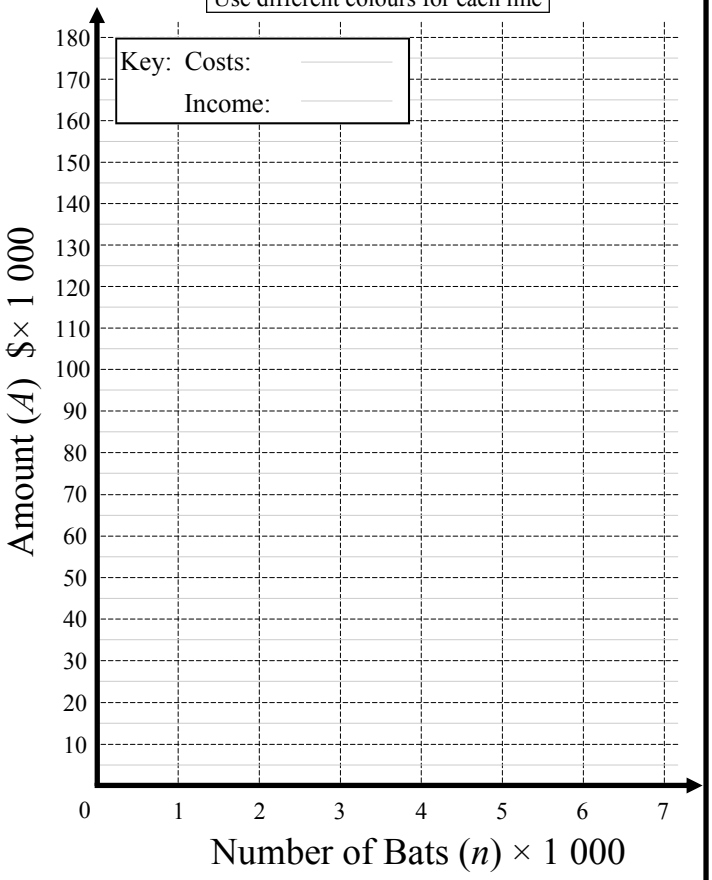
9 *Operation Costs:* $A = 10n + 90\,000$

n	1 000	2 000	3 000	4 000	5 000
A					

10 *Income Sales:* $A = 25n$

n	1 000	2 000	3 000	4 000	5 000
A					

Use different colours for each line



11 How many bats have to be sold to break even? _____

12 How much does it cost to make 2 500 bats? _____

13 What was the income on selling 7 000 bats? _____

14 What was the profit on selling 7 000 bats? _____

"Mite-e-Willow" sold 38 000 bats for the year, calculate:

15 The cost of production

$A = 10n + 90\,000$

$= 10 \times \square + 90\,000$

$A =$

16 The sales income

$A =$

17 The profit. _____

Straight Lines - Positive Quadrant & Cost Functions

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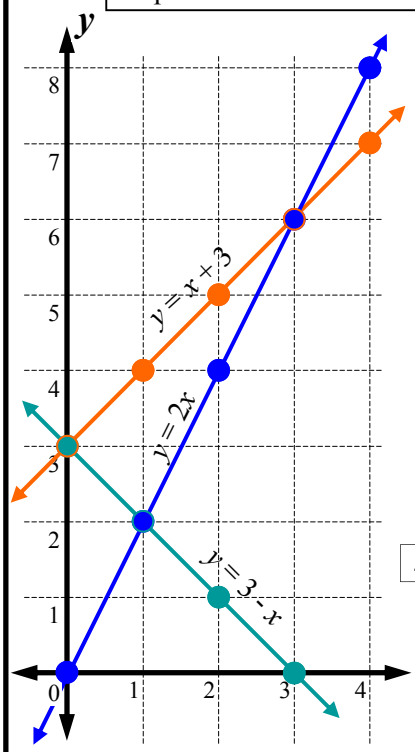
Complete the table of values then plot the points. Draw the line and remember to show arrow heads and the name of the line. List the coordinates for the points of intersection when required.



1 $y = 2x$

x	0	1	2	3	4
y	0	2	4	6	8

2 $y = x + 3$

x	0	1	2	3	4
y	3	4	5	6	7

3 $y = 3 - x$

x	0	1	2	3
y	3	2	1	0

4 Points of Intersection: ([0], [3]), ([1], [2]), ([3], [6]).

Use the lower grid for these

5 $y = x - 3$

x	3	4	5	6	7
y	0	1	2	3	4

6 $y = 3x - 5$

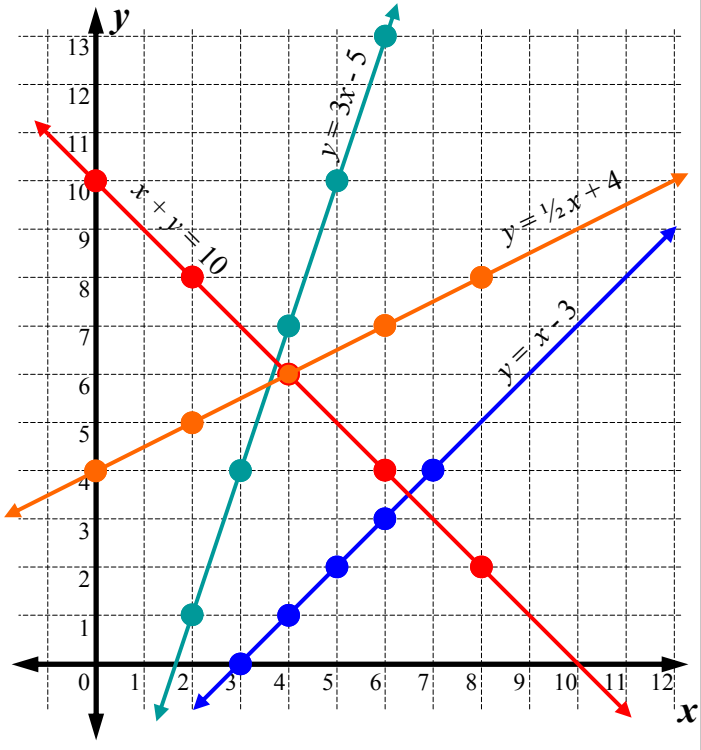
x	2	3	4	5	6
y	1	4	7	10	13

7 $y = \frac{1}{2}x + 4$

x	0	2	4	6	8
y	4	5	6	7	8

8 $x + y = 10$

x	0	2	4	6	8
y	10	8	6	4	2



"Mite-e-Willow" manufacture cricket bats. The manufacture cost function and sales function are shown. Complete the table of values and plot the lines. The intersection is the break even point.

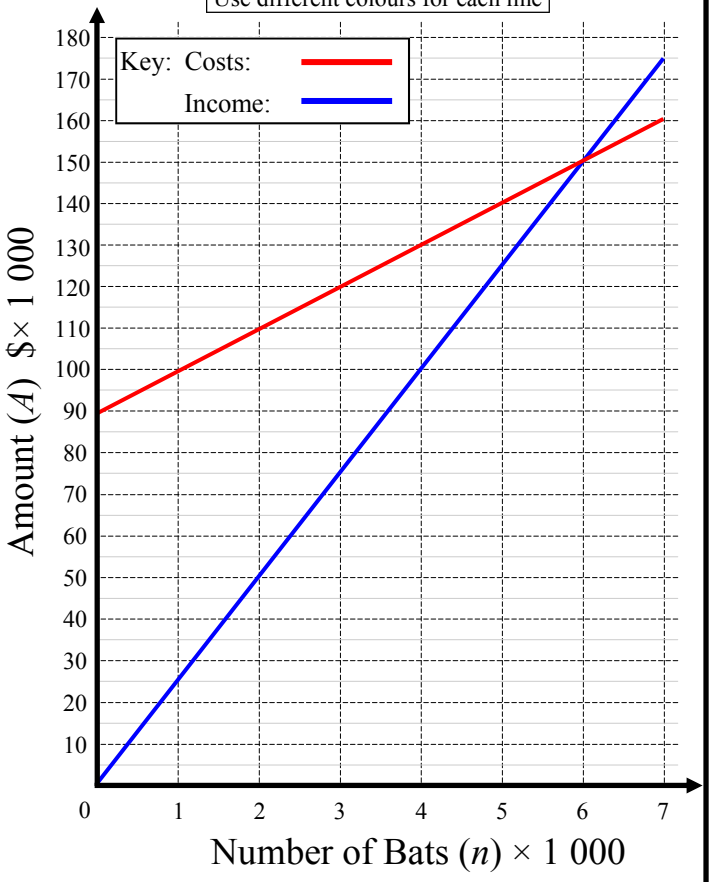
9 *Operation Costs:* $A = 10n + 90\,000$

n	1 000	2 000	3 000	4 000	5 000
A	100 000	110 000	120 000	130 000	140 000

10 *Income Sales:* $A = 25n$

n	1 000	2 000	3 000	4 000	5 000
A	25 000	50 000	75 000	100 000	125 000

Use different colours for each line



11 How many bats have to be sold to break even? 6 000

12 How much does it cost to make 2 500 bats? $115 000

13 What was the income on selling 7 000 bats? $175 000

14 What was the profit on selling 7 000 bats? $15 000

"Mite-e-Willow" sold 38 000 bats for the year, calculate:

15 The cost of production

$A = 10n + 90\,000$

$= 10 \times$ [38 000] $+ 90\,000$

$A =$ $470 000

16 The sales income

$A = 25n$

$= 25 \times 38\,000$

$A =$ $950 000

17 The profit. Profit = 950 000 - 470 000 = $480 000

Straight Lines - All Quadrants

This sheet deals with plotting straight lines with points being plotted in all quadrants. The most difficult aspect of this sheet is with the use of negative numbers. You have to remember that when you multiply numbers with a different sign that the answer is negative, or when the signs are both negative then the answer would be positive.

Example

$y = -2x + 3$

x	-4	-2	0	2	4
y	11	7	3	-1	-5

$-2 \times 4 + 3 = -8 + 3 = -5$

$-2 \times 2 + 3 = -4 + 3 = -1$

$-2 \times 0 + 3 = 0 + 3 = 3$

$-2 \times -2 + 3 = 4 + 3 = 7$

$-2 \times -4 + 3 = 8 + 3 = 11$

It is very easy to make a mistake with negative number operations. But remember that the y-values in the Table of Values increase or decrease in a pattern.

$y = -2x + 3$

x	-4	-2	0	2	4
y	11	7	3	-1	-5

-4 -4 -4 -4

The y-values decrease by 4, so if a point wasn't 4 less than the previous number, you know to check for an error.

IMPORTANT - It may be that you use the pattern to complete the table after you have used substitution for two values. If you do this the check system won't work.

Then plot the points and draw a line through them. Remember arrow heads and to label the line. Label the line in pencil so that you can rub it out later if another line gets in the way.

Important features of lines are:

Slope (or gradient) - is the line going up to the left or up to the right? Is the line more or less steep than the other lines?

y-intercept - where is the line cutting the y-axis, is this point visible in the line equation?

Straight Lines - All Quadrants

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Complete the table of values then plot the points. Draw the line with arrow heads and the equation on the line. With values in the table that can't be plotted, construct the line with the points that you can plot.

1 $y = -2x$

x	-4	-2	0	2	4
y					

2 $y = x + 1$

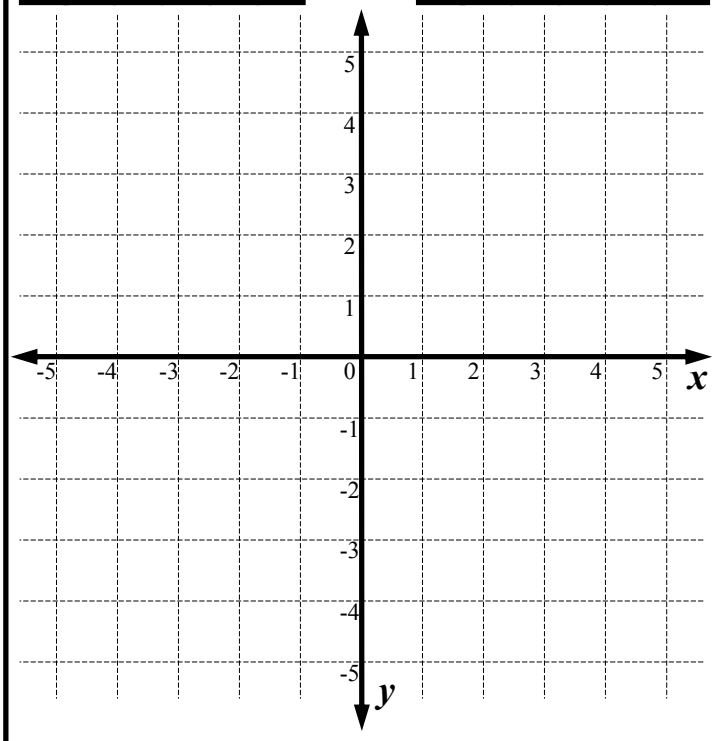
x	-4	-2	0	2	4
y					

3 $y = -\frac{1}{2}x + 1$

x	-4	-2	0	2	4
y					

4 $y = -2x - 5$

x	-4	-2	0	2	4
y					



5 Lines in Q.1 and Q.4 both have '-2' in front of the x. Look at the lines and describe a common feature. _____

6 Lines in Q.2 and Q.3 both have '+1' in the equation. Look at the lines, describe a common feature. _____

Use the top grid, in the next column for Q.7, 8 & 9. below

7 $y = 3x - 6$

x	-4	-2	0	2	4
y					

8 $y = 2x + 1$

x	-4	-2	0	2	4
y					

9 $y = x$

x	-4	-2	0	2	4
y					

Use the lower grid, in the next column for these below

10 $y = -2x + 1$

x	-4	-2	0	2	4
y					

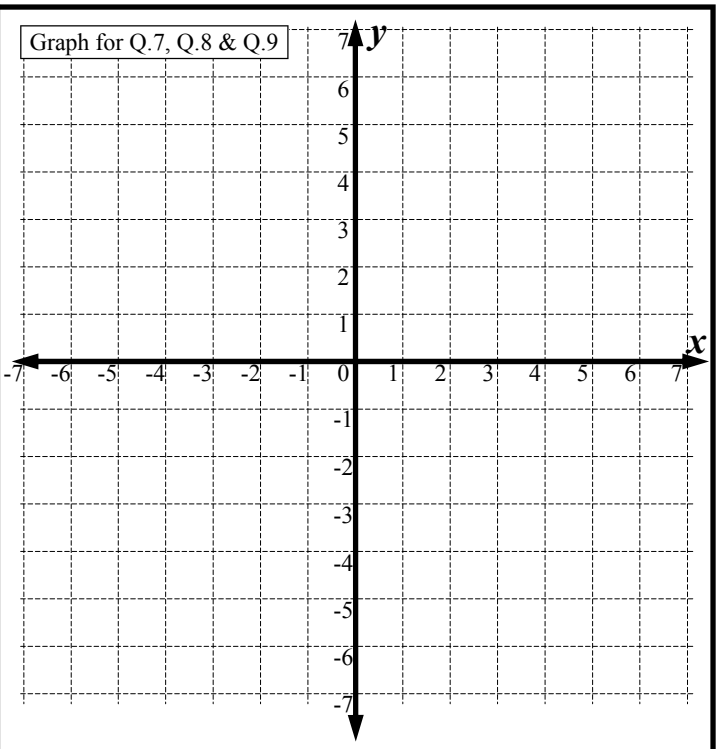
11 $y = -3x - 6$

x	-4	-2	0	2	4
y					

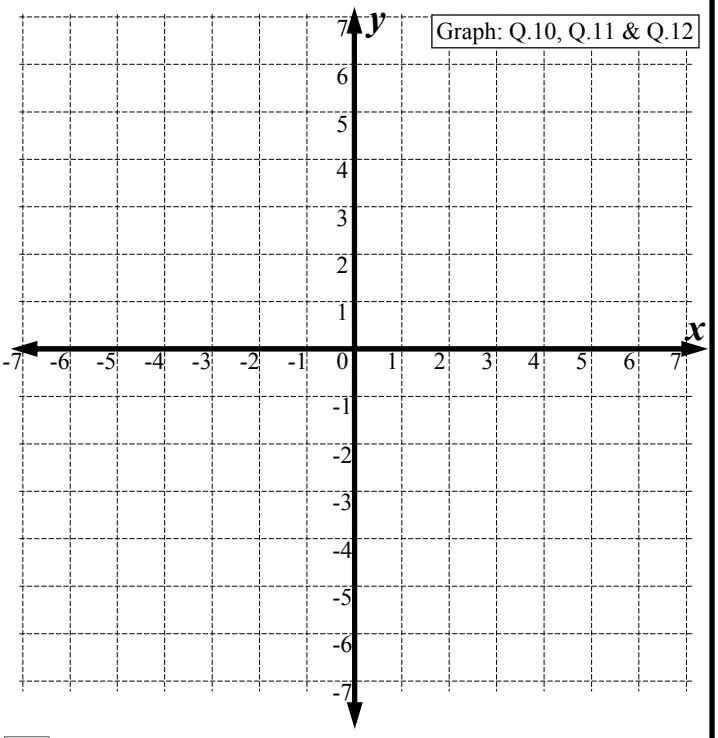
12 $y = -x$

x	-4	-2	0	2	4
y					

Graph for Q.7, Q.8 & Q.9



Graph: Q.10, Q.11 & Q.12



13 What do all the lines in the top grid have in common? _____

14 What do all the lines in the lower grid have in common? _____

15 Complete: A negative number in front of the x means that the line 'goes up' to the _____

16 Complete: A positive number in front of the x means that the line 'goes up' to the _____

17 Some lines are more steep than others. Which is more steep … $y = 4x$ or $y = 3x$? _____

Straight Lines - All Quadrants

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Complete the table of values then plot the points. Draw the line with arrow heads and the equation on the line. With values in the table that can't be plotted, construct the line with the points that you can plot.

1 $y = -2x$

x	-4	-2	0	2	4
y	8	4	0	-4	-8

2 $y = x + 1$

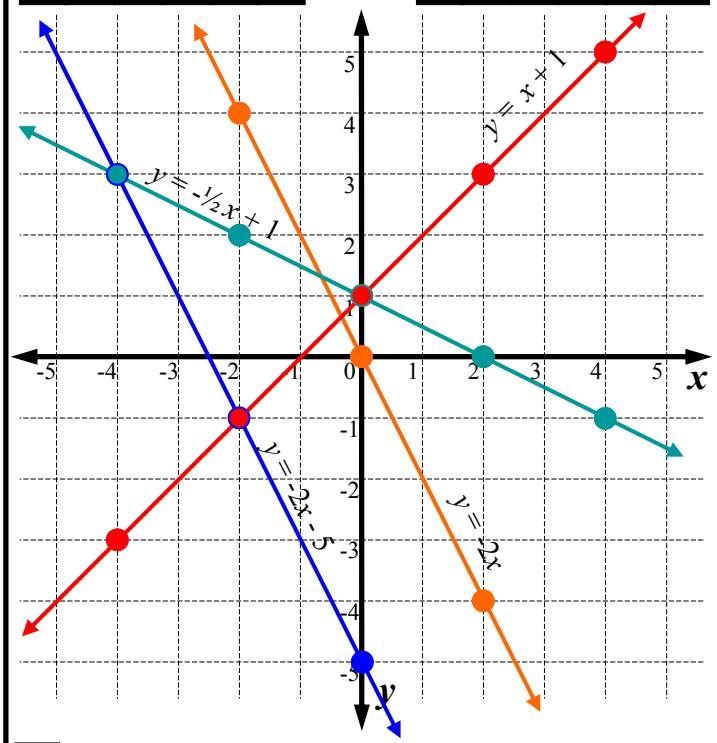
x	-4	-2	0	2	4
y	-3	-1	1	3	5

3 $y = -\frac{1}{2}x + 1$

x	-4	-2	0	2	4
y	3	2	1	0	-1

4 $y = -2x - 5$

x	-4	-2	0	2	4
y	3	-1	-5	-9	-13



5 Lines in Q.1 and Q.4 both have '-2' in front of the x. Look at the lines and describe a common feature. The lines are parallel. (Same gradient)

6 Lines in Q.2 and Q.3 both have '+1' in the equation. Look at the lines, describe a common feature. Both cut y axis at 1. (Same y-intercept)

Use the top grid, in the next column for Q.7, 8 & 9. below

7 $y = 3x - 6$

x	-4	-2	0	2	4
y	-18	-12	-6	0	6

8 $y = 2x + 1$

x	-4	-2	0	2	4
y	-7	-3	1	5	9

9 $y = x$

x	-4	-2	0	2	4
y	-4	-2	0	2	4

Use the lower grid, in the next column for these below

10 $y = -2x + 1$

x	-4	-2	0	2	4
y	9	5	1	-3	-7

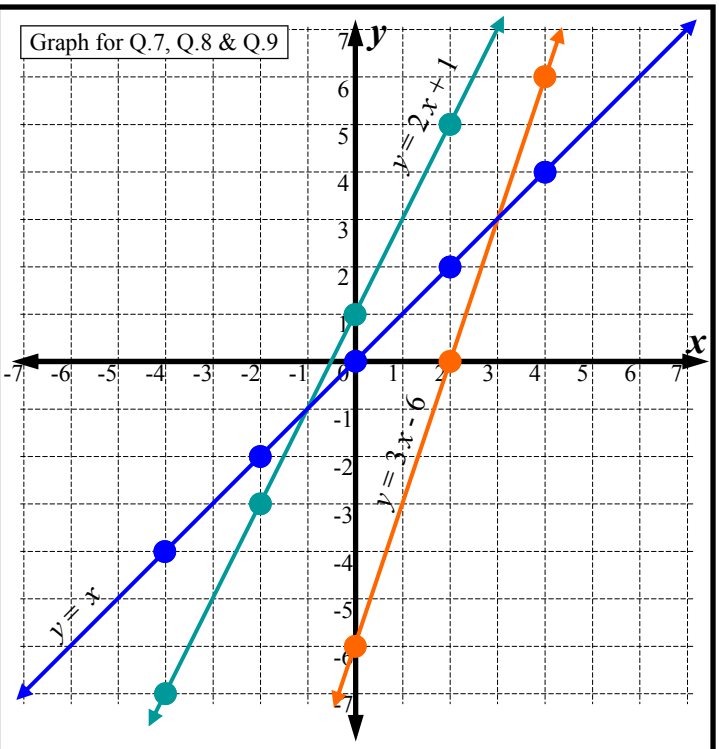
11 $y = -3x - 6$

x	-4	-2	0	2	4
y	6	0	-6	-12	-18

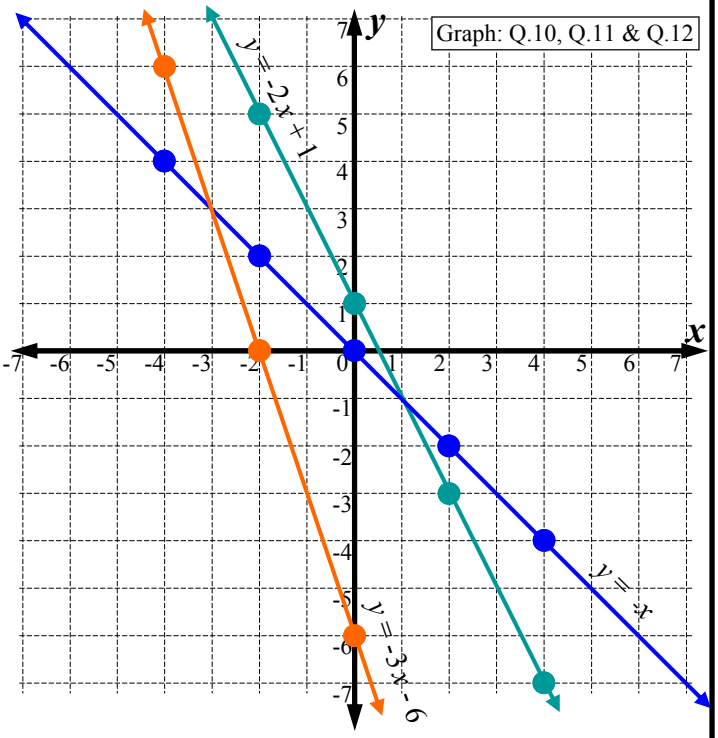
12 $y = -x$

x	-4	-2	0	2	4
y	4	2	0	-2	-4

Graph for Q.7, Q.8 & Q.9



Graph: Q.10, Q.11 & Q.12



13 What do all the lines in the top grid have in common? All go up to the right. (or down to left)

14 What do all the lines in the lower grid have in common? All go up to the left. (or down to right)

15 Complete: A negative number in front of the x means that the line 'goes up' to the left.

16 Complete: A positive number in front of the x means that the line 'goes up' to the right.

17 Some lines are more steep than others. Which is more steep … $y = 4x$ or $y = 3x$? $y = 4x$

Finding the Equation of a Line

Imagine being given a table of values and you have to find the equation of the line. The process changes from a substitution process to an extraction process. In other words, the information is removed from the table.

Two values are required to form a straight line equation. The coefficient of the x is required (called the gradient) and the constant (number by itself) is also required, called the y-intercept. So two numbers needed and they can be either positive or negative. The process has examples on the worksheet but a step by step process is shown below.

Equation of a line is in the form:

Gradient y-intercept

$y = \square\, x + \square$

Example

STEP 1 A typical table of values. It is completed but has no equation.

x	0	1	2	3	4
y	5	7	9	11	13

STEP 2 Find the value of y, when $x = 0$.

x	0	1	2	3	4
y	5	7	9	11	13

At $x = 0$
$y =$ 5

STEP 3 Now find the co-efficient, the value written in front of the x. How much do the values increase by?

x	0	1	2	3	4
y	5	7	9	11	13

+2 +2 +2 +2

Gradient: +2

STEP 4 Write the equation.

$y =$ 2 x + 5

Column 1 asks you to find the value of y at $x = 0$, (called the y-intercept) and the x co-efficient called the gradient. Column 2 then has the extra step of finding the line equation itself.

Column 3 features a graph of the line rather than a table. Extract the co-ordinates of the points from the graph and write them in the table. The point furthest to the left (smallest x-value) should be the first point entered into the table, then work to the right. Once the points are in the table …. same process as before.

Finding the Equation of a Line

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Find the y-value at $x = 0$, then find the Gradient amount between y values.

Example

x	0	1	2	3	4
y	5	7	9	11	13

+2 +2 +2 +2

At $x = 0$ $y =$ +5 Gradient: +2

1

x	0	1	2	3	4
y	-2	1	4	7	10

At $x = 0$ $y =$ ____ Gradient: ____

2

x	0	1	2	3	4
y	0	5	10	15	20

At $x = 0$ $y =$ ____ Gradient: ____

3

x	0	1	2	3	4
y	-9	-10	-11	-12	-13

At $x = 0$ $y =$ ____ Gradient: ____

4

x	-2	-1	0	1	2
y	5	6	7	8	9

At $x = 0$ $y =$ ____ Gradient: ____

5

x	-2	-1	0	1	2
y	-2	-1	0	1	2

At $x = 0$ $y =$ ____ Gradient: ____

Now find the equations of the lines given the table of values.

Example

x	0	1	2	3	4
y	-1	-3	-5	-7	-9

-2 -2 -2 -2

At $x = 0$ $y =$ -1 Gradient: -2

The y-value goes at the back, including the sign.

Gradient goes in front of the x including the sign.

Equation: $y =$ -2 x − 1

Answer these using your Column 1 answers.

6 Equation: $y =$ ____ x +/- ____

For Q.6 Use Gradient and y-value from Q.1

7 Equation: $y =$ ____ x

For Q.7 Use Gradient and y-value from Q.2

8 Equation: $y =$ ____ x +/- ____

For Q.8 Use Gradient and y-value from Q.3

9 Equation: $y =$

For Q.9 Use Gradient and y-value from Q.4

10 Equation: $y =$

For Q.10 Use Gradient and y-value from Q.5

11

x	-3	-2	-1	0	1
y	32	22	12	2	-8

At $x = 0$ $y =$ ____ Gradient: ____

Equation: $y =$

12

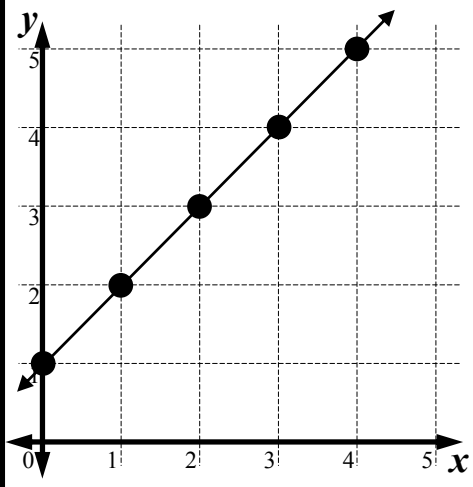
x	0	1	2	3	4
y	8	10	12	14	16

At $x = 0$ $y =$ ____ Gradient: ____

Equation: $y =$

Change the points on the graph to a table then find the line equation by the same method.

13

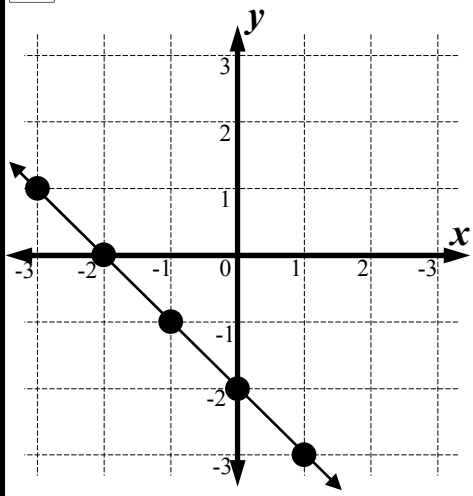


x						
y						

At $x = 0$ $y =$ ____ Gradient: ____

Equation: $y =$

14



x						
y						

At $x = 0$ $y =$ ____ Gradient: ____

Equation: $y =$

Finding the Equation of a Line

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Find the y-value at $x = 0$, then find the Gradient amount between y values.

Example

x	0	1	2	3	4
y	5	7	9	11	13

+2 +2 +2 +2

At $x = 0$ $y =$ +5 Gradient: +2

1

x	0	1	2	3	4
y	-2	1	4	7	10

+3 +3 +3 +3

At $x = 0$ $y =$ -2 Gradient: +3

2

x	0	1	2	3	4
y	0	5	10	15	20

+5 +5 +5 +5

At $x = 0$ $y =$ 0 Gradient: +5

3

x	0	1	2	3	4
y	-9	-10	-11	-12	-13

-1 -1 -1 -1

At $x = 0$ $y =$ -9 Gradient: -1

4

x	-2	-1	0	1	2
y	5	6	7	8	9

+1 +1 +1 +1

At $x = 0$ $y =$ 7 Gradient: +1

5

x	-2	-1	0	1	2
y	-2	-1	0	1	2

+1 +1 +1 +1

At $x = 0$ $y =$ 0 Gradient: +1

Now find the equations of the lines given the table of values.

Example

x	0	1	2	3	4
y	-1	-3	-5	-7	-9

-2 -2 -2 -2

At $x = 0$ $y =$ -1 Gradient: -2

The y-value goes at the back, including the sign.

Gradient goes in front of the x including the sign.

Equation: $y =$ -2 x − 1

Answer these using your Column 1 answers.

6 Equation: $y =$ 3 x − 2

For Q.6 Use Gradient and y-value from Q.1

7 Equation: $y =$ 5 x

For Q.7 Use Gradient and y-value from Q.2

8 Equation: $y =$ - x +/− 9

For Q.8 Use Gradient and y-value from Q.3

9 Equation: $y = x + 7$

For Q.9 Use Gradient and y-value from Q.4

10 Equation: $y = x$

For Q.10 Use Gradient and y-value from Q.5

11

x	-3	-2	-1	0	1
y	32	22	12	2	-8

-10 -10 -10 -10

At $x = 0$ $y =$ 2 Gradient: -10

Equation: $y = -10x + 2$

12

x	0	1	2	3	4
y	8	10	12	14	16

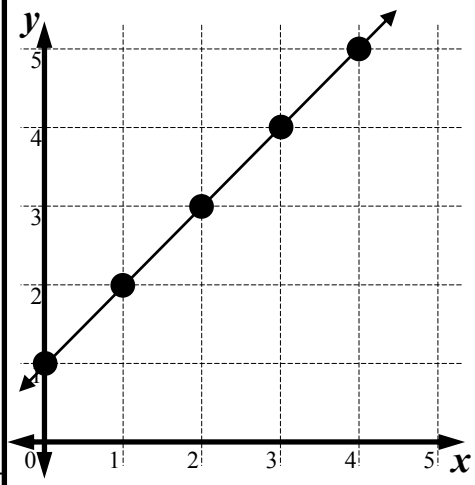
+2 +2 +2 +2

At $x = 0$ $y =$ 8 Gradient: +2

Equation: $y = 2x + 8$

Change the points on the graph to a table then find the line equation by the same method.

13



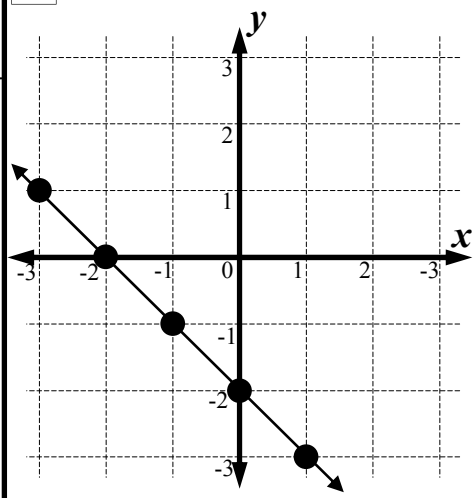
x	0	1	2	3	4
y	1	2	3	4	5

+1 +1 +1 +1

At $x = 0$ $y =$ 1 Gradient: +1

Equation: $y = x + 1$

14



x	-3	-2	-1	0	1
y	1	0	-1	-2	-3

-1 -1 -1 -1

At $x = 0$ $y =$ -2 Gradient: -1

Equation: $y = -x - 2$

Further Finding the Equation of a Line

This sheet builds on the previous sheet and shouldn't be attempted before the other sheet has been completed.

It may be that the table supplied in a question doesn't have '$x = 0$' featured in the top row of the table. If this is the case then the solution is to extend the table until '0' is featured. An example is below, which is the method used to solve Column 1.

Example

STEP 1 A typical table of values. It is completed but has no equation.

x	2	3	4	5	6
y	5	7	9	11	13

STEP 2 Extend the table to assist you to find the value of y, when $x = 0$.

0	1	x	2	3	4	5	6
		y	5	7	9	11	13

STEP 3 Now find the co-efficient, the value written in front of the x. How much do the values increase by?

0	1	x	2	3	4	5	6
		y	5	7	9	11	13

+2 +2 +2 +2

Gradient: +2

STEP 4 Now take the y-values back by subtracting by the gradient. This gives you y when $x = 0$.

0	1	x	2	3	4	5	6
1	3	y	5	7	9	11	13

-2 -2

At $x = 0$ $y =$ 1

STEP 5 Write the equation.

$y =$ 2 x + 1

Column 2 deals with questions that have x-values increasing in steps larger than 1. An example is below, which is the method used to solve Column 2.

Example

STEP 1 A typical table of values. It is completed but has no equation.

x	0	2	4	6	8
y	-1	9	19	29	39

STEP 2 Note that the x-values are going up in steps of 2. So calculate the gradient and divide by 2.

+2 +2 +2 +2

x	0	2	4	6	8
y	-1	9	19	29	39

STEP 3 To find the gradient, how much do the values increase by? Then divide by 2, due to Step 2.

x	0	2	4	6	8
y	-1	9	19	29	39

+10 +10 +10 +10

Gradient: [+10] ÷ [2]

Gradient: [+5]

STEP 4 Read off the y-value at $x = 0$.

x	0	2	4	6	8
y	-1	9	19	29	39

At $x = 0$

$y =$ [-1]

STEP 5 Write the equation.

$y =$ [5] x - [1]

Column 3 is a combination of both skills, so you have to extend the table of values and then also allow for the x-values being in steps and affecting the gradient.

Further Finding the Equation of a Line

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Extend the tables below so that $x = 0$ is shown. Then solve for the line equation.

Example

x	3	4	5	6	7
y	9	11	13	15	17

Rewrite taking x back to zero.

0	1	2	x	3	4	5	6	7
3	5	7	y	9	11	13	15	17

-2 -2 -2 +2 +2 +2 +2

At $x = 0$ Gradient: +2

$y =$ +3

Equation: $y = 2x + 3$

1

		x	2	3	4	5	6
		y	2	5	8	11	14

At $x = 0$ Gradient:

$y =$

Equation:

2

				x	4	5	6	7
				y	11	13	15	17

At $x = 0$ Gradient:

$y =$

Equation:

3

x	-6	-5	-4	-3			
y	-9	-6	-3	0			

At $x = 0$ Gradient:

$y =$

Equation:

These tables have x-values that increase in larger steps. So divide to find the gradient.

4

x	-4	-2	0	2	4
y	-19	-11	-3	5	13

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

5

x	0	5	10	15	20
y	-3	22	47	72	97

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

6

x	-6	-3	0	3	6
y	8	-1	-10	-19	-28

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

7

x	0	2	4	6	8
y	-2	-14	-26	-38	-50

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

8

x	-40	-30	-20	-10	0
y	195	155	115	75	35

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

These are harder, they require the table to be extended and the x-value steps to be considered.

9

		x	4	6	8	10	12
		y	17	23	29	35	41

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

10

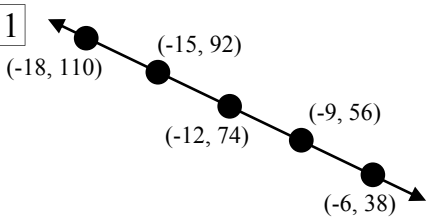
				x	12	15	18	21
				y	16	25	34	43

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

11



x							
y							

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

12

			x	15	20	25	30	35
			y	20	30	40	50	60

At $x = 0$ Gradient: ÷

$y =$ Gradient:

Equation:

Further Finding the Equation of a Line

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Extend the tables below so that $x = 0$ is shown. Then solve for the line equation.

Example

x	3	4	5	6	7
y	9	11	13	15	17

Rewrite taking x back to zero.

0	1	2	x	3	4	5	6	7
3	5	7	y	9	11	13	15	17

-2 -2 -2 +2 +2 +2 +2

At $x = 0$ Gradient: +2

$y =$ +3

Equation: $y = 2x + 3$

1

0	1	x	2	3	4	5	6
-4	-1	y	2	5	8	11	14

-3 -3 +3 +3 +3 +3

At $x = 0$ Gradient: +3

$y =$ -4

Equation: $y = 3x - 4$

2

0	1	2	3	x	4	5	6	7
3	5	7	9	y	11	13	15	17

-2 -2 -2 -2 +2 +2 +2

At $x = 0$ Gradient: +2

$y =$ 3

Equation: $y = 2x + 3$

3

x	-6	-5	-4	-3	-2	-1	0
y	-9	-6	-3	0	3	6	9

+3 +3 +3 +3 +3 +3

At $x = 0$ Gradient: +3

$y =$ 9

Equation: $y = 3x + 9$

These tables have x-values that increase in larger steps. So divide to find the gradient.

4

x	-4	-2	0	2	4
y	-19	-11	-3	5	13

+8 +8 +8 +8

At $x = 0$ Gradient: 8 ÷ 2

$y =$ -3 Gradient: 4

Equation: $y = 4x - 3$

5

x	0	5	10	15	20
y	-3	22	47	72	97

+5 +5 +5 +5

At $x = 0$ Gradient: 5 ÷ 5

$y =$ -3 Gradient: 1

Equation: $y = x - 3$

6

x	-6	-3	0	3	6
y	8	-1	-10	-19	-28

-9 -9 -9 -9

At $x = 0$ Gradient: -9 ÷ 3

$y =$ -10 Gradient: -3

Equation: $y = -3x - 10$

7

x	0	2	4	6	8
y	-2	-14	-26	-38	-50

-12 -12 -12 -12

At $x = 0$ Gradient: -12 ÷ 2

$y =$ -2 Gradient: -6

Equation: $y = -6x - 2$

8

x	-40	-30	-20	-10	0
y	195	155	115	75	35

-40 -40 -40 -40

At $x = 0$ Gradient: -40 ÷ 10

$y =$ 35 Gradient: -4

Equation: $y = -4x + 35$

These are harder, they require the table to be extended and the x-value steps to be considered.

9

0	2	x	4	6	8	10	12
5	11	y	17	23	29	35	41

-6 -6 +6 +6 +6 +6

At $x = 0$ Gradient: 6 ÷ 2

$y =$ 5 Gradient: 3

Equation: $y = 3x + 5$

10

0	3	6	9	x	12	15	18	21
-20	-11	-2	7	y	16	25	34	43

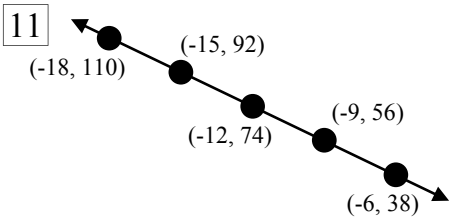
-9 -9 -9 -9 +9 +9 +9

At $x = 0$ Gradient: 9 ÷ 3

$y =$ -20 Gradient: 3

Equation: $y = 3x - 20$

11



x	-18	-15	-12	-9	-6	-3	0
y	110	92	74	56	38	20	2

-18 -18 -18 -18 -18 -18

At $x = 0$ Gradient: -18 ÷ 3

$y =$ 2 Gradient: -6

Equation: $y = -6x + 2$

12

0	5	10	x	15	20	25	30	35
-10	0	10	y	20	30	40	50	60

-10 -10 -10 +10 +10 +10 +10

At $x = 0$ Gradient: 10 ÷ 5

$y =$ -10 Gradient: 2

Equation: $y = 2x - 10$

Special Lines - Horizontal and Vertical Line Equations

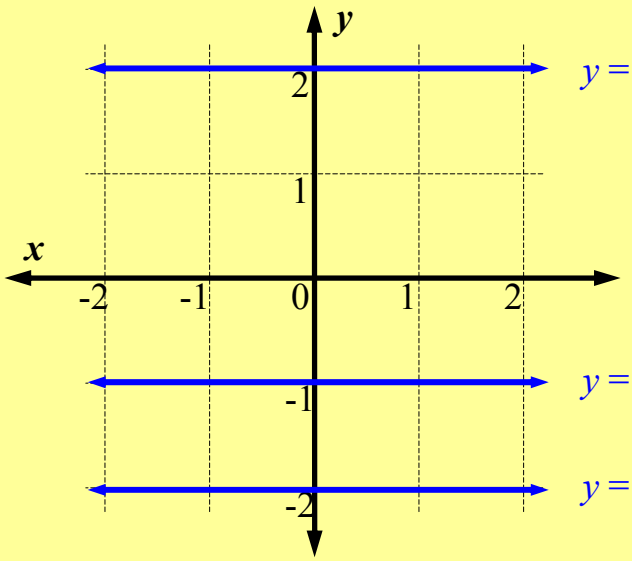
These lines are special because they are horizontal or vertical and only have one letter in them, instead of two. They are also unusual because they seem to be the opposite to what you think.

Special Lines are in the form:

$y = b$ where b is the number where the line cuts the y-axis

$x = a$ where a is the number where the line cuts the x-axis

Here are some examples, first $y = b$:



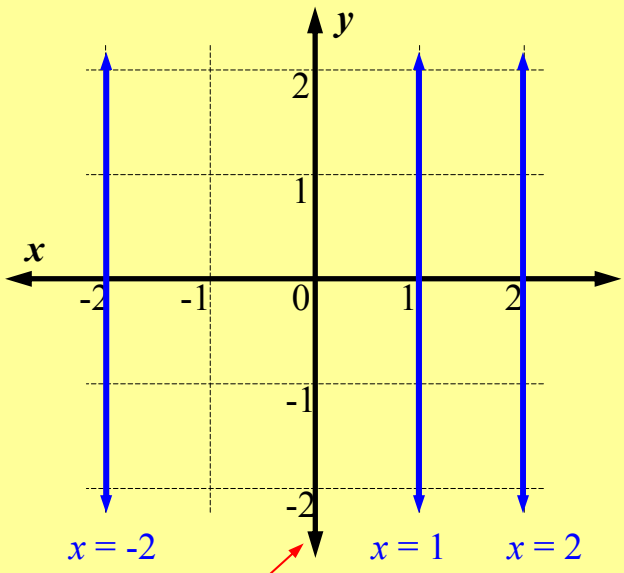
$y = 2$ (cuts the y-axis at 2)

$y = 0$ (cuts the y-axis at 0), so the equation for the x-axis is $y = 0$.

$y = -1$ (cuts the y-axis at -1)

$y = -2$ (cuts the y-axis at -2)

Here are some examples for $x = a$:



$x = -2$ $x = 1$ $x = 2$

$x = 0$ (cuts the x-axis at 0), so the equation for the y-axis is $x = 0$.

So these lines are simple to plot, the only problem is that they seem the opposite to what you would think.

Lines parallel to the y-axis aren't in the format '$y =$' they are '$x =$'. Likewise for the lines parallel to the x-axis, being '$y =$'.

If you can remember this, these are the easiest lines to plot.

Special Lines - Horizontal and Vertical Line Equations

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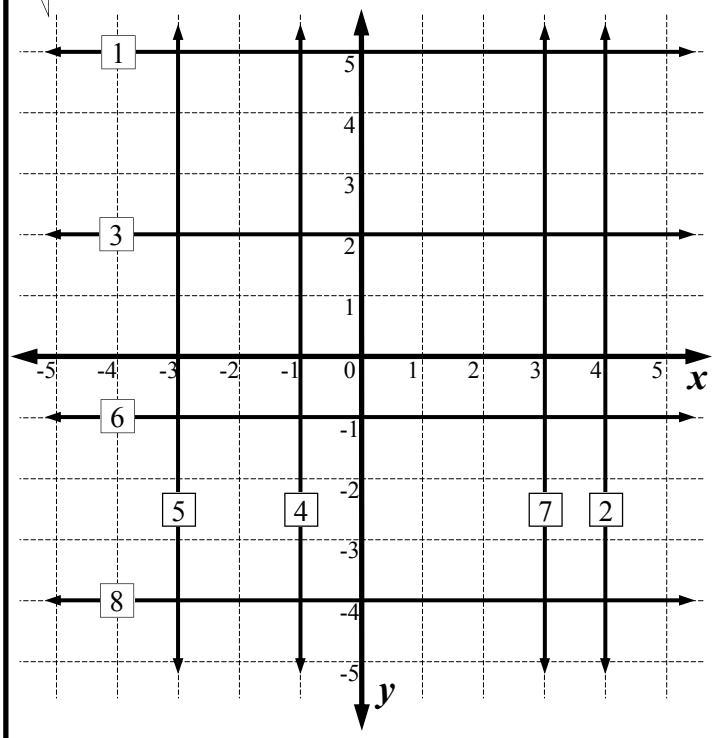
Name the lines shown. Use the spaces below the number plane then label the lines on the plane also.



1 $y =$ ____ 2 $x =$ ____ 3 _____ 4 _____

5 _____ 6 _____ 7 _____ 8 _____

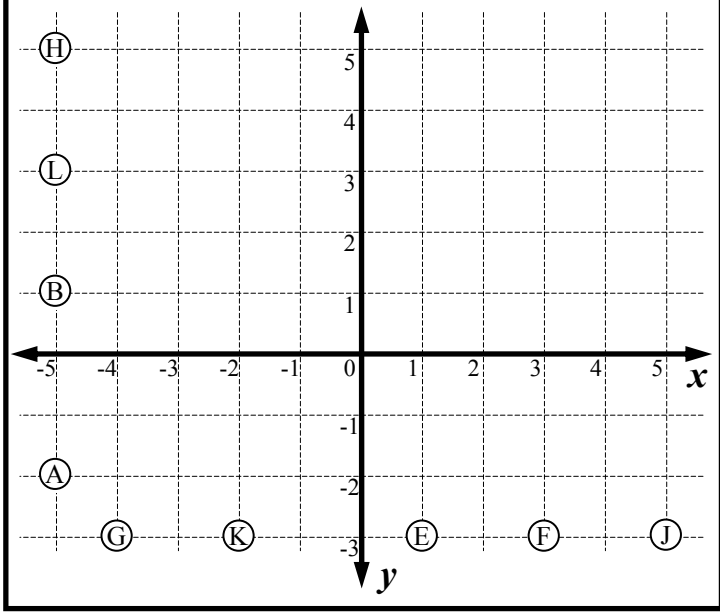
9 What is the equation for the y-axis? _____

10 What is the equation for the x-axis? _____

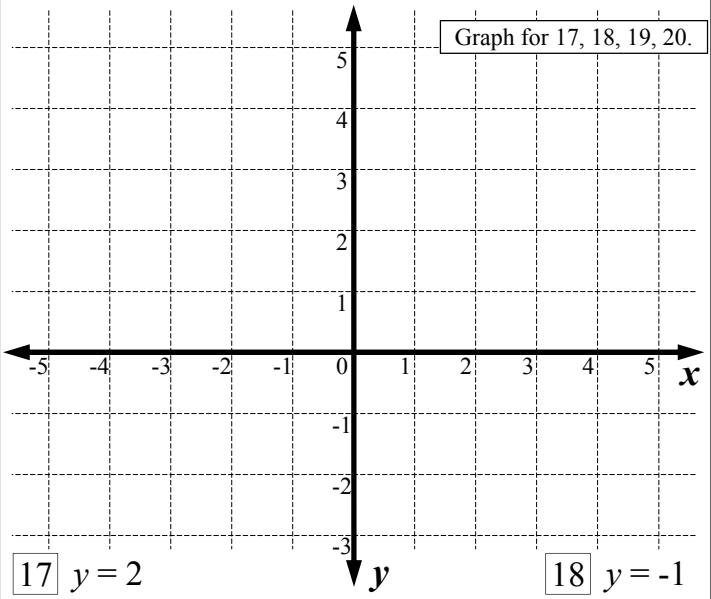
Draw these lines on the Cartesian plane, show arrow heads and labels. After you draw the line it will pass through a letter, write it in the box.

11 $y = -2$ ____ Letter 12 $x = 3$ ____ Letter 13 $x = -4$ ____ Letter

14 $y = 5$ ____ Letter 15 $x = 1$ ____ Letter 16 $y = 3$ ____ Letter



Draw the lines on the grid. Once drawn there will be five intersection points, give the coordinates for these points as well as the name of the two shapes formed.



17 $y = 2$ 18 $y = -1$

19 $y = x + 4$

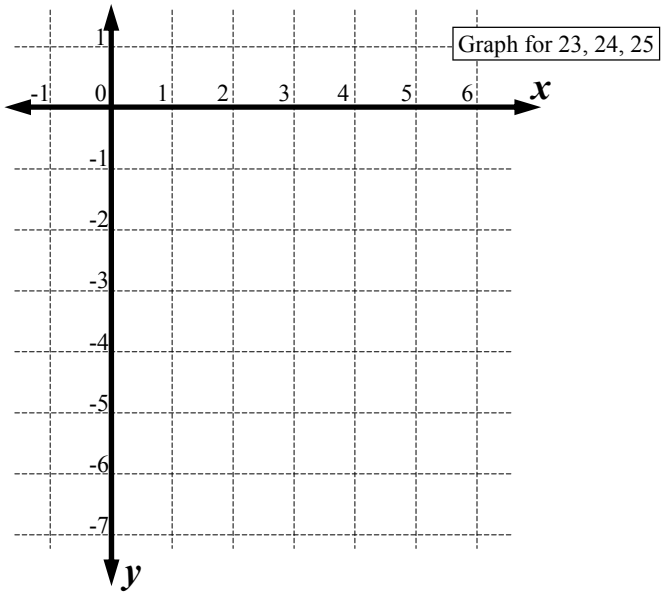
x	-4	-3	-2	-1	0
y					

20 $x + y = 4$

x	-1	0	1	2	3
y					

21 Points: _____

22 Shapes formed: _____



23 $y = -2x + 3$

x	1	2	3	4	5
y					

24 $y = -5$

25 $x = 2$

26 Points: (□, □), (□, □), (□, □).

27 Shape formed: _____

Special Lines - Horizontal and Vertical Line Equations

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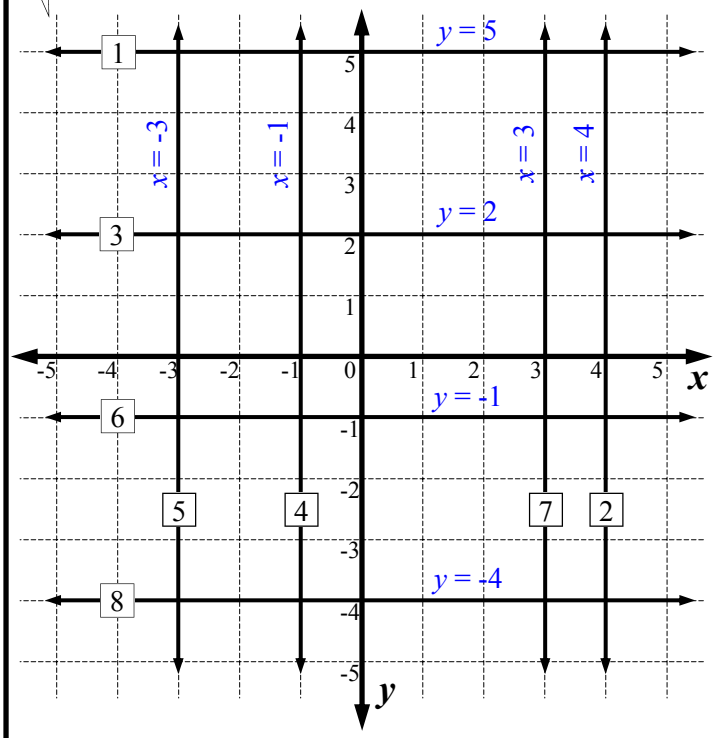
Name the lines shown. Use the spaces below the number plane then label the lines on the plane also.



1 $y =$ 5 2 $x =$ 4 3 $y = 2$ 4 $x = -1$

5 $x = -3$ 6 $y = -1$ 7 $x = 3$ 8 $y = -4$

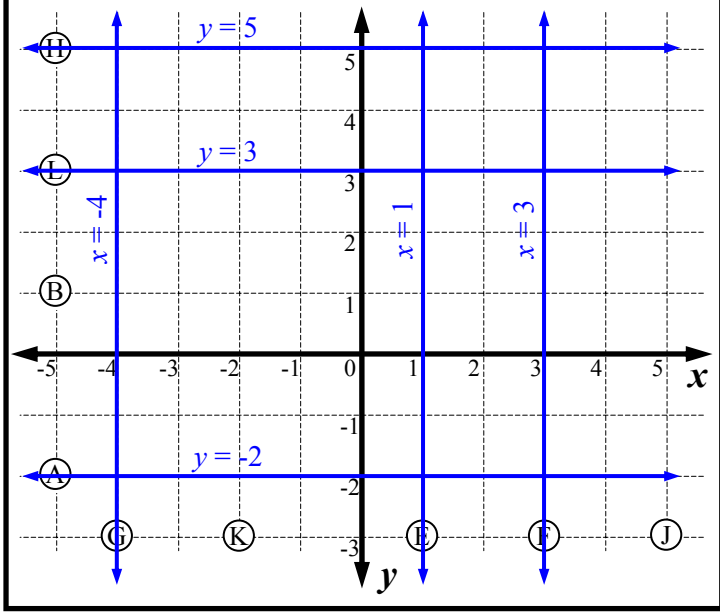
9 What is the equation for the y-axis? $x = 0$

10 What is the equation for the x-axis? $y = 0$

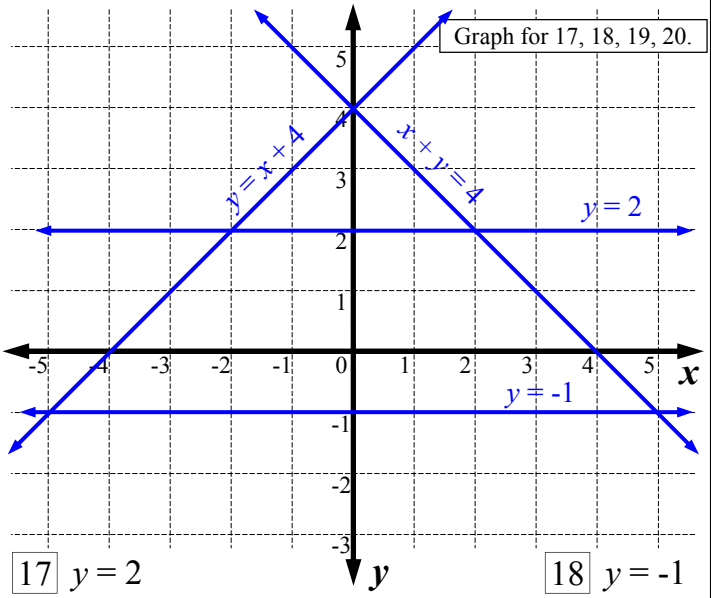
Draw these lines on the Cartesian plane, show arrow heads and labels. After you draw the line it will pass through a letter, write it in the box.

11 $y = -2$ A Letter 12 $x = 3$ F Letter 13 $x = -4$ G Letter

14 $y = 5$ H Letter 15 $x = 1$ E Letter 16 $y = 3$ L Letter



Draw the lines on the grid. Once drawn there will be five intersection points, give the coordinates for these points as well as the name of the two shapes formed.



17 $y = 2$ 18 $y = -1$

19 $y = x + 4$

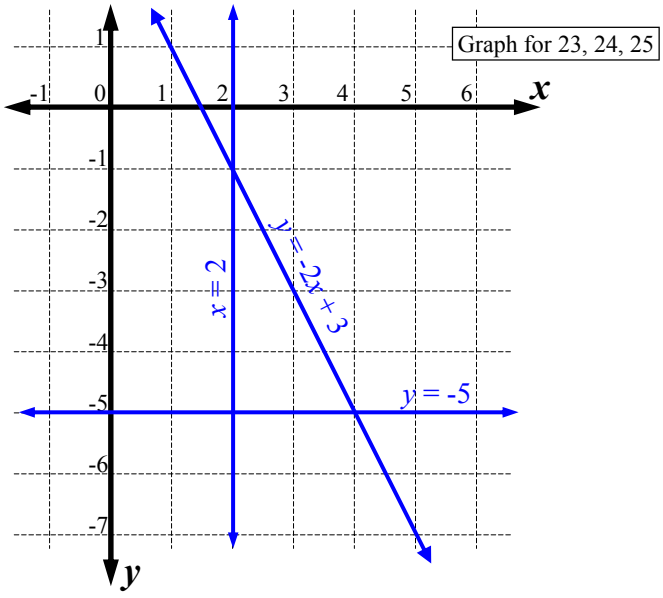
x	-4	-3	-2	-1	0
y	0	1	2	3	4

20 $x + y = 4$

x	-1	0	1	2	3
y	5	4	3	2	1

21 Points: (0, 4), (-2, 2), (2, 2), (-5, -1), (5, -1).

22 Shapes formed: Isosceles triangle, trapezium.



23 $y = -2x + 3$

x	1	2	3	4	5
y	1	-1	-3	-5	-7

24 $y = -5$

25 $x = 2$

26 Points: (2, -1), (2, -5), (4, -5).

27 Shape formed: Right angled triangle

Number Plane & Cost Function Templates

The following are blanks for printing.

Positive Quadrant 1

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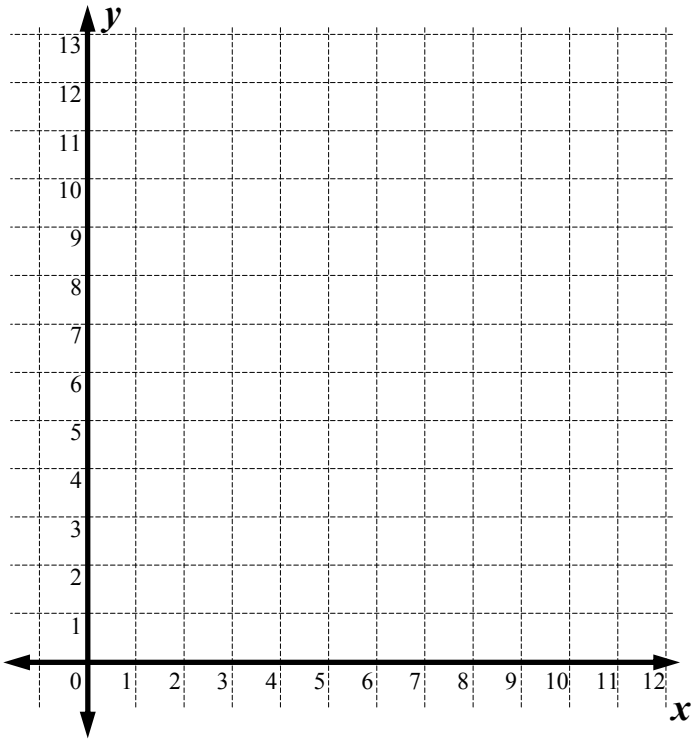
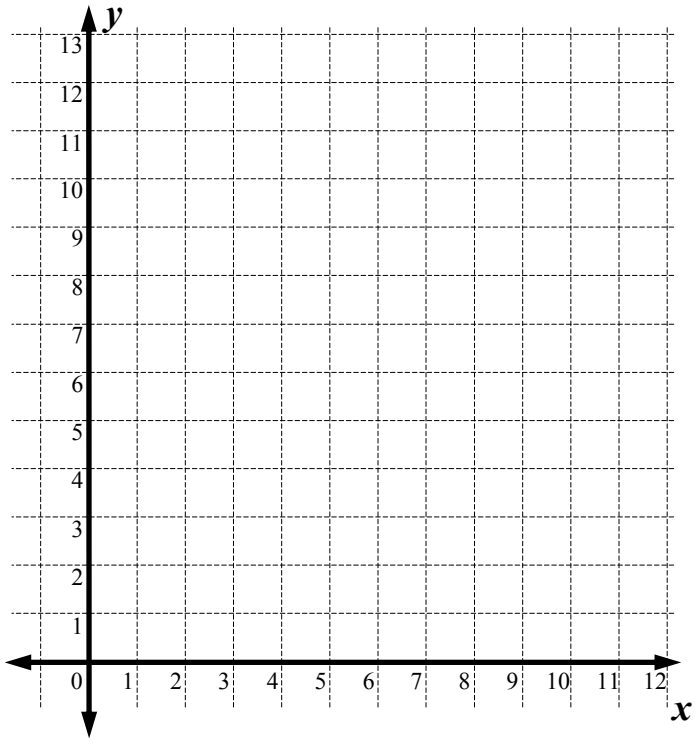
x					
y					

x					
y					



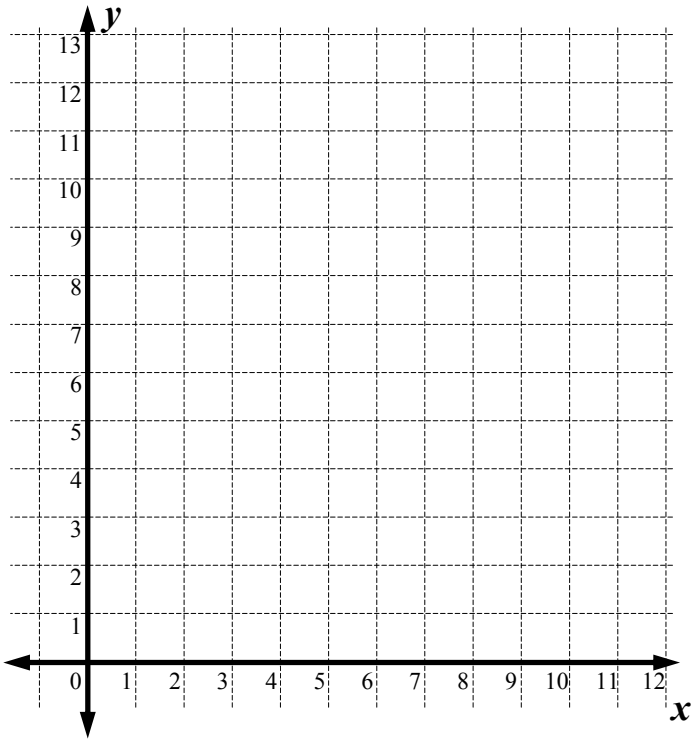
x					
y					

x					
y					



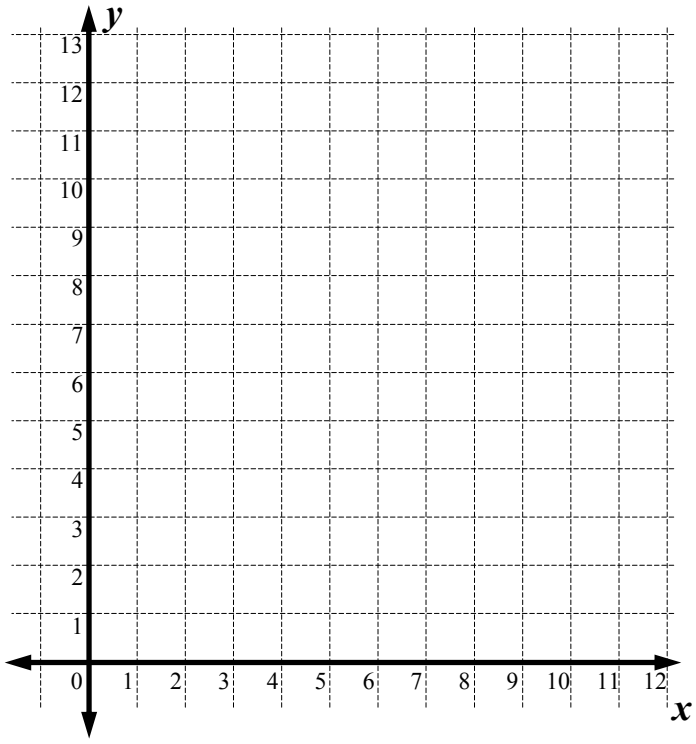
x					
y					

x					
y					



x					
y					

x					
y					



Positive Quadrant 2

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x					
y					

x					
y					

x					
y					

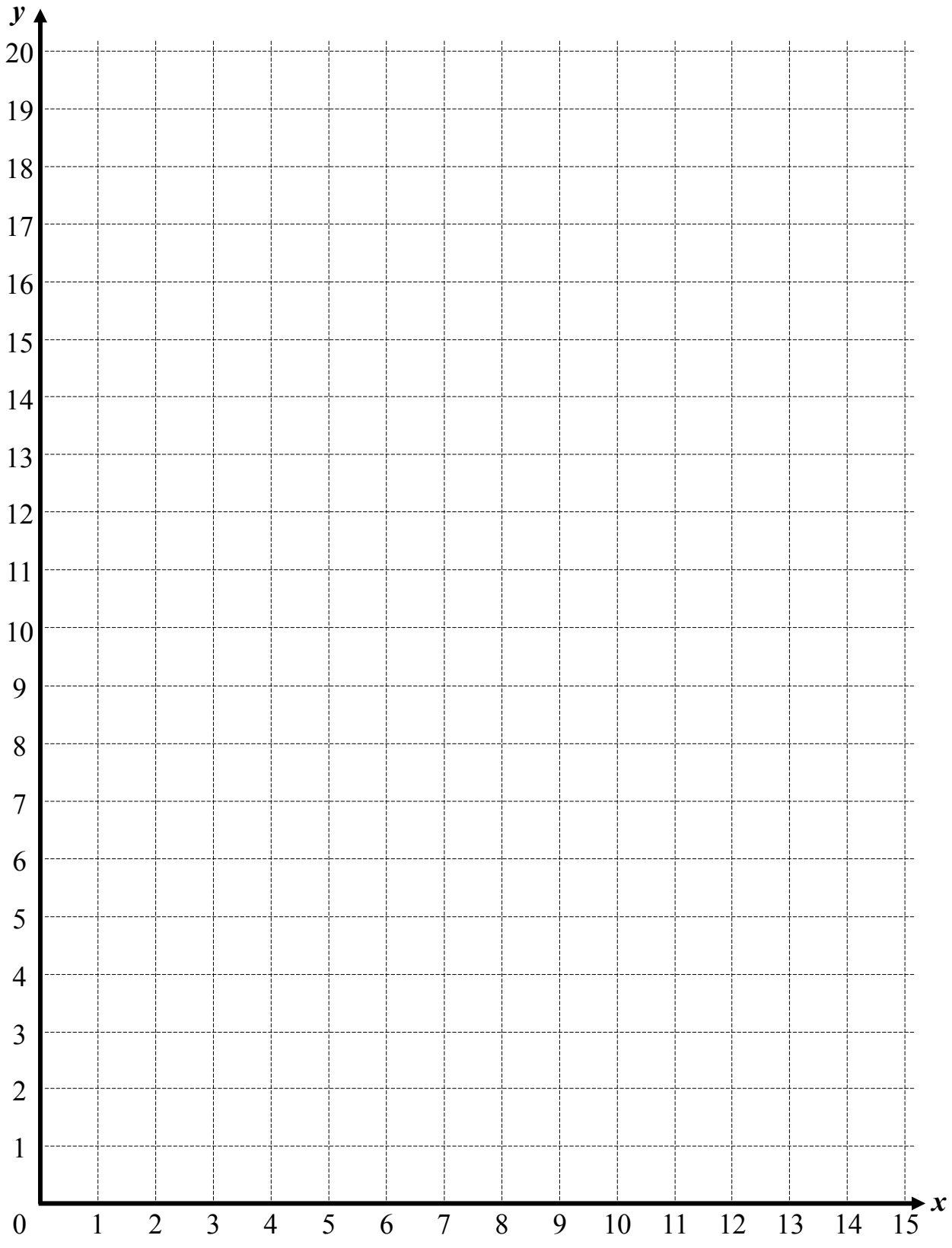
x					
y					

x					
y					

x					
y					

x					
y					

x					
y					



Cost Functions 1

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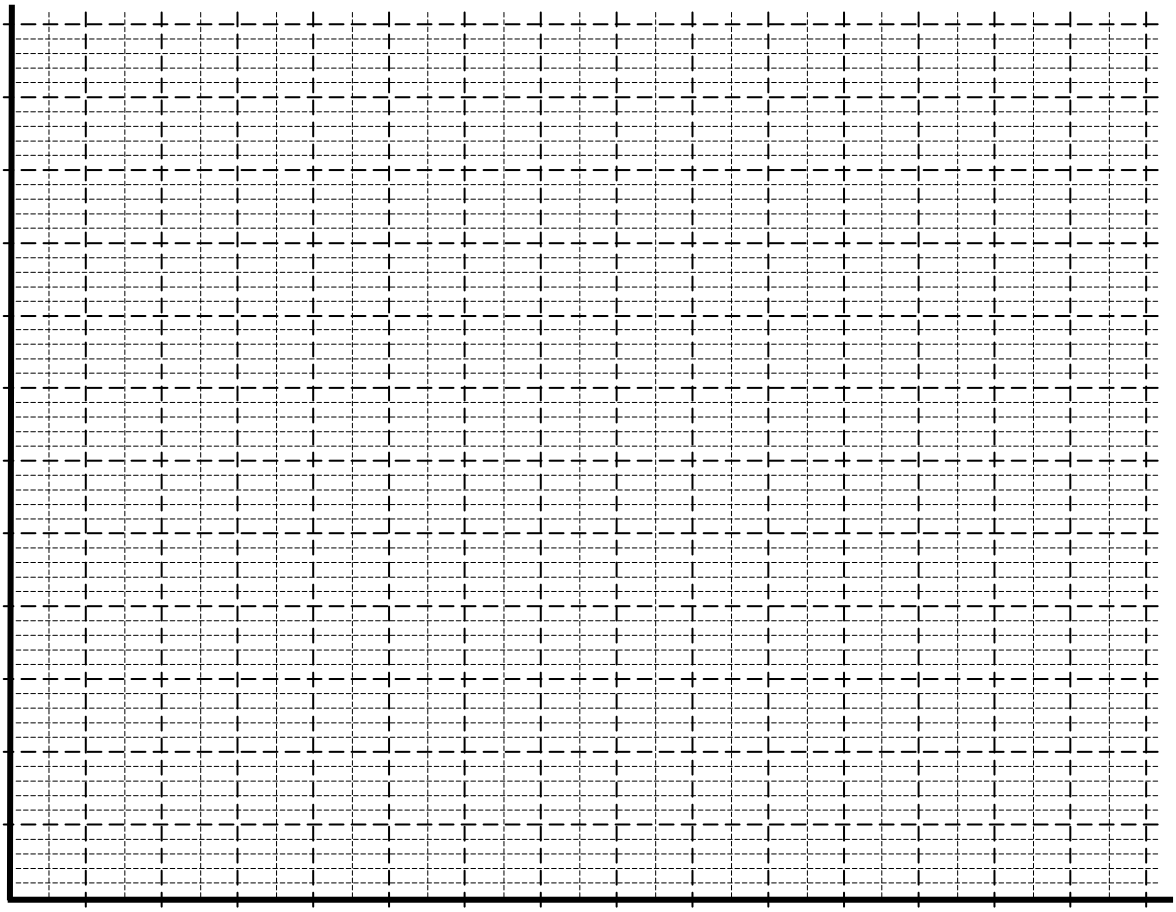

Cost Functions 2

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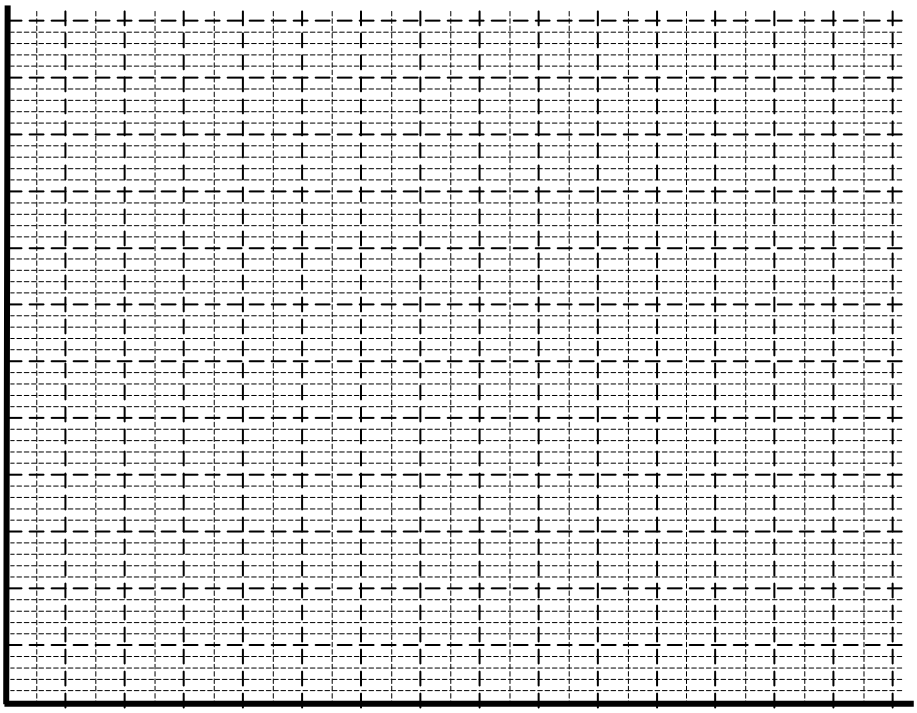

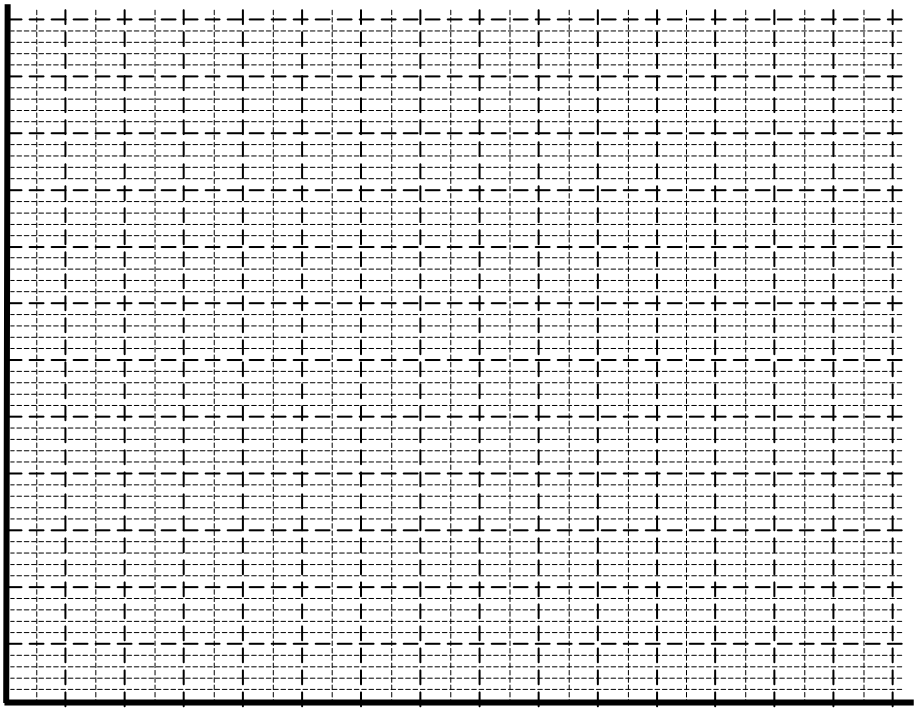

All Quadrants 1

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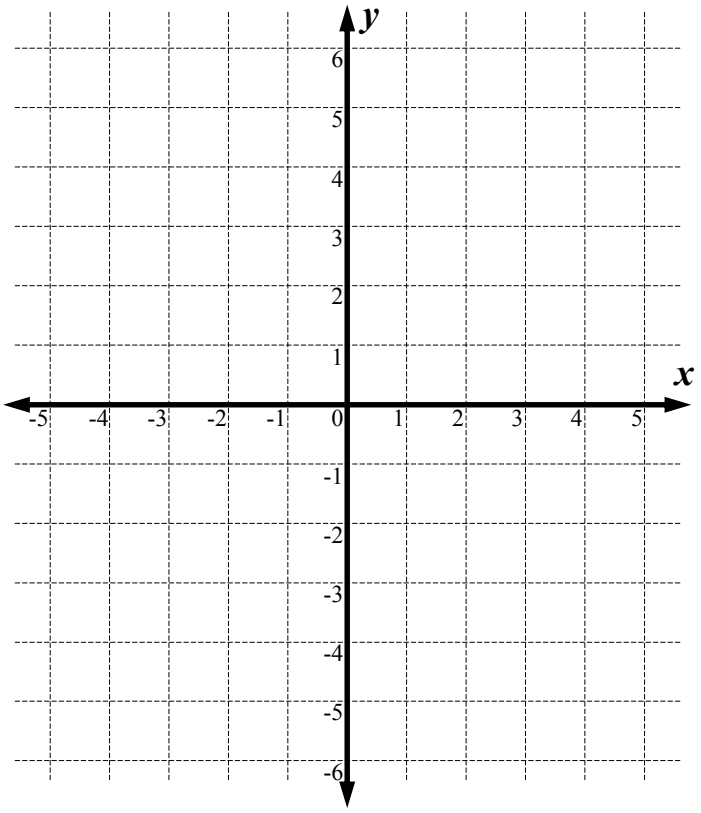
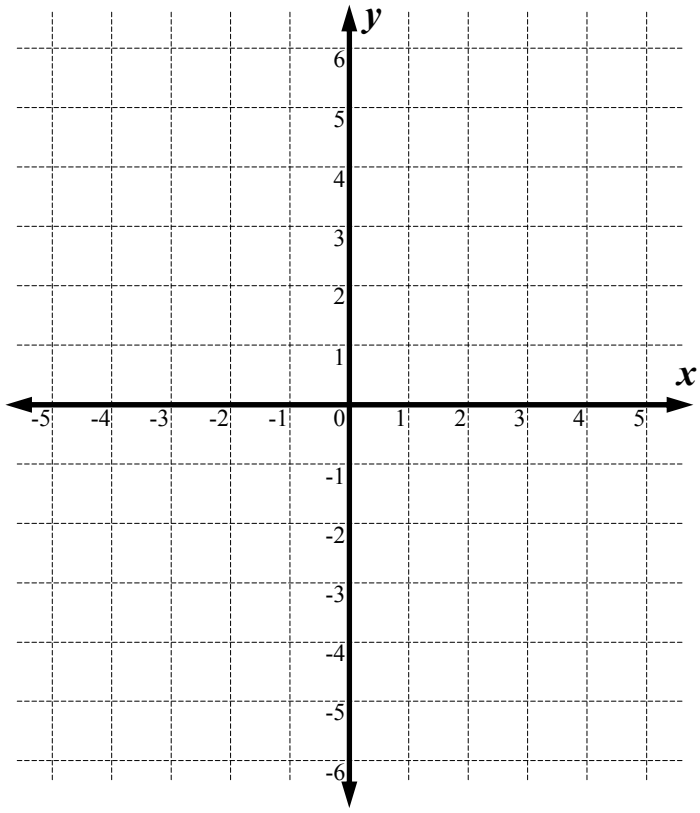
x					
y					

x					
y					



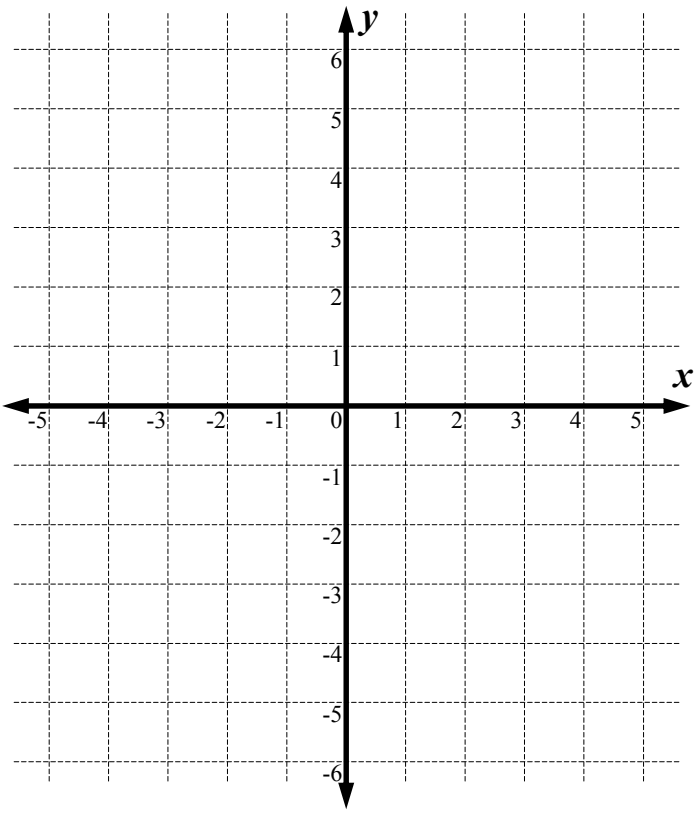
x					
y					

x					
y					



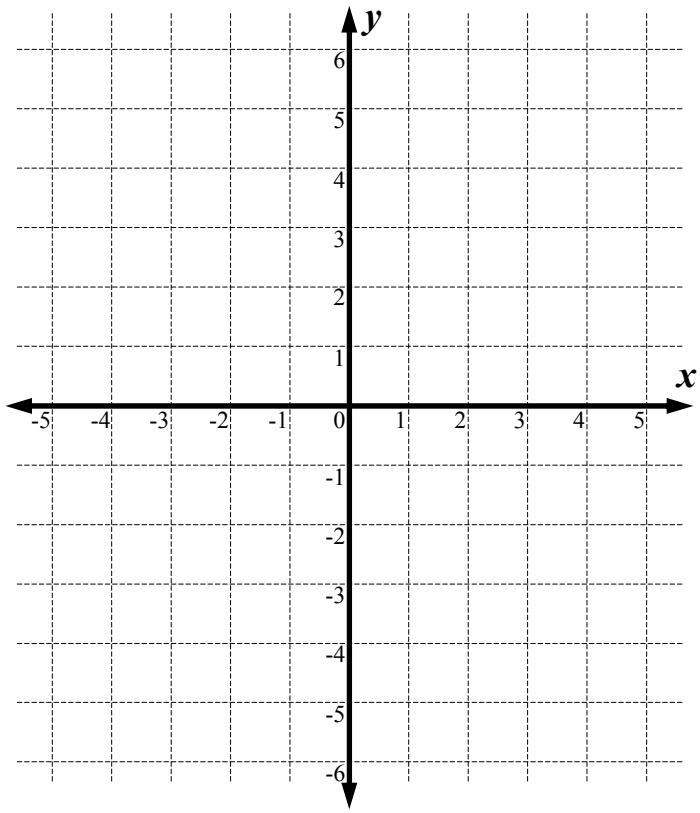
x					
y					

x					
y					



x					
y					

x					
y					



All Quadrants 2

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x					
y					

x					
y					

x					
y					

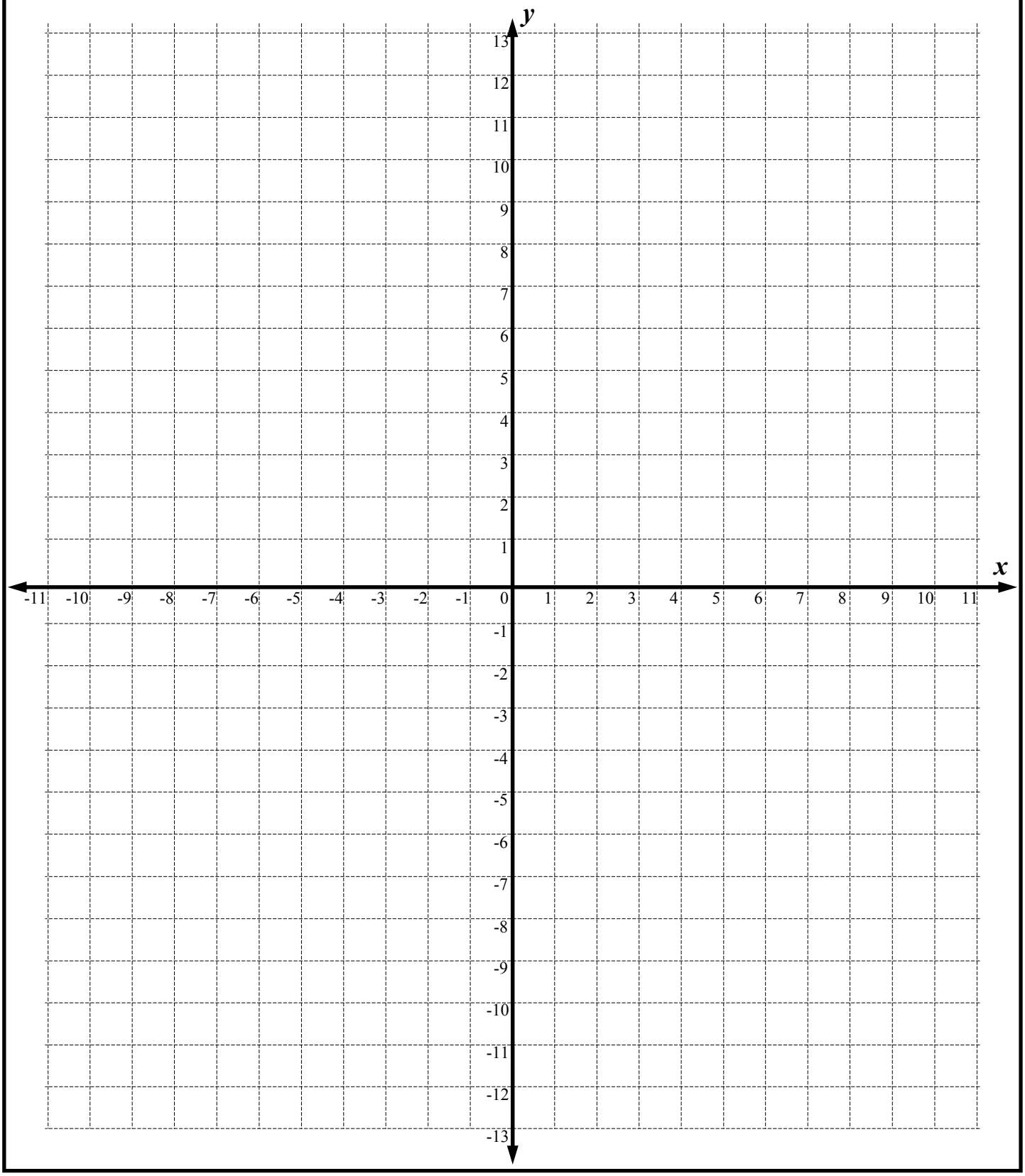
x					
y					

x					
y					

x					
y					

x					
y					

x					
y					



Specific Quadrants

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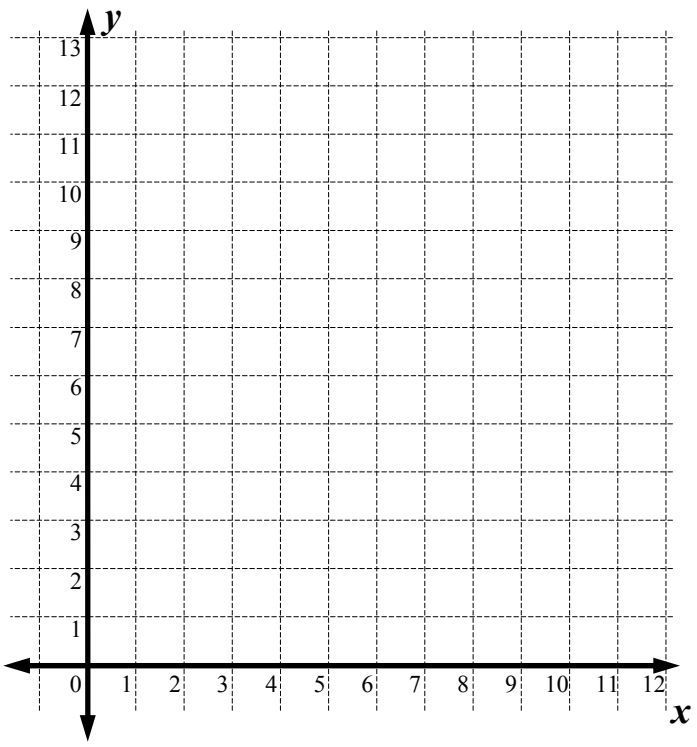
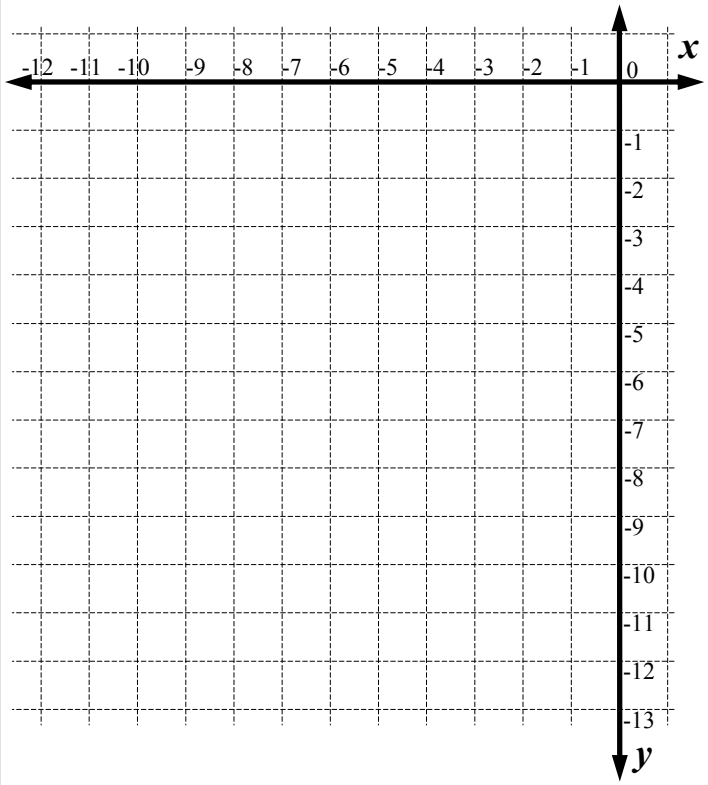
x					
y					

x					
y					



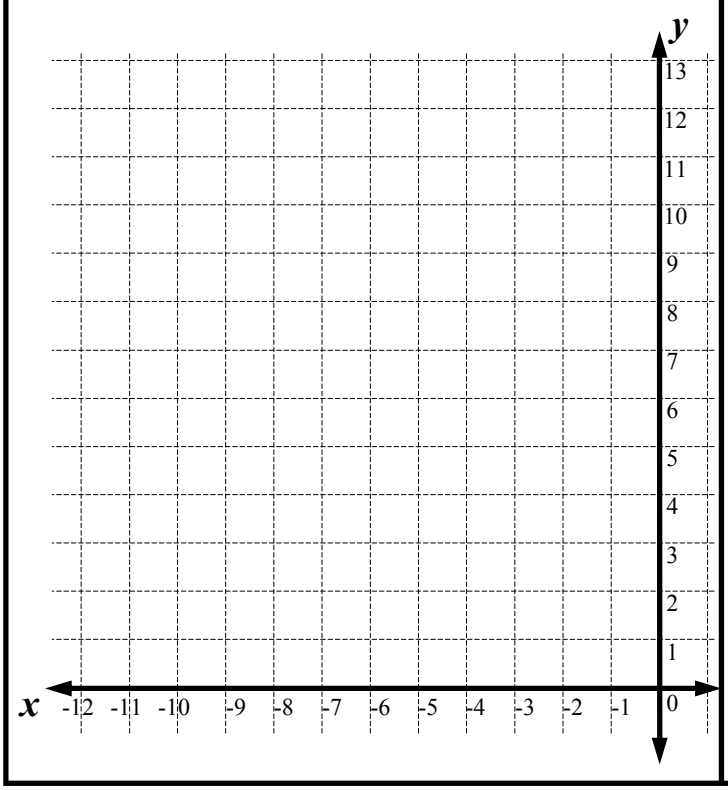
x					
y					

x					
y					



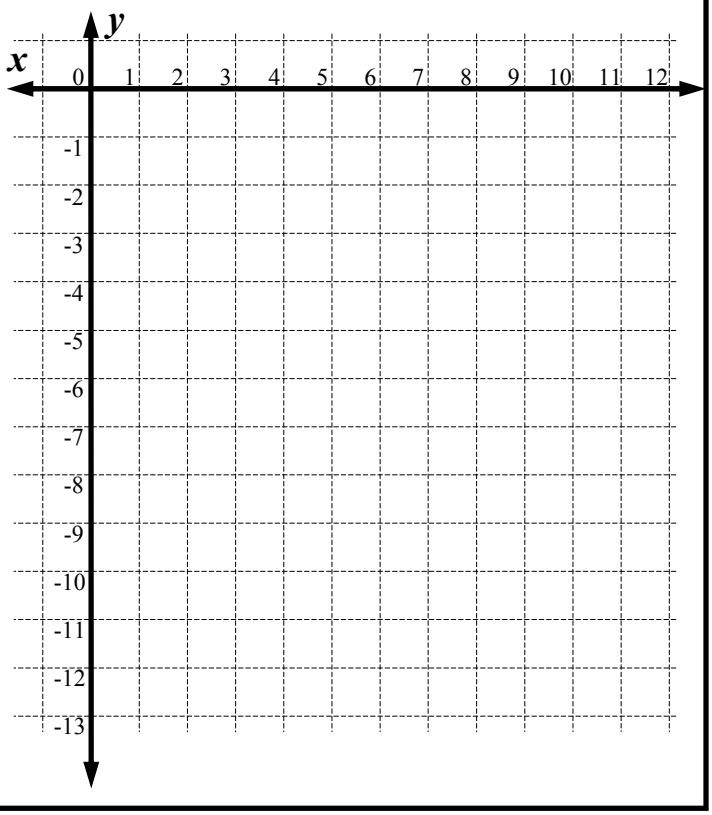
x					
y					

x					
y					



x					
y					

x					
y					



Linear Relationships - Find a Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don't colour it in too dark (using a texta for example) so that you can still read it.

Linear Relationships - Find a Word

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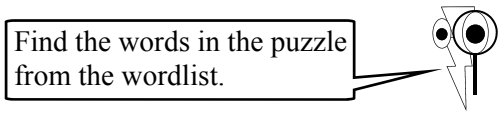

P	A	I	N	B	L	T	Q	S	X	N	Z	T	A	L
A	U	Q	I	E	P	U	S	W	B	O	X	E	U	Q
R	X	B	G	L	V	B	L	O	L	I	R	Q	U	E
A	Q	T	I	Z	X	E	R	R	C	T	V	T	P	A
L	E	T	R	P	Q	A	K	R	T	C	A	O	X	X
L	T	W	O	L	E	E	N	A	V	E	L	L	T	E
E	A	Q	A	N	E	T	A	B	E	S	U	P	I	S
L	N	U	I	R	O	U	P	A	I	R	E	R	F	L
A	I	L	R	P	D	T	A	B	L	E	B	Q	O	U
C	D	X	H	O	R	I	Z	O	N	T	A	L	R	Q
I	R	N	E	G	A	T	I	V	E	N	X	Z	P	X
T	O	U	Q	R	T	S	T	R	A	I	G	H	T	X
R	O	U	A	I	I	B	N	O	I	T	A	U	Q	E
E	C	A	Q	D	X	U	N	O	I	T	C	N	U	F
V	A	U	N	A	I	S	E	T	R	A	C	Q	U	Q

WORDLIST

HORIZONTAL	DRAW	NEGATIVE	STRAIGHT	ORIGIN
COORDINATE	TABLE	EQUATION	PLOT	PROFIT
SUBSTITUTE	AXES	LINEAR	VERTICAL	VALUE
INTERSECTION	PARALLEL	GRID	FUNCTION	CARTESIAN
BREAK-EVEN	PAIR	COST	SLOPE	ARROWS

No Hyphen in grid

Linear Relationships - Find a Word

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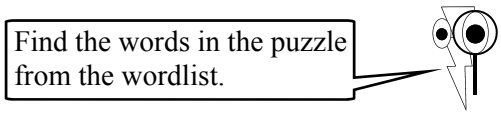

P	A	I	N	B	L	T	Q	S	X	N	Z	T	A	L
A	U	Q	I	E	P	U	S	W	B	O	X	E	U	Q
R	X	B	G	L	V	B	L	O	L	I	R	Q	U	E
A	Q	T	I	Z	X	E	R	R	C	T	V	T	P	A
L	E	T	R	P	Q	A	K	R	T	C	A	O	X	X
L	T	W	O	L	E	E	N	A	V	E	L	L	T	E
E	A	Q	A	N	E	T	A	B	E	S	U	P	I	S
L	N	U	I	R	O	U	P	A	I	R	E	R	F	L
A	I	L	R	P	D	T	A	B	L	E	B	Q	O	U
C	D	X	H	O	R	I	Z	O	N	T	A	L	R	Q
I	R	N	E	G	A	T	I	V	E	N	X	Z	P	X
T	O	U	Q	R	T	S	T	R	A	I	G	H	T	X
R	O	U	A	I	I	B	N	O	I	T	A	U	Q	E
E	C	A	Q	D	X	U	N	O	I	T	C	N	U	F
V	A	U	N	A	I	S	E	T	R	A	C	Q	U	Q

WORDLIST

~~HORIZONTAL~~ ~~DRAW~~ ~~NEGATIVE~~ ~~STRAIGHT~~ ~~ORIGIN~~

~~COORDINATE~~ ~~TABLE~~ ~~EQUATION~~ ~~PLOT~~ ~~PROFIT~~

~~SUBSTITUTE~~ ~~AXES~~ ~~LINEAR~~ ~~VERTICAL~~ ~~VALUE~~

~~INTERSECTION~~ ~~PARALLEL~~ ~~GRID~~ ~~FUNCTION~~ ~~CARTESIAN~~

~~BREAK-EVEN~~ ~~PAIR~~ ~~COST~~ ~~SLOPE~~ ~~ARROWS~~

No Hyphen in grid

FREEFALL MATHEMATICS

ALTITUDE

DATA ANALYSIS

Census, Sampling and Questioning

Gathering information is done regularly. Businesses often wish to gather an opinion of their operation through the eyes of their customers. But asking everyone can be very costly or even impossible, so instead they question a smaller body of people whose responses are then considered to reflect the opinion of all the customers.

Some terms:

Population: means all people relevant to the survey
Sample: means a portion of the population being surveyed
Census: is a when the entire population is surveyed

Column 1 starts with questions asking you to identify when a type of survey is a census or a sample. If everyone connected is surveyed, then it is a census. If only some of the population are involved, it is a sample.

Questions can be open-ended, which means that the respondent can offer an opinion and lengthy response to a question. Closed-ended questions are either questions that are answered with a 'yes' or 'no' answer or are multiple choice. This allows results to be easily collected and analysed. The second part of column 1 gives you an open-ended question and asks you to suggest some closed-end questions.

Column 2 introduces Stratified sampling. This means that if groups of people are interviewed you can suggest how many people from each group should be surveyed. This is essentially the same as 'Relative Frequency' which you may have encountered already in the 'Probability' chapter.

Imagine 2 000 people are to be interviewed by an insurance company, represented in proportion to the size of the age groups below. How many from each group should be interviewed?

STEP 1 Find the total for the frequency column.

Client Age	*Frequency*	*No. of Clients Interviewed*
0 - 20	38 900	
20 - 40	145 618	
40 - 60	196 752	
60 - 80	76 513	
	457 783	

STEP 2 Create a fraction and multiply by the sample amount.

Client Age	*Frequency*	*No. of Clients Interviewed*
0 - 20	38 900	$\frac{38\,900}{457\,783} \times 2\,000 = 170$
20 - 40	145 618	$\frac{145\,638}{457\,783} \times 2\,000 = 636$
40 - 60	196 752	$\frac{196\,712}{457\,783} \times 2\,000 = 900$
60 - 80	76 513	$\frac{76\,533}{457\,783} \times 2\,000 = 334$
	457 783	Note that due to rounding the total of the above column may not be 2000.

Census, Sampling and Questioning

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Suggest whether the following are examples of a census or sample. Explain why.

1 The School Representative members survey every year 7 student about the school uniform policy.

2 Students arriving on Bus 3 are surveyed for their opinion on school bus services.

3 All ticket buyers to a "Budget Tuesday" movie screening are surveyed about ticket prices.

4 All school staff, students and parents are asked about the content of a new canteen menu.

Analysing open-ended question responses is difficult. Suggest "Yes" or "No" questions to make analysing the response data easier.

Example

Open-ended Question: What do you think of your school?

Alternatives: Do you like school?
Would you make changes to your school?
Do you prefer another school?

5 What is your opinion of fast food?

-
-

6 Where did you go on your last holiday?

-
-
-

7 How are young driver fatalities best avoided?

-
-

50 people are to be interviewed from Year 8 through to Year 11. How many people from each year should be interviewed using a stratified sampling approach?

8

Year	*No. of Students*	*No. of Students Interviewed*
8	108	$\frac{108}{\square} \times 50 = \square$
9	125	
10	140	
11	92	

A survey was completed by a radio station. If a 200 song marathon is to be played, suggest the number of songs from each music type to be played, according to the votes received.

9

Genre	*Votes*	*200 Song Marathon Content*
Rock	480	
Metal	238	
Rap	109	
Dance	137	
R & B	86	
Hip Hop	95	
Techno	55	

Sampling must be adequate to reflect the population, if the population is n, then sample size should be $\sqrt{n}$ at the least. Find the minimum sample size for these.

10 A school of 220 students

11 A town of 17 000 residents

12 A plane carrying 350 passengers

13 A magazine with 55 000 readers

Census, Sampling and Questioning

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Suggest whether the following are examples of a census or sample. Explain why.

1 The School Representative members survey every year 7 student about the school uniform policy. It is a sample, as other years weren't represented in a matter that affects them.

2 Students arriving on Bus 3 are surveyed for their opinion on school bus services.
Sample, as other bus travelers aren't included.

3 All ticket buyers to a "Budget Tuesday" movie screening are surveyed about ticket prices.
Sample. Only covers ticket buyers on one night and response will be biased.

4 All school staff, students and parents are asked about the content of a new canteen menu.
Census. All affected people are included in the process.

Analysing open-ended question responses is difficult. Suggest "Yes" or "No" questions to make analysing the response data easier.

Example

Open-ended Question: What do you think of your school?

Alternatives: Do you like school?
Would you make changes to your school?
Do you prefer another school?

5 What is your opinion of fast food?

- Is fast food a problem for a healthy society?
- Do you eat fast food too often?

6 Where did you go on your last holiday?

- Did you travel overseas on your last holiday?
- Did you travel domestically?
- Did you travel by plane?

7 How are young driver fatalities best avoided?

- Would introducing a curfew improve safety?
- Would increasing starting age improve safety?

50 people are to be interviewed from Year 8 through to Year 11. How many people from each year should be interviewed using a stratified sampling approach?

8

Year	*No. of Students*	*No. of Students Interviewed*
8	108	$\frac{108}{465} \times 50 = 12$
9	125	$\frac{125}{465} \times 50 = 13$
10	140	$\frac{140}{465} \times 50 = 15$
11	92	$\frac{92}{465} \times 50 = 10$
	$\Sigma = 465$	

A survey was completed by a radio station. If a 200 song marathon is to be played, suggest the number of songs from each music type to be played, according to the votes received.

9

Genre	*Votes*	*200 Song Marathon Content*
Rock	480	$\frac{480}{1\,200} \times 200 = 80$
Metal	238	$\frac{238}{1\,200} \times 200 = 40$
Rap	109	$\frac{109}{1\,200} \times 200 = 18$
Dance	137	$\frac{137}{1\,200} \times 200 = 23$
R & B	86	$\frac{86}{1\,200} \times 200 = 14$
Hip Hop	95	$\frac{95}{1\,200} \times 200 = 16$
Techno	55	$\frac{55}{1\,200} \times 200 = 9$
	$\Sigma = 1\,200$	

Sampling must be adequate to reflect the population, if the population is n, then sample size should be $\sqrt{n}$ at the least. Find the minimum sample size for these.

10 A school of 220 students 15 students

11 A town of 17 000 residents 130 residents

12 A plane carrying 350 passengers 19 passengers

13 A magazine with 55 000 readers 234 readers

Data Analysis - Range and Mode

The range of a data set is found by subtracting the smallest score from the largest score. So look through the data for the largest score, circle it if you like, then find the smallest score.

Example

Data: 15 , 33 , 21 , 18 , 11 , 15 .

Data: 15 , (33) , 21 , 18 , (11) , 15 .

Find the largest and the smallest scores. Circle them if you like.

Range = 33 - 11 = 22

Subtract them for the range.

Negative numbers can also occur in data sets, the same method is used but when you subtract then the two minus signs become a plus sign.

Example - Negative numbers

Data: -8 , 11 , -3 , 12 , -5 , 7.

Data: (-8) , 11 , -3 , (12) , -5 , 7.

Find the largest and the smallest scores. If circling them helps you, do it!

Range = 12 - -8 = 20

12 - - 8 becomes 12 + 8 which = 20.

Data can also be assembled in Stem and Leaf Plots

Example

Tens	Units
0	5 5 7 8
1	0 0 3 3
2	8 8 9

The smallest score is '05' which is 5

The largest score is '29' which is 29.

Range = 29 - 5 = 24

Calculate the range by the same method.

The mode is the score that occurs the most frequently in a data set. Note that the word 'mode' is similar to the word 'most'. If the data contains no numbers that are repeated then there is 'No Mode', note that you write 'No Mode' not Mode = 0. If one number occurs the most then the mode is that number. If two numbers occur the most and with the same frequency then list both numbers (this is called Bi-modal). However if there are three modes (or more) then the data set is said to have "No Mode". So it is either No mode, a single score, or two scores.

15, 17, 16, 3.	17, 3, 17, 3, 9, 9, 6.	17, 15, 17, 15, 17.	17, 11, 17, 15, 11.
Mode = No Mode	Mode = No Mode	Mode = 17.	Mode = 11, 17.
No score occurs more than once	3, 9, 17 occur the most … No Mode	17 occurs more frequently than 15	Bi-Modal as 11 and 17 both occur twice

Data Analysis - Range and Mode

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Find the smallest and largest numbers. Subtract them and you have the range.

1 10 , 53 , 19 , 21 , 33 , 40 .

Range = ☐ - ☐ = ☐

2 71 , 66 , 57 , 42 , 42 , 25 .

Range = ☐ - ☐ = ☐

3 13, 38, 23, 46, 27, 19, 54, 62, 83, 50, 39, 44, 23, 11, 6, 79.

Range =

4 812, 1 076, 909, 694, 552, 773, 1 121, 905.6, 541.7, 890.

Watch out when you subtract negative numbers, - - = + !

5 10, 3, -4, 17, 0, 8, -9, 6, 11.

Range = ☐ - ☐ = ☐

6 52, -37, 65, 2.4, 39, -7.2, 19.

7 38, 12, 19, 0, 58, -21, 83, 9, 41, -65, 42, -33, 91, 50, -18.9.

8

Tens	Units
3	2 2 5 8
4	0 1 1
5	3 7 9 9

Range = ☐ - ☐ = ☐

9

Tens	Units
8	1 3 3 5 9
9	7 7 8
10	0 6 7 8

The MOde is the score that occurs the MOst. You can have no mode, one mode or two modes. No more than two. 3 Modes = No Mode!

10 32 , 42 , 16 , 28 , 14 , 16 .

Mode =

11 25, 13, 12, 18, 12, 16, 13, 5.

Mode =

12 11, 15, 9, 14, 17, 25, 23, 6.

13 95, 97, 86, 95, 90, 84, 86.

14 -6, 23, 17, 5, 11, 17, 5, 17.

15 6, 7, 5, 2, 7, 2, 3, 8, 9, 1, 5.

16

Tens	Units
0	3 5 5 7 8
1	2 2 2 6
2	0 1 1
3	3 5 5 5

17

Tens	Units
5	0 0 1 4
6	2 6 7 9
7	4 5

18

Tens	Units
10	0 4 7 8
11	3 4 6
12	1 7 8

Find both the range and the mode for these.

19 18, 31, 43, 20, 24, 8, 17, 55, 29, 33, 17, 39, 40, 19, 20, 16.

Range = Mode =

20 34, 27, 16, 19, 48, 35, 24, 13, 16, -5, 22, 6, 35, 18, 20, 16.

Range = Mode =

21

Tens	Units
97	8 8
98	3 5 5 7
99	0 7 9
100	1 1

Its time to work backwards. Find the missing score.

22 22 , ☐ , 16 , 19 , 28 , 19 .

Range = 14

23 82 , 30 , 34 , -7 , ☐ , 34 .

Range = 100

24 ☐ , 18 , 35 , 21 , 11 , 7.

Range = 28 Mode = 11

25 ☐ , 10 , ☐ , 19 , 19 , 10.

Range = 26 Mode = 19

26

Tens	Units
5	2 3 6 6
6	2 4 4 4 7
7	0 5 8 8 ☐
8	0 2 2 ☐

Range = 35 Mode = 64

Data Analysis - Range and Mode

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Find the smallest and largest numbers. Subtract them and you have the range.

1 10 , 53 , 19 , 21 , 33 , 40 .

Range = 53 - 10 = 43

2 71 , 66 , 57 , 42 , 42 , 25 .

Range = 71 - 25 = 46

3 13, 38, 23, 46, 27, 19, 54, 62, 83, 50, 39, 44, 23, 11, 6, 79.

Range = 83 - 6 = 77

4 812, 1 076, 909, 694, 552, 773, 1 121, 905.6, 541.7, 890.

Range = 1 121 - 541.7 = 579.3

Watch out when you subtract negative numbers, - - = + !

5 10, 3, -4, 17, 0, 8, -9, 6, 11.

Range = 17 - -9 = 26

6 52, -37, 65, 2.4, 39, -7.2, 19.

Range = 65 - - 37 = 102

7 38, 12, 19, 0, 58, -21, 83, 9, 41, -65, 42, -33, 91, 50, -18.9.

Range = 91 - - 65 = 156

8

Tens	Units
3	2 2 5 8
4	0 1 1
5	3 7 9 9

Range = 59 - 32 = 27

9

Tens	Units
8	1 3 3 5 9
9	7 7 8
10	0 6 7 8

Range = 108 - 81 = 27

The MOde is the score that occurs the MOst. You can have no mode, one mode or two modes. No more than two. 3 Modes = No Mode!

10 32 , 42 , 16 , 28 , 14 , 16 .

Mode = 16

11 25, 13, 12, 18, 12, 16, 13, 5.

Mode = 12, 13.

12 11, 15, 9, 14, 17, 25, 23, 6.

Mode = No Mode.

13 95, 97, 86, 95, 90, 84, 86.

Mode = 86, 95.

14 -6, 23, 17, 5, 11, 17, 5, 17.

Mode = 17.

15 6, 7, 5, 2, 7, 2, 3, 8, 9, 1, 5.

Mode = No Mode. (2,5,7)

16

Tens	Units
0	3 5 5 7 8
1	2 2 2 6
2	0 1 1
3	3 5 5 5

Mode = 12, 35.

17

Tens	Units
5	0 0 1 4
6	2 6 7 9
7	4 5

Mode = 50.

18

Tens	Units
10	0 4 7 8
11	3 4 6
12	1 7 8

Mode = No Mode.

Find both the range and the mode for these.

19 18, 31, 43, 20, 24, 8, 17, 55, 29, 33, 17, 39, 40, 19, 20, 16.

Range = 55 - 8 = 47 Mode = 17, 20.

20 34, 27, 16, 19, 48, 35, 24, 13, 16, -5, 22, 6, 35, 18, 20, 16.

Range =48 - - 5 = 53 Mode = 16

21

Tens	Units
97	8 8
98	3 5 5 7
99	0 7 9
100	1 1

Range = 1 001 - 978 = 23

Mode = No Mode.

Its time to work backwards. Find the missing score.

22 22 , 14 , 16 , 19 , 28 , 19 .

Range = 14

23 82 , 30 , 34 , -7 , -18 , 34 .

Range = 100 (or 93)

24 11 , 18 , 35 , 21 , 11 , 7.

Range = 28 Mode = 11

25 19 , 10 , 36 , 19 , 19 , 10.

Range = 26 (or -7) Mode = 19

26

Tens	Units
5	2 3 6 6
6	2 4 4 4 7
7	0 5 8 8 9
8	0 2 2 7

Range = 35 Mode = 64

Data Analysis - The Mean

The Mean is the mathematical term for "Average". So when you find the mean you are finding the average. The mean is found by adding all the scores then dividing the total by the number of scores. As a formula it is written:

The short way of writing the mean is $\bar{x}$. *'x bar'*

$$\bar{x} = \frac{\Sigma x}{n}$$

Σx means the sum of x. So the total of all the scores in the data set.

n means the number of scores in the data set.

Example - Round to 1 decimal place.

Data: 14, 19, 35

$$\bar{x} = \frac{68}{3} = 22.7$$

68 ← 14 + 19 + 35 = 68

3 ← Three scores

22.7 ← Round to 1 d.p.

Calculator Steps:

6 8 ÷ 3 =

Calculator Screen:

22.6666666….

Column 1 also includes questions involving units. These questions have to be in the same units so look at the question and check what units the answer must have, then change all the data to those units first. Then add and divide for the mean.

Column 2 features Stem and Leaf plots, remember that the scores are found by joining the tens column with the units column, an example is below.

Example - Stem and Leaf Plots

Tens	Units
0	3 5
1	0 5 6
2	2 2

$$\bar{x} = \frac{93}{7} = 13.3$$

93 ← 3 + 5 + 10 + 15 + 16 + 22 + 22 = 93

7 ← Seven scores.

13.3 ← Round to 1 d.p.

Column 2 then features a table showing the prices for 6 items purchased from 5 different stores. Total the price list for each store. One shop will be cheaper than the others, but does that mean it is the cheapest store to purchase goods from? To answer this the mean price for each product is calculated, then the store price is given a star for being under the average price. If the price is higher than the average price it is given a cross.

Comparing mean prices can give a different idea of which store is the better store for lower prices. Which is better to use, the total shopping cost or the mean price? I feel a class discussion coming.

Data Analysis - The Mean

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Find the mean for these, the answers are integers.

1 16, 24, 41.

Add the scores

$\bar{x} = \frac{\square}{3} = \square$

Three scores = 3

Divide total by 3

2 35, 47, 64, 31, 53.

$\bar{x} = \frac{\square}{\square} = \square$

3 8, 13, 17, 23, 15, 19, 31.

$\bar{x} = \frac{\square}{\square} = \square$

4 -6, -2, 5, -7, 3, 0, -4, 6, -9, 4.

$\_\_ = \frac{\square}{\square} = \square$

These questions include units. Find $\bar{x}$ to 1 d.p.

5 846 m, 277 m, 567 m, 308 m.

$\_\_ = \frac{\square}{\square} = \_\_\_\_$

6 1.2 kg, 2.3 kg, 805 g.

$\_\_ = \frac{\square}{\square} = \_\_\_\_$

Answer in grams

7 25 s, 1 min 41 s, 82 s, 2 min, 1 m 17 s, 19 s, 1 min 53 s, 34 s, 41 s.

$\_\_ = \frac{\square}{\square} = \_\_\_\_$

Answer in min & s

8 56 cm, 25 cm, 1.2 m, 38 cm, 1.07 m, 94 cm, 1.9 m, 74 cm.

$\_\_ = \frac{\square}{\square} = \_\_\_\_$

Answer in cm

9 0.7 L, 1.05L, 217 ml, 1.2 L, 850 mL, 1.5 L, 3.03 L

$\_\_ = \frac{\square}{\square} = \_\_\_\_$

Answer in L

Find the mean for these Stem and Leaf plots. Round to 1 decimal place.

10

Tens	Units
0	5 7 8 8
1	3 8
2	0 0 0 3 9
3	3 4 6

$\_\_ = \frac{\square}{\square} = \_\_\_\_$

$\_\_ = \frac{\square}{\square} = \_\_\_\_$

11

Tens	Units
9	1 4 8 8 8
10	3 5 9
11	0 4 6 6

$\_\_ = \frac{\square}{\square} = \_\_\_\_$

12

Tens	Units
78	0 0 1 4
79	2 6 7 9
80	4 5

Liam performs market research for supermarkets. He prices 6 products at 5 stores to base his research, shown in the table. Total each column.

Product	Shop A	Shop B	Shop C	Shop D	Shop E
Window Cleaner	$3.25	$3.07	$4.15	$3.35	$3.56
Baked Beans	$1.82	$1.96	$2.19	$1.75	$1.99
Eggs	$5.35	$4.83	$6.10	$5.25	$5.85
Laundry Powder	$14.60	$15.45	$14.95	$14.90	$10.75
Pasta	$3.11	$3.20	$3.55	$2.96	$3.87
BBQ Chicken	$9.00	$10.55	$12.17	$11.60	$10.85
13 Total					

Key: ⊠ Price Above Store Average ☆ Price Equal to, or Below Store Average

Find the average price for a product from all the shops. Then colour a star if the shop price equals or is less than the average price for the product. Colour the cross if the price is above the average shop price.

14 Window Cleaner

Add all shop prices for Window Cleaner

$\bar{x} = \frac{\square}{5} = \$\square$

15 Baked Beans

$\_\_ = \frac{\square}{\square} = \square$

16 Eggs

$\_\_ = \frac{\square}{\square} = \square$

17 Laundry Powder

$\_\_ = \frac{\square}{\square} = \square$

18 Pasta

$\_\_ = \frac{\square}{\square} = \square$

19 Barbequed Chicken

$\_\_ = \frac{\square}{\square} = \square$

20 Discuss Shop E result in class.

Data Analysis - The Mean

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Find the mean for these, the answers are integers.

1 16, 24, 41.

Add the scores

$\bar{x} = \frac{81}{3} = 27$

Divide total by 3

Three scores = 3

2 35, 47, 64, 31, 53.

$\bar{x} = \frac{230}{5} = 46$

3 8, 13, 17, 23, 15, 19, 31.

$\bar{x} = \frac{126}{7} = 18$

4 -6, -2, 5, -7, 3, 0, -4, 6, -9, 4.

$\bar{x} = \frac{-10}{10} = -1$

These questions include units. Find $\bar{x}$. (1 d.p.)

5 846 m, 277 m, 567 m, 308 m.

$\bar{x} = \frac{1\,998}{4} = 499.5$ m

6 1.2 kg, 2.3 kg, 805 g.

$\bar{x} = \frac{4\,305}{3} = 1\,435$ g

Answer in grams

7 25 s, 1 m 41 s, 82 s, 1 m 17 s, 19 s, 2 m, 1 m 53 s, 34 s, 41 s.

$\bar{x} = \frac{612}{9} = 1$ min 8 s

Answer in m & s

8 56 cm, 25 cm, 1.2 m, 38 cm, 1.07 m, 94 cm, 1.9 m, 74 cm.

$\bar{x} = \frac{704}{8} = 88$ cm

Answer in cm

9 0.7 L, 1.05L, 217 ml, 1.2 L, 850 mL, 1.5 L, 3.03 L

$\bar{x} = \frac{8.547}{7} = 1.2$ L

Answer in L

Find the mean for these Stem and Leaf plots. Round to 1 decimal place.

10

Tens	Units
0	5 7 8 8
1	3 8
2	0 0 0 3 9
3	3 4 6

$\bar{x} = \frac{274}{14} = 19.6$

$\bar{x} = \frac{7\,918}{10} = 791.8$

11

Tens	Units
9	1 4 8 8 8
10	3 5 9
11	0 4 6 6

$\bar{x} = \frac{1\,252}{12} = 104.3$

12

Tens	Units
78	0 0 1 4
79	2 6 7 9
80	4 5

Liam performs market research for supermarkets. He prices 6 products at 5 stores to base his research, shown in the table. Total each column.

Product	Shop A	Shop B	Shop C	Shop D	Shop E
Window Cleaner	$3.25	$3.07	$4.15	$3.35	$3.56
Baked Beans	$1.82	$1.96	$2.19	$1.75	$1.99
Eggs	$5.35	$4.83	$6.10	$5.25	$5.85
Laundry Powder	$14.60	$15.45	$14.95	$14.90	$10.75
Pasta	$3.11	$3.20	$3.55	$2.96	$3.87
BBQ Chicken	$9.00	$10.55	$12.17	$11.60	$10.85
13 Total	$37.13	$39.06	$43.11	$39.81	$36.87

Key: ✖ Price Above Store Average ★ Price Equal to, or Below Store Average

Find the average price for a product from all the shops. Then colour a star if the shop price equals or is less than the average price for the product. Colour the cross if the price is above the average shop price.

14 Window Cleaner

Add all shop prices for Window Cleaner

$\bar{x} = \frac{17.38}{5} = \$3.48$

15 Baked Beans

$\bar{x} = \frac{9.71}{5} = \$1.94$

16 Eggs

$\bar{x} = \frac{27.38}{5} = \$5.48$

17 Laundry Powder

$\bar{x} = \frac{70.65}{5} = \$14.13$

18 Pasta

$\bar{x} = \frac{16.69}{5} = \$3.34$

19 Barbequed Chicken

$\bar{x} = \frac{54.17}{5} = \$10.83$

20 Discuss Shop E result in class.

Data Analysis - Further Mean

The previous sheet is about calculating the mean from a data set. This sheet works backwards asking you to calculate a score given the mean of a set of scores. Below is an example.

Example - Finding a score given the mean

Data: 46, 75, 19, 55, x. $\bar{x} = 47$.

$5 \times 47 = 235$ ← Multiply the mean by the number of scores (including x)

$x = 235 - 46 - 75 - 19 - 55$

or

$x = 235 - 195$

← Subtract the given scores from the total, calculated above. Use either method, the bottom method is better when there is a larger numbers of scores.

$x = 40$ ← The missing score is 40, calculate the mean to check.

Column 2 onwards uses algebra to solve the questions. Two step equations are formed and solved to find the missing scores.

Example

If Tom scored 9% more than Rebecca and Sienna scored double Rebecca's score, find their scores if together they have an average of 65%.

If Rebecca's score is called 'x' and Tom scored 9% more, then Tom's score is '$x + 9$'.
If Sienna scored double Rebecca's score then Sienna scored $2x$.

$x + (x + 9) + 2x = 3 \times 65$ ← Add the scores and it is equal to the number of scores × the mean.

$4x + 9 = 195$ ← Collect like terms and simplify

-9 -9

$4x = 186$ ← Remove the '+ 9' by subtracting 9 from each side of the equation.

÷4 ÷4

$x = 46.5$ ← Divide both sides of the equation by 4 to get x. Remember x is Rebecca's score.

Rebecca 46.5% Tom 55.5% Sienna 93%

Tom is 46.5 + 9

Sienna is 2 × 46.5

Data Analysis - Further Mean

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Given the mean, find the missing score 'x'.

Example

Data: 16, 24, 27, x. $\bar{x} = 21$

$4 \times 21 = 84$

The sum of the scores = 84.

$x = 84 - 16 - 24 - 27$

$x = 17$

1 Data: 45, 25, x. $\bar{x} = 26$

2 Data: 105, 38, 64, x. $\bar{x} = 83$

3 Data: 78, 93, 31, 24, 16, x. $\bar{x} = 44$

4 Jane has finished her 4th test in English. Her earlier test results are 62%, 75% and 68%. What result does she need in her 4th test for an average of 75%?

5 After two tests Ron has an average of 86%. What mark is required in his last test to end with 90% overall.

It's time to solve questions using algebra. Call the unknown score x.

Example

Three consecutive scores have a mean of 144, find the scores.

$x + (x + 1) + (x + 2) = 3 \times 144$

$3x + 3 = 432$

-3 -3

$3x = 429$

$\div 3$ $\div 3$

$x = 143$

Scores are 143, 144 and 145.

6 Four consecutive even scores have a mean of 87, find the scores. Use x, $x + 2$, $x + 4$ ...

7 Five consecutive odd scores have a mean of 41, find the scores. Use x, $x + 2$, $x + 4$ ...

8 John scored x in his test, Ben scored 10 marks more, while Jodi scored 14 more than John. If the average score is 67% find each person's score.

John ☐ Ben ☐ Jodi ☐

9 Joe is 10 years younger than Ian, while Jess is twice as old as Joe. Liam is 20 years older than Ian. If the average age of all the group is 50, calculate each person's age.

Joe ☐ Ian ☐

Jess ☐ Liam ☐

Data Analysis - Further Mean

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Given the mean, find the missing score 'x'.

Example

Data: 16, 24, 27, x. $\bar{x} = 21$

$4 \times 21 = 84$

The sum of the scores = 84.

$x = 84 - 16 - 24 - 27$

$x = 17$

1 Data: 45, 25, x. $\bar{x} = 26$

$3 \times 26 = 78$

$x = 78 - 45 - 25$

$x = 8$

2 Data: 105, 38, 64, x. $\bar{x} = 83$

$4 \times 83 = 332$

$x = 332 - 105 - 38 - 64$

$x = 125$

3 Data: 78, 93, 31, 24, 16, x. $\bar{x} = 44$

$6 \times 44 = 264$

$x = 264 - 78 - 93 - 31 - 24 - 16$

$x = 22$

4 Jane has finished her 4th test in English. Her earlier test results are 62%, 75% and 68%. What result does she need in her 4th test for an average of 75%?

$4 \times 75 = 300$

$x = 300 - 62 - 75 - 68$

$x = 95\%$

5 After two tests Ron has an average of 86%. What mark is required in his last test to end with 90% overall.

$3 \times 90 = 270$

$x = 270 - 86 - 86$

$x = 98\%$

It's time to solve questions using algebra. Call the unknown score x.

Example

Three consecutive scores have a mean of 144, find the scores.

$x + (x + 1) + (x + 2) = 3 \times 144$

$3x + 3 = 432$

(-3) (-3)

$3x = 429$

$(\div 3)$ $(\div 3)$

$x = 143$

Scores are 143, 144 and 145.

6 Four consecutive even scores have a mean of 87, find the scores. Use x, $x + 2$, $x + 4$ ...

$x + x + 2 + x + 4 + x + 6 = 348$

$4x + 12 = 348$

(-12) (-12)

$4x = 336$

$(\div 4)$ $(\div 4)$

$x = 84$

Numbers are 84, 86, 88 & 90.

7 Five consecutive odd scores have a mean of 41, find the scores. Use x, $x + 2$, $x + 4$ ...

$x + x + 2 + x + 4 + x + 6 + x + 8$

$= 5x + 20$

$5 \times 41 = 205$

$5x + 20 = 205$

(-20) (-20)

$5x = 185$

$(\div 5)$ $(\div 5)$

$x = 37$

Numbers are 37, 39, 41, 43 & 45.

8 John scored x in his test, Ben scored 10 marks more, while Jodi scored 14 more than John. If the average score is 67% find each person's score.

$x + x + 10 + x + 14 = 3 \times 67$

$3x + 24 = 201$

(-24) (-24)

$3x = 177$

$(\div 3)$ $(\div 3)$

$x = 59$

John 59% Ben 69% Jodi 73%

9 Joe is 10 years younger than Ian, while Jess is twice as old as Joe. Liam is 20 years older than Ian. If the average age of all the group is 50, calculate each person's age.

$x + x + 10 + 2x + x + 30$

$= 5x + 40$

$5x + 40 = 4 \times 50$

$5x + 40 = 200$

(-40) (-40)

$5x = 160$

$(\div 5)$ $(\div 5)$

$x = 32$

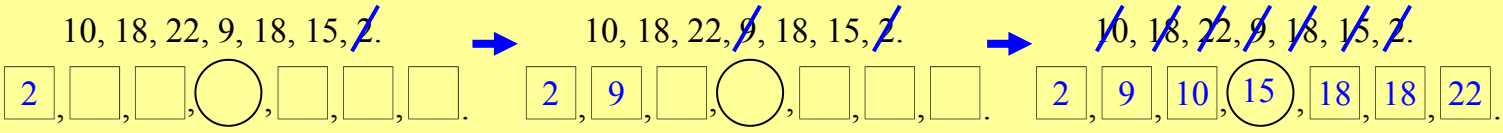
Joe	32	Ian	42
Jess	64	Liam	62

Data Analysis - The Median

The Median is the middle score in a data set. That is if there are an odd number of scores. Just imagine a see-saw pivoting on the median. Write the scores in order from lowest to highest and circle the middle number … that is the median.

Example - Finding the Median (Odd Number of Scores)

Data: 10, 18, 22, 9, 18, 15, 2.



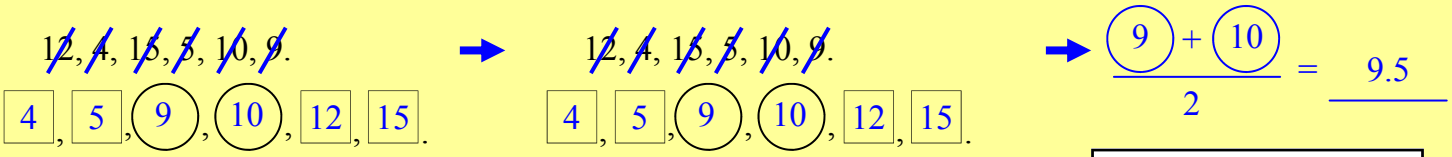
Find the smallest number in the data set and write it down first, then strike out the number to avoid mistakes.

Find the next smallest number and write it down.

Repeat the process until all the data is in ascending order. The middle score (circled) is the Median.

The above is for an odd number of scores. If there are an even number of scores then the Median is halfway between the two middle scores. So find the two middle scores and average them to find the Median.

Example - Finding the Median (Even Number of Scores)



Write the data in ascending order as with odd numbered data sets.

This time there are two middle numbers. Find the average, that is the median.

Calculator Steps:

9 + 1 0 = ÷ 2 =

Data can also be displayed in Ordered Stem and Leaf plots. These are easier to answer because there are already in ascending order. An example is below.

Example - Finding the Median in an Ordered Stem and Leaf Plot

Stem	Leaf
0	5 7 7
1	2 5 9
2	3 4 8

Cross out the first and last number.

Stem	Leaf
0	5 7 7
1	2 5 9
2	3 4 8

Now the 2nd and 2nd last numbers

Stem	Leaf
0	5 7 7
1	2 5 9
2	3 4 8

Repeat until left with one central number.

The median is 15.
(As the Stem is 1 and Leaf is 5).

If there are an even number of scores then you have to average the two middle scores, just like with a data set.

Data Analysis - The Median

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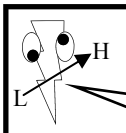

Rewrite these scores in ascending order. Circle the median. (Middle)

1 8, 7, 10, 9, 6, 5, 4.

□, □, □, ◯, □, □, □.

2 16, 6, -5, 8, 11.

□, □, ◯, □, □.

3 15, 9, 12, 3, 7, 2, 6, 6, 7.

4 8, 8, 6.

An even number of scores has its median between the two centre numbers. Average the two middle numbers.

5 3, 11, 4, 8.

Rewrite Ascending □, ◯, ◯, □.

$$\frac{\bigcirc + \bigcirc}{2} = \square$$

6 12, 23, 15, 24, 21, 14.

□, □, ◯, ◯, □, □.

$$\frac{\bigcirc + \bigcirc}{2} = \square$$

7 5, 3, 2, 7, 6, 4, 7, 4, 6, 8.

$$\frac{\bigcirc + \bigcirc}{\square} = \square$$

8 8, 3, 14, -5, 0, 6, -2, -7.

$$\frac{\bigcirc + \bigcirc}{\square} = \square$$

Find the median for these scores. Answer using the working spaces below.

9 8, 3, 2, 1, 9.

10 24, 30, 14, 10.

11 156, 72, 83, 28, 40, 133.

12 16, 7, 12, -3, 7, 10, 0.

13 0, -2, 6, -4, 8, 4, -4, 6.

14 -8, 2, 10, -6, 4, -8, 6, -4, -5.

15 2.4, 4.1, 2.9. 4.3, 3.6, 7.8.

Answer below, write the question number answered in the box below.
3 spaces for odd numbered data sets.
4 spaces for even numbered data sets.

□

The median is □

□

The median is □

□

□

□

$$\frac{\bigcirc + \bigcirc}{\square} = \square$$

□

$$\frac{\bigcirc + \bigcirc}{\square} = \square$$

□

$$\frac{\bigcirc + \bigcirc}{\square} = \square$$

□

$$\frac{\bigcirc + \bigcirc}{\square} = \square$$

Find the median for these Stem and Leaf plots.

16

Stem	Leaf
0	3 3 7
1	0
2	5 6 6 6 7
3	2 4

The median is □

17

Tens	Units
8	3 5 8
9	4 6 7 9
10	0 5 5
11	1 2

$$\frac{\bigcirc + \bigcirc}{\square} = \square$$

18

Tens	Units
97	0 1 1
98	4 5 5 5
99	0 0 2
100	2 3 4 4

$$\frac{\bigcirc + \bigcirc}{\square} = \square$$

19

Stem	Leaf
-4	0 0 2 5 8
-3	2 2 2
-2	4 6 7 8
-1	1 1
-0	6 7 7 7 9
0	0 0 3 9 9
1	2 6 6 6 8

□

Data Analysis - The Median

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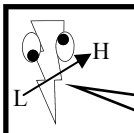

Rewrite these scores in ascending order. Circle the median. (Middle)

1 ~~8~~, ~~7~~, ~~10~~, ~~9~~, ~~6~~, ~~5~~, ~~4~~.

4, 5, 6, (7), 8, 9, 10.

2 ~~16~~, ~~6~~, ~~-5~~, ~~8~~, ~~11~~.

-5, 6, (8), 11, 16.

3 ~~15~~, ~~9~~, ~~12~~, ~~3~~, ~~7~~, ~~2~~, ~~6~~, ~~6~~, ~~7~~.

2, 3, 6, 6, (7), 7, 9, 12, 15

4 ~~8~~, ~~8~~, ~~6~~.

6, (8), 8.

An even number of scores has its median between the two centre numbers. Average the two middle numbers.

5 ~~3~~, ~~11~~, ~~4~~, ~~8~~.

Rewrite Ascending: 3, (4), (8), 11.

$\frac{4 + 8}{2} = 6$

6 ~~12~~, ~~23~~, ~~15~~, ~~24~~, ~~21~~, ~~14~~.

12, 14, (15), (21), 23, 24.

$\frac{15 + 21}{2} = 18$

7 ~~5~~, ~~3~~, ~~2~~, ~~7~~, ~~6~~, ~~4~~, ~~7~~, ~~4~~, ~~6~~, ~~8~~.

2, 3, 4, 4, (5), (6), 6, 7, 7, 8.

$\frac{5 + 6}{2} = 5.5$

8 ~~8~~, ~~3~~, ~~14~~, ~~-5~~, ~~0~~, ~~6~~, ~~-2~~, ~~-7~~.

-7, -5, -2, (0), (3), 6, 8, 14.

$\frac{0 + 3}{2} = 1.5$

Find the median for these scores. Answer using the working spaces below.

9 8, 3, 2, 1, 9.

10 24, 30, 14, 10.

11 156, 72, 83, 28, 40, 133.

12 16, 7, 12, -3, 7, 10, 0.

13 0, -2, 6, -4, 8, 4, -4, 6.

14 -8, 2, 10, -6, 4, -8, 6, -4, -5.

15 2.4, 4.1, 2.9. 4.3, 3.6, 7.8.

Answer below, write the question number answered in the box below. 3 spaces for odd numbered data sets. 4 spaces for even numbered data sets.

9 1, 2, (3), 8, 9.

The median is 3

12 -3, 7, 0, (7), 10, 12, 16

The median is 7

14 -8, -8, -6, -5, (-4), 2, 4, 6, 10.

The median is -4.

10 10, (14), (24), 30

$\frac{14 + 24}{2} = 19$

11 28, 40, (72), (83), 133, 156.

$\frac{72 + 83}{2} = 77.5$

13 -4, -4, -2, (0), (4), 6, 6, 8.

$\frac{0 + 4}{2} = 2$

15 2.4, 2.9, (3.6), (4.1), 4.3, 7.8.

$\frac{3.6 + 4.1}{2} = 3.85$

Find the median for these Stem and Leaf plots.

16

Stem	Leaf
0	~~3~~ ~~3~~ ~~7~~
1	~~0~~
2	~~5~~ 6 ~~6~~ ~~6~~ ~~7~~
3	~~2~~ ~~4~~

The median is 26

17

Tens	Units
8	~~5~~ ~~5~~ ~~8~~
9	~~4~~ ~~6~~ 7 9
10	~~0~~ ~~5~~ ~~5~~
11	~~1~~ ~~2~~

$\frac{97 + 99}{2} = 98$

18

Tens	Units
97	~~0~~ ~~1~~ ~~1~~
98	~~4~~ ~~5~~ ~~5~~ 5
99	0 ~~0~~ ~~2~~
100	~~2~~ ~~3~~ ~~4~~ ~~4~~

$\frac{985 + 990}{2} = 987.5$

19

Stem	Leaf
-4	~~0~~ ~~0~~ ~~2~~ ~~5~~ ~~8~~
-3	~~2~~ ~~2~~ ~~2~~
-2	~~4~~ ~~6~~ ~~7~~ ~~8~~
-1	~~1~~ ~~1~~
-0	6 ~~7~~ ~~7~~ ~~7~~ ~~9~~
0	~~0~~ ~~0~~ ~~3~~ ~~9~~ ~~9~~
1	~~2~~ ~~6~~ ~~6~~ ~~6~~ ~~8~~

The median is -6.

Data Analysis - Median Position

Crossing off the first and last score, then the 2nd and 2nd last score ….. and so on. This is the manual way to find the median. But the faster way, especially with larger amounts of data is to use the formula:

$$\text{Median Position} = \frac{n+1}{2}$$

Where n = number of scores in a data set. Note the answer can be a whole number or end with 0.5.

Example - Odd number of scores

Data: 1, 5, 7, 10, 14, 19, 23.

There are 7 scores

$$\text{Position} = \frac{7+1}{2} = 4^{th} \text{ score.}$$

So for the data above, the median is the 4th score.

Data: 1, 5, 7, 10, 14, 19, 23.

Median = 10

Example - Even number of scores

Data: 1, 5, 7, 10, 14, 19, 23, 37, 37, 40.

There are 10 scores

$$\text{Position} = \frac{10+1}{2} = 5.5^{th} \text{ score.}$$

So for the data above, the median is the 5.5th score. In other words it is between the 5th and 6th scores.

Data: 1, 5, 7, 10, 14, 19, 23, 37, 37, 40.

$$\text{Median} = \frac{14+19}{2} = 16.5$$

Example Stem and Leaf Plots

Tens	Units
0	1 7
1	0 0 5
2	1 8 8 8 9
3	3 3

There are 12 scores

$$\text{Position} = \frac{12+1}{2} = 6.5^{th} \text{ score.}$$

$$\text{Median} = \frac{21+28}{2} = 24.5$$

Data Analysis - Median Position

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Find the position of the median and the median itself, the data is in ascending order.

1 3, 4, 7, 12, 12, 15, 18, 25, 26, 30, 38, 38, 45, 48, 55, 63, 70.

Position = $\frac{\square + 1}{2}$ (Count the scores)

$= \underline{\square^{th} \text{ score.}}$

The median is $\square$

2 63, 82, 83, 87, 88, 94, 96, 98, 99, 100, 104, 110, 115, 122, 122, 127, 129, 135, 142, 154, 179.

Position = $\frac{\square + 1}{2}$

$= \underline{\square^{th} \text{ score.}}$

The median is $\square$

3 -19, -12, -5, -2, -2, 0, 0, 3, 4, 12, 13, 13, 18, 23, 24, 29, 29, 30, 31, 37, 39, 42, 46, 55, 57, 59, 94, 123, 128, 152.

Position =

Median = $\frac{\square + \square}{2} = \square$

4 105, 127, 185, 209, 234, 276, 362, 368, 391, 412, 419, 438, 446, 502, 535, 572, 590, 590, 612, 616, 623, 655, 662, 720, 739, 750, 750, 773, 781, 782, 810, 834, 880, 881.

Position =

Median =

Arrange the data in ascending order then find both the position of the median and its value.

5 53, 26, 40, 45, 52, 31, 76, 42, 50, 64, 39, 43, 24, 33, 60, 59, 78, 25, 10, 18, 32, 40, 57, 62, 20, 25, 55, 19, 28.

Position = Median =

Find the position and value for the median in the stem and leaf plots.
For Q.8, 9 place the data in the raw and ordered S&L plots first.

6

Tens	Units
0	5 7 7 8 9
1	4 4
2	1 1 1 4 4
3	0 0 3

Position =

Median =

7

Tens	Units
8	3 5
9	2 4 9
10	0 3 3 4 7
11	2 6

Position =

Median =

8 16, 25, 11, 24, 8, 12, 24, 20, 23, 15, 19, 4, 13, 8, 2, 9, 3.

RAW

Stem	Leaf
$\square$	.....
$\square$	.....
$\square$	.....

ORDERED

Stem	Leaf
$\square$	.....
$\square$	.....
$\square$	.....

9 58, 42, 67, 70, 73, 53, 68, 32, 49, 50, 62, 76, 61, 75, 41, 62, 44, 34, 78, 51, 57, 33, 69, 72, 40, 35, 52, 66, 74, 49, 63, 67.

RAW

Tens	Units
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....

ORDERED

Tens	Units
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....
$\square$	.....

Data Analysis - Median Position

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Find the position of the median and the median itself, the data is in ascending order.

1 3, 4, 7, 12, 12, 15, 18, 25, 26, 30, 38, 38, 45, 48, 55, 63, 70.

Position = $\frac{17 + 1}{2}$ (Count the scores)

$= 9^{th}$ score.

The median is 26

2 63, 82, 83, 87, 88, 94, 96, 98, 99, 100, 104, 110, 115, 122, 122, 127, 129, 135, 142, 154, 179.

Position = $\frac{21 + 1}{2}$

$= 11^{th}$ score.

The median is 104

3 -19, -12, -5, -2, -2, 0, 0, 3, 4, 12, 13, 13, 18, 23, 24, 29, 29, 30, 31, 37, 39, 42, 46, 55, 57, 59, 94, 123, 128, 152.

Position = $\frac{30 + 1}{2}$

$= 15.5^{th}$ score

Median = $\frac{24 + 29}{2} = 26.5$

4 105, 127, 185, 209, 234, 276, 362, 368, 391, 412, 419, 438, 446, 502, 535, 572, 590, 590, 612, 616, 623, 655, 662, 720, 739, 750, 750, 773, 781, 782, 810, 834, 880, 881.

Position = $\frac{34 + 1}{2}$

$= 17.5^{th}$ score

Median = $\frac{590 + 590}{2}$

$= 590$

Arrange the data in ascending order then find both the position of the median and its value.

5 53, 26, 40, 45, 52, 31, 76, 42, 50, 64, 39, 43, 24, 33, 60, 59, 78, 25, 10, 18, 32, 40, 57, 62, 20, 25, 55, 19, 28.

10, 18, 19, 20, 24, 25, 25, 26, 28, 31, 32, 33, 39, 40, 40, 42, 43, 45, 50, 52, 53, 55, 57, 59, 60, 62, 64, 76, 78.

Position = $\frac{29 + 1}{2} = 15^{th}$ score

Median = 40.

Find the position and value for the median in the stem and leaf plots. For Q.8, 9 place the data in the raw and ordered S&L plots first.

6

Tens	Units
0	5 7 7 8 9
1	4 4
2	1 1 1 4 4
3	0 0 3

Position = $\frac{15 + 1}{2} = 8^{th}$ score

Median = 21

7

Tens	Units
8	3 5
9	2 4 9
10	0 | 3 3 4 7
11	2 6

Position = $\frac{12 + 1}{2} = 6.5^{th}$ score

Median = $\frac{100 + 103}{2} = 101.5$

8 16, 25, 11, 24, 8, 12, 24, 20, 23, 15, 19, 4, 13, 8, 2, 9, 3.

RAW

Stem	Leaf
0	848293
1	612593
2	54403

ORDERED

Stem	Leaf
0	234889
1	123569
2	03445

Position = $\frac{17 + 1}{2}$

$= 9^{th}$ score

Median = 13

9 58, 42, 67, 70, 73, 53, 68, 32, 49, 50, 62, 76, 61, 75, 41, 62, 44, 34, 78, 51, 57, 33, 69, 72, 40, 35, 52, 66, 74, 49, 63, 67.

RAW

Tens	Units
3	2435
4	291409
5	830172
6	782129637
7	0365824

ORDERED

Tens	Units
3	2345
4	012499
5	012378
6	122367789
7	0234568

Position = $\frac{32 + 1}{2}$

$= 16.5^{th}$ score

Median = $\frac{58 + 61}{2}$

$= 59.5$

Data Analysis - Mode, Median, Range and Mean.

These sheets are review sheets and so the method of finding the Mode, Mean, Range and Median won't be outlined below. But here is a reminder:

Mode - Is the score that occurs the most, there can be 1 mode, 2 modes (bi-modal) or No Mode. Write "No Mode" not "0" if there is no mode. If there are 3 or modes then the data is considered to have no mode.

Mean - Is the average of the scores. So add the scores then divide by the number of scores.

Range - Is the highest score minus the lowest score. If the lowest score is negative then when you subtract the two negative signs change to a plus '+'.

Median - Is the middle score when the scores are arranged in ascending order. If there are an odd number of scores it is the middle score. If there are an even number of scores then the median is the average of the two middle scores. So add them and divide by two.

The first worksheet is for Mean and Range and is designed to be completed with the second worksheet which is for Mode and Median.

The third worksheet is all four, Mode, Mean, Median and Range, being completed for each question.

Data Analysis - Mean and Range

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The MEAN is the average of the scores. Add the scores and divide by the number of scores. Calculate the mean rounding to 1 d.p. if required.

Example

3, 2, 4, 1, 5. $\bar{x} = \frac{15}{5} = 3$ ← 3 + 2 + 4 + 1 + 5 = 15; 5 ← There are 5 scores

1 9, 3, 6, 1, 3. $\bar{x} = \frac{\square}{\square} = \_\_\_$

2 2, 6, 8, 3, 9, 7, 5. $\bar{x} = \frac{\square}{\square} = \_\_\_$

3 -5, 2, -4, 2. $\bar{x} = \frac{\square}{\square} = \_\_\_$

4 8, 2, 9, 4, 5, 3, 7, 6. $\bar{x} = \frac{\square}{\square} = \_\_\_$

5 895, 403, 89. $\frac{\square}{\square} = \_\_\_$

6 2, 3, 3, 8, 7, 8, 9, 10, 5. $\frac{\square}{\square} = \_\_\_$

7 1 056, 695. $\frac{\square}{\square} = \_\_\_$

8 2, 3, 3, 5, 8, 4, 7, 7, 1, 2. $\frac{\square}{\square} = \_\_\_$

9 56, 13, 11, 7. $\frac{\square}{\square} = \_\_\_$

10 306, 712, 36, 112, 377. $\frac{\square}{\square} = \_\_\_$

11 55, -81, 163. $\frac{\square}{\square} = \_\_\_$

12 1 166, 3 992, 2 001. $\frac{\square}{\square} = \_\_\_$

13 13, 30, 25, 19. $\frac{\square}{\square} = \_\_\_$

14 7, -16, 5, 24, -35, -48. $\frac{\square}{\square} = \_\_\_$

15

Tens	Units
0	4 5
1	3 6 8 8
2	7 7 7

$\frac{\square}{\square} = \_\_\_$

16

Stem	Leaf
9	0 0 3 7
10	2 2
11	8 8 8 9

$\frac{\square}{\square} = \_\_\_$

The RANGE is the highest score minus the lowest score. Calculate the range.

Example

3, 2, 4, 1, 5. Range = 5 - 1 = 4 (Highest score is 5. Lowest score is 1.)

17 9, 3, 6, 1, 3. Range = ☐ - ☐ = ___

18 4, 5, 7, 7, 3, 2. Range = ☐ - ☐ = ___

19 9, 4, 1, 1, 0, 3. Range = ☐ - ☐ = ___

20 8, 3, -4, 7. Range = ☐ - ☐ = ___

21 -10, -5, -6, -8, -15. ___

22 35 076, 887, 21 302. ___

23 45, 11, 9, 15, 5, 16. ___

24 167, 93, 205, 199. ___

25 -5, 2, 17, -4, 0, 10. ___

26 8, 8, 8, 8, 8. ___

27

Stem	Leaf
5	0 1 3 3
6	2 4
7	5 5 6

28

Stem	Leaf
-0	7 9
0	3 5 7 8
1	1 1 3

29 7.2, 2.7, 7.22, 2.72. ___

30 3, 0.3, 3.03, 0.03, 3.3. ___

31 $3^1/_4$, $5^2/_3$, $6^5/_7$, $3^2/_5$. ___

32 $1^1/_4$, $1^3/_5$, $1^1/_{10}$, $1^3/_4$. ___

33

Tens	Units
7	0 5
8	4 4 9
9	3
10	2 9 9
11	1 1 4

34

Stem	Leaf
89	4
90	1 7 2
91	6
92	0 0
93	0 9

Data Analysis - Mean and Range

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The MEAN is the average of the scores. Add the scores and divide by the number of scores. Calculate the mean rounding to 1 d.p. if required.

Example

3, 2, 4, 1, 5. $\bar{x} = \frac{15}{5} = 3$ ← 3 + 2 + 4 + 1 + 5 = 15; 5 ← There are 5 scores

1 9, 3, 6, 1, 3. $\bar{x} = \frac{22}{5} = 4.4$

2 2, 6, 8, 3, 9, 7, 5. $\bar{x} = \frac{40}{7} = 5.7$

3 -5, 2, -4, 2. $\bar{x} = \frac{-5}{4} = -1.3$ (1 d.p.)

4 8, 2, 9, 4, 5, 3, 7, 6. $\bar{x} = \frac{44}{8} = 5.5$

5 895, 403, 89. $\bar{x} = \frac{1\,387}{3} = 462.3$ (1 d.p.)

6 2, 3, 3, 8, 7, 8, 9, 10, 5. $\bar{x} = \frac{55}{9} = 6.1$ (1 d.p.)

7 1 056, 695. $\bar{x} = \frac{1\,751}{2} = 875.5$

8 2, 3, 3, 5, 8, 4, 7, 7, 1, 2. $\bar{x} = \frac{42}{10} = 4.2$

9 56, 13, 11, 7. $\bar{x} = \frac{87}{4} = 21.8$ (1 d.p.)

10 306, 712, 36, 112, 377. $\bar{x} = \frac{1\,543}{5} = 308.6$

11 55, -81, 163. $\bar{x} = \frac{137}{3} = 45.7$ (1 d.p.)

12 1 166, 3 992, 2 001. $\bar{x} = \frac{7\,159}{3} = 2\,386.3$ (1 d.p.)

13 13, 30, 25, 19. $\bar{x} = \frac{87}{4} = 21.8$ (1 d.p.)

14 7, -16, 5, 24, -35, -48. $\bar{x} = \frac{-63}{6} = -10.5$

15

Tens	Units
0	4 5
1	3 6 8 8
2	7 7 7

$\bar{x} = \frac{155}{9} = 17.2$ (1 d.p.)

16

Stem	Leaf
9	0 0 3 7
10	2 2
11	8 8 8 9

$\bar{x} = \frac{1\,047}{10} = 104.7$

The RANGE is the highest score minus the lowest score. Calculate the range.

Example

3, 2, 4, 1, 5 Range = 5 - 1 = 4 (Highest score is 5. Lowest score is 1.)

17 9, 3, 6, 1, 3. Range = 9 - 1 = 8

18 4, 5, 7, 7, 3, 2. Range = 7 - 2 = 5

19 9, 4, 1, 1, 0, 3. Range = 9 - 0 = 9

20 8, 3, -4, 7. Range = 8 - -4 = 12

21 -10, -5, -6, -8, -15. -5 - (-15) = 10

22 35 076, 887, 21 302. 35 076 - 887 = 34 189

23 45, 11, 9, 15, 5, 16. 45 - 5 = 40

24 167, 93, 205, 199. 205 - 93 = 112

25 -5, 2, 17, -4, 0, 10. 17 - (-5) = 22

26 8, 8, 8, 8, 8. 8 - 8 = 0

27

Stem	Leaf
5	0 1 3 3
6	2 4
7	5 5 6

76 - 50 = 26

28

Stem	Leaf
-0	7 9
0	3 5 7 8
1	1 1 3

13 - (-9) = 22

29 7.2, 2.7, 7.22, 2.72. 7.22 - 2.7 = 4.52

30 3, 0.3, 3.03, 0.03, 3.3. 3.3 - 0.03 = 3.27

31 $3\frac{1}{4}, 5\frac{2}{3}, 6\frac{5}{7}, 3\frac{2}{5}$. $6\frac{5}{7} - 3\frac{1}{4} = 3\frac{13}{28}$

32 $1\frac{1}{4}, 1\frac{3}{5}, 1\frac{1}{10}, 1\frac{3}{4}$. $1\frac{3}{4} - 1\frac{1}{10} = \frac{13}{20}$

33

Tens	Units
7	0 5
8	4 4 9
9	3
10	2 9 9
11	1 1 4

114 - 70 = 44

34

Stem	Leaf
89	4
90	1 7 2
91	6
92	0 0
93	0 9

939 - 894 = 45

Data Analysis - Mode and Median

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The MODE is the score that occurs the most. There can be one mode, two modes or no mode. If three or more scores are 'modes', then its 'No Mode'.



Examples

3, 7, 4, 7, 5. Mode = 7 1, 2, 3, 4. No Mode

2, 7, 2, 7, 5. Modes = 2, 7

1 12, 9, 6, 3, 12, 4, 7. Mode = ____

2 7, 6, 19, 3, 8, 6, 5, 3, 9, 6. Mode = ____

3 5, 9, 5, 7, 3, 8, 8, 7, 6, 5, 8. Mode = ____

4 3, 7, 2, 9, 9, 2, 2. ____

5 12, 16, 11, 12, 13, 11, 9, 16. ____

6 8, 9, 3, 6, 3, 1, 5, 10. ____

7 7, 0, 3, 6, 5, 2, 2, 7, 3. ____

8 8, -5, 7, 10, 6, 3, 2, 5, 9. ____

9

Tens	Units
0	6 6
1	3 4 4 4
2	0 1 1 1

10

Tens	Units
9	0 1 2 2
10	0 1 2
11	0 1 2

11

Tens	Units
-1	3 7 7
-0	2 2
0	6 9
1	0 3
2	4

12

Stem	Leaf
97	6
98	2 2 7
99	0 5
100	7 7
101	0 5

13 2.5, 5.2, 2.25, 5.22, 2.2, 5.2. ____

14 0.44, 4.4, 4.4, 0.44, 4.04, 4. ____

15 16, 23, 19, 15, 12, 14. ____

16 -8, -6, 0, -7, -3, -6, -3, -2, -3. ____

The MEDIAN is the middle score when the scores are arranged in order (odd number of scores). If there are an even number of scores, average the two middle scores to find the median.

Examples

~~3~~, ~~7~~, ~~4~~, ~~7~~, ~~6~~. → 3, 4, 6, 7, 7. Median = 6.
In ascending order

~~3~~, ~~7~~, ~~2~~, ~~7~~. → 2, 3, 7, 7. Median = $\frac{3+7}{2} = 5$
In ascending order

17 6, 2, 1, 9, 6, 3, 4. Median = ____
In Order → ____

18 2, 7, 6, 9, 3, 2. Median = ____
In Order → ____

19 10, 2, 6, 4, 9, 3, 4, 1. Median = ____
In Order → ____

20 8, 3, 2, 1, 9, 6, 7, 2, 7. Median = ____
____

21

Tens	Units
5	4 8 8
6	3 6
7	7 9

22

Tens	Units
0	3
1	0 5 9
2	3 3

23 19, 7, 23, 41, 33. ____
____

24 154, 212, 166, 250. ____
____

25 -14, 22, -19, 23. ____
____

26 -7, -12, -16, 5, 0, -2, 3. ____
____

Data Analysis - Mode and Median

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The MODE is the score that occurs the most. There can be one mode, two modes or no mode. If three or more scores are 'modes', then its 'No Mode'.

Examples

3, 7, 4, 7, 5. Mode = 7 1, 2, 3, 4. No Mode

2, 7, 2, 7, 5. Modes = 2, 7

1 12, 9, 6, 3, 12, 4, 7. Mode = 12

2 7, 6, 19, 3, 8, 6, 5, 3, 9, 6. Mode = 6

3 5, 9, 5, 7, 3, 8, 8, 7, 6, 5, 8. Modes = 5, 8

4 3, 7, 2, 9, 9, 2, 2. Mode = 2

5 12, 16, 11, 12, 13, 11, 9, 16. No Mode

6 8, 9, 3, 6, 3, 1, 5, 10. Mode = 3

7 7, 0, 3, 6, 5, 2, 2, 7, 3. No Mode

8 8, -5, 7, 10, 6, 3, 2, 5, 9. No Mode

9

Tens	Units
0	6 6
1	3 4 4 4
2	0 1 1 1

Modes = 14, 21

10

Tens	Units
9	0 1 2 2
10	0 1 2
11	0 1 2

Mode = 92

11

Tens	Units
-1	3 7 7
-0	2 2
0	6 9
1	0 3
2	4

Modes = -17, -2

12

Stem	Leaf
97	6
98	2 2 7
99	0 5
100	7 7
101	0 5

Modes = 982, 1 007

13 2.5, 5.2, 2.25, 5.22, 2.2, 5.2. Mode = 5.2

14 0.44, 4.4, 4.4, 0.44, 4.04, 4. Modes=0.44,4.4

15 16, 23, 19, 15, 12, 14. No Mode

16 -8, -6, 0, -7, -3, -6, -3, -2, -3. Mode = -3

The MEDIAN is the middle score when the scores are arranged in order (odd number of scores). If there are an even number of scores, average the two middle scores to find the median.

Examples

3, 7, 4, 7, 6. → 3, 4, 6, 7, 7. (In ascending order) Median = 6.

3, 7, 2, 7. → 2, 3, 7, 7. (In ascending order) Median = $\frac{3+7}{2} = 5$

17 6, 2, 1, 9, 6, 3, 4.

In Order → 1, 2, 3, 4, 6, 6, 9.

Median = 4

18 2, 7, 6, 9, 3, 2.

In Order → 2, 2, 3, 6, 7, 9

Median = $\frac{3+6}{2} = 4.5$

19 10, 2, 6, 4, 9, 3, 4, 1.

In Order → 1, 2, 3, 4, 4, 6, 9, 10

Median = $\frac{4+4}{2} = 4$

20 8, 3, 2, 1, 9, 6, 7, 2, 7.

1, 2, 2, 3, 6, 7, 7, 8, 9

Median = 6

21

Tens	Units
5	4 8 8
6	3 6
7	7 9

Median = 63

22

Tens	Units
0	3
1	0 5 9
2	3 3

Median = $\frac{15+19}{2} = 17$

23 19, 7, 23, 41, 33.

7, 19, 23, 33, 41

Median = 23

24 154, 212, 166, 250.

154, 166, 212, 250

Median = $\frac{166+212}{2} = 189$

25 -14, 22, -19, 23.

-19, -14, 22, 23

Median = $\frac{-14+22}{2} = 4$

26 -7, -12, -16, 5, 0, -2, 3.

-16, -12, -7, -2, 0, 3, 5

Median = -2

Data Analysis - Mode, Median, Range and Mean.

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Calculate the mode, range, median and mean, for the data sets below. Round to 1 d.p. if required.

1 19 , 34 , 22 , 16 , 27 , 35 , 19 , 12 , 16

Write the scores out in ascending order on this line

Mode = _____ (Mode = Most)

Median = _____

_____ Line if even number of scores

Range = ☐ - ☐ = _____

Mean = ☐/☐ = _____

2 68 , 54 , 23 , 42 , 38 , 52 , 65 , 42 .

Mode = _____

Median = (☐ + ☐)/2 = _____

Range = _____ = _____

Mean = ☐/☐ = _____

3 240, 310, 205, 296, 240, 256, 205, 282, 310.

Mode = _____

Median = _____ = _____

Range = _____ = _____

Mean = ☐/☐ = _____

4 15, 11, -3, 12, 19, -7, 4, -10, 6, 7, 0, -3, 2, -8.

Mode = _____

Median = _____ = _____

Range = _____ = _____

Mean = ☐/☐ = _____

5 $375, $80, $210, $195, $145, $95, $110, $99.

Mode = _____

Median = _____ = _____

Range = _____ = _____

Mean = ☐/☐ = _____

Calculate the mode, range, median and mean (1 d.p.) for the ordered stem and leaf plot below.

6

Tens	Units
0	0 7 7
1	2 5 9 9
2	2 3 7
3	1
4	2 6

Mode = _____

Range = _____ = _____

Median = _____ = _____

Mean = ☐/☐ = _____

7

Stem	Leaf
7	5 5 9 9
8	0 0 2
9	6 6 6 7
10	
11	3 4 8
12	1 3 5 5

Mode = _____

Range = _____ = _____

Median = _____ = _____

Mean = ☐/☐ = _____

8

Stem	Leaf
29	5 8
30	9
31	6 6 8 8
32	0 1 1 4
33	3 3 8

Mode = _____

Range = _____ = _____

Median = _____ = _____

Mean = ☐/☐ = _____

9

Tens	Units
-2	0 6
-1	3 9 9
-0	4
0	7
1	0 5 5
2	0 2
3	6 8

Mode = _____

Range = _____ = _____

Median = _____ = _____

Mean = ☐/☐ = _____

Data Analysis - Mode, Median, Range and Mean.

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Calculate the mode, range, median and mean, for the data sets below. Round to 1 d.p. if required.

1 19 , 34 , 22 , 16 , 27 , 35 , 19 , 12 , 16

12, 16, 16, 19, 19, 22, 27, 34, 35.

Mode = 16, 19. (Mode = Most)

Median = 19

Line if even number of scores

Range = 35 - 12 = 23

Mean = 200 / 9 = 22.2 (1 d.p.)

2 68 , 54 , 23 , 42 , 38 , 52 , 65 , 42 .

23, 38, 42, 42, 52, 54, 65, 68

Mode = 42

Median = (42 + 52) / 2 = 47

Range = 68 - 23 = 45

Mean = 384 / 8 = 48

3 240, 310, 205, 296, 240, 256, 205, 282, 310.

205, 205, 240, 240, 256, 282, 296, 310, 310

Mode = No Mode

Median = 256

Range = 310 - 205 = 105

Mean = 2 344 / 9 = 260.4 (1 d.p.)

4 15, 11, -3, 12, 19, -7, 4, -10, 6, 7, 0, -3, 2, -8.

-10, -8, -7, -3, -3, 0, 2, 4, 6, 7, 11, 12, 15, 19

Mode = -3

Median = (2 + 4) / 2 = 3

Range = 19 - (-10) = 29

Mean = 45 / 14 = 3.2 (1 d.p.)

5 $375, $80, $210, $195, $145, $95, $110, $99.

$80, $95, $99, $110, $145, $195, $210, $375

Mode = No Mode

Median = (110 + 145) / 2 = 127.5

Range = 375 - 80 = $295

Mean = 1 309 / 8 = 163.6 (1 d.p.)

Calculate the mode, range, median and mean (1 d.p.) for the ordered stem and leaf plot below.

6

Tens	Units
0	0 7 7
1	2 5 9 (9)
2	2 3 7
3	1
4	2 6

Mode = 7, 19.

Range = 46 - 0 = 46

Median = 19

Mean = 270 / 13 = 20.8 (1 d.p.)

7 Mode = 96

Stem	Leaf
7	5 5 9 9
8	0 0 2
9	6 6 6 7
10	
11	3 4 8
12	1 3 5 5

Range = 125 - 75 = 50

Median = (96 + 96) / 2 = 96

Mean = 1 774 / 18 = 98.6 (1 d.p.)

8

Stem	Leaf
29	5 8
30	9
31	6 6 8 (8)
32	0 1 1 4
33	3 3 8

Mode = No Mode

Range = 338 - 295 = 43

Median = (318 + 320) / 2 = 319

Mean = 4 460 / 14 = 318.6 (1 d.p.)

9

Mode = -19, 15

Range = 38 - (-26) = 64

Median = (7 + 10) / 2 = 8.5

Mean = 62 / 14 = 4.4 (1 d.p.)

Tens	Units
-2	0 6
-1	3 9 9
-0	4
0	7
1	0 5 5
2	0 2
3	6 8

Frequency Tables - The Mean

Larger amounts of data is often 'tabulated' (presented in a frequency table) rather than in data sets. Finding the mean from a frequency table is the focus of this sheet. The formula for the calculation is:

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

Where: $\bar{x}$ is the mean (or average) of the scores
Σfx is the sum of the fx column
Σf is the sum of the f column

Example - Finding the mean from a frequency table

STEP 1 Frequency table, find the mean.

Score (x)	*Frequency (f)*
0	6
2	2
4	5
6	8
8	15
10	7

STEP 2 Add an fx column and 'total' boxes at the base.

Score (x)	*Frequency (f)*	*fx*
0	6	
2	2	
4	5	
6	8	
8	15	
10	7	

STEP 3 Multiply the f and the x to get fx.

Score (x)		*Frequency (f)*		*fx*
0	×	6	=	0
2	×	2	=	4
4	×	5	=	20
6	×	8	=	48
8	×	15	=	120
10	×	7	=	70

STEP 4 Add the f column for Σf and the fx column for Σfx.

Score (x)	*Frequency (f)*	*fx*
0	6 +	0 +
2	2 +	4 +
4	5 +	20 +
6	8 +	48 +
8	15 +	120 +
10	7 =	70 =
	$\Sigma f = 43$	$\Sigma fx = 262$

STEP 5 Calculate the mean. (1 d.p.)

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{262}{43} = 6.1 \text{ (1 d.p.)}$$

Frequency Tables - The Mean

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Add an *fx* column to the tables. Find the totals Σf and Σfx then calculate the mean. (1 d.p.)

1 Use a ruler to go over the dotted lines.

Score (x)		*Frequency (f)*		*fx*
1	×	3	=	
2	×	4	=	
3	×	2	=	
4	×	7	=	
5	×	4	=	
6	×	3	=	
Sum the *f* column →		$\Sigma f =$		$\Sigma fx =$ Sum the *fx* column

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{\quad}{\quad} = \quad$$

2

Score (x)	*Frequency (f)*	*fx*
0	17	
2	25	
4	16	
6	8	
	$\Sigma f =$	$\Sigma fx =$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{\quad}{\quad} = \quad$$

3

Score (x)	*Frequency (f)*	
1	2	
2	4	
3	4	
4	3	
5	7	
6	13	
7	18	
8	15	
9	12	
10	6	

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{\quad}{\quad} = \quad$$

We're taking data … raw data. A Tally Column is required, then find *f*, Σf, *fx*, Σfx and $\bar{x}$.

4 Data: 8, 3, 9, 7, 5, 8, 6, 9, 7, 6, 8, 3, 6, 8, 4, 9, 10, 7, 6, 8, 9, 9, 7, 6, 5, 4, 6, 4, 7, 8, 10, 7, 6, 5, 7, 4, 8, 9, 8, 9.

x	*Tally*	*f*	

$$\bar{x} = \text{——} = \frac{\quad}{\quad} = \quad$$

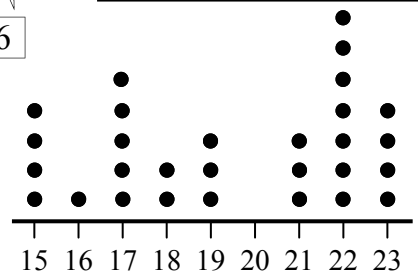
5 Data: 25, 26, 31, 27, 29, 25, 28, 23, 24, 26, 20, 22, 25, 24, 28, 25, 25, 26, 25, 26, 28, 30, 29, 22, 28, 31, 29, 22, 20, 23, 24, 25, 23, 28, 24, 26, 23, 22, 25, 24, 24, 26, 28, 25, 24, 27, 30.

x	*Tally*	*f*	

$$\bar{x} = \text{——} = \frac{\quad}{\quad} = \quad$$ Round to 1 d.p.

Convert these to frequency tables, then find *f*, Σf, *fx*, Σfx and $\bar{x}$, to 1 d.p.

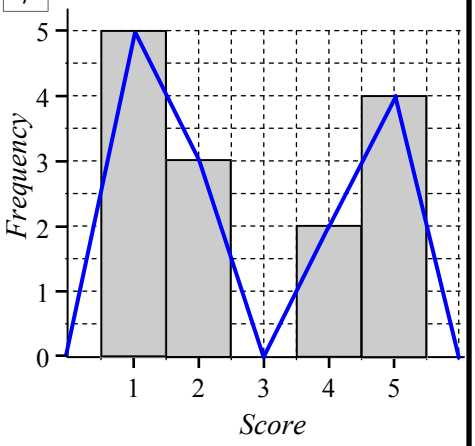
6



Score (x)	*Frequency (f)*	*fx*

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{\quad}{\quad} = \quad$$

7



Score (x)	*Frequency (f)*	*fx*

$$\bar{x} = \text{——} = \frac{\quad}{\quad} = \quad$$

Frequency Tables - The Mean

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Add an *fx* column to the tables. Find the totals Σf and Σfx then calculate the mean. (1 d.p.)

1 Use a ruler to go over the dotted lines.

Score (x)		*Frequency (f)*		*fx*
1	×	3	=	3
2	×	4	=	8
3	×	2	=	6
4	×	7	=	28
5	×	4	=	20
6	×	3	=	18
		$\Sigma f =$ 23		$\Sigma fx =$ 83

Sum the *f* column → $\Sigma f = 23$

Sum the *fx* column → $\Sigma fx = 83$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{83}{23} = 3.6 \text{ (1 d.p.)}$$

Round to 1 d.p.

2

Score (x)	*Frequency (f)*	*fx*
0	17	0
2	25	50
4	16	64
6	8	48
	$\Sigma f =$ 66	$\Sigma fx =$ 162

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{162}{66} = 2.5 \text{ (1 d.p.)}$$

3

Score (x)	*Frequency (f)*	*fx*
1	2	2
2	4	8
3	4	12
4	3	12
5	7	35
6	13	78
7	18	126
8	15	120
9	12	108
10	6	60
	$\Sigma f =$ 84	$\Sigma fx =$ 561

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{561}{84} = 6.7 \text{ (1 d.p.)}$$

We're taking data … raw data. A Tally Column is required, then find *f*, Σf, *fx*, Σfx and $\bar{x}$.

4 Data: 8, 3, 9, 7, 5, 8, 6, 9, 7, 6, 8, 7, 6, 8, 4, 9, 10, 7, 6, 8, 9, 9, 7, 6, 5, 4, 6, 4, 7, 8, 10, 7, 6, 5, 7, 4, 8, 9, 8, 9.

x	*Tally*	*f*	*fx*
3	||	2	6
4	||||	4	16
5	|||	3	15
6	𝍸 ||	7	42
7	𝍸 ||	7	49
8	𝍸 |||	8	64
9	𝍸 ||	7	63
10	||	2	20
		$\Sigma f =$ 40	$\Sigma fx =$ 275

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{275}{40} = 6.875$$

5 Data: 25, 26, 31, 27, 29, 25, 28, 23, 24, 26, 20, 22, 25, 24, 28, 25, 25, 26, 25, 26, 28, 30, 29, 22, 28, 31, 29, 22, 20, 23, 24, 25, 23, 28, 24, 26, 23, 22, 25, 24, 24, 26, 28, 25, 24, 27, 30.

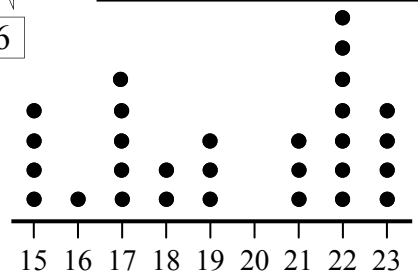
x	*Tally*	*f*	*fx*
20	||	2	40
21		0	0
22	||||	4	88
23	||||	4	92
24	𝍸 ||	7	168
25	𝍸 ||||	9	225
26	𝍸 |	6	156
27	||	2	54
28	𝍸 |	6	168
29	|||	3	87
30	||	2	60
31	||	2	62
		$\Sigma f =$ 47	$\Sigma fx =$1200

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1\,200}{47} = 25.5 \text{ (1 d.p.)}$$

Round to 1 d.p.

Convert these to frequency tables, then find *f*, Σf, *fx*, Σfx and $\bar{x}$, to 1 d.p.

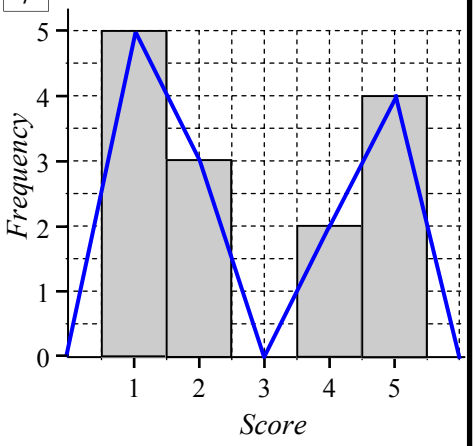
6



Score (x)	*Frequency (f)*	*fx*
15	4	60
16	1	16
17	5	85
18	2	36
19	3	57
20	0	0
21	3	63
22	7	154
23	4	92
	$\Sigma f =$ 29	$\Sigma fx =$ 563

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{563}{29} = 19.4 \text{ (1 d.p.)}$$

7



Score (x)	*Frequency (f)*	*fx*
1	5	5
2	3	6
3	0	0
4	2	8
5	4	20
	$\Sigma f =$ 14	$\Sigma fx =$ 39

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{39}{14} = 2.8 \text{ (1 d.p.)}$$

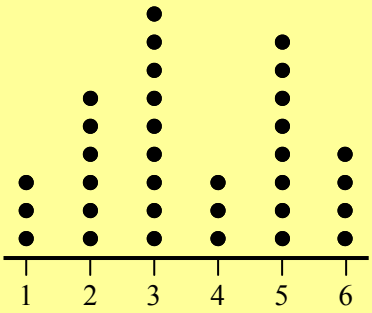
Graphs/Dot Plots - The Mean

This sheet builds on skills learned in the previous sheet, so it shouldn't be attempted unless the earlier sheet has been completed. This sheet shows a faster method of finding the mean when presented with data displayed in a dot plots, frequency histograms and cumulative frequency graphs.

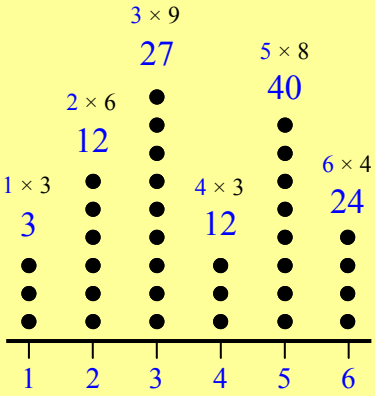
The value for *fx* is found by multiplying the score and the frequency directly on the graph.

Example - Dot Plots

STEP 1 Here is a dot plot, find the mean.



STEP 2 Calculate *fx* for each column by multiplying *f* & *x*.



STEP 3 Calculate Σ*fx* by adding all the column *fx* together.

3 + 12 + 27 + 12 + 40 + 24

Σ*fx* = 118

STEP 4 Calculate Σ*f* by counting all the dots.

3 + 6 + 9 + 3 + 8 + 4

Σ*f* = 33

STEP 5 Find $\bar{x}$ to 2 d.p.

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{118}{33}$$

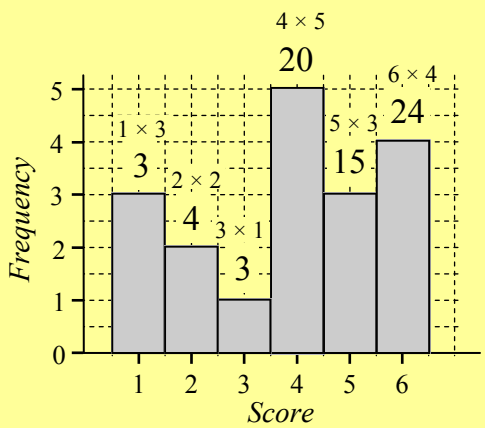
$\bar{x} = 3.58$ (2 d.p.)

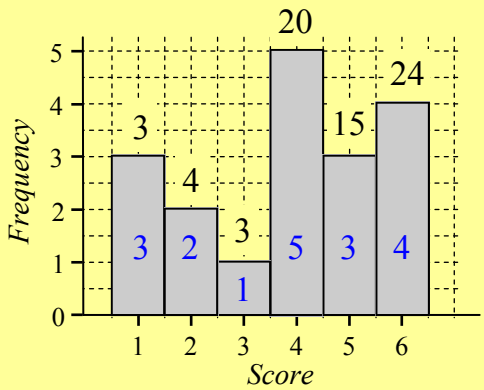
Example - Frequency Histograms

STEP 1 As with dot plots, multiply the frequency and score.

STEP 2 Write the frequency in each column, as they are easier to add.

STEP 3 Solve as with dot plots.





$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$\bar{x} = \frac{69}{18}$$

$\bar{x} = 3.83$ (2 d.p.)

Graphs/Dot Plots - The Mean

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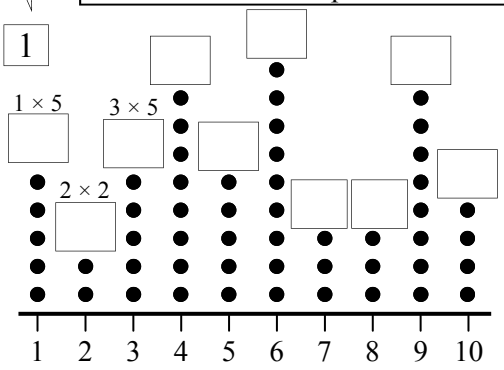
Write the *fx* total at the top of each column of dots. Add the totals for Σfx. Count the dots for Σf. Now find the mean to 2 d.p. without a frequency table.

1



$\Sigma fx =$ ☐

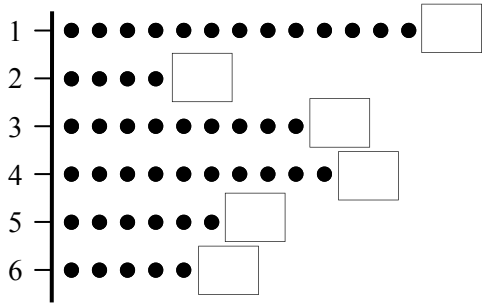
$\Sigma f =$ ☐

Count dots for Σf

$\bar{x} =$ ☐/☐

$\bar{x} =$ _____

2



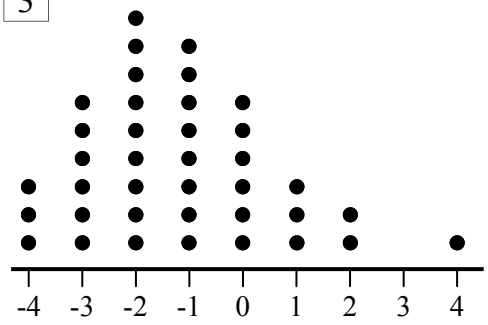
$\Sigma fx =$ ☐

$\Sigma f =$ ☐

$\bar{x} =$ ☐/☐

$\bar{x} =$ _____

3



$\Sigma fx =$ ☐

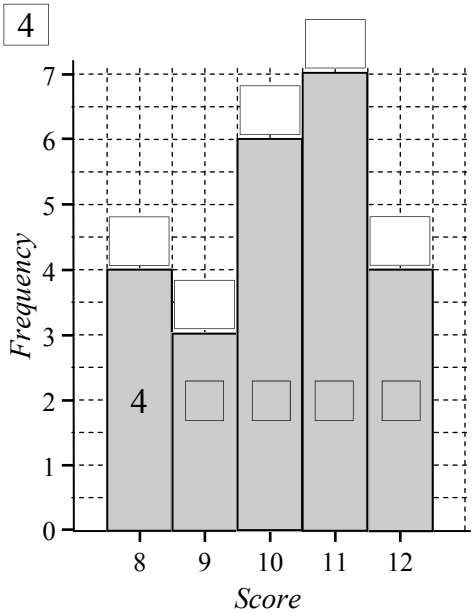
$\Sigma f =$ ☐

$\bar{x} =$ ☐/☐

$\bar{x} =$ _____

Use the same process but this time it is a frequency histogram. Mean to 1 d.p.

4



$\Sigma fx =$ ☐

$\Sigma f =$ ☐

$\bar{x} =$ ☐/☐

$\bar{x} =$ _____

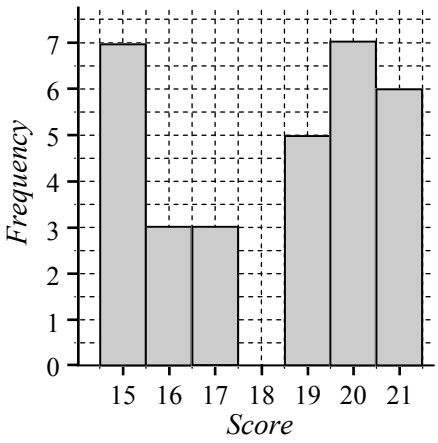
5

$\Sigma fx =$ ☐

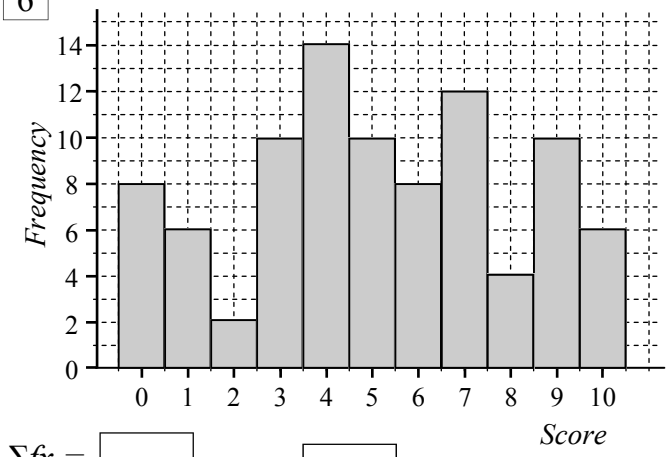
$\Sigma f =$ ☐

$\bar{x} =$ ☐/☐

$\bar{x} =$ _____



6



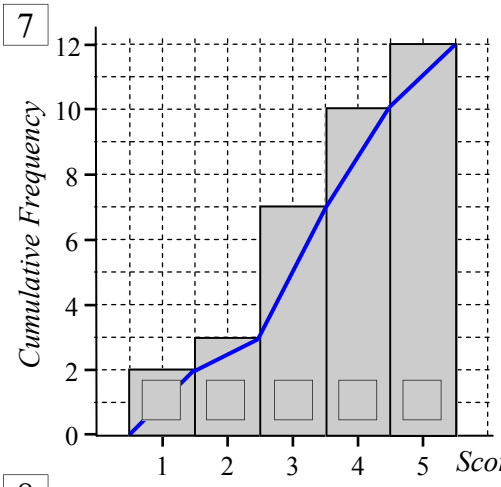
$\Sigma fx =$ ☐

$\Sigma f =$ ☐

$\bar{x} =$ ☐/☐ $=$ _____

These are cumulative frequency graphs. So calculate the frequency for each score **first**, then use the same method to find $\bar{x}$. (1 d.p.)

7



$\Sigma fx =$ ☐

$\Sigma f =$ ☐

$\bar{x} =$ ☐/☐

$\bar{x} =$ _____

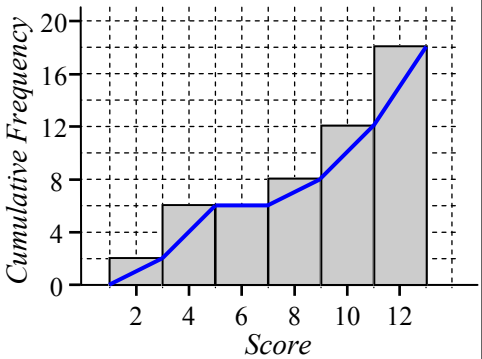
8

$\Sigma fx =$ ☐

$\Sigma f =$ ☐

$\bar{x} =$ ☐/☐

$\bar{x} =$ _____



Graphs/Dot Plots - The Mean

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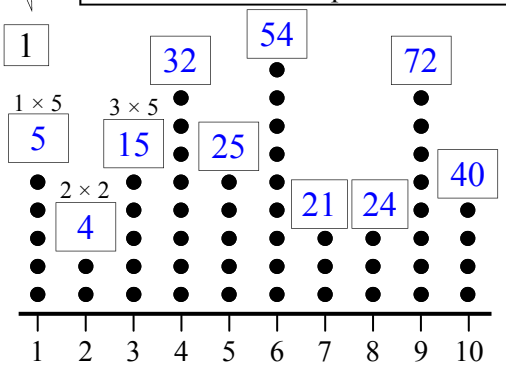
Write the *fx* total at the top of each column of dots. Add the totals for Σfx. Count the dots for Σf. Now find the mean to 2 d.p. without a frequency table.

1



$\Sigma fx =$ 292

$\Sigma f =$ 52

Count dots for Σf

$\bar{x} = \frac{292}{52}$

$\bar{x} = 5.62$ (2 d.p.)

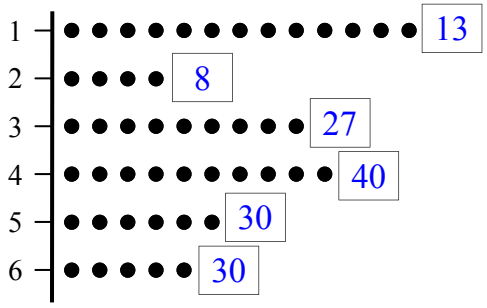
2

$\Sigma fx =$ 148

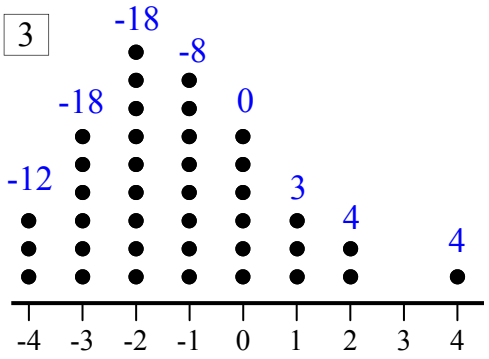
$\Sigma f =$ 47

$\bar{x} = \frac{148}{47}$

$\bar{x} = 3.15$ (2 d.p.)



3



$\Sigma fx =$ -45

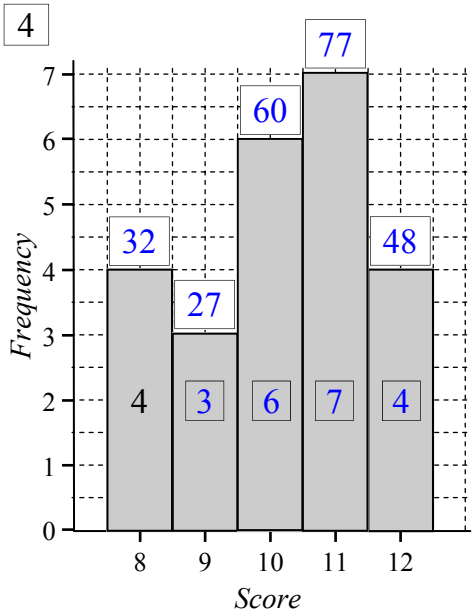
$\Sigma f =$ 38

$\bar{x} = \frac{-45}{38}$

$\bar{x} = -1.18$ (2 d.p.)

Use the same process but this time it is a frequency histogram. Mean to 1 d.p.

4



$\Sigma fx =$ 244

$\Sigma f =$ 24

$\bar{x} = \frac{244}{24}$

$\bar{x} = 10.17$ (2 d.p.)

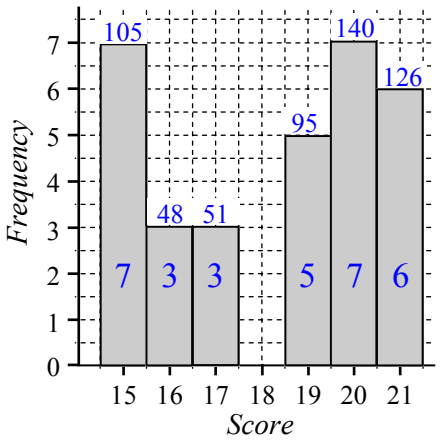
5

$\Sigma fx =$ 565

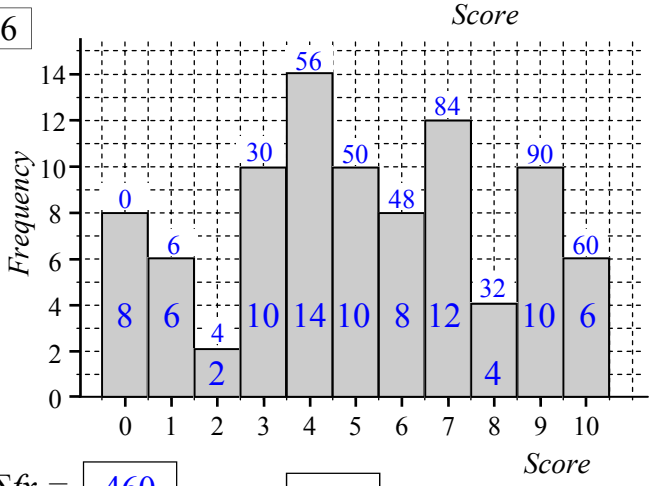
$\Sigma f =$ 31

$\bar{x} = \frac{565}{31}$

$\bar{x} = 18.23$ (2 d.p.)



6



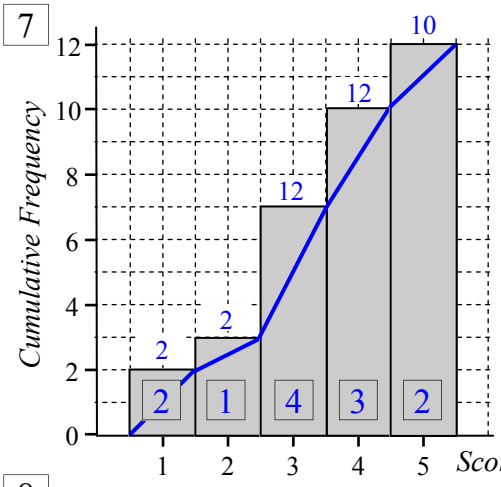
$\Sigma fx =$ 460

$\Sigma f =$ 90

$\bar{x} = \frac{460}{90} = 5.11$ (2 d.p.)

These are cumulative frequency graphs. So calculate the frequency for each score **first**, then use the same method to find $\bar{x}$. (1 d.p.)

7



$\Sigma fx =$ 38

$\Sigma f =$ 12

$\bar{x} = \frac{38}{12}$

$\bar{x} = 3.17$ (2 d.p.)

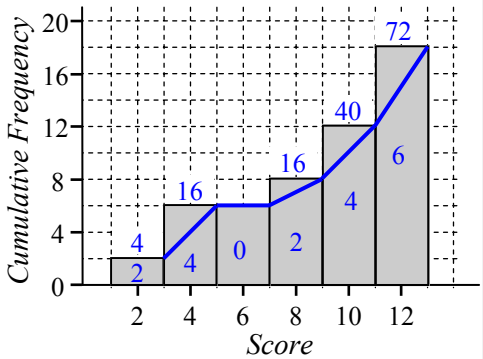
8

$\Sigma fx =$ 148

$\Sigma f =$ 18

$\bar{x} = \frac{148}{18}$

$\bar{x} = 8.22$ (2 d.p.)



Frequency Tables/Dot Plots - The Median

When data is arranged in ascending order the median is the middle, or average of the two middle scores. When data is displayed in a frequency table or dot plot, then the median is found in a different way.

Column 1 starts with two frequency tables, both of which contain a low number of scores. The median can be calculated by removing the data from the table, writing it in ascending order and solving like a data set. This is always an option if you forget the other skills outlined in this sheet.

A Cumulative Frequency (c.f.) column is added to the table, which allows you to find the median, steps are below.

Example - Finding the Median from a Frequency Table

STEP 1 Complete the *c.f.* column.

Score	*f*	*c.f.*
1	3	3
2	7	10
3	2	12
4	3	15
5	5	20

3 goes straight across

3 + 7 = 10

10 + 2 = 12

12 + 3 = 15

15 + 5 = 20

STEP 2 Sum the frequency column, it should match the last number in the *c.f.* column.

Score	*f*	*c.f.*
1	3	3
2	7	10
3	2	12
4	3	15
5	5	20
$\Sigma f =$	20	

check

STEP 3 Calculate the position of the median using:

$$\text{Position} = \frac{n + 1}{2}$$

$$= \frac{20 + 1}{2}$$

Position = 10.5^{th} score

STEP 4 Find the 10.5th score using the *c.f.* column. Find the 10th score, 11th score, then average them.

Score	*f*	*c.f.*
1	3	3
2	7	10
3	2	12
4	3	15
5	5	20
$\Sigma f =$	20	

1st, 2nd and 3rd scores are 1

4th - 10th scores are 2

11th & 12th scores are 3

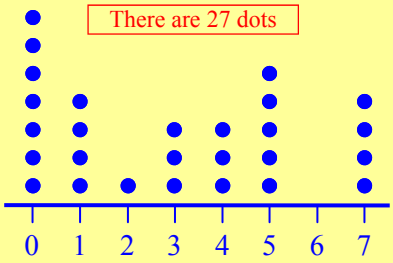
So the 10th score is 2
&
The 11th score is 3.

STEP 5 Calculate the median.

$$\text{Median} = \frac{2 + 3}{2} = 2.5$$

Example - Dot Plots, finding the Median.

STEP 1 Count the dots.

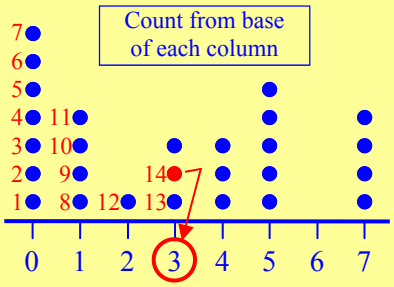


STEP 2 Position the median

$$\text{Position} = \frac{27 + 1}{2}$$

$$= 14^{th} \text{ score.}$$

STEP 3 Find the median



Median = 3

If the position of the median is between two dots then average the value of the two dots. If the dots are in the same score column, the value of the median is the same as both dots. If in different score columns then the median will be a halfway between the two scores.

Frequency Tables/Dot Plots - The Median

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Extract the data from the table and write the data in ascending order. Find the Median.

1

Score	*Frequency*
0	2
1	3
2	5
3	1

2

x	*f*
2	4
4	2
6	1
8	3
10	2

1 ☐, ☐, ☐, ☐, ☐, ◯, ☐, ☐, ☐, ☐, ☐.

Median is _____

2 ☐, ☐, ☐, ☐, ☐, ◯, ◯, ☐, ☐, ☐, ☐, ☐.

Median is $\frac{\square + \square}{2}$ = _____

Complete the cumulative frequency column, find the position of the median then the median itself.

3

Score	*f*	*c.f.*
0	2	2
2	3	5
4	1	
6	6	
8	5	
10	2	
	Σf = ☐	check

Σf

Position = $\frac{\square + 1}{2}$

= ☐ th score.

Median = _____

4

x	*f*	*c.f.*
1	1	
2	4	
3	12	
4	9	
5	12	
6	9	
	Σf = ☐	

Position = $\frac{\square + 1}{2}$

= ☐ th score.

Median = _____

5

Score	*Frequency*	*c.f.*
-3	9	
-2	15	
-1	32	
0	38	
1	22	
2	13	
3	7	
	☐	

Position = $\frac{\square + 1}{2}$

= ☐ th score.

Median = $\frac{\square + \square}{2}$

Median = _____

6

Score	*Frequency*	*c.f.*
10	14	
20	8	
30	5	
40	8	
50	6	
60	3	
70	10	
	☐	

Position = _____

= _____

Median = _____

= _____

8

x	*f*	*c.f.*
1	3	
2	5	
3	11	
4	9	
5	13	
6	0	
7	11	
8	18	
9	7	
10	5	
	☐	

Position = _____

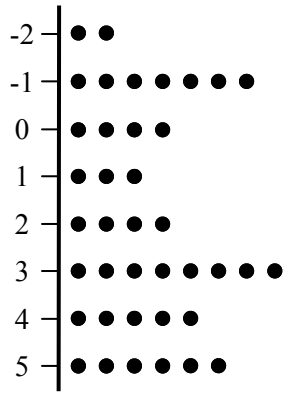
= _____

Median = _____

= _____

This time it's dot plots. Find the position and value for the median.

9



Count those dots!

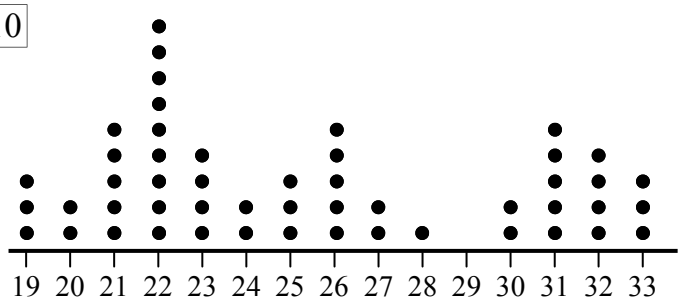
Position = $\frac{\square + 1}{2}$

= ☐ th score.

Count from LHS of each column

Median = _____

10



Position = _____

= _____

Median = _____

= _____

Frequency Tables/Dot Plots - The Median

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Extract the data from the table and write the data in ascending order. Find the Median.

1

Score	*Frequency*
0	2
1	3
2	5
3	1

1 0, 0, 1, 1, 1, (2), 2, 2, 2, 2, 2, 3.

Median is 2

2

x	*f*
2	4
4	2
6	1
8	3
10	2

2 2, 2, 2, 2, 2, 4, (4), (6), 8, 8, 8, 10, 10.

Median is $\frac{4 + 6}{2} = 5$

Complete the cumulative frequency column, find the position of the median then the median itself.

3

Score	*f*	*c.f.*
0	2	2
2	3	5
4	1	6
6	6	12
8	5	17
10	2	19
	$\Sigma f =$ 19	check

Σf

Position = $\frac{19 + 1}{2}$

= 10th score.

Median = 6

4

x	*f*	*c.f.*
1	1	1
2	4	5
3	12	17
4	9	26
5	12	38
6	9	47
	$\Sigma f =$ 47	check

7th - 12th

18th - 26th

Position = $\frac{47 + 1}{2}$

= 24th score.

Median = 4

5

Score	*Frequency*	*c.f.*
-3	9	9
-2	15	24
-1	32	56
0	38	94
1	22	116
2	13	129
3	7	136
	$\Sigma f = 136$	check

57th - 94th

Position = $\frac{136 + 1}{2}$

= 68.5th score.

Median = $\frac{0 + 0}{2}$

Median = 0

6

Score	*Frequency*	*c.f.*
10	14	14
20	8	22
30	5	27
40	8	35
50	6	41
60	3	44
70	10	54
	$\Sigma f = 54$	check

27th

28th

Position = $\frac{54 + 1}{2}$

= 27.5th score

Median = $\frac{30 + 40}{2}$

= 35

8

x	*f*	*c.f.*
1	3	3
2	5	8
3	11	19
4	9	28
5	13	41
6	0	41
7	11	52
8	18	70
9	7	77
10	5	82
	$\Sigma f = 82$	check

41st

42nd

Position = $\frac{82 + 1}{2}$

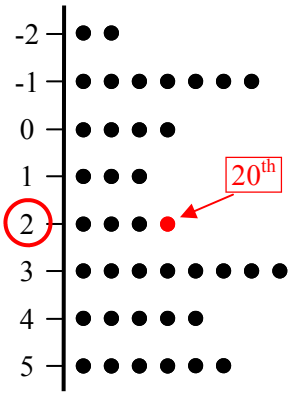
= 41.5th score

Median = $\frac{5 + 7}{2}$

= 6

This time it's dot plots. Find the position and value for the median.

9



Count those dots!

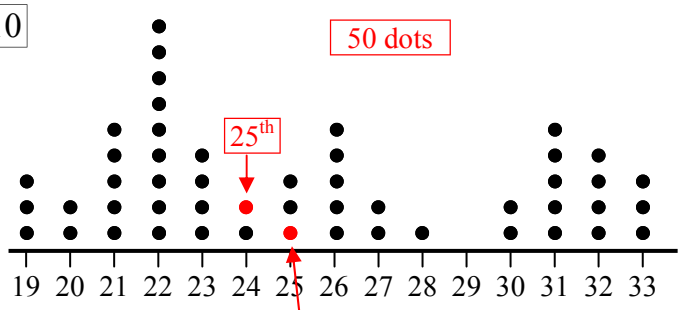
Position = $\frac{39 + 1}{2}$

= 20th score.

Count from LHS of each column

Median = 2

10



Position = $\frac{50 + 1}{2}$

= 25.5th score

Median = $\frac{24 + 25}{2}$

Count from base of eacn column

= 24.5

Frequency Tables - The Median (Graphical Solution)

The Median can be found from a Frequency Table using a Cumulative Frequency Histogram. The process is shown below.

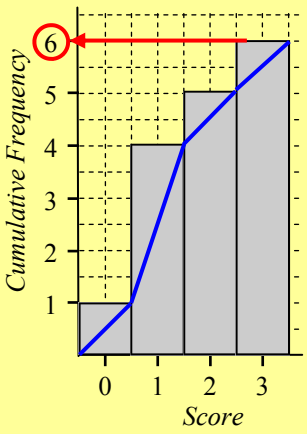
Example - Histogram Supplied

STEP 1 Read the height of the last column.

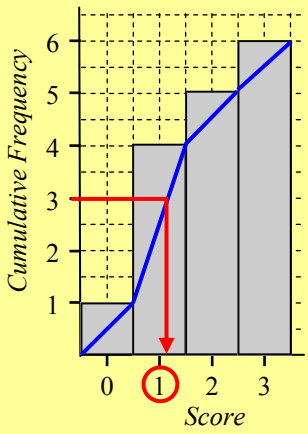
STEP 2 Calculate 50% of the column height

$0.5 \times 6 = 3$

STEP 3 Rule a line across to the ogive (from 3) and drop it down to the Score axis. Read off the median.



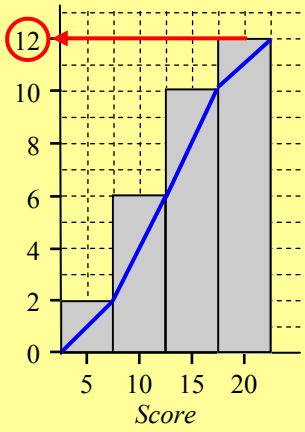
Column Height is 6



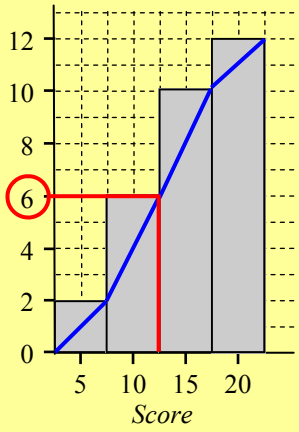
Median is 1

What happens if the 50% line hits the top of a block?

STEP 4 Below is another example. It shows the 50% line hitting the meeting point of two blocks. When this occurs the median is found by averaging the score of each of the blocks. Just like an even numbered data set.



Column Height is 12



50% line = 0.5 × 12 = 6

$$\text{Median} = \frac{10 + 15}{2} = 12.5$$

The other questions on the sheet are answered the same way as above, the difference being that you need to complete a cumulative frequency column first, then construct the Cumulative Frequency Histogram and Ogive yourself. The outline sheet is on the next page if you need a refresher.

Example - Adding a Cumulative Frequency Column (or *c.f.* for short)

Score	*f*	*c.f.*
0	4	4
1	2	6
2	7	13
3	4	17
$\Sigma f =$	17	check

BEFORE YOU START TOTAL THE *f* COLUMN

Write the '4' from the *f* column straight across into the *c.f.* column. This means so far there have been 4 scores.

A score of '1' occurs twice. So add '2' to the '4' from the *c.f.* column. This means so far there have been 6 scores.

A score of '2' occurs seven times. So add '7' to the '6' from the *c.f.* column. This means so far there have been 13 scores.

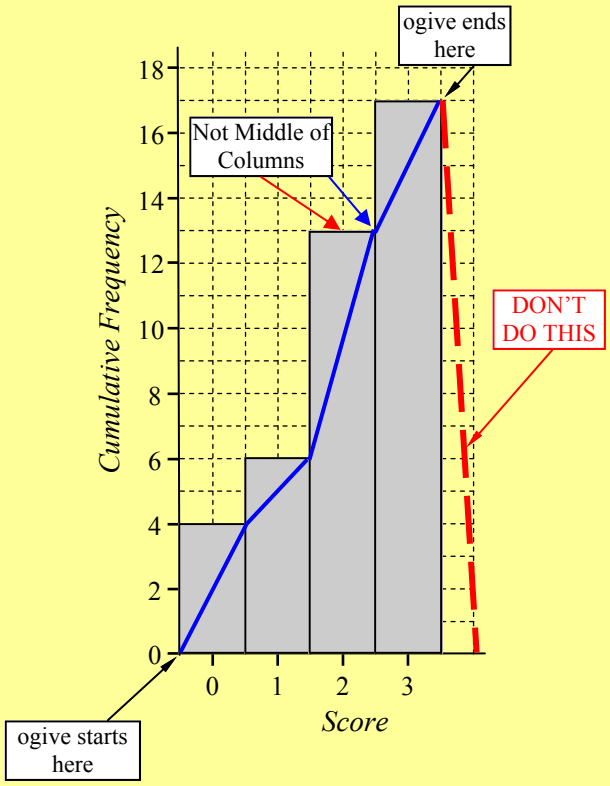
A score of '3' occurs four times. So add '4' to the '13' from the *c.f.* column. This means there has been 17 scores.

As a final check, make sure that Σf is equal to the last number in the *c.f.* column. If they don't match, look for an error in your calculation.

When you display a cumulative frequency graph, you use a 'Cumulative Frequency Histogram'. You also include a 'Cumulative Frequency Polygon', also known as an 'Ogive'.

There are a couple of differences between a Frequency Histogram/Polygon and a Cumulative Frequency Histogram and Ogive:

- A half column gap at the start isn't required, unlike the Frequency Histogram.
- The columns always increase or stay the same height, they never reduce in height.
- The ogive interconnects the top right hand corner of the columns, not their centres.
- The ogive stops at the top right hand corner of the last column. It doesn't drop down.



Frequency Tables - The Median (Graphical Solution)

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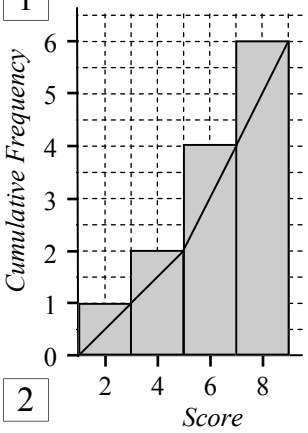
Draw a line at 50% of the final column height, rule to the ogive, drop it down and find the median.

1



50% line = 0.5 × ☐

= ☐

1

Median = _____

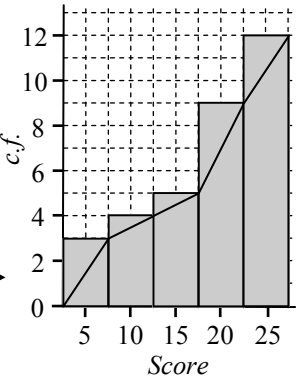
2

50% line = 0.5 × ☐

= ☐

2

Median = _____

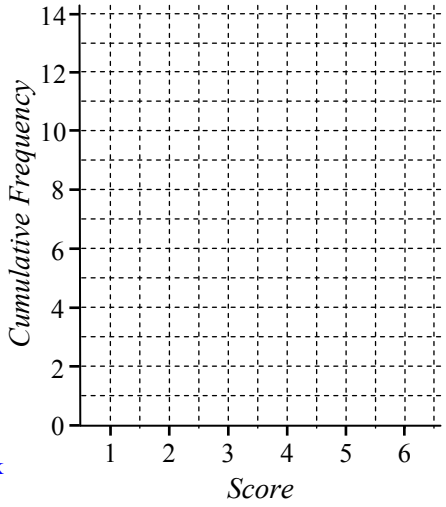


Create a cumulative frequency histogram to find the median.

3

Score	f	c.f.
1	2	2
2	1	
3	3	
4	3	
5	1	
6	4	
Σf =		

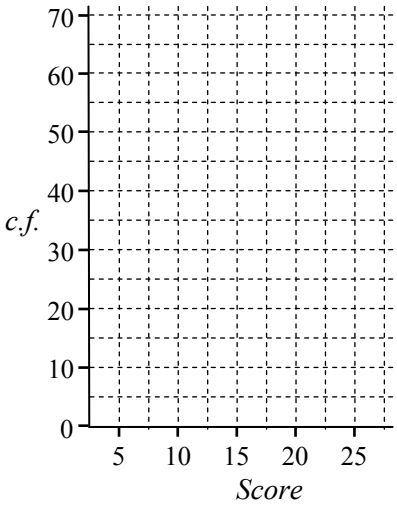
check



50% line = 0.5 × ☐ = ____ Median = _____

4

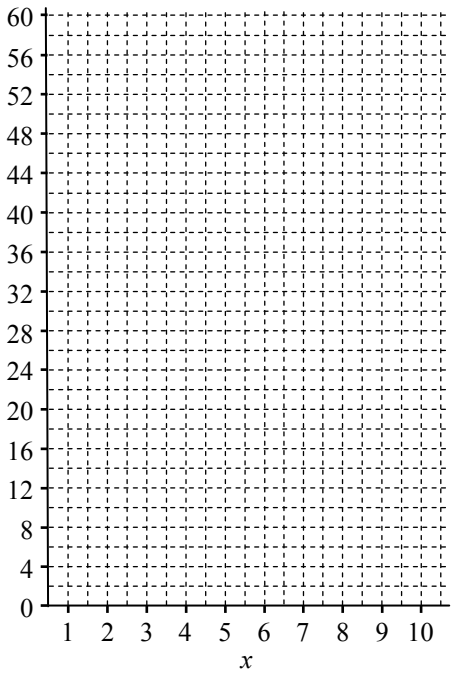
Score	f	c.f.
5	15	
10	10	
15	7	
20	11	
25	21	



50% line: _____ Median = _____

5

x	f	c.f.
1	2	
2	2	
3	3	
4	5	
5	8	
6	11	
7	9	
8	7	
9	4	
10	3	



50% line = _____ Median = _____

The data is raw, so a tally column first then use the same approach. Find the median.

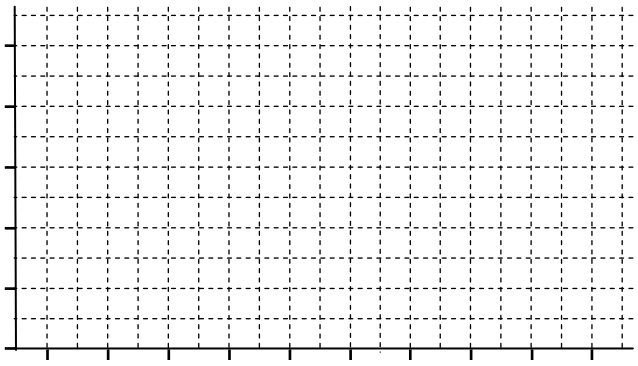
6 Customer Service Ranking Survey.

Data:

7, 4, 3, 1, 2, 8,
7, 9, 6, 10, 8,
5, 5, 8, 9, 8, 7,
6, 10, 7, 6, 8,
9, 7, 6, 10, 3,
10, 7, 8, 9, 5,
8, 4, 8, 8, 9, 7,
8, 10, 5, 6, 4,
8, 9, 10, 8, 5,
9, 7.

x	Tally	f	c.f.

Customer Service Ranking Survey.



50% line = _____ Median = _____

Frequency Tables - The Median (Graphical Solution)

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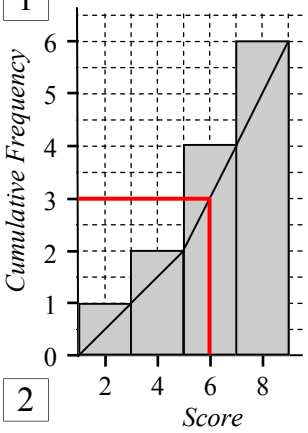
Draw a line at 50% of the final column height, rule to the ogive, drop it down and find the median.

1



50% line = 0.5 × 6

= 3

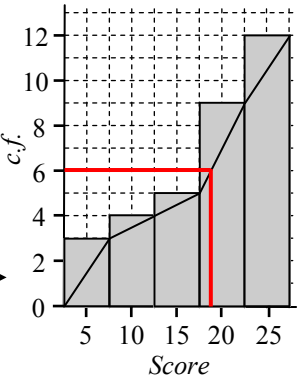
Median = 6

2

50% line = 0.5 × 12

= 6

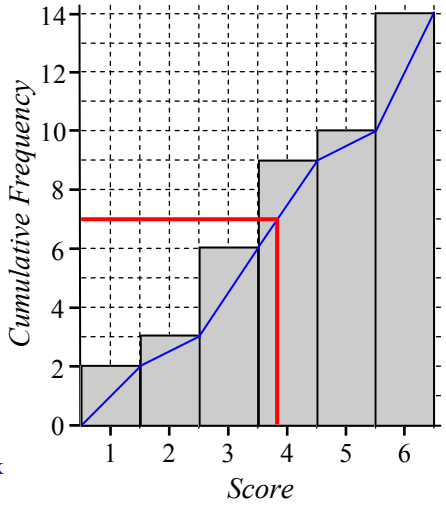
Median = 20



Create a cumulative frequency histogram to find the median.

3

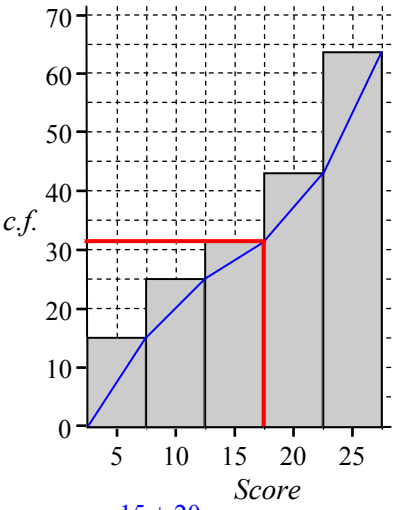
Score	f	c.f.
1	2	2
2	1	3
3	3	6
4	3	9
5	1	10
6	4	14
Σf =	14	check



50% line = 0.5 × 14 = 7 Median = 4

4

Score	f	c.f.
5	15	15
10	10	25
15	7	32
20	11	43
25	21	64
Σf =	64	check

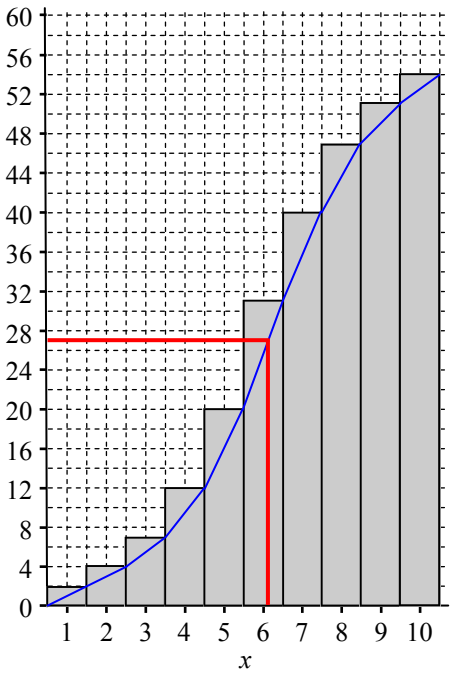


50% line:

0.5 × 64 = 32 Median = $\frac{15+20}{2}$ = 17.5

5

x	f	c.f.
1	2	2
2	2	4
3	3	7
4	5	12
5	8	20
6	11	31
7	9	40
8	7	47
9	4	51
10	3	54
Σf =	54	check



50% line = 0.5 × 54 = 27 Median = 6

The data is raw, so a tally column first then use the same approach. Find the median.

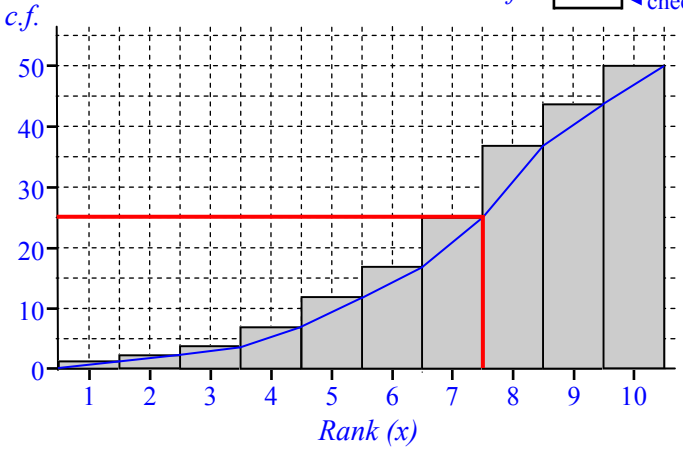
6 Customer Service Ranking Survey.

Data:

7, 4, 3, 1, 2, 8, 7, 9, 6, 10, 8, 5, 5, 8, 9, 8, 7, 6, 10, 7, 6, 8, 9, 7, 6, 10, 3, 10, 7, 8, 9, 5, 8, 4, 8, 8, 9, 7, 8, 10, 5, 6, 4, 8, 9, 10, 8, 5, 9, 7.

x	Tally	f	c.f.
1	|	1	1
2	|	1	2
3	||	2	4
4	|||	3	7
5	𝍸	5	12
6	𝍸	5	17
7	𝍸 |||	8	25
8	𝍸 𝍸 ||	12	37
9	𝍸 ||	7	44
10	𝍸 |	6	50
Σf =		50	check

Customer Service Ranking Survey.



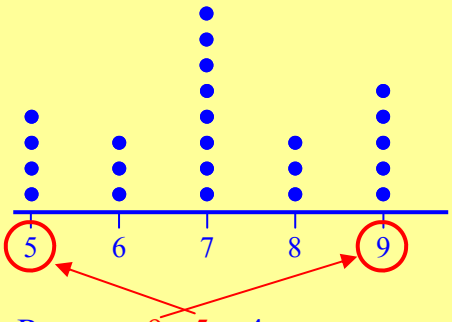
50% line = 0.5 × 50 = 25 Median = 7.5

Frequency Tables/Dot Plots - Range and Mode

The range is the highest score minus the lowest score, the mode is the score that occurs the most (has the highest frequency).

Data can be displayed in dot plots, frequency tables and frequency histograms. Below are some examples.

Dot Plots



Range = 9 - 5 = 4

Mode is 7 (7 has most dots)

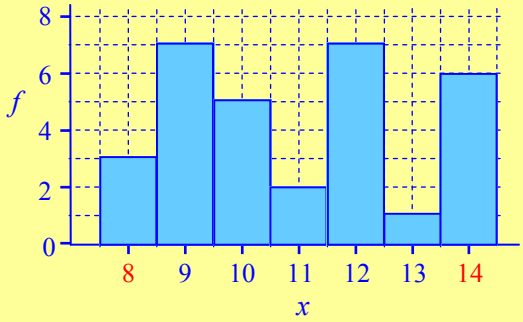
Frequency Tables

Score	*Frequency*
0	4
2	9
4	2
6	9
8	8
10	3

Range = 10 - 0 = 10

Mode is 2, 6 (highest frequency)

Frequency Histograms



Range = 14 - 8 = 6

Mode is 9, 12 (highest frequency)

Column 3 introduces Frequency Tables with Class Intervals. These questions are answered the same way except:

The Range is found by subtracting the lowest possible score from the highest possible score.

The **Modal class** is found by the class or classes with the highest frequency.

Example - find the Modal Class and the Range.

Class	*f*
1 - 20	15
21 - 40	27
41 - 60	16
61- 80	52
81 -100	18

Modal Class is 61- 80.
(Highest Frequency)

Range is 100 - 1 = 99

Frequency Tables/Dot Plots - Range and Mode

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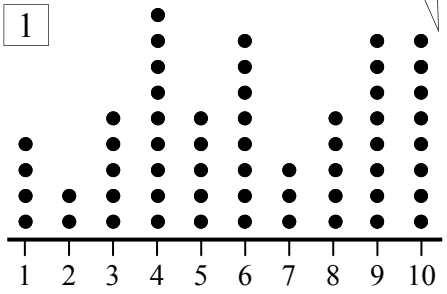
Find the Range and the Mode for the following.

1



Range= ☐ - ☐ = ☐

Mode is _____

2

Score	*Frequency*
5	9
10	11
15	13
20	5
25	9
30	3

3

x	*f*
2	3
4	7
6	2
8	5
10	3

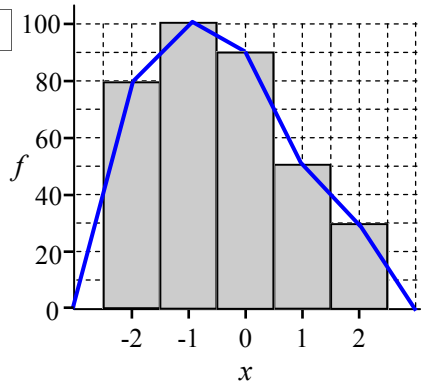
2 Mode is _____

Range= ☐ - ☐ = ☐

3 Mode is _____

Range= _____

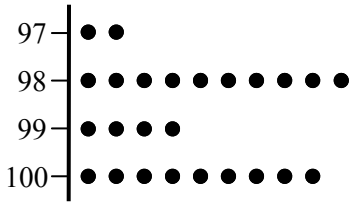
4



Mode is _____

Range= _____

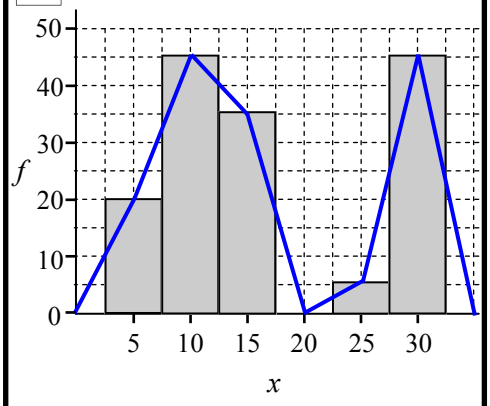
5



Mode is _____

Range= _____

6



Mode is _____

Range= _____

7

x	0	1	2	3	4	5	6
f	2	6	0	7	3	9	3

Mode is _____

Range= _____

Place the data in a frequency table, then calculate the range and mode.

8 15, 12, 19, 18, 11, 16, 14, 13, 12, 19, 14, 12, 11, 14, 15, 12, 16, 11, 13, 15, 11, 12, 14, 12, 11, 13, 13, 14, 11, 12, 19, 18, 12, 11, 12, 13, 16, 12, 15, 15, 11, 13, 12, 14, 16, 18, 11, 12, 14, 13, 12, 11, 12.

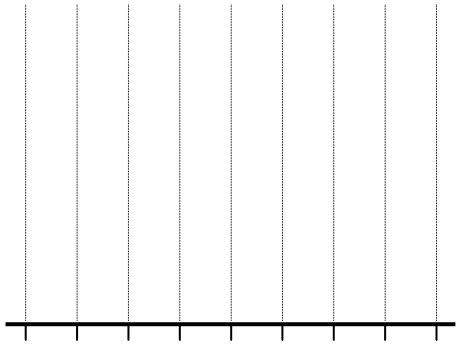
x	*Tally*	*f*

Mode is _____

Range= _____

Place the data in a dot plot then find the range and mode.

9 6, 2, 7, 3, 2, 9, 6, 4, 7, 8, 3, 4, 4, 6, 6, 8, 3, 2, 5, 1, 3, 6, 2, 2, 4, 3, 5, 8, 1, 6, 4, 7, 2, 3, 6, 1, 1, 4, 2, 8, 5, 4, 8, 3, 9, 7, 4, 8, 7, 5, 6.



Mode: _____ Range: _____

Find the range and modal class for the tabulated data below.

10

Class	*f*
0 - 9	4
10 - 19	7
20 - 29	9
30 - 39	12
40 - 49	9

11

Class	*f*
0 - 4	3
5 - 9	7
10 - 14	5
15 - 19	2
20 - 24	7
25 - 29	7

10 Modal class is _____

Range= _____

11 Modal class is _____

Range= _____

12

Class	*f*
1 - 2	5
3 - 4	13
5 - 6	7
7 - 8	11
9 - 10	11
11 - 12	5
13 - 14	9
15 - 16	13

Modal class is _____

Range= _____

Frequency Tables/Dot Plots - Range and Mode

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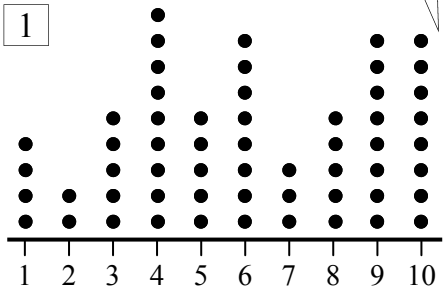
Find the Range and the Mode for the following.

1



Range= 10 - 1 = 9

Mode is 4

2

Score	Frequency
5	9
10	11
15	13
20	5
25	9
30	3

3

x	f
2	3
4	7
6	2
8	5
10	3

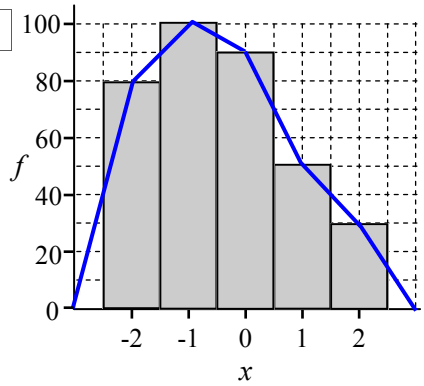
2 Mode is 15

Range= 30 - 5 = 25

3 Mode is 4

Range= 10 - 2 = 8

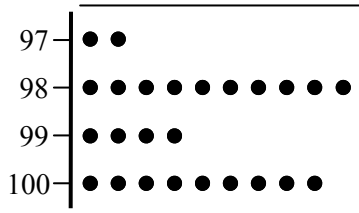
4



Mode is -1

Range= 2 - (-2) = 4

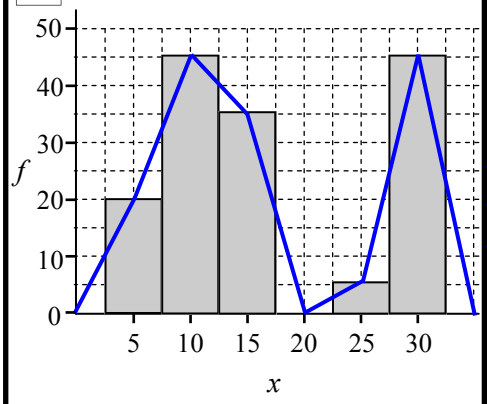
5



Mode is 98

Range= 100 - 97 = 3

6



Mode is 10 and 30

Range= 30 - 5 = 25

7

x	0	1	2	3	4	5	6
f	2	6	0	7	3	9	3

Mode is 5

Range= 6 - 0 = 6

Place the data in a frequency table, then calculate the range and mode.

8 15, 12, 19, 18, 11, 16, 14, 13, 12, 19, 14, 12, 11, 14, 15, 12, 16, 11, 13, 15, 11, 12, 14, 12, 11, 13, 13, 14, 11, 12, 19, 18, 12, 11, 12, 13, 16, 12, 15, 15, 11, 13, 12, 14, 16, 18, 11, 12, 14, 13, 12, 11, 12.

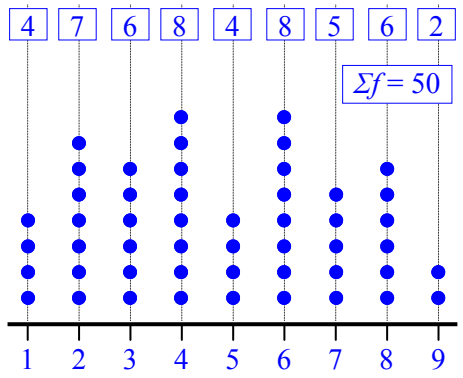
x	Tally	f
11	𝍸 𝍸	10
12	𝍸 𝍸 ||||	14
13	𝍸 ||	7
14	𝍸 ||	7
15	𝍸	5
16	||||	4
17		0
18	|||	3
19	|||	3
		Σf=53

Mode is 12

Range= 19 - 11 = 8

Place the data in a dot plot then find the range and mode.

9 6, 2, 7, 2, 9, 6, 4, 7, 8, 3, 4, 4, 6, 6, 8, 3, 2, 5, 1, 3, 6, 2, 2, 4, 3, 5, 8, 1, 6, 4, 7, 2, 3, 6, 1, 1, 4, 2, 8, 5, 4, 8, 3, 9, 7, 4, 8, 7, 5, 6.



Mode: 4, 6. Range: 9 - 1 = 8

Find the range and modal class for the tabulated data below.

10

Class	f
0 - 9	4
10 - 19	7
20 - 29	9
30 - 39	12
40 - 49	9

11

Class	f
0 - 4	3
5 - 9	7
10 - 14	5
15 - 19	2
20 - 24	7
25 - 29	7

10 Modal class is 30 -39.

Range= 49 - 0 = 49

11 Modal class is No Mode

Range= 29 - 0 = 29

12

Class	f
1 - 2	5
3 - 4	13
5 - 6	7
7 - 8	11
9 - 10	11
11 - 12	5
13 - 14	9
15 - 16	13

Modal class is 3 - 4, 15 -16.

Range= 16 - 1

Range= 15

Frequency Tables - Mean, Mode, Range & Median

This sheet is a review sheet and so the method of finding the Mode, Mean, Range and Median won't be outlined below. But here is a reminder:

Mode - Is the score that occurs the most, the score with the highest frequency. There can be 1 mode, 2 modes (bi-modal) or No Mode. Write "No Mode" not "0" if there is no mode. If there are 3 or modes then the data is considered to have no mode.

Mean - Is the average of the scores. Found using: $x = \frac{\Sigma fx}{\Sigma f}$

Range - Is the highest score minus the lowest score.

Median - Is the middle score when the scores are arranged in ascending order. If there are an odd number of scores it is the middle score. If there are an even number of scores then the median is the average of the two middle scores. Use the cumulative frequency column to find the median, after finding its position using $\frac{n+1}{2}$

Frequency Tables - Mean, Mode, Range & Median

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Complete a *fx* and *c.f.* column then find the range, mode, mean and median for the frequency tables.

1

x	*f*	*fx*	*c.f.*
5	2		
10	6		
15	18		
20	11		
25	2		
30	12		
	Σf =	Σfx =	

Mode is _____

Range = ☐ - ☐ = ☐

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ = ☐/☐ = _____ (1 decimal place)

Position (median) = $\frac{☐ + 1}{2}$ = ☐ th score.

Median = _____

2

x	*f*	*fx*	*c.f.*
0	8		
2	19		
4	10		
6	5		
8	19		
10	13		
	Σf =	Σfx =	

Mode is _____

Range = ☐ - ☐ = ☐

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ = ☐/☐ = _____ (1 decimal place)

Position = $\frac{☐ + 1}{2}$ = ☐ th score.

Median = $\frac{☐ + ☐}{2}$

Median = _____

3

x	*f*		
14	13		
16	5		
18	7		
20	14		
22	0		
24	16		
26	13		
28	10		

Mode is _____

Range = ☐ - ☐ = ☐

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ = ☐/☐ = _____ (1 decimal place)

Position = $\frac{☐ + 1}{2}$ = ☐ th score.

Median = $\frac{☐ + ☐}{2}$

Median = _____

Now its raw data. Use a tally column to enter the data, then complete as before. Mean to 1 d.p.

4 Data: 4, 9, 10, 7, 6, 8, 9, 9, 7, 6, 5, 4, 6, 4, 7, 8, 10, 7, 4, 6, 5, 7, 4, 8, 9, 8, 9, 8, 3, 9, 7, 5, 8, 6, 9, 7, 6, 8, 3, 6, 8, 3, 5.

x	*Tally*	*f*	*fx*	*c.f.*

Mode is _____

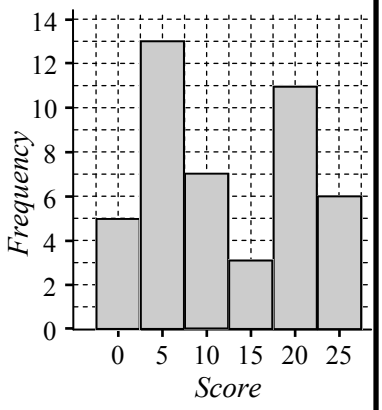
Range = ☐ - ☐ = ☐

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ = ☐/☐ = _____

Position = ☐/☐ = ☐ nd score.

Median = _____

5 Complete the Frequency Table, using the Frequency histogram. Then as before.



Mode is ____

Range = ☐ - ☐ = ☐

$\bar{x} = \frac{\Sigma fx}{\Sigma f}$ = ☐/☐ = _____

Position = ☐/☐ = ☐ rd score.

Median = _____

Frequency Tables - Mean, Mode, Range & Median

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Complete a *fx* and *c.f.* column then find the range, mode, mean and median for the frequency tables.

1

x	*f*	*fx*	*c.f.*
5	2	10	2
10	6	60	8
15	18	270	26
20	11	220	37
25	2	50	39
30	12	360	51
	$\Sigma f =$ 51	$\Sigma fx =$ 970	

Mode is 15

Range = 30 - 5 = 25

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{970}{51} = 19$ (1 decimal place)

Position (median) $= \frac{51 + 1}{2} = 26^{\text{th}}$ score.

Median = 15

2

x	*f*	*fx*	*c.f.*
0	8	0	8
2	19	38	27
4	10	40	37
6	5	30	42
8	19	152	61
10	13	130	74
	$\Sigma f =$ 74	$\Sigma fx =$ 390	

Mode is 2, 8

Range = 10 - 0 = 10

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{390}{74} = 5.3$ (1 decimal place)

Position $= \frac{74 + 1}{2} = 37.5^{\text{th}}$ score.

Median $= \frac{4 + 6}{2}$

Median = 5

3

x	*f*	*fx*	*c.f.*
14	13	182	13
16	5	80	18
18	7	126	25
20	14	280	39
22	0	0	39
24	16	384	55
26	13	338	68
28	10	280	78
	$\Sigma f =$ 78	$\Sigma fx =$ 1 670	

Mode is 24

Range = 28 - 14 = 14

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{1\,670}{78} = 21.4$ (1 decimal place)

Position $= \frac{78 + 1}{2} = 39.5^{\text{th}}$ score.

Median $= \frac{20 + 24}{2}$

Median = 22

Now its raw data. Use a tally column to enter the data, then complete as before. Mean to 1 d.p.

4 Data: 4, 9, 10, 7, 6, 8, 9, 9, 7, 6, 5, 4, 6, 4, 7, 8, 10, 7, 4, 6, 5, 7, 4, 8, 9, 8, 9, 8, 3, 9, 7, 5, 8, 6, 9, 7, 6, 8, 3, 6, 8, 3, 5.

x	*Tally*	*f*	*fx*	*c.f.*
3	|||	3	9	3
4	𝍸	5	20	8
5	𝍸|	4	20	12
6	𝍸 ||	7	42	19
7	𝍸 ||	7	49	26
8	𝍸 |||	8	64	34
9	𝍸 ||	7	63	41
10	||	2	20	43
	$\Sigma f =$ 43	$\Sigma fx =$ 287		

Mode is 8

Range = 10 - 3 = 7

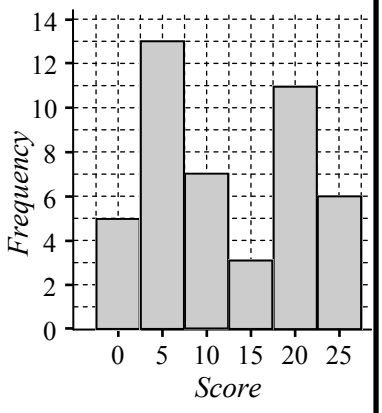
$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{287}{43} = 6.7$ (1d.p.)

Position $= \frac{43 + 1}{2} = 22^{\text{nd}}$ score.

Median = 7

5 Complete the Frequency Table, using the Frequency histogram. Then as before.

x	*f*	*fx*	*c.f.*
0	5	0	5
5	13	65	18
10	7	70	25
15	3	45	28
20	11	220	39
25	6	150	45
	$\Sigma f =$ 45	$\Sigma fx =$ 550	



Mode is 5

Range = 25 - 0 = 25

$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{550}{45} = 12.2$ (1 d.p.)

Position $= \frac{45 + 1}{2} = 23^{\text{rd}}$ score.

Median = 10

Frequency Tables with Classes - The Mean

When data is grouped into class intervals there is no score to use for calculations, instead an average is taken and a 'class centre' is created. This class centre is then used in calculations, the formula is:

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f}$$

Where: $\bar{x}$ = the mean for the table
$\Sigma f \times c.c.$ = is the sum of the $f \times c.c.$ (frequency × class centre) column
Σf = is the sum of the f (frequency) column

Example - Finding the Mean for a Frequency Table with Classes

STEP 1 Calculate the class centre (*c.c.*), if not already in the column.

Class	*c.c.*	*f*	*f*×*c.c.*	
0 - 2	1	8		(0 + 2) ÷ 2 = 1
3 - 5	4	11		(3 + 5) ÷ 2 = 4
6 - 8	7	5		(6 + 8) ÷ 2 = 7
9 - 11	10	3		(9 + 11) ÷ 2 = 10
12 - 14	13	9		(12 + 14) ÷ 2 = 13
15 - 17	16	4		(15 + 17) ÷ 2 = 16
	Σf=		*Σf*×*c.c.*=	

STEP 2 Multiply the *c.c.* and *f* columns to calculate the *f* × *c.c.* column.

Class	*c.c.*	*f*	*f*×*c.c.*	
0 - 2	1	8	8	1 × 8 = 8
3 - 5	4	11	44	4 × 11 = 44
6 - 8	7	5	35	7 × 5 = 35
9 - 11	10	3	30	10 × 3 = 30
12 - 14	13	9	117	13 × 9 = 117
15 - 17	16	4	64	16 × 4 = 64
	Σf=		*Σf*×*c.c.*=	

STEP 3 Sum the *f* and *f* × *c.c.* columns. Sum means 'total', or 'add them up'.

Class	*c.c.*	*f*	*f*×*c.c.*
0 - 2	1	8	8
3 - 5	4	11	44
6 - 8	7	5	35
9 - 11	10	3	30
12 - 14	13	9	117
15 - 17	16	4	64
	Σf=	40	*Σf*×*c.c.*= 298

STEP 4 Calculate the mean.

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{298}{40} = 7.45$$

Frequency Tables with Classes - The Mean

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Complete the $f \times c.c.$ column by multiplying f and $c.c.$ Find the mean. (1 d.p.)



1

Class	*c.c.*	f	$f \times c.c.$
0 - 9	4.5	8	
10 - 19	14.5	11	
20 - 29	24.5	5	
30 - 39	34.5	3	
40 - 49	44.5	9	
50 - 59	54.5	4	
	$\Sigma f =$		$\Sigma f \times c.c. =$

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{\quad}{\quad} = \underline{\quad}$$

2

Class	*c.c.*	f	$f \times c.c.$
1 - 5	3	7	
6 - 10	8	21	
11 - 15	13	17	
16 - 20	18	3	
21 - 25	23	9	
26 - 30	28	4	

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{\quad}{\quad} = \underline{\quad}$$

3

Class	*c.c.*	f	$f \times c.c.$
1 - 20	10.5	15	
21 - 40	30.5	9	
41 - 60	50.5	13	
61 - 80	70.5	20	
81 - 100	90.5	8	
101 - 120	110.5	11	
121 - 140	130.5	6	

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{\quad}{\quad} = \underline{\quad}$$



A cinema complex has surveyed its customers entering the complex in one day. Complete the table's Class Centre ($c.c.$) column then the $f \times c.c.$ column. Calculate the average customer age (1 d.p.).

4

Age (years)	*c.c.*	f	$f \times c.c.$
1 - 10		93	
11 - 20		352	
21 - 30		385	
31 - 40		277	
41 - 50		150	
51 - 60		66	
61 - 70		23	
71 - 80		11	
81 - 90		2	
91 - 100		0	

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{\quad}{\quad} = \underline{\quad}$$



Display the data above in different classes, calculate the mean. So add the frequencies for 1-10 and 11-20 above, put the total in the 1-20 class below.

5

Age (years)	*c.c.*	f	$f \times c.c.$
1 - 20			
21 - 40			
41 - 60			
61 - 80			
81 - 100			

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{\quad}{\quad} = \underline{\quad}$$



6 Below are a car yard's sales for the month. Calculate the average car price, using the table to calculate the mean, then without the table.

Data ($): 15 900, 34 500, 23 450, 9 999, 25 133, 47 900, 12 100, 28 888, 37 080, 55 600, 20 300, 47 122, 36 510, 14 068, 33 500, 45 770, 25 900, 65 000, 42 800, 11 550, 27 279, 53 211, 39 010, 17 800, 20 116, 37 080, 36 510, 39 888, 52 665, 19 995, 40 750.

Car Price Range ($)	*c.c. ($)*	*Tally*	f	$f \times c.c.$
0 - 9 999				
10 000 - 19 999				
20 000 - 29 999				
30 000 - 39 999				
40 000 - 49 999				
50 000 - 59 999				
60 000 - 69 999				

Table

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{\quad}{\quad} = \underline{\quad}$$

Actual

$$\bar{x} = \frac{\text{Total value of sales}}{\text{Number of Sales}} = \frac{\quad}{\quad} = \underline{\quad}$$

Frequency Tables with Classes - The Mean

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Complete the $f \times c.c.$ column by multiplying f and $c.c.$ Find the mean. (1 d.p.)

1

Class	*c.c.*	f	$f \times c.c.$
0 - 9	4.5	8	36
10 - 19	14.5	11	159.5
20 - 29	24.5	5	122.5
30 - 39	34.5	3	103.5
40 - 49	44.5	9	400.5
50 - 59	54.5	4	218
	$\Sigma f =$	40	$\Sigma f \times c.c. = 1\,040$

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{1\,040}{40} = 26$$

2

Class	*c.c.*	f	$f \times c.c.$
1 - 5	3	7	21
6 - 10	8	21	168
11 - 15	13	17	221
16 - 20	18	3	54
21 - 25	23	9	207
26 - 30	28	4	112
	$\Sigma f =$	61	$\Sigma f \times c.c. = 783$

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{783}{61} = 12.8 \text{ (1 d.p.)}$$

3

Class	*c.c.*	f	$f \times c.c.$
1 - 20	10.5	15	157.5
21 - 40	30.5	9	274.5
41 - 60	50.5	13	656.5
61 - 80	70.5	20	1 410
81 - 100	90.5	8	724
101 - 120	110.5	11	1 215.5
121 - 140	130.5	6	783
	$\Sigma f =$	81	$\Sigma f \times c.c. = 5\,221$

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{5\,221}{81} = 64.5 \text{ (1 d.p.)}$$

A cinema complex has surveyed its customers entering the complex in one day. Complete the table's Class Centre ($c.c.$) column then the $f \times c.c.$ column. Calculate the average customer age (1 d.p.).

4

Age (years)	*c.c.*	f	$f \times c.c.$
1 - 10	5.5	93	511.5
11 - 20	15.5	352	5 456
21 - 30	25.5	385	9 817.5
31 - 40	35.5	277	9 833.5
41 - 50	45.5	150	6 825
51 - 60	55.5	66	3 663
61 - 70	65.5	23	1 506.5
71 - 80	75.5	11	830.5
81 - 90	85.5	2	171
91 - 100	95.5	0	0
	$\Sigma f =$	1 359	38 614.5 $(\Sigma f \times c.c.)$

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{38\,614.5}{1\,359} = 28.4 \text{ y (1 d.p.)}$$

Display the data above in different classes, calculate the mean. So add the frequencies for 1-10 and 11-20 above, put the total in the 1-20 class below.

5

Age (years)	*c.c.*	f	$f \times c.c.$
1 - 20	10.5	445	4 672.5
21 - 40	30.5	662	20 191
41 - 60	50.5	216	10 908
61 - 80	70.5	34	2 397
81 - 100	90.5	2	181
	$\Sigma f =$	1 359	38 349.5 $(\Sigma f \times c.c.)$

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{38\,349.5}{1\,359} = 28.2 \text{ y (1 d.p.)}$$

6

Below are a car yard's sales for the month. Calculate the average car price, using the table to calculate the mean, then without the table.

Data ($): 15 900, 34 500, 23 450, 9 999, 25 133, 47 900, 12 100, 28 888, 37 080, 55 600, 20 300, 47 122, 36 510, 14 068, 33 500, 45 770, 25 900, 65 000, 42 800, 11 550, 27 279, 53 211, 39 010, 17 800, 20 116, 37 080, 36 510, 39 888, 52 665, 19 995, 40 750.

Car Price Range ($)	*c.c. ($)*	*Tally*	f	$f \times c.c.$
0 - 9 999	4 999.5	|	1	4 999.5
10 000 - 19 999	14 999.5	𝍸 |	6	89 997
20 000 - 29 999	24 999.5	𝍸 ||	7	174 996.5
30 000 - 39 999	34 999.5	𝍸 |||	8	279 996
40 000 - 49 999	44 999.5	𝍸	5	224 997.5
50 000 - 59 999	54 999.5	|||	3	164 998.5
60 000 - 69 999	64 999.5	|	1	64 999.5
		$\Sigma f =$	31	1 004 984.5 $(\Sigma f \times c.c.)$

Table

$$\bar{x} = \frac{\Sigma f \times c.c.}{\Sigma f} = \frac{1\,004984.5}{31} = \$32\,418.85$$

Actual

$$\bar{x} = \frac{\text{Total value of sales}}{\text{Number of Sales}} = \frac{1\,017\,374}{31} = \$32\,818.52$$

Frequency Table with Classes - The Median (Graphical Solution)

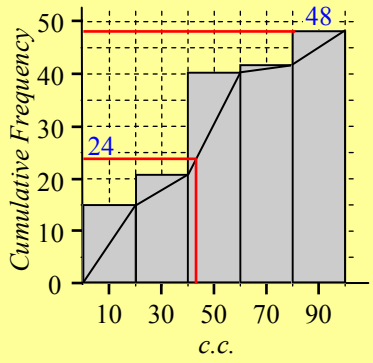
The process of finding the median from a frequency table with classes is very similar to that used with frequency tables (without classes). The differences being:

- you have to calculate a class centre
- The median is estimated from the horizontal scale using the class centres as a reference.
- The median class, is the class that contains the median. It can be found from the table using the *c.f.* column or using the median value itself to locate the class.

Column 1 starts with frequency histograms/polygons that are already constructed. Calculate 50% of the last column height rule across and drop the line down. Estimate the median value.

Example

STEP 1 Read off last column height and calculate 50%.

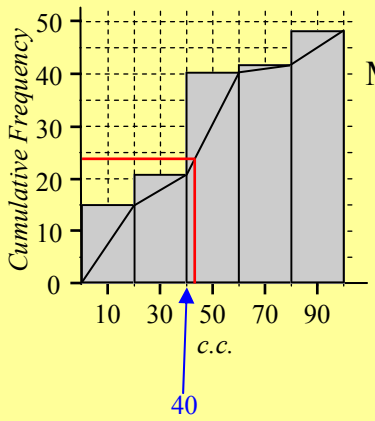


50% line:

$0.5 \times \boxed{48}$

$= \boxed{24}$

STEP 2 Estimate the median value



Median = 43

The second column of the sheet uses exactly the same method, but the frequency table has to be completed first. First step is to calculate the class centre, then the c.f. column. Once you have done that construct the graph and use the above method to calculate the median. The median class is found by finding the class that contains the median, otherwise use the *c.f.* column to locate the class that contains the median.

Frequency Table with Classes - The Median (Graphical Solution)

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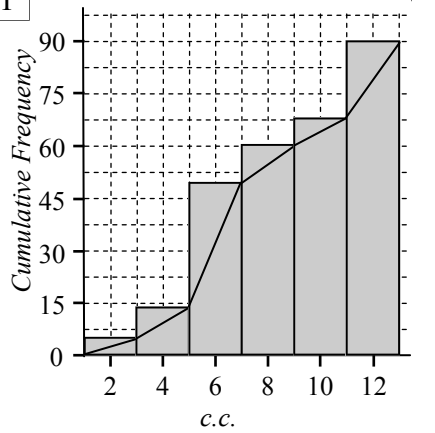
Estimate the median from the C. F. Histograms below.

1

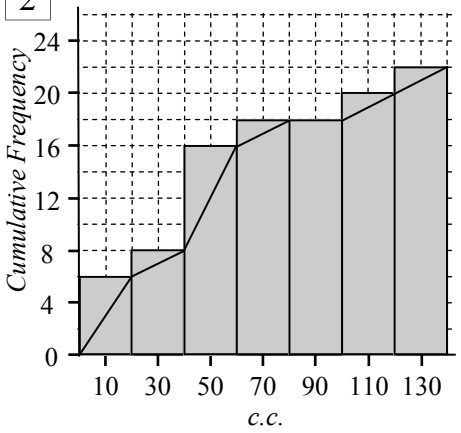


50% line:

0.5 × ☐ Median = _____

= ☐

2



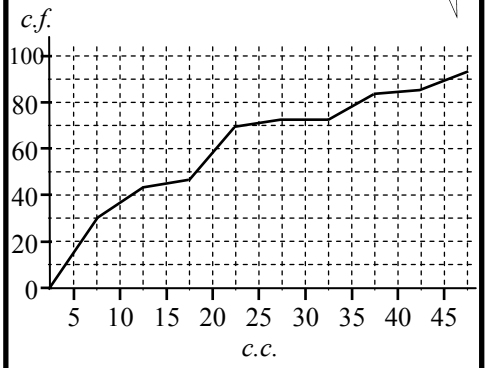
50% line:

0.5 × ☐ Median = _____

= ☐

Now just a Cumulative Frequency Polygon. Find the median.

3



50% line:

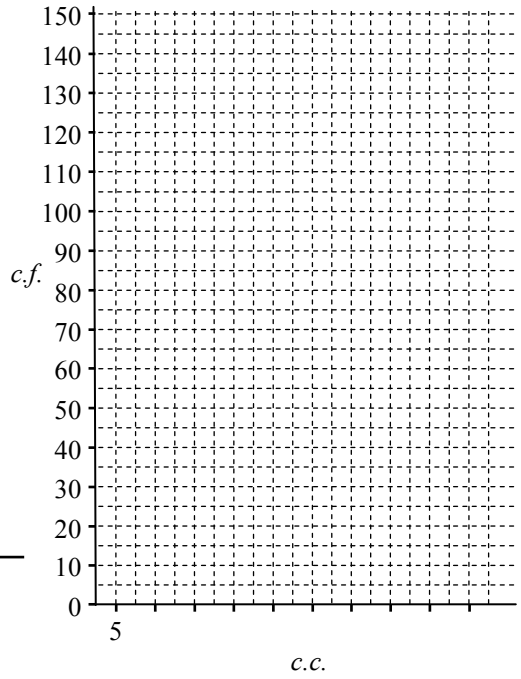
0.5 × ☐ Median = _____

= ☐

4

150 students sat a yearly exam, the results are displayed in the table. Calculate *c.c.* and the *c.f.* Then draw a *c.f.* histogram with an ogive. Find the median score (graph), median class.

Mark	*c.c.*	*f*	*c.f.*
0 - < 10		2	
10 - < 20		5	
20 - < 30		11	
30 - < 40		17	
40 - < 50		29	
50 - < 60		34	
60 - < 70		25	
70 - < 80		14	
80 - < 90		9	
90 - < 100		4	
		☐	



50% line = _____

Median = _____

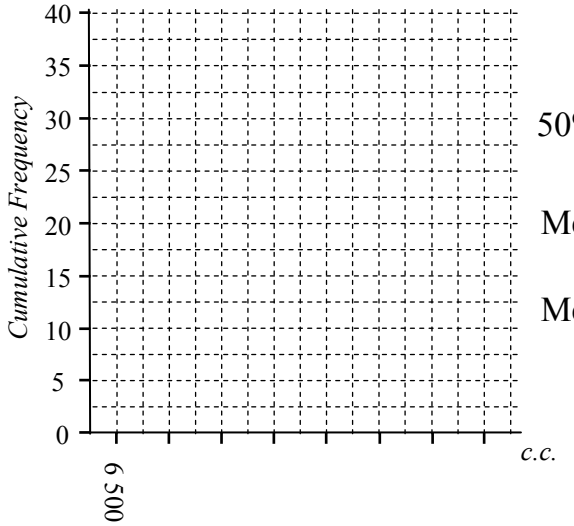
Median Class = _____

A company manufactures light bulbs. It claims the bulbs have a life of 10 000 hrs. Systematic testing of 40 bulbs resulted in the data below. Tabulate then graph the results using a C.F. Histogram/polygon and estimate the median life for the bulbs.

5 Data:

11 600	12 900	12 500
12 200	8 600	11 300
9 800	11 700	10 400
8 500	12 300	10 200
13 700	9 600	9 600
11 200	6 700	9 800
10 400	10 500	11 300
13 300	7 200	13 400
12 100	11 000	12 900
10 600	13 900	11 700
9 400	9 400	9 500
11 300	13 600	7 700
13 500	10 100	12 700
		9 900

Duration (hrs)	*c.c.*	*Tally*	*f*	*c.f.*
6 000 - < 7 000				
7 000 - < 8 000				
8 000 - < 9 000				
9 000 - < 10 000				
10 000 - < 11 000				
11 000 - < 12 000				
12 000 - < 13 000				
13 000 - < 14 000				
			☐	



50% line = _____

Median = _____

Median Class : _____

Frequency Table with Classes - The Median (Graphical Solution)

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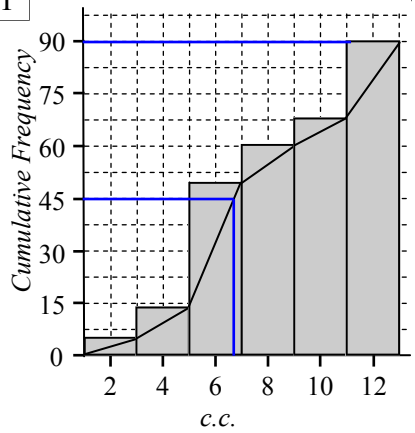
Estimate the median from the C. F. Histograms below.

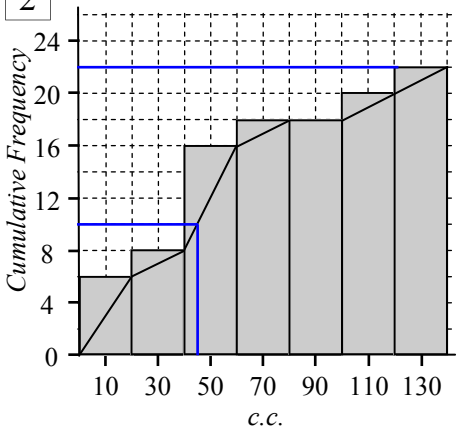
1



50% line:

0.5 × 90 = 45 Median = 6.7

2

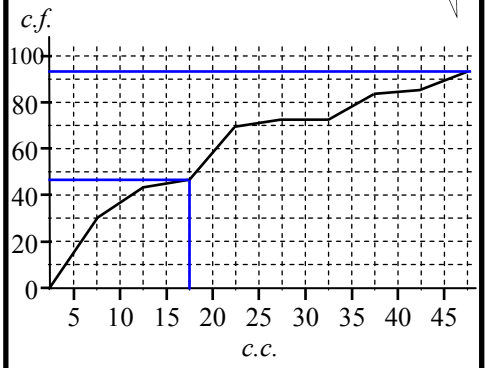


50% line:

0.5 × 22 = 11 Median = 45

Now just a Cumulative Frequency Polygon. Find the median.

3



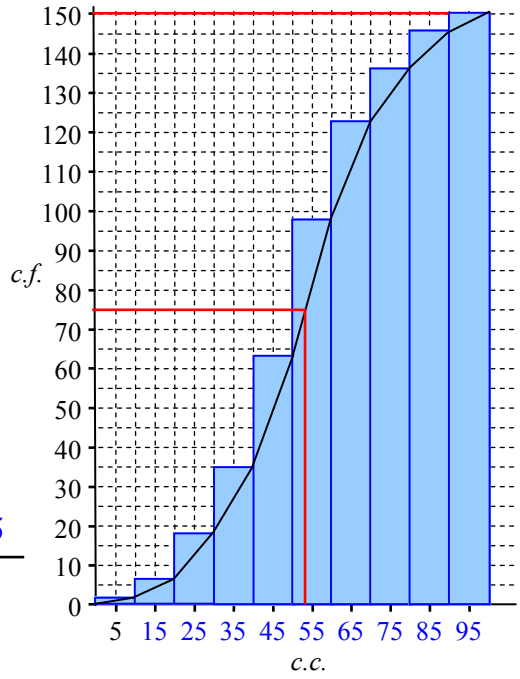
50% line:

0.5 × 94 = 47 Median = 17 .5

4

150 students sat a yearly exam, the results are displayed in the table. Calculate *c.c.* and the *c.f.* Then draw a *c.f.* histogram with an ogive. Find the median score (graph), median class.

Mark	*c.c.*	*f*	*c.f.*
0 - < 10	5	2	2
10 - < 20	15	5	7
20 - < 30	25	11	18
30 - < 40	35	17	35
40 - < 50	45	29	64
50 - < 60	55	34	98
60 - < 70	65	25	123
70 - < 80	75	14	137
80 - < 90	85	9	146
90 - < 100	95	4	150
		Σ*f* =	150



50% line = 0.5 × 150 = 75

Median = 53

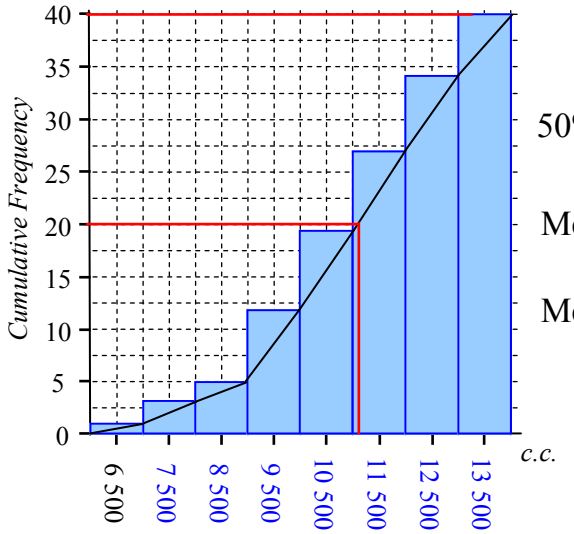
Median Class = 50 -<60

A company manufactures light bulbs. It claims the bulbs have a life of 10 000 hrs. Systematic testing of 40 bulbs resulted in the data below. Tabulate then graph the results using a C.F. Histogram/polygon and estimate the median life for the bulbs.

5 Data:

11 600	12 900	12 500
12 200	8 600	11 300
9 800	11 700	10 400
8 500	12 300	10 200
13 700	9 600	9 600
11 200	6 700	9 800
10 400	10 500	11 300
13 300	7 200	13 400
12 100	11 000	12 900
10 600	13 900	11 700
9 400	9 400	9 500
11 300	13 600	7 700
13 500	10 100	12 700

Duration (hrs)	*c.c.*	*Tally*	*f*	*c.f.*
6 000 - < 7 000	6 500	|	1	1
7 000 - < 8 000	7 500	||	2	3
8 000 - < 9 000	8 500	||	2	5
9 000 - < 10 000	9 500	𝍸 ||	7	12
10 000 - < 11 000	10 500	𝍸 ||	7	19
11 000 - < 12 000	11 500	𝍸 |||	8	27
12 000 - < 13 000	12 500	𝍸 ||	7	34
13 000 - < 14 000	13 500	𝍸 |	6	40
		Σ*f* =	40	



50% line = 0.5 × 40 = 20

Median = 11 100 hours

Median Class :

11 000 - < 12 000

Data Analysis - Find A Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Data Analysis - Find a Word

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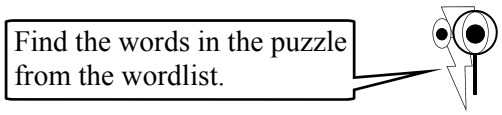

G	R	A	R	R	A	N	G	E	U	Q	A	T	A	D
T	M	B	D	E	I	F	I	T	A	R	T	S	X	R
F	O	T	E	E	Y	C	L	S	U	S	N	E	C	A
E	D	O	P	G	D	M	O	U	Q	N	A	E	M	N
T	N	Q	U	A	C	Y	C	Y	O	M	F	R	I	G
F	A	K	O	R	D	I	A	I	D	O	T	T	D	E
S	R	Q	R	E	P	M	T	D	D	D	P	H	D	G
Q	U	E	G	V	X	A	I	A	F	E	O	I	L	N
L	C	Y	Q	A	L	M	O	P	M	U	S	G	E	I
A	S	W	T	U	Y	C	N	Q	E	E	I	H	T	D
T	A	G	P	O	E	D	G	R	D	C	T	E	C	N
O	M	O	T	I	I	N	T	C	I	Y	I	S	Y	E
T	P	M	C	E	R	O	C	S	A	K	O	T	Y	C
T	L	Q	M	I	D	D	U	Y	N	C	N	C	Y	S
M	E	F	R	L	O	W	E	S	T	S	I	G	M	A

WORDLIST

LOWEST	RANGE	SIGMA	MODE	ASCENDING
RANDOM	AVERAGE	POPULATION	CENSUS	SCORE
SUM	SAMPLE	MEDIAN	POSITION	DATA
MIDDLE	STRATIFIED	LOCATION	HIGHEST	TOTAL
GROUPED	MEAN	ARRANGE	FREQUENCY	SYSTEMATIC

Data Analysis - Find a Word

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Find the words in the puzzle from the wordlist.



G	R	A	R	R	A	N	G	E	U	Q	A	T	A	D
T	M	B	D	E	I	F	I	T	A	R	T	S	X	R
F	O	T	E	E	Y	C	L	S	U	S	N	E	C	A
E	D	O	P	G	D	M	O	U	Q	N	A	E	M	N
T	N	Q	U	A	C	Y	C	Y	O	M	F	R	I	G
F	A	K	O	R	D	I	A	I	D	O	T	T	D	E
S	R	Q	R	E	P	M	T	D	D	D	P	H	D	G
Q	U	E	G	V	X	A	I	A	F	E	O	I	L	N
L	C	Y	Q	A	L	M	O	P	M	U	S	G	E	I
A	S	W	T	U	Y	C	N	Q	E	E	I	H	T	D
T	A	G	P	O	E	D	G	R	D	C	T	E	C	N
O	M	O	T	I	I	N	T	C	I	Y	I	S	Y	E
T	P	M	C	E	R	O	C	S	A	K	O	T	Y	C
T	L	Q	M	I	D	D	U	Y	N	C	N	C	Y	S
M	E	F	R	L	O	W	E	S	T	S	I	G	M	A

WORDLIST

~~LOWEST~~ ~~RANGE~~ ~~SIGMA~~ ~~MODE~~ ~~ASCENDING~~

~~RANDOM~~ ~~AVERAGE~~ ~~POPULATION~~ ~~CENSUS~~ ~~SCORE~~

~~SUM~~ ~~SAMPLE~~ ~~MEDIAN~~ ~~POSITION~~ ~~DATA~~

~~MIDDLE~~ ~~STRATIFIED~~ ~~LOCATION~~ ~~HIGHEST~~ ~~TOTAL~~

~~GROUPED~~ ~~MEAN~~ ~~ARRANGE~~ ~~FREQUENCY~~ ~~SYSTEMATIC~~

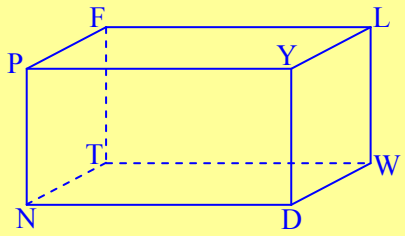
FREEFALL MATHEMATICS

ALTITUDE

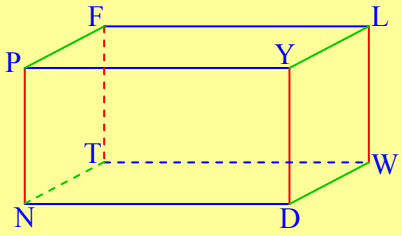
SURFACE AREA

Parallel, Perpendicular and Skew

Parallel means the same distance apart and not touching. This sheet deals with parallel faces and parallel edges. The rectangular prism shown below has three pairs of parallel faces.



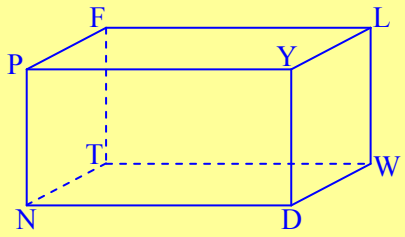
The top face PFLY is parallel to the bottom face NTWD.
The front face PYDN is parallel to the back face FLWT.
The LHS face PFTN is parallel to the RHS face YLWD.



There are parallel edges as well. On the diagram to the right, the edges that are the same colour are all parallel. You can show this using // :

PN // FT // LW // YD ND // PY // FL // TW PF // YL // DW // NT

Perpendicular means ‘at right angles to’ and touching. Again you can have perpendicular faces and also perpendicular edges. Faces first:

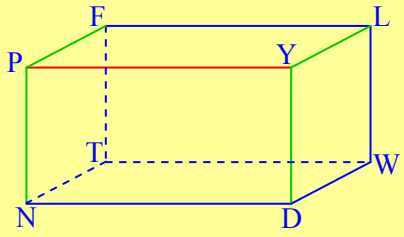


The top face PFLY and the bottom face NTWD are perpendicular to FLWT, YLWD, PYDN and PFTN.
The front face PYDN and the back face FLWT are perpendicular to PFLY, YLWD, WDNT and PFTN.
The side faces PFTN and YLWD are perpendicular to PFLY, FLWT, NTWD and PYDN.

Edges can also be parallel. With rectangular prisms and cubes there will be two perpendicular edges and the end of each edge.

So the edge PY (in red) has perpendicular edges (in green) YL, YD, PF and PN.

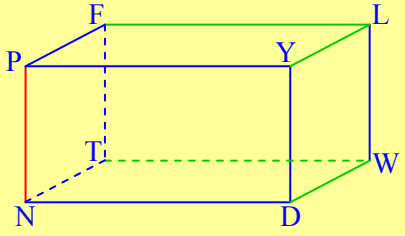
Not the perpendicular symbol is ⊥ .



Skew edges are edges that aren’t parallel and aren’t perpendicular. So if an edge is skew it won’t be parallel and it isn’t going to touch the given edge.

Name the skew edges to PN (shown in red).
The skew edges are shown in green and are FL, TW, YL and DW.

Note that none touch PN, and none are parallel to PN.



Parallel, Perpendicular and Skew

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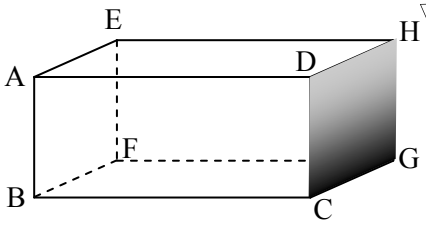
Name the face(s) that ….



1 Is shaded. _____

2 Is parallel to BFGC.

BFGC // _____

3 Is parallel to the shaded face.

_____ // _____

4 Is parallel to ABCD.

_____

5 Is parallel to AEFB.

_____

6 Is perpendicular to ABCD & has an edge EH.

_____ ⊥ _____

7 Are perpendicular to EHGF & have an edge BF, (2 faces).

_____ ⊥ _____ ⊥ _____

8 Are perpendicular to BFGC & perpendicular to AEFB.

[ ] , [ ]

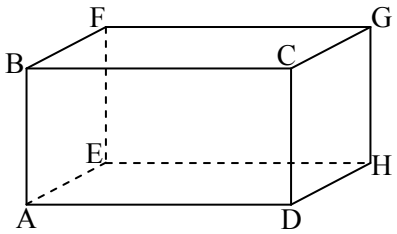
9 Are perpendicular to the shaded face and have edge GF.

10 Is perpendicular to ABCD & has vertices C & F.

11 Is perpendicular to DAEH & has edge BC.

_____ ⊥ _____

Name the edges that are parallel to, & perpendicular to, the edges on the rectangular prisms below.



12 3 edges parallel to AB are:

AB // [ ] // [ ] // [ ]

13 3 edges parallel to GC are:

GC // [ ] // [ ] // [ ]

14 3 edges parallel to EH are:

_____

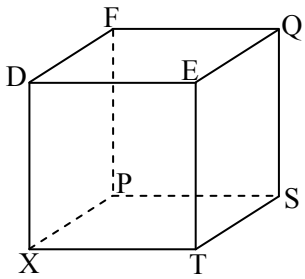
15 4 edges perpendicular to AB

[ ] , [ ] , [ ] , [ ]

16 4 edges perpendicular to GC

_____

Now use the cube shown below.



17 Name the face parallel to QFPS. _____

18 Name the edges parallel to FP. _____

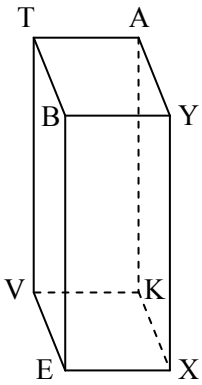
19 Name the horizontal faces perpendicular to DFPX.

_____

20 Name the vertical faces perpendicular to PFQS.

_____

What's skew with you? Name the skew edges in these questions.



21 Name 4 edges that are skew to YB: [ ] , [ ] , [ ] , [ ] .

22 Name 4 edges that are skew to VT: _____

23 Name 4 edges that are skew to KX: _____

24 Name two edges that are skew to EX and parallel to EB.

_____

25 Name two edges that are skew to AK and parallel to AT.

_____

26 Name an edge that is skew to VT and parallel to BY. _____

27 Name an edge that is skew to KV and parallel to YX. _____

28 Name an edge that is skew to EX and is on the face TBEV.

_____

29 Name two edges that are skew to AK and perpendicular to VT. _____

30 Name two edges that are skew to KV and perpendicular to EX. _____

31 Name two edges that are skew to XY and are on the face ATVK. _____

Parallel, Perpendicular and Skew

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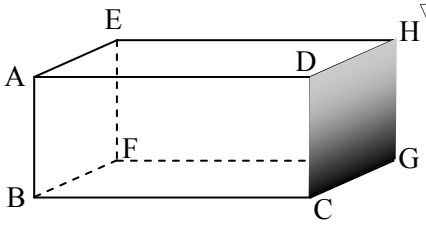
Name the face(s) that ….



1 Is shaded. GCDH

2 Is parallel to BFGC.

BFGC // AEHD

3 Is parallel to the shaded face.

GCDH // FBAE

4 Is parallel to ABCD.

ABCD // EFGH

5 Is parallel to AEFB.

AEFB // DHGC

6 Is perpendicular to ABCD & has an edge EH.

ABCD ⊥ AEHD

7 Are perpendicular to EHGF & have an edge BF, (2 faces).

EHGF ⊥ AEFB ⊥ BFGC

8 Are perpendicular to BFGC & perpendicular to AEFB.

ABCD, EHGF

9 Are perpendicular to the shaded face and have edge GF.

CDHG ⊥ CBFG ⊥ GFEH

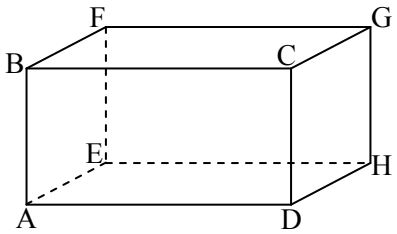
10 Is perpendicular to ABCD & has vertices C & F.

ABCD ⊥ CBFG

11 Is perpendicular to DAEH & has edge BC.

DAEH ⊥ ABCD

Name the edges that are parallel to, & perpendicular to, the edges on the rectangular prisms below.



12 3 edges parallel to AB are:

AB // EF // HG // DC

13 3 edges parallel to GC are:

GC // HD // EA // FB

14 3 edges parallel to EH are:

EH // AD // BC // FG

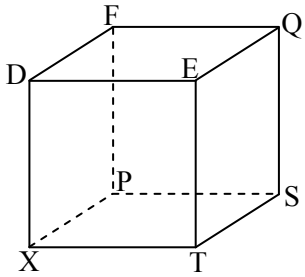
15 4 edges perpendicular to AB

AD , AE , BC , BF

16 4 edges perpendicular to GC

CB , CD , GH , GF

Now use the cube shown below.



17 Name the face parallel to QFPS. EDXT

18 Name the edges parallel to FP. FP // DX // ET // QS

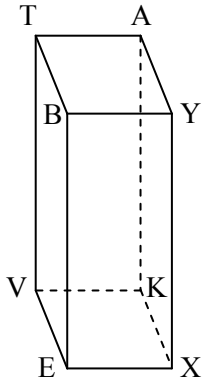
19 Name the horizontal faces perpendicular to DFPX.

DFQE, XPST

20 Name the vertical faces perpendicular to PFQS.

EQST, DFPX

What's skew with you? Name the skew edges in these questions.



21 Name 4 edges that are skew to YB: TV , AK , VE , KX .

22 Name 4 edges that are skew to VT: BY, EX, AY, KX.

23 Name 4 edges that are skew to KX: BY, TA, TV, BE.

24 Name two edges that are skew to EX and parallel to EB.

VT, KA.

25 Name two edges that are skew to AK and parallel to AT.

BY, EX.

26 Name an edge that is skew to VT and parallel to BY. EX

27 Name an edge that is skew to KV and parallel to YX. BE

28 Name an edge that is skew to EX and is on the face TBEV.

TB or TV

29 Name two edges that are skew to AK and perpendicular to VT. TB, VE.

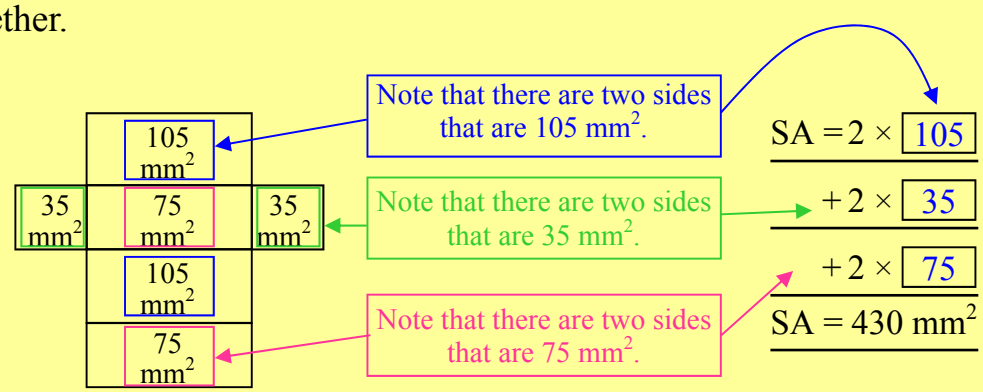
30 Name two edges that are skew to KV and perpendicular to EX. BE, YX.

31 Name two edges that are skew to XY and are on the face ATVK. VK, TA.

Surface Area of Prisms - Adding Face Areas

Surface area (S.A.) is the outside area of the faces/curved surfaces of a solid. So to calculate the S.A. of a solid you add all the areas of the individual shapes/surfaces that make up the solid. This sheet also shows the **nets** of popular solids. A net is like a solid that has been opened up, like it was constructed from paper and unfolded.

Column 1 starts with showing nets that have the area for each shape that makes the solid labeled with its area. Note that there are no side lengths … just areas. So the S.A. is found by adding all these areas together. But because some areas are repeated, these are gathered together.



Columns 2 and 3 show solids with the areas labeled on them. But they aren't shown as nets so you can't see all the faces of the solid. But the faces are repeated and you have all the numbers you need, just ask …. How many of each area is there? Then you solve the question just the same as Column 1.

Surface Area of Prisms - Adding Face Areas

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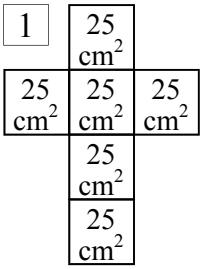
Find the surface area of the nets by adding the separate areas. But use × too!

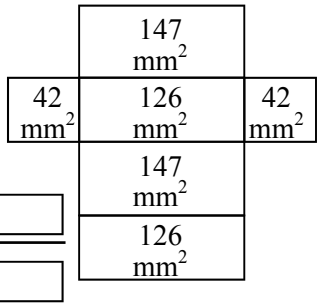
1



SA = 6 × ☐

SA = cm²

2



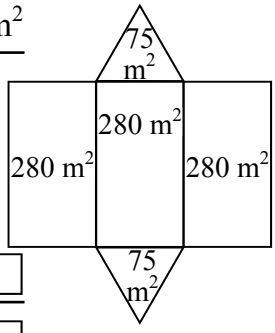
SA = 2 × ☐

+ 2 × ☐

+ 2 × ☐

SA = mm²

3

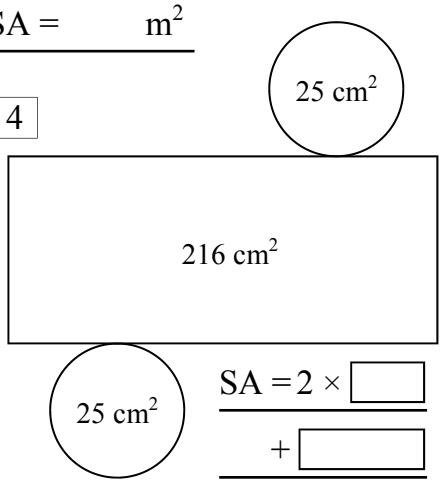


SA = 2 × ☐

+ 3 × ☐

SA = m²

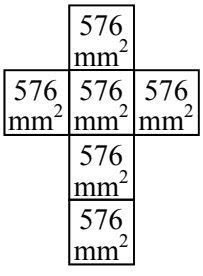
4



SA = 2 × ☐

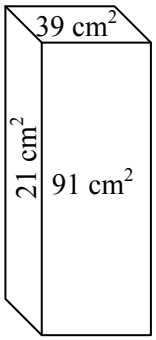
+ ☐

5



Remember the matching sides, find the SA!

6



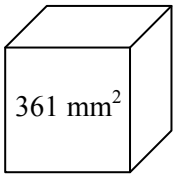
SA = 2 × ☐

+ 2 × ☐

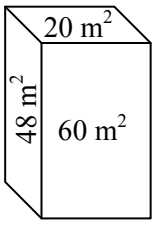
+ 2 × ☐

SA = cm²

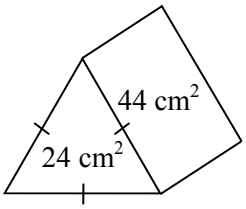
7 Cube



8



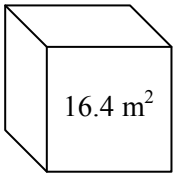
9



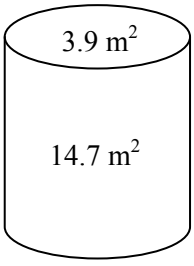
SA = 2 ×

+ 3 ×

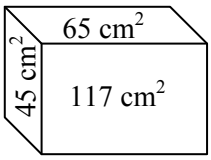
10 Cube



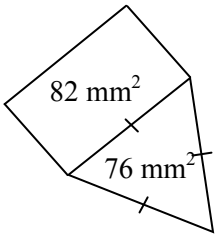
11



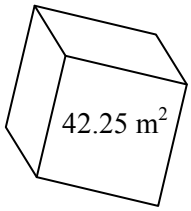
12



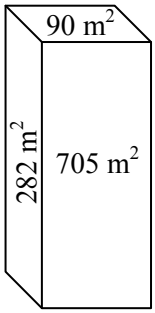
13



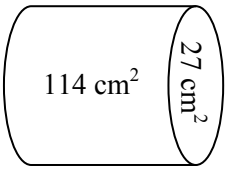
14 Cube



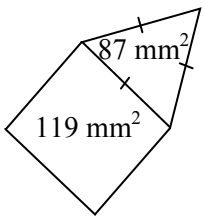
15



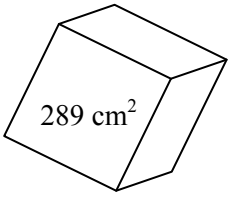
16



17



18 Cube



Surface Area of Prisms - Adding Face Areas

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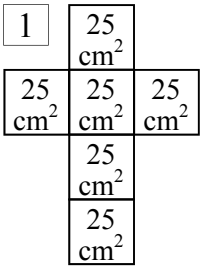
Find the surface area of the nets by adding the separate areas. But use × too!

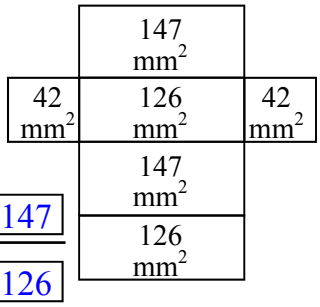
1



SA = 6 × 25

SA = 150 cm²

2



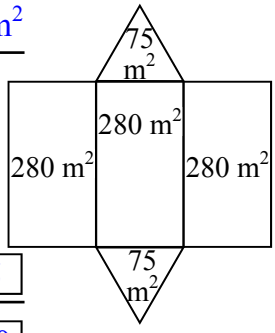
SA = 2 × 147

+ 2 × 126

+ 2 × 42

SA = 630 mm²

3

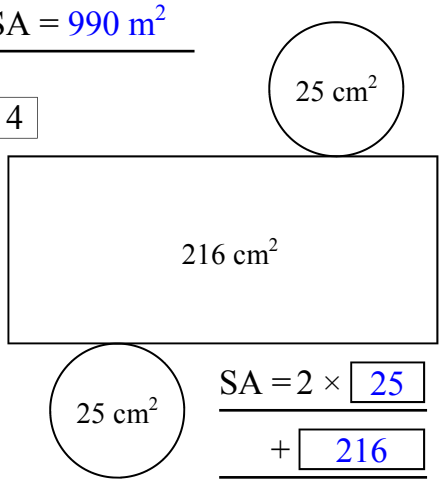


SA = 2 × 75

+ 3 × 280

SA = 990 m²

4

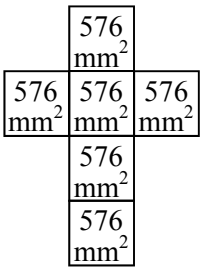


SA = 2 × 25

+ 216

SA = 266 cm²

5

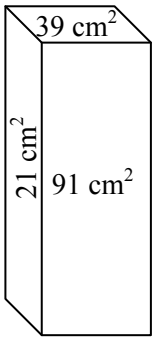


SA = 6 × 576

SA = 3 456 mm²

Remember the matching sides, find the SA!

6



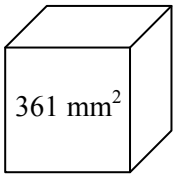
SA = 2 × 39

+ 2 × 21

+ 2 × 91

SA = 302 cm²

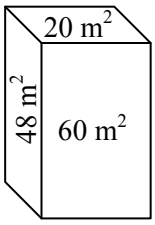
7 Cube



SA = 6 × 361

SA = 2 166 mm²

8



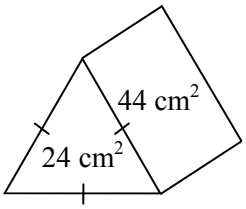
SA = 2 × 20

+ 2 × 48

+ 2 × 60

SA = 256 m²

9

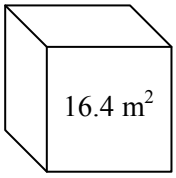


SA = 2 × 24

+ 3 × 44

SA = 180 cm²

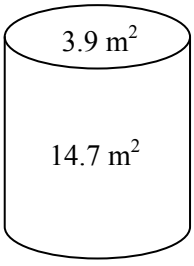
10 Cube



SA = 6 × 16.4

SA = 98.4 m²

11

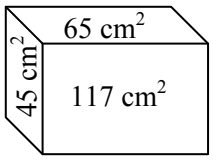


SA = 2 × 3.9

+ 14.7

SA = 22.5 m²

12



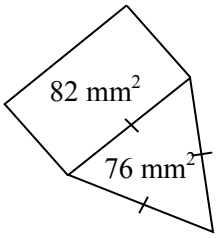
SA = 2 × 65

+ 2 × 45

+ 2 × 117

SA = 454 cm²

13

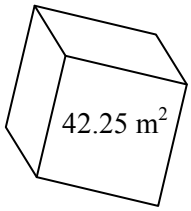


SA = 2 × 76

+ 3 × 82

SA = 398 mm²

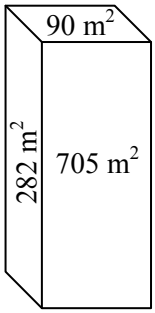
14 Cube



SA = 6 × 42.25

SA = 253.5 m²

15



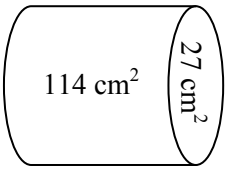
SA = 2 × 90

+ 2 × 282

+ 2 × 705

SA = 2 154 m²

16

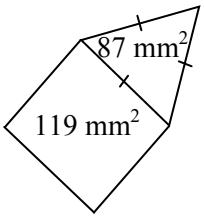


SA = 2 × 27

+ 114

SA = 168 cm²

17

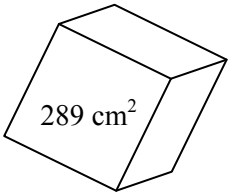


SA = 2 × 87

+ 3 × 119

SA = 531 mm²

18 Cube



SA = 6 × 289

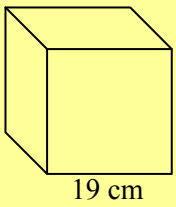
SA = 1 734 cm²

Surface Area - Cubes and Rectangular Prisms

The previous sheet gave you the areas of each face of a solid. This exercise gives you the length of the edges of the solids. So you have to calculate the areas yourself.

Column 1 asks you to find the S.A. of cubes. A cube has six identical square faces, so calculate the area of one square face, then multiply (times) by 6. But there is a formula as well and it is: S.A. = $6l^2$, where 'l' is the side length of the square face.

Example



$SA = 6l^2$ ← Write out the formula.

$= 6 \times 19^2$ ← Substitute the side length into the formula.

$SA = 2\ 166\ cm^2$ ← Use your calculator and write the answer showing the units, which are square units.

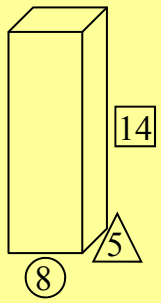
S.A. of a Cube Calculator Steps:

To Find 6×19^2

6 × 1 9 x^2 =

Column 2 deals with finding the S.A. of rectangular prisms. These are more complicated than cubes because they have 3 pairs of identical face areas. So you have to perform three separate calculations. This column assists you by putting shapes around the edge lengths, this matches the shapes in the working out area …. so just rewrite the number in the correct position and so on. The lengths are all cm, so your final answer should have cm^2 in it.

Example



$SA = 2 \times \boxed{14} \times \triangle 5$ ← The square is 14 and the triangle is 5.

$+ 2 \times \boxed{14} \times (8)$ ← The square is 14 and the circle is 8.

$+ 2 \times (8) \times \triangle 5$ ← The circle is 8 and the triangle is 5.

$SA = 444\ cm^2$ ← Use your calculator and write the answer showing the units, which are all cm^2.

S.A. of a Rectangular Prism Calculator Steps:

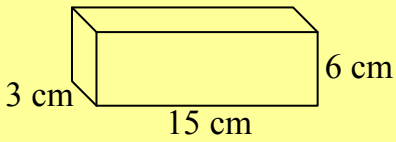
2 × 1 4 × 5 + 2 × 1 4 × 8 + 2 × 8 × 5 =

Column 3 is a mix of cubes and rectangular prisms. But this time the rectangular prisms don't have the shapes around the lengths. So you have to make sure that you use two of each number when you calculate the S.A.

A step-by step process is on the next page.

When you select the sides you are going to use, you can place a stroke through them. Once a number has a cross through it, you don't use it again.

Example



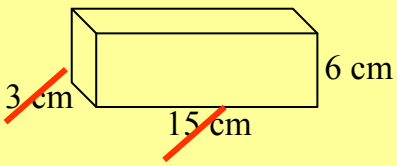
SA = 2 × ×
+ 2 × ×
+ 2 × ×
SA =

STEP 1

Write SA = 2 × ×
+ 2 × ×
+ 2 × ×
SA =

This is always the same, this sets you up to write the side lengths in.

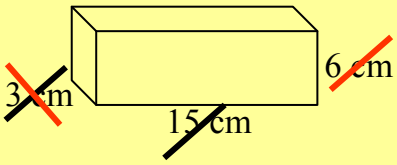
SA = 2 × **3** × **15**
+ 2 × ×
+ 2 × ×
SA =



STEP 2

Pick two side lengths, whichever two you like. Write them in the top line and put a stroke through the numbers on the diagram.

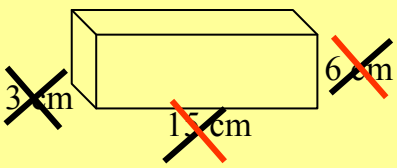
SA = 2 × 3 × 15
+ 2 × **3** × **6**
+ 2 × ×
SA =



STEP 3

Pick the side you haven't used (6 cm) and one side that you have (we used 3 cm). Write them in the second line and put a stroke through the numbers on the solid. Note that the 3 cm side has a cross through it, it is no longer able to be used.

SA = 2 × 3 × 15
+ 2 × 3 × 6
+ 2 × **15** × **6**
SA =



STEP 4

The sides with only one stroke are now used, so use 15 cm and 6 cm. Write these in the third line and you are now ready to calculate the answer.

2	×	3	×	1	5	+	2	×	3	×	6	+	2	×	1	5	×	6	=

SA = 306 cm^2

STEP 5

Calculate the answer and write the square units on the answer. You are done!

Surface Area - Cubes and Rectangular Prisms

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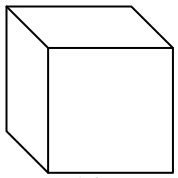
Use $SA = 6l^2$ to find the SA of these cubes. Your answer should show units.

1



$SA = 6l^2$

$= 6 \times \square^2$

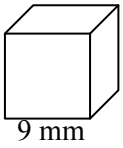
$SA =$ cm^2

2

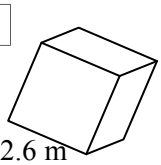
$SA = 6l^2$

$=$

$SA =$ mm^2



3

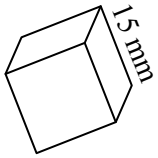


$SA =$

$=$

$SA =$

4



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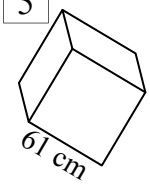
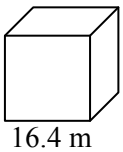
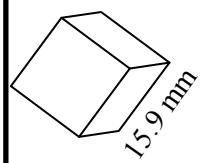


Round these surface areas to 1 d.p.

6



7



Use the Δ, □, and ○ to help you with the SA of these rectangular prisms. All side lengths are in cm.

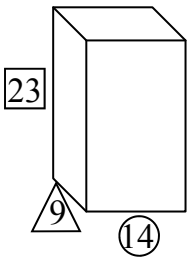
8

$SA = 2 \times \square \times \triangle$

$+ 2 \times \square \times \bigcirc$

$+ 2 \times \bigcirc \times \triangle$

$SA =$ cm^2



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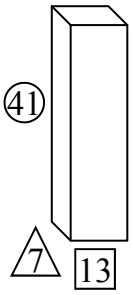


$SA = 2 \times \square \times \triangle$

$+ 2 \times \square \times \bigcirc$

$+ 2 \times \bigcirc \times \triangle$

$SA =$ cm^2

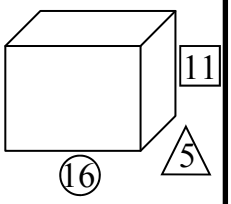
10

$SA = 2 \times \square \times \triangle$

$+ 2 \times \square \times \bigcirc$

$+ 2 \times \bigcirc \times \triangle$

$SA =$



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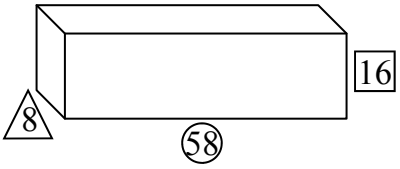


$SA = 2 \times \square \times \triangle$

$+ 2 \times \square \times \bigcirc$

$+ 2 \times \bigcirc \times \triangle$

$SA =$

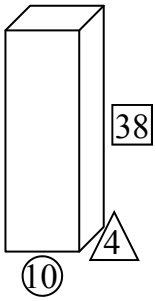
12

$SA = 2 \times \square \times \triangle$

$+ 2 \times \square \times \bigcirc$

$+ 2 \times \bigcirc \times \triangle$

$SA =$



Now try these, round to 1 d.p. if required.

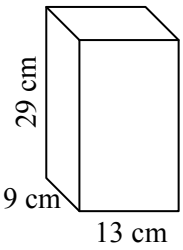
13

$SA = 2 \times$

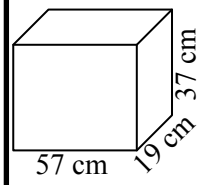
$+ 2 \times$

$+ 2 \times$

$SA =$ cm^2



14



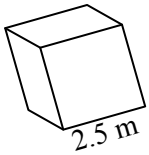
$SA = 2 \times$

$+ 2 \times$

$+ 2 \times$

$SA =$ cm^2

15 Cube



16 A rectangular prism has sides 2.4 m, 3.8 m and 4.3 m

$=$

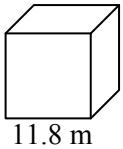
$+$

$+$

$=$

17 A rectangular prism has sides 78 mm, 14 mm and 23 mm

18 Cube



Surface Area - Cubes and Rectangular Prisms

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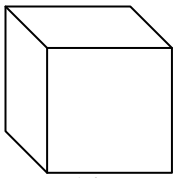
Use $SA = 6L^2$ to find the SA of these cubes. Your answer should show units.

1



$SA = 6l^2$

$= 6 \times 14^2$

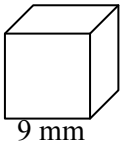
$SA = 1\ 176\ cm^2$

2

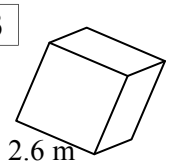
$SA = 6l^2$

$= 6 \times 9^2$

$SA = 406\ mm^2$



3



$SA = 6l^2$

$= 6 \times 2.6^2$

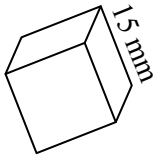
$SA = 40.56\ m^2$

4

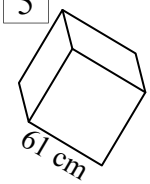
$SA = 6l^2$

$= 6 \times 15^2$

$SA = 1\ 350\ mm^2$



5



$SA = 6l^2$

$= 6 \times 15^2$

$SA = 1\ 350\ cm^2$

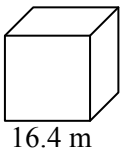
Round these surface areas to 1 d.p.

6

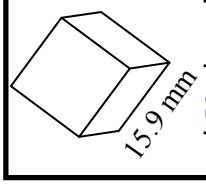
$SA = 6l^2$

$= 6 \times 16.4^2$

$SA = 1\ 613.8\ m^2$



7



$SA = 6l^2$

$= 6 \times 15.9^2$

$SA = 1\ 516.9\ mm^2$

Use the Δ, □, and ○ to help you with the SA of these rectangular prisms. All side lengths are in cm.

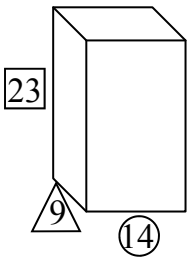
8

$SA = 2 \times \boxed{23} \times \triangle 9$

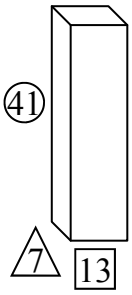
$+ 2 \times \boxed{23} \times (14)$

$+ 2 \times (14) \times \triangle 9$

$SA = 1\ 310\ cm^2$



9



$SA = 2 \times \boxed{13} \times \triangle 7$

$+ 2 \times \boxed{13} \times (41)$

$+ 2 \times (41) \times \triangle 7$

$SA = 1\ 822\ cm^2$

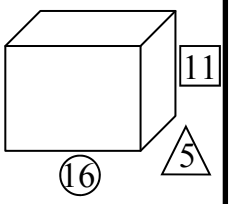
10

$SA = 2 \times \boxed{11} \times \triangle 5$

$+ 2 \times \boxed{11} \times (16)$

$+ 2 \times (16) \times \triangle 5$

$SA = 622\ cm^2$



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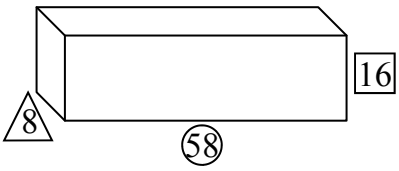


$SA = 2 \times \boxed{16} \times \triangle 8$

$+ 2 \times \boxed{16} \times (58)$

$+ 2 \times (58) \times \triangle 8$

$SA = 3\ 040\ cm^2$

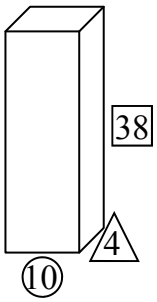
12

$SA = 2 \times \boxed{38} \times \triangle 4$

$+ 2 \times \boxed{38} \times (10)$

$+ 2 \times (10) \times \triangle 4$

$SA = 1\ 144\ cm^2$



Now try these, round to 1 d.p. if required.

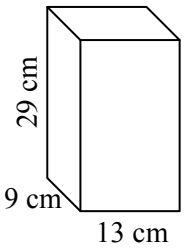
13

$SA = 2 \times 9 \times 13$

$+ 2 \times 9 \times 29$

$+ 2 \times 13 \times 29$

$SA = 1\ 510\ cm^2$



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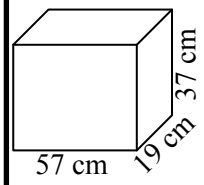


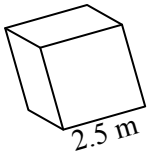
$SA = 2 \times 57 \times 19$

$+ 2 \times 57 \times 37$

$+ 2 \times 19 \times 37$

$SA = 7\ 790\ cm^2$

15 Cube



$SA = 6l^2$

$= 6 \times 2.5^2$

$SA = 37.5\ m^2$

16 A rectangular prism has sides 2.4 m, 3.8 m and 4.3 m

$SA = 2 \times 2.4 \times 3.8$

$+ 2 \times 2.4 \times 4.3$

$+ 2 \times 3.8 \times 4.3$

$SA = 71.56\ m^2$

17 A rectangular prism has sides 78 mm, 14 mm and 23 mm

$SA = 2 \times 78 \times 14$

$+ 2 \times 78 \times 23$

$+ 2 \times 14 \times 23$

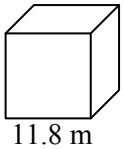
$SA = 6\ 416\ mm^2$

18 Cube

$SA = 6l^2$

$= 6 \times 11.8^2$

$SA = 835.4\ m^2$



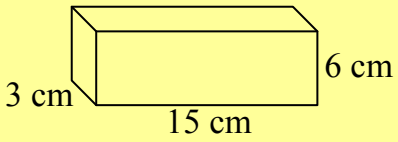
Surface Area - Rectangular Prisms

This sheet deals with calculating the surface area (S.A.) of a Rectangular Prism. The previous sheet should be completed before attempting this sheet.

The process for completing the sheet is shown below. Column 2 uses the same process as Column 1 but before you start you have to convert all the lengths into cm. Cross out the non-cm lengths and write them again, but in cm. Then away you go!

Example

When you select the sides you are going to use, you can place a stroke through them. Once a number has a cross through it, you don't use it again.



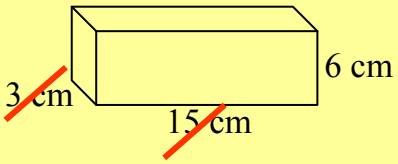
SA = 2 × ×
+ 2 × ×
+ 2 × ×
SA =

STEP 1

Write SA = 2 × ×
+ 2 × ×
+ 2 × ×
SA =

This is always the same, this sets you up to write the side lengths in.

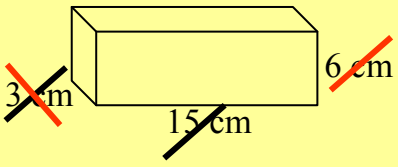
SA = 2 × **3** × **15**
+ 2 × ×
+ 2 × ×
SA =



STEP 2

Pick two side lengths, whichever two you like. Write them in the top line and put a stroke through the numbers on the diagram.

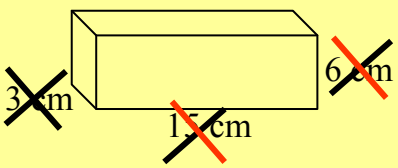
SA = 2 × 3 × 15
+ 2 × **3** × **6**
+ 2 × ×
SA =



STEP 3

Pick the side you haven't used (6 cm) and one side that you have (we used 3 cm). Write them in the second line and put a stroke through the numbers on the solid. Note that the 3 cm side has a cross through it, it is no longer able to be used.

SA = 2 × 3 × 15
+ 2 × 3 × 6
+ 2 × **15** × **6**
SA =



STEP 4

The sides with only one stroke are now used, so use 15 cm and 6 cm. Write these in the third line and you are now ready to calculate the answer.

2	×	3	×	1	5	+	2	×	3	×	6	+	2	×	1	5	×	6	=

STEP 5

Calculate the answer and write the square units on the answer. You are done!

SA = 306 cm^2

Surface Area - Rectangular Prisms

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Find the surface area of these rectangular prisms. Remember to show the units.

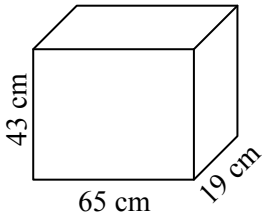
1

SA = 2 ×

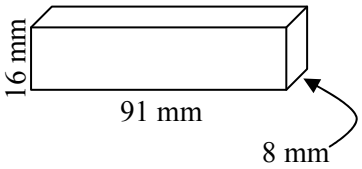
\+ 2 ×

\+ 2 ×

SA = cm^2



2



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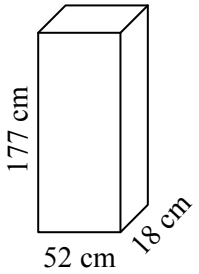

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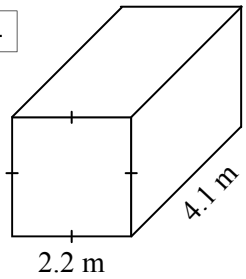

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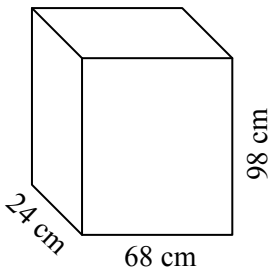

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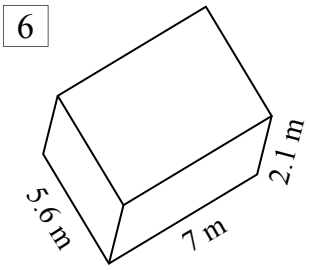
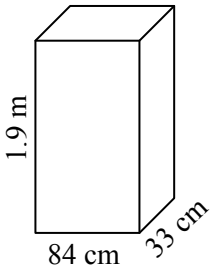


Give the surface area in cm^2. So change all units to cm first, then perform the SA calculation. Round to 1 d.p. if needed.

7



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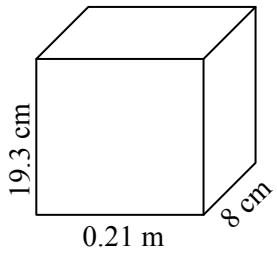

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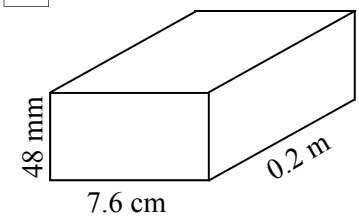

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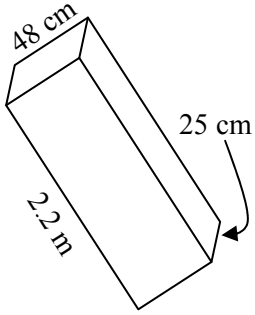

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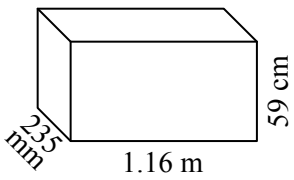

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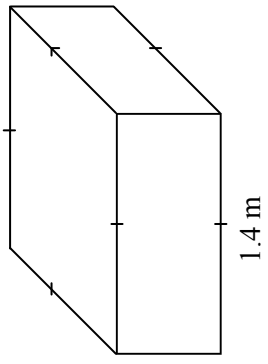

Surface Area - Rectangular Prisms

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Find the surface area of these rectangular prisms. Remember to show the units.

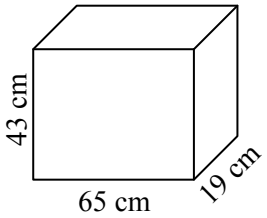
1

$SA = 2 \times 43 \times 65$

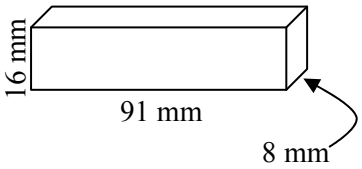
$+ 2 \times 43 \times 19$

$+ 2 \times 65 \times 19$

$SA = 9\,694\ cm^2$



2



$SA = 2 \times 16 \times 91$

$+ 2 \times 16 \times 8$

$+ 2 \times 91 \times 8$

$SA = 4\,624\ mm^2$

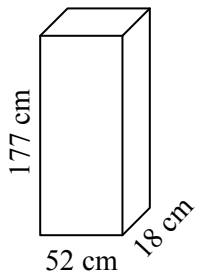
3

$SA = 2 \times 52 \times 177$

$+ 2 \times 52 \times 18$

$+ 2 \times 177 \times 18$

$SA = 26\,652\ cm^2$



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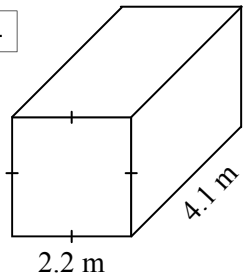


$SA = 2 \times 2.2 \times 4.1$

$+ 2 \times 2.2 \times 2.2$

$+ 2 \times 4.1 \times 2.2$

$SA = 45.76\ m^2$

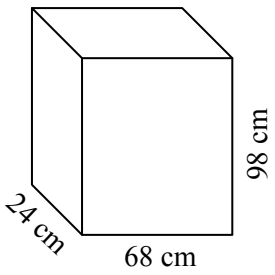
5

$SA = 2 \times 24 \times 68$

$+ 2 \times 24 \times 98$

$+ 2 \times 68 \times 98$

$SA = 21\,296\ cm^2$



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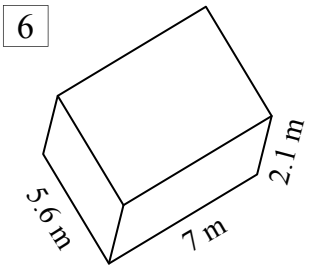


$SA = 2 \times 5.6 \times 2.1$

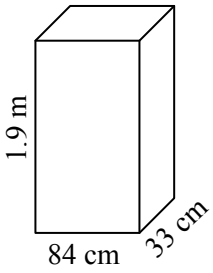
$+ 2 \times 5.6 \times 7$

$+ 2 \times 2.1 \times 7$

$SA = 131.32\ m^2$

Give the surface area in cm^2. So change all units to cm first, then perform the SA calculation. Round to 1 d.p. if needed.

7



$SA = 2 \times 190 \times 84$

$+ 2 \times 190 \times 33$

$+ 2 \times 84 \times 33$

$SA = 50\,0004\ cm^2$

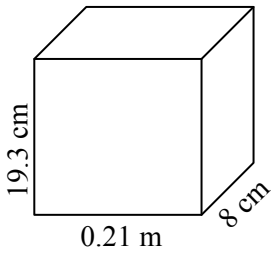
8

$SA = 2 \times 19.3 \times 21$

$+ 2 \times 19.3 \times 8$

$+ 2 \times 21 \times 8$

$SA = 1\,455.4\ cm^2$



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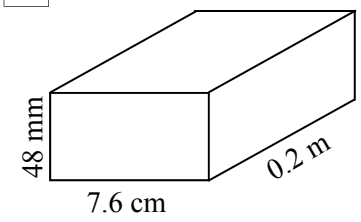


$SA = 2 \times 4.8 \times 7.6$

$+ 2 \times 4.8 \times 20$

$+ 2 \times 7.6 \times 20$

$SA = 569\ cm^2$

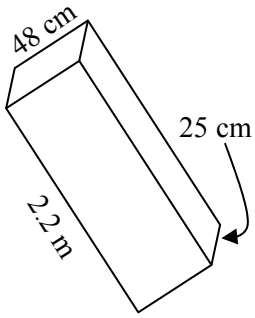
10

$SA = 2 \times 220 \times 48$

$+ 2 \times 220 \times 25$

$+ 2 \times 48 \times 25$

$SA = 34\,520\ cm^2$



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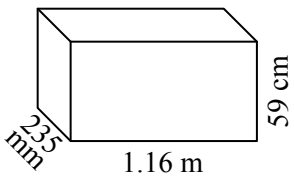


$SA = 2 \times 23.5 \times 116$

$+ 2 \times 23.5 \times 59$

$+ 2 \times 116 \times 59$

$SA = 21\,193\ cm^2$

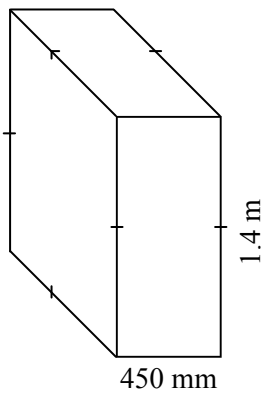
12

$SA = 2 \times 140 \times 45$

$+ 2 \times 140 \times 140$

$+ 2 \times 45 \times 140$

$SA = 64\,400\ cm^2$



Surface Area of Rectangular Prisms - Problems.

This worksheet takes your skills with finding the surface area of a rectangular prism and applies it to different situations.

Column 1 starts with constructing a steel tool box. The sides are then increased by 20%, and the SA is calculated again. Would a 20% increase on the side lengths result in a 20% increase in SA? If you can't remember how to increase by a percentage, below is an example of increasing 60 cm by 20%.

Example - Increasing 60 cm by 20%.

Using Decimals		Using Fractions
$60 \times 0.2 = 12$ cm	This is the amount of the increase	$60 \times \frac{20}{100} = 12$ cm
$60 + 12 = 72$ cm	This is the final length	$60 + 12 = 72$ cm

Column 2 has Bob renovating his bathroom. The room is shown with a doorway, the doorway size is shown in Question 8, where it is needed. First find the SA of the entire room, then calculate the individual areas that are tiled, painted or having laminated flooring installed. Each area has a different cost per m^2.

Column 3 starts with a question that is related to Bob's bathroom, this is essentially changing a fraction to a percentage. The theme then turns to toasters and how the rate of heat loss by a toaster casing increases as its SA increases.

There are two different toasters calculate the SA for both, then use an experimental formula created by the manufacturer to consider if a toaster's heat dissipation is considered safe.

$$T = 230 - 0.1 \times SA$$

where T = safety constant (must be below 50 to be considered safe)
SA = surface area of the rectangular prism shaped toaster in cm^2

Example - Applying the formula to a toaster that has an SA of 1 500 cm^2.

$T = 230 - 0.1 \times SA$	Write the formula
$= 230 - 0.1 \times 1500$	Substitute the SA into the formula
$T = 80$	Solve for the value of T
The toaster is unsafe.	As T = 80 the toaster is unsafe as 80 > 50.

Surface Area of Rectangular Prisms - Problems.

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Find the SA of these tool boxes!

A steel rectangular shaped toolbox is 35 cm long, 20 cm high and 25 cm wide, draw a diagram and find the SA

Diagram

1

2 SA = 2 ×

\+ 2 ×

\+ 2 ×

SA = cm^2

Increase each of the sides of the tool box by 20%.

3 35:

20:

25:

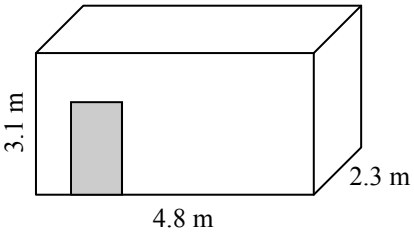
Draw a diagram and then find the SA of the enlarged tool box.

Diagram

4

5

Bob's bathroom is shown below, he's painting the ceiling, tiling the walls and fitting laminate to the floor. Find the SA and costs.



6 Find the SA of the room.

7 Calculate the floor area and the cost to fit laminate, when it costs \$24/m^2.

8 Calculate the tiled area and the cost to tile it, when tiles cost \$37/m^2. There is a doorway that is 2.1 m × 1.2 m, that isn't tiled.

9 If painting costs \$3.50/m^2 find the total cost for the ceiling.

10 Calculate, as a percentage of the total SA, the area of the room that is tiled. (to 2 d.p.)

These toasters are all rectangular prisms.

A company finds toasters are safer if they cool faster. Cooling time is found by the formula:

$T = 230 - 0.1 \times SA$.

If T is less than 50 then they are safe.

12 Find the SA for a toaster of size 17 cm × 14 cm × 20 cm.

13 Apply the formula to find if it is considered safe.

Write formula

Sub in SA value

Solve for T

Worded response

14 Repeat for a different model sized 18 cm × 23 cm × 16 cm.

Surface Area of Rectangular Prisms - Problems.

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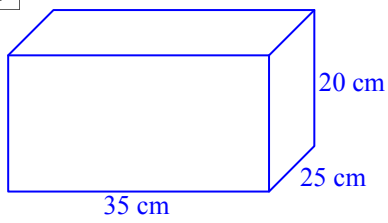
Find the SA of these tool boxes!

A steel rectangular shaped toolbox is 35 cm long, 20 cm high and 25 cm wide, draw a diagram and find the SA

Diagram

1



2 SA = $2 \times 35 \times 25$

$+ 2 \times 35 \times 20$

$+ 2 \times 25 \times 20$

SA = $4\,150\ cm^2$

Increase each of the sides of the tool box by 20%.

3 35: $35 \times 0.2 = 7$ cm

$35 + 7 = 42$ cm

20: $20 \times 0.2 = 4$ cm

$20 + 4 = 24$ cm

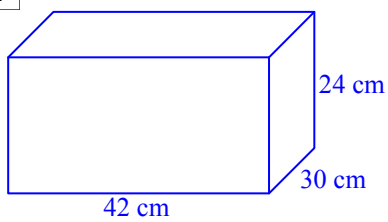
25: $25 \times 0.2 = 5$ cm

$25 + 5 = 30$ cm

Draw a diagram and then find the SA of the enlarged tool box.

Diagram

4



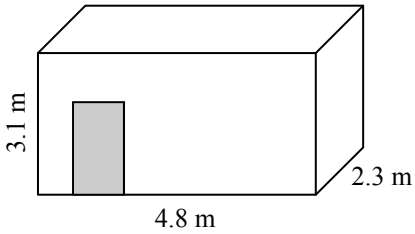
5 SA = $2 \times 42 \times 30$

$+ 2 \times 42 \times 24$

$+ 2 \times 30 \times 24$

SA = $5\,976\ cm^2$

Bob's bathroom is shown below, he's painting the ceiling, tiling the walls and fitting laminate to the floor. Find the SA and costs.



6 Find the SA of the room.

SA = $2 \times 4.8 \times 2.3$

$+ 2 \times 4.8 \times 3.1$

$+ 2 \times 3.1 \times 2.3$

SA = $66.1\ m^2$

7 Calculate the floor area and the cost to fit laminate, when it costs \$24/m^2.

A = $4.8 \times 2.3 = 11.04\ m^2$

Cost = 11.04×24

Cost = \$264.96 ($12\ m^2$ = \$288)

8 Calculate the tiled area and the cost to tile it, when tiles cost \$37/m^2. There is a doorway that is 2.1 m × 1.2 m, that isn't tiled.

SA = $2 \times 4.8 \times 3.1$

$+ 2 \times 3.1 \times 2.3$

SA = $44.02\ m^2$

Tiled Area = $44.02 - 2.1 \times 1.2$

Tiled Area = $41.5\ m^2$

Cost = 41.5×37 = \$1 535.50

Or 42×37 = \$1 554

9 If painting costs \$3.50/m^2 find the total cost for the ceiling.

Area = Floor = $11.04\ m^2$

Cost = 11.04×3.5

Cost = \$38.64 ($12\ m^2$ = \$42)

10 Calculate, as a percentage of the total SA, the area of the room that is tiled. (to 2 d.p.)

$\frac{41.5}{66.1} \times 100$

$= 62.78\%$

These toasters are all rectangular prisms.

A company finds toasters are safer if they cool faster. Cooling time is found by the formula:

$T = 230 - 0.1 \times SA$.

If T is less than 50 then they are safe.

12 Find the SA for a toaster of size 17 cm × 14 cm × 20 cm.

SA = $2 \times 17 \times 14$

$+ 2 \times 17 \times 20$

$+ 2 \times 14 \times 20$

SA = $1\,716\ cm^2$

13 Apply the formula to find if it is considered safe.

$T = 230 - 0.1 \times SA$ — Write formula

$= 230 - 0.1 \times 1\,716$ — Sub in SA value

$T = 58.4$ — Solve for T

The toaster is not safe. — Worded response

14 Repeat for a different model sized 18 cm × 23 cm × 16 cm.

SA = $2 \times 18 \times 23$

$+ 2 \times 18 \times 16$

$+ 2 \times 23 \times 16$

SA = $2\,140\ cm^2$

$T = 230 - 0.1 \times SA$

$= 230 - 0.1 \times 2\,140$

$T = 16$

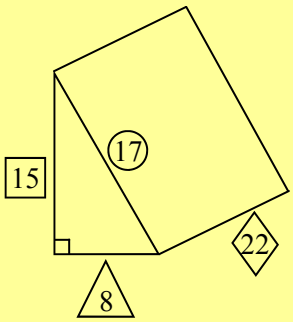
The toaster is safe.

Method Sheet - Surface Area of Triangular Prisms

Finding the surface area of a triangular prism can be confusing but this sheet is designed to introduce the method and make the process easier.

A triangular prism consists of two triangular ends and the three rectangles around the shape, all added together. Because there are four numbers and it is important which is multiplied by which, this sheet uses shapes around the numbers that match their correct position in the calculation. Just take the numbers off the shape and place them into the calculation and away you go!

Example - Finding the SA of the triangular prism shown below



The shape has a circle, square, triangle and rhombus around its dimensions. Place each into their correct position in the calculation below.

SA = $2 \times \frac{1}{2} \times$ 8 $\times$ 15

The shape has two identical triangles on the front and back. This line calculates their combined area. Remember that ½ = 0.5, easier on your calculator and maybe you know what 2 × 0.5 is? Is it necessary?

$+$ 8 $\times$ 22

This is calculating the rectangular base of the triangular prism.

$+$ 15 $\times$ 22

This is calculating the rectangular side of the triangular prism. (The vertical face on the left side of the shape).

$+$ 17 $\times$ 22

This is calculating the rectangular side of the triangular prism. (The slanted face on the right side of the shape).

SA = 1 000 cm^2

Method Sheet - Surface Area of Triangular Prisms

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Use the Δ, □, ◊ and ○ to help you to find the SA of these triangular prisms. All measurements are in cm.

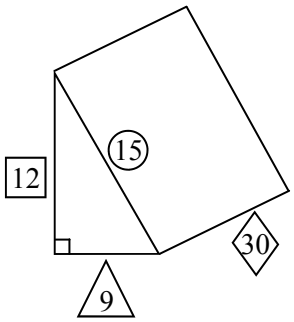
1 SA = 2×½× △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____ cm²



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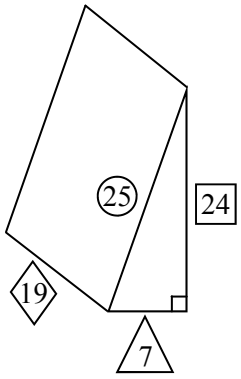


SA = 2×½× △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____ cm²

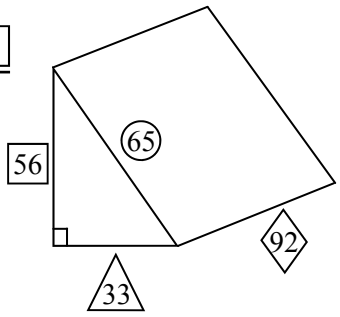
3 SA = 2×½× △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____



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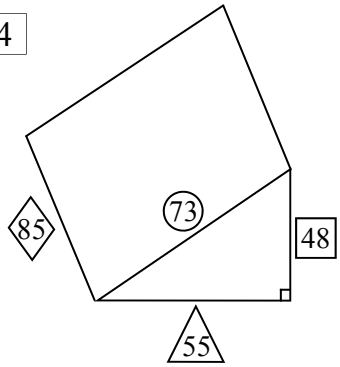


SA = × × △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____

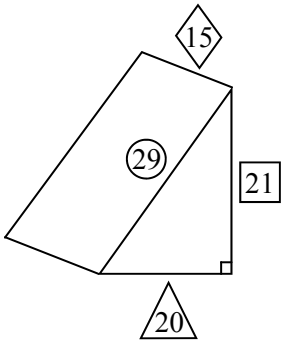
5 SA = × × △ × □

\+ △ × ◊

\+ □ × ◊

\+ ○ × ◊

SA = _____



6 SA = 2×½× △ × □

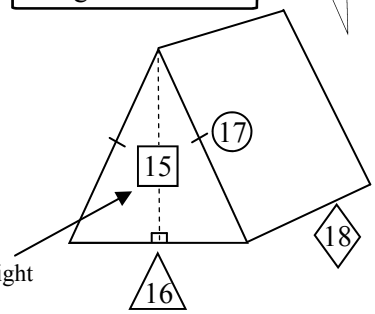
\+ △ × ◊

\+2 × ○ × ◊

SA = _____ cm²

15 is the perpendicular height

These are isosceles triangles. Careful!



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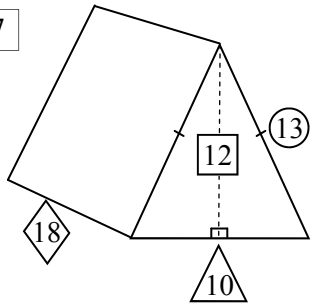


SA = × × △ × □

\+ △ × ◊

\+2 × ○ × ◊

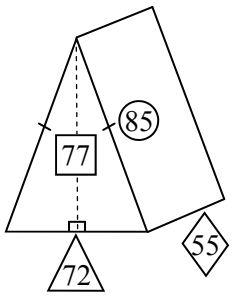
SA = _____

8 SA = × × △ × □

\+ △ × ◊

\+ × ○ × ◊

SA = _____



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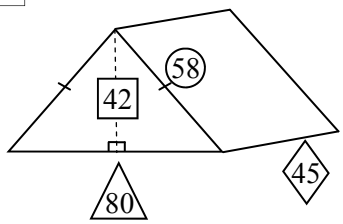


SA = × × △ × □

\+ △ × ◊

\+ × ○ × ◊

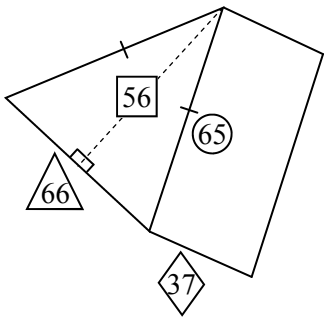
SA = _____

10 SA = × × △ × □

\+ △ × ◊

\+ × ○ × ◊

SA = _____



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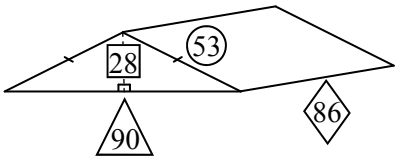

SA = × × △ × □

\+ △ × ◊

\+ × ○ × ◊

SA = _____

Method Sheet - Surface Area of Triangular Prisms

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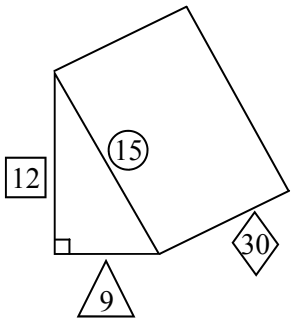
Use the Δ, □, ◊ and ○ to help you to find the SA of these triangular prisms. All measurements are in cm.

1

SA = 2×½× △ 9 × □ 12
+ △ 9 × ◊ 30
+ □ 12 × ◊ 30
+ ○ 15 × ◊ 30
SA = 1 188 cm²



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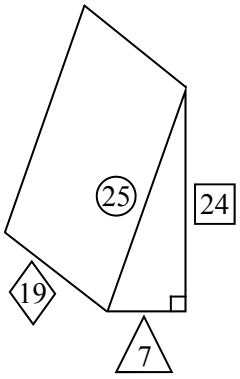
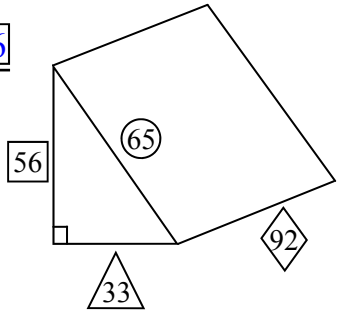


SA = 2×½× △ 7 × □ 24
+ △ 7 × ◊ 19
+ □ 24 × ◊ 19
+ ○ 25 × ◊ 19
SA = 1 232 cm²

3

SA = 2×½× △ 33 × □ 56
+ △ 33 × ◊ 92
+ □ 56 × ◊ 92
+ ○ 65 × ◊ 92
SA = 16 016 cm²



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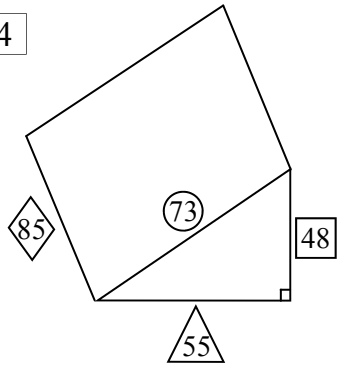
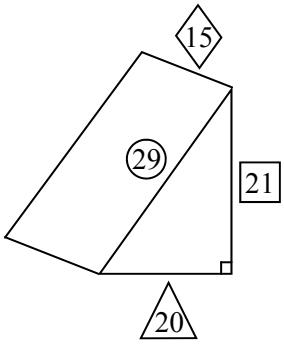


SA = 2×½× △ 55 × □ 48
+ △ 55 × ◊ 85
+ □ 48 × ◊ 85
+ ○ 73 × ◊ 85
SA = 17 600 cm²

5

SA = 2×½× △ 20 × □ 21
+ △ 20 × ◊ 15
+ □ 21 × ◊ 15
+ ○ 29 × ◊ 15
SA = 1 470 cm²

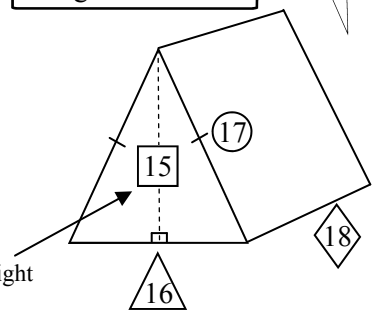


6

SA = 2×½× △ 16 × □ 15
+ △ 16 × ◊ 18
+2 × ○ 17 × ◊ 18
SA = 1 140 cm²

□ 15 is the perpendicular height

These are isosceles triangles. Careful!



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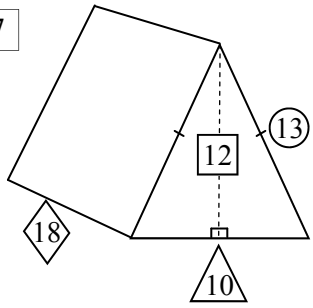
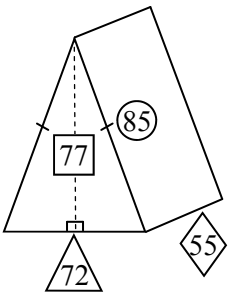


SA = 2×½× △ 10 × □ 12
+ △ 10 × ◊ 18
+2 × ○ 13 × ◊ 18
SA = 768 cm²

8

SA = 2×½× △ 72 × □ 77
+ △ 72 × ◊ 55
+2 × ○ 85 × ◊ 55
SA = 18 854 cm²



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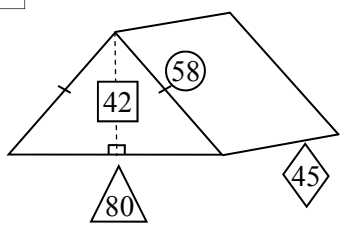
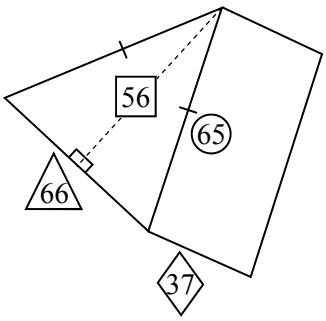


SA = 2×½× △ 80 × □ 42
+ △ 80 × ◊ 45
+2 × ○ 58 × ◊ 45
SA = 12 180 cm²

10

SA = 2×½× △ 66 × □ 56
+ △ 66 × ◊ 37
+2 × ○ 65 × ◊ 37
SA = 10 948 cm²



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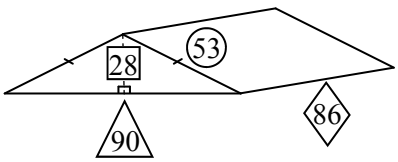


SA = 2×½× △ 90 × □ 28
+ △ 90 × ◊ 86
+2 × ○ 53 × ◊ 86
SA = 19 376 cm²

Stepped Procedure - Surface Area of Triangular Prisms

When you calculate the Surface Area of a Triangular Prism you are adding 5 areas together. There are the two triangular ends of the solid, then there are the three rectangles that wrap around the solid.

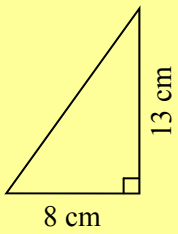
This sheet takes you through the procedure in parts to assist you to understand the steps involved in the process.

Column 1 deals with finding the area of a triangle. The formula for this is

$A = \frac{1}{2}bh$ (which is ½ × base length × perpendicular height of triangle)

There is an example at the top of the column, use the calculator fraction key to make the ½ or just use 0.5, which is the better choice otherwise you may get answers with fractions.

Example



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 13$

$A = 52 \text{ cm}^2$

Using 0.5 : 0 . 5 × 8 × 1 3 =

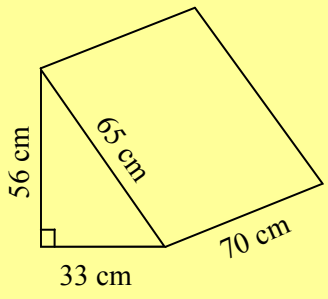
Using ½ : 1 $a^b/_c$ 2 × 8 × 1 3 = $a^b/_c$

Then use again if your answer contains a fraction.

Column 2 isn't finding the Surface Area, it is finding the area of the rectangular faces of the solid. So you are finding the area of three rectangles. Note that if the triangular face of the prism is isosceles then two of the triangles are identical. There is an example at the top of the column.

Column 3 is finding the surface area, so the two triangular faces plus the three rectangles added together. The first line is the two triangles, you should show 2×½ in the working out, but do you need to do this with the calculator? Then add the three rectangles and add get a total. Remember the square units and it is done.

Example



base and height of triangle

$SA = 2 \times \frac{1}{2} \times 33 \times 56$

$+ 33 \times 70$

$+ 56 \times 70$

$+ 65 \times 70$

Note the 'depth' of the solid is used three times

$SA = 12\,628 \text{ cm}^2$

Stepped Procedure - Surface Area of Triangular Prisms

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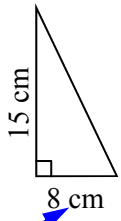
Find the area of these triangles. Use A = ½*bh*, using either 0.5 or ½.

Example

A = ½*bh*

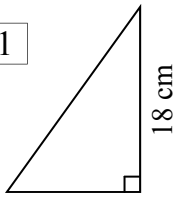
= ½×8×15

A = 60 cm²



units² must match

1

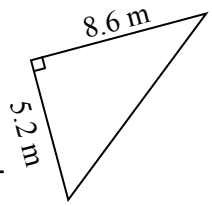


A = ½*bh*

= ½× □ × □

A = cm²

2

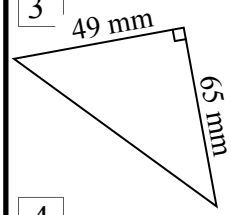


A = ½*bh*

= ½×

A =

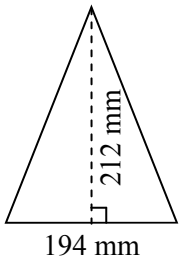
3



A =

A =

4



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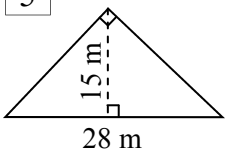

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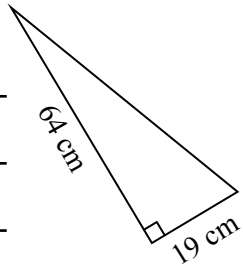


Find the rectangular wall surface area for these triangular prisms.

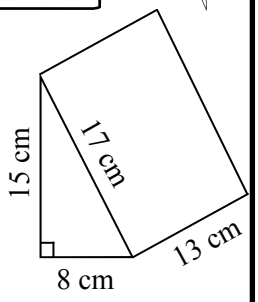
Example

SA = 8 × 13

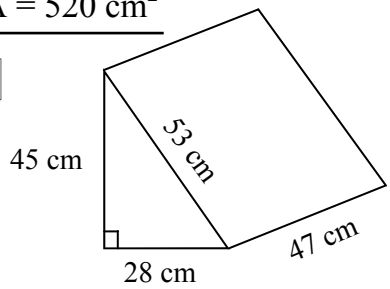
+ 15 × 13

+ 17 × 13

SA = 520 cm²



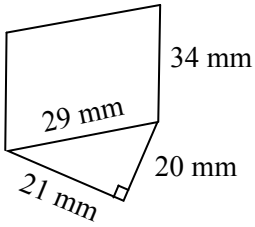
7



SA = □×□+□×□+□×□

SA = cm²

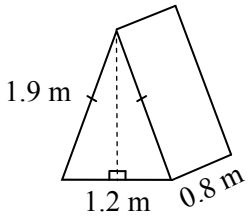
8



SA = □×□+□×□+□×□

SA =

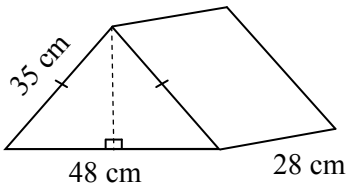
9



SA = □×□+ 2 × □×□

SA =

10

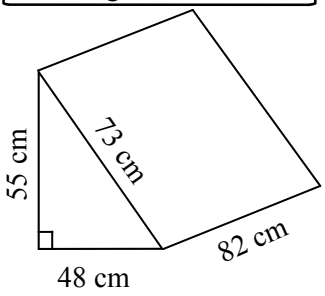


SA =

SA =

Now add two triangular faces and the rectangular walls to get the total SA.

11



SA = 2 × ½ × □ × □

+ □ × □

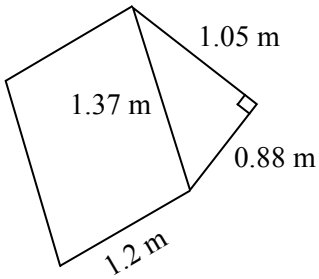
+ □ × □

+ □ × □

base and height of triangle

SA =

12



SA = 2 × ½ × □ × □

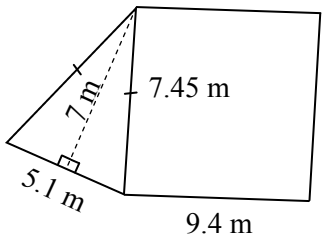
+ □ × □

+ □ × □

+ □ × □

SA =

13



SA = 2 × ½ × □ × □

+ □ × □

+ 2 × □ × □

SA =

Stepped Procedure - Surface Area of Triangular Prisms

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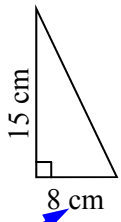
Find the area of these triangles. Use A = ½*bh*, using either 0.5 or ½.

Example

$A = \frac{1}{2}bh$

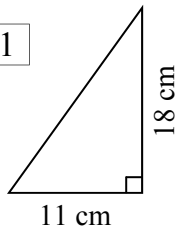
$= \frac{1}{2} \times 8 \times 15$

$A = 60 \text{ cm}^2$



units² must match

1

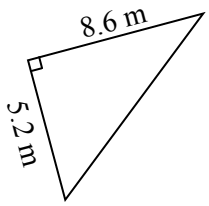


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times \boxed{11} \times \boxed{18}$

$A = 99 \text{ cm}^2$

2

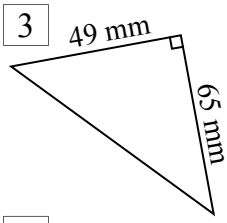


$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 5.2 \times 8.6$

$A = 22.36 \text{ m}^2$

3



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 49 \times 65$

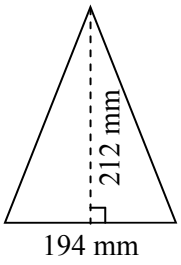
$A = 1\,592.5 \text{ mm}^2$

4

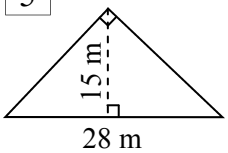
$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 194 \times 212$

$A = 20\,564 \text{ mm}^2$



5



$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 28 \times 15$

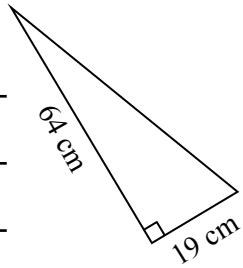
$A = 210 \text{ m}^2$

6

$A = \frac{1}{2}bh$

$= \frac{1}{2} \times 19 \times 64$

$A = 608 \text{ cm}^2$



Find the rectangular wall surface area for these triangular prisms.

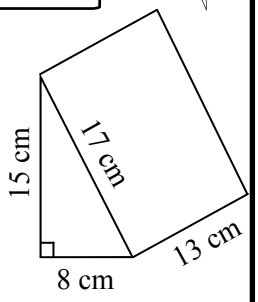
Example

$SA = 8 \times 13$

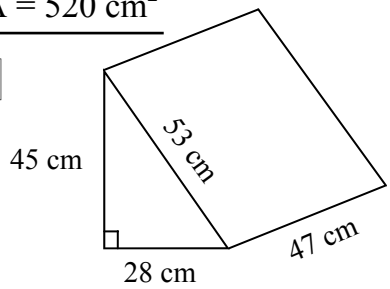
$+ 15 \times 13$

$+ 17 \times 13$

$SA = 520 \text{ cm}^2$



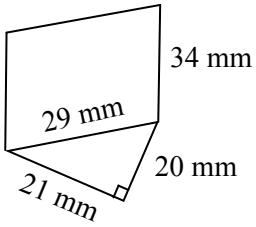
7



$SA = \boxed{28} \times \boxed{47} + \boxed{45} \times \boxed{47} + \boxed{53} \times \boxed{47}$

$SA = 5\,922 \text{ cm}^2$

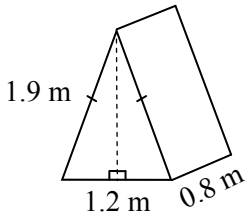
8



$SA = \boxed{20} \times \boxed{34} + \boxed{21} \times \boxed{34} + \boxed{29} \times \boxed{34}$

$SA = 2\,380 \text{ mm}^2$

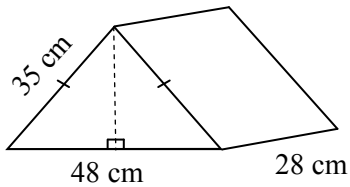
9



$SA = \boxed{1.2} \times \boxed{0.8} + 2 \times \boxed{1.9} \times \boxed{0.8}$

$SA = 4 \text{ m}^2$

10

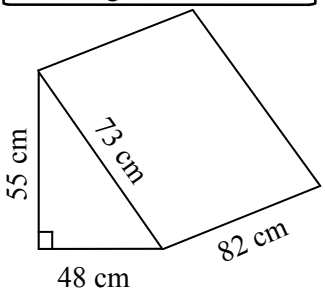


$SA = 48 \times 28 + 2 \times 35 \times 28$

$SA = 3\,304 \text{ cm}^2$

Now add two triangular faces and the rectangular walls to get the total SA.

11



$SA = 2 \times \frac{1}{2} \times \boxed{48} \times \boxed{55}$

$+ \boxed{48} \times \boxed{82}$

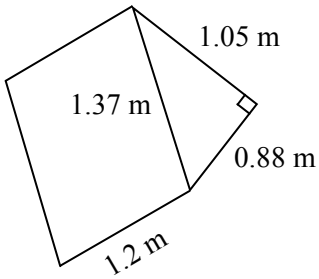
$+ \boxed{55} \times \boxed{82}$

$+ \boxed{73} \times \boxed{82}$

base and height of triangle

$SA = 17\,072 \text{ cm}^2$

12



$SA = 2 \times \frac{1}{2} \times \boxed{0.88} \times \boxed{1.05}$

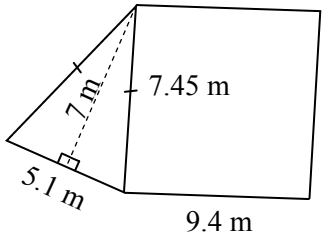
$+ \boxed{0.88} \times \boxed{1.2}$

$+ \boxed{1.37} \times \boxed{1.2}$

$+ \boxed{1.05} \times \boxed{1.2}$

$SA = 4.884 \text{ m}^2$

13



$SA = 2 \times \frac{1}{2} \times \boxed{5.1} \times \boxed{7}$

$+ \boxed{5.1} \times \boxed{9.4}$

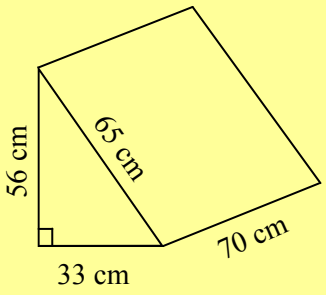
$+ 2 \times \boxed{7.45} \times \boxed{9.4}$

$SA = 223.7 \text{ m}^2$

Surface Area - Triangular Prisms

To calculate the surface area of a solid, the area of all the faces of the solid are calculated and added together. So with a triangular prism there are two triangular ends and three rectangles around the middle. If the triangular end is isosceles then two of the rectangles are the same size, the next sheet will deal with triangular prisms that feature equilateral triangle faces.

Example 1 - Scalene Triangular face



base and height of triangle

$SA = 2 \times \frac{1}{2} \times 33 \times 56$

$+ 33 \times 70$

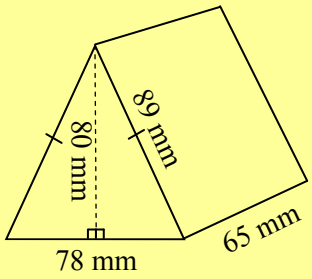
$+ 56 \times 70$

$+ 65 \times 70$

Note the 'depth' of the solid is used three times

$SA = 12\,628 \text{ cm}^2$

Example 2 - Isosceles Triangular face



base and height of triangle

$SA = 2 \times \frac{1}{2} \times 78 \times 80$

$+ 78 \times 65$

$+ 2 \times 89 \times 65$

Note the 'depth' of the solid is used three times, but two of the areas are the same

$SA = 12\,628 \text{ mm}^2$

The sheet is attempted as above for all questions, except for the last two questions being problems that require some additional steps.

Surface Area - Triangular Prisms

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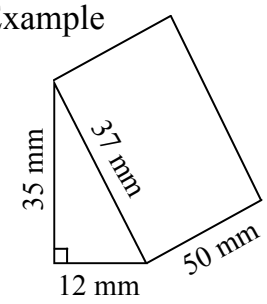
Find the surface area of these triangular prisms.

Example



$SA = 2 \times \frac{1}{2} \times 12 \times 35$

$+ 12 \times 50$

$+ 35 \times 50$

$+ 37 \times 50$

$SA = 4\,620 \text{ mm}^2$

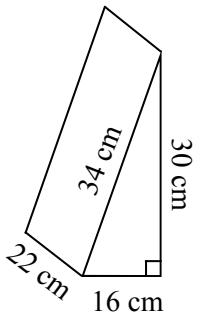
1 $SA = 2 \times \frac{1}{2} \times \square \times \square$

$+ \square \times \square$

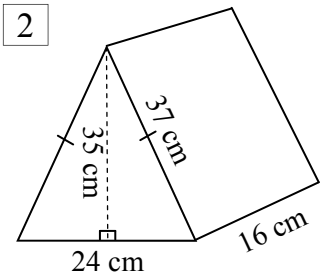
$+ \square \times \square$

$+ \square \times \square$

$SA =$ cm^2



2



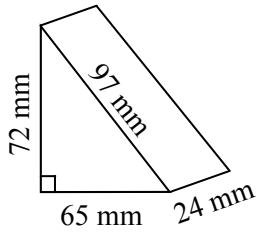
$SA = 2 \times \frac{1}{2} \times \square \times \square$

$+ \square \times \square$

$+ 2 \times \square \times \square$

$SA =$

3



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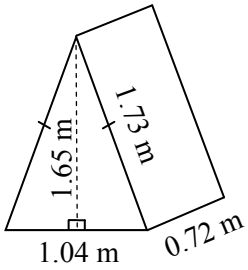

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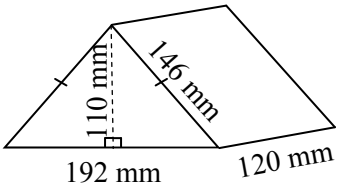

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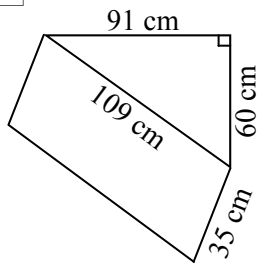
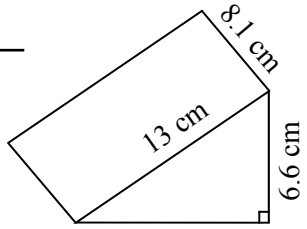
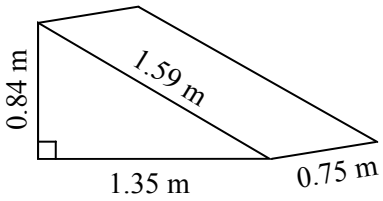


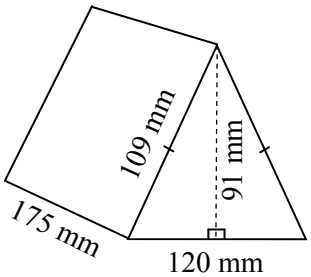
7



8 A skateboard ramp is to be constructed from plate steel. If steel costs \$28.20 per m^2, find the cost to build the skate ramp shown. The ramp has a steel base (no missing sides) and assume metal is supplied cut to size and charged to nearest m^2.



9 A chocolate box in the shape of a triangular prism is shown below. If the wrapper has a white area of 14 000 mm^2, with the rest being purple, calculate the percentage of the wrapper area that is purple. Answer to the nearest whole number.



Surface Area - Triangular Prisms

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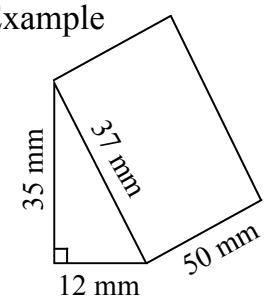
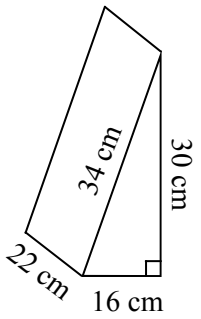
Find the surface area of these triangular prisms.

Example

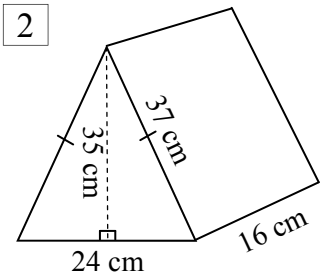


SA = 2 × ½ × 12 × 35
+ 12 × 50
+ 35 × 50
+ 37 × 50
SA = 4 620 mm^2

1 SA = 2 × ½ × 16 × 30
+ 16 × 22
+ 34 × 22
+ 30 × 22
SA = 2 240 cm^2

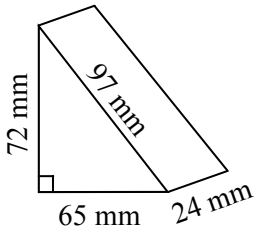


2

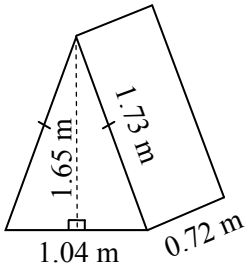


SA = 2 × ½ × 24 × 35
+ 24 × 16
+ 2 × 37 × 16
SA = 2 408 cm^2

3 SA = 2 × ½ × 65 × 72
+ 65 × 24
+ 72 × 24
+ 97 × 24
SA = 10 296 mm^2

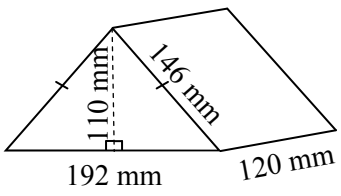


4

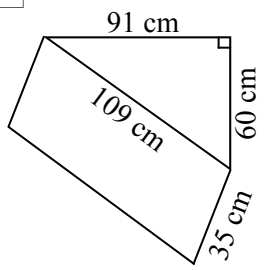


SA = 2×½× 1.04 × 1.65
+ 1.04 × 0.72
+2 × 1.73 × 0.72
SA = 4.956 m^2

5 SA = 2× ½ × 192 × 110
+ 192 × 120
+ 2 × 146 × 120
SA = 79 200 mm^2

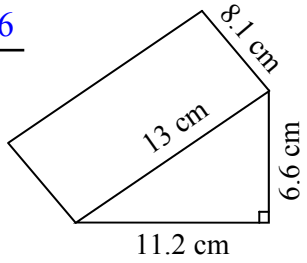


6



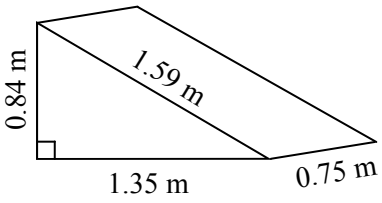
SA = 2 × ½ × 60 × 91
+ 60 × 35
+ 109 × 35
+ 91 × 35
SA = 14 560 cm^2

7 SA = 2× ½ × 11.2 × 6.6
+ 6.6 × 8.1
+ 11.2 × 8.1
+ 13 × 8.1
SA = 323.4 cm^2



8 A skateboard ramp is to be constructed from plate steel. If steel costs $28.20 per m^2, find the cost to build the skate ramp shown. The ramp has a steel base (no missing sides) and assume metal is supplied cut to size and charged to nearest m^2.

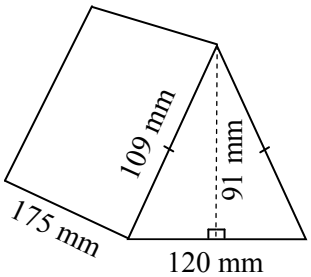
SA = 2×½×1.35×0.84
+ 0.84 × 0.75
+ 1.35 × 0.75
+ 1.59 × 0.75
SA = 3.969 m^2



Cost = 4 × 28.20 = $112.80

9 A chocolate box in the shape of a triangular prism is shown below. If the wrapper has a white area of 14 000 mm^2, with the rest being purple, calculate the percentage of the wrapper area that is purple. Answer to the nearest whole number.

SA = 2 × ½ × 120 × 91
+ 120 × 175
+ 2 × 109 × 175
SA = 70 070 mm^2



Purple area = 70 070 - 14 000 = 56 070 mm^2

$\frac{56\,070}{70\,070}$ × 100 = 80% (nearest whole number)

Surface Area - Triangular Prisms using Pythagoras Theorem

This sheet requires the ability to perform Pythagoras Theorem, if you haven't learned P. T. then don't attempt this worksheet. This worksheet covers finding the surface area of a triangular prism but you need to find either a missing side length or the perpendicular height first.

Column 1 has an example, note that this uses P. T. addition, some questions will use subtraction. This column features questions that don't require rounding, from Column 2 rounding to 2 d. p. is required.

From Column 2 units may have to be converted. Some questions have units used that are different from the square units required in the final answer. So change the measurements **FIRST**, before you perform any calculations.

Surface Area - Triangular Prisms using Pythagoras Theorem

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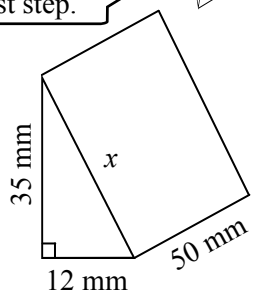
Find the surface area of these triangular prisms. Pythagoras Theorem is required as a first step.

Example



$x^2 = 12^2 + 35^2$

$x^2 = 1\ 369$

$x = \sqrt{1\ 369}$

$x = 37$ mm

SA = 2 × ½ × 12 × 35

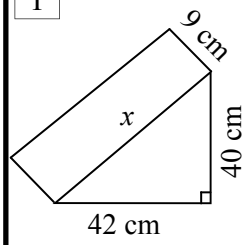
+ 12 × 50

+ 35 × 50

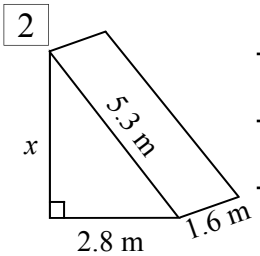
+ 37 × 50

SA = 4 620 mm^2

1

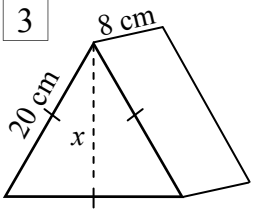


2

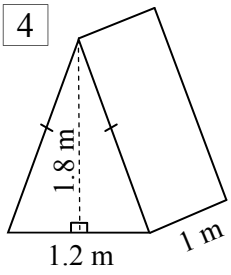


Round to 2 d.p when required.

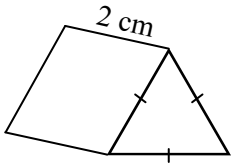
3



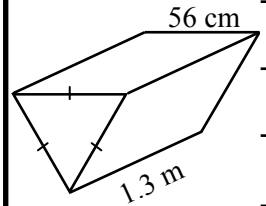
4



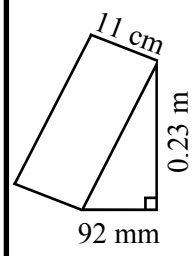
5 *Answer in mm^2



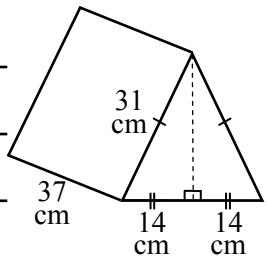
6 *Answer in m^2



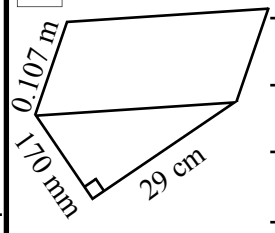
7 *Answer in cm^2



8



9 *Answer in cm^2



Surface Area - Triangular Prisms using Pythagoras Theorem

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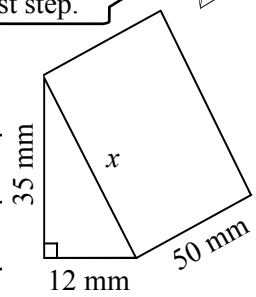
Find the surface area of these triangular prisms. Pythagoras Theorem is required as a first step.

Example



$x^2 = 12^2 + 35^2$

$x^2 = 1\ 369$

$x = \sqrt{1\ 369}$

$x = 37$ mm

SA = 2 × ½ × 12 × 35

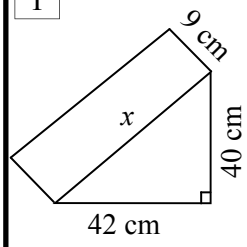
+ 12 × 50

+ 35 × 50

+ 37 × 50

SA = 4 620 mm^2

1



$x^2 = 42^2 + 40^2$

$x^2 = 3\ 364$

$x = \sqrt{3\ 364}$

$x = 58$ cm

SA = 2 × ½ × 42 × 40

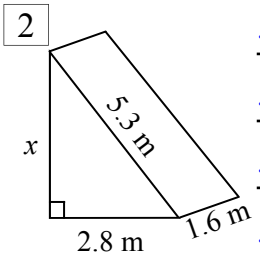
+ 42 × 9

+ 58 × 9

+ 40 × 9

SA = 2 940 cm^2

2



$x^2 = 5.3^2 - 2.8^2$

$x^2 = 20.25$

$x = \sqrt{20.25}$

$x = 4.5$ m

SA = 2 × ½ × 2.8 × 4.5

+ 2.8 × 1.6

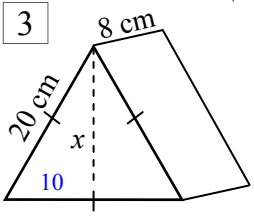
+ 4.5 × 1.6

+ 5.3 × 1.6

SA = 32.76 m^2

Round to 2 d.p when required.

3



$x^2 = 20^2 - 10^2$

$x^2 = 300$

$x = \sqrt{300}$

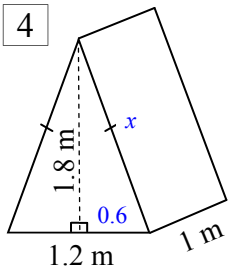
$x = 17.32$ cm

SA = 2 × ½ × 20 × 17.32

+ 3 × 20 × 8

SA = 826.4 cm^2

4



$x^2 = 1.8^2 - 0.6^2$

$x^2 = 2.88$

$x = \sqrt{2.88}$

$x = 1.70$ m

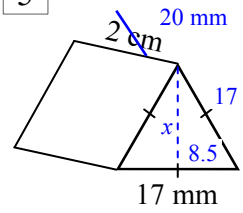
SA = 2 × ½ × 1.2 × 1.8

+ 1.2 × 1

+ 2 × 1.7 × 1

SA = 6.76 m^2

5 *Answer in mm^2



$x^2 = 17^2 - 8.5^2$

$x^2 = 216.75$

$x = \sqrt{216.75}$

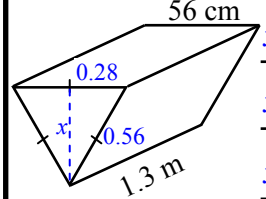
$x = 14.72$ mm

SA = 2 × ½ × 17 × 14.72

+ 3 × 17 × 20

SA = 1 270.24 mm^2

6 *Answer in m^2



$x^2 = 0.56^2 - 0.28^2$

$x^2 = 0.2352$

$x = \sqrt{0.2352}$

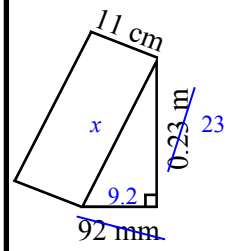
$x = 0.48$ m (or 48.50 cm)

SA = 2 × ½ × 0.56 × 0.48

+ 3 × 0.56 × 1.3

SA = 2.45 m^2 or 2.46 m^2 (if 0.485 m used)

7 *Answer in cm^2



$x^2 = 9.2^2 + 23^2$

$x^2 = 613.64$

$x = \sqrt{613.64}$

$x = 24.77$ cm

SA = 2 × ½ × 9.2 × 23

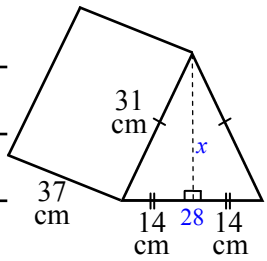
+ 9.2 × 11

+ 23 × 11

+ 24.77 × 11

SA = 838.27 cm^2

8



$x^2 = 31^2 - 14^2$

$x^2 = 765$

$x = \sqrt{765}$

$x = 27.66$ cm

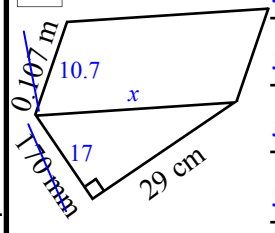
SA = 2 × ½ × 28 × 27.66

+ 28 × 37

+ 2 × 31 × 37

SA = 4 104.48 cm^2

9 *Answer in cm^2



$x^2 = 17^2 + 29^2$

$x^2 = 1\ 130$

$x = \sqrt{1\ 130}$

$x = 33.62$ cm

SA = 2 × ½ × 29 × 17

+ 17 × 10.7

+ 33.62 × 10.7

+ 29 × 10.7

SA = 1 344.93 cm^2

Surface Area - Square & Rectangular Pyramids

A Square (based) Pyramid consists of its square base and four identical triangles. So to find the S.A. of a Square Pyramid:

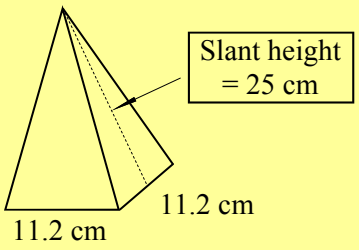
$$SA = l^2 + 4 \times \tfrac{1}{2}bh$$

where:

l = side length of the square base
b = the base length of the triangles (which is equal to l)
h = the perpendicular height of the triangles (which is called the slant height)

You don't have to remember the formula, just remember to add the area of the square and the four triangles together.

Example



4 × ½ = 2 Quicker

$SA = 11.2^2 + 4 \times \tfrac{1}{2} \times 11.2 \times 25$

$SA = 685.44 \text{ cm}^2$

Square units

Calculator Steps: 1 1 . 2 x^2 + 4 × 0 . 5 × 1 1 . 2 × 2 5 =

Note that some questions show the net of the solid, these are calculated the same as if the net were folded up to construct the solid.

Rectangular (based) Pyramids consist of a rectangular base and two pairs of identical triangles. So to find the S.A. of a Rectangular Pyramid:

$$SA = lb + 2 \times \tfrac{1}{2}b_1h_1 + 2 \times \tfrac{1}{2}b_2h_2$$

where:

l, b = side lengths of the rectangular base
b_1 = the base length of the first of the triangles (which is equal to b)
h_1 = the perpendicular height of the first triangle (which is called the slant height)
b_2 = the base length of the second of the triangles (which is equal to l)
h_2 = the perpendicular height of the second triangle (which is called the slant height)

Would you remember the above formula? Don't bother, just realise that the SA is found by adding the rectangle with the two pairs of triangles. Question 5 has the setting out for you to follow.

Surface Area - Square & Rectangular Pyramids

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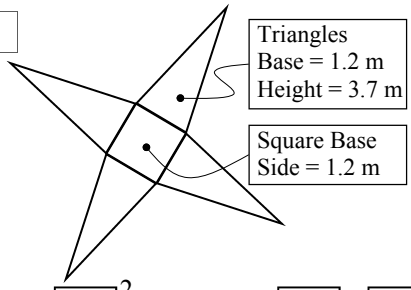
Find the surface area of the Square Pyramids first shown as nets.

1

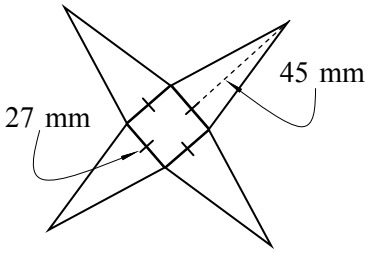


SA = $\square^2 + 4 \times \frac{1}{2} \times \square \times \square$

Area of Square + Area of 4 triangles

SA = $\qquad m^2$

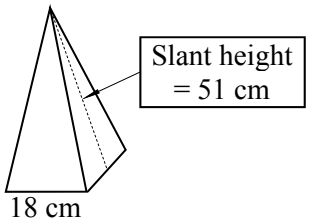
2



SA =

SA = $\qquad mm^2$

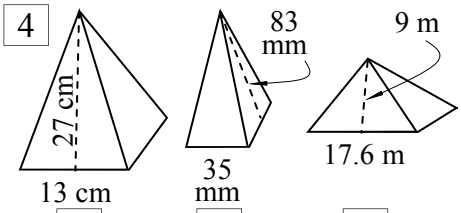
3



SA =

SA =

4



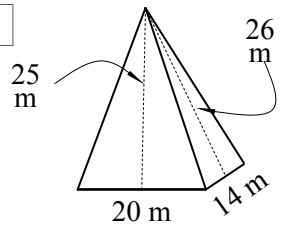
A

B

C

Rectangular pyramids have two matching pairs of triangles and a rectangular base.

5



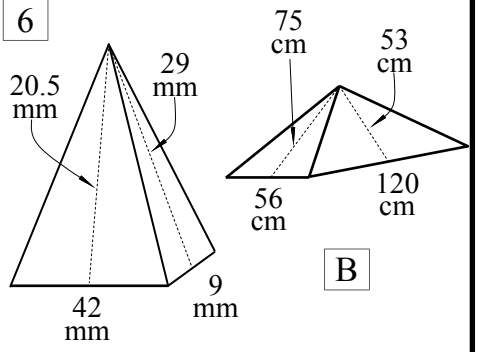
SA = $\square \times \square$ Area of Rectangular Base

$+ 2 \times \frac{1}{2} \times \square \times \square$ Area of 1st pair of Δ's

$+ 2 \times \frac{1}{2} \times \square \times \square$ Area of 2nd pair of Δ's

SA =

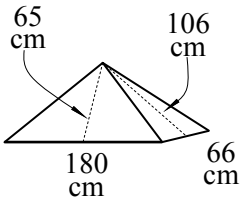
6



A

B

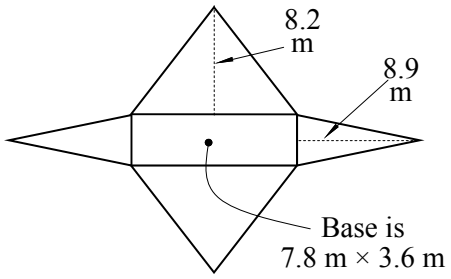
7



These are a mix of Square and Rectangular based pyramids. Find the SA.

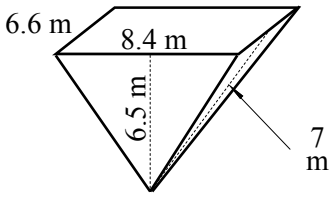
8 Square Pyramid with a slant height of 19 cm and a base side length of 8 cm.

9 Net - Find assembled SA.



10 Square Pyramid with a slant height of 46 m and a base side length of 27 m.

11 Find the cost to anodise the Aluminium seed hopper shown if the process costs \$11.35/m^2 of the external SA. (To nearest m^2)



Surface Area - Square & Rectangular Pyramids

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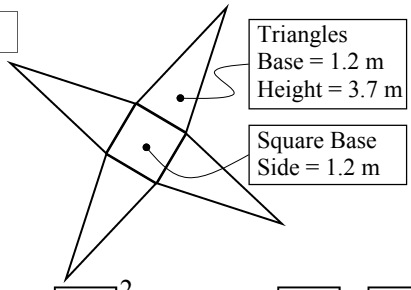
Find the surface area of the Square Pyramids first shown as nets.

1

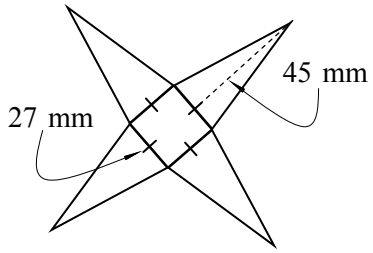


SA = $1.2^2 + 4 \times ½ \times 1.2 \times 3.7$

Area of Square + Area of 4 triangles

SA = 10.32 m^2

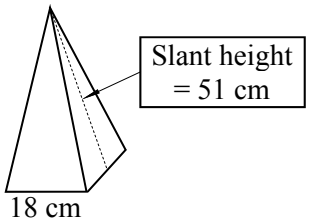
2



SA = $27^2 + 4 \times ½ \times 27 \times 45$

SA = 3 159 mm^2

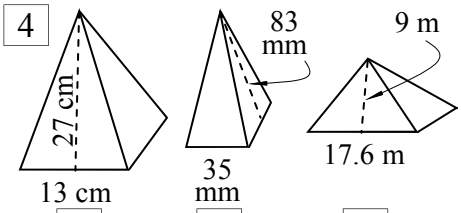
3



SA = $18^2 + 4 \times ½ \times 18 \times 51$

SA = 2 160 cm^2

4



A SA = $13^2 + 4 \times ½ \times 13 \times 27$

SA = 871 cm^2

B SA = $35^2 + 4 \times ½ \times 35 \times 83$

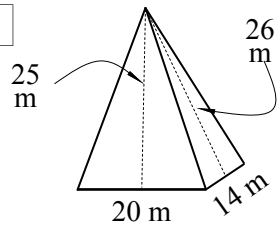
SA = 7 035 mm^2

C SA = $17.6^2 + 4 \times ½ \times 17.6 \times 9$

SA = 626.56 m^2

Rectangular pyramids have two matching pairs of triangles and a rectangular base.

5



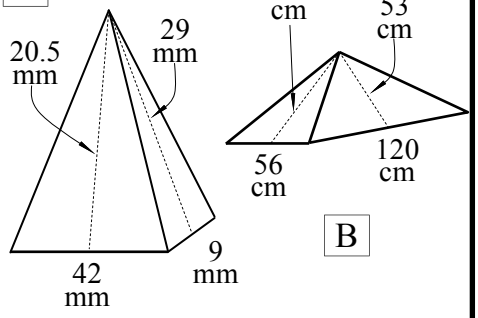
SA = 20×14 Area of Rectangular Base

$+ 2 \times ½ \times 20 \times 25$ Area of 1st pair of Δ's

$+ 2 \times ½ \times 14 \times 26$ Area of 2nd pair of Δ's

SA = 1 144 m^2

6



A SA = 42×9

$+ 2 \times ½ \times 42 \times 20.5$

$+ 2 \times ½ \times 9 \times 29$

SA = 1 500 mm^2

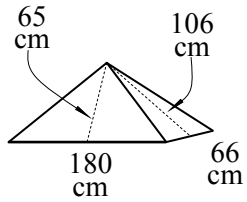
B SA = 56×120

$+ 2 \times ½ \times 56 \times 75$

$+ 2 \times ½ \times 120 \times 53$

SA = 17 280 cm^2

7



SA = 180×66

$+ 2 \times ½ \times 180 \times 65$

$+ 2 \times ½ \times 66 \times 106$

SA = 30 576 cm^2

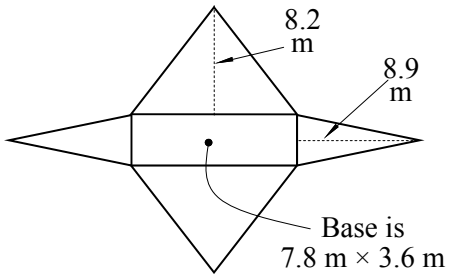
These are a mix of Square and Rectangular based pyramids. Find the SA.

8 Square Pyramid with a slant height of 19 cm and a base side length of 8 cm.

SA = $8^2 + 4 \times ½ \times 8 \times 19$

SA = 368 cm^2

9 Net - Find assembled SA.



SA = 7.8×3.6

$+ 2 \times ½ \times 3.6 \times 8.9$

$+ 2 \times ½ \times 7.8 \times 8.2$

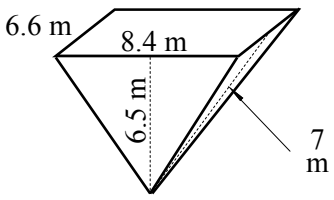
SA = 124.08 m^2

10 Square Pyramid with a slant height of 46 m and a base side length of 27 m.

SA = $27^2 + 4 \times ½ \times 27 \times 46$

SA = 3 213 m^2

11 Find the cost to anodise the Aluminium seed hopper shown if the process costs \$11.35/m^2 of the external SA. (To nearest m^2)



SA = 6.6×8.4

$+ 2 \times ½ \times 6.6 \times 7$

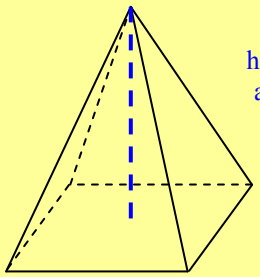
$+ 2 \times ½ \times 8.4 \times 6.5$

SA = 156.24 m^2 (156 nearest m^2)

Cost = 156×11.35 = \$1 770.60

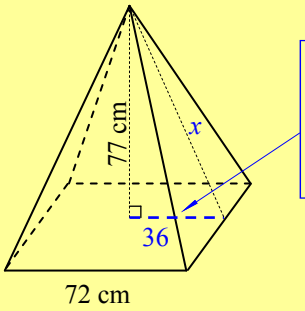
Surface Area - Square Pyramids - Using Pythagoras

This sheet should be attempted after the previous sheet has been completed. It deals with finding the surface area of a Square Pyramid but rather than being given the slant height you are given the altitude of the pyramid. The altitude is the vertical height of the pyramid, from the centre of the base to the apex.



The method is the same as the previous sheet, except Pythagoras Theorem has to be applied first to obtain the slant height.

Example - Find the Surface Area of the Square Pyramid.



Note that this length is half the width of the square base.

$72 \div 2 = 36$ cm

Find the slant height first

$x^2 = 36^2 + 77^2$

$x^2 = 7\,225$

$x = \sqrt{7\,225}$

$x = 85$ cm

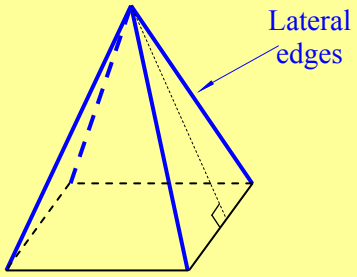
Then calculate the surface area

$SA = 72^2 + 4 \times \frac{1}{2} \times 72 \times 85$

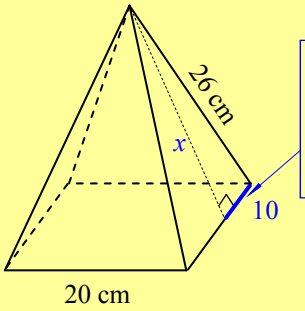
square + 4 triangles

$SA = 17\,424$ cm^2

Another length you could be given is the length along the 'lateral edge'. These are shown in blue on the diagram (right). If you are given this length then you need to use Pythagoras Theorem to find the slant height, but you will subtract when using Pythagoras Theorem.



Example - Find the Surface Area of the Square Pyramid.



Note that this length is half the width of the square base.

$20 \div 2 = 10$ cm

Find the slant height first

$x^2 = 26^2 - 10^2$

$x^2 = 576$

$x = \sqrt{576}$

$x = 24$ cm

Then calculate the surface area

$SA = 20^2 + 4 \times \frac{1}{2} \times 20 \times 24$

square + 4 triangles

$SA = 17\,424$ cm^2

Surface Area - Square Pyramids - Using Pythagoras

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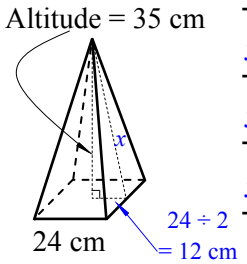
Given the height (altitude) of these square pyramids use P. T. to calculate the slant height then the SA.

Example



$x^2 = 12^2 + 35^2$

$x^2 = 1\ 369$

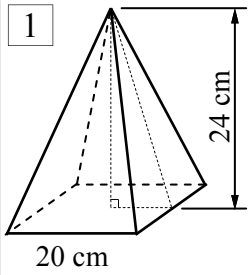
$x = \sqrt{1\ 369}$

$x = 37$ cm

$SA = 24^2 + 4 \times ½ \times 24 \times 37$

$SA = 2\ 352\ cm^2$

1



$x^2 =$

$x^2 =$

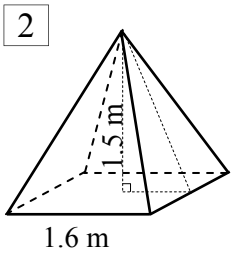
$x =$

$x =$ cm

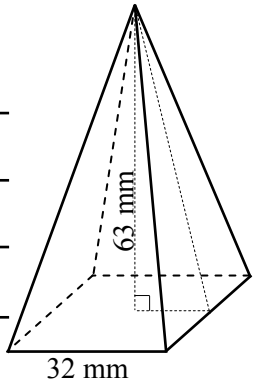
SA =

SA =

2

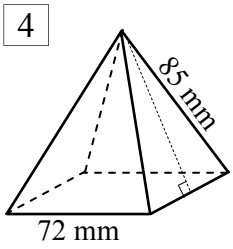


3

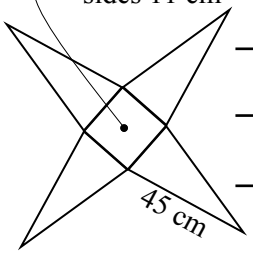


Now you are given the length along the lateral edge. Use P.T ***subtraction***, find the SA and round to 1 d.p. if needed.

4

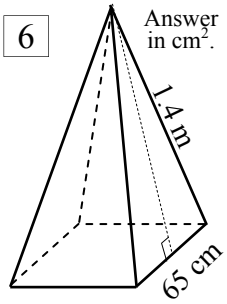


5

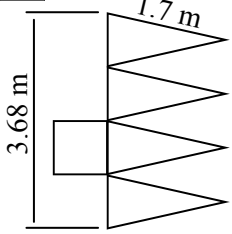


Find SA when the solid is assembled.

6



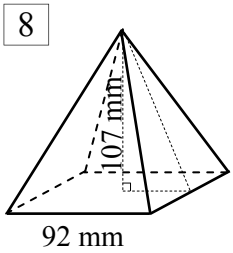
7



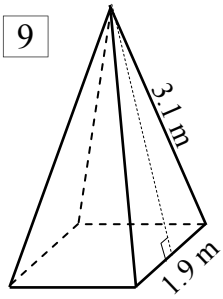
Find SA when the solid is assembled.

These are mixed, find the slant height and then the SA to 1 decimal place.

8



9



10 The Pyramid of Khufu has a square base with side lengths of 230 m. It has a height (attitude) of 138 m. Estimate the SA of its 4 exposed triangular faces. (m^2)

Sketch the pyramid, showing measurements

Note that the SA is an estimation because the surface of the faces are stepped rather than flat

Surface Area - Square Pyramids - Using Pythagoras

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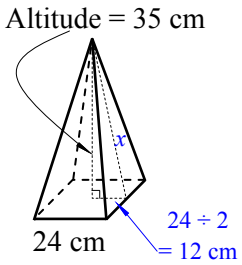
Given the height (altitude) of these square pyramids use P. T. to calculate the slant height then the SA.

Example



$x^2 = 12^2 + 35^2$

$x^2 = 1\ 369$

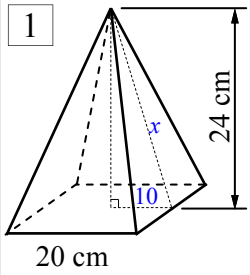
$x = \sqrt{1\ 369}$

$x = 37$ cm

$SA = 24^2 + 4 \times ½ \times 24 \times 37$

$SA = 2\ 352$ cm^2

1



$x^2 = 24^2 + 10^2$

$x^2 = 676$

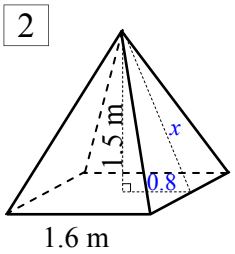
$x = \sqrt{676}$

$x = 26$ cm

$SA = 20^2 + 4 \times ½ \times 20 \times 26$

$SA = 1\ 440$ cm^2

2



$x^2 = 0.8^2 + 1.5^2$

$x^2 = 2.89$

$x = \sqrt{2.89}$

$x = 1.7$ m

$SA = 1.6^2 + 4 \times ½ \times 1.6 \times 1.7$

$SA = 8$ m^2

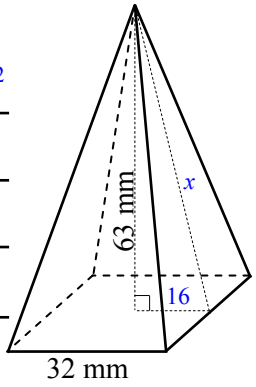
3

$x^2 = 16^2 + 63^2$

$x^2 = 4\ 225$

$x = \sqrt{4\ 225}$

$x = 65$ mm

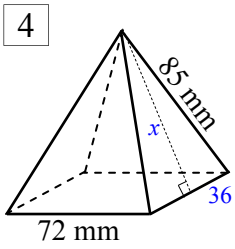


$SA = 32^2 + 4 \times ½ \times 32 \times 65$

$SA = 5\ 184$ mm^2

Now you are given the length along the lateral edge. Use P.T ***subtraction***, find the SA and round to 1 d.p. if needed.

4



$x^2 = 85^2 - 36^2$

$x^2 = 5\ 929$

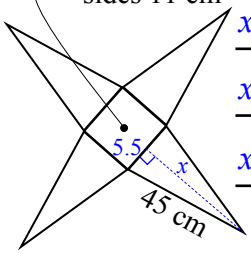
$x = \sqrt{5\ 929}$

$x = 77$ mm

$SA = 72^2 + 4 \times ½ \times 72 \times 77$

$SA = 16\ 272$ mm^2

5



$x^2 = 45^2 - 5.5^2$

$x^2 = 1\ 994.75$

$x = \sqrt{1\ 994.75}$

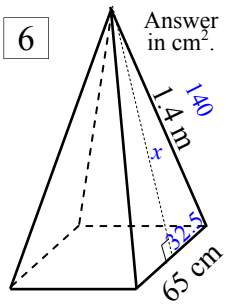
$x = 44.7$ cm

Find SA when the solid is assembled.

$SA = 11^2 + 4 \times ½ \times 11 \times 44.7$

$SA = 1\ 104.4$ cm^2

6



$x^2 = 140^2 - 32.5^2$

$x^2 = 18\ 543.75$

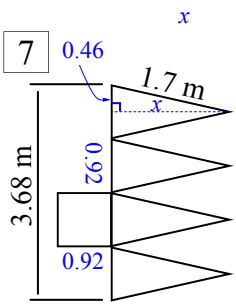
$x = \sqrt{18\ 543.75}$

$x = 136.2$ cm

$SA = 65^2 + 4 \times ½ \times 65 \times 136.2$

$SA = 21\ 931$ cm^2

7



$x^2 = 1.7^2 - 0.46^2$

$x^2 = 2.6784$

$x = \sqrt{2.6784}$

$x = 1.6$ m

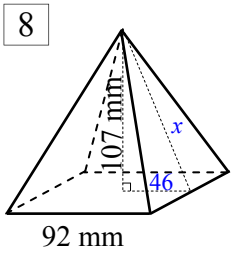
Find SA when the solid is assembled.

$SA = 0.92^2 + 4 \times ½ \times 0.92 \times 1.6$

$SA = 3.8$ m^2

These are mixed, find the slant height and then the SA to 1 decimal place.

8



$x^2 = 107^2 + 46^2$

$x^2 = 13\ 565$

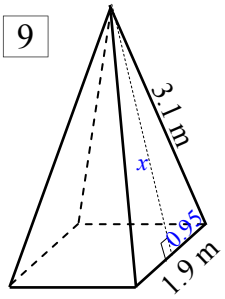
$x = \sqrt{13\ 565}$

$x = 116.5$ mm

$SA = 92^2 + 4 \times ½ \times 92 \times 116.5$

$SA = 29\ 900$ mm^2

9



$x^2 = 3.1^2 - 0.95^2$

$x^2 = 9.1875$

$x = \sqrt{9.1875}$

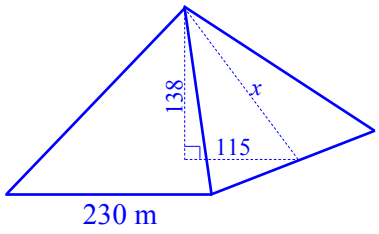
$x = 3$ m

$SA = 1.9^2 + 4 \times ½ \times 1.9 \times 3$

$SA = 15$ m^2

10 The Pyramid of Khufu has a square base with side lengths of 230 m. It has a height (attitude) of 138 m. Estimate the SA of its 4 exposed triangular faces. (m^2)

Sketch the pyramid, showing measurements



$x^2 = 138^2 + 115^2$

$x^2 = 32\ 269$

$x = \sqrt{32\ 269}$

$x = 179.6$ m

Note that the SA is an estimation because the surface of the faces are stepped rather than flat

$SA = 4 \times ½ \times 230 \times 179.6$

$SA = 82\ 616$ m^2

Surface Area - Mixed Exercises

This is a review sheet covering previous work completed, but it is mixed together.

The first worksheet has no rounding, no Pythagoras Theorem and like units in each question. There are also hints to help you get started.

The second sheet requires rounding, unit conversion and Pythagoras Theorem to be applied and contains no hints. So the second sheet is more difficult.

Surface Area - Mixed Exercises (Sheet 1)

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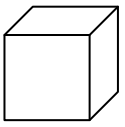
Find the surface area of these solids, make sure you show all working out and the units

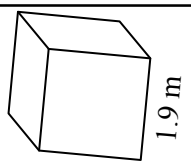
1 Cube



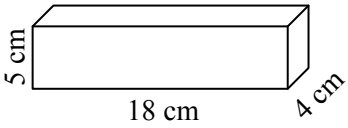
$SA = 6l^2$

$SA =$ cm^2

2 Cube



3

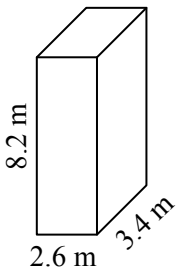


SA = 2 × ☐ × ☐

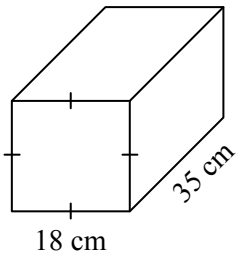
+ 2 × ☐ × ☐

+ 2 × ☐ × ☐

4



5

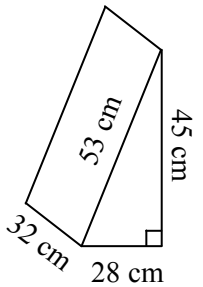


6 SA = 2 × ½ × ☐ × ☐

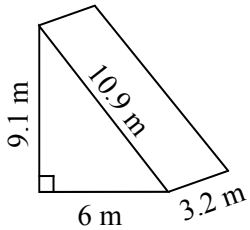
\+ ☐ × ☐

\+ ☐ × ☐

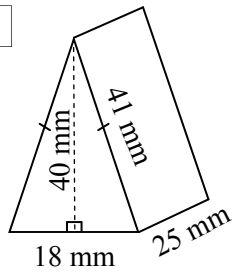
\+ ☐ × ☐



7



8

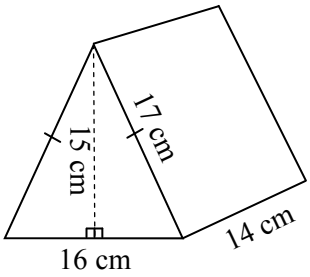


SA = 2 × ½ × ☐ × ☐

+ 2 × ☐ × ☐

\+ ☐ × ☐

9



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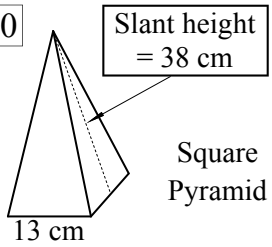
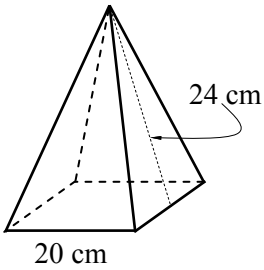


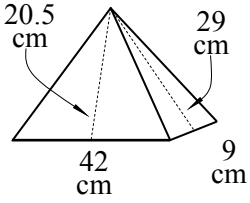
SA = ☐²

\+ 4 × ½ × ☐ × ☐

11



12 Rectangular Pyramid

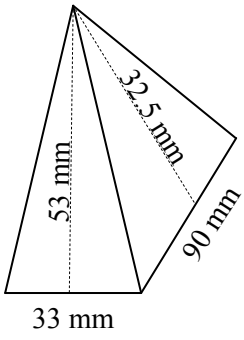


SA = ☐ × ☐

\+ 2 × ½ × ☐ × ☐

\+ 2 × ½ × ☐ × ☐

13 Rectangular Pyramid



Surface Area - Mixed Exercises (Sheet 1)

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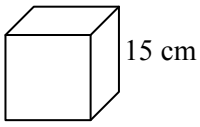
Find the surface area of these solids, make sure you show all working out and the units

Cube



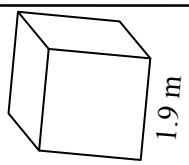
1

$SA = 6l^2$

$= 6 \times 15^2$

$SA = 1\ 350\ cm^2$

Cube



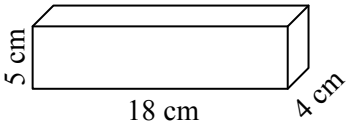
2

$SA = 6l^2$

$= 6 \times 1.9^2$

$SA = 21.66\ m^2$

3



$SA = 2 \times \boxed{5} \times \boxed{18}$

$+ 2 \times \boxed{5} \times \boxed{4}$

$+ 2 \times \boxed{18} \times \boxed{4}$

$SA = 364\ cm^2$

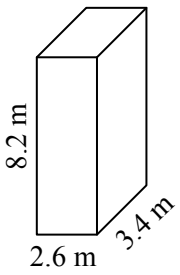
4

$SA = 2 \times 2.6 \times 8.2$

$+ 2 \times 2.6 \times 3.4$

$+ 2 \times 3.4 \times 8.2$

$SA = 116.08\ m^2$



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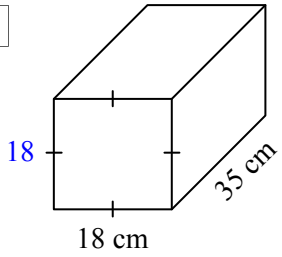


$SA = 2 \times 18 \times 18$

$+ 2 \times 18 \times 35$

$+ 2 \times 18 \times 35$

$SA = 3\ 168\ cm^2$

6

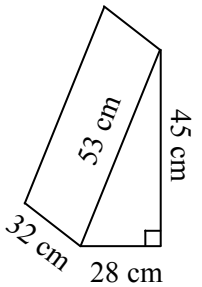
$SA = 2 \times ½ \times \boxed{28} \times \boxed{45}$

$+ \boxed{28} \times \boxed{32}$

$+ \boxed{45} \times \boxed{32}$

$+ \boxed{53} \times \boxed{32}$

$SA = 5\ 292\ cm^2$



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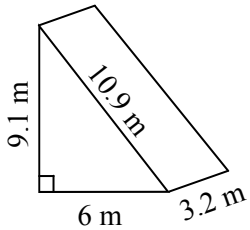


$SA = 2 \times ½ \times 6 \times 9.1$

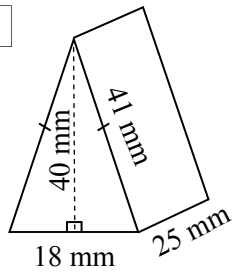
$+ 6 \times 3.2$

$+ 9.1 \times 3.2$

$+ 10.9 \times 3.2$

$SA = 137.8\ m^2$

8



$SA = 2 \times ½ \times \boxed{18} \times \boxed{40}$

$+ 2 \times \boxed{41} \times \boxed{25}$

$+ \boxed{18} \times \boxed{25}$

$SA = 3\ 220\ mm^2$

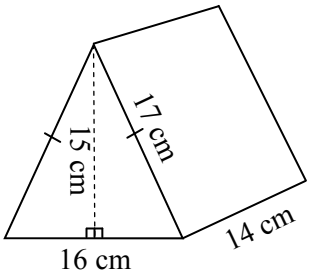
9

$SA = 2 \times ½ \times 16 \times 15$

$+ 2 \times 17 \times 14$

$+ 16 \times 14$

$SA = 940\ cm^2$



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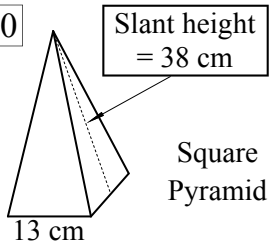


Square Pyramid

$SA = \boxed{13}^2$

$+ 4 \times ½ \times \boxed{13} \times \boxed{38}$

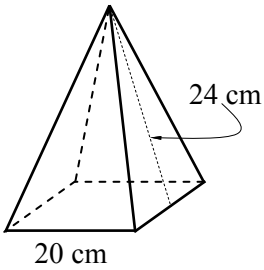
$SA = 1\ 157\ cm^2$

11

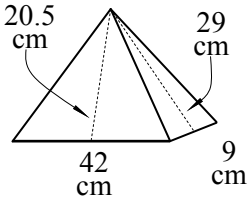
$SA = 20^2$

$+ 4 \times ½ \times 20 \times 24$

$SA = 960\ cm^2$



12 Rectangular Pyramid



$SA = \boxed{42} \times \boxed{9}$

$+ 2 \times ½ \times \boxed{42} \times \boxed{20.5}$

$+ 2 \times ½ \times \boxed{9} \times \boxed{29}$

$SA = 1\ 500\ cm^2$

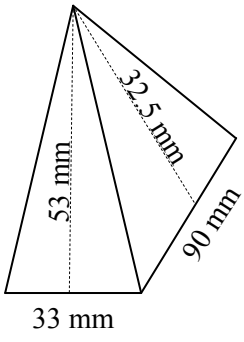
13 Rectangular Pyramid

$SA = 33 \times 90$

$+ 2 \times ½ \times 33 \times 53$

$+ 2 \times ½ \times 90 \times 32.5$

$SA = 7\ 644\ mm^2$



Surface Area - Mixed Exercises (Sheet 2)

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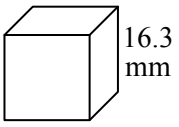
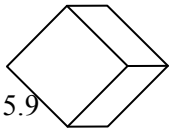
Find the surface area of these solids, round your answer to 1 decimal place.

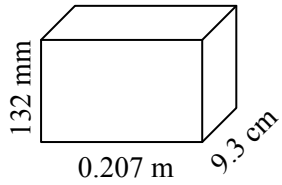
1 Cube



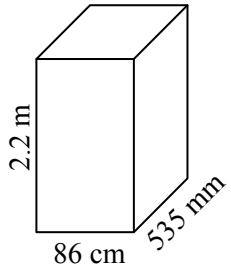
2 Cube



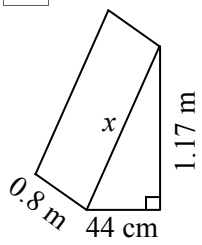
3 Answer in cm^2



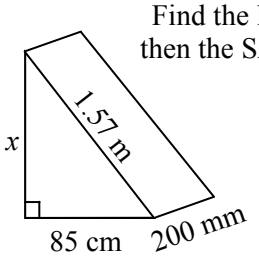
4 Answer in m^2



5 Find the length *x*, then the SA in m^2.



6 Find the length *x*, then the SA in cm^2.



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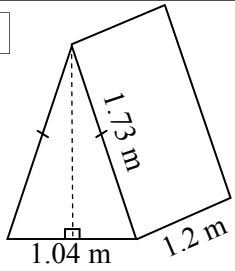

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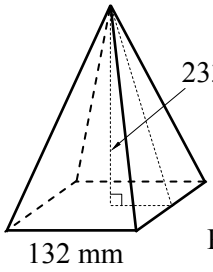

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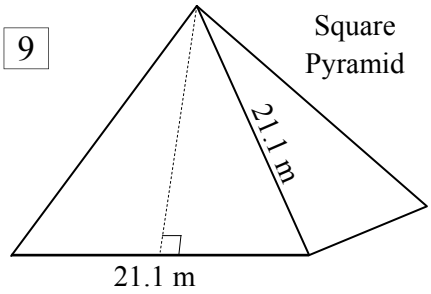

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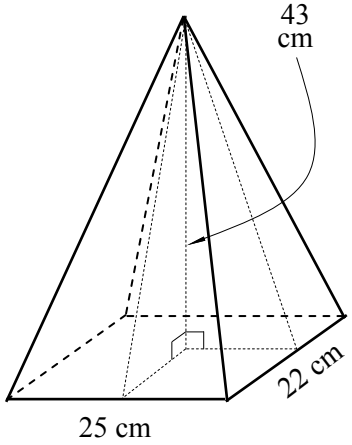

Surface Area - Mixed Exercises (Sheet 2)

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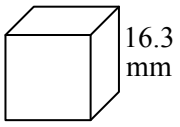
Find the surface area of these solids, round your answer to 1 decimal place.

1 Cube

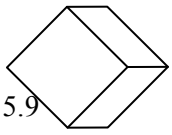


$SA = 6l^2$

$= 6 \times 16.3^2$

$SA = 1\ 594.1\ mm^2$

2 Cube

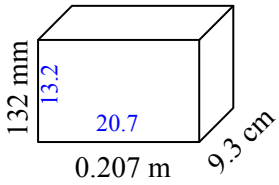


$SA = 6l^2$

$= 6 \times 5.9^2$

$SA = 208.9\ units^2$

3 Answer in cm^2



$SA = 2 \times 20.7 \times 13.2$

$+ 2 \times 20.7 \times 9.3$

$+ 2 \times 13.2 \times 9.3$

$SA = 116.1\ m^2$ (1 d.p.)

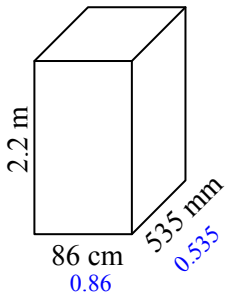
4 Answer in m^2

$SA = 2 \times 0.86 \times 2.2$

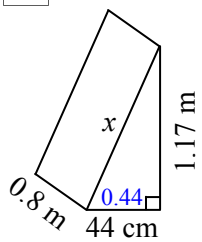
$+ 2 \times 0.86 \times 0.535$

$+ 2 \times 2.2 \times 0.535$

$SA = 7.1\ m^2$ (1 d.p.)



5 Find the length x, then the SA in m^2.



$x^2 = 0.44^2 + 1.17^2$

$x^2 = 1.5625$

$x = \sqrt{1.5625}$

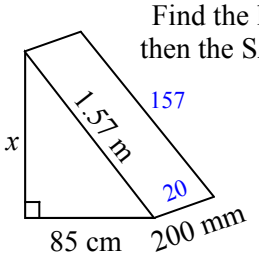
$x = 1.25$ m

$SA = 2 \times ½ \times 0.44 \times 1.17$

$+ 0.44 \times 0.8 + 1.25 \times 0.8 + 1.17 \times 0.8$

$SA = 2.8\ m^2$ (1 d.p.)

6 Find the length x, then the SA in cm^2.



$x^2 = 157^2 - 85^2$

$x^2 = 17\ 424$

$x = \sqrt{17\ 424}$

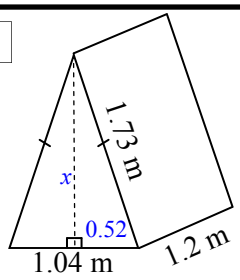
$x = 132$ cm

$SA = 2 \times ½ \times 85 \times 132$

$+ 132 \times 20 + 157 \times 20 + 85 \times 20$

$SA = 18\ 700\ cm^2$

7



$x^2 = 1.73^2 - 0.52^2$

$x^2 = 2.7225$

$x = \sqrt{2.7225}$

$x = 1.65$ m

$SA = 2 \times ½ \times 1.04 \times 1.65$

$+ 1.04 \times 1.2 + 2 \times 1.73 \times 1.2$

$SA = 7.1\ m^2$ (1 d.p.)

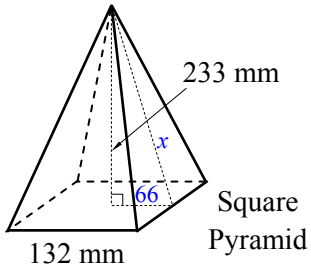
8

$x^2 = 233^2 + 66^2$

$x^2 = 58\ 645$

$x = \sqrt{58\ 645}$

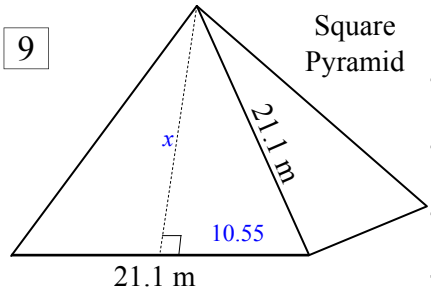
$x = 242.2$ mm (1 d.p.)



$SA = 132^2 + 4 \times ½ \times 132 \times 242.2$

$SA = 81\ 364.8\ mm^2$

9



$x^2 = 21.1^2 - 10.55^2$

$x^2 = 333.9075$

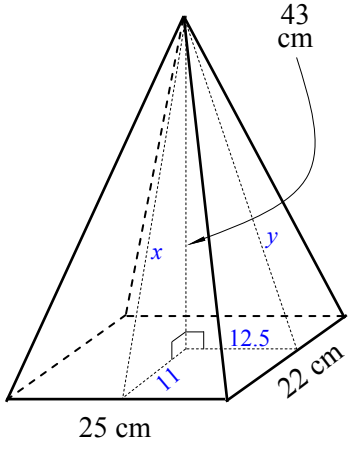
$x = \sqrt{333.9075}$

$x = 18.3$ m

$SA = 21.1^2 + 4 \times ½ \times 21.1 \times 18.3$

$SA = 1\ 217.5\ m^2$

10



$x^2 = 43^2 + 11^2$

$x^2 = 1\ 970$

$x = \sqrt{1\ 970}$

$x = 44.4$ cm

$y^2 = 43^2 + 12.5^2$

$y^2 = 2\ 005.25$

$y = \sqrt{2\ 005.25}$

$y = 44.8$ cm

$SA = 25 \times 22 + 2 \times ½ \times 25 \times 44.4$

$+ 2 \times ½ \times 22 \times 44.8$

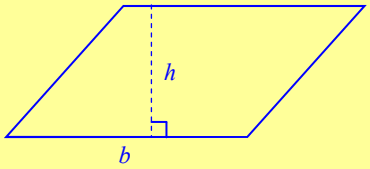
$SA = 2\ 645.6\ cm^2$

Surface Area - Other Prisms

Prisms are solids that have a uniform cross-section. This uniform face can be of any shape, but common plane shapes include squares, rectangles, triangles and circles. There are also other plane shapes such as kites, rhombi (plural of rhombus), parallelograms and trapezia (plural of trapezium).

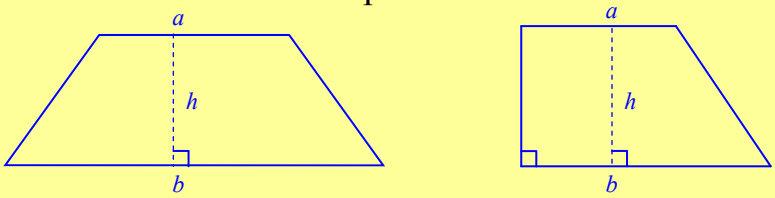
Area Formulae

Parallelogram



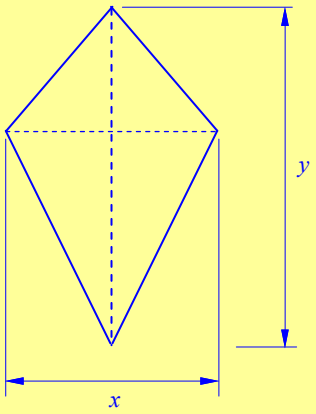
$A = bh$

Trapezium



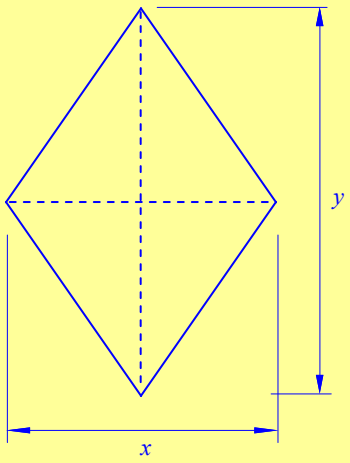
$A = \frac{1}{2}(a + b)\, h$

Kite



$A = \frac{1}{2}xy$

Rhombus



$A = \frac{1}{2}xy$

The process is similar to finding the SA of rectangular prisms. Find double the area of the front face then add all the side faces. Sometimes there will be two pairs of matching faces, with parallelograms/kites, four matching faces with rhombi and a different possibilities with Trapezia, depending on the sides of the shape. The first 3 questions have the working out set up for you, the last two questions (third column) require Pythagoras Theorem to be applied before the SA can be found.

Surface Area - Other Prisms

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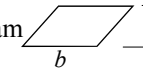
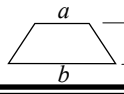
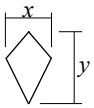
Area Formulae

Parallelogram  $A = bh$

Trapezium  $A = \frac{1}{2}(a + b)\,h$

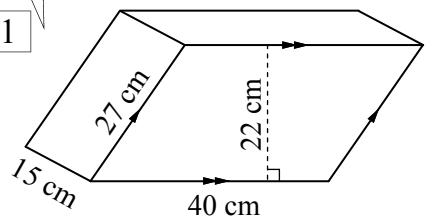
Kite  $A = \frac{1}{2}xy$

Rhombus 

Find the S.A., these require Pythagoras Theorem to be used to find the length x first.

Find the surface area of these solids.

1



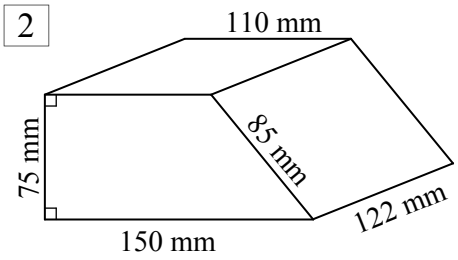
SA = 2 × ☐ × ☐ Area of front and back

+ 2 × ☐ × ☐ Area of top and base

+ 2 × ☐ × ☐ Area of left and right sides

SA =

2



SA = ☐ × ☐ + ☐ × ☐

Area of top Area of base

+ ☐ × ☐ + ☐ × ☐

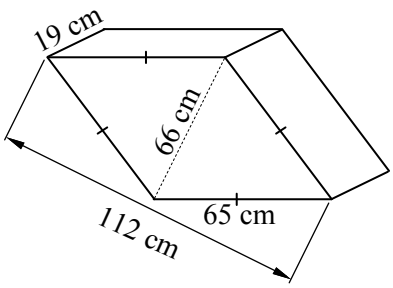
Area of LHS face Area of slanted face

+ 2 × ½ × (☐ + ☐) × ☐

Area of the front and back faces (trapezia)

SA =

3



SA = 2 × ½ × ☐ × ☐

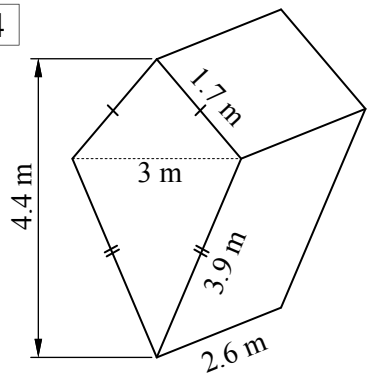
Front and Rear faces (Rhombi)

+ 4 × ☐ × ☐

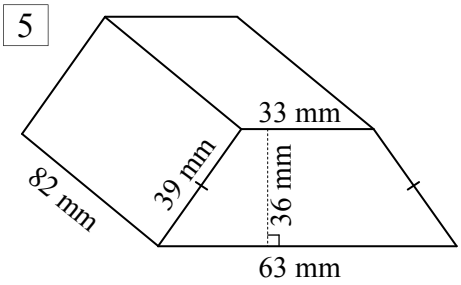
Four sides

SA =

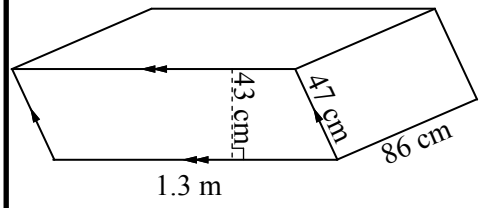
4



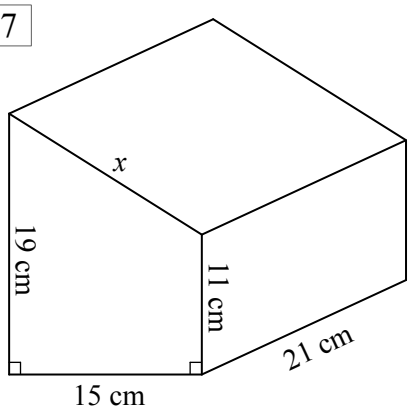
5



6 Answer in m^2



7



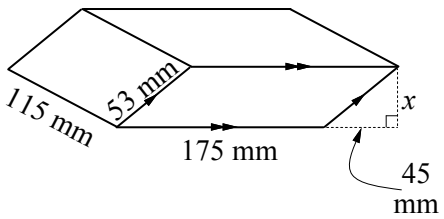
$x^2 =$

$x^2 =$

$x =$

$x =$

8



Surface Area - Other Prisms

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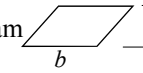
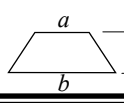
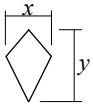
Area Formulae

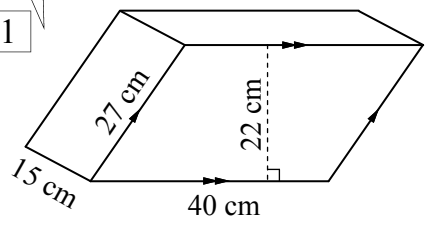
Parallelogram  $A = bh$

Trapezium  $A = \frac{1}{2}(a + b)\,h$

Kite  $A = \frac{1}{2}xy$ Rhombus 

Find the surface area of these solids.

1



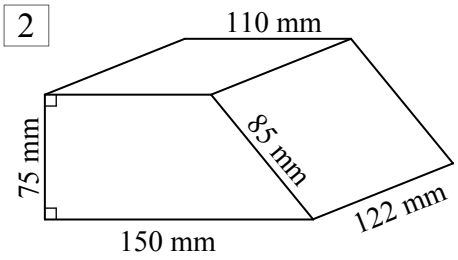
SA = $2 \times \boxed{40} \times \boxed{22}$ Area of front and back

$+ 2 \times \boxed{40} \times \boxed{15}$ Area of top and base

$+ 2 \times \boxed{15} \times \boxed{27}$ Area of left and right sides

SA = 3 770 cm^2

2



SA = $\boxed{110} \times \boxed{122} + \boxed{150} \times \boxed{122}$

Area of top Area of base

$+ \boxed{75} \times \boxed{122} + \boxed{85} \times \boxed{122}$

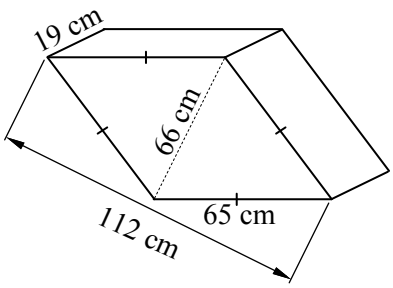
Area of LHS face Area of slanted face

$+ 2 \times \frac{1}{2} \times (\boxed{150} + \boxed{110}) \times \boxed{75}$

Area of the front and back faces (trapezia)

SA = 70 740 mm^2

3



SA = $2 \times \frac{1}{2} \times \boxed{66} \times \boxed{112}$

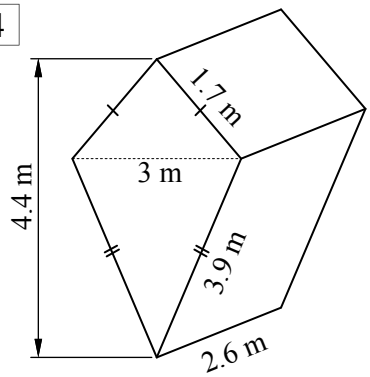
Front and Rear faces (Rhombi)

$+ 4 \times \boxed{65} \times \boxed{19}$

Four sides

SA = 12 332 cm^2

4



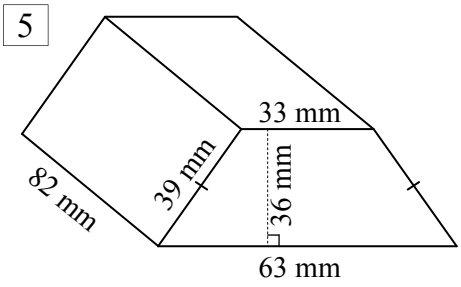
SA = $2 \times \frac{1}{2} \times 3 \times 4.4$

$+ 2 \times 1.7 \times 2.6$

$+ 2 \times 3.9 \times 2.6$

SA = 42.32 m^2

5



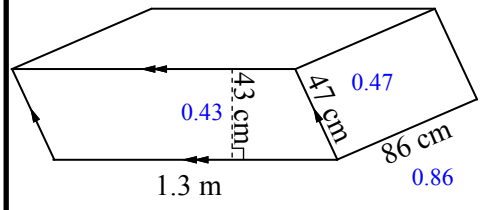
SA = $2 \times \frac{1}{2} \times (33 + 63) \times 36$

$+ 2 \times 39 \times 82$

$+ 33 \times 82 + 63 \times 82$

SA = 17 724 mm^2

6 Answer in m^2



SA = $2 \times 1.3 \times 0.43$

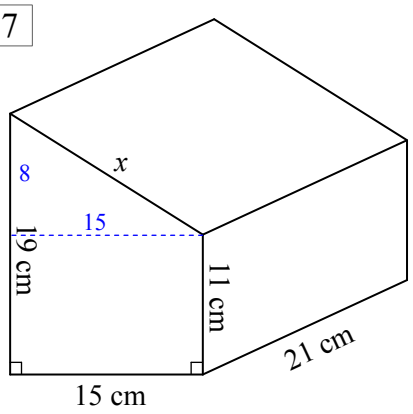
$+ 2 \times 1.3 \times 0.86$

$+ 2 \times 0.47 \times 0.86$

SA = 4.1624 m^2

Find the S.A., these require Pythagoras Theorem to be used to find the length x first.

7



$x^2 = 8^2 + 15^2$

$x^2 = 289$

$x = \sqrt{289}$

$x = 17$ cm

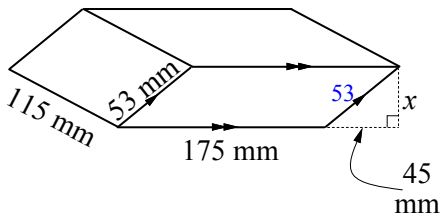
SA = $2 \times \frac{1}{2} \times (19 + 11) \times 15$

$+ 15 \times 21 + 19 \times 21$

$+ 17 \times 21 + 11 \times 21$

SA = 1 752 cm^2

8



$x^2 = 53^2 - 45^2$

$x^2 = 784$

$x = \sqrt{784}$

$x = 28$ mm

SA = $2 \times 175 \times 28$

$+ 2 \times 175 \times 115$

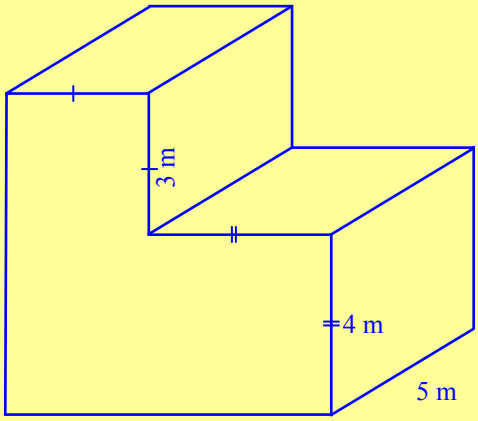
$+ 2 \times 53 \times 115$

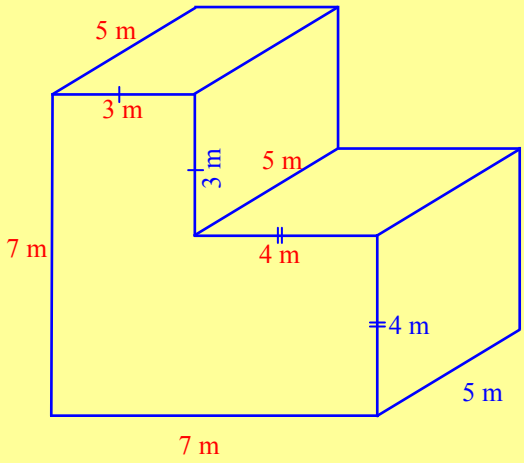
SA = 62 240 mm^2

Surface Area - Composite Solids

Finding the SA of a composite solids can be difficult as it requires visualizing the solid, identifying all the face areas and then a complicated process of adding them all. But there are two ways to approach the questions, the longer but more easy to follow approach or the quicker method. It is easier to lose track of where you are up to and make mistakes using the quicker method. Sheet 1 is structured with full setting out ready to go. Sheet 2 has no setting out and harder questions, including Pythagoras Theorem.

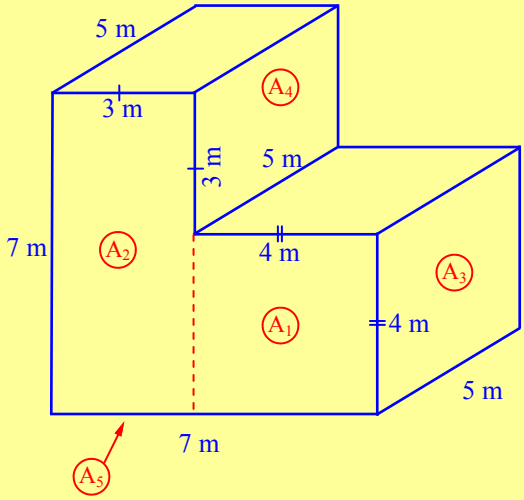
Example - Find the SA of the solid below - Method 1





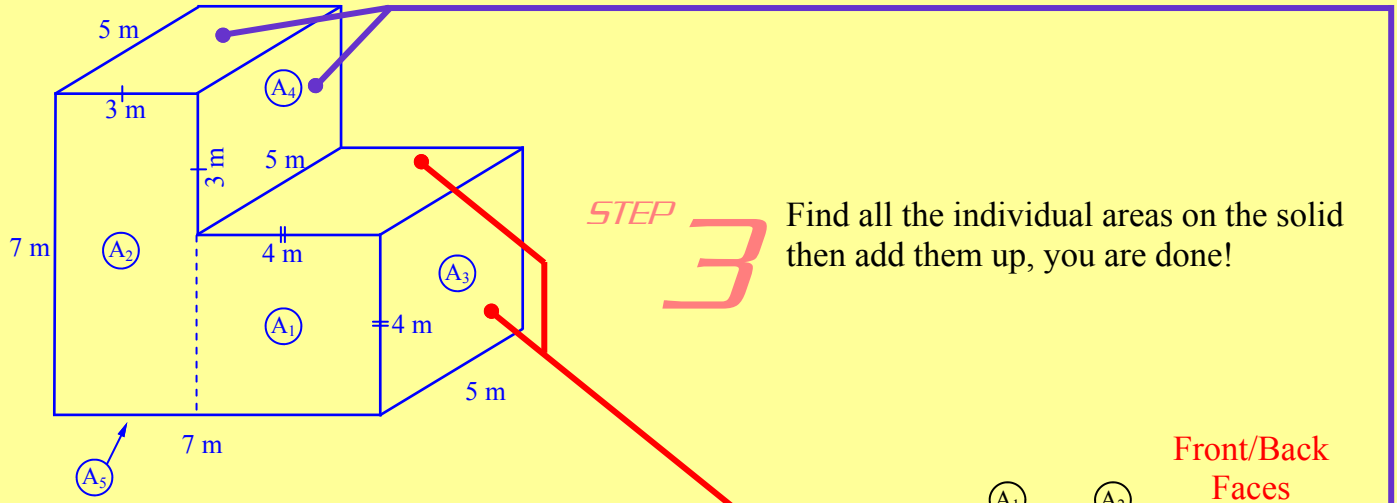
STEP 1

Write distances on all sides, including the two missing sides '7 m' which had to be calculated.



STEP 2

Break the front face up into 2 areas, label all faces note that some faces are repeated, there are four matching pairs of faces with this question, the front/rear face and three matching pairs around the solid.



STEP 3

Find all the individual areas on the solid then add them up, you are done!

Front Face

Side Faces

$A_1 = 4 \times 4 = 16\ m^2$

$A_2 = 7 \times 3 = 21\ m^2$

$A_3 = 4 \times 5 = 20\ m^2$

$A_4 = 3 \times 5 = 15\ m^2$

$A_5 = 7 \times 5 = 35\ m^2$

Front/Back Faces

$SA = 2 \times (16 + 21)$

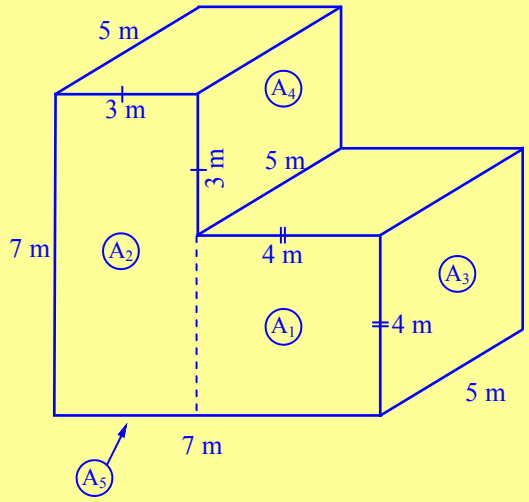
$+ 2 \times 20 + 2 \times 15$

LHS Face & Base

$+ 2 \times 35$

$SA = 214\ m^2$

Example - Find the SA of the solid below - Method 2



STEP 1

Write distances on all sides, including the missing side '7 m' which had to be calculated.

STEP 2

Break the front face up into 2 areas, label all faces note that some faces are repeated, there are four matching pairs of faces with this question, the front/rear face and three matching pairs around the solid.

STEP 3

Use either of the two methods below.

$SA = 2 \times 4 \times 4 + 2 \times 7 \times 3$ Front/Back Faces

$+ 2 \times 5 \times 4 + 2 \times 3 \times 5$ $2 \times A_3 + 2 \times A_4$

$+ 2 \times 7 \times 5$ LHS Face & Base

$SA = 214\ m^2$

OR

$SA = 2 \times 4 \times 4 + 2 \times 7 \times 3$ Front/Back Faces

$+ 5 \times (4 + 4 + 3 + 3 + 7 + 7)$ Perimeter Front Face × Depth

$SA = 214\ m^2$

Surface Area - Composite Solids (Sheet 1)

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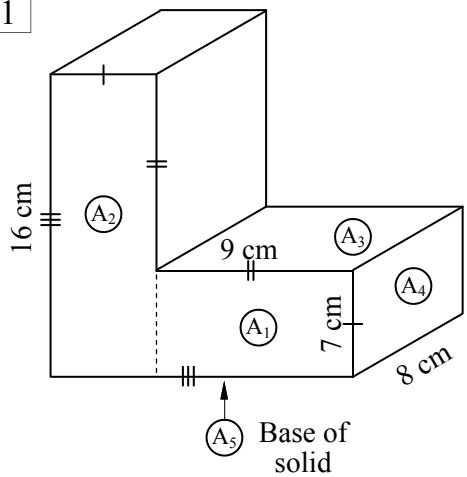
Find the individual areas on the solid, then place those areas in the SA template supplied to find the surface area for the solid.

1



$A_1 =$ ☐ × ☐ =

$A_2 =$ ☐ × ☐ =

$A_3 =$ ☐ × ☐ =

$A_4 =$ ☐ × ☐ =

$A_5 =$ ☐ × ☐ =

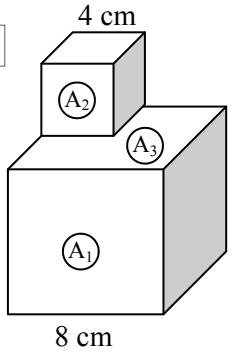
SA = 2 × (A_1 ☐ + A_2 ☐)

+ 2 × A_3 ☐ + 2 × A_4 ☐

+ 2 × A_5 ☐

SA =

2



Cube with sides 4 cm length sits on top of a cube with sides 8 cm.

$A_1 =$ ☐ × ☐ =

$A_2 =$ ☐ × ☐ =

$A_3 =$ = A_1 - A_2

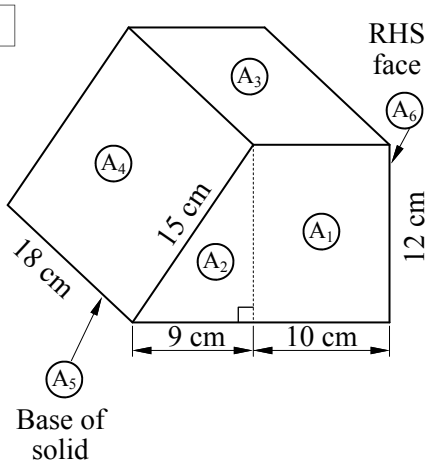
= ☐ - ☐ =

SA = 5 × A_1 ☐ + 5 × A_2 ☐

+ A_3 ☐

SA =

3



$A_1 =$ ☐ × ☐ =

$A_2 =$ ½ × ☐ × ☐ =

$A_3 =$ ☐ × ☐ =

$A_4 =$ ☐ × ☐ =

$A_5 =$ ☐ × ☐ =

$A_6 =$ ☐ × ☐ =

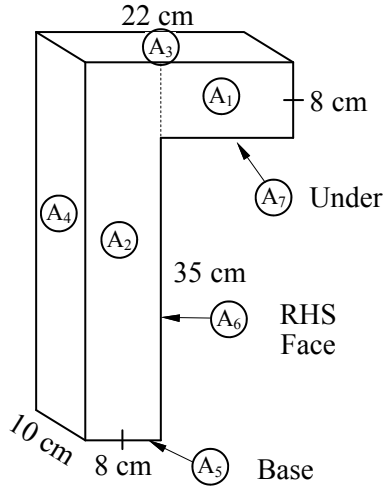
SA = 2 × (A_1 ☐ + A_2 ☐)

+ A_3 ☐ + A_4 ☐

+ A_5 ☐ + A_6 ☐

SA =

4



$A_1 =$ ☐ × ☐ =

$A_2 =$ ☐ × ☐ =

$A_3 =$ ☐ × ☐ =

$A_4 =$ ☐ × ☐ =

$A_5 =$ ☐ × ☐ =

$A_6 =$ ☐ × ☐ =

$A_7 =$ ☐ × ☐ =

SA = 2 × (A_1 ☐ + A_2 ☐)

+ A_3 ☐ + A_4 ☐

+ 2 × A_5 ☐ + A_6 ☐

+ A_7 ☐

SA =

Surface Area - Composite Solids (Sheet 1)

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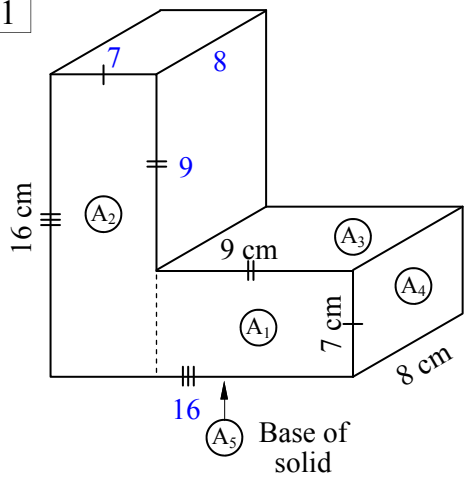
Find the individual areas on the solid, then place those areas in the SA template supplied to find the surface area for the solid.

1



$A_1 = 9 \times 7 = 63 \text{ cm}^2$

$A_2 = 16 \times 7 = 112 \text{ cm}^2$

$A_3 = 9 \times 8 = 72 \text{ cm}^2$

$A_4 = 8 \times 7 = 56 \text{ cm}^2$

$A_5 = 16 \times 8 = 128 \text{ cm}^2$

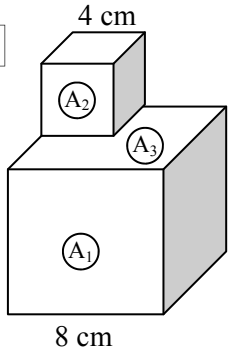
$SA = 2 \times (A_1\ 63 + A_2\ 112)$

$+ 2 \times A_3\ 72 + 2 \times A_4\ 56$

$+ 2 \times A_5\ 128$

$SA = 862 \text{ cm}^2$

2



Cube with sides 4 cm length sits on top of a cube with sides 8 cm.

$A_1 = 8 \times 8 = 64 \text{ cm}^2$

$A_2 = 4 \times 4 = 16 \text{ cm}^2$

$A_3 = A_1 - A_2$

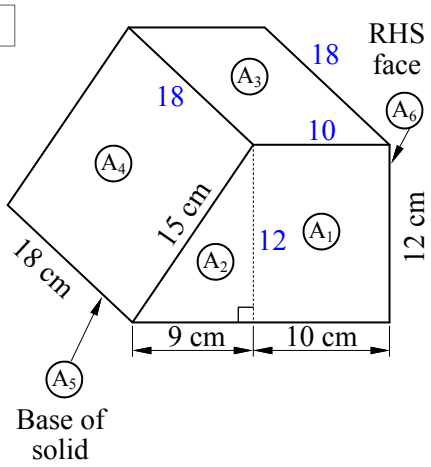
$= 64 - 16 = 48 \text{ cm}^2$

$SA = 5 \times A_1\ 64 + 5 \times A_2\ 16$

$+ A_3\ 48$

$SA = 448 \text{ cm}^2$

3



$A_1 = 10 \times 12 = 120 \text{ cm}^2$

$A_2 = \frac{1}{2} \times 9 \times 12 = 54 \text{ cm}^2$

$A_3 = 18 \times 10 = 180 \text{ cm}^2$

$A_4 = 18 \times 15 = 270 \text{ cm}^2$

$A_5 = 19 \times 18 = 342 \text{ cm}^2$

$A_6 = 18 \times 12 = 216 \text{ cm}^2$

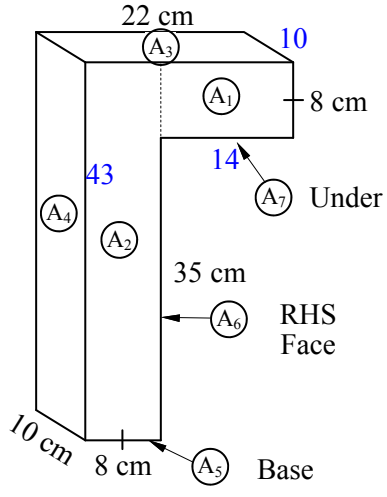
$SA = 2 \times (A_1\ 120 + A_2\ 54)$

$+ A_3\ 180 + A_4\ 270$

$+ A_5\ 342 + A_6\ 216$

$SA = 1\ 356 \text{ cm}^2$

4



$A_1 = 14 \times 8 = 112 \text{ cm}^2$

$A_2 = 43 \times 8 = 344 \text{ cm}^2$

$A_3 = 22 \times 10 = 220 \text{ cm}^2$

$A_4 = 43 \times 10 = 430 \text{ cm}^2$

$A_5 = 10 \times 8 = 80 \text{ cm}^2$

$A_6 = 35 \times 10 = 350 \text{ cm}^2$

$A_7 = 14 \times 10 = 140 \text{ cm}^2$

$SA = 2 \times (A_1\ 112 + A_2\ 344)$

$+ A_3\ 220 + A_4\ 430$

$+ 2 \times A_5\ 80 + A_6\ 350$

$+ A_7\ 140$

$SA = 2\ 212 \text{ cm}^2$

Surface Area - Composite Solids (Sheet 2)

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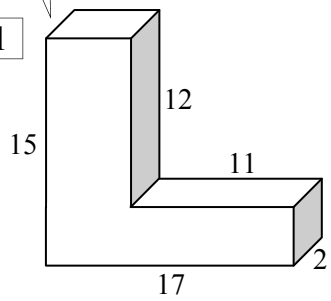
Find the SA for the composite solids shown. All measurements are in cm.

1



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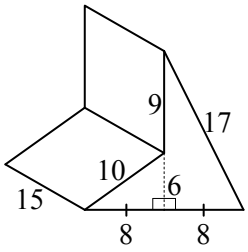

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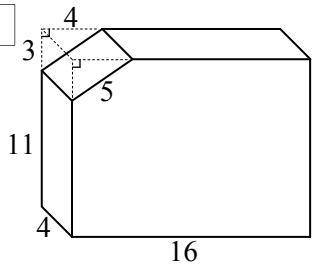

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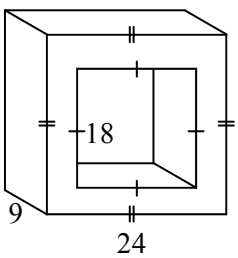

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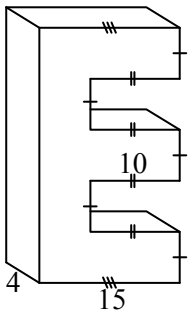
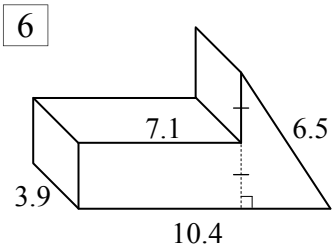


These require Pythagoras Theorem to be applied. All units are in metres.

6



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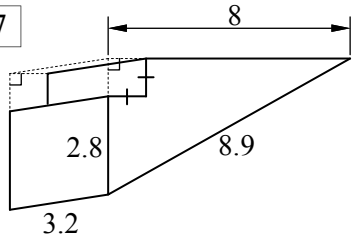

Surface Area - Composite Solids (Sheet 2)

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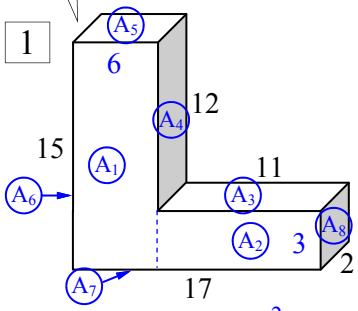
Find the SA for the composite solids shown. All measurements are in cm.

1



$A_1 = 15 \times 6 = 90 \text{ cm}^2$

$A_2 = 3 \times 11 = 33 \text{ cm}^2$

$A_3 = 11 \times 2 = 22 \text{ cm}^2$

$A_4 = 12 \times 2 = 24 \text{ cm}^2$

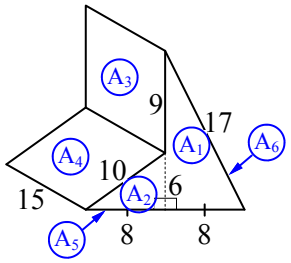
$A_5 = 6 \times 2 = 12 \text{ cm}^2$ $A_6 = 15 \times 2 = 30 \text{ cm}^2$

$A_7 = 17 \times 2 = 34 \text{ cm}^2$ $A_8 = 3 \times 2 = 6 \text{ cm}^2$

$SA = 2 \times (90 + 33) + 22 + 24 + 12 + 30 + 34 + 6$

$SA = 374 \text{ cm}^2$

2



$A_1 = \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$

$A_2 = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

$A_3 = 9 \times 15 = 135 \text{ cm}^2$

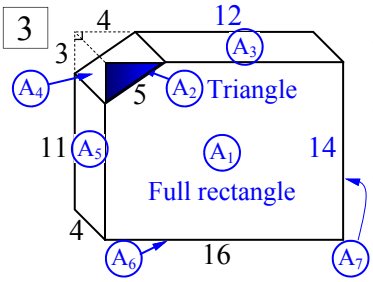
$A_4 = 10 \times 15 = 150 \text{ cm}^2$

$A_5 = 16 \times 15 = 240 \text{ cm}^2$ $A_6 = 17 \times 15 = 255 \text{ cm}^2$

$SA = 2 \times (60 + 24) + 135 + 150 + 240 + 255$

$SA = 948 \text{ cm}^2$

3



$A_1 = 16 \times 14 = 224 \text{ cm}^2$

$A_2 = \frac{1}{2} \times 3 \times 4 = 6 \text{ cm}^2$

$A_3 = 12 \times 4 = 48 \text{ cm}^2$

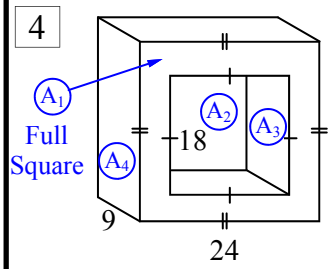
$A_4 = 5 \times 4 = 20 \text{ cm}^2$

$A_5 = 11 \times 4 = 44 \text{ cm}^2$ $A_6 = 16 \times 4 = 44 \text{ cm}^2$

$A_7 = 14 \times 4 = 56 \text{ cm}^2$

$SA = 2 \times (224 - 6) + 48 + 20 + 44 + 44 + 56 = 648 \text{ cm}^2$

4



$A_1 = 24^2 = 576 \text{ cm}^2$

$A_2 = 18^2 = 324 \text{ cm}^2$

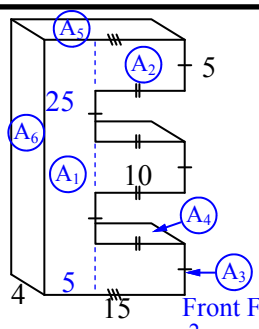
$A_3 = 18 \times 9 = 162 \text{ cm}^2$

$A_4 = 24 \times 9 = 216 \text{ cm}^2$

$SA = 2 \times (576 - 324) + 4 \times 162 + 4 \times 216$

$SA = 2\,016 \text{ cm}^2$

5



$A_1 = 25 \times 5 = 125 \text{ cm}^2$

$A_2 = 10 \times 5 = 50 \text{ cm}^2$

$A_3 = 5 \times 4 = 20 \text{ cm}^2$

$A_4 = 10 \times 4 = 40 \text{ cm}^2$

$A_5 = 15 \times 4 = 60 \text{ cm}^2$ $A_6 = 25 \times 4 = 100 \text{ cm}^2$

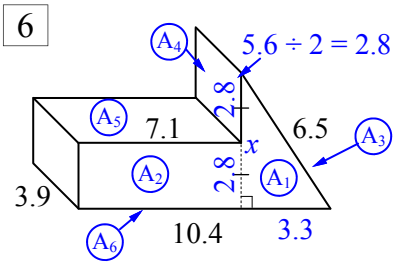
$SA = 2 \times (125 + 3 \times 50) + 5 \times 20 + 4 \times 40$

$+ 2 \times 60 + 100$

$SA = 1\,030 \text{ cm}^2$

These require Pythagoras Theorem to be applied. All units are in metres.

6



$x^2 = 6.5^2 - 3.3^2$

$x^2 = 31.36$

$x = \sqrt{31.36}$

$x = 5.6 \text{ m}$

$A_1 = \frac{1}{2} \times 3.3 \times 5.6 = 9.24 \text{ m}^2$

$A_2 = 7.1 \times 2.8 = 19.88 \text{ m}^2$ $A_3 = 6.5 \times 3.9 = 25.35 \text{ m}^2$

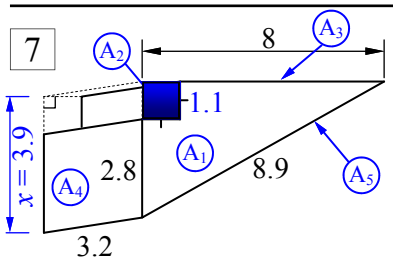
$A_4 = 2.8 \times 3.9 = 10.92 \text{ m}^2$ $A_5 = 7.1 \times 3.9 = 27.69 \text{ m}^2$

$A_6 = 10.4 \times 3.9 = 40.56 \text{ m}^2$

$SA = 2 \times (9.24 + 19.88) + 25.35 + 2 \times 10.92$

$+ 27.69 + 40.56 = 173.68 \text{ m}^2$

7



$x^2 = 8.9^2 - 8^2$

$x^2 = 15.21$

$x = \sqrt{15.21}$

$x = 3.9 \text{ m}$

$A_1 = \frac{1}{2} \times 3.9 \times 8 = 15.6 \text{ m}^2$

$A_2 = 1.1^2 = 1.21 \text{ m}^2$ $A_3 = 8 \times 3.2 = 25.6 \text{ m}^2$

$A_4 = 3.9 \times 3.2 = 12.48 \text{ m}^2$

$A_5 = 8.9 \times 3.2 = 28.48 \text{ m}^2$

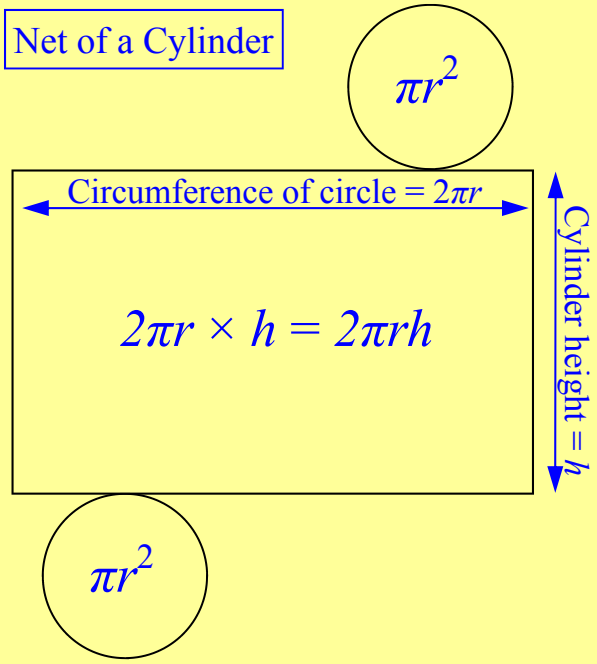
$SA = 2 \times (15.6 - 1.21) + 25.6 + 12.48 + 28.48$

$SA = 95.34 \text{ m}^2$

Estimation/Terms of π - Surface Area of Cylinders

When you calculate the SA of a cylinder, or the area of a cylinder wall (the curved part of a cylinder) you will generally use a calculator. A calculator uses an approximation for π because π is irrational, its decimal component goes on forever. So if an exact value is required, the answer must be expressed 'in terms of π'.

Net of a Cylinder



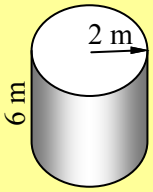
As the net shows, each circle has an area of πr^2. So the two circles added gives an area of $2\pi r^2$.

The cylinder wall, the rectangle in the net, wraps around the circle so one of its sides is the same length as the circumference of the circular ends, which is $2\pi r$. Its height is the height of the cylinder, which is h. As this area is a rectangle, its area is one side times the other, or $2\pi r \times h = 2\pi rh$.

SA of a Cylinder = $2\pi r^2 + 2\pi rh$

SA of a Cylinder Wall = $2\pi rh$

Example - Exact Area of a Cylinder Wall & Estimation of area (using π =3).



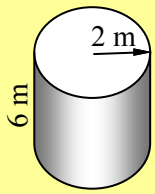
$A = 2\pi rh$	Write the formula
$= 2\times\pi\times2\times6$	Substitute r and h
$A = 24\pi$ m^2	Find area

Estimate:

$A = 24\pi$ m^2	
$\approx 24 \times 3$	Replace π with 3
$A \approx 72$ m^2	

Example - Exact Surface Area of a Cylinder & Estimation of area (using π =3).

$SA = 2\pi r^2 + 2\pi rh$	Write the formula
$= 2\times\pi\times2^2 + 2\times\pi\times2\times6$	Substitute r and h
$= 8\pi + 24\pi$	Simplify parts
$SA = 32\pi$ m^2	Find SA



Estimate: using $\pi = 3$

$SA = 32\pi$ m^2
$\approx 32 \times 3$
$SA \approx 96$ m^2

Note that as π = 3.14 to 2 d.p., using π = 3 will always give an answer that is below the actual surface area/area. But it is a simple estimate and a quick way to check your calculations.

Estimation/Terms of π - Surface Area of Cylinders

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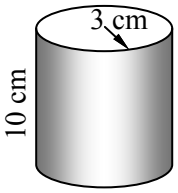
Find the area of the cylinder walls in terms of π, then estimate the area using $\pi = 3$

Example



$A = 2\pi rh$

$= 2\times\pi\times3\times10$

$A = 60\pi$ cm^2

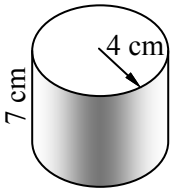
units^2 must match

Estimate: $A = 60\pi$ cm^2

$\approx 60 \times 3$

$A \approx 180$ cm^2

1



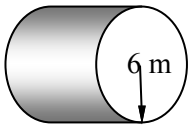
$A = 2\pi rh$

$=$

$A =$

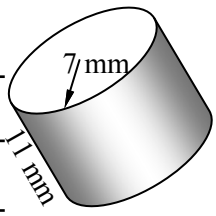
Estimate:

2



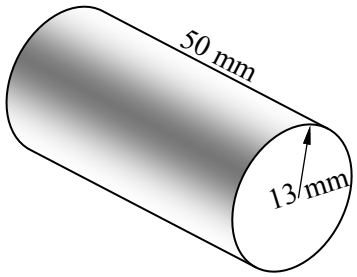
Estimate:

3



Estimate:

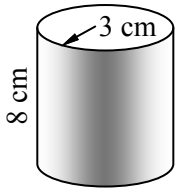
4



Estimate:

Now repeat the same process except this time for the SA of a cylinder.

5



$SA = 2\pi r^2 + 2\pi rh$

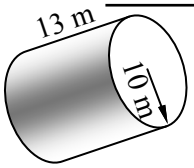
$= 2\times\pi\times\square^2 + 2\times\pi\times\square\times\square$

$= \square\ \pi + \square\ \pi$

SA = $\square\ \pi$ cm^2

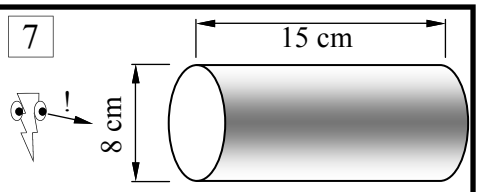
Estimate: using $\pi = 3$

6



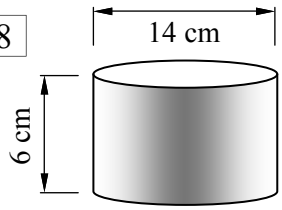
Estimate: using $\pi = 3$

7



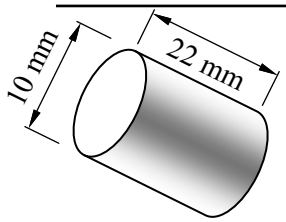
Estimate: using $\pi = 3$

8



Estimate: using $\pi = 3$

9



Estimate: using $\pi = 3$

Estimation/Terms of π - Surface Area of Cylinders

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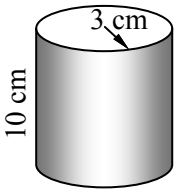
Find the area of the cylinder walls in terms of π, then estimate the area using $\pi = 3$

Example



$A = 2\pi rh$

$= 2\times\pi\times3\times10$

$A = 60\pi$ cm^2

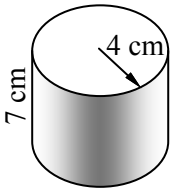
units^2 must match

Estimate: $A = 60\pi$ cm^2

$\approx 60 \times 3$

$A \approx 180$ cm^2

1



$A = 2\pi rh$

$= 2\times\pi\times4\times7$

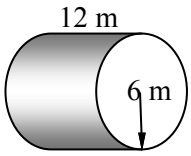
$A = 56\pi$ cm^2

Estimate: $A = 56\pi$ cm^2

$\approx 56 \times 3$

$A \approx 168$ cm^2

2



$A = 2\pi rh$

$= 2\times\pi\times6\times12$

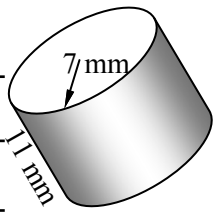
$A = 144\pi$ m^2

Estimate: $A = 144\pi$ m^2

$\approx 144 \times 3$

$A \approx 432$ m^2

3



$A = 2\pi rh$

$= 2\times\pi\times7\times11$

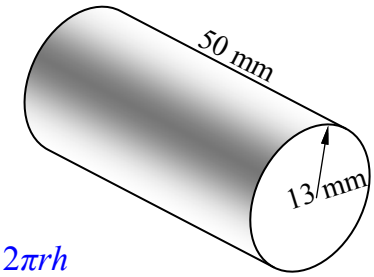
$A = 154\pi$ mm^2

Estimate: $A = 154\pi$ mm^2

$\approx 154 \times 3$

$A \approx 462$ mm^2

4



$A = 2\pi rh$

$= 2\times\pi\times13\times50$

$A = 1\,300\pi$ mm^2

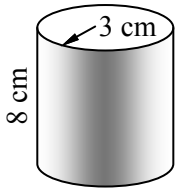
Estimate: $A = 1\,300\pi$ mm^2

$\approx 1\,300 \times 3$

$A \approx 3\,900$ mm^2

Now repeat the same process except this time for the SA of a cylinder.

5



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times\boxed{3}^2 + 2\times\pi\times\boxed{3}\times\boxed{8}$

$= \boxed{18}\ \pi + \boxed{48}\ \pi$

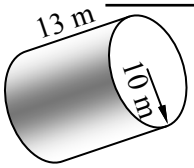
$SA = \boxed{66}\ \pi$ cm^2

Estimate: $SA = 66\pi$ cm^2

$\approx 66 \times 3$

$SA \approx 198$ cm^2

6



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times10^2 + 2\times\pi\times10\times13$

$= 200\pi + 260\pi$

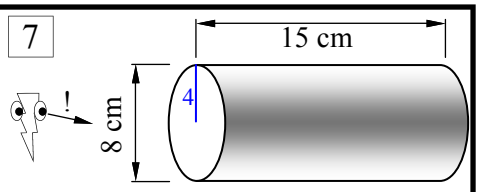
$SA = 460\pi$ m^2

Estimate: $SA = 460\pi$ m^2

$\approx 460 \times 3$

$SA \approx 1\,380$ m^2

7



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times4^2 + 2\times\pi\times4\times15$

$= 32\pi + 120\pi$

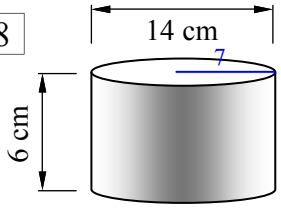
$SA = 152\pi$ cm^2

Estimate: $SA = 152\pi$ cm^2 using $\pi = 3$

$\approx 152 \times 3$

$SA \approx 456$ cm^2

8



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times7^2 + 2\times\pi\times7\times6$

$= 98\pi + 84\pi$

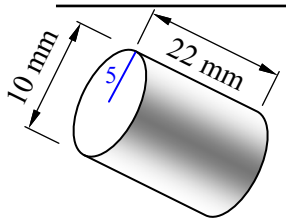
$SA = 182\pi$ cm^2

Estimate: $SA = 182\pi$ cm^2 using $\pi = 3$

$\approx 182 \times 3$

$SA \approx 546$ cm^2

9



$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times5^2 + 2\times\pi\times5\times22$

$= 50\pi + 220\pi$

$SA = 270\pi$ mm^2

Estimate: $SA = 270\pi$ mm^2 using $\pi = 3$

$\approx 270 \times 3$

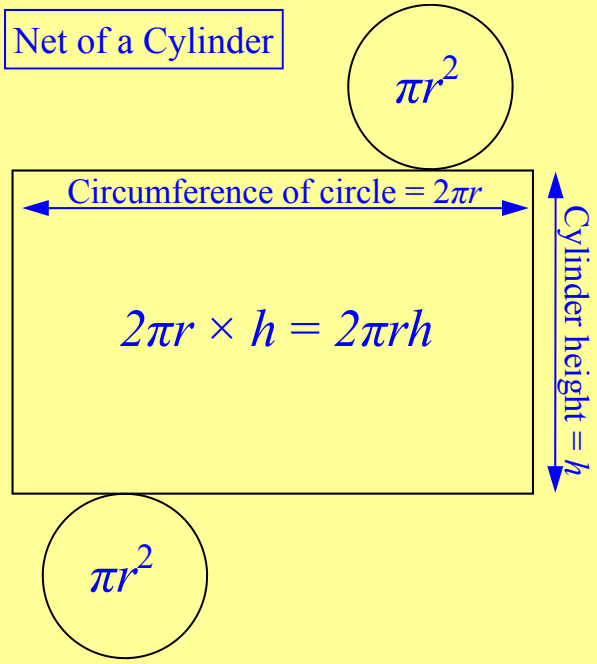
$SA \approx 810$ mm^2

Surface Area of Cylinders

The surface area of a cylinder is the sum of the areas of the solid. As the net of a cylinder below shows, each circle has an area of πr^2. So the two circles added together gives an area of $2\pi r^2$. The cylinder wall, (also called the curved surface) is the rectangle in the net, it wraps around the circles so one of its sides is the same length as the circumference of the circular ends, which is $2\pi r$. Its height is the height of the cylinder, which is h.

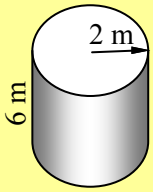
As this area is a rectangular area, its area is one side times the other, or $2\pi r \times h = 2\pi rh$.

Net of a Cylinder



SA of a Cylinder = $2\pi r^2 + 2\pi rh$

Example - SA of a cylinder - Using $\pi = {}^{22}/_{7}$



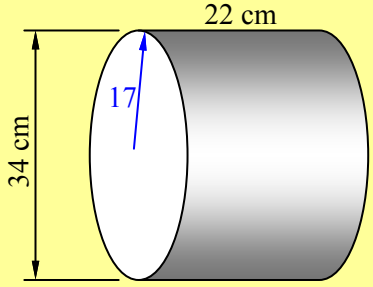
$SA = 2\pi r^2 + 2\pi rh$	Write the formula
$= 2\times{}^{22}/_{7}\times 2^2 + 2\times{}^{22}/_{7}\times 2\times 6$	Substitute r and h, Change π to ${}^{22}/_{7}$.
$SA = 100.57\ m^2$ (2 d.p.)	Find surface area

Calculator 2 × 2 2 $a^b/_c$ 7 × 2 x^2 + 2 × 2 2 $a^b/_c$ 7 × 2 × 6 =

Answer on Screen 100⌟4⌟7

Calculator $a^b/_c$ Answer on Screen 100.5714286 Round to 2 decimal places $100.57\ m^2$ (2 d.p.)

Example - SA of a cylinder - Given the diameter instead of radius - Using π key



Divide the diameter by two. Then on the diagram draw the radius.

Use that number in your calculation. It may be a decimal.

$SA = 2\pi r^2 + 2\pi rh$	Write the formula
$= 2\times\pi\times 17^2 + 2\times\pi\times 17\times 22$	Substitute r and h
$SA = 4\ 165.75\ cm^2$ (2 d.p.)	Find surface area

Calculator 2 × π × 1 7 x^2 + 2 × π × 1 7 × 2 2 =

Answer on Screen 4165.751859 Round to 2 decimal places $4\ 165.75\ cm^2$ (2 d.p.)

Surface Area of Cylinders

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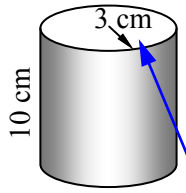
Find the surface area of these cylinders.
Use π = 3.14, not the π key. Round to 2 dp.

Example

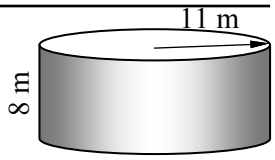


$SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times3^2 + 2\times3.14\times3\times10$

$SA = 244.92 \text{ cm}^2$ (2 d.p.)

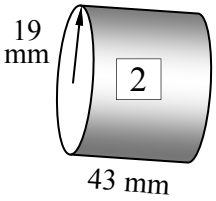
units² must match

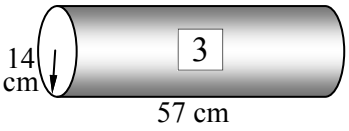


1 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times$ [ ]$^2 + 2\times3.14\times$ [ ] $\times$ [ ]

$SA =$ m^2





2 $SA = 2\pi r^2 + 2\pi rh$

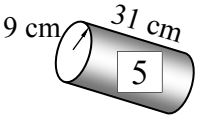
$=$

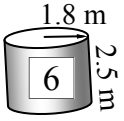
$SA =$

3

Now use π = $^{22}/_{7}$. Use the fraction key after '=' to change to a decimal, round to 1 d.p.





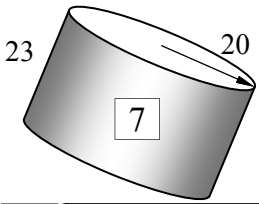


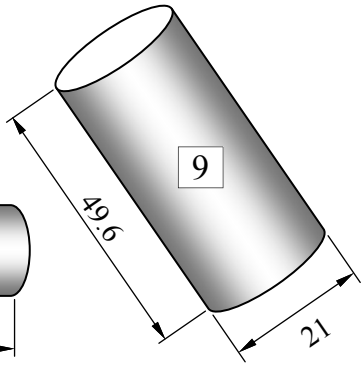
4

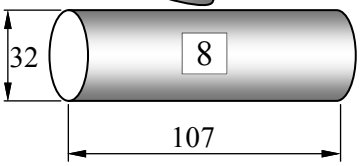
5

6

Now use your calculator's π key. These questions can include diameters. All measurements are in cm. Round to 1 d.p.







7 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times$ [ ]$^2 + 2\times\pi\times$ [ ] $\times$ [ ]

8

9

10 Bubblin' Bobs cylindrical water tanks are 4.2 m high and 5.1 m in diameter. Metal sheeting costs \$9.10/m², find the cost to build 4 tanks.

11 Slow & Slim's steam rollers have a cylinder for a front wheel that is 1.9 m diameter and 3.1 m wide (height). The roller needs painting, if a tin of paint covers 11 m² how many tins are required to paint the roller.

Surface Area of Cylinders

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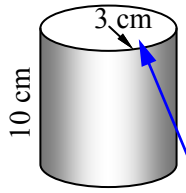
Find the surface area of these cylinders.
Use $\pi = 3.14$, not the π key. Round to 2 dp.

Example

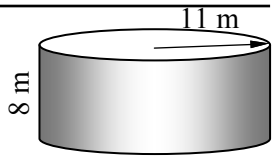


$SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times3^2 + 2\times3.14\times3\times10$

$SA = 244.92 \text{ cm}^2$ (2 d.p.)

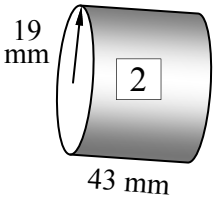
units² must match

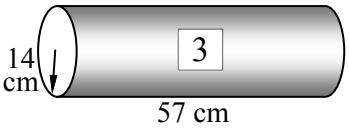


1 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times\boxed{11}^2 + 2\times3.14\times\boxed{11}\times\boxed{8}$

$SA = 1\,312.52 \text{ m}^2$ (2 d.p.)





2 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times3.14\times19^2 + 2\times3.14\times19\times43$

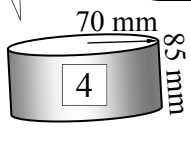
$SA = 7\,397.84 \text{ mm}^2$ (2 d.p.)

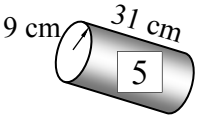
3 $SA = 2\pi r^2 + 2\pi rh$

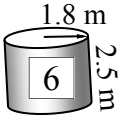
$= 2\times3.14\times14^2 + 2\times3.14\times14\times57$

$SA = 7\,121.52 \text{ mm}^2$ (2 d.p.)

Now use $\pi = {}^{22}/_{7}$. Use the fraction key after '=' to change to a decimal, round to 1 d.p.







4 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times{}^{22}/_{7}\times70^2 + 2\times{}^{22}/_{7}\times70\times85$

$SA = 68\,200 \text{ mm}^2$

5 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times{}^{22}/_{7}\times9^2 + 2\times{}^{22}/_{7}\times9\times31$

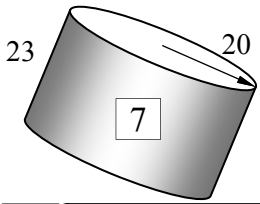
$SA = 2\,262.9 \text{ cm}^2$ (1 d.p.)

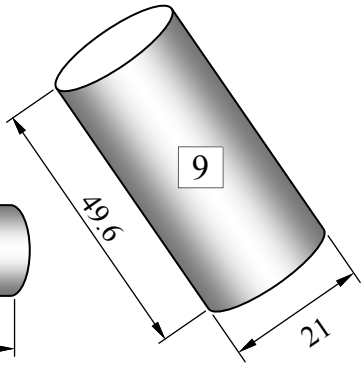
6 $SA = 2\pi r^2 + 2\pi rh$

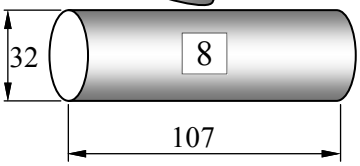
$= 2\times{}^{22}/_{7}\times1.8^2 + 2\times{}^{22}/_{7}\times1.8\times2.5$

$SA = 48.7 \text{ m}^2$ (1 d.p.)

Now use your calculator's π key. These questions can include diameters. All measurements are in cm. Round to 1 d.p.







7 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times\boxed{20}^2 + 2\times\pi\times\boxed{20}\times\boxed{23}$

$SA = 5\,403.5 \text{ cm}^2$ (1 d.p.)

8 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times16^2 + 2\times\pi\times16\times107$

$SA = 12\,365.3 \text{ cm}^2$ (1 d.p.)

9 $SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times10.5^2 + 2\times\pi\times10.5\times49.6$

$SA = 3\,965 \text{ cm}^2$ (1 d.p.)

10 Bubblin' Bobs cylindrical water tanks are 4.2 m high and 5.1 m in diameter. Metal sheeting costs \$9.10/m², find the cost to build 4 tanks.

$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times2.55^2 + 2\times\pi\times2.55\times4.2$

$SA = 108.15 \text{ m}^2$ (2 d.p.)

$Cost = 108.15 \times 4 \times 9.10 = \$3\,936.66$ \$3 936.64 No rounding

11 Slow & Slim's steam rollers have a cylinder for a front wheel that is 1.9 m diameter and 3.1 m wide (height). The roller needs painting, if a tin of paint covers 11 m² how many tins are required to paint the roller.

$SA = 2\pi r^2 + 2\pi rh$

$= 2\times\pi\times0.95^2 + 2\times\pi\times0.95\times3.1$

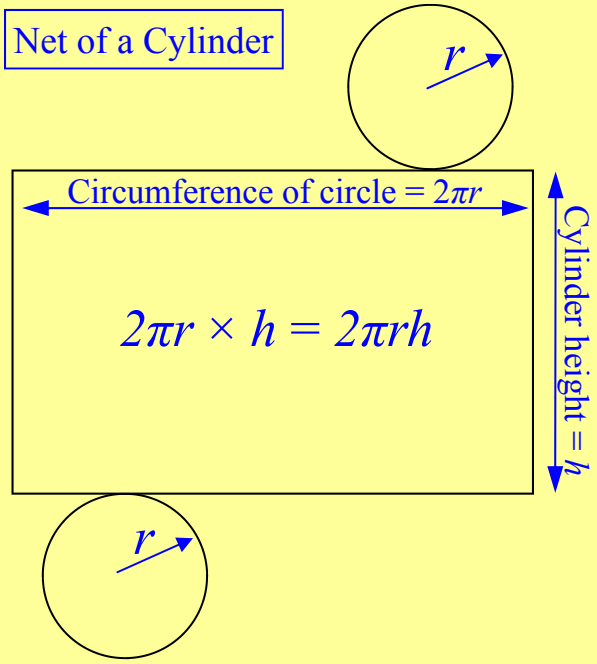
$SA = 24.17 \text{ m}^2$ (2 d.p.)

$Cost = 24.17 \div 11 = 2.19 \ldots = 3$ tins required

Area of Curved Surface - Cylinders

The cylinder wall of a cylinder, (also called the curved surface) is the rectangle shown in the net below, it wraps around the circular ends so one of its sides is the same length as the circumference of the circular ends, which is $2\pi r$. Its height is the height of the cylinder, which is h.

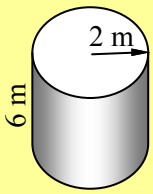
Net of a Cylinder



As this area is a rectangular area, its area is one side times the other,
or $2\pi r \times h = 2\pi rh$.

Area = $2\pi rh$

Example - Area of a cylinder wall - Using $\pi = {}^{22}/_{7}$



SA = $2\pi rh$	Write the formula
= $2\times{}^{22}/_{7}\times2\times6$	Substitute r and h, Change π to ${}^{22}/_{7}$.
SA = 75.4 m^2 (1 d.p.)	Find area of the curved surface

Calculator 2 × 2 2 $a^b/_c$ 7 × 2 × 6 =

Answer on Screen 75⌟ 3⌟ 7

Calculator $a^b/_c$ Answer on Screen 75.42857143 Round to 1 decimal place 75.4 m^2

Column 1 uses $\pi = 3.14$ first. So use the formula $A = 2\pi rh$ and calculate the area. The lower part of the sheet uses $\pi = {}^{22}/_{7}$, solve as above in the example.

Column 2 uses the π key on your calculator, also you may be given the radius or the diameter. If you are given the diameter then halve it (÷ 2) to change the length to the radius, then solve as before. There is a table at the bottom of the column, this doesn't require working out, just answers. The last two questions are different and require division and brackets, see how you go.

Column 3 features problem style questions, they require an extra step or two to complete.

Area of Curved Surface - Cylinders

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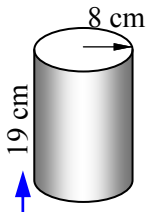
Find the area of the curved surface of these cylinders. Use $\pi = 3.14$. Round to 2 d.p.

Example



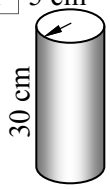
$A = 2\pi rh$

$= 2 \times 3.14 \times 8 \times 19$

$A = 954.56$ cm^2

units^2 must match

1



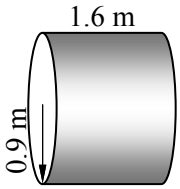
$A = 2\pi rh$

$= 2 \times 3.14 \times \square \times \square$

$A =$ cm^2

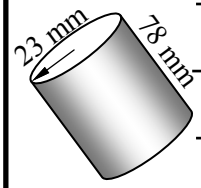
2

$A = 2\pi rh$



$A =$ m^2

3



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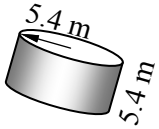
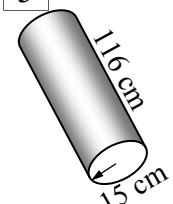


Now use $\pi = {}^{22}/_{7}$. Round to 1 decimal place.

5



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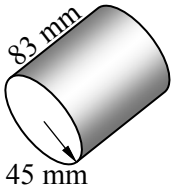
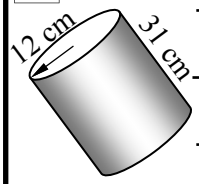


Use the π key for these and round to 1 d.p. Divide the diameter by 2 for the radius.

7



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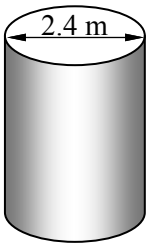

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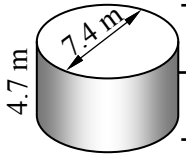

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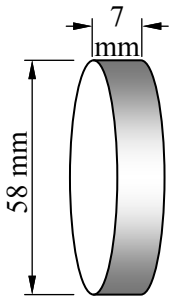

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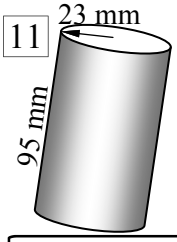
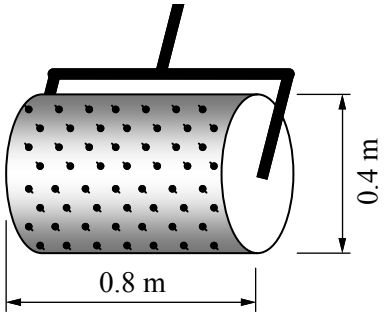


Use the π key and fill all the gaps, round to 1 d.p. All lengths are in cm.

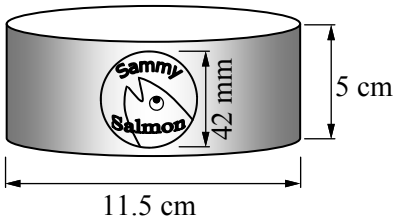
Q	r	d	h	$A = 2\pi rh$
12	8		6	
13		18	25	
14		14	19	
15	17		20	
16		12		565.5 cm^2
17			13	735.1 cm^2

Use the π key for these and round to 2 d.p.

18 A turf roller is 0.8 m wide with a diameter of 0.4 m. Find the area of lawn covered, in m^2, if the roller rotates 20 times.



19 A tin of salmon has a label covering its curved surface. If the label has a circular fish logo, what percentage of the label's area is used by the fish logo?



Area of Curved Surface - Cylinders

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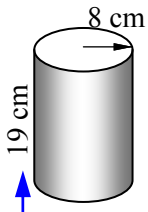
Find the area of the curved surface of these cylinders. Use $\pi = 3.14$. Round to 2 d.p.

Example



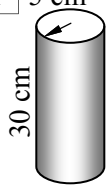
$A = 2\pi rh$

$= 2\times3.14\times8\times19$

$A = 954.56$ cm^2

units^2 must match

1

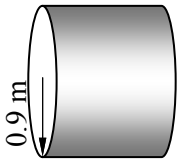


$A = 2\pi rh$

$= 2\times3.14\times$ [5] $\times$ [30]

$A = 942$ cm^2

2

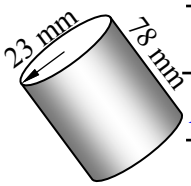


$A = 2\pi rh$

$= 2\times3.14\times0.9\times1.6$

$A = 9.04$ m^2

3

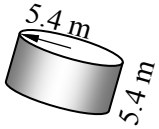


$A = 2\pi rh$

$= 2\times3.14\times23\times78$

$A = 11\,266.32$ mm^2

4



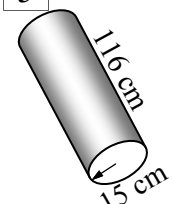
$A = 2\pi rh$

$= 2\times3.14\times5.4^2$

$A = 183.12$ m^2

Now use $\pi = {}^{22}/_{7}$. Round to 1 decimal place.

5

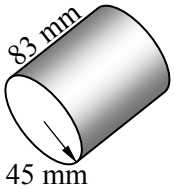


$A = 2\pi rh$

$= 2\times{}^{22}/_{7}\times15\times116$

$A = 10\,937.1$ cm^2

6



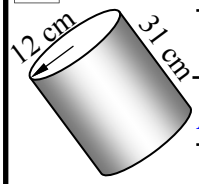
$A = 2\pi rh$

$= 2\times{}^{22}/_{7}\times45\times83$

$A = 23\,477.1$ mm^2

Use the π key for these and round to 1 d.p. Divide the diameter by 2 for the radius.

7

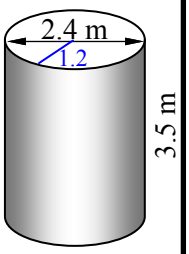


$A = 2\pi rh$

$= 2\times\pi\times12\times31$

$A = 2\,337.3$ cm^2

8

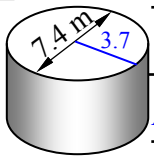


$A = 2\pi rh$

$= 2\times\pi\times1.2\times3.5$

$A = 26.4$ m^2

9

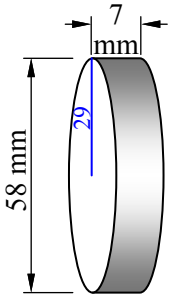


$A = 2\pi rh$

$= 2\times\pi\times3.7\times4.7$

$A = 109.3$ m^2

10

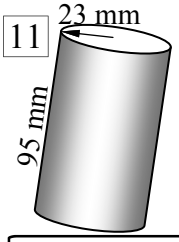


$A = 2\pi rh$

$= 2\times\pi\times29\times7$

$A = 1\,275.5$ mm^2

11



$A = 2\pi rh$

$= 2\times\pi\times23\times95$

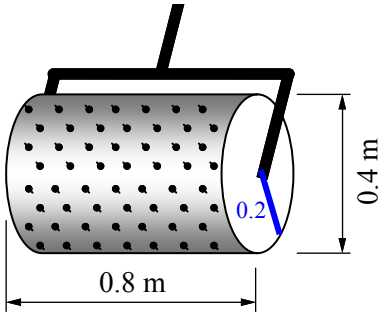
$A = 13\,728.8$ mm^2

Use the π key and fill all the gaps, round to 1 d.p. All lengths are in cm.

Q	r	d	h	$A = 2\pi rh$
12	8	16	6	301.6 cm^2
13	9	18	25	1 413.7 cm^2
14	7	14	19	835.7 cm^2
15	17	34	20	2 136.3 cm^2
16	6	12	15	565.5 cm^2
17	9	18	13	735.1 cm^2

Use the π key for these and round to 2 d.p.

18 A turf roller is 0.8 m wide with a diameter of 0.4 m. Find the area of lawn covered, in m^2, if the roller rotates 20 times.



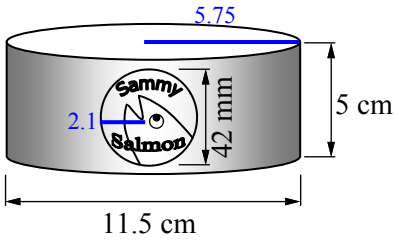
$A = 2\pi rh$

$= 2\times\pi\times0.2\times0.8$

$A = 1.01$ m^2

Area $= 20 \times 1.01 = 20.2$ m^2

19 A tin of salmon has a label covering its curved surface. If the label has a circular fish logo, what percentage of the label's area is used by the fish logo?



$A = 2\pi rh$

$= 2\times\pi\times5.75\times5$

$A = 189.64$ cm^2

$A = \pi r^2$

$= \pi\times2.1^2$

$A = 13.85$ cm^2

Percentage $= \frac{13.85}{189.64} \times 100$

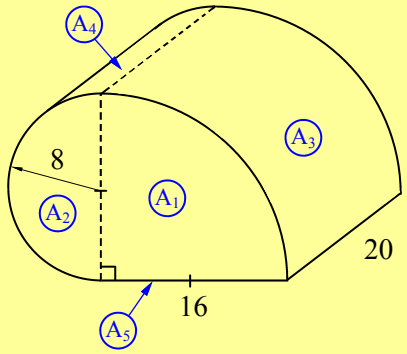
Percentage $= 7.30$ %

Surface Area of Composite Solids - Circles

Finding the SA of a composite solids can be difficult as it requires visualizing the solid, identifying all the face/curved surface areas and then a complicated process of adding them all. The questions on this worksheet all prisms except for question 2. The method is to:

- Find the area of the front face and double it (as the rear face is the same - prisms only)
- Find the face/curved surface areas around the sides of the solid
- Add the areas together

Example - Find the SA of the solid below, measurements are in cm.



STEP 1

Break the front face up into 2 areas, label all faces/curved surfaces around the outside of the solid.

$A_1 = \frac{1}{4} \times \pi \times 16^2$

$A_1 = 201.06 \text{ cm}^2$

$A_2 = \frac{1}{2} \times \pi \times 8^2$

$A_2 = 100.53 \text{ cm}^2$

$A_3 = \frac{1}{4} \times 2 \times \pi \times 16 \times 20$

$A_3 = 502.65 \text{ cm}^2$

$A_4 = \frac{1}{2} \times 2 \times \pi \times 8 \times 20$

$A_4 = 502.65 \text{ cm}^2$

$A_5 = 16 \times 20 = 320 \text{ cm}^2$

STEP 2

Calculate the individual areas.

$SA = 2 \times (201.06 + 100.53) + 502.65 + 502.65 + 320$

$SA = 1\,568.48 \text{ cm}^2$

STEP 3

Find the SA by adding each of the areas.

Surface Area of Composite Solids - Circles

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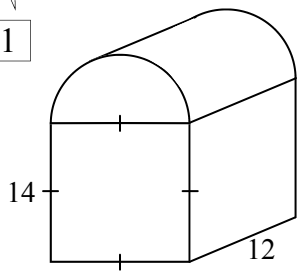
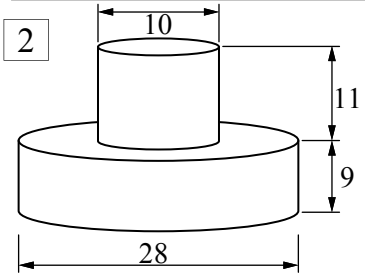
Find the SA for the composite solids. All measurements are in cm. Round to 2 d.p.

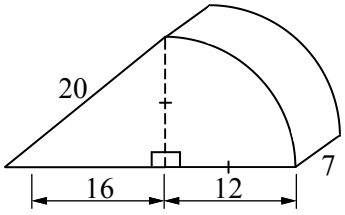
1



2



3



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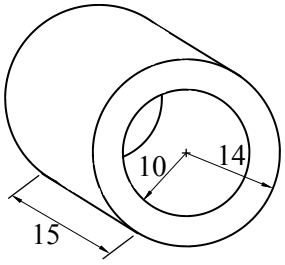
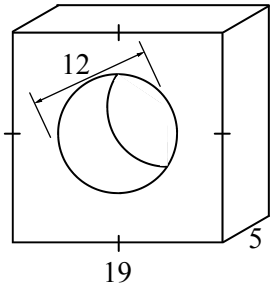
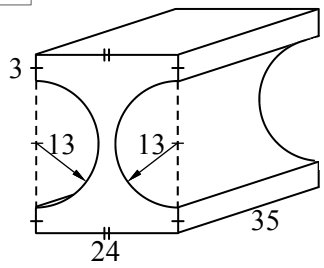


5

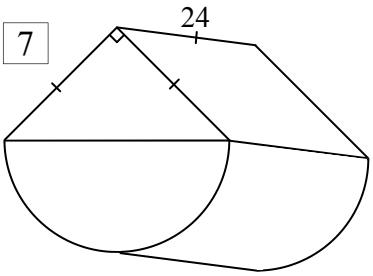


6



Pythagoras needed!

7



Surface Area of Composite Solids - Circles

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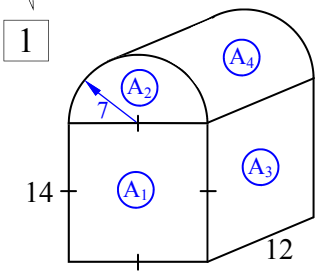
Find the SA for the composite solids. All measurements are in cm. Round to 2 d.p.

1



$A_1 = 14^2 = 196$ cm^2

$A_2 = \frac{1}{2}\pi r^2$

$= \frac{1}{2} \times \pi \times 7^2 = 76.97$ cm^2

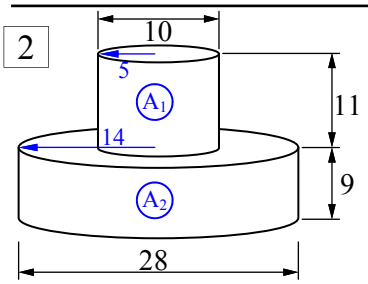
$A_3 = 12^2 = 144$ cm^2

$A_4 = \frac{1}{2} \times 2\pi rh = \frac{1}{2} \times 2 \times \pi \times 7 \times 12 = 263.89$ cm^2

SA = 2 × (196 + 76.97) + 3 × 144 + 263.89

SA = 1 241.83 cm^2

2



$A_1 = 2\pi rh$

$= 2 \times \pi \times 5 \times 11$

$= 345.58$ cm^2

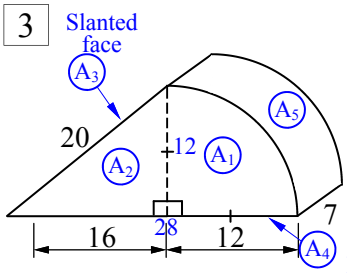
$A_2 = 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times 14^2 + 2 \times \pi \times 14 \times 9 = 2\,023.19$ cm^2

SA = 345.58 + 2 023.19

SA = 2 368.77 cm^2 (or 2 368.76 cm^2 in 1 step)

3



$A_1 = \frac{1}{4} \times \pi \times 12^2$

$= 113.10$ cm^2

$A_2 = \frac{1}{2} \times 16 \times 12 = 96$ cm^2

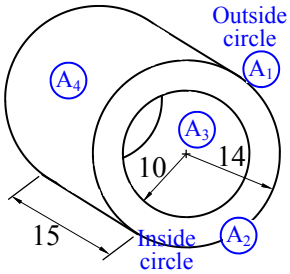
$A_3 = 20 \times 7 = 140$ cm^2

$A_4 = 28 \times 7 = 196$ cm^2 A_5: $\frac{1}{4} \times 2 \times \pi \times 12 \times 7 = 131.94$ cm^2

SA = 2 × (113.10 + 96) + 140 + 196 + 131.94

SA = 886.14 cm^2

4



$A_1 = \pi \times 14^2 = 615.75$ cm^2

$A_2 = \pi \times 10^2 = 314.16$ cm^2

$A_3 = 2 \times \pi \times 10 \times 15$

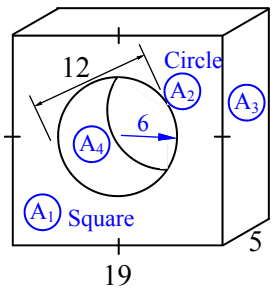
$= 942.48$ cm^2

$A_4 = 2 \times \pi \times 14 \times 15 = 1\,319.47$ cm^2

SA = 2 × (615.75 - 314.16) + 942.48 + 1 319.47

SA = 2 865.13 cm^2 (or 2 865.14 cm^2 in 1 step)

5



$A_1 = 19^2 = 361$ cm^2

$A_2 = \pi r^2 = \pi \times 6^2$

$= 113.10$ cm^2

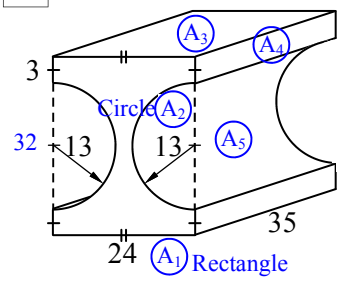
$A_3 = 19 \times 5 = 95$ cm^2

$A_4 = 2\pi rh = 2 \times \pi \times 6 \times 5 = 188.50$ cm^2

SA = 2 × (361 - 113.10) + 4 × 95 + 188.50

SA = 1 064.30 cm^2

6



$A_1 = 24 \times 32 = 768$ cm^2

$A_2 = \pi r^2 = \pi \times 13^2$

$= 530.93$ cm^2

$A_3 = 24 \times 35 = 840$ cm^2

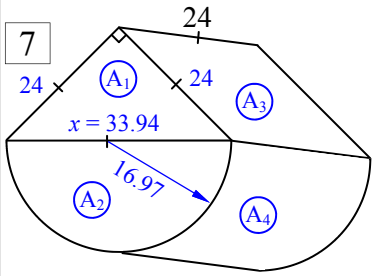
$A_4 = 3 \times 35 = 105$ cm^2

$A_5 = 2\pi rh = 2 \times \pi \times 13 \times 35 = 2\,858.85$ cm^2

SA = 2×(768 - 530.93)+2×840+4×105+2 858.85

SA = 5 432.99 cm^2

7



Pythagoras needed!

$x^2 = 24^2 + 24^2$

$x^2 = 1\,152$

$x = \sqrt{1\,152}$

$x = 33.94$ cm (2 d.p.)

$A_1 = \frac{1}{2} \times 24 \times 24 = 288$ cm^2

$A_2 = \frac{1}{2}\pi r^2 = \frac{1}{2} \times \pi \times 16.97^2 = 452.36$ cm^2

$A_3 = 24^2 = 576$ cm^2

$A_4 = \frac{1}{2} \times 2 \times \pi \times 16.97 \times 24 = 1\,279.51$ cm^2

SA = 2 × (288 + 452.36) + 2 × 576 + 1 279.51

SA = 3 912.23cm^2

Further Surface Area Applications

These are complex questions that require multiple steps and care.

No Help! No Hints! See how you go!

Further Surface Area Applications

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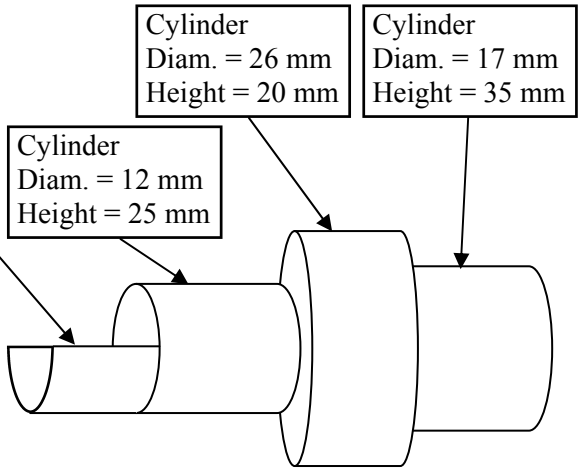
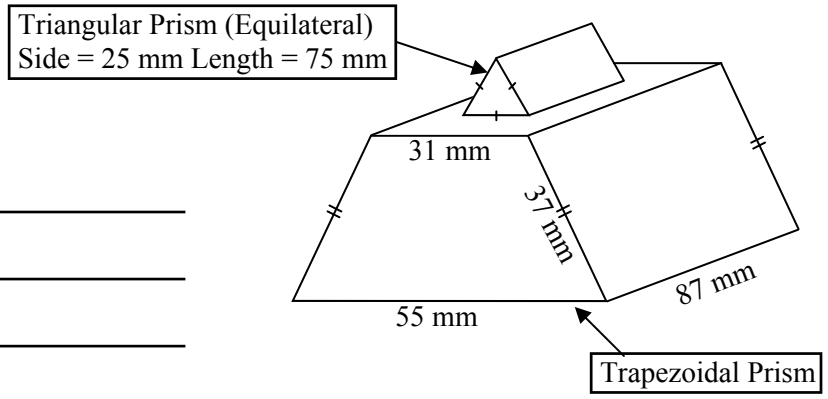
Calculate the surface area, round answers to 2 decimal places.

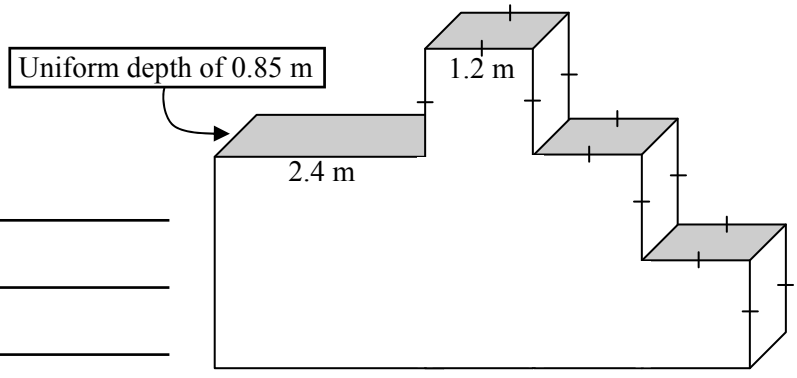
1 Collared Roller Spindle



2 Rubber impact block



3 Concrete loading dock



Further Surface Area Applications

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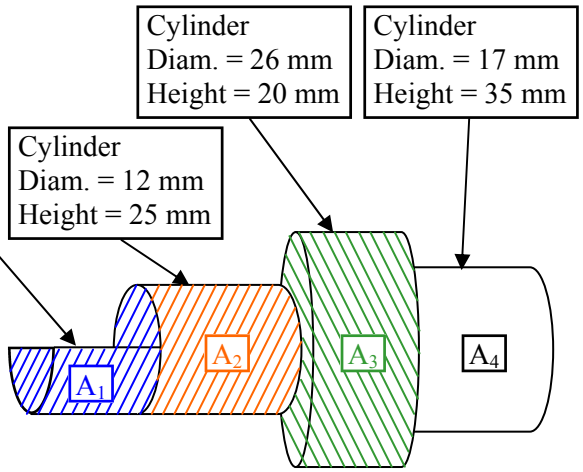
Calculate the surface area, round answers to 2 decimal places.

1 Collared Roller Spindle



$A_1 = \pi r^2 + \pi rh + 2rh$

$= \pi \times 6^2 + \pi \times 6 \times 20 + 2 \times 6 \times 20$

$A_1 = 730.09 \text{ mm}^2$

$A_2 = 2\pi rh$

$= 2 \times \pi \times 6 \times 25$

$A_2 = 942.48 \text{ mm}^2$

$A_3 = \pi R^2 - \pi r^2 + 2\pi Rh$

$= \pi \times 13^2 - \pi \times 6^2 + 2 \times \pi \times 13 \times 20$

$A_1 = 2\,051.46 \text{ mm}^2$

$A_4 = 2\pi rh + \pi R^2$

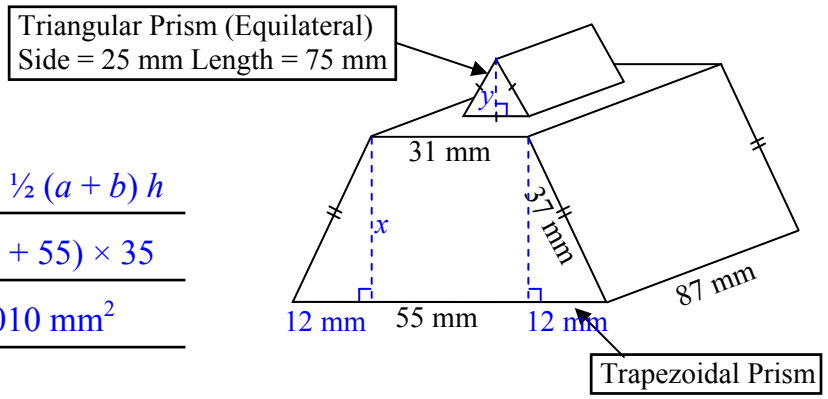
$= 2 \times \pi \times 8.5 \times 35 + \pi \times 13^2$

$A_4 = 2\,400.18 \text{ mm}^2$

Cylinder wall A_4 + hidden A_3 face

$TSA = 6\,124.21 \text{ mm}^2$

2 Rubber impact block



$x^2 = 37^2 - 12^2$

$x^2 = 1\,225$

$x = \sqrt{1\,225} = 35 \text{ mm}$

$y^2 = 25^2 - 12.5^2$

$y^2 = 468.75$

$y = \sqrt{468.75} = 21.65 \text{ mm}$ (2 dp)

$A_{TRAPS} = 2 \times \frac{1}{2} (a + b)\, h$

$= (31 + 55) \times 35$

$A_{TRAPS} = 3\,010 \text{ mm}^2$

$A_{SIDE} = 2 \times 37 \times 87 + 55 \times 87 + 31 \times 87 - 25 \times 75$

$A_{SIDE} = 12\,045 \text{ mm}^2$

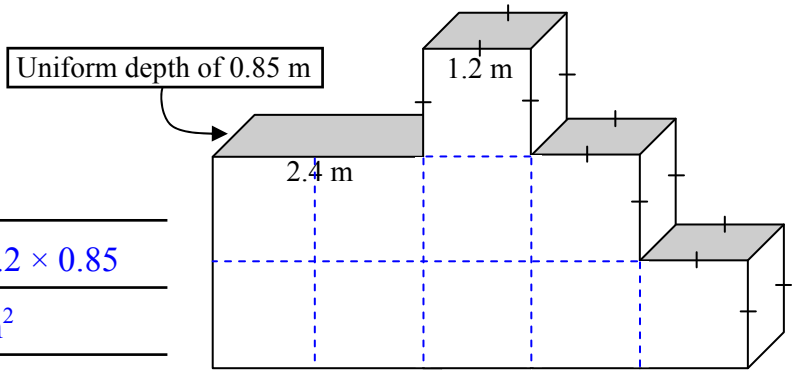
$A_{TRI} = 2 \times \frac{1}{2} \times 25 \times 21.65 + 2 \times 25 \times 75$

$A_{TRI} = 4\,291.25 \text{ mm}^2$

$TSA =$

$19\,346.25 \text{ mm}^2$

3 Concrete loading dock



Front face has 10 square faces with side length of 1.2 m. Front + Back = 20

$A_{F+B} = 20l^2$

$= 20 \times 1.2^2$

$A_{F+B} = 28.8 \text{ m}^2$

$A_{BASE} = 5lb$

$= 5 \times 1.2 \times 0.85$

$A_{BASE} = 5.1 \text{ m}^2$

$A_{2.4} = 2lb$

$= 2 \times 2.4 \times 0.85$

$A_{2.4} = 4.08 \text{ m}^2$

$A_{1.2} = 7lb$

$= 7 \times 1.2 \times 0.85$

$A_{1.2} = 7.14 \text{ m}^2$

$TSA = 45.12 \text{ m}^2$

Surface Area - Find a Word

Look for words in the list at the bottom of the grid. Once you find a word cross it off the list.

A letter could be used more than once so don’t colour it in too dark (using a texta for example) so that you can still read it.

Surface Area - Find a Word

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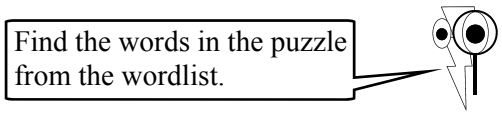

R	X	C	A	Q	E	B	S	S	U	R	F	A	C	E
E	E	L	C	R	I	C	C	U	E	R	A	U	Q	S
C	B	B	Q	U	A	R	E	T	I	B	E	U	U	A
T	U	E	R	O	B	L	I	X	R	D	A	A	A	B
A	C	Q	E	S	U	S	U	Q	T	C	A	E	R	A
N	G	E	T	U	O	T	O	G	Y	E	X	R	X	U
G	N	X	E	P	P	X	S	L	N	X	R	N	E	Q
U	I	P	M	X	E	A	I	I	I	A	E	N	E	U
L	H	O	A	X	X	N	I	E	D	D	I	K	A	T
A	C	S	I	E	D	C	U	R	V	E	D	R	N	L
R	T	E	D	E	T	X	N	T	A	M	P	A	T	M
X	A	D	R	T	H	G	I	E	H	U	L	T	A	S
E	M	Q	L	E	N	G	T	H	X	S	Q	T	A	I
D	I	M	A	R	Y	P	S	E	L	L	A	W	X	R
U	Q	R	A	L	U	C	I	D	N	E	P	R	E	P

WORDLIST

COMPOSITE	EXTERNAL	CIRCLE	NET	TRIANGULAR
WALL	HEIGHT	LENGTH	EXPOSED	SURFACE
SQUARE	RADIUS	RECTANGULAR	SLANT	SUM
CURVED	MATCHING	AREA	BASE	PERPENDICULAR
PYRAMID	CUBE	DIAMETER	UNITS	CYLINDER
	OUTSIDE	SOLID	PRISM	

Surface Area - Find a Word

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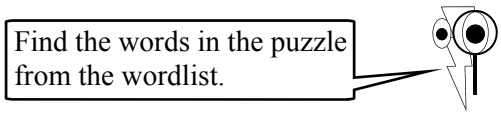

R	X	C	A	Q	E	B	S	S	U	R	F	A	C	E
E	E	L	C	R	I	C	C	U	E	R	A	U	Q	S
C	B	B	Q	U	A	R	E	T	I	B	E	U	U	A
T	U	E	R	O	B	L	I	X	R	D	A	A	A	B
A	C	Q	E	S	U	S	U	Q	T	C	A	E	R	A
N	G	E	T	U	O	T	O	G	Y	E	X	R	X	U
G	N	X	E	P	P	X	S	L	N	X	R	N	E	Q
U	I	P	M	X	E	A	I	I	I	A	E	N	E	U
L	H	O	A	X	X	N	I	E	D	D	I	K	A	T
A	C	S	I	E	D	C	U	R	V	E	D	R	N	L
R	T	E	D	E	T	X	N	T	A	M	P	A	T	M
X	A	D	R	T	H	G	I	E	H	U	L	T	A	S
E	M	Q	L	E	N	G	T	H	X	S	Q	T	A	I
D	I	M	A	R	Y	P	S	E	L	L	A	W	X	R
U	Q	R	A	L	U	C	I	D	N	E	P	R	E	P

WORDLIST

~~COMPOSITE~~
~~WALL~~
~~SQUARE~~
~~CURVED~~
~~PYRAMID~~
~~EXTERNAL~~
~~HEIGHT~~
~~RADIUS~~
~~MATCHING~~
~~CUBE~~
~~OUTSIDE~~
~~CIRCLE~~
~~LENGTH~~
~~RECTANGULAR~~
~~AREA~~
~~DIAMETER~~
~~SOLID~~
~~NET~~
~~EXPOSED~~
~~SLANT~~
~~BASE~~
~~UNITS~~
~~PRISM~~
~~TRIANGULAR~~
~~SURFACE~~
~~SUM~~
~~PERPENDICULAR~~
~~CYLINDER~~

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